Housing Bubbles and Income Inequality

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ABSTRACT

ANDREW C. GRACZYK: Housing Bubbles and Income Inequality
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After a century of decline, income inequality has grown dramatically over the last several decades and is now one of the principal economic realities of our time. The recent housing bubbles and global financial crises have shown us that the Great Moderation may be over, and income inequality may be a problem with which the next generation of economists will have to contend. There is a growing body of literature showing that income inequality has the potential to fundamentally alter the very shape of the economic landscape. In this small collection of papers I show just a few ways that the presence of income inequality can alter our expectations about how a market should act in response to shocks, and the ways that shocks to a market in turn can accelerate the growth of inequality. The first paper, Regressive Welfare Effects of Housing Bubbles, shows that, in an economy with income inequality, a bubble attached to a durable, utility-yielding good can cause harm to low income borrowers even before the bubble bursts by reducing their lifetime acquisition of housing and consumption through higher prices and interest rates. This is in contrast to the standard bubble literature in which bubbles generally only cause harm when they collapse. In the second paper, Spatial Heterogeneity in Employment and Wage Growth After the Housing Crisis, I show evidence that the spatial differences (across MSAs and occupations) in the post-crisis period do not follow many of the general patterns that characterized the spatial differences of occupational wage and employment growth over much of the preceding two decades. This may be evidence of a new pattern of spatial economic differences at the MSA level after the housing crisis.

This dissertation is dedicated to my wife, Barbara, without whose support and affection I would never have been able to complete this project.

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CHAPTER 1

Regressive Welfare Effects of Housing Bubbles

1.1 Introduction

Many countries have experienced several episodes of bubble-like booms in asset prices. Examples include the real estate and stock price booms in Japan in the 1980s and South East Asia in the 1990s; the housing price booms in Ireland, Spain, and the U.S. in the 2000s; and the current housing price boom in China (27, 33, 21). In general, when there is a high demand for savings but limited investment outlet, the rates of returns from investment are depressed and real estate investment can serve as a prominent store of value. Thus, a low interest rate environment, as seen in the recent decade, provides a fertile ground for the emergence of asset bubbles, especially in real estate. Given the prevalence of bubble episodes, a central question arises for academics and policymakers: What are the welfare effects of asset bubbles, especially bubbles in real estate?

In this paper, we highlight the nuanced welfare effects of bubbles that are attached to housing. We develop a simple overlapping generations (OLG) model of bubbles with intra-generation heterogeneity and financial friction. As described in section 1.2 of the paper, households have identical preferences over a perishable consumption good and a durable and perfectly divisible housing asset in fixed supply. Young agents receive endowments, and a fraction of them are *savers*, who are born with high endowments, and the remaining fraction are *borrowers*, who are born with low endowments. Young borrowers, given their low endowment, need to borrow to purchase the desired amount of housing that maximizes their utility. In contrast, young savers, given their high endowment, do not need to borrow and instead save income for old age. Thus, for savers, housing not only yields utility dividend but also serves as a savings vehicle.

In an economy without financial friction, households can achieve their first best allocations by borrowing and lending in the credit market. However, in the presence of financial friction, such as imperfect contract enforcement, borrowers face a binding credit constraint, modeled as an exogenous limit on borrowers' debt capacity, as in Huggett (26), Aiyagari (1), and Eggertsson and Krugman (20).

In equilibrium, the constraint effectively limits how much savers can store their income by investing in the credit market. As we show in section 1.3, in an economy with high income inequality, there is a shortage of storage for savers, which can lead to an equilibrium interest rate that is below the economy's growth rate. The low interest rate environment in turn facilitates the emergence of asset bubbles.

In section 1.4, the main part of our paper, we study housing bubbles. In a housing bubble equilibrium, the price of one unit of housing consists of a fundamental component equal to the net present value of the stream of utility dividends, and a bubble component, which grows at the interest rate. The housing bubble causes two macroeconomic effects: it raises the equilibrium interest rate and it raises the equilibrium housing price (relative to the bubble-less benchmark).

We then show that the housing bubble has opposite effects on borrowers and savers. On the one hand, the housing bubble increases the return from real estate investment for high-income savers, who demand storage of value, and hence increases their welfare (relative to the bubble-less benchmark). On the other hand, by raising the interest rate on debt and raising the housing price, the bubble *reduces* the wealth and hence the welfare of low-income borrowers, who in equilibrium have a relatively high marginal utility from housing. By positively affecting high-income savers and negatively affecting low-income borrowers, the housing bubble thus has *regressive welfare effects*. Overall, the results so far imply a feedback loop on inequality: high income inequality depresses the interest rate, thereby facilitating the existence of housing bubbles, which in turn have regressive welfare effects.

In comparison, section 1.5 shows that the regressive welfare implications are lessened if the model considers pure bubbles, which are widely used in the rational bubble literature for their simplicity. A pure bubble is an asset that has no fundamental value,[1] but which is traded at a positive price. The pure bubble provides an additional investment vehicle for savers: besides investing in the credit market and the housing market, savers can invest in the bubble market by purchasing the bubble asset when young and reselling it when old. However, the bubble provides no useful service for borrowers who do not want to save. Thus, unlike the housing bubble equilibrium, the pure bubble equilibrium is characterized by an endogenous *segmentation* in the bubble market, as only savers purchase the bubble asset. However, the pure bubble does crowd out savers' investment in household debt of borrowers, which drives up the interest rate on debt and restricts the amount of resources available to borrowers in young age (relative to an equilibrium with no bubbles). This means that the borrowers will experience lower lifetime welfare in the pure bubble equilibrium than in the equilibrium without bubbles. But, since the pure bubble will not increase the price of housing, the negative effect on borrowers' welfare relative to the equilibrium without bubbles is smaller in the pure bubble than in the housing bubble equilibrium.

Related literature. Our paper is related to the rational bubble literature, which has a long heritage dating back to Samuelson (39), Diamond (17), and Tirole (41). For a survey of this literature, see Miao (34).[2] Much of the literature has focused on a positive analysis of bubbles. A common theme in this literature is that rational bubbles emerge to reduce some inefficiency in the financial market, such as an aggregate shortage of assets for storage or a credit market imperfection, as in Hirano and Yanagawa (25), Miao and Wang (35), Martin and Ventura (32), and Ikeda and Phan (28).

By departing from the pure bubble assumption and modeling a bubble as attached to a fundamentally useful durable asset such as housing, our paper is related to Arce and López-Salido (2), Miao and Wang (36), Wang and Wen (42), Hillebrand and Kikuchi (23), Zhao (43) and Basco (9).

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[1]Examples include such as tulips, fiat money, or stocks of an unproductive firm. In fact, the literature often uses the boom in the stock prices of many dot com firms in the U.S. in the late 1990s and early 2000s as an example of a pure bubble (see, inter alia, 32).

[2]There is another bubble literature that focuses on the role of information in coordinating agents' actions to purchase and sell bubbles. See, inter alia, Brunnermeier (15), Doblas-Madrid (18), Barlevy (8), and Doblas-Madrid and Lansing (19).

A common theme among Arce and López-Salido (2), Zhao (43), and Basco (9) is that they focus on setups where agents have heterogeneous preferences for housing, and they define a housing bubble as an equilibrium where some agents, who derive no direct utility or use from an asset, purchase the asset purely as a store of value. In contrast, we focus on setups where agents have identical preferences and all derive utility from housing, and we define a housing bubble as an equilibrium where the price of the housing asset has a bubble component. Our model of fundamental bubbles is thus more related to Blanchard and Watson (12), Wang and Wen (42), and Hillebrand and Kikuchi (23). The main difference is our paper's focus on a welfare analysis. A second difference is our emphasis on income inequality.

To the best of our knowledge, among papers that analyze the welfare effects of bubbles, ours is the first to document regressive welfare effects of a housing bubble. Saint-Paul (38), Grossman and Yanagawa (22), and King and Ferguson (30) show that if there is a positive externality in the accumulation of capital, the emergence of bubbles on an unproductive asset would inefficiently divert resources from investment. Similarly, Hirano et al. (24) show that oversized bubbles inefficiently crowd out productive investment. On the other hand, Miao et al. (37) show that bubbles can crowd in too much investment. Caballero and Krishnamurthy (16) show that bubbles can marginally crowd out domestic savings and cause a shortage of liquid international assets in a small open economy framework. Focusing instead on risk, Ikeda and Phan (29) show that rational bubbles financed by credit can be excessively risky. The regressive welfare effects that we highlight are complementary to the effects highlighted by these papers.

1.2 Model

Consider an endowment economy with overlapping generations of agents who live for two periods. Time is discrete and infinite, with dates denoted by $t = 0, 1, 2, \ldots$. The population of young households in each period is constant with population $L_t = 1$ for all t. There is a consumption good and a housing asset. The consumption good is perishable and cannot be stored. The housing asset is durable, perfectly divisible. The supply of housing is fixed to one. The consumption good is the numeraire and the market price of a unit of the housing asset is denoted by p_t.

Heterogeneity. Each generation consists of two groups of households, savers and debtors/borrowers, denoted by $i \in \{s, d\}$, with equal measure of each group. Each young household is endowed with e^i of the consumption good, where $e^s > e^d$. In addition, each household receives an endowment of $\epsilon > 0$ when old. Let $e^s + e^d + = e$ be the total endowment the young.[3]

Preferences. Households derive utility from the housing asset and from the consumption good, consumed both when young and old. Denote their utility function by $U(h_t^i, c_{t,y}^i, c_{t+1,o}^i)$, where h_t^i denotes the housing and $c_{t,y}^i$ and $c_{t+1,o}^i$ denote consumption in young and old age of a household of type $i \in \{s, d\}$ born in period t. We assume utility from consumption and housing separable such that:

$$U(h_t^i, c_{t,y}^i, c_{t+1,o}^i) = v(h_t^i) + u(c_{t,y}^i) + \beta u(c_{t+1,o}^i),$$

where β is the discount factor, and u and $v : (0, \infty) \to \mathbf{R}$ satisfy the usual conditions ($u', v' > 0, u'', v'' < 0, \lim_{c \to 0+} u'(c) = \lim_{h \to 0+} v'(h) = \infty$).

Credit market and credit friction. Households can borrow and lend to each other via a credit market. Let $1 + r_t$ denote the gross interest rate for debt between period t and $t + 1$. As in Bewley (11), Huggett (26) and Aiyagari (1), we model credit friction in the simplest possible way: an agent can commit to repay at most $\bar{d}$ units of the consumption good, where $\bar{d} > 0$ is an exogenous debt limit. This imperfection in the financial market will lead to a constraint on households' ability to borrow, as manifested in the optimization problem below.

A household purchases housing, consumes, and borrows or lends when young, and then sells their housing asset and consumes when old. As in Lorenzoni (31) and Hillebrand and Kikuchi (23), we assume a per-unit maintenance cost $\kappa > 0$ on the durable housing asset, which the household has to pay before re-selling the asset.[4] The optimization problem of a young household of type

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[3]We thus effectively set the economic growth rate to be zero, but the results extend easily to any exogenous growth setting.

[4]This cost prevents the housing price from exploding when the interest rate falls below the growth rate of the economy.

$i \in \{s, d\}$ born in period t consists of choosing housing asset position h_t^i, net financial asset position a_t^i, and old-age consumption c_{t+1}^i to maximize lifetime utility:

$$\max_{h_t^i, c_{t,y}^i, c_{t+1,o}^i, a_t^i} U(h_t^i, c_{t,y}^i, c_{t+1,o}^i) \tag{1.1}$$

subject to a budget constraint in young age:

$$p_t h_t^i + \frac{1}{1 + r_t} a_t^i + c_{t,y}^i = e^i,$$

a budget constraint in old age (taking into account the maintenance cost of housing):

$$c_{t+1,o}^i = (p_{t+1} - \kappa) h_t^i + a_t^i + \epsilon,$$

a short-selling constraint on the housing asset:

$$h_t^i \geq 0,$$

non-negativity constraints on consumption in both periods of life:

$$c_{t,y}^i, c_{t+1,o}^i \geq 0$$

and the credit constraint:

$$a_t^i \geq -\bar{d}. \tag{1.2}$$

Finally, to close the model, without loss of generality assume that the old savers own the entire supply of housing in the initial period $t = 0$. We define an equilibrium as follows:

Definition 1. An *equilibrium* consists of allocation $\{h_t^i, c_{t,y}^i, c_{t+1,o}^i, a_t^i\}_{t \geq 0}$ and prices $\{p_t, r_t\}_{t \geq 0}$ such that:

1. Given prices, the allocations solve the optimization problem (1.1) for all $i \in \{s, d\}$ and $t \geq 0$.

2. The consumption good market clears:

$$c_{t,y}^s + c_{t,y}^d + p_t h_t = e + \epsilon, \forall t \geq 0;$$

3. The credit market clears:

$$a_t^s + a_t^d = 0, \forall t \geq 0;$$

4. And the housing market clears:

$$h_t^s + h_t^d = 1, \forall t \geq 0.$$

We will be mainly focusing on *stationary* equilibria, which are equilibria where quantities and prices are time-invariant.

Throughout the paper we assume $\bar{d}$ is sufficiently small so that the credit constraint (1.2) is always binding in any equilibrium. Then, the credit market clearing condition implies that savers must be lending and hence are not credit constrained. From first order conditions, we know this will be the case when young age consumption for each household type obeys:

$$u'(c_{t,y}^s) = \beta(1 + r_t)u'(c_{t+1,o}^s), \tag{1.3}$$

$$u'(c_{t,y}^d) > \beta(1 + r_t)u'(c_{t+1,o}^d). \tag{1.4}$$

Therefore, both the equilibrium housing price and interest rate are determined by the first order conditions of savers. In particular, the equilibrium housing price is given by:[5]

$$p_t = \frac{v'(h_t)}{u'(c_t)} + \frac{p_{t+1} - \kappa}{1 + r_t}. \tag{1.5}$$

Asset pricing equation (1.5) states that the price of one unit of housing in period t is equal to the sum of the marginal utility from housing, plus the discounted resale value p_{t+1}, net the maintenance cost, and discounted by the gross interest rate $1 + r_t$. It is convenient for exposition to rewrite (1.5) in terms of the price *net* of the maintenance cost:

$$p_t - \kappa = \frac{v'(h_t)}{u'(c_t)} - \kappa + \frac{p_{t+1} - \kappa}{1 + r_t}. \tag{1.6}$$

Recursively substituting the equations for $p_{t+1} - \kappa, p_{t+2} - \kappa$, and so on yields:

$$p_t - \kappa = \sum_{j \geq 0} \frac{1}{\Pi_{k=1}^{j}(1 + r_{t+k})} \left(\frac{v'(h^s_{t+j})}{u'(c^s_{t+j})} - \kappa \right) + \lim_{j \to \infty} \frac{p_{t+j+1} - \kappa}{\Pi_{k=1}^{j}(1 + r_{t+k})}, \tag{1.7}$$

which shows that the price of housing (net maintenance cost) is equal to its discounted net dividend stream, which we will call the *fundamental value of housing*, plus a pricing residual, which we call the *bubble component*. We now proceed to show that this setup can support multiple equilibria: a bubble-less equilibrium (where the bubble component is zero) and a housing bubble equilibrium (where the bubble component is positive).

[5]This equation is derived from savers' first order conditions:

$$p_t \frac{\partial U^s}{\partial c^s_{t,y}} = \frac{\partial U^s}{\partial h^s_t} + (p_{t+1} - \kappa) \frac{\partial U^s}{\partial c^s_{t+1,o}}.$$

Because savers are not credit constrained, we have $\frac{\partial U^s}{\partial c^s_{t,y}} = (1 + r_t) \frac{\partial U^s}{\partial c^s_{t+1,o}}$, or equivalently, $u'(c^s_{t,y}) = \beta(1 + r_t) u'(c^s_{t+1,o})$, which yields (1.5).

1.3 Bubble-less Equilibrium

We start with the bubble-less benchmark. In this case, the housing price that solves asset pricing equation (1.5) is simply the net present value of the net dividend stream:

$$p_t - \kappa = \sum_{j \geq 0} \frac{1}{\Pi_{k=0}^{j}(1+r_{t+k})} \left(\frac{v'(h_{t+j}^s)}{u'(c_{t+j}^s)} - \kappa \right). \tag{1.8}$$

We can then see that a stationary equilibrium exists with $p_t = p_{t+1} = \kappa$:

$$p_t = \kappa = \frac{v'(h_t^s)}{u'(c_{t,y}^s)} = \frac{v'(h_t^d)}{u'(c_{t,y}^d)}. \tag{1.9}$$

In this case, using the consumption rule for savers we can determine the interest rate in the stationary equilibrium as:

$$1 + r = \frac{v'(h^s)}{\beta \kappa u'(c_o^s)}. \tag{1.10}$$

We call this the *bubble-less equilibrium*. The equilibrium interest rate r_n and housing price p_n (where the subscript n stands for no-bubble) satisfy the steady state version of (1.8) with $p_n = \kappa$, as formalized in the following lemma:

Lemma 1 (Bubble-less equilibrium)**.** *There exists a stationary equilibrium with the price of housing p_n equal to the maintenance cost on housing κ in all periods. The interest rate in this equilibrium will be $r_n < 0$ for e^s sufficiently large.*

The stationary bubble-less equilibrium is characterized by the first order conditions of the savers:

$$\begin{aligned} p_n &= \kappa = \frac{v'(h_n^s)}{u'(c_{n,y}^s)}, \\ u'(c_{n,y}^s) &= \beta(1+r_n)u'(\epsilon + \bar{d}), \end{aligned}$$

the first order conditions of the borrowers, who are credit constrained:

$$\begin{aligned} p_n &= \kappa = \frac{v'(h_n^d)}{u'(c_n^d)} \\ u'(c_{y,n}^d) &> \beta(1+r_n)u'(\epsilon-\bar{d}), \end{aligned}$$

the budget constraints:

$$\begin{aligned} c_{y,n}^s &= e^s - \frac{\bar{d}}{1+r_n} - p_n h_n^s \\ c_{y,n}^d &= e^d + \frac{\bar{d}}{1+r_n} - p_n h_n^d, \end{aligned}$$

and market clearing conditions: $h_n^s + h^d = 1$, $c_{y,n}^s + c_{y,n}^d + p_n = e$.

Proof. Appendix 3.0.1. □

Furthermore, it is straightforward to show that r_n is increasing in debt limit $\bar{d}$, and decreasing in savers' endowment e^s. Thus, either an increase in credit friction (a lower $\bar{d}$) or an increase in inequality will lower the interest rate. As is well known, a low interest rate environment is a fertile ground for bubbles to arise.

1.4 Housing bubble

We now construct an equilibrium, where in the asset pricing equation (1.7), the bubble component in the price of one unit of housing (net of maintenance) is positive:

$$p_t - \kappa = \underbrace{\sum_{j\geq 0} \frac{1}{\Pi_{k=0}^{j}(1+r_{t+k})}\left(\frac{v'(h_{t+j}^s)}{u'(c_{t+j,y}^s)} - \kappa\right)}_{\text{fundamental component } f_t} + \underbrace{\lim_{j\to\infty} \frac{p_{t+j+1}-\kappa}{\Pi_{k=0}^{j}(1+r_{t+k})}}_{\text{bubble component } b_t>0}.$$

As we focus on a stationary equilibrium, the asset equation above simplifies to:

$$p_h = f_h + b_h,$$

where we use the subscript h to denote variables in the stationary housing bubble equilibrium. The bubble term on the right of this equation converges to a finite, positive number if and only if $r_h = 0$. This means that the fundamental value can only converge to a finite, positive value if:

$$\kappa = \frac{v'(h_t^s)}{u'(c_{t,y}^s)},$$

which means that the fundamental value of the marginal unit of housing purchased must be equal to the maintenance cost on housing ($f_h = \kappa$), with the price of housing now given by:

$$p_h = \kappa + b_h. \tag{1.11}$$

1.4.1 Existence and characteristics

Lemma 2 below shows that the housing bubble equilibrium can exist if and only if the bubble-less interest rate is smaller than the housing bubble interest rate:

$$1 + r_n < 1 + r_h = 1.$$

Intuitively, when savers are sufficiently wealthy and maintenance costs on housing are sufficiently high, the economy is dynamically inefficient and there is a shortage of storage for savers. A bubble that arises in the value of housing helps reduce the dynamic inefficiency by raising the returns from housing investment for savers. The housing bubble improves the storage service provided by the durable housing asset.

The lemma below summarizes the existence condition and characterizes the allocations and prices of the housing bubble steady state.

Lemma 2 (Housing Bubble Equilibrium)**.** *The housing bubble can exist with $b_h > 0$ in a steady state if and only if the bubble-less equilibrium interest rate has $r_n < 0$. If $r_n < 0$ in the bubble-less equilibrium then there will exist a unique value of b_h at which a housing bubble steady state can occur.*

The housing bubble steady state is then characterized by the first order conditions of savers:

$$u'(c^s_{y,h})(\kappa + b_h) = v'(h^s_h) + \beta u'(b_h h^s_h + \epsilon + \bar{d})b_h, \quad (1.12)$$

$$u'(c^s_{y,h}) = \beta u'(c^s_{o,h}), \quad (1.13)$$

the first order conditions of borrowers, who are credit constrained:

$$u'(c^d_{y,h})(\kappa + b_h) = v'(h^d_h) + \beta u'(c^d_{o,h})(\kappa + b_h), \quad (1.14)$$

$$u'(c^d_{y,h}) > \beta u'(b_h h^d_h + \epsilon - \bar{d}), \quad (1.15)$$

and budget constraints:

$$c^s_{y,h} = e^s - \bar{d} - p_h h^s_h, \quad (1.16)$$

$$c^d_{y,h} = e^d + \bar{d} - p_h h^d_h, \quad (1.17)$$

and market clearing conditions: $h^s_h + h^d_h = 1$, $c^s_{y,h} + c^d_{y,h} + p_h = e$.

Proof. Appendix 3.0.2. □

Corollary 3 (The bubble raises the price of housing)**.** *The price of housing in the housing bubble steady state is greater than the price of housing in the bubble-less steady state:*

$$p_h > p_n.$$

This must be the case since, for any $b_h > 0$, $p_h = b_h + \kappa > \kappa = p_n$. *Therefore, as long as* $b_h > 0$, *the bubble attached to housing* raises *the price of housing.*

1.4.2 Welfare analysis

We can now address the main question of interest: What are the welfare effects of the housing bubble? We define the welfare of a household of type $i \in \{s, d\}$ in the housing bubble steady state as $U_h^i = U(h_h^i, c_{y,h}^i, c_{o,h}^i)$, where h_h^i, $c_{y,h}^i$ and $c_{o,h}^i$ are given in lemma 2. Similar for the welfare U_n^i in the bubble-less steady state.

The bubble has heterogeneous effects on savers and borrowers. For savers, who want to save for old age and use the housing asset in part as a savings vehicle, the housing bubble improves welfare. This is because the bubble raises the interest rate (from $1 + r_n$ to 1), hence increasing the return from lending. This effect is summarized in the following lemma:

Lemma 4. *The housing bubble improves savers' welfare: $U_h^s > U_n^s$.*

Proof. Appendix 3.0.3. □

In contrast, the housing bubble has an *unambiguously negative* effect on the welfare of borrowers. This is because it increases the interest rate paid on debt, hence reducing the amount that borrowers can borrow when young. At the same time, it increases the price of housing, hence reducing the amount of housing that borrowers purchase and consequently their housing utility. This effect is summarized in the following lemma:

Lemma 5. *The housing bubble reduces borrowers' welfare: $U_h^d < U_n^d$.*

Proof. Appendix 3.0.4. □

The combination of lemmas 4 and 5 yields the main result of our paper:

Proposition 6 (Regressive welfare effects of housing bubble)**.** *The housing bubble improves welfare for savers but reduces welfare for borrowers (relative to the bubble-less steady state).*

This proposition highlights the regressive welfare effects of a housing bubble. The housing bubble improves welfare for savers by providing them with a more efficient way to store value. However, by raising the cost of debt and the price of housing, the bubble reduces borrowers' ability to purchase housing. Therefore, an interesting implication arises on the bi-directional relationship between inequality and bubble: high income inequality depresses the interest rate, facilitating the existence of housing bubbles, which in turn have regressive welfare effects.

1.5 Pure bubble equilibrium

To appreciate the welfare results established in the previous section, we compare them against the welfare effects of a pure bubble, which is an asset that pays no dividend but has a positive market price. Common interpretations for this type of asset include a positive price on the stock of an unproductive firm, fiat money, or the famous ”tulip mania” in the Netherlands. This asset can be useful as a savings instrument. However, unlike housing, the pure bubble asset does *not* give households any direct utility. As a consequence, there will be an endogenous *segmentation* of the pure bubble market: only savers purchase the asset to store income for old age. This leads to another important distinction between the housing asset and the pure bubble asset: it is never optimal for borrowers to go into debt to acquire the pure bubble.

Formally, assume there is an asset that pays no dividend but is traded at price $\tilde{b}_t$ per unit (assuming that the bubble has not collapsed). Given prices, each household of type i chooses its holding $x_t^i \geq 0$ of the bubble asset. Their optimization problem is:

$$\max_{h_t^i, c_{t,y}^i, c_{t+1,o}^i, x_t^i, a_t^i} \quad U(h_t^i, c_{t,y}^i, c_{t+1,o}^i), \tag{1.18}$$

The addition of the bubble asset will change the aggregate market clearing condition:

$$p_t h_t + \tilde{b}_t + c_{y,t} = e,$$

where $\tilde{b}_t$ represents the total value of the bubble asset in time t. We assume a fixed supply of the bubble, b_0=1.

Budget constraints for young and old households respectively in this setup are given as follows:

$$e^i = p_t h_t^i + \tilde{b}_t x_t^i + a_t^i \frac{1}{1+r_t} + c_{y,t}^i, \tag{1.19}$$

$$c_{o,t+1}^i = (p_{t+1} - \kappa) h_t^i + \tilde{b}_{t+1} x_t^i + a_t^i, \tag{1.20}$$

Households then solve their maximization problem (1.18) subject to the budget constraints, the credit constraint:

$$a_t^i \geq -\bar{d},$$

non-negativity constraints on consumption in both periods of life:

$$c_{t,y}^i, c_{t+1,o}^i \geq 0,$$

and no short-selling constraints on housing and the bubble asset:

$$x_t^i, h_t^i \geq 0.$$

To close the model, assume that old savers own the entire supply of housing and the bubble in the initial period $t = 0$. A pure bubble equilibrium is defined as follows:

Definition 2. An *equilibrium* consists of allocation $\{h_t^i, c_{t,y}^i, c_{t+1,o}^i, a_t^i, x_t^i\}_{t\geq 0}$ and prices $\{p_t, r_t, b_t\}_{t\geq 0}$ such that:

1. Given prices, the allocations solve the optimization problem (1.18) for all $i \in \{s, d\}$ and $t \geq 0$.

2. The consumption good market clears:

$$c_{t,y}^s + c_{t,y}^d + p_t + \tilde{b}_t = e, \forall t \geq 0;$$

3. The credit market clears:

$$a_t^s + a_t^d = 0, \forall t \geq 0;$$

4. The housing market clears:

$$h_t^s + h_t^d = h_t, \forall t \geq 0;$$

5. And the bubble market clears:

$$x_t^s + x_t^d = 1.\forall t \geq 0.$$

We focus on *asymptotic* pure bubble equilibria, where the bubble does not vanish, i.e., $\lim_{t\to\infty} \tilde{b}_t > 0$. A pure bubble *steady state* is an asymptotic pure bubble equilibrium where prices and quantities are time-invariant.

1.5.1 Existence and characteristics

We now formalize the existence and characteristics of a pure bubble equilibrium. The first order conditions of savers imply that the following no-arbitrage condition must hold for the bubble asset:

$$\tilde{b}_{t+1} = (1 + r_t)\tilde{b}_t. \tag{1.21}$$

This equation equates the return from lending in the credit market and the return from speculating in the bubble market for savers. It is a standard equation stating that in any pure bubble equilibrium, the bubble price must grow at the interest rate. Intuitively, if this condition does not hold, then either the bubble yields a greater expected return than lending (which means savers would never lend in equilibrium) or lending would have a greater return than the bubble (which means savers would have no demand for the bubble). It also implies a standard identity that the interest rate in any pure bubble steady state must be equal to the growth rate of the economy (which we have normalized to zero):

$$1 + r_p = 1,$$

as in the housing bubble case (the subscript p stands for pure-bubble).

The first order conditions of savers and borrowers yield pricing equations similar to the bubble-less equilibrium:

$$\begin{aligned} p_t &= \frac{v'(h_t^s)}{u'(c_{t,y}^s)} + \frac{p_{t+1} - \kappa}{1 + r_t}, \\ &= u'(c_{t,y}^d)^{-1} \left(v'(h_t^d) + \beta(1 + r_t) u'(c_{t+1,o}^d)(p_{t+1} - \kappa) \right), \end{aligned}$$

which equate the price with discounted dividends of housing for savers and borrowers respectively. In steady state, since $1 + r = 1$, these equations become:

$$p_p = \frac{v'(h_p^s)}{u'(c_{p,y}^s)} + p_p - \kappa, \tag{1.22}$$

$$= u'(c_{t,y}^d)^{-1}(v'(h_t^d) + \beta(1 + r_t)u'(c_{p,o}^d)(p_p - \kappa)). \tag{1.23}$$

Equation (1.22) reveals that the price of housing must once again be equal to the maintenance cost κ in the pure bubble steady state. Since only savers invest in the pure bubble, we can use the savers' young-age budget constraint to find an equation that determines the size of the bubble in equilibrium:

$$\tilde{b} = e^s - \bar{d} - c_{p,y}^s - \kappa h_p^s, \tag{1.24}$$

which confirms that the amount that savers invest in the bubble asset is simply whatever income they have left over after consuming the optimal amount in young age, lending to borrowers, and purchasing housing to satisfy utility demands.

The subsequent lemma shows that $\tilde{b} > 0$ if and only if $1 + r_n < 1$, as in the case of the housing bubble. As in lemma 2, it also characterizes the equilibrium allocations and prices.

Lemma 7 (Pure Bubble Equilibrium). *The pure bubble can exist with $\tilde{b} > 0$ in a steady state if and only if the bubble-less equilibrium interest rate has $r_n < 0$. If $r_n < 0$ in the bubble-less equilibrium then there will exist a unique value of $\tilde{b}$ at which a pure bubble steady state can occur.*

The pure bubble *steady state is then characterized by the first order conditions of savers:*

$$u'(c^s_{y,p})\kappa = v'(h^s_h), \tag{1.25}$$

$$u'(c^s_{y,p}) = \beta u'(\tilde{b}_p + \epsilon + \bar{d}), \tag{1.26}$$

the first order conditions of borrowers, who are credit constrained:

$$u'(c^d_{y,p})\kappa = v'(h^d_h), \tag{1.27}$$

$$u'(c^d_{y,p}) > \beta u'(\epsilon - \bar{d}), \tag{1.28}$$

and budget constraints:

$$c^s_{p,y} = e^s - \bar{d} - \kappa h^s_p - \tilde{b}_p, \tag{1.29}$$

$$c^d_{y,p} = e^d + \bar{d} - \kappa h^d_p, \tag{1.30}$$

and market clearing conditions: $h^s_p + h^d_p = 1$, $x^s = 1$, $c^s_{y,p} + c^d_{y,p} + p_p = e$.

Proof. Appendix 3.0.5. □

1.5.2 Welfare analysis

Are the welfare implications of a pure bubble different from those of a housing bubble? As before, the welfare in the pure bubble steady sate is defined as $U^i_p = U(h^i_p, c^i_{y,p}, c^i_{o,p})$, where h^i_p, $c^i_{y,p}$, and $c^s_{o,p}$ are given in lemma 7. In a low interest rate environment, a standard result is that a pure bubble allows savers to store their income into old age more efficiently and hence improves their welfare relative to the bubble-less benchmark. This result also holds in our environment:

Lemma 8. *The pure bubble improves welfare for savers:* $U^s_p > U^s_n$.

Proof. Appendix 3.0.6 □

On the other hand, borrowers face a higher interest rate in the pure bubble equilibrium than in the bubble-less equilibrium ($r_n < 0 = r_p$), which limits their ability to borrow when young. We also see that the price of housing is unchanged from the bubble-less equilibrium. Therefore, borrowers will experience less lifetime utility under the pure bubble equilibrium than in the bubble-less equilibrium. We can then say that the pure bubble, like the bubble on housing, generates *regressive welfare effects*.

Lemma 9. *The pure bubble harms the welfare of borrowers: $U_p^d < U_n^d$.*

Proof. Appendix 3.0.7. □

But, there remains an important distinction between the two bubbles: the housing bubble, in addition to increasing the interest rate, also increases the price of housing.

1.5.3 Welfare comparison across bubble steady states

We can also compare household welfare across bubble equilibria since both bubbles can exist under the same condition (that $1 + r_n < 1$). We can show that borrowers have higher lifetime welfare in the pure bubble equilibrium than in the housing bubble equilibrium. This is because the housing bubble raises the interest rate and the price of housing for borrowers. The pure bubble also raises the interest by the same amount, but does not raise the housing price. Therefore, borrowers are always better able to acquire housing (and consume when young) in a pure bubble equilibrium than in a housing bubble equilibrium.

Savers also prefer the pure bubble to the housing bubble. The housing bubble improves savers' welfare over the bubble-less equilibrium by raising the interest rate and increasing the housing price, but still requires savers to save excess wealth in housing. Since housing requires maintenance at cost κ, the efficiency of housing as a store of value is always inferior to that of the pure bubble. The pure bubble then allows savers to more efficiently allocate wealth across periods of life in the pure bubble equilibrium than housing does in the housing bubble equilibrium.

Corollary 10. *The pure bubble steady state Pareto dominates the housing bubble steady state.*

Proof. Appendix 3.0.8 □

1.6 Conclusion

We have shown that a housing bubble, or, more generally, a bubble attached to a fundamentally useful asset, has heterogeneous welfare effects on borrowers and savers. By providing an additional investment vehicle, it raises the returns from investment for savers and thus improves their welfare. However, by raising the interest rate on debt and raising the housing price, the housing bubble negatively affects the welfare of borrowers, who need debt to finance their purchase of housing. Overall, our model implies a feedback loop on inequality: high income inequality leads to an environment with low interest rates, which facilitate housing bubbles, which in turn have regressive welfare effects.

Even though we model income inequality in an overly simple way, our model suggests an interesting possibility that, in an economy with sufficient income inequality and credit friction, consumption and welfare inequality can be exacerbated by the housing bubble. This result is relevant for the ongoing debate about inequality, especially in the U.S.. Future research can explore the interaction between bubbles and inequality in a more sophisticated framework of endogenous income/wealth inequality (e.g., with longer-lived overlapping generations).

CHAPTER 2

Spatial Heterogeneity in Employment and Wage Growth After the Housing Crisis

2.1 Introduction

In this paper I examine the structure of employment of different occupations across MSAs in the United States, and find that there are very different patterns to occupational employment and wage changes in the dot-com recession recovery period and the recovery from the housing bubble recession, both in terms of which MSAs experience the highest growth and which and what kinds of occupation groups recovered employment and wages during the crisis recovery.

I find that, in keeping with the theory of labor demand proposed in Beaudry et al. (10), occupations with generically higher cognitive task requirements experience greater wage and employment growth than other occupations in most metropolitan areas between 1999 and 2004. However, the model in Beaudry et al. (10) predicts that employment and income of cognitive-focused occupations (for example: managerial occupations) should rise and fall in conjunction with other cognitively demanding occupations, which is shown to be false in the post-crisis period: employment and wages of different cognitive tasks are highly dissociated from one another in the 2006-2014 period.

This general finding presents a puzzle that has not, until now, been revealed in the data. This is because this change in heterogeneity only reveals itself when looking at MSA-level occupation data over all occupations and in the particular time periods in question *without* looking at long run trends or aggregating to national trends. For example, (Autor et al.) analyzes and attempts to estimate the widespread employment loss and wage stagnation among certain (tradionally non-cognitive) occupations. But, their data sample does not capture the potential difference in post-crisis employment and wage growth among those occupations as they specifically omit any data from 2008 onwards from the sample. Autor et al. (3) examines much of the same post-crisis wage trends, but focuses only on occupations that have lost wages (proportional to total sector income) and not on all occupations.

Using the Occupation and Employment Statistics (OES) from the US Bureau of Labor Statistics I look at employment and income distribution data for over six hundred occupation categories across nearly four hundred metropolitan areas. Since the OES data is estimated from employer-side samples of employment and income distributions among workers in a metropolitan area, it provides a focused estimate of occupational employment and income distributions within a particular MSA.

With this data, I compare employment and income distributions within and across metropolitan areas, across regions, and nationwide over time using a simple difference-in-difference method. The analyses are broken into two time periods: 1999-2004 and 2006-2014. With these two time periods we can analyze both "normal" post-1980 changes to income and employment distributions and those changes that coincide with the 2008 financial crisis and its aftermath. The 1999-2004 period also contains the dot-com recession, which can further help us to identify the differences in changes to employment and income structures coinciding with a "normal" recession compared to a financial crisis.

By highlighting structural changes to employment and income throughout the United States at the metropolitan area level I can determine what common factors unite MSAs and occupation groups that experienced the highest average employment and wage growth from 1999 to 2004 and from 2006 to 2014 and determine what, if anything, is systematically different between the two patterns of post-recession recovery.

The period from 1999 to 2004, was characterized by growth proportional to MSA wealth/population levels across MSAs nationwide. Growth was not even across all MSAs, but it was similar across MSAs for a given wealth level nationwide, and varied little with geography or other measured factors. Most occupations experienced average real wage increases, though high-income occupations were disproportionately likely to see large increases in real wages.

Employment growth generally favored cognitive tasks over non-cognitive tasks: some of the largest gains in employment between 1999 and 2004 occurred in computational fields, particularly those devoted to integrating computers and technology in business, and business administration. But, employment and wages also tended to grow for non-cognitive tasks, just at a slower rate.

These results are generally consistent with historical post-recession employment recovery/growth as documented in (14) and suggest that the 1999-2000 recession was not a vast deviation from historical norms. This, alone, is interesting since the period was marked by rapidly increasing income inequality nationwide, which some have argued reflects a substantial change in the labor market (40) (7), specifically in the bargaining power of traditionally middle-class laborers as suggested by Bound and Johnson (13), which could suggest deviations from historical patterns.

In the 2006-2014 sample, however, historical norms of recession recovery (as well as patterns from the 1999-2004 sample itself) do not seem to apply. Neither occupational nor MSA average employment growth follow any clear patterns: wealthy MSAs may attain higher employment growth on average, but there is a high degree of variance, in large part driven by regional and state fixed effects that were nor present in the 1999-2004 sample; cognitive and non-cognitive occupation groups no longer appear to move as units; some cognitive tasks appear to become non-cognitive tasks (judging by MSA-level employment and wage growth patterns), but only in certain States or geographic regions. In this period, the patterns, where they exist at all, become quite different than in the previous period, and there is no clear explanation of why this might be the case.

These findings show a significant change in growth patterns at the MSA level which may represent some structural change in the way labor in different occupation groups are valued.

The rest of the paper is structured as follows: Section 2 details data and empirical methods while sections 3 and 4 detail results and patterns for the 1999-2004 and 2006-2014 time frames respectively. All graphs and figures are located in the Figures section at the end of the paper.

2.2 Data

The principal data that I use in this paper is the Occupation and Employment Statistics (OES) data from the Bureau of Labor Statistics (BLS).

I examine two time periods with this data to find the structure of changes in employment and income nationwide across MSAs in two difference economic paradigms. First, there is the time period from 1999-2004, which captures the dot-com recession and illustrates post-1980 growth and a ”normal” recession. I use this time period as a baseline, as the employment recovery and wage growth in this period closely reflects theories presented in (10). The 2006-2014 time period captures the end of the housing bubble and the recovery from the 2008 financial crisis, which displays few of the same patterns as the 1999-2004 time period at the MSA level.

I use the 2006 sample as the base year in my 2008 crisis recovery sample because, since the 2008 crisis and recession was, possibly, the result of the burst of a very large credit-backed asset bubble (specifically, on housing), it is likely that 2007 or early 2008 employment and wage numbers reflect the peak of an unsustainable economic boom and do not reflect the actual, relevant numbers to which we should compare a stable recovery.

Data from the OES goes back to 1997 for most occupation groups, and back to 1988 for some industry-level occupation and income data. But, observations before 1999 use different occupation classifications for many occupations from observations in 1999 and later. While the major occupation groups are nominally the same between 1997/1998 and 1999, major occupation groups are missing employment and occupational income distribution data for most metropolitan areas, making it difficult to compare occupational income distributions from 1997 and 1998 with those from 1999 and later even for major occupation groups. In the interest of consistency, I restricted my sample to 1999 and later years to avoid the problem of comparing occupations that were not entirely similar. Where required, tasks for occupations are determined as ”cognitive” or ”non-cognitive” following the methods in (4) and (10)[1]

To ensure that all wages are comparable, I adjust all income data by regional CPI data from the BLS. Using regional CPI rather than national allows for income in different metropolitan areas to be adjusted more appropriately for the actual buying power of the region. Otherwise, tests for regional effects and inter-regional comparisons would be skewed, as some regions have much higher costs

[1]Specifically, ”Cognitive” occupations include: Management, Financial, Architectural/Engineering, Mathematical/Computation, Health Care, Life, Physical, and Social Sciences, Legal Occupations, Arts and Design, and Education and Training. All other BLS designations are considered ”Non-cognitive” in nature for the purposes of this analysis.

of living on average than others. While municipality-level CPI data is available for a few cities, the municipality-level data only counts consumers and purchases within the city itself, while the metropolitan areas of the OES data generally encompass a larger area. All wages and dollar values given in this paper are in terms of 2006 dollars as valued in the Northeast region of the United States.

To examine employment recovery and income growth in each period I treat the locational occupation data as a panel and perform a series of difference-in-difference measures to assess the relative rates of employment and income growth of occupations across metropolitan areas and regions of the United States. I compare the employment and income changes of each occupation group in each metropolitan area against equivalent occupation groups in other metropolitan areas and attempt to determine what, if any, patterns across metropolitan areas with better employment and wage growth can be isolated.

2.3 Employment and Income Growth Distributions: 1999 to 2004

2.3.1 MSA Characteristics

In general, the metropolitan areas that experienced the highest average employment recovery after the dot-com recession of 1999-2000 and wage growth between 1999 and 2004 were the largest, most populous MSAs with the largest total gross income for all workers in 1999. These MSA's experienced higher average occupational employment growth and wage growth for all measured occupation groups, with the exception of Manufacturing Occupations, for which employment was very slow to recover during the sample period. There was also very little heterogeneity in employment and wage growth patterns among the wealthiest third of MSAs: there was very little regional dependence or spatial inequality within the wealthiest third of MSAs.

For example, the financial occupations, which experienced the largest average increases in income nationwide, gained, on average, 4% *more* income in the top third wealthiest MSAs than in other MSAs. Most other occupation groups experienced greater increases in average wage in the wealthiest MSAs than in less wealthy MSAs. However, this wage effect may simply reflect the higher costs of living that wealthier MSAs often impose on their residents (relative to regional price levels). What's likely more important is the employment recovery in these wealthier MSAs relative to less wealthy MSAs.

So, while there were changes in wage inequality and substantial heterogeneity in employment recovery and growth across occupation groups, the growth (or lack thereof) of occupational employment and wages was generally consistent across MSAs (See figures 2.6 and 2.5).

These findings fit the general theoretical expectations of Beaudry et al. (10) and Autor et al. (6), which posit that occupational employment and wage growth are driven by a model of variable task demand and capital accumulation. We would then expect that MSAs with similar overall wealth levels would exhibit similar overall patterns of occupational employment and wage growth and that similar occupations would be in high demand in most areas, which is the general pattern we can observe in the data.

2.3.2 Cross-MSA Heterogeneity

While there was some substantial heterogeneity in average occupational employment changes, the occupations that saw occupational employment and average wage growth were largely consistent across MSAs. For most occupation groups, the growth in employment and wages in an MSA was roughly proportional to MSA population and total MSA income, though a few occupation groups, specifically Production occupations and Transportation occupations, did not seem to depend on MSA income.

This MSA size effect seemed to dominate most other effects that I can identify: there was a slight difference in average MSA employment growth and occupational employment growth across regions, but that was mostly driven by the fact that the top ten percent of wealthiest and most populous MSAs are mostly concentrated in the Northeast and West Coast regions. Once differences in MSA wealth in a region were accounted for, the region or State in which the MSA lies did not have a very large impact on average or overall occupational employment over the 1999-2004 period.

The major exception to this was, again, Production and Transportation occupations, which saw a major occupational decline and very limited wage growth in the South and Midwest regions, especially among lower-income MSAs. This is likely because these were the MSAs in which these occupations still had large levels of occupational employment, however, as the occupation levels of these occupations in these MSAs seemed to converge to national average location quotients.

A proximity effect, that is, a boost to wage growth for many of the more cognitive occupations, especially Mathematical, Computational, and Medical occupations, was observable in MSAs within 80 miles of wealthier MSAs. Distances greater than 80 miles did not yield a systematic, noticeable impact. This effect was present even after accounting for the wealth/population of the measured MSA, but was small at only about 2-3 % additional wage growth on average for MSAs within 80 miles of one of the top 10% of wealthiest MSAs. There were some marginal wage growth benefits to MSAs within 80 miles of the wealthiest third of MSAs, as well, but these average benefits were even smaller (between .8% and 1.3% on average, with the proximity effect being slightly more pronounced in the South). Below the top third wealthiest MSAs it did not appear that proximity to other MSAs provided extra wage growth. Some, occupations, however, most notably Retail and Food Service occupations, had a slight *negative* effect on wage growth from MSA proximity to wealthier MSAs. For the wealthier MSAs, the proximity to less wealthy MSAs did not appear to have any systematic, noticeable impact on employment or wage growth of any occupation groups.

These effect likely captures the competition that employers face from more wealthy MSAs for traditionally high cognition tasks if MSAs are sufficiently close that commuting between them is feasible. Similarly, it seems that workers in some of the less cognitive-focused occupations (like Retail or Food Service occupations) may have faced some increased competition in their own MSAs from more wealthy and populous neighbors, which could have slowed wage growth.

There was also a slight, noticeable Coastal Effect. That is, MSAs with boundaries that are within 60 miles of an ocean experienced, on average, 4% more total employment growth (mostly concentrated in Retail and Food Services occupations), even after adjusting for MSA population/income and proximity to high population/income MSAs. Great Lakes, rivers, and other bodies of water did not have a measurable impact on total employment growth or on any particular occupation group's employment growth.

Overall, the employment and wage growth for occupations in the 1999 to 2004 time period was largely homogeneous across MSAs of similar population and total income levels, with very little regional, state or other variations that were not correlated with the wealth of the MSA. There were differences in how employment and wages for different occupation groups changed nationwide, but those changes were largely consistent across MSAs after accounting for MSA population/income.

2.3.3 Occupational Employment Changes

On average, occupation groups in the 1999 to 2004 sample moved somewhat consistently in two general categories as recognized in the literature: Cognitive and Non-cognitive tasks. Occupations generally associated with cognitive tasks experienced greater proportional employment and wage growth in most MSAs (naturally impacted by MSA wealth effects) than their non-cognitive counterparts.

There were, however, some outliers: the largest declines in employment nationwide were among managers/executives and production occupations, both of which saw employment declines of nearly 11% and 18% respectively. These occupation groups were clear outliers in employment changes among occupation groups: Engineering occupations suffered the next-greatest loss in employment, but employment in those occupations declined by only 3% on average nationwide.

The loss of production jobs affected metropolitan areas nationwide, but was especially pronounced in poorer metropolitan areas and in the South. Production job loss in the south was also more concentrated in textile and garment production and machinist occupations, while nationwide there was a more even spread of proportional employment loss across all production types.

Management and executive employment loss also occurred nationwide, and, while it occurred evenly across regions, wealthy metropolitan areas were much more likely to see large proportional losses in employment for this occupation group, even after controlling for the relatively high concentrations of managers/executives in those wealthy metropolitan areas.

While there was wage growth across many different occupation types, executives and financial occupations saw the greatest increases in real income. Workers in these professions experienced an increase of locational average income of 17% across all metropolitan areas between 1999 and 2004. But, this wage increase was not enough to offset the loss of employment that these occupations experienced; the total income going to these occupations declined by approximately 20%. A lot of this decline in upper-executive and financial employment and total income coincided with shocks associated the dot-com bubble and 2001-2002 recession from which employment in those sectors did not recover by 2004.

These high-income occupations were most likely to see large gains to average income in wealthy metropolitan areas. Nearly all of the metropolitan areas that did not see a net increase in average income for executives and financial occupations were in the poorer 50% of metropolitan areas nationwide and most were located in the South or Midwest. Income changes for most other occupation groups were more even across regions and metropolitan areas, with the exception of education occupations and personal care and service occupations, which grew disproportionately slowly in poorer metropolitan areas, particularly in the South.

2.4 Employment and Income Growth Distributions: 2006 to 2014

2.4.1 MSA Characteristics

At first glance, the general pattern of occupational employment and wage growth for the 2006-2014 period resembles that of the 1999-2004 period: the wealthiest MSAs did, on average, experience the highest degrees of average occupational employment and wage growth. However, the degree of heterogeneity across MSAs was much greater between 2006 and 2014 than between 1999 and 2004, both within the set of wealthiest MSAs and between the wealthiest MSAs and lower-income MSAs. The degree of heterogeneity in employment and wage growth between the same occupation groups in different MSAs was much greater, as well.

Overall, there was less consistency in which occupation groups experienced substantial recovery and growth in employment across MSAs in the 2006-2014 period than in the 1999-2004 period, and the wealth/population of an MSA seemed to have a lesser impact on occupational employment and average wage growth within the MSA than in the 1999 to 2004 sample. Instead, This heterogeneity, however, would only become apparent upon looking at the MSA-level data after 2006. In a national level study like Autor et al. (6) or a longer time-horizon study these peculiarities would likely not be detectable since they only appear to manifest at the MSA level during the recovery from the 2008 recession.

2.4.2 Cross-MSA Heterogeneity

MSAs which experienced relatively higher average wage growth or occupational employment shared far fewer predictable characteristics in the 2006 to 2014 sample than in the 1999 to 2004 sample. For example, total MSA population/income, which seemed responsible for the majority of cross-MSA heterogeneity in occupational employment and wage growth, had a much smaller impact on employment and wage growth for most occupations in he 2006 to 2014 sample. While MSA population and income were, on average, correlated with higher employment growth over the period, the impact on average wage growth was negligible. Furthermore, there was a high degree of inconsistency in occupational employment growth for many occupations even across MSAs with similar total income levels. This inconsistency was not nearly as prevalent in the 1999 to 2004 sample.

For example, wealthy MSAs, on average, saw greater percent employment and wage growth for traditionally cognitive occupations than less wealthy MSAs from 2006 to 2014. But, this effect was driven almost completely by Financial and Management occupations' employment and wage growth in wealthy MSAs in the sample period. Other cognitive occupations were either much less impacted by MSA wealthy on average or not impacted at all. This contrasts with the 1999-2004 sample period, in which most traditionally cognitive occupations benefited from higher percent employment and wage increases in wealthier MSAs more uniformly. Traditionally non-cognitive occupations also retained the average employment benefit from wealthy MSAs, but with a similar decrease in consistency across particular occupations relative to the 1999 to 2004 sample.

The proximity effects for MSAs in close proximity to wealthier MSAs and the coastal effects observable in the 1999-2004 sample do not appear to exist in the 2006 to 2014 sample. But, in the 2006-2014 sample regional and state effects are identifiable for MSAs that impact the average employment and wage growth for some occupation groups. Region effects for the South and Midwest regions are significant and negative: MSAs in the South and Midwest regions gained, on average, approximately 3% and 2 % less employment across all occupation groups respectively and experienced roughly 4 % less overall wage growth each compared to their similarly wealthy counterparts in the Southwest, Northeast, and West regions.

In addition, within the South there were several states with significant additional negative fixed effects on MSAs: Louisiana, Georgia, Mississippi, and Florida all had varying but significant state effects on MSA employment growth (but not wage growth). There were some state fixed effects in the Midwest, as well, but these were generally smaller and positive: MSAs in Illinois, Michigan, and Minnesota had, on average, slightly higher occupational employment and wage growth than their similar Midwestern counterparts in other states.

Finally, there were some regional and state effects in the Western and Northeastern regions in which MSAs in those regions, especially in California, Washington, Massachusetts, and Connecticut experienced higher average wage growth than similarly wealthy MSAs in other states and regions.

This reveals that there are clear differences between the characteristics of MSAs that experienced the highest average employment and wage growth rates in the 1999 and 2004 sample and in the 2006 and 2014 sample. For the 1999 to 2004 sample, the most important factors in determining average occupational and wage growth in an MSA were the starting wealth and population levels of the MSAs themselves, with some minor effects from proximity to high-wealth MSAs and coastal areas. Regional and state effects were generally insignificant. In the 2006-2014 sample, wealthier MSAs did experience greater average employment and wage growth, but there was a lot more inconsistency in employment and wage growth among MSAs of similar population and income levels than between 1999 and 2004. In particular, regional and state effects seemed to impact MSA employment and wage growth to an extent that did not occur from 1999 to 2004.

2.4.3 Occupational Employment Changes

In the 1999-2004 sample, there was, in most MSAs, a clear connection between the cognitive nature of an occupation and the employment and wage growth of that occupation over the sample: cognitive-based occupations experienced faster employment growth and wage growth (as measured by percent changes in employment or average income in an MSA) than their non-cognitive counterparts on average. This fits with the theories of cognitive labor attachment found in Beaudry et al. (10) and also makes sense given the high potential for increased competition in global markets and with automated processes faced by many non-cognitive occupations over the time period (Autor et al.).

On average, at the national level, the general result that cognitive occupation groups experience employment and income growth greater than their non-cognitive counterparts hold in the 2006-2014 time frame. But, just as the 2006 to 2014 time frame exhibited different average patterns for MSAs compared to the 1999-2004 sample, so, too, did the occupation groups. Over the 1999-2004 sample period, most occupation groups moved generally uniformly according to their typical cognitive vs. non-cognitive designations (as in (6)) at the national and MSA level. But, in the 2006-2014 time frame, the cognitive vs. non-cognitive designation seems less significant.

The first and most obvious major difference between the 1999-2004 and 2006-2014 samples is, however, the size of the average difference in employment and income growth between cognitive and non-cognitive occupations. If we look at the two categories together, the impact of being a cognitive occupation is much smaller than in the 1999-2004 sample. So, *on average* cognitive occupations within an MSA did not grow in income or employment by as much relative to non-cognitive occupations. The same general result appears if we look at the aggregation of all MSAs.

In the 2006 to 2014 time frame, the traditional categorization of occupation (Cognitive vs. Non-cognitive) seemed to matter less for employment changes than the average wage of the occupation in 2006. Though there is some correlation between the cognitive nature of the occupation and the average wage, in the 2006-2014 time frame some cognitive occupation groups (most notably

Education, Architectural, and Mathematical/Engineering occupations) saw employment and wage growth more similar to non-Cognitive occupation groups with similar average income levels than with other Cognitive occupation groups in the same MSA.

One possible explanation could be that, through the evolution of technology between 1999 and 2014, some occupations transitioned between involving generally cognitive vs. non-cognitive tasks. But, even this inconsistency was not highly consistent within the sample period. There were some regions and states in which the cognitive vs. non-cognitive distinction *did* serve as a significant predictor of relative employment and wage growth within MSAs.

Incidentally, the regions and states where traditionally cognitive occupations exhibited patterns more similar to their non-cognitive counterparts had substantial overlap with the regions and states in which the MSAs experienced below average employment and wage growth: the South and Midwest, Florida, Georgia, Louisiana, Mississippi, and Oklahoma, in particular[2], tended to have labor markets at the MSA level in which traditionally cognitive occupations experienced wage and employment growth more similar to non-cognitive occupations than in other parts of the country. Nationwide, however, Education occupations, particularly public school K-12 educators, were the ”cognitive” occupation most likely to have employment and wages move alongside their non-cognitive counterparts in an MSA rather than other cognitive occupations.

The idea that some tasks have changed from cognitive to non-cognitive over the total sample period likely cannot explain the drastic change in patterns that some occupation groups demonstrate in the 2006-2014 time period (compared to the 1999-2004 period), at least not entirely. There is no independent reason to believe that an architecture, education, or engineering job is cognitive in one location and non-cognitive in another.

While overall employment nationwide increased by around 3% between 2006 and 2014, the employment structure in most locations and in the national average shows a decline in employment

[2] West Virginia also, but since Charlottesville, WV was the only MSA from West Virginia in the sample that could be something particular to that MSA instead of the state itself.

in occupations with real[3] average annual incomes between \$60,000 and \$80,000, most of which would be classified as Cognitive tasks. On the national level, employment in these occupations declined by approximately 7%. Loss in employment in Architectural and Engineering occupations, Computing and Mathematical occupations, and Business and Financial occupations drove the decline in Cognitive-task employment for most metropolitan areas.

There was also a decline in nationwide employment for occupations making less than \$35,000 annually of approximately 6%. Nearly all metropolitan areas saw a decline in employment in occupation groups with annual incomes in this range. This may be because the OES data only factors in full-time employment, and the workers making \$35,000 or less in 2006, many of whom were concentrated in retail or food service occupations, may have been replaced with part-time workers by 2014. Nationwide, this decline in full-time employment among workers making \$35,000 or less annually was driven by declines in the Sales and Retail and Food Service occupational employments. Approximately one third of metropolitan areas, however, saw larger declines in employment in Transportation and Material Moving and Production occupations than in Sales/retail and Food Service. This pattern of employment loss was most common in the South.

Occupations with average real incomes between \$35,000 and \$50,000 saw the largest net gains in employment nationwide, increasing by approximately 12%, primarily in the Office and Administrative Support occupations. Employment in occupations making more than \$90,000 on average annually rose by approximately 6% as well, though this is mostly due to increasing average wages for Management occupations and Legal occupations in the wealthier 50% of metropolitan areas and is not totally driven by changes in employment in any particular occupational group.

Locational income dispersion within an occupation group in 2006 (here measured as the difference between the top decile and median incomes for that occupation group in a given metropolitan area) was negatively correlated with employment loss in an occupation within a metropolitan area, even when average income of the occupation group was taken into account. This means that the occupation groups that lost the most employment nationwide, like sales and retail or business and

[3] All wages and prices are given in 2006 dollars as valued in the Northeast region of the United States.

financial occupations, were less likely to lose employment in metropolitan areas where there was significant wage dispersion within the occupational group. Note that this correlation only exists when looking at net employment changes between 2006 and 2014. Occupation groups with high locational wage divergence in 2006 were actually more likely to lose employment between 2007 and 2009, but employment in those groups recovered much more quickly than those groups where wage divergence was low in 2006.

Employment for the majority of occupation groups in metropolitan areas in the south failed to return to 2006 levels by 2014. This problem was particularly pronounced in the Gulf Coast states (Louisiana, Mississippi, Alabama, Georgia, Florida, Texas) where the majority of measured metropolitan areas lost 5% or more of total 2006 employment by 2014. Notable exceptions to this are the Atlanta, GA, Houston, TX, and Abilene, TX metropolitan areas, whose total employment between 2006 and 2014 grew significantly above the national average.

The Northeast and West regions of the United States lost much less employment in their metropolitan areas' occupation groups than the South or Midwest did. Employment in metropolitan areas in the Northeast and West regions grew by equivalent percentages (approximately 5 % on average), but employment growth in the Northeast was highly concentrated in a few major cities (New York, NY, Boston, MA, Philadelphia, PA), with smaller metropolitan areas growing much more slowly or losing employment between 2006 and 2014. Growth in the West was much more consistent, but spread out. metropolitan areas in the southern and eastern sections of the West (Wyoming, Nevada, Utah, New Mexico, Arizona, Montana) experienced slower average employment growth than the Pacific coast state metropolitan areas on average, but even in those states the majority of occupation groups returned to within 2% of 2006 employment levels by 2014.

We can see from these statistics that, between 2006 and 2014, full-time employment became much more concentrated nationwide in occupations where the average wage was between $35,000 and $50,000 annually and much less concentrated in occupations making between $60,000 and $90,000 annually. National employment also became more concentrated in the Northeast and Western regions of the US, particularly on the Pacific coast (California, Washington, Oregon) and the largest cities in the Northeast and less concentrated in the South and Midwest.

2.5 Conclusion

The MSA and occupation group patterns in the 1999-2004 and 2006-2014 samples present a strange series of patterns to explain. The dot-com recession and recovery of 1999-2004 exhibits occupation group patterns that align with cognitive labor market theories as in Beaudry et al. (10) (or a general DSGE model with differential capital accumulation across MSAs) and the general wage and employment patterns identified in (6) or (40) at both the national and MSA level. Specifically, MSAs with higher initial income/population levels tend to experience higher percentage growth in income and employment than their less wealthy counterparts, with some marginal proximity and coastal effects, while workers in cognitive-based tasks experience higher occupational employment and wage growth on average than their non-cognitive peers.

The housing crisis recovery period, 2006 to 2014, however, has very different observable patterns to its occupation growth and MSA growth: occupation groups' employment and income growth cannot be neatly summarized according to whether or not the occupation is traditionally a cognitive or non-cognitive task. Some cognitive occupations even demonstrate occupation and wage dynamics more similar to non-cognitive tasks in the same MSA than other cognitive task occupations (or even the same cognitive task occupation in a different MSA). But, even these patterns vary by geographic state and region, for reasons that I cannot identify in this analysis. The behavior or occupation wage and employment growth for occupation groups does not seem nearly as neatly predictable by prevailing cognitive-task theories of labor as does the 1999-2004 sample.

Even the MSA patterns in the 2006 to 2014 sample are much harder to solidly identify and are not as easily intuitive to explain as the behavior of MSA patterns from 1999 to 2004. The wealth/population effect still predisposes some MSAs to higher average employment growth and income levels, but now there is a great deal of heterogeneity in average impact of MSA wealth on employment and wage growth, and regional and state fixed effects appear much more significant.

It is possible that the increased duration and severity of the 2008 recession caused its recovery to behave wholly differently from that of the (relatively) recent dot-com bubble burst, but there are likely other factors that have changed the labor market, as well. For example, given that regional and state effects appear much more significant to both average MSA and occupational employment/wage growth, it is possible that some policies were enacted or changed to alter labor dynamics in some regions. Or, some of the cognitive requirements for certain occupations and tasks may have changed between 1999 and 2014.

This work is not intended to definitively express any causal source of these patterns. Neither is it truly intended to fully specify the exact structure in which the differences in spatial and occupational heterogeneity emerge across the examined time periods: there are no structural estimations for any of the variables in this paper. In fact, it was something that I intentionally avoided since, even now, I am not certain of the full extent of the patterns that the data is exhibiting and do not feel comfortable assuming any particular structure for estimation. But, after a great deal of examination of the data, the patterns of spatial and occupational heterogeneity detailed in this paper appear simultaneously strikingly different from what has been observed in the past *and* subtle enough that they could easily be missed in aggregate data or in MSA-level data not specifically looking at the post-2008 crisis recovery.

It will take more work, and likely future data, to fully determine what, if any, fundamental changes occurred in the US labor market across MSAs to explain the observable differences in the recovery from the 2008 housing crisis, and additional theoretical work to explain how those changes came about. I hope this paper can serve as at least a piece of evidence that something, at least, appears drastically different at the MSA level for the housing crisis recovery that, at least as of now, appears wholly inconsistent with labor market theory and established patterns of recession recovery in the United States.

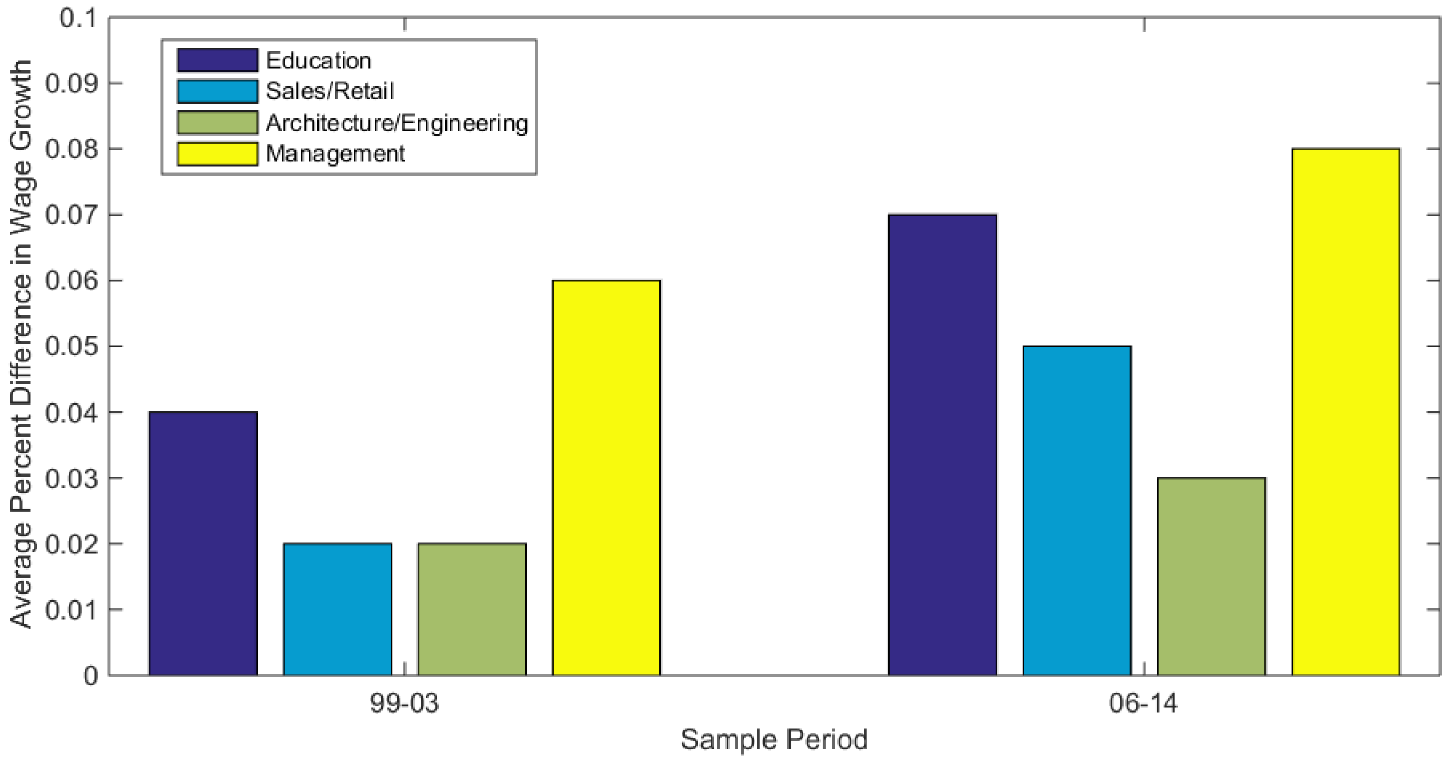


Figure 2.1: Difference in Employment: South vs. Other Regions (Unadjusted)

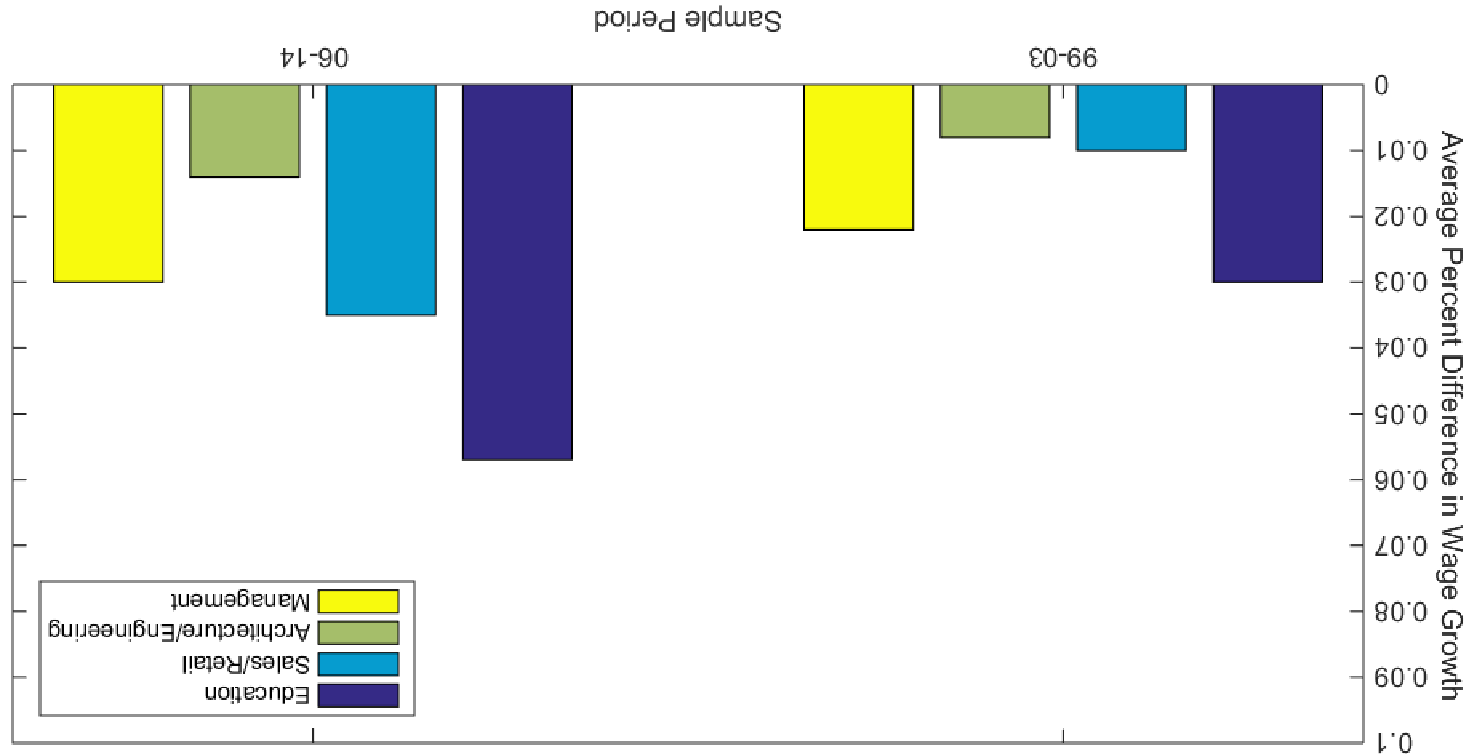
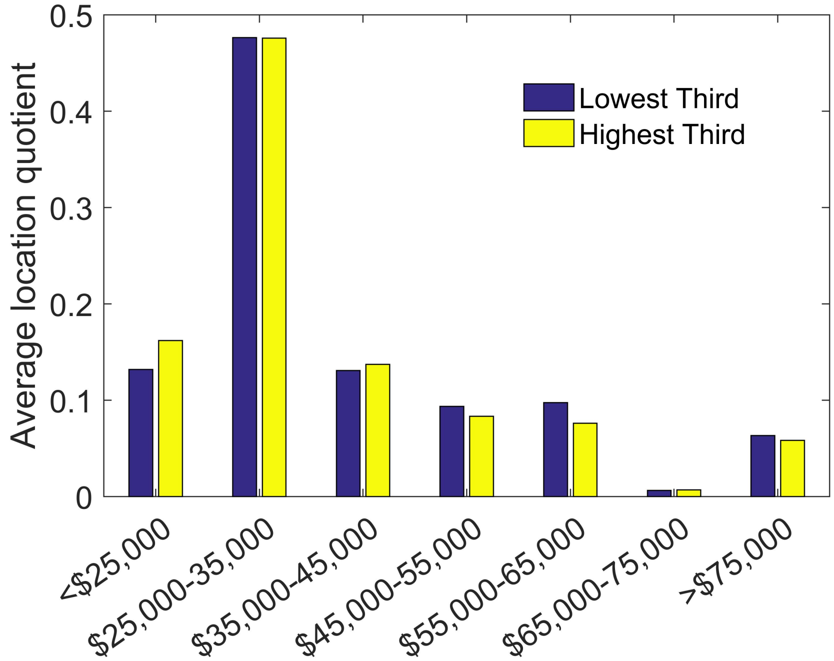


Figure 2.2: Average Percent Difference in Employment in Other Regions vs. South: Adjusted for MSA wealth

Figure 2.3: Average occupational income distribution for MSAs, 1999



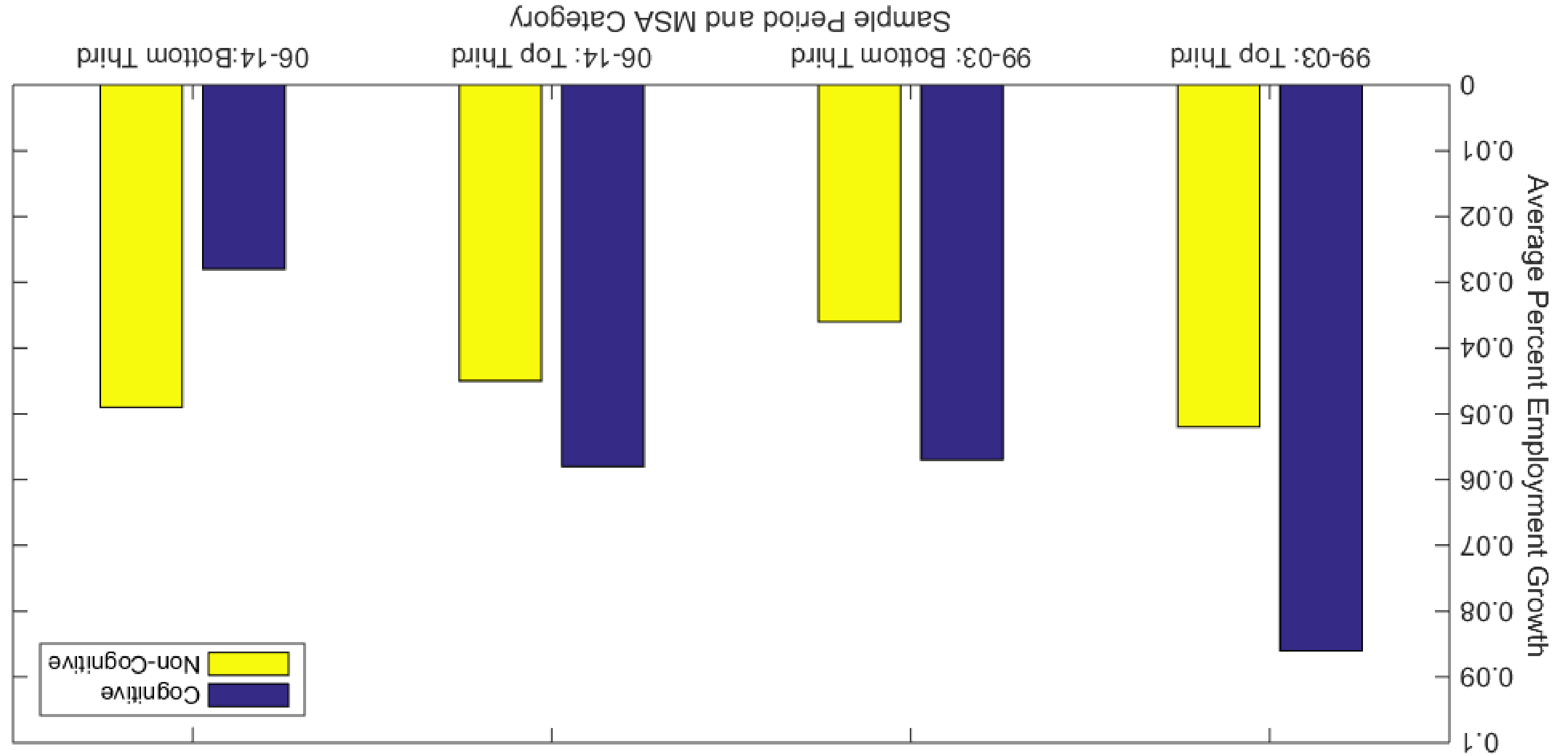


Figure 2.4: Employment Growth for Cognitive vs. Non-cognitive Occupations

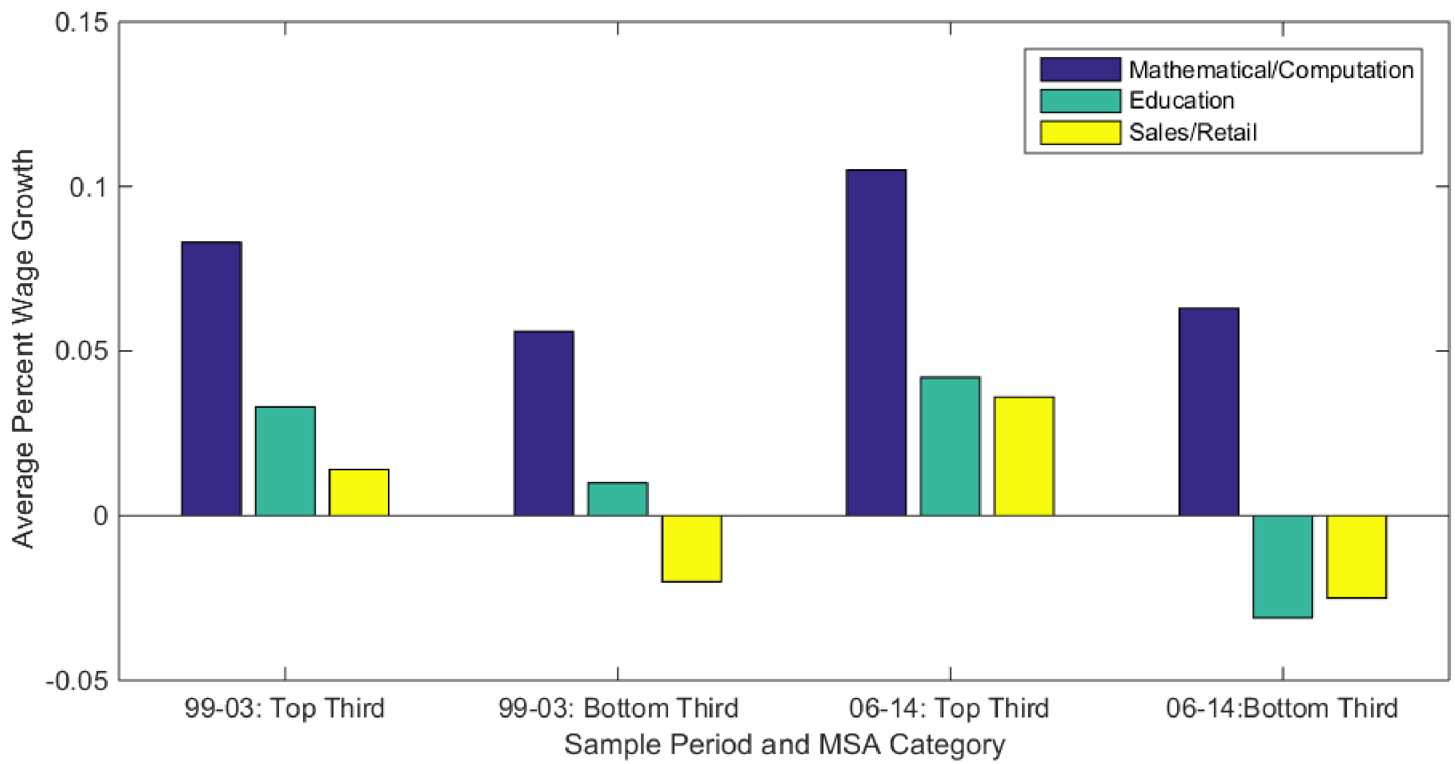


Figure 2.5: Wage Growth for Selected Occupations

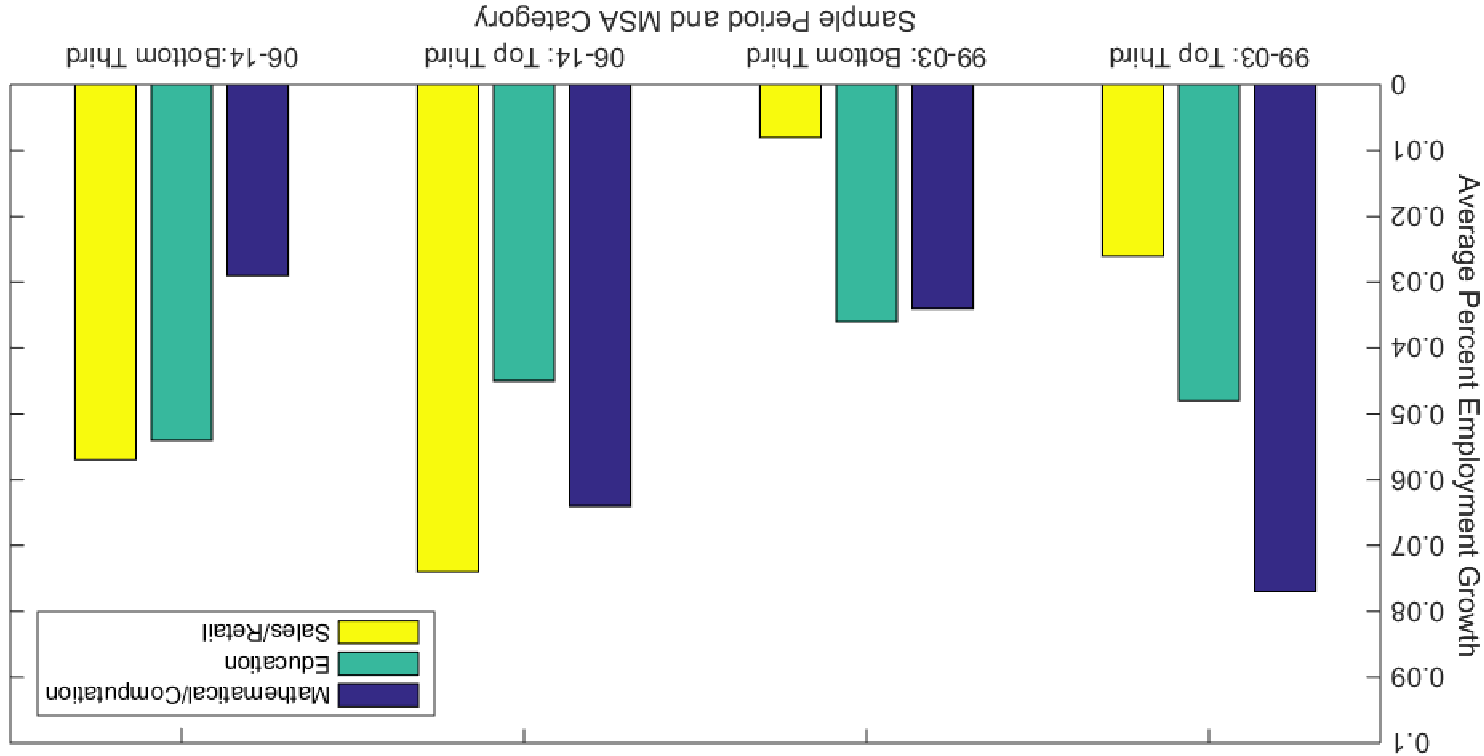


Figure 2.6: Average Employment Growth across Sample Periods and MSA wealth categories for Selected Occupations

Figure 2.7: Average occupational income distribution for MSAs, 2004

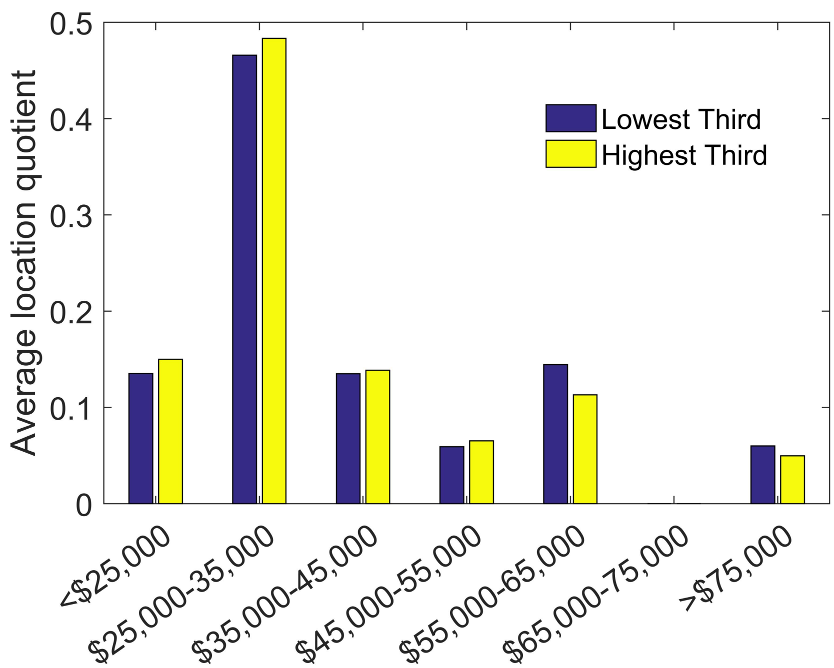


Figure 2.8: Average occupational income distribution for MSAs, 2006

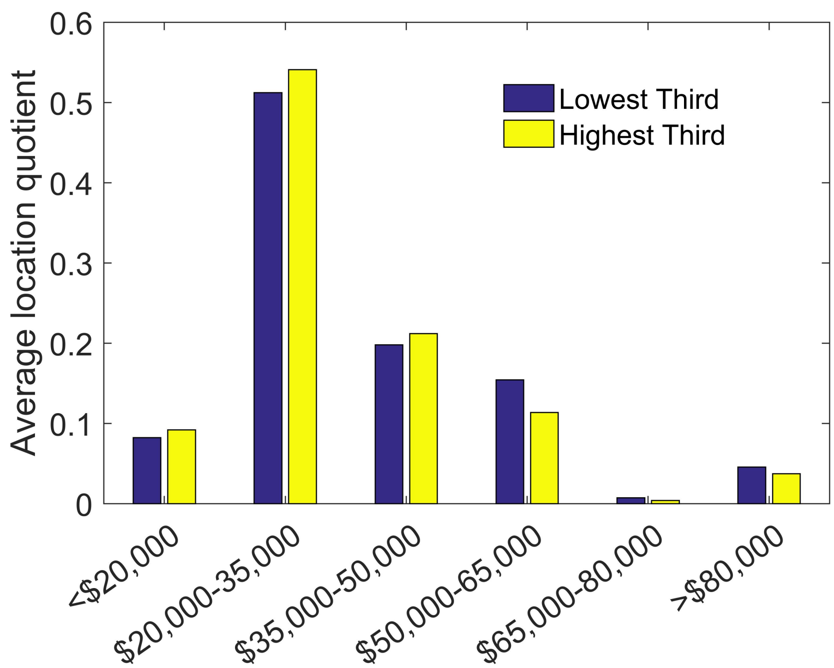


Figure 2.9: d
istributions]Average occupational income distribution for MSAs, 2014

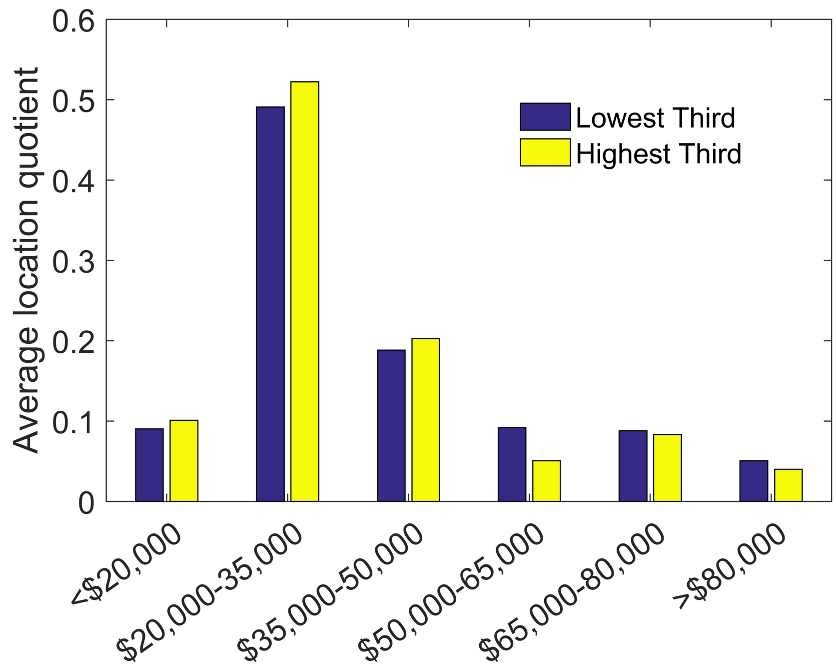


Figure 2.10: Average percent change in occupational employment by occupational income, 1999-2004

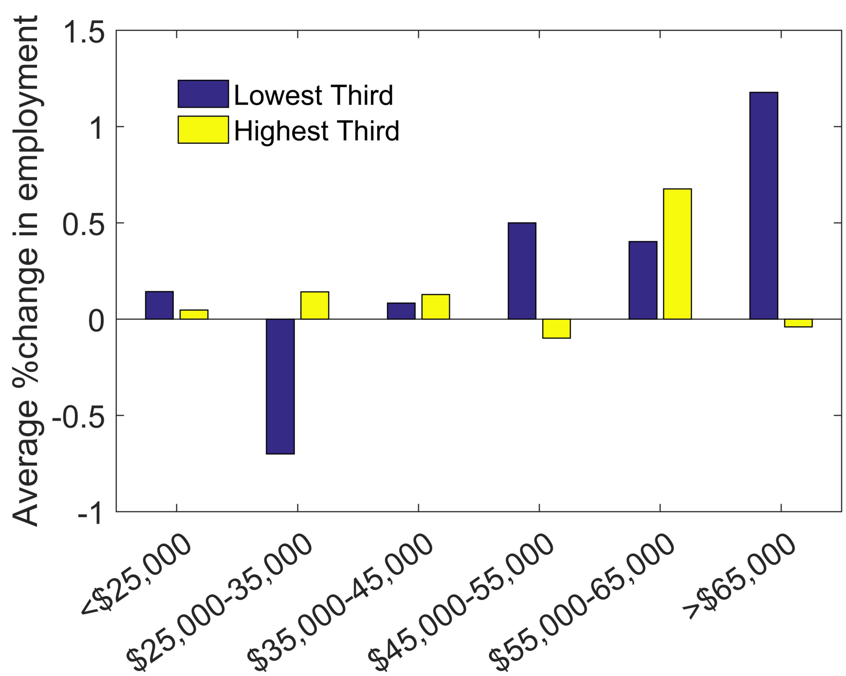
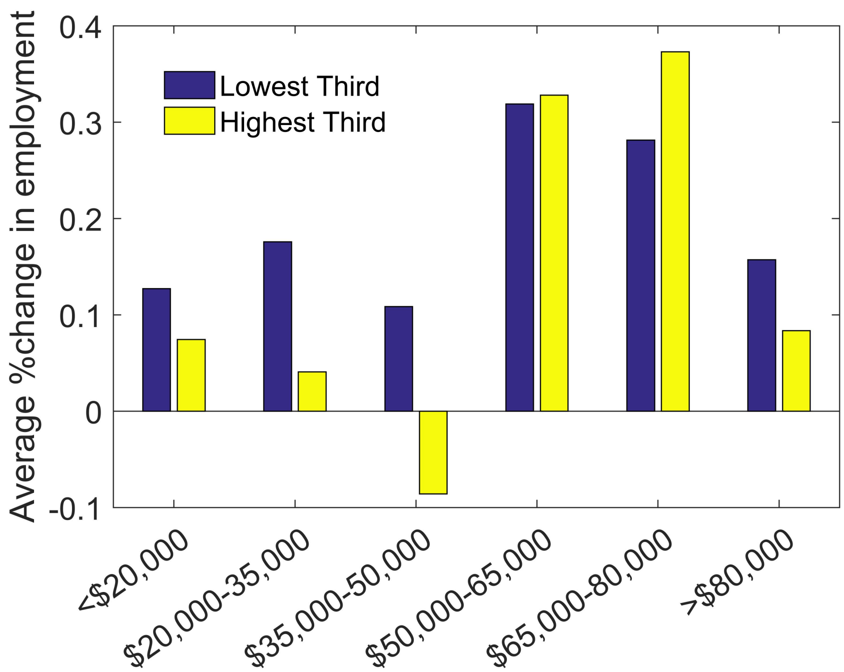


Figure 2.11: Average percent change in occupational employment by occupational income, 2006-2014



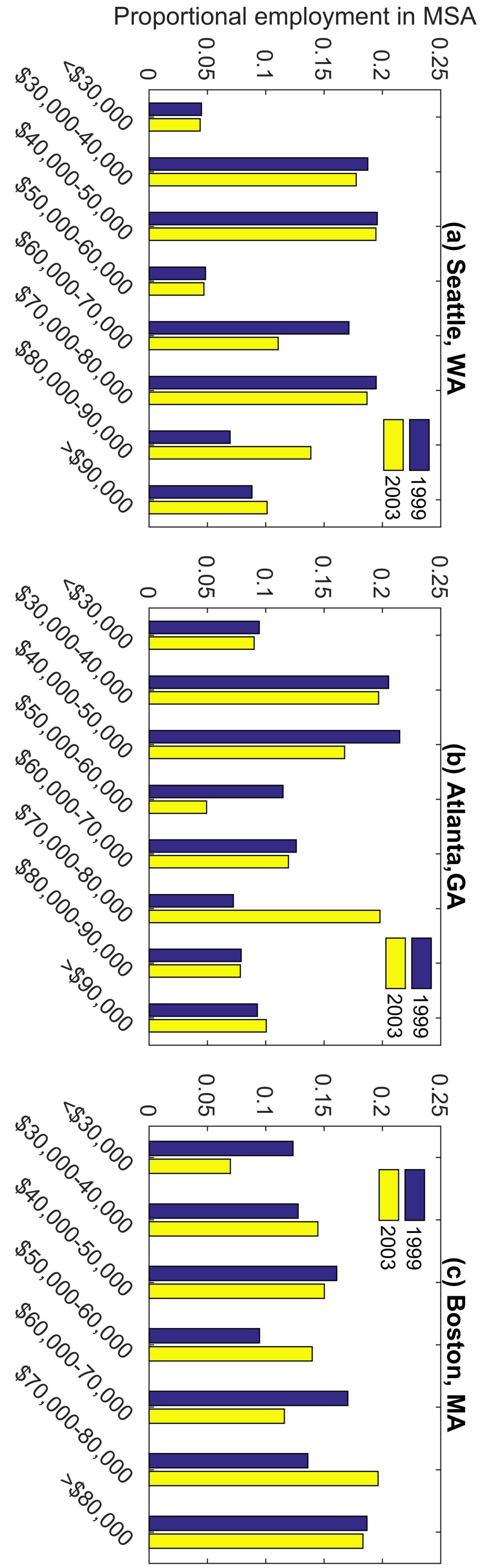


Figure 2.12: Sample income distributions by average occupational employment, 1999-2004

Figure 2.13: Sample income distributions by average occupational employment, 2006-2014

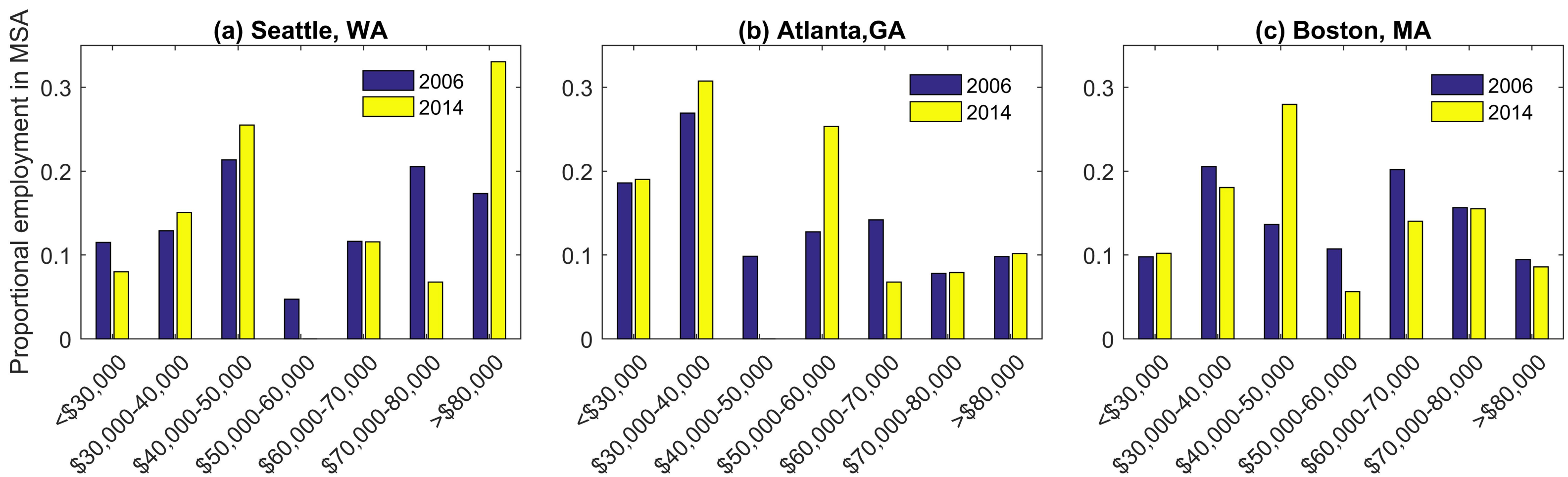


Figure 2.14: Total national employment for selected occupation groups, 1999-2003

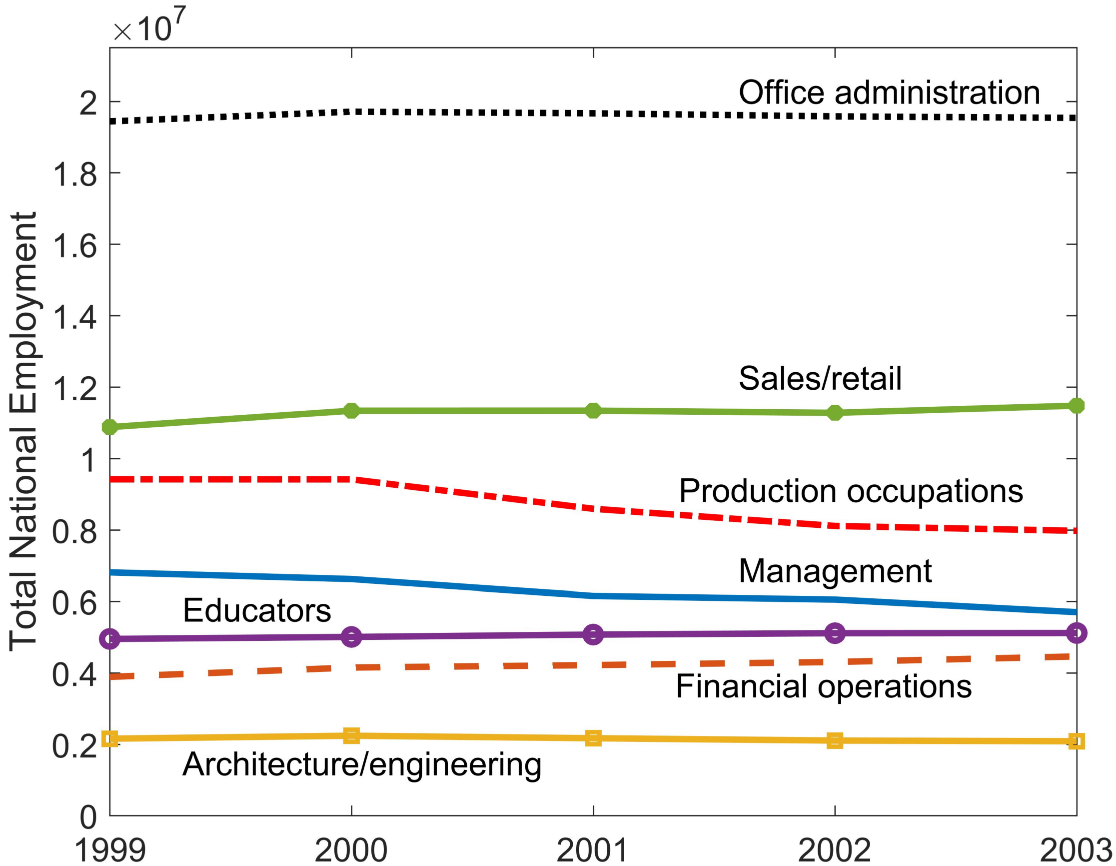
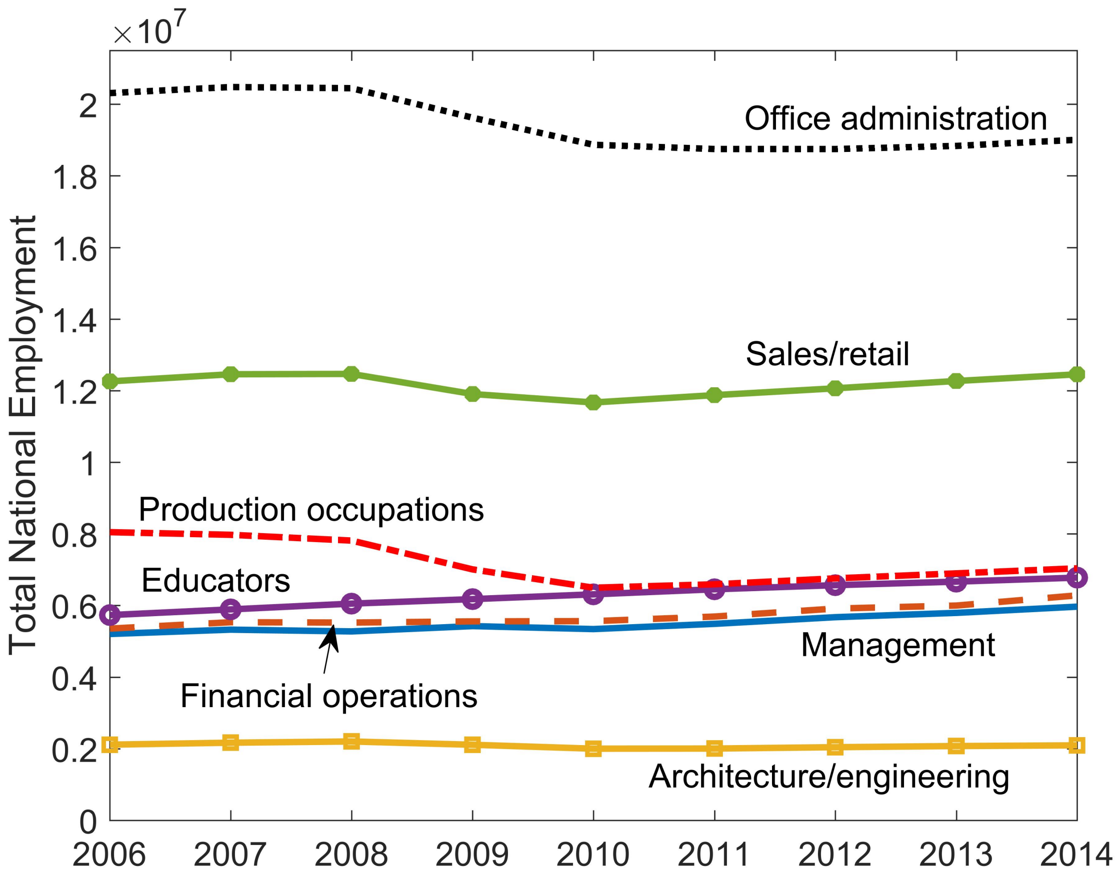


Figure 2.15: Total national employment for selected occupation groups, 2006-2014



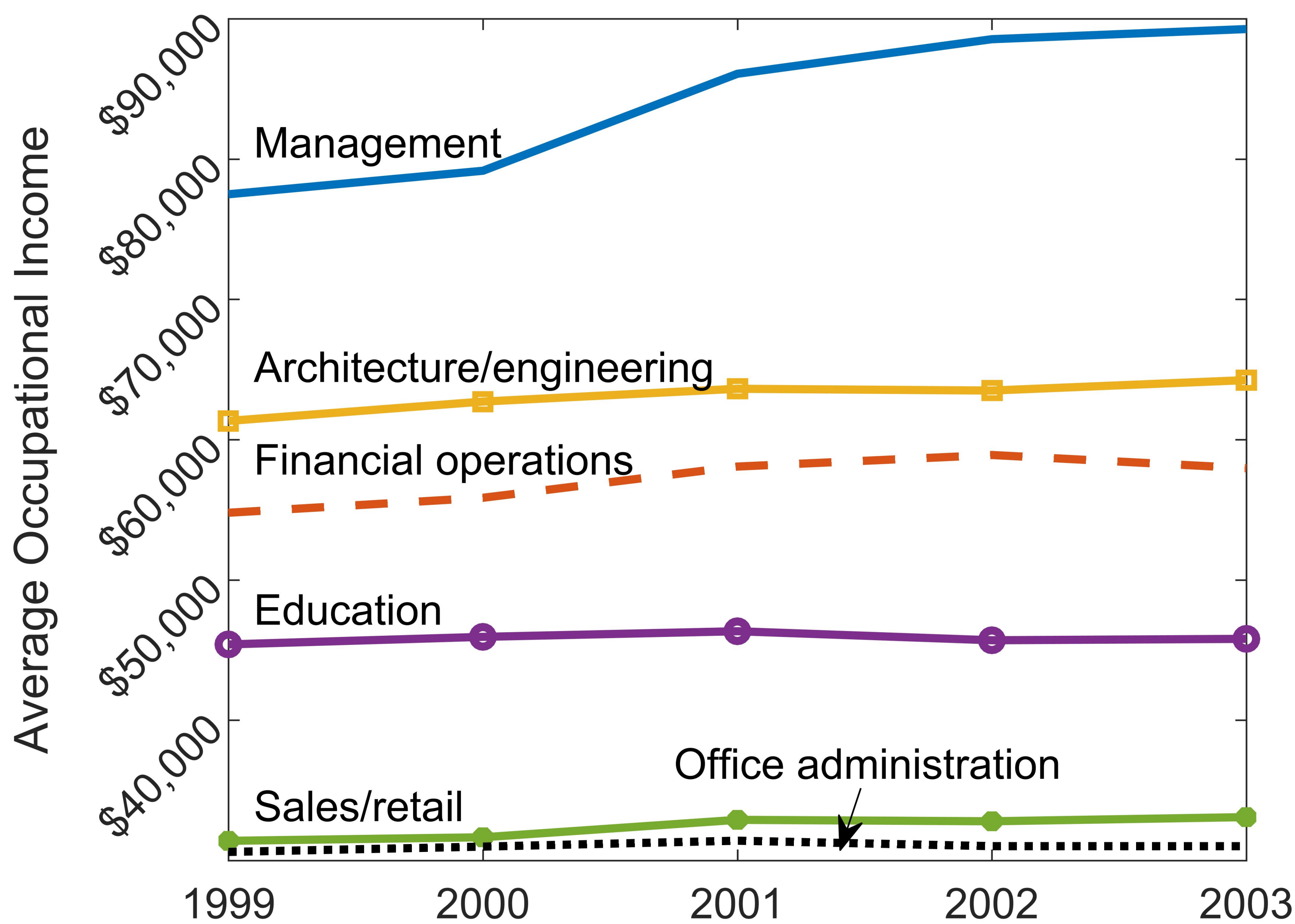


Figure 2.16: National Average Income for Selected Occupations, 1999-2003

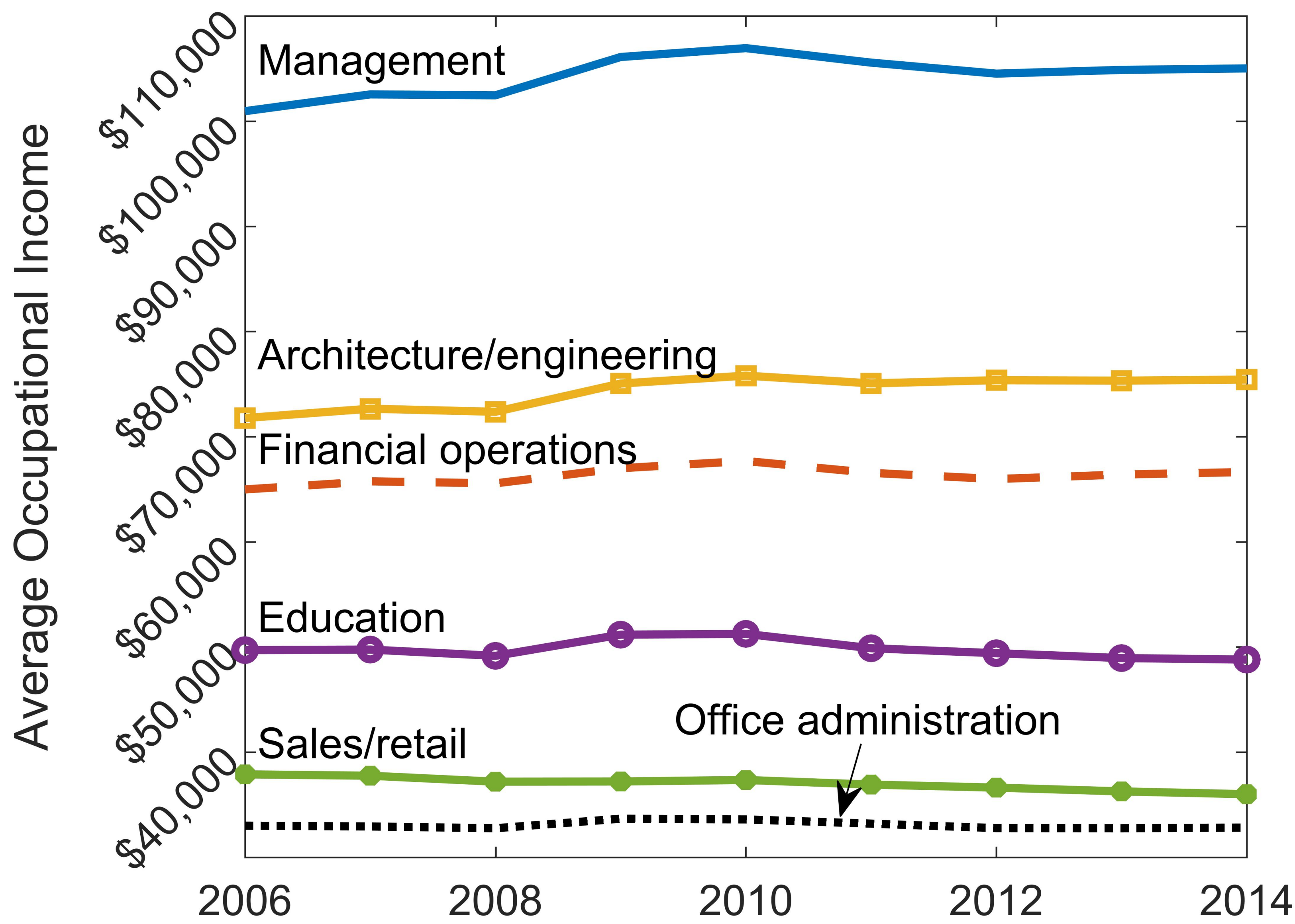


Figure 2.17: National Average Income for Selected Occupations, 2006-2014

CHAPTER 3

Appendix: Proofs

3.0.1 Proof of Lemma 1

In any equilibrium without bubbles it must be the case that the price of housing obeys:

$$p_t - \kappa = \sum_{j \geq 0} \frac{1}{\Pi_{k=0}^{j}(1 + r_{t+k})} \left(\frac{v'(h^s_{t+j})}{u'(c^s_{t+j})} - \kappa \right), \tag{3.1}$$

Specifically, as demonstrated in the main text, there must exist a stationary equilibrium with the housing equal to the maintenance cost κ in all periods. The asset pricing equation for savers then gives a rule for the interest rate:

$$1 + r_n = \frac{v'(h^s_n)}{\beta \kappa u'(c^s_{n,o})}. \tag{3.2}$$

In any bubble-less equilibrium with $p = \kappa$, $c^s_{o,n} = \epsilon + \bar{d}$. So, $c^s_{o,n}$ is an exogenous constant. This means that, for a given interest rate, $u'(c^s_{n,y}) = \beta(1 + r_n)u'(c^s_{n,o})$ is constant, as well. This means that, for a given e^s,

$$v'(h^s_n) = v' \left(\frac{e^s - c^s_{n,y} - \bar{d}\frac{1}{1+r_n}}{\kappa} \right),$$

and the equilibrium with $p = \kappa$ must be an equilibrium with $r_n < 0$ if e^s, κ sufficiently large.

3.0.2 Proof of Lemma 2

Part 1: Existence

We will now show that the bubble exists with $b > 0$ if and only if $1 + r_n < 1$. First, assume $1 + r_n < 1$. We know from budget constraints that the bubble exists with $b > 0$ in the steady state if and only if:

$$e^s > \bar{d} + c^s_{h,y} - \kappa v'^{-1}(u'(c^s_{h,y})\kappa). \tag{3.3}$$

This implies that the housing bubble can exist only if savers have some endowment left over after lending, consuming the optimal amount in young age, and purchasing housing to satisfy utility purposes. Note, however, that we know from budget constraints in the bubble-less equilibrium that:

$$e^s = \bar{d}\frac{1}{1+r_n} + c^s_{y,n} - \kappa v'^{-1}(u'(c^s_{n,y})\kappa), \tag{3.4}$$

which is the same condition as (3.3) only in the bubble-less equilibrium. Since the housing bubble does not increase the *fundamental* value of housing ($\frac{v'(h^s_h)}{u'(c^s_{h,y})}$ is equal to κ in the housing bubble equilibrium), with $1 + r_n < 1$, we know that:

$$e^s > \bar{d} + c^s_{h,y} - v'^{-1}(u'(c^s_{h,y}))\kappa) \cdot \kappa,$$

Now, suppose $b > 0$ and $1 + r_n \geq 1$. Then, it *must* be the case that that savers find it optimal to drive interest rates down below r_n, purchase less housing, and consume less when young (relative to the bubble-less equilibrium) to preserve $\frac{v'(h^s)}{u'(c^s_y}= \kappa$. Since the bubble-less allocation is always feasible for savers (if savers choose $b = 0$), this must mean that there exists an allocation for savers $\{h^s < h^s_n, c^s_y > c^s_{y,n}, c^s_o \geq c^s_{0,n}, \bar{d}\}$ with equilibrium housing price $p = p_n$ and $r < r_n$ that is feasible in the bubble-less equilibrium that yields strictly greater utility for savers than the bubble-less equilibrium bundle. This cannot be the case since we know the bubble-less equilibrium bundle to be optimal for unconstrained savers. Therefore: $b > 0 \Rightarrow 1 + r_n < 1$. Thus, we have shown $b > 0 \Leftrightarrow 1 + r_n < 1$.

With these two results, we have shown $b > 0 \Leftrightarrow r_n < n$, as desired. Intuitively, this result shows that there can only be a housing bubble in equilibrium if there is insufficient storage in the economy for savers to efficiently store wealth for old age in the equilibrium without a bubble.

Part 2: Allocations and prices

With $b_h > 0$ and $1 + r_n < 1$, housing allocations follow from the household budget constraints as given in the Lemma.

Borrowers are credit constrained while savers are not:

$$a^d = -\bar{d} = -a^s.$$

Borrowers' consumption and housing asset holdings are given by:

$$\begin{aligned} h_h^d &= \frac{e^d + \bar{d} - c_{h,y}^d}{p_h}, \\ c_{h,y}^d &= e - p_h - c_{h,y}^s, \\ c_{o,h}^d &= b_h h_h^d - \bar{d} + \epsilon \end{aligned}$$

Savers' consumption and housing asset holdings are given by:

$$\begin{aligned} h_h^s &= \frac{e^s - \bar{d} - c_{h,y}^s}{p_h}, \\ u'(c_{h,y}^s) &= \beta u'(c_{h,o}^s), \\ c_{h,o}^s &= b_h h_h^s + \bar{d} + \epsilon, \end{aligned}$$

with the housing price given by:

$$p_h = \kappa + b_h. \tag{3.5}$$

Part 3: Convergence

We know from the asset pricing equation for savers that the price of housing must obey:

$$p_t - \kappa = \sum_{j \geq 0} \frac{1}{\Pi_{k=0}^{j}(1 + r_{t+k})} \left(\frac{v'(h_{t+j}^s)}{u'(c_{t+j}^s)} - \kappa \right) + \lim_{j \to \infty} \frac{p_{t+j+1} - \kappa}{\Pi_{k=1}^{j}(1 + r_{t+k})},$$

which means that the only way the price of housing can converge to a steady state with a positive bubble component is when:

$$
\begin{aligned}
1 + r_h &= \frac{v'(h_h^s)}{\beta \kappa u'(c_{h,o}^s)} = 1, \\
\kappa &= \frac{v'(h_h^s)}{u'(c_{h,y}^s)}.
\end{aligned} \tag{3.6}
$$

Since there is a unique interest rate at which the bubble can exist and, for a given interest rate, a unique size of the bubble that allows for $\kappa = \frac{v'(h_h^s)}{u'(c_{h,y}^s)}$, and all equilibria in this model must be stationary, there must exist a unique bubble value in equilibrium given by:

$$
b_h = p - \kappa.
$$

3.0.3 Proof of Lemma 4

Savers must have greater purchasing power when young in a steady state with a housing bubble than in a steady state without one since $1 + r_n < 1$. Also, since savers will be spending fewer resources on lending $(\kappa + b)h_h^s > \kappa h_n^s$, and savers can consume more when old under a housing bubble than when there are no bubbles (since reselling housing now yields consumption in old age net of the maintenance cost on housing).

Formally, denote lifetime welfare for savers with net housing price and interest rate $p = \kappa + b_h$ and $r = 0$ (as in the housing bubble equilibrium) as $U_h^s(h^s, c_y^s, c_o^s, a^s)$. Suppose savers choose to acquire the bubble-less equilibrium bundle when young: $h^s = h_n^s, c_y^s = c_{y,n}^s, a^s = \bar{d}$. This bundle is feasible under housing bubble equilibrium prices and leads to old age consumption $c_o^s = bh_n^s + \bar{d}$. Comparing c_o^s with $c_{o,n}^s$, we see that $c_o^s - c_{o,n}^s = bh_n^s > 0$. Therefore, the bundle for the bubble-less equilibrium is feasible under the housing bubble and

$$
U_h^s(h_h^s, c_{y,h}^s, c_{o,h}^s, \bar{d}) \geq U_h^s(h_n^s, c_{y,n}^s, c_o^s, \bar{d}) > U_n^s(h_n^s, c_{y,n}^s, c_{o,n}^s, \bar{d}),
$$

where $U_n^s(h, c_y, c_o, a)$ is savers' welfare with the price and interest rate of the bubble-less equilibrium with bundle (h, c_y, c_o, a).

The housing bubble thus improves savers' lifetime utilities in the steady state by raising the interest rate on debt and making housing a more efficient store of value. Intuitively, savers are better able to consume and acquire housing when young due to the increased interest rate, while the bubble component of the housing price allows them to more efficiently store wealth for old age consumption.

3.0.4 Proof of Lemma 5

The total purchasing power of young borrowers in any equilibrium is given by $e^d + \bar{d}\frac{1}{1+r}$. Therefore, young borrowers have more purchasing power in a bubble-less steady state compared to a housing bubble steady state if and only if $e^d + \bar{d}\frac{1}{1+r_n} > e^d + \bar{d}$. This inequality is always satisfied if $1 + r_n < 1$. Since borrowers have access to less wealth in the housing bubble steady state than in the bubble-less steady state, it must also be that: $c_{y,h}^d + p_h h_h^d < c_{y,n}^d + p_n h_n^d$.

Consider the two possibilities: $c_{y,h}^d \geq c_{y,n}^d$ or $c_{y,h}^d < c_{y,n}^d$. If $c_{y,h}^d < c_{y,n}^d$, the borrowers' fist order condition shows that, if $c_{y,h}^d < c_{y,n}^d$, then $h_h^d < h_n^d$. The only way borrowers could experience greater welfare in this case is if the bubble helps borrowers save for old-age consumption through their housing acquisition. But, if sacrificing young-age consumption and housing for old-age consumption is optimal for the borrower, then borrowers could take on less debt in the bubble-less steady state, acquire $h^d < h_n^d$, $c_y^d \leq c_{y,n}^d$, and consume more when old. Formally, there must exist $\phi > 0$ such that:

$$U_n^d\left(h_n^d - \frac{\phi}{(1+r_n)p_n}, c_y^d, , c_{o,n}^d + \phi, \bar{d} - \phi\right) > U_n^d(h_n^d, c_{y,n}^d, c_n^d, \bar{d}),$$

where $U_n^d(h, c_y, c_o, a)$ denotes the borrowers' welfare with price $p = p_n$ and interest rate $r = r_n$. This is impossible since the bundle $(h_n^d, c_{y,n}^d, c_{o,n}^d, \bar{d})$ is optimal for borrowers in the bubble-less equilibrium.

Similarly, if $c^d_{y,h} \geq c^d_{y,n}$, then $p_h h^d_h < p_n h^d_n$, which implies that $h^d_h < h^d_n$. If it is optimal to sacrifice housing (and, hence, old age consumption) for young age consumption in a bundle that is feasible in the bubble-less equilibrium, then there must exist $\phi > 0$ such that:

$$U^d_n\left(h^d_n - \frac{\phi}{(1+r_n)p_n}, c^d_{y,n} + \phi, c^d_n - \phi, \bar{d}\right) > U^d_n(h^d_n, c^d_n, c^d_n, \bar{d}),$$

which is impossible since $(h^d_n, c^d_n, c^d_n, \bar{d})$ is the optimal bundle for borrowers in the bubble-less equilibrium. Therefore, the housing bubble *reduces* welfare for borrowers.

Intuitively, the bubble on housing introduces two difficulties for credit-constrained borrowers: first, the interest rate on debt increases, which makes borrowing to acquire housing and consume when young more expensive, second, the bubble increases the cost of acquiring housing. This means that, when there is a housing bubble, borrowers must pay more over their lifetimes in debt to pay for housing at the inflated cost with the bubble. This leads to less lifetime consumption and less housing for borrowers. The bubble does make housing a more efficient store of value, but, if borrowers are already debt constrained, sacrificing young age consumption and housing for old age consumption cannot possibly improve their welfare.

It is important to remember that this unambiguously negative impact of the housing bubble on borrower welfare is brought about first by the bubble's increase in the interest rate, and second by its attachment to an object from which the households derive utility. If the bubble were attached to something that only yielded consumption in old age or was a store of value, as in the pure bubble case, borrowers could ignore the bubble asset completely, or would care only about the amount of wealth stored in the bubble asset instead of the amount of the bubble asset acquired.

3.0.5 Proof of Lemma 7

Part 1: Existence

We will now show that the bubble exists with $\tilde{b} > 0$ if and only if $1 + r_n < 1$. First, assume $1 + r_n < 1$. We know from (1.24) that the bubble exists with $\tilde{b} > 0$ in the steady state if and only if:

$$e^s > \bar{d} + c^s_{p,y} - \kappa v'^{-1}(u'(c^s_{p,y})\kappa). \tag{3.7}$$

This implies that the pure bubble can exist only if savers have some endowment left over after lending, consuming the optimal amount in young age, and purchasing housing to satisfy utility purposes. Note, however, that we know from (1.22) in the bubble-less equilibrium that:

$$e^s = \bar{d}\frac{1}{1 + r_n} + c^s_{y,n} - \kappa v'^{-1}(u'(c^s_{n,y})\kappa), \tag{3.8}$$

which is the same condition as (3.7) only in the bubble-less equilibrium. Since the pure bubble cannot increase the price of housing, with $1 + r_n < 1$, we know that:

$$e^s > \bar{d} + c^s_{p,y} - v'^{-1}(u'(c^s_{p,y}))\kappa) \cdot \kappa,$$

Now, suppose $\tilde{b} > 0$ and $1 + r_n \geq 1$. Then, it *must* be the case that that savers find it optimal to drive interest rates down below r_n, purchase less housing, and consume less when young (relative to the bubble-less equilibrium) to preserve $\frac{v'(h^s)}{u'(c^s_y}= \kappa$. Since the bubble-less allocation is always feasible for savers (if savers choose $\tilde{b} = 0$), this must mean that there exists an allocation for savers $\{h^s < h^s_n, c^s_y > c^s_{y,n}, c^s_o \geq c^s_{0,n}, \bar{d}\}$ with equilibrium housing price $p = p_n$ and $r < r_n$ that is feasible in the bubble-less equilibrium that yields strictly greater utility for savers than the bubble-less equilibrium bundle. This cannot be the case since we know the bubble-less equilibrium bundle to be optimal for unconstrained savers. Therefore: $\tilde{b} > 0 \Rightarrow 1 + r_n < 1$. Thus, we have shown $\tilde{b} > 0 \Leftrightarrow 1 + r_n < 1$.

Part 2: Allocations and prices

First, we show that the debt constraint and short-selling constraint on the bubble asset must either both bind or both be nonbinding. First order conditions from the household's problem yield the following rule for debt and bubble acquisition:

$$\lambda^i_{d,t}\tilde{b} = \lambda^i_{b,t},$$

where $\lambda_{d,t}$ and $\lambda_{b,t}$ are the Lagrange multipliers associated with the binding credit constraint and binding short-selling constraint on the bubble respectively and $i \in \{s, d\}$. Given $\tilde{b} > 0$, the debt constraint is non-binding if and only if the bubble asset short-selling constraint is non-binding. This means that a credit constrained household will never invest in the bubble asset, while a non-credit constrained household always will. Intuitively, the same household will never find it optimal both to save with the bubble and take on debt. Therefore, with p_p and $\tilde{b}$ as determined by (1.23) and (1.24) respectively, all allocations are as given in the lemma.

Part 3: Convergence

In any period t, the no arbitrage condition on the bubble asset requires that the bubble grows at the interest rate:

$$\tilde{b}_{t+1} = (1 + r_t)\tilde{b}_t.$$

This no arbitrage condition gives $\tilde{b}_{t+1}$ as a function of r_t and $\tilde{b}_t$: $\tilde{b}_{t+1} = g(r_t, \tilde{b}_t)$. Note that $\frac{\partial g}{\partial r_t} = \tilde{b}_t \geq 0$, i.e., the bubble size in $t + 1$ is an increasing function in the interest rate r_t. Furthermore, savers' first order conditions yield:

$$1 + r_t = \frac{v'(h^s_t)}{\kappa u'(c^s_{t+1,o})}.$$

From the savers' young-age budget constraint we know that h^s_t is decreasing in $\tilde{b}_t$. We also know that an increase in the bubble price $\tilde{b}_t$ will increase old-age consumption for savers. Therefore, we can express the interest rate as a function of $\tilde{b}_t$: $r_t = f(\tilde{b}_t)$ with $\frac{\partial f}{\partial \tilde{b}_t}$ given by:

$$\frac{\partial f}{\partial \tilde{b}_t} = \frac{v''(h_t^s)}{\kappa u'(c_{t+1,o}^s)} \frac{\partial h_t^s}{\partial \tilde{b}_t} - \frac{v'(h_t^s)}{\kappa (u'(c_{t+1,o}^s))^2} u''(c_{t+1,o}^s) \frac{\partial c_{t+1,o}^s}{\partial \tilde{b}_t} > 0. \tag{3.9}$$

This implies that the interest rate $r_t = f(\tilde{b}_t)$ is increasing in the size of the bubble $\tilde{b}_t$. Therefore the equilibrium dynamics can be characterized by the following equations, with $\tilde{b}_t$ being the state variable:

$$\begin{aligned} r_t &= f(\tilde{b}_t), \\ \tilde{b}_{t+1} &= g(r_t, \tilde{b}_t), \end{aligned}$$

with f and g both monotonic in $\tilde{b}_t$ and r_t, respectively. Note that $r_t = 0$ and $\tilde{b}_t = \tilde{b}$ is the unique steady state of this system. Now suppose that the initial bubble size is small: $\tilde{b}_0 < \tilde{b}$. Then, $r_0 = f(\tilde{b}_0) < f(\tilde{b}) = 0$. This means that, in period $t = 1$:

$$\tilde{b}_1 = g(r_0) = (1 + r_0)\tilde{b}_0 < \tilde{b}_0.$$

This inequality implies that $r_1 = f(\tilde{b}_1) < r_0 < 0$, and $\tilde{b}_2 = g(r_1) = (1 + r_1)\tilde{b}_1 < \tilde{b}_1$. By induction, we can prove that $\{\tilde{b}_t\}_{t=0}^{\infty}$ and $\{r_t\}_{t=0}^{\infty}$ are decreasing sequences. Therefore, at any period t:

$$\tilde{b}_t = \left[\Pi_{s=0}^{t-1} (1 + r_s)\right] \tilde{b}_0 \leq (1 + r_0)^t \tilde{b}_0.$$

Since $1+r_0 < 1$, it follows that $\lim_{t\to\infty} \tilde{b}_t \leq \lim_{t\to\infty} (1 + r_0)^t \tilde{b}_0 = 0$. Thus, in any pure equilibrium with initial $\tilde{b}_0 < \tilde{b}$, it must be that the bubble vanishes: $\lim_{t\to} \tilde{b}_t = 0$.

Now, suppose $\tilde{b}_0 > \tilde{b}$. Then $r_0 = f(\tilde{b}_0) > f(\tilde{b}) = n$ and

$$\tilde{b}_1 = g(r_o) = (1 + r_0)\tilde{b}_0 > \tilde{b}_0,$$

Which means that $r_1 = f(\tilde{b}_1) > r_0$ and $\tilde{b}_2 = g(r_1) = (1 + r_1)\tilde{b}_1 > \tilde{b}_1$. By induction, we can prove that $\{\tilde{b}_t\}_{t=0}^{\infty}$ and $\{r_t\}_{t=0}^{\infty}$ are increasing sequences. Therefore, at any time t, $\tilde{b}_t$ given by

$$\tilde{b}_t = \left[\Pi_{s=0}^{t-1}(1+r_s)\right]\tilde{b}_0 \geq (1+r_0)^t\,\tilde{b}_0.$$

Since $1+r_0>1$, it follows that the bubble explodes to infinity: $\lim_{t\to\infty}\tilde{b}_t=\infty$. Therefore, the bubble economy converges to the bubble steady state only when the initial bubble is $\tilde{b}_0=\tilde{b}$.

3.0.6 Proof of Lemma 8

Since we assume e^s sufficiently large that $r_n<0$ we know that the bubble-less allocation is feasible but not optimal in the pure bubble equilibrium. Formally, we can define b' such that $0<b'<\tilde{b}$. We can then define h' as the housing and $h_p^s\leq h'$, and let $c_y'\leq c_{y,n}^s$. With a bubble of size $b'>0$, it must then be that $c_o'\geq c_{o,n}^s$ since $r_n<n$. Therefore there exists a feasible bundle of housing under the pure bubble equilibrium that strictly dominates the optimal bundle for the bubble-less equilibrium and :

$$U(h_n^s,c_{y,n}^s,c_{o,n}^s)<U(h',c_{y,n}^s,c_o')\leq U(h_p^s,c_{y,p}^s,c_{o,p}^s).$$

3.0.7 Proof of Lemma 9

We show in this proof that borrowers experience lower lifetime welfare under a pure bubble than in the bubble-less equilibrium. Since we know that credit constrained borrowers will not purchase the bubble asset in equilibrium, the budget constraints for borrowers in the bubble-less and pure bubble equilibria respectively will be:

$$e^d+\bar{d}\frac{1}{1+r_n} = \kappa h_n^d+c_{n,y}^d, \tag{3.10}$$

$$e^d+\bar{d} = \kappa h_p^d+c_{p,y}^d, \tag{3.11}$$

which, when $r_n<0$, implies that $\kappa h_n^d+c_{n,y}^d>\kappa h_p^d+c_{p,y}^d$. This means that the borrowers' pure bubble bundle of housing and consumption must be feasible but non-optimal in the bubble-less equilibrium and:

$$U_p^d < U_n^d.$$

3.0.8 Proof of Corollary 10

Part 1: Comparison of savers' welfare

Since both bubble equilibria must have the same interest rate, the preferable equilibrium for savers will be the equilibrium that allows savers to most efficiently save for old age consumption. Old age budget constraints reveal:

$$c_{h,o}^s = b_h h_h^s + \bar{d} + \epsilon, \tag{3.12}$$

$$c_{p,o}^s = \tilde{b} + \bar{d} + \epsilon, \tag{3.13}$$

which means that, per unit size of the bubble, the pure bubble must allow for a *greater* increase in old age consumption (relative to the bubble-less equilibrium) since it will never be the case that $h^s = 1$ in any equilibrium. Intuitively, this is because of two factors: first, it is feasible (and optimal) for the savers to acquire *all* of the bubble for storage in the pure bubble equilibrium, while in the housing bubble equilibrium borrowers will be in competition for the bubble, as well, as it is attached to housing which they need to satisfy utility demand. Second, when housing is the bubble asset there is still the maintenance cost κ that must be spent before resale of housing, which makes housing a relatively inefficient means of storage (compared to the pure bubble).

Therefore, for bubbles of equal size, as long as $1 > h_h^s$, savers have greater utility in the pure bubble equilibrium than in the housing bubble equilibrium and $U_p^s > U_h^s$.

Part 2: Comparison of borrowers' welfare

In any equilibrium with credit constrained borrowers, welfare of borrowers is determined by how much housing and consumption borrowers are able to acquire when young. Since borrowers can take on the same amount of debt in either bubble equilibrium ($1 + r = 1$ in both equilibria), we know that:

$$\kappa h_d^s + c_{y,p}^d = (\kappa + b_h)h_h^s + c_{y,p}^d,$$

From this equation we can see that the housing bubble bundle for young borrowers is feasible but non-optimal in the pure bubble equilibrium since we know that, as long as the housing bubble exists with $b_h > 0$, $\kappa = p_p = p_n < p_h$. Therefore, borrowers experience higher welfare under the pure bubble than in a housing bubble and $U_p^d > U_h^d$.

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