TWO ESSAYS ON LIFE CYCLE MODELS

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ABSTRACT

DAVER CUNEYT KAHVECIOGLU: Two Essays on Life Cycle Models
(Under the direction of David Blau)

This dissertation consists of two essays. In “A Life-Cycle Model: Retirement, Savings, and Portfolio Allocation”, I analyze the interrelationships among retirement, saving, and asset allocation. I show that optimal portfolio allocation can be very sensitive to plans about retirement timing and to the presence and characteristics of Social Security. I also show how portfolio decisions have impact on when to retire. In order to take into account these important effects, Social Security reform proposals should be evaluated with models that incorporate portfolio choice.

In the second essay, “Asset Allocation, Bequests, and Wealth Dynamics of the Elderly”, I build and estimate a life cycle model of asset allocation and saving with a fixed cost of stock market entry, medical expenditure risk, and a general bequest function that captures risk preferences over bequests. The estimates imply that there is no operative bequest motive, and that medical expenditure risk is a very strong motivation for saving at older ages. Even though the model explains reasonably well both the observed average age profiles and the heterogeneity in saving and portfolio allocation, the cross-equation restrictions that are required for internal consistency are strongly rejected. This is mainly due to the fact that, in constant relative risk aversion utility, a single parameter - coefficient of relative risk aversion - governs both wealth and portfolio paths. While a very high level of risk aversion is required to explain portfolio choices, a very low level of risk aversion is required to explain wealth dynamics. Despite its internal inconsistency, the model is one of the richest versions of the standard model and I use the estimates to do some policy simulations with caution. I used the estimates to simulate the impact of reducing the Social Security benefit of a 70-year old female retiree by a specific amount, and giving her a lump sum equal to the expected present discounted value of the benefit cut, which is to be invested in her individual account. I find that this reform would be undesirable. Main reason behind this is that, most retirees are either optimally annuitized or under-annuitized.

To my late grandparents, Ayse and Ali Usta...
...who are missed dearly.

To mom and dad, Melahat and Sabri Kahvecioglu...
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CHAPTER I

A Life Cycle Model: Retirement, Savings, and Portfolio Allocation

1. Introduction

There is a large and growing literature in finance on life-cycle household portfolio allocation. Even though this literature[1] recently acknowledges that labor income may have a big role to play, it ignores the fact that retirement is a choice and treats it as given. On the other hand, in the economics literature, the portfolio mix has been treated as given or has been ignored altogether in the models of labor supply and retirement behavior.[2]

In this paper, I analyze the interrelationships among retirement behavior, savings, and portfolio mix by modeling these decisions as being made jointly. In particular, I am interested in how the optimal patterns of life-cycle portfolio allocation are affected by introducing retirement as a choice variable. That information will help assess the validity of asset allocation advice provided by portfolio managers to the people who are saving for retirement. Another interesting question that I will explore is whether and to what extent the presence and characteristics of Social Security affect optimal household portfolio allocations. This is quite important in view of the current debate about incorporating individual accounts into Social Security. The model proposed in this paper could be used to analyze the impact of a Social Security reform that introduces individual accounts, while models that treat the timing of retirement or asset allocation as given would be poorly suited for such an analysis. Predicting baby boomers' portfolio allocations will also be helpful in forecasting the future trends in the stock market since they will be holding a significant amount of national wealth.

I have solved and simulated a relatively simple version of the model. I find similar results to the previous literature when I consider retirement as given. However, if retirement is a choice, I demonstrate that the model predicts richer and different life-cycle patterns. It is shown that the optimal portfolio allocation can be very sensitive to expectations about the timing of retirement. Additionally, I show that the presence and characteristics of Social Security can be very influential on optimal portfolio allocations as well.

[1]Gomes and Michaelides, 2003; Bertaut and Haliassos, 1997; Cocco, Gomes, and Maenhout, 2001; Svensson 1988; Viceira 2001; Vissing-Jorgensen 1999 to name a few.

[2]One exception is MaCurdy (1985). He analyses hours of work (in contrast, my paper analyses labor force participation decision).

The rest of the paper is as follows: In section 2, I review the previous work done on the household life-cycle portfolio allocation. The theoretical model is described in section 3. Model's solution and its implications are presented and discussed in the same section. Section 4 concludes.

2. Literature Review

I summarize the relevant literature below. Note that none of the papers discussed here treats retirement, savings, and portfolio choice as decisions being made jointly.

Addressing the problem of portfolio choice over the life-cycle dates back to the seminal papers, Samuelson (1969) and Merton (1969). They showed that optimal portfolio allocation is independent of both age and wealth. The agents in their model should hold a positive fraction of their wealth in risky assets and this ratio is fixed over the lifetime and over the level of wealth. It is now known that this result is sensitive to the papers' assumptions (some of them were implicitly assumed in the papers). As explained in Ameriks and Zeldes (2001), among those assumptions are: (1) asset returns are independently and identically distributed over time, (2) households have utility functions that exhibit constant relative risk aversion (CRRA) and that are time-invariant and additively separable over time, (3) markets are frictionless and complete.

There was no labor income in Samuelson's (1969) model. However, without violating any of the above 3 assumptions Merton (1971) adds a labor income process to the model and the qualitative results do not change. Introduction of labor income to portfolio choice models was somewhat trivial if the following two conditions are satisfied as in Merton (1971): 1) Labor income is treated as income from ***traded*** [3] human wealth, that is investors are allowed to borrow against their human capital. 2) Human wealth is assumed to be nontraded but traded assets provide perfect hedges against labor income. In other words, investors are able to fully insure

[3]The existence of nontraded assets can be the result of various market imperfections which may be caused by transactions cost, moral hazard, legal restrictions like capital controls. As explained in Svensson (1988), an individual cannot trade claims to his future wages (his human capital) for obvious moral hazard reasons.

their labor income risk. Note that labor supply was not endogenous in Merton (1971). Bodie, Merton and Samuelson (1991) extend that model by including leisure as a second good and hence endogenizing labor supply, but they assume that retirement occurs at a fixed age and that households can borrow against their future labor income. As I will explain below, both of these assumptions are dropped in this dissertation since they are unrealistic. They focus only on the relationship between labor supply flexibility and portfolio choice.[4] They find that flexibility of labor supply leads to higher shares of the risky asset in the optimal portfolio allocation.

The more recent articles solve for optimal portfolio allocation patterns with at least one of those assumptions relaxed. Some articles assess whether the observed household behavior is attributable to the more realistic models. Cocco, Gomez and Maenhout (2001) points out that many households cannot capitalize future labor income and hence face borrowing constraints due to moral hazard issues. Moreover, they also face uninsurable labor income risk since explicit insurance markets for labor income risk are not well-developed. The authors consider a finitely-lived investor facing mortality risk, borrowing and short-sale constraints, and receiving labor income. The agent can invest in a risky asset or a riskless asset. Using the PSID they estimate the labor income profile and its risk characteristics and then calibrate and solve numerically for the optimal portfolio and savings decisions. Stock returns are allowed to be correlated with labor income shocks. They find that the optimal share of stocks in the portfolio goes down as agents age and as wealth increases, in contrast to the findings of Samuelson and Merton. This is very similar to my model except in my model retirement is endogenous. I will show that if I take retirement as given then I obtain similar results as theirs, but treating retirement as a choice variable changes the monotonicity of the share of risky assets with respect to wealth and age.

Ameriks and Zeldes (2001) examine the empirical relationship between age and portfolio choice using new panel data from TIAA-CREF and pooled cross-sectional data from the Surveys of Consumer Finances. They document significant non-stockownership, wide-ranging heterogeneity in allocation choices, and the infrequency of active portfolio allocation changes.

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[4]They propose that labor supply flexibility can be measured by the number of adults in the household, or having an occupation that offers opportunities for working extra hours, taking extra jobs, or delaying retirement. Benitez-Silva (2003) tests their hypothesis using panel data from the HRS.

Most of the rest of the literature describing and analyzing the dependency of portfolio allocation on age has a common motivation: Solving the micro-level equity premium puzzle. There are patterns in data that are inconsistent with portfolio theory: There is a very sizable fraction of the U.S. population[5] that does not hold any equities. Papers in this literature have added uninsurable labor income (which may or may not be correlated with asset returns), permanent and temporary wage shocks, non i.i.d. asset returns, borrowing constraints, short selling constraints, fixed costs for stock market entry, transaction costs for stock trading, and others in attempts to try to solve the puzzle.

Using data from the PSID and other data sets Vissing-Jorgensen (2002) estimates a dynamic panel data model of stock market participation and equity share in portfolios controlling for unobserved heterogeneity and the endogeneity of initial conditions. She focuses on the effects of the first and second moments of non-financial income and costs of stock market participation on stock market participation and on equity shares in household portfolios. She finds that both of them contribute to the explanation of the puzzle. She finds evidence of a positive effect of mean non-financial income on the probability of stock market participation and on the proportion of wealth invested in stocks conditional on being a participant. Variance of non-financial income is found to have the opposite effect on those two variables. She also finds evidence of state dependence in the stock market participation decision, a result that is consistent with the theory that small fixed costs for stock market entry may deter stockholding. In her study, labor income is treated as given.

Haliassos and Michaelides (2001) show that the puzzle is robust to relaxations of the benchmark assumptions of Samuelson (1969). They find that assuming that there is a relatively small fixed costs for stock market participation may help explain the puzzle. They state that such costs can arise from informational considerations, sign-up fees, and investor inertia.

The timing of retirement, and thus, to some extent, labor income is not given. Households may adjust their labor supply as events unfold over the life-cycle. Social insurance programs such as Social Security,

[5] This is true for European countries too. See Guiso, Haliassos, and Jappelli (2001).

Disability Insurance, Medicaid, Medicare and the others have the potential to influence savings, retirement timing, and portfolio choice behavior because they affect the riskiness and amount of current and future income. Hence it is crucial to model these three decisions jointly. As I will demonstrate in the next section, optimal portfolio allocation can be very sensitive to plans about retirement timing and to the presence and characteristics of the Social Security.

3. Theoretical Model

The agent in the model has a finite horizon, T with discrete time t = 1,...,T. Each period corresponds to one year. The agent dies at T+1 with certainty and there is no risk of death prior to T+1. There is no bequest motive. Suppose T is 10.

There are three decision variables: Employment, consumption/saving, and portfolio allocation. There is one risk-free, and one risky asset over which the portfolio decision is made. Wage offers and portfolio returns are stochastic.

At the beginning of each period, the agent observes the return shock that applies to his assets carried over from the previous period and so determines his beginning-of-period assets. At the same time he observes the wage offer for current period employment. He then decides whether to be employed or not, how much to consume, and if he decides to hold financial assets, what the portfolio allocation is. Note that I do not consider the hours of work decision. Since there is one risky and one risk-free asset, I define his portfolio decision to be simply the ratio of his financial wealth held in the risky asset to his total financial wealth.

<u>Choice Variable 1: Employment</u>

$$j_t = \begin{cases} 1 & \text{employed in period t} \\ 0 & \text{not employed in period t} \end{cases}$$

Choice Variable 2: Consumption

$C_t = A_t + I_t - A_t^*$ where C_t is consumption, I_t is income, A_t is assets at the beginning of period t, and A_t^* is assets at the end of t.

Choice Variable 3: Portfolio Allocation

In the following definition, x_t is the fraction of financial wealth held in the risky asset. That is a measure of the riskiness of the portfolio.

$$x_t = \frac{Z_t}{Z_t + M_t} = \frac{Z_t}{A_t^*}$$

where $Z_t = x_t A_t^*$ is the amount of risky asset held and $M_t = (1-x_t)A_t^*$ is the amount of risk-free asset held.

RETURNS:

$$A_t = \underbrace{[r_t(1 - x_{t-1}) + z_t x_{t-1}]}_{R_t} A_{t-1}^*$$

The returns on the risky asset (z) and on the risk-free asset (r) are realized at the beginning of the next period.

$$z_t = \overline{z} + \theta_t \qquad \theta_t \sim \text{i.i.d.}N(0, \sigma_\theta^2)$$

r_t is fixed and non-stochastic

For simplicity, I assume that return shocks and wage shocks are uncorrelated.

INCOME:

If the agent works, he earns labor income that depends on experience. If the agent does not work, he may be eligible for pension payments that depend on his labor market experience. In the current model, an agent cannot collect benefits and work simultaneously. Hence income is

$$I_t = j_t W_t + (1 - j_t) B_t$$

where labor income is $W_t = \beta_0 + \beta_1 e_t + \eta_t$ where e_t is experience at the beginning of period t, and η_t ~ i.i.d. $N(0,\sigma_\eta^2)$.

Social Security/Pension Income is deterministic and collected only if not employed:

$$B_t = B(e_t, t)$$

In the simulations, I set B(e,t) = 0 for e<6; B(6,t) = 5, B(7,t) = 10, B(8,t) = 12, and B(9,t) = 15. Benefits are in $000's. Note that this schedule is a crude approximation to the Social Security benefit rules by requiring a minimum number of work periods for any benefit, providing a relatively low benefit for early retirement (t = 7, e = 6), and increasing benefits with experience beyond the experience required for normal retirement (t=8, e=7). Social Security taxes are not modeled.

There is a consumption floor $\overline{C}$, which is provided by welfare programs to guarantee $\overline{C}$ units of consumption if the agent does not have enough cash-in-hand (assets + income).

if $A_t + I_t < \overline{C}$, then

$$C_t = \overline{C}$$
$$A_t^* = 0$$

Hubbard, Skinner, Zeldes (1995) show that the presence of means-tested social insurance policies designed to maintain consumption has a large negative effect on saving for lower-lifetime-income groups.

CONSTRAINTS:

For $A + I \geq \overline{C}$	For $A + I < \overline{C}$
$C \geq 0$ $A^* \geq 0$ $x \in [0,1]$ $A_1 = A$	$C = \overline{C}$ $A^* = 0$ $A_1 = A$

x in [0,1]: no short-selling of the risky asset; no borrowing

$\Rightarrow$ $Z \geq 0$, and $M \geq 0$.

where Z is the amount of the risky asset and M is the amount of the risk-free asset.

A in the above equations is a given constant. I used A = 0 in my simulations, hence I assume that individuals start period 1 with no assets.

UTILITY FUNCTION:

$$U_{jt} = \frac{C_{jt}^{1-\alpha}}{1-\alpha} + (\gamma_0 + \gamma_1 t) j_t$$

CRRA with parameter α

Most of the studies use CRRA utility functions including the seminal article of Samuelson in 1969. Choosing the same type of function facilitates the comparison of my results to the other results documented in the literature.

Being employed gives a negative utility to the agent ($\gamma_0 < 0$), which increases with age ($\gamma_1 < 0$). Dislike for work is the reason agents retire in this model. Inclusion of disutility of work provides a motive for the agents to save: saving for retirement. At some point in time they will choose not to work anymore because of the disutility of working, and they will save for retirement since they are forward-looking. They will also save some amount due to precaution and intertemporal substitution.

The utility function is additively separable in utility from consumption and leisure. It is straightforward to modify the function so that it becomes non-separable.

Agents maximize the expected present discounted value of lifetime utility by choosing j (employment), C (consumption), and x (portfolio mix) at each period t from 1 to T. They discount the future with the discount factor δ.

3.1. Solving the Model

The model is solved by backward recursion starting from the last period, T. Since there is no analytic solution of the model I solve it numerically. There are 3 state variables: t (age), e_t (beginning-of-period experience), and COH_t (beginning-of-period cash-on-hand). Cash-on-hand is defined as assets plus income (either labor or non-labor). If both of them were serially correlated, then we would have to have 2 additional state variables: the wage shock and return shock. As a special case, I also solve the model assuming that there is no Social Security and that wages do not depend on experience. Then, experience is no longer a state variable.

Starting in period T, going backwards to period 1, the model must be solved at every point in the state space, that is, for every feasible combination of t, e, COH, and j[6]. However, since COH is a continuous variable,

[6]Note that, actually, j is not a state variable, yet it is treated as if it is one. That is done to save computation time. Instead of looping over beginning-of-period assets ***and*** labor income, I only loop over cash-on-hand. The downside is that I have to loop over employment too, because given cash-on-hand we do not know whether the agent is employed or not. Let's say

I discretize it as COH[n], n = 1,...,N. I partitioned the interval of feasible cash-on-hand into 200 subintervals. The lower end point of the interval is 0. To calculate the other end point of the grid, I calculate the maximum possible cash-on-hand. I find this number by assuming that the agent receives the highest wage shock (out of 10,000 draws) at current and all previous periods, then saves it all in the form of risky assets and gets the highest possible return (out of 10,000 draws) in the current and all previous periods. Then I divide this range into 200 intervals that are smaller towards the lower end but get larger towards the upper end.

It is very unlikely that the agents will have cash-on-hand that is close to the maximum possible one. Hence the grid is set in such a way that there are more intervals close to the left end point, 0, that is, the grid is coarser toward the end. Whenever I need to calculate the value of a variable (for example, value function) at some COH value, which is not at one of COH grid points, I use interpolation.

<u>Period T:</u>

In the last period, the model is solved at every point in the state space (that is, for all feasible combinations of t, e, n, and j). Since there is no bequest and the agent is going to die at the end of this period with certainty, there is neither a savings nor portfolio decision to make. Optimal consumption equals cash-on-hand.

<u>Period T-1:</u>

For each e, n, and j we calculate the consumption and portfolio mix that maximizes the value function:

$$\boldsymbol{V}_{\boldsymbol{T}-1}^{\boldsymbol{j}_{\boldsymbol{T}-1}}(\boldsymbol{e},\boldsymbol{n}) = \max_{\substack{\boldsymbol{C}_{\boldsymbol{T}-1}\in[0,\boldsymbol{A}_{\boldsymbol{T}-1}+\boldsymbol{I}_{\boldsymbol{T}-1}] \\ \boldsymbol{x}_{\boldsymbol{T}-1}\in[0,1]}} \{\boldsymbol{U}_{\boldsymbol{T}-1}^{\boldsymbol{j}_{\boldsymbol{T}-1}}(\boldsymbol{C}_{\boldsymbol{T}-1}) + \delta \boldsymbol{E}_{\boldsymbol{T}-1}[\max_{\boldsymbol{j}_{\boldsymbol{T}}} \boldsymbol{U}_{\boldsymbol{T}}^{\boldsymbol{j}_{\boldsymbol{T}}}(\boldsymbol{C}_{\boldsymbol{T}})]\}$$

where,

there are N grid points for COH (asset+income), P grid points for income (if it was used as a state variable), and 2 points for j. As long as Nx2 is smaller than NxP, using only COH as a state variable will save significant computation time.

$$C_T = \max\{A_T + I_T, \overline{C}\}, \qquad \boldsymbol{A_T} = [\boldsymbol{r_T}(1-\boldsymbol{x_{T-1}}) + \boldsymbol{z_T}\,\boldsymbol{x_{T-1}}]\boldsymbol{A^*_{T-1}},$$

$$I_T = j_T W_T + (1-j_T)B_T, \qquad A^*_{T-1} = A_{T-1} + I_{T-1} - C_{T-1}.$$

The last term in the above optimization expression is the discounted expected value of last period value function in T-1. It is,

$$E_{T-1}V_T = E_{T-1}\max_{j_T} V_T^{j_T} = \iint \max_{j_T}\{V_T^{j_T}\} f(\eta_T, \theta_T)\, d\eta\, d\theta$$

where f(.,.) is a joint probability density function. This integral is evaluated using monte carlo simulation. $V_T^{j_T}$ is a function of the period-T shocks, and the previous period consumption and portfolio mix decisions. Once those decisions are made, we know the value of end-of-period assets that is carried over to period T. Then for a number of random draws from the distributions of wage and return shocks, we can calculate the amount available for consumption. Since agents consume all cash-on-hand in period T, we calculate the period-T-value function by plugging cash-on-hand in place of C. Averaging over all of the random draws of shocks, we obtain an approximation of the expected value of the period-T-value function. As the number of draws increases, the approximation converges to the true value.

The above algorithm is applied for an equally partitioned two-dimensional grid of consumption and portfolio mix values. [7] Then we search for the pair for which the T-1 value function is maximized. Hence, we obtain optimal consumption and portfolio decisions for each point in (e_{T-1}, n_{T-1}, j_{T-1}) space. I also calculate the percentage saved (variable ***sper***) out of cash-on-hand using the consumption values. The variable ***sper*** will be

[7] I use this simple grid search instead of solving the euler equations for the exact optimal values of C and X for the following two reasons: 1) To numerically solve the euler equation, we need to solve two highly nonlinear equations simultaneously. I have tried numerous algorithms and none of them produced reliable results. 2) Solving the model at every grid point and saving the results enables us to gain further insight into the dynamics of the model since we can graph the value functions for their entire domain. This feature of grid search and the use of Monte Carlo integration over distributions came in handy when I encountered seemingly odd decision rules such as jumps in optimal consumption and portfolio allocation.

used in order to interpolate for consumption during simulation. The results are saved to be used in period T-2 calculations and in simulations.

Although in similar studies the number of draws used in evaluating integrals is less than 100, I used 10,000 draws. Particularly, the portfolio decision is very sensitive to the number of draws used. There were substantial differences between the portfolio allocation solutions (not consumption or employment solutions) when I used 100, 1,000, or 10,000 draws.

Periods T-k, k = 2, … ,T-1:

As in period T-1, for each e, n, and j we calculate the consumption and portfolio mix that maximizes the value function:

$$V_{T-k}^{j_{T-k}}(e,n) = \max_{\substack{C_{T-k}\in[0,A_{T-k}+I_{T-k}] \\ x_{T-k}\in[0,1]}} \{U_{T-k}^{j-k}(C_{T-k}) + \delta E_{T-k}[\max_{j_{T-k+1}} V_{T-k+1}^{j_{T-k+1}}(C_{T-k+1})]\}$$

In contrast to value function calculations in T-1, for periods T-k, k>1, we do not readily know what the next period optimal choices are given current period optimal choices. In period T-1, next period is the last period, and we already know that optimal consumption is all of cash-on-hand and there is no portfolio allocation decision. In periods T-2 and earlier, however, we do not know the next period optimal decision for all the possible current period decisions. We have calculated and saved only a grid of them. We use an interpolation algorithm to find the values that are not on the grid points but somewhere in between. For the value function (especially for power utility functions), an interpolation algorithm based on weighted geometric mean proves to be “accurate”. For given trial values of consumption and X in T-k, I first find the cash-on-hand available in the next period. Then find the subinterval that includes this value. Then I interpolate for the value of the value function at this point by using the already calculated and saved function values at the end points of this interval. The distances of this point to the end points are used as weights in the weighted geometric average calculations.

I have tried a number of interpolation algorithms, and found out that this is most appropriate. This method is better than interpolation based on weighted arithmetic means because for power utility functions it can be shown that the absolute error for weighted geometric mean is always smaller than the one for weighted arithmetic mean. I have tried all the interpolation routines in the IMSL package and found out that none of them are “true” enough to the shape of this kind of utility function.

4. Simulations

Simulation of the solution of the model will be useful to get a sense of what kind of behavior the agents exhibit during their life courses. Analyzing the solution at each period separately will of course reveal the most amount of information about the model, but this is more tedious. So I left analyzing the solution to the next section. In this section, I only focus on the simulation of the solution keeping in mind that we get a limited insight into the model but a good overview of the model. I present 2 sets of simulation results: One without Social Security and one with Social Security. In the simulations, I tried to choose all the parameters in a way such that we have a chance to observe the roles played by the main features of the model.

Using the solution that is described above, the model is simulated for 10,000 agents who are identical at the beginning of the first period. I describe and explain the solution path for the decision variables over the life cycle.

During simulation, for the values of state variables that fall somewhere in between the grid points that were used in the solution, appropriate interpolation methods are used. First, given the beginning-of-period assets, return shock, and wage offer, cash-on-hand is calculated for both for employment and non-employment. Value functions are calculated for both values of employment. I use the same weighted geometric average interpolation algorithm that I used in the solution. Then, the employment value that gives a higher value to the agent is selected. Now that we know cash-on-hand, we go back to the solution where optimal consumption (or optimal ***sper***, i.e. percentage saved out of cash-on-hand) and portfolio decisions were calculated for the grid of all feasible cash-on-hand values. The ***sper*** value corresponding to the closest point on the cash-on-hand grid is the interpolated ***sper***. As will be explained later, true sper may display "jumps" at some cash-on-hand values. Those jumps are important (they are presented and explained below) and by interpolating sper using the simple scheme described above we preserve the jumps. Had I used a weighted average of the 2 closest points, I may have undermined the size of the jumps. We do not need to worry about the size of the error this method causes because sper is smooth except at those "jump" points, I use a fine grid, and using percentages instead of values already has a smoothing effect. Given ***sper***, we calculate optimal consumption. To find the interpolated portfolio mix, we find the portfolio mix value corresponding to the closest point on the cash-on-hand grid. That value is assigned as the optimal portfolio mix. The same method is applied in calculating the probabilities of future employment sequences.

Here is a chart that illustrates these steps more clearly:

Table 1: How the Simulation Is Done

Starting Values:	$A_1 = A_0^*$ $x_1 = x_0$ $e_1 = 0$		Starting values are given. In my simulations A_0^*=0 and x_0=0.5
For each t	θ_t η_t		Rate of return shock and wage offer shock are drawn
	$z_t = \bar{z}_t + \theta_t$ $R_t = r_t(1-x_t) + z_t x_{t-1}$ $W_t = \beta_0+\beta_1 e_t+\eta_t$		Rate of return on the risky asset is determined Overall portfolio rate of return is calculated Wage offer is calculated
	For each j (employment status)	$I_t^j=W_t j_t+B(e_t,t)(1-j_t)$ $COH_t^j = A_t + I_t^j$ $V_t^j(COH_t^j)$	Income and Cash-on-hand are determined Value function values are interpolated using the results saved from solution
	$j_t = \begin{cases} 1 \text{ if } V_t^{j=1} \geq V_t^{j=0} \\ 0 \text{ otherwise} \end{cases}$		**Employment Status** is determined
	$sper(COH_t)$ $x_t(COH_t)$ $A_t^*=sper \cdot COH_t$ $C_t = (1 - sper).COH_t$		Percentage saved (sper) and **portfolio allocation** (x_t) are interpolated using the results saved from solution **End-of-period asset,** **Consumption are determined**

4.1. Simulations without Social Security

Horizon: T = 10

Constant relative risk aversion parameter: $\alpha = 4$

Time preference: 0.98 (~1/1.02)

Consumption floor: $\overline{C}$ = 1 (this is set to a very small value in order to concentrate on understanding how the model works without the complication that the consumption floor introduces.)

Wage function: $\beta_0 = 20$, $\beta_1 = 0$, $\sigma_\eta = 1$ (Average wage is $20,000, wages do not depend on experience, and the standard deviation of wages is $1,000. I am modeling the decisions of 50+ year olds and since their wage-experience profile should be relatively flat $\beta_1=0$ is not a bad assumption. Besides, I get rid of one state variable (experience) in the case where there is no social security since nothing depends on experience.)

Disutility of work: $\gamma_0 = -0.0004$, $\gamma_1 = -0.00002$ (These values are selected to be able to see realistic life-cycle patterns among the agents. If these were too high in absolute value, then the agent would work only if he receives an unrealistically big wage shock. In this case almost of them will be on welfare. On the other hand, if these were too low in absolute value then almost all of the agents would work all the time.)

Risky asset returns: $\overline{z} = 1.04$, $\sigma_\theta = 0.2$ (hence, most of the time: z is in [0.54,1.54]. There were a total of 100,000 draws in the simulations (t=10)x10,000, and the maximum realized asset return was 186% and the minimum realized asset return was –85%.

Risk-free asset return: r = 1.03 (The values selected for the asset returns are somewhat atypical compared to the similar studies. Usually the average rate of return is taken to be around 8% for the risky asset and around 2% for risk-free asset. I have chosen the returns of these assets to be unrealistically close to each other. Had I chosen them to be further away from each other almost all the agents would be investing 100% of their portfolios in the risky asset with such a low value for the constant relative risk aversion parameter (4). ***That is the equity premium puzzle.*** I get similar "puzzling" results if I set the equity premium high. Since analyzing life-cycle patterns for portfolio allocation is the main objective of this study, I set the means of the two assets to be close to each other.

Number of points in the cash-on-hand grid: 200

Since I have searched for the optimum for consumption and portfolio allocation in an equally partitioned grid (201 grid points for consumption in the interval (0,Cash-on-hand], and 101 grid points in the interval [0,1] for portfolio allocation, we know that the errors cannot be greater than twice the distance of neighboring grid points:

Maximum possible error for consumption solution (C): 0.5% of cash-on-hand

Maximum possible error for portfolio allocation solution (X): 0.01 (1%)

Number of draws for monte carlo simulations: 10,000

Initial Assets: 0

There is no serial or cross-sectional correlation within or among the error terms.

Below, Figures 1,2, and 3 show the paths of the important variables: employment rate, average consumption, assets, income, and portfolio allocation by age.

4.1.1. Retirement

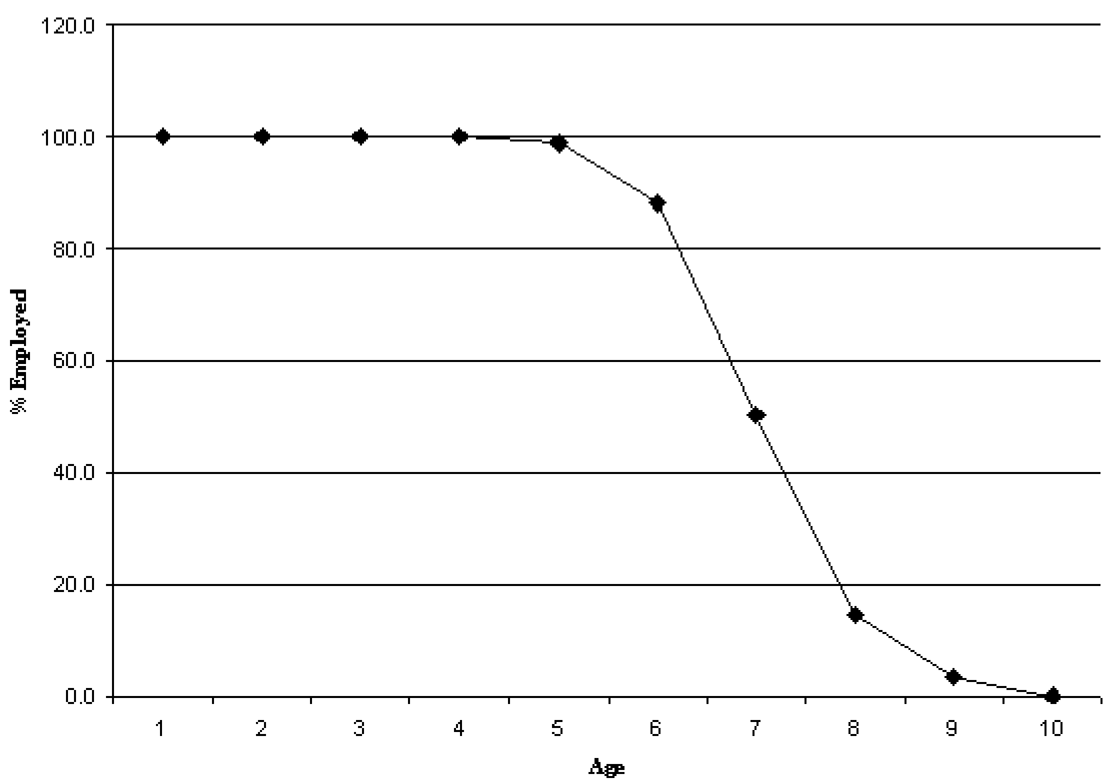
Retirement patterns show a great deal of variation although most of the simulated agents follow a "normal" life-cycle employment sequence: They work until a certain age and they do not work thereafter. Some of the agents, however, become temporarily unemployed after they work for a while, and then they go back to employment to retire at a later age. This pattern is mostly seen after period 5, because in the early periods, no matter how low wages they draw, they do not have the "luxury" not to work for a while since they have not yet accumulated assets to be substituted for labor income. As Table 2 shows, around 40% retire at period 8, 34% retire at period 7. 9.61% of them are temporarily unemployed at period 7, employed the next period, and then retire at period 9. The agents are all identical except that they may realize different wage and return shocks along their lives, and hence retire at different ages. In the finance literature studying household portfolio choices, it is always assumed that retirement age is fixed. As we can see in this model (and in the real data), it is not true. As will be made clear shortly, portfolio allocation decisions are sensitive to plans about retirement. If the agents' retirement patterns show such variability then a significant amount of variation in the household portfolios in data could be attributed to different retirement patterns. Hence assuming retirement age to be given may not be very appropriate.

Table 2: Timing of Retirement without Social Security

Employment Sequence	As a percentage of the population
1111111000	39.69
1111110000	33.63
1111110**1**00	9.61
111110**1**000	5.61
11111100**1**0	2.25
1111100000	2.22
111110**11**00	2.05
1111111100	1.25
1111100**1**00	1.25
Other	2.44

In Figure 1, we see that employment rate is 100% in the first 3 periods, and then it falls gradually to 88% in period 6. Then we see a sharp decline: 50.3% in period 7, 14.6% in period 8, 3.6% in period 9, and 0.1% in period 10.

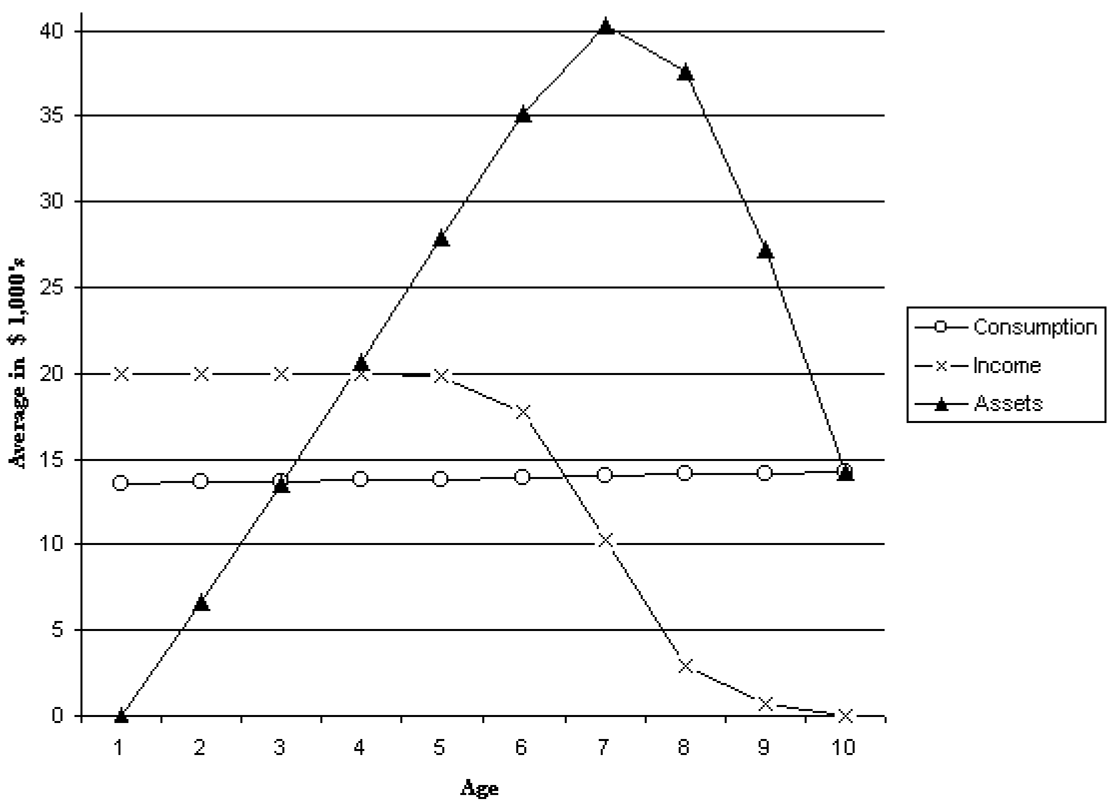
Figure 1
Employment Rate by Age



4.1.2. Consumption and Assets

Consumption grows with age since the average rate of return on assets (between 1.03 and 1.04, depending on the portfolio composition) is higher than the rate of time preference (1.02). As a typical life-cycle model would suggest, the agents accumulate wealth while they are working and decumulate wealth during retirement. (See Figure 2)

Figure 2
Average Consumption, Income, and Assets by Age



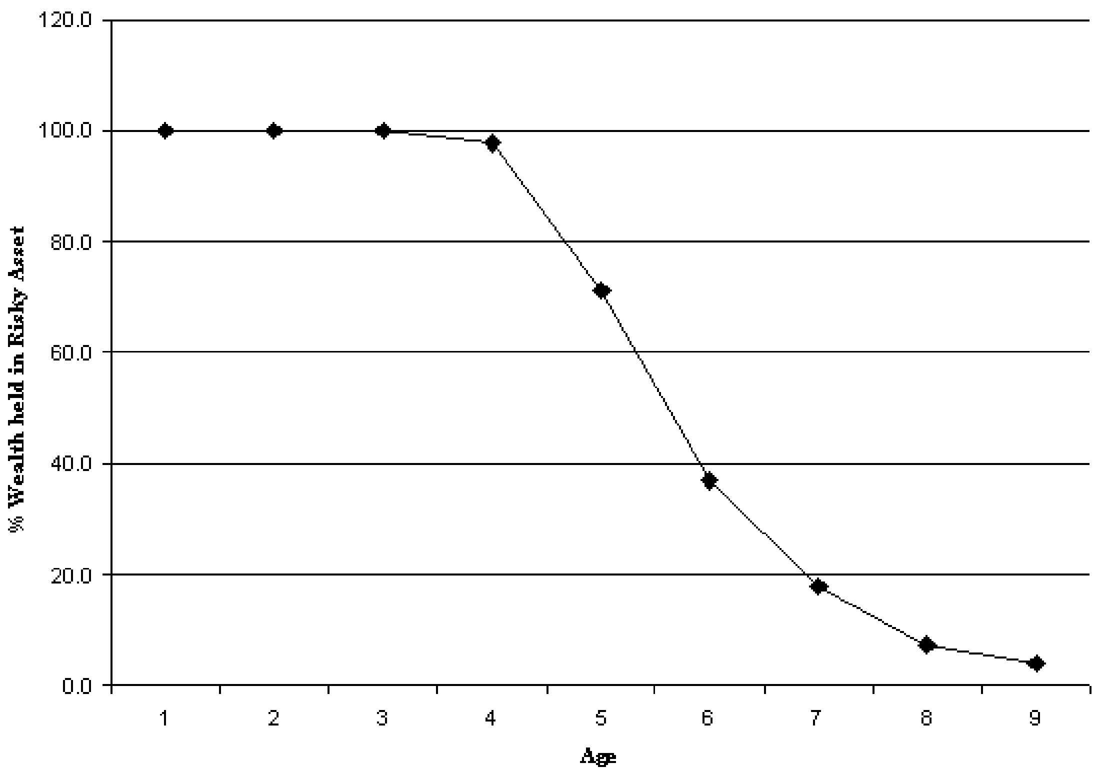
4.1.3. Portfolio Allocation

As we will see more clearly in the next section where solution results are presented, portfolio allocation depends on current cash-on-hand and expected future labor income. If cash-on-hand is very low, then the agents may prefer not to save, and we have no portfolio decision. If the agent decides to save, then there are a couple of ways the magnitude of cash-on-hand affects portfolio choice. As explained in Bodie, Merton, Samuelson (1991), total wealth can be thought of as the sum of human wealth and financial wealth. Human wealth is the expected total future labor income and financial wealth is the financial assets held. The relative magnitudes and risks of these two portions of wealth determine current portfolio choice.[8]

Figure 3 shows the evolution of the average percentage share of risky assets in total financial wealth. Portfolios are composed of 100% risky asset in the first 3 periods. Starting with the 4th period, we see a continuous decline until death. This is consistent with the well-known advice given by portfolio managers to individual investors: Hold a higher percentage of risky assets before retirement, and monotonically decrease the percentage of risky assets after retirement. The rationale behind that advice is the following: If the expected future labor income is relatively less risky compared to risky assets held, or if the correlation between future labor income shocks and return shocks is not significantly positive, then in the earlier stages of life total expected lifetime wealth has a big less risky component. As the agent approaches the end of the life cycle, expected labor income, that is, his less risky component of total expected financial wealth, shrinks. Hence the agents should hold riskier portfolios early in the life cycle and less risky portfolios towards the end of the life cycle. In the earlier periods of the life-cycle the agents can mitigate the effect of any big adverse return shock by dissipating its effect to longer future time periods (i.e. by working longer). On the other hand, in the later stages of life, there are not many periods left.

[8]For simplicity of interpretation I select the parameters in such a way that human wealth is "less risky" than financial wealth. We can easily obtain the results where wage offers are drawn from a distribution that has much bigger variance.

Figure 3
Average Portfolio Allocation by Age



4.1.4. Behavior of Groups of Various Retirement Ages:

I have picked the most populous groups in Table 2 and graphed their consumption, saving, income, and portfolio allocation paths. These groups are named as follows:

Ret8: The group that consists of simulated agents who work for the first 7 periods, then retire at period 8 and never return to work. This group constitutes around 40% of the population.

Ret7: The group that consists of simulated agents who work for the first 6 periods, then retire at period 7 and never return to work. This group constitutes around 34% of the population.

Ret7Back8: The group that consists of simulated agents who work for the first 6 periods, then temporarily retire at period 7, go back to work at period 8, and finally retire at period 9. This group constitutes around 10% of the population.

In the Figures 4, 5, 6, 7 and 8 below, I depict consumption, assets, income, return shocks, and portfolio allocation in order to demonstrate how identical agents exhibit different behaviors depending on the shocks (particularly on return shocks since I have kept the standard deviation of wages very small).

In Figure 4, we see that consumption drops in the period where Ret7 agents retire. This is not because of a strong adverse shock (See Figure 7). These agents are lucky in the sense that they received big positive return shocks throughout the first 6 periods, and they were able to save more than the other groups (see Figure 5). Having a lot of savings, realizing an above average return shock in period 7, and the desire not to work contributes to their decision that they would sacrifice some current consumption in order not to work thereafter. Almost the opposite scenario applies to Ret8 and Ret7Back8 so that they cannot afford to retire, and they have to work one more period. Once they decide to work, they have around 20 thousand dollars more, and hence we see a big increase in consumption. The difference between Ret8 and Ret7Back8 is that the latter group has a little more assets and they temporarily retire at period 7 only to receive a big adverse return shock in period 8 and go back to employment.

In Figure 8 we see that the Ret7Back8 group has a higher share of risky assets in their portfolio at period 6, and especially at period 7. Even though Ret7Back8 group has almost same amount of cash-on-hand as the Ret7 group, they decide to hold more risky financial wealth in period 7 compared to the Ret7 group. Why? That is because they are more likely to work in period 8 than the Ret7 group. They do not have enough assets saved to withstand a big adverse return shock in period 8. But it is worth putting a little bit more money on the risky asset with the hope that a good return will allow them to retire sooner (if they get a good return they would have been in the Ret7 group).

Figure 4
Average Consumption by Age
by Groups of Various Retirement Ages

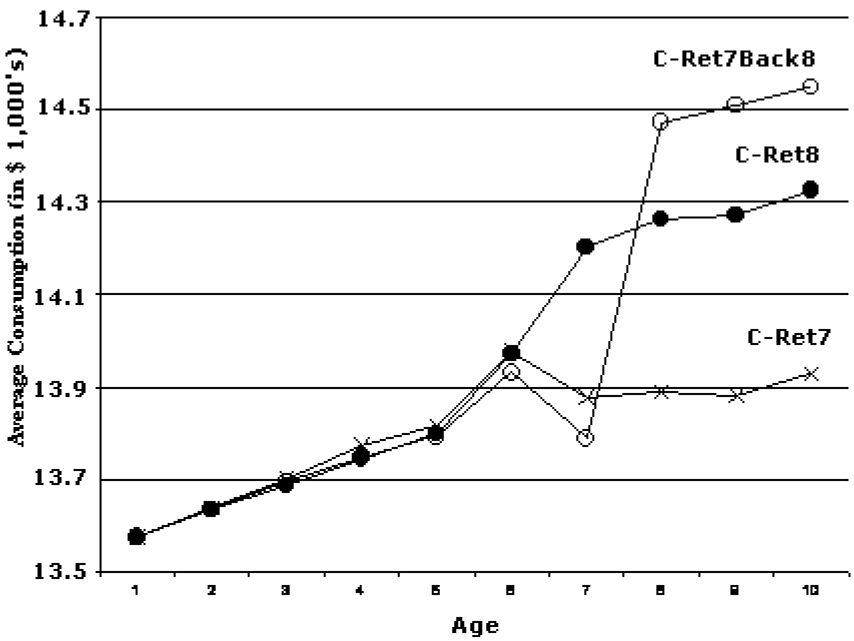


Figure 5
Average Wealth by Age
by Groups of Various Retirement Ages

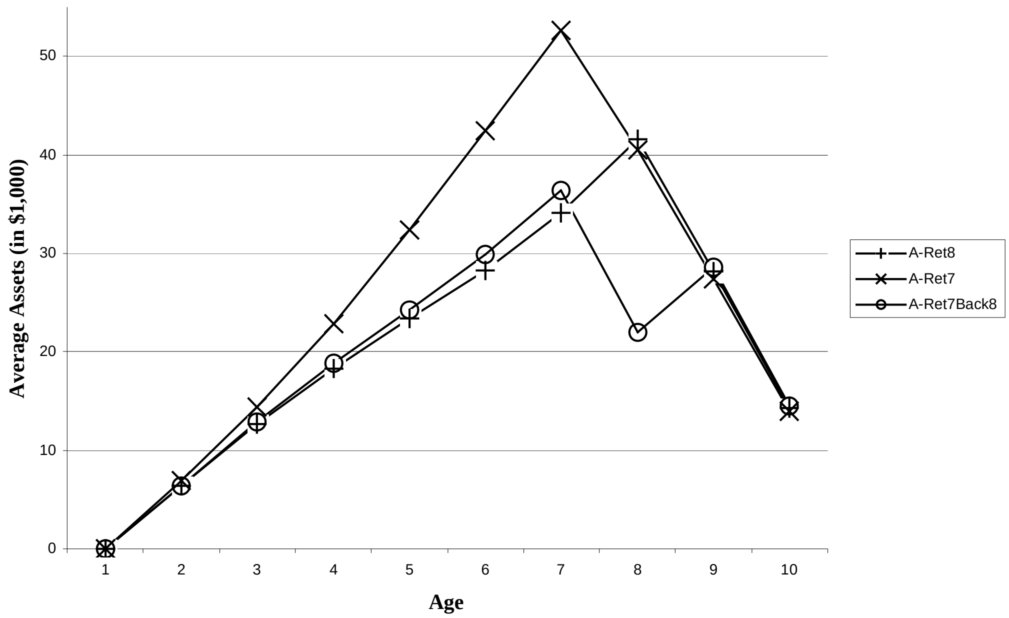


Figure 6
Average Incomes by Age
by Groups of Various Retirement Ages

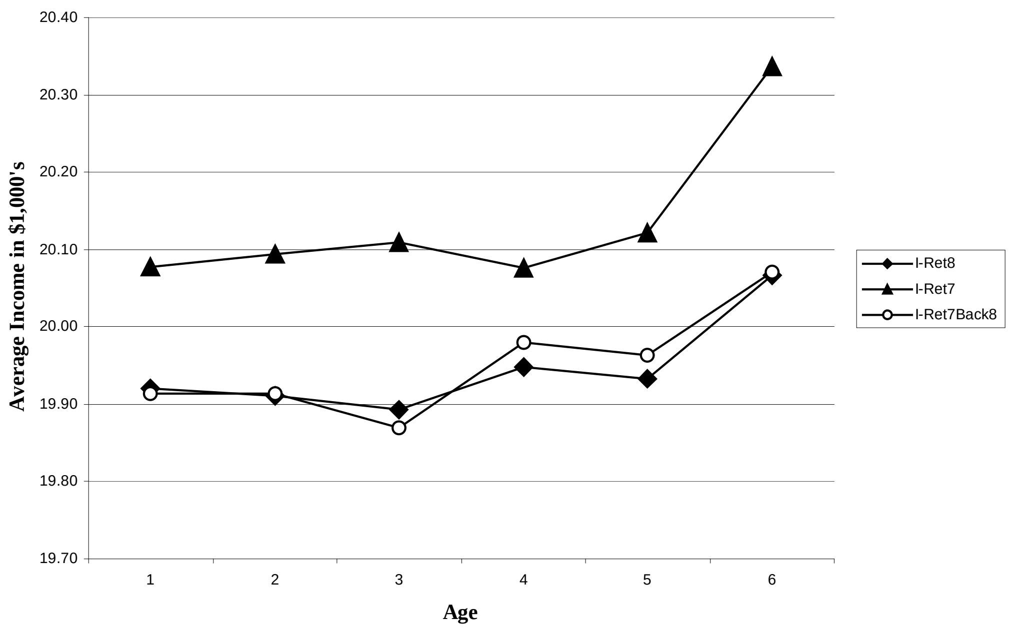


Figure 7
Average Returns on Risky Asset Holdings by Age
by Groups of Various Retirement Ages

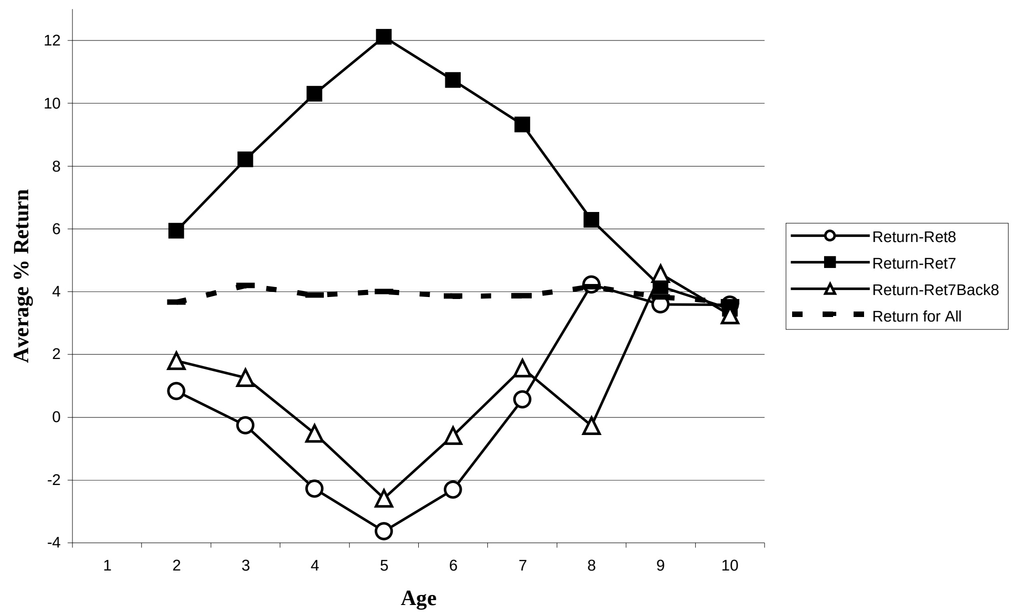
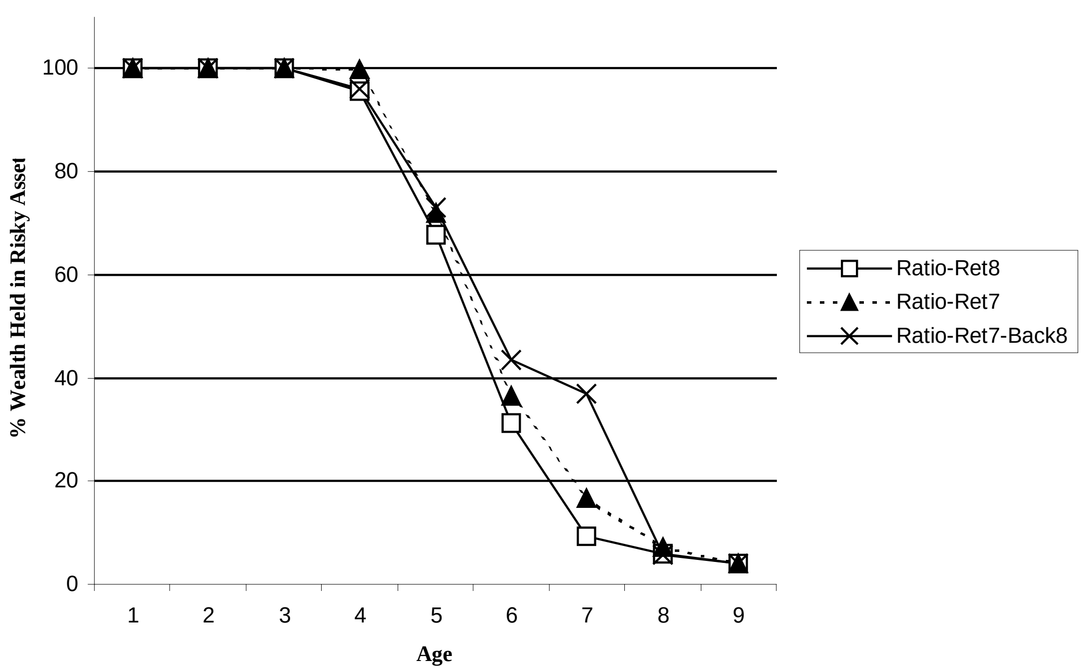


Figure 8
Average Portfolio Allocation by Age
by Groups of Various Retirement Ages



4.2. Simulations with Social Security

I use the same parameters as simulation 1 other than the social insurance parameters. I used the following benefit schedule:[9]

t\e	0	1	2	3	4	5	6	7	8	9
1	***0***									
2	***0***	***0***								
3	***0***	***0***	***0***							
4	***0***	***0***	***0***	***0***						
5	***0***	***0***	***0***	***0***	***0***					
6	***0***	***0***	***0***	***0***	***0***	***0***				
7	***0***	***0***	***0***	***0***	***0***	***0***	***5***			
8	***0***	***0***	***0***	***0***	***0***	***0***	***5***	***10***		
9	***0***	***0***	***0***	***0***	***0***	***0***	***5***	***10***	***12***	
10	***0***	***0***	***0***	***0***	***0***	***0***	***5***	***10***	***12***	***15***

Table 3: Timing of Retirement with Social Security

Employment Sequence	Percent
1111111000	9.54
1111110000	90.10
1111110100	0.36

Table 3 shows that employment patterns are more regular with Social Security because of the incentives to retire at certain times provided by the system. Around 90% of the agents retire at period 7, when they are eligible for a relatively small early retirement benefit.

Figure 9 shows average consumption and saving paths. We see that this social insurance program depresses savings, and average consumption in the Social Security case is higher by about $1,000 at every point in time compared to the one with no Social Security.

[9] Note that this is not an actuarially fair system since the agents become eligible for benefit even though they do not contribute at all.

Figure 10 shows that until period 8, optimal portfolios are more risky without the social insurance system. One factor contributing to this should be that for 90% of the simulated agents future income in the simulation with Social Security is 5 for every period after 6, and that is small relative to assets held. Hence the inherent risk in total wealth is bigger in simulations with Social Security, and thus less risky portfolios are optimal. In the last 2 periods optimal portfolios in the SS case are a little more risky than the ones in no SS case. That could be because there is almost no chance to be working in the last 2 periods in no SS case. Hence there is less non-risky human wealth remained in no SS case compared to SS case. Thus agents may be holding less risky portfolios in no SS case compared to SS case.

Figure 9
Average Consumption and Assets by Age
with and without Social Security

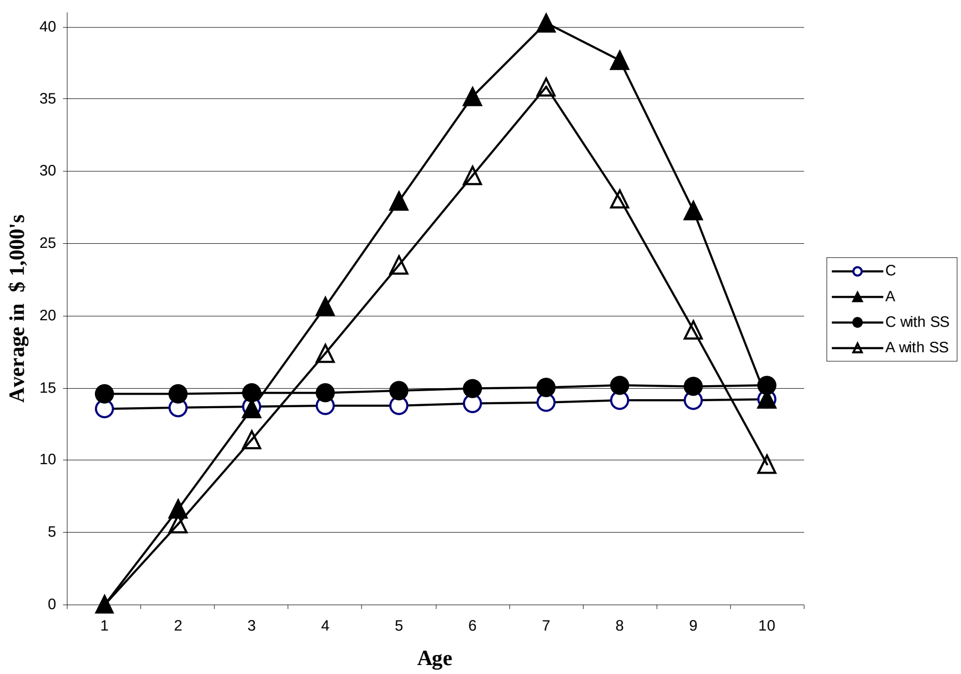
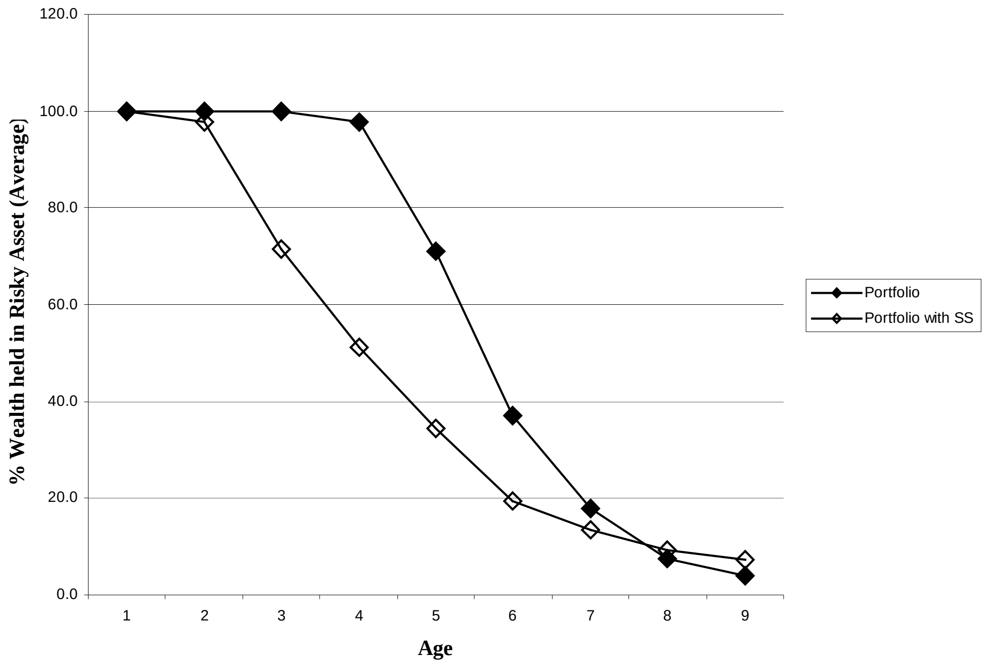


Figure 10
Average Portfolio Allocation by Age
with and without Social Security



5. Analyzing the Solution in More Detail

Analyzing the solution of the model we can get a more detailed idea about how the model works. In the literature, generally simulations are presented but examining the solution patterns gives the most detailed information on the model dynamics.

The following 3 graphs depict how the solution (optimal consumption and portfolio allocation) changes with respect to cash-on-hand in the last three periods (period 9, 8, and 7 respectively). For period 9, I also show the value function to point out its non-concavity.

Figure 11

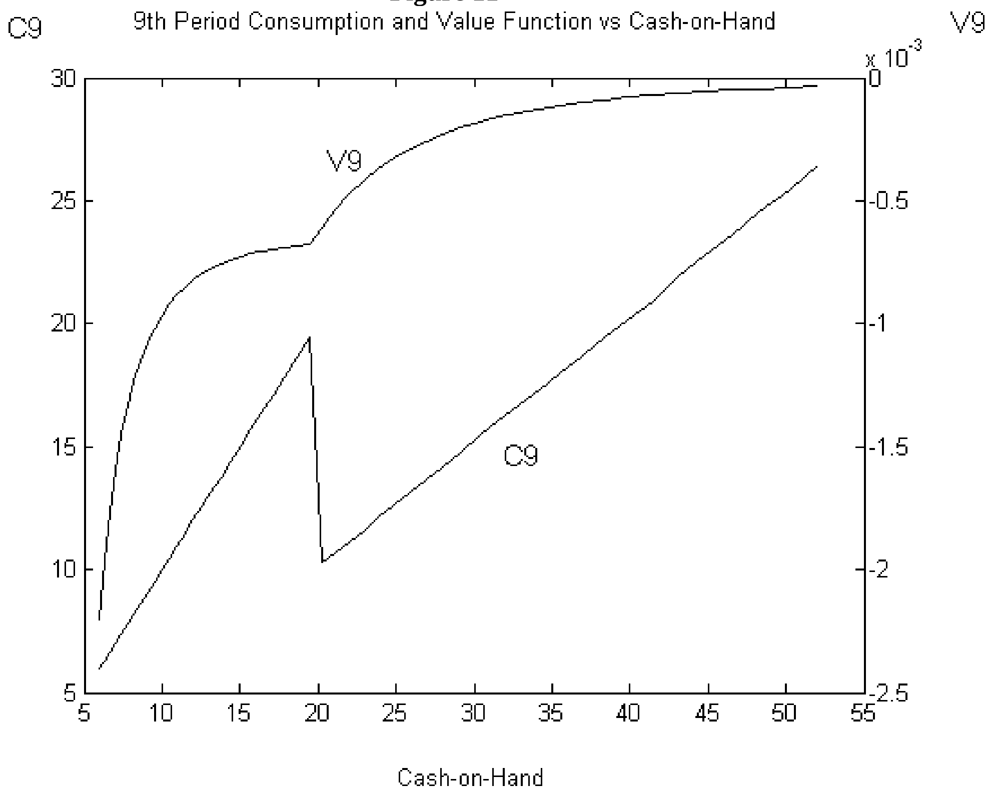


Figure 12

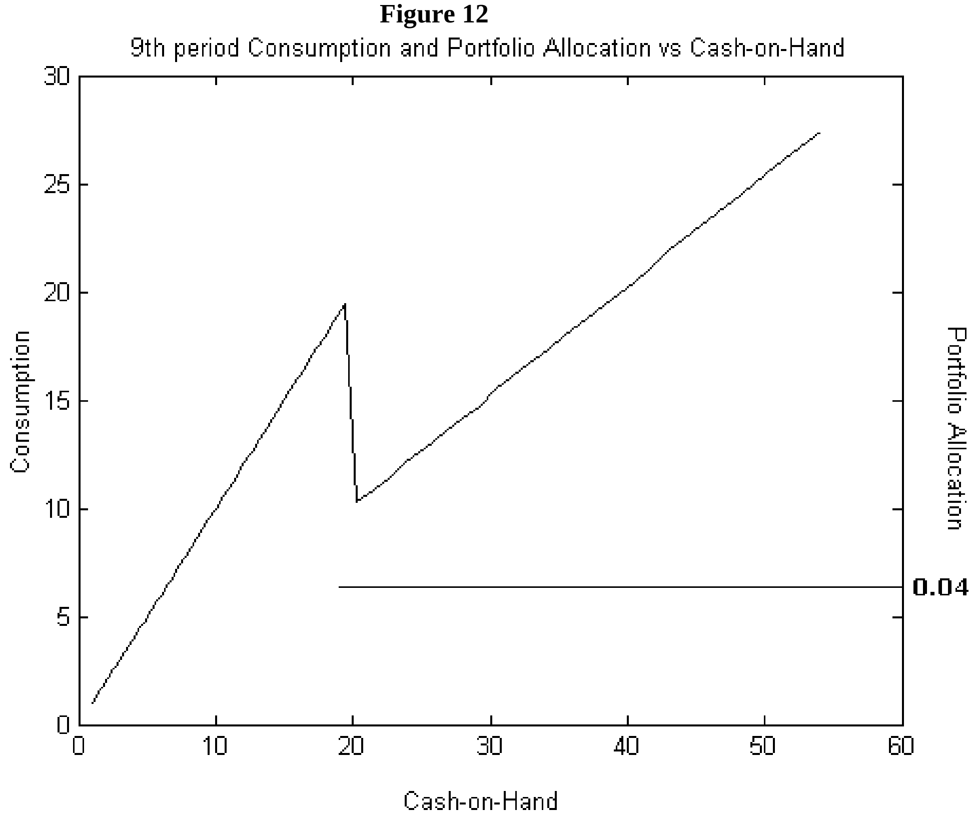


Figure 13

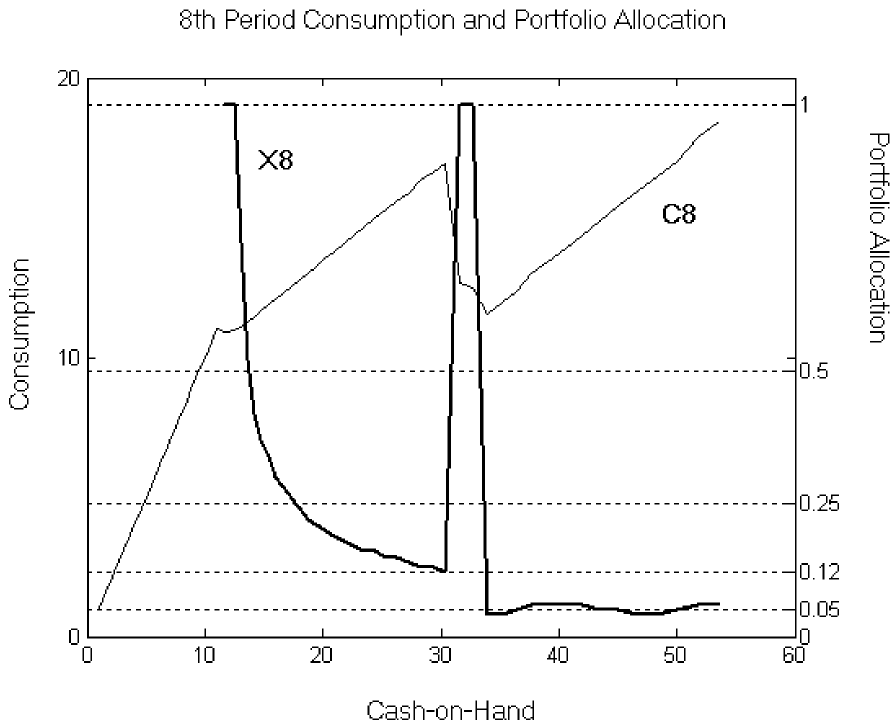
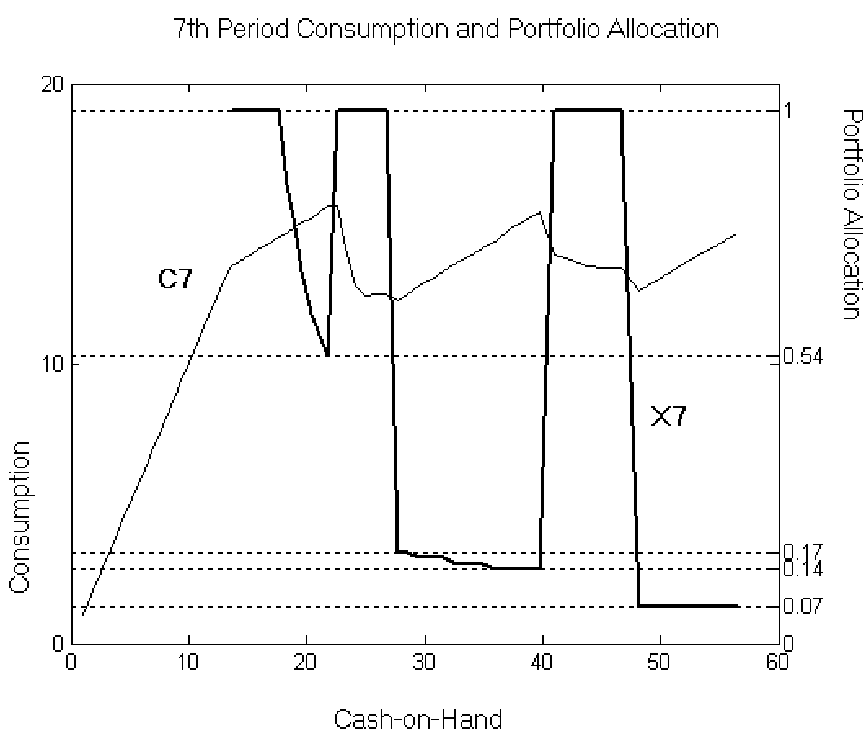


Figure 14



When we examine the graphs we make the following observations:

1) Consumption: There are abrupt consumption drops. The number of consumption drops is equal to the number of periods left.

2) Portfolio Allocation: There is a general downward trend, and there are some spikes in between. The number of spikes is same as the number of periods left.

I will show that the reason behind the jumps in the portfolio allocation and consumption is the "sudden"[10] changes in expected future income.[11]

The following two factors have important roles in the sudden big drops in consumption: 1) The strong desire not to work: Individuals either work or don't work. Once they decide to work, they face a big negative utility loss due to the additive marginal disutility of work component of the utility function. 2) The utility function, which is very steep at low levels of consumption and flat at higher levels of consumption. So, although the benefit of working (through increased income) can be very low at relatively higher levels of consumption, the utility cost of working may be very high, which does not depend on consumption level. Hence, the agents are willing to forego a big chunk of consumption today in order to afford not to work next period.

To illustrate that reasoning let us consider Figure 12 and trace optimal consumption relative to cash-on-hand. For small values of cash-on-hand (up until about $19,500) the agent does not save at all. He consumes everything. That is because the cash-on-hand he has is too small to maintain minimal consumption levels in period 9 ***and*** period 10. Since the utility function is very steep at those relatively small levels of consumption,

[10]The word "sudden" is used in the sense that for a small change in cash-on-hand, we may see abrupt changes in optimal consumption and portfolio allocation.

[11]We will see that the sudden changes in portfolio allocation and also in consumption happen at the points where the probability of future employment dramatically changes. The graphs of probabilities of future employment are provided in the upcoming pages.

he realizes that he will have to work next period: The penalty for not working next period (that is, getting very low levels of utility from consumption in this and the future period) is even worse than the stiff utility penalty for working next period. Since he knows he will be working next period, there is no need to save now. Enjoy consumption today, and tomorrow's consumption will be completely financed by tomorrow's labor income.

If we keep increasing cash-on-hand, at some point the agent will have enough cash-on-hand to afford not to work next period. At that point savings has to jump from 0 to a not-so-small positive number since, due to the shape of the utility function, miniscule levels of next period consumption cannot be optimal.

To help understand the patterns in portfolio allocation solutions consider the following two possible cases:

Case 1: Corner Solution. Financial wealth can be a very small fraction of total wealth (especially in early periods). Then the total wealth is dominated by human wealth. If future labor earnings are not very risky, then the overall riskiness of total wealth may be too low and the agents may find it optimal to invest all their financial wealth in the risky asset.

Case 2: If the share of financial wealth in total wealth gets bigger, and we keep everything else equal, then we would expect a smaller percentage of risky assets held compared to the previous case.

If we examine the solutions carefully, we can see these two effects in play both within a period and across periods.

Within a given period, changing cash-on-hand changes the share of financial wealth in total wealth. For the moment let's focus on each smooth part of the portfolio allocation that are between the spikes separately. All of them are smoothly declining. Explanation: If the expected future employment sequence is roughly constant, then increasing cash-on-hand (hence increasing the more risky component of wealth, financial wealth) should lead the agent to decrease the share of risky assets in his portfolio.

Now notice that the lowest point of each smooth part (in between the spikes) is lower than its counterparts, which are to the left. Explanation: The magnitude of cash-on-hand determines the probabilities of expected future employment sequences. If we keep increasing cash-on-hand, at some point, the forward looking agent will realize that he may have enough cash-on-hand to retire one period early. At this point we may observe a fast change of probabilities of future employment sequences. Portfolio allocation responds to this change of future employment probabilities: If it is likely now that the agent is going to work for one less period, then he has to hold a less risky portfolio since the share of risky wealth in total wealth has grown.

Across periods: The same level of cash-on-hand implies different portfolio allocations in different periods simply because the composition of the total changes over time.

Now it is time for the spikes: Above, I have described some of the main channels through which portfolio allocation is affected. The results that we see are far richer than that. The interaction between the magnitude of cash-on-hand and probabilities of future employment creates possibilities for optimal portfolio allocation to vary a lot. For example in addition to the cases described above, think of the "gray" areas where the agent has such an amount of cash-on-hand such that he is neither very likely to work nor very unlikely to work in a future period. In those cases the agent may gamble now for a chance to avoid working next period. He may prefer to hold a riskier financial portfolio hoping that, next period, he may get returns high enough that he will not have to work. If he is lucky he will get a huge utility boost. If not, he may not "suffer" too much in the sense that it was already probable that he would work next period, and now he will have more cash-on-hand (because of the labor income) to be spent on consumption.

In the light of the explanations above, now let's reexamine the portfolio allocation decisions by looking at the above 3 figures: Portfolio allocation (x) versus cash-on-hand in period 9, period 8, and then 7:

In period 9, x is constant just the way it is in the simplest form of dynamic portfolio choice model of Samuelson. With the constant relative risk aversion utility function, a fixed proportion of wealth is invested in the risky asset. Our model is more general than his, but period 9 is a special case since the future horizon is only 1 period and the agents are certain that they are not going to work if they decide to save.

Note that for all periods the number of consumption drops is same as the number of periods left. That is not a coincidence. At those points where consumption drops abruptly, probabilities of future employment change abruptly too. Once the agent thinks that saving more aggressively allows him to retire one period earlier, he is willing to take a big consumption drop. This is very clear in period 9 since only 1 period is ahead. But, the earlier periods we analyze are more complicated since there is now interaction between possible future employment sequences.

In periods 8 and 7 we see two things: The optimal share of risky assets is a monotonic decreasing function of cash-on-hand, and the optimal share suddenly jumps to a higher value two times in period 8 and 3 times in period 7. Again, this is because of the way the probabilities of future employment sequences change. Consider period 8: For small values of cash-on-hand agents do not save. For a little higher value agents save and plan to work for the next 2 periods. If they have a little more cash-on-hand they may plan to work for 1 period less, and finally if they have a huge amount of savings they figure they will never work. These are where we see the jumps in x.

To see whether the jump in consumption is the result of using a "not fine enough" grid, I solve the model with 10 times more grid points for cash-on-hand, that is I use 2,000 grid points rather than 200. We still see the jumps in consumption and portfolio choice.

To better understand the reason behind the jump, I calculate and report the probability of next period employment at period 9. I report this for period 9 because at this period there are only two possible future employment patterns: work and not work. If I were to use previous periods, then the number of possible future employment patterns increase exponentially. As I have explained previously, the reason of the jump is the "sudden" big changes in the probability of next period employment. I report the relationship between the probability of next period employment and current cash-on-hand. Note that the probability is endogenous in that it is calculated after the agents optimally choose consumption and portfolio allocation given cash-on-hand. We will see below that as cash-on-hand increases, at some point the probability jumps down from 100% to 0%. To help visualize how that probability jumps, I also present the relationship between the probability of next period employment and consumption-portfolio allocation pairs at a given cash-on-hand. There we will see, in some regions, how extremely sensitive that probability is to changes in either consumption or portfolio choice.

The way I calculate the probabilities is simple: Due to the nature of dynamic programming, we already had to take into account every possible future event in calculating optimal decisions. I just modified the program to keep track of whether the agent works or not in every case. Then I simply calculate the ratio of the number of cases in which the agent works to the number of total cases. The number of total cases is 10,000 since it is the number of draws I use in Monte Carlo integrations.

Figure 15

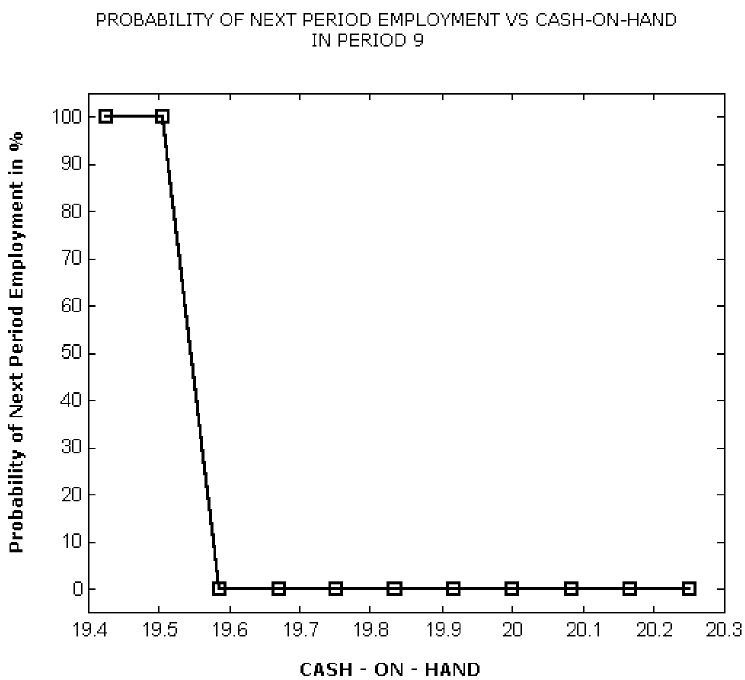
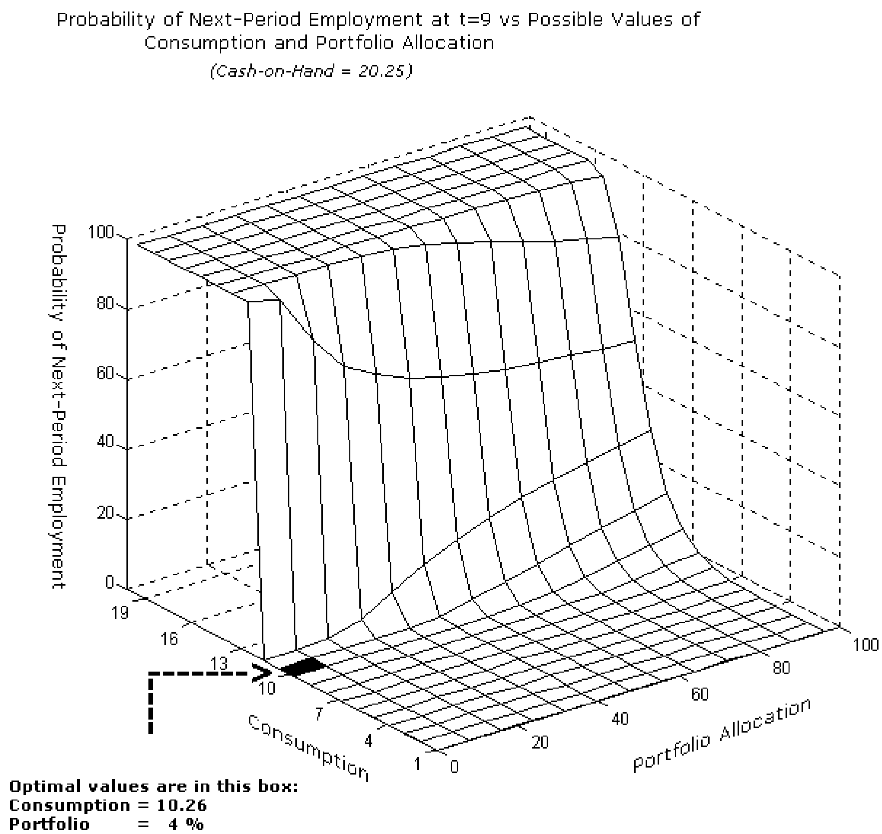
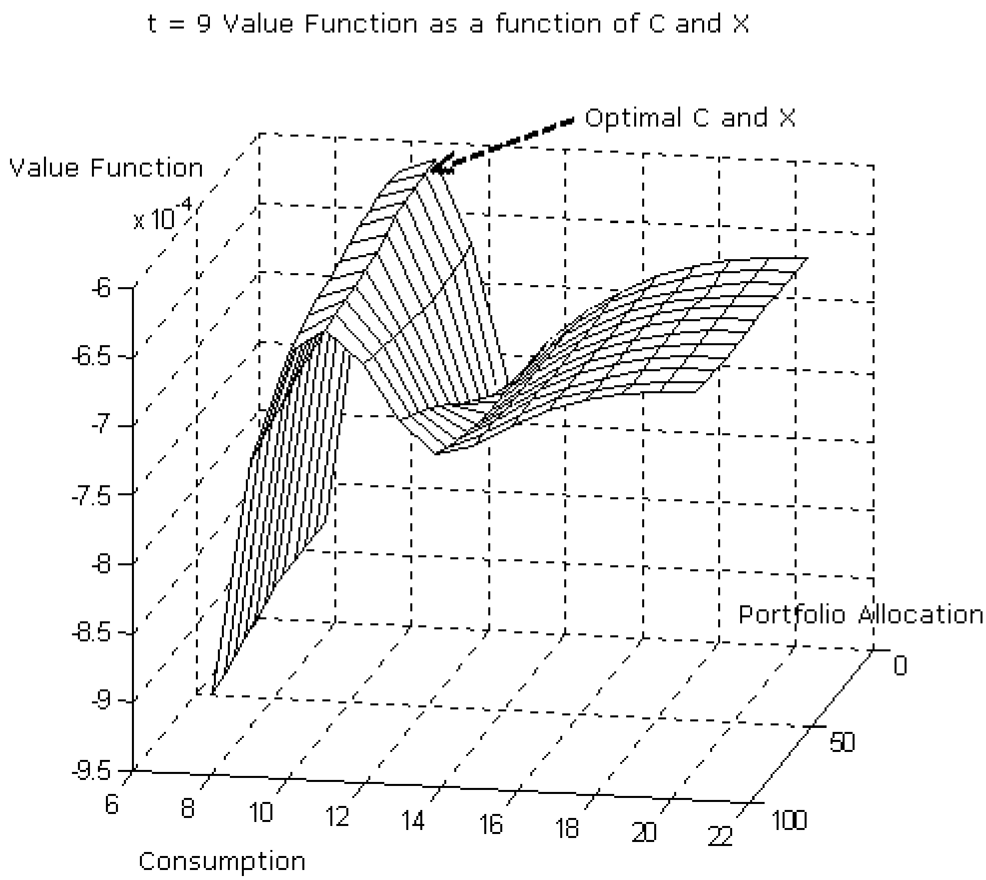


Figure 16



Probability of Next-Period Employment at t=9 vs Possible Values of
Consumption and Portfolio Allocation
(Cash-on-Hand = 20.25)
Probability of Next-Period Employment
100
80
60
40
20
0
19
16
13
10
7
4
1
Consumption
0
20
40
60
80
100
Portfolio Allocation
Optimal values are in this box:
Consumption = 10.26
Portfolio = 4 %

Figure 17



Note that the probability of next period employment is very sensitive to both consumption and portfolio allocation in the region where consumption is around 11 and 12. We see abrupt changes in the probability of future work even if the change in consumption or portfolio allocation is very small. In the most sensitive part of the grid, (where consumption takes the values 12.30 and 12.34, and portfolio allocation takes the values 0%, 1%, and 2%) here are the magnitudes of the sensitivities:

<u>Sensitivity to Consumption:</u>

At x = 0, a tiny increase in consumption from 12.30 to 12.34 causes the probability of next-period employment to jump from 0% to 82.53%.

Sensitivity to Portfolio Allocation:

At C = 12.30, 1% point increase in X from 1% to 2% causes the probability of next period employment to increase by 12.07% points from 2.48% to 14.55%.

At C = 12.34, 1% point increase in X from 0% to 1% causes the probability of next period employment to decrease by 18.62% points from 82.53% to 63.91%.

A case where we do not see any "jumps" neither in consumption nor in portfolio allocation policy rules is when labor supply is not a choice (retirement age is fixed):

What happens when retirement age is treated as given?

I also solved the model with the disutility of employment equal to 0. Then all agents worked for all the periods of course. As can be seen in Figure 18, we do not see any of these jumps in optimal portfolio allocations[12] because the plan of future employment does not depend on the magnitude of cash-on-hand: Agents always work. Two important patterns in Figure 18 should be explained here:

Pattern 1: We still see the monotonically decreasing portfolio allocation with cash-on-hand.
Pattern 2: As agent ages, he holds less and less risky financial portfolios at each level of cash-on-hand. That is, X9 lies below X5, X5 lies below X4, and X4 lies below X2.

In order to understand those patterns, we should keep in mind that expected future labor income is a risky asset that is held implicitly. That asset may serve as a substitute for the risky or the riskfree financial asset.

[12] Even though I do not present a figure for consumption here, we do not see any jumps in optimal consumption either.

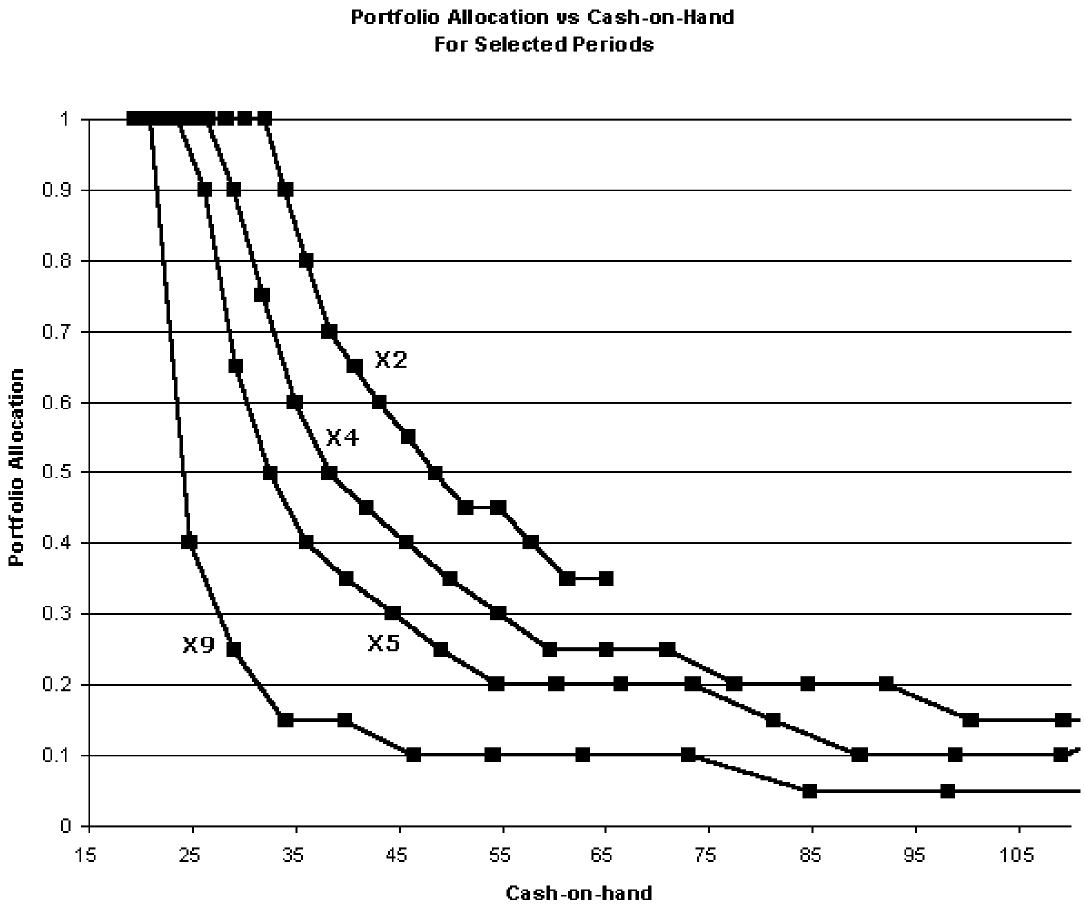
Explanation of pattern 1: More cash-on-hand means a higher share of financial wealth and thus a lower share of human wealth in total wealth. As the share of human wealth decreases, the share of riskfree financial wealth increases. Hence, expected future labor income stream and riskfree financial asset serve as substitutes for each other.

Explanation of pattern 2: As agent gets older, the share of his human wealth in total wealth naturally decreases. As the share of human wealth decreases (due to aging), he holds a higher share of riskfree financial wealth at each level of cash-on-hand. Again, this is consistent with the intuition that expected future labor income stream and riskfree financial asset serve as substitutes for each other.

Cocco, Gomes, Maenhout (2001) reports the same patterns. Observing the same patterns as theirs is not a surprise because the assumption of no disutility of employment in my model corresponds to their assumptions that the agents work until they retire and the retirement age is fixed. They also show that these patterns prevail when the correlation between labor income risk and stock market risk has a small positive value.[13]

[13]The correlation is set to 0 in the solutions presented throughout my study.

Figure 18



In summary the theoretical model presented here implies that as agents age they should choose less risky financial portfolios. This result is consistent with the recent literature on household portfolios. The innovation in the model is the treatment of labor supply as a dichotomous choice variable. In the household portfolio literature it was always treated as given. I show that introducing labor supply as a choice variable changes decision rules. We may see big "jumps" in the consumption and portfolio allocation decision rules for a given set of model parameters. One other innovation in the model is the presence of a social insurance program. The simulations show that the presence and generosity of social insurance programs may greatly alter household portfolio choice. In their working years, households hold much less risky financial portfolio in the presence of Social Security than the absence of Social Security. The opposite result holds when the households are retired.[14]

6. Conclusion

In this study, I analyze the interrelationships among retirement, saving, and asset allocation. I demonstrate that optimal portfolio allocation can be very sensitive to plans about retirement timing and to the presence and the characteristics of Social Security. I also show that portfolio decisions have impact on when to retire. Social Security reform proposals - especially the ones that include individual accounts - should be evaluated with models that incorporate portfolio choice. We would want to predict how individuals' portfolios would change after the reform, so that we would have a better idea of the returns they would experience on their wealth. In return, we would predict more accurately how their wealth would evolve and when they would retire.

[14]Without doubt, this result depends on model parameters. However, the important point here is to demonstrate how Social Security can influence portfolio allocations.

CHAPTER II

Asset Allocation, Bequests, and Wealth Dynamics of the Elderly

1. Introduction

There has been growing interest among economists and policy makers in the saving and portfolio allocation behavior of older individuals. As the aging U.S. population threatens the financial solvency of the Social Security system, policy makers have been considering reducing benefits and adding individual accounts. However, two key issues arise: First, with lower Social Security benefits and the recent decline in private pension coverage, older individuals will be more dependent on their personal savings. But the household saving rate is very low in the U.S., and it is not clear how saving behavior will respond to changes in Social Security and pensions. Second, individuals will have to make investment decisions as they take control of their retirement assets. Many individuals have little experience with portfolio allocation, and there is considerable concern that poor investment decisions could eliminate the potential advantage of increased individual savings. Moreover, a sizable fraction of retirees do not hold any stocks. If this behavior is an indication of how they would allocate the assets in their Social Security individual retirement accounts, then the benefits from having access to high stock returns will be limited. The interest in how older individuals manage their assets is also spurred by macroeconomic considerations. Older individuals hold an increasingly larger share of the U.S. financial wealth. Thus, how wealth and its composition change with age in the last stage of life has important implications for asset prices in the near future when the baby boom generation retires and then reaches the end of life the cycle. A particular concern is the possibility of an "asset meltdown" - whether the baby boomers will liquidate their sizable assets to finance consumption during retirement, thus causing a plunge in asset prices.[1] I develop and estimate a life cycle model of asset allocation, bequests, and saving in order to understand these complex behaviors and how they interact with each other, and to find the implications for the important issues stated above.

Life cycle models provide an ideal framework to address these issues. Economics of aging and finance are two broad strands of literature that have successfully used the life cycle framework to provide an explicit structure that reflects the complex and uncertain environments individuals face in making important decisions that have future consequences. This framework naturally captures the important dynamic link between current

[1]See Poterba, 2004 for a review and analysis of this issue.

and future behavior. In the economics of aging literature, life cycle models have been estimated structurally to analyze important decisions such as wealth and bequests of the elderly (Hurd, 1989), wealth of the elderly in the presence of medical expenditure uncertainty (Palumbo, 1999), pre-retirement consumption (Gourinchas and Parker, 2002), retirement (Gustman and Steinmeier, 1986), Rust and Phelan, 1997), and retirement and saving (French, 2005; and van der Klaauw and Wolpin, 2005).[2] However, this literature has not yet explored asset allocation behavior and its relation to saving in a structural framework. On the other hand, in the finance literature, there has been an extensive amount of work on portfolio choice, since portfolio choice generates the demand side of one of the fundamental topics in finance: asset pricing. However, they do not focus on the savings behavior of the elderly, and their method of choice for finding the key behavioral parameters is fixing them (i.e., calibration) instead of estimating them.[3] I combine these two approaches from economics and finance by estimating the behavioral parameters of a joint model of saving and asset allocation.

Specifically, I extend the seminal work of Hurd (1989) on saving behavior of the elderly by generalizing his life cycle model to include an asset allocation decision, stochastic asset returns, a fixed stock market entry cost, medical expenditure risk, and a more general bequest function. This is the first study to estimate a life cycle model of asset allocation and savings using longitudinal micro data by explicitly solving a stochastic dynamic programming model for the elderly.

A stock market entry cost is included in the model to capture the fact that a significant fraction of the elderly do not hold stocks in the U.S. Even though U.S. household stock market participation has steadily increased, about half of the households still do not own stocks (Ameriks and Zeldes, 2004). The non-participation rate is even higher for the elderly. 73% of the elderly sample I use for this study do not own stocks. This empirical fact is commonly rationalized by monetary and other costs of holding stocks. One must spend time to gather information about the stock market, and pay a brokerage fee to set up an account.[4] Inadequate understanding of the financial system, distrust in the institutions facilitating transactions, and excess worrying

[2]This is only a partial list of related literature.

[3]Gomes and Michaelides (2005) is a recent example among many others.

[4]See Vissing-Jorgensen (2002) for how different types of costs might affect the stock market participation decision.

about the possibility that lifetime savings could evaporate are some of the other possible explanations for the reluctance of even the wealthy to hold stocks.[5] I estimate a dollar denominated stock market entry cost within this framework for the first time in the literature.[6]

There has been growing evidence that the elderly might not intend to completely deplete their wealth by the time they die.[7] In my sample, individuals who were 90 years or older in 1995 spent down only 6.2% of their 1995 net worth by 2002. There are a few potential explanations for this phenomenon: First, there is a significant out-of-pocket medical expenditure risk that increases with age. Retirees may be holding on to wealth in case they need it for their medical problems. Second, the presence of a strong bequest motive may also cause the elderly to keep their wealth. A simple linear bequest function has been successfully used to match wealth profiles in models of saving. However, this relatively simple bequest function is too restrictive when one is interested in analyzing portfolio choice. This is a result of the implicit assumption built into a linear bequest function that individuals are risk neutral with respect to their bequests. This implies that if the individual's saving decision is mainly driven by a strong bequest motive, then she should be aggressively investing in risky assets. This is not what the data show. To overcome that limitation, I specify a general bequest function similar to the one suggested by Carroll (2000), with an additional behavioral parameter, capturing attitudes towards bequest risk. Estimating such a general bequest function is one of the original contributions of this work. A third possible explanation for the slow observed rate of wealth decumulation is the run-up in the stock market (for example in the 1990's). The studies analyzing data sets from these periods suffer from not taking into account the actual returns each individual earns. These returns depend on the individual's portfolio choices and one needs to model this choice in order to measure asset returns accurately. In this study, I incorporate all three explanations within a structural framework. Thus, I will be able to identify the effect of each of these factors.

[5]The non-economic factors thought to influence the decision to own stocks are usually named "psychic" costs. See Curcuru, Heaton, Lucas, and Moore (2004) for the current state of the finance literature in explaining the large heterogeneity in observed portfolio choice.

[6]Alan (2005) also estimates this fixed cost, but she only fits stock market participation profiles whereas I fit wealth and asset allocation profiles.

[7]See Kopczuk and Lupton (in press) for evidence of a bequest motive in the same data set that I use.

The estimation result indicate that the bequest motive is economically almost nonexistent. The estimated model fits the asset and portfolio allocation data reasonably well, both in sample and out of sample. The model explains both the observed average age profiles and the heterogeneity in saving and portfolio allocation. Furthermore, the model explains the apparently puzzling portfolio allocation behavior of the elderly: only a minority of the elderly with a significant amount of wealth hold stocks, and of those who do hold stocks, the share of stocks in the portfolio is relatively small. The model captures these patterns with very large estimates for the stock market fixed entry cost and the degree of risk aversion.

I use the estimates of the behavioral parameters to simulate the impact of a reform of Social Security to include individual accounts. What would happen if the Social Security benefit of a 70-year old retiree was cut by a given amount and she was given a lump sum equal to the expected present discounted value of the benefit cut? Will she be better off, since she can invest it in assets that have high average returns? The answer provided by the model is an unequivocal no. This answer does not depend on whether a private annuity market is available to the retirees that could be used to ***undo*** the reform. I find that the typical retiree would be willing to pay about 10% of her wealth at age 70 to avoid this change if she does not have access to the annuity market. Put differently, to keep her welfare constant the government would need to pay 16% more than the actuarially fair amount in converting the benefit cut into a lump sum. Furthermore, the less well-to-do retirees are predicted to be hurt more than the well-to-do retirees. Having access to the annuity market makes retirees slightly better off, but not enough to make the reform desirable, due to the actuarially very unfair pricing of annuities. The model predicts that this reform is welfare reducing because the flow of financial resources provided by the risk-free Social Security lifetime annuity cannot be easily matched by the returns on the lump sum payment when the date of death is uncertain. This is especially true for retirees who do not have a private pension to supplement Social Security benefit payments.

The paper has 6 remaining sections. Section 2 details the theoretical model of saving and asset allocation and how I solve it. Section 3 discusses the implications of the model for retiree behavior. Section 4 describes the data on the elderly that are used for estimating the model. Section 5 explains the empirical model.

The results, the fit of the model, and the policy simulations are presented in Section 6. Section 7 concludes with the limitations and the planned extensions for future work.

2. The Model

Consider an unmarried individual who is and will be out of the labor force until death. Her only source of income is from public and private pension benefits and annuities. [8] Suppose her income is constant in real terms. Age of death is uncertain: The probability of dying at age t conditional on being alive until age t is m_t. Each year, she faces a random out-of-pocket medical expense. The distribution of this expense is age-dependent and is estimated from the data. This expense is deducted from her available cash-on-hand. Each year, she decides how much to consume, C_t, and whether to pay the stock market entry cost if she hasn't already paid it. If she has paid the entry cost before or if she decides to pay it now, she also chooses the fraction of her wealth allocated to risky assets, x_t. Otherwise, all of her wealth is kept in a risk-free asset. She is not allowed to borrow or sell either of her assets short: $x_t \in [0,1]$. She derives utility from consumption and the amount of wealth held at death, which is bequeathed to her heirs. At each age, the retiree makes these decisions so as to maximize the EPDV of remaining lifetime utility.

2.1. For Retirees Who Have Already Paid the Entry Cost

For a retiree who has already paid the stock market entry cost, the value function at age t, given non-asset income I, and wealth W_t is

[8] I have made two assumptions about income from pensions and annuities that contributed significantly to the feasibility of estimating the model: They continue to be paid until the respondent dies, and they are constant in real terms (that is, cost-of-living adjusted). These assumptions are true for payments from Social Security, but not for some types of pensions and annuities.

$$V_t^{paid}(W_t, I) \;=\; \max_{C_t, x_t} \left\{ u(C_t) + m_t E_t\left[b(W_{t+1})\right] + (1-m_t)\,\delta\, E_t[V_{t+1}^{paid}(W_{t+1}, I)] \right\}.$$

(Equation 1)

She maximizes this value function subject to

$$W_{t+1} = (1+R_{t+1})W_t^* - M_{t+1},$$

$x_t \in [0,1]$, and $C_t \geq \overline{C}$,

where u is the consumption utility function, b is the bequest function, δ is the discount factor, W^*_t is the end-of-period wealth, which is equal to $COH_t - C_t$, and R_{t+1} is the total return on period t end-of-year wealth that is realized at the beginning of period t+1.

Cash-on-hand, $COH_t = W_t + I - M_t$ is the total net resources available for consumption or saving in period t. Once consumption is made, end-of-period wealth, $W_t^* = COH_t - C_t$ is invested in a risky and risk-free asset. The return on W_t^* is:

$$R_{t+1} = (1-x_t)(1+\bar{r}) + x_t(1+r_{t+1}).$$

$\bar{r}$ and r_{t+1} are the rates of return for the riskless and the risky assets, respectively. Note that R_{t+1} is stochastic since r_t is stochastic. Further note that the amount of the bequest in the event of death is uncertain because it depends on the return realized at the beginning of period t+1, r_{t+1}. M_{t+1} is the out-of-pocket medical expenditure, which is realized at the beginning of period t+1.

$\overline{C}$ is the minimum consumption level guaranteed by the government. Individuals with low income may find it optimal not to save if they are likely to utilize asset-tested government transfers (Hubbard, Skinner and Zeldes, 1994). A simple way of modeling this is to include a minimum guaranteed consumption level (consumption floor), $\overline{C}$. I assume that consumption never falls below $\overline{C}$. Whenever cash-on-hand falls below $\overline{C}$, the government transfers cash to the individual in order to increase cash-on-hand to $\overline{C}$, and requires the individual to spend all of his assets.

2.2. For Retirees Who Have ***Not*** Paid the Entry Cost

We can think of this optimization problem in two steps: In the first step, the retiree computes the optimal plan for each of the two possible choices: pay and do not pay the entry cost. In step two, she chooses the one that gives her the higher lifetime utility.

Step 1: Since the returns are not stochastic in this case, the value function if she chooses not to pay the entry cost is:

$$V_t^{\text{not paid}}(W_t, I) = \max_{c_t} \left\{ u(c_t) + m_t b(W_{t+1}) + (1-m_t)\,\delta E_t[V_{t+1}(W_{t+1}, I)] \right\}$$

subject to $W_{t+1} = (W_t + I - C_t)(1+\overline{r}) - M_{t+1}$ and $C_t \geq \overline{C}$. $V_{t+1}(W_{t+1}, I)$ is defined below.

The value function if she chooses to pay the fixed entry cost, FEC is:

$$V_t^{\text{pays now}}(W_t, I) = V_t^{paid}(W_t - FEC, I),$$

where V_t^{paid} was specified above.

Step 2: The value function for this individual is the maximum of these two choice-specific value functions:

$$V_t(W_t, I) = \max\{\ V_t^{\text{not paid}}(W_t, I)\ ,\ V_t^{\text{paid}}(W_t - FEC, I)\ \}$$

2.3. Preferences and Asset Returns

The utility from consumption in period ***t*** is $u_t(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, where ***u*** is a constant-relative-risk-aversion (CRRA) function and γ measures the degree of risk aversion towards consumption. Expected utility derived during period t from wealth held at the beginning of ***t***+1 if the individual dies at the end of ***t*** is: $\boldsymbol{E_t}[\boldsymbol{b_t}(\boldsymbol{W_{t+1}})] = \boldsymbol{E_t}[\alpha \frac{\boldsymbol{W_{t+1}}^{1-\gamma_b}}{1-\gamma_b}]$, where W_{t+1} is next period wealth, γ_b measures the degree of risk aversion towards bequests, and α is a parameter that measures the strength of the bequest motive. Note that we do not exclude the possibility that risk aversion may differ for bequests and consumption. For example, individuals may be more worried about the volatility in their own future consumption than about volatility in the amount of wealth they bequeath. In this case, $\gamma > \gamma_b$. This will also imply that wealthier individuals have higher saving rates and a higher share of risky assets in their portfolios (Carroll 2000).

The risk-free asset has a fixed rate of return, $\bar{\boldsymbol{r}}$. The risky asset return is independently and identically distributed through time. I approximate the real annual return of U.S. equities by a discrete distribution with three points of support using real S&P500 splits/dividends adjusted historical series. ***r*** can take the values $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ with probabilities $\boldsymbol{p}_1$, $\boldsymbol{p}_2$, and 1-$\boldsymbol{p}_1$-$\boldsymbol{p}_2$, respectively.[9]

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[9] $\boldsymbol{r}_1$=0.315, $\boldsymbol{r}_2$=0.04, $\boldsymbol{r}_3$=-0.26, $\boldsymbol{p}_1$=0.35, $\boldsymbol{p}_2$=0.45, $\boldsymbol{p}_3$=0.20. ***r*** has 7.625% mean and 20.77% standard deviation.

2.4. Out-of-Pocket Medical Expenditure

We have data on total out-of-pocket medical expenses (OMX) within the past 2 years of each interview date. Assume that these two years correspond to ages t and t+1. I define $OMX_t = OMX_{t+1}$ to be half of what the reported total is. A categorical variable is defined such that the value 1 corresponds to a low expense, value 2 corresponds to a medium expense, and value 3 corresponds to a high expense. The cut-off points are $1,000 and $11,000. A multinomial logit of this variable is regressed on age. Then, probabilities of belonging in categories are predicted using the estimated parameters. Separate least squares regressions of level of expenses are run for each category. Mean level of expenses for each category is predicted using the OLS parameters. See Tables 12 and 13 in the Appendix for the estimated probabilities and levels of medical expenses.

2.5. Solution Method

The model does not have an analytic solution. It is solved numerically by backward recursion from the last period to the first.[10] ***Step 1***: In the last period, the probability of surviving to the next period is zero and hence, given the annuity income stream and beginning-of-period wealth, the individual's problem simplifies to:

$$\max_{c_T, x_T} \left\{ u(c_T) + E_T\left[b(W_{T+1})\right]\right\}$$

where $W_{T+1} = (W_T + I - c_T)(x_T r + (1 - x_T)\bar{r})$.

Given that the return distribution has been discretized, we can write $E_t[b(W_{t+1})] = \sum_{i=1}^{3} p_i b([W_t + I - C_t][x_t r_i + (1 - x_t)\bar{r}])$ where $\boldsymbol{r}_i$ is the $\boldsymbol{i}^{th}$ possible value of the asset return and $\boldsymbol{p_i}$ is the probability of this realization. Wealth and annuity income are continuous variables and hence it is not

[10] I outline the solution method for a retiree who has paid the stock market entry cost. The method extends to the other case easily.

feasible to solve this equation for every possible beginning-of-period wealth and annuity. Instead, I solve the model for a two dimensional grid of wealth and income values.[11] ***Step 2***: Solve Equation 1 for ***t*** = ***T***-1. The expected value of the next period value function is calculated by integrating over the return distribution, as illustrated above for the terminal period. We have already calculated the next-period value function on a grid of wealth and income in step 1. The off-grid values of the next-period value function are calculated by linear interpolation. This procedure is repeated until the period-1 equation is solved. I use a two-dimensional grid search algorithm to find consumption and the share of risky assets at each step.

3. Implications of the Model for Asset Allocation and Saving Behavior of Retirees[12],[13]

To see the behavioral implications of the model, I simulate its solution for a 70-year old female who has already paid the stock market entry fee and has $200,000 net wealth and a $15,000 real annuity. The simulated solutions are calculated by solving the model 10,000 times – each for a random draw from the return distribution at each age – and averaging over these solutions. Simulated wealth and stock shares without and with bequests are depicted in Figures 19 and 20, respectively, for specific values of the behavioral parameters described below. The discount factor is 0.98, the risk-free asset has 2% mean real return, and the stock market has the return distribution that was described in the previous section.

Figure 19 shows the optimal wealth (left panel) and the stock share (right panel) trajectories at low (γ = 0.5), moderate (γ = 4.0), and high (γ = 15.0) levels of risk aversion for the above retiree who does ***not*** have a bequest motive. There are some notable patterns in this figure. On the left panel, the absence of a bequest motive manifests itself with wealth being completely depleted before death. Also, greater risk aversion leads to slower wealth decumulation (see Hurd, 1989). On the right panel, we see that the share of risky assets is

[11]During estimation, I use 500 grid points for beginning-of-period wealth and 6 grid points for annuity income.

[12]Throughout this section, I assume there is no medical expense risk to better focus on the effect of a bequest motive. Later, I will include the cases when there is medical expense risk.

[13]Throughout this section, I assume that there is no medical expenditure risk. This allows me discuss the effects of a bequest motive on behavior in isolation from other factors that might also induce saving (ie, medical expenditure risk).

inversely related to the degree of risk aversion. This ordering is directly implied by the definition of risk aversion. More risk averse individuals keep lower shares of stocks at every age than do less risk averse individuals. An important thing to note is that low to moderate risk aversion levels imply stock shares that are much higher than what we observe in the data. In the elderly sample I use for this study, most stockholders hold little stocks and the mean share of stocks is about 1/3 among stock holders. These observations imply that only high risk aversion can possibly explain the data within this framework.

A striking observation in the right panel of Figure 19 is that no matter how risk averse an individual is, the optimal share of risky assets ***increases*** as she ages (except for the low risk aversion, $\gamma = 0.5$, where the solution is on the boundary). This result is in stark contrast to advice from many financial planners, which is that people should hold about (100-Age) percent of their wealth in stocks (Malkiel, 1996). This seemingly counterintuitive pattern needs explanation.[14] A retiree's financial resources consist of wealth and income. Wealth can be spent or accumulated and can be held in risky or risk free assets. However, income of retirees is fixed and risk free since it is a real annuity. Portfolio rebalancing as the retiree ages can only be done by adjusting wealth. Consider a particular portfolio configuration and wealth at a given age, ***t***. At age ***t***+1, the retiree will spend some of her wealth since there is by assumption no bequest motive in the model used to generate Figure 19. Thus, total resources decline but the risk-free source (income) does not change. The portfolio is rebalanced by putting a greater share of wealth in the risky asset to keep the risk exposure level at the optimal level. Kahvecioglu and Hurd (2005) show that the reason financial planners' advice is different than optimal behavior found here is that financial planners do not take into account the significant decline in wealth after retirement.[15] As a matter of fact, Ameriks and Zeldes (2004) find evidence that individuals do not decrease their stock shares as they age. There is also evidence in the AHEAD sample I use for this study (as will be seen in the section on data) that stock shares increase slightly with age, which would be consistent with the model presented here. Introducing bequests may change this conclusion as discussed later in this section.

[14]See Kahvecioglu and Hurd (2005) for more on this.

[15]They show that, if optimal wealth were constant until death, the optimal percentage of wealth held in stocks would roughly be in agreement with the financial planners' advice.

Introducing a bequest motive will make the wealth profiles flatter (Hurd, 1989).[16] The wealth profile for $\gamma = 4$ in Figure 19 flattens in Figure 20 on the left panel after including a bequest function. The dotted and long-dashed lines correspond to bequest functions that are linear and exhibit the same degree of risk aversion as the consumption-utility function, respectively. The dashed line is for a bequest function that exhibits risk aversion that is in between the other two. Notice how similar the three wealth profiles, in the presence of bequests, are for most of the remaining life. This is not a coincidence: I picked those three bequest parameter pairs (γ_b and α) deliberately so as to produce similar wealth profiles. Even the simple linear bequest function (the dotted line) can produce the same profile. As a matter of fact, the linear bequest function has been successfully used in the savings literature to help match the wealth data. However, if we wish to match ***both*** asset allocation and wealth data, we will need a bequest function with at least two parameters, one for risk aversion, (γ_b) and one for strength (α), as I will show later in this section. Even though I do not demonstrate it here, it is worth mentioning that the way I model bequests implies that wealthy individuals are more likely to have operative bequest motives if risk aversion for bequests is lower than risk aversion for consumption. Then, bequests become luxury goods and the wealthy individual invests a higher fraction of her wealth in the risky asset and her saving rate will be higher (see Carroll (2000)). However, note that, in estimating the model, I will allow the possibility that risk aversion for bequests could be equal to or higher than risk aversion for consumption. Even though there is a recent surge of articles on life cycle portfolios, none to my knowledge has allowed this level of generality in risk aversion toward bequests. They usually use bequest functions that have the same risk aversion parameter value as the consumption-utility function (Cocco, Gomes, and Maenhout (2005) is a recent example). Given that the wealthy are thought to have an operative bequest motive, and thus a significant fraction of wealth is held to be bequeathed, it is worth freeing up the parameter that governs the risk aversion toward bequests in order to better understand the demand for stocks. As a matter of fact, I show below that this in fact seems necessary in order to understand the joint saving and portfolio behavior of the elderly.

There are two major channels through which the bequest motive affects the optimal portfolio allocation path. The first channel is risk aversion toward bequests. If an agent is less risk averse for bequests than she is for consumption, and if she wishes to bequeath some fraction of her wealth, then she will be willing to expose

[16]Even an increasing wealth profile is possible if the bequest motive is strong enough.

more of her wealth to risk than she would without a bequest motive. Furthermore, as she ages and the risk of death increases, utility from bequests will be discounted less. As the strength of the bequest motive relative to utility from consumption increases, there will be upward pressure on the riskiness of the portfolio. Consider the extreme case in which the agent is risk neutral toward bequests ($\gamma_B = 0$) but risk averse toward consumption ($\gamma_B > 0$). In the last period of her life, she would invest 100% of her end-of-period wealth in the risky asset (see Figure 20). The second channel is an indirect one whereby the optimal portfolio allocation is affected through the change in the wealth profile caused by the bequest motive. In the presence of annuity income, an increase in wealth holdings due to a bequest motive will lead to shifting of the portfolio toward the risk free asset.[17] Thus a bequest motive has an ambiguous net effect on asset allocation. The net effect depends on age, wealth, income, and preferences.

On the right panel of Figure 20, the values of the parameter for the strength of the bequest motive, α, is chosen so that the implied wealth profiles are very similar. This allows us to isolate the first major channel mentioned above through which the bequest motive affects the portfolio allocation path: risk aversion toward bequests. Risk aversion toward consumption (γ) is set at 4. Three levels of risk aversion toward bequests are depicted: no risk aversion ($\gamma_b = 0$, linear bequest function), the same risk aversion as for consumption ($\gamma_b = 4$), and a risk aversion in between ($\gamma_b = 2$). It is useful to remember that in the no-bequest case (solid line), the optimal share of stocks increases with age since wealth - which is the risky portion of total financial resources whereas the risk-free portion is income - decreases. It will be very informative to compare the three bequest functions that differ by risk aversion, to the case of no bequests: First, in the presence of a linear bequest motive (dotted line, risk neutrality), the optimal stock share is even higher than in the no-bequest case (shown in Figure 19, right panel, solid line). Despite the fact that wealth is higher, she finds it optimal to increase her stock share because she does not care about the risk for the additional wealth that she is holding due to a bequest motive. Second, once we introduce mild risk aversion toward bequests ($\gamma_b = 2$, dashed line) the optimal stock shares are smaller than in the no-bequest case. Third, when we increase the risk aversion up to the level exhibited by the

[17]This follows the same logic that was explained in the second paragraph of this section; only this time wealth is increasing rather than decreasing.

utility from consumption (γ_b = 4, long-dashed line), we get a much lower stock share profile than in any of the other cases.

The lesson we take from this comparison is that, if we model bequests by either a function exhibiting no risk aversion, or the same level of risk aversion as the consumption utility function[18], then we would be severely restricting the life cycle asset allocation behavior the model can to generate. This will introduce serious biases in the estimates of the key structural parameters in a study that estimates a joint savings and portfolio choice model. To demonstrate this, suppose that the actual portfolio profiles look like the short dashed line in Figure 20: They are almost flat and may be slightly increasing. We actually have empirical evidence that this might be the case. (Ameriks and Zeldes (2004) and the in the AHEAD sample I use, which will be shown in the next section) If we force a linear bequest function or a bequest function that has identical risk aversion as the consumption-utility function to fit the life cycle portfolio data, then the only way for the model to fit the data would be by estimating an insignificant bequest motive. Furthermore, without bequests, the only way to match the flat wealth profile would then be to overestimate risk aversion and/or overestimate the discount factor.

4. Data

The data used to estimate the model are from the Assets and Health Dynamics Among the Oldest Old (AHEAD) sub-sample of the Health and Retirement Study (HRS) sample. The AHEAD subsample consists of individuals who were born in or before 1924, and were not institutionalized at the time of their first interview in 1993-4. The respondents were reinterviewed in 1995-6, 1998, 2000, and 2002.[19] There are a total of 8,397 respondents of whom 5,322 are female.

The analysis in this paper uses retired unmarried female respondents who appear in at least two consecutive waves. The model solves for next period wealth and asset allocation given current period wealth, so

[18] This is the assumption of almost all of the life cycle asset allocation studies in the finance literature.
[19] The actual time period between two consecutive interviews varies between 12 and 39 months. Data from the 2004 interview are not yet available.

a minimum of two observations per person is needed. I exclude observations in which the woman was younger than 70 or older than 102. The 1993 wave is dropped for reasons explained below.[20] After these restrictions are applied the sample contains 5,094 observations on 2,384 women. Table 4 tabulates the women by the number of observations they contribute to the sample.

The change in mean wealth from 1993 to 1995 is qualitatively and quantitatively strikingly different than the changes in other periods. From 1993 to 1995 mean wealth went up by 27% (from $117K to $148K), while it went down by 9% between 1995 and 1998, barely changed between 1998 and 2000, and went down by 6% between 2000 and 2002. This is an important issue since in this paper we are trying to explain wealth changes and asset allocation and we cannot do so properly with questionable data. After doing extensive calculations, I found that underreporting of assets in 1993 data is the likely reason behind this anomaly and I decided not to use 1993 data to explain 1995 data.[21, 22] My calculations showed that the high stock market returns cannot explain the 27% increase in wealth from 1993 to 1995. Hence, I estimate only 1998, 2000, and 2002 wealth and asset allocation using data from the respective previous waves.

I assume that income is constant in real terms. When we follow respondents over time, we see large variation in reported income for quite a few of them. This is most likely a result of measurement error, since the elderly are not subject to large swings in their income from Social Security, pensions, and annuities. Averaging over as many income observations as we have will reduce the measurement error. To do this, I dropped extreme observations on the components of income. Then, I calculated averages for each of these components. Finally, I added them up to come up with the constant real measure of income I use.

[20]Further sample selection criteria eliminated a few more observations. These criteria are: nonnegative current and previous total wealth, total annual income not more than $74,000, and equity share of wealth less than or equal to 100%. Wealth and equity share restrictions are due to the model: An individual can never have negative net worth and he/she cannot be short or long on equities. The annual income restriction is to reduce the computational burden, and will be explained later. I also dropped extreme observations for total wealth (> $10,000,000) and annual income from pensions (>$70,000).

[21]I do not completely throw out the 1993 data. Income data for 1993 do not have the same problems as the wealth data, and are used in income calculations as described below.

[22]See Hurd and Rohwedder (2004) for a detailed analysis of the problems with the 1993 data. Kopczuk and Lupton (in press) also dropped the 1993 wealth data in their analysis of bequest behavior.

Table 5 provides a first glimpse of the data. The column labeled "1998" shows that there were 1,826 AHEAD respondents who were interviewed both in 1995 and 1998, were at least 70 years old in 1995, were single in 1995 and remained single through the interview in 1998, and did not have any attachment to the labor force between 1995 and 1998. All dollar amounts are reported in 2002 dollars using the Bureau of Labor Statistics' all urban consumer price index. In the tables, "previous" refers to the previous interview. For example, previous total wealth in the 1998 column refers to total wealth in 1995.

4.1. Are the Assets of the Elderly "Melting Down"?

In Table 5, we see that mean wealth declines by 11% (from $162.9K to $144.5K) during 1995-1998, and does not change much during 1998-2000 and 2000-2002. Table 6 reports the 7-year changes in mean wealth from 1995 to 2002 by age groups. The annual change in mean wealth is between 0.5 and -2.9%. At face value, these figures provide evidence that the elderly are dissaving, but at a slower rate than a standard life cycle model with neither bequests nor medical expenditure would predict, especially given the fact that these people are very old (in 2002 average age is over 85).

The means might be heavily influenced by the wealthy minority of retirees who hold a disproportionately high share of total wealth in the sample. Looking at the changes in the median may give us a better idea about how the majority of the sample is doing. Tables 5 and 7 report 2-year and 7-year changes in median wealth, respectively. The decumulation of wealth is much more pronounced and consistent in these tables. Table 5 shows that median wealth declined significantly in each period, and the fraction of the sample with zero wealth increased in each period between any consecutive waves. The increase in the number of people with no wealth was consistently between 10 and 20%. In Table 7, we see that between one 12 to 76% of median wealth is depleted within 7 years depending on age in 1995. The discrepancy between the changes in mean and median wealth suggests that the relatively poor majority and the wealthy minority have different wealth profiles. Two major reasons causing this discrepancy would be that saving behavior is different for these groups and that the wealthy group might be experiencing much different realized returns on their wealth. Wealthy individuals are the main stockholders and, as will be explained in the next section, the real annual return on the S&P500 index was 25% between 1995 and 1998, 12% between 1998 and 2000, and -19% between 2000 and 2002. The first reason suggests an operative bequest motive for the wealthy, while the second reason suggests differences in realized returns might be the cause. This points out the importance of controlling for realized returns in order to draw robust conclusions about the presence and the strength of a bequest motive. Additionally, there is no support for a wealth decumulation rate that is increasing by age in the means, although there is support in the medians.

4.2. How Do the Stock Shares Vary with Age?

As they age, do the elderly behave in a way consistent with the common financial planners' advice (gradually move out of stocks) or with the model solution presented in this paper (increase stock shares if they are not as risk averse toward bequests as they are toward consumption)? This question is relevant only for a minority of the sample (about 20.0-23.1%, see Table 5) since most of the elderly do not own stocks. Table 5 shows that the mean share of stocks in wealth is only about 10%, yet about a fifth of total wealth is held in stocks. This is because the wealth distribution is skewed toward the right and it is mostly the people in the right tail of the wealth distribution who are the stock-owners. As for the stock-owners in consecutive waves, we see that about a third of their wealth is held in stocks. The mean share of stocks increased from 28.4% to 36.9% between 1995 and 1998, stayed constant between 1998 and 2000, and increased again from 31.8% to 33.9% between 2000 and 2002. So there is no evidence for moving away from stocks while aging. Table 8 depicts how mean stock shares differ by age groups for those with nonzero current and previous wealth (the top panel), for those holding stocks in both waves (second panel), and for those holding stocks in at least one wave (bottom panel). There is no evidence that the elderly follow the advice of financial planners. Over the 7 years from 1995 to 2002, either there is either a flat or increasing portfolio profile. Furthermore, the older the individuals are in the initial year, the higher their stock share increase (panel 2), which is consistent with the model.

4.3. Stock Returns

I used the Standard & Poor 500 index (taking into account dividends and splits) in constant dollars as an external data source to approximate the distribution for stock returns and to calculate the realized equity returns for the period when AHEAD respondents were interviewed. Assuming that the respondent buys $1 worth of the S&P500 index on the day of a given interview (which varies by individual) and keeps it until the day of the next interview day, the first row of Table 9 displays the mean return between adjacent interviews.[23] Figure 21 shows the distribution of returns across all respondents and all interviews. Notice the large variation in stock market returns. Figure 22 depicts the evolution of the S&P 500 index and the periods when the AHEAD interviews were conducted. We note two things in Figure 22: First, the S&P 500 index is highly variable over this period. Second, the interview dates are scattered throughout the calendar year. These two facts and the fact that the periods between interviews differ substantially from respondent to respondent (from about 1 year to about 3 years with mean of about 2 years), make it no surprise that the individual stock market gains show the large variation that we see in Figure 21. In Figure 21, the observations between -50% and -25% belong to the period 2000-2002 when the stock market crashed. The lump around 65% corresponds to the period 1995-1998 when the stock market gains were highest. The biggest lump, which is around 25%, is for two periods, 1993-1995 and 1998-2000. As explained in the previous section, the theoretical model is solved every year for the respondents. Due to high variability in stock returns it may be important for a model that is explaining wealth to take into account different interval lengths between interviews. I prorate consumption if the next period is not a whole year away.

The second row of Table 9 presents the annualized returns that the respondents had realized at the current interview since the previous interview. I calculated the annualized return for individual ***i*** as follows:

$$\text{Annualized Return}^{i} = \left(\frac{\text{S \& P500 index}^{i}\ \text{(current interview)}}{\text{S \& P500 index}^{i}\ \text{(previous interview)}}\right)^{12/\#\,\text{months between interviews}^{i}}$$

[23]The actual time period between two consecutive interviews varies between 12 and 39 months.