Role of Environmental Legislations and Firm-Level Strategies on Product Take Back

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ABSTRACT

GÖKÇE ESENDURAN: Role of Environmental Legislations and Firm-Level Strategies on Product Take Back
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In the last two decades, an increasing number of companies provide take-back programs. It is therefore essential to understand the drivers and influencers of take-back strategies. In the second chapter, we examine an original equipment manufacturer (OEM) under take-back legislation that holds manufacturers financially responsible for collecting and treating products discarded by customers. We characterize the manufacturer's optimal collection and remanufacturing policies when she has in-house remanufacturing and when she faces competition from an independent remanufacturer. We show that legislation on collection levels never decreases remanufacturing levels if the OEM remanufactures; however, it might cause remanufacturing level to decrease if the third-party remanufactures. We also find that legislation creates incentives for designing environmentally-friendly products regardless of the existence of competition. Our research has also implications for policy makers. We find that in order to achieve higher remanufacturing levels policy makers might consider subsidizing third-party remanufacturers rather than imposing take-back legislations. While we consider take-back legislation that stipulates individual responsibility in the second chapter, some implementations of legislation allow manufacturers to fulfill their obligation either individually or by joining a collective scheme. In the third chapter, we explore a company's optimal strategy in complying with take-back legislations and compare the individual versus collective schemes in terms of cost effectiveness and environmental benefits they achieve. We show that which compliance scheme yields the lowest cost depends on the collection rate maintained by the government and the market shares of partner firms. Apart from legislations, there might

be a number of different economic and/or marketing concerns driving product take-back. In the fourth chapter, we consider a manufacturer with a dual distribution channel, i.e. rental and sales channel, and study the profitability of buyback program, a form of product take-back motivated by the goal of managing distribution channels better. We characterize how the profitability of the buyback program changes depending on the uncertainty in demand and the terms of buyback contract. We show that committing to the buyback price at the time of initial sales always leads to lower manufacturer profits; however, it enhances a buyback program's ability in resolving channel conflicts under uncertainty.

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CHAPTER 1

Introduction

1.1 Motivation

Traditionally, a producer's role in the supply chain ended with the sale of the product. However, today an increasing number of firms are making product take-back an important part of their business strategy. Product take-back, which is a form of extended producer responsibility, requires firms to organize strategies to take their used products back from customers. In this dissertation, we aim to develop theoretical insights to help firms effectively manage their take-back strategies.

One of the main drivers of product take-back is environmental legislation. Several countries now impose product take-back legislation with the objectives of both reducing amounts and the environmental impacts of waste as well as creating incentives for environmentally-friendly business decisions by shifting the cost of waste product management from society to producers. Although fundamental legislative requirements vary from one country to another, take-back legislations basically hold companies financially responsible for taking back their used products and treating them properly. Assessing the profitability and efficiency of operations/strategies launched in order to comply with environmental legislations is one of the main objectives of this dissertation. In particular, we intend to understand how different implementations of product take-back legislations affect the optimal strategies of companies. In addition, we provide insights for policy makers on how to design effective take-back legislations.

Some companies (e.g., Xerox, Motorola) launch product take-back programs even in the absence of legislations. Apart from legislations, there might be a number of different economic and/or marketing concerns driving product take-back. For instance, companies with dual distribution channels, i.e. rental and sales channels, set up channel structures that facilitate product buyback, which is a form of take-back, in order to manage their distribution channels better. One of the key goals of this dissertation is examining the profitability of the buyback channel structure in comparison to other possible channel structures.

The rest of this dissertation is organized as follows: In Chapter 2, we examine how product take-back legislations affect a firm's collection and remanufacturing decisions when she has in-house remanufacturing and when she faces competition from an independent remanufacturer. In Chapter 3, we study a firm's compliance decision and identify the situations where complying collectively is more cost efficient than complying individually. In Chapter 4, we consider a manufacturer with dual distribution channels and intend to shed light on how the buyback program should be designed to mitigate the conflicts between the rental and sales channel. We conclude and provide a brief overview of each chapter of the dissertation in Chapter 5.

1.2 Dissertation Overview

1.2.1 Chapter 2

As more countries are imposing increasingly stricter product take-back legislations that hold producers responsible for financing collection and proper treatment of their products (e.g. tires, batteries, electronic products such as computers, etc.) discarded by customers, product take-back for environmental reasons becomes even more important. Even though take-back legislations exist in many countries, the form of legislation as well as the specific targets that are set on collection/recycling levels might differ. It is clear that the legislation's requirements affect companies' responses, which in turn, determine the success of the legislation in terms of achieving two main objectives: reducing the amount of waste by increasing product take-back levels and creating incentives for environmentally-friendly product designs.

Some argue that existing legislations may in fact hinder remanufacturing, which is a more efficient way of capturing the value in discarded products than recycling, by granting original equipment manufacturers (OEM) first access to discarded products through collection/recycling targets and thus limiting the availability of those products to third-party remanufacturers. Hence it is not clear whether existing take-back legislations will be a driving force for remanufacturing or not (Gray and Charter, 2007). In the second chapter, we investigate how existing forms of take-back legislations with different collection/recycling targets affect remanufacturing decisions and if there is any additional benefit in imposing remanufacturing targets in terms of meeting the objectives of such legislations.

Using a stylized model of take-back legislation in a two-period setting, we study how existing forms of legislations as well as extended forms with additional remanufacturing targets affect optimal production, take-back and remanufacturing decisions. First, we study the decision of a monopolist OEM with in-house remanufacturing capabilities. We then consider an OEM with no in-house remanufacturing but facing competition from a third-party remanufacturer, and formulate a Cournot competition where the OEM is Stackelberg leader. As take-back legislations grant the OEM first access to cores (discarded products) we assume that first the OEM collects cores and then the independent remanufacturer collects among the remaining cores left in the market. We also assume that cores that are collected but not remanufactured must be disposed of properly or recycled at a cost. We solve the game by backward induction and characterize the optimal policies analytically.

Among others, our results show that legislation that imposes the same targets across a wide range of products might induce very different behavior for each of the different products it covers. While it induces no further increase in remanufacturing levels for products with high production costs, a positive level of remanufacturing might be achieved for products with low manufacturing costs. As for the concern regarding the effect of take-back legislations on remanufacturing levels, we prove that legislation never causes a decrease in remanufacturing levels under monopoly. In the presence of competition, we show that legislation may indeed cause a decrease in remanufacturing levels and hence the concerns raised by environmentalists are not groundless. But surprisingly, while comparing the effect of take-back legislation in a monopolistic versus competitive environment, we find that remanufacturing levels may be higher in the latter when faced with the same level of legislation. In addition, we provide examples where the remanufacturing level achieved by a third-party remanufacturer, who is in competition with an OEM and facing no take-back legislation, may be higher than that achieved by an OEM with in-house remanufacturing facing legislation. This suggests that, in order to achieve higher remanufacturing levels policy makers might indeed want to consider subsidizing third-party remanufacturers rather than imposing take-back legislations.

One of the main objectives of take-back legislations is to create incentives for designing environmentally-friendly products. When we analyze the ability of legislation to create incentives for designing products that are cheaper to remanufacture at the expense of increasing manufacturing cost, we find that such incentives exists not only for a monopolist OEM but also for an OEM facing competition from a third-party remanufacturer.

Finally, we study the impact of take-back legislation on consumers and find that, in general, legislation reduces the consumer surplus. Surprisingly, we also show that, except in rare cases, consumer surplus is larger when the OEM remanufactures than when a third-party does.

1.2.2 Chapter 3

In an industry regulated with product take-back legislations, each firm meets her obligations through a *compliance scheme*. Choosing a compliance scheme entails decisions regarding the choice of collection channel, treatment processes, and partner firms. Firms need to choose a compliance scheme that ensures environmentally-conscious management of discarded products in the most cost effective way. In most implementations of take-back legislation one or more of the following three compliance schemes are available/allowed: (i) Individual scheme, where the company collects and treats only discarded products of her brand-name and pays for the actual collection and treatment cost of her own products; (ii) Collective scheme with cost allocation by market share (MS), where a subset of companies collectively take care of collection and treatment of their discarded products without making a distinction between the brands and the total compliance cost is allocated among the companies with respect to their market shares; (iii) Collective scheme with individual financial responsibility (IFR), where a subset of companies collectively take care of collection and treatment of their discarded products but each company pays for the treatment and collection of her brand-name products.

Among the three compliance schemes, individual compliance is likely to be the most expensive choice because the firm pays hundred percent of the collection and treatment costs. Collective scheme with MS, on the other hand, is claimed to be less costly due to economies of scale in the collection and treatment activities. To this end, one of our main goals in the third chapter is to identify the key market and operating parameters that make one scheme preferable to the others from the firm's point of view.

We compare the three compliance schemes by using a stylized model of each compliance structure. We model the total compliance cost in detail and the two decision variables in our stylized models are the percentage of sold products collected and the product's treatability level where higher treatability level implies an environmentally-friendly product design and a lower treatment cost. When we compare the compliance schemes, we find that collective compliance with IFR is in general the most cost effective alternative. Although IFR within a collective compliance is clearly the best form of compliance, it is not easy to implement in practice because it requires sorting discarded products by brand and tracking them through the treatment process in order to record the true cost of treatment. For that reason, individual scheme and collective scheme with MS are the two prevalent forms of compliance that we encounter in practice. We find that which scheme (of these two) gives lower compliance cost depends on the base collection rate maintained by government and non-profits as well as the market shares of partner firms. Some implementations of take-back legislations mandates a minimum amount of collection as a percentage of the firm's previous period sales. When we explore the implications of collection targets on a firm's compliance scheme choice, we find that imposing high collection targets might alter firms' compliance scheme choice in favor of collective scheme.

Even though we analyze firms' responses to take-back legislation, our research has implications for policy makers as well. While it is easy to show that the collective scheme with IFR achieves superior environmental benefits in terms of higher collection rates and treatment levels, as mentioned previously it is hard to implement in practice. When we compare the other two compliance schemes, we find that high collection rates and treatability levels may be hard to achieve simultaneously unless governments are willing to partially incur the collection costs. Our research also suggests that before imposing stricter legislation, policy makers should be aware of the trade-offs involved. High collection targets imposed by policy makers push more firms to choose the collective compliance scheme with MS and result in a degradation of treatability levels.

1.2.3 Chapter 4

The fourth chapter deals with product take-back motivated by the goal of managing distribution channels better. Growing number of firms with dual distribution channels, i.e., rental and sales channel, set up buyback programs, a form of product take-back, to buy used rental products back from rental agencies and sell them through their sales channel. For example, through most of the 1980s U.S. auto manufacturers were managing two *separate* channels where rental agencies were permitted only to rent while dealers were permitted only to sell cars. After experiencing a drastic decrease in the sales, manufacturers started to experiment with different channel structures. First, they adopted an *overlapping* channel structure, under which rental agencies sell used rental cars in the sales market. However this structure triggered a competition between rental agencies and dealers. As dealers opposed the overlapping channel structure, some manufacturers launched an alternative channel structure, called *buyback*, and started buying back program cars, i.e., used rental cars, from rental agencies at a guaranteed price and selling them through dealers.

While a buyback channel evidently alleviates channel conflicts, its profitability depends on two characteristics of the buyback program: (i) timing of the announcement of buyback prices; and (ii) quality standards for the products repurchased by manufacturers. In this chapter we investigate how a buyback channel's profitability -for both the manufacturer and the intermediaries- as well as its ability in resolving channel conflicts change depending on the uncertainty in consumer demand and the two aforementioned characteristics of the buyback program. We develop a two-period model to analyze the relationship not only between the manufacturer and the intermediaries but also among the intermediaries. Under each channel structure, in each period the manufacturer decides on the wholesale prices and then the intermediaries simultaneously decide on the quantities. We study the problem under deterministic demand as a benchmark.

Among other results, we find that compared to setting the buyback price at the time of repurchase (which we call *no price commitment*), as in (Purohit, 1997), early commitment of buyback price at the time of initial sales (which we call *price commitment*) always reduces the manufacturer's profit under no demand uncertainty. That brings up the question of why buyback price commitment is still being implemented in practice by automobile manufacturers. Our main aim in this chapter is not characterizing the most profitable channel structure for the manufacturer but rationalizing manufacturer's decision in launching a buyback program with price commitment. To this end, we explore different effects that rationalize the choice of price commitment under buyback channel structure. First, our results suggest that price commitment enhances a buyback channel's ability in resolving channel conflicts in the presence of demand uncertainty. Moreover, when faced with demand uncertainty, buyback price commitment may lead to higher total sales for the manufacturer than no price commitment. In contrast, in the absence of uncertainty, buyback price commitment always leads to the smallest total sales among the possible channel structures. Therefore, with demand uncertainty, we identify sufficiently different effects that support the attractiveness of price commitment strategy for a buyback program. Finally, we characterize how the firm can use the quality standard as an operational lever to improve profits and show that her profit generally increases in the quality standard under demand uncertainty.

CHAPTER 2

The Impact of Take-Back Legislation on Remanufacturing

2.1 Introduction

An increasing number of countries are enacting take-back legislations that hold manufacturers responsible for collecting and properly disposing of their products when their useful lives end. The best-known such legislation is the Waste Electrical and Electronic Equipment (WEEE) Directive by the European Union. Similarly, Japan has the Specified Home Appliances Recycling Law and in the U.S. twenty states and New York City have passed legislation imposing take-back and recycling of e-waste (ETC, 2010). The requirements as well as the implementations of take-back legislations can be quite different, but a common characteristic of these legislations is the target levels on the amount of end-of-life products that should be collected/recycled (e.g. the WEEE Directive) where recycling is defined as the reprocessing of waste materials -reprocessed waste materials could be used in manufacturer's own production process or could be sold into recycling market-. Another treatment option for end-of-life products that are taken back is reuse, under which whole appliances or components are used for the same purpose for which they were conceived. In this chapter without losing generality we confine ourselves to the context of remanufacturing, a form of reuse which brings the whole appliance to "as new" condition. Hence, we analyze certain types of products for which remanufacturing is a viable option. Although not all used products can be remanufactured, remanufacturing -where viable- is usually perceived as being environmentally-friendlier than recycling because it extends product's life time and curtails its potential ecological impact (Rose and Stevels, 2001). However, some argue that existing legislations may in fact hinder remanufacturing by granting original equipment manufacturers (OEM) first access to discarded products through collection/recycling targets and thus limiting the availability of those products to third-party remanufacturers. Hence it is not clear whether existing take-back legislations will be a driving force for remanufacturing or not (Gray and Charter, 2007), and our main goal in this chapter is to clarify the impact of take-back legislations on companies' remanufacturing decisions.

Even in the absence of take-back legislation, companies have very different product take-back and treatment strategies. There are companies like Xerox and IBM who voluntarily take their products back and remanufacture them. They sell refurbished products under their brand-names and remanufacturing is a profitable part of their business. Other companies like Motorola take their products back even though they do not remanufacture in-house and not even sell refurbished products; instead Motorola sends collected phones to ReCellular Inc, a for-profit buyer and seller of cell phones. ReCellular pays for these phones based on their estimated value, refurbishes and sells them (Most, 2003). It is also a common policy to use refurbished items as warranty replacements or maintenance parts (e.g. Dell). And there are still others (e.g. JVC) who have no voluntary take back programs in the absence of legislation. These examples illustrate that, depending on product and market characteristics, companies follow different strategies when it comes to establishing take-back programs. In this chapter we aim to understand how take-back legislation impacts the way companies handle their end-of-life product management and identify the situations where legislation is not adequate by itself to promote remanufacturing. Motivated by the industry examples, we group producers into two categories with respect to their ability to remanufacture in-house. We first consider an OEM who remanufactures in-house (a company like Xerox) and characterize her optimal response to different levels of take-back legislation. Next we analyze the case where the OEM does not/cannot remanufacture and faces competition from a third party remanufacturer. With this second model we want to identify if take-back legislation makes the third-party remanufacturer more or less competitive vis-a-vis the OEM. Since take-back legislations grant the OEM first access to cores we assume that the OEM collects cores first and then the third party remanufacturer collects among the remaining cores left in the market. We also assume that cores that are collected but not remanufactured must be disposed of properly or recycled at a cost.

Another question we are interested in concerns policy makers. Most take-back legislations impose the same requirements across a group of products with different cost structures. For example, Minnesota specifies a single collection and recycling rate of 80% for all the products covered by law (Revisor of Statutes, 2007). We question if this approach achieves the same environmental benefits in terms of higher collection and remanufacturing levels for all products covered by legislation, and when the extra administrative burden and cost of imposing different levels of legislation for different types of products is justified. In addition, we investigate if there is any benefit of introducing remanufacturing targets.

Among other results, we find that the same level of legislation may induce very different behavior depending on the underlying cost structure of the product. For products with higher manufacturing costs and favorable remanufacturing environment, legislation may be redundant; while for products with low manufacturing costs, there is no remanufacturing even under legislation. These suggest that take-back legislation that imposes the same take-back levels across a wide range of products in an industry may be inefficient. How legislation affects remanufacturing levels depends also on whether legislation is being imposed in a market where OEMs remanufacture versus one where third parties remanufacture. We show that legislation never causes a decrease in remanufacturing levels in a market where OEMs remanufacture. However in a market where third parties remanufacture, we find that legislation may indeed cause a decrease in remanufacturing levels and hence the concern raised regarding the effect of legislation on remanufacturing levels is not groundless.

When we compare the effect of legislation on an OEM with in-house remanufacturing versus one that faces competition from a third party, we find some counterintuitive results. One might expect that optimal remanufacturing level achieved by an OEM who remanufactures in-house would be higher than that achieved by a third-party remanufacturer. In fact, we identify situations where this is not necessarily true. In addition, we provide examples where the remanufacturing level achieved by a third-party remanufacturer, who is in competition with an OEM and facing no take-back legislation, may be higher than that achieved by an OEM with in-house remanufacturing facing legislation. Therefore, for some products, simply the competition between an OEM and an independent remanufacturer may be more effective than take-back legislation. This suggests that, in order to achieve higher remanufacturing levels government might indeed want to consider subsidizing third party remanufacturers rather than imposing take-back legislations.

An underlying motivation for take-back legislation is to create incentives for environmentally friendly business decisions such as designing products that are easier and cheaper to recyle/remanufacture. We find that an OEM with in-house remanufacturing and, surprisingly, even an OEM who faces competition from a third-party remanufacturer have incentive to decrease remanufacturing cost at the expense of increasing manufacturing cost unless the latter is too low to begin with. Especially under take-back legislation, reducing the remanufacturing cost may benefit the OEM facing competition because then the remanufacturer will be willing to buy more cores, which effectively reduces the cost of the legislation for the OEM.

We also study the impact of take-back legislation on consumers and find that, in general, legislation reduces the consumer surplus. As legislation increases the effective cost for the producers, some of that cost is passed onto consumers and consumer surplus goes down. Surprisingly, we also find that, except in rare cases, consumer surplus is larger when the OEM remanufactures than when a third party does.

The rest of this chapter is organized as follows: In §2.2 we review the relevant literature, in §2.3 we define our assumptions and introduce our model. In §2.4 and §2.5 we respectively analyze the case where the OEM remanufactures in-house and the one where third-party does the remanufacturing and we compare our insights regarding these two cases in §2.7. We study the impact of take-back legislation on the consumer surplus in §2.8. In §2.8.1 we discuss our insights from a numerical study where we model life cycles longer than two periods and allow the potential demand for the product to change over its life cycle. We conclude in §2.9. All proofs are relegated to Appendix A1. We provide a separate appendix, Appendix B1.2, where we derive the optimal regions and their respective bounds for the two models we analyze.

2.2 Related Literature

This chapter connects two important streams of research in operations management literature, namely remanufacturing under competition and environmental legislations. The competition between an OEM and another OEM or a third-party remanufacturer is studied in a number of papers. Majumder and Groenevelt (2001) and Ferrer and Swaminathan (2006) study the competition between an OEM and an independent remanufacturer while Heese et al. (2005) investigate the competition between two OEMs. Ferrer and Swaminathan (2010) also model the competition between new and remanufactured products but the same firm sells both. Debo et al. (2005) model competition between an OEM and N independent remanufacturers. Ferguson and Toktay (2006) show that an OEM may deter the entry of an independent remanufacturer by either introducing a remanufactured product or collecting cores but not remanufacturing. Finally Groenevelt and Majumder (2001) model the competition between an OEM and a remanufacturer on the procurement of cores. Our research builds on the models developed in this literature and explores the effects of take-back legislation on an OEM in a competitive environment.

A number of recent papers model new environmental legislation, especially the WEEE (e.g. Zuidwijk and Krikke (2007)), and study their environmental impact and effects on supply chain strategies. Atasu et al. (2009a) explore the efficiency of the WEEE legislation and argue that industry-wide, weight-based WEEE legislation must be adjusted to meet the particular circumstances of each industry and sector. In this chapter, we do not specifically model weight-based legislation; rather we study minimum collection targets that are set in term's of percentages of the previous period's sales (motivated by take-back laws in Minnesota and NYC). Unlike Atasu et al. who consider a benefit model but do not explicitly model remanufacturing, we model that and the operational decisions of the OEM and the remanufacturer in greater detail.

While Webster and Mitra (2007) also aim to generate insights about take-back legislation, our modeling assumptions (among other differences we model quantity competition rather than price competition and the OEM is the Stackelberg leader in the competition with the remanufacturer), research questions, and the resulting insights are significantly different. In the absence of take-back legislation, Webster and Mitra solve the price competition problem between a remanufacturer and an OEM with no in-house remanufacturing capability and characterize the Nash equilibrium. Under legislation, they do not model collection/reuse targets explicitly and generate insights numerically by changing the values of cost parameters to mimic different types of legislation. By modeling the collection/reuse targets explicitly we are able to generate insights regarding the undesirable consequences (e.g. reduced remanufacturing levels) of ad hoc targets levels. Through their numerical experiments Webster and Mitra find that both the OEM and the remanufacturer may benefit from legislation (higher profits when compared to the no take-back scenario) if the government is responsible for collecting the returns. However this observation is made under the assumption that OEMs do not have access to cores in the absence of take-back legislation and overlooks the possibility that the OEM may preemptively collect cores to prevent the remanufacturer's entry (we indeed find that preemptive collection may be optimal). Another underlying assumption is that the OEM does not incur a cost on the cores bought by the remanufacturer if the government is doing the collection. However under many take-back legislations the OEM is responsible for the collection cost even when the collection is done by a government agency (e.g as in Connecticut) or the OEM pays a flat fee to finance such collection (e.g. as in Maryland).

Finally another important research question is how the competition between an OEM and a third-party remanufacturer affects the outcomes of legislation. To that end we separately model an OEM with in-house remanufacturing and compare our insights (in-house remanufacturing is not considered in Webster and Mitra). To our knowledge ours is the first research to make such a comparison. Hammond and Beullens (2007) model the competition between m manufacturers with in-house remanufacturing under take-back legislation. Through numerical examples they make observations on whether increasing landfill costs and collection targets induces recycling.

In many implementations of take-back legislation, the producer is the only supply chain party that incurs legislation-related costs and we study such an implementation; see Jacobs and Subramanian (2007) for an analysis where costs are split between a supplier and a producer. Plambeck and Wang (2010) and Plambeck and Taylor (2007) also examine the consequences of environmental legislation on supply-chain decisions, but their research questions are significantly different. Plambeck and Wang address new product introduction under e-waste laws while Plambeck and Taylor model RoHS, which regulates the use of hazardous substances in electronics products.

2.3 Modeling Framework

2.3.1 Legislative Scenarios

In this chapter we focus on the form of legislation that entails individual *financial* responsibility and assume that the producers also have *physical* responsibility of their returned products either because they have set up their own take-back schemes or because the collective scheme they have joined sorts returns with respect to brand names (such as Maine's e-waste law (http://www.maine.gov/dep/rwm/ewaste/)). We consider three different levels of this form of legislation:

1. No regulation: No legislation is imposed.

2. Partial regulation: The legislation specifies a lower bound (specified as a percentage of the sales of the company) on the cores to be collected and disposed of properly

(proper disposal also entails recycling). We denote this percentage by β. For example, in Minnesota the target β is set at 80%. In addition, proponents of take-back legislations specify that collection/recycling targets are needed under every implementation (http://www.rreuse.org/t3/).

3. Full regulation: In addition to β, there is a lower bound β_R (again specified as a percentage) on the amount of cores to be remanufactured ($\beta_R \leq \beta$). Cores collected but not remanufactured must be disposed of properly or recycled. Remanufacturing/reuse targets are not imposed in current implementations of take-back legislations but proponents advocate for their inclusion in future revisions (http://www.rreuse.org/t3/).

2.3.2 Assumptions

We analyze the effects of these three different legislative scenarios on two different types of producers. In the first case (which we call the *monopoly model*) we consider an OEM who remanufactures in-house and controls the sales of both the new and remanufactured products[1]. In the second case (which we call the *competition model*), the OEM is a monopolist in the new product market, but does not have remanufacturing capability and there is an independent remanufacturer in the market. Hence the new products by the OEM face competition from the remanufacturer's products.

Following the common assumption in the literature (e.g. Majumder and Groenevelt (2001), Ferrer and Swaminathan (2006), Ferguson and Toktay (2006)), we use a two-

[1]The same model could be used to analyze outsourced remanufacturing where the OEM sells the remanufactured products under her own brand name.

period formulation (In section 2.8.1 we relax this assumption). We assume that every item sold in the first period is available for collection and suitable for remanufacturing in the second period. This assumption could easily be relaxed by modeling the available amount of cores as a percentage of items sold in the first period. In addition, we assume that the items sold in the second period cannot be remanufactured. This is reasonable if the product changes so significantly (i.e. new technology) over the course of two periods that the old cores cannot be remanufactured.

Cores that are collected but not remanufactured must be disposed of properly at a disposal cost. We model cost of manufacturing (c_M), cost of remanufacturing (c_R), cost of disposal/recycling (c_D), cost of collection to the manufacturer (c_C), and cost of collection to the third-party remanufacturer (c_C^R). Note that recycling might be an option instead of a cost. In our model there is no restriction on the sign of c_D but in order for our analysis to hold $c_D + c_C \geq 0$ should be satisfied. Like Ferguson and Toktay (2006), we assume that $c_C^R > c_C$, where c_C is the collection cost of the OEM. This is reasonable because the OEM can utilize her well-established forward logistics channel for collection and incur a lower cost. Note that all costs are linear with no economies of scale. In addition, costs are constant with no learning curve or process improvement effect. Although we do not include fixed set-up cost (e.g. costs incurred to initiate remanufacturing process), impact of such costs on our findings would easily be identified.

If there is take-back legislation, the producer is responsible for the collection and proper disposal of 100β percent of the items sold in the second period. In the competition model, we assume that the remanufacturer does not face collection/disposal costs on the remanufactured products sold in the second period. This is consistent, for example with the WEEE directive where only the producers who put the product on the market for the first time are liable under legislation (Gray and Charter, 2007).

2.3.3 Demand Model

We utilize a deterministic demand model with different consumer valuations for new and remanufactured products. We assume that there are Q potential customers whose valuations for the new product, v is distributed uniformly in the interval $[0, Q]$ and each customer's valuation for the remanufactured product is a fraction (α) of his valuation for the new product. When a customer with valuation v purchases a new (remanufactured) product, his utility is $U_{iM}(v) = v - p_{iM}$ $(U_{iR}(v) = \alpha v - p_{iR})$ where p_{iM} (p_{iR}) is the price of the new (remanufactured) product in period i. In any period each customer purchases the product, if any, that provides him with the highest utility. In the first period, the linear inverse demand function for the first period is $p_{1M} = Q - q_{1M}$ where q_{1M} is the quantity of the new product and for the second period, the inverse demand functions for the new product is $p_{2M} = Q - q_{2M} - \alpha q_{2R}$ and that for the remanufactured product is $p_{2R} = \alpha(Q - q_{2M} - q_{2R})$ where p_{2M} and p_{2R} are the prices of the new and remanufactured products respectively (For derivations interested reader might see Ferguson and Toktay (2006) or Ferrer and Swaminathan (2006)).

2.4 The OEM as a Monopolist

2.4.1 Formulation and Analysis

In this section we consider the OEM as the sole decision maker. In the first period, she determines how many units of the new product to produce (q_{1M}). In the second period, the OEM decides how many cores to collect (q_{2C}), of the collected cores, how many to remanufacture (q_{2R}), and how many units of the new product to produce (q_{2M}). Throughout the chapter we use the superscript (*) to denote the optimal value of the corresponding decision variable. We provide only the formulation for the full regulation case and the formulations for the other two legislative scenarios are obtained by setting the relevant parameter(s) to zero (i.e. $\beta = 0$ in case of partial regulation and both $\beta = 0$ and $\beta_R = 0$ in case of no regulation). The monopolist OEM's problem is formulated as follows:

$$
\begin{aligned}
(P1) \quad \underset{q_{1M}\geq 0,\, q_{2M}\geq 0,\, q_{2R},\, q_{2C}}{Max} \quad \Pi_M \quad = \quad & q_{1M}\left(Q - q_{1M} - c_M\right) + \phi\left[q_{2M}\left(\left(Q - q_{2M} - \alpha q_{2R}\right) - c_M\right)\right. \\
& + q_{2R}\left(\alpha\left(Q - q_{2M} - q_{2R}\right) - c_R\right) - q_{2C}c_C - \left(q_{2C} - q_{2R}\right)c_D \\
& \left. - \left(c_C + c_D\right)\phi\beta q_{2M}\right]
\end{aligned}
$$

$$
s.t. \qquad \beta q_{1M} \leq q_{2C} \leq q_{1M} \tag{2.1}
$$

$$
\beta_R q_{1M} \leq q_{2R} \leq q_{2C} \tag{2.2}
$$

where ϕ is the discount factor.

Constraint (2.1) ensures that the amount of collected cores is at least as much as required by legislation and less than what is produced in the first period while (2.2) imposes that the amount of remanufacturing is at least as much as required by legislation but no more than the amount of collected cores. To avoid the trivial situations where neither manufacturing nor remanufacturing is profitable in the second period, we assume that $Q - c_M - \phi\beta(c_C + c_D) \geq 0$ and $\alpha Q - c_R - c_C \geq 0$. One can show that the first-order conditions are necessary and sufficient. In Theorem 1 we characterize the optimal regions for the OEM under different legislative settings.

Theorem 1 *In the absence of take-back legislation, the optimal strategy of the OEM corresponds to one of the four regions in Table 2.1, where the boundaries of the regions are defined with respect to the values of* c_M *and* c_R *(See Figure 2.1a). Under partial (full) regulation, the optimal strategy of the OEM corresponds to one of the eight regions in Table 2.2, where the boundaries of regions are defined with respect to the values of* c_M *and* c_R *(See Figures 2.1b and 2.1c).*

Under each legislative setting, the regions of optimal strategies are characterized by c_M and c_R; and therefore, the optimal strategy is specific to each product and each firm. When there is no take-back legislation, regions 1-4 in Figure 2.1a are defined as in Table 2.1. The different regions in which the OEM's optimal strategy may lie are differentiated with respect to the optimal amount of second-period manufacturing and remanufacturing. The optimal values of q_{1M}, q_{2C} and q_{2R} in each region and the bounds on c_M that characterize the regions are in Table A1 in Appendix A1. The characterization is similar to that of Ferrer and Swaminathan (2010).

Under take-back legislation (partial or full), regions 1-8 in Figure 2.1b-c are defined

Region	1	2	3	4
q_{2R}^*	0	> 0	q_{1M}^*	q_{1M}^*
q_{2M}^*	> 0	> 0	> 0	0

TABLE 2.1: The regions that characterize the feasible strategies for the OEM under no regulation

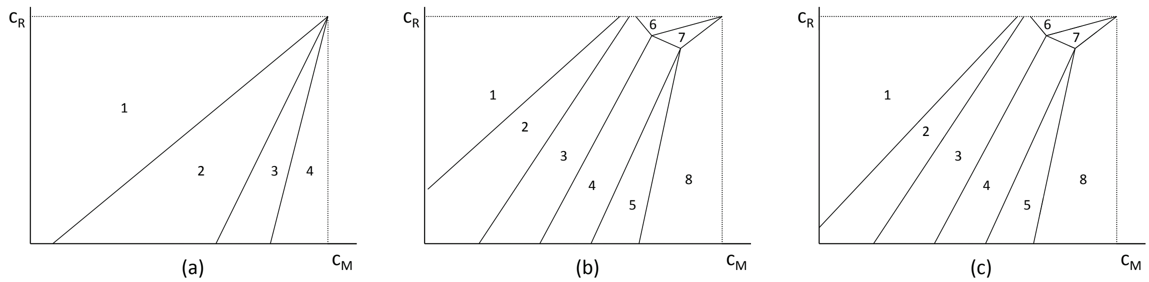

FIGURE 2.1: A possible characterization of the optimal policies for the monopolist OEM (a)in the absence of regulations and (b)under partial regulation and (c) under full regulation

as in Table 2.2. Note that under partial legislation β_R in Table 2.2 is set to zero. The optimal values of q_{1M}, q_{2C} and q_{2R} in each region and the bounds on c_M that characterize the regions for partial and full regulation are given in Tables A2 and A3 respectively in Appendix A1.

Region	1	2	3	4	5	6	7	8
q_{2R}^*	$\beta_R q_{1M}^*$	$> \beta_R q_{1M}^*$	βq_{1M}^*	$> \beta q_{1M}^*$	q_{1M}^*	βq_{1M}^*	$> \beta q_{1M}^*$	q_{1M}^*
q_{2M}^*	> 0	> 0	> 0	> 0	> 0	0	0	0

TABLE 2.2: The regions that characterize the feasible strategies for the OEM under regulation

Comparing Figures 2.1a-c, one might identify the regions that would be optimal for a firm when partial regulation is imposed, if initially region i is optimal under no regulation. However this comparison does not provide the complete picture of how the levels of manufacturing and remanufacturing change as legislation is imposed. In the next section we explore these questions.

2.4.2 The Effect of Legislation on a Monopolist OEM

We study how legislation affects optimal prices and quantities of new and remanufactured products. First, we analyze the case of going from no regulation to partial regulation.

Theorem 2 *As the legislation structure changes from no regulation to partial regulation, the directions of change in q^*_{1M}, q^*_{2R} and p^*_{2R} depend on the value of c_M. If $c_M \geq \frac{Q(1-\alpha)(1+\alpha\phi)+\alpha\phi(c_R+c_C)}{1-\alpha+\alpha\phi}$ partial legislation does not cause a change in the optimal values of q^*_{1M}, q^*_{2R} and p^*_{2R}. Otherwise,*

1. *There exist two threshold levels κ_1 and κ_2 ($\kappa_1 < \kappa_2$, full expressions provided in the proof of the theorem) such that q^*_{1M} and p^*_{2R} go down if $c_M < \kappa_1$, remain the same if $\kappa_1 < c_M < \kappa_2$, and go up if $\kappa_2 < c_M$,*

2. *q^*_{2R} remains the same if $c_M < \kappa_3 = \frac{-\alpha\beta\phi(c_C+c_D)+c_R-c_D}{\alpha}$ and goes up if $c_M > \kappa_3$.*

An immediate observation from Theorem 2 is that legislation on collection levels never causes the amount of remanufacturing to go down. This is intuitive, because as the OEM is forced to incur the collection cost, remanufacturing becomes more attractive and the amount of remanufacturing level remains the same or goes up. However, even though the remanufacturing level goes up, this does not imply that the price of the remanufactured product goes down. While surprising this result can be explained as follows: Due to the competition between the new and remanufactured products in the second period, if the new product manufacturing goes down too much in the second period, the price of the remanufactured product may go up due to dampened competition.

Another interesting observation is that first-period manufacturing levels might go up as a response to legislation. If the cost of manufacturing is low relative to the cost of remanufacturing, manufacturing is very profitable due to low cost and the additional cost due to legislation dampens the production level in the first period. As the cost of manufacturing increases, there is a region where the first-period manufacturing levels are the same regardless of the legislative scenario. Then for higher values of manufacturing cost, first-period manufacturing is higher under partial regulation. In this region, remanufacturing is comparatively more profitable than manufacturing from scratch and in order to make money on the remanufactured products in the second period, the OEM increases first-period production.

Theorem 2 says that if it is very cheap to manufacture new products then partial regulation alone is not sufficient to induce any additional remanufacturing. On the other hand, for products that are costly to manufacture -relative to remanufacture-, the optimal decisions of the OEM do not depend on the legislative scenario, because in that case economic incentives alone induce product take-back and remanufacturing, i.e., the legislation is "redundant". Therefore our results clearly show that imposing legislation with target levels chosen in an ad hoc fashion without taking the underlying cost structures into account can lead to inefficiencies.

When partial regulation alone is unable to induce remanufacturing, a lower bound on the level of remanufacturing may be required, i.e., full regulation may be imposed. The following theorem identifies how first-period manufacturing and remanufacturing levels as well as the price of the remanufactured product change as full regulation is imposed.

Theorem 3 *As the legislation structure changes from partial to full the directions of change in q_{1M}^*, q_{2R}^* and p_{2R}^* depend on the value of c_M. If $c_M \geq \kappa_4$, where*

$$\kappa_4 = \frac{\alpha(1-\alpha)\beta_R Q - (1+\beta_R(1-\alpha))\alpha\beta\phi(c_C+c_D) + c_R - c_D}{(1+\beta_R(1-\alpha))\alpha},$$

full legislation does not cause a change in q_{1M}^*, q_{2R}^* and p_{2R}^*. For lower c_M values, q_{1M}^* and p_{2R}^* go down while q_{2R}^* goes up.

The above theorem shows that the OEM responds to legislation by decreasing the first-period new-product manufacturing level and increasing the remanufacturing level only if c_M is low. Therefore we conclude that full regulation may only be necessary to induce remanufacturing of those products that are very cheap to produce new.

Our results in this section clearly shows that, if the aim is to increase remanufacturing levels, the same level of β may lead to very different outcomes depending on the underlying cost structures. Next we analyze how β and β_R should be set in order to refrain from introducing redundant legislations.

2.4.3 How Should the Target Levels in Legislation Be Set?

The way the legislation is structured it is clear that increasing β will (weakly) increase collection levels while increasing β_R will (weakly) increase remanufacturing levels. The more interesting question is since legislation aims to increase reuse/remanufacturing levels (because remanufacturing is perceived as environmentally friendlier than manufacturing), how high should β be? If prior to legislation, it is optimal for an OEM not to remanufacture at all, how should β be set to induce some remanufacturing? Next theorem answers this question.

Proposition 1 *If the optimal level of remanufacturing is zero prior to legislation then imposing partial regulation with* $\beta \geq \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)}$ *induces some level of remanufacturing. If* $\beta \geq \frac{\alpha(1-\alpha)Q + c_R + c_C - (2-\alpha)\alpha c_M}{\alpha\phi(c_C + c_D)}$ *and* $\alpha c_M - c_R - c_C \geq 0$ *then all first period cores are remanufactured under partial regulation.*

How the threshold on β changes with the other parameters is intuitive. If the c_R is high, a higher β is needed to induce remanufacturing. Similarly if c_D, c_C, c_M and/or the customers' acceptance of the remanufactured product α is high then remanufacturing is induced even at lower levels of β. For the second threshold in Proposition 1 to be meaningful we need it to be less than 1, which is likely to hold when α and ϕ are close to 1 and $c_D \gg c_C$. This implies that when the consumers value the remanufactured product highly and c_D is very high, imposing large enough β may induce a high level of remanufacturing.

One reason to impose full regulation is to increase the level of remanufacturing by appropriately setting β_R. So how high should β_R be? From Proposition 1 we know that below a threshold value of β, partial regulation is not sufficient to induce remanufacturing, hence the remanufacturing target level, β_R, is always binding when β is low. For higher values of β, our next proposition states that unless β_R can be set high enough (how "high" depends on the other parameters), full regulation (i.e. β_R) is redundant.

Proposition 2 *If* $\beta \geq \beta^* = \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)}$ *then imposing full regulation does not increase the level of remanufacturing unless* $\beta_R > \frac{\phi(c_C + c_D)(\beta - \beta^*)}{(1-\alpha)(Q - c_M - \phi\beta(c_C + c_D))}$.

Proposition 2 is important because it shows that β_R should be set at an appropriate level in relation to β and the higher β is, the higher β_R should be. Otherwise implementing full regulation simply increases administrative burden without resulting in any additional benefit.

2.4.4 Does Legislation Incentivize Cheaper to Remanufacture Products?

Gray and Charter (2007) argue that the remanufacturing cost of a product is a product characteristic that is mainly locked at the design stage and how the product is manufactured affects the remanufacturing cost. From an environmental benefit point of view, an interesting question is whether take-back legislation that only imposes collection targets induces better design for remanufacturing, i.e., easier- and cheaper-to-remanufacture products. Such reduction in the remanufacturing cost, c_R, may increase the manufacturing cost because of use of more durable materials, more modular design, or designing easy to assemble products as opposed to using an integrated design etc. Proposition 3 explores if the OEM has incentive to reduce c_R at the expense of increasing c_M.

Proposition 3 *Consider two (c_M, c_R) cost pairs $A = \left(c_M^A, c_R^A\right)$ and $B = \left(c_M^B, c_R^B\right)$ such that $c_M^B > c_M^A$, $c_R^B < c_R^A$ and at optimality there is no remanufacturing at A and some remanufacturing at B. Then there exists a threshold $\theta(c_C, c_D, c_M^B, c_R^B, \alpha, \beta)$ such that if $c_M^A > \theta$ then $\Pi_M^*\left(c_M^B, c_R^B\right) > \Pi_M^*\left(c_M^A, c_R^A\right)$ and θ is decreasing in β.*

Proposition 3 states that if c_M is very low, decreasing c_R is not profitable, but if c_M is higher than a threshold, it is profitable to increase c_M and decrease c_R. The fact that the threshold is decreasing in β implies that the OEM indeed has more incentive for better design under legislation.

2.5 Competition Between the OEM and a Third-Party Remanufacturer

2.5.1 Formulation and Analysis

In order to understand the effects of different take-back legislations on markets where the OEM does not remanufacture, we model an OEM, who does not remanufacture in-house but faces competition from a third-party remanufacturer. The OEM is the only player offering a single product in the first period. In the second period the OEM and the remanufacturer compete for cores and we allow the OEM to move first (i.e. the OEM is the Stackelberg leader). Modeling the OEM as the first-mover captures the fact that OEMs have first access to collected cores under take-back legislation. We want to explicitly analyze how this affects the remanufacturer because some remanufacturers worry that the amount of cores available to them will decrease as a result of take-back laws due to cores going directly to collection centers where it is easier for the OEMs to pick them up (http://www.techworld.com/green-it/news/index.cfm?newsid=9595). Since the OEM does not have in-house remanufacturing, she can either dispose of the cores and/or sell them to the remanufacturer. In the case of the latter, the OEM decides on the price she will charge for the cores. Then the remanufacturer decides on the quantity of cores to buy from the OEM and/or collect from the market. While it is common for remanufacturers to collect cores themselves or buy from collectors, an example where the remanufacturers get cores from the OEMs is the case of toner cartridge remanufactures in Sweden: in order to comply with Swedish take-back laws predating WEEE, the OEMs handed over end-of-life products to remanufacturers (Sundin, 2004). Finally the OEM decides on the production level while the remanufacturer decides on the remanufacturing level.

The two-period competition is formulated as a Cournot model as is common in the closed loop supply chain literature (e.g. Atasu et al. (2009a), Ferguson and Toktay (2006) and Majumder and Groenevelt (2001)). We first define the remanufacturer's problem. Let q_{2S} denote the amount of cores purchased from the OEM, q_{2C}^R the amount of cores collected by the remanufacturer, p_{2S} the price at which the OEM sells the cores to the remanufacturer. Since there is no incentive to acquire cores unless remanufacturing is profitable, the remanufacturer remanufactures all the cores on hand, namely the total amount remanufactured is given by $q_{2R} = q_{2S} + q_{2C}^R$. Then regardless of the take-back scenario under which the problem is analyzed the remanufacturer's problem is

$$
\begin{aligned}
(P2) \quad \underset{q_{2C}^R,\, q_{2S} \geq 0}{Max} \; \Pi_{2R} \;&=\; \left(q_{2C}^R + q_{2S}\right)\left(\alpha\left(Q - q_{2M} - \left(q_{2C}^R + q_{2S}\right)\right) - c_R\right) - q_{2S}p_{2S} - q_{2C}^R c_C^R \\
s.t. \quad & \quad 0 \leq q_{2C}^R \leq q_{1M} - q_{2C}
\end{aligned}
$$

The only constraint on the remanufacturer's problem is that the quantity collected by the remanufacturer cannot exceed the amount left in the market after the OEM collects.

Next we define the OEM's second period problem. While we separately solve the OEM's problem under no regulation, partial regulation and full regulation, here we only present the problem formulation under full regulation. The formulations for the other two scenarios follow by setting β_R and β to zero respectively.

$$
\begin{aligned}
(P3) \quad \underset{q_{2M}\geq 0, q_{2C}, p_{2S}}{Max} \quad & \Pi_{2OEM} = q_{2M}\left(\left(Q - q_{2M} - \alpha\left(q_{2C}^{R*} + q_{2S}^{*}\right)\right) - c_M\right) - q_{2C}c_C + q_{2S}^{*}p_{2S} \\
& - \left(q_{2C} - q_{2S}^{*}\right)c_D - \left(c_C + c_D\right)\phi\beta q_{2M} \\
s.t. \quad & \beta_R q_{1M} \leq q_{2C}^{R*} + q_{2S}^{*}, \quad \beta q_{1M} \leq q_{2C}^{R*} + q_{2C}, \quad q_{2S}^{*} \leq q_{2C} \leq q_{1M}
\end{aligned}
$$

The OEM decides on the amount of cores to collect, q_{2C}, the price to charge per core, p_{2S}, and the amount of new products to manufacture, q_{2M}. The constraints guarantee that at least $100\beta_R$ percent of first-period sales is being remanufactured, at least 100β percent of first-period sales is collected, the amount of collected cores does not exceed first-period production and is no less than the amount sold to the remanufacturer. Note that there is no nonnegativity constraint on p_{2S}, so our model allows the case where the OEM pays the remanufacturer to take the cores. In the first period the OEM is the only player and chooses the amount to manufacture, q_{1M}, to maximize her two-period profit.

$$
(P4) \quad \underset{q_{1M}\geq 0}{Max} \ \Pi_{OEM} = q_{1M}\left(\left(Q - q_{1M}\right) - c_M\right) + \phi\,\Pi_{2OEM}^{*}
$$

The Stackelberg game is solved by backward induction and standard application of first order conditions on the Lagrangian. Before we fully characterize the possible optimal strategies of the players, we provide two propositions that help us eliminate some policies.

Proposition 4 *It is never optimal for the OEM to allow the remanufacturer to collect from the market. The OEM either collects* ***some*** *cores and sells them to the remanufacturer at a price lower than or equal to the remanufacturer's cost of collection or collects*

all *cores.*

The first strategy in Proposition 4, namely collecting some cores and selling them to the remanufacturer at a price lower than the remanufacturer's collection cost, is optimal when remanufacturing is not very profitable. However if remanufacturing is so profitable that the remanufacturer is willing to collect cores in addition to those he can buy from the OEM, to prevent the remanufacturer's access, the OEM collects all the available cores and sells only a fraction to him. We call this "preemptive collection" as do Ferguson and Toktay (2006). When preemptive collection is optimal for the OEM, she may still sell cores to the remanufacturer, but as the next proposition shows, she charges a price that is strictly higher than the remanufacturer's cost of collection.

Proposition 5 *If the OEM follows the preemptive collection strategy then she prices the cores strictly higher than the remanufacturer's cost of collection, i.e.* $p^*_{2S} > c^R_C$.

Due to Propositions 4 and 5, the remaining possible policies that the OEM may want to play as the Stackelberg leader are listed in Table 2.3.

Policy A	$p^*_{2S} < c^R_C,\ q^*_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha},\ q^{R*}_{2C} = 0$
Policy B	$p^*_{2S} = c^R_C,\ q^*_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha},\ q^{R*}_{2C} = 0$
Policy C	$p^*_{2S} > c^R_C,\ q^*_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha},\ q^{R*}_{2C} = q_{1M} - q_{2C} = 0$

TABLE 2.3: Possible optimal policies for the competition problem

The policies in Table 2.3 are differentiated with respect to the optimal price the OEM charges for the cores. Due to Proposition 4, the remanufacturer only buys cores from the OEM and it is clearly optimal for him to remanufacture all the cores he buys; hence Table 2.3 (which also lists the optimal quantities the remanufacturer buys from the OEM) completely characterizes the possible actions the remanufacturer may take. For the OEM we still need to characterize the optimal actions she may take on the remaining decision variables, i.e., first and second period manufacturing levels and second period collection level. In Theorem 1 we identified the regions that the monopolist OEM's optimal actions may lie in and numbered them with respect to the percentage of first-period cores that are remanufactured – there are 4 such regions in the absence of legislation (listed in Table 2.1) and there are 8 under legislation (listed in Table 2.2). For Policies A-C, each such region constitutes a possible (but not necessarily feasible) region where the OEM's optimal actions in q_{1M}, q_{2M} and q_{2C} may lie in. If there is no legislation, for Policies A and C all 4 regions are feasible, for Policy B the feasible regions are 1 and 2. If there is take-back legislation, the feasible regions are: For A, 1-8; for B, 1-4,6 and 7; for C, 1, 2, 5, and 8. To complete the analysis we check if the feasible regions defined by Policies A-C may overlap. The complete details of the analysis through which we derive the possible optimal regions and the bounds on c_M that characterize these regions are provided in Appendix B1.2.

We next provide an example to illustrate the possible orderings of the policies in different parameter regions. Figure 2.2 depicts how the optimal policy changes as the cost of manufacturing and the cost of remanufacturing change. For example, in area A_1 of Figure 2.2b, Policy A from Table 2.3 and Region 1 from Table 2.2 characterize the optimal actions of the OEM and the remanufacturer. Figure 2.2 shows that, as in the case of monopoly, as the cost of manufacturing increases percentage of first-period products that are remanufactured goes up. As the cost of remanufacturing increases, the

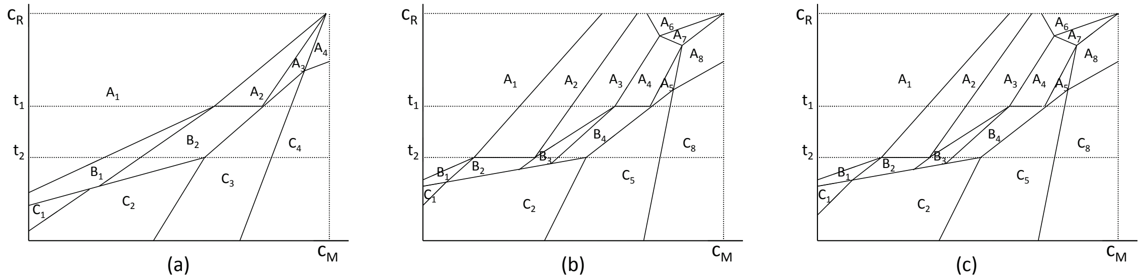

FIGURE 2.2: A possible characterization of the optimal policies under competition where $t_1 = \alpha Q + c_C - 2c_C^R$ and $t_2 = \alpha Q - c_D - 2c_C^R$ (a)in the absence of regulations and (b)under partial regulation and (c) under full regulation

OEM starts to price the cores lower (when compared to the remanufacturer's collection cost).

Figure 2.2 demonstrates that preemptive collection to restrict remanufacturing (characterized by regions C_1 and C_2 in which the OEM collects all the available cores but sells only a fraction to the remanufacturer) may be optimal for the OEM both before and after legislation. In Figure 2.2 we observe that preemptive collection is never optimal when the cost of remanufacturing is high and in Proposition 6 we derive a threshold on c_R, above which preemptive collection is never optimal.

Proposition 6 *Regardless of the level of legislation, preemptive collection is not optimal when* $c_R \geq \alpha Q - c_D - 2c_C^R$.

When the remanufacturing cost is high, the competitive power of the remanufacturer is low. In that case, the OEM does not need to preemptively collect cores in order to restrict the remanufacturer's actions. Also observe that preemptive collection is less likely to be optimal under low customer valuation for the remanufactured product (low α) – another situation where the remanufacturer's competitive power is low. The OEM follows the Preemptive collection strategy to make it harder for the remanufacturer to get cores. So does take-back legislation make the OEM more or less likely to follow this strategy?

Proposition 7 *Preemptive collection strategy is optimal for a larger range of cost parameters under partial regulation than it is under no regulation.*

Earlier we mentioned that some remanufacturers are concerned that fewer cores will be available to them under take-back legislation. Proposition 7 shows that this fear may not be unfounded as the OEM is indeed more likely to follow the preemptive collection strategy. But how does this impact the level of remanufacturing? We next analyze this question.

2.5.2 The Effect of Legislation on an OEM Facing Competition

A concern raised regarding take-back legislation (especially when there are third-party remanufacturers in the market) is whether legislation hinders remanufacturing by granting the OEM first access to cores through the collection targets. We find that the remanufacturing level may indeed go down after legislation is imposed, but, first, with the following proposition we identify a condition under which the amount of remanufacturing *cannot* decrease with the introduction of take-back regulation.

Proposition 8 *When partial (full) regulation is imposed, if it is still optimal to play the policy (as given in Table 2.3) that was optimal under no (partial) regulation, the amount of remanufactured products remains the same or goes up.*

Corollary 1 provides a condition under which Proposition 8 holds: when the remanufacturing cost is high, take-back legislation never causes a decrease in the level of remanufacturing.

Corollary 1 *When $c_R > \alpha Q + c_C - 2c_C^R$ the amount of remanufactured products remains the same or goes up when take-back legislation is imposed.*

While Proposition 8 and Corollary 1 provide sufficient conditions under which partial regulation does not cause the remanufacturing level to go down, they do not rule out the possibility. Next we provide an example where the remanufacturing level goes down as a result of legislation.

Example 1 *Let $Q = 800$, $\phi = 0.95$, $c_M = 210$, $c_C = 10$, $c_D = 10$, and $c_C^R = 15$. The cost of remanufacturing is 30% of the cost of manufacturing, i.e., $c_R = 63$. The customer valuation for the remanufactured product is $\alpha = 0.2$. In the absence of regulations, the remanufacturing level is 53.2 whereas under partial regulation with $\beta = 0.7$ the remanufacturing level is 0.*

For this specific example when there is no regulation, it is optimal to collect some cores and sell all to the remanufacturer at a price that is equal to his collection cost. When take-back legislation with a high collection target (in this example 70%) is imposed, the collection and disposal costs increase significantly for the OEM and the relative competitiveness of the branded products vis-a-vis the remanufactured products decreases. When the high collection target is coupled with a low remanufacturing-level target (in this example $\beta_R = 0$), the OEM switches to the preemptive collection strategy where she collects all the cores, and sells just as much as required by law at a very high price. Since the cores are priced high, the amount of remanufacturing goes down. This example demonstrates that government should especially be careful about imposing legislation and setting collection targets in markets where OEMs and third-party remanufacturers are competing.

2.5.3 How Should the Target Levels in Legislation Be Set?

In this section, by Proposition 9 we show that imposing partial regulation with the *correct* bounds can induce remanufacturing.

Proposition 9 *If policy A or C is optimal both before and after partial regulation is imposed, imposing partial regulation with* $\beta \geq \beta^* = \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)}$ *induces some level of remanufacturing. On the other hand, if policy B is optimal both before and after partial regulation is imposed, imposing partial regulation with* $\beta \geq \beta^* - \frac{(2-\alpha)(\alpha Q - c_R - c_D - 2c_C^R)}{2\alpha\phi(c_C + c_D)}$ *induces some level of remanufacturing.*

Note that the first threshold on β given in Proposition 9 is the same as the one given in Proposition 1 for the case of monopoly. Hence if either policy A or C is optimal both before and after partial regulation, imposing the same legislation level in the monopoly and competition environments is enough to ensure some level of remanufacturing. However, note that even when the same β is imposed, the amount of remanufacturing is lower under competition when compared to monopoly. The second condition is interesting in that policy B is optimal for low to moderate values of c_R (see Figure 2.2) which suggests a relatively strong remanufacturer and thus a smaller β is enough to ensure some level of remanufacturing.

2.6 Does Legislation Incentivize Cheaper to Remanufacture Products?

As we stated before, according to Gray and Charter (2007) the remanufacturing cost of a product is mainly locked at the design stage and hence is mostly determined by the OEM. Recall that under the monopoly model we showed that unless the manufacturing cost is too low, it may be profitable to invest in the product (at additional cost to the OEM) and decrease the cost of remanufacturing such that remanufacturing becomes feasible. Under competition, intuition suggests that the OEM is better off if the remanufacturing cost is high and the OEM does not have any incentive to reduce it. Here we assume that if the OEM engages in activity to reduce the remanufacturing cost such as through Design for Remanufacturing (DfR), the cost of manufacturing increases. Surprisingly, the OEM's profits may increase due to a reduction in remanufacturing cost even if such a reduction comes at the expense of higher manufacturing cost.

Proposition 10 *Consider two $(c_M,\, c_R)$ cost pairs $A = \left(c_M^A, c_R^A\right)$ and $B = \left(c_M^B, c_R^B\right)$ such that $c_M^B > c_M^A$, $\alpha Q - c_D - 2c_C^R < c_R^B < c_R^A$ and at optimality there is no remanufacturing at A and some remanufacturing at B. There exists a threshold $\theta_C(c_C, c_D, c_M^B, c_R^B, \alpha, \beta)$ such that if $c_M^A > \theta_C$ then $\Pi^*_{OEM}\left(c_M^B, c_R^B\right) > \Pi^*_{OEM}\left(c_M^A, c_R^A\right)$. In addition, θ_C is decreasing in β and is higher than the corresponding θ derived in Proposition 3 under monopoly.*

Proposition 10 states that the OEM can make more profit as c_M increases and c_R decreases as long as the initial cost of manufacturing is not too low. Since the threshold on c_M is decreasing in β, the proposition implies that the incentive to decrease c_R is higher under legislation. The following example illustrates that unless there is take-back legislation, the OEM may not have any incentive to reduce the remanufacturing cost.

Example 2 *Consider the competition model for the case where* $Q = 2000$, $\phi = 0.95$, $\alpha = 0.4$, $c_C = 90$, $c_D = 80$, $c_C^R = 100$, $\beta = 0.8$ *and* $\beta_R = 0$. *We consider two scenarios: under the base scenario* $c_M = 1470$ *and* $c_R = 720$ *and under the DfR Scenario* $c_M = 1500$ *and* $c_R = 550$. *Under base scenario,* $q_{2S}^* = 0$, $p_{2S}^* = -0.16$, $\Pi_{OEM}^* = 78312.3$. *Under DfR Scenario* $q_{2C} = q_{2S}^* = 141.9$, $p_{2S}^* = 85$, $\Pi_{OEM}^* = 79276.6$. *In the absence of take-back legislation under base scenario* $\Pi_{OEM}^* = 136,938.8$ *while under DfR scenario* $\Pi_{OEM}^* = 116,650.0$.

Under partial regulation, the OEM's profit is higher under the DfR scenario, where c_R is lower while c_M is higher than that under the base scenario. When there is no regulation, the profit is higher under the base scenario. This implies that unless take-back legislation is imposed, the OEM will not redesign the product to reduce the remanufacturing cost. Under legislation the OEM is responsible for the collection and proper disposal of cores unless they are remanufactured and thus the effective cost of manufacturing is higher. Therefore the OEM has more incentive to sell the cores to the remanufacturer, which is an alternative way to increase her profits. Under reduced cost of remanufacturing the OEM can sell more cores and this creates additional incentive to reduce the remanufacturing cost. This tendency is more pronounced when the OEM does not face intense competition from the remanufacturer as is the case in this example (Note that the valuation for the remanufactured products (α) is low).

2.7 Comparison between Monopoly and Competition Scenarios

In this section we compare the effects of legislation on a monopolist OEM versus one that faces third-party competition. We are especially interested in identifying whether the environmental benefits (e.g. amount of product remanufactured) of take-back legislation are more significant under one versus the other. We first compare the remanufacturing levels. Intuition suggests that regardless of the legislation level, the amount of remanufacturing should be higher when the OEM has in-house remanufacturing. We find that this is indeed true under some conditions.

Proposition 11 *When faced with the same level of legislation, the remanufacturing level achieved by a monopolist OEM is higher than that achieved by a third-party remanufacturer as long as the optimal price of the cores is less than the remanufacturer's cost of collection.*

Another way of phrasing Proposition 11 is that the remanufacturing level under competition can only exceed that under monopoly if policy B or C is optimal. Under policy A, the OEM sells the cores to the third-party remanufacturer in order to save from the disposal cost and/or to comply with the legislation. Otherwise, selling cores to the third-party creates a competition and thus she prefers to keep the competition under control. On the other hand, policy B is optimal when the remanufacturer's competitive power is higher. In that case, if the remanufacturer had no access to cores, the OEM would have priced the cores higher than the remanufacturer's cost of collection. However, knowing that the remanufacturer can access, the OEM sets the price equal to the remanufacturer's cost of collection. As the OEM prices the cores lower than that she would have priced in the case of no remanufacturer access, the remanufacturer purchases more cores. Therefore remanufacturing level might increase even beyond the level of remanufacturing level achieved by a monopolist OEM. Similarly remanufacturing level under policy C might be higher than that under monopoly. In that case, the OEM collects all the cores in order to preempt remanufacturer's access and makes profit out of selling the cores at a very high price. When manufacturing is costly while remanufacturing is profitable but not strictly enforced by the legislation, i.e., c_M is high but c_R and β are low, OEM facing competition makes profit out of selling a significant amount of cores to a third-party remanufacturer and thus remanufacturing level achieved under competition exceeds that under monopoly.

To illustrate that the remanufacturing level may be higher under competition, we provide an example. Consider Example 1 with the following regulation levels: $\beta_R = 0.1$ and $\beta \in \{0.25, 0.5, 0.75\}$. The detailed solutions for the monopoly and the competition scenarios are given in Table 2.4. Note that the amount of remanufactured products

	Monopoly					Competition				
β	Π^*_M	q^*_{2R}	q^*_{2C}	q^*_{1M}	p^*_{1M}	Π^*_{OEM}	q^*_{2R}	q^*_{2C}	q^*_{1M}	p^*_{1M}
0	169,698.7	0	0	295.0	505	166,812.4	53.2	53.2	295.0	505
0.25	166,568.6	29.2	72.9	291.7	508.3	165,114.9	57.3	73.2	292.6	507.4
0.5	163,899.7	28.9	144.7	289.4	510.6	162,403.4	58.6	145.1	290.3	509.7
0.75	161,252.3	28.7	215.3	287.1	512.9	159,740.7	28.4	284.1	284.1	515.9

TABLE 2.4: Optimal solution for the OEM under different β values (when $\beta > 0$, $\beta_R = 0.1$ and when $\beta = 0$, $\beta_R = 0$)

is higher under competition when $\beta \in \{0.25, 0.5\}$. The regulations on both collection and remanufacturing levels are tight under monopoly, whereas under competition, the optimal policy is to collect as much as the regulation requires and sell more than what the regulation requires at a price equal to the remanufacturer's cost of collection. The difference in policies stems from the following: Since α is small, the remanufactured product is perceived as low quality and the monopolist remanufactures only as much as the regulation requires. In the competition model, low α implies that the competitive power of the remanufacturer is not high. Therefore the OEM finds it optimal to sell more than what the regulation requires to the remanufacturer and make a profit out of selling cores.

The preceding result shows that when facing the *same* level of legislation, a third-party remanufacturer may produce more remanufactured products than a monopolist OEM does. Can the level of remanufacturing under competition and *no* legislation be higher than the remanufacturing level achieved by the monopolist OEM under *full* regulation? Surprisingly the answer is yes.

We compare the first row of Table 2.4 under Competition column with the results under Monopoly column. Observe that under competition the amount of collected and remanufactured products is 53.2 in the absence of regulations while the amount of remanufactured products is much lower for a monopolist even under regulation. Hence the government can attain even higher amounts of remanufacturing and collection just by introducing competition rather than by imposing legislation.

As a result of a more extensive numerical study, we find that competition alone is likely to increase the level of remanufacturing for moderate values of remanufacturing cost. When cost of remanufacturing is low, under competition, the OEM follows preemptive collection and sells some or none to the remanufacturer. On the other hand for moderate values of c_R the OEM makes more profit out of collecting and selling the cores at a price equal to the cost of collection of the remanufacturer rather than following preemptive collection. Because of competition, the prices of new and remanufactured products decrease while the remanufacturing level goes up. Finally as the potential profitability of remanufacturing decreases with higher c_R values, there is no remanufacturing under competition and no regulation.

It is good news that competition alone can induce remanufacturing. We also observe that the increase in the new product price due to legislation may be lower under competition: When there is no legislation, the price of the new product in the first period is 505 under both monopoly and competition. After legislation is imposed, the price goes up in both cases indicating that legislation-related cost is inevitably passed on to the consumers, however the increase is more pronounced under monopoly. Although the increase in the new product prices can be interpreted as a negative impact of legislation on the consumers, in order to fully understand how legislation affects customers, in the next section we analyze how the total consumer surplus changes under take-back legislation for both the monopoly and competition scenarios.

2.8 Impact of Take-back Legislation on Consumer Surplus

The total consumer surplus S is the surplus of consumers who buy the new product in periods 1 and 2 plus the surplus of those who buy the remanufactured product in period 2:

$$\begin{aligned} S &= \int_{p_{1M}}^{Q} (v - p_{1M}) dv + \phi \left(\int_{\frac{p_{2M}-p_{2R}}{1-\alpha}}^{Q} (v - p_{2M}) dv + \int_{\frac{p_{2R}}{\alpha}}^{\frac{p_{2M}-p_{2R}}{1-\alpha}} (\alpha v - p_{2R}) dv \right) \\ &= \frac{q_{1M}^2}{2} + \phi \left(\frac{\alpha q_{2R}^2 + q_{2M}^2 + 2\alpha q_{2R} q_{2M}}{2} \right). \end{aligned}$$

Our next proposition states that when a monopolist OEM remanufactures, the total consumer surplus decreases as the collection and/or the remanufacturing target in the legislation increases.

Proposition 12 *In the monopoly model, as the collection and/or the remanufacturing target increases, consumer surplus decreases.*

Proposition 12 shows that under both partial and full regulation, a monopolist OEM will always pass some of the compliance cost to the consumers, resulting in a reduction in consumer welfare. From a numerical study, we observe that this is also the case in the competition model under partial regulation, i.e., when there is competition between an OEM and a third-party remanufacturer, the total consumer surplus decreases as β increases. However, surprisingly, under full regulation, the total consumer surplus may increase as β_R increases when there is competition between an OEM and an independent remanufacturer. This happens when stricter legislation forces the OEM to give up the preemptive collection strategy. The following example illustrates how total consumer surplus may change as β_R increases.

Example 3 *Let* $Q = 1600$, $\phi = 0.95$, $\alpha = 0.20$, $c_M = 105$, $c_C = 10$, $c_D = 10$, *and* $c_C^R = 15$. *We consider two levels of cost of remanufacturing:* $c_R = 42$ *(low) and* $c_R = 84$ *(high). Figure 2.3 depicts total consumer surplus under monopoly and competition versus different* β_R *values.*

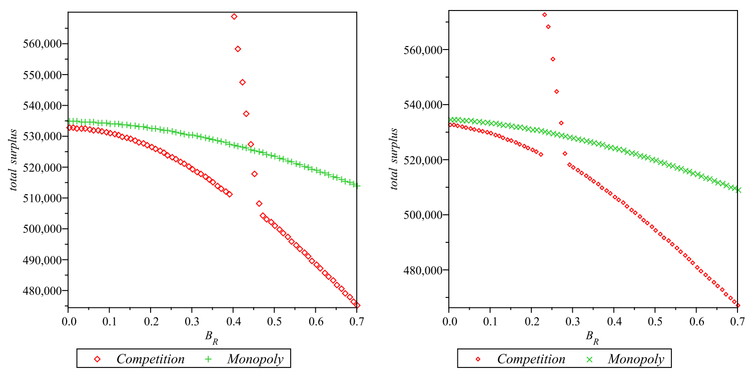

FIGURE 2.3: Total consumer surplus versus β_R when cost of remanufacturing is (a) low and (b) high

In Figure 2.3 we observe that there exists a β_R (e.g., 0.4 for $c_R = 42$) at which total consumer surplus under competition increases significantly. For lower β_R values, the OEM follows the preemptive collection strategy and as β_R increases, consumer surplus decreases. However when β_R reaches the threshold, preemptive collection is no longer profitable and the OEM switches to the policy of pricing the cores at the level of remanufacturer's collection cost. As preemptive collection is abandoned, remanufacturing and manufacturing levels increase and thus the total consumer surplus increases significantly. As β_R increases beyond the threshold, the OEM continues to price the cores equal to the remanufacturer's collection cost and as β_R increases further, she prices them below the remanufacturer's collection cost. However, in either case, consumer surplus decreases as β_R increases. As proven in Proposition 12, Figure 2.3 also shows that consumer surplus under monopoly decreases as β_R increases. Hence, while consumers of a monopolist OEM are always worse off as take-back legislation gets stricter, consumers in an industry where there is competition between an OEM and a third-party remanufacturer may benefit from higher remanufacturing targets.

As a result of a more extensive numerical study (also see Figure 2.3a and b), we find that the threshold β_R, at which the consumer surplus jumps up to a higher level, is lower for higher values of c_R. Recall that high remanufacturing cost is an indication of low competitive power for the third-party remanufacturer. When the third-party remanufacturer is not competitive vis-a-vis the OEM, even a low remanufacturing target makes the OEM abandon the preemptive collection strategy and benefits the consumers in terms of higher consumer surplus.

Finally, when we compare the total consumer surplus under the monopoly and competition models, we find that consumers of a monopolist OEM generally generate higher surplus.

Proposition 13 *When faced with the same level of legislation (partial or full), the total consumer surplus under monopoly is higher than that under competition as long as the optimal price of the cores is less than the remanufacturer's cost of collection.*

Proposition 13 shows that if the OEM prices the cores lower than the remanufacturer's collection cost (i.e., policy A is optimal) then total consumer surplus under competition is always lower than that under monopoly. In Proposition 11 we show that when policy A is optimal, the remanufacturing level is lower under competition than under monopoly. In addition, the new product prices are always higher than the monopoly prices in the first period (because the OEM manufactures fewer products due to expected competition). As a result, the total consumer surplus under competition is lower than that under monopoly. Our numerical study shows that the total consumer surplus is higher under monopoly under both partial and full regulation also when the optimal price of the cores is *more* than the remanufacturer's collection cost (i.e., policy C is optimal). In this case, the OEM collects all the cores to preempt the remanufacturer's access to the cores, and the first period prices are significantly higher than the monopoly prices because the cost of collecting all the cores is passed onto the customers. As a result, the total consumer surplus is lower than that under monopoly. Finally, our numerical study indicates that total consumer surplus might be *higher* under competition when the OEM prices the cores at the remanufacturer's collection cost (i.e., policy B is optimal). In Figure 2.3a the region where $\beta_R \in \{0.4 \ldots 0.44\}$ and in Figure 2.3b the region where $\beta_R \in \{0.23 \ldots 0.27\}$ are examples under full regulation and we can find similar examples under partial regulation. When policy B is optimal, the remanufacturing level under competition may be higher than that under monopoly (see the example in Section 2.7) because the OEM is forced to sell the cores at a price lower than what is optimal for her (otherwise, the remanufacturer would collect cores himself and would not buy from the OEM) and tries to increase her profits by manufacturing more in the first period and selling a larger quantity of cores to the remanufacturer. Hence, under policy B, it is not feasible for the OEM to severely restrict the remanufacturer's access to cores. As a result, a higher level of competition occurs between the OEM and the remanufacturer and the consumer benefits in the form of higher consumer surplus.

2.8.1 Life-Cycle Effects and the Length of the Planning Horizon

Up to this section, we have assumed that the demand for the product lasts only two periods, after which the product becomes obsolete, and that the potential market size is the same in each period. In this section we relax these assumptions, which makes the model intractable and we base our insights on an extensive numerical study. We consider a multi-period planning horizon with remanufacturing in the second and subsequent periods. We assume that each product sold can be remanufactured only once, which is a common assumption in the literature (Debo et al., 2005; Lebreton, 2007) as most products are not remanufactured more than once (e.g. mobile phones, computers, diesel motors, etc.) and that consumers keep the product for only one period, i.e., a product sold in period t becomes available for remanufacturing in period $t+1$ (Relaxing this assumption does not generate any additional insights.). Among others, Geyer et al. (2007) and Debo et al. (2006) study remanufacturing in the presence of life cycle effects. Different from these papers, we aim to understand the interaction between take-back legislations and product life cycle effects.

To facilitate comparison with our earlier results, we choose the cost parameters to be consistent with Example 1 and such that we observe all possible feasible solutions . We pick values for (c_M, c_R) such that $c_M \in \{105, 210, 315, 420, 525, 630\}$ and $c_R \in$

$\{42, 63, 84, 105, 126\}$. For each (c_M, c_R) pair, we solve the problem for life-cycle lengths of three and four periods under both the monopoly and competition scenarios. We vary the potential market size in each period by multiples of 100 while keeping the total market size over the product's life constant at 2400 (this is similar to how Geyer et al. (2007) model life cycle effects). For a product with a life of three periods, we use $[Q_1, Q_2, Q_3] \in \{[800, 800, 800], [700, 1000, 700], ..., [100, 2200, 100]\}$ and for four periods we use $[Q_1, Q_2, Q_3, Q_4] \in \{[600, 600, 600, 600], [500, 700, 700, 500], ..., [100, 1100, 1100, 100]\}$. We call products with stable demand throughout their life cycles *mature* products and those that exhibit more distinct life cycle effects, *innovative* products.

2.8.2 Monopolist OEM under Life Cycle Effects

For the multi-period problem, we find that Theorem 1 characterizes the optimal strategy of the OEM in any period with respect to the previous period's production. The only exception is that for innovative products, if c_M is relatively high and $Q_{t-1} \gg Q_t$ then the remanufacturing level in period t can be lower than the collection level even when there is no manufacturing in period t, which is never the case for mature products.

In Section 2.4 we show that when remanufacturing is carried out by the OEM, if β is higher than a threshold level β^*, imposing partial regulation by itself induces some level of remanufacturing and stricter take-back legislation (achieved by increasing β) never causes a decrease in the level of remanufacturing (Theorem 2). Our numerical results indicate that the same insights hold true for the three- and four-period problems. However, while the same β^* induces *some* remanufacturing for a range of products as long as their cost structures are similar, the *amount* of remanufacturing depends on the life cycle length and the potential demand pattern as seen in Figure 2.4. On the figure, the levels of remanufacturing induced by partial regulation alone (at $\beta = 0.7$) are the data points corresponding to $\beta_R = 0$. When the product's demand is stable or close to stable (e.g., Q_2=Q_3=600 or 700) throughout its life, the collection target alone induces a relatively high level of remanufacturing. However, for products where demand starts low, peaks steeply and then drops again (e.g. Q_2=Q_3=900), the total remanufacturing levels are low. This is because a high level of remanufacturing is not profitable either in the period when the demand first peaks (due to low availability of cores since demand is low prior to the peak) or towards the end of the product life cycle (plenty of cores are available but the potential market size is small). As a result, the total remanufacturing levels are lower when compared to a product with stable demand.

A separate remanufacturing target, β_R, will secure a minimum level of remanufacturing and by Proposition 2 we show that such β_R should be set appropriately when compared to β (the higher β is, the higher β_R should be). When we take longer product life cycles and changing demand patterns into account, our numerical study shows that β_R should also depend on how fast the diffusion is, i.e., when and by how much the demand peaks over the product's life. As seen in Figure 2.4, for a product, whose demand first increases and then decreases very steeply (e.g. Q_2=Q_3=900), even a very low β_R may trigger a large increase in the remanufacturing level while for a product with completely stable demand (i.e. Q_2=Q_3=600), β_R of any value does not change the remanufacturing level at all. For products whose demands first increase and then decrease very steeply, remanufacturing is not profitable in either the period where demand peaks

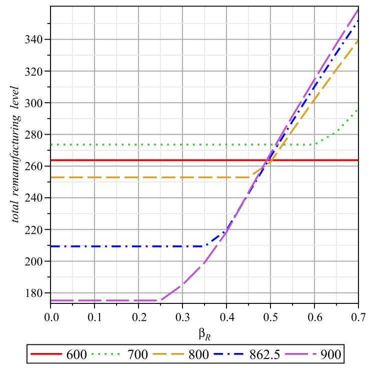

FIGURE 2.4: Total remanufacturing level over the product's 4-period life cycle as β_R changes for different levels of demand diffusion identified by the values of $Q_2 = Q_3$ ($c_M = 315$, $c_R = 42$ and $\beta = 0.7$)

or the last period where the potential market size is very small. Even a small β_R forces the OEM to do some remanufacturing in these periods, which results in a huge increase in the total remanufacturing level. But this finding also indicates that β_R should be chosen carefully, especially for products whose demands exhibit clear life cycle effects. If the potential market size declines significantly in the last period, unrealistically high remanufacturing targets might be impossible to meet because remanufacturing is unprofitable and will upset the manufacturers. Figure 2.4 also illustrates that when we take life cycle effects into account, it is difficult to find a single β_R that can increase remanufacturing levels beyond what is already achieved by partial regulation for a wide range of products. Hence, imposing the same target level across one or more product categories may not increase remanufacturing by an amount high enough to justify the additional administrative burden associated with implementing more complex legislation.

Finally, we investigate how the incentive to decrease the remanufacturing cost (at the expense of higher manufacturing cost) changes with life cycle effects with the following example.

	Π_M^* under Base Scenario			Π_M^* under DfR Scenario		
$[Q_1, Q_2, Q_3]$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$	$\beta = 0.25$	$\beta = 0.5$	$\beta = 0.75$
[1200,1200,1200]	159,799.8	148,772.5	138,139.5	158,783.7	152,272.2	145,067.2
[1000,1600,1000]	216,706.5	205,683.4	195,054.4	216,355.4	209,434.3	201,129.5
[800,2000,800]	387,663.1	376,644.2	366,019.4	387,234.4	377,963.3	368,137.0
[700,2100,700]	448,222.6	437,204.7	426,534.6	447,335.4	436,810.4	426,236.2

TABLE 2.5: The OEM's profits under base and DfR scenarios for different levels of potential market sizes and β values

Example 4 *Consider the monopoly model for the case where* $\phi = 0.95$, $\alpha = 0.15$, $c_C = 15$, $c_D = 55$, *and* $\beta_R = 0$. *We consider two scenarios: under the base scenario* $c_M = 710$ *and* $c_R = 185$ *and under the DfR scenario* $c_M = 720$ *and* $c_R = 55$. *Table 2.5 summarizes the OEM's profits for different levels of potential market sizes and* β *values.*

Results from Example 4 presented in Table 2.5 is representative of our insights. As in the two-period scenario, the incentive to decrease the remanufacturing cost increases as β increases. In addition, for high levels of β (e.g., $\beta = 0.75$), the incentive is highest for mature products with stable demand throughout the product's life cycle because the OEM can recoup the benefits of lower remanufacturing cost by satisfying a larger percentage of demand using remanufactured products. For moderate levels of β (e.g., $\beta = 0.50$), the incentive is highest for innovative products with a moderate diffusion rate, i.e., moderate levels of Q_2. For the same β (and assuming that β is binding, which is the case in this example), the amount of cores collected up to the last period is higher in the presence of life cycle effects than under a stable demand pattern. As long as the demand for the product does not go down steeply towards the end of its life cycle, the OEM can reap the benefits of lower remanufacturing cost by remanufacturing and selling a large percentage of the cores collected. If the market size goes down steeply in the last period (e.g., 700 in the example), the OEM has no incentive to reduce the remanufacturing cost.

2.8.3 An OEM Facing Competition under Life Cycle Effects

We can show that Propositions 4 and 5 still characterize the optimal policies for an OEM facing competition under life cycle effects. In this section, using this analytical characterization and a numerical study, we test the robustness of our results from Section 2.5.

When remanufacturing is carried out by a third party, the OEM may preemptively collect cores to prevent high levels of remanufacturing, but in Section 2.5, we show that OEM does not do this if c_R is high. Similarly, for life cycles longer than two periods, preemptive collection is not optimal in any period t if $c_R > \alpha Q_t - c_D - 2c_C^R$. When $Q_t = Q$ $\forall t$, this implies that preemptive collection is never optimal throughout the product's life if the remanufacturing cost c_R is high enough. However, when the potential market size changes over the product's life, even if c_R is high, the OEM may preemptively collect in a period with a large enough potential market in order to be the exclusive seller and make a large profit.

As in the two-period model, through our numerical study we find that preemptive collection is optimal for the OEM for a wider range of cost parameters under partial regulation— in other words, the OEM may switch to the preemptive collection strategy after partial regulation is imposed. The only exception is products whose demands exhibit a *very* steep increase in the second period, remain high one or more periods, and finally exhibit a *very* steep decrease. In periods of high demand, the OEM follows preemptive collection for a wide range of cost parameters even in the absence of legislation. Hence imposing legislation does not change the behavior of the OEM. And in the last period, the OEM does not follow preemptive collection with or without legislation because the demand is low and the remanufacturer is not a strong competitive threat.

That preemptive collection is optimal for a wider range of parameters under legislation suggests that legislation may negatively impact the total remanufacturing levels. To investigate that, we directly look at how the remanufacturing levels change as legislation becomes stricter (i.e. as β increases from $\beta = 0$) and observe that the total remanufacturing level may go down if β exceeds a threshold. Legislation's effect on remanufacturing levels is the most negative for innovative products where the potential market is *moderately* large in the second *and* third periods. In this case, the remanufacturer is a competitive threat in the third period due to ample availability of cores accompanied by a high market potential and the threshold β above which the OEM collects preemptively is *low*.

2.9 Conclusions

This chapter contributes to the important and growing literature on supply chain operations under environmental legislation. Manufacturers are likely to face take-back legislation in an increasing number of industries as the amount of waste grows in developed and developing countries alike. Even though such legislation in general receives enthusiasm from environmentalists, there is debate on implementation details and it is not clear exactly what type of legislation will achieve the ultimate goals of reducing waste, inducing better design and reuse. By analyzing a stylized model of take-back legislation we provide insights on how such legislation affects an OEM's decisions on manufacturing and remanufacturing[2] levels and the corresponding prices.

Considering an OEM with in-house remanufacturing capability, a question of interest is whether forcing the manufacturer to take her products back at end-of-life induces higher levels of remanufacturing. If the cost of manufacturing is very low, not only is the answer no, but legislation does not create incentives to reduce remanufacturing costs either. However we find that targets on collection levels may induce the remanufacturing of a high percentage of the collected cores *if* the collection targets are correctly chosen given the product's cost characteristics and the customers' valuation of remanufactured products. In that case, legislation also gives incentive to the OEM to reduce the remanufacturing cost unless the cost of manufacturing is too low. On the other hand, when remanufacturing is profitable as is (low cost, high customer valuation), legislation is redundant.

When the OEM is competing with a third-party remanufacturer, legislation may be more successful than expected. We find examples where remanufacturing levels are higher under competition than under monopoly. Hence introducing competition rather than imposing legislation can result in higher remanufacturing levels. In other words, in order to increase the level of remanufacturing policy makers might subsidize the third-party remanufacturers rather than imposing take-back legislation on the OEMs. In terms of legislation's ability to induce the OEM to reduce remanufacturing cost, similar to the case of monopoly, we find that such an incentive exists unless the cost of manufacturing

[2]In our model we focus on a specific form of reuse which is remanufacturing. In addition, our model does not explain the situations where recycling is a very profitable option.

is too low. While, in general, take-back legislation can be implemented with positive outcomes in industries where third parties remanufacture, our results also indicate that in these types of industries, legislators should be especially cautious about setting the collection and/or remanufacturing targets—ad hoc legislation may lead to undesirable results such as a drop in remanufacturing levels.

While take-back legislation diverts end-of-life products from landfills and may increase remanufacturing levels, in general, it hurts consumer welfare. We find that legislation costs are passed onto the consumers and the consumer surplus goes down as a result of legislation. Except in rare cases, consumers are more negatively impacted in industries where there is competition between an OEM and a third-party remanufacturer; their surplus is lower than that of the consumers in an industry where an OEM both manufactures and remanufactures.

When the product's life cycle is longer than two periods and the potential market size changes over the product's life, remanufacturing is not a very profitable business and legislation that only stipulates product take-back and disposal (for the most part) fails to induce higher levels of remanufacturing or reduction in remanufacturing costs. Especially if there is competition between an OEM and a remanufacturer, legislation might incentivize the OEM to push the remanufacturer out of the market through preemptive collection in an effort to sell as many new products as possible while the demand for the product lasts.

CHAPTER 3

Complying with Take-Back Legislation: A Cost Comparison and Benefit Analysis of Three Compliance Schemes

3.1 Introduction

Several countries around the world now impose take-back legislations that hold producers responsible for financing the collection, proper treatment/recycling and recovery of their products (e.g., tires, batteries, electronic products such as computers) discarded by customers at the end of their useful lives. Although the implementations of take-back legislations vary from one country to another, the common fundamental objectives are reducing the amounts as well as the environmental impact of end-of-life (EOL) products. The best-known such legislation is the Waste Electrical and Electronic Equipment (WEEE) directive by the European Union. Similarly, Japan and twenty states (plus New York City) in the U.S. have passed laws creating e-waste take-back programs.

In an industry regulated with product take-back legislations, each producer meets her obligations through a *compliance scheme*. Choosing a compliance scheme entails decisions regarding the choice of collection channel, treatment processes, and partner firms. Both policy makers and producers see compliance as a valid cost of business, and since the true cost of EOL product management is internalized by producers under legislation, they seek ways to reduce the cost of compliance (FES, 2003; WEEEForum, 2008). Therefore, a producer's goal is to choose a compliance scheme that ensures environmentally-conscious management of discarded products in the most cost effective way.

In most implementations of take-back legislation, one or more of the following three compliance schemes are available/allowed: First, the producer may set up an *individual scheme* by collecting and treating only his brand-name products either himself or by contracting with a third party. Larger companies such as Cisco operate individual compliance schemes and thus only pay for their own collection and treatment costs. Second, a subset of producers may form a collaboration and set up their *collective scheme* that collectively takes care of discarded products on behalf of its producer members. In 2002, Braun, Electrolux, HP and Sony, set up the European Recycling Platform (ERP) as a compliance scheme in response to the WEEE Directive. A special case of the collective scheme is the *national collective scheme*, where all producers who are covered by legislation are members. National collective schemes usually operate in small countries (Bohr, 2007). For instance, Recupel is the national collective body for the collection and treatment of WEEE in Belgium. Similar national schemes also exist in the Netherlands and Sweden. Under a collective scheme, the total compliance cost of the collaboration is recorded by a non-profit central authority without distinguishing brand names and then allocated among the members with respect to a previously agreed-on rule. Finally, a subset of producers may set up their *collective scheme with individual financial responsibility* where discarded products are collected and treated collectively; however, each producer member pays for the actual treatment and collection costs of her brand-name products. This compliance scheme requires an elaborate sorting of EOL products with respect to brands and models as well as keeping track of the treatment processes each model goes through. The Dutch take-back program for information and communication technology (ICT) equipments that was run until 2002 is an example (Tojo, 2003). Similarly, under Japan's Specified Home Appliances Recycling Law (SHARL), brands treated in each plant are separately recorded and thus, it possible to calculate the exact treatment cost for individual products (Tojo, 2003).

Among the three compliance schemes, individual compliance is likely to be the most expensive choice as some components of compliance cost may be proportionately higher. Still, some companies (e.g., Cisco) prefer individual compliance because it gives them greater control over EOL product management and they are in a better position to recoup the benefits of their environmentally-friendlier products, for example, in the form of lower treatment costs. On the other hand, it is well documented that recycling and treatment activities become economically feasible only when a certain volume of items is processed collectively (Tojo, 2003). When a firm sets up her own compliance scheme and collects/treats her own brand-name products alone (either herself or through a third-party), total volume might be small and thus recycling/treatment may not be economically viable. For that reason, collective schemes are established to take advantage of the economies of scale inherent in the treatment activities and the total compliance costs of producers, who participate in such schemes, may be lower.

In this chapter, we address the question of (given the choice) which compliance scheme should a producer follow in order to minimize the total cost of compliance. For example, in Maine, producers are given two options: (i) If the total return share is more than 5%, they can start an individual/collective scheme, (ii) otherwise they must join the collective scheme to manage the collection and treatment of their discarded electronic products. In response to the alternative compliance schemes allowed by legislations, companies may choose different ones at different locations. For example, Dell opted to be a member of collective program in Maine while reserving the right to update her decision. Dell's senior compliance manager explains that the reason behind this decision is Maine's relatively small population (Toto, 2007). However, elsewhere (e.g., in Maryland) Dell established her own individual take-back program.

One of our goals is to identify the key market and operating parameters (e.g., relative market shares of the producers, different components of compliance cost such as cost of collection, treatment, etc.) that make one scheme preferable to the others from the producer's point of view. Take-back legislations aim to divert as many EOL products as possible from landfills to environmentally sound treatment options. For that reason, some implementations of take-back legislations, such as the WEEE Directive, impose target levels on the amount of EOL products that should be collected and treated properly. While this seems reasonable from an environmental point of view, meeting the collection targets specified by the legislation might affect the total compliance cost as well as a compliance scheme's relative cost effectiveness. Hence, collection targets may force a producer to switch to a different compliance scheme and indirectly impact other environmental outcomes of legislation in addition to collection levels. To address this question, we also analyze if and how a producer's compliance scheme preference changes in the presence of collection targets.

Even though we specifically model and analyze producers' responses to take-back legislation, our research has implications for policy makers as well. Given the choice, the producers will pick the compliance scheme that minimizes their total compliance costs and may fall short on the environmental benefits that the legislation is trying to achieve. By identifying the conditions under which producers are likely to choose a specific compliance scheme, our research provides insights on when governments should impose or provide incentives for a scheme with higher environmental benefits. Specifically we answer the following question that concerns policy makers: Depending on the market size and initial return rate of EOL products which compliance scheme achieves the highest collection rates?

By shifting the cost of EOL product management from society to producers, an underlying motivation for take-back legislations is to create incentives for designing products that are easier and cheaper to treat/recycle. However, it is argued that since producers do not directly bear the true costs of their own products, collective schemes do not give incentives for designing products that are easier and cheaper to treat at the end of their lives. Hence, the choice of compliance scheme has a direct impact on how well the objectives of legislation will be met in terms of creating incentives for designing greener products. We compare the three compliance schemes we analyze with respect to the treatment levels achieved under each and specifically address the following question: Depending on the market size and initial return rate of EOL products which compliance scheme induces higher treatment levels through creating incentives for products that are easier and cheaper to treat?

Among other results, we find that collective scheme with individual financial responsibility in general yields lower cost than the other two alternatives. Since individual financial responsibility under a collective scheme is not easy to implement in practice, we also compare collective scheme -where the total cost is allocated with respect to a previously agreed rule- and individual scheme in terms of cost effectiveness and conclude that which scheme performs better depends on the initial collection rate maintained by government and the market shares of other partner firms. If the partner firms have large market shares then the collective scheme, in general, yields lower cost. Thus, when deciding which compliance scheme yields lower compliance cost a producer should consider the individual market shares of the partner firms in addition to the total market share of the collective scheme.

An underlying motivation for take-back legislation is to achieve higher environmental benefits in terms of achieving higher collection rates and creating more incentives for designing environmentally-friendlier products. We find that collective scheme with individual financial responsibility achieves superior environmental outcomes than the other two possible compliance schemes. When we compare the other schemes we find that, in general, the incentive for designing environmentally-friendlier products is higher but the collection rate is lower under individual scheme. Finally, if policymakers impose very high collection targets, collective scheme might become more cost effective and the incentive to design environmentally-friendlier products might decrease. Our findings suggests that, policymakers should be careful while imposing collection targets or a particular compliance scheme as these might result in conflicting environmental benefits.

The rest of the chapter is organized as follows: In §3.2 we review the relevant literature, in §3.3 we introduce our model as well as our assumptions. We characterize the optimal policy under the individual scheme, the collective scheme and the collective scheme with individual financial responsibility (IFR) in §3.4, §3.5, and §3.6, respectively. In §3.7, we compare the three compliance schemes with respect to cost effectiveness and the environmental benefits achieved. We conclude in §3.8. All proofs are relegated to Appendix C1.

3.2 Literature Review

The literature on the impact of different environmental legislations/standards on operational decisions is growing. Some examples are Atasu et al. (2009a) who explore the efficiency of the WEEE legislation, Plambeck and Taylor (2007) who model RoHS which regulates the use of hazardous substances in electronics products, Subramanian et al. (2007) who study compliance strategies under emission trading programs, and Corbett and Kirsch (2001) who identify the drivers behind the diffusion of ISO 14000.

Of this body of work, our research is related to the research stream on product take-back legislations, especially those papers that study product design decisions, collection channel choice, and compliance decisions. Whether legislations can provide incentives for environmentally-safer products has been studied in a number of papers. Zuidwijk and Krikke (2007) study two strategic responses to product returns under take-back legislations, namely product eco-design versus new recovery process technologies, and conclude that the former performs better. They also find that under the WEEE Directive, more incentives are needed to reward product eco-design. Subramanian et al. (2009) study a manufacturer who invests in two design attributes (performance and remanufacturability) that affect the product's cost. They analyze the impact of extended producer responsibility (EPR) and supply chain coordination on product design decisions and propose contracts that can be used to achieve coordination. In other work, Jacobs and Subramanian (2009) study a two-period two-echelon model to identify the effects of EPR programs on design incentives. They show that when the cost of EPR programs is shared between the two echelons, the performance of the supply chain improves in terms of approaching the coordinated profit benchmark. Esenduran et al. (2010) explore the impact of take-back legislations on remanufacturing decisions and find that a manufacturer has incentive to decrease the remanufacturing cost at the expense of increasing the manufacturing cost both when she remanufactures in-house as well as when a competing third-party remanufacturer does. Finally, Plambeck and Wang (2010) study new product introduction under take-back legislation and find that legislations enforcing fee-upon-disposal such as individual EPR creates incentives for design for recyclability, while legislations enforcing fee-upon-sale fail to do so. Until now, researchers have identified and studied various settings and situations where design incentives exist under take-back legislations. Our research builds on the assumptions/models developed in these papers and explores how the compliance scheme choice affects the optimal treatment level of products.

Our research is also related to the literature on the procurement of EOL products (e.g., Guide et al. (2003), Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), Atasu et al. (2009b)). In particular, Savaskan et al. (2004) consider a manufacturer's choice between three collection channel options, i.e., collecting directly from customers, collecting through an existing retailer, or subcontracting to a third party. Savaskan and Van Wassenhove (2006), on the other hand, extends the previous research by incorporating competing retailers. We build our collection model on this literature; however, in contrast to these papers, we are not concerned about who is collecting the EOL products, because in the presence of take-back legislations it is the producers' responsibility to pay for the collection of EOL products regardless of who is collecting.

Compliance scheme choice under take-back legislations is one of the issues that has not been explored in depth in the literature. A notable exception is Boyaci et al. (2007) who study the compliance scheme choice in the presence of competing recyclers. In order to identify the impact of consolidation of the recycling industry as well as the intensity of competition in the recycling market, they model a two-stage game with two manufacturers and two recyclers and compare two different settings: one where the recyclers are independent (competitive scheme) versus one where a non-profit body allocates the waste products to the recyclers (monopolistic scheme). They conclude that the former often performs better as it achieves lower product prices as well as higher recycler and manufacturer profits. Although Boyaci et al. study compliance-related issues assuming that EOL products are treated at an additional cost to the producers, we take much different perspectives in generating insights about the compliance scheme choice. In contrast to Boyaci et al., we do not model the profit maximizing recycling industry but we allow the producers to choose the treatment level and collection rate of EOL products as well as with whom to collaborate in order to minimize the compliance cost. Another paper that studies compliance scheme choice is Atasu and Subramanian (2009). Modeling two profit maximizing manufacturers differentiated in terms of consumer preferences for their brands, the authors explore how individual and collective systems affect manufacturers' recyclability choices. While Atasu and Subramanian (2009) also aim to generate insights about individual and collective compliance, our assumptions, research questions and the resulting insights are notably different. In addition to modeling only two manufacturers, they assume that these manufacturers have no effect on the collection rate but they pay a treatment cost for every product sold in the market which implies that the collection rate is 100%. Another distinguishing assumption they use is that there is no economies of scale in treatment cost. Therefore, their model does not account for the fact that manufacturers might aim to increase the collection rates, which are generally less than 100%, and thus benefit from the economies of scale in treatment costs (an example is Nokia). They do not specifically model cost allocation by market share or individual financial responsibility but assume that each manufacturer pays a unit weighted treatment cost (using an exogenous weight). They compare the manufacturer profits under individual versus collective compliance and find that the low-end manufacturer always prefers individual compliance whereas the high-end manufacturer might benefit from collective compliance if the brand differentiation is high and its influence on treatment cost is low. In our study, by explicitly modeling n producers with different market shares, we generate insights on which compliance scheme is more cost effective depending on the collection rates and how market shares of the firms affect the results. By allowing the collection rates to be endogenous in the model, we are also able to generate insights regarding when collective compliance may generate superior incentives for environmentally-safer product design.

3.3 The Model

In order to compare the three compliance schemes, we model total compliance cost in detail. The major cost components are collection cost, treatment cost, recycling and disposal costs of EOL products. Stevels (2004) states that collection and sorting comprise 30% of total cost while treatment (treatment includes recycling and environmentally-sound disposal) accounts for 50-55%. In addition to the collection and treatment costs, we also model a "cost of increasing the treatability level" to capture the fact that producers can lower the treatment costs of their EOL products by making costly changes in design, materials, manufacturing methods, etc. In what follows we introduce the notation and the different components of compliance cost in detail.

1. Costs Related to Collection Activities: Here we capture the cost components that are common to collection activities regardless of legislation structure, firm level policies, and the size of the collected product, etc..

1.a. Cost of Maintaining a Collection Rate: Collection rate is the amount of EOL products collected (as a percentage of the products sold in the market in the previous sales period) and it depends on how the collection network and facilities are set up (Kokkinaki et al., 2004). Since the rate of collection depends on how convenient collection is to consumers, how well the producer facilitates product returns, etc., the producer needs to incur a cost to achieve and maintain a collection rate. In the literature it is argued that there is diminishing returns to investment to increase collection rates (Savaskan and Van Wassenhove, 2006) and the total investment to achieve a collection rate of τ is modeled as $I = \eta\tau^2$ where η is a scaling parameter that indicates how difficult/costly it is to increase the collection rate. The scaling parameter η depends, among other things, on population size, geographic region of interest, and how willing the consumers are to return the product.

Under product take-back legislations, typically it is the producer's responsibility to finance and/or provide accessible and efficient collection facilities as well as to publicize adequate information to the customers. In order to maintain a collection rate of τ, producers invest in running collection/sorting centers and raising customers awareness of the collection programs. For example, Nokia who recycles old cell phones, invests to increase their collection rate by expanding the accessibility of their take-back channels, developing awareness-building programs and putting up more take-back bins (Nokia, 2006). Similarly, LG has increased the number of mobile phone drop-off points in order to increase the accessibility to her take-back channels (www.lg.com/global/sustainability/environment/take-back-recycling/mobile-phones.jsp).

However, in many developed countries, some level of product take-back already exists even before the legislation comes into force, being facilitated by non-profits, municipalities, and local organizations, and therefore the producers do not need to incur additional costs to achieve and maintain this initial return rate, which we call the *base collection rate*, τ_0 (which can be set to zero). For example, before the Norwegian WEEE legislation came into force in 1999, the collection rate was around 8.2% in Norway (Ronningen, 2005). Similarly, when the Japanese SHARL[1] was introduced in 2001, the collection rate was 2.5 kg/capita (Van Rossem, 2008). After accounting for the cost of the base collection rate τ_0 (on which the producer does not incur a cost), the producer incurs a total cost of

$$TCC_1(\tau) = \eta\left(\tau_0 + \tau\right)^2 - \eta\tau_0^2 = 2\eta\tau_0\tau + \eta\tau^2$$

to maintain a final collection rate of $\tau_0 + \tau$. The reason we assume incremental cost structure as opposed to marginal cost structure is the fact that if the base collection rate is higher, it is more difficult and thus costlier to increase the collection rate further.

1.b. Cost of Collection and Sorting: In addition to the cost of maintaining a collection rate, the producers also incur a unit cost of collection and sorting for each EOL product collected. Collection and sorting activities include the transportation from customers to collection facilities (unless customers drop off the products at a collection facility), the transportation from collection facilities to treatment/processing centers as well as the sorting of collected items at the collection facilities. As a result, the cost of collec-

[1]Japanese SHARL has fines for consumers who do not place the products in the collection bins and this increases the collection rate

tion/sorting is a function of the total volume collected. If the total collection rate is τ_T and the total number of items sold by producer i is $\alpha_i Q$ where α_i is the market share of producer i and Q is the total market size, the producer pays for the collection and sorting of $\alpha_i Q \tau_T$ units of EOL product at a per-unit collection cost of c_C. Therefore, when the total collection rate is τ_T the total collection and sorting cost is $c_C \alpha_i Q \tau_T$.

However, even when the producer targets achieving a total collection rate of $\tau_0 + \tau$, the realized collection rate may be lower due to the uncertainty in customer return rates. While the base collection rate τ_0 is the well-established return rate in the market, it is not certain how the customers will react to the producer's efforts to increase the collection rate by τ. When the producer makes an investment to reach τ percent more customers, not every target customer responds to these efforts and eventually the producer may realize an additional collection rate of τx (and a total collection rate of $\tau_T = \tau_0 + \tau x$). We assume that x is a random variable distributed between x_L and x_H with pdf $f(x)$, and mean μ and standard deviation σ. The random variable x is interpreted as the customer's likelihood of returning the EOL product. This also explains the rationale behind the multiplicative uncertainty model. For the very same reason it is also plausible to assume $\mu \leq 1$. Among others, Toktay et al. (2000) and Wojanowski et al. (2007) have also explicitly modeled the probability that a customer will eventually return his EOL product.

To sum up, when producer i targets a collection rate of $\tau_0 + \tau$ the cost of collection and sorting she incurs is $TCC_2(\tau, x) = c_C \alpha_i Q(\tau_0 + \tau x)$ while the total collection-related cost is

$$TCC(\tau, x) = TCC_1(\tau) + TCC_2(\tau, x) = 2\eta\tau_0\tau + \eta\tau^2 + (\tau_0 + x\tau)\,\alpha_i Q c_C.$$

2. Cost of Treatment: The cost of treatment mainly comprises of the cost of disassembly, the cost of environmentally sound disposal of toxic and hazardous substances or components, and the cost of shredding/processing of remaining metals, plastics, glass, and circuitry. We assume that the unit cost of treatment is c_R and the total cost of treatment is a function of the total amount treated, i.e. $\alpha_i Q(\tau_0 + \tau x)$. Since some treatment processes such as smelting/shredding are capital intensive, it is argued that the cost of treatment decreases due to economies of scale (GAO, 2005; Hageluken, 2006). To capture this in our cost model, similar to Boyaci et al. (2007), we subtract $\theta(\alpha_i Q(\tau_0 + \tau x))^2$ from the total cost of treatment, i.e. $c_R \alpha_i Q(\tau_0 + \tau x)$, where θ is the economies of scale factor.

One way to reduce the cost of treatment activities is designing products that are easier to dismantle and treat. For that reason, some producers make costly design changes to facilitate dismantling. Nokia, for example, puts effort in improving the treatability level of mobile phones in order to facilitate the extraction of valuable materials in an efficient manner (Nokia, 2006). Similarly, producers might remove the toxic substances from their product designs and thus decrease the cost associated with disposal. For example, HP eliminated mercury fluorescent tubes and made the treatment of the display screens easier and cheaper (HP, 2008). Since the cost of treatment processes depend on the treatability level of the product (Stevels, 2003) and the associated costs might be brought down by increasing the product's treatability level, we model the unit treatment cost as

$c_R - \beta\xi$ where β is the savings achieved per treatability level. Note that, products treatability level ξ is an index where higher treatability level implies an environmentally-friendly product design and a lower treatment cost. Similar to Subramanian et al. (2009) our formulation implies that unit cost of treatment decreases linearly with the level of treatability ξ.

Putting everything together, given the realized number of units collected is $\alpha_i Q(\tau_0 + \tau x)$, the total cost of treatment is

$$TCT(\tau, \xi, x) = (c_R - \beta\xi)\alpha_i Q(\tau_0 + \tau x) - \theta\alpha_i^2 Q^2(\tau_0 + \tau x)^2.$$

Even though we call c_R "a cost", depending on product characteristics, the revenue generated from selling recycled materials may outweigh the total cost associated with treatment. That is why we allow c_R to be negative, implying that treatment of the product may be profitable. For example, the revenue received from the precious metals concentrated in electronics products such as cell phones, CPUs, and laptops[2] might indeed exceed the cost of treatment, while for televisions the revenue does not offset the cost of treatment (HECC, 2005; Sodhi and Reimer, 2001).

Generally, regardless of the compliance scheme choice, treatment processes are carried out by third-party companies. When treatment is carried out by third-parties our cost structure captures the price charged for treatment by the these companies. In this chapter, neither the pricing decision of third-party companies nor the possible competition between the these companies are modeled. Since we want to focus on how different com-

[2]For example, 1 metric ton of electronic scrap from obsolete computers contains more gold than 17 metric ton of gold ore (Bleiwas and Kelly, 2001).

pliance schemes affects the collection and treatability decisions of a firm, we do not model the treatment industry in detail. While a company who contracts with a third-party on her own might be quoted a higher price, in order to facilitate the comparison between the compliance schemes we assume that the treatment cost structure/parameters does not differ with the compliance scheme choice.

3. Cost of increasing product's treatability level: Take-back legislations aim to make easily-treatable products more cost effective by establishing feedback loops from downstream (EOL product management level) to the upstream producer. If a producer prefers to, she can increase the treatability level of her products and incurs an additional cost to achieve that level. We call this cost component *the cost of increasing the treatability level* and it depends on product design, the materials, the processes followed to manufacture the product, etc. For example, HP uses common fasteners and snap-in features and avoids screws, adhesives and welds to make the dismantling of products easier (HP, 2008). When HP spends \$1 more on the design cost to reduce the number of different screws in each computer, she saves Noranda Recycling Inc., who recycles used electronics for HP, approximately \$4 in disassembly costs (GAO, 2005).

We assume that the cost to achieve treatability level ξ for producer i is convex and increasing in ξ. Therefore for a producer who produces $\alpha_i Q$ units of product, the total cost of manufacturing products at treatability level ξ is $TCG(\xi) = k\xi^2\alpha_i Q$ where k is the cost scaling factor. Increasing marginal cost for achieving higher levels of environmental-friendliness is a common assumption both in the operations (e.g., Subramanian et al. (2009)) and economics (e.g., Gonzalez and Fumero (2002), Nordhaus (1991)) literatures.

The rationale is that each additional unit of effort put in reducing the product's environmental impact is more difficult and thus costlier to execute. In addition, beyond a threshold, the cost of achieving lower environmental impact might exceed the price that customers are willing to pay for the product and therefore, decreasing the product's environmental impact below some threshold level may be infeasible; our cost structure captures this effect.

For the remainder of this chapter, we use subscripts S, C, F to denote firm i's cost, decisions or problem parameters under the individual scheme, the collective scheme, and the collective scheme with IFR, respectively. In addition, we use superscripts UB and LB to denote the upper and lower bounds on the problem parameters. First, we analyze the problem of minimizing the total compliance cost under the individual compliance scheme.

3.4 Individual Scheme

When producer i sets up her own individual compliance scheme, the total compliance cost is

$$\begin{aligned}
TC_S(\tau_S, \xi_S, x) &= TCC_S(\tau_S, x) + TCT_S(\tau_S, \xi_S, x) + TCG_S(\xi_S) \\
&= 2\eta\tau_0\tau_S + \eta\tau_S^2 + c_C\alpha_i Q\left(\tau_0 + x\tau_S\right) + (c_R - \beta\xi_S)\alpha_i Q(\tau_0 + \tau_S x) \\
&\quad -\theta\alpha_i^2 Q^2(\tau_0 + \tau_S x)^2 + \alpha_i Q k \xi_S^2 \\
&= 2\eta\tau_0\tau_S + \eta\tau_S^2 + (c_C + c_R - \beta\xi_S)\alpha_i Q(\tau_0 + \tau_S x) - \theta\alpha_i^2 Q^2(\tau_0 + \tau_S x)^2 \\
&\quad +\alpha_i Q k \xi_S^2.
\end{aligned}$$

Under the individual scheme, the producer chooses both the optimal treatability level ξ_S and the collection rate τ_S for her products. As the uncertainty in terms of product returns is not resolved at the time of decision, the producer minimizes her expected total cost of compliance:

$$(P1) \quad \underset{1-\tau_0 \geq \tau_S \geq 0,\, \xi_S \geq 0}{Min} E_x(TC_S)$$

where

$$\begin{aligned}
E_x(TC_S) &= \int_{x_L}^{x_H} TC_S(\tau_S, \xi_S, x) f(x)\, dx \\
&= 2\eta\tau_0\tau_S + \eta\tau_S^2 + \alpha_i Q(c_C + c_R - \beta\xi_S)\left(\tau_0 + \tau_S \int_{x_L}^{x_H} x f(x)\, dx\right) \\
&\quad -\theta\alpha_i^2 Q^2 \left(\tau_0^2 + 2\tau_0\tau_S \int_{x_L}^{x_H} x f(x)\, dx + \tau_S^2 \int_{x_L}^{x_H} x^2 f(x)\, dx\right) + \alpha_i Q k \xi_S^2 \\
&= 2\eta\tau_0\tau_S + \eta\tau_S^2 + \alpha_i Q(c_C + c_R - \beta\xi_S)\left(\tau_0 + \tau_S\mu\right) \\
&\quad -\theta\alpha_i^2 Q^2 \left(\tau_0^2 + 2\tau_0\tau_S\mu + \tau_S^2(\mu^2 + \sigma^2)\right) + \alpha_i Q k \xi_S^2.
\end{aligned}$$

3.4.1 Characterization of Optimal Policy

In order to guarantee positive semidefiniteness of the Hessian corresponding to the Lagrangian function of the problem $(P1)$, we assume that condition $(A1)$ holds.

$$4k(\eta - \theta\alpha_i^2 Q^2(\mu^2 + \sigma^2)) - \alpha_i Q \mu^2 \beta^2 \geq 0 \quad (A1)$$

Condition $(A1)$ characterizes a lower bound on η such that $\eta \geq \eta_S^{LB}$ where $\eta_S^{LB} = \frac{\alpha_i Q(4k\theta\alpha_i Q(\mu^2+\sigma^2)+\beta^2\mu^2)}{4k}$ and implies that when it is very cheap to increase the collection rate further ($\eta \leq \eta_S^{LB}$) then the Hessian is negative semidefinite and the optimum solution is observed at the corner points, i.e., either $\tau_S^* = 0$ or $\tau_S^* = 1-\tau_0$. Therefore, by assuming $(A1)$, we restrict attention to the region where $\eta \geq \eta_S^{LB}$ and avoid the trivial solutions. In Proposition 14 we characterize producer i's optimal solution under the individual compliance scheme.

Proposition 14 *If* $\eta \geq \frac{\alpha_i Q\mu(4k\theta\alpha_i Q+\beta^2-2k(c_R+c_C))}{4k} = \eta_S^{UB}$ *then* $\tau_S^* = 0$ *regardless of* τ_0*. If* $\eta_S^{UB} \geq \eta \geq \eta_S^{LB}$ *then there exist two threshold levels* κ_S^I *and* κ_S^{II} *on* τ_0 *(expressions for* κ_S^I *and* κ_S^{II} *are provided in the proof in Appendix C1) such that*

1. *if* $0 \leq \tau_0 \leq \kappa_S^I$ *then* $\tau_S^* = 0$
2. *if* $\kappa_S^I \leq \tau_0 \leq \kappa_S^{II}$ *then* $0 < \tau_S^* = \frac{\tau_0(4k\theta\alpha_i^2 Q^2\mu+\alpha_i Q\mu\beta^2-4k\eta)-2k\alpha_i Q\mu(c_C+c_R)}{4k(\eta-\theta\alpha_i^2 Q^2(\mu^2+\sigma^2))-\alpha_i Q\mu^2\beta^2} < 1-\tau_0$*, and*
3. *if* $\kappa_S^{II} \leq \tau_0$ *then* $\tau_S^* = 1-\tau_0$

The optimal treatability level in terms of τ_S^* *is* $\xi_S^* = \frac{\beta(\tau_0+\mu\tau_S^*)}{2k}$*. For a possible characterization of* τ_S^* *and* ξ_S^* *with respect to* τ_0 *see Figure 3.1.*

Proposition 14 shows that the optimal level of treatability is higher when the collection rate and the savings (β) achieved through higher treatability levels are higher. For example, one characteristic that increases β is high metal content because using metal parts instead of their plastic counterparts facilitates recycling and reduces treatment cost (Tojo, 2003). Although it is difficult to measure a product's treatability level, one possible measure is its recycling level, i.e., the percentage of product's total weight that

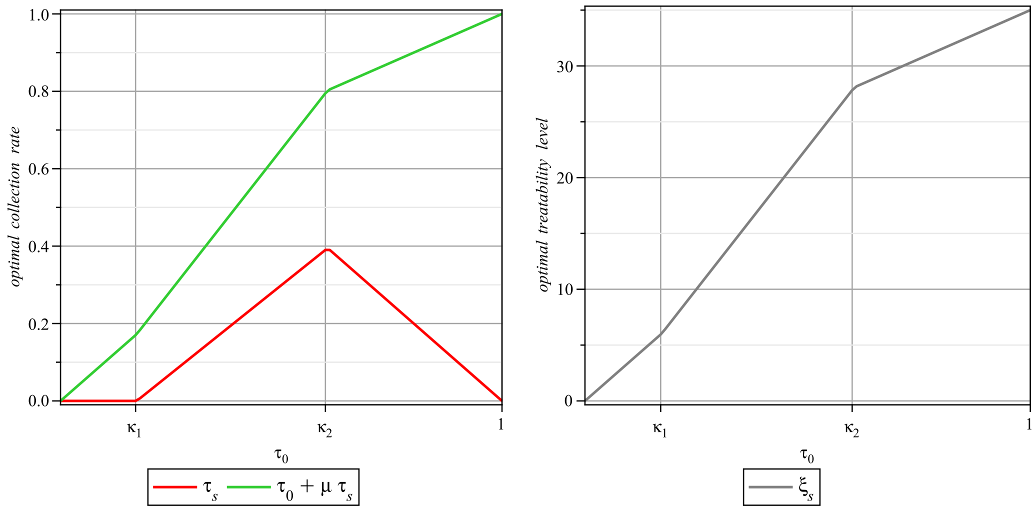

FIGURE 3.1: A possible characterization of optimal (a) collection rate and (b) treatability level with respect to the base collection rate τ_0 under individual compliance

can be recycled. In the member EU states, it is estimated that the recycling level is only 4% for coffee machines while it is as high as 43% and 63% for vacuum cleaners and white goods respectively (Abele et al., 2005). It is also known that both the collection rate and the metal content are low for coffee machines and high for vacuum cleaners and, especially, for white goods. Therefore, Proposition 14 may explain why recycling level is low for coffee machines but much higher for white goods.

As for the optimal collection rate, Proposition 14 implies that when the unit cost of collection and treatment $(c_R + c_C)$ is higher, optimal collection rate is lower. In addition, if it is expensive to maintain a collection rate, i.e., $\eta \geq \eta_S^{UB}$, then regardless of the base collection rate maintained by government and non-profits, the producer never invests in increasing the collection rate further. On the other hand, when it is not costly to maintain a collection rate, i.e., η is low, the producer may choose any feasible collection rate, i.e., $0 \leq \tau_S^*(\tau_0) \leq 1 - \tau_0$, depending on the base collection rate. Next, with Corollary 2, we summarize how optimal collection rate τ_S^* changes with respect to the base collection rate τ_0.

Corollary 2 *If $\eta \geq \eta_S^{UB}$ then $\tau_S^* = 0$ regardless of τ_0. Otherwise (i) if $\tau_0 \leq \kappa_S^I$ then $\tau_S^* = 0$ and does not depend on τ_0, (ii) if $\kappa_S^I \leq \tau_0 \leq \kappa_S^{II}$ then τ_S^* is increasing in τ_0 and (iii) if $\tau_0 \geq \kappa_S^{II}$ then τ_S^* is decreasing in τ_0.*

Corollary 2 together with Proposition 14 implies that the expected optimal collection rate $(\tau_0 + \mu\tau_S^*)$ is increasing in the base collection rate, τ_0. This implies that the optimal treatability level is increasing in the base collection rate, too. Although policy makers may choose τ_0 low and delegate all the cost and responsibility of increasing the collection rate to producers in order to decrease the costs incurred by the society, they need to consider the tradeoff between incentivizing higher treatability levels and the cost of maintaining a collection rate.

With Corollary 3, we summarize how optimal collection rate changes with η (the parameter that measures how costly it is to maintain a collection rate), σ (variability in the collection rate), and α_i (firm i's market share).

Corollary 3 *If $\eta \geq \eta_S^{UB}$ then $\tau_S^* = 0$ regardless of η, σ, or α_i. Otherwise,*

- *as η increases, κ_S^I and κ_S^{II} increase and τ_S^* weakly decreases,*
- *as α_i increases, κ_S^I and κ_S^{II} decrease and τ_S^* weakly increases,*
- *as μ increases, κ_S^I and κ_S^{II} decrease and τ_S^* weakly increases,*
- *as σ increases, κ_S^I remains the same while κ_S^{II} decreases and τ_S^* weakly increases.*

Corollary 3 shows that when it is not costly to maintain a collection rate and the firm's market share is high, the optimal collection rate is higher. For example, Nokia, the market leader in mobile phones in the UK and a follower of the individual compliance scheme, promotes higher collection rates by raising consumer awareness and helping the customers find the nearest site for product returns (Canning, 2006). Since Nokia phones are portable, they are easily dropped-off at the nearest store and the cost of maintaining a collection rate is quite low (http://artsresearch.brighton.ac.uk/research/projects/if-lab/rubbish). Thus, Corollary 3 explains why Nokia aims to achieve higher collection rates.

Finally, Corollary 3 shows that variability does not affect the threshold on τ_0 above which τ_S^* is positive. Hence, the minimum base collection rate that would incentivize investing in higher collection rates does not depend on the variability of customer returns. However, once the optimal additional collection rate τ_S^* is positive then higher variability results in a higher collection rate. When τ_S^* is positive, the firm aims to exploit the economies of scale. Therefore, when the variability in customer returns is high, she targets a higher collection rate knowing that, due to high variability, the realized collection rate may be much lower than the target rate and thus she may fail to benefit from the scale economies in treatment activities.

3.5 Collective Scheme with Cost Allocation by Market Share

In order to comply with take-back legislations, producers may act collectively and set up collective compliance schemes. Collective schemes are usually run by a non-profit producer responsibility organization (PRO) that manages the collection and treatment activities collectively on behalf of its producer members (Toffel, 2003). For example, The European Recycling Platform (ERP) set up by Braun, Electrolux, HP and Sony ensures that its members are in compliance with the WEEE directive.

Under collective schemes, when the collective collection and treatment cost is realized, it is allocated among the members according to a previously-agreed-on rule. In practice, the most frequently used rule is allocating the total cost among the members with respect to their current market shares (Van Rossem, 2008). Therefore, in the rest of this section we focus our discussion on the market share model and in the remainder of the chapter, by *collective scheme* we imply *collective scheme with cost allocation by market share* unless otherwise stated.

We model the relationships among the n producers and a non-profit central authority, a so-called producer responsibility organization (PRO), under the collective scheme as a two-stage Nash game. In the first stage, each producer j decides on the treatability level ξ_j of her product as all n producers do the same by simultaneously minimizing their compliance costs given a collection rate. In the second stage, the PRO decides on the additional collection rate τ_C by minimizing the total cost of collection and treatment, i.e., $E(TCC_C) + E(TCT_C)$. Finally, the uncertainty on the product returns is resolved and the total costs are realized. The total collection and treatment cost is divided among the members of scheme according to their market shares.

Under collective systems, products under each category (for example, the WEEE Directive covers ten categories of EEE) are collected separately and the actual cost of treatment for each category is recorded. Fees from a specific category of products are used exclusively to cover the costs of that category and product categories do not cross subsidize other product categories (www.weee-forum.org). Therefore, we consider a collective scheme for products under a single category. We assume that the base collection rate, τ_0, and the additional collection rate is specific to a product category and thus every brand in the same category—every product in the collective scheme—achieves the same collection rate. We base our assumptions on many reviews for take-back legislations that report category-wise return rates. For instance, in the UK the collection rate is reported to be 56% for fridges and 77% for displays (www.360environmental.co.uk/ news/45/). In addition, we assume that when the investment $2\tau_0\tau\eta + \tau^2\eta$ is made, the increase in the collection rate of each product is the same. In other words, this lump sum investment raises the realized collection rate to the level of $\tau_0 + x\tau$ for each firm in the collaboration. Also, note that the cost of collection does not vary much among products of the same category. Hence, differentiation of the collection cost between the brands is not necessary and c_C does not depend on the brand.

3.5.1 Characterization of Optimal Solution

Collaborative Decision: Optimal Collection Rate

When the collective scheme aims to increase the total collection rate by τ_C, the total collection cost is

$$TCC_C(\tau_C, x) = 2\tau_0\tau_C\eta + \tau_C^2\eta + c_C A_n Q(x\tau_C + \tau_0),$$

whereas the total treatment cost is

$$TCT_C(\tau_C, \vec{\xi}, x) = \left(\sum_{j=1}^{n}(c_R - \beta\xi_j)\alpha_j Q(\tau_0 + \tau_C x)\right) - \theta A_n^2 Q^2(\tau_0 + \tau_C x)^2,$$

where A_n is the total market share of n firms, i.e., $A_n = \sum_{j=1}^{n}\alpha_j$. The non-profit PRO decides on the optimal collection rate, τ_C^* by minimizing the total expected cost of collection and treatment for the collaboration:

$$\underset{1-\tau_0 \geq \tau_C \geq 0}{Min} E_x \left(TCC_C(\tau_C, x) + TCT_C(\tau_C, \vec{\xi}, x)\right)$$

where $E_x(TCC_C(\tau_C, x)+TCT_C(\tau_C, \vec{\xi}, x)) = \int_{x_L}^{x_H} \left(TCC_C(\tau_C, x) + TCT_C(\tau_C, \vec{\xi}, x)\right) f(x)\, dx$

$$\begin{aligned}
&= 2\eta\tau_0\tau_C + \eta\tau_C^2 + \sum_{j=1}^{n} \alpha_j Q(c_C + c_R - \beta\xi_j) \left(\tau_0 + \tau_C \int_{x_L}^{x_H} x f(x)\, dx\right) \\
&\quad -\theta A_n^2 Q^2 \left(\tau_0^2 + 2\tau_0\tau_C \int_{x_L}^{x_H} x f(x)\, dx + \tau_C^2 \int_{x_L}^{x_H} x^2 f(x)\, dx\right) \\
&= \underbrace{2\eta\tau_0\tau_C + \eta\tau_C^2 + A_n Q c_C \left(\tau_0 + \tau_C\mu\right)}_{E_x(TCC_C(\tau_C, x))} \\
&\quad +A_n Q c_R \left(\tau_0 + \tau_C\mu\right) - \sum_{j=1}^{n} (\alpha_j\xi_j) Q\beta \left(\tau_0 + \tau_C\mu\right) \\
&\quad \underbrace{-\theta A_n^2 Q^2 \left(\tau_0^2 + 2\tau_0\tau_C\mu + \tau_C^2(\mu^2 + \sigma^2)\right)}_{E_x(TCT_C(\tau_C, \vec{\xi}, x))}. \qquad (3.1)
\end{aligned}$$

In the formulation, the expected total collection cost is $E_x(TCC_C(\tau_C, x))$ and the expected total treatment cost is $E_x(TCT_C(\tau_C, \vec{\xi}, x))$.

Individual Decision: Optimal Treatability Level

Consider n producers acting collectively and sharing the cost with respect to their market shares. The producers are the Stackelberg leaders and thus each producer simultaneously decides on her product's optimal treatability level by minimizing her share of expected total compliance cost and the cost of increasing the product's treatability level

$$\underset{\xi_i \geq 0}{Min}\, E(TC_{C,i}) = \frac{\alpha_i}{A_n} \left(E_x \left(TCC_C(\tau_C^*) + TCT_C(\tau_C^*, \vec{\xi})\right)\right) + k\xi_i^2 \alpha_i Q$$

where $TC_{C,i}$ is the total compliance cost for firm i when n firms collaborate.

The Optimal Policy

The first stage game where the individual decisions are made and the optimal treatability levels are determined is submodular and thus a pure strategy Nash equilibrium exists. To show the uniqueness of the equilibrium, we use the dominant diagonal condition in Milgrom and Roberts (1990). We find that uniqueness is guaranteed as long as condition $(A2)$ holds.

$$4k(\eta - \theta Q^2(\mu^2 + \sigma^2)A_n^2) - Q\mu^2\beta^2\alpha_i \geq 0 \quad \forall i.(A2)$$

In the rest of the chapter we assume that condition $(A2)$, which also guarantees the convexity of objective function, holds. Condition (A2) characterizes a lower bound on η such that $\eta \geq \frac{Q(4k\theta Q(\mu^2+\sigma^2)A_n^2+\mu^2\beta^2\alpha_k)}{4k} = \eta_C^{LB}$ where $\alpha_k = \max\{\alpha_1, .., \alpha_n\}$. Note that if condition (A2) holds for the firm with the biggest market share then it holds for all the other member firms. In the following proposition, we characterize the optimal policy for the PRO and each producer i with respect to the base collection rate τ_0.

Proposition 15 *If* $\eta \geq \frac{4k\theta Q^2\mu A_n^3+Q\mu\beta^2 S_{2n}-2kQ\mu(c_R+c_C)A_n^2}{4k} = \eta_C^{UB}$ *where* $S_{2n} = \sum_{i=1}^{n} \alpha_i^2$, *then* $\tau_C^* = 0$ *for every* τ_0*. Otherwise, there exist two threshold levels* κ_C^I *and* κ_C^{II} *on* τ_0 *(expressions for* κ_C^I *and* κ_C^{II} *are provided in the proof in Appendix C1) such that*

1. *if* $0 \leq \tau_0 \leq \kappa_C^I$ *then* $\tau_C^* = 0$,
2. *if* $\kappa_C^I \leq \tau_0 \leq \kappa_C^{II}$ *then* $0 < \tau_C^* = \frac{(Q\mu\beta^2 S_{2n}-4k(\eta-\theta Q^2\mu A_n^2)A_n)\tau_0-2kQ\mu(c_R+c_C)A_n^2}{4kA_n(\eta-Q^2\theta A_n^2(\mu^2+\sigma^2))-S_{2n}Q\mu^2\beta^2} < 1 - \tau_0$, *and*
3. *if* $\kappa_C^{II} \leq \tau_0$ *then* $\tau_C^* = 1 - \tau_0$.

Optimal treatability level of firm i is $\xi_i^* = \frac{\beta\alpha_i(\tau_0+\mu\tau_C^*)}{2kA_n}$. For a possible characterization of τ_C^* and ξ_i^* with respect to τ_0 see Figure 3.2.

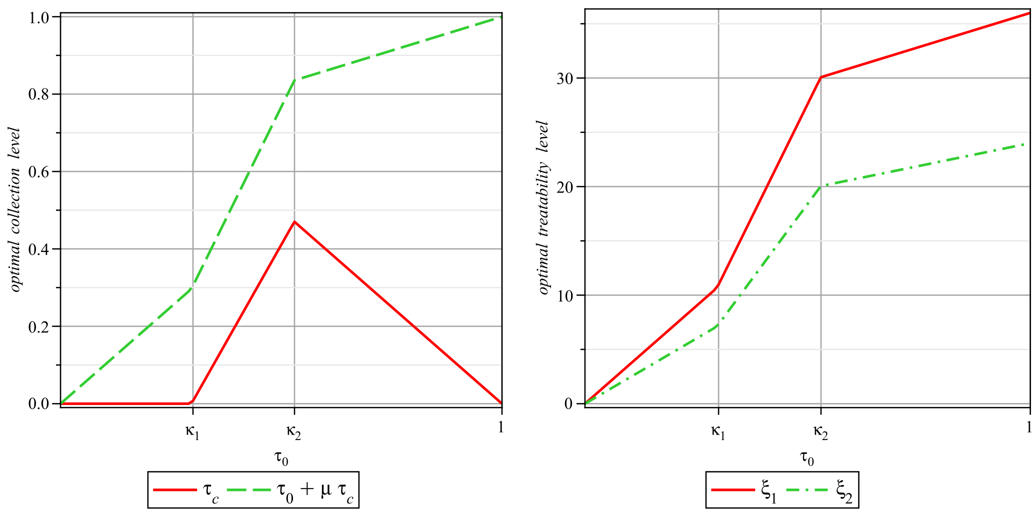

FIGURE 3.2: A possible characterization of optimal (a) collection rate and (b) treatability level with respect to the base collection rate for two firms under collective compliance with $\alpha_1 = 0.12$, $\alpha_2 = 0.08$, $\eta = 7000, c_R = 6$, $c_C = 0.5$, $\beta = 3$, $Q = 1500$, $\theta = 2 \cdot 10^{-3}$, $\mu = 0.65$, $\sigma = 0.2$, $k = 0.025$.

Similar to our findings under the individual compliance scheme, one can easily show that optimal additional collection rate τ_C^* increases as the base collection rate τ_0 increases. In addition, optimal treatability level for each member in the collective scheme increases as the total collection rate increases. In a collective scheme, a big concern for the member companies is potential free-riders—companies who do not invest enough in treatability and benefit from the investments of their partner firms (they benefit because the total cost is shared). Proposition 15 shows that the optimal treatability level is higher for member firms with higher market shares (note that for any two firms k and i $\xi_k^* = \frac{\xi_i^*}{\alpha_i}\alpha_k$). Hence, producers' concern regarding potential free-riders is not groundless, especially if some partner firms have small market shares. In addition, our results indicate that the treatability level is increasing in the final collection rate in proportion to the producer's market share. As the collection rate increases, the improvement in the treatability level of a producer with a small market share will be less than that of a producer with a high market share. Thus, as the base collection rate increases, the difference between the optimal treatability levels widens (see Figure 3.2(b)).

3.6 Collective Scheme with Individual Financial Responsibility (IFR)

As we stated in Section 3.5, in most implementations of the collective compliance scheme, the total cost is allocated among members according to their market shares. However, in their original statements most take-back legislations aim for IFR (which do not hold when the legislation is implemented through a collective scheme with cost allocation by market share). For example, the WEEE Directive clearly states that each producer should be financially responsible for managing the waste from her own EOL products. The excerpt from Article 8.2 reads:

For products placed on the market later than 13 August 2005, each producer shall be responsible for financing the operations referred to in paragraph 1 relating to the waste from his own products. The producer can choose to fulfill this obligation either individually or by joining a collective scheme.

Article 8.2 requires IFR for producers' own EOL products while providing a choice in how to fulfil their responsibility, i.e., through an individual or a collective scheme. Therefore even when the producers become a member of a collective scheme, they are still responsible for financing the waste of their own brand name products. However, when member EU states transposed the WEEE Directive into law, most failed to clarify IFR and cost allocation with respect to market share has been allowed. The reason why companies prefer cost allocation by market share is that implementing IFR within a collective compliance scheme requires sorting products by brand and tracking them through the treatment process in order to record the true cost of treatment. This is a complicated and costly endeavor. Still, IFR *can* be implemented within a collective scheme and the Japanese SHARL where producers are financially responsible for collection and treatment of their EOL products is an example. In the future, with new technologies such as radio frequency identification (RFID), sorting and tracking of EOL products will be much easier (Bohr, 2007). Still, RFID tags would be extremely expensive and it brings the question of who would be responsible for incurring the cost of tagging and tracking.

If we ignore the additional costs incurred due to brand-based tracking, the collective scheme with IFR is an attractive option for producers: the producer members exploit the economies of scale in compliance cost and also benefit from better incentives for improved design since each producer pays only for the collection and treatment of her own products and there is no free-riding. That is why we analyze collective schemes with IFR for comparison purposes even though they are not prevalent in implementation.

We model the relationships among the n producers and a non-profit PRO similar to that under the *collective scheme* in section 3.5. As producer members are collectively responsible for collection, the non-profit PRO decides on the optimal collection rate τ_F^* by minimizing the total cost of collection and treatment for the collaboration as is the case under the *collective scheme* (cf. Section 3.5.1). And each producer simultaneously decides on her product's optimal treatability level by minimizing her true compliance cost:

$$\underset{\xi_i \geq 0}{Min}\, E(TC_{F,i}) = E_x\left(TCC_F(\tau_F^*) + TCT_F(\tau_F^*, \xi)\right) + k\xi_i^2\alpha_i Q$$

where $TCC_F(\tau_F^*) = \frac{\alpha_i}{A_n}\left(2\tau_0\tau_F^*\eta + (\tau_F^*)^2\eta\right) + c_C\alpha_i Q(x\tau_F^* + \tau_0)$ and $TCT_F(\tau_F^*, \vec{\xi}) = (c_R - \beta\xi_i)\alpha_i Q(\tau_0 + \tau_F^* x) - \frac{\alpha_i}{A_n}\left(\theta A_n^2 Q^2(\tau_0 + \tau_F^* x)^2\right)$.

We assume that the cost of maintaining the collection rate τ_F^* ($i.e.$, $2\tau_0\tau_F^*\eta + (\tau_F^*)^2\eta$) is allocated among the members according to their markets shares. Under a collective system, members share the same physical infrastructure and publicize information about the collection activities collectively. Therefore, they are collectively responsible for financing the collection points and the activities required to increase the customer awareness. And since these costs depend on the total volume treated and not on the individual product characteristics, cost allocation by market share is reasonable. Similarly, we assume that the decrease in the total treatment cost due to economies of scale ($i.e.$, $\theta A_n^2 Q^2(\tau_0 + \tau_F^* x)^2$) is also allocated among the members according to their market shares. Recall that the economies of scale in treatment cost arises from the capital intensive treatment processes such as shredding or smelting through which all products of the same category go through. Therefore, the member firms benefit from the economies of scale collectively regardless of brand or product characteristics.

3.6.1 Characterization of Optimal Policy

The first stage game where the individual decisions are made and the optimal treatability levels are determined is submodular and thus a pure strategy Nash equilibrium exists. To show the uniqueness of the equilibrium, we once again use the dominant diagonal condition in Milgrom and Roberts (1990) and find that uniqueness is guaranteed as long as condition $(A3)$ holds.

$$4k(\eta - \theta Q^2(\mu^2 + \sigma^2)A_n^2) - Q\mu^2\beta^2 A_n \geq 0 \ \ (A3)$$

In the rest of our analysis we assume that condition $(A3)$, which also guarantees the convexity of objective function, holds. Condition (A3) characterizes a lower bound on η such that $\eta \geq \frac{QA_n(4k\theta Q(\mu^2+\sigma^2)A_n+\mu^2\beta^2)}{4k} = \eta_F^{LB}$. Proposition 16 characterizes the optimal policy for the PRO and the producer members with respect to the base collection rate τ_0.

Proposition 16 *If* $\eta \geq \frac{4k\theta Q^2\mu A_n^2+Q\mu\beta^2 A_n-2kQ\mu(c_R+c_C)A_n}{4k} = \eta_F^{UB}$*, then* $\tau_F^* = 0$ *for every* τ_0*. If* $\eta_F^{UB} \geq \eta \geq \eta_F^{LB}$*, there exist two threshold levels* κ_F^I *and* κ_F^{II} *on* τ_0 *(expressions for* κ_F^I *and* κ_F^{II} *are provided in the proof in Appendix C1) such that*

1. *if* $0 \leq \tau_0 \leq \kappa_F^I$ *then* $\tau_F^* = 0$*,*

2. *if* $\kappa_F^I \leq \tau_0 \leq \kappa_F^{II}$ *then* $0 < \tau_F^* = \frac{(QA_n\mu\beta^2-4k(\eta-\theta Q^2\mu A_n^2))\tau_0-2kQ\mu(c_R+c_C)A_n}{4k(\eta-\theta Q^2A_n^2(\mu^2+\sigma^2))-QA_n\mu^2\beta^2} < 1-\tau_0$*,*

3. *if* $\kappa_F^{II} \leq \tau_0$ *then* $\tau_F^* = 1-\tau_0$*.*

Optimal treatability level of firm i *is* $\xi_i^* = \frac{\beta(\tau_0+\mu\tau_F^*)}{2k}$*.*

3.7 Comparison of Compliance Schemes

3.7.1 Producers' Preference for a Collective Scheme over the Individual Scheme

Even though producers are occasionally restricted by governments to comply with legislation through a particular compliance scheme, more often they are free to choose the most cost effective one among the alternatives. In this section, we characterize a producer's preference regarding the most cost-effective form of compliance. In doing the comparisons, we assume that when firm i joins a collective scheme (with IFR or with cost allocation by market share) of market share $A_n - \alpha_i$ (with $n-1$ members) the PRO calculates the updated optimal collection rate. Since the PRO aims to provide the most cost effective compliance for its members, once the new member joins the scheme, she may modify the collection rate by changing the effort she puts in collection activities. Evidence suggests that PROs do vary their collection rates over time. For example, Repic, a collective compliance scheme in the UK, recently contacted with additional collection sites in order to increase the collection rate (Gyekye, 2009). In addition, we assume that when firm i becomes a member of the collective scheme, each existing member calculates the updated optimal treatability level. Since the total volume collected goes up and, furthermore, the collection rate may be updated by the PRO once a new firm becomes a member, existing members may find it optimal to adjust their products' treatability levels in response. The following example demonstrates that how willing companies are to invest in treatability levels depends on both the collection rate and the total volume collected: HP's take-back compliance manager states that there are design changes they can make but they do not get enough products back (BBCNews, 2007) to justify them.

First, we characterize how cost effective it is to follow a collective compliance scheme with IFR in comparison to the other two compliance schemes we consider.

Proposition 17 *A firm with market share α_i always prefers to join a collective scheme of size $A_n - \alpha_i$ that follows IFR rather than setting up her own individual compliance scheme. She prefers to join a collective scheme with IFR rather than the one with cost allocation by market share if and only if $2S_{2n} - A_n^2 - \alpha_i^2 \leq 0$.*

As intuition suggests, a collective scheme with IFR yields lower compliance cost when compared to an individual compliance scheme because the members share the cost of maintaining the collection level and benefit from economies of scale in the treatment cost. On the other hand, surprisingly implementing IFR within a collective scheme does not always yield lower cost than allocating the cost by market share. That is the case only if $2S_{2n} - A_n^2 - \alpha_i^2 \leq 0$, which is more likely to hold when small producers are members of the collective scheme. Since members with small market shares may free-ride, treatability levels are lower under cost allocation by market share, which increases the total cost for all members. For a firm with moderate of large market share, collective scheme with cost allocation by market share is preferred only when the scheme consists of firms with large market shares. By the same token, collective scheme with cost allocation by market share is preferred only by small firms and only when the scheme consists of firms with large market shares. Since the partner firms with large market shares will invest in treatability and thus the total treatment cost will be low, market share allocation rule yields smaller cost for the smaller firm.

Next, we compare the collective scheme with cost allocation by market share and the individual scheme with respect to the minimum cost of compliance achieved under each. First, we let $\tau = 0$ and define the difference between the costs of compliance under each scheme as

$$\Omega = E(TC_S)(\tau = 0) - E(TC_C)(\tau = 0) = \frac{4k\theta Q A_n^2 (A_n - \alpha_i) + \beta^2 (2S_{2n} - A_n^2 - \alpha_i^2)}{4kA_n^2}. \tag{3.2}$$

Note that $\Omega \geq 0$ if and only if

$$\sum_{j=1}^{n-1} \alpha_j^2 \geq \Omega_0 = \frac{\beta^2 (A_n^2 - \alpha_i^2) - 4k\theta Q A_n^2 (A_n - \alpha_i)}{2\beta^2}. \tag{3.3}$$

The next proposition characterizes the producer's optimal decision regarding choosing the compliance scheme that yields lower compliance cost.

Proposition 18 *Consider firm i with market share α_i. We compare her compliance cost under an individual scheme with that under a collective scheme where she collaborates with $n-1$ other producer members of total market size $A_n - \alpha_i$ and we find that for firm i*

1. *If $\Omega \geq 0$ then collective scheme yields lower cost of compliance unless $\tau_1 \leq \tau_0 \leq \tau_2$, $\eta \leq \eta_1$, and $\Omega \leq \Omega_1$ hold simultaneously, where*

$$\begin{aligned}
\eta_1 &= \frac{\alpha_i Q \mu (\beta^2 (A_n^2 - S_{2n}) - 4A_n^2 k\theta Q (A_n - \alpha_i))}{4kA_n (A_n - \alpha_i)}, \\
\Omega_1 &= \frac{A_n^2 \mu (4k(\alpha_i^2 Q^2 \mu\theta - \eta) + \alpha_i Q \mu \beta^2)^2}{\alpha_i \mu Q (4k\alpha_i^2 \theta Q^2 (\mu^2 + \sigma^2) - 4\eta k + \alpha_i Q \mu^2 \beta^2)}.
\end{aligned}$$

2. *If $\Omega \leq 0$ then*

    (a) *If $\eta \geq \eta_1$ then*

        i. *If $\Omega \leq \Omega_2(\tau_0, \eta, \theta, \alpha_i, A_n, k, \mu, \sigma, Q)$ then the individual scheme yields lower cost regardless of τ_0.*

        ii. *Otherwise the collective scheme yields lower cost if $\tau_3 \leq \tau_0 \leq \tau_4$.*

    (b) *Otherwise the collective scheme yields lower cost if and only if $\tau_5 \leq \tau_0 \leq \tau_6$.*

The bounds τ_i for $i = 1, .., 6$ on τ_0 and Ω_2 are provided in Appendix C1.

Which compliance scheme yields lower cost depends, among other things, on the sign of Ω. One may interpret Ω as an indicator of how costly it is to set up an individual scheme compared to joining a collective scheme when the base collection rate is maintained as the final collection rate (i.e., the additional collection rate $\tau^* = 0$). Using the equivalence of $\Omega \geq 0$ to expression (3.3), we observe that $\Omega \geq 0$ is more likely to hold when the existing members of the collective scheme have large market shares. Thus if the collective scheme already consists of firms with big market shares, then a firm with market share α_i is more likely to pay a lower compliance cost under the collective compliance scheme. Especially, when $\Omega \geq 0$, the individual scheme yields a lower cost if and only if $\eta \leq \eta_1$, $\Omega < \Omega_1$, and τ_0 is in the right interval.

Let us take a closer look at the conditions that need to be satisfied for the individual scheme to yield a lower cost when $\Omega \geq 0$. The first condition is that $\eta \leq \eta_1$, which means that the market environment is such that it is relatively cheap to maintain a high collection rate. Under this condition, a firm complying under the individual scheme may find it optimal to set the collection rate high, to the extent that $\tau_S^* \geq \tau_C^*$ (one can show that $\tau_S^* \geq \tau_C^*$ is a necessary condition for the individual scheme to yield a lower cost). One reason for choosing the individual scheme is to be able to invest in treatability and reap 100% of the benefits. However, for this investment to pay off, the firm needs to achieve a high collection rate and be able to collect a critical volume of products and this is possible when $\eta \leq \eta_1$. Hence, a low η favors the individual scheme.

The second condition is that $\Omega < \Omega_1$ and a closer inspection of this condition reveals that this condition is more likely to hold when the market sizes of the existing members of the collective scheme are not too large. When the existing members are really big, the collective scheme benefits from economies of scale and regarding the treatability levels, the incentive to free-ride is low. Hence, when the firms in the collective scheme are really big, it is unlikely that the individual scheme will yield a lower cost.

Finally, when $\Omega \geq 0$, which collective scheme yields the lower cost depends on the base collection rate τ_0. When τ_0 is low, the collective scheme yields lower cost because it is expensive for individual firms to increase the total collection rate and they join their volumes to benefit from economies of scale. When τ_0 is of moderate values, the optimal collection rate under individual compliance may be higher than that under the collective scheme (because firms would like to increase their products' treatability levels and this investment is economical only if they are above a critical collection rate rate). As a result, firm i enjoys both economies of scale benefits and lower treatability costs and only then individual scheme yields lower compliance cost. Finally, when the base collection rate is high, the collective scheme is the less costly alternative again because treating the large volumes returned individually is very expensive.

We also observe that if $\beta = 0$ (recall that β is the coefficient for possible savings in unit treatment cost) then the collective scheme always yields a lower cost for firm i because there is no incentive to set up an individual compliance scheme. This finding is aligned with the expectation that small appliances such as electric toothbrushes, hand-held vacuum cleaners, toasters and irons will be treated together and their treatment will be jointly financed (CECED, 2002). These kind of appliances have low metal content which, as discussed in section 3.4, is an indication of a small β.

Corollary 4 *If* $\alpha_i \mu^2 \beta^2 Q(2S_{2n} - A_n^2 - \alpha_i^2) + 4\alpha_i k A_n^2 \theta Q^2(\mu^2+\sigma^2)(A_n - \alpha_i) + 4kA(A_n - \alpha_i)\eta \leq 0$ *then the firm incurs lower compliance cost under the individual scheme regardless of* τ_0.

Observe that the condition in Corollary 4 holds only if $A_n^2 \gg S_{2n}$, i.e., $A_{n-1}^2 + 2A_{n-1}\alpha_i \gg S_{2(n-1)}$. Hence when $S_{2(n-1)}$ is sufficiently small and A_{n-1} is sufficiently high (this means that the collective scheme consists of many small firms before firm i joins), the individual scheme yields a lower compliance cost for firm i. In other words, for a given firm, it is not cost effective to join a collective scheme if it consists of many firms with small market shares.

When $\Omega \leq 0$, unless $\tau_C^* = 0$ or Corollary 4 holds, the base collection rate τ_0 determines the most cost-effective compliance scheme. The individual scheme yields a lower cost when the base collection rate is low or very high. When τ_0 is small, $\tau_C^* = \tau_S^* = 0$ (or both are very small). Hence, the total volume collected is small and the firm prefers the individual scheme and invests in higher treatability levels in order to reduce the total compliance cost (as opposed to joining the collective scheme with small members and trying to exploit economies of scale). On the other hand, as τ_0 increases, the cost of collection increases. This decreases her willingness to go individual and invest more in higher treatability levels. Rather, she prefers the collective scheme to benefit from economies of scale. Eventually when τ_0 is very high, because of high volume of returns which come at no cost to the firm, the firm again prefers the individual scheme and benefits from both economies of scale and higher treatability levels.

For a possible characterization of the optimal compliance decision given a base collection rate see Figure 3.3 which plots $\delta = E(TC_S) - E(TC_C)$. The figure confirms our analytical insights that for sufficiently low and high base collection levels, individual compliance scheme yields lower compliance cost while for moderate levels of base collection rate, the collective scheme yields a lower cost.

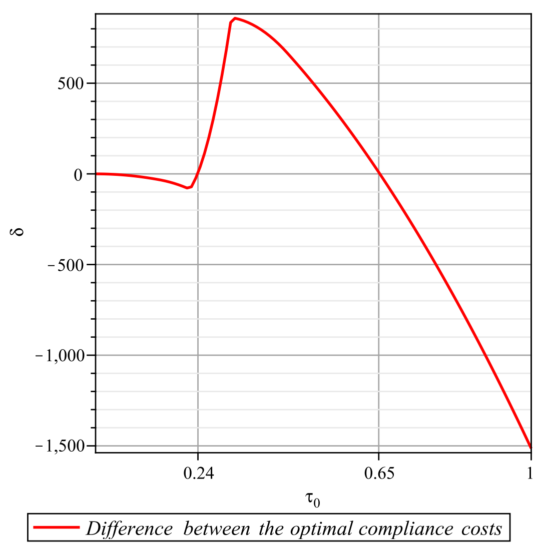

FIGURE 3.3: A possible characterization of compliance decision respect to base collection rate: When $\delta = E(TC_S) - E(TC_C) \geq 0 (\leq 0)$ the optimal decision is joining the collective scheme (setting up an individual scheme)

Effect of return uncertainty on the Producer's Preference for a Collective Scheme: When the base collection rate τ_0 is small, which compliance scheme yields lower cost depends on the sign on Ω. It is apparent that the sign of Ω depends neither on the mean collection rate μ nor on how much the collection rate deviates σ. Therefore, if the base collection rate is small then the uncertainty in collection rate does not affect a producer's preference for a collective scheme. On the other hand, this might not be the case when the base collection rate maintained by government and non-profits is relatively high. Next we provide an example where the producer's preference for the collective scheme changes as μ increases.

`EXAMPLE 1.` Assume that the total market size is $Q = 1000$ and a firm of size $\alpha_1 = 0.15$ considers setting up a collective scheme with another firm of size $\alpha_2 = 0.20$. The treatment/collection costs are $c_R = 6$ and $c_C = 0.1$. The other parameters are as follows: $\beta = 5$, $k = 0.1$, $\theta = 5.5\,10^4$, $\eta = 3150$, $\sigma = 0.2$. When $\mu = 0.3$, the individual scheme yields lower cost regardless of the base collection rate τ_0. When $\mu = 0.5$ the compliance decision with respect to τ_0 is as depicted in Figure 3.3.

This example illustrates that even the individual scheme might yield lower cost when the mean collection rate μ is small, collective scheme might be the less costly alternative -if the base collection rate is of moderate values- when the mean collection rate is high. When μ is high, expected collection rate is higher and thus the firm benefits from economies of scale under a collective scheme. Thus, the firm's willingness to set up an individual scheme and to invest in higher treatability levels decrease. Still, when τ_0 is very high she does not need to join the collective scheme to benefit from economies of scale because a high volume of returns is already maintained by government/non-profits.

Impact of Collection Targets on Producer's Preference for a Collective Scheme

Assume that the take-back legislation specifies a lower bound τ_L (specified as a percentage of the total sales of the company) on the EOL products to be collected and disposed of or treated properly. For example, in their take-back legislation for video display devices, the state of Minnesota sets τ_L to 70% for the first year and increases it to 80% after that. In New York City, the collection target is set at 25%. Not all forms of take-back legislation specify the collection rate as a minimum percent of sales in the previous year. WEEE sets the collection targets as a percentage of the weight of EEE put on the market. For example, for large household appliances, the collection target is 80% of the total weight of appliances sold in the previous period. Since we are modeling an economy with a single category of products, if the products under the same category weigh almost the same then even these different types of collection targets can be represented as some τ_L percent of previous period's sales. On the other hand, if the products under the same category do not weigh same, then we can tweak the model by using *adjusted market shares* which are obtained by multiplying the market shares with the corresponding product's weight.

In this section, we analyze how a firm's compliance scheme preference is affected by exogenous collection targets. We restrict attention to cases where τ_L is higher than the optimum collection rates the firms would otherwise choose since there would be no need for the governments to set such targets if the firms were voluntarily meeting them. First, as Proposition 17 shows, imposing a collection target does not influence a firm's preference for the collective system with IFR over the other schemes. But as Proposition 19 demonstrates, it *may* impact a firm's preference for the individual scheme over the collective scheme. Most importantly, comparing Proposition 18 with Proposition 19 reveals that an exogenous collection target of τ_L makes the collective scheme more attractive to producers. This is intuitive; if legislation forces a producer to collect more than she otherwise would, she joins the collective scheme to benefit from possible scale economies.

Proposition 19 *If the government imposes a lower bound $\tau_L = \tau_0 + \mu\bar{\tau}$ on the percentage of goods sold that should be collected and treated properly, which compliance scheme yields lower cost for firm i depends on Ω (given by (3.2)) as follows:*

- *If $\Omega \geq 0$ then the collective scheme yields lower compliance cost.*
- *If $\Omega \leq 0$ then the collective scheme yields lower compliance cost if and only if $\eta \geq \eta_5$ and τ_L is sufficiently large for a given τ_0. Otherwise, individual scheme yields lower cost.*

3.7.2 Policymaker's Perspective: Environmental Benefits

In this section, we take the policymaker's perspective and assume that firm i with market share α_i is told to follow either the individual scheme or the collective scheme. In other words, at this point we are not concerned about the firm's preference regarding the compliance scheme that yields lower compliance cost but assume that the compliance scheme is already imposed. Although producers are free to choose individual compliance scheme under the WEEE Directive, some governments impose strict administrative requirements such as providing additional waste plans and financial guarantees for individual compliance and thus individual compliance becomes a highly unattractive option. For that reason, in countries such as Greece or Slovenia, no firm has yet applied for individual compliance (Perchards, 2007). In this section, we aim to understand if the policy maker's decision to impose additional requirements for individual compliance and thus turn it into an infeasible option is useful in achieving higher environmental benefits. In order to identify which compliance scheme achieves higher environmental benefits, we compare the collection rate and the treatability level achieved when the firm complies individually versus when she joins a collective scheme. We first compare the total collection rates achieved under each possible compliance scheme.

Proposition 20 *Consider firm i with market share α_i and a collective scheme with total market share $A_n - \alpha_i$. Then*

1. $\tau_F^* \geq \max\{\tau_C^*, \tau_S^*\}$.

2. *Depending on η, τ_S^* and τ_C^* compare as follows:*

   (a) *If $\eta \geq \max\{\eta_C^{UB}, \eta_S^{UB}\}$ then $\tau_C^* = \tau_S^* = 0$ for every τ_0.*

   (b) *If $\eta_C^{UB} \geq \eta \geq \eta_S^{UB}$ ($\eta_S^{UB} \geq \eta \geq \eta_C^{UB}$) then $\tau_S^* = 0$ ($\tau_C^* = 0$) and $\tau_C^* \geq \tau_S^*$ ($\tau_S^* \geq \tau_C^*$) for every τ_0.*

   (c) *If $\min\{\eta_C^{UB}, \eta_S^{UB}\} \geq \eta$ then*

      i. *If $\eta \geq \eta_1$ then $\tau_C^* \geq \tau_S^*$ for every τ_0.*

ii. If $\eta_2 \leq \eta \leq \eta_1$ where

$$\eta_2 = \frac{\alpha_i Q \mu \begin{pmatrix} (4kA_n^2\theta Q(\mu-\mu^2-\sigma^2)(A_n-\alpha_i))(c_R+c_C) \\ -(\mu\beta^2(A_n^2-S_{2n})(1-\mu))(c_R+c_C) \\ +2Q\beta^2\sigma^2\theta(A_n^3-\alpha_i S_{2n}) \end{pmatrix}}{2((1-\mu)(A_n\alpha_i-S_{2n})\mu\beta^2+4kA_n\theta Q(A_n^2-\alpha_i^2)(\mu^2+\sigma^2-\mu))}$$

then $\tau_C^* \geq \tau_S^*$ ($\tau_C^* \leq \tau_S^*$) when $\tau_0 \geq \kappa_3$ ($\tau_0 \leq \kappa_3$).

iii. If $\eta \leq \eta_2$ then $\tau_C^* \leq \tau_S^*$ for every τ_0.

The expression for κ_3 is provided in the proof in Appendix C1.

As intuition suggests, the total collection rate is highest under collective compliance with IFR. On the other hand, among the other two compliance schemes which one gives higher environmental benefits in terms of higher collection rates depends on how expensive it is to maintain the collection rate, (i.e., on η). When η is high (e.g., large country, population dispersed, customers are not willing to return the products) collective schemes result in higher collection rates because it is feasible to increase the collection rate only when producers share the costs. On the other hand, when η is low (e.g., small and less populated country) higher collection rates are achieved if the government enforces the individual compliance scheme. This result implies that imposing additional requirements for individual compliance and thus turning collective compliance into the more attractive alternative in smaller countries such as Greece or Slovenia may be a flawed strategy and may result in lower collection rates when compared to what would be achieved under individual compliance. For moderate values of η, the base collection rate τ_0 should be chosen carefully: For higher values of τ_0, i.e., $\tau_0 \geq \kappa_3$, collective scheme yields higher collection rates. On the other hand, for lower values of τ_0, i.e., $\tau_0 \leq \kappa_3$, individual scheme achieves higher collection rates. The conditions under which individual scheme yields higher collection rates, i.e., $\eta_2 \leq \eta \leq \eta_1$ and $\tau_0 \leq \kappa_3$, are satisfied if β is high and there are relatively small firms in a big collective scheme, i.e., total market share of the collective scheme is big enough but member shares are relatively small. Therefore, when they act collectively they benefit from economies of scale and do not need to increase the collection rate for small values of τ_0. On the other hand, when the firm with market share α_i (a relatively large market share) sets up an individual scheme she benefits from increasing the collection rate since β is high.

Finally, note that if the base collection rate is very low or very high, i.e., $\tau_0 \leq \min\{\kappa_S^I, \kappa_C^I\}$ or $\tau_0 \geq \min\{\kappa_S^{II}, \kappa_C^{II}\}$, then both schemes achieve the same level of collection rate. In the former case, the optimal additional collection rate is zero for both schemes, while in the latter case it is $1 - \tau_0$. Thus, in either case, the policy maker does not need to worry about imposing a particular compliance scheme in order to achieve higher collection rates.

Proposition 21 *Consider firm i with market share α_i and a collective scheme with total market share $A_n - \alpha_i$. Then, $\xi_F^* \geq \max\{\xi_C^*, \xi_S^*\}$. The optimal treatability level under collective scheme is higher than that under individual scheme, i.e., $\xi_C^* \geq \xi_S^*$ if and only if $\eta_2 \leq \eta \leq \eta_3 = \frac{Q\mu^2(4kA_n^3\theta Q+\beta^2 S_{2n})+4kA_n^2\theta Q^2\sigma^2(A_n-\alpha_i)}{4k(A_n-\alpha_i+\alpha_i\mu)}$, and $\tau_7 \leq \tau_0 \leq \tau_8$. The bounds τ_i for $i = 7, 8$ on τ_0 are provided in Appendix C1.*

With Proposition 21, we establish the intuitive result that the treatability level is highest under the collective scheme with IFR. In addition, we conclude that if η is small, i.e., $\eta \leq \eta_2$, the optimal treatability level achieved under the individual scheme is higher than that under the collective scheme regardless of the base collection rate. When η is small, it is cheap to increase the collection rate further and the additional collection rate is higher under the individual scheme. Since the treatability level is linear in the collection rate (see Propositions 14 and 15), the treatability level is higher under the individual scheme. On the other hand, when η is large, i.e., $\eta \geq \eta_3$, the treatability level is higher under the individual scheme for completely different reasons. Note that for higher values of η, the difference between collective and individual collection rates decreases since increasing the collection rate is very costly in either case. Therefore, collective scheme can never achieve high enough collection rates that is necessary for higher treatability levels. Finally, only when both η and τ_0 are in the right interval, optimal treatability level under the collective scheme is higher than the individual treatability level. The treatability level under the collective scheme increases as the collection rate and/or the market share of the firm increases. That is why we observe a higher treatability level under the collective scheme only if the collective collection rate is significantly higher than the individual collection rate.

Another condition under which the difference between the two collection rates is particularly high is when the market shares of each of the firms in the collective scheme are large. Therefore, the collective scheme may yield higher treatability levels than the individual scheme when it comprises of firms with large market shares. The treatability level under the collective scheme is generally lower than that under the individual scheme since firms do not invest as much when the other firms free-ride on their investment

(Tojo, 2004). However, our result shows that this fear is somewhat mitigated when the collective scheme members have large market shares and are less likely to free-ride (since the optimal action for all members is to invest in high levels of treatability).

Proposition 21 implies that even when the collective collection rate is higher, treatability levels may be higher under the individual scheme. Therefore, policy makers should be particularly careful when imposing a specific compliance scheme because there may be conflicting environmental benefits. Next we provide an example to illustrate this.

`EXAMPLE 2.` Assume that the total market size is $Q = 1000$ and a firm of size $\alpha_1 = 0.23$ considers setting up a collective scheme with another firm of size $\alpha_2 = 0.3$. The treatment/collection costs are $c_R = 6$ and $c_C = 0.5$. The other parameters are as follows: $\beta = 3.5$, $k = 0.039$, $\theta = 0.01$, $\eta = 9500$, $\sigma = 0.5$ and $\mu = 0.6$. The optimal collection and treatability levels under collaborative and individual scheme are depicted in Figure 3.4.

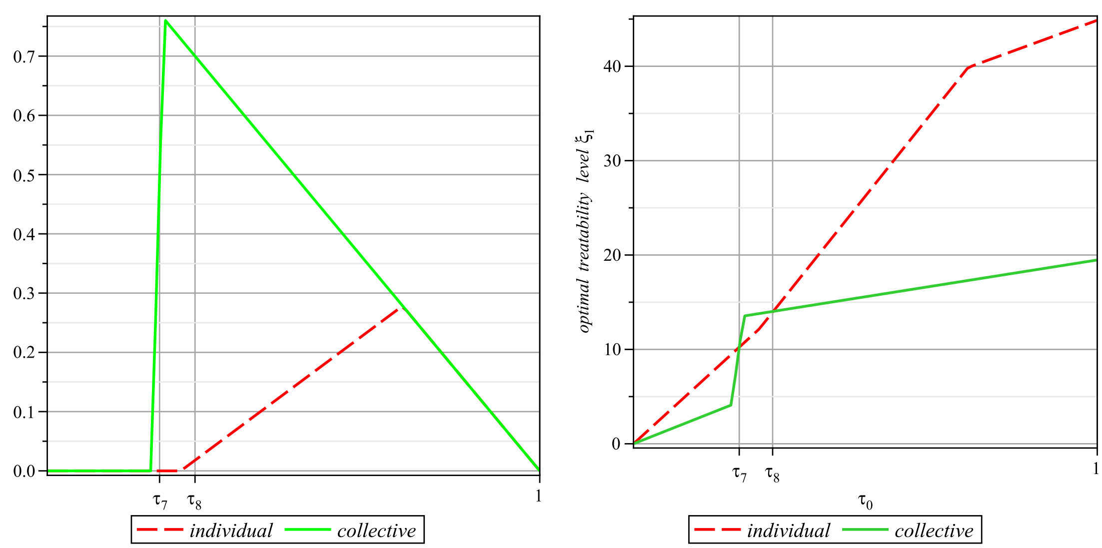

FIGURE 3.4: Conflicting environmental benefits unless τ_0 is in the right interval

For this specific example, even when the collective collection rate is higher, individual scheme gives a higher treatability level unless τ_0 is in the right interval and there are conflicting environmental benefits for a large range of base collection rates.

Recall that in Section 3.7 we show that if a minimum collection level is imposed by the government, a producer, who would otherwise choose the individual scheme, may end up joining a collective compliance scheme. In that context, our results suggest that if such an exogenous collection target causes the firm to switch compliance schemes, it may result in a degradation in the treatability levels and our next example demonstrates that this is indeed possible.

`EXAMPLE 3.` Assume that the total market size is $Q = 100$ and a firm of market share $\alpha_1 = 0.3$ considers setting up a collective scheme with another firm of market share $\alpha_2 = 0.3$. The treatment/collection costs are $c_R = 7$ and $c_C = 1$. The other parameters are as follows: $\beta = 5$, $k = 0.1$, $\theta = 0.05$, $\eta = 1400$, $\sigma = 0.2$ and $\mu = 0.8$. In the absence of collection targets, individual scheme yields lower compliance cost regardless of τ_0. However, under legislation with a collection target of $\tau_L = 0.95$, collective scheme yields lower compliance cost when $\tau_0 \leq 0.695$ (for the difference between the compliance costs under individual and collective schemes see Figure 3.5(a)). The optimal treatability levels are plotted on Figure 3.5(b).

When the legislation imposes collection targets, both collective and individual schemes achieve the same level of collection rate, i.e., $\tau_0 + \mu\tau_L$, which is higher than the collection rate achieved in the absence of collection targets. While the collection rate increases, in this specific example, we observe that the treatability level under the optimal compliance

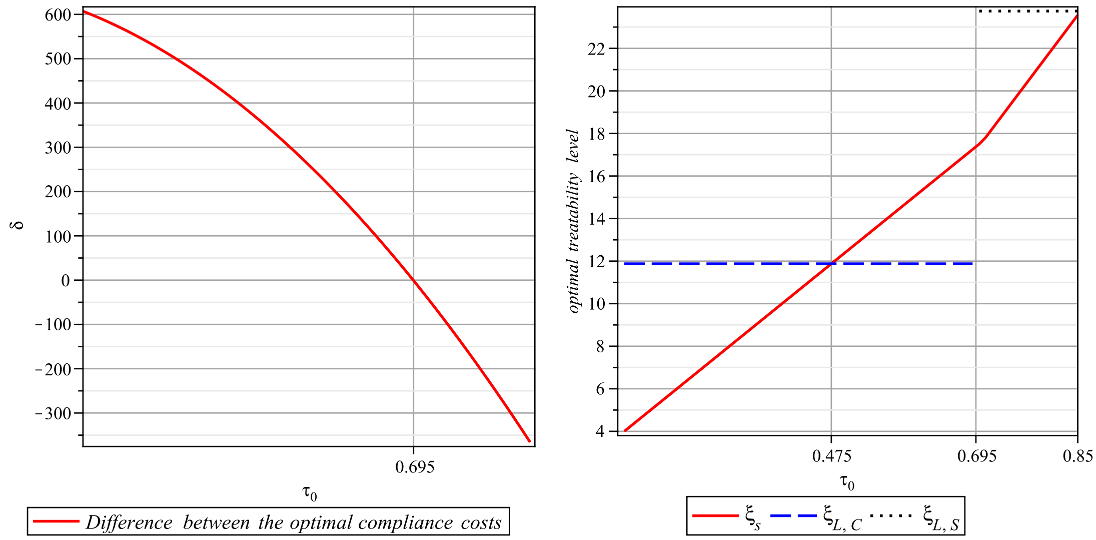

FIGURE 3.5: For Example 3, (a)the collective scheme yields lower compliance cost under τ_L (b)$\xi_{L,k}$ $k \in \{S, C\}$ is the optimal treatability level under the compliance scheme that yields lower cost under τ_L and ξ_S the optimal treatability level in the absence of τ_L.

decision may be lower than that would be achieved in the absence of collection targets. We observe a degradation in treatability levels only when the base collection rate is of moderate levels. When the base collection rate τ_0 is low, even though the optimal compliance decision changes from individual scheme to collective scheme, optimal treatability does not decrease. In the absence of collection targets, the producer does not have much incentive neither to increase the collection rate nor the treatability level. Recall that the collective scheme achieves higher treatability levels than the individual scheme only when the collective collection rate is significantly higher than the individual collection rate. Here, inducing very high collection targets causes the firm to switch to collective compliance, increases the collection rate drastically and thus yields higher treatability levels. For moderate levels of base collection rate, in the absence of collection targets, the individual scheme is optimal and achieves higher collection rates and treatability levels than the collective scheme. The target collection rate causes a switch to the collective scheme, but the overall collection rate is not high enough to ensure higher treatability levels under the collective scheme. Finally, when base collection rate is high, optimal compliance decision remains the same, i.e. individual scheme yields lower cost, and therefore optimal treatability level increases as the legislation imposes high collection rates.

3.8 Conclusions

As an increasing number of countries act to require producers to finance the collection and treatment of their end-of-life products through legislation, it is important for producers to fully understand the compliance-related costs and how they depend on the particular compliance scheme followed. Even though research on take-back legislations is growing, no previous work looked at the question of which compliance scheme yields the lowest cost for a producer and our research aims to fill this gap.

Given the choice, some producers (e.g., Cisco) choose individual compliance whereas others prefer collective compliance (e.g., ERP by Braun, Electrolux, HP, Sony). In order to characterize the market and operating conditions that make one form of compliance more cost effective than the others, we first compare the three compliance schemes with respect to the compliance cost a producer incurs under each. We find that, as intuition suggests, collective compliance with IFR is in general the most cost effective alternative since it allows producers to exploit the economies of scale inherent in a collective system as well as recoup the benefits of their environmentally-friendlier products. However, the collective scheme with IFR is not easy to implement in practice because it requires sorting EOL products by brand and tracking them through the treatment process in order to record the true cost of treatment. For that reason, individual scheme and collective scheme with cost allocation by market share are the two prevalent forms of compliance in practice. We find that which scheme (of these two) gives lower compliance cost depends on the base collection rate maintained by the government and non-profits as well as the market shares of partner firms. If the partner firms have large market shares then collective compliance, in general, yields lower cost for a producer. The producer not only benefits from economies of scale but also invests in increasing the treatability level—and thus decreases the treatment cost—with no fear that the others will free-ride on her investment. On the other hand, if the partner firms have small market shares, a producer pays lower compliance cost under collective scheme only if the initial collection rate maintained by the government is of moderate levels.

Our research has policy implications. We analyze how the choice of a particular compliance scheme affects the success of legislation in terms of increasing the collection rate and creating incentives for designing environmentally-friendlier products. One lever policymakers have in enacting legislation is to specify the compliance scheme producers are allowed to follow. To that end, we find that collective scheme with IFR provides superior environmental outcomes than the others, but as evidence from the implementation of the WEEE Directive suggests, collective scheme with IFR is difficult to implement and enforce. When we compare the individual scheme with the collective scheme with cost allocation by market share, we find that, in general, the treatability level is higher in the former and the collection rate is higher in the latter. However, if the government is willing to maintain a moderate level of collection rate and incur the related cost, both the collection rate and the treatability level may be higher under the collective scheme. Hence, policymakers should think twice about shifting the cost of EOL products completely to producers. Another lever the policymakers have is the possibility of imposing collection rate targets. We find that high collection rate targets cause producers to switch to the collective scheme and result in lower treatability levels. Therefore, policymakers need to be cognizant about the trade-off between the collection rates achieved and the incentives producers have to increase the treatability levels of their products.

CHAPTER 4

The Impact of Buyback Price Commitment and Demand Uncertainty on Channel Conflicts

4.1 Introduction

This chapter deals with product take-back motivated by the goal of managing distribution channels better. When manufacturers distribute products through different channel intermediaries, channel competition may rise. Effectively managing the relationships with and mediating the conflicts between intermediaries is of key importance to manufacturers' profitability. For that reason, a growing number of companies with dual distribution channels, i.e. rental and sales channels, set up buyback programs, a form of product take-back. For instance, U.S. automobile manufacturers sell their products through dealers and rental agencies. Until the late 1980s, these two channels were *separate*: dealers were franchised to sell products in the sales market and rental agencies to rent in the rental market. Low sales in the consumer market prompted manufacturers to experiment with new channel structures. Initially, they adopted an *overlapping* channel in which rental agencies were allowed to sell used rental cars to consumers. This channel arrangement led to a large number of slightly used rental cars entering the sales market and thus a competition between rental agencies and dealers. Due to dealers' strong opposition and even law suits against the manufacturers, this channel strategy failed in action (Auto Rental News, 1990). In order to mitigate the channel conflicts between the intermediaries, some manufacturers launched *buyback* channel structure. Under buyback programs manufacturers sell the so-called *program* cars to rental agencies and then repurchase them at a guaranteed price after a period of time in order to redistribute them through dealers. Figure 4.1 illustrates the three channel structures in the U.S. automobile industry.

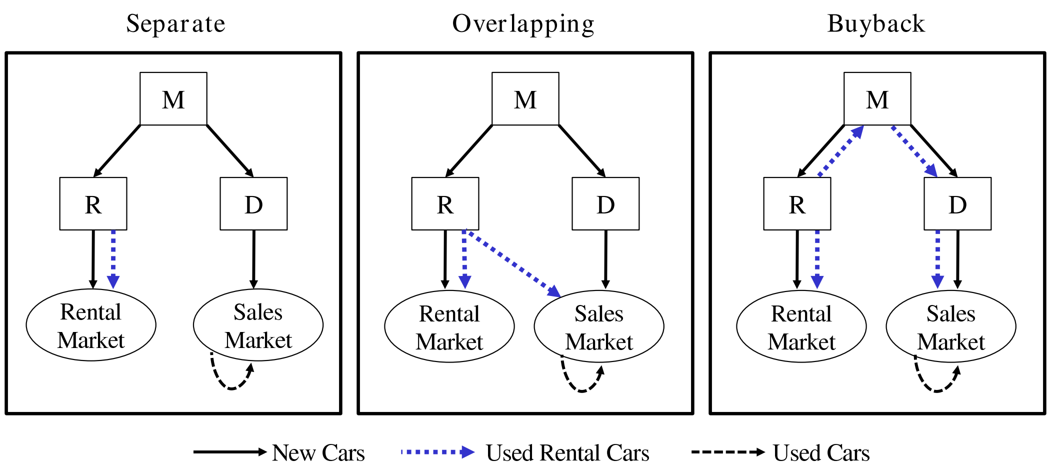

FIGURE 4.1: Three Channel Structures in the U.S. Automobile Industry

The evolvement of channel structures in the U.S. automobile industry shows that when setting channel policies manufacturers have to take intermediaries' profitability into consideration in addition to their own. While buyback channel clearly avoids direct competition between dealers and rental agencies, how valuable a buyback channel is in mediating channel conflicts seems to depend on the quality of used cars[1] Highly durable used cars increases the competition between new and used cars and thus the competition between dealers and rental agencies in an overlapping channel structure. Durability of used cars is often reflected in their residual value, or resale prices. However, price depreciation of used cars can fluctuate significantly depending on different factors such as the economy, gas price, or sales of new cars. For example, average resale prices of used rental cars ranged from $10500 to $13000 in 2005 (Auto Rental News, 2006). Uncertainty in the residual value of used cars is a source of demand uncertainty that affects the market size of both new and used cars. In this chapter, we incorporate uncertainty in residual value and explore how it affects buyback channel's ability in mediating channel conflicts and manufacturers' channel policies.

While a buyback channel alleviates channel conflicts, it does not necessarily maximize manufacturers' profits. Their profitability depends on two characteristics of the buyback program: (i) timing on the announcement of buyback prices; and (ii) quality standards for the cars repurchased by manufacturers (also called *turn-back* standards). There are two alternatives related to timing of setting the buyback price: buyback prices could be committed by manufacturers at the time of the initial purchase or determined at the time of repurchase. For example, Ford currently uses a fixed price depreciation rate in their daily rental repurchase program, under which the buyback price is calculated using projected rental time (https://www.fleet.ford.com). Using a fixed price depreciation rate is equivalent to committing the buyback price at the time of initial purchase for a given

[1] Although our research applies to any durable good industry where rental and sales market coexist, in the remainder of this chapter we refer to this durable good as car.

length of usage. One of our main goals in this chapter is to identify how the timing of price announcement in a buyback program impacts the profitability of manufacturers and intermediaries.

In a buyback channel, manufacturers set turn-back standards on repurchased program cars, i.e., used rental cars. To be eligible for repurchase, program cars need to meet these turn-back standards which include regular maintenance service records, original factory-installed equipments, completed warranty and recall repairs, etc. These turn-back standards help to explain the evidences suggesting that program cars typically have a lower depreciation rate and thus a higher resale price than used cars owned by consumers (Purohit, 1997). Thus turn-back standards affect how much program cars depreciate which in turn affect the intensity of competition between rental agencies and dealers. In this chapter, we aim to understand how manufacturers can use turn-back standards as an operational lever to mediate channel conflicts and to improve their own profits.

Among other results, we find that the timing strategy on buyback prices affects not only total channel profits but also the allocation of profits among the channel members. Compared to setting it at the time of repurchase, as in (Purohit, 1997), early commitment of the buyback price under no uncertainty always reduces total channel profit and the manufacturer's profit. Moreover, buyback price commitment leads to the lowest profit for both the channel and the manufacturer among the three channel structures. This is in stark contrast to the profit rankings under no price commitment with which a buyback channel always dominates a separate channel in terms of total channel profits and the manufacturer's profit. Even in the presence of residual value uncertainty, the manufacturers' profit is always the lowest in a buyback channel when the buyback price is committed early.

Our results suggest that the timing policy on the buyback price can be used as a profit lever for the manufacturer. Instead of committing it at the initial time of purchase, the manufacturer can improve profits by setting the buyback price at the time of repurchase. That brings up the question of why buyback price commitment is still being implemented in practice by automobile manufacturers. Our main aim in this chapter is not characterizing the most profitable channel structure for the manufacturer but rationalizing manufacturer's decision in launching buyback channel with price commitment. To this end, we explore different effects that rationalize the choice of price commitment under buyback channel structure. First, our results suggest that commitment of buyback price enhances a buyback channel's ability in resolving channel conflicts in the presence of residual value uncertainty. The dealer always achieves the highest profit under a buyback channel with price commitment. Buyback price commitment also improves the rental agency's profit when the depreciation rate differential between program cars and used cars is sufficiently small. Moreover, in the presence of uncertainty the manufacturer achieves higher total sales under buyback price commitment than that under no price commitment. In contrast, in the absence of uncertainty, buyback price commitment always leads to the smallest total sales among the channel structures. With residual value uncertainty, we find sufficiently different effects from those obtained with a deterministic model thereby emphasizing the importance of taking uncertainty into account.

While the buyback channel never maximizes the manufacturer's profitability among the three channel structures, our analysis shows that the manufacturer can use the turn-back standard as an operational lever to improve profits. A higher turn-back standard leads to lower depreciation of program cars relative to that of used cars. We show that the manufacturer's profit generally increases in the turn-back standard in the presence of residual value uncertainty.

The rest of this chapter is organized as follows: In §4.2 we review the relevant literature, in §4.3 we lay out the model. We analyze the impact of buyback price commitment and turn-back standards on the performance of the three channel structures in §4.4 under no uncertainty. In §4.5, we explore the same questions under residual value uncertainty. We conclude in §4.6. All proofs are relegated to Appendix D1.

4.2 Related Literature

This chapter studies profitability of the buyback channel structure by connecting two important streams of research in operations management literature: Channel competition and resolution of channel conflicts in a durable good market. Prior academic research falling in the intersection of these two research streams has been limited. The literature on durable goods is vast but has primarily concerned a manufacturer's decision on leasing vs. selling, e.g. Stokey (1981), Bulow (1982), Purohit and Staelin (1994), Purohit (1997), Desai and Purohit (1999), Bhaskaran and Gilbert (2009).

Only a few papers study competition among intermediaries and its effects on manufacturers' channel decisions. Bhaskaran and Gilbert (2009) study how product durability affects interactions between the manufacturer and dealers. They show that when the competition is intense among dealers, the manufacturer prefers so-called lease brokering arrangement, where the dealers earn a margin for brokering leases between manufacturer and customers, instead of selling her product to dealers. Purohit and Staelin (1994), on the other hand, study the three aforementioned channel structures, namely, separate, overlapping, and buyback, focusing on the manufacturer maximizing profits through the dealer channel in a setting where the rental market is fixed. They compare manufacturer's total sales under each channel structure and conclude that total sales of new cars are greatest under overlapping channel structure.

This chapter is closely related to Purohit (1997), who studies the three channel structures under no demand uncertainty. Purohit examines the form of buyback channel structure where buyback price is determined in the second period, i.e. buyback with no price commitment, and finds that a buyback channel serves to mediate channel conflicts between dealers and rental agencies. In this chapter, by incorporating uncertainty in the residual value of used cars, buyback price commitment, and turn-back standards, we introduce important operational elements into channel management. Our main contribution is showing how residual value uncertainty and an operational policy such as turn-back standard affect attractiveness of a buyback program and channel competition

Although most papers in the literature on durable goods do not consider uncertainty, a notable exception is Desai et al. (2007), who study the role of demand uncertainty in a manufacturer's production and marketing decisions on a durable product. However, their paper does not consider intermediaries and channel interactions. In contrast, we focus on how uncertainty in the residual value of used products affects channel competition and a manufacturer's channel policies.

4.3 The Model

We consider a monopolist manufacturer who makes a durable good and distributes it to consumers through a dealer and a rental agency. The durable good lasts for two periods. Although our analysis applies to any durable-good industry where rental and sales market coexist, we will refer to this durable good as car. There are three types of cars: *new*, *program*, and *used* cars. Program cars refer to those used rental cars while used cars refer to those used ones owned by consumers. In the first period only new cars are available in the market, whereas in the second period new, program, and used cars might coexist.

Consumers are heterogenous and their valuation for a new car in each period is represented by $\phi = [0, 1]$ where ϕ is distributed uniformly between 0 and 1. The total mass of consumers is normalized to 1. We use subscript n, p and u to stand for new, program and used cars, respectively. As discussed in §4.1, program cars owned by the rental agency depreciate less than used cars owned by consumers. Thus $0 \leq \theta_p \leq \theta_u \leq 1$ where θ_i is the depreciation rate of type i car, $i \in \{p, u\}$. Without loss of generality we assume that $\theta_p = \gamma\theta_u$ where $\gamma \in [0, 1]$. Consumer valuation for program (used) cars is $v = (1 - \gamma\theta_u)\phi$ ($v = (1 - \theta_u)\phi$). When a consumer with valuation v purchases a car, his utility is $v - p$ where p is the price. In any period each consumer purchases a car, if any, that provides him with the highest nonnegative utility.

For the remainder of this chapter, we use subscript M, D, R, and SC to denote manufacturer, dealer, rental agency, and supply chain, respectively. In addition we will use superscript S, O, BC, and BN to denote separate, overlapping, buyback with price commitment, and buyback with no price commitment, respectively.

Notations.

q_{ij} : quantities in the sales market in period i for $j \in n, p, u$ type of car

p_{ij} : prices in period i for $j \in n, p, u$ type of car

$\bar{q}_{ij}$: rental quantities in period i for $j \in n, p$ type of car

$\bar{p}_{ij}$: rental prices in period i for $j \in n, p$ type of car

q_{2p}^{D} : quantity of program cars that the dealer purchases from the manufacturer in a buyback channel in the second period

q_{2p}^{R} : quantity of program cars that the rental agency sells back to the manufacturer in a buyback channel in the second period

w_i : wholesale price for new cars charged to the dealer in period i

$\bar{w}_i$: wholesale price for new cars charged to the rental agency in period i

w_b : buyback price for program cars

w_p : wholesale price for program cars charged to the dealer (only relevant in period 2)

Π_{ij}^{l} : the total profit to go for $j \in \{M, D, R, SC\}$ in period i under channel structure $l \in \{S, O, BC, BN\}$. We assume $\Pi_{3j}^{l} = 0$, i.e., the terminal value at the end of period 2 is zero.

The inverse demand functions can be derived from solving a consumer's purchasing decision in the two periods and are listed as follows (For derivations interested reader might see Purohit (1997)).

$$
\begin{aligned}
p_{1n} &= p_{2u} + (1 - q_{1n}), \\
p_{2n} &= p_{2p} + \gamma\theta_u(1 - q_{2n}), \\
p_{2p} &= p_{2u} + (1 - \gamma)\theta_u(1 - q_{2n} - q_{2p}), \\
p_{2u} &= (1 - \theta_u)(1 - q_{2n} - q_{2p} - q_{2u}), \\
\bar{p}_{1n} &= 1 - \bar{q}_{1n}, \\
\bar{p}_{2n} &= \bar{p}_{2n} + \gamma\theta_u(1 - \bar{q}_{2n}), \\
\bar{p}_{2p} &= (1 - \gamma\theta_u)(1 - \bar{q}_{2n} - \bar{q}_{2p}).
\end{aligned}
$$

We first solve the problem with no uncertainty. Then in §4.5 we introduce uncertainty by assuming that θ_u, the rate at which consumer valuation of used cars depreciates, is distributed uniformly between a and b where $0 \leq a \leq b \leq 1$. We assume that the uncertainty is resolved at the beginning of the second period before all price and quantity decisions. Because depreciation reduces consumer valuation of used cars, random realizations of θ_u capture demand uncertainty, specifically uncertainty in the allocation of demand between new and used cars. Since $\theta_p = \gamma\theta_u$ uncertainty exits not only in the depreciation of used cars but also in the depreciation of program cars. We use uncertainty in the depreciation rates to capture the fact that the residual value of used cars is stochastic and to introduce the mixed demand uncertainty for new, program, and used cars. Our model, however, does not capture uncertainty in total market demand, and thus holds the maximum demand size to be constant at 1. Because the competition between new, program, and used cars is the main source of channel conflicts, incorporating the mixed demand uncertainty enables our model to focus on the impact of uncertainty on channel competition.

4.3.1 Channel Structures

We first describe the common features of the three channel structures. Regardless of the channel structure, the manufacturer maximizes her profit in each period by choosing the optimal wholesale prices. We assume that the manufacturer is the Stackelberg leader and the intermediaries compete on quantities. Therefore based on the wholesale prices announced by the manufacturer, the intermediaries maximize their individual profits by choosing the optimal quantities. We solve for the subgame perfect equilibrium using backward induction.

The two-period competition between intermediaries is formulated as a Cournot model as is commonly used in the literature, e.g. Purohit (1997). While price competition is appropriate where capacity and production quantity can be adjusted easily (e.g. information goods), quantity competition is appropriate in modeling capital-intensive industries where production capacity is relatively fixed. For durable goods, manufacturing facility is expensive and can be adjusted only after considerable lead time. Thus modeling quantities as the decision variables of intermediaries is a reasonable approach. Next we describe each channel structure in detail.

Separate Channel

Under a separate channel, the dealer is franchised to sell cars in the sales market and the rental agency to rent in the rental market. Therefore, in period i the dealer maximizes his profit $\Pi_{iD}^{S} = q_{in}(p_{in} - w_i) + \Pi_{(i+1)D}^{S}$ by choosing how many new cars to sell in the sales market (q_{in}) while the rental agency maximizes his profit $\Pi_{iR}^{S} = \bar{q}_{in}(\bar{p}_{in} - \bar{w}_i) + \Pi_{(i+1)R}^{S}$ by choosing how many rental cars to rent in the rental market ($\bar{q}_{in}$). The manufacturer maximizes her profit $\Pi_{iM}^{S} = (q_{in}^{*} w_i + \bar{q}_{in}^{*} \bar{w}_i) + \Pi_{(i+1)M}^{S}$ by choosing the optimal wholesale prices ($w_i, \bar{w}_i$) in period i given the intermediaries' optimal quantity decisions. Here note all rental cars bought in period 1 remain in the rental market, and thus, are rented again in period 2, i.e. $\bar{q}_{2p} = \bar{q}_{1n}$.

Overlapping Channel

Under an overlapping channel, the rental agency is allowed to sell some of the used rental cars in the sales market in period 2. Therefore, the profit maximization problem for each channel member in each period is the same as the one under the separate channel with the exception of the rental agency's second period problem. In period 2, the rental agency maximizes his profit $\Pi_{2R}^{O} = \bar{q}_{2p}\bar{p}_{2p} + \bar{q}_{2n}(\bar{p}_{2n} - \bar{w}_2) + q_{2p}p_{2p}$ by choosing how many program cars to sell in the sales market (q_{2p}) as well as how many rental cars to rent in the rental market ($\bar{q}_{2n}$). Here note that in period 2 the rental agency rents the remaining used rental cars after selling some in the sales market, i.e. $\bar{q}_{2p} = \bar{q}_{1n} - q_{2p}$.

Buyback Channel with Price Commitment

Under a buyback channel with price commitment, the manufacturer repurchases a certain number of used rental cars from the rental agency at a guaranteed price and sells them through the dealer in the sales market. Therefore, in the second period the dealer maximizes his profit, $\Pi_{2D}^{BC} = q_{2n}(p_{2n} - w_2) + q_{2p}^D(p_{2p} - w_p)$ by choosing quantities (q_{2n}, q_{2p}^D) whereas the rental agency maximizes his profit $\Pi_{2R}^B = \bar{q}_{2p}\bar{p}_{2p} + \bar{q}_{2n}(\bar{p}_{2n} - \bar{w}_2) + q_{2p}^R w_b$ by choosing quantities $(\bar{q}_{2n}, q_{2p}^R)$. Here note that the rental agency rents out the remaining program cars after selling q_{2p}^R units back to the manufacturer, i.e. $\bar{q}_{2p} = \bar{q}_{1n} - q_{2p}^R$.

We assume that all program cars repurchased from the rental agency are sold to the dealer, i.e., the manufacturer does not withhold any program cars. Therefore the wholesale price for the program cars charged to the dealer, w_p, comes from the market equilibrium where $(q_{2p}^D)^* = (q_{2p}^R)^*$. The manufacturer maximizes her profit $\Pi_{2M}^{BC} = q_{2n}^* w_2 + \bar{q}_{2n}^* \bar{w}_2 + (q_{2p}^D)^* w_p^* - (q_{2p}^R)^* w_b$ by choosing wholesale prices $(w_2, \bar{w}_2)$.

The formulation of the first period problem for the intermediaries is identical to the one under the separate channel. The manufacturer, however, maximizes her profit $\Pi_{1M}^{BC} = \Pi_{2M}^{BC} + \bar{q}_{1n}^*(\bar{w}_1, w_1, w_b)\,\bar{w}_1 + q_{1n}^*(\bar{w}_1, w_1, w_b)\,w_1$ by choosing the buyback price w_b in addition to the wholesale prices in the first period $(\bar{w}_1, w_1)$.

Buyback Channel with no Price Commitment

Under a buyback channel without price commitment, the manufacturer repurchases a certain number of used rental cars from the rental agency and sells them through the dealer in the sales market. Here, the buyback price is not committed at the time of initial purchase but rather determined through market equilibrium in the second period. Therefore the intermediaries' problems are identical to those provided in §4.3.1. However, the buyback price for program cars paid to the rental agency, w_b, comes from the equilibrium where $(q_{2p}^D)^* = (q_{2p}^R)^*$. The manufacturer, in the second period, maximizes her profit $\Pi_{2M}^{BN} = q_{2n}^* w_2 + \bar{q}_{2n}^* \bar{w}_2 + (q_{2p}^D)^* w_p - (q_{2p}^R)^* w_b^*$ by choosing wholesale prices $(w_2, \bar{w}_2)$ as well as the wholesale price charged to the dealer w_p. In the first period, the manufacturer does not commit to a buyback price and thus maximizes her profit $\Pi_{1M}^{BN} = \bar{q}_{1n}^*(\bar{w}_1, w_1)\,\bar{w}_1 + q_{1n}^*(\bar{w}_1, w_1)\,w_1 + \Pi_{2M}^{BN}$ by choosing wholesale prices $(\bar{w}_1, w_1)$ alone. This channel structure is identical to the buyback channel in Purohit (1997).

4.4 Channel Behavior Under No Uncertainty

In this section, we compare the profitability of different channel structures under no residual value uncertainty. This allows us to isolate the effect of buyback price commitment.

We are able to derive closed-form solutions for all equilibrium prices and quantities but the solutions are very complicated functions of θ_u and γ. As our focus is on profit comparisons, the equilibrium solutions are omitted here. When comparing the profits, we conduct numerical analysis by scanning the feasible regions of θ_u and γ. Given that both parameters have bounded support $[0, 1]$, our numerical study is designed to cover the entire parameter space and thus is comprehensive. All proofs are relegated to Appendix D1.

4.4.1 Impact of Buyback Price Commitment on Profitability

We compare the profitability of the channel structures and the results are as follows.

Proposition 22 *The profit rankings under no uncertainty are:*

i) (***Manufacturer***) $\Pi^O_{1M} \geq \Pi^{BN}_{1M} \geq \Pi^S_{1M} \geq \Pi^{BC}_{1M}$.

ii) (***Rental Agency***) *Depending on the value of* γ, *the profit rankings are:*

1. $\Pi^O_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^S_{1R} \geq \Pi^{BC}_{1R}$ *if and only if* $0 \leq \gamma \leq \bar{\gamma}_1(\theta_u)$ *where* $\bar{\gamma}_1(\theta_u)$ *is a decreasing function of* θ_u. *There exists a* $\bar{\theta}_u$ *such that* $\bar{\gamma}_1(\theta_u) = 1$ *if and only if* $\theta_u \leq \bar{\theta}_u$.

2. $\Pi^O_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^{BC}_{1R} \geq \Pi^S_{1R}$ *if and only if* $\bar{\gamma}_1(\theta_u) \leq \gamma \leq \bar{\gamma}_2(\theta_u)$ *where* $\bar{\gamma}_2(\theta_u)$ *is a convex function of* θ_u. *There exists a* $\bar{\bar{\theta}}_u$ *such that* $\bar{\gamma}_2(\theta_u) = 1$ *if and only if* $\theta_u \leq \bar{\bar{\theta}}_u$.

3. $\Pi^O_{1R} \geq \Pi^{BC}_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^S_{1R}$, *otherwise.*

iii) (***Dealer***) *Depending on the value of* γ *the profit rankings are:*

1. $\Pi^{BC}_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^S_{1D} \geq \Pi^O_{1D}$ *if and only if* $0 \leq \gamma \leq \bar{\bar{\gamma}}_1(\theta_u)$ *where* $\bar{\bar{\gamma}}_1(\theta_u)$ *is an increasing function of* θ_u. *There exits a* $\tilde{\theta}_u$ *such that* $\bar{\bar{\gamma}}_1(\theta_u) = 0$ *if and only if* $\theta_u \leq \tilde{\theta}_u$.

2. $\Pi^{BC}_{1D} \geq \Pi^S_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^O_{1D}$ *if and only if* $\bar{\bar{\gamma}}_1(\theta_u) \leq \gamma \leq \bar{\bar{\gamma}}_2(\theta_u)$ *where* $\bar{\bar{\gamma}}_2(\theta_u)$ *is an increasing function of* θ_u. *There exits a* $\tilde{\tilde{\theta}}_u \leq \tilde{\theta}_u$ *such that* $\bar{\bar{\gamma}}_2(\theta_u) = 0$ *if and only if* $\theta_u \leq \tilde{\tilde{\theta}}_u$.

3. $\Pi^S_{1D} \geq \Pi^{BC}_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^O_{1D}$ *if and only if* $\bar{\bar{\gamma}}_2(\theta_u) \leq \gamma \leq \bar{\bar{\gamma}}_3(\theta_u)$ *where* $\bar{\bar{\gamma}}_3(\theta_u)$ *is an increasing function of* θ_u.

4. $\Pi^S_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^{BC}_{1D} \geq \Pi^O_{1D}$, *otherwise.*

iv) (***Supply Chain***) $\Pi^O_{1SC} \geq \Pi^{BN}_{1SC} \geq \Pi^S_{1SC} \geq \Pi^{BC}_{1SC}$.

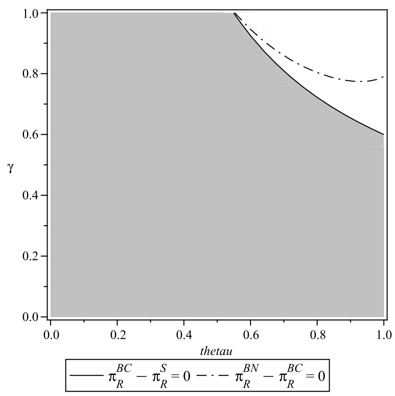

FIGURE 4.2: Three regions from left to right are (i) $\Pi^{BN}_{1R} \geq \Pi^S_{1R} \geq \Pi^{BC}_{1R}$, (ii) $\Pi^{BN}_{1R} \geq \Pi^{BC}_{1R} \geq \Pi^S_{1R}$, (iii) $\Pi^{BC}_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^S_{1R}$

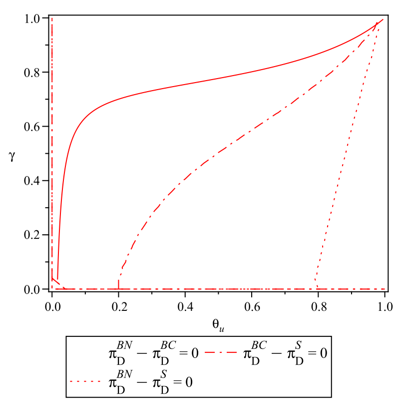

FIGURE 4.3: Four regions from left to right are (i) $\Pi^S_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^{BC}_{1D}$, (ii) $\Pi^S_{1D} \geq \Pi^{BC}_{1D} \geq \Pi^{BN}_{1D}$, (iii)$\Pi^{BC}_{1D} \geq \Pi^S_{1D} \geq \Pi^{BN}_{1D}$, (iv) $\Pi^{BC}_{1D} \geq \Pi^{BN}_{1D} \geq \Pi^S_{1D}$

First, we identify the most and the least profitable channel structures. Proposition 22 shows that the overlapping channel is the most profitable one for the supply chain, the manufacturer, and the rental agency but is the least profitable one for the dealer. The other profit rankings depend on the values of γ and θ_u. Figure 4.2 and 4.3 show the regions for the various cases described in Proposition 22.

The buyback channel, regardless of when the buyback price is set, can improve the dealer's profit at the expense of the rental agency's profit. This result is consistent with the findings of Purohit (1997) in terms of a buyback channel's ability to mediate the conflicts introduced by an overlapping channel. (Note that the buyback channel in Purohit (1997) is equivalent to our buyback channel with no price commitment, i.e., BN) The result is intuitive as a buyback channel avoids direct competition between the dealer and the rental agency that exists in an overlapping channel and benefits the dealer.

Second, Proposition 22 demonstrates that the attractiveness of a buyback channel for each party critically depends on whether the buyback price is committed at the time of initial purchase of new rental cars, i.e., in period 1. For sufficiently small γ, i.e., $\min\{\bar{\gamma}_1(\theta_u), \bar{\bar{\gamma}}_2(\theta_u)\}$, price commitment makes the buyback channel the most profitable one for the dealer but the least profitable one for the rental agency. In other words, when program cars depreciate much less than used cars, buyback price commitment benefits the dealer but hurts the rental agency. The intuition is as follows. The inclusion of slightly depreciated program cars in period 2 intensifies the competition between new cars and program cars, i.e., the competition between the two intermediaries. Early announcement of the buyback price by the manufacturer allows the dealer to better gauge the rental agency's decision on the number of program cars returning to the manufacturer and to be able to better adjust his quantity decision on new cars in both periods. Therefore, when channel competition is most intense between the intermediaries, early commitment of buyback price allows the manufacturer to shift channel profits to the dealer.

Third, Proposition 22 shows that for the manufacturer the buyback channel with price commitment is the least profitable one and thus is dominated by the separate channel. If, however, the buyback price is set in period 2, the buyback channel then always dominates the separate channel in terms of the manufacturer's profit, as shown by Purohit (1997). Our results suggest that by committing the buyback price early, the manufacturer gives the intermediaries a strategic advantage to adjust their first-period order quantities, which ends up hurting the manufacturer's profit.

It is important to understand why a buyback channel with price commitment gives the lowest profit for the manufacturer. At first glance, one would think that the manufacturer can always set a buyback price such that no program cars is returned by the rental agency in period 2, thus mimic the separate channel equilibrium. While this is true if the buyback price is set in period 2 according to the market clearing price such that the intermediaries choose $(q_{2p}^D)^* = (q_{2p}^R)^* = 0$, which is indeed the case for BN, it is not true when the manufacturer commits the buyback price in period 1. Let us denote the committed buyback price that would make the intermediaries choose $(q_{2p}^D)^* = (q_{2p}^R)^* = 0$ as w_b^{zero}. Assume the manufacturer chooses w_b^{zero} instead of the optimal w_b in period 1. The separate channel equilibrium cannot be mimicked because

1. If the manufacturer chooses to optimize the wholesale prices in period 1, then the optimal wholesale prices are smaller than those under the separate channel while the first period new car sales are higher and rental car sales are lower.

2. If the manufacturer does not optimize the first period wholesale prices but chooses the optimal separate channel wholesale prices, i.e. $(\bar{w}_1^S, w_1^S, w_b^{zero})$, then again

the dealer orders more and the rental agency orders less compared to the optimal quantities under the separate channel, i.e. $q_{1n}^B(\bar{w}_1^S, w_1^S, w_b^{zero}) \geq q_{1n}^S$ and $\bar{q}_{1n}^B(\bar{w}_1^S, w_1^S, w_b^{zero}) \leq \bar{q}_{1n}^S$.

We conclude that once the manufacturer commits a buyback price, regardless of how the manufacturer chooses the wholesale prices in the first period, the rental agency orders less than the optimal order quantity than he would under the separate channel. We can easily show that the wholesale price for rental cars in the second period $\bar{w}_2$ increases as q_{1n} and/or $\bar{q}_{1n}$ increase under the buyback channel with price commitment, while it decreases as $\bar{q}_{1n}$ increases under the separate channel. Hence, if the rental agency increases his first period order quantity under the buyback channel with price commitment, the manufacturer punishes him in the second period by increasing the wholesale price. Knowing that he would face a higher $\bar{w}_2$, the rental agency orders less in the first period, namely keeps $\bar{q}_{1n}$ low compared to that under the separate channel. In summary, price commitment changes the channel interactions.

Proposition 22 identifies the difference between the two versions of the buyback channel (i.e., BC and BN): price commitment leads to the worst profit while no price commitment leads to the second best for the manufacturer. The difference is a result of how the wholesale prices in period 1 and the buyback price are related. When the buyback price is determined in period 2, i.e., in the case of BN, it does not affect the wholesale prices and quantities in period 1 but rather it is a function of first-period decisions. As the wholesale prices in period 1 increase, the number of used cars in period 2 decreases and thus program cars face less competition from used cars and become more valuable.

This explains why in the buyback channel with no price commitment, as the wholesale prices increase in period 1 the buyback price in period 2 increases as well. In the buyback channel with price commitment, however, the relationship between the buyback price and the first period wholesale prices is different as shown by the next proposition.

Proposition 23 Π_{1M}^{BC} *is submodular in* (w_b, w_1) *and supermodular in* $(w_b, \bar{w}_1)$*.*

Proposition 23 implies that a higher w_b^* implies a higher $\bar{w}_1^*$ but a lower w_1^*. This suggests that as the manufacturer charges more for new cars, i.e., as the wholesale price of new cars increases, her incentive to make profit by buying and reselling program cars in period 2 decreases and so does the buyback price. To further understand why this occurs under price commitment, we compare the wholesale prices and the profit breakdowns of the manufacturer under the two versions of the buyback channel in Table 4.1 and Table 4.2, respectively. Note that the only place the manufacturer makes more profit under BC is from the rental car sales in period 2. Table 4.1 also suggests that price commitment may hurt the rental agency's profit by increasing the wholesale price of rental cars in the second period $\bar{w}_2$, as we discussed earlier. In contrast, price commitment may improve the dealer's profit by lowering w_p, i.e., the price charged to the dealer for the program cars.

	w_1	$\bar{w}_1$	w_2	$\bar{w}_2$	w_b	w_p
BN	+/-	+				+
BC			+	+	+	

TABLE 4.1: Comparison of wholesale prices under the buyback channel with no price commitment (BN) and the buyback channel with price commitment (BC).

We have seen how price commitment impacts profitability. It is not ex ante clear how it would affect the manufacturer's total sales. Let $Q = \sum_{i=1}^{2}(q_{in} + \bar{q}_{in})$ denote the total

	Manufacturer's Profit				
	New Cars in Period 1	Rental Cars in Period 1	New Cars in Period 2	Rental Cars in Period 2	Buyback Program
BN	+	+/-	+		+
BC				+	

TABLE 4.2: Comparison of profits under the buyback channel with no price commitment (BN) and the buyback channel with price commitment (BC).

new car sales in both periods.

Proposition 24 *The total sales ranking is* $Q^O \geq Q^{BN} \geq Q^S \geq Q^{BC}$.

Proposition 24 suggests that the total sales ranking is consistent with the ranking of the manufacturer's profits. In other words, under no uncertainty, larger sales leads to higher profits for the manufacturer.

In summary, buyback price commitment may shift channel profits from the manufacturer and the rental agency to the dealer. The shift of profit is most significant when program cars depreciate much less than used cars, i.e., when channel competition is potentially most severe. Therefore, the manufacturer can use the timing strategy on buyback price as a lever to redistribute channel profits and to alleviate channel conflicts caused by competition between program cars and new cars in the sales market.

4.4.2 Channel Conflict Resolution

We have shown how buyback price commitment may shift profits from the rental agency to the dealer when channel competition is intense. How would this affect a buyback channel's ability to resolve channel conflicts? To answer this question, it is useful to define a measure for channel conflicts and a measure for conflict resolution. Because there is no competition between the intermediaries under the separate channel, we use it as the *benchmark channel structure* under which there is no channel conflicts. Let $\delta_j = \Pi^O_{1j} - \Pi^S_{1j}$ for $j \in \{D, R\}$. Note that $\delta_D \leq 0$ while $\delta_R \geq 0$ for every θ_u and γ as suggested by Proposition 22, which indicates channel conflicts always exist under the overlapping channel.

To measure a buyback channel's ability to resolve channel conflicts, we define $\Delta^l_j = \Pi^l_{1j} - \Pi^S_{1j}$ for $j \in \{D, R\}$ and $l \in \{BC, BN\}$. If both $\Delta^l_D \geq 0$ and $\Delta^l_R \geq 0$ then we say that channel conflict is resolved by channel structure l. Although we have chosen the separate channel as the benchmark for no channel conflict for the rest of our analysis, one might set the standard for conflict resolution differently by requiring $\Delta^l_D \geq x$ and $\Delta^l_R \geq y$ where x and y are positive or negative constants. The choices of x and y reflect how the manufacturer would like to allocate channel profits between the intermediaries. The choices of x and y will lead to larger or smaller feasible regions for resolving channel conflicts in the parameter space of θ_u and γ.

Using the definition of channel conflict and conflict resolution, we first analyze how well the channel conflict is resolved by a buyback channel without price commitment. From Proposition 22 we know that for the rental agency this channel structure is always more profitable than the separate channel, i.e. $\Delta^{BN}_R \geq 0$ for all θ_u and γ. Therefore the channel conflict between the two intermediaries is resolved as long as the buyback channel is more profitable than the separate channel for the dealer, i.e. $\Delta^{BN}_D \geq 0$.

Proposition 25 *Channel conflict is resolved in a buyback channel with no price commitment, i.e. both $\Delta^{BN}_D \geq 0$ and $\Delta^{BN}_R \geq 0$, if and only if $0 \leq \gamma \leq \gamma^{BN}_{UB}(\theta_u)$ where $\gamma^{BN}_{UB}(\theta_u)$*

is an increasing function of θ_u. Moreover, $\gamma_{UB}^{BN}(\theta_u) = 0$ if and only if $\theta_u \leq 0.783$.

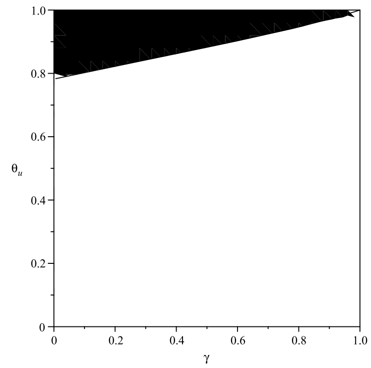

FIGURE 4.4: Shaded region shows the area where channel conflict is resolved under no price commitment, i.e. both $\Delta_R^{BN} \geq 0$ and $\Delta_D^{BN} \geq 0$.

Although this result has been demonstrated in Purohit (1997) (see Figure 4 in his paper), we also formalize it here for the purpose of comparing the two versions of the buyback channel. From Proposition 25 and Figure 4.4, we conclude that channel conflict is resolved under no price commitment if the depreciation rate for used cars is sufficiently high, i.e. $\theta_u \geq 0.783$. Moreover, if program cars depreciate similarly as used cars, i.e. γ is high, then channel conflict is resolved only if used car depreciation is remarkably high. For example if $\gamma = 0.85$ then channel resolution is achieved only for $\theta_u \geq 0.95$. Therefore the buyback channel with no price commitment provides a means for channel resolution only for cars with very high depreciation rates. For cars with lower depreciation rates, the separate channel is more profitable for the dealer and thus the buyback channel with price commitment cannot solve channel conflict according to our definition.

Now the question is how committing buyback price early would impact a buyback channel's ability to solve channel conflicts. To answer this question we first characterize the region where the buyback channel with price commitment resolves the channel conflict between the intermediaries, as shown in Proposition 26.

Proposition 26 *In a buyback channel with price commitment, if* $\theta_u < 0.7785$ *then the channel conflict is not resolved regardless of* γ*. Otherwise, the conflict is resolved, i.e. both* $\Delta_D^{BC} \geq 0$ *and* $\Delta_R^{BC} \geq 0$*, if and only if* $\gamma_{LB}^{BC}(\theta_u) \leq \gamma \leq \gamma_{UB}^{BC}(\theta_u)$ *where* $\gamma_{LB}^{BC}(\theta_u)$ *is decreasing in* θ_u *and* $\gamma_{UB}^{BC}(\theta_u)$ *is increasing in* θ_u*. In addition* $\gamma_{LB}^{BC}(\theta_u = 1) = 0.5997$ *and thus channel conflict is never resolved if* $\gamma \leq 0.5997$*.*

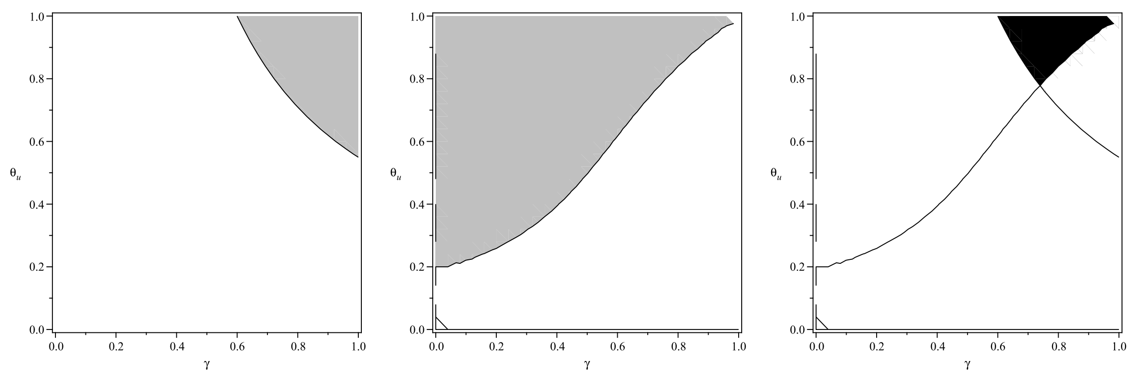

FIGURE 4.5: Shaded region shows where (a) $\Delta_R^{BC} \geq 0$ (b) $\Delta_D^{BC} \geq 0$ (c) both $\Delta_R^{BC} \geq 0$ and $\Delta_D^{BC} \geq 0$.

From Proposition 26 and Figure 4.5 we conclude that the buyback channel with price commitment actually provides a middle ground for both intermediaries as long as θ_u is sufficiently large and γ is in an appropriate interval. More specifically, we show that channel conflict is never resolved if $\theta_u \leq 0.7785$ or $\gamma \leq 0.5997$. Note that $\Delta_R^{BC} = 0$ on the lower bound $\gamma_{LB}^{BC}(\theta_u)$, whereas $\Delta_D^{BC} = 0$ on the upper bound $\gamma_{UB}^{BC}(\theta_u)$. Comparing Figure 4.5 (b) with Figure 4.4, we see that the dealer gains in profit in the buyback channel for a much larger region under price commitment than under no price commitment. However, Figure 4.5 (a) shows that the rental agency may become worse off in the buyback channel with price commitment when both θ_u and γ are sufficiently large (i.e., the white region in the lower left corner of the figure). Recall that, on the contrary the rental agency always has higher profit under a buyback channel with no price commitment than under the separate channel.

The differences in the shaded regions of Figure 4.4 and 4.5 point out the impact of price commitment on a buyback channel's ability to resolve channel conflicts. Committing buyback price early shifts channel profits from the rental agency to the dealer and improves the manufacturer's ability to resolve channel conflicts when program cars depreciate much less than used cars. Our results show that if γ is high then price commitment solves the channel conflict for a wider range of θ_u. On the other hand, γ is low then no price commitment solves the channel conflict for a wider range of θ_u.

4.4.3 Turn-Back Standard as a Profit Lever

The last two propositions show that a buyback channel's ability in resolving channel conflicts highly depends on the depreciation characteristics of the products, i.e. θ_u and γ. As lower γ implies smaller depreciation of program cars in comparison to the depreciation of used cars, the manufacturer can influence γ by changing turn-back standards on repurchased cars. To be eligible for repurchase, program cars need to meet certain turn-back standards such as regular maintenance service records, completed warranty and recall repairs, etc. Note that, higher turn-back standards might mean more criteria or higher levels of a set of criteria. Therefore if the manufacturer requires high turn-back standards then γ is low, i.e. program cars have a lower depreciation rate compared to used cars. Our following analysis demonstrates how the manufacturer's profitability depends on γ and suggests that the manufacturer can use turn-back standard as a lever to improve profits. Although a manufacturer cannot fully control γ through turn-back standards alone, in the following discussion we use γ interchangeably with turn-back standards. Still, our results continue to hold if γ is a decreasing linear function of turn-back standards.

Although the buyback channel structure is never the most profitable one for the manufacturer, by setting turn-back standards optimally the manufacturer might increase her profit under a buyback channel. First, we consider the buyback channel with no price commitment and characterize γ^* that would give highest manufacturer profit while solving the channel conflicts. We answer this question by solving problem (P1). We first characterize the behavior of the manufacturer's profit with respect to depreciation differential γ in Proposition 27.

$$\begin{aligned} (P1)\ Max_{0\leq\gamma\leq 1} \quad & \Pi_{1M}^{BN}(\gamma,\theta_u) \\ s.t. \quad & \Delta_R^{BN}(\gamma,\theta_u) \geq 0 \\ & \Delta_D^{BN}(\gamma,\theta_u) \geq 0 \end{aligned}$$

Proposition 27 *In a buyback channel with no price commitment, the manufacturer's profit function Π_{1M}^{BN} is convex in γ. In addition, if $\theta_u \leq 0.690$ then Π_{1M}^{BN} is strictly decreasing in γ.*

Since Proposition 27 shows the convexity of the manufacturer's objective function in γ, we can easily characterize the solution of the unconstrained profit maximization problem, $Max_{0\leq\gamma\leq 1}\Pi_{1M}^{BN}(\gamma,\theta_u)$ and then find the maximum profit is obtained at $\gamma^*(\theta_u) = 0 \ \ \forall\theta_u$. Therefore, if the manufacturer is not bound by the constraint of solving channel conflicts then she should set turn-back standards as high as possible. However, if she is, Proposition 28 provides the characterization of the optimal solution for (P1).

Proposition 28 *If $\theta_u \leq 0.783$ then problem (P1) has no feasible solution. Otherwise, the optimal γ that maximizes the manufacturer's profit under no price commitment while solving channel conflicts is $\gamma^*(\theta_u) = 0$, for $\forall\theta_u$.*

With Proposition 28 we show that optimal $\gamma^*(\theta_u) = 0$. Therefore, whether the manufacturer is constrained by solving channel conflicts or not, she should set turn-back standards as high as possible. However, she should also keep in mind that if θ_u is low, i.e. $\theta_u \leq 0.79$, then setting turn-back standards alone would not be sufficient for solving channel conflicts.

Similarly, the manufacturer can use turn-back standards as a profit lever in a buy-back channel with price commitment. Again, we first characterize the behavior of the manufacturer's profit function with respect to θ_u and γ.

Proposition 29 *The manufacturer's profit under price commitment Π_{1M}^{BC} is a convex function of γ. Besides, if $\theta_u \leq 0.608$ then Π_{1M}^{BC} is a decreasing function of γ.*

Proposition 29 implies that the manufacturer should keep γ as small as possible if $\theta_u \leq 0.608$. On the other hand if θ_u is greater than 0.608 the manufacturer should choose either the smallest or the highest *possible* γ whichever provides the highest profit. After characterizing the behavior of manufacturer's profit, now it is straightforward to solve the unconstrained profit maximization problem, $Max_{0\leq\gamma\leq 1}\Pi_{1M}^{BC}(\gamma,\theta_u)$, for the manufacturer. Again we can show that the maximum profit is obtained at $\gamma^*(\theta_u)=0\ \ \forall\theta_u$. As in the case of no price commitment, if the manufacturer were not to consider the channel conflicts under price commitment then she would set the turn-back standards as high as possible. However she is. Thus, we next solve problem (P2) and provide the characterization of the optimal solution in Proposition 30.

$$
\begin{aligned}
(P2)\ Max_{0\leq\gamma\leq 1} \quad & \Pi_{1M}^{BC}(\gamma,\theta_u) \\
s.t. \quad & \Delta_R^{BC}(\gamma,\theta_u)\geq 0 \\
& \Delta_D^{BC}(\gamma,\theta_u)\geq 0
\end{aligned}
$$

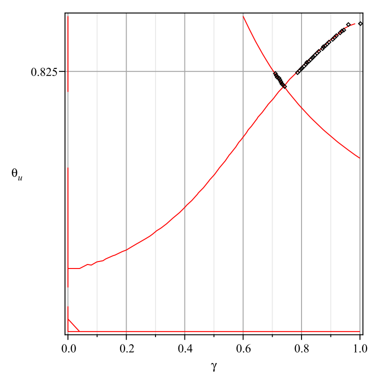

FIGURE 4.6: Optimal γ^* that solves the channel conflicts is depicted in black dots

Proposition 30 *If* $\theta_u < 0.778$ *the problem (P2) has no feasible solution. Otherwise the optimal* γ *for problem (P2) is as follows: (i) If* $0.778 \leq \theta_u \leq 0.825$ *then choose* $\gamma^*(\theta_u)=\gamma_{LB}^{BC}(\theta_u)$*, (ii) otherwise, choose* $\gamma^*(\theta_u)=\gamma_{UB}^{BC}(\theta_u)$ *(see Figure 4.6).*

First Proposition 30 shows that the channel conflict can never be resolved if $\theta_u <$

0.778. Second, if $0.778 \leq \theta_u \leq 0.825$ the optimal policy for the manufacturer is setting the turn-back standards as high as possible, i.e. keeping γ as small as possible. Figure 4.6 shows that for this case $\gamma^*(\theta_u)$ is identified by the condition where $\Delta_R^{BC} = 0$ (recall the two shaded regions from Figure 4.5.) Hence, for moderate values of θ_u, the manufacturer chooses γ^* so that the rental agency makes the same profit as he would make under the separate channel while the dealer makes more profit than he would under the separate channel.

Finally, if $\theta_u \geq 0.825$ the optimal policy is to set turn-back standards as low as possible, i.e. $\gamma^*(\theta_u)$ as high as possible. Figure 4.6 shows that for this case $\gamma^*(\theta_u)$ is identified by the condition where $\Delta_D^{BC} = 0$. Thus, when θ_u is high, it is optimal to set turn-back standards low so that the dealer makes as much profit as he would make under the separate channel while the rental agency makes more profit than he would under the separate channel.

In conclusion, when using a buyback channel to mediate channel conflicts, the manufacturer prefers high turn-back standards (i.e., low γ) when the depreciation of used cars is not too high. This is because high turn-back standards reduce the depreciation rate of program cars and thus alleviates the competition between used and program cars. This avoids a sharp decrease in the prices of cars in the used market and in turn helps keep new car prices high in both periods.

4.5 Channel Behavior under Uncertainty

In this section we compare the performance of the channel structures under residual value uncertainty. The way we model residual value uncertainty is by assuming that θ_u is distributed uniformly over the region $[a, b]$ where $0 \leq a \leq b \leq 1$. The uncertainty is resolved at the beginning of the second period. Accordingly the second period problem formulations remain the same as the case of no uncertainty. In the first period, on the other hand, we maximize the expected profit to go $E_{\theta_u}\left[\Pi_{1j}^{l}\right]$ for player $j \in \{M, D, R\}$ under channel structure $l \in \{S, O, BN, BC\}$. We first analyze a special case where the support of θ_u is $[0, 1]$ and later relax the assumption.

4.5.1 Impact of Buyback Price Commitment on Profitability

Proposition 31 compares the profitability of the channel structures under uncertainty.

Proposition 31 *If $\theta_u \sim U[0,1]$ then the profit rankings are:*

i) (***Manufacturer***) $\Pi_{1M}^{O} \geq \Pi_{1M}^{BN} \geq \Pi_{1M}^{S} \geq \Pi_{1M}^{BC}$.

ii) (***Rental Agency***) *Depending on the value of γ, the profit rankings are as follows:*

1. $\Pi_{1R}^{O} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S} \geq \Pi_{1R}^{BC}$ *if* $0 \leq \gamma \leq 0.549$.

2. $\Pi_{1R}^{O} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{S}$ *if* $0.549 \leq \gamma \leq 0.572$.

3. $\Pi_{1R}^{O} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S}$ *if* $0.572 \leq \gamma \leq 0.765$.

4. $\Pi_{1R}^{BC} \geq \Pi_{1R}^{O} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S}$, *otherwise.*

iii) (***Dealer***) $\Pi_{1D}^{BC} \geq \Pi_{1D}^{S} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{O}$.

iv) (***Supply Chain***) *Depending on the value of γ, the profit rankings are as follows:*

1. $\Pi^{O}_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^{S}_{1R} \geq \Pi^{BC}_{1R}$ *if* $0 \leq \gamma \leq 0.962$.

2. $\Pi^{O}_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^{BC}_{1R} \geq \Pi^{S}_{1R}$ *if* $0.962 \leq \gamma \leq 0.964$.

3. $\Pi^{O}_{1R} \geq \Pi^{BC}_{1R} \geq \Pi^{BN}_{1R} \geq \Pi^{S}_{1R}$, *otherwise.*

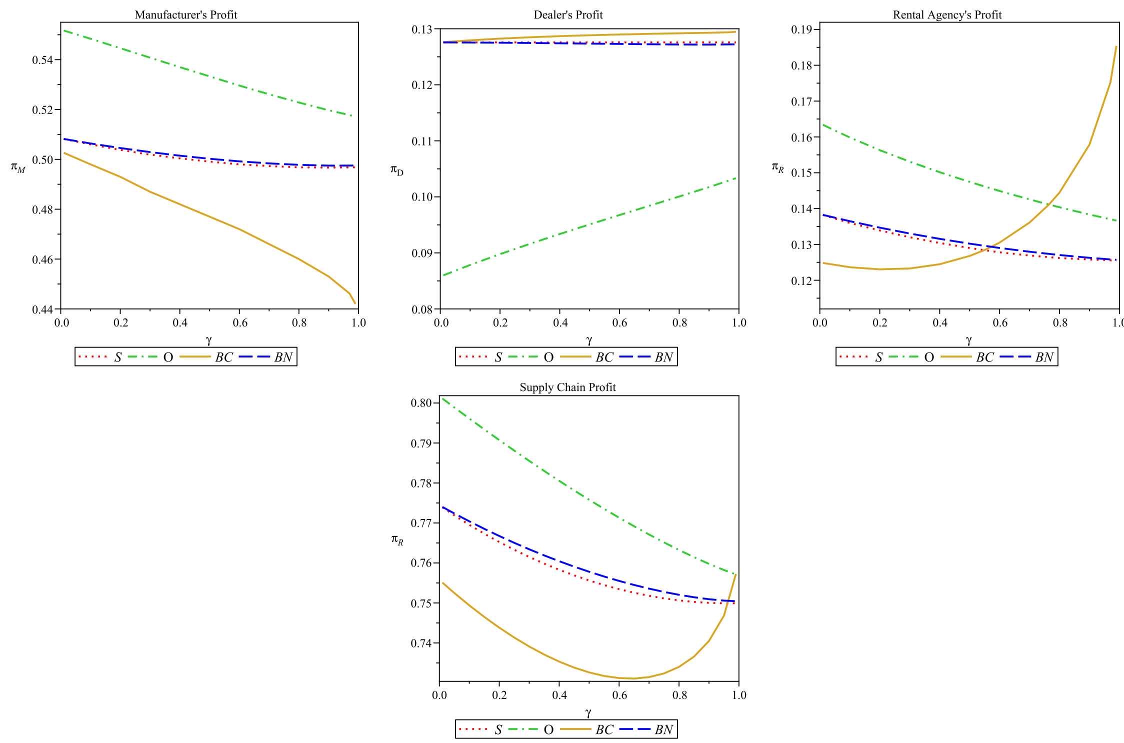

FIGURE 4.7: Profit rankings of (a) the manufacturer, (b) the dealer, (c) the rental agency, and (d) the supply chain when $\theta_u \sim U[0, 1]$

First we identify if and how uncertainty affects the relative profitability of channel structures. Figure 4.7 illustrates the results stated in Proposition 31. As in the case of no uncertainty, the overlapping channel is the most profitable one for the supply chain and the manufacturer. However, the rental agency's profitability in the overlapping channel decreases under uncertainty. When program cars depreciate similarly as used cars, i.e. when γ is high, the buyback channel with price commitment becomes the most profitable one for the rental agency. Price commitment under uncertainty improves profits not only for the rental agency but also for the dealer and the supply chain. Even though the separate channel might be the most profitable one for the dealer for sufficiently high γ under no uncertainty, the buyback channel with price commitment is always the most profitable one for the dealer under uncertainty regardless of γ. The manufacturer, however, still makes the least profit under the buyback channel with price commitment.

Note that under price commitment as γ increases channel profits get transferred from the manufacturer to the rental agency. For higher γ values, i.e., when program cars depreciate similarly as used cars, by committing to a buyback price early the manufacturer bears the residual value risk and thus improves the rental agency's profit. Next we compare total sales across the channel structures.

Proposition 32 *Under residual value uncertainty, the rankings of total sales are as follows:*

1. $Q^O \geq Q^{BN} \geq Q^S \geq Q^{BC}$ *if and only if* $0 \leq \gamma \leq 0.915$.

2. $Q^O \geq Q^{BN} \geq Q^{BC} \geq Q^S$ *if and only if* $0.915 \leq \gamma \leq 0.918$

3. $Q^O \geq Q^{BC} \geq Q^{BN} \geq Q^S$, *otherwise.*

Proposition 32 suggests that uncertainty increases the attractiveness of price commitment in terms of total sales. When γ is sufficiently high, the buyback channel with price commitment can achieve the second highest total sales. This is in stark contrast to the ranking under no uncertainty where the buyback channel with price commitment always

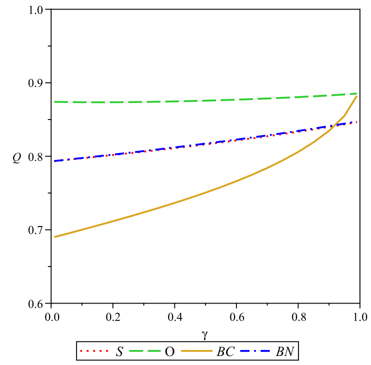

FIGURE 4.8: Total sales achieved under various distribution channel structures

gives the lowest total sales. Total sales is crucial as auto manufacturers are struggling with the problem of excess capacity and are still constrained to keep their factories open. Hence the widespread strategy in the auto industry has been keeping their plants running even if the profits are not high (Henry (2008)). Our result suggests that manufacturers' preference for the buyback channel with price commitment might also be motivated by keeping sales high in the face of uncertainty.

4.5.2 Channel Conflict Resolution

Even though the overlapping channel is not always the most profitable one for the rental agency, it still introduces channel conflicts under uncertainty by Proposition 31. Under no uncertainty we know that a buyback channel, regardless of the timing of buyback price announcement, can be used to mediate channel conflicts. Therefore the next question is, how does uncertainty affect a buyback channel's ability in resolving channel conflicts?

Proposition 33 *Suppose $\theta_u \sim U[0,1]$. The buyback channel with no price commitment cannot solve the channel conflict. The buyback channel with price commitment solves the* *channel conflict if and only if* $0.5485 \leq \gamma \leq 1$.

Proposition 33 shows that if the buyback price is not committed early then uncertainty reduces a buyback channel's ability in solving channel conflicts. However, if the buyback price is committed early then the channel conflict may be resolved when program cars' depreciation rate is not much less than used cars', i.e. $\gamma \geq 0.5485$. This result is also illustrated in Figure 4.9. Note that in the case of *no uncertainty* channel conflicts can be resolved using a buyback channel with price commitment only if $\gamma \geq 0.5997$ and θ_u is in the appropriate interval. In comparison, uncertainty enlarges the feasible region of γ for conflict resolution. In other words, uncertainty enhances a buyback channel's ability in conflict resolution when the buyback price is committed.

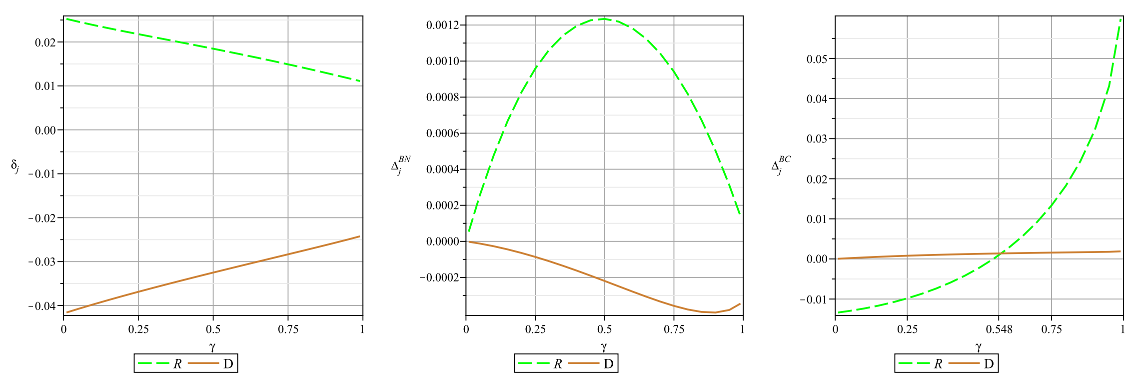

FIGURE 4.9: (a)Channel conflicts in the overlapping channel (b) No resolution is achieved by the buyback channel without price commitment (c) Conflict resolution under the buyback channel with price commitment when $\gamma \geq 0.5485$

4.5.3 Turn-Back Standard as a Profit Lever

As we discussed earlier, turn-back standards can be used as a lever to improve profits. We now explore this effect under uncertainty. On the one hand, the manufacturer's profit is monotone decreasing in γ as shown in Figure 4.7(a). On the other hand, channel conflicts cannot be resolved unless γ is sufficiently high as shown in Figure 4.9 (c). Therefore the manufacturer has to set γ appropriately considering the trade-off between resolving channel conflicts and maximizing her profit. Taken together, it is straightforward to see that for $\theta_u \sim U[0,1]$ the optimal $\gamma^* = 0.5485$.

Next we relax our assumption on the distribution of θ_u by considering a series of uniform distributions with different mean and range values. We consider nine mean values $\in \{0.1, 0.2, .., 0.9\}$ and various ranges that are obtained by 0.2 increments. Hence our experimental setting roughly spans all possible uniform distributions for θ_u bounded between 0 and 1. The results are presented in Table 4.3. Note that if no γ value can resolve channel conflicts for that a particular mean and range combination it is shown with "$-$". For the feasible ones, we provide the lower and upper bound, i.e. $[\gamma_{LB}, \gamma_{UB}]$, for the γ value range within which channel conflicts can be resolved.

Mean	Range				
	0.2	0.4	0.6	0.8	1.0
0.9	[0.624,1]				
0.8	[0.684,0.804]	[0.615,1]			
0.7	-	[0.671,0.844]	[0.595,1]		
0.6	-	-	[0.647,1]	[0.571,1]	
0.5	-	-	[0.731,0.862]	[0.619,1]	[0.548,1]
0.4	-	-	-	[0.677,1]	
0.3	-	-	-		
0.2	-	-			
0.1	-				

TABLE 4.3: Mean and range values for θ_u and the feasible region $[\gamma_{lb}, \gamma_{ub}]$ with which conflict is resolved in a buyback channel with price commitment

From Table 4.3 we first observe that if the mean is below a threshold (less than 0.3 in our experiments) then channel conflict is never resolved regardless of the value of γ.

Otherwise, channel conflict is resolved if and only if the range is sufficiently high and γ is in the right interval, i.e. $\gamma_{lb} < \gamma < \gamma_{ub}$.

Note that for the same mean, as the range increases γ_{lb} decreases while γ_{ub} increases or remains the same. In other words, as the residual value of used cars becomes more volatile, channel conflict is resolved for a larger range of γ values. For the same range, on the other hand, as the mean increases γ_{lb} decreases while γ_{ub} increases or remains the same, and thus as the average depreciation rate of used cars increases, channel conflict is resolved for a larger range of γ values. Finally, the largest feasible region for conflict resolution is achieved when $\theta_u \sim U[0, 1]$ with mean 0.5.

Mean	Range				
	0.1	0.2	0.3	0.4	0.5
0.9	1				
0.8	0.684	0.615			
0.7	-	0.671	0.595		
0.6	-	-	0.647	0.571	
0.5	-	-	0.731	0.619	0.548
0.4	-	-	-	0.677	
0.3	-	-	-		
0.2	-	-			
0.1	-				

TABLE 4.4: The optimal γ values maximizing the manufacturer's profit while solving the channel conflicts under the buyback channel with price commitment, i.e. γ_M^*

We further compute the optimal γ^* that maximizes the manufacturer's profit over the feasible region of conflict resolution and present the results in Table 4.4. We observe that γ_M^*, i.e. the γ value that maximizes the manufacturer's profit in the buyback channel with price commitment while resolving channel conflicts, is equal to γ_{lb} in many cases. However when the mean of θ_u is 0.9 we observe that γ_{ub} maximizes the profit. Thus if the average depreciation rate of used cars is very high then the manufacturer should impose lower maintenance standards and otherwise she should impose higher maintenance standards. Recall that this result is consistent with those obtained for the no uncertainty case.

Mean	Range				
	0.2	0.4	0.6	0.8	1.0
0.9	[0,0.635]				
0.8	[0,0.126]	[0,0.260]			
0.7	-	-	-		
0.6	-	-	-	-	
0.5	-	-	-	-	-
0.4	-	-	-	-	
0.3	-	-	-		
0.2	-	-			
0.1	-				

TABLE 4.5: Mean and range values for θ_u and the feasible region $[\gamma_{lb}, \gamma_{ub}]$ with which conflict is resolved in a buyback channel with no price commitment

Next we analyze how the buyback channel with no price commitment performs in terms of solving channel conflicts under uncertainty. The results are displayed in Table 4.5. Comparing Table 4.3 and Table 4.5 we observe that under uncertainty price commitment allows a buyback channel to achieve conflict resolution for a larger set of mean and range values.

To summarize, residual value uncertainty enhances a buyback channel's ability to resolve channel conflicts when the buyback price is committed early. Moreover, price commitment helps the manufacturer to achieve higher total sales in a buyback channel when the depreciation rate of program cars and used cars are sufficiently close. Finally, under uncertainty, the manufacturer, in most cases, should set turn-back standards as high as possible to improve profits in a buyback channel.

4.6 Conclusions

Channel conflicts between dealers and rental agencies make it a challenging task for durable-good manufacturers to set channel policies. While competition among intermediaries leads to higher profits for manufacturers, they have to keep the competition under control so as not to face opposition from the intermediaries. Prior academic research shows that to mitigate channel conflicts manufacturers can implement a take-back strategy, so-called buyback program, under which they repurchase used products from rental agencies and resell them to dealers.

We show that with no uncertainty, early commitment of the buyback price reduces the total channel profit and the manufacturer's profit. In addition, price commitment benefits the dealer but hurts the rental agency when program cars depreciate much less than used cars. However, our results suggest that under residual value uncertainty, manufacturers can pull two levers to adjust the performance of a buyback channel: the timing policy on when to set the buyback price and the turn-back standards on the repurchased used products. Under uncertainty price commitment enhances a buyback channel's ability to alleviate channel conflicts and may lead to higher total sales for the manufacturer. Finally, setting a higher turn-back standard in general increases the manufacturer's profit in the presence of uncertainty.

CHAPTER 5

Conclusions and Future Research

In recent times, an increasing number of companies are making product take-back an important part of their business strategy. Product take-back is mostly motivated for two different reasons: While some firms are forced by environmental legislations that hold them responsible for financing collection and treatment of discarded products by the customers, others are driven by economic and/or marketing concerns. In this dissertation, we study the profitability and efficiency of various product take-back programs.

The first two essays of this dissertation examine take-back legislation's impacts on companies' collection and treatment decisions as well as the efficacy of various implementations of take-back legislations. The second chapter of this dissertation titled "The Impact of Take-Back Legislation on Remanufacturing" investigates how existing forms of take-back legislations with different collection/recovery targets affect remanufacturing decisions and if there is any additional benefit in imposing remanufacturing targets in terms of meeting the objectives of legislations. First, we consider a monopolist OEM with in-house remanufacturing capabilities (or a monopolist OEM who outsources remanufacturing but sells the remanufactured products under her own brand name). We then consider an OEM who does not sell remanufactured products but faces competition from a third-party remanufacturer. Using a stylized model of take-back legislation in a two-period setting, we study how existing forms of legislations as well as extended forms with additional remanufacturing targets affect optimal production, take-back and remanufacturing decisions. Among other results, our key findings are as follows: First, we explore the effect of take-back legislations on remanufacturing levels and we find that legislation never causes a decrease in remanufacturing levels of a monopolist OEM. In the presence of competition, we show that legislation may indeed cause a decrease in remanufacturing levels and hence imposing take-back legislations in an industry where remanufacturing is carried out by independent third-party remanufacturers might hurt remanufacturing industry. Moreover, we compare the effect of legislation on an OEM with in-house remanufacturing versus one competing with a third-party remanufacturer. Surprisingly, we find that the remanufacturing level achieved by a third-party remanufacturer, who is in competition with an OEM, may be higher than that achieved by a monopolist OEM facing legislation. This suggests that, in order to achieve higher remanufacturing levels government might indeed want to consider subsidizing third party remanufacturers rather than imposing take-back legislations. An underlying motivation for take-back legislation is to create incentives for environmentally-friendly business decisions such as designing products that are easier and cheaper to remanufacture. Our results suggest that, legislation creates incentive for designing products that are cheaper to remanufacture at the expense of increasing manufacturing cost not only for a monopolist OEM but also for an OEM facing competition from a third-party remanufacturer.

A firm launching a product take-back program have a number of different options to utilize the cores: She might remanufacture and sell the product, remanufacture and use the product for warranty purposes, reuse the cores -as it is- for warranty replacements or in production processes, recycle and use recycled materials in production process or recycle and sell recycled materials in the recycling market, etc. Although we study the impact of legislations on company's remanufacturing decisions in the second chapter, we acknowledge that not all returned products can be remanufactured. It depends on the type of the products, its age, condition and core technology. In some cases, remanufacturing is not viable and recycling or reuse -as it is- might be the only option. This limitation of our research leads to opportunities for future research. The next step would be to investigate how take-back legislations affect companies' environmentally-friendly business decisions when remanufacturing is not an option. In such a setting, collected cores might be used for warranty replacements, recycling, etc. Analyzing these situations would also assess the robustness of our results. Another limitation of our research is that we assume that all items sold in a particular period are available for collection and remanufacturing in the following period. Although this assumption facilitates the analysis, another research opportunity is to consider the situation where used products become available for collection and remanufacturing after some period of time.

In an industry regulated with product take-back legislations, each producer meets her obligations through a compliance scheme. While we consider take-back legislation that stipulates individual responsibility in the second chapter, some implementations of legislation allow manufacturers to fulfill their obligation either individually or by joining a collective scheme. In the third chapter of the dissertation titled "Complying with Take-Back Legislation: A Cost Comparison and Benefit Analysis of Three Compliance Schemes", we study a firm's compliance scheme choice. Complying with legislation is a valid cost to business and each firm chooses the most cost effective compliance scheme. In most implementations of take-back legislation, individual and/or collective schemes (with cost allocation by market share and with IFR) are available/allowed. We consider n manufacturers and a non-profit PRO who manages the collection and treatment activities on behalf of its members, and compare the three possible compliance schemes in terms of cost effectiveness and the environmental benefits they achieve. Among other results, our key findings are as follows: As intuition suggests, collective scheme with IFR is in general the most cost effective alternative. However, the question is how feasible it is to implement a collective scheme with IFR that requires sorting EOL products by brand and tracking them through the treatment process? Although in the future with RFID technology, sorting and tracking of EOL products would be much easier (but still costly) and thus IFR within a collective scheme might be easier to implement; individual scheme and collective scheme with cost allocation by market share are the two prevalent forms of compliance that we encounter in practice. We find that which scheme (of these two) gives lower compliance cost depends on the initial collection rate maintained by government and non-profits as well as the market shares of partner firms. If the partner firms have high market shares then collective compliance, in general, yields lower cost for a producer. On the other hand, if the partner firms have small market shares then a producer pays lower compliance cost under collective scheme only if the initial collection rate maintained by the government is of moderate levels. Hence, which compliance scheme is more cost effective depends not only on the total market share of the collective scheme but also on the market shares of individual members. From policy maker's perspective we analyze how the choice of a particular compliance scheme affects the environmental benefits, i.e., collection rate and treatability level, achieved. Our results suggest that, before imposing a particular compliance scheme, the policy maker should be aware of the trade-offs involved and should recognize that even when the collection level is higher under the collective scheme, the individual scheme, in general, gives higher treatability levels. Therefore, our results suggest that high collection rates and treatability levels may be hard to achieve simultaneously unless governments are willing to partially incur the collection costs. We also show that high collection targets imposed by governments push more producers to choose the collective compliance scheme and result in a degradation of treatability levels.

When comparing the three compliance schemes we assume that the cost parameters are the same under each compliance scheme. However, treatment activities are usually carried out by third-party treatment companies and third-party companies might quote a lower price for the collective schemes due to various reasons including the bargaining power of collective schemes. Our research does not capture this potential difference in cost parameters. Therefore, a possible extension would be to consider the cost parameter differentials between individual and collective schemes. Another interesting extension would be to consider alternative cost sharing mechanisms such as Shapley value and explore how different cost sharing mechanisms affect the goals of the legislations as well as the compliance cost for the producers.

Some companies launch product take-back programs even in the absence of legislations. In the fourth chapter of the dissertation titled “The Impact of Buyback Price Commitment and Demand Uncertainty on Channel Conflicts”, we study product take-back motivated by the goal of managing distribution channels better and mitigating the channel conflicts between the intermediaries. We consider a durable goods manufacturer with dual distribution channels, i.e., rental and sales channel, and compare the buyback channel structure with two other channel structures, i.e., separate channel and overlapping channel. Although it is argued in the literature that buyback channel does not maximize the manufacturer's profits (Purohit, 1997), buyback programs are still commonly implemented in practice. In this chapter, using a two-period model of the relationship between the manufacturer and the intermediaries, we rationalize a manufacturer's decision in launching buyback channel even if it does not provide the most profitable alternative among the three channel structures. We consider two characteristics that affect the profitability of the buyback program: (i) timing on the announcement of buyback prices; and (ii) turn-back (quality) standards for the repurchased cars. We find that the manufacturer can improve profits by setting the buyback price at the time of repurchase instead of committing it at the time of initial purchase. Then, why is buyback price commitment still being implemented in practice by durable goods manufacturers? The answer becomes clear under demand uncertainty considerations. We show that in the presence of demand uncertainty, commitment of buyback price enhances a buyback channel's ability in resolving channel conflicts –which are introduced by overlapping channel structure. In addition, in the presence of uncertainty the manufacturer achieves higher total sales under buyback price commitment than that under no price commitment. In contrast, in the absence of uncertainty, buyback price commitment always leads to the smallest total sales among the channel structures. Finally, we identify operational levers that might improve the manufacturer's profitability under buyback channel and show that the manufacturer's profit generally increases in the turn-back standard.

Although we consider the three different channel structures, the current prevailing channel structure is close to the hybrid of a buyback channel and an overlapping channel. An interesting extension to the fourth chapter would be to consider a hybrid channel structure where the manufacturer announces an upper bound on the amount of items that she would buyback from the rental agency and then the rental agency decides on how many items to sell back to manufacturer and how many items to sell in the sales market.

While this dissertation contributes to the better management of take-back programs, there are still several different avenues for future research. For example, while some companies take back only their own brand-name products, some others (e.g. HP) take back products regardless of the products' brand name. When a company accepts all product returns without distinguishing brand name, the total volume of products collected would be higher and thus the company might benefit from economies of scale in treatment cost or if the treatment is a profitable business she might make more profit by collecting competitors' products. However, she also faces an uncertainty in the condition (treatability level) of other brand-name products. Thus, an interesting research question would be to explore how a manufacturer should design the take-back program when she takes back products regardless of brand-name. Another question that arises in this setting is how the competition in collection activities affect the take-back decisions when the treatment of discarded products is a profitable business.

A1 Appendix for Chapter 2: Proofs

In this section we provide proofs of theorems and propositions in Chapter 2. Note that throughout the analysis we assume that $Q-c_M-\phi\beta(c_C+c_D) \geq 0$ and $\alpha Q-c_R-c_C \geq 0$ to avoid the trivial situations where neither manufacturing nor remanufacturing is profitable in the second period. We define $c_M^{max} = Q - \phi\beta(c_C + c_D)$ and $c_R^{max} = \alpha Q - c_C$.

Proof of Theorem 1. *Characterization of the optimal regions in the absence of regulations:* The problem in the absence of regulations is obtained by setting both $\beta_R = 0$ and $\beta = 0$ in (P1). The Lagrangian function is $\Pi_M^{Lagr} = \Pi_M + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2R}) + \gamma_3 q_{2M} + \gamma_5 q_{2R}$. Then the first order conditions are

$$
\begin{aligned}
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2M}} &= \phi\left(Q - c_M - 2\alpha q_{2R} - 2q_{2M}\right) + \gamma_3 = 0\,, && \text{(A-1)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - 2q_{1M} = 0\,, && \text{(A-2)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2C}} &= \gamma_2 - \gamma_1 - \phi\left(c_C + c_D\right) = 0\,, && \text{(A-3)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2R}} &= \phi\left(\alpha\left(Q - q_{2M} - q_{2R}\right) - c_R + c_D - \alpha(q_{2R} - q_{2M})\right) - \gamma_2 + \gamma_5 = 0. && \text{(A-4)}
\end{aligned}
$$

We can shot that the Hessian is negative semidefinite and the first order conditions guarantee optimality.

From the first order condition (A-3), $\gamma_2 = \gamma_1 + \phi\left(c_C + c_D\right) > 0$ and hence $q_{2C} = q_{2R}$. As long as $q_{2M} > 0$, i.e. $\gamma_3 = 0$, the first order condition (A-1) can be written as $q_{2M} = \frac{Q-c_M}{2} - \alpha q_{2R}$. On the other hand, since $q_{1M} - \alpha q_{2R} > 0$ should hold, from the first order conditions (A-1) and (A-2), we conclude that $q_{1M} - \alpha q_{2R} = \frac{-\gamma_3/\phi + 2q_{2M} + \gamma_1}{2} > 0$. Hence when $q_{2M} = 0$, i.e. $\gamma_3 > 0$, $q_{1M} = q_{2C}$. Using these properties we find that four different regions are feasible. After the redundant optimality conditions (OCs) are eliminated (For the identification of redundant OCs see Appendix B1.2.) we rewrite the remaining conditions characterizing each feasible region as bounds on c_M where bound $Bi_{M,N}$ stands for the i^{th} bound for the monopoly model under no regulation. Table A1 and Table 2.1 summaries our results.

(a) Bounds characterizing the optimal regions

Bound	**Expression**
$B1_{M,N}$	$\frac{c_C+c_R}{\alpha}$
$B2_{M,N}$	$\frac{(1-\alpha)\alpha Q+c_C+c_R}{2\alpha-\alpha^2}$
$B3_{M,N}$	$\frac{Q(1-\alpha)(1+\alpha\phi)+\alpha\phi(c_R+c_C)}{1-\alpha+\alpha\phi}$

(b) Optimal regions and the corresponding optimal quantities

Reg.	**Condition**	q_{1M}^*	q_{2R}^*	q_{2M}^*
1	$c_M \leq B1_{M,N}$	$\frac{Q-c_M}{2}$	0	$\frac{Q-c_M}{2}$
2	$B1_{M,N} \leq c_M \leq B2_{M,N}$	$\frac{Q-c_M}{2}$	$\frac{\alpha\, c_M-c_C-c_R}{2\alpha(1-\alpha)}$	$\frac{Q-c_M}{2} - \alpha q_{2R}^*$
3	$B2_{M,N} \leq c_M \leq B3_{M,N}$	$\frac{Q-\phi(c_C+c_R)-c_M(1-\alpha\phi)}{2(1+\alpha\phi(1-\alpha))}$	q_{1M}^*	$\frac{Q-c_M}{2} - \alpha q_{2R}^*$
4	$B3_{M,N} \leq c_M$	$\frac{Q(1+\phi\alpha)-\phi(c_R+c_C)-c_M}{2(1+\alpha\phi)}$	q_{1M}^*	0

TABLE A1: The characterization of the optimal regions for the monopolist OEM in the absence of regulation

In Region 1, the OC $\gamma_5 = \phi\left(-\alpha c_M + c_R + c_C\right) \geq 0$ implies $c_M \leq B1_{M,N}$.

In Region 2, the OC $q_{2R} \geq 0$ implies $c_M \geq B1_{M,N}$ while $q_{1M} - q_{2R} \geq 0$ implies $c_M \leq B2_{M,N}$.

In Region 3, the OC $q_{2M} = \frac{\alpha\phi(c_R+c_C)-(1+\alpha\phi-\alpha)c_M+(1-\alpha+\alpha\phi(1-\alpha))Q}{2(1+\phi\alpha(1-\alpha))} \geq 0$ implies $c_M \leq B3_{M,N}$ while $\gamma_1 = \phi\frac{-(c_R+c_C)+(2-\alpha)\alpha c_M-\alpha(1-\alpha)Q}{(1+\phi\alpha(1-\alpha))} \geq 0$ implies $c_M \geq B2_{M,N}$.

In Region 4, the OC $\gamma_3 = \phi\frac{(1-\alpha(1-\phi))c_M-\phi\alpha(c_C+c_R)-((1+\phi\alpha)(1-\alpha))Q}{(1+\phi\alpha)} \geq 0$ can be expressed as $c_M \geq B3_{M,N}$.

Observe that the bounds that characterize the regions are ordered as $B1_{M,N} \leq$

$B2_{M,N} \leq B3_{M,N}$. The ordering implies that if c_M is increasing while the other parameters are held constant, the optimal regions change in the following order: 1-2-3-4.

Finally, note that in the absence of regulations when $\alpha Q - c_C - c_R < 0$ and $Q - c_M < 0$, neither manufacturing nor remanufacturing is profitable in the second period; however we eliminate this region from analysis due to our earlier-stated assumptions on the values of the cost parameters.

Characterization of the optimal regions under partial regulation: The problem under partial regulation is obtained by setting $\beta_R = 0$ in (P1). The Lagrangian function is $\Pi_M^{Lagr} = \Pi_M + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2R}) + \gamma_3(q_{2M}) + \gamma_4 (q_{2C} - \beta q_{1M}) + \gamma_5 q_{2R}$. Then the first order conditions are

$$\begin{aligned}
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2M}} &= \phi \left(Q - c_M - \phi\beta \left(c_C + c_D\right) - 2\alpha q_{2R} - 2q_{2M}\right) + \gamma_3 = 0\,, && \text{(A-5)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - \gamma_4\beta - 2q_{1M} = 0\,, && \text{(A-6)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2C}} &= \gamma_4 + \gamma_2 - \gamma_1 - \phi \left(c_C + c_D\right) = 0\,, && \text{(A-7)}\\
\frac{\partial \Pi_M^{Lagr}}{\partial q_{2R}} &= \phi \left(\alpha \left(Q - q_{2M} - q_{2R}\right) - c_R - \alpha q_{2R} - q_{2M}\alpha + c_D\right) - \gamma_2 + \gamma_5 = 0. && \text{(A-8)}
\end{aligned}$$

The Hessian is still negative semidefinite, therefore the first order conditions guarantee optimality.

Note that as long as $q_{2M} > 0$ which implies that $\gamma_3 = 0$, the first order condition (A-5) can be written as $q_{2M} = \frac{Q - c_M - \phi\beta(c_C + c_D)}{2} - \alpha q_{2R}$. Now consider the first order condition given in (A-7). If $q_{2C}^* > q_{2R}^*$, then the corresponding Lagrange multiplier γ_2 should be zero. Then the first order condition (A-7) becomes $\gamma_4 = \gamma_1 + \phi \left(c_C + c_D\right)$. Hence $\gamma_4^* > 0$

holds and $q_{2C}^* = \beta q_{1M}^*$ at the optimality. On the other hand, if it is optimal for the OEM to collect more than the regulation requires, i.e. $\beta q_{1M}^* < q_{2C}^*$, then $\gamma_4^* = 0$ and the first order condition becomes $\gamma_2 = \gamma_1 + \phi\left(c_C + c_D\right) > 0$ which implies $q_{2C}^* = q_{2R}^*$. Hence the amount of cores collected by the OEM is either equal to the optimum level of remanufacturing or the level of collection dictated by legislation, whichever is larger. On the other hand, since $q_{1M} - \alpha q_{2R} > 0$ should hold, from the first order conditions (A-5), (A-6), and (A-7) we conclude that $q_{1M} - \alpha q_{2R} = \frac{-\gamma_3/\phi + 2q_{2M} + (1-\beta)\gamma_1 + \beta\gamma_2}{2} > 0$. Hence when $q_{2M} = 0$, i.e. $\gamma_3 > 0$, $\gamma_2 = \gamma_1 = 0$ cannot hold which means that $q_{1M} > q_{2C} > q_{2R}$ cannot be optimal if there is no second period manufacturing. Using these properties we find that eight different regions are feasible. After the redundant OCs are eliminated (For the identification of redundant OCs see Appendix B1.2.) we rewrite the remaining conditions characterizing each feasible region as bounds on c_M where bound $Bi_{M,P}$ ($Bi_{M,F}$) stands for the i^{th} bound for the monopoly model under partial regulation (under partial regulation when full regulation is redundant). Table A2 and Table 2.2 (with $\beta_R = 0$) summarize our results.

In Region 1, the OC $\gamma_5 = \phi\left(-\alpha c_M - \alpha\beta\phi\left(c_C + c_D\right) + \left(c_R - c_D\right)\right) \geq 0$ can be written as $c_M \leq B1_{M,P}$.

In Region 2, the OC $q_{2R} \geq 0$ can be written as $c_M \geq B1_{M,P}$ while and $\beta q_{1M} - q_{2R} \geq 0$ implies $c_M \leq B3_{M,F}$.

In Region 3, the OC $\gamma_4 = \phi\frac{(c_R+c_C)-\alpha\beta\phi(c_C+c_D)-(1+\beta(1-\alpha))\alpha c_M+\alpha\beta(1-\alpha)Q}{(1+\phi\beta^2\alpha(1-\alpha))} \geq 0$ implies $c_M \leq B5_{M,F}$ and $q_{2M} = \frac{\beta^2\alpha\phi(c_R+c_C)-(1+\beta^2\alpha\phi-\beta\alpha)c_M-(\beta^2\alpha\phi+1)\beta\phi(c_C+c_D)+\left(1-\beta\alpha+\alpha\phi\beta^2(1-\alpha)\right)Q}{2(1+\phi\beta^2\alpha(1-\alpha))} \geq 0$ can be written as $c_M \leq B4_{M,F}$ while $\gamma_2 \geq 0$ implies $c_M \geq B3_{M,F}$.

(a) Bounds characterizing the optimal scenarios

Bound	**Expression**
$B1_{M,P}$	$\frac{-\alpha\beta\phi(c_C+c_D)+c_R-c_D}{\alpha}$
$B3_{M,F}$	$\frac{\alpha\beta(1-\alpha)Q-\alpha\beta\phi(1+\beta(1-\alpha))(c_C+c_D)+c_R-c_D}{\alpha(1+\beta(1-\alpha))}$
$B4_{M,F}$	$\frac{Q(1+\phi\beta^2\alpha(1-\alpha)-\alpha\beta)-(1+\phi\beta^2\alpha)\phi\beta(c_C+c_D)+\alpha\beta^2\phi(c_C+c_R)}{1-\alpha\beta(1-\phi\beta)}$
$B5_{M,F}$	$\frac{\alpha(1-\alpha)\beta Q-\alpha\beta\phi(c_C+c_D)+c_R+c_C}{\alpha(1+\beta(1-\alpha))}$
$B6_{M,F}$	$\frac{\alpha(1-\alpha)Q+c_R+c_C-\alpha\beta\phi(c_C+c_D)}{2\alpha-\alpha^2}$
$B7_{M,F}$	$Q(1-\alpha)-\beta\phi(c_C+c_D)+c_C+c_R$
$B8_{M,F}$	$\frac{Q(1-\alpha)(1+\alpha\phi)+\alpha\phi(c_C+c_R)-\beta\phi(c_C+c_D)(1+\phi\alpha)}{1-\alpha+\alpha\phi}$
$B10_{M,F}$	$\frac{-\alpha Q(1-\beta)+c_R+c_C}{\alpha\beta}$
$B11_{M,F}$	$\frac{c_C+c_R}{\alpha}$

(b) Optimal regions and the corresponding optimal quantities

Reg.	**Condition**	q^*_{1M}
1	$c_M \leq B1_{M,P}$	$\frac{Q-c_M-\phi\beta(c_C+c_D)}{2}$
2	$B1_{M,P} \leq c_M \leq B3_{M,F}$	$\frac{Q-c_M-\phi\beta(c_C+c_D)}{2}$
3	$B3_{M,F} \leq c_M \leq \min(B4_{M,F}, B5_{M,F})$	$\frac{Q-c_M+\beta\phi((\alpha c_M-c_C-c_R)+\beta\alpha\phi(c_C+c_D))}{2(1+\phi\alpha\beta^2(1-\alpha))}$
4	$B5_{M,F} \leq c_M \leq \min(B6_{M,F}, B7_{M,F})$	$\frac{Q-c_M}{2}$
5	$B6_{M,F} \leq c_M \leq B8_{M,F}$	$\frac{Q-(1-\alpha\phi)c_M+\phi^2\alpha\beta(c_C+c_D)-\phi(c_R+c_C)}{2(1+\phi\alpha(1-\alpha))}$
6	$B4_{M,F} \leq c_M \leq B10_{M,F}$	$\frac{(1+\phi\beta\alpha)Q-\phi\beta(c_R+c_C)-c_M}{2(1+\phi\beta^2\alpha)}$
7	$\max(B10_{M,F}, B7_{M,F}) \leq c_M \leq B11_{M,F}$	$\frac{Q-c_M}{2}$
8	$\max(B8_{M,F}, B11_{M,F}) \leq c_M$	$\frac{(1+\phi\alpha)Q-c_M-\phi(c_R+c_C)}{1+\phi\alpha}$

(c) Optimal regions and the corresponding optimal quantities

Reg.	**Condition**	q^*_{2C}	q^*_{2R}
1	$c_M \leq B1_{M,P}$	βq^*_{1M}	0
2	$B1_{M,P} \leq c_M \leq B3_{M,F}$	βq^*_{1M}	$\frac{\alpha c_M-c_R+c_D+\alpha\beta\phi(c_C+c_D)}{2\alpha(1-\alpha)}$
3	$B3_{M,F} \leq c_M \leq \min(B4_{M,F}, B5_{M,F})$	βq^*_{1M}	βq^*_{1M}
4	$B5_{M,F} \leq c_M \leq \min(B6_{M,F}, B7_{M,F})$	$\frac{\alpha c_M-c_R-c_C+\alpha\beta\phi(c_C+c_D)}{2\alpha(1-\alpha)}$	q^*_{2C}
5	$B6_{M,F} \leq c_M \leq B8_{M,F}$	q^*_{1M}	q^*_{1M}
6	$B4_{M,F} \leq c_M \leq B10_{M,F}$	βq^*_{1M}	βq^*_{1M}
7	$\max(B10_{M,F}, B7_{M,F}) \leq c_M \leq B11_{M,F}$	$\frac{\alpha Q-c_R-c_C}{2\alpha}$	q^*_{2C}
8	$\max(B8_{M,F}, B11_{M,F}) \leq c_M$	q^*_{1M}	q^*_{1M}

TABLE A2: Characterization of the optimal regions for the monopolist OEM under partial regulation

In Region 4, the OC $q_{2R} - \beta q_{1M} \geq 0$ implies $c_M \geq B5_{M,F}$ while $q_{1M} - q_{2R} \geq 0$ can be written as $c_M \leq B6_{M,F}$ and $q_{2M} = \frac{Q(1-\alpha)-c_M-\beta\phi(c_C+c_D)+(c_R+c_C)}{2(1-\alpha)} \geq 0$ can be written as $c_M \leq B7_{M,F}$.

In Region 5, the OC $q_{2M} = \frac{\alpha\phi(c_R+c_C)-(1+\alpha\phi-\alpha)c_M-(\alpha\phi+1)\beta\phi(c_C+c_D)+(1-\alpha+\alpha\phi(1-\alpha))Q}{2(1+\phi\alpha(1-\alpha))} \geq 0$ can be written as $c_M \leq B8_{M,F}$ while $\gamma_1 = \phi\frac{-(c_R+c_C)+(2-\alpha)\alpha c_M+\alpha\beta\phi(c_C+c_D)-\alpha(1-\alpha)Q}{(1+\phi\alpha(1-\alpha))} \geq 0$ implies $c_M \geq B6_{M,F}$.

In Region 6, the OC $\gamma_4 = \phi\frac{(c_C+c_R)-\beta\alpha c_M-\alpha(1-\beta)Q}{1+\phi\beta^2\alpha} \geq 0$ can be written as $c_M \leq B10_{M,F}$ while $\gamma_3 = \phi\frac{-\phi\beta^2\alpha(c_R+c_C)+\beta\phi\left(1+\phi\beta^2\alpha\right)(c_C+c_D)+(1-\beta\alpha(1-\beta\phi))\alpha c_M-\left(1-\alpha\beta+\phi\beta^2\alpha(1-\alpha)\right)Q}{1+\phi\beta^2\alpha} \geq 0$ can be written as $c_M \geq B4_{M,F}$.

In Region 7, the OC $q_{1M} - q_{2R} \geq 0$ can be written as $c_M \leq B11_{M,F}$ while $q_{2R} - \beta q_{1M} \geq 0$ can be written as $c_M \geq B10_{M,F}$ and $\gamma_3 = (\alpha-1)Q + c_M + \beta\phi\left(c_D + c_C\right) - (c_R + c_C) \geq 0$ can be written as $c_M \geq B7_{M,F}$.

In Region 8, the OC $\gamma_3 = \phi\frac{\beta\phi(1+\phi\alpha)(c_C+c_D)+(1-\alpha(1-\phi))c_M-\phi\alpha(c_C+c_R)-((1+\phi\alpha)(1-\alpha))Q}{(1+\phi\alpha)} \geq 0$ can be written as $c_M \geq B8_{M,F}$ while $\gamma_1 = \phi\frac{\alpha c_M-c_R-c_C}{1+\phi\alpha} \geq 0$ can be written as $c_M \geq B11_{M,F}$.

After eliminating the redundant OCs and rearranging the remaining ones, we obtain the following possible orderings:

I. $B1_{M,P} \leq B3_{M,F} \leq B5_{M,F} \leq B6_{M,F} \leq B8_{M,F}$ which implies that if c_M is increasing while the other parameters are held constant, the optimal regions change in the following order: 1-2-3-4-5-8.

II. $B1_{M,P} \leq B3_{M,F} \leq B5_{M,F} \leq B7_{M,F} \leq B11_{M,F}$ which implies that if c_M is increasing while the other parameters are held constant, the optimal regions change in the following order: 1-2-3-4-7-8.

III. $B1_{M,P} \leq B3_{M,F} \leq B4_{M,F} \leq B10_{M,F} \leq B11_{M,F}$ which implies that if c_M is increasing while the other parameters are held constant, the optimal regions change in the following order: 1-2-3-6-7-8.

Finally, note that in the presence of regulations when $\alpha Q + c_D - c_R < 0$ and $Q - c_M - \phi\beta(c_C + c_D) < 0$, neither manufacturing nor remanufacturing is profitable in the second period; however we eliminate this region from analysis due to our earlier-stated assumptions on the values of the cost parameters.

Characterization of the optimal regions under full regulation: The monopolist OEM's problem under full regulation is given in (P1). Then the Lagrangian function becomes:

$$\begin{aligned}\Pi_M^{Lagr} &= \Pi_M + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2R}) + \gamma_3(q_{2M}) \\ &\quad + \gamma_4\left(q_{2C} - \beta q_{1M}\right) + \gamma_5\left(q_{2R} - \beta_R q_{1M}\right) \qquad \text{(A-9)}\end{aligned}$$

The first order conditions are

$$\frac{\partial \Pi_M^{Lagr}}{\partial q_{2M}} = \phi\left(Q - c_M - \beta\phi\left(c_C + c_D\right) - 2\alpha q_{2R} - 2q_{2M}\right) + \gamma_3 = 0\,, \qquad \text{(A-10)}$$

$$\frac{\partial \Pi_M^{Lagr}}{\partial q_{1M}} = Q - c_M + \gamma_1 - \beta\gamma_4 - \beta_R\gamma_5 - 2q_{1M} = 0\,, \qquad \text{(A-11)}$$

$$\frac{\partial \Pi_M^{Lagr}}{\partial q_{2C}} = \gamma_4 + \gamma_2 - \gamma_1 - \phi\left(c_C + c_D\right) = 0\,, \qquad \text{(A-12)}$$

$$\frac{\partial \Pi_M^{Lagr}}{\partial q_{2R}} = \phi\left(\alpha\left(Q - q_{2M} - q_{2R}\right) - c_R - \alpha q_{2R} - q_{2M}\alpha + c_D\right) - \gamma_2 + \gamma_5 = 0. \qquad \text{(A-13)}$$

Again the first order conditions ensure optimality. With similar arguments to those given under partial regulation, we can see that the amount of cores collected by the

OEM is either equal to the optimum level of remanufacturing or the level of collection dictated by legislation, whichever is larger. On the other hand $q_{1M} > q_{2C} > q_{2R}$ cannot be optimal if there is no second period manufacturing. Note that as long as $\gamma_5 = 0$, the first order conditions are the same as the ones under partial regulation. In other words, if $q^*_{2R} > \beta_R q^*_{1M}$ under partial regulation imposing full regulation is redundant. Hence under full regulation Regions 3-8 are the same as the corresponding ones under partial regulation whereas Regions 1 and 2 may change. Table A3 and 2.2 summarize our results. After the redundant OCs are eliminated (For the identification of redundant OCs see Appendix B1.2.) we rewrite the remaining conditions for Regions 1 and 2 as bounds on c_M and the bounds on c_M which characterize each feasible region are provided in Table A3.

(a) Bounds characterizing the optimal regions

Bound	**Expression**
$B1_{M,F}$	$\frac{(c_R-c_D)+\beta_R\alpha(1-\alpha)Q-(1+\beta_R(1-\alpha))\alpha\beta\phi(c_C+c_D)}{(1+\beta_R(1-\alpha))\alpha}$

(b) Optimal regions and the corresponding optimal quantities

Reg.	**Condition**	q^*_{1M}
1	$c_M \leq B1_{M,F}$	$\frac{Q+\phi\beta_R\alpha(c_M+\phi\beta(c_C+c_D))-(c_M+\phi\beta(c_C+c_D))-\phi\beta_R(c_R-c_D)}{2(1+\phi\beta_R^2\alpha(1-\alpha))}$
2	$B1_{M,F} \leq c_M \leq B3_{M,F}$	$\frac{Q-c_M-\phi\beta(c_C+c_D)}{2}$

(c) Optimal regions and the corresponding optimal quantities

Reg.	**Condition**	q^*_{2C}	q^*_{2R}
1	$c_M \leq B1_{M,F}$	βq^*_{1M}	$\beta_R q^*_{1M}$
2	$B1_{M,F} \leq c_M \leq B3_{M,F}$	βq^*_{1M}	$\frac{\alpha(c_M+\phi\beta(c_C+c_D))-c_R+c_D}{2\alpha(1-\alpha)}$

TABLE A3: The characterization of different optimal regions for the monopolist OEM under full regulation

In Region 1, the OC $\gamma_5 = \phi\frac{-(\beta_R(1-\alpha)+1)\beta\phi\alpha(c_C+c_D)+(c_R-c_D)-(1+\beta_R(1-\alpha))\alpha c_M+\beta_R\alpha(1-\alpha)Q}{\left(1+\phi\beta_R^2\alpha(1-\alpha)\right)} \geq 0$ implies $c_M \leq B1_{M,F}$. In Region 2, the OCs $\beta q_{1M} - q_{2R} \geq 0$ can be written as $c_M \leq B3_{M,F}$

and $q_{2R} - \beta_R q_{1M} \geq 0$ can be written as $c_M \geq B1_{M,F}$.

Observe that under full regulation, the possible orderings of the regions are the same as those under partial regulation. ■

Proof of Theorem 2. For each region that can be optimal under no regulation, we study in detail what happens as partial regulation is imposed. It is easy to see how q^*_{1M}, q^*_{2R} and p^*_{2R} change by checking the the expressions provided in the proof of Theorem 1:

Region 4 Under No Regulation. Since $\max\{B8_{M,F}, B11_{M,F}\} \leq B3_{M,N}$ holds, if Region 4 is optimal under no regulation, i.e. $B3_{M,N} < c_M$, Region 8 becomes optimal after imposing partial regulation and q^*_{1M}, q^*_{2R} and p^*_{2R} stay the same.

Region 3 Under No Regulation. Since $\max\{B6_{M,F}, B11_{M,F}\} < B2_{M,N}$ holds, Region 3 under no regulation corresponds to Region 5 or 8 under partial regulation. In either case, q^*_{1M}, q^*_{2R} and p^*_{2R} increase.

Region 2 Under No Regulation. Observe that $B1_{M,P} < B1_{M,N}$ which implies that Region 1 can be optimal under partial regulation only if Region 1 is optimal under no regulation. Hence if Region 2 is optimal under no regulation, i.e. $B1_{M,N} = B11_{M,N} < c_M < B2_{M,N}$, either one of Regions 2, 3, 4, 5 or 8 can be optimal as partial regulation is imposed.

If Region 2 or 3 becomes optimal, q^*_{1M} and p^*_{2R} decrease but q^*_{2R} increases. If Region 4 becomes optimal, q^*_{1M} and p^*_{2R} stay the same but q^*_{2R} goes up. Finally, if Region 5 or 8 becomes optimal q^*_{1M}, q^*_{2R} and p^*_{2R} increase.

Region 1 Under No Regulation. Note that if Region 1 is optimal under no regulation, i.e. $c_M < B1_{M,N}$, then region 5 and 8 where $c_M > \max\{B6_{M,F}, B11_{M,F}\} > B11_{M,F} = B1_{M,N}$ can never be optimal as partial regulation is imposed. Therefore if Region 1 (where

$q^*_{2R} = 0$) is optimal under no regulation, either one of Regions 1, 2, 3, 4, 6 or 7 can be optimal under partial regulation.

If Region 1 becomes optimal as partial regulation is imposed, q^*_{1M} decreases while $q^*_{2R} = 0$ stays the same. Note that unless Region 1 is optimal under partial regulation, q^*_{2R} increases. In the absence of regulations q^*_{1M} in Region 1 is the same as q^*_{1M} in Region 2 (see proof of Theorem 1). Hence the results given above for *Region 2 under no regulation* regarding how q^*_{1M} changes when Region 2, 3 or 4 becomes optimal as partial regulation is imposed, also hold here. On the other hand, if Region 6 is optimal q^*_{1M} decreases, but if Region 7 becomes optimal q^*_{1M} stays the same.

Let us summarize the analysis: If $B3_{M,N} < c_M$, then q^*_{1M}, q^*_{2R} and p^*_{2R} stay the same as partial regulation is imposed. Otherwise, as partial regulation is imposed

1. q^*_{1M} and p^*_{2R} go down if $c_M < \max\{B5_{M,F}, B10_{M,F}\} = \kappa_1$, i.e. region 1-3 or 6 becomes optimal under partial regulation, remain the same if $\kappa_1 < c_M < \kappa_2 = \max\{B6_{M,F}, B11_{M,F}\}$, i.e. region 4 or 7 becomes optimal under partial regulation, and go up if $\kappa_2 < c_M$, i.e. region 5 or 8 becomes optimal under partial regulation.

2. q^*_{2R} remains the same if $c_M < B1_{M,P}$, i.e. region 1 becomes optimal under partial regulation and goes up if $c_M > B1_{M,P}$, i.e. region 2-8 becomes optimal under partial regulation.

■

Proof of Theorem 3. Under full regulation Regions 3-8 are the same as the ones under partial regulation. For Regions 1 and 2, we study what happens as full regulation is imposed. Observe that $B1_{M,F} - B1_{M,P} = \frac{\beta_R(1-\alpha)(\alpha Q - c_R + c_D)}{\alpha(1+\beta_R-\alpha\beta_R)} \geq 0$ because $\alpha Q - c_R - c_C \geq 0$

holds in the parameter region of interest. Hence Region 1 and part of Region 2 under partial regulation corresponds to Region 1 under full regulation. It is easy to see how q^*_{1M}, q^*_{2R} and p^*_{2R} change by checking the expressions provided in the proof of Theorem 1.

Region 1 Under Partial Regulation. Region 1 remains optimal under full regulation; q^*_{2R} goes up while q^*_{1M} goes down.

Region 2 Under Partial Regulation. When $c_M < B1_{M,F}$, Region 1 becomes optimal as full regulation is imposed. Then q^*_{1M} and p^*_{2R} decrease while q^*_{2R} increases. Finally when $c_M > B1_{M,F}$, Region 2 remains optimal as full regulation is imposed and the decision variables of interest do not change. ∎

Proof of Proposition 1. Under partial regulation Region 1 is optimal when $c_M \leq B1_{M,P}$ (see proof of Theorem 1) which can be written as $\beta \leq \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)} = \beta^*$. On the other hand, if $c_M \geq B1_{M,P}$, i.e. $\beta \geq \beta^*$ Region 1 cannot be optimal. Note, that Region 1 is the only region with no remanufacturing under partial regulation. Hence as long as $\beta \geq \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)}$, there is some level of remanufacturing under partial regulation.

The only two regions where all products from the first period are remanufactured are Regions 5 and 8. These regions are optimal when $c_M \geq \max\{B11_{M,F}, B6_{M,F}\}$. We can write $c_M \geq B6_{M,F}$ as $\beta \geq \frac{\alpha(1-\alpha)Q + c_R + c_C - (2-\alpha)\alpha c_M}{\alpha\phi(c_C + c_D)} = \beta_1$ and $c_M \geq B11_{M,F}$ as $\alpha c_M - c_C - c_R \geq 0$. Hence when both $\alpha c_M - c_C - c_R \geq 0$ and $\beta \geq \beta_1$ all first period cores are remanufactured under partial regulation. ∎

Proof of Proposition 2. With Proposition 1, we show that when $c_M \leq B1_{M,P}$ which can be written as $\beta \leq \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)} = \beta^*$, Region 1 is optimal and there is no remanufacturing under partial regulation; hence β_R is always binding if $\beta \leq \beta^*$. When $c_M \geq B1_{M,P}$, i.e. $\beta \geq \beta^*$, one of Regions 2-8 is optimal. When Region 2 is optimal under partial regulation, the additional regulation on remanufacturing level is redundant only if $q_{2R}^* > \beta_R q_{1M}^*$ which can be rewritten as $q_{2R}^* - \beta_R q_{1M}^* = \frac{\alpha c_M + \alpha\beta\phi(c_C+c_D) - (c_R - c_D)}{2\alpha(1-\alpha)} - \beta_R \frac{Q - c_M - \phi\beta(c_C+c_D)}{2} \geq 0$ which we further simplify to $\beta_R \leq \frac{\alpha c_M + \alpha\beta\phi(c_C+c_D) - (c_R - c_D)}{\alpha(1-\alpha)(Q - c_M - \phi\beta(c_C+c_D))} = \frac{\phi(\beta-\beta^*)(c_C+c_D)}{(1-\alpha)(Q-c_M-\phi\beta(c_C+c_D))}$. Finally, when one of Regions 3-8 is optimal and thus $q_{2R}^* \geq \beta q_{1M}^*$ under partial regulation, the additional regulation on remanufacturing level is redundant.

■

Proof of Proposition 3. First, we find the threshold θ under partial regulation: Assume that at point $A = \left(c_M^A, c_R^A\right)$ remanufacturing is not profitable, i.e. Region 1 is optimal under partial regulation, hence the optimal production quantities are $q_{1M}^A = q_{2M}^A = \frac{Q - c_M^A - \phi\beta(c_C+c_D)}{2}$. The profit at point A is

$$\Pi_M^A = \frac{(Q - c_M^A - \phi\beta\left(c_C + c_D\right))^2(1+\phi)}{4}.$$

Now compare the profit at point $B = \left(c_M^B, c_R^B\right)$ where $c_R^B < c_R^A$, $c_M^B > c_M^A$ and remanufacturing is profitable, i.e. Region 2 is optimal, with profit at $A = \left(c_M^A, c_R^A\right)$. The optimal production quantities at point B are

$$\begin{aligned}
q_{1M}^B &= \frac{Q - c_M^B - \phi\beta\left(c_C + c_D\right)}{2}, \\
q_{2M}^B &= \frac{Q\left(1-\alpha\right) - c_M^B - \beta\phi\left(c_C + c_D\right) + \left(c_R^B - c_D\right)}{2\left(1-\alpha\right)}, \\
q_{2R}^B &= \frac{\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - \left(c_R^B - c_D\right)}{2\alpha\left(1-\alpha\right)}.
\end{aligned}$$

The profit at point B is

$$\Pi_M^B = \frac{\phi\left(\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - \left(c_R^B - c_D\right)\right)^2}{4\alpha\left(1-\alpha\right)} + \frac{(Q - c_M^B - \phi\beta\left(c_C + c_D\right))^2(1+\phi)}{4}.$$

Now let us compare the profit values obtained at point A where remanufacturing is not profitable and at point B where remanufacturing becomes profitable:

$$\begin{aligned}\Pi_M^B - \Pi_M^A = \Delta \;\; &= \;\; \frac{\phi\left(\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - \left(c_R^B - c_D\right)\right)^2}{4\alpha\left(1-\alpha\right)} \\ &\quad - \frac{\left(c_M^B - c_M^A\right)\left(2Q - c_M^B - c_M^A - 2\beta\phi(c_D + c_C)\right)(1+\phi)}{4}\end{aligned}$$

Simplifying the difference we get a second order polynomial of c_M^A, $\Delta = c_M^A - c_M^B + k\frac{\left(c_M^B - B1_{M,P}\right)^2}{(2X - c_M^B - c_M^A)}$ where $k = \frac{\phi\alpha}{(1+\phi)(1-\alpha)}$ and $X = 2Q - 2\beta\phi c_D - 2\beta\phi c_C$. Solving the polynomial we find two roots, $r_{1,2} = X \pm \sqrt{(c_M^B - X)^2 + k(c_M^B - B1_{M,P})^2}$. Note that the larger root, r_2 is greater than c_M^B. The sign of smaller root, r_1 depends on $\Delta(c_M^A = 0) = \frac{(X - c_M^B)^2 + k(c_M^B - B1_{M,P})^2 - X^2}{2X - c_M^B}$. There are two possibilities: (i) If $\Delta(c_M^A = 0) < 0$ then $r_1 > 0$ and Δ is positive in the interval (r_1, c_M^B). (ii) If $\Delta(c_M^A = 0) > 0$ then $r_1 < 0$ and Δ is positive in the interval $(0, c_M^B)$. Hence the profit at point B is higher when $\theta = \max\{0, r_1\} < c_M^A < c_M^B$. Finally the lower bound, r_1 decreases as the legislation becomes stricter:

$$\frac{\partial r_1}{\partial \beta} = -\frac{k\alpha\phi(c_C + c_D)\left(\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - \left(c_R^B - c_D\right)\right)}{\sqrt{k(\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - (c_R^B - c_D))^2 + \alpha^2(c_M^B - X)^2}} < 0\,.$$

■

Proof of Proposition 4. The Lagrangian function for the remanufacturer's problem (P2) can be written as $L_{2R} = \Pi_{2R} + \eta_1 q^R_{2C} + \eta_2 \left(q_{1M} - q_{2C} - q^R_{2C}\right) + \eta_3 q_{2S}$. Then the OCs are:

$$\begin{aligned}
\frac{\partial L_{2R}}{\partial q^R_{2C}} = \alpha \left(Q - q_{2M} - 2\left(q^R_{2C} + q_{2S}\right)\right) - c_R - c^R_C + \eta_1 - \eta_2 &= 0 \qquad \text{(A-14)}\\
\frac{\partial L_{2R}}{\partial q_{2S}} = \alpha \left(Q - q_{2M} - 2\left(q^R_{2C} + q_{2S}\right)\right) - c_R - p_{2S} + \eta_3 &= 0 \qquad \text{(A-15)}\\
\eta_1 \left(q^R_{2C}\right) &= 0 \qquad \text{(A-16)}\\
\eta_2 \left(q_{1M} - q_{2C} - q^R_{2C}\right) &= 0 \qquad \text{(A-17)}\\
\eta_3 \left(q_{2S}\right) &= 0 \qquad \text{(A-18)}
\end{aligned}$$

In addition, $0 \leq q^R_{2C} \leq q_{1M} - q_{2C}$, $q_{2S} \geq 0$, $\eta_1 \geq 0$, $\eta_2 \geq 0$, $\eta_3 \geq 0$ should hold. Solving OCs (A-14)-(A-18) we find the five possible solutions given in Table A5. The corresponding Lagrangian multipliers are given in Table A4.

Region	η_1	η_2	η_3
R1	$-\alpha Q + \alpha q_{2M} + c_R + c_C^R$	0	$-\alpha Q + \alpha q_{2M} + c_R + p_{2S}$
R2	0	0	$p_{2S} - c_C^R$
R3	0	$\alpha Q - \alpha q_{2M} - c_R - c_C^R - 2\alpha\left(q_{1M} - q_{2C}\right)$	$-\alpha Q + \alpha q_{2M} + c_R + p_{2S} + 2\alpha\left(q_{1M} - q_{2C}\right)$
R4	$c_C^R - p_{2S}$	0	0
R5	0	$p_{2S} - c_C^R$	0

TABLE A4: The Lagrangian multipliers corresponding to each feasible strategy of the remanufacturer

Region	q_{2C}^R	q_{2S}
R1	0	0
R2	$\frac{\alpha Q - \alpha q_{2M} - c_R - c_C^R}{2\alpha}$	0
R3	$q_{1M} - q_{2C}$	0
R4	0	$\frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha}$
R5	$q_{1M} - q_{2C}$	$\frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S} - 2\alpha(q_{1M} - q_{2C})}{2\alpha}$

TABLE A5: The feasible strategies for the remanufacturer

Note that for a region to be optimal, the corresponding Lagrangian multipliers must be nonnegative. Therefore, nonnegativity of Lagrangian multipliers from the remanufacturer's problem become constraints in the OEM's problem. Next we show that Region R2 and Region R3 can never be optimal and Region R5 can only be optimal if $q_{1M} = q_{2C}$. In each case the proof is by contradiction.

Elimination of Region R2: Assume that Region R2 is optimal, the following OCs should be satisfied by the OEM's solution: $q^*_{1M} - q^*_{2C} \geq q^{R*}_{2C} = \frac{\alpha Q - \alpha q^*_{2M} - c_R - c^R_C}{2\alpha} \geq 0$ and $\eta_3 = p^*_{2S} - c^R_C \geq 0$.

If the OEM sets $p_{2S} = c^R_C - \delta$ holding q^*_{1M} and q^*_{2M} the same, it becomes optimal for the remanufacturer to purchase the cores from the OEM, i.e. $\left(q^{R*}_{2C}\right)^{new} = 0$, $\left(q^*_{2S}\right)^{new} > 0$. The new optimal amount of remanufacturing is

$$\left(q^*_{2S}\right)^{new} = \frac{\alpha Q - \alpha q^*_{2M} - c_R - c^R_C + \delta}{2\alpha} = q^{R*}_{2C} + \frac{\delta}{2\alpha} > q^{R*}_{2C}$$

whereas the optimal amount of collected items is $\left(q^*_{2C}\right)^{new} = \max\left(\left(q^*_{2S}\right)^{new}, q^*_{2C}\right)$.

Note that the new solution is already feasible. The OEM's first period profit does not change and the change in the second period's profit is

$$\begin{aligned}\Delta\Pi_{2OEM} \quad = \quad & -q^*_{2M}\frac{\delta}{2} + \left(c^R_C - \delta\right)\left(q^{R*}_{2C} + \frac{\delta}{2\alpha}\right) - c_C \max\left(q^{R*}_{2C} + \frac{\delta}{2\alpha}, q^*_{2C}\right) \\ & -c_D\left(\max\left(q^{R*}_{2C} + \frac{\delta}{2\alpha}, q^*_{2C}\right) - q^{R*}_{2C} - \frac{\delta}{2\alpha}\right) + \left(c_C + c_D\right) q^*_{2C}\,.\end{aligned}$$

When $(q_{2C}^*)^{new} = \max\left((q_{2S}^*)^{new}, q_{2C}^*\right) = (q_{2S}^*)^{new}$, we have

$$\Delta\Pi_{2OEM} = -q_{2M}^*\frac{\delta}{2} + \left(c_C^R - \delta - c_C\right)\left(q_{2C}^{R*} + \frac{\delta}{2\alpha}\right) + \left(c_C + c_D\right)q_{2C}^*.$$

We observe that the change in profit is positive as δ goes to zero: $\lim_{\delta\to 0}\Delta\Pi_{2OEM} = \left(c_C^R - c_C\right)q_{2C}^{R*} + \left(c_C + c_D\right)q_{2C}^* > 0$.

When $(q_{2C}^*)^{new} = \max\left((q_{2S}^*)^{new}, q_{2C}^*\right) = q_{2C}^*$, we have:

$$\Delta\Pi_{2OEM} = -\alpha q_{2M}^*\frac{\delta}{2\alpha} + \left(c_C^R - \delta + c_D\right)\left(q_{2C}^{R*} + \frac{\delta}{2\alpha}\right).$$

Similarly, the change in profit is positive as δ goes to zero:

$$\lim_{\delta\to 0}\Delta\Pi_{2OEM} = \left(c_C^R + c_D\right)q_{2C}^{R*} > 0.$$

Hence the OEM can make more profit by collecting the items herself (and setting the price so that the remanufacturer purchases the cores from the OEM) rather than letting the remanufacturer collect cores himself.

Elimination of Region R3: Assume that Region R3 is optimal, so that the remanufacturer collects all the remaining cores but does not buy from the OEM, i.e. $q_{2C}^{R*} = q_{1M}^* - q_{2C}^* \geq 0$ and $q_{2S}^* = 0$. The OEM's solution should satisfy the following conditions:$\eta_2^* = \alpha Q - \alpha q_{2M}^* - 2\alpha q_{1M}^* + 2\alpha q_{2C}^* - c_R - c_C^R \geq 0$ and $\eta_3^* = -\alpha Q + \alpha q_{2M}^* + 2\alpha q_{1M}^* - 2\alpha q_{2C}^* + c_R + p_{2S}^* \geq 0$.

Note that $\eta_2^* + \eta_3^* = p_{2S}^* - c_C^R \geq 0$. Keeping q_{1M}^*, q_{2M}^* and q_{2C}^* the same, the OEM can set $p_{2S} = c_C^R + \delta$ such that $\frac{\alpha Q - \alpha q_{2M}^* - c_R - c_C^R - \delta}{2\alpha} = q_{1M}^* - q_{2C}^*$. Then η_3^* becomes zero, but the optimal solution remains the same. If the OEM collects all the available cores, i.e. $\left(q_{2C}^*\right)^{new} = q_{1M}^*$, and sells just $q_{1M}^* - q_{2C}^*$ to the remanufacturer at a price $p_{2S} = c_C^R + \delta$, she makes more profit. Moreover, the remanufacturer still makes positive profit. Hence the regions where the OEM collects some of the cores and lets the remanufacturer collect and remanufacture the rest cannot be optimal.

*Elimination of Region R5:*Region 5 can be optimal only if the OEM collects all the available cores, i.e. $q_{1M}^* = q_{2C}^*$. Assume that Region R5 is optimal, and $q_{1M}^* > q_{2C}^*$. Note that the total remanufactured amount does not depend on $q_{1M}^* - q_{2C}^*$, i.e. $q_{2C}^{R*} + q_{2S}^* = \frac{\alpha Q - \alpha q_{2M}^* - c_R - p_{2S}^*}{2\alpha}$. Hence the OEM makes more profit by collecting all the available items, i.e. $\left(q_{2C}{}^*\right)^{new} = q_{1M}^*$ and increasing q_{2S} by $q_{1M}^* - q_{2C}^*$ so that $\left(q_{2S}^*\right)^{new} = \frac{\alpha Q - \alpha q_{2M}^* - c_R - p_{2S}^*}{2\alpha}$ while decreasing q_{2C}^R to zero. ■

Proof of Proposition 5. In the proof of Proposition 4, we show that if the OEM collects cores to restrict the availability of cores to the remanufacturer, it is profitable for her to collect all the cores. Now we show that the OEM never sets $p_{2S} = c_C^R$ under this situation. First, assume that the OEM prices the remanufactured product at the collection cost of the remanufacturer. Then the OEM's second period profit can be written as

$$\begin{aligned}
\Pi_{2OEM} &= q_{2M}\left(\left(Q - q_{2M} - \alpha\left(q_{2C}^{R*} + q_{2S}^*\right)\right) - c_M - \phi\beta\left(c_C + c_D\right)\right) \\
&\quad - q_{2C}c_C + q_{2S}^* p_{2S} - \left(q_{2C} - q_{2S}^*\right)c_D \\
&= q_{2M}\left(\left(Q - q_{2M} - \alpha q_{2S}^*\right) - c_M - \phi\beta\left(c_C + c_D\right)\right) \\
&\quad - q_{1M}c_C + q_{2S}^* c_C^R - \left(q_{1M} - q_{2S}^*\right)c_D
\end{aligned}$$

where $q_{2S}^{*}=\frac{\alpha Q-\alpha q_{2M}-c_R-c_C^R}{2\alpha}\geq 0$ and $q_{2C}^{R*}=q_{1M}-q_{2C}=0$.

Now assume that the OEM keeps q_{1M} and q_{2M} constant but decreases the price of the remanufactured products by an infinitesimal amount δ, namely $p_{2S}^{new}=c_C^R-\delta$. Then purchasing the cores from the OEM becomes more profitable for the remanufacturer compared to collecting the cores himself from the market. Hence by setting $p_{2S}^{new}=c_C^R-\delta$ the OEM does not need to collect all the items from the market, i.e. $q_{2C}^{new}\leq q_{1M}$. Note that the amount remanufactured becomes $q_{2S}^{new}=\frac{\alpha Q-\alpha q_{2M}-c_R-c_C^R+\delta}{2\alpha}=q_{2S}+\frac{\delta}{2\alpha}$ and also $q_{2S}^{new}\leq q_{2C}^{new}\leq q_{1M}$ holds. The second period profit of the OEM is

$$\begin{aligned}\Pi_{2OEM}^{new} &= q_{2M}\left(Q-q_{2M}-\alpha\left(q_{2S}^{*}+\frac{\delta}{2\alpha}\right)-c_M-\phi\beta\left(c_C+c_D\right)\right)-q_{2C}^{new}c_C\\ &\quad+\left(q_{2S}^{*}+\frac{\delta}{2\alpha}\right)\left(c_C^R-\delta\right)-\left(q_{2C}^{new}-q_{2S}^{*}-\frac{\delta}{2\alpha}\right)c_D\,.\end{aligned}$$

Hence as δ goes to zero the increase in the profit function can be written as $\Delta\Pi_{2OEM}=\Pi_{2OEM}^{new}-\Pi_{2OEM}=\left(q_{1M}-q_{2C}^{new}\right)\left(c_D+c_C\right)\geq 0$.

Therefore the OEM does not set the price of the cores equal to the collection cost of the remanufacturer under Policy C. Since preemptive collection only happens under Policy C, the result follows. ■

Proof of Proposition 6. Under preemptive collection $q_{1M}=q_{2C}>q_{2S}$ and $q_{2C}>\beta q_{1M}$ by definition and hence the corresponding Lagrangian multipliers are zero i.e., $\gamma_2=0$, $\gamma_4=0$ (see the OEM's Lagrangian function given in equation (B-3) in Appendix B1.2). With Proposition 5 we show that $p_{2S}>c_C^R$ should hold and hence the corresponding Lagrangian multiplier is also zero ($\gamma_7=0$). Setting $\gamma_2=0$, $\gamma_4=0$, $\gamma_7=0$ and $q_{2C}=q_{1M}$ and solving the first order conditions given in equations (B-4)-(B-5) for p_{2S}

we obtain $p_{2S} = \frac{(\phi\alpha Q - \phi c_R - \phi c_D - \gamma_5)}{2\phi}$. Note that under Policy C, $p_{2S} > c_C^R$, and hence $p_{2S} - c_C^R = \frac{(\phi\alpha Q - \phi c_R - \phi c_D - \gamma_5 - 2\phi c_C^R)}{2\phi} > 0$ implies that $(\alpha Q - c_D - \gamma_5/\phi - 2c_C^R) > c_R$. Therefore, preemptive collection cannot be optimal when $c_R \geq \alpha Q - c_D - 2c_C^R$. ■

Proof of Proposition 7. First we show that preemptive collection strategy is feasible over a larger region of cost parameters under partial regulation compared to no regulation. For the bounds under no regulation and partial regulation please refer to Tables B1 and B2-B3 respectively in Appendix B1.2.

Note that the regions C_1 and C_2 define the preemptive collection region. In the absence of regulations C_1 is optimal if $B2_{C,N} \leq c_M \leq B7_{C,N}$ and C_2 is optimal if $B7_{C,N} \leq c_M \leq B8_{C,N}$ and $B2_{C,N} \leq B7_{C,N} \leq B8_{C,N}$. Under partial regulation C_1 is optimal if $B3_{C,P} \leq c_M \leq B2_{C,P}$ and C_2 is optimal if $B2_{C,P} \leq c_M \leq B33_{C,F}$ and $B3_{C,P} \leq B2_{C,P} \leq B33_{C,F}$. Observe that the lower bound is lower under partial regulation, i.e. $B2_{C,N} - B3_{C,P} = \beta\phi(c_C + c_D) > 0$; while the upper bound is also lower, i.e. $B8_{C,N} - B33_{C,F} = \frac{\beta\phi(c_C + c_D)}{(3-\alpha)} > 0$. The magnitude of decrease on the upper bound is smaller and hence the overall feasible region becomes larger.

Since the feasible region of preemptive collection strategy overlaps with the regions of Policy B, we need to compare the optimal profits in order to understand whether partial regulation favors preemptive collection. In the proof of Proposition 8, we show that the formulations of no regulation and partial regulation scenarios differ only by the cost of manufacturing in the second period and the lower bound on the collection level. Since the OEM already collects all available cores under preemptive collection (Policy C), the lower bound on the collection level is not binding and thus this constraint does not affect the profit. On the other hand, under Policy B the profit may go down due to the constraint. Under no regulation, the cost of manufacturing in the second period, c_{M2} is equal to c_M, while under partial regulation it becomes $c_M + \beta\phi\,(c_C + c_D)$. It is sufficient to show that the difference between the profits under Preemptive Collection Strategy and Policy B, i.e. $\Pi^C_{OEM} - \Pi^B_{OEM}$ increases as c_{M2} increases. According to how the regions C_1 and C_2 and Policy B can overlap, we analyze four cases:

Case i. Region C_1 and Region B_1 overlap and Region C_1 is optimal under no regulation. The difference between the profits is

$$\Pi^{C1}_{OEM} - \Pi^{B1}_{OEM} = -\frac{(2(Q - c_M) - \phi(c_C + c_D))(c_C + c_D)}{4\phi} + \frac{\phi(\alpha Q - 2c^R_C - 2c_C + \alpha c_{M2})^2}{4\alpha^2}.$$

The difference increases as c_{M2} increases because

$$\frac{\partial(\Pi^{C1}_{OEM} - \Pi^{B1}_{OEM})}{\partial c_{M2}} = \frac{\phi(\alpha Q - 2c^R_C - 2c_C + \alpha c_{M2})}{2\alpha} = \frac{\phi(p^{C1}_{2S} - c^R_C)}{\alpha} > 0.$$

Case ii. Region C_1 and Region B_2 overlap and Region C_1 is optimal under no regulation. The difference between the profits increases as c_{M2} increases because

$$\frac{\partial(\Pi^{C1}_{OEM} - \Pi^{B2}_{OEM})}{\partial c_{M2}} = \frac{\phi(c_R + c_C - \alpha c_{M2})}{2(2-\alpha)} = \frac{\gamma^{C1}_5 + (c_C + c_D)\phi}{2(2-\alpha)} > 0.$$

Case iii. Region C_2 and Region B_1 overlap and Region C_2 is optimal under no regulation.

The difference between the profits increases as c_{M2} increases because

$$\begin{aligned}\frac{\partial(\Pi_{OEM}^{C2}-\Pi_{OEM}^{B1})}{\partial c_{M2}} &= \frac{\phi((2-\alpha)\alpha Q-2(2-\alpha)c_C^R-(4-\alpha)c_R+2\alpha c_{M2}+\alpha c_D)}{2\alpha(2-\alpha)}\\ &= \frac{\alpha\gamma_7^{B1}+2\phi\alpha q_{2S}^{C1}}{2}>0.\end{aligned}$$

Case iv. Region C_2 and Region B_2 overlap and Region C_1 is optimal under no regulation. The difference between the profits increases as c_{M2} increases because

$$\frac{\partial(\Pi_{OEM}^{C2}-\Pi_{OEM}^{B2})}{\partial c_{M2}}=\frac{\phi(c_D+c_C)}{2(2-\alpha)}>0.$$

■

Proof of Theorem 8. First we give the proof for partial regulation:

From No Regulation to Partial Regulation: In Part a, we give the proof for Policies A-B, then in Part b we give the proof for Policy C.

Part a. When either Policy A or B is optimal under no regulation, the OEM's problem is

$$\begin{aligned}(P5)\ Max\ \Pi_{OEM} &= \phi(q_{2M}\left(Q-q_{2M}-\alpha\left(q_{2C}^R+q_{2S}\right)-c_M\right)\\ &\quad +p_{2S}q_{2S}-c_Cq_{2C}-c_D\left(q_{2C}-q_{2S}\right))+q_{1M}\left(Q-q_{1M}-c_M\right)\\ s.t. &\quad q_{1M}\geq q_{2C}\geq q_{2S}\\ &\quad q_{2S}+q_{2C}^R\geq 0\\ &\quad q_{2M}\geq 0,\ \ p_{2S}\leq c_C^R,\ \ q_{2S}=\frac{\alpha Q-\alpha q_{2M}-c_R-p_{2S}}{2\alpha},\ \ q_{2C}^R=0\end{aligned}$$

whereas under partial regulation, the problem is

$$
\begin{aligned}
(P6)\ Max\ \Pi_{OEM} \quad &= \quad \phi(q_{2M}\left(Q - q_{2M} - \alpha\left(q_{2C}^R + q_{2S}\right) - \left(c_M + \phi\beta\left(c_C + c_D\right)\right)\right) \\
&\quad +p_{2S}q_{2S} - c_C q_{2C} - c_D\left(q_{2C} - q_{2S}\right)) + q_{1M}\left(Q - q_{1M} - c_M\right) \\
s.t. \quad & \quad q_{2C} + q_{2C}^R \geq \beta q_{1M} \qquad\qquad \text{(A-19)} \\
& \quad q_{1M} \geq q_{2C} \geq q_{2S} \\
& \quad q_{2S} + q_{2C}^R \geq 0 \\
& \quad q_{2M} \geq 0,\ \ p_{2S} \leq c_C^R,\ \ q_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha},\ \ q_{2C}^R = 0
\end{aligned}
$$

Note that (P5) and (P6) differ by the cost of manufacturing in the second period and the constraint (A-19). We show that as c_{M2} increases q_{2S} increases under no regulation and that imposing constraint (A-19) never causes a decrease in q_{2S}. For the OCs and the values of the corresponding Lagrangian multipliers under no regulation and partial regulation please refer to Appendix B1.2. In the remainder of the proof, we use the following notation: $c_{M2}^{I} = c_M$ and $c_{M2}^{II} = c_M + \beta\phi\left(c_C + c_D\right)$.

Under no regulation when Region A_1 is optimal $q_{2S}^* = 0$, $\frac{\partial \gamma_5^{A1}}{\partial c_{M2}} = -\phi\alpha < 0$ and $\frac{\partial\left(c_C^R - p_{2S}\right)}{\partial c_{M2}} = \frac{-\alpha}{2} < 0$. Hence as c_{M2} increases while the amount of remanufacturing stays the same, the region in which Region A_1 stays optimal shrinks. Since the total remanufacturing level is already zero when Region A_1 is optimal under no regulation, imposing partial regulation cannot decrease the remanufacturing level further.

Under no regulation when Region A_2 is optimal, as c_{M2} increases, the amount of remanufacturing increases, i.e $\frac{\partial q_{2S}^{A2}}{\partial c_{M2}} = \frac{1}{2(2-\alpha)} > 0$, whereas one of the upper bounds characterizing the optimal region of Region A_2 decreases, i.e. $\frac{\partial(q_{1M}^{A2}-q_{2S}^{A2})}{\partial c_{M2}} = -\frac{1}{2(2-\alpha)} < 0$, while the second one does not change, i.e. $\frac{\partial(c_C^R-p_{2S})}{\partial c_{M2}} = 0$. If Region A_2 remains optimal as c_{M2} increases then either $\beta q_{1M} < q_{2C}$ or $\beta q_{1M} \geq q_{2C}$ holds at $c_{M2} = c_{M2}^{II}$. Under the former case, constraint (A-19) is already redundant. We check what happens when the constraint is binding. Under partial regulation, the optimal remanufacturing level can be written as $q_{2S}^{partial} = \frac{(\phi\alpha c_{M2}-\gamma_3\alpha-\phi c_R+\gamma_5-\phi c_C+\gamma_4)}{2\alpha\phi(2-\alpha)}$ in terms of the Lagrangian multipliers while under no regulation $q_{2S}^{A2} = \frac{(\alpha c_{M2}-c_R-c_C)}{2\alpha(2-\alpha)}$. Hence the difference in the remanufacturing levels is $q_{2S}^{partial} - q_{2S}^{A2} = \frac{(\gamma_5^{partial}+\gamma_4^{partial}-\gamma_3^{partial}\alpha)}{2\alpha\phi(2-\alpha)}$ which is positive as long as $\gamma_3^{partial} = 0$. When $\gamma_3^{partial} > 0$ which also implies that $q_{2M}^{partial} = 0$, the difference is $q_{2S}^{partial} - q_{2S}^{A2} = \frac{(\gamma_5^{partial}+\gamma_4^{partial}-\alpha\gamma_3^{partial})}{2\alpha\phi(2-\alpha)} > \frac{(\gamma_5^{partial}+\gamma_4^{partial}-2\gamma_3^{partial})}{2\alpha\phi(2-\alpha)} = \frac{(2Q-\alpha Q-2c_{M2}+c_R+c_C)}{2\alpha(2-\alpha)} = \frac{q_{2M}^{A2}}{\alpha} > 0$. Hence, imposing partial regulation cannot decrease the remanufacturing level.

On the other hand when Region A_2 is optimal at c_{M2}^{I} but Region A_3 is optimal at c_{M2}^{II},

$$\gamma_1^{A3}(c_{M2}^{II}) = \frac{\phi(2\alpha c_M + \alpha c_{M2}^{II} + \alpha^2 Q - 2\alpha Q - \alpha^2 c_M - c_R - c_C)}{1 + 2\phi\alpha - \phi\alpha^2} > 0$$

which implies $c_{M2}^{II} > \frac{2\alpha Q-\alpha^2 Q-2\alpha c_M+\alpha^2 c_M+c_R+c_C}{\alpha} = Y$ should hold under Region A_3 at c_{M2}^{II}. The change in the remanufacturing level is positive when $q_{2S}^{A3}(c_{M2}^{II}) - q_{2S}^{A2}(c_{M2}^{I}) = \frac{c_R+c_C-\alpha(3-\alpha+\phi\alpha(2-\alpha))c_M+\alpha(2-\alpha)(Q+\phi\alpha c_{M2}^{II})}{2(1+2\phi\alpha-\phi\alpha^2)\alpha(2-\alpha)} > 0$ which can be written as

$$c_{M2}^{II} > \frac{-c_R - c_C + \alpha(3-\alpha+\phi\alpha(2-\alpha))c_M - \alpha(2-\alpha)Q}{\phi\alpha^2(2-\alpha)} = X.$$

Since $q_{1M}^{A2}(c_M) - q_{2S}^{A2}(c_M) = \frac{2\alpha Q-\alpha^2 Q-3\alpha c_M+\alpha^2 c_M+c_R+c_C}{2\alpha(2-\alpha)} > 0$ should hold under Region A_2 at c_{M2}^{I}, we conclude that $Y - X = \frac{(1+2\phi\alpha-\phi\alpha^2)(2\alpha Q-\alpha^2 Q-3\alpha c_M+\alpha^2 c_M+c_R+c_C)}{\alpha^2(2-\alpha)\phi} > 0$. Since

$Y > X$, ${c_M}_2^{II} > Y$ implies ${c_M}_2^{II} > X$. Hence the remanufacturing level goes up as increasing c_M causes a move from Region A_3 from Region A_2. Since all available cores are collected and remanufactured under Region A_3 and A_4, the lower bound on the collection level is redundant for this particular situation and also in the rest of the analysis for Policy A in Part a.

When Region A_3 remains optimal, the amount of remanufacturing increases, i.e. $\frac{\partial q_{2S}^{A3}}{\partial c_{M2}} = \frac{\phi\alpha}{2(1+2\phi\alpha-\phi\alpha^2)} > 0$, whereas the upper bound characterizing the region in which Region A_3 is optimal decreases, i.e. $\frac{\partial q_{2M}^{A3}}{\partial c_{M2}} = -\frac{1+2\phi\alpha}{2(1+2\phi\alpha-\phi\alpha^2)} < 0$. When Region A_3 is optimal at ${c_M}_2^I$ but Region A_4 is optimal at ${c_M}_2^{II}$, the change in the remanufacturing level is positive because $q_{2S}^{A4}({c_M}_2^{II}) - q_{2S}^{A3}({c_M}_2^{I}) = \frac{\phi\alpha\left(\left(1-\phi\alpha^2+2\phi\alpha-\alpha\right)Q+\phi\alpha c_R+\phi\alpha c_C+(-2\phi\alpha+\alpha-1)c_M\right)}{2(1+2\phi\alpha-\phi\alpha^2)(1+2\phi\alpha)} = \frac{\alpha\phi q_{2M}^{A3}({c_M}_2^I)}{(1+2\alpha\phi)} \geq 0$. Hence the amount of remanufacturing goes up as the increase in the cost of manufacturing results in a move to Region A_4 from Region A_3. When Region A_4 remains optimal, the amount of remanufacturing does not depend on the cost of manufacturing in the second period.

Next we analyze in detail how the remanufacturing level changes under Policy B. For the OCs and the values of the corresponding Lagrangian multipliers under no regulation and partial regulation please refer to Appendix B1.2.

When Region B_1 is optimal under no regulation, since $q_{2S}^{B1} = 0$, the amount of remanufacturing cannot decrease. On the other hand, as c_{M2} increases, the upper bound characterizing the region in which Region B_I is optimal decreases, i.e. $\frac{\partial \gamma_5^{B1}}{\partial c_{M2}} = -2\alpha < 0$. When Region B_1 is optimal under ${c_M}_2^I$ but Region B_2 is optimal under ${c_M}_2^{II}$, the remanufacturing level goes up.

When Region B_2 is optimal under no regulation, as c_{M2} increases, the amount of remanufacturing increases, i.e $\frac{\partial q_{2S}^{B2}}{\partial c_{M2}} = \frac{1}{2(2-\alpha)} > 0$, whereas the upper bound characterizing the optimal region of Region B_2 decreases, i.e. $\frac{\partial(q_{1M}-q_{2S})}{\partial c_{M2}} = -\frac{1}{2(2-\alpha)} < 0$. When Region B_2 stays optimal as c_{M2} increases, either $\beta q_{1M} < q_{2C}$ or $\beta q_{1M} \geq q_{2C}$ holds at $c_{M2} = c_{M2}^{II}$. Under the former case, constraint (A-19) is redundant. We next analyze what happens when the constraint is binding, i.e. $\gamma_1 = 0$ and $\gamma_4 > 0$: Under partial regulation, the optimal remanufacturing level can be written as $q_{2S}^{partial} = \frac{(2\alpha Q+\alpha c_R+2\alpha c_{M2}-\alpha^2 Q-\alpha c_C-4c_R-4c_C^R+2\alpha c_C^R)\phi+\alpha\gamma_4-2\gamma_3\alpha+\gamma_5\alpha}{4\alpha\phi(2-\alpha)}$ in terms of the Lagrangian multipliers while under no regulation $q_{2S}^{B2} = \frac{(2\alpha Q+\alpha c_R+2\alpha c_{M2}-\alpha^2 Q-\alpha c_C-4c_R-4c_C^R+2\alpha c_C^R)}{4\alpha(2-\alpha)}$. Hence, the difference between the remanufacturing levels is $q_{2S}^{partial} - q_{2S}^{B2} = \frac{(\gamma_5+\gamma_4-2\gamma_3)}{4\phi(2-\alpha)}$ which is positive as long as $\gamma_3 = 0$. When $\gamma_3 > 0$ which also implies that $q_{2M}^{partial} = 0$, the difference is $q_{2S}^{partial} - q_{2S}^{B2} = \frac{(2Q-\alpha Q-2c_{M2}+c_R+c_C)}{4(2-\alpha)} = \frac{q_{2M}^{B2}}{2\alpha} > 0$. Hence, imposing partial regulation cannot decrease the remanufacturing level.

Part b. When Policy C is optimal under no regulation the OEM's problem is

$$
\begin{aligned}
(P7)\ Max\ \Pi_{OEM} \quad = \quad & \phi\left(q_{2M}\left(Q-q_{2M}-\alpha\left(q_{2C}^R+q_{2S}\right)-c_M\right)\right) \\
& +\phi\left(p_{2S}q_{2S}-c_C q_{2C}-c_D\left(q_{2C}-q_{2S}\right)\right)+q_{1M}\left(Q-q_{1M}-c_M\right) \\
s.t. \qquad & q_{1M} \geq q_{2C} \geq q_{2S} \\
& q_{2S} = \frac{\alpha Q-\alpha q_{2M}-c_R-p_{2S}}{2\alpha}-\left(q_{1M}-q_{2C}\right) \geq 0 \\
& q_{2C}^R = q_{1M}-q_{2C}, \;\; q_{2M} \geq 0, \;\; p_{2S} \geq c_C^R
\end{aligned}
$$

whereas under partial regulation the problem is

$$
\begin{aligned}
(P8)\ Max\ \Pi_{OEM} \quad = \quad & \phi\left(q_{2M}\left(Q - q_{2M} - \alpha\left(q_{2C}^{R} + q_{2S}\right) - \left(c_M + \phi\beta\left(c_C + c_D\right)\right)\right)\right) \\
& +\phi\left(p_{2S}q_{2S} - c_C q_{2C} - c_D\left(q_{2C} - q_{2S}\right)\right) + q_{1M}\left(Q - q_{1M} - c_M\right) \\
s.t. \qquad & q_{2C} + q_{2C}^{R} \geq \beta q_{1M} \\
& q_{1M} \geq q_{2C} \geq q_{2S} \\
& q_{2S} + q_{2C}^{R} \geq 0 \\
& q_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha} - \left(q_{1M} - q_{2C}\right) \\
& q_{2C}^{R} = q_{1M} - q_{2C},\ \ q_{2M} \geq 0,\ \ p_{2S} \geq c_C^{R}
\end{aligned}
$$

For the OCs and the values of the corresponding Lagrangian multipliers under no regulation and partial regulation please refer to Appendix B1.2.

In Proposition 4 we show that under Policy C, the OEM collects all the available cores at the beginning of the second period. Therefore as long as Policy C stays optimal, imposing partial legislation on the first period's production is redundant. On the other hand, collecting β percent of the second period production in order to comply with the legislation increases the manufacturing cost in the second period.

When Region C_1 is optimal under no regulation, the amount of remanufacturing does not change while the upper bound characterizing the optimal region decreases as c_{M2} increases, i.e. $\frac{\partial \gamma_5^{C1}}{\partial c_{M2}} = \frac{-\phi\alpha}{2} < 0$. When Region C_1 is optimal under ${c_M}_2^I$ but Region C_2 is optimal under ${c_M}_2^{II}$, the remanufacturing level goes up.

When Region C_2 is optimal under no regulation, the amount of remanufacturing increases as c_{M2} increases, i.e. $\frac{\partial q_{2S}^{C2}}{\partial c_{M2}} = \frac{1}{2(2-\alpha)} > 0$ while the upper bound characterizing the feasible region decreases, i.e. $\frac{\partial (q_{1M}^{C2} - q_{2S}^{C2})}{\partial c_{M2}} = -\frac{1}{2(2-\alpha)} < 0$.

On the other hand when Region C_2 is optimal at $c_{M_2}^{I}$ but Region C_3 is optimal at ${c_{M_2}}^{II}$, $\gamma_2^{C3}({c_{M_2}}^{II}) = \frac{\phi(-(2-\alpha)\alpha Q + \alpha {c_{M_2}}^{II} + (2-\alpha)\alpha c_M + 2\phi\alpha c_C - \phi\alpha^2 c_C - c_R + c_D + 2\phi\alpha c_D - \phi\alpha^2 c_D)}{1 + 2\phi\alpha - \phi\alpha^2} > 0$ which implies

$$ {c_{M_2}}^{II} > \frac{\phi((2-\alpha)\alpha Q - \alpha(2-\alpha)c_M - (2-\alpha)\phi\alpha(c_C + c_D) + c_R - c_D}{\alpha} = Y $$

should hold under Region C_3 at ${c_{M_2}}^{II}$. Besides,

$$ (q_{1M}^{C2} - q_{2S}^{C2})(c_{M_2}^{I}) = \frac{((2-\alpha)\alpha Q - \phi\alpha(2-\alpha)(c_C + c_D) - (3-\alpha)\alpha c_M + c_R - c_D)}{2\alpha(2-\alpha)} > 0 $$

should hold under Region C_2 at $c_{M_2}^{I}$.

The change in the remanufacturing level is positive when

$$ q_{2S}^{C3}({c_{M_2}}^{II}) - q_{2S}^{C2}(c_{M_2}^{I}) = \frac{\begin{array}{c}(2-\alpha)\phi\alpha(\alpha {c_{M2}}^{II} - (c_C + c_D)) \\ -(3 - \alpha + \phi\alpha(2-\alpha))\alpha c_M - c_D + c_R + (2-\alpha)\alpha Q\end{array}}{(1 + 2\phi\alpha - \phi\alpha^2)2\alpha(2-\alpha)} > 0 $$

which implies ${c_{M_2}}^{II} > \frac{(2-\alpha)\phi\alpha(c_C + c_D) + (3 - \alpha + \phi\alpha(2-\alpha))\alpha c_M + c_D - c_R - (2-\alpha)\alpha Q}{(2-\alpha)\phi\alpha^2} = X$. Since $q_{1M}^{C2}(c_M) - q_{2S}^{C2}(c_M) > 0$, we can conclude that

$$ Y - X = \frac{(1 + \phi\alpha(2-\alpha))((2-\alpha)(\alpha Q - \phi\alpha(c_C + c_D)) - (3-\alpha)\alpha c_M + c_R - c_D)}{\phi\alpha^2(2-\alpha)} > 0. $$

Therefore ${c_{M_2}}^{II} > Y$ implies ${c_{M_2}}^{II} > X$. Hence remanufacturing level goes up as the increase in the cost of manufacturing results in moving to Region C_3 from Region C_2.

When Region C_3 remains optimal, the amount of remanufacturing increases, i.e. $\frac{\partial q_{2S}^{C3}}{\partial c_{M2}} = \frac{\phi\alpha}{2(1+2\phi\alpha-\phi\alpha^2)} > 0$, whereas the upper bound characterizing the region in which Region C_3 is optimal decreases, i.e. $\frac{\partial q_{2M}^{C3}}{\partial c_{M2}} = -\frac{1+2\phi\alpha}{2(1+2\phi\alpha-\phi\alpha^2)} < 0$. When Region C_3 is optimal at ${c_M}_2^I$ but Region C_4 is optimal at ${c_M}_2^{II}$, the change in the remanufacturing level is $q_{2S}^{C4}({c_M}_2^{II}) - q_{2S}^{C3}({c_M}_2^{I}) = \frac{\phi\alpha\left((1+2\phi\alpha-\phi\alpha^2-\alpha)Q-(1+2\phi\alpha){c_M}_2^I+\alpha c_M+\phi\alpha c_R+\phi\alpha c_C\right)}{2(1+2\phi\alpha)(1+2\phi\alpha-\phi\alpha^2)} = \phi\alpha\frac{q_{2M}^{C3}({c_M}_2^I)}{(1+2\phi\alpha)} \geq 0$. Hence the amount of remanufacturing goes up as the increase in the cost of manufacturing results in moving to Region C_4 from Region C_3.

Finally, when Region C_4 stays optimal, the amount of remanufacturing does not depend on the cost of manufacturing in the second period. Therefore the increase in the cost of manufacturing and the additional bound on the amount of collected items do not cause a decrease in the amount of remanufacturing as long as imposing partial regulation does not cause switching between policies.

From Partial Regulation to Full Regulation We complete the proof by analyzing how q_{2S} changes as full regulation is imposed: In Appendix B1.2, we show that the OCs for regions where $q_{2S}^* \geq \beta q_{1M}^*$ are the same under partial and full regulation. When in a region where $q_{2S}^* = 0$ is optimal under partial regulation, amount of remanufacturing definitely goes up as full regulation is imposed. Hence analyzing the regions where $0 < q_{2S}^* < \beta q_{1M}^*$ is optimal, i.e. regions A_2, B_2 and C_2 under partial regulation is sufficient. For the OCs and the values of the corresponding Lagrangian multipliers for each policy under full regulation, refer Appendix B1.2.

i. Under partial regulation when A_2 is optimal, if $q_{2S}^{partial} > \beta_R q_{1M}^{partial}$ then imposing full regulation does not change q_{2S}. If $q_{2S}^{partial} < \beta_R q_{1M}^{partial}$ which we can rewrite as $c_M < B2_{C,F}$, subregion A_1 becomes optimal under full regulation. The difference $q_{2S}^{full} - q_{2S}^{partial}$ is positive because $q_{2S}^{full} - q_{2S}^{partial} > 0 \Leftrightarrow c_M < B2_{C,F}$ which holds in the region of interest.

ii. Under partial regulation when B_2 is optimal, if $q_{2S}^{partial} > \beta_R q_{1M}^{partial}$ then imposing full regulation does not change q_{2S}. If $q_{2S}^{partial} < \beta_R q_{1M}^{partial}$ which we can rewrite as $c_M < B21_{C,F}$ region B_1 becomes optimal under full regulation. Then the difference $q_{2S}^{full} - q_{2S}^{partial}$ is positive because $q_{2S}^{full} - q_{2S}^{partial} > 0 \Leftrightarrow c_M < B21_{C,F}$ which already holds in the region of interest.

iii. Under partial regulation when C_2 is optimal, if $q_{2S}^{partial} > \beta_R q_{1M}^{partial}$ then imposing full regulation does not change q_{2S}. On the other hand, if $q_{2S}^{partial} < \beta_R q_{1M}^{partial}$ which we can rewrite as $c_M < B30_{C,F}$ region C_1 becomes optimal under full regulation. Then the difference $q_{2S}^{full} - q_{2S}^{partial}$ is positive because $q_{2S}^{full} - q_{2S}^{partial} > 0 \Leftrightarrow c_M < B30_{C,F}$ which holds in the region of interest.

Hence, under full regulation the amount of remanufactured products remains the same or goes up as long as it is still optimal to play the Policy that was optimal under partial regulation. ■

Proof of Corollary 1. When $c_R > \alpha Q + c_C - 2c_C^R$ either Policy A or region C_5 or region C_8 is optimal. By Proposition 8 as long as Policy A stays optimal as partial (full) regulation is imposed, the level of remanufacturing remains the same or goes up. Since $q_{2S}^{C5} = q_{2S}^{A5}$ and $q_{2S}^{C8} = q_{2S}^{A8}$ (for optimal quantities under partial regulation see Appendix B1.2), the result follows from the analysis provided for regions A_5 and A_8 in the proof of Proposition 8. ■

Proof of Proposition 9. The only regions under which remanufacturing is not feasible under partial regulation are A_1, B_1 and C_1.

i. Consider regions A_1 and C_1. In Appendix B1.2 we show that if $c_M \leq B2_{C,P}$, either region A_1 or C_1 is optimal. Hence some remanufacturing is imposed as long as $c_M \geq B2_{C,P}$ which can be written as $\beta \geq \frac{-\alpha c_M + c_R - c_D}{\alpha\phi(c_C + c_D)} = \beta^*$.

ii. Consider region B_1. In Appendix B1.2 we show that region B_1 is optimal when $B2_{C,P} \leq c_M \leq B21_{C,P}$. Thus if policy B stays optimal as legislation is introduced, some remanufacturing is imposed as long as $c_M \geq B21_{C,P}$ which can be written as $\beta \geq \frac{(-2\alpha+\alpha^2)Q + (4-\alpha)c_R - 2\alpha c_M - \alpha c_D + (4-2\alpha)c_C^R}{2\alpha\phi(c_C + c_D)}$. ■

Proof of Proposition 10. Note that when $\alpha Q - c_D - 2c_C^R < c_R$, A_1 is the only region under which there is no remanufacturing. First, we find the threshold θ^c under partial regulation: Assume that at point $A = \left(c_M^A, c_R^A\right)$ remanufacturing is not profitable, i.e. Region A1 is optimal under partial regulation, hence the optimal production quantities are $q_{1M}^A = q_{2M}^A = \frac{Q - c_M^A - \phi\beta(c_C + c_D)}{2}$. The profit at point A is $\Pi_{OEM}^A = \frac{(Q - c_M^A - \phi\beta(c_C + c_D))^2(1+\phi)}{4}$. Now consider point $B = \left(c_M^B, c_R^B\right)$ where $c_R^B < c_R^A$, $c_M^B > c_M^A$ and remanufacturing is profitable, i.e. Region 2 is optimal. The optimal production quantities at point B are

$$
\begin{aligned}
q_{1M}^B &= \frac{Q - c_M^B - \phi\beta\left(c_C + c_D\right)}{2}, \\
q_{2M}^B &= \frac{Q\left(2 - \alpha\right) - 2(c_M^B + \beta\phi\left(c_C + c_D\right)) + \left(c_R^B - c_D\right)}{2\left(2 - \alpha\right)}, \\
q_{2R}^B &= \frac{\alpha c_M^B + \alpha\beta\phi\left(c_C + c_D\right) - \left(c_R^B - c_D\right)}{2\alpha\left(2 - \alpha\right)},
\end{aligned}
$$

and the profit at point B is

$$\Pi^B_{OEM} = \frac{\phi\left(\alpha c^B_M + \alpha\beta\phi\left(c_C + c_D\right) - \left(c^B_R - c_D\right)\right)^2}{4\alpha\left(2-\alpha\right)} + \frac{(Q - c^B_M - \phi\beta\left(c_C + c_D\right))^2(1+\phi)}{4}.$$

Now let us compare the profit values obtained at point A where remanufacturing is not profitable and at point B where remanufacturing becomes profitable:

$$\begin{aligned}\Pi^B_{OEM} - \Pi^A_{OEM} = \Delta \; &= \; \frac{\phi\left(\alpha(c^B_M + \beta\phi\left(c_C + c_D\right)) - \left(c^B_R - c_D\right)\right)^2}{4\alpha\left(2-\alpha\right)} \\ &\quad - \frac{\left(c^B_M - c^A_M\right)\left(2Q - c^B_M - c^A_M - 2\beta\phi c_D - 2\beta\phi c_C\right)(1+\phi)}{4}\end{aligned}$$

Simplifying the difference we get a second order polynomial of c^A_M, $\Delta = c^A_M - c^B_M + k_c\frac{\left(c^B_M - B2_{C,P}\right)^2}{(2X - c^B_M - c^A_M)}$ where $k_c = \frac{\phi\alpha}{(1+\phi)(2-\alpha)}$, $B2_{C,P} = \frac{c^B_R - c_D - \alpha\beta\phi(c_C + c_D)}{\alpha}$ (from in Table B2 in Appendix B1.2) and $X = 2Q - 2\beta\phi c_D - 2\beta\phi c_C$. Solving the polynomial we find two roots: $r^c_{1,2} = X \pm \sqrt{(c^B_M - X)^2 + k_c(c^B_M - B2_{C,P})^2}$. Note that the larger root, r^c_2 is greater than c^B_M. The sign of smaller root, r^c_1 depends on $\Delta(c^A_M = 0) = \frac{(X - c^B_M)^2 + k_c(c^B_M - B2_{C,P})^2 - X^2}{2X - c^B_M}$. There are two possibilities: (i) If $\Delta(c^A_M = 0) < 0$ then $r^c_1 > 0$ and Δ is positive in the interval (r^c_1, c^B_M). (ii) If $\Delta(c^A_M = 0) > 0$ then $r^c_1 < 0$ and Δ is positive in the interval $(0, c^B_M)$. Hence the profit at point B is higher than the profit at point A when $\theta^C = \max\{0, r^c_1\} < c^A_M < c^B_M$.

In addition r^c_1 decreases as the legislation becomes stricter, i.e.

$$\frac{\partial r^c_1}{\partial \beta} = -\frac{k_c\alpha\phi(c_C + c_D)\left(\alpha c^B_M + \alpha\beta\phi\left(c_C + c_D\right) - \left(c^B_R - c_D\right)\right)}{\sqrt{k_c(\alpha c^B_M + \alpha\beta\phi\left(c_C + c_D\right) - (c^B_R - c_D))^2 + \alpha^2(c^B_M - X)^2}} < 0\,.$$

Now, consider the proof of Proposition 3 and observe that $B1_{M,P} = B2_{C,P}$ and $k > k_c$. Hence r_1 evaluated at $k = k_c$ is equal to r_1^c and r_1 increases as k decreases and thus we conclude that the threshold under competition is higher compared to the threshold under monopoly. ■

Proof of Proposition 11. Under Policy A, $q_{2S} = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha}$, $q_{2C}^R = 0$, and the total remanufacturing level $q_{2R} = q_{2S} + q_{2C}^R = \frac{\alpha Q - \alpha q_{2M} - c_R - p_{2S}}{2\alpha}$. Solving for p_{2S} to get $\alpha \left(Q - q_{2M} - 2q_{2R}\right) - c_R$ and plugging this expression into the OEM's problem we obtain:

$$
\begin{aligned}
(P9) \quad \underset{q_{2M},\, q_{2R},\, q_{2C}}{Max} \; \Pi_{2OEM} \quad &= \quad q_{2R} \left(\alpha \left(Q - q_{2M} - 2q_{2R}\right) - c_R\right) \\
&\quad + q_{2M} \left(\left(Q - q_{2M} - \alpha q_{2R}\right) - c_M\right) \\
&\quad - q_{2C} c_C - \left(q_{2C} - q_{2R}\right) c_D - \left(c_C + c_D\right) \beta \phi q_{2M} \\
s.t. \qquad & \beta q_{1M} \leq q_{2C} \leq q_{1M}, \; \beta_R q_{1M} \leq q_{2R} \leq q_{2C}, \; q_{2M} \geq 0
\end{aligned}
$$

We show that problem (P9) where q_{2R} is a variable (instead of p_{2S}) is equivalent to OEM's original problem under Policy A. Compare (P9) to the problem of the monopolist OEM:

$$
\begin{aligned}
\underset{q_{2M},\, q_{2R},\, q_{2C}}{Max} \; \Pi_{2M} \quad &= \quad q_{2R} \left(\alpha \left(Q - q_{2M} - q_{2R}\right) - c_R\right) + q_{2M} \left(\left(Q - q_{2M} - \alpha q_{2R}\right) - c_M\right) \\
&\quad - q_{2C} c_C - \left(q_{2C} - q_{2R}\right) c_D - \left(c_C + c_D\right) \beta \phi q_{2M} \\
s.t. \qquad & \beta q_{1M} \leq q_{2C} \leq q_{1M}, \; \beta_R q_{1M} \leq q_{2R} \leq q_{2C}, \; q_{2M} \geq 0
\end{aligned}
$$

Observe that the problem of the monopolist OEM and the problem of the OEM facing competition differ only in the objective function and $\Pi_{2OEM} = \Pi_{2M} - q_{2R}^2 \alpha$. As the coefficient of q_{2R}^2 is smaller in the competition model, at the optimality $q_{2S}^{Competition} < q_{2R}^{Monopoly}$ holds and thus the optimal amount of remanufacturing for an OEM facing competition under Policy A can never be higher than that under monopoly. ■

Proof of Theorem 12. In Theorem 1 we provide eight regions that characterize the feasible strategies for a monopolist OEM under partial regulation. Using Implicit Function Theorem, we show that in each of these eight regions, total consumer surplus decreases as the collection target β increases. Let $J(q_{1M}, q_{2M}, q_{2R}, q_{2C}, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5)$ denote the Jacobian of the first order conditions given in (A-5)-(A-8), i.e. $f_1 = \frac{\partial \Pi_M^{Lagr}}{\partial q_{1M}} = 0$, $f_2 = \frac{\partial \Pi_M^{Lagr}}{\partial q_{2M}} = 0$, $f_3 = \frac{\partial \Pi_M^{Lagr}}{\partial q_{2R}} = 0$, and $f_4 = \frac{\partial \Pi_M^{Lagr}}{\partial q_{2C}} = 0$, along with the complementary slackness conditions, i.e. $f_5 = \gamma_1(q_{1M} - q_{2C}) = 0$, $f_6 = \gamma_2(q_{2C} - q_{2R}) = 0$, $f_7 = \gamma_3(q_{2M}) = 0$, $f_8 = \gamma_4 (q_{2C} - \beta q_{1M}) = 0$, and $f_9 = \gamma_5 q_{2R} = 0$. Then using implicit function theorem we obtain the derivatives as

$$\begin{bmatrix} \frac{\partial q_{1M}^*}{\partial \beta} & \frac{\partial q_{2M}^*}{\partial \beta} & \frac{\partial q_{2R}^*}{\partial \beta} & \frac{\partial q_{2C}^*}{\partial \beta} & \frac{\partial \gamma_1^*}{\partial \beta} & \frac{\partial \gamma_2^*}{\partial \beta} & \frac{\partial \gamma_3^*}{\partial \beta} & \frac{\partial \gamma_4^*}{\partial \beta} & \frac{\partial \gamma_5^*}{\partial \beta} \end{bmatrix}$$
$$= -J(q_{1M}, q_{2M}, q_{2R}, q_{2C}, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5)^{-1} \cdot M^T$$

where $M = \begin{bmatrix} \frac{\partial f_1}{\partial \beta} & \frac{\partial f_2}{\partial \beta} & \frac{\partial f_3}{\partial \beta} & \frac{\partial f_4}{\partial \beta} & \frac{\partial f_5}{\partial \beta} & \frac{\partial f_6}{\partial \beta} & \frac{\partial f_7}{\partial \beta} & \frac{\partial f_8}{\partial \beta} & \frac{\partial f_9}{\partial \beta} \end{bmatrix}$ is

$$\begin{bmatrix} -\gamma_4 & -\phi^2(c_C + c_D) & 0 & 0 & 0 & 0 & 0 & -\gamma_4 q_{1M} & 0 \end{bmatrix}.$$

Note that the change in surplus $S = \frac{q_{1M}^2}{2} + \left(\frac{\alpha q_{2R}^2 + q_{2M}^2 + 2\alpha q_{2R} q_{2M}}{2}\right)\phi$ with respect to β is

$$\frac{\partial S}{\partial \beta} = \phi \left((q_{2M}^* + \alpha q_{2R}^*) \frac{\partial q_{2M}^*}{\partial \beta} + \alpha \frac{\partial q_{2R}^*}{\partial \beta} (q_{2M}^* + q_{2R}^*) \right) + q_{1M}^* \frac{\partial q_{1M}^*}{\partial \beta}.$$

By plugging in the values of decision variables, we obtain the change in surplus in each feasible region as in Table A6.

Region	**Change in Surplus** $\frac{\partial S}{\partial \beta}$	**Region**	**Change in Surplus** $\frac{\partial S}{\partial \beta}$
1	$\frac{-q_{2M}^* \phi^2 (c_C + c_D) - \gamma_4^* q_{1M}^*}{2}$	5	$\frac{-q_{2M}^* \phi^2 (c_C + c_D)}{2}$
2	$\frac{-q_{2M}^* \phi^2 (c_C + c_D) - \gamma_4^* q_{1M}^*}{2}$	6	$\frac{-\gamma_4^* q_{1M}^*}{2}$
3	$\frac{-q_{2M}^* \phi^2 (c_C + c_D) - \gamma_4^* q_{1M}^*}{2}$	7	0
4	$\frac{-q_{2M}^* \phi^2 (c_C + c_D)}{2}$	8	0

TABLE A6: The change in consumer surplus in each feasible region

Finally we calculate the surplus S in each region i at $c_M = Bi_{M,P}$ and show that the surplus is continuous at the boundaries. Since by IFT, we know that as β increases surplus decreases in each region, we conclude that as β increases surplus goes down under partial regulation.

Under full regulation, the optimal strategies are the same as those under partial regulation except the strategy characterized by Region 1 and the upper bound characterizing that region. Applying IFT we obtain the change in surplus (as β increases) in Region 1 under full regulation as $\frac{-q_{2M}^* \phi^2 (c_C + c_D) - \gamma_4^* q_{1M}^*}{2} \leq 0$. Moreover, the surplus is continuous at the boundary, $c_M = B1_{M,F}$.

Similarly, we analyze how the change in remanufacturing target β_R affects total consumer surplus. Under full regulation, in Region 1 applying IFT we obtain the change in surplus (as β_R increases) as $\frac{-q_{1M}^* \gamma_5^*}{2} \leq 0$. In the other regions, the change in surplus is zero as the surplus does not depend on β_R. ■

Proof of Proposition 13. We compare the total consumer surplus under monopoly (S^M) and under competition when Policy A is optimal (S^A). The difference $\Delta_S = S^M - S^A = \frac{(q_{1M}^M)^2 - (q_{1M}^A)^2}{2} + \left(\frac{\alpha (q_{2R}^M)^2 + q_{2M}^M (q_{2M}^M + 2\alpha q_{2R}^M)}{2} - \frac{\alpha (q_{2S}^A)^2 + q_{2M}^A (q_{2M}^A + 2\alpha q_{2S}^A)}{2} \right) \phi$.

In order to show that Δ_S is always nonnegative, we first prove that $q_{1M}^M \geq q_{1M}^A$: In Section B1.2 and B1.2 we derive the bounds on c_M characterizing the feasible regions for the OEM's policies. Using the bounds, we identify the ordering of eight possible regions under Policy A as follows: **I.** $B2_{C,P}$ ($B2_{C,F}$ under full regulation) $\leq B5_{C,F} \leq B7_{C,F} \leq B10_{C,F} \leq B11_{C,F}$ which implies that if c_M is increasing, the optimal regions change in the following order: $A_1 - A_2 - A_3 - A_4 - A_5 - A_8$, **II.** $B2_{C,P} \leq B5_{C,F} \leq B6_{C,F} \leq B9_{C,F} \leq B18_{C,F}$ which implies that if c_M is increasing, the optimal regions change in the following order: $A_1 - A_2 - A_3 - A_4 - A_7 - A_8$, **III.** $B2_{C,P} \leq B5_{C,F} \leq B6_{C,F} \leq B16_{C,F} \leq B18_{C,F}$ which implies that if c_M is increasing, the optimal regions change in the following order: $A_1 - A_2 - A_3 - A_6 - A_7 - A_8$. In the proof of Theorem 1 we show that there are eight different feasible regions under monopoly and we also provide the bounds that characterize each region. It is easy to show that for each one of the eight regions, the lower bound of Region i under monopoly is smaller than the lower bound that characterizes Region A_i under competition. This implies that if Region i is optimal under monopoly then only Region A_j where $j \leq i$ may be optimal for $i \leq 5$, and Region A_j where $5 < j \leq i$ or $1 \leq j \leq j - 3$ may be optimal for $i \geq 6$, under competition. In Section B1.2 and B1.2 we provide how q_{1M} is expressed in terms of the nonzero Lagrangian multipliers in region each A_i. It is easy to verify that the same expression (in terms of the corresponding Lagrangian multipliers) hold in Region i for all i under monopoly. Since we know how the feasible regions under monopoly scenario would overlap with those under competition scenario (Policy A), for any Region i under monopoly that may overlap with Region A_j under competition, we compare the expressions and deduce that q_{1M}^M in Region i is always (weakly) greater than q_{1M}^A in Region A_j. Therefore $q_{1M}^M \geq q_{1M}^A$ and thus the consumer surplus in the first period is higher under monopoly.

Next we compare the consumer surplus under second period: From the first order condition given in (A-10), we know that under monopoly $q_{2M}^M = \frac{\phi(Q-c_M-\beta\phi(c_C+c_D))+\gamma_3^M-2\alpha\phi q_{2R}^M}{2\phi}$. Similarly, solving the first order conditions given in (B-1) and (B-2) in Appendix B1.2 for q_{2M}^A after plugging in $\gamma_7^A = 0$ (Lagrangian multiplier of the constraint $p_{2S} \geq c_C^R$ which holds as strict inequality under Policy A) and $q_{2S}^A = \frac{\alpha Q-\alpha q_{2M}-c_R-p_{2s}}{2\alpha}$ (see Table 2.3), we obtain $q_{2M}^A = \frac{\left(\phi(Q-c_M-\beta\phi(c_C+c_D))+\gamma_3^A\right)-2\alpha\phi q_{2S}^A}{2\phi}$. Therefore the difference between the total consumer surplus in the second period is

$$
\begin{aligned}
\Delta_{S2} &= \phi\left(\frac{\alpha(q_{2R}^M)^2+q_{2M}^M(q_{2M}^M+2\alpha q_{2R}^M)}{2}-\frac{\alpha(q_{2S}^A)^2+q_{2M}^A(q_{2M}^A+2\alpha q_{2S}^A)}{2}\right) \quad \text{(A-20)}\\
&= \frac{\phi}{2}\left(\alpha(1-\alpha)(q_{2R}^M)^2+\left(\frac{\left(\phi(Q-c_M-\beta\phi(c_C+c_D))+\gamma_3^M\right)}{2\phi}\right)^2\right)\\
&\quad -\frac{\phi}{2}\left(\alpha(1-\alpha)(q_{2S}^A)^2+\left(\frac{\left(\phi(Q-c_M-\beta\phi(c_C+c_D))+\gamma_3^A\right)}{2\phi}\right)^2\right). \quad \text{(A-21)}
\end{aligned}
$$

Next we analyze four cases depending on the second period manufacturing levels, keeping in mind that $(q_{2R}^M)^2 \geq (q_{2S}^A)^2$ (see Proof of Proposition 8): *Case i.* If $q_{2M}^M > 0$ and $q_{2M}^A > 0$ then $\gamma_3^M = 0$ and $\gamma_3^C = 0$. Therefore from Equation (A-21) we deduce that second period surplus is higher under monopoly, i.e. $\Delta_{S2} \geq 0$. *Case ii.* If $q_{2M}^M = 0$ and $q_{2M}^A = 0$ then Equation (A-20) implies that $\Delta_{S2} \geq 0$. *Case iii.* If $q_{2M}^M = 0$ and $q_{2M}^A > 0$ then $\gamma_3^M > 0$ and $\gamma_3^C = 0$. Therefore from Equation (A-21) we deduce that $\Delta_{S2} \geq 0$. *Case iv.* The above analysis regarding how optimal regions under monopoly overlap with those under competition indicate that q_{2M}^A can never be zero unless $q_{2M}^M = 0$. Thus the case where

$q_{2M}^{M} > 0$ and $q_{2M}^{A} = 0$ is not possible.

Therefore both the first period and the second period surplus is higher under monopoly when compared to those under Policy A of competition. ■

B1 Appendix for Chapter 2: Optimality Conditions

In this section we provide the details of how the optimal solutions to the monopoly and competition scenarios are derived for the problems presented in Chapter 2.

In the proof of Theorem 1 we characterize the optimal regions and the corresponding optimality conditions (OCs) for a monopolist OEM under different levels of take-back legislation; however, we do not explain how the redundant OCs are eliminated. In Section B1.1 we give details of how we identify those redundant OCs. For each one of these conditions, we show that either it can be expressed in terms of the other necessary OCs (that are provided in the proof of Theorem 1) or it never assumes a negative value in the region where $c_M \leq c_M^{max}$ and $c_R \leq c_R^{max}$, and thus it can be eliminated.

In Section B1.2 we characterize the optimal regions and the corresponding OCs for an OEM facing competition under no, partial and full regulation, respectively. As provided in Table 2.3, there are three possible optimal policies for the OEM. For each of the three policies we solve the OEM's problem by setting up the Lagrangian function and then solving for the first order conditions and the complementary slackness conditions. After deriving the OCs, we eliminate the redundant OCs using the approach as in monopoly problem, as explained above. Then we rewrite the remaining OCs characterizing each feasible region as bounds on c_M. Since the feasible region for Policy C might overlap with the feasible regions of the other two policies, we also compare the profit of OEM under Policy C with the profit under Policy A or B in order to figure out which Policy is optimal in the regions where more than one policy is feasible.

B1.1 Elimination of the Redundant OCs for the Monopolist OEM's Problem

Redundant OCs under No Regulation:

Region 1 $(q_{2R} = q_{2C} = 0, q_{2M} > 0)$**:** In this region, $\gamma_2 = \phi\left(c_C + c_D\right) \geq 0$ is redundant.

Region 2 $(0 < q_{2C} = q_{2R} < q_{1M}, q_{2M} > 0)$**:** In this region, $q_{2M} = \frac{Q(1-\alpha)-c_M+(c_R+c_C)}{2(1-\alpha)} = (q_{1M} - q_{2R}) + (1-\alpha)\, q_{2R} \geq 0$ and $\gamma_2 = \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Region 3 $(q_{2R} = q_{2C} = q_{1M}, q_{2M} > 0)$**:** In this region, $q_{1M} = \frac{Q-\phi(c_R+c_C)-(1-\phi\alpha)c_M}{2(1+\phi\alpha(1-\alpha))} = \frac{Q-c_M+\gamma_1}{2} \geq 0$ and $\gamma_2 = \phi\frac{(c_D-c_R)+(2-\alpha)\alpha c_M+\alpha\phi(1-\alpha)(c_C+c_D)-\alpha(1-\alpha)Q}{(1+\phi\alpha(1-\alpha))} = \gamma_1 + \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Region 4 $(q_{2R} = q_{1M} = q_{2C}, q_{2M} = 0)$**:** In this region, $q_{1M} = \frac{(1+\phi\alpha)Q-c_M-\phi(c_R+c_C)}{2(1+\phi\alpha)} = \frac{Q-c_M+\gamma_1}{2} \geq 0$, $\gamma_2 = \phi\frac{\alpha c_M-(c_R-c_D)+\alpha\phi(c_C+c_R)}{(1+\phi\alpha)} = \gamma_1 + \phi\left(c_C + c_D\right) \geq 0$, and $\gamma_1 = \frac{\alpha c_M-c_R-c_C}{1+\phi\alpha} = \frac{(1-\alpha)(Q-c_M)+\gamma_3}{\phi\alpha} \geq 0$ are redundant.

Redundant OCs under Partial Regulation:

Region 1 $(0 = q_{2R} < \beta q_{1M} = q_{2C},\ q_{2M} > 0)$**:** In this region, $q_{2M} = \frac{Q-c_M-\phi\beta(c_C+c_D)}{2} \geq 0$ and $\gamma_4 = \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Region 2 $(0 < q_{2R} < \beta q_{1M} = q_{2C},\ q_{2M} > 0)$**:** In this region, $q_{2M} = q_{1M} - \alpha q_{2R} \geq 0$ and $\gamma_4 = \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Region 3 $(q_{2R} = \beta q_{1M} = q_{2C},\ q_{2M} > 0)$**:** In this region, $q_{1M} = \frac{q_{2M}+\beta\gamma_2/2}{(1-\alpha\beta)} \geq 0$ is redundant.

Region 4 $(\beta q_{1M} < q_{2C} = q_{2R} < q_{1M},\ q_{2M} > 0)$**:** In this region, $\gamma_2 = \phi\left(c_C + c_D\right)$ is redundant.

Region 5 ($q_{2R} = q_{1M} = q_{2C},\ q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M+\gamma_1}{2} \geq 0$ and $\gamma_2 = \phi\frac{(c_D-c_R)+(2-\alpha)\alpha c_M+\alpha\phi(\beta+1-\alpha)(c_C+c_D)-\alpha(1-\alpha)Q}{(1+\phi\alpha(1-\alpha))} = \gamma_1 + \phi\,(c_C + c_D) \geq 0$ are redundant.

Region 6 ($q_{2R} = \beta q_{1M} = q_{2C},\ q_{2M} = 0$): In this region, $q_{1M} = \frac{\gamma_3+\phi(Q-c_M-\phi\beta(c_C+c_D))}{2\phi\alpha\beta} \geq 0$ and $\gamma_2 = \frac{\gamma_3+\phi(1-\alpha\beta)(Q-c_M-\phi\beta(c_C+c_D))}{\phi\alpha\beta^2} \geq 0$ are redundant.

Region 7 ($\beta q_{1M} < q_{2C} = q_{2R} < q_{1M},\ q_{2M} = 0$): In this region, $\gamma_2 = \phi\,(c_C + c_D)$ is redundant.

Region 8 ($q_{2R} = q_{1M} = q_{2C},\ q_{2M} = 0$): In this region, $q_{1M} = \frac{Q-c_M+\gamma_1}{2} \geq 0$ and $\gamma_2 = \gamma_1 + \phi\,(c_C + c_D) \geq 0$ are redundant.

Redundant OCs under Full Regulation:

Region 1 ($\beta_R q_{1M} = q_{2R} < \beta q_{1M} = q_{2C},\ q_{2M} > 0$): In this region, $q_{1M} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_M} < 0$ and $\frac{\partial q_{1M}}{\partial c_R} < 0$ and $q_{1M}(c_M = c_M^{max}, c_R = c_R^{max}) = \frac{\phi\beta_R(c_C+c_D)}{2\left(1+\phi\beta_R^2\alpha(1-\alpha)\right)} > 0$. In addition, $q_{2M} = \frac{(1-\beta_R\alpha-\phi\beta_R^2\alpha(\alpha-1))Q+(-1+\beta_R\alpha(1-\phi\beta_R))(c_M+\phi\beta(c_C+c_D))+\phi\beta_R^2\alpha(c_R-c_D)}{2(1+\phi\beta_R^2\alpha(1-\alpha))} = (1-\alpha\beta_R)\,q_{1M} + \beta_R\gamma_5/2 \geq 0$ and $\gamma_4 = \phi\,(c_C + c_D)$ are also redundant.

Region 2 ($\beta_R q_{1M} < q_{2R} < \beta q_{1M} = q_{2C},\ q_{2M} > 0$): The redundant OCs are the same as the ones in Region 2 under Partial regulation.

B1.2 Solution of the OEM's Problem facing Competition

No regulation: Problem Formulation

In Proposition 4, we show that the OEM chooses the quantities and prices so that either *Region R4* or *R5* (see Table A5) is optimal for the remanufacturer. If the remanufacturer's optimal solution is in *Region R4*, the OEM's problem is given by (P5) (see the proof of Proposition 8). The Lagrangian function is $\Pi_{OEM}^{Lagr} = \Pi_{OEM} + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2S}) + \gamma_3 q_{2M} + \gamma_5 q_{2S} + \gamma_7(c_C^R - p_{2S})$. Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$
\begin{aligned}
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\phi\left((2-\alpha)\,Q - 2c_M + c_R - c_D - 2\,(2-\alpha)\,q_{2M}\right) + 2\gamma_3 + \gamma_2 - \gamma_5}{2} = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - 2q_{1M} = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= \gamma_2 - \gamma_1 - \phi\,(c_C + c_D) = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\,(\alpha Q - c_R - c_D - 2p_{2S}) - 2\alpha\gamma_7 + \gamma_2 - \gamma_5}{2\alpha} = 0\,.
\end{aligned}
$$

We can show that the Hessian is negative semidefinite and the first order conditions ensure optimality.

From the first order condition $\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} = 0$, $\gamma_2 = \gamma_1 + \phi\,(c_C + c_D) > 0$ and hence $q_{2C} = q_{2S}$. On the other hand, since $q_{1M} - \alpha q_{2S} > 0$, from the first order conditions, we conclude that $2\phi\,(q_{1M} - \alpha q_{2S}) = -\gamma_3 - \alpha\gamma_7 + 2\phi q_{2M} + \gamma_1\phi > 0$. Hence when $q_{2M} = 0$, i.e. $\gamma_3 > 0$, $q_{1M} = q_{2C}$ should hold. Using these properties we solve the OEM's problem under no regulation for each region. Note that when $c_C^R > p_{2S}^*$ we obtain Policy A solutions whereas when the constraint is tight, i.e. $c_C^R = p_{2S}^*$ we obtain Policy B solutions.

Solving the OEM's problem under *Region R5* of remanufacturer's solution we obtain Policy C solutions. Given the remanufacturer plays *R5* the OEM's problem can be written (P7) (see the proof of Proposition 8). Then the Lagrangian function is $\Pi_{OEM}^{Lagr} = \Pi_{OEM} + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2S}) + \gamma_3 q_{2M} + \gamma_5 q_{2S} + \gamma_7(p_{2S} - c_C^R)$. Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$\begin{aligned}
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\phi\left(\left(2-\alpha\right)Q - 2c_M + c_R - c_D - 2\left(2-\alpha\right)q_{2M}\right) + 2\gamma_3 + \gamma_2 - \gamma_5}{2} = 0\,,\\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - 2q_{1M} - \phi p_{2S} - \phi c_D + \gamma_2 - \gamma_5 = 0\,,\\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= \gamma_5 - \gamma_1 + \phi\left(p_{2S} - c_C\right) = 0\,,\\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\left(\alpha Q - c_R - c_D - 2p_{2S}\right) + 2\alpha\gamma_7 + \gamma_2 - \gamma_5 - 2\alpha\phi(q_{1M} - q_{2C})}{2\alpha} = 0\,.
\end{aligned}$$

From the first order condition $\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} = 0$ we deduce that $\gamma_1 = \gamma_5 + \phi\left(p_{2S} - c_C\right) > 0$ and hence $q_{2C} = q_{1M}$. On the other hand, since $q_{1M} - \alpha q_{2S} > 0$ should hold, from the first order conditions, we can conclude that $2\phi\left(q_{1M} - \alpha q_{2S}\right) - 2\phi q_{2M} + \phi^2(c_C + c_D) = -\gamma_3 + \alpha\gamma_7 + \gamma_2\phi > 0$. With proposition 5 we show that $p_{2S} > c_C$, i.e. $\gamma_7 = 0$ under Policy C. Hence when $q_{2M} = 0$, i.e. $\gamma_3 > 0$, $q_{2S} = q_{2C}$ should hold. Using these properties we solve the OEM's problem under no regulation for region C.

The OCs that should be satisfied under each region are given in sections B1.2-B1.2 below. After the redundant OCs are eliminated we rewrite the remaining conditions as bounds on c_M and the bounds on c_M characterizing the optimal policies for the OEM are provided in Table B1.

OCs for No Regulation: Policy A where $p_{2S}^* < c_C^R$:

Region A$_1$ ($q_{2S} = q_{2C} = 0, q_{2M} > 0$): In this region, $q_{1M} = q_{2M} = \frac{Q - c_M}{2} \geq 0$ and $\gamma_2 = \phi\left(c_C^R + c_D\right) \geq 0$ are redundant. Remaining OCs are $\gamma_5 = \phi\left(-\alpha c_M + c_R + c_C\right) \geq 0$ which can be written as $c_M \leq B1_{C,N}$ and $c_C^R - p_{2S} = \frac{-\alpha(Q + c_M) + 2\left(c_R + c_C^R\right)}{2} \geq 0$ which can

Bound	Expression
$B1_{C,N}$	$\frac{c_R+c_C}{\alpha}$
$B2_{C,N}$	$\frac{-\alpha Q+2\left(c_R+c_C^R\right)}{\alpha}$
$B3_{C,N}$	$\frac{\alpha(2-\alpha)Q+c_C+c_R}{\alpha(3-\alpha)}$
$B4_{C,N}$	$\frac{(1-\alpha+\phi\alpha(2-\alpha))Q+\phi\alpha(c_C+c_R)}{(1-\alpha+2\phi\alpha)}$
$B5_{C,N}$	$\frac{(1-(2-\alpha)\phi\alpha-\alpha)\alpha Q+(2+\phi\alpha(2-\alpha))c_R+2(1+\phi\alpha(2-\alpha))c_C^R-(2-\alpha)\phi\alpha c_C}{(3-\alpha)\alpha}$
$B6_{C,N}$	$\frac{-\phi\alpha^2 Q-\alpha\phi c_C+(1+\alpha\phi)c_R+(1+2\phi\alpha)c_C^R}{\alpha}$
$B7_{C,N}$	$\frac{c_R-c_D}{\alpha}$
$B8_{C,N}$	$\frac{(2-\alpha)\alpha Q+c_R-\phi\alpha(2-\alpha)c_C-(1+\phi\alpha(2-\alpha))c_D}{(3-\alpha)\alpha}$
$B9_{C,N}$	$\frac{-\alpha(2-\alpha)Q+(4-\alpha)c_R+2(2-\alpha)c_C^R+\alpha c_C}{2\alpha}$
$B10_{C,N}$	$\frac{\alpha(2-\alpha)Q+(4-\alpha)c_R+2(2-\alpha)c_C^R+\alpha c_C}{2\alpha(3-\alpha)}$

TABLE B1: Bounds characterizing the optimal policies for the OEM facing competition under no regulation

be written as $c_M \leq B2_{C,N}$. Since $B2_{C,N} - B1_{C,N} = \frac{\left(c_R-\alpha Q+2c_C^R+c_C\right)}{\alpha}$ we conclude that if $c_R > \alpha Q - 2c_C^R + c_C$ then $c_C^R - p_{2S} > 0$ is redundant. On the other hand, when $c_R < \alpha Q - 2c_C^R + c_C$ holds, $\gamma_5 \geq 0$ is redundant.

Region $\mathbf{A}_2$ $(0 < q_{2S} = q_{2C} < q_{1M}, q_{2M} > 0)$: In this region $q_{1M} = \frac{Q-c_M}{2} \geq 0$, $\gamma_2 = \phi\left(c_C^R + c_D\right) \geq 0$, and $q_{2M} = \frac{(2-\alpha)Q-2c_M+c_C+c_R}{2(2-\alpha)} = (q_{1M} - q_{2S}) + (1-\alpha)\, q_{2S} \geq 0$ are redundant. Remaining OCs are $q_{2S} = \frac{\alpha c_M-c_C-c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B1_{C,N}$, $q_{1M} - q_{2S} = \frac{\alpha(2-\alpha)Q-\alpha(3-\alpha)c_M+c_C+c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B3_{C,N}$ and $c_C^R - p_{2S} = \frac{-\alpha Q-c_C+c_R+2c_C^R}{2} \geq 0$ which can be written as $c_R > \alpha Q + c_C - 2c_C^R$.

Region $\mathbf{A}_3$ $(q_{2S} = q_{2C} = q_{1M}, q_{2M} > 0)$: In this region, $q_{1M} = \frac{Q-c_M+\phi(\alpha c_M-c_C-c_R)}{2(1+2\phi\alpha-\phi\alpha^2)} = \frac{2q_{2M}+\gamma_1}{2(1-\alpha)} \geq 0$ and $\gamma_2 = \frac{\left(-2\alpha+\alpha^2\right)\phi Q-\phi c_R+\left(3\alpha-\alpha^2\right)\phi c_M+\left(2\phi\alpha-\phi\alpha^2\right)\phi c_C+\left(1+2\phi\alpha-\phi\alpha^2\right)\phi c_D}{(1+2\phi\alpha-\phi\alpha^2)} = \gamma_1 + \phi\left(c_C^R + c_D\right) \geq 0$ are redundant. Remaining OCs are

$$q_{2M} = \frac{(1-\alpha+\phi\alpha\,(2-\alpha))\,Q - (1-\alpha+2\phi\alpha)\,c_M + \phi\alpha\,(c_C+c_R)}{2\,(1+2\phi\alpha-\phi\alpha^2)} \geq 0$$

which can be written as $c_M \leq B4_{C,N}$, $\gamma_1 = \frac{\left(-2\alpha+\alpha^2\right)\phi Q+\phi\left(3\alpha-\alpha^2\right)c_M-\phi c_C-\phi c_R}{(1+2\phi\alpha-\phi\alpha^2)} \geq 0$ which can be written as $c_M \geq B3_{C,N}$, and

$$c_C^R - p_{2S} = \frac{\begin{array}{l}(1-(2-\alpha)\phi\alpha-\alpha)\alpha Q+(2+\phi\alpha(2-\alpha))c_R+2(1+\phi\alpha(2-\alpha))c_C^R \\ -(3-\alpha)\alpha c_M-(2-\alpha)\phi\alpha c_C\end{array}}{2(1+2\phi\alpha-\phi\alpha^2)} \geq 0$$

which can be written as $c_M \leq B5_{C,N}$.

Region $\mathbf{A}_4$ ($q_{2S} = q_{1M} = q_{2C}, q_{2M} = 0$): In this region $q_{1M} = \frac{(1+\phi\alpha)Q-c_M-\phi c_C-\phi c_R}{2(2\phi\alpha+1)} = \frac{Q-c_M+\gamma_1}{2} \geq 0$ and $\gamma_2 = \frac{-\phi(\alpha Q-2\alpha c_M-(1+2\alpha\phi)c_D+c_R-2\alpha\phi c_C)}{(1+2\phi\alpha)} = \gamma_1 + \phi(c_C + c_D) \geq 0$ and $\gamma_1 = \frac{-\phi(\alpha Q-2\alpha c_M+c_C+c_R)}{(1+2\phi\alpha)} = \frac{(1-\alpha)\phi(Q-c_M)+\gamma_3}{\phi\alpha} \geq 0$ are redundant. Hence $\gamma_3 \geq 0$ and $c_C^R - p_{2S} \geq 0$ are enough to ensure the optimality of this region. Remaining OCs are $\gamma_3 = \frac{-(1-\alpha+\phi\alpha(2-\alpha))\phi Q+(1-\alpha+2\phi\alpha)\phi c_M-\phi^2\alpha(c_C+c_R)}{(1+2\phi\alpha)} \geq 0$ which can be expressed as $c_M \geq B4_{C,N}$ and $c_C^R - p_{2S} = \frac{-\phi\alpha^2 Q-\alpha c_M-\alpha\phi c_C+(1+\alpha\phi)c_R+(1+2\phi\alpha)c_C^R}{(1+2\phi\alpha)} \geq 0$ which can be written as $c_M \leq B6_{C,N}$.

OCs for No Regulation: Policy B where $p_{2S}^* = c_C^R$:

Before proceeding with the details of possible regions, we prove that in this region $q_{2C}^* < q_{1M}^*$. Assume that $q_{2C} = q_{1M}$, then $q_{2S} = q_{2C} = q_{1M} = \frac{\alpha Q-\alpha q_{2M}-c_R-c_C^R}{2\alpha}$ should hold. Then the OEM's profit is

$$\Pi_{OEM}^{B5} = \frac{\begin{array}{l}\alpha^2(-1+2\phi\alpha-4\phi)q_{2M}^2 \\ -2\alpha\left(\phi\alpha(\alpha-2)Q+\alpha(2\phi-1)c_M-\alpha\phi c_C+(1-\alpha\phi)c_R+c_C^R\right)q_{2M} \\ +\left(\alpha Q-c_R-c_C^R\right)\left(\alpha Q-2\alpha\phi c_C+2\alpha\phi c_C^R-2\alpha c_M+c_R+c_C^R\right)\end{array}}{4\alpha^2}.$$

If the OEM increases p_{2S} such that the new price is $p_{2S}^{new} = p_{2S} + \delta = c_C^R + \delta$ while keeping q_{2M} the same, the optimal production amounts become $q_{2S}^{new} = q_{2C}^{new} = q_{1M}^{new} = \frac{\alpha Q - \alpha q_{2M} - c_R - c_C^R - \delta}{2\alpha}$. Then the profit $\Pi_{OEM}^{B5^{new}}$ is

$$\Pi_{OEM}^{B5} + \frac{\delta\left(-\delta - 2\alpha(\phi\delta + q_{2M}) + 2\alpha(\phi\alpha Q + c_M) - 2(1+2\alpha\phi)c_C^R - 2c_R - 2\alpha\phi(c_R - c_C)\right)}{4\alpha^2}.$$

Hence the difference $\Pi_{OEM}^{B5^{new}} - \Pi_{OEM}^{B5}$ is positive as long as $0 \leq \delta \leq \delta_{UB}$ where

$$\delta_{UB} = 2\frac{\left(\phi\alpha^2 Q - \alpha q_{2M} + \alpha c_M - (1+2\alpha\phi)c_C^R - (1+\alpha\phi)c_R + \phi\alpha c_C\right)}{1+2\phi\alpha}.$$

Setting $\gamma_5 = 0$, $p_{2S} = c_C^R$ and solving the OCs for γ_7, we get

$$\gamma_7 = \frac{\phi\alpha^2 Q - c_C^R - 2\alpha\phi c_C^R - \alpha\phi c_R - c_R - \alpha q_{2M} + \alpha c_M + \phi\alpha c_C}{2\alpha^2}.$$

Hence $\delta_{UB} \geq 0$ and also $\Pi_{OEM}^{B5^{new}} - \Pi_{OEM}^{B5} \geq 0$ for any $\delta \leq \delta_{UB}$. Finally note that the proof does not depend on the value of q_{2M} and also goes through when $q_{2M} = 0$.

Region B$_1$ ($q_{2S} = q_{2C} = 0, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M}{2} \geq 0$, $\gamma_2 = \phi\left(c_C^R + c_D\right) \geq 0$, and $q_{2M} = \frac{\alpha Q - c_R - c_C^R}{\alpha} = \frac{(2\gamma_7\alpha + \gamma_4) + \phi(c_C^R - c_C)}{\phi\alpha} \geq 0$ are redundant. Remaining OCs are $\gamma_7 = \frac{\phi\left(\alpha Q + \alpha c_M - 2c_R - 2c_C^R\right)}{\alpha^2} \geq 0$ which can be written as $c_M \geq B2_{C,N}$ and $\gamma_5 = \frac{\phi\left(-\alpha(2-\alpha)Q - 2\alpha c_M + (4-\alpha)c_R + 2(2-\alpha)c_C^R + \alpha c_C\right)}{\alpha} \geq 0$ which can be written as $c_M \leq B9_{C,N}$. In this region $B9_{C,N} - B2_{C,N} = \frac{\left(\alpha Q - c_R - 2c_C^R + c_C\right)}{2} \geq 0$ and thus $c_R < \alpha Q - 2c_C^R + c_C$ should.

Region B$_2$ ($0 < q_{2S} = q_{2C} < q_{1M}, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M}{2} \geq 0$, $\gamma_2 = \phi\left(c_C^R + c_D\right) \geq 0$, and $q_{2M} = \frac{Q(2-\alpha) - 2c_M + c_R + c_C}{2(2-\alpha)} = (q_{1M} - q_{2S}) + (1-\alpha)\, q_{2S} + \frac{\gamma_7\alpha}{2\phi} \geq 0$ are redundant. Remaining OCs are $q_{2S} = \frac{\alpha(2-\alpha)Q+2\alpha c_M-(4-\alpha)c_R-2(2-\alpha)c_C^R-\alpha c_C}{4\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B9_{C,N}$, $q_{1M} - q_{2S} = \frac{\alpha(2-\alpha)Q-2\alpha(3-\alpha)c_M+(4-\alpha)c_R+2(2-\alpha)c_C^R+\alpha c_C}{4\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B10_{C,N}$ and $\gamma_7 = \frac{\phi\left(\alpha Q-c_R-2c_C^R+c_C\right)}{2\alpha} \geq 0$ which can be written as $c_R < \alpha Q + c_C - 2c_C^R$.

OCs for No Regulation: Policy C where $p_{2S}^* > c_C^R$:

Recall that $q_{2C} = q_{1M}$ under Policy C by Proposition 4.

Region C$_1$ ($q_{2S} = 0, q_{2C} = q_{1M}, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M-\phi(c_C+c_D)}{2} \geq 0$ is satisfied as long as $\gamma_1 \geq 0$ and $q_{2M} = \frac{Q-c_M}{2} \geq 0$ is also redundant. In addition $\gamma_1 = \frac{\phi\alpha(Q-c_M)-2\phi(c_C+c_D)}{2} = \phi\left(p_{2S} - c_C\right) + \gamma_5 \geq 0$ is redundant $p_{2S} - c_C > p_{2S} - c_C^R \geq 0$. Remaining OCs are $\gamma_5 = -\phi\left(\alpha c_M - c_R + c_D\right) \geq 0$ which can be written as $c_M \leq B7_{C,N}$ and $p_{2S} - c_C^R = \frac{\alpha(Q+c_M)-2\left(c_R+c_C^R\right)}{2} \geq 0$ which can be written as $c_M \geq B2_{C,N}$. In this region $B7_{C,N} - B2_{C,N} = \frac{\left(\alpha Q-c_R-2c_C^R-c_D\right)}{\alpha} > 0$ and thus $c_R < \alpha Q - 2c_C^R - c_D$ should hold.

Region C$_2$ ($0 < q_{2S} < q_{2C} = q_{1M}, q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M-\phi(c_C+c_D)}{2} \geq 0$ and $\gamma_1 = \frac{\phi\alpha Q-\phi(c_R+c_D-2c_C)}{2} = \phi\left(p_{2S} - c_C\right) > \phi\left(p_{2S} - c_C^R\right) \geq 0$ are redundant. In addition $q_{2M} = \frac{(2-\alpha)Q-2c_M-c_D+c_R}{2(2-\alpha)} \geq 0$ is also redundant because $q_{1M} - q_{2M} - \alpha q_{2S} = -\left(c_C + c_D\right)\phi/2 < 0$, so $q_{1M} - q_{2M} - \alpha q_{2S} < 0$ and $q_{1M} - q_{2S} < q_{1M} - \alpha q_{2S} < q_{2M}$ and thus $\left(q_{1M} - q_{2S}\right) \geq 0$ implies $q_{2M} \geq 0$. Remaining OCs are $q_{2S} = \frac{\alpha c_M+c_D-c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B7_{C,N}$, $q_{1M} - q_{2S} = \frac{(2-\alpha)\alpha Q-(3-\alpha)\alpha c_M+c_R-\phi\alpha(2-\alpha)c_C-(1+\phi\alpha(2-\alpha))c_D}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B8_{C,N}$, and $p_{2S} - c_C^R = \frac{\alpha Q-c_R-c_D-2c_C^R}{2} > 0$ which can be written as $c_R < \alpha Q - 2c_C^R - c_D$. Note that $B8_{C,N} - B7_{C,N} > 0$ can be written as $c_R < -\phi c_C\alpha - \phi c_D\alpha + \alpha Q + c_D$, which is already satisfied when $c_R < \alpha Q - 2c_C^R - c_D$.

Region C$_3$ ($q_{2S} = q_{2C} = q_{1M}, q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M+\phi(\alpha c_M-c_C-c_R)}{2(1+2\phi\alpha-\phi\alpha^2)} =$

$\frac{2\gamma_1+(Q-c_M)(2-\phi\alpha)}{2(2+\phi\alpha(2-\alpha))} \geq 0$ and $\gamma_1 = \phi\left(p_{2S}-c_C\right) > \phi\left(p_{2S}-c_C^R\right) \geq 0$ are redundant.

Remaining OCs are $q_{2M} = \frac{(1-\alpha+2\phi\alpha)(Q-c_M)-\phi\alpha(\alpha Q-c_C-c_R)}{2(1+2\phi\alpha-\phi\alpha^2)} \geq 0$ which can be written as $c_M \leq B4_{C,N}$, $\gamma_2 = \frac{-\alpha(2-\alpha)\phi Q-\phi c_R+(3-\alpha)\phi\alpha c_M+\phi\alpha(2-\alpha)\phi c_C+(1+\phi\alpha(2-\alpha))\phi c_D}{(1+\phi\alpha(2-\alpha))} \geq 0$ which can be written as $c_M \geq B8_{C,N}$, and $p_{2S}-c_C^R > 0$ which can be written as $c_M \geq B5_{C,N}$. Since $B5_{C,N}-B8_{C,N} = \frac{\left(1+2\phi\alpha-\phi\alpha^2\right)\left(-\alpha Q+c_R+2c_C^R+c_D\right)}{\alpha(3-\alpha)}$, $c_M \geq B5_{C,N}$ is active when $c_R > \alpha Q-2c_C^R-c_D$.

Region $\mathbf{C}_4$ ($q_{2S} = q_{1M} = q_{2C}, q_{2M} = 0$): In this region, $q_{1M} = \frac{(1+\phi\alpha)Q-c_M-\phi c_C-\phi c_R}{2(2\phi\alpha+1)} = \frac{Q-c_M+\gamma_1}{2(1+\phi\alpha)} \geq 0$ and $\gamma_1 = \frac{\phi^2\alpha^2 Q+\phi\alpha c_M-\phi(\phi\alpha+1)c_C-\phi(\phi\alpha+1)c_R}{(1+2\phi\alpha)} = \phi\left(p_{2S}-c_C\right) > \phi\left(p_{2S}-c_C^R\right) \geq 0$ are redundant. In addition, $\gamma_2 = \frac{-\phi(\alpha Q-2\alpha c_M-(1+2\alpha\phi)c_D+c_R-2\alpha\phi c_C)}{(1+2\phi\alpha)} \geq 0$ is redundant because $\gamma_3-\phi\alpha\gamma_2 = -\phi\alpha\left(c_C+c_D\right)-(1-\alpha)\left(Q-c_M\right) < 0$ which implies that $\gamma_3 < \phi\alpha\gamma_2 < \gamma_2$. Hence $\gamma_3 \geq 0$ and $p_{2S}-c_C^R \geq 0$ are enough to ensure the optimality of this region. Remaining OCs are $\gamma_3 = \frac{-(1-\alpha+\phi\alpha(2-\alpha))\phi Q+(1-\alpha+2\phi\alpha)\phi c_M-\phi^2\alpha(c_C+c_R)}{(1+2\phi\alpha)} \geq 0$ which can be expressed as $c_M \geq B4_{C,N}$ and $p_{2S}-c_C^R = \frac{\phi\alpha^2 Q+\alpha c_M+\alpha\phi c_C-(1+\alpha\phi)c_R-(1+2\phi\alpha)c_C^R}{(1+2\phi\alpha)} \geq 0$ which can be written as $c_M \geq B6_{C,N}$.

Comparison of Profits of Regions $\mathbf{B}_1$ and $\mathbf{B}_2$ with $\mathbf{C}_1$ and $\mathbf{C}_2$:

Note that Policy B is feasible in the region where $B2_{C,N} \leq c_M \leq B10_{C,N}$ and $c_R \leq \alpha Q-2c_C^R+c_C$. When we compare the bounds of regions of Policies B and C, at $c_R = \alpha Q-2c_C^R+c_C$ we find that $B10_{C,N}-B8_{C,N} = \frac{(c_D+c_C)(1+4\phi-2\phi\alpha)}{2(3-\alpha)} > 0$ and $B9_{C,N}-B7_{C,N} = (c_D+c_C)/2 > 0$. Hence Region B_1 overlaps with at least Region C_1 and C_2 whereas Region B_2 overlaps with at least Region C_2 and C_3. Observe that as long as C_3 or C_4 is feasible, the profit from that region is higher than the profit under any other possible feasible region at the same point. Hence we proceed with comparison of profits under

Regions B_1 and B_2 with profits under Regions C_1 and C_2.

Under both B_1 and B_2, $\gamma_1 = 0$, $\gamma_3 = 0$, $p_{2S} = c_C^R$, and $q_{2C} = q_{2S}$ hold. Plugging these into the first order conditions and solving for q_{2M} we get $q_{2M} = \frac{Q-\alpha Q+c_R-c_M+c_C^R+\gamma_7\alpha/\phi}{2-\alpha}$. Also note that $q_{1M}^{B1} = q_{1M}^{B2} = \frac{Q-c_M}{2}$.

Under both C_1, C_2 $\gamma_3 = 0$, $\gamma_2 = 0$, $p_{2S} = c_C^R + \delta > c_C^R$, and $q_{2C} = q_{1M}$ hold. Plugging these into first order conditions and solving for q_{2M} we get $q_{2M} = \frac{Q-\alpha Q+c_R-c_M+c_C^R+\delta}{2-\alpha}$. Also note that $q_{1M}^{C1} = q_{1M}^{C2} = \frac{Q-c_M-\phi(c_C+c_D)}{2}$. Plugging these values into the profit function, we find the condition under which B regions would give higher profit.

$$
\begin{aligned}
\Pi_{OEM}^B - \Pi_{OEM}^C &= \frac{\phi\delta\left(-\alpha Q + 2c_C^R + c_D + c_R + \delta\right)}{\alpha\left(2-\alpha\right)} \\
&+ \frac{\gamma_7^B\alpha\left(\phi\alpha Q - \gamma_7^B\alpha - 2\phi c_C^R + \phi c_C - \phi c_R\right)}{\phi 2\left(2-\alpha\right)} \\
&+ \frac{\phi\left(c_C + c_D\right)\left(-\alpha(2-\alpha)(c_D + c_C)\phi\right)}{4\alpha(2-\alpha)} \\
&+ \frac{\phi\left(c_C + c_D\right)\left(-2\alpha(3-\alpha)c_M + 4c_R + 2\alpha(1-\alpha)Q + 4c_C^R\right)}{4\alpha(2-\alpha)} \\
&= \frac{-\delta^C(\gamma_5^C + \phi\delta^C)}{\alpha\left(2-\alpha\right)} + \frac{\gamma_7^B\alpha(\gamma_5^B + \alpha\gamma_7^B)}{\phi 2\left(2-\alpha\right)} \\
&+ \frac{\phi\left(c_C + c_D\right)\left(-\phi\alpha c_C - \phi c_D\alpha - 2\alpha c_M + 2\alpha q_{2M}^{un} + 2c_C^R + 2c_R\right)}{4\alpha} > 0
\end{aligned}
$$

where $q_{2M}^{un} = \frac{\left(Q-\alpha Q+c_R-c_M+c_C^R\right)}{2-\alpha}$

Note that when $c_C + c_D$ is high, Policy B is more likely to be optimal.

Partial regulation: Problem Formulation

Given the remanufacturer plays *R4* (see Table A5), the OEM's problem is given by (P6) (see the proof of Proposition 8). Then the Lagrangian function is $\Pi_{OEM}^{Lagr} = \Pi_{2M} +$

$\gamma_1(q_{1M}-q_{2C})+\gamma_2(q_{2C}-q_{2S})+\gamma_3(q_{2M})+\gamma_4(q_{2C}^R+q_{2C}-\beta q_{1M})+\gamma_5(q_{2C}^R+q_{2S})+\gamma_7(c_C^R-p_{2S})$. Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$\begin{aligned}
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\begin{array}{c}\phi\left(\left(2-\alpha\right)Q-2c_M-2\beta\phi(c_C+c_D)\right)\\ \phi\left(c_R-c_D-2\left(2-\alpha\right)q_{2M}\right)+2\gamma_3+\gamma_2-\gamma_5\end{array}}{2}=0\,, && \text{(B-1)}\\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q-c_M-\gamma_4\beta+\gamma_1-2q_{1M}=0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= \gamma_2-\gamma_1+\gamma_4-\phi\left(c_C+c_D\right)=0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\left(\alpha Q-c_R-c_D-2p_{2S}\right)-2\alpha\gamma_7+\gamma_2-\gamma_5}{2\alpha}=0\,. && \text{(B-2)}
\end{aligned}$$

Note that the Hessian is still negative semidefinite, so first order conditions ensure optimality. From the first order condition $\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}}=0$, $\gamma_2+\gamma_4=\gamma_1+\phi\left(c_C+c_D\right)>0$ and hence $q_{2C}=\max(q_{2S},\beta q_{1M})$. On the other hand, since $q_{1M}-\alpha q_{2R}>0$ should hold, from the first order conditions we conclude that $2\phi(q_{1M}-\alpha q_{2R}-q_{2M})=-\alpha\gamma_7-\gamma_3+(1-\beta)\,\phi\gamma_1+\beta\phi\gamma_2>0$ holds. Hence when $q_{2M}=0$, i.e. $\gamma_3>0$, $\gamma_2=\gamma_1=0$ cannot hold which means that $q_{1M}>q_{2C}>q_{2R}$ cannot be optimal if there is no second period manufacturing. Using these properties, we solve the OEM's problem under no regulation for each region. When $c_C^R>p_{2S}^*$ we obtain Policy A solutions whereas when the constraint is tight, i.e. $c_C^R=p_{2S}^*$ we obtain Policy B solutions.

On the other hand, solving the OEM's problem under *Region R5* (see Table A5) of remanufacturer's solution we obtain Policy C solutions. Under *Region R5* the OEM's problem is given in (P8) (see the proof of Proposition 8). The Lagrangian function is

$\Pi_{OEM}^{Lagr} = \Pi_{OEM} + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2S}) + \gamma_3(q_{2M}) + \gamma_4(q_{2C}^R + q_{2C} - \beta q_{1M}) + \gamma_5(q_{2C}^R + q_{2S}) + \gamma_7(c_C^R - p_{2S})$. Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$\begin{aligned}
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\begin{array}{l}\phi\left((2-\alpha)\,Q - 2c_M - 2\phi\beta(c_C + c_D) + c_R - c_D - 2\,(2-\alpha)\,q_{2M}\right) \\ +2\gamma_3 + \gamma_2 - \gamma_5\end{array}}{2} = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - 2q_{1M} - \phi p_{2S} - \phi c_D + \gamma_2 - \gamma_5 + \gamma_4 - \beta\gamma_4 = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= \gamma_5 - \gamma_1 + \phi\,(p_{2S} - c_C) = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\,(\alpha Q - c_R - c_D - 2p_{2S}) + 2\alpha\gamma_7 + \gamma_2 - \gamma_5 - 2\alpha(q_{1M} - q_{2C})}{2\alpha} = 0\,.
\end{aligned}$$

From the first order conditions, $\gamma_1 = \gamma_5 + \phi\,(p_{2S} - c_C) > 0$ and hence $q_{2C} = q_{1M}$. On the other hand, since $q_{1M} - \alpha q_{2S} > 0$, from the first order conditions, we conclude that $2\phi\,(q_{1M} - \alpha q_{2S} - q_{2M} + (1-\beta)\phi(c_C + c_D)/2) = -\gamma_3 + \gamma_2\phi > 0$. Hence when $q_{2M} = 0$, i.e. $\gamma_3 > 0$, $q_{2S} = q_{2C}$. Using these properties, we solve the OEM's problem under no regulation for region C.

The OCs that should be satisfied under each region are given in sections B1.2-B1.2 below. After the redundant OCs are eliminated we rewrite the remaining conditions as bounds on c_M and the bounds on c_M characterizing the optimal policies for the OEM facing competition under partial regulation are provided in Table B2 and B3.

OCs for Partial Regulation: Policy A where $p^*_{2S} < c^R_C$:

Region A$_1$ ($0 = q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M-\phi\beta c_C-\beta\phi c_D}{2} \geq 0$, $q_{2M} = \frac{Q-c_M-\phi\beta c_C-\phi\beta c_D}{2} \geq 0$, and $\gamma_4 = \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Remaining OCs are $\gamma_5 = \phi\left(-\alpha c_M + c_R - c_D - \alpha\beta\phi\left(c_C + c_D\right)\right) \geq 0$ which can be written as $c_M \leq B2_{C,P}$ and $c^R_C - p_{2S} = \frac{2c^R_C-\alpha Q-\alpha c_M+2c_R-\alpha\beta\phi c_C-\alpha\beta\phi c_D}{2} \geq 0$ which can be written as $c_M \leq B3_{C,P}$. Note that $B3_{C,P} - B2_{C,P} = \frac{c_R-\alpha Q+2c^R_C+c_D}{\alpha}$. Hence if $c_R > \alpha Q - 2c^R_C - c_D$ then $c^R_C - p_{2S} > 0$ is redundant. On the other hand, when $c_R < \alpha Q - 2c^R_C - c_D$ holds, $\gamma_5 \geq 0$ is redundant.

Region A$_2$ ($0 < q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M-\phi\beta c_C-\phi\beta c_D}{2} \geq 0$, $q_{2M} = \frac{(2-\alpha)Q-2(c_M+\phi\beta c_C+\phi\beta c_D)+c_R-c_D}{2(2-\alpha)} = q_{1M} - \alpha q_{2S} \geq 0$, and $\gamma_4 = \phi\left(c_C + c_D\right) \geq 0$ are redundant.

Remaining OCs are $q_{2S} = \frac{\alpha c_M+\alpha\beta\phi c_C+\alpha\beta\phi c_D-c_R+c_D}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B2_{C,P}$, $\beta q_{1M} - q_{2S} = \frac{(2-\beta)\beta\alpha Q-(\beta(2-\alpha)+1)\alpha c_M-(\beta(2-\alpha)+1)\alpha\beta\phi c_C-((\beta(2-\alpha)+1)\beta\alpha\phi+1)c_D+c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B5_{C,F}$, and $c^R_C - p_{2S} = \frac{\left(-\alpha Q+2c^R_C+c_R+c_D\right)}{2\alpha} \geq 0$ which can be written as $c_R > \alpha Q - 2c^R_C - c_D$.

Region A$_3$ ($q_{2S} = \beta q_{1M} = q_{2C}, q_{2M} > 0$): In this region, $q_{1M} = \frac{Q-c_M-\beta\phi(c_C+c_D)+\beta\gamma_2}{2} = \frac{Q-cm}{2} - \frac{\beta\gamma_4}{2} \geq 0$ is redundant.

Remaining OCs are $q_{2M} \geq 0$ which can be written as $c_M \leq B6_{C,F}$, $\gamma_2 \geq 0$ which can be written as $c_M \geq B5_{C,F}$, $\gamma_4 \geq 0$ which can be written as $c_M \leq B7_{C,F}$, and $c^R_C - p_{2S} \geq 0$ which can be written as $c_M \leq B8_{C,F}$. As $B8_{C,F} - B7_{C,F} = -\frac{\left(1+2\phi\alpha\beta^2-\phi\alpha^2\beta^2\right)\left(\alpha Q-2c^R_C-c_R+c_C\right)}{\alpha(1+\beta(2-\alpha))}$, if $c_R > \alpha Q - 2c^R_C + c_C$ then $c^R_C - p_{2S} > 0$ is redundant. On the other hand, when $c_R < \alpha Q - 2c^R_C + c_C$ holds, $\gamma_4 \geq 0$ is redundant. Also note that in order for this region to be feasible $B8_{C,F} - B5_{C,F} = -\frac{\left(1+2\phi\alpha\beta^2-\phi\alpha^2\beta^2\right)\left(\alpha Q-2c_C^R-c_R-c_D\right)}{\alpha(1+\beta(2-\alpha))} > 0$ should be satisfied. Therefore this region can be optimal only if $\alpha Q - 2c_C^R - c_D < c_R$ is satisfied.

Region $\mathbf{A}_4$ ($\beta q_{1M} < q_{2S} = q_{2C} < q_{1M}, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M}{2} \geq 0$ and $\gamma_2 = \phi\,(c_C + c_D) \geq 0$ are redundant. In addition, $q_{2S} = \frac{\alpha(c_M+\beta\phi c_C+\beta\phi c_D)-c_C-c_R}{2\alpha(2-\alpha)} \geq 0$ is redundant because $q_{2S} - \beta q_{1M} \geq 0$ holds already.

Remaining OCs are $q_{2M} = \frac{(2-\alpha)Q-2(c_M+\beta\phi c_C+\beta\phi c_D)+c_C+c_R}{2(2-\alpha)} \geq 0$ which can be written as $c_M \leq B9_{C,F}$, $q_{1M} - q_{2S} = \frac{\alpha(2-\alpha)Q-\alpha(3-\alpha)c_M+(1-\alpha\beta\phi)c_C-\alpha\beta\phi c_D+c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B10_{C,F}$, $q_{2S} - \beta q_{1M} = \frac{-\beta\alpha(2-\alpha)Q+\left(\alpha+2\beta\alpha-\beta\alpha^2\right)c_M-(1-\alpha\beta\phi)c_C+\alpha\beta\phi c_D-c_R}{2\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B7_{C,F}$, and $c_C^R - p_{2S} = \frac{\left(-\alpha Q+2c_C^R-c_C+c_R\right)}{2\alpha} \geq 0$ which can be written as $c_R > \alpha Q - 2c_C^R + c_C$.

Region $\mathbf{A}_5$ ($q_{2S} = q_{1M} = q_{2C}, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M+\gamma_1}{2} \geq 0$ and $\gamma_2 = \gamma_1 + (c_C + c_D)\,\phi \geq 0$ are redundant.

Remaining OCs are $c_C^R - p_{2S} \geq 0$ which can be written as $c_M \leq B12_{C,F}$, $q_{2M} \geq 0$ which can be written as $c_M \leq B11_{C,F}$, and $\gamma_1 \geq 0$ which can be written as $c_M \geq B10_{C,F}$. In order for this region to be feasible $B12_{C,F} - B10_{C,F} = -\frac{\left(1+2\phi\alpha-\phi\alpha^2\right)\left(\alpha Q-2c_C^R-c_R+c_C\right)}{\alpha(3-\alpha)} > 0$ and thus $\alpha Q - 2c_C^R + c_C < c_R$ should be satisfied.

Region $\mathbf{A}_6$ ($q_{2S} = \beta q_{1M} = q_{2C}, q_{2M} = 0$): In this region $q_{1M} = \frac{(1+\alpha\beta\phi)Q-c_M-\beta\phi(c_C+c_R)}{2(2\alpha\beta^2\phi+1)} = \frac{Q-c_M-\beta\phi(c_C+c_D)+\beta\gamma_2}{2} = \frac{Q-c_M-\beta\gamma_4}{2} \geq 0$ and $\gamma_2 = \frac{\phi\alpha(1-2\beta)Q+2\alpha\beta\phi(c_M+\phi\beta(c_C+c_D))+\phi(c_D-c_R)}{2\alpha\beta^2\phi+1} = \frac{\gamma_3+\phi(1-\alpha\beta)(Q-c_M-\beta\phi(c_C+c_D))}{\phi\alpha\beta^2} \geq 0$ are redundant.

Remaining OCs are $\gamma_3 \geq 0$ which can be written as $c_M \geq B6_{C,F}$, $\gamma_4 \geq 0$ which can be written as $c_M \leq B16_{C,F}$, and $c_C^R - p_{2S} \geq 0$ which can be written as $c_M \leq B17_{C,F}$. As $B17_{C,F} - B16_{C,F} = -\frac{\left(1+2\phi\alpha\beta^2\right)\left(\alpha Q-2c_C^R-c_R+c_C\right)}{2\alpha\beta}$, if $c_R > \alpha Q - 2c_C^R + c_C$ then $c_C^R - p_{2S} > 0$

is redundant. On the other hand, when $c_R < \alpha Q - 2c_C^R + c_C$ holds, $\gamma_4 \geq 0$ is redundant.

Region $\mathbf{A}_7$ ($\beta q_{1M} < q_{2C} = q_{2S} < q_{1M}, q_{2M} = 0$): In this region, $q_{1M} = \frac{Q-c_M}{2} \geq 0$ is redundant. In addition $q_{2S} = \frac{\alpha Q - c_C - c_R}{4\alpha} \geq$ is redundant because $q_{2S} - \beta q_{1M} \geq 0$ holds already.

Remaining OCs are $\gamma_3 = -\frac{(2-\alpha)Q - 2(c_M + \phi\beta c_C + \phi\beta c_D) + c_C + c_R}{2} \geq 0$ which can be written as $c_M > B9_{C,F}$, $q_{1M} - q_{2C} = \frac{\alpha Q - 2\alpha c_M + c_C + c_R}{4\alpha} \geq 0$ which can be written as $c_M \leq B18_{C,F}$, $q_{2C} - \beta q_{1M} = \frac{\alpha(1-2\beta)Q + 2\alpha\beta c_M - c_C - c_R}{4\alpha} \geq 0$ which can be written as $c_M \geq B16_{C,F}$, and $c_C^R - p_{2S} = \phi \frac{\left(-\alpha Q + 2c_C^R - c_C + c_R\right)}{2}$ which can be written as $c_R > \alpha Q - 2c_C^R + c_C$.

Region $\mathbf{A}_8$ ($q_{2S} = q_{1M} = q_{2C}, q_{2M} = 0$): In this region $q_{1M} = \frac{(1+\alpha\phi)Q - c_M - \phi c_R - \phi c_C}{2(2\alpha\phi+1)} = \frac{Q - c_M + \gamma_1}{2} \geq 0$ and $\gamma_2 = \gamma_1 + (c_C + c_D)\,\phi \geq 0$ are redundant.

Remaining OCs are $\gamma_3 = \phi \frac{-(1-\alpha+\phi\alpha(2-\alpha))Q + (1-\alpha+2\phi\alpha)c_M + (1+2\phi\alpha)\beta\phi(c_C+c_D) - \phi\alpha(c_C+c_R)}{(1+2\phi\alpha)} \geq 0$ which can be written as $c_M \geq B11_{C,F}$, $\gamma_1 = \frac{-\phi(\alpha Q + c_C - 2\alpha c_M + c_R)}{2\alpha\phi+1} \geq 0$ which can be written as $c_M \geq B18_{C,F}$, and $c_C^R - p_{2S} = \frac{(2\alpha\phi+1)c_C^R - \alpha^2\phi Q - \alpha c_M + (1+\alpha\phi)c_R - \alpha\phi c_C}{2\alpha\phi+1} \geq 0$ which can be written as $c_M \leq B19_{C,F}$. In order for this region to be feasible $B19_{C,F} - B18_{C,F} = -\frac{(1+2\phi\alpha)\left(\alpha Q - 2c_C^R - c_R + c_C\right)}{2\alpha} > 0$ and thus $\alpha Q - 2c_C^R + c_C < c_R$ should be satisfied.

OCs for Partial Regulation: Policy B where $p_{2S}^* = c_C^R$:

First we prove that under Policy B, $q_{2C}^* < q_{1M}^*$. Assume that it is optimal to collect and remanufacture all the items, i.e. $q_{2S} = q_{2C} = q_{1M} = \frac{\alpha Q - \alpha q_{2M} - c_R - c_C^R}{2\alpha}$ and $p_{2S} = c_C^R$. Then the profit can be written as follows:

$$\Pi_{OEM}^{B5} = \frac{\begin{array}{l} \alpha^2\left(2\phi\alpha - 1 - 4\phi\right)q_{2M}^2 \\ +\left(\alpha Q - c_R - c_C^R\right)\left(\alpha(Q - 2\phi\left(c_C - c_C^R\right) - 2c_M) + c_R + c_C^R\right) \\ -2\alpha\left(\phi\alpha^2 Q - \alpha c_M - \alpha\phi c_C - \alpha\phi c_R + c_R + c_C^R\right)q_{2M} \\ +4\alpha^2\phi\left(Q - c_M - \beta\phi c_C - \beta\phi c_D\right)q_{2M} \end{array}}{4\alpha^2}$$

If the OEM increases p_{2S} such that the new price is $p_{2S}^{new} = p_{2S} + \delta = c_C^R + \delta$ while keeping q_{2M} the same, the optimal production levels become $q_{2S}^{new} = q_{2C}^{new} = q_{1M}^{new} = \frac{\alpha Q - \alpha q_{2M} - c_R - c_C^R - \delta}{2\alpha}$. Then the OEM's profit is

$$\Pi_{OEM}^{B5^{new}} = \frac{\begin{array}{l} \left(\alpha Q - c_R - c_C^R - \delta\right)\left(\alpha(Q - 2\phi\left(c_C - c_C^R - \delta\right) - 2c_M) + c_R + c_C^R + \delta\right) \\ -2\alpha\left(\phi\alpha^2 Q - \alpha c_M - \alpha\phi c_C - \alpha\phi c_R + c_R + c_C^R + \delta\right)q_{2M} \\ +4\alpha^2\phi\left(Q - c_M - \beta\phi c_C - \beta\phi c_D\right)q_{2M} + \alpha^2\left(2\phi\alpha - 1 - 4\phi\right)q_{2M}^2 \end{array}}{4\alpha^2}.$$

The change in profit $\Pi_{OEM}^{B5^{new}} - \Pi_{OEM}^{B5}$ is

$$\frac{\delta\left(2\phi\alpha^2 Q + 2\alpha c_M - 2(1+\phi\alpha)c_R - 2(1+2\phi\alpha)c_C^R + 2\alpha\phi c_C - 2\alpha q_{2M} - (1+2\phi\alpha)\,\delta\right)}{4\alpha^2}.$$

Note that it is positive as long as $0 \leq \delta \leq \delta_{UB}$ where

$$\delta_{UB} = 2\frac{\phi\alpha^2 Q + \alpha c_M - \alpha q_{2M} - c_R - \phi\alpha(c_R - c_C + 2c_C^R) - c_C^R}{1 + 2\phi\alpha} = \frac{4\alpha^2\gamma_7}{1 + 2\phi\alpha}.$$

Hence $\delta_{UB} \geq 0$ and also $\Pi_{OEM}^{B5^{new}} - \Pi_{OEM}^{B5} \geq 0$ for any $\delta \leq \delta_{UB}$. Finally note that the proof does not depend on the value of q_{2M} and also goes through when $q_{2M} = 0$.

Region B$_1$ **(**$0 = q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$**):** In this region, $q_{1M} = \frac{Q - c_M - \phi\beta c_C - \phi\beta c_D}{2} \geq 0$ and $\gamma_4 = (c_C + c_D)\,\phi \geq 0$ are redundant.

Remaining OCs are $\gamma_5 = \frac{-\alpha(2-\alpha)Q + (4-\alpha)c_R + 2(2-\alpha)c_C^R - 2\alpha c_M - \alpha(2\beta\phi+1)c_D - 2\alpha\beta\phi c_C}{\alpha}$ which can be written as $c_M \leq B21_{C,P}$ and $\gamma_7 = \phi\frac{-2c_C^R + \alpha Q + \alpha c_M - 2c_R + \alpha\beta\phi c_C + \alpha\beta\phi c_D}{\alpha^2} \geq 0$ which can be written as $c_M \geq B3_{C,P}$. In order for this region to be feasible $B21_{C,P} - B3_{C,P} = \frac{\alpha Q - 2c_C^R - c_R - c_D}{2} > 0$ and thus $c_R < \alpha Q - c_D - 2c_C^R$ should be satisfied. Hence $q_{2M} = \frac{\alpha Q - c_C^R - c_R}{\alpha} \geq 0$ is also redundant.

Region B$_2$ **(**$0 < q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$**):** In this region, $q_{1M} = \frac{Q - c_M - \phi\beta(c_C + c_D)}{2} \geq 0$ and $q_{2M} = \frac{(2-\alpha)Q + c_R - c_D - 2c_M - 2\beta\phi c_C - 2\beta\phi c_D}{2(2-\alpha)} = \frac{(2q_{1M}\phi - 2q_{2S}\alpha\phi + \gamma_7\alpha)}{2\phi} \geq 0$ are redundant.

Remaining OCs are $q_{2S} = \frac{2(2-\alpha)Q - (4-\alpha)c_R - 2(2-\alpha)c_C^R + 2\alpha c_M + 2\alpha\beta\phi c_C + \alpha(1+2\beta\phi)c_D}{4\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \geq B21_{C,P}$, $\gamma_7 = \frac{\phi\left(\alpha Q - 2c_C^R - c_R - c_D\right)}{2\alpha} \geq 0$ which can be written as $c_R < \alpha Q - c_D - 2c_C^R$, and $\beta q_{1M} - q_{2S} \geq 0$ which can be written as $c_M \leq B23_{C,F}$.

Region B$_3$ **(**$q_{2S} = \beta q_{1M} = q_{2C}, q_{2M} > 0$**):** In this region, $q_{1M} = \frac{Q - c_M - \beta\phi(c_C + c_D) + \beta\gamma_2}{2} \geq 0$ is redundant.

Remaining OCs are $\gamma_2 \geq 0$ which can be written as $c_M \geq B23_{C,F}$, $q_{2M} \geq 0$ which can be written as $c_M \leq B24_{C,F}$, $\gamma_4 \geq 0$ which can be written as $c_M \leq B25_{C,F}$, and $\gamma_7 \geq 0$ which can be written as $c_M \geq B8_{C,F}$. As $B23_{C,F} - B8_{C,F} = \frac{\left(1+2\phi\beta^2(2-\alpha)\right)\left(\alpha Q - 2c_C^R - c_R - c_D\right)}{1+\beta(2-\alpha)}$, if $c_R < \alpha Q - 2c_C^R - c_D$ then $\gamma_7 \geq 0$ is redundant. On the other hand, when $c_R > \alpha Q - 2c_C^R - c_D$ holds, $\gamma_2 \geq 0$ is redundant. Also in order for this region to be feasible $B25_{C,F} - B8_{C,F} = \frac{\left(1+2\phi\beta^2(2-\alpha)\right)\left(\alpha Q - 2c_C^R - c_R + c_C\right)}{1+\beta(2-\alpha)} > 0$ and thus $c_R < \alpha Q + c_C - 2c_C^R$ should be satisfied.

Region B$_4$ ($\beta q_{1M} < q_{2C} = q_{2S}, q_{2M} > 0$): In this region $q_{1M} = \frac{Q-c_M}{2}$, $\gamma_2 = \phi\left(c_C + c_D\right)$, and $q_{2S} = \frac{(2-\alpha)\alpha Q-(4-\alpha)c_R-2(2-\alpha)c_C^R+2\alpha c_M-\alpha(1-2\phi\beta)c_C+2\alpha\phi\beta c_D}{4\alpha(2-\alpha)} \geq 0$ are redundant.

Remaining OCs are $q_{2M} = \frac{(2-\alpha)Q+c_R+c_C-2c_M-2\beta\phi c_C-2\beta\phi c_D}{2(2-\alpha)} \geq 0$ which can be written as $c_M \leq B9_{C,F}$, $q_{1M} - q_{2C} = \frac{Q\alpha(2-\alpha)+(4-\alpha)c_R+2(2-\alpha)c_C^R-2\alpha(3-\alpha)c_M+\alpha(1-2\beta\phi)c_C-2\alpha\beta\phi c_D}{4\alpha(2-\alpha)} \geq 0$ which can be written as $c_M \leq B27_{C,F}$, $q_{2C} - \beta q_{1M} \geq 0$ which can be written as $c_M \geq B25_{C,F}$, and $\gamma_7 = \phi\frac{\left(\alpha Q-2c_C^R+c_C-c_R\right)}{2\alpha} \geq 0$ which can be written as $c_R < \alpha Q - 2c_C^R + c_C$.

Region B$_6$ ($q_{2S} = \beta q_{1M} = q_{2C}, q_{2M} = 0$): Under this region, $q_{1M} = \frac{\alpha Q-c_R-c_C^R}{2\beta\alpha} \geq 0$ is redundant because this region can be optimal only if $c_R < \alpha Q + c_C - 2c_C^R$ which guarantees that q_{1M} is positive. In addition, $\gamma_2 = \frac{(1-\beta)\alpha Q+\beta^2\alpha\phi(c_C+c_D)-c_C^R-c_R+\beta\alpha c_M}{\alpha\beta^2} = \frac{\gamma_3+\alpha\gamma_7+2\phi(1-\alpha\beta)q_{1M}}{\beta\phi} \geq 0$ is also redundant.

Remaining OCs are $\gamma_7 = \frac{\left(\alpha+\alpha^2\beta^2\phi-\alpha\beta\right)Q+\alpha\beta c_M-\left(1+2\alpha\beta^2\phi\right)c_C^R-\left(1+\alpha\beta^2\phi\right)c_R+\alpha\beta^2\phi c_C}{2\alpha^2\beta^2} \geq 0$ which can be written as $c_M \geq B17_{C,F}$, $\gamma_4 = \frac{-(1-\beta)\alpha Q-\beta\alpha c_M+c_R+c_C^R}{\alpha\beta^2} \geq 0$ which can be written as $c_M \leq B28_{C,F}$, and $\gamma_3 \geq 0$ which can be written as $c_M \geq B24_{C,F}$. Also in order for this region to be feasible $B28_{C,F} - B17_{C,F} = \phi\beta(\alpha Q - 2c_C^R - c_R + c_C) > 0$ and thus $c_R < \alpha Q + c_C - 2c_C^R$ should be satisfied.

Region B$_7$ ($\beta q_{1M} < q_{2C} = q_{2S}, q_{2M} = 0$) In this region, $q_{1M} = \frac{Q-c_M}{2} \geq 0$ and $q_{2S} = \frac{\alpha Q-c_R-c_C^R}{2\alpha} \geq 0$ are redundant.

Remaining OCs are $\gamma_3 = \frac{-(2-\alpha)Q-c_R-c_C+2c_M+2\beta\phi c_C+2\beta\phi c_D}{2} \geq 0$ which can be written as $c_M \geq B9_{C,F}$, $q_{1M} - q_{2C} = \frac{-\alpha c_M+c_R+c_C^R}{2\alpha}$ which can be written as $c_M \leq B29_{C,F}$, $\gamma_7 = \phi\frac{\left(\alpha Q-2c_C^R+c_C-c_R\right)}{2\alpha}$ which can be written as $c_R < \alpha Q - 2c_C^R + c_C$, and $q_{2C} - \beta q_{1M} = \frac{(1-\beta)\alpha Q+\beta\alpha c_M-c_R-c_C^R}{2\alpha} \geq 0$ which can be written as $c_M \geq B28_{C,F}$.

OCs for Partial Regulation: Policy C where $p_{2S}^{*} > c_C^R$:

In this region $q_{2C} = q_{1M}$ by Proposition 4. The remaining regions and the corresponding OCs are provided below:

Region C$_1$ $(0 = q_{2S} < q_{2C} = q_{1M}, q_{2M} > 0)$: In this region, $q_{1M} = \frac{Q-c_M-\phi c_D-\phi c_C}{2} \geq 0$ is redundant because it is satisfied as long as $\gamma_1 > 0$ is satisfied. In addition, $q_{2M} = \frac{Q-c_M-\beta\phi(c_D+c_C)}{2} \geq 0$ and $\gamma_1 = \phi\frac{(\alpha Q-\alpha c_M-(2+\alpha\beta\phi)(c_C+c_D))}{2} = \phi\left(p_{2S}-c_C\right) > \phi\left(p_{2S}-c_C^R\right) \geq 0$ are also redundant.

Remaining OCs are $\gamma_4 = \phi\left(c_R - c_D - \alpha c_M - \alpha\beta\phi\left(c_C + c_D\right)\right) \geq 0$ which can be written as $c_M \leq B2_{C,P}$ and $p_{2S} - c_C^R = \frac{\alpha Q+\alpha c_M+\alpha\beta\phi(c_C+c_D)-2c_R-2c_C^R}{2} \geq 0$ which can be written as $c_M \geq B3_{C,P}$. In order for this region to be feasible $B2_{C,P} - B3_{C,P} = \frac{\left(\alpha Q-2c_C^R-c_R-c_D\right)}{\alpha} > 0$ and thus $\alpha Q - 2c_C^R - c_D > c_R$ should be satisfied.

Region C$_2$ $(0 < q_{2S} < q_{2C} = q_{1M}, q_{2M} > 0)$: In this region, $q_{1M} = \frac{Q-c_M-\phi(c_D+c_C)}{2}$ is redundant because $q_{1M} - q_{2S} \geq 0$ holds. In addition, $q_{2M} = \frac{(2-\alpha)Q-2c_M-(1+2\beta\phi)c_D-2\beta\phi c_C+c_R}{2(2-\alpha)} = q_{1M} - \alpha q_{2S} + (1-\beta)\phi(c_C + c_D)/2 \geq 0$ and $\gamma_1 = \frac{\phi(\alpha Q-c_D-2c_C-c_R)}{2} = \phi\left(p_{2S} - c_C\right) > \phi\left(p_{2S} - c_C^R\right) \geq 0$ are also redundant

Remaining OCs are $q_{2S} = \frac{\alpha c_M+(1+\alpha\beta\phi)c_D+\alpha\beta\phi c_C-c_R}{2\alpha(2-\alpha)}$ which can be written as $c_M \geq B2_{C,P}$ and $q_{1M} - q_{2S} = \frac{\left(2\alpha-\alpha^2\right)Q+\left(\alpha^2-3\alpha\right)c_M+\left(\phi\alpha^2-2\phi\alpha-\alpha\beta\phi-1\right)c_D+\left(\phi\alpha^2-2\phi\alpha-\alpha\beta\phi\right)c_C+c_R}{2\alpha(2-\alpha)}$ which can be written as $c_M \leq B33_{C,F}$. Besides, $p_{2S} - c_C^R = \frac{\alpha Q-c_R-c_D-2c_C^R}{2} > 0$ is satisfied when $\alpha Q - c_D - 2c_C^R > c_R$.

Region C$_5$ $(q_{2S} = q_{2C} = q_{1M}, q_{2M} > 0)$: In this region, $q_{1M} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_M} < 0$ and $\frac{\partial q_{1M}}{\partial c_R} < 0$ and $q_{1M}(c_M = c_M^{max}, c_R = c_R^{max}) = \frac{\phi\beta(c_C+c_D)}{2(1+2\phi\alpha-\phi\alpha^2)} > 0$. Besides, $\gamma_1 \geq 0$ as long as $p_{2S} - c_C^R > 0$ is satisfied.

Remaining OCs are $q_{2M} = \frac{(1-\alpha+\phi\alpha(2-\alpha))Q-(1+2\phi\alpha)(c_M+(c_C+c_D))+\alpha(c_M+\phi(c_C+c_R))}{2(1+2\phi\alpha-\phi\alpha^2)} \geq 0$ which can be written as $c_M \leq B11_{C,F}$, $\gamma_2 = \frac{(-2\alpha+\alpha^2)\phi Q-\phi c_R+(3\alpha-\alpha^2)\phi c_M+(2\alpha-\alpha^2+\alpha\beta)\phi^2(c_C+c_D)+\phi c_D}{1+2\phi\alpha-\phi\alpha^2} \geq 0$ which can be written as $c_M \geq B33_{C,F}$, and finally $p_{2S} - c_C^R \geq 0$ which can be written as $c_M \geq B12_{C,F}$.

Region C$_8$ **(**$q_{2S} = q_{2C} = q_{1M}, q_{2M} = 0$**):** Under this region, $q_{1M} = \frac{(1+\phi\alpha)Q-c_M-\phi(c_C+c_R)}{2(2\phi\alpha+1)} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_M} < 0$ and $\frac{\partial q_{1M}}{\partial c_R} < 0$ and $q_{1M}(c_M = c_M^{max}, c_R = c_R^{max}) = \frac{\phi\beta(c_C+c_D)}{2(2\phi\alpha+1)} > 0$. In addition, $\gamma_2 = \frac{\phi(-\alpha Q+2\alpha c_M+(1+2\alpha\phi)c_D-c_R+2\alpha\phi c_C)}{(1+2\phi\alpha)} = 2(1-\alpha)q_{1M} + \gamma_3/\phi + (1-\beta)(c_C+c_D)\phi \geq 0$ is redundant and $\gamma_1 = \frac{\phi^2\alpha^2 Q+\phi\alpha c_M-\phi(\phi\alpha+1)c_C-\phi(\phi\alpha+1)c_R}{(1+2\phi\alpha)} \geq 0$ as long as $p_{2S} - c_C^R \geq 0$ is satisfied.

Remaining OCs are $\gamma_3 \geq 0$ which can be written as $c_M \geq B11_{C,F}$ and $p_{2S} - c_C^R = \frac{\phi\alpha^2 Q+\alpha c_M+\alpha\phi c_C-(1+\alpha\phi)c_R-(1+2\phi\alpha)c_C^R}{(1+2\phi\alpha)} \geq 0$ which can be written as $c_M \geq B19_{C,F}$.

Bound	**Expression**
$B2_{C,P}$	$\frac{c_R-c_D-\alpha\beta\phi(c_C+c_D)}{\alpha}$
$B3_{C,P}$	$\frac{2c_C^R-\alpha Q+2c_R-\alpha\beta\phi c_C-\alpha\beta\phi c_D}{\alpha}$
$B5_{C,F}$	$\frac{(2-\alpha)\beta\alpha Q-(\beta(2-\alpha)+1)\beta\alpha\phi c_C-\left((2-\alpha)\beta^2\alpha\phi+\beta\alpha\phi+1\right)c_D+c_R}{(\beta(2-\alpha)+1)\alpha}$
$B6_{C,F}$	$\frac{-\left(\beta\alpha-1-2\beta^2\phi\alpha+\beta^2\phi\alpha^2\right)Q-\left(1-\beta\alpha+2\beta^2\phi\alpha\right)\beta\phi c_C-\left(1+2\beta^2\phi\alpha\right)\beta\phi c_D+\beta^2\phi\alpha c_R}{(1+2\beta^2\phi\alpha-\beta\alpha)}$
$B7_{C,F}$	$\frac{\phi\beta\alpha(2-\alpha)Q+\phi(1-\beta\alpha\phi)c_C-\phi^2\beta\alpha c_D+\phi c_R}{\phi\alpha(1+\beta(2-\alpha))}$
$B8_{C,F}$	$\frac{(-1+\beta(1-\phi\beta\alpha)(2-\alpha))\alpha Q-(1+\beta(2-\alpha))\phi\alpha\beta c_C-\beta\alpha\phi c_D+\left(2+\phi\beta^2\alpha(2-\alpha)\right)c_R+2\left(1+\phi\beta^2\alpha(2-\alpha)\right)c_C^R}{(1+\beta(2-\alpha))\alpha}$
$B9_{C,F}$	$\frac{(2-\alpha)Q-2\beta\phi(c_C+c_D)+c_C+c_R}{2}$
$B10_{C,F}$	$\frac{\alpha(2-\alpha)Q+(1-\alpha\beta\phi)c_C-\alpha\beta\phi c_D+c_R}{\alpha(3-\alpha)}$
$B11_{C,F}$	$\frac{(1-\alpha+\phi\alpha(2-\alpha))Q-(1+2\phi\alpha)\beta\phi(c_C+c_D)+\phi\alpha(c_C+c_R)}{(1-\alpha+2\phi\alpha)}$
$B12_{C,F}$	$\frac{(1-\alpha-\phi\alpha(2-\alpha))\alpha Q+(2+\phi\alpha(2-\alpha))c_R+2(1+\phi\alpha(2-\alpha))c_C^R-(\phi(2-\alpha)+\beta\phi)\alpha c_C-\alpha\beta\phi c_D}{(3-\alpha)\alpha}$
$B16_{C,F}$	$\frac{-\alpha(1-2\beta)Q+c_R+c_C}{2\alpha\beta}$
$B17_{C,F}$	$\frac{-\left(\alpha^2\beta^2\phi+\alpha-\alpha\beta\right)Q-\alpha\beta^2\phi c_C+\left(1+\alpha\beta^2\phi\right)c_R+\left(1+2\alpha\beta^2\phi\right)c_C^R}{\beta\alpha}$
$B18_{C,F}$	$\frac{\alpha Q+c_C+c_R}{2\alpha}$
$B19_{C,F}$	$\frac{(2\alpha\phi+1)c_C^R-\alpha^2\phi Q+(1+\alpha\phi)c_R-\alpha\phi c_C}{\alpha}$

TABLE B2: Bounds characterizing the optimal policies for the OEM facing competition under partial regulation (where $p_{2S} < c_C^R$)

Bound	**Expression**
$B21_{C,P}$	$\frac{-\alpha(2-\alpha)Q+(4-\alpha)c_R+2(2-\alpha)c_C^R-\alpha(2\beta\phi+1)c_D-2\alpha\beta\phi c_C}{2\alpha}$
$B23_{C,F}$	$\frac{\alpha(2\beta-1)(2-\alpha)Q+(4-\alpha)c_R+2(2-\alpha)c_C^R-2\alpha\beta\phi(1+(2-\alpha)\beta)c_C-\alpha\left(1+2\beta\phi+\beta^2\phi 2(2-\alpha)\right)c_D}{2\alpha(1+(2-\alpha)\beta)}$
$B24_{C,F}$	$\frac{-\alpha\left(1-\beta+\phi\beta^2(2-\alpha)\right)Q+c_C^R+\left(1-\phi\beta^2\alpha\right)c_R+2\phi^2\beta^3\alpha c_D-\phi\alpha\beta^2(1-2\beta\phi)c_C}{\beta\alpha(2\phi\beta-1)}$
$B25_{C,F}$	$\frac{\alpha(2\beta-1)(2-\alpha)Q+(4-\alpha)c_R+2(2-\alpha)c_C^R+\alpha(1-2\beta\phi)c_C-2\alpha\beta\phi c_D}{2\alpha(1+(2-\alpha)\beta)}$
$B27_{C,F}$	$\frac{Q\alpha(2-\alpha)+(4-\alpha)c_R+2(2-\alpha)c_C^R+\alpha(1-2\beta\phi)c_C-2\alpha\beta\phi c_D}{2\alpha(3-\alpha)}$
$B28_{C,F}$	$\frac{-(1-\beta)\alpha Q+c_R+c_C^R}{\beta\alpha}$
$B29_{C,F}$	$\frac{c_R+c_C^R}{\alpha}$
$B33_{C,F}$	$\frac{\left(2\alpha-\alpha^2\right)Q+\left(\phi\alpha^2-2\phi\alpha-\alpha\beta\phi-1\right)c_D+\left(\phi\alpha^2-2\phi\alpha-\alpha\beta\phi\right)c_C+c_R}{\alpha(3-\alpha)}$

TABLE B3: Bounds characterizing the optimal policies for the OEM facing competition under partial regulation

Full regulation: Problem Formulation

Given the remanufacturer plays *R4* (see Table A5), the OEM's problem is the same as the one under partial regulation with the additional constraint that $q_{2S} + q_{2C}^R \geq \beta_R q_{1M}$.

Then the Lagrangian function is $\Pi_{OEM}^{Lagr} = \Pi_{OEM} + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2S}) + \gamma_3 q_{2M} + \gamma_4(q_{2C}^R + q_{2C} - \beta q_{1M}) + \gamma_5(q_{2C}^R + q_{2S} - \beta_R q_{1M}) + \gamma_7(c_C^R - p_{2S})$. Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$
\begin{aligned}
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q - c_M - \gamma_5 \beta_R - \gamma_4 \beta + \gamma_1 - 2q_{1M} = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\begin{array}{c}[\phi((2-\alpha)\,Q - 2c_M - 2\beta\phi(c_C + c_D)) \\ +\phi(c_R - c_D - 2\,(2-\alpha)\,q_{2M}) + 2\gamma_3 + \gamma_2 - \gamma_5]\end{array}}{2} = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= \gamma_2 - \gamma_1 + \gamma_4 - \phi\,(c_C + c_D) = 0\,, \\
\frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\,(\alpha Q - c_R - c_D - 2p_{2S}) - 2\alpha\gamma_7 + \gamma_2 - \gamma_5}{2\alpha} = 0\,.
\end{aligned}
$$

The Hessian is negative semidefinite, so first order conditions ensure optimality. From the first order conditions, $q_{2C} = \max(q_{2S}, \beta q_{1M})$. Following the same arguments as under partial regulation, we can show that $q_{1M} > q_{2C} > q_{2R}$ cannot be optimal if there is no second period manufacturing. Using these properties we solve the OEM's problem under full regulation for each region. As before, when $c_C^R > p_{2S}^*$ we obtain Policy A solutions whereas when the constraint is tight, i.e. $c_C^R = p_{2S}^*$ we obtain Policy B solutions.

Given the remanufacturer plays *R5* (see Table A5), the OEM's problem is the same as the one under partial regulation with the additional constraint on the lower bound of remanufacturing level, i.e. $q_{2S} + q_{2C}^R \geq \beta_R q_{1M}$. Then the Lagrangian function is

$$\begin{aligned}\Pi_{OEM}^{Lagr} &= \Pi_{OEM} + \gamma_1(q_{1M} - q_{2C}) + \gamma_2(q_{2C} - q_{2S}) + \gamma_3 q_{2M} + \gamma_4(q_{2C}^R + q_{2C} - \beta q_{1M}) \\ &\quad + \gamma_5(q_{2C}^R + q_{2S} - \beta_R q_{1M}) + \gamma_7(p_{2S} - c_C^R)\,. \qquad \text{(B-3)}\end{aligned}$$

Plugging in q_{2C}^R and q_{2S} in the Lagrangian function and taking the first order derivatives, we obtain the following OCs:

$$\begin{aligned}\frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2M}} &= \frac{\begin{array}{c}\phi\left(\left(2-\alpha\right)Q - 2c_M - 2\phi\beta(c_C + c_D)\right) \\ +\phi\left(c_R - c_D - 2\left(2-\alpha\right)q_{2M}\right) + 2\gamma_3 + \gamma_2 - \gamma_5\end{array}}{2} = 0\,, \\ \frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{1M}} &= Q - c_M + \gamma_1 - 2q_{1M} - \phi p_{2S} - \phi c_D + \gamma_2 - \beta_R\gamma_5 + \gamma_4 - \beta\gamma_4 = 0\,, \qquad \text{(B-4)} \\ \frac{\partial \Pi_{OEM}^{Lagr}}{\partial q_{2C}} &= -\gamma_1 + \phi\left(p_{2S} - c_C\right) = 0\,, \\ \frac{\partial \Pi_{OEM}^{Lagr}}{\partial p_{2S}} &= \frac{\phi\left(\alpha Q - c_R - c_D - 2p_{2S}\right) + 2\alpha\gamma_7 + \gamma_2 - \gamma_5 - 2\alpha(q_{1M} - q_{2C})}{2\alpha} = 0 \qquad \text{(B-5)}\end{aligned}$$

The new bounds on c_M characterizing the optimal policies for the OEM facing competition under full regulation are provided in Table B4. Under full regulation the regions 3-8 are the same as the corresponding ones under partial regulation. Solving the OEM's problem under full regulation for regions 1 and 2 the following OCs are obtained.

OCs for Full Regulation: Policy A where $p_{2S}^* < c_C^R$:

Region A$_1$ ($\beta_R q_{1M} = q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): In this region, $q_{1M} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_M} < 0$ and $\frac{\partial q_{1M}}{\partial c_R} < 0$ and $q_{1M}(c_M = c_M^{max}, c_R = c_R^{max}) = \frac{\phi\beta_R(c_C+c_D)}{2\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)} > 0$. In addition, $q_{2M} = (1-\beta_R\alpha)\, q_{1M} + \beta_R\gamma_5/2 \geq 0$ is also redundant.

Remaining OCs are $\gamma_5 = \frac{(\beta_R\alpha(2-\alpha))\phi Q+\phi(c_R-c_D)-((2-\alpha)\beta_R+1)\alpha\phi(c_M+\phi\beta(c_C+c_D))}{\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)} \geq 0$ which can be written as $c_M \leq B2_{C,F}$ and $c_C^R - p_{2S} \geq 0$ which can be written as $c_M \leq B3_{C,F}$. As $B3_{C,F} - B2_{C,F} = \frac{\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)\left(c_R-\alpha Q+2c_C^R+c_D\right)}{\alpha(1+2\beta_R-\beta_R\alpha)}$, if $c_R > \alpha Q - 2c_C^R - c_D$ then $c_C^R - p_{2S} > 0$ is redundant. On the other hand, when $c_R < \alpha Q - 2c_C^R - c_D$, $\gamma_5 \geq 0$ is redundant.

Region A$_2$ ($\beta_R q_{1M} < q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): The OCs are the same as the ones in Region 2 under partial regulation except $q_{2S} \geq 0$. Under full regulation $q_{2S} - \beta_R q_{1M} \geq 0$ can be written as $c_M \geq B2_{C,F}$.

OCs for Full Regulation: Policy B where $p_{2S}^* = c_C^R$:

Region B$_1$ ($\beta_R q_{1M} = q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): In this region, $q_{1M} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_R} < 0$ and q_{1M} is positive at both extreme points, i.e. $q_{1M}(c_M = B21_{C,F}, c_R = \alpha Q - c_D - 2c_C^R) = q_{1M}(c_M = B3_{C,F}, c_R = \alpha Q - c_D - 2c_C^R) = \frac{(c_C^R+c_D)}{\alpha(1+2\beta_R-\alpha\beta_R)} > 0$. In addition $q_{2M} = (1-\beta_R\alpha)\, q_{1M} + \beta_R\gamma_5/2 + \alpha\gamma_7/\,(2\phi) \geq 0$ is also redundant.

Remaining OCs are $\gamma_5 = \frac{2\alpha(1+\beta_R(2-\alpha))(Q-c_M-\beta\phi(c_C+c_D))-(4-\alpha)(\alpha Q-c_R-c_D)-4c_D+2(2-\alpha)c_C}{\left(1+2\phi\beta_R^2(2-\alpha)\right)\alpha} \geq 0$ which can be written as $c_M \leq B21_{C,F}$ and $\gamma_7 \geq 0$ which can be written as $c_M \geq B3_{C,F}$.

In order for this region to be feasible $B21_{C,F} - B3_{C,F} = \frac{\left((1+2\phi\beta_R^2(2-\alpha))\right)\left(\alpha Q-2c_C^R-c_R-c_D\right)}{1+\beta_R(2-\alpha)} > 0$ and thus $c_R < \alpha Q - c_D - 2c_C^R$ should be satisfied.

Region B$_2$ ($\beta_R q_{1M} < q_{2S} < \beta q_{1M} = q_{2C}, q_{2M} > 0$): The OCs are the same as the ones in Region 2 under Partial regulation except $q_{2S} > 0$. The new OC $q_{2S} - \beta_R q_{1M} \geq 0$ can be written as $c_M \geq B21_{C,F}$.

OCs for Full Regulation: Policy C where $p_{2S}^* > c_C^R$:

In this region $q_{2C} = q_{1M}$ by Proposition 4. The remaining regions and the corresponding OCs are provided below:

Region C$_1$ ($q_{2S} = \beta_R q_{1M} < q_{2C} = q_{1M}, q_{2M} > 0$): In this region, $q_{1M} \geq 0$ is redundant because $\frac{\partial q_{1M}}{\partial c_M} < 0$ and $\frac{\partial q_{1M}}{\partial c_R} < 0$ and $q_{1M}(c_M = B30_{C,F}, c_R = c_R^{max}) = \frac{(1-\alpha\phi+\phi\beta\alpha)(c_C+c_D)}{2\alpha(1+2\beta_R-\alpha\beta_R)} > 0$. In addition, $q_{2M} = (1-\beta_R\alpha)q_{1M} + \beta_R\gamma_5/2 + (c_D + c_C)(1-\beta)\phi/2 \geq 0$ and $\gamma_1 = \phi\left(p_{2S} - c_C\right) > \phi\left(p_{2S} - c_C^R\right) \geq 0$ are also redundant.

Remaining OCs are $\gamma_5 \geq 0$ which can be written as $c_M \leq B30_{C,F}$ and $p_{2S} - c_C^R \geq 0$ which can be written as $c_M \geq B31_{C,F}$. In order for this region to be feasible $B30_{C,F} - B31_{C,F} = \frac{\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)\left(\alpha Q - 2c_C^R - c_R - c_D\right)}{\alpha(1+2\beta_R-\beta_R\alpha)} > 0$ and thus $\alpha Q - 2c_C^R - c_D > c_R$ should be satisfied.

Region C$_2$: ($\beta_R q_{1M} < q_{2S} < q_{2C} = q_{1M}, q_{2M} > 0$): The OCs are the same as the ones in Region 2 under Partial regulation except $q_{2S} > 0$. The condition on q_{2S}, $q_{2S} - \beta_R q_{1M} \geq 0$ can be written as $c_M \geq B30_{C,F}$.

Comparison of Profits from Policy A and Policy C:

Feasible regions of Policy A and Policy C may overlap. If this happens the profit from Policy A is always higher. Consider the problem formulation for Policy A. When region A is feasible $p_{2S} < c_C^R$ holds. On the other hand, when Policy C is feasible $p_{2S} > c_C^R$ holds. Compare the two problem formulations ignoring the constraint on p_{2S}, because we know that p_{2S} will satisfy the constraint in the feasible region anyway. Then the formulation of Policy C is same as the formulation of Policy A with the exception of an additional constraint which is $q_{2C} = q_{1M}$. Hence profit cannot be higher under Policy C.

Comparison of Profits from Policy B and Policy C:

As long as C_5 or C_8 is feasible, the profit from that region will be higher than the profit under any other possible feasible region (a region of Policy B) at the same point. Hence we compare the profits from regions B_1, B_2, B_3, B_4, B_6, and B_7 with the profit from regions C_1 and C_2:

Under Policy B, the optimal production levels can be calculated in terms of the Lagrangian multipliers as follows:

$$
\begin{aligned}
q_{1M}^B &= \frac{Q - c_M - \phi\beta c_D - \phi\beta c_C + \beta\gamma_2^B - \beta_R\gamma_5^B}{2}, \\
q_{2M}^B &= \frac{2\phi Q - \phi\alpha Q + \phi c_R - 2\phi c_M - \phi c_D + 2\gamma_3^B - \gamma_5^B + \gamma_2^B - 2\phi^2\beta c_D - 2\phi^2\beta c_C}{2\phi\,(2-\alpha)}, \\
q_{2S}^B &= \frac{\begin{array}{l}\phi\alpha Q\,(2-\alpha) + 2\phi\alpha(\phi\beta c_C + \phi\beta c_D + c_M) + \phi\alpha(c_R + c_D + 2c_C^R) \\ -\gamma_2^B\alpha + \gamma_5^B\alpha - 2\gamma_3^B\alpha - 4\phi(c_R + c_C^R)\end{array}}{4\alpha\phi\,(2-\alpha)}.
\end{aligned}
$$

Note that $q_{2C}^B = \max\left(q_{2S}^B, \beta q_{1M}^B\right)$. When $q_{2C}^B = \max\left(q_{2S}^B, \beta q_{1M}^B\right) = q_{2S}^B$ we can easily calculate

$$
\Pi_{OEM}^{B^{unc}} = \Pi_{OEM}\left(q_{1M} = q_{1M}^B, q_{2M} = q_{2M}^B, q_{2S} = q_{2S}^B, q_{2C} = q_{2S}^B, p_{2S} = c_C^R\right).
$$

On the other hand, when $q_{2C}^B = \max\left(q_{2S}^B, \beta q_{1M}^B\right) = \beta q_{1M}^B$, the profit can be written as

$$\begin{aligned}
\Pi_{OEM}^{B^{cons}} &= \Pi_{OEM}\left(q_{1M} = q_{1M}^B, q_{2M} = q_{2M}^B, q_{2S} = q_{2S}^B, q_{2C} = \beta q_{1M}^B, p_{2S} = c_C^R\right) \\
&= \Pi_{OEM}^{B^{unc}} - \phi\left(c_C + c_D\right)\left(\beta q_{1M}^B - q_{2S}^B\right).
\end{aligned}$$

Hence, in general $\Pi_{OEM}^B = \Pi_{OEM}^{B^{unc}} - \phi\left(c_C + c_D\right)\left(\beta q_{1M}^B - q_{2S}^B\right)^+$. Let us define $\Pi_{OEM}^{B^R} = \Pi_{OEM}^{B^{cons}}(\gamma_2^B = 0, \gamma_3^B = 0, \gamma_5^B = 0)$. Under Policy C, we know that $p_{2S} = c_C^R + \delta$ where $\delta > 0$. Then the optimal amount of production quantities can be written in terms of the Lagrangian multipliers as follows:

$$\begin{aligned}
q_{1M}^C &= \frac{Q - c_M - \phi c_C - \phi c_D - \beta_R \gamma_5^C}{2}, \\
q_{2M}^C &= \frac{2\phi Q - \phi\alpha Q + \phi c_R - 2\phi c_M - \phi c_D + 2\gamma_3^C - \gamma_5^C - 2\phi^2\beta c_D - 2\phi^2\beta c_C}{2\phi\left(2-\alpha\right)}, \\
q_{2S}^C &= \frac{\phi\alpha c_M + \gamma_5^C + \phi^2\alpha\beta c_D - \gamma_3^C\alpha - \phi c_R + \phi c_D + \phi^2\alpha\beta c_C}{2\alpha\phi\left(2-\alpha\right)}, \\
\delta &= \frac{\phi\alpha Q - \phi c_D - \phi c_R - 2\phi c_C^R - \gamma_5^C}{2\phi}.
\end{aligned}$$

Under Policy C all the available cores are collected, i.e. $q_{2C} = q_{1M}$, hence we can calculate

$$\Pi_{OEM}^C = \Pi\left(q_{1M} = q_{1M}^C, q_{2M} = q_{2M}^C, q_{2S} = q_{2S}^C, q_{2C} = q_{1M}^C, p_{2S} = c_C^R + \delta\right).$$

Let us define $\Pi_{OEM}^{C^R} = \Pi_{OEM}^C\left(\gamma_3^C = 0, \gamma_5^C = 0\right)$. Then we can calculate the difference as follows:

$$\Pi^B_{OEM} - \Pi^C_{OEM} = \begin{array}{l} \frac{{\gamma_3^C}^2 - {\gamma_3^B}^2}{2\phi(2-\alpha)} - \frac{\gamma_5^C \gamma_3^C}{2\phi(2-\alpha)} + \frac{\left(1+\phi\alpha\beta_R^2(2-\alpha)\right){\gamma_5^C}^2}{4\alpha\phi(2-\alpha)} - \frac{\left(1+2\phi\beta^2(2-\alpha)\right){\gamma_2^B}^2}{8\phi(2-\alpha)} \\ + \left(-\frac{\gamma_3^B}{2\phi(2-\alpha)} + \frac{(1+2\phi\beta\beta_R(2-\alpha))\gamma_5^B}{4\phi(2-\alpha)}\right)\gamma_2^B + \frac{\gamma_5^B \gamma_3^B}{2\phi(2-\alpha)} - \frac{\left(1+2\phi\beta_R^2(2-\alpha)\right){\gamma_5^B}^2}{8\phi(2-\alpha)} \\ + \left(\Pi^{B^R}_{OEM} - \Pi^{C^R}_{OEM}\right) + \phi\left(c_C + c_D\right)\left(\beta q^B_{1M} - q^B_{2S}\right)^+ \end{array}$$

Note that γ_5^B is positive only in region B_1 where both γ_2^B and γ_3^B are zero. Besides both under regions C_1 and C_2, $\gamma_3^C = 0$. Hence the difference can be written as follows:

$$\begin{aligned} \Pi^B_{OEM} - \Pi^C_{OEM} &= \begin{array}{l} \frac{-{\gamma_3^B}^2}{2\phi(2-\alpha)} + \frac{\left(1+\phi\alpha\beta_R^2(2-\alpha)\right){\gamma_5^C}^2}{4\alpha\phi(2-\alpha)} - \frac{\left(1+2\phi\beta^2(2-\alpha)\right){\gamma_2^B}^2}{8\phi(2-\alpha)} \\ -\frac{\gamma_3^B \gamma_2^B}{2\phi(2-\alpha)} - \frac{\left(1+2\phi\beta_R^2(2-\alpha)\right){\gamma_5^B}^2}{8\phi(2-\alpha)} + \left(\Pi^{B^R}_{OEM} - \Pi^{C^R}_{OEM}\right) \\ +\phi\left(c_C + c_D\right)\left(\beta q^B_{1M} - q^B_{2S}\right)^+ \end{array} \\ &= K_{B,C}\left(\alpha, Q, c_M, c_R, c_C, c_D, c_C^R, \phi, \beta, \beta_R\right) \end{aligned}$$

where

$$\begin{aligned} \Pi^{B^R}_{OEM} - \Pi^{C^R}_{OEM} &= \frac{\phi\left(c_C + c_D\right)(1-\beta)\left(2(Q - c_M) - \phi(1+\beta)\left(c_C + c_D\right)\right)}{4} \\ &\quad - \frac{\phi\left(\alpha Q - c_D - 2c_C - c_R\right)^2}{8\alpha} \\ &= \frac{\phi\left(c_C + c_D\right)(1-\beta)\left(q^{B^R}_{1M} + q^{C^R}_{1M}\right)}{4} - \frac{\phi\left(\alpha Q - c_D - 2c_C - c_R\right)^2}{8\alpha}. \end{aligned}$$

We can deduce that for higher values of $c_C + c_D$ preemptive collection, i.e. Policy C is less likely to be optimal, whereas for lower values of c_R Policy B is less likely to be optimal.

Bound	**Expression**
$B2_{C,F}$	$\frac{\left(2\beta_R\alpha-\beta_R\alpha^2\right)Q+c_R+\left(-\alpha\beta-2\phi\beta_R\alpha\beta+\phi\beta_R\alpha^2\beta\right)c_C+\left(-2\phi\beta_R\alpha\beta-\alpha\beta\phi+\phi\beta_R\alpha^2\beta-1\right)c_D}{(2\beta_R\alpha+\alpha-\beta_R\alpha^2)}$
$B3_{C,F}$	$\frac{-(\alpha-\beta_R\alpha(1-\phi\beta_R\alpha)(2-\alpha))Q+\left(2+\phi\beta_R^2\alpha(2-\alpha)\right)c_R-\phi(2\beta_R\alpha(\beta-\beta_R)(2-\alpha)+\alpha\beta)c_D+2\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)c_C^R-\phi(\alpha\beta+\beta_R\alpha\beta(2-\alpha))c_C}{(1+\beta_R(2-\alpha))\alpha}$
$B21_{C,F}$	$\frac{2(2-\alpha)c_C^R+(4-\alpha)c_R+\alpha(-2\beta\phi-4\phi\beta_R\beta-1+2\phi\beta_R\alpha\beta)c_D-\alpha(1-2\beta_R)(2-\alpha)Q-2\alpha\beta\phi(1+\beta_R(2-\alpha))c_C}{2\alpha(1+\beta_R(2-\alpha))}$
$B30_{C,F}$	$\frac{\beta_R\alpha(2-\alpha)Q+c_R+\left(-2\beta_R\alpha\phi-1+\beta_R\alpha^2\phi-\alpha\beta\phi\right)c_D+\alpha(\beta_R\alpha-2\beta_R-\beta)\phi c_C}{\alpha(1-\beta_R\alpha+2\beta_R)}$
$B31_{C,F}$	$\frac{\alpha\left(-\beta_R\alpha+\beta_R^2\alpha^2\phi-1+2\beta_R-2\phi\beta_R^2\alpha\right)Q-\left(\beta_R^2\alpha^2\phi-2\phi\beta_R^2\alpha-2\right)c_R-\alpha(-\beta_R\alpha+\beta+2\beta_R-2\beta_R^2+\beta_R^2\alpha)\phi c_D+2\left(1+\phi\beta_R^2\alpha(2-\alpha)\right)c_C^R+\alpha\phi(-\beta-\beta_R(2-\alpha))c_C}{\alpha(1-\beta_R\alpha+2\beta_R)}$

TABLE B4: Bounds characterizing the optimal policies for the OEM facing competition under full regulation

C1 Appendix for Chapter 3

Lemma C1 *Assume that the following cost structures hold under individual compliance scheme: (*i*) Cost of maintaining a collection rate* of $\tau + \tau_0$, $TCC^I(\tau, \tau_0)$ is a convex and increasing function of τ; and base collection rate and additional collection rate are complementary:

$$TCC^I_\tau(\tau, \tau_0) \geq 0 \;\; TCC^I_{\tau\tau}(\tau, \tau_0) \geq 0 \;\; TCC^I_{\tau\tau_0}(\tau, \tau_0) \geq 0$$

*(*ii*)Cost of collection and sorting,* $TCC^{II}(\tau, \tau_0)$ is an increasing and linear function of τ and τ_0:

$$TCC^{II}_\tau(\tau, \tau_0) \geq 0 \;\; TCC^{II}_{\tau\tau}(\tau, \tau_0) = 0 \;\; TCC^{II}_{\tau_0}(\tau, \tau_0) \geq 0 \;\; TCC^{II}_{\tau_0\tau_0}(\tau, \tau_0) = 0$$

*(*iii*)Cost of treatment* $TCT(\tau, \tau_0)$ benefits from economies of scale and from treatability. Hence we assume that $TCT(\tau, \tau_0) = TCT^I(\tau, \tau_0) + TCT^{II}(\tau, \tau_0, \xi)$ where $TCT^I(\tau, \tau_0)$ captures the economies of scale effect while $TCT^{II}(\tau, \tau_0, \xi)$ captures the benefit from higher treatability levels. Note that $TCT^I(\tau, \tau_0)$ is a concave and increasing function of τ and $TCT^{II}_\tau(\tau, \tau_0, \xi)$ is decreasing in τ (because it depends on total volume) and ξ linearly:

$$TCT^I_\tau(\tau, \tau_0) \geq 0 \;\; TCT^I_{\tau\tau}(\tau, \tau_0) \leq 0 \;\; TCT^I_{\tau\tau_0}(\tau, \tau_0) \leq 0$$

$$TCT^{II}_\tau(\tau, \tau_0, \xi) \leq 0 \;\; TCT^{II}_\xi(\tau, \tau_0, \xi) \leq 0 \;\; TCT^{II}_{\xi\tau}(\tau, \tau_0, \xi) \leq 0$$

(iv) *Cost of increasing product's treatability level,* $TCG(\xi)$ is convex and increasing in the product's treatability level:

$$TCG_{\xi}(\xi) \geq 0 \quad TCG_{\xi\xi}(\xi) \geq 0$$

When producer i minimizes the total compliance cost

$$\begin{aligned} \underset{1-\tau_0 \geq \tau \geq 0,\, \xi \geq 0}{Min} TC(\tau_0, \tau, \xi) &= TCC^{I}(\tau, \tau_0) + TCC^{II}(\tau, \tau_0) + TCT^{I}(\tau, \tau_0) \\ &\quad + TCT^{II}(\tau, \tau_0, \xi) + TCG(\xi) \end{aligned}$$

then (i) if τ_0 is small then $\tau^* = 0$; (ii) if τ_0 of moderate values then $\tau^*(\tau_0)$ is an increasing function of τ_0 and (iii) if τ_0 is high then $\tau^* = 1 - \tau_0$.

Proof of Lemma C1. First, consider the unconstrained problem. The first order conditions are

$$\frac{\partial TC}{\partial \tau} = TCC^{I}_{\tau}(\tau, \tau_0) + TCC^{II}_{\tau}(\tau, \tau_0) + TCT^{I}_{\tau}(\tau, \tau_0) + TCT^{II}_{\tau}(\tau, \tau_0, \xi) = 0, \quad \text{(C-1)}$$

$$\frac{\partial TC}{\partial \xi} = TCT^{II}_{\xi}(\tau, \tau_0, \xi) + TCG_{\xi}(\xi) = 0. \quad \text{(C-2)}$$

For the Hessian, $\begin{pmatrix} TCG_{\xi\xi}(\xi) & TCT^{II}_{\xi\tau}(\tau,\tau_0,\xi) \\ TCT^{II}_{\xi\tau}(\tau,\tau_0,\xi) & TCC^{I}_{\tau\tau}(\tau,\tau_0)+TCT^{I}_{\tau\tau}(\tau,\tau_0) \end{pmatrix}$, to be positive semidefinite

$$(TCG_{\xi\xi}(\xi))(TCC^{I}_{\tau\tau}(\tau, \tau_0) + TCT^{I}_{\tau\tau}(\tau, \tau_0)) - (TCT^{II}_{\xi\tau}(\tau, \tau_0, \xi))^2 \geq 0,$$

which also implies that $G_1 = TCC^I_{\tau\tau}(\tau, \tau_0) + TCT^I_{\tau\tau}(\tau, \tau_0) \geq 0$, should hold.

Let subscript UC denote solution to unconstrained problem. From (C-2), $\xi^*_{UC}(\tau)$ is solution to $TCG_\xi(\xi) = -TCT^{II}_\xi(\tau, \tau_0, \xi)$. Note that $TCT^{II}_\xi(\tau, \tau_0, \xi)$ depends on τ and τ_0 but not ξ. In addition,

$$\frac{\partial^2 TC}{\partial\tau\partial\tau_0} = TCC^I_{\tau\tau_0}(\tau, \tau_0) + TCT^I_{\tau\tau_0}(\tau, \tau_0) + TCT^{II}_{\tau\tau_0}(\tau, \tau_0, \xi = \xi^*_{UC}(\tau)) = G_2$$

Note that (C-1) might be rewritten as $\frac{\partial TC}{\partial\tau} = \int_0^\tau G_1 d\tau + \int_0^{\tau_0} G_2\tau_0 + C_1 = 0$ where $C_1 = TCC^{II}_\tau$ is a positive constant. Note that G_1 is positive (due to convexity) and this implies that G_2 is negative. Therefore, $\tau^*_{UC}(\tau_0)$ increases as τ_0 increases. Since there is a linear lower bound and a linear upper bound on τ^* under the original constrained problem, τ^* might take one of three values depending on τ_0: (i) if $\tau^*_{UC} \leq 0$ (could be the case when τ_0 is small) then $\tau^* = 0$; (ii) if $0 \leq \tau^*_{UC} \leq 1 - \tau_0$ (could be the case for moderate values of τ_0) $\tau^* = \tau^*_{UC}(\tau_0)$ is an increasing function of τ_0 and (iii) if $\tau^*_{UC} \geq 1 - \tau_0$ (could be the case for high values of τ_0) then $\tau^* = 1 - \tau_0$. ■

Proof of Proposition 14. From the Lagrangian function $L = E(TC_S) - \gamma_1\xi_S - \gamma_2\tau_S - \gamma_3(1 - \tau_0 - \tau_S)$, we get the first order conditions

$$\frac{\partial L}{\partial\tau_S} = 2\eta(\tau_0 + \tau_S) + (c_C + c_R - \beta\xi_S)\alpha_i Q\mu - 2\theta\alpha_i^2 Q^2(\mu\tau_0 + \tau_S(\mu^2 + \sigma^2)) \quad \text{(C-3)}$$

$$-\gamma_2 + \gamma_3 = 0, \quad \text{(C-4)}$$

$$\frac{\partial L}{\partial\xi_S} = -\beta\alpha_i Q(\mu\tau_S + \tau_0) + 2k\alpha_i Q\xi_S - \gamma_1 = 0. \quad \text{(C-5)}$$

For the Hessian, $\left(\begin{smallmatrix} 2k\alpha_i Q & -\alpha_i\beta Q\mu \\ -\alpha_i\beta Q\mu & 2(\eta-\theta\alpha_i^2 Q^2(\mu^2+\sigma^2)) \end{smallmatrix}\right)$, to be positive semidefinite $4k(\eta-\theta\alpha_i^2Q^2(\mu^2+\sigma^2))-\alpha_iQ\beta^2\mu^2 \geq 0$ should hold. To ensure positive semidefiniteness we impose assumption (A1) and thus first order conditions guarantee optimality. Solving the optimality conditions (C-4)-(C-5) we find the optimal ξ_S^* and τ_S^* as functions of the base collection rate τ_0:

Region 1 ($\tau_S^* = 0$): The optimality condition

$$\gamma_2^* = \frac{-\tau_0(\alpha_iQ\mu(4k\theta\alpha_iQ+\beta^2)-4\eta k)+2k\alpha_iQ\mu(c_C+c_R))}{2k} \geq 0$$

can be written as $\tau_0 \leq \kappa_S^I$ where

$$\kappa_S^I = \frac{2k\alpha_iQ\mu(c_C+c_R)}{4k\theta\alpha_i^2Q^2\mu+\alpha_iQ\mu\beta^2-4k\eta}.$$

The optimal treatability level is $\xi_S^* = \frac{\beta\tau_0}{2k} \geq 0$. In this region $\xi_S^* > 0$ unless $\tau_0 = 0$.

Region 2 ($0 < \tau_S^* < 1-\tau_0$): The decision variables are

$$\begin{aligned}
\xi_S^* &= \frac{\beta(2\tau_0(\eta-\mu\eta-\sigma^2Q^2\alpha_i^2\theta)-\alpha_iQ\mu^2(c_C+c_R))}{4k(\eta-\theta\alpha_i^2Q^2(\mu^2+\sigma^2))-\alpha_iQ\mu^2\beta^2}, \\
\tau_S^* &= \frac{\tau_0(4k\theta\alpha_i^2Q^2\mu+\alpha_iQ\mu\beta^2-4k\eta)-2k\alpha_iQ\mu(c_C+c_R)}{4k(\eta-\theta\alpha_i^2Q^2(\mu^2+\sigma^2))-\alpha_iQ\mu^2\beta^2}.
\end{aligned}$$

The optimality conditions $0 \leq \tau_S^* = \frac{\tau_0(4k\theta\alpha_i^2Q^2\mu+\alpha_iQ\mu\beta^2-4k\eta)-2k\alpha_iQ\mu(c_C+c_R)}{4k(\eta-\theta\alpha_i^2Q^2(\mu^2+\sigma^2))-\alpha_iQ\mu^2\beta^2} \leq 1-\tau_0$ can be written as $\kappa_S^I \leq \tau_0 \leq \kappa_S^{II}$ where

$$\kappa_2 = \frac{4k(\eta - \theta\alpha_i^2 Q^2(\mu^2 + \sigma^2)) - \alpha_i Q\mu^2\beta^2 + 2Qk\alpha_i\mu(c_C + c_R)}{\alpha_i Q(4k\theta\alpha_i Q(\mu - \mu^2 - \sigma^2) + \mu\beta^2(1 - \mu))}.$$

Since $\frac{\tau_S^*}{2k} - \frac{\xi_S^*}{\mu\beta} = -\frac{\tau_0}{2\mu k} \leq 0$, $\xi_S^* > 0$ as long as $0 < \tau_S^* < 1 - \tau_0$.

Region 3 ($\tau_S^* = 1 - \tau_0$): The optimality condition

$$\gamma_3^* = \frac{\begin{array}{c}\alpha_i Q(4k\alpha_i\theta Q(\mu - \mu^2 - \sigma^2) + \mu\beta^2(1 - \mu))\tau_0 \\ -4k(\eta - \theta\alpha_i^2 Q^2(\mu^2 + \sigma^2)) + \alpha_i Q\mu^2\beta^2 - 2\alpha_i Q\mu k(c_C + c_R)\end{array}}{2k} \geq 0$$

can be written as $\tau_0 \geq \kappa_S^{II}$. Optimal treatability level $\xi_S^* = \frac{\beta(\mu - \mu\tau_0 + \tau_0)}{2k}$ is always positive in this region.

Note that in each region optimal treatability level is $\xi_S^* = \frac{\beta(\tau_0 + \mu\tau_S^*)}{2k}$. Finally, when $-4k(\eta - \theta\alpha_i^2 Q^2\mu) + \alpha_i Q\mu\beta^2 - 2kQ\mu\alpha_i(c_R + c_C) \leq 0$, i.e. $\eta \geq \eta_S^{UB}$, $\tau_S^* = 0$ regardless of τ_0. ∎

Proof of Corollary 2. If $\eta \leq \eta_S^{UB}$ and $\kappa_S^I \leq \tau_0 \leq \kappa_S^{II}$ (region 2 in the proof of Proposition 14) then $\frac{\partial\tau_S^*}{\partial\tau_0} = \frac{(4k\theta\alpha_i^2 Q^2\mu + \alpha_i Q\mu\beta^2 - 4k\eta)}{4k(\eta - \theta\alpha_i^2 Q^2(\mu^2 + \sigma^2)) - \alpha_i Q\mu^2\beta^2} = \frac{4k(\eta_S^{UB} - \eta) + 2k\alpha_i Q\mu(c_C + c_R)}{4k(\eta - \eta_S^{LB})} \geq 0$, because $\eta_S^{UB} \geq \eta \geq \eta_S^{LB}$ in the region of interest. The sign of $\frac{\partial\tau_S^*}{\partial\tau_0}$ in regions 1 and 3 follows from their definitions provided in the proof of Proposition 14. ∎

Proof of Corollary 3. From Proposition 14, as η increases κ_S^I and κ_S^{II} increases. In addition, in Region 2, $\frac{\partial\tau_S^*}{\partial\eta} = \frac{-(\tau_0 + \tau_S^*)}{\eta - \eta_S^{LB}} \leq 0$.

From Proposition 14, as σ increases τ_S^* increases in Region 2, while κ_S^I remains the same. On the other hand $\frac{\partial\kappa_2}{\partial\sigma} = -\frac{32k^2\theta\sigma(\eta_S^{UB} - \eta)}{(4k\theta\alpha_i Q(\mu - \mu^2 - \sigma^2) + \mu\beta^2(1 - \mu))^2} \leq 0$.

From Proposition 14, the denominator of τ_S^* in Region 2 is decreasing in α_i. The direction of change in $numerator = \tau_0(4k\theta\alpha_i^2Q^2\mu + \mu\beta^2\alpha_i Q - 4k\eta) - 2k\mu Q\alpha_i(c_C + c_R)$ is positive: $\frac{\partial numerator}{\partial \alpha_i} = Q\mu(8k\theta\alpha_i\tau_0 Q - 2k(c_C + c_R) + \beta^2\tau_0)$ is positive because $\tau_0 \geq \frac{2k(c_C+c_R)}{(8k\theta\alpha_i Q+\beta^2)}$ as $\tau_0 \geq \kappa_S^I$. Thus $\frac{\partial \tau_S^*}{\partial \alpha_i} \geq 0$ which also implies that $\frac{\partial \kappa_2}{\partial \alpha_i} \leq 0$. In addition $\frac{\partial \kappa_S^I}{\partial \alpha_i} = \frac{-8Q\mu k^2(c_C+c_R)(\eta+\theta\alpha_i^2Q^2\mu)}{(-4k\eta+4k\theta\alpha_i^2Q^2\mu+\alpha_i Q\mu\beta^2)^2} < 0$.

Finally, in regions 1 and 3, τ_S^* does not depend on η, σ, or α_i. ■

Proof of Proposition 15. The sequence of events are as follows: *(i)* Each firm decides on the treatability ξ_i (Game R), *(ii)* the PRO decides on τ on behalf of members (Game C), *(iii)* total compliance cost is realized and divided among the members of the collective scheme according to their market shares. We solve the problem by backwards induction. First the collective scheme finds the optimal τ_C by minimizing the total collection and treatment cost (Game C). The total collection and treatment cost is

$$\begin{aligned} E\left(TCC_{C_T} + TCT_{C_T}\right) &= \left(\eta - XA_n^2\right)\tau_C^2 + (2\tau_0(\eta - ZA_n^2) - Q\mu\beta W_n + YA_n)\tau_C \\ &\quad + \frac{(YA_n - ZA_n^2\tau_0 - Q\beta\mu W_n)\tau_0}{\mu} \end{aligned}$$

where

$$\begin{aligned} Y &= Q\mu(c_R + c_C) \\ Z &= \theta Q^2 \mu \\ X &= \theta Q^2(\mu^2 + \sigma^2) \\ T &= Q\mu\beta^2 \\ W_n &= \sum_{i=1}^{n} \alpha_i \xi_i. \end{aligned}$$

The Lagrangian function is $L = E\left(TCC_{C_T} + TCT_{C_T}\right) - \zeta_1 \tau_C - \zeta_2(1 - \tau_0 - \tau_C)$. Convexity is ensured as long as $\frac{\partial^2 L}{\partial \tau_C^2} = 2\eta - 2\theta A_n^2 Q^2(\mu^2 + \sigma^2) = 2(\eta - X A_n^2) \geq 0$. Solving the FOC, $\frac{\partial L}{\partial \tau_C} = 0$ we find the optimal τ_C^* in terms of ξ_js. Next we characterize three possible policies:

Region 1 ($\tau_C^* = 0$): Solving the FOC when $\tau_C = 0$, $\zeta_2 = 0$, and $\zeta_1 > 0$ we get the optimality condition $\zeta_1^* = -Q\mu\beta \sum_{i=1}^{n} (\alpha_i \xi_i) + Y A_n + 2\tau_0(\eta - Z A_n^2) \geq 0$. Plugging in $\tau_C^* = 0$ in $E(TCC_{C_T} + TCT_{C_T})$, the total collection and treatment cost is

$$E\left(TCC_{C_T} + TCT_{C_T}\right) = \frac{(Y A_n - Z A_n^2 \tau_0 - Q\beta\mu \sum_{i=0}^{n} (\alpha_i \xi_i))\tau_0}{\mu} = \Gamma(\vec{\xi}). \quad \text{(C-6)}$$

When n firms acts collectively, they play the Nash game and each firm decides on optimal treatability level by minimizing her expected compliance cost $E(TC_C)$:

$$\underset{\zeta_1^* \geq 0,\, \xi_i \geq 0}{Min} E(TC_C)(\tau_C^* = 0) = \frac{\alpha_i}{A_n} \Gamma(\vec{\xi}) + kQ\alpha_i \xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_C) - \nu_{i1}\xi_i - \nu_{i2}\zeta_1^*$. Solving the first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ for $\nu_{i1} = 0$ and $\nu_{i2} = 0$, we find that the optimal treatability level is

$$\xi_i = \frac{\tau_0 \beta \alpha_i}{2kA_n}, \ \forall i.$$

Plugging optimal values in, optimality condition is $\zeta_1^* = \frac{-T\tau_0 S_{2n}}{2kA_n} + YA_n + 2\tau_0(\eta - ZA_n^2) \geq 0$ which can be written as

$$\tau_0 \leq \frac{2kYA_n^2}{S_{2n}T - 4k(\eta - ZA_n^2)A_n} = \kappa_C^I.$$

Under this region it is trivial to show that the problem is *submodular* with linear constraints and thus there exists a pure strategy NE. It is also trivial to show that the solution is *unique* here. Note that when $\zeta_1^* \geq 0$, ξ_i^* cannot be zero unless $\tau_0 = 0$.

Region 2 ($0 < \tau_C^* < 1 - \tau_0$): Solving the FOC when $\zeta_1 = 0$ and $\zeta_2 = 0$, we get the conditions under which $\tau_C^* > 0$. The optimal τ_C^* is given by

$$\tau_C^* = \frac{Q\mu\beta W_n - YA_n - 2\tau_0(\eta - ZA_n^2)}{2(\eta - XA_n^2)}. \tag{C-7}$$

Note that τ^* increases as the weighted treatability W_n increases. Plugging in τ_C^* in

$E(TCC_{C_T} + TCT_{C_T})$ we obtain the total cost as

$$\begin{aligned} E\left(TCC_{C_T} + TCT_{C_T}\right) &= -\frac{\left(2\tau_0(\eta - ZA_n^2) - Q\mu\beta\sum_{i=0}^{n}\left(\alpha_i\xi_i\right) + YA_n\right)^2}{4(\eta - XA_n^2)} \\ &+\frac{(YA_n - ZA_n^2\tau_0 - Q\beta\mu\sum_{i=0}^{n}\left(\alpha_i\xi_i\right))\tau_0}{\mu} = \Gamma(\vec{\xi}) \quad \text{(C-8)} \end{aligned}$$

Then each firm decides on the level of treatability by minimizing his total cost:

$$(P2) \quad \underset{1-\tau_0 \geq \tau_C^* \geq 0,\, \xi_i \geq 0}{Min} E(TC_C)(\tau = \tau_C^*) = \frac{\alpha_i}{A_n}\Gamma(\vec{\xi}) + k\alpha_i Q\xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_C) - \gamma_{i1}\xi_i - \gamma_{i2}\tau_C^* - \gamma_{i3}(1 - \tau_0 - \tau_C^*)$. The first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ should be satisfied at the optimality.

Now let us show that problem (P2) is **submodular** with linear constraints. Assume $\boldsymbol{\xi^1}$ and $\boldsymbol{\xi^2}$ are two feasible vectors such that:

$$\boldsymbol{\xi^1} = \begin{bmatrix} \xi_1^1 \\ \xi_2^1 \\ \ldots \\ \xi_n^1 \end{bmatrix}, \quad \boldsymbol{\xi^2} = \begin{bmatrix} \xi_1^2 \\ \xi_2^2 \\ \ldots \\ \xi_n^2 \end{bmatrix}.$$

Then it is easy to show that

$$\begin{aligned} E(TC_C)(\boldsymbol{\xi^1}) + E(TC_C)(\boldsymbol{\xi^2}) &\geq E(TC_C)(\boldsymbol{\xi^1} \wedge \boldsymbol{\xi^2}) + E(TC_C)(\boldsymbol{\xi^1} \vee \boldsymbol{\xi^2}) \\ \Gamma(\boldsymbol{\xi^1})\frac{\alpha_i}{A_n} + k\alpha_i Q(\xi_i^1)^2 + \Gamma(\boldsymbol{\xi^2})\frac{\alpha_i}{A_n} + k\alpha_i Q(\xi_i^2)^2 &\geq \begin{array}{l} \Gamma(\boldsymbol{\xi^1} \wedge \boldsymbol{\xi^2})\frac{\alpha_i}{A_n} + k\alpha_i Q(\xi_i^1 \wedge \xi_i^2)^2 + \\ \Gamma(\boldsymbol{\xi^1} \vee \boldsymbol{\xi^2})\frac{\alpha_i}{A_n} + k\alpha_i Q(\xi_i^1 \vee \xi_i^2)^2 \end{array} \end{aligned}$$

Keeping in mind that $(\xi_i^1 \vee \xi_i^2) + (\xi_i^1 \wedge \xi_i^2) = \xi_i^1 + \xi_i^2$ and $(\xi_i^1 \vee \xi_i^2)^2 + (\xi_i^1 \wedge \xi_i^2)^2 = (\xi_i^1)^2 + (\xi_i^2)^2$ for every i, the above condition simplifies to

$$\left(\sum_{j=1}^{n} \alpha_j (\xi_j^1 \vee \xi_j^2)\right)^2 + \left(\sum_{j=1}^{n} \alpha_j (\xi_j^1 \wedge \xi_j^2)\right)^2 \geq \left(\sum_{j=1}^{n} \alpha_j \xi_j^1\right)^2 + \left(\sum_{j=1}^{n} \alpha_j \xi_j^2\right)^2 .$$

Let $(\xi_i^1 \vee \xi_i^2) = \xi_i^{max}$ and $(\xi_i^1 \wedge \xi_i^2) = \xi_i^{min}$. Then we have

$$\left(\sum_{j=1}^{n} \alpha_j \xi_j^{min}\right)^2 - \left(\sum_{j=1}^{n} \alpha_j \xi_j^2\right)^2 \ \geq \ \left(\sum_{j=1}^{n} \alpha_j \xi_j^1\right)^2 - \left(\sum_{j=1}^{n} \alpha_j \xi_j^{max}\right)^2$$

which is rewritten as

$$\begin{aligned} &\left(\sum_{j=1}^{n} \alpha_j \left(\xi_j^{min} - \xi_j^2\right)\right) \left(\sum_{j=1}^{n} \alpha_j \left(\xi_j^{min} + \xi_j^2\right)\right) \\ \geq &\left(\sum_{j=1}^{n} \alpha_j \left(\xi_j^1 - \xi_j^{max}\right)\right) \left(\sum_{j=1}^{n} \alpha_j \left(\xi_j^1 + \xi_j^{max}\right)\right) \end{aligned}$$

Without loss of generality we define sets $N = \{j : \xi_j^{max} = \xi_j^1\} = \{j : \xi_j^{min} = \xi_j^2\}$ and $K = \{j : \xi_j^{min} = \xi_j^1\} = \{j : \xi_j^{max} = \xi_j^2\}$. Then we can rewrite the above inequality as follows:

$$\begin{aligned} &\left(\sum_{j\in K} \alpha_j \left(\xi_j^{min} - \xi_j^{max}\right)\right) \left(\sum_{j\in N} 2\alpha_j \xi_j^{min} + \sum_{j\in K} \alpha_j \left(\xi_j^{min} + \xi_j^{max}\right)\right) \\ \geq &\left(\sum_{j\in K} \alpha_j \left(\xi_j^{min} - \xi_j^{max}\right)\right) \left(\sum_{j\in N} 2\alpha_j \xi_j^{max} + \sum_{j\in K} \alpha_j \left(\xi_j^{min} + \xi_j^{max}\right)\right) \end{aligned}$$

Note that the above inequality always holds, therefore we can conclude that the game is submodular and thus pure strategy NE exists.

For the **uniqueness** of the equilibrium we use the dominant diagonal condition in Milgrom and Roberts (1990). For our problem the dominant diagonal condition is written as

$$\begin{aligned}\frac{\partial^2 E(TC_C)}{\partial \xi_i^2} &\geq \left| \sum_{j\neq i} \frac{\partial^2 E(TC_C)}{\partial \xi_j \partial \xi_i} \right| \\ \frac{4k(\eta - XA_n^2)A_n - \alpha_i^2 \mu T}{2(\eta - XA_n^2)A_n} &\geq \sum_{j\neq i} \frac{T\mu\alpha_j\alpha_i}{2(\eta - XA_n^2)A_n} = \frac{T\mu(A_n - \alpha_i)\alpha_i}{2(\eta - XA_n^2)A_n}\end{aligned}$$

Therefore as long as the following condition holds

$$4k(\eta - XA_n^2) - T\mu\alpha_i \geq 0 \quad (A2)$$

for every i, we guarantee uniqueness. In the rest of the analysis we assume that condition (A2) holds. Note that this condition also guarantees the convexity of $E(TCC_{C_T} + TCT_{C_T})$. For convexity of $E(TC_C)$ we need to have

$$\frac{\partial^2 E(TC_C)}{\partial \xi_i^2} = \frac{\alpha_i Q(4k(\eta - XA_n^2)A_n - \alpha_i^2 \mu T)}{2(\eta - XA_n^2)A_n} \geq 0$$

for every i which already holds under assumption (A2). Assumption (A2) also guarantees that $4k(\eta - XA_n^2)A_n - T\mu S_{2n} \geq 0$.

When $0 < \tau^* < 1 - \tau_0$, the optimal treatability level is

$$\xi_i^* = \frac{\beta\alpha_i\left(2(\eta(1-\mu) + (Z\mu - X)A_n^2)\tau_0 - \mu Y A_n\right)}{4k(\eta - XA_n^2)A_n - \mu T S_{2n}} \tag{C-9}$$

where $S_{2n} = \sum_{i=1}^{n} \alpha_i^2$. *Note that firms with higher α_i choose higher treatability level.* If $(\eta(1-\mu) + (Z\mu - X)A_n^2) \leq 0$ then $\xi_i^* = 0$ and therefore $\tau^* = 0$ from equation (C-7). Also note that $(\eta(1-\mu) + (Z\mu - X)A_n^2) \geq 0$ implies that $\mu \leq 1$.

As ξ_i^* increases, the optimal treatability level of the other members increase too, i.e. $\frac{\partial^2 E(TC_C)}{\partial \xi_i \partial \xi_j} = -\frac{Q^2\mu^2\beta^2\alpha_j\alpha_i^2}{2(\eta - XA_n^2)A_n} \leq 0 \ \ \forall j$.

Weighted treatability level is

$$W_n = \sum_{j=1}^{n} \alpha_j \xi_j^* = \frac{\beta S_{2n}\left(2(\eta(1-\mu) + (Z\mu - X)A_n^2)\tau_0 - \mu Y A_n\right)}{4k(\eta - XA_n^2)A_n - \mu T S_{2n}}.$$

and thus the optimal collection rate is

$$\tau_C^* = \frac{(TS_{2n} - 4k(\eta - ZA_n^2)A_n)\tau_0 - 2kYA_n^2}{4k(\eta - XA_n^2)A_n - \mu T S_{2n}}.$$

Hence $0 < \tau^* < 1 - \tau_0$ when τ_0 is in the following interval:

$$\kappa_C^I = \frac{2kYA_n^2}{TS_{2n} - 4k(\eta - ZA_n^2)A_n} \ \ < \ \ \tau_0 < \frac{4k(\eta - XA_n^2)A_n - \mu T S_{2n} + 2kYA_n^2}{4k(Z - X)A_n^3 + (1-\mu)TS_{2n}} = \kappa_C^{II}.$$

Here note that for any firm $\xi_j = \frac{\xi_i}{\alpha_i}\alpha_j$ and therefore $W_n = \frac{\xi_i}{\alpha_i} S_{2n}$. Besides we have $\xi_j = \frac{\beta\alpha_j(\tau_0 + \mu\tau_C^*)}{2kA} \ \forall j$ and thus $W_n = \frac{\beta S_{2n}(\tau_0 + \mu\tau_C^*)}{2kA}$.

Region 3 ($\tau_C^* = 1 - \tau_0$): Solving the FOC when $\tau_C^* = 1 - \tau_0$, $\zeta_2 > 0$, and $\zeta_1 = 0$, we get the conditions under which $\tau_C^* = 1 - \tau_0$:

$$\zeta_2^* \ \ = \ \ -2\eta + 2XA_n^2(1-\tau_0) + 2\tau_0 ZA_n^2 + Q\beta\mu \sum_{i=0}^{n} (\alpha_i \xi_i) - YA_n \geq 0 \qquad \text{(C-10)}$$

Plugging in $\tau_C^* = 1 - \tau_0$ in $E(TCC_{C_T} + TCT_{C_T})$ we obtain the total collection and treatment cost as

$$\begin{aligned} E\left(TCC_{C_T} + TCT_{C_T}\right) &= (\eta - XA_n^2)(1-\tau_0)^2 + 2(\eta - ZA_n^2)\tau_0(1-\tau_0) \\ &\quad - Q\beta(\tau_0 + \mu - \mu\tau_0)\sum_{i=0}^{n}(\alpha_i\xi_i) \\ &\quad + \frac{YA_n(\tau_0 + \mu - \mu\tau_0) - ZA_n^2\tau_0^2}{\mu} \end{aligned}$$

Then each firm decides on the level of treatability by minimizing her total cost:

$$\underset{\zeta_2^* \geq 0,\, \xi_i \geq 0}{Min} E(TC_C)(\tau_C^* = 1 - \tau_0) = \frac{\alpha_i}{A_n}\Gamma(\vec{\xi}) + k\alpha_i Q\xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_C) - \nu_1\xi_i - \nu_2\zeta_2^*$. Solving the first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ for $\nu_1 = 0$ and $\nu_2 = 0$, we find the optimal treatability level as

$$\xi_i^* = \frac{\alpha_i\beta(\mu - \mu\tau_0 + \tau_0)}{2kA_n}, \;\; \forall i.$$

Then $\zeta_2^* = -2\eta + 2XA_n^2(1-\tau_0) + 2\tau_0 ZA_n^2 + \frac{TS_{2n}(\mu - \mu\tau_0 + \tau_0)}{2kA_n} - YA_n \geq 0$ which we rewrite as [1]

$$\kappa_C^{II} = \frac{4k(\eta - XA_n^2)A_n - \mu TS_{2n} + 2kYA_n^2}{4k(Z-X)A_n^3 + (1-\mu)TS_{2n}} < \tau_0 < 1.$$

In this region it is trivial to show that the problem is *submodular* with linear constraints and thus there exists a pure strategy NE. It is also trivial to show that the

[1] $\eta \leq \eta_C^{UB}$ together with Condition (A2) ensure that $4k(Z-X)A_n^3 + (1-\mu)TS_{2n} \geq 0$.

solution is *unique* here.

Note that when $\zeta_2^* \geq 0$, ξ_i^* cannot be zero because $\mu - \mu\tau_0 + \tau_0 \geq 0$.

In order these regions to be meaningful we need to ensure that κ_i's are no greater than 1. We write the differences as

$$\begin{aligned}
1 - \kappa_C^I &= \frac{D_C^0}{D_C^{II}} \\
1 - \kappa_C^{II} &= \frac{D_C^0}{D_C^I + D_C^{II}} \\
\kappa_C^{II} - \kappa_C^I &= \frac{D_C^0 D_C^I}{D_C^{II}(D_C^I + D_C^{II})},
\end{aligned}$$

where

$$\begin{aligned}
D_C^0 &= TS_{2n} - 4k(\eta - ZA_n^2)A_n - 2YkA_n \\
D_C^I &= 4k(\eta - XA_n^2)A_n - \mu TS_{2n} \\
D_C^{II} &= TS_{2n} - 4k(\eta - ZA_n^2)A_n.
\end{aligned}$$

Note that D_C^I is positive due to (A2). On the other hand, when D_C^0 is negative, i.e. $\eta \geq \frac{4kZA_n^3 + S_{2n}T - 2YkA_n}{4k} = \eta_C^{UB}$ then $\tau^* = 0$ regardless of τ_0 (and regardless of the sign of D_C^{II}).

When D_C^0 is nonnegative (which also implies that D_C^{II} is nonnegative), i.e. $\eta \leq \eta_C^{UB}$ then $0 \leq \kappa_1 \leq \kappa_2 \leq 1$ in the interval of interest, i.e. where $0 \leq \tau_0 \leq 1$. ■

Proof of Proposition 16. The sequence of events are as given in the proof of Proposition 15. We solve the problem by backwards induction. The collective scheme finds the optimal τ_F by minimizing the total collection and treatment cost as in the proof of Proposition 15. We next characterize three possible policies:

Region 1 ($\tau_F^* = 0$): In this region, total cost of collection and treatment for firm i is

$$E_x\left(TCC_F(\tau_F^*) + TCT_F(\tau_F^*, \xi)\right) \;=\; \frac{\tau_0\alpha_i(Y - ZA_n\tau_0 - \beta\xi_i Q\mu)}{\mu} = \Gamma(\xi_i) \quad \text{(C-11)}$$

When n firms form a collaboration, they play a Nash game and each firm decides on optimal treatability level by minimizing her compliance cost:

$$\underset{\zeta_1^*\geq 0,\, \xi_i\geq 0}{Min}\; E(TC_F)(\tau_F^* = 0) = \Gamma(\xi_i) + kQ\alpha_i\xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_F) - \nu_{i1}\xi_i - \nu_{i2}\zeta_1^*$. Solving the first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ for $\nu_{i1} = 0$ and $\nu_{i2} = 0$, we find the optimal treatability level as

$$\xi_i \;=\; \frac{\tau_0\beta}{2k}, \;\; \forall i.$$

Plugging optimal values in, optimality condition is $\zeta_1^* = -\frac{T\tau_0 A_n}{2k} + YA_n + 2\tau_0(\eta - ZA_n^2) \geq 0$ which can be written as

$$\tau_0 \leq \frac{2kYA_n}{A_nT - 4k(\eta - ZA_n^2)A_n} = \kappa_F^I$$

In this region we can show that the problem is *submodular* with linear constraints and thus there exists a pure strategy NE. We can also show that the solution is *unique* here.

Note that when $\zeta_1^* \geq 0$, ξ_i^* cannot be zero unless $\tau_0 = 0$.

Region 2 ($0 < \tau_C^* < 1-\tau_0$): The optimal collection rate τ_F^* is given in (C-7). We plug τ_F^* in $\Gamma(\vec{\xi}) = E_x\left(TCC_F(\tau_F^*) + TCT_F(\tau_F^*, \xi)\right)$ and get

$$\begin{aligned}
\Gamma(\vec{\xi}) &= \frac{\alpha_i(\eta - XA_n^2)(\tau_C^*)^2}{A} + \frac{((2\eta - 2ZA_n^2)\tau_0 - Q\mu\beta\xi_iA_n + YA_n)\alpha_i(\tau_C^*)}{A} \\
&\quad - \frac{\tau_0\alpha_i(\tau_0ZA_n - Y + Q\mu\beta\xi_i)}{\mu} \\
&= \frac{\alpha_i(Q\mu\beta\sum_{j=1}^n \alpha_j\xi_j - YA_n - 2(\eta - ZA_n^2)\tau_0)^2}{4(\eta - XA_n^2)A_n} \\
&\quad + \frac{(2(\eta - ZA_n^2)\tau_0 - Q\mu\beta\xi_iA_n + YA_n)\alpha_i(Q\mu\beta\sum_{j=1}^n \alpha_j\xi_j - YA_n - 2(\eta - ZA_n^2)\tau_0)}{2(\eta - XA_n^2)A_n} \\
&\quad - \frac{\tau_0\alpha_i(\tau_0ZA_n - Y + Q\mu\beta\xi_i)}{\mu}
\end{aligned}$$

Each firm decides on the level of treatability by minimizing her total cost simultaneously:

$$\underset{1-\tau_0 \geq \tau_F^* \geq 0,\, \xi_i \geq 0}{Min} E(TC_F)(\tau = \tau_F^*) = \Gamma(\vec{\xi}) + k\alpha_iQ\xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_F) - \gamma_{i1}\xi_i - \gamma_{i2}\tau^* - \gamma_{i3}(1 - \tau_0 - \tau^*)$. The first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ should be satisfied at the optimality.

Now let us show that the problem is **submodular** with linear constraints. Assuming that $\xi^{\mathbf{1}}$ and $\xi^{\mathbf{2}}$ are two feasible vectors, it is easy to show that

$$\begin{aligned}
E\left(TCC_F(\xi^{\mathbf{1}})\right) + E\left(TCT_F(\xi^{\mathbf{1}})\right) + k\alpha_iQ(\xi_i^1)^2 &+ \\
E\left(TCC_F(\xi^{\mathbf{2}})\right) + E\left(TCT_F(\xi^{\mathbf{2}})\right) + k\alpha_iQ(\xi_i^2)^2 &\geq \\
E\left(TCC_F(\xi^{\mathbf{1}} \wedge \xi^{\mathbf{2}})\right) + E\left(TCT_F(\xi^{\mathbf{1}} \wedge \xi^{\mathbf{2}})\right) + k\alpha_iQ((\xi_i^1 \wedge \xi_i^2))^2 &+ \\
E\left(TCC_F(\xi^{\mathbf{1}} \vee \xi^{\mathbf{2}})\right) + E\left(TCT_F(\xi^{\mathbf{1}} \vee \xi^{\mathbf{2}})\right) + k\alpha_iQ((\xi_i^1 \vee \xi_i^2))^2 &
\end{aligned}$$

Keeping in mind that $(\xi_i^1 \vee \xi_i^2) + (\xi_i^1 \wedge \xi_i^2) = \xi_i^1 + \xi_i^2$, $(\xi_i^1 \vee \xi_i^2)^2 + (\xi_i^1 \wedge \xi_i^2)^2 = (\xi_i^1)^2 + (\xi_i^2)^2$, and $\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\vee\xi_j^2)\right)+\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\wedge\xi_j^2)\right)=\sum_{j=1}^{n}\alpha_j\xi_j^1+\sum_{j=1}^{n}\alpha_j\xi_j^2$ for every i, the above condition simplifies to

$$\left(\left(\sum_{j=1}^{n}\alpha_j\xi_j^1\right)^2-2\xi_i^1A_n\sum_{j=1}^{n}\alpha_j\xi_j^1\right)+\left(\left(\sum_{j=1}^{n}\alpha_j\xi_j^2\right)^2-2\xi_i^2A_n\left(\sum_{j=1}^{n}\alpha_j\xi_j^2\right)\right)\geq$$
$$\left(\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\vee\xi_j^2)\right)^2-2(\xi_i^1\vee\xi_i^2)A_n\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\vee\xi_j^2)\right)\right)$$
$$+\left(\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\wedge\xi_j^2)\right)^2-2(\xi_i^1\wedge\xi_i^2)A_n\left(\sum_{j=1}^{n}\alpha_j(\xi_j^1\wedge\xi_j^2)\right)\right).$$

Let $(\xi_i^1\vee\xi_i^2)=\xi_i^{max}$ and $(\xi_i^1\wedge\xi_i^2)=\xi_i^{min}$. Then we have

$$2\xi_i^{max}A_n\sum_{j=1}^{n}\alpha_j\xi_j^{max}+2\xi_i^{min}A_n\sum_{j=1}^{n}\alpha_j\xi_j^{min}-2\xi_i^1A_n\sum_{j=1}^{n}\alpha_j\xi_j^1-2\xi_i^2A_n\sum_{j=1}^{n}\alpha_j\xi_j^2\geq$$
$$+\left(\sum_{j=1}^{n}\alpha_j\xi_j^{max}\right)^2+\left(\sum_{j=1}^{n}\alpha_j\xi_j^{min}\right)^2-\left(\sum_{j=1}^{n}\alpha_j\xi_j^2\right)^2-\left(\sum_{j=1}^{n}\alpha_j\xi_j^1\right)^2.$$

Without loss of generality, let us assume that $\xi_i^1=\xi_i^{max}$. Then the inequality further simplifies to

$$2\xi_i^{max}A_n\sum_{j=1}^{n}\alpha_j\xi_j^{max}+2\xi_i^{min}A_n\sum_{j=1}^{n}\alpha_j\xi_j^{min}-2\xi_i^1A_n\sum_{j=1}^{n}\alpha_j\xi_j^1-2\xi_i^2A_n\sum_{j=1}^{n}\alpha_j\xi_j^2\geq$$
$$\left(\sum_{j=1}^{n}\alpha_j\xi_j^{max}\right)^2+\left(\sum_{j=1}^{n}\alpha_j\xi_j^{min}\right)^2-\left(\sum_{j=1}^{n}\alpha_j\xi_j^2\right)^2-\left(\sum_{j=1}^{n}\alpha_j\xi_j^1\right)^2,$$

which is rewritten as

$$2(\xi_i^{max}-\xi_i^{min})A_n\geq\sum_{j=1}^{n}\alpha_j(\xi_j^{max}+\xi_j^1-\xi_j^{min}-\xi_j^2)=2\sum_{j\in N}\alpha_j(\xi_j^{max}-\xi_j^{min})$$

where $N = \{j : \xi_j^{max} = \xi_j^1\}$. Note that the above inequality holds for every i on a lattice S that is a subset of R^n where $\xi_i = \xi_j \; \forall i, j$. Therefore the game is submodular on lattice S and pure strategy NE exists.

For the **uniqueness** of the equilibrium we use the dominant diagonal condition in Milgrom and Roberts (1990). In our problem the dominant diagonal condition can be written as

$$\begin{aligned}
\frac{\partial^2 E(TC_F)}{\partial \xi_i^2} &\geq \left| \sum_{j \neq i} \frac{\partial^2 E(TC_F)}{\partial \xi_j \partial \xi_i} \right| \\
\frac{4k(\eta - XA_n^2)A_n - (2A_n\alpha_i - \alpha_i^2)\mu T}{2(\eta - XA_n^2)A_n} &\geq \sum_{j \neq i} \frac{T\mu\alpha_j(A_n - \alpha_i)}{2(\eta - XA_n^2)A_n} = \frac{T\mu(A_n - \alpha_i)^2}{2(\eta - XA_n^2)A_n}
\end{aligned}$$

Therefore as long as the condition

$$4k(\eta - XA_n^2) - T\mu A_n \geq 0 \quad (A3)$$

holds we guarantee uniqueness. In the rest of the analysis we assume that (A3) holds.

When $0 < \tau^* < 1 - \tau_0$, the optimal treatability level is

$$\xi_i^* = \frac{\beta\left(2(\eta(1-\mu) + (Z\mu - X)A_n^2)\tau_0 - \mu Y A_n\right)}{4k(\eta - XA_n^2) - \mu T A_n} \tag{C-12}$$

and the optimal collection rate is

$$\tau_F^* = \frac{(TA_n - 4k(\eta - ZA_n^2))\tau_0 - 2kYA_n}{4k(\eta - XA_n^2) - \mu T A_n}.$$

Hence we have $0 < \tau_F^* < 1 - \tau_0$ when τ_0 satisfies

$$\kappa_F^I = \frac{2kYA_n}{TA_n - 4k(\eta - ZA_n^2)} \quad < \quad \tau_0 < \frac{4k(\eta - XA_n^2) - \mu TA_n + 2kYA_n}{4k(Z-X)A_n^2 + (1-\mu)TA_n} = \kappa_F^{II}$$

Region 3 ($\tau_F^* = 1-\tau_0$): Solving the FOC when $\tau_F^* = 1-\tau_0$, $\zeta_2 > 0$, and $\zeta_1 = 0$ we get optimality condition (C-10). We plug $\tau_F^* = 1-\tau_0$ in $\Gamma(\vec{\xi}) = E_x\left(TCC_F(\tau_F^*) + TCT_F(\tau_F^*, \xi)\right)$ and get

$$\begin{aligned}\Gamma(\vec{\xi}) \quad = \quad & \frac{\alpha_i\left((\eta - XA_n^2)(1+\tau_0)^2 + (2\eta - ZA_n^2))\tau_0(1-\tau_0)\right)}{A_n} \\ & -\alpha_i Q\xi_i\beta(\mu - \mu\tau_0 - \tau_0) + \frac{\alpha_i(Y(\mu - \mu\tau_0 - \tau_0) - \tau_0^2 ZA_n)}{\mu}\end{aligned}$$

Then each firm decides on the level of treatability by minimizing her total cost:

$$\underset{\zeta_2^* \geq 0,\, \xi_i \geq 0}{Min} E(TC_F)(\tau_F^* = 1-\tau_0) = \Gamma(\vec{\xi}) + k\alpha_i Q\xi_i^2$$

For firm i the Lagrangian is written as $L_i = E(TC_F) - \nu_1\xi_i - \nu_2\zeta_2^*$. Solving the first order conditions $\frac{\partial L_i}{\partial \xi_i} = 0$ for $\nu_1 = 0$ and $\nu_2 = 0$, we find the optimal treatability level as

$$\xi_i^* \quad = \quad \frac{\beta(\mu - \mu\tau_0 + \tau_0)}{2k}, \ \ \forall i.$$

Then $\zeta_2^* = \frac{(4kA_n^2(Z-X) + TA_n(1-\mu))\tau_0 - 2YkA_n - 4\eta k + 4XA_n^2 k + \mu TA_n}{2k} \geq 0$ which we rewrite as

$$\kappa_F^{II} = \frac{4k(\eta - XA_n^2) - \mu TA_n + 2kYA_n}{4k(Z-X)A_n^2 + (1-\mu)TA_n} < \tau_0 < 1.$$

In this region we show that the problem is *submodular* with linear constraints and thus there exists a pure strategy NE. The solution is also *unique* here. Note that when $\zeta_2^* \geq 0$, ξ_i^* cannot be zero because $\mu - \mu\tau_0 + \tau_0 \geq 0$.

In order these regions to be meaningful we need to ensure that κ_F^i's are no greater than 1. We write the differences as

$$\begin{aligned} 1 - \kappa_F^I &= \frac{D_3^F}{D_2^F} \\ 1 - \kappa_F^{II} &= \frac{D_3^F}{D_1^F + D_2^F} \\ \kappa_F^{II} - \kappa_F^I &= \frac{D_3^F D_1^F}{D_2^F(D_1^F + D_2^F)}, \end{aligned}$$

where

$$\begin{aligned} D_1^F &= 4k(\eta - A_n^2 X) - A_n \mu T \\ D_2^F &= 4ZkA_n^2 + A_n T - 4k\eta \\ D_3^F &= 4ZkA_n^2 + A_n T - 4k\eta - 2A_n Y k. \end{aligned}$$

Note that D_1^F is positive due to (A3). On the other hand, when D_3^F is negative, i.e. $\eta \geq \frac{4kZA_n^2 + A_n T - 2YkA_n}{4k} = \eta_C$ $\tau^* = 0$ regardless of τ_0. When D_3^F is nonnegative, i.e. $\eta \leq \eta_C$ then $0 \leq \kappa_1 \leq \kappa_2 \leq 1$ in the interval of interest, i.e. where $0 \leq \tau_0 \leq 1$. ■

Proof of Proposition 17. Let us define

$$\begin{aligned} D_1^F &= 4k(\eta - XA_n^2) - \mu T A_n \\ D_2^F &= TA_n - 4k(\eta - ZA_n^2) \\ G_1^F &= 4ZkA_n^2 + TA_n\,. \end{aligned}$$

Then total cost is written in terms of τ as

$$E(TC_F) = \frac{(D_1^F\mu\tau^2 - 2\mu(D_2^F\tau_0 - 2YA_nk)\tau - \tau_0(G_1^F\tau_0 - 4YA_nk))\alpha_i}{(4kmuA_n)} \tag{C-13}$$

The total cost under individual scheme $E(TC_S)$ is given in (C-14) (see the proof of Proposition 18). Then the difference

$$E(TC_F) - E(TC_S) = \frac{-(A_n - \alpha_i)(An\alpha_i\tau_0^2 Z + 2\mu\tau(A_n\alpha_i Z + \eta)\tau_0 + \mu\tau^2(X\alpha_i A_n + \eta))}{\mu A_n}$$

is always negative. Therefore $E(TC_F)(\tau^* = \tau_F^*) < E(TC_F)(\tau^* = \tau_S^*) < E(TC_S)(\tau^* = \tau_S^*)$.

The difference between the total cost for firm i under individual financial responsibility $E(TC_F)$ and the total cost under collective financial responsibility $E(TC_C)$ (see (C-15) in the proof of Proposition 18) is

$$E(TC_F) - E(TC_C) = \frac{\alpha_i T(\mu\tau + \tau_0)^2(2S_{2n} - A_n^2 - \alpha_i^2)}{4k\mu A_n^2}.$$

The difference is calculated at the same τ and it is always positive as long as $(2S_{2n} -$

$A_n^2 - \alpha_i^2) \geq 0$. Thus $E(TC_F)(\tau^* = \tau_F^*) > E(TC_C)(\tau^* = \tau_F^*) > E(TC_C)(\tau^* = \tau_C^*)$ if $(2S_{2n} - A_n^2 - \alpha_i^2) \geq 0$ and $E(TC_F)(\tau^* = \tau_F^*) < E(TC_C)(\tau^* = \tau_C^*) < E(TC_C)(\tau^* = \tau_C^*)$ if $(2S_{2n} - A_n^2 - \alpha_i^2) \leq 0$. ■

Proof of Proposition 18. Let us define

$$\begin{aligned}
D_S^I &= 4k(\eta - X\alpha_i^2) - \mu T\alpha_i \\
D_S^{II} &= T\alpha_i - 4k(\eta - Z\alpha_i^2) \\
G_S^I &= 4Zk\alpha_i + T\,.
\end{aligned}$$

For a firm following individual scheme we show in Proposition 14 that $\xi_S^* = \frac{\beta(\tau_0 + \mu\tau)}{2k}$ for any τ (in all three regions). Thus the expected total cost can be written in terms of any given τ (and also for τ^*) as

$$E(TC_S) = \frac{(D_S^I \mu\tau^2 + (4\mu Y\alpha_i k - 2\mu\tau_0 D_S^{II})\tau - \alpha_i\tau_0(G_S^I\tau_0 - 4kY))}{4k\mu}. \tag{C-14}$$

Under collective scheme we show in Proposition 15 that $\xi_C^* = \frac{\beta\alpha_i(\tau_0 + \mu\tau)}{2kA_n}$ for any τ (in all three regions). Thus if the same firm decides to join a collaboration of size $A_n - \alpha_i$ with $n-1$ members then her expected total cost in terms of any given τ (and also for τ^*) is

$$E(TC_C) = \frac{\alpha_i \left(\begin{array}{l} -2\mu((D_2 + T(S_{2n} - \alpha_i^2))\tau_0 - 2YA_n^2 k)\tau \\ +(D_1 - T\mu(S_{2n} - \alpha_i^2))\mu\tau^2 - \tau_0^2 G_1 + 4YA_n^2 k\tau_0 \end{array} \right)}{4kA_n^2\mu} \tag{C-15}$$

where

$$\begin{aligned}
D_1 &= 4k(\eta - XA_n^2)A_n - \mu T S_{2n} \\
D_2 &= TS_{2n} - 4k(\eta + ZA_n^2)A_n \\
G_1 &= 4kA_n^3 Z + (2S_{2n} - \alpha_i^2)T \\
S_{2n} &= \sum_{j=0}^{n-1} \alpha_j^2 + \alpha_i^2.
\end{aligned}$$

Calculating $\Delta_0 = E(TC_C)(\tau) - E(TC_S)(\tau)$ at the same τ level we get

$$\Delta_0 = \frac{\mu a \tau^2 + 2\mu\tau_0 b\tau + \alpha_i \tau_0^2 c}{4kA_n^2 \mu} \tag{C-16}$$

where

$$\begin{aligned}
c &= A_n^2 G_S^I - G_1 \\
b &= T\alpha_i^3 - \alpha_i D_2 + A_n^2 D_S^{II} - \alpha_i S_{2n} T = \alpha_i c - 4kA_n\eta(A_n - \alpha_i) && \text{(C-17)} \\
a &= \mu T\alpha_i^3 + \alpha_i D_1 - A_n^2 D_S^I - \mu\alpha_i S_{2n} T = \alpha_i \mu c - 4kA_n(A_n - \alpha_i)(\eta + \alpha_i A_n(X - \mu Z)) \\
&= \alpha_i \mu c - 4kA_n(A_n - \alpha_i)(\eta + \alpha_i A_n \theta Q^2 \sigma^2). && \text{(C-18)}
\end{aligned}$$

Let us denote the optimal individual collection rate as τ_S and the optimal collective collection rate as τ_C. Consider the regions provided in Proposition 14 and 15. There are three regions under both collective and individual schemes. When $\tau_0 \leq \min\{\kappa_S^I, \kappa_C^I\}$, both $\tau_C = 0$ and $\tau_S = 0$. In this region the difference between the optimal total costs of firm i is

$$E(TC_C)(\tau = \tau_C = 0) - E(TC_S)(\tau = \tau_S = 0) = \Delta_0(\tau = 0) = \frac{\alpha_i \tau_0^2 (A_n^2 G_S^I - G_1)}{4kA_n^2 \mu} = \frac{\alpha_i \tau_0^2 c}{4kA_n^2 \mu}$$

Let us analyze the following cases with respect to the sign of $c = (A_n^2 G_S^I - G_1)$. In the rest of the analysis we denote the difference between the optimal cost values as $\Delta = E(TC_C)(\tau = \tau_C) - E(TC_S)(\tau = \tau_S)$.

Case 1. If $c = (A_n^2 G_S^I - G_1) \leq 0$ then both $b \leq 0$ and $a \leq 0$ and thus $\Delta_0 \leq 0$ for every τ. Therefore, if $\tau_C^* = \tau_S^* = 0$ or $\tau_C^* = \tau_S^* = 1 - \tau_0$ then $\Delta = \Delta_0 \leq 0$. First let us identify the possible cases where $\tau_C^* \neq \tau_S^*$: If $c \leq 0$ then $N_2 \leq 0$ (for the definition of N_1 and N_2 please see the proof of Proposition 20) because $\alpha\mu c - N_2 = 4kA_n(A_n - \alpha_i)(\eta + A_n\alpha_i(X - \mu Z)) - \alpha_i T\mu(S_{2n} - \alpha_i^2) \geq 0$. The last inequality holds because under the initial collaboration (with total market share $A_n - \alpha_i$) in order to have a unique solution $4k(A_n - \alpha_i)(\eta - X(A_n - \alpha_i)^2) - (S_{2n} - \alpha_i^2)T\mu \geq 0$ (corresponding to the assumption (A2)) should already be satisfied and therefore $4k(A_n - \alpha_i)A_n\eta - (S_{2n} - \alpha_i^2)T\mu\alpha_i \geq 4k(A_n - \alpha_i)\eta - (S_{2n} - \alpha_i^2)T\mu \geq 0$. On the other hand N_1 can take both positive and negative values. Hence we can observe either Case 2 (i.e. $N_1 \geq 0$ and $N_2 \leq 0$) or Case 3 (i.e. $N_1 \leq 0$ and $N_2 \leq 0$) provided in the proof of Proposition 20. Let us analyze these two cases:

Case 1.1. If both $N_1 \leq 0$ and $N_2 \leq 0$, then we know that $\tau_S \leq \tau_C$. Then along the span of τ_0 we can observe two different regions where $\tau_S \neq \tau_C$: (i)$1 - \tau_0 \geq \tau_C > 0$ and $\tau_S = 0$, and (ii) $1 - \tau_0 \geq \tau_C > 0$ and $1 - \tau_0 > \tau_S > 0$. Let us analyze the two cases keeping in mind that the optimal collection rate can be written as $\tau_C = \frac{D_C^{II}\tau_0 - 2YA_n^2 k}{D_C^I} - x$ where

$x = 0$ in Region 2 and $x > 0$ in Region 3.

Case 1.1.i. The expected cost of collaborative compliance for firm i in terms of τ_C and D_C^I is

$$E(TC_C) = \frac{\alpha_i \left(\begin{array}{l} \mu(-T\mu(S_{2n} - \alpha_i^2) - D_C^I)(\tau_C)^2 - 2\mu(xD_C^I + T\tau_0(S_{2n} - \alpha_i^2))\tau_C \\ +\tau_0(-G_C^I\tau_0 + 4YA_n^2 k) \end{array} \right)}{4kA_n^2\mu}.$$

while the expected cost of individual compliance at $\tau_S = 0$ (in Region 1), is $E(TC_S)(\tau = \tau_S = 0) = \frac{\alpha_i\tau_0(4kY - G_S^I\tau_0)}{4k\mu}$. Thus the difference between the optimal compliance costs, i.e. $\Delta = E(TC_C)(\tau = \tau_C) - E(TC_S)(\tau = 0)$ is

$$\frac{\alpha_i(\mu(-\mu T(S_{2n} - \alpha_i^2) - D_C^I)(\tau_C)^2 + 2\mu(-T\tau_0(S_{2n} - \alpha_i^2) - xD_C^I)\tau_C + \tau_0^2(A_n^2 G_S^I - G_1))}{4kA_n^2\mu}.$$

The difference is negative because $D_C^I \geq 0$, $S_{2n} \geq \alpha_i^2$ and $c = (A_n^2 G_S^I - G_1) \leq 0$.

Case 1.1.ii. The expected cost of collaborative compliance for firm i in terms of τ_C and D_C^{II} is

$$E(TC_C) = \frac{\alpha_i \left(\begin{array}{l} -T\tau_C\mu(2\tau_0 + \tau_C\mu)(x + \tau_C)(S_{2n} - \alpha_i^2) \\ -\tau_0(G_C^I\tau_0(x + \tau_C) + \tau_C\mu D_C^{II}(\tau_C + 2x)) \\ +2k\tau_C\mu YA_n^2(\tau_C + 2x) + 4kYA_n^2\tau_0(x + \tau_C) \end{array} \right)}{4(x + \tau_C)kA_n^2\mu}.$$

On the other hand when $0 < \tau_S = \frac{\tau_0 D_S^{II} - 2\alpha_i Yk}{D_S^I} < 1 - \tau_0$ (in Region 2), the expected cost of individual compliance in terms of D_S^{II} and τ_S is

$$E(TC_C) = \frac{((-\tau_0 D_S^{II} + 2\alpha_i Y k)\tau_S \mu - \alpha_i \tau_0 (G_S^I \tau_0 - 4kY))}{4k\mu}.$$

Thus the difference between optimal compliance costs in terms of τ_S is

$$\Delta = \frac{\begin{array}{l} \alpha_i \tau_0^2 (x + \tau_S + \delta) c - \alpha_i \mu^2 T (S_{2n} - \alpha_i^2)(\tau_S + \delta)^3 \\ + \mu(-T\alpha_i (S_{2n} - \alpha_i^2)(\mu x + 2\tau_0) + \tau_0 N_1)(\tau_S + \delta)^2 - \mu A_n^2 D_S^I \tau_S \delta^2 \\ + \mu(2\alpha_i(-\tau_0 D_C^{II} + 2Y A_n^2 k - \tau_0 T(S_{2n} - \alpha_i^2))x - A_n^2 \tau_S D_S^I \tau_S)\delta \\ \tau_S x \mu(-2\alpha_i T \tau_0 (S_{2n} - \alpha_i^2) + \alpha_i(-\tau_0 D_C^{II} + 2Y A_n^2 k) + \tau_0 N_1) \end{array}}{(4(x + \tau_S + \delta))k A_n^2 \mu}. \quad \text{(C-19)}$$

The difference is negative because $c < 0$, $S_{2n} > \alpha_i^2$, $N_1 < 0$, $D_S^I > 0$, and $\tau_0 D_C^{II} - 2Y A_n^2 k > 0$.

Hence we conclude that if both $N_1 \leq 0$ and $N_2 \leq 0$, then $E(TC_C)(\tau_C) - E(TC_S)(\tau_S) \leq 0$.

Case 1.2. If $N_1 \geq 0$ and $N_2 \leq 0$, then we know that $\tau_S \geq \tau_C$ if $\kappa_C^{II} \geq \kappa_S^{II}$. On the other hand, if $\kappa_C^{II} \leq \kappa_S^{II}$ then $\tau_S \geq \tau_C$ ($\tau_S \leq \tau_C$) if $\tau_0 \leq \kappa_3$ ($\tau_0 \geq \kappa_3$). Thus we can observe the following cases:

1. If $0 \leq \kappa_S^I \leq \kappa_S^{II} \leq 1 \leq \min\{\kappa_C^I, \kappa_C^{II}\}$ (i.e. $\tau_C = 0$ for every τ_0) then see *Case 1.2.ii.* and *Case 1.2.iii.*

2. If $0 \leq \kappa_S^I \leq \kappa_C^I \leq \kappa_S^{II} \leq \kappa_C^{II} \leq 1$ then see *Case 1.2.ii.*, *Case 1.2.iv.*, and *Case 1.2.v.*

3. If $0 \leq \kappa_S^I \leq \kappa_S^{II} \leq \kappa_C^I \leq \kappa_C^{II} \leq 1$ then see *Case 1.2.ii.*, *Case 1.2.v.*, and *Case 1.2.v.*

4. If $0 \leq \kappa_S^I \leq \kappa_C^I \leq \kappa_3 \leq \kappa_C^{II} \leq \kappa_S^{II} \leq 1$ then see *Case 1.2.ii.*, *Case 1.2.iv.*, *Case 1.2.i.*

It is easy to show that Δ is a piecewise function of τ_0 and continuous at the endpoints of subdomains. Next we analyze how Δ changes in the remaining subdomains:

Case 1.2.i. (The case where $\tau_C \geq \tau_S$) First consider the region where $\tau_0 \in [\kappa_3, \kappa_S^{II}]$ under ordering 4. Since $0 < \tau_C \leq 1 - \tau_0$ (in Region 2 or 3), $0 < \tau_S = \frac{\tau_0 D_S^{II} - 2\alpha_i Y k}{D_S^I} < 1 - \tau_0$ (in Region 2) and $\tau_C = \tau_S + \delta$ the difference between the optimal compliance costs, Δ is as given in (C-19). Since $\alpha_i c - 4k\eta A_n(A_n - \alpha_i) = N_1 - \alpha_i T(S_{2n} - \alpha_i^2) \leq 0$, the expression in equation (C-19) is negative.

On the other hand, if $0 \leq \tau_0 \leq \kappa_S^I$ or $\max\{\kappa_S^{II}, \kappa_C^{II}\} \leq \tau_0 \leq 1$ then $\tau_S = \tau_C$ and thus $\Delta \leq 0$. Therefore we conclude that if $\tau_C \geq \tau_S$ then $E(TC_C)(\tau_C) - E(TC_S)(\tau_S) \leq 0$.

If $\tau_C < \tau_S$ then the sign of Δ depends on τ_0. Next, we analyze the sign of Δ in each possible region of τ_0:

Case 1.2.ii. Consider the region where $\tau_0 \in [\kappa_S^I, \min\{\kappa_S^{II}, \kappa_C^I\}]$. Since $\tau_C = 0$ (in Region 1) and $0 < \tau_S < 1 - \tau_0$ (in Region 2) the difference between the optimal compliance costs in terms of τ_0 is

$$\begin{aligned}\Delta_1 &= \frac{(\alpha_i D_S^I(G_S^I A_n^2 - G_C^I) + A_n^2\mu(D_S^{II})^2)\tau_0^2 + 4kA_n^2\mu\alpha_i Y(Yk\alpha_i - \tau_0 D_S^{II})}{4kA_n^2\mu D_S^I} \\ &= a_1(\tau_0)^2 + b_1\tau_0 + c_1\end{aligned}$$

where $b_1 \leq 0$ and $c_1 \geq 0$.

- If $a_1 \leq 0$ then Δ_1 has one positive and one negative root. Since $\Delta_1(\tau_0 = 0) \geq 0$

and $\Delta_1(\tau_0 = \kappa_S^I) = \Delta_0(\tau = 0, \tau_0 = \kappa_S^I) \leq 0$, the positive bound is smaller than κ_S^I. Thus $\Delta_1 \leq 0$ for every $\tau_0 \in [\kappa_S^I, 1]$.

- If $a_1 \geq 0$ then Δ_1 has two positive roots. Since $\Delta_1(\tau_0 = 0) \geq 0$ and $\Delta_1(\tau_0 = \kappa_S^I) = \Delta_0(\tau = 0, \tau_0 = \kappa_S^I) \leq 0$, the smaller bound is smaller than κ_S^I. Thus $\Delta_1 \leq 0$ if and only if $\tau_0 \leq \max\{B_1 = \frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1}, \min\{\kappa_S^{II}, \kappa_C^I\}\}$.

Case 1.2.iii. Consider the region where $\tau_0 \in [\kappa_S^{II}, 1]$ under ordering 1 or the region where $\tau_0 \in [\kappa_S^{II}, \kappa_C^I]$ under ordering 3. Since $\tau_C = 0$ (in Region 1) and $\tau_S = 1 - \tau_0$ (in Region 3) the difference between the optimal compliance costs in terms of τ_0 is

$$\begin{aligned}
\Delta_2 &= \frac{\begin{array}{c}(-A_n^2\mu(D_S^I + 2D_S^{II}) + \alpha_i(G_S^I A_n^2 - G_C^I))\tau_0^2 \\ +2A^2\mu(D_S^I + 2Yk\alpha_i + D_S^{II})\tau_0 - A_n^2\mu(4Yk\alpha_i + D_S^I)\end{array}}{4kA_n^2\mu} \\
&= a_2(\tau_0)^2 + b_2\tau_0 + c_2
\end{aligned}$$

where $a_2 \leq 0$, $b_2 \geq 0$, and $c_2 \leq 0$. Hence Δ_2 has two positive roots. Also note that $\Delta_2 - \Delta_1 = \frac{-(D_S^I+2Yk\alpha_i-\tau_0(D_S^I+D_S^{II}))^2}{4kD_S^I} \leq 0$.

- If $a_1 \leq 0$ then $\Delta_1 \leq 0$ and thus $\Delta_2 \leq 0$ as well. Hence $\Delta \leq 0$ for every τ_0 under ordering 1.

- If $a_1 \geq 0$ and $B_1 \leq \kappa_S^{II}$ then $\Delta_1(\tau_0 = \kappa_S^{II}) = \Delta_2(\tau_0 = \kappa_S^{II}) \geq 0$. We also know that $\Delta_2(\tau_0 = 0) \leq 0$ and $\Delta_2(\tau_0 = 1) = \frac{\alpha_i c}{(4k\mu A_n^2} \leq 0$. Thus the larger bound of the polynomial Δ_2, $B_2 = \frac{-b_2-\sqrt{b_2^2-4a_2c_2}}{2a_2} \in [\kappa_S^{II}, 1]$. Hence under ordering 1, Δ is positive if $B_1 \leq \tau_0 \leq B_2$ and negative otherwise. For ordering 3, Δ_2 is positive if $B_1 \leq \tau_0 \leq \min\{B_2, \kappa_C^I\}$ and negative otherwise.

- If $a_1 \geq 0$ and $B_1 \geq \kappa_S^{II}$ then $\Delta_1 \leq 0$ in the region where $\tau_0 \in [\kappa_S^I, \kappa_S^{II}]$. Then $\Delta_2(\tau_0 = \kappa_S^{II}) = \Delta_1(\tau_0 = \kappa_S^{II}) \leq 0$. Keeping in mind that $\Delta_2(\tau_0 = 1) \leq 0$ and $\Delta_2 \leq \Delta_1$, we conclude that under ordering 1, $\Delta \geq 0$ if $B_2' \leq \tau_0 \leq B_2$ where $B_2' = \frac{-b_2+\sqrt{b_2^2-4a_2c_2}}{2a_2}$ and negative otherwise. Under ordering 3, $\Delta_2 \geq 0$ if $B_2' \leq \tau_0 \leq \min\{B_2, \kappa_C^I\}$ and negative otherwise.

Case 1.2.iv. Consider the region where $\tau_0 \in [\kappa_C^I, \kappa_S^{II}]$ under ordering 2 or the region where $\tau_0 \in [\kappa_C^I, \kappa_3]$ under ordering 4. Since $0 < \tau_C = \frac{\tau_0 D_C^{II} - 2YA_n^2 k}{D_C^I} < \tau_S$ (in Region 2) and $0 < \tau_S = \frac{\tau_0 D_S^{II} - 2\alpha_i Y k}{D_S^I} < 1 - \tau_0$ (in Region 2) the difference between the optimal costs is

$$\Delta_3 \quad = \quad \frac{a_3\tau_0^2 + 4kA_n^2\mu\alpha_i Y b_3\tau_0 + 4k^2A_n^2\alpha_i\mu Y^2 c_3}{4(D_C^I)^2 A_n^2 k\mu D_S^I} \tag{C-20}$$

where

$$\begin{aligned}
a_3 \quad &= \quad \begin{aligned}&-\mu T\alpha_i D_C^{II} D_S^I(\mu D_C^{II} + 2D_C^I)(S_{2n} - \alpha_i^2) \\ &+\mu D_C^I((D_S^{II})^2 A_n^2 D_C^I - \alpha_i D_S^I (D_C^{II})^2) + (D_C^I)^2\alpha_i D_S^I(A_n^2 G_S^I - G_C^I)\end{aligned} \\
b_3 \quad &= \quad TD_S^I(\mu D_C^{II} + D_C^I)(S_{2n} - \alpha_i^2) - D_C^I(-D_S^I D_C^{II} + D_C^I D_S^{II}) \\
c_3 \quad &= \quad -TA_n^2 D_S^I\mu(S_{2n} - \alpha_i^2) - D_C^I(A_n^2 D_S^I - \alpha_i D_C^I) = -TA_n^2 D_S^I\mu(S_{2n} - \alpha_i^2) + D_C^I N_2
\end{aligned}$$

Note that $b_3 \geq 0$ (see the proof of Proposition 20 for the discussion regarding the sign of $(-D_S^I D_C^{II} + D_C^I D_S^{II})$) and $c_3 \leq 0$. Also note that

$$\Delta_1 - \Delta_3 = \frac{\alpha_i(\tau_0 D_C^{II} - 2YkA_n^2)}{4(D_C^I)^2 A_n^2 k} \left(\begin{array}{l} 2TD_C^I(S_{2n} - \alpha_i^2)\tau_0 \\ +(D_C^I + T\mu(S_{2n} - \alpha_i^2))(\tau_0 D_C^{II} - 2YkA_n^2) \end{array} \right) \geq 0$$

as long as $\tau_0 \geq \kappa_C^I$.

- If $a_1 \leq 0$ then $\Delta_1 \leq 0$ and then $\Delta_3 \leq 0$ as well. Thus $\Delta \leq 0$ for every τ_0 under ordering 4.

- If $a_1 \geq 0$ and $\Delta_1(\tau_0 = \kappa_C^I) = \Delta_3(\tau_0 = \kappa_C^I) \geq 0$ then we observe one of the following two situations depending on the sign of a_3:

  - If $a_3 \geq 0$ then Δ_3 has one positive and one negative root. Since $\Delta_3(\tau_0 = 0) \leq 0$, the positive root of the polynomial Δ_3, i.e. $B_3 = \frac{-b_3+\sqrt{b_3^2-4a_3c_3}}{2a_3}$ is less than κ_C^I. Thus $\Delta_3 \geq 0$ in the region where $\tau_0 \in [\kappa_C^I, \kappa_S^{II}]$ under ordering 2. On the other hand this situation cannot occur under ordering 4, because $\kappa_3 \geq \kappa_C^I$ and $\Delta_3(\kappa_3) \leq 0$ (due to continuity and the discussion under case *Case 1.2.i.*).

  - If $a_3 \leq 0$ then Δ_3 has two positive roots. Since $\Delta_3(\tau_0 = 0) \leq 0$, the smaller root of the polynomial Δ_3 is less than κ_C^I. Thus $\Delta_3 \geq 0$ if and only if $\kappa_C^I \leq \tau_0 \leq \min\{\kappa_S^{II}, B_3'\}$ (where $B_3' = \frac{-b_3-\sqrt{b_3^2-4a_3c_3}}{2a_3}$) and $\Delta_3 \leq 0$ otherwise.

- If $a_1 \geq 0$ and $\Delta_1(\tau_0 = \kappa_C^I) = \Delta_3(\tau_0 = \kappa_C^I) \leq 0$ then we observe one of the following two situations depending on the sign of a_3:

  - If $a_3 \geq 0$ then $\Delta_3 \leq 0$ if and only if $\tau_0 \leq \min\{B_3, \kappa_S^{II}\}$ under ordering 2. On the other hand $\Delta_3 \leq 0$ under ordering 4 in the region of interest.

  - If $a_3 \leq 0$ then $\Delta_3 \geq 0$ if and only if $\max\{B_3, \kappa_C^I\} \leq \tau_0 \leq \min\{\kappa_S^{II}, B_3'\}$ and $\Delta_3 < 0$ otherwise. (Under ordering 4, $\Delta_3 \geq 0$ if and only if $\max\{B_3, \kappa_C^I\} \leq \tau_0 \leq \min\{\kappa_3, B_3'\}$ and $\Delta_3 < 0$ otherwise.)

Case 1.2.v. Consider the region where $\max\{\kappa_C^I, \kappa_S^{II}\} \leq \tau_0 \leq \kappa_C^{II}$ under ordering 2 and 3. Since $0 < \tau_C = \frac{\tau_0 D_C^{II} - 2YA_n^2 k}{D_C^I} < 1 - \tau_0$ (in Region 2) and $\tau_S = 1 - \tau_0$ (in Region 3) the difference between the optimal costs is

$$\Delta_4 = \frac{a_4\tau_0^2 + 2A_n^2\mu b_4\tau_0 + A_n^2\mu c_4}{4(D_C^I)^2 A_n^2 k\mu} \tag{C-21}$$

where

$$\begin{aligned}
a_4 &= \mu(\alpha_i T D_C^{II}(\mu D_C^{II} + 2D_C^I)(-S_{2n} + \alpha_i^2) - D_C^I(A^2 D_S^I D_C^I + 2D_C^I A^2 D_S^{II} + \alpha_i (D_C^{II})^2)) \\
&\quad + (D_C^I)^2\alpha_i(A^2 G_S^I - G_C^I) \\
b_4 &= 2Yk\alpha_i(T(\mu D_C^{II} + D_C^I)(S_{2n} - \alpha_i^2) + D_C^I D_C^{II} + (D_C^I)^2) + (D_C^I)^2(D_S^I + D_S^{II}) \\
c_4 &= 4Tk^2A^2\alpha_i\mu Y^2(-S_{2n} + \alpha_i^2) - 4k\alpha_i D_C^I Y(D_C^I + YkA^2) - (D_C^I)^2 D_S^I
\end{aligned}$$

where $a_4 \leq 0$, $b_4 \geq 0$ and $c_4 \leq 0$. Thus the polynomial Δ_4 has two positive roots. Note that $\Delta_4(\tau_0 = 0) \leq 0$ and $\Delta_4(\tau_0 = \kappa_C^{II}) \leq 0$ because $\Delta_4(\tau_0 = \kappa_C^{II}) = \Delta_0(\tau = 1 - \tau_0, \tau_0 = \kappa_C^{II}) \leq 0$. Besides $\Delta_4 - \Delta_3 = -\frac{(-\tau_0 D_S^{II} + 2Yk\alpha_i + D_S^I - D_S^I\tau_0)^2}{4kD_S^I} \leq 0$.

- If $a_1 \leq 0$ then $\Delta_1 \leq 0$, $\Delta_2 \leq 0$, $\Delta_3 \leq 0$ and $\Delta_4 \leq 0$. Thus $\Delta \leq 0$ for every τ_0 under ordering 2 and 3.

- If $a_1 \geq 0$ and $\Delta_4(\tau_0 = \max\{\kappa_C^I, \kappa_S^{II}\}) \geq 0$ then the greater root of polynomial Δ_4, i.e. $B_4 = \frac{-b_4 - \sqrt{b_4^2 - 4a_4c_4}}{2a_4}$, is in the interval of $\max\{\kappa_C^I, \kappa_S^{II}\} \leq \tau_0 \leq \kappa_C^{II}$. Therefore if $\max\{\kappa_C^I, \kappa_S^{II}\} \leq \tau_0 \leq B_4$ then $\Delta_4 \geq 0$, otherwise $\Delta_4 \leq 0$.

- If $a_1 \geq 0$ and $\Delta_4(\tau_0 = \max\{\kappa_C^I, \kappa_S^{II}\}) \leq 0$ then we conclude that $\Delta_4 \geq 0$ if and only if $\max\{\kappa_C^I, \kappa_S^{II}, B_4'\} \leq \tau_0 \leq \min\{\kappa_C^{II}, B_4\}$, where $B_4' = \frac{-b_4+\sqrt{b_4^2-4a_4c_4}}{2a_4}$, and $\Delta_4 < 0$ otherwise.

Case 2. If $c = (A_n^2 G_S^I - G_1) \geq 0$ then $\Delta = E(TC_C)(\tau_C = 0) - E(TC_S)(\tau_S = 0) = \frac{\alpha_i \tau_0^2 c}{4kA_n^2 mu} \geq 0$ (when both are in Region 1). However knowing that $c = (A_n^2 G_S^I - G_1) \geq 0$ does not guarantee any further information regarding the sign of N_1 and N_2. Also we can numerically observe every possible combination regarding the sign of N_1 and N_2 when $c \geq 0$. Hence if $c \geq 0$ we can observe every case provided in Proposition 20.

If $\tau_0 \leq \kappa_C^I$ then $\tau_C = 0$. Note that we can write $\tau_S = \frac{\tau_0 D_S^{II} - 2\alpha_i Yk}{D_S^I} - x$ where $x = 0$ in Region 2 and $x > 0$ in Region 3. Then the difference $\Delta = E(TC_C)(\tau_C = 0) - E(TC_S)(\tau_S)$between the optimal cost values is written in terms of D_S^{II} and τ_S as follows:

$$\Delta = \frac{\tau_S A_n^2 \mu(\tau_0 D_S^{II} - 2\alpha_i Yk)(\tau_S + 2x) + \alpha_i \tau_0^2 (A_n^2 G_S^I - G_S^I)(x + \tau_S)}{4A_n^2 k\mu(x + \tau_S)}$$

The difference is positive because $c = (A_n^2 G_S^I - G_C^I) \geq 0$ and $(\tau_0 D_S^{II} - 2\alpha_i Yk) = (\tau_S + x)D_S^I \geq 0$. Thus we show that $\Delta \geq 0$ when $\tau_C = 0$. In order to complete the analysis we need to identify the sign of Δ under other possible situations. Next we analyze *signum*(Δ) in detail with respect to N_1.

Case 2.1. When $N_1 \leq 0$ (we know that $N_2 \leq 0$ and $\tau_C \geq \tau_S$ hold when $N_1 \leq 0$) we observe one of the following three orderings:

1. If $0 \leq \kappa_C^I \leq \kappa_C^{II} \leq 1 \leq \min\{\kappa_S^I, \kappa_S^{II}\}$ (i.e. $\tau_S = 0$ for every τ_0) then see *Case 2.1.i,Case 2.1.ii.*

2. If $0 \leq \kappa_C^I \leq \kappa_S^I \leq \kappa_C^{II} \leq \kappa_S^{II} \leq 1$ then see *Case 2.1.i,Case 2.1.iii,Case 2.1.iv,Case*

2.1.v.

3. If $0 \leq \kappa_C^I \leq \kappa_C^{II} \leq \kappa_S^I \leq \kappa_S^{II} \leq 1$ then see *Case 2.1.i*,*Case 2.1.ii*,*Case 2.1.iv*,*Case 2.1.v.*

Case 2.1.i Consider the region where $\tau_0 \in [\kappa_C^I, \min\{\kappa_S^I, \kappa_C^{II}\}]$. Since $0 < \tau_C < 1 - \tau_0$ (in Region 2) and $\tau_S = 0$ (in Region 1) the difference between the optimal costs in terms of τ_0 is

$$\begin{aligned}
\Delta_5 &= \frac{\alpha_i \left(\begin{array}{l} (T\mu D_C^{II}(\alpha_i^2 - S_{2n})(\mu D_C^{II} + 2D_C^I) + (D_C^I)^2(-G_C^I + G_S^I A_n^2))\tau_0^2 \\ -4k\mu A_n^2 Y(T(\alpha_i^2 - S_{2n})(\mu D_C^{II} + D_C^I) - D_C^I D_C^{II})\tau_0 - \mu D_C^I (D_C^{II})^2 \tau_0^2 \\ +4\mu Y^2 k^2 A_n^4 (T\mu(\alpha_i^2 - S_{2n}) - D_C^I) \end{array} \right)}{4(D_C^I)^2 A_n^2 k\mu} \\
&= a_5(\tau_0)^2 + b_5\tau_0 + c_5
\end{aligned}$$

where $b_5 \geq 0$ and $c_5 \leq 0$.

- If $a_5 \geq 0$ then Δ_5 has one positive and one negative root. Since $\Delta_5(\tau_0 = 0) \leq 0$ and $\Delta_5(\tau_0 = \kappa_C^I) = \Delta_0(\tau = 0, \tau_0 = \kappa_C^I) \geq 0$, the positive root is smaller than κ_C^I. Thus $\Delta_5 \geq 0$ for every $\tau_0 \in [\kappa_C^I, 1]$.

- If $a_5 \leq 0$ then Δ_5 has two positive roots. Since $\Delta_5(\tau_0 = 0) \leq 0$ and $\Delta_5(\tau_0 = \kappa_C^I) = \Delta_0(\tau = 0, \tau_0 = \kappa_C^I) \geq 0$, the smaller root is smaller than κ_C^I. Thus $\Delta_5 \geq 0$ in the region of interest if and only if $\tau_0 \leq \min\{B_5 = \frac{-b_5 - \sqrt{b_5^2 - 4a_5c_5}}{2a_5}, \kappa_S^I, \kappa_C^{II}\}\}$.

Case 2.1.ii. Consider the region where $\tau_0 \in [\kappa_C^{II}, 1]$ under ordering 1 or the region where $\tau_0 \in [\kappa_C^{II}, \kappa_S^I]$ under ordering 3. Since $\tau_C = 1 - \tau_0$ (in Region 3) and $\tau_S = 0$ (in Region 1) the difference between the optimal costs in terms of τ_0 is

$$\begin{aligned}
\Delta_6 &= \frac{\alpha_i \begin{pmatrix} (T\mu(S_{2n}-\alpha_i^2)(2-\mu)+\mu(2D_C^{II}+D_C^I)+G_S^I A_n^2 - G_C^I)\tau_0^2 \\ +(-2T\mu(S_{2n}-\alpha_i^2)(1-\mu)-2\mu(D_C^I+D_C^{II}+2YkA_n^2))\tau_0 \\ +\mu(-T\mu(S_{2n}-\alpha_i^2)+D_C^I+4YkA_n^2) \end{pmatrix}}{4kA_n^2\mu} \\
&= a_6(\tau_0)^2 + b_6\tau_0 + c_6
\end{aligned}$$

where $a_6 \geq 0$, $b_6 \leq 0$. If $c_6 \geq 0$ then there are two positive roots, otherwise there is one positive and one negative root. Also note that

$$\Delta_6 - \Delta_5 = \frac{\alpha_1 x(x((S_{2n}-\alpha_i)T\mu + D_C^I) + 2TD_C^I(-\tau_0-\mu+\mu\tau_0)(S_{2n}-\alpha_1^2))}{4(D_C^I)^2A^2k} \geq 0$$

where $x = (-D_C^I\tau_0 + 2YkA^2 + D_C^I - \tau_0 D_C^{II}) \leq 0$ in the region of interest (because $1-\tau_0-\frac{\tau_0 D_C^{II}-2YA_n^2k}{D_C^I} = -\frac{x}{D_C^I} \geq 0$ in Region 3).

- If $a_5 \geq 0$ then $\Delta_5 \geq 0$ and thus $\Delta_6 \geq 0$. Hence $\Delta \leq 0$ for every τ_0 under ordering 1.

- If $a_5 \geq 0$ and $\Delta_5(\tau_0 = \kappa_C^{II}) \leq 0$ then $\Delta_6 \geq 0$ if and only if $B_6 \leq \tau_0 \leq \min\{1, \kappa_S^I\}$ where $B_6 = \frac{-b_6+\sqrt{b_6^2+4a_6c_6}}{2a_6}$.

- If $a_5 \geq 0$ and $\Delta_5(\tau_0 = \kappa_C^{II}) \geq 0$ then in the region of interest $\Delta_6 \leq 0$ if and only if $B_6' \leq \tau_0 \leq B_6$ where $B_6' = \frac{-b_6-\sqrt{b_6^2+4a_6c_6}}{2a_6}$. Note that if $c_6 \leq 0$ then $B_6 \leq \kappa_C^{II}$ and thus $\Delta_6 \geq 0$ in the region of interest.

Case 2.1.iii. Consider the region where $\kappa_S^I \leq \tau_0 \leq \kappa_C^{II}$ under ordering 2. Since both τ_S

and τ_C are in Region 2 the difference between optimal compliance costs is given in (C-20) in terms of τ_0.

Note that $b_3 \geq 0$ and $c_3 \leq 0$. Also note that

$$\Delta_3 - \Delta_5 = \frac{(\tau_0 D_S^{II} - 2Yk\alpha_i)^2}{4kD_S^I} \geq 0$$

- If $a_5 \geq 0$ then $\Delta_5 \geq 0$ and thus $\Delta_3 \geq 0$ as well.
- If $a_5 \leq 0$ and $\Delta_5(\tau_0 = \kappa_S^I) = \Delta_3(\tau_0 = \kappa_S^I) \geq 0$ then we observe one of the following two subcases depending on the sign of a_3:
  - If $a_3 \geq 0$ then $\Delta_3 \geq 0$ in the region of interest.
  - If $a_3 \leq 0$ then Δ_3 has two positive roots. Since $\Delta_3(\tau_0 = 0) \leq 0$, the smaller root of the polynomial Δ_3 is less than κ_S^I. Thus $\Delta_3 \geq 0$ if and only if $\kappa_S^I \leq \tau_0 \leq \min\{\kappa_C^{II}, B_3'\}$.
- If $a_5 \leq 0$ and $\Delta_5(\tau_0 = \kappa_S^I) = \Delta_3(\tau_0 = \kappa_S^I) \leq 0$ then we observe one of the following two subcases depending on the sign of a_3:
  - If $a_3 \geq 0$ then $\Delta_3 \leq 0$ if and only if $\tau_0 \leq \min\{B_3, \kappa_C^{II}\}$.
  - If $a_3 \leq 0$ then $\Delta_3 \geq 0$ if and only if $B_3 \leq \tau_0 \leq B_3'$ in the region of interest.

Case 2.1.iv. Consider the region where $\max\{\kappa_S^I, \kappa_C^{II}\} \leq \tau_0 \leq \kappa_S^{II}$ under ordering 2 and 3. Since $\tau_C = 1 - \tau_0$ (in Region 3) and $0 < \tau_S < 1 - \tau_0$ (in Region 2) the difference between optimal compliance costs in terms of τ_0 is

$$\Delta_7 = \frac{a_7\tau_0^2 + 2\mu\alpha_i b_7\tau_0 + \mu\alpha_i c_7}{4A_n^2 k\mu D_S^I} \tag{C-22}$$

where

$$\begin{aligned}
a_7 &= \mu(\alpha_1 T D_S^I(S_{2n} - \alpha_1^2)(2-\mu) + (D_S^{II})^2 A_n^2 + 2\alpha_1 D_S^I D_C^{II} + \alpha_1 D_S^I D_C^I) \\
&\quad + \alpha_1 D_S^I(G_S^I A_n^2 - G_C^I) \\
b_7 &= -TD_S^I(S_{2n} - \alpha_1^2)(1-\mu) - D_S^I D_C^I - D_S^I D_C^{II} - 2YkA_n^2(D_S^I + D_S^{II}) \\
c_7 &= -T\mu D_S^I(S_{2n} - \alpha_1^2) + D_S^I D_C^I + 4kA_n^2 Y(Yk\alpha_1 + D_S^I)
\end{aligned}$$

Note that $a_7 \geq 0$, $b_7 \geq 0$ and $\Delta_7 - \Delta_3 = \frac{\alpha_i x(x((S_{2n}-\alpha_i^2)T\mu + D_C^I) + 2TD_C^I(-\tau_0-\mu+\mu\tau_0)(S_{2n}-\alpha_i^2))}{4(D_C^I)^2 A_n^2 k} \geq 0$ where $x = -D_C^I\tau_0 + 2YkA_n^2 - \tau_0 D_C^{II} + D_C^I \leq 0$ and $\Delta_7 - \Delta_6 = \frac{(-\tau_0 D_S^{II} + 2Yk\alpha_i)^2}{4kD_S^I} \geq 0$.

- If $a_5 \geq 0$ then $\Delta_5 \geq 0$ and thus $\Delta_6 \geq 0$, $\Delta_3 \geq 0$, and $\Delta_7 \geq 0$.
- If $a_5 \leq 0$ and $\Delta_7(\tau_0 = \max\{\kappa_S^I, \kappa_C^{II}\}) \leq 0$ then $\Delta_7 \leq 0$ if and only if $\tau_0 \leq \min\{B_7, \kappa_S^{II}\}$ where $B_7 = \frac{-b_7 + \sqrt{b_7^2 - 4a_7c_7}}{2a_7}$.
- If $a_5 \leq 0$ and $\Delta_7(\tau_0 = \max\{\kappa_S^I, \kappa_C^{II}\}) \geq 0$ then in the region of interest $\Delta_7 \leq 0$ if and only if $B_7' \leq \tau_0 \leq B_7$ where $B_7' = \frac{-b_6 - \sqrt{b_6^2 + 4a_6c_6}}{2a_6}$. Note that if $c_7 \leq 0$ then $B_7 \leq \max\{\kappa_S^I, \kappa_C^{II}\}$ and thus Δ_7 is positive in the interval of interest.

Case 2.1.v. Consider the region where $\kappa_S^{II} \leq \tau_0 \leq 1$ under ordering 2 and 3. Since $\tau_C = \tau_S = 1 - \tau_0$ (both are in Region 3) the difference between the optimal costs in terms of τ_0 is

$$\Delta_8 = \frac{a_8\tau_0^2 + 2\mu b_8\tau_0 + \mu c_8}{4A_n^2 k\mu} \tag{C-23}$$

where

$$\begin{aligned}
a_8 &= \alpha_i T\mu(\alpha_i^2 - S_{2n})(\mu - 2) + \mu(\alpha_i D_C^I - A_n^2 D_S^I) + 2\mu(-A_n^2 D_S^{II} + \alpha_i D_C^{II}) \\
&\quad -\alpha_i(G_1 - G_S^I A_n^2) \\
b_8 &= \alpha_i T(S_{2n} - \alpha_i^2)(\mu - 1) - \alpha_i D_C^I + A_n^2 D_S^{II} + A_n^2 D_S^I - \alpha_i D_C^{II} \\
c_8 &= \alpha_i T\mu(\alpha_i^2 - S_{2n}) + \alpha_i D_C^I - A_n^2 D_S^I = \alpha_i T\mu(\alpha_i^2 - S_{2n}) + N_2.
\end{aligned}$$

Note that $\Delta_8(\tau_0 = 0) = c_8 \leq 0$ and $\Delta_8(\tau_0 = 1) = \frac{\alpha_i(G_S^I A_n^2 - G_C^I)}{A_n^2 k\mu} \geq 0$. Therefore Δ_8 has only one root, B_8 in the interval where $\tau_0 \in [0, 1]$. Also observe that $\Delta_7 - \Delta_8 = \frac{(D_S^I - D_S^I\tau_0 - \tau_0 D_S^{II} + 2Yk\alpha_i)^2}{4kD_S^I} \geq 0$.

- If $a_5 \geq 0$ then $\Delta_7 \geq 0$ and thus $\Delta_8 \geq 0$. Therefore $\Delta \geq 0$ under ordering 2 and 3 for every τ_0.

- If $a_5 \leq 0$ and $\Delta_8(\tau_0 = \kappa_S^{II}) \leq 0$ then $\Delta_8 \leq 0$ if and only if $\tau_0 \leq B_8$.

- If $a_5 \leq 0$ and $\Delta_8(\tau_0 = \kappa_S^{II}) \geq 0$ then Δ_8 is positive in the interval of interest.

Case 2.2.iii. Consider the region where $\max\{\kappa_S^{II}, \kappa_C^{II}\} \leq \tau_0 \leq 1$ under ordering 2,3 and 4. Since $\tau_C = \tau_S = 1 - \tau_0$ (both are in Region 3) the difference between optimal compliance costs in terms of τ_0 is Δ_8 as given in equation (C-23).

- If $\Delta_8(\tau_0 = \max\{\kappa_S^{II}, \kappa_C^{II}\}) \geq 0$ then $\Delta_8 \leq 0$ if and only if $B_8' \leq \tau_0 \leq B_8$ in the region of interest.

- If $\Delta_8(\tau_0 = \max\{\kappa_S^{II}, \kappa_C^{II}\}) \leq 0$ then $\Delta_8 \geq 0$ if and only if $B_8 \leq \tau_0 \leq 1$.

Case 2.2.iv. Consider the region where $\kappa_C^{II} \leq \tau_0 \leq \kappa_S^{II}$ under ordering 4. Since $\tau_C = 1-\tau_0$ (in Region 3) and $0 < \tau_S < 1-\tau_0$ (in Region 2) the difference between optimal compliance costs in terms of τ_0 is Δ_7 (see equation (C-22)) where $a_7 \geq 0$ and $b_7 \geq 0$. Besides $\Delta_7 \geq \Delta_3$ in the region of interest. Thus we observe one of the following cases:

- If $\Delta_3(\tau_0 = \kappa_C^{II}) \geq 0$ then $\Delta_7 \geq 0$ in the region of interest (because $\Delta_7(\tau_0 = \kappa_C^{II}) \geq 0$ -continuity– and $\Delta_7 \geq \Delta_3$)

- If $\Delta_3(\tau_0 = \kappa_C^{II}) \leq 0$ then $\Delta_7 \geq 0$ if $B_7 \leq \tau_0 \leq \kappa_S^{II}$ and $\Delta_8 \leq 0$ otherwise.

■

Proof of Corollary 4. If $c = (A_n^2 G_S^I - G_C^I) \geq 0$ does not give any information on the signs of a and b in equation (C-16). Note that $a \geq 0$ and $b \leq 0$ cannot be observed simultaneously because $a - \mu b = -4kA_n(A_n - \alpha_i)(\eta(1-\mu) + \alpha_i A_n \theta Q^2 \sigma^2) \leq 0$. On the other hand, if $a \geq 0$ and $b \geq 0$ then $\Delta \geq 0$ at any τ value. Since $\Delta \geq 0$ for every τ, the following inequality also holds at the optimal collective collection rate:

$$\Delta(\tau = \tau_C) = E(TC_C)(\tau = \tau_C) - E(TC_S)(\tau = \tau_C) \geq 0$$

■

Proof of Proposition 19. If the government imposes a binding lower bound, $\tau_T = \tau_0 + \mu\bar{\tau}$, on the percentage of items that needs not be collected/treated properly, then

$\tau_S = \tau_C = \bar{\tau} = \frac{\tau_T - \tau_0}{\mu}$. Firm i as well as the other firms in the collaboration then decide on their optimal treatment levels. Note that $\xi_S^* = \frac{\beta(\tau_0 + \mu\tau_S)}{2k} = \frac{\beta\tau_T}{2k}$ and $\xi_C^* = \frac{\alpha_i\beta(\tau_0+\mu\tau_C)}{2kA_n} = \frac{\beta\alpha_i\tau_T}{2kA_n}$. In Proposition 18 we calculate the difference between the cost under individual and collective schemes under same τ in equation (C-16). Remember that $\Delta_0 = E(TC_C)(\bar{\tau}) - E(TC_S)(\bar{\tau}) = \frac{\mu a\bar{\tau}^2 + 2\mu\tau_0 b\bar{\tau} + \alpha_i\tau_0^2 c}{4k\alpha_i^2\mu}$ where $c = A_n^2 G_S^I - G_1$, $b = \alpha_i c - 4kA_n\eta(A_n - \alpha_i)$ and $a = \alpha_i\mu c - 4kA_n(A_n - \alpha_i)(\eta + \alpha_i A_n(X - \mu Z))$. We need to analyze the two following cases with respect to c in order to complete the analysis:

Case 1. If $c \leq 0$ then both $a \leq 0$ and $b \leq 0$. Thus $\Delta_0 \leq 0$, i.e. firm i decides to join the collaboration, as long as $c \leq 0$.

Case 2. We write Δ_0 in terms of τ_0 and τ_T after plugging in $\bar{\tau} = \frac{\tau_T - \tau_0}{\mu}$ as $\Delta_0 = x_1\tau_0^2 + x_2\tau_0\tau_T + x_3\tau_T^2$ where

$$
\begin{aligned}
x_1 &= \alpha_1 T\mu(S_{2n} - \alpha_i^2) + N_2 - 2\mu N_1 + \alpha_i\mu(A_n^2 G_S^I - G_C^I) \\
x_2 &= 2(\mu N_1 - N_2) \\
x_3 &= N_2 - \alpha_i T\mu(S_{2n} - \alpha_i^2).
\end{aligned}
$$

First note that $\tau_0 \leq \tau_T$ and the difference is positive at the lower bound of τ_T, i.e. $\Delta_0(\tau_T = \tau_0) = \alpha_i\mu\tau_0 c > 0$.

Next we analyze the sign of Δ_0 with respect to the possible signs of coefficients. In Proposition 20 we show that $\mu N_1 \geq N_2$ and thus $x_2 \geq 0$. Depending on the sign of x_3 we observe two cases:

Case 2.a: $x_3 \leq 0$ If $x_1 > 0$ then the polynomial $\Delta_0(\tau_T)$ has one positive and one negative root. As $\Delta(\tau_T = 0) = x_1\tau_0^2 > 0$ and $\Delta(\tau_T = \tau_0) > 0$ the positive root,

$\bar{\tau}_T = \tau_0 \frac{-2x_2 - \sqrt{x_2^2 - 4x_3x_1}}{2x_3}$, is greater than τ_0. Therefore $\Delta_0 \leq 0$, i.e. firm i decides to join if and only if $\bar{\tau}_T(\tau_0) \leq \tau_T$.

If $x_1 < 0$ then the polynomial $\Delta_0(\tau_T)$ has two positive roots. As $\Delta(\tau_T = 0) = x_1\tau_0^2 < 0$ and $\Delta(\tau_T = \tau_0) > 0$ the greater positive root, $\bar{\tau}_T = \tau_0 \frac{-2x_2 - \sqrt{x_2^2 - 4x_3x_1}}{2x_3}$, is greater than τ_0. Therefore $\Delta_0 \leq 0$, i.e. firm i decides to join if and only if $\bar{\tau}_T(\tau_0) \leq \tau_T$.

To summarize our findings, we conclude that if $x_3 \leq 0$ which is rewritten as

$$\eta \geq \eta_5 = \frac{\alpha_i Q \mu^2(\alpha_i^2 - 2S_{2n} + A_n^2)\beta^2 - 4A^2 k\theta Q^2 \alpha_i(\mu^2 + \sigma^2)(A - \alpha_i)}{4kA_n(A_n - \alpha_i)}$$

then firm i decides to join if and only if $\bar{\tau}_T(\tau_0) \leq \tau_T$.

Case 2.a: $x_3 \geq 0$ If $x_1 > 0$ then $\Delta_0(\tau_T) \geq 0$ regardless of τ_T and thus the decision is not to join. If $x_1 < 0$ then the polynomial $\Delta_0(\tau_T)$ has one positive and one negative root. As $\Delta(\tau_T = 0) = x_1\tau_0^2 < 0$ and $\Delta(\tau_T = \tau_0) > 0$ the positive root is smaller than the lower bound of τ_T, τ_0 and thus in the region of interest $\Delta_0(\tau_T) \geq 0$. Therefore if $x_3 \geq 0$ which is rewritten as $\eta \leq \eta_5$ then the decision is not to join. ■

Proof of Proposition 20. By comparing numerators with numerators and denominators by denominators we show that $\kappa_F^I < \kappa_S^I$, $\kappa_F^I < \kappa_C^I$, $\kappa_F^{II} > \kappa_S^{II}$ and $\kappa_F^{II} > \kappa_C^{II}$. Then we compare τ^* in the second region of each compliance scheme and find that $\tau_F^* > \tau_S^*$ and $\tau_F^* > \tau_C^*$.

Next we compare τ_C^* and τ_S^*. Let us define

$$
\begin{aligned}
\kappa_S^I &= \frac{2Yk\alpha_i}{D_S^{II}} \\
\kappa_S^{II} &= \frac{2Yk\alpha_i + D_S^I}{D_S^I + D_S^{II}} \\
\kappa_C^I &= \frac{2YkA_n^2}{D_C^{II}} \\
\kappa_C^{II} &= \frac{2YkA_n^2 + D_C^I}{D_C^I + D_C^{II}}.
\end{aligned}
$$

The difference between the upper bounds of first regions is $\kappa_C^I - \kappa_S^I = \frac{2YkN_1}{D_C^{II}D_S^{II}}$ where

$$N_1 = D_S^{II}A_n^2 - \alpha_i D_C^{II} = \alpha_i(A_n^2 - S_{2n})T - 4kA_n(A_n - \alpha_i)(\eta + \alpha_i A_n Z).$$

Let us denote the optimal collection rate in the second region under individual(collective) scheme as τ_S (τ_C) and $\Delta = \tau_C - \tau_S$. Then we have $\Delta(\tau_0 = 0) = \frac{2YN_2k}{D_C^I D_S^I}$ where

$$N_2 = \alpha_i D_C^I - A_n^2 D_S^I = \mu\alpha_i(A_n^2 - S_{2n})T - 4kA_n(A_n - \alpha_i)(\eta + \alpha_i A_n X).$$

Note that $N_2 - \mu N_1 = -4kA_n(A_n - \alpha_i)(\alpha_i A_n(X - Z\mu) + \eta(1 - \mu)) \leq 0$. Therefore if $N_1 \leq 0$ then $N_2 \leq 0$ should hold.

Case 1. $N_1 \geq 0$, $N_2 \geq 0$

Then the slope of τ_C is smaller than the slope of τ_S and the lines cross in the fourth quadrant, thus $\tau_S \geq \tau_C$ in the region of interest, i.e. the first quadrant. Note that in this case $\kappa_C^{II} \geq \kappa_S^{II}$.

Case 2. $N_1 \geq 0$, $N_2 \leq 0$

In this case we need to compare the upper bounds of the second regions:

Case 2.i. If $\kappa_C^{II} \geq \kappa_S^{II}$ then it is obvious that $\tau_S \geq \tau_C$.

Case 2.ii. If $\kappa_C^{II} \leq \kappa_S^{II}$ then we need to identify the point where $\Delta = 0$.

First, let us identify the sign of $N_1 D_C^I + N_2 D_C^{II} = A^2(D_S^{II} D_C^I - D_S^I D_C^{II})$ when $N_1 \geq 0$ and $N_2 \leq 0$. Observe that $N_1 D_C^I + N_2 D_C^{II}$ is negative (positive) when evaluated at η that makes $N_1 = 0$ ($N_2 = 0$). Since both N_1 and N_2 are decreasing in η and $N_1 \geq N_2$, η value that makes $N_1 = 0$ is greater than the η value that makes $N_2 = 0$. Hence if $N_1(\eta = \eta_1) = 0$ and $N_2(\eta = \eta_2) = 0$ then $\eta_1 \geq \eta_2$. Hence as η increases $N_1 D_C^I + N_2 D_C^{II}$ decreases, i.e. $\frac{\partial(N_1 D_C^I + N_2 D_C^{II})}{\partial \eta} \leq 0$. Hence $N_1 D_C^I + N_2 D_C^{II} = \frac{\partial(N_1 D_C^I + N_2 D_C^{II})}{\partial \eta} A^2 \eta - 4kT\alpha_i A^2 (X - Z\mu)(A_n^3 - S_{2n}\alpha_i) \leq 0$.

Note that $\Delta = \frac{\tau_0(-D_S^{II} D_C^I + D_S^I D_C^{II}) - 2kY(-\alpha_i D_C^I + A_n^2 D_S^I)}{D_C^I D_S^I} = 0$ at $\tau_0 = \frac{2kYN_2}{D_S^{II} D_C^I - D_S^I D_C^{II}} = \kappa_3 = \frac{2A_n^2 kYN_2}{N_1 D_C^I + N_2 D_C^{II}}$ ($\kappa_3 \geq 0$ because both $N_2 \leq 0$ and $N_1 D_C^I + N_2 D_C^{II} \leq 0$ here). Hence if $\tau_0 \leq \kappa_3$ then $\tau_S \geq \tau_C$ and if $\tau_0 \geq \kappa_3$ then $\tau_S \leq \tau_C$.

Case 3. $N_1 \leq 0$, $N_2 \leq 0$

Then the slope of τ_C is greater than the slope of τ_S and the lines cross in the fourth quadrant, thus $\tau_S \leq \tau_C$ in the region of interest, i.e. in the first quadrant. Note that in this case $\kappa_C^{II} \leq \kappa_S^{II}$.

We can summarize the findings as follows:

i. If $N_1 \leq 0$ (see Case 3 above), i.e. $\eta \geq \frac{\alpha_i(T(A^2 - S_{2n}) - 4A_n^2 kZ(A_n - \alpha_i))}{4kA_n(A_n - \alpha_i)} = \eta_1$, then $\tau_S \leq \tau_C$.

ii. If $N_1 \geq 0$ and $\kappa_C^{II} - \kappa_S^{II} = \frac{2kA_n^2(N_1 - N_2)Y + (N_1 D_C^I + N_2 D_C^{II})}{(D_C^I + D_C^{II})(D_S^I + D_S^{II})A_n^2} \leq 0$ (see Case 2.ii. above), i.e.

$\eta_2 \le \eta \le \eta_1$ where

$$\eta_2 = \frac{\alpha_i \left((4kA_n^2(X-Z)(A-\alpha_i) + T(A_n^2 - S_{2n})(1-\mu))Y + 2T(\alpha_i S_{2n} - A_n^3)(X - Z\mu)\right)}{2(1-\mu)(A_n\alpha_i - S_{2n})T - 8kA_n(A^2-\alpha_i^2)(Z-X)}$$

then $\tau_C \ge \tau_S$ ($\tau_C \le \tau_S$) when $\tau_0 \ge \kappa_3$ ($\tau_0 \le \kappa_3$).

If $N_1 \ge 0$ and $\kappa_C^{II} \ge \kappa_S^{II}$ (see Case 1 and 2.i. above), i.e. $\eta \le \eta_2$ then $\tau_S \ge \tau_C$. ■

Proof of Proposition 21. From Proposition 14 and 16, $\xi_S^*(\tau^*) = \xi_F^*(\tau^*) = \frac{\beta(\tau_0+\mu\tau^*)}{2k}$ whereas from Proposition 15 $\xi_C^* = \frac{\alpha_i\beta(\tau_0+\mu\tau^*)}{2A_nk}$. These together with the ordering of optimal collection rates, i.e. $\tau_F^* > \max\{\tau_S^*, \tau_C^*\}$ (see Proposition 16) prove that $\xi_F^* > \max\{\xi_S^*, \xi_C^*\}$.

Next we compare ξ_S^* and ξ_C^*. We show that $\xi_S^* = \frac{\beta(\tau_0+\mu\tau_S^*)}{2k}$ and $\xi_C^* = \frac{\alpha_i\beta(\tau_0+\mu\tau_C^*)}{2kA_n}$ in Proposition 14 and Proposition 15 respectively. We analyze the following three cases depending on η:

Case A: As long as $\tau_S^* \ge \tau_C^*$, the optimal treatability level under individual scheme is higher, i.e. $\xi_S^* \ge \xi_C^*$. In Proposition 20, we show that $\tau_S^* \ge \tau_C^*$ and thus $\xi_S^* \ge \xi_C^*$ regardless of τ_0 as long as $\eta \le \eta_2$.

Case B: If $\tau_S^* \le \tau_C^*$ regardless of τ_0, i.e. $\eta \ge \eta_1$, then we observe one of the following orderings:

1. If $0 \le \kappa_C^I \le \kappa_C^{II} \le 1 \le \min\{\kappa_S^I, \kappa_S^{II}\}$ (i.e. $\tau_S^* = 0$ for every τ_0) then see *Case B.1*,*Case B.4*.

2. If $0 \le \kappa_C^I \le \kappa_S^I \le \kappa_C^{II} \le \kappa_S^{II} \le 1$ then see *Case B.1*,*Case B.2*,*Case B.3*.

3. If $0 \le \kappa_C^I \le \kappa_C^{II} \le \kappa_S^I \le \kappa_S^{II} \le 1$ then see *Case B.1*,*Case B.4*,*Case B.3*.

Case B.1. If $\kappa_C^I \leq \tau_0 \leq \min\{\kappa_C^{II}, \kappa_S^I\}$ then $0 < \tau_C^* < 1 - \tau_0$ and $\tau_S^* = 0$. Then $\Delta_1 = \xi_{c,i}^* - \xi_{s,i}^* = \frac{\beta((\alpha_i D_C^I - D_C^I A_n + \alpha_i \mu D_C^{II})\tau_0 - 2\alpha_i \mu Y k A_n^2)}{2D_C^I k A_n} = \frac{\beta(x_1\tau_0 - 2\alpha_i\mu Y k A_n^2)}{2D_C^I k A_n} \geq 0$ if and only if $\tau_0 \geq \frac{2\alpha_i \mu Y k A_n^2}{\alpha_i \mu D_C^{II} - (A_n - \alpha_i) D_C^I}$. Keep in mind that $\Delta_1(\tau_0 = \kappa_C^I) \leq 0$. If $x_1 = \alpha_i \mu D_C^{II} - (A_n - \alpha_i) D_C^I \leq 0$ then $\xi_{c,i}^* \leq \xi_{s,i}^*$ in the region of interest (or for any $\tau_0 \geq 0$).

Case B.2. If $\kappa_S^I \leq \tau_0 \leq \kappa_C^{II}$ then $0 < \tau_S^* < \tau_C^* < 1 - \tau_0$. Then

$$\Delta_2 = \xi_{c,i}^* - \xi_{s,i}^* = \frac{\beta((x_1 D_S^I - D_C^I A_n \mu D_S^{II})\tau_0 - 2Y k \alpha_i A_n \mu (D_S^I A_n - D_C^I))}{2D_C^I k A_n D_S^I}.$$

We summarize the findings in the following cases:

- If $x_1 \leq 0$ (which also implies $\Delta_1(\tau_0 = \kappa_S^I) = \Delta_2(\tau_0 = \kappa_S^I) \leq 0$) then the coefficient of τ_0 is negative. If $(D_S^I A_n - D_C^I) \geq 0$ then $\Delta_2 \leq 0$ in the region of interest. In order to understand what happens when $(D_S^I A_n - D_C^I) \leq 0$ keep in mind that $\Delta_2(\tau_0 = \kappa_S^I) \leq 0$ (because $x_1 \leq 0$) and $\Delta_2(\tau_0 = 0) > 0$. Thus when $x_1 \leq 0$, $\Delta_2 \leq 0$ in the region of interest regardless of the sign of $(D_S^I A_n - D_C^I)$.

- If $x_1 \geq 0$ and $\Delta_1(\tau_0 = \kappa_S^I) = \Delta_2(\tau_0 = \kappa_S^I) \geq 0$ then check the sign of $(D_S^I A_n - D_C^I)$. If it is positive (i.e. $\Delta_2(\tau_0 = 0) \leq 0$) then $\Delta_2 \geq 0$ in the region of interest. If it is negative then, $\Delta_2 \geq 0$ if and only if $\tau_0 \leq \frac{2Y k \alpha_i A_n \mu (D_S^I A_n - D_C^I)}{\beta((x_1 D_S^I - D_C^I A_n \mu D_S^{II})}$.

- If $x_1 \geq 0$ and $\Delta_1(\tau_0 = \kappa_S^I) = \Delta_2(\tau_0 = \kappa_S^I) \leq 0$ then check the sign of $(D_S^I A_n - D_C^I)$. If it is positive then $\Delta_2 \geq 0$ if and only if $\tau_0 \leq \frac{2Y k \alpha_i A_n \mu (D_S^I A_n - D_C^I)}{\beta((x_1 D_S^I - D_C^I A_n \mu D_S^{II})}$. If it is negative then $\Delta_2 \leq 0$ in the region of interest.

Case B.3. If $\max\{\kappa_S^I, \kappa_C^{II}\} \leq \tau_0 \leq \kappa_S^{II}$ then $\tau_C^* = 1 - \tau_0$ and $0 < \tau_S^* < 1 - \tau_0$. Then $\Delta_3 = \xi_{c,i}^* - \xi_{s,i}^* = \frac{\beta(-(D_S^I(A - \alpha_i) + \mu(\alpha_i D_S^I + D_S^{II} A_n))\tau_0 + \alpha_i \mu (D_S^I + 2A_n Y k))}{2A_n k D_S^I}$. Note that $\Delta_3(\tau_0 = 0) \geq 0$

and $\Delta_3(\tau_0 = \kappa_S^{II}) \leq 0$ (because of continuity). If $\Delta_3(\tau_0 = \max\{\kappa_S^I, \kappa_C^{II}\}) \leq 0$ then $\Delta_3 \leq 0$ in the region of interest. If $\Delta_3(\tau_0 = \max\{\kappa_S^I, \kappa_C^{II}\}) \geq 0$ then $\Delta_3 \geq 0$ if $\tau_0 \leq \frac{\alpha_i \mu (D_S^I + 2A_n Y k)}{D_S^I(A-\alpha_i)+\mu(\alpha_i D_S^I + D_S^{II} A_n)}$.

Case B.4. If $\kappa_C^{II} \leq \tau_0 \leq \min\{\kappa_S^I, 1\}$ then $\tau_C^* = 1-\tau_0$ and $\tau_S^* = 0$. Then $\Delta_4 = \xi_{c,i}^* - \xi_{s,i}^* = \frac{\beta(-(A_n-(1-\mu)\alpha_i)\tau_0+\alpha_i\mu)}{2kA_n}$. Note that $\Delta_4(\tau_0 = 0) \geq 0$ and $\Delta_4(\tau_0 = 1) \leq 0$ (due to continuity). If $\Delta_4(\tau_0 = \kappa_C^{II}) \leq 0$ then $\Delta_4 \leq 0$ in the region of interest. If $\Delta_4(\tau_0 = \kappa_C^{II}) \geq 0$ then $\Delta_4 \geq 0$ if $\tau_0 \leq \min \kappa_S^I, \frac{\alpha_i \mu}{(A_n-(1-\mu)\alpha_i)}$.

Case C: If $\eta_2 \leq \eta \leq \eta_1$ then we show in Proposition 20 that $\tau_S^* \geq \tau_C^*$ if and only if $\tau_0 \leq \kappa_3$. Besides when $\tau_0 \geq \kappa_S^{II}$ we have $\tau_S^* = \tau_C^* = 1-\tau_0$. Hence if $\tau_0 \leq \kappa_3$ or $\tau_0 \geq \kappa_S^{II}$ then $\tau_S^* \geq \tau_C^*$ and thus $\xi_{s,i}^* \geq \xi_{c,i}^*$. For the region where $\kappa_3 \leq \tau_0 \leq \kappa_S^{II}$ see the analysis under Case B.2 (keep in mind that $\Delta_2(\tau_0 = \kappa_3) \leq 0$) and B.3. ■

D1 Appendix for Chapter 4

Proof of Proposition 22. Let $\Delta_j^{(l_1-l_2)}$ represent the difference between the profits for $j(=M,D,R)$ under channel structure $l_1=(S,O,BN,BC)$ and $l_2=(S,O,BN,BC)$. In this proof we plot the differences $\Delta_j^{(l_1-l_2)}$ with respect to γ and θ_u. If the difference is always positive or negative, which is immediate from the plots, then the ranking between l_1 and l_2 does not depend on γ or θ_u. If the sign of difference $\Delta_j^{(l_1-l_2)}$ depends on the parameters, then we plot the line $\Delta_j^{(l_1-l_2)}=0$ on a $\gamma-\theta_u$ coordinate in the region of interest, i.e. $0\le\gamma\le1$ and $0\le\theta_u\le1$, and identify the points where the line $\Delta_j^{(l_1-l_2)}=0$ cut the boundaries of the coordinate space.

1. Manufacturer: From Figure D1 we see that (a) $\Delta_M^{(O-BN)}=\Pi_{1M}^{O}-\Pi_{1M}^{BN}\ge0$, (b)$\Delta_M^{(BN-S)}=\Pi_{1M}^{BN}-\Pi_{1M}^{S}\ge0$, and (c) $\Delta_M^{(S-BC)}=\Pi_{1M}^{S}-\Pi_{1M}^{BC}\ge0$.

2. Rental Agency: From Figure D2 we see that (a) $\Delta_R^{(O-BN)}=\Pi_{1R}^{O}-\Pi_{1R}^{BN}\ge0$, (b) $\Delta_R^{(BN-S)}=\Pi_{1R}^{BN}-\Pi_{1R}^{S}\ge0$, and (c)$\Delta_R^{(O-BC)}=\Pi_{1R}^{O}-\Pi_{1R}^{BC}\ge0$.

From Figure D3, on the other hand, we see that $\Delta_R^{(BC-S)}=\Pi_{1R}^{BC}-\Pi_{1R}^{S}\le0$ and $\Delta_R^{(BC-BN)}=\Pi_{1R}^{BC}-\Pi_{1R}^{BN}\le0$ if and only if γ is sufficiently small. We conclude that $\Pi_{1R}^{O}\ge\Pi_{1R}^{BN}\ge\Pi_{1R}^{S}\ge\Pi_{1R}^{BC}$ if and only if $0\le\gamma\le\bar{\gamma}_1(\theta_u)$ where $\bar{\gamma}_1(\theta_u)$ is a decreasing function of θ_u (see the solid line in Figure D3(c)). There exists a $\bar{\theta}_u=0.549$ such that $\bar{\gamma}_1(\theta_u)=1$ if and only if $\theta_u\le\bar{\theta}_u$. Note that $\Delta_R^{(BC-S)}(\theta_u=0.549,\gamma=1)=0$.

Similarly, from Figure D3 we see that $\Delta_R^{(BC-S)}=\Pi_{1R}^{BC}-\Pi_{1R}^{S}\ge0$ and $\Delta_R^{(BC-BN)}=\Pi_{1R}^{BC}-\Pi_{1R}^{BN}\le0$ if and only if γ is in moderate levels. We conclude that $\Pi_{1R}^{O}\ge\Pi_{1R}^{BN}\ge\Pi_{1R}^{BC}\ge\Pi_{1R}^{S}$ if and only if $\bar{\gamma}_1(\theta_u)\le\gamma\le\bar{\gamma}_2(\theta_u)$ where $\bar{\gamma}_2(\theta_u)$ is a convex function of θ_u (see the dashed line in Figure D3(c)) and $\bar{\gamma}_2(\theta_u)=1$ if and only if $\theta_u\le\bar{\theta}_u=0.553$. Note

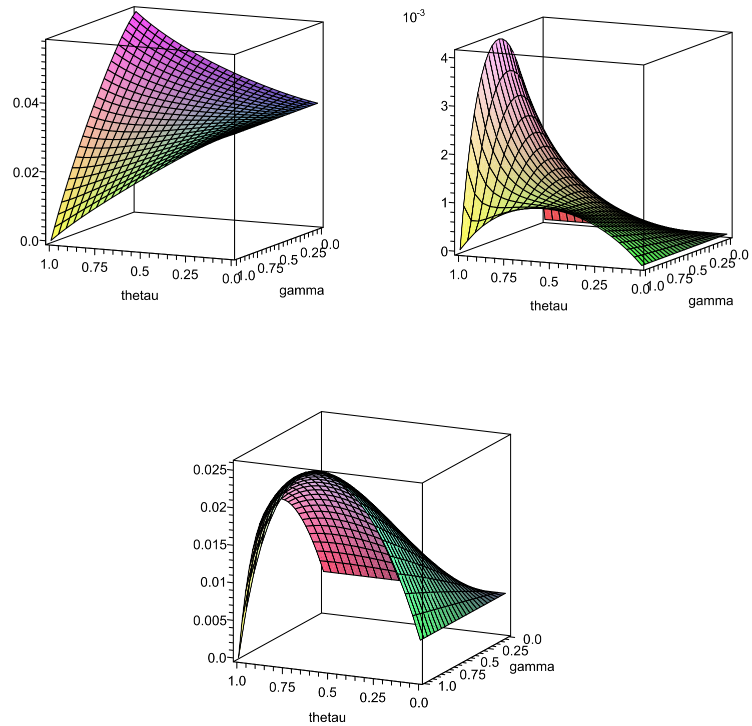

FIGURE D1: (a) $\Delta_M^{(O-BN)} \geq 0$ (b)$\Delta_M^{(BN-S)} \geq 0$ (c)$\Delta_M^{(S-BC)} \geq 0$

that $\Delta_R^{(BC-BN)}(\theta_u = 0.553, \gamma = 1) = 0$.

Finally, from Figure D3 we see that $\Delta_R^{(BC-S)} = \Pi_{1R}^{BC} - \Pi_{1R}^{S} \geq 0$ and $\Delta_R^{(BC-BN)} = \Pi_{1R}^{BC} - \Pi_{1R}^{BN} \geq 0$ if and only if γ is sufficiently large. Therefore $\Pi_{1R}^{O} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S}$ if and only if $\bar{\gamma}_2(\theta_u) \leq \gamma \leq 1$ (Figure D3(c)).

3. Dealer: From Figure D4 we see that (a) $\Delta_D^{(S-O)} = \Pi_{1D}^{S} - \Pi_{1D}^{O} \geq 0$, (b) $\Delta_D^{(BC-O)} = \Pi_{1D}^{BC} - \Pi_{1D}^{O} \geq 0$, and (c) $\Delta_D^{(BN-O)} = \Pi_{1D}^{BN} - \Pi_{1D}^{O} \geq 0$.

From Figure D5, on the other hand, we see that $\Delta_D^{(BC-S)} = \Pi_{1D}^{BC} - \Pi_{1D}^{S} \geq 0$, $\Delta_D^{(BN-S)} = \Pi_{1D}^{BN} - \Pi_{1D}^{S} \geq 0$, and $\Delta_D^{(BN-BC)} = \Pi_{1D}^{BN} - \Pi_{1D}^{BC} \leq 0$ if γ is sufficiently small. Therefore we conclude that $\Pi_{1D}^{BC} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{S} \geq \Pi_{1D}^{O}$ if and only if $0 \leq \gamma \leq \bar{\bar{\gamma}}_1(\theta_u)$ where $\bar{\bar{\gamma}}_1(\theta_u)$ is an increasing function of θ_u (see the dotted line in Figure D5(d)).

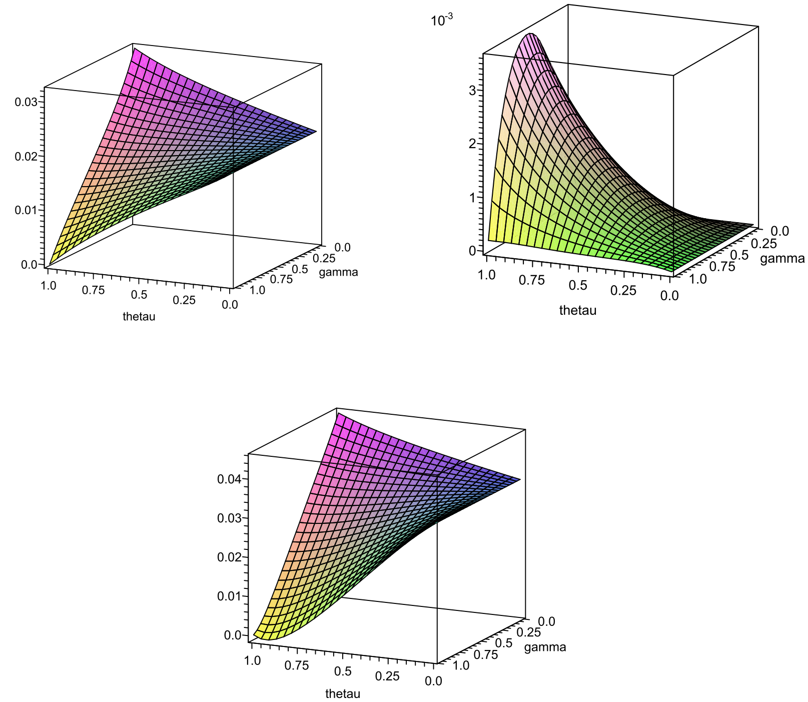

FIGURE D2: (a) $\Delta_R^{(O-BN)} \geq 0$ (b)$\Delta_R^{(BN-S)} \geq 0$ (c)$\Delta_R^{(O-BC)} \geq 0$

There exits a $\bar{\bar{\theta}}_u = 0.783$ such that $\bar{\bar{\gamma}}_1(\theta_u) = 0$ if and only if $\theta_u \leq \bar{\bar{\theta}}_u$. Note that $\Delta_D^{(BN-S)}(\theta_u = 0.783, \gamma = 0^+) = 0$.

For moderate values of γ, from Figure D5 we see that $\Delta_D^{(BC-S)} = \Pi_{1D}^{BC} - \Pi_{1D}^{S} \geq 0$, $\Delta_D^{(BN-S)} = \Pi_{1D}^{BN} - \Pi_{1D}^{S} \leq 0$, and $\Delta_D^{(BN-BC)} = \Pi_{1D}^{BN} - \Pi_{1D}^{BC} \leq 0$. Therefore, we identify that $\Pi_{1D}^{BC} \geq \Pi_{1D}^{S} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{O}$ if and only if $\bar{\bar{\gamma}}_1(\theta_u) \leq \gamma \leq \bar{\bar{\gamma}}_2(\theta_u)$ where $\bar{\bar{\gamma}}_2(\theta_u)$ is an increasing function of θ_u (see the dashed line in Figure D5(d)). There exits a $\bar{\bar{\bar{\theta}}}_u \leq \bar{\bar{\theta}}_u$ such that $\bar{\bar{\gamma}}_2(\theta_u) = 0$ if and only if $\theta_u \leq \bar{\bar{\bar{\theta}}}_u$.

For higher values of γ, from Figure D5 we see that $\Delta_D^{(BC-S)} = \Pi_{1D}^{BC} - \Pi_{1D}^{S} \leq 0$, $\Delta_D^{(BN-S)} = \Pi_{1D}^{BN} - \Pi_{1D}^{S} \leq 0$, and $\Delta_D^{(BN-BC)} = \Pi_{1D}^{BN} - \Pi_{1D}^{BC} \leq 0$. Therefore, we conclude that $\Pi_{1D}^{S} \geq \Pi_{1D}^{BC} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{O}$ if and only if $\bar{\bar{\gamma}}_2(\theta_u) \leq \gamma \leq \bar{\bar{\gamma}}_3(\theta_u)$ where $\bar{\bar{\gamma}}_3(\theta_u)$ is an

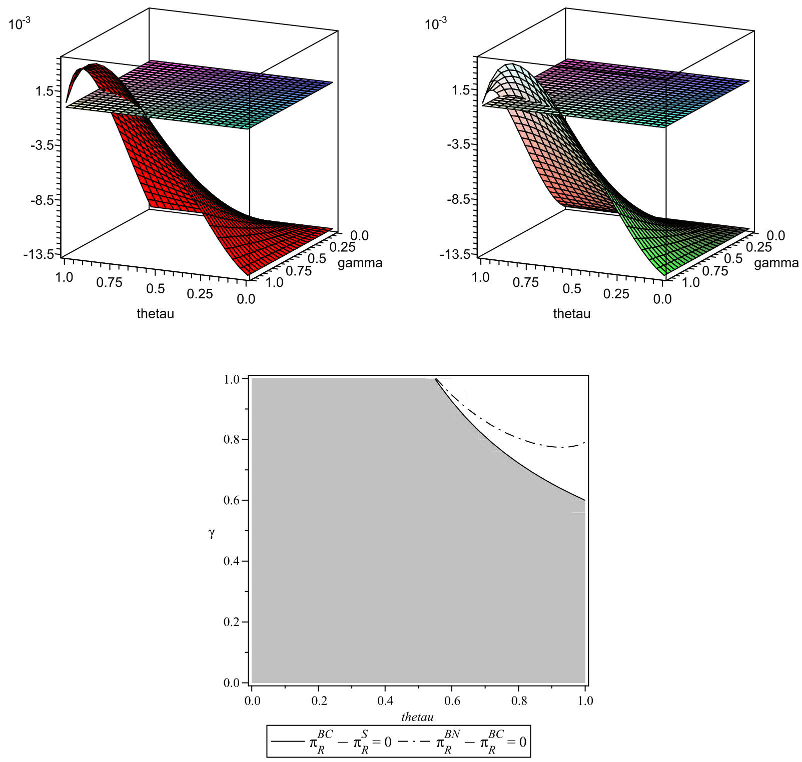

FIGURE D3: (a) $\Delta_R^{(BC-S)}$, (b) $\Delta_R^{(BC-BN)}$, (c) Three regions from left to right are (i) $\Delta_R^{(BC-S)} \leq 0$ and $\Delta_R^{(BC-BN)} \leq 0$, (ii) $\Delta_R^{(BC-S)} \geq 0$ and $\Delta_R^{(BC-BN)} \leq 0$, (iii) $\Delta_R^{(BC-S)} \geq 0$ and $\Delta_R^{(BC-BN)} \geq 0$

increasing function of θ_u (see the solid line in Figure D5(d)).

Finally, for even higher values of γ, from Figure D5 we see that $\Delta_D^{(BC-S)} = \Pi_{1D}^{BC} - \Pi_{1D}^{S} \leq 0$, $\Delta_D^{(BN-S)} = \Pi_{1D}^{BN} - \Pi_{1D}^{S} \leq 0$, and $\Delta_D^{(BN-BC)} = \Pi_{1D}^{BN} - \Pi_{1D}^{BC} \geq 0$. Therefore we conclude that $\Pi_{1D}^{S} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{BC} \geq \Pi_{1D}^{O}$ if and only if $\bar{\bar{\gamma}}_3(\theta_u) \leq \gamma \leq 1$.

3. Supply Chain: From Figure D6 we see that (a) $\Delta_{SC}^{(O-BN)} = \Pi_{1SC}^{O} - \Pi_{1M}^{BN} \geq 0$, (b)$\Delta_{SC}^{(BN-S)} = \Pi_{1SC}^{BN} - \Pi_{1M}^{S} \geq 0$, and (c) $\Delta_{SC}^{(S-BC)} = \Pi_{1SC}^{S} - \Pi_{1SC}^{BC} \geq 0$. ■

Proof of Proposition 23. In order to show that higher w_b^* implies lower w_1^* we observe that $\Delta_1 = \frac{\partial^2 \Pi_{1M}^B}{\partial w_b \partial w_1}$ is always negative (see Figure D7(a)). On the other hand, higher w_b^* implies higher $\bar{w}_1^*$ because $\Delta_2 = \frac{\partial^2 \Pi_{1M}^B}{\partial w_b \partial \bar{w}_1}$ is always positive (see Figure D7(b)) regardless

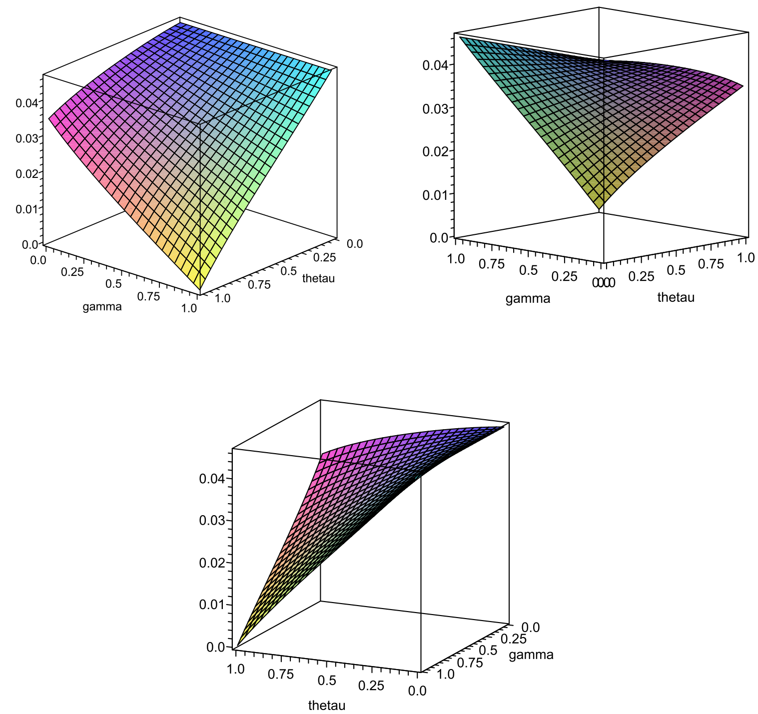

FIGURE D4: (a) $\Delta_D^{(S-O)} \geq 0$, (b)$\Delta_D^{(BC-O)} \geq 0$, (c)$\Delta_D^{(BN-O)} \geq 0$

of θ_u and γ. ■

Proof of Proposition 24. We define the total sales as $Q^l = \sum_i (q_{in} + \bar{q}_{in})$ for $l = S, O, BN, BC$. In Figure D8 the differences between total sales are plotted. ■

Proof of Theorem 25. Starting from $\gamma = 0$ and by incrementing γ with 0.01 we solve $\Delta_D^{BN}(\gamma) = 0$ for θ_u. And we obtain the line $\gamma_{UB}^{BN}(\theta_u)$ in Figure D9. We also verify our calculation by using Maple 11's implicitplot command that computes the two-dimensional plot of an implicitly defined curve. From Figure D9 we deduce that $\Delta_D^{BN} \geq 0$, and thus channel conflict is resolved, only if $0 \leq \gamma \leq \gamma_{UB}^{BN}(\theta_u)$ where $\gamma_{UB}^{BN}(\theta_u)$ is an increasing function of θ_u. We find that at $\gamma = 0$, $\gamma_{UB}^{BN}(\theta_u)(\theta_u = 0.783, \gamma = 0) = 0$, and thus $\gamma_{UB}^{BN}(\theta_u) = 0$ if and only if $\theta_u \leq 0.783$. ■

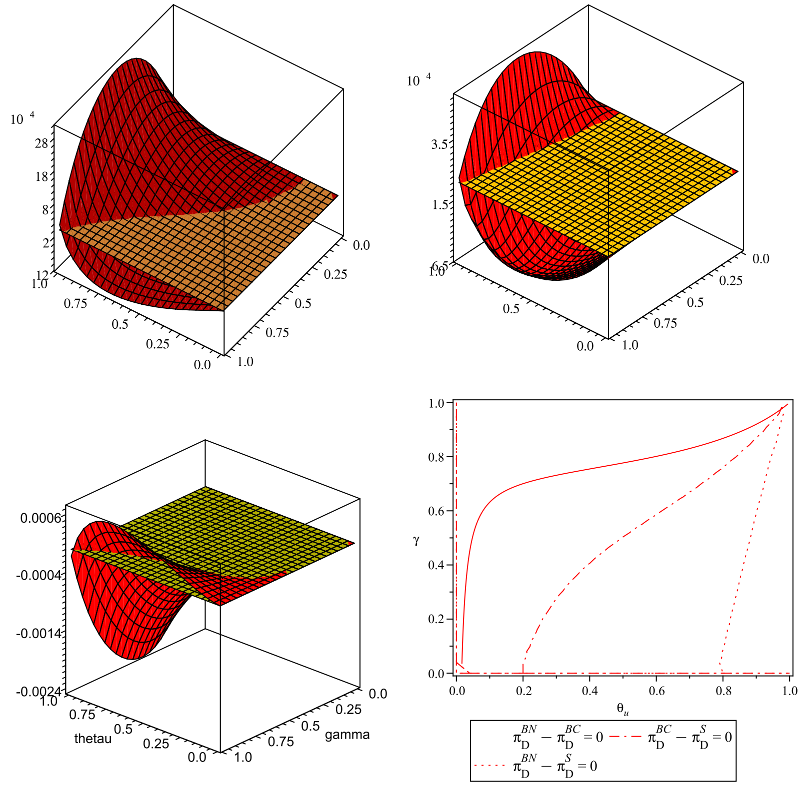

FIGURE D5: (a) $\Delta_D^{(BC-S)}$ (b) $\Delta_D^{(BN-S)}$ (c)$\Delta_D^{(BN-BC)}$ (d)Four regions from left to right are (i) $\Pi_{1D}^{S} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{BC}$, (ii) $\Pi_{1D}^{S} \geq \Pi_{1D}^{BC} \geq \Pi_{1D}^{BN}$, (iii)$\Pi_{1D}^{BC} \geq \Pi_{1D}^{S} \geq \Pi_{1D}^{BN}$, (iv) $\Pi_{1D}^{BC} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{S}$

Proof of Proposition 26. Using a method similar to that in the Proof of Proposition 25, we characterize the region where both $\Delta_D^{BC} \geq 0$ and $\Delta_R^{BC} \geq 0$ and plot that region in Figure 4.5. We find that $\Delta_D^{BC} = \Delta_R^{BC} = 0$ only at $\theta_u = 0.778$ and $\gamma = 0.739$. Keeping Figure 4.5 also in mind we deduce that if $\theta_u \leq 0.778$ then the channel conflict is not resolved regardless of γ. Otherwise, the conflict is resolved if and only if $\gamma_{LB}^{BC}(\theta_u) \leq \gamma \leq \gamma_{UB}^{BC}(\theta_u)$ where $\gamma_{LB}^{BC}(\theta_u)$ is decreasing in θ_u while $\gamma_{UB}^{BC}(\theta_u)$ is increasing in θ_u (see Figure 4.5). In addition we find that $\gamma_{LB}^{BC}(\theta_u = 1) = 0.599$ and thus channel conflict is never resolved if $\gamma \leq 0.599$. ■

Proof of Proposition 27. The plot of the manufacturer's objective function under

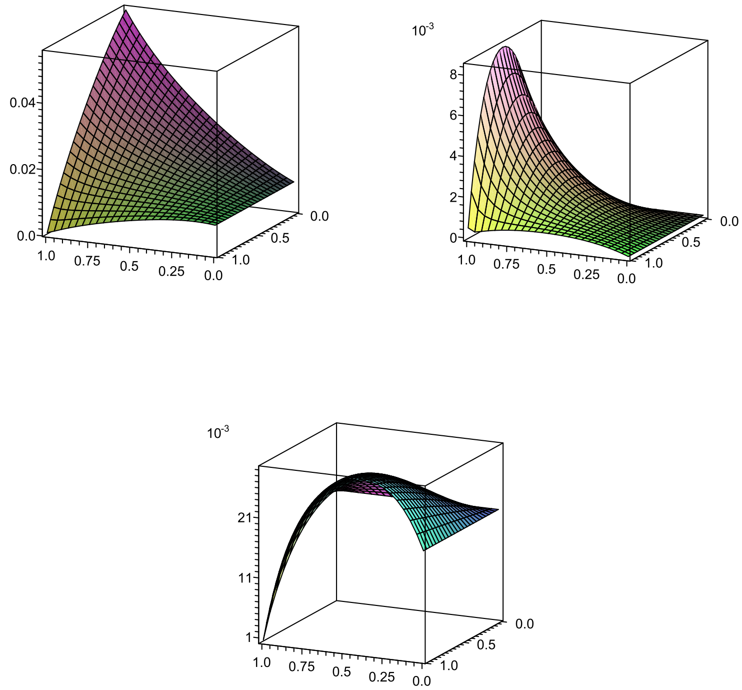

FIGURE D6: (a) $\Delta_{SC}^{(O-BN)} \geq 0$ (b)$\Delta_{SC}^{(BN-S)} \geq 0$ (c)$\Delta_{SC}^{(S-BC)} \geq 0$

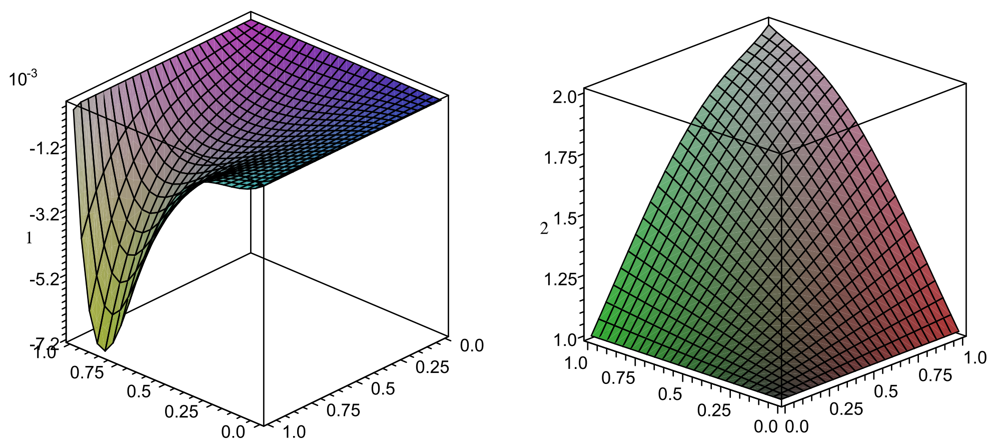

FIGURE D7: (a)Δ_1 and (b)Δ_2 with respect to γ and θ_u

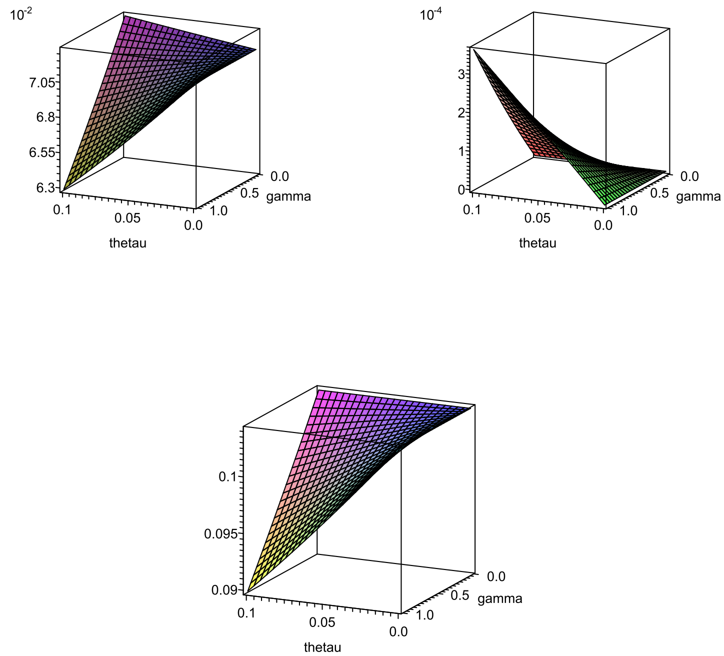

FIGURE D8: (a) $Q^O - Q^{BN} \geq 0$ (b)$Q^{BN} - Q^S \geq 0$ (c)$Q^S - Q^{BC} \geq 0$

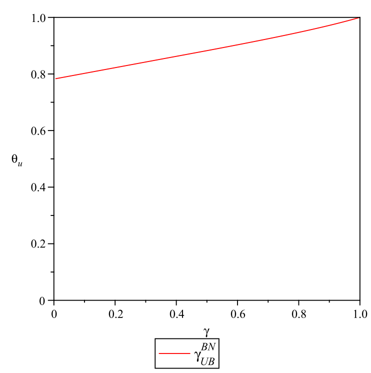

FIGURE D9: The area where channel conflict is resolved under buyback channel with no price commitment is defined by $\gamma_{UB}^{BN}(\theta_u)$

buyback channel without price commitment, Π_{1M}^{BN}, in Figure D10(a), shows the convexity of Π_{1M}^{BN} in γ. Besides, if $\theta_u \leq 0.69047$ then the manufacturer's objective function is strictly decreasing in γ from Figure D10(b). ■

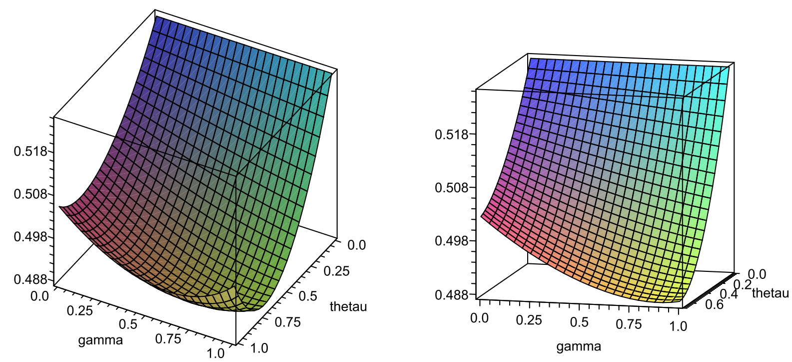

FIGURE D10: (a) Manufacturer's objective function with respect to θ_u and γ under buyback channel without price commitment (b) A cut of objective function for $\theta_u \leq 0.690$

Proof of Proposition 28. We solve problem (P1) first using Maple 11's solver tool called NLPSolve. In order to verify the solutions from Maple we use a brute force method. Since in Proposition 27 we show that the objective function is convex in γ, we know that the optimal point will be on one of the boundaries of feasible region defined by $\Delta_D^{BN} \geq 0$. Therefore by comparing the objective function values at $\gamma = 0$ and $\gamma = \gamma_{UB}^{BN}$ and picking the one that gives the maximum we obtain the optimal γ^*. Both methods give $\gamma^*(\theta_u) = 0$.

■

Proof of Proposition 29. The plot of the manufacturer's objective function under buyback channel with price commitment, Π_{1M}^{BC}, in Figure D11(a), shows the convexity of Π_{1M}^{BC} in γ. Besides, if $\theta_u \leq 0.608$ then the manufacturer's objective function is strictly decreasing in γ from Figure D11(b). ■

Proof of Proposition 30. We solve Problem (P2) with Maple 11's optimization tool NLPSolve and also with the alternative method explained in the proof of Proposition 28. We plot the optimal γ^* in Figure 4.6. We find that if $0.778 \leq \theta_u \leq 0.825$ then optimal γ^* is on the lower bound of the feasible region defined by $\Delta_D^{BC} \geq 0$ and $\Delta_R^{BC} \geq 0$,

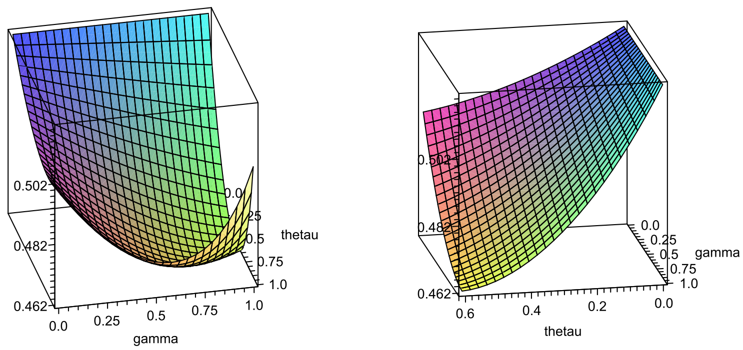

FIGURE D11: (a) Manufacturer's objective function with respect to θ_u and γ under buyback channel with price commitment (b) A cut of objective function for $\theta_u \leq 0.608$

i.e.$\gamma^*(\theta_u) = \gamma_{LB}^{BC}(\theta_u)$. However, if $\theta_u \geq 0.825$ then the optimal γ^* is on the upper bound of the feasible region, i.e.$\gamma^*(\theta_u) = \gamma_{UB}^{BC}(\theta_u)$. Finally, if $\theta_u \leq 0.778$ then the feasible region is empty as stated in Proposition 26. ■

Proof of Proposition 31. We plot the expected optimal profit functions $E_{\theta_u}(\Pi_{1j}^l)$ for $j = (M, R, D)$ and $l = (O, S, BN, BC)$ in the range of interest, i.e. $0 \leq \gamma \leq 1$ in Figure 4.7. In the rest of our discussion we drop the expectation operation and use Π_{1j}^l alone for the sake of simplicity.

1. Manufacturer: From Figure 4.7(a), it is immediate that for the manufacturer the ranking is $\Pi_{1M}^O \geq \Pi_{1M}^{BN} \geq \Pi_{1M}^S \geq \Pi_{1M}^{BC}$.

2. Rental Agency: From Figure 4.7(b) we identify that there are four possibilities for the rental agency's profit ranking depending on γ. (i) If $0 \leq \gamma \leq 0.549$ then $\Pi_{1R}^O \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^S \geq \Pi_{1R}^{BC}$. Also $\Pi_{1R}^{BC}(\gamma = 0.549) = \Pi_{1R}^S(\gamma = 0.549)$. (ii) If $0.549 \leq \gamma \leq 0.572$ then $\Pi_{1R}^O \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^S$. Also $\Pi_{1R}^{BN}(\gamma = 0.572) = \Pi_{1R}^{BC}(\gamma = 0.572)$. (iii) If $0.572 \leq \gamma \leq 0.765$ then $\Pi_{1R}^O \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^S$. Also $\Pi_{1R}^O(\gamma = 0.765) = \Pi_{1R}^{BC}(\gamma = 0.765)$. (iv) If $\gamma \geq 0.765$ then $\Pi_{1R}^{BC} \geq \Pi_{1R}^O \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^S$.

3. Dealer: From Figure 4.7(c), it is immediate that for the dealer the ranking is $\Pi_{1D}^{BC} \geq \Pi_{1D}^{S} \geq \Pi_{1D}^{BN} \geq \Pi_{1D}^{O}$.

4. Supply Chain: From Figure 4.7(d), we see that depending on the value of γ, the profit rankings may change as follows: (i) If $0 \leq \gamma \leq 0.962$ then $\Pi_{1R}^{O} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S} \geq \Pi_{1R}^{BC}$. (ii) If $0.962 \leq \gamma \leq 0.964$ then $\Pi_{1R}^{O} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{S}$. (iii) If $\gamma \geq 0.964$ then $\Pi_{1R}^{O} \geq \Pi_{1R}^{BC} \geq \Pi_{1R}^{BN} \geq \Pi_{1R}^{S}$. ■

Proof of Proposition 32. We plot the total sales Q^l for each channel structure l with respect to γ. From Figure 4.8, we observe that (i) If $0 \leq \gamma \leq 0.915$ then $Q^O \geq Q^{BN} \geq Q^S \geq Q^{BC}$ and $Q^S(\gamma = 0.915) = Q^{BC}(\gamma = 0.915)$. (ii) If $0.915 \leq \gamma \leq 0.918$ then $Q^O \geq Q^{BN} \geq Q^{BC} \geq Q^S$ and $Q^{BN}(\gamma = 0.918) = Q^{BC}(\gamma = 0.918)$. (iii) If $\gamma \geq 0.918$ then $Q^O \geq Q^{BC} \geq Q^{BN} \geq Q^S$. ■

Proof of Proposition 33. We plot Δ_R^{BC} and Δ_D^{BC} in Figure 4.9(b). From the figure it is immediate that it can never be the case where both $\Delta_R^{BC} \geq 0$ and $\Delta_D^{BC} \geq 0$ and thus the conflict can never be resolved.

We plot Δ_R^{BN} and Δ_D^{BN} in Figure 4.9(c). We find that both $\Delta_D^{BC} \geq 0$ for all γ. On the other hand, Δ_R^{BC} is positive only if $0.5485 \leq \gamma \leq 1$. Therefore the buyback channel with price commitment solves the channel conflict if and only if $0.5485 \leq \gamma \leq 1$. ■

REFERENCES

Abele, E., R. Anderl, H. Birkhofer. 2005. *Environmentally-friendly product development: methods and tools*. Springer.

Atasu, A., M. Sarvary, L.V. Van Wassenhove. 2009a. Efficient take-back legislation. *Production and Operations Management* **18** 243–258.

Atasu, A., R. Subramanian. 2009. Competition under product take-back laws: Individual or collective systems? Working Paper, College of Management, Georgia Institute of Technology, Atlanta, GA.

Atasu, A., B. Toktay, L.N. Van Wassenhove. 2009b. The impact of cost structure on reverse channel choice. Working Paper, College of Management, Georgia Institute of Technology, Atlanta, GA.

Auto Rental News. 1990. Buyback programs being questioned. *Auto Rental News* .

Auto Rental News. 2006. Rental companies favorably weather used-car trends. *Auto Rental News* .

BBCNews. 2007. New e-waste recycling laws begin URL `http://news.bbc.co.uk/go/pr/fr/-/2/hi/science/nature/6250080.stm, last accessed June 25, 2010`.

Bhaskaran, Sreekumar R., Stephen M. Gilbert. 2009. Implications of channel structure for leasing or selling durable goods. *Marketing Science* **28**(5) 918–934.

Bleiwas, D.I., T. Kelly. 2001. Obsolete computers, gold mine, or high-tech trash? Resource recovery from recycling. Tech. rep., U.S. Geological Survey. URL `http://pubs.usgs.gov/fs/fs060-01/fs060-01.pdf, last accessed June 25, 2010`.

Bohr, P. 2007. The economics of electronics recycling: New approaches to extended producer responsibility. Ph.D. thesis, der Technischen Universitat Berlin.

Boyaci, T., F. Toyasaki, V. Verter. 2007. An analysis of monopolistic and competitive take-back schemes for WEEE recycling. Working Paper, McGill University, Montreal, Quebec.

Bulow, J. 1982. Durable goods monopolists. *Journal of Political Economy* **90**(2) 314–332.

Canning, L. 2006. Rethinking market connections: mobile phone recovery, reuse and recycling in the UK. *Journal of Business and Industrial Marketing* **21**(5) 320–329.

CECED. 2002. Frequently asked questions about the domestic appliance industry's position on key issues of the WEEE Directive. Tech. rep. European Committee of Manufacturers of Domestic Appliances.

Corbett, C.J., D.A. Kirsch. 2001. International diffusion of ISO 14000 certification. *Production and Operations Management* **10**(3) 327–342.

Debo, L.G., L.B. Toktay, L.N. Van Wassenhove. 2005. Market segmentation and technology selection for remanufacturable products. *Management Science* **51**(8) 1193–1205.

Debo, L.G., L.B. Toktay, L.N. Van Wassenhove. 2006. Joint life-cycle dynamics of new and remanufactured products. *Production and Operations Management* **15**(4) 498–513.

Desai, Preyas, D. Purohit. 1999. Competition in durable goods markets: The strategic consequences of leasingand selling. *Marketing Science* **18**(1) 42–58.

Desai, Preyas S., Oded Koenigsberg, Devavrat Purohit. 2007. The role of production lead time and demand uncertainty in marketing durable goods. *Management Science* **53**(1) 150–158.

Esenduran, G., E. Kemahlioglu-Ziya, J.M. Swaminathan. 2010. Impact of take-back legislation on remanufacturing. Working Paper, University of North Carolina, Kenan-Flagler Business School, Chapel Hill, NC.

ETC, Electronics TakeBack Coalition. 2010 URL `http://www.electronicstakeback.com/legislation/federal_legislation.htm, last accessed March 15, 2010`.

Ferguson, M., L. B. Toktay. 2006. The effect of competition on recovery strategies. *Production and Operations Management* **15**(3) 351–368.

Ferrer, G., J. Swaminathan. 2006. Managing new and remanufactured products. *Management Science* **52**(1) 15–26.

Ferrer, G., J. Swaminathan. 2010. Managing new and differentiated remanufactured products. *European Journal of Operational Research* **203**(2) 370–379.

FES. 2003. Study into European WEEE schemes. Tech. rep., Future Energy Solutions, U.K. Department of Trade and Industry. URL `http://www.berr.gov.uk/files/file30305.pdf, last accessed June 25, 2010`.

GAO. 2005. Electronic wastes: Strengthening the role of the federal government in encouraging recycling and reuse. Tech. rep., United States Government Accountability Office.

Geyer, R., L. Van Wassenhove, A. Atasu. 2007. Economics of remanufacturing under limited component durability and finite life cycles. *Management Science* **53**(4) 88–100.

Gonzalez, J.L.M., N.P. Fumero. 2002. Environmental policy in a green market. *Environmental and Resource Economics* **22** 419–447.

Gray, C., M. Charter. 2007. Remanufacturing and product design: Designing for the 7th generation. Tech. rep., The Center for Sustainable Design, Farnham, UK.

Groenevelt, H., P. Majumder. 2001. Procurement competition in remanufacturing. Working Paper, Simon School of Business, University of Rochester, Rochester, NY.

Guide, Jr., V.D.R., R.H. Teunter, L.N. Van Wassenhove. 2003. Matching demand and supply to maximize profits from remanufacturing. *Manufacturing and Service Operations Management* **5**(4) 303–316.

Gyekye, L. 2009. WEEE PCSs work together to solve evidence trading problems URL `http://www.mrw.co.uk/3004951.article, last accessed June 25, 2010`.

Hageluken, C. 2006. Improving metal returns and eco-efficiency in electronics recycling a holistic approach to interface optimization between pre-processing and integrated metal smelting and refining. *IEEE International Symposium on Electronics and the Environment*. San Francisco, CA, 218–223.

Hammond, D., P. Beullens. 2007. Closed-loop supply chain network equilibrium under legislation. *European Journal of Operational Research* **183** 895–908.

HECC. 2005. Electronic waste: An examination of current activity, implications for environmental stewardship, and the proper federal role. Tech. rep., House of Committee on Energy and Commerce. URL `http://www.access.gpo.gov/congress/house/pdf/109hrg/22988.pdf, last accessed June 25, 2010`.

Heese, H. S., K. Cattani, G. Ferrer, W. Gilland, A.V. Roth. 2005. Competitive advantage through take-back of used products. *European Journal of Operational Research* **164** 143–157.

Henry, J. 2008. GM signals its getting out of the volume game. *BNET Auto* URL `http://industry.bnet.com/auto/100031/`.

HP. 2008. Testimony of Hewlett-Packard company before the House Committee on Science and Technology for the hearing on "electronic waste: Can the nation manage modern refuse in the digital age?". Tech. rep. URL `http://www.hp.com/hpinfo/abouthp/government/us/pdf/Sciencecommitteetestimony.pdf, last accessed June 25, 2010`.

Jacobs, B., R. Subramanian. 2009. Sharing responsibility for product recovery across the supply chain. Working Paper.

Jacobs, B. W., R. Subramanian. 2007. Sharing responsibility for product recovery across the supply chain. Working Paper, College of Management, Georgia Institute of Technology, Atlanta, GA.

Kokkinaki, A., R. Zuidwijk, J. van Nunen, R. Dekker. 2004. *Reverse Logistics: Quantitative Models for Closed-Loop Supply Chains*. Springer Verlag.

Lebreton, B. 2007. *Strategic closed-loop supply chain management*. Springer.

Majumder, P., H. Groenevelt. 2001. Competition in remanufacturing. *Production and Operations Management* **10**(2) 125–141.

Milgrom, Roberts. 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica* **58**(6).

Most, Eric. 2003. Calling all cell phones: Collection, reuse, and recycling programs in the US. Tech. rep., INFORM.

Nokia. 2006. Nokia CR Report. Tech. rep. URL `www.nokia.com/NOKIA_COM_1/Results/nokia_cr_2006.pdf, last accessed June 25, 2010`.

Nordhaus, W.D. 1991. To slow or not to slow: The economics of the greenhouse effect. *The Economic Journal* **101** 920–937.

Perchards. 2007. WEEE legislation and compliance in the 27 EU member states. Tech. rep., Perchards. URL `http://www.enviro.gov.sk/servlets/files/22184, last accessed June 25, 2010`.

Plambeck, E.L., T.A. Taylor. 2007. Green production through competitive testing Working Paper.

Plambeck, E.L., Q. Wang. 2010. Effects of e-waste regulation on new product introduction. *Management Science* **5**(3) 333–347.

Purohit, D. 1997. Dual distributions channels: The competition between rental agencies and dealers. *Marketing Science* **16**(3) 228–245.

Purohit, D., R. Staelin. 1994. Rentals, sales, and buybacks: Managing secondary distribution channels. *Journal of Marketing Research* **31**(3) 325–338.

Revisor of Statutes, Minnesota Office. 2007. Minnesota Session Laws 2007 - Chapter 48 H.F.No. 854 URL `https://www.revisor.mn.gov/bin/getpub.php?type=law&year=2007&sn=0&num=48, last accessed March 15, 2010`.

Ronningen, B. 2005. Competitors together in PRO (producers responsibility organization) a case study of the pro-system in Norway. *Proc. Int. Symposium on Electronics and the Environment*. May 16-19, New Orleans, US.

Rose, C. M., A. Stevels. 2001. Metrics for end-of-life strategies (ELSEIM). *Proceedings of the 2001 IEEE International Symposium on Electronics and the Environment*. Denver, CO, 100–105.

Savaskan, R.C., S. Bhattachaya, L.N. Van Wassenhove. 2004. Closed loop supply chain models with product remanufacturing. *Management Science* **50**(2) 239–252.

Savaskan, R.C., L.N. Van Wassenhove. 2006. Reverse channel design: The case of competing retailers. *Management Science* **52**(1) 1–14.

Sodhi, M.S., B. Reimer. 2001. Models for recycling electronics end-of-life products. *OR Spectrum* 97–115.

Stevels, A. 2003. Is the WEEE Directive ecoefficient? *IEEE Symposium on Electronics and the Environment* 7–12.

Stevels, A. 2004. Take-back and recycling of consumer electronics at work in the Netherlands. *In s.n. (Ed.), Take it back! WEST 2004. College Park: Raymond Communications.* 1–13.

Stokey, Nancy. 1981. Rational expectations and durable goods pricing. *Bell Journal of Economics* **12** 112–128.

Subramanian, R., S. Gupta, B. Talbot. 2007. Compliance strategies under permits for emissions. *Production and Operations Management* **16** 763–779.

Subramanian, R., S. Gupta, B. Talbot. 2009. Product design and supply chain coordination under extended producer responsibility. *Production and Operations Management* **18** 259–277.

Sundin, Erik. 2004. Product and process design for successful remanufacturing. Ph.D. thesis, Department of Mechanical Engineering, Linkopings Universitet.

Toffel, M.W. 2003. The growing strategic importance of end-of-life product management. *California Management Review* **45**(3) 102–129.

Tojo, N. 2003. EPR programmes: Individual vs. collective responsibility. Tech. rep., Industrial Institute for Industrial Environmental Economics, Lund University, Sweden.

Tojo, N. 2004. Extended producer responsibility as a driver for design change - Utopia or reality? Ph.D. thesis, Industrial Institute for Industrial Environmental Economics, Lund University, Sweden.

Toktay, L.B., L.M. Wein, S.A. Zenios. 2000. Inventory management of remanufacturable products. *Management Science* **46**(11) 1412–1426.

Toto, DeAnne. 2007. The second wave. *Recycling Today* URL `http://www.recyclingtoday.com/articles/article.asp?ID=6641, last accessed June 25, 2010`.

Van Rossem, Chris. 2008. Individual producer responsibility in the WEEE directive: From theory to practice? Ph.D. thesis, The International Institute for Industrial Environmental Economics, Lund University.

Webster, S., S. Mitra. 2007. Competitive strategy in remanufacturing and the impact of take-back laws. *Journal of Operations Management* **25**(6) 1123–1140.

WEEEForum. 2008. WEEE Forum guidance document on compliance with Directive 2002/96/EC on waste electrical and electronic equipment (WEEE). Tech. rep., European Association of Electric and Electronic Waste Take-Back Systems. URL `http://www.weee-forum.org/doc/WEEE_Forum_Guidance_for_Compliance_En_v2, last accessed June 25, 2010`.

Wojanowski, R., T. Boyaci, V. Verter. 2007. Retail-collection network design under deposit-refund. *Computers and Operations Research* **34** 324–345.

Zuidwijk, R., H. Krikke. 2007. Strategic response to EEE returns: Product eco-design or new recovery process? *European Journal of Operational Research* **191**(3) 1206–1222.