GUIDANCE IN THE DESIGN AND IMPLEMENTATION OF AN ONLINE MATHEMATICS EDUCATION COURSE

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ABSTRACT

Bryan Fede: Guidance in the Design and Implementation of an Online Mathematics Education Course
(Under the direction of Susan N. Friel)

The number of students taking online distance education (ODE) courses as part of their programs of study has steadily increased since 2012 (Seaman, Allen, & Seaman, 2018). This has led to a proliferation of resources related to online teaching pedagogy (Dennen, 2013). In many cases, instructors of ODE courses are left to sort through this information on their own as they decide how to design and implement ODE experiences for their students. The overarching goal of this research is to develop *A Field Guide for Mathematics Educators in the Design and Implementation of Learning Environments in Online Distance Education* that incorporates what is known about the best practices in teaching ODE courses as well as in applying high leverage teaching practices in mathematics education.

The study addresses two questions:

(1) In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the *Field Guide* in course planning and implementation?

(2) What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance?

The researcher followed one instructor as she applied advice from the *Field Guide* through one iteration an algebra course in a K-5 Elementary Mathematics Add-on Licensure (EMAoL) Program. After providing the instructor with a copy of the *Field Guide*, research analyzed the ways that that the instructor applied the guidance in the *Field Guide* to the

design and implementation of her course and collected ‘instances of practice’ that might be used to illustrate principles contained in the *Field Guide*.

The goal of this research is to inform a revision of the *Field Guide* that provides more detailed support for mathematics educators in design and implementation of ODE learning experiences. The work highlights the need for instructors to adjust their teaching practices in an ODE environment and resist the temptation to translate what is done in a face-to-face class setting and “put it online” (Pollock, 2013, p. 3).

To the friends and family that supported me along the way. I have missed many of the important events in *your* lives as I completed this degree. Thank you for your understanding and I promise to make it up to you all.

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Lastly, I am grateful to my parents, Linda and Joe for your continued love and support as I have navigated careers and degrees in my search for happiness. I love you both.

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LIST OF ABBREVIATIONS

AIC	Akaike information criterion
BIAT	bringing it all together
BIC	Bayesian information criterion
CCK	common content knowledge
CCSSM	Common Core State Standards for Mathematics
CFI	comparative fit index
CoI	community of inquiry
EMAoL	elementary mathematics add-on licensure
EMS	elementary mathematics specialist
LMS	learning management system
MKT	mathematics knowledge of teaching
NCLB	No Child Left Behind’
NCTM	National Council of Teachers of Mathematics
PCK	pedagogical content knowledge
PTD	perceived transactional distance
PTDBLE	perceived transactional distance in blended learning environments
QM	quality matters
QSWA	questioning and student work analysis
QUASAR	Quantitative Understanding: Amplifying Student Achievement and Reasoning
SCK	specialized content knowledge
SG3	small group discussion 3
SMK	subject matter knowledge
SOL	Satisfaction of Online Learning

SRMR	standardized root mean residual
TLI	Tucler-Lewis index
UNC	University of North Carolina

CHAPTER 1: INTRODUCTION

Ernst Haeckel (1834 – 1919) was a German naturalist who is considered by many to be the father of the field of ecology. For Haeckel, ecology was the study of the natural environment, including the relationship of organisms to one another and their surroundings (Haeckel, 1869 in Odum & Barrett, 2004). Although Haeckel's definition emphasizes the *natural* environment, a closer look at the origin of the word 'ecology' uncovers a close connection to a more humanistic definition. A look at the etymology of the word reveals that it stems from the Greek word *oikos*, meaning house, dwelling place, or habitation (Online Etymology Dictionary, 2018). This definition implies that ecology, at least to some extent, lends insight into how individuals interact with one another in specific settings, given the various cultural tools and technologies at their disposal. This dissertation aims to take an ecological view of the design and implementation of a blended distance education learning environment that is designed to deliver professional development for practicing elementary teachers who intend to become elementary mathematics specialist professionals.

Online Distance Education

Online distance education may be defined as "institution-based, formal education where the learning group is separated, and where interactive telecommunications systems are used to connect learners, resources, and instructors" (Schlosser & Simonson, 2010, p. 1). Tracing roots back at least 160 years, distance education has evolved as communication technologies have gradually advanced. Originating as 'correspondence study' and facilitated through the postal service, distance education in the mid-1800s provided the opportunity for individuals who might not have had access to the classical curriculum to study from home via the mail through monthly interactions with instructors (Simonson, Smaldino, & Zvacek, 2014). Today, online distance education serves a similar purpose. This educational medium allows individuals who are either geographically isolated or temporally restricted by family or profession to further their own educational goals.

Although technology is fundamental to the delivery of online distance education, each new technology brings with it unique pedagogical challenges that compel instructors to adjust the teaching and learning environment in which they engage students (Christopher, Thomas, & Tallent-Runnels, 2004). However, the presence of technology, in and of itself, does not enhance teaching or learning in the classroom. Instead, technology is a 'vehicle' (Clark, 1983) for the delivery of instruction. Clark notes that technological innovations "do not influence student achievement any more than the truck that delivers our groceries causes changes in our nutrition" (Clark, 1983, p. 445). The implication of this statement is that it is the quality of the content that drives changes. Technology can be leveraged to deliver content, but it must be used deliberately and appropriately.

Online distance education has seen a marked increase in popularity, not only in colleges of education, but also across universities as a whole. A report released by the Babson Survey Research Group produced in conjunction with the College Board (Allen & Seaman, 2013) shows that as of 2012, 6.7 million college students had enrolled in at least one distance education online course. The report further demonstrates that the number of college students interested in education that is delivered online steadily increased over the ten-year span between 2003 and 2013, a trend that already has and will undoubtedly increase in the future. This expansion of student interest has initiated renewed attention in the nature of the ecology of online distance learning, including the kinds of interactions that occur between participants in this environment and how these interactions may be fostered (Moore, 2013d).

Despite this growth in its popularity, in many ways online distance education continues to remain an uncharted frontier for university faculty. In many cases, an ad hoc approach often is taken to the development and implementation of these types of courses as designers and instructors attempt to simply translate face-to-face courses directly to a distance education format. An oft-cited reason for poor planning and implementation of an online distance education course is the lack of instructor support in the development phase (Simonson et al., 2014). Moore and Kearsley (2005) note that one element that is often missing in the planning phase of an online distance education class is the presence of an instructional designer with intimate knowledge of best practices in online learning. In fact, in a comprehensive review of the literature, Tallent-Runnels et al. (2006) found no comprehensive theory or model to guide the creation of online courses and few guidelines to aide instructors in course implementation, a conclusion that Simonson et al. (2014) note remains a critical weakness of the field.

Certification of Elementary Mathematics Specialists

Given the varied demands and expertise needed by teachers in the elementary grades, there has been a movement in support of training the elementary mathematics specialist (EMS) to lead and provide support for their colleagues who teach math in the elementary grades. These EMS professionals are teacher-leaders with strong preparation and background in mathematics knowledge for teaching that includes specialized knowledge of mathematics content and pedagogy and school-based leadership. Typically, such a position is held by "a knowledgeable colleague who has pedagogical expertise and an understanding of mathematics and of how students learn and that this person is qualified and capable of serving as an on-site resource and leader for teachers, providing school-based and content-specific professional development" (Campbell & Malkus, 2014, p. 214). Although the demand for EMS has increased, programs that prepare these professionals lag behind. Fennell (2011) notes that, in many cases, EMS are appointed by school districts without proper vetting with regard to their knowledge of content, pedagogy, or leadership. In an effort to alleviate this problem, many states have begun to offer credentialing programs and licenses for EMS professionals. Currently, nineteen states offer a licensure endorsement for EMS professionals, with a number of additional states finalizing plans for endorsements. One of the states that offer this sort of certification is North Carolina, which is a focus of this research.

In 2009, the North Carolina Board of Education approved a program of study that allows approved universities in the University of North Carolina (UNC) system to offer teachers who have an existing North Carolina elementary school teaching license an option for an add-on Elementary Mathematics Specialist endorsement. This program of study requires individuals to complete 18 credit hours (six courses) that are designed to address mathematics knowledge for teaching and school-based leadership skills. Although the content of the program has been refined and agreed upon by all participating universities, the mode of delivery varies among universities. Courses may be delivered face-to-face, completely online and asynchronous, or in some hybrid format that uses either both asynchronous and synchronous or asynchronous and face-to-face learning experiences. For many teachers who might be limited by their busy professional and personal lives or by geography, the ability to take classes online allows them to pursue this form of professional advancement.

Purpose of this Study

Drawing on a combination of the current research into best practices in online distance education as well as in mathematics education, this study addresses ways that instructors might create online educational experiences around mathematical content that are equivalent to learning in a physical classroom. In order to assist instructors in the creation of high quality online experiences, a draft field guide was developed for this research study with the intent to remind instructors of these best practices and to suggest ways that these practices may be implemented in the online distance education environment. This document, *A Field Guide for Mathematics Educators in the Design and Implementation of Learning Environments in Online Distance Education* (referred to hereafter as the *Field Guide*), is organized into two main sections and is intended to provide guidance to instructors about the setup of online distance education courses and the design of mathematical activities to be implemented in an online distance education environment.

This study followed one instructor's application of material that is addressed in the *Field Guide* in one iteration of the algebra content course for the K-5 Elementary Mathematics Add-on Licensure (EMAoL) program. The study utilized a combination of qualitative description (Sandelowski, 2000, 2010) and exploratory case study methodology (Yin, 2009) to investigate two areas of interest. The first area of interest concerns the way that one mathematics teacher educator, experienced in online course delivery, responded to suggested best practices for implementing online teaching as well as high leverage mathematics teaching practices and how these practices were evident in her planning and implementation of an algebra course for in-service elementary mathematics teachers. The second area of interest concerns the way that participants[1] received the online learning experiences. The expectation was that the course, which was designed and implemented with high leverage mathematics teaching practices and best practices in implementing online teaching in mind, would offer participants satisfying learning experiences. The ultimate goal of this research is to inform a revision of the *Field Guide* that provides more detailed support for mathematics educators based on the design and implementation of online learning experiences.

The research questions that guide the investigation that is related to these two areas of interest are as follows:

1. In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the *Field Guide* in course planning and implementation? This includes a consideration of:
   a. In what ways are the use of best practices in online teaching strategies that support student learning evident in the course planning and implementation?
   b. In what ways is the use of high leverage mathematics teaching practices that support student learning evident in the course planning and implementation?
   c. In what ways are the uses of best practices in online teaching strategies and of effective teaching practices in mathematics education that support student learning related to the components of Transactional Distance in the course planning and implementation?
2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance? This includes a

[1] 'Participants' refers to the teachers who were enrolled in this course. See Chapter 3 for a detailed description.

consideration of:

a. Using measure of student satisfaction with the course, what is a description of participants' response to the course?
b. Using a measure of transactional distance (see Appendix A), what is a description of participants' perceptions of transactional distance experienced at the end of the course?
c. In what ways are participant satisfaction and perceptions of transactional distance related in this distance education learning experience?

Both questions involve the evaluation and revision of the original *Field Guide*. The first question specifically addresses the degree to which the *Field Guide* is helpful to the instructor as she prepares for and delivers online distance instruction. The second question looks at participants' perception of perceived psychological distance between themselves and other participants in the course as well as their satisfaction with the course as a whole, which informs the section on high leverage mathematics teaching practices in general.

Organization of this Dissertation

Chapter 2 of this dissertation reviews research into theories that are relevant to the creation of the *Field Guide* and includes theory that drives the development of both best practices in online distance education, namely transactional distance theory and communities of inquiry research, as well as high leverage mathematics teaching practices. The chapter also addresses the relationship between these practices and participant satisfaction in online learning settings.

Chapter 3 discusses the elements of qualitative description and case study research methodologies used in the investigation. This chapter includes a description of the data collected and the instruments used in the study.

Findings from the study are presented in Chapter 4. This chapter includes a description of events as they transpired over the online distance education course. For the purpose of this study, events are defined as online modules that include both synchronous and asynchronous online interactions. Selected modules from the beginning, middle, and end of the course were selected for analysis in an effort to provide evidence of ways that the implementation of the course changed over the semester. Results for the student satisfaction survey and student interviews are also presented in order to infer participant experience in the class.

Finally, Chapter 5 discusses implications of the results of the study. Specific emphasis is placed on the effects of teacher actions on perceived participant learning and participant satisfaction. This chapter also discusses implications of the research for future versions of *A Field Guide for Mathematics Educators in the Design and Implementation of Learning Environments in Online Distance Education.*

CHAPTER 2: LITERATURE REVIEW

Distance education originally emerged as a viable alternative to in-person university study sometime in the mid- to late-19th century (Wiesner, 1983). Initially facilitated by correspondence through the postal service, the field of distance education grew during the early part of the 20th century as broadcast technologies (radio and television) began to emerge as viable and reliable communication media (Diehl, 2013). Throughout the 1950s, 1960s, 1970s, and 1980s, the popularity of distance education, known at the time as 'correspondence study', grew along with the ability of telecommunications technologies to provide relevant and reliable content. In 1992, the invention of the world-wide web began a new era in distance education as the Internet allowed for increased accessibility to content and the emergence of new pedagogical models for remote content delivery (Harasim, 2000).

By the turn of the 21st century, distance education, facilitated by computer-mediated technologies, began to be a significant force in education. In the preface to the first *Handbook of Distance Education* (Moore & Anderson, 2003), Moore referred to distance learning as "arguably the most important development in education in the past quarter century" (Moore, 2003a, p. ix). Theory development in distance education, however, was merely in its infancy. Much of the attention paid to distance education in the 1980s and 1990s was not on theory but on the technology side of instruction as new and exciting computer-based communications technologies emerged, making interactions faster and more personal than ever before. Moore recognized this "frenzy of activity" (p. ix) and urged the field to reconsider its intellectual base by refocusing the attention of scholarship away from technology itself and towards "the consequences of separating learners and teachers" (Moore, 2003c, p. xiii). Moore recognized that, although exciting, the technology part of distance education was a relatively simple consideration compared to the pedagogical, organizational, and policymaking (Moore, 2003c, p. xiii) challenges that were created by teaching and learning from a distance.

The overarching agenda of this research is to provide assistance for mathematics/mathematics education instructors in the design and implementation of online courses. The focus of the research is not necessarily on features of the technological aspects of online mathematics instruction, but rather on the consequences of the separation of teachers and learners in online mathematics classes. This focus led to the development of a field guide that mathematics instructors might use to assist in the creation and execution of their online courses. *A Field Guide for Mathematics Educators in the Design and Implementation Online Distance Education Learning Experiences* addresses pedagogical issues that lie at the convergence of mathematics education and distance education. Using the *Field Guide* as a lens, this research centered on observations of the ways one instructor of a semester-long online distance education course in mathematics education set up her online environment, fostered a collegial learning environment, selected and organized mathematics/mathematics education content, and implemented instruction within a blended synchronous/asynchronous online setting. The literature that is discussed in this chapter was used in the creation of the initial *Field Guide* as well as to address the following research questions:

1. In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the *Field Guide* in course planning and

implementation? This includes a consideration of:

   a. In what ways is the use of best practices in online teaching strategies that support student learning evident in the course planning and implementation?
   b. In what ways is the use of effective teaching practices in mathematics education that support student learning evident in the course planning and implementation?
   c. In what ways are the uses of best practices in online teaching strategies and of effective teaching practices in mathematics education that support student learning related to the components of transactional distance in the course planning and implementation?

2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance? This includes a consideration of:
   a. Using a measure of student satisfaction with the course, what is a description of participants' response to the course?
   b. Using a measure of transactional distance (see Appendix A), what is a description of participants' perceptions of transactional distance experienced at the end of the course?
   c. In what ways are participant satisfaction and perceptions of transactional distance related in this distance education learning experience?

This literature review provides insights into the overarching framework that was used to construct the *Field Guide* that is at the heart of this study. The literature selected is centered on themes that are relevant to the best practices of online learning as well as effective teaching practices in mathematics education. With regard to online education, the selected literature includes a discussion of Moore's theory of transactional distance (Moore, 2013b) as well as effective teaching practices in mathematics education that support student learning and are inherent of the community of inquiry model of participant interaction (Garrison, Anderson, & Archer, 2000). In addition to these theoretical constructs for distance learning classrooms, this review presents a collection of literature that concerns effective teaching practices specific to mathematics education. Finally, a discussion of elementary mathematics specialists (EMS) provides context for the participants in the particular class that was investigated in this study.

Theoretical Underpinnings of the *Field Guide*

As distance education has become increasingly popular, a significant evolution in theoretical perspectives related to the design and implementation of online courses has occurred. Such theories have evolved at three different levels of thought. High-end distance education theory is concerned with the structural complexities of distance education. Transactional distance theory (Moore, 1993, 2013c), originally conceived as the 'theory of independent learning' (Moore, 1973, 1977), serves as a broad theoretical framework that incorporates the complex interplay of course structure, interpersonal dialogue, and student control. For example, the idiosyncrasies of online courses that allow for varying degrees of structure, dialogue, and student autonomy can create psychological spaces of misinterpretation between student and instructor. In the middle range of distance education theory, the 'community of inquiry' framework focuses on the design of context-specific collaborative educational experiences (Garrison & Akyol, 2013; Garrison et al., 2000). At the low end of distance education theory are the actionable principles that govern everyday online educational transactions between students and teachers. Taken together, these three levels of theory provide solid grounding for instructors as they design their class, facilitate activities, and interact with students.

Transactional Distance: High End Distance Education Theory

Transactional distance describes the physical and cognitive space that is an inherent part of the distance education environment. The 'distance' portion of transactional distance is not solely determined by geography but is also influenced by the way instructors and learners interact with one another in the learning environment (Sandoe, 2005). Transactional distance is defined as "a psychological and communication space to be crossed, a space of potential misunderstanding between the inputs of instructor and those of the learner" (Moore, 1993, p. 22). It is interesting to note that misunderstandings may occur just as frequently in traditional face-to-face classrooms as they do in online classrooms (Rumble, 1986). In the online classroom, however, the physical and temporal separation of learner and instructor requires specialized strategies and techniques to accommodate learning and minimize potential miscommunication.

Conceptual definitions. The concept of 'transaction' in common usage usually implies an exchange of goods and services between consumer and producer. A transaction in an educational context, however, might be thought of as an intellectual exchange between the instructor and the learner – in other words, an exchange of ideas (Shearer, 2009). Moore's work that defines transactional distance (Moore, 1973, 1977, 1997, 2013d) provides a framework within which we can monitor how this exchange (dialogue) is influenced in light of the manner in which the course is constructed (its structure) and the level of autonomy that individual students bring to the transaction (Shearer, 2009). The modification of dialogue, structure, and learner autonomy allows for flexibility that is a key factor in making distance education attractive to students (Simonson et al., 2014). When examining the interplay of these three dimensions of the ecology of the online environment, the pedagogical complexity of distance education is realized (Moore, 2013).

Dialogue. The concept of 'dialogue' has perhaps been the most difficult of the three main aspects of transactional distance to define. Moore (1993) describes dialogue as a particular kind of interpersonal interaction where teachers and learners exchange words and symbols, with the goal being the creation of knowledge in the mind of the learner. Shearer (2009) raises the question of how narrowly or broadly dialogue might be defined. On the one hand, Burbles (1993) suggests a rather narrow conception of dialogue that limits it to the construction of knowledge where dialogic exchanges assist in the restructuring of one's mental schema of a construct or topic. Shearer (2009), on the other hand, suggests that dialogue might be thought of as broader sets of interactions that not only lead directly to knowledge production, but also support the building of community in the group. For the purposes of this dissertation, the author follows Shearer (2009) and defines dialogue as:

> [a]n educational exchange that involves two or more interlocutors. It is marked by a climate of open participation, and is an interaction or series of interactions that are positive. These interactions are purposeful, constructive, and valued by each party and lead to improved understanding of the students. Dialogic interactions are a series of alternating statements (including questions, responses, redirections, and building statements) that are continuous and developmental, and where the interaction persists in the face of disagreement, confusion and misunderstanding. The direction of dialogue in an educational exchange or transaction is guided by a spirit of discovery and is towards improved knowledge, insight, or sensitivity of the student (p. 159).

The definition of dialogue offered by Shearer is advantageous for this study because it also incorporates some of the elements of mathematical discourse. Mathematical discourse involves the use of dialogue (both verbal and non-verbal) to share ideas, clarify understanding, and make convincing arguments as students engage in mathematical practices (Moschkovich, 2012; NCTM, 2014).

Structure. A second dimension of transactional distance that affects educational transaction is the structure of a course. A typical course consists of one or more lessons or units. Each unit contains of a number of elements (course objectives, activities, exercises, discussion questions, etc.) that serve as the foundation for the construction of knowledge (Moore, 1993). Moore suggests that some of these elements might be rigidly defined by the course designer or instructor, whereas others might allow more flexibility based on the needs of the learners in a particular class. Moore (1993) further contends that, although a high degree of structure might be appropriate for some sorts of experiences (technical training, etc.), in many cases, freedom over educational experiences is more appropriate. Structure, then, refers to the elements of the course design, or "the ways in which the teaching programme [sic] is structured so that it can be delivered through the various communications media" (Moore, 1993, p. 26). Additionally, structure implies the latitude that students are given with regard to learning objectives, educational activities, and assessment of knowledge.

Student autonomy. Student autonomy is an individual characteristic of the learner and refers to a student's ability, or lack thereof, to be self-directed in his or her own learning (Shearer, 2009). The course structure directly influences the construct of autonomy, as a highly-structured course obscures an individual's ability to be self-directed in his or her learning. A low-structure educational experience, however, may not allow enough scaffolding or direction for a student to complete the objectives of a course or assignment. Although it is directly linked to the structure of the course, the amount of dialogue also exerts influence on student autonomy in that, within a highly dialogic structure, regular interaction with the instructor allows for the clarification of goals and objectives. In highly structured experiences, highly autonomous students may interpret for themselves the intent of an activity whereas a learner with a low degree of self-direction may become easily frustrated and give up.

The interplay of dialogue, structure, and learner autonomy. Moore (2013c) emphasizes that teaching/learning programs are not dichotomous; that is, they are not either ‘distant’ or ‘not distant’. Rather, such programs range from more distant to less distant depending on the makeup of the three critical elements of structure, dialogue, and learner autonomy. Transactional distance does not refer to a fixed specified space but varies as the result of the interplay of structure, dialogue, and learner autonomy (Peters, 1998). Scenarios that are both highly structured and contain a high degree of dialogue exhibit low levels of transactional distance (e.g., see Figure 2.1).

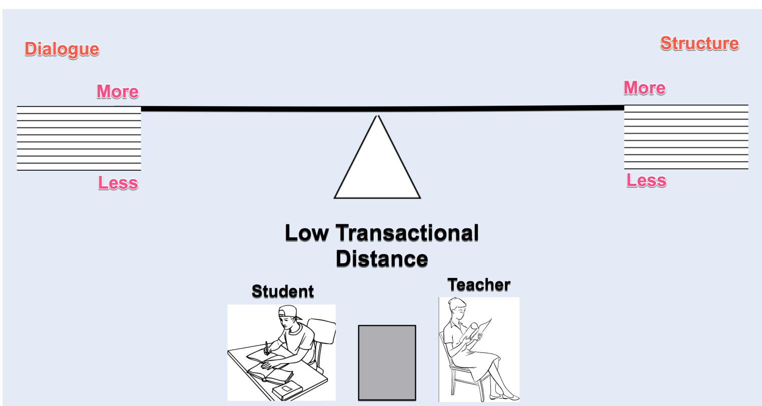


Figure 2.1. Low transactional distance situation.

If dialogue remains high but the structure of a class is more open, the degree of transactional distance increases (see Figure 2.2).

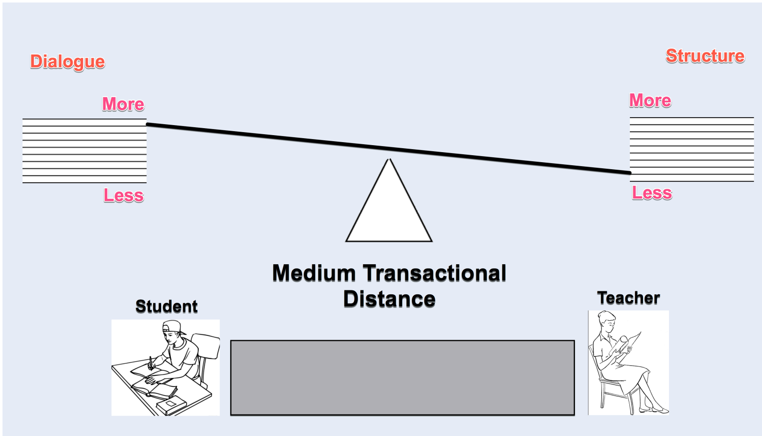


Figure 2.2. Intermediate levels of transactional distance.

Students tend to experience high-level transactional distance in a course that employs little dialogue and low structure (see Figure 2.3). Many of the correspondence classes conducted through the exchange of assignments via the postal service are good examples of such a model.

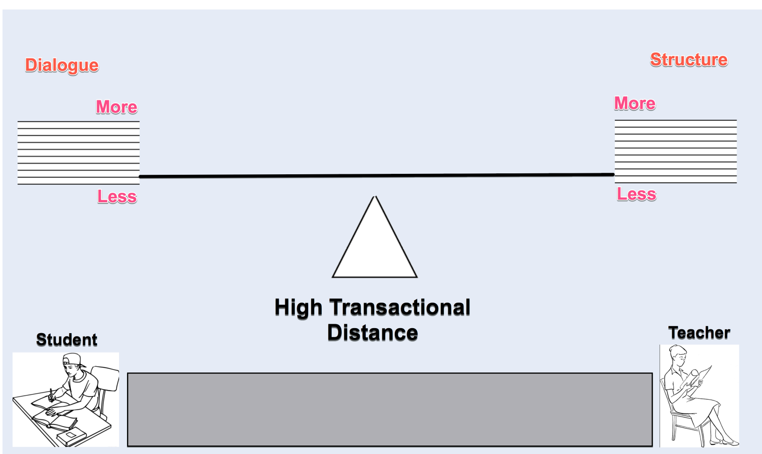


Figure 2.3. High transactional distance.

It is important to note that distinguishing the levels of transactional distance does not necessarily dictate the quality of the learning experience; i.e., courses that exhibit high levels of transactional distance versus those that exhibit less transactional distance cannot be characterized as 'bad' or 'good' learning experiences. Rather, the goal is to bring awareness of the implication of structural components of online coursework as designers and instructors construct learning environments.

Moore (1993) suggests that there are fundamental structural supports that can affect the level of transactional distance in the online classroom. Building on Moore's ideas, Benson and Samarawickrema (2009) have suggested general learning supports that manage the effects of transactional distance (Figure 2.4).

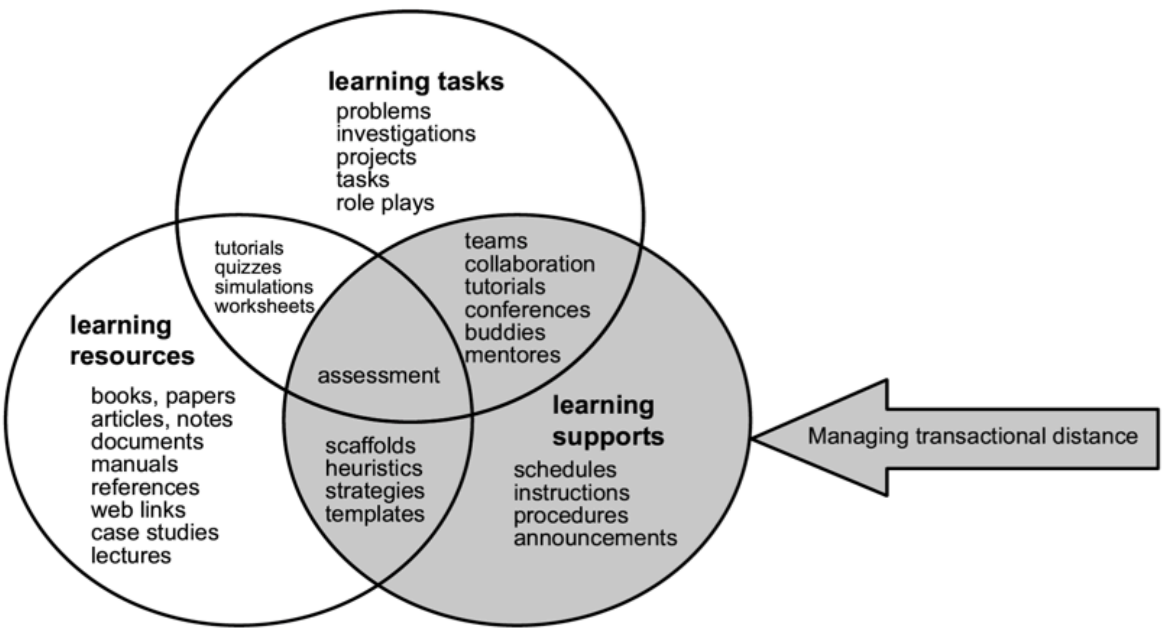


Figure 2.4. Learning supports that mediate transactional distance as students utilize learning resources and implement learning tasks. From: Benson, R., & Samarawickrema, G. (2009). Addressing the context of e-learning: Using transactional distance theory to inform design. *Distance Education, 30*(1), 5-21.

The diagram presented in Figure 2.4 serves as an important reminder that learning supports play an integral role in the architecture of a course. Along with learning resources and learning tasks, learning supports allow for finer control of dialogue in the course. Although instructors may have little control over the learning resources and learning tasks in highly structured classes, they often have greater control over the learning supports in the class. The learning supports that instructors introduce will differ from class to class. Younger students and students who are new to distance education might need more structural supports (online group meetings, schedules and calendars, exemplar material, etc.) than those who have previously experienced this mode of education. As students gain experience, many of these structural supports might be decreased or removed altogether to fit the needs of learners by giving them more control over their learning environment.

Communities of Inquiry: Middle-range Theory

If transactional distance describes the exchange of educational ideas between the student and the instructor, community of inquiry theory (Garrison, Anderson, & Archer, 1999) represents the marketplace in which this transaction takes place. The community of inquiry framework is situated in specific learning theory that addresses learning processes from a collaborative-constructivist point of view to foster critical discourse within a group. An educational community of inquiry is defined as "a group of individuals who collaboratively engage in purposeful critical discourse and reflection to construct meaning and confirm mutual understanding" (Garrison, 2011, p. 2). Within the community of inquiry framework, the experience of learning is mediated through the development of three interdependent elements, referred to as 'presences': social presence, cognitive presence, and teaching presence. Figure 2.5 illustrates how these elements interact to construct an educational experience. The following sections explain each element in detail.

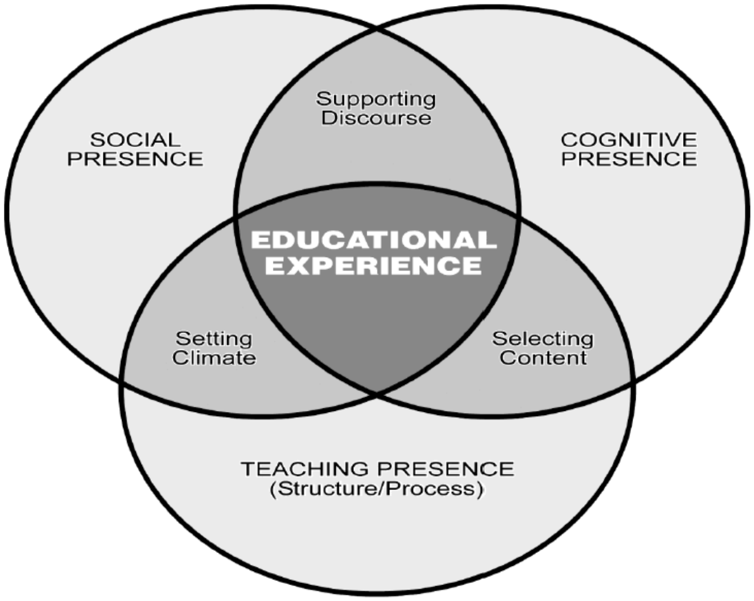


Figure 2.5. Creation of the educational experience within the Community of Inquiry framework. From: Garrison, R., Anderson, T., & Archer, W. (2000). Critical inquiry in a text-based environment: Computer conferencing in higher education. *The Internet and Higher Education, 2*(2), 87-105.

Social Presence

Garrison (2009) defines 'social presence' as "the ability of participants to identify with the group or course of study, communicate purposefully in a trusting environment, and develop personal and affective relationships progressively by way of projecting their individual personality" (Garrison & Akyol, 2013, p. 107). According to this definition, the development of social presence goes beyond mere support for the establishment of purely social relationships, but also fosters group cohesion in an environment that encourages probing questions, skepticism, and expressing ideas (Garrison & Akyol, 2013). Although social presence tends to develop naturally over time, instructors have the ability to facilitate its growth. Effort that is put into social presence pays off considerably. Simple suggestions to nurture social presence include encouraging the use of inclusive pronouns such as 'we' and 'our' as well as addressing others directly by name (Garrison & Akyol, 2013). It might be noted that students with different levels of experience in online classes may require different scaffolding with regard to social presence depending on the instructional design of the course, the nature and use of the technology involved, or the level instructor mediation (Garrison & Akyol, 2013). Swan and Shih (2005) provide evidence that students who perceive the highest social presence project themselves more into online discussions and reveal meaningful differences in perceptions of the usefulness and purpose of online discussions than students who perceive themselves to be socially distant. Other researchers have noted that social interaction is a critical factor in student course completion and retention (Boston et al., 2009). The increase in group cohesion that is fostered by social presence leads to a greater capacity for group collaboration, which in turn optimizes the learning experience.

Cognitive Presence

'Cognitive presence' is defined as "the extent to which the participants in any particular configuration of a community of inquiry are able to construct meaning through sustained communication" (Garrison et al., 2000, p. 89). Grounded in critical thinking literature, cognitive presence promotes critical thinking, which in turn both authenticates existing knowledge and generates new knowledge. Cognitive presence is enacted through the practical inquiry model, which is rooted in Dewey's (1933) model of reflective thinking. The practical inquiry model (Figure 2.6) represents an inquiry process by which students, in collaboration with others, make sense and draw meaning from complex situations and confusion.

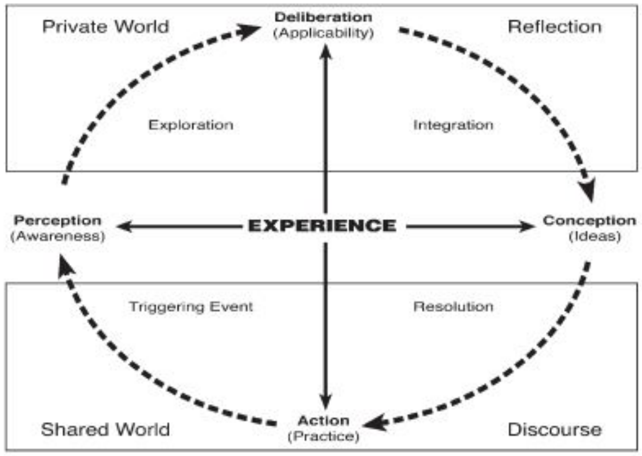


Figure 2.6. The Model of Practical Inquiry. From: Garrison, R., Anderson, T., & Archer, W. (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *American Journal of Distance Education, 15*(1), 7-23.

Practically speaking, cognitive presence is perhaps the most elusive of the three presences to operationalize because it relates to the path students take through the learning process. Related to Moore's (2013) concept of student autonomy, cognitive presence is affected by the student's approach to the course, which includes how the student approaches problems and seeks to understand difficult content material. However, a student's approach to a course incorporates issues over which the instructor exercises relatively little control. The instructor can, however, facilitate cognitive presence by the methods that they choose to use to present material and the goals they establish for learners (Garrison, Anderson, & Archer, 2000).

Teaching Presence

The third element of the CoI framework is teaching presence. Teaching presence is defined as "the design, facilitation and direction of cognitive and social processes for the purposes of realizing personally meaningful and educationally worthwhile learning outcomes (Anderson, Liam, Garrison, & Archer, 2001, p. 5). Following this definition, teaching presence plays a critical role in realizing the intended educational outcome of a course. Teaching presence also plays a crucial part with regards to establishing social and cognitive presence in a classroom as the main responsibilities of teaching presence overlap the other two in the following areas: course design and organization, facilitating discourse, and direct instruction (Garrison & Akyol, 2013).

The CoI framework flows evenly from the higher-level transactional distance model from which it evolved. Where Transactional Distance theory defined the realms in which perceived distance that can impede student success, CoI theory offers an idea of the types of behaviors and levels of cognitive though that govern these realms. In this model, the educational experience is personalized as different levels of cognitive, social and teaching presence are enacted in a classroom. It becomes important as a theory of distance education in that teaching presence, social presence and cognitive presence career focus in Mathematics Education

Online Best Practices: Low-End Theory

Whereas transactional distance theory is concerned with the psychological space between students and instructors and the community of inquiry framework concerns the design of context-related educational experiences, distance education theory at the low end is defined by actionable day-to-day interactions between instructors and students. Teaching online is different from teaching in a physical classroom, and it is not easy. Planning online coursework is more demanding and the student-teacher relationship is more complex than in the traditional classroom (Dykman & Davis, 2008a). Online educators must adapt a highly socialized process of teaching in the physical classroom to the online classroom where social interactions are limited and made possible only with technological assistance (Dykman & Davis, 2008b).

Coordinating online classwork requires significant effort on the part of the instructor – perhaps more effort than a first-time instructor expects. In addition to developing course activities, the instructor must perform a variety of supporting actions throughout the semester. One of the more time-consuming of these actions is simply creating and maintaining a course page within a learning management system (LMS) and orchestrating the outside technologies that will be used to deliver content. The time and skill needed to effect this work efficiently underscores the need for a careful and deliberate approach to planning the course. Time that is spent planning activities, choosing technologies, and preparing the course page during the semester is time that the instructor cannot use to monitor his/her student's progress, manage online discussions, or build intimate relationships with students. As such, the closest student-teacher relationships are likely to develop when planning and other elements of course construction are completed before the start of the semester.

Ultimately, a successful online educational experience requires precise communication. In the face-to-face classroom, there is often more room for error in this regard as students have a variety of opportunities to have expectations reinforced and to clarify misunderstandings (Dykman & Davis, 2008b). In addition to having opportunities for direct exchanges with the instructor, students in a physical classroom have the ability to check with their peers when they are confused about assignments or timelines. Shearer (2013) notes the existence of a variety of social media, social networking, and mobile learning sites that have the potential to bring some of the immediacy of classroom interactions to the distance learning environment. Despite the appeal of these technologies, however, Shearer (2013) warns of taking the 'tool of the day' approach to their implementation. Instead, he urges designers and instructors to give careful consideration to a mix of dialogue, learner autonomy, and structure that are present in the course in addition to the learning curve associated with use of the technology when considering adopting one of these online approaches (Shearer, 2013).

Regardless of the technology used to approach communication-related problems, instructors need to plan for communication and understand that communicating effectively with students is likely to consume a considerable amount of their time in the implementation phase of the course. Ragan (2008) offers thoughts on teacher actions in the online classroom that constitute the best practices in online learning. Many of these actions are directed towards developing the sorts of communication strategies that are necessary for successful online experiences. These actions address elements of communication, such as monitoring student progress, identifying and encouraging students who fall behind, providing feedback and student support, and dealing with conflicts that may arise online. This list of practices may not seem much different than the sorts of practices expected of instructors in face-to-face classrooms, but because all of these communicative skills need to be mediated through technology, they may require significantly more effort than expected.

Effective Teaching Practices in Mathematics Education that Support Student Learning

The literature on best practices in distance education promotes the use of learning design that is situated in meaningful contexts and is active in nature (Naidu, 2013). A similar approach is found in mathematics education with regard to the development and implementation of tasks. Research over the last two decades suggests that student achievement is greater in classrooms that routinely require high-level thinking and reasoning than in classrooms that characteristically require students to engage in procedurally oriented activities (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane, 1996). Although the vast majority of this research has been conducted in physical classrooms, there is nothing to suggest that an online ecology alters the nature of the connection between task and achievement. The question, then, is not so much whether tasks that demand high-level thinking and reasoning are appropriate in online contexts, but what are the specific factors of implementation that must be modified for online delivery.

The mathematics reform movement of the 1980s and 1990s directed a curricular shift away from the memorization of facts and procedures and towards one aimed at a conceptual understanding of mathematics. Documents published by the National Council of Teachers of Mathematics (NCTM, 1989, 1991), National Research Council (1989), and the Mathematical Association of America (Leitzel, 1991) promote classroom interactions to help students achieve a complete understanding of mathematics by allowing students to engage in the process of 'doing math' rather than merely executing algorithms and memorizing procedures. These documents were followed by NCTM's *Principles and Standards for School Mathematics* (2000), which not only outlined the mathematics that should be included in school curricula, but also made an effort to articulate a fundamental set of principles to describe high-quality mathematics education. Currently, many of these principles are still evident in the *Common Core State Standards Initiative's Standards for Mathematical Practice* (National Govonors Association Center for Best Practices, 2010) as well as in NCTM's own *Principles to Action* (NCTM, 2014) that revisits its 2000 standards and offers assistance to teachers, schools, and districts with regard to implementing national standards outlined in *Common Core*.

This quest for conceptual understanding, which is sometimes referred to as 'doing mathematics' (Smith & Stein, 1998), requires students not only to be able to perform mathematical operations, but also to participate in the framing and solving of mathematical situations by making conjectures, looking for patterns and connections amongst various mathematical representations, abstracting beyond the immediate situation, and communicating and justifying their conclusions to others (Stein, Grover, & Henningsen, 1996). Doing mathematics has a purpose well beyond mere accurate calculation, which is to encourage math exploration for its own sake by encouraging students in the process of doing mathematics to work like a mathematician. Often, this work is done by exploring mathematics that are 'real-world relevant'

Much work has been undertaken during recent decades to identify the characteristics of mathematical tasks that allow for conditions to do mathematics effectively in the classroom. Doyle (1988) proposed four essential components of tasks that define academic tasks in the classroom:

> (a) a goal state or end product to be achieved, (b) a problem space or set of conditions and resources available to accomplish the task, (c) the operations used in assembling and using resources to reach the goal state or generate the product, and (d) the importance of the task in the overall work system of the class (Doyle 1988, p. 169).

Doyle notes that teachers effect tasks by setting expectations for the product of student work and explaining how task goals might be accomplished.

Starting with a high-level task for instruction is generally recognized in the teaching community as important (Stein & Lane, 1996). Building on Doyle's work (Doyle, 1983, 1986, 1988), Smith and Stein (1998) emphasized the cognitive demand associated with instructional tasks. They note that, whereas high-level tasks do not guarantee high-level student engagement, low-level tasks almost never achieve high engagement levels. This idea suggests that a high-level task may be a prerequisite for promoting thinking, reasoning, and problem-solving skills. Smith and Stein (1998) further offer a tool to assist instructors in analyzing the level of cognitive demand that is demanded of students by an instructional task.

Smith and Stein (1998) define tasks in terms of being either low or high cognitive demand (Figure 2.7). Low demand cognitive tasks rely heavily on a procedural understanding of mathematics. These tasks are usually straightforward in nature and require little cognitive effort on the part of the student. The goals of a low-level task are usually aimed at the production of 'correct' answers and require little explanation by the student beyond stating the procedure that was used. low-level demand tasks usually fall into one of two categories. Memorization tasks involve rote memorization of material and simply ask students to regurgitate previously discovered fact. Tasks that go beyond simple memorization and require students to execute procedures like those required for an algorithm are deemed 'procedures without connections'.

Low Cognitive Demand Tasks	High Cognitive Demand Tasks
Memorization Tasks • Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous-such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures *with* Connections Tasks** • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures without Connections Tasks • Are algorithmic. Use of procedure is either specifically called or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedures being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge and experiences and make appropriate use of them in working with the task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Figure 2.7. Matrix for determining the cognitive demand of an instructional task.
From: Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*: Teachers College Press.

Higher-level tasks may require considerable effort on the part of the student. These tasks may require the use of procedures that have close connections to broader mathematical concepts. Solution paths through these problems may be opaque or entirely obscured, thus allowing the student to explore different entry points into the problem as well as a variety of problem-solving methods. Like lower-level tasks, higher-level tasks generally fall into one of two categories. Tasks defined as 'procedures with connections' may make use of algorithms; however, unlike lower-demand tasks, algorithmic thinking cannot be used mindlessly to produce a result. Higher-level tasks may rise to the level of 'doing mathematics'. As discussed previously, doing mathematics involves students mimicking the work of mathematicians. This work includes uncovering patterns, making and testing conjectures, constructing viable mathematical arguments, and critiquing the work of others with the goal to strengthen an argument.

As stated previously, the identification of a higher-level task does not guarantee a high level of student engagement. Henningsen and Stein (1997) note that the intended demand of a task may be lowered as the task is implemented. This reduction in cognitive demand can be caused by a variety of factors, including how the instructor sets up the task, the classroom conditions, and the way that the task is perceived by the student. Using data collected as part of the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project (Silver & Stein, 1996), Henningsen and Stein developed a conceptual framework to illustrate the progression of a mathematical task through classroom implementation (Figure 2.8). This framework includes the definition of variables that may affect the setup and implementation of the task that lower the intended cognitive demand of the task.

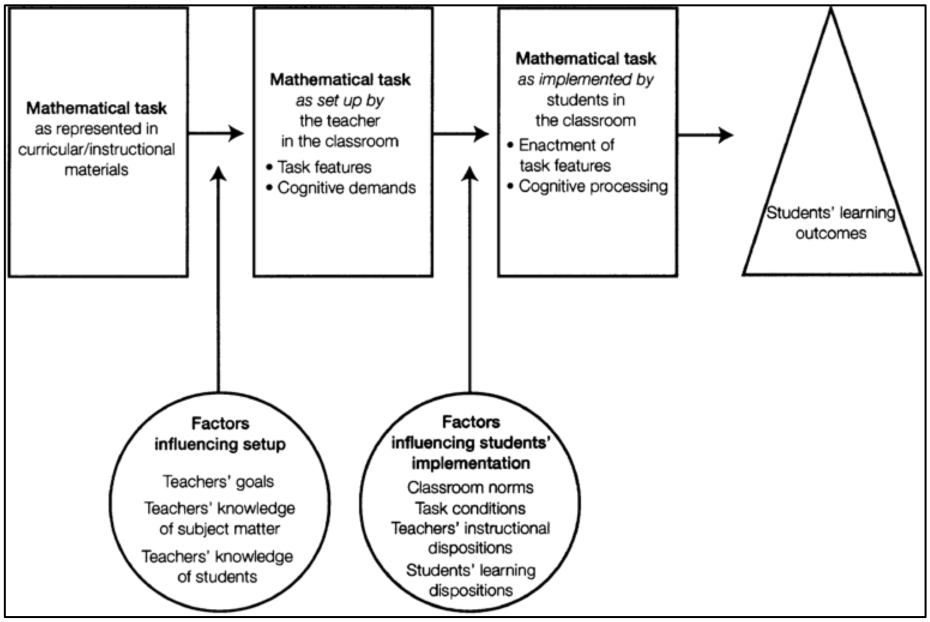


Figure 2.8. Various task-related factors that effect learning outcomes. Adapted from Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 524-549.

Figure 2.2 shows that factors that influence the setup of the problem as well as the students' implementation of the task can easily alter the task's cognitive demand and have an effect on student learning outcomes.

Even the best designed tasks do not always become quality learning experiences in the classroom. Tasks that are set up as high-demand cognitive assignments sometimes devolve into low-demand cognitive experiences (Doyle, 1983, 1986, 1988). When faced with challenging tasks that put them in a state of cognitive disequilibrium, students often demand help that pressures the instructor to decrease or remove the demanding features of a task. This phenomenon can happen either when the instructor provides additional explanation or reduces expectations for the outcome of the task (Doyle, 1983). Another threat to the cognitive demand of a task occurs when a mismatch is evident between the task and students' prior mathematical understanding or motivation (Henningsen & Stein, 1997).

Lastly, the cognitive demand of the task may be affected when an instructor applies pressure on students to perform with accuracy and speed to the detriment of their conceptual understanding (Doyle, 1988).

Given the potential perils to the cognitive demands associated with such tasks, instructors must find ways to support students in the classroom that do not jeopardize the demands placed on them by the task. The establishment of sociomathematical norms that govern teacher and student interactions in the classroom may be of particular importance to the ecology of the classroom (Doyle, 1988; Kazemi & Stipek, 2001). 'Sociomathematical norms' refers to the rules that govern both teacher and student behaviors and expectations in the classroom. Teachers need to be acutely aware of the extent to which they emphasize construction of mathematical meaning as part of the outcome of a classroom event. This emphasis can be managed when instructors make explicit connections between the mathematical ideas addressed in the task and the activity in which students engage (Doyle, 1988; Henningsen & Stein, 1997; Smith & Stein, 2011).

Cognitive demand might also be supported after the task is completed but before the class moves on from one mathematical idea to the next. Demand may be accelerated or maintained when students are expected to communicate their findings and justify their results to others in the classroom (Kazemi & Stipek, 2001; Otten, 2010). In many classrooms, students are asked to describe the steps that they took to solve a problem. However, this public explanation of their method may not include the reason that their method worked or the reason that the student chose that particular mathematical path to a solution (Kazemi & Stipek, 2001). When these explanations are not addressed by the students themselves, the teacher must press the student for these explanations. When engaged in 'high-press' teacher questioning (Kazemi & Stipek, 2001), students are better able to justify their results by triangulating their arguments using a mix of verbal, graphical, and numerical strategies. In addition to making clearer arguments, students in such high-press classrooms are also reported to be better able to identify similarities and differences amongst various student representations and to verify the validity of responses presented to them by others.

Although many of the factors that support the establishment and maintenance of cognitive demand hold for online environments as well as for face-to-face classrooms, additional online factors that affect cognitive demand are less well understood. Many researchers in the field of distance education view the connection between learning goals and tasks and activities in online settings as important as it is for face-to-face instruction (Dennen, 2013; Naidu, 2013; Ragan, 2008; Simonson et al., 2014). However, many of the factors that influence the synchronicity of an event, the management of elapsed time, and the media through which interactions occur might also affect the maintenance of demand throughout the task (Dennen, 2013). Online instructors will need to have an additional understanding of content and their students to ensure that selected media will transmit key understandings and that students will have the ability to be self-regulated in their learning (Kim, Kozan, Kim, & Koehler, 2013; Ragan, 2008).

Asynchronous Interactions

Asynchronous interactions, often in the form of discussions, are frequently used to complement learning in a variety of online and blended format classes. As such interactions impact the teaching of mathematics, constructing asynchronous discussions that are centered on high-demand mathematical tasks may be the most productive means of facilitating mathematical discourse online. In fact, online asynchronous mathematical discussions may encourage cognitive engagement better than discussions in a physical classroom. Whereas face-to-face discussions are often spontaneous and ephemeral in nature, online discussions allow participants the time to process information and carefully compose responses that become part of a permanent record of student thinking (Wang & Chen, 2008). Asynchronous online discussions become spaces where students can exchange initial thoughts, discuss mathematical issues, compare results, and collaboratively work towards solutions to problems. These features of asynchronous discussions emphasize issues that surround effective mathematics teaching and learning that are highlighted in *Guiding Principles of School Mathematics* (NCTM, 2014) as well as in *Common Core State Standards for Mathematical Practices* (National Govonors Association Center for Best Practices, 2010).

However, quality asynchronous mathematical discussions can be difficult to create. Promoting discussion online is more complicated than merely finding and posting a high-demand mathematical task. A number of problems that are unique to the asynchronous environment can arise. For instance, participation may not be as easily definable as it is in a face-to-face setting. In traditional classroom discussions, visual cues and body language can give hints regarding student engagement in a topic. Such engagement is harder to detect online and is usually measured by behaviors that mark student presence, such as constructing posts. Listening, which is a large part of face-to-face engagement, also is less easily detected. Once participation is defined, then the issue of facilitation arises. What part does a facilitator play in an online discussion? How 'present' should this facilitator be? Related to the instructor's role in the conversation is the issue of instructor feedback. Providing students with prompt, personalized feedback on their discussion posts can demand more of the instructor's time in asynchronous environments than in face-to-face settings. This section of the literature review attempts to look at some of these issues in more depth.

Participation

Participation is generally considered to be an essential prerequisite to student learning (Wenger, 1998). In face-to-face classrooms, visual cues and other noticeable student behaviors can alert instructors as to the level of student participation in a class. A key challenge for online learning is to define both the notion of online participation and the optimal conditions in which participation can occur. In a review of the literature, Hrastinski (2008b) developed a classification scheme to classify the nature and complexity of interactions in online environments. This six-level classification scheme (Figure 2.9) is constructed on a continuum that ranges from mere student access to an e-learning environment to students' full participation in a meaningful dialogue with peers.

Level 1: Participation as Accessing e-learning environments
Level 2: Participating as writing
Level 3: Participation as quality writing
Level 4: Participation as writing and reading
Level 5: Participation as actual and perceived writing
Level 6: Participation as taking part and joining in a dialogue

Figure 2.9. Levels of online participation. From: Hrastinski, S. (2008). What is online learner participation? A literature review. *Computers & Education, 51*(4), 1755-1765.

Hrastinski's conclusions and subsequent classification scheme are based on a breakdown of the 'units of analysis' used by other researchers to identify participation. These units, listed from most frequently used to least frequently used, are: number of messages, message quality, learner perceptions, message lengths, system logins, messages read, and time spent (Hrastinski, 2008b). The author notes that these units of analysis differ significantly in their ability to be measured, and that, as the complexity of the level of participation increases in an e-learning environment, measuring participation becomes more difficult. Hrastinski's classification scheme as well as the work of others who have attempted to identify and measure online participation (Hrastinski, 2008a; Romiszowski & Mason, 1996; Vonderwell & Zachariah, 2005), challenge long-held notions of online participation as writing and suggest that students in these settings might exhibit other behaviors apart from posting messages that constitute participation.

Although posting messages is perhaps the most visible evidence of student participation, other, less evident, behaviors may also contribute to student participation. Romiszowski and Mason (1996) established a construct that they refer to as 'lurking' as a valid form of participation in online classrooms. The same way that traditional classroom students who are reluctant to speak up in class may benefit from open discussion, students in online classes may benefit from forms of passive participation. Romiszowski and Mason (1996) point out that, even in face-to-face classrooms, students spend most of their time as passive consumers (with varying degrees of engagement) and only occasionally make verbal contributions to the conversation. Romiszowski and Mason (1996) suggest that such behavior is likely a feature of online student conduct as well.

'Lurking' may be an unfortunate term for this behavior, given the negative connotations it provokes, but many teachers have touted its role in group learning. Although sometimes viewed by teachers as 'freeloading' behavior, Lave and Wenger suggest that lurking may be a form of legitimate peripheral participation (Lave & Wenger, 1991). Traces of this participation, however, may be harder to detect online than in the traditional classroom. As Dennen (2008) points out, although posting to message boards is a more highly valued behavior than lurking, there would be no value to engagement if participants engaged solely in posting.

A study by Dennen (2008) looks at students' perceptions of lurking behavior. Dennen's findings show that students self-report engaging in reading behaviors more frequently than engaging in posting behaviors. When they did engage in posting behaviors, however, students reported feeling more actively engaged in the discussion than when reading. Dennen (2008) suggests that this outcome may be the result of some students' belief that the purpose of the discussion was to show the instructor what they know or to earn participation points rather than to engage in discussion with classmates, and suggests that students are "likely to look to the instructor to specify the motivation and purpose" (Dennen, 2008, p. 1631) of discussions.

Factors that Support or Inhibit Participation in Asynchronous Discussions

Literature examples that include asynchronous collaborative discussions suggest that such discussions provide a variety of benefits to the learner. The benefit that has gained the most attention is scheduling flexibility. Asynchronous online discussions allow students to 'check into' the discussion from virtually anywhere at any time, which is a feature of online learning that is attractive to students with busy schedules outside of school. Apart from convenience, a number of benefits might be linked more directly to student learning. One feature that is unique to the asynchronous discussion forum is that the forum structure creates a permanent record of the dialogue. In a face-to-face classroom, discussions are fleeting moments that are difficult for students to capture and process, let alone review at a later date. Asynchronous discussion forums create written records of discussion materials so that learners have the ability to control how quickly they go through the material as well as to rewind the discussion to review material at a later date (Hew, Cheung, & Ng, 2010). Although students in asynchronous discussions can benefit from a number of cognitive and social benefits (see Murphy & Coleman, 2004), Mazzolini and Maddison (2003) remind us that students only reap the benefits of asynchronous discussions when they are willing to participate in the discussion in the first place.

In a review of the literature, Hew et al. (2010), summarized a variety of factors that influence student participation in asynchronous online discussions. Among the challenges of participation is the student's lack of belief in the need for discussion. If students feel that they are posting as a mere marker of presence, rather than being asked to discuss a meaningful prompt, students might not feel properly motivated to participate. Dennen (2008) notes that, often times, the assignment of grades is the primary motivator for student participation in asynchronous online discussions. She further notes that, although grades may instigate posting behavior, they do not necessarily promote true interaction and dialogue amongst students. Dennen (2008) concludes that, if interaction and dialogue are not the primary objectives of the activity, then the premise for requiring an asynchronous discussion must be reconsidered.

Hew et al. (2010) note that, in addition to students' lack of motivation to participate in a discussion, often the behavior of other participants in the forum affect how students interact with the group. Some students reported that they lost interest in the discussion between the time they posted and when they received peer or instructor reaction to their posts. In this case, the feature of online education that may have drawn them to the class in the first place became a demotivating factor as the students did not receive instantaneous feedback to their thoughts, which ultimately left them with the sensation of speaking into a vacuum (Feenberg, 1987). This outcome suggests that, when the instructor and other participants do not frequently monitor and acknowledge contributions to the discussion space, interest for the discussion ultimately wanes (Xie, Debacker, & Ferguson, 2006).

Hew et al. (2010) also suggest that students react to the real and perceived personality traits of other members of the discussion group. They note that students tend to disengage from discussion when they perceive others to be pontificating on a topic. Hewitt (2005) remarks that this situation can be exacerbated when the conversations become emotional and parties feel threatened or overtly challenged. This outcome relates, in part, to the varied personalities of the members who are participating in the forum. Chen and Caropreso (2004) investigated three personality traits (extraversion, agreeableness, and openness) and found that students who tended towards these traits featured more prominently in asynchronous discussions. Students who scored at the low ends of scales of these personality traits participated less frequently and tended to contribute posts that were tangential or only marginally related to the topic of discussion compared to students who reflected these traits.

In addition to intrinsic motivation and personality traits, Hew et al. (2010) also discuss a variety of design aspects that influence participation in asynchronous online discussion forums. First, the structure of the discussion platform itself may make it easier or more difficult to follow threads. Conversations with multiple threads and multiple subdiscussions within a thread can initiate "information overload" (Whittaker, Terveen, Hill, & Cherny, 2003) and lead to decreased participation. Other technical aspects of the interface can frustrate students as well. Murphy and Coleman (2004) found that design quirks, such as the inability to edit posts easily or toggle between posts, can lead to student frustration and a lack of participation. Lastly, when discussion prompts are not sufficiently deep, or require only a single, fact-based answer, students report that they do not know what to contribute (Dennen, 2005), which, in turn, results in superficial student posts.

Related to the research collected by Hew et al. (2010), Kim (2013) discusses the influence of the size of the discussion group on student participation in online asynchronous forums. Kim suggests that it is often difficult for students to maintain focus on important topics in large classes. Here, the setup of the space may be a factor in the derailment of the conversation. Kim states that, in many discussion spaces, the newest unread posts are featured in bold at the top of the screen, with older posts further down in the feed. This structure may have the unintentional effect of featuring ideas expressed in newer posts rather than relevant ideas, which can have the result of moving the group discussion off topic. Related to the setup of the space, Kim (2013) also suggests that the hierarchical structure of the space deemphasizes the intricacies of the overall conversation. In most asynchronous discussion spaces, replies can relate to only one initial post. Kim notes that this reply may in fact be related and relevant to other threads as well. This nuance of the discussion can therefore get lost in the posting structure itself. An important overall point mentioned by Kim (2013) is that, without proper facilitation, online asynchronous discussion groups are, by their very nature, divergent discussions. As individuals reply to the initial topic, conversations can easily become branched. Once such dispersion happens, it is difficult to converge the discussion and synthesize ideas.

Instructor Facilitation and Feedback in Asynchronous Forums

Most teachers agree that feedback plays an important role in students' construction of knowledge. How that feedback is given, however, plays an important role in its effectiveness (Hattie & Timperley, 2007). When it comes to providing feedback for online assignments, such as found in asynchronous forums, little research has been undertaken that describes what useful feedback looks like (Conrad & Dabbagh, 2015; Getzlaf, Perry, Toffner, Lamarche, & Edwards, 2009). Also debated is the directness of the role that the instructor plays in facilitating feedback within the discussion itself (An, Shin, & Lim, 2009; Guo, Chen, Lei, & Wen, 2014). This section briefly addresses these debates.

Different facilitation approaches may impact the cognitive engagement of students in asynchronous discussions. In one study of online discussion forums in an education course for elementary teacher candidates, An et al. (2009) examined the effects of different facilitation approaches on students' sense of social presence. They varied two elements of the feedback process. The first variable altered the frequency with which instructors responded to initial student posts. The second variable that was manipulated was the presence of a requirement that asked students to respond to posts by their peers. Three different sections of the same online course were defined. In the first section, the instructor responded to each student's initial post and required students to respond to at least two of their classmates' posts. In the second section, the instructor responded to each student's initial post but did not require additional peer posts. In the third section, the instructor made infrequent responses to student' initial posts but did require at least two additional posts. The study cites two significant findings. First, voluntary posts were infrequent. In the section where additional posts were not required, very few additional posts were made. Second, and perhaps more significantly, in the section where instructor intervention was minimal, students tended to express their thoughts and opinions more freely and more cues related to the building of social presence were present in discussions. These findings indicate that students may prefer a discussion environment where public instructor feedback is minimal, at least as far as building social presence is concerned.

Guo, Chen, Lei, and Wen (2014) obtained slightly different results when measuring discussions for cognitive engagement. In their study, two instructor participation patterns were defined. In the experimental group, a tutor was assigned to facilitate the discussion by scoring and giving feedback for each post. Feedback included pointing out the strengths and weaknesses of an argument, probing the student's thought process, assigning bonus points for additional posts, and providing praise. The treatment group, by contrast, had no tutor assigned. The results of the study found no major differences in the number of posts made in either group; however, in the experimental group, cognitive engagement was significantly higher than in the control group. Levels of cognitive engagement also improved over the course of the semester in the experimental group.

Although the results from the An et al. (2009) and Guo et al. (2014) studies may seem a little at odds with one another, a couple of generalizations might be made. First, as demonstrated by Guo et al. (2014), feedback in the form of instructor responses to online posts may help students self-regulate their learning and keep them on task (Conrad & Dabbagh, 2015). Second, when facilitators step back and require students to provide feedback to their peers, closer social bonds are formed, potentially positively impacting student satisfaction. It is likely, however, that a mixed facilitation strategy may provide the best of both worlds. In other words, a heavy or light facilitation strategy may be employed by the instructor, depending on the goal of the task.

Synchronous Interactions

While asynchronous online activities in the form of blogs and forums have played a featured role in online education, exciting new possibilities for synchronous interactions have become available with the improvement of video conferencing technologies and other forms of computer assisted communication. As compared to interactions that happen in the physical classroom, asynchronous, text-based events are sometimes characterized as less sociable due in part to the fact that non-verbal visual cues are rarely discernable in this context. Asynchronous discussions, which are often focused on subject matter content, leave little space for marginal classroom interactions like chatting with peers before or after class. With the advent and development of computer assisted communication technology, online classroom interaction has grown beyond a reliance on asynchronous activities to the point where students can interact in "real time" from wherever they are. These synchronous learning activities show promise in increasing students' sense of social presence – the ability to sense and relate to others in online environments – and thus improve the online educational experience.

In a review of the literature on the effects of social presence on students' online experiences, Richardson, Maeda, Lv, and Caskura (2017) note that social presence has been shown to impact student motivation, participation, actual and perceived learning, course satisfaction and student retention in online courses. Despite this correlation, little research has been done looking across various measures of social presence, the effects of social presence in various disciplines, or the features of an online course that promote social presence. The meta-analysis conducted by Richardson, et al. (2017) suggests that that these factors are "significant moderators in determining the strength of the correlation between social presence and course satisfaction" (p. 409-410). One finding of their research suggests that the correlation between social presence and course satisfaction increases with the length of the educational experience. The correlation is weaker for shorter classes (6 weeks or less) than classes that span a longer period of time. The research also shows that the correlation between social presence and course satisfaction varies by content area. For instance, in an example offered by the authors of the study, social presence and student satisfaction showed a stronger correlation in education classes than those in the business school. This research suggests that, while social presence is important in all online contexts, it's effect is stronger in some contexts than others and that efforts to construct social presence should be moderated by factors such as the audience and the intended length of the course.

In the contexts where social presence is more important, it can be difficult to achieve online. Synchronous events however, show great potential for cultivating social presence in online courses. Synchronous online activities offer instructors the potential to have meaningful, in-the-moment interactions with students and provide students with opportunities to communicate directly with their peers while maintaining the geographical distance from campus that many online students find appealing (McBrien, Cheng, & Jones, 2009). In a study of six undergraduate and graduate classes facilitated by way of synchronous online communication (Elluminate Live!), McBrien et al. (2009) found that synchronous interaction promoted a feeling of greater social interaction amongst most participants and made them more willing to participate in class. Among the reasons for their increased participation, students noted that the synchronous mode of participation allowed them increased wait time to formulate their thoughts. The relative anonymity of the event also seemed to help provide shy students with the confidence to participate. Students that reported being reluctant to speak out in face-to-face classes noted that they became more willing participants in the synchronous events.

Social presence may not be the only element enhanced by the synchronous learning environment. Studies by Szeto (2015) and Kılıç, Horzum, and Çakıroğlu (2016) note that synchronous interactions may strengthen all three presences defined by the community of inquiry online teaching framework. Szeto (2015) suggests that innovative instructional approaches facilitated through computer mediated communication technologies challenge "ways through which students and instructors' experiences can be shaped in different forms of online, face-to-face, or blended learning instruction" (p. 191). Szeto conducted a case study that applied the community of inquiry framework to investigate instructional approaches across various forms of leaning (face-to-face, synchronous, and blended learning). The community of inquiry online teaching framework proposes that the online learning experience occurs at the intersection of three teaching presences; cognitive presence, teaching presence and social presence (Garrison, et al. 2000). Szeto notes that the results of this study suggest that while all three presences affect the educational experience, the presences may have disproportional impact. Results from Szeto's case study suggest that teaching presence may have the greatest impact on the attainment of the desired learning outcome of an online activity. Synchronous online activities allow for more immediate teacher presence than do asynchronous online events suggesting that synchronous teaching and learning events may ultimately enhance students' online experiences.

Given that synchronous experiences appear to make a valuable contribution to the educational experience, the quest becomes how to construct them. Woodcock, Siso, and Eady (2015) conducted a study that looked to identify prerequisite factors necessary for successful synchronous experiences. To this end, Woodcock et al. (2015) asked students in an online primary-teacher education course to self-assess their online learning experience. Results of their study found that course participants were generally receptive to (and in some cases in favor of) online learning experiences. However, their satisfaction was predicated on four hierarchical conditions: (a) ease of use of the online platform, (b) the presence of a psychologically safe online learning environment, (c) a willingness to participate as students in online events, and (d) the instructor's e-learning self-efficacy.

Mykota (2017) also studied factors affecting the success in synchronous environments. Mykota surveyed 273 graduate students in nine online courses over two years. Survey results indicate that that learner interaction is greatest amongst individuals with greater amounts of experience with computer mediated communication as a whole. While previous course experience was a factor, Mykota found that it was not necessarily these learning experiences that were most important, but rather general experiences with social interaction tools (Facebook, Skype, etc.) that was correlated with interactivity. Additionally, Mykota noted higher rates of class interaction and collaboration amongst students in enrolled in courses where the instructor provided pre-course instructional activities related to the communication platform.

Synchronous online learning is a relatively recent addition to the online learning experience and, as such, warrants additional study. Currently, the evidence is at least suggestive that synchronous online learning events may positively impact levels of social presence in online classes. While synchronous experiences do not provide the same experience as working in the face-to-face classroom, synchronous communication technologies provide at least a modicum of interactions. As technology improves more research will be necessary to determine the critical elements of a successful synchronous learning experience.

Elementary Mathematics Specialist Professionals

One key element to the success of any educational experience is aligning the goals and structure of the course with the needs of the students who are likely to engage in the experience. Aligning the educational experience for prospective students may be a particular concern for online courses, given the student-oriented nature of distance learning. The specific course that is at the heart of this dissertation is a mathematically focused education course that targets practicing elementary school teachers who are looking to become mathematical leaders in their respective schools and districts. This part of the literature review frames the role of elementary mathematics specialists (EMS) that is at the heart of this study.

Prompted by legislation, such as *No Child Left Behind* (2001) and similar reports, educators and policymakers alike have attempted to address high stakes accountability requirements for increased student achievement in mathematics. One way to address these concerns has been to employ EMS within districts and schools as a means to provide recurring, on-site professional development for teachers within a variety of coaching models. To date, little quantitative data are available that correlate increased student achievement and the presence of EMS within a school; however, it has been proposed that these professionals have the ability to strengthen mathematics teaching and learning in the elementary grades (Fennell, 2011; NCTM, 2009). Although they appear under different names and have various responsibilities, EMS serve as the 'go-to' people in their schools and districts to answer mathematics-related questions.

Although the specific duties of EMS may vary from school district to school district, the characteristics of these specialists remain consistent across placements. Reyes and Fennell (2003) define this professional educator as "a teacher whose interest and special preparation in mathematics content and pedagogy are matched with special teaching or leadership assignments" (p. 280). Campbell and Malkus (2011) describe this specialist as "a knowledgeable colleague with a deep understanding of how students learn, as well as pedagogical expertise, to serve as an on-site resource and leader for teachers" (p. 431). Although both of these definitions serve as adequate descriptions of EMS, this research project adopts a broader definition of EMS as individuals who have "a deep and broad knowledge of mathematics content, expertise in using and helping others use effective practices, and the ability to support efforts that help students learn important mathematics" (Association of Mathematics Teacher Educators, 2010).

Although widely assumed to be effective, the mechanisms that link educational supports such as teacher professional development to student achievement are currently poorly understood (Desimone, Smith, Hayes, & Frisvold, 2005; Fennell, 2011; NCTM, 2009). It has long been suggested, however, that efforts that focus on a teacher's understanding of content knowledge and how students learn lie at the heart of educational reform efforts (Sykes, 1996). Although numerous professional development opportunities are available to teachers today, these offerings vary in quality and, thus, their potential ability to affect student achievement likewise varies. One-shot workshops and short duration professional development conferences have proven to be an ineffective agent of teacher learning (Ball & Cohen, 1999; Desimone, 2009; Sykes, 1996). In their place, long-term, sustained efforts directed towards changing teachers' knowledge base and beliefs in the context of their own classroom practices have shown promising influences on student achievement (Desimone, 2009).

In mathematics education in particular, these long-term, sustained efforts that are aimed at improving professional development are increasingly focused on providing teacher support through coaching that follows a variety of different, although equally worthwhile, models (Campbell, Ellington, Haver, & Inge, 2013; Campbell & Malkus, 2011). Most of these models fall into one of two categories: (1) the teacher-leader model and (2) the specialized-teaching-assignment model (Reys & Fennell, 2003). In the first of these models, an individual elementary teacher is identified for the role of math specialist and released from teaching duties to mentor other teachers, organize building resources, and orchestrate instructional and programmatic change across the school or district (Campbell & Malkus, 2011). In the second of these models, a school or district organizes its teaching staff by subject matter (Reys & Fennell, 2003). In other words, one or two third-grade teachers might be identified to teach mathematics, whereas other third-grade teachers focus on history, literacy, science, etc.

Regardless of the model, the mathematics specialist is tasked with becoming an agent of change in a school or district and assumes a variety of responsibilities that require careful preparation and training. As demand for these positions has increased, however, individuals have been appointed to these positions without proper vetting that is related to their knowledge of content, pedagogical skills, or leadership ability (Fennell, 2011). As early as the 1980s, the NCTM and NCTM President John Dossey called for elementary mathematics specialists and recommended that states provide a credential endorsement for such individuals (Dossey, 1984; Fennell, 2011).

Summary

This literature review began by framing the theoretical influences on the construction of the *Field Guide*. Here, three levels of theory in distance education were identified and their relevance to the *Field Guide* was explicated. Then, the literature on effective teaching practices in mathematics education that support student learning was discussed. Together, theoretical considerations of the best practices of online learning and effective teaching practices in mathematics education that support student learning were used as a foundational base for the development of the *Field Guide*. Discussion of this theoretical base was followed up with a general discussion of issues related to designing and facilitating asynchronous discussions, as these sorts of activities play a prominent role in the specific mathematics distance education course that is at the center of this dissertation. The chapter ends with brief consideration of EMS. Analysis of this literature served as a base of knowledge for the collection and analysis of teaching episodes in this online course and ultimately will serve to inform future versions of the *Field Guide*.

CHAPTER 3: METHODOLOGY

This study focuses on ways that instructors create online distance education experiences to address mathematics/mathematics education content. Specifically, in order to assist instructors in the creation of high-quality online distance education experiences, a draft field guide was developed to support instructors' use of suggested best practices and to recommend ways to implement these practices in an online distance education environment. This document, *A Field Guide for Mathematics Educators in the Design and Implementation of Online Distance Education Learning Experiences* (hereafter referred to as the *Field Guide*), is organized into two main sections. The first section is intended to provide guidance to the instructor about the setup of his/her online distance education course. The second section assists in the design of mathematical activities to be implemented in an online distance education environment.

The focus of this dissertation is on ways that one instructor of a semester-long online distance education course (one of the six courses offered in the EMAoL program) set up her online teaching/learning environment, fostered a collegial learning environment, selected and organized mathematics/mathematics education content, and implemented instruction within a blended synchronous/asynchronous online setting. The research for the dissertation focused on the instructor's understanding of the content in the *Field Guide*, her previous experiences conducting online distance education courses, and the students' experiences in the course. The intended outcome of this study is to provide direction for developing a revised version of the *Field Guide* that reflects the instructor's perspectives and course activities, and the students' experiences in the online distance education course.

Case study methodology was used in this study to address the following research questions:

1. In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the *Field Guide* in course planning and implementation?

   This includes a consideration of:

   a. In what ways are the use of best practices in online teaching strategies that support student learning evident in the course planning and implementation?
   b. In what ways is the use of effective teaching practices in mathematics education that support student learning evident in the course planning and implementation?
   c. In what ways are the uses of best practices in online teaching strategies and of effective teaching practices in mathematics education that support student learning related to the components of transactional distance in the course planning and implementation?

2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance?

   This includes a consideration of:

   a. Using a measure of student satisfaction with the course, what is a description of participants' response to the course?
   b. Using a measure of transactional distance (see Appendix A), what is a

description of participants' perceptions of transactional distance experienced at the end of the course?

c. In what ways are participant satisfaction and perceptions of transactional distance related in this distance education learning experience?

The case study methodology employed in this study draws on both quantitative and qualitative approaches in order to investigate the elements of the blended online distance education course as they relate to the *Field Guide* material that is needed to optimize the delivery of the content and facilitate student learning.

Case Study Methodology

Studying fast-paced, real-world phenomena over which the researcher has little control is often challenging. Case study methodology allows the researcher to deal with the ever-evolving contexts of distinctive situations (Yin, 2009), or what MacDonald and Walker (1975) describe as "the examination of instance in action" (p. 1). Because this method calls for extensive study and description of the case, there is potential for uncovering an endless number of variables. Utilization of case study methodology allows the researcher to address "the technically distinctive situation in which there will be many more variables of interest than data points" (Yin, 2009, p. 17).

Yin (1981) describes three different forms that a case study can take, namely descriptive, explanatory, and exploratory. The purpose of a descriptive case study is to describe particular phenomena and the real-life contexts in which they occurred. Descriptive case studies are characterized by rich descriptions that seek to reveal patterns and connections relative to a theoretical construct (Tobin, 2010). In this sense, descriptive case studies do not attempt to make causal conclusions or to describe previously unexplored phenomena. Explanatory case studies are employed when attempting to explicate a particular phenomenon (Yin, 1981). This type of case study is exemplified by an accurate accounting of observed facts, considerations regarding alternative explanations for the phenomenon, and drawing conclusions based on the most plausible justification for the observed behavior.

This study falls into the third case study category described by Yin (1981). Exploratory case studies are often applied in research contexts that lack detailed preliminary research and where the research environment limits the researcher's choice of methodology. As has been noted by Tallent-Runnels et al. (2006) as well as Simonson et al. (2014), little progress has been made in developing comprehensive theories or models that guide the creation of online distance education courses. This lack of theoretical models of implementation extends to the adaptation and implementation of the best practices in mathematics education in the online environment.

Participants

Students

This study was conducted with a group of 27 teachers of kindergarteners through fifth-graders from various school districts across North Carolina. Most participants in the study were part of a cohort enrolled in the EMAoL program at one university that offers this program. The remaining students were part of a funded project that this university had with a local school district. As part of this project (headed by the primary course instructor), teachers from this district were participating in some or all of the EMAoL courses.

The goal of the EMAoL program is to offer high-quality professional development instruction that is focused on mathematics content and pedagogy to practicing elementary school teachers in the state. The program enrolls cohorts of students who take a sequence of six mathematics-focused, graduate-level university courses that are offered through several institutions across the state. The original program was developed jointly by faculty members from these universities and, eventually, received state board approval for an elementary math add-on endorsement for teachers with pre-existing elementary teacher certification. The actual delivery of the program varies at different universities. One program delivers content exclusively in a face-to-face setting. One program offers all courses asynchronously online. One site offers a hybrid of asynchronous online and face-to-face learning. Finally, three universities offer the courses collaboratively with a blend of asynchronous and synchronous online offerings. This last delivery configuration is the one studied in this dissertation.

The students in the cohort studied were enrolled in the class through one of three universities in North Carolina. Due to limited enrollment in the program, the three universities shared students as well as teaching responsibilities for the six-course sequence through a memorandum of agreement approved by the deans of each of the three universities. Nineteen of the 27 students in the class were enrolled members of the cohort. Seven students were part of an outside grant procured by the lead instructor assigned to the course. Although these seven students were not part of the official cohort, all seven had participated in prior online classes in the EMAoL program. One student was neither a part of the cohort nor of the grant program. This individual was an instructor at a local community college who had taught fundamental mathematics similar to the mathematics addressed in the class. Because this student was not seeking the math K-5 add-on license, this participant was excluded from in-depth interviews and his/her student work was not included in analysis. However, because the class was focused heavily on discussion, this student’s participation in small group activities was included when it was relevant to the study.

Teaching Team

Algebraic Reasoning: K-5 Discourse & Questioning is a course that was originally conceived in 2008 as part of program planning through collaboration with seven state colleges and universities in the southeastern United States. Since 2008, this course has been taught several times by different faculty members among these institutions. Members of the partner colleges and universities meet frequently to revise the content of the course and discuss issues related to implementation. The most recent meeting occurred in the summer of 2015 and included faculty members from six of the original seven universities.

Dr. Kerry Spencer (pseudonym) (instructor of record for *Algebraic Reasoning: K-5 Discourse & Questioning*). Dr. Spencer is an Associate Professor of Mathematics Education in the College of Education and is well known for teacher training in her state. Dr. Spencer has held an appointment at this university since 2008 and has been an integral member of the team that developed and revised this course. Before obtaining her doctorate in curriculum and instruction, Dr. Spencer was employed as an elementary school teacher. She has published and presented on a number of topics related to mathematics education and teacher education specifically. At the time of this study, she had taught the *Algebraic Reasoning* course multiple times. Dr. Spencer was the contact person for students in the class and was the sole individual who was responsible for assessing student work and assigning grades.

In addition to Dr. Spencer, two individuals participated in the planning of synchronous and asynchronous activities. These individuals participated in bi-weekly planning meetings with Dr. Spencer and participated in debriefing sessions following each asynchronous event. While participating in the planning of the class, these researchers had minimal direct content with students, particularly in the course of normal classroom activities.

Dr. Susan N. Friel. Dr. Friel is a Professor of Mathematics Education at a university in the southeast United States where she has been a faculty member since 1990. Dr. Friel has extensive experience in teacher training and curriculum design, is an original member of the EMAoL program development team, and has taught all six courses in the EMAoL sequence multiple times. She is also co-author of the *Field Guide for Mathematics Educators in the Design and Implementation of a Learning Environment in Distance Education*, the document that is the focus of this dissertation. For this study, Dr. Friel participated as dissertation advisor to the author as well as in bi-weekly planning and debriefing sessions. Dr. Friel served as the instructor of record for an EMAoL class that approximately half of the students in the Algebraic Reasoning class had taken the previous semester. As such, she had an existing relationship with some of the student participants. She also participated as a lecturer/facilitator for one of the synchronous sessions in the Algebraic Reasoning course.

Bryan Fede (researcher). Mr. Fede is a doctoral candidate and co-author of *Field Guide for Mathematics Educators in the Design and Implementation of a Learning Environment in Distance Education.* Mr. Fede has ten years of experience as a former high school teacher and elementary school mathematics coach. He served as the main researcher for this dissertation. Also, he served primarily as observer and data collector and had minimal interaction with students over the course of normal classroom events. Mr. Fede conducted all interviews, collected all survey data, and participated in bi-weekly planning meetings and debriefing sessions.

The Field Guide

Despite concerns regarding whether or not online distance education is in the best interest of the learner, recent discussions regarding e-learning have shifted from *whether* online classes should be offered to *how* they might be delivered. When looking at best practices in the delivery of online education, it is relatively easy to find exemplars of common practices as well as well-articulated arguments for the need for institutional support for the changing landscape. Often missing from these discussions, however, are recommendations for action that can form the basis for strong online pedagogy within a discipline.

Developing Best Practices for Online Courses

The present study attempts to connect suggested best practices in online learning with high-leverage practices in mathematics education through the development of a field guide. The purpose of this particular *Field Guide* is to help instructors facilitate high-quality mathematical work and foster mathematical discussions and learning in a blended synchronous/asynchronous online learning environment. The *Field Guide* is divided into two sections. The first section outlines the elements that an instructor must incorporate in the setup and execution of any online distance education classroom. With the understanding that there is variability in the needs of students who take online classes, the *Field Guide* provides several checklists of instructor behaviors (Ragan, 2008) to help the instructor prepare students for class ahead of time and monitor the needs of students as the course is implemented. These checklists are designed to bring awareness of particular elements of a student's experience to the instructor's attention and are not meant to be prescriptive. In other words, the checklists provide recommendations that an instructor may or may not implement depending on his/her perception of the needs of the class.

The second section of the *Field Guide* outlines high-leverage practices in mathematics/mathematics education. Research work over the last three decades has attempted to address teaching and learning in the mathematics classroom. Most of this work addressed teaching and learning in the traditional face-to-face classroom. Beginning in 1986, the National Council of Teachers of Mathematics (NCTM) commissioned a board to draft a document that would establish a broad framework to guide reform in school mathematics into the 1990s. As a result, the commission made recommendations for mathematics education reform that would be summarized in the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989).

Although the NCTM was pleased with the *Curriculum and Evaluations Standards* document, the organization knew that in order to remain influential, the standards must be periodically evaluated, tested, and revised. Recognizing this need, the NCTM appointed a Commission on the Future of the Standards and, by April of 1996, had approved the creation of the *Standards 2000* project which was engaged in updating and revising the original *Standards* document. In spring 1997, the *Standards 2000* Writing Group was appointed and directed to create a new set of standards that (a) built on the foundation of the original *Standards* document, (b) integrated the classroom-related portions of the *Curriculum and Evaluation Standards for School Mathematics, Professional Standards for Teaching Mathematics*, and *Assessment Standards for School Mathematics*, and (c) was organized into four grade bands: pre-kindergarten through grade 2, grades 3-5, grades 6-8, and grades 9-12 (NCTM, 2000, p. x).

Whereas *Curriculum and Evaluation Standards* had focused on the mathematics that should be included in the school curriculum, the *Principles and Standards for School Mathematics* document was an additional effort to articulate a fundamental set of principles to describe high-quality mathematics education. These principles addressed not only the content features of a robust mathematics curriculum, but also the mathematical dispositions necessary to develop strong, mathematically literate students of mathematics. Organized into six overarching themes, i.e., (a) equity, (b) curriculum, (c) teaching, (d) learning, (e) assessment, and (f) technology (NCTM, 2000, p. 11), the principles included in the document, which are distinct and separate from the content standards, describe vital issues that are related to school mathematics programs.

Shortly after the release of the *Principles and Standards* document, the 107th Congress of the United States reauthorized the Elementary and Secondary Education Act (Pub. L. 89-10) under the short title ‘No Child Left Behind’ (NCLB) (Pub. L. 107-110; https://www2.ed.gov/policy/elsec/leg/esea02/index.html). This act required that all schools receiving public funding administer an annual, statewide, standardized examination. Continued federal funding required students in the district to make adequate yearly progress towards measurable objectives developed by the state. State-defined measurable objectives, as measured by the state-administered standardized examination, were targeted towards improved achievement by all students and subcategories of students. Additionally, state objectives must have as their ultimate goal grade-level proficiency for all students within twelve years of the passing of the legislation.

The federal government did not define national achievement standards. Instead, states were left to develop both the standards and the assessments for themselves. The NCLB legislation also required that states provide highly qualified teachers for all of their students, although, similar to the standards for academic achievement, the standards for ‘highly qualified’ were also left to the states’ discretion. With schools struggling to make sense of the NCLB legislation and meet adequate yearly progress standards, the United States Department of Education offered grant money as part of the American Recovery and Reinvestment Act of 2009 (Pub. L. 111-5). Similar to the NCLB legislation before it, the grant program, which became known as ‘Race to the Top’, was intended to hold local schools accountable for the academic achievement of students based on student standardized test scores and performance-based teacher evaluations.

Confusion regarding these standards for students and teachers remained, as different states applied different measures. In 2014, the NCTM stepped in once again to attempt to clarify the situation. This time, the goal of the NCTM Writing Group was not to write the standards or suggest underlying principles. Rather, the goal was to articulate a “unified vision” that would ensure quality mathematics instruction for all students “under any standards or in any educational setting” (NCTM, 2014, p. vii). *Principles to Actions: Ensuring Mathematical Success for All* set forth this vision as it served as a collection and synthesis of the best of research-based mathematics teaching practices from the previous three decades.

Creating the *Field Guide*

In an effort to better align assessment with instruction, the mathematics education community has conducted extensive research into teacher in-classroom behaviors that have an effect on student achievement. The original version of the *Field Guide* included a potpourri of these research-informed mathematical teaching practices and attempted to extend discussion of these behaviors beyond the physical classroom to the online distance education classroom. Research-informed practices in the original version of the *Field Guide* included practices that addressed task development (Smith & Stein, 1998), fostered mathematical discourse (Hufferd-Ackles, Fuson, & Sherin, 2004; Smith & Stein, 2011), and developed teacher questioning strategies (Chapin, O'Connor, & Anderson, 2009).

Bryan Fede (researcher of the present study) and Susan N. Friel developed the *Field Guide* over the course of approximately 12 months at the University of North Carolina at

Chapel Hill. The idea for a *Field Guide* was the outgrowth of a faculty reading group in the spring semester of 2015. The focus of the reading group, *Teaching and Learning at a Distance: Foundations of Distance Education* (Simonson et al., 2014) provided a starting place for the development of the *Field Guide*. As a result of the reading group, the authors of the *Field Guide* narrowed the focus to two theoretical but complementary frameworks: transactional distance (Moore, 1997) and communities of inquiry (Garrison et al., 2000).

The Course

Description

Algebraic Reasoning: K-5 Discourse & Questioning was taught as the third course in the EMAoL program for this K-5 cohort. The EMAoL program was developed through a coordinated effort with the North Carolina Department of Public Instruction (NCDPI), the University System General Administration, statewide local educational agency representation at the school and district levels, and university faculty representation from colleges of education and arts and sciences in the participating universities. Each of the six courses in the program has two objectives: (1) a mathematical content domain that aims to develop the profound understanding of fundamental mathematics that teachers will need to demonstrate through the program of study and (2) the development of high-leverage teaching practice.[2] Over the course of the semester, the content domain acts as a context for exploring the high-leverage teaching practice.

The content focus of the *Algebraic Reasoning: K-5 Discourse & Questioning* course is to develop participants' mathematics knowledge of teaching (MKT) by simultaneously

[2] For more information on high-leverage teaching practices, refer to the following site: http://www.teachingworks.org/work-of-teaching/high-leverage-practices.

fostering participants' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Deborah Loewenberg Ball, Thames, & Phelps, 2008). This course "aims to develop the content and pedagogical content knowledge of in-service elementary teachers in the areas of Algebra and Algebraic Reasoning" (see Syllabus – Appendix B). The content is paired with high-leverage teaching practices that foster student reasoning through discourse and questioning. Together, this content domain and associated high-leverage practices assist practicing elementary mathematics teachers in engaging their students in tasks that foster algebraic reasoning and preparing them for the more formal algebra courses that they will see later in their educational careers.

Course objectives. Objectives of the Algebraic Reasoning course address two categories. The first set of objectives addresses SMK and the second set addresses PCK (Figure 3.1). SMK consists of both common content knowledge (CCK), or the knowledge expected of any reasonably educated individual, and specialized content knowledge (SCK), which is specific knowledge of content that is useful for teaching mathematics. SMK further includes knowledge of mathematical horizons. Knowledge of the mathematical horizon allows teachers to view mathematics holistically and gives them a perspective that allows them to connect content across mathematical domains. The second set of course objectives addresses the participants' knowledge of mathematics and student development. PCK is focused on the curriculum as a whole, interpretation of a student's location within the curriculum, and techniques of teaching that foster student progression in the curriculum.

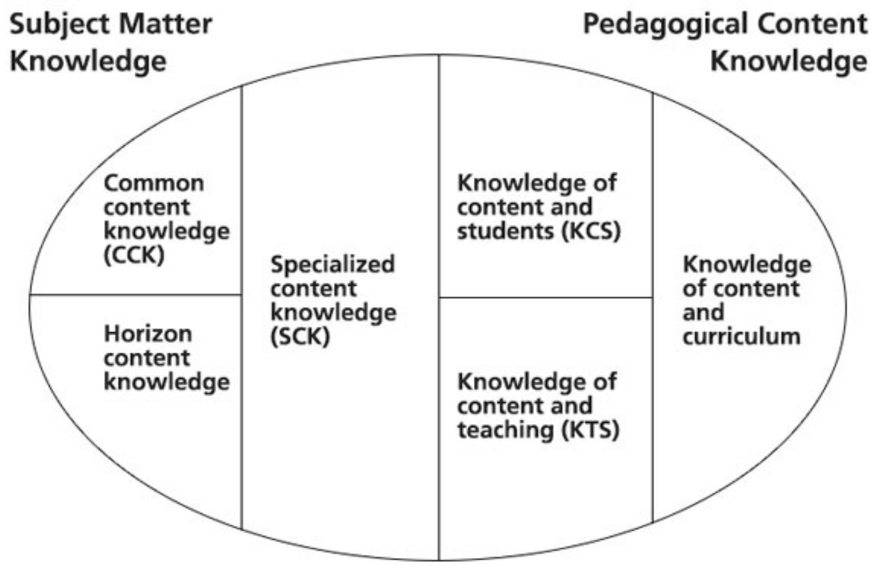


Figure 3.1. Domains of Mathematical Knowledge for Teaching. From Content Knowledge for Teaching: What Makes It Special? Ball, D. L., Thames, M. H., & Phelps, G. (2008). *Journal of Teacher Education, 59*(5), 389-407.

According to the syllabus (See Appendix B) for *Algebraic Reasoning: K-5 Discourse & Questioning*, the SMK goals for this course are to:

- Understand patterns, relations, and functions from a variety of perspectives.
- Understand the role of properties in number systems and their use in computation.
- Represent mathematical situations and structures using algebraic symbols.
- Prove mathematical conjectures.
- Use mathematical models to represent and understand quantitative relationships.

The PCK goals are to:

- Implement a variety of appropriate instructional strategies to assist elementary children in constructing algebraic ideas.
- Demonstrate an understanding of the assessment of algebraic reasoning in elementary classrooms through questioning and listening to students, analyzing students' written

work, documenting patterns in student thinking, and planning appropriate student/teacher interactions.

- Demonstrate an understanding of ways to facilitate discourse to elicit algebraic reasoning in elementary classrooms.
- Demonstrate content knowledge in K-8 algebraic thinking based upon national standards (i.e. Common Core State Standards, NCTM – National Council of Teachers of Mathematics Process and Content standards).

Course resources. The class material includes two main resources. The first resource, *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School* (Carpenter, Franke, & Levi, 2003), provides insight into student thinking and reasoning about algebraic situations. This text provides information that allows the reader to engage students in substantial mathematics and evaluate their students' level of reasoning about algebraic situations. The second text, *Algebra and the Elementary Classroom* (Blanton, 2008), provides participants with ways that algebra might be interwoven into the curricular fabric of the classroom. In addition to these two required texts, participants were encouraged to read *Classroom Discussions: Using Math Talk to Help Students Learn, Grades, K-6* (Chapin et al., 2009) in which the course's high-leverage practice of fostering student discourse through questioning is explored. Finally, *Elementary and Middle School Mathematics: Teaching Developmentally* (Van de Walle, Karp, Bay-Williams, Wray, & Rigelman, 2012) is used in all six EMAoL classes to anchor elementary grades' mathematical content. Additional readings and resources (see Syllabus, Appendix B for a complete list) were assigned throughout the semester and made available for download to participants.

Course structure.

Course dates. The Algebraic Reasoning course followed a blended distance education model that includes a combination of both synchronous and asynchronous learning experiences. Although the cohort was populated by students from multiple universities, the calendar for the class followed the dates used at Dr. Spencer's university. The course was offered in the spring semester of 2015; classes started January 11, 2016 and ran through April 26, 2016. The final examination was given during the regularly scheduled exam period between April 28, 2016 and May 5, 2016.

The course was broken into eight units (referred to as *modules*). Each module (See Appendix C for screenshots of all course modules) consisted of a variety of synchronous and asynchronous online activities. Asynchronous activities, including problem-solving forums, blogs, journals, and individual assignments, had flexible due dates that allowed students to participate in their own time over the course of the module. Synchronous sessions, conducted via teleconferencing meeting software (Saba Meeting), were scheduled for alternating Wednesday evenings beginning January 13, 2016 from 5:00 p.m. to approximately 7:50 p.m. Figure 3.2 shows the beginning and closing dates for each module as well as for the scheduled synchronous session.

Module Number	**Mathematical Topic**	**Asynchronous Dates**	**Synchronous Session**	**Notes**
Module 1	What is Algebra?	January 11 - 13	January 13	
Module 2	Equality	January 13 - 27	January 27	The synchronous session for this class was rescheduled for February 1 due to a scheduling conflict with the instructor's schedule.
Module 3	Relational Thinking	January 27 – February 10	February 10	
Module 4	Properties (algebra as generalized arithmetic)	February 10 - 24	February 24	This synchronous session was rescheduled for February 29 due to severe weather that affected connectivity across the state.

Module 5	Variables	February 24 – March 9	March 9	
Module 6	Functional Thinking Part 1	March 9 - 23	March 23	
Module 7	Functional Thinking Part 2	March 23 – April 6	April 26	This synchronous session was cancelled due to instructor illness.
Module 8	Course Wrap-up	April 6 – April 20	April 20	

Figure 3.2. Modular schedule for the course.

Blackboard learning management system. Asynchronous activities for this class were facilitated through the Blackboard Learning Management System. Students in the course were provided with log-in credentials that allowed them to access course content. Although some of the students were familiar with the Blackboard system, for others, it was a new experience. When students accessed the 'Blackboard Algebraic Reasoning: K-5 Discourse & Questioning' page, they were greeted with a home screen that defaulted to recent announcements. Along the left side of the page was a variety of menu options (Figure 3.3).

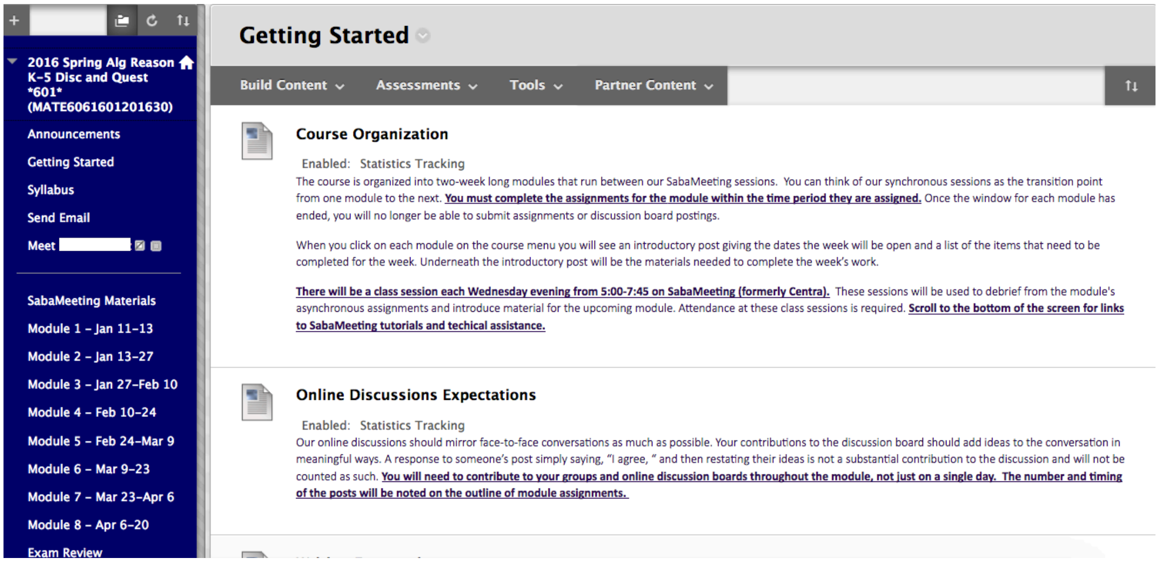


Figure 3.3. The Blackboard home screen. Student options are displayed in the blue panel on the right of the page.

Options for students included the following:

- An 'Announcements' tab. Announcements are typically one-way correspondence from the instructor to the students as a group. In this class, postings in the Announcements section included links to join Saba Meeting sessions, class cancellations, and assignment clarifications.
- A 'Getting Started' tab (Figure 3.4). This tab provides pertinent information about a class as well as links to student help resources. Information provided by the instructor under the 'Getting Started' tab is generally broad and general (see example below regarding course organization). For this class, the instructor included the following information for getting started:
    - A course organization statement
    - Expectations for online discussions
    - More general expectations for student written work
    - Guidelines for showing mathematical work

- A statement regarding the naming of digital files for submission
- A link to Saba Meeting tutorials
- A link to Saba Meeting frequently asked questions (FAQs)

- A tab linked to the syllabus.
- A tab linked to email that can be sent to a customized group of class participants.

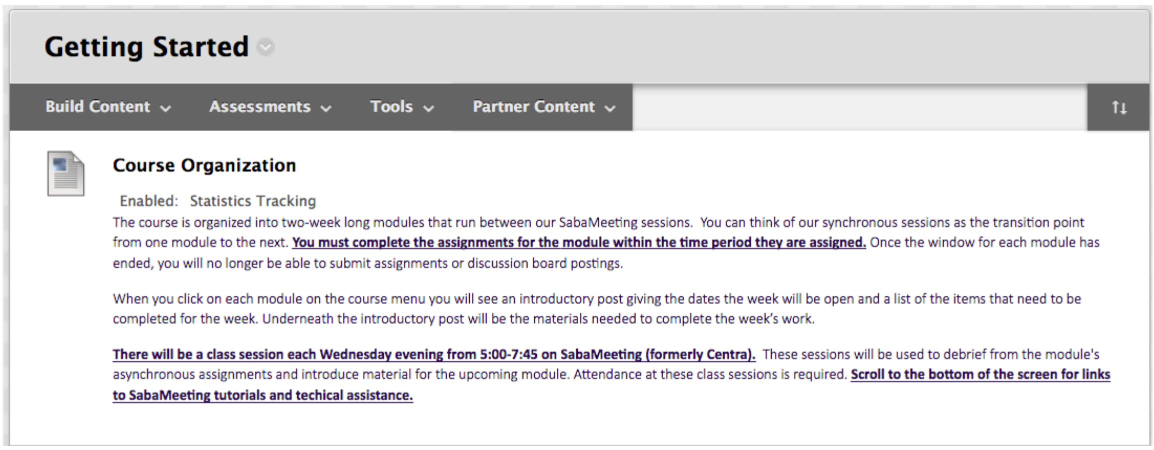


Figure 3.4. Screenshot of the information contained under the ‘Getting Started’ menu.

New content links (organized by module) appeared approximately every two weeks below the main menu options described above. Each module (see Appendix C) of the course (8 total) contained a variety of asynchronous activities that students were expected to complete over the course of two weeks. Links to specific assignments were accessible within the module itself along with a table that listed the task with the due dates.

In addition to readings and course assignments, blogs are frequently used as a forum to facilitate conversations between participants. A blog is a personal online journal that is intended to be shared with others. Three different types of blogs were used for the Algebraic Reasoning course.

- Course blogs in which only the instructor determines the topic to be addressed. All course members can add blog entries and add comments to blog entries.
- Group blogs in which all group members can add blog entries and make comments on blog entries, building upon one another. Other course members (not assigned to the group) can view group blogs, but can only add comments. A group blog is different from a threaded discussion as each entry does not need to continue the discussion of the previous entry, but can be a complete thought on its own.
- Individual blogs are created by the instructor for a specific course member to use. Individuals are able to create entries in their personal blog space. Other members of the course can read the blogs and make comments at the end, but they cannot contribute additional threads within the blog.

Module structure. The course content for *Algebraic Reasoning: K-5 Discourse & Questioning* was created using a modular structure. The module structure allows the instructor to maintain related course material in one place and organize content by subject, level, time period, etc. In the case of this class, the module was used to contain a two-week collection of course information. Each of the two-week content packets was centered on a particular mathematical concept within the domain of algebra. Assignments with a pedagogical focus on questioning and fostering student discourse were interwoven into each of the modules where the specific algebraic topic being addressed provided a context to demonstrate a high-level teaching practice. Online synchronous sessions were conducted every other week as students transitioned from one module to the next. Each synchronous session was designed to bridge content from one module to the next, thus allowing the instructor to wrap up discussions from the previous module and launch discussion and exploration of the upcoming module.

The first module and the eighth (last) module contained different sorts of activities than the middle six modules, as the first module served as a course introduction and the eighth module attempted to summarize the course material. The middle six modules (Modules 2, 3, 4, 5, 6, and 7) shared a similar organizational structure that focused on four elements: (1) a team problem-solving exercise, (2) an individual activity (the format of which varied from week to week), (3) a questioning and student work analysis (QSWA) activity, and (4) a 'bringing it all together' (BIAT) blog. The following sections provide a description of each element.

Team problem-solving. Team problem-solving activities provided participants with opportunities to engage in algebraic tasks as students (as opposed to as teachers). Tasks selected as team problem-solving activities aligned generally with the mathematical content goal of the module. For each activity, students were assigned to a group of three to five members and given a mathematical task to complete (see Figure 3.5 for an example prompt). Participants worked collaboratively to solve the problem and explain their thinking to one another.

> You have a balance scale and you are trying to weigh 40 packages of meat ranging in weight from 1 kg to 40 kg. You have only four weights with which to work – a 1 kg, 3 kg, 9 kg, and 27 kg weight. How can you weigh each package of meat with just these four weights? Look for shortcuts in finding solutions to this problem by using previous work when you can to arrive at solutions.

Figure 3.5. Module 3 Team Problem-Solving Activity: The Sheep Problem

Participants were asked to post to the group blog at least three times over the course of a module. In an attempt to even out participation, each participant's first post was required approximately five days after the module started. The other two posts could be posted any time between the beginning and the end of the module. Interactions were evaluated on a three-point scale (Figure 3.6) as described in the syllabus. Of the four major module elements, team problem-solving was employed the most inconsistently, appearing in four of the six modules within the 'heart' (Modules 2-7) of the class.

3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response.

2 – Response shares ongoing work the problem with explanations of your thinking.

1 – Response submitted either does not include your work on the problem or does not explain your thinking.

***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.)

Figure 3.6. Rubric for Team Problem-Solving

Individual activities. Although the group problem-solving activities encouraged participants to work together to solve mathematical tasks, most of the modules also contained some sort of individual task that was related to the mathematical content focus for the module. These activities explicitly focused participants' attention on their own MKT, often highlighting SMK. The expectations for each activity varied by assignment. Generally speaking, students were given a problem set or reading and asked to answer a series of questions that were related to the activity. Some activities had an application component that required teachers to try an algebraic activity with a group of their own students, collect data, and report to the whole group via a blog or presentation during the upcoming synchronous session.

Questioning and student work analysis (QSWA). The QSWA assignments were cumulative, ongoing assignments that participants worked on in stages throughout the semester. The cumulative project was worth 25 percent of a participant's overall grade in the course. As part of the assignment, teachers were expected to spend time interacting with a small group of children (grades K-5) to work on algebraic thinking tasks. Each task was broken into three parts that spanned Modules 2 through 7. QSWA 1, which spanned Modules 2 and 3, was situated in participants' work with students around the idea of equality. QSWA 2, which spanned Modules 4 and 5, was grounded in the context of an algebraic task. Lastly, QSWA 3 was dedicated to tasks that addressed functional thinking.

For each of the QSWA assignments, participants were asked to choose a lesson or exercise from Carpenter et al. (2003) or Blanton (2008), complete a Questioning Planning Grid (Appendix D) prior to executing the lesson, audio/video record the lesson and record questions and prompts that were used in the lesson, and complete an Analysis of Student Written Work Chart (Appendix E). Participants were asked to bring the two charts to the Saba Meeting bridge session in the middle of the two sessions (synchronous sessions 2, 4, and 6). After discussion in the synchronous session, participants were asked to further analyze their questioning techniques as well as their use of 'talk moves' (Chapin et al., 2009) and to write a reflection regarding common questioning mistakes that they made in their own practice. The instructor asked that participant reflections should include:

- Examples (i.e., short exchanges transcribed from audio or an attached audio or video, etc.)

- Data (number of times participant had a student comment on another student's thinking; wait-time seconds, etc.)
- A general take-away message about the participant's current status of his/her questioning and what he/she will try to accomplish in the next lesson.

Participants were asked to submit a portfolio of their work that related to all three QSWA assignments and a final reflection that addressed how the three QSWA assignments had impacted their teaching.

Bringing it all together (BIAT) blog. The BIAT blog was organized as a course blog. In a course blog, the instructor sets the general topic for the space and participants can make original blog posts or respond to the post of others within one online space. The intent of this blog was for participants to synthesize their reflections regarding the various experiences and readings from the module. The prompts from module to module were similar, but they were tailored to the teaching of the mathematical topic that was the focus of the module (see Figure 3.7 for an example). Similar to the case of the team problem-solving activities, participants were encouraged to contribute to the course BIAT blog a minimum of three times over the course of the module. The instructor further suggested that the three posts be made at different times across the module and that the first post be constructed in the first week of the module.

Discuss teaching and learning of relational thinking throughout the module. Why is relational thinking so important in elementary school?

Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

Figure 3.7. Module 3 Bringing It All Together prompt.

Synchronous online sessions. Bi-weekly online synchronous sessions were conducted throughout the semester. Sessions were scheduled in advance and usually occurred regularly (every other Wednesday). Two sessions needed to be rescheduled (one due to a conflict in the instructor's schedule and the other due to instructor illness) and one session was cancelled due to severe weather in parts of the state (tornado and severe thunderstorm warnings). In total, six synchronous sessions were held.

Participants attended the synchronous sessions using the online web-conferencing software Saba Meeting. Students were sent a link to a virtual meeting room that was set up by the instructor before the start of class. This link allowed the students to attend the event. Each session lasted approximately three hours and consisted of a combination of large group (whole group) discussion and smaller breakout discussions. Breakout discussions typically had between two and five participants and were held in special breakout rooms that were set up by the instructor using the Saba Meeting software. In general, Saba Meeting sessions were seen as bridging sessions where students debriefed elements from the prior week's asynchronous activities and launched events that were scheduled for the upcoming week.

Synchronous sessions were recorded using the Saba Meeting software, which allowed students who were absent from class to view a recording of the class, although they could not interact in the class. Also, only whole group interactions could be recorded; thus, students who were absent could not view the small group breakout sessions. In order to collect as many interactions as possible for research purposes, Saba Meeting sessions were recorded from five different computers using Camtasia screen-capture software. Interactions were recorded from the perspective of each member of the research team (three recordings) as well as from two additional machines (designated as Guest 1 and Guest 2). This format allowed the researcher to capture a significant number of small group synchronous interactions as well as interactions in the whole group. Over the span of the semester, 63 of these small group episodes were recorded for potential analysis.

Data Collection

Data collection methods for this study included surveys, interviews, class recordings, planning meetings, and student work exemplars. Two types of surveys were conducted for this project. A Likert-based student online course satisfaction survey (Davis, 2014) and a blended learning transactional distance survey (Horzum, 2011) were administered to gauge students' reactions to the class. A smaller subsample of student participants was chosen for in-depth interviews about their experiences in the class. Synchronous online classes as well as asynchronous discussions were monitored and recorded to capture the frequency and nature of student–instructor interactions as well as the context and resolution of the event.

Planning meetings and debriefing sessions of synchronous classes were also recorded with the intent to capture rationales for decisions, either in anticipation or as a result of synchronous group meetings. Figures 3.8 and 3.9 present 'research crosswalks' that connect the data collection methods with the research questions.

Research Crosswalk	Student Satisfaction Survey	Transactional Distance Survey	Student Interviews	Instructor Interviews	Class Recordings	Planning Meetings	Student Work Exemplars
Question 1. In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the ***Field Guide*** in course planning and implementation?							
a. In what ways are three components of Transactional Distance (structure, dialogue and autonomy) present in the course planning and implementation?		X		X	X	X	
b. In what ways is the use of suggested best practices for online courses evident in the course planning and implementation?				X	X	X	
c. In what ways is the use of high leverage practices for mathematics education pedagogy evident in the course planning and implementation?				X	X	X	X
d. In what ways are the use of suggested best practices for online courses and high leverage practices for mathematics education pedagogy related to the components of Transactional Distance in the course planning and implementation?		X		X	X	X	

Figure 3.8. Research crosswalk that aligns data collection with research question 1.

Research Crosswalk	Student Satisfaction Survey	Transactional Distance Survey	Student Interviews	Instructor Interviews	Class Recordings	Planning Meetings	Student Work Exemplars
Question 2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance?							
a. Using a demographic survey, what is a description of the students in the course?							
b. Using measure of student satisfaction with the course, what is a description of students' response to the course?	X		X				
c. Using a measure of transactional distance (see Appendix 1), what is a description of students' perceptions of transactional distance experienced at the in the middle and the end of the course?		X	X				
d. In what ways are student satisfaction and perceptions of transactional distance related in this distance education learning experience?	X	X	X		X		X

Figure 3.9. Research crosswalk that aligns data collection with research question 2.

Survey Instruments

Two different survey instruments were used near the conclusion of this study in an attempt to gauge student responses to and satisfaction with the course. The first survey, the Satisfaction of Online Learning (SOL) instrument (Davis, 2014), was used as a valid and reliable instrument to assess student satisfaction. Although an end-of-semester survey was administered and collected by the university, the researcher administered the SOL instrument because this assessment was designed specifically to target the unique psychological and emotional aspects of taking an online course. The second survey, the Perceived Transactional Distance in Blended Learning Environments (PTDBLE) scale (Horzum, 2011), was used to assess students' perceptions of transactional distance in this course. For ease of collection, both the SOL and the PTDBLE results were transcribed to a web-based survey using Qualtrics, an online survey collection program. Students were sent a link to the Qualtrics survey following the last online synchronous session. Access to the results of the Qualtrics survey was available only to the principal researcher for this project.

Satisfaction of Online Learning (SOL) instrument. The SOL instrument was developed as a valid and reliable measure of student satisfaction with online mathematics classes. This instrument consists of 24 Likert-type survey items that fit into an eight-factor model where the factors include (1) effectiveness of feedback, (2) timeliness of feedback, (3) use of discussion boards, (4) instructor student dialogue, (5) perceptions of online experiences, (6) instructor characteristics, (7) feeling of a learning community, and (8) computer-mediated communication. This eight-factor model (Figure 3.10) was compared against a null model and a one-factor model (Figure 3.11) to determine the degree of fit of the model in question.

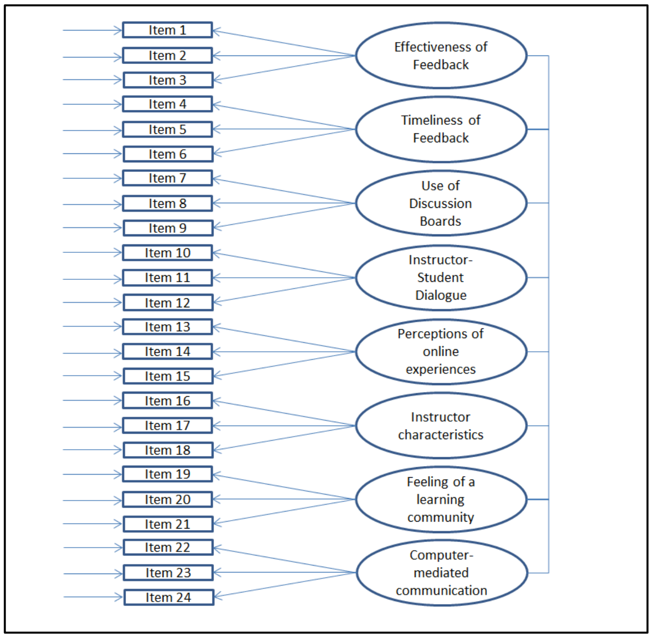


Figure 3.10. Eight Factor Comparison Model. Reprinted from *Measuring Student Satisfaction in Online Math Courses* (p. 58), by A. M. Davis, 2014, Lexington, KY: University of Kentucky Press.

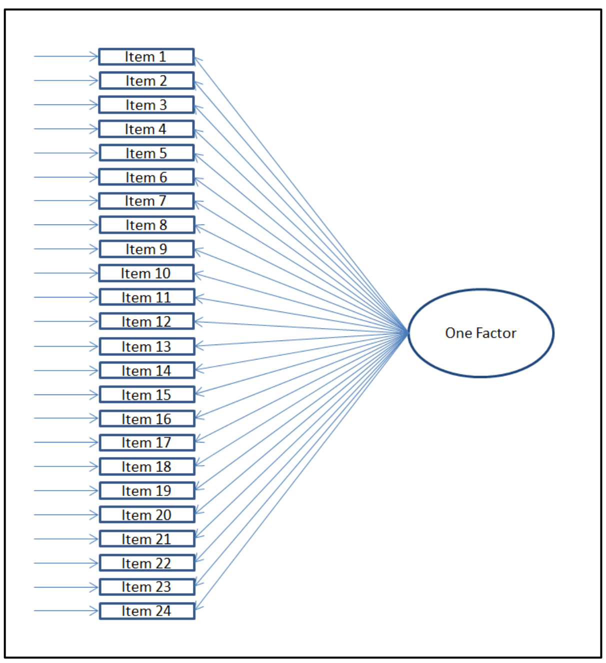


Figure 3.11. One Factor Comparison Model. Reprinted from *Measuring Student Satisfaction in Online Math Courses* (P. 57), by A. M. Davis, 2014, Lexington, KY: University of Kentucky Press.

A χ^2 statistic indicates that, compared to the null and one-factor models, the eight-factor model leads to a considerably better fit to the data. Confirmatory factor analyses, including standardized root mean residual (SRMR), a comparative fit index (CFI), the Tucler-Lewis index (TLI), Akaike information criterion (AIC), and Bayesian information criterion (BIC), all confirm the superior fit of the eight-factor model over either the null or the one-factor model. Chronbach's alpha was used to assess the internal consistency of each scale and the instrument as a whole. It was found that all scales were internally highly reliable as all alpha coefficients were above .98 for the scales as well as the instrument as a whole.

Perceived transactional distance (PTD) scale. The PTD scale (Horzum, 2011) is a survey tool that consists of 38 items measured on a Likert-based scale from 0 to 5. Exploratory factor analysis revealed the presence of five subcategories or factors. The factors are referred to as dialogue, autonomy, structure flexibility, content organization, and student control. These subcategories loosely fit variables suggested by the hypothetical theoretical model of transactional distance (Moore, 1972, 2013c) as well as subsequent iterations of the model (Dron, 2007a, 2007b; Saba & Shearer, 1994).

Participant Interviews. In addition to the two surveys previously described, a subsample of the class was chosen to participate in three in-depth, open-ended interviews. Questions were formulated prior to the interview and the prompts loosely asked the participants to respond to or elaborate on their experiences in both the online synchronous and online asynchronous experiences. Participants for this portion of the study volunteered to be interviewed for three thirty-minute interviews. Six students volunteered to be interviewed. Four of the six students completed all three interviews, and one student completed the first two interviews but did not respond to requests for a third interview. The remaining participant had to withdraw from the class for personal reasons after completing the first interview.

Interviews with participants were conducted at three points during the course of the semester. One interview was conducted at the beginning of the semester (January, 4 – 18), one in the middle of the semester (around spring break, March 11 - 25), and one after the completion of the final synchronous online event April 27th – May 6). Interviews were generally open ended. Prior to each round of interviews, a short list of general prompts were generated. These questions were situated in the principal features of Moore's (2013) transactional distance theory and centered around students' perceptions of the structure of the course, the levels of dialogue present between themselves and their peers and instructor, and self-assessment of their preparedness for the course and their ability to self-monitor their progress in the course (student autonomy). Follow-up questions varied from student to student and attempted to take advantage of emerging themes that were of interest to the researcher. All interviews ended with a chance for students to add any additional comments that were not raised in the interviews but had not been already raised in the course of the interview.

Interviews primarily were carried out online using a variety of web-conferencing products, including Saba Meeting, Blackboard Collaborate, Skype, and FaceTime. The final interview for one student was conducted face-to-face. All interviews were audio recorded in their entirety and sent out for transcription. Although all interviews began with a list of the same questions, these interviews often took a more conversational tone and followed participant's line of thinking.

Participants gave permission to use their information via the eSignature software *HelloSign*. A letter that described the research (see Appendix F) was sent to each student's university email address. At the end of the letter, participants were given a variety of options to grant permission to collect various forms of information. Students also indicated their willingness to participate in one-on-one interviews. All students were assigned pseudonyms and all data downloaded or collected were stored on an external hard drive that was in the possession of the primary researcher.

CHAPTER 4: FINDINGS

The goal of this research was to address how to assist instructors in the design, development and facilitation of mathematics education courses taught using online distance education. Drawing on a combination of current research about best practices in online distance education as well as in mathematics/mathematics education, this study addresses how instructors may develop online education experiences involving mathematics content. To assist instructors in the creation of high quality online experiences, a draft document called the *Field Guide for Mathematics Educators in the Design and Implementation of Online Distance Education Learning Experiences* (*Field Guide*) was developed. The intent of the *Field Guide* was to provide guidance in using online teaching strategies and effective teaching practices in mathematics education that support student learning during both the development and implementation phases of a course. Through an application of Case Study methodology, a description using snapshots of an instructor's practice throughout the development and implementation phases of the course is provided. The findings are presented in relation to the following research questions:

1. In what ways does the instructor respond to, interpret, and apply the underlying premises and guidance provided in the *Field Guide* in course planning and implementation? This includes a consideration of:
    a. In what ways are the use of best practices in online teaching strategies that support student learning evident in the course planning and implementation?
    b. In what ways is the use of effective teaching practices in mathematics

education that support student learning evident in the course planning and implementation?

c. In what ways are the uses of best practices in online teaching strategies and of effective teaching practices in mathematics education that support student learning related to the components of Transactional Distance in the course planning and implementation?

2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance? This includes a consideration of:

   a. Using measure of student satisfaction with the course, what is a description of participants' response to the course?

   b. Using a measure of transactional distance (see Appendix A), what is a description of participants' perceptions of transactional distance experienced at the end of the course?

   c. In what ways are participant satisfaction and perceptions of transactional distance related in this distance education learning experience?

The chapter is organized into two main sections based on the two questions posed. Section one addresses the foci of Question one and discusses the ways in which use of the *Field Guide* was evidenced in the instructor's practice in the design and implementation of the course. The potential implications of these decisions with regards to student perceptions of Transactional Distance in the course is also considered. Section two of the chapter addresses the foci of Question two. It addresses how instruction was received by participants in the class as measured through student satisfaction and Transactional Distance surveys.

Section 1: In What Ways Does the Instructor Respond to, Interpret, and Apply the Underlying Premises and Guidance Provided in the *Field Guide* in Course Planning and Implementation?

Section 1 provides an in-depth look at the mechanics of instructor practice in the course studied. The researcher and the instructor of record had multiple discussions prior to the beginning of the semester in which the *Field Guide* was discussed along with potential actions that the instructor might take as she developed the course site on the LMS and prepared for the first weeks of class. This analysis interprets the actions that were or were not present in the delivery of the course. The researcher cannot say for certain that these discussions played a role in the instructor's decisions to include or omit the various suggestions for practice provided in the *Field Guide*. The purpose of this section is to give a snapshot overview of the actions taken by the instructor in the course and feature places where recommended actions were not present. This snapshot of the course is then used in conjunction with student feedback from surveys and interviews to highlight what the researcher perceived as critical episodes of success as well as illustrate opportunities that may have been missed by the instructor in the implementation of the course.

Section 1 is organized into three parts. Part A focuses on evidences related to the ten online teacher practices (referred to as components) recommended in the *Field Guide*. The researcher provides a brief description highlighting the importance of each of the components and providing a rationale for its inclusion in the *Field Guide*. Each component in the *Field Guide* contains a checklist of various actions that might be taken by the instructor to achieve the overall goal of addressing a component. The researcher took a holistic view of the semester and attempted to determine which of the recommended actions were addressed by the instructor and which of the recommended actions appear not to have been addressed. Evidence actions related to the component being considered were compiled in summary tables. An abbreviated example of one of these tables is shown below in Figure 4.1. The complete set of tables (one for each of the ten components considered) is available in Appendix G of this dissertation.

Component 1: Preparing the Students for Online Learning

Teacher Action	Example	Evidence Source
Post a welcome message to help students get started	Welcome to [illegible]! You should already be in our Blackboard site. Those of you coming from another university should have received an email last night with an invitations and instructions. If you did not receive this email, please let me know over the weekend so I can be sure to follow-up with our Blackboard Technical Support. In the meantime, I wanted to go ahead and send the syllabus and the assignments in this email that will be due before our first SabaMeeting Session on Wednesday, January 13 at 5:00. This information will all be posted on Blackboard as well. Also attached is a letter from Bryan Fede about a research study that he is conducting in conjunction with this course. Please take the time to read the letter and respond to him. **I will be en route to a conference on January 27 so I would like to move our second SabaMeeting session to Monday, February 1, from 5:00-7:50.** I am hoping that with a few weeks notice most of you can make this date work, but if not, please let me know, and we will make arrangements for you to listen to the recording after the session. I look forward to spending this semester thinking deeply about algebraic reasoning and questioning and discourse. Please feel free to contact me by email or phone anytime. We can also set up video conferencing appointments if you prefer to speak "Face to Face." I will look for you on Blackboard early next week and see you in SabaMeeting on Wednesday! Best,	Group email sent Friday, January 8 for class beginning on Wednesday the 13th.
Include a brief orientation for students to get familiar with the terminology and tools used in your CMS.	Not Present	N/A
Provide contact information (email, phone number, etc.) for technical help in different ways: post in syllabus, group email messages, or by course announcement.	SabaMeeting Tutorials The tutorials at this link can help you get started if you have never used SabaMeeting before this class. SabaMeeting FAQ's If you are having technical difficulties, check this link first. Many problems and possible solutions are address.	Found under the "Getting Started" tab on the front page of the Blackboard class site

Figure 4.1. Example of the table used to compile evidences related to component 1: Preparing students for online learning. The full table for component 1 as well as the tables for the other nine components is provided in Appendix G of this dissertation. Notice the table is organized into 3 columns: Teacher Action (from checklist), Examples found in the teaching of the course, and the Sources of the evidence (where the evidence was found, e.g., syllabus, email, course page on LMS, etc.)

Part B of Section 1addresses issues related to the use of high leverage mathematics teaching practices in the planning and implementation of the course. This section takes an in-depth look at two online teaching episodes. The first teaching episode involves an asynchronous online discussion forum revolving around a high cognitive demand mathematical task. Interactions were observed between course participants in problem solving groups. The researcher attempts to note instructor actions related to the setup and implementation of the group task. Notes are also made suggesting how students may have engaged in the task during implementation. The second teaching episode describes a synchronous online discussion about the nature of odd and even numbers. Participant interactions were observed in synchronous, small group discussions. The researcher, again, looked at the ways group interactions were facilitated by the instructor, this time in a synchronous session conducted with group meeting software using teleconnection technologies.

Part C of Section 1 situates the possible effects of the implementation of online best practices and effective teaching practices in mathematics education to support student leaning in the context of this course in the broader context of a Transactional Distance framework as well as student satisfaction in the course. Analysis of data involves the Satisfaction of Online Learning (SOL) tool (Davis, 2014), the Perceived Transactional Distance (PTD) scale (Horzum, 2011), as well as in-depth interviews with participants from the course. The interviews with class participants use students “own words” to highlight aspects relevant to student satisfaction and perception of distance in the course.

Part A: In What Ways is the Use of Best Practices for Online Teaching Strategies that Support Student Learning Evident in the Course Planning and Implementation?

Part A presents evidences related to the ten online teacher practices (referred to as components) recommended by the *Field Guide*. Each component appears detailed in a checklist format. A rationale for the inclusion of each component is presented followed by a summary of the evidences collected relating to the component[3]. These descriptions are intended to provide the reader with an in-context "snapshot" overview of the ways in which the elements of each component were carried out in this class. Where relevant, commentary from interviews with participants is included to provide further context regarding student reaction to teacher actions.

Component 1: Preparing students for online learning (See Summary Table in Appendix G)**.** One of the first tasks related to online teaching involves preparing students for learning online (Ragan, 2008). Learning done online requires students to use additional study skills that may be different from those they practice in traditional classroom settings. Many of these additional skills reflect the unique relational dimension created by distance and the separation of students from the instructor and one another. While the dimension of distance can create various learning roadblocks not found in the traditional classroom environment, online distance education allows for the creation of spaces that make it possible for students to participate in educational opportunities that they might otherwise be prohibited from by either restraints on their time or their geography (Simonson et al., 2014). Given that both physical and temporal distance are inherent parts of the landscape of the

[3] As a reminder, a complete collection of the tables used in the analysis for this section can be found in Appendix G of this dissertation. These tables show the teacher actions listed in the field guide accompanied by examples of the action in practice.

online distance education classroom, instructors must make significant efforts to prepare students for the experience of learning online. These efforts include the creation of supports that lay out the overall structures of the course (orientation to the navigation of the course site, etc.), begin to foster a collaborative learning environment (posting of introductions, providing contact information, etc.), and describe how learning online is different from learning face-to-face learning.

Summary of analysis and researcher observations. The checklist for component 1 as detailed in the *Field Guide* presents nine teacher actions that address the preparation of the student for the online distance education learning experience. Of these nine actions, six actions were observed over the course of the semester in the course studied. The instructor began by sending an email to all participants of the class. The email was sent out on the 8^{th} of January, just prior to the first synchronous session on the January 13^{th}. The email contained a copy of the syllabus, a preliminary list of assignments, and directed students to the course Blackboard LMS page for specifics about attending the first synchronous session. This email partially satisfied recommendations suggested on the checklist for component 1 of the *Field Guide*; however, the nature of the instructor actions seemed minimal and included much of the same information that might be included in any "first day" offering in a face-to-face class. While the email provided the "when's and where's" of attendance in the first synchronous session, it failed to comprehensively outline ways in which the students might prepare themselves as they approach the first day of class taken online.

Once students navigated their way to the course Blackboard LMS page, they were met with some more preliminary information about the class. A "Getting Started" tab helped orient students to the class by providing a link to the course syllabus, some basic guidelines for participating in online discussions, and links to pages provided by Blackboard LMS and Saba Meeting (meeting software used for synchronous sessions) regarding frequently asked questions about the technology. In addition to the "Getting Started" tab, students were provided a link to "Module 1" to be completed between the first day of the semester (January 11) and the first synchronous session (January 13). This directed students to read the syllabus, submit a headshot for a class picture directory, and complete an introductory post introducing themselves to the class. The researcher notes that there was no follow-up by the instructor on the course picture directory. Additionally, while the students were asked to introduce themselves to one another, the instructor did not introduce herself. Both of these were missed opportunities for creation of social presence at the beginning of class.

Three instructor behaviors were not evident. First, there was no orientation to the LMS itself. Fortunately for students, Blackboard is a relatively well structured LMS and tabs for the syllabus Module 1 were prominently displayed in the left margin on the front page so navigation of these features was clear. Second, a reminder to set up email forwarding was also absent from the introductory email, the Blackboard site, and the syllabus. Many of the students enrolled in the class were working professionals that were not full-time university students. As such, they may not have been accustomed to checking their university assigned email. As a result, any email sent through the Blackboard system to a student's university account had the potential of not being read unless students specifically accessed their university email; hence the reason for students to forward messages sent to the university email address to an alternative email address. Finally, while general FAQs were provided for Blackboard and Saba Meeting, no such FAQ was provided for issues more closely associated with the university (financial aid, registration, library access, etc.).

Overall, evidences related to component 1 of the *Field Guide* were of mixed quality and usefulness. The primary concern of much of the information provided to students by the instructor seemed focused on the logistics of site navigation and participation. Little time was spent orienting students to the online experience or orienting students to the distance dimension of the class. The beginning of the course may have benefited from suggestions from the instructor regarding issues like time management in the course. While the syllabus contained some information about student participation in the course, this information read more like "course policy" rather than suggestions for participating online, a skill with which many online students struggle. Instructor follow-through on assignments in course module one may have had a significant impact on the students' mindset at the beginning of the semester. The course picture directory was never completed, and course introductions were never debriefed. While students created introductions, it was not clear that these introductions were ever accessed by other members of the class or the instructor. Instead, this introduction became merely another assignment to "get done".

Component 2: Specify goals, expectations, and policies (See Summary Table in Appendix G)**.** A significant problem facing online education has been low course completion rates among students (Allen & Seaman, 2008; Boton & Gregory, 2015; Harrell, 2008; Simonson et al., 2014). It has been suggested that one reason for these high attrition rates is a mismatch between the level of student autonomy demanded by the course and the level of autonomy of typical participants in the course (Allen & Seaman, 2008). Student autonomy requires that the individual both initiate and sustain cognitive behaviors that are concentrated on achieving a desired learning outcome (Schunk, 1990). Clear delineation of the goals of these outcomes are a key component to student success in an online class, as these elements become the focus of students' efforts (West, Hannifan, Hill, & Song, 2013). Component 2 of the *Field Guide* emphasizes the need for clear student outcomes by providing a checklist of teacher actions that ensure that goals and objectives are clearly stated, and students understand what is expected of them in the course.

Summary of analysis and researcher observations. The usual method for communicating information about the course to the students is through a course syllabus. Typically the course syllabus needs to contain enough information so students can do what you want them to do, while at the same time providing them with information regarding how course structures operate (Fink, 2013). The syllabus for the class under study (see Appendix B) looks much like the syllabus that that one would get in many traditional face-to-face classes. The syllabus includes the instructor's name and contact information, a brief course overview, the instructional material needed for class, an outline of the grading policy for the course, and a sense of the expectations for course assignments. While this syllabus serves as an acceptable example for a face-to-face class, it may be in need of some adjustments to adapt it for an online setting.

As stated previously, Dr. Spencer is an accomplished teacher with ample experience as a mathematics teacher educator. She has taught extensively in both face-to-face and online settings. In planning meetings with Dr. Spencer, it was clear that she had a strategy and a direction for the course. During these meetings, she revealed her plan for both the topics that would be covered as well as the timing for the topics. Dr. Spencer also had a general sense of the timing of the major assignments for the semester. Despite having these items outlined for herself, these details were not clearly shared with the students. The syllabus contained nine broad mathematical and pedagogical objectives; however students were not given much information on how the course would unfold across the semester. One glaring omission from the syllabus and the course site was a course schedule outlining the details of what content would be covered in what order and which assessments (projects, discussion forums, quizzes, etc.) would connect to the content with a rough idea of due date deadlines. The lack of an overall course schedule lead to an impression that the course was unfolding “week-to-week”. As stated previously, the instructor did, in fact, have a reasonably well-organized plan for the course. Sharing of this plan, even in a provisional document, may have helped alleviate the sensation that the course was evolving “week-to-week”.

A strength of the course in relation to Component 2 was the synchronous online class session that was held on the third day of the semester. Many of the vagaries in the syllabus were clarified in this first session, leaving participants with a clearer idea of the purpose and direction of the class. After a brief introduction of herself and the course, Dr. Spencer moved students into a brainstorming activity around the concept of algebra. The question “What is algebra?” was posed, and the participants were separated into small group discussion meeting rooms consisting of between three and five students. Participants spent approximately 18 minutes in small groups brainstorming the question and recording their thoughts on virtual whiteboards (Figure 4.2). After completion of the brainstorming session students were pulled back from their meeting rooms into the large group setting (the software permits the instructor to ‘dissolve’ the small groups and return to a large group) to debrief their discussions with the whole class.

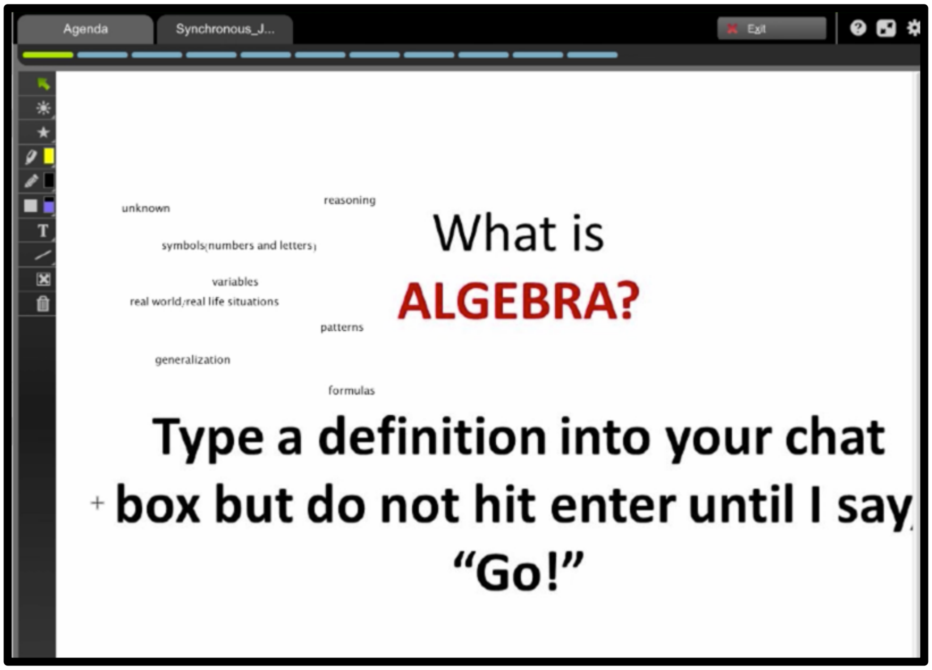


Figure 4.2. Example of a whiteboard created by small groups during the first synchronous online activity.

As class progressed during the first synchronous session, students were returned to their small groups two more times to discuss mathematical solutions for a variety of algebraic equations. As the session ended, Dr. Spencer held a final debriefing session with the whole group in which she looked across the three exercises for general emerging themes. The instructor presented a slide titled "Big Ideas in this Course" (Figure 4.3) that presented students with the evolution of mathematical topics for the course. Dr. Spencer also used this slide to connect the mathematical goals for the class to the pedagogical goals relating to the high leverage teaching practices addressed in the course. This "Big Ideas" slide, while planned ahead of time by the instructor, nicely summarized the topics that would addressed over the course of the semester, while at the same time gave the participants the feeling that they had participated in the creation of these goals.

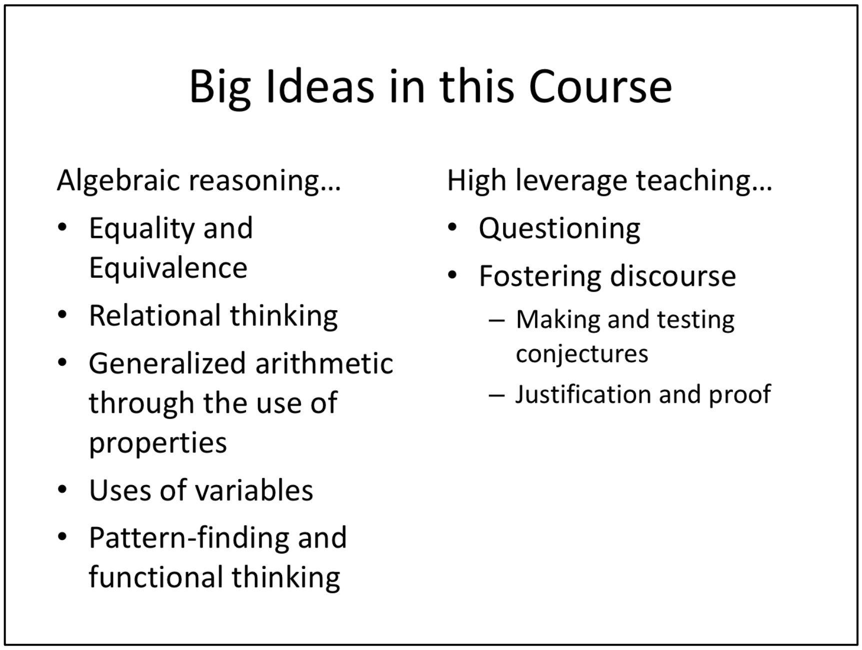


Figure 4.3. Slide outlining course objectives presented in synchronous online session 1, January 13, 2016.

Overall, while elements of component 2 were largely met on a superficial level, the detail in the syllabus was insufficient to ground students in how the course was to operate. The synchronous session helped in this respect, but a lack of a course schedule still had students wondering what was coming next. Stated differently, the problems observed in this course with regards to component 2 were not about missing elements so much as missing alignment across the elements. Alignment is a concept that is critical for course goals, assessments and class policies to work together to ensure that participants achieve the desired learning outcomes (QM, 2014). In this course, it was unclear at the outset how instructional materials, course assessments and learning activities related to the goals and objectives of the course. The timing of course events was also unclear, making it difficult for students to plan their time purposefully and efficiently.

Component 3: Foster the creation of a warm and inviting atmosphere to build a learning community (See Summary Table in Appendix G)**.** The evolution of technologies that allow for both synchronous and asynchronous online distance education experiences has brought with it a renewed focus on providing collaborative and constructivist learning to the distance classroom (Garrison & Archer, 2000). These technological advancements also allow for the possibility to create and sustain communities of learners at a distance (Garrison & Akyol, 2013). However, because students are not directly interacting in "real time" and lack visual cues such as body language and facial expressions, students can sometimes feel like they are not interacting with real people. Thus, instructors need to make a deliberate effort to foster a learning environment where individuals feel free to collaboratively engage in purposeful critical discourse and reflection to construct personal meaning and confirm mutual understanding (Garrison, 2011, p. 2). While the creation of a learning community is as important in a face-to-face classroom as it is in an online distance education classroom, the element of distance and the lack of direct human contact in the online distance education setting can make it more difficult for students to feel connected to other participants in the class (Hung & Chou, 2015; Symeonides & Childs, 2015). The intent of component 3 of the *Field Guide* is to alert the instructor to actions that may help mitigate the sense of artificial interaction and create a warm and inviting atmosphere for the online distance education student (See Summary Table in Appendix G).

Summary of analysis and researcher observations. In many cases, artifacts from a course serve multiple purposes that span multiple components in the Field Guide. One such artifact in this class is the welcome email sent out by the instructor prior to the beginning of the semester. Dr. Spencer made contact with students on January 8th, two days prior to the beginning of the semester and five days prior to the first synchronous meeting. This email welcomed students to the class and directed them to the course Blackboard LMS page. Once arriving on the course page, students were directed to an ice-breaker assignment in Module One that asked them to introduce themselves to the class. This prompt asked participants to share their names, their locations, and the grade-levels they teach or their positions within the school system (coach, principal, etc.). Some participants in the class were already familiar with one another as they were from the same schools or districts. Others had taken a previous class together in the EMAoL online program. Still others had never met in any capacity. The introduction activity served to provide background information about participants that they would be working closely with throughout the semester.

The researcher found reading through the introductions a good way of familiarizing himself with the demographic profile of the class. In many cases, participants shared more information than asked about themselves, giving further context to their current situations and motivations for taking the course. While the researcher found this valuable, it is unclear what value that this had for the participants in the class. While implied, participants were never asked to read other participant's introductory posts, nor were they asked to reply to the postings of others for this assignment. The lack of a follow-up exercise or requirement to respond to posts may have made it seem to students that they were speaking into a vacuum rather than directly to their classmates. Unsurprisingly, few conversations emerged from this activity and the assignment may have actually contributed to the very feelings of isolation that the assignment was designed to address.

In addition to encouraging introductions, the checklist for component 3 of the *Field Guide* suggests that the instructor provide constructive feedback to students early in the course. This is partially to give the participants a sense that "someone is listening". This early feedback also serves to reassure students that the work they are producing is on point. Dr. Spencer was able to provide this feedback in the first few weeks of the semester. In student's first content-based assignment (the "algebraic reasoning" journal assignment in Module 2), Dr. Spencer directly responded to 24 of the 27 students who submitted responses[4]. The deadline for the submission of this assignment was February 2nd. The instructor responded to posts in two rounds. She responded to approximately half of the participants on February 7th, and the other half on February 16th. This two week time window seems acceptable for an assignment of this type. While feedback on this assignment came back in a timely fashion, the instructor had difficulty maintaining this pace. While there was considerable overlap from comment to comment, Dr. Spencer took time to personalize many of the comments by using students names or mentioning something from their journal posts.

The overall sense of the researcher was that activities and interactions in the first two modules set a positive tone for the semester and encouraged the fostering of a respectful learning community. Some elements from component 3 of the *Field Guide* were not addressed and seemed like missed opportunities to further the goal of establishing the learning community. First, While Dr. Spencer successfully introduced students to the content of the course and their peers, she never really introduced herself. Even in the first synchronous online session Dr. Spencer gave a brief "hello" and went right into the first activity. It is probable that this was an unintentional oversight on her part. The fact that she

[4] Examples of Dr. Spencer's responses to students for the "algebraic reasoning" journal assignment can be found in Appendix G under component three of the *Field Guide* evidence table.

had previous face-to-face interaction with about a third of the participants in the class may have made it feel like the class already knew who she was. The second missed opportunity was providing a mechanism by which students would be encouraged to interact with one another around the introductions as discussed previously. Finally, while Dr. Spencer was able to keep up a proficient pace responding to student posts and providing feedback for assignments in the first few weeks of the course, her rate of response precipitously declined after module two. This decline in feedback becomes a common theme in relation to the success of the course and will be discussed further when addressing components 5 though 9 of the *Field Guide*.

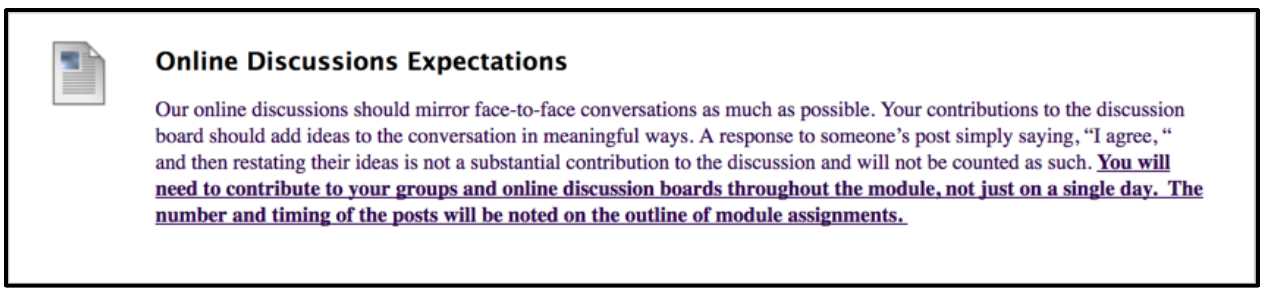
Component 4: Promote active learning (See Summary Table in Appendix G)**.** In the distance education classroom, it can be a challenge to design, promote and support activity-based learning. Active learning is a critical component of a constructivist approach that promotes "learning by doing"; an idea that focuses attention on the activity of learning rather than on the presentation of material through the one-way medium of lecture or some other form of "telling' that allows few opportunities for student participation (Moore, 2013a). The general consensus is that students learn more in an active learning environment than from the more passive approaches of information delivery. Despite positive learning outcomes, the mere implementation of active learning in the classroom is not a remedy for poor instructional technique. Instructors must carefully consider the learning goals and the context when matching a given activity to a learning environment.

There are two issues to consider for distance education. First, the spatial and temporal separation of students as well as the instructor adds an element to the learning context that is not present in the face-to-face classroom. As a result, some successful cooperative activities from the classroom may not be easily adaptable online. Second, students must be trained to understand what active and productive collaboration looks like in the online classroom. In a traditional classroom, students can focus all of their attention on a topic or activity for a finite period. In the online environment, where students may be logging on at different times from different locations, it is often more difficult to maintain a true interaction, especially when having discussions. Students who participate in online discussions early or late in the unit, or students that complete the required number of posts in one sitting, may not be fully engaged in the discussion and thus may only receive limited benefits from the conversation. Component 4 of the *Field Guide* serves to remind instructors that cooperative activities like discussion forums must be crafted to meet particular educational goals and that students must be reminded of what it means to be active in an online setting.

Summary of analysis and researcher observations. One strength of this course was the emphasis that was placed on active learning activities. It was evident to the researcher that emphasis was built into the overall structure of the course from the start. This general commitment to active learning was first evidenced in the course syllabus where 20% of the course grade was dedicated to class participation. The syllabus further elaborated on what expected in terms of class participation. These expectations included intellectual risk taking, making connections, thinking clearly on paper, contributing to the community, commitment to developing listening and speaking skills, and commitment to exploring new ways to think about teaching and learning mathematics (see syllabus Appendix B).

In order to accomplish these community objectives, Dr. Spencer had clearly stated overall expectations for asynchronous participation in group discussions (Figure 4.4).

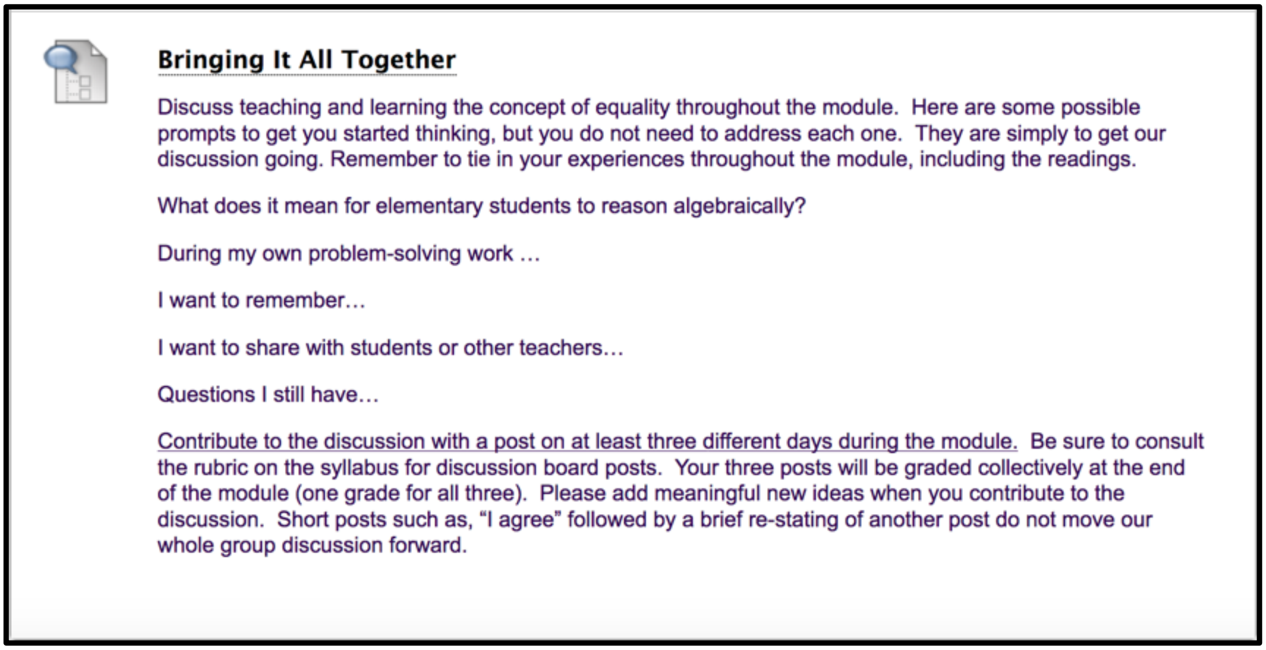
Generally speaking, participants in asynchronous discussions were expected to contribute to the assigned forum at least three different days across the duration of the module, with at least one post required within a few days of the opening of the module. Dr. Spencer explains that part of her rationale for the "three different day" requirement was to distribute participation over the length of the module instead of visiting the site 1 time in order to complete the assignment. Online discussion forums were a consistent feature on the landscape of this course occurring at least once in seven of the eight modules. The eighth module was designated for the final exam and completion of the course portfolio and thus did not contain a discussion forum.


Online Discussions Expectations

Our online discussions should mirror face-to-face conversations as much as possible. Your contributions to the discussion board should add ideas to the conversation in meaningful ways. A response to someone's post simply saying, "I agree, " and then restating their ideas is not a substantial contribution to the discussion and will not be counted as such. **You will need to contribute to your groups and online discussion boards throughout the module, not just on a single day. The number and timing of the posts will be noted on the outline of module assignments.**

Figure 4.4. Expectations for participation in online discussion forums. These expectations could be found by navigating to the "Start Here" tool on the front page of the Blackboard course site.

Modules typically contained between one and three asynchronous online discussions. Sessions two through seven contained an assignment titled "Bringing It All Together" (BIAT). The apparent purpose of the BIAT prompt was to provide a forum for participants in the class to connect the readings and activities prescribed over the course of the module to their classrooms and their students. Dr. Spencer left the prompt relatively open to the participants so that they might emphasize aspects of the module that were most important to them and their teaching practice. In addition to a general prompt to connect their posts to their experiences throughout the module, Dr. Spencer also provided some suggested prompts in order to assist students in getting started (Figure 4.5). These suggested prompts consisted of sentence starters that participants might elaborate on. The same sentence starters were used in many of the BIATs across modules.



Bringing It All Together

Discuss teaching and learning the concept of equality throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

What does it mean for elementary students to reason algebraically?

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

Figure 4.5. "The Bringing It All Together" (BIAT) prompt from Module 2. Prompts for the BIAT activities in modules three through seven were similar, with the only difference being the mathematical focus of each module.

In addition to the BIATs, which were designed to connect content to practice, Dr. Spencer frequently assigned participants to group problem-solving activities. These problem-solving forums consisted of between three and five students and were centered around high-demand tasks that mirrored the mathematical goal for the module. While the BIAT assignment was designed as a way for participants to interact with all members of the class, group problem-solving activities focused discussion with a small group of participants, thus making it easier to track individual thinking over the course of the conversation. The researcher found many of these group problem-solving forums a rich source of student thinking with regards to mathematical topics despite the challenges of sharing mathematical work digitally. One specific problem-solving forum will be analyzed and unpacked later in this chapter.

Not all of the learning activities in this course were constrained to asynchronous discussion sessions. As stated earlier, one strength of the course was the holding of bi-weekly synchronous online sessions that utilized Saba Meeting teleconferencing software. In these synchronous online sessions, the instructor had the opportunity to interact with class participants directly in real time. Frequently, Dr. Spencer would divide the whole group into three to six smaller groups and send them into "breakout rooms" provided by the Saba Meeting software. Here, in breakout rooms, students might be directed to discuss a variety of topics chosen by the instructor. These breakout sessions typically lasted less than 20 minutes and were followed up by a debrief and sharing of small group results with the entire class.

These synchronous online sessions were important to the overall structure of the class for a variety of reasons. First, the Saba Meeting software allowed students the ability to get auditory, real-time feedback from the instructor as well as their peers. This had the potential effect of reducing the feelings of isolation that can often accompany an online class. Second, the synchronous sessions served as a platform for providing immediacy to topics important to students. In the live sessions, students could ask "housekeeping" questions about assignments and due dates and get an immediate answer from the instructor rather than waiting for an answer in an email exchange. Participants could also ask "in the moment" questions of the instructor or their peers. In other words, questions which students had difficulty expressing in writing could be asked in these sessions verbally and clarified immediately when necessary. Lastly, the instructor used these synchronous sessions as a "bridge" between asynchronous topics. Synchronous sessions often served as a space where one topic could be wrapped up as well as a springboard into the next topic. More about the phenomena that Dr. Spencer referred to as "bridging" will be presented in Chapter 5 of this dissertation.

Overall, the combination of synchronous and asynchronous activities made it possible for students to work collaboratively exploring important mathematical concepts as well as share teaching experiences that gave their coursework meaning and value. Unlike many online courses where students are simply asked to read and respond, this course utilized a variety of technologies that allowed students to co-construct knowledge by engaging in rich mathematical problems and providing feedback regarding these problems with their peers and the class. These interactions truly made the participants the focus of learning in the course and was ultimately one of the richest features of the course. Two of these learning episodes will be discussed in a later part of this chapter.

Component 5: Monitor student progress and encourage lagging students (See Summary Table in Appendix G)**.** As mentioned previously (see Component 2), one of the greatest challenges facing online education is a high degree of attrition in many online courses. Students in a face-to-face environment are in some sense encouraged to participate by the mere act of attending class, making it perhaps a little easier for the instructor to track progress and identify students at risk for falling behind. In online classes, the instructor must be proactive in their approach to supporting students as many student actions in this setting are invisible to the teacher. Moore, (1989) notes that instructors will vary in the amount of support they offer to online learners based on the prior experience level of the student, personality of the instructor, and philosophical stance of the learning institution.

Nonetheless, learner-instructor dialogue is important to the successful completion of the course by the student. Component 5 of the *Field Guide* addresses the public and private actions that an instructor might take in order to keep all participants on task and working productively throughout the semester.

Summary of analysis and researcher observations. Many of the elements of component 5 are private interactions that are difficult to evaluate as present or not present. Generally, students enrolled in the EMAoL license program tend to be highly motivated individuals with a high degree of autonomy as students. Despite motivations and intentions, the job of teaching can often be overwhelming as demands on their time are numerous and often extend beyond the school day (Hoerr, 2005). In private conversations and planning meetings with the researcher, it was evident that Dr. Spencer was aware of her students' special needs and flexible when considering timelines for assignments. Dr. Spencer seemed to be aware of absences from synchronous online sessions ahead of time and accommodated for these absences in her planning for the session. In interviews with participants, the researcher was given numerous examples where Dr. Spencer responded quickly to email requests for assistance or clarifications indicating that she was in close contact with students. Besides anecdotal reports from students, there is little evidence of the degree to which Dr. Spencer was monitoring student progress in the course. That is not to say that she did not track students, but rather that evidence of this type of interaction was not collected by the researcher.

While Dr. Spencer seemed to communicate well one-on-one with students that requested her assistance, outside of the synchronous online sessions she rarely addressed the class as a whole. Mid-module updates on the progress of assignments, or questions that may have arisen between modules were not frequently given. In an interview, Chelsea, K-5 mathematics coach in the class, noted the lack of mid-module contact in an interview conducted about a month into the class. She stated:

> I really liked how Dr. Anonymous last semester, would give us in-between emails, between our Wednesday sessions as to, "Hi, this is just a reminder, blah-blah-blah-blah-blah-blah-blah-blah. Hope everybody's doing great." Maybe contact once a week would be nice, just, and then I would respond. I found myself responding to her and asking questions several times. I don't think it has to be a lot. I just think once a week, through an email, especially on the off week of the course of the online section, would be helpful, or just to touch basis with everybody.

In this quote, Chelsea is referring to the first course in the EMAoL sequence where the instructor had sent out occasional updates between synchronous sessions.

Dr. Spencer did do a number of things to assist students with keeping track of their work and ensuring they didn’t fall behind. For example, the last part of each synchronous session was dedicated to previewing the upcoming module and allowing for questions. When previewing the module, Dr. Spencer would conduct a screen-share of her desktop as she discussed the upcoming module. This screen-share allowed students to follow along with the instructor as she navigated through the class Blackboard page and previewed assignments. Another feature of the class that students found helpful was the assignment checklist (Figure 4.6) provided by the instructor. These checklists consisted of two columns. The first column displayed the due date for the assignment while the second column listed provides the short name of the activity to be completed. This checklist was prominently featured at the top of each new module page so that students could easily return to reference as they progressed through the module.

Module 2 Overview

Attached Files: Module 2.doc (40.5 KB)

Due Date	Task
February 1	Individual Journal
Post at least three different days by February 1. First post due by Monday, January 18.	Team Problem-Solving – The coin problem
February 1	Teaching Equality Blog
February 1	Questioning and Student Work Analysis 1 (Turn in on Blackboard and have it available to use/discuss during our SabaMeeting session).
On your own (No separate assignment is due)	Algebra Potpourri – Equality (Blendspace)
Post at least three different days by February 1. First post due by Monday, January 18.	Bringing It All Together Blog

Figure 4.6. Screen snapshot example of the overview checklist provided at the top of each page of the new module.

Component 6: Assess students' messages in online discussions (See Summary Table in Appendix G)**.** Online discussion forums hold great promise for collaborative knowledge construction in online environments (Kent, Laslo, & Rafaeli, 2016). Asynchronous discussions are a key factor in in developing learning communities and supporting peer interaction online (Gao, Zhang, & Franklin, 2013; Yang, Yeh, & Wong, 2010). Despite their apparent importance to the online community, there are conflicting opinions on what participation in these forums looks like and how these interactions should be assessed. Variables like the length of the post, frequency of postings, and quality of posts have all been considered as essential elements in assessing student participation in discussion forums (Hrastinski, 2008b). Component 6 of the *Field Guide* attempts to capitalize on research that has been done around student interaction in online discussion forums and focuses on the quality, quantity and frequency of student postings in discussions.

Summary of analysis and researcher observations. Dr. Spencer entered into the semester with guidelines and rubrics for online asynchronous discussions. These guidelines generally addressed the quality and frequency of posts to discussion topics. This was evident in the rubrics and instructions that she provided to students in the course syllabus as well as in discussion prompts. Each student response was assigned a grade between zero and three. Because there were multiple types of discussion forums that were assigned, two different rubrics were provided to the students; one for responses to the BIAT assignments and a second for responses to the mathematical problem-solving forums. The quality of posts was the major emphasis of both rubrics. For the BIAT assignments, Dr. Spencer placed an emphasis on responses that integrated the reading assignments within student opinions, observations, and past experiences. For the mathematical problem-solving assignments, the emphasis was on explanations of mathematical reasoning. In addition to the quality element, students were asked to make three posts to each forum. A twist to this requirement was that these posts needed to be submitted on different days, thus encouraging active participation across multiple visits, rather than simply completing all three responses at once.

Students in the course did an admirable job in both the mathematical problem-solving forums as well as the BIAT responses. As a result, the vast majority of students received twos and threes for these assignments. It is unclear, however, how closely the instructor stuck to the rubric while grading. While reviewing grades, the researcher found multiple instances where the student participated the requisite three times, but not on three different days, and still received full credit for the assignment. The requirement for "three different day" rule was prominently featured as a requirement of each module's BIAT and according to the syllabus, students that did not fulfil this requirement were to receive a one-point penalty. The researcher could not find one case across the BIAT or the mathematical problem-solving assignments where students lost points by virtue of the "three different day" rule.

In general, the instructor for this class provided students with adequate rubrics to produce a reasonable level of interaction among participants. Students were advised on the quantity of post that they should provide along with minimal suggestions on appropriate levels with regards to quality posts. The instructor even made some attempt to distribute participation over the entirety of the module with her "three different day" rule, although it is unclear what effect this policy had on the way that students participated in discussions. An element of component 6 of the *Field Guide* for which there was no evidence was the extent to which the instructor considered providing students with additional points for extra posts over and above the requirements in an attempt to encourage additional student postings.

Component 7: Sustain students' motivation (See Summary Table in Appendix G)**.** Richard Clark, Director of the Center for Cognitive Technology at the University of Southern California's Rossier School of Education has long been interested in the impact of media on learning. Clark has been outspoken in his contention that media, itself does not influence learning. It is educational methods rather than the use of technology that is the main influence on learning (Clark, 1983, 1994, 2012). Some argue that the increased technical and interpersonal complexity of the distance learning environment may in fact actually decrease student motivation (West et al., 2013). Therefore, it is important that instructors offer well-designed and supported instruction to assist in the maintenance of student motivation. Without these supports students are more likely to feel the "distance" part of distance education (Bolliger, Supanakorn, & Boggs, 2010). In order to combat this feeling of distance, Garrison, Anderson & Archer (Garrison et al., 2000) propose course supports that develop student's social presence within a learning community. This involves fostering their online identity as "real people". In the absence of visual cues (eye contact, gestures, etc.) a conscious effort must be made on the part of the teacher to mark the instructor's presence in the classroom. Component 7 of the *Field Guide* discusses ways in which instructors might design and facilitate activities that engage students and maintain their motivation throughout the course.

Summary of analysis and researcher observations. Unlike other components of the *Field Guide*, component 7 consists of instructor actions that must be replicated numerous times throughout the semester. Other instructor actions related to previous components are one-time actions or actions that the instructor can phase out doing as participants become more familiar with the online format. In essence, this component addresses suggestions for what might be labeled *course facilitation*. Of course, instructors will have their own styles when presenting and implementing course material, but they must also be aware of the uniqueness of the online environment. In the same way that students must adapt to learn differently, instructors must accept that they may need to facilitate differently than they are used to in face-to-face classes. This may mean participating more frequently and more deliberately in student led activities online.

The evidence of instructor actions related to this component for the algebra class was mixed. Generally speaking, most of the recommended instructor actions were displayed at some point in the semester. However, while many of the actions were evident, they were not sustained throughout the semester, especially after about the halfway point of the class.
As discussed previously, one of the highlights of the course was the emphasis on task design. Dr. Spencer did a nice job selecting engaging tasks and thought-provoking discussion prompts. This was undoubtedly a result of the time and effort she put into the planning of the course prior to the beginning of the semester. The instructor showed diligence in the early

planning of the course as evidenced in the numerous meetings with the planning team for the course. These meetings began as soon as early November, a full two months before the beginning of the semester. A half dozen meetings and numerous communications via email and text were had over the holiday season and into the New Year before the beginning of class on the 11^{th} of January.

Component 8: Provide feedback and support (See Summary Table in Appendix G**).** One of the foremost thoughts in a student's mind is how their work will be assessed and what sorts of support will be provided. Naidu (2013) suggests that instructors in distance education need to be particularly vigilant in providing feedback in a fair, consistent and timely manner. This can be difficult and is certainly time consuming for the instructor of an online course. In addition to fair and consistent, feedback might be personalized for students. It is not always feasible to provide a unique response to each and every student for each and every assignment. Sometimes a "stock" response might be used as cut-and-paste feedback to a number of students. Using this technique too frequently, however, increases the transactional distance felt by students in the class. Some effort to personalize responses should be made for each and every activity.

Fortunately, there are things the instructor can do to provide personalized feedback without a lot of additional effort on their part. Peer assessment can be a valuable learning experience for both the participants involved and saves the instructor from having to play the role of editor early in the writing process. A second advantage is that the final product, which the instructor does grade, often arrives in a much more polished form having already been reviewed by multiple students. An unintended consequence of this peer editing procedure is that it begins to build student awareness to the fact that they are in fact interacting with other humans rather than simply a faceless entity on the other side of a computer.

Perhaps the most important aspect of feedback is that it gets to students in a timely fashion. What this means in an online course is quite a bit different than in traditional classrooms. In the traditional classroom feedback is being given immediately and constantly. The teacher can use a quick glance, a nod of approval, or simple eye contact to indicate to the student that they are listening and that their message has been received. Also, an instructor can listen to multiple group conversations at the same time to assess the overall understanding of the class and address student work in “real time”. Effective classroom monitoring can catch misunderstandings about the task, diagnose misconceptions about the material, and redirect off-task behaviors as they happen. In the virtual classroom, the same problems are likely to arise. While the instructor cannot make themselves available 24/7, it is clear that the instructor must be often-present in the virtual class space to provide feedback; perhaps more often than they realize. Component 8 of the field guide offers suggestions with regards to the monitoring of student work during the semester and the offering of feedback in ways that allow students to sense the instructor’s presence in the class and engagement in student work.

Summary of analysis and researcher observations. One of the most glaring concerns about the course was the lack of consistent feedback to students by the instructor. Over the course of the semester, the 27 students in the class produced nearly 1,500 pieces of work that needed to be graded and returned. Including the final exam, 495 assignments, or nearly one third of the total assignments for the course were not graded and returned to students until after May 1. Given that much of the work turned in by students did not get graded until after the completion of the semester, many students found it difficult to get a sense for how well they were doing in the course. It also made it difficult for them to make adjustments to improve their work as the semester unfolded. This lack of feedback detracted from the value of many of the semester's otherwise well-designed assignments. This was particularly true for the recurring Questioning and Student Work Analysis (QSWA) assignments that were a central feature of the course and constituted a quarter of students' overall grade.

The QSWA assignments were tasks designed to link the mathematical goals of the course (algebraic reasoning) with the course pedagogical focus on questioning that elicits students' thinking. This assignment was broken into three parts with each part spanning two modules (approximately 4 weeks). Each part of the QSWA was designed to allow participants to reflect on and analyze their own questioning techniques in relation to various lenses provided by the instructor. In the first half of each QSWA assignment, participants were asked to select an algebraic task to use with students. Using the task as the context, participants were asked to complete a planning guide intended to be used in the implementation of the teaching of the lesson with their students. This planning document was focused on designing questions that that would elicit student thinking in the execution of the lesson. Teachers then audio or video recorded their lessons and compiled a list of questions that were actually asked during implementation. Participants also collected artifacts from their students related to the lesson. Course participants were asked to bring this material to a synchronous session where they shared data on their lesson with other participants in a small group synchronous discussion. In part two of the assignment, participants used their own experience teaching the lesson, readings from the course, lecture material from the synchronous session, and feedback from their peers to analyze and revise their student questioning grid. They were then asked to write a reflection of the process to turn into the instructor.

The assignment included many elements that made it a potentially rewarding online learning experience. The QSWA was an active learning experience for the students that allowed them to directly relate their work in class to their work in the classroom and reflect on their experiences. It also provided opportunities for participants to share stories of their practice with one another thus encouraging a sense of connectedness to others in the class. Lastly, the QSWA provided an opportunity for students to capitalize on multiple lines of feedback to improve their questioning skills in their teaching practice. Ultimately, however, the researcher fears that the QSWA assignment merely became just another assignment to turn in as it became unclear the amount of attention it was going to be given by the instructor.

The lack of feedback on the QSWA assignments was disconcerting to some as they navigated the semester. In an interview in the last weeks of the class, Julie, a second-grade teacher, expressed some of this concern in the following statement regarding her thoughts as she prepared to turn in the third and final QSWA assignment:

> When we were getting ready to do our last... I guess it was the student analysis [QSWA three]. I was kind of frustrated because I was like “I've got no feedback on the first two and I'm not even really sure if I'm doing this right”! And then, lo and behold of course, she [the instructor] must have heard me in my mind because I had some feedback and some grades posted that afternoon. So that really helps… It's [QSWA three] the same format and it is kind of clear-cut, but then at the same time it's like “Does she want less detail? Does she want more? Am I going deep enough?” and things like that. So there was kind of that hesitation like “wing and a prayer”. “Hope this is what she's looking for!” But I’m still not really sure, and here we go... round three... I mean it's not as if I thought I was "failing" or really doing poorly, but just to have that feedback is kind of nice and reassuring. Just to know, as you move forward, like okay I'm on the right track.

Julie has confidence that she is not doing poorly in the class, but at the same time seems to have no sense for how her assignments are being received by the instructor until late in the semester.

Component 9: Encourage students to regulate their own learning (See Summary Table in Appendix G). Not all of the responsibility for student learning falls on the instructor. Students must take responsibility for this also. As an online course develops and students become more familiar with the format of online learning, it is appropriate for the instructor to begin to cede a modicum of control to the learner. This relinquishing of responsibility not only benefits the student, as it allows them to tailor the educational experience to meet their needs and desires, but also the instructor, as it decreases the amount of time and energy that it takes to moderate the experience. Academic independence is not a skill that comes easy to many students. Instead it must be fostered through deliberate actions on the part of the instructor early in the semester. Component 9 of the field guide suggests actions that foster academic independence amongst students in an online setting.

Summary of analysis and researcher observations. Actions that result from this section of the field guide are often difficult to identify as they are often subtle actions that gradually culminate in independent students. The Bringing It All Together forum prompts and Questioning and Student Work Analysis assignments provided a basis for the majority of the evidence collected in this section. Bringing It All Together prompts, in particular, allowed students to take conversations in a multitude of directions (for an example of a BIAT prompt see Figure 4.5). As the example shows, these forum prompts allowed students to share their own opinions and classroom anecdotes that helped deepen other's knowledge and understanding of various topics. The BIAT prompts also allowed students to voice their own opinions of what they felt was most important from the module as well as identify particulars from the readings and assignments that they did not understand.

This course had a number of built-in opportunities for self-regulated learning. This was evident in both the asynchronous sessions as well as the synchronous online sessions. Activities like the Team Problem Solving and the BIATs were almost entirely driven by students themselves. While Dr. Spencer provided the mathematical task in the case of Team Problem Solving, and loose guidelines for sharing in the BIATs, students were given the flexibility to take these assignments in just about any direction they wished. This flexibility bled over into the synchronous sessions where initial small group discussions in the bridge sessions were focused on ideas that came up in the asynchronous activities.

In addition to debriefing asynchronous activities, synchronous small group sessions also allowed students to make their opinions heard outside of online message boards. This is not insignificant as Dr. Spencer viewed synchronous class time as extremely valuable "real estate". She views synchronous meeting sessions as space for live student-student interaction rather than a forum for her to lecture and present. In an early planning session, Dr. Spencer noted her philosophy on small groups as follows:

> It is the only time you get to interact with them and have real discussion versus just written discussion forums. So, that's the most important thing that goes on there. And so, I tried to figure out – how am I going to use that time. Doing some sort of lecture – or, information presentation – just doesn't feel like a good use of that time. Because it felt like that was something that could be done in a screen shot or a screen cast or a PowerPoint. It's just not a good use of that real estate. So, I started out using mostly small group discussions – some whole group – to give them time to process during that time.

Dr. Spencer remained true to this philosophy throughout the semester dedicating approximately one third of synchronous class time to small groups discussions. While there were general goals and guidelines for these small group discussions, the prompts were generally broad enough to give students significant leeway to take up ideas that they want to process through further with colleagues.

Peer review was also an important feature of this class. In some ways, the Team Problem Solving activities served as peer reviews of mathematical ideas. The most apparent example of the built-in peer review mechanism of the course was in the Questioning and Student Work Analysis (QSWA) assignments. Specifics of these activities are discussed later in this section but one relevant aspect of these activities is mentioned here. While student expectations for this assignment may have been vague and feedback may not have been prompt, students were never left on their own to complete these important assignments. Built into these assignments were benchmarks that required students to bring data back to the synchronous sessions to share. There was also a peer review process for these assignments whereby students exchanged writing with one another to make comments. This peer review process served two important goals. First, it gave students the opportunity to see at least one other exemplar of student work before turning in their assignment, thereby allowing them to determine for themselves if they were on the right track or not. Second, it offered the potential to improve the overall writing of the final product.

Component 10: Deal with conflicts promptly (See Summary Table in Appendix G)**.** Just as in a traditional classroom, conflict is inevitable. The instructor must be prepared ahead of time for inevitable conflict, but also be aware that this sort of interaction may be a symbol of growth and group cohesion amongst participants. While it is best to monitor potentially problematic situations closely, it is sometimes best to let students work through their problems on their own as it may be through negotiation of this conflict that students learn. Component 10 of the *Field Guide* addresses instructor actions to prevent and respond to conflict online.

Summary of analysis and researcher observations. Much of the suggestions made by this checklist were either not applicable to this class or elements to which the researcher was not privy. The researcher was not aware of any incidents of students posting inappropriately, nor was he aware of any actions that violated academic integrity. That being said, there were two recommendations that this checklist provides that were not actualized in the course. First, a simple way to avoid many situations is to provide guidelines for web etiquette. Most students will know a lot of this material, but they should be reminded that some posting behaviors that are ok for social media are not necessarily so for an online academic community. A brief guide to these differences might be placed in a prominent place on the course LMS page for easy access. Second, a simple peer evaluation function that assesses how the group is functioning should be frequently employed, especially if the groups are kept the same throughout the semester.

Part B: In What Ways is the Use of Effective Teaching Practices in Mathematics Education that Support Student Learning Evident in the Course Planning and Implementation?

In Part A, elements of the best practices for online courses generally were addressed. It is clear that there are a number of non-negotiable characteristics that must be met in an online course to mitigate the effects of transactional distance on students' experience. This section of the dissertation looks beyond the non-negotiables core elements of any online class and attempts to investigate practices specific to the context of a mathematics education experience.

Construction of High-Leverage Practices in Field Guide Version 1.0.

Research on the best practices in in online teaching supported Part A of the initial *Field Guide* and guided recommendations regarding the setup of courses and preparation of students for engaging with online classes in general. Part B of the initial *Field Guide* that address online mathematics teaching practices was not as well-developed, due mainly to the paucity of research in conducting online distanced education mathematics content courses. Part B of the *Field Guide* serves to remind online mathematics educators of important research-informed, high leverage teaching practices related to mathematics teaching and learning. More importantly, it's intent is to pinpoint important teacher actions that might need to be approached differently in online distance education settings and provide recommendations on ways to modify these actions.

Field Guide version 1.0 draws on four, high-leverage teaching practices (see Chapter 2). The first of these practices serves to guide mathematics educators on the selection, design and use of mathematical tasks. This includes the identifying features of high cognitive demand tasks and the factors that can increase or decrease cognitive demand as a task is implemented (Henningsen & Stein, 1997). Second, this version of the *Field Guide* proposes suggestions for orchestrating classroom interactions through the development of a math talk learning community (Hufferd-Ackles et al., 2004) as well as recommendations for encouraging productive math talk in the classroom (Chapin et al., 2009). Third, the *Field Guide* highlights teacher actions that foster class discussions in mathematics classrooms (Smith & Stein, 2011). Finally, guidance on the planning of online distance education (synchronous and asynchronous) teaching events using the Thinking Through A Lesson Protocol (Smith, Bill, & Hughes, 2008) is included.

These four areas of focus, while helpful as reminders, are skills that most mathematics educators would most likely be aware of when thinking about their practice. In an interview with Dr. Spencer before the start of the semester, she confirmed that she had been aware of most of the information addressed in this part of the *Field Guide*. The challenge for her was to take these guidelines/principles and adapt them to distance education. As an example, Dr. Spencer noted maintaining the cognitive demand of a task. Henningsen and Stein (1997) point out that high cognitive demand mathematical tasks can often decrease in demand depending on how the task is set up by the instructor, the particulars of the classroom setting, and how students engage with the task. These factors clearly change depending on whether the class is taught face-to-face or in an online distance education environment. Dr. Spencer felt that the *Field Guide* might better outline these differences.

One change being made to future iterations of the *Field Guide* is a shift from these four somewhat disjoint frameworks to one overarching framework built on the National Council of Teachers of Mathematics updated Guiding Principles for School Mathematics found in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). The core mathematics teaching practices (Figure 4.7) found in this publication integrate the important aspects from the initial version of the *Field Guide* into a more unified framework that includes examples of actions that teachers and students are performing in productive mathematics face-to-face classrooms.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Figure 4.7. Description of the eight core mathematics teaching practices identified by the NCTM (2014)

In the *Principles to Actions*, a teaching framework (Figure 4.8) is characterized that describes actions in the context of a traditional face-to-face classroom. The *Field Guide* revisions will highlight these eight actions and explore their use for online distance education interactions. Further discussion of this will be provided in Chapter 5 of this dissertation.

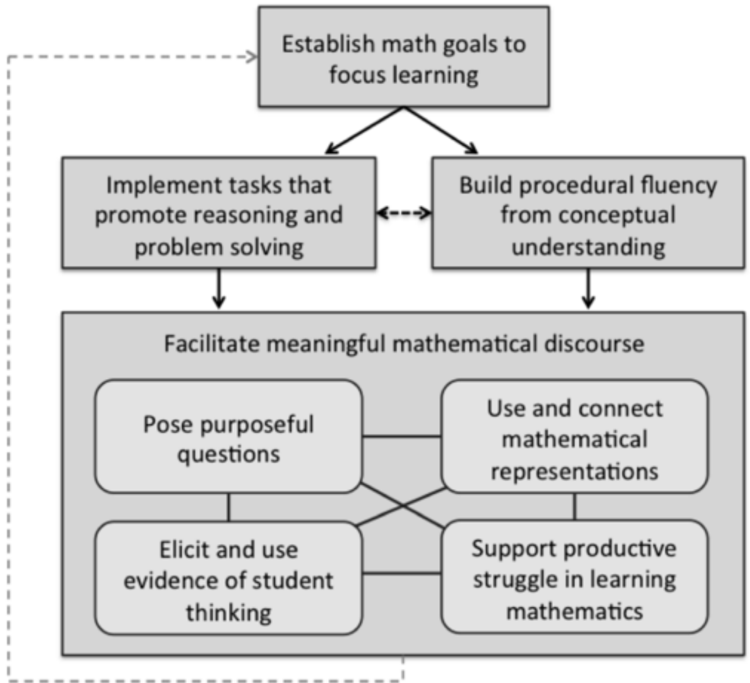


Figure 4.8. Teaching framework for mathematics highlighting relationships between the eight effective teaching practices

Description of selected course activities. This section looks at two course activities in the context of the teaching practices outlined in the original *Field Guide* that was reviewed by the course instructor, Dr. Spencer, prior to the beginning of course planning. These two activities, one asynchronous and one synchronous, exemplify the types of activities assigned in this class. An asynchronous activity that is referred to as "The Coin Problem" was a small group problem solving activity that required participants of the group to co-construct and explain a solution to a high cognitive demand mathematical task. The second activity, a small group synchronous discussion, asked participants to make conjectures regarding the nature of operations with odd and even numbers, and to justify their assertions. Both activities draw from events occurring in the first half of the semester.

Case 1 - asynchronous discussion problem: the coin problem.

> You have nine blocks. Eight of them weigh the same. The ninth one is lighter than the others. The difference is only perceptible using a balance scale and only the blocks themselves can be weighed. Is it possible to figure out which block is lighter with only two weighings on the scale? How? (Copied verbatim from the Team Problem Solving section of the Blackboard course website)

This section explores what the instructor referred to as "The Coin Problem". While the context given to the students includes blocks, the title indicates that it was most likely an adapted version of the counterfeit coin problem, a popular logic puzzle whose origin is unknown but seems to have gained popularity in the United States in the mid 1940's (Smith, 1947; Goldstein, 1945; Schell & Durnham, 1945). The presentation of the problem varies from source to source, and the original problem generally is similar to the following:

> You have eight similar coins and a balance beam. At most, one coin is counterfeit and hence underweight. How can you determine if there is an underweight coin, and if so, which one, using the balance only twice (Schell & Durnham, 1945 p. 397).

Variables in the presentation of the problem include the number of coins in the original batch, the number of weighings that can be made, and the certainty of a counterfeit coin is actually in the batch. In this version of the task, participants were presented with 9 hypothetical blocks. Of the nine blocks, one block is assumed to be counterfeit. Participants were asked to identify the counterfeit block in two weighings. Conversations among group members were conducted asynchronously to solve the problem.

Task conditions. The instructor set up groups using the "group blog" structure in the Blackboard LMS. Group blog structure allows participants to add blog entries and comment on blog entries made by other group members. It also allows participants of the whole class to review the work of each group. However, participants not assigned to a given group cannot make contributions to another group's blog. The group blog structure in Blackboard differs from individual blogs and course blogs that control the privacy of the original post and the ability for other class participants to respond. There was a range of group sizes for this problem from four and to six participants.

The “Module 2” tab on the Blackboard homepage contained the activity prompt and a link to the blogspace (see Figure 4.9).

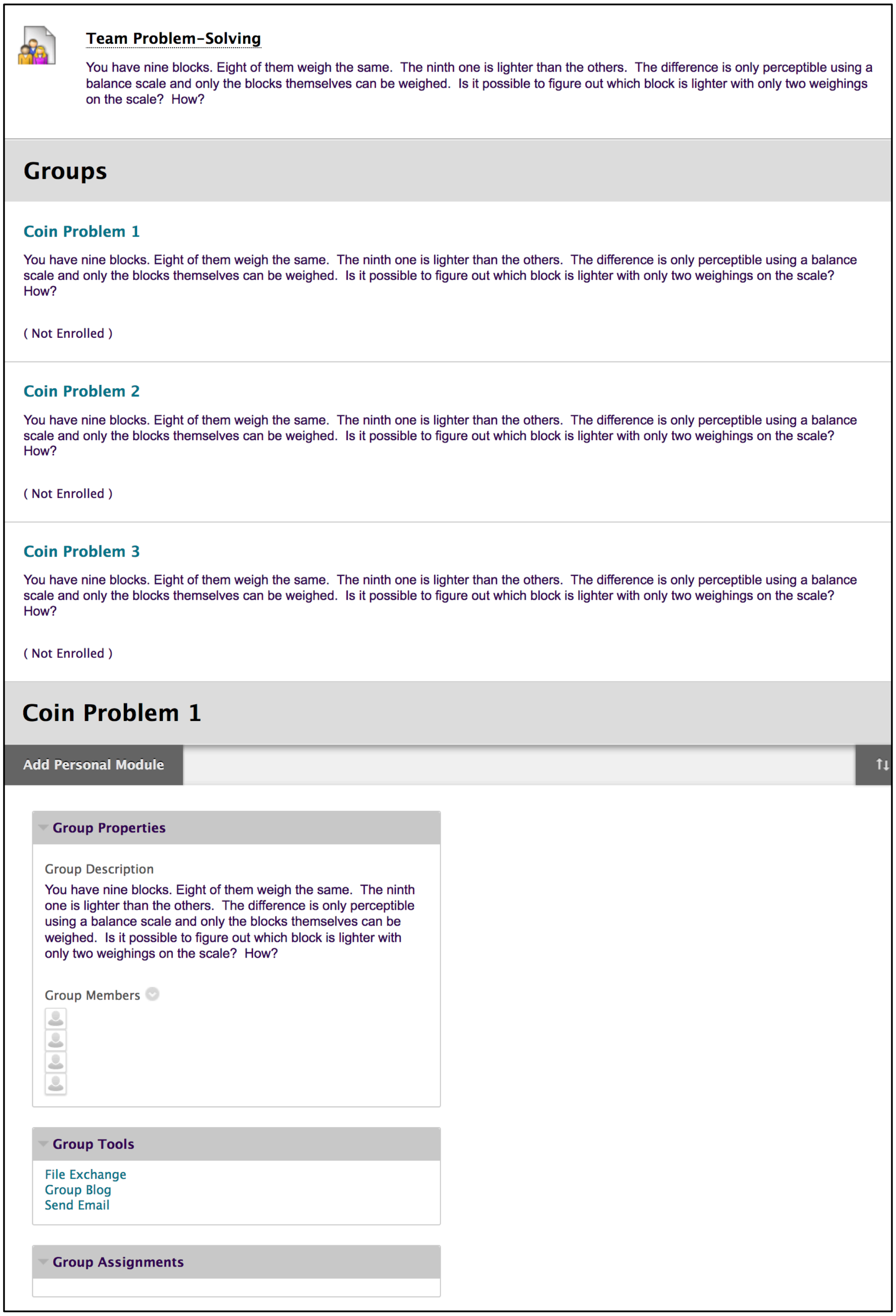


Figure 4.9. Screenshots of pages encountered by students as they entered the group blog to complete the Team Problem Solving activity. The top screenshot shows the statement of the problem on the Module 2 homepage. The middle screenshot shows the screen where students select their assigned groups. The bottom screenshot shows what students saw when they entered the group blog.

Once in the group blogspace, a “group properties” dropdown menu detailed both a re-statement of the problem and a list of the members of the group. On the same page, a “group tools” dropdown menu was available which listed the various tools through which students could interact asynchronously. These tools included a “file exchange” where group members could upload documents (.docx, .pdf, .xlsx, etc.), allowing all group members access to their work. A second tool available to the group was a “group blog” blogspace. This space allowed students to create conversation threads and potentially build off of each other’s work. This space offered a “send email” tool that allowed participants to send messages to select other members of the group. A lasting record of the “group blog” and “file exchange” was created in Blackboard; however, the researcher had no way to track private email messages between students.

Task expectation and evaluation. A chart on the main page for Module 2 outlined the timeframe for group interaction (see Figure 4.10).

Module 2 – Jan 13-27

Module 2 Overview

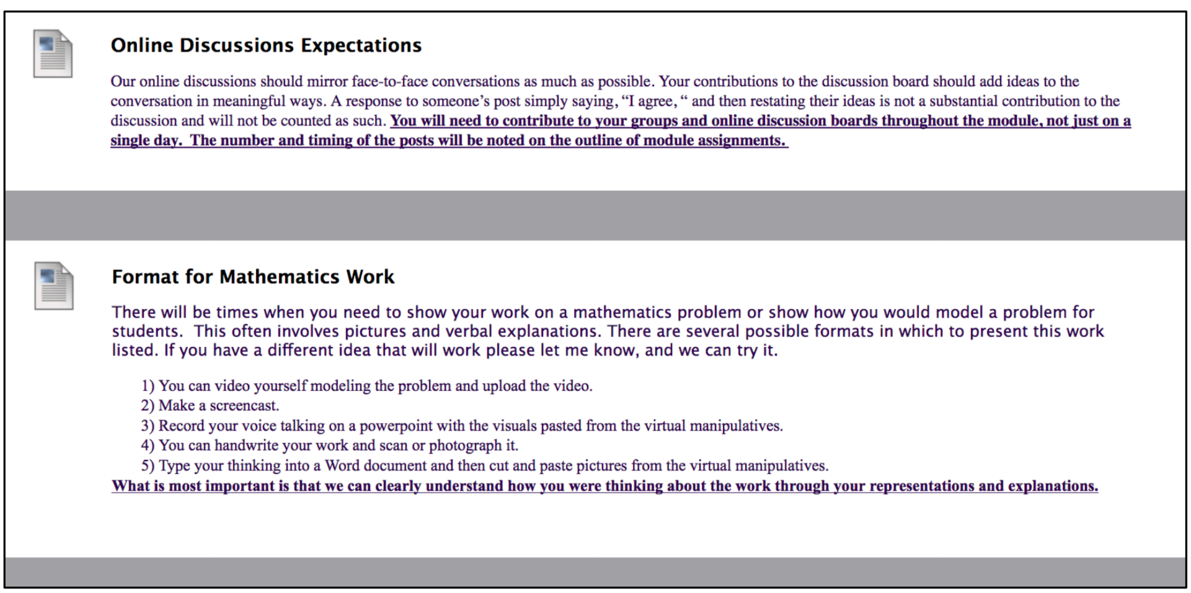
Attached Files: Module 2.doc (40.5 KB)

Due Date	Task
February 1	Individual Journal
Post at least three different days by February 1. First post due by Monday, January 18.	Team Problem-Solving – The coin problem
February 1	Teaching Equality Blog
February 1	Questioning and Student Work Analysis 1 (Turn in on Blackboard and have it available to use/discuss during our SabaMeeting session).
On your own (No separate assignment is due)	Algebra Potpourri – Equality (Blendspace)
Post at least three different days by February 1. First post due by Monday, January 18.	Bringing It All Together Blog

Figure 4.10. Screenshot of the timeline chart provided by Dr. Spencer at the beginning of Module 2.

The module opened after the conclusion of the first synchronous class on January 13th. In the synchronous session, Dr. Spencer noted that not all students had been assigned a group (due to participants adding and dropping the course) but would be later that evening. Dr. Spencer assigned all students to groups by the following morning. Participants were asked to engage with the group on three different days before February 1st, the date of the second synchronous class meeting. They were also instructed that their first post or "initial thoughts" should be made by January 18th. Additional posts could be made anytime between the students' first posts and the February 1st deadline so long as all three posts were made on different days.

Expectations for the assignment were not posted on either the Module 2 front page or The Coin Problem group blogspace. They were, however provided in a number of other locations on the Blackboard LMS course page (see figure 4.11). The "Getting Started" tab located on the left and side of the page above the list of modules contained a number of helpful instructions.



Online Discussions Expectations

Our online discussions should mirror face-to-face conversations as much as possible. Your contributions to the discussion board should add ideas to the conversation in meaningful ways. A response to someone's post simply saying, "I agree, " and then restating their ideas is not a substantial contribution to the discussion and will not be counted as such. **You will need to contribute to your groups and online discussion boards throughout the module, not just on a single day. The number and timing of the posts will be noted on the outline of module assignments.**

Format for Mathematics Work

There will be times when you need to show your work on a mathematics problem or show how you would model a problem for students. This often involves pictures and verbal explanations. There are several possible formats in which to present this work listed. If you have a different idea that will work please let me know, and we can try it.

1) You can video yourself modeling the problem and upload the video.
2) Make a screencast.
3) Record your voice talking on a powerpoint with the visuals pasted from the virtual manipulatives.
4) You can handwrite your work and scan or photograph it.
5) Type your thinking into a Word document and then cut and paste pictures from the virtual manipulatives.

What is most important is that we can clearly understand how you were thinking about the work through your representations and explanations.

Figure 4.11. Student response expectations for the group problem solving tasks. Both of these statements were found on the "Getting Started" page of the Blackboard LMS course site.

This page contained a paragraph on “online discussion expectations”. Directions found in this paragraph stated that “online discussions should mirror face-to-face discussions as much as possible” and that asynchronous posts should “add ideas to the conversation in meaningful ways”. Directions in this paragraph also reiterate the need to contribute to the discussion on multiple days as opposed to writing all posts on a single day.

Since problem solving groups were intended to discuss mathematical problems, the section regarding “Format for Mathematics Work” (see Figure 4.16) was also relevant to Group Problem Solving responses. This section acknowledged the difficulty of presenting and sharing mathematical thoughts online and offered a number of a number of suggestions. These suggestions included:

- Video yourself modeling the problem and upload the video;
- Make a screencast;
- Record your voice talking on a PowerPoint with visuals posted from the virtual manipulatives;
- Handwrite your work and scan or photograph it; or
- Type your thinking into a word document and cut and paste pictures from the virtual manipulatives.

Of these suggestions, students most frequently typed responses directly into Blackboard, used the cut and paste function from a word document, or took photos and scans of handwritten work to upload to the blog as an attachment. A rubric located in the syllabus (see Appendix B) provided basic guidelines to the participants regarding how they would be graded for this task. The instructor assigned each individual a score between 0 and 3 depending on the quality of their posts. The syllabus also stipulated that one point (total) would be taken from the grade if the posting requirements were not met. This included the timing of posts as well as the number of overall posts.

The Coin Problem was assigned to groups following the conclusion of the first asynchronous meeting on January 13. Students were required to make their initial posts by January 18 and complete their responses by February 1. The table below (Figure 4.12) shows the timing of group participation throughout this assignment.

COIN PROBLEM JANUARY 13 - FEBRUARY 1	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	1	LATE
GROUP 2				X X X	X X X X X	X O X X X X X		X		X			X	X X X	X		X		X X X	X	
GROUP 2			X			O X X	X X X	X X		X		X X X	X X X	X X	X X	X		X X X X O	X X	X	
GROUP 3					X	O O X X X	X	O O					X X X X X						X	X X X X	
GROUP 4				X	X X X X	O X X X		O	X				X	X				X	X		X X
GROUP 5				X	X	O O X X X X	X	O	X	X	X X X			X				O X X	X X X X	X	

Figure 4.12. Coin problem participation by group for January 13 to February 1. The Xss represents responses by a group’s participants. The Os represent comments made by the instructor. Marks within the green bar represent comments made during the core of the conversation. Marks in the orange represent comments that were possibly made too early or too late to contribute to the rest of the group’s understanding of the problem. Marks in the red were made after the February 1st deadline.

Of the 26 students participating in the discussions, all but three participants received a score of 3. Two participants received a score of 2, and one participant received a score of 1. One student did not participate in the thread due to a family emergency. This student was omitted from any analysis of the problem. While the task was due on February 1, the instructor did not score or provide student feedback to individual students until February 16th. This means that students were unaware if they had met the expectations of the professor with regards to Team Problem Solving assignment until mid-way through the fourth module (3rd Team Problem Solving activity). As stated previously, the majority of students received full credit for the assignment. Two of the three students that did not receive full credit lost points due to missing posts.

In addition to a numerical grade, many students were given short, personalized feedback from the instructor. The following figure is an example of general feedback given to many of the participants (Figure 4.13).

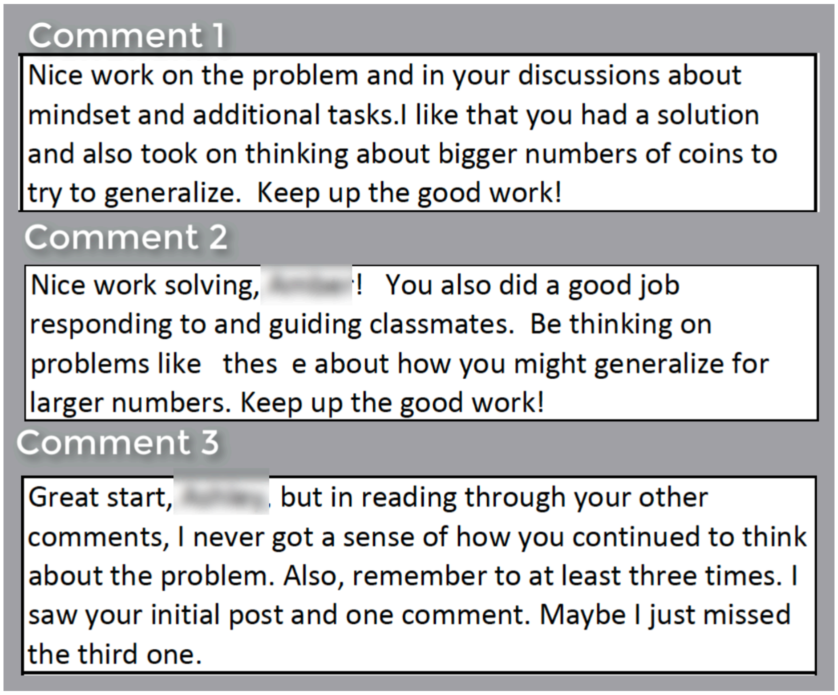


Figure 4.13. Examples of comments given by the instructor in reaction to student responses to the coin problem. Comment 1 is an example of general student feedback to a student receiving full credit for a response. Comment 2 shows an example of more specific student feedback for a full credit response. Comment 3 is an example of specific feedback to a student who did not receive full credit.

Seventeen of the participants got one of three different standardized responses like the top comment in Figure 4.19. The other students that received responses that were personalized to the students' work. The middle comment in the figure shows an example of personalized feedback given to a participant that received full credit. The bottom example in the figure shows personalized feedback to a participant that received reduced credit for their response.

Cognitive level of the task through implementation. As mentioned earlier, the coin task has a rich history as a problem for discussion, and numerous individuals have written on the possible variants and solutions to this problem. If a group were to complete the task with minimal discussion, there are an endless number of more challenging extension questions that might be generated from the original context. For instance, the number of coins could be increased to 12, or any a larger multiple of three. In this scenario, students might recognize that it would not be possible to discover the counterfeit coin in merely two weighings as nine is the maximum number of coins for which one counterfeit coin can be identified. A follow up question to this realization might be to determine the number of weighings necessary to positively identify a fake amongst 12 coins, 15 coins, 18 coins, or 3n coins. If students can generalize for any number of the form $3n$ (where n is whole number), they might then determine a generalization for any whole number n of coins.

Applying the cognitive demand matrix suggested by Henningsen and Stein (1997), the coin task could be considered a high-level cognitive demand task that possesses many of the characteristics of "doing mathematics" (see Figure 2.7). The thinking required to complete the task is non-algorithmic in nature, and it requires students to think deeply about mathematical concepts like equivalence. Students have a variety of ways of thinking about the problem and can participate on a continuum of skills from direct modeling of the problem to formal logic statements.

Implementation of the task in the asynchronous online setting differed from how the task might be delivered in a face-to-face classroom. While the set-up portions of the implementation model were not directly affected by the medium of instruction, implementation was heavily impacted. Taking into account the technologies available, the instructor made the decision to run this task as an asynchronous, small-group discussion rather than a small or large group synchronous discussion during the bi-weekly Saba Meeting session. This allowed students a number of advantages. First, it gave participants opportunities to try the problem on their own before engaging in discussion. Instead of one person taking the lead and constructing a discussion, all participants needed to reflect on what was a challenging mathematical task. Second, running the discussion asynchronously allowed those with weaker knowledge of the content the ability to "lurk" within the blogspace and carefully construct a response rather than simply having to react within the conversation. This had the potential to increase the mathematical confidence of students that might be reluctant to share. Lastly, the blogspace created a written record of the evolution of student thinking giving all students a tangible product to reference later if needed.

Given the nature of asynchronous discussions, the instructor made a number of decisions regarding the setup of the problem which influenced how the participants engaged in their work. First, the staggered requirement for posting dates encouraged students to engage with the problem multiple times over the course of the two-week period. Further, the added requirement of an early posting date, in this case 5 days after the assigning of the task, compelled students to begin their work earlier in the two-week period than they might otherwise have done. Lastly, actualizing the task in an asynchronous online setting provides the opportunity for the instructor to monitor the conversation closer than would be possible either face-to-face or in a synchronous online environment.

As in any classroom, there are factors that influence how the task unfolds. In this case, some effort was made to support students in taking up the task. First, the instructor provided a rubric that was intended to provide students with a framework for asynchronous interaction. While, in retrospect, this rubric may have been too general for students to use as a self-evaluation tool, it did provide a place for them to start. This was the first asynchronous group problem solving activity in this class; however, nearly all of the students had previously engaged in asynchronous mathematical discussions in a previous course in the EMAoL program, albeit in a different LMS. However, not all of these factors were positive.

The instructor of the course showed her presence infrequently in the conversation. Furthermore, the task was never summarized in a "wrap-up" post or during the synchronous session to address the students' learning outcomes.

Planning, initiating and facilitating mathematical discussion. Dr. Spencer's planning for the coin problem was deliberate and connected to the module's focus on the mathematical concept of equality. Her plans for the coin problem had been constructed before the beginning of the semester for the second module, and it was to be the first Team Problem Solving assignment of the semester. Students were added to Team Problem Solving groups for this first activity randomly. As students were assigned to the course through the Blackboard system, they were assigned to groups of five. The first five students enrolled became Group One, the second five became Group Two, and so on. Because there were 27 total students enrolled, and Dr. Spencer did not want to have six students in problem solving groups, the instructor made minor adjustments to the constitution of the groups to even out the numbers before the start of the task.

One problem that the instructor had experienced in the past was that students often default to their knowledge of formulas and solving equations when initially introduced to an 'equality task'. Dr. Spencer hoped to avoid that by assigning this particular task. In a planning meeting approximately one week before the start of the semester, Dr. spencer gave the following rationale for the inclusion of the coin problem in Module 2:

> I picked this one [task] deliberately because I think it takes away their tendency to go straight for numbers and they really do start thinking about relationships. I played with some other problems over the years but they went straight to equations and trying to figure it out, and this one seems to help them avoid that. (January 4th Planning Meeting)

As stated previously, the instructor planned for students to participate in this assignment asynchronously on three different days throughout the course of module two.

In the same planning meeting, Dr. Spencer spoke of her plans to monitor the groups. Figure 4.12 shows Dr. Spencer's participation across groups for the coin problem. Figure 4.14 shows an example of the type of interaction Dr. Spencer had with the groups in the coin problem.

Monday, January 18, 2016 11:38:49 AM
[redacted]

Hi All,

I'm enjoying reading your thinking about this problem! Several of you have proposed using sets of 3 to determine in two weighings. If there is a consensus about this approach, then how would it apply if there were more coins? Say 12 coins instead of nine. What is the least number of weighings you can do? Does your sets of 3 idea still work?

[redacted]

Figure 4.14. Example of instructor interaction in the Coin Problem.

In this interaction, Dr. Spencer is responding to the group thread after nine exchanges between the group members. In these prior exchanges, the group shared ideas, revised thoughts and came to a consensus of how to solve the problem. Dr. Spencer's extension question prompted the group to continue their conversation and consider a number of other initial coins. It also pushed the group to attempt to generalize about the minimum number of weighings needed to identify a counterfeit coin in any number of coins, although they never quite got to a formal understanding of this.

In addition to extending the problem, Dr. Spencer was also able to use the forum space to redirect student work (See Figure 4.15). After a shaky start to the problem, Dr. Spencer restated the task to clarify the groups' misunderstanding of "two weighings". While some members of the group were on the right track and had solved the initial problem, other members of the group had interpreted the problem differently and were heading off task. Note that in addition to redirecting the wayward conversation, Dr. Spencer also makes a comment similar to the one made to Group One that extends the problem for those that are ready.

Monday, January 18, 2016 1:30:05 PM

Hi All,

I'm enjoying reading your thinking about this problem so far. A couple of thoughts. Some of you read "two weighings" as you can put items on the balance two times. The first interpretation was the intention of the problem.

For those of you that proposed using sets of 3 to determine in two weighings, how would it apply if there were more coins? Say 12 coins instead of nine. What is the least number of weighings you can do? Does your sets of 3 idea still work?

Figure 4.15. Example of instructor participation in Group Two redirecting the focus of the problem.

In all, "The Coin Problem" forum seemed to achieve a high number of quality interactions between participants. Having said that, more research is needed to thoroughly evaluate what is meant by quality interaction. In most cases, participants added to group forums the requisite number of times. While it seemed like there was distributed participation amongst group members, the researcher wonders about individual students' posting patterns as the timing of posts may have some effect on value added to the conversation. One way, however, of mitigating the effect of distributed participation (the spacing of posts and participation across a longer period of time) is to supplement asynchronous course material with regular synchronous sessions. This next heading describes aspects of one such synchronous episode from this course.

Case 2 - synchronous discussion problem: conjecturing about operations with odds and even numbers. This discussion was conducted as the third discussion session during a synchronous session. Typically, the third discussion of synchronous sessions served as a launching activity for the upcoming module. This small group session centered on the idea of conjecturing. Conjecturing relates to the course Big Idea of "Generalized Arithmetic Through the Use of Properties". For the purposes of this discussion, a conjecture is defined as "a general mathematical statement that is either true or false on a specified domain" (Blanton, 2008). Operations related to adding and multiplying odd and even numbers was used as a context to foster participants making conjectures and justifying their statements (see Figure 4.16).

Room 1 Prompt: What happens when you add an odd number and an even number? Is the result even or odd? Is this always true? How do you know?

Room 2 Prompt: What happens when you add three odd numbers? Four odd numbers? Is the result even or odd? Is this always true? How do you know?

Room 3 prompt (unrecorded): Suppose I told you that I was going to add a lot of odd numbers together but didn't tell you how many. What could you say about whether my result would be odd or even? How do you know your conjecture is true? Will it always work?

Room 4 Prompt (unrecorded): What happens when you multiply an odd number and an even number? Is the result even or odd? Is this always true? How do you know?

Room 5 Prompt (Same as the unrecorded group 3 prompt): Suppose I told you that I was going to add a lot of odd numbers together but didn't tell you how many. What could you say about whether my result would be odd or even? How do you know your conjecture is true? Will it always work?

Room 6 Prompt: What happens when you multiply three odd numbers? Four odd numbers? Is the result even or odd? Is this always true? How do you know?

Figure 4.16. Prompts for Small Group Discussion 3 (SG3) held as part of Synchronous Session 3. One way the instructor set up groups was to assign specific prompts, as here. Notice that each prompt is different. Four prompts focus on adding odd/even numbers and two prompts focus on multiplying even/odd numbers. There are connections across prompts. It is possible for each group to contribute to a larger framework of working with odd/even numbers with this arrangement of prompts.

Each group was given a slightly different prompt and asked to record their results on the virtual whiteboard in their breakout room. Groups were instructed that they would be sharing conjectures and justifications when the whole group re-convened. The small group discussion session itself lasted approximately 25 minutes and the whole group debriefing, approximately another 25 minutes.

Task Conditions. Synchronous class discussions were held using Saba Meeting telecommunication software. Saba Meeting allows instructors to present information to a number of students at one time in an online setting. Using this software, the instructor can share computer applications (PowerPoint, etc.), share their entire desktop to the group, take students through a tour of a website, or simply use a virtual whiteboard to exchange ideas.

One drawback to the Saba Meeting system is that only four microphones can be active at any one time. In recognition of the microphone limitation, the large group was divided into groups of four and sent to breakout rooms so that all members could participate in their discussion at the same time (i.e., all members could have their microphones turned on[5]).

The 25 students present for this class (two absences) were separated into a six small groups with between 3 and 5 members in a group. They were moved to separate breakout rooms to discuss their respective prompts. Breakout rooms were virtually identical to the large group setting but the smaller number of participants in each room allowed for increased participant participation in the discussion. One member of each group was assigned as a moderator. The moderator in a discussion room was allowed certain privileges such as advancing slides and saving whiteboard work. Otherwise the moderator had the same role as other group members. Each of the members of the group had access to a virtual whiteboard tools (Figure 4.17). These tools allow students to take a number of different actions including highlighting text, creating shapes, adding text, and creating freehand drawings.

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[5] Access and use of microphones is an important condition when conducting synchronous sessions using this type of meeting software. If there were five people in a group, in order for each member to access the conversation, they would deliberately have to turn off their microphones when not talking. Since students are used to just ‘hanging out’ and talking in face-to-face settings, this kind of control is problematic; they just forget to turn on/off mics. So we choose group size based on having all microphones left on during these sessions. In the whole class, the instructor can make sure microphones are off when not in use.

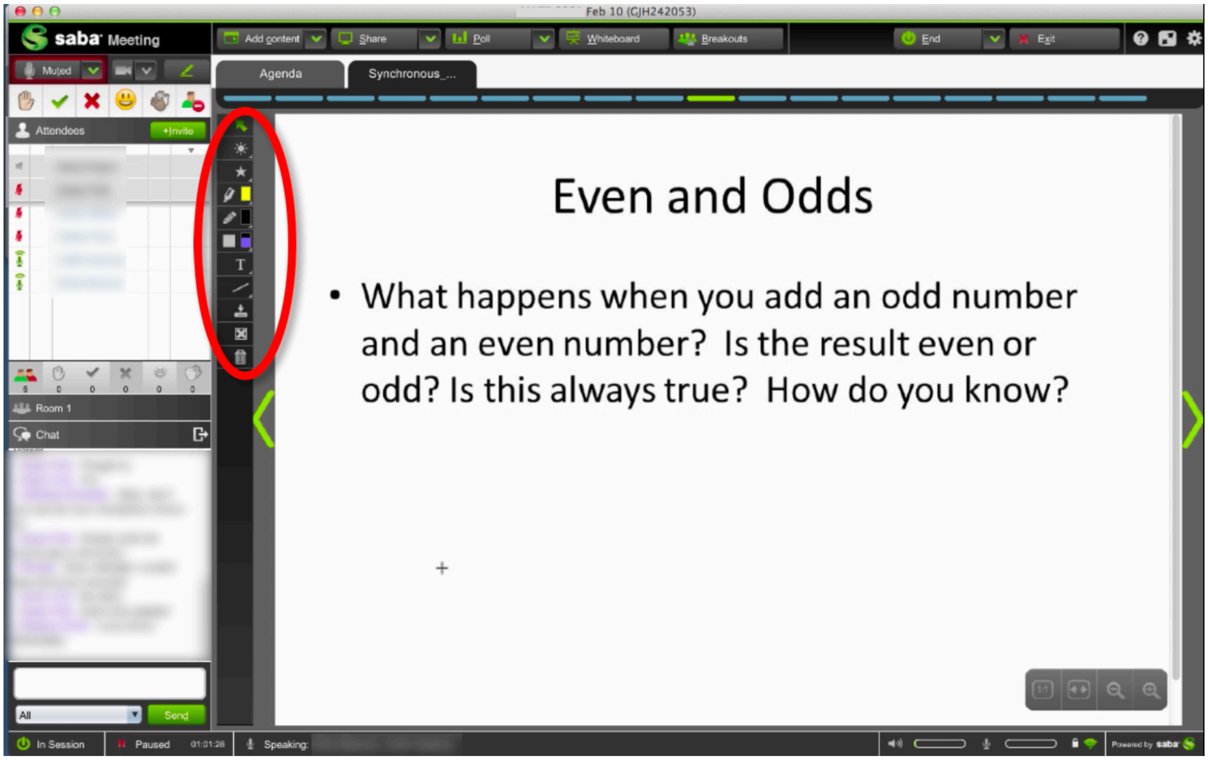


Figure 4.17. Example of the Saba Meeting screen during small group breakout sessions. The area on the left circled in red shows the whiteboard toolbar.

In addition to supporting discussion through the use of whiteboard tools, Saba Meeting also allows for communication via a chat function (Figure 4.18). The chat box allows members of the group to send messages to the group without interrupting the speaker. This is also true in the main whole class setting.

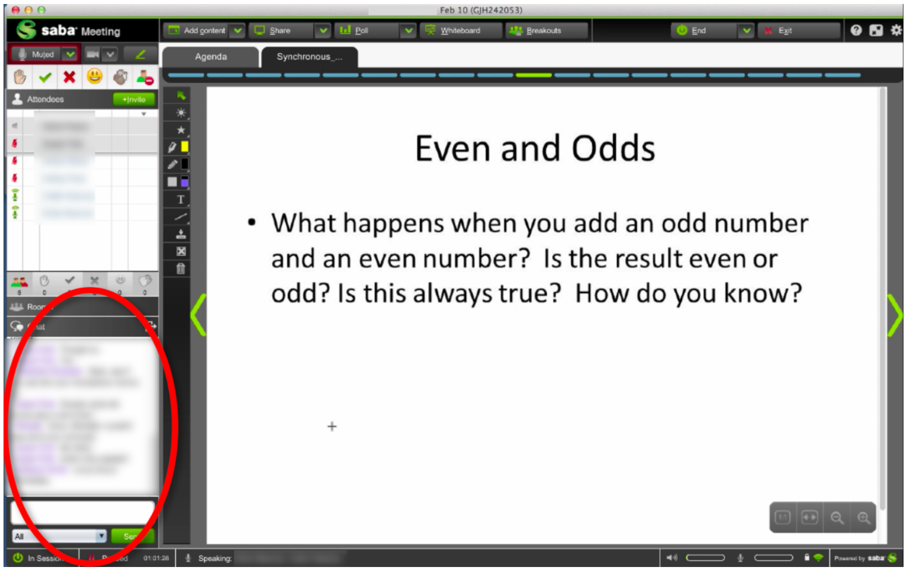


Figure 4.18. Example of the Saba Meeting interface during small group breakout sessions. The area on the left circled in red is the location of the chat box.

Task expectations and evaluation. Unlike the coin task which had rubrics regarding participation in Group Problem Solving activities, synchronous small group discussions in Saba Meeting had few guidelines. Grading for this assignment likely fell under the category of participation outlined in the syllabus (see Appendix B), however, no formal grade was recorded in the gradebook for this exercise.

Cognitive level of the task through implementation. Like the Coin Problem, this synchronous discussion, which revolved around the properties of odd and even numbers, can be thought of as a high cognitive demand mathematical task. Elements of the prompt across the groups require participants in the discussion to think deeply about the concept of odd and even numbers and explore relationships among them. By expanding the task to investigate any number of combinations of odd and even numbers, participants are asked to wrestle with the idea of possible constraints to their conjectures.

Participants in these discussions had a fair amount of knowledge of odd and even numbers. All participants could identify odds and evens and generate multiple examples of each. They also often knew the “rules” for addition and multiplication as they relate to odd and even numbers in that they could predict the result of a simple combination of any two numbers. When pushed beyond operating with only two numbers, confidence within the group began to waiver; however, all of the recorded groups quickly came to a consensus regarding their conjecture about odd and even numbers in relation to their prompts.

For many groups, this is where the intellectual challenge began. The groups’ first justification attempts typically tried to demonstrate their conjecture through the presentation of multiple examples. Another common way to explain was through “teacher talk”, or how they might introduce it to their students. The following is a portion of the start of the conversation in group one in which Maddie relays an anecdote from her classroom that attempts to explain why the sum of an odd number and an even number is always odd.

Mandy: Okay, so, my initial thoughts are that when you add an odd number to an even number, it's always gonna be odd, because an odd number is an even number plus one. And so, if you add two even numbers together, it's always gonna be even, but then you have to plus one to make it odd. So, I'm thinking that the result is always odd.

Maddie: I agree, I actually did this conjecture with my kids two weeks ago. And what one of my kids said that I think helped to really have a lot of other people understand was that they said, "Oh, it's just like pairs of shoes". So if you are even number, then everything is paired up. It's like a pair of shoes. But if it's an odd number, then there's, like, one shoe that's left over. And if you add an even to an odd number, and the even number, all the shoes are already paired up, so there's not, there's nothing you can do with that. Like you said, the plus one. There's nothing you can do with that extra shoe.

Claire: Yeah, I like that explanation. That's interesting. I agree with both of you saying that "it's gonna be an odd number always". But I think that shoe example is really interesting. I like that.

Maddie: I know this isn't what our conjecture is about, but in that same example, that lets kids say, "oh, well if I have an odd plus an odd, then that's always gonna be an even, because that odd shoe out is always gonna have a partner, if it's added to another odd number".

In this exchange, it appears that Maddie is drawing on her classroom experiences to aid her own knowledge of odd and even numbers. Conversations regarding conjecture justification were initiated in much the same way. When discussion the result of adding numerous odd numbers, Chelsea had the following explanation.

> Grace: So how do we know besides this, the fact that we've tried it numerous times? Or is that a good enough reason? Well, we do know two odd numbers always make an even number. Is that right? So it would make sense that four, which is just double on two, will always make an even number.
>
> Chelsea: And I actually went back to my little first graders, and I drew circles, and I buddied them up, so everybody had a partner when I did three odd numbers. But there is always one guy left out, but when I did the four odd numbers, everybody had a partner, so there was nobody left out. I don't know about using that to help or not.
>
> Okay, let me piggyback on it, okay? So, when you have an odd number, there's always going to be one left out. One odd man. So on three, there's a set of two, and then one odd. Then five, there's two sets of two and one odd. So if you're adding an even set of numbers, you're always going to be able to have a partner for that odd man out. But if you're adding an odd set of numbers, then you're going to always have one left out. I don't know. I don't know about that. Odd man out, there you go.

In the previous examples, both Maddie and Chelsea are engaged in maintaining the level of cognitive demand required by the task.

While these anecdotal explanations got the groups off to a good start, they were still insufficient to justify all cases. In Maddie's group a discussion emerged regarding the constraints of their justification. One group member suggested that the justification didn't work for negative numbers. Discussion ensued regarding whether negative numbers could be classified as odd or even. The conversation slowed until Dr. Spencer made a comment to the room.

Dr. Spencer: … I wanted to just point you back, … earlier when you were talking about an odd number being even plus one, and I said, "What is the definition of even and odd?". I think that will help you both with that and with your negative question. So, or negative number question, it's not a negative question.

Claire: Okay, so even numbers are divisible by two. Right? Is that what we're going with for the definition?

Maddie: I would agree.

Mandy: Well, if that's the case, then negative numbers can be because a negative two can be divided by two.

Claire: Right, I think negative numbers can be even and odd. And then, so when you are kind of looking at it without the shoe, I don't know, I'm thinking of doubling a number and then adding or subtracting one. If you have even groups and you combine them. If you have two numbers that are divisible by two, and you combine them they're still gonna be divisible by two. But if you have one that is and one that isn't, it's not.

Maddie: Yeah and I kind of have a, I don't know if this is a good example, it's kind of a crazy example. But if you think of the negative numbers like, what if you imagine how much money someone owed you and then for some reason you said and they had to pay you in 2 dollar bills? And if it was an even sum and an odd sum, and you said "if it was an odd amount then you wouldn't be able to pay in 2 dollar bills and it was an even amount they owed you, you would be able to pay in 2 dollar bills'. Then the idea of matching and pairing because you could make groups of two I think could still work. Do you guys have any thoughts about that?

Claire: I agree with you I guess we should probably put something on the white board about negative numbers?

In this exchange, it is unclear if the group would have continued to push on the idea of negative numbers had Dr. Spencer not intervened. Had the instructor not been in the room at that exact moment, the issue may never have been taken up.

Overall, all four of the recorded groups maintained levels of cognitive demand required by the initial task. At the end of the small group discussion, all groups were prepared to share their conjectures and justifications with the class. It appears, however, that the groups who dug deepest into the task were the ones in which the instructor was present and pushed on particular topics. This comes as no surprise as this is true in face-to-face classroom situations as well. In the online classroom, however, it is more difficult for the instructor to monitor all conversations at once. While the idea of negatives was raised and resolved in one group, the instructor was not present in other groups to push on the issue. Looking across the task, however, the goal of the lesson was for participants to practice making conjectures. The concept of odd and even numbers was initially intended to provide context for the investigation. Towards this end, it seems that most groups were successful in being able to produce a conjecture. As Dr. Spencer facilitated discussions in the small groups, she was able to begin to get participants to offer rudimentary justifications for their conjectures.

Planning, initiating and facilitating mathematical discussion. This group discussion that centered on making and testing conjectures about odd and even numbers was the third discussion of the evening of the third synchronous session. The synchronous session was seen by the instructor as a “bridge” connecting one module to another.

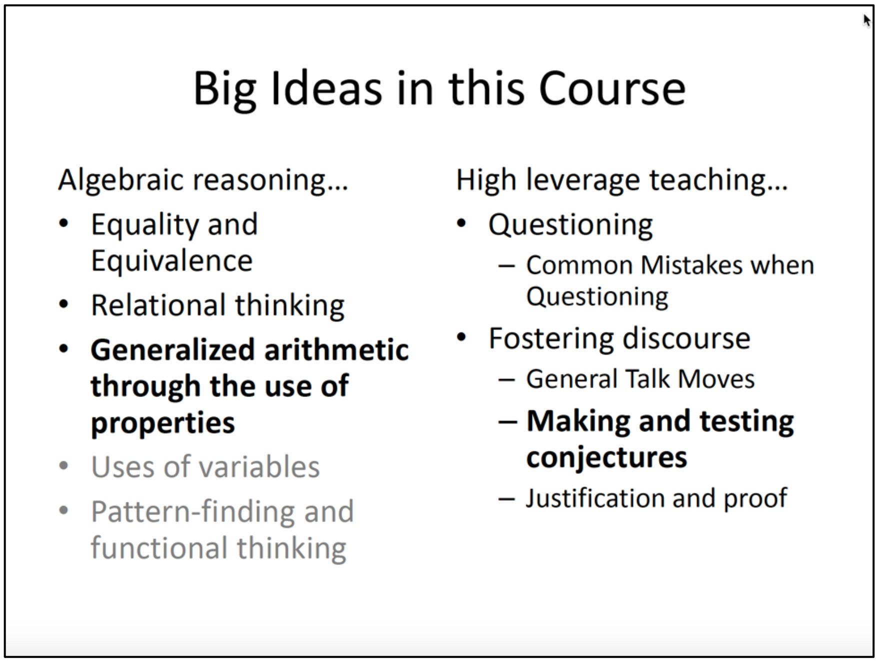


Figure 4.19. Big Ideas for the course. Bold type face indicated the topics of focus for the third synchronous online session.

In early planning discussions, Dr. Spencer noted that she wanted to use the synchronous sessions to build continuity form module to module. She attempted to sequence mathematical topics to overlap to some degree so that as one idea was "fading out", another would "fade in". The instructor designed this third synchronous online session to serve as a bridge in between both mathematical and pedagogical topics in the course. Mathematically, the focus of this session was on the transition between relational thinking and generalized arithmetic through the use of properties (Figure 4.19). In terms of the high leverage teaching practices, the third synchronous session served as a transition from general classroom questioning strategies to more specific questioning strategies designed for making, testing, and justifying mathematical conjectures.

Both of the earlier discussions in the third synchronous session provided the foundation for the third small group discussion for the evening. The first small group discussion attempted to clear up lingering questions leftover form Module 2. Participants were presented with a slide containing statements and questions made by peers over the course of the module (Figure 4.20).

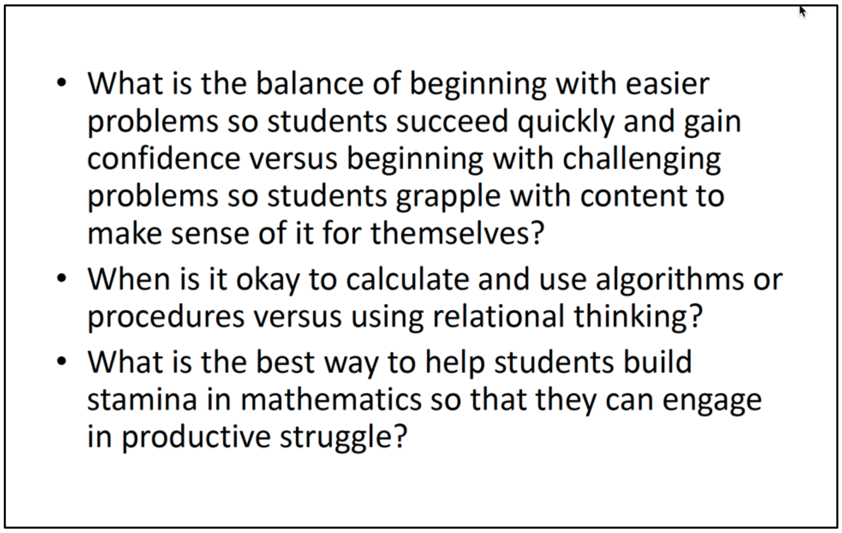


Figure 4.20. Points of discussion for the first small group breakout discussion of the third synchronous session (February 10, 2016).

Groups convened for approximately 15 minutes to discuss these peer generated ideas. The discussion served as a wrap-up focused on relational thinking and primed participant to begin to think about generalized arithmetic.

After wrapping up discussion around Module 2, Dr. Spencer held a 15-minute math talk centered on the multiplication problem 25 x 16. Class participants were given some time to think about the problem and then volunteers shared their answer and strategy with the whole group. A number of strategies were elicited from participants and recorded on the Saba Meeting virtual whiteboard. At that point, the whole class was sent back into breakout rooms to have discussions about the mathematical properties evident in student solutions.

This small group discussion set the stage for the third small group discussion about conjectures. While the second small group session had students conjecturing about the fundamental properties related to numbers and operations, the third small group breakout session had them exploring properties within a specific subset of numbers, namely odd and even numbers. This activity fit into the trajectory of the class in two ways. First, it built upon the fundamental properties of numbers that was the focus of the discussion in small group two and would be the topic under consideration for the upcoming module. Second, it allowed participants of the class to transition from thinking about the high leverage teaching skill of questioning in general to more purposeful mathematical questioning.

As stated earlier, the goal of the discussion was for students to practice making and justifying conjectures. Discussion around this topic was rich mathematically and may have gone in a number of directions that were not anticipated by the instructor. Dr. Spencer did a nice job refocusing the whole group on the mathematics important to the lesson (making conjectures and providing justifications) in the post-breakout debrief session that followed. This debrief lasted approximately 12 minutes and gave groups the opportunity to restate their group's conclusions in front of the class. Dr. Spencer sequenced these discussions nicely, starting with the group that worked on specific cases (what happens when you add two odd/even numbers) to the groups that had more abstract prompts (What would the results be if I asked you to add 'a lot' of even/odd numbers). Figure 4.21 shows a screenshot of the work produced by group three in their small group.

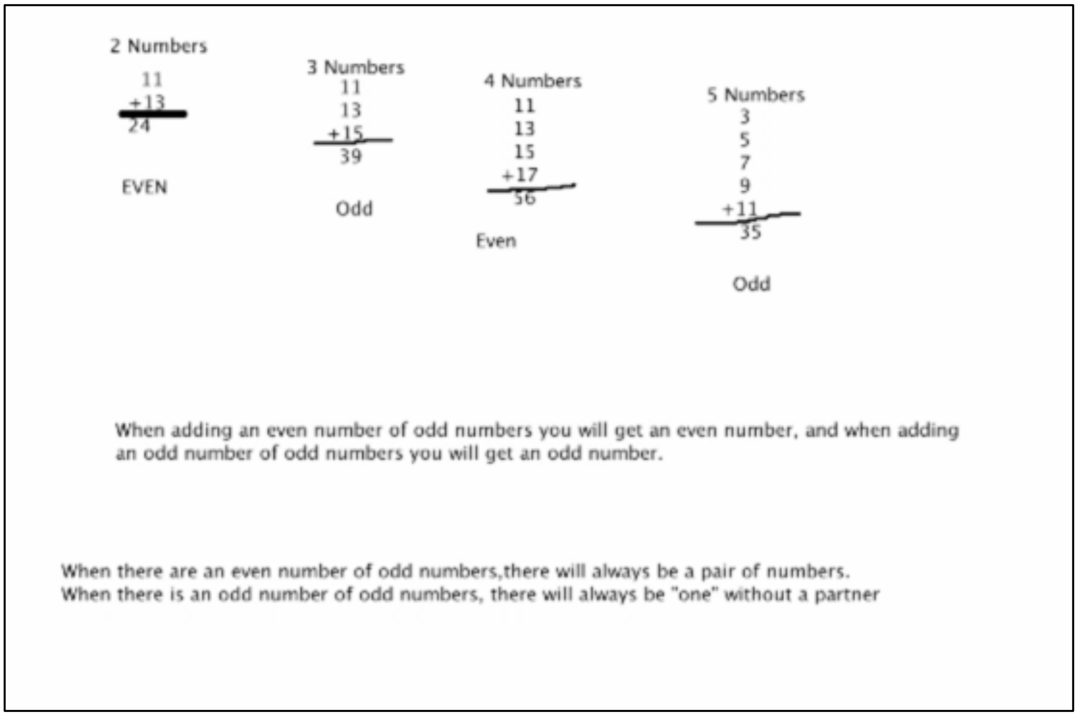


Figure 4.21. Screenshot example of the work produced by group three in the small group discussion centering on conjecturing about odd and even numbers.

Notice the group progress from concrete examples (two odd numbers, to three odd numbers, etc), to a conjecture about the nature of odd and even numbers. Finally, they arrive at a justification for their reasoning. Dr. Spencer joined group three about midway through the investigation while monitoring. At this point, the group had arrived at the conjecture but appeared to make no attempt at justification. Dr. Spencer joined the group asking the question "Is this always true? How do you know?" in the group chat window. This prompt ignited additional discussion amongst the group regarding how they might justify their conjecture.

Summary. The preceding pages attempt to share two "instructional moments" in a strategically blended (i.e., synchronous and asynchronous sessions) online distance education classroom. The purpose of these two episodes is to highlight the instructor's use of effective teaching practices in mathematics education that support student learning across various delivery methods. In the coin problem, the instructor used a high-demand mathematical task as a basis for a discussion of mathematical equality. This activity was delivered as an asynchronous forum discussion. The asynchronous nature of the forum allowed students the time to work independently on the task and formulate their thoughts before having to compose an entry in the forum. The delayed nature of the discussion also allowed ample processing time for participants to read others' responses carefully. These features may have allowed participants to get more out of the assignment than if they had been asked to work on the same problem in a synchronous discussion or even face-to-face.

The synchronous discussion of odds and evens, on the other hand, was an excellent way to allow students to brainstorm together about a mathematical topic. In this discussion, the focus of debate was not necessarily focused on the true nature of odd and even numbers; rather the instructor took a mathematical topic which she anticipated many student were already familiar with (odd and even numbers) and used it as a context for developing an algebraic idea in which they were less comfortable (conjecturing). The fast-paced nature of the synchronous discussion had the net result of getting numerous issues out on the table for discussion quickly. The familiar nature of the topic provided students with the opportunity to think about the evidence needed to support conjectures. Both the coin problem and odd-even discussion capitalized on the delivery medium and available technology while at the same time highlighting elements of best practice in the mathematics classroom.

Part C: In What Ways are the Uses of Best Practices in Online Teaching Strategies and of Effective Teaching Practices in Mathematics Education that Support Student Learning Related to the Components of Transactional Distance in the Course Planning and Implementation?

Given the relativistic nature of Transactional Distance, and the nature of the data collected in this dissertation, it is impossible to determine a cause-and-effect relationship between best practices in online teaching strategies and high leverage mathematics teaching practices and Transactional Distance. In many cases, both the online best practices and the mathematics teaching practices are sensitive to the elements of transactional distance; however, each student's reaction and resulting sense of distance is unique. While the *Field Guide* makes suggestions and provides guidance to the instructor, much of how these actions are perceived will be unique to the individual student. Still, each teacher action is intended to target at least one of the elements of Transactional Distance. This section outlines the potential impact of the various teaching practices on the elements of transactional distance.

Overall course structure. The structure of a course includes a variety of different components that make the course pliable from student to student and from semester to semester. Classes with high structure are the same for every student in a particular class, across the history of the course. Classes with low structure, on the other hand, are easily adapted to a particular student's needs. In a course with low structure, students have the flexibility to follow multiple curricular paths and may even have the ability to negotiate their own path through the content.

The course under study displays a balance of both high and low structure. As described in Chapter 3, the course as studied has evolved over time with input across a number of universities. These collaborators have made a commitment to following a general structure that will prepare course participants for a mathematics leadership position in their schools. Major course content is not intended to vary greatly from semester to semester or university to university. In each iteration of the course, the mathematical focus is on elementary algebra and the pedagogical focus is on student questioning in the classroom. The course concludes with a common final assessment that stays relatively stable from year to year.

When viewed from the outside, the course may seem relatively structured and rigid. When looked at closer, however, many of the internal structures of the course are quite flexible. The same general themes in algebraic thinking may be addressed from semester to semester; however, internal course structures allow for some degree of personalization to the interests of the class and the individual. One example of this is the Bringing It All Together (BIAT) blog. The BIAT blog prompt is an assignment that is frequently used across the 6-course EMAoL program. The writing prompts do not vary much from module-to-module (See Figure 4.22 for an example of the BIAT from Module 2).

Bringing It All Together

Discuss teaching and learning the concept of equality throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

What does it mean for elementary students to reason algebraically?

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

Figure 4.22. Bringing it All Together Prompt for Module 2

Participants in the blog have a high degree of flexibility with regards to the issues that they can emphasize. In the case of the BIAT for Module 2, a few common questions arose with regards to the module's reading assignment. This issue became the point of discussion for the first small group breakout session of the synchronous online session for that module. Had this question not arisen in the BIAT, it may not have been addressed. In all, while the programmatic structure of the course is high, it is not so high that students cannot modify material to their interests.

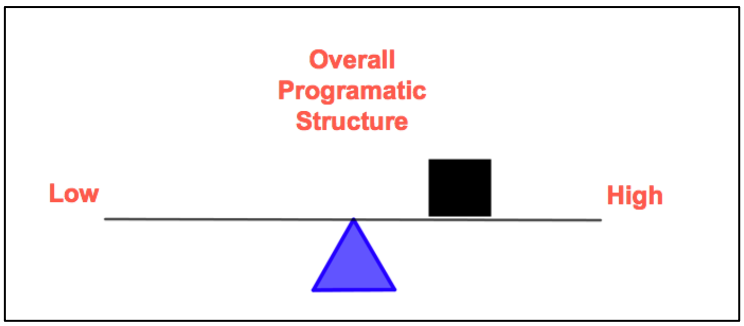


Figure 4.23. Level of overall structure for the course.

Overall course dialogue. Like structure, courses can vary greatly with regard to inter-participant dialogue. Some courses allow for almost constant dialogue between participants, and others, nearly none. While there are a variety of factors that affect levels of dialogue in a course, course structure is an overarching factor. In the course under study, opportunities for dialogue were extremely high (see Figure 4.24). Student-to-student dialogue was encouraged across most of the course assignments. For example, Participants were asked to respond to each other's BIAT blogs, Group problem solving activities were assigned to discuss mathematics content, and students were assigned partners for peer review of Student Questioning assignments.

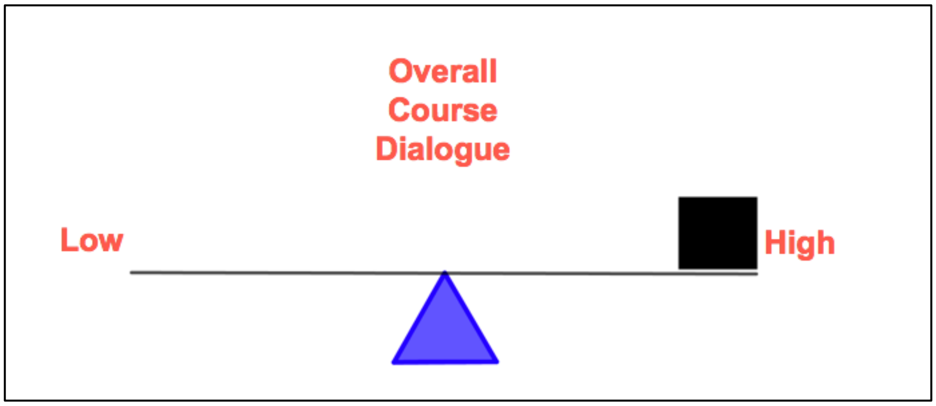


Figure 4.24. Level of overall dialogue for the course.

When added to the bi-weekly synchronous sessions, there was the opportunity for an extraordinary amount of student-to-student dialogue.

The same level of opportunity was present for student-to-instructor dialogue. In addition to the synchronous online sessions which allowed for direct student-to-instructor interaction, various other structures were in place to provide feedback and encouragement to students. Rubrics for discussion posts and group problem solving activities were present in the syllabus. This gave students some sense of the expectation of the instructor with regards to these interactions. The instructor was also both approachable and available for students via email communication.

Despite the opportunities for dialogue that were built into the structure of the course, sometimes, high levels of dialogue in the course were not achieved. In small group discussions, for instance, students were asked to respond to a high cognitive demand task and build on each other's work towards a group understanding or solution. It is unclear if this was the mindset of students as they approached the problems. Particularly in early asynchronous group activities, participants seemed to be producing initial posts that were independent of each other. It is unclear if they were taking into account other's work before posting their own.

In addition to a lack of student-to-student dialogue at times, there was also a paucity of asynchronous dialogue from instructor-to student. The instructor rarely engaged in asynchronous discussion with students. Even dialogue in the form of feedback was lacking through most of the course as the majority of assignments were graded and returned after completion of the class. The bright spot of the course was the dialogue that was encouraged in synchronous online sessions.

Overall student autonomy. Unlike course structure and dialogic level, levels of autonomy in a course is more complicated than a simple high-low continuum. Instead, autonomy is determined by a three-element matrix (Figure 4.25) that takes into account the flexibility of students to (1) determine what it is they learn, (2) how they learn, and (3) how they are evaluated.

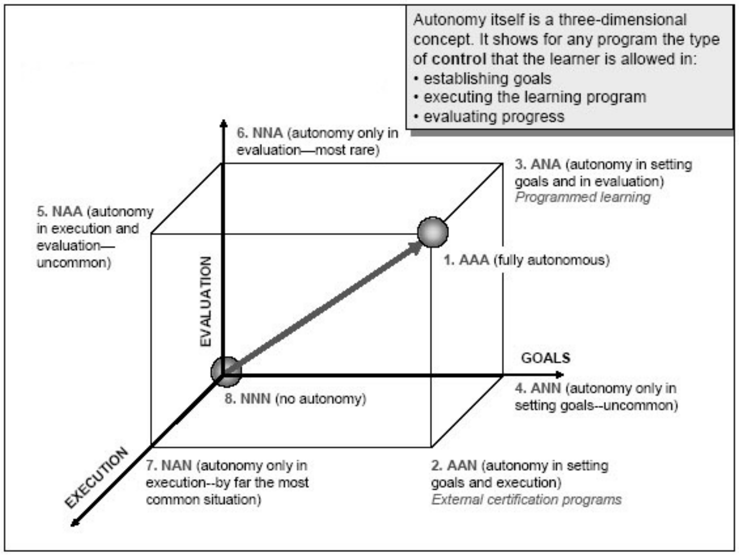


Figure 4.25. 3-dimensions of autonomy. In the figure A stands for autonomous and N stands for not-autonomous. The order of the letters governs autonomy in (1) determine what is learned (goals), (2) how learning occurs (execution), and (3) how student knowledge is assessed. For example, AAN represents autonomy in goal setting, autonomy in how information is learned, and no autonomy in how students are evaluated.

In the course under study, students are not given autonomy over what they learned or how they were assessed. There were clear parameters set for what was to be covered in the course in terms of both mathematics and pedagogical skill. They were also given directions on how they were to be assessed. They were, however given autonomy over how they learned.

Section 2 (Question 2) What are Students' Perceptions of the Distance Education Classroom in Terms of any Impact They Consider It Makes on their Sense of Transactional Distance?

Section 2 of this chapter shifts the focus from the suggested actions of the instructor in the design and implementation of the course to how the course was received by students. Like Section 1, this section is broken into three parts. Part A offers results from a student satisfaction survey administered at the end of the course. Part B discusses the results of a transactional distance survey, also given at the end of the course. Part C discusses the ways in which participant satisfaction may be related to Transactional Distance in this course.

Part A: Using Measure of Student Satisfaction with the Course, What is a Description of Participants' Response to the Course?

The *Satisfaction of Online Learning* (SOL) instrument was administered upon conclusion of the course. Of the 27 students in the class, 20 students returned this survey. This Likert-based, 24 question survey measures student satisfaction along eight different factors of student satisfaction where the factors include (1) effectiveness of feedback, (2) timeliness of feedback, (3) use of discussion boards, (4) instructor student dialogue (5) perceptions of online experiences, (6) instructor characteristics, (7) feeling of a learning community, and (8) computer-mediated communication. In addition to the survey, interview transcripts were examined for themes that generally addressed these 8 factors.

Research indicates that much of what motivates and satisfies students in online distance education classes is external to the learner (Bekele, 2010). This implies that factors like technology, course content, teaching methods, and teaching support play a significant role in the overall satisfaction that a student feels in a class. Results of the SOL survey combined with results from the one-on-one survey allowed some sense of student satisfaction in this course.

Factor 1: Effectiveness of feedback. Feedback is an essential aspect of the learning process. Whereas assessment indicates how well a student has performed on a given task, the purpose of feedback is to provide the learner with information about how close they are to achieving a given learning goal or outcome. The SOL instrument asks students to assess their level of satisfaction with regards to the feedback received from the instructor in two dimensions. First, students are asked about the effectiveness of the feedback that they received. In other words, did the feedback provided in the course make an impact on the final outcome of their work. Second, students were asked questions relating to the degree to which feedback was provided in a timely manner. Feedback that is provided too late, even when providing clear and relevant suggestions, lacks impact, as students do not have the chance to incorporate these comments into their final product.

One of the most important ways that instructors can demonstrate their presence in a classroom is by providing feedback to students. In this study, the instructor provided feedback in a variety of ways. Asynchronously, the instructor provided sporadic comments on course assignments as they were graded. For instance, in Module 2, students were asked to produce an individual journal entry around the general topic of "algebraic reasoning" after completing some course reading (See Figure 4.26). The instructor briefly responded with a sentence or two in response to each student's journal post.

Individual Journal – Algebraic Reasoning

Enabled: Statistics Tracking

Follow up on our class discussion of what algebraic reasoning is by reading chapters one and two of the Blanton book. Respond to the question on page 12 (2.1). What percentage of your math lessons focus on learning arithmetic skills and procedures? What percentage focuses on generalizing arithmetic or some other form of algebraic reasoning? (If you are a curriculum specialist, answer this question in terms of how you perceive the mathematics lessons taught school-wide.) What are the implications of your answers?

Figure 4.26. Algebraic reasoning journal prompt.

In synchronous sessions, the instructor provided timely and immediate feedback in much the same way that one would expect in a face-to-face class. With the exception of a couple of individual journal exercises like the one described previously, the instructor did not participate or comment frequently in the group problem solving or the BIAT blog posts. Instead, the instructor appeared to look for themes or common threads that showed up throughout the course of the asynchronous discussions. Points that needed clarification or additional emphasis were often the focus of a small group synchronous "wrap-up" discussion at the beginning of asynchronous classes.

Towards the conclusion of a synchronous session, the instructor devoted time to previewing the upcoming module. In this preview, the instructor introduced the upcoming group problem solving forum as well as any additional assignments that would be due in the coming weeks. Most importantly, towards the end of this preview the instructor opened up the class for questions regarding submission dates, assignment clarifications. Students frequently took advantage of this opportunity to ask specific questions regarding what was expected for a particular assignment.

Reviewing the results of the SOL, it appears that students were reasonably satisfied with the effectiveness of the feedback being offered by the instructor (Table 4.1).

Table 4.1. *Results of questions 1-3 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to the Effectiveness of Feedback.*

Question	Mean	Standard Deviation	Variance
1. I am satisfied with my online experience because effective feedback related to my classwork is consistently provided to me in terms of clarification for my questions about the course (e.g. assignments).	3.90	0.94	0.89
2. I am satisfied with my online learning experience because effective feedback related to my classwork is consistently provided to me in terms of instruction on how to fix incorrect problems in assignments.	3.45	1.32	1.75
3. I am satisfied with my online learning experience because effective feedback related to my classwork is consistently provided to me in terms of sufficient explanations on my specific questions related to my classwork.	3.95	0.86	0.75

Question 1 provides evidence that students felt the instructor was able to clarify student queries regarding course assignments. Question 3 suggests that students were further happy with the quality of the comments that were made by the instructor and felt that feedback was relevant and helpful. Question 2, however suggests that students felt less satisfied with the feedback in terms of how to correct future mistakes. As a whole, students seemed to be happy with the quality and clarity of the feedback received. They were less sure about how to use the feedback provided to push their own work forward.

Students made comments that reinforced the views captured by the SOL. When ask specifically about the perceived value of the feedback given by Dr. Spencer one student responded as follows:

> I think that she's a very intelligent woman so when she does give specific feedback, I do read it. I'll think about it, whereas I have had other professors that give feedback and I'm just like, "Whatever. Okay." Maybe on blog posts, when she'll respond to a comment, it'll actually be something that's worth thinking about. I'll post something and she'll give me a suggestion or try this or think about that. I feel like it's worthy feedback so I'll give it consideration or apply it.

In this quote, Claire expresses her appreciation for Dr. Spencer's feedback. More than simply the written feedback on assignments, Claire voiced appreciating Dr. Spencer's ability to push her thinking on mathematical concepts by moving the group forward with questions.

A recurring theme across many of the interviews was anxiousness with regards to the expectation of the instructor on various assignments. Despite the provided grading rubrics, students in the online course were concerned that they may not be completing the assignment the way that the instructor intended. This may stem from the lack of student-to-student contact at the margins of class. In a face-to-face class, students have time to informally discuss such issues with their peers before and after class, as well as lulls in the action of the class. Thus, students appreciated the ability to ask clarifying questions at the end of synchronous online classes.

Not all comments about the level of feedback were positive. One student in the class, Julie, noted that the feedback provided in activities like the Team Problem Solving was insufficient to affect her thinking.

> … I don't know really where else she might envision us going, so it's been a little bit kind of frustrating that we haven't really followed up in class at all on some of the Team Problem Solving because I would have really loved to see well how did other people think about it and attack it and approach it and what can I learn from their strategies to become more efficient and think more algebraically. That's probably kind of my only thing that I really, really feel like I need to share right this split second.

Synchronous small groups were rarely held around Team Problem Solving activities. So long as resolution was achieved within the group, Team Problem Solving strategies rarely became the focus of synchronous sessions. Julie makes the point that she would have liked more debriefing time to look at solutions produced by other groups as well as her own. While there was a perception that Dr. Spencer's comments were valuable, it seems that students may not have necessarily been able to use these comments to improve their work for one reason or another.

Factor 2: Timeliness of feedback. Related to the previous theme of the *effectiveness* of the feedback in the course is the *timeliness* of the feedback. While students seemed to agree that the feedback provided by Dr. Spencer was effective, some students reported not being able to use feedback to fix incorrect problems in their assignments. A large part of the reason for this appears to have been related to the timeliness with which the instructor delivered feedback on assignments. Of the eight aspects measured by the SOL, timeliness of feedback received the lowest scores (See Table 4.2).

Table 4.2. *Results of questions 4-6 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to the Timeliness of Feedback.*

Question	Mean	Standard Deviation	Variance
4. I am satisfied with my online learning experience because timely feedback related to my classwork is consistently provided to me so that I am able to complete my assignments efficiently.	3.30	1.49	2.21
5. I am satisfied with my online learning experience because timely feedback related to my classwork is consistently provided to me so that I am able to improve my assignments for better grades.	2.80	1.54	2.36
6. I am satisfied with my online learning experience because timely feedback related to my classwork is consistently provided to me so that I am more focused on learning.	3.05	1.40	1.95

Studies have shown that for feedback to be effective it must be delivered in a timely manner to the student (Jonassen, 2004; Ramsden, 2003; Sopina & McNeill, 2015). Students are unlikely to accommodate feedback into future assignments when it is not delivered in a timely fashion. In fact, late-arriving feedback runs the risk of not being read at all (Higgins, Hartley, & Skeleton, 2002).

Some of the lowest scores received on the SOL were related to this category of timely feedback. While it seems that students valued the feedback that they got from Dr. Spencer, it looks as though this feedback was not delivered consistently enough to make a large impact on student work. Formal feedback in the form of comments and grades on course assignments were often late in being returned to students. This was particularly problematic for the second and third Questioning and Student Work Analysis (QSWA) assignments as constructive comments on previous QSWA assignments might have been used to improve the quality of later assignments.

Factor 3: Use of discussion boards. Asynchronous discussions are often utilized in online courses to provide a venue for students to openly communicate and build shared understanding. While discussions can be effective tools for learning in the online classroom, they are not effective if not optimally designed and skillfully facilitated (deNoyelles, Zydney, & Chen, 2014). Despite their prominence, online discussions pose challenges. Discussions must be structured in such a way as to allow students the room to co-construct knowledge, but over and above the structuring of the event, discussions must be skillfully facilitated in order to ensure that student conversations are on track and that there is even participation across the group (Mazzolini & Maddison, 2007).

In the class under study, discussions were present in many forms. Asynchronously, students used Group Problem Solving forums to work on focused mathematical goals for a given module. Students also asynchronously reflected on and discussed their work in the Bringing It All Together forums. In the synchronous, online sessions there were both whole group discussions debriefings as well as more intimate group discussions consisting of between 3-5 students. Questions 7-9 addressed by the SOL survey instrument (Table 4.3) were intended to address the asynchronous forums.

Table 4.3. *Results of questions 7-9 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) Related to the Use of Discussion Boards.*

Questions	Mean	Standard Deviation	Variance
7. I am satisfied with my online learning experience because asynchronous discussions (where I can post my discussion at any time of the day) are more convenient to my schedule are more convenient to my schedule than traditional discussions.	4.58	0.49	0.24
8. I am satisfied with my online learning experience because discussion boards make me more comfortable participating than traditional modes of discussion.	3.50	1.02	1.05
9. I am satisfied with my online learning experience because I have plenty of time to think and draft my responses for online discussion.	4.00	0.95	0.90

Results from these questions were encouraging. Students seemed to react well to asynchronous discussion boards in general, but they were particularly satisfied with the convenience and think time that discussion boards allowed. Whereas students' schedules are constrained by face-to-face classes that are scheduled for them, asynchronous discussions allowed students the convenience of participating around their already busy schedules at a time of their choosing. A second benefit for students was the time that asynchronous discussions afforded them in terms of reflecting on the question and crafting a response.

Question 8 of the survey yielded interesting results. While the score for this item was high, and students indicated a comfort level with asynchronous postings, the score for this item was lower than the other two discussion board questions. While asynchronous discussion boards were more convenient and allowed the participant time to carefully plan a response, still, some students reported feeling uncomfortable making asynchronous posts as compared to the traditional discussions that occur in a face-to-face setting. The interview data revealed several possible reasons for this discomfort.

First, some students may have felt as if their ideas were overlooked or ignored. One student, Claire, attributed this to the lack of a natural flow to the discussion.

> I guess what I was thinking about really was doing group discussion board where I type something, you type something, I type something. It never really seems to work out a nice flow. For an example, we had an assignment recently where I did it and I typed something with the solution to the assignment and instead of feeding off what I had already done, people started posting incorrect solutions. I just don't understand, it's a group discussion… we're supposed to be going off each other. I was completely skipped over what was posted and everyone just posted an original post. It was really not working on what others had done so it just seemed a little pointless I guess.

In this quote, Claire is making several points. First, what she has posted to the forum has not been valorized, or perhaps even read, by her co-contributors in the small group. Second, there was no discussion happening. In her mind, the thread was a group of loosely associated posts connected by a single prompt instead of a conversation where one post builds of the work of the previous poster. Third, the messages being posted were redundant in nature, often covering the same material or making the same points. When asked to elaborate on this point, Claire offered this response:

> I think it's that people feel that I need to participate and show that I'm being active. I'm going to find another way to say something. It ends up being the same thing that was already said, but just reworded because that person wants to make sure they're participating. That's when it gets a bit tedious.

While in a face-to-face classroom students can show their engagement and participation in a discussion in ways that are non-verbal in nature (nods of agreement, eye contact, etc.), this is not possible in the asynchronous forum. In her quote, Claire suggests that in order to prove participation, students might feel the need to take a more active presence in the thread, even if their point has already been made.

A second potential reason for dissatisfaction with the asynchronous discussion board may have been the posting requirements. The posting requirements for Team Problem Solving necessitated three separate posts, the first of which had an earlier deadline, after which the other two posts could be made. The intent was to get students thinking about the material early in the module and then to distribute participation over the 2-week period. Like the Team Problem Solving, students were asked to make postings to the Bringing it All Together blog forum on 3 different days. One student, Julie, noted feeling like three postings was "excessive":

> The one thing, and I don't mean for it to sound like gripe session, but the one thing that I just think could kind of be improved upon is I kind of, like when we were talking… blog posts seems to be kind of a recurring thing we've talked about today and it's kind of… sometimes 3 times feels a little bit excessive. Like the initial post to get your thoughts down, I like that, and then like maybe 1 more post but sometimes I feel like the third one …

Julie found ways, however, to circumvent this requirement. Later in her second interview she notes this strategy:

> So like I would sit down on like a Sunday afternoon and I'd read what people were saying, to the two [posts] that I would like to respond to, type out my two responses right then and there, and then when I would go and post them, like say one on Monday, the first one on Monday, the second one on Tuesday, depending if no one else had already posted and commented on that person's particular post. It's kind of like I'm still following the parameters but I'm still kind of working ahead in a little bit of a more purposeful way I guess. In managing her time, Julie is conforming to the assignment and posting on three different days, but she is not necessarily participating in a discussion.

In managing her time, Julie is conforming to the assignment and posting on three different days, but she is not necessarily participating in a discussion.

Factor 4: Dialogue between instructor and students. While the course was designed with ample opportunities for dialogue between instructor and student, often times this potential was not fully realized. The instructor for the course was generally quick to respond to student emails and allowed ample time in synchronous sessions for students to clarify assignment expectations. Students also reported that the feedback received from the instructor was valuable and could be used to accomplish future assignments. As seen previously, however, students found the timeliness of feedback to be a problem. In many cases the feedback from one assignment did not make it back to the students to impact the following assignment.

In asynchronous blogs and problem-solving groups, dialogue between students and the instructor was relatively rare. Students were generally left to their own devices to solve problems in the group problem solving activities. In Bringing It All Together Blogs, comments were also infrequent. Students, however, were generally satisfied in the level of instructor dialogue in the course. Most felt like communication between the student and instructor was effective throughout the course of the semester and that the instructor was helpful when learning the material. A smaller proportion of students, however felt that the level of dialogue with the instructor in the course made them feel less distant or better connected to the class.

Table 4.4. *Results of questions 10-12 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to the Dialogue between Instructor and Students.*

Question	Mean	Standard Deviation	Variance
10. I am satisfied with my online learning experience because I am able to communicate effectively with my instructor throughout the semester.	3.95	1.02	1.05
11. I am satisfied with my online learning experience because online dialogue with my instructor helps me as I learn in the course.	3.85	0.96	0.93
12. I am satisfied with my online learning experience because I feel less distant in my online learning due to online dialogue with my instructor	3.60	1.11	1.24

Factor 5: Perceptions of online distance education experiences. Students seemed decidedly split in their overall perceptions of online learning. Most students agreed that there were elements to the online experience that they found enjoyable. Julie reported the following:

> One of the things that I kind of really enjoy, It is just kind of it's nice to just come home and to kind of take class in the comfort of your own home, like change when it comes to clothes and just kind of be in your own environment and not have to drive to [town] another time for the week. That kind of a bonus that I had found. One of the things that I also really like about the online schedule is kind of like every other week. It's kind of like you can self-please yourself in what you need to get done. Say if you have prior plans or something going on at school and it's nice to kind of know that you don't have class every single week, but since it's online you can kind of build your own schedule on the off weeks per say.

In essence, it seems that Julie is appreciative of the convenience of scheduling that is part of the online experience.

There was some disagreement as to whether their needs as learners were being met. Emalee notes:

> When you're sitting in a class at least three times a week for an hour, you get a whole lot more of the information auditorily [sic]. Then here, sometimes you have to read it, and you have to re-read it, and you go, "What does that mean? Let me read it again."

There's nobody to say, "What's that means is…" You kind of have to work a little bit harder on your own.

Throughout the course, Emalee found it challenging that the majority of the content came in the form of reading material and felt that in a face-to-face class there was a better mix of lecture material and reading.

The class was somewhat divided on their preference for online distance education courses. Question 15 of the SOL (See Table 4.) indicates that many of the students in the class may prefer taking the course face-to-face or in a blended format.

Table 4.5. *Results of questions 13-15 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to the Perceptions of Online Experiences.*

Question	Mean	Standard Deviation	Variance
13. I am satisfied with my online learning experience because my personal needs as a student are met in an online environment.	3.80	1.17	1.36
14. I am satisfied with my online learning experience because many aspects (features) of online education are enjoyable to me as a learner.	4.20	0.81	0.66
15. I am satisfied with my online learning experience because overall, I would rather take online courses than traditional courses.	3.45	1.36	1.85

Factor 6: Instructor characteristics. Students seemed satisfied with the characteristics of the instructor. Most felt satisfied that they got the same explanations and received the same amount of assistance from the instructor as compared with a face-to-face class. Students were further satisfied with the creativity shown by the instructor throughout the semester when presenting content.

Table 4.6. *Results of questions 16-18 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to Instructor Characteristics.*

Question	Mean	Standard Deviation	Variance
16. I am satisfied with my online learning experience because I still get the same explanation from online instructors that I do from traditional instructors.	3.90	1.22	1.49
17. I am satisfied with my online learning experience because online instructors and traditional instructors offer the same amount of help with my learning issues.	3.70	1.23	1.51
18. I am satisfied with my online learning experience because technology makes online instructors more creative in teaching than a more traditional classroom.	3.70	1.14	1.31

Factor 7: Feel of a learning community. The defining characteristic of distance education is the physical and/or temporal separation of the student from the instructor and other members of their cohort. Because students are in different locations than the instructor and their peers, the instructor faces the challenge of figuring out ways to make students feel connected and be able to succeed in the online environment. The key to developing a feeling of connectedness in the online classroom is transitioning students from the position of isolated learner to that of a member of a learning community (Haythornthwaite, Kazmer, Robbins, & Shoemaker, 2000). It seems that students in this class were satisfied with the learning community that developed (Figure 4.7). While there were elements of the class that may have contributed to the development of an online learning community, it is more probable than not that the leaning community was fostered as much by the program as it was by this class specifically.

Table 4.7. *Results of questions 19-21 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to the Feel of a Learning Community.*

Question	Mean	Standard Deviation	Variance
19. I am satisfied with my online learning experience because the online environment is like a community where I can communicate with other students.	4.05	0.74	0.55
20. I am satisfied with my online learning experience because the online environment promotes sufficient sharing and caring among students.	3.95	0.86	0.75
21. I am satisfied with my online learning experience because the online environment is a safe place where I can be confident in completing group work with other students in the class.	3.79	0.89	0.80

While the EMAoL program does not have a face-to-face component, some students appear to meet each other simply by engaging in common interests. For instance, in the following clip Emalee notes having run into numerous classmates at a professional conference and relays that it is nice to “put the face with the name”:

Emalee: They're still about the same. I think everybody's starting to get the blues. It always seems the slumps come around this time of the course, but, I still… like, ... I have a colleague, at work, and she and I bounce ideas off of each other. Like, "What did that say?". There's probably nine of us together this weekend, at the Eastern Regional NCCTM Math Conference, and we were having this conversation ... "Don't forget…", and "I didn't know I was supposed to be doing that.". It's kind of nice when you know people, outside of just virtually.

Bryan: Right. How about the people that you haven't know before? Have you started up relationships, at all? Not where you have to contact them a lot, or anything like that. Are there relationships that you feel have developed over this course?

Emalee: Oh, yeah. In fact, I've met several of the people in our class.

Bryan: Talk a little bit more about that. Where did you meet them?

Emalee: Lori (Pseudonym), I met her at the NCCTM Math Conference, in Greensboro. We met her back in ... She was in our class last semester. We met her back in… I think it was October. We had a conversation with her. Come to find out, she's ... I think her son goes to school in our same county, at the private school. Then, we were supposed to meet ... Jason (Pseudonym) was supposed to be at the conference this weekend, but we never did run across him.

Bryan: What have those ... How do you feel about those sorts of interactions? Are they something that you'd like to see included in some sort of a class like this? I know it's hard, because again, part of this is convenience, and you don't want to have to factor in those sorts of face-to-face interactions. How has the interaction enhanced your experience in the class?

Emalee: I don't really know that it has, it's just sometimes nice to put a face with the name.

While Emalee was appreciative of the opportunity to meet members of her cohort and used those meetings to develop relationships wither classmates, she was also clear that she was not always trusting that members of the class were engaged in online large group discussions. She indicated that she was aware of times where she, herself and other cohort members were off doing other things while supposedly present in class.

Emalee: It's really hard, because you can't read people. You've got those kids in any class, that will sit back and listen, and they are engaged, but they're just not ... They don't want to speak. Then, of course, there is a possibility that whoever you're dealing with may not even be in the room.

Bryan: Do you think that happens, or do you know that happens?

Emalee: I'm pretty sure it happens.

Bryan: I can't say that I'm not guilty of that sometimes, where I'm like… "I'm just going to quickly go to the bathroom here.", or "I'm going to run downstairs, real quick, and check my mail.".

Emalee: I know one of our ... One of the girls I know, she's like, "Well, I check into the class, and I had to go pick my son up from ball practice.", so she was gone. Not even in the building, she was gone. (Interview 2)

Factor 8: Computer mediated communication. While much of the communication in this course happened asynchronously, synchronous online discussions were a key element to the success of this class. While asynchronous exchanges allow for increased think time and flexible scheduling of the workload, nothing can replace the immediacy of a real-time discussion. Baker (2004) found a strong, positive correlation between perceived instructor immediacy and affective learning, and a moderate, positive correlation between immediacy and cognitive learning further reporting that verbal immediacy behaviors such as asking questions, using humor, addressing individuals by name, and initiating discussion increases psychological closeness. While these behaviors can be replicated in the asynchronous environment to some extent, synchronous interactions allow for a wider range of these behaviors in real time.

This course utilized 180-minute synchronous meeting sessions every other week. Synchronous meetings allowed for direct student-to-instructor interaction as well as provided a space for student-to-student interaction to happen. Real time communications were conducted using the educational video conferencing software Blackboard Collaborate. While actual video of the students and instructor were never used in the course, Blackboard Collaborate offered a platform through which to talk and share ideas without the delays experienced in the asynchronous forums. While the synchronous session reduced the flexibility of the asynchronous discussions that was greatly valued by students, it was hoped that these sessions would provide the immediacy lacking in the asynchronous discussions.

On the whole, students responded well to the Blackboard Collaborate platform (Table 4.8). Students in the study reported a feeling of "closeness" with the instructor and other students in the class. Despite this feeling of closeness, students reported it difficult to form the same types of meaningful relationship as in a face-to-face class.

Table 4.8. *Results of questions 22-24 from the Satisfaction of Online Learning (SOL) Tool (Davis, 2014) related to Computer Mediated Communication.*

Question	Mean	Standard Deviation	Variance
22. I am satisfied with my online learning experience because computer-mediated communication makes me feel like a real person when I communicate in the online environment.	3.90	0.77	0.59
23. I am satisfied with my online learning experience because computer-mediated communication makes it easier to form meaningful relationships among students in the online environment.	3.25	1.04	1.09
24. I am satisfied with my online learning experience because computer-mediated communication allows me to feel the presence of my instructor and other students in the online environment.	3.70	0.78	0.61

In the interviews, many of the comments relating to computer mediated communication tends to be focused on classroom decorum or the technical limitations of the software. One student, Chelsea, reported being unsure how to participate in the whole group synchronous discussions:

Chelsea: I had the question last week about… I know it's a silly thing, but just how the discussions are going to work. Because in my first two classes, we raised our hand, and in this class, everybody just jumps in. I'm just like, "Okay." I don't want to step on somebody's toes, and I don't want to... In a classroom situation… I don't know, that's just… I'll get used to it ... (Interview 1)

Chelsea struggled with understanding the norms of the virtual classroom. While students are accustomed to interacting in the face-to-face classroom, there is no reason to think that they tacitly understand the norms of the virtual classroom. While verbal cues and the arrangement of material culture (desks, chairs, chalkboard, etc.) can provide students with hints of how to act and behave in the face-to-face classroom, no such cues exist in the virtual classroom. Absent of explicit instructions about participation behaviors in synchronous discussion, the student is left on their own to determine how to participate online.

In addition to the norms of the classroom, students in the interviews expressed that they often did not form the same sorts of relationships with their peers that they might in face-to-face classes. The following excerpt from the initial interview with Julie demonstrates this:

Julie: Yeah, this is a tough thing. A lot of questions… a lot to think about. I think one of the things that I kind of miss when I think about my Tuesday night class or my prior classes at (University) that were all at the CCEE face to face, I kind of feel like those relationships that you form with your fellow colleagues and peers in the classes with you is really special and really important and you can build them. I guess with the online cohort as well, but it's not quite to the same in-depth kind of comfort level that it is with people that you meet face to face. Just kind of on a more collaborative and a cooperative way of thinking about it, that's been something that's been a little tricky for me.

Then I also sometimes feel like, I don't know, maybe not being able to see everyone's reaction… If I had a question or was confused about it, I think I'm more hesitant to ask it in an online setting than I would be if it were a face-to-face interaction class. Those are the few things that kind of just pop right into my mind with that question. …

Bryan: When you say you might be more hesitant to ask the question is it because you are not sure if you are the only person who has the question? ... I don't want to put words in your mouth, but is that sort of where you are going with that?

Julie: Yeah. It's kind of like, "Oh! Well, I'm I going to be the only one that's either remotely thinking that… or people could be thinking behind their computer screens as they are listening, 'Wow! She really doesn't know that? That's surprising!" I sometimes... I guess I feel like if it was face-to-face I don't think I would be as hesitant to ask a question and I think it's also… because again it's that familiarity and comfort level with your peers and knowing them on a face-to-face personal level.

Bryan: It sounds like this happens definitely sometimes in the whole group where you are not sure if you want to interject? That's a hard thing to do in an online course anyway, but how about the smaller group interactions? Are you more comfortable in there?

Julie: Definitely much more comfortable asking questions and there has been times I think especially in Dr. Fisher's (Pseudonym) class this past semester there were a couple of instances where I vividly remember… I said to my team members, I said, "Well, can someone paraphrase this? Can someone explain that to me in a different way?" Like X-Mania comes to mind. That took me a while to kind of process through that. I did stop my small group and I was like, "Can you all ..." Because they immediately, just boom! They knew exactly what to do and I think it was also because they had taught older grades and I think someone in the group I think was actually teaching community college level. I think I am definitely more comfortable and willing to ask a question in small group more so than in the whole group setting. (Interview 1)

In this selection, Julie describes discomfort related to asking questions in the whole group setting because the online setting does not allow her to "read" the expressions of other members of the class. She notes that in her face-to-face experiences she would be more apt to ask those questions because she had a different level of comfort and familiarity with her peers. It is interesting to note that Julie expresses more comfort in the small group interactions suggesting that perhaps these types of interactions are an important aspect of building community in virtual classroom and fostering more intimate relationships between members of an online class.

Part B: Using a Measure of Transactional Distance, What is a Description of Participants' Perceptions of Transactional Distance Experienced at the End of the Course?

One of the main road blocks with regard to distance education has been student follow-through in online courses. High dropout levels have plagued the discipline. It has been reported that dropout rates for online classes have historically been between 30% and 50% (Moore & Kearsley, 2011). In highly interactive classes, a dropout rate near 50% would be detrimental to the overall learning community, even the students who are successful in completing the course. Given the interactivity of this particular algebra course, the author was interested in investigating students perceived level of distance in the course as measured by an instrument designed to establish transactional distance among students (Horzum, 2011).

Transactional Distance is not defined as a physical distance, although physical distance may play a role in perceptions of transactional distance. Instead, Transactional Distance is a psychological space that exists between students and instructors (Moore & Kearsley, 2011). It is in this "space" that instructors and students communicate about various elements of the course and its content. According to Moore (Moore, 1973, 1977, 2013d) Transactional Distance is related to two variables. The first element, distance, refers to course structure and dialogue. Structure and dialogue work inversely to create perceptions of distance. As dialogue increases, structure necessarily decreases. Conversely as the structure of a course increases, there are fewer chances for dialogue that may take class in a different direction. The second element, student autonomy, refers to the participation of the student in participating in the direction of their own learning activities and establishment of course learning criteria (Horzum, 2011).

In this study, the *Perceived Transactional Distance in Blended Learning Environments* instrument, developed by Horzum (2011), was used to gain a sense of students' perceptions regarding Transactional Distance upon completion of the course. The survey was created using the Qualtrics Research Suite software and distributed to students via a weblink. This survey was sent to students along with a link to the *Satisfaction of Online Learning* survey discussed earlier in this chapter. Both surveys were voluntary. While most of the students in the class completed the *Satisfaction of Online Learning* survey only about half of the students completed the *Perceived Transactional Distance in Blended Learning Environments* survey. While it would be interesting to see how students perceived learning in this course, no conclusions could be drawn given the low completion rate of the survey.

Part C: In What Ways are Participant Satisfaction and Perceptions of Transactional Distance Related in this Distance Education Learning Experience?

Moore's (2013) theory of Transactional Distance has provided distance educators with a framework within which to provide content to students. The theory views learning as an educational exchange between students and instructors. The efficiency and effectiveness of this exchange is dependent on the interplay of three key variables: (a) dialogue, (b) structure, and (c) learner autonomy (Shearer, 2009). The current study had hoped to be able to evaluate the efficiency and effectiveness of the educational transaction between participants and the instructor in MATH 307 and perhaps link this effectiveness to elements discussed in the *Field Guide*.

Ultimately, the researcher was unable to draw conclusions regarding this research question. As discussed in the previous section, response to the *Perceived Transactional Distance in Blended Learning Environments* survey was sparse. Twenty-seven students were asked to complete this survey. Of the 27, only 13 returned the online questionnaire. Irrespective of the limited response to the survey, as the study progressed, the researcher began to question the value of attempting to make a measurement of Transactional Distance at all. Moore, himself, notes that Transactional Distance is, by its nature, a relative rather than absolute measure (Moore, 2013c). Instead of something to be measured, the author began to see Transactional Distance as a window into the pedagogical complexity of the online learning environment where dialogue, structure, and learner autonomy are variables rather than fixed quantities (Peters, 1998). Instead of a focus on the variables that effect the online learning environment, perhaps it would be better to shift focus from the nature of the transaction generally, to the building of the learning community. In other words, given that structure, dialogue and learner autonomy are variables which the instructor has varying degrees of control over, how might instruction be planned for and analyzed. In the chapter that follows, the author presents a different framework for evaluating the online experience. This framework incorporates practical elements of design (Quality Matters course evaluation) in the building of an online community of learners (Community of Inquiries framework).

CHAPTER 5: DISCUSSION

"*In case you do not already know it, teaching online is different from teaching in the traditional classroom. You cannot just take what you do in a face-to-face class and "put it online." In fact, because online learning is so different from traditional classroom learning, the online learning revolution has forced us to look more closely at how courses are constructed and how students learn. Ultimately, this forced self-reflection of sorts will hopefully result in better instruction in* ***of both the concrete and online kind***".

(Pollock, 2013, p. 3, emphasis added)

This exploratory case study examined the design and implementation of an online distance education class in mathematics teacher education which was taught in a weekly class format for one full semester. The purpose of this exploratory case study has been to assess the impact of and provide directions for modifications to *A Field Guide for Mathematics Educators in the Design and Implementation of Learning Environments in Online Distance Education*, a document used to provide guidance to the instructor who was teaching the course. Using the *Field Guide* as a lens through which to observe instructor and student interactions, the intent of the study was to document and record as exemplar some of the behaviors present in this course and outlined in the *Field Guide* so that others who are teaching similar courses online might benefit from using the *Field Guide* as a resource to help in their design and delivery of online distance education learning experiences.. The research questions for this exploration were as follows:

1. In what ways does the instructor respond to, interpret, and apply the underlying

premises and guidance provided in the *Field Guide* in course planning and implementation? This includes a consideration of:

   a. In what ways are the use of best practices in online teaching strategies that support student learning evident in the course planning and implementation?
   b. In what ways is the use of high leverage mathematics teaching practices that support student learning evident in the course planning and implementation?
   c. In what ways are the uses of best practices in online teaching strategies and of effective teaching practices in mathematics education that support student learning related to the components of Transactional Distance in the course planning and implementation?

2. What are students' perceptions of the distance education classroom in terms of any impact they consider it makes on their sense of transactional distance? This includes a consideration of:
   a. Using measure of student satisfaction with the course, what is a description of participants' response to the course?
   b. Using a measure of transactional distance (see Appendix A), what is a description of participants' perceptions of transactional distance experienced at the end of the course?
   c. In what ways are participant satisfaction and perceptions of transactional distance related in this distance education learning experience?

Findings and initial interpretations related to these questions were presented in Chapter 4 . In this chapter, implications for specific revisions of the *Field Guide* are discussed and a model for iterative course improvement is presented and discussed. This chapter ends with a discussion of the limitations of this work and future directions for research.

Adapting to the Ecology of Online Learning

It is acknowledged that there are differences in the ecological landscape of online distance education when compared to the more traditional face-to-face classroom. Over the course of the research, data analysis, and writing of this dissertation, the researcher has been frequently reminded less about the differences between these two environments, but rather in their similarities. In the Preface to the first *Handbook of Distance Education*, Michael Moore noted a "frenzy of activity" (Moore, 2003b, p. ix) related to the emergence of new computer-mediated communication technologies. Today, there is still a great deal of interest in these ever-emerging technological advances that make computer-mediated communication faster and ultimately improve our ability to deliver educational content across distances. But, as Lerreamendy-Jones and Lienhardt (2006) note, "one of the most important promises of online education is not so much in the quality of the resulting products as in how online environments allow educators who develop courseware to enhance the status of their pedagogical practice" (Larreamendy-Joerns & Leinhardt, 2006, p. 596-97).

The issue is not so much that online educators must transform or re-create educational content to be delivered online. Instead, the issue is focused on adaptation of existing curricula to a new ecological landscape of learning. When living organisms evolve they change their look or behavior to make themselves more suitably adapted to a particular environment. Basic structures, however, remain intact. This may serve as a relevant analogy for the field of education as the landscape of learning changes. Essential pedagogical structures should continue to guide instruction even as the anatomy of learning begins to change around us. It is with this in mind that the researcher discusses findings and suggests revisions to the *Field Guide*. This new vision focuses on the strong pedagogical structures situated around course planning and mathematics instruction that currently exist and uses episodes from data to illustrate these points.

Strengthening Course Design and Implantation in the Field Guide

A Field Guide for Mathematics Educators in the Design and Implementation of Learning Environments in Online Distance Education was initially intended as a document that might guide novice instructors of mathematics/mathematics education courses through some of the basic steps of designing and implementing online distance education instruction. The authors of the *Field Guide* recognized themselves and their colleagues 'reinventing the wheel', course after course, with little in the way of a theoretical or conceptual model as guidance. Over multiple semesters of designing and implementing asynchronous/synchronous blended online courses in mathematics teacher education, the authors of the *Field Guide* collected their thoughts and experiences, combined with emerging knowledge of the literature in online distance education, to create the *Field Guide version 1.0.* This document was created with the understanding that it would not be perfect, but might be useful for our colleagues teaching online distance education courses who have a strong knowledge base in mathematics/mathematics education content and teaching, but seek guidance in moving to teaching using online distance education environments.

After observing the algebra course using the *Field Guide* as a lens for analysis over the course of a complete semester, the *Field Guide's* foundations appear to be strong. The first half of the *Field Guide* was written to offer instructors of online mathematics/mathematics education courses recommendations regarding the use of technology to deliver instruction as they design the course, prepare themselves and their students for the learning experience, and facilitate their class. These recommendations were largely drawn from the book *Best Practices in Online Teaching* by Larry Ragan (2008). This resource offered what the authors of the *Field Guide* considered to be a *minimum* set of general online actions that might be played out by the instructor of a course over a semester and touched upon issues related to both the design of the course and its implementation. The Ragan (2008) checklists provide a good starting point to help guide novice online instructors in the setup and delivery of their courses. However, while these checklists may have the ability to prepare the instructor for designing a course and offering advice on facilitation, they do not measure instructor performance in light of some set of standards. In this study, the researcher found that many of the items from the checklist were carried out. What was unclear is if these instructor actions were sufficient to satisfy the components of the *Field Guide* checklists, or if they were merely present.

The second half of the *Field Guide* attempted to address the best practices in mathematics education teaching that support student learning. While most mathematics teacher-educators would be familiar with these practices, the *Field Guide* is intended to illuminate how these mathematical practices may be enacted in online environments. The mathematical practices offered in this section provide some good advice to novice online distance education instructors. As research unfolded around the algebra course, it became clear, however, that two elements were missing. The first element missing from this section of the *Field Guide* was an overall framework that ties together the mathematics practices offered in Section One of the *Field Guide*. The second feature that is missing are clear exemplars regarding how these practices might be enacted online and what that might look like.

Shifting the theoretical lens of the *Field Guide*. The research conducted in this project relied heavily on Moore's (1980, 2013c) theory of Transactional Distance. Research Question Two of this dissertation attempted to tie various aspects of course design to students and their sense of transactional distance. This was in some ways naive on the part of the researcher as Moore himself describes transactional distance as relativistic rather than absolute in nature (Moore, 2013c), thus suggesting that it might be difficult or even inappropriate to measure. Moore's theory, however remains as a significant influence on the discipline and should remain as a part of any design framework. Additional theories and models have emerged within the discipline, however, that can effectively direct instructors in the process of design and implementation of an online course. One such model is the Community of Inquiry (CoI) framework (Garrison, Anderson, & Archer, 2001).

The CoI model targets the elements of transactional distance that affect one's sense of psychological connection to the community which in turn may lead to increased motivation and satisfaction with the educational experience (Shearer, 2013). Building on Moore's (1989) work on interaction types, the CoI model proposes that learning online is supported by three presences: social presence, teaching presence, and cognitive presence[6]. Research conducted around these presences has, in fact, linked social presence (Swan & Shih, 2005), teaching presence (Shea, Li, Swan, & Pickett, 2005), and cognitive presence (Garrison & Cleveland-Innes, 2005) to course satisfaction, sense of community and perceived learning. In 2008, a group of researchers developed and validated a survey (Swan et al., 2008) designed to measure student perceptions of the extent to which the three presences are

[6] For a more in-depth discussion of these three presences, please see the Communities of Inquiry section in chapter 2 of this dissertation.

evident in an online distance education course. While transactional distance still provides a suitable overall framework for online distance education, the Communities of Inquiry model is better suited to assist instructors with the 'nuts and bolts' of course construction and implementation.

Strengthening design using quality matters. While the Regan (2008) checklists provided in part one of the *Field Guide* remain a solid starting point for the novice online distance education instructor, some effort needs to be made to ensure that the actions suggested in this section are not simply performed, but, rather, are performed well. There are a number of resources available for peer or institutional review of online courses. Among these options is the *Quality Matters (QM) Higher Education Rubric Workbook* (Quality Matters, 2014). Quality matters is a peer-based review process of the design of online distance education courses. While the Ragan (2008) checklists included in the initial draft of the *Field Guide* address both course design and course facilitation, the QM rubric, guided by standards, focuses solely on the organization and clarity of structural components of a course. Research centered on course improvement using the QM rubric suggests that higher overall achievement (measured by course grades), greater levels of interaction, and higher students satisfaction are present after QM course redesign (Legon, 2015; Swan, Day, Bogle, & Matthews, 2014; Swan, Matthews, Bogle, Boles, & Day, 2012).

The QM review is a peer-review process that can be used to evaluate the quality of an online or blended distance education course. This process of evaluation is carried out using a rubric based in principles of instructional design (Quality Matters, 2014). The rubric itself is grounded in eight general course standards – (1) course overview and introduction, (2) learning objectives (competencies), (3) assessment and measurement, (4) instructional materials, (5) course activities and learner interactions, (6) course technology, (7) learner support, and (8) accessibility and usability. Within the eight categories there are a total of 43 standards that must be addressed. These 43 standards are ranked in terms of relative importance and assigned point totals in the rubric. Twenty-one of the 43 standards are designated as 'essential' and assigned a 3-point maximum. Fourteen standards are deemed 'very important' and assigned a 2-point maximum. Finally, eight standards are noted as 'important' and assigned a total of one point each. In order to meet the expectations of the QM rubric, all of the 3-point standards must achieve their maximum value, and the review as a whole must achieve a score of 84 points out of a possible 99 total. While an official QM review is conducted by a 3-person team consisting of trained QM Course Reviewers and a QM-certified Course Review Manager, self-evaluations using the rubric may also be conducted.

Strengthening course implementation in mathematics education. The Quality Matters rubric discussed above is a relatively straightforward process of standardizing course design. The rubric and associated peer feedback process provides designers and instructors with straightforward, tangible directives on *what* to do to improve the design of the course, as well as *how* to do it. Strengthening the implementation of a course is quite different and presents instructors of mathematics/mathematics education courses significant challenges. In an interview with Dr. Spencer prior to the first online session in the algebra class, she was asked about her perceptions of the usefulness of the original *Field Guide*, which she had a chance to review. At this time, she indicated that she indicated that she had found the Ragan (2008) checklists from the first part of the *Field Guide* easy to use for checking off tasks as she went along. When it came to the mathematical practices component, Dr. Spencer agreed that the *Field Guide* addressed critical issues, but noted that she had trouble envisioning how she might enact these practices in an online distance education environment. In order to help better illustrate how these actions might be carried out online, two adjustments might be made. First, a reorganization of the mathematical practices to show how the practices relate to each other. The first *Field Guide* contained much of this advice but was not organized into a cohesive framework. *The Principles to Actions: Ensuring Mathematical Success for All* published by the NCTM (2014) summarizes a set of eight mathematics teaching practices that provides a framework for strengthening the teaching and learning of mathematics in general. These principles may be as applicable to the online environment as they are in the face-to-face classroom. A second component of strengthening the online implementation of a mathematics/mathematics education course is the incorporation of the CoI framework (Garrison et al., 2000) discussed above.

Principles to actions. The National Council of Teachers of Mathematics has a long history in advocating for high-quality mathematics teaching and learning. In 2000, the NCTM published *Principles and Standards for School Mathematics* (NCTM, 2000). Among other things, this document established six guiding principles that, together, illuminated the organizations vision for teaching and learning in the 21st Century. In the *Principles to Actions: Ensuring Mathematical Success for All*, the NCTM (2014) updates these standards (Figure 5.1) in light of the changing landscape in mathematics education. Prominent among these guiding principles is the *Teaching and Learning Principle* (NCTM, 2014). While all of NCTM's principles have bearing on mathematics instruction, the *Teaching and Learning Principle* might be the most applicable to the discussion of learning mathematics online.

Guiding Principles for School Mathematics
Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote the ability to make sense of mathematical ideas and reason mathematically.
Access and Equity. An excellent mathematics program requires that all students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.
Curriculum. An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study between mathematics and the real world.
Tools and Technology. An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking
Assessment. An excellent mathematics program assures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.
Professionalism. In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every students and for their personal and collective growth towards effective teaching and learning of mathematics.

Figure 5.1. Guiding Principles for School Mathematics. Adapted from: National Council of Teachers of Mathematics (NCTM). (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM.

The Teaching and Learning Principle has become the centerpiece of NCTM's Guiding Principles of School Mathematics. The last 10 years has seen a push by all states to expand and implement rigorous mathematical standards headlined by the Common Core State Standards for Mathematics (CCSSM) (National Govonors Association Center for Best Practices, 2010). While these new principles and directions have succeeded in providing a general blueprint for mathematics achievement, instructors have had little guidance in how to implement instruction that fosters these ambitious learning goals. The goal of the *Principles to Actions* is to "fill this gap between the between the development and adoption of CCSSM and other standards, and the enactment of practices, policies, programs, and actions required for their widespread and successful implementation (NCTM, 2014, p. 4). The Teaching and Learning Principle offers a set of eight teaching practices (Figure 5.2) that guide teachers in their actions as they implement curriculum. Again, these teaching practices suggested by the NCTM were developed in the context of the traditional classroom but may be relevant to mathematics instruction delivered via online distance education as well.

8 Essential Mathematics Teaching Practices For Effective Teaching and Learning
Establish mathematics goals to focus learning. Effective teaching of mathematics established clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics constantly provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understandings and to adjust instruction continually in ways that support and extend learning

Figure 5.2. High-leverage teaching practices outlining effective teaching of mathematics. Adapted from: National Council of Teachers of Mathematics (NCTM). (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: NCTM.

Equally as important as the teaching practices themselves, is the teaching framework that interrelates these practices (Figure 5.3). Notice that goal setting sits atop this instructional pyramid. Regardless of the landscape (face-to-face or online distance education), the importance of this element is the same in both settings. Goals set the purpose of instruction.

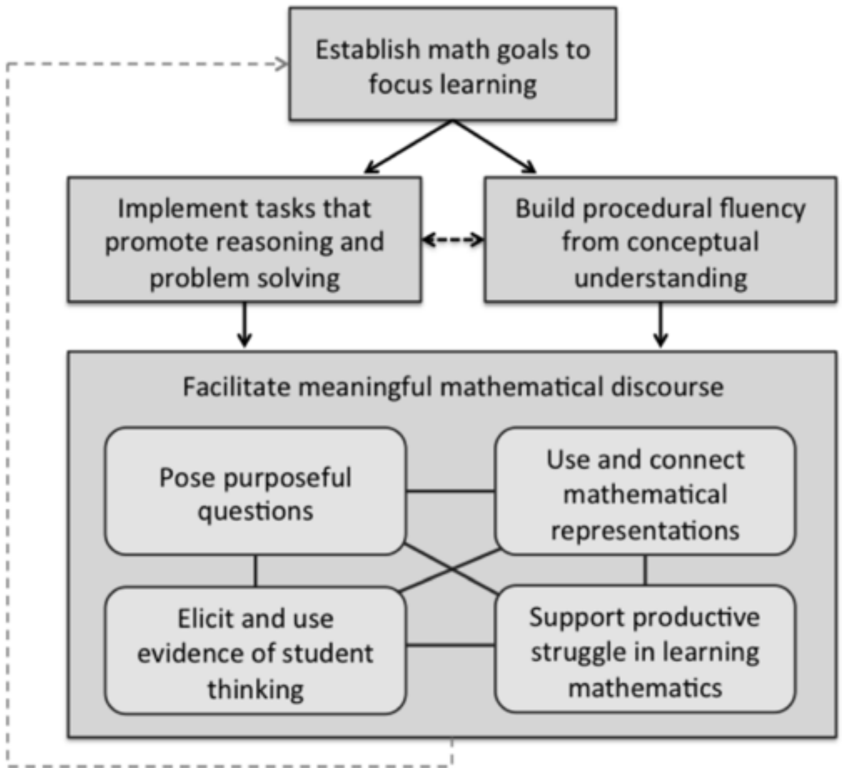


Figure 5.3. Teaching framework for mathematics highlighting the relationship between the eight effective teaching practices. Adapted from: Huinker, D., & Bill, V. (2017). *Taking action: Implementing effective mathematics teaching practices in K-5*. Reston, VA: NCTM.

The Quality Matters (2014) rubric can assist in helping make sure goals are present and help instructors align these goals throughout their course; however, the nature of mathematical goals requires additional thought on how goals should be constructed. Here, the instructor can turn to the *Principles to Actions* (NCTM, 2014) for guidance. In establishing math goals to focus learning, the NCTM offers guidance on what instructors need to pay attention to including:

- Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.

- Identifying how goals fit within a mathematics learning progression.
- Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how their current work contributes to their learning. And,
- Using the mathematics goals to guide lesson planning and reflection to make in-the-moment decisions during instruction (NCTM, 2014, p. 16).

The phrase "in-the-moment" may be the difficult part of the last bullet in this list as "in-the-moment" has a different meaning in online distance education than it does in the face-to-face classroom.

The second level of the framework addresses task development. Mathematical tasks provide the context through which students work to achieve the goals of the lesson. The landscape of online learning does play an important part in instructional decisions regarding the selection of tasks. This can be a particularly challenging aspect for online instructors. Dennen (2013) notes that recent scholarship has provided a proliferation of strategies, philosophies, and examples regarding the design of motivating and pedagogically sound online activities. She notes, however, that while this may seem like a good thing at first, variation muddies the waters around task selection and activity design. Instead of converging on and refining one set of strategies, diversity may make it more difficult for instructors to select appropriate tasks and tools. The *Principles to Actions* (NCTM, 2014) utilizes Stein and colleagues' (Smith & Stein, 1998; Stein et al., 1996) taxonomy of mathematical tasks to address the initial construction of tasks and the Mathematics Task Framework (Stein, Smith, Henningsen, & Silver, 2000) to remind instructors of the importance of presenting students with tasks that promote high-level reasoning and problem solving, and also allow students to build procedural fluency in mathematics by utilizing their conceptual understandings.

The teacher actions suggested by the NCTM form the basis of what a good mathematical task is and how it plays out in the classroom, but it is ignorant of the vagaries of online distance education settings. Dennen (2013) offers general advice that may prove helpful when adapting mathematical tasks to online settings. Speaking generally about the design of activities, Dennen notes that the temporal and special dimensions of online learning impact the appropriateness of particular pedagogical approaches and suggests that when considering the use of an online task, the instructor must consider the following questions:

- What are the desired learning outcomes?
- What types of interactions do I hope to foster?
- What are the desired outcomes of those interactions?
- How will both chronological and elapsed time be managed in the course?
- What level of synchronicity will be present?
- What tools might I use, taking into consideration access, learning curve, and user comfort? (Dennen, 2013, p. 283)

Considering both the mathematical task from the perspectives of the *Principles to Actions* in conjunction with the advice offered by Dennen may be a useful construct for mathematics educators teaching via online distance education.

The lower portion of the teaching framework involves discourse. Discourse has become the heart of the contemporary mathematics classroom, but is exceedingly complex to orchestrate. Fostering discussion in the mathematics classroom allows teachers to recognize and address areas of student misunderstanding, motivates students to become more interested in their peer's conjectures and justifications, and ultimately pushes students to deeper understanding of mathematical content by making them aware of inconsistencies between their own thinking and the thinking of others (Chapin et al., 2009). This teaching framework, situated around the essential mathematical teaching practices suggested by the NCTM, provides a serviceable model with which to facilitate meaningful discourse in mathematics classroom, face-to-face or via online distance education. The skills that teachers need to have to orchestrate discussion online are the same as those needed for the classroom. The online instructor, however, may need to be more deliberate in their actions. Again, the advice from Dennen seems apropos; the online mathematics instructor needs to think about the elements that foster mathematical discourse and the special and temporal dimensions added by the online distance education classroom.

A Framework for Online Course Improvement

Despite all of the advice offered by the *Principles to Actions* (2014), Dr. Spencer's question still remains: what does this look like online? Dennen's (2013) advice is a good start, but still does not provide the sort of detail that an instructor might need to improve their online course. To get a better sense of the 'hows' I return to the Communities of Inquiry (CoI) framework. With it's focus on the process of learning, the CoI framework compliments the QM framework that focuses on design to provide a complete learning experience for students. Swan and colleagues (Swan et al., 2014; Swan et al., 2012) have offered what may be a useful model for course development and improvement that instructors of mathematics/mathematics education may wish to use and adapt to their needs. This process involves a two-pronged approach whereby course design is evaluated using the QM rubric and then revised after teaching with a post-evaluation of the levels of teaching, cognitive, and social presence with a CoI measurement survey (Swan et al., 2008).

In their work, Swan and colleagues (Swan et al., 2014; Swan et al., 2012) use a design-based research study of their own online practice to systematically improve their courses using an iterative process of design, development, implementation, and analysis in a real-world setting. The initial purpose of the research team was to investigate how course design might result in an improved learning process, which in turn would enhance student learning outcomes. Ultimately the researchers realized that design and process are orthogonal to learning outcomes as they approach learning from different perspectives. As such they suggest a trajectory of incremental improvements that begin with a QM review and then adjust course activities based on the results from a post-course CoI survey. In mathematics/mathematics education post course CoI redesign might be guided by the eight essential teaching practices suggested in NCTM's (2014) *Principles to Actions*.

Promising Practice

Looking across Dr. Spencer's algebra course, there were many instances where her practice is exemplar of what online practice could look like in mathematics/mathematics education. While she was not explicitly using the frameworks presented in this chapter, she had an implicit knowledge of the mathematical frameworks. Additionally, she did a nice job leveraging the technologies she had available to deliver content. Of course, there were places that she could improve, but the purpose of this investigation is to point out what she did well so that it might provide a model of practice for other instructors. To this end, I mention three episodes of practice that exemplify some of the principles and actions discussed previously in this chapter. First, the algebra course utilized a unique combination of synchronous and asynchronous interactions. The inclusion of the synchronous sessions seemed to play a critical role in the overall flow of the course as they 'linked' content from one asynchronous module to another. This notion of linking modules together through the use of synchronous sessions, which the instructor referred to as "bridging sessions", will be explored here. The second episode discusses some features of task design that were present in Dr. Spencer's selection of synchronous and asynchronous course activities. Lastly, the third episode describes the way in which the instructor attempted to manage elapsed time over the course of a module in ways that had the potential to enhance online mathematical discussions. When applicable the researcher will interject suggestions for actions that may enhance what is already good practice.

"Bridging": Making use of Important Instructional Real Estate

The course at the focus of this dissertation utilized a combination of asynchronous discussions and activities combined with synchronous 'bridging' sessions. The inclusion of both asynchronous and synchronous activities has tradeoffs in a distance education course. One of the elements of distance education that students seem most attracted to is the freedom from a traditional course schedule (E. Allen & J. Seaman, 2013). The addition of synchronous activities constrains the freedoms that some online students are looking for. In the algebra course, students were required to set aside a three-hour chunk of their time, approximately every other week. The trade-off being that they still did not have to travel to campus for class, a feature that multiple students in this class told the researcher would have made their participation in the course unlikely. Aside from issues of convenience and not having to physically attend class, the synchronous sessions provide advantages on the theoretical side of the educational experience. While the synchronous sessions do not provide the levels of intimacy that face-to-face experiences often do, by participating 'ear-to-ear' students are still getting many of the benefits of real time interaction that they would in a classroom. First, students have the ability to ask the instructor in-the-moment questions about material and assignments reducing the chance for misunderstanding. Second, synchronous sessions also allow for the development of social presence in the online classroom, an element that some suggest translates to greater levels of cognitive presence (Wang & Chen, 2008) and ultimately a more satisfying online educational experience (Swan et al., 2014; Swan et al., 2012).

Just like time in the classroom, time in synchronous sessions is valuable and fleeting. The synchronous sessions conducted during the algebra course occurred on time every other week for approximately 3 hours. Given the relatively small direct contact the instructor had with participants during these synchronous sessions, Dr. Spencer had to make instructional decisions for these valuable three-hour sessions. In an interview prior to the first synchronous online session, Dr. Spencer expressed what she felt was so valuable about these sessions.

> The question really became (pause) these [synchronous sessions] seems like prime real estate. It is the only time you get to interact with them and have real discussion versus just written discussion forums. So, what's the most important thing that goes on there? And so, I tried to figure out – how am I going to use that time? Doing some sort of lecture – information presentation – just doesn't feel like a good use of that time. Because it felt like that was something that could be done in a screen shot or a screen cast or a PowerPoint. It's just not a good use of that real estate.

Dr. Spencer stayed pretty true to her pre-course thoughts with regards to synchronous sessions and, while there were isolated times where she would use synchronous online time to present new material, the vast majority of the time in these sessions was dedicated either to small group breakout discussions or the whole group debrief of these events. With the level of interaction set, Dr. Spencer turned to the content of these online sessions.

To get a sense of what to do in these synchronous sessions, Dr. Spencer turned to her previous experiences teaching online. Many online teaching resources advocate for instructors to modularize course content as part of an effort to standardize course design

(Dykman & Davis, 2008a). This is advice that Dr. Spencer had found helpful in the past and was, in fact, the way she organized the algebra course. Dr. Spencer noted, however that past courses taught with the modular structure seemed choppy. Referring to the various modules, she stated:

> ... it didn't feel connected. It felt like, now we are on to something else... and now we are on to something else... and now we are on to something else... and I needed a way (pause) to help students to make the connection. To be sure that – at least to be sure that they were somehow making connections to the next part.

In this interview, Dr. Spencer noted a variety of ways that she planned to try to reduce this sensation over the course of the semester, but the 'bridging' sessions stood out as particularly successful episodes and seemed to really capture the essence of adapting instruction online.

The goals for the bridging sessions themselves were student oriented in the sense that they often focused on the work that students had completed in the last module and looked forward to the sorts of activities that they would do in the upcoming module. Dr. Spencer felt that this reflective look back and projective look forward was important:

> ...because if it is just a series of isolated assignments, even if I see how they are related, I'm not always sure my students do – and I can step back and (inaudible). So, I really do see that Saba Meeting session as a place to take what I have seen of their work in the modules and do any – course adjustments ... or bring up something that maybe I wasn't even hitting, ...or highlight things that were really neat that I hadn't thought of. A way to finish that all off and then say how that (inaudible) so that I'm making sure they're seeing all of the connections but also framing where I'd like to get to.

Bridging session hit on a variety of topics, but tended to be chances for students to make connections to readings or assignments. For example, Dr. Spencer always included a 'bridging' discussion related to participants' Questioning and Student Work Analysis (QSWA) assignment in the synchronous session between the module where they completed their planning chart and taught their lesson, and the module where they were required to submit the reflection of their experience. This allowed students to share some of their in-class observations and gather advice from their peers on the experience.

Issues in Task Design: Choosing the Synchronicity of an Activity

It is easy to forget that synchronous tasks and asynchronous tasks serve vastly different purposes. Too often, a synchronous task becomes asynchronous when we "can't fit it into the class period". In the face-to-face setting, we may be tempted to assign the task for homework. Online, we may decide to make it a forum discussion or blog. This is not always appropriate in either context. The synchronicity of the event plays an important part in how students engage in the task (Dennen, 2013). Aviv (2000) found that the design and structure of asynchronous discussion events has an effect on the cognitive engagement of students. In asynchronous engagements without the proper design and structure, students took a more passive role in discussions and their cognitive engagement was low. Thus, the synchronicity of the event should be prominently considered in the design of the course.

In the execution of the algebra course, Dr. Spencer seemed to have an intuitive grasp of the sorts of prompts appropriate for synchronous and asynchronous activities. An example of this was her incorporation of the Team Problem Solving events as featured asynchronous activities (see Chapter 4 for an in-depth description of the "Coin Problem", a Team Problem Solving event from Module 2). Team Problem Solving prompts were designed as small-group asynchronous discussions featuring a mathematical investigation related to the module's goal. In these Team Problem Solving events, students in the algebra class were encouraged to add to the discussion by expressing their initial thoughts and evolving ideas, argue for or against points being made by others, answer questions posed to the group by their peers, and offer alternative mathematical perspectives to the discussion.

The asynchronous nature of the Team Problem Solving events seemed to have a variety of positive effects for both participants and the instructor. First, because participants could post responses at any time of the day, they could choose to do so at a time when they were most free from distractions. While many face-to-face and synchronous discussions can manifest as spontaneous or transitory in nature (Wang & Chen, 2008), the asynchronous Team Problem Solving events allowed participants the time to absorb the opinions expressed by others in the thread as well as formulate their own thoughts before posting themselves. Because Team Problem Solving questions contained the ‘heaviest’ mathematics topics, many of the participants may have needed the extra processing time afforded by the asynchronous nature of the forum discussion.

Of benefit to both participants and the instructor was the permanent nature of the asynchronous discussion. Keeping discussion around these mathematical problems in an online forum created a permanence to the conversation that is often not possible in a face-to-face discussion. This permanence creates a record of all student thinking that went into the solving of the problem. This allowed participants the opportunity to review their thinking and the thinking of others at any point in the discussion. For the instructor, creating this record slowed the pace of the discussion providing her the time to better monitor the event. Because the instructor has so many things to orchestrate in synchronous sessions, monitoring what is going on in groups can be difficult. By reserving challenging material for asynchronous, Dr. Spencer could spend more time to get a sense of student thinking related to the question. Issues arising in the asynchronous sessions could then be brought into the synchronous sessions and discussed further with the whole group.

Synchronous activities in the algebra course, on the other hand, often took a more spontaneous nature. When mathematical topics were addressed, they were more exploratory in nature. Often this might mean investigating some sort of topic intended to launch or preview the upcoming module. These explorations did not require so much a resolution as they did a starting point to begin to think about a topic. Often, however, the synchronous small group discussions and activities allowed participants to have spontaneous interactions that were more casual in nature. These sorts of interactions were important in developing a sense of comradery in the group, an issue closely related to social presence.

Managing Participation in Asynchronous Discussions

It is often suggested that participation has a positive effect on perceived learning (Fredericksen, Pickett, Shea, Pelz, & Swan, 2000; Swan, 2001), grades (Davies & Graff, 2005), and student satisfaction in online classes. Participation, however, is a difficult construct to define. What constitutes participation is currently an issue of great debate as many of the behaviors that we think of as participation are obscured or invisible in the online environment. Dr. Spencer had an interesting approach to student participation in asynchronous discussions that may have some merit moving forward. While many instructors have minimum posting requirements, Dr. Spencer combined minimum number of posts with guidelines for the distribution of student posts over the course of the two-week module. Students in the algebra course were expected to make a minimum of three posts over the course of the module, however, those three posts had to occur on three different days.

One of the suggestions for activity design suggested by Dennen (2013) is that instructors and designers need to consider “how both chronological and elapsed time are managed in the course” (p. 283). When students participate in online discussions all at once, there is the risk that they really do not engage in true conversations at all. If the only directive that students are given is that they have to participate in a forum three times, they might possibly make a series of posts on one day, at one time. This allows participants to circumvent the spirit of the discussion forum and allows the student to present monologues about an idea or topic rather than dialogue with their peers. The inclusion of a requirement to post on different days attempts to ensure that turn-taking, the hallmark of discussions, occurs between participants of the conversation. Of course, this does not ensure that students will engage in true discussions. As described in Chapter 4, some students found creative ways to get around this requirement. One student constructed a series of posts in one sitting and then released the posts on three different days.

Dr. Spencer's attempt to distribute participation may not have achieved its goal with all students, but it brings up the important issue of "what constitutes participation?". When comparing online discussions to those that happen in a classroom, it should be noted that in face-to-face discussions, instructors do not require all students to 'say something' in order to earn participation credit. Active listening in discussions is often just as valorized as showing interest in the thoughts of others is an important part of non-verbal communication. The idea of showing simple forms of interest in what others say online has been near-perfected by social media where one person can show interest or empathize with another person with a click of a 'like' button. The social media site Facebook has added the ability to 'react' to the posts of others through a number of easily accessible emoticons that express emotion such as displeasure, laughter, and sadness. In some cases this might be a welcome addition to the asynchronous discussion forums used in class.

Other forms of non-verbal communication might be considered participation as well. In a study by Dennen (2008), students reported that they benefited from both the reading of and construction of posts. However, since posting behaviors are the most visible, they are often what counts as participation online. The Dennen study also noted that students who only posted to meet course requirements and focused on posting behaviors more than reading the post of peers had less positive impressions of the class experience. In the algebra class, Dr. Spencer's 'three different day' rule may have added value to the discussion for some, however, clearly others merely posted to fulfil the requirements of the course. Nonetheless, Dr. Spencer's attempt at distributing participation highlights an important issue in facilitating discussions.

Limitations, Future Research, and Conclusions

One limitation of this research was the lack of the ability of the researcher to control various aspects of course implementation. For example, the number of course participants was a variable that had deleterious implications on the implementation of course content. Twenty-seven participants may have been too many for one instructor to facilitate effectively. As such, the instructor seemed unable to return assignments in a timely manner. The large number of students also appeared to make managing discussions difficult, both synchronously and asynchronously. In addition to the large number of course participants, inconsistency in the course schedule may have had a negative impact on the course. There had been eight planned synchronous 'bridging' sessions scheduled for the course. Of these eight scheduled events, only six actually occurred. One synchronous session was cancelled due to severe weather across the state while another was cancelled due to an instructor illness. A third synchronous session was rescheduled from its original date, thus preventing a number of participants from attending that class as well.

An additional potential limitation to this study was the choice of methodology. This study represented an exploratory case study on the design and implementation of courses in mathematics/mathematics education. Case study was chosen for this work because of its adaptability to changing conditions over which the researcher has little control. Due to its exploratory nature, the choice of methodology allowed the researcher to explore and describe 'what was happening' in the course being studied. The methodology was not rich enough to discuss systematic changes that may or may not have been made by the instructor during the course of teaching, nor does the methodology address how or why particular actions took place.

As an alternative, future investigations might consider design research as a viable methodology for this type of research. McKenny and Reeves (2013) describe design research as "a genre of researcher in which the iterative development of solutions to practical and complex educational problems also provides the context for empirical investigation, that yields theoretical understanding that can inform the work of others" (McKenney & Reeves, 2013, p. 7). When approaching revisions to the *Field Guide* document, an iterative design spanning multiple semesters focusing on more than one case may be preferable to be able to make substantive improvements.

Emerging from this case study, however, is a critical issue worthy of future consideration. As the popularity of online education and professional development grows, the issue of how to design and systematically improve online distance education experiences in mathematics/mathematics education will become forefront. Swan suggests (Swan et al., 2014; Swan et al., 2012) a method for iterative course design that has merit. The challenge for mathematics/mathematics education is to look at the ways in which mathematical tasks fit within the Communities of Inquiry framework, particularly with respect to cognitive presence. To help in this task, course designers and instructors might look to the Principles to Actions framework offed by the NCTM as these principles and the CoI framework share much in common.

An important part of making these two frameworks work together will be in the way that instructors approach online discourse. This includes factors such as (1) developing a clearer understanding of how the synchronicity of a task affects the type of dialogue that is created, (2) developing a clearer understanding of the meaning of online participation, and (3) gaining a better grasp on how discourse is facilitated by the online instructor. To solve some of these dilemmas, we might turn to work that has already been done in the classroom and attempt to adapt this work to the online classroom. For example, Chapin, O'Connor, and Anderson (2009) have suggested a set of five fundamental teacher actions (dubbed *Talk Moves*) that promote discourse in the face to face classroom. One wonders what this set of *Talk Moves* might look like in the asynchronous online discussions for instance.

In conclusion, all directions point to the fact that online distance education has become a permanent feature of the educational landscape. There is no denying that teaching online with continuously emerging technology is complex and offers the instructor a number of challenges. Although teaching online is clearly different from face-to-face teaching, there are many similarities between the two. In fact, teaching online might highlight important pedagogical practices that can ultimately help instructors in both settings. The focus of this research has been to highlight the ways that one mathematics education instructor, with an expertise in high leverage mathematical teaching practices, adapted her instruction for use in an online course.

APPENDIX A: TD SURVEY

Default Question Block

Interpersonal dialog contributed to my learning.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I could easily contact the other students when I've ever felt the need.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Instructors and other students supported me when I couldn't understand something.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Interpersonal dialogues motivated me in this class.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to get feedback on my work related to courses.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to share my knowledge with the instructor and other students.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Instructor encouraged students to collaborate with others.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Instructors and other students encouraged and supported me to join activities.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Structure Flexibility

Courses included more flexible and adaptable ways of learning.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to access the courses over again when I wanted

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to follow learning content in the order which I wanted.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Synchronous and asynchronous classes were well organized with regards to flow.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Synchronous classes were inconvenient and hampered my ability to learn.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to access the entire contents of the course at any time.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Content Organization

Presentation of the course content was appropriate for my learning approach.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

The instructor provided a sufficient amount of context to the content of the course

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Content was well organized in the on-line environment

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

The content of the course included a good mix of class activities, "real-world" applications and assessment.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Course content met my needs as a learner

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to access all learning resources from the system when I wanted

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I often felt confused about what to do when I logged onto the Blackboard site.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

The course content made it easy to participate in the learning process

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Learner Control

I have the study skills that I need to be successful in a class like this.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I have the ability to manage my time wisely in a course like this.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was given the opportunity to select the people that I would work with and take responsibility in those groups.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I was able to motivate myself to learn in this class

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I had the chance to suggest topics that I wanted to learn more about in this class

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Autonomy - Some of these questions are about you as a learner

I am able to follow a personalized study-plan which I set for myself

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

In the past, I have been successful learning without the need for interpersonal communication.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

In this class, I was able to learn at my own pace

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

In this class, I was able to direct my own learning

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I did not need much guidance in this course

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

I did not need to rely on my peers in this course due to clear "troubleshooting" protocols.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

Synchronous class sessions prevented me from working independently.

Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Disagree
○	○	○	○	○

APPENDIX B: SYLLABUS

Department of Mathematics, Science, & Instructional Technology Education
[redacted]
Spring 2016

[redacted] Algebraic Reasoning: K-5 Discourse & Questioning

Instructor: [redacted]
Office: [redacted]
Telephone: [redacted]
Email: [redacted]u (This is the best way to reach me.)
Office Hours: By appointment; daily on via email, Skype, or phone (allow 48 hours on weekends)

Textbooks:
Required:
Blanton, M. L. (2008). *Algebra and the elementary classroom*. Heinemann: Portsmouth, NH.
Carpenter T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in the elementary school*. Heinemann: Portsmouth, NH.

Recommended:
Chapin, S., O'Connor, C., & Anderson, N. (2009). Classroom Discussions: Using Math Talk to Help Students Learn, Grades, K-6 (Second Edition). Sausalito, CA: Math Solutions.

Meeting Times:
Synchronous Online (SabaMeeting): Jan. 13, Feb. 1 (Monday), 10, 24, March 9, 23, April 6, 20
Asynchronous Online (Blackboard): Ongoing

Course Description: This course aims to develop the content and pedagogical content knowledge of in-service elementary teachers in the areas of Algebra and Algebraic Reasoning. Algebra is a content standard listed by the National Council of Teachers of Mathematics beginning at the Pre-K level and extending through grade 12. *Early Algebra* is of particular importance to children and is foundational in helping them to reason and problem-solve – especially in the areas of arithmetic and functions. By engaging in and analyzing activities which emphasize algebraic reasoning, students of this course will develop an understanding of the essential approaches vital in teaching algebra effectively. This course is also designed to help teachers examine and improve their questioning and classroom discourse.

Course Objectives:
1. Implement a variety of appropriate instructional strategies to assist elementary children in constructing algebraic ideas.
2. Demonstrate content knowledge in K-8 algebraic thinking based upon national standards (i.e. Common Core State Standards, NCTM – National Council of Teachers of Mathematics Process and Content standards).
3. Understand patterns, relations, and functions from a variety of perspectives.
4. Understand the role of properties in number systems and their use in computation.
5. Represent mathematical situations and structures using algebraic symbols.
6. Prove mathematical conjectures.
7. Use mathematical models to represent and understand quantitative relationships.
8. Demonstrate an understanding of how to facilitate discourse to elicit algebraic reasoning in elementary classrooms.
9. Demonstrate an understanding of the assessment of algebraic reasoning in elementary classrooms through questioning and listening to students, analyzing students' written work, documenting patterns of students' thinking and planning appropriate student/teacher interactions.

MAJOR COURSE COMPONENTS

All written items should include a professional standard of spelling, grammar and punctuation. Cohesion of thought, clarity of expression, depth of reading, analysis of issues and relevance of discussion will need to be evident. Use of appropriate referencing style, use of headings and subtitles if necessary and reference list will be standard requirements for each assignment.

Grades will be determined in the following manner:

1.	Homework Assignments	30%
2.	Class Participation/Attendance	20%
3.	Questioning & Discourse Assessment	25%
4.	Final Assessment (content exam/course portfolio)	25%

Grading Scale:	A 90-100	C 70-79
	B 80-89	D 60-69

Homework Assignments (30 %)

Each module you will complete a variety of activities including readings from the text, watching video, problem-solving activities, interactions or lessons with students, and writing responses. During the online portion of the course, these assignments will be opened every two weeks. **You must complete the assignments for the module within the time frame they are assigned.** Once the window for each module has ended, you will no longer be able to submit assignments.

When you click on each module on the course menu you will see an introductory post giving the dates the module will be open and a list of the items that need to be completed for the week. Underneath the introductory post will be the materials needed to complete the week's work.

Regularly, you are expected to post to Blogs or Discussion Forums about assigned reading or problems. These comments are graded on a 3-point rubric as follows:

Reading Responses

- 3 – Response addresses the prompt with references to the readings for the week, and when appropriate, earlier readings. Opinions, observations, and/or past experiences are thoughtfully related to the readings throughout the response.
- 2 – Response addresses the prompt with references to the readings for the week. Connections between the readings and opinions, observations, and/or past experiences are present.
- 1 – Response submitted either does not address the prompt or does not refer to the readings.

Problem-Solving Responses

- 3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response.
- 2 – Response shares ongoing work the problem with explanations of your thinking.
- 1 – Response submitted either does not include your work on the problem or does not explain your thinking.

***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.)

Class Participation/Attendance (20%)
This course is an opportunity for you to become part of a community of learners who are committed to learning about teaching and learning through reading, writing, discussing, and collaborating. Your overall involvement in the course includes the following:

- ***Intellectual risk taking:*** demonstrated willingness to offer and pursue ideas and suggestions that go beyond the ordinary
- ***Making connections:*** demonstrated ability to connect the theoretical and the practical, to relate specific ideas to larger themes
- ***Thinking clearly on paper:*** demonstrated proficiency in expressing ideas, organizing information, and communicating in writing
- ***Contributing to the community:*** demonstrated willingness to share information and ideas with the group and to support others in their efforts to build understanding
- ***Commitment to developing listening and speaking skills:*** demonstrated effort to develop effective speaking skills and active listening and responding skills
- ***Commitment to exploring new ways to think about teaching and learning mathematics:*** demonstrated willingness to being open to trying out new ways of teaching mathematics and to allowing children opportunities to make sense of mathematics.

Attendance Policy: Class attendance and participation are required. The classes are designed to be a forum for us to engage ideas through discussions and in-class activities that supplement rather than reiterate the readings. Since much of the course is collaborative in nature, your presence is needed (and desired)!

If you are absent (not present for an online session), you are unable to participate or support your colleagues in the course, and this necessarily impacts your final course grade. You are welcome to complete any activities that can be done outside of class, of course, but you cannot make-up for missed conversations or the insights you might have provided the class during our discussions. If you must miss a synchronous class session, please inform me beforehand and arrange to make-up work missed. Missing more than one class may affect your final course grade significantly, such as dropping a letter grade.

I am aware that we are all adults with busy lives and that family, work, or medical emergencies will unfortunately happen. I am willing to discuss these situations with you and consider possible alternative arrangements.

Questioning & Discourse Assessment (25%)
During the course you are to spend time interacting with a small group of children (Grades K-5) to work on algebraic thinking tasks. In each of the modules, you will work on a task that you choose from your textbooks. We will analyze the students' algebraic reasoning and how your questioning both elicits student thinking and provides opportunities for student learning. Prior to each SabaMeeting session you are to complete a questioning planning grid, teach the lesson (either small or whole group), and complete the chart, "Analyzing Students' Written Work". In addition, *you will record the session in order to analyze your questioning*. Bring your work back to our synchronous class discussions to share ideas with others and raise questions about what you noticed. You will submit a written reflection of the student assessments to be included in your final portfolio.

Final Assessment (25%)
The final assessment consists of two parts, a content exam and a course portfolio.

Final Content Exam --The final exam will cover algebraic content including generalized arithmetic (from number and operations to algebra) and functional thinking. This exam will be given on Blackboard as a take home exam.

Course Portfolio --The portfolio provides opportunity for you to demonstrate your achievement of the Algebraic Reasoning Course goals. The portfolio has 3 parts: Learning, Implementing, and Leading. It should include a collection of reflections, classroom lessons, and other evidences that document your growth in understanding and practice as a result of participating in the Algebraic Reasoning Course.

In the Learning section you should include two reflections addressing any problem-solving/activities encountered during the class that were challenging or engaging for you. The reflections should describe what you learned about your own algebraic reasoning and the implications the experience has for your future teaching.

The Implementing part of your portfolio should include artifacts from your teaching. Choose one of the four activities you completed with students through the course to teach again with a different group of students. Revise your questioning planning grid for the activity. Complete another student work analysis after the lesson. In this section of your portfolio, include the activity, the completed planning grids, the student work, your analysis, and a reflection discussing your use of discourse and questioning and its impact on the algebraic reasoning of your students. Also, describe how and why you changed the activity from the first time you taught it.

Finally, as a math specialist, explain how you will lead in the future by considering the course as a whole. The Leading portion of the portfolio should give an overview of how you have grown as a result of the algebraic reasoning class, how it has impacted your teaching, and how you might lead in the future. Some questions to consider are: *How has this class changed your approach to teaching? What impact has it had on student learning? How can you support other teachers in your school or district in their mathematics teaching?*

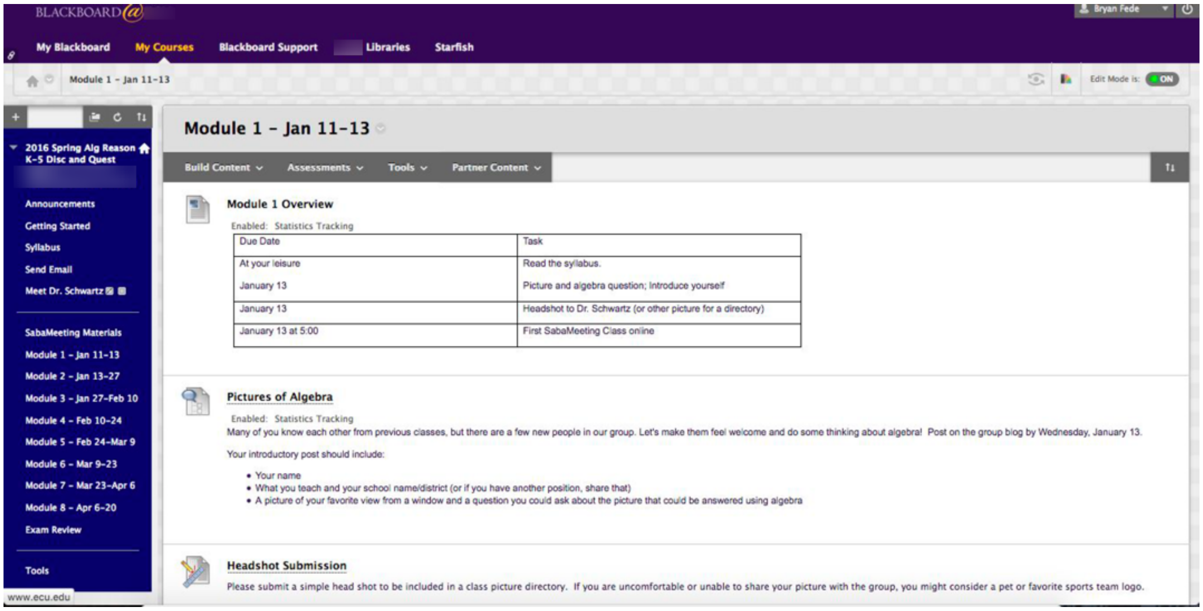
The portfolio must include typed reflective papers (size 12 font, Times New Roman, double-spaced).

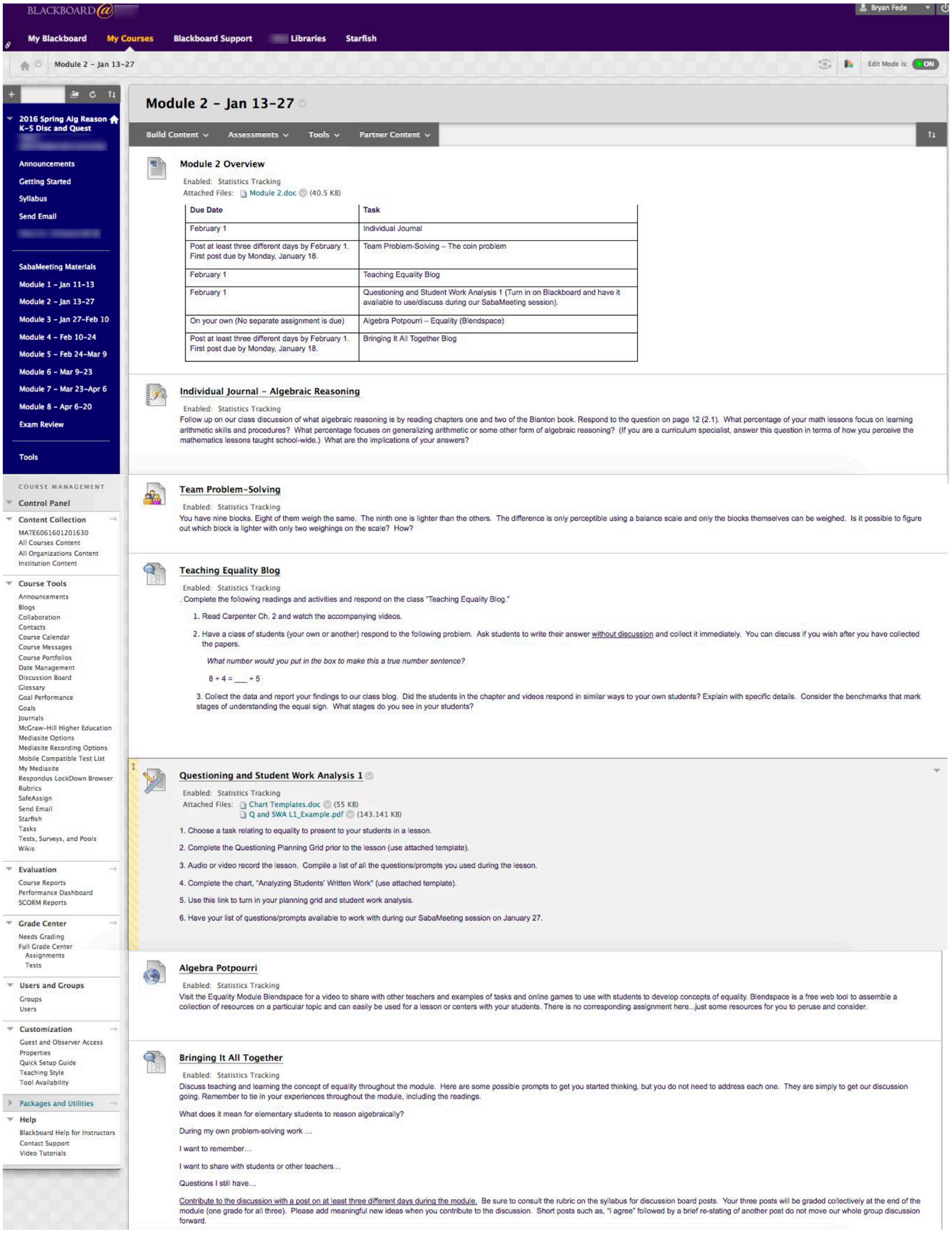
OTHER INFORMATION

Academic Integrity: In keeping with [redacted] policy on Academic Integrity found in the Code of Student Conduct, students are expected to be the sole contributor to work bearing their name, except where group projects have been assigned. Students are expected to follow the University's honor code. Please see the website [redacted] ***Any violation of the academic integrity policy will result in a grade of F for the course, regardless of the student's current average in the class.***

Academic Accommodations: [redacted] University seeks to comply fully with the Americans with Disabilities Act (ADA). Students requesting accommodations based on a disability must be registered with the Department for Disability Support Services located in [redacted] ([redacted] (Voice) / [redacted]). More information for students is also located at https://www.[redacted]/studentlife/dss/For_Students.asp.

APPENDIX C: MODULE SCREENSHOTS





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Bryan Fede

Module 2 - Jan 13-27

Edit Mode is: ON

2016 Spring Alg Reason K-5 Disc and Quest

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Mediasite Options
Mediasite Recording Options
Mobile Compatible Test List
My Mediasite
Respondus LockDown Browser
Rubrics
SafeAssign
Send Email
Starfish
Tasks
Tests, Surveys, and Pools
Wikis

Evaluation
Course Reports
Performance Dashboard
SCORM Reports

Grade Center
Needs Grading
Full Grade Center
Assignments
Tests

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Users

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Guest and Observer Access
Properties
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Teaching Style
Tool Availability

Packages and Utilities

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Video Tutorials

Module 2 - Jan 13-27

Build Content | Assessments | Tools | Partner Content

Module 2 Overview

Enabled: Statistics Tracking
Attached Files: Module 2.doc (40.5 KB)

Due Date	Task
February 1	Individual Journal
Post at least three different days by February 1. First post due by Monday, January 18.	Team Problem-Solving – The coin problem
February 1	Teaching Equality Blog
February 1	Questioning and Student Work Analysis 1 (Turn in on Blackboard and have it available to use/discuss during our SabaMeeting session).
On your own (No separate assignment is due)	Algebra Potpourri – Equality (Blendspace)
Post at least three different days by February 1. First post due by Monday, January 18.	Bringing It All Together Blog

Individual Journal - Algebraic Reasoning

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Follow up on our class discussion of what algebraic reasoning is by reading chapters one and two of the Blanton book. Respond to the question on page 12 (2.1). What percentage of your math lessons focus on learning arithmetic skills and procedures? What percentage focuses on generalizing arithmetic or some other form of algebraic reasoning? (If you are a curriculum specialist, answer this question in terms of how you perceive the mathematics lessons taught school-wide.) What are the implications of your answers?

Team Problem-Solving

Enabled: Statistics Tracking

You have nine blocks. Eight of them weigh the same. The ninth one is lighter than the others. The difference is only perceptible using a balance scale and only the blocks themselves can be weighed. Is it possible to figure out which block is lighter with only two weighings on the scale? How?

Teaching Equality Blog

Enabled: Statistics Tracking

. Complete the following readings and activities and respond on the class "Teaching Equality Blog."

1. Read Carpenter Ch. 2 and watch the accompanying videos.
2. Have a class of students (your own or another) respond to the following problem. Ask students to write their answer without discussion and collect it immediately. You can discuss if you wish after you have collected the papers.

   What number would you put in the box to make this a true number sentence?

   8 + 4 = ___ + 5

3. Collect the data and report your findings to our class blog. Did the students in the chapter and videos respond in similar ways to your own students? Explain with specific details. Consider the benchmarks that mark stages of understanding the equal sign. What stages do you see in your students?

Questioning and Student Work Analysis 1

Enabled: Statistics Tracking
Attached Files: Chart Templates.doc (55 KB)
Q and SWA L1_Example.pdf (143.141 KB)

1. Choose a task relating to equality to present to your students in a lesson.
2. Complete the Questioning Planning Grid prior to the lesson (use attached template).
3. Audio or video record the lesson. Compile a list of all the questions/prompts you used during the lesson.
4. Complete the chart, "Analyzing Students' Written Work" (use attached template).
5. Use this link to turn in your planning grid and student work analysis.
6. Have your list of questions/prompts available to work with during our SabaMeeting session on January 27.

Algebra Potpourri

Enabled: Statistics Tracking

Visit the Equality Module Blendspace for a video to share with other teachers and examples of tasks and online games to use with students to develop concepts of equality. Blendspace is a free web tool to assemble a collection of resources on a particular topic and can easily be used for a lesson or centers with your students. There is no corresponding assignment here...just some resources for you to peruse and consider.

Bringing It All Together

Enabled: Statistics Tracking

Discuss teaching and learning the concept of equality throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

What does it mean for elementary students to reason algebraically?

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

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Module 3 – Jan 27–Feb 10 | Edit Mode is: ON

2016 Spring Alg Reason K-5 Disc and Quest

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COURSE MANAGEMENT

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Course Tools: Announcements, Blogs, Collaboration, Contacts, Course Calendar, Course Messages, Course Portfolios, Date Management, Discussion Board, Glossary, Goal Performance, Goals, Journals, McGraw-Hill Higher Education, Mediasite Options, Mediasite Recording Options, Mobile Compatible Test List, My Mediasite, Respondus LockDown Browser, Rubrics, SafeAssign, Send Email, Starfish, Tasks, Tests, Surveys, and Pools, Wikis

Evaluation: Course Reports, Performance Dashboard, SCORM Reports

Grade Center: Needs Grading, Full Grade Center, Assignments, Tests

Users and Groups: Groups, Users

Customization: Guest and Observer Access, Properties, Quick Setup Guide, Teaching Style, Tool Availability

Packages and Utilities

Help: Blackboard Help for Instructors, Contact Support, Video Tutorials

Module 3 – Jan 27–Feb 10

Build Content | Assessments | Tools | Partner Content

Module 3 Overview

Enabled: Statistics Tracking

Attached Files: Module 3.pdf (93.928 KB)

Due Date	Task
February 9	Analyze Questioning from Lesson 1
February 9	Relational Thinking Assignment
February 9 (Post at least three times with first post by Feb. 6)	Team Problem-Solving
February 7	Relational Thinking Interview Assessment with Individual Student
February 9 (Post at least three times with first post by Feb. 6)	Bringing It All Together 2
February 16	Questioning and Student Work Analysis 2

Analyze Questioning from Lesson 1

Enabled: Statistics Tracking

Attached Files: Moyer_Milewicz_2002.pdf (112.115 KB)
talk_moves_map_oconnor_4-27-13 copy.pdf (1.008 MB)

Look back at your Questioning and Student Work Analysis 1 that you submitted in the last module to further analyze your questioning and talk moves during the implementation of the lesson (see Power Point from SabaMeeting 2/1/16 and attached documents).

Write a reflection discussing the common questioning mistakes or talk moves that you saw in your own teaching. Be sure your reflection includes:

- examples (i.e. a short exchange transcribed from your audio or an attached audio or video, etc.)
- data (# of times you had a student comment on another's thinking; wait time seconds, etc.)
- a general take away message about where you are now in your questioning and what you will try to accomplish in your next lesson.

Link to turn in Questioning Analysis

Enabled: Statistics Tracking

Relational Thinking Assignment

Enabled: Statistics Tracking

Read Chapter 3 in the Carpenter book and watch the corresponding videos.

Complete the following exercises from the end of the chapter: #1, 3, 4, and 5. For number five, write a set that would be appropriate for the grade in which you work (if you span multiple grades, you may choose). For example, with Kindergarten, the sentences might focus on the one more/one less relationship and use pictures instead of/in addition to symbols. Upper elementary teachers may want to focus on multiplication or division or fractions. Grades one through three might choose addition facts or place value concepts.

Team Problem-Solving -- Sheep Problem

Enabled: Statistics Tracking

One shepherd says to another, "Give me 4 sheep, and then we will have an equal number." The other answers, "No, you give me 4 sheep and then I will have twice as many as you." How many sheep did each shepherd have? How do you know? (In a few days an eight year old student's solution will be posted.)

Sonya's Solution to the shepherd problem (age eight)

If one were to give the other 4 sheep and they would then have an equal number, that means they have a difference of 8 sheep. If, on the other hand, the other gives away 4, then the difference becomes 16 (since one loses 4 and the other gains 4 sheep). And then we get that one has 2 times as many, or 16 sheep more. This means there will be 16 and 32, and before the exchange there were 20 and 28.

Solved in 40 seconds as reported by V.A. Kruetetskii in the Psychology of Mathematical Abilities in School Children, 1976

Relational Thinking Interview Assessment

Enabled: Statistics Tracking

Attached Files: Relational_Thinking_baseline_assessment.doc (322.5 KB)

Find one child in grades 3-7 to interview. Complete the relational thinking interview assessment with the student. Turn in scan of the student's sheet and your scoring sheet on Blackboard. If the student was in a grade above 3-5, please note that somewhere on the recording sheet. I will compile our class results to share at our next synchronous session.

Bringing It All Together 3

Discuss teaching and learning of relational thinking throughout the module. Why is relational thinking so important in elementary school?

Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

Looking ahead…Questioning and Student Work Analysis 2

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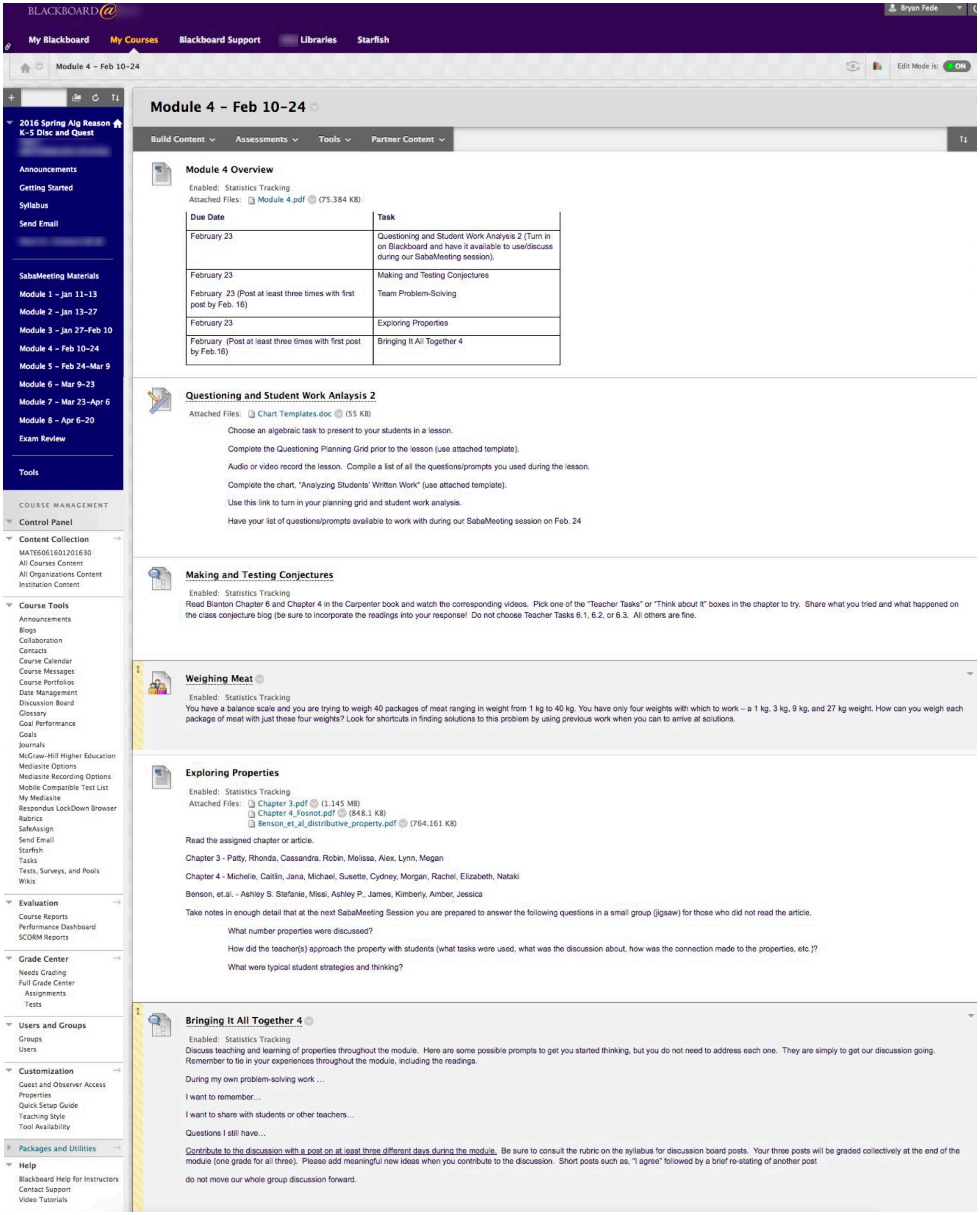
You will complete Questioning and Student Work Analysis 2 during Module 4. You may go ahead and start on this, but be sure to complete your analysis from the first lesson before doing the second questioning grid so you can use your analysis to inform any changes you might want to make.



Module 4 – Feb 10–24

Build Content Assessments Tools Partner Content

Module 4 Overview

Enabled: Statistics Tracking
Attached Files: Module 4.pdf (75.384 KB)

Due Date	Task
February 23	Questioning and Student Work Analysis 2 (Turn in on Blackboard and have it available to use/discuss during our SabaMeeting session).
February 23 February 23 (Post at least three times with first post by Feb. 16)	Making and Testing Conjectures Team Problem-Solving
February 23	Exploring Properties
February (Post at least three times with first post by Feb.16)	Bringing It All Together 4

Questioning and Student Work Anlaysis 2

Attached Files: Chart Templates.doc (55 KB)

Choose an algebraic task to present to your students in a lesson.

Complete the Questioning Planning Grid prior to the lesson (use attached template).

Audio or video record the lesson. Compile a list of all the questions/prompts you used during the lesson.

Complete the chart, "Analyzing Students' Written Work" (use attached template).

Use this link to turn in your planning grid and student work analysis.

Have your list of questions/prompts available to work with during our SabaMeeting session on Feb. 24

Making and Testing Conjectures

Enabled: Statistics Tracking
Read Blanton Chapter 6 and Chapter 4 in the Carpenter book and watch the corresponding videos. Pick one of the "Teacher Tasks" or "Think about It" boxes in the chapter to try. Share what you tried and what happened on the class conjecture blog (be sure to incorporate the readings into your response! Do not choose Teacher Tasks 6.1, 6.2, or 6.3. All others are fine.

Weighing Meat

Enabled: Statistics Tracking
You have a balance scale and you are trying to weigh 40 packages of meat ranging in weight from 1 kg to 40 kg. You have only four weights with which to work – a 1 kg, 3 kg, 9 kg, and 27 kg weight. How can you weigh each package of meat with just these four weights? Look for shortcuts in finding solutions to this problem by using previous work when you can to arrive at solutions.

Exploring Properties

Enabled: Statistics Tracking
Attached Files: Chapter 3.pdf (1.145 MB)
Chapter 4_Fosnot.pdf (848.1 KB)
Benson_et_al_distributive_property.pdf (764.161 KB)

Read the assigned chapter or article.

Chapter 3 - Patty, Rhonda, Cassandra, Robin, Melissa, Alex, Lynn, Megan

Chapter 4 - Michelle, Caitlin, Jana, Michael, Susette, Cydney, Morgan, Rachel, Elizabeth, Nataki

Benson, et.al. - Ashley S. Stefanie, Missi, Ashley P., James, Kimberly, Amber, Jessica

Take notes in enough detail that at the next SabaMeeting Session you are prepared to answer the following questions in a small group (jigsaw) for those who did not read the article.

What number properties were discussed?

How did the teacher(s) approach the property with students (what tasks were used, what was the discussion about, how was the connection made to the properties, etc.)?

What were typical student strategies and thinking?

Bringing It All Together 4

Enabled: Statistics Tracking
Discuss teaching and learning of properties throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post

do not move our whole group discussion forward.

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Module 5 - Feb 24-Mar 9

2016 Spring Alg Reason K-5 Disc and Quest

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Module 5 - Feb 24-Mar 9

Build Content | Assessments | Tools | Partner Content

Module 5 Overview - Using Variables

Enabled: Statistics Tracking
Attached Files: Module 5.pdf (142.749 KB)

Due Date	Task
March 8	Analyze Questioning from Lesson 2
March 8	Read Carpenter Ch. 5 and 6. Watch corresponding videos. For your own understanding (not to turn in), complete Carpenter pp. 76-77 #1, 5, and either 7 or 8
March 8 March 5 (Turn in on Blackboard March 5; Give partner feedback March 8)	Read Usiskin chapter on uses of variables Complete properties problem set; Give feedback to a partner
March 8	Bringing it All Together – Including response to required prompt
March 22	Looking Ahead Q and SWA 3 – Lesson on Functional Thinking

Questioning Analysis 2

Read the article by Schielack, Chancellor, & Childs (2000). Look back at your Questioning and Student Work Analysis 2 that you submitted in the last module to further analyze your questioning using this article.

Write a reflection discussing the types of questioning/prompts from the article that you saw in your own teaching. Be sure your reflection includes:

- examples (i.e. a short exchange transcribed from your audio or an attached audio or video, etc.)
- data (# of times you had a student comment on another's thinking; wait time seconds, etc.)
- a general take away message about where you are now in your questioning and what you will try to accomplish in your next lesson.

Schielack, J. F., Chancellor, D., & Childs, K. M. (2000). Designing questions to encourage children's mathematical thinking. *Teaching Children Mathematics*, 6(6), 398-402

Shielack, et.al. Article

Enabled: Statistics Tracking
Attached Files: Schielack_et_al.pdf (3.921 MB)

Module 5 Readings

Enabled: Statistics Tracking
Attached Files: Usiskin-Conceptions of School Algebra.pdf (9.039 MB)

Read Carpenter Ch. 5 and 6. Watch corresponding videos. For your own understanding (not to turn in), complete Carpenter pp. 76-77 #1, 5, and either 7 or 8

Read Usiskin, Z. (1988). Conceptions of school algebra and uses of variable. In A.F. Coxford, & A.P. Shulte (Eds.). *The ideas of algebra, K-12* (pp. 8-19). Reston, VA: National Council of Teachers of Mathematics.

Properties Problem Set

Enabled: Statistics Tracking
Attached Files: Properties Problem Set.pdf (134.222 KB)

Complete the properties problem set. You may use the computer or write by hand and scan or take a picture of your work. Send your problem set to your partner by March 5. Review your partners' work and make comments/suggestions in writing or by phone, Skype, etc. Turn in your final problem set using the link.You can email your partner directly through Blackboard by clicking "Send email" in the menu to the left.

Partners:

Bringing It All Together 5

Enabled: Statistics Tracking

Here are some prompts for your discussion. The starred items must be included. Others are just to get you started in your thinking and we'll see where the discussion takes us. Remember that you can post any time during the module. Your emerging thoughts at the beginning are just as important as your more refined thoughts at the end. Double check the rubric on the syllabus for class discussions.

**The interpretation of variable I use most often in my classroom is…(Explain)

My students think variables are used….

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Looking Ahead…Questioning and Student Work Analysis 3

Enabled: Statistics Tracking

You will complete Questioning and Student Work Analysis 3 during Module 6. You may go ahead and start on this, but be sure to complete your analysis from the second lesson before doing the third questioning grid so you can use your analysis to inform any changes you might want to make. Also, this lesson should focus on functional thinking. There are plenty of potential tasks in the Blanton book.

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Module 6 – Mar 9-23 Edit Mode is: ON

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Glossary
Goal Performance
Goals
Journals
McGraw-Hill Higher Education
Mediasite Options
Mediasite Recording Options
Mobile Compatible Test List
My Mediasite
Respondus LockDown Browser
Rubrics
SafeAssign
Send Email
Starfish
Tasks
Tests, Surveys, and Pools
Wikis

Evaluation
Course Reports
Performance Dashboard
SCORM Reports

Grade Center
Needs Grading
Full Grade Center
Assignments
Tests

Users and Groups
Groups
Users

Customization
Guest and Observer Access
Properties
Quick Setup Guide
Teaching Style
Tool Availability

Packages and Utilities

Help
Blackboard Help for Instructors
Contact Support
Video Tutorials

Module 6 – Mar 9-23

Build Content | Assessments | Tools | Partner Content

Module 6 Overview

Enabled: Statistics Tracking
Attached Files: Module 6.pdf (169.502 KB)

Due Date	Task
March 22	Questioning and Student Work Analysis 3 (Turn in on Blackboard and have it available to use/discuss during our SabaMeeting session).
March 22 March 21 (Work with group to set schedule to complete parts 1 and 2)	Read Blanton Chapter 3 Complete tasks on group blog or file exchange.
March 22 (Post at least three different days with first post by March 14)	Bringing It All Together 6

Questioning and Student Work Analysis 3

Enabled: Statistics Tracking
Attached Files: Chart Templates.doc (55 KB)

Complete Questioning and Student Work Analysis 3 during Module 6. This lesson should focus on functional thinking. There are plenty of potential tasks in the Blanton book in addition to the activities on the task cards shared in the SabaMeeting session. When you do your student work analysis, really look at how students are making sense of the patterns.

Questioning and Student Work Analysis 3

Enabled: Statistics Tracking
Attached Files: Chart Templates.doc (55 KB)

Complete Questioning and Student Work Analysis 3 during Module 6. This lesson should focus on functional thinking. There are plenty of potential tasks in the Blanton book in addition to the activities on the task cards shared in the SabaMeeting session. When you do your student work analysis, really look at how students are making sense of the patterns.

Link to Turn in Questioning Student Work Analysis 3

Module 6 Readings

Enabled: Statistics Tracking
Read Blanton Chapter 3. Incorporate your thoughts about the chapter into your Bringing It All Together post.

Functional Thinking

Enabled: Statistics Tracking

Part 1:

Each person in your group should choose a different task card and complete the set of prompts below. Then share your work with the other members of the group.

1. What do you notice about the pattern? Circle patterns you see and describe the pattern in words.
2. What is another way to show and describe how the pattern grows?
3. Build and show the 4th stage of the pattern. Describe how you know if follows the pattern.
4. Complete the chart for the pattern.

Stage Number	Our Thinking	Number of Pattern Blocks
1		
2		
3		
4		
10		
37		
100		

Part 2:

Next, think about sorting the figures into two groups: simple patterns and more complex patterns.

- Which patterns would you consider simple? Why?
- Which patterns would you consider complex? Why?
- What are the similarities and differences among the patterns?
- How might these similarities and differences be highlighted to promote different figural reasoning strategies?

You may want to sort separately and then compare or sort together through a phone/video conversation. Either way is fine, just so it is clear on the blog or wiki how you sorted and your rationale so we can share it at the next SabaMeeting session.

Patterns for Sorting

Enabled: Statistics Tracking
Attached Files: Pattern Sort (Friel) 1-21.doc (301.5 KB)

Bringing It All Together 6

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Discuss teaching and learning of properties throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

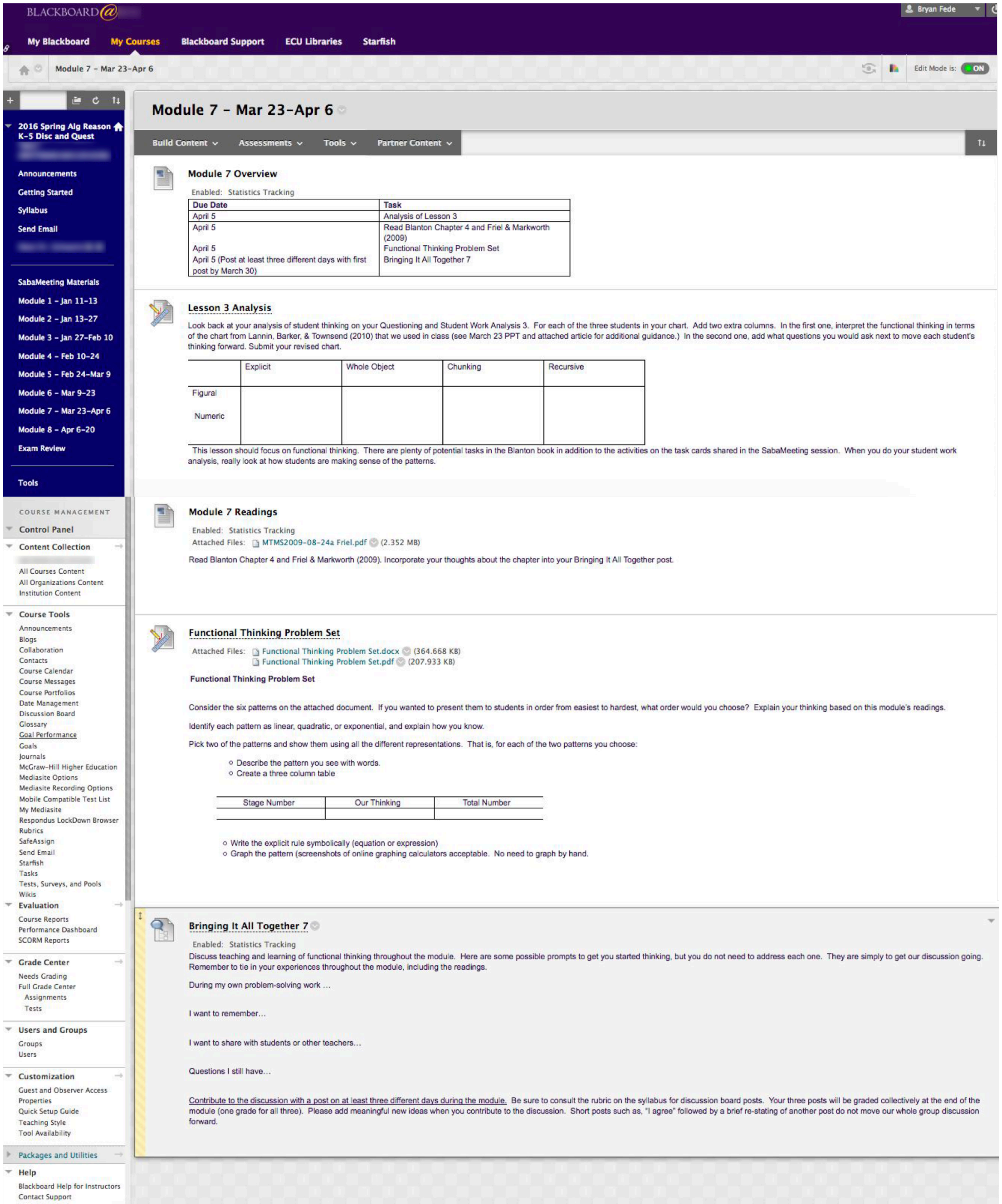
During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.



Module 7 – Mar 23–Apr 6

Build Content | Assessments | Tools | Partner Content

Module 7 Overview

Enabled: Statistics Tracking

Due Date	Task
April 5	Analysis of Lesson 3
April 5 April 5 April 5 (Post at least three different days with first post by March 30)	Read Blanton Chapter 4 and Friel & Markworth (2009) Functional Thinking Problem Set Bringing It All Together 7

Lesson 3 Analysis

Look back at your analysis of student thinking on your Questioning and Student Work Analysis 3. For each of the three students in your chart. Add two extra columns. In the first one, interpret the functional thinking in terms of the chart from Lannin, Barker, & Townsend (2010) that we used in class (see March 23 PPT and attached article for additional guidance.) In the second one, add what questions you would ask next to move each student's thinking forward. Submit your revised chart.

	Explicit	Whole Object	Chunking	Recursive
Figural Numeric				

This lesson should focus on functional thinking. There are plenty of potential tasks in the Blanton book in addition to the activities on the task cards shared in the SabaMeeting session. When you do your student work analysis, really look at how students are making sense of the patterns.

Module 7 Readings

Enabled: Statistics Tracking
Attached Files: MTMS2009-08-24a Friel.pdf (2.352 MB)

Read Blanton Chapter 4 and Friel & Markworth (2009). Incorporate your thoughts about the chapter into your Bringing It All Together post.

Functional Thinking Problem Set

Attached Files: Functional Thinking Problem Set.docx (364.668 KB)
Functional Thinking Problem Set.pdf (207.933 KB)

Functional Thinking Problem Set

Consider the six patterns on the attached document. If you wanted to present them to students in order from easiest to hardest, what order would you choose? Explain your thinking based on this module's readings.

Identify each pattern as linear, quadratic, or exponential, and explain how you know.

Pick two of the patterns and show them using all the different representations. That is, for each of the two patterns you choose:

- Describe the pattern you see with words.
- Create a three column table

Stage Number	Our Thinking	Total Number

- Write the explicit rule symbolically (equation or expression)
- Graph the pattern (screenshots of online graphing calculators acceptable. No need to graph by hand.

Bringing It All Together 7

Enabled: Statistics Tracking

Discuss teaching and learning of functional thinking throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings.

During my own problem-solving work …

I want to remember…

I want to share with students or other teachers…

Questions I still have…

Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.

APPENDIX D: QUESTION PLANNING GRID

Question Planning Grid

<table>
<tr><td colspan="3">Algebraic Concept:</td></tr>
<tr><td colspan="3">Things I want my students to know (goal of the lesson):</td></tr>
<tr><td>Questions I might ask
to focus attention on the algebraic content of the lesson</td><td>Responses I anticipate from my students</td><td>Follow-up questions I might need to ask</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="3">Documenting Things My Students Know (What evidence will you collect?)</td></tr>
<tr><td colspan="3">Documenting Things My Students Need to Know (What evidence will you collect?)</td></tr>
</table>

APPENDIX E: ANALYZING STUDENTS WRITTEN WORK

Analyzing Students Written Work

Student	Evidence:	Interpretation:	Other Interpretations:	Needs:
	What I notice about understanding of algebraic concept	What I think this evidence means about the student's level of reasoning	What else the evidence could mean	What instruction does the student need to move to the next level of thinking and other algebraic concepts

APPENDIX F: HELLOSIGN CONSENT FORM

Dear Members [redacted]

My name is Bryan Fede and I am a graduate student at the University of North Carolina at Chapel Hill. I am doing a research study entitled *Designing and Implementing Distance Education in Preparing Elementary Mathematics Specialists.* The purpose of this study is to refine the ways in which both mathematical and pedagogical content is delivered online (both synchronously and asynchronously) to professionals like you. Specifically, I am interested in the kinds of interactions that are fostered in online classes and ultimately how these interactions affect your success and satisfaction in the course.

As part of this research, I would like to follow your progress throughout the semester. This includes monitoring your participation in forums, blogs, and other asynchronous activities. In addition to these asynchronous activities, I am also interested in how you and your classmates engage with each other around issues of mathematics and pedagogy in the bi-weekly synchronous sessions. The results from this project will be used to enhance the future experiences of in-service teachers in this program and other like it.

There are a number of different ways that you might participate in this project. In general, participation requires no additional effort on your part. You have the potion to participate in part, all or none of the following:

- Allow the monitoring of posts in online forums, blogs, and other asynchronous interactions on Blackboard.
- The monitoring of assignments completed over the course of the semester.
- The monitoring of correspondence between me, other students and the instructor as it relates to this class.
- The monitoring (audio recording) of your participation in the bi-monthly Saba Meeting sessions.
- Participation in one or more interviews if selected.

This project has met the guidelines for an exemption through the University of North Carolina's Office of Human Research Ethics. Exempt status means that, after reviewing the research proposal, there is minimal risk of individual harm inherent in participation. Despite minimal risk of harm to the individual, it is still your choice as to whether you wish to participate in the study. If you chose to participate, your personal information will be kept secure at all times and participation will in no way affect your grade in this course.

If you agree to participate in the study as a whole, or the various elements of the study, please initial on the lines provided. You may also choose not to participate at all if you wish (see last line).

Thank you for your time.

Bryan Fede
Doctoral Candidate
School of Education
University of North Carolina at Chapel Hill

_____ I agree to allow you access to posts in online forums, blogs, and other asynchronous interactions on Blackboard

_____ I agree to allow you access to the assignments completed over the course of the semester.

_____ I agree to allow monitoring of correspondence between me, other students and the instructor as it relates to this class.

_____ I agree to allow audio-records that include my participation in the bi-weekly Saba Meeting sessions.

_____ I am willing to participate in one or more interviews if selected.

_____ I do not wish to participate in any part of this study

_____________________________ __________
Signature of participant Date

APPENDIX G: EVIDENCE TABLES

Component 1: Preparing students for online learning

Teacher Action	Example	Evidence Source
Post a welcome message to help students get started	Welcome to [redacted]! You should already be in our Blackboard site. Those of you coming from another university should have received an email last night with an invitations and instructions. If you did not receive this email, please let me know over the weekend so I can be sure to follow-up with our Blackboard Technical Support. In the meantime, I wanted to go ahead and send the syllabus and the assignments in this email that will be due before our first SabaMeeting Session on Wednesday, January 13 at 5:00. This information will all be posted on Blackboard as well. Also attached is a letter from Bryan Fede about a research study that he is conducting in conjunction with this course. Please take the time to read the letter and respond to him. **I will be en route to a conference on January 27 so I would like to move our second SabaMeeting session to Monday, February 1, from 5:00-7:50.** I am hoping that with a few weeks notice most of you can make this date work, but if not, please let me know, and we will make arrangements for you to listen to the recording after the session. I look forward to spending this semester thinking deeply about algebraic reasoning and questioning and discourse. Please feel free to contact me by email or phone anytime. We can also set up video conferencing appointments if you prefer to speak "Face to Face." I will look for you on Blackboard early next week and see you in SabaMeeting on Wednesday! Best,	Group email sent Friday, January 8 for class beginning on Wednesday the 13th.
Include a brief orientation for students to get familiar with the terminology and tools used in your CMS.	Not Present	N/A
Provide contact information (email, phone number, etc.) for technical help in different ways: post in syllabus, group email messages, or by course announcement.	**SabaMeeting Tutorials** The tutorials at this link can help you get started if you have never used SabaMeeting before this class. **SabaMeeting FAQ's** If you are having technical difficulties, check this link first. Many problems and possible solutions are address.	Found under the "Getting Started" tab on the front page of the Blackboard class site
Remind students to set up email forwarding to their preferred accounts.	Not Present	N/A
Provide your contact information, standard response time, and preferred communication methods.	**MATE 6061 Algebraic Reasoning: K-5 Discourse & Questioning** **Instructor:** [redacted] **Office:** [redacted] **Telephone:** [redacted] **Email:** [redacted] (This is the best way to reach me.) **Office Hours:** By appointment; daily on via email, Skype, or phone (allow 48 hours on weekends)	Found in the syllabus (online and downloadable print version)
Provide online office hours as needed.	**Office Hours:** By appointment; daily on via email, Skype, or phone (allow 48 hours on weekends)	Found in the syllabus (online and downloadable print version)

Structure the course by providing guidelines for participation and other class policies to help students learn more effectively.	Example 1: **Homework Assignments (30 %)** Each module you will complete a variety of activities including readings from the text, watching video, problem-solving activities, interactions or lessons with students, and writing responses. During the online portion of the course, these assignments will be opened every two weeks. **<u>You must complete the assignments for the module within the time frame they are assigned.</u>** Once the window for each module has ended, you will no longer be able to submit assignments. When you click on each module on the course menu you will see an introductory post giving the dates the module will be open and a list of the items that need to be completed for the week. Underneath the introductory post will be the materials needed to complete the week's work. Regularly, you are expected to post to Blogs or Discussion Forums about assigned reading or problems. These comments are graded on a 3-point rubric as follows: *Reading Responses* 3 – Response addresses the prompt with references to the readings for the week, and when appropriate, earlier readings. Opinions, observations, and/or past experiences are thoughtfully related to the readings throughout the response. 2 – Response addresses the prompt with references to the readings for the week. Connections between the readings and opinions, observations, and/or past experiences are present. 1 – Response submitted either does not address the prompt or does not refer to the readings. *Problem-Solving Responses* 3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response. 2 – Response shares ongoing work the problem with explanations of your thinking. 1 – Response submitted either does not include your work on the problem or does not explain your thinking. ***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.)	Example 1 is from the course syllabus (online and downloadable print version). The "Reading Response" rubric and "Problem- Solving Response" Rubric were also available in the "Course Tools" section in Blackboard under "Rubrics"
Structure the course by providing guidelines for participation and other class policies to help students learn more effectively. (Cont.)	Example 2: **Class Participation/Attendance (20%)** This course is an opportunity for you to become part of a community of learners who are committed to learning about teaching and learning through reading, writing, discussing, and collaborating. Your overall involvement in the course includes the following: • ***Intellectual risk taking:*** demonstrated willingness to offer and pursue ideas and suggestions that go beyond the ordinary • ***Making connections:*** demonstrated ability to connect the theoretical and the practical, to relate specific ideas to larger themes • ***Thinking clearly on paper:*** demonstrated proficiency in expressing ideas, organizing information, and communicating in writing • ***Contributing to the community:*** demonstrated willingness to share information and ideas with the group and to support others in their efforts to build understanding • ***Commitment to developing listening and speaking skills:*** demonstrated effort to develop effective speaking skills and active listening and responding skills • ***Commitment to exploring new ways to think about teaching and learning mathematics:*** demonstrated willingness to being open to trying out new ways of teaching mathematics and to allowing children opportunities to make sense of mathematics. *Attendance Policy*: Class attendance and participation are required. The classes are designed to be a forum for us to engage ideas through discussions and in-class activities that supplement rather than reiterate the readings. Since much of the course is collaborative in nature, your presence is needed (and desired)! If you are absent (not present for an online session), you are unable to participate or support your colleagues in the course, and this necessarily impacts your final course grade. You are welcome to complete any activities that can be done outside of class, of course, but you cannot make-up for missed conversations or the insights you might have provided the class during our discussions. If you must miss a synchronous class session, please inform me beforehand and arrange to make-up work missed. Missing more than one class may affect your final course grade significantly, such as dropping a letter grade.	Found in the syllabus (online and downloadable print version)
Provide resources and strategies for online learning, and explain how learning online is different than learning in a classroom.	**Online Discussions Expectations** Our online discussions should mirror face-to-face conversations as much as possible. Your contributions to the discussion board should add ideas to the conversation in meaningful ways. A response to someone's post simply saying, "I agree," and then restating their ideas is not a substantial contribution to the discussion and will not be counted as such. **<u>You will need to contribute to your groups and online discussion boards throughout the module, not just on a single day. The number and timing of the posts will be noted on the outline of module assignments.</u>**	Found under the "Getting Started" tab on the front page of the Blackboard class site
Include a Student FAQ. (e.g. common questions about courses, registration, tuition, financial aid, course materials and software.)	Not Present	N/A

Component 2: Specify goals, expectations, and policies.

Teacher Action	Example	Evidence Source
Course name and overview	**Course Description:** This course aims to develop the content and pedagogical content knowledge of in-service elementary teachers in the areas of Algebra and Algebraic Reasoning. Algebra is a content standard listed by the National Council of Teachers of Mathematics beginning at the Pre-K level and extending through grade 12. *Early Algebra* is of particular importance to children and is foundational in helping them to reason and problem-solve – especially in the areas of arithmetic and functions. By engaging in and analyzing activities which emphasize algebraic reasoning, students of this course will develop an understanding of the essential approaches vital in teaching algebra effectively. This course is also designed to help teachers examine and improve their questioning and classroom discourse.	Found in the syllabus (online and downloadable print version)
Instructor's name and contact information	**MATE 6061 Algebraic Reasoning: K-5 Discourse & Questioning** **Instructor:** [redacted] **Office:** [redacted] **Telephone:** [redacted] **Email:** [redacted] (This is the best way to reach me.) **Office Hours:** By appointment; daily on via email, Skype, or phone (allow 48 hours on weekends)	Found in the syllabus (online and downloadable print version)
Course goals and learning objectives	**Course Objectives:** 1. Implement a variety of appropriate instructional strategies to assist elementary children in constructing algebraic ideas. 2. Demonstrate content knowledge in K-8 algebraic thinking based upon national standards (i.e. Common Core State Standards, NCTM – National Council of Teachers of Mathematics Process and Content standards). 3. Understand patterns, relations, and functions from a variety of perspectives. 4. Understand the role of properties in number systems and their use in computation. 5. Represent mathematical situations and structures using algebraic symbols. 6. Prove mathematical conjectures. 7. Use mathematical models to represent and understand quantitative relationships. 8. Demonstrate an understanding of how to facilitate discourse to elicit algebraic reasoning in elementary classrooms. 9. Demonstrate an understanding of the assessment of algebraic reasoning in elementary classrooms through questioning and listening to students, analyzing students' written work, documenting patterns of students' thinking and planning appropriate student/teacher interactions.	Found in the syllabus (online and downloadable print version)
A description of course structure, including how online courses work generally as well as specifics	Not Present	N/A
Required and optional course materials or textbooks.	**Textbooks:** *Required:* Blanton, M. L. (2008). *Algebra and the elementary classroom.* Heinemann: Portsmouth, NH. Carpenter T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in the elementary school.* Heinemann: Portsmouth, NH. *Recommended:* Chapin, S., O'Connor, C., & Anderson, N. (2009). Classroom Discussions: Using Math Talk to Help Students Learn, Grades, K-6 (Second Edition). Sausalito, CA: Math Solutions.	Found in the syllabus (online and downloadable print version)
Course schedule, including lessons, reading assignments, assignments and deadlines, projects, quizzes, exams or papers, and/or other learning activities planned	Not Present	N/A

Clear and specific grading policies and academic integrity policies	Example 1: *Reading Responses* 3 – Response addresses the prompt with references to the readings for the week, and when appropriate, earlier readings. Opinions, observations, and/or past experiences are thoughtfully related to the readings throughout the response. 2 – Response addresses the prompt with references to the readings for the week. Connections between the readings and opinions, observations, and/or past experiences are present. 1 – Response submitted either does not address the prompt or does not refer to the readings. *Problem-Solving Responses* 3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response. 2 – Response shares ongoing work the problem with explanations of your thinking. 1 – Response submitted either does not include your work on the problem or does not explain your thinking. ***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.) Example 2: **Academic Integrity**: In keeping with [illegible] policy on Academic Integrity found in the Code of Student Conduct, students are expected to be the sole contributor to work bearing their name, except where group projects have been assigned. Students are expected to follow the University's honor code. Please see the website. http://www.[illegible]du/studenthandbook/III.htm ***Any violation of the academic integrity policy will result in a grade of F for the course, regardless of the student's current average in the class.***	Both examples are drawn from the course syllabus (online and downloadable print version)
Guidelines for online communication, such as posting messages to online discussion board, responding to messages posted by others, sending course emails, and working in teams in the online course;	**Writing Expectations** We are in an academic setting, so the expectation is that you will use academic writing. This means writing in complete sentences and using correct grammar, punctuation, and spelling. Don't be tempted to abbreviate to "text message" style language -- LOL (Laugh Out Loud). For most assignments you can type directly into the assignment box provided on Blackboard. If you choose to attach a file instead, please use Times New Roman or Arial 12 pt. font with one inch margins. **Online Discussions Expectations** Our online discussions should mirror face-to-face conversations as much as possible. Your contributions to the discussion board should add ideas to the conversation in meaningful ways. A response to someone's post simply saying, "I agree, " and then restating their ideas is not a substantial contribution to the discussion and will not be counted as such. **You will need to contribute to your groups and online discussion boards throughout the module, not just on a single day. The number and timing of the posts will be noted on the outline of module assignments.** **Format for Mathematics Work** There will be times when you need to show your work on a mathematics problem or show how you would model a problem for students. This often involves pictures and verbal explanations. There are several possible formats in which to present this work listed. If you have a different idea that will work please let me know, and we can try it. 1) You can video yourself modeling the problem and upload the video. 2) Make a screencast. 3) Record your voice talking on a powerpoint with the visuals pasted from the virtual manipulatives. 4) You can handwrite your work and scan or photograph it. 5) Type your thinking into a Word document and then cut and paste pictures from the virtual manipulatives. **What is most important is that we can clearly understand how you were thinking about the work through your representations and explanations.**	Found under the "Getting Started" tab on the front page of the Blackboard class site
Policy for assignment submission and grading (e.g. by Drop Box or by email)	**Saving Files** When you attached a file, please be sure that the file name includes your name, the module, and the assignment name. For example, a lesson plan for the first module would be named [illegible]*_1_lessonplan*	Found under the "Getting Started" tab on the front page of the Blackboard class site
Netiquette guidelines for the online course and/or additional netiquette resources	**Writing Expectations** We are in an academic setting, so the expectation is that you will use academic writing. This means writing in complete sentences and using correct grammar, punctuation, and spelling. Don't be tempted to abbreviate to "text message" style language -- LOL (Laugh Out Loud). For most assignments you can type directly into the assignment box provided on Blackboard. If you choose to attach a file instead, please use Times New Roman or Arial 12 pt. font with one inch margins.	Found under the "Getting Started" tab on the front page of the Blackboard class site

Component 3: Foster the creation of a warm and inviting atmosphere to build a learning community.

Teacher Action	Example	Evidence Source
Welcome students before the course begins via email or course announcement.	Welcome to [illegible]! You should already be in our Blackboard site. Those of you coming from another university should have received an email last night with an invitations and instructions. If you did not receive this email, please let me know over the weekend so I can be sure to follow-up with our Blackboard Technical Support. In the meantime, I wanted to go ahead and send the syllabus and the assignments in this email that will be due before our first SabaMeeting Session on Wednesday, January 13 at 5:00. This information will all be posted on Blackboard as well. Also attached is a letter from Bryan Fede about a research study that he is conducting in conjunction with this course. Please take the time to read the letter and respond to him. **I will be en route to a conference on January 27 so I would like to move our second SabaMeeting session to Monday, February 1, from 5:00-7:50.** I am hoping that with a few weeks notice most of you can make this date work, but if not, please let me know, and we will make arrangements for you to listen to the recording after the session. I look forward to spending this semester thinking deeply about algebraic reasoning and questioning and discourse. Please feel free to contact me by email or phone anytime. We can also set up video conferencing appointments if you prefer to speak "Face to Face." I will look for you on Blackboard early next week and see you in SabaMeeting on Wednesday! Best,	Group email sent Friday, January 8 for class beginning on Wednesday the 13th.
Post a personal introduction with an informal tone.	Not Present	N/A
Provide lots of encouragement and support, particularly in the beginning of the course. This includes positive feedback administered to students privately by email.	Example 1: *Really, a very thoughtful response, Chelsea. Take heart! You will learn strategies for your struggling students as you grow in your new role. Do you have materials that are particularly for strugglers? Many are just procedure workbooks which are not that helpful, but the math recovery resources by Wright, Martland, Stafford, and Stanger are helpful.* Example 2: *Kristin, I love hearing about your experiences with the LEAP curriculum. Are their any links you could share for the Algebra Potpourri board so others could take a look if interested!* Example 3: *Nice response, Amy. It will be interesting to see if you feel like your percentages are the same at the end of the semester! Good job incorporating the readings from Blanton into your response.*	Three typical comments from the algebraic reasoning journal assignment (Module 2)
Commend students privately by email	Unable to Determine	N/A
Encourage students to post a short self-introduction to the discussion forum or user profile.	**Pictures of Algebra** Many of you know each other from previous classes, but there are a few new people in our group. Let's make them feel welcome and do some thinking about algebra! Post on the group blog by Wednesday, January 13. Your introductory post should include: • Your name • What you teach and your school name/district (or if you have another position, share that) • A picture of your favorite view from a window and a question you could ask about the picture that could be answered using algebra **Headshot Submission** Please submit a simple head shot to be included in a class picture directory. If you are uncomfortable or unable to share your picture with the group, you might consider a pet or favorite sports team logo.	Module 1 assignment. Due prior to the first synchronous class session.
Upload any relevant pictures to the course site and encourage students to do so as well.	Not Present	N/A

Component 4: Promote active learning.

Teacher Action	Example	Evidence Source
Emphasize to students the importance of learning by playing an active role in the learning process, a role, which differs from the direct instruction or lecture in traditional classrooms.	**Class Participation/Attendance (20%)** This course is an opportunity for you to become part of a community of learners who are committed to learning about teaching and learning through reading, writing, discussing, and collaborating. Your overall involvement in the course includes the following: • ***Intellectual risk taking:*** demonstrated willingness to offer and pursue ideas and suggestions that go beyond the ordinary • ***Making connections:*** demonstrated ability to connect the theoretical and the practical, to relate specific ideas to larger themes • ***Thinking clearly on paper:*** demonstrated proficiency in expressing ideas, organizing information, and communicating in writing • ***Contributing to the community:*** demonstrated willingness to share information and ideas with the group and to support others in their efforts to build understanding • ***Commitment to developing listening and speaking skills:*** demonstrated effort to develop effective speaking skills and active listening and responding skills • ***Commitment to exploring new ways to think about teaching and learning mathematics:*** demonstrated willingness to being open to trying out new ways of teaching mathematics and to allowing children opportunities to make sense of mathematics. *Attendance Policy*: Class attendance and participation are required. The classes are designed to be a forum for us to engage ideas through discussions and in-class activities that supplement rather than reiterate the readings. Since much of the course is collaborative in nature, your presence is needed (and desired)!	Found in the syllabus (online and downloadable print version)
Provide opportunities for students to critique and reflect upon certain course topics.	Bringing it All Together (BIAT) recurring assignment in each module Example: **Bringing It All Together** Discuss teaching and learning the concept of equality throughout the module. Here are some possible prompts to get you started thinking, but you do not need to address each one. They are simply to get our discussion going. Remember to tie in your experiences throughout the module, including the readings. What does it mean for elementary students to reason algebraically? During my own problem-solving work … I want to remember… I want to share with students or other teachers… Questions I still have… Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.	BIAT assignment from Module 2. Similar assignments exist for modules 3 through 7
Encourage students to be proactive learners by regularly logging into the course site, submitting assignments on time, participating in discussions within required timeframe, and cooperating with teammates.	*A lot of the dates are just the end of the module date, January twenty-sixth. You'll see for the team problem solving and also the bringing it all together blog, I have posted at least three different days, so that's three different calendar dates by the end of the module, with the first post due by January eighteenth. A lot of people ask me why it's set up that way. It's because my experience is that if that's not the case, no matter how well intentioned we are we all get very, very, very busy because that's the way life is. If you have to post three different times you do it all the night before the module's due and the point of those two things is to have an actual discussion. It's hard to discuss with somebody if they haven't posted until two hours before it's due. By having that first post due on January eighteenth, that gives people at least something to respond to on all the other days and it makes it more of a discussion like you might have in a class. That's the rationale behind that because I always get that question.*	Portion of transcript of synchronous online session 1 (January 13, 2016)

Component 5: Monitor Student Progress and Encourage Lagging Students.

Teacher Action	Example	Evidence Source
Be aware that students who fall behind are in jeopardy of not completing the course.	Unable to Determine	N/A
Use available educational technology tools, such as course management systems, to track student progress in course activities.	Unable to Determine	N/A
Contact students who have not logged in for over a week to inquire whether they are experiencing technical difficulties or problems with course content/activities. If students can't participate due to technical problems, connect them immediately to provide technical help.	Unable to Determine	N/A
Contact students who have not completed assignments by email or phone.	Unable to Determine	N/A
Send a weekly email summarizing course activities as a general reminder to the whole class near the end of the week.	Not Present	N/A
Introduce a new week with an overview of upcoming events and deadlines	Previews of upcoming modules were held at the end of synchronous sessions and typically lasted approximately 15 minutes. At this time the professor would often do a "screen share" and preview the class Blackboard website.	Synchronous Online Sessions
Include flexibility in grading if possible (e.g. allow students to drop lowest grade, give choice in assignments, etc.).	Not Present	N/A

Component 6: Assess students' messages in online discussions.

Teacher Action	Example	Evidence Source
Make sure the assessment criteria measure both the quantity and quality of the online message.	Example 1: (Response Rubrics) *Reading Responses* 3 – Response addresses the prompt with references to the readings for the week, and when appropriate, earlier readings. Opinions, observations, and/or past experiences are thoughtfully related to the readings throughout the response. 2 – Response addresses the prompt with references to the readings for the week. Connections between the readings and opinions, observations, and/or past experiences are present. 1 – Response submitted either does not address the prompt or does not refer to the readings. *Problem-Solving Responses* 3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response. 2 – Response shares ongoing work the problem with explanations of your thinking. 1 – Response submitted either does not include your work on the problem or does not explain your thinking. ***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.)	
	Example 2: Contribute to the discussion with a post on at least three different days during the module. Be sure to consult the rubric on the syllabus for discussion board posts. Your three posts will be graded collectively at the end of the module (one grade for all three). Please add meaningful new ideas when you contribute to the discussion. Short posts such as, "I agree" followed by a brief re-stating of another post do not move our whole group discussion forward.	Instructions for BIAT (Module 2)
Consider assigning points to messages that encourage additional posting.	Not Present	
Make use of recommended rubrics that include: promptness and initiative in delivery of post, relevance of the post, expression within the post, and contribution to the learning community.	*Reading Responses* 3 – Response addresses the prompt with references to the readings for the week, and when appropriate, earlier readings. Opinions, observations, and/or past experiences are thoughtfully related to the readings throughout the response. 2 – Response addresses the prompt with references to the readings for the week. Connections between the readings and opinions, observations, and/or past experiences are present. 1 – Response submitted either does not address the prompt or does not refer to the readings. *Problem-Solving Responses* 3 – Response shares ongoing work the problem with explanations of your thinking, and when appropriate, connections to other mathematics you have done yourself or with your students. Opinions, observations, and/or past experiences are thoughtfully related throughout the response. 2 – Response shares ongoing work the problem with explanations of your thinking. 1 – Response submitted either does not include your work on the problem or does not explain your thinking. ***One point will be subtracted if the posting requirements are not met (number of posts/days, etc.)	

Component 7: Sustain students' motivation.

Provide opportunities for student collaboration and facilitate their collaborative learning processes using available tools.	Example 1: **Questioning & Discourse Assessment (25%)** During the course you are to spend time interacting with a small group of children (Grades K-5) to work on algebraic thinking tasks. In each of the modules, you will work on a task that you choose from your textbooks. We will analyze the students' algebraic reasoning and how your questioning both elicits student thinking and provides opportunities for student learning. Prior to each SabaMeeting session you are to complete a questioning planning grid, teach the lesson (either small or whole group), and complete the chart, "Analyzing Students' Written Work". In addition, *you will record the session in order to analyze your questioning*. Bring your work back to our synchronous class discussions to share ideas with others and raise questions about what you noticed. You will submit a written reflection of the student assessments to be included in your final portfolio.	Found in the syllabus (online and downloadable print version)
Choose a conversational tone that makes students feel comfortable in the online learning environment and establishes trust in communication while building a learning community.	Unable to Determine	
Provide meaningful feedback to all assignments with recognition of good work as well as specific suggestions for improvement.	Example 1: *It be interesting to see if your percentages are different by the end of the semester! Remember to reference the readings in your responses.*	Individual Journal – Algebraic Reasoning, Module 2
	Example 2: *Lindsey, you have a good start, but you were missing some follow-up questions and a whole column in the student work analysis chart. In addition, you included very little specifics about actual evidence and suggestions when discussing student thinking. I put lots of comments on the attached document to give you some direction. If you want to revise your second assignment before I grade it, please let me know. Also, I loved reading about all your students, but you only need to report on three to complete the requirements of the assignment. Looking forward to reading your next plan.*	Questioning and Student Work Analysis 1, Module 2
Provide a weekly summary of discussion topics to demonstrate your participation, and assess messages for both quantity and quality.	Not Present	

Provide encouragement to students to articulate their confusion or difficulty with course content, projects, requirements, or instructions for activities.	Time was reserved at the end of each synchronous online sessions for student questions/concerns	Audio recordings of SabaMeeting synchronous online sessions
Take an active role in helping your students think and learn actively through careful task structuring, questioning, and scaffolding.	**Example:** **Team Problem-Solving -- Sheep Problem** One shepherd says to another, "Give me 4 sheep, and then we will have an equal number." The other answers, "No, you give me 4 sheep and then I will have twice as many as you." How many sheep did each shepherd have? How do you know? (In a few days an eight year old student's solution will be posted.) Sonya's Solution to the shepherd problem (age eight) If one were to give the other 4 sheep and they would then have an equal number, that means they have a difference of 8 sheep. If, on the other hand, the other gives away 4, then the difference becomes 16 (since one loses 4 and the other gains 4 sheep). And then we get that one has 2 times as many, or 16 sheep more. This means there will be 16 and 32, and before the exchange there were 20 and 28. *Solved in 40 seconds as reported by V.A. Kruetetskii in the Psychology of Mathematical Abilities in School Children, 1976*	Initial problem posed Jan 27. "Sonia's Response" <u>was added</u> as fodder for discussion on Feb. 7 ahead of synchronous session on February 10
In online discussions, consider: • Designing thought-provoking questions to elicit student discussions on the topics of your focus	Example Team Problem Solving Prompt: **Weighing Meat** You have a balance scale and you are trying to weigh 40 packages of meat ranging in weight from 1 kg to 40 kg. You have only four weights with which to work – a 1 kg, 3 kg, 9 kg, and 27 kg weight. How can you weigh each package of meat with just these four weights? Look for shortcuts in finding solutions to this problem by using previous work when you can to arrive at solutions.	Module 4
• Providing a weekly summary of discussion topics to demonstrate your participation	Not Present	
• Redirecting off-topic discussion through gentle reminders or a recast of the question	Example 1: **Thoughts so far...** *Posted by* [illegible] *(Not a group member) at Monday, January 18, 2016 1:30:05 PM* Hi All, I'm enjoying reading your thinking about this problem so far. A couple of thoughts. Some of you read "two weighings" as you can put items on the balance two times. The first interpretation was the intention of the problem. For those of you that proposed using sets of 3 to determine in two weighings, how would it apply if there were more coins? Say 12 coins instead of nine. What is the least number of weighings you can do? Does your sets of 3 idea still work? [illegible]	Response to group 2 (Coin Problem Module 2)
• Assessing messages by both quantity and quality (For more information about assessing online messages.	**Online Discussions Expectations** Our online discussions should mirror face-to-face conversations as much as possible. Your contributions to the discussion board should add ideas to the conversation in meaningful ways. A response to someone's post simply saying, "I agree, " and then restating their ideas is not a substantial contribution to the discussion and will not be counted as such. <u>**You will need to contribute to your groups and online discussion boards throughout the module, not just on a single day. The number and timing of the posts will be noted on the outline of module assignments.**</u>	Getting Started tab of course page Blackboard

Component 8: Provide feedback and support.

Encourage students to articulate their confusion or difficulty with course content, projects, requirements, or instructions for activities	Opportunities were consistently provided for students to ask questions or clarify assignments.	Recordings of synchronous online meetings.
Provide meaningful feedback on graded assignments with recognition of good work as well as specific suggestions for improvement	Example 1: *Brielle, great job on your questioning grid and student work analysis. I put a few comments on your to help you think a little about what you are listing as evidence. See attached and let me know if you have questions!* Example 2: *Lindsey, please see attached with comments. You really needed a lot more focus on conceptual understanding and algebraic thinking and more detail on students' strategies. I've put lots of comments.*	Questioning and Student Work Analysis 1, Module 2 Questioning and Student Work Analysis 2, Module 4
Respond to students' concerns or technical difficulties quickly and provide contact information of tech support	Individual from IT was invited for the first 30 minutes of the first synchronous session to assist with technical issues. Links to SabaMeeting help and FAQs	Synchronous Session 1 "Getting Started" tab in course Blackboard site
Consider using peer assessment to provide additional feedback to students while reducing faculty workload.	Students were partnered for the three Questioning and Student Work Analysis assignments for peer editing and comments	Questioning and Student Work Analysis 1, 2, 3

In online discussions, your students will feel motivated to participate and learn when you:		
• Encourage openness in online discussions or collaborative assignment and allow different opinions to exist	Unable to Determine	
• Diagnose misconception without delay to avoid further misunderstanding or confusion, but explain with background information	Not Present	
• Provide timely feedback to comment, confirm, evaluate, or to question	Not Present	
• Provide additional important resources for further study	Not Present	
• Use gentle reminders to carry the discussion further or redirect discussions	**So far so good!** *Posted by* [redacted] *(Not a group member) at Monday, January 18, 2016 1:42:13 PM* Hi All, Some good thinking here so far, but as you have indicated, if you start with 4 on each side, you could get it in two weighings, but only if you are lucky! What could you start with that would work even if your luck has run out?! :)	Team Problem Solving, Module 3, Group 5
• Encourage your students to use examples, real cases, or literature to support their views	Questioning and Student Work Analysis assignments were focused on participant's own classrooms.	Questioning and Student Work Analysis 1, 2, 3

Component 9: Encourage students to regulate their own learning.

Encourage students to become "process managers" in the online course by giving up some of the traditional power of teachers. For example, students may be directed to take turns leading online learning experiences.	Not Present	
Encourage students' reflection and feedback through the inclusion of an introductory survey with questions on student expectations for the course and engagement in students' course evaluation.	Not Present	
Encourage students to take responsibility for their peers' learning as well as their own through discussion forums.	This sentiment was fostered in the Bringing It All Together prompts where student were encouraged to share points that they thought were important from the readings as well as share anecdotal stories regarding connections to the classroom. Example: [redacted] said... Tuesday, February 9, 2016 7:33:24 PM EST [redacted] I face the same challenge sometime when trying to explain what I've learned in our classes. It doesn't always quite come out and make the point I want it to. I think your point on using 2 x 7 + 7 is equal to 3 x 7 is one way to show teachers how algebra is integrated into our standards. Another idea would be how students use the associative property and distributive property in multiplication. These properties are generalizations in algebra that strengthen students' fluency and ability to flexibly solve multiplication problems. Today, in a 4th grade PLC, we discussed this fifth grade problem: How can the powers of ten help you solve 4 x 23 x 25? The solution is to use the associative property to solve instead 25 x 4 x 23, so that students can solve 100 x 23. This thinking is based in algebraic properties. I am just realizing myself how vital algebra is to our students' learning and creating connections in mathematics. Good for you for taking this thinking to the 3rd and 4th grade teams!	Bringing It All Together Prompts Student Response:Bringing It All Together, Module 3 Heather - Maddie
Provide opportunities for peer review.	Students were partnered for the three Questioning and Student Work Analysis assignments for peer editing and comments	Questioning and Student Work Analysis 1, 2, 3

Component 10: Deal with conflicts promptly.

Provide guidelines for web etiquette.	Not Present	
Offer private communications with students who are posting inappropriately, and contact the appropriate department if you suspect that a student has violated integrity policies.	Unable to Determine	
Provide a regular peer evaluation function so that students can communicate their impressions of how the group is functioning.	Not Present	
Intervene only when conflicts escalate to a point where students can no longer work through the issue on their own. Otherwise, conflict should be welcomed as a sign that a learning community is developing.	Not Applicable	

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