AN EXAMINATION OF THE RELATIONSHIPS BETWEEN KINDERGARTEN
TEACHERS’ BELIEFS, MATHEMATICAL KNOWLEDGE FOR TEACHING, AND
INSTRUCTIONAL PRACTICES

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ABSTRACT

AMY S. SCRINZI: An Examination of the Relationships between Beliefs, Mathematical Knowledge of Teaching, and Instructional Practices of Kindergarten Teachers (Under the direction of Dr. Barbara Day)

This study was designed to address the numerous calls for research in the field of mathematics and early childhood education, providing important information regarding influences on teaching practices used with young children. Mathematical pedagogical beliefs, mathematical knowledge for teaching, and instructional practices of 29 kindergarten teachers were examined. The Self-Report Survey (Ross, McDougall, Hogoboam-Gray, & LeSage, 2003), the Mathematical Knowledge for Teaching (Hill, Schilling, & Ball, 2004) measure, and the FirstSchool Snapshot classroom observation tool (Ritchie, Weiser, Kraft-Sayre, Mason, Crawford, & Howes, 2010) were used to investigate these variables.

The research study employed a non-experimental quantitative research design. Descriptive statistics provided insight into each of the three variables and correlational statistics were used to determine possible relationship among them. Results indicated that the sample population favored more reform-oriented, constructivist based beliefs regarding mathematics than traditional practices; performed statistically significantly better on geometry items than number items; and used constructivist teaching practices, with respect to mathematics, an average of 15% of the time observed.

Correlational statistics were used to determine possible correlations among the variables and the strength of those relationships. A significant positive correlation of \( r = \)
.384 ($p = .05$) was found to exist between beliefs and mathematical knowledge for teaching (MKT). When examining correlations between MKT and the nine domains of pedagogical beliefs, a statistically significant positive correlation ($p < .05$) between Program Scope, Student Tasks and Teacher’s Role and mathematical knowledge for teaching was found. These results led the researcher to believe that teachers who have a stronger mathematical knowledge for teaching tend to believe that the role of a teacher is that of a co-learner, favor the use of complex, use open-ended problems embedded in real life contexts, and believe that the breadth mathematics extends beyond number and operations. No other statistically significant correlations were found among the variables.
DEDICATION

To teachers of young mathematicians, may you revel in your own journey of learning.
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CHAPTER 1

INTRODUCTION

Background

The world in which we live is fast-paced and undergoing astonishing change. New information and expanding technology mean that being mathematically proficient is of utmost importance, recognizing that “those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures” (NCTM, 2000, p. 3). Building mathematical competency is seen as a key strategy to keep America competitive, creative, innovative, and successful (Bush, 2006).

In today’s economy, employers seek those who are problem-solvers, effective communicators, and proficient in mathematics (U.S. Department of Education, n.d.). The National Science Board (2008) indicated that the growth of jobs in the mathematics-intensive science and engineering workforce was outpacing overall job growth by 3:1. This increase in need was met by a shortage of qualified job candidates, particularly with the increased retirement of the current science and engineering workforce (Grossman, 2008). This “mathematical ignorance of our citizenry seriously handicaps our nation in a competitive and increasingly technological global marketplace” (Battista, 1999, p. 426). Educational mediocrity in mathematics and science remains the most certain long-term national threat (Steen, 2003). The United States government has cautioned the public that “America’s schools are not producing the mathematics excellence required for global economic
leadership and homeland security in the 21st century” (U.S. Department of Education, 2000, p. 1). Without considerable and continual changes to mathematics education, “the United States will relinquish its leadership in the 21st century” (National Mathematics Advisory Panel, 2008, p. xi). In order to maintain leadership in our global world, mathematics must be a central focus.

Researchers and educators recognize that all students must have the opportunity and support to learn significant mathematics with depth and understanding (NCTM, 2000). However, according to international and national data, U.S. students have been consistently outperformed in mathematics by numerous, often less economically-developed nations (U.S. Department of Education, 2009b). The 2007 Trends in International Mathematics and Science Study (TIMSS) (U.S. Department of Education, 2009a) reports that the average U.S. fourth-grade mathematics score in 2007 was lower than 8 of the 35 countries tested and the average eighth grade score was lower than 5 of the 47 countries tested. While the fourth- and eighth-grade average scores were above the international mean, only 10% of fourth-grade and 6% of eighth-grade students reached the TIMSS advanced international benchmark. In comparison, 41% of the fourth-grade students in Singapore and 40% of the fourth-grade students in Hong Kong scored at the advanced benchmark. In addition, TIMSS noted a significant difference between boys and girls scores for the U.S., with boys in both fourth and eighth grades outscoring girls.

The National Assessment of Educational Progress (NAEP) reported that the U.S. average mathematics score for fourth-graders in 2009 was unchanged from the score in 2007 while eighth graders continued their upward trend with a 2-point increase. Yet, only 39% of fourth graders and 34% of eighth graders performed at a “proficient” level, which indicates
solid academic performance, “demonstrating competency over subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter” (NAEP, 2009b, p. 1)

All students can and should be mathematically proficient (Kilpatrick, Swafford, & Findell, 2001), beginning with our youngest students. The difference between the mathematics ability of the youth of the U.S. and the youth from other countries begins as early as preschool or Kindergarten (Ginsburg, Lee, & Boyd, 2008). Using the Early Childhood Longitudinal Study (ECLS-K) information, the National Center for Education Statistics (NCES) studied the mathematics achievement of 22,625 nationally representative kindergarten students as they progressed through school from 1998-2004 (U.S. Department of Education, National Center for Education Statistics, 2004). While the average mathematics scale score increased 63 points from 1998-2002, Black children demonstrated smaller gains in mathematics than White, Hispanic, and Asian/Pacific Islander children. NCES also noted that Black children started kindergarten with a lower mean achievement score in mathematics than other racial/ethnic groups. Children from families with risk factors (e.g., household below poverty level, mother’s highest education less than a high school diploma/GED) also had scores in mathematics that were lower than their peers. The gaps between these various groups proved critical, as they continued to increase from the kindergarten year in 1998 to the end of third grade in 2002.

The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) affirm the importance of mathematics and clearly acknowledge the power of the early childhood years as a “vital foundation for future mathematics learning” (National Association for the Education of
Young Children, 2002, p. 1). Research has shown that school-entry mathematics is a strong predictor of later success (National Mathematics Advisory Panel, 2008), that mathematics has been found to be a better predictor of later success than early reading ability, and is linked to later success in both mathematics and reading (Duncan et al., 2007; Ginsburg et al., 2008; U.S. Department of Education, 2008). In 2005, the American Institutes for Research strongly recommended to the U.S. Department of Education, Policy and Program Studies Service that the “U.S. mathematics system must do a better job of establishing a strong foundation of students’ initial mathematical knowledge in the early grades” (p. 25). It is critical that all children have a strong foundation in mathematics.

**The Power of the Classroom Teacher**

In order for students to be mathematically proficient, they must have competent teachers (National Mathematics Advisory Panel, 2008; National Research Council, 2000). An effective teacher can have a stronger influence on student achievement than poverty, language background, class size, and minority status (Aaronson, Barrow, & Sander, 2007; Darling-Hammond, 2000; Jacob, Lefgren, & Sims, 2008; Kane & Staiger, 2008; Nye, Konstantopoulos, & Hedges, 2004; Rivkin, Hanushek, & Kain, 2005; Rockoff, 2004; Rothstein, 2010). Research conducted using the Tennessee Student-Teacher Achievement Ratio (STAR) data from Grades 1, 2, and 3 found that gains in students’ mathematics achievement during an elementary school year can increase 12%-14% with a capable and knowledgeable teacher of mathematics (Nye et al., 2004). Rockoff (2004) found that one-standard-deviation increase in teacher quality increased elementary student reading and math test scores by approximately 0.1 standard deviations on nationally standardized distributions of achievement.
In addition, numerous studies have shown that student gains can compound over the years depending on the teacher’s effectiveness (Hanuskek, 2002; Haycock, 1998; National Mathematics Advisory Panel, 2008; Rivkin et al., 2005; Sanders & Rivers, 1996; Schacter & Thum, 2004). Sanders and Rivers (1996) estimated that elementary students who were enrolled in highly effective teachers’ classes would have attained fifth-grade mathematics scores that were as much as 50 percentile points higher than students who were not in similar classrooms but had similar beginning mathematics scores. Furthermore, “if a bad year is compounded by other bad years, it may not be possible for the student to recover” (Hanushek, 2010). Highly effective teachers are crucial to the successful achievement of all students.

Unfortunately, research also indicates the lack of ability and competence of teachers to teach mathematics. Studies reveal that elementary teachers and preservice teachers possess a limited knowledge of mathematics, including the mathematics that they are responsible for teaching (Ball, 1990b; Kilpatrick et al., 2001; Ma, 1999). Although early childhood teachers are able to create a classroom environment that includes mathematics (Lee, 2005), they are poorly trained in mathematics instruction, fear mathematics, question the importance of mathematics in the early years, and are inept at mathematics instruction (Clements & Sarama, 2007; Copley, 1999; Lee, 2005; Ginsburg et al., 2008; Schram, Wilcox, Lapan, & Lanier, 1988). Many of the nation’s teachers are not sufficiently prepared to teach mathematics “using standards-based approaches and in ways that bolster student learning and achievement” (National Research Council, 2000, p. 31).

Recognizing the importance of the classroom teacher, the Bush administration established The No Child Left Behind Act of 2002, the reauthorization of the Elementary and
Secondary Education Act (ESEA). This Act called for all teachers to be “highly qualified” by the end of the 2005-2006 school year. According to Title IX, section 9101 of the ESEA, elementary teachers deemed highly qualified must demonstrate competence by passing a rigorous state test of the basic elementary curriculum, hold at least a bachelor’s degree, and have a state license. The U.S. Department of Education acknowledged the importance of classroom teachers, who “are the key to unlocking the potential in every child and finally closing the staggering achievement gap” (U.S. Department of Education, 2004, p. v).

In March 2010 the Obama administration released *A Blueprint for Reform: The Reauthorization of the Elementary and Secondary Education Act*. This Act acknowledged that “every child in America deserves a world-class education” (U.S. Department of Education, 2010, p. 1), with the goal of every student graduating from high school to be “well prepared and college ready” (U.S. Department of Education, 2010, p. 1). With an emphasis on “great teachers and leaders in every school” (U.S. Department of Education, 2010, p. 1), the Obama administration recognized the power of the classroom teacher’s interactions with students as “the primary determinant of student success” (U.S. Department of Education, 2010, p. 13). As a result, individual states were required to define “effective teacher” and develop measures of effectiveness. As states transition to the new requirements, the Obama administration stated that the provisions of the NCLB law relating to “Highly Qualified Teachers” would be maintained “but with additional flexibility” (U.S. Department of Education, 2010, p. 14).

However, the definition of a “highly qualified” teacher is more complex than that described by NCLB criteria. Teachers who are mathematically competent demonstrate many
core elements, three of which are the focus of this study: pedagogical beliefs, mathematics content knowledge, and instructional practices.

**Teacher Competence: Pedagogical Beliefs**

Each teacher holds beliefs that strongly affect behavior (Abelson, 1979; Bandura, 1986; Eisenhart, Shrum, Harding, & Cuthbert, 1988; Nespor, 1985). Research has established that teachers’ beliefs about mathematics teaching and learning are related to their instructional practices, and ultimately, student learning (Beilock, Gunderson, Ramirez, & Levine, 2010; Capraro, 2001; Philipp, 2007; Potari & Georgiadou-Kabouridis, 2008; Prawat, 1992; Ziccardi-Priselac, 2009; Stipek & Byler, 1997; Staub & Stern, 2002; Thompson, 1992; Wright, 1992). Staub and Stern (2002) found that second- and third-grade students whose teachers believed in constructivist instructional practices made larger achievement gains in mathematical word problems than students whose teachers believed in a “direct transmission” view and were as proficient with computation as the comparison group. Fennema et al. (1996) studied changes in teacher beliefs, teacher practices, and student gains over a 4-year period of 21 primary-grade teachers while participating in a professional development program on Cognitively Guided Instruction. The study showed that 17 of the 21 teachers’ final ratings regarding beliefs and instruction were higher, and that the changes in instructional practices were directly related to gains in students’ concepts and problem-solving performance.

Beliefs, a “driving force” (Raymond & Santos, 1995, p. 58) of teacher actions, influence the thousands of decisions made and instructional practices used on a daily basis by a classroom teacher. Within the “entangled domain” (Nespor, 1987, p. 311) of the classroom, impulse and intuition often guide a teacher’s decision—more so than knowledge
Impulse and intuition are grounded in beliefs and are “implicit in teacher discourse, teacher objectives, and teacher practices” (Capraro, 2001, p. 4).

**Teacher Competence: Mathematics Content Knowledge**

Competent teachers must also have a deep understanding of mathematics. Research studies indicate a strong relationship between a teachers’ mathematical knowledge and students’ achievement (National Mathematics Advisory Panel, 2008). Yet, the mathematical knowledge needed for teaching is quite different from the mathematics needed in other relevant professions, such as accounting or engineering. Teachers must know the mathematics content in ways that can be understood by all students.

Shulman’s (1986) presidential address to the American Educational Research Association highlighted that teachers need “pedagogical content knowledge” in order to best support student learning: this includes the ability to understand the mathematics content and ability to teach the mathematics to a diverse population of students. This specialized “mathematical knowledge for teaching” (Ball & Bass, 2000, p. 390) acknowledges that teachers must be able to solve student problems, understand the content in the particular ways applicable for teaching, predict how students are likely to interpret content, and design instruction that takes into consideration both students and mathematical content (Hill, Sleep, Lewis, & Ball, 2007).

**Teacher Competence: Instructional Practices for Young Children**

Many research studies have shown that teacher quality is the key to student achievement. Although these studies have also highlighted the issue that specific characteristics of teachers that are reliably related to student outcomes are difficult to identify (Hanushek, 2010), NCTM (2000) advocates that effective teachers require knowledge and
understanding of mathematics, students, and instructional practices. In years past, there have been various philosophies regarding mathematics instruction which were seen as in conflict one another. Dubbed by the media as “The Math Wars,” these conflicts represent a philosophical discourse about how children should be taught and what children should learn (DeMott, 1962; Schoenfeld, 2004). On one side of the war are those who believe in more traditional instructional practices, based on the behaviorist theory. Students are explicitly instructed in why, when, and how to use the strategies modeled, and they are provided independent practice of the skills taught (Esqueda, 2008). On the other side of the war are those who believe in standards-based reform practices, based on the constructivist theory. Students are provided opportunities to learn through exploration, discourse, and interaction as the teacher guides learning rather than teaches by telling.

Many research studies have been conducted to learn more about the benefits of different types of practices based on these conflicting philosophies (Boaler, 1998; Brewer & Daane, 2002; Buchanan, Burts, Bidner, White, & Charlesworth, 1998; Cobb, Wood, Yackel & Perlwitz, 1992; Simon & Schifter, 1993, Steffe & Wiegel, 1992; Warfield, 2001). Cobb et al. (1992) conducted a follow-up assessment of a second grade project that was based on a constructivist theory of knowledge. They found that students expressed a greater understanding of mathematics and experienced more success in constructivist classrooms than students in traditional classrooms. Buchanan et al. (1998) found that heavy use of didactic methods in kindergarten did not translate into achievement in later primary grades. In 2005, Malofeeva conducted a meta-analysis of mathematics instructional practices implemented in preschool and kindergarten classrooms. This analysis found that preschool
and kindergarten children appear to learn better from a guided child-centered approach than from a direct teacher-centered approach.

Unfortunately, despite research that acknowledges successful instructional practices, classroom instruction has changed very little (Cuban, 1993; Hoetker & Ahlbrand, 1969). Based on research from classroom observations, recitation continues to be the most common form of teaching mathematics. That is, teachers begin a math lesson by reviewing homework, asking yes/no questions in succession while listening for right or wrong answers, present new information through telling and demonstrating, and end by having students independently complete an assignment, practicing the skills that were just shown (Kirkpatrick, Swafford, & Findell, 2001). If we are to foster the development of mathematically proficient citizens, then instruction must move beyond demonstrations of procedures and repeated practice to “creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding” (NCTM, 2000, p. 18).

Statement of the Problem

The importance of mathematics is clear. Calls for research have come from the U.S. Department of Education (National Mathematics Advisory Panel, 2008), the Society for Research in Child Development (The Social Policy Report, 2008), and the National Research Council (Committee on Early Childhood Mathematics, 2009) as follows:


1. “Support research on teacher knowledge and how to enrich it. Research is needed to illuminate how teachers think about learning, how they interpret the individual child’s behavior, how they think critically about their teaching efforts and
children’s learning, and what they understand of both the curriculum and the mathematics underlying it” (p. 16)

2. “Support research on teaching mathematics, specifically teaching mathematics to 4 and 5 year old children” (p. 16)


3. “Research on direct assessments of teachers’ actual mathematical knowledge” (p. 37)

4. “More precise measures should be developed to uncover in detail the relationships among teachers’ knowledge, their instructional skill, and students’ learning, and to identify the mathematical and pedagogical knowledge needed for teaching” (p. 38)

_National Research Council: Committee on Early Childhood Mathematics, 2009_

5. “Research on the role of teachers in providing effective instruction, with special attention to the early childhood setting” (p. 348)

All reports have made specific recommendations for future studies. In particular, early childhood settings have been a focus, particularly since “scant attention to the special challenges of teaching 4- and 5-year-olds” has been addressed by researchers (Ginsburg et al., 2008, p. 16).

Until we make mathematics learning a priority, and until we invest in preparing early childhood educators to be effective math teachers, we can expect avoidance and ineffective practices to continue, and we will continue to be embarrassed by the poor performance of children in the country that has been the world leader in innovation (Stipek, as cited in Ginsberg et al., 2008, p. 13).
Purpose of the Study

The purpose of this study was to describe kindergarten teachers’ pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice and to examine the relationships among them. This study responded to specific requests for research from the National Mathematics Advisory Panel (2008), the Society for Research in Child Development (2008), and the National Research Council, Early Childhood Mathematics Committee (2009).

Research Questions and Hypotheses

This research was investigated using the following research questions and hypotheses:

Research Question 1: What pedagogical beliefs about mathematics do kindergarten teachers hold?

Research Question 2: What mathematical knowledge for teaching do kindergarten teachers possess?

Research Question 3: What instructional practices do the kindergarten teachers use that promote mathematical understanding?

Research Question 4: What are the relationships among beliefs about mathematics, mathematical knowledge for teaching, and instructional practices for the kindergarten teachers?

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.

Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a positive relationship with beliefs.
Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a positive relationship with instructional practices.

Definition of Terms

Beliefs: Beliefs have typically been defined in accordance with the researcher’s purpose of the study, using words such as attitudes, opinions, perceptions, perspectives, and personal theories (Pajares, 1992). For the purpose of this study, definitions from Kagan (1992) and Speer (2002) will be used:

Teacher belief is defined as tacit, often unconsciously held assumptions, conceptions, personal ideologies, and worldviews about students, classrooms, and the academic material to be taught that shape practices and orient knowledge.

Mathematical Knowledge for Teaching: For the purpose of this study, the term “mathematical knowledge for teaching” as defined by Ball, Thames, and Phelps (2008) will be used:

Mathematical knowledge for teaching is the mathematical knowledge needed to carry out the work of teaching mathematics; the ability to understand the math content and ability to teach the math to a diverse population of students. It enables teachers to solve student problems, “understand the content in the particular ways needed for teaching it, understand what students are likely to make of the content, and to craft instruction that takes into account both students and the mathematics” (as cited in Hill et al., 2007, p. 125)

As illustrated in Figure 1.1, the Mathematical Knowledge for Teaching Model is comprised of two types of knowledge, Subject Matter Knowledge and Pedagogical Content Knowledge, and six domains. For the purpose of this study, the Common Content Knowledge domain and the Specialized Content Knowledge domain are of primary focused and are defined here.

Common Content Knowledge (CCK): As defined by Ball et al. (2008):

The mathematical knowledge and skill used in settings other than teaching. This knowledge is used in a wide variety of settings and is not unique to teaching.
Specialized Content Knowledge (SCK): As defined by Ball et al. (2008):

The mathematical knowledge and skill unique to teaching. It involves unique unpacking of mathematics that is not needed—or even desired—in settings other than teaching. This unique mathematical understanding and reasoning enables teachers to make features of particular content visible to and learnable by students, to talk explicitly about how mathematical language is used; to choose, make and use mathematical representations effectively; and to explain and justify one’s mathematical ideas.

Conceptual Framework

Instructional decisions are not made in isolation. Each decision is influenced by multiple factors: including teachers’ beliefs, mathematical knowledge for teaching, and instructional practices. For example, what one believes about how mathematics is best taught will likely affect instructional practices used (Ma, 1999; National Research Council, 2009; Philipp, 2007; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wright, 1992). The knowledge a teacher has about content or a particular concept will likely correlate with how that content or concept is taught (Cai, 2005; Fennema, Franke, Carpenter, & Carey, 1993; Lehrer & Franke, 1992; Hill et al., 2008; Ma, 1999; Raymond, 1997; Spillane, 2000; van den Kieboom, 2008; Warfield, 2001; Weiss & Miller, 2006; Wilkins, 2002). Beliefs about the importance of mathematics will likely affect the allotment of instructional time (Pajares, 1992; Prawat, 1992; Wilkins, 2008). It is also possible that the success (or lack of success) of an instructional practices will influence teacher beliefs (Buzeika, 1996). The conceptual framework in Figure 1.2 illustrates possible interactions among the three factors of teacher beliefs, mathematics content knowledge for teaching, and instructional practices.

Summary

This study responded to the specific requests for research regarding the mathematical and pedagogical knowledge needed for teaching, direct assessments of teachers’ actual mathematical knowledge, and mathematics instructional practices for 4 and 5 year old children. The following review in Chapter II will provide an examination of the research conducted on teacher beliefs, mathematical knowledge for teaching, and instructional practices as well as correlational research conducted among each of these variables.
Figure 1.2. Conceptual Framework

Teacher Competency

- Teacher Beliefs
- Mathematical Knowledge for Teaching
- Instructional Practices
CHAPTER 2
LITERATURE REVIEW

Purpose of the Study

The purpose of this study was to describe kindergarten teachers’ pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice and to examine the relationships among them (see Figure 1.2). This study responded to specific requests for research from the National Mathematics Advisory Panel (2008), the Society for Research in Child Development (2008), and the National Research Council, Early Childhood Mathematics Committee (2009).

Research Questions and Hypotheses

Four major research questions were asked in this study:

1. What pedagogical beliefs of mathematics do kindergarten teachers hold?
2. What mathematical knowledge for teaching do kindergarten teachers possess?
3. What instructional practices do the kindergarten teachers use that promote mathematical understanding?
4. What are the relationships among pedagogical beliefs of mathematics, mathematical knowledge for teaching, and instructional practices for the kindergarten teachers?

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.
Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a positive relationship with beliefs.

Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a positive relationship with instructional practices.

The first part of this chapter will review the literature for pedagogical beliefs, mathematical knowledge for teaching, and instructional practices and examine studies that relate to each of those areas. The second part of this chapter will examine studies that explore the relationships between these variables.

**Variables of Teacher Competency**

**Pedagogical Beliefs**

“Beliefs are the best indicators of the decisions individuals make throughout their lives” (Pajares, 1992, p. 307). They have been regarded as “the most valuable psychological construct to teacher education” (Fenstermacher, 1979, p. 174) and have become widely recognized as a significant influential factor of the teaching and learning of mathematics (Capraro, 2001; Grant, Hiebert, & Wearne, 1994; Kagan, 1992; Pajares, 1992; Quillen, 2004; Speer, 2002; Wilins, 2008; Yates, 2006). Teacher beliefs “can inform educational practice in ways that prevailing research agendas have not and cannot” (Pajares, 1992, p. 307).

Each teacher holds beliefs which strongly affect behavior (Abelson, 1979; Bandura, 1986; Eisenhart et al., 1988; Nespor, 1985). Experiences throughout childhood, from the first time mathematics was introduced, form a personal belief system about what it means to learn and teach mathematics (Fosnot, 1989; Liljedahl, 2008; Uusimaki & Nason, 2004). It is these beliefs upon which teachers build their own practice, filtering new information and experiences through this belief system, and “absorb[ing] it into their unique pedagogies”
A belief system is comprised of information that is considered true, whether actually true or not. Entrenched beliefs are maintained “even in the face of contradictions” (Pajares, 1992, p. 317).

An “elaborate personal belief system” (Kagan, 1992, p. 65) is recognized as a necessary construct for a classroom teacher. Isolation, uncertainty, and need to maintain control are characteristics that teachers face on a daily basis, and beliefs allow teachers to take control, make decisions and solve problems (Ashton & Webb, 1986; Kagan, 1992). Since beliefs are “significant forces affecting teaching” (Speer, 2002, p. 651), it is important that teachers are aware of their beliefs, realize the impact their beliefs make on daily instruction, and challenge their beliefs when changes are needed (Kagan, 1992; Liljedahl, 2008; Prawat, 1992; Thompson, 1992).

Defining beliefs. A clear definition of beliefs does not exist among researchers and educators (Philipp, 2007). The threads of beliefs, values and knowledge are tightly woven, and making distinctions has proved challenging over the years, often resulting in different interpretations and meanings. Values and knowledge are sometimes found to be defined by beliefs, with values viewed as a “type of belief to which one is deeply committed” (Philipp, 2007, p. 268). Beliefs have been described as a true-false dichotomy while values present a desirable-undesirable dichotomy which is viewed as more internalized than beliefs, thus harder to change (Bishop & Seah, 2003). Philipp (2007) takes the stance: “a belief that’ is about beliefs, but ‘a belief in’ is about values” (p. 265).

Although there is not one accepted definition, beliefs have typically been defined in accordance with the researcher’s purpose of the study, using words such as attitudes, opinions, perceptions, perspectives, and personal theories (Pajares, 1992). To believe,
according to John Dewey (1906) was “to ascribe value, impute meaning, assign import” (p. 113) and the stronger the belief, the more it “is held to, asserted, affirmed, acted upon” (p. 113). Beliefs are often described as something that is understood as true (Eisenhart et al., 1988; Richardson, 1996; Strawhecker, 2004); relational to a situation depending on attitude (Eisenhart et al., 1988; Carter & Yackel, 1989); and “mental representations of reality that guide thought and behavior” (Capraro, 2005, p. 3). For the purpose of this study, teacher beliefs are defined as tacit, often unconsciously held assumptions, conceptions, personal ideologies, and worldviews about students, classrooms, and the academic material to be taught that shape practice and orient knowledge (Kagan, 1992; Speer, 2002).

Measuring beliefs. Case studies, self-reported surveys, and questionnaires are often used by researchers to uncover teacher beliefs, making inferences from what is said or done (Pajares, 1992). While case studies capture rich information on a small scale, they are not conducive to large scale studies. Therefore, self-report surveys and questionnaires are often used with larger sample sizes. Self-report surveys have been used with pre-service teachers, such as Hart’s (2002) three-part beliefs survey and the Mathematics Teaching Efficacy Belief Instrument (Enochs, Smith, & Huinker, 2000); in-service teachers, such as the Standards Belief Instrument (SBI) (Zollman & Mason, 1992) and the Elementary Teacher’s Commitment to Mathematics Education Reform: The Self-Report Survey (Ross, McDougall, Hogaboam-Gray, & LeSage, 2003); or both pre-service and in-service teachers, such as The Mathematics Belief Scale (Fennema, Carpenter, & Loef, 1990).

While researchers agree that teacher beliefs play an important role in the classroom, beliefs can be difficult to accurately capture on self-reported questionnaires due to a concern regarding the accuracy and validity of such types of measures (Philipp, 2007). The
interpretation of words, importance of issues, and little context within self-report questionnaires raise concerns for current researchers (Philipp, 2007). When documenting similarities and differences between stated beliefs and instructional practices, it is possible that the surveys or questionnaires used to gather beliefs do not explicitly articulate the understanding and definitions of words used creating a “lack of shared understanding” (Speer, 2002, p. 654). Thus, researchers and participants may have very different understanding of the proffered statements or questions and different meanings for the terms used.

Video clips are an alternative means for assessing beliefs. Speer (2001) determined that video clips provided an avenue through which shared understanding could be built and more accurate attributions of beliefs could be produced as observed classroom practices were discussed. The use of video-clip interviews could be used to “illicit teachers’ thinking and reasoning about their teaching practices [and] provide access to information that is not possible to obtain using typical interviews and/or observations” (Speer, 2002, p. 658).

Ambrose, Philipp, Chauvot, and Clement (2003) developed a web-based survey in an attempt to assess pre-service teachers’ beliefs about mathematics and mathematics understanding and learning as part of the federally funded Integrating Mathematics and Pedagogy (IMAP) project. This tool is comprised of seven segments, using a combination of student work, teaching scenarios and video clips. Pre-service teachers were asked to respond to each context using self-generated words rather than selecting from provided options. Rubrics were designed for scoring responses, thus creating quantitative data using qualitative means.
While the use of video clips for capturing teacher beliefs is promising, it is only in the beginning stages. Although Speer (2002) advocates for using videos to guide conversations, a collection of videos from which researchers can pull is not available from Speer. Video availability from Ambrose et al. (2003) is minimal due to funding issues, personnel changes, and lack of staff to support video clip creation, which have impacted the availability of clips and the growth of the IMAP project.

Overall, best practice indicates that in order to truly understand teacher beliefs, opportunities for open-ended dialogue such as in interviews, classroom observations and/or scenario-style questions provides rich information in addition to data gathered from belief questionnaires (Pajares, 1992).

**Research findings regarding mathematics reform-based beliefs.** Beginning in the early 1980’s, NCTM led a movement to reform mathematics education. This reform addressed the issues regarding traditional mathematics education, which focused primarily on methods and procedures, and advocated for mathematics instruction that focused on sense making through exploration, discourse, and connections based on the constructivist learning theory. As the mathematics community moved towards reform-based mathematics, researchers began to study the beliefs of teachers regarding reform-based mathematics. Recent studies have revealed that teacher beliefs regarding mathematics instruction are often aligned with reform mathematics education (Anderson & Bobis, 2005; Chval, Grouws, Smith, Weiss, & Ziebarth 2006; Paterson, 2009).

Anderson and Bobis (2005) developed a survey and used interviews and observations to explore the beliefs and reform-based teaching practices of K-6 teachers in Australia. The survey consisted of three parts: (a) background information; (b) adaptation of Ross et al.’s
(2003) *Self-Report Survey*; and (c) four open-ended questions. From the forty surveys that were returned, responses indicated that a majority of the teachers supported reform-oriented teaching approaches that promote working mathematically in elementary classrooms.

Chval et al. (2006) also found similar results in their study of 528 K-5 teachers and their reform-oriented beliefs about student learning. Data were collected from a questionnaire developed from items from the 2000 National Survey of Science and Mathematics Education Mathematics Teacher Questionnaire (Weiss, Banilower, McMahon, & Smith, 2001) and the *Self Report Survey* (Ross et al., 2003). Analysis indicated generally reform oriented views about student learning. Although the teachers were from three different school districts, there were no significant differences among the districts.

Paterson (2009) studied survey results from 174 elementary, middle, and high school mathematics teachers to determine their beliefs regarding reform oriented mathematics instruction and instructional practices used. A modified version of the *Teacher Survey* by the Rand Corporation (2003) was developed to capture beliefs (part one) and practices (part two). Results indicated that the teachers in this study held a strong belief in reform oriented mathematics teaching, with the teachers’ responses mean near to or above “5” on the 6-point Likert scale, demonstrating “agree moderately” or “agree strongly” beliefs. Although the teachers were from different grade spans (K-5, 6-8, 9-12), there were no significant differences between the different spans.

While studies indicate that teachers believe in mathematics teaching that is grounded in constructivist theory, research also indicates that there can be inconsistencies between professed beliefs and actual instructional practices used in the classroom. These inconsistencies will be discussed in Part Two of this chapter.
Mathematical Knowledge for Teaching

As focus is placed on the mathematical ability of America’s children, attention is also placed on the mathematical capacity of America’s teachers. New curriculum continues to be developed, and standards have become streamlined and nationalized in an effort to increase student outcomes. However, “little improvement is possible without direct attention to the practice of teaching” (Ball, Hill, & Bass, 2005, p. 14). Mathematics curriculum and standards must be implemented through the ability of competent teachers. Teachers’ understanding of mathematics “is central to their capacity to use instructional materials wisely, to assess students’ progress, and to make sound judgments about presentation, emphasis, and sequencing” (Ball et al., 2005, p. 14). The mathematical competence of a teacher is a key factor that impacts the quality of classroom instruction (Elbaz, 1993; Hill & Lubienski, 2007; Ma, 1999; Koency & Swanson, 2000).

In 2008, the Mathematics and Science Partnership Knowledge Management and Dissemination project examined the benefits of a knowledgeable teacher of mathematics (Miller, Schiavo, & Busey, 2008). The project reported during the March 2008 American Educational Research Association meeting that mathematical knowledge for teaching influences student engagement with mathematics, the evaluation and use of instructional materials, and student mathematical understanding and ability (Heck, 2008).

The project found that teachers with strong mathematical knowledge for teaching were able to engage students in the content by carefully selecting problems that were linked to previous learning and presented in contexts that were familiar to the students (Bright, Bowman, & Vacc, 1998; Fennema et al., 1996; Ma, 1999; Schwartz & Riedesel, 1994; Warfield, 2001). Materials, such as manipulatives and models, were appropriately selected
and furthered students’ understanding of the concept (Chao, Stigler, & Woodward, 2000; Clements & Sarama, 2002, 2003; Fennema, Carpenter, & Lamon, 1991; Geary, 2006; Griffin, Case, & Capodilupo, 1995; Kamii, Lewis, & Kirkland, 2001). Student questions were addressed from a mathematical point of view and turned into learning experiences by discussing them together collaboratively (Malofeeva, 2005; Rowan & Robles, 1998). In Warfield’s (2001) qualitative research of a kindergarten teacher’s experience implementing Cognitively Guided Instruction, the teacher was able to use children’s thinking to carefully select problems and choose subsequent activities to further student learning. In contrast, Warfield (2001) found that teachers who were less knowledgeable tended to focus on algorithms and finding correct answers rather than deepening the mathematical concept at hand.

Evaluation and effective use of instructional materials are also strengths of mathematically competent teachers (Lloyd, 2002; Manouchehri, 1998; Manouchehri & Goodman, 2000). Teachers are able to recognize big ideas associated with a mathematics concept and select appropriate resources, activities and experiences to help teach mathematics with a coherent storyline. Manouchehri and Goodman (2000), in a qualitative study of two seventh grade middle school mathematics teachers, found that teacher mathematical knowledge had the greatest influence on the evaluation and implementation of a seventh-grade standards-based textbook. On the other hand, less competent teachers struggled to identify and understand the big ideas of a mathematical concept and had difficulty unpacking the content into sub-components.

Research projects are beginning to place more emphasis on studying the correlation between mathematical knowledge for teaching and student outcomes. In Hill, Rowan, and
Ball’s (2005) study, statistically significant positive relationships were found for two cohorts of elementary grade students over a three-year period. They found that teachers’ mathematical knowledge for teaching in these two cohorts was significantly related to student achievement gains in both first and third grades. Furthermore, mathematical knowledge was found to be the strongest teacher-level predictor, larger than teacher background variables and average class time for mathematics.

Unfortunately, research studies have indicated that teachers are not sufficiently knowledgeable, particularly those who work with young children (Ma, 1999; Moseley, 2005).

Studies over the past 15 years consistently reveal that the mathematical knowledge of many teachers is dismayingly thin . . . This is to be expected because most teachers—like most other adults in this country—are graduates of the very system that we seek to improve . . . We are simply failing to reach reasonable standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers. This is a big problem, and a challenge to our desire to improve. (Ball et al., 2005, p. 14)

The National Council of Teachers of Mathematics (NCTM) and the National Association for the Education of Young Children (NAEYC) affirm the importance of mathematics and clearly acknowledge the power of the early childhood years as a “vital foundation for future mathematics learning” (NAEYC, 2002). Research has shown that mathematics ability upon entry to kindergarten is a strong predictor of later success, has been found to be a better predictor of later success than early reading ability, and is linked to later success in both mathematics and reading (Duncan et al., 2007; Ginsburg et al., 2008; Wright, Horn, & Sanders, 1997). It is critical that all children have a strong foundation in mathematics from the beginning. Evidence has been provided that the mathematical competence of a teacher is
a key factor that impacts the quality of classroom instruction (Hill & Lubienski, 2007; Koency & Swanson, 2000; Ma, 1999).

In an attempt to address teachers’ lack of knowledge, various solutions have been proposed. These have included more coursework within a degree, a subject-matter major, revamped coursework and professional development that focus on the mathematics within the classroom, or more intelligent teachers from highly selective colleges. However, “the effects of these advocated changes in teachers’ mathematical knowledge on student achievement are unproven or, in many cases, hotly contested” (Ball et al., 2005, p. 16). Simply knowing mathematical content does not ensure good instruction or student learning and requiring more mathematics of prospective teachers will not increase their understanding of the mathematics needed for teaching (Ball, 1990a). Teachers must also know the content in ways that helps students, influences activity and task selection, and meets the needs of all students (Ball & Bass, 2000). In studies that compared the knowledge of elementary teachers versus secondary teachers, preservice versus inservice teachers, and U.S. teachers versus teachers from other countries (Ball, 1991; Ball & Wilson, 1990; Fuller, 1997, Ma, 1999), mathematical concept knowledge of all populations was uniformly low, despite slight differences between various populations (Mewborn, 2001).

The importance of mathematical knowledge for teaching is not a new concern. In fact, “teachers’ knowledge of mathematics was one of the first variables investigated by researchers on teaching in the 1960s” (Ball, Lubienski, & Mewborn, 2001, p. 440; Begle, 1972). However, pinpointing what teachers need to know and its impact on student learning has been continually evolving as researchers’ focus on pure mathematics content knowledge shifted to mathematics content for teaching.
Defining mathematical knowledge for teaching. In 1986, Shulman’s presidential address to the American Educational Research Association introduced the importance of a special kind of mathematics content knowledge that teachers needed to have, known as “pedagogical content knowledge.” This type of knowledge was more than knowing a subject sufficiently. Shulman (1986) stressed the importance of teachers having the knowledge to understand the mathematics content and the knowledge of how to teach it. Since this introduction, researchers have provided various definitions for this term for mathematics and other disciplines, including Ma (1999), who used the term “knowledge packages” (p. 124) and “concept knots” (p. 114) to describe the complexity of teacher knowledge. For the purpose of this study, Ball’s (1990) term, “mathematical knowledge for teaching” will be used.

As defined by Ball and Bass (2000), “Mathematical Knowledge for Teaching” is the mathematical knowledge needed to carry out the work of teaching mathematics. It is the ability to understand the mathematics content and the ability to teach the mathematics to a diverse population of students. This special type of mathematical knowledge for teaching is more detailed than what is typically needed for everyday experiences, and is not considered less knowledge than that needed by other adults. It is a different kind of mathematics knowledge, specialized for the art of teaching (Hill et al., 2005). Mathematical knowledge for teaching acknowledges that teachers must be able to solve student problems, understand the content in the particular ways needed for teaching it, predict how students are likely to interpret the content, and to design instruction that takes into consideration both students and the mathematics (Hill et al., 2007). It enables teachers to be skillful planners, facilitators, and evaluators and it supports the teachers’ ability to do this work “rapidly, often on the fly,
because in a classroom, students cannot wait as a teacher puzzles over the mathematics himself” (Hill et al., 2005, p. 397).

Mathematical Knowledge for Teaching is comprised of two domains: Subject Matter Knowledge and Pedagogical Content Knowledge (Hill et al., 2005) (see Figure 1.1, on page 14). The Subject Matter Knowledge domain is comprised of sub-domains: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Knowledge. These sub-domains address the difference between the mathematics content and skill that is widely known, not unique to teaching, and needed by teachers and non-teachers (CCK) and the mathematical knowledge and skill that is unique to teaching (SCK). This specialized content knowledge enables teachers to unpack mathematics concepts and highlight features of particular content to students; to talk explicitly about how mathematical language is used; to choose, make and use mathematical representations effectively; and to explain and justify one’s mathematical ideas (Ball et al., 2008). Horizon Content Knowledge comprises the ability to envision the connections mathematics makes over time and the ability to make those connections for students (Ball et al., 2008).

Pedagogical Content Knowledge is comprised of sub-domains as well: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). As defined by Ball et al. (2008), KCS combines the knowledge one has about students and the knowledge one has about mathematics. This knowledge enables teachers to anticipate students’ interpretations and misinterpretations of concepts, to select experiences that the students would find interesting and appealing, and to listen carefully to students in order to better understand their thinking. KCT combines the knowledge one has about teaching with the knowledge about mathematics. This knowledge
provides the teacher the ability to carefully plan and sequence instruction; to purposefully select examples, activities, and experiences that will teach specific ideas; to discern appropriate materials and models to use; and to balance the mathematics, knowledge of child development, and effective instructional practices. KCC combines the knowledge about mathematics with the knowledge about curriculum development. This knowledge enables teachers to consider the selection of content, the order in which concepts are presented, and the relevancy of the content for students.

From research work that Ball and colleagues have been conducting for over ten years, certain teacher characteristics necessary for student learning of mathematics have been identified (Ball et al., 2005). Teachers with mathematical knowledge for teaching know the topics and procedures they teach. They know why certain procedures work, particularly standard algorithms, and represent meaning to the students using examples, models, and words in a way that students can understand and make connections. Mathematically proficient teachers also have “specialized fluency with mathematical language” (Ball et al., 2005, p. 21), enabling teachers to use words carefully and symbols accurately. Overall, teaching mathematics “involves mathematical reasoning as much as it does pedagogical thinking” (Ball et al., 2005, p. 21).

**Measuring mathematical knowledge for teaching.** In today’s world, the need to better understand teachers’ mathematical knowledge is part of the current culture, particularly with the call for “highly qualified” teachers, the need to provide evidence of practices, knowledge and skill, and the continued need for the field of education to be viewed as a professional career (Hill et al., 2007). Yet, understanding teacher knowledge has proven
to be more complicated than learning about basic mathematics skills a teacher may have mastered.

Researchers over the years have used both large and small qualitative and quantitative studies in an attempt to capture mathematical knowledge for teaching. Techniques included observations and analysis of teaching practices (Leinhardt & Smith, 1985; Borko et al., 1992); interviews and open-ended tasks, including the use of video clips (Kersting, 2008); multiple choice questions (Ball et al., 2001; Saderholm, Ronau, Brown, & Collins, 2010; University of Louisville, 2008); self reports (Desimone, Porter, Garet, Yoon, & Birman, 2002); and discourse analyses (Sowder, Phillip, Armstrong, & Schappelle, 1998). However, results were mixed. From the 1960’s and 1970’s when teacher knowledge was determined by the number of mathematics courses, majors, and grade point averages to the mixed methods studies in the 1960’s-1980’s that verified the lack of mathematical knowledge of teachers, to the comparison studies during the 90’s, researchers attempting to measure teacher content knowledge realized the “multifaceted and complex issues surrounding teacher’s knowledge” (Mewborn, 2001, p. 29).

With attention on the complexities of mathematical knowledge, Ball and colleagues (2004) created the Mathematical Knowledge for Teaching measure (MKT) through their work with the Learning Mathematics for Teaching (LMT) project at the University of Michigan. Recognizing that there was not a perfect test, the creators sought to devise a measure that separated the “quality of mathematics instruction” from the “quality of the mathematics in instruction” (Hill et al., 2007, p. 150). In addition, they sought to create a measure that was equitable, particularly to people of color, aligned with the research question being investigated, was specific to the knowledge used in teaching children, and was
rigorously validated. As a result, this multiple-choice measure was designed to assess the elementary and middle school mathematical knowledge for teaching accurately and efficiently and provide reliable and valid inferences about individuals’ or groups’ mathematical knowledge for teaching elementary mathematics (Hill, Schilling, & Ball, 2004). Items on number concepts and operations, geometry and patterns, and functions and algebra were written, refined and piloted, and underwent factor analyses and scaling techniques (Hill et al., 2004).

In various studies, the LMT measures were found to link with the mathematical quality of instruction as well as to student achievement (Blunk, 2007; Hill et al., 2005; Hill et al., 2004). During the analysis of almost 3,000 first- and third-grade students, and their respective teachers,

we found that teachers’ performance on our knowledge for teaching questions—including both common and specialized content knowledge—significantly predicted the size of student gain scores, even though we controlled for things such as student SES, student absence rate, teacher credentials, teacher experience, and average length of mathematics lessons (Ball et al., 2005, p. 44).

Results indicated that a direct measure of teacher’s content knowledge for teaching “trumps proxy measures such as courses taken or experience” (Hill et al., 2005, p. 400). Teachers who were able to correctly answer more questions on this measure had students who gained more over the course of a year than students from classrooms with teachers who did not answer as many correctly (Hill et al., 2005). Students whose teachers scored in the top quartile of the measure showed gains that “were equivalent to that of an extra two to three weeks of instruction” (Ball et al., 2005, p. 44); thus, teachers with strong mathematical knowledge for teaching may help close the socioeconomic achievement gap (Ball et al., 2005). Furthermore, because this study had a positive effect on student gains in the first
grade suggests that “teachers’ content knowledge plays a role even in the teaching of very elementary mathematics content” (Hill et al., 2005, p. 399).

When attempting to research mathematical knowledge of teaching, Hill et al. (2007) remind researchers that an in-depth research study must embrace a multiple-approach to the investigation. When a possibility, a variety of measures help to capture the intricacies of teacher’s knowledge and abilities and for establishing possible relationships to other domains and student learning (Hill et al., 2007, p. 151). With careful consideration, planning, and implementation, research is able to capture mathematical knowledge for teaching through measures that “honor and define the work of teaching, ratify teachers’ expertise, and help to ensure that every child has a qualified teacher” (Hill et al., 2007, p. 150).

In order for an elementary teacher to be a competent teacher of mathematics, it is clear that mathematical knowledge for teaching is critical for student success. “Whether students will become proficient in mathematics depends in large part on the instruction they receive” (Kilpatrick et al., 2001, p. 25). Although a majority of the mathematical knowledge studies have been focused on grades other than kindergarten, evidence indicates that student learning, regardless of the grade, is highly dependent on the highly-qualified teachers’ capacity as an instructor, facilitator, and evaluator of students, curriculum, and content.

**Research findings regarding mathematical knowledge for teaching.** Many of the research studies measuring mathematical knowledge for teaching compare knowledge to other factors, such as instructional practices and student outcomes. Such studies will be further discussed in Part Two of this chapter. Fewer studies have described teacher knowledge in relationship to strands of mathematics, such as number and geometry (Lee, 2010; McCray, 2008; Schwartz & Reidesel, 1994).
Schwartz and Reidesel (1994) explored the relationship between mathematics content knowledge and pedagogical beliefs in teaching among 140 K-5 pre and in-service teachers. Mathematical content knowledge was assessed using a 30-item measure that focused on conceptual understandings. Statistically significant results indicated that the easiest items on the content measure consisted of geometry concepts. Those that were most difficult were those that addressed number sense. In direct opposition, Lee (2010) studied the pedagogical knowledge of mathematics of 81 kindergarten teachers. His analysis from data collected from *The Survey of Pedagogical Content Knowledge (PCK) in Early Childhood Mathematics* (Smith, 1998, 2000) indicated that the teachers’ knowledge about number sense was greater than that of geometry.

McCray (2008) studied mathematics pedagogical content knowledge of 26 Head Start preschool teachers from a large Midwestern city. Using an interview measure she created, she found no significant relationships regarding number and operations items and shape and spatial relations items. The teachers’ scores indicated that they did not significantly perform better or worse on number and geometry concepts.

Mewborn (2001) consolidated findings of numerous studies conducted to explore mathematical knowledge for teaching of elementary pre-service teachers.’ Her collection of research revealed that conceptual understandings of number was often lacking, although they typically had a grasp of facts and algorithms. Specifically, studies found that teachers struggled with quotitive division (as opposed to partitive division), rational numbers including the difference between ratios and fractions, and area and perimeter.

**Conclusion.** While direct comparisons cannot be made between these studies, particularly since different assessment measures were used, they do highlight the fact that
additional research is needed to understand the content knowledge of teachers. By analyzing teachers’ strengths and weaknesses of the various mathematics strands, teacher educators and curricula developers would be better informed as they work with and support the development of teachers’ mathematical knowledge for teaching.

**Instructional Practices for Young Children**

Intertwined within “mathematical knowledge for teaching” is knowledge of instructional practices to support mathematics learning. While NCTM (2000) recognizes that effective teaching requires knowledge and understanding of mathematics, students, and instructional strategies, it also recognizes that there are various philosophies regarding math instruction which often oppose one another. These contradicting beliefs, known as “The Math Wars,” represent a philosophical discourse about how children should be taught and what children should learn. These “wars” are often pitted as a dichotomous relationship, with one side as the traditionalists and the other side as the constructivists. However, it is important to note that there is a wide continuum of practices across the views. For the purpose of this research, a general definition of the two extremes of the continuum will be provided and attention will be placed on practices that research have found to benefit children’s understanding of mathematics.

**Defining: Traditionalist theory.** It is the traditionalists’ belief that students learn best from the teacher, the expert who ultimately knows what the less educated students should know and be able to do (Kohn, 1996; Marshall, 2002). Often referred to as teacher-centered, teacher-directed, and didactic instruction, information deemed relevant by the teacher is broken down into parts and disseminated to the students through models and explanations (Mikusa & Lewellen, 1999). Students are explicitly instructed why, when, and
how to use the strategies modeled and provided independent practice of the skills taught (Esqueda, 2008).

Traditionalists base their beliefs on the behaviorist theory. This theory is concerned with the proper use of “shaping” with reinforcement to promote the development of new behaviors (Zanden, Crandell, & Crandell, 2007). Gagne’s instructional theory is also present in the traditionalists’ philosophy of education. Central to his instructional theory is the view that intellectual skills or tasks are broken down into smaller parts, organized in a hierarchy according to complexity, and presented as instructional steps with pre-requisite skills (Zanden et al., 2007). Based on these theories, traditionalists advocate for the creation of behavioral goals and objectives, tightly sequenced curriculum, and assessment that is focused on measurement of knowledge and skills.

The traditionalist’s classroom has often been analogous to a factory, where instruction is catered to mass production, teaching is rote, and rigid academic agendas are followed (Askew, 2005/2006; Day, 1999). Control is established by isolating children by lining desks up in rows, prohibiting communication and collaboration between students (Cooper & White, 2004), and using rewards and punishments (Kohn, 1996; Marshall, 2002). Student choice and the use of hands-on concrete learning experiences are absent (Askew, 2005/2006).

The traditionalist teachers are “dispensers of disconnected knowledge in the form of facts and procedures” (Prevost, 1996) as knowledge is directly transmitted from the teacher to the student as sets of established facts, skills, and concepts (Clements & Battista, 2002; Mikusa & Lewellen, 1999). Use of repetition, rote memorization, drill-and-practice, use of workbooks and worksheets are commonly used practices (Buchanan et al., 1998; Stipek, 1993). The learning progress is examined regularly in tests designed by the teacher (Cronje,
The role of the student is to conform to teacher expectations, listen to the expertise, respond appropriately when asked, and to learn by rote (Atherton, 2009).

**Defining: Constructivist theory.** On the other side of the “war” are educators, scholars and researchers who base their opinions on the theories of constructivism. While “there are as many varieties of constructivism as there are researchers” (Ernest, 1995, p. 459), the central idea of constructivism is that human learning is actively created and built upon previous learning (Atherton, 2009). Often referred to as student-centered, child-centered, and learner-centered, constructivism is based on the notion that children build their own understanding of the world by the things that they do (Mooney, 2000). Two most regularly recognized types of constructivism are cognitive and social constructivism.

Cognitive constructivists, originating with Dewey, Piaget, and Montessori, believe that a child’s interaction with the environment creates learning (Mooney, 2000). Rather than being given explanations by adults, cognitive constructivists believe that children learned best by satisfying their curiosity by actively doing the work themselves. Using real-life materials and experiences, children create understanding by working with materials and/or concepts, experimenting and thinking independently. These experiences construct and reconstruct schema gradually (Wadsworth, 2003) as children move in and out of a state of equilibrium as new information is adapted into current understanding.

As children learn from these experiences, Piaget also believed that children’s cognitive and intellectual changes progressed through a series of developmental stages. These four stages of cognitive development addressed behaviors of children of particular ages and how these behaviors related to the development of cognition. Moving from the sensorimotor stage at birth to 18 months, when children learn primarily through senses,
reflexes and materials, children 18 months to six years of age are of the preoperational stage. At this stage, of which kindergarten students are a part, children often form ideas based on their perceptions. They typically focus on one variable at a time and can over-generalize concepts, forming conclusions based on their limited experiences (Mooney, 2000). Because direct experiences are such an important part of children of this stage, Piaget and other constructivists stress the importance of providing large blocks of uninterrupted time to allow students to become absorbed, interested and involved in their work. As children work, constructivists also emphasize the importance of supplying open-ended activities and strategic questioning to foster inquiry, wonder, and thinking (Wadsworth, 2003).

Vygotsky, the father of social constructivism, also believed in learning by doing. But he found that personal and social experience could not be separated (Mooney, 2000). He believed that children’s understanding of the world came from their social surroundings, particularly from other children. Vygotsky viewed interaction through play as the vehicle through which young children construct understanding. “Language becomes the tool for play” (Bodrova & Leong, 1996, p. 57) with meanings and understanding emerging from these social encounters (Atherton, 2009). Vygotsky also differed from Piaget’s belief about developmental stages. He believed that children were able to learn skills or ideas that they had not yet come to learn on their own though careful scaffolding by adults or peers (Mooney, 2000). He viewed learning as a continuum, rather than a scale, and the zone of proximal development identified when a child could work independently, when assistance was needed, and when a concept was out of the reach of the learner.

Constructivism is the foundation of standards-based reform mathematics (NCTM, 2000). The reform framework recognizes that students must be able to understand
mathematics and be presented different learning opportunities in order to succeed (Stiff, 2001). In order for students to “solve problems, reason logically, compute fluently, and use it to make sense of their world” (Kilpatrick et al., 2001, p. 432), teachers use reform-based instruction to provide opportunities to learn through exploration, discourse, and interaction, with solid tasks that are well planned and implemented.

The constructivist philosophy is also the foundation of “developmentally appropriate practice,” the term NAEYC and the early childhood field use to described high quality education for children birth through eight years of age (Maxwell, McWilliam, Hemmeter, Ault, & Schuster, 2001). The developmentally appropriate practice (DAP) framework “outlines practice that promotes young children’s optimal learning and development” (Copple & Bredekamp, 2009, p. 1). Grounded in constructivist thought, the framework articulates best practices for young children, with the overall goal to provide rich and engaging experiences which ultimately establishes a solid foundation for the future.

A constructivist, developmentally appropriate, classroom is child-centered. An engaging environment of respect, acceptance, compassion, and dignity provide the framework for a caring community of learners (Brown & Campione, 1994; Cooper & White, 2004; Copple & Bredekamp, 2009; Knight, 2001; Patrick, Turner, Meyer, & Midgley, 2003). Within this community, all domains of development are fostered through socially and culturally rich experiences that shape developing dispositions towards learning. Through a guided approach, children are provided opportunities to construct new ideas and are led to new understandings. Using play as a vehicle, children are encouraged to interact, explore, discuss, observe, predict, discover, and revisit ideas and concepts in a variety of ways (Banks, 2001; Bowman, 1998; Day, 1995) while being challenged to achieve at a level just
beyond their reach (Copple & Bredekamp, 2009). Time to allow mathematical concepts to
develop is provided through both focused mathematics times and through everyday
encounters (Copple & Bredekamp, 2009) providing opportunities to reason, problem-solve,
communicate and foster understanding of all strands mathematics with particular attention to
number, geometry and measurement (Copple & Bredekamp, 2009).

Constructivist teachers are facilitators of student inquiry and thinking, often following
the children’s interests (Wakefield, 1998). As a guide, the teacher supports students’
“invention of viable mathematical ideas rather than transmitting “correct” adult ways of
doing mathematics” (Clements & Battista, 1990, p. 7). Because the constructivist teacher
recognizes that students learn in unique and individual ways, curriculum is interpreted with
student needs in mind, “build[ing] on children’s intuitive, informal notions and encounters
relating to math” (Copple & Bredekamp, 2009, p. 239), with student understanding as the
focus of assessment (Atherton, 2009; Day, 1999; Wright, 2006). An importance on the
learning and development of the whole child is placed as instruction is integrated across all
domains of learning (physical, emotional, cognitive, etc.) (Copple & Bredekamp, 2009).

Challenging each student at his/her level, the teacher understands that learning and
development occurs through well established sequences, but at different rates. “Teachers
reflect on the developmental progression of children’s thinking to understand the wide range
of thinking patterns of students in a class and to plan tasks for groups and individuals”
(Clements, 1997, p. 2). Taking advantage of optimal times for growth, the classroom teacher
plans curriculum to achieve important goals using data from assessments, families, and other
educators involved with the children’s development and learning (Copple & Bredekamp,
2009; Day, 1999). Students are given autonomy in selecting and completing tasks, which are
challenging but achievable (Atherton, 1999), supported by guiding questions to focus students’ attention on a particular concept or idea (Clements & Battista, 2002). Active learning, social experiences, and choice are common practices used by a constructivist teacher.

The role of the student in a constructivist classroom is to share responsibility for learning with the teacher. The student participates in a classroom learning environment, listens to others, and uses ideas of others to inform personal ideas and decisions. “Learning occurs as an individual constructed act in a milieu of social interaction and negotiation” (Atherton, 2009, slide 8).

**Research findings regarding traditional and constructivist instruction.** In light of the Math Wars, numerous studies have addressed these philosophical differences. Although the traditionalists believe that the constructivist views “ignore the importance of fundamental building blocks, are not sufficiently rigorous, do not cover aspects of mathematical content that are necessary for proficiency, and over-generalize the role of curiosity and discovery as core principles of mathematics learning” (Whitehurst, 2004, p. 1), a majority of the research results favor a constructivist philosophy (Burts, Hart, & Charlesworth, 1993; Cobb et al., 1992; Neuharth-Pritchett, 2001; Raths, 2001; Steffe & Wiegel, 1992; Stipek, Feiler, Daniels, & Milburn, 1995; Zambo & Zambo, 2007). Research suggests that young children learn better through a guided student-centered approach rather than a direct teacher-centered approach (Guarino, Hamilton, Lockwood, Rathbun, & Hausken, 2006; Malofeeva, 2005), which positively correlates with later grade-level achievement, displaying long-term effects (Burts et al., 1993; Charlesworth, Hart, Burts, & DeWolf, 1993).
Students in this type of classroom had mathematics achievement scores that were significantly better than teachers not using standards-based reform methods, had a greater understanding of mathematics, and were more successful in mathematics than those in traditional classrooms (Campbell, 2009; Cobb et al., 1992). Students, who consistently learned from mathematics reform curricula, significantly outperformed students learning from traditional curricula with respect to conceptual understanding and problem solving (Schoenfeld, 2002). Furthermore, there were no significant differences between both sets of students on tests of basic skills (Schoenfeld, 2002).

Overall, constructivist developmentally appropriate practices have “more positive academic, motivational, emotional and behavioral outcomes” (Bryant, Clifford, & Peisner, 1991; Stipek et al., 1995) and less recommended retentions (Neuharth-Pritchett, 2001). Students in developmentally appropriate classrooms had higher expectations for their success in school, chose more challenging mathematical problems to solve, showed less dependency on adults for permission and approval, exhibited more pride in their accomplishments and worried less about school (Stipek et al., 1995).

Although some studies have noted benefits of a traditionalist approach with low-performing children and children with special needs (Baker, Gersten, & Lee, 2002; Bodovski & Farkas, 2007; Kroesbergen, Van Luit, & Mass, 2004), other studies examining specific instructional practices often found traditionalist classrooms not beneficial to mathematics learning. Educators implementing mathematics reform projects found that practicing procedures and asking students to solve many problems did not support the development of understanding (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Fennema et al., 1996; Hiebert & Wearne, 1993). This is
further supported by a substantial body of research demonstrating that children are able to solve mathematical problems, including word problems, without direct instruction (Carpenter et al., 1993).

Other studies have highlighted concerns about children’s learning and development in a traditionalist setting. Hyson, Hirsh-Pasek, and Rescorla (1990) and Stipek (1993) found long-term negative effects of didactic practices on motivation and personality. The didactic practices used negatively impacted some children’s self-confidence and desire to engage in learning activities (Stipek, 1993) and a “loss of creative expression and emotional well-being” (Hyson et al., 1990, p. 67) despite any academic growth that may or may not have occurred (Hyson et al., 1990). Male students in didactic classrooms displayed more stress than males in developmentally appropriate classrooms, and African Americans exhibited more stress in didactic classrooms during transitions, waiting, and whole-group activities (Burts et al., 1993). White students exhibited more stress during group story time in didactic classrooms (Burts et al., 1993). In addition, Zambo and Zambo (2007) discovered that when a teacher uses “rote and dry” learning, “the brain will habituate . . . become used to . . . the input and tune it out, firing brain cells less frequently” (p. 268).

Schweinhart and Weikart (1997) found a lack of long-term benefits of direct instruction. Children who attended a traditionalist preschool outperformed children in a child-initiated program during and up to a year after the preschool program but did not maintain that performance beyond that time. In fact, children who attended academic preschools earned significantly lower grades by the end of sixth grade than those students in developmentally appropriate classrooms. In addition, heavy use of academic, didactic methods in kindergarten did not translate into achievement in later primary grades.
With “considerable evidence that the promises of reform mathematics are real and the
fears of the anti-reformers unjustified” (Swafford, 2003, p. 458), many researchers and
mathematics experts view constructivism as the way that mathematics education could be
transformed (Hiebert, 2003; Steffe & Wiegel, 1992; Swafford, 2003). “Presuming that
traditional approaches have proven to be successful is ignoring the largest database we have”
(Hiebert, 2003, p. 13). Recognizing that there is significant research to inform instruction,
NCTM and the National Research Council also stress that instructional practices “should not
be based on extreme positions that students learn, on the one hand, solely by internalizing
what a teacher or book says or, on the other hand, solely by inventing mathematics on their
own” (Kilpatrick et al., 2001, p. 409). Instead, classroom instruction should support the
development of mathematical proficiency for all (Kilpatrick et al., 2001). The teacher must
consider various factors including the child’s background knowledge and culture, the
mathematical concepts and skills, and sound instructional practices that best meet the
individual needs of the students. Children, rather than labels, should drive instruction.
Research based mathematically-rich instructional practices should be at the core of those
instructional decisions.

**Defining: Mathematically-rich instructional practices.** Classroom teachers are
pivotal to student success. “Substantial differences in mathematics achievement of students
are attributable to differences in teachers” (National Mathematics Advisory Panel, 2008, p.
35). Research studies have found that effective teachers of mathematics make intentional
instructional decisions, engage students in learning, foster dialogue and communication, and
provide time for practice and reflection.
Mathematics proficiency is promoted when intentional instructional decisions are based on the teacher’s knowledge and understanding of each student in the class (Copley, 2000; Shouse, 2001). This knowledge is often gathered during instruction as teachers use various assessment strategies designed to identify strengths and needs (Baker et al., 2002; Charles, 2005; Clements & Sarama, 2003; Copley, 2000; Day, 1999; Fennema et al., 1991; Geary, 1994; Griffin et al., 1995). Plans are carefully and strategically made based on gathered data, addressing particular objectives while building understanding of the big ideas of mathematics (Hiebert & Carpenter, 1992; Ma, 1999; NCTM, 2000). The complexity of selected activities and experiences are considered and the sequence of objectives and tasks is deliberate, impacting student achievement at all levels (Baker et al., 2002; Curtis, 2000; Shouse, 2001). In addition, a sufficient amount of time spent on mathematics is also correlated with gains in student achievement, depending on the quality of instruction that occurs during the allotted time (Anderson & Walberg, 1994; Bodovski & Farkas, 2007; Guarino et al., 2006; Walberg, Niemiec, & Frederick, 1994). Strong teachers of mathematics seize opportunities to practice and apply concepts and skills throughout the day by integrating learning into routines and daily activities (Arnold, Fisher, Doctoroff, & Dobbs, 2002).

Competent teachers of mathematics engage students in learning by acknowledging preconceptions and building on existing knowledge (National Research Council, 2005) by providing learning experiences that are “concrete and relevant to their own lives” (Day, 1995, p. 316). In addition, competent teachers use meaningful resources to support such engagement. Strategies effective teachers use to engage students include beginning a lesson or experience using the various senses of sight, sound, touch, smell, and taste to align with
the sensory cortex of the brain where processing begins (Zambo & Zambo, 2007); using situations that are personally meaningful to children (Anderson, 1997; Clements & Sarama, 2003; Fennema et al., 1991; Geary, 1994; Griffin et al., 1995; Zambo & Zambo, 2007); and utilizing multiple grouping settings, such as small group, whole group, multiage grouping, and mixed achievement (Copley, 2000; Day, 1999; Guarino et al., 2006). Particular resources and materials have been found to benefit student learning by engaging children’s curiosity and interest, allowing for practice and exploration, and solidifying mathematical understandings. The use of manipulatives, when used to help students actively construct knowledge, has enormous empirical support in fostering student learning (Chao et al., 2000; Clements & McMillen, 1996; Clements & Sarama, 2002, 2003; Day, 1995; Fennema et al., 1991; Geary, 1994; Griffin et al., 1995; Kamii et al., 2001; Shouse, 2001). Blocks, often found in preschool and kindergarten classrooms (Taylor-Cox, 2009), technology (Clements & Sarama, 2003; Fennema et al., 1991; Geary, 1994; Griffin et al., 1995) and the use of children’s literature (Sarama & Clements, 2006; Whitin, Mills, & O’Keefe, 1994; Wilburne et al., 2007) are also useful tools to engage, support, and develop conceptual understanding and learning of mathematics.

Student engagement is also high when a problem-solving approach is used during instruction. Solving problems encourages children to make choices and use a variety of strategies, requiring students to use a greater degree of reflective and analytic thought (Hiebert & Wearne, 1993). “Classrooms where mathematics is taught as anything less bore this part of the brain” (Zambo & Zambo, 2007, p. 269). However, the “nature of the discourse around a problem is critical” (Franke, Kazemi, & Battery, 2007, p. 229). Research indicates that the use of appropriate, well-designed and timely questions help children make
important connections needed for understanding (Charles, 2005; Malofeeva, 2005; Rowan & Robles, 1998). This type of questioning extends beyond the use of recall questions and encourages the student to explain his/her thinking, express ideas and actions verbally, describe strategies in detail, justify decisions made, and draw connections across strategies (Charles, 2005; Copley, 2000; Fraivillig, Murphy, & Fuson, 1999; Franke et al., 2007; Yackel et al., 1990). When carefully asked, questions become an important tool to help scaffold student learning (Baker et al., 2002; Curtis, 2000). They provoke students to reason and reflect about their actions, decisions, and strategies (Van de Walle, 2005). The use of effective questioning techniques allows the intricacy of the mathematics within problems to emerge.

In addition to careful questioning, knowledgeable teachers of mathematics orchestrate classroom discourse (Hufferd-Ackles, Fuson, & Sherin, 2004), fostering dialogue and communication among the students and with the teacher. The teacher carefully sets the stage for encouraging students to participate, promotes understanding through scaffolding and questioning, uses mistakes as learning opportunities, and encourages students to question and clarify (Stein, 2007). Such opportunities enhance students’ mathematical reasoning and problem-solving abilities as they share solutions, explain thoughts, challenge ideas, and provide explanations (Kilpatrick et al., 2001). When teachers create a mathematical community that provides such experiences, students learn “the essence of what it means to do mathematics” (Stein, 2007, p. 288).

Allowing time for practice, transfer, reflection, and further explanation are also characteristics of excellent mathematics instruction (Clements & Sarama, 2003; Fennema et al., 1991; Geary, 1994; Griffin et al., 1995; Zambo & Zambo, 2007). Appropriate practice
leads students to automaticity, enabling students to tackle more complex tasks due to the less mental effort on particular parts of a task (Kilpatrick et al., 2001, p. 351). This occurs as peers interact with one another and exchange ideas, predictions, and outcomes applying knowledge and skills as problems are solved. (Baker et al., 2002; Clements & Sarama, 2003; Curtis, 2000; Fennema et al., 1991; Geary, 1994; Malofeeva, 2005; Weidinger, 2006).

**Measuring instructional practices for young children.** Effective teaching impacts student success. Therefore, the need to identify strengths and areas of need for teachers is important in improving instructional practices in the classroom (Danielson & McGreal, 2000; Shinkfield & Stufflebean, 1995). Formative and summative assessments of teacher evaluations “can identify and measure the instructional strategies, professional behaviors, and delivery of content knowledge that affect student learning” (Mathers, Oliva, & Laine, 2008, p. 1). Measures used to investigate instructional practices used by teachers of young children have varied over the years. Principals and other school administrators have typically used lesson plans, direct classroom observations, self-assessments, portfolio assessments, student achievement data, and student work-sample reviews (Mathers et al., 2008). The focus of this study will be on observation based measures for kindergarten classrooms.

Classroom observations can provide a researcher with rich information about instructional practices (Mujis, 2006). However, in order for observations to be effective, they must be conducted by trained observers (Shannon, 1991) for a relatively long amount of time in order to have stronger validity (Cronin & Capie, 1986). They must also be aligned with the purpose of the observation. The selection of an observational measure is thus dependent on the student population, purpose for the observations, and area of interest (Snow & Van Hemel, 2008).
Most existing measures assess the social environment well and the learning environment at a very general level, but only a few adequately assess practices designed to teach academic or social skills specifically” (Snow & Van Hemel, 2008, p. 149). In addition, most early childhood measures address the birth through five-year-old preschool population (Halle & Vick, 2007), and elementary school measures are typically the state designated evaluation measure leaving Kindergarten in a unique position in education, straddling the worlds of preschool and elementary school (Graue, 2006). Furthermore, while there is a measure for early language and literacy for the K-3 classroom (ELLCO; Smith, Brady, & Clark-Chiarelli, 2008), there is not a measure that is designed to specifically address mathematics instruction in the kindergarten classroom.

Fortunately, there are several observation tools that can be beneficial to gaining information about the general classroom environment, teacher-student interactions, and instructional practices used in kindergarten programs. The Assessment of Practices in Early Elementary Classrooms (APEEC) is designed to evaluate the developmentally appropriate practices in the K-3 classroom (Hemmeter, Maxwell, Ault, & Schuster, 2001). This instrument measures the physical environment, instructional context, and social context with a scale of 1 (inadequate) to 7 (adequate). While it provides general information about K-3 classrooms, it does not measure specific curriculum content or in-depth teacher-child interactions (Halle & Vick, 2007).

The K-3 version of the Classroom Assessment Scoring System (CLASS) addresses quality instruction with an emphasis on the teacher-student interactions that occur in the classroom rather than the physical environment (Smith et al., 2008). Ten dimensions are organized into three domains of interaction: Emotional Support, Classroom Organization,
and Instructional Support. For three hours, trained users observe and score occurs in 20-minute cycles for a minimum of two hours. Although previous versions did include numeracy and mathematics instruction as one of the constructs, the current measure does not.

The FirstSchool Snapshot (Ritchie et al., 2010), formerly known as the Emerging Academics Snapshot, attends to the social and academic experiences of individual children in early childhood classrooms. Using time-sampling, it is an observational instrument “designed to describe children’s exposure to instruction and engagement in academic activities, as well as to describe activities and adult responsive involvement” (Snow & Van Hemel, 2008, p. 166). Trained data collectors randomly select four children for sampling, two boys and two girls. The snapshot sampling occurs in succession, as the data collector observes student A for 30 seconds and then codes for 40 seconds, followed with student B, student C, etc. When data are collected on the selected children, the observer begins again with student A and continues throughout the school day. In a typical six hour day, this results in 90 observations of each child. This instrument identifies specific curriculum content, including four strands of mathematics: number, geometry, algebra, and time. Through analysis of the observation data, a researcher can pinpoint the activity settings, child behaviors, and teacher-child engagement observed in relationship to a specific content area, such as mathematics.

Research findings regarding instructional practices for young children.

Numerous studies have examined the types of instructional practices and teacher-child interactions used in early childhood classrooms (Bryant et al., 1991; Early et al., 2010; La Paro et al., 2009; Pianta, La Paro, Payne, Cox, & Bradley, 2002; Schwartz & Reidesel, 1994). Using observational data collected from the National Center for Early Development
and Learning’s (NCEDL) Multi-State Study of Pre-Kindergarten, Paro et al. (2009) studied
the quality of learning opportunities in 730 kindergarten classrooms in six states, one of
which was the state of the current study. Three measures were used to collect a varying
degree of information regarding learning opportunities: the Early Childhood Environment
Rating Scale–Revised (ECERS-R, Harms, Clifford, & Cryer, 1998); the Classroom
Assessment Scoring System (CLASS; Pianta, La Paro, & Hamre, 2008), and the Emerging
Academics Snapshot (Ritchie, Howes, Kraft-Sayre, & Weiser, 2001). Overall, the findings
indicated that high-quality learning opportunities in kindergarten are minimal. The students
experienced more didactic instruction which incorporated a large amount of whole group
instruction, seatwork, recall practices, and worksheets and less use of small group or
Choice/Centers type situations which provided opportunities for critical thinking, prediction
and the development of understanding. Of the amount of time in which mathematics was
addressed ($M = .11, SD = .05, Range = .00 - .32$), the study found that teachers typically
taught numeracy and literacy concepts in isolation, relying on these described traditional
instructional practices. In addition, classrooms in which the teacher-child ratio was larger,
less percentage of time was spent in math and small groups. Furthermore, the study found
that 39% of the average kindergartener’s day was spent with no instructional opportunity.
The authors note that even if children were washing hands or walking down the hallway,
opportunities for learning could occur if the teacher supported them. However, for a large
percentage of the day, kindergarten students were found to not be engaged with any
intentional learning.

In a study of 223 kindergarten classrooms in three states, one of which was the state
of the current study, Pianta et al. (2002) studied the classroom environment, classroom
activities, and teacher-child interactions. Although the classroom observations revealed a wide range of percentages for each category (at times, 0-100%), overall results from the data collected using the *Classroom Observation System for Kindergarten* (COS-K) found that the average kindergarten student spent 44% of the observation time in teacher-directed activities and whole group instruction. 18% of the time was spent in Centers while 17% of the time was spent on seatwork. The researchers discussed the variance across the classrooms observed in three states, noting that “there was no typical kindergarten classroom” (p. 235), indicating large inconsistencies between kindergarten programs.

Bryant et al. (1991) studied the use of developmentally appropriate practices in 103 randomly selected kindergarten classrooms from the state in which the current study was conducted. Using data collected from ECERS-R observations and NAEYC guidelines, they found that only 20% of the classrooms were developmentally appropriate. Observers noted large use of time spent in “whole-group, didactic instruction, frequent use of worksheets and rote learning exercises, and little emphasis on small-group or individualized instruction or hands-on and child-chosen activities” (p. 798).

Schwartz and Reidesel (1994) explored the relationship between mathematics content knowledge and pedagogical beliefs in teaching among 140 K-5 pre- and in-service teachers. Data regarding teacher practice was collected from self-reports in which teachers identified vignettes that most reflected their classroom instruction. Data from the instructional self-report survey indicated that while teachers typically embraced the vignettes that were most reform-oriented, the teachers were less likely to discount the vignettes that were more didactic. In particular, the didactic vignettes that were appealing to the teachers contained examples of the teacher showing how to use key words to solve problems and how to find
common denominators by following a carefully-worded list of steps. Thus, while teachers expressed approval of some reform-oriented practices, they also expressed approval of non-reform practices.

Early et al. (2010) studied how time was spent in 652 state funded pre-kindergarten programs across 11 states. Data were collected from results on a family questionnaire and from the *Emerging Academics Snapshot* (Richie, Howes, Kraft-Sayre, & Weiser, 2001) during two major studies: the National Center for Early Development and Learning’s (NCEDL) Multi-State Study of Pre-Kindergarten and the Study of State-Wide Early Education Programs (SWEEP). Findings indicated that children (*n* = 2061) spent more time with language/literacy, social studies and art experiences and less time in mathematics activities. Mathematics experiences were observed mostly during teacher-assigned settings than during Free Choice opportunities. Overall, didactic teaching interactions were more than three times more likely to occur than scaffolded interactions. During the teacher-assigned settings, of which mathematics was most commonly found, didactic teacher interactions “were especially prevalent.” The authors acknowledge that effective teachers use a variety of strategies, of which didactic is one. However, the authors also acknowledge the importance of scaffolding interactions and the need to have a high proportion of scaffolding occurring in a variety of settings. Yet, the data in this study indicated that this was not occurring in the classrooms studied.

**Conclusion.** Although numerous research studies describe instructional practices that are beneficial to student learning of mathematics, studies indicate that teachers of young children typically do not implement such practices. Continued focus on barriers to applying
research based practices, and strategies for supporting teachers in implementing such practices are needed in order to help move students toward mathematical competency.

Relationships between Teacher Competency Variables

Beliefs about the teaching of mathematics, mathematical knowledge for teaching, and instructional practices are important constructs of teacher competency. They form the basis for decision making, inform instruction, and impact student learning. While each on its own is an important construct, they do not exist in isolation of one another. Rather, one influences the other, and all are brought into play in some degree by the classroom teacher every moment of every day.

Relationships between Beliefs and Instructional Practices

Teacher beliefs directly affect the actions taken in the classroom regardless whether they are consciously acknowledged or not (Fazio, 1986; Fisbein & Ajzen, 1975; Quillen, 2004; Raymond & Santos, 1995). They “lie at the very heart of teaching” (Kagan, 1992, p. 85). Few would disagree that teacher beliefs influence perceptions, resonate in judgments, and affect behavior in the classroom. Yet, beliefs are a “messy construct” (Pajares, 1992, p. 329), complex (Phillip, 2007), and not easily defined, identified, or linked to specific instructional practices (Pajares, 1992).

Studies have revealed the complexity of teacher beliefs, finding inconsistencies of beliefs (Anderson, 1998; Brown, 2005; Hoyles, 1992; Raymond, 1997; Skott, 2001; Yates, 2006) and inconsistencies of reported beliefs regarding instruction and instructional practices observed (Brown, 2005; Cooney, 1985; Staub & Stern, 2002; Thompson, 1992). However, numerous studies have found correlations between stated beliefs, also known as “professed beliefs” (Speer, 2002, p. 649), and practices based on those beliefs, referred to as “attributed
beliefs” (Speer, 2002, p. 649) (Beilock et al., 2010; Bryant et al., 1991; Charlesworth et al., 1993; Grant et al., 1994; Hollon, Anderson, & Roth, 1991; Morine-Dershimer, 1983; Nespor, 1987; Oakes & Caruso, 1990; Philipp, 2007; Pianta et al., 2005; Potari & Georgiadou-Kabouridis, 2008; Prawat, 1992; Raths, 2001; Raymond, 1993; Richardson, 1992; Ross et al., 2003; Schwartz & Riedesel, 1994; Staub & Stern, 2002; Stein, Baxter, & Leinhardt, 1990; Stipek & Byler, 1997; Stipek et al., 2001; Thompson, 1992; Thorton, 1985; Vartulli, 1999; Wilkins, 2008; Wright, 1992; Ziccardi-Priselac, 2009).

In 1994, Grant et al. studied the relationships between the use of alternative instruction for instructing place value concepts and multi-digit addition and subtraction and teacher beliefs about how mathematics is defined. Beliefs regarding a mathematics definition and beliefs of how mathematics should be taught were closely related. These beliefs teachers held about mathematics and about teaching and learning mathematics were then related to how each teacher responded to implementing alternative instruction.

Ross et al. (2003) also found correlations between professed beliefs and classroom practices. They developed a 20-item survey that was designed to determine if standards-based teaching was being implemented in classrooms. The 6-point scale provided a continuum, differentiating between those who believed in traditional approaches to teaching mathematics (Level 1) to those who believed in constructivist reform mathematics (Level 6). They found that teachers whose stated beliefs were closer to the constructivist reform mathematics also used instructional practices in this manner.

Yates (2006) conducted a study among 127 experienced Australian elementary teachers five years after a constructivist curriculum reform began. Based on survey results, Yates found that teacher beliefs regarding the beauty of mathematics and constructivism
were related significantly to some child-centered practices, including the use of manipulatives and less use of worksheets and tests. Teacher age, qualifications, and the number of years teaching mathematics were not significantly related to the reported teaching practices.

Wilkins’ (2008) study found that the 481 in-service K-5 elementary teachers’ beliefs regarding effective mathematics instruction were positively correlated to the frequency with which the teachers used inquiry-based instruction. Furthermore, her study indicated that teacher beliefs had the strongest effect on teachers’ practice.

Some studies found a disconnect between what is reported and what is observed (Anderson, 1998; Brown, 2005; Paterson, 2009; Raymond, 1997; Stipek & Byler, 1997; Vacc & Bright, 1999; Willis, 2010). In a study conducted by Vacc and Bright (1999), the beliefs and practices of 34 pre-service elementary teachers were examined during a two-year intensive professional development on Cognitively Guided Instruction with a focus on children’s mathematical thinking. Results indicated that while the beliefs of the teachers changed to embrace more reform-oriented practices such as CGI, they were unable to apply these beliefs and knowledge to their planning or instruction.

Anderson (1998) studied primary teachers’ beliefs about the role of problem-solving and problem-solving strategies used in the classroom using self-reported surveys. From the 132 returned surveys, the data revealed that there was not a significant relationship between reported beliefs and practices. Brown (2005) used two self-report questionnaires to determine 94 preschool teachers’ beliefs about the importance of mathematics and sense of self-efficacy and observed 20 teachers’ mathematics instructional practices. She did not find a correlation between teachers’ instructional practices and beliefs or self-efficacy. Both of
these studies suggested the use of additional qualitative data, such as interviews, to better understand how teachers make instructional decisions regarding mathematics instruction and to explore personal and external factors that relate to mathematics instruction.

In Paterson’s (2009) mixed methods study, a survey (n = 174), in-class observations (n = 10), and post-observation teacher interviews (n = 10) were used to study relationships between beliefs and reform-oriented teaching methods of K-12 teachers from three different school districts. Results of her study indicated that although the teachers expressed a strong belief in reform-oriented mathematics teaching, there was only a marginal relationship between their professed beliefs and their observed instructional practices. Teachers cited pressures regarding student performance on standardized tests as one of the main reasons why there was a discrepancy between beliefs and practices.

Raymond’s (1997) qualitative study investigated relationships between six first and second year elementary teachers’ beliefs and mathematics instructional practices. Data were collected from multiple interviews, classroom observations, lesson plan samples, lesson plans, concept-mapping, and questionnaires. Focusing on one fourth-grade teacher, Raymond found that the teacher’s beliefs about mathematics content, which were quite traditional, were more aligned with her instructional practices than beliefs about mathematics pedagogy, which were quite non-traditional. When discussing the difference between beliefs and practices, the teacher identified the content topic of focus and the behavior of the students as reasons for the inconsistencies between beliefs and practices.

Researchers, such as Anderson (1998) and Brown (2005) acknowledged the value of adding additional information to their study, including qualitative data and classroom observations. This suggestion aligns with Pajares’ (1992) recommendation for using direct,
long-term observations and interviews in addition to self-reports to capture a deeper understanding of how teachers make decisions and judgments about mathematics instruction. Conclusions that other researchers have reached regarding the inconsistency between professed and attributed beliefs address extraneous issues such as time constraints, scarcity of resources, concerns over standardized tests, and students’ behavior (Raymond, 1997). Thus, the different environments in which teachers work may contribute to the consistency or inconsistency of beliefs and practices (Skott, 2001).

**Relationships between Beliefs and Mathematical Knowledge for Teaching**

Teachers enter the classroom, as a student and as a teacher, with varying degrees of experiences with mathematics, knowledge of mathematics, and beliefs about the importance of mathematics and how mathematics should be taught. Like children, teachers have preconceived notions about what mathematics should look like in their classroom (Ball, 1988). In order for teachers to learn and develop new strategies for instruction, these beliefs and experiences need to be explored and then built upon in various ways. Some ideas and beliefs will be “firmly rooted in tendencies or habits . . . [and] not readily shed” (Ball, 1988, p. 15) while other ideas do not need to be changed, but extended (Ball, 1998). The relationship between beliefs and knowledge is an important one; “teacher beliefs about what mathematics is and what it means to know, do and teach mathematics may be driving forces in instruction of mathematical ideas” (Raymond & Santos, 1995).

A few research studies have examined the relationship between mathematical content knowledge and reform-oriented beliefs. Some have identified a relationship between teacher beliefs and mathematical content knowledge (Aubrey, 1996; Ball, 1989; Grant et al., 1994; Grossman, Wilson, & Shulman, 1989; Prawat, 1992; Raymond, 1993; Schwartz & Riedesel,
In 1989, Peterson, Fennema, Carpenter, and Loef investigated the relationships between beliefs, content knowledge, and student outcomes. Thirty-nine first-grade teachers, along with a control group, participated in the study. Beliefs were reported on a 48-item questionnaire, and interviews of teachers and student assessments were conducted. The researchers found that there was a relationship between teacher’s pedagogical content beliefs and pedagogical knowledge and classroom actions. This research project encouraged future studies to explore these relationships in order gain a better understand of how teachers’ pedagogical content beliefs underlie classroom practices and discern how pedagogical content beliefs may affect instruction.

Other studies have found that teachers’ mathematics knowledge and beliefs shape how curriculum materials (textbooks, manuals, etc.) are interpreted, and then used, in the classroom (Ball et al., 2001). Remillard (1999) examined teachers’ interactions with a new textbook and its contribution to mathematics reform instruction. Remillard discovered that beliefs were the factors that most influenced how the teachers read the curriculum text. In addition, the curriculum also influenced their beliefs and reading, particularly when new challenging ideas about the teaching and learning of mathematics was presented (Remillard, 1999). Collopy’s study of two elementary teachers use of textbooks as professional development tools (2003) found that beliefs were “integral to teachers’ identity” and must be considered when the use of curriculum materials to foster change is sought.

Wilkins (2002, 2008) did not find a significant correlation between mathematics beliefs and knowledge. Wilkins (2002) found that the 407 in-service elementary teachers with higher content knowledge were less likely to believe in the effectiveness of reform-oriented instruction and less likely to use such instruction in their classrooms. Again, in
Wilkins found content knowledge to be negatively related to beliefs with regards to inquiry-based instruction when studying 481 K-5 in-service teachers.

Mathematical knowledge for teaching does not solely guarantee the use of reform-oriented, constructivist teaching practices (Lubinski, Otto, Rich, & Jaberg, 1998; Ma, 1999; Mewborn, 2000; Voss, 2006). Other factors, such as beliefs, influence instructional practices (Ashton & Webb, 1986; Harper & Daane, 1998; Ma, 1999; Peterson et al., 1989; Stipek & Byler, 1997) which are considered as the “driving forces in instruction of mathematical ideas” (Raymond & Santos, 1995, p. 58). Yet, few studies have explored specific relationships between pedagogical beliefs and mathematical content knowledge. Liljedahl (2008) encourages future research to acknowledge the interconnectedness of mathematical knowledge for teaching and beliefs and discourages studies that solely focus on knowledge without the consideration of beliefs.

**Relationships between Knowledge and Practices**

It is widely accepted that what a teacher knows influences what and how it is taught (Bright et al., 1998; Cai, 2005; Chi-chung, Yun-peng, & Ngai-ying, 1999; Fennema et al., 1996; Fennema et al., 1993; Grossman et al., 1989; Hill et al., 2008; Hill et al., 2005; Lehrer & Franke, 1992; Leung & Park, 2002; Ma, 1999; Prawat, Remillard, Putnam, & Heaton, 1992; Raymond, 1997; Schwartz & Reidesel, 1994; Spillane, 2000; Stein et al., 1990; Swafford, Jones, & Thornton, 1997; van den Kieboom, 2008; Warfield, 2001; Weiss & Miller, 2006; Wilkins, 2002). Consistent findings report that teachers’ mathematics content knowledge influences how teachers engage students with the content and how they evaluate and use materials for instruction (Math and Science Partnership, 2010). When teachers have a “profound understanding of fundamental mathematics” (Ma, 1999, p. 120), they “do not
invent connections between and among mathematical ideas, but reveal and present them in terms of mathematics teaching and learning” (p. 122).

Ma (1999) used items from Ball’s 1988 dissertation to compare the ability of the United States teachers and Chinese teachers to explain why particular mathematics procedures worked. As a result of this study, *Knowing and Teaching Elementary Mathematics: Teachers’ Understanding of Fundamental Mathematics in China and the United States*, a profound understanding of fundamental mathematics was found to be absent among the U.S. teachers. They were able to do a particular procedure, but the U.S. teachers were unable to explain to students why the procedures worked. Ma (1999) found that teachers needed to understand a topic with both depth and breadth, enabling teachers to connect topics with similar concepts and other big ideas of mathematics. Leung and Park (2002) investigated East Asian teachers’ competence in mathematics by replicating Ma’s study. The researchers found that Hong Kong and Korean teachers lacked a deep understanding of the mathematics content, were unable to deconstruct concepts in a way that they could teach them to students, and relied on procedural processes rather than conceptual processes when teaching.

Fennema et al. (1993) studied a first grade teacher’s knowledge, beliefs and accessing and using knowledge of children’s thinking in the context of Cognitively Guided Instruction. The teacher acknowledged that the knowledge she had gained regarding children’s thinking allowed her to structure her instruction in ways that her students could “learn more than any children she had taught before” (p. 579).

Hill et al. (2008) examined the relationships between mathematical knowledge for teaching and the quality of instruction of five classroom teachers in grades second through
sixth. Ten teachers completed a 2002 version of the MKT measure that had 34 problems, 12 of which were number and operations items, 14 geometry items, and 8 algebra items. In addition, nine lessons from each of the ten teachers were videotaped at three different times during the school year. Analysis results indicated a significant, strong, and positive association between mathematical knowledge and instruction presenting an “inescapable conclusion of this study that there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction” (Hill et al., 2008, pp. 486-497).

Hill et al. (2005) studied the effects that teachers’ mathematical knowledge for teaching and instructional practices had on student achievement for 1,190 first-grade and 1,773 third-grade students. Mathematical knowledge for teaching data were collected from the MKT measure. Student assessment data were collected from student assessments (CTB/McGraw-Hill’s Terra Nova Complete Battery, the Basic Battery, and the Survey) and parent interviews. Classroom instructional practice data were collected from a questionnaire that included questions about mathematics teaching and a highly structured self-reporting log. From their sample of 334 first-grade and 365 third-grade teachers, Hill et al. found that teachers who scored lower on the content measure made more errors in their mathematics instruction than teachers with higher content measure scores. Teachers with higher content scores engaged their students in more rich mathematics, particularly through the use of representations, explanations and justifications. Furthermore, when controlling for student and teacher level covariates, results of this study indicated that teachers’ mathematical knowledge was significantly related to student achievement for both grade levels. With a significant relationship between content knowledge, instruction, and student outcomes, the
researchers emphasized the importance that the role of teachers’ content knowledge plays at the earliest grade levels and the importance of improving teacher’s mathematical knowledge in order to improve students’ mathematics achievement.

In a literature review conducted by Math and Science Partnership (2010), all but two of the eighteen studies selected for review reported a “direct link between teacher content knowledge and teaching practice” (p. 1). They found that with respect to mathematics, more knowledgeable teachers were more likely to present problems in contexts that were familiar to the students and to connect problems to what students already learned, more likely to use multiple representations to extend students’ understanding, and more likely to respond to student questions in a manner that facilitated group learning (Math and Science Partnership, 2010).

However, some studies have not found positive relationships between content knowledge and instructional practices (Borko et al., 1992; Schwartz & Reidesel, 1994; Thompson & Thompson, 1994; Wilkins, 2008). Borko et al. (1992) studied a pre-service teacher who was majoring in mathematics and education. While the teacher demonstrated knowledge of the subject, she demonstrated great difficulty in answering a student’s question regarding the division of fractions and expanding the students’ understanding. Thompson and Thompson (1994) studied one teacher’s instruction with a student as they talked about rates on a conceptual level. Although the teacher had a strong knowledge base about rate as determined by a paper-pencil test, he was unable to use helpful language outside of using numbers and operations in order to talk about it with a student.

Schwartz and Reidesel (1994) explored the relationship between mathematics content knowledge and pedagogical beliefs with respect to teaching practices among 140 K-5 pre-
and in-service teachers. Data regarding teacher practice was collected from self-reports in which teachers identified vignettes that most reflected their classroom instruction. Correlation analyses indicated that there was not a relationship between mathematics content knowledge and instruction.

Wilkins (2008) surveyed 481 in-service elementary teachers to determine if relationships existed between mathematical content knowledge, attitudes towards mathematics, beliefs about the effectiveness of inquiry-based instruction, and practices that used inquiry methods. Mathematical knowledge, as determined by a 44-item survey using items that were determined by the author to be found in grades K-8, was found to have a negative significant relationship with the use of inquiry-based instruction. Thus, results indicated that teachers with higher content knowledge were less likely to use inquiry-based instruction (and vice versa).

A “deep, vast, and thorough understanding” (Ma, 1999, p. 122) of mathematics influences how teachers engage students with mathematical concepts (Weiss & Miller, 2006). This relationship between mathematics knowledge and instruction has been identified as a means for ensuring equitable opportunities for all children to learn mathematics: “One important contribution we can make toward social justice is to ensure that every student has a teacher who comes to the classroom equipped with the mathematical knowledge needed for teaching” (Ball et al., 2005, p. 44).

Relationships among Beliefs, Knowledge, and Practices

The importance of beliefs, knowledge and instruction is well grounded in research. However, research on the relationships among all three areas is scarce. The three research studies that examined correlations among these three constructs found various relationships.
Borko et al. (1992) studied a student teacher’s inability to explain the concept behind the algorithm taught for the division of fractions. Their study found that while the student teacher had beliefs about mathematics instruction that reflected current views of effective mathematics teaching, she did not have a strong conceptual understanding of fractions nor motivation to use available resources to understand the concept behind the algorithm before teaching it. The researchers recognized that the teacher’s methods course did not challenge the student teacher to confront existing beliefs with her knowledge base. They also acknowledged that the mathematics courses the teacher took for two years as a mathematics major focused on rote procedures rather than meaningful understanding of the mathematics. As a result, a belief that mathematics is something to practice in a rote manner and a possible false sense of mathematical mastery may have influenced her lack of need or desire to seek additional information in order to teach the beyond the procedural algorithm to her students. The researchers urged future college programs to “challenge [student teachers’] fundamental beliefs about learning, teaching, and learning to teach” (Borko et al., 1992, p. 220).

Schwartz and Riedesel (1994) explored 140 K-5 pre-and in-serve teachers’ beliefs about elementary mathematics, their understanding of elementary mathematics, and their professed teaching practices. Peterson et al.’s (1989) Belief Scales, Riedesel and Callahan’s (1977) Elementary Mathematics Tests for Teachers, and classroom instruction self-reports were used. Findings indicated that there was a correlation between professed instructional practices and beliefs regarding mathematics reform. In addition, a greater mathematical content knowledge was found as a possible variable that influenced beliefs regarding mathematics understanding.
Wilkins (2008) surveyed 481 in-service elementary teachers to determine if relationships existed between mathematical content knowledge, attitudes towards mathematics, beliefs about the effectiveness of inquiry-based instruction, and practices that used inquiry methods. The study found that there was a consistent relationship between beliefs about what good mathematics teaching entails and the reported use of inquiry based teaching used in the classrooms. Teachers’ attitudes towards inquiry learning also had a positive relationship with the professed use of inquiry practice. However, mathematical knowledge, as determined by a 44-item survey using items that were determined by the author to be found in grades K-8, was found to have a negative relationship with the use of inquiry-based instruction. Overall, beliefs in the effectiveness of inquiry learning were found to have the strongest effect on the use of inquiry-based instruction.

Research has identified the importance of teacher beliefs, mathematical knowledge for teaching, and instructional practices. Yet, after exhaustive search, only three studies were found which explored the relationships among these three elements. Nonetheless, all three studies offer insight into relationships between and among beliefs, knowledge, and instruction and provide a base from which the proposed study can compare findings.

Conclusion

Teaching is complex. It consists of numerous constructs that affect each decision made. Pedagogical beliefs, mathematical knowledge for teaching, and instructional practices form the basis of who a teacher is, why the teacher thinks certain things, what tools, materials, and practices will be used, and how concepts will be taught and assessed. Although research recognizes the importance of each of these three constructs, and has identified relationships among them, few studies have explored these relationships with
kindergarten teachers. Significant national reports stress the importance of early mathematics and have requested specific research studies to be conducted regarding mathematics instruction in the early years. Among the needs identified, requests for research that carefully examines how early childhood teacher beliefs, mathematical knowledge for teaching, and instructional practices impact the quality of instruction have been requested. This study sought to address those needs. Chapter 3 will describe the research methods used in order to address such requests. Chapter 4 and 5 will discuss the findings and implications of the results.
CHAPTER 3
RESEARCH METHODS

Purpose of the Study

The purpose of this study was to describe kindergarten teachers’ pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice and to examine the relationships among them. This study responded to specific requests for research from the National Mathematics Advisory Panel (2008), the Society for Research in Child Development (2008), and the National Research Council, Early Childhood Mathematics Committee (2009).

Research Questions and Hypotheses

Four major research questions were asked in this study:

1. What pedagogical beliefs of mathematics do kindergarten teachers hold?
2. What mathematical knowledge for teaching do kindergarten teachers possess?
3. What instructional practices do the kindergarten teachers use that promote mathematical understanding?
4. What are the relationships among pedagogical beliefs of mathematics, mathematical knowledge for teaching, and instructional practices for the kindergarten teachers?

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.
Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a positive relationship with beliefs.

Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a positive relationship with instructional practices.

Design of Study

The research study employed a non-experimental quantitative research design that addressed the descriptive and associational research questions (Creswell, 2008). Descriptive statistics were calculated to provide details about the each of the variables. Correlational procedures were applied to measure associations between the variables. As with similar studies, correlational analyses cannot establish possible cause and effect between variables. However, such statistics do describe potential relationships among the identified population, using the particular measures within the distinct circumstances of the study.

Participants

Data for this study were gathered from an established group of 29 teachers from 27 different school districts representing all eight State Board regions of a southeastern state. This group of teachers was participating in the third year of a three-year state initiative with the State Department of Public Instruction designed to impact pedagogy and leadership ability of kindergarten teachers. As one of the two project leaders for this state initiative during the past three years, relationships were established with the teachers and their principals and access to the teachers and their schools was already in place.

Selected from 219 applicants, this group of teachers exhibited an interest to address reform for kindergarten programs by implementing developmentally appropriate practices and leading the implementation of such practices across the state. Of the thirty-six original
members, three teachers moved into administrator roles, one became a resource teacher, one left on a year-long maternity leave, one moved to a different state, and one passed away. Although two teachers “looped” with their students (they taught the same group of students for kindergarten and first grade) and were teaching first grade at the time of the study, they were included in this study due to their long-term involvement in the initiative, the fact that two of the three years of involvement in this initiative the teachers were teaching in kindergarten, and the appropriate transferability of practices addressed during this initiative from kindergarten to first grade.

One of the twenty-nine teachers represented in this study was male. Two teachers were African-American and one teacher was Indian. Fifty-seven percent of the teachers in this study had National Board Certification. Seventy percent of the teachers had a Master’s degree. Forty percent were in Elementary Education, 33% were in Early Childhood Education, 6% were in Child Development, 6% were in School Administration, 6% were in Reading/Literacy, 3% were in Special Education, 3% were in Mathematics Education, and 3% were in Educational Media. The mean number of years the teachers had taught Kindergarten was 12.7 (SD=5.53) with a range from 3 to 26 years. Thirteen teachers had taught preschool, seventeen teachers had taught first grade, and eleven teachers had taught above first grade.

Two teachers taught in a public charter school, with the remaining teachers located in regular public elementary schools of which 65% were classified as Title I schools. Eighty percent of the teachers have a full-time teacher assistant. The teachers had an average of 20 students in their class, with as few as 14 and as many as 25. On average, the classes were diverse, with 62.5% White, 17.5% Black, 14% Hispanic/Latino, 3% Asian, and 3%
American Indian, Mixed, or unknown. Fifty-six of the 587 students (about 10%) were considered limited English proficient (LEP) whose native language was other than English and whose skills in listening, speaking, reading, or writing English were such that they had difficulty understanding school instructions in English. Ninety percent of the classrooms had students with an active IEP, with 20 of the 30 teachers anticipating additional referrals during this school year.

With respect to mathematics, seven of the twenty-nine teachers stated that they had a fear of mathematics and felt uncomfortable teaching mathematical concepts beyond the primary years. Twenty-three teachers identified a specific mathematics curriculum adopted for use the school year in which this study took place. Twenty-three percent of the teachers had not received training specific to the mathematics curricula identified. Of those who did receive training, seven teachers described the training either as brief, limited, “wasn’t very good” or “terrible.” Outside support dedicated to the mathematics program varied from non-existing (17%), professional organization membership/materials (13%), professional learning communities (33%), district-wide professional development (53%), and school level professional development (57%). Mathematics assessments used varied from the state suggested assessments, district created assessments, assessments from the adopted curriculum, and a mixture of all. Two teachers reported not having district mandated mathematics assessments at the time of the survey. Two teachers belonged to the National Council of Teachers of Mathematics (NCTM) and the state affiliate (NCCTM).
Research Instruments

The Self-Report Survey

Case studies, self-reported surveys, and questionnaires are often used in studies to research teacher beliefs. While case studies capture rich information on a small scale, they are not conducive to larger scale studies due to time and financial resources. Therefore, self-report surveys and questionnaires are often used and have been found to provide insight into actual classroom practice. The *Self-Report Survey: Elementary Teacher’s Commitment to Mathematics Education Reform* (Ross et al., 2003) was used to collect data regarding teachers’ beliefs about mathematics instruction for this study. The *Self-Report Survey* (see Appendix A) is a well established elementary teacher beliefs instrument (Desimone, 2009; Groth, 2007; Pang, 2005) and has been used to determine reform-based beliefs of hundreds of teachers, including 528 K-5 teachers and 130 sixth-twelfth grade teachers (Chval et al., 2006), 40 primary age teachers (Anderson & Bobis, 2005), and 23 novice teachers who were a part of a Master’s degree cohort (Schafer, 2008).

This 20-item self-report survey is designed to determine whether standards-based teaching is being implemented in elementary classrooms. Using NCTM standards as a blueprint for its design, the survey consists of twenty items which fall into one of nine dimensions of elementary mathematics reform: program scope, student tasks, discovery, teacher’s role, use of manipulatives and tools, student-student interaction, student assessment, teacher’s conceptions of mathematics as a discipline, and student confidence (See Appendix B for details). In an attempt to counteract the tendency to agree rather than disagree or the tendency to make extreme responses (response sets), 35% of the items on the survey are worded negatively. This instrument uses a 6-point Likert scale, ranging from 6
(strongly agree) to 1 (strongly disagree). Higher scores on the measure indicate more reform-oriented beliefs and lower scores indicate more traditional oriented beliefs. This survey takes approximately fifteen minutes to complete.

Internal consistency was determined with this measure on two separate occasions using Cronbach’s α (Ross et al., 2003). With 517 Grade K-8 teachers from a single district, a reliability coefficient of $\alpha = .81$, a mean rating of 4.48 (out of 6), and a standard deviation of 0.53 was found. On a separate occasion with 2,170 K-8 teachers from a different district, researchers found a coefficient $\alpha = .81$, a mean rating of 4.64, and a standard deviation of 0.20. The typically accepted level for internal reliability is .7 (Nunnally, 1978), therefore, the reliability of this measure was found to be highly reliable within the parameters of that study’s design.

Content validity, the extent in which the questions and scores represent all possibilities available regarding the topic (Creswell, 2008) was also measured. The survey questions were created based on a review of standards-based teaching research. The questions were then judged by experts to determine whether they were representative of standards-based teaching. These tests indicated that the measure had evidence of adequate content validity.

Additionally, the instrument’s authors reported three other types of validity: predictive validity, concurrent validity, and construct validity. In order to test predictive validity, the authors tested their hypothesis that schools with higher scores of reform based teaching beliefs would have students with higher mathematics achievement than schools with low scores on the survey. Results from the teacher survey were aggregated and compared to the results from a mandated student performance assessment which was comprised of open-
ended problems. Family income was also taken into account. Findings indicated that the survey was reliable ($\alpha = .96$) and that the survey results, taken at one point in time, correlated to student achievement, which was assessed at a different point in time.

Concurrent validity was sought to determine current beliefs and skills at one point in time. Therefore, survey responses of a small sample of teachers were compared to classroom practices. Observations, teacher interviews, artifacts, and student focus groups were used to develop a rubric which summarized the differences among the selected teachers among four levels of implementation of standards-based teaching. Results suggested that the scores on the survey accurately depicted recorded standards-based practices.

Construct validity determines whether an instrument’s scores are “significant, meaningful, useful, and have a purpose” (Creswell, 2008, p. 173). In order to determine construct validity of the survey, the authors compared the use of an assigned standards-based textbook to high- and low–reform teachers, as determined by the survey. Interviews and classroom observations were used to collect data. Results indicated that the survey was able to identify teachers who were using the same assigned text. Teachers who were identified as high-reform teachers from the survey results used the textbook to support standards-based teaching. Teachers, who were identified as low-reform teachers from the survey results, used the text to support traditional teaching practices. Therefore, the authors concluded that the Self-Report Survey had construct validity.

**Mathematical Knowledge for Teaching**

Researchers have used both large and small qualitative and quantitative studies in an attempt to capture mathematical knowledge for teaching. For the purpose of this study, the
Mathematical Knowledge for Teaching measure (MKT) was used (see released examples Appendix C). The MKT is a set of survey-based teaching problems designed to represent various components of knowledge of mathematics needed for teaching (Hill et al., 2004). The measure captures teachers’ understanding of the mathematics problems they may assign to students and how teachers may solve particular issues that arise in the teaching of mathematics, “including evaluating unusual solution methods, using mathematical definitions, representing mathematical content to students, and identifying adequate mathematical explanations” (University of Michigan, 2010, para. 4). Highly recognized among the mathematics community, the MKT helps researchers understand what constitutes mathematical knowledge, identifies the role of teachers’ content knowledge in student performance, and guides teacher development. It is also a valuable tool for making statements about how content knowledge differs among groups of elementary and middle school teachers or how a group of teachers performs at a point in time (Hill et al., 2004). The MKT is not a criterion referenced test producing a “passing” score. Instead, the raw scores from the teachers are converted to standardized, or one parameter IRT scale scores, so that each teacher’s score is expressed as the distance from the average score in this population. Thus, the instrument is designed to measure the range of teacher ability reliably so that growth over time can be evaluated and relationships between scores and/or instructional practices can be explored.

Each elementary (K-6) item has been piloted with hundreds of elementary teachers, measuring content knowledge for teaching in number and operations, patterns, functions, and algebra, and geometry, yielding information about item characteristics and overall scale reliabilities. The MKT has been validated with multiple methods, including interviews,
connections to instructional practices, and student achievement (Blunk, 2007; Hill et al., 2005, Schilling & Hill, 2007).

The Learning to Teach Mathematics Project maintains legal copyright of the MKT. Therefore, prior to using the *Mathematical Knowledge for Teaching* measure, the researcher attended the required one-day training on the use and implementation of this instrument. Guidelines provided by the authors during this training were followed. Although pre-equated forms were available for use, the authors acknowledged that the pre-equated forms may ask kindergarten teachers to answer questions on content unlikely to be taught in a typical kindergarten classroom. Therefore, as recommended by the research committee, a measure designed specifically for kindergarten teachers was created.

In order to create a measure for the study, the teacher population was considered when selecting items for both content and difficulty as stated in the procedure manual. Due to NCTM’s (2006) emphasis on the importance of Number and Operations and Geometry and early childhood specialists’ attention to number and geometry as a “particularly important . . . focus for mathematics instruction in the early years” (National Research Council, 2009, p. 337), questions regarding number sense, addition and subtraction, and basic geometry concepts were selected from the item pool. Thirteen elementary items were ultimately selected to address number and operations concepts and sixteen items to address geometry concepts. Although the content of every question was not specific to a typical kindergarten curriculum, the items selected from the available pool best represented problems that this sample of teachers would most likely encounter when teaching young children according to the state’s mathematics standards.
The authors of the MKT created a rigorous tool to help measure the range of teacher
ability reliably. Therefore, each item in the MKT item bank had two Item Response Theory
parameters: slope and difficulty. The slope indicated the probability of a participant getting
the item correct, discriminating between those who do well on the test and those who do not.
Thus, if a particular item was easy to guess and get correct, regardless of the teacher’s
understanding, the slope for that item was low (close to 0). However, if the item was
difficult to guess and required mathematical understanding to answer correctly, then the
slope of that particular item was high (close to 1). As recommended by the authors, each
item selected for this study had a slope of at least .5 or higher (see Appendix D). Three sub-
parts from the questions selected were not used in the analysis since the slope of each of
these responses fell below .5 (see shaded boxes in Appendix D). However, they were
included in the measure since they were a sub-part of an overall question and the removal of
the question may have negatively impacted the participants’ ability to respond to subsequent
sub-parts.

The difficulty score indicated how difficult a particular item was. An item that had a
difficulty of 1 was an item that participants, one standard deviation above the mean, had a
50/50 chance of getting right. Typically, the number of items correct and the difficulty are
very closely related. With respect to this study, the average teacher in the study population
was expected to be .5 standard deviations below the participants used in the scales provided
by the author. This assumption was based on this sample’s previously expressed fear of
mathematics, the limited depth of mathematical content taught in the kindergarten year, and
the under-provided access to mathematics professional development and limited course
requirements expressed in previous survey responses collected during the initiative.
Therefore, the average teacher of the population was expected to answer items with -.5 and lower values correctly. The selected questions provided a good distribution of difficulty across the ability spectrum, with a range from -1.876 (easiest) to 1.77 (hardest) with an average difficulty score of -.28. With a good distribution of difficulty, the measure was better able to discriminate knowledge among teachers and more accurately capture a teacher’s mathematical understanding.

In addition to content and difficulty, guidelines regarding the length of the measure were also followed. Thirteen questions, rendering 32 responses, were selected from the item pool. Because three responses from the items selected were not used in the analysis since the slope of each of these responses fell below .5, thirteen questions with a total of 29 responses were used in the analysis. With a sample size of thirty, the guidelines recommended that the measure included at least 20-25 items to provide an approximately reliability score of .85. Therefore, the measure for this study was on the longer side in an attempt to ensure that there were enough items for sufficient reliability.

FirstSchool Snapshot

Many studies, particularly those that explore beliefs in relationship to instruction, stress the importance of actual classroom observations for gathering a truer picture of classroom instruction (Ball & Bass, 2000; Parajes, 1992). Because it is possible that the reported practices are not reflective of actual practices used, classroom observations can provide a researcher with valuable information about instructional practice (Mujis, 2006). The FirstSchool Snapshot (Ritchie et al., 2010) was used in this study to capture rich and detailed descriptions of children’s learning experiences and interactions with teachers. The FirstSchool Snapshot (see Appendix E), formally known as the Emergent Academic
Snapshot (Ritchie et al., 2001) is an established tool that has been used in numerous state and national studies, including the Multi-State Study of Pre-Kindergarten (2001) with over 2,900 pre-kindergarten children as well as the Study of State-Wide Early Education Programs (SWEEP) with over 1,775 children (2003). The version available for use for this study, The FirstSchool Snapshot, has been refined to focus on older children and on elementary school practices.

The Snapshot is a time sampling observation instrument designed to describe pre-K through third grade children’s exposure to instruction and engagement in academic activities as well as to describe teaching practices used throughout the course of a school day. During the full-day classroom observation, a trained data collector samples the behavior of four randomly-selected children: A, B, C, D. The data collector observes student A for 30 seconds and then codes for 40 seconds, followed with student B, student C, etc. When data are collected on the four selected children, the observer begins again with student A and continues in this succession over the course of the day, sampling each child approximately 90 times to ensure reliability. In addition to providing specific information about student engagement and teacher-child interactions, the data collected also pinpoint specific times and settings when students are working with mathematics content- specifically number, geometry, algebra, and time.

The data collected fell within 41 sub-composites of five categories: Activity Setting, Child Engagement, Child Behavior, Teacher Behavior, and Teacher-Child Engagement. Because I was interested in capturing information about constructivist-based teaching practices, the data collected within the following sub-composites were analyzed (see Appendix F for description):
As indicated in the literature review (Chapter 2), these particular sub-composite criteria are well documented and research-based strategies that are deemed as effective constructivist practices. Constructivist teachers use a variety of grouping settings to effectively engage students and meet varying abilities and learning styles (Clements, 1997; Copley, 2000; Day, 1999; Guarino et al., 2006; Zemelman, Daniels, & Hyde, 2005). In addition, they provide choices that reflect children’s knowledge, abilities, and interests, allowing students opportunities to take risks, experiment with ideas and think independently (Copley, 2000; Copple & Bredekamp, 2009; Day, 1995; Gullo, 2006; Richardson, 2002; Wadsworth, 2003). In a “math-talk learning community” (Hufferd-Ackles et al., 2004, p. 82) constructivist teachers promote peer interactions by providing opportunities to interact, exchange ideas, make predictions, and solve problems (Baker et al., 2002; Clements & Sarama, 2003; Curtis, 2000; Fennema et al., 1991; Geary, 1994; Malofeeva, 2005; Stein, 2007; Weidinger, 2006; Whitehurst, 1993). In this environment, the teacher carefully uses questions to probe and scaffold understanding in order to help children understand, explain and extend their own thinking; provides opportunities for students to make connections needed for understanding; and allows the mathematics to unfold through this scaffolding technique (Baker et al., 2002; Charles, 2005; Copley, 2000; Curtis, 2000; Franke et al., 2007; Hufferd-Ackles et al., 2004; Malofeeva, 2005; Stein, 2007; Yackel et al., 1990; Van de Walle, 2005). Constructivist teachers also provide time for reflection so that the students may think critically, draw conclusions, and transfer knowledge (Alro & Skovsmose, 2002; Clements & Sarama, 2003;
Fennema et al., 1991; Geary, 1994; Griffin et al., 1995; Richardson, 2002; Wheatley, 1992; Zambo & Zambo, 2007).

Inter-rater reliability of the Snapshot is established and monitored for consistency across time. Using Kappa, a measure of agreement among raters that is corrected for chance, the instrument met or exceeded the 0.6 level to establish reliability. Results also indicate predictive validity, as children’s engagement in academic activities and child assessments in language and literacy were positively associated in fall and spring of pre-kindergarten (Early et al., 2005); and concurrent validity, as teacher engagement of the children and children’s engagement with academic activities have modest and positive associations with the Early Childhood Environment Rating Scale-Revised (Howes et al., 2008).

Data Collection Methods

Recruitment

Once Internal Review Board approval was granted (see Appendix G), each of the 29 teachers received written information by email recruiting them to volunteer to be a part of this study. The recruitment letter (see Appendix H) and consent form (see Appendix I) asked for permission to complete the Self-Report Survey and the Mathematical Knowledge for Teaching measure and to participate in a one-day classroom observation using the FirstSchool Snapshot instrument. If a teacher volunteered, then the teacher’s principal received a written invitation asking permission for their teacher to be a part of the study and to determine if further permission needs to be sought from the school district (see Appendix J). Two school districts required local approval, which was granted following application procedures. Parent permission was not needed from the parents of the four randomly selected children, since the data collected from each child were not identifiable; there was not
a true interest in those children as individuals; and the data obtained in each classroom were pooled into one composite to gain information about the classroom, not the individual children.

**The Self-Report Survey and Mathematical Knowledge for Teaching**

When the sample population was finalized, those participating in the study were asked to complete *The Self-Report Survey* and the *Mathematical Knowledge for Teaching* measures during part of their February professional development meeting previously established by the project leaders. A label with the teacher’s name and a seven-digit number was half-affixed on the cover page of these two measures. Once each teacher received his/her measure, s/he removed the name portion of the label, leaving only the assigned number affixed to the measure. All directions were provided in written format and read aloud by the researcher. Participants were asked to answer all of the questions on each measure. As indicated in the *Content Knowledge for Teaching Mathematics Measures* instructions, participants were asked to spend only a few minutes on each question, deciding upon an answer that best reflected the decision the participant would make at that moment if teaching. Each instrument administration was proctored to ensure that each teacher worked independently. One teacher was unable to attend the meeting but completed the two surveys at the beginning of the following professional development meeting.

**Instructional Practices**

In addition, each teacher was contacted by one of two trained data collectors hired by the project leaders to arrange a full-day classroom observation using the *FirstSchool Snapshot* during the months of March, April, and May. During the full-day classroom observation, the trained data collector used the *FirstSchool Snapshot* instrument to sample
the behavior of four randomly selected children. The data collector observed the children during all parts of the school day, including transitions, classes held outside the main classroom (e.g., Art Class, Physical Education), lunch time and recess. The researcher obtained the raw observation data from the data collectors over the course of the three-month period.

**Data Confidentiality**

To protect against a possible breach of confidentiality, all data were identified only with an ID number, and were kept in a locked file cabinet and in password protected electronic files. The linking list between name and ID was needed to be maintained after data collection to allow for the one-to-one sharing of individual teachers’ results with them if requested. Individual data were not shared with administrators or any other persons by the researcher.

The *Terms of Use* for the MKT measure were followed, including the request that raw frequencies or number correct were not publicly discussed nor that the participants’ raw frequencies or number correct were compared to other participants. Furthermore, the items from the *MKT* measure were not used for any other purposes than as part of the analysis for this study, and as something to share with individual teachers who requested that information.

Identifiers (name and location and email address) were maintained until the completion of the one-to-one consultations which provided feedback to the participants, if desired. The contact information was then destroyed, leaving only ID codes linked to data, without linkage to identity participants.

**Timeline**
Table 3.1 illustrates the procedures and sequence of events of the data collection.

**Table 3.1. Instrument Sequence of Events**

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What pedagogical beliefs of mathematics do kindergarten teachers hold?</td>
<td>The Self-Report Survey</td>
<td>The Self-Report Survey</td>
<td>Make-Up Date</td>
<td>Make-Up Date</td>
</tr>
<tr>
<td>2. What mathematical knowledge for teaching do kindergarten teachers possess?</td>
<td>Mathematical Knowledge for Teaching Measure (MKT)</td>
<td>MKT Make-Up Date</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. What instructional practices do the kindergarten teachers use that promote</td>
<td>Observation Dates Established</td>
<td>FirstSchool Snapshot</td>
<td>FirstSchool Snapshot</td>
<td>FirstSchool Snapshot</td>
</tr>
<tr>
<td>mathematical understanding?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Data Analysis Methods**

Three sets of data were collected from each individual as previously indicated. Item responses from the beliefs survey, MKT, and the Snapshot were scored and entered by hand onto an Excel spreadsheet and imported into the Statistical Package for the Social Sciences (SPSS) and carefully re-checked by the researcher to ensure accurate entries. All statistical analyses were performed using SPSS. Although the Self-Report Survey and the Mathematical Knowledge for Teaching instruments had produced results that were reliable and valid for their particular studies, it did not guarantee that they would produce similar results for this study. Therefore, the reliability for the scores in this proposed study were computed in the course of the data analysis by calculating Cronbach’s Alpha with a goal of .7 reliability or higher.
Scoring Procedures

The Self-Report Survey consisted of twenty items based on a 6-point Likert scale. Once all of the Self-Report Survey data were collected, the seven negatively worded items were reversed-scored. Individual scores were entered into SPSS.

The Mathematical Knowledge for Teaching measure originally consisted of thirteen questions, rendering 32 responses. Three items selected were not used in the analysis since the slope of each of these responses fell below .5. Therefore, thirteen questions with a total of 29 responses were used in the analysis. Answers from each test were counted either correct or incorrect, according to the supplied answer key. If an item was left blank, it was assumed that the teacher did not know the answer, and the item was counted incorrect. Individual scores were entered into SPSS.

The data collected from the FirstSchool Snapshot consisted of 41 separate codes within five categories: Activity Setting, Child Engagement, Child Behavior, Teacher Behavior, and Teacher-Child Engagement. All codes were entered once by the researcher, and a second time by one of the project leaders, onto two separate excel spreadsheets. If discrepancies were detected between the two data entries, then the researcher used the raw data to correct the discrepancy. Therefore, the researcher is confident that the numerous raw data for each teacher were entered accurately.

For the purposes of the study, only data pertaining to the classroom teacher were of interest. Therefore, all data pertaining to all other adults (assistant teacher, parent, or other school staff) as indicated by the Adult ID, were not included in the analysis.

In order to capture information about constructivist-based teaching practices, the data collected within Choice, Station, Small Group, Oral Language, Peer, Collaboration,
Reflection, and Scaffold were analyzed. Constructivism incorporates numerous strategies to reach a variety of students’ needs, interests, and learning styles (Banks, 2001; Clements & Battista, 2002; Copple & Bredekamp, 2009; National Mathematics Advisory Panel, 2008; Stiff, 2001). The compilation of strategies, rather than simply one instructional practice, creates the entity “constructivism.” Therefore, the constructivist codes (Choice, Station, Small Group, Oral Language, Peer, Collaboration, Reflection, and Scaffold) were equally weighted, receiving a score of “1” for each instance the data indicated that it was observed. It is important to note that codes within Child Engagement, Child Behavior, Teacher Behavior, and Teacher-child engagement were not mutually exclusive. Therefore, a student who was talking with a peer (Peer) may have also been negotiating a solution to a problem together (Collaboration), thus two sub-composites were coded during that 30-s interval. In order to take into account the non-mutually exclusive codes, only 1 point was given for each interval, regardless of the number of codes recorded during that interval (> 0). The incidences of constructivist-based teaching practices were then totaled and divided by the length of the entire school day, creating a proportion of the day each child spent for each of the four categories. The four proportions for each category were then averaged to achieve a classroom proportion for constructivist practices.

A mathematics constructivist score was created when constructivist codes were co-coded with mathematics (Numbers, Geometry, Algebra, and Time). Because it was possible that a teacher used constructivist practices with some content and not others, these two scores provided an overall picture of whether constructivist practices were being used and if they were being used with mathematics. These codes were similarly averaged to achieve a classroom math-constructivist code.
Statistical Processes

Data were analyzed to provide a description of each of the three variables and to determine the degree of association between the variables. Table 3.2 indicates the data analysis method used with each research question.

Table 3.2. Data Analysis Methods

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Analysis Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What pedagogical beliefs of mathematics do kindergarten teachers hold?</td>
<td>Internal Consistency analysis (Goal: Cronbach’s alpha of .7 or higher)</td>
</tr>
<tr>
<td></td>
<td>Descriptive Statistics</td>
</tr>
<tr>
<td></td>
<td>• Determine mean, standard deviation, variance, range, minimum, maximum</td>
</tr>
<tr>
<td></td>
<td>• Determine skewness, kurtosis, and normal distribution</td>
</tr>
<tr>
<td>2. What mathematical knowledge for teaching do kindergarten teachers possess?</td>
<td>Internal Consistency analysis (Goal: Cronbach’s alpha of .7 or higher)</td>
</tr>
<tr>
<td></td>
<td>Descriptive Statistics</td>
</tr>
<tr>
<td></td>
<td>• Z-score*</td>
</tr>
<tr>
<td></td>
<td>• Determine mean, standard deviation, variance, range, minimum, maximum</td>
</tr>
<tr>
<td></td>
<td>• Determine skewness, kurtosis, and normal distribution</td>
</tr>
<tr>
<td></td>
<td>Inferential Statistics</td>
</tr>
<tr>
<td></td>
<td>• Paired samples t-test b/t number and geometry</td>
</tr>
<tr>
<td></td>
<td>• Two-Tailed Pearson correlation b/t number and geometry</td>
</tr>
<tr>
<td></td>
<td>*Raw frequencies or number correct may not be publicly discussed (MKT Terms of Use)</td>
</tr>
<tr>
<td>3. What instructional practices do the kindergarten teachers use during the day</td>
<td>Inter-rater Reliability</td>
</tr>
<tr>
<td>that promote mathematical understanding?</td>
<td>Descriptive Statistics (Mathematics Minutes, Overall Constructivist, Mathematics</td>
</tr>
<tr>
<td></td>
<td>Constructivist)</td>
</tr>
<tr>
<td></td>
<td>• Determine mean, standard deviation, variance, range, minimum, maximum</td>
</tr>
<tr>
<td></td>
<td>• Determine skewness, kurtosis, and normal distribution</td>
</tr>
<tr>
<td>4. What are the relationships among pedagogical beliefs of mathematics,</td>
<td>Inferential Statistics*</td>
</tr>
<tr>
<td>mathematical knowledge for teaching, and instructional practices for the</td>
<td>• Two-tailed Pearson Product-moment correlations</td>
</tr>
<tr>
<td>kindergarten teachers?</td>
<td>*Mathematical constructivism score was used</td>
</tr>
</tbody>
</table>
CHAPTER 4

RESULTS

The purpose of this study was to describe kindergarten teachers’ pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice and to examine the relationships among the variables. This study responded to specific requests for research from the National Mathematics Advisory Panel (2008), the Society for Research in Child Development (2008), and the National Research Council, Early Childhood Mathematics Committee (2009).

This chapter describes the quantitative findings regarding the four major research questions asked in this study:

1. What pedagogical beliefs of mathematics do kindergarten teachers hold?
2. What mathematical knowledge for teaching do kindergarten teachers possess?
3. What instructional practices do the kindergarten teachers use that promote mathematical understanding?
4. What are the relationships among pedagogical beliefs of mathematics, mathematical knowledge for teaching, and instructional practices for the kindergarten teachers?

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.
Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with beliefs.

Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with instructional practices.

The Statistical Software Package for the Social Sciences (SPSS) was used to analyze the data collected in order to answer the four research questions. In this chapter, the results from the statistical analyses are organized by each research question.

**Research Question 1: What Pedagogical Beliefs of Mathematics Do Kindergarten Teachers Hold?**

**Summary of Statistical Analysis**

The 29 kindergarten teachers completed the *Self Report Survey* (Ross et al., 2003) indicating their beliefs about mathematics pedagogy and content. Statistical analyses indicate that this sample favors more reform-based, constructivist-type practices with respect to mathematics instruction than non-reform, traditional practices. While beliefs were most similar with regards to student-student interaction, student tasks, and student confidence, there was a large amount of variance with respect to teachers’ conceptions of math as a discipline.

**Internal Consistency**

Internal consistency establishes how well the items on a measure align with one another as a group (Salkind, 2004). The stronger the reliability, or the closer the Cronbach’s alpha is to 1.00, the more consistent the items on the measure relate to the concept of interest. While internal consistency had been established for the *Self-Report Survey* instrument from two separate occasions, .81 for 517 K-8 teachers and .81 for 2,170 K-8 teachers, these data do not guarantee that the measures would produce similar results for this study. Therefore,
the internal consistency of *The Self-Report Survey* was computed by calculating Cronbach’s alpha with a goal of .7 reliability or higher (Nunally, 1978). The alpha for the *Self-Report Survey* instrument was .733 indicating that the *Self-Report Survey* had acceptable internal consistency for this sample population. An Item-Total Statistics analysis was performed to determine whether the deletion of particular survey items would increase the reliability of the measure. This analysis determined that the removal of items would not greatly impact the Cronbach’s alpha, therefore, all twenty questions of the survey were used in the analyses.

**Descriptive Statistics**

Participants answered twenty questions using a 6-point Likert scale from strongly agree (6) to strongly disagree (1). For each teacher, average sum scores were computed. Higher scores (5, 6) indicated more constructivist-type beliefs on the part of the teacher while lower scores (1, 2) indicated more traditional-type beliefs. Measures of central tendency were computed to summarize the data for the composite score for the beliefs instrument, and measures of dispersion were computed to understand the variability of scores. The average score for the sample population (\(n = 29\)) was 4.92 (\(SD = .40\)). The minimum score was 4.10 (Slightly Agree) and the maximum score was 5.70 (Agree), with a range of 1.60. Thirteen of the twenty-nine participants scored a level 5 or higher, indicating that they agreed with statements that were constructivist in nature. Sixteen participants scored a level 4 (Slightly Agree), indicating that they somewhat agreed with statements that conveyed beliefs about constructivist instructional practices for mathematics. There were not any final scores that fell within level 1, 2 or 3 (Disagree). Thus, on average, teachers tended to be fairly constructive with regard to their teaching beliefs.

The ratio of skewness to its standard error was .25, indicating that this sample distribution was slightly positively skewed. The ratio of kurtosis to its standard error was -
.47, indicating that this sample distribution was slightly platykurtic. Because both ratios are less than two, these ratios do not indicate a cause for concern because they indicate a relatively normal distribution.

**Nine Dimensions of the Self-Report Survey**

The *Self-Report Survey* is based on a blueprint of standards-based teaching generated by the authors based on nine-dimensions of reform oriented mathematics: (D1) Program Scope, (D2) Student Tasks, (D3) Discovery, (D4) Teacher’s Role, (D5) Manipulatives and tools, (D6) Student-Student Interaction, (D7) Student Assessment, (D8) Teacher’s Conceptions of math as a discipline, and (D9) Student Confidence (see Appendix B for Dimension Descriptions). These dimensions provide sub-constructs of the overall beliefs survey which display a deeper look at the categories of beliefs of the participants. As with the overall beliefs scores, average scores of the items within that dimension were computed. Table 4.1 illustrates the descriptive statistics for each of these dimensions.

**Table 4.1. Descriptive Statistics for Nine Dimensions of the Self-Report Survey**

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Range</th>
<th>Minimum</th>
<th>Maximum</th>
<th>M</th>
<th>SD</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>29</td>
<td>3.00</td>
<td>3.00</td>
<td>6.00</td>
<td>4.540</td>
<td>.669</td>
<td>.448</td>
</tr>
<tr>
<td>D2</td>
<td>29</td>
<td>2.00</td>
<td>4.00</td>
<td>6.00</td>
<td>5.092</td>
<td>.604</td>
<td>.364</td>
</tr>
<tr>
<td>D3</td>
<td>29</td>
<td>4.00</td>
<td>2.00</td>
<td>6.00</td>
<td>4.552</td>
<td>.948</td>
<td>.899</td>
</tr>
<tr>
<td>D4</td>
<td>29</td>
<td>2.50</td>
<td>3.50</td>
<td>6.00</td>
<td>5.103</td>
<td>.726</td>
<td>.507</td>
</tr>
<tr>
<td>D5</td>
<td>29</td>
<td>3.00</td>
<td>3.00</td>
<td>6.00</td>
<td>4.609</td>
<td>.855</td>
<td>.731</td>
</tr>
<tr>
<td>D6</td>
<td>29</td>
<td>1.00</td>
<td>5.00</td>
<td>6.00</td>
<td>5.609</td>
<td>.309</td>
<td>.096</td>
</tr>
<tr>
<td>D7</td>
<td>29</td>
<td>3.00</td>
<td>3.00</td>
<td>6.00</td>
<td>4.483</td>
<td>.726</td>
<td>.526</td>
</tr>
<tr>
<td>D8</td>
<td>29</td>
<td>5.00</td>
<td>1.00</td>
<td>6.00</td>
<td>3.828</td>
<td>1.311</td>
<td>1.719</td>
</tr>
<tr>
<td>D9</td>
<td>29</td>
<td>2.00</td>
<td>4.00</td>
<td>6.00</td>
<td>5.603</td>
<td>.699</td>
<td>.489</td>
</tr>
</tbody>
</table>

Valid N (listwise) 29
Participants had similar beliefs regarding how the formation of the classroom promotes student-student interaction (D6), the use of open-ended problems that typically have multiple solutions (D2), and the focus on raising student self-confidence in mathematics (D9). Thus, as a group, the teachers agreed with the belief that math is best taught by providing student opportunities to explain thoughts and ideas with one another as they work on open-ended problems with peers in mixed ability groups. In addition, the teachers believed in building student confidence as mathematical learners.

The dimensions with the largest range and variance illustrate the difference in beliefs regarding conceptions of mathematics as a dynamic subject or fixed body of knowledge (D8). Thus, while the teachers’ indicated a strong belief (M=5.092) about using open-ended problems that encouraged multiple strategies to solve rather than using problems with a fixed procedure for solving, thirteen teachers indicated that they disagreed or strongly disagreed with the belief that math is a fixed body of knowledge in which “a lot of things in math must simply be accepted as true and remembered” (Q15).

**Summary of Statistical Analysis**

Overall, these data indicate that this sample favors more reform-based, constructivist-type practices with respect to mathematics instruction than non-reform, traditional practices. While beliefs were most similar with regards to student-student interaction, student tasks, and student confidence, there was a large amount of variance with respect to teacher’s conceptions of math as a discipline.
Research Question 2: What Mathematical Knowledge for Teaching Do Kindergarten Teachers Possess?

Summary of Statistical Analysis

Participants completed a *Mathematical Knowledge for Teaching* (MKT) measure (Hill et al., 2004), answering 30 questions concerning number and geometry content. Statistical analyses indicate that the sample’s mathematical knowledge for teaching varied from -1.60 (approximately 8 out of 27 correct) to 1.62 (approximately 24 out of 27 correct). The teachers performed significantly better on geometry questions than number sense questions. In addition, a positive significant correlation was found between geometry and number sense items.

Internal Consistency

While internal consistency had been established for various versions of the MKT measure, the measure used for this study was created using items from an MKT item bank for the study’s specific sample. Therefore, the internal consistency of the measure was computed by calculating Cronbach’s alpha with a goal of .7 reliability or higher (Nunally, 1978). The alpha the MKT measure was .78 indicating that the MKT measure had acceptable internal consistency reliability for this sample population.

An Item-Total Statistics analysis was performed to determine whether the deletion of particular items would increase the reliability of the measure. This analysis identified that the removal of items 6a, 6b and 6d would greatly impact the Cronbach’s alpha (see Table 4.2). When considering the removal of this question, several issues were taken into account. Each of the four parts in Question 6 asked teachers to evaluate the correct thinking of student’s statements regarding the recording of a number sentence that addressed positive and negative temperatures. Although kindergarten teachers build the foundation for addition
and subtraction and the meaning behind the symbols used in such situation, this question was as directly related to a typical kindergarten scenario as other questions included on the measure.

**Table 4.2. Item-Total Statistics**

<table>
<thead>
<tr>
<th></th>
<th>Scale Mean if Item Deleted</th>
<th>Scale Variance if Item Deleted</th>
<th>Corrected Item Total Correlation</th>
<th>Cronbach’s Alpha if Item Deleted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q6a</td>
<td>16.86</td>
<td>24.909</td>
<td>-.170</td>
<td>.799</td>
</tr>
<tr>
<td>Q6b</td>
<td>17.00</td>
<td>25.429</td>
<td>-.278</td>
<td>.803</td>
</tr>
<tr>
<td>Q6d</td>
<td>16.76</td>
<td>22.975</td>
<td>.225</td>
<td>.779</td>
</tr>
</tbody>
</table>

In addition, Part C of this question was identified by the authors as an item with a slope less than .5 and therefore was previously identified by the researcher as an item that would not be a reliable item according to the authors’ recommendations. Furthermore, the remaining number sense questions addressed basic number concepts that kindergarten teachers typically need in order to teach addition and subtraction. Therefore, question six was removed, and Cronbach’s alpha was re-computed. As a result of the deletion of this question, the internal consistency increased from .78 to .81, indicating that the items in the measure were consistent with one another.

**Descriptive Statistics**

Twenty-seven of the 30 items answered were used in the statistical analyses of the MKT measure for reasons stated earlier. As requested by the authors of the measure, raw units are not reported. Instead, all raw scores were standardized by way of SPSS so that each person’s score is expressed as a Z-score, the standardized distance of each score from the average score in this population. Table 4.3 provides a summary of the descriptive statistics for the MKT measure.
Table 4.3. MKT Descriptive Statistics

<table>
<thead>
<tr>
<th>No. of Survey Items</th>
<th>N</th>
<th>Min</th>
<th>Median</th>
<th>M</th>
<th>SD</th>
<th>Max</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>29</td>
<td>-1.60*</td>
<td>.22</td>
<td>.00**</td>
<td>1.00</td>
<td>1.62***</td>
<td>3.22</td>
</tr>
</tbody>
</table>

Note: Reliability of the MKT instrument (Cronbach’s alpha) was 0.81.
* 8 out of 27 responses correct
** approximately 15 out of 27 responses correct
*** 24 out of 27 responses correct

As illustrated, the ratio of skewness to its standard error was -.54. The ratio of kurtosis to its standard error was -1.43, indicating that this sample distribution was slightly platykurtic. Because both ratios are less than two, these ratios do not indicate a cause for concern because they indicate a relatively normal distribution.

MKT Sub-parts: Number and Geometry

The 27 items used in the statistical analyses consisted of eleven Number Concepts and Operations items and sixteen Geometry items. The teachers’ z-scores for the Number and Operations strand ranged from -1.79 (1 out of 11 responses correct) to 1.45 (9 out of 11 responses correct), with a mean (.00) of approximately 5 out of 11 response items correct. With a ratio of skewness to its standard error of -.73 and a ratio of kurtosis to its standard error of -1.23, a relatively normal distribution is indicated.

With respect to Geometry portion of the measure, the teachers’ z-scores ranged from -2.42 (3 out of 16 responses correct) to 1.45 (15 out of 16 responses correct), with a mean (.00) of approximately 10 out of 16 response items correct. The ratio of skewness to its standard error was -1.01, and the ratio of kurtosis to its standard error was -.74 indicating a relatively normal distribution.

Although these results were used in correlational analyses later discussed, there was a desire to learn additional information about the teachers’ knowledge by the researcher for...
descriptive purposes. Therefore, a paired-samples t-test was conducted to compare the means of number content knowledge and geometry content knowledge to determine if the average difference was significantly different from zero. The scores on the geometry portion of the measure ($M = .66, SD = .19$) were significantly greater than the scores on the number portion ($M = .49, SD = .23; t(28) = -4.62, p = 0.000, d = .81$) as indicated by the higher mean proportion for that subtest. Thus, the teachers obtained a higher score on geometry than on number sense items.

A two-tailed Pearson correlation was also conducted to determine how the number portion of the measure correlated with the geometry portion. A positive significant correlation of .586 was found ($p = .001$), indicating that the teachers’ number sense was not mutually exclusive of their geometric sense. Therefore, an increase in the score for number correlated with an increased score for geometry.

**Summary of Statistical Analysis**

Overall, these data indicate that the kindergarten teachers’ mathematical knowledge for teaching varied from -1.60 (approximately 8 out of 27 correct) to 1.62 (approximately 24 out of 27 correct). The data analysis also indicates that this sample performed significantly better on questions regarding geometry concepts than number concepts. In addition, a positive significant correlation was found between geometry and number sense items.

**Research Question 3: What Instructional Practices Do the Kindergarten Teachers Use that Promote Mathematical Understanding?**

**Summary of Statistical Analysis**

The instructional practices used by the teachers in this sample were captured through the use of the *FirstSchool Snapshot* (Ritchie et al., 2010) classroom observation tool. Statistical analyses indicate that the kindergarten teachers used constructivist practices an
average of 61% of the time. Of the time children spent on mathematical topics ($M = .22$, $SD = .08$, $min = .07$, $max = .41$), the average percentage of time teachers spent using constructivist practices was 15%.

**Inter-rater Reliability**

Inter-rater reliability is a measure used to report the level of agreement on different data collectors’ judgments of an outcome (Salkind, 2004). The authors of the *FirstSchool Snapshot* use Cohen’s Kappa to measure inter-rater reliability of the data collectors. Generally, the closer the Cohen’s Kappa score is to 1, the better the reliability. Scores .60 and greater are typically considered having substantial agreement (Landis & Koch, 1977), yet researchers typically agree that a number of factors influence the magnitude of the score and caution the use of one value of kappa as universally acceptable (Sim & Wright, 2005). The authors of the *FirstSchool Snapshot* agreed that kappa values of .6 and above are acceptable for this, and previous, studies. Before collecting data in classrooms of this sample population, the data collectors had to attain an overall kappa across all 28 codes of .60 on each item with the correct codes with one of the measure’s authors during live observations. Only data collectors who met or exceeded the required kappa were given permission to collect data using the *Snapshot* instrument for this study. The measure’s authors acknowledged that higher kappa values are desired and will be addressed in future trainings.

**Descriptive Statistics: Math Minutes**

Teachers in the sample were observed by a trained data collector for an entire school day. Data were collected during all parts of the day including time spent on transitions, meals, and enrichment specials (e.g., Music, PE). However, only data that pertained to the classroom teacher were analyzed. Thus, any experiences that the observed children had with
a teacher other than the classroom teacher were not included in the analysis. Due to the variance in length of each school day, the number of total minutes observed differed from school to school. In order to standardize the number of overall minutes, as well as math minutes observed, a proportion for the amount of time observed was created for each teacher. On average, children spent .22 of their day with mathematics ($SD = .08$). The least amount of time spent on mathematics was .07, with a maximum of .41.

**Descriptive Statistics: Overall Constructivist Scores**

Because I was interested in capturing information about constructivist-based teaching practices of the classroom teacher with respect to mathematics, the sub-composites *Choice, Station, Small Group, Oral Language, Peer, Distracted, Collaboration, Reflection,* and *Scaffold* were selected from the *Snapshot* for analysis. An overall constructivist score, in the form of a proportion, was determined for each teacher by looking at these practices over the course of the entire day. Teachers’ overall use of constructivist practices averaged .61 ($SD = .07$) with a minimum proportion of .42 and a maximum proportion of .78. The ratio of skewness to its standard error was -.53 and the ratio of kurtosis to its standard error was -.82. Both ratios are less than two, indicating a relatively normal distribution.

**Descriptive Statistics: Math Constructivist Scores**

A mathematics constructivist score, in the form of a proportion, was determined for each teacher by looking at the sub-composites *Choice, Station, Small Group, Oral Language, Peer, Distracted, Collaboration, Reflection,* and *Scaffold* only when *Numbers, Geometry, Algebra* and *Time* were co-coded. On average, teachers spent .15 ($SD = .10$) of the math coded time using constructivist practices, with a minimum proportion of .03 and a maximum proportion of .27. The ratio of skewness to its standard error was .79, and the ratio of
kurtosis to its standard error was -.60. Both ratios are less than two, indicating a relatively normal distribution.

Summary of Statistical Analysis

Overall, teachers in this sample used constructivist practices an average of 61% of the time. Of the time children spent on mathematical topics ($M = .22, SD = .08$, min = .07, max = .41), teachers spent an average of 15% of that time using constructivist practices. Thus, while teachers used constructivist practices for over half of the entire school day, only 15% of the time spent on mathematics was constructivist in nature.

Research Question 4: What Are the Relationships among Pedagogical Beliefs of Mathematics, Mathematical Knowledge for Teaching, and Instructional Practices for the Kindergarten Teachers?

Summary of Statistical Analysis

Correlation statistics were conducted to determine relationships among and between the different variables, testing the four hypotheses:

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.

Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with beliefs.

Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with instructional practices.

A statistically significant positive correlation was found between content knowledge and overall beliefs, thus strongly favoring Hypothesis 2. When examining the correlation
between content knowledge and the sub-dimensions of beliefs on a deeper level, three statistically significant positive correlations were found: Program Scope (D1), Student Tasks (D2), and Teacher’s Role (D4). A statistically significant correlation was not found between instructional practices and content knowledge or between instructional practices and beliefs, thus results are against Hypotheses 1 and 3.

**Correlation Results**

Correlational statistics describe and measure potential relationships between two or more variables in order to examine how the variables influence one another (Creswell, 2008). While this method cannot establish cause and effect between variables, it does identify possible relationships among the sample population, using the particular measures within the distinct circumstances of the study. For the purpose of this study, the degree of association between overall beliefs and content; overall beliefs and constructivist practices; overall beliefs and math constructivist practices; content and constructivist practices; and content and mathematics constructivist practices were examined. Correlational statistics were also computed to examine possible relationships between the nine dimensions of the beliefs survey ((D1) Program Scope, (D2) Student Tasks, (D3) Discovery, (D4) Teacher’s Role, (D5) Manipulatives and tools, (D6) Student-Student Interaction, (D7) Student Assessment, (D8) Teacher’s Conceptions of math as a discipline, and (D9) Student Confidence and content knowledge, constructivist practices and mathematics constructivist practices. Table 4.4 provides the results of the correlational analyses. Results are then discussed in order of the three hypotheses.

**Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.**
### Table 4.4. Correlation Matrix of MKT Dimensions

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<tr>
<th></th>
<th>MKT Content</th>
<th>Beliefs</th>
<th>(D1)</th>
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<th>Math Construct Practices</th>
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<td>Beliefs</td>
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<td>(D1)</td>
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<td>.782**</td>
<td>.541**</td>
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<td>.521**</td>
<td>.549**</td>
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<td>0.274</td>
<td>0.107</td>
<td>-0.030</td>
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<td>0.151</td>
<td>0.580</td>
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<td>Pearson Correlation</td>
<td>0.223</td>
<td>0.471**</td>
<td>0.083</td>
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<td>-0.007</td>
<td>0.249</td>
<td>0.437*</td>
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<td>Sig. (2-tailed)</td>
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<tr>
<td>Pearson Correlation</td>
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<td>0.039</td>
<td>-0.042</td>
<td>0.065</td>
<td>-0.167</td>
<td>-0.104</td>
<td>0.316</td>
<td>-0.043</td>
<td>0.155</td>
<td>-0.058</td>
<td>-0.147</td>
<td>1</td>
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<tr>
<td>Sig. (2-tailed)</td>
<td>0.764</td>
<td>0.842</td>
<td>0.829</td>
<td>0.738</td>
<td>0.387</td>
<td>0.593</td>
<td>0.095</td>
<td>0.825</td>
<td>0.423</td>
<td>0.766</td>
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Note: D1=Beliefs Program Scope  
D2=Beliefs Student Tasks  
D3=Beliefs Discovery  
D4=Beliefs Teacher’s Role  
D5=Beliefs Manipulatives & Tools  
D6=Beliefs Student-Student Interaction  
D7=Beliefs Student Assessment  
D8=Beliefs Teacher's Conceptions of Math as a Discipline  
D9=Beliefs Student Confidence
A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between constructivist beliefs about mathematics and constructivist practices used in the classroom with respect to mathematics. A statistically significant correlation was not found between the two variables, \( r = -0.008, n = 29, p = 0.968 \) (\( p > 0.05 \)). The test was not in favor of the research hypothesis, and it was concluded that the teachers with stronger beliefs about reform-oriented, constructivist-based mathematics do not necessarily use constructivist based instructional practices in the classroom when teaching mathematics.

**Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a positive relationship with beliefs**

A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between constructivist beliefs about mathematics and mathematical content knowledge. There was a statistically significant positive correlation (\( p < 0.05 \)) between the two variables, \( r = 0.384, n = 29, p = 0.04 \). The test was significant in favor of the research hypothesis. It was concluded that a statistically significant correlation was found to exist between mathematics beliefs and mathematical knowledge for teaching. This result leads the researcher to believe that when teachers have stronger content knowledge about mathematics, they tend to favor more reform-oriented beliefs regarding mathematics content and pedagogy.

A two-tailed Pearson product-moment correlation coefficient was also computed to assess the relationship between the nine dimensions of the Beliefs Survey and content knowledge to determine which specific beliefs correlated with mathematical knowledge for teaching. Of the nine belief dimensions, three were found to have a statistically significant
correlation with mathematical knowledge for teaching: *Program Scope* (D1), *Student Tasks* (D2), and *Teacher’s Role* (D4).

There was a statistically significant positive correlation (p<.05) between *Program Scope* and content knowledge, $r = .384$, $n=29$, $p=.04$. This result leads the researcher to believe that teachers who tend to believe that the role of a teacher is that of a co-learner tend to have a stronger mathematical knowledge for teaching.

There was a statistically significant positive correlation (p<.05) between *Student Tasks* and content knowledge, $r = .415$, $n=29$, $p=.025$. This result leads the researcher to believe that teachers who favor the use of complex, open-ended problems embedded in real life contexts have a stronger mathematical knowledge for teaching.

In addition, there was a statistically significant positive correlation (p<.05) between *Teacher’s Role* and content knowledge, $r = .401$, $n = 29$, $p = .031$. This result leads the researcher to believe that teachers with beliefs that the breadth mathematics extends beyond number and operations tend to have a stronger mathematical knowledge for teaching.

**Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a positive relationship with instructional practices**

A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between mathematical content knowledge and math constructivist practices. There was not a correlation between the two variables, $r = -.058$, $n = 29$, $p = .764 (p > .05)$. The test was not in favor of the research hypothesis, and it was concluded that there was no evidence of an association between teachers’ mathematical knowledge for teaching and use of constructivist instructional practices when teaching mathematics.
Summary of Correlation Results

Four significant correlations were found, as illustrated in Figure 4.1.

Figure 4.1. Summary of Correlation Results

A statistically significant positive correlation was found between content knowledge and overall beliefs. Three statistically significant positive correlations were also found between content knowledge and three beliefs’ dimensions: Program Scope (D1), Student Tasks (D2), and Teacher’s Role (D4). No statistically significant correlations were found with Instructional Practices.

Significance of the Study

This study addressed numerous requests for research studies from the US Department of Education: National Mathematics Advisory Panel (2008), the Society for Research in Child Development: The Social Policy Report (2008), and the National Research Council: Committee on Early Childhood Mathematics (2009). In particular, this study placed a focus
on kindergarten teachers, particularly since “scant attention to the special challenges of teaching 4- and 5-year-olds” has been addressed by researchers (Social Policy Report, 2008). The categorization and examination of beliefs, mathematics knowledge for teaching, and instructional practices guided the content, pacing, and support for the teachers participating in this study. In addition, the data collected informed state leaders’ work with the revision of mathematics support documents and materials for the new mathematics standards and provided insight into possible needs and areas of focus for the K-2 state leaders guiding future professional development, resources and state support offered. Most importantly, the findings from the study provided the participants valuable insight into instructional practices used and an avenue for which to pursue future conversation about the role that beliefs and mathematical knowledge for teaching play in the daily decisions they make. While “research can never provide prescriptions . . . it can be used to help guide skilled teachers in crafting methods that will work in their particular circumstances” (Kilpatrick et al., 2001, p. 26).
CHAPTER 5
SUMMARY OF RESULTS AND DISCUSSION

Overview

The purpose of this study was to describe kindergarten teachers’ pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice and to examine the relationships among them. This study responded to specific requests for research from the National Mathematics Advisory Panel (2008), the Society for Research in Child Development (2008), and the National Research Council, Early Childhood Mathematics Committee (2009).

This chapter will review and discuss the results based on the quantitative data from The Self-Report Survey, the Content Knowledge for Teaching Mathematics measure, and the FirstSchool Snapshot. First, a description of the sample population will be provided. An overview, summary of statistical analysis, discussion, and recommendations will be organized by the four major research questions and the hypotheses:

1. What pedagogical beliefs of mathematics do kindergarten teachers hold?
2. What mathematical knowledge for teaching do kindergarten teachers possess?
3. What instructional practices do the kindergarten teachers use that promote mathematical understanding?
4. What are the relationships among pedagogical beliefs of mathematics, mathematical knowledge for teaching, and instructional practices for the kindergarten teachers?

Hypothesis 1: Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.

Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with beliefs.

Hypothesis 3: Teachers’ mathematical knowledge for teaching will have a statistically significant relationship with instructional practices.

The chapter will conclude with the significance of the study, limitations within the study, suggestions for future research, and final remarks.

**Sample Population**

Data for this study were gathered from twenty-nine teachers participating in a three-year professional development initiative designed to increase kindergarten teacher’s early childhood pedagogy and leadership. In order to become a part of this professional development opportunity, kindergarten teachers applied by completing an application discussing their current beliefs and practices. Of the 119 applications, 36 teachers were selected to participate in the three-year initiative. It is from these 36 teachers that the sample of 29 teachers for this research study was formed. During the first two years of the project, the teachers learned about and experimented with research-based, developmentally appropriate instructional practices for young children, primarily focusing on literacy. At the time of the study, the participants were in their third, and last, year of the study. Table 5.1 provides descriptive statistics for the sample population.
When queried, seven of the 29 teachers stated that they had a fear of mathematics and felt uncomfortable teaching mathematical concepts beyond the primary years. Twenty-three teachers identified a specific mathematics curriculum adopted for use during the school year in which this study took place, however twenty-three percent of the teachers had not received training specific to the mathematics curricula identified. Of those who did receive training, seven teachers described the training either as brief, limited, “wasn’t very good” and “terrible.” Outside support dedicated to the mathematics program varied from non-existing (17%), professional organization membership/materials (13%), professional learning communities (33%), district-wide professional development (53%), and school level
professional development (57%). Mathematics assessments varied from the state suggested assessments, district created assessments, assessments from the adopted curriculum, and a mixture of all. Two teachers reported having no district mandated mathematics assessments at the time of the survey. Two teachers belonged to the National Council of Teachers of Mathematics (NCTM) and the state affiliate (NCCTM).

**Discussion of Results**

**Research Question 1: What Pedagogical Beliefs of Mathematics Do Kindergarten Teachers Hold?**

**Overview.** In order to assess teacher beliefs, participants completed the *Self Report Survey* (Ross et al., 2003). In general, findings from this survey indicated that the teachers held beliefs that were in line with constructivist practices but may still hold some traditional beliefs about mathematics content. Understanding the beliefs of teachers, and the differences between the beliefs of instruction and content has practical implications for teacher educators as they design teacher educator courses, instruct reform-oriented courses and provide in-service professional development. In order to move pre-service and in-service teachers from traditional to reform-oriented mathematics, they need opportunities to become aware of their beliefs and the role and impact that these beliefs play in the teaching and learning of mathematics.

**Summary of statistical analysis.** Participants completed the *Self Report Survey* (Ross et al., 2003), answering twenty questions using a 6-point Likert scale from strongly agree (6) to strongly disagree (1). Ultimately, the closer the score was to 6, the more math reform constructivist type beliefs the teacher held. Each teacher’s score was totaled and then averaged to create a final score from 1 to 6. For the sample population, the maximum score
was 5.70 and the minimum score was 4.1, with a mean of 4.92 ($SD = .40$). Thirteen of the twenty-nine participants scored a level 5 or higher, indicating that they agreed to more constructivist type beliefs with regards to mathematics instruction. The remaining participants scored a level 4, indicating that they slightly agreed to more constructivist type beliefs with regards to mathematics instruction. There were not any final scores that fell within level 1, 2 or 3, indicating that the sample favored a more reform-based, constructivist approach to mathematics instruction than non-constructivist methods.

**Discussion.** Results from the teachers’ overall beliefs were not surprising. The twenty-nine teachers in this sample were participating in a three-year professional development initiative to which they voluntarily applied. Thus, while the teachers varied in experience, location, resources, degrees, and honors, each teacher selected indicated an interest to address developmentally appropriate kindergarten practices and a desire to impact those practices across the state.

In addition, the twenty-nine participants in this sample were entering the third and last year of the professional development program. Prior to completing the survey, the teachers had completed two years of learning about and experimenting with research-based, developmentally appropriate instructional practices for young children, primarily focused on literacy. Taking into account their interest and dedication to learning about and using practices that were developmentally appropriate, it was not surprising to find that this sample’s overall beliefs favored more reform-based constructivist practices with regard to mathematics, even though mathematics professional development had yet to occur at the time that the survey was conducted.
Results of the current study are similar to findings from Anderson and Bobis (2005), Chval et al. (2006), and Paterson (2009) who found that teachers favored a more reform-based constructivist approach to teaching mathematics. It is possible that the results from these recent studies, including the current study, are due to NCTM’s leadership and development of the *Principles and Standards of Mathematics* (2000). This publication was an effort to outline a national vision for school mathematics by identifying the essential components of a high-quality school mathematics program. Since its first publication in 1989, mathematics educators have promoted the beliefs and practices of the organization and documents within teacher education programs, teacher professional development initiatives, and curricula development. It is possible that teachers were educated in the philosophy and research behind the reform, thus impacting personal beliefs about the teaching of mathematics.

Teacher beliefs influence the decisions teachers make on a daily basis (Kagan, 2002; Pajares, 1992; Speer, 2002). Beliefs about mathematics influence what mathematics is taught and how it is taught. These decisions are ultimately internalized by the students they are teaching, sculpting their beliefs about what mathematics is and how it is learned (Liljedahl, 2008). If reform-oriented mathematics is to be successful, teachers need opportunities to identify their beliefs, recognize the influence they have on teaching practices, and reflect on the alignment between their beliefs and their teaching practices.

**Summary of statistical analysis.** The 20 questions from the beliefs survey fell into one of nine dimensions of elementary mathematics reform as identified by the author of the measure: program scope (D1), student tasks (D2), discovery (D3), teacher’s role (D4), use of manipulatives and tools (D5), student-student interaction (D6), student assessment (D7),
teacher’s conceptions of mathematics as a discipline (D8), and student confidence (D9) (see Appendix B for details). Participants had similar beliefs regarding how the classroom environment promotes student-student interaction (D6), the use of open-ended problems that typically have multiple solutions, (D2) and the focus on raising student self-confidence in mathematics (D9). The greatest variance among responses was found for beliefs regarding conceptions of mathematics as a dynamic subject or fixed body of knowledge (D8). Table 5.2 provides the descriptive statistics for these dimensions.

Table 5.2. Descriptive Statistics for Selected Dimensions

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Range</th>
<th>Minimum</th>
<th>Maximum</th>
<th>M</th>
<th>SD</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2</td>
<td>29</td>
<td>2.00</td>
<td>4.00</td>
<td>6.00</td>
<td>5.092</td>
<td>.604</td>
<td>.364</td>
</tr>
<tr>
<td>D6</td>
<td>29</td>
<td>1.00</td>
<td>5.00</td>
<td>6.00</td>
<td>5.609</td>
<td>.309</td>
<td>.096</td>
</tr>
<tr>
<td>D9</td>
<td>29</td>
<td>2.00</td>
<td>4.00</td>
<td>6.00</td>
<td>5.603</td>
<td>.699</td>
<td>.489</td>
</tr>
<tr>
<td>D8</td>
<td>29</td>
<td>5.00</td>
<td>1.00</td>
<td>6.00</td>
<td>3.828</td>
<td>1.311</td>
<td>1.719</td>
</tr>
</tbody>
</table>

Discussion. The data analyses of the nine dimensions of the survey highlight differences between beliefs of pedagogy regarding mathematics instruction and beliefs about mathematics content. The teachers in this sample collectively and strongly agreed that student-student interactions, open-ended problems embedded in real life contexts, and raising student self-confidence in mathematics were important to the teaching of mathematics. All three of these dimensions can be interpreted as part of the instructional practices for mathematics. However, the teachers varied greatly in their beliefs regarding mathematics content as either a dynamic subject or a fixed body of knowledge. This dimension focuses on the content of mathematics rather than the instructional side of mathematics.

This distinction between beliefs regarding mathematics instruction and mathematics content is similar to the findings of Raymond’s (1997) qualitative study of pre-service
elementary teachers which also focused on beliefs and practices. Results indicated that teachers’ beliefs about mathematics pedagogy were more constructivist in nature while beliefs about mathematics content were quite traditional. In particular she found that the participants of the study believed that mathematics was a fixed body of knowledge with rigid rules and procedures and became frustrated when the instructor did not provide the final answers to the problems they were solving or tell them what they needed to know.

This passive process to learning is deeply embedded in our culture (Ball et al., 2001; Cohen, 1989; Jackson, 1986; Stigler & Hiebert, 1999). Most Americans typically view mathematics as fixed knowledge which is learned by following steps and directions with arriving at the correct answer as the main objective (NRC, Mathematics Learning in Early Childhood, 2009). Memorization and imitation are encouraged and expected as students are told to master specific procedures and practices. This strong belief is perpetuated as teachers who were formally instructed in this manner year after year continue to do so in their own classrooms. This phenomenon needs continued focus by researchers, educators and policy makers in order to move the understanding of mathematics beyond rote memorization to rich conceptual understanding.

**Recommendations.** In order to help pre-service and in-service teachers progress from traditional to reform-oriented mathematics, teachers need numerous opportunities to become aware of their beliefs and the impact that their beliefs have on daily instructional decisions. They also need opportunities to continually reflect on their beliefs as new information is learned and new practices are explored. Teacher educators need to understand the importance that beliefs play, and must ensure that attention to beliefs is purposefully included in teacher education courses, professional development, and reform-oriented
initiatives. Further study is needed to explore the differences between beliefs of mathematics instructional practices and mathematics content with particular attention to the types of beliefs that teachers possess and the relationship each type influences teaching practices.

**Research Question 2: What Mathematical Knowledge for Teaching Do Kindergarten Teachers Possess?**

**Overview.** In order to assess mathematical knowledge for teaching, participants completed the *Mathematical Knowledge for Teaching* measure (Hill et al., 2004) that was specifically designed for this sample population. In general, findings indicated that there were varying levels of content knowledge among the teachers, from very weak to very competent. In addition, results showed that the teachers’ content knowledge of geometry was significantly better than their content knowledge of number and their knowledge of geometry and number were significantly correlated with one another. Understanding the mathematical content knowledge of teachers, specifically those who teach our youngest students, has implications for teacher educators as decisions are made regarding mathematics content and delivery within courses and professional development experiences. In addition, policies affecting the various early childhood licensure options offered need to be considered in order for early childhood teachers to become highly qualified teachers of mathematics.

**Summary of statistical analysis.** Data analyses indicated that the mathematical knowledge for teaching of the study’s teachers varied from -1.60 (8 out of 27 correct) to 1.62 (24 out of 27 correct) with a mean score of 0 (approximately 15 out of 27 correct). Ten teachers performed below the mean, four teachers performed at the mean, and fifteen teachers performed above the mean. The data analysis indicates that this sample performed significantly better on questions regarding geometry concepts ($M = .66$, $SD = .19$) than
number concepts ($M = .49, SD = .23; t(28) = -4.62, p = 0.000, d = .81$). Furthermore, correlational analysis indicated that there was a relationship between geometry and number ($r = .586; p < .01$).

**Discussion: Overall results.** The MKT measure is a rigorous assessment, designed for the average teacher to answer 50% of the items correctly. The assessment created for this specific population was created from a pool of MKT items in order to create a measure in which items that were most likely to be encountered in a Kindergarten classroom were presented. The researcher carefully followed protocols set by the authors as described in Chapter 3. The data analysis indicates that the created measure functioned as intended, creating a normal distribution. These results lent themselves to correlational analyses with instructional practices and beliefs which will be discussed later in this chapter.

**Overall discussion.** Numerous studies have focused on the mathematical knowledge of teaching of both pre-service and in-service teachers (e.g., Anders, 1995; Burton, 2006; Fennema et al., 1996; Heaton, 1992; Kajander, 2007; Lundin, 2007; Matthews, Rech, & Grandgenett, 2010; Mullens, Murnane, & Willet, 1996; Roy, 2008; Safi, 2009; Spillane, 2000; van den Kieboom, 2008); yet, fewer studies have included kindergarten teachers in their sample population (Ball et al., 2005; Bright et al., 1998; Chi-chung et al., 1999; Hill et al., 2005; Lee, 2010; Leung & Park, 2002; Ma, 1999; Schwartz & Reidesel, 1994; Warfield, 2001; Wilkins, 2002). Several of these studies will be the focus of discussion here with regards to number and geometry content and measures for early childhood teachers. Additional reviews of the studies will also be addressed in sub-sections of Research Question 4 with attention to the results of correlational analyses.
Discussion: Geometry vs. number. It was interesting to the researcher to find that the teachers’ geometry knowledge was significantly stronger than number knowledge. Number is considered a foundational strand of mathematics for the primary years, and most of the state standards, curriculum materials, and professional development focus on this particular strand. This significant difference in knowledge seems to bring forth the issue of the complexity of number sense, and the depth of understanding beyond that of memorization and rote procedures. While the concepts presented in the test items represented typical content that for the elementary years, the format of the questions required teachers to analyze student work, consider unusual situations, and think about number sense in ways that require a conceptual understanding of number. Although the geometry questions were similar in difficulty, and in an area in which American students continue to perform poorly (NAEP, 2009a), it was clear from the analysis that this group of teacher’s understanding of number sense was less than that for geometry.

Studies of early childhood teachers regarding number and geometry knowledge of teaching have been mixed. As with the findings in this current study, Schwartz and Reidesel (1994) found that teachers were more knowledgeable in geometry than number. McCravy (2008) found no significant relationships regarding number and operations items and shape and spatial relations items. Lee’s (2010) results indicated that the teachers’ knowledge about number sense was greater than that of geometry. While a direct association cannot be made between these studies and the current study, particularly since different assessment measures were used, the studies do highlight the fact that additional research is needed to understand the content knowledge of teachers. By analyzing teachers’ strengths and weaknesses of the various mathematics strands, teacher educators and curricula developers would be better
informed as they work with and support the development of teachers’ mathematical knowledge for teaching.

**Discussion: Mathematics measures for early childhood teachers.** While studies have begun to identify teachers’ mathematical knowledge for teaching, Mewborn, McCray, and Lee bring forth issues regarding assessing mathematical knowledge for teaching of early childhood teachers. In her analysis of studies focused on preservice teachers’ mathematical knowledge, Mewborn (2001) noted the lack of research focused on essential concepts for teachers who teach kindergarten and first grade. She addressed the need for measures and studies of early childhood teachers on concepts such as whole numbers, patterns, and counting, which are “fundamental topics” (Mewborn, 2001, p. 33) for these grades.

Due to a lack of content knowledge assessments that used examples commonly found in preschool classrooms, McCray (2008) created an interview measure in order to study preschool teachers’ content knowledge. Basing this measure on methodology by Ball and colleagues, McCray used interview questions along with teaching scenarios to capture the teacher’s ability to contextualize questions in situations they would most likely recognize. McCray surmised that the scenarios embedded in common practices similar to those the teachers experience on a daily basis provided a “powerful window” (McCray, 2008, p. 122) into a preschool teacher’s mathematical content knowledge for teaching.

Due to the fact that “there was no single instrument found in a comprehensive database (EBSCO) to measure early childhood teachers’ PCK in mathematics” (Lee, 2010, p. 32), Lee (2010) selected *The Survey of Pedagogical Content Knowledge (PCK) in Early Childhood Mathematics* (Smith, 1998, 2000) to measure content knowledge of his kindergarten sample. Associate professor Dr. Mosvold, at the University of Stavanger,
Norway, questioned Lee’s lack of reference to Ball, her colleagues, and the MKT measure, asking “how it is possible to [conduct a study] about teachers’ pedagogical content knowledge of mathematics without making any reference to the MKT or any of the work done by Deborah Ball and her colleagues at the University of Michigan. I understand that this [study] has a focus on Kindergarten, but still…” (Mosvold, 2/26/2010 blog). He goes on to agree with Lee regarding the lack of research done on teachers’ knowledge of mathematics in kindergarten, and acknowledges the focus as a “good thing” (Mosvold, 2/26/10 blog).

The lack of measurements for this particular population is something to consider. The researcher’s committee and the authors of the MKT measure acknowledged that none of the current forms of the MKT measure lent themselves directly to kindergarten teachers due to the fact that many of the situations and scenarios in the items were not commonly found in kindergarten classrooms. Perhaps this is why Lee (2010) did not use this measure with his population. Future studies are needed in order to consider the types of measures of mathematical content that most accurately assess preschool and kindergarten teachers’ knowledge.

**Recommendations.** Research studies over the past decade have highlighted the fact that teachers’ mathematical knowledge is “dismaying thin” (Ball et al., 2005, p. 14). Calls for research, particularly focusing on the early years, have been made by numerous committees. However, research has only just begun to help identify what mathematical knowledge teachers actually need to know. Future studies are needed, particularly for teachers who work in the field of early childhood (birth – 8 years of age), to better understand and measure the “balance of knowledge of content and knowledge of pedagogy, the nature of content knowledge useful for teaching, and the content of pedagogical
knowledge” (Hill et al., 2007, p. 149). In addition, policies affecting the mathematics coursework required for the various early childhood certification options offered need to be considered for early childhood teachers. Teachers of young children are also teachers of complex mathematics, and rich mathematics courses are needed for all teachers, regardless of the ages on which the teacher program is focused.

**Research Question 3: What instructional practices do the kindergarten teachers use that promote mathematical understanding?**

**Overview.** In order to assess the constructivist-based instructional practices of the teachers, full day classroom observations were conducted using the *FirstSchool Snapshot* measure (Ritchie et al., 2010). When taking into account instructional practices that were used over the course of an entire day, across all subject matter, findings, on average, indicated that constructivist practices were observed over half of the day. However, when constructivist practices were examined with respect to mathematics, only a marginal amount of time was spent using constructivist practices. Understanding the types of practices that are used in general, and whether they are incorporated into mathematics instruction, has practical implications for teacher educators and reform-oriented curricula developers. Teachers need the support of universities, professional development initiatives, administrators, policy makers, and curriculum materials that help translate the philosophy of constructivism into actual practices in the classroom.

**Summary of statistical analysis.** Data were collected for an entire school day by observing four randomly-selected students and coding their experiences and interactions using the *FirstSchool Snapshot* observation instrument. In order to address the specific research questions of this study, only data that related directly to the classroom teacher were
analyzed. Because this study was interested in capturing information about constructivist-based teaching practices of the classroom teacher with respect to mathematics, the sub-composites *Choice*, *Station*, *Small Group*, *Oral Language*, *Peer*, *Distracted*, *Collaboration*, *Reflection*, and *Scaffold* were analyzed with and without cross-references with the mathematics sub-categories from the Child Engagement sub-composite: *Numbers*, *Geometry*, *Algebra*, and *Time*. Results are summarized in Table 5.2.

**Table 5.3. Constructivist Practices and Time Spent on Mathematics**

<table>
<thead>
<tr>
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<th>M</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use of constructivist practices</td>
<td>.61</td>
<td>.07</td>
<td>.42</td>
<td>.78</td>
</tr>
<tr>
<td>Use of constructivist practices with respect to mathematics</td>
<td>.15</td>
<td>.10</td>
<td>.03</td>
<td>.27</td>
</tr>
<tr>
<td>Time Spent on Mathematics Topics</td>
<td>.22</td>
<td>.08</td>
<td>.07</td>
<td>.41</td>
</tr>
</tbody>
</table>

**Discussion.** Although the teachers in this sample used constructivist type practices during over half of their day, they only applied those practices to mathematics an average of 15% of the time. It is eye opening to realize that only 15% of the time spent with mathematics incorporated small group, center-based settings with opportunities for oral language, collaboration, and engagement with peers in which the teacher scaffolds learning and encourages reflection. 85% of the mathematics time was spent in a whole group setting or individual work with didactic instruction. Although the mathematics community has researched, advocated, and encouraged the use of reform-based instruction of mathematics, it is clear that the teachers in the sample utilized a more traditional approach to mathematics instruction.

The findings from the current study are similar to studies that have examined the types of instructional practices and teacher-child interactions used in early childhood.
classrooms (Bryant et al., 1991; Early et al., 2010; La Paro et al., 2009; Pianta et al., 2002; Schwartz & Reidesel, 1994). Although findings indicated variance among kindergarten classrooms (Pianta et al., 2002), these studies found that high-quality learning opportunities in kindergarten were minimal (La Paro et al., 2009). A frequent focus on didactic instruction was observed in the early childhood settings, particularly with mathematics instruction (Early et al., 2002; Paro et al., 2009). In addition, mathematics concepts were taught in isolation using traditional instructional practices (La Paro et al., 2009), particularly teacher-directed activities and whole group instruction (Bryant et al., 1991; Early et al., 2010; Pianta et al., 2002). Despite state and national research, recommendations, and literature that describes, advocates, and supports meaningful mathematics instruction, the use of ineffective practices in the teaching of mathematics continue to be prevalent in early childhood classrooms.

**Recommendations.** The lack of consistent, high-quality kindergarten programs is a national concern. Kindergarten is the first formal year of school for most children and one of the foundational years for children’s development. Researchers, teacher educators and policymakers need to focus on identifying, articulating, and expecting quality experiences for kindergarten, particularly with a focus on mathematics. Further study is needed to explore ways to best prepare teachers to provide high-quality, mathematically rich experiences as well as to identify strategies for administrators and policy makers to support the increase of quality during these early years.

**Research Question 4: What are the Relationships among Pedagogical Beliefs of Mathematics, Mathematical Knowledge for Teaching, and Instructional Practices for the Kindergarten Teachers?**
The discussion of the analyses of this research question will be organized by each of the four hypotheses of the study.

**Hypothesis 1:** Teachers’ pedagogical beliefs about mathematics will have a positive relationship with instructional practices.

**Overview.** Correlational statistical analyses were conducted in order to determine if a relationship existed between teachers’ pedagogical beliefs about mathematics and the instructional practices they use. Results indicated that there was not a significant correlation between the two variables which contradict numerous research studies in which significant correlations were found between beliefs and teaching practices. Willis’ (2010) qualitative study of ten of the teachers in the current study offers some insight into why expressed beliefs were not aligned with the practices observed. Results from this study have practical implications for teacher educators and administrators as they help teachers recognize the relationship between beliefs and practices, identify possible barriers that may impede upon their beliefs, and find strategies for aligning beliefs and practices.

**Summary of statistical analysis.** It was hypothesized that there would be a positive relationship between teacher beliefs and instructional practices. A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between pedagogical beliefs about mathematics and math constructivist practices. Correlation analysis indicated no statistically significant differences, $r = .039, n = 29, p = .842 (p > .05)$. Therefore, I must conclude that there is no significant relationship between professed beliefs and actual mathematics instructional practices used in the classroom for this sample population.


**Discussion.** These results contradict previous research studies in which correlations between mathematics beliefs and practices were demonstrated (e.g., Beilock et al., 2010; Philipp, 2007; Pianta et al., 2005; Potari & Georgiadou-Kabouridis, 2008; Staub & Stern, 2002; Stipek & Byler, 1997; Wilkins, 2008; Wright, 1992; Ziccardi -Priselac, 2009). However, inconsistencies between reported beliefs and observed instructional practices have also been noted in the research (e.g., Anderson, 1998; Brown, 2005; Paterson, 2009; Vacc & Bright, 1999; Willis, 2010).

Willis’ (2010) qualitative research study supports thinking about why there was not a correlation between beliefs and practices for this specific sample. Willis, studying ten of the 29 kindergarten teachers from this study, sought to understand the experiences of these kindergarten teachers as they worked to establish, sustain, or improve developmentally appropriate practices in their classrooms. Based upon Festinger’s theory of cognitive dissonance (Festinger, 1957), Willis used qualitative data to help identify barriers to the alignment between beliefs and practices. This theory conceives that when a person faces two cognitions—ideas, knowledge, beliefs, values, or practices- that are associated with each other but are also conflicting, dissonance is created. “The individual is then compelled to find a way to resolve the dissonant situation” (Willis, 2010, p. 94), most typically done by compliance.

Willis found that all ten teachers strongly believed in developmentally appropriate practices, which she characterized as the use of learning centers, movement, exploration, meaningful hands-on learning experiences, and projects that support children’s curiosity, interests, and natural eagerness to learn in which the teachers serve as “both guides to and directors of learning” (p. 108). However, the teachers expressed concern that particular
barriers prevented them from teaching in a constructivist manner and forced conformity regardless of their beliefs. The predominant barriers included “incidents of nonsupport for the teachers and developmentally appropriate practices in general, developmentally inappropriate mandates related to curriculum, lack of meaningful professional development, professional relationships with colleagues, expectations from first grade teachers and parents, as well as availability of resources” (Willis, 2010, p.70). It was clear that the ten teachers in this study experienced obstacles that prevented them from acting on their professed beliefs.

Willis’ findings support and contribute to previous research studies (Anderson, Sullivan & White, 2004; Bobis, 2000; Paterson, 2009; Raymond, 1997; Skott, 2001, Stipek & Byler, 1997), which found that time constraints, lack of resources, mandated curricula, concerns over standardized tests, assessment and reporting practices, student’s needs, students’ behavior, classroom management, parent’s expectations, and school environment contribute to the inconsistency between beliefs and practice. Ziccardi-Priselac (2009) also identifies professional development experiences that neglect teacher beliefs as a reason for little change in practices.

**Recommendations.** Beliefs impact the instructional decisions teachers make, yet many times teachers are unable to align their beliefs with their practices. Teacher educators and administrators need to help teachers identify their beliefs, challenge their beliefs, determine if their beliefs and practices align, and recognize the barriers between their beliefs and practices. Further study is needed to identify the ways in which teachers have overcome barriers in order to align beliefs and practices.

**Hypothesis 2: Teachers’ mathematical knowledge for teaching will have a positive relationship with beliefs.**
Overview. Correlational statistical analyses were conducted to determine if a relationship existed between teachers’ mathematical knowledge for teaching and pedagogical beliefs. Results indicated that there was a statistically significant correlation between the two variables. In particular, beliefs about Program Scope, Student Tasks, and Teacher’s Role were significantly correlated with mathematical knowledge for teaching. Although correlations do not ensure causation, these results led the researcher to believe that teachers who have a stronger mathematical knowledge for teaching tend to believe that the role of a teacher is that of a co-learner, favor the use of complex, open-ended problems embedded in real life contexts, and believe that the breadth mathematics extends beyond number and operations. Results from this study could help guide the development of course work and professional development initiatives, indicating that focus on both beliefs and mathematical knowledge may support the development of reform-based mathematical understandings. Furthermore, due to the lack of studies specifically related to these two variables, additional research is needed to further clarify the relationships between beliefs and mathematical knowledge for teaching.

Summary of statistical analysis. It was hypothesized that there would be a positive relationship between teacher beliefs and mathematical knowledge for teaching. A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between mathematical content knowledge and pedagogical beliefs about mathematics. Correlation analysis indicated a statistically significant positive correlation ($p < .05$) between the two variables, $r = .384$, $n = 29$, $p = .04$. The test was significant in favor of the research hypothesis. It was concluded that a statistically significant correlation was found to exist between mathematics beliefs and mathematical knowledge for teaching. Although
significant correlations imply association, not causation, this result led the researcher to believe that when teachers have stronger content knowledge about mathematics, they tend to favor more reform-oriented beliefs regarding mathematics content and pedagogy, and inversely.

A two-tailed Pearson product-moment correlation coefficient was also computed to assess the relationship between the nine dimensions of the Beliefs Survey and content knowledge to determine which specific beliefs correlated with mathematical knowledge for teaching. Of the nine belief dimensions, three were found to have a statistically significant correlation (p < .05) with mathematical knowledge for teaching: *Program Scope* (D1), \( r = .384, p = .04 \); *Student Tasks* (D2), \( r = .415, p = .025 \); and *Teacher’s Role* (D4), \( r = .401, p = .03 \). These results led the researcher to believe that teachers who have a stronger mathematical knowledge for teaching tend to believe that the role of a teacher is that of a co-learner, favor the use of complex, open-ended problems embedded in real life contexts, and believe that the breadth mathematics extends beyond number and operations.

**Discussion.** Results indicate that a teacher who is more knowledgeable of specialized mathematics content is more likely to believe in reform-oriented teaching of mathematics. These results align with Schwartz and Riedesel’s (1994) findings of elementary teachers in which greater mathematical understanding was identified as an enhancer of beliefs of mathematics understanding. These results seem logical to the researcher for several reasons. First of all, it seems reasonable to consider that teachers who are more knowledgeable of mathematics content are more comfortable with the belief that mathematics is best taught in ways that encourage dialogue, construction of understanding, and the use of open-ended problems and manipulatives. Such knowledge could enable the teacher to have the
conceptual understanding needed to foster such dialogue, carefully plan and provide experiences that encourage students to construct meaning, and use materials appropriately. In addition, it would appear that teachers who are not as competent in specialized mathematical knowledge tend to believe in more traditional practices such as telling, procedures, closed problems, and engage in little dialogue because the teacher his/herself is unable to extend the mathematics beyond these more rote and procedural strategies.

Another possibility for the correlation is that teachers who possess greater mathematical knowledge for teaching, learned mathematics through constructivist ways themselves. Perhaps that is why they have a stronger understanding of the mathematics needed to teach because they themselves learned mathematics in such a way that allowed them to make sense of mathematical concepts. Perhaps teachers who are less knowledgeable learned mathematics by memorizing and following procedures, and came to believe that mathematics was a fixed body of knowledge to be memorized rather than a dynamic subject of which to make sense or understand.

Little research has been conducted with regards to identifying relationships between these two variables. Thus, a follow up study with this sample may help to discern why mathematical knowledge for teaching and beliefs do indeed correlate, and validate or contradict the suggestions made here.

**Recommendations.** Results indicate that there is a relationship between reform-based beliefs regarding mathematics and mathematical knowledge for teaching. Teacher educators need to be aware of this possible relationship, design coursework and trainings that avoid isolating, or ignoring either of these variables, and helps teachers understand how these two variables may relate to one another as they continue to reflect on their practices. Additional
research studies need to be conducted in order to gain a better perspective around this possible relationship.

**Hypothesis 3:** Teachers’ mathematical knowledge for teaching will have a positive relationship with instructional practices.

**Overview.** In order to determine if there was a significant correlation between mathematical knowledge for teaching and instructional practices, correlational statistical analyses were conducted. Results indicated that there was not a significant correlation between the two variables. This contradicts numerous research studies in which significant correlations were found between content knowledge and teaching practices. However, it is important to note that most of these studies focused on small samples of teachers and did not typically include kindergarten in the sample populations. The findings in this current study implies a need for researchers to continue to identify the mathematical knowledge for teaching of teachers of young children and to explore relationships between knowledge and the practices used in an early childhood classroom. Such studies would inform teacher educators and policy makers as certification requirements, courses and professional development opportunities are designed and implemented.

**Summary of statistical analysis.** It was hypothesized that there would be a positive relationship between mathematical knowledge for teaching and instructional practices. A two-tailed Pearson product-moment correlation coefficient was computed to assess the relationship between mathematical content knowledge and mathematics constructivist practices. Correlation analysis indicated no statistically significant differences, \( r = -.058, n = 29, p = .764 \) \((p > .05)\). Therefore, the researcher concludes that there is no significant
relationship between mathematical knowledge for teaching and actual instructional practices used in the classroom.

**Discussion.** It is commonly accepted that teacher content knowledge influences the types of instructional practices used (Math and Science Partnership, 2010). So, it was surprising to find that there was not a significant relationship between knowledge and practices with this sample population. These results contradict numerous research studies in which a relationship between content knowledge and instruction has been found (Fennema et al., 1993; Hill et al., 2008; Ma, 1999; Spillane, 2000; van den Kieboom, 2008; Warfield, 2001; Weiss & Miller, 2006; Wilkins, 2002). It is possible that the results from the current study were influenced by the mathematical knowledge for teaching measure created for this study’s population since there was not one that specifically addressed kindergarten content. Although items selected for this measure were the best representation of what kindergarten teachers would encounter in their classroom, it is possible that a more specific measure designed specifically for this sample would help to create a more accurate picture of their mathematical knowledge for teaching.

It is also possible that the findings of the current study contradict numerous studies due to the fact that many of the research studies conducted with these particular variables did not typically include kindergarten as part of the sample population. With a national focus on the early years, continued work needs to investigate content knowledge, instructional practices and relationships between these two variables in order for teacher educators, administrators and policy makers to best support early childhood programs.

**Recommendations.** Effective teachers of mathematics require knowledge and conceptual understanding of mathematics, students, and instructional practices (NCTM,
Studies have begun to examine teacher content knowledge regarding mathematics, and many studies have found correlations between mathematical knowledge for teaching and instructional practices as indicated earlier. Yet many of these studies, while they offer insight into variables that attributed to teaching behaviors, focused on one or a few teachers. Additional research with larger sample sizes is needed in order to generalize “what” content and “how” that content influences teaching decisions (Adler, Ball, Krainer, Lin, & Novotna, 2005). Furthermore, most of the studies conducted did not include kindergarten in the sample. With a national focus on the early years, continued studies need to investigate content knowledge, instructional practices and the relationships between these two variables in order for teacher educators, administrators and policy makers to best support kindergarten teachers and programs.

Suggestions for Future Research

This study presents several opportunities for future exploration. First of all, the sample in this study was a small select group of kindergarten teachers who volunteered to participate in a long-term professional development initiative that focused on pedagogy and leadership with respect to kindergarten. Research related to the current study with teachers outside of this initiative would add to the literature.

While the population of this study was a sample of convenience, it was also diverse in respect of school culture, location, school socio-economic status, and local district expectations. Similar to Hill and Lubienski’s (2007) study, it would be interesting to compare the teachers’ mathematical knowledge to the demographics of the schools in which they taught. Hill and Lubienski found that a decrease in K-8 teachers’ content scores correlated with schools with a higher proportion of low-SES and Hispanic students. Would
similar results be found with this population, particularly given the diverse population and socio-economic status of this state?

In addition, the amount of time spent on mathematics content varied across the sample. Is there a correlation between the amount of instructional time and the teachers’ content knowledge or pedagogical beliefs? Is there a difference between the proportion of time children spent with mathematics to the amount of time spent with literacy? While more time on a subject does not ensure that quality experiences are occurring, it is interesting to discover if beliefs or knowledge are hindering mathematics opportunities for students.

Several of the teachers in this study indicated high anxiety regarding mathematics in written responses to the initiative’s leaders several months prior to this study. Early childhood teachers have often expressed their dislike and/or anxiety regarding mathematics content and have even contributed the selection of teaching in the primary grades to this mathematics anxiety. Additional exploration into the possible relationships of their affect towards mathematics and their overall beliefs, their content knowledge, and their instructional practices would bring forth valuable information regarding the complexity of being a competent mathematics teacher and the difficulty of changing practices when anxieties and fears are prevalent.

In addition, the beliefs of the teachers in the current study seemed to differ between beliefs about mathematics instruction versus the general construct of mathematics content. Future studies designed to explore beliefs with specific attention to these variables would help teacher educators to better understand the role that beliefs play in daily decision making.

Furthermore, research has only just begun to solidify the actual mathematical content of pedagogical knowledge for teachers. Future studies are needed, particularly for teachers
who work in the field of early childhood, to address questions such as: What mathematical knowledge for teaching impacts understanding of mathematics of young children? Do teachers of preschool and kindergarten need different content knowledge for teaching than fourth or fifth grade teachers? Mathematical competency of students begins in the early years. If the mathematical development of our future generation is to be supported, then continued research needs to address how the mathematical knowledge of teachers can be best understood, fostered and developed.

**Limitations**

Although this research provided a deeper understanding of kindergarten teacher’s beliefs, content knowledge and instructional practice with respect to mathematics, the population did not represent the general kindergarten population of the state. Since each of the teachers volunteered to be a part of the project focusing on best practices for young children, it was possible that the beliefs and instructional practices captured were not as representational as a random sampling of teachers. In addition, teacher mathematical knowledge is difficult to decipher (Ball, Bass, & Hill, 2004). While the MKT measure is a well established instrument, it is not nuanced. It is unable to provide a fine grain report of teacher knowledge, but, instead, provides a general view of overall mathematical knowledge for teaching. Furthermore, the sample size of this study was somewhat small, although it was the total number of classroom teachers participating in the initiative and was within suggested minimal count for correlational studies (Creswell, 2008).

**Conclusion**

This study focused on the mathematics beliefs, content knowledge, and instructional practices of kindergarten teachers. The participants in the study were involved in their third
and last year of a three-year professional development initiative led by the state to address pedagogy and leadership of kindergarten teachers. In order to learn about this sample’s beliefs, mathematical knowledge for teaching, and constructivist-based instructional practices, and the relationships that these three variables had to one another, quantitative data were collected using three separate instruments. Results indicated that the teachers mostly favored reform-oriented constructivist beliefs, performed significantly better on geometry concepts than number concepts, and used constructivist oriented practices a marginal amount of time with respect to mathematics. Furthermore, results indicated that there was a positive and significant correlation between the beliefs of teachers and their content knowledge. However, a significant relationship between instructional practices and beliefs or content knowledge was not found. This study indicates that the beliefs and content knowledge do not necessarily translate into daily instructional practices. The researcher encourages future studies, specifically addressing the kindergarten year, in order to learn more about the beliefs, knowledge and practices of teachers and addressing ways to support teachers with the translation of beliefs and knowledge into the classroom in order to move to a more reform-based mathematics culture. Students deserve the most capable and competent teacher. The researcher hopes that this study will contribute to the body of research that impacts the development of mathematically proficient early childhood teachers.
Appendix A

Self-Report Survey: Elementary Teacher’s Commitment to Mathematics Education Reform

Table 2

<table>
<thead>
<tr>
<th>Item number</th>
<th>Item</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I like to use math problems that can be solved in many different ways.</td>
<td>D2</td>
</tr>
<tr>
<td>2</td>
<td>I regularly have my students work through real-life math problems that are of interest to them.</td>
<td>D2</td>
</tr>
<tr>
<td>3</td>
<td>When two students solve the same math problem correctly using two different strategies I have them share the steps they went through with each other.</td>
<td>D6</td>
</tr>
<tr>
<td>4</td>
<td>I tend to integrate multiple strands of mathematics within a single unit.</td>
<td>D1</td>
</tr>
<tr>
<td>5</td>
<td>I often learn from my students during math time because my students come up with ingenious ways of solving problems that I have never thought of.</td>
<td>D4</td>
</tr>
<tr>
<td>6*</td>
<td>It is not very productive for students to work together during math time.</td>
<td>D6</td>
</tr>
<tr>
<td>7</td>
<td>Every child in my room should feel that mathematics is something he/she can do.</td>
<td>D9</td>
</tr>
<tr>
<td>8</td>
<td>I integrate math assessment into most math activities.</td>
<td>D7</td>
</tr>
<tr>
<td>9</td>
<td>In my classes, students learn math best when they can work together to discover mathematical ideas.</td>
<td>D6</td>
</tr>
<tr>
<td>10</td>
<td>I encourage students to use manipulatives to explain their mathematical ideas to other students.</td>
<td>D5</td>
</tr>
<tr>
<td>11*</td>
<td>When students are working on math problems, I put more emphasis on getting the correct answer than on the process followed.</td>
<td>D2</td>
</tr>
<tr>
<td>12</td>
<td>Creating rubrics for math is a worthwhile assessment strategy.</td>
<td>D7</td>
</tr>
<tr>
<td>13</td>
<td>In my class it is just as important for students to learn data management and probability as it is to learn multiplication facts.</td>
<td>D1</td>
</tr>
<tr>
<td>14</td>
<td>I don’t necessarily answer students’ math questions but rather let them puzzle things out for themselves.</td>
<td>D3</td>
</tr>
<tr>
<td>15*</td>
<td>A lot of things in math must simply be accepted as true and remembered.</td>
<td>D8</td>
</tr>
<tr>
<td>16*</td>
<td>I like my students to master basic mathematical operations before they tackle complex problems.</td>
<td>D1</td>
</tr>
<tr>
<td>17</td>
<td>I teach students how to explain their mathematical ideas.</td>
<td>D4</td>
</tr>
<tr>
<td>18*</td>
<td>Using computers to solve math problems distracts students from learning basic math skills.</td>
<td>D5</td>
</tr>
<tr>
<td>19*</td>
<td>If students use calculators they won’t master the basic math skills they need to know.</td>
<td>D5</td>
</tr>
<tr>
<td>20*</td>
<td>You have to study math for a long time before you see how useful it is.</td>
<td>D9</td>
</tr>
</tbody>
</table>

* Denotes negatively worded item.

Appendix B

*Self-Report Survey: Descriptions of Dimensions (D) of Elementary Mathematics Reform*

**D1: Program scope**  A broader scope (e.g., multiple mathematics strands with increased attention on those less commonly taught such as probability, rather than an exclusive focus on numeration and operations) with all student having access to all forms of mathematics.

**D2: Student tasks**  Students tasks are complex, open-ended problems embedded in real life contexts; many of these problems do not afford a single solution. In contrast in traditional mathematics students work on routine applications of basic operations in decontextualized, single solution problems.

**D3: Discovery**  Instruction in reform classes focuses on the construction of mathematical ideas through student discovery contrasting with the transmission of canonical knowledge through presentation, practice, feedback, and remediation in traditional programs.

**D4: Teacher’s role**  The teacher’s role in reform settings is that of co-learner and creator of a mathematical community rather than sole knowledge expert.

**D5: Manipulatives and tools**  Mathematical problems are undertaken in reform classes with the aid of manipulatives and with ready access to mathematical tools (i.e., calculators and computers). In traditional programs such tools are not available or their use is restricted to teacher presentations of new ideas.

**D6: Student-student interaction**  In reform teaching the classroom is organized to promote student-student interaction, rather than to discourage it as an off task distraction.

**D7: Student assessment**  Assessment in the reform class is authentic (i.e., relevant to the lives of students), integrated with everyday instruction, and taps multiple-levels of performance. In contrast, assessment in traditional programs is characterized by end of week and unit tests of near transfer.

**D8: Teacher’s conceptions of math as a discipline**  The teacher’s conception of mathematics in the reform class is that of a dynamic subject rather than a fixed body of knowledge.

**D9: Student confidence**  Teachers in the reform setting strive to raise student self-confidence in mathematics rather than impede it.

Appendix C

Content Knowledge for Teaching Mathematics Measures: Sample of Released Items

Study of Instructional Improvement/Learning Mathematics for Teaching
Content Knowledge for Teaching Mathematics Measures (MKT measures)
Released Items, 2008

ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>I'm not sure</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0 is an even number.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>b) 0 is not really a number. It is a placeholder in writing big numbers.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>c) The number 8 can be written as 008.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

2. Ms. Chambreaux’s students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

For additional released items, see:
http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf
## Appendix D

**Content Knowledge for Teaching Mathematics Measures: Slope & Difficulty**

<table>
<thead>
<tr>
<th>Math Strand</th>
<th>Test Item</th>
<th>Original Test Item #</th>
<th>Slope</th>
<th>Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>GEO 12a2</td>
<td>2004B-19a2</td>
<td>0.417</td>
<td></td>
<td>2.262</td>
</tr>
<tr>
<td>GEO 12a1</td>
<td>2004B-19a1</td>
<td>0.755</td>
<td></td>
<td>-1.876</td>
</tr>
<tr>
<td>GEO 10b</td>
<td>2004B-17b</td>
<td>1.076</td>
<td></td>
<td>-1.713</td>
</tr>
<tr>
<td>NCOP CK 1a</td>
<td>2008-3a</td>
<td>0.615</td>
<td></td>
<td>-1.317</td>
</tr>
<tr>
<td>GEO 10d</td>
<td>2004B-17d</td>
<td>0.659</td>
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<td>-1.271</td>
</tr>
<tr>
<td>NCOP KSC 5b</td>
<td>2008-15b</td>
<td>0.619</td>
<td></td>
<td>-1.216</td>
</tr>
<tr>
<td>NCOP KSC 8</td>
<td>2008-15</td>
<td>0.824</td>
<td></td>
<td>-1.815</td>
</tr>
<tr>
<td>NCOP CK 4</td>
<td>2008-12</td>
<td>0.63</td>
<td></td>
<td>-1.157</td>
</tr>
<tr>
<td>GEO 10c</td>
<td>2004B-17c</td>
<td>0.816</td>
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<td>-1.151</td>
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<tr>
<td>GEO 12b1</td>
<td>2004B-19b1</td>
<td>0.993</td>
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<tr>
<td>GEO 12c1</td>
<td>2004B-19c1</td>
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<tr>
<td>GEO 12c2</td>
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<td>NCOP KSC 5a</td>
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<td>0.572</td>
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<tr>
<td>GEO 12a2</td>
<td>2004B-19a2</td>
<td>0.866</td>
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<td>GEO 13</td>
<td>2004B-31</td>
<td>0.774</td>
<td></td>
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<td>NCOP CK 1c</td>
<td>2008-3c</td>
<td>0.506</td>
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<tr>
<td>NCOP 6d</td>
<td>2008B-13d</td>
<td>0.735</td>
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<tr>
<td>NCOP 6a</td>
<td>2008B-13a</td>
<td>0.783</td>
<td></td>
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<tr>
<td>NCOP 6b</td>
<td>2008B-13b</td>
<td>0.561</td>
<td></td>
<td>-0.501</td>
</tr>
<tr>
<td>GEO 8</td>
<td>2004B-15</td>
<td>0.838</td>
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<tr>
<td>GEO 10a</td>
<td>2004B-17a</td>
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<tr>
<td>GEO 9c</td>
<td>2004B-17a</td>
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<td>0.125</td>
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<td>NCOP CK 7</td>
<td>2008-13</td>
<td>0.661</td>
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<td>0.28</td>
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<tr>
<td>GEO 9b</td>
<td>2004B-16b</td>
<td>0.724</td>
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<td>NCOP CK 3</td>
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<td>0.607</td>
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<tr>
<td>GEO 9a</td>
<td>2004B-16a</td>
<td>0.922</td>
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<td>0.69</td>
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<tr>
<td>NCOP CK 4a</td>
<td>2008-2d</td>
<td>0.560</td>
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<td>0.702</td>
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<td>NCOP CK 1b</td>
<td>2008-3b</td>
<td>0.589</td>
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<tr>
<td>NCOP CK 2</td>
<td>2008-5</td>
<td>0.683</td>
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<tr>
<td>GEO 11</td>
<td>2004B-18</td>
<td>0.509</td>
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<td>NCOP KSC 5c</td>
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<td>0.529</td>
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<td>1.542</td>
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<tr>
<td>GEO 9d</td>
<td>2004B-16d</td>
<td>0.738</td>
<td></td>
<td>1.77</td>
</tr>
</tbody>
</table>

NCOP = Number Concepts & Operations  
GEO = Geometry  
CK = Common Knowledge  
KSC = Knowledge of Students & Content
Appendix E

*FirstSchool Snapshot*

Source: Ritchie, Weiser, Kraft-Sayre, Mason, Crawford, & Howes, 2010

<table>
<thead>
<tr>
<th>Teacher names &amp; ID#________________________</th>
<th>Date: _____________________</th>
</tr>
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<tbody>
<tr>
<td>School &amp; ID#____________________</td>
<td>Observer ______________ Cycle ______</td>
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<table>
<thead>
<tr>
<th>Activity Setting</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>Basics</td>
<td>meals/snacks</td>
<td>Whole group</td>
<td>Choice</td>
<td>Station</td>
<td>Small group</td>
<td>Individual</td>
<td>Outside</td>
<td>Can’t Watch</td>
<td>Specials</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<table>
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<th>Child Engagement</th>
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<th>3</th>
<th>4</th>
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<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read to</td>
<td>Whole language</td>
<td>Phonics</td>
<td>Oral language</td>
<td>Vocabulary</td>
<td>Compose</td>
<td>Copy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<table>
<thead>
<tr>
<th>Child Behavior</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<th>2</th>
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<tbody>
<tr>
<td>Peer</td>
<td>Distracted</td>
<td>Flexible</td>
<td>Collaboration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>Teacher Behavior</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
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<th>4</th>
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<th>2</th>
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<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>Negative</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Teacher-child Eng.</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Literate</td>
<td>Reflection</td>
<td>Scaffolds</td>
<td>Didactic</td>
<td>Strategies</td>
<td>Second Language</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adult ID: Teacher=1; Assistant=2; Student teacher=3; Other school adults=4; Parent=5
### Appendix F

**FirstSchool Snapshot Codebook Details**

<table>
<thead>
<tr>
<th>Category</th>
<th>Sub-Composite</th>
<th>Codebook Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity Setting</strong></td>
<td>Choice</td>
<td>Code when a student is engaged in activities of his/her choosing. During this time students are able to select what and where they would like to play or learn. Activities can include individual art projects, blocks, fantasy play, puzzles, reading, puppets, computers, science experiments, etc. The key here is that students have chosen their activities. Often these choices will be made from Centers that are set up for Blocks, Pretend Play, Science, Writing and/or Art.</td>
</tr>
<tr>
<td></td>
<td>Station</td>
<td>Code when a student is engaged in small groups that are teacher organized and assigned, but are NOT teacher led. Students encounter set tasks or assignments designed by the teacher. Often this will be coded during literacy or math blocks when students go to stations while the teacher works with a specific group. Teachers may check in on children while they are in stations.</td>
</tr>
<tr>
<td><strong>Student Behavior</strong></td>
<td>Peer</td>
<td>Code when a student is engaged with a peer in talking, playing, learning, laughing, experimenting, negotiating, arguing. This is at least a two way interaction. This can be a physical interaction - students imitating each other, purposefully sharing materials, etc.</td>
</tr>
<tr>
<td><strong>Teacher-Student Engagement</strong></td>
<td>Reflection</td>
<td>Code when the teacher is engaging the students to remember, reflect on their thought processes-explain what they learned or how they solved a problem, or represent their learning visually. This may be evident during ‘conferencing’ on the basal or free choice reading-teachers ask students what they have read, what they learned, what happened in the story. May be evident when students are being asked to explain how they solved math problems.</td>
</tr>
<tr>
<td></td>
<td>Scaffolds</td>
<td>The defining characteristic is if the teacher shows an awareness of an individual student’s needs and responds in a manner that supports and expands the student’s learning. Code if the teacher: • is utilizing the curiosity or interest of the student • uses student’s initiations as an opportunity to add to his/her learning • asks open-ended questions • patiently waits in order for a student to work out their thoughts or demonstrate capability • helps student expand on his answers and thoughts • works to link classroom activities to student’s life and experiences. • asks the student questions or poses problems that have multiple solutions, including conflict resolution. • is actively engaged in listening to student • engages in reciprocal conversation Code if the child: is motivated by teacher engagement and participation</td>
</tr>
<tr>
<td><strong>Student Engagement</strong></td>
<td>Numbers</td>
<td>Code when students are representing, ordering numbers (rote counting, counting with 1:1 correspondence, skip counting, adding, subtracting, multiplying, dividing, identifying written numerals, matching numbers to pictures, reading graphs, working with fractions (part to whole) playing counting games (e.g.: dice, dominoes, Candyland, Chutes and Ladders), playing Concentration or Memory with numbers).</td>
</tr>
<tr>
<td>Category</td>
<td>Codebook Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td>Code when students are looking at position in space and identifying shapes; talking about the properties of shapes (e.g. how many sides); finding shapes in the room; building pictures and designs with two and three dimensional shapes; discussing relative position (above, below, next to), working on puzzles.</td>
<td></td>
</tr>
<tr>
<td><strong>Algebra</strong></td>
<td>Code when students are looking at attributes: (red, blue, big red, small blue, big green triangle etc.) sorting, classifying, identifying same and different, comparing (most/least, big/small, greater than) or discerning patterns, measuring for size, weight or quantity; (can be traditional or non-traditional measurement (how many inches or how many steps), playing a memory game (ie. turning over cards to match symbols, letters, animals, etc.)</td>
<td></td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>Code when students are discussing elements of time. Often, this will be when they are talking about the calendar (days, weeks, months, year) May also include conversations about WHEN something has or will happen. This also includes lessons about clocks, telling time.</td>
<td></td>
</tr>
</tbody>
</table>

Appendix G

Institutional Review Board Approval

To: Amy Scrinzi
5426 Pageford Dr Durham, NC 27703

From: Behavioral IRB

Approval Date: 3/12/2010
Expiration Date of Approval: 3/11/2011

RE: Notice of IRB Approval by Expedited Review (under 45 CFR 46.110)
Study #: 10-0464

Study Title: Kindergarten Mathematics: An Examination of the Relationships between Kindergarten Teacher's Beliefs, Mathematical Knowledge for Teaching, and Instructional Practices

This submission has been approved by the above IRB for the period indicated. It has been determined that the risk involved in this research is no more than minimal.

Study Description:

Purpose: To describe kindergarten teachers' pedagogical beliefs about mathematics, their mathematical knowledge for teaching, and their instructional practice, and to examine the relationships between and among each of these three areas.

Participants: ~30 kindergarten teachers who previously participated in a state wide initiative called "Power K."

Procedures: Secondary data analysis of previously collected administrative data about teacher beliefs and mathematical content knowledge. In addition, participating teachers will be observed in their classroom using the Emerging Academic Snapshot Instrument.

Investigator's Responsibilities:

Federal regulations require that all research be reviewed at least annually. It is the Principal Investigator’s responsibility to submit for renewal and obtain approval before the expiration date. You may not continue any research activity beyond the expiration date without IRB approval. Failure to receive approval for continuation before the expiration date will result in automatic termination of the approval for this study on the expiration date.

When applicable, enclosed are stamped copies of approved consent documents and other recruitment materials. You must copy the stamped consent forms for use with subjects unless you have approval to do otherwise.

You are required to obtain IRB approval for any changes to any aspect of this study before they can
be implemented (use the modification form at ohre.unc.edu/forms). Any unanticipated problem involving risks to subjects or others (including adverse events reportable under UNC-Chapel Hill policy) should be reported to the IRB using the web portal at https://irbis.unc.edu/irb.

Researchers are reminded that additional approvals may be needed from relevant "gatekeepers" to access subjects (e.g., principals, facility directors, healthcare system).

This study was reviewed in accordance with federal regulations governing human subjects research, including those found at 45 CFR 46 (Common Rule), 45 CFR 164 (HIPAA), 21 CFR 50 & 56 (FDA), and 40 CFR 26 (EPA), where applicable.

CC:
Sharon Ritchie, Fpg Child Development Inst
Seung Yu, School Of Education
Crystal Daniel, (School of Education), Non-IRB Review Contact
Appendix H

Recruitment Letter (Via E-mail)

Dear (Insert Power of K teacher’s name here),

As a graduate student in the School of Education at the University of North Carolina- Chapel Hill, I would like to invite you to participate in my research study: Kindergarten Mathematics. The purpose of this study is to learn about what kindergarten teachers believe about the teaching of mathematics, what content knowledge kindergarten teachers have about number and geometry, and the types of instructional practices used in kindergarten classrooms. You may participate if you are a teacher currently part of the Power of K initiative.

You will be asked to give permission for the researcher to access previously collected data from your responses to the Self-Report Survey and the Mathematical Knowledge for Teaching measure. These data were collected for administrative purposes as part of the February, 2010 Power of K mathematics professional development.

You will also be asked to give consent to being observed by a trained data collector, using the Emergent Academic Snapshot, for an entire school day during the month of March, April, or May 2010. A copy of the consent form for this study is attached for you to review. The person who will conduct the observation will ask you to sign a copy that will be provided on the day of the observation. You should keep this emailed version for your records. I will also get permission for this classroom observation from your principal, and from your school district, if necessary.

If you choose (and this is totally optional), you may also ask to have an individual meeting with Dr. Ritchie and me to discuss your individual data collected from each of these three measures. This information may be interesting and useful to you. You can request a meeting to discuss your individual data at any point after the observation in your classroom has been completed. We will then find a convenient time to get together once all your individual data have been obtained and analyzed.

Please respond to this email by indicating either YES or NO to the following:

I give consent to participate in the Kindergarten Mathematics study. I understand that I am giving consent to be observed by a trained data collector for an entire school day in March, April, or May 2010. I also acknowledge that I am giving permission for the researcher to access my previously collected data from the Self-Report Survey and the Mathematical Knowledge for Teaching measure.

If you have any additional questions or want additional information, please feel free to contact me, Amy Scrinzi, by email (ocracokey@earthlink.net). A response by March 15, 2010 is appreciated.

Thank you for your consideration,

Amy Scrinzi
Appendix I

Consent to Participate in a Research Study

University of North Carolina-Chapel Hill
Consent to Participate in a Research Study
Adult Participants
Social Behavioral Form

________________________________________________________________________

IRB Study # 10-0464
Consent Form Version Date: March 5, 2010

Title of Study: Kindergarten Mathematics: An examination of the relationships between kindergarten teacher’s beliefs, mathematical knowledge for teaching, and instructional practices

Principal Investigator: Amy S. Scrinzi
UNC-Chapel Hill Department: School of Education: Curriculum and Instruction
Faculty Advisor: Dr. Sharon Ritchie, 919-843-2779; sharon.ritchie@unc.edu

Study Contact telephone number: 919-807-3852
Study Contact email: ocracokey@earthlink.net

_________________________________________________________________

What are some general things you should know about research studies?
You are being asked to take part in a research study. To join the study is voluntary. You may refuse to join, or you may withdraw your consent to be in the study, for any reason, without penalty.

Research studies are designed to obtain new knowledge. This new information may help people in the future. You may not receive any direct benefit from being in the research study. There also may be risks to being in research studies.

Details about this study are discussed below. It is important that you understand this information so that you can make an informed choice about being in this research study. You will be given a copy of this consent form. You should ask the researchers named above, or staff members who may assist them, any questions you have about this study at any time.

What is the purpose of this study?
The purpose of this research study is to learn about what kindergarten teachers believe about the teaching of mathematics, what content knowledge kindergarten teachers have about number and geometry, and the types of instructional practices used in kindergarten classrooms.

You are being asked to be in the study because you are a kindergarten teacher.

How many people will take part in this study?
If you decide to be in this study, you will be one of approximately 30 kindergarten teachers in this
research study.

**How long will your part in this study last?**
You will be actively involved in the study for one full-day classroom observation. The data collector will arrive at your classroom approximately 10 minutes before the first student arrives and will stay for the entire school day, leaving approximately 10 minutes after the last child has left.

**What will happen if you take part in the study?**
If you decide to take part in this study, you will be asked to give permission for the researcher to access previously collected data from the Self-Report Survey and the Mathematical Knowledge for Teaching measure. These data were collected for administrative purposes as part of the February, 2010 Power of K mathematics professional development.

You will also be asked to give consent to being observed by a trained data collector, using the Emergent Academic Snapshot, for an entire school day during the month of March or April, 2010.

If you choose (and this is totally optional), you may also ask to have an individual meeting with the researcher to discuss your individual data collected from each of these three measures. This information may be interesting and useful to you. You can request a meeting with the researcher to discuss your individual data at any point after the observation in your classroom has been completed. You and the researcher will then find a convenient time to get together once all your individual data have been obtained and analyzed by the researcher.

**What are the possible benefits from being in this study?**
Research is designed to benefit society by gaining new knowledge. Allowing the researcher to have access to previously collected administrative data, and being observed, will not provide any real benefit to you. However, if you choose to meet with the researcher after data collection and analysis, you may receive some benefit from this study by being able to learn about specific types of instructional practices you use in the classroom.

**What are the possible risks or discomforts involved from being in this study?**
There are not any known risks to you should you choose to participate in this study. However, there may be uncommon or previously unknown risks. You should report any problems to the researcher.

**How will your privacy be protected?**
Your privacy and the confidentiality of your data will be protected. All data will be kept in a locked file cabinet and in password protected electronic files with the researcher. Your name will be removed from all collected data and replaced by a number which will be kept on a secured master list and kept separately from the raw data. Only the researcher will have access to this data. Individual data will not be shared with any other persons unless you choose to share that information on your own.

Participants will not be identified in any report or publication about this study.
What if you want to stop before your part in the study is complete?
You can withdraw from this study at any time, without penalty.

Will you receive anything for being in this study?
You have the option for arranging a one-to-one conference with the researcher to review your individual data that was obtained as a result of your participation in this study.

Will it cost you anything to be in this study?
There will be no costs for being in the study.

What if you have questions about this study?
You have the right to ask, and have answered, any questions you may have about this research. If you have questions, complaints, concerns, or if a research-related injury occurs, you should contact the researchers listed on the first page of this form.

What if you have questions about your rights as a research participant?
All research on human volunteers is reviewed by a committee that works to protect your rights and welfare. If you have questions or concerns about your rights as a research subject, or if you would like to obtain information or offer input, you may contact the Institutional Review Board at 919-966-3113 or by email to IRB_subjects@unc.edu.

Title of Study: An examination of the relationships between kindergarten teacher’s beliefs, mathematical knowledge for teaching, and instructional practices

Principal Investigator: Amy S. Scrinzi

Participant’s Agreement:

I have read the information provided above. I have asked all the questions I have at this time. I voluntarily agree to participate in this research study. I understand that participation involves being observed for one full school day and permission for the researcher to access and analyze previously collected administrative data about me.

_________________________________________________ _________________
Signature of Research Participant  Date

_________________________________________________
Printed Name of Research Participant
Appendix J

Principal Permission (Via E-Mail)

Dear (Insert Principal’s name here),

Your Power of K teacher is being invited to participate in the research study: Kindergarten Mathematics. The purpose of this study is to learn about what kindergarten teachers believe about the teaching of mathematics, what content knowledge kindergarten teachers have about number and geometry, and the types of instructional practices used in kindergarten classrooms. If you grant permission for the classroom observation to occur, and your Power of K teacher gives consent to participate in the study, then your Power of K teacher will be one of approximately 30 kindergarten teachers in this research study.

Once permission is granted from you and consent is obtained from your Power of K teacher, your Power of K teacher will be contacted by one of two trained data collectors to make arrangements for a one-day classroom observation to occur during March, April, or May 2010. During this one-day observation, the data collector will use the Emerging Academic Snapshot observation tool to gather information about instructional practices used in the classroom. The data collector will arrive at your teacher’s classroom approximately 10 minutes before the first student arrives and will stay for the entire school day, leaving approximately 10 minutes after the last child has left. No change in classroom activities is expected, nor desired. The goal is to capture a typical day in your Power of K kindergarten classroom. Please know that there will not be any identifiable data collected about the students- the only identifying information used may be “red-shirt” or “braids.”

The privacy and the confidentiality of the collected data will be protected. Only the researcher will have access to these data which will be kept in a locked file cabinet and in password protected electronic files. An opportunity to review a particular teacher’s data with him/her will be provided. Individual data will not be shared with any other persons unless the teacher chooses to share that information him/herself. Also know that participants will not be identified in any report or publication about this study.

Please respond to this email by indicating either YES or NO to the following:

I give permission for the Power of K teacher in my school to be observed for one day as part of the Kindergarten Mathematics study.

Please know that my research staff will not be conducting any observations until I have signed consent from the teacher. In addition, even if you grant permission, teachers can still choose to not be a part of the study before and during the observation. All research participation is voluntary.

A copy of the teacher consent form is attached for your reference. Please let me know if you have any questions or need additional information regarding this study. Also, please let me know if there is any other procedure I need to follow, such as asking permission at the district level, to conduct this one-day observation with teacher consent. A response by March 15, 2010 is appreciated.

Thank you for your support.

Amy Scrinzi
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