ABSTRACT

MING-TA HSIEH Preliminary Study of the Growth of Metallic Aerosols under Reduced Pressure Environment. (Under the direction of Dr. PARKER C. REIST and Dr. PHIL A. LAWLESS)

The growth behavior of metallic aerosols under normal and reduced pressure environments was investigated. Particles grown at both environments exhibit very irregular profiles. Particles' growth, in general, can be categorized into three types in this study, they are: cluster-cluster aggregation, particle-cluster aggregation, and ballistic aggregation.

A descriptor, called fractal dimension, is applied to quanti fying the effect of reducing the pressure on the growth of particles. According to the measurements of fractal dimension, at higher pressure, 0.1 and 1.0 atm., cluster-cluster aggregation occurred at the early stage of growth; while particle-cluster aggregation occurred at the later stage of growth. Opposite phenomenon was found when pressure decreased to 0.01 atm.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to his advisor, Dr. Parker C. Reist, for his guidance and direction. Gratitude is also due to Dr. Phil A. Lawless, Research Triangle Institute, for his valuable advices. Appreciation is likewise expressed to my dear friends for their encouragement during writing this report.

Ming-Ta Hsieh

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I. Introduction

Aerosol growth is an area of study most interesting to aerosol technologists. For the past few decades much of research has been concentrated on particle coagulation due to Brownian motion.[1-9] However, almost all studies considered the atmospheric pressure parameter as a constant having a value of one atmosphere, so that these studies were only applicable to the atmosphere near the surface of the earth. The pressure in the upper level of the atmosphere is less than 1 atm, decreasing exponentially as the altitude increases. Thus in the upper reaches of the atmosphere the gas mean free path is no longer the same as used in the traditional growth model. In other words, to study aerosol growth in the upper atmosphere, it is necessary to take into account the parameter of gas mean free path, i.e. gas pressure.

One of the major goals of this report is to investigate the growth behavior of metallic aerosols at several reduced pressure environments. To do this we have explored the use of a recently developed descriptor to characterize the shape of grown aerosols, called the fractal dimension of the particle.

Traditionally, the term most often used in characterizing aerosols is aerodynamic diameter. In many cases, however, aerodynamic diameter is hard to describe the irregular aggregates very well and cannot tell much characteristics of the irregular aggregates. It is the fractal dimension that can describe certain rugged profiles.

Fractal dimensions are numbers between the classical whole number dimensions of 1, 2, and 3. This term was first introduced by Mandelbrot in 1977.[10] He suggested that the concept of dimension can be extended from integer values of 1, 2, and 3 by the addition of a fractional value describing the space filling power of a boundary or a surface. Thus a line of fractal dimension 1.4 would fill space more efficiently than an extremely straight line, whose fractal dimension is 1.0, even though both would be topologically of dimension 1. Similarly, a rugged surface with fractal dimension 2.4 would fill space more than a surface of fractal dimension 2.0 or 2.2.

I.1 Growth of Metallic Aerosols

Metallic aerosols investigated in this report were generated by the Exploding Wire Generator (EWG), which will be described in Section II. The first step of growth is the nucleation of aerosols from the vapor phase, the second step is the liquid metal coagulation, and the third step is the growth of solid aerosol particles. The time needed for the first two steps to go to completion is extremely short, so that observation is very difficult. Only the growth of solid aerosols could be "seen" via filter sampling at different times (see electron micrographs).

According to the literature,[11-22] formation of aggregates can be characterized in two categories: particle-cluster growth model, and cluster-cluster growth model.

For more than two decades, computer simulations have been applied to the formation of aggregates in a wide variety of systems. This approach has been particularly successful in developing a better understanding of aggregation phenomena. In the 1960s, the earliest model for computer simulation of floc formation in colloidal systems were carried out by Vold[23,24] and Sutherland et al.[25,26] In these studies, particles were added to growing clusters of particles via randomly oriented trajectories, without including the effects of Brownian motion. A reasonable model of cluster formation in the colloidal systems should include the effects of long- and short-range interactions, particle size distribution and irregular shapes, hydrodynamic interactions, clustering of clusters, etc.[12] Recently, Witten and Sander[20] have introduced a particle-cluster aggregation model where the effects of Brownian motion were included.

Witten and Sander start with a single-seed particle at the origin of a lattice. A second particle is added a long distance from the origin and undergoes a random walk on the lattice until it reaches a site adjacent to the seed and becomes part of the growing cluster. Then, a third particle is introduced at a random distant point and undergoes a random walk until it also becomes part of the growing cluster. The procedure is repeated until a cluster of sufficiently large size is formed.

As the case of particle-cluster aggregation, the earliest models for cluster-cluster aggregation were carried out also using linear trajectories. In recent years, Paul Meakin has explored a series of investigations[11-17] regarding the diffusion-limited aggregation in two- and three-dimensional simulations, in which both linear and random walk (Brownian) trajectories are considered and in which both particle-cluster growth model and cluster-cluster growth model are taken into consideration.

In the original Witten-Sander model, all growth originates from a single immobile growth site and only one particle is allowed in the vicinity of the growing cluster at any time. These features are unrealistic for many real colloidal systems. [14] In Meakin's model, clusters of particles, as well as single particles, are allowed to diffuse; clusters of all sizes stick together on contact.[14]

Meakin uses two-dimensional simulations on a simple square lattice with periodic boundary conditions. At the start of the simulation, a fraction of the lattice sites are picked at random and occupied (avoiding multiple occupancy). Particles at nearest neighbor positions are considered to belong to the same cluster. Clusters, including single-particle clusters, are picked randomly and moved with a probability proportional to their mobility by one lattice spacing in one of four equally probable directions. If a cluster contacts other clusters, they are merged to form a single cluster. In this manner, the clusters grow larger and larger until only one large cluster remains.

I.2 Fractal Dimension

Before discussing the fractal dimension, it is necessary to know what "fractals" are. Fractals are mathematical entities which have been developed to describe the geometrical similarities between irregular systems. This is a relatively new branch of mathematics and is increasingly applied to the problems dealing with irregular shapes.

According to Mandelbrot [27], fractals can be categorized into two kinds: mathematical fractals and natural fractals. An ideal fractal mathematical curve has two important properties. First, it has an indeterminate boundary magnitude approaching to infinity. Secondly, it is mathematically self-similar at any two different scales of scrutiny; in other words, the boundary looks the same whatever magnification is used in the examination of the structure of the curve. A natural fractal is a curve whose structure appears indeterminate at a series of resolutions but may ultimately exhibit other significant geometrical behavior when measured at a sufficiently high level of resolution. Methodology for measuring the fractal dimension will be discussed in Section III.

An approach to the description of rugged profiles using the concepts of fractal dimension to describe irregular profile of a fine particle has been widely developed by Brian H. Kaye and co-workers.[28-33] The concept of the fractal dimension of a non-Euclidean boundary is introduced by Mandelbrot.[10] The origin of "fractal" comes from the Latin adjective "fractus", meaning "irregular" or "fragmented". In his book Mandelbrot describes the ruggedness of a line as its space filling ability and assigns a dimension between one and two to the structure of a line depending on its ability to fill space.[27] A simple example is a straight line of traditional Euclidean geometry, which has a fractal dimension of 1; the trajectory of Brownian motion on a flat plane has a fractal dimension of 2.

Those studies dealing with computer simulations of growth models, as discussed previously, also worked out the measurements of fractal dimension of corresponding growth model. Table 1.1 summarizes these characteristic values of fractal dimension for different growth models.

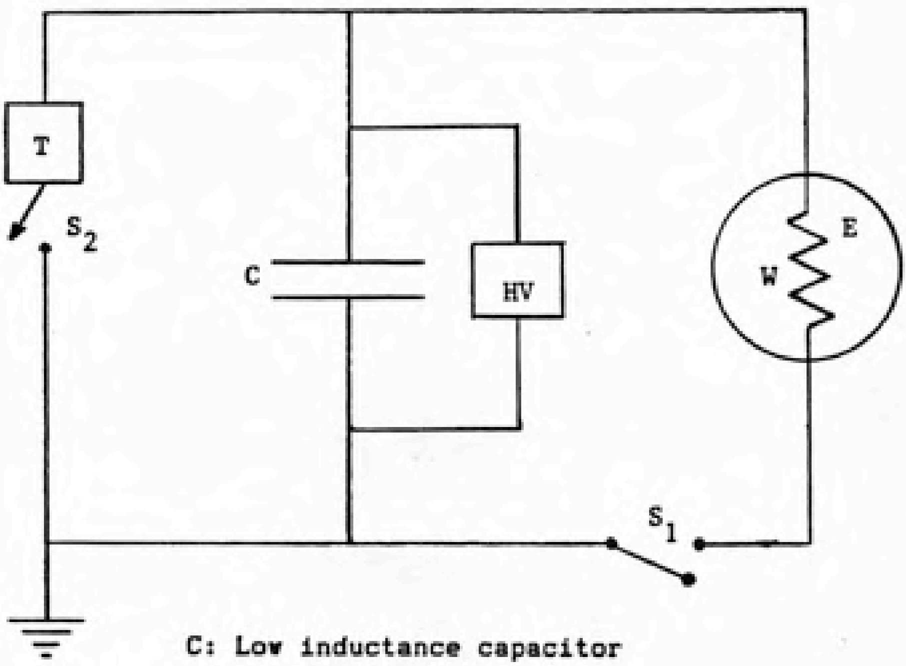
Table 1.1 Fractal Dimensions Obtained from Two-Dimensional Aggregation Models

Model	D	
Linear trajectory, particle-cluster	1.95 ± 0.002	[12]
Brownian trajectory, particle-cluster	5/3	[20]
Brownian trajectory, particle-cluster	1.73 ± 0.06	[12,13]
Linear trajectory, cluster-cluster	1.50 ± 0.05	[15]
Brownian trajectory, cluster-cluster	1.44 ± 0.02	[15]

II. Experimental Apparatus

II.1 Introduction to the Exploding Wire Generator

Metallic aerosols to be studied in this report are generated by the Exploding Wire Generator (EWG). The principle of aerosol generation is that the EWG supplies enough energy to vaporize a piece of wire mounted on two electrodes. After the wire is vaporized, i.e. exploded, condensation proceeds right away, and aerosols are condensed from the vapor phase. Figure 2.1 shows the basic scheme of an EWG.[34]



C: Low inductance capacitor

HV: High voltage power supply

S1: Trigger switch

W: Wire inside exploding chamber E

T: Timer for automatic dumping switch

S2: Dumping switch

Fig. 2.1 Basic Scheme of An Exploding Wire Generator

The physics of the exploding wire phenomena is well investigated in the literature.[34-37] In general, the previous studies indicated that aerosols generated by the EWG have the following characteristics:

1) The primary particles form chain-like structures of smooth spherical particles, whose size distribution can be characterized by a log normal distribution.

2) The mean diameter of these primary particles are between 0.01 and 0.1 micrometers in size, depending on the parameters used in their generation, such as electrical energy or diameter of wire.

3) The aerosols are reproducible if they are generated under the same operating conditions.

The EWG used for this study is a Tobe Deutschmann Laboratories Model ESB-118 energy Storage System. It is a self-contained 9 kilojoule system consisting of three main assemblies: Capacitor Bank, Power and Relay Tank, and Control Console.

This system is designed around a 45 μf, 20 KV capacitor and a high voltage trigger unit. The capacitor may be charged up to 25 KV as desired. A push-button switch is used to remotely trigger the capacitor to discharge the stored energy through a wire mounted across two electrodes. A dumping switch can be used to discharge the capacitor without exploding the wire when the test is to be aborted. Pushing the "Start Charge" button connects the high voltage power supply to the capacitor and starts the two-minute time delay automatic dump circuit which is provided for safety purposes. This assures that no charge will accidentally be left on the Capacitor Bank. The circuit is designed so that it allows 15 seconds for the operator to trigger the capacitor for explosion before the system automatically dumps the charges on the capacitor. After wire explosion, the circuit will automatically dump the residual charge left on the capacitor through a grounded wire. The distance between two electrodes is adjustable; the mass of wire exploded may be altered by changing the relative positions of electrodes, because sometimes it is necessary to obtain higher mass concentration or number concentration of particles.

II.2 Sampling Devices

1. Laser Aerosol Spectrometer

This is an optical particle counter manufactured by Particle Measuring System, Inc. It is used to monitor particle size distribution and, roughly, number concentration. A size range of 0.09 to 3.0 micrometers is covered by this counter.

2. 47-mm Filter Holder

A 47-mm, 0.05 micrometer nuclepore filter is used to collected particles for Scanning Electron Microscope (SEM) analyses, and then image analyses on computer.

3. Main Chamber (Exploding Chamber)

This is a large stainless steel pipe tee, amounted on the top of the Exploding Wire Generator, with a volume of 74.6 liters. Fig. 2.2 shows a side-view of this chamber.

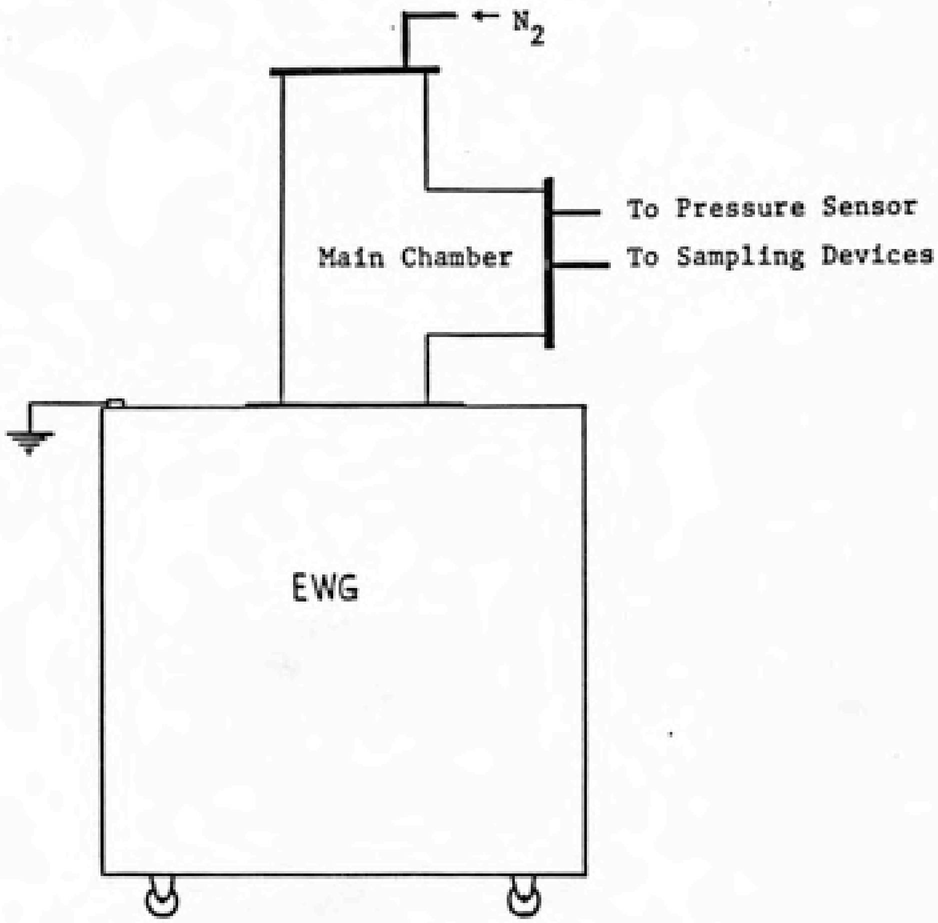


Fig. 2.2 Side-View of the Main Chamber

4. Extractive Sampling Chamber

This is a small chamber, with a volume of 145 cc, located near the center of the exploding chamber (main chamber). Extractive sampling is used because of the difficulties of collecting particles without disturbing the gas pressure in exploding chamber. Particles captured in this chamber can be quickly raised to atmospheric pressures and flushed through several sampling devices described above.

5. In Situ Sampler

When the operating pressure is reduced to lower values, the problem of particle settling would become more severe, and extractive sampling would become more difficult also. Therefore, an in situ sampler is designed to overcome this difficulty. This sampler has been added, but not been applied, to the bottom of exploding chamber to collect timed particles samples directly on SEM substrates by settling. By this sampler, however, the time resolution may not be so good as with other samplers. Fig. 2.3 shows the outline of this in situ sampler.

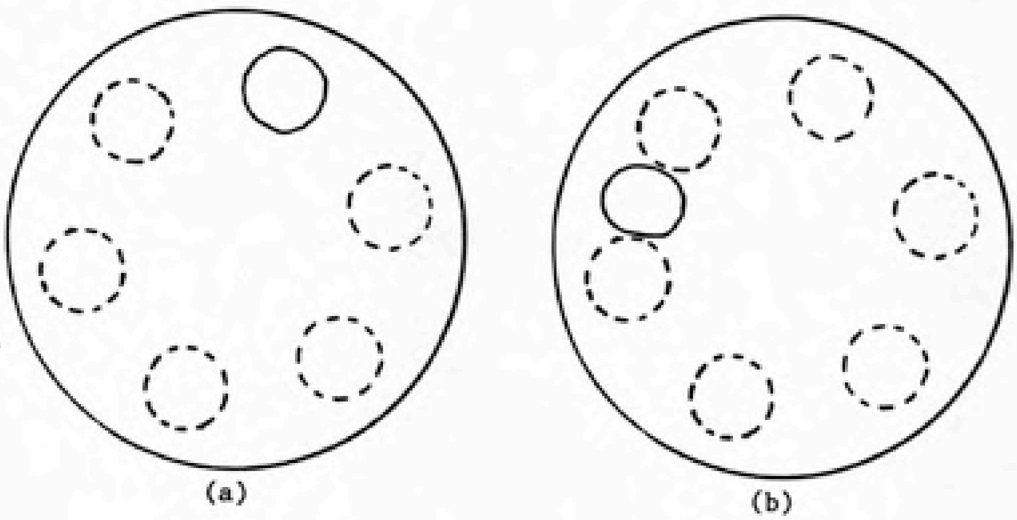


Fig. 2.3 In Situ Sampler. (a) Opened (b) Closed

There are six positions, where SEM substrates are located, on the sampler to collect settled particles, i.e. this sampler is designed such that only six samples can be obtained in an experiment. Essentially, this sampler consists of two round plates; one of them has one hole on it, and the other one has six. The one, having one hole, amounts atop the other one. When sample is to be taken, turn the plate on the top so that the hole on the top coincides one of six holes on the bottom plate; otherwise, turn it to the "blind" area as illustrated in Fig. 2.3.

II.3 Pumping and Pressure Measuring System

The pumping and pressure measuring system are interfaced to the exploding chamber through vacuum-tight flanges on the arms of this chamber. A mechanical vacuum pump, direct drive vacuum pump, manufactured by Precision Scientific Group, is applied to provide reduced pressure. This pump can provide an ultimate low pressure down to 0.1 μm Hg absolute. System pressure is monitored by the Barocel Electronic Manometer which can be calibrated by a Mcleod gauge if necessary.

II.4 Other Equipment

1. Faraday Cage Filter and Electrometer

This assembly was used in the early stage of experiment to measure total charges on aerosols. In practice, a metallic aerosol which would have a low residual electrostatic charge is desired.

2. Video Capture System --- PC-EYE, Computer Software

This system has been set up in order to analyze the fractal dimensions of particles. It consists of a video camera, an image analyzer board and a software package for the IBM-PC. A particle image on an electron micrograph is digitized by this system and then stored as a file to the floppy disk for further analyses. This video capture system, called PC-EYE, is a commercial product distributed by Chorus Data Systems, Inc.

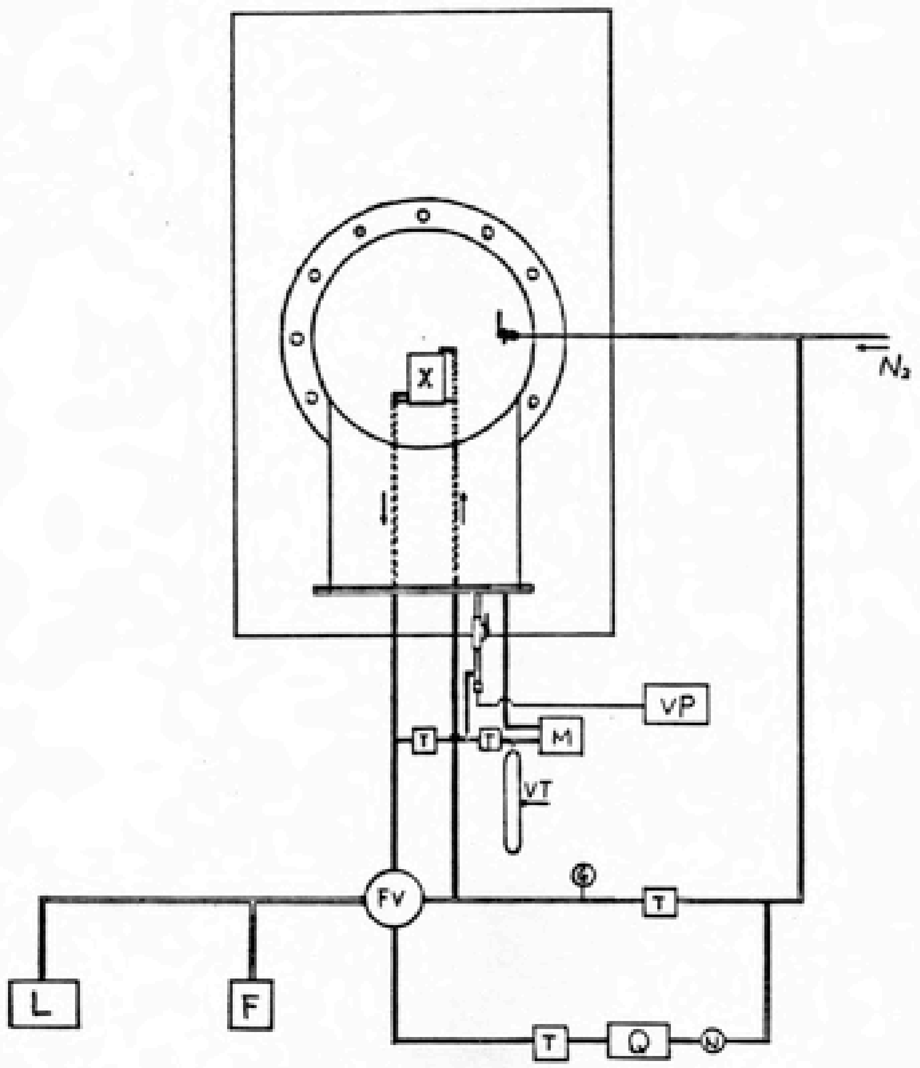
II.5 System Assembly

Figure 2.4 shows the assembly of entire chamber and monitoring system (top view).

II.6 Sampling Technique

It was readily known that aerosols generated from EWG were oxidized in normal air. In this study oxidized metallic aerosols are not desired, so that pure nitrogen is used as the gas medium to prevent metallic aerosols from being oxidized. Before the wire is exploded, it is required to "clean" the exploding chamber such that no particles other than exploded aerosols exist. This was done by evacuating the exploding chamber and filling it with pure, filtered nitrogen at least three times, and then adjusting the pressure to the desired value.

After the wire has exploded, particles diffuse into the open extractive sampling chamber. After several seconds, which is enough time for particles to enter this sampling chamber, the



F : 47-mm Filter Holder

FV: Four-way Valve

G : Vacuum Gauge

L : Laser Spectrometer

M : Manometer

N : Needle Valve

Q : Flow Meter

T : Toggle Valve

VP: Vacuum Pump

VT: Vacuum Tank

X : Extractive Sampling Chamber (Inside Main Chamber)

Fig 2.4 Assembly of Experimental Apparatus chamber's door is closed, the pressure is raised with nitrogen gas, and then nitrogen flushes the particles out to the sampling devices, laser spectrometer, and nuclepore filter. The nitrogen flushing rate is 10 cc/sec, of which 1 cc/sec is applied to laser counter; the remaining 9 cc/sec of aerosol is collected by the filter.

After all of the particles in the sampling chamber have been flushed out (this can be monitored by the laser counter), the sampling chamber is pumped out to the pressure of main chamber. Then sampling chamber's door is opened to receive particles again.

III. Analyses

III.1 Scanning Electron Microscope

Every filter sample is examined by scanning electron microscopy (SEM). On the SEM in use, the maximum magnification for a picture with good resolution is about 20,000x. On certain electron micrographs, the primary particles are hard to identify, but appears to be about 0.04 - 0.05 micrometer spheres. Even so, this is not a major limitation, because in this study the measurement of size distribution of primary particles is not crucial.

For each filter sample, cut off about 1 square centimeter of this sample. Stick it on a specimen stub with carbon substrate, and then examine it by the scanning electron micrometer.

III.2 Fractal Analysis

There are two ways applicable to fractal analysis: perimeter method (or compass walk method), and dilation method (or covering squares method).

1. Perimeter (Compass Walk) Method

The basic concept of this method is to estimate the perimeter of a profile by drawing a polygon around this profile. If P represents the estimated perimeter of the profile, i.e. exact perimeter of corresponding polygon, and S represents the length of each side of polygon; the plot of ln(P) versus ln(S) would be a straight line with a slope of m. The fractal dimension of this profile is then calculated by

$$D = 1 - m$$

To measure it practically, take a drafting compass and open it to a stride length S. Starting at one end of Ferets diameter, swing the legs of the compass from the outside inwards to meet the profile, and then make repeated swings, until the compass returns to the starting point. There is a brief discussion of alternate ways to swing the compasses.[28] It should be noted, however, that a consistent method must be used throughout the entire analyses.

This analytical method seems easier than dilation method (to be described in the next section), but is a tedious process and will consume much more time than the dilation method because it can not be easily computerized.

2. Dilation (Covering Squares) Method

Consider an arbitrary planar profile, as illustrated in Fig. 3.1 outlined by the heavy line. The figure is then covered with a plane covering pattern of different scales, such as regular triangles, squares, pentagons, or hexagons (squares are usually used), until the figure is either completely covered or one pattern covers the figure exactly. The smaller the covering cell (triangle, square, etc.), the more the number of cells will be. Similar to perimeter method, the logarithmic plot of the number required to cover versus the length of the covering cell will show a straight line, whose negative slope can be used to express the fractal dimension. That is, the negative slope of the line will be:

$$D = -\frac{d \ln (N)}{d \ln (b)}$$

where N is the number of covering squares, b is the length of the side of the corresponding square, and D will be termed as the fractal dimension.

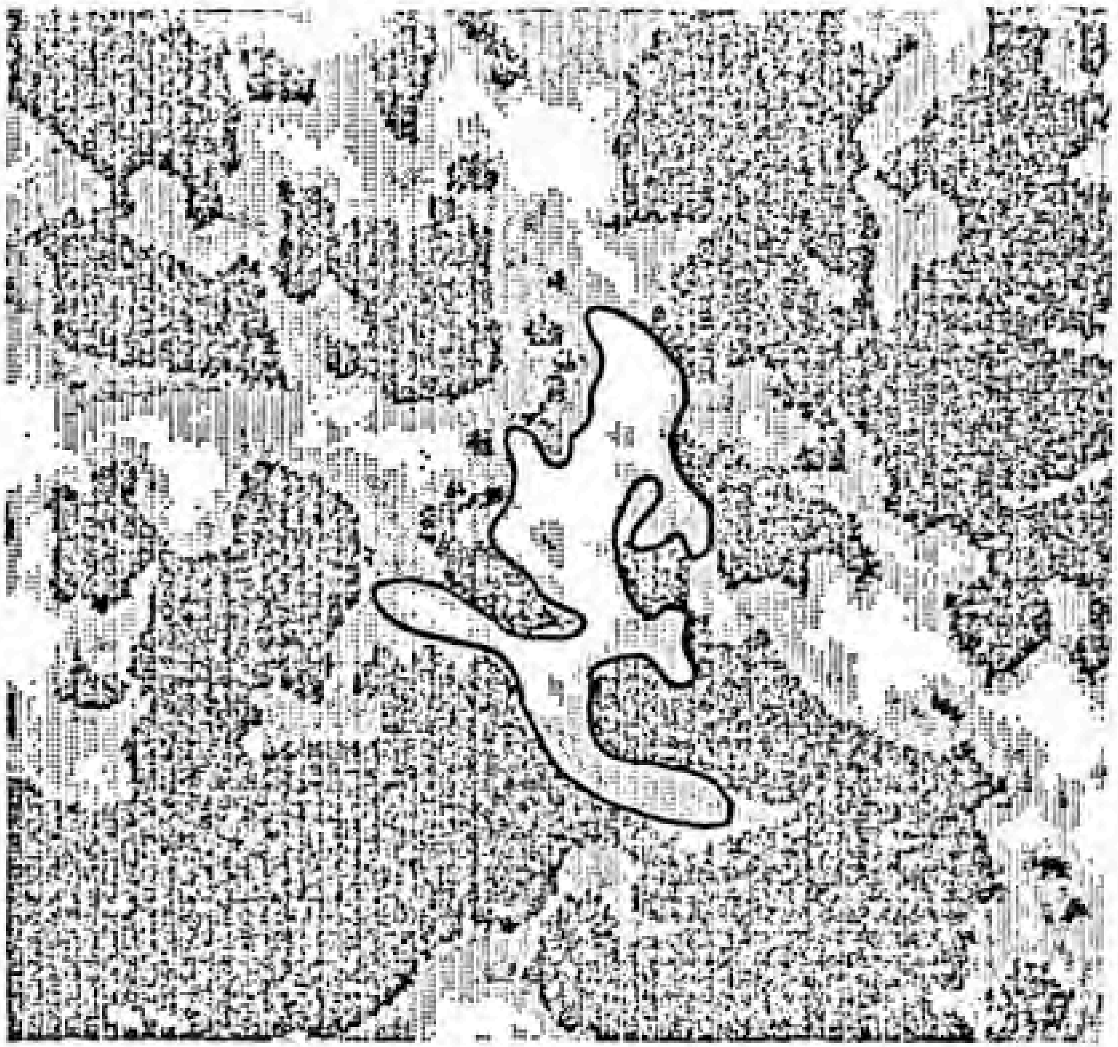
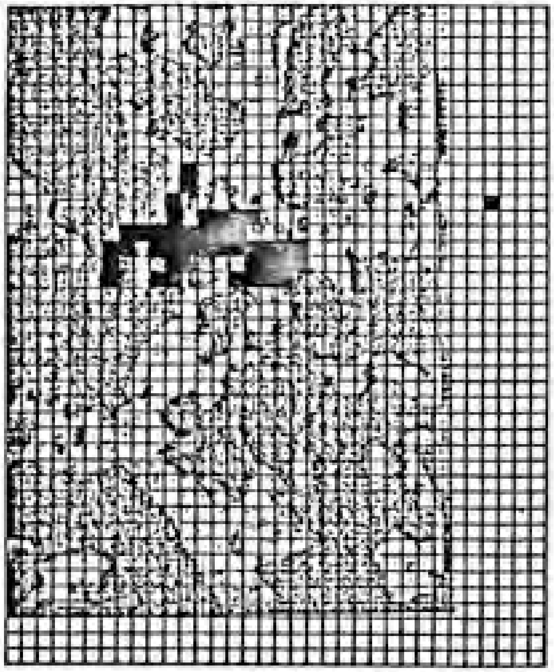
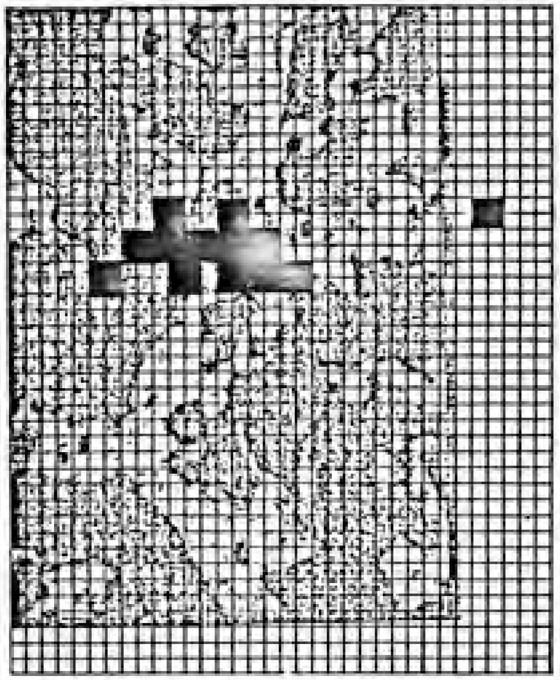


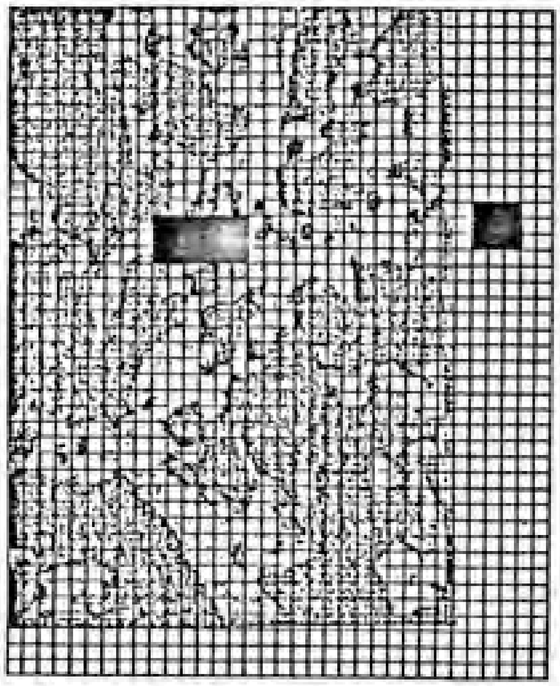
Fig. 3.1 Shape Representing an Image of a Particle for Fractal Analysis



(a)



(b)



(c)

Fig. 3.2 Tiling of Shape by (a) Unit Square, (b) Squares of Side 2, (c) Squares of Side 3

Fig. 3.2 shows the processes for three sizes of squares. It is possible for this method to be applied to computer. But it was found that a fractal analysis for a single picture would require more than two hours on the computer available. This leads to the consideration of applying another dilation method described below.

This alternative dilation method was addressed by S. R. Forrest and T. A. Witten in 1979.[38] A digitized electron micrograph presents a matrix of "ones" and "zeros" (or "blanks"), the "ones" ("zeros") corresponding to the presence (absence) of one point of a particle. This digitized image is analyzed by computer (IBM PC); the computer program is written by BetterBASIC, an improved version of the BASIC language, and is listed in the Appendix. When analyzing, a smallest box was picked such that its geometric center is near the center of mass of particle, then a series of nested squares of different sizes was placed around it and the number of "ones" in each square was counted. This analysis would yield a powerlaw relationship between the length B of the square and the number of pixels Np within it, i.e. $Np \propto B^D$, where D is fractal dimension.[38] Forrest and Witten also found that results were most reproducible when squares were chosen whose centers of mass coincided with their geometric centers.

In order to verify the availability and accuracy of this procedure, a Koch curve (see, e.g., Mandelbrot[27]) with known fractal dimension of 1.500 was analyzed. A mean value for this test was 1.509, within 1% of accuracy. There are two limitations for this method, which were also stated by Forrest and Witten. One is due to the finite total number of pixels in the image analyzed, the other is due to the finite size of digitized image. The first limitation will cause the exponent of the power law, D (fractal dimension), approaching 0. The second limitation results in the data span being limited to a range of B. Despite these two limitations, this approach spends much less time than the one described at the beginning of this section.

IV. Results

IV.1 Selection of Metallic Wire

The earliest experiments were engaged in the selection of metallic wire. Two criteria are required: low melting point and low residual charge on the particles generated from wire explosion. The higher melting point metals appear to become thermally ionized during the explosion, while the low melting point metals could be easily dispersed without ionization. Besides, it is also necessary to have a wire that low residual electrostatic charges can be built up on the particles after explosion, such that electrically dominant forces would not influence the growth of particles. The charges on numerous metal aerosols were measured with a Faraday cage filter. After trying Cu, Ag, Mo, and Al, Ag wire was chosen. Table 4.1 lists the measurements of the charge levels on different aerosols.

Table 4.1 Measurements of the Charge Levels on Different Aerosols

Material	Charge/Mass (C/g)	Space Charge (C/m^3)
Ag	3.5 E-8	2.9 E-9
Ag	3.9 E-8	2.1 E-9
Ag	1.1 E-8	7.5 E-10
Cu	4.3 E-9	1.0 E-9

IV.2 Electron Micrographs Made by SEM

Fig. 4.1 through 4.10 show the electron microscope (SEM) pictures of particles at different pressures and times. Four different pressures were conducted: 1 atm, 0.1 atm, 0.01 atm, and 0.001 atm. There is no problem to collecting particles by using extractive sampler when the pressure was equal to or higher than 0.01 atm, although particles settled more quickly at the lower pressures. Even with repeated attempts, no particles were observed at 0.001 atm. Beside the pressure, other parameters were fixed. They are: wire material, Ag; wire diameter, 0.012"; electric energy, 15 KV; distance between two electrodes, 3 cm.

After realizing that no particles could be collected at the pressure of 0.001 atm, it was desired to know the lower limit for particle formation. Another two pressure settings were performed, 0.004 atm and 0.002 atm. In these two tests, it was only desired to know the "threshold" of pressure, and no pictures were taken. The PMS counter did show the presence of particles within one hour after wire explosion for these two pressures.

From the electron microscope pictures (Fig. 4.1 to Fig. 4.10), the primary particles reveal a diameter of 0.04 - 0.05 micrometer at all levels of pressure. It was also seen that some primary particles remained molten long enough to coalesce into larger spheres at 0.01 atm (not shown in the figures).

IV.3 Fractal Analysis

Fig. 4.11 to Fig. 4.20 are the plots of fractal analysis for the particles grown at different pressures (1.0 atm, 0.1 atm, and 0.01 atm). The fractal dimension revealed in the figures, as determined by one of dilation methods, do show a pressure dependent as well as time dependent tendencies.

For the particles grown at one atmosphere, the fractal dimension in the range of 0.07 to 0.5 micrometer is about 1.3 at short times after wire explosion, and increased to about 1.45 two hours later. The value of 1.45 is characteristic of cluster-cluster agglomeration,[11] i.e. the particle growth is due largely to the coagulation of clusters of particles of comparable sizes. Also in one atmosphere, particles grown above 0.5 micrometer have a fractal dimension of 1.7, which is close to the value of 5/3 expected for particle-cluster agglomeration.

In 0.1 atmosphere, particles exhibited the same general trend; where the range of cluster-cluster growth was from 0.08 to 0.3 micrometer, and the particle-cluster growth range was from 0.5 micrometer and higher.

In 0.01 atmosphere, however, there was an opposite trend. The smaller size range (from 0.07 to 0.5 micrometer) showed a fractal dimension in a range from 1.4 to almost 1.7. While at size of 0.5 micrometer and up to about 1.0 micrometer, the fractal dimension was about 1.35. This will be discussed later.

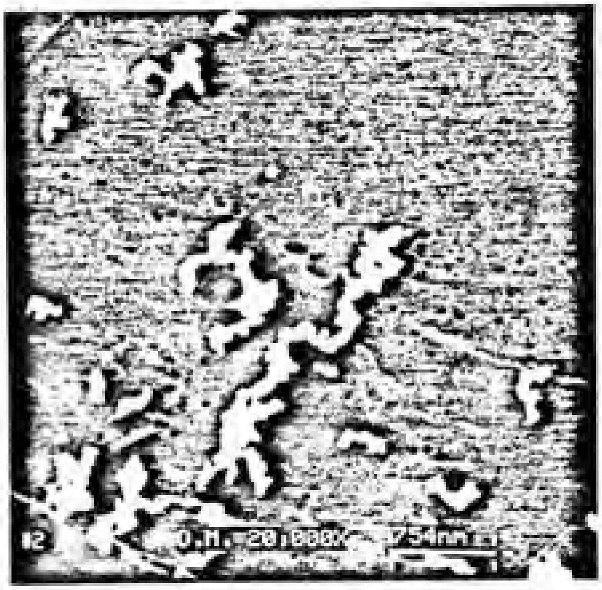


Fig. 4.1 Electron Micrograph of Agglomerates, 1 atm.
Sample Taken at 37 sec. after Explosion. 20,000x

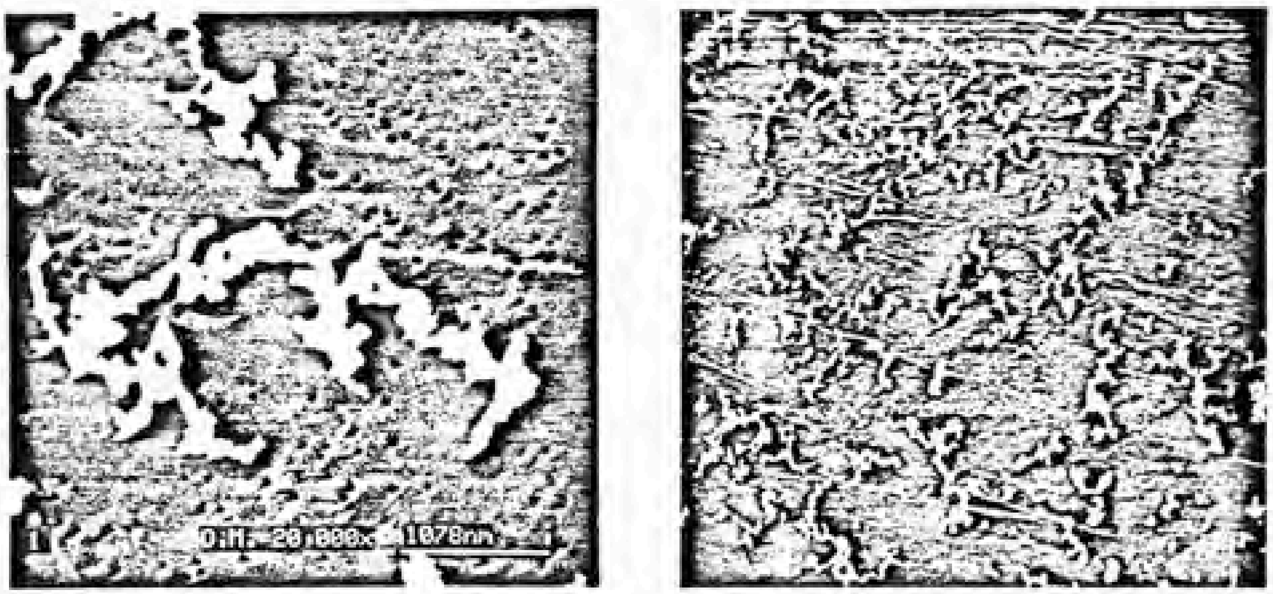


Fig. 4.2 Electron Micrograph of Agglomerates, 1 atm.
Sample Taken at 26.3 min. after Explosion.
(a) 20,000x, (b) 2,000x

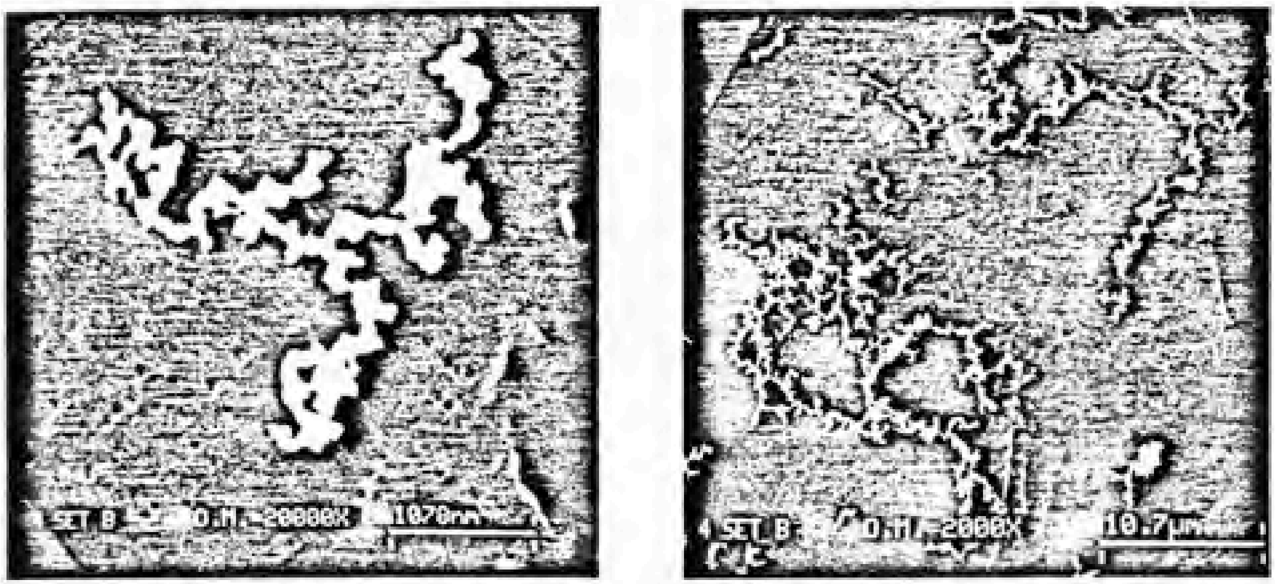


Fig. 4.3 Electron Micrograph of Agglomerates, 1 atm. Sample Taken at 129.5 min. after Explosion. (a) 20,000x, (b) 2,000x

Fig. 4.4 Electron Micrograph of Agglomerates, 1 atm. Sample Taken at 194.8 min. after Explosion. 20,000x

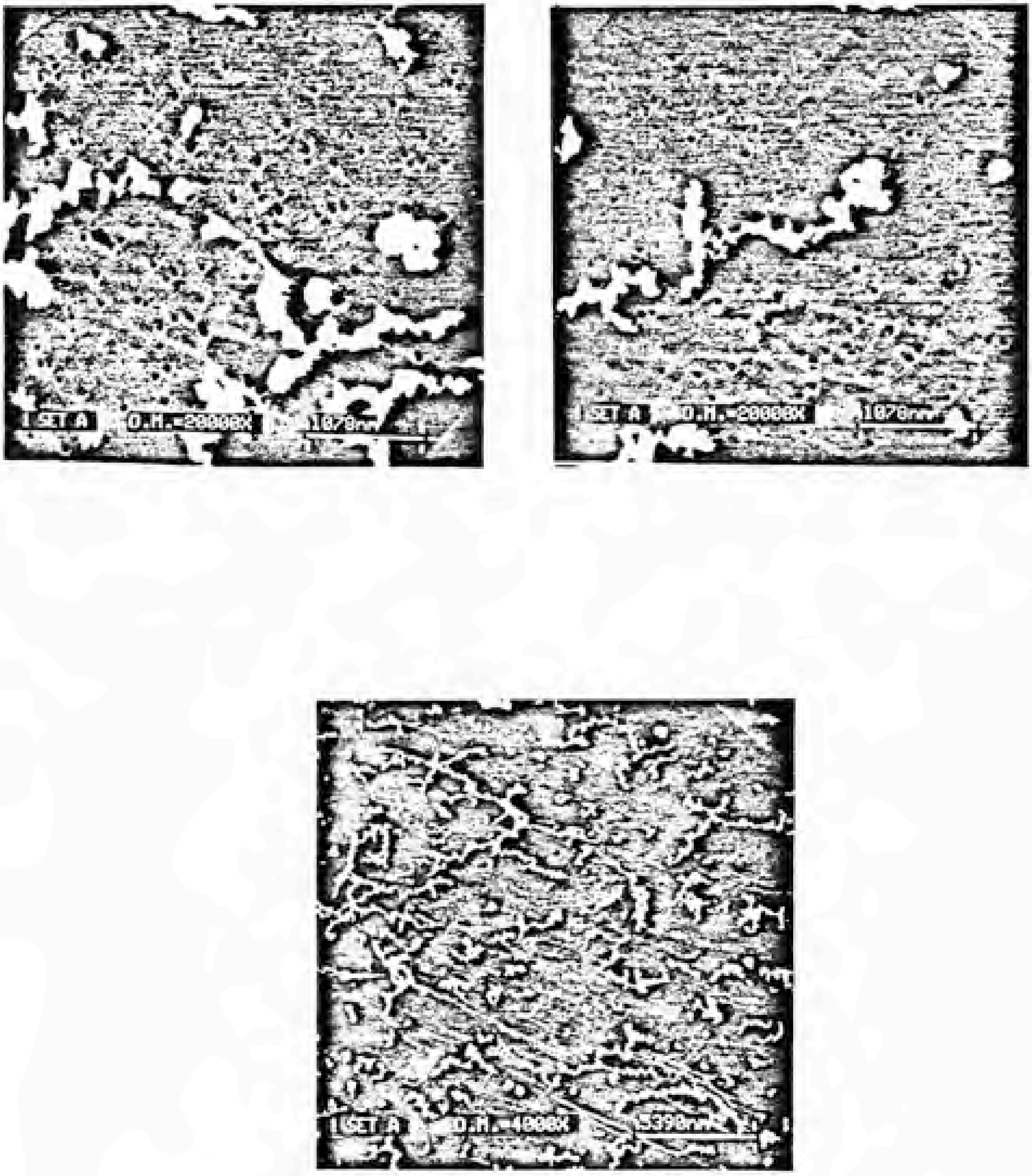


Fig. 4.5 Electron Micrograph of Agglomerates, 0.1 atm.
Sample Taken at 30 sec. after Explosion.
(a),(b) 20,000x, (c) 4,000x

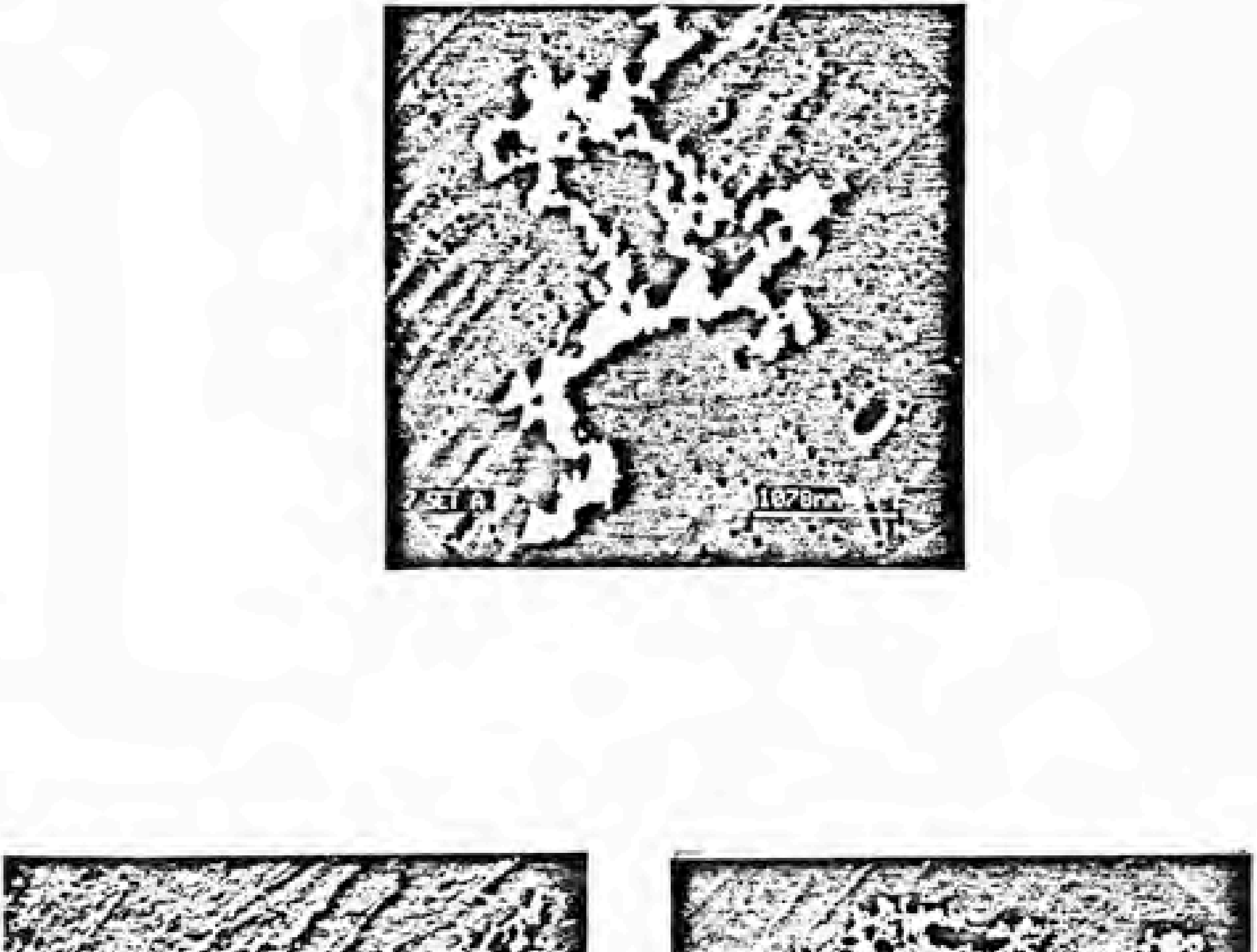


Fig. 4.6 Electron Micrograph of Agglomerates, 0.1 atm.
Sample Taken at 30 min. after Explosion.
(a) 20,000x, (b),(c) 4,000x

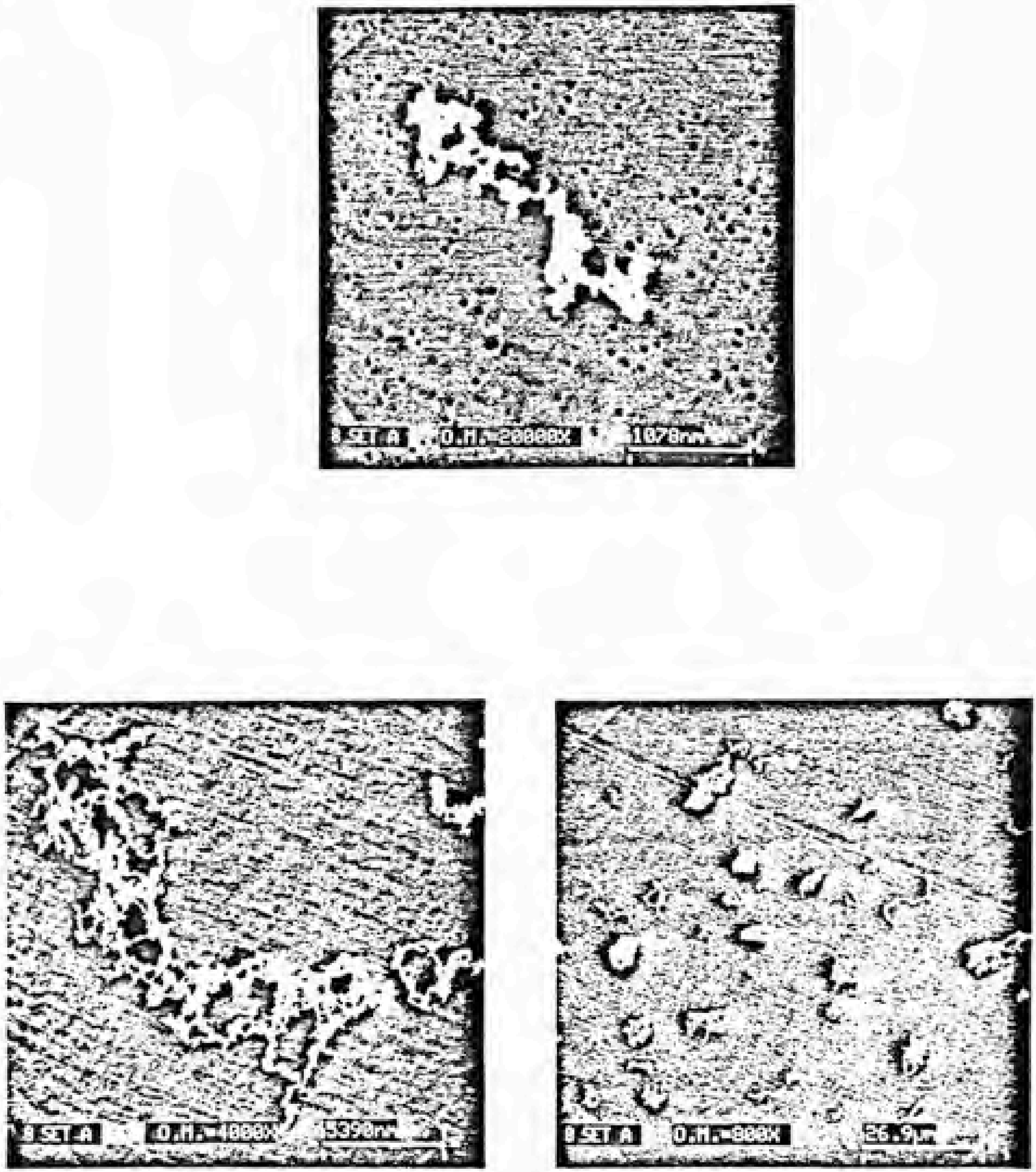


Fig. 4.7 Electron Micrograph of Agglomerates, 0.1 atm. Sample Taken at 60 min. after Explosion. (a) 20,000x, (b) 4,000x, (c) 800x

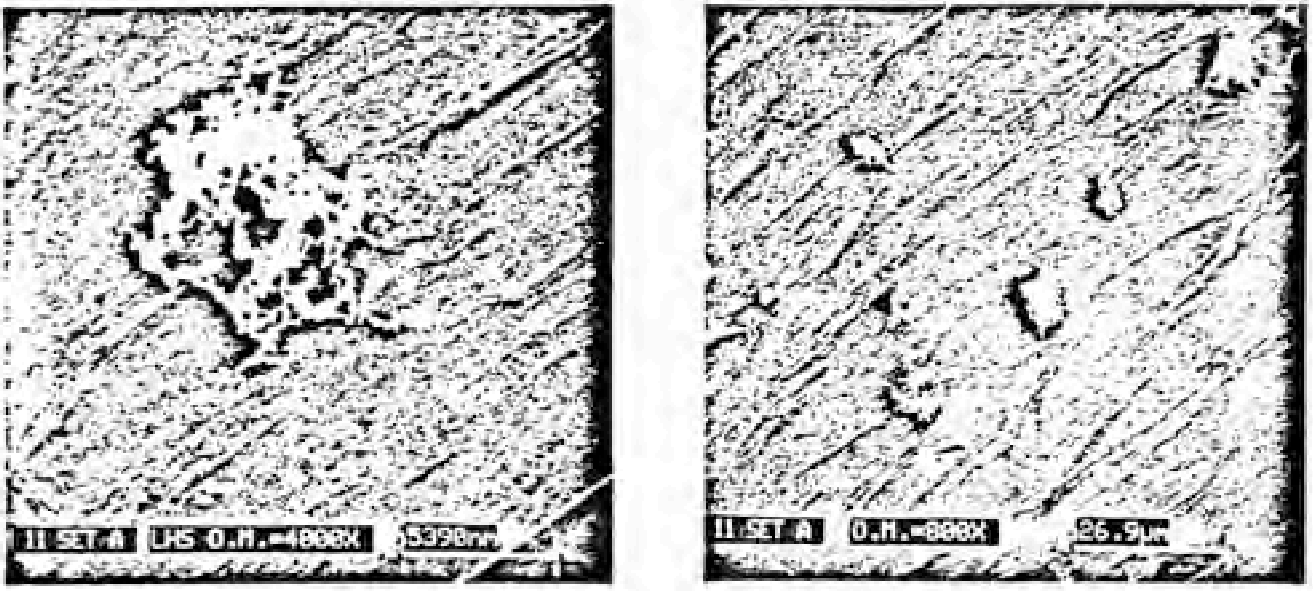


Fig. 4.8 Electron Micrograph of Agglomerates, 0.1 atm. Sample Taken at 120 min. after Explosion. (a) 4,000x, (b) 800x

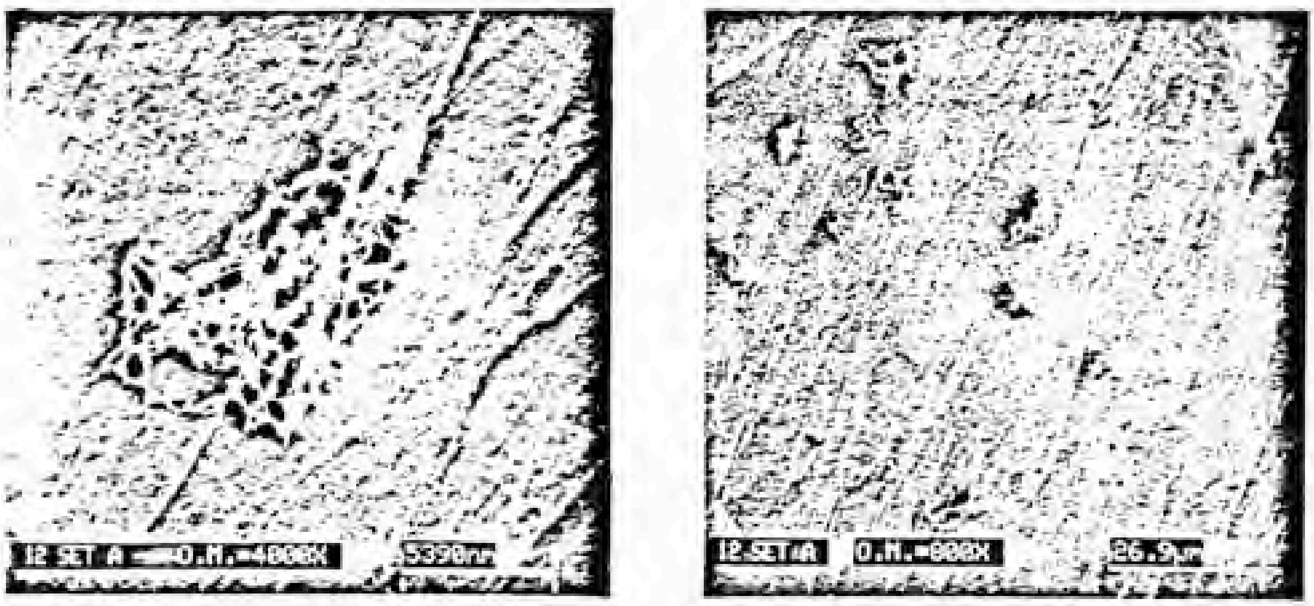


Fig. 4.9 Electron Micrograph of Agglomerates, 0.1 atm. Sample Taken at 180 min. after Explosion. (a) 4,000x, (b) 800x

Fig. 4.10 Electron Micrograph of Agglomerates, 0.01 atm. Sample TAken ar (a) 26 sec., 20,000x, (b) 29.8 min., 12,000x, after Explosion.

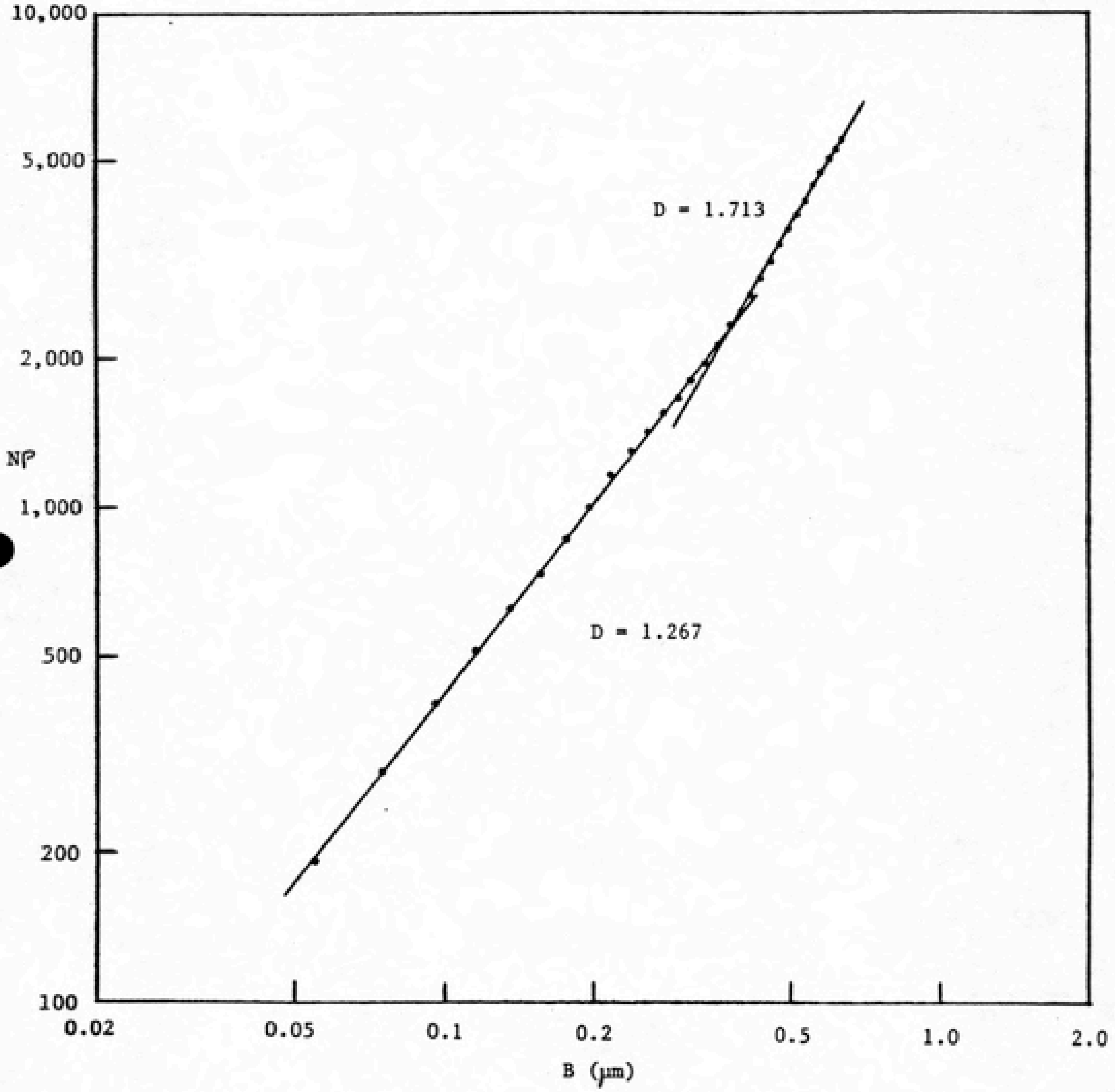


Fig. 4.11 Fractal Analysis for the Particle in Fig. 4.1

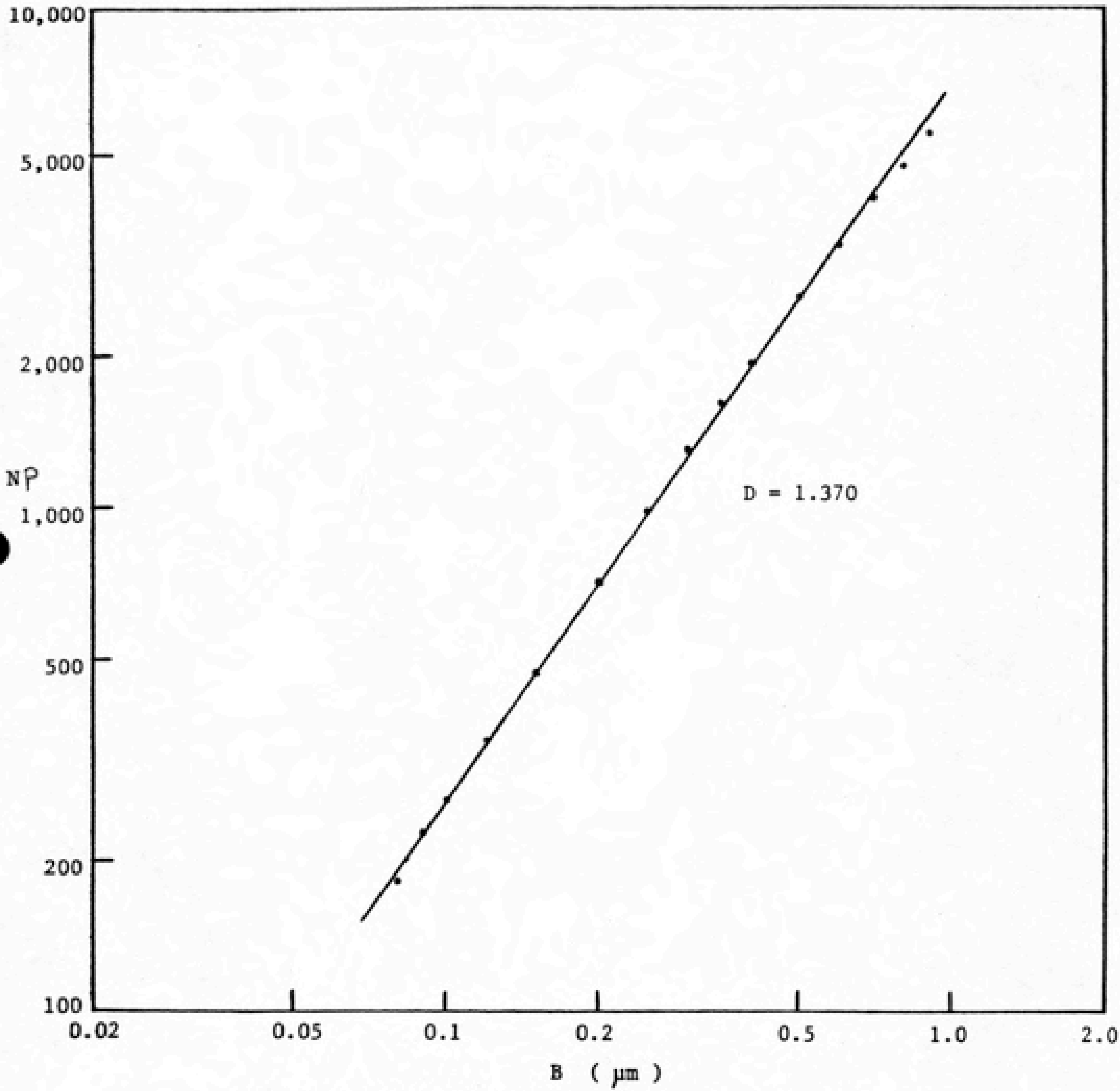


Fig. 4.12 Fractal Analysis for the Particle in Fig. 4.2(a)

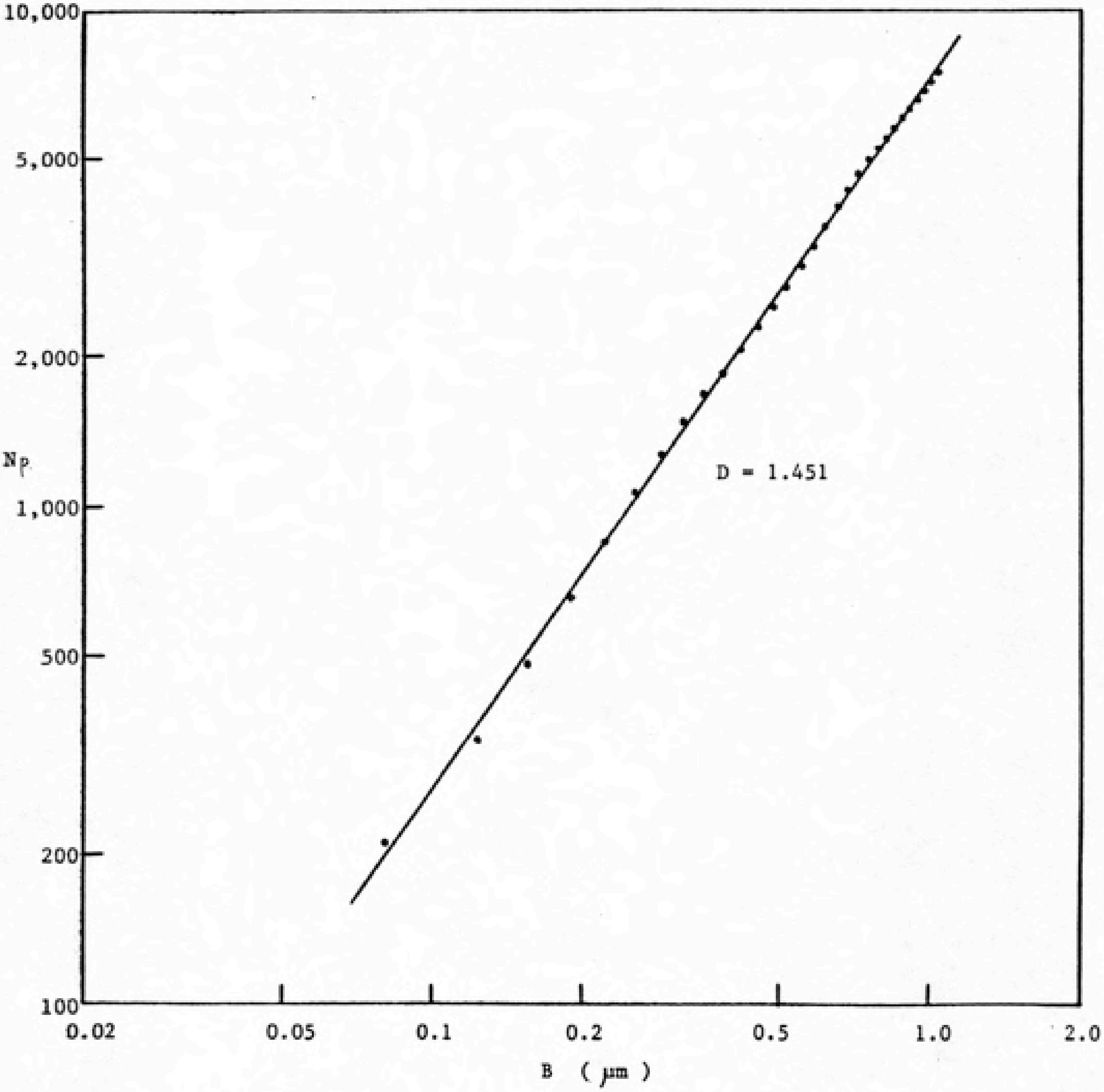


Fig. 4.13 Fractal Analysis for the Particle in Fig. 4.3(a)

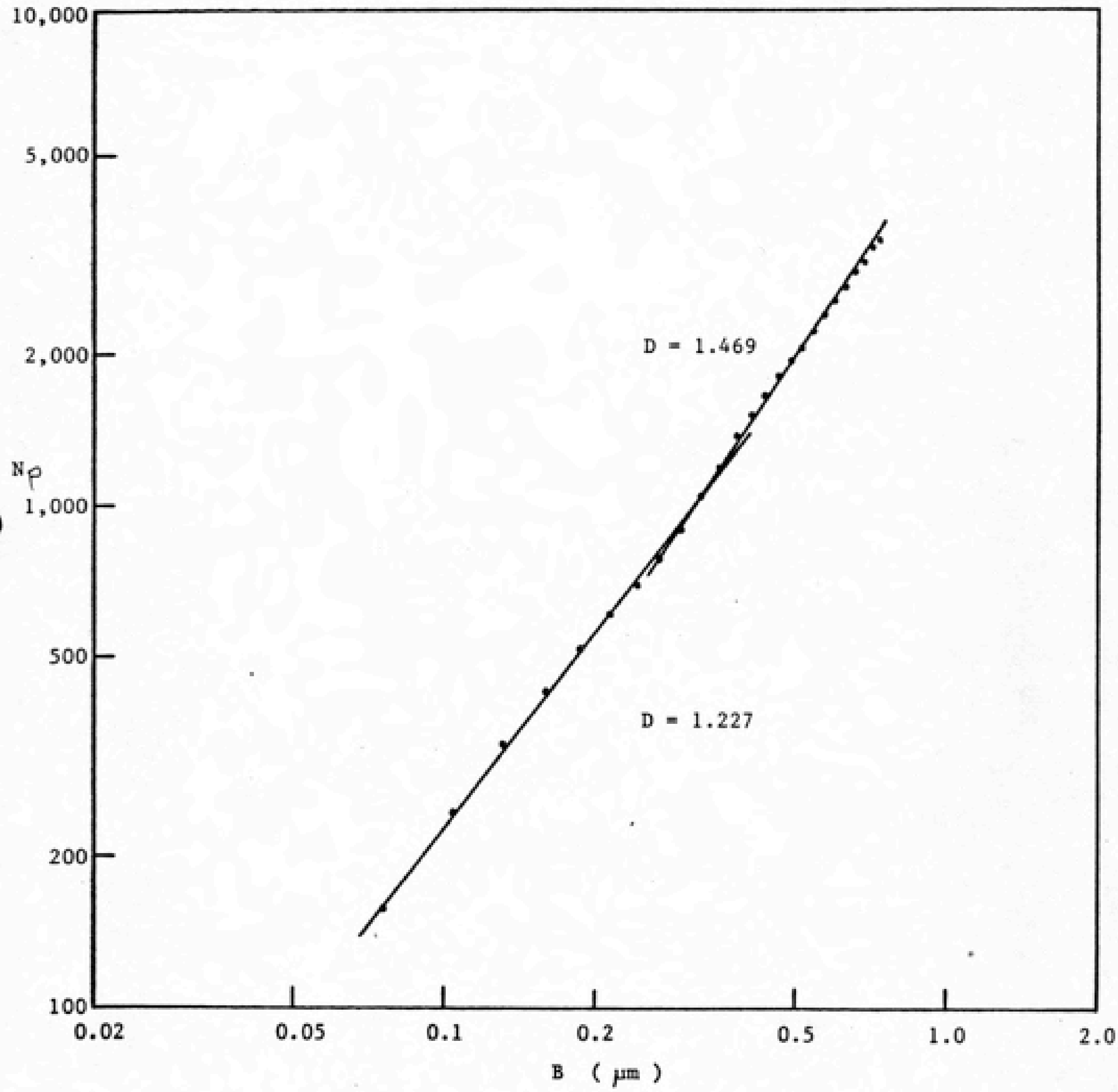


Fig. 4.14 Fractal Analysis for the Particle in Fig. 4.5(b)

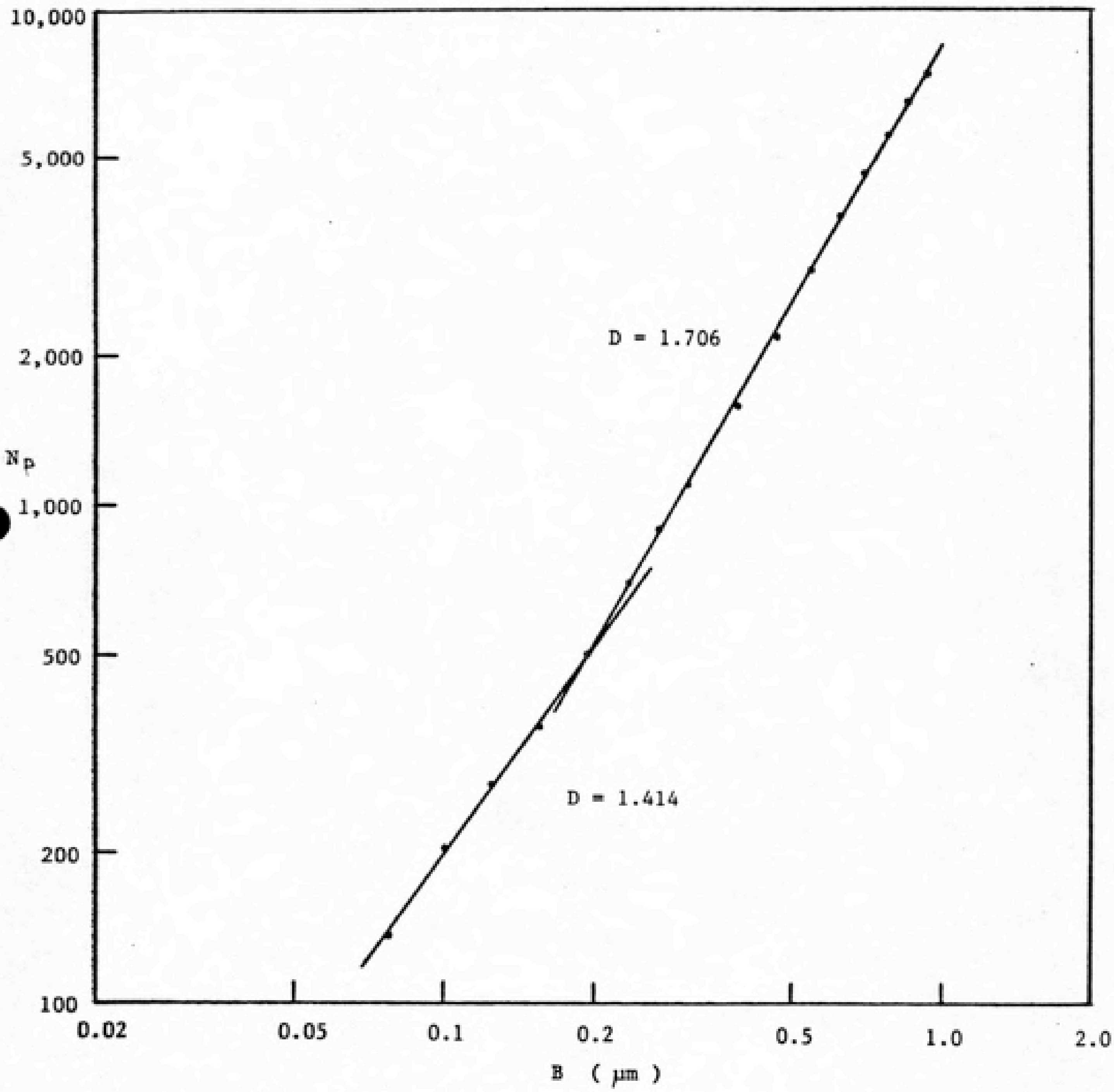


Fig. 4.15 Fractal Analysis for the Particle in Fig. 4.6(a)

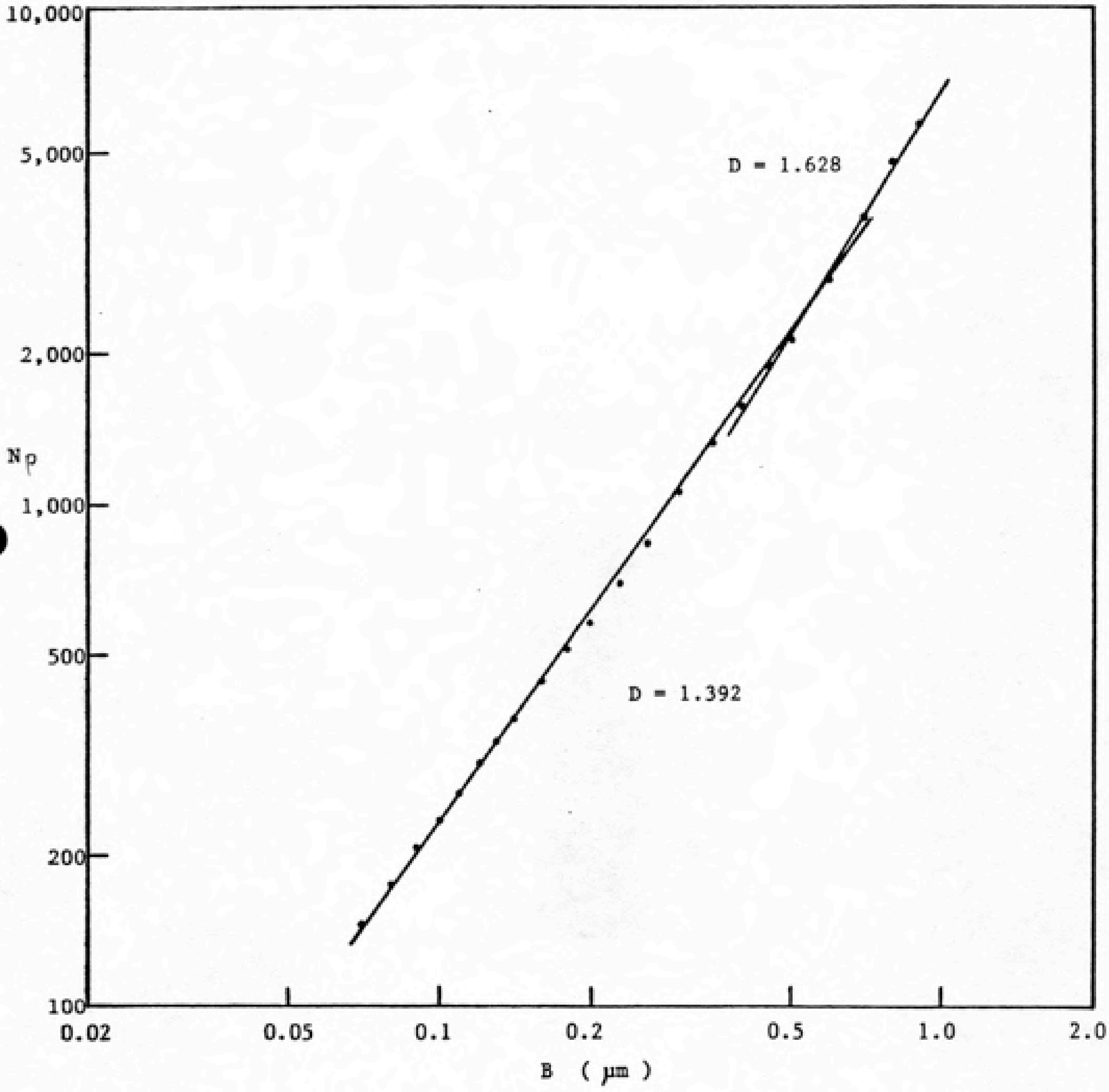


Fig. 4.16 Fractal Analysis for the Particle in Fig. 4.7(a)

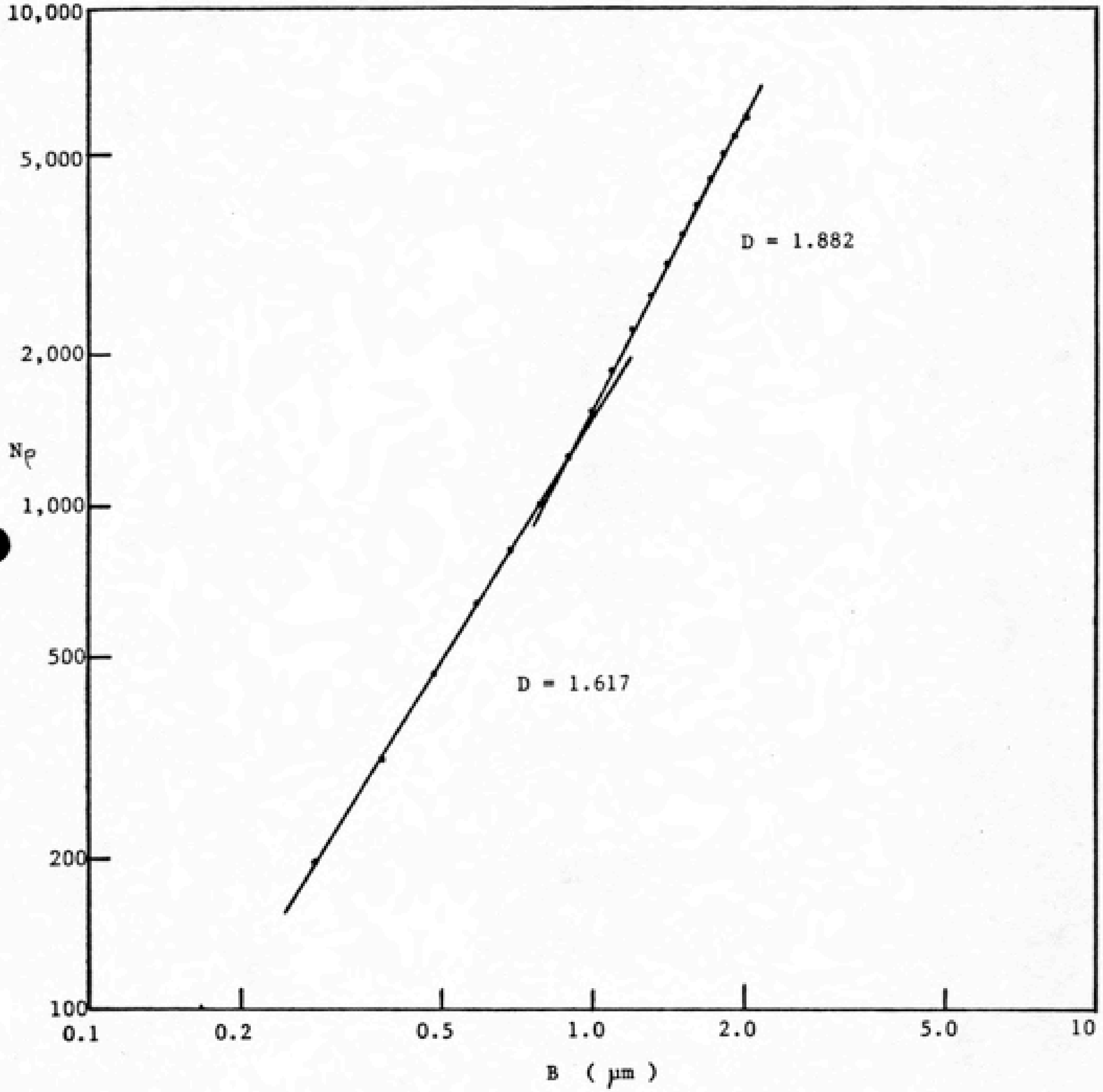


Fig. 4.17 Fractal Analysis for the Particle in Fig. 4.9(a)

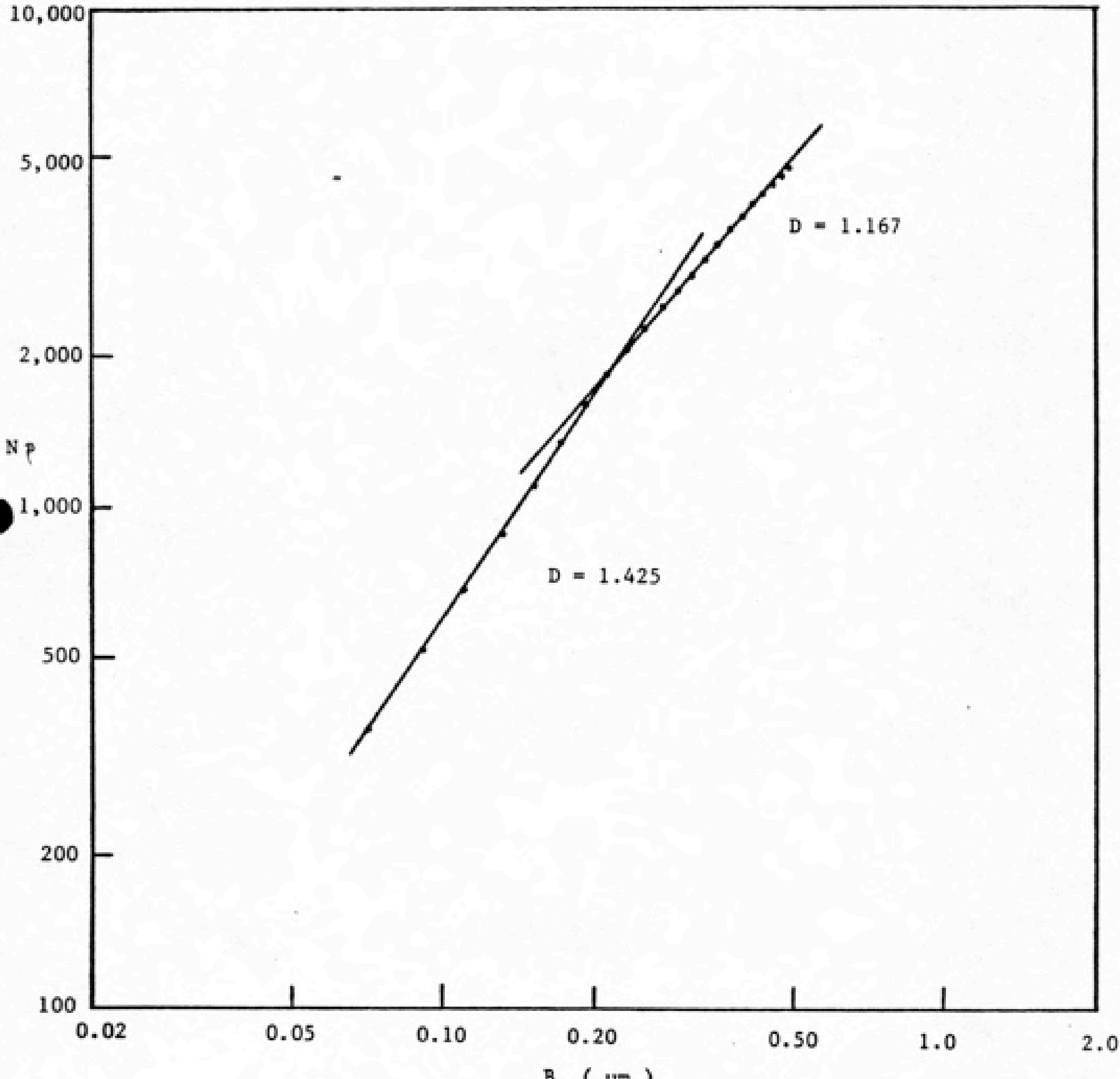


Fig. 4.18 Fractal Analysis for the Particle in Fig. 4.10(a)

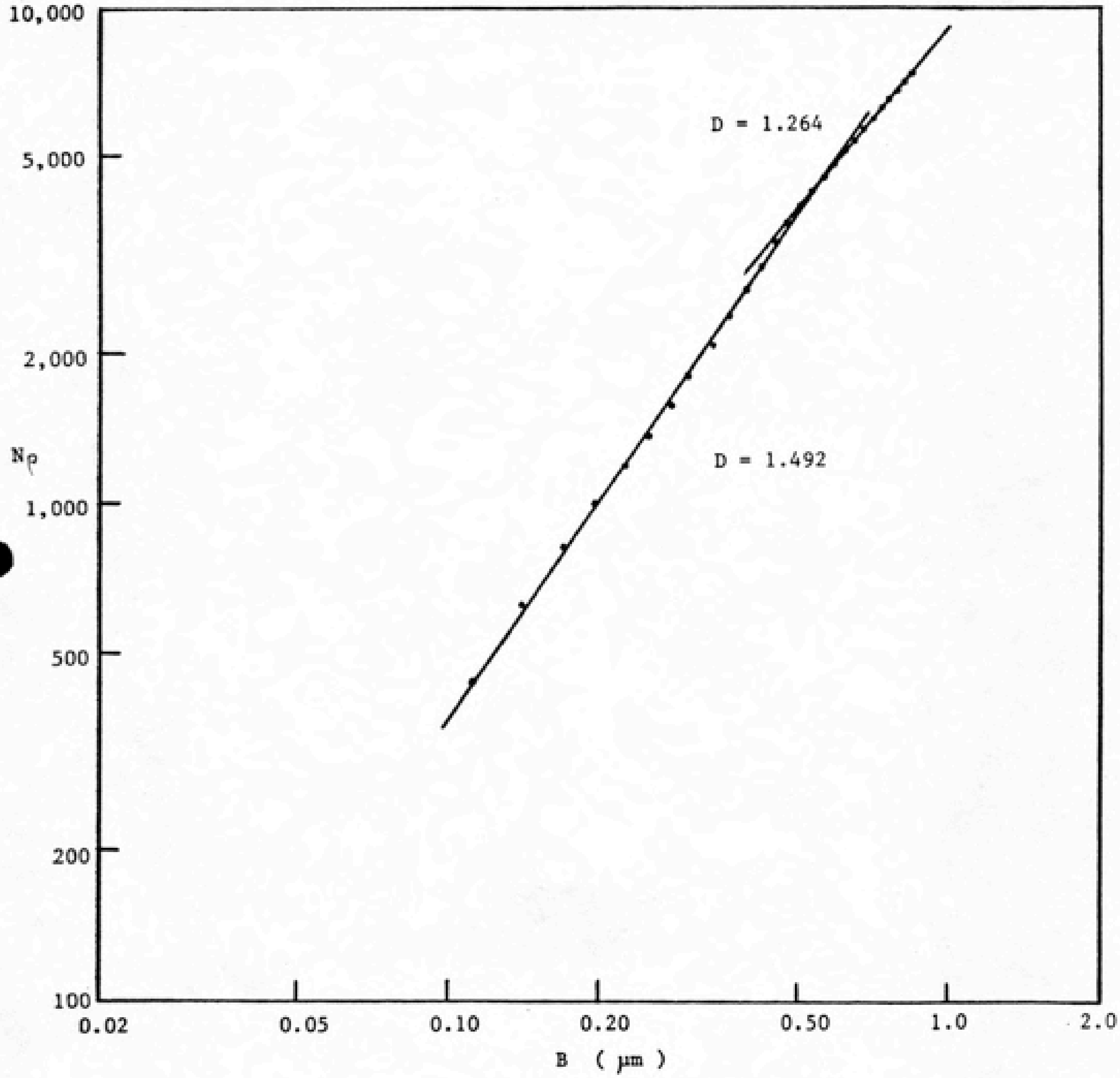


Fig. 4.19 Fractal Analysis for the Particle in Fig. 4.10(b)

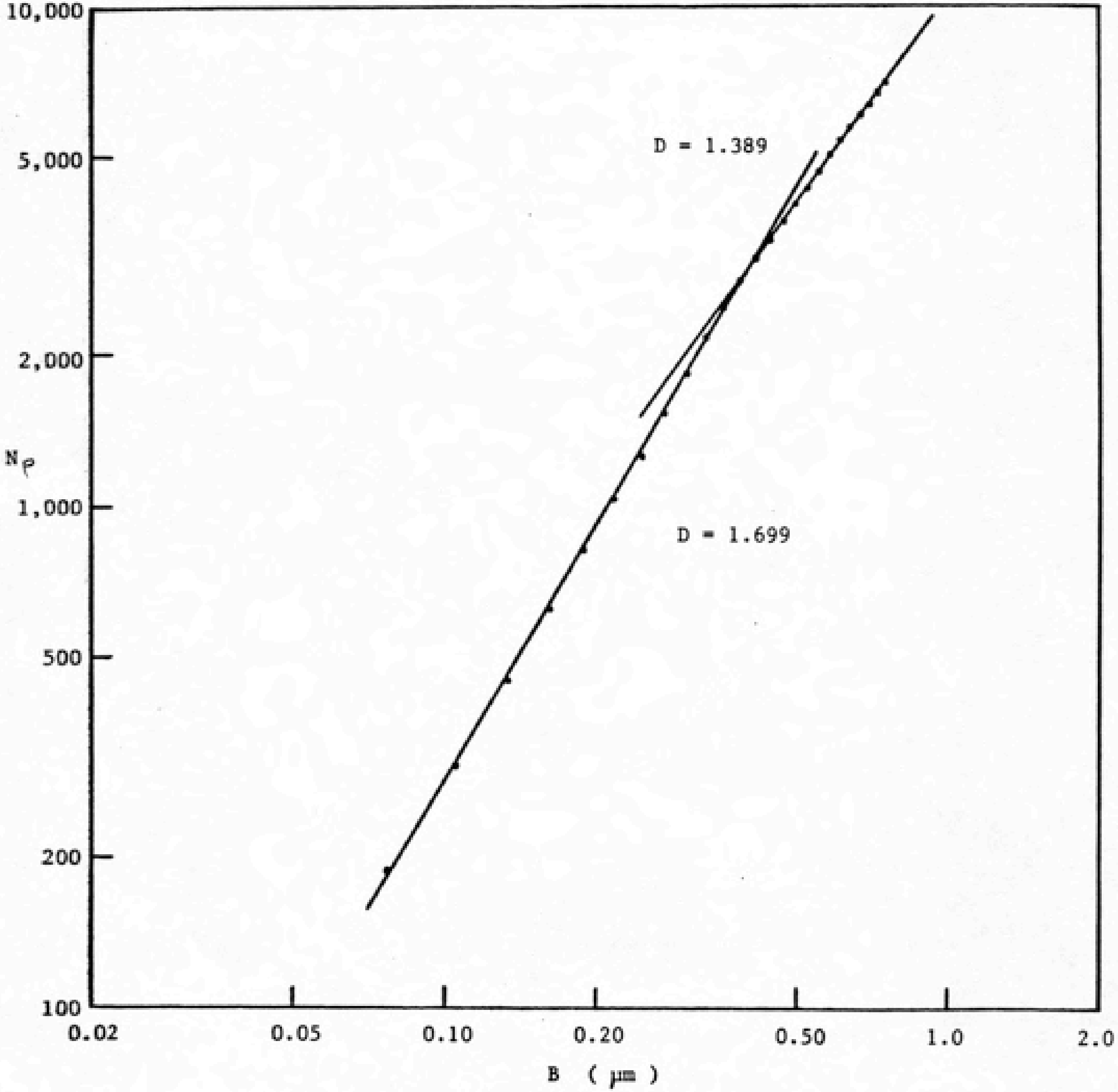


Fig. 4.20 Fractal Analysis for the Particle Taken at 66.7 min. after Explosion at 0.01 atm

V. Discussion

From the electron micrographs (Fig. 4.1 to Fig. 4.10), it is seen that at normal pressure (1 atm) particles tend to form a plane, chain-like agglomerate (Fig.4.1 to Fig. 4.4). At short time after wire explosion, the agglomerates exhibit an outline of linear chain (Fig. 4.1). At the later stage of growth, branched chain agglomerates appeared (Fig. 2 to Fig. 4). these branched chain agglomerates, obviously, are built up from a linear chain base. Besides, those branches are not long compared to their parent chain. Comparing these chain-like agglomerates it is found that the chains formed at earlier stage of growth are opened and smaller than those formed at later stage of growth, in that closed chains were observed (Fig. 4.1 and Fig. 4.2 versus Fig. 4.3 and Fig. 4.4).

As the pressure was reduced, distinct differences were observed. At 0.1 atm (Fig. 4.5 to Fig. 4.9), agglomerates are still chain-like. Nontheless, these chain-like agglomerates are more compact, or "thicker" (i.e. no longer in two dimensional pattern but three dimensional), than those formed at 1 atm; and the formation of closed chain agglomerates seemed to be earlier. As pressure went even lower, at 0.01 atm, grown particles were still in the form of chain; however, they became more and more compact, i.e. space among particles was less, than those grown at higher pressures (see Fig. 4.10). The reason is that the growth at lower pressure, say 0.01 atm, is due largely to ballistic collision of particles rather than Brownian motion. This leads to the fact that either individual particles or clusters of particles are more likely to penetrate inside a growing cluster of particles than to stick around it, such that a very compact structure of aggregate is formed. On the other hand, at higher pressure, say 0.1 atm and 1.0 atm, the interaction of the gas molecules with the particles is essential in the aggregation process in which particles are more likely to stick around the periphery of a growing aggregate than to penetrate into it, such that an open structured aggregate is formed.

From the SEM microgarghs, it is obvious that the major effect of reducing gas pressure on particles' growth is that particles will grow into more compact structures. To quantify this effect, a descriptor named fractal dimension is applied.

Before measuring fractal dimensions, it should be noted that fractal dimension can be used to describe the ruggedness of an object only when this object exhibits fractal characteristics. It is necessary to realize whether or not the grown particles generated in this study reveal fractal characteristics.

Consider, for example, Fig. 4.6. Fig. 4.6(a) is a magnified picture of the particle laid on the left side of Fig. 4.6(b). Compare these two pictures, it is found that the structure of the particle shown on Fig. 4.6(a) is similar to that of the particle laid on right side of Fig. 4.6(b). They are constructed by the base of closed-chain structure. If we take a closer look at this particle, however, the fundamental structure of this particle would be changed. It will be found that this agglomerate is added up with spherical primary particles even though the resolution gets higher and higher. Beyond this magnification, probably 25,000x or more, the boundary of the agglomerate becomes a smooth finite profile and exhibits Euclidean structure rather than fractal structure, since spheres are Euclidean objects. By definition (see Section I) particles exhibiting the above described behavior are characterized as natural fractals, provided they also exhibit scale invariance of some properties.

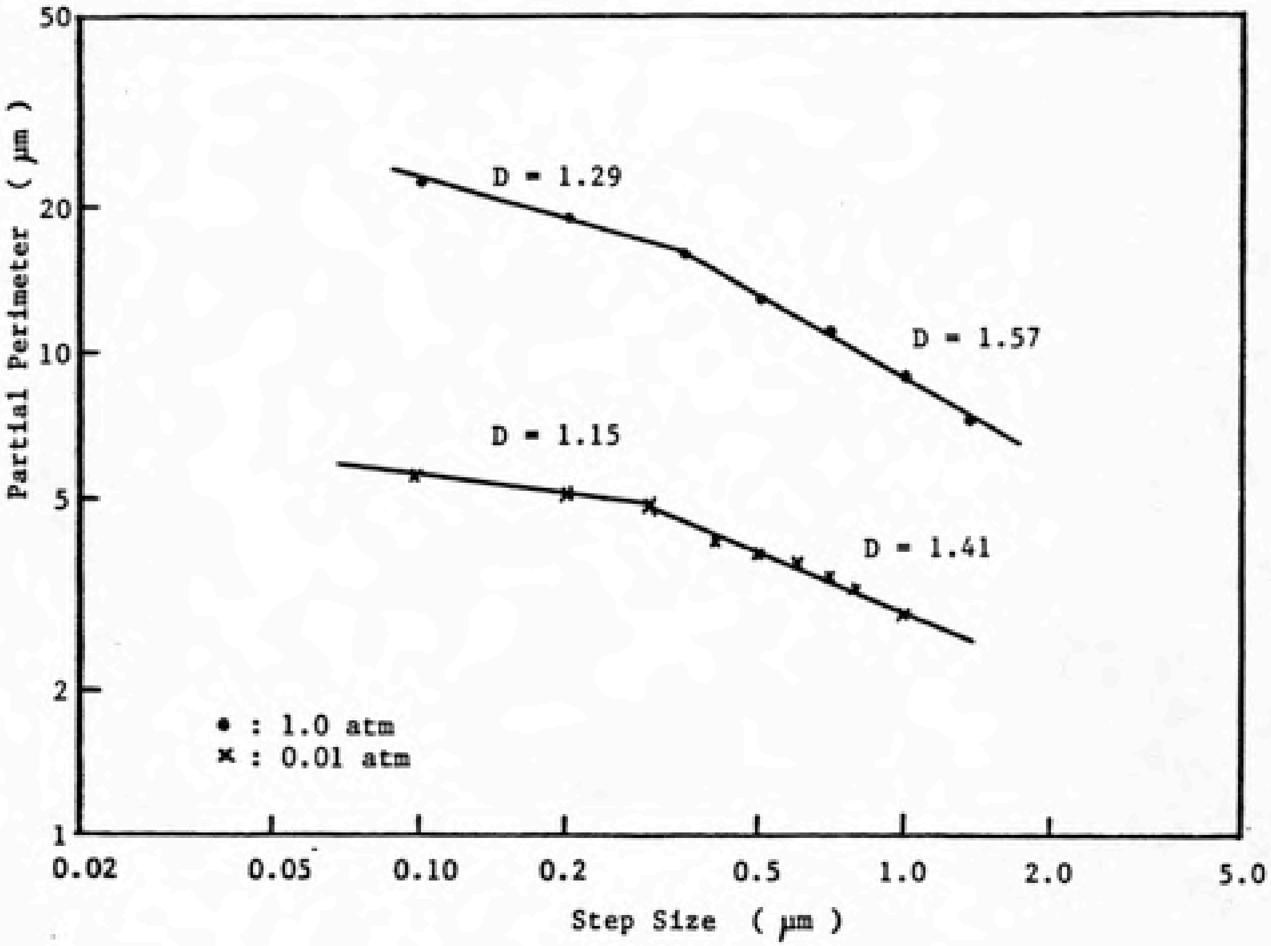


Fig. 5.1 Fractal Dimension Measured by Compass Walk Method

If we use compass walk technique (perimeter method) to measure the fractal dimension of a natural fractal, the plot on log-log scale would show two linear regions (e.g. see Fig. 5.1). The linear region at higher resolution describes the general structure of the profile, the other region at lower resolution describes the packing texture of the subunits (i.e. primary particles). Kaye suggests the former be described as structural fractal and the latter as textural fractal. [28] Furthermore, the break-point between these two linear regions should be theoretically at the dimension of the discernible subunit. [30]

The method used to measure fractal dimension in this study is dilation method which is described in Section III.2. Fig. 4.11 through Fig. 4.20 are log-log plots derived from the analysis of the digitized images of SEM micrographs. Unlike the plot shown in Fig. 5.1 which is obtained from the analysis using compass walk method, these plots show positive slopes. This is true because they are based on the relation described in Section III.2: $N_p \propto B^D$. More actually, this relation can be written into

$$M \propto R^D \qquad [39]$$

where M is the mass enclosed within a sphere of radius R. In this relation it assumes that the center of mass of the particle be the center of the circle.

According to the experience derived from analyzing those digitized images, there are some problems to which one has to pay attention. First and the most important, the boxes enclosing the object should be expanded around the center of mass of the object. This requires at least an evaluation of the center of mass. Secondly, the slope of the plot is very sensitive to the initial location of the smallest box. In other words, small changes in the location of the smallest box will result in fluctuating slopes. This occurs even though the largest boxes contain the whole image; in that case the total number of pixels (which are coded "1") is the same. To overcome these problems, an averaging technique has been applied. Several expansion centers were chosen in order to derive an averaged plot. That is to say, Fig. 4.11 to Fig. 4.20 are all averaged plots.

Table 5.1 Fractal Dimensions Determined by Dilation Method

1.0 atm		0.1 atm		0.01 atm	
Time*	D	Time	D	Time	D
0.6	1.267 (0.07-0.5)**	0.5	1.227 (0.08-0.3)	0.5	1.425 (0.07-0.2)
	1.713 (> 0.5)		1.469 (> 0.3)		1.167 (> 0.2)
26.3	1.370	30	1.414 (0.08-0.2)	30	1.492 (0.1-0.5)
129.5	1.451		1.706 (> 0.2)		1.264 (> 0.5)
		60	1.392 (0.06-0.5)	67	1.699 (0.1-0.4)
			1.628 (> 0.5)		1.389 (> 0.4)
		180	1.617 (0.3-0.9)		
			1.882 (> 0.9)		

* Time is counted after wire explosion and is in a unit of minute.

** Unit in the parentheses is in micrometers.

Table 5.1 lists the measures of fractal dimensions determined by the dilation method. In this table, most of the values of fractal dimensions exhibit two regions. There is no evidence showing that, like the plot of compass walk method, the break point is near the size of primary particles. However, this can be described as two stages of growth. For example, let us take a look at the fractal dimension of a grown particle at 30 minutes after explosion under 0.1 atmosphere of pressure. At the early stage of growth (growing from 0.08 to 0.2 micrometers), the fractal dimension of 1.414 manifests a cluster- cluster aggregation. While at the later stage of growth (growing from 0.2 micrometers and larger), the fractal dimension of 1.717 reveals that particle growing at this stage is in particle-cluster aggregation.

For the particles grown at one atmosphere, at short time after explosion, the fractal dimension in the range of 0.07 to about 0.5 micrometer is about 1.3, and is about 1.7 for the range above 0.5 micrometers. Two hours later, it increases to about 1.45 which is a characteristic value of cluster-cluster aggregation (see Table 1.1). The value of 1.7 is close to the value of 5/3 which is an anticipated value for particle-cluster aggregation (see also Table 1.1). The trend of growth at this pressure manifests that cluster-cluster agglomeration will be dominated for long-term coagulation.

At 0.1 atmosphere, particles' growth reveals similar features as they grow at one atmosphere. The data presented in Table 5.1 show that there is a transition to particle-cluster aggregation during the coagulation times. The cluster-cluster aggregation occurs for the size ranging from 0.08 to 0.3 micrometers, and from 0.5 micrometers and larger reveals particle-cluster aggregation.

Unlike 0.1 and 1 atmosphere, particles grown at 0.01 atmosphere exhibit another character. For the size ranging from 0.07 to 0.5 micrometers the fractal dimension is from 1.43 to 1.70, and is from 1.2 to 1.4 for the size of 0.5 micrometers and larger. This tells us that at the early stages of growth cluster-cluster aggregation is dominated within 30 minutes after wire explosion; and particle-cluster aggregation begins dominating at one hour after explosion and later. At the later stages of growth, the growth will approach to cluster-cluster aggregation with longer coagulation time, say, one hour after explosion.

The reason that growth switches from cluster-cluster aggregation to particle-cluster aggregation can be described as follows. As the size of cluster increases, its Brownian motion will become slow because this cluster is large enough that the bombardments of gas molecules on it could not affect its motion; that is to say, this cluster is "stable". In that case, the motion of this cluster is much less than the background particles or small clusters of much smaller size. Therefore those background particles and small clusters are more likely to collide with the relatively large clusters rather than stick themselves. This is the characteristics of particle-cluster aggregation.

As gas density is low, for example, 0.01 atmosphere, the fractal dimension for early stage of growth is larger than that for later stage of growth for the entire sampling time. The micrographs of the particles grown at 0.01 atmosphere (Fig. 4.10) can demonstrate these measurements. In the figures, the structure of the particles shows that it becomes more compact and solid as looking from the periphery inward. From another point of view, particles collide together by cluster-cluster aggregation at the early stage of growth; since the gas density is low these clusters of particles can contact one another tightly, such that a compact structure forms. In the limit of purely ballistic aggregation, a nearly solid structure will have a fractal dimension of 2.0.

The method used to measure fractal dimension in this study is dealing with two-dimensional projecting shapes which are electron micrographs of the particles generated. However, the real particles are three-dimensional, which is apparent in low angle micrographs or stereo-pairs. However, in the literature reported by Weitz and Huang,[40] it was demonstrated that as long as the particles are sparse enough, the two dimensional projection of a three-dimensional particle will have the same fractal characteristics as the particle itself. Therefore, it is unnecessary to do a three-dimensional analysis, except perhaps in the ballistic limit.

VI. Summary

In this report, the growth behavior of metallic aerosols under normal and reduced pressure environments was investigated. Particles grown at both environments exhibit very irregular outlines, as shown in the electron microscope pictures from Fig. 4.1 to Fig. 4.10. It was found that the basic structure of agglomerates in all three pressure conditions performed is a chain with some branches. At normal pressure, the agglomerates look like several branched chains stuck together end to end and, constructing an open-chain agglomerate. As pressure decreases, particles' profile is changing from open-chain to closed-chain and their structure becomes more compact and solid.

The growth of particles is characterized into three types in this study, they are: cluster-cluster aggregation, particle-cluster aggregation, and ballistic aggregation, according to the measured fractal dimensions.

According to the observations in this study, the agglomerates under normal and reduced pressure exhibit fractal characteristics, and can be categorized as natural fractals.

A descriptor, called fractal dimension which is used to describe a fractal object, is applied to quantifying the effect of reducing the pressure on the growth of particles. The measures of fractal dimensions of experimentally generated and grown particles are summarized in Table 5.1.

Particles grown at 1.0 atmosphere exhibit cluster-cluster aggregation in the early stage of growth, and particle-cluster aggregation for the later stage of growth at short time after wire explosion. Then the growth tends to be a cluster-cluster aggregation.

At 0.1 atmosphere, like at one atmosphere, the growth is attributed to forming small clusters of particles initially. After that, these small clusters of particles aggregate one another by cluster-cluster aggregation to form a larger cluster with a characteristic fractal dimension of about 1.4. The later stage of growth is dominated by particle-cluster aggregation and is characterized by a fractal dimension of about 1.7.

At even lower pressure, 0.01 atmosphere, the initial growth is changing from cluster-cluster aggregation to particle-cluster aggregation. While the later stage of growth will approach to a cluster-cluster aggregation, but will presumably change to particle-cluster aggregation at longer times.

Throughout this report, it is obvious that gas mean free path is an important factor to the growth of particles. Although the pressure does not decrease to very low value, it seems clear that the aggregation of particles will tend toward forming more compact structures with fractal dimensions approaching to a limit value of 2. The most important point in this report is the concept and application of fractal dimension. The fractal analysis is able to interpret the structural informations of agglomerates about their growth history.

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APPENDIX

```
SOURCE
PRECISION 6
PROCS=44
BYTE ARRAY(16384): Ba/X
BYTE ARRAY(320,200): A/X
INTEGER ARRAY(100): X1,X2,X3,X4,Y1,Y2,Y3,Y4
INTEGER ARRAY(100): W
REAL ARRAY(100): P
BYTE: Inc,Boxnum
INTEGER: Rb,Lb,Ul,Ll

PROCEDURE: Acqpic
END PROCEDURE

PROCEDURE: Shrink
END PROCEDURE

PROCEDURE: SetWidth
END PROCEDURE

PROCEDURE: Anykey
END PROCEDURE

PROCEDURE: Showpic
END PROCEDURE

PROCEDURE: Counting
END PROCEDURE

PROCEDURE: Definewindow
END PROCEDURE

PROCEDURE: Plot
END PROCEDURE

PROCEDURE: Setbound
END PROCEDURE

PROCEDURE: Message1
END PROCEDURE

PROCEDURE: Resul
END PROCEDURE

PROCEDURE: Acqpic
   EXTERNAL: Ba,Anykey
   INTEGER: I
   STRING: File$[16]

   PROCEDURE: Mes
   END PROCEDURE

   PROCEDURE: Mes
      EXTERNAL: File$,Anykey
```

```
        10 CLS
        20 SET CURSOR 10,25:INPUT "Picture file name ? ";File$
        30 CLS
        40 SET CURSOR 2,19:PRINT "For picture to appear it needs 22
           seconds."
        50 SET CURSOR 5,27:COLOR LTGREEN:PRINT "When picture appears
           ....."
        60 SET CURSOR 6,13:COLOR YELLOW:PRINT "Use arrow keys and
           hit certain keys to set boundaries"
        70 SET CURSOR 9,25:COLOR LTGREEN:PRINT "After setting
           boundaries ....."
        80 SET CURSOR 10,17:COLOR YELLOW:PRINT "The picture will
           then be stored into an array."
        90 SET CURSOR 11,28:COLOR YELLOW:PRINT "(It needs a few
           minutes)"
        100 SET CURSOR 14,19:COLOR LTCYAN:PRINT "Hit the following
           keys to set boundaries."
        110 SET CURSOR 16,13:COLOR YELLOW
        120 PRINT "'L' : Left boundary";SPC(15) "'R' : Right
            boundary"
        130 SET CURSOR 17,13:COLOR YELLOW
        140 PRINT "'U' : Upper boundary";SPC(14) "'D' : Lower
            boundary"
        142 SET CURSOR 18,29:COLOR YELLOW
        144 PRINT "<ESC> : Finish setting"
        150 SET CURSOR 21,28:COLOR LTRED:PRINT "Type anykey to
            continue.";
        160 Anykey
        170 SET CURSOR 21,28:COLOR LTRED:PRINT "Acquiring Picture
            ......";
    END PROCEDURE
      10 Mes
      30 OPEN File$ AS 1 LEN 16384
      40 READ RECORD #1,1,Ba
      50 CLOSE 1
      60 FOR I = 4 TO 16003 : Ba(I) = Ba(I+252) : NEXT
      70 Ba(0)=128:Ba(1)=2:Ba(2)=200:Ba(3)=0
END PROCEDURE

PROCEDURE: Shrink
    EXTERNAL: A,Ul,Ll,Rb,Lb,Definewindow,Anykey
    INTEGER: I,J

    PROCEDURE: Modify
    END PROCEDURE

    PROCEDURE: Mes
    END PROCEDURE

    PROCEDURE: Modify
       EXTERNAL: Rb,Lb
          10 IF Rb MOD 2 = 1 THEN Rb=Rb+1
          20 IF Lb MOD 2 = 1 THEN Lb=Lb-1
    END PROCEDURE
```

```
    PROCEDURE: Mes
       EXTERNAL: Definewindow,Anykey
          10 Definewindow
            20 SET CURSOR 10,14:PRINT "The picture has been stored into
             a 320 x 200 array."
               30 SET CURSOR 12,12:PRINT "Next step is to set width and
             number of counting boxes."
                 40 SET CURSOR 15,27:COLOR LTRED:PRINT "Type any key to
             continue."
          50 Anykey
    END PROCEDURE
       10 Modify
       20 CLEAR(A)
       30 FOR J = Ul TO Ll
       40   FOR I = Lb TO Rb
       50     A(I/2,J) = POINT(I,J) + POINT(I+1,J)
       60   NEXT
       70 NEXT
       80 SOUND 1000,2
       90 Mes
 END PROCEDURE

 PROCEDURE: SetWidth
    INTEGER: X,Y
    BYTE ARRAY(5): Ptr
    EXTERNAL: W,X1,X2,X3,X4,Y1,Y2
    EXTERNAL: Y3,Y4,Showpic,Inc,Definewindow,Boxnum,Anykey
    STRING: K$[5]
    EXTERNAL: Ba

    PROCEDURE: Init
    END PROCEDURE

    PROCEDURE: MovePtr
    END PROCEDURE

    PROCEDURE: Boxinit
    END PROCEDURE

    PROCEDURE: Box
    END PROCEDURE

    PROCEDURE: Mes
    END PROCEDURE

    PROCEDURE: Valreset
    END PROCEDURE

    PROCEDURE: Mes1
    END PROCEDURE

    PROCEDURE: Init
       EXTERNAL: Ptr
```

```
      10 PSET(10,10),1
      20 GET(10,10)-(10,10),Ptr
      30 PRESET(10,10)
END PROCEDURE

PROCEDURE: MovePtr
   STRING: K$[5],R[5],L[5],U[5],D[5]
   EXTERNAL: X,Y,Ptr,Showpic
   EXTERNAL: Boxinit,Init
      10 Showpic
      20 Init
      30 R=CHR$(0)+CHR$(77):L=CHR$(0)+CHR$(75):
         U=CHR$(0)+CHR$(72):D=CHR$(0)+CHR$(80)
      40 X = 300 : Y = 50
      50 PUT(X,Y),Ptr,XOR
      60 DO
      70   K$=INKEY$:IF K$="" THEN GOTO 70
      80   IF K$ = CHR$(27) THEN Boxinit
      90   IF K$ = " " THEN EXIT
     100   IF K$  =  R  THEN  PUT(X,Y),Ptr,XOR  :  X  =  X  +  5  :
              PUT(X,Y),Ptr,XOR
     110   IF K$  =  L  THEN  PUT(X,Y),Ptr,XOR  :  X  =  X  -  4  :
              PUT(X,Y),Ptr,XOR
     120   IF K$  =  U  THEN  PUT(X,Y),Ptr,XOR  :  Y  =  Y  -  4  :
              PUT(X,Y),Ptr,XOR
     130   IF K$  =  D  THEN  PUT(X,Y),Ptr,XOR  :  Y  =  Y  +  5  :
              PUT(X,Y),Ptr,XOR
     140 REPEAT
END PROCEDURE

PROCEDURE: Boxinit
   EXTERNAL: X,Y,X1,X2,X3,X4,Y1,Y2
   EXTERNAL: Y3,Y4
   STRING: K$[5]
      10 K$=INKEY$ : IF K$ = "" THEN GOTO 10
      20 IF K$ = "1" THEN X1(0) = X : Y1(0) = Y
      30 IF K$ = "2" THEN X2(0) = X : Y2(0) = Y
      40 IF K$ = "3" THEN X3(0) = X : Y3(0) = Y
      50 IF K$ = "4" THEN X4(0) = X : Y4(0) = Y
END PROCEDURE

PROCEDURE: Box
   EXTERNAL: W,X1,X2,X3,X4,Y1,Y2,Y3
   EXTERNAL: Y4,Inc,Boxnum
   INTEGER: I
      10 CLS
      20 W(0)=Y2(0)-Y1(0)
      30 X3(0)=X1(0)+2*W(0) : Y3(0)=Y2(0)
      40 X4(0)=X3(0) : Y4(0)=Y1(0)
      50 FOR I = 0 TO Boxnum
      60   X1(I+1)=X1(I)-2*Inc : Y1(I+1)=Y1(I)-Inc
      70   X2(I+1)=X2(I)-2*Inc : Y2(I+1)=Y2(I)+Inc
      80   X3(I+1)=X3(I)+2*Inc : Y3(I+1)=Y3(I)+Inc
      90   X4(I+1)=X4(I)+2*Inc : Y4(I+1)=Y4(I)-Inc
```

```
      100   W(I)=Y2(I)-Y1(I)
      110   LINE(X1(I),Y1(I))-(X3(I),Y3(I)),1,B
      120 NEXT
END PROCEDURE

PROCEDURE: Mes
   EXTERNAL: Inc,Boxnum
   STRING: K$[5]
      10 CLS : COLOR YELLOW
       20 SET CURSOR 8,12:PRINT "Hit <CR> to choose number of boxes
         and increment value."
              30 SET CURSOR 9,23:PRINT "Other keys will use default
         values"
          40 SET CURSOR 13,19:PRINT "Default values are:  Number of
         boxes = 20"
               50  SET CURSOR 14,19:PRINT "                          Box
         increment = 4"
            60 SET CURSOR 18,24:COLOR LTGREEN:PRINT "<ESC> will use
         earlier values."
      70 K$=INKEY$:IF K$ = "" THEN GOTO 70
      80 DO IF K$=CHR$(13)
      90   CLS : COLOR YELLOW
     100   SET CURSOR 9,16:INPUT "Number of boxes wanted ?";Boxnum
     110   SET CURSOR 11,16:INPUT "Box increment ? ";Inc
     120 END DO
     130 DO IF K$<>CHR$(13) AND K$<>CHR$(27)
     140   Boxnum = 20 : Inc = 4
     150 END DO
END PROCEDURE

PROCEDURE: Valreset
   EXTERNAL: Inc,Boxnum,Definewindow
   STRING: K$[5]
      10 Definewindow
      20 SET CURSOR 10,8
      30 PRINT "Want to reset number of  boxes  and  box  increment
               values ? (Y/N)"
      40 K$=INKEY$:IF K$="" THEN GOTO 40
      50 DO IF K$="y" OR K$="Y"
      60   CLS
      70   SET CURSOR 9,29:INPUT "Number of boxes ?";Boxnum
      80   SET CURSOR 11,29:INPUT "Box increment ?";Inc
      90 END DO
END PROCEDURE

PROCEDURE: Mes1
   EXTERNAL: Anykey
      10 CLS : COLOR YELLOW
      20 SET CURSOR 3,24:PRINT "Use arrow keys to move pointer."
       30 SET CURSOR 5,15:PRINT "The pointer should be moved to the
         position where"
        40 SET CURSOR 6,15:PRINT "is one of the four corners of the
         most inner box."
         50 SET CURSOR 7,15:PRINT "The corners can be set by doing
```

```
                the following way"
                60 SET CURSOR 9,12:PRINT "Hit <ESC> and 1 to set the upper
                left corner of that box."
               70 SET CURSOR 10,12:PRINT "Hit <ESC> and 2 to set the lower
                left corner of that box."
               80 SET CURSOR 12,15:PRINT "The upper and lower left corners
                should be set"
                90 SET CURSOR 13,15:PRINT "before viewing the boxes. After
                setting these"
                100 SET CURSOR 14,15:PRINT "corners, hit SPACE bar to view
                how those boxes"
                 110 SET CURSOR 15,15:PRINT "look like and how do you like
                them. You could"
               120 SET CURSOR 16,15:PRINT "either reset box number and box
                increment values"
             130 SET CURSOR 17,15:PRINT "or remain these values but change
                the positions"
            140 SET CURSOR 18,15:PRINT "of those boxes."
            150 SET  CURSOR  21,27:COLOR  LTRED:PRINT  "Type  any  key  to
                continue.";
            160 Anykey
   END PROCEDURE
      10 Mes
      20 CLS:COLOR YELLOW
      30 SET CURSOR 10,27:PRINT "Need instructions ? (Y/N)"
      40 K$=INKEY$:IF K$="" THEN GOTO 40
      50 IF K$="y" OR K$="Y" THEN Mes1
      60 MovePtr
      70 Box
      80 PUT(0,0),Ba,XOR
      90 LOCATE 24,65:PRINT "OK ? (Y/N)     ";
     100 K$=INKEY$:IF K$="" THEN GOTO 100
     110 IF K$<>"Y" AND K$<>"y" THEN Valreset : GOTO 60
     120 LOCATE 24,65:PRINT "Computing ....";
END PROCEDURE

PROCEDURE: Anykey
   STRING: S[5]
      10 S = INKEY$ : IF S = "" THEN GOTO 10
END PROCEDURE

PROCEDURE: Showpic
   EXTERNAL: Ba
      10 SCREEN 2
      15 'LINE(2,0)-(639,191),1,BF
      20 PUT(0,0),Ba,XOR
END PROCEDURE

PROCEDURE: Counting
   EXTERNAL: A,X1,X2,X3,X4,Y1,Y2,P
   EXTERNAL: Boxnum
   INTEGER: I,J,K

   PROCEDURE: Modify
```

```
   END PROCEDURE

   PROCEDURE: Modify
      INTEGER: I
      EXTERNAL: X1,X2,X3,X4,Boxnum
         10 DO IF X1(0) MOD 2 = 1
         20   FOR I = 0 TO Boxnum-1
         30     X1(I) = X1(I) - 1
         40     X2(I) = X2(I) - 1
         50     X3(I) = X3(I) - 1
         60     X4(I) = X4(I) - 1
         70   NEXT
         80 END DO
   END PROCEDURE
      10 Modify
      20 CLEAR(P)
      30 FOR J = Y1(0) TO Y2(0)-1
      40   FOR I = X1(0)/2 TO X4(0)/2-1
      50     P(0) = P(0) + A(I,J)
      60   NEXT
      70 NEXT
      80 SOUND 1000,0.5
      90 FOR K = 1 TO Boxnum-1
     100   P(K) = 0
     110   FOR J = Y1(K) TO Y1(K-1)-1
     120     FOR I = X1(K)/2 TO X4(K)/2-1
     130       P(K) = P(K) + A(I,J)
     140     NEXT
     150   NEXT
     160   FOR J = Y2(K-1) TO Y2(K)-1
     170     FOR I = X1(K)/2 TO X4(K)/2-1
     180       P(K) = P(K) + A(I,J)
     190     NEXT
     200   NEXT
     210   FOR J = Y1(K-1) TO Y2(K-1)-1
     220     FOR I = X1(K)/2 TO X1(K-1)/2-1
     230       P(K) = P(K) + A(I,J)
     240     NEXT
     250   NEXT
     260   FOR J = Y1(K-1) TO Y2(K-1)-1
     270     FOR I = X4(K-1)/2 TO X4(K)/2-1
     280       P(K) = P(K) + A(I,J)
     290     NEXT
     300   NEXT
     310   P(K) = P(K) + P(K-1)
     320   SOUND 1000,0.5
     330 NEXT
     340 SOUND 1000,0.5
END PROCEDURE

PROCEDURE: Definewindow
      10 CLS : SCREEN 0 : STATUSLINE OFF
      20 DEFINE WINDOW 1,0,0,23,79,YELLOW,ON BLUE
      30 FRAME WINDOW 1,10
```

```
        40 COLOR BORDER RED
END PROCEDURE

PROCEDURE: Plot
   EXTERNAL: W,P,Anykey,Boxnum,Definewindow
   INTEGER: L,Vc,Hc,Pmin

   PROCEDURE: Coord
   END PROCEDURE

   PROCEDURE: Scale
   END PROCEDURE

   PROCEDURE: Regression
   END PROCEDURE

   PROCEDURE: Plot4x3
   END PROCEDURE

   PROCEDURE: Plot3x3
   END PROCEDURE

   PROCEDURE: Plot3x2
   END PROCEDURE

   PROCEDURE: Plot2x2
   END PROCEDURE

   PROCEDURE: Plot2x3
   END PROCEDURE

   PROCEDURE: Coord
      EXTERNAL: Hc,Vc

      PROCEDURE: FourxThree
      END PROCEDURE

      PROCEDURE: ThreexThree
      END PROCEDURE

      PROCEDURE: ThreexTwo
      END PROCEDURE

      PROCEDURE: TwoxTwo
      END PROCEDURE

      PROCEDURE: TwoxThree
      END PROCEDURE

      PROCEDURE: FourxThree
         INTEGER: I,J
            10 LINE(170,0)-(470,180),1,B
            20 FOR I = 0 TO 1
            30   LINE(270+100*I,0)-(270+100*I,4),1
```

```
      40   LINE(270+100*I,176)-(270+100*I,180),1
      50 NEXT
      60 FOR I = 0 TO 2
      70   LINE(170,135-45*I)-(177,135-45*I),1
      80   LINE(463,135-45*I)-(470,135-45*I),1
      90 NEXT
     100 FOR J = 2 TO 5 STEP 3
     110   FOR I = 0 TO 2
             120            LINE(170+100*(I+LOG(J)/2.3),0)-
              (170+100*(I+LOG(J)/2.3),4),1
             130           LINE(170+100*(I+LOG(J)/2.3),176)-
              (170+100*(I+LOG(J)/2.3),180),1
     140   NEXT
     150   FOR I = 0 TO 3
         160        LINE(170,180-45*(I+LOG(J)/2.3))-(177,180-
              45*(I+LOG(J)/2.3)),1
         170        LINE(463,180-45*(I+LOG(J)/2.3))-(470,180-
              45*(I+LOG(J)/2.3)),1
     180   NEXT
     190 NEXT
END PROCEDURE

PROCEDURE: ThreexThree
   INTEGER: I,J
      10 LINE(125,0)-(515,180),1,B
      20 FOR I = 0 TO 1
      30   LINE(255+130*I,0)-(255+130*I,4),1
      40   LINE(255+130*I,176)-(255+130*I,180),1
      50 NEXT
      60 FOR I = 0 TO 1
      70   LINE(125,60+60*I)-(132,60+60*I),1
      80   LINE(508,60+60*I)-(515,60+60*I),1
      90 NEXT
     100 FOR J = 2 TO 5 STEP 3
     110   FOR I = 0 TO 2
             120            LINE(125+130*(I+LOG(J)/2.3),0)-
              (125+130*(I+LOG(J)/2.3),4),1
             130           LINE(125+130*(I+LOG(J)/2.3),176)-
              (125+130*(I+LOG(J)/2.3),180),1
     140   NEXT
     150   FOR I = 0 TO 2
         160        LINE(125,180-60*(I+LOG(J)/2.3))-(132,180-
              60*(I+LOG(J)/2.3)),1
         170        LINE(508,180-60*(I+LOG(J)/2.3))-(515,180-
              60*(I+LOG(J)/2.3)),1
     180   NEXT
     190 NEXT
END PROCEDURE

PROCEDURE: ThreexTwo
   INTEGER: I,J
      10 LINE(190,0)-(450,180),1,B
      20 LINE(320,0)-(320,4),1
      30 LINE(320,176)-(320,180),1
```

```
      40 FOR I = 0 TO 1
      50   LINE(190,120-60*I)-(197,120-60*I),1
      60   LINE(443,120-60*I)-(450,120-60*I),1
      70 NEXT
      80 FOR J = 2 TO 5 STEP 3
      90   FOR I = 0 TO 1
             100          LINE(190+130*(I+LOG(J)/2.3),0)-
              (190+130*(I+LOG(J)/2.3),4),1
             110          LINE(190+130*(I+LOG(J)/2.3),176)-
              (190+130*(I+LOG(J)/2.3),180),1
     120   NEXT
     130   FOR I = 0 TO 2
         140        LINE(190,180-60*(I+LOG(J)/2.3))-(197,180-
              60*(I+LOG(J)/2.3)),1
         150        LINE(443,180-60*(I+LOG(J)/2.3))-(450,180-
              60*(I+LOG(J)/2.3)),1
     160   NEXT
     170 NEXT
END PROCEDURE

PROCEDURE: TwoxTwo
   INTEGER: I,J
      10 LINE(120,0)-(520,180),1,B
      20 LINE(320,0)-(320,4),1
      30 LINE(320,176)-(320,180),1
      40 LINE(120,90)-(127,90),1
      50 LINE(513,90)-(520,90),1
      60 FOR J = 2 TO 5 STEP 3
      70   FOR I = 0 TO 1
              80          LINE(120+200*(I+LOG(J)/2.3),0)-
             (120+200*(I+LOG(J)/2.3),4),1
              90          LINE(120+200*(I+LOG(J)/2.3),176)-
             (120+200*(I+LOG(J)/2.3),180),1
     100   NEXT
     110   FOR I = 0 TO 1
         120        LINE(120,180-90*(I+LOG(J)/2.3))-(127,180-
             90*(I+LOG(J)/2.3)),1
         130        LINE(513,180-90*(I+LOG(J)/2.3))-(520,180-
             90*(I+LOG(J)/2.3)),1
     140   NEXT
     150 NEXT
END PROCEDURE

PROCEDURE: TwoxThree
   INTEGER: I,J
      10 LINE(20,0)-(620,180),1,B
      20 FOR I = 0 TO 1
      30   LINE(220+200*I,0)-(220+200*I,4),1
      40   LINE(220+200*I,176)-(220+200*I,180),1
      50 NEXT
      60 LINE(20,90)-(27,90),1
      70 LINE(613,90)-(620,90),1
      80 FOR J = 2 TO 5 STEP 3
      90   FOR I = 0 TO 2
```

```
                    100             LINE(20+200*(I+LOG(J)/2.3),0)-
                     (20+200*(I+LOG(J)/2.3),4),1
                    110             LINE(20+200*(I+LOG(J)/2.3),176)-
                     (20+200*(I+LOG(J)/2.3),180),1
             120   NEXT
             130   FOR I = 0 TO 1
                   140          LINE(20,180-90*(I+LOG(J)/2.3))-(27,180-
                     90*(I+LOG(J)/2.3)),1
                  150           LINE(613,180-90*(I+LOG(J)/2.3))-(620,180-
                     90*(I+LOG(J)/2.3)),1
             160   NEXT
             170 NEXT
        END PROCEDURE
           10 DO IF Vc=4 AND Hc=3
           20   FourxThree
           30 END DO
           40 DO IF Vc=3 AND Hc=3
           50   ThreexThree
           60 END DO
           70 DO IF Vc=3 AND Hc=2
           80   ThreexTwo
           90 END DO
          100 DO IF Vc=2 AND Hc=2
          110   TwoxTwo
          120 END DO
          130 DO IF Vc=2 AND Hc=3
          140   TwoxThree
          150 END DO
     END PROCEDURE

     PROCEDURE: Scale
        EXTERNAL: Vc,Hc,W,Pmin

        PROCEDURE: FourxThree
        END PROCEDURE

        PROCEDURE: ThreexThree
        END PROCEDURE

        PROCEDURE: ThreexTwo
        END PROCEDURE

        PROCEDURE: TwoxTwo
        END PROCEDURE

        PROCEDURE: TwoxThree
        END PROCEDURE

        PROCEDURE: FourxThree
           EXTERNAL: Pmin
              10 DO IF INT(LOG(Pmin)/2.3)=1
              20    LOCATE  1,14:PRINT "100,000";
              30    LOCATE  6,15:PRINT "10,000";
              40    LOCATE 12,16:PRINT "1,000";
```

```
        50    LOCATE 17,18:PRINT "100";
        60    LOCATE 23,19:PRINT "10";
        70 END DO
        80 DO IF INT(LOG(Pmin)/2.3)=0
        90    LOCATE  1,15:PRINT "10,000";
       100    LOCATE  6,16:PRINT "1,000";
       110    LOCATE 12,18:PRINT "100";
       120    LOCATE 17,19:PRINT "10";
       130    LOCATE 23,20:PRINT "1";
       140 END DO
       150 LOCATE 24,22:PRINT "1";
       160 LOCATE 24,34:PRINT "10";
       170 LOCATE 24,46:PRINT "100";
       180 LOCATE 24,58:PRINT "1000";
END PROCEDURE

PROCEDURE: ThreexThree
   EXTERNAL: Pmin
        10 DO IF INT(LOG(Pmin)/2.3)=2
        20    LOCATE  1, 9:PRINT "100,000";
        30    LOCATE  8,10:PRINT "10,000";
        40    LOCATE 16,11:PRINT "1,000";
        50    LOCATE 23,13:PRINT "100";
        60 END DO
        70 DO IF INT(LOG(Pmin)/2.3)=1
        80    LOCATE  1,10:PRINT "10,000";
        90    LOCATE  8,11:PRINT "1,000";
       100    LOCATE 16,13:PRINT "100";
       110    LOCATE 23,14:PRINT "10";
       120 END DO
       130 LOCATE 24,16:PRINT "1";
       140 LOCATE 24,32:PRINT "10";
       150 LOCATE 24,48:PRINT "100";
       160 LOCATE 24,63:PRINT "1,000";
END PROCEDURE

PROCEDURE: ThreexTwo
   EXTERNAL: W,Pmin
        10 DO IF INT(LOG(Pmin)/2.3)=2
        20    LOCATE  1,17:PRINT "100,000";
        30    LOCATE  8,18:PRINT "10,000";
        40    LOCATE 16,19:PRINT "1,000";
        50    LOCATE 23,21:PRINT "100";
        60 END DO
        70 DO IF INT(LOG(Pmin)/2.3)=1
        80    LOCATE  1,18:PRINT "10,000";
        90    LOCATE  8,19:PRINT "1,000";
       100    LOCATE 16,21:PRINT "100";
       110    LOCATE 23,22:PRINT "10";
       120 END DO
       130 DO IF INT(LOG(W(0))/2.3)=1
       140    LOCATE 24,24:PRINT "10";
       150    LOCATE 24,40:PRINT "100";
       160    LOCATE 24,55:PRINT "1,000";
```

```
      170 END DO
      180 DO IF INT(LOG(W(0))/2.3)=0
      190   LOCATE 24,25:PRINT "1";
      200   LOCATE 24,41:PRINT "10";
      210   LOCATE 24,57:PRINT "100";
      220 END DO
END PROCEDURE

PROCEDURE: TwoxTwo
   EXTERNAL: W,Pmin
       10 DO IF INT(LOG(Pmin)/2.3)=3
       20   LOCATE  1, 8:PRINT "100,000";
       30   LOCATE 12, 9:PRINT "10,000";
       40   LOCATE 23,10:PRINT "1,000";
       50 END DO
       60 DO IF INT(LOG(Pmin)/2.3)=2
       70   LOCATE  1, 9:PRINT "10,000";
       80   LOCATE 12,10:PRINT "1,000";
       90   LOCATE 23,12:PRINT "100";
      100 END DO
      110 DO IF INT(LOG(W(0))/2.3)=1
      120   LOCATE 24,15:PRINT "10";
      130   LOCATE 24,39:PRINT "100";
      140   LOCATE 24,63:PRINT "1,000";
      150 END DO
      160 DO IF INT(LOG(W(0))/2.3)=0
      170   LOCATE 24,16:PRINT "1";
      180   LOCATE 24,40:PRINT "10";
      190   LOCATE 24,65:PRINT "100";
      200 END DO
END PROCEDURE

PROCEDURE: TwoxThree
   EXTERNAL: Pmin
       10 DO IF INT(LOG(Pmin)/2.3)=3
       20   LOCATE 12, 5:PRINT "10,000";
       30 END DO
       40 DO IF INT(LOG(Pmin)/2.3)=2
       50   LOCATE 12, 5:PRINT "1,000";
       60 END DO
       70 LOCATE 24, 3:PRINT "1";
       80 LOCATE 24,27:PRINT "10";
       90 LOCATE 24,52:PRINT "100";
      100 LOCATE 24,75:PRINT "1,000";
END PROCEDURE
   10 DO IF Vc=4 AND Hc=3
   20   FourxThree
   30 END DO
   40 DO IF Vc=3 AND Hc=3
   50   ThreexThree
   60 END DO
   70 DO IF Vc=3 AND Hc=2
   80   ThreexTwo
   90 END DO
```

```
     100 DO IF Vc=2 AND Hc=2
     110   TwoxTwo
     120 END DO
     130 DO IF Vc=2 AND Hc=3
     140   TwoxThree
     150 END DO
END PROCEDURE

PROCEDURE: Regression
   EXTERNAL: W,P,Boxnum
   INTEGER: I
   REAL ARRAY(100): RegX,RegY
   REAL: Xsum,Ysum,U,Xysum,X2sum
   REAL: Y2sum,Numer,Denom,R,B
      10 CLEAR(RegX,RegY,U,Xsum,Ysum,Xysum,X2sum,Y2sum)
      20 FOR I = 0 TO Boxnum-1
      30   DO IF W(I) > 0 AND P(I) > 0
      40     RegX(I) = LOG(W(I))/2.3 : RegY(I) = LOG(P(I))/2.3
      50   END DO
      60   Xsum = Xsum + RegX(I) : Ysum = Ysum + RegY(I)
      70   U = RegX(I)*RegY(I) : Xysum = Xysum + U
      80   U = RegX(I)^2 : X2sum = X2sum + U
      90   U = RegY(I)^2 : Y2sum = Y2sum + U
     100 NEXT
     110 Numer = Xysum - Xsum*Ysum/Boxnum
             120  Denom  =  (SQR(X2sum-Xsum^2/Boxnum))*(SQR(Y2sum-
                 Ysum^2/Boxnum))
     130 R = Numer/Denom
     140 B = (Xysum-Xsum*Ysum/Boxnum)/(X2sum-Xsum^2/Boxnum)
     150 DO IF ABS(R) > 0.98
     160   LOCATE 21,42:PRINT "Slope =";:PRINT USING "#.###";B
     170   LOCATE 22,42:PRINT "    R =";:PRINT USING "#.####";R
     180 END DO
END PROCEDURE

PROCEDURE: Plot4x3
   EXTERNAL: W,P,L,Pmin
      10 DO IF INT(LOG(Pmin)/2.3)=1
           20   CIRCLE(170+100*LOG(W(L))/2.3,180-45*(LOG(P(L))/2.3-
                1)),2,1
      30 END DO
      40 DO IF INT(LOG(Pmin)/2.3)=0
      50   CIRCLE(170+100*LOG(W(L))/2.3,180-45*LOG(P(L))/2.3),2,1
      60 END DO
END PROCEDURE

PROCEDURE: Plot3x3
   EXTERNAL: W,P,L,Pmin
      10 DO IF INT(LOG(Pmin)/2.3)=2
           20   CIRCLE(125+130*LOG(W(L))/2.3,180-60*(LOG(P(L))/2.3-
                2)),2,1
      30 END DO
      40 DO IF INT(LOG(Pmin)/2.3)=1
           50   CIRCLE(125+130*LOG(W(L))/2.3,180-60*(LOG(P(L))/2.3-
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```
            1)),2,1
   60 END DO
END PROCEDURE

PROCEDURE: Plot3x2
  EXTERNAL: W,P,L,Pmin
    10 DO IF INT(LOG(Pmin)/2.3)=1 AND INT(LOG(W(0))/2.3)=1
    20     CIRCLE(190+130*(LOG(W(L))/2.3-1),180-60*(LOG(P(L))/2.3-
            1)),2,1
    30 END DO
    40 DO IF INT(LOG(Pmin)/2.3)=1 AND INT(LOG(W(0))/2.3)=0
     50      CIRCLE(190+130*LOG(W(L))/2.3,180-60*(LOG(P(L))/2.3-
            1)),2,1
    60 END DO
    70 DO IF INT(LOG(Pmin)/2.3)=2 AND INT(LOG(W(0))/2.3)=1
    80     CIRCLE(190+130*(LOG(W(L))/2.3-1),180-60*(LOG(P(L))/2.3-
            2)),2,1
    90 END DO
   100 DO IF INT(LOG(Pmin)/2.3)=2 AND INT(LOG(W(0))/2.3)=0
    110      CIRCLE(190+130*LOG(W(L))/2.3,180-60*(LOG(P(L))/2.3-
            2)),2,1
   120 END DO
END PROCEDURE

PROCEDURE: Plot2x2
  EXTERNAL: W,P,L,Pmin
    10 DO IF INT(LOG(Pmin)/2.3)=2 AND INT(LOG(W(0))/2.3)=1
     20    CIRCLE(120+200*(LOG(W(L))/2.3-1),180-90*(LOG(P(L))/2.3-
            2)),2,1
    30 END DO
    40 DO IF INT(LOG(Pmin)/2.3)=2 AND INT(LOG(W(0))/2.3)=0
        50   CIRCLE(120+200*LOG(W(L))/2.3,180-90*(LOG(P(L))/2.3-
            2)),2,1
    60 END DO
    70 DO IF INT(LOG(Pmin)/2.3)=3 AND INT(LOG(W(0))/2.3)=0
        80   CIRCLE(120+200*LOG(W(L))/2.3,180-90*(LOG(P(L))/2.3-
            3)),2,1
    90 END DO
   100 DO IF INT(LOG(Pmin)/2.3)=3 AND INT(LOG(W(0))/2.3)=1
    110    CIRCLE(120+200*(LOG(W(L))/2.3-1),180-90*(LOG(P(L))/2.3-
            3)),2,1
   120 END DO
END PROCEDURE

PROCEDURE: Plot2x3
  EXTERNAL: W,P,L,Pmin
    10 DO IF INT(LOG(Pmin)/2.3)=2
      20       CIRCLE(20+200*LOG(W(L))/2.3,180-90*(LOG(P(L))/2.3-
            2)),2,1
    30 END DO
    40 DO IF INT(LOG(Pmin)/2.3)=3
      50       CIRCLE(20+200*LOG(W(L))/2.3,180-90*(LOG(P(L))/2.3-
            3)),2,1
    60 END DO
```

```
   END PROCEDURE
      10 FOR L = 0 TO Boxnum-1
      20   DO IF P(L) > 0
      30     Vc = INT(LOG(P(Boxnum-1))/2.3)-INT(LOG(P(L))/2.3)+1
      40     Pmin = P(L) : EXIT 2 LEVELS
      50   END DO
      60 NEXT
      70 Hc = INT(LOG(W(Boxnum-1))/2.3) - INT(LOG(W(0))/2.3) + 1
      80 CLS : SCREEN 2
      90 Coord
     100 Scale
     110 DO IF Vc=4 AND Hc=3
     120   FOR L = 0 TO Boxnum-1
     130     DO IF P(L)>0
     140       Plot4x3
     150     END DO
     160   NEXT
     170 END DO
     180 DO IF Vc=3 AND Hc=3
     190   FOR L = 0 TO Boxnum-1
     200     DO IF P(L)>0
     210       Plot3x3
     220     END DO
     230   NEXT
     240 END DO
     250 DO IF Vc=3 AND Hc=2
     260   FOR L = 0 TO Boxnum-1
     270     DO IF P(L)>0
     280       Plot3x2
     290     END DO
     300   NEXT
     310 END DO
     320 DO IF Vc=2 AND Hc=2
     330   FOR L = 0 TO Boxnum-1
     340     DO IF P(L)>0
     350       Plot2x2
     360     END DO
     370   NEXT
     380 END DO
     390 DO IF Vc=2 AND Hc=3
     400   FOR L = 0 TO Boxnum-1
     410     DO IF P(L)>0
     420       Plot2x3
     430     END DO
     440   NEXT
     450 END DO
     460 Regression
     470 Anykey
     480 Definewindow
END PROCEDURE

PROCEDURE: Setbound
   BYTE ARRAY(200): B,C
   EXTERNAL: Rb,Lb,Ul,Ll
```

```
PROCEDURE: Init
END PROCEDURE

PROCEDURE: Leftright
END PROCEDURE

PROCEDURE: Updown
END PROCEDURE

PROCEDURE: Init
   EXTERNAL: B,C
      10 LINE(0,25)-(0,175),1 : GET(0,25)-(0,175),B
      20 LINE(0,25)-(0,175),0
      30 LINE(170,0)-(470,0),1 : GET(170,0)-(470,0),C
      40 LINE(170,0)-(470,0),0
END PROCEDURE

PROCEDURE: Leftright
   EXTERNAL: B,Rb,Lb
   STRING: R[5],L[5],A[5]
   INTEGER: X
      10 R=CHR$(0)+CHR$(77) : L=CHR$(0)+CHR$(75)
      20 X=50
      30 PUT(X,25),B,XOR
      40 DO
      50   A=INKEY$:IF A="" THEN GOTO 50
      60   IF A=CHR$(27) THEN PUT(X,25),B,XOR : EXIT
             70       IF  A=R  THEN  PUT(X,25),B,XOR : X = X + 5 :
           PUT(X,25),B,XOR
             80       IF  A=L  THEN  PUT(X,25),B,XOR : X = X - 4 :
           PUT(X,25),B,XOR
      90   IF A="R" THEN Rb=X
     100   IF A="L" THEN Lb=X
     110 REPEAT
END PROCEDURE

PROCEDURE: Updown
   EXTERNAL: C,U1,L1
   STRING: U[5],D[5],A[5]
   INTEGER: Y
      10 U=CHR$(0)+CHR$(72) : D=CHR$(0)+CHR$(80)
      20 Y=0
      30 PUT(170,Y),C,XOR
      40 DO
      50   A=INKEY$:IF A="" THEN GOTO 50
      60   IF A=CHR$(27) THEN PUT(170,Y),C,XOR : EXIT
             70       IF  A=D  THEN PUT(170,Y),C,XOR : Y = Y + 5 :
           PUT(170,Y),C,XOR
             80       IF  A=U  THEN PUT(170,Y),C,XOR : Y = Y - 4 :
           PUT(170,Y),C,XOR
      90   IF A="U" THEN U1=Y
     100   IF A="D" THEN L1=Y
     110 REPEAT
```

```
   END PROCEDURE
      10 Init
      20 Leftright
      30 Updown
END PROCEDURE

PROCEDURE: Message1
   EXTERNAL: Definewindow
   STRING: K$[5]

   PROCEDURE: Directory
   END PROCEDURE

   PROCEDURE: Directory
      STRING: K$[5]
         10 CLS
                  20  PRINT "Which dirve ? (A/B)";SPC(20) "Hit <CR> to
                   continue"
         30 K$=INKEY$:IF K$="" THEN GOTO 30
         40 IF K$=CHR$(13) THEN EXIT
           50 IF K$<>"A" AND K$<>"a" AND K$<>"B" AND K$<>"b" THEN GOTO
               10
         60 IF K$="A" OR K$="a" THEN FILES"a:*.*" ELSE FILES"b:*.*"
         70 PRINT : PRINT : PRINT "Once Again ? (Y/N)"
         80 K$=INKEY$:IF K$="" THEN GOTO 80
         90 IF K$="Y" OR K$="y" THEN GOTO 10
   END PROCEDURE
      10 Definewindow
           20 SET CURSOR 10,14:PRINT "Hit 'd' for directory, any other
         keys to continue"
      30 K$=INKEY$:IF K$="" THEN GOTO 30
      40 IF K$="D" OR K$="d" THEN Directory
END PROCEDURE

PROCEDURE: Resul
   EXTERNAL: Definewindow,Anykey,P,W,Boxnum
   INTEGER: I
   STRING: K$[5]
      10 Definewindow
      20 SET CURSOR 9,28:PRINT "Counting has completed."
      30 SET CURSOR 11,26:COLOR  LTRED:PRINT  "Type  any  key  to  see
         result."
      40 SET CURSOR 18,25:COLOR LTGREEN:PRINT "<ESC>  will  bypass  to
         the plot.";
      50 K$=INKEY$:IF K$="" THEN GOTO 50
      60 IF K$=CHR$(27) THEN EXIT
      70 CLS : COLOR YELLOW
      80 FOR I = 0 TO Boxnum-1
      90   PRINT SPC(24) "W(";I;" ) =";W(I),"P(";I;" ) = ";P(I)
     100 NEXT
     110 SET CURSOR 21,27:COLOR  LTRED:PRINT  "Type  any  key  to  see
         plot.";
     120 Anykey
END PROCEDURE
```

'MAIN Program:

```
 10 '****  This is a Dilation Fractal Analysis.  ****
 20 Message1
 30 Acqpic
 40 Showpic
 50 Setbound
 60 Shrink
 70 SetWidth
 80 Counting
 90 Resul
100 Plot
110 GOTO 70
```

ENDFILE