Descartes' Philosophical Grounds for Algebra and Geometry

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A thesis submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Master of Arts in the Department of Philosophy.

Chapel Hill
2008

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ABSTRACT

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(Under the direction of Alan Nelson)

Though neither Descartes nor Fermat developed analytic geometry as we understand the mathematical discipline today, they each developed a different central feature of it. But history of mathematics has not explained their differences in mathematical aims and practice in terms of philosophical commitments. Generally, Descartes' metaphysics has not played a role in the explication of his mathematical emphasis. While it is undisputed that Descartes placed a lot of emphasis on the geometrical aspect of analytic geometry over the algebraic component, I explain his mathematical practice by appealing to his philosophical views. I argue that Descartes views geometry to be more basic than algebra. Algebra, and its significance, depended on geometrical objects. I have defend this view by appealing to Descartes metaphysical and epistemological commitments, but I also defend this view against a possible alternative interpretation of Descartes' philosophy that would allow algebra not to depend on geometry.

Acknowledgements

I would like to express my gratitude to my committee members, Alan Nelson, Marc Lange and Ram Neta, for their patient and generous guidance. From Alan, I have benefited greatly from hours and hours of discussions that have formed the foundations of this thesis. His feedback and comments have helped me immensely, and I cannot express how grateful I am for his support and encouragement. I am also indebted to Ram and Marc for their time and interest in this project; they have both been very supportive and encouraging, and have helped steer me and keep me on track. I also would like to thank you to Penelope Maddy who played an instrumental part in the selection of my thesis topic. My acknowledgements would not be complete without acknowledging my gratitude to Kent Johnson whose unwavering support has been vital for the completion of this thesis.

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Introduction

The development of analytic geometry arose from the problem of the correlation between number and geometrical magnitude. The problem with numbers and magnitudes of geometrical extension was the disputed nature of their relationship. It was not clear if the numbers had any meaning that was not dependent on the geometrical objects. Variations of this problem have also been expressed with associating numeric relationships with spatial configurations, or associating number with time. One way to understand the difference between Descartes' and Fermat's mathematical developments of early analytic geometry is in terms of their different views regarding the relationship between numbers and geometric extension.

Descartes and Fermat developed different features of what we would call elementary analytic geometry. Though each inherited some aspect of their project from the classical mathematicians and other influences like Viete, each went in a different direction toward different mathematical aims. Fermat developed the study of the properties of curves of linear and quadratic equations. He sought to show that algebraic equations could have geometrical interpretations: linear equations can represent straight lines, and quadratic equations can represent conics. But in contrast to Fermat, Descartes developed the derivation of equations of loci. Descartes begins with lines and curves and then derives an equation that can represent them. Descartes' method is opposed to Fermat's method of beginning with an equation and then deriving the properties of the curve described by it. Fermat begins with an algebraic equations and then shows it can

be regarded as defining a locus of points with respect to a coordinate system. There is a difference in aim and emphasis between the two men. While Fermat only admits curves into geometry if it is possible to find its equation, Descartes studies curves defined by equations. In Descartes' mathematical practices, the significance of geometry was more predominate than the algebraic procedures he used to solve problems. Insofar as any algebra was to be used at all, it was only in terms of the geometrical interpretation that was given to the numeric quantities and the algebraic operations.

Often the explanation given for explaining the difference between Descartes and Fermat's mathematical aims is to assume Descartes to be overly-influenced by the existing geometrical construction problems of the ancients and scholastics. On this account the geometrical emphasis of Descartes' mathematical work was merely the result of the types of unsolved problems he happened to want to solve. Mathematical accounts of Descartes' emphasis on geometric construction do not, and could not, explain why he was so committed to the geometrical dominance over his algebraic methods; nor did they seek to find possible explanations from elsewhere. Consider the following passage from Boyer's *History of Analytic Geometry*[1]:

> Some attention has been given to the status of analytic geometry vis-à-vis other branches of mathematics; but the impact of the wider intellectual milieu has been referred to only where it was regarded as of particular significance. It is of interest to note in this connection that the development of coordinate geometry was not to any great extent bound up with general philosophical problems. The discoveries of Descartes and Fermat in particular are relatively free of any metaphysical background. Indeed, *La Geometrie* was in many respects an isolated episode in the career of Descartes – one suggested by a classical problem of Greek geometry.

[1] Boyer (1956), viii.

Pace Boyer, I think there are philosophical issues that significantly influence the discoveries and practices of Descartes. While I cannot speak for Fermat, I do think the metaphysical views of Descartes are a source of grounding for his mathematics. It is a mistake to think that Descartes' mathematical views are orthogonal to his systematic philosophical account of the world. I will lay out an interpretation of Descartes' metaphysics and epistemology that will show the geometrical priority of his mathematical practice as a consequence of his philosophical views.

In the following, I will argue that Descartes views geometry as prior to algebra, and that algebra is a derivative of, or dependent on, geometry. His view about the relationship between algebra and geometry is a direct result of his philosophy. Because Descartes metaphysically grounds mathematics in extension, geometry—the mathematical discipline that makes the most fundamental distinctions of extended objects—is prior to algebra. Before I begin my argument, I would like to make clear the domain of the project at hand. In this paper I will set aside the question of Descartes' views concerning whether or not algebra and arithmetic really differ[2]. This is a complicated issue that I will not, due to the limited scope of this paper, be able to address. Instead, I will avoid this issue by taking for granted the interchangeability of the two sub-disciplines of mathematics for present purposes. This means that I will assume algebra to be a generalized form of arithmetic, and will discuss the priority of geometry to *any* form of arithmetic (*e.g.*, in cases where the number is known and the quantities are specified as numbers, such as '*5*'; or in cases where they are unknown and the quantities are expressed generally with variables). Since I mean both the specific

[2] I will also not be addressing a similar version of this issue: What is the status of universal mathematics in Descartes' philosophy, and what is its relation to algebra and arithmetic?

and general cases, I will use the term 'algebra' instead of 'arithmetic'. This distinction between geometry and algebra, however, will not be discussed until after I lay down some of the metaphysical and epistemological framework of the interpretation of Descartes that I am using. Again, due to the scope of the topic at hand, rather than argue for this particular interpretation of Descartes, I will instead lay out the basics of the interpretation by examining and explaining some key passages in order to give the reader the gist of the view I will be exploiting.

The advantages of this particular reading will be three-fold. First, the metaphysical view will be helpful for delimiting exactly what we have knowledge of when we are engaged in mathematical activity. Second, the epistemological view will help make clear the difference between geometry and algebra. And, third, as a result of understanding the difference between geometry and algebra, the sense in which geometry is prior to algebra will also be better understood.

This view that I argue is Descartes' is very different from the view of mathematics we have now. Especially since the development of pure mathematics, it is common to believe that mathematical disciplines work with objects or concepts that have no useful spatial or geometrical interpretation. Various branches of mathematics have developed without concern for our geometrical intuitions. But Descartes' mathematical views and practices seems to just the reverse of our current ones. Rather than find algebraic relations any spatial interpretation, Descartes gives our Euclidian spatial relations an algebraic representation. It is not until algebraic operations like addition, subtraction, multiplication, division and the extraction of roots are made intelligible in a intuitive geometrical way that they acceptable for use. Descartes' view of the significance of all other forms of mathematics as geometrically grounded is very

much opposed to our view of mathematics' independence from geometry. We do not now think, as Descartes' did, that the geometry of space constrains the development of other mathematical realms.

1.0 Descartes' Metaphysics and Epistemology

1.1 Finding Substance

Perfect knowledge, for Descartes, is of that which is so clear and distinct its truth is evident and beyond doubt or skepticism. He takes, "as a general rule that the things we conceive very clearly and very distinctly are all true,"[3] for, as he explains, that is what certainty consists in: being able to clearly see the necessity of a truth, or what is distinct in the thought. For example, even if I were to doubt my own existence, I can recognize the truth that I, insofar as I am a thing that thinks, exist. It is evident and clear from thinking the doubt that the doubt-thought exists. The doubt, when considered in a clear and distinct way so that I perceive that it is a thought, leads me to something that is true and certain: that I, some substance whose nature is thought, exist.

The existence of a substance whose nature is thought is the Archimedean point that Descartes is looking for in the *Mediations* in order to have some certain truth to help him out of his skepticism. He needed to find one firm and certain point from which he could, as it were, "move the entire earth from one place to another"[4]. Descartes' close inspection of his thoughts reveals that thought is the essential nature of thinking substance. He discovers this when he considers his understanding of "body" to be:

> …all that is suitable for being bounded by some space, for being enclosed in some place, and thus for filling up space, so that it excludes every other

[3] Discourse on Method, Part 4, AT 33, CSM I, p. 127,

[4] Meditations on First Philosophy (hereafter 'Meditations'), meditation 2, AT 24, CSM II.

> body from that space; for being moved in several ways, not surely by itself, but by whatever else that touches it. For [he] judge[s] that the power of self-motion, and likewise of sensing or *of thinking, in no way pertains to the nature of the body*."[5]

"From this," Descartes writes, "I knew I was a substance whose whole essence or nature is simply to think"[6], and also, "I am therefore precisely only a thing that thinks; that is, a mind, or soul, or intellect, or reason…"[7]. Thought is an attribute of the substance Descartes calls mind, in fact, it is the principal attribute of the substance mind.

1.2 Identifying the Nature and Essence of a Substance

For Descartes, a substance can be thought of as just a thing that has attributes[8]. Whenever we identify an attribute, we have found a substance. While a substance may be known through any attribute, each kind of substance, according to Descartes, has a unique attribute that differentiates it from all the other substances. Descartes calls this attribute a substance's **principal attribute**[9]. This attribute constitutes the nature and essence of the substance, and since it is unique to a single substance, each substance is distinguished by its essence or nature from the other by its principal attribute. For Descartes, the principal attribute's uniqueness to a single substance means we can identify the substance by its principal attribute. When we think of these substances as the substance that has its particular essence or nature, we are able to clearly and distinctly perceive and know them. For example, the substance "mind"—when

[5] Meditations, mediation 2, AT 26: emphasis added, CSM II

[6] Discourse, Part 4, AT 33, CSM I, p. 127

[7] Meditations, mediation 2, AT 27, CSM II

[8] c.f. Principles I, §52, AT 25, CSM I, p.210

[9] c.f. Principles I, §53, AT 25, CSM I, p 210

considered simply as the substance whose particular principal attribute is thought—can be identified as the type of substance that thinks. Likewise, a "body" is nothing more than an extended substance since extension is the principal attribute of the substance "body".

If two substances share the same principal attribute, then they are the same type of substance for they have the same essence and nature. Principal attributes will also indicate when two things are not the same substances: when a single principal attribute pertains to one, and only one, of the two of them. For example, when Descartes clearly and distinctly perceives that thought is the unique, principal attribute of his mind, he has a clear and distinct perception of his mind as a substance through its principal attribute. Whatever body is, it is not the same substance as mind, for as Descartes already discovered, thinking in no way pertains to the nature of body. Body and mind are not the same, identical substance: mind is really distinct from body[10].

1.3 Clearly and Distinctly Perceiving More Substances

It is by making "real distinctions" that Descartes is able to work up the rest of his foundations of knowledge, as he famously does in the *Meditations on First Philosophy,* and recounts in *Discourse on Method*. By discerning principal attributes, he is able to work up to some of the other substances, such as God or body. A **real distinction**, Descartes writes, "exists only between two or more substances; and we can perceive that two substances are really distinct simply from the fact that we can clearly and distinctly

[10] At this point in the Meditations, we do not know even if body exists, or if it did, if body is a substance or some feature or attribute of a substance. What we can say is that body (whether it exists nor not, whether it's nature really is extension or not) does not have the same nature as the mind, *i.e.,* thinking substance, and we can further say that it is not an attribute or mode of thinking substance.

understand one apart from the other."[11] For example, our cognition or knowledge of the substance mind, or thinking-thing, as explained above is cognition through our clear and distinct perception of its essence: thought. But in order to understand the nature of mind, *i.e.*, thought, we do not have to have an understanding of body or its nature. We can understand ourselves simply as thinking things through any of our various thoughts, *e.g.*, our doubts, denials, affirmations, wills, etc. Likewise, we can understand the substance, body, through a clear and distinct perception of its principal attribute, extension, without having to understand the substance of mind. When we consider the substance body as its principal attribute of extension through its various extended qualities, *e.g.*, the position, place, space it takes up, etc., we can do so without needing to understand it with mind, and thereby understand body to be something that is distinct and separate from mind Thus, mind, *i.e.*, thinking substance, and body, *i.e.*, extended substance, are really distinct substances.

1.4 The Correlative Relation between Principal Attributes and Real Distinctions

Real distinctions and principal attributes are very closely related. From the above discussion, we can see that principal attributes play a central role in making real distinctions. When we can have a clear and distinct perception of a substance through its principal attribute—the attribute that is the substance's unique essence and nature—we can very easily make a real distinction. By noticing that he is precisely a thing that thinks, Descartes is able to make a real distinction between thinking substance and extended substance. We can go in the reverse direction of this relationship as well: from real distinctions to principal attributes. Descartes notices that thinking substance and

[11] Principles I, § 60, AT28, CSM I, p. 213

extended substance are really distinct, and then Descartes arrives at the clear and distinct perception of thought alone as the attribute that cannot be detached from his mind. Extension is the principle attribute of corporeal substance.

It is extension that the knowledge of mathematics takes as its subject matter. Mathematics concerns the nature and essence of extension, body, matter, or even space[12]. The reason mathematics concerns extension is because, as I will show below, its nature, unlike the essences of the other substances, is exactly that which is necessary for mathematical thought. The nature of corporeal substances is spatial, extended, with magnitude, and thus quantifiable. But I wish to postpone this discussion a little longer because it can be better explained and filled out after we understand Descartes' general account of the various ways we can think of thought or extension. Thus, let us continue with the account of Descartes' metaphysics and epistemology, and then, once we have that picture laid out, return to the topic of mathematics as the discipline concerned with extension.

1.5 Ways We Think of Substances and Attributes: Rational Distinctions

Above, while explaining principal attributes, I mentioned two features of attributes and substances that I'd like to return to and discuss more carefully. Substance, for Descartes, is something that has attributes, so that determining if a substance exists or not can be resolved by determining if some attribute exists. Descartes writes, "A substance may indeed be known through any attribute at all; but each substance has one principal property which constitutes its nature and essence, and to which all its other

[12] I do not mean to imply here an empty space or vacuum, but rather the spatial or space-iness of extended substance.

properties are referred."[13] When we find that an attribute really exists, we can infer that some substance that has the attribute exists. But we do not need the attribute we find to be the principal attribute of the substance in order to conclude that a substance exists. The principal attribute is only necessary for identifying what type of substance it is that exists. But Descartes in this stretch of the *Principles* also says that principal attributes are more than just a unique attribute of the substance among the many attributes the substance can have. He says that the principal attribute is the one which all its other properties are referred. He gives an example using body and extension. Extension constitutes the nature of corporeal or bodily substance. Everything else that could be said of extended things, what Descartes calls modes of an extended thing, presupposes extension. A shape is only intelligible insofar as it is thought of as an extension of some length, breadth, or depth. "Shape is unintelligible except in an extended thing"[14]. A two dimensional plane figure's shape can only be thought in terms of as some sort of extension of length, and breadth. A natural question to ask here is how different and distinct the principal attribute is from the other attributes or modes. A good place to start would be to look at what other distinction might pertain to attributes and substances besides real distinctions.

When Descartes draws a distinction between a substance and its affections, he further distinguishes between two types of affections: attributes and modes. I follow the interpretation of attributes and modes that other commentators have argued[15], I think we should understand Descartes' attributes and modes not as properties that inhere in

[13] Principles I, § 53, AT 25, CSM I 210.

[14] Principles I, § 53, AT 25, CSM I 210.

[15] C.f. NOLAN (1997)

the substance, but as ways in which we regard or think of the substance. Attributes should be understood as a mere distinction we make with our reason. This interpretation of attributes as a **rational distinction**[16] is motivated by passages in the *Principles*[17] where Descartes claims, as Nolan argues[18], that there is a mere distinction of reason or *distincto rationis* between "a substance and some attribute of that substance without which the substance is unintelligible; alternatively, it is a distinction between two such attributes of a single substance."[19] We are to understand attributes as something that is only distinguishable from the substance (and one another) by our reason.

Take two rationally distinct attributes, the size and shape of an extended substance like a single solid figure, specifically a cube. Any two attributes of the cube, the size and shape of its extension, is only distinguished as two different attributes of extension by our reason, but actually they are attributes of only one, numerically identical extended body: the cube. By numerically identical, I mean that it is ontologically one and the same thing that I am conceiving of through its various rationally distinguished attributes. This is true because if the two attributes were two attributes of two different substances and not a single, numerically identical substance, then the two substances would have different principal attributes. But the two attributes are not really distinct from the same extended substance (and thereby also its principal

[16] "Rational distinction" is the literal translation of "*distincto rationis*" which is translated as "conceptual distinction" in CSM.

[17] Principles I, § 62, AT 30

[18] Nolan, (1998) page 164

[19] Principles 1, § 62, AT 30, CSM I, p.214

attribute, extension) for we cannot understand what size or shape of the cube is except as the size and shape of the extension of the cube. A size or shape that is not an extended quality is unintelligible. This, I take it, is what Descartes means when he says that the principal attribute is the one that is referred to by all the other attributes: that it is the way in which the other attributes or modes are made intelligible. But this is not to say that having extension as an attribute is really different from having a shape or a size; nor is having a shape really different from having extension or size. When we conceive of the size of an extended thing, like the solid cube figure, we think of the size of the cube's extension, but that does not mean the cube's extension does not also have a shape that the size takes. We can then think of the shape of the extension, but this does not exclude that the size of it is a different "thing" or substance. Both the size and shape are ways in which we think of the extended solid cube. The only way we can differentiate or identify which substance we are thinking of is by characterizing it through its principal attribute—the attribute that is the essence and nature of the substance. Thus we can identify that the solid figure, the cube, is an extended substance because we can understand its nature to be extended when we consider its size, its shape, or even other attributes like its position, or motion. What makes these attributes distinct is only that we are thinking of that one extended substance, the cube, as extended in different ways: now through its size, now through its shape, but in both cases, always as extended.

In a related fashion, if two substances were different, their principal attributes would be really distinct and different, and not merely distinct by reason. Going in the other direction, if there are two attributes such that they are not merely rationally distinct but really distinct from each other, it would follow that they are attributes of two different substances. For example, the attribute of a thinking substance like doubt is

more than merely rationally distinct from the attribute of an extended substance like size. In the first case, doubt is understood as a thought, and in the latter case size is only intelligible as extended (in some length, breadth, or depth). Body is just its extendedness, and the other attributes or modes are just the ways in which we can attend to the extendedness of the substance. So far, in most of my examples I discuss cases concerning extended substances because I will be mainly focused on extension later, but the same holds true for all the other substances and their natures, or attributes. The substance just is its attributes.

Descartes says just what you would think he would, or should, say, given the reading that I am suggesting we use. Consider the following passages of text:

> By 'extension' we mean whatever has length, breadth and depth, leaving aside the question whether it is a real body or merely a space…So we must point out to [people who employ distinctions that obscure even that which is perfectly clear] that by the term 'extension' we do not mean here something distinct and separate from the subject itself… For although someone may convince himself that it is not self-contradictory for extension *per se* to exist all on its own even if everything extended in the universe were annihilated, he would not be employing a corporeal idea in conceiving this, but merely an incorrect judgment of the intellect alone.[20]

Here, Descartes says that you cannot think that the attribute of extension is different or separate from the thing which has extension, and he goes on to give the example of not being able to think that in a universe where all extended things were annihilated, there could still exist extension. This is all to suggest that the attribute is not a separate thing from the substance that has it. To talk of having "it", this attribute, is deceptive in this way: such that you might be tempted to think the attribute is something that is not identical with the subject, or substance, that it is being predicated of. A little further down, still in the same discussion of Rule 14, Descartes considers the sentence, "Body

[20] RULE 14, AT 442-443, CSM I, p. 59

possesses extension". He says here that we do not form two distinct ideas with that sentence; instead, we have just the single idea of extended body. "So far as the fact of the matter is concerned I might just as well have said, 'Body is extended', or better still, 'That which is extended is extended.' This is a peculiarity of those entities which exist only in something else, and which can never be conceived apart from a subject."[21] It is through the subject, thing, or substance that our understanding of them and their particular qualities, modes, or attributes, are made intelligible. The last issue we will examine before we move on to Descartes' mathematics is what it means to understand a substance through an attribute, or to "make intelligible" an attribute in a type of substance. These are what may be called "cognitive routes".

1.6 Ways We Think With Substances and Attributes: Cognitive Routes

If, as I have claimed above, attributes and modes are to be thought of as "the way in which" we think of a substance, then **cognitive routes** may be thought of as "the process by which" we shift or direct our mental attention and cognitions from one "way-in-which" to another. A cognitive route is how we move from attending to a particular feature of a substance or attribute, to attending to another. It gives a "route" or various possible routes we can move along. We can think of the cognitive route as being either the recorded path that was actually taken, or the paths that are open to us to take from one thought to another. Examples of cognitive routing can be found above, and before revisiting a couple of them, I will first quickly review what was covered.

Rational distinctions of attributes are just various ways in which we can regard a substance. We only have clear and distinct ideas of substances when we can distinguish

[21] Rule 14, AT 444, CSM I, p. 60

the substance via a principal attribute of it, an attribute that distinguishes it from other types of substances. This attribute that we distinguish is what we turn our attention to when we consider the nature and essence of the substance through it: we are recognizing that the particular attribute we have our attention on is the one which we are using to consider this substance; and that helps us tell this substance apart from the others, *i.e.,* make real distinctions. Which substance we have the idea of depends on what attribute we are distinguishing (*e.g.,* whether it is one that belongs to thinking substances or extended substances). In the case of extended substances, its attributes are the things like length, breadth, depth, size, shape, motion, etc.

The reason we regard the substance via different attributes that are only rationally distinct from the substance itself is because this single, simple substance is something that we can think in many ways. There are various modes of thinking of the substance that can be the particular way we think of it. However, this is just another way we can direct ourselves to think about the other attributes of the same substance, since the attributes are only rationally distinct from each other in a single substance. But we think of these things in particular ways, or modes of thought, like when we have ideas of particular shapes like triangles, and square. In this case, we could be thinking of the extended substance, say a solid figure like a square based pyramid, through one of its attributes, shape, particularly a triangular shape of one of the solid's faces. The substance that we are thinking about is one extended body, but when we think about a triangular face of it, we are delimiting the face or the polyhedron from the rest of the figure in our thought. Furthermore, we can either be delimiting the triangular face as one of the sides of the pyramid, or we could be thinking of it as a three-sided polygon. The difference is that in the first case, we pick out the triangle as a face of the pyramid,

not especially attending to the triangularity of it in our thought. In the latter case, we pick out the triangle as a three-sided polygon, not attending in our thought to it being a face rather than the base of the pyramid. In both cases, they pick out the same planar region on the same solid figure. The thoughts of the different attributes of the pyramid are only rationally distinct from the thought of the pyramid. The modes are to be understood as the particular ways in which the substance is conceived of through one of its attributes. All the transitions in this series of thoughts are instances where we have used cognitive route. *Cognitive routes are the various ways by which we are able to use what we are attending to in our thought to get to a different thing we can attend to within our thought*. We can think of the cognitive route as either a historical record or account of the whole process or path our thought took, *e.g.*, from attending to the substance as a solid figure to attending to the shape the solid figure's face has. Or we can think of the cognitive route as a way, or point at which a shift can made in what is being attended to in our thought, *e.g.*, seeing that the same triangular shape can be viewed either as a face of the pyramid, or as a three-sided polygon.

I think we can view real distinctions, rational distinctions, and principal attributes all as ways in which we can cognitively route or direct our attention. When we notice, as we did earlier, that doubting is a thought, and that there existed thinking, we could attend to the attribute of thought as existing, and route our attention from there to the existence of our mind (like Descartes did in the *Meditations*). But we could also have routed our attentions so that we were attending to the thought-attribute being the principal attribute of the substance mind (like Descartes does when he notices he is precisely a thing that thinks in the *Meditations*, or that he is a thing whose sole nature and essence is thinking as he does in the *Rules for the Direction of the Mind*). Using yet

another route, we could have attended to our doubt as being only rationally distinct from what we take to be the thought-like substance, or the other type of rational distinction can route us when we notice that doubting is one of many ways in which we can think, *i.e.*, that doubt is one of many different modes that our mind can take. We could also understand, affirm, deny, will or refuse: doubt is united with all of these modes as being ways in which we could think, or be a Mind. This would be a cognitive route to a principal attribute being the way in which other properties are referred (as it is done in the *Principles*) or to the merely rational distinction that we make between two attributes of the same substance, or a substance and its attribute (also as it is done in the *Principles*).

If we were to add more to the inventory of thoughts, we can expand the things we can attend to or notice. For example, we can cognitively route ourselves using a real distinction when we form an idea that thought in no way pertains to the nature of body to an idea that body and minds have different principal attributes and are different in their essences and natures. Alternatively, we could have been routed to an idea of thought being the principal attribute of the mind. This should seem familiar from our earlier discussion of the close relationship between real distinctions and principal attributes. The whole discussion showed various ways we could be cognitively routed.

Before I continue, I wish to pause for a moment so that I can summarize what has been laid out thus far in the Cartesian picture. Substances are really distinct from one another because each has a unique nature or essence. The essences that distinguish one substance from another are the principal attributes. Substances are not really distinct from their rationally distinct attributes. The various attributes pick out one and the same substance, and two attributes that are only merely rationally distinct from each

other and from the substance are really, ontologically, numerically identical to each other. We think of a substance through some particular mode of one of the substances' attributes. That is to say, we regard, or turn our attention to some particular aspect of an idea we have of the substance through some attribute of it. For example we think of, pay attention to, or regard, the particular length of the square we are thinking of as the shape of the base of the solid pyramid of extended substance. In this case we are thinking of the square base's length as a length of extension of the solid substance. But we could have also thought that particular length of the base of the square as being a side of the shape that makes the base of the pyramid. The first case is an example of attending to the solid figure in a way that would allow us to understand its extension through length. In the second case, we view the same line segment of the square base as a side or edge of the square shaped polygon that the pyramid's base takes. The shifting of the attention of our cognition of the solid figure allows us to understand its extension as a length or as a shape. With this basic view in hand, we can now turn to Descartes account of mathematics.

2.0 Geometry and Algebra considered in relation to Attributes and Modes

2.1 Mathematics concerns Comparing [Extended] Magnitudes

In the *Rules for the Direction of the Mind*, Descartes writes, "I came to see that the exclusive concern of mathematics is with questions of *order and measure* and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds, or any other object whatever."[22] The only concern of mathematics is the questions that regard order and measure, or *things of magnitude*. I say that Descartes' view is that we are concerned with magnitudes in mathematics because when we are engaged in the practice of mathematics, we are comparing two or more objects. An example would be comparing a part to the whole, or a discrete object with another. In order to compare two or more things, we have to compare them along, or with respect to, some common feature that they both share. Sometimes we require the additional assistance of their relations or proportions to other objects because the thing common nature sought is not present equally in the things being compared. Descartes' method in this case is to reduce the additional relations and proportions down until an equality between what is sought and what is already known is found. But here Descartes notes that, "nothing can be reduced to such equality except *what admits of differences of degree, and everything covered by the term 'magnitude'*." [23] If we abstract away all the extraneous aspects such

[22] Rule 4, AT 378, CSM I, p. 19: emphasis mine

[23] Rule14 AT 440-441, CSM I, p. 57-58: emphasis mine

that only what is at issue in the mathematical problem remains, we see we are dealing only with magnitudes in general. But we should also be wary not to mistakenly take Descartes to be implying that magnitudes are things that are really distinct and separate from the things that have magnitudes. This would be to incorrectly think the things whose magnitude we are regarding are somehow excluded in our cognition of the magnitudes when considered generally in mathematics, as if they somehow were a different substance. What Descartes emphasizes is that the magnitude—or what admits of difference of degree—are modes of attributes: mere rational distinctions of extended substances.

Descartes points out that even mathematicians make the mistake of thinking that the magnitudes abstracted from objects in their thoughts by reason are really distinct things that are separate from the objects. Sometimes arithmeticians think that they have really abstracted numbers from the subjects numbered. Geometers, thinking lines have no breadth, or surfaces no depth, create a surface using the extended quality of the line, not realizing that they are all modes, mere rational distinctions, of Body, or extension[24]. If we attend to the idea of magnitude carefully, we shall see that our idea of magnitude is also an idea of extended substance: extended substance understood as that which has magnitude. When we are engaged with mathematical problems, we are concerned with an extended object that we think of solely in terms of its extensive magnitues.

2.2 Three Aspects that Help Us Understand Extension as Magnitudes

Descartes believes that once every problem has been reduced to the point where we can compare the extensions of what we know in order to find the extension of what

[24] Rule 14, AT 446, CSM I, p. 61

we are looking for, all we do is reduce and simplify the proportions until we get an equality between what is known and what is sought. Thus, according to Descartes, "it is enough for our purposes if we consider all the characteristics of extension itself which may assist us in elucidating differences in proportion. There are only three such characteristics, *viz*. dimension, unity and shape."[25] He then goes on to explain each in turn.

By **dimension**, Descartes means "simply a mode or aspect in respect of which some subject is considered to be measurable."[26] This would include not only length, breadth and depth as you might expect, (though he also notes that these dimensions are only nominally different[27]), but also other dimensions such as weight, speed, etc. In one of his examples[28], Descartes makes a rational distinction between counting and measuring. When we are considering the order of the parts in relation to the whole, we are **counting**. When we consider the whole as something that is divided up into parts, we are **measuring**. The important point, for Descrates, is that they are all dimensions along which something is measurable and thereby also counted when considered or regarded in the reverse direction. We can count the days or weeks in a century; or we can measure centuries in terms of years or days. It is the same magnitude we consider, first in a way that attends to the order of the parts to the whole, then in a way that attends to the measure of the whole in terms of its parts. The important point is that

[25] Rule 14, AT 447, CSM I, p.62

[26] Ibid.

[27] Rule 14, AT 449, CSM I, p.63

[28] Rule 14, AT 448, CSM I, p.62

counting and measuring are different directions of the relationship between parts and wholes, *i.e.,* different ways in which we can attend to the same thing.

There can be, within the same subject, many different dimensions, and each one is not to be considered really distinct from the extended body. Descartes stressed this point by example. Though some geometers may mistakenly think the line, plane, and solid to be different species of quantity, Descartes reminds us that they are not really distinct from one another. They only are rationally distinguished through the abstractions we make with our reason.[29] "Indeed," he goes on to say, "if they are thought of without respect to anything else, as abstractions of the intellect, then they are no more different species of quantity than 'animal' and 'living' in man are different species of substances." We do not think that there are two separate parts of man, one that is 'animal', and the other that is 'living'. Instead, we view them as two different ways in which we can regard a man. This is similarly the case with extension and quantity. A length of a line or a side is a different way to regard a quantity of the same extended object. The area of a plane or surface is a different way to regard a quantity of the same extended region. The volume of the solid is yet another way to regard a quantity of that same object. All are ways in which we can regard the same extended substance and its extended nature.

We can think of an extended quantity of corporeal substance like a cube by thinking of the volume of that cube. We can further delimit our thought of that cube by attending to the magnitude of its surface area. Both these quantities of extension are not things that are really distinct from the cube. But suppose now we further delimit our thought of the cube and consider one face of it, or we can consider a particular part of its

[29] Rule 14, AT 448-449, CSM I, p. 63

surface area, *i.e.*, a square polygon in the first instance, or the area of the square face of it in the latter instance. From here we can delimit the cube even further in our thoughts by either thinking of a particular side or edge of the square, or the two triangular parts that we could divide the square into along its diagonal. There are a number of different ways we could attend or cognitively route our thoughts of the cube using any number of rational distinctions. However, in each instance, we never stop thinking of the same extended cube with some quantity of extension.

The next characteristic of extension Descartes discusses is "unity". **Unity**, according to Descartes, is "the common nature which,…, all the things which we are comparing must participate in equally."[30] So long as no unity is specified in the problem, we can take any magnitude, either given in the problem or not, and use it as the unit, or common, measure, of all the other magnitudes. It just needs to have as many dimensions in it that the most extreme term that we are comparing has in it. So if the most extreme magnitude sought has two dimensions, the magnitude we take up as our common measure should have two dimensions, and it should be two dimensions that all the other magnitudes can have in common. Note here that unity, or determining our common unit of measure, already assumes that some common dimension has been selected. It is only after we have made distinct what type of magnitude or dimension that we are interested in that we can set ourselves the task of determining what will be a single unit of that measure. To repeat, the dimension or type of magnitude along which that measure (or count/order) is sought needs to be already specified.

When Descartes discusses "**shape"**, or figure, a further distinction is drawn. When we are concerned with mathematical issues, only two types of figures are useful

[30] Rule 14, AT 450, CSM I, p. 63

for comparing relations or proportions of multitudes and magnitudes: figures that represent sets, and figures that illustrate continuous magnitudes. These two figures can both represent either of the two different types of relations mentioned earlier that obtain between the same kinds or of the same dimensions: that of order, or that of measure. Figures that represent sets or **multitudes** are discrete like a collection of points. Figures that illustrate continuous **magnitudes** can be a polyhedron like a cube or a polygon like a square, or a curve or straight line. A **relation of order** differs from a relation of measure in the direct, immediate way in which its terms are related: they do not require an extra term to mediate between them. The parts that are in a relation of order are related, or ordered, with respect to themselves. We can compare which line segment is longer or which area is larger, or that there are three sides to a figure with three angles. This is not the case with relations of measure. **Relations of measure** require the mediation of an extra term: the specified unit of measure. Without knowing what the unit of measure is, I do not know the quantity of measure along some specified dimension of extension;

Whereas we can know the relation of order between two things without having to know anything more than the two things being compared, we would need to know the common unit of measure if we were to know the relation of measure of the two things along some common dimension. We can know the relation of order between a line segment and a congruent line segment by comparing them side by side. We could be able to tell if a third line segment was longer than the other two put together if we concatenated the first two and compared it against the third. But we cannot know the relations of measure between the lines without considering them using a common unit of measure. We can only consider ourselves to know a quantity of measure, like the

length of the hypotenuse of a particular triangle, when it is measured in terms of a known quantity of the shared dimension, like length of one of the legs of a right-triangle.

To bring together our current discussion of ways we can attend to magnitudes with our earlier discussion of Descartes metaphysics and epistemology, let us examine how dimension, unity, and shape work together in mathematics. Descartes highlights that once we have established a unit, "it is sometimes possible completely to reduce continuous magnitudes to a set and that this can always be done partially at least."[31] This, as I mentioned above, is a result of being able to use our cognitive routes to shift our attentions to different ways to think about the same things. We can think of the number of parts in a continuous whole, what we above called the count, thus yielding a set, or multitude of the unit part we specify. This can be done because once we have used the unit to help us conceive of our continuous magnitude in terms of it, we will have supplied the common measure so that we can conceive of the parts in that continuous whole as a multitude of those parts. Thus, we can apprehend the measures of the figure by working with the orders. In this way, a figure can either represent sometimes a continuous magnitude, or sometimes a set, a number, or multitude.

For Descartes the continuous magnitudes, and the discrete multitudes, are not really distinct but only rationally distinct. Using our cognitive routes, we can use either multitude or magnitude to consider the quantity of extension. Of course, this all can only happen if we specify the dimension along which the magnitudes are to be considered in the first place. Descartes gives a nice example in Rules[32]. We can investigate a problem using general terms but also may want to re-express it in numbers.

[31] Rule 14, AT452, CSM I, p. 65

[32] Rule 16, AT 457-458, CSM I, p. 68-69

For example, take the case of finding the hypotenuse (the dimension specified here is length) of a right triangle with sides a and b to be equivalent to the $\sqrt{(a^2+b^2)}$ [33]. We might not want to always express the relation as a relation among the order of the parts, but instead as a measure. If we substitute 81 for a^2 and 144 for b^2, their sum would 225. The root of 225, or the mean proportional between the unit [which we designated to be the common measure along the dimension of length] and 225, is 15. These are two ways in which we can understand the same problem regarding the same triangle. We can understand the relation as a measure of some continuous magnitude, or we can understand the relation as an order of the unit parts.

2.3 The Difference between Geometry and Algebra

When we regard the figure as a set or number, a unit measure is required. But in order to determine a unit of measure, we need it to be along some dimension. This means that before we can think of a multitude or a quantity of measure along some dimension of the extended body, we need to determine, or delimit, in what respect (*i.e.*, through which attribute) we are going to regard it. This is selected out of the many dimensions or attributes that are available for us to consider. We can, for example, chose to direct our focus and attention to the entire surface of a solid square pyramid and pick out the dimension of surface area, which will pick out the quantity of measure of an area. Alternatively we can chose to regard just one edge of a face of the pyramid, thus picking out the dimension of length, in which case we will attend to the quantity of measure of length. But in order to delimit what the dimension of measure of the figure

[33] For Descartes this would be interpreted as the length given as the first proportional of the sum of the second proportional of a with the second proportional of b.

we are interested in, we must first delimit the object of interest. This would entail thinking of the whole extended object, or some aspect of the extended object. In doing so, (like when we consider the intersection of a solid cone with a plane, and think of the conic section), we are delimiting in our thought the particular way in which we will be thinking of particular extended objects. We delimit a mode of the substance considered through one of its attributes. To give a quick example, we can consider the cube as the object we are interested in, or we can delimit some part of the cube, like one of its square polygon faces, as the object we wish to consider. If we take the object of interest to be the cube we can delimit a dimension of measure for the cube, but if we take the object of interest to be a square face of the cube, we would delimit a dimension of measure for the square. The dimension of measure we use to consider the quantity of an extended object will differ depending on how we consider it. It is important to not forget that though we may think of the square as a different object from the cube for the purposes of determining a dimension of measure, they are only separated in our reason, and are in fact one and the same extended substance. We are only attending to the same polyhedron by using its different attributes to think of it in different ways.

This activity of determining, or delimiting a figure corresponds to the subject and activity of **geometry**. Geometry concerns the *relations of the aspects or attributes* that we use to rationally distinguish, or delimit, our figure. Geometry takes as its subject matter the "geometric" magnitudes, *i.e.,* dimensions of measure (size, shape, length, area, etc), and then considers them as relations and proportions of order.

When we want to consider the particular measure, or quantity of these magnitudes, and their relations, then we enter the realm of **algebra**. This is distinguished from geometry because it will involve *relations of measures* which will all

require some unit of measure being specified. If we are counting (considering the relation of order of the parts to the whole), we will need to determine our unit measure and then we can determine how many there are. For example we can take as our unit of measure the distance from the ground to the tip of my finger on my fully raised right arm[34]. Then we can give a quantity of the number of those units that are in the perimeter of a parcel of land. If we are measuring, (considering the whole with respect to some divided part of it), we will need to determine the unit of measure in order to know the quantity of measure in relation to the unit measure specified. For example, if we were to give a measure of a particular given distance, we would want to specify what unit our quantity of measure is given in. Suppose someone asked us how far the destination point was far from our starting place, and we replied 100. This would not be helpful, for we could have meant 100 feet or 100 miles. Note that by asking how far some place was, the dimension of distance or a length is specified.

2.4 The Priority of Geometry to Algebra

Now, we are in the position to say why geometry is prior to algebra for Descartes, and to what that priority really amounts to. Algebra concerns the relations of orders of measures; however, we cannot specify a measure unless we do so in relation to a unit of measure. This unit of measure, this common measure, is given along a particular dimension of measure. The dimensions—along which there can be measures—is the concern of geometry. We engage in geometry when we consider and determine

[34] This was one of many units used by the Aztecs to survey a tract of land for the purpose of calculating the tribute the landowner had to pay.

dimensions and their relations of order. Only then we can be in a position to even consider the particular quantities of measure along those dimensions or count the parts.

Mathematics concerns only the things that can be quantified. The only things that can be quantified are extended things. The subject matter of geometry is the relations and proportions of order of extension considered through its attributes *i.e.,* the dimensions along which there can be quantity (*e.g.,* motion and length) and the particular object we are interested in quantifying. The subject matter of algebra is the relations of the particular quantities of which are counted, measured, or determined along some specified or delimited dimension or attribute of extension (*e.g.,* particular measures of speed and particular measures of length as determined though a given or arbitrarily specified unit measure.

3.0 A Possible Objection

3.1 The General Structure of the Objection

An objection that someone might try to make against the interpretation of Descartes philosophy of mathematics we have just examined is to claim that algebra can be generated some other way that does not depend on geometry and extended objects. All she would have to do is show that we can, according to Descartes, arrive at numbers or quantities another way that does not require extended objects. It would have to be in a way that does not assume some magnitude or dimension of extension that is quantified. She might search Descartes' corpus for a way to generate quantity through another substance. And indeed, she may find what seems to be some problematic text that seems to suggest we can generate the requisite numbers through thinking substances, and expand counting into algebra without requiring geometry or extension to be prior. The problem texts can be found in both the *Meditations* and the *Principles*. In the third meditation, Descartes writes that, "I perceive that I exist now and recall that I have previously existed for some time, and when I have several thoughts and know the number of these thoughts, I acquire the ideas of duration and number, which I then apply to everything else." [35] In the other text, the *Principles,* Descartes writes, "…substance, duration, order, number and any other items of this kind… extend to all

[35] Mediations, meditation 3, AT 44-45

classes of things."[36] If we can count other minds or modes of our own thought without appealing to extended substances, then Descartes cannot be interpreted to hold the view that geometry prior to algebra in the way that I have argued. It would also indicate that Descartes' mathematics does not take extension as its sole subject matter. I will examine each way the objection can be made below. First I will discuss what I consider to be the easier case of counting or numbering minds, and then I will discuss the more difficult cases of counting or numbering thoughts.

3.2 Regarding the Counting of Distinct Minds.

If we make clear what would be necessary in order for us to count the number of minds it will become clear to us why Descartes does not believe that we can do so without the assistance of extension. This would show that counting minds cannot undermine algebra's dependency on extension. To start, we should notice that we have earlier made certain of the existence of our mind as a substance when we noticed that our mind is a substance whose principal attribute is to think. But now suppose we attempt to count other individuated minds, *e.g.*, my mind, your mind, and so on. In order to do this, we need to be able to find a way to distinguish individual minds from one another. As it turns out, the separation of my mind from your mind is hard to make without appealing to extension. The intuitive way we might attempt to do this is to think of my mind being in one place and your mind not occupying the same one. But here notice that having a place is an attribute of extension. The place or space occupied is conceived of as what is possessed by a shaped thing that is given in relation to other

[36] Principles I, § 48, AT 23, CSM I, p. 208

shaped things. Shape arises from a limit of extension in length, breadth, depth[37]. Another way we might have thought we could individuate different minds was to think of the separate, distinct bodies that belong to different minds: different body, different mind. This, again, runs into the same problem. It seems we still need to appeal to extended substances in order to individuate the different minds. Trying to count the number of other minds is not really possible without extension.

Another method we might attempt is to distinguish my mind and its thoughts from not-my-mind, and not-my-thoughts. This is a weaker version of attempting to count the number of other minds. We would only aim to perceive that there are at least two minds: my mind and a mind that is not the same as mine. We are not be able to tell if what is not my mind is a single other mind or many other minds without appealing to extended bodies. We must avoid using spatial determinations to draw the distinction between what counts as my mind as "inside" my body, and other minds as "outside" my body to avoid appealing to extended substances. We might attempt to make the distinction we are after by working up a thought of my mind and its thoughts as being active and under my will. Then we can think about other minds and how their thoughts will seem passive, or not under my will. We can think of how our imagination works, or how we can use cognitive routing to actively direct the attention of our thoughts to particular features. This would be compared to some representation we have of other minds and thoughts not being under our will in the same way. The thoughts of other minds are thoughts that we cannot cognitively route or direct according to our will. The problem with this attempt is that any and all representations we have or take to be of other thoughts and other minds is only mediated to us as such through our own mind,

[37] Meditations, meditation 3, AT 42

as a thought we have. We only can know the thoughts, even if the thought is of thoughts not belonging to us, as belonging to us. So it seems not even this weaker way of distinguishing individual minds is going to succeed.

3.3 Regarding the Counting of Distinct Thoughts

Though we have shown that we are not able to count minds, we are not yet safe from the objection. The objection can still be made successfully if it turns out we can generate numbers and quantities from thoughts of our mind. This seems like an easier task than individuating other minds without extension. Since we are only concerned with the thoughts of our own mind, we need not worry about individuating them from other minds. All we have to do is find a way to generate the ideas of numbers from our thoughts. There are a couple of ways we might try to count our own thoughts.

No matter how we try to count our thoughts, we need to first determine the dimension of measure along which we are quantifying. There has to be some common unit or feature that we are comparing, as was discussed earlier. One way that we might try to quantify our thoughts is though the dimension of duration. A measure of duration is a quantity of time. But it is not clear how we can successfully go about the task of measuring our thoughts this way. If we were to measure the duration of our thoughts we would need some unit measure of time. The difficulty lies in how we can get this temporal measure without extension, because the concept of time relies on a physical process. For Descartes, getting an idea of time will ultimately depend on there being motion.[38] Since motion is an attribute of extended substances, it depends on the

[38] c.f. Principles I, §57, AT 27, CSM I p.212

existence of extended objects.[39] We can only conceive of motion as the change in position or space of an extended body. We cannot make motion, and thus time, intelligible without appealing to extension.

Perhaps we can try a different dimension of measure, but what kinds of measure are available to us? We cannot divide any mode of thought into parts (like a left part or a top part, or a half) because the nature of thought is different from the nature of extension. Extension is divisible into parts but thoughts are not. We have to think of each mode of though at a whole, indivisible unity.

We can try to count whole discrete units of thoughts. One worry we might have would be that every time we are counting our instances of assenting thoughts we might be counting the same one. In order to not count the same assenting thought over and over again if it is the same one that is appearing to us, we might require the total number of assent-thoughts we are attempting to count to be present to our mind simultaneously. This would be to attempt something that resembles the part/whole, count/measure relations of extension. The idea is to take all the times the assent-thought occurs and think of them as parts of the whole. Each occurrence would be a part that we can consider in relation to the total number of them. If there were identical assent-thoughts that had multiple appearances, they would be the same numerically identical "piece" when we considered them simultaneously. But it would be strange to talk of all the thoughts of a particular type, like assent-thoughts, being simultaneously thought. It would be as if we have laid them out, and concatenated them in a way that is space-like. There are spatial implications to the notion of simultaneity that we would

[39] c.f. Principles II, §25, AT 53-54, CSM I p.233

want to avoid because we want to generate numbers without appealing to features of extension.

It seems a better strategy, and perhaps the best strategy, for distinguishing thoughts so that we can quantify them is to use real or rational distinctions. Both are familiar ways we have used to identify something about a thought that is different from another thought. When we make a real distinction in our thought, we recognize that some thought of a substance is really distinct from a thought of something else: that they are not thoughts of the same thing, and not the same thought. We might take this to be a way to get the idea of two, or at least, the idea of more-than-one. Since we want to avoid appealing to extension, we will avoid making a real distinction that involves extended substances. For example our clear and distinct perception God is distinct from our clear and distinct perception of our mind. By being able to distinguish those perceptions from each other, we would be able to count some of our perceptions that way. When we know that the thoughts are distinct then we know that the thoughts are not the same one. Similarly, we can use our thoughts of rationally distinct things. We can recognize our thoughts are not thoughts of the same rationally distinct thing and thus not the same thought. It seems we have here a very troublesome objection to the view that I am defending.

3.4 Possible Replies to the Objection

I have three considerations in reply to the objection in defense of my interpretation of Descartes' view on mathematics. The first thing is that for Descartes, numbers are modes of thinking when considered in the abstract, and not in created things. Descartes explains abstractions of universals, the sort of mode a number is, in

the Principles[40]. His examples of the process by which we get the idea of the number, 2, all use extended objects such as birds, stones, or trees. We form the idea of the (two) trees, and when we consider the fact that there are two of them, we call *that* mode in which we regard them, "two". Then when we later see two birds, and consider that there are two of them, we can go back to the idea we had formed before of the number we call "two". How we get a number idea, two, may first have to be generated by some extension much in the way I laid out earlier, and only later once we have the idea of two, can we use the abstract idea, the universal, or mode of thinking to consider our thoughts.[41]

There is a second consideration to take into account. When we make the distinctions between our different thoughts such that we identify that they are not of the same natures, what we do is to see, or judge that there is some repugnance, or contrariety. However, the idea of bare repugnance is not the same as an idea of two, or the idea of at-least-greater-than-the-one. The metaphysical concept of repugnance and the concept of the number two as a quantity or measure are far from the same thing. This is perhaps more easily seen in a simpler case: the case of metaphysical unity, and the number one (as in first in the series, or a unit measure). The number one requires that there first be an idea of unity. From the idea of unity we can then get the idea of a unit, and after that a numeric quantity of one is possible. Similarly, the number two is a concept that requires there first be some idea of repugnance, but it is not the case that an idea of repugnance immediately gives us the idea of two. To get from repugnance to the idea of two requires further steps. At the very least it seems to require us to notice that

[40] Principles I, §59, AT 27-28, CSM I, p. 212

[41] c.f. Nolan, 1998.

we can apply the abstraction, the number idea, to the repugnant things. This assumes we have the abstract idea of two, which is something, I argue that can only be arrived at through cognitive routes that involve extension. I make this claim based on what Descartes says about numbers as unit measure in relation to a measure of some continuous magnitude (*i.e.,* quantity, or dimension) or to an order, count or multitude of parts to the whole.

The further steps that are required between having an idea of unity or repugnance and having an idea of a numeric quantity is the subject of my third consideration. The concept of needing further steps in our process of cognitive routing implies a successive order. There is something in the notion of a "first" that comes before and "after" that requires the sequence or series to be in time. And time in Descartes' view, as I have argues above depends on extension.

Though I think the objection is serious, I do not think that defeats the view I am arguing. While I have not fully explored the objection or given a reply that has fully satisfied or laid the objection to rest, I do think I have shown my interpretation of Descartes' philosophy of mathematics to be a plausible one. The view that the subject matter of mathematics is extension, and that through extended substances and our ideas of them we must first engage in some geometric activity prior to even the possibility of engaging in arithmetic or algebraic activity remains a plausible account of Descartes mathematics.

4.0 Conclusions

I wish to conclude by returning to where I started, back to the history of analytic geometry. Though neither Descartes nor Fermat developed analytic geometry as we understand the mathematical discipline today, they each developed a different central feature of it. But not much of the literature on the history of mathematics has explained their differences in mathematical aims and practice in terms of philosophical commitments. Generally, Descartes' metaphysics has not played a role in the explication of his mathematical emphasis. While it is undisputed that Descartes placed a lot of emphasis on the geometrical aspect of analytic geometry over the algebraic component, I have gone a step further and explained his mathematical practice by appealing to his philosophical views. I argued that Descartes views geometry to be more basic than algebra. Algebra, and its significance, depended on geometrical objects. I have defended this view by appealing to Descartes metaphysical and epistemological commitments, but I have also defended this view against a possible alternative interpretation of Descartes' philosophy that would allow algebra not to depend on geometry.

I think especially in cases like Descartes, it would be worth while both for the history of the development of mathematical ideas and for the history of philosophy to examine how the philosophy and mathematics are related. One benefit is that examining philosophical views in relation to mathematical practices can offer reasons for mathematical practices in a way that merely examining mathematical practice cannot

provide. A second benefit is that examining mathematical practices may indicate a philosophical view of mathematics that would be more likely than another when potentially problematic texts are considered. Because all of Descartes' mathematics is grounded in extension, the branch of mathematics that concerns the most fundamental distinctions of extension will underwrite and guide the other mathematical branches. It is for this reason Descartes emphasized the geometrical significance over his algebraic methods in his analytic geometry.

I have demonstrated the effectiveness of the first case by using an argument that grounds Descartes' mathematical work in his metaphysics and epistemology. I have explained Descartes' mathematical practice in a way that looking merely at his mathematical practice cannot. The philosophical account provides an explanation for his mathematical views. Given my picture of Descartes, one where his metaphysics is intimately related to his mathematics, we have an explanation that can provide us with a better understanding of his mathematical choices, views, and practices.

Additionally, I have demonstrated the effectiveness of the second case by using an interpretation of Descartes philosophical views such that it would allow a unifying account of his philosophy and mathematical aims. This is more preferable to an interpretation of Cartesian philosophy that did not unify his philosophy and mathematics so well. His mathematical practice, in addition to the philosophical reasons, provides a further motivation for why we should not think that algebra can be generated without appealing to extension. Any interpretation of Descartes that thinks geometry is not necessary for grounding algebra will have to come up with a further explanation for his mathematical practice.

References

Bos, H. J. M. (2001). *Redefining geometrical exactness :Descartes' transformation of the early modern concept of construction*. New York: Springer.

Boyer, C. B. (1956). *History of analytic geometry*. New York: Scripta Mathematica.

Boyer, C. B., & Merzbach, U. C. (1989). *History of mathematics*. New York: Wiley.

Cottingham, J., Robert S., and Murdoch D., trans. (1985-86). *The philosophical writings of Descartes, Vol.1-2.* New York: Cambridge University Press.

Cottingham, John, Robert Stoothoff, Dugald Murdoch, and Anthony Kenny, trans. (1991). *The philosophical writings of Descartes: The correspondence, vol. 3.* New York: Cambridge University Press.

Grattan-Guinness, I. (1998). *The Norton history of the mathematical sciences :The rainbow of mathematics* [Fontana history of the mathematical sciences] (1st American ed.). New York: W.W. Norton.

Klein, J. (1968). *Greek mathematical thought and the origin of algebra* [Griechische Logistik und die Entstehung der Algebra. English] . Cambridge, Mass.: M.I.T. Press.

Kline, M. (1959). *Mathematics and the physical world*. New York: Crowell.

Kline, M. (1964). *Mathematics in western culture* (1st pbk. ed.). London; New York: Oxford University Press.

Kline, M. (1972). *Mathematical thought from ancient to modern times*. New York: Oxford University Press.

Macbeth, D. (2004). Viete, Descartes, and the emergence of modern mathematics. *Graduate Faculty Philosophy Journal, 25*(2), 87-117.

Mancosu, P. (1996). *Philosophy of mathematics and mathematical practice in the seventeenth century*. New York: Oxford University Press.

Nelson, A. J. (2005). *A companion to rationalism*. Malden, MA; Oxford: Blackwell Pub.

Nolan, L. (1997). Reductionism and nominalism in Descartes's Theory of Attributes. *Topoi, 16*(2), 129-140.

Nolan, L. (1998). Descartes' theory of universals. *Philosophical Studies, 89*(2-3), 161-180.

Sepper, D. L. (1996). *Descartes's imagination :Proportion, images, and the activity of thinking*. Berkeley: University of California Press.