# Relative Wealth Concerns and Complementarities in Information Acquisition\*

Diego García<sup>†</sup> and Günter Strobl<sup>‡</sup> Kenan-Flagler Business School University of North Carolina at Chapel Hill

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#### Abstract

This paper studies how relative consumption effects, in which a person's satisfaction with their own consumption depends on how much others are consuming, affect investors' incentives to acquire information. We find that such consumption externalities can generate complementarities in information acquisition within the standard rational expectations paradigm. When agents are sensitive to the wealth of others, they herd on the same information, trying to mimic each other's trading strategies. We show that there can be multiple herding equilibria in which some assets receive considerable attention while others with similar characteristics are ignored. Further, different communities of agents may specialize in different assets. This multiplicity of equilibria also generates jumps in asset prices: an infinitesimal shift in fundamentals can lead to a discrete price movement.

JEL classification: D82, G14.

*Keywords*: Keeping up with the Joneses; Consumption externalities; Complementarities in information acquisition; Jumps in asset prices.

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<sup>&</sup>lt;sup>†</sup>Diego García, Kenan-Flagler Business School, University of North Carolina at Chapel Hill, McColl Building, C.B. 3490, Chapel Hill, NC, 27599-3490. Tel: 1-919-962-8404; Fax: 1-919-962-2068; Email: diego\_garcia@unc.edu; Webpage: http://www.unc.edu/~garciadi

<sup>&</sup>lt;sup>‡</sup>Günter Strobl, Kenan-Flagler Business School, University of North Carolina at Chapel Hill, McColl Building, C.B. 3490, Chapel Hill, NC 27599-3490. Tel: 1-919-962-8399; Fax: 1-919-962-2068; Email: strobl@unc.edu

"Men do not desire merely to be rich, but to be richer than other men."

John Stuart Mill

### 1 Introduction

Economists have long believed that relative consumption effects, in which a person's satisfaction with their own consumption depends on how much others are consuming, are important (Veblen, 1899). Indeed, the growing literature on happiness in economics points to relative wealth concerns as one of the main explanations for why the growth in GDP over the last fifty years has not been accompanied by a similar increase in life satisfaction. Starting with Abel (1990) and Galí (1994), such relative wealth concerns have been formally modeled in the asset pricing literature. To date, the theoretical literature has primarily focused on the price implications that these consumption externalities have in a symmetric information environment. In this paper, we identify an additional channel through which relative wealth concerns affect asset prices. By incorporating relative wealth concerns into a rational expectations equilibrium (REE) model, we examine how such consumption externalities influence the production of information in the economy.

The economic setting we use extends the model developed by Hellwig (1980) and Verrecchia (1982) to account for "keeping up with the Joneses" (KUJ) preferences. In particular, we adopt a preference specification similar to Galí (1994), in which an investor's marginal utility of consumption increases in the average consumption of the other investors in the economy. This allows for the idea that investors care not only about their own wealth, but also about how their wealth compares with that of others, in the spirit of John Stuart Mill's quote. In all other respects, our model is standard: agents decide whether or not to acquire costly information about asset payoffs before trading, and, based on that information, trade a riskly and a riskless asset in a competitive market.

<sup>&</sup>lt;sup>1</sup>For two recent surveys of the topic, see Frey and Stutzer (2002) and Clark, Frijters, and Shields (2008).

Our main result is to show that consumption externalities resulting from a KUJ preference specification can generate complementarities in information acquisition. In the standard model, an investor's expected benefit from collecting information is decreasing in the number of informed agents. The reason is that as more agents acquire and act on their information, prices become more informative and uninformed agents can free-ride on the learning of others. If agents are sensitive to the wealth of others, this information revelation effect is counteracted by the investors' desire to keep up with their peers. A larger number of informed agents increases the expected trading profit of the average agent and, hence, reduces an uninformed agent's relative position in the economy. The disutility associated with a higher average wealth level therefore raises the value of information. We find that if the number of informed investors is not too high, relative wealth concerns dominate the information revelation effect, and the marginal value of information increases in the number of agents who acquire it. This creates incentives for agents to strategically coordinate their information production activities. Such coordination entails forming beliefs about the actions of other agents. These beliefs may be self-fulfilling, introducing the possibility of multiple "herding" equilibria with the property that some assets receive considerable attention while others with similar characteristics are ignored.

These complementarities in information acquisition have a number of important implications. First, they lead to an increase in informed trading, which reduces the risk that investors have to bear and, hence, lowers the equilibrium risk premium. Allowing investors to endogenously choose their information structure reinforces the effect of relative wealth concerns on the equity premium that has been identified in previous studies: in addition to the direct effect due to negative consumption externalities, as agents are more willing to hold risky assets if their peers hold them too, KUJ preferences have an indirect effect due to the increased value of private information to investors, which further reduces the equity premium.

Another important implication relates to price discontinuities. Extending our model to

a dynamic setting in which the distribution of asset payoffs is linked to past realizations, we show that an infinitesimal shift in fundamentals can lead to a discrete jump in asset prices. The mechanism responsible for these price jumps is complementarities in the demand for information that, together with small changes in fundamental risk, make investors switch between no-information and high-information equilibria. The same forces that generate price jumps can also cause excess volatility. Price jumps can be viewed as an extreme form of excess volatility in the sense that these are price movements that are unrelated to fundamentals.

The existence of complementarities in the information market can lead to a particularly strong form of herding. By giving agents a choice between perfectly correlated signals and signals that are independently distributed conditional on the asset's payoff, we demonstrate that if relative wealth considerations are sufficiently strong, agents prefer the former signals. This inefficient allocation of research effort can arise because the gain from trading on new information may be more than offset by the disutility that an agent incurs when her consumption falls short of the average level.<sup>2</sup> This result is in stark contrast to the standard REE model (Grossman and Stiglitz, 1980; Hellwig, 1980), in which agents always prefer to acquire conditionally independent signals. As prices partially reveal the agents' private information, traders are better off with information that is orthogonal to prices.

Most of our analysis is conducted under the assumption that relative wealth concerns are global, in the sense that agents care about their relative position with respect to the entire economy. However, the empirical literature on investors' portfolio choice (discussed below) has also documented some anomalies that are local in nature. In order to address these issues, we extend our model by grouping agents into different communities and by introducing relative wealth considerations within each community. Interestingly, we find that, in many cases, symmetric equilibria in which different communities of agents follow the

<sup>&</sup>lt;sup>2</sup>We want to emphasize that the information inefficiencies that we discuss in this paper pertain to the agents' information acquisition decisions: relative wealth concerns induce agents to focus on the same source of information, rather than on a diverse set of signals. However, the market is efficient at the pricing stage. Once a set of information has been acquired, market prices incorporate this information in a Bayesian fashion.

same information acquisition strategy are unstable. There exist, however, stable equilibria with the property that each community of agents specializes on gathering information about particular assets. Thus, consistent with the growing literature studying community effects and social interactions, we show that agents overinvest in some assets (and neglect others), relative to the predictions of modern portfolio theory.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 discusses the preferences we use to model consumption externalities and formally defines an equilibrium. Section 4 describes the equilibrium at the trading stage and the information acquisition stage. Section 5 shows how complementarities in information acquisition can generate jumps in asset prices. Section 6 demonstrates that consumption externalities resulting from a KUJ preference specification can lead to an inefficient allocation of the agent's research effort. Section 7 introduces the notion of communities and studies the implications of local relative wealth concerns. Section 8 summarizes our contribution and concludes. All proofs are contained in the Appendix.

### 2 Related Literature

Our paper is related to several strands of the literature. First, it is related to the literature that studies relative wealth concerns. This literature examines if and to what extent people's happiness depends on the consumption of others. Frey and Stutzer (2002) and Clark, Frijters, and Shields (2008) provide an excellent review of the empirical evidence on the relationship between relative position and well-being. In the finance literature, Abel (1990) was the first to introduce relative considerations with his "catching up with the Joneses" preference specification. Abel (1990) and Galí (1994) consider these consumption externalities as a potential resolution to the equity premium puzzle. Bakshi and Chen (1996) study their impact on stock price volatility; Gómez, Priestley, and Zapatero (2006, 2008) study the cross-sectional implications of relative wealth concerns. More recently, DeMarzo, Kaniel, and

Kremer (2004) present a model in which relative wealth considerations arise endogenously. They demonstrate that when investors care about their consumption relative to their local community, there may be a community effect whereby investors under-diversify and over-invest in local firms. We differ from this literature in that we examine the consequences that relative wealth concerns have on information acquisition. Rather than exogenously imposing an allocation of information, we endogenously derive the investors' incentives to engage in information collection activities. In contrast to symmetric information models, our model also predicts that local investors will outperform non-local investors, which is consistent with the empirical literature.<sup>3</sup>

Our paper also contributes to the literature on the home bias puzzle and on other local or community biases in portfolio choice.<sup>4</sup> Our model uses similar preferences to those endogenously derived in DeMarzo, Kaniel, and Kremer (2004) to explain the concentration of holdings. However, the actual mechanism is rather different and works via complementarities in the acquisition of information: agents have similar portfolios because they mimic each other's efforts to learn about securities. Nieuwerburgh and Veldkamp (2008) also present a model in which informational asymmetries are used to explain the home equity bias. In contrast to their paper, in which information is a strategic substitute, agents in our model may buy information that others already have, even when other information is available to them, since this ensures that their wealth will be highly correlated with that of their peers.

A number of papers have studied complementarities in information acquisition. Froot, Scharfstein, and Stein (1992) show that if agents have short horizons, the value of an informative signal about asset payoffs can increase in the number of agents owning the signal. This occurs because short-term speculators who have to liquidate their position before any public news arrives can only profit from their information if it is subsequently reflected in

<sup>&</sup>lt;sup>3</sup>See, for example, Coval and Moskowitz (2001) and Ivković and Weisbenner (2005b).

<sup>&</sup>lt;sup>4</sup>For empirical evidence on this topic, see French and Poterba (1991), Coval and Moskowitz (2001), Grinblatt and Keloharju (2001), Duflo and Saez (2002), Feng and Seasholes (2004), Hong, Kubik, and Stein (2004a,b), Bae, Stulz, and Tan (2005), Ivković and Weisbenner (2005b, 2007), Brown, Ivković, Weisbenner, and Smith (2008), and Dorn, Huberman, and Sengmueller (2008).

the price through the trades of similarly informed investors. Veldkamp (2006a,b) generates complementarities by incorporating information markets in which agents can acquire informative signals at endogenously determined prices into the REE model developed by Grossman and Stiglitz (1980). This paper expands on this literature by showing that complementarities in information acquisition can arise rather naturally as a consequence of relative wealth considerations.<sup>5</sup>

Finally, our paper is related to the literature on bubbles and crashes in financial markets (see Brunnermeier, 2001, for an excellent survey of this literature). Whereas early papers by Gennotte and Leland (1990) and Jacklin, Kleidon, and Pfleiderer (1992) focus on the role of portfolio insurance, our paper is more closely related to the literature that links the existence of crashes to informational considerations (Barlevy and Veronesi, 2003; Bai, Chang, and Wang, 2006; Mele and Sangiorgi, 2008). In a dynamic setting with symmetric information, DeMarzo, Kaniel, and Kremer (2008) show that endogenous relative wealth concerns can create bubble-like deviations in asset prices. Our paper complements theirs by providing an alternative mechanism that generates crashes via endogenous information acquisition decisions.

### 3 The Model

This section introduces a model which extends the rational expectations model developed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982) to allow for consumption externalities. In particular, we assume that agents care not only about their own consumption, but also about that of their peers.

<sup>&</sup>lt;sup>5</sup>Within the REE paradigm, Barlevy and Veronesi (2000, 2008), Chamley (2008), Ganguli and Yang (2007) and Mele and Sangiorgi (2008) provide other mechanisms that make information a complementary good. See Hellwig and Veldkamp (2008) and Goldstein, Ozdenoren, and Yuan (2008) for an analysis of complementarities in information acquisition in global games.

### 3.1 Preferences and assets

We study a three-period economy with consumption taking place only in the last period (i.e., at t = 3). At t = 2, there is a round of trade, whereas at t = 1, agents make a decision as to whether to become informed or not. These two stages are described in more detail below. Since consumption takes place only at the final date, we shall use the terms wealth and consumption interchangeably.

We assume that agent i has preferences of the form  $\mathbb{E}\left[u(W_i, \bar{W})\right]$ , where  $W_i$  denotes her terminal wealth, and  $\bar{W}$  denotes the average wealth in the economy. Specifically, we assume that the agents' utility function is given by:

$$u(W_i, \bar{W}) = -\exp(-\tau(W_i - \gamma \bar{W})). \tag{1}$$

The particular functional form we have chosen captures the notion that agents care about the consumption of others in the most parsimonious way. We note that the utility function satisfies the usual conditions with respect to an agents own consumption  $W_i$ : it is increasing and concave in  $W_i$ , and the coefficient of absolute risk-aversion is  $-u_{11}/u_1 = \tau$ . The parameter  $\gamma$  captures the extent of the consumption externality, i.e., how much agent i cares about other agents' wealth. The utility specification in (1) satisfies  $u_{12}/u_1 = \gamma \tau$ , which implies that an increase in the average wealth in the economy raises the marginal utility of consumption when  $\gamma$  is positive, as an agent tries to "keep up with the Joneses."

The preferences we introduce are essentially the CARA version of the standard KUJ preferences with CRRA utility.<sup>6</sup> A crucial feature of the specification in (1) is that agent i receives a negative utility shock when average wealth  $\bar{W}$  is high and  $\gamma > 0$ . In order to mitigate such a shock, she will trade in the same direction as the average agent, in order

<sup>&</sup>lt;sup>6</sup>The properties of the utility function in (1) are identical to those discussed on page 3 of Galí (1994), with the exception that there is no scaling by consumption. The additive structure we use in (1), as opposed to the multiplicative structure used in most of the asset pricing literature in conjunction with CRRA preferences, is more natural when coupled with CARA preferences, which are standard in the REE literature (Ljungqvist and Uhlig (2000) use a similar formulation).

to induce a high correlation between her wealth and that of others. This is the source of complementarities in the agents' decisions that will drive our results.

We want to emphasize that our contribution is to study the effect of consumption externalities in information acquisition activities in financial markets. The particular interpretation of the utility function introduced above is not critical. For example, one could interpret  $\gamma$  as measuring jealousy (our preferences satisfy both the definition of jealousy and that of KUJ introduced in Dupor and Liu, 2003). Further, although we focus on the case where  $\gamma$  is positive, the model formally also allows for the case where agents view the consumption of others as a substitute for their own consumption. Finally, we should remark that the consumption externalities we consider are global instead of local, in the sense that agent i cares about the average consumption, not the consumption of their neighbors (see, for example, Glaeser and Scheinkman, 2002, and Bisin, Horst, and Özgüra, 2006, for other utility specifications). We also want to point out that the type of preferences we study can be constructed from a purely axiomatic approach (Maccheroni, Marinacci, and Rustichini, 2008).

Rather than stemming form investors' preferences, relative wealth concerns in financial markets can also be motivated by the current compensation contracts in the mutual fund industry. Indeed, most fund managers get compensated not just based on total performance, but rather on their performance relative to a peer group (i.e., growth funds, small-cap funds, ...). If we interpret  $\bar{W}$  in (1) as the value of the reference portfolio, letting  $\gamma = 1$  corresponds to a setting where agents maximize the portfolio returns relative to this reference portfolio. Further, given that professional fund managers are the most likely investors to possess private information, and to exert effort to acquire such private signals, KUJ preferences may actually be even more relevant for them than the standard ones.

We study a competitive market that is populated by a continuum of agents, represented by the set [0,1]. There are two assets available for trading in the market: a riskless asset in perfectly elastic supply with a price and payoff normalized to 1, and a risky asset with a random final payoff of  $X \in \mathbb{R}$ . We assume that all random variables belong to a linear space of jointly normally distributed random variables. In particular, we assume that the risky asset pays off  $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$ . The aggregate supply of the risky asset is random and equals  $Z \sim \mathcal{N}(\mu_z, \sigma_z^2)$ . Such supply shocks are a typical ingredient of rational expectations models. The noise that they create prevents equilibrium prices from fully revealing the informed agents' private information.

A fraction  $\lambda$  of the agents receive a private signal prior to trading, whereas the rest are uninformed and must base their trading decision solely on their priors and what they may learn from prices. Specifically, we assume that, by incurring a cost of c, agent i can observe the signal  $Y_i = X + \epsilon_i$ , where  $\epsilon_i \sim \mathcal{N}(0, \sigma_{\epsilon}^2)$ . The error terms  $\{\epsilon_i\}$  are assumed to be independent across agents (we relax this assumption in section 6). We use  $\mathcal{F}_i$  to denote the information set of agent i at the time of trading.<sup>7</sup> Without loss of generality, we can label the informed agents with the subscripts  $i \in [0, \lambda]$ . We also let  $\theta_i$  denote the number of shares of the risky asset bought by agent i, so that, assuming zero initial wealth, we have  $W_i = \theta_i(X - P)$ , where P denotes the price of the risky asset. Finally, average wealth equals  $\bar{W} = \int_0^1 \theta_i(X - P) di$ .

#### 3.2 Definition of equilibrium

Fixing the fraction of informed agents,  $\lambda$ , a rational expectations equilibrium is characterized by a set of trading strategies  $\{\theta_i\}$ ,  $i \in [0, 1]$ , and a price function P, such that:

(1) Each agent *i* chooses her trading strategy  $\theta_i$  so as to maximize her expected utility conditional on her information set  $\mathcal{F}_i$ , i.e.,  $\theta_i$  solves:

$$\max_{\theta_i} \quad \mathbb{E}\left[-\exp(-\tau(W_i - \gamma \bar{W}))|\mathcal{F}_i\right], \qquad i \in [0, 1]. \tag{2}$$

<sup>&</sup>lt;sup>7</sup>Note that  $\mathcal{F}_i = \sigma(Y_i, P)$  for informed agents  $i \in [0, \lambda]$ , and  $\mathcal{F}_i = \sigma(P)$  for uninformed agents  $i \in [\lambda, 1]$ , where  $\sigma(X)$  denotes the  $\sigma$ -algebra generated by a random variable X, and P is the price of the risky asset.

<sup>&</sup>lt;sup>8</sup>We consider the information acquisition cost c a non-pecuniary cost that does not affect the average wealth  $\bar{W}$ . However, subtracting the aggregate cost  $\lambda c$  from  $\bar{W}$  would not change our results, since this reduction in average wealth is the same for informed and uninformed investors.

### (2) Markets clear, i.e.:

$$\int_0^1 \theta_i \, di = Z. \tag{3}$$

As is customary in the literature, we restrict our attention to linear equilibria. Thus, we postulate that the equilibrium price is a linear function of the average signal and the aggregate stock supply, such that:

$$P = a + bX - dZ. (4)$$

In the ensuing analysis, we derive a linear equilibrium in which this conjecture is confirmed to be correct.

At the ex-ante stage (i.e., at t = 1), agents must decide whether to spend c in order to get a private signal c prior to trading. Letting  $\hat{\theta}_i$  denote the optimal trading strategy of agent c, the certainty equivalent of wealth gross of information acquisition costs, c is defined by the equation:

$$\mathbb{E}\left[-\exp\left(-\tau\left(\mathcal{U}_{i}-\gamma\bar{W}\right)\right)\right] = \mathbb{E}\left[-\exp\left(-\tau\left(\hat{\theta}_{i}(X-P)-\gamma\bar{W}\right)\right)\right]. \tag{5}$$

An equilibrium at the information acquisition stage is defined by a fraction of agents  $\lambda \in [0,1]$  such that (i) all informed agents  $i \in [0,\lambda]$  are better off spending c in order to obtain information, taking all other agents' actions as given; (ii) all uninformed agents  $j \in [\lambda,1]$  are better off staying uninformed than purchasing information, taking all other agents' actions as given. To be more precise, let  $\mathcal{V}_I(\lambda) \equiv \mathcal{U}_i$  for any informed agent  $i \in [0,\lambda]$ , and let  $\mathcal{V}_U(\lambda) \equiv \mathcal{U}_j$  for any uninformed agent  $j \in [\lambda,1]$ . Then, an interior equilibrium at the information acquisition stage is a fraction  $\lambda \in (0,1)$  such that  $\mathcal{V}_I(\lambda) - c = \mathcal{V}_U(\lambda)$ . The noninterior equilibria are defined in the natural way:  $\lambda = 0$  is an equilibrium if  $\mathcal{V}_I(0) - c \leq \mathcal{V}_U(0)$ , and  $\lambda = 1$  is an equilibrium if  $\mathcal{V}_I(1) - c \geq \mathcal{V}_U(1)$ .

The model outlined thus far reduces to a symmetric version of Verrecchia (1982) if  $\gamma = 0$ . Diamond (1985) solves such a model in closed form, showing, among other things, that the equilibrium at the information acquisition stage is unique, i.e., there is a unique  $\lambda \in [0,1]$  that satisfies the above definition. The focus of our analysis is to see how consumption externalities change the equilibrium at the trading stage, as well as the incentives to acquire information.

# 4 Characterization of Equilibrium

In this section, we solve for the equilibrium defined above by backward induction. First, we conjecture that a fraction  $\lambda$  of agents become informed and solve for the equilibrium asset prices at the trading stage at t=2. Then, we study the ex-ante information acquisition decision of agents at t=1, given that they anticipate the equilibrium in the asset market at t=2.

When agents have relative wealth concerns, they must form beliefs about the trading strategies of all other traders, since their utility is affected directly by the average wealth of other investors. We start by assuming that a fraction  $\lambda$  of the agents are informed, i.e., they receive signals of the form  $Y_i = X + \epsilon_i$ . We conjecture that in equilibrium an informed agent's trading strategy will be of the affine form  $\theta_i = \alpha + \beta Y_i - \delta P$ , whereas an uninformed agent's strategy is given by  $\theta_i = \zeta - \nu P$ , for some constants  $\alpha, \beta, \delta, \zeta, \nu \in \mathbb{R}$ . This implies that the aggregate demand is given by:

$$\bar{\theta} = \int_0^1 \theta_i di = \xi + \lambda \beta X - \kappa P, \tag{6}$$

with  $\kappa = \lambda \delta + (1 - \lambda)\nu$  and  $\xi = \lambda \alpha + (1 - \lambda)\zeta$ .

<sup>&</sup>lt;sup>9</sup>Here, we have assumed that  $\int_0^1 \epsilon_i di = 0$ . Of course, with a continuum of agents, any appeal to a law of large numbers will encounter some well-known technical problems. For a discussion of these problems and solutions to them, see Judd (1985).

We first note that average wealth  $\bar{W} = \bar{\theta}(X-P)$  is a quadratic function in X, which makes the investment problem in (2) non-standard: the relevant payoff variable  $W_i - \gamma \bar{W}$  is not normally distributed, conditional on the information set of either informed or uninformed agents. This is due to the fact that agents are asymmetrically informed. As they try to tilt their portfolios closer to their peers, they need to forecast the trades of other agents. The following proposition shows that the optimal investment problem is nonetheless fairly tractable and that a rational expectations equilibrium exists under mild conditions.

**Proposition 1.** Suppose that  $\gamma < 1/(\tau \sigma_x \sigma_z)$ . Then, for a given fraction of informed agents  $\lambda$ , an equilibrium exists. The optimal investment by agent i in the risky asset is given by:

$$\theta_i = \frac{\mathbb{E}\left[X - P|\mathcal{F}_i\right]}{\tau \operatorname{var}(X|\mathcal{F}_i)} + \gamma \left(\xi - P(\kappa - \lambda \beta)\right),\tag{7}$$

where  $\mathcal{F}_i$  denotes the information possessed by agent i.

Equilibrium prices are as in (4); the equilibrium price coefficients are given in the Appendix.

The rational expectations equilibrium presented in Proposition 1 shares many of the properties of the standard model. As in Hellwig (1980), price aggregates the disperse information possessed by agents, while the stochastic supply Z prevents prices from fully revealing the payoff X. The informational content of prices is given by:<sup>10</sup>

$$\operatorname{var}(X|P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{\lambda}{\tau \sigma_\epsilon^2}\right)^2. \tag{8}$$

We note that price informativeness is exactly as in Hellwig (1980). The only difference in the information content of prices between the standard model and the one with KUJ preferences comes through a different fraction of informed agents,  $\lambda$ , which we endogenize below.

Other equilibrium price properties are directly affected by the preference externality pa-

<sup>&</sup>lt;sup>10</sup>See the proof of Proposition 1.

rameter  $\gamma$ . For example, the equity premium, defined as  $\mathbb{E}[X-P]$ , is given by:<sup>11</sup>

$$\mathbb{E}\left[X - P\right] = \frac{\tau(1 - \gamma)\mu_z}{\operatorname{var}(X|P)^{-1} + \frac{\lambda(1 - \gamma)}{\sigma_{\epsilon}^2}}.$$
(9)

In the absence of asymmetric information (i.e., when  $\lambda = 0$ ), the risk premium on the stock is given by  $\mathbb{E}[X - P] = \tau(1 - \gamma)\mu_z\sigma_x^2$ . The KUJ parameter  $\gamma$  therefore lowers the equilibrium risk premium, just as in the standard model with CRRA preferences (Galí, 1994). This result follows from an increased demand for the risky asset caused by relative wealth concerns: when  $\gamma$  is positive, the investors' marginal utility of consumption is higher in "good times" (i.e., for high realizations of X and, hence, of  $\overline{W}$ ), and lower in "bad times" (i.e., for low realizations of X). The presence of informed agents leads to a further reduction in the equity premium: as more information is impounded in prices, agents demand a smaller compensation for the lower (conditional) risk that they bear.

Proposition 1 shows that an agent's optimal trading strategy contains the standard meanvariance term plus a term that depends explicitly on the consumption externality parameter  $\gamma$ . As expected, traders try to mimic the trading strategy of the average agent: the term depending on  $\gamma$  in equation (7) mimics the average trade given by equation (6). Thus, Proposition 1 formalizes the intuition that traders tilt their portfolio to imitate the P-measurable portion of the average portfolio.

The existence condition in the above proposition,  $\gamma < 1/(\tau \sigma_x \sigma_z)$ , comes from the secondorder condition of the agents' optimization problem.<sup>12</sup> Intuitively, agents' expected utility will not be well-defined, if they put too much weight on their peers' wealth. Although the condition is simple, it is far from necessary. The necessary and sufficient condition for the existence of the rational expectations equilibrium is provided in the proof of the proposition

<sup>&</sup>lt;sup>11</sup>This expression follows immediately from the equilibrium price coefficients specified by equations (43) and (44) in the Appendix.

<sup>&</sup>lt;sup>12</sup>Galí (1994) also needs to constrain the amount of consumption externalities in order to guarantee existence of an equilibrium in his model.

(see equation (45)).

We next endogenize the fraction of traders that become informed. Intuitively, we expect the incentives to acquire information to differ from the standard model, since agents with KUJ preferences also care about the information that other agents possess. The following proposition solves for the equilibria at the ex-ante information acquisition stage in closed form.

**Proposition 2.** Let  $\hat{C} = 1/(\sigma_{\epsilon}^2(e^{2\tau c} - 1)) - 1/\sigma_x^2$  and assume that  $\gamma < 1/(\tau \sigma_x \sigma_z)$ .

1. If  $\hat{C} > 0$ , then there exists a unique equilibrium at the information acquisition stage. The fraction of agents that become informed is given by  $\lambda = \min(\lambda^*, 1)$ , where:

$$\lambda^* = \tau \sigma_{\epsilon}^2 \sigma_z^2 \left( \tau \gamma + \sqrt{\tau^2 \gamma^2 + \hat{C}/\sigma_z^2} \right). \tag{10}$$

- 2. If  $\hat{C} \leq 0$ , then two cases are relevant:
  - (a) If  $\tau^2 \gamma^2 + \hat{C}/\sigma_z^2 \geq 0$ , then there are three equilibria at the information acquisition stage:  $\lambda = \min(\lambda^*, 1)$  as given by (10),  $\lambda = 0$ , and  $\lambda = \min(\lambda^{**}, 1)$ , where:

$$\lambda^{**} = \tau \sigma_{\epsilon}^2 \sigma_z^2 \left( \tau \gamma - \sqrt{\tau^2 \gamma^2 + \hat{C}/\sigma_z^2} \right). \tag{11}$$

(b) If  $\tau^2 \gamma^2 + \hat{C}/\sigma_z^2 < 0$ , the unique equilibrium is for all agents to stay uninformed, i.e.,  $\lambda = 0$ .

Proposition 2 shows that there are three different types of equilibria, depending on the value of  $\hat{C}$ .<sup>13</sup> When information is very costly,  $\hat{C}$  is negative and large, and consequently in the unique equilibrium no agent becomes informed. When the cost of gathering information is sufficiently low,  $\hat{C}$  is positive, and the unique equilibrium involves a fraction of agents

<sup>&</sup>lt;sup>13</sup>Note that  $\hat{C}$  is decreasing in the cost of gathering information, c.

 $\lambda^* > 0$  gathering information. These are the only two regimes that arise in the standard model, i.e., when there are no consumption externalities ( $\gamma = 0$ ).

The novel regime occurs when  $\gamma$  is sufficiently large relative to  $\hat{C}$ , and  $\hat{C}$  is negative (i.e., for intermediate values of the cost parameter c). In this case, there are three equilibria at the information acquisition stage: one where no agent becomes informed, and two others where a positive fraction of agents become informed. Intuitively, when agents expect other agents to purchase information, they have an incentive to purchase information as well, since they want to keep up with their peers, even if it is expensive to do so. At the same time, if no one expects others to purchase information, not acquiring information is an optimal strategy. This multiplicity of equilibria arises naturally from KUJ preferences and drives the core of the results discussed below.

It is worth noting that  $\lambda^*$  is increasing in the KUJ parameter  $\gamma$ . Thus, the fraction of informed investors in the presence of negative consumption externalities exceeds the one in the standard model.<sup>14</sup> This increase in information production has several important implications. First, it makes prices more informative about the asset's fundamentals. As can be seen from equation (8), the residual payoff uncertainty, conditional on the equilibrium price P, is strictly decreasing in  $\lambda$ . Second, and more importantly, more informed trading reduces the risk premium that investors require to hold the risky asset in equilibrium (see equation (9)). Thus, by endogenizing the agents' information acquisition decision, we identify a new channel through which relative wealth concerns affect the equity premium. In addition to the direct channel discussed above, there is an indirect channel that operates through changes in the value of information to investors.

Figure 1 plots the equilibrium values of  $\lambda$  as a function of the cost of gathering information. When there are no consumption externalities ( $\gamma = 0$ ), the unique equilibrium is represented by the solid line. In this case, a positive fraction of agents acquire information if and only if

<sup>&</sup>lt;sup>14</sup>Note that in the standard model where  $\gamma = 0$ , the unique equilibrium is characterized by  $\lambda = 0$  whenever there are multiple equilibria under KUJ preferences.

the cost c is smaller than 0.55. Once investors care about their peers' wealth, multiplicity of equilibria arises. For example, when  $\gamma = 0.4$ , Figure 1 shows that when the cost parameter c takes on intermediate values, namely when  $c \in (0.55, 0.62)$ , there are three different equilibria, two of which have a positive fraction of informed agents. When c is lower than 0.55, there is a unique equilibrium that involves some agents gathering information. For costs higher than 0.62, the unique equilibrium coincides with the one in the absence of consumption externalities, since no agent becomes informed.

As the proof of Proposition 2 shows, agent i's ex-ante certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left( \operatorname{var}(X|\mathcal{F}_i)^{-1} - \frac{2\lambda\gamma}{\sigma_{\epsilon}^2} \right) + H, \tag{12}$$

where H is independent of the information that the agent possesses. This expression shows that, in contrast to the standard model, the agents' utility may be decreasing in the fraction of informed investors when  $\gamma > 0.15$  The reason is that under KUJ preferences, informed agents impose a negative externality on the other agents, because they earn on average higher profits. Furthermore, this externality generates complementarities in information acquisition decisions. In order to see this, let  $\mathcal{R}(\lambda) = \mathcal{V}_I(\lambda) - \mathcal{V}_U(\lambda)$  denote the marginal value of information (gross of the cost of obtaining the information). An interior equilibrium is given by the condition  $\mathcal{R}(\lambda) = c$ . One can easily verify that  $d\mathcal{R}(\lambda)/d\gamma \geq 0$ , which means that the value of information is increasing in how much agents care about consumption externalities. Moreover, we have:

$$\operatorname{sign}\left(\frac{d\mathcal{R}(\lambda)}{d\lambda}\right) = \operatorname{sign}\left(\gamma - \frac{\lambda}{\tau^2 \sigma_{\epsilon}^2 \sigma_z^2}\right). \tag{13}$$

The value of information can therefore increase in the number of informed agents, as long as agents care about each other's wealth. In the standard case ( $\gamma = 0$ ), increasing the number of informed agents lowers the value of information, because prices become more informative.

<sup>15</sup> Note, however, that the conditional precision  $var(X|\mathcal{F}_i)^{-1}$  is increasing in  $\lambda$  and, hence, in  $\gamma$ .

When  $\gamma$  is positive, we find that as long as  $\lambda$  is low enough so that the information revelation effect does not dominate, there are complementarities in information acquisition in the sense that the marginal value of information is increasing in the number of agents who buy the information. Intuitively, if a trader's neighbors buy information, consumption externalities will increase the incentives of this trader to gather information herself.

As Proposition 2 shows, multiple equilibria at the information acquisition stage exist when the cost of gathering information is in an intermediate range such that  $\hat{C} \in [-\tau^2 \gamma^2 \sigma_z^2, 0]$ . In order to assess the plausibility of the three different equilibria that arise in this case, we employ a refinement criteria based on dynamic stability. The definition of stability relies on an iterative process in which agents react to last period's outcome. An equilibrium is stable if it is the limiting outcome of such a process. A simple method to determine whether an equilibrium is stable is to analyze the agents' optimal response to small deviations in the equilibrium outcome.

Figure 2 plots an investor's optimal information acquisition decision as a function of the fraction of informed agents in the economy (the parameter values correspond to case 2(a) in Proposition 2). An equilibrium at the information acquisition stage is defined by the condition that the fraction of informed agents equals the probability that any agent acquires information. In our numerical example, the three points at which an investor's optimal response function crosses the 45° line are characterized by  $\lambda = 0$ ,  $\lambda^{**} = 0.39$ , and  $\lambda^* = 0.81$ . Figure 2 clearly illustrates that information can be a complementary good in our model. Agents have no incentive to purchase the private signal  $Y_i$  if  $\lambda < \lambda^{**}$ ; however, as  $\lambda$  increases to a level between  $\lambda^{**}$  and  $\lambda^*$ , the value of the signal goes up and investors find it optimal to acquire it. This observation establishes the following result.

**Proposition 3.** Suppose that  $\hat{C} < 0 < \tau^2 \gamma^2 + \hat{C}/\sigma_z^2$  and  $\lambda^* < 1$ . Then, the only stable equilibria at the information acquisition stage are  $\lambda = 0$  and  $\lambda = \lambda^*$ .

<sup>&</sup>lt;sup>16</sup>For a formal definition, see, e.g., Mas-Colell, Whinston, and Green (1995), chapter 17.

Stability rules out the equilibrium in which a fraction  $\lambda = \lambda^{**}$  of the agents become informed. As can be seen in Figure 2, any perturbation of this equilibrium will make agents switch to one of the other two equilibria. We will therefore ignore the unstable equilibrium in the ensuing analysis.

# 5 Asset Price Jumps

In order to discuss movements in asset prices, it is necessary to extend our model to a dynamic setting. Following Veldkamp (2006b), we take the most parsimonious approach, and simply consider a sequence of one-period economies, where the model primitives are allowed to vary from period to period. In particular, we assume that the asset payoff in period t satisfies  $X_t \sim \mathcal{N}\left(\mu_x(X_{t-1}), \sigma_x^2(X_{t-1})\right)$ . The aggregate supply is given by  $Z_t \sim \mathcal{N}\left(\mu_z, \sigma_z^2\right)$ . We assume that the functions  $\mu_x$  and  $\sigma_x$  are continuous, so that a small change in the realization of  $X_{t-1}$  only causes a small change in the distribution of  $X_t$ . To guarantee some non-trivial dynamics, we further assume that the function  $\sigma_x$  is strictly increasing, and that its range is the entire positive real line, i.e.,  $\sigma_x : \mathbb{R} \to \mathbb{R}_+$ . For simplicity, we let the preference parameters measuring risk-aversion,  $\tau$ , and consumption externalities,  $\gamma$ , be time-invariant.

Letting  $\lambda_t$  and  $P_t$  denote the fraction of informed agents and the asset price at date t, one can obtain an equilibrium path in this dynamic economy for both  $\lambda_t$  and  $P_t$  simply by using the results from Propositions 1 and 2. In particular, we assume that the realization of  $X_t$  is publicly observable at date t, so that traders can predict the moments  $\mu_x(X_t)$  and  $\sigma_x(X_t)$ . Using these parameters in Proposition 2 yields the measure of informed agents  $\lambda_t$ . In the case of multiple equilibria, we follow the natural convention that if in the previous period we had  $\lambda_{t-1} = 0$  and  $\lambda_t = 0$  is an admissible equilibrium, then agents will coordinate

<sup>&</sup>lt;sup>17</sup>In Veldkamp (2006b), the asset payoff  $X_t$  is assumed to follow an AR(1) process with mean  $\mu_x$  and proportional shocks  $\eta$ :  $X_t = (1 - \rho)\mu_x + \rho X_{t-1}(1 + \eta_t)$ , where  $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ . This specification corresponds to the special case  $\mu_x(X_t) = (1 - \rho)\mu_x + \rho X_t$  and  $\sigma_x^2(X_t) = \rho^2 \sigma_\eta^2 X_t^2$  in our setting.

 $<sup>^{18}</sup>$ We want to point out that allowing for time variation in any of the other model primitives that affect the endogenous measure of informed traders would lead to the same conclusion as the one stated in Proposition  $^{4}$ 

on that equilibrium. Similarly, if  $\lambda_{t-1} = \lambda^*$ , then agents will again coordinate on  $\lambda_t = \lambda^*$ , if possible. Proposition 1 then yields the equilibrium prices at date t. Given the dependence of prices at date t on the time t-1 realization of the random variable  $X_{t-1}$  (via the functions  $\mu_x$  and  $\sigma_x$ ), as well as on the realizations of  $X_t$  and  $Z_t$ , we write  $P(X_t, Z_t; X_{t-1})$ .

There are several definitions of price jumps, crashes, and bubbles in the literature.<sup>19</sup> In the above economy, prices will change in each period because of shocks to the fundamentals  $X_t$  and to the aggregate supply  $Z_t$ . However, since the payoff variance  $\sigma_x^2$  is a continuous function of  $X_{t-1}$ , small changes in fundamentals will lead to small changes in prices when there are no consumption externalities. The following definition of price jumps therefore seems natural in our setting:

$$\lim_{|\epsilon| \downarrow 0} P_t(X_t, Z_t; X_{t-1} + \epsilon) - P_t(X_t, Z_t; X_{t-1}) > 0.$$
(14)

An economy exhibits price jumps when small changes in fundamentals at date t-1 can produce large changes in prices at date t.<sup>20</sup> The following result is a consequence of the multiplicity of equilibria in Proposition 2.

**Proposition 4.** The prices in the dynamic model exhibit jumps (in the sense defined in (14)), if and only if  $\gamma > 0$ .

In order to obtain an intuitive understanding of this result, we plot the stable equilibria from Proposition 2 as a function of the payoff variance  $\sigma_x^2$  in Figure 3. Over the range  $\sigma_x^2 \in (0.87, 1.01)$ , we have two stable equilibria: one where no agent becomes informed, and one with a positive fraction of informed agents. No matter what equilibrium one starts at, it is clear that changes in the payoff variance will induce discrete price jumps, as the measure of informed agents differs across these two equilibria. For example, suppose that we start with an equilibrium such that  $\lambda > 0$  (i.e.,  $\sigma_x^2$  is sufficiently large). Then, as  $\sigma_x^2$  falls below

<sup>&</sup>lt;sup>19</sup>For an excellent survey of this literature, see Brunnermeier (2001).

<sup>&</sup>lt;sup>20</sup>Our definition of jumps is similar in spirit to a continuous time definition where the sample paths of prices exhibit discontinuities.

0.87, the fraction of informed agents jumps from  $\lambda = 0.42$  to  $\lambda = 0$ . An inspection of the price coefficients in Proposition 1 shows that this change in  $\lambda$  causes a discontinuity in prices in the sense defined in (14). The model also predicts jumps in the other direction. If the economy is in an equilibrium where  $\lambda = 0$ , then, as  $\sigma_x^2$  increases above 1.01, the fraction of informed agents jumps from  $\lambda = 0$  to  $\lambda = 0.80$ .

The assumption that  $\sigma_x$  is increasing in  $X_t$  implies that the value of information is positively related to the expected payoff. Thus, when the state of the economy improves, we expect to see a jump up in prices, as the agents move from the no-information to the high-information equilibrium. Similarly, when the fundamentals go down, asset prices experience a discrete jump down (i.e., a "crash") as agents stop acquiring information. An important implication of our model is that crashes are accompanied by an increase in the conditional asset variance and, hence, by an increase in the equity premium.

# 6 Herding and Informational Inefficiencies

In this section, we show that complementarities in the information market can lead to an inefficient allocation of the agents' research effort. In particular, we demonstrate that when relative wealth concerns are sufficiently strong, agents prefer to acquire perfectly correlated signals, even though their incremental value in predicting future asset payoffs is low. To this end, we extend our basic model by giving agents a choice between acquiring a conditionally independent signal  $Y_i = X + \epsilon_i$  and a perfectly correlated signal  $Y = X + \delta$ . The collection of error terms  $\{\epsilon_i\}$  and  $\delta$  are assumed to be independently normally distributed with zero means and variances  $\sigma_{\epsilon}^2$  and  $\sigma_{\delta}^2$ , respectively. At the information acquisition stage, agents can acquire the signal  $Y_i$  at a cost of  $c_{\epsilon}$ , and the signal Y at a cost of  $c_{\delta}$ . To distinguish the two types of informed agents, we refer to agents who choose the former (latter) signal as  $\epsilon$ -informed ( $\delta$ -informed). All other aspects of the model are the same as in Section 3.

As in the previous sections, we restrict our attention to linear equilibria and conjecture

that the equilibrium price function is of the following form:

$$P = a + b_x X + b_y Y - dZ. (15)$$

We further postulate that the optimal trading strategy of  $\epsilon$ -informed ( $\delta$ -informed) agents is a linear function of the signal  $Y_i$  (Y) and the equilibrium price P. Letting  $\beta_{\epsilon}$  ( $\beta_{\delta}$ ) denote the coefficient of the signal  $Y_i$  (Y) in the linear demand function of  $\epsilon$ -informed ( $\delta$ -informed) investors, we can therefore write the aggregate demand as follows:

$$\bar{\theta} = \int_0^1 \theta_i di = \xi + \lambda_\epsilon \beta_\epsilon X + \lambda_\delta \beta_\delta Y - \kappa P, \tag{16}$$

where  $\lambda_{\epsilon}$  ( $\lambda_{\delta}$ ) denotes the fraction of  $\epsilon$ -informed ( $\delta$ -informed) agents, and  $\xi$  and  $\kappa$  are constants.

The following proposition characterizes the equilibrium at the trading stage for a given number of  $\epsilon$ -informed and  $\delta$ -informed agents.

**Proposition 5.** There exists a  $\bar{\gamma} > 0$  such that for any  $\gamma < \bar{\gamma}$ , an equilibrium at the trading stage exists. The optimal investment by agent i in the risky asset is given by:

$$\theta_{i} = \frac{\mathbb{E}\left[X - P|\mathcal{F}_{i}\right]}{\tau \operatorname{var}(X|\mathcal{F}_{i})} + \gamma \left(\xi - P(\kappa - \lambda_{\epsilon}\beta_{\epsilon})\right) + \gamma \lambda_{\delta}\beta_{\delta} \left(\mathbb{E}\left[Y|\mathcal{F}_{i}\right] - \frac{\operatorname{cov}(X, Y|\mathcal{F}_{i})}{\operatorname{var}(X|\mathcal{F}_{i})}\mathbb{E}\left[X - P|\mathcal{F}_{i}\right]\right)$$
(17)

where  $\mathcal{F}_i$  denotes the information possessed by agent i.

Equilibrium prices are as in (15); the relevant price coefficients are given in the Appendix.

In contrast to the case analyzed in Section 4, the agents' optimal trading strategy contains an additional term that is proportional to  $\lambda_{\delta}$ . As agents try to mimic the average demand  $\bar{\theta}$ because of their relative wealth concerns, they now have to forecast the signal Y, since the error term  $\delta$  has a non-negligible effect on  $\bar{\theta}$  if  $\lambda_{\delta} > 0$ .  $\delta$ -informed agents directly observe Yand thus demand an additional  $\gamma \lambda_{\delta} \beta_{\delta} Y$  shares to make sure that their wealth is close to the average level.  $\epsilon$ -informed agents, on the other hand, have to forecast Y based on their own signal  $Y_i$  and the equilibrium price P.

The proof of Proposition 5 reveals that the equilibrium trading intensities  $\beta_{\epsilon}$  and  $\beta_{\delta}$  of  $\epsilon$ -informed and  $\delta$ -informed agents are given by:

$$\beta_{\epsilon} = \frac{1}{\tau \sigma_{\epsilon}^2} \quad \text{and} \quad \beta_{\delta} = \frac{1}{\tau \sigma_{\delta}^2 \left(1 - \gamma \lambda_{\delta} + \frac{\lambda_{\delta} \lambda_{\epsilon}}{\tau^2 \sigma_{\epsilon}^2 \sigma_z^2}\right)}.$$
(18)

The trading intensity of  $\epsilon$ -informed agents is not affected by the presence of  $\delta$ -informed investors and does not depend on the KUJ parameter  $\gamma$ . It is identical to the expression derived in Section 4. The trading intensity of  $\delta$ -informed agents, however, is influenced by their relative wealth considerations: as  $\gamma$  increases, they care more about each other's wealth and, hence, trade more aggressively on their common signal Y. It is also worth noting that  $\beta_{\delta}$  is decreasing in the fraction of  $\epsilon$ -informed agents. This is due to the fact that prices become more informative as  $\lambda_{\epsilon}$  increases, making the signal Y less valuable to agents.

The above discussion has assumed that both types of informed agents coexist in the market. In order to demonstrate that this can indeed be the case, we have to calculate the expected utility of  $\epsilon$ -informed,  $\delta$ -informed, and uninformed agents at the information acquisition stage. The proof of Proposition 6 shows that agent i's certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_{i} = \frac{1}{2\tau} \log \left( \operatorname{var}(X|\mathcal{F}_{i})^{-1} - 2\tau\gamma \left( \lambda_{\epsilon}\beta_{\epsilon} + \lambda_{\delta}\beta_{\delta} \frac{\operatorname{cov}(X,Y|\mathcal{F}_{i})}{\operatorname{var}(X|\mathcal{F}_{i})} \right) - \tau^{2}\gamma^{2}\lambda_{\delta}^{2}\beta_{\delta}^{2} \operatorname{var}(Y|X,\mathcal{F}_{i}) \right) + H,$$
(19)

where H is independent of the agent's information set  $\mathcal{F}_i$ .

Comparing the above expression to equation (12) in Section 4, we find that in this generalized framework, there are two additional terms that are negatively related to  $\lambda_{\delta}$ . The first term, which is proportional to the linear regression coefficient of the signal Y on the asset payoff X (conditional on the equilibrium price P), captures the intuition that a more precise

signal Y improves the trading profit of  $\delta$ -informed agents, and thus hurts other investors if they care about their relative wealth. The second term is related to the fact that, because of the common error term in their signals,  $\delta$ -informed agents increase the variance of the average wealth level. This imposes a negative externality on agents who do not observe this error term.

The definition of an equilibrium at the information acquisition stage in this extended model is analogous to the one given in Section 3.2. In order to calculate the equilibrium number of  $\epsilon$ -informed,  $\delta$ -informed, and uninformed agents, we have to compare the ex-ante expected utility of the different investor types to each other. We will say that an equilibrium exhibits "weak herding" if  $\lambda_{\delta} > 0$ . We will use the term "strong herding" to refer to equilibria for which  $\lambda_{\delta} > 0$  and  $\lambda_{\epsilon} = 0$ . These equilibria are characterized by the fact that agents herd on the same information, even though private signals with orthogonal error terms are available to them. Of course, the existence of such herding equilibria depends on the signal-to-noise ratio of the two signals captured by the parameters  $\sigma_{\epsilon}$  and  $\sigma_{\delta}$ , as well as on the cost parameters  $c_{\epsilon}$  and  $c_{\delta}$ . The following proposition presents results for the case in which both types of signals have the same precision and are equally costly.

**Proposition 6.** Suppose that the two signals  $Y_i = X + \epsilon_i$  and  $Y = X + \delta$  have the same precision (i.e.,  $\sigma_{\epsilon} = \sigma_{\delta}$ ) and are equally costly to investors (i.e.,  $c_{\epsilon} = c_{\delta}$ ). Then, in the absence of relative wealth concerns (i.e., when  $\gamma = 0$ ), there are no weak herding equilibria. If, on the other hand, the agents' relative wealth concerns are sufficiently strong, there exist equilibria that exhibit strong herding.

The first result in Proposition 6 establishes the non-existence of herding equilibria in standard REE models without consumption externalities. In such models, the returns to acquiring information fall as the number of identically informed agents increases. These negative informational externalities encourage investors to acquire signals that are orthogonal to the information revealed by prices. Thus, when given a choice between the two equally

informative (and equally costly) signals Y and  $Y_i$ , agents always prefer the conditionally independent signal  $Y_i$  when relative wealth considerations are not important to them.

The second result shows that strong herding equilibria, in which all informed agents acquire the same signal, can exist when agents care about their peer's consumption.<sup>21</sup> In fact, if  $\gamma$  is sufficiently large, relative wealth concerns dominate the information effect, and investors are better off gathering perfectly correlated information. This can be seen from equation (19). While the incremental value of the signal  $Y_i$  exceeds that of the signal Y (i.e.,  $\text{var}(X|Y_i, P) \leq \text{var}(X|Y, P)$  under our assumption that  $\sigma_{\epsilon} = \sigma_{\delta}$ ), knowing Y allows agents to eliminate the uncertainty about the average wealth level caused by the common error term  $\delta$ . By knowing what others know, agents can make sure not to fall behind their peers. In that sense, the signal's value to agents goes beyond its usefulness in predicting future asset payoffs. In fact, equation (19) reveals that agents with KUJ preferences may herd on information that is completely unrelated to fundamentals.

Strong herding equilibria are clearly inefficient. Rather than acquiring signals that complement the information revealed by prices, agents exert costly effort to duplicate the information that is available to their peers. This inefficient allocation of research effort reduces the informational content of asset prices, which can affect social welfare through two distinct channels. First, it leads to less informed portfolio decisions and, hence, lowers the agents' expected utility from trading. Second, in a broader framework in which firms use asset prices to guide their production decisions, this informational inefficiency may also lead to a suboptimal allocation of investment resources.

<sup>&</sup>lt;sup>21</sup>We want to point out, however, that strong herding equilibria are typically not unique. There are other equilibria in which agents prefer to acquire conditionally independent signals or to stay uninformed. This is not surprising, given our results in Section 4.

# 7 Community Effects and Local Interactions

Our analysis so far has been conducted under the assumption that relative wealth concerns are global, in the sense that agents care about their relative position with respect to the entire economy. More realistic social interactions suggest a more local take on relative wealth considerations. The most natural interpretation is that of communities, where each agent has relative wealth concerns only with respect to other agents in her community.<sup>22</sup> We incorporate this idea into our model by generalizing our KUJ preference specification in the following way:

$$u(W_i, \bar{W}_k) = -\exp(-\tau(W_i - \gamma \bar{W}_k)), \tag{20}$$

where  $\bar{W}_k$  denotes the average wealth of agents that belong to the same community k as agent i. To keep the notational burden to a minimum, we assume that there are only two communities, a and b. Each community consists of a continuum of agents with measure 1/2. As before, we conjecture that the average demand function in community  $k \in \{a, b\}$  is linear in the asset payoff X and the equilibrium price P:  $\bar{\theta}_k = \xi_k + \lambda_k \beta_k X - \kappa_k P$ . All other aspects of the model are the same as in Section 3.

The equilibrium at the trading stage is similar to the one characterized in Proposition 1. The only difference is that, rather than trying to mimic the average trade in the economy, investors now only care about trades executed by agents in their own community. Thus, while the total number of informed agents across both communities,  $\lambda_a + \lambda_b$ , influences an investor's demand function through its effect on the conditional payoff variance  $\operatorname{var}(X|\mathcal{F}_i)$ , the demand effect due to our KUJ preference specification only depends on the number of

<sup>&</sup>lt;sup>22</sup>Going back to our motivation based on relative performance contracts in Section 3.1, we can interpret different communities as different classes of mutual funds, each compensated relative to its own benchmark, so that relative wealth considerations arise with respect to other fund managers that invest in the same asset class.

informed agents in community  $k, \lambda_k$ :

$$\theta_i = \frac{\mathbb{E}\left[X - P|\mathcal{F}_i\right]}{\tau \operatorname{var}(X|\mathcal{F}_i)} + \gamma \left(\xi_k - P(\kappa_k - \lambda_k \beta_k)\right). \tag{21}$$

In order to derive the equilibrium number of informed agents in communities a and b, one has to compare the ex-ante expected utility of informed and uninformed investors in each community. Simple calculations (analogous to the ones in the proof of Proposition 2) show that an agent's certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left( \operatorname{var}(X|\mathcal{F}_i)^{-1} - \frac{2\lambda_k \gamma}{\sigma_{\epsilon}^2} \right) + H, \tag{22}$$

where H is independent of the agent's information set  $\mathcal{F}_i$ . It is important to note that  $\mathcal{U}_i$  is a function of the information acquired in both communities, since the informativeness of the equilibrium price P, which is contained in the information set  $\mathcal{F}_i$ , depends on the total number of informed agents in the economy.<sup>23</sup> The following proposition characterizes all stable equilibria at the information acquisition stage for the two-community case.

**Proposition 7.** Let 
$$\hat{C} = 1/(\sigma_{\epsilon}^2(e^{2\tau c} - 1)) - 1/\sigma_x^2$$
 and assume that  $\gamma < 1/(\tau \sigma_x \sigma_z)$ .

1. If  $\hat{C} > 0$ , then there exist three stable equilibria at the information acquisition stage: one symmetric equilibrium with the same fraction  $\lambda_k = \min(\lambda^*, 1)$  of informed agents in both communities, where  $\lambda^*$  is given by equation (10), and two asymmetric equilibria in which a fraction  $\lambda_k = \min(\lambda_c^*, 1)$  of agents become informed in one community and all agents stay uninformed in the other community, where:

$$\lambda_c^* = 2\tau \sigma_\epsilon^2 \sigma_z^2 \left( 2\tau \gamma + \sqrt{4\tau^2 \gamma^2 + \hat{C}/\sigma_z^2} \right). \tag{24}$$

$$\operatorname{var}(X|P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{\lambda_a + \lambda_b}{2\tau\sigma_\epsilon^2}\right)^2.$$
 (23)

<sup>&</sup>lt;sup>23</sup>One can easily verify that the conditional payoff variance is given by:

- 2. If  $\hat{C} \leq 0$ , then two cases are relevant:
  - (a) If  $4\tau^2\gamma^2 + \hat{C}/\sigma_z^2 \geq 0$ , then there are three stable equilibria at the information acquisition stage: two asymmetric equilibria with a positive fraction  $\lambda_k = \min(\lambda_c^*, 1)$  of informed agents in one community and no informed agents in the other community, and one symmetric equilibrium that involves no information acquisition, i.e.,  $\lambda_a = \lambda_b = 0$ .
  - (b) If  $4\tau^2\gamma^2 + \hat{C}/\sigma_z^2 < 0$ , the unique equilibrium is for all agents to stay uninformed, i.e.,  $\lambda_a = \lambda_b = 0$ .

Similar to the single-community case, there are three different types of equilibria, depending on the value of  $\hat{C}$ . Figure 4 illustrates these equilibria for a representative set of parameter values. When information is cheap (c < 0.347 in Figure 4), then  $\hat{C} > 0$  and we have three stable equilibria: one is symmetric with the same number of informed agents in both communities (solid line); the other two are asymmetric with a positive number of informed agents in one community (dashed line) and no informed agents in the other community (dotted line). Not surprisingly, when the cost of gathering information is sufficiently high (c > 0.357), it is optimal for all agents to stay uninformed. Thus, the unique equilibrium is characterized by  $\lambda_a = \lambda_b = 0$  in this case.

For intermediate values of the information acquisition cost (i.e., when  $0.347 \le c \le 0.357$ , so that  $\hat{C} \le 0 \le 4\tau^2\gamma^2 + \hat{C}/\sigma_z^2$ ), there are multiple symmetric and asymmetric equilibria. Interestingly, all symmetric equilibria in which a positive fraction of agents from both communities acquire information are unstable.<sup>24</sup> This can be seen from Figure 5, which plots the optimal response of agents in community b as a function of the fraction of informed agents in this community, setting the fraction of informed agents in community a to its equilibrium value. In our numerical example, the two symmetric equilibria are given

One can show that for  $\hat{C} < 0 < 4\tau^2\gamma^2 + \hat{C}/\sigma_z^2$ , there are always two unstable symmetric equilibria corresponding to the fractions  $\lambda^*$  and  $\lambda^{**}$  of informed agents specified in Proposition 2.

by  $\lambda_a = \lambda_b = \lambda^* = 0.274$  (left panel) and  $\lambda_a = \lambda_b = \lambda^{**} = 0.126$  (right panel). Figure 5 shows that neither of them is a stable equilibrium, since the agents' optimal response function crosses the 45° line from below at these points, implying that a small increase (decrease) in the number of informed traders makes it optimal for the agent (not) to acquire information herself.

To obtain an intuitive understanding for why the equilibrium with  $\lambda_k = \lambda^*$  is stable in the single-community case, but fails to be stable when we divide the population of agents into two communities (and  $\hat{C} < 0$ ), consider the effect that a small increase in  $\lambda_k$  has on an agent's expected utility in these two cases. The reduction in the agent's certainty equivalent of wealth caused by the increase in the average wealth in her community is the same in both cases (see equations (12) and (22)). However, the positive effect due to the improved price informativeness (i.e., the increase in the precision  $\text{var}(X|P)^{-1}$ ) is smaller in the case with two communities, since an increase in  $\lambda_k$  by  $\Delta$  increases the total fraction of informed agents in the economy only by  $\Delta/2$ . This can also be seen from equations (8) and (23). Thus, compared to the single-community case, relative wealth concerns play a more prominent role when there are two communities, making information a complementary good whenever  $\lambda_k$  exceeds  $\lambda^*$ . The opposite holds when  $\lambda_k$  falls below  $\lambda^*$ .

The above discussion indicates that introducing relative wealth concerns at the community level dramatically changes how agents optimally allocate their research effort. While symmetric equilibria in which different communities of agents follow the same information acquisition strategy may exist, it is quite natural to expect agents in different communities to specialize in different assets. This is consistent with the empirical findings of the growing literature on community effects and social interactions. For example, Hong, Kubik, and Stein (2004a) show that investment decisions are related to social interaction, which is naturally linked to communities. A number of studies, including Grinblatt and Keloharju (2001), Feng and Seasholes (2004), and Ivković and Weisbenner (2005a), also document that investors are more likely to invest in firms that are geographically close to them. Our model of relative

wealth concerns can generate these patterns, and, for some parameterizations, predicts them as the only stable outcome.

### 8 Conclusion

This paper extends the standard REE model with endogenous information acquisition developed by Hellwig (1980) and Verrecchia (1982) to account for relative wealth considerations. In particular, we examine how consumption externalities resulting from a KUJ preference specification affect investors' incentives to acquire information. Our analysis shows that such consumption externalities can generate complementarities in information acquisition. In the absence of relative wealth concerns, an agent's benefit from acquiring information is decreasing in the number of informed investors. The reason is that as more agents become informed, more information is revealed through asset prices and uninformed agents can free-ride on the learning of others. If agents are sensitive to the wealth of others, this effect is counteracted by the agents' concern about their relative position in the economy. A larger number of informed investors increases the average wealth, which imposes a negative externality on uninformed agents. We demonstrate that if the number of informed investors is not too high, relative wealth concerns dominate the information revelation effect, and the marginal value of information increases in the number of informed agents.

These complementarities in information acquisition can generate multiple herding equilibria. An agent's optimal decision as to whether she should gather information depends on her beliefs about the behavior of other agents. If she believes that most of her peers acquire information, she has an incentive to acquire information as well in order to keep up with them. On the other hand, if she expects others not to be informed, she may not find it worthwhile to spend resources on collecting information. In equilibrium, these beliefs are self-fulfilling. Some of these herding equilibria involve an inefficient allocation of the agents' research effort. In particular, we show that when relative wealth concerns are sufficiently strong, agents

ignore signals about fundamentals in favor of signals that are informative about their peers' trades.

We also consider the implications of relative wealth concerns for asset returns. We find that these consumption externalities lead to an increase in informed trading, which reduces the risk that investors have to bear and thus lowers the risk premium. Thus, by endogenizing the agents' information acquisition decision, we identify a new channel through which KUJ preferences affect the equity premium. In addition to the direct channel discussed in previous studies (Abel, 1990; Galí, 1994), there is an indirect channel that operates through an increase in the value of information to investors.

Further, we demonstrate that the multiplicity of equilibria at the information acquisition stage can cause price discontinuities. Extending our model to a dynamic setting in which the distribution of asset payoffs is linked to past realizations, we show that small changes in fundamentals can lead to large changes in asset prices. These price jumps are caused by changes in the risk premium as agents switch from an equilibrium with many informed traders to an equilibrium with few informed traders (and vice versa).

Finally, we discuss how relative wealth concerns can help explain recent empirical findings regarding the home bias or other local biases in portfolio choice. By introducing relative wealth concerns at the community level, we show that in many cases, only equilibria in which different communities follow different information acquisition strategies exist. Thus, consistent with the empirical evidence, our results indicate that it is quite natural for agents in different communities to specialize on different assets.

# **Appendix**

The following lemma is a standard result on multivariate normal random variables (see, e.g., Marin and Rahi (1999)) and is used to calculate the agents' expected utility:

**Lemma 1.** Let  $X \in \mathbb{R}^n$  be a normally distributed random vector with mean (vector)  $\mu$  and covariance matrix  $\Sigma$ . Then:

$$\mathbb{E}\left[\exp\left(X^{\top}AX + b^{\top}X\right)\right] = |I - 2\Sigma A|^{-1/2} \exp\left(b^{\top}\mu + \mu^{\top}A\mu\right) + \frac{1}{2}(b + 2A\mu)^{\top}(I - 2\Sigma A)^{-1}\Sigma(b + 2A\mu),$$
(25)

where  $A \in \mathbb{R}^{n \times n}$  is a symmetric matrix and  $b \in \mathbb{R}^n$  is a vector.

Note that (25) is only well-defined if the matrix  $I - 2\Sigma A$  is positive definite.

#### **Proof of Proposition 1**

Let  $V_i$  denote agent i's "relative payoff"  $V_i = W_i - \gamma \bar{W}$ . Simple calculations show that:

$$V_i = (\theta_i + \gamma (P(\kappa - \lambda \beta) - \xi))(X - P) - \gamma \lambda \beta (X - P)^2.$$
(26)

The agent's expected utility is therefore given by the exponential of a quadratic function of the normally distributed random variable X - P. Using Lemma 1, we have:

$$\mathbb{E}\left[-e^{-\tau V_i}|\mathcal{F}_i\right] = -\Psi_i^{-1/2} \exp\left(-\tau \left(\Upsilon_i + \theta_i \eta_i - \frac{\tau}{2\Psi_i} \Gamma_i(\theta_i)^2 \Sigma_i\right)\right),\tag{27}$$

where:

$$\Psi_i = 1 - 2\tau\gamma\lambda\beta\Sigma_i,\tag{28}$$

$$\Upsilon_i = \gamma (P(\kappa - \lambda \beta) - \xi - \lambda \beta \eta_i) \eta_i,$$
 (29)

$$\Gamma_i(\theta_i) = \theta_i + \gamma (P(\kappa - \lambda \beta) - \xi - 2\lambda \beta \eta_i),$$
 (30)

with  $\eta_i = \mathbb{E}[X - P|\mathcal{F}_i]$  and  $\Sigma_i = \text{var}(X - P|\mathcal{F}_i)$ .

Since  $\Psi_i$  and  $\Upsilon_i$  are independent of  $\theta_i$ , maximizing (27) with respect to  $\theta_i$  is equivalent to maximizing  $\theta_i \eta_i - \tau \Gamma_i(\theta_i)^2 \Sigma_i / (2\Psi_i)$ . Simple algebra shows that the optimal trading strategy is given by (7). The second-order condition reduces to  $\Psi_i > 0$ .

Furthermore, one can verify that:

$$\operatorname{var}(X|Y_i, P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\xi^2} + \frac{1}{\sigma_z^2} \left(\frac{b}{d}\right)^2,$$
 (31)

$$\operatorname{var}(X|P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{b}{d}\right)^2,$$
 (32)

$$\mathbb{E}\left[X|Y_i,P\right] = \mu_x + \frac{\operatorname{var}(X|Y_i,P)}{\sigma_{\epsilon}^2}(Y_i - \mu_x) + \frac{b\operatorname{var}(X|Y_i,P)}{d^2\sigma_z^2}(P - \mathbb{E}\left[P\right]), \quad (33)$$

$$\mathbb{E}[X|P] = \mu_x + \frac{b \operatorname{var}(X|P)}{d^2 \sigma_z^2} (P - \mathbb{E}[P]). \tag{34}$$

Substituting these conditional moments into the agent's demand function given by (7) yields the following expressions for the coefficients  $\alpha$ ,  $\beta$ ,  $\delta$ ,  $\zeta$ , and  $\nu$ :

$$\alpha = \zeta = \frac{\mu_x}{\tau \operatorname{var}(X|P)} - \frac{b}{\tau d^2 \sigma_z^2} \mathbb{E}[P] + \gamma \xi, \tag{35}$$

$$\beta = \frac{1}{\tau \sigma_{\epsilon}^2},\tag{36}$$

$$\delta = \gamma(\kappa - \lambda \beta) - \frac{b}{\tau d^2 \sigma_z^2} + \frac{1}{\tau \operatorname{var}(X|Y_i, P)}, \tag{37}$$

$$\nu = \gamma(\kappa - \lambda\beta) - \frac{b}{\tau d^2 \sigma_z^2} + \frac{1}{\tau \operatorname{var}(X|P)}.$$
 (38)

The market clearing condition can therefore be written as:

$$\int_{0}^{1} \theta_{i} di = \alpha + \lambda \beta X - \kappa P = Z. \tag{39}$$

This implies that the equilibrium price coefficients a, b, and d are characterized by the following three equations:  $\kappa a = \alpha$ ,  $\kappa b = \lambda \beta$ , and  $\kappa d = 1$ . From these equations, it immediately follows that  $b/d = \lambda/(\tau \sigma_{\epsilon}^2)$ , which pins down the variances var(X|P) and  $\text{var}(X|Y_i, P)$ .

From the definition of  $\kappa$  and the above expressions for  $\delta$  and  $\nu$ , we have:

$$\kappa = \lambda \delta + (1 - \lambda)\nu \tag{40}$$

$$= \gamma(\kappa - \lambda\beta) - \frac{\lambda\beta}{\tau\sigma_z^2} \left(\frac{1}{d}\right) + \frac{1}{\tau} \text{var}(X|P)^{-1} + \frac{\lambda}{\tau\sigma_\epsilon^2}$$
(41)

$$= \frac{\frac{1}{\tau} \operatorname{var}(X|P)^{-1} + \frac{\lambda}{\tau \sigma_{\epsilon}^{2}} - \gamma \lambda \beta - \frac{\lambda \beta}{\tau \sigma_{z}^{2}} \left(\frac{1}{d}\right)}{1 - \gamma}.$$
 (42)

This expression together with the equilibrium condition  $\kappa d = 1$  can be used to solve for the

price coefficient d. Further, the expression for a can be derived from the condition  $\kappa a = \alpha$ . One can readily verify that the price coefficients can be expressed in terms of the model's primitives as follows:

$$d = \frac{\tau(1-\gamma) + \frac{\lambda}{\tau\sigma_{\epsilon}^2\sigma_z^2}}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{\lambda}{\tau\sigma_{\epsilon}^2}\right)^2 + \frac{\lambda(1-\gamma)}{\sigma_{\epsilon}^2}},$$
(43)

$$\frac{a}{d} = \frac{\frac{\mu_x}{\sigma_x^2} + \frac{\mu_z \lambda}{\tau \sigma_\epsilon^2 \sigma_z^2}}{\tau (1 - \gamma) + \frac{\lambda}{\tau \sigma_\epsilon^2 \sigma_z^2}}, \qquad \frac{b}{d} = \frac{\lambda}{\tau \sigma_\epsilon^2}.$$
 (44)

Finally, we note that if the second-order condition  $\Psi_i = 1 - 2\tau\gamma\lambda\beta\Sigma_i > 0$  is satisfied for an uninformed agent, then it is also satisfied for an informed agent. Therefore, a necessary and sufficient condition for a linear equilibrium with  $\lambda > 0$  to exist is that  $\Psi_i > 0$  holds for uninformed agents, which reduces to:

$$\frac{\sigma_{\epsilon}^2}{\sigma_x^2} + \frac{\lambda^2}{\tau^2 \sigma_{\epsilon}^2 \sigma_z^2} - 2\lambda\gamma > 0. \tag{45}$$

It is easy to check that  $\gamma < 1/(\tau \sigma_x \sigma_z)$  is a sufficient condition for the above inequality to hold for all  $\lambda \in [0, 1]$ . This completes the proof.

#### Proof of Proposition 2

Substituting the agent's optimal demand  $\theta_i$  given by equation (7) into the expression for the interim expected utility given by equation (27), we have:

$$\mathbb{E}\left[u(V_i)|\mathcal{F}_i\right] = -|\Psi_i|^{-1/2} \exp\left(-\frac{\eta_i^2}{2\Sigma_i}\right),\tag{46}$$

where, as before,  $\eta_i = \mathbb{E}[X - P|\mathcal{F}_i]$  and  $\Sigma_i = \text{var}(X - P|\mathcal{F}_i)$ . The ex-ante expected utility (before P and  $Y_i$  are observed) is therefore given by the expectation of an exponential function of  $\eta_i^2$ . Since  $\eta_i$  is a normal random variable, it follows from Lemma 1 that:

$$\mathbb{E}\left[\mathbb{E}\left[u(V_i)|\mathcal{F}_i\right]\right] = -|\Psi_i|^{-1/2} \left|1 + \frac{\operatorname{var}(\eta_i)}{\Sigma_i}\right|^{-1/2} \exp\left(-\frac{\mathbb{E}\left[\eta_i\right]^2}{2\Sigma_i} \left(1 - \frac{\operatorname{var}(\eta_i)}{(\Sigma_i + \operatorname{var}(\eta_i))}\right)\right)$$
(47)  
$$= -\left|\Psi_i \frac{\operatorname{var}(X - P)}{\Sigma_i}\right|^{-1/2} \exp\left(-\frac{(\mu_x - \mathbb{E}\left[P\right])^2}{2\operatorname{var}(X - P)}\right),$$
(48)

where we have used the fact that for normally distributed random variables, the variance satisfies  $var(X) = var(\mathbb{E}[X|\mathcal{F}_i]) + var(X|\mathcal{F}_i)$ .

The certainty equivalent of wealth for informed agents, gross of information acquisition costs, is therefore given by:

$$\mathcal{V}_I(\lambda) = \frac{1}{2\tau} \log \left( \operatorname{var}(X|Y_i, P)^{-1} - \frac{2\lambda\gamma}{\sigma_{\epsilon}^2} \right) + H, \tag{49}$$

where:

$$H = \frac{1}{2\tau} \log \left( \operatorname{var}(X - P) \right) + \frac{(\mu_x - \mathbb{E}[P])^2}{2\tau \operatorname{var}(X - P)} + \frac{1}{\tau} \log \left( \mathbb{E}\left[ \exp\left(\tau \gamma \bar{W}\right) \right] \right). \tag{50}$$

For uninformed agents, we have:

$$\mathcal{V}_{U}(\lambda) = \frac{1}{2\tau} \log \left( \operatorname{var}(X|P)^{-1} - \frac{2\lambda\gamma}{\sigma_{\epsilon}^{2}} \right) + H.$$
 (51)

Further, recall from the proof of Proposition 1 that:

$$\operatorname{var}(X|Y_i, P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_z^2} \left(\frac{\lambda}{\tau \sigma_\epsilon^2}\right)^2, \tag{52}$$

$$\operatorname{var}(X|P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{\lambda}{\tau \sigma_\epsilon^2}\right)^2.$$
 (53)

Now consider the case where  $\hat{C} \equiv 1/(\sigma_{\epsilon}^2(e^{2\tau c}-1))-1/\sigma_x^2>0$ . An interior equilibrium is defined by  $\lambda \in (0,1)$  such that  $\mathcal{V}_I(\lambda)-c=\mathcal{V}_U(\lambda)$ . Using the expressions derived above, one can easily verify that such an interior equilibrium is given by the solution to the following quadratic equation:

$$\frac{\lambda^2}{\tau^2 \sigma_{\epsilon}^4 \sigma_z^2} - \frac{2\lambda \gamma}{\sigma_{\epsilon}^2} - \hat{C} = 0. \tag{54}$$

The discriminant of this quadratic equation is always positive when  $\hat{C} > 0$ . Furthermore, if  $\hat{C} > 0$ , it has only one positive root.<sup>25</sup> Thus, the unique interior equilibrium is given by  $\lambda = \lambda^*$  as defined by equation (10). We are left to check whether there exist any corner equilibria. Clearly,  $\lambda = 0$  cannot be an equilibrium when  $\hat{C} > 0$ . However,  $\lambda^*$  can exceed 1. In this case, the unique equilibrium is  $\lambda = 1$ .

Next, consider the case where  $\hat{C} \leq 0$ . In this case, the quadratic equation in (54) has real roots if and only if  $\tau^2 \gamma^2 + \hat{C}/\sigma_z^2 \geq 0$ . If this condition is not met, the unique equilibrium is therefore for all agents to stay uninformed (i.e,  $\lambda = 0$ ). On the other hand, if this inequality

<sup>&</sup>lt;sup>25</sup>This follows immediately from Descartes' rule of sign.

holds, then there are two positive solutions. The corresponding equilibria are given by  $\lambda = \min(\lambda^*, 1)$  and  $\lambda = \min(\lambda^{**}, 1)$ , where  $\lambda^*$  and  $\lambda^{**}$  are defined in Proposition 2. Note, however, that since  $\hat{C} \leq 0$ , we have  $\mathcal{V}_I(0) - c \leq \mathcal{V}_U(0)$ . Thus,  $\lambda = 0$  is an equilibrium as well.

#### **Proof of Proposition 3**

Let  $\mathcal{R}(\lambda) = \mathcal{V}_I(\lambda) - \mathcal{V}_U(\lambda)$  denote the difference between the certainty equivalent of informed and uninformed agents. Then, a necessary (sufficient) condition for an interior equilibrium characterized by  $\mathcal{R}(\hat{\lambda}) = c$  to be stable is that  $d\mathcal{R}(\hat{\lambda})/d\lambda \leq 0$  ( $d\mathcal{R}(\hat{\lambda})/d\lambda < 0$ ). Substituting the expressions for  $\mathcal{V}_I(\lambda)$  and  $\mathcal{V}_U(\lambda)$  derived in the proof of Proposition 2 into the function  $\mathcal{R}(\lambda)$  and using the fact that  $\operatorname{sign}(df(x)/dx)) = \operatorname{sign}(de^{f(x)}/dx)$ , we have:

$$\operatorname{sign}\left(\frac{d\mathcal{R}(\lambda)}{d\lambda}\right) = \operatorname{sign}\left(\frac{d}{d\lambda}\left(\frac{1}{\sigma_{\epsilon}^{2}\sigma_{x}^{2}} + \frac{\lambda^{2}}{\tau^{2}\sigma_{\epsilon}^{2}\sigma_{z}^{2}} - 2\lambda\gamma\right)^{-1}\right). \tag{55}$$

The above inequality can therefore be written as follows:

$$\gamma - \frac{\hat{\lambda}}{\tau^2 \sigma_{\epsilon}^2 \sigma_z^2} \le 0. \tag{56}$$

If  $\hat{C} < 0 < \tau^2 \gamma^2 + \hat{C}/\sigma_z^2$  and  $\lambda^* < 1$ , the two interior equilibria are given by  $\lambda = \lambda^*$  and  $\lambda = \lambda^{**}$ . From the expressions in equations (10) and (11), it follows immediately that  $\lambda^* > \gamma \tau^2 \sigma_\epsilon^2 \sigma_z^2$  and that  $\lambda^{**} < \gamma \tau^2 \sigma_\epsilon^2 \sigma_z^2$ . This proves that only the equilibrium given by  $\lambda = \lambda^*$  is stable. Clearly, the corner solution  $\lambda = 0$  is a stable equilibrium as well.

#### **Proof of Proposition 4**

We first argue that if  $\gamma = 0$ , condition (14) cannot hold. In this case, the only interior equilibrium at the information acquisition stage is characterized by  $\lambda_t = \lambda^*(\sigma_x(X_t))$ , where  $\lambda^*$  is defined in Proposition 2. Note that  $\lambda^*$  is a continuous function of  $X_t$ , since  $\sigma_x$  is continuous in  $X_t$  and  $\lambda^*$  is continuous in  $\sigma_x$ . Further, since  $\lambda_t = 0$  ( $\lambda_t = 1$ ) if  $\lambda^*(\sigma_x(X_t)) \leq 0$  ( $\lambda^*(\sigma_x(X_t)) \geq 1$ ), it follows immediately that  $\lambda_t$  is a continuous function of  $X_t$ . Thus, price jumps as defined in (14) cannot occur if  $\gamma = 0$ .

Next, consider the case where  $\gamma > 0$ . Let  $X^*$  denote the value of  $X_t$  such that  $\hat{C}(\sigma_x(X_t)) = 0$ , where  $\hat{C}$  is defined in Proposition 2. Further, let  $X^{**}$  denote the value of  $X_t$  such that  $\tau^2 \gamma^2 + \hat{C}(\sigma_x(X_t))/\sigma_z^2 = 0$ . We claim that if  $\lambda_{t-1} = 0$ , then  $X_t = X^*$  satisfies condition (14). The argument goes as follows. At  $X_t = X^*$ , we have  $\hat{C} = 0$ , which implies that  $\lambda_t = 0$  is an equilibrium (Proposition 2). Thus, by our convention, the equilibrium fraction

of informed agents is equal to  $\lambda_t = 0$  when  $X_t = X^*$  and  $\lambda_{t-1} = 0$ . However, for any  $\epsilon > 0$ ,  $\hat{C}(\sigma_x(X^* + \epsilon)) > 0$  and thus the unique equilibrium is given by  $\lambda_t = \min(\lambda^*(\sigma_x(X^* + \epsilon)), 1) > 0$ . This follows again from Proposition 2. An infinitesimal change in  $X_t$  therefore causes a discrete change in  $\lambda_t$ , which in turn causes a discrete jump in prices as defined by condition (14).<sup>26</sup>

Similarly, if  $\lambda_{t-1} > 0$ , then  $X_t = X^{**}$  satisfies condition (14). In this case, the equilibrium at the information acquisition stage is characterized by  $\lambda_t > 0$ , since  $\hat{C} < 0 = \tau^2 \gamma^2 + \hat{C}/\sigma_z^2$  at  $X_t = X^{**}$  and  $\lambda_{t-1} > 0$ . However, for any  $\epsilon > 0$ , we have  $\tau^2 \gamma^2 + \hat{C}(\sigma_x(X^{**} - \epsilon))/\sigma_z^2 < 0$ , which implies that the unique equilibrium is given by  $\lambda_t = 0$  (Proposition 2). Thus, a small change in fundamentals leads again to a large change in prices.

## **Proof of Proposition 5**

The calculations involved in this proof are analogous to those in the proof of Proposition 1. The relevant payoff variable of agent i,  $V_i = W_i - \gamma \bar{W}$ , is given by:

$$V_i = (\theta_i + \gamma (P(\kappa - \lambda_\epsilon \beta_\epsilon) - \xi)) (X - P) - \gamma \lambda_\delta \beta_\delta Y (X - P) - \gamma \lambda_\epsilon \beta_\epsilon (X - P)^2, \tag{57}$$

which is a quadratic function of the normal random vector (X - P, Y). Using Lemma 1, we can therefore rewrite the agent's conditional expected utility as follows:

$$\mathbb{E}\left[-e^{-\tau V_i}|\mathcal{F}_i\right] = -\Psi_i^{-1/2} \exp\left(-\tau \left(\Upsilon_i + \theta_i \eta_{i,1} - \frac{\tau}{2\Psi_i} Q_i(\theta_i)\right)\right), \tag{58}$$

where:

$$\Psi_i = 1 - 2\tau\gamma \left(\lambda_{\epsilon}\beta_{\epsilon}\Sigma_{i,11} + \lambda_{\delta}\beta_{\delta}\Sigma_{i,12}\right) - \tau^2\gamma^2\lambda_{\delta}^2\beta_{\delta}^2|\Sigma_i|, \tag{59}$$

$$\Lambda_i = -\gamma \lambda_\delta \beta_\delta \eta_{i,1}, \tag{60}$$

$$\Upsilon_i = \gamma \left( P(\kappa - \lambda_{\epsilon} \beta_{\epsilon}) - \xi - \lambda_{\epsilon} \beta_{\epsilon} \eta_{i,1} - \lambda_{\delta} \beta_{\delta} \eta_{i,2} \right) \eta_{i,1}, \tag{61}$$

$$\Gamma_i(\theta_i) = \theta_i + \gamma \left( P(\kappa - \lambda_\epsilon \beta_\epsilon) - \xi - 2\lambda_\epsilon \beta_\epsilon \eta_{i,1} - \lambda_\delta \beta_\delta \eta_{i,2} \right), \tag{62}$$

$$Q_i(\theta_i) = \Gamma_i(\theta_i)^2 \Sigma_{i,11} + 2\Gamma_i(\theta_i) \Lambda_i(\Sigma_{i,12} + \tau \gamma \lambda_\delta \beta_\delta |\Sigma_i|) + \Lambda_i^2(\Sigma_{i,22} - 2\tau \gamma \lambda_\epsilon \beta_\epsilon |\Sigma_i|), (63)$$

with  $\eta_i = \mathbb{E}[(X - P, Y) | \mathcal{F}_i]$  and  $\Sigma_i = \text{var}((X - P, Y) | \mathcal{F}_i)$ .  $\eta_{i,m}$  ( $\Sigma_{i,mn}$ ) denotes the *m*th (*mn*th) element of the vector  $\eta_i$  (matrix  $\Sigma_i$ ).

The agent's optimal trading strategy in (17) follows immediately from the first-order

 $<sup>^{26}</sup>$ It can easily be verified that the equilibrium price  $P_t$  is strictly increasing in  $\lambda_t$  as long as  $\gamma < 1$  and  $\mu_z > 0$ . This can also be seen from the risk premium given by equation (9).

condition. The second-order condition is given by  $\Psi_i > 0.^{27}$  Equation (59) shows that this inequality holds for sufficiently small values of  $\gamma$ . The conditional moments  $\eta_i$  and  $\Sigma_i$  can be calculated from the projection theorem. Substituting these conditional moments into the agent's demand function and imposing the market clearing condition  $\bar{\theta} = Z$ , we obtain the following expressions for the demand coefficients  $\beta_{\epsilon}$  and  $\beta_{\delta}$ :

$$\beta_{\epsilon} = \frac{1}{\tau \sigma_{\epsilon}^2},\tag{64}$$

$$\beta_{\delta} = \frac{1}{\tau \sigma_{\delta}^2} - \frac{\lambda_{\delta} \beta_{\delta} \lambda_{\epsilon} \beta_{\epsilon}}{\tau \sigma_{z}^2} + \gamma \lambda_{\delta} \beta_{\delta}. \tag{65}$$

Finally, rearranging the market clearing condition, we find that the equilibrium price coefficients satisfy the restrictions  $b_x/d = \lambda_{\epsilon}\beta_{\epsilon}$  and  $b_Y/d = \lambda_{\delta}\beta_{\delta}$ .<sup>28</sup>

## Proof of Proposition 6

Analogous calculations to those in the proof of Proposition 2 show that the agents' ex-ante expected utility is given by:

$$\mathbb{E}\left[u(V_i)\right] = -\left|\Psi_i \frac{\operatorname{var}(X-P)}{\operatorname{var}(X|\mathcal{F}_i)}\right|^{-1/2} \exp\left(-\frac{(\mu_x - \mathbb{E}\left[P\right])^2}{2\operatorname{var}(X-P)}\right),\tag{66}$$

where  $\Psi_i$  is defined in equation (59). The certainty equivalent of wealth, gross of information acquisition costs, can therefore be written as:

$$\mathcal{U}_{i} = \frac{1}{2\tau} \log \left( \operatorname{var}(X|\mathcal{F}_{i})^{-1} - 2\tau \gamma \left( \lambda_{\epsilon} \beta_{\epsilon} + \lambda_{\delta} \beta_{\delta} \frac{\operatorname{cov}(X, Y|\mathcal{F}_{i})}{\operatorname{var}(X|\mathcal{F}_{i})} \right) - \tau^{2} \gamma^{2} \lambda_{\delta}^{2} \beta_{\delta}^{2} \operatorname{var}(Y|X, \mathcal{F}_{i}) \right) + H,$$
(67)

where we have used the fact that:

$$var(Y|X, \mathcal{F}_i) = var(Y|\mathcal{F}_i) - \frac{cov(X, Y|\mathcal{F}_i)^2}{var(X|\mathcal{F}_i)}.$$
 (68)

As before, the H term is independent of agent i's information set  $\mathcal{F}_i$ .

For a weak herding equilibrium to exist, investors must be indifferent between acquiring

<sup>&</sup>lt;sup>27</sup>It can easily be verified that this condition is also sufficient for the agent's conditional expected utility in (58) to be well-defined.

Note that the normalized price  $\tilde{P} = P/d$  is informationally equivalent to P. Thus, in order to characterize the equilibrium at the information acquisition stage, we only need to know the ratios  $b_x/d$  and  $b_Y/d$ .

the signal Y and the signal  $Y_i$ . Thus, under the assumption that  $c_{\epsilon} = c_{\delta}$ , we must have:

$$\operatorname{var}(X|Y,P)^{-1} = \operatorname{var}(X|Y_i,P)^{-1} - 2\tau\gamma\lambda_{\delta}\beta_{\delta} \frac{\operatorname{cov}(X,Y|Y_i,P)}{\operatorname{var}(X|Y_i,P)} - \tau^2\gamma^2\lambda_{\delta}^2\beta_{\delta}^2 \operatorname{var}(Y|X,Y_i,P).$$
(69)

When  $\gamma = 0$ , this equation simplifies to:

$$\frac{1}{\sigma_{\epsilon}^2} + \frac{(\lambda_{\epsilon}\beta_{\epsilon})^2}{\sigma_z^2} = \frac{1}{\sigma_{\delta}^2} + \frac{(\lambda_{\epsilon}\beta_{\epsilon} + \lambda_{\delta}\beta_{\delta})^2}{\sigma_z^2 + (\lambda_{\delta}\beta_{\delta})^2 \sigma_{\delta}^2},\tag{70}$$

where we have used the projection theorem to calculate the conditional moments:

$$\operatorname{var}(X|Y,P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\delta^2} + \frac{(\lambda_\epsilon \beta_\epsilon)^2}{\sigma_z^2}, \tag{71}$$

$$\operatorname{var}(X|Y_i, P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{(\lambda_\epsilon \beta_\epsilon + \lambda_\delta \beta_\delta)^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2}.$$
 (72)

Substituting the equilibrium trading intensities  $\beta_{\epsilon}$  and  $\beta_{\delta}$  given by equation (18) into equation (70) and setting  $\sigma_{\epsilon} = \sigma_{\delta} = \sigma_{s}$  yields:

$$\lambda_{\delta}^{2} \lambda_{\epsilon}^{2} + \lambda_{\delta} (\lambda_{\delta} + 2\lambda_{\epsilon}) \tau^{2} \sigma_{s}^{2} \sigma_{z}^{2} = 0.$$
 (73)

Clearly, the above equation only holds for  $\lambda_{\delta} = 0$ . This proves that, in the absence of relative wealth concerns, there are no herding equilibria.

In order to prove the existence of strong herding equilibria, we have to show (i) that investors are indifferent between acquiring the signal Y and staying uninformed, and (ii) that investors strictly prefer to acquire the signal Y rather than the signal  $Y_i$  (i.e.,  $\beta_{\epsilon} = 0$ ):

$$\operatorname{var}(X|Y,P)^{-1}e^{-2\tau c} = \operatorname{var}(X|P)^{-1} - 2\tau\gamma\lambda_{\delta}\beta_{\delta}\frac{\operatorname{cov}(X,Y|P)}{\operatorname{var}(X|P)} - \tau^{2}\gamma^{2}\lambda_{\delta}^{2}\beta_{\delta}^{2}\operatorname{var}(Y|X,P), \quad (74)$$

$$\operatorname{var}(X|Y,P)^{-1} > \operatorname{var}(X|Y_i,P)^{-1} - 2\tau\gamma\lambda_{\delta}\beta_{\delta} \frac{\operatorname{cov}(X,Y|Y_i,P)}{\operatorname{var}(X|Y_i,P)} - \tau^2\gamma^2\lambda_{\delta}^2\beta_{\delta}^2 \operatorname{var}(Y|X,Y_i,P), \tag{75}$$

where we have again assumed that  $c_{\epsilon} = c_{\delta} = c$ . Using the projection theorem, one can show that:

$$\frac{\operatorname{cov}(X,Y|\mathcal{F}_i)}{\operatorname{var}(X|\mathcal{F}_i)} = \frac{\sigma_z^2 - \lambda_\epsilon \beta_\epsilon \lambda_\delta \beta_\delta \sigma_\delta^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2}$$
(76)

$$\operatorname{var}(Y|X,\mathcal{F}_i) = \frac{\sigma_z^2 \sigma_\delta^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2}$$
 (77)

for both  $\epsilon$ -informed and uninformed investors. Substituting these expressions and the conditional moments derived above into (75), and setting  $\sigma_{\epsilon} = \sigma_{\delta} = \sigma_{s}$ , yields:

$$\lambda_{\delta}\beta_{\delta} - 2\tau\gamma\sigma_{z}^{2} - \tau^{2}\gamma^{2}\lambda_{\delta}\beta_{\delta}\sigma_{z}^{2}\sigma_{\delta}^{2} < 0. \tag{78}$$

Since  $\beta_{\delta}$  is decreasing in  $\gamma$ , this inequality holds for sufficiently large values of  $\gamma$ . The equilibrium fraction of  $\delta$ -informed investors can then be derived from equation (74), which is a quadratic equation in  $\lambda_{\delta}$ , since:

$$\operatorname{var}(X|Y,P)^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\delta^2} + \frac{(\lambda_\epsilon \beta_\epsilon)^2}{\sigma_z^2}, \tag{79}$$

$$\operatorname{var}(X|P)^{-1} = \frac{1}{\sigma_x^2} + \frac{(\lambda_\epsilon \beta_\epsilon + \lambda_\delta \beta_\delta)^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2}.$$
 (80)

It can easily be verified that equation (74) has a positive root, if the information acquisition cost c is not too large. This completes the proof.

## **Proof of Proposition 7**

Analogous calculations to those in the proof of Proposition 1 show that the optimal trading strategy of agent i in community k is given by:

$$\theta_i = \frac{\mathbb{E}\left[X - P|\mathcal{F}_i\right]}{\tau \operatorname{var}(X|\mathcal{F}_i)} + \gamma \left(\xi_k - P(\kappa_k - \lambda_k \beta_k)\right),\tag{81}$$

where  $\beta_k = 1/(\tau \sigma_{\epsilon}^2)$ . From the market clearing condition, it thus follows that equilibrium prices are as in (4) with  $b/d = (\lambda_a + \lambda_b)/(2\tau \sigma_{\epsilon}^2)$ .

The derivation of the equilibrium at the information acquisition stage follows the steps of the proof of Proposition 2. Substituting the agent's optimal trading strategy into the expression for the interim expected utility and applying Lemma 1, we can write the certainty equivalent of wealth, gross of information acquisition costs, as follows:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left( \operatorname{var}(X|\mathcal{F}_i)^{-1} - \frac{2\lambda_k \gamma}{\sigma_{\epsilon}^2} \right) + H, \tag{82}$$

where H is independent of the agent's information set  $\mathcal{F}_i$ . Comparing this expression to the certainty equivalent in the single-community case given by equation (12) immediately reveals that  $\lambda_a = \lambda_b = \min(\lambda^*, 1)$  is an equilibrium when there are two communities. In addition to this symmetric equilibrium, there are also asymmetric equilibria in which only agents in one of the two communities gather information. If  $\lambda_b = 0$ , it can be shown that  $\lambda_a = \lambda_c^*$ , as

defined in the proposition, makes agents in community a indifferent between acquiring and not acquiring information (assuming that  $\lambda_c^* \in (0,1)$ ), whereas agents in community b have no incentive to acquire information. Of course,  $\lambda_a = 0$  and  $\lambda_b = \lambda_c^*$  is also an equilibrium. When  $\hat{C} > 0$ , these are the only equilibria, since the other root of the quadratic equation—given by  $\lambda^{**}$  for the symmetric equilibrium, and by  $\lambda_c^{**}$  (defined below) for the asymmetric equilibrium—is negative. It can easily be verified that all three equilibria are stable in this case.

When  $\hat{C} \leq 0 \leq 4\tau^2\gamma^2 + \hat{C}/\sigma_z^2$ , four additional equilibria exist: two symmetric equilibria with  $\lambda_a = \lambda_b = 0$  and  $\lambda_a = \lambda_b = \lambda^{**}$ , where  $\lambda^{**}$  is defined in Proposition 2; and two asymmetric equilibria characterized by  $\lambda_a = 0$  and  $\lambda_b = \lambda_c^{**}$ , and by  $\lambda_a = \lambda_c^{**}$  and  $\lambda_b = 0$ , where  $\lambda_c^{**} = 2\tau\sigma_\epsilon^2\sigma_z^2\left(2\tau\gamma - \sqrt{4\tau^2\gamma^2 + \hat{C}/\sigma_z^2}\right)$ . It is straightforward to show that all equilibria involving  $\lambda^{**}$  or  $\lambda_c^{**}$  are unstable for the reasons discussed in Section 4. The asymmetric equilibria involving  $\lambda_c^{*}$ , on the other hand, are easily verified to be stable.

Finally, when  $4\tau^2\gamma^2 + \hat{C}/\sigma_z^2 < 0$ , it follows from (10) and (24) that the quadratic equations characterizing  $\lambda^*$  and  $\lambda_c^*$  have no real roots. Thus, the unique equilibrium is given by  $\lambda_a = \lambda_b = 0$ .

Interestingly, the symmetric equilibrium  $\lambda_a = \lambda_b = \lambda^*$  turns out to be unstable as well, if  $\hat{C} < 0$ . To see this, let  $\mathcal{R}(\lambda_a, \lambda_b)$  denote the difference between the certainty equivalent of informed and uninformed agents in community a, i.e.:

$$\mathcal{R}(\lambda_a, \lambda_b) = \mathcal{V}_{Ia}(\lambda_a, \lambda_b) - \mathcal{V}_{Ua}(\lambda_a, \lambda_b), \tag{83}$$

where:

$$\mathcal{V}_{Ia}(\lambda_a, \lambda_b) = \frac{1}{2\tau} \log \left( \operatorname{var}(X|Y_i, P)^{-1} - \frac{2\lambda_a \gamma}{\sigma_{\epsilon}^2} \right) + H, \tag{84}$$

$$\mathcal{V}_{Ua}(\lambda_a, \lambda_b) = \frac{1}{2\tau} \log \left( \operatorname{var}(X|P)^{-1} - \frac{2\lambda_a \gamma}{\sigma_{\epsilon}^2} \right) + H.$$
 (85)

A necessary condition for the symmetric equilibrium to be stable is that  $\partial \mathcal{R}(\lambda^*, \lambda^*)/\partial \lambda_a \leq 0$ . Tedious but straightforward calculations show that this inequality holds if and only if  $\lambda^* \geq 2\tau^2\gamma\sigma_\epsilon^2\sigma_z^2$ . It is obvious from the definition of  $\lambda^*$  in Proposition 2 that this can only be the case if  $\hat{C} \geq 0$ . Thus, for negative values of  $\hat{C}$ , the symmetric equilibrium  $\lambda_a = \lambda_b = \lambda^*$  is unstable.

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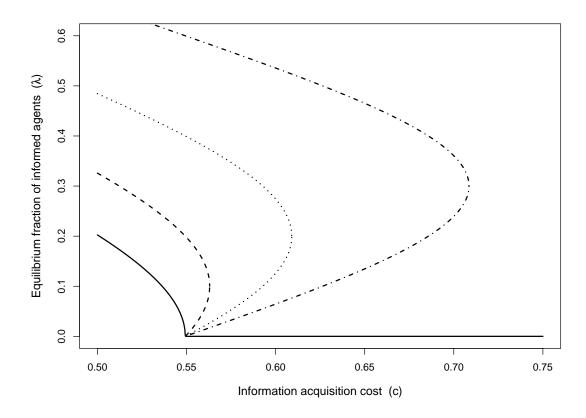


Figure 1: The graph presents the equilibrium fraction of informed agents,  $\lambda$ , as a function of the cost of gathering information, c. The solid line corresponds to the standard model with  $\gamma=0$ . The dotted and dashed lines correspond to equilibria with  $\gamma=0.2,\,0.4,\,$  and 0.6, respectively. Other parameter values are  $\sigma_{\epsilon}^2=0.5,\,\sigma_x^2=\sigma_z^2=\tau=1.$ 

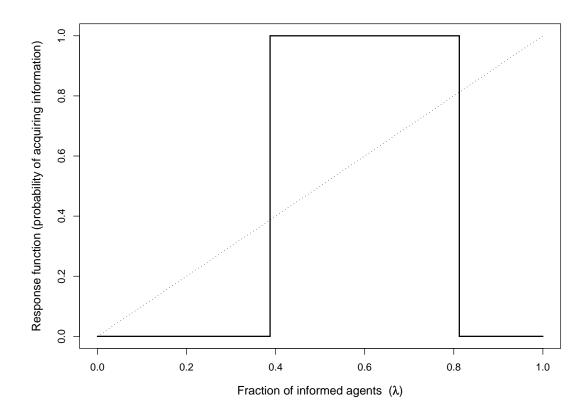


Figure 2: The graph presents the optimal information acquisition decision of an agent as a function of the fraction of informed agents. Parameters correspond to those satisfying  $\hat{C} < 0 < \tau^2 \gamma^2 + 4 \hat{C}/\sigma_z^2$  in Proposition 2:  $\sigma_x^2 = \sigma_\epsilon^2 = \sigma_z^2 = \tau = 1$ , c = 0.45, and  $\gamma = 0.6$ .

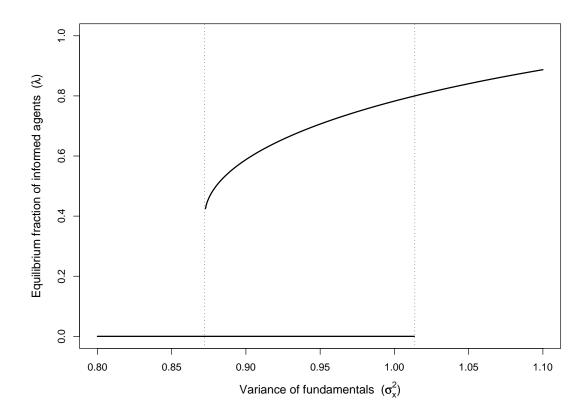


Figure 3: The graph presents the equilibrium fraction of informed agents,  $\lambda$ , at the two stable equilibria as a function of the payoff variance,  $\sigma_x^2$ . The dotted lines correspond to the points where the equilibrium switches from uniqueness to multiplicity. The parameter values used in the graph are  $\sigma_\epsilon^2 = \sigma_z^2 = \tau = 1$ , c = 0.3, and  $\gamma = 0.2$ .

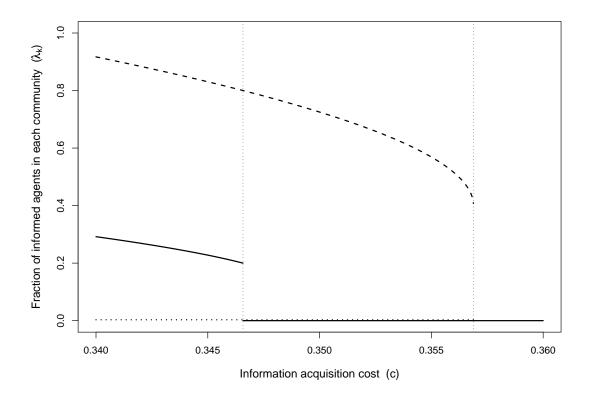


Figure 4: The graph presents the equilibrium fraction of informed agents in community  $k, \lambda_k, k = a, b$ , as a function of the cost of gathering information, c. The solid line corresponds to the stable symmetric equilibrium where  $\lambda_a = \lambda_b$ . The dashed and the dotted line characterize the stable asymmetric equilibria. The dashed line indicates the fraction of informed agents in one community when all agents in the other community are uninformed (dotted line). The parameter values used in the graph are  $\sigma_x^2 = \sigma_\epsilon^2 = \sigma_z^2 = \tau = 1$  and  $\gamma = 0.1$ .

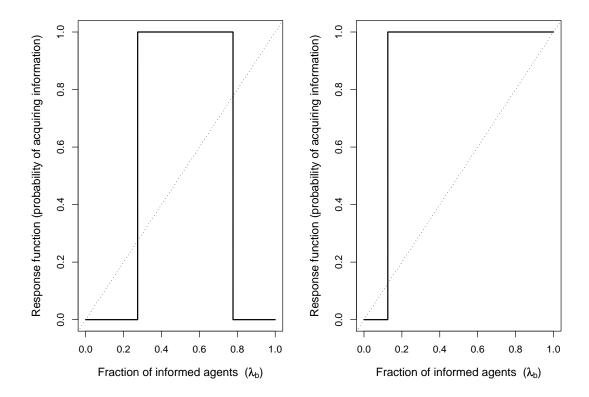


Figure 5: The graph presents the optimal information acquisition decision of an agent in community b as a function of the fraction of informed agents in community b. The fraction of informed agents in community a is set to its equilibrium value:  $\lambda_a = 0.274$  in the left panel, and  $\lambda_a = 0.126$  in the right panel. The parameter values used are  $\sigma_x^2 = \sigma_z^2 = \tau = 1$ ,  $\sigma_\epsilon^2 = 1/2$ , c = 0.6, and  $\gamma = 0.4$ .