TEACHERS’ IN-THE-MOMENT NOTICING OF STUDENTS’ MATHEMATICAL THINKING: A CASE STUDY OF TWO TEACHERS

Yanjun Liu

A dissertation submitted to the faculty at the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the School of Education.

Chapel Hill
2014

Approved by:
Susan N. Friel
Rita O'Sullivan
Lynda Stone
Xue Lan Rong
Marta Civil
ABSTRACT

Yanjun Liu: Teachers’ In-the-Moment Noticing of Students’ Mathematical Thinking: A Case Study of Two Teachers
(Under the direction of Susan N. Friel)

The purpose of this research is to access teachers’ in-the-moment noticing of students’ mathematical thinking, in the context of teaching a unit from a reform-based mathematics curriculum, i.e., *Covering and Surrounding* from *Connected Mathematics Project*. The focus of the study is to investigate the following research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?
2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

Conceptualized as a set of interrelated components in this study, the construct of teachers’ in-the-moment noticing of students’ mathematical thinking includes attending to students’ strategies, interpreting students’ understandings, deciding how to respond on the basis of students’ understandings, and responding in certain ways.

A review of literature reveals that much of the research on teacher noticing does not examine teacher noticing as it occurs in the midst of instruction. Rather, it involves asking teachers to analyze and reflect on videos outside the context and pressure of in-the-moment instruction. Thus, in order to access teachers’ in-the-moment noticing in a more explicit and direct way,
the researcher in this study applied a new technology to explore teacher noticing, enabling two teacher participants to capture their noticing through their own perspectives while teaching in real time.

Findings indicate that teacher participants noticed for a variety of reasons, including student thinking, instructional adaptations, assessment, content, and student characteristics, focusing primarily on student thinking and instructional adaptations. Furthermore, these participants noticed student thinking in the midst of instruction to different extents, and made adjustments to instruction in different ways.

Examination of the data also suggests that teachers’ noticing of student thinking was shaped by teachers’ beliefs, knowledge, and goals. Therefore, influenced by these constructs, teachers noticed student thinking to different extents, influencing students’ opportunities to think mathematically in different ways. A diagram that illustrates the paths through which teachers traveled in the process of noticing is presented, as one of the findings.
ACKNOWLEDGMENTS

Although my name alone stands as author of this document, it is through the work of many that it has been completed. Each deserves a note of thanks.

First and foremost, to my committee chair and advisor, Dr. Susan N. Friel: Thank you for the vision that inspired me to conceive this project. Without your support, guidance, and endless patience and encouragement, this research would not have been possible. You ignited my passion for an adventure, cheered me on as I ran my race, and always believed in me as I fumbled my way on this exploratory journey. Thank you also for my opportunities to both teach and learn during my doctoral program at the University of North Carolina at Chapel Hill that helped to shape me into the person I will become as a mathematics educator. It has been an honor to learn from your wisdom and enthusiasm.

To Dr. Rita O'Sullivan: Thank you for having taken on a heavy load of responsibility in helping me in undertaking this dissertation, and in bringing it to a successful end. Thank you for your insights and encouragement throughout work on this project.

To my dissertation committee members, and my former advisor-- Dr. Lynda Stone, Dr. Xue Lan Rong, Dr. Marta Civil, and Dr. Olof Steinthorsdottir: Thank you for your persistent support, many learning experiences, and valuable advice during my doctoral program at the University of North Carolina at Chapel Hill. Your listening ears and caring hearts were invaluable in nurturing my growth as I was maturing professionally.
To the two teacher participants: I am greatly indebted to you for your generosity with both time and energy. Thank you for grappling with this technology, staying late after school sharing your thoughts and insights, and welcoming me as a visitor and researcher. To your students, it was a pleasure to work with them.

To my family: Thank you for your patience and understanding. You have not always understood what I have been doing, but your support has been unwavering. Special thanks go to my aunt, Jianqiong Mo, and my cousin, Stephie Jia: The word “support” does not convey what you have provided. This could not have been done without your endless patience, timely reassurances, and countless hours of being a wonderful grandaunt and an aunt of Steffi’s respectively. Finally to Mason and Steffi, thank you. I am blessed that you joined us. You made this all worthwhile. Thank you both for lighting the way.
# TABLE OF CONTENTS

LIST OF TABLES.............................................................................................................................................. xi

LIST OF FIGURES............................................................................................................................................... xii

CHAPTER 1: INTRODUCTION ......................................................................................................................... 1

The Problem..................................................................................................................................................... 1

The Purpose of the Study ................................................................................................................................. 6

CHAPTER 2: REVIEW OF THE LITERATURE .............................................................................................. 9

Introduction..................................................................................................................................................... 9

History of Teacher Noticing............................................................................................................................. 9

Researchers’ Conceptions of Mathematics Teacher Noticing......................................................................... 12

Overview of Current Research Areas of Mathematics Teacher Noticing...................................................... 14

What Teachers Notice................................................................................................................................... 15

Differences in Novice and Expert Teachers................................................................................................... 15

Enhancement of Teachers’ Practice................................................................................................................ 17

Overview of Current Methodologies for Studying Mathematics Teacher Noticing..................................... 19

Professional Noticing of Children’s Mathematical Thinking....................................................................... 22

Children’s Mathematical Thinking................................................................................................................ 23

Reform-Based Mathematics Instruction....................................................................................................... 25

A Clarification.................................................................................................................................................. 27

CHAPTER 3: METHODOLOGY.................................................................................................................... 31
Case Study...................................................................................................31

The Curriculum..............................................................................................................33

The Connected Mathematics Project..............................................................................33

Covering and Surrounding...........................................................................................35

Overview of Methods.........................................................................................................36

Participants..........................................................................................................................36

Teacher Participants............................................................................................................38

Student Participants...........................................................................................................39

Field Work...........................................................................................................................40

Part 1: Initial Interview.........................................................................................................41

Part 2: Observations and Videotaping.................................................................................45

Standard video camera: rationale and design......................................................................47

New technology: rationale and design.................................................................................47

Part 3: Follow-up Review of Videotapes..............................................................................51

Analysis.................................................................................................................................54

Part 1: informal analysis......................................................................................................55

Part 2: formal analysis..........................................................................................................56

Summary.................................................................................................................................60

CHAPTER 4: FINDINGS.........................................................................................................62

Introduction..............................................................................................................................62

Meeting the Teachers............................................................................................................63

A Profile of Joe Marshall......................................................................................................64

A Vignette...............................................................................................................................64
Teacher Beliefs, Knowledge, and Goals.................................................................121
Limitations and Future Research........................................................................126
Limitations...........................................................................................................126
Recommendations...............................................................................................129
APPENDIX A - TEACHER RECRUITMENT SCRIPT.............................................134
APPENDIX B - TEACHER CONSENT FORM.......................................................135
APPENDIX C - STUDENT RECRUITMENT SCRIPT...........................................139
APPENDIX D - STUDENT ASSENT FORM...........................................................140
APPENDIX E - PARENTAL PERMISSION FORM...............................................143
APPENDIX F - PROTOCOLS FOR INITIAL INTERVIEW......................................148
APPENDIX G - PROTOCOLS FOR DAILY FOLLOW-UP DISCUSSION.....................153
APPENDIX H - INNOVATION CONFIGURATION MAPS........................................155
APPENDIX I - CODING SCHEME......................................................................160
APPENDIX J - DAILY CLIP CAPTURE.................................................................169
APPENDIX K - CMP INVESTIGATION.................................................................171
REFERENCES.....................................................................................................174
LIST OF TABLES

Table 3.1 - Student demographics for participating classrooms.................................................39
Table 3.2 - Student participants in participating classrooms..........................................................40
Table 3.3 - Data collection process and instruments used..............................................................41
Table 3.4 - Classroom observation and follow-up interview schedule..........................................52
Table 3.5 - Clip capture and selection information.........................................................................58
Table 4.1 - Percentage of Mr. Marshall’s reasons reported as themes for clip capture.................87
Table 4.2 - Percentage of Ms. Goldberg’s reasons reported as themes for clip capture.............87
LIST OF FIGURES

Figure 4.1 - Student’s work on finding perimeter of a rectangle........................................72
Figure 4.2 - Student’s work on finding perimeter of a rectangle........................................80
Figure 4.3 - Using “surrounding rectangle strategy” for area and perimeter of triangles.........100
Figure 4.4 - Melody’s presentation.........................................................................................106
Figure 4.5 - Greg’s thinking..................................................................................................109
Figure 5.1 - Diagrammatic representation of what takes place when teachers notice.............125
CHAPTER 1: INTRODUCTION

The Problem

Tell me to what you pay attention and I will tell you who you are.
José Ortega y Gasset (Sherin, Jacobs, & Philipp, 2011)

To pay attention implies to notice. Noticing is a part of everyday life, indicating the act of observing or recognizing something. Individuals who embrace similar goals and experiences tend to display similar patterns of noticing (Jacobs, Philipp, & Sherin, 2011). For instance, professional chess players and radiologists develop particular ways of noticing in their worlds. Teacher noticing, then, is a practice essential to the domain for which teachers are responsible, as teachers are a group of professionals who hold particular goals and experiences (Ball, 2011).

Bombarded with “the blooming, buzzing confusion of sensory data” (Sherin & Star, 2011, p. 69) in the classroom, the teacher not only explains content, formulates questions, and reacts to student answers, but responds to the broader demands of the classroom environment (Doyle, 1977). According to Doyle, multidimensionality, simultaneity, and unpredictability are the most salient features of the classroom environment for teachers. Thus the noticing of teachers is conceptualized as a professional skill of processing complex classroom events (Fernandez, Coles, & Brown, 2013). The phrase teacher noticing depicts an active rather than a static process, through which teachers attend to the ongoing information presented in an environment that is multidimensional and unpredictable to some extent (Sherin, Jacob, et al., 2011).

Although a teacher is often faced with the “blooming, buzzing confusion of sensory data” (Sherin & Star, 2011, p. 69), it is not realistic for the teacher to attend to all information
presented in the classroom. When two incidents occurred within about 0.5 seconds of each other, one incident was found to cause a decrement in the recognition of the other incident (Schneider & Shiffrin, 1977). Thus, attention is selective, whether conscious or unconscious, due to focusing and capacity limitations (Jacobs et al., 2010; van Es & Sherin, 2002). Attention is also subjective instead of objective. It is highly influenced by individuals’ knowledge, beliefs, and prior experiences. Bartlett (1932) talked about “an imaginative reconstruction” (p. 213) of incidents that had just happened, describing attention and memory as personal reconstruction flavored by instincts, interests, ideals, etc. The consciously or unconsciously selected information in the classroom, therefore, opens up a window to the “subjective worlds” (Erickson, 2011, p.21) inhabited by the teacher, telling a unique story of the teacher as the authority of the classroom, who brings experiences, resources, and perspectives to the process of noticing.

As researchers in teacher education have begun to describe their work as being about teacher noticing, it is important to ask the primary question: Why does teacher noticing matter? Schoenfeld (2011) believes that people act on what they notice. Thus, noticing is consequential, and action is a natural consequence of noticing. Erickson (2011) proposes that teachers notice in order to take action. Noticing is considered to be a fundamental element of expertise in the act of teaching (Mason, 2002). Ball (2011) points out that the noticing required of teachers is specialized, like noticing is in any other profession. Professional noticing is described as cognitive practices that highlight the perceptual field so that relevant phenomena are made salient, while other phenomena fade into the background (Goodwin, 1994). For example, a doctor develops sensitivities to variations in sound, frequency, and duration of coughs, and attending to these important details in particular ways is a critical component of the ability to reason about the different causes, and then provide cures of diseases.
In the field of teaching, teachers attend to classroom happenings in particular ways in order to help students learn. Van Es and Sherin (2008) propose that the skill of noticing for teaching includes three main aspects: (a) identifying what is important in a teaching situation; (b) using what one knows about the context to reason about a situation; and (c) making connections between specific events and broader principles of teaching and learning. Of particular interest is teachers’ noticing of student thinking, considered to be powerful evidence of effective teaching and linked to documented gains in student achievement (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; van Es, 2011). However, there are classrooms where students’ thinking simply goes unnoticed, when teachers do not necessarily know how to “highlight” (Goodwin, 1994, p. 606) the situation when faced with the diversity of information. Although a teacher may or may not always have the resources to deal successfully with what has been noticed, there is a better chance for meaningful connections to be made, actions to be taken, and learning to happen in a classroom if signs of potential progress or problems are observed. Thus, noticing is a critical component of teaching expertise, and the ability to see and respond to student thinking is indispensable for effective teaching (Ball & Cohen, 1999). A better understanding of teacher noticing of student thinking will contribute to improving mathematics teaching and learning.

The emphasis in current mathematics education reform also calls for a flexible approach to instruction that is responsive to student ideas (Ball & Cohen, 1999). A fundamental principle is the creation of classroom environments in which teachers make pedagogical decisions in the midst of instruction. Borko, Livingston, McCaleb, and Mauroet (1988) talked about teaching as improvisational performance, emphasizing interactive lessons that were designed to be flexible and responsive to what the students knew and could do (see also Borko & Livingston, 1989). Such emphasis encourages teachers to center their instruction partially on the lesson plan as it
unfolds in the classroom while paying particular attention to the ideas that students raise. This ability to attend to important aspects of classroom interactions and then adapt instruction in the midst of teaching requires teachers to learn to notice student thinking, and also reason about it in new ways (van Es, 2011).

Therefore, in mathematics classrooms, teachers and students are expected to attend carefully to one another’s ideas, with teachers adapting their instruction, at least in part, based on interpretation of the ideas that students raise (Smith, 1996). As Smith states, students learn well when they construct their own mathematical understanding. Rather than playing the role of knowledge source and mathematical authority, teachers must recognize students’ thinking through students’ arguments, explanations, and questions; make meaning of it, and take in-the-moment instructional actions in response to what they recognize (Luna, Russ, & Colestock, 2009). Rodgers (2002) also emphasizes that teachers need to develop their ability “to see students learning, to discern, differentiate, and describe the elements of that learning, to analyze the learning and to respond” (p. 231), given the current context of reform in the U.S. According to van Es and Sherin (2002), requisite skills that reformers have in mind when they call for flexible teaching involve first noticing what is significant in a classroom interaction, making sense of these data, and then using interpretations to inform pedagogical decisions.

However, there are studies that highlight teachers’ difficulties in attending to the details of children’s strategies and lack of understanding of children’s thinking and reasoning in mathematics. For example, through examining teacher behavior, Even and Wallach (2004) suggested a list of issues against the current expectation that teachers evaluate their students’ understanding by attending to their talk and actions. Some of the problems included teachers’
inability to make unplanned changes, lack of knowledge about common student conceptions and
their possible sources, and failure to ascribe values to students’ ideas in the midst of instruction.

Some researchers attributed these problems partially to teachers’ own traditional schooling
experience. Nicol (1999), for example, talked about how teachers themselves were successful
graduates of schools with mathematics classrooms and curricula that tended to focus on the
learning and application of routine procedural skills, lacking adaptability and responsiveness to
learners’ thinking. Others focused on teachers’ personal resources, for instance, teachers’
personal knowledge of mathematics, beliefs about the nature of mathematics, expectations about
how mathematical knowledge should be communicated, experience with and expectations of
students and classrooms in general, and their cultural backgrounds (Morgan & Watson, 2002).
Some researchers highlighted programs of teacher education. van Es and Sherin (2002) pointed
out that these programs often did not stress helping teachers learn to notice or interpret
classroom interactions. Instead, they focused on helping teachers learn to act, with instruction
regarding new pedagogical techniques and new activities or tools that teachers could use. Despite
the importance of these techniques and activities as resources for teachers, they did not
necessarily ensure that teachers would learn to notice or interpret classroom interactions in ways
that would allow for flexibility and adaptability in their approach to teaching mathematics.
Therefore, it is a big challenge to encourage teachers to perceive the teaching of mathematics
differently from how they once learned it, from how their cooperative teacher taught it, from
how their students will most likely have learned it, and from how other teachers in their schools
may teach it (Nicol).

In the context of mathematics education reform, a focus on investigating what teachers are
noticing and how they are making sense of important events and interactions to inform teaching
decisions is particularly relevant, and also key to provide a starting point for teachers to understand and embrace the reform vision (Rodgers, 2002). The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the *Principles and Standards for School Mathematics* (NCTM, 2000) help to clarify the aspects of reform-based practice in mathematics education, pointing to a focus on classroom discourse, an adaptive style of teaching, and changes in instructional or curriculum materials (Sherin, 2002). In response to the NCTM standards, teachers are expected to use new curricula that have been designed with these goals and standards in mind (Sherin). Consistent with the call for reform, these curricula emphasize mathematical thinking and reasoning, conceptual understanding, and problem solving in realistic contexts (Remillard, 2005). As a result, they require teachers to play a substantially different role in the mathematics classrooms than what has been typically assumed (Remillard). Thus, there exists a need for mathematics teachers to take the opportunity to examine their own practice, to analyze student learning, and to explore the relationship between teaching moves and the learning that results in the midst of everything that is happening in the classroom (van Es, 2011). In order to help teachers focus on student thinking and reasoning, opportunities may need to be created for teachers to slow the pace of their instruction so they can attend closely to what they say and do, in addition to what students say and do. This study provides a window into potential patterns of teacher noticing of student thinking in mathematics classrooms.

**The Purpose of the Study**

The purpose of this study is to investigate a particular focus for noticing, i.e., students’ mathematical thinking, in the context of the use of a reform-based mathematics curriculum. The focus of the study is to better understand mathematics teachers’ in-the-moment noticing. Sherin, Russ, and Colestock (2011) examined the variety of what teachers noticed in the midst of
instruction, including student thinking, classroom discourse, teacher moves, teacher strategies, and student engagement. However, rather than attend to the specific kinds of things teachers notice, the present focus is a specialized type of noticing, i.e., teachers’ professional noticing of children’s mathematical thinking. Jacobs, Lamb, and Philipp (2010) defined professional noticing of children’s mathematical thinking as a set of interrelated skills, including attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings. According to Jacobs et al. (2010), if instruction is to build on children’s thinking, teachers must be able to execute these three skills almost simultaneously before responding to the thinking.

This study is organized around the following two research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

The first question involves the patterns of teacher noticing of students’ mathematical thinking, and the extent to which student thinking is observed in the midst of other noticeable events during instruction. The second question focuses on the pathways on which relations of teacher noticing to instruction occur.

In the next chapter, I survey research on teacher noticing, including its history, the current state of teacher noticing research, and a focus on professional noticing of student thinking. In the third chapter, I detail the methodology employed in this study, describing why a case study is appropriate for addressing the research questions, selection of teacher participants, and methods of data collection and analysis. In this chapter, I also provide an overview and description of the
Connected Mathematics Project and the unit Covering and Surrounding. I present the findings of the study in the fourth chapter.
CHAPTER 2: REVIEW OF THE LITERATURE

Introduction

The purpose of this study is to investigate mathematics teachers’ in-the-moment noticing of students’ mathematical thinking in the context of the use of a reform-based mathematics curriculum. This study is organized around the following two research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

Chapter 2 provides a survey of the current field of teacher noticing and is organized into four sections. First, a historical perspective on teacher noticing is provided, followed by examination of three general conceptions held by researchers studying teacher noticing. This is then followed by an overview of current studies of mathematics teacher noticing in the context of teaching and learning, and by an account of current methodologies for studying this construct. The chapter ends with a special focus on a specialized type of mathematics teacher noticing, i.e., professional noticing of children’s mathematical thinking. Although the state of research on teacher noticing is young, an overview of the field provides the background for this study that grows out of the existing body of knowledge.

History of Teacher Noticing

As Sherin, Jacobs, et al. (2011) state, the image of teacher noticing is not completely novel.
In 1904, John Dewey wrote an essay, “The Relation of Theory to Practice in Education” (Dewey, 1904). In this essay, Dewey made a distinction between children’s outer attention and inner attention, bringing up the question of whether the teacher was attending more to students’ moving pencils or their moving thoughts. He seems to be reminding today’s researchers of how much we have forgotten about paying attention to what and how teachers notice (Erickson, 2011). Then, during the early years of the twentieth century, teachers were encouraged to become acute observers of children’s behavior, under the influence of the child study movement (Stern, 1930). Later, as research on cognition began to develop and thrive in the sixties and seventies, research attention also returned to the study of teacher thinking (Erickson). What Doyle (1977) described as the “most salient features of the classroom” (p. 52), that is, multidimensionality, simultaneity, and unpredictability, revealed the demand for teachers’ ability to attend to and respond to signs of potential progress or hurdles, filtered out of quickly passing data, in the act of teaching (Hammer & van Zee, 2006).

Then, in the nineties, different terms were used to explicitly emphasize the importance of skilled viewing in professions, including teaching, that require the individual’s ability to act as an effective, timely decision maker when operating within dynamic systems that involve complex cognitive tasks. For instance, Endsley (1995) used situation awareness to depict the need for crews of military aircraft, air traffic controllers, and other professionals to make decisions across a fairly narrow space of time, depending on an ongoing, up-to-date analysis of the environment. According to Endsley, this construct involved perception of meaningful elements in an environment, comprehension of their meaning, and projection of their status in the near future. Likewise, Goodwin (1994) coined the term professional vision to characterize the specialized way that members of a professional group look while performing duties, or their way of seeing
that is sufficient to complete the job at hand. Similarly, Stevens and Hall (1998) used *disciplined perception* to describe the visual practices typical of particular professions or disciplines. Finally Mason (2002, 2011) introduced the construct of *intentional noticing*, distinguishing professional noticing from everyday noticing. In short, learning to notice in particular ways is part of the development of expertise in a profession.

Building upon the idea of the noticing expertise of professionals, a teacher’s professional vision is concerned with the ability to notice and interpret significant interactions in a classroom (Sherin, Russ, Sherin, & Colestock, 2008). In other words, a key aspect of a teacher’s visual practices is quickly perceiving student behavior and making sense of what the behavior means in terms of student understanding and engagement (Miller, 2011). Prior research in the field of psychology has examined what people attend to as they interact with their surroundings and explored the phenomenon known as “inattentional blindness” (Simons, 2000). In essence, it is concerned with the question of how likely subjects are to notice something salient and potentially relevant that they do not expect. It has been found that, quite often, these unexpected objects or events fail to capture attention (Simons). Individuals do not see everything in a situation. Goodwin (1994) uses “highlighting”, that is, making specific phenomena salient by marking them in some fashion, to describe the act of deciding what is noteworthy and deserves further attention in a complex perceptual field. In a similar way, teacher noticing is a term that depicts the selective act of attending to certain elements in a teaching situation, as it is impossible to see everything (Sherin, Jacob, et al., 2011). Although teacher noticing is important to teaching in any domain, this dissertation is focused on noticing as a component of teaching expertise in mathematics. The mathematical pedagogy of current reform efforts emphasizes how teachers
listen to students and what they listen for and the ways in which they respond to the students (Nicol, 1999).

The work on mathematics teacher noticing has ties to several other research efforts in mathematics education. Sherin, Jacobs, et al. (2011) believe that there are three areas of mathematics education research that support the idea that teacher noticing is likely to be an important and fruitful focus. These include adaptive and responsive teaching, learning from teaching, and decomposing practice. Reform mathematics promotes teaching that is attentive and adaptive to students’ interests and thinking processes (see, e.g., Remillard, 2005); learning from teaching focuses on the effort to help teachers analyze their teaching for improvement (see, e.g., Ball, Sleep, Boerst, & Bass, 2009; Hiebert, Morris, Berk, & Jansen, 2007); decomposing practice is centered on the idea that teaching can be enhanced through decomposing the complexity of mathematics teaching into specific activities, for instance, teacher noticing, and feasibly discussing these activities or constructs (see, e.g., Lampert, 2001). Linked to and supported by these research areas, the noticing of mathematics teachers has been explored in recent years to understand how teachers make sense of complex classroom environments in which they cannot see or respond to everything that is occurring (Jacobs et al., 2010).

**Researchers’ Conceptions of Mathematics Teacher Noticing**

In an effort to make sense of how and what mathematics teachers notice, researchers have developed various perspectives on this construct. Sherin and Star (2011) listed three general perspectives adopted by researchers studying teacher noticing. What distinguish these perspectives from each other are the underlying conceptions of what teacher noticing means, that is, what researchers attend to when they study teacher noticing.
According to Sherin and Star (2011), the first conception defines noticing as recognizing classroom phenomena that occur occasionally or infrequently. For example, a teacher might notice that a group of students who normally do not work well together are having a good mathematics conversation. These phenomena are seen as existing outside and independent of the teacher. Researchers who embrace this notion would thus likely ignore a broad range of more routine events that take place relatively frequently.

The second conception focuses on selecting a subcomponent of the larger system. In other words, through noticing, some elements are selected over others in some manner. These selected elements do not need to be particularly surprising or interesting but are just the elements filtered out of the “blooming, buzzing confusion of sensory data” (Sherin & Star, 2011, p. 69) in some manner. Inspired by the dual-process model of cognition to some extent, Sherin and Star (2011) define noticed things or elements as information that is passed from automatic to controlled process. According to Feldon (2007), the dual-process model of cognition asserts that information processing occurs simultaneously on two pathways, i.e., automatic and controlled pathways. The pathways operate independently but intersect at certain points to produce human performance. The former are fast, effortless, and unconscious; the latter are slow, effortful, and conscious. Researchers who hold this view may be interested in the process through which information is passed from the unconscious to conscious pathway. For example, these researchers may focus on the trajectory of development in teachers’ abilities to recognize and react to complex cognitive stimuli consciously.

The third conception is centered on capturing surfacing features of teachers’ reasoning about noticed classroom events. Researchers embracing this notion focus on investigating the noticed things for the purpose of understanding teacher thinking or teacher belief behind the reasoning.
For instance, van Es and Sherin (2008) examined teacher thinking through studying teachers’ comments about and interpretations of noticed elements while watching teaching videos in a video club.

Among the three conceptions, the second one is the more widely adopted perspective of studying teacher noticing, embraced by most of the researchers referred to in this review. Central to this perspective is the idea that noticing is a collection of practices designed to capture the moments when information is passed from the unconscious to conscious pathway. The purpose of studying noticing is to sensitize and enable oneself to avoid the habitual and to act freshly rather than automatically out of habit (Mason, 2011). Furthermore, according to Sherin and Star (2011), the teacher is not separated from or outside of the data that are presented. Being a part of the multidimensional and complex environment, the teacher can play an active role in shaping and arranging what takes place in the classroom. Consequently, the teacher can influence the classroom dynamics to enable certain data to be generated and, hence, noticed. What teachers notice ends up affecting what students notice, too. On one hand, teachers are continuously making meaning in the act of teaching; on the other hand, students can be seen to be making meaning interpretively in the same time as their teachers (Erickson, 2011). Through the conjoined effort of teachers and students, desired learning outcomes may or may not be achieved in the end, depending on what elements are noticed and how they are noticed.

**Overview of Current Research Areas of Mathematics Teacher Noticing**

Approaching the construct of teacher noticing from these various perspectives, researchers in this field have generally attempted to focus on three aspects. These include what teachers tend to notice, differences in noticing found between novice and expert teachers, and enhancement of
teachers’ practice through noticing (Luna, Russ, & Colestock, 2009). What follows is an overview of current studies of mathematics teacher noticing from these different aspects.

**What Teachers Notice**

Some researchers have been engaged in identifying what teachers actually notice, that is, the specific kinds of things that teachers tend to notice while teaching or watching videotapes of teaching and learning (Luna et al., 2009). For example, an early study conducted by Erickson et al. (1986) enabled researchers to look at how and what five teacher education students noticed throughout their first year of teaching. More recently, some researchers conducted exploratory studies to capture the kinds of events that stood out to the teachers during instruction (Luna et al.; Sherin et al., 2008). Jacob et al. (2010) explored teachers’ noticing of children’s mathematical thinking, through asking the teachers to view and react to a classroom video clip and a set of written student work, attempting to identify what these teachers noticed and how they interpreted the events concerning children’s thinking. Schoenfeld (2011) summarized that all teachers engage in multiple noticing activities, including classroom management, implementing activities, and engaging in diagnostic teaching, with various time allocations observed for beginning, typically accomplished, and highly accomplished teachers.

**Differences in Novice and Expert Teachers**

In addition to paying attention to what teachers notice, some researchers have attempted to characterize the differences in how and what teachers notice, generally adopting a novice/expert model (Luna et al., 2009). Miller (2011) credited this work to David Berliner. In the article titled “In Pursuit of the Expert Pedagogue” (Berliner, 1986), the author elaborated on the significance of studying novice/expert differences. For example, these kinds of studies would inform us of the routines and schemata characteristic of novice and expert teachers; they would promote thinking
about the nature of expert systems in pedagogy; they would help to identify exemplary performances for novices to analyze; they would help to train cooperative teachers to articulate their knowledge in ways that might truly educate their novice apprentices (Berliner).

In the field of teacher noticing, researchers have contributed to this novice/expert work in various ways. Erickson et al. (1986) examined how new teachers and veteran teachers observed and made practical sense of classroom interactions daily for an extended period of time, comparing their ways of seeing. Other researchers described how expert teachers relied more on teaching principles when discussing viewed videos and recalled and analyzed classroom events in more detail (see, e.g., Clarridge & Berliner, 1991; Glaser & Chi, 1988; Peterson & Comeaux, 1987). Miller (2011) summarized that expert teachers’ noticing was likely to differ from novice teachers’ in three aspects. Expert teachers were able to maintain attention to student understanding in the midst of everything that was occurring; they showed more systematic scanning patterns of the whole group of students; they were quicker to identify situations that required intervention, whether it was misbehavior or lack of understanding. The differences were considered an indication that expert teachers had better developed knowledge structures or schemata for phenomena related to classroom teaching and learning. Expert teachers were found to have more cognitively complex schemata, leading to better perceptions and understanding of classroom events. According to Hogan, Rabinowitz, and Craven (2003), as a teacher’s schemata gained greater complexity, focus of the classroom environment shifted from a more teacher-centered approach to an environment where student thinking and understanding became the primary concerns.
Enhancement of Teachers’ Practice

Beyond describing differences in novice and expert teachers, a large number of researchers have focused on influencing what preservice or inservice teachers attend to, using videos or other technologies, with the ultimate goal of changing and improving their ways of seeing (Luna et al., 2009). They have found that teachers can improve their noticing by changing what they notice and how they reason. For instance, Rosaen, Lundeberg, Cooper, Fritzen, and Terpstra (2008) investigated to what extent and in what ways using video-based, instead of memory-based written reflection, might help preservice teachers reflect on their discussion-based teaching in a more complex manner. They reported that video-based reflection helped interns attend more to instruction and children’s classroom discussions rather than classroom management, while reflecting on their teaching. In a similar way, Star and Strickland (2008) explored what preservice teachers noticed when watching a video lesson and whether their ability to notice improved after a methods course. They found that at the beginning of the semester, preservice teachers were not particularly observant of many types of classroom events. However, after a semester of a methods course that involved watching and discussing classroom videos, field observation, peer-teaching laboratory work, and so on, there were significant increases in preservice teachers’ observation skills and ability to notice teacher and student communication during a lesson.

van Es and Sherin (2002) used a multimedia tool to help inservice teachers learn to notice. They reported that use of a video analysis support software helped the teachers to analyze videos from their own classrooms and to develop new ways of noticing. The participants began to identify particular events as noteworthy, to more frequently use specific evidence to discuss these events, and to provide their own interpretations of these events. Besides, in other studies,
researchers focused on what and how inservice teachers noticed while participating in a video club or a series of video club meetings, in which each teacher shared clips from his or her own classroom (Sherin & Han, 2004; van Es, 2011; van Es & Sherin, 2008). For instance, van Es & Sherin (2008) identified three paths along which teachers learned to notice: direct, cyclical, and incremental. According to these researchers, a direct path could be characterized as a single qualitative shift in noticing; a cyclical path could feature a teacher cycling between perspectives over time; an incremental path could be characterized as gradual development in noticing. Their data also suggested evidence of change in these teachers’ discourse, shifting from a primary focus on the teachers to increased attention to students’ actions and ideas, and moving from simple restatements of students’ ideas to detailed analyses of student thinking. Teachers began to frame their discussions of pedagogical issues in terms of student thinking. They also found that video clubs had the potential to support teacher learning in ways that would extend to classroom instruction (Sherin & van Es, 2009).

An overview of current studies of mathematics teacher noticing reveals that researchers in this field have been interested in what teachers notice, the differences between noticing of novice and expert teachers, and how noticing expertise changes over the course of a particular intervention or over the career paths of teachers (Sherin, Jacobs, et al., 2011). Erickson (2011) summarizes that noticing is highly variable across individual teachers, implying that different teachers do not inhabit identical “subjective worlds” (p. 21) even engaging in similar teaching and learning activities. However, through research and professional development, teachers are seeing and making sense of things related to classroom interactions in a more similar, skillful way. Thus new and enhanced teacher noticing takes place (Sherin, Jacobs, et al.).
Overview of Current Methodologies for Studying Mathematics Teacher Noticing

While engaging in studies that have a variety of focuses, researchers have indicated that investigation of teacher noticing posed formidable methodological challenges (Sherin, Russ, & Colestock, 2011). Ericsson and Simon (1993) elaborated on how “think aloud” was used to observe and examine a subject’s behavior while performing a task in the field of psychology. However, as Sherin, Russ, et al. (2011) stated, it was simply unfeasible to ask teachers to “think aloud” in the midst of instruction because of the ongoing nature of teaching. Therefore, instead of asking teachers to verbalize their thinking while teaching, researchers rely mainly on three alternatives for accessing teacher noticing (Sherin, Russ, et al., 2011).

The first approach involves scenario methodology, presenting slides or video clips of other teachers’ teaching and asking the viewers to discuss their perceptions about and reactions to visual stimuli and information (see, e.g., Carter, Cushing, Sabers, Stein, & Berliner, 1988; Kersting, 2008; Sherin & Han, 2004). A central view of this approach is that teachers’ understanding and ability to analyze others’ practice might be reflective of the teachers’ own teaching knowledge and what and how they generally notice when presented with similar instructional situations. However, Sherin, Russ, et al. (2011) expressed concern about this approach, in that teachers’ reaction to the visual information might not be representative of their actual teaching performance when they were faced with real demands of the classroom environment. Sherin and Han (2004) found that participating in video clubs help teachers redefine practice by engaging themselves in an activity that is very different from their usual classroom practices. In contrast to usual classroom instruction, teachers do not have to respond immediately to the situation that they view. Thus, unlike teaching, viewing classroom interactions via video is a time for reflection rather than both reflection and action. Furthermore,
this reflection can take place in the form of repeated viewings and analysis of an excerpt of classroom interaction, techniques that are not available to teachers during real-time instruction. For example, in a related study conducted by Levin, Hammer and Coffey (2009), there was an obvious discrepancy between what a student teacher was able to notice in terms of student thinking during a methods class discussions and what she actually attended to and reflected upon during her instruction. In addition, filtered by people who actually recorded the information, these videos might not offer teachers the same level of knowledge as that of the recorders, or provide the same level of information these teachers would have about their own classrooms, for example, student characteristics, prior learning experiences, etc.

The second approach involves asking teachers to retrospectively recall what they were noticing and thinking during their own instruction, individually or in video clubs (Sherin, Russ, et al., 2011). For instance, Borko and Livingston (1989) investigated three student teachers’ and their cooperative teachers’ thinking and actions during instruction, through analyzing their immediate postlesson reflections. Ainley and Luntley (2007) studied experienced teachers’ attentional skills using videos of these teachers’ own teaching. After watching these videos, teachers were interviewed about what they were noticing through simulated recall strategies (see also Rosaen et al., 2008). This retrospective-recall approach provides teachers the opportunity to reflect on classroom interactions and articulate what they notice, with the assistance of prescribed reflection tools or recorded videos. However, according to Sherin, Russ, et al. (2011), this approach may not generate accurate data about teachers’ in-the-moment experiences, as they “have been removed from the demands of the classroom” (p. 82). Using this approach, researchers are investigating a particular aspect of teachers’ noticing, that is, the way that teachers notice and interpret classroom interactions after the fact as they appear on videos.
Luna et al. (2009) suspected that teacher noticing as it occurred after the fact was likely significantly different than what was noticed in the teacher’s very act of teaching. Distortion could result in a time and place removed from the occurrence of teachers’ in-the-moment noticing, interpretation, and instantaneous reaction during class (Sherin et al., 2008). In addition, Thomas, Wineburg, Grossman, Oddmund, and Woolworth (1998) found that teachers tended to expect viewing video to be an evaluative instead of a learning activity in a video club. The teachers discussed in their study came to use video not as a resource for trying to better understand the process of teaching and learning, but rather as a resource for evaluating each other’s practices.

Third, rather than relying on self-reports, this approach concerns making inferences from teaching videos captured and viewed by researchers (Sherin, Russ, et al., 2011). The essential idea is that researchers have the expertise to collect evidence about what teachers notice through inferring from visible actions taken by the teacher in videos of their instruction. For instance, in the study conducted by Levin et al. (2009), researchers focused on interns’ attention using field notes and videos. These researchers considered it evidence of attention to student thinking when the intern appeared to notice and respond to a student’s idea, relying on researchers’ own interpretations of the episodes. Rosenberg, Hammer and Phelan (2006) also inferred what the teacher and her students noticed from watching videos of classroom interactions. Although it might be the case that researchers have the expertise to make certain claims from videos of instruction (Linsenmeier & Sherin, 2007), these videos by no means precisely represent every instructional move, as teachers may attend to interactions and events in a way that is not manifested in visible actions. In addition, the typical approach to videotaping in a classroom represents a somewhat distorted view of what a teacher actually sees, involving setting up a
camera in the back of the room, showing the teacher’s face from a frontal view, and viewing whole-class interactions from a fairly wide angle (Sherin et al., 2008). Therefore, it is difficult to accurately describe what teachers are noticing based on their observable behaviors and responses, when relying on videos captured from researchers’ perspectives.

Combined, researchers have attempted to study teacher noticing using the scenario methodology, the retrospective-recall approach, and making inferences from teaching videos captured and viewed by researchers themselves. In all of this work, they do not examine teacher noticing as it occurs in the midst of instruction. Rather, they ask teachers to analyze and reflect on videos outside the context and pressure of in-the-moment instruction.

**Professional Noticing of Children’s Mathematical Thinking**

Despite the variety of conceptions, foci, and methodologies adopted by researchers who study teacher noticing, this growing body of work has underscored the same idea that emphasis in current mathematics education reform calls for a flexible approach to instruction that attends to and is responsive to student ideas (Ball & Cohen, 1999). According to Ball, Lubienski, and Mewborn (2001), “sizing up students’ ideas and responding” (p. 453) has been recognized as one of the core activities of teaching that is built on children’s mathematical thinking. However, researchers indicate many teachers do not actively involve students’ thinking in learning tasks or making that thinking public (Black, Harrison, Lee, Marshall, & Wiliam, 2004). They may listen for whether the students know memorized facts or can do what has been shown or explained at a superficial level; however, they rarely request or probe for further information (Black et al.). Some teachers simply may not recognize that children have their own mathematical ideas and strategies that can be different from teachers’ own thinking about mathematics (Empson & Jacobs, 2008). Thus, in response to the call for a flexible approach to instruction that attends to
and is responsive to student ideas, merit can be found in investigating teachers’ noticing of children’s mathematics thinking. Explicit noticing is critical to change for better teaching because if teachers do not notice, they cannot choose to teach differently.

**Children’s Mathematical Thinking**

Researchers who examine children’s mathematics thinking have documented how children’s intuitive or informal approaches to mathematical tasks demonstrated conceptually sound strategies (e.g., Carpenter, Fennema, Franke, Levi, & Empson, 1999; Empson, Junk, Dominguez, & Turner, 2006; Lesh & Harel, 2003). According to Empson and Jacobs (2008), children’s mathematics implied “the existence of a coherent and logical approach to reasoning that differs in important ways from that of mathematicians and other adults” (p. 260). When children were able to make sense of learning situations based on their own personal knowledge and experiences, and were thoughtfully guided to express their ways of thinking, they often invented or significantly extended powerful mathematical ideas and strategies (Lesh & Harel).

Although children’s mathematical reasoning or thinking is not always completely accurate, it is more powerful and productive than many teachers realize (Empson & Jacobs, 2008). An understanding of different facets of children’s mathematical thinking, for instance, conceptually sound alternatives to traditional reasoning, milestones in children’s thinking, and the evolution of mathematical intuitions with age in general, informed teachers about what to attend to in their classrooms and fostered the learning called for by reform (Empson & Jacobs). Children’s mathematical thinking also included errors and misconceptions. An understanding of the types of misconceptions held or produced by children helped teachers quickly diagnose and remediate these errors during instruction (Fischbein & Schnarch, 1997). Empson and Jacobs state that listening to children’s thinking during instruction appeared to improve children’s understanding
and intellectual autonomy, provide opportunities for formative assessment, increase teachers’ mathematical knowledge, and support teachers’ generative learning process.

A variety of studies have shown that a focus on noticing of students’ mathematical thinking promoted both teaching and learning mathematics for understanding and led to improved student achievement. For example, Rodgers (2002) suggested that good teaching was a response to students’ learning rather than the cause of students’ learning. This researcher presented a framework for reflective inquiry into student thinking that helped teachers to change the ways they thought about their teaching and their students’ learning. In an examination of research on contemporary professional development, Wilson and Berne (1999) also described how professional development centered on noticing of students’ thinking influenced teachers’ ways of seeing their own thinking, and teaching and learning. In addition, Franke, Carpenter, Levi, and Fennema (2001) documented teachers’ generative growth supported by professional development that focused on analyzing children’s thinking. In the study, teachers who learned to emphasize and analyze their children’s thinking created learning communities that included their students and colleagues, resulting in improved teaching as well as student learning (see also Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

Research conducted by Empson and Jacobs (2008) suggested that learning to listen to children’s mathematics involved a pathway by which teachers moved from directive to observational to responsive listening, as children’s mathematics became progressively more central. Directive listening focused on seeking a match between a child’s thinking and an expected response, and eliciting that expected response even when it contradicted the child’s understandings (see also Crespo, 2000). Observational listening referred to an attempt to hear the child’s thinking with emerging but sporadic conceptualizations about what was heard and little
active support for extending that thinking. Lastly, responsive listening implied an effort to not only listen carefully to the child’s thinking but also work actively to support and extend that thinking, through drawing out, making explicit, and building on the details of the child’s thinking (see also Ball & Cohen, 1999; Sherin, 2002). In this last type of listening, student thinking played a central role in the interactions between teachers and students.

Reform-Based Mathematics Instruction

In the current context of mathematics education reform, learning to notice students’ mathematical thinking becomes particularly relevant in teacher-student interactions (Ball & Cohen, 1999). The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the *Principles and Standards for School Mathematics* (NCTM, 2000) help to clarify the aspects of reform-based practice, emphasizing a focus on an adaptive style of teaching (Sherin, 2002). Teacher noticing of student thinking is recognized as an expertise that is required for responsive and improvisational teaching in the context of reform (e.g., Empson & Jacobs, 2008; Jacobs et al., 2010; Sherin & van Es, 2003). The mathematics reform movement puts forward an ambitious set of outcome goals for student learning (Stein, Grover, & Henningsen, 1996). Documents published by the National Council of Teachers of Mathematics (1989, 2000), Mathematical Association of America (1991), and National Research Council (1989) all point to the importance of developing students’ deep and interconnected understandings of mathematical concepts, procedures, and principles, rather than simply memorizing formulas or applying procedures. Emphasis is also put on students’ capacity to *do mathematics*, that is, the ability to engage in the process of mathematical thinking, “framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on” (Stein et al., 1996, p. 456). As
Romberg (1992) stated, students should be guided to view mathematics learning as a dynamic process of “gathering, discovering and creating knowledge in the course of some activity having a purpose” (p. 61), instead of seeing it as a static process of absorbing fixed facts, concepts, and procedures.

Increased emphasis on noticing students’ capacity to not only understand the substance of mathematical concepts but also to do mathematics places great demands on teachers. The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and the *Principles and Standards for School Mathematics* (NCTM, 2000) also call for productive classroom discourse and changes in instructional or curriculum materials (Sherin, 2002). In order to produce these kinds of student outcomes and move student thinking from a procedural understanding to a relational and conceptual understanding, teachers are expected to create instructional environments in which students have sufficient time for exploration of mathematical ideas and are encouraged to discuss their ideas with one another and reflect on them (Sherin). Students should also be exposed to meaningful and worthwhile mathematical tasks that are truly problematic rather than disguised ways for students to practice a known algorithm or procedure (Stein et al., 1996). Teaching in such an environment, teachers are expected to embrace the notion of teaching as listening and interpreting rather than merely telling and explaining, to make noticing of students’ thinking one of the central tasks, and be respectful of students’ own sense-making, invention, and intellectual autonomy. Learning to notice students’ mathematical thinking, as a result, also promotes and sustains a learning environment that is conductive to students’ capacity to both understand and do mathematics (Crespo, 2000). Furthermore, teachers can learn as they notice, with their existing content knowledge being modified and new knowledge being generated (Sherin; see also Empson & Jacobs, 2008).
A Clarification

All of the research discussed in this review is predicted on the belief that teacher noticing is a crucial part of mathematics teaching expertise (Sherin, Russ, et al., 2011). This belief indicates that among a variety of what teachers notice, teacher noticing of student thinking is key to success of national reform of mathematics teaching and learning. Although there is agreement about what teachers should notice in mathematics classrooms, researchers who study teacher noticing do not necessarily share a common definition for this term. They conceptualize the construct, teacher noticing, in a variety of ways, mainly differing in the processes included in noticing. On the one hand, Star, Lynch, and Perova (2011) define teacher noticing as identifying what is important or noteworthy about a classroom situation. Star and Strickland (2008) consider the ability to notice important classroom features as the most foundational. They are interested in determining what teachers do and do not attend to when viewing a classroom lesson. On the other hand, van Es and Sherin (2002) propose three key aspects of noticing: (a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions.

What distinguishes the first definition from the second is that the definition given by Star et al. (2011) is limited to Part (a) of the definition provided by van Es and Sherin (2002). Although it is foundational to attend to what is noteworthy about a classroom situation, teachers’ interpretations of that noticing are substantial as well, according to van Es and Sherin’s (2002) notion that the act of noticing and sense making is an interrelated process. How teachers reason about what they notice is as important as the particular events they notice (van Es & Sherin,
Finally, *teacher noticing of children’s mathematical thinking* is introduced as a particular focus of noticing, providing a way to begin to unpack the decision making that lays the foundation for the teaching and learning endorsed in the current context of reform mathematics. Situated in prior research on teacher reflection, this focus of noticing resonates with Rodgers’s (2002) framework for reflection, the “reflective cycle” that consists of several stages. In this cycle, teachers first describe in detail selected noteworthy situations from their classrooms, then ascribe meaning to those events, and then decide a course of action to take.

Jacobs, Lamb, and Philipp (2010) conceptualized this particular focus of mathematics teacher noticing, i.e., *teacher noticing of children’s mathematical thinking*, as *professional noticing of children’s mathematical thinking*, recognized as a set of three interrelated skills, involving attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings. Jacobs et al. (2010) suggested that the three skills, i.e., attending, interpreting, and deciding how to respond, take place almost simultaneously in the background, before the teacher responds. In order to make explicit the three skills that provide the foundation for teachers’ responses, these researchers developed two assessments to capture participants’ professional noticing of students’ mathematical thinking involving a classroom video clip and a set of written student work. Researchers analyzed participants’ written responses to prompts about attending, interpreting, and deciding how to respond in an effort to provide resources for supporting the development of professional noticing expertise for these prospective and practicing teachers. They identified the significance of using particular discussion prompts to assess the expertise and developed growth indicators that could help professional developers recognize and celebrate shifts in teachers’ professional noticing of children’s mathematical thinking.
In this study, teachers’ professional noticing of children’s mathematical thinking not only includes attending, interpreting, and deciding how to respond, but also extends to responding. The construct is conceptualized as a process of attending, interpreting, deciding how to respond and also responding in certain ways, echoing Erickson’s (2011) notion that noticing is usually highly instrumental. That is, teachers notice in order to act or respond. Teachers often notice and do something about what is noticed, on the basis of their interpretation. The way teachers respond then shapes subsequent events, resulting in new episodes to be noticed, interpreted, and to which teachers respond. Thus, the process of attending, interpreting, and deciding how to respond is not the end but provides starting points for responding. Similar to what Lampert (1990) notes, being able to notice, interpret, and decide how to respond does not imply that one will be able to produce such teaching. Therefore, it is necessary to look at the actual response following noticing. In light of Erickson’s assumption that a teacher’s noticing is highly instrumental, this present study aims to investigate teachers’ noticing of children’s mathematical thinking as they are in the act of teaching, involving attending, interpreting, deciding how to respond, and also responding as introduced above. The focus of this study is to better understand teachers’ in-the-moment noticing, that is, how and to what extent teachers notice student thinking and how and to what extent that noticing influences teachers’ instruction, in the context of the use of a reform-based mathematics curriculum. This study provides a window into understanding whether mathematics teachers pay attention to student thinking and reasoning as reformers want them to. If they do, then how do they describe and respond to such moments? If they don’t, then what other aspects of classroom moments capture their attention and why? The answers to these questions are necessary in order to understand how to support teachers in shifting their teaching practice towards a reform pedagogy. This study intends to contribute to
the above scholarship by exploring new ways of examining teacher noticing. While many of the studies focus solely on what and how teachers tend to notice outside the context and pressure of in-the-moment instruction, the specific contribution of this study is to investigate a particular focus for noticing, i.e., teachers’ *in-the-moment* noticing of children’s mathematical thinking, in the hope of helping to better understand teachers’ in-the-moment noticing from their own perspectives.

In summary, the review of the literature in the dissertation has focused on four areas. First, a historical perspective on teacher noticing was explored. Second, three general conceptions held by researchers studying teacher noticing were described. Third, an overview of current studies of mathematics teacher noticing was provided, focusing on three most studied areas. Fourth, an account of current methodologies was presented. Last, a particular aspect of mathematics teacher noticing, i.e., *professional noticing of children’s mathematical thinking*, was introduced. Rather than focusing on identifying gaps in the literature, this review has attempted to clarify what is currently known and done in order to inform the study that is aimed to add knowledge to the field, using a new way of exploring teacher noticing.
CHAPTER 3: METHODOLOGY

Introduction

This research is conducted and reported as a case study that explores teachers’ in-the-moment noticing of students’ mathematical thinking, in the context of teaching a unit from a reform-based mathematics curriculum, i.e., Covering and Surrounding from Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2011, second edition). The focus of the study is to investigate how and to what extent teachers notice students’ mathematical thinking in the midst of instruction, and how and to what extent that noticing influences the teacher’s instruction. Specifically, this study is organized around the following two research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

This chapter begins with a discussion of case study research. Second, it provides a description of the reform-based curriculum and unit used by the teachers participating in this research. Next, the specific methods and the study design are outlined. Lastly, data collection and the analysis process is described.

Case Study

A case study is “an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context” (Yin, 2009, p. 18). Procedures for conducting a case study
involve detailed, in-depth data collection, including the use of multiple sources of information, i.e., observations, interviews, audiovisual materials, and documents. In a case study, data converge in a triangulating fashion, and both data collection and analysis phases are guided by the prior development of theoretical propositions or frameworks (Yin). In the final phase, researchers report the meaning of the case and case-based themes (Creswell, 2007). Lincoln and Guba (1985) state that the researchers tell the lessons learned from the case or cases.

Case studies have been widely used to examine teaching and learning of mathematics. For instance, Putnam (1992) presented a case study of a fifth grade teacher, describing how her knowledge and beliefs manifested themselves in her teaching by describing two lessons about averages. Remillard (2005) provided an extensive review of case studies designed to investigate teachers’ varied use of reform mathematics curricula. A key characteristics of such case studies is that the phenomena studied are embedded in the context with all its complexity, and researchers aim to make meaning of multifaceted teaching and learning experiences within the larger context.

Researchers have used small sample sizes as a means to study what teachers notice in the midst of instruction, capitalizing on a new technology. Although these studies generally do not focus specifically on noticing of children’s mathematical thinking, the studies in this area do investigate teachers’ in-the-moment noticing. For example, Luna et al. (2009) used Camwear 100 to capture in real time what one high school biology teacher actually attended to during instruction and conducted reflection interviews with the teacher after instruction to look at why she noticed the things that she did. Sherin et al. (2008) also used this technology to answer questions about what kinds of events stood out to a mathematics teacher during instruction, and to what extent the teacher was able to articulate why he noticed those events.
Although these studies generally do not focus specifically on noticing of children’s mathematical thinking, the findings do suggest teachers vary in the kinds of things they notice and the extent to which they notice student thinking during instruction (Colestock, 2009; Luna et al., 2009; Sherin et al., 2008). For instance, Luna et al. found that a large percentage of the clips captured by the teacher participant concerned student characteristics or personality. Colestock recognized that the mathematics teacher participating in his study focused almost exclusively on students’ thinking. These studies are examples of how small sample sizes (i.e., number of teachers studied) can be used to explore how and what teachers notice in the midst of instruction. Research techniques that are used in constructing such studies, for example, conducting interviews and observations, have been used to explore how teachers pay attention to student thinking and make decisions about how to respond (e.g., Jacobs & Philipp, 2010; Sherin, Russ, et al., 2011). Additionally, other characteristics of these studies, including in-depth description and analysis of a few individuals and extensive observation, are congruent with the call for research on what teachers are noticing and how they are making sense of important events and interactions to inform teaching decisions (Rodgers, 2002).

**The Curriculum**

This section provides a description of the reform-inspired curriculum and the unit that was taught during this study. Situated within the context of a problem-centered curriculum, this study was designed to reveal classroom practices from a particular perspective, that is, teacher noticing of student thinking. First, a description of the curriculum is presented.

**The Connected Mathematics Project**

This research studied two middle school teachers, each teaching the same unit from the *Connected Mathematics Project* (Lappan et al., 2011, second edition), development of which
was originally funded by the National Science Foundation. The mathematics reform movement calls for exposing students to meaningful and worthwhile mathematical tasks and reforming the teaching of school mathematics around problem solving, sense-making, discourse, and teaching and learning for understanding. This middle school curriculum (i.e., Connected Mathematics Project) was developed for grades six, seven, and eight, focusing on in-depth investigation of mathematically rich problems and situations (Rickard, 1995; Stein et al., 1996). In such tasks, students need to make decisions about what to do and how to do it, come up with more than one solution strategy, and communicate and justify their actions and solutions in written and oral forms.

This curriculum was designed to support the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (NCTM) in 1989. The title, *Connected Mathematics Project (CMP)*, reflects the commitment to a complete middle school mathematics curriculum that attempts to make connections among mathematical topics, to other subject areas, between the elementary and secondary grade levels, and to the real world (Rickard, 1995). CMP devotes special attention to connections among important mathematical ideas (e.g., relationships between perimeter and area), in contrast to typical spiral curricula that tend to be disjointed, fragmented, and superficial (*Connected Mathematics Project*, 2009). Emphasizing problem centered mathematics instruction, mathematical thinking and reasoning, and conceptual understanding, this curriculum provides opportunities for students to develop higher-order skills in mathematics rather than application of routine procedural skills. In CMP classrooms, teachers are expected to center on modeling problem solving and giving students time to hypothesize, discuss, and investigate. As students explore a series of connected and interesting problems, they develop understanding of the embedded mathematical ideas,
problem-solving strategies, and ways of thinking. They learn mathematics and learn how to learn mathematics, with the aid of the teacher (Connected Mathematics Project, 2009).

**Covering and Surrounding**

The CMP unit *Covering and Surrounding* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2011, second edition) is the second of two sequenced units that together constitute the geometry strand of the sixth-grade CMP curriculum as located in a pilot version of the program. The overarching goal of this unit is to help students begin to understand measurement. In particular, the unit is aimed at exploring the relationship between perimeter and area, and effects of each of these measures on the other (Connected Mathematics Project, 2009). As the title of the unit indicates, area is a measure of the number of the square units needed to *cover* a shape; perimeter is a measure of the length or distance needed to *surround* a shape. A sub-theme throughout the unit focuses on questions of what is the greatest and what is the least, helping students to begin developing the notions of maximum and minimum. Preceded by units on number theory and fractions in sixth grade, *Covering and Surrounding* also reinforces students’ comprehension of these previously learned concepts.

The second edition of *Covering and Surrounding* is divided into four major investigations, each of which includes two to four carefully sequenced real-world problems (Lappan et al., 2011). During the instructional process, the teacher launches a problem, then the students explore the problem individually, in groups, or as a whole class, with the aid of the teacher. During exploration, students engage in problem-solving, making conjectures and discussions. A summary phase occurs at the end of each problem. During the summary, the teacher helps students share thoughts and strategies and explicitly describe the mathematics of the problem (see Appendix K for a complete Problem in Investigation 1 from *Covering and Surrounding*).
Overview of Methods

In what follows, an overview of methods is provided. It begins with an explanation of teacher and student participants. Then, it describes the phases of field work, followed by procedures for data analysis.

Participants

One of the first challenges of this study was selection of teacher participants. Teachers needed to teach a unit from a reform-based mathematics curriculum, i.e., *Covering and Surrounding* from *Connected Mathematics Project* (CMP). They needed to be open to new ideas, be willing to use a particular videotaping technology, and agree to engage in discussions of their noticing. Sixth grade teachers and students in Callaway Local School District in the Midwestern United States were selected for recruitment for this study. This district was chosen because the reform-based middle grades mathematics curriculum (i.e., *Connected Mathematics Project*) had been implemented for more than ten years, and the majority of middle grade mathematics teachers (grade 6-8) had previously been involved in professional development related to the use of this curriculum. Teachers and students in the sixth grade were chosen because the unit involved in the study (i.e., *Covering and Surrounding*) was designed to be taught in sixth grade.

The school district superintendent and three middle school principals were approached via a blind, generic email first for permission to conduct the study. Each was told the purpose for the study, the methods to be used, and the time requirements for participants, along with attachments of teacher and student recruitment scripts and consent documents. Once permission was granted, the researcher requested the superintendent to forward a generic, blind teacher recruitment email along with the consent form to all sixth grade mathematics teachers in the district, explaining the
study and requesting information on number of years of teaching experience, number of years teaching this curriculum, and training experience in teaching this curriculum from those who were interested. The initial email contained the recruitment script approved by the University of North Carolina at Chapel Hill Institutional Review Board (IRB) (Appendix A). The original plan was to select two to four teachers from the pool of those qualified teachers willing to participate. However, only two teachers from two different schools expressed an interest in participating and were recruited based on two criteria, including teaching and training experiences.

Teaching and training experiences were considered significant for two reasons. First, Chase and Simon (1973) studied the seeing of grandmasters in chess and concluded that expertise in chess required approximately 10,000 hours in practicing, which equaled to roughly five years of full-time work. Miller (2011) connected this assertion to the domain of teaching, implying that teachers needed at least five years of full-time experience to achieve this level of expertise. Next, the researcher took into account information about these teachers’ amount of professional development experience. As stated by Jacobs et al. (2010), although many important aspects of teaching improve with experience, teachers need more than teaching experience alone to learn to teach mathematics in ways suggested in reform documents. Wilson and Berne (1999) also emphasized the importance of professional development in teacher learning and reform movement in mathematics education. These two teachers had at least seven years of teaching experience, and received ongoing training on teaching this reform-based curriculum. Therefore, they were both selected, and provided IRB-approved consent forms (Appendix B).

One class was selected from each teacher participant’s schedule. For Joe Marshall, the only sixth grade class he taught was in the morning. For Jennifer Goldberg, the class was chosen based on two primary criteria. First, the class was considered “average”, meaning that students
were enrolled in a standard sixth grade mathematics class. There were also no students requiring Exceptional Children services, and every student in the class demonstrated adequate proficiency with English. Second, the class was chosen based on schedule, such that the researcher would have adequate time to arrive, set up, and transition between schools.

Students from these two classes were also recruited to participate in the study. There was a face-to-face meeting with each class for the researcher to explain the study and the student assent/parent permission forms to the students, using the recruitment script approved by the IRB (Appendix C). At the end of the meeting, each student was presented with copies of a student assent form and a parent permission form (Appendix D & E). Students were asked to discuss participation with parent(s) or guardians. Both the parent and the student needed to say “YES” for the student to be in the study and to agree to the specific components of the study, that is, to be observed, videotaped, and/or asked to provide written work. The IRB-approved parental assent and consent forms allowed students to participate in certain aspects of the study and decline participation in others. For the purposes of the study, class seating was rearranged and the cameras were positioned so that any video-recording of teaching and learning activities could include only those students who had agreed to be video-recorded.

**Teacher participants.** This study was carried out at East Park School and Daisy School, in the classrooms of Joe Marshall and Jennifer Goldberg, respectively. Mr. Marshall is a Caucasian man, who has taught fifth to eighth grade mathematics for 23 years, and this particular curriculum for 14 years. Ms. Goldberg is a Caucasian woman, who has taught sixth grade mathematics for 7 years, and this curriculum for 7 years. East Park School has an enrollment of 180 students in grades one through eight (GreatSchools Incorporated, 2013). The student population is 2% African American, 2% two or more races, and 96% White. There are 26% of
the student population that are eligible for free or reduced-price lunch program. Daisy School has an enrollment of 385 students in grades one through eight (GreatSchools Incorporated, 2013). The student population is 1% African American, 2% Hispanic, 1% Asian, 3% two or more races, and 93% White. There are 44% of the student population that are eligible for free or reduced-price lunch program.

**Student participants.** The sixth grade class taught by Mr. Marshall was his third period and met daily from 10:44 AM - 11:37 AM. The demographics of this class appeared to be representative of the larger school population and are provided in Table 3.1. The class selected from Ms. Goldberg’s schedule was her sixth period class and met daily after lunch and recess from1:18 PM to 2:05 PM. Although dominated by a majority of girls (see Table 3.1), this class was also adequately representative of the Daisy School student population.

<table>
<thead>
<tr>
<th></th>
<th>Girls</th>
<th>Boys</th>
<th>African American</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshall</td>
<td>11</td>
<td>14</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>25 students</td>
<td>44%</td>
<td>56%</td>
<td>4%</td>
<td>96%</td>
</tr>
<tr>
<td>Goldberg</td>
<td>12</td>
<td>6</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>18 students</td>
<td>66.7%</td>
<td>33.3%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

More than half of the students in each class chose to participate in the study (see Table 3.2). The total number of students who submitted the appropriate forms for participation included: Marshall, n = 18, 72%; Goldberg, n = 15, 83.3%, involving only White students. The IRB-approved assent and consent forms allowed students to participate in certain aspects of the study and decline participation in others, that is, to be observed, videotaped, and/or asked to provide written work. Only two student participants from Mr. Marshall’s class participated partially in
the study, that is, they agreed to be observed and videotaped, but did not allow collection of their work. Although some students chose not to participate in the study, they were all part of the class, instruction, and tests. Accommodations were made so that non-participants were not included in the study’s data collection methods, i.e., observation, video-recording, and collection of student work.

Table 3.2: Student participants in participating classrooms

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Total Students in Class</th>
<th>Number of Student Participants</th>
<th>Number of Participants Allowing:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Observations</td>
</tr>
<tr>
<td>Marshall</td>
<td>25</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Goldberg</td>
<td>18</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Field Work

This project consisted of three main parts in the field work stage. The first part involved separate initial interviews. The second part included a series of classroom observations as well as researcher-generated and teacher-generated videotaping. The third part involved daily follow-up interviews with teachers about the video-tapes taken by them, in each classroom. There were mainly six types of data collected in this study. These data were from

- initial interviews of teacher participants,
- daily researcher classroom observations,
- daily researcher-generated whole-class video-recording,
- daily teacher-generated individual/small group video-recording,
- daily follow-up interviews of teacher participants,
- collection of student classwork.
The data served various purposes and were collected at different points at the field work stage. Table 3.3 illustrates the process during which each type of data was collected, and instruments used.

Table 3.3: Data collection process and instruments used

<table>
<thead>
<tr>
<th></th>
<th>Part One</th>
<th>Part Two</th>
<th>Part Three</th>
<th>Instrument Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Interviews</td>
<td>x</td>
<td></td>
<td></td>
<td>Interview Protocols</td>
</tr>
<tr>
<td>Classroom Observations</td>
<td></td>
<td>x</td>
<td></td>
<td>Observation Protocols (Innovation Configuration Maps)</td>
</tr>
<tr>
<td>Researcher-Generated Video-</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Recordings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher-Generated Video-</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recordings</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Follow-Up Interviews</td>
<td></td>
<td></td>
<td>x</td>
<td>Interview Protocols</td>
</tr>
<tr>
<td>Artifact Collection</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

An overview of the data collection process is provided below. Each part is detailed, including the purpose, design, and data collection procedures.

**Part 1: initial interview.** During this part of the study, the researcher interviewed both teacher participants, separately. The purpose of the interviews was to acquire general background information on the teacher participants to get an idea of teachers’ mathematics knowledge related to teaching perimeter and area, and the role children’s mathematical thinking played in teachers’ decision making. These areas represent regions that the research literature indicates influence what teachers notice and how they act on it in the midst of instruction.
For instance, Even and Wallach (2004) suggested lack of knowledge about common student conceptions and their possible sources and failure to ascribe values to students’ ideas were preventing teachers from attending to children’s mathematical thinking in the midst of instruction. Morgan and Watson (2002) also attributed how and to what extent teachers noticed children’s mathematical thinking to teachers’ personal knowledge of mathematics, and expectations of students. As identified by Jacobs and Philipp (2010), instead of focusing on students’ mathematical thinking, some teachers tended to use their own mathematical thinking or general teaching moves that could be applied to any problem and any child to guide their noticing and decision making. Therefore, through interviewing the participants at the beginning of the study, the researcher hoped to gain general background information to better gauge issues emerging in these teachers’ in-the-moment noticing of children’s mathematical thinking. These interviews were by no means designed to achieve information to predict teacher behavior.

This part involved two steps, both of which took place in the initial interview. In step one, the researcher provided teachers with three items and asked them to respond to some interview protocols verbally, in order to assess mathematics knowledge of teaching perimeter and area, during initial visit to the sites (see Appendix F for items 1, 2, and 3). These exact items were used in Rickard’s (1996) study to evaluate teachers’ teaching knowledge of perimeter and area, in an effort to examine teachers’ use of the CMP curriculum materials. Item 1 concerned knowledge of the relationship between perimeter and area as well as understanding of proof. Item 2 was intended to find out if teachers think conceptually about area as the number of square units needed to cover a shape. Item 3 asked teachers to make sense of a student’s response to perimeter problems. In this case, correct answers were obtained in a non-standard way. This item hoped to gain insight on teachers’ flexibility in their understanding of finding perimeter.
Using interview items that deal with the concepts of perimeter and area was consistent with the focus of the *Covering and Surrounding* unit. Centered around the same topics as the unit, these items were expected to help the researcher make better sense of the data collected from other sources, for instance, classroom observations, video-recordings, and interviews, compared with items that deal with concepts different from perimeter and area. Further, this interview was designed to assess teachers’ teaching knowledge. Thus, items framed in the context of classroom scenarios helped the researcher look at the interaction of teachers’ subject-matter knowledge with teaching knowledge. Since there was lack of information on validity and reliability of the instrument, the researcher consulted with two university mathematics educators prior to field work to ensure quality and content validity of the three interview items.

In step two, the researcher focused on collecting information on the role children’s mathematical thinking plays in teachers’ decision making. First, a short video clip was shared with each teacher participant. This clip involved a second-grade class in a suburban public school in the Midwest exploring the concept of measurement, in particular, rules of linear measure, without using conventional tools. A focus on measurement was consistent with the *Covering and Surrounding* unit. Then, after showing the video to the teacher participants, the researcher asked them to respond to the following prompts verbally. Only one prompt was given out at a time, in the following order. Conversations were audio-recorded with permission:

- What did you notice/stood out to you about this video in general?
- What did you notice about the teaching behavior from this video?
- What did you notice about student thinking from this video?
- Now that you told me what you had noticed, can you tell me what noticing means, in your opinion? In terms of teaching?
This interview part was designed to examine the role children’s mathematical thinking plays in teachers’ noticing. In this video, the mathematically rich classroom discourse on measurement served as an example of how a student-centered classroom might look, that is, when the teacher questioned, listened, and responded to student thinking constantly. This video clip afforded a window into noticing on the part of the viewer. The intent in this interview part was not to gain information to predict teacher behavior. Through examining the participants’ noticing using a teaching video, the researcher hoped to better evaluate future issues emerging in the participants’ noticing of children’s thinking in real time. In order for the video to serve as a substitute for the actual instructional situation where children often share their thinking verbally and “a rewind button does not exist” (Jacobs, Lamb, & Philipp, 2010, p. 180), the prompts were presented to the participants after the video was played only once. These open-ended prompts allowed the researcher to tap the participants’ understanding of teacher noticing, and their interpretation of student-centered instruction exemplified in a teaching video.

Using a teaching video to investigate teacher thinking is supported by the research literature that endorses video-based activities for teaching and learning. Videos of classrooms or a particular moment allow viewers the time and place to reflect and to hone their abilities to be observers of classroom interactions (see, e.g., Star & Strickland, 2008; van Es & Sherin, 2002). What teachers offer might be reflective of the teachers’ own teaching knowledge and what and how they generally notice when presented with similar instructional situations. In this interview, the researcher hoped to gain information on participants’ skills of noticing, and interpretation of a learning environment that encouraged students’ mathematical inquiry, understanding, and sense-making. Data from this part was intended to shed light on the future phenomena observed in these participants’ own classrooms.
Combined, teachers’ verbal responses to the three items and also prompts after viewing the video provided information about teachers’ mathematics knowledge of teaching perimeter and area, and the role children’s mathematical thinking played in teachers’ noticing. By asking participants open-ended questions throughout the interview parts, the researcher tried to allow them room to articulate their own reasoning and teaching knowledge of the mathematics, and their own observation of student thinking and student-centered instruction. Interviews were audio-recorded and partially transcribed. In what follows, part two of field work is explained.

**Part 2: observations and videotaping.** In this part, the researcher conducted on-site, daily observations of the teacher participants as they were teaching the *Covering and Surrounding* unit. The original plan was to spend one class period per day in each teacher’s classroom for nine days. However, on Day 3, due to a scheduling conflict caused by some unexpected school-wide activities at East Park School, the researcher decided to have Mr. Marshall videotape his entire room and also wear the headset camera without the researcher in the room (as the researcher was observing Ms. Goldberg during the same time when Mr. Marshall taught his 6th grade class). As a result, the researcher spent eight days in Mr. Marshall’s classroom, and nine days in Ms. Goldberg’s room. For the purpose of understanding how and to what extent teachers noticed children’s mathematical thinking during instruction, and in particular, how and to what extent that noticing influenced their instruction, the researcher chose to observe the same class periods throughout the study in which related learning objectives were emphasized. As stated by Sherin, Jacobs, et al. (2011), the phrase *teacher noticing* depicts an active rather than a static process through which the teacher attends to the ongoing information presented in a complex, dynamic system, and the teacher’s analysis of that noticing might produce continuous change in the practice of teaching by the individual. Therefore, observation of the same class periods for
approximately two weeks provided the opportunity to capture the changes made in response to what had been noticed during instruction by the teacher, enabling the researcher to trace the influence of that noticing on subsequent teaching. This decision is also supported by a study conducted by Borko et al. (1988). In that study, several secondary student teachers reported revising lesson plans and making adjustments between class periods during which they taught the same objective.

Observational data were collected during classroom observation via detailed field notes. The researcher assumed the role of a complete observer and took notes on teacher-student interactions during group work. For the purpose of tackling memory lapses and potential technical issues, the researcher decided to follow the teachers’ noticing during the second week, and spent more time recording observational situations. This part of the field notes helped teachers better recall captured moments and provided information on the video clips recorded incorrectly as well. Field notes were mainly divided into two segments, including descriptive and reflective notes (i.e., notes about the physical setting, classroom routines, events and activities, along with the researcher’s own hunches or reactions). The researcher expanded the field notes the same day as the classes were observed or the next day at the latest. Besides, in the beginning, middle, and end of the field work stage, the researcher used a valid and reliable CMP-related instrument (i.e., Innovation Configuration Maps developed by Huntley, 2009) to measure the quality of participants’ reform-based practice in teaching CMP. This part of the data collection was designed to provide some information on the quality of reform-based instruction being implemented in these classrooms, and helped the researcher better gauge and interpret issues emerging in participants’ in-the-moment noticing of children’s thinking.
In addition to taking observational notes, the researcher also recorded the lessons using a standard video camera and a new technology concurrently with on-site observations. In what follows, there is an explanation of why and how these two technologies were employed. Discussion of the new technology is substantially longer than the first.

**Standard video camera: rationale and design.** Several attributes of video indicate that it might be a valuable media for exploring teachers’ in-the-moment noticing. First, to some extent, video has the potential of capturing the complexity of classroom interaction that takes place simultaneously as it is. Second, video provides a permanent record that can be reviewed and analyzed repeatedly. Last, video affords the time for viewers to engage in extended reflection on what has taken place in a lesson, enabling them to notice things they may have missed before (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Sherin et al., 2008; van Es, 2011). Therefore, the researcher decided to videotape the lessons using a standard video camera stationed in the back of the room, providing mostly distant shots of the entire lesson. The researcher tried to keep the distractions caused by videotaping to a minimum, through practicing using it once in these classrooms, prior to beginning official field work. The video was reviewed, and summarized following each visit. Later during analysis, the whole class videos were fully transcribed.

**New technology: rationale and design.** Although particular aspects of a traditional video camera make it useful to richly represent classroom environments, such a classroom video only takes on an observer perspective, in which a main focus is on watching the teacher and the environment. Recorded from the researcher’s perspective, a traditional classroom video represents a somewhat distorted view of what a teacher actually sees, involving setting up a camera in the back of the room, showing the teacher’s face from a frontal view, and viewing whole-class interactions from a fairly wide angle (Sherin et al., 2008). In order to overcome this
shortage, in a study conducted by Stigler, Gonzales, Kawanaka, Knoll, and Serrano (1999), the videographers were explicitly instructed to assume the perspective of an ideal student and point the camera toward what the ideal student should focus on at any given time. The study demonstrated an effort to videotape from a perspective other than the researcher’s. However, such an attempt still did not allow a research participant’s perspective.

Miller (2011) talked about how a teacher-perspective video might have a more direct role to play in teacher professional development. He maintained that video from this point of view could be strikingly different from a standard classroom video. The motion of the teacher could lead to a far more complex and dynamic scene than what was captured by a traditional video. Therefore, to study teachers’ in-the-moment noticing, researchers should take into account teacher-perspective video. Through using videos captured from teachers’ perspective, researchers put the video in the hands of the teachers and allow classroom interactions to be represented from teachers’ perspective. In addition, a review of literature reveals that traditional approaches do not examine teacher noticing as it occurs in the midst of instruction. Rather, they ask teachers to analyze and reflect on videos outside the context and pressure of in-the-moment instruction. Thus, in order to tackle these methodological disadvantages and access teachers’ in-the-moment noticing in a more explicit and direct way, the researcher in this study applied a new technology to explore teacher noticing, in addition to using a standard video camera.

Adopted by Sherin, Russ, et al. (2011) in a recent study of teachers’ in-the-moment noticing, this new technology involves a portable video camera with selective-archiving capability. It enables the user to select moments to capture immediately after they occur. As the camera is designed to record moments of informal interaction occurring in natural settings, users typically
do not find recording burdensome or interfering with the ongoing nature of activities in the settings.

The Deja View Camwear 100 has two components (Reich, Goldberg, & Hudek, 2004). One of them is an approximately 1-inch long wearable camera that can be affixed to the teacher’s glasses or to the bill of a hat; the other one is a small recording module that can be attached to a belt. The camera streams video continuously, recording over previously recorded material after a short time. This process can be interrupted when the “record” button is pressed, saving the previous 30 seconds of video in a digital file that can be stored on a video card.

In particular, the feature that makes this video camera significant for this study is that it allows the teacher to record instruction from the teacher’s point of view. In their groundbreaking study, Sherin, Russ, et al. (2011) found that capturing moments with this camera during instruction was both a sensible and a feasible task. Several findings are as follows. First, thirteen teachers who participated were able to notice and also become conscious of the fact that they were noticing. Second, teachers were aware of both their noticing and their thinking about those noticed events approximately at the same time. Third, during interviews that took place later on the same day of videotaping, most teachers were able to quickly recall what they had noticed and why they had chosen to capture certain events, simply from being shown the still image or only a few seconds of video. Some teachers were even able to predict what was captured in a subsequent video before viewing it. These findings, therefore, distinguished this methodology from the traditional retrospective-recall approach, indicating the potential of a method to record teachers’ in-the-moment noticing. Overall, these participants reported that wearing the camera did not interfere with the ongoing nature of activities in the classrooms. Thus, this method allowed these researchers to examine teacher noticing as it occurred in the midst of instruction,
instead of asking teachers to analyze and reflect on videos outside the context and pressure of in-the-moment instruction.

In the above-mentioned study, the researchers were mainly interested in trying out this new technology and understanding the variety of what teachers noticed in the midst of instruction. These noticed elements included student thinking, classroom discourse, teacher moves, teacher strategies, and student engagement. Importantly and differently, for the present dissertation project, rather than attend to the specific kinds of things teachers noticed, the researcher chose to focus on a specialized type of noticing, i.e., teachers’ noticing of children’s mathematical thinking. In the current context of mathematics education reform, learning to notice student thinking becomes particularly relevant (Ball & Cohen, 1999). Teachers cannot decide how to respond to students’ ideas until they have also attended to their strategies and interpreted the understandings reflected in those strategies (Jacobs, Lamb, et al., 2011). In addition to focus on teachers’ attending, interpreting, and decision making, this study tried to tap teachers’ next moves that might be influenced by their noticing, through exploring the question like “How and to what extent does that noticing influence the teacher’s instruction?”

Therefore, the researcher implemented a similar method used by Sherin, Russ, et al. (2011), with necessary modifications. First, the researcher provided teachers the opportunity to try the camera prior to beginning official field work. Then, before each class period, the teacher was outfitted with the camera and instructed to press the record button on the camera when something noteworthy happened, in terms of student thinking. Originally the term “student thinking” was not included in the instruction, in order to minimize the potential influence it could have on the types of classroom moments captured. However, after discussion with the dissertation committee, the researcher decided to include the term, as it was possible that
teachers’ understanding of “student thinking” differed from what had been described in reform documents, and making it explicit would also ensure that appropriate data was collected. No limit was given regarding the number of moments the teacher could capture. Further, as indicated above, students who did not want to be videotaped sat in places outside of the camera’s range. At the conclusion of field work, there were 165 episodes correctly captured on camera. In addition, there were 2 episodes incorrectly captured on camera but recorded in field notes.

**Part 3: follow-up review of videotapes.** The researcher conducted approximately 30-minute follow-up interviews with the teachers separately, regarding teacher-generated videotapes. Once these recordings were uploaded, the researcher selected one to nine episodes based on the extent of mathematics thinking involved and/or how representative they were of the teacher’s overall noticing pattern, and discussed them with teachers, as it was not practical to review all episodes within 30 minutes. The original plan was to interview each teacher later on the same day of each visit. However, certain scheduling difficulties and unexpected situations (e.g., parent conferences) required adjustment of the plan. Table 3.4 illustrates the adjusted interview schedule.
Table 3.4: Classroom observation and follow-up interview schedule

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Day1</th>
<th>Day2</th>
<th>Day3</th>
<th>Day4</th>
<th>Day5</th>
<th>Day6</th>
<th>Day7</th>
<th>Day8</th>
<th>Day9</th>
<th>Day10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshall</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Follow-Up Interview</td>
</tr>
<tr>
<td>Goldberg</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Classroom Observation</td>
<td>Follow-Up Interview</td>
</tr>
</tbody>
</table>
The researcher interviewed the teachers using a standard protocol adopted from Sherin, Russ, et al. (2011), with modifications based on the focus of this project (See Appendix G). Although the teacher-captured clips allowed access to teachers’ in-the-moment noticing, they could not fully represent the thinking that teachers had engaged in while noticing these events in the midst of instruction. Thus, interviews provided information on teachers’ interpretations of the captured moments, rather than the researcher’s inference from watching these clips. Another purpose of the interviews was to overcome, at least partially, the potential logistical difficulties of capturing noticed moments using the camera, as viewing just a portion of the video during interviews was usually sufficient to cue the entire moment for teachers who did not remember the moment captured, or who had difficulty timing their capture. Third, the interviews helped the researcher understand how and to what extent teachers’ noticing of student thinking influenced their instruction. As it was not always the case that teachers would respond to certain noticed events immediately or explicitly during instruction, these interviews afforded them the opportunities to describe how that noticing would influence their future instruction. Finally, during some interviews, the researcher was able to ask the teachers to talk about particular occurrences documented with field notes during the researcher’s previous visit(s). During each interview, the teacher was asked to describe the reasons the moments had been captured, after watching each of the selected episodes. At the end of each interview, the researcher asked whether the captured clips represented what the teacher had intended to capture, and whether all of the clips together overall reflected what the teacher had found interesting or important in terms of student thinking. Interviews were videotaped, summarized, and fully transcribed.

In addition to the three main parts of data collection, the researcher also collected data from relevant documents throughout the field work stage, for example, teachers’ lesson plans,
students’ class work, the CMP materials, information on the schools and their communities, and district curriculum objectives and policies. Student work was photocopied, participants’ identification numbers were placed on the work, and names were removed. At times the researcher asked the teachers to react to and discuss specific student work as a way of better understanding the classroom context and student understanding. All of these resources were aimed at a better understanding of teachers’ in-the-moment noticing. Following is a detailed description of the data analysis process.

**Analysis**

The analytic process is described below. The discussion begins with a description of the informal analysis process. Then formal analysis process is described.

A wide range of data sources included initial interview, on-site observations, videotapes recorded using two technologies, follow-up interviews (i.e., interviews conducted after the video clips were captured), and relevant documents, for instance, teachers’ lesson plans, copies of students’ work, and the CMP materials. All data were prepared and organized both by type and participant for analysis, i.e., all interviews, all observations, all teacher-generated whole class video recordings, all teacher-generated group work video recordings, and all documents for each participant. With regard to data security, all data were password-protected and stored on the researcher’s password protected computer. All contact information and completed permission forms were kept in a locked space. Every effort was made to keep individual school identities and individual teacher identities confidential. Then, after being organized and secured, data were reduced into themes through coding, condensing the codes, and finally representing the data in diagrams, tables, or discussions. This inductive form of data analysis took place concurrently with and after field work, too; analysis of the data was ongoing in a spiral and iterative manner.
**Part 1: informal analysis.** In more detail, the data analysis phase mainly consisted of two parts. It involved first a more informal form of analysis, rather than the official qualitative coding and counting that occurred once all data had been collected. During the entire data collection phase, information previously collected was constantly reviewed. For example, the researcher compared some observational data with teachers’ responses to the initial interview items. The intent was not to predict or check the behavior of the teacher participants regarding their teaching of the unit, but was to help draw attention to issues that were occurring or not occurring in their actual teaching practice. The researcher also made constant comparisons among data collected through observations, the standard video camera, and the new technology. Particular attention was paid to teachers’ discussions with the researcher during follow-up interviews. A critical component of analysis during this phase was the researcher’s selection of teacher-generated videos to be discussed with participants, as it was not feasible to talk about all of them during 30 minutes. Drawing on multiple sources of data, such as initial interviews, lesson objectives, and previous follow-up interviews, the researcher determined the videos worthy of discussion, and the order in which they were to be discussed, on a daily basis, based on the extent of mathematics thinking involved and/or how representative these videos were of the teacher’s overall noticing pattern. Throughout the data collection process, the researcher wrote memos in the margins of fieldnotes to record ideas and hunches occurring to the researcher.

Such informal forms of analysis were critical for processing the teacher-generated videotaping and making the best of the 30-minute follow-up interviews, as such analyses helped the researcher not only decide on certain episodes to be discussed but also form questions and look for major ideas to be explored in the remainder of the data collection process. The second
part in the analysis process is the formal analysis phase, in which all data were compiled and analyzed as a whole. The analysis procedures are detailed below.

**Part II: formal analysis.** At the conclusion of field work, all data were compiled for formal analysis. A more thorough analysis was conducted. This phase occurred over a period of 12 months. First, it included a thorough organization of all data, and extensive transcribing, coding, and analysis. Through reading data from multiple sources in their entirety several times, immersing in the details, and trying to get a sense of the data as a whole before organizing them into parts, the researcher maintained a transparent interpretive presence throughout the analytical process (Agar, 1980). In addition to organizing data by type and participant, the researcher created an Excel spreadsheet and entered information on each teacher-generated episode for organizational purposes. That is, a number was assigned to each episode, followed by information regarding the teacher who had recorded the episode, whether it was selected for follow-up interviews, what day it was recorded, and later the type of noticing (i.e., code) once coded.

Second, transcription took place. Recordings from initial interviews were reviewed on an ongoing basis throughout field work, and selected portions of these recordings were transcribed at the conclusion of the field work phase. Other audio-visual materials, including researcher-generated whole class videos and recordings from follow-up interviews, were fully transcribed after field work was completed. Each transcript was individually reviewed. This review included a substitution of pseudonyms for all students and both teachers, references to the schools, the district, etc. In addition, attempts were made to fill in inaudible gaps or clarify the content of both teacher and student contributions. At the conclusion of this review, there were 18 complete transcripts of whole-class videos recorded by the researcher, and 11 complete transcripts of
follow-up interviews. Transcription of 165 episodes recorded by teacher participants was neither necessary nor practicable for this volume of data. Instead, the videos were scanned, and notes were recorded on topics of interest. It was decided that portions of these group work video-recordings could be transcribed at a later time as needed. Throughout the data analysis phase, transcripts of recordings from initial interviews, researcher-generated whole class videos, and follow-up interviews were reviewed for evidence of how teachers’ noticing of student thinking had impacted the progression of instruction.

Third, data from teacher-generated video-recordings and follow-up interview video-recordings underwent qualitative coding. A general description of the qualitative coding procedures is provided here. A more thorough description of the coding scheme is provided in Appendix I. This includes categories of codes, individual codes and their descriptions, and some examples where practical.

As previously mentioned, only one to nine recorded episodes were selected for each follow-up interview, resulting in 50 selected episodes out of a total of 167 episodes. Table 3.5 illustrates the number of episodes recorded and selected for each participant each day. On some days, the researcher was able to interview the teachers the same day when clips were captured. On other days, the researcher had to discuss with the teachers clips captured across a few days, due to scheduling difficulties (see table 3.4 for interview schedule).
Table 3.5: Teacher clip capture and selection information

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Day1</th>
<th>Day2</th>
<th>Day3</th>
<th>Day4</th>
<th>Day5</th>
<th>Day6</th>
<th>Day7</th>
<th>Day8</th>
<th>Day9</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marshall</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td>13</td>
<td>2</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>77</td>
</tr>
<tr>
<td>Goldberg</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
<td>T</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>10</td>
<td>5</td>
<td>13</td>
<td>4</td>
<td>14</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0</td>
<td>7</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>17</td>
<td>3</td>
<td>90</td>
<td>27</td>
</tr>
</tbody>
</table>

Note. “T” represents the total number of episodes captured by each participant. “S” represents the number of selected episodes for follow-up interviews.

Transcriptions of these interviews served as the primary sources of interpretation of the selected episodes. As well, data from other sources were frequently referenced during analysis and coding of these teacher-generated videos and related transcripts of follow-up interviews. These included the transcriptions of initial interviews and whole-class instruction, observation protocols and field notes on classroom observation, and relevant documents, such as, copies of student work. They were frequently reviewed during interpretation of the teacher-generated episodes and related interviews in order to discern and comprehend patterns of these teachers’ noticing.

Generally, a chronological approach was used to review and code the episodes and follow-up interviews. For example, initially all Day 1 episodes, transcriptions of follow-up interviews, field notes, and whole class videos were analyzed for each participant, then all Day 2 episodes and related transcriptions. With regard to episodes that were not selected for discussion, the researcher’s inference from watching the clips and examining related data (e.g., field notes and transcripts of whole class videos) became the primary source of interpretation of the clips.

Coding occurred using the approach of emergent coding. According to Stemler (2001), “with emergent coding, categories are established following some preliminary examination of the data.” Therefore, codes were created exclusively based on analysis after data had been...
collected. The strategy for coding was quite straightforward and encompassed four stages: initial review, second round review, reliability check and revision (if necessary), and consolidation. The first step was simply to review all data and come up with a set of codes to form a checklist according to which each episode could be coded. In the second step, the researcher attempted to approach the data with a blank slate, assigning codes for a second time without referring to the original checklist for each episode. Third, for inter rater reliability purposes (i.e., the researcher served as both first coder and second coder, given the time lapse between the first and second codings), the researcher compared two sets of codes for each episode, and reconciled any differences showing up in the codes. A reliability score of 0.8 was established to show the extent of consensus between two rounds of coding. This involved adding up the number of episodes that were coded the same way through two rounds of coding and dividing by the total number of episodes. Last, the coding scheme was reviewed to meet Marshall and Rossman’s (2006) recommendation that “the categories should be internally consistent but distinct from one another” (p. 159). Codes that were slightly distinct were counted as different codes, in order to capture the variety of things teacher noticed, and the different reasons for noticing them. For example, student conjecture and student misconjecture were counted as two different codes. A few codes were consolidated. For example, student work ethic and student engagement were not distinctive codes, so the episodes coded as student work ethic were re-coded as student engagement. As well, vocabulary and specific content were not distinctive codes, so the episodes coded as vocabulary were re-coded as specific content.

In addition, since this study involved two different classrooms, it was important that data collected at these two sites be compared in an ongoing manner in order to identify common and different themes. Thus, a table was created to categorize all episodes by teacher, including the
frequency of each type of noticing (i.e., code) for each teacher. A second table was designed to categorize all episodes by week, showing the frequency of each type of noticing (i.e., code) for each week. Another table was established to show a frequency count for different categories of codes.

In summary, as described by Frykholm (2004), the analysis and interpretation of data consisted of a continuous reading of field and observational notes, a memoing process that helped capture potential lines of inquiry during data collection and preliminary analysis, a systematic breakdown and coding of the episodes and transcriptions of follow-up interviews, which led to the identification of key categories of codes, and patterns appearing across data sets and case descriptions, and finally saturation or a point where themes were fully developed and no new information could be added to the list of existing themes. Following Frykholm’s recommendations, the researcher utilized this wide range of data sources to investigate the questions guiding this study, including how and to what extent teachers notice students’ mathematical thinking in the midst of instruction, and how and to what extent that noticing influences their instruction.

Summary

Thus far a discussion of case study has been provided, explaining the rationale for choosing this approach. Moreover, the methods and the study design have been outlined, and rationales and procedures for recruiting participants and conducting field work have been elaborated. Data analysis, for instance, coding and procedures for checking the accuracy of the study, has been discussed as well. Drawn from Jacobs et al. (2010) conceptualization of what is involved in professional noticing of children’s mathematical thinking, the researcher structured analysis of participants’ noticing in terms of the process teachers experienced in their efforts to respond to
their students. In this study, teachers’ professional noticing of children’s mathematical thinking not only includes attending, interpreting, and deciding how to respond, but also extends to responding. As a matter of fact, Jacobs et al. (2010) recognized that the ultimate utility of the construct, i.e., teachers’ in-the-moment noticing of children’s thinking, would depend on the ways in which future studies would connect this construct with the actual execution of teaching moves in response to that noticing. This study sought to do just that. The findings are presented in the next chapter. In the final chapter, the conclusions and implications of these findings are discussed, and avenues for further research on mathematics teacher noticing of student thinking are proposed.
CHAPTER 4: FINDINGS

Introduction

In the previous chapter, the methodology for accessing two teachers’ in-the-moment noticing of students’ mathematical thinking, in the context of teaching a unit from a reform-based mathematics curriculum was detailed and analysis procedures discussed. Data were collected from multiple sources:

- initial interviews of teacher participants,
- daily researcher classroom observations,
- daily researcher-generated whole-class video-recording,
- daily teacher-generated individual/small group video-recording,
- follow-up interviews of teacher participants,
- collection of student classwork.

These data were collected throughout the field work to be used to answer the following research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?
2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

In this chapter, the findings are presented. The two guiding research questions provide an overall framework. Highly interrelated with each other, these questions are explored and discussed in an
integrated way, based upon the premise that mathematics teachers’ noticing of students’ thinking is recognized as a set of four interrelated components, involving attending to student thinking, interpreting students’ understandings, deciding how to respond on the basis of interpretation of students’ understandings, and responding to students in particular ways. Instead of discussing findings about these questions separately from one to the other, this chapter addresses the research questions using an integrated approach and is organized into four main sections. In the first section, descriptions of each teacher, including teaching practices in the context of a reform-based curriculum, beliefs, and knowledge related to the unit are presented that draw on a variety of sources of data, such as recordings of classroom instruction and initial interviews. In the second section, findings on what teachers noticed are presented, including explanation of procedures for data collection and analysis, and a general description of what teachers noticed. In the third section, each teacher’s noticing pattern is described that involves the process of attending to student thinking, interpreting students’ understandings, deciding how to respond on the basis of interpretation, and responding to students in certain ways. In the last section, a comparison between the ways of noticing of the two teachers is provided that elaborates on how and what they notice and how they act upon it is a function of the teachers’ beliefs and knowledge.

**Meeting the Teachers**

This study was carried out at East Park School and Daisy School in Callaway Local School District, located in the Midwestern United States, in the sixth-grade classrooms of Joe Marshall and Jennifer Goldberg, respectively. This section presents a profile of each teacher participant, followed by a description and interpretation of his or her reformed-based teaching using the
CMP curriculum unit, based on the Innovation Configuration Maps developed by Huntley (2009).

Mr. Marshall is a Caucasian man who has taught fifth to eighth grade mathematics for 23 years, and this particular curriculum for 14 years. Ms. Goldberg is a Caucasian woman who has taught sixth grade mathematics for 7 years, and this curriculum for 7 years. Both teachers are considered to be experienced teachers who are interested in mathematics education reform but exhibit different orientations toward accomplishing mathematics teaching and learning in their classrooms. Mr. Marshall appears to attend to understanding and working with knowledge students bring with them to any mathematical situation to frame his instruction, whereas Ms. Goldberg seems to frame her instruction by laying out content and explaining it to students in clear and straightforward ways. While both teachers profess belief in a reform-oriented approach, their behaviors focus on the nature of mathematics and teaching and learning mathematics in different ways. This section begins with a vignette from Mr. Marshall’s lesson on Day 1, when the unit of Covering and Surrounding was launched. Then a profile of the teacher, including a description of his teaching is provided, followed by a discussion of his beliefs and knowledge about mathematics teaching and learning.

A Profile of Joe Marshall

A vignette. With his students sitting in small groups, Mr. Marshall addressed the class:

Marshall: I want your group to write down what you know about perimeter and area. We are gonna start by just getting some thoughts out...

[Students engaged in small group discussion for approximately three minutes. Then the teacher called the class back together.]

Marshall: Let’s stop there. We’re gonna have people talking. If you want to come up and show your paper, you can...

After allowing students to engage in substantive whole class conversations with peers on what they knew about perimeter and area, the teacher introduced the investigation problem
that involved designing floor plans of bumper car rides that have an area of 36 square meters and a perimeter of 26 meters (see Appendix K for Problem 1.1 from Investigation 1 in its entirety, in the final published version, as the teacher was using a pilot version of this problem during the study). Then he placed four meter sticks in an open area in the back of the room, forming a square shape on the floor and addressed the class with his students facing the back of the room:

Marshall: (pointing to the floor) This is just an example of what one piece of [bumper car] flooring looks like. The area of this shape is how many meters, or square meters?...

Holland: Four.

Marshall: Does everybody agree? The area of this is four. Go ahead and talk: I agree, or disagree...Corey?

Corey: The sides are all the same. And you times (i.e., multiply) them by themselves, and that’s the area of the shape.

Marshall: So you think the area is how much?

Corey: Four. That’s four, that’s four, that’s four... (pointing to each side of the shape)

Marshall: Do you guys agree with that? The side [of the square] is four? Four of what?

Newton: I agree. Each side is one meter, and you have four sides around, so you add them up to get four square meters.

Marshall: Does everybody agree with that? You add them up and you get four square meters?

Thia: I disagree. There is only one square [inside the sticks], so it’s one square meter.

Marshall: So you think it’s one square meter because there’s only one square in it. Any other opinions?

Tora: Each side is one meter, and there is one square [inside the sticks], so it’s one square meter.

Marshall: I agree. Newton, when you were counting, you were counting the outside [edge] of it, and you were saying, ‘One meter, two meters...’

Newton: I was counting the perimeter.

Marshall: Can you say that again?

Newton: I was counting the perimeter.

Marshall: Awesome. Can somebody repeat that? If I did one meter plus one meter plus one meter plus one meter, what am I counting? And how much is it? Helen?

Helen: Four.

Marshall: Four what?

Helen: (hesitantly) Square meters?

Marshall: Is that what the distance is around the shape? Four square meters?
Joe: Four meters.
Marshall: So that’s a big issue. Is it four meters around this, or is it four square meters? That’s very important to me when you write answers that you are trying to write the right units. I’m not gonna tell you my opinion yet, but I want you to think about it... I’m worried about your counting: Are you counting the right things? Now go for it.

After spending approximately 30 minutes investigating Problem 1.1 (see Appendix K) in small groups, the students reconvened for a summary of the lesson. Mr. Marshall stated that he had “noticed something happening that needs to be addressed,” and demonstrated one of the most common student misconceptions noticed for finding perimeter on the overhead transparency projector, which is what to count for perimeter and area. He then showed the correct way of counting the perimeter, leaving the students with the question of what to count for area.

This vignette from Mr. Marshall’s lesson on Day 1 is taken from the launch of Problem 1.1, before students started designing floor plans that have an area of 36 square meters and a perimeter of 26 meters. It presents a messy picture of student confusion and misconceptions of area and perimeter; however, there appears to be a clear focus on the specific learning goals of the lesson. Eliciting students’ prior knowledge of area and perimeter through small group discussion and whole-class conversation, Mr. Marshall made explicit some of the important ideas involved in understanding the lesson goals, such as, what counts for area and perimeter, units of measurement, and vocabulary. His introduction of the scenario of the problem situation was relatively short, but provided a foundation for student exploration. Through commenting on the common shapes of a bumper car floor and stating that, “it would be really cool if your company designs a bumper car floor besides a square and a rectangle,” Mr. Marshall provided just enough information for students to tackle the problem without taking away the challenge, as he possibly foresaw the issue that some students might limit their designs to squares and rectangles only.
Building on initial students’ comments and knowledge on the task, Mr. Marshall supplemented the lesson with a teacher-made activity (i.e., the meter stick model and discussion) in order to reveal and foster more student thinking centered on the lesson goals. With the knowledge of potential student misconceptions, Mr. Marshall left the initial question open-ended during the whole-class activity by including both correct and incorrect units of measure, i.e., square meters and meters, in his question. Once Holland offered an incorrect answer, instead of making any indication regarding whether or not it was correct, the teacher invited the class to converse with each other, and speak out loud about their thoughts. Then Corey expressed his thinking, appearing to be using the area formula for rectangles learned in lower grades. However, rather than make any assumption, the teacher continued to probe. The dialogue then became even messier, as Corey started to talk about each side being four. Without correcting the student directly or immediately, the teacher kept on investigating the student’s thinking. Finally Newton spoke out and expressed his reasoning, making the misconception obvious. Instead of pointing out the mistake right away, Mr. Marshall waited and allowed more students to speak. Having allowed the students opportunity to hear some correct thinking from their peers, i.e., Thia and Tora, the teacher offered his own opinion, leading Newton to self-correct eventually. In a similar way, the teacher allowed Helen to reveal her misunderstanding of the units of measure towards the end of the conversation, through probing instead of making any assumptions about her thinking. At the end of the discussion, Mr. Marshall made explicit the primary focus of the lesson, motivating the students to make effort to sort things out on their own, rather than trying to set them straight by telling them exactly what they were supposed to do.

This excerpt occurred on the first day of the unit, Covering and Surrounding. As in the other lessons the teacher taught throughout the study, Mr. Marshall appeared to have kept his students
focused on the core of the mathematics topic at hand, for example, what to count for perimeter and area in the lesson described above. As his students worked on problems in small groups, Mr. Marshall moved briskly about the classroom, answering questions, probing understanding, and redirecting and extending students’ thinking. Reflecting on his instruction, the teacher said, “I knew my goals... I want people to [be able to] measure perimeter; I want them to be able to measure area. And I know what they (i.e., the range of approaches to learning the topics of perimeter and area, and the knowledge and misconceptions students bring with them to this particular mathematical situation) are.” Appearing to be a passionate teacher who frequently uses the phrase “get fired up [about math]” and a strong leader in the classroom, Mr. Marshall emphasizes problem solving, making connections, and reciprocal interaction between students.

Mr. Marshall’s instruction on launching the topics of area and perimeter above foreshadows some important features of teaching practice endorsed by the reform-based Connected Mathematics Project and described in the Innovation configuration Maps developed by Huntley (2009). The teacher

(1) Makes sure students are prepared for the mathematical investigation by activating necessary prior knowledge during the launch phase;

(2) Values and encourages active participation and reciprocal interaction of students throughout the lesson;

(3) Consistently capitalizes on “teachable moments” with students and frequently uses student errors to further discussion and enhance understanding;

(4) Provides lesson closure through reviewing key ideas and asking thought-provoking questions (see Appendix H for the complete Innovation Configuration Maps instrument).
The excerpt above highlights Mr. Marshall’s classroom as being structured as a mathematics community, where the teacher brings students’ attention to ways of working together, through responding to students’ answers by asking other students questions (e.g., “Does everybody agree?”). Students have the opportunities to debate the correctness of answers and revise their thinking. They speak primarily to the class, rather than just to the teacher, and appear comfortable putting forth alternative answers.

A national board certified teacher with 23 years of teaching experience in mathematics and nominee for “the Teacher of the School Year 2011-2012,” Mr. Marshall has taught middle school mathematics throughout his career, bringing tremendous enthusiasm and energy to his sixth-grade mathematics instruction. Having taught CMP for 14 years, the teacher states that he has learned the importance of quality Launch-Explore-Summary components to every lesson, and also developing mathematical ideas around story lines that are connected within a unit and across the curriculum from the students’ perspective. The majority of students enrolled in Mr. Marshall’s sixth grade class were white, with a slightly greater percentage of boys. The demographics of this class appeared to be representative of the larger school population. The class was considered “average,” meaning that students were enrolled in a standard sixth grade mathematics class. There were three students, or 12% of the class being placed in the Individualized Education Program (IEP), and there was one student, or 4% of the class, being considered “gifted.” A special education teacher worked mainly with the three students who had IEPs.

**Teacher beliefs and knowledge.** The above excerpt appears to provide a snapshot of a reform classroom environment where Mr. Marshall and the students share a culture of collaboration that appears to be student and problem centered. This subsection elaborates on
what it suggests in terms of the teacher’s pedagogical beliefs and knowledge, which are utilized throughout this chapter to conceptualize his in-the-moment noticing of students’ mathematical thinking. First, the episode discussed above provides a window into Mr. Marshall’s beliefs about teaching and learning mathematics and about how students learn. Derived from classroom observations and interviews, reform beliefs appear to be consistent with how the teacher behaves. For example, a feature of the teacher’s beliefs appears to be a focus on the real life meaning of doing mathematics, and mathematics learning as a dynamic, collaborative process of problem solving, according to the teacher’s comments during interviews. Mr. Marshall believes mathematics is everyday life, which is all about problem solving, rather than “something that just magically came down,” and students learn by constructing their own understanding, and by engaging in conversations with each other and with the teacher. In the follow-up interview on Day 1, he stated,

...There are two goals [in my class]-one is to learn math, the other one is to problem solve, learn to think and reason. That’s the two goals. It’s life. It’s with the real world...putting them (i.e., students) in problem solving situations every day; giving them the encouragement they need to try to figure out these problems, but letting them struggle too. That’s even a bigger goal than the math we are doing, because this is what they are going to do in life. Doesn’t matter what they are going to do in life: getting married, having kids, writing a paper in language arts, doing science, friendships. You are going to be in problem situations. How do you handle them? That’s what we are trying to help them with.

Also during the initial interview, Mr. Marshall commented on what noticing means in terms of teaching, and said,

I also believe we should allow, and every day we should have opportunities for kids to not know answers and to have to think, ponder, and be unsure of what’s correct. And it’s ok. I think more learning occurs when we put kids in those positions. The more we can do it, have them fail it, and try to work through it-that’s a good thing, because that’s what life is about. It’s solving problems. It’s enjoying the good times but also trying to figure out things that you don’t know. The better we get in dealing with those situations, the better we will succeed in life.
The above statements provide a picture of the teacher’s beliefs about the nature of mathematics, teaching and learning and about how students learn. They reflect a perspective of mathematics that appears to emphasize problem-solving and real-life meaning. Mr. Marshall provides opportunities for students to engage in the mathematics using each other and the teacher as resources, demonstrates confidence in students’ ability to explore and learn from messy, problematic situations, and appears to embrace a flexible approach to instruction that attends to and is responsive to student ideas, and to the unpredictability of classroom events. The teacher comments on believing that students “want to do the right thing” and will “start to search for that reasoning” if teachers can facilitate to make the reasoning become more apparent and teach in a more meaningful way moving from the concrete to the abstract. His orientations toward mathematical sense making appear to focus on understanding and working with the knowledge students bring with them to any mathematical situation. Students in Mr. Marshall’s class tend to be willing to speak out, make personal connections to real world situations, and also make connections to concepts learned in previous units in the curriculum. Drawing on personal knowledge and experience, the teacher appears to focus on using this reform curriculum in flexible and meaningful ways. Rather than rely solely on and cover the instructions provided in the textbook, he creates his own activities and materials to uncover student misconceptions, suit local circumstances, and unlock much of the potential embedded in the materials.

Second, the episode discussed above may indicate that Mr. Marshall’s knowledge of perimeter and area is rich and connected, including a blend of subject-matter knowledge and pedagogy, or pedagogical content knowledge. He appears to have a clear understanding of the conceptual relationships between area and perimeter, including how they differ from and connect with each other. It appears that his subject-matter knowledge enables him to launch the topics
from students’ perspectives, through careful preparation and sequencing of the instructional activities. In addition, his subject-matter knowledge appears to help him to unpack and make sense of a non-standard technique for finding perimeters of rectangles. This may have been demonstrated in his response to a question about a student’s non-standard way of calculating the perimeter of a rectangle during the initial interview, prior to teaching *Covering and Surrounding* (see item 3 in Appendix F). In this case, the student finds perimeter through dividing the area amount of the rectangle by its length and then width, and calculating and doubling the sum of one length and one width.

![Figure 4.1: Student’s work on finding perimeter of a rectangle](image)

\[
\text{Perimeter} = 2 \times (\frac{24}{4} + \frac{24}{6}) = 2 \times (\frac{72}{12} + \frac{48}{12}) = 2 \times \frac{120}{12} = \frac{120}{6} = 20
\]

While going through the process of trying to make sense of the method and constructing a mathematical explanation of what the student was doing, Mr. Marshall stated,

...Why did you (i.e., the student) do it? I would have an issue as a teacher that you are thinking that you need an area amount to help you get a perimeter, because you don’t. There’s no reason for why we would. You could always figure out a way to make your area number become the side length of the shape... (sudden long pause) uhm, this is making me think now. I’m seeing, yes, it would always work. On this one, 24/4 gives you one edge, and 24/6 gives you the other edge. Wow! I totally get it now. I apologize. Wow! I see it now. Good for them. So the child realizes that to get one edge, you just divide area by the other side that you know. So wow...Now it would be fun then to say, ‘Here’s another rectangle. Here’s the A, L, and W. Let’s write [the pattern] out.’ This is pretty cool. This is definitely a new one for me, the way it’s written.

Being asked to interpret the student’s thinking, Mr. Marshall replied,
This is extremely powerful. I see it as inverse thinking. For area, it’s four times six and equals 24. But the inverse of that is if I know the area and one side length, how do I get the other one? I can do the inverse operation to get that...This child basically generalized the whole thing: In any situation, I take the area, divide it by one side, and I get one length; I take the area, divide it by the perpendicular side, and I get the other length. Then I add those together, and I double that. That’s a high level of reasoning. But I was having a hard time seeing that. So in my classroom...I wouldn’t have a problem with a child doing this. I would definitely let it be shared, but I would also ask the child, ‘Could’ve you written it in a different way too? You don’t have to write it this way if you don’t want to...You could write in less complicated ways...’

It appears that Mr. Marshall’s subject-matter knowledge supported his ability to “think out of the box” and uncover how the student was most likely thinking about finding perimeter of rectangles using area. Initially disapproving of the student’s method, Mr. Marshall focused on the algorithm, or numerical relationships, and stated that, “you could always figure out a way to make your area number become the side length of the shape.” However, his subject-matter knowledge of the relationship between area and perimeter appeared to enable him to be flexible in his thinking and to quickly recognize the mathematical essence of the strategy as “inverse thinking” and obtain a general expression for how the student was computing perimeter, i.e., \( P = 2 \times \left( \frac{A}{L} + \frac{A}{W} \right) \). Moreover, Mr. Marshall drew on his pedagogical content knowledge and crafted extension questions to scaffold student learning and guide the student to move from concrete to abstract thinking, stating that it would be fun then to ask the student to generalize the pattern using an algebraic expression, and suggesting that the student try a less complicated way to compute perimeter. He also made connections to his own classroom instruction, describing the significance of valuing often incomplete and messy student strategies and making sense of them from the student’s perspective. Rather than allowing his initial attention to algorithm to dominate the process of interpreting the student’s thinking, Mr. Marshall demonstrated his flexibility in understanding the mathematics and student thinking. His comments above and the process he went through to make sense of the student thinking may have provided some evidence of his
conceptual understanding of the relationship between area and perimeter, as well as beliefs about a student-centered approach to teaching mathematics.

Taken together, the above excerpt and interview response appear to suggest that Mr. Marshall’s beliefs about teaching and learning mathematics and about the students, and his knowledge of perimeter and area influence the way he teaches. His beliefs and knowledge about teaching and learning mathematics appear in his classroom instruction and seem to shape the teacher’s in-the-moment noticing of students’ mathematical thinking. The subsection below starts with a vignette from Ms. Goldberg’s lesson on Day 1, focusing on similar learning goals of what counts for area and perimeter.

This subsection presents a description and interpretation of Ms. Goldberg’s reform-based teaching also using the Innovation Configuration Maps. Then the teacher’s beliefs and knowledge are discussed in relation to her teaching. First, a vignette from Ms. Goldberg’s lesson is provided.

**A Profile of Jennifer Goldberg**

**A vignette.** After a routine bell work activity in which students worked on a worksheet called “math minute” for a few minutes, Ms. Goldberg addressed the class:

Goldberg: All right, *Covering and Surrounding*. Anybody have any idea what two measurements those two words might be talking about? Owen?

Owen: Area and perimeter.

Goldberg: You are correct. Now tell me which word matches which. Jo?

Jo: Area is covering. Perimeter is surrounding.

Goldberg: Very good. Area is covering; perimeter is surrounding. Everybody look down at the floor. We have these beautiful tiles on our wonderful new floor. What shapes are the tiles? Bryan?

Bryan: Squares.

Goldberg: When we talk about area, what units are we talking about? (nodding) Bryan?

Bryan: Square units.

Goldberg: Square units, right? If Ms. Goldberg is going to put a nice, colorful artistic border around the top of walls all the way around the room, is that area or perimeter? Anna?
Anna: (pause) Area?
Goldberg: No. Kate?
Kate: Perimeter.
Goldberg: Yes. I’m not covering the walls with the nice border, right? I’m just surrounding it. Every year I see kids getting them mixed up. So keep it straight in your head. The easiest way is to look down at Ms. Goldberg’s floor, (pause) area; think about the artistic border, (pause) perimeter. Ok?

After a whole-class discussion directed by Ms. Goldberg about what area and perimeter are, Ms. Goldberg turned on the overhead projector, showing a picture of a bumper-car floor plan provided on page one in Investigation 1.1 (see Appendix K). She provided directions to the class about using the procedures for finding perimeter and area of the rectangle-shaped floor plan, or what the students should count (i.e., number of liner units vs square units) when finding perimeter and area of the floor plan. Next, group work began.

After spending approximately 30 minutes investigating Problem 1.1 (see Appendix K for Problem 1.1 from Investigation 1 in its entirety) in small groups, the students were reconvened for the homework assignment and then the class was dismissed. Resembling the structure of Mr. Marshall’s lesson described earlier, the excerpt above occurred during the launch of Problem 1.1, prior to the small group activity involving sketching floor plans that have an area of 36 square meters and a perimeter of 26 meters. Unlike the “messier” picture presented in Mr. Marshall’s class, this excerpt portrays a “clean” and structured form of instruction, appearing to resonate with the features of the Initiation-Response-Feedback (IRF) pattern described by Herbel-Eisenmann and Breyfogle (2005). Within this pattern, “the teacher asks a question, a student provides a response, and the teacher offers evaluative feedback” (Herbel-Eisenmann & Breyfogle, 2005, p. 484). During the IRF process, the students are usually directed through a pre-determined solution path in order to reach the desired conclusion. Normally the teacher does not focus on inviting students to explain their thinking or react to other students’ responses.
Providing the thinking for the students, the teacher uses language cues, whether consciously or not, to direct students to respond in certain ways. According to Herbel-Eisenmann and Breyfogle, “although this form of interaction was identified and described over twenty-five years ago, it is still prevalent in classrooms today” (p. 484).

To some extent, Ms. Goldberg appeared to be following a form of the IRF process. She began by asking a question, called on a student to answer it, and repeated or expanded the answer based on what appeared to reflect her own thinking or reasoning vs. a focus on developing content as emerged from students’ thinking. The cycle was repeated throughout this particular interaction. The students appeared to be less active participants in the learning process; the teacher accepted one-word or incomplete student responses without requesting elaboration to help make ideas clear, and appeared to carry out a pre-determined instructional plan. This excerpt suggests a discourse that appeared to be dominated by a pattern of teacher-student-teacher-student...interactions, instead of teacher-student 1-student 2-student 3-student 4-student 5-teacher...interactions (Huntley, 2009). In this interaction, the teacher’s thinking was made explicit, and little was clarified about what the students were actually thinking or understanding. For example, Ms. Goldberg launched a discussion of the topics of area and perimeter by leading a whole-class discussion that involved only the students who had the correct answers. Rather than conversing at times with each other, students primarily spoke to the teacher. Appearing to be passive listeners, students did little to elaborate their thinking, or to consider alternative strategies to distinguish between area and perimeter, besides using the teacher-suggested examples or “the easiest way.” There was no summary of the current lesson; only assigned homework was addressed at the end of the class. Rather than allowing time within the lesson to “uncover” (Boaler & Humphreys, 2005, p. 105) what the students understood or did not
understand, Ms. Goldberg tended to focus on covering the curriculum during whole-class instruction, providing mathematical directions for the students, or calling on students who would offer desired responses, resulting in a more teacher-centered lesson that appeared to unfold in a pre-determined way that was consistent with the teacher’s instructional plan. The possible messiness that might have occurred along the way did not emerge; given the goal of the lesson, the teacher focused her discussion, leading the content development rather than orchestrating its emergence from students per se.

This excerpt provides a snapshot of Ms. Goldberg’s teaching in the context of a reform-based curriculum. Students enrolled in Ms. Goldberg’s sixth grade class were all white. There were twice as many girls as boys. The demographics of this class appeared to be representative of the larger school population to some degree. The class was also considered “average,” meaning that students were enrolled in a standard sixth grade mathematics class. There were no students requiring Exceptional Children services.

**Teacher beliefs and knowledge.** An experienced and dedicated middle school mathematics teacher, Ms. Goldberg has taught sixth grade mathematics using CMP for seven years. An increasing interest in teaching was the main factor that eventually led her to switch her career from being a lawyer for ten years to teaching middle school mathematics. Having been mentored by Mr. Marshall and also meeting with Mr. Marshall regularly on Sunday afternoons to plan lessons for the semester, Ms. Goldberg appears to have been greatly influenced by Mr. Marshall’s enthusiasm and confidence in reforming middle school mathematics instruction. Ms. Goldberg appears to embrace the spirit of reform and speaks highly of the reform-based curriculum (i.e., *Connected Mathematics Project*), praising the mathematics teaching and
learning opportunities afforded by this curriculum. For example, in the follow-up interview on Day 1, Ms. Goldberg stated,

> It’s that spiral curriculum which I don’t agree with, because it is kind of getting them (i.e., students) in something and out something, in something and out something. They (i.e., students) don’t make the connections with that. And so I think Connected Math is incredibly multi-layered...It is like peeling an onion. To get to an answer in a Connected Math problem, you may have to peel four layers of that onion, to get to it.

In the interview on Day 2, she remarked:

> ...the kids, they need time to explore. They need time to figure, because there is no one way to do a math problem. No matter what concept it is, there are probably between 5 to 100 ways to do a particular concept. And who am I to tell them my way? That’s just a memorization game at that point...I guess, (long pause) it makes me think that with all the political stuff in education, we are doing the right thing, the right curriculum.

The above comments provide a snapshot of the teacher’s professed beliefs of mathematics teaching and learning reflecting that of a reform-oriented teacher. Ms. Goldberg comments on the vision that there are many related ideas, procedures, and skills associated with any important mathematical concept, and she states that students should engage in learning of mathematical content embedded in problems in relation to other important mathematical ideas. She also talks about the significance of allowing students to explore, make conjectures, come up with more than one strategy, and take ownership of their own learning. During the initial interview, Ms. Goldberg commented on the importance of noticing students going down the wrong path, “I think sometimes that can be powerful.”

Acknowledging the professed reform-oriented beliefs, the researcher then examines Ms. Goldberg’s practice, reflected in the vignette described above, and sees that the teacher appears to use a more traditional approach to teaching and learning mathematics, implemented in such a way as to suggest a belief system that includes different perspectives on teaching and learning mathematics, i.e., there appears to be a discrepancy between what the teachers professes to
believe and how she implements instruction with students. As the findings in Aguirre’s (1995) teacher belief study suggest, on the one hand, Ms. Goldberg’s traditional beliefs appear well developed from her personal history of learning mathematics and years of teaching mathematics. Consistent with the behavior of the teacher that is observed during instructional interactions described in above vignette, Ms. Goldberg appears to hold the belief that mathematics instruction is the means for transferring information from teacher to student, as students “just don’t want to peel it (i.e., the ‘onion’)” (Goldberg, Day 2), and the teacher plays the role of authority of knowledge in the learning process. Ms. Goldberg appears to be involved in classroom interactions that do not highlight the use of collaborative exploration and students joint problem solving. Students are observed to be learning mainly by attentively watching the teacher demonstrate procedures and methods for performing mathematical tasks, and by following and responding to the teacher’s thinking. Ms. Goldberg’s orientations toward mathematical sense making appear to involve laying out content in clear ways, and following a didactic, step-by-step approach to instruction, as described in the vignette above. However, it is evident that the professed reform beliefs are emerging, appearing to be unstable, and fragile to some degree.

Possibly influenced by Mr. Marshall’s teaching and her own positive experience with the reform-base curriculum, Ms. Goldberg appears to have developed a reform perspective in theory on mathematics instruction. Situated alongside with her traditional beliefs, these somewhat unstable reform beliefs may be in conflict with the traditional beliefs, as described in a study conducted by Aguirre and Speer (1996). The two belief systems could be perceived to be in different states of activation or priority, depending on the given constraints and resources, and upon occurrence of any new event. For example, in the situation described in the vignette, the following questions may come into play: Is there enough time to deal with the new event? Will
the teacher need to march off into unfamiliar territory to deal with the issue? What knowledge
gets activated? In light of these issues, in the next few paragraphs, Ms. Goldberg’s knowledge of
perimeter and area is discussed in an attempt to understand the role teacher beliefs and
knowledge may play in her in-the-moment noticing of students’ thinking.

Analysis of multiple sources of data appears to show that Ms. Goldberg tends to focus on
certain topics involving area and perimeter and does not pay attention to other mathematics
relationships, when faced with certain new events during instruction. For example, Ms. Goldberg
responded to a question in which she had been asked to help a colleague make sense of a
student’s non-standard way of finding perimeter during the initial interview (shown below):

4          4
6

Perimeter=2×(24/4+24/6)=2×(72/12+48/12)=2×120/12=120/6=20

First, I would definitely tell the fellow teacher that it is definitely a valid method. All they
were doing was converting the whole numbers into equivalent, improper fractions. I don’t
know if I will ever have a child who would like to move from whole numbers to fractions
though! So he was just taking a different form of length and width, the improper fraction
form, and he was getting the common denominator, adding together the length and width,
and multiplying that by two. That will give you the perimeter. It definitely looks like
that’s what he was doing. And that’s definitely the right answer... The procedure the child
is using is to add the length and width together, and double it. My question is why are
you changing the whole number of 6 into 24/4, and why are you changing the whole
number of 4 into 24/6? I don’t understand why they are making those conversions. (long
pause) So the child knows the relationship between 1/6 and 1/12. The child knows every
two 1/12s equal 1/6, and to double 120/12, you just leave the numerator alone, and
double 1/12. You [end up] being 1/6 instead of 1/12. Yea.
In trying to follow the student’s thinking, Ms. Goldberg’s response focused on the student’s computational skills, going into depth with description and analysis of the student’s number sense. She did not comment on the general strategy the student might have been using to make sense of the problem or give a mathematical explanation of what she thought the student might have been doing.

Later, when she was teaching, one of her students came up with a similar idea for finding perimeter using the area amount during her lesson on Day 3. She responded immediately, stating that “area is not directly related to perimeter, because remember area is covering, perimeter is surrounding.” Ms. Goldberg might be less comfortable exploring these kinds of relationships between area and perimeter and, so, did not appear to attend to student thinking and respond taking into account what might have been students’ own perspectives. In a sense, what Ms. Goldberg appeared not to acknowledge about mathematics in this unit “protected” (Cohen, 1990, p. 323) her from experiences that might have provoked uncertainty and messiness in the middle of instruction. The teacher’s response above to the student’s work during initial interview did demonstrate her willingness and desire to accept and make sense of the student’s non-standard way of finding perimeter. In a similar way, Ms. Goldberg’s comment during a follow-up interview reflected both her struggle and success in teaching this reform-base curriculum. She said,

I will be honest with you. I think every year I teach Connected Math, I see more and more ways and places the formula [for finding area of triangle] comes from. Whereas you know, I am 42, so when I learned math, it was drill ’n kill. I learned math back when the teacher wrote the formula on the board, ‘One half of base times height, and here is the worksheet, so go practice it.’ So when I teach it conceptually, since I didn’t learn it conceptually, often times, I don’t see where the formula is coming from. Like every year, I see it more and more and more. I just think the first few years, I am not sure that the first few years, I understood whether the non-right triangles worked [using the ‘surrounding rectangle strategy’ to find area]. I mean, I knew they did, I even taught they
did...Whereas now, I have been teaching it long enough, I know why they work. And I know the kids need to see that...

Taken together, the above excerpts and interview responses reveal Ms. Goldberg’s potential belief in a traditional approach to teaching and learning mathematics that may conflict with her reform vision, suggesting a potential dual belief system that consists of different perspectives on teaching and learning mathematics. Ms. Goldberg’s potential dual belief system and what she acknowledges and does not acknowledges about mathematics involving perimeter and area may influence the way she teaches, consciously or unconsciously. Her differing beliefs and activated knowledge about teaching and learning mathematics occur in her classroom instruction and may shape the teacher’s in-the-moment noticing of students’ mathematical thinking. In the section that follows, general findings on what teachers noticed are presented that include an explanation of procedures for data collection and analysis, and a general description of what teachers noticed.

**What Teachers Notice**

An examination of the literature suggests that teachers vary in the kinds of things they notice and the extent to which they notice student thinking during instruction (Colestock, 2009; Luna et al., 2009; Sherin et al., 2008). The description of findings on what teachers noticed is organized into two subsections. First, the procedure for data collection and analysis is detailed. Second, a summary of what teachers noticed is presented.

**Data Collection and Analysis**

In this study, data from different sources were collected to consider the kinds of things teachers had noticed and the extent to which they had noticed student thinking amongst a variety of things. One crucial source of these data was the daily teacher-generated individual/small group video-recording using the Deja View Camwear 100; these data are referred to as teacher clip captures. Teachers were instructed to capture clips of student-student or student-teacher
interactions during instruction, using the new technology. Before each class period, the teacher was outfitted with the camera and instructed to “press the record button on the camera when something noteworthy happened during whole-class instruction or small group activity, in terms of student thinking.” After each class, the researcher uploaded the episodes to a computer, reviewed all of them, selected one to nine episodes based on the extent of mathematics thinking involved and/or how representative they were of the teacher’s overall noticing pattern, and played back and discussed these episodes with the teachers during follow-up interviews. The researcher interviewed the teachers using a standard protocol adapted from Sherin, Russ, et al. (2011), with modifications made based on the focus of this particular project (see Appendix G). The interviews were relatively unstructured and conversational in style. During each interview, the teacher was asked to describe the reasons the moments had been captured, after watching each of the selected episodes. At the end of each interview, the researcher asked whether the captured clips represented what the teacher had intended to capture, and whether all of the clips together overall reflected what the teacher had found interesting or important in terms of student thinking. Interviews were videotaped, summarized, and fully transcribed.

With regard to analysis, a crucial component was coordination of multiple records of the lesson surrounding each episode to come up with an emergent coding scheme. More precisely, the researcher first reviewed the lesson plan and any accompanying materials related to each episode to gain an understanding of the teacher’s planning, or envisioning of the lesson. Second, the researcher reviewed the teacher-generated whole class video and field notes in relation to each episode. A review of the whole class video was essential in that it provided the information about who had spoken or what ideas had recently been raised or discussed in class, prior to or after the captured episode, in order to make better sense of the episode that was situated in the
lesson but stood out among the rest of the lesson segments. At times, multiple episodes and/or whole class videos were reviewed to help understand one particular episode. Third, the researcher analyzed the interview data, and identified a set of preliminary reasons or codes for capturing each episode using the approach of emergent coding. This preliminary analysis took place in a cyclical manner in which the set of reasons or codes was refined as needed. In addition, for inter rater reliability purposes, the researcher attempted to approach the data with a blank slate, assigning codes or reasons for a second time (after a delay of approximately two months) without referring to the original checklist for each episode. The researcher then compared the two sets of codes or reasons for each episode and reconciled any differences showing up in the codes. A reliability score of 0.8 was established to show the extent of consensus between two rounds of coding. This involved adding up the number of episodes that were coded the same way through two rounds of coding and dividing by the total number of episodes. Finally, the 40 codes or reasons were categorized into a set of 5 stable themes to represent what the teachers had noticed in real time instruction throughout the study, including student thinking, instructional adaptations, assessment, content, and student characteristics (see Appendix I). In more detail, the teacher was characterized as capturing an episode because of student thinking when his or her reflection focused on the substance of the ideas raised by students. For instructional adaptations, the teacher’s reflection focused on the teacher’s in-the-moment instructional approaches. An assessment reflection focused on the teacher’s in-the-moment actions of determining what students had learned and where instruction needed to be adjusted and adapted by assessing. For content, the teacher was characterized as capturing an episode because of content when his or her reflection focused on specific mathematics content, task, or the curriculum. Finally, a student characteristics reflection focused on specific attributes
of a student or group of students, reflected in certain student behaviors. Four of these themes are
more related to teaching and learning the mathematics content: student thinking, instructional
adaptations, assessment, and content. The other theme concerns student characteristics. This
theme encompasses noticing activities that are not specific to the mathematics instruction.

When determining themes for each episode, original codes were constantly reviewed. For
example, a teacher’s comments on one episode during follow-up interview indicated student
difficulty solving a problem and teacher facilitation were the reasons for capture; the teacher’s
comments on another episode suggested student conjecture and teacher decision making were
the reasons for capture. As both student difficulty solving a problem and student conjecture are
categorized into the theme of student thinking, and both teacher facilitation and teacher decision
making are included in the theme of instructional adaptations, these two episodes are considered
to have been captured for the same reasons (i.e., student thinking and instructional adaptations),
in spite of the different codes assigned to them (see Appendix I). During analysis, the original 40
codes such as student difficulty solving a problem and student conjecture were often referenced
for a more detailed and specific description of the reasons for clip capture as needed. With regard
to episodes that were not selected for discussion, the researcher’s inference from watching the
clips and triangulating among related data sources (e.g., field notes and transcripts of whole class
videos) became the primary source of interpretation of the clips. Other sources of data, such as,
initial interviews and daily classroom observations, were frequently referenced in order to better
understand teachers’ noticing activities within the instructional contexts, the day-to-day “flow”
of teaching, and the relationship among individual noticing episodes, whole-class instruction,
and follow-up interviews. Next, a summary of the findings is presented.
A Summary of Findings

The total number of episodes captured by each teacher participant included: Marshall, n=77; Goldberg, n=90. Each teacher incorrectly captured one episode, recorded in field notes and included in the total number of episodes. More than 90% of the episodes captured by both teachers were when students were working in small groups. The total number of episodes selected for discussion during follow-up interviews included: Marshall, n=23; Goldberg, n=27. Table 4.1 and 4.2 below provide a summary of each teacher’s reasons for clip capture as coded using the five broad themes, including both percentage and frequency count for each theme (see Appendix I for descriptions of emergent coding scheme). Four of these themes are more related to teaching and learning the mathematics content: student thinking, instructional adaptations, assessment, and content. The other theme concerns student characteristics.

Each table consists of two rows, representing either all episodes captured or episodes selected for follow-up interviews. Each percentage represents the proportion of episodes captured for a particular reason (reported as a theme) in relation to the total number of episodes captured within a particular row. For example, out of a total of 77 episodes captured by Mr. Marshall, there were 41 episodes or 53% of them captured for student thinking, and 38 episodes or 49% of them captured for instructional adaptations. Likewise, out of a total of 90 episodes captured by Ms. Goldberg, there were 35 episodes or 39% of them captured for student thinking, and 61 episodes or 68% of them captured for instructional adaptations. With regard to episodes selected for follow-up interviews, out of a total of 23 episodes discussed with Mr. Marshall, there were 17 episodes or 74% of them captured for student thinking, and 15 episodes or 65% of them captured for instructional adaptations. Likewise, out of a total of 27 episodes discussed with Ms. Goldberg, there were 17 episodes or 63% of them captured for student thinking, and 25 episodes...
or 93% of them captured for *instructional adaptations*. In Table 4.1 and 4.2, each row adds up to more than 100% because the majority of episodes were coded as being captured for multiple reasons.

Table 4.1: Percentage of Mr. Marshall’s reasons reported as themes for clip capture

<table>
<thead>
<tr>
<th>Marshall</th>
<th>Student Characteristics</th>
<th>Content</th>
<th>Assessment</th>
<th>Instructional adaptations</th>
<th>Student Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Episodes Captured 77</td>
<td>23%</td>
<td>27%</td>
<td>49%</td>
<td>49%</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>21</td>
<td>38</td>
<td>38</td>
<td>41</td>
</tr>
<tr>
<td>Episodes Discussed 23</td>
<td>52%</td>
<td>43%</td>
<td>43%</td>
<td>65%</td>
<td>74%</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4.2: Percentage of Ms. Goldberg’s reasons reported as themes for clip capture

<table>
<thead>
<tr>
<th>Goldberg</th>
<th>Student Characteristics</th>
<th>Content</th>
<th>Assessment</th>
<th>Instructional adaptations</th>
<th>Student Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Episodes Captured 90</td>
<td>24%</td>
<td>37%</td>
<td>22%</td>
<td>68%</td>
<td>39%</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>33</td>
<td>20</td>
<td>61</td>
<td>35</td>
</tr>
<tr>
<td>Episodes Discussed 27</td>
<td>52%</td>
<td>44%</td>
<td>15%</td>
<td>93%</td>
<td>63%</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>12</td>
<td>4</td>
<td>25</td>
<td>17</td>
</tr>
</tbody>
</table>
On the one hand, in looking at teachers’ reasons for capturing all clips, as represented in the top rows of the tables, it appears that student thinking and instructional adaptations are motivating reasons for capturing the episodes. In more detail, Mr. Marshall was found to have attended to student thinking to a greater degree in the midst of instruction, compared with Ms. Goldberg. He also captured a good number of moments for assessment, focusing on formative assessment, or activities that the teacher undertook to gain an understanding of what students knew and did not know in order to make responsive changes; whereas Ms. Goldberg attended to instructional adaptations to a large extent, centered on teacher remediation, or her act of correcting a fault or deficiency in students’ responses or strategies. The top rows of these two tables reveal a similar pattern between teachers with regard to the distribution of teachers’ reasons for capturing the events, focusing on student thinking, instructional adaptations, assessment, and content. However, a closer examination of the frequency counts of individual codes demonstrates that within content, Mr. Marshall appeared to tend toward specific content, i.e., specific mathematics content that had been taught in previous units or was to be taught in the future; Ms. Goldberg instead focused on task management, i.e., the act of managing tasks or how instructions were being carried out by the students.

On the other hand, in looking at both teachers’ reasons for capturing only clips discussed during follow-up interviews, as represented in the bottom rows, one sees that student thinking and instructional adaptations also appear to be the most primary reasons, similar to the patterns for all episodes captured but with higher percentages. It is no surprise that both teachers attended to student thinking to a greater degree in bottom rows, compared with data represented in top rows, as the episodes discussed were intentionally selected for their potential focus of student thinking. In addition, both teachers attended to student characteristics to a great extent,
demonstrated in the episodes discussed in bottom rows; however, a closer look at the frequency distribution of original codes suggests that Mr. Marshall’s noticing differed from Ms. Goldberg’s in that he focused on student discourse, or mathematical conversations between students, reflected in the vignette above; whereas Ms. Goldberg focused on student attributes, or specific characteristics of a student or group of students (see Appendix I for explanation of original codes). In addition, Tables included in Appendix J provide a more detailed summary of the distribution for each teacher participant, showing the number of episodes captured daily for each theme throughout the two weeks. It suggests that both teachers noticed a variety of things in each lesson every day. On the majority of days, both teachers attended to certain moments for the following reasons, including student thinking, instructional adaptations, assessment, content, and student characteristics, allocating the most time to student thinking and instructional adaptations. On Day 6, a test was given for the majority of the lesson, resulting in a limited number of episodes captured by both teachers.

In summary, analysis of teachers’ reported reasons for clip capture shows both teachers noticed for a variety of reasons, similar to the teachers studied by other researchers using traditional methods in the field (see Colestock, 2009; Luna et al., 2009; Sherin, Russ, et al., 2008). A comparison between these two teachers reveals that Mr. Marshall and Ms. Goldberg focused on different issues when capturing their moments of noticing. Mr. Marshall demonstrated a focus on both students’ mathematical thinking and instructional adaptations at the same time, whereas Ms. Goldberg appeared concerned primarily with her own practices while attending to student thinking to a lesser extent. In the section that follows, findings about the following research questions are presented, in light of teachers’ beliefs and knowledge:
1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?
Teacher Noticing of Student Thinking

In this study, both teachers captured certain moments for the following reasons, including student thinking, instructional adaptations, assessment, content, and student characteristics, appearing to allocate the most time to student thinking and instructional adaptations. The focus of this study was to investigate two middle school teachers’ in-the-moment noticing of students’ mathematical thinking. By identifying a special focus for noticing, the researcher attended less to the variety of what teachers had noticed, and more to how and the extent to which teachers had noticed students’ mathematical thinking in the midst of instruction. In this study, the teachers were characterized as capturing an episode because of student thinking when their reflection focused on the substance of the ideas raised by students, such as, student misconception, student strategy, student error, insightful mathematical question, and so on (see Appendix I for explanation of the codes). Teachers’ professional noticing of students’ mathematical thinking not only includes attending to student thinking, interpreting that thinking, and deciding how to respond, according to Jacobs et al. (2010), but also extends to responding in this study. The construct is conceptualized as a set of skills including attending, interpreting, deciding how to respond, and also responding in certain ways, echoing Erickson’s (2011) notion that noticing is usually highly instrumental. Built upon the work of Jacobs et al. (2010), each skill is conceptualized in a way that is consistent with the reform vision: Attending to student thinking refers to attending to the mathematical details in students’ strategies; interpreting is concerned with teachers’ interpretation of students’ understandings as reflected in student strategies; deciding how to respond is the reasoning that teachers use when deciding how to respond, based on interpretation of students’ understandings from the specific situation and research on children’s mathematical development; responding refers to teachers’ act of responding in accordance with their decisions. Focusing on each teacher’s noticing that involves the process of
attending, interpreting, deciding how to respond, and responding, this section of the chapter is organized into three subsections. In the first two subsections, drawing upon some teacher-generated episodes, the researcher presents each teacher’s typical noticing pattern. In the third subsection, the researcher focuses on a comparison between the noticing of the two teachers, in light of teachers’ beliefs and knowledge. First, each teacher’s pattern of noticing is described, drawing on some teacher-generated episodes. For each teacher, the episode is presented and then analyzed, focusing on the four components involved in the process of noticing. It begins with the case of Mr. Marshall.

The Case of Joe Marshall

Mr. Marshall appears to have noticed student thinking to a great degree in the midst of instruction. He also captured a number of moments for instructional adaptations and assessment (see Table 4.1). Generally, Mr. Marshall’s noticing of student thinking was accompanied by his noticing of instructional adaptations and/or assessment. Out of the 41 episodes captured for student thinking, there were 33 or 80% that were also captured for either instructional adaptations or assessment, or both. The first two components involved in the process of noticing are attending to the mathematical details in students’ strategies and interpreting understandings as reflected in student strategies. First, the teacher appeared to attend to primarily the mathematically important details in students’ strategies, or details that could inform the teacher’s instruction. Then, reasoning through what he had attended to in these details, the teacher drew inferences about or generated productive and evidence-based evaluation of student understandings, in a way that appeared to be consistent with the details of the specific strategies and the research on students’ mathematical development. Consider the following episode from Day 3 that provides a window into the teacher’s in-the-moment noticing of student thinking.
**An episode.** In this episode, students were working on finding rectangles that have a constant area of 24 square meters. Gratia was arguing with Helen about the issue of whether a square would work, as Helen insisted that a square does not meet the requirement that it needs to be a rectangle because “squares are not rectangles,” whereas Gratia believed “it can be a square.”

Reflecting on this episode during follow-up interview, Mr. Marshall stated,

...this [argument] is a great thing, because we said they have to be rectangles. Gratia says it can be a square. Then Helen says it has to look like a rectangle. And it is a dilemma for 6th and 7th grade kids, and 8th grade kids to struggle with-Is a rectangle square? Because the conversation was that if we can draw a square, are we going to count it? ...the conversation was there, and I just wanted to capture it. I didn’t say anything. We’ve had it in our last unit. But it is such a hard concept. So I heard them talking, and I was like, I want to capture that conversation, because that was a typical [issue]...Can we draw a square when the direction says to draw a rectangle? We are not going to be able to draw a square [to meet the requirement of an area of 24 in this case], so Gratia was just talking in theory with Helen. So [it tells me] they weren’t having an issue with [what counts as] area...

In this episode, Mr. Marshall first attended to the mathematical essence of student thinking revealed in the argument between Gratia and Helen, using the details from students’ conversation to support what he had observed. For instance, appearing to refrain from making any comments, the teacher allowed the students to express their confusion about an important issue: what defines a rectangle. His observation of Gratia’s “just talking in theory with Helen” without drawing any square was used as evidence for the students’ potential understanding of what counts as area, as the students appeared to understand it is impossible to draw a square with whole number dimensions that has an area of 24. Consistent with the research on children’s mathematical development, Mr. Marshall interpreted the students’ confusion as revealing a typical “dilemma” or struggle for middle school students, connecting it to students’ prior experience with the same issue in the last unit. He considered the argument “a great thing” that had exposed the students’ struggle with an important topic. In a word, the teacher attended to the mathematically important
aspects in the conversation, made sense of the aspects, and noted how they reflected what the students did and did not understand.

This particular part of the follow-up discussion on Day 3 serves as an example of how Mr. Marshall appeared to have attended to the mathematical details of student thinking, and used the mathematically significant aspects of the conversation to reason through what he had observed and to interpret students’ understanding, in a way that appeared to be consistent with middle school students’ characteristics of mathematical development. Research conducted in static geometry environments has shown that many children, even in the middle school grades, “rely on static, visual prototypes when identifying geometric shapes and that formal definitions, even if known, play no role in this process” (Uttal, Scudder, & Deloache, 1997, p. 37). In this case, Helen’s statement that a shape has to look like a rectangle to be considered a rectangle demonstrated a typical misconception found in middle school mathematics classrooms. Discerning the mathematical essence embedded in the students’ conversation, Mr. Marshall acknowledged that it was “a hard concept” and the argument was “a great thing”, without offering any comments on the spot.

The other two components involved in the process of noticing are the reasoning that teachers use when deciding how to respond, based on observation and interpretation of students’ understandings from the specific situation and research on children’s mathematical development, and also responding in accordance with their decisions. Mr. Marshall not only attended to the details of students’ mathematical thinking and interpreted what he had observed in accordance with students’ mathematical development, he also extended his noticing to quick analysis of possible teaching moves in light of the current specific learning goal, the structure of the curriculum, and broader principles of teaching and learning, deciding how to respond on the
basis of his analysis. Teacher beliefs and knowledge also possibly came into play during this process. Consider the following comments on the same episode from Mr. Marshall’s lesson on Day 3:

I wanted to capture that conversation, because that was a typical [issue]. For sure it is going to come up [again]: Can they be squares? Can we draw a square when the direction says to draw a rectangle? ...That will happen in the constant perimeter lesson, in the next lesson (i.e., the lesson on Day 5), because we can’t get a square today [due to the constraints of this particular problem]... Yeah, I am going to go around [on Day 5]. I am going to see if everybody else has the same kind of concern. Do I think it is important to bring it up? Yes, I do. But we were still in the beginning stage, you know... [I wanted to make sure] most groups had two to four of the drawings done [today]. It (i.e., this same particular issue) will come out better in the next lesson [on Day 5]...

In looking at what took place on Day 5, one then sees the potential path the teacher went through from attending to student thinking two days earlier, to the actual execution of teaching moves in response to that noticing two days later. The following excerpt is taken from a whole-class discussion of what counts for a rectangle on Day 5. Prior to the discussion, students were working on Problem 1.3 in small groups, sketching different rectangles of whole number dimensions that have a constant perimeter of 24 meters. Kaine then motioned to the teacher and asked, “Will that (pointing to a 6 by 6 square) count as a rectangle?” Immediately after the teacher listened to Kaine’s question, he turned to the whole class:

Hey guys, time out for a minute. This question has come up in several groups, so we are going to talk about it. (walking up to the overhead transparency projector) Several groups were trying to make this shape (sketching a 6 by 6 square on the projector, and highlighting the sides of the shape) Does it fit the criteria that it has to be a rectangle? I want to hear you guys argue about it a little bit. Put your hand up if you want to talk about this idea. You need to remember the [assigned] order [of speaking]...

After bringing together the class, Mr. Marshall launched the conversation, and then called on four students volunteering to speak. He arranged the students to talk in certain order, asking them to listen to the previous person carefully so as not to repeat what had already been said. Gratia
was one of the four students. Later during follow-up interview, Mr. Marshall reflected on this episode:

...yeah, I have been to a couple tables that had conversations about square: Is a square a rectangle, or is a rectangle a square? That whole conversation. And I decided to stop the lesson for a minute. Because I have been to a couple tables that were there, and I figured it was going on with all the tables. And it is good math. It reviews the last geometry book we were in. We talked about it there, but kids are still unconfident. So I wanted to capture that because I feel like it was a place where I could stop class, and say, ‘Hey we are going to talk about this with everybody.’ And I tried not to get the answer upfront. I said, ‘We are going to have this kid talk, then this kid, then this kid...’ And I was trying to get them to talk to each other rather than to me. So it was a big math idea, and it is going to help us get one more rectangle, or one less, in this problem. And it just so happens that with the square [in this case], it is going to be the maximum area you could have. So I wanted to do it.

During the same interview above, Mr. Marshall also recalled the previous episode captured on Day 3 involving Gratia, who had been selected to be one of the four speakers during the whole-class discussion described above on Day 5, “She (i.e., Gratia) was arguing with Helen the other day. And she was making a statement that a square was a rectangle...”

**Analysis.** According to the coding scheme described in Appendix I, the previous episode on Day 3 involved student thinking, content, student characteristics, and instructional adaptations. In more detail, the reasons for capture included the teacher’s noticing of students’ insightful question on related content, i.e., formal definitions of rectangle and square, student discourse, and the teacher’s process of decision making regarding how to act on the noticing. Appearing to be attuned to seeing and responding to mathematics content from a perspective of connectedness, Mr. Marshall used student discourse to help gauge student understandings of an important topic, and to decide upon appropriate action to take, reflected in his behaviors and comments.

A close look at the potential path the teacher has gone through from attending to responding may indicate the role teacher beliefs and knowledge play in the teacher’s in-the moment noticing of students’ mathematical thinking, suggesting that the teacher’s pedagogical beliefs and
pedagogical content knowledge may have a fundamental impact on his noticing. Mr. Marshall appears to believe the importance for students to engage in the mathematics using each other and the teacher as resources, have confidence in students’ ability to explore and learn from uncertain situations, and embraces a flexible approach to instruction that attends to and is responsive to student ideas, and also connects to concepts learned in previous units in the curriculum.

Reconsider Mr. Marshall’s comments on his observation of Helen’s misconception captured on Day 3. Because a square would not satisfy the constraints of the problem, Mr. Marshall possibly did not expect a conversation like that to occur. However, the teacher’s pedagogical content knowledge might have allowed him to immediately attend to the mathematical essence of the issue brought up in the conversation between Gratia and Helen, recognizing it as “good math” that “reviews the last geometry book” he had taught. Consistent with the findings reflected in Table 4.1, Mr. Marshall appeared to value the opportunity of helping students see mathematical connections between current content and specific topics that had been taught in previous units. Drawing upon his understanding of the relationships between a rectangle and a square, and knowledge of associated student misconceptions, the teacher reasoned about what needed to take place, and when and how to make it happen, reflected in his comments during Day 3’s follow-up interview. Guided by the current instructional goal that most groups should have two to four of the drawings completed at the end of the lesson on Day 3, Mr. Marshall decided not to offer any feedback or take any action right away, allowing Gratia and Helen the opportunity to argue with each other. His knowledge of the sequence of lessons and understanding of the future instructional goals and content might have led him to have the confidence that the same issue would “come out better in the next lesson”. Then on Day 5, immediately after hearing a similar question on this topic, Mr. Marshall was possibly prompted about the need to bring up the issue.
As he stated, “Because I have been to a couple tables that were there, and I figured it was going on with all the tables.” The teacher’s comment demonstrated how he had used “sampling”, or his noticing of two groups’ thinking, starting with Helen’s on Day 3, then Kaine’s on Day 5, to gauge the significance of this issue with regard to student learning as a whole class. Finally, in accordance with his decision on Day 3 that “it is important to bring it up,” the teacher stopped the lesson and had a whole-class discussion of this issue, offering students the opportunity to converse with each other. In light of his noticing of Gratia’s thinking two days earlier, Mr. Marshall selected her to be one of the four speakers. Again, the teacher’s pedagogical content knowledge might have come into play, influencing his decision of having students argue about the issue and trying to get them to talk to each other rather than to the teacher. Consistent with his professed beliefs about teaching and learning, a student-led discussion was considered the most useful way of presenting and addressing this particular issue. As illustrated by these episodes, Mr. Marshall’s noticing of student thinking appears to be accompanied by his analysis of instructional adaptations and assessment to a great degree. He extends his noticing from student thinking to assessment of student understanding, and considers possible teaching moves on the basis of his analysis of a variety of particulars involved in the process, such as, the specific learning goals and sequence of lessons.

Investigation of the potential path the teacher went through from attending to student thinking to the actual execution of teaching moves provides some evidence that multiple factors may have worked together to produce the improvised teaching episode on what counts for a square on Day 5. Serving as an example of the improvisational work of teaching, this episode illustrates a path that appears to be highly interactive and multifaceted, rather than a straightforward process as may be assumed. The teacher’s goals, including general goals for
helping students see mathematical connections, and specific goals and plans for components of a particular lesson, appear to have taken an active role in the path of what the teacher notices, i.e., his noticing of Helen’s misconception on Day 3, and how he acts on it, in a way consistent with the teacher’s professed beliefs and knowledge. Next, Ms. Goldberg’s pattern of noticing is described, drawing on a teacher-generated episode.

**The Case of Jennifer Goldberg**

Ms. Goldberg noticed *instructional adaptations* to a greater degree in the midst of instruction, i.e., took more clips of these episodes. Different from Mr. Marshall’s focus on both students’ mathematical thinking and instructional adaptations at the same time, Ms. Goldberg’s focus on her own practices was accompanied with attention to student thinking occurring to a lesser extent. Out of the total of 90 episodes captured by Ms. Goldberg, 61 or 68% of them captured for *instructional adaptations*, whereas only 35 or 39% of them captured for the reason of student thinking. With regard to the 35 episodes involving student thinking, there were 27 or 77% of the episodes that were also captured for *instructional adaptations*. The first two components involved in the process of noticing, i.e., *attending* and *interpreting*, involve *attending to* the mathematical details in students’ strategies and *interpreting* understandings as reflected in student strategies. Ms. Goldberg primarily attended to her instructional adaptations with some attention to some details in students’ strategies. The teacher demonstrated some evidence of interpretation of students’ understandings and behaviors reflected in students’ strategies, highlighting noteworthy moments and providing primarily evaluative with some interpretive comments. The teacher described the students’ understandings, often in broad terms that were sometimes vague and undefined and with limited differentiation of comments about the various students. The teacher’s observations tended to highlight *student characteristics* rather
than student thinking to some extent.

**An episode.** In this episode, students were working on finding the areas of some triangles through drawing the smallest possible rectangle around each triangle. The goal of this particular lesson was to allow students to use their knowledge for finding area of a rectangle to find the area of a triangle, in order to begin to develop understanding of the area formula for a triangle (Lappan et al., 2011). The particular problem is provided below. The yellow and red line segments were added by the researcher to make the problem clear to the readers.

Problem 2.1

A.2. Find the area of each triangle. Describe the strategies you used for finding the areas.

*Figure 4.3:* Using “surrounding rectangle strategy” for finding area and perimeter of triangles

Prior to the captured moment, students in Amy’s group, including Amy, Taylor, Jo, and Bryan, worked on finding the areas of right triangles $a$, $b$, and $c$ through drawing the smallest possible rectangles around the figures and dividing the areas of the rectangles by two, referred to as the “surrounding rectangle strategy.” In this captured episode, they were arguing about the feasibility of finding the area of triangle $d$ using the same strategy, as it differs from triangles $a$, $b$, and $c$ in that it is not a right triangle. Amy and Jo were talking about surrounding it with trapezoids ABDF or BCDF.
Taylor: But you can’t divide a trapezoid.
Amy: Oh... Never mind.
Goldberg: So can we do the same thing as we did with a, b, and c?
Amy: But you have to have [a] right angle to make rectangle [around it].
Taylor: And you couldn’t really divide it [after drawing a rectangle around the figure], because it wouldn’t be equal sections.
Amy: Yeah. Because it’s like half of it [needs to be] a right triangle, and half of it is a square round triangle. And if it doesn’t do that, you can’t [use the same strategy].

As they were arguing, Bryan then proposed to use the original strategy, suggesting drawing the red line segments using his fingers and commenting that the area of triangle d or BFD should equal to the sum of areas of triangle ABF and triangle BCD. However, without paying any attention to Bryan’s idea, the rest of the group resumed the conversation on surrounding the triangle with a trapezoid. Ms. Goldberg then immediately said: “I think maybe you should revisit what Bryan just said.” After Bryan explained his strategy, Taylor began counting the number of unit squares for the area of triangle BCD, appearing to be struggling with the concept of dividing triangle d and regrouping all triangles inside rectangle ACDF to find the area of triangle d.

Reflecting on this episode during a follow-up interview, Ms. Goldberg stated:

...So Amy was saying, you can’t do it anymore because there was no right angle there. And Taylor said he can’t do it because there were three triangles instead of two and you can’t divide it by three. I hear that a lot. You know, I think for a 6th grader, that’s at least something to test... When they do these non-right triangles...it is definitely a different pattern. And I think instead of sitting there, really exploring it for a minute, they jumped quickly to the conclusion that, ‘Oh that doesn’t work...’ So, then it was when Bryan said,’ I am wondering if these two pieces are the same as the triangle.’ And they totally skipped right over that. And then Taylor started talking about [a] trapezoid and just brushed over what Bryan had said. It is funny [that] Bryan will say things, but if a lot of people are talking about other things, he is not real pushy. Then I was sitting there, I waited as long as I thought I could. And I said, ‘Let’s revisit what Bryan just said.’...I wanted them to go back to what Bryan was saying, so I kind of led them back that way... You know, if I can spend 3 days, I won’t say a word the first day when I walk around. You know, I wish I didn’t have to guide them back to Bryan’s method in 5 minutes. I wish they can talk about trapezoid for 20 minutes, and realize by themselves that there is no trapezoid in there. You know, the problem is it is not practical. I wish we have more time to let them explore more. I wish they have time to go down wrong avenues, without having me have
to quickly turn them back around. But we still have to take state tests like everybody else. We still have to hit everything...

In this episode, the teacher attended to the instructional adaptations, while paying attention to some details in students’ strategies. The teacher appeared to focus on decisions and efforts to redirect students to the desired instructional pathway or strategy. The teacher attended to general features of the students’ strategies, giving a general account of how each student had contributed to the conversation. The teacher mentioned that Amy’s and Taylor’s ideas were frequently heard and were “something to test,” but the mathematical essence of student thinking revealed in the conversation was not made explicit. The difficulties those students had in applying the same strategy for triangle $d$ demonstrated their understanding of characteristics of a rectangle versus a trapezoid, but also revealed a possible deficiency in flexibility of thinking, implying a potential lack of understanding or even recognition of the relationship between the area of the surrounding rectangle and that of the triangle inside, that is, the area of the triangle is always half of the area of the smallest surrounding rectangle. Understanding of the relationship is key to achieving the specific goal of the lesson. While highlighting noteworthy student behaviors in broad terms, the teacher demonstrated some evidence of interpretation of students’ understandings as they may have been reflected in their strategies and struggles, without providing differentiation of comments about the various students in the episode. For instance, Taylor’s thinking possibly varied from Amy’s in that Taylor appeared to demonstrate a better and more flexible understanding of the “surrounding rectangle strategy,” showing mathematically important distinctions in his understanding of how to inscribe a figure in a rectangle. Without making a note of the distinction, the teacher provided general and evaluative comments that, “[W]hen they do these non-right triangles...it is definitely a different pattern. And I think instead of sitting there, really exploring it for a minute, they jumped quickly to the conclusion that, ‘Oh that
doesn’t work...” The teacher included commentary on the students’ characteristics but not on their understandings, in a way consistent with the findings reflected in Table 4.2. For instance, the teacher commented on how Bryan was not “pushy” enough when his idea differed from that of the others, but did not say anything about the student’s understanding or how it related to the goal of the lesson. Although she did indicate an approval for Bryan’s strategy, mathematics embedded in his thinking was not clarified. That is, Bryan’s idea of dividing triangle $d$ through drawing the line segment $BE$ contained the seeds of understanding of the base and height of a triangle. Ms. Goldberg’s reflection on the episode included a general description of the students’ difficulty solving the problem, without referring to the mathematical essence of student thinking on an individual level, and leaving open the question of how well the students understood the “surrounding rectangle strategy” and the concept of inscribing a figure in a rectangle. Ms. Goldberg’s comments suggested an example of how she may have focused on her teaching practices while attending to student thinking to a lesser extent in the midst of instruction, demonstrating limited evidence of integrating an interpretation of students’ understandings and behaviors reflected in students’ strategies in her instruction.

The other two components involved in the process of noticing are the reasoning that teachers use when deciding how to respond, based on observation and interpretation of students’ understandings from the specific situation and research on children’s mathematical development, and responding in accordance with their decisions. In this episode, Ms. Goldberg’s decision to wait first and then intervene in response to Taylor’s disregard of Bryan’s idea reflected the dilemma between allowing students time to explore and being pressed for time due to pacing considerations. The teacher’s reasoning involved deciding when to intervene, referring to student thinking in a general and vague fashion. For instance, the teacher mentioned that “for a 6th
grader, that’s at least something to test,” without including what she had learned about the students’ understandings from the specific situation or how consistent their reasoning was with the research on students’ mathematical development. It is not clear that the teacher was able to consider the students’ understandings reflected in their conversations or how those understandings may be utilized to tailor instruction for achieving the learning goal, upon the teacher’s proposal to revisit Bryan’s strategy. When deciding on how to respond and responding to her noticing of student thinking, Ms. Goldberg’s actions and comments provide less visible evidence of using students’ understandings, with alternative foci being her instructional adaptations.

**Analysis.** The above episode involved *student thinking, student characteristics, and instructional adaptations,* described in the coding scheme in Appendix I. In more detail, the reasons for capture included Ms. Goldberg’s noticing of students’ difficulty solving a problem, student attributes or personality, and the teacher’s act of redirecting the group’s attention. With a focus on teaching practices, the teacher made limited references to student thinking when reflecting on her reasons for capture of a video episode.

Examination of the path Ms. Goldberg has gone through from attending to responding in this episode indicates the role teacher beliefs and knowledge may be playing in the teacher’s in-the-moment noticing of students’ mathematical thinking. Ms. Goldberg appears to understand the significance of allowing students to explore, make conjectures, and take ownership of their own learning. For instance, she states that she wants to “let them explore more” without having to “quickly turn them back around.” Holding a belief system that may consist of different perspectives on teaching and learning mathematics, the teacher appears to be skeptical in practice of the reform perspective of teaching and learning mathematics and less confident in the
approach of more learning with less teaching, or an increase of student-driven inquiry and exploration with a reduction of teacher-driven instruction. What Ms. Goldberg does not acknowledge about the breakdown of the mathematics embedded in the problem appears to play a role in her noticing of students’ mathematical thinking in the episode and may interfere with her recognition and interpretation of the various student ideas reflecting a range of understandings in this group. For instance, the teacher makes no reference to Amy’s lack of understanding in inscribing a figure in a rectangle and how it may influence the student’s achieving the goal of the lesson, resulting in a particular piece of student thinking left not addressed. It appears that the teacher’s combination of beliefs and knowledge influences the teacher’s general goal of setting students right by quickly suggesting the right way to think about things and correcting deficiency in students’ responses or strategies, in terms of mathematics teaching, in a way consistent with the findings reflected in Table 4.2. Next the researcher focuses on a comparison between the noticing of the two teachers, in light of teachers’ beliefs and knowledge.

**Marshall and Goldberg: A Comparison**

The cases of Mr. Marshall’s and Ms. Goldberg’s noticing of students’ mathematical thinking indicate how teacher beliefs and knowledge may have shaped teacher noticing. Next, drawing upon one episode captured by each teacher that shares similar learning goals, the researcher aims to compare the two teachers’ noticing, focusing on teacher beliefs and knowledge.

**The case of Joe Marshall.** To begin to address this topic, first consider the following reflective comments made by Mr. Marshall immediately after the lesson on Day 7. In that lesson, students worked on Problem 2.1, and were shown the “surrounding rectangle strategy” during the summary at the end of class to find both area and perimeter for each triangle. For area, the
strategy involves drawing the smallest possible rectangle on the grid lines around each triangle and dividing the rectangle’s area by two, or multiplied it by one half to get the area of the triangle. During a casual conversation with the researcher immediately after class, Mr. Marshall stated, “...tomorrow [when we get to the formulas for area and perimeter] there will be kids who will say the perimeter of the triangle is also half of that of the rectangle...We will address that.” Foreseeing the potential misconception of what counts for the perimeter of a triangle, Mr. Marshall brought up the issue during student presentation time the next day, on Day 8. At that point Melody was presenting her solution to the problem in Figure 4.4, using the overhead transparency projector. The teacher initiated the following conversation immediately after Melody’s presentation of Figure 4.4. The yellow line segments were added by Melody. The students had been instructed to measure the hypotenuse (i.e., line segment BC) using a ruler:

![Figure 4.4: Melody’s presentation](image)

**Figure 4.4: Melody’s presentation**

Marshall: How did you get the perimeter again?  
Melody: (quickly pointing to the sides of ΔABC) Seven plus ten plus twelve equals 29.  
Marshall: (slowly pointing to each side of ΔABC) Seven, (long pause) plus ten, (long pause) plus twelve. (long pause) How much was that again?  
Melody: Twenty-nine.  
Marshall: I’m just curious here-What was the perimeter of the big [surrounding] rectangle up there? (pause) The book didn’t ask it. I was just wondering what it was. Tora?  
Tora: Thirty four.  
Marshall: (pointing to each side of ΔABC) Uhmm, Melody said this (i.e., perimeter of ΔABC) was 29. Is this perimeter half of the rectangle’s perimeter? Is 29 half
of 34? (long pause) Cal?

Cal: No.

Marshall: No, it’s not. And remember that, ok? Every year we have kids who misuse these formulas. The perimeter of the triangle is not (pause) half of the perimeter of the rectangle. Just keep it in the back of your head as we do more [formulas].

Reflecting on this episode later, Mr. Marshall stated that he had expected some of the students would start dividing the surrounding rectangle’s perimeter by two to get the triangle’s perimeter, intuitively. However, as he did not notice any student making this type of mistake during real-time instruction, Mr. Marshall decided to bring it up at the end of the class period.

This episode is an illustration of how a teacher’s pedagogical content knowledge, focusing on student misconceptions in this case, may influence noticing of student thinking. According to Ball, Thames, and Phelps (2008), a particular interest of pedagogical content knowledge acknowledges that accounting for how students rather than teachers understand a content domain is a key feature of the work of teaching that content. Teachers must draw upon both their knowledge of subject matter to select appropriate topics and their knowledge of students’ prior knowledge and conceptions, or misconceptions, to formulate appropriate and provocative representations of the content to be learned (Grossman, 1990). In Mr. Marshall’s case, the teacher’s knowledge of the content and potential student misconception leads to a purposeful plan for instruction. The teacher’s understanding of how students might perceive the relation between a triangle and a rectangle first drove his active search for such a misconception during instruction. Then based on his observation of the absence of the misconception, the teacher drew inferences about student understanding, commenting that “that means they really know the perimeter is the distance around an object.” Through extending his noticing to quick analysis of possible teaching moves in light of the current learning goal and research on children’s mathematical development, the teacher made a decision on appropriate changes to the plan:
Rather than use student errors to open a conversation according to plan, Mr. Marshall brought up the issue through careful questioning based on Melody’s correct answer. During a follow-up interview, he commented on how he had decided on bringing up the issue at the end of the period, having attended to and interpreted the absence of an expected student misconception. Such a pattern resonates with the finding from Colestock’s (2009) study, in which the teacher participant was found to intentionally introduce the necessary ideas into the classroom at certain points in time during instruction, when a particular conceptual idea or solution strategy needed was noticed as absent.

In this case, Mr. Marshall’s pedagogical content knowledge enabled him to address a potential misconception that did not reveal itself to the teacher during instruction. Such a direct conversation about the issue was no doubt needed at that point in time, as it was simply a daunting task to be able to always notice even a trace of this type of misconception in a busy classroom, with numerous distractions. In fact the researchers’ own observation showed that at least one student had attempted to divide the surrounding rectangle’s perimeter by two to get the triangle’s perimeter, prior to the teacher-initiated conversation on this misconception. In a sense, Mr. Marshall’s pedagogical content knowledge compensated for focusing and capacity limitations hindering him from attending to some important student thinking or misconceptions in real-time instruction. The teacher’s knowledge of student perceptions drove what he was seeing during instruction and shaped what occurred in the above episode. Mr. Marshall’s strategy resonates with Colestock’s (2009) description of how teachers may react during teaching if they notice the absence of a particular mathematical idea that is considered necessary at some point during the lesson. Colestock identifies two possibilities for how teachers generally proceed when faced with this dilemma, involving either simply providing the key conceptual insight at times,
or extending a lesson segment with an improvised teaching episode to help the students voice the required insight at other times. In Mr. Marshall’s case, because the teacher was not informed of the student misconception that had been observed by the researcher, he believed it was not an issue for this particular group of students and decided to “supply” (Colestock, 2009, p. 1464) the needed insight himself through extending Melody’s presentation, at the very end of the class period, rather than actively working towards developing a scenario that would lead to and work for this situation throughout the lesson.

The case of Jennifer Goldberg. Next, what is worth discussing, in light of Mr. Marshall’s example, is an episode captured by Ms. Goldberg regarding the same issue. This episode was recorded while students were working on finding the perimeter of each triangle in Problem 2.1 on Day 8. Instead of finding both area and perimeter at the same time, the students were instructed to find each on two separate days, that is, Day 7 and Day 8.

In this episode, Ms. Goldberg noticed that none of Greg’s answers was correct. Rather than add all three sides for each right triangle, Greg only included the base and height (i.e., the length and width of the surrounding rectangle). Figure 4.5 illustrates what Greg did to find the perimeter of one of the right triangles. The yellow line segments were added by Greg:

![Figure 4.5: Greg’s thinking](image)

Analysis of transcription of the whole-class video on Day 8 helped the researcher make better sense of the student’s thinking. Below is an excerpt taken from the beginning of the lesson,
while the teacher was leading the conversation on how to find the perimeter of a triangle. She called on students to speak by drawing names randomly.

Goldberg: Somehow we have to include all the sides of the rectangle (to find its perimeter), right? That’s what perimeter is. So can I find the perimeter of a triangle by doing something similar? Greg? How do I find the perimeter of a triangle?

Greg: You split it in half...

Goldberg: (shaking head) No. We don’t split anything...

After quickly disapproving of Greg’s response, the teacher called on a few other students and directed the class to the conclusion, “So, no matter what shape it is, all you do is making sure you include all sides, and add them up.”

A close look at both Greg’s answer on grid paper and his response during class reveals the student’s potential misconception of what counts for a triangle’s perimeter. Possibly influenced by the “surrounding rectangle strategy” of splitting the rectangle in half for the triangle’s area, Greg developed an “intuitive” misunderstanding about finding the triangle’s perimeter through splitting the rectangle’s perimeter in half. Different from Mr. Marshall’s purposeful instruction, Ms. Goldberg’s dismissal of an important student misconception appears to indicate that the teacher might be less comfortable exploring these kinds of relationships with her activated pedagogical content knowledge. As a result, Ms. Goldberg did not appear to recognize the seeds of a possible issue in Greg’s comment, although it had revealed itself during both whole class instruction and individual work. In fact, the teacher’s conversation with another group in an earlier episode also appeared to show a similar misconception that had gone unnoticed.

Reflecting on the episode regarding Greg’s thinking, Ms. Goldberg stated,

He was just adding two sides. It’s what he was doing. He said the perimeter was 11. I think he was just doing 5 plus 6. He got 11. Then he thinks we are done with perimeter. I know that there is something like, not connecting quite right yet. I can tell he is not there... He is not really understanding it completely. So you as a teacher make a mental note that thankfully, I’ve got Greg five days a week for the intervention class: I need to
make sure that during intervention, he is continuing practicing getting perimeters of shapes. It’s a mental note that I need to keep giving him different kinds of shapes to find the perimeters. Because I don’t want him to get stuck on trying to memorize his formulas, because he is not really understanding what he is doing... You have got to do it a few times, before you really get it.

The teacher’s comments on Greg’s thinking appear to support what has been identified by Jacobs and Philipp (2010), that is, instead of focusing on students’ mathematical thinking, some teachers tend to use their own mathematical thinking or general teaching moves that could be applied to any problem and any child to guide their noticing, with limited specificity and customization in their reasoning. In this particular case, without making reference to the mathematical essence of Greg’s strategy and potential reasons that Greg had failed to include the diagonal (i.e., \( \overline{AC} \)) for the triangle’s perimeter, Ms. Goldberg made vague comments about the student’s thinking, such as, “I know that there is something like, not connecting quite right yet. I can tell he is not there,” leaving the specific misconception undefined, and suggested a general intervention strategy for addressing the issue. Instead of focusing on the details of how the student might have understood the mathematics, the teacher tended toward her instructional adaptations, commenting on her own in-the-moment thought process in response to the student error. Thus this episode provides another snapshot of how Ms. Goldberg appeared to attend to student thinking to a limited extent, and demonstrated little evidence of using students’ understandings when interpreting, deciding how to respond, and responding to her noticing of student thinking.

A comparison. Investigation of the same student misconception reflects two different pathways of noticing. In Mr. Marshall’s case, pedagogical content knowledge compensated for missed opportunities; whereas in Ms. Goldberg’s case, what the teacher did not acknowledge about a particular type of student misconception appeared to lead to important student thinking unnoticed. Furthermore, Ms. Goldberg’s activated knowledge might have made it difficult for
her to gain knowledge about an important student misconception from this particular teaching experience. These examples show how pedagogical content knowledge may play an important role in teachers’ noticing of student thinking, as well as interpretation of and response to that noticing. In order to capitalize on student thinking, teachers have to recognize thinking that can be productive, and also be able to navigate the mathematical territory at a level appropriate for the students (Schoenfeld, 2011). This is particularly important in the case of the absence of a specific mathematical idea or issue that is considered necessary at some point during the lesson, such as in Mr. Marshall’s case. When such an idea or issue is not revealing itself to the teacher, the teacher’s pedagogical content knowledge makes it possible for the teacher to either provide the key conceptual insight or clarification, or extend a lesson with an improvised teaching episode to help the students voice the desired insight or foreseen misconception.

In addition to providing a snapshot of the teachers’ pedagogical content knowledge, a close look at the episodes also reveals the role teacher beliefs may play in teachers’ in-the-moment noticing of student thinking. As described earlier, Ms. Goldberg decided to have students work on finding area and perimeter of triangles on two separate days. Deviating from the instruction provided in the Teacher’s Edition of CMP, the teacher expressed her hesitation to let the students get into messy situations while working on both area and perimeter at the same time. During the follow-up interview, the teacher talked about her decision based on the goal of maintaining the same pacing and staying “cohesive” on pacing for all classes, as dealing with both topics simultaneously would be “confusing”, challenging, and a lot “messier”. Therefore, the teacher explained: “I knew I had to pick one or the other...With perimeter, we have to get into the issue of measuring the diagonals. It was just a coin flip really. I just decided [to work on] one [concept] one day, one the next day. I picked area today.” Deviating from her professed belief in
learning of mathematical content in relation to other important mathematical ideas, Ms. Goldberg appeared to be more concerned with keeping the class in order and laying out content in clear ways, rather than providing the opportunity for students to be exposed to two different but closely related ideas simultaneously in order for better learning to take place. Consistent with the findings reflected in Table 4.2, Ms. Goldberg tended to focus on managing tasks or how certain strategies used to solve problems were being carried out by the students. It demonstrates how the teacher’s professed belief in student exploration appears to differ from her actual instruction. It resulted in directing students down a pre-determined path, revealing a structured image that was centered on what the teacher had planned on doing and how she had expected the students to respond. The teacher’s hesitation to allow room for confusion and struggle may suggest her lack of experience with and confidence in the long-term benefits of investing time in meaningful although messy exploration.

This is in contrast to Mr. Marshall’s decision to include both topics in one lesson. Reflecting on this issue, Mr. Marshall explained: “The neat thing about doing both is you are forcing the issue about what’s perimeter. Right? ... I think that’s why the authors [of CMP] did it [this way]: Let’s have both of them (i.e., area and perimeter) out there [because students tend to use square units instead of linear units when looking for perimeter]...” Commenting on the option of dealing with the topics in separate lessons, the teacher said: “It takes away some thinking...If we do it that way, then we are not going to have these kids wrestle with [the issue of] ‘what I am looking for.’ ‘Am I looking for a line, or am I looking for a square?’ They may not be thinking about that.” Thus Mr. Marshall decided to allow students the opportunity to struggle and get confused, appearing to reflect his image of what mathematics instruction looks like, shaped by his belief in making connections in mathematics and constructing meaning through exploration.
The cases above provide examples of how a mixture of teacher beliefs and knowledge may shape teachers’ in-the-moment noticing of students’ mathematical thinking. What teachers believe about teaching and learning strongly drives their practice, supported by their subject-matter or pedagogical content knowledge about the mathematics topics. Furthermore, teachers’ practice leads to certain kinds of observations, which teachers may notice and for which they may establish particular interpretations, in a way consistent with the teacher’s currently activated beliefs and knowledge. For example, Ms. Goldberg’s underlying beliefs that appear to focus on teacher-driven instruction, coupled with her knowledge about perimeter and area and how children learn this content, may have supported teaching that potentially contributed to development of certain student misconceptions. Furthermore, her traditional beliefs and knowledge may have also influenced her noticing and interpreting the part of Greg’s thinking that possibly contained the seeds of sense making, suggesting her orientation toward misconceptions as things to be overridden for the sake of clarifying content. Whereas Mr. Marshall’s belief of student-driven exploration, supported by his rich and connected knowledge of perimeter and area and students’ thinking, may have led to a learning experience that minimized the chance of self-developed student misconceptions. His knowledge of the common misconceptions held by students also compensated for focusing and capacity limitations hindering him from noticing some important student thinking in the midst of instruction.

Of course, what teachers notice is also influenced by teachers’ goals, or what teachers are trying to accomplish in a lesson. Depending on their beliefs and knowledge, teachers who embrace different beliefs may have goals that appear the same on paper but imply vastly different practices (see, e.g., Aguirre & Speer, 1996; Putnam, 1992; Thompson, 1984). For example, in the above cases, despite the same professed goal of using the “surrounding rectangle
strategy” to find area and perimeter for a triangle, as described by the teacher’s manual of the curriculum, the teachers appeared to demonstrate different foci in practice. Mr. Marshall emphasized helping students see mathematical connections and explore the range of possibilities embedded in the problem, whereas Ms. Goldberg’s purpose in introducing the problem was to provide students with practice finding area and perimeter using a certain strategy. As a result, students in Mr. Marshall’s class were able to “wrestle” with the issue of what the two measures are about; however, some of Ms. Goldberg’s students showed a tendency to develop an incomplete, intuitive misunderstanding of the two measurements. The point is that one must distinguish between the professed goals and the goals that underlie actual teaching practice, or what one professes and what one does (see, e.g., Cooney, 1985; Putnam, 1992). In Mr. Marshall’s case, in the service of the goal of investigating mathematical connections, the teacher’s noticing led to purposeful instruction on an important idea; in Ms. Goldberg’s case, shaped by the goal of practicing a particular strategy, the teacher’s noticing resulted in possible unawareness of an “intuitive” misunderstanding of the problem.

In sum, noticing is essential, as what a teacher notices and does not notice shapes what a teacher does and does not do. However, noticing “does not suffice by itself (Schoenfeld, 2011, p. 233). Taking place within the context of teachers’ beliefs and knowledge, what teachers notice, and how they act upon their noticing, is a function of the teachers’ beliefs and knowledge. In order for teachers to notice in a way that fosters the kind of mathematical learning environment espoused by a reform agenda, it would require some rather fundamental shifts in teachers’ beliefs about the nature of mathematical knowledge, and how it is learned and taught, and a richer knowledge of the mathematics being taught (Putnam, 1992). In Mr. Marshall’s case, shaped by his reform-oriented beliefs, and rich and connected pedagogical content knowledge, he appears
to be able to notice where opportunities to help students see mathematical connections exist in student ideas and behaviors, and in the curriculum, and also act upon his noticing to bring the class into his reform agenda. In Ms. Goldberg’s case, driven by her teacher-centered beliefs, and particular ways of perceiving mathematical connections, she appears to be attuned to seeing and responding to student thinking through a didactic, step-by-step approach, and as a result, some things, such as some elements of understanding and pedagogical possibilities, may remain invisible or go unnoticed, and may not be acted on.

Summary

The findings in this chapter have been presented in response to the research questions, including how and to what extent teachers notice students’ mathematical thinking in the midst of instruction, and how and to what extent teachers’ in-the-moment noticing of students’ mathematical thinking influences teachers’ instruction. Highly interrelated with each other, these questions are explored and discussed in an integrated way. First, descriptions of each teacher, including the teaching practices in the context of a reform-based curriculum, and their potential beliefs and knowledge related to the unit are presented, drawing on a variety of sources of data, such as recordings of classroom instruction and initial interviews. Second, findings on what teachers noticed are presented, including explanation of procedures for data collection and analysis, and a general description of what teachers noticed. Third, each teacher’s noticing pattern is described, involving the process of attending, interpreting, deciding how to respond, and responding. Last, a comparison between the noticing of two teachers is provided, in light of the power of beliefs and knowledge. An interpretation of these findings is offered in the final chapter of this dissertation. Implications for this research, limitations of this research, and areas for future research are discussed as well.
CHAPTER 5: CONCLUSION

Introduction

The guiding purpose of this research is to access teachers’ in-the-moment noticing of students’ mathematical thinking, in the context of teaching a unit from a reform-based mathematics curriculum, i.e., Covering and Surrounding from Connected Mathematics Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2011, third edition). The focus of the study is to investigate the following research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

This research process begins with an exploration of the current field of teacher noticing in Chapter 2. Although the state of research on teacher noticing is young, a review of the field provides a background for this study that has grown out of the existing body of knowledge. Based on a thorough review of the literature, the construct of teachers’ in-the-moment noticing of students’ mathematical thinking is conceptualized as a set of interrelated components in this study, including attending to students’ strategies, interpreting students’ understandings, deciding how to respond on the basis of students’ understandings, and responding in certain ways. It is conjectured that attending, interpreting, and deciding how to respond all happen in the background, almost simultaneously, “as if constituting a single, integrated teaching move” (Jacobs et al., 2010, p. 173), before the teacher responds.
A review of literature reveals that much of the research on teacher noticing does not examine teacher noticing as it occurs in the midst of instruction. Rather, the research involves asking teachers to analyze and reflect on videos outside the context and pressure of in-the-moment instruction. Thus, in order to tackle the different methodological disadvantages, and access teachers’ in-the-moment noticing in a more explicit and direct way, the researcher in this study applied a new technology to explore teacher noticing, enabling teacher participants to capture their noticing through their own perspectives while teaching in real time. There were six types of data collected in this study. These data were from: initial interviews, daily classroom observations, daily researcher-generated whole-class video-recording, daily teacher-generated individual/small group video-recording, follow-up interviews of teacher participants, and collection of student classwork.

Findings related to these data were presented in the previous chapter. In this chapter, these findings are interpreted and discussed. This is organized around the research questions:

1. How and to what extent do teachers notice students’ mathematical thinking in the midst of instruction?

2. How and to what extent does teachers’ in-the-moment noticing of students’ mathematical thinking influence teachers’ instruction?

Findings about these questions are intertwined and presented in conjunction with each other, based upon the premise that mathematics teachers’ noticing of students’ thinking is recognized as involving a set of four interrelated components. Finally, implications for this research, limitations of this research, and areas for future research are discussed.
Teacher Noticing of Student Thinking

This section is organized into two subsections. First, findings about the research questions are presented using an integrated approach. Second, constructs that are found to play a potential role in teachers’ noticing are discussed, including teacher beliefs, knowledge, and goals. A diagram is also presented to illustrate the potential paths through which teachers travel in the process of noticing, upon a new occurrence in the midst of instruction.

A Synopsis of Findings

Analysis of teacher-generated video-recording and related data from other sources suggests that teacher participants appeared to notice for a variety of reasons (see Table 4.1 and 4.2 in the previous chapter), similar to the teachers studied by other researchers in the field (see Colestock, 2009; Luna et al., 2009; Sherin, Russ, et al., 2008). Tables included in Appendix J show a similar pattern on a daily basis for each teacher. Furthermore, these participants appeared to notice student thinking in the midst of instruction to different extents. Based on Table 4.1, more than half of Mr. Marshall’s episodes focused on student thinking, whereas less than half of Ms. Goldberg’s episodes concerned student thinking. A closer look at the data also indicates that both teachers appeared to notice instructional adaptations in the midst of instruction to a great degree, resembling the noticing accessed and described by van Es (2011) in a sense, who studied how a group of teachers noticed student thinking in the context of a video club.

In the present study, Mr. Marshall’s noticing of student thinking appeared to be accompanied by his noticing of instructional adaptations and/or assessment. Conceptualized as a set of interrelated skills, the teacher’s noticing included first attending to primarily the mathematically important details in students’ strategies. Then, reasoning through what he had attended to in these details, the teacher drew inferences about or generated productive and
evidence-based evaluation of student understandings, in a way that was consistent with the
details of the specific strategies. Next, the teacher extended his noticing to quick analysis of
possible teaching moves in light of the current specific learning goal, the structure of the
curriculum, and broader goals and principles of teaching and learning, deciding whether and how
to respond on the basis of his analysis. Last, the teacher took some action based on his decision.
Mr. Marshall’s noticing of the essence of important mathematical details in students’ strategies
led to adjustment or change in his practices, such as, teacher remediation or facilitation, in order
for the teacher to address what had been noted essential.

In contrast to Mr. Marshall’s focus on both students’ mathematical thinking and instructional
adaptations at the same time, Ms. Goldberg appeared to focus primarily on her own practices
while attending to student thinking to a lesser extent. This was similar to van Es’s (2011)
findings regarding some sorts of mixed practice demonstrated by teachers who had just begun to
attend to students’ mathematical thinking, in addition to teacher pedagogy, transiting from
primarily attending to whole class environment and student participation, in the context of a
video club. In Ms. Goldberg’s case, the teacher appeared primarily to focus on her own
instructional adaptations while attending to some details in students’ strategies. Then the teacher
gave an account of noteworthy moments and provided limited interpretive comments on the
basis of the details in students’ strategies. The teacher described the students’ understandings to
some extent, often in broad terms that were sometimes undefined and with limited differentiation
of comments about the various students. Next, when deciding how to respond to her noticing, the
teacher made limited reference to the students’ existing understandings, with alternative foci
such as her instructional adaptations. Typically Ms. Goldberg’s absence of evidence in noticing
important mathematical details in students’ strategies led to non-execution of instructional
responses in some situations, resulting in student thinking with promising mathematical potential possibly going unnoticed and not acted upon. Ms. Goldberg’s case may indicate that a teacher could notice student thinking in the midst of instruction, but not in a way that would always foster the learning called for by reform.

Teacher Beliefs, Knowledge, and Goals

Examination of the data also suggests that teachers’ noticing of student thinking appears to take place within the context of their beliefs, knowledge, and goals. What teachers notice in the midst of instruction and how their noticing influences their instruction appears to be shaped by teachers’ beliefs, knowledge, and goals. Thompson (1984) found that teachers’ beliefs about mathematics and teaching play a significant role in shaping their behavior. Schoenfeld (1998) noted that “people’s beliefs shape what they perceive in any set of circumstances, what they consider to be possible or appropriate in those circumstances, the goals they might establish in those circumstances, and the knowledge they might bring to bear in them (p. 19).” In the current context, what teachers notice, and whether they notice student thinking in a way that is consistent with the reform vision appear to depend on teachers’ beliefs about mathematics teaching and learning, and students, supported by available knowledge, in the service of certain goals established upon occurrence of a new event in the midst of instruction. If the teacher believes in helping students see mathematics connections through capitalizing on what students bring with them to the learning situation, then pursuing a range of possibilities involved in a particular problem at a level appropriate for the students might be established as a goal. Supported by the teacher’s pedagogical content knowledge about the mathematics topics, the teacher attends to mathematically significant details in student thinking that could inform instruction, interprets student understandings using these details, decides how to respond based on what the teacher has
learned about the students’ understandings and the research on students’ mathematical development of understanding, and responds in ways that could help the students make meaningful connections and build a solid knowledge structure for the mathematical territory. If the teacher believes in a didactic, step-by-step approach to instruction, and that misconceptions or non-standard student strategies are things to be undone or overridden for the sake of clarifying content, then providing students with predetermined instruction might be established as a goal. Consistent with the teacher’s beliefs and goals, particular knowledge is activated in the teacher’s act of noticing in the midst of instruction. The teacher attends to student thinking in a way that supports a teacher-centered agenda, interprets or sizes up students’ ideas using a preset standard, decides how to respond and responds to certain student thinking based on what has been prescribed in a lesson plan. Such a combination of beliefs, goals, and knowledge may lead to teachers’ focus on teaching a lesson that appears to stay on preset paths and unfold in a way consistent with the teacher’s plan, instead of making unplanned changes or being responsive to the unpredictability of classroom events in the midst of instruction, and as a result alternative ways of thinking about and making sense of a particular mathematical problem may go unnoticed, and not acted upon, consciously or unconsciously.

Derived from interviews and classroom observations, an overview of both teachers’ beliefs shows that Mr. Marshall’s noticing appears to be consistent with his professed beliefs in making connections and learning mathematics through problem solving; whereas Ms. Goldberg’s noticing may be shaped by a potential dual belief system that consists of different perspectives on teaching and learning mathematics. Depending on the constraints and resources, a particular belief may be prioritized upon suspension of the other belief, leading to the formulating and carrying out of goals consistent with the prioritized belief. Even though Ms. Goldberg may have
attempted to deal with student thinking that contains seeds of sense making, the teacher’s well-developed traditional beliefs, in the combination of a knowledge base that may not be centered on awareness of student perceptions and responding to them, appear to exert a stronger influence on formulating goals that demonstrate a traditional perspective. For instance, in the case of using the “surrounding rectangle strategy” to find area and perimeter for triangles in the previous chapter, it appears that Mr. Marshall’s belief of student-driven exploration, supported by his rich and connected knowledge of perimeter and area, has enabled him to provide a learning experience that minimized the chance of self-developed student misconceptions; whereas in Ms. Goldberg’s case, although the teacher sensed that “there is something like, not connecting quite right yet,” her potential belief of undoing rather than working with student misconceptions for the sake of clarifying content, and what she did not acknowledge about the particular student perception appeared to be employed at the time, in the service of the goal of providing students with practice using a standard strategy to solve a particular type of problems.

Figure 5.1 presents a diagram that illustrates how teachers’ in-the-moment noticing of students’ mathematical thinking may take place within the context of their beliefs, knowledge, and goals. It shows that teachers’ in-the-moment noticing of students’ mathematical thinking does not suffice by itself. Instead, it is shaped by other constructs, resonating with Schoenfeld’s (1998) notion of how and why people do what they do in any set of circumstances. What a teacher actually does at any moment is driven by the teacher’s goals. More precisely, decisions about what to do and how to do it are shaped by the set of currently active, high priority goals (Schoenfeld, 1998). Thus, in the service of the currently active goals reflecting certain beliefs, the teacher activates relevant pedagogical content knowledge in the act of noticing, including attending, interpreting, deciding how to respond, and responding. Hence, happening in the
background implicitly and almost simultaneously, the combination of beliefs, goals, and knowledge results in certain moments being observed, interpreted, and acted upon, and brings forth certain instructional responses in the end. The paths represented in Figure 5.1 show that four constructs, i.e., teacher beliefs, pedagogical content knowledge, mathematics knowledge, and instructional goals, all play a role in the teacher’s noticing of students’ mathematical thinking, involving attending, interpreting, deciding how to respond, and responding. Upon occurrence of a new event at any particular time during instruction, all four constructs may come into play in shaping the teacher’s noticing; however, depending on the given constraints and/or resources available, and/or an event that has recently taken place, it is also reasonable to conjecture that any individual construct or combination of constructs may influence the teacher’s noticing, resulting in certain moments noticed and actions taken, in ways consistent with the teacher’s activated knowledge, beliefs, and goals at any particular moment. None of these constructs necessarily has priority over any of the others, as they interact with each other in shaping the teacher’s in-the-moment noticing of students’ mathematical thinking. Resonating with Schoenfeld’s (1998) theoretical description of teaching-in-context, the diagram provides a window into teachers’ moment-to-moment decision-making and actions during instruction through the perspective of their in-the-moment noticing of students’ mathematical thinking, based on findings of an empirical inquiry.
In summary, rather than functioning on its own, teachers’ *in-the-moment* noticing is influenced by multiple constructs, including teacher beliefs, goals, and knowledge. Shaped by these constructs, teachers notice student thinking to different extents, influencing students’ opportunities to think mathematically in different ways. In the final section of this chapter, limitations of the study, and area for future research are discussed.
Limitations and Future Research

The guiding purpose of this research is to access teachers’ in-the-moment noticing of students’ mathematical thinking, in the context of teaching a unit from a reform-based mathematics curriculum. This research process began with an exploration of the current field of teacher noticing, from which a case study involving two teacher participants was developed, for the purpose of adding knowledge to the field using a new way of exploring teacher noticing. In this section, limitations of the study, and area for future research are discussed. First, limitations of the study are presented. With these limitations in mind, the next step for research is suggested. Finally, the possibility of this methodology as an effective design for professional development is put forward.

Limitations

There are several limitations to this study that must be addressed before considering future research. The first two limitations focus on conceptual issues, and the other three concern methodological issues. First, findings presented in the previous chapter evolved from analysis of two White teacher participants and 33 White student participants in the Midwestern United States. Both teachers were considered experienced mathematics teachers who had taught the reform-based curriculum for at least seven years and had been involved in ongoing learning for teaching the curriculum. In analysis of another group of teachers of diverse ethnicities or backgrounds or different levels of expertise in a different region, one might identify alternative patterns for teachers’ in-the-moment noticing of students’ mathematical thinking. Even within the present study, findings could have been interpreted in alternative ways, taking into account the issues of gender, personality, personal approach to mathematics, experience with teacher education programs, and so on. In addition, it is reasonable to conjecture that the particular unit
and curriculum taught, i.e., *Covering and Surrounding* from *Connected Mathematics Project*, might also play a role in the nature of episodes that were able to be captured by teachers, and the specific kinds of student thinking noticed.

Second, data concerning both teachers’ beliefs were derived from interviews and classroom observations, rather than from using a formal assessment instrument on teachers’ beliefs prior to conducting fieldwork. According to Remillard (2005), a key characteristic of most case studies is that the phenomena studied are embedded in the context with all its complexity, and researchers often aim to make meaning of them *during* or *after* data collection. In this case, *teacher belief* did not surface as a construct to be considered for studying teachers’ in-the-moment noticing until after data collection and, therefore, was not formally assessed throughout the study. Researchers conducting future research of a similar nature should, however, consider using a formal assessment instrument on teachers’ beliefs, in order to gain better insight on how teachers are noticing students’ mathematical thinking in the midst of instruction.

Third, the change of interview schedule brought some methodological challenges. Due to practical reasons such as scheduling difficulties, the researcher was not able to conduct each follow-up interview on the same day when the lesson was taught. The longest interval of time was found between the lesson on Day 2 and related follow-up interview on Day 4 for Mr. Marshall. Although viewing the teacher-generated episodes was usually sufficient for providing good memory cues, there were times when teachers had difficulty articulating their reasons for capture due to memory lapses. This change of interview schedule from the original plan was unavoidable, leading to a limited number of episodes being discussed for each lesson. As a result, the researcher had to code more than half of the episodes based on the researcher’s interpretation of possible reasons for capture, rather than the teacher participants’ own
interpretations. Although great care was taken to make sense of the episodes not discussed through utilizing multiple sources of data collected, in an effort to ensure that the researcher was accurately describing the phenomena and the meanings made of them by participants, teachers’ own explanations would still remain the most ideal source of interpretation.

Fourth, there were some issues concerning the use of the technology. On the one hand, at times Mr. Marshall reported simply failing to use the camera because he was so involved in his everyday practices. For example, he stated,

I was thinking about a million things. It (i.e., remembering to press the record button) was the last thing on my mind... [I told myself] I will be concentrating on what kids know, [and] not knowing, where am I going from here [for the lesson]...

Mr. Marshall’s comments show that it might be the case that he simply forgot to use the camera, or he was unable to use it because of the cognitive intensity of teaching at a particular moment. On the other hand, Ms. Goldberg reported that she had tried to press the record button upon walking away from each table during small group activity, and remarked: “I was like, whatever we talked about [at the table], it might be important [to record]. I don’t know.” As a potential result of the different ways of using the camera, Ms. Goldberg captured 17% more episodes than Mr. Marshall did. Therefore, it might be worth looking into the issue regarding the influence of “logistics of clicking” on the quantity and nature of episodes captured. Additionally, in order to tackle memory lapses and technical issues, the researcher decided to follow the teachers’ noticing during the second week. Although this strategy assisted in the process of recalling captured moments for teachers, and provided information on the video clips recorded incorrectly, it brought challenges for observing the rest of the class. The researcher was not able to take rather extensive notes on all groups to some extent, while following the teacher’s noticing at the same time.
Last, both teachers’ comments during some interviews indicated potential influence of use of the cameras on their teaching. Mr. Marshall expressed his concerns about his performance while being videotaped by the researcher, and Ms. Goldberg talked about her tendency to spend less time with each table sometimes, because of her intention to visit all tables with the camera recording continuously. These comments suggested potential influence of this part of the field work on teachers’ everyday practices.

Therefore, researchers conducting future research of a similar nature should make every effort to ensure daily follow-up interviews with each teacher, so that more episodes are discussed in a timely and more cohesive manner. Furthermore, it is important to allow participants sufficient time to practice using the technology and get used to being videotaped while videotaping. Next, in light of the findings reported, it is suspected that the following areas may point to a productive direction for research.

**Recommendations**

This study does not examine how teacher beliefs, knowledge, and goals may influence each other. More empirical research can be done along these lines to explore how these constructs may affect each other in playing a role in teachers’ in-the-moment noticing of student thinking, and how in turn teachers’ noticing influences these constructs. For instance, research using traditional methods has identified that teachers can learn as they listen to student thinking, with their existing content knowledge being modified and new knowledge being generated (Sherin; see also Empson & Jacobs, 2008). Therefore, it may be worthwhile to investigate how teachers’ in-the-moment noticing of student thinking may affect teacher beliefs, knowledge, and goals, in order to provide even deeper analysis and better explanation of the dynamic, moment-to-moment decisions and actions of teachers in real time teaching. Moreover, this study does not examine
the development of learning to notice student thinking or the paths teachers follow while they learn to notice student thinking. In the following paragraphs, the possibility for professional development is put forward, opening up a potential line of research that would be centered on investigating how teachers learn to notice students’ mathematical thinking in the midst of instruction.

In more detail, an important product of this study is an understanding of the process involved in teachers’ in-the-moment noticing of student thinking, as an interaction of teacher beliefs, knowledge, and goals. The methods used in this work could be used for teacher professional development. In fact, an important result of the study appears to be the positive influence of reflection on these teachers’ noticing. For example, throughout the follow-up interviews in this study, Mr. Marshall demonstrated active and ongoing effort in questioning his practices and conjecturing about alternative approaches to teaching the concepts, using the phrase “I could have done...” on average ten times during each interview. He was provoked to make adjustments to lesson plans or attend to and address student thinking that had gone unnoticed or misinterpreted previously, as a result of discussion of the related teacher-generated episodes during follow-up interviews. Reflecting on this part of the study, Mr. Marshall stated,

I have loved the opportunity to have to reflect upon what was going on in the classroom, because you forget...So I’ve learned a lot by doing this personally about teaching, because there were several different areas where I would have not dug in as deep as I did afterwards, if you weren’t here, watching and asking questions...To hear what I said, to see what the kids said and what was there, it’s definitely good to look at that (i.e., teacher-generated video recording) again...The conversations we had [during follow-up interviews], if we could do that on a daily basis, we are gonna learn a lot.

Also, in Ms. Goldberg’s case, although the influence of reflection was not found evident in her everyday practice throughout the study, her comment on one episode during the last follow-up interview revealed a trace of the influence reflection had on her noticing:
I thought about one thing [on the spot], because you ask, you usually ask the question of how does this [noticing], you know, influence your teaching down the road-I got an aha moment right there [during the interaction]. I was like OMG, I should’ve done that! I’m going to do it from now on. From now on, I am going to throw up four triangles at the very beginning of that lesson before we talk about anything else...

The above comment was made at the conclusion of a follow-up discussion of one episode involving a group of students working on families of triangles that are congruent. The teacher reflected on what she had thought about in terms of future instruction in the very act of capturing the moment on the spot. The example suggested that the cycle of noticing followed by immediate reflection had made its way in influencing Ms. Goldberg’s way of noticing, showing that she had begun to become more aware of her own noticing in taking the initiative of trying to make sense of it even in the very midst of instruction.

Thus, in light of the findings described above, a single teacher’s noticing could be documented using this type of technology and reflected upon immediately. That is, one might start with a teacher-generated episode, asking why the teacher pressed the button, what the teacher noticed about student thinking, and how the teacher’s awareness/noticing of this moment influenced his or her decision making in the midst of instruction, and so on. The episode and related analysis could be used by the individual, fellow teachers, researchers, and/or professional development people for improvement on content and pedagogy or as the focus of attention in a teacher video club (Aguirre & Speer, 1996). The intention would not be to evaluate the teacher’s practice, but instead to provide a tool for focusing the discussion/reflection on how teachers notice within the context of beliefs, knowledge, and goals, in light of the diagram presented in Figure 5.1. It is argued by Jacobs et al. (2010) that how individuals analyze what they notice is as important as what they notice. The process could promote and develop teachers’ self-reflections on their in-the-moment noticings, sensitizing and enabling them to avoid the habitual, and to act
“freshly” (Mason, 2011, p. 37) upon noticing, with the aid of a sort of framework for analysis and identification of potential areas of improvement through addressing each construct described in this study, i.e., beliefs, knowledge, or goals. Although a teacher cannot be expected to anticipate everything that might come up in a mathematics class, establishing an instructional environment in which reflection is central may be a way to alleviate some of the difficulties associated with the moment-to-moment noticing of students’ mathematical thinking. Such suggestions echo findings by Thompson (1984), who attributed one of his teacher participants’ consistency between what she says she believes and what she does in the classroom to her being reflective about her practice, or reflectiveness, i.e., teachers’ tendency to think about their actions in relation to their beliefs, students, and the subject matter. As suggested by Stipek, Givvin, Salmon, and MacGyvers (2001), teachers’ beliefs tend not to change much from experiences with their pre-service training programs or simply reading findings of educational research. However, reflection on classroom experiences has been shown effective in influencing teachers’ beliefs. Therefore, the present study appears to point to a promising direction for professional development efforts that would motivate a shift in teachers like Ms. Goldberg’s beliefs, and noticing.

In conclusion, as a first step in examining the complex process of teachers’ in-the-moment noticing of student thinking, this study offers insight on how teachers are noticing student thinking in the midst of instruction and how that noticing influences instruction. The consciously or unconsciously selected information in the classroom opens up a window to the “subjective worlds” (Erickson, 2011, p.21) inhabited by the teacher, telling a unique story of the teacher as the authority of the classroom, who brings experience, resources, and perspectives to the process of noticing. Further investigation is necessary, as the present study was limited in its potential for
finding answers to some important questions mentioned earlier; however, it provides a foundation for unpacking the potential constructs playing a role in shaping teachers’ *in-the-moment* noticing of student thinking. Furthermore, as stated by José Ortega y Gasset, “tell me to what you pay attention and I will tell you who you are.” This dissertation illustrates a viable methodology for professional development that is centered on supporting teacher change as envisioned by the reforms, through changing the way teachers notice in the midst of instruction.
APPENDIX A: TEACHER RECRUITMENT SCRIPT – EMAIL

Good morning! My name is Yanjun Liu, and I am a graduate student in the School of Education at UNC. I am studying with Dr. Susan N. Friel. I have become interested in the domain of mathematics teacher noticing, indicating mathematics teachers’ particular ways of observing or recognizing something in the classrooms. In particular, I am interested in examining teachers’ ways of noticing students’ mathematical thinking in the midst of instruction. I am very excited about the potential of what I learn in this domain for contributing to mathematics teaching and learning, and I am looking for middle grades classrooms where I can study teachers’ noticing of student thinking during instruction. I would like to invite you to be a participant in my dissertation research.

If you decide to take part in this research study, you will allow me, the principal investigator, to conduct an initial interview of you prior to any classroom observations. You will be asked to respond to interview protocols, and then watch a thirteen-minute video clip, and respond to prompts. Both the interview protocols and video clip involve understanding of perimeter and area. The conversations will be audio-recorded, with your permission. You may choose not to answer a question for any reason. Then you will allow me to videotape and observe you teach ten consecutive class periods, in one of your sixth grade mathematics classrooms. In addition, during the same class periods, you will be instructed to videotape certain moments of noticing student thinking while teaching a CMP unit, i.e., the Covering and Surrounding unit, using a portable video camera with selective-archiving capability. You will also allow me to conduct daily, follow-up discussions with you regarding videotapes of each lesson observed. These discussions will be videotaped, with your permission.

Your students will also be invited to be in this research, but they will be asked for separate assent and parental permission. With their parents’ permission and their own assent, students’ activities and participation will be video-recorded by both you and me for later transcription and analysis. Student written work may also be collected to examine their conceptual understanding of the learning objectives.

Participation in this study is completely voluntary. You may choose not to be in the study or to stop being in the study before it is over at any time. Your participation and any data collected will be kept confidential. Pseudonyms for participants, school, and school district will be used in publications or presentations. Additionally, other identifiers will be removed, masked, or changed. You will have access to transcripts, recordings, and reports at any point. You will also have the opportunity to review the final reports and/or publications and make requests for changes to any potential identifying information. Not every teacher who is interested will be included in the study.

Additional information about the study is provided on the consent form. I am also happy to answer any questions you may have. Thank you! If you are interested in participating, please reply to this email as soon as possible, but do NOT fill out or send me the consent form yet. Please simply include the following information in your response: number of years of teaching experience, number of years of teaching the CMP curriculum, and training experience in teaching this curriculum, if you are willing. I look forward to hearing from you!
APPENDIX B: TEACHER CONSENT FORM

University of North Carolina at Chapel Hill

Consent to Participate in a Research Study
Adult Participants
Social Behavioral Form - TEACHERS

Consent Form Version Date: November 25, 2011
IRB Study # 102686
Title of Study: Teachers In-the-Moment Noticing of Students Mathematical Thinking: A Case Study
Principal Investigator: Yanjun Liu
Principal Investigator Department: School of Education
Principal Investigator Phone number: 919-259-3146
Principal Investigator Email Address: yanjunl@email.unc.edu
Faculty Advisor: Dr. Susan N. Friel
Faculty Advisor Contact Information: sfriel@email.unc.edu; 919-962-6605

What are some general things you should know about research studies?
You are being asked to take part in a research study. To join the study is voluntary. You may refuse to join, or you may withdraw your consent to be in the study, for any reason, without penalty. Research studies are designed to obtain new knowledge. This new information may help people in the future. You may not receive any direct benefit from being in the research study. There also may be risks to being in research studies. Details about this study are discussed below. It is important that you understand this information so that you can make an informed choice about being in this research study. You will be given a copy of this consent form. You should ask the researchers named above, or staff members who may assist them, any questions you have about this study at any time.

What is the purpose of this study?
The purpose of this study is to investigate how teachers are noticing students’ mathematical thinking in the midst of teaching a lesson, and how that noticing influences teachers' instruction, in the context of the use of a reform-based mathematics curriculum. A better understanding of noticing will contribute to improving mathematics teaching and learning. In this study, I will videotape and observe you teaching one of your sixth grade mathematics classes for two weeks, after an initial interview, and also ask you to video-record your own moments of noticing student thinking using a portable video camera with selective-archiving capability, during the same class periods, followed by daily discussions of the captured videotapes.

How many people will take part in this study?
If you decide to be in this study, you will be one of approximately 4 middle grades mathematics teachers in this study.

How long will your part in this study last?
Your time commitment in this study will entail approximately 45 minutes of initial interview and 8 hours of discussion with me during a two-week study period. You will also be videotaped and
asked to videotape certain moments described above while teaching 10 lessons during regular instruction time.

**What will happen if you take part in the study?**
If you participate in this study, you will allow me, the principal investigator, to conduct an initial interview of you prior to any classroom observations. You will be asked to respond to interview protocols, and then watch a thirteen-minute video clip, and respond to prompts. Both the interview protocols and video clip involve understanding of perimeter and area. The conversations will be audio-recorded, with your permission. You may choose not to answer a question for any reason. Then you will allow me to videotape and observe you teach ten consecutive class periods, in one of your sixth grade mathematics classrooms. In addition, during the same ten class periods, you will be instructed to videotape certain moments of noticing student thinking while teaching a CMP unit, i.e., the Covering and Surrounding unit, using a portable video camera with selective-archiving capability. You will also allow me to conduct daily, follow-up discussions with you regarding videotapes of each lesson observed. These discussions will be videotaped, with your permission.

**What are the possible benefits from being in this study?**
Research is designed to benefit society by gaining new knowledge. Your participation in this study may be a valuable activity for you as well. For example, wearing the camera during instruction may heighten your awareness of important moments taking place during teaching. The process of making decisions about what to capture, pressing the save button, and revisiting/discussing those captured moments at a later time on the same day may help you slow the pace of instruction, make your noticing behaviors more captured moments at a later time on the same day may help you slow the pace of instruction, make your noticing behaviors more explicit, and provide needed support and opportunity for you to hone noticing skills and respond to student thinking in more meaningful and productive ways.

**What are the possible risks or discomforts involved from being in this study?**
There are no known or anticipated risks for participation in this research study. There may be uncommon or previously unknown risks. You should report any problems to the researcher.

**What if we learn about new findings or information during the study?**
You will be given any new information gained during the course of the study that might affect your willingness to continue your participation.

**How will your privacy be protected?**
Consent forms and the pre-assigned participant number identification sheet that links study ID codes to names will be kept in a locked file cabinet in my dead-bolted apartment, and will only be identifiable to me. Care will be taken to ensure that all identifying information is removed and replaced with the assigned participant number upon artifact collection or during data transcription. Pseudonyms for participants, school, and school district will be used in publications or presentations. Additionally, other identifiers will be removed, masked, or changed. Thus, the investigator will abstract the data needed for the research in such a way that the information can no longer be connected to your identity. All documents will be shredded following transcription. All audio-recordings will be password protected on a computer laptop and external hard drive until transcription, after which they will be destroyed. Videos will be either password protected on a computer laptop and external hard drive (for digital video-recordings) or stored in a locked cabinet in my apartment until they are destroyed after transcription.
Your phone number will not be saved into my cell phone contact list. Your email address will be kept in my private email address book. There will be no identification of you as a study subject. At the end of the study, your email address will be deleted from the address book, and your phone number will be deleted from the cell phone.

You will be referred to in transcription of the audio-recordings or video-recordings of our interview and discussions after each meeting (if you give permission for audio-recordings and/or video-recordings) by your pre-assigned participant number. All names of people or places stated in conversation will be replaced with pseudonyms or participant numbers during transcription. Prior to transcription, all notes, artifacts, documents, video-recordings, and audio-recordings will be stored either on my password protected computer laptop or in a locked cabinet in my apartment. All data collected, when transcribed, will be stored on a laptop and an external hard drive in my home. The data will be password protected. Field notes and other written documentation will be shredded after transcription into conventional Word documents.

You will be asked at the end of this form whether you are willing to be recorded. There is a slight possibility for deductive disclosure, which means that other people such as the staff at the schools might be able to figure out which class or which teacher is being discussed in a research report. Because of that, we are using pseudonyms for participants, school, and school district in all reports, publications or presentations. Additionally, other identifiers will be removed, masked, or changed. Participants will not be identified in any report or publication about this study.

In addition, teacher participants will have access to transcripts, recordings, and reports at any point. Teacher participants will also have the opportunity to review the final reports and/or publications and make requests for changes to any potential identifying information. Participants will not be identified in any report or publication about this study. Although every effort will be made to keep research records private, there may be times when federal or state law requires the disclosure of such records, including personal information. This is very unlikely, but if disclosure is ever required, UNC-Chapel Hill will take steps allowable by law to protect the privacy of personal information. In some cases, your information in this research study could be reviewed by representatives of the University, research sponsors, or government agencies (for example, the FDA) for purposes such as quality control or safety.

**What if you want to stop before your part in the study is complete?**
You can withdraw from this study at any time, without penalty. The investigator also has the right to stop your participation at any time. This could be because you have had an unexpected reaction, or have failed to follow instructions, or because the entire study has been stopped.

**Will you receive anything for being in this study?**
You will not receive anything for taking part in this study.

**Will it cost you anything to be in this study?**
It will not cost you anything to be in this study.

**What if you have questions about this study?**
You have the right to ask, and have answered, any questions you may have about this research. If you have questions, complaints, concerns, you should contact the researchers listed on the first page of this form.

**What if you have questions about your rights as a research participant?**
All research on human volunteers is reviewed by a committee that works to protect your rights and welfare. If you have questions or concerns about your rights as a research subject, or if you
would like to obtain information or offer input, you may contact the Institutional Review Board at 919-966-3113 or by email to IRB_subjects@unc.edu.

Check the line that best matches your choice:

_____ OK to audio-record me during the study (during initial interview)

_____ Not OK to audio-record me during the study

Check the line that best matches your choice:

_____ OK to video-tape me during the study (during instruction and follow-up discussions)

_____ Not OK to video-tape me during the study

Participant’s Agreement:

I have read the information provided above. I have asked all the questions I have at this time. I voluntarily agree to participate in this research study.

_________________________________________  ___________________________  ___________________________
Signature of Research Participant                      Date                          

______________________________________________________
Printed Name of Research Participant

_________________________________________  ___________________________  ___________________________
Signature of Research Team Member Obtaining Consent                      Date                          

______________________________________________________
Printed Name of Research Team Member
Obtaining Consent

Please return the signed copy of this form to the researcher using the self-addressed stamped envelope, as soon as possible. Keep the other copy of the form for your records.
Good morning/afternoon. My name is Yanjun Liu. Now I am a graduate student at UNC, but I used to teach 8th grades and high school math. You are being asked to participate in a research study. I am doing this study to learn more about how something that we call “teacher noticing” helps middle grades students learn math. The reason for doing this research is to develop knowledge that can be used to help students like you to learn math better.

Your time commitment for this research will be small. I will observe and videotape your teacher teaching 10 lessons in your mathematics class, with the camera placed in the back of the room focusing mostly on the entire room. It is possible that you might be on the videotape, if you are willing. Each lesson, which includes observation and videotaping, should last about one hour. Your teacher will video-record student activities or responses that demonstrate mathematical thinking that happens during the lesson. This is part of the one-hour lesson. It is possible that your participation and comments might be videotaped, if you are willing. Your class work and/or homework might be collected from the observed lessons. I will make copies of it so I can study it later, and return the original work to your teacher.

Everyone in the class will be taught the same lessons, do the same activities, and do the same assessments. Your participation in this study is completely voluntary. You may choose not to be in the study or to stop being in the study before it is over at any time. If you choose not to be in the study, it will not affect your mathematics grade in any way. Your participation and any data collected will be kept confidential. Fake names for participants, schools, and the school district will be used in publications or presentations. Additionally, other information that might identify you will be removed or changed. If you are not in the study, then I will not take observation notes about you or collect copies of your work.

There is an assent form that you will have to sign if you would like to be a part of the study. There is also a parent permission form for your parent or guardian. Both your parent and you need to say “YES” for you to be in the study, but even if your parent says you may, you can still choose not to be in the study, or to quit being in the study at any time. Additional information about the study is provided on the parent permission form and on your assent form. There are two copies of both of these forms—one for your parent and you to sign and return, and one copy of each for your family to keep for your records.

Please take them home and discuss participation with your parents, and then bring the signed copies back to school using the sealable envelope provided, as soon as possible, whether or not you want to be in the study. I do hope that many of you will be interested in being in my study.

I will return on __dd/mm__ to collect the parent permission and assent forms from your teacher, whether you say yes or no. I am happy to answer any questions you may have. Thank you very much!
APPENDIX D: STUDENT ASSENT FORM

University of North Carolina at Chapel Hill

Assent to Participate in a Research Study
Minor Subjects (7-14 yrs)

Consent Form Version Date: November 25, 2011
IRB Study # 102686
Title of Study: Teachers In-the-Moment Noticing of Students Mathematical Thinking: A Case Study

Person in charge of study: Yanjun Liu
Where they work at UNC-Chapel Hill: School of Education
Other people working on this study: Dr. Susan N. Friel, sfriel@email.unc.edu; 919-962-6605 The people named above are doing a research study.

These are some things we want you to know about research studies:
Your parent needs to give permission for you to be in this study. You do not have to be in this study if you don’t want to, even if your parent has already given permission.
You may stop being in the study at any time. If you decide to stop, no one will be angry or upset with you.
Sometimes good things happen to people who take part in studies, and sometimes things happen that they may not like. We will tell you more about these things below.

Why are they doing this research study?
I am doing this study to learn more about how teachers notice student mathematical thinking during a lesson.
The reason for doing this research is to help teachers pay better attention to student mathematical thinking so students like you can learn mathematics better.

Why are you being asked to be in this research study?
You are being asked to be in this study because your teacher is allowing me to do research with one of the mathematics classes. You are a member of that class.

How many people will take part in this study?
If you decide to be in this study, you will be one of about 100 middle grades students in this research study. This includes students in your school and other schools.

What will happen during this study?
Please note that all students in this class, whether they participate in the study or not, will receive all the same lessons and do all the same activities and assessments.

If you are in the study, your participation in this study will involve:

Classroom Observation: I will observe and videotape your teacher teaching 10 lessons in your mathematics class, with the camera placed in the back of the room focusing mostly on the entire lesson. It is possible that you might be on the videotape, if you are willing. Each lesson, which includes observation and videotaping, should last about one hour.
Your teacher will video-record student activities or responses that demonstrate mathematical thinking that happens during the lesson. This is part of the one-hour lesson. It is possible that you might be videotaped, if you are willing.

Your class work and/or homework might be collected from the observed lesson. I will make copies of it so I can study it later, and return the original work to your teacher.

You will be asked at the end of this form whether you are willing to be recorded.

You can decide not to be videotaped, but still be in the study to be observed, and to have your classwork and/or homework analyzed by me. Students who do not want to be videotaped will sit in places outside of the camera's range.

A fake name will be created for you in the research report or presentations. All documents (e.g., your class work and homework) will be shredded after the information is sorted out. All videotapes, consent forms, and field notes will be password protected on my computer laptop or locked in a cabinet in my apartment until the information is sorted out, and then they will be destroyed.

This study will take place at your school and will last for two weeks, but all of this time is what would normally happen in your class.

Who will be told the things we learn about you in this study?
The researcher will be the only person who has access to your information in the study. Your teacher may choose to video-record you during the lessons, but will not have any access to other information about you if any.

We will not tell anyone what you tell us without your permission unless there is something that could be dangerous to you or someone else.

What are the good things that might happen?
People may have good things happen to them because they are in research studies. These are called "benefits." There is little chance you will benefit from being in this research study, but I expect that what I learn in this study will help teachers and students like you in the future.

What are the bad things that might happen?
Sometimes things happen to people in research studies that may make them feel bad. These are called “risks.” There are no known risks for your participation in this study. However, You should report any problems to the researcher.

Will you get any money or gifts for being in this research study?
You will not receive any money or gifts for being in this research study.

Who should you ask if you have any questions?
If you have questions, you or your parents should ask the people listed on the first page of this form. If you have other questions, complaints or concerns about your rights while you are in this research study, you may contact the Institutional Review Board at 919-966-3113 or by email to IRB_subjects@unc.edu.

Please indicate if you want to be in the study, and if you are willing to be recorded.

_____YES, I want to be in the study.

Check the line that best matches your choice:
_____ OK to video-record me during the study  
_____ Not OK to video-record me during the study  

Check the line that best matches your choice:  
_____ OK to collect copies of my work  
_____ Not OK to collect copies of my work  

OR  

_____ NO, thanks, I do not want to be in the study at all, so the researcher should NOT collect copies of my work or observe me.

If you sign your name below, it means that you agree to take part in this research study.

_____________________________  __________________________  
Sign your name here if you want to be in the study  
Date

_____________________________  
Print your name here if you want to be in the study

_____________________________  __________________________  
Signature of Research Team Member Obtaining Assent  
Date

_____________________________  
Printed Name of Research Team Member Obtaining Assent

Please return the signed copy of this form to your mathematics teacher, along with your parent's signed form using the sealable envelope provided, as soon as possible.

Keep the other copy of the form for your family's records.
APPENDIX E: PARENTAL PERMISSION FORM

University of North Carolina at Chapel Hill
Parental Permission for a Minor Child to Participate in a Research Study

Consent Form Version Date: November 25, 2011
IRB Study # 102686
Title of Study: Teachers In-the-Moment Noticing of Students Mathematical Thinking: A Case Study

Principal Investigator: Yanjun Liu
Principal Investigator Department: School of Education
Principal Investigator Phone number: 919-259-3146
Principal Investigator Email Address: yanjunl@email.unc.edu
Faculty Advisor: Dr. Susan N. Friel
Faculty Advisor Contact Information: sfriel@email.unc.edu; 919-962-6605

What are some general things you and your child should know about research studies? You are being asked to allow your child to take part in a research study. To join the study is voluntary.
You may refuse to give permission, or you may withdraw your permission for your child to be in the study, for any reason, without penalty. Even if you give your permission, your child can decide not to be in the study or to leave the study early.
Research studies are designed to obtain new knowledge. This new information may help people in the future. Your child may not receive any direct benefit from being in the research study.
There also may be risks to being in research studies.
Details about this study are discussed below. It is important that you and your child understand this information so that you and your child can make an informed choice about being in this research study.
You will be given a copy of this consent form. You and your child should ask the researchers named above, or staff members who may assist them, any questions you have about this study at any time.

What is the purpose of this study?
The purpose of this research study is to investigate how teachers’ noticing of students' mathematical thinking in the midst of teaching a lesson influences teachers' instruction, in the context of the use of a reform-based mathematics curriculum (i.e., the Connected Mathematics Project). In this study, I will videotape and observe your child's mathematics teacher teaching the Covering and Surrounding unit, and also ask the teacher to video-record his moments of noticing student thinking during instruction, using a portable video camera with selective-archiving capability.

The phrase teacher noticing describes an active process through which teachers attend to the ongoing, multidimensional information presented in the classroom. As it is not realistic for the teacher to attend to all information, attention becomes selective. Although a teacher may or may not always have the resources to deal successfully with what has been noticed, there is a better chance for meaningful actions to be taken and learning to take place in a classroom where signs
of potential problems or progress are noticed. The emphasis in current mathematics education reform calls for instruction that is responsive to student thinking, for example, student ideas and strategies. Therefore, a better understanding of how teachers are noticing student thinking in a reform-based classroom will contribute to improving mathematics teaching and learning.

Before entering the doctoral program at UNC Chapel Hill, I successfully taught middle grades and upper secondary grades mathematics for 3 years. My prior experience as a mathematics teacher, as well as my graduate work and supervisory experience at UNC Chapel Hill, helped me to realize the importance and possibility of examining teachers' noticing behavior during real-time instruction. I am confident that a better understanding of teachers' noticing of student thinking can help teachers be better prepared to respond to and build on student responses and ideas, in order to make appropriate adjustments during the course of the lesson(s). It is my aim to study how teachers are noticing student thinking, and how their noticing behavior may influence their instruction, through interviews, observations, videotaping, and follow-up discussions with the teachers.

Your child is being asked to be in this study because his/her teacher is allowing me to do research with one of his/her mathematics classes. Your child is a member of that class.

**How many people will take part in this study?**
If your child is in this study, your child will be one of approximately 100 middle grades students in this research study. This includes students at your child's school as well as other schools.

**How long will your child’s part in this study last?**
Observations and videotaping will take place in your child's mathematics class at your child's school, and will last approximately one hour each day for 10 days. All of this time will be spent on activities which would normally happen in your child's class. These are the activities that will occur in your child's math class:

- **Classroom Observation:** I will observe and videotape your child's teacher teaching 10 lessons in your child's mathematics class, with the camera placed in the back of the room focusing mostly on the entire lesson. It is possible that your child might appear in the videotape, if you and your child are willing. Each lesson, which includes observation and videotaping, should last approximately one hour each day, for 10 days.
- Your child's teacher will video-record student activities or responses that demonstrate mathematical thinking that happens during the lesson. This is part of the one-hour lesson time listed above. It is possible that your child's comments and participation might be videotaped, if you and your child are willing.
- Your child's class work and/or homework might be collected from each observed lesson. This will not take any additional time. I will make copies of the work, and return the originals to your child's mathematics teacher.

**What will happen if your child takes part in the study?**
If you give permission for your child to participate in this study, he or she will be observed in the mathematics classroom for ten class periods. During each lesson, I will take observational notes on the teacher's instruction and teacher-student interaction. The lesson will be video-taped using a standard video camera stationed in the back of the room, providing mostly distant shots of the entire lesson. It is possible that your child might be on the videotape, if you and your child are willing. If not, then the camera will be positioned and seating arranged so that your child will not be on camera.
Your child's teacher will also video-record student activities and responses that demonstrate mathematical thinking that happens during each lesson. It is possible that your child's comments and participation might be recorded, if you and your child are willing. If not, then the teacher will choose not to aim the camera at your child, or your child can request to be in groups that are not recorded. Student work from the observed lessons may also be collected to analyze; copies will be made, and the original work will be returned to the teacher.

You and your child can decide not to allow your child to be video-recorded, but still be in the study. Students who do not want to be recorded can still appear in the hand-written observational notes about the lessons, using only their participant codes, and have their work analyzed by me to help me understand how everyone is learning. If students are not in the study at all, then they will not appear in the notes and they will not have their work collected for analysis by the researcher.

**What are the possible benefits from being in this study?**
Research is designed to benefit society by gaining new knowledge. There is little chance your child will benefit from being in this research study; however, what I learn will help teachers and students your child’s age in the future.

**What are the possible risks or discomforts involved from being in this study?**
There are no known or anticipated risks for participation in this research study. There may be uncommon or previously unknown risks. You should report any problems to the researcher.

**What if we learn about new findings or information during the study?**
You and your child will be given any new information gained during the course of the study that might affect your willingness to continue your child’s participation in the study.

**How will your child’s privacy be protected?**
The only written documentation indicating the identities of the participants will be the parent permission and student assent forms and a pre-assigned participant number identification sheet; these will be kept in a locked cabinet in my dead-bolted apartment, and will only be identifiable to me. Care will be taken to ensure that all identifying information is removed upon document collection or during data transcription. The researcher will be the transcriber of all recordings; data will be transcribed from audio and video files into conventional written documents. If your child is mentioned in the transcription, your child will be referred to only by his or her pre-assigned participant number. All names of people or places stated in conversation will be replaced with pseudonyms or participant numbers during transcription. Prior to transcription, all notes, artifacts, documents, video-, and audio-recordings will be either password protected on a computer laptop and external hard drive (for digital video-recordings), or stored in a locked cabinet in my apartment until they are transcribed. Original observation notes and other written documentation will be shredded after transcription. Video recordings will be stored for possible analysis beyond what can be recorded through transcription (i.e., body language). Videos will not be used in future studies. Videos, after transcription, will be stored in a locked cabinet in my apartment until they are destroyed. You will be asked about your preferences for recording at the end of this form. Participants will not be identified in any report or publication about this study. Pseudonyms (fake names) for participants, their school, and the school district will be used in publications or presentations. Additionally, other possible identifiers will be removed or changed.
Although every effort will be made to keep research records private, there may be times when federal or state law requires the disclosure of such records, including personal information. This is very unlikely, but if disclosure is ever required, UNC-Chapel Hill will take steps allowable by law to protect the privacy of personal information. In some cases, your child’s information in this research study could be reviewed by representatives of the University, research sponsors, or government agencies (for example, the FDA) for purposes such as quality control or safety.

**What if you or your child wants to stop before your child’s part in the study is complete?**
You can withdraw your child from this study at any time, without penalty. The investigators also have the right to stop your child’s participation at any time. This could be because your child has had an unexpected reaction, or has failed to follow instructions, or because the entire study has been stopped.

**Will your child receive anything for being in this study?**
Neither you nor your child will receive anything for being in this study.

**Will it cost you anything for your child to be in this study?** It will not cost anything to be in this study.

**What if you or your child has questions about this study?**
You and your child have the right to ask, and have answered, any questions you may have about this research. If there are questions, complaints, concerns, or if a research-related injury occurs, contact the researchers listed on the first page of this form.

**What if there are questions about your child’s rights as a research participant?**
All research on human volunteers is reviewed by a committee that works to protect your child’s rights and welfare. If there are questions or concerns about your child’s rights as a research subject, or if you would like to obtain information or offer input, you may contact the Institutional Review Board at 919-966-3113 or by email to IRB_subjects@unc.edu.

Parent’s Agreement:
I have read the information provided above. I have asked all the questions I have at this time.

_____ YES, I voluntarily give permission to allow my child to participate in this research study.

Check the line that best matches your choice:

_____ OK to video-record my child during the study

_____ Not OK to video-record my child during the study

Check the line that best matches your choice:

_____ OK to collect work from my child during the study

_____ Not OK to collect work from my child during the study
OR

____ NO, thanks, I am not interested in my child being included in the study at all, so my child will not appear in observation notes with an ID code, and my child's work will not be included in the study.

______________________________
Printed Name of Research Participant (child)

______________________________
Signature of Parent Date

______________________________
Printed Name of Parent

______________________________
Signature of Research Team Member Obtaining Permission Date

______________________________
Printed Name of Research Team Member Obtaining Permission

Please return the signed copy of this form, whether you give permission or you do not, to your child's teacher using the sealable envelope provided, as soon as possible. If you do not return the form, your child will not participate in the study.

If you have decided NOT to give permission, then you do not need to include your child's own form. If you DO GIVE PERMISSION, your child may still say either "Yes" or "No" so your child's signed form should be returned too.

Keep the other copies of the forms for your records.
APPENDIX F: PROTOCOLS FOR INITIAL INTERVIEW

Below are the interview protocols used for Items 1, 2, and 3 in the teacher initial interview to assess teachers’ teaching knowledge of perimeter and area. Following the protocols are the individual items provided for teachers to react:

**Item 1:**
Suppose that while you are teaching your class about perimeter and area, one of your students come up to you. She says that she has discovered a new theory that you never told the class. She says that she has found that as the perimeter of a figure increases its area also increases. She shows you these pictures she has made, and says that they prove her theory (Show item 1 sheet to the teacher participant).  
*How would you respond to this student?*

**Item 2:**
Suppose that during your instruction on perimeter and area, a student raises his hand and is very excited. He says that he has figured out how to convert between units used to measure area. He shows you his solution to this area problem (Show item 2 sheet to the teacher participant). He says that the area of the triangle is 24 square yards. He also says that since there are 3 feet in a yard, he can convert the area of the triangle to square feet.  
*How would you respond to this student?*

**Item 3:**
Suppose that one day after school a fellow sixth-grade teacher stops by your room. She has a student’s assignment on perimeter with her and shows it to you (Show item 3 to the teacher participant). She says that she isn’t sure if the student’s method is valid, or what the student’s method is, even though he got right answers. She would like you to help her make sense of the student’s work.  
*How would you try and explain the student’s work to her?*  
Probe: *Can you generalize the student’s rule?*  
and/or  
*Can you show whether the student’s method is valid or invalid for finding the perimeter of rectangles?*
At the conclusion of the interview, ask: *How was your experience teaching this unit last year?*  
*and/or*  *Do you recall any similar situations?*
Item 1

Perimeter=14; Area=12

Perimeter=18; Area=20

Perimeter=12; Area=6

Perimeter=36; Area=54
Item 2

A = \frac{1}{2} \times B \times H = \frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 48 = 24 \text{ square yards}

3 \text{ feet} = 1 \text{ yard}

Area = 24 \times 3 = 72 \text{ square feet}
Item 3

Perimeter = 2 × (24/4 + 24/6) = 2 × (72/12 + 48/12) = 2 × 120/12 = 120/6 = 20

Perimeter = 2 × (18/9 + 18/2) = 2 × (36/18 + 162/18) = 2 × 198/18 = 198/9 = 22
APPENDIX G: PROTOCOLS FOR DAILY FOLLOW-UP DISCUSSION

**After Class, Before Discussion:**

1) Download clips onto the camera.

2) View clips if possible to get a sense of their content.

3) Set up the camera for the discussion – Make sure I can see their profile and computer.

**Protocols:**

“First I want to talk just about how it was to use the camera today. Then we’re going to talk about the clips individually.”

*Before Viewing the Clips – approx 5 minutes only. This part can be partially skipped after the teacher has used the camera a few times.*

1) First I want to ask about wearing the camera today. How did it feel to have the camera on in class today?

   a. Were you constantly aware that you were wearing the camera? Was the camera in your way? Was it bothering you that it was on a hat/visor or on your belt today?

   b. Was it hard to tell if you captured a clip?

   c. Was pushing the button difficult?

2) How did wearing the camera affect your teaching?

   a. Did your students behave differently?

   b. Did wearing it change the way you taught at all?

   c. Did trying to capture a moment get in the way of your teaching?

3) Was the 30-second timing an issue? How did you decide to push the button in the moment of teaching?

4) How many clips do you think you captured today?
“Now we’re going to talk about the times you pressed the button. For each episode I’m hoping you can tell me why you thought it was an interesting or important moment while you were teaching. It’s okay if you don’t remember or aren’t sure. Just let me know if that’s the case.”

For Each Clip

1) Show a good early still image of the clip. -- “This is the Xth clip you captured. Do you remember why you pushed the button? What did you notice about student thinking?”

2) If no – “Let’s watch the clip and when you remember why you pressed the button, you can stop the clip and we’ll talk about it.”

3) When they stop it or at the end of the clip – “Okay. When you were teaching earlier today, why did you press the button? What did you notice about student thinking?”

4) If they don’t watch the entire clip (and if I need it to get the context of what’s happening) – “Now I want to just watch the whole clip so you can give me a sense for what’s going on in the classroom at this time.”

5) Can you talk about what happened prior to and/or beyond the 30-second clip?

6) How did your awareness/ noticing of this moment influence your decision making while teaching? Do you think such awareness/ noticing matters in terms of instruction? How?

After All the Clips: This part can be partially skipped after the teacher has used the camera a few times.

1) Overall, did you capture what you intended to capture?
   a. Did each individual clip get the action that you intended to capture?
   b. Did all of the clips together overall reflect what you thought you had found interesting or important in terms of student thinking during class?

2) Overall, what criteria were you using when deciding what to capture? What does it mean to notice student thinking in terms of teaching?

3) What might you want to do differently with this camera?
APPENDIX H: INNOVATION CONFIGURATION MAPS

Innovation Configuration Map: Textbook Use

1. Textbook Use During Instruction
   A. Problems from the textbook serve as the basis for classroom instruction. There is no (or very minimal) supplementation or omission of material from the textbook. Some problem contexts may be altered by the teacher to suit local circumstances, but the mathematical content (i.e., learning goals) of the altered problems remains the same. The textbook is followed from one page to the next in order, and students work on the problems in the same order as they are presented in the textbook.

   B. Problems from the textbook serve as the basis for classroom instruction. There is some minor supplementation or omission of material from the textbook. Some problem contexts may be altered by the teacher to suit local circumstances, but the mathematical content (i.e., learning goals) of the altered problems remains the same. The textbook is followed from one page to the next in order, and students work on the problems in the same order (or with slight variation) as they are presented in the textbook.

   C. Some problems from the textbook are used for classroom instruction. However, there is considerable supplementation from other sources and/or omission of material from the textbook. Up to 50% of instructional time is used for mathematical learning goals different than the ones outlined for the lesson.

   D. The textbook is used minimally, accounting for less than 50% of classroom instructional time.

2. Textbook Use for Homework
   A. Problems from the textbook are assigned for homework. While problem contexts may be altered by the teacher to suit local circumstances, the assigned problems have content that is consistent with the learning goals of the lesson and/or is a review of previously-learned content.

   B. The majority of problems assigned for homework are from the textbook. On problems from the textbook, the contexts may be altered by the teacher to suit local circumstances, the assigned problems have content that is consistent with the learning goals of the lesson and/or is a review of previously-learned content.

   C. Some problems from the textbook are assigned for homework. Some problems may have contexts that are altered by the teacher to suit local circumstances. Some problems may come from other sources. Overall, problems that account for up to 69% of students' homework address learning goal(s) different than the ones outlined for the lesson, or address learning goals that are beyond what students have previously learned (i.e., those are problems for which students are unlikely to be prepared, for instance, problems intended to be done in class during the Explore).

   D. One of the following situations exists:
      - Few, if any problems from the textbook are assigned for homework. Overall, problems that account for more than 50% of students' homework address learning goals different than the ones outlined for the lesson, or address learning goals that are beyond what students have previously learned (i.e., these are problems for which students are unlikely to be prepared).
      - The teacher assigns no homework.
### Innovation Configuration Map: Standards-Based Instructional Practice

#### 2. Student Engagement

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Widespread engagement: Most students, most of the time, are on task pursuing the substance of the lesson. Most students seem to take the work seriously.</td>
<td>sporadic or episodic engagement: Most students are engaged in the lesson, but this engagement might be inconsistent or dependent on frequent teacher prodding.</td>
<td>Passive disengagement: Students appear lethargic and are only occasionally on task. For substantial portions of time many students are either clearly off task or only nominally on task.</td>
<td>Disruptive disengagement: Many students are frequently off task as evidenced by gross inattention or serious disruptions.</td>
</tr>
</tbody>
</table>

#### 4. Teachable Moments

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The teacher consistently capitalizes on &quot;teachable moments&quot; with students and frequently uses student errors to enhance understanding.</td>
<td>The teacher sometimes uses &quot;teachable moments&quot; and/or student errors to enhance understanding.</td>
<td>Many &quot;teachable moments&quot; are unrealized or not exploited. The teacher makes little use of student errors to enhance understanding.</td>
<td>There is no evidence of the teacher using &quot;teachable moments&quot; or student errors to enhance understanding.</td>
</tr>
</tbody>
</table>

#### 5. Student-Student Discourse

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Most students engage in substantive conversations with peers. Interaction is reciprocal - students listen carefully to others' ideas in order to understand them, build conversations on them, or extend ideas to a new level. Students discuss alternative strategies, question how peers arrive at solutions, and explain their thinking.</td>
<td>Student conversations only occasionally encourage further discussion or ideas. At times these discussions focus on reporting facts or procedures for solving problems rather than elaborating their thinking and solution path.</td>
<td>Student exchanges between peers reflect little or no substantive conversation of mathematical ideas. Much student discourse involves asking for clarification of directions given by the teacher.</td>
<td>There is very little (or perhaps no) student-student discourse of mathematical ideas.</td>
</tr>
</tbody>
</table>

#### 6. Teacher-Student Discourse

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The teacher seems to value and encourage active participation of students throughout the lesson. The teacher asks students for explanations and responses; responds to students' answers by asking other students questions (e.g., &quot;Does everyone agree with this?&quot; or &quot;Would anyone like to comment on this response?&quot;); focuses on supporting students' responsibility for listening and participating (e.g., by not repeating what a student says, by giving directions or asking questions only once, by not interrupting students' explanations).</td>
<td>The teacher seems to value and encourage active participation of students. The teacher invites students to explain their thinking but at times tunnels their discourse toward correct answers rather than their reasoning processes. In whole-class discussions students are seldom invited to react to other students' responses (discourse is dominated by T-St.T-St... vs T-St.St-T-St-St-St-St...). To a limited extent the teacher focuses on supporting students' responsibility for listening and participating.</td>
<td>The teacher asks students questions but seems to be primarily interested in correct answers. The majority of the teacher's remarks about students' responses are neutral short comments such as: &quot;Okay,&quot; &quot;All right,&quot; or &quot;Fine.&quot; Little or no attempt is made to use students' responses to further discussion.</td>
<td>Throughout the lesson the teacher asks students few or no questions.</td>
</tr>
</tbody>
</table>
## Innovation Configuration Map: Connected Mathematics Launch

### 7. Duration

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Launch is of relatively short duration (e.g., 5-10 min.).</td>
<td>The Launch is 10-15 min. in duration.</td>
<td>The Launch is &gt; 15 min. in duration.</td>
<td>There is no Launch.</td>
</tr>
</tbody>
</table>

### 8. Teacher Activity

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher introduces the mathematical problem and leaves the mathematical challenge intact by not cutting off the rich variety of strategies that may evolve.</td>
<td>The teacher introduces the mathematical problem but takes away from students some of the challenge of solving the problem.</td>
<td>The teacher introduces the mathematical problem but then takes most of the challenge of solving the problem away from students. The teacher does one or more of the following:  - Models how to solve the problem  - Questions away the problem (asks so many questions that students are not left with any mathematical challenge)  - Provides insufficient information for students to tackle the problem  - Essentially teaches the lesson in the Launch.</td>
<td>Either there is no Launch, or if there is one, the teacher reads aloud from the text and there is no discussion before students begin working on the problems.</td>
</tr>
</tbody>
</table>

### 9. Student Activity

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most students appear to be actively engaged in the discussion. In the discussion their thinking and ideas are solicited (as appropriate).</td>
<td>Students seem to be actively engaged to a limited extent. At times they appear focused on the discussion and other times they appear to be passive listeners.</td>
<td>Most students appear to be disengaged from the discussion.</td>
<td>Students read aloud (or silently) from the text and then begin working on the problems.</td>
</tr>
</tbody>
</table>

### 10. Coherence

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher makes sure students are prepared for the mathematical investigation by activating necessary prior knowledge, and making explicit the connections between this lesson and their prior mathematical experiences (as appropriate).</td>
<td>The teacher pays limited attention to students' prior mathematical learning needed to solve the problem. Previous mathematical ideas are revisited and connections to the new content are weak.</td>
<td>The teacher pays little attention to students' prior mathematical learning needed to solve the problem.</td>
<td>The teacher makes no attempt to activate students' prior knowledge. New content is isolated from students' prior mathematical experiences.</td>
</tr>
</tbody>
</table>
### Innovation Configuration Map: Connected Mathematics Explore

#### 11. Group Work

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>While the teacher may give students some alone “think time,” the Explore phase is predominantly pairs/small group work, with students being involved with their group members solving problems and making sure that others are caught up and understand a problem before moving on to the next. The teacher visits every group. If time runs out during class, some part(s) of the problems may be assigned for homework.</td>
</tr>
<tr>
<td>B</td>
<td>Students are given considerable time to work on problems in their groups. The teacher might frequently interrupt students’ work, perhaps to clarify directions, give hints, or to conduct &quot;mini-summarizes&quot; after each part of a problem is solved (staccato instruction). It may appear that the teacher lacks trust that students will stay on task working in groups. The teacher may or may not visit every group.</td>
</tr>
</tbody>
</table>
| C      | Students are given some time to work on problems with peers but one or more of the following occurs:  
  - For an extended period of time students work individually; their work with peers seems non-substantive (they simply compare answers or clarify directions)  
  - Students are expected to do many of the problems for HW  
  - The teacher may or may not visit every group |
| D      | Students do not work in groups on the problems. The teacher does one of the following:  
  - Uses direct instruction  
  - Conducts a whole-class discussion  
  - Expects students to work on the problems individually (in class or at home) |

#### 12. Teacher Activity

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
</table>
| A      | The teacher helps students stay on task, become responsible for their own learning, and scaffolds students’ thinking in one or more of the following ways:  
  - Asking questions about anticipated difficulties or to probe understanding  
  - Redirecting students’ thinking (without giving away the solution)  
  - Helping students abstract, generalize, and make explicit the mathematical ideas  
  - Extending students’ thinking |
| B      | The teacher helps students stay on task and becomes responsible for their own learning to a limited extent (e.g., the teacher may answer individual students’ questions without asking them to check first with their partner). At times the teacher asks students open-ended questions but frequently guides/leads students to the correct answers by de-problematizing the tasks (e.g., taking the mathematical challenge away from students). |
| C      | The teacher does one or more of the following:  
  - Focuses students’ attention on getting correct answers (perhaps by calculating using routines/procedures) rather than focusing on solution strategies or the reasonableness of their answers  
  - Does the mathematical thinking for the students  
  - Asks students low-level questions, but instead tells students what to do or tells them the answers |
| D      | The teacher does not interact with students when solving the problems. |

#### 13. Student Activity

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Students do the mathematical thinking. They look for patterns, make generalizations, choose appropriate tools/strategies, challenge questions of each other, investigate the validity of their ideas, and record problem solutions.</td>
</tr>
<tr>
<td>B</td>
<td>At times the students seem to do the mathematical thinking, and at other times it appears they rely on the teacher to provide considerable scaffolding.</td>
</tr>
</tbody>
</table>
| C      | Students grapple with the problems to a limited extent as evidenced by one or more of the following:  
  - Students direct their questions to the teacher rather than to peers  
  - Students look to the teacher for validity of their strategies and/or solutions |
| D      | One of the following occurs:  
  - Students do not work on problems in class  
  - Students appear to use strategies modeled by the teacher and tools selected by the teacher. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong> Discourse centers on the mathematical learning goal(s) of the lesson.</td>
<td><strong>B</strong> The teacher leads a whole-class discussion by asking students for their answers/explanations, but there are limited opportunities for them to consider alternative strategies. The following may appear during the discussion:</td>
<td><strong>A</strong> Students ask questions and challenge others mathematically by creating the correctness and similarity of strategies/answers.</td>
<td><strong>A</strong> The teacher provides lesson closure in one or more of the following ways:</td>
</tr>
<tr>
<td><strong>B</strong> Discourse centers on goal(s) that are related to the learning goal(s) but is service to, or tangential to, the learning goals.</td>
<td><strong>C</strong> The teacher summarizes the mathematics in the lesson by telling students the answers and/or showing how the problems should have been solved.</td>
<td><strong>B</strong> Students participate in the discussion to a limited extent. The focus is more on making sense of mathematics rather than giving (receiving) answers and understanding strategies, rather than their reflecting on their thinking.</td>
<td><strong>B</strong> The teacher provides only limited lesson closure. For instance:</td>
</tr>
<tr>
<td><strong>C</strong> There is no Summary.</td>
<td><strong>D</strong> The teacher does not conduct a Summary.</td>
<td><strong>C</strong> Students participate in the discussion only to a limited extent. The focus is more on their giving (receiving) answers and understanding strategies, rather than their reflecting on their thinking.</td>
<td><strong>C</strong> There are two learning goals but the teacher only focuses on one</td>
</tr>
<tr>
<td><strong>D</strong> Discourse centers on goal(s) tangential to or unrelated to the mathematical learning goal(s). The focus might be on problem context.</td>
<td></td>
<td><strong>D</strong> Students do not participate in a Summary.</td>
<td><strong>D</strong> There is superficial treatment (or discussion) of the learning goals.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>D</strong> The teacher re-states the goals of the lesson without active engagement of the students</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>D</strong> The teacher closes the lesson with a discussion that focuses on problem context rather than mathematics.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>D</strong> The teacher assigns homework or moves on to the next lesson with no closure of the current lesson.</td>
</tr>
</tbody>
</table>
APPENDIX I: CODING SCHEME

Coding occurred using the approach of emergent coding. Stebler (2001) states that “with emergent coding, categories are established following some preliminary examination of the data.” Therefore, codes were created exclusively based on analysis after data had been collected. The strategy for coding was quite straightforward and encompassed four stages: initial review, second round review, reliability check and revision if necessary, and consolidation. The first step was simply to review all data and come up with a set of codes to form a checklist according to which each episode could be coded. In the second step, the researcher attempted to approach the data with a blank slate, assigning codes for a second time without referring to the original checklist for each episode. Third, for inter rater reliability purpose, the researcher compared two sets of codes for each episode, and reconciled any differences showing up in the codes. Last, the coding scheme was reviewed to meet Marshall and Rossman’s (2006) recommendation that “the categories should be internally consistent but distinct from one another” (p. 159). Codes that are slightly distinct are counted as different codes, in order to capture the variety of things teacher noticed, and the different reasons for noticing them. For example, student conjecture and student misconjecture were counted as two different codes. A few codes were consolidated. For example, student work ethic and student engagement were not distinctive codes, so the episodes coded as student work ethic were re-coded as student engagement. As well, vocabulary and specific content were not distinctive codes, so the episodes coded as vocabulary were re-coded as specific content.

The following table provides a list of the codes that emerged during data analysis. Once these codes had been determined and edited, the researcher sought emergent themes for these codes. The 40 codes were grouped into 5 broad themes. Four of these themes are more related to
teaching and learning the mathematics content: student thinking, instructional adaptations, assessment, and content. The other theme concerns student characteristics, and it encompasses noticing activities that are not specific to the mathematics instruction. These codes were applied to 167 teacher-generated episodes and 11 transcriptions of related follow-up interviews. Other sources of data, such as, initial interviews and daily classroom observations, were frequently referenced in order to better understand teachers’ noticing activities within the instructional contexts, the day-to-day “flow” of teaching, and the relationship among individual noticing episodes, whole-class instruction, and follow-up interviews. The 40 emergent codes have been grouped in the following table according to these 5 emergent themes. Descriptions of the codes are provided, and examples are included when applicable.
### Emergent Coding Scheme

<table>
<thead>
<tr>
<th>Broad Theme</th>
<th>Emergent Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Misconception</td>
<td>Student Misconception</td>
<td>Self-developed or school-made concepts held by students that do not match up with correct understanding of mathematical concepts. Mr. Marshall captures a moment of a student counting squares instead of line segments for perimeter, and explains, “She was telling me she was counting the squares.”</td>
</tr>
<tr>
<td>Student Misconjecture</td>
<td>Student Misconjecture</td>
<td>Wrong conjectures or guesses. Ms. Goldberg captures a moment of a student conjecturing about a pattern of number pairs that make up the sides of all rectangles with a perimeter of 26 and an area of 36, “…he was talking about taking the factors of the perimeter…(when) it doesn’t make any sense to take the factor pairs of the perimeter.”</td>
</tr>
<tr>
<td>Student Error</td>
<td>Student Error</td>
<td>Mistakes as the result of students’ partial understanding. Ms. Goldberg describes a moment when Owen counts the number of dots instead of line segments between dots while measuring the height of a triangle on grid paper (see note 1), “But when I put my ruler up there, it is not five centimeters long, it is only four centimeters long.”</td>
</tr>
<tr>
<td>Student Confusion</td>
<td>Student Confusion</td>
<td>The state of being confused while engaging with the material. Ms. Goldberg captures a moment when a student becomes confused about finding the area of a rectangle, “So I am trying to show (the student) if you ever get confused, you can go back to how you developed the formula in class…Yeah, she was confused on how to get back to the area formula.”</td>
</tr>
<tr>
<td>Student Conjecture</td>
<td>Student Conjecture</td>
<td>Guesses or hypotheses that are conscious and uncertain. Ms. Goldberg describes a moment of a student conjecturing about finding the perimeter of a rectangle, “…he was basically saying if I could figure out what the length and width is, I can just double it.”</td>
</tr>
<tr>
<td>Student Strategy</td>
<td>Student Strategy</td>
<td>A plan or method for achieving a specific mathematics goal. Mr. Marshall captures a moment and explains, “I pushed the button because when I walked over there, they had a clever way showing the perimeter…”</td>
</tr>
<tr>
<td>Student Thinking Through a Problem</td>
<td>Student Thinking Through a Problem</td>
<td>Students working through what they think they should do to solve a problem. Ms. Goldberg describes a moment of a group of students working through a problem situation, “I was listening to what they are talking about… I think at that point, one of the kids says, ‘Maybe you can divide it by three….’ (Another kid says), ‘I am wondering if these two pieces (put together) are the same as (this) triangle…”</td>
</tr>
<tr>
<td>Student Thinking</td>
<td>Instructional Adaptations</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>Student Justification of Solution</strong></td>
<td><strong>Teacher Focusing</strong></td>
<td></td>
</tr>
<tr>
<td>The act of justifying or proving a solution to a problem. Ms. Goldberg recalls a moment when she notices a student’s correct response, and says to the student, “I am not sure (about) what I was seeing when you were talking about this other one. Can you explain it to me?”</td>
<td>A focusing-interaction questioning pattern that requires the teacher to listen to students’ responses and guide them based on what the students are thinking rather than how the teacher would solve the problem. Mr. Marshall reflects on his questioning skill while noticing how Gratia is counting the square units instead of linear units for perimeter of an irregular shape (see note 3), and says, “So I have to figure out a way, to, I think as a teacher, we have to figure out ways to ask questions (to find out about student understandings). First to see, what were they doing? How were they doing it? Whatever our task was, how does that fit in with what we were trying to do?”</td>
<td></td>
</tr>
<tr>
<td><strong>Student Difficulty Solving a Problem</strong></td>
<td><strong>Teacher Making Sense of Student Thinking</strong></td>
<td></td>
</tr>
<tr>
<td>Students struggling with a problem situation. Ms. Goldberg captures a moment of a student working on finding the area of a non-right triangle (see note 2), and says, “I can still tell Madeline was really struggling with the whole concept of surrounding (the triangle) with the rectangle.”</td>
<td>Teachers’ efforts to understand student thinking. Mr. Marshall captures a moment of attempting to understand student strategy in finding the perimeter formulas for rectangles, and explains, “I notice the answer is correct...but I (also) want to know the reason (or strategy) they did to get those answers. I wasn’t sure what they meant (after hearing them talk about their strategy). So I said, ‘Tell me what you meant with the numbers you had there.’ ”</td>
<td></td>
</tr>
<tr>
<td><strong>Insightful Mathematical Question</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students’ questions exhibiting insight. Mr. Marshall captures a student-student conversation on whether a square is also considered a rectangle, and explains, “Because the conversation was, ‘If we can draw a square, are we going to count it (as a rectangle when the requirement says “to build a rectangular pen for a dog”)?’ And it is always a good, always a great argument...”</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Absence of Certain Ideas</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher’s noticing of the absence of certain ideas needed at certain points in the lesson. Ms. Goldberg recalls a moment when a student is working on all the number pairs that make up the sides of rectangles with a constant perimeter of 24, and says, “She was just, she was not using half of the perimeter strategy... she had all the combinations, but there weren’t in any kind of order (i.e., the number pairs were not organized from least to greatest, or vice versa). I can tell they were all random.”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instructional Adaptations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>----------------------------------</td>
<td></td>
</tr>
<tr>
<td>Teacher Clarification</td>
<td>Teachers’ interpretation that removes obstacles to understanding. In reflecting on a captured moment of a student who is confused about the perimeter formula for rectangles, Ms. Goldberg remarks, “...the perimeter formula just...I almost think sometimes, they hinder the kids from getting (the right answer). So what is it? It is the distance around an object. Ok. Let’s just go back to it, not worrying about the formula. Just how can we do it?”</td>
<td></td>
</tr>
<tr>
<td>Teacher Funneling</td>
<td>Funneling occurs when the teacher asks a series of questions that guide the students through a procedure or to a desired end that the teacher has in mind. Ms. Goldberg reflects on a moment when she is found directing the student to a desired strategy, “I was trying to get her to say, ‘Now they added to 12.’”</td>
<td></td>
</tr>
<tr>
<td>Teacher Re-Directing</td>
<td>Teachers’ efforts to direct students to alternative pathways or strategies. Ms. Goldberg captures a moment of students not understanding the problem, and explains, “So I really wanted to point out to them what the question asked. (It asked) what pattern you see in the graph (not what kind of shape the graph is like).”</td>
<td></td>
</tr>
<tr>
<td>Teacher Guiding</td>
<td>The role of teachers in providing support and guidance to assist students in getting to the desired solution path. Mr. Marshall recalls an episode of a group of high performing students not making meaningful connections when exploring the area formula for triangles, and says, “It took a lot. I had to ask a lot of questions, to get them to get that. I had to ask a lot of questions. That was interesting.”</td>
<td></td>
</tr>
<tr>
<td>Teacher Facilitation</td>
<td>The act of assisting or making easier the progress of student activity (e.g., small group work). Ms. Goldberg comments on an episode of orchestrating a group conversation, “...it is my job. My job is to be the facilitator of the group. It is not my job to say this is how you do it. But it is my job to facilitate.”</td>
<td></td>
</tr>
<tr>
<td>Teacher Prompting</td>
<td>The strategy of assisting learning with a reminder or a cue. Mr. Marshall captures a moment of a student struggling with units of linear measurement, and explains, “I don’t even want to say...meters or square meters. I don’t want to even say that. I want him to think about it, so I said dogs, or cats. Was it 24 dogs? (I was) just trying to say something silly...so he will at least think about it.”</td>
<td></td>
</tr>
<tr>
<td>Teacher Remediation</td>
<td>The act of correcting a fault or deficiency in students’ responses or strategies. Ms. Goldberg captures a moment of helping a student who makes a mistake about differentiating between perimeter and area, and explains, “As a math department, we are trying to come up with strategies to deal with kids, that, you know, make this kind of mistakes.”</td>
<td></td>
</tr>
<tr>
<td>Instructional Adaptations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Decision Making</td>
<td>The process of making and implementing instructional decisions to increase the probability of learning. Mr. Marshall describes his decision making process while noticing a student error, “Because that was, what am I going to do? She doesn’t know what half plus half is. What am I going to do? ... And it was like, am I really going to do this? Yeah, I am... but it was like, that means I wasn’t going to (visit) the back table, who was still struggling with...”</td>
<td></td>
</tr>
<tr>
<td>Follow-Up Noticing</td>
<td>The act of following up as to reinforce or review previous noticing. Ms. Goldberg reflects on a moment of a student restating a previously discovered pattern, and explains, “I saw him talking about those factors again. That’s why I went over there, because I was like, I want to talk about this because yesterday you were saying...”</td>
<td></td>
</tr>
<tr>
<td>Purposeful Noticing</td>
<td>Intentional Noticing. Mr. Marshall reflects on the reason of capturing a moment, “We are trying to find some groups who were actually doing the task that was asked... They were right there just doing one part wrong...”</td>
<td></td>
</tr>
<tr>
<td>Sampling</td>
<td>The attempt to understand or assess progress of the larger group of students through monitoring the progress of a small number of students. Mr. Marshall captures a moment of a high-performing student struggling with a problem, and says, “Well she was still confused. When I saw it there, that was probably going on all around this room. So when I got that sense, I went around the room, I was like, yes, that’s kind of what’s going on.”</td>
<td></td>
</tr>
<tr>
<td>Extending Learning</td>
<td>The efforts to enrich student learning experience. Mr. Marshall captures a moment of challenging a high-performing student through extending a problem from the textbook, and remarks, “...if I wasn’t challenging Terri sometimes, Terri would be angry at me. If Terra is bored all the time, she will be angry.”</td>
<td></td>
</tr>
<tr>
<td>Aha Moment for Teacher</td>
<td>Teachers’ moments of inspiration or insight. Ms. Goldberg captures a moment of sudden inspiration while listening to a student-student conversation, and remarks, “I got an aha moment right there when I said that (to the two students). I was like OMG, I should have done that! I am going to do it from now on.”</td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>Activities that teachers undertake to gain an understanding of what students know (and don’t know) in order to make responsive changes in teaching and learning. Mr. Marshall captures a moment of observing a student analyzing a graph, and explains, “I wanted to see what they could read from the graph... that helps me, remember, how difficult it is, this was going to be, for children, to analyze the graph.”</td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>Monitoring Instructional and/or Student Progress</td>
<td>The teacher’s ongoing efforts to gather information about how a lesson is proceeding and/or how students are performing. Mr. Marshall reflects on moments of checking instructional and student progress during a lesson, and explains, “The ultimate goal for me is to get what is our formula for the area for triangle, and the perimeter for triangle... you saw me going to two or three groups when we were getting close to finish (and ask), ‘...can you guys get the perimeter of that, can you guys tell me how to get the area of that?’”</td>
</tr>
<tr>
<td>CMP Issue</td>
<td>Reference to potential issues of the Connected Mathematics Project. Ms. Goldberg captures a moment of students being confused about the requirement of coming up with different triangles with the same area, “I love the new book (i.e., Connected Mathematics Project) for the most part, but like anything else, there can be improvements to be made. I really think we need to talk about that, that question...like what is a different triangle...is it a triangle with the same dimensions that we just turn, or is a different triangle really a different triangle.”</td>
<td></td>
</tr>
<tr>
<td>CMP Feature</td>
<td>Reference to features of the Connected Mathematics Project. Ms. Goldberg captures a moment of helping a student visualize how long a centimeter is, and remarks, “I love that key on the bottom (of the CMP worksheet), because I think, I think visually when they see that, I think it really show them, because I used to put my fingers, then my fingers go away, when I go away. That key is there. It is not moving. And they can keep going back to it.”</td>
<td></td>
</tr>
<tr>
<td>Future Instructional Content</td>
<td>Reference to future math content for what students should be able to accomplish in middle school or high school. Mr. Marshall explains selection of a particular moment, “I wanted just to push that, that the units...If you are doing the labeling the units correctly, and get better and better out of it...Then it is going to be the foundation. It is going to allow you to go, way further, much more deep into this unit. And when we do volume and surface area, it is going to be... more units...”</td>
<td></td>
</tr>
<tr>
<td>Specific Content</td>
<td>Reference to specific math content that has been taught in previous units or is to be taught in the future. Mr. Marshall explains selection of a particular moment, “…I did want to capture that, because it is about orientation, and it is important...That’s orientation. Does orientation matter... how things are orientated.”</td>
<td></td>
</tr>
<tr>
<td>Student Characteristics</td>
<td>Student Attributes</td>
<td>Specific attributes of a student or group of students. Ms. Goldberg commented on an episode, “Greg is one of those kids... but he got frustrated really easily. Like he gives up really easily. I have to really work out the frustration level with him.”</td>
</tr>
<tr>
<td>-------------------------</td>
<td>--------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Student Discourse</td>
<td>Mathematical conversations between students. Ms. Goldberg recalls the reason for capturing an episode, “I was listening to what they were talking about, and that’s why I was trying to ask them...”</td>
</tr>
<tr>
<td></td>
<td>Student Interaction</td>
<td>Interactions between students. Ms. Goldberg captures a group interaction, and remarks, “The problem is, Kelly and Greg were in the middle of building their own (rectangles), and they were kind of half (way through). Greg wasn’t looking at Alice’s at all, and Kelly was kind of half way looking at it. So I made Greg stop, and said, “Alice thinks she has something (that met the requirements of the problem), and I want you to look and see if (you agree) she has it.’”</td>
</tr>
<tr>
<td></td>
<td>Student Improvement</td>
<td>Student change for the better. Ms. Goldberg captures a moment of a student performing better, “…when I went over there, she shot that answer back to me like that. She has made leaps and strides from August to now, so I am pretty proud of her. It was very good, from where she was.”</td>
</tr>
<tr>
<td></td>
<td>WOW! Moments</td>
<td>Moments of unexpected events that are impressive or rewarding. Ms. Goldberg captures a moment of a student explaining a solution strategy for the constant perimeter problem, and explains, “I have never seen a child do that before. That’s why I thought it was pretty neat.”</td>
</tr>
<tr>
<td></td>
<td>Disciplining</td>
<td>Develop or correct students’ behavior by instruction and practice. Ms. Goldberg captures a moment of correcting a student’s behavior when the student is not actively participating in group discussion.</td>
</tr>
<tr>
<td></td>
<td>Surprise</td>
<td>Unexpected events or ideas. Mr. Marshall commented on an episode, “…they answered all those questions, (and) they had the table made correctly, but they didn’t know the big picture. They didn’t know to get the area of the triangle, you can do base times height, then divide it by 2...I was totally astonished.”</td>
</tr>
</tbody>
</table>
1. In this problem, students were required to construct four triangles that have a base of 6 centimeters and a height of 4 centimeters. Owen counted the number of dots instead of line segments between dots while measuring the height below, and thought the height was 5.

2. Madeline was struggling with the idea of surrounding triangle d with the smallest possible rectangle for finding the area of triangle d, as it is not a right triangle. The problems are provided below:

   Find the area of each triangle. Describe the strategies you used for finding the areas.

3. Gratia’s counting for perimeter of the shape below
APPENDIX J: DAILY CLIP CAPTURE

Table J.1: Mr. Marshall’s reasons reported as themes for daily clip capture

<table>
<thead>
<tr>
<th>Week</th>
<th>Day</th>
<th>Student Characteristics</th>
<th>Content</th>
<th>Assessment</th>
<th>Instructional Adaptations</th>
<th>Student Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Week 2</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>9</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table J.2: Ms. Goldberg’s reasons reported as themes for daily clip capture

<table>
<thead>
<tr>
<th>Week</th>
<th>Day</th>
<th>Student Characteristics</th>
<th>Content</th>
<th>Assessment</th>
<th>Instructional Adaptations</th>
<th>Student Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>Week 2</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>4</td>
<td>8</td>
<td>6</td>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>
APPENDIX K: CMP INVESTIGATION

Investigation

1

Designing Bumper Cars: Extending and Building on Area and Perimeter

Most people enjoy rides at amusement parks and carnivals such as merry go rounds, Ferris wheels, roller coasters, and bumper cars. A company called Midway Amusement Rides (MARS for short) builds rides for amusement parks and carnivals. To do well in their business, MARS designers have to use mathematical thinking.

1.1 Designing Bumper-Car Rides
Area and Perimeter

Bumper cars are a popular ride at amusement parks and carnivals. Bumper cars ride on a smooth floor with bumper rails surrounding it. MARS makes their bumper-car floors from 1 meter-by-1 meter square tiles. The bumper rails are built from 1 meter sections.

Common Core State Standards

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane . . .
6.EE.A.3 Apply the properties of operations to generate equivalent expressions.
6.EE.C.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
Also 6.EE.A.2, 6.EE.A.2a, 6.EE.A.2c, 6.EE.B.6, 6.G.A.1
Two measures tell you important facts about the size of the bumper-car floor plans. The number of tiles needed to cover the floor is a measure of area. The number of rail sections needed to surround the floor is a measure of perimeter. The bumper-car floor you just saw had the shape of a rectangle. Next, you will see bumper-car floors that are not rectangles.

Problem 1.1

When a customer places an order, the designers at MARS use square tiles to model possible floor plans. MARS receives the customer orders below. Experiment with square tiles and then sketch some designs on grid paper for the customer to consider.

1. Lone Star Carnivals in Texas wants a bumper-car ride that covers 36 square meters of floor space and has lots of rail sections. Sketch two or three possible floor plans.

2. Badger State Shows in Wisconsin requests a bumper-car ride with 36 square meters of floor space and 26 meters of rail sections. Sketch two or three floor plans for this request.

The designers at MARS created four designs for bumper-car rides.

1. Find the area and perimeter of each bumper-car floor plan. Record your data in a table such as the one shown. You will use the “Cost” column of the table in part (3).

2. Which of the designs can be made from the same number of floor tiles? Will those designs have the same number of rail sections? Explain.

3. The designers at MARS charge $25 for each rail section and $30 for each floor tile. For the designs with the same floor area, which design costs the most? Which design costs the least? Explain.

4. Rearrange the tiles in Design B to form a rectangle. Can you make more than one rectangle? If so, are the perimeters the same? Explain.

continued on the next page >
Problem 1.1 continued

Riverview School orders a bumper-car ride in the shape of a rectangle for their fundraising festival. The MARS company sends the school Designs I, II, and III.

1. What is the area of each design? Explain how you found the area.
2. What is the perimeter of each design? Explain how you found the perimeter.

3. The dimensions of a rectangle are called length \( \ell \) and width \( w \). Look for patterns throughout Problem 1.1 to help you answer the questions below.

   \( \text{w} \)

   \( \text{\ell} \)

   a. Use words to describe a formula for finding the perimeter of a rectangle. Write the formula using symbols. Explain why it works.
   b. Use words to describe a formula for finding the area of a rectangle. Write the formula using symbols. Explain why it works.
   c. Find the perimeter and area of a rectangle with a width of 6 centimeters and a length of 15 centimeters.

ACE Homework starts on page 14.
REFERENCES


Pajares, Frank M. (1992). Teachers’ beliefs and educational research: Cleaning up a messy


