

UTILIZING MULTILEVEL EVENT HISTORY ANALYSIS TO MODEL TEMPORAL  
CHARACTERISTICS OF FRIENDSHIPS UNFOLDING IN DISCRETE-TIME SOCIAL  
NETWORKS

Danielle O. Dean

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Approved by:

Daniel Bauer

Kathleen Gates

David Thissen

Mitch Prinstein

Donglin Zheng

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## **ABSTRACT**

Danielle O. Dean: Utilizing multilevel event history analysis to model temporal characteristics of friendships unfolding in discrete-time social networks  
(Under the direction of Daniel Bauer)

A social network perspective can bring important insight into the processes that shape human behavior. Longitudinal social network data, measuring relations between individuals over time, has become increasingly common – as have the methods available to analyze such data. Adding to these methods, a modeling framework utilizing discrete-time multilevel survival analysis is proposed in this dissertation to answer questions about temporal characteristics of friendships, such as the processes leading to friendship dissolution or how long it takes an individual to reciprocate a friendship. While the modeling framework is introduced in terms of understanding friendships, it can be used to understand micro-level dynamics of a social network more generally, such as the duration of reciprocated ties (or undirected relations) and the timing of reciprocal actions. Similar to the model proposed by de Nooy (2011), these models can be fit with standard generalized linear mixed model software, after transforming the data from a network representation to a pair-period dataset. Two main models are introduced as part of the framework, and a simulation study and empirical example are proposed for each. The first empirical example concerns friendship duration in high school students and the second concerns the timing of reciprocal “following” actions on the social network site Twitter. Advantages of the modeling framework are highlighted, and potential limitations and future directions are discussed.

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## **LIST OF ABBREVIATIONS**

AIP	Alternating Imputation Posterior estimation
FD	Friendship Duration
GLMM	Generalized Linear Mixed Model
ICC	Intraclass correlation
MFQ	Mood & Feelings Questionnaire
MCMC	Markov Chain Monte Carlo
PQL	Penalized quasi-likelihood
RT	Reciprocal Timing
WMM	Weighted multiple membership

## **Chapter 1: Introduction**

In recent years psychologists and other social scientists have become increasingly interested in analyzing network data, recognizing that social networks play a key role in people's lives (Borgatti, Mehra, Brass, & Labianca, 2009; Wasserman & Faust, 1994; Snijders, 2005a). For example, attributes of adolescent peer networks are important predictors of an individual's substance use (Ennett, et al., 2006) and socialization and social selection within networks play important roles in shaping adolescents' religious beliefs and behaviors (Cheadle & Schwadel, 2012). The rapid rise and influence of virtual social networks such as Facebook and Twitter have also highlighted the existence of network structures and have become a focus of research as well as a vehicle for studying network effects for many researchers (Cha, Haddadi, Benevenuto, & Gummadi, 2010; Romero & Kleinberg, 2010).

Although a variety of models have been developed to identify and study the basic structure of social networks, many fundamental questions remain stubbornly difficult to address using current analysis approaches. Example questions may include those surrounding friendship dissolution such as, "What are the processes leading to a friendship ending and what's the role of the individuals' depression in this process?" as well as "How long does a typical friendship last and does this differ between girls and boys?" Other example questions may surround reciprocal tendencies, such as "Do some individuals reciprocate the actions of others more quickly than other individuals?" as well as "Do people reciprocate actions more quickly if they are similar in status within the social network?"

One common aspect of all of these questions is that they share a concern with dyadic bonds within a network (e.g., specific friendships) rather than a concern with the global network structure. Another is that they focus on understanding the processes leading to – as well as timing of – an event occurring between two individuals, such as friendship formation or dissolution. The overarching aim of this dissertation is to provide a modeling framework for addressing such questions.

## **Goals**

The first goal of this dissertation is to propose the use of multilevel event history models to understand the processes leading to an event occurring between two individuals embedded within a social network. This proposed modeling framework enables analysis of how long friendships last and whether and when people reciprocate actions as well as the processes leading to these events, for example. The second goal is to test the use of these models under representative conditions that would be found in social network data. The last goal is to demonstrate the application of the modeling framework through two empirical analyses.

To accomplish the goals outlined above, I first provide some background and preliminary developments for the models to be investigated in this dissertation in the remainder of this chapter. First, I describe shortcomings in extant modeling approaches for addressing questions concerning event occurrence among dyads embedded in the network. Second, I discuss the use of event history models for social network data, leading to the general modeling framework to be investigated within this dissertation: a survival analysis approach that takes account of the special features (e.g., dependencies) present within network data. Subsequent chapters elaborate on the application and testing of this modeling framework.

## Models for Network Data

Statistical models with a social network perspective are often concerned with modeling the processes leading to a network's structure, known as social selection processes (Daraganova & Robins, 2013; Anderson, Wasserman, & Crouch, 1999). In other words, the entire network is often the outcome of interest. For example, network structure – specifically the formation of ties between nodes such as friendship nominations between peers – can be modeled using exponential random graph models (ERGMs), commonly referred to as  $p^*$  models (Wasserman & Robins, 2005; Frank & Strauss, 1986). ERGMs are built on the idea that social networks are stochastic and that the patterns evident within a given network can be seen as evidence of specific local processes, such as reciprocity, transitivity, and homophily<sup>1</sup> (Wasserman & Pattison, 1996). In addition to these social selection models, models of social influence are also common; these models aim to understand how a network's structure may constrain an individual's behavior (Daraganova & Robins, 2013). This dissertation focuses only on models for social selection processes, both for tie formation and tie dissolution, rather than on models for social influence.

Importantly, social network analysis provides a formal framework for testing ideas about the structure of relationships and an alternative to the assumption that individuals are independent (Wasserman & Faust, 1994). Multilevel models have a strong relationship to social network data due to their ability to account for the non-independence of individuals (Snijders & Bosker, 2012; Goldstein, 2011; Raudenbush & Bryk, 2002). In fact, multilevel models – also

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<sup>1</sup> Reciprocity is the tendency for nodes to form directed relationships to alters who have initiated a directed relationship to the node. Transitivity is the tendency for a “friend of a friend” to become a friend. Homophily is the tendency of nodes to form relations with those who are similar to themselves.

referred to as random effect models – have been applied to network data for several purposes in the past. The “ $p_2$ ” model for example allows the incorporation of nodal and dyadic attributes into a model of network structure for binary network data, utilizing random effects to represent the remaining variability in the actor level structural parameters (van Duijn, Snijders, & Zijlstra, 2004; Holland & Leinhardt, 1981). Multilevel models are also very useful as applied to connections in personal “ego” networks (Snijders, Spreen, & Zwaagstra, 1995; van Duijn, Snijders, & Zijlstra, 2004) and to understand directed dyadic data as in the Social Relations Model which can be estimated with a cross-classified multilevel model (Snijders & Kenny, 1999).

In addition to social network models as applied to cross-sectional network data, longitudinal social network models have become increasingly popular in recent years to understand network evolution. Many of these models are based on the assumption that observed networks are the result of a continuous-time Markov process (Snijders, 2005a). Longitudinal network data generally enables researchers to better understand the dynamics leading to an observed network at a point in time than cross-sectional data as one can condition on the first observation of the network to better understand the subsequent changes. For example, the actor-oriented model as implemented in the software program SIENA is a general and flexible framework that allows the probabilities of relational changes to depend on the entire network structure, with actors assumed to change their ties in order to optimize an objective function (Snijders, 2005a). Alternatively, Hanneke, Fu, and Xing (2010) build directly on cross-sectional ERGMs to model longitudinal network data with a model referred to as a Temporal ERGM (TERGM), adding an exponential family function for the transition probability of a network from one time period to the next.

While these models provide a framework from which to evaluate the processes leading to the network's structure, they are not structured to understand whether and when a specific type of event occurs between individuals embedded within the network. They are thus also limited with respect to answering questions regarding the processes leading to the event, such as those questions listed at the start of this chapter.

### **Event History Analysis**

In contrast to standard social selection network models, the proposed modeling framework was developed to answer research questions about temporal characteristics of friendships – whether and when an event occurs between individuals embedded within a network – and is not a general purpose social selection model. The proposed modeling framework does not aim to understand the processes leading to the entire network structure, but rather the processes leading to the occurrence and timing of specific events between dyads within a network through the use of multilevel event history analysis.

Event history analysis is at the core of the proposed framework as it is useful for understanding both whether and when events occur (event history models are also known as survival models, duration models, or failure-time models). In the social and behavioral sciences, the timing of an event can often be considered to be a discrete variable as data is often collected from a panel study or the year of an event is measured rather than the exact timing, as is the case with the timing of the events in the empirical applications in this dissertation. This paper thus focuses on discrete-time event history models, which can easily handle “tied” event times where two or more people have the same event time (Singer & Willett, 1993; Freedman, Thomson, Cambum, Alwin, & Young-DeMarco, 1988; Blossfeld, Hamerle, & Mayer, 1989). Discrete time models can be used as an approximation to the results of a continuous-time survival analysis if

continuous-time data were indeed available (Vermunt, 1997). Also important for the purposes here, discrete-time methods extend to incorporate time-varying covariates and a multilevel structure relatively easily (Steele, 2008; de Nooy, 2011).

To formalize discrete-time univariate event history analysis, let us assume for now that the event under study is non-repeatable in that an individual may only experience the event once.<sup>2</sup> An example event within directed social network data could be the timing of reciprocity between individuals from when the first individual within the pair initiates a relationship. Let the random variable  $E$  denote the event time and  $t$  the discrete time point, with  $t = 1, 2, \dots T$ . Due to censoring where the event time is not known for all pairs, the probability mass function  $f_t$  of  $E$  is not straightforward to compute. However, a function known as the hazard can be utilized instead by building on the conditional nature of event occurrence. The hazard function is the conditional probability of event occurrence at a certain point of time, given the event had not occurred at earlier time periods (Singer & Willett, 2003). The hazard is thus the unique risk of event occurrence at a given time period, for those eligible to experience the event:

$$h_t = P(E = t | E \geq t) = P(E = t | E > t-1) = \frac{P(E = t)}{P(E > t-1)}. \quad (1)$$

The hazard of event occurrence is a conditional probability bounded between 0 and 1 and can be modeled with a generalized linear model (GLM) (Hedeker, 2005; Agresti, Booth, Hobert, & Caffo, 2000; Singer & Willett, 1993). With the unit of observation denoted  $i$ , a model with a vector of predictors  $\mathbf{x}$  and associated vector of fixed effects  $\boldsymbol{\beta}$  may be written as:

---

<sup>2</sup> This assumption can be relaxed, through additional random effects for example.



$$\begin{aligned} \text{logit}(h_{it}) &= \mathbf{x}_{it}'\boldsymbol{\beta} \\ y_{it} &\sim \text{Bernoulli}(h_{it}) \end{aligned} \quad (2)$$

For example, suppose we aim to predict whether an individual will reciprocate a friendship after a friendship is initiated toward him or her (ignoring for now the dependencies of social network data). Further, suppose that there is an effect of gender and time such that females are more likely to reciprocate across all time periods and the hazard of reciprocating decreases over time. This model can be written (with example regression weights):

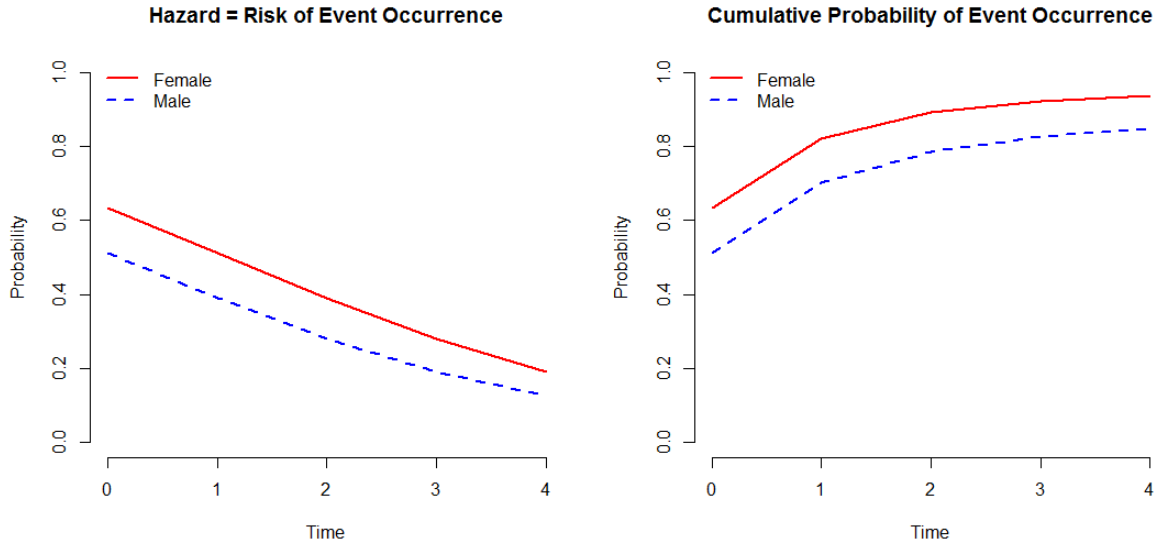
$$\begin{aligned} \text{logit}(h_{it}) &= -0.25 + 0.5 \cdot \text{female}_i - 0.5 \cdot \text{time}_{it} \\ y_{it} &\sim \text{Bernoulli}(h_{it}) \end{aligned} \quad (3)$$

Through simple algebra, the hazard function can then be used to find the cumulative probability of event occurrence, which may be more intuitive to interpret (Singer & Willett, 2003).<sup>3</sup> The equation above would result in a cumulative probability of 0.89 of reciprocating the friendship by four time periods for a female and 0.77 for a male, for example, as visualized in Figure 1 below.

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<sup>3</sup> The cumulative probability of event occurrence is one minus the survival probability, which is an algebraic function of the discrete-time hazard.

Figure 1: Example event history functions for reciprocating a friendship.  
*Simple example with effects of gender and time of 1) hazard functions – the conditional probability of reciprocating a friendship over time given the friendship was not reciprocated at earlier time periods and 2) the cumulative probability of reciprocating.*



## Event History Analysis for Social Networks

While event history analysis is commonly used in medical research, relatively few applications exist of event history analysis to social network data and questions. Focusing on models for social selection processes, applications are especially rare and nearly exclusively rely on continuous-time measurement of the network (Krempel, 1990; Hu, Kaza, & Chen, 2009; Kossinets & Watts, 2009; Butts, 2008). Others have focused on the propensity to be involved in ties but not specific tie formations (Robinson & Smith-Lovin, 2001; Tsai, 2000; Kim & Higgins, 2007). However, the vast majority of applications involving event history analysis and network data are social influence applications where network features are used as predictors in an event history analysis of another outcome, such as understanding the impact of the composition of a person's network on their health (Adams, Madhavan, & Simon, 2002; Villingshoj, Ross, Thomsen, & Johansen, 2006).

Event history models have rarely been used for social selection processes because of the complexities arising from the dependencies in network data. Researchers applying event history models to longitudinal social network data have largely either ignored dependencies (as was done in the simple example in Figure 1), sampled pairs from an extremely large network in order to obtain independent observations, or used a fixed effects approach by for example adding a dummy intercept in the model for all persons except one (Krempel, 1990; Kossinets & Watts, 2009; Butts, 2008; Brandes, Lerner, & Snijders, 2009). Ignoring dependencies can result in biased parameter estimates and standard errors, potentially leading to incorrect conclusions (Guo & Zhao, 2000). Also, while a fixed effects approach does adjust for all unmeasured covariates at the person-level, it is not parsimonious, nor does it allow for inference beyond the individuals within the sample and makes it difficult to examine person-level predictors (Snijders, 2005b, p. 665).

Fortunately, multilevel event history analysis can provide a parsimonious framework for accounting for dependencies inherent in social network data (Steele, 2011; Goldstein, 2011; Rabe-Hesketh & Skrondal, 2012). These models allow examination of both individual and dyad-level covariates simultaneously as well as time-varying network context effects and permit inference to the population from which the sample is drawn (Raudenbush & Bryk, 2002). De Nooy (2011) recently applied a multilevel event history model to model the dynamics of longitudinal network data on the timing of book reviews, utilizing cross-classified random effects to account for the individual tendencies of book authors and reviewers within the directed social network. Predictors within this model could include attributes of the book authors, the book reviewers, as well as the pair together (e.g. similarity of the two individuals on an attribute in order to test homophily). These predictors could also include characteristics of local network

structure such as transitivity – the tendency for a “friend of a friend” to become a friend.<sup>4</sup>

Discrete-time multilevel event history models are part of the generalized linear mixed model (GLMM) family and can thus be fit using software that fits GLMMs such as SAS, MLwiN, and R (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006).<sup>5</sup>

### **Proposed Modeling Framework**

The proposed modeling framework builds upon the prior application of the multilevel event history model for social network data by de Nooy (2011) to study more generally the processes leading to the occurrence and timing of events for individuals embedded within a network, expanded to include undirected networks and outcomes such as reciprocity and dissolution. The proposed modeling framework is sensitive to the fact that processes leading to the creation of a symmetric relationship from an asymmetric one may be very different from the processes leading to a tie forming within the pair at all (Cheng, Romero, Meeder, & Kleinberg, 2011; Luo, Tang, Hopcroft, Fang, & Ding, 2013). Also intuitively, processes leading to friendship dissolution could be very different from the processes leading to tie formation.

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<sup>4</sup> Network context covariates can be calculated by counting subnetworks created by the formation (or dissolution) of the tie being modeled from links that precede this tie (Wasserman & Pattison, 1996). The time ordering of the network thus allows network effects beyond the dyad to be included in the model in a straightforward manner without the “circular dependencies” that prohibit their inclusion in cross-sectional models of networks (de Nooy, 2011; Wasserman & Robins, 2005). One approach to including network context is to limit the context considered to lines appearing in the previous time period through a retrospective sliding window approach (Moody, McFarland, & Bender-deMoll, 2005). Alternatively, a decay function may be used to weigh network context by the length of time passed.

<sup>5</sup> A review of estimation options as well as estimation issues related to GLMMs is outside the scope of this dissertation (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006; Rodríguez & Goldman, 2001; Zeger & Karim, 1991; Raudenbush & Bryk, 2002). The methods within this dissertation are especially related to estimation of GLMMs with cross-classified and multiple membership structures (Cho & Rabe-Hesketh, 2011; Rasbash & Goldstein, 1994; Rasbash & Browne, 2008). For a comparison of different statistical packages for fitting logistic random effects models, see Li, Lingsma, Steyerberg, and Lesaffre (2011) who note that different software implementations ranging across both Frequentist and Bayesian approaches produce similar results for large datasets but not necessarily for smaller datasets.

For example, suppose one was interested in the tendencies for individuals to reciprocate friendships within a high school social network. A general purpose social selection model may reveal the effect of gender on the likelihood of an edge forming between two individuals within a network, controlling for the effect of reciprocity. For example, there could be a positive reciprocity effect, revealing that individuals are more likely to consider an alter to be a friend if the alter already considered them a friend; controlling for this effect, females may be more likely to consider other individuals to be a friend than males. Individuals are assumed to have the same underlying tendencies to reciprocate and the covariates are predicting tie formation generally rather than reciprocity.<sup>6</sup> In contrast, the proposed modeling framework allows one to study the effect of gender on the actual outcome of interest – the likelihood of reciprocating. It allows individuals to vary in their underlying tendencies to reciprocate and be the product of reciprocation; the modeling framework also answers questions about the timing of reciprocity, such as the median time to reciprocation, in a straightforward manner as it actually models the risk of reciprocating over time for each dyad.

Within this broader framework, this dissertation develops two specific models of network events as examples: one for friendship duration in undirected networks, and one for the timing of reciprocity in directed networks. For ease of exposition, the models that are introduced are mainly described as models for understanding friendship, as this is the research domain within which the models are applied in the empirical applications. However, these specific models aim

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<sup>6</sup> This may be relaxed somewhat by including an interaction term between gender and reciprocity, to understand how gender interacts with reciprocity tendencies to influence the formation of an edge. For example, one may find that females could have an even stronger reciprocity tendency than males. Then to understand whether females may reciprocate at different rates than males, even higher order interactions would be needed. Note that within this standard framework, the timing of reciprocity is not a straightforward product of the model.

to be examples within a general framework through which other outcomes could be analyzed, such as the timing of friendship duration in a directed network (e.g. perceived friendship duration) or different types of relationships other than friendship (e.g. organizational ties). When the outcome of interest is the timing of events occurring between individuals embedded within a network, the proposed modeling framework provides a more straightforward path than global network models for analyzing the data and evaluating the hypotheses of interest.

In addition to answering specific research questions which are important in their own right, the modeling framework discussed in this dissertation has an advantage of being easily applied to networks without clearly defined bounds (e.g., changing node set, sample from larger network) and can easily handle censored event times (e.g. missing ties). Also, model fitting of sparse data that is a ubiquitous problem for social network data – due to most dyads being unconnected versus connected – is mitigated as the focus remains on specific dyadic processes rather than the entire network.

In the remaining chapters, I formalize this modeling approach as well as test and apply the framework. Within Chapter 2, the model for friendship duration is developed and then subsequently tested with a simulation study and applied to high school friendship nomination data. Within Chapter 3, the model for the timing of reciprocity is developed, tested under a simulation study similar to Chapter 2 and then the use of the model is demonstrated with an application to user followings on the social network site Twitter. Chapter 4 concludes with discussion of the strengths and limitations of the proposed framework and directions for future research.

## **Chapter 2: Friendship Duration (FD) Model**

As described in Chapter 1, this dissertation proposes the use of multilevel event history analysis to study the processes leading to an event occurring between individuals within a dyad of a social network as well as the timing of that event. Toward that goal, the “Friendship Duration” (FD) Model is developed in this chapter to answer questions about the length of friendships and processes leading to friendship dissolution. This model development was motivated by an aim to model the processes leading to high school friendship dissolutions, specifically to understand the role of gender and depression in friendship dissolution, which is examined in the empirical example later in this chapter.

For the purposes of this model, friendship can be defined to start when both individuals nominate each other as a friend in a directed network or a friendship is recorded for a pair in an undirected network.<sup>7</sup> More generally, this proposed model is useful for understanding the duration of reciprocated ties in a directed network, or the duration of a tie in an undirected network. As the model is concerned with the duration of the dyadic relationship, the process being studied is dissolution rather than selection (e.g. examining whether and when the relationship ends rather than why the relationship began in the first place). In this chapter, I first develop the modeling framework including a description of how to structure the data, followed by a simulation study and an empirical application.

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<sup>7</sup> This model does not cover “perceived” friendship duration considering the direction of the tie in a directed network (this would require a cross-classified model rather than multiple membership). Instead, the reciprocal timing model could be used for this purpose.

For this model, consider the event process to begin when the ties are mutually reciprocated (or in the case of an undirected network, when the tie first appears). Let the outcome  $y$  in this case be a binary indicator, equal to 0 for time periods during which the ties remain mutual and 1 for the time period when the ties are no longer mutual. For simplicity here, it is assumed that once the mutual ties are dissolved, the ties remain in this state, although it is possible to extend the model for repeatable events.

Right censoring is very likely to occur in data measuring friendship dissolution events, as some individuals will remain friends throughout the observation period.<sup>8</sup> However, unlike most event history analyses, left censoring may also occur relatively often depending on how the network is sampled – as individuals may be friends before observation begins.<sup>9</sup> If start times of the friendships are known, these left censored event times may be incorporated by the conditional likelihood approach as outlined by Guo (1993). Thus one potential solution, which must be considered before sampling data, is to ask participants to recall when the friendship first began in addition to asking about their current friendships. Alternatively, careful selection of the observation period may allow one to define the friendship for specific time periods. For example, when sampling a friendship network from the beginning of high school to the end of high school, no left-censored data would exist if the event process under study is “how long do high school friendships last,” as the event process is thus defined to begin at the start of high school at the earliest irrespective of whether individuals were friends prior to high school; this is

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<sup>8</sup> Right censoring is when the event of interest does not occur during the time frame of the study (in the friendship duration case, this occurs when a friendship does not end during the study period).

<sup>9</sup> Left censoring occurs when the event process begins before the observation period. Unlike right censoring which is easily dealt with in the formulation of the likelihood, left censoring is not as straightforward to accommodate unless the hazard function is assumed to be constant over time (Guo, 1993). However, erroneously assuming a constant hazard can lead to severe bias in parameter estimates (Heckman & Singer, 1986).



the approach proposed for the empirical example in this chapter. Other alternatives include simply deleting left censored observations – although this risks a potential selection bias (Cain, et al., 2011), or incorporating the left-censored observations through more sophisticated techniques, such as imputing event times (Karvanen, Saarela, & Kuulasmaa, 2010), which are not discussed in this paper.

The dataset can be arranged in a “pair-period” structure where there is a row for each pair by time combination during which the pair is eligible to experience friendship dissolution.

Table 1: Construction of pair-period dataset.

*Time for a dyad begins when the relationship starts. “Time=0” is wave 3 for pair 1,2 but wave 2 for pair 1,3 for example.*

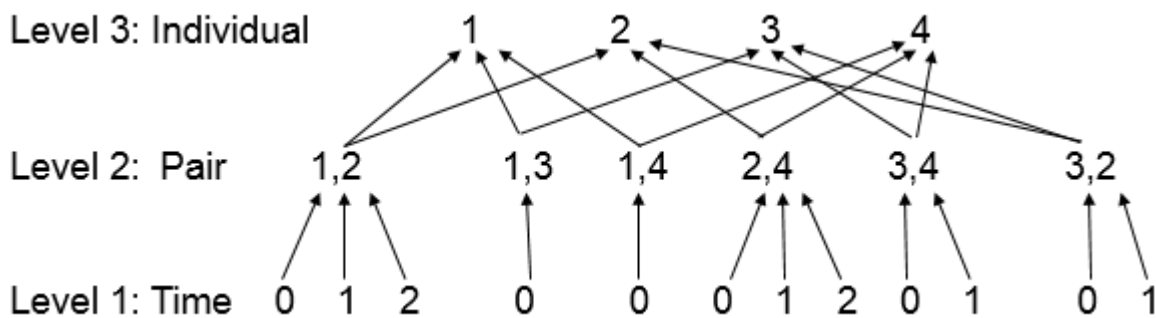
Pair	Friend 1	Friend 2	Time	Event	Wave
1,2	1	2	0	0	3
1,2	1	2	1	0	4
1,2	1	2	2	1	5
1,3	1	3	0	1	2
1,4	1	4	0	0	4
2,4	2	4	0	0	1
2,4	2	4	1	0	2
2,4	2	4	2	0	3
3,4	3	4	0	0	3
3,4	3	4	1	1	4
3,2	3	2	0	0	5
3,2	3	2	1	0	6

An example dataset for the model is given above in Table 1, where friend 1 and friend 2 are an arbitrary numbering of individuals within the pair. A time of 0 signifies the first time period that a friendship is eligible to dissolve after relationship formation and the outcome of interest is a binary indicator  $y$  of whether or not the relationship ends at time  $t$  as discussed above. Thus, time in the model is dyad dependent and may represent different actual waves of data collection

for different dyads. While the time scores need not be equidistant, the distance between the same time periods for different pairs should be consistent.<sup>10</sup>

Often in multilevel models, the individuals or occasions within individuals are the lowest level units of the analysis. However, the model described here is built in order to understand the relationship's duration, an outcome of the pair of individuals. We thus have pairs nested within individuals (Goldstein, 2011, p. 260). Each pair is measured over time, creating a three level structure. The model thus aims to understand whether and when the pair's relationship ends, accounting for the dependency that occurs from the fact a pair is nested within two individuals, and these individuals contribute to multiple pairs.

Figure 2: FD Model Nesting Diagram.  
*Multiple membership structure.*



This three level structure is visualized above in Figure 2, where the arrows represent the nesting structure and to which higher level unit the lower level unit belongs (Rasbash & Browne, 2008). Here, we have time nested within pairs which are then nested within individuals; for example,

<sup>10</sup> However, the distance between the same time periods for different pairs is unlikely to be consistent in practice unless data collection is specific to the timing within pairs, such as the data collection discussed in the empirical example in Chapter 3. For collection of data in waves where multiple pairs are examined around the same date, the time difference between waves should be consistent.

friendship between individuals 1 and 2 is recorded by “pair 1,2” over 3 time periods. While pairs are made up of only two individuals, individuals may be involved in numerous friendship pairs.

To formalize the model, let  $i$  and  $i'$  be two individuals within a pair  $p$ , with the total number of individuals  $I$  across pairs  $P$ . The hazard  $h$  of relationship duration, which represents the risk or conditional probability of friendship dissolution given the friendship had not yet dissolved, is modeled as a function of a vector of predictors  $\mathbf{x}$ . These predictors could be time as well as pair-level covariates, any of which may be allowed to vary over time (e.g. there could be an effect of time, pair’s average depression, and an interaction between time and the pair’s average depression). These predictors have an associated vector of fixed effects  $\boldsymbol{\beta}$ . A random effect  $u$  is added for each individual within the pair, making the model a multiple membership multilevel model, in order to account for each individual’s underlying propensity for their friendships to end (Goldstein, 2011; Rasbash & Browne, 2008). The random effects for all individuals are assumed to be normally distributed with mean 0 and common variance  $\sigma^2$ . The equations for the model are:

$$\begin{aligned} \text{logit}(h_{pt}) &= \mathbf{x}_{pt}'\boldsymbol{\beta} + u_i + u_{i'}, \\ y_{pt} &\sim \text{Bernoulli}(h_{pt}) \\ u_i &\sim N(0, \sigma^2) \end{aligned} \quad (4)$$

A high random effect would imply that an individual’s friendships are more likely to dissolve than a low random effect. Unlike most multiple membership models, there is no need to add

different weights for different dyads to the random effects as each individual is assumed to be an equal member of the dyad.<sup>11</sup>

## **Simulation Study**

The goal of this simulation study is to test the practicality of the proposed models, under a set of non-exhaustive but informative conditions. While multilevel event history models are by no means unique, nor are multilevel models with multiple membership structures, these models have yet to be tested together in the types of conditions that would arise in social network data such as in the empirical example in the next section. The conditions are chosen to reflect conditions that might be seen in practice, influenced in large part by preliminary analyses of the data from the empirical applications in this chapter and Chapter 3. The main goals of the simulation are to understand how the hazard rate and sample size – both in terms of the number of individuals as well as the number of friendships per individual – influence the ability to recover the fixed and random effects of the model. Additionally, the simulation study is designed to reveal the influence of the estimation method chosen and the amount of dependence in terms of the magnitude of the random effect variance.

The hazard function for this simulation study is constant in both the population-generating model and fitted model (i.e. single intercept), and the number of time periods is held constant at five to keep the scope of the simulation reasonable while focusing on more interesting conditions. This number of time periods is chosen to be close to the empirical examples, as the empirical example in this chapter has six time periods and the next chapter has five time periods. One binary pair-specific true effect is included in all conditions to test

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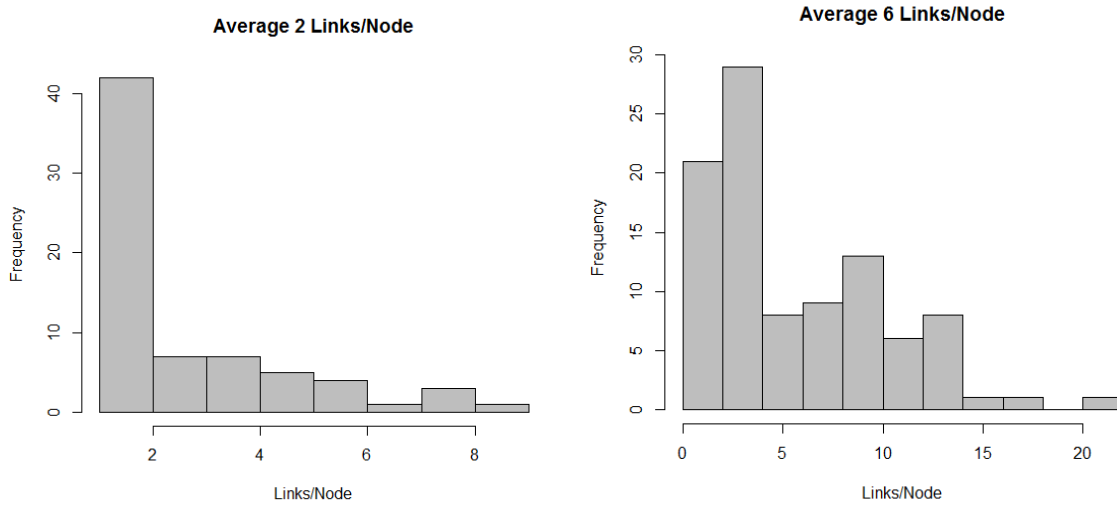
<sup>11</sup> For example, a multiple membership model with school effects may have a weight which approximates the length of time spent in each school for students who transition between schools.

recovery, with a regression coefficient of 0.5 and a balanced mix of the two groups in the population-generating model (e.g. in practice, this could be the effect of ‘same-gender’ versus ‘different-gender’ friendship pair with half of the pairs being one type versus another). One pair-specific null effect is included to test false recovery with the predictor generated from the mean of each individual’s predictor coming from a  $N(0,1)$  distribution (e.g. in practice, this could be testing for an effect of the mean depression of the pair where no effects exists).

The number of individuals is varied at 100 or 500, reflecting reasonable sample sizes for relatively small and relatively large network studies that are seen in psychology and related disciplines (Schaefer, Light, Fabes, Hanish, & Martin, 2010; Ojanen, Sijtsema, & Rambaran, 2013). The average number of links per node is varied at 2 and 6, again aiming to reflect a reasonable low and high range of ties per node seen in practice (Dijkstra, Cillessen, & Borch, 2013; Ahn & Rodkin, 2014). Both of these types of conditions reflect similar values to what is seen in the empirical applications in this chapter and the next chapter. Whether a link is ever to occur between nodes is generated randomly until the average links per node in the network reach this number; example distributions of the number of links per node are shown below in Figure 3. The timing of the link dissolution is then simulated with the logit link function as specified in Equation (4).

Figure 3: Links per Node Simulation Factor.

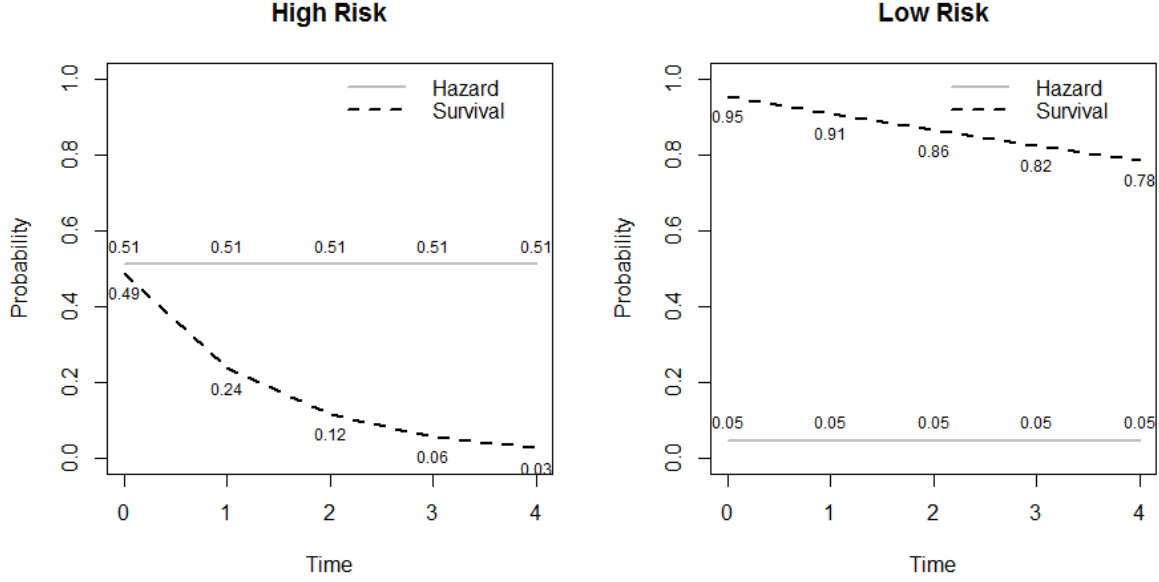
*Example distributions of links per node across individuals in two datasets, one with an average of two links per node and the other with an average of six links per node, with frequency representing the number of individuals.*



The hazard risk is varied between “low” and “high” risk. Low risk is defined specifically as an intercept within the logit function of -3, while high risk is an intercept of 0.05. These values were chosen after preliminary analyses of the empirical applications in this and the next chapter, as the “high” risk condition reflects a hazard function similar to what the friendship duration data reveal in the next section of this chapter while the “low” risk condition reflects a hazard function similar to the data analyzed in Chapter 3. These parameter choices result in the hazard and survival functions plotted in Figure 4, when random effects are held at the population average. High risk implies a probability of 0.51 of the event occurring at any time period for pairs that are eligible to experience the event and only a 0.03 probability of the event not occurring by the last time period, and low risk implies a conditional probability of 0.05 of the event occurring at any time period, with a cumulative probability of 0.78 of the event not occurring by the last time period.

Figure 4: Hazard factor definition.

*Resulting hazard and survival functions at “high” and “low” risk, keeping random effects at the population average.*



The random effect variance is varied at two levels ( $\sigma^2 = \{0.35, 1.40\}$ ). Previous research has often found a downward bias in the estimated variance components when fitting GLMMs to binary outcomes, the magnitude of which depends on a number of factors including the estimation method as well on the size of the random effect variance which is why the random effect variance is a factor in this simulation. With the two individuals in the pair  $p$  denoted  $i$  and  $i'$  respectively and an individual outside the pair denoted  $j$ , a latent variable conceptualization of the binary response variable can be used such that the friendship dissolution event occurs if  $y_p^* > 0$  and does not otherwise, to define an intraclass correlation coefficient. With the FD Model equation for the simulation then written as:

$$y_{pt}^* = \beta_0 + 0.5 \cdot \text{true} + 0 \cdot \text{null} + u_{1i} + u_{2i'} + \varepsilon_{pt} \quad , \quad (5)$$

where  $\beta_0$  denotes the intercept, true denotes the binary pair-level predictor that has a true effect, null denotes the continuous pair-level predictor that has no effect,  $u_{1i}$  denotes the latent variable for the first individual and  $u_{2i}$  denotes the latent variable for the second individual in the pair. Assuming the error  $\varepsilon_{pt}$  in Equation (5) follows a logistic distribution, an intraclass correlation (ICC) can be defined for the multiple membership random effect as the correlation among latent responses for the same person, conditional on the latent variable – underlying tendency to dissolve friendships in the FD Model – of other individuals:

$$Corr(y_{ii}^*, y_{ij}^* | u_{2i}, u_{2j}) = \frac{\sigma^2}{\sigma^2 + \frac{\pi^2}{3}} . \quad (6)$$

A random effect variance of 0.35 results in an ICC of 0.1 while a random effect variance of 1.4 result in an ICC of 0.3. An ICC of 0.1 was chosen because preliminary analyses of the empirical data in this chapter indicate an ICC close this value, without covariates in the model. The higher ICC of 0.3 was chosen to test the model with a stronger dependence structure, but low enough to reflect an ICC that might be seen in practice with similar network data, and is close to the dependence seen in preliminary analyses of the empirical data in the next chapter.

The FD model is fit using pseudo-likelihood (PQL)<sup>12</sup> through SAS GLIMMIX and using MCMC techniques through the MCMCglmm package in R (Hadfield, 2010).<sup>13</sup> Previous research

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<sup>12</sup> Although multiple membership and cross-classified models can be reformulated as multilevel models with nested random effects (Rasbash & Browne, 2008), the approach requires the evaluation of integrals with high dimensions and is computationally burdensome. Numerical quadrature is not available as an option for multiple membership and cross-classified random effects using GLIMMIX, as “METHOD=QUAD” requires that model can be processed by subjects (SAS/STAT(R) 9.3 User's Guide, 2015).

<sup>13</sup> Burn-in was set to 50,000 and the number of iterations at 100,000 with a thinning interval of 50 across all replications after trying different values and examining the autocorrelation between successive stored iterations for a few replications. The random effect was specified to have a univariate inverse Wishart prior with the variance at the limit set to 1 (V=1) and varying degree of belief parameter set to 0.002 (nu=0.002) as is commonly used for variance



has revealed that PQL is computationally efficient and is robust to starting values, performing better with larger cluster sizes and smaller variance components, but has been found to result in fixed and random effect estimates biased toward zero when modeling binary outcomes (Rodríguez & Goldman, 1995; Rasbash & Goldstein, 1994; Bauer & Sterba, 2011). Maximum likelihood approaches and MCMC perform best with a large number of clusters such that asymptotic properties hold, as biased and highly variable estimates may result otherwise (Tuerlinckx, Rijmen, Verbeke, & De Boeck, 2006; Bauer & Sterba, 2011).

In summary, the FD model is tested under 5 factors, together constituting 32 total simulation conditions ( $2 \times 2 \times 2 \times 2 \times 2$ ), and are summarized in Table 2 below. The models are replicated 1000 times between the first four factors below and the same data is then used to fit the model separately by PQL and MCMC.

Table 2: FD Model – Simulation conditions summary.

Number of Nodes	Links per Node	Hazard	Random Effect Variance	Estimation
100	2	low	0.35	PQL
500	6	high	1.40	MCMC

## ***Results***

Performance is assessed by examining bias of the different parameter estimates (Table 3), the standard deviation of the estimates compared to the standard error of the estimates (Table 4), Type I error of the null effect, and power of the pair-level effect (Burton, Altman, Royston, & Holder, 2006). Results are listed in Table 3 and Table 4 and then summarized after the tables by

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priors (Hadfield J. , 2014). A small set of sensitivity analyses revealed little substantive change in results with minor change in these values.

examining specific conditions as well as aggregating across factors to determine how well the model is able to recover the fixed and random effect estimates.

Table 3: FD Model Bias and Relative Bias of Parameter Estimates.  
*Recovery of the parameters across different conditions (factors italicized), with relative bias greater than 25% bolded.*

Estimation	Nodes	Links	TRUE	$\beta_0$		$\beta_1 = 0.5$		$\beta_2 = 0$	$\sigma^2$		Prop. *	
				Bias	Relative Bias	Bias	Relative Bias	Bias	TRUE	Bias		Relative Bias
MCMC	100	2	-3	-0.71	0.24	0.13	<b>0.27</b>	0.00	0.35	0.56	<b>1.61</b>	NA
MCMC	100	2	-3	-0.79	<b>0.26</b>	0.15	<b>0.29</b>	0.02	1.4	1.47	<b>1.05</b>	NA
MCMC	100	2	0.05	0.05	<b>1.01</b>	0.09	0.19	0.00	0.35	0.12	<b>0.36</b>	NA
MCMC	100	2	0.05	0.14	<b>2.71</b>	0.15	<b>0.30</b>	-0.02	1.4	1.05	<b>0.75</b>	NA
MCMC	100	6	-3	-0.45	0.15	0.05	0.09	0.01	0.35	0.07	0.20	NA
MCMC	100	6	-3	-0.48	0.16	0.09	0.17	0.00	1.4	0.49	<b>0.35</b>	NA
MCMC	100	6	0.05	0.03	<b>0.66</b>	0.10	0.20	0.00	0.35	0.17	<b>0.48</b>	NA
MCMC	100	6	0.05	0.06	<b>1.11</b>	0.09	0.17	0.01	1.4	0.63	<b>0.45</b>	NA
MCMC	500	2	-3	-0.43	0.14	0.06	0.11	0.00	0.35	0.05	0.15	NA
MCMC	500	2	-3	-0.47	0.16	0.06	0.13	-0.01	1.4	0.49	<b>0.35</b>	NA
MCMC	500	2	0.05	0.02	<b>0.33</b>	0.10	0.21	0.00	0.35	0.15	<b>0.44</b>	NA
MCMC	500	2	0.05	0.03	<b>0.54</b>	0.09	0.17	-0.01	1.4	0.63	<b>0.45</b>	NA
MCMC	500	6	-3	-0.43	0.14	0.05	0.10	-0.01	0.35	0.07	0.19	NA
MCMC	500	6	-3	-0.44	0.15	0.07	0.13	0.00	1.4	0.37	<b>0.27</b>	NA
MCMC	500	6	0.05	0.01	0.24	0.10	0.20	0.00	0.35	0.15	<b>0.43</b>	NA
MCMC	500	6	0.05	0.02	<b>0.39</b>	0.09	0.18	0.00	1.4	0.56	<b>0.40</b>	NA
PQL	100	2	-3	0.14	-0.05	-0.01	-0.02	0.00	0.35	-0.04	-0.11	0.88
PQL	100	2	-3	0.45	-0.15	-0.08	-0.15	0.02	1.4	-0.62	<b>-0.44</b>	1.00
PQL	100	2	0.05	0.00	-0.05	-0.05	-0.10	0.00	0.35	-0.09	<b>-0.26</b>	0.97
PQL	100	2	0.05	-0.09	<b>-1.87</b>	-0.08	-0.17	-0.01	1.4	-0.59	<b>-0.42</b>	1.00
PQL	100	6	-3	0.16	-0.05	-0.02	-0.05	0.01	0.35	-0.05	-0.14	0.99
PQL	100	6	-3	0.37	-0.12	-0.04	-0.08	0.00	1.4	-0.40	<b>-0.29</b>	1.00
PQL	100	6	0.05	0.01	<b>-0.87</b>	-0.03	-0.06	0.00	0.35	-0.05	-0.14	1.00
PQL	100	6	0.05	-0.04	<b>-0.87</b>	-0.07	-0.13	0.01	1.4	-0.40	<b>-0.28</b>	1.00
PQL	500	2	-3	0.22	-0.07	-0.02	-0.05	0.00	0.35	-0.10	<b>-0.29</b>	1.00
PQL	500	2	-3	0.52	-0.17	-0.10	-0.20	0.00	1.4	-0.69	<b>-0.49</b>	1.00
PQL	500	2	0.05	-0.04	<b>-0.76</b>	-0.04	-0.08	0.00	0.35	-0.10	<b>-0.30</b>	1.00
PQL	500	2	0.05	-0.13	<b>-2.58</b>	-0.10	-0.21	0.00	1.4	-0.64	<b>-0.46</b>	1.00
PQL	500	6	-3	0.17	-0.06	-0.02	-0.03	0.00	0.35	-0.06	-0.18	1.00
PQL	500	6	-3	0.38	-0.13	-0.05	-0.10	0.00	1.4	-0.43	<b>-0.31</b>	1.00
PQL	500	6	0.05	-0.02	<b>-0.46</b>	-0.03	-0.06	0.00	0.35	-0.07	-0.20	1.00
PQL	500	6	0.05	-0.06	<b>-1.29</b>	-0.06	-0.13	0.00	1.4	-0.41	<b>-0.29</b>	1.00

\*Proportion of simulations where the random effect variance parameter was greater than 0.

Table 4: FD Model Standard Error Recovery.

Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.

Estimation	Nodes	Links	TRUE	$\beta_0$			$\beta_1 = 0.5$			$\beta_2 = 0$			$\sigma^2$		
				SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	TRUE	SD	M(SE)
MCMC	100	2	-3	1.08	0.55	<b>0.51</b>	0.76	0.54	<b>0.71</b>	0.47	0.45	0.95	0.35	7.10	1.37
MCMC	100	2	-3	0.94	0.76	0.81	0.71	0.65	0.92	0.67	0.63	0.94	<i>1.40</i>	4.06	<b>0.75</b>
MCMC	100	2	<i>0.05</i>	0.37	0.38	1.01	0.46	0.44	0.97	0.39	0.37	0.95	0.35	0.43	0.55
MCMC	100	2	<i>0.05</i>	0.59	0.59	1.00	0.61	0.59	0.96	0.56	0.57	1.03	<i>1.40</i>	1.47	1.81
MCMC	100	6	-3	0.27	0.27	1.00	0.27	0.27	0.99	0.26	0.24	0.94	0.35	0.26	0.31
MCMC	100	6	-3	0.44	0.41	0.93	0.31	0.30	0.96	0.39	0.38	0.97	<i>1.40</i>	0.66	0.84
MCMC	100	6	<i>0.05</i>	0.25	0.24	0.98	0.23	0.24	1.06	0.25	0.24	0.97	0.35	0.20	0.27
MCMC	100	6	<i>0.05</i>	0.37	0.38	1.04	0.29	0.28	0.96	0.38	0.38	0.99	<i>1.40</i>	0.58	0.77
MCMC	500	2	-3	0.21	0.20	0.93	0.21	0.21	1.00	0.16	0.16	1.01	0.35	0.25	0.28
MCMC	500	2	-3	0.26	0.27	1.03	0.26	0.25	0.97	0.23	0.23	1.01	<i>1.40</i>	0.53	0.70
MCMC	500	2	<i>0.05</i>	0.17	0.16	0.96	0.19	0.19	1.02	0.16	0.16	0.99	0.35	0.16	0.21
MCMC	500	2	<i>0.05</i>	0.24	0.24	0.99	0.24	0.24	1.00	0.22	0.23	1.04	<i>1.40</i>	0.44	0.57
MCMC	500	6	-3	0.12	0.12	0.99	0.11	0.12	1.07	0.11	0.11	0.97	0.35	0.09	0.12
MCMC	500	6	-3	0.18	0.18	0.98	0.13	0.13	1.00	0.16	0.16	1.01	<i>1.40</i>	0.26	0.33
MCMC	500	6	<i>0.05</i>	0.11	0.11	0.97	0.10	0.11	1.08	0.10	0.11	1.05	0.35	0.09	0.11
MCMC	500	6	<i>0.05</i>	0.17	0.17	0.99	0.13	0.12	0.95	0.16	0.16	1.02	<i>1.40</i>	0.23	0.32
PQL	100	2	-3	0.35	0.35	1.01	0.41	0.43	1.05	0.34	0.34	1.00	0.35	0.25	0.30
PQL	100	2	-3	0.40	0.39	0.97	0.44	0.44	1.00	0.39	0.38	0.98	<i>1.40</i>	0.32	0.40
PQL	100	2	<i>0.05</i>	0.27	0.27	0.98	0.35	0.35	1.00	0.29	0.28	0.97	0.35	0.16	0.18
PQL	100	2	<i>0.05</i>	0.35	0.35	1.00	0.39	0.40	1.02	0.36	0.36	1.00	<i>1.40</i>	0.30	0.33
PQL	100	6	-3	0.22	0.22	1.00	0.23	0.24	1.03	0.22	0.21	0.96	0.35	0.14	0.14
PQL	100	6	-3	0.31	0.29	0.94	0.24	0.25	1.02	0.29	0.28	0.98	<i>1.40</i>	0.27	0.29
PQL	100	6	<i>0.05</i>	0.17	0.17	0.98	0.15	0.16	1.06	0.17	0.17	0.99	0.35	0.09	0.09
PQL	100	6	<i>0.05</i>	0.26	0.27	1.05	0.21	0.22	1.04	0.28	0.27	0.97	<i>1.40</i>	0.25	0.26
PQL	500	2	-3	0.15	0.15	0.98	0.18	0.18	1.01	0.14	0.14	0.99	0.35	0.10	0.11
PQL	500	2	-3	0.16	0.17	1.03	0.18	0.19	1.04	0.16	0.16	1.01	<i>1.40</i>	0.13	0.16
PQL	500	2	<i>0.05</i>	0.12	0.12	0.97	0.15	0.15	1.01	0.12	0.12	1.00	0.35	0.07	0.07
PQL	500	2	<i>0.05</i>	0.15	0.15	1.01	0.17	0.17	1.01	0.15	0.15	1.02	<i>1.40</i>	0.12	0.14
PQL	500	6	-3	0.10	0.10	0.98	0.10	0.10	1.04	0.09	0.09	1.02	0.35	0.06	0.06
PQL	500	6	-3	0.13	0.13	1.00	0.11	0.11	0.98	0.12	0.12	1.04	<i>1.40</i>	0.11	0.12
PQL	500	6	<i>0.05</i>	0.08	0.08	1.01	0.08	0.09	1.08	0.08	0.08	1.03	0.35	0.05	0.05
PQL	500	6	<i>0.05</i>	0.12	0.12	1.01	0.09	0.10	1.07	0.12	0.12	1.00	<i>1.40</i>	0.10	0.11

First examining recovery of the intercept (see Figure 5 and Figure 6 below), both MCMC and PQL estimation methods have small absolute bias for the high hazard condition (when  $\beta_0=0.05$ )<sup>14</sup> and larger absolute bias for the low hazard condition (when  $\beta_0= -3$ ) with MCMC biased away from zero and PQL biased toward zero. While bias is relatively large in some cases for the low hazard condition (as large as -0.79 with relative bias of 26%), this condition provides a rather stringent test as the event occurs extremely rarely. For both the high and low intercept conditions, MCMC estimation has larger variability across simulations than PQL ( $\overline{SD} = 0.36$  versus  $\overline{SD} = 0.21$ ). The worst estimation of the standard error of the estimates is for MCMC estimation for 100 nodes and 2 links per node where the standard deviation of the estimates is approximately twice the mean standard error of the estimates; however, the standard deviation of the estimates and the mean standard error of the estimates is nearly identical for other conditions, including all conditions with PQL estimation.

Aggregating across estimation methods, there is small or negligible bias in recovering the intercept for other factors but a clear pattern in variability where the smaller number of nodes and number of links per node result in greater variability in the estimates of the intercept; a larger random effect variance also results in greater estimate variability. Examining results within estimation method however, MCMC results in less average absolute bias when increasing from 100 to 500 nodes (0.39 versus 0.23) while the average absolute bias actually increases slightly

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<sup>14</sup> Although the relative bias in the high hazard condition is often quite large, the absolute bias is small enough that one can argue the model performs reasonably well in this case.

for PQL estimation when going from 100 to 500 nodes (0.16 versus 0.19). Both MCMC and PQL result in less average absolute bias when increasing from 2 to 6 links per node.

Figure 5: FD Model – Intercept Recovery, High Hazard Condition.

*Boxplot of parameter estimates across simulations, aggregated by each factor, with the red dotted line indicating the true parameter value.*

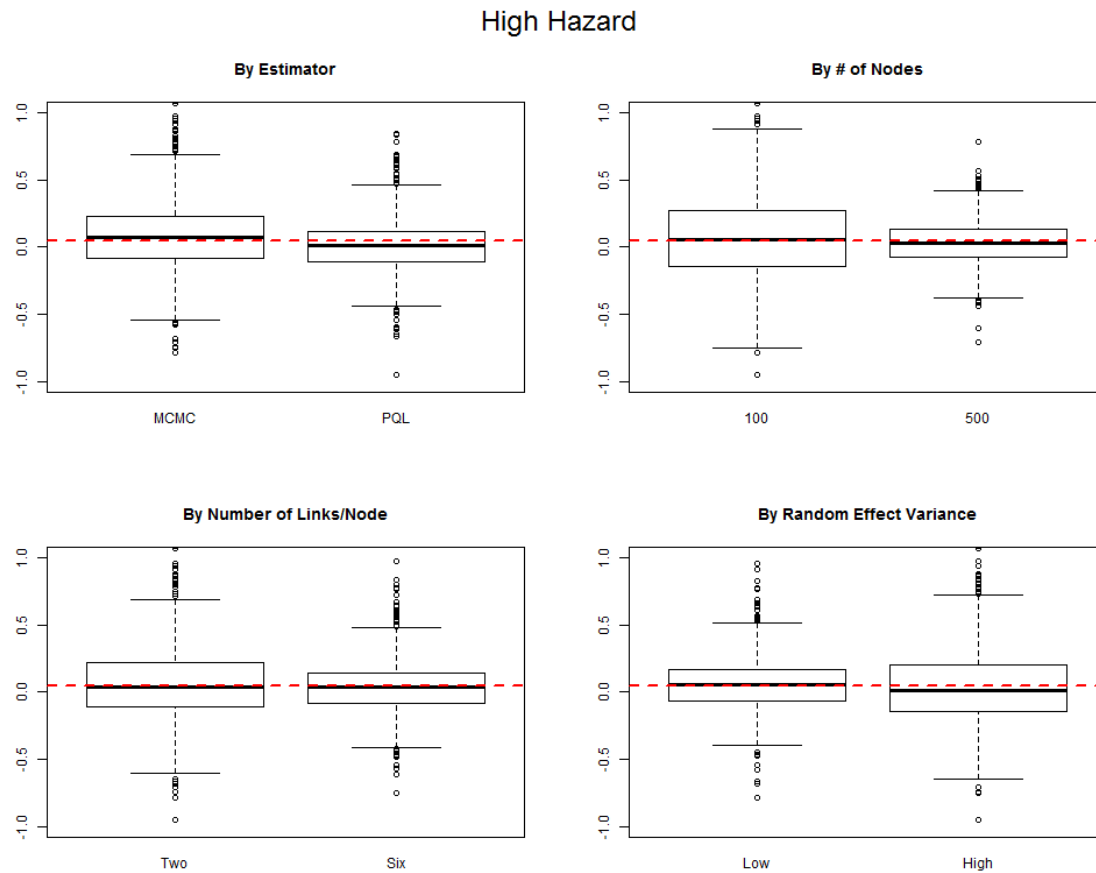
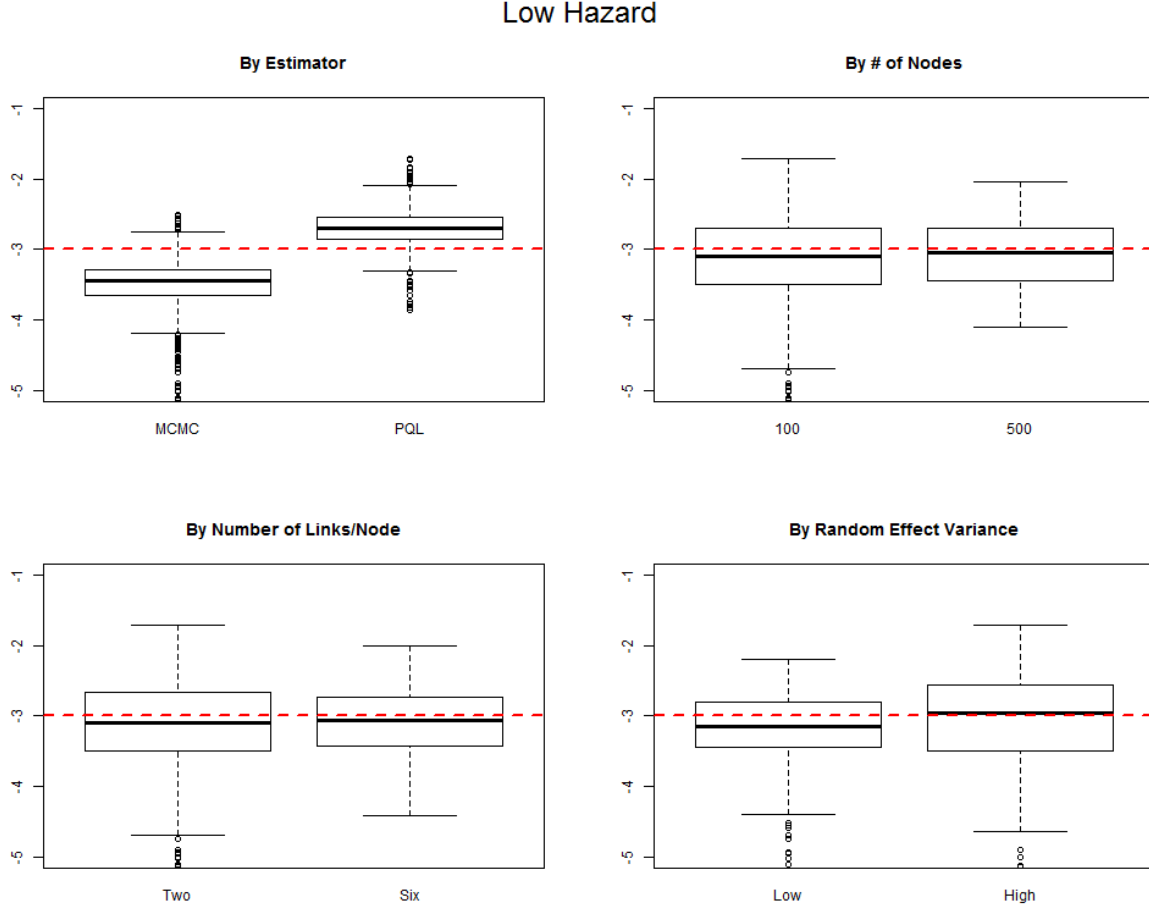


Figure 6: FD Model – Intercept Recovery, Low Hazard Condition.

*Boxplot of parameter estimates across simulations, aggregated by each factor, with the red dotted line indicating the true parameter value.*



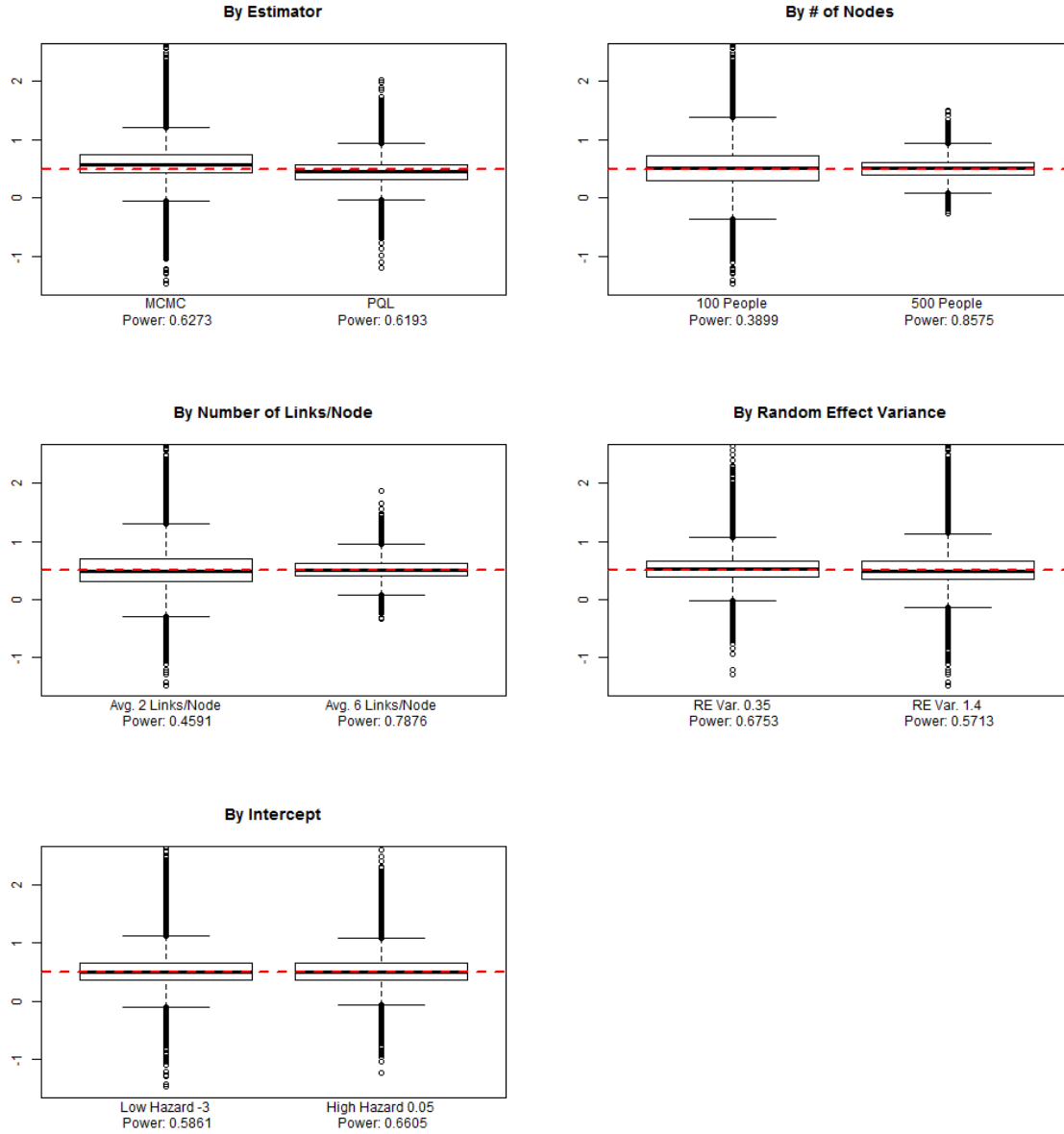
Recovery of the true effect (pair-level covariate,  $\beta_1=0.5$  across all conditions) follows a similar pattern, where MCMC estimation, as well as a smaller number of nodes, smaller average links per node, and larger random effect variance result in greater variability in the estimates (see Figure 7 below). The difference between 100 and 500 nodes makes the largest impact on the power to detect a significant effect (on average 0.39 versus 0.86), although power is higher for larger number of links per node, smaller random effect variance, and for the high intercept condition. Similar to intercept recovery, MCMC estimation is biased away from zero while PQL is biased toward zero, while each other factor reveals little bias when aggregating across the

other factors. MCMC tends to result in larger relative bias than PQL (MCMC with average relative bias of 18% versus PQL with average relative bias of 10%), with the difference especially apparent for small sample sizes and with smaller links per node. MCMC estimation again tends to result in less average absolute bias when increasing from 100 to 500 nodes (0.11 to 0.08) while the average bias for PQL between the two levels is similar on average (0.05 for both levels). Despite these differences, PQL and MCMC result in similar power to detect a significant effect, as well as the same pattern for power across the different factors. The difference between a high and low intercept has little impact on bias and variability of recovering the true pair-level covariate effect.



Figure 7: FD Model – True Effect (Pair-level Covariate) Recovery.

*Boxplot of parameter estimates across simulations, aggregated by each factor, with the red dotted line indicating the true parameter value.*



Type I error of the null effect (pair-level covariate,  $\beta_2=0$  across all conditions) is as expected, around 0.05 for  $\alpha=0.05$  for all conditions. There is negligible bias in the estimates across all conditions and similar to recovery of the intercept and true effect, the simulation

reveals that MCMC estimation, a smaller number of nodes, smaller average links per node, and larger random effect variance result in greater variability in the estimates, while the intercept factor has little impact.

The standard deviation of the random effect (Figure 8) is consistently overestimated by MCMC (average relative bias of 50%) and underestimated by PQL (average relative bias of 29%), for both low and high random effect conditions. The bias is quite severe with a smaller number of nodes and links per node, especially for MCMC; however, we again see an improvement in average absolute bias for MCMC when moving from 100 to 500 nodes (0.57 versus 0.39) but not for PQL (0.28 versus 0.31). The standard error of the random effect variance also tends to be overestimated by MCMC, with the exception of small number of nodes and links per node and low hazard when the standard error is largely underestimated (Table 4). The bias in estimating the random effect variance is consistent with previous research on binary outcome models with random effects, especially considering the number of links per node (equivalent to the number of objects per cluster) is so small in such network data (Rodríguez & Goldman, 2001; Rodríguez & Goldman, 1995). There is less variability in the estimates when the number of nodes, as well as the number of links per node, is higher and also when the intercept is higher (higher risk of event occurrence).

Figure 8: FD Model – Low Random Effect Standard Deviation Recovery.

*Boxplot of parameter estimates across simulations, aggregated by each factor, with the red dotted line indicating the true parameter value.*

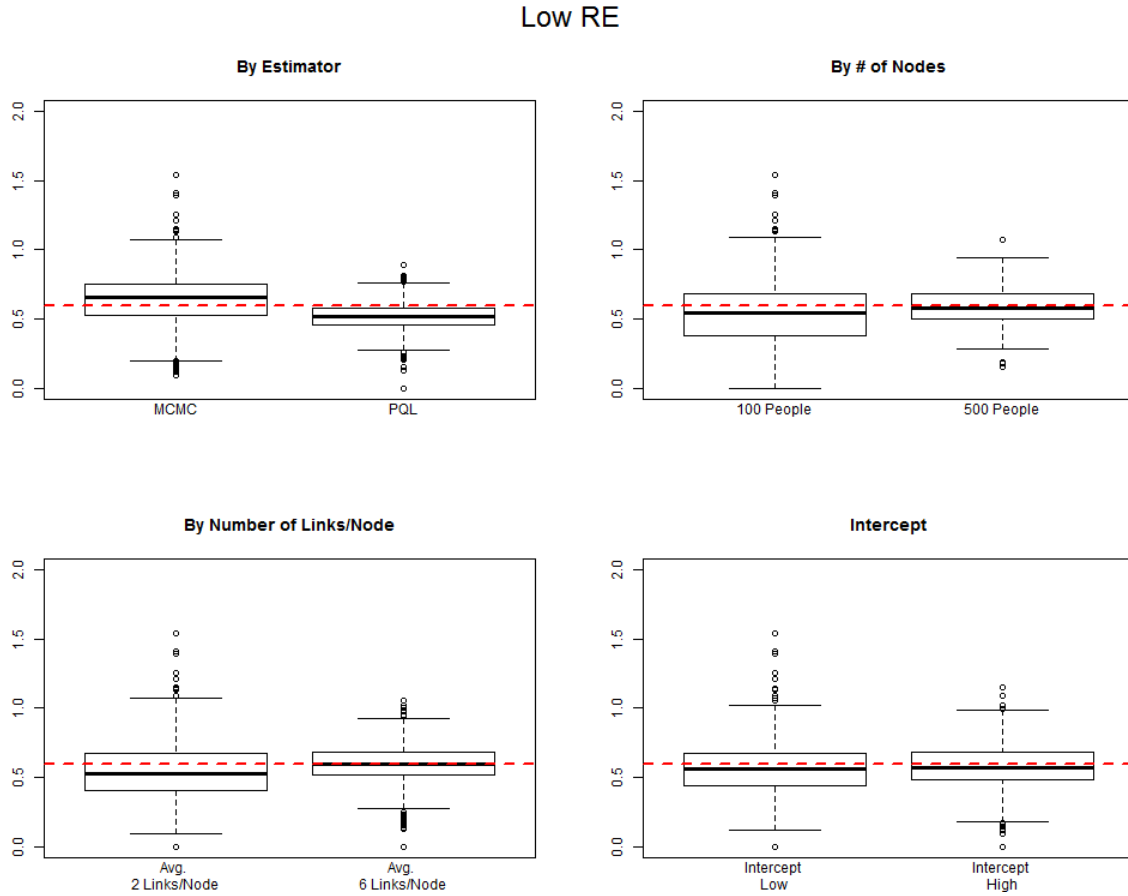
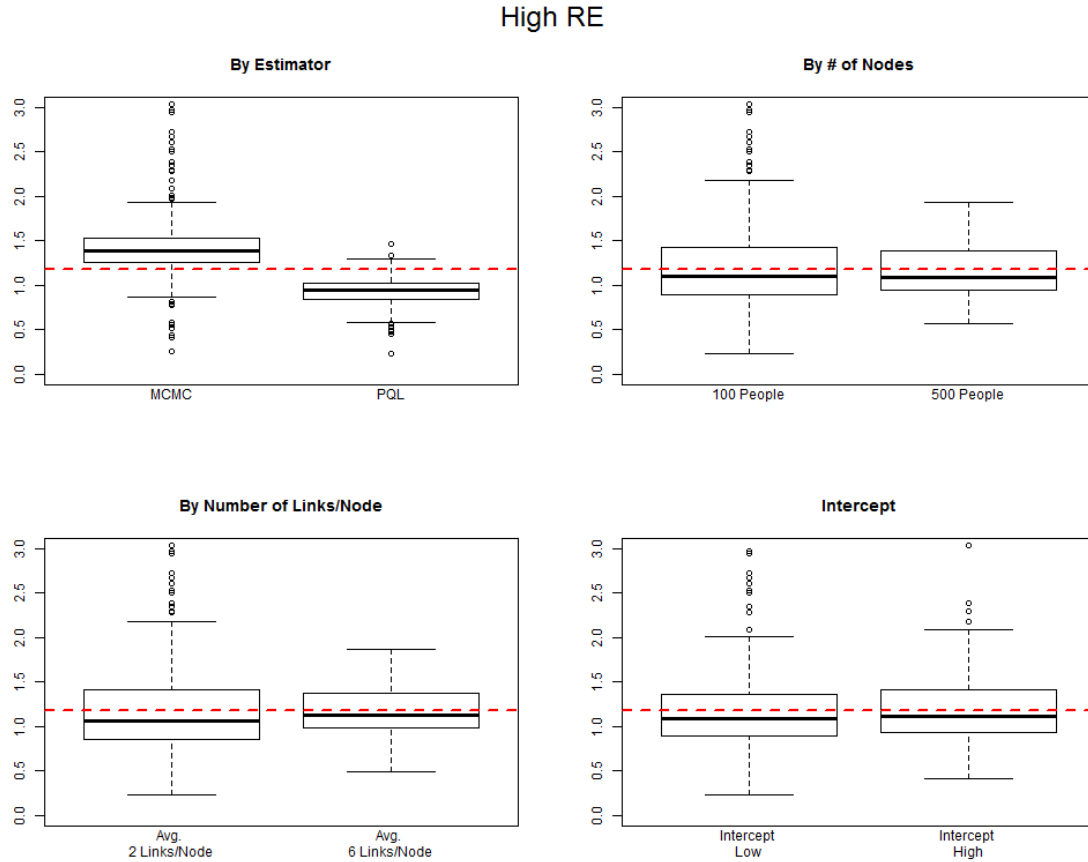


Figure 9: FD Model – High Random Effect Standard Deviation Recovery.  
*Boxplot of parameter estimates across simulations, aggregated by each factor, with the red dotted line indicating the true parameter value.*



In summary, although recovery of the random effect tends to be poor, the model recovers the fixed effect parameter estimates relatively well – especially when the number of nodes and number of links per node is larger. Increasing the number of nodes has the largest impact on the power to detect a significant pair-level effect and also improves the bias of MCMC estimation results although not PQL estimation results. An increase in the number of links per node also has a large influence on power, and improves results in terms of both bias and variability for both MCMC and PQL estimation, as does a smaller random effect variance. Finally, PQL estimation tends to have less variability in the conditions of this study than MCMC, and recovery tends to

be better for the high intercept condition when the risk of event occurrence is higher. Now that the model has been tested under relevant conditions to real social network data, we next apply the FD Model to high school friendship duration data, the empirical example that motivated the development of this model.

### **Application to High School Network Data**

Friendship dissolution occurs when the relationship ceases to exist and is often experienced with “considerable distress” (Baumeister & Leary, 1995, p. 503). Researchers have found that friendship serves different purposes for men and woman; from as early as preschool to adulthood, gender differences within friendships have been noted (Maccoby, 1990; Johnson & Aries, 1983). Females have been found to develop closer and more intimate relationships in general, and some researchers argue dissolution tends to be more significant for females as a result (Jalma, 2008). While dissolution is an important phase of many relationships, most research has focused solely on the mechanisms leading to friendship formation. Schaefer, Kornienko, and Fox (2011), for example, found that depression homophily could be created in a network through a withdrawal mechanism even in the absence of preference. However, comparatively little research has analyzed the factors leading to friendship dissolution and that research which has focused on dissolution has largely focused on romantic relationships (Sprecher, 1988; Felmlee, Sprecher, & Bassin, 1990).

In this empirical example, the processes leading to high school friendship dissolutions and the length of high school friendships are examined, especially in relation to the influence of gender and depression. Students were recruited at the beginning of high school, in the fall of their ninth grade. A total of 423 students were recruited from three different schools in North Carolina. All ninth graders within the three schools had the opportunity to consent. Data

collection began in February of ninth grade (in 2009), and collections took place at approximately six-month intervals through the spring of their twelfth grade year, with seven waves of data collection total.<sup>15</sup>

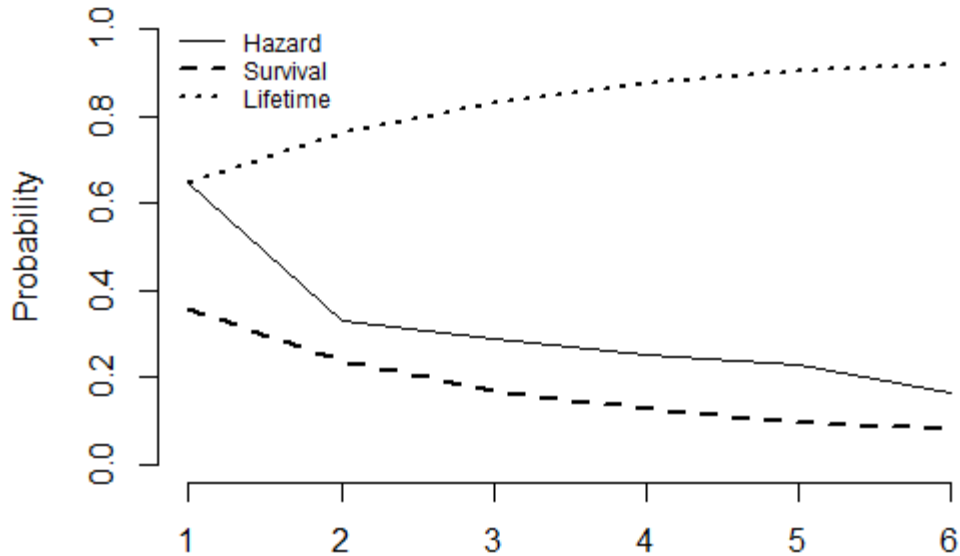
At each wave of data collection, students were given a current roster and allowed to nominate an unlimited number of individuals that they considered to be their friends. A friendship pair was considered to exist when both individuals within the pair nominated each other as friends. A total of 814 friendship pairs existed for at least two time periods of which 194 were male friendship pairs, 391 were female friendship pairs, and 229 were mixed gender friendship pairs. The friendship pairs were drawn from a total of 339 individuals (42% male). For each pair, time was coded to indicate when the friendship began (rather than the wave of data collection), and a binary indicator of friendship ending was created if either or both of the individuals failed to nominate the other individual within the pair as a friend. Friendship pairs were included for all time periods when both individuals within the pair provided nominations until the first friendship dissolution or censoring occurred.

The sample observed hazard, survival, and lifetime distribution functions are plotted in Figure 10, without modeling or accounting for the multiple membership dependence structure. The hazard function indicates friendships are most likely to end at the first time period after the friendship began, but if the friendship continues to exist it becomes less likely to end over time. The sample observed functions also indicate that high school friendships in this sample are highly fickle, in that 64% end after the first time period and only 8% are estimated to survive from the start of high school until the end.

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<sup>15</sup> Although there are seven waves of collection, there are only six time periods for purposes of the data structure as friendship cannot start and end at the same time period with this discrete-time collection.

Figure 10: Sample estimated functions for high school friendship data.  
*Sample estimated hazard, survival, and lifetime distribution functions without a formal statistical model.*



The FD Model is applied to longitudinal friendship nomination data using PQL through SAS Glimmix, as the simulation study revealed a tendency for PQL to have less variable parameter estimates albeit similar absolute bias for fixed and random effect recovery. Gender and depression enter into the model as the main predictors of friendship dissolution. Specifically, type of pair (males, females, mixed) is entered as two dummy-coded variables with mixed as the reference group and depression is investigated both in terms of the average depression of the pair as well as absolute difference in depression of the pair. Depression is measured at each time period through a mood and feelings questionnaire (MFQ) as described in Table 13 in Appendix A.1 (Costello & Angold, 1988).

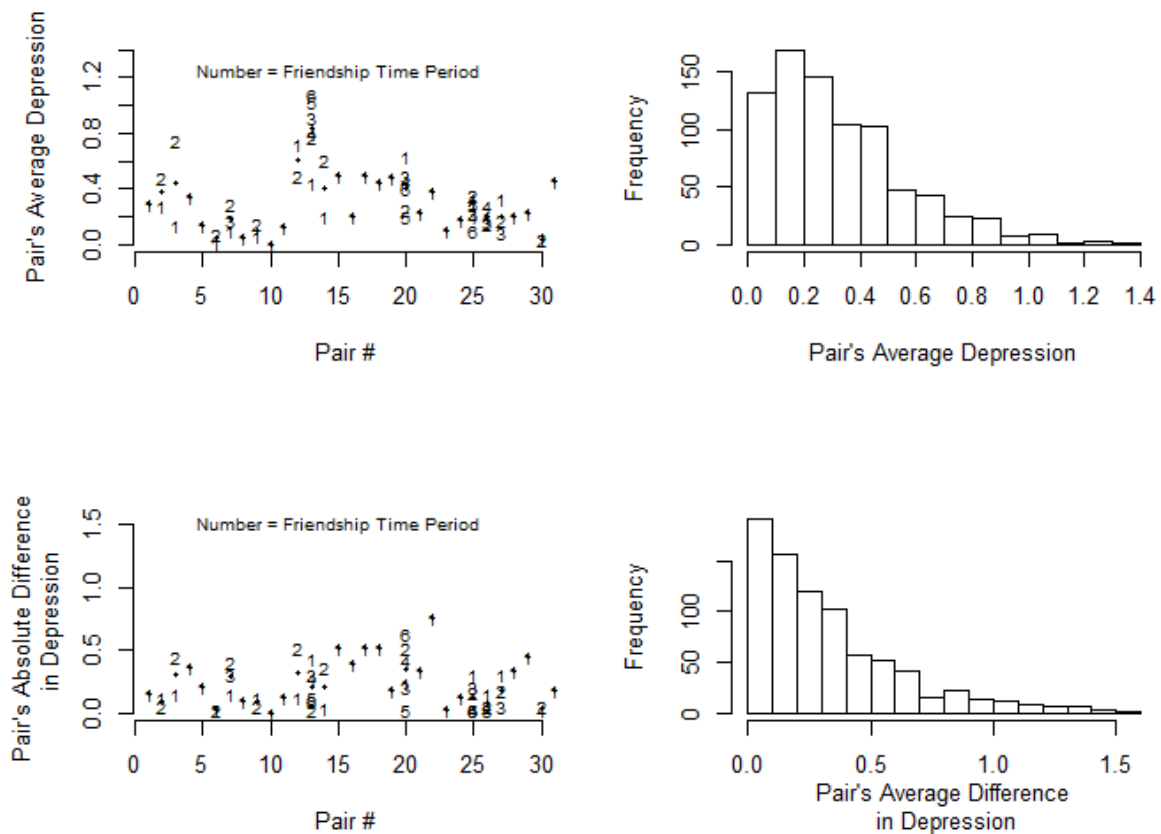
As each individual has a time-varying depression score, and there are two individuals within a pair, the effect of depression can be investigated multiple ways. Here, the average depression of the pair as well as the absolute difference in depression in the pair is investigated.

Each pair has a mean depression score at each time period as well as an absolute difference in depression at each time period, and there is variability both between pairs as well as within pairs.

In Figure 11, a sample of pairs' scores are plotted where the numbers in the plots on the left indicate the pair's mean depression score (top) and the pair's difference in depression score (bottom) at each time period, and the dot indicates the pair's average across time periods.

Figure 11: Depression.

*Depression scores within and between pairs*



To investigate both within and between pair effects of depression, the pair's mean depression across time periods as well as each pair's individual time period's deviation from the pair's mean depression is entered into the model. Similarly, pair-mean centering is used to investigate the influence of the absolute difference in the two individual's depression scores. Thus, there are



formally four depression effects in the model: 1) average depression of the pair across time periods, 2) pair's time-specific deviation from their average depression, 3) average absolute difference in the two individuals' depression, and 4) pair's time-specific deviation from their average absolute difference in depression. The complexity of breaking down the effect of depression in this way allows a more detailed view of the different ways depression of the two individuals influence the pair's likelihood of dissolution.

### ***Results***

As expected from the sample observed hazard functions, the model finds a high risk of dissolution, yet a decreasing risk of dissolution over time. Thus, high school friendships in this sample are very fickle, yet if they continue to last, they are less likely to end over time. For a mixed gender pair with zero depression scores, the probability of dissolution by the first time period after the friendship begins is 0.682 and although the risk of dissolving decreases over time, the cumulative probability of remaining friends through six time periods is only 0.03. The fixed effects are displayed below in Table 5 and the multiple membership effect is displayed below in Table 6.

Table 5: FD Model Application – Fixed Effects.

*Results from applying the FD Model to the high school friendship data.*

Effect	Standard		DF	t Value	Pr >  t
	Estimate	Error			
<b>Intercept</b>	<b>1.269</b>	<b>0.190</b>	<b>612</b>	<b>6.670</b>	<b>&lt;.0001</b>
<b>Time</b>	<b>-0.507</b>	<b>0.062</b>	<b>1280</b>	<b>-8.180</b>	<b>&lt;.0001</b>
Male Pair (Mixed Gender Reference)	-0.333	0.191	611	-1.740	0.0825
<b>Female Pair (Mixed Gender Reference)</b>	<b>-0.579</b>	<b>0.162</b>	<b>540</b>	<b>-3.580</b>	<b>0.0004</b>
Pair Mean Depression	-0.332	0.353	1280	-0.940	0.3481
Within Pair Mean Depression	0.482	0.551	1280	0.870	0.3818
<b>Pair Mean – Absolute Difference Depression</b>	<b>0.629</b>	<b>0.292</b>	<b>1280</b>	<b>2.150</b>	<b>0.0316</b>
Within Pair Difference Depression	-1.516	0.900	1280	-1.690	0.0922

Significant effects at  $\alpha=0.05$  are in bold.

Table 6: FD Model Application – Covariance Parameter Estimates.

Cov. Parameter	Estimate	Standard Error
Multiple Membership Effect	0.0738	0.0464

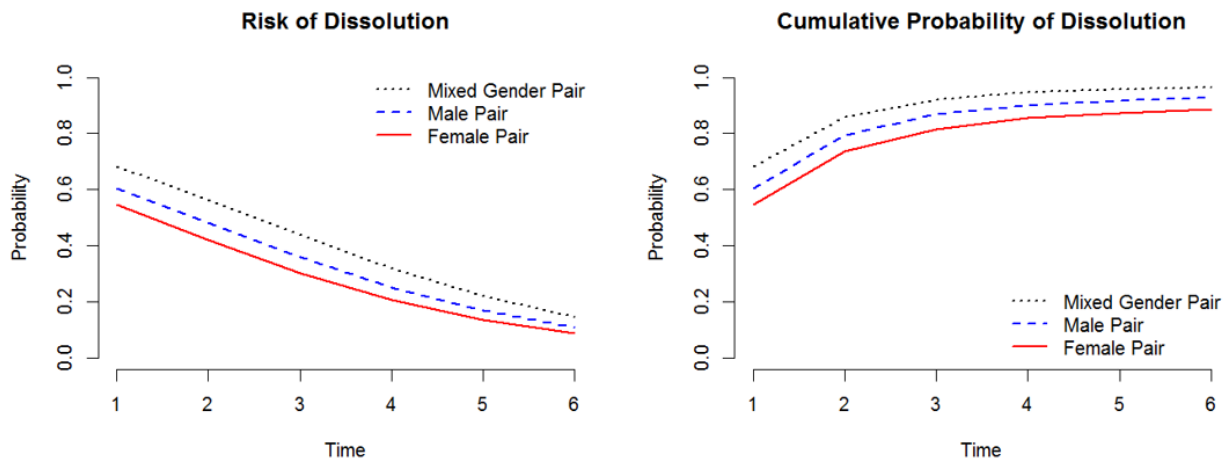
First examining the impact of depression, the average absolute difference in the two individuals' depression is found to have a positive relationship with the likelihood of dissolution,  $F(1, 1280) = 4.63, p = 0.03$ . Thus pairs who have a larger absolute difference in depression across time periods are more likely to end friendships across time than pairs who are more similar in depression. However, no significant effect is found for the within-pair effect of the difference in depression signifying there is not enough information to conclude a pair's absolute difference in depression changing over time relative to their average absolute difference is related to their likelihood of dissolution when controlling for the other effects. Given so many friendships end so quickly in this sample, it may be difficult for the model to pick out within-pair

effects.<sup>16</sup> Similarly, there is no significant effect of average depression (within or between pairs) on the likelihood of the friendship dissolving.

Holding depression constant, gender is found to have an influence on the likelihood of dissolution,  $F(2, 255.6) = 6.43, p < 0.01$ . Mixed-gender pairs have the highest likelihood of dissolution, while female pairs are the least likely to dissolve friendships over time, as visualized in Figure 12 below. The difference between the rate of dissolution for mixed-gender and female pairs is significant,  $F(1, 540.1) = 12.85, p < 0.01$ ; this implies that females have 0.56 times as high of odds of high school friendships ending at each time period than mixed-gender pairs, holding the random effect at the population average. However, no significant difference between male and female pairs is found,  $F(1, 127.8) = 1.96, p = 0.16$ , nor between male and mixed-gender pairs,  $F(1, 610.8) = 3.02, p = 0.08$ .

Figure 12: Friendship Dissolution by Gender.

*Mixed gender pairs are the most likely to dissolve across time periods, while female pairs are the least likely to dissolve (holding all depression effects at 0).*



<sup>16</sup> Entering depression without separating into within and between pair effects results in a similar pattern as found here, in that there is a significant and positive effect of the pair's absolute difference in depression on the likelihood of dissolution but no significant effect is found for the pair's average depression.

Higher-level interactions between gender and time, gender and depression, and depression and time were tested but not found to have significant effects. Accounting for the effect of depression and gender in the model, the ICC as defined in the simulation study was only 0.02, revealing a very small correlation between the latent response variable for an individual conditional on other individual's friendship dissolution tendency. Given the findings of the simulation study, however, it is likely that the random effect is actually underestimated here.

## **Conclusion**

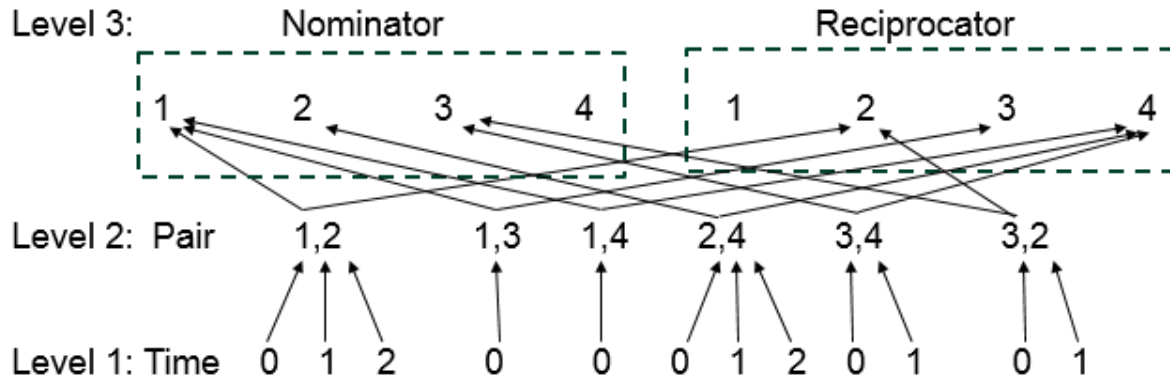
The FD Model introduced in this chapter allows an investigation of the processes leading to friendship dissolution and gives insight into the typical length of friendships. The simulation study revealed the model was able to recover the fixed effects reasonably well, with better recovery with a larger number of nodes, a larger number of links per node, and smaller random effect variance. The empirical application revealed insights into the timing of friendship dissolution for a high school friendship network, leading to straightforward answers about how long high school friendships typically last and the role of gender and depression in this process.

The FD Model includes a multiple membership random effect structure to account for dependencies inherent in a social network in that individuals are members of multiple relationships. Although the dependence structure found in the empirical example after accounting for gender and depression effects was relatively weak, the FD Model is able to account for this to allow investigation of the processes leading to friendship dissolution. In the chapter that follows, another multilevel event history model is proposed, but this one with a cross-classified membership structure to allow an investigation of the timing of reciprocity, as there is now a distinction between the two members of the pair.

### **Chapter 3: Reciprocal Timing (RT) Model**

The motivation for the second proposed model is to answer questions about the processes determining whether and when people reciprocate and will be denoted the “Reciprocal Timing” (RT) Model. This model was motivated by an aim to understand the processes leading to reciprocity and the timing of reciprocity on the social network site Twitter, which will be investigated in the empirical example later in this chapter. The second model thus concerns the timing of reciprocal actions and in this case, we have pairs nested within a cross-classification of nominators and reciprocators. The nominator is the individual who initiated the relationship between the individuals of the pair and thus started the event duration process, and the reciprocator has the opportunity to reciprocate the relationship. The duration between nomination and reciprocity is studied, formally placing time at level 1, pairs at level 2, and individuals (separated into nominator and reciprocators) at level 3, as visualized in Figure 13, where time is conditional on the relationship formation. Some or all individuals will be nominators for multiple relationships, as well as reciprocators in different relationships.

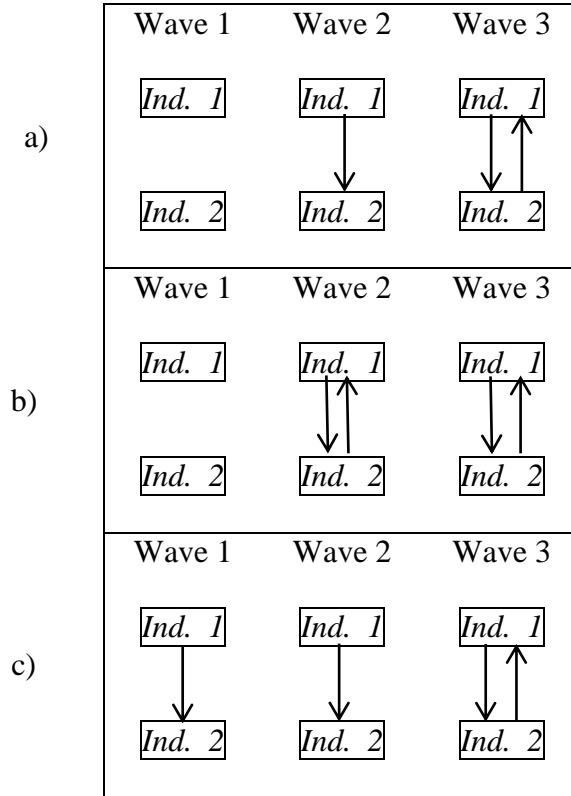
Figure 13: RT Model Nesting Diagram.  
*Cross-classified data structure.*



In this case, let dyad pair  $(n,r)$  contain two individuals, one of whom is the nominator  $n$  and one of which is the reciprocator  $r$ . We are interested in the timing of reciprocity, or the length of time from nomination to reciprocation. For example, in Figure 14a below, individual 1 is the nominator and individual 2 is the reciprocator. The duration to reciprocity in this pair is 1 (Wave 3 – Wave 2). The analysis in this case will often be complicated when collecting data in discrete-time in that the identity of the nominator and the reciprocator will not always be known. For example, in Figure 14b below, reciprocity happens at the same wave collection – therefore it is unknown who the nominator is and who the reciprocator is. Left censoring, as discussed for the FD Model, may commonly occur in the RT Model as well – for a nomination may occur before the observation period begins, as in Figure 14c. Approaches outlined previously for left-censoring may be used here as well, such as obtaining information on the start time of the nominations in addition to current nominations or reframing the definition of the start time of the event processes (Guo, 1993; Singer & Willett, 2003). In the empirical example later in this chapter, left censoring is avoided by only sampling nominations that occur after data collection has started.

Figure 14: Dyad ties over time when studying reciprocation.

*When measuring the network in discrete-time, in some cases we can distinguish between the “nominator” and the “reciprocator” (a), while we cannot distinguish between them in other cases (b), and event times may be masked by left-censored data (c).*



Let the binary indicator of reciprocity  $y$  be 0 for all time periods after nomination and before reciprocity occurs; if applicable for pair  $(n,r)$ , let the indicator be 1 for the time period of reciprocity. Time is dyad-dependent and is scored as the time elapsed from nomination, such that nomination begins the scoring of time for a dyad. Pairs only contribute data for time periods during which the reciprocator is eligible to reciprocate, as it is assumed for simplicity that individuals can only reciprocate once. However, this assumption may be relaxed in practice by extending to a repeated event model (Steele, 2008; Steele, Goldstein, & Browne, 2004; Willett & Singer, 1995). Similar to the model in Chapter 2, the analysis requires the data to be arranged into a “pair-period” dataset, only here there is a clear distinction between the roles of the two

individuals within the pair. See Table 7 for an example dataset for this model, when both the nominator and the reciprocator are known.

Table 7: Construction of pair-period dataset, known nominator and reciprocator. *Dataset is similar to the example in Chapter 2 only there is a differentiation between individuals.*

Pair	Nominator	Reciprocator	Time	Event	Wave
1,2	1	2	0	0	3
1,2	1	2	1	0	4
1,2	1	2	2	1	5
1,3	1	3	0	0	2
1,4	1	4	0	0	4
2,4	2	4	0	0	1
2,4	2	4	1	0	2
2,4	2	4	2	0	3
3,4	3	4	0	0	3
3,4	3	4	1	1	4
3,2	3	2	0	0	5
3,2	3	2	1	0	6

If the nominator and reciprocator for all pairs are known, the timing of reciprocity can be modeled as a function of time with standard cross-classified random effects; a random effect  $u_{1n}$  is added for the nominator, and a random effect  $u_{2r}$  is added for the reciprocator, each of which is assumed to be normally distributed. The hazard  $h$  of reciprocating is then modeled as a function of a vector of predictors  $\mathbf{x}$ , which could be time as well as person and pair-level covariates, any of which may be allowed to vary over time. These predictors have an associated vector of fixed effects  $\boldsymbol{\beta}$ . Using a logit link function, the equations for the model are:

$$\begin{aligned}
\text{logit}(h_{nrt}) &= \mathbf{x}'_{nrt} \boldsymbol{\beta} + u_{1n} + u_{2r} \\
u_{1n} &\sim N(0, \sigma_1^2), u_{2r} \sim N(0, \sigma_2^2) , \\
y_{nrt} &\sim \text{Bernoulli}(h_{nrt})
\end{aligned} \tag{7}$$

where  $\mathbf{x}$  could for example include gender, with potentially an effect for each role – such as nominators' gender, reciprocators' gender, and the gender of the pair (i.e. mixed, female, male).



Notice that in contrast to the FD Model, there is an opportunity to add person-level variables separately to tease out how attributes of individuals within the different roles of the pair affect the timing of reciprocity. There are also two distinct random effects, each with their own variance as there could be a different amount of variability between individuals in their tendency to reciprocate versus individuals' tendency to be the object of reciprocation.

### **Indistinguishable Nominator and Reciprocator**

Unfortunately, as mentioned above, it is likely that network data measured in discrete-time will not be able to distinguish between the nominator and the reciprocator when both occur within the same time period. Two potential approaches to this problem are outlined below, and are tested in the simulation study later in this chapter.

#### ***Weighted Multiple Membership Approach (WMM approach)***

The first potential approach is to consider both individuals within the pair as having some probability of being both the nominator and the reciprocator. The probabilities are equal to 0 or 1 for pairs where the identity of the nominator and reciprocator is known, but estimated for pairs where these identities are unknown. As described shortly, these probabilities function to define weights for the nominator and reciprocator random effects. This approach builds on ideas proposed by Hill and Goldstein (1998) to account for missing school identifiers in multilevel models of student outcomes. The model is similar to what Browne, Goldstein, and Rasbash (2001) term a 'multiple membership multiple classification' model, as the pair is technically modeled as being composed of multiple nominators (with probabilities that sum to 1) and multiple reciprocators (with probabilities opposite of the nominators, again which sum to 1).

Before considering the multiple membership structure for the two random effects, consider first a model with a single random effect (e.g. reciprocation only), with a multiple

membership structure. If there were two higher level units and the lower unit was a member of each equally, one might intuitively expect a weight of 0.5 would be used for each. However, the multiple membership structure results in smaller variation than if the lower level unit was a member of a single higher level unit:

$$\text{If } w_{i1} = w_{i2} = 0.5, \text{ var} \left( \sum_{i \in p} w_i u_i \right) = \sigma^2 / 2. \quad (8)$$

To obtain the correct variance, the weights must be entered as the square root of the probabilities.

Now extending to the full model with two random effects, let  $\pi_{i=n(p)}$  be equal to the probability that individual  $i$  is the nominator of pair  $p$  composed of  $(i, i')$ .

$$\begin{aligned} \text{logit} \left( h_{(i,i')t} \right) &= \beta_0 + u_{1i} \sqrt{\pi_{i=n(p)}} + u_{1i'} \sqrt{1 - \pi_{i=n(p)}} + u_{2i} \sqrt{1 - \pi_{i=n(p)}} + u_{2i'} \sqrt{\pi_{i=n(p)}} \\ u_{1n} &\sim N(0, \sigma_1^2), u_{2r} \sim N(0, \sigma_2^2) \\ y_{(i,i')t} &\sim \text{Bernoulli} \left( h_{(i,i')t} \right) \end{aligned} \quad (9)$$

where  $\pi_{i=n(p)} = 1$  if individual  $i$  is the nominator of pair  $p$ . The square root of the probabilities are used in order to ensure the level 3 variation remains the same as when all identifiers are known:

$$\begin{aligned} \text{var} \left( u_{1i} \sqrt{\pi_{i=n(p)}} + u_{1i'} \sqrt{1 - \pi_{i=n(p)}} \right) &= \sigma_1^2 \\ \text{var} \left( u_{2i} \sqrt{1 - \pi_{i=n(p)}} + u_{2i'} \sqrt{\pi_{i=n(p)}} \right) &= \sigma_2^2. \end{aligned} \quad (10)$$

In the examples that follow, the probability of each individual within the pair being the nominator is entered as 0.5, consistent with the notion of complete ignorance of identities (Hill & Goldstein, 1998; Goldstein, 2011). The dataset thus holds a weight of  $\sqrt{0.5}$  for each potential nominator and each potential reciprocator within the pair, such as pairs 5,7 and 7,6 in the example dataset in Table 8.

Table 8: Construction of pair-period dataset, weighted approach.

*In the case of an unknown nominator and reciprocator for some time periods, one can utilize a multiple membership cross-classified model*

Pair	Nominator				Reciprocator				Time	Event
	First		Second		First		Second			
	Id	Weight	Id	Weight	Id	Weight	Id	Weight		
5,6	5	1	6	0	5	0	6	1	1	0
5,6	5	1	6	0	5	0	6	1	2	0
5,6	5	1	6	0	5	0	6	1	3	1
5,7	5	$\sqrt{0.5}$	7	$\sqrt{0.5}$	5	$\sqrt{0.5}$	7	$\sqrt{0.5}$	1	1
5,8	5	1	8	0	5	0	8	1	1	0
5,8	5	1	8	0	5	0	8	1	2	0
6,8	6	0	8	1	6	1	8	0	1	0
6,8	6	0	8	1	6	1	8	0	2	0
6,8	6	0	8	1	6	1	8	0	3	0
7,8	7	1	8	0	7	0	8	1	1	0
7,8	7	1	8	0	7	0	8	1	2	1
7,6	7	$\sqrt{0.5}$	6	$\sqrt{0.5}$	7	$\sqrt{0.5}$	6	$\sqrt{0.5}$	1	1

The approach has been used with a single random effect (e.g. students are members of multiple schools, but the schools are assumed to follow a single distribution) (Hill & Goldstein, 1998). However, this approach has not previously been tested in similar circumstances to these – binary outcomes, applying to multiple random effects, and in social network data – motivating the simulation study later in this chapter.

#### ***Data Augmentation Approach (AIP approach)***

The alternating imputation posterior (AIP) algorithm has been utilized for approximate maximum likelihood estimation of cross-classified random effect models, with each individual's random effects being treated as missing data that the method approximates in order to find the optimal model parameters (Cho & Rabe-Hesketh, 2011; Clayton & Rasbash, 1999). The second proposed approach accommodating the indistinguishable nominators and reciprocators builds

upon the ideas of this estimation method, treating the identifiers as another piece of missing information in the optimization.

More specifically, the AIP algorithm alternates between a data augmentation step and a step in which the posterior distribution is updated. In the RT Model with partially unknown identifiers, the AIP algorithm alternates between what can be denoted 1) a “nominator wing” in which the fixed effect parameters and random effect variance of the nominators is estimated treating the current reciprocator random effect estimates as known, and 2) a “reciprocator wing” in which the fixed effects and random effect variance of the reciprocators is estimated treating the current estimates of the nominator random effects as known. Rather than treating only the random effects as missing data as has been done with the AIP algorithm in previous research, the proposed approach is to treat also the identification of the nominator and the reciprocator within the pair as missing data. Each individual’s classification as the nominator or the reciprocator in the pair is imputed by borrowing information from the individual’s other relationships and observed characteristics of the individuals within the pair. An imputation step is added after the traditional wings of the algorithm to impute the distinction between the nominator and the reciprocator given the current random effect estimates and covariates. After setting initial values for the random effects and identity of the nominator and reciprocator with each pair, the algorithm iterates between wings until convergence. An outline of a single iteration is given below.

Consider the RT Model with a single intercept (constant hazard) and cross-classified random effects, given by  $\text{logit}(h_{nr}) = \beta_0 + u_{1n} + u_{2r}$  where  $u_{1n} \sim N(0, \sigma_1^2)$ ,  $u_{2r} \sim N(0, \sigma_2^2)$ .

### *Nominator wing*

1. Treat the random effect estimates of the reciprocator of the pair  $\hat{\mathbf{u}}_2^{k-1}$  from the previous iteration as known and obtain maximum likelihood estimates  $\hat{\boldsymbol{\vartheta}}_1^k$  with estimated covariance matrix  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\vartheta}}^k$  by fitting the model:

$$\begin{aligned} \text{logit}(h_{nrt}) &= \beta_0 + u_{1n} + u_{2r}^{k-1} \\ u_{1n} &\sim N(0, \sigma_1^2) \end{aligned} \quad (11)$$

where the parameters are denoted  $\boldsymbol{\vartheta}_1 = \{\beta_0, \log(\sigma_1^2)\}$ .

2. Sample model parameters from their estimated sampling distribution,

$$\boldsymbol{\vartheta}_1^k | \mathbf{u}_2^{k-1} \sim N(\hat{\boldsymbol{\vartheta}}_1^k, \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\vartheta}}^k) . \quad (12)$$

3. Sample random effect estimates of the nominators of the pair  $\mathbf{u}_1^k$  from its conditional posterior distribution with parameters  $\hat{\boldsymbol{\vartheta}}_1^k$ .

### *Reciprocator wing*

1. Treat the random effect estimates of the nominator of the pair  $\hat{\mathbf{u}}_1^k$  from the nominator wing as known and obtain maximum likelihood estimates  $\hat{\boldsymbol{\vartheta}}_2^k$  with estimated covariance matrix  $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\vartheta}}^k$  by fitting the model:

$$\begin{aligned} \text{logit}(h_{nrt}) &= \beta_0 + u_{1n}^k + u_{2r} \\ u_{2r} &\sim N(0, \sigma_2^2) \end{aligned} , \quad (13)$$

where the parameters are denoted  $\boldsymbol{\vartheta}_2 = \{\beta_0, \log(\sigma_2^2)\}$ .

2. Sample model parameters from their estimated sampling distribution,

$$\boldsymbol{\vartheta}_2^k | \mathbf{u}_1^k \sim N(\hat{\boldsymbol{\vartheta}}_2^k, \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\vartheta}}^k) . \quad (14)$$

3. Sample random effect estimates of the nominators of the pair  $\mathbf{u}_{1n}$  from its conditional posterior distribution with parameters  $\hat{\boldsymbol{\theta}}_2^k$ .

### *Identification Step*

Impute the nominator and reciprocator identification within each pair with missing distinctions using a conditional probability model for the distinction between the two based on the sampled model parameters, given current estimates of the random effects,  $\hat{\mathbf{u}}_1^k, \hat{\mathbf{u}}_2^k$  and observed characteristics of the individuals within the pair, if applicable. A model, such as a logistic regression model using current estimates of the random effects as predictors, is used to predict the probability that individual  $i$  is the nominator of the pair and individual  $i'$  is the reciprocator. The proposed approach is thus to modify the AIP algorithm and impute  $i$  as the nominator and  $i'$  as the reciprocator within each wing of the algorithm at a given iteration if the model reveals that:

$$\hat{P}(i = n(p)) > \hat{P}(i' = n(p)) \quad (15)$$

or to impute  $i$  as the reciprocator and  $i'$  as the nominator in the reverse case, or finally to randomly impute one as the reciprocator and the other as the nominator in the case these probabilities are equal.

As the algorithm iterates over the nominator wing, reciprocator wing, and imputation step, it is possible that the individual imputed as the nominator may fluctuate between iterations. The idea of using other information in the data to impute the missing data of the nominator or reciprocator identity builds intuitively on the ideas of the AIP algorithm, but is assessed empirically in the simulation study in the next section of this chapter.

The models at step 1 of each wing of the AIP algorithm may be fit with any GLMM estimation method, including ML by adaptive quadrature – as the estimation occurring during a given wing of the algorithm includes simply one random effect and is no longer cross-classified. Convergence may be checked by running at least two independent chains, such as giving different starting values for the random effects as is done in this dissertation or starting one of the chains with the reciprocator wing instead (Gelman & Rubin, 1992; Clayton & Rasbash, 1999). Procedures outlined by Cho and Rabe-Hesketh (2011) are used in this dissertation for determining the length of burn-in required. The marginal posterior means and covariance matrix are estimated using the means and the variance of the conditional posterior distributions,  $\{\hat{\boldsymbol{\theta}}_1^k, \hat{\boldsymbol{\Sigma}}_g^k\}$  for the first random effect for example, after discarding those estimated before burn-in (Gelfand & Smith, 1990; Rubin, 2004).

## **Simulation Study**

The simulation study follows the structure of the study in Chapter 2, in terms of the structure of the hazard and the number of time periods, as well as the conditions under study for the number of nodes and the number of links per node. One binary pair-specific true effect is again included in all conditions with a regression coefficient of 0.5 to test recovery of effects and one pair-specific null effect is included to test false recovery. However, the simulation contains an extra factor for the potential approaches to accounting for an unknown distinction between the nominator and the reciprocator of the pair which was not applicable for the FD Model.

The RT Model is fit three different ways: 1) known identifiers, 2) some unknown identifiers – weighted multiple membership approach, and 3) some unknown identifiers – data augmentation approach. The data for the unknown identifiers models is taken exactly from the data simulated for the known identifiers model, except the identity of the nominator and

reciprocator is set to unknown for reciprocations that occur at the same time period as the nomination, just as would occur in practice. The random effect variance is varied at three levels (low:  $\sigma_1^2 = 0.35, \sigma_2^2 = 0.35$ ; unbalanced:  $\sigma_1^2 = 0.35, \sigma_2^2 = 1.40$ , and high:  $\sigma_1^2 = 1.40, \sigma_2^2 = 1.40$ ). While the RT Model is fit using both PQL and MCMC when the identifiers are known, only PQL (using SAS Glimmix) is used for the models dealing with missing identifiers in order to keep the scope of the study more reasonable.<sup>17</sup> The simulation conditions are summarized in Table 9.

Table 9: RT Model – Simulation conditions summary.

Model	Number of Nodes	Links per Node	Hazard	Random Effect Variance	Estimation
RT Model – known identifiers	100	2	low	Low: 0.35 (u1) & 0.35 (u2)	PQL
	500	6	high	Unbalanced: 0.35 (u1) & 1.40 (u2) High: 1.40 (u1) & 1.40 (u2)	MCMC
RT Model – WMM approach	100	2	low	Low: 0.35 (u1) & 0.35 (u2)	PQL
	500	6	high	Unbalanced: 0.35 (u1) & 1.40 (u2) High: 1.40 (u1) & 1.40 (u2)	
RT Model – AIP approach	100	2	low	Low: 0.35 (u1) & 0.35 (u2)	PQL (through AIP)
	500	6	high	Unbalanced: 0.35 (u1) & 1.40 (u2) High: 1.40 (u1) & 1.40 (u2)	

<sup>17</sup> The WMM approach may not be fit with maximum likelihood by numerical quadrature, due to the non-nested structure as noted in footnote 12. The AIP approach may be fit with maximum likelihood by numerical quadrature as the model at each iteration can be processed by subjects. However, PQL using SAS Glimmix was chosen due to the high computational burden of the AIP algorithm which would be even higher for numerical quadrature, and also to allow a comparison between methods where the base estimator was the same (known vs. WMM vs. AIP).



The models fit with the AIP algorithm were replicated 100 times per condition<sup>18</sup>, while others were replicated 1000 times per condition.

## ***Results***

Similar to Chapter 2, performance is assessed by examining bias of the different parameter estimates, the standard deviation of the estimates compared to the standard error of the estimates, Type I error of the null effect, and power of the pair-level effect. The results are summarized by each of the model types, first the RT Model with known identifiers, followed by the weighted multiple membership approach, and finally the AIP estimation approach.

For the RT Model with known identifiers, the findings are very similar to the simulation study in Chapter 2 and thus are only be briefly summarized here. For full results, see Table 14 to Table 21 in Appendix A.2. The estimation method has the most influence on the bias of the estimated intercept, with PQL biased toward zero and MCMC biased away from zero for the low hazard condition similar to the results of the FD Model (see Figure 15 below). The number of nodes, average links per node, and random effect variance factors each results in small bias when aggregating across other factors, but these factors have a large influence on the variability of the estimates. The variability in the estimates across replications is smallest with a larger number of nodes, larger average links per node, and smaller random effect variance. The “unbalanced” random effect variance condition (with a small nominator random effect and large reciprocator random effect) results in a variability in the estimates of the intercept in between the “low” and “high” random effect variance conditions. Again, the average absolute bias for MCMC

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<sup>18</sup> Due to the high computational burden of the AIP algorithm, as has been done in other simulation studies with the AIP algorithm (Cho & Rabe-Hesketh, 2011).

decreases with a larger number of nodes (0.34 versus 0.22) but does not for PQL (0.22 versus 0.25), while the average absolute bias for both methods decreases with a larger number of links per node.

Figure 15: RT Model – Intercept Recovery, High Hazard.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

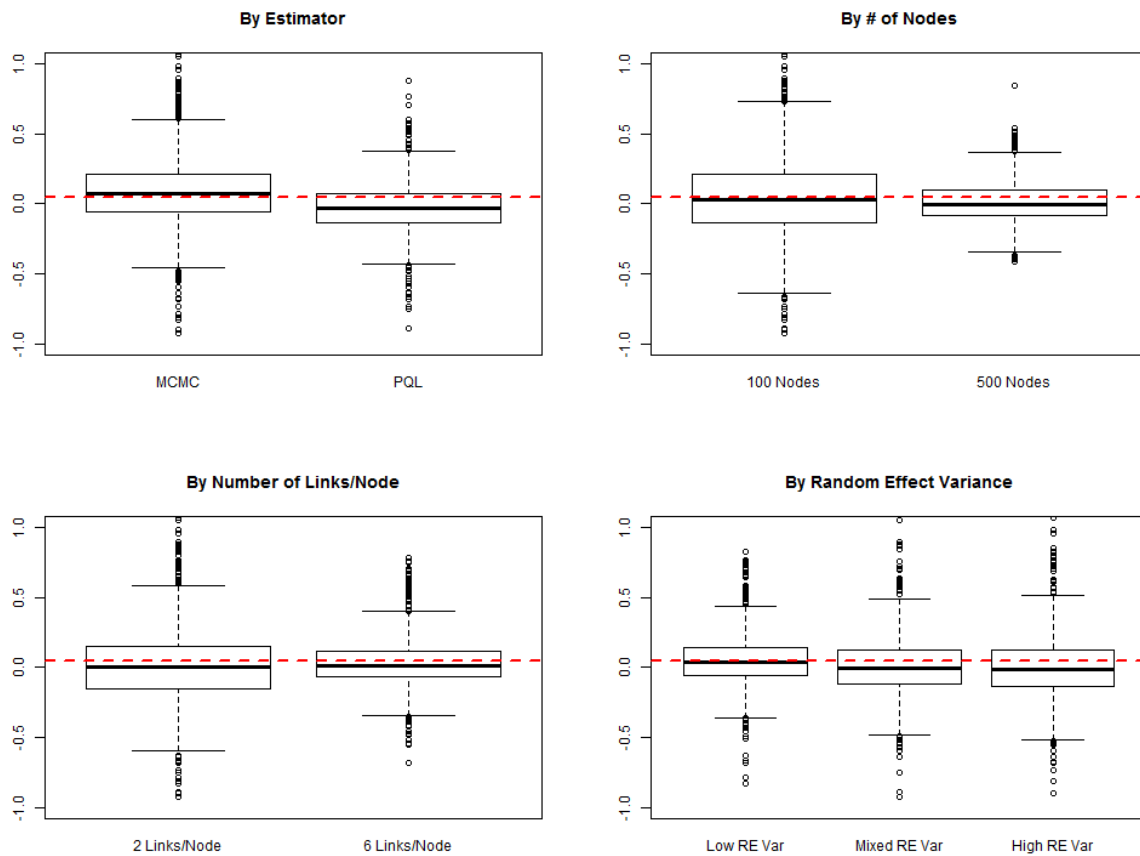
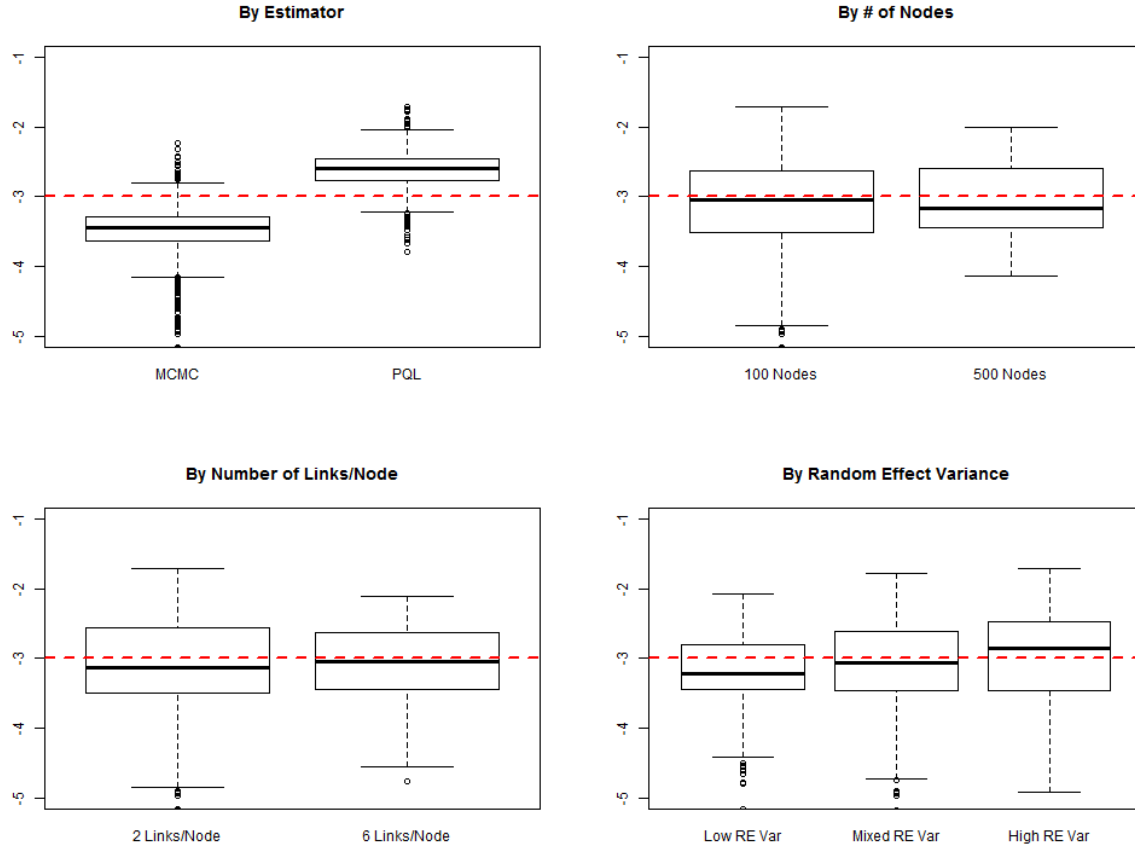


Figure 16: RT Model – Intercept Recovery, Low Hazard.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

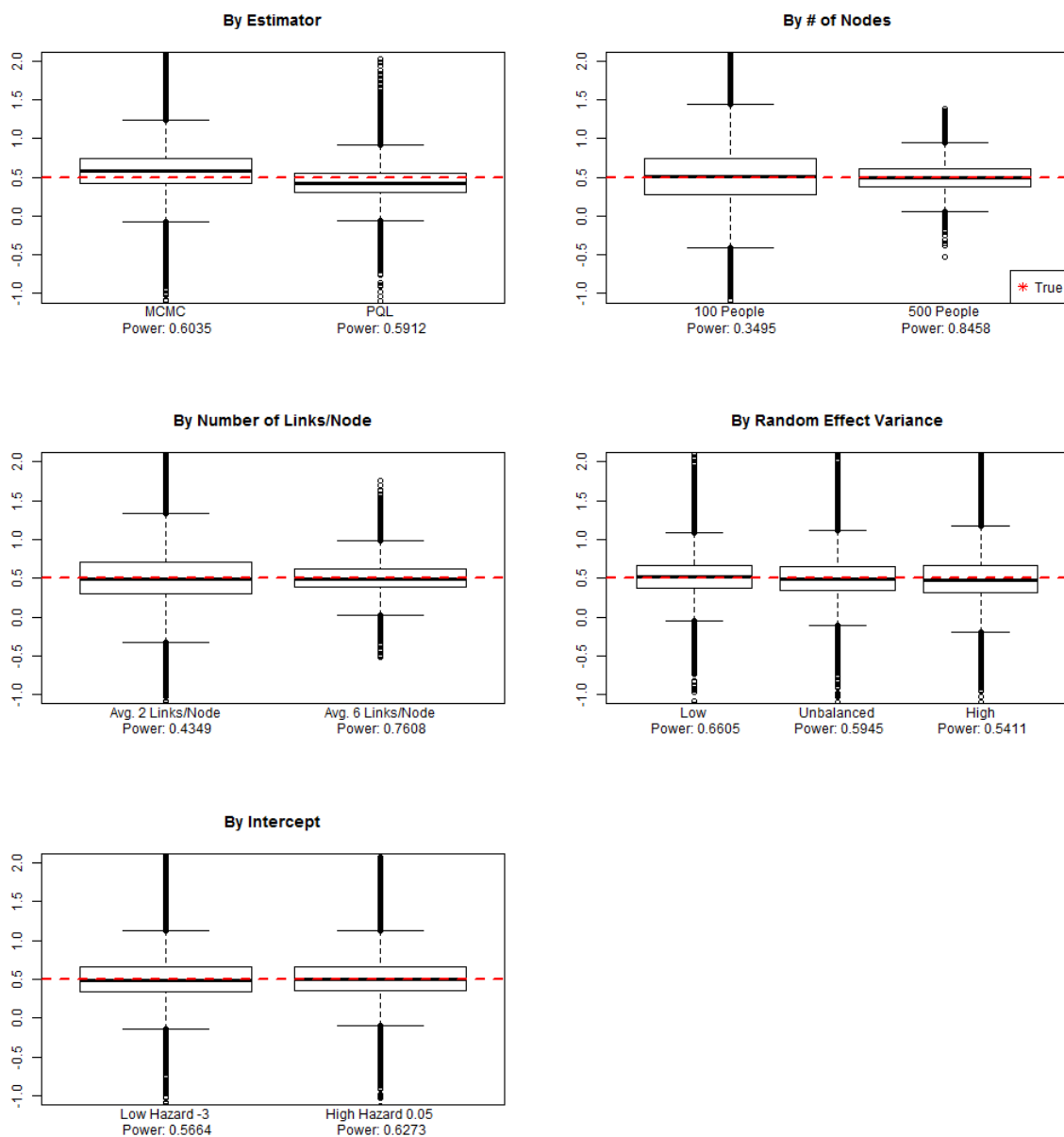


The true pair-level covariate effect is recovered well, with little bias as a result of the number of nodes, average links per node, random effect variance, and intercept when aggregating across other factors (see Figure 17 below). However, we again find that MCMC estimation results in an overestimation of the true effect of  $\beta = 0.50$  (on average across conditions  $\hat{\beta} = 0.59$ ), and PQL results in an underestimation ( $\hat{\beta} = 0.43$ ). The power to detect the true effect is most influenced by the number of nodes (power of 0.85 for 500 nodes versus 0.35 for 100 nodes when aggregating across conditions). The variability in the estimates by factor follow the same pattern as recovery of the intercept for the number of nodes, links per

node, estimation method, and random effect variance; in addition, the high intercept factor results in slightly smaller variability of the estimates. Estimates of the null pair-level covariates show negligible bias and follow the same pattern in terms of estimate variability, and the Type I error rate is close to the expected 0.05 for all conditions.

Figure 17: RT Model – True Effect Recovery.

*Boxplot of parameter estimates across simulations, aggregated by each factor*



Recovery of the standard deviation of each of the random effects is poor in most conditions – especially when the number of links per node is small and the number of nodes is small, similar to the findings of Chapter 2 for recovery of the multiple membership random effect (see Figure 23 to Figure 26 in Appendix B). Given the small number of links per node results in very little information in the data for the model to be able to infer individuals' underlying propensity to reciprocate or be the object of reciprocation, it is not surprising that recovery is poor in this case. In these conditions here, MCMC tends to result in an overestimation of the random effects, while PQL estimation results in an underestimation of the random effects. PQL estimation resulted in a negative or zero random effect variance in a proportion of the simulations for at least one of the random effects and constraining it there (from as many as 60% of cases – when the number of links per node was 2, nodes was 100, low hazard, and low random effect variances – to 0% in many other conditions; see Table 19).<sup>19</sup> Recovery is better for small random effects, also when the number of nodes and links per node is larger.

The second model, RT Model with a weighted multiple membership structure, generally had a similar pattern of results to the PQL estimation results discussed above (full results in Appendix A.3, Table 22 to Table 25). First, fixed effect estimates as well as the variance estimates for the random effects tended to be biased toward zero, with a decrease in variability of the estimates with larger number of nodes, links per node, and smaller random effect variance. The standard deviation of the estimates and the average standard error of the estimates was

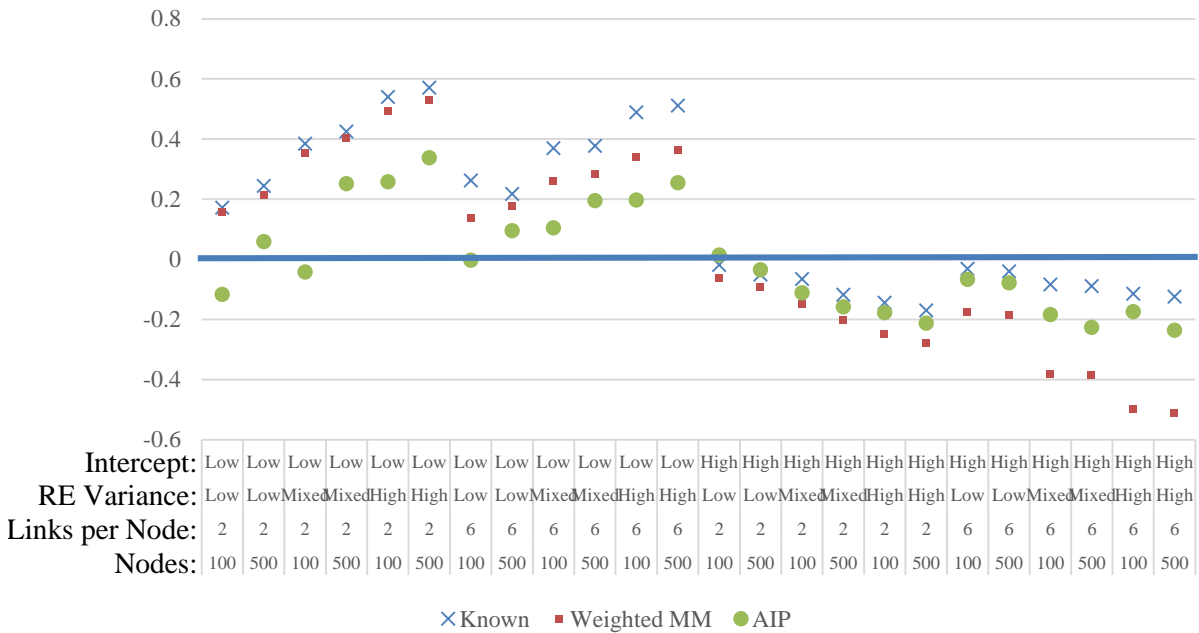
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<sup>19</sup> The figures and tables within this section reference results across all conditions (regardless of whether the covariance matrix was non-positive definite). Sensitivity analyses examining results only from simulations with a positive definite covariance matrix of the random effects revealed very little difference in the patterns of results.

nearly identical between the two models for all conditions. However, the bias in the estimate of the intercept had a different pattern for this model, in that for the high hazard condition, the bias was much higher when the number of links per node was larger than when the number of links per node was smaller (see Figure 18 below). This was opposite from the pattern in the low hazard condition, where the bias of the intercept was smaller when the number of links per node is higher. When the average number of links per node was only two, the two models produced nearly identical results.

Figure 18: Intercept bias comparison of different approaches.

*Raw bias of the intercept for the different approaches, where “known” is the results with PQL estimation with known identifiers, in order to enable comparisons across methods.*



The third model, RT Model estimated with AIP accounting for indistinguishable pairs, also generally had a similar pattern to the RT Model with known identifiers estimated with PQL (full results in Appendix A.4, Table 26 to Table 29). Similar to the WMM approach, the AIP approach also interacted with the hazard factor, in that it performed worse than the known

identifiers model when the hazard was high in terms of absolute bias (see Figure 18 above) but actually better than the known identifiers and WMM approach when the hazard was low.

Using the WMM approach, estimates of the null effect showed negligible bias and the Type I error rate was as expected around 0.05 across all of the conditions, indistinguishable from the RT Model with known identifiers estimated with PQL. The AIP approach, however, had a higher level of Type I error than the other approaches (average 0.09 across conditions) but again the estimates showed negligible bias across all of the conditions and the same pattern of variability for the different factors. For the true pair-level effect, all three approaches had a very similar and nearly indistinguishable pattern of raw bias as well as power, variability of the estimates, and mean standard error of the estimates.

Last, the pattern of recovery of the random effects was also generally similar between the three approaches in terms of the effects of the different factors on bias and variability of the estimates across replications. However, a large difference is evident between the raw bias for the RT model with known identifiers compared to the AIP approach, as AIP approach actually overestimated the nominator random effect variance in the ‘unbalanced’ random effect, but generally had smaller bias when the reciprocator random effect variance was low and high (see Figure 19). The AIP approach resulted in smaller bias for the reciprocator random effect when both the reciprocator and nominator random effects were high (Figure 20). The WMM approach resulted in similar raw bias to the RT Model with known identifiers, although the standard error of the random effect variances was overestimated in the WMM approach. Conversely, the AIP approach resulted in the standard error of the random effect variances being underestimated.

*Raw bias of the nominator random effect variance for the different approaches, where “known” is the results with PQL estimation with known identifiers, in order to enable comparisons across methods*

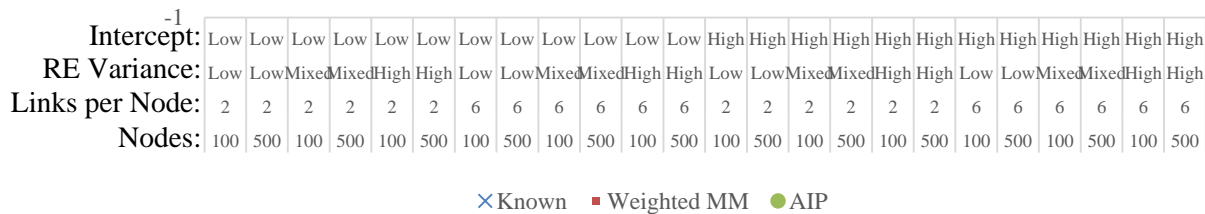
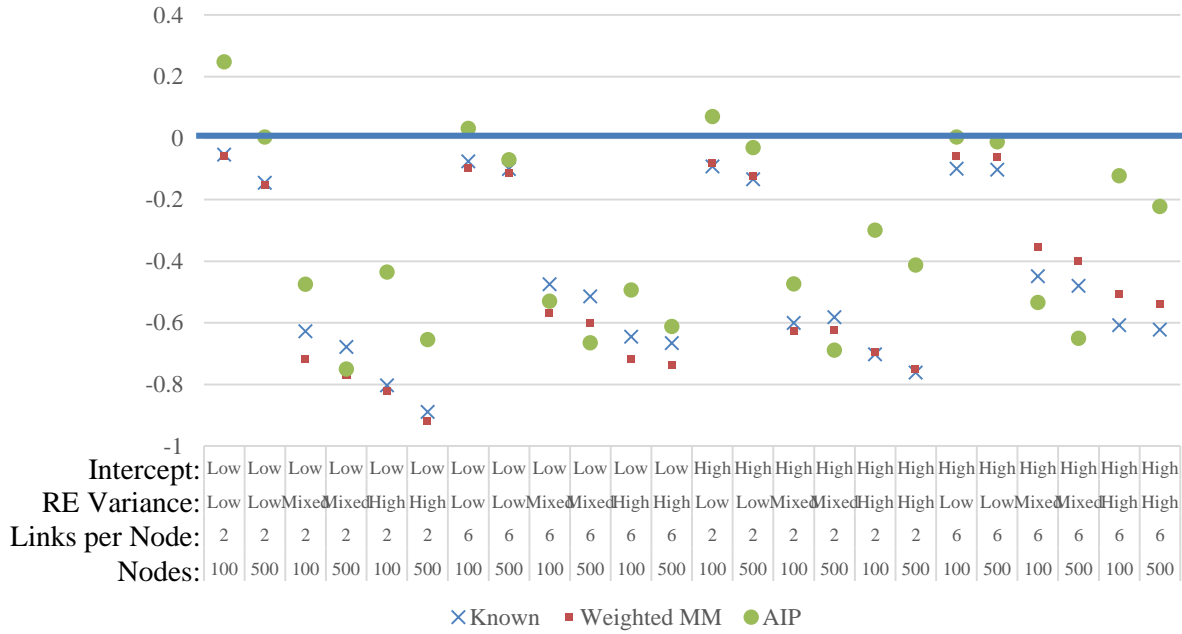




Figure 20: Reciprocator random effect bias comparison of different approaches.  
*Raw bias of the reciprocator random effect variance for the different approaches, where “known” is the results with PQL estimation with known identifiers, in order to enable comparisons across methods*



In conclusion, while the three approaches demonstrate some differences especially in recovery of the random effects, all three approaches tend to result in similar patterns to those found in Chapter 2 in terms of the number of nodes and number of links per node, with reasonable recovery of the fixed effects. Next, the RT Model is applied to empirical data on reciprocity within the social network site Twitter.

### Application to Reciprocity on Twitter

Reciprocity is a basic tendency within many social networks, in that a directed link forming between two nodes often leads to a symmetric, or reciprocated, interaction. In this empirical example, the timing of reciprocity is examined using data from Twitter. Twitter is a free micro-blogging platform that allows users to send and receive public messages known as tweets. Tweets are limited to 140 characters and can include text, as well as links to webpages,

images, and videos for example. The social network of Twitter is directed in that users can follow other users without being followed back.

While much of the research on Twitter has focused on the content of tweets (Chew & Eysenbach, 2010; Sriram, Fuhry, Demir, Ferhatosmanoglu, & Demirbas, 2010; Jansen, Zhang, Sobel, & Chowdury, 2009), several researchers have examined the network structure in terms of relationships between users. Cha et al. (2010) found that the most popular users in terms of high numbers of followers were not necessarily the most influential. Hutto et al. (2013) examined the relationship between factors such as message content and network structure in a longitudinal study of the follower count of users. They found behavior influenced follower count as much as network structure; for example, positive sentiment messages tended to promote follower growth.

Subprocesses of network relationships have also been investigated. For example, Cheng et al. (2011) used decision trees and logistic regression models to predict reciprocity of directed “@” messages on Twitter, and noted that features observed to work well for predicting edges generally were not the most effective in predicting reciprocity. Considering only whether the message had been reciprocated or not, they found individuals with similar status participate in reciprocated interactions more often. Romero and Kleinberg (2010) investigated “link copying” analogous to triadic closure where users tend to copy the links of the individuals they follow.

Similar in many ways to the study by Cheng et al. (2011), this empirical example concerns modeling reciprocal actions on Twitter. In this case, however, the relationship examined is the following network itself rather than “@” messages within tweets and the following network is studied over time rather than predicting reciprocity at a single time point. Specifically, after a source user starts to follow another user, both whether and when the target user reciprocates the follow by following the source user is examined. While the Twitter

network is dynamic in that users may both add and remove links over time, the RT Model will be applied assuming individuals are only able to reciprocate once, as the vast majority of changes to ties are ties being formed rather than dissolved (one study estimated at least 95% of tie changes were new links being formed, Lou et al. (2013)).

Data is collected from the Twitter API. As the Twitter API is rate-limited and only a certain number of requests can be made in a given period of time, a limited quantity of data is available. Users are selected by collecting user id's from the Twitter API live stream of tweets and then collecting information on who they are following and who is following them back. This process is repeated over time in order to determine at what time period a follow-back occurs, given that the Twitter API does not divulge at what time relationships between users are formed.

More specifically, a user collected from the live stream of tweets is examined once a minute to see their current collection of friends and followers (Twitter REST API calls to 'GET friends/ids' and 'GET followers/ids'). Calls to the API are done once a minute because examining the relationships at too coarse of a time scale would result in a large number of "indistinguishable" relationships where it would be unknown who is the nominator and who is the reciprocator in the relationship; preliminary data collection demonstrated a large percentage of reciprocal followings are completed within an hour. However, calls as often as once per minute has the disadvantage that due to rate limits, only one user may be examined at a time. To ensure collection on enough individuals, each user is examined until 1) at least five new friends and five new followers were recorded, or 2) one day has passed, and then a new user is examined. Once a new link occurs, the relationship between the specific pair of users is examined to determine whether the reciprocal following had occurred at one hour, one day, one

week, and one month after nomination (resulting in a 5 wave collection considering the first minute as the first time period; Twitter API calls to ‘GET friendships/show’<sup>20</sup>). For purposes of the model, time is entered as a numeric variable from 0 to 4.<sup>21</sup> Data was collected for three months in 2014, resulting in collection from 890 individuals – 604 nominators and 416 reciprocators (130 individuals who were both nominators and reciprocators) from 912 pairs – with an average of 2 links per individual, and 3,277 rows of data when arranged into a pair-period dataset.

In addition to monitoring the reciprocation status, several covariates are recorded. When the new link occurs for the first time, the number of mutual connections is recorded (by making additional calls to Twitter REST API calls to ‘GET friends/ids’ and ‘GET followers/ids’ for the second user). At each of the five waves of data collection for a pair, the number of friends (expansiveness), the number of followers (popularity), and the number of tweets is recorded, as well as whether the user has a ‘protected’ account (Twitter API calls to ‘GET users/lookup’). A ‘protected’ account signifies a user must accept the new follower before the follower may read the user’s tweets.

After data collection, the RT Model is fit to the data using PQL through SAS GLIMMIX. The structure of the hazard is investigated, and predictors arising from both the network structure as well as from the attributes of the nodes are considered for inclusion. The list of predictors to be investigated are below in Table 10.

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<sup>20</sup> Calls about the relationship between the pair are rate limited separately from calls regarding a user’s collection of friends and followers.

<sup>21</sup> While time was entered as a numeric variable, it was investigated as both a categorical (dummy-coded) and numeric variable for purposes of the model.

Table 10: Predictors definition for RT model of Twitter data.

Type	Object	Predictor	Definition
Network Structure	Node	Popularity	Number of followers
		Expansiveness	Number of friends (user this node is following)
	Dyad	Mutual Connections	The intersection between the nominator's connections and the reciprocator's connections, where the connections for a user are defined as the union between the user's friends and followers.
		Similar Status	Absolute value of the difference in followers (reversed so higher value implies more similarity)
Nodal Attributes	Node	Engagement	Number of tweets
		Protected Account	Binary indicator of whether the account is protected (user must approve new follow requests)

The node-level predictors are added separately for both the nominator and the reciprocator, as it is possible and even likely that these effects are different for the nominator and the reciprocator in predicting the timing of reciprocity. All predictors – except the binary indicator of protected account – are logged and then centered before entering into the model. Histograms of the variables, as well as the mean and variance, are displayed in Figure 28 in Appendix B. For both the nominator and the reciprocator, the binary indicator of protected account is entered with true as the reference category.

## Results

A quadratic hazard function was utilized based on sample observed functions and finding a significant quadratic effect when fitting the model to the data.<sup>22</sup> The model reveals the risk of reciprocating from one minute to one hour increases ( $h_{\text{minute}} = 0.292$ ,  $h_{\text{hour}} = 0.591$ ), but that the risk declines from one hour to one day to one week to one month ( $h_{\text{day}} = 0.585$ ,  $h_{\text{week}} = 0.277$ ,  $h_{\text{month}} = 0.028$ ), for pairs with attributes and network structure effects at the population average

<sup>22</sup> The same pattern of effects described below was found when time was entered as a categorical (dummy-coded) variable.

and with random effects held at the population average and where both the nominator and the reciprocator have a protected account. Fixed effects are listed below in Table 11, with some example predictions based on the model plotted in Figure 21 and Figure 22.<sup>23</sup>

Table 11: RT Model Application – Fixed Effects.

*Results from applying the RT Model to the reciprocity of links on Twitter.*

Effect	Estimate	Standard Error	DF	t Value	Pr >  t
Intercept	-0.89	0.65	306	-1.37	0.1709
<b>Time</b>	<b>1.89</b>	<b>0.23</b>	<b>2354</b>	<b>8.10</b>	<b>&lt;.0001</b>
<b>Time<sup>2</sup></b>	<b>-0.64</b>	<b>0.08</b>	<b>2354</b>	<b>-7.92</b>	<b>&lt;.0001</b>
<b>Nominator Expansiveness</b>	<b>-0.96</b>	<b>0.14</b>	<b>2354</b>	<b>-6.77</b>	<b>&lt;.0001</b>
<b>Nominator Popularity</b>	<b>0.92</b>	<b>0.13</b>	<b>2354</b>	<b>6.85</b>	<b>&lt;.0001</b>
Nominator Tweets	0.19	0.10	2354	1.89	0.059
<b>Reciprocator Expansiveness</b>	<b>1.16</b>	<b>0.17</b>	<b>2354</b>	<b>6.64</b>	<b>&lt;.0001</b>
<b>Reciprocator Popularity</b>	<b>-1.01</b>	<b>0.15</b>	<b>2354</b>	<b>-6.64</b>	<b>&lt;.0001</b>
<b>Reciprocator Tweets</b>	<b>0.29</b>	<b>0.11</b>	<b>2354</b>	<b>2.77</b>	<b>0.0056</b>
<b>Mutual Connections</b>	<b>0.42</b>	<b>0.15</b>	<b>2354</b>	<b>2.91</b>	<b>0.0036</b>
Similarity Status	0.18	0.12	2354	1.51	0.1302
Reciprocator Protected (False)	-0.47	0.46	2354	-1.03	0.3027
<b>Nominator Protected (False)</b>	<b>-3.12</b>	<b>0.47</b>	<b>2354</b>	<b>-6.65</b>	<b>&lt;.0001</b>

Significant effects at  $\alpha=0.05$  are in bold.

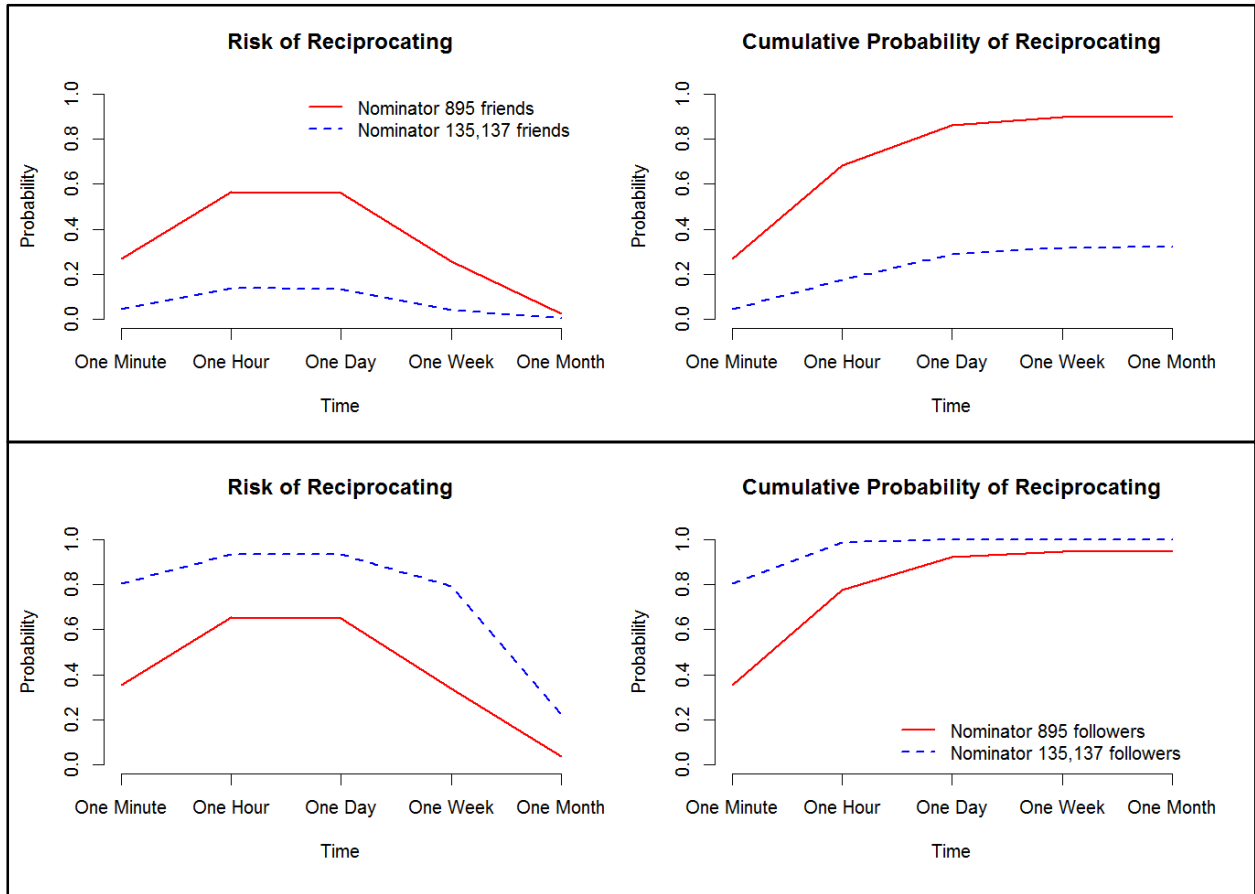
The model reveals that an increase in the nominator's expansiveness is associated with a decreased likelihood of reciprocity,  $F(1, 2354) = 0.92, p < 0.0001$ , while an increase in the nominator's popularity is associated with an increased likelihood of reciprocity,  $F(1, 2534) = 0.85, p < 0.0001$ . See Figure 21 for example predicted hazard and cumulative probability of reciprocating for two different levels of expansiveness (i.e. number of followees) and popularity (i.e. number of followers). The opposite pattern applies for the reciprocator, in that an increase in the reciprocator's expansiveness is associated with an increased likelihood of reciprocity,  $F(1,$

<sup>23</sup> Interactions were also investigated, such as the interaction between mutual connections and time, but the model was not able to converge with a larger number of terms with this data.

2534) = 1.35,  $p < 0.0001$ , while an increase in the reciprocator's popularity is associated with a decrease in the likelihood of reciprocity,  $F(1, 2354) = 1.02$ ,  $p < 0.0001$ . An increase in the reciprocator's number of tweets is associated with an increased risk of reciprocating,  $F(1, 2354) = 0.08$ ,  $p = 0.0056$ , as is an increase in the number of mutual connections,  $F(1, 2354) = 0.18$ ,  $p = 0.0036$ . The nominator's number of tweets is not found to be related to likelihood of reciprocity, nor is the similarity in status as defined in Table 10.

Figure 21: Effects of the Nominator's Number of Friends and Followers.

*Predicted hazard and cumulative probability of reciprocating depending on two levels of the nominator's number of friends, users the nominator is following (top); and predicted hazard and cumulative probability of reciprocating depending on two levels of the nominator's number of followers, users who are following the nominator (bottom).*

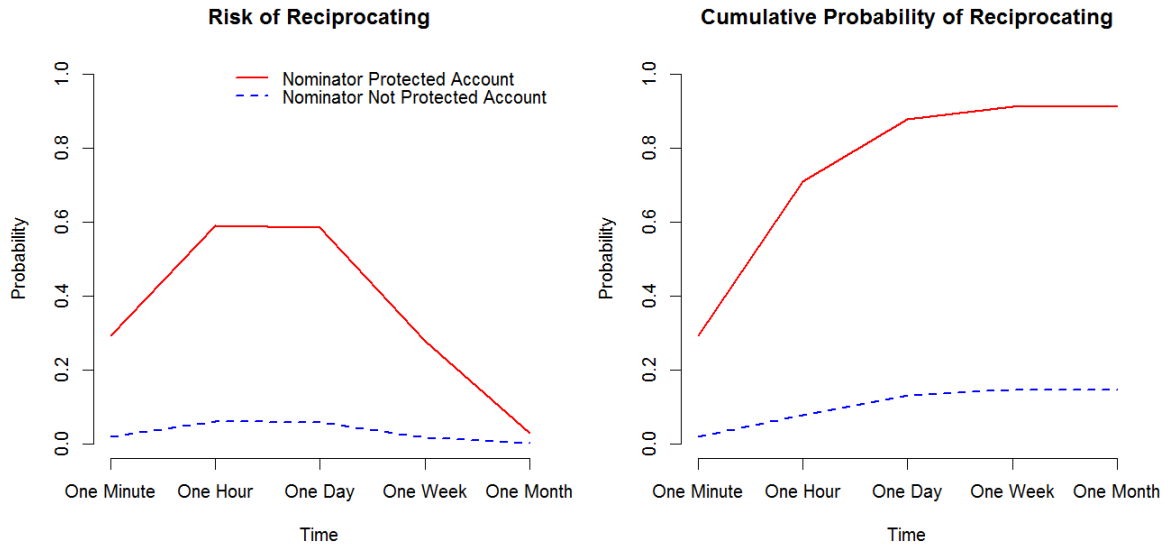


Nominators without a protected account are associated with relationships that are less likely to be reciprocated,  $F(1, 2354) = 9.73, p < 0.0001$  (Figure 22).



Figure 22: Effect of Protected Account of the Nominator.

*Predicted hazard and cumulative probability of reciprocating for nominators with and without a protected account, holding other predictors at the average level and the random effects at the population average.*



The dependence structure was quite large in this data even after controlling for the predictors in the model, with an ICC of 0.11 for nominators and an ICC of 0.21 for reciprocators (Table 12). The correlation between the latent response variable of reciprocation for the same reciprocator holding the nominator random effects constant is thus found to be larger than the correlation between the latent response variable for the same nominator holding the reciprocator random effects constant.

Table 12: RT Model Application – Random Effects.

*Cross-classified random effect structure estimates.*

Covariance Parameter	Estimate	Standard Error
Nominator	0.4138	0.2186
Reciprocator	0.8831	0.2620

## Conclusion

The RT Model developed in this chapter allows an investigation of the timing of reciprocity, defined as the length of time between an individual nominating another as a friend

and the potential reciprocation. To account for the dependence found in such data, in that individuals can be involved in multiple friendship pairs and have underlying tendencies to both be the object of reciprocation and to reciprocate, a cross-classified random effect structure is utilized. The simulation study revealed a similar pattern of results to the FD Model in Chapter 2, as we see PQL resulted in fixed and random effect estimates biased toward zero, with MCMC biased in the opposite direction, but with PQL resulting in less variability in estimates across replications. A higher number of nodes and links per node are also associated with less variability in estimates, with the number of nodes having the largest influence on the power to recover a true pair-level effect. Recovery is better both in terms of bias and variability of the estimates when the random effect variances are smaller.

With discrete-time data collection, it may be impossible to distinguish between the nominator and the reciprocator, and two approaches are proposed to account for this. In the simulation study, both approaches estimated with PQL are found to work reasonably well for recovering the fixed effects, with similar patterns of result to the RT Model with known identifiers estimated with PQL. However, the results should not be used to generalize results to other conditions such as larger links per node or for person-level covariates, as will be discussed in Chapter 4.

Finally, the RT Model was applied to individuals' followings on Twitter, and an individual's relative position within the network as well as the reciprocator's engagement was found to be strongly related to the likelihood of a link being reciprocated. Also, a nominator with a protected account was found to generate links that are much more likely to be reciprocated than nominators without a protected account. One possible explanation is that users with protected accounts, who must accept other users before they can read their tweets, are more

selective about to who they follow. For example, nominators with protected accounts may generally only follow other individuals that they actually know in person, who would be more likely to reciprocate the relationship online. However, the current data does not reveal why a nominator's protected account leads to a greater likelihood of reciprocation, but requires a follow-up study. While there are several directions for future study as will be discussed further in the next chapter, the simulation studies and empirical example demonstrate that the RT Model developed in this chapter allows researchers to investigate the processes leading to reciprocity for individuals embedded within a network.

## **Chapter 4: Conclusion**

Individuals are embedded in a complex network of relationships with others around them. While social scientists are interested in studying various aspects of individual nature and society, individuals are often studied in isolation from one another. Standard statistical techniques assume that individuals are randomly sampled from a population, and typically an individual's outcomes are modeled as a function of their own characteristics, with the assumption that the behavior of one individual in the sample does not influence another. Yet by focusing on the relationships between individuals and explicitly recognizing the interdependencies between individuals inherent in everyday life, a social network perspective can bring new insight into human nature and processes that influence behavior (Abbot, 1997; White, Boorman, & Breiger, 1976). Social network analysis has given us insight into the effects of communication network structures on a group's ability to complete a task, how position in a social network can influence access to resources, and the influence of urbanism on psychological well-being, for example (Borgatti, Mehra, Brass, & Labianca, 2009).

Although various models have been developed to study how network structures arise, many fundamental questions surrounding the processes leading to specific types of events between dyads within a network remain stubbornly difficult to address. Example questions may include those surrounding friendship dissolution or reciprocal tendencies such as "how does depression influence friendship dissolution?" and "does someone's popularity include the likelihood a tie they initiate will be reciprocated?" To address these types of questions, in this dissertation, I have proposed the use of discrete-time multilevel event history models to

understand temporal characteristics of friendships, consisting of a model for friendship duration (FD Model) and the timing of reciprocity (RT Model). The FD Model contains a multiple membership random effect structure, while the RT Model consists of a simple cross-classified random effect structure when the identity of the nominator and the reciprocator are both known. Two proposed approaches for dealing with unknown identifiers when fitting the RT Model were also proposed.

The processes leading to specific types of network events – whether and when friendships end or reciprocal ties occur, for example – can be very different from more general tie formation processes, which can be modeled in more general longitudinal social network social selection process models. In addition to answering specific research questions, the models considered in this dissertation have the advantage of being easily applied to networks without clearly defined boundaries and can easily handle censored event times (e.g. missing ties). As the modeling framework focuses on specific events between individuals within the network, the ubiquitous sparseness problem of social network data is also not present here, making modeling more straightforward. The simulation study and empirical examples are designed to give more insight into the potential advantages as well as limitations of the modeling framework.

In Chapter 2, the FD Model was developed and investigated in a simulation study. The fixed effects were generally found to be recovered reasonably well, except that PQL resulted in estimates biased toward zero while MCMC resulted in estimates biased away from zero. Recovery was better for the larger number of nodes in terms of bias, variability, and power to detect a true pair-level effect; this was especially true for MCMC which revealed smaller average absolute bias for the different parameters for 500 nodes than 100 nodes as compared to PQL which was similar on average between the two levels. PQL is not a consistent estimator, and

these results are consistent with prior research in terms of the pattern of bias (Bauer & Sterba, 2011). A larger number of links per node improved both bias and variability for MCMC and PQL estimation methods, and improved the power to detect a significant pair-level effect. Recovery was also better both for fixed and random effects when the random effect variance was smaller. Recovery of the random effect variance was poor both for MCMC and PQL estimation, consistent with prior research on binary outcomes, especially given the number of links per node was so small in the kind of social network data investigated in the simulation study (Rodríguez & Goldman, 1995).

The FD Model was then applied to high school friendship data, to investigate the processes leading to friendship dissolution and the timing of dissolution over time. The model reveals high school friendships are very fickle, with a majority ending after one semester; but given that the friendship continues, the likelihood of dissolution decreases over time. Both gender and depression are found to affect dissolution. Mixed-gender pairs have the highest likelihood of dissolving with female pairs having the lowest likelihood, and pairs with similar depression are associated with a decreased likelihood of dissolution.

In Chapter 3, the RT Model both for known and indistinguishable pairs was investigated in a simulation study and the pattern of results was generally similar across all approaches to the conclusions for the FD Model described above. The RT model with all three developed approaches was able to capture the fixed effects reasonably well, but had large bias for the random effects where the pattern of recovery was different for the three models. Researchers should be careful not to generalize the simulation results in Chapter 2 and 3 without further study. For example, a larger number of links per node might not work well for the RT Model with a WMM approach, as we saw that recovery of the intercept was actually worse with 6 links

per node rather than only 2 links per node. The WMM approach model is imputing wrong information purposely (weighted identities when there is only one true person in each role) in order to still use the information to model and thus may not work well in all situations. Also, results from MCMC estimation may be heavily dependent upon the specification of the prior distributions (Cho & Rabe-Hesketh, 2011), and only a single specification was done in this dissertation, so additional work is needed before general conclusions can be made on the ability for MCMC estimation to accurately recover parameters for the general type of social network data and models discussed in this dissertation.

Future research should investigate the ability of the AIP algorithm, both for known and indistinguishable pairs to recover effects when using maximum likelihood with adaptive quadrature, which was not a condition in the simulation studies covered here. This approach, while computationally burdensome, is worthy of future investigation given sometimes problematic performance of PQL and MCMC. Last, the ability of the WMM approach and the AIP approach to recover the parameter estimates when there are person-level covariates in the model was not investigated, and is an area for future research to understand whether covariates at this level may increase or decrease the ability to accurately recover the model parameters.

Finally, the RT Model was applied to the timing of reciprocity of link followings on the social network site Twitter. The model revealed a large dependence structure for nominators and reciprocators, as well as a large number of effects that are related to the likelihood of reciprocating a following link. The nominator's as well as reciprocator's expansiveness and popularity, as well as the reciprocator's engagement and the nominator's protected account status were some of the effects that were found to be related to reciprocity. An increase in the number of mutual connections between the two individuals was found to increase the likelihood of

reciprocation, however the similarity in status defined in terms of popularity was not found to be related. With a larger collection of data, more predictors could be investigated such as similarity defined in other ways (e.g. engagement, expansiveness) as well as higher-order interactions (e.g. similarity in status by time).

In this dissertation, there was limited inclusion of network effects beyond the dyad, both in the simulation studies and the empirical examples, with the exception of investigating the number of mutual connections on the timing of reciprocity in the Twitter network. Higher level effects might have an impact on the ability to recover the lower level effects that are often of interest such as the effect of depression on the pair. For example, high school friendships may be impacted by the friendships connected to the dyad; even the attributes of surrounding friends not within the dyad may have an impact and is another aspect of the network that may be considered in relation to the effects on friendship dissolution.

Another limitation is that any potential measurement error in recording friendships was not investigated or accounted for in the model, which has the potential to impact our ability to understand the processes leading to the friendship events of interest. Measurement error would likely decrease our effectiveness in recovering the effects, lowering power and increasing variability in estimates. Future research should investigate how network effects beyond the dyad as well as error in recording friendships may influence results to understand this in more detail.

Both the FD Model and the RT Model can be extended in future work to examine repeated events, such as pairs who become friends again after dissolution. Additionally, models could be formulated to understand other events such as perceived friendship dissolution (cross-classified structure) for example, or even events beyond the dyad (e.g. triad closure). While there are several possible ways to extend the framework presented here, the two models



developed in this dissertation aim to be example multilevel event history models that researchers can utilize. With this broader framework, researchers can directly model an event between individuals embedded within a network, answering questions about the processes leading to the event as well as the timing of that event.

## APPENDIX A: Tables

### A.1: FD MODEL APPLICATION

Table 13: MFQ Scale.

*Depression scale given to students.*

This form is about how you might have been feeling or acting recently. For each question, please circle how much you have felt or acted this way **IN THE PAST TWO WEEKS**.

		NOT TRUE	SOMETIMES TRUE	MOSTLY TRUE
1.	I felt miserable or unhappy.....	0	1	2
2.	I didn't enjoy anything at all .....	0	1	2
3.	I was less hungry than usual .....	0	1	2
4.	I ate more than usual .....	0	1	2
5.	I felt so tired I just sat around and did nothing .....	0	1	2
6.	I was moving and walking more slowly than usual .....	0	1	2
7.	I was very restless .....	0	1	2
8.	I felt I was no good anymore .....	0	1	2
9.	I blamed myself for things that weren't my fault .....	0	1	2
10.	It was hard for me to make up my mind .....	0	1	2
11.	I felt grumpy and cross with my parents .....	0	1	2
12.	I felt like talking less than usual .....	0	1	2
13.	I was talking more slowly than usual .....	0	1	2
14.	I cried a lot .....	0	1	2
15.	I thought there was nothing good for me in the future .	0	1	2
16.	I thought that life wasn't worth living .....	0	1	2
17.	I thought about death or dying .....	0	1	2
18.	I thought my family would be better off without me ....	0	1	2
19.	I thought about killing myself .....	0	1	2
20.	I didn't want to see my friends .....	0	1	2
21.	I found it hard to think properly or concentrate .....	0	1	2
22.	I thought bad things would happen to me .....	0	1	2
23.	I hated myself .....	0	1	2
24.	I felt I was a bad person .....	0	1	2
25.	I thought I looked ugly .....	0	1	2
26.	I worried about aches and pains .....	0	1	2
27.	I felt lonely .....	0	1	2
28.	I thought nobody really loved me .....	0	1	2
29.	I didn't have any fun at school .....	0	1	2
30.	I thought I could never be as good as other kids .....	0	1	2
31.	I did everything wrong .....	0	1	2
32.	I didn't sleep as well as I usually sleep .....	0	1	2
33.	I slept a lot more than usual .....	0	1	2

## A.2: RT MODEL WITH KNOWN IDENTIFIERS

Table 14: RT Model with known identifiers – MCMC Simulation Fixed Effects Bias.  
Factors italicized and conditions with relative bias above or below 25% of parameter bolded.

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$		$\beta_2 = 0$	
					Bias	Relative Bias	Bias	Relative Bias ( $\alpha=0.05$ )	Power ( $\alpha=0.05$ )	Bias	Type I ( $\alpha=0.05$ )
1	100	2	Low	-3	-0.75	<b>0.25</b>	0.15	<b>0.30</b>	0.19	0.00	0.03
2	100	2	Mixed	-3	-0.72	0.24	0.12	<b>0.25</b>	0.15	-0.01	0.03
3	100	2	High	-3	-0.69	0.23	0.14	<b>0.29</b>	0.13	-0.02	0.04
4	100	2	Low	0.05	0.06	<b>1.11</b>	0.13	<b>0.26</b>	0.28	0.00	0.06
5	100	2	Mixed	0.05	0.11	<b>2.20</b>	0.09	0.19	0.17	0.01	0.05
6	100	2	High	0.05	0.10	<b>1.93</b>	0.14	<b>0.28</b>	0.17	0.01	0.06
7	100	6	Low	-3	-0.45	0.15	0.06	0.13	0.55	-0.01	0.05
8	100	6	Mixed	-3	-0.47	0.16	0.08	0.15	0.51	0.01	0.04
9	100	6	High	-3	-0.50	0.17	0.08	0.15	0.41	0.01	0.04
10	100	6	Low	0.05	0.00	-0.03	0.09	0.18	0.68	0.00	0.04
11	100	6	Mixed	0.05	0.01	0.10	0.10	0.19	0.57	-0.01	0.05
12	100	6	High	0.05	0.03	<b>0.69</b>	0.10	0.21	0.50	0.01	0.05
13	500	2	Low	-3	-0.41	0.14	0.03	0.07	0.73	0.00	0.05
14	500	2	Mixed	-3	-0.45	0.15	0.06	0.12	0.66	0.00	0.05
15	500	2	High	-3	-0.45	0.15	0.08	0.15	0.59	0.00	0.05
16	500	2	Low	0.05	0.00	0.06	0.09	0.19	0.86	0.00	0.06
17	500	2	Mixed	0.05	0.01	0.22	0.11	0.21	0.75	0.00	0.05
18	500	2	High	0.05	0.02	<b>0.47</b>	0.09	0.18	0.63	0.00	0.04
19	500	6	Low	-3	-0.42	0.14	0.04	0.08	1.00	0.00	0.05
20	500	6	Mixed	-3	-0.42	0.14	0.05	0.10	0.99	0.00	0.04
21	500	6	High	-3	-0.44	0.15	0.05	0.10	0.97	0.00	0.05
22	500	6	Low	0.05	0.01	0.15	0.10	0.20	1.00	0.00	0.05
23	500	6	Mixed	0.05	0.00	0.05	0.09	0.19	1.00	-0.01	0.06
24	500	6	High	0.05	0.01	0.22	0.09	0.18	0.99	0.00	0.06

Table 15: RT Model with known identifiers – PQL Simulation Fixed Effects Bias.  
*Factors italicized and conditions with relative bias above or below 25% of parameter bolded.*

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$		$\beta_2 = 0$	
					Bias	Relative Bias	Bias	Relative Bias	Power ( $\alpha=0.05$ )	Bias	Type I ( $\alpha=0.05$ )
1	100	2	Low	-3	0.17	-0.06	0.00	0.00	0.23	0.00	0.04
2	100	2	Mixed	-3	0.38	-0.13	-0.07	-0.14	0.16	0.00	0.04
3	100	2	High	-3	0.54	-0.18	-0.10	-0.19	0.14	-0.01	0.05
4	100	2	Low	0.05	-0.02	<b>-0.39</b>	-0.03	-0.06	0.27	0.00	0.06
5	100	2	Mixed	0.05	-0.07	<b>-1.32</b>	-0.09	-0.19	0.17	0.00	0.06
6	100	2	High	0.05	-0.14	<b>-2.89</b>	-0.09	-0.19	0.16	0.01	0.06
7	100	6	Low	-3	0.26	-0.09	-0.02	-0.03	0.53	0.00	0.06
8	100	6	Mixed	-3	0.37	-0.12	-0.04	-0.09	0.49	0.01	0.04
9	100	6	High	-3	0.49	-0.16	-0.09	-0.18	0.37	0.01	0.05
10	100	6	Low	0.05	-0.03	<b>-0.64</b>	-0.05	-0.09	0.66	0.00	0.04
11	100	6	Mixed	0.05	-0.08	<b>-1.68</b>	-0.06	-0.13	0.53	-0.01	0.05
12	100	6	High	0.05	-0.11	<b>-2.29</b>	-0.09	-0.18	0.47	0.01	0.05
13	500	2	Low	-3	0.24	-0.08	-0.04	-0.09	0.73	0.00	0.05
14	500	2	Mixed	-3	0.43	-0.14	-0.08	-0.16	0.63	0.00	0.05
15	500	2	High	-3	0.57	-0.19	-0.11	-0.22	0.57	0.00	0.05
16	500	2	Low	0.05	-0.05	<b>-1.03</b>	-0.05	-0.10	0.85	0.00	0.06
17	500	2	Mixed	0.05	-0.12	<b>-2.36</b>	-0.08	-0.15	0.74	0.00	0.05
18	500	2	High	0.05	-0.17	<b>-3.41</b>	-0.12	-0.24	0.61	0.00	0.04
19	500	6	Low	-3	0.22	-0.07	-0.04	-0.07	1.00	0.00	0.05
20	500	6	Mixed	-3	0.38	-0.13	-0.06	-0.13	0.99	0.00	0.04
21	500	6	High	-3	0.51	-0.17	-0.10	-0.21	0.97	0.00	0.05
22	500	6	Low	0.05	-0.04	<b>-0.82</b>	-0.04	-0.09	1.00	0.00	0.05
23	500	6	Mixed	0.05	-0.09	<b>-1.78</b>	-0.07	-0.13	1.00	-0.01	0.06
24	500	6	High	0.05	-0.13	<b>-2.50</b>	-0.09	-0.19	0.99	0.00	0.05

Table 16: RT Model with known identifiers – MCMC Simulation Fixed Effect Standard Error Recovery.  
*Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$			$\beta_2 = 0$		
					SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD
1	100	2	Low	-3	0.77	0.58	<b>0.75</b>	0.62	0.57	0.92	0.42	0.43	1.01
2	100	2	Mixed	-3	0.77	0.64	0.84	0.68	0.62	0.92	0.51	0.49	0.96
3	100	2	High	-3	0.86	0.70	0.81	0.69	0.68	0.97	0.58	0.54	0.94
4	100	2	Low	0.05	0.39	0.37	0.93	0.48	0.46	0.96	0.38	0.35	0.94
5	100	2	Mixed	0.05	0.46	0.46	0.99	0.53	0.54	1.01	0.45	0.43	0.95
6	100	2	High	0.05	0.53	0.53	1.01	0.62	0.61	0.98	0.53	0.50	0.95
7	100	6	Low	-3	0.25	0.25	0.99	0.27	0.27	1.01	0.22	0.21	0.95
8	100	6	Mixed	-3	0.31	0.30	0.97	0.30	0.29	0.98	0.25	0.25	1.00
9	100	6	High	-3	0.37	0.35	0.95	0.33	0.32	0.99	0.31	0.31	0.99
10	100	6	Low	0.05	0.21	0.21	0.99	0.25	0.25	0.97	0.20	0.20	1.02
11	100	6	Mixed	0.05	0.26	0.26	1.02	0.28	0.27	0.97	0.25	0.25	0.97
12	100	6	High	0.05	0.32	0.31	0.98	0.33	0.31	0.94	0.31	0.31	0.99
13	500	2	Low	-3	0.20	0.20	1.00	0.21	0.21	0.98	0.15	0.15	1.02
14	500	2	Mixed	-3	0.24	0.23	0.98	0.25	0.24	0.96	0.18	0.18	0.98
15	500	2	High	-3	0.26	0.26	1.00	0.26	0.26	1.00	0.20	0.20	1.05
16	500	2	Low	0.05	0.17	0.15	0.93	0.20	0.20	0.99	0.15	0.15	0.99
17	500	2	Mixed	0.05	0.19	0.19	0.99	0.23	0.23	0.97	0.18	0.18	1.00
18	500	2	High	0.05	0.21	0.22	1.04	0.26	0.26	0.99	0.21	0.20	0.99
19	500	6	Low	-3	0.11	0.11	0.99	0.12	0.12	1.01	0.10	0.09	0.96
20	500	6	Mixed	-3	0.13	0.13	1.02	0.12	0.13	1.04	0.11	0.11	1.01
21	500	6	High	-3	0.15	0.15	0.99	0.14	0.14	1.01	0.13	0.13	1.01
22	500	6	Low	0.05	0.09	0.09	1.02	0.11	0.11	1.01	0.09	0.09	1.00
23	500	6	Mixed	0.05	0.12	0.12	1.00	0.12	0.12	1.05	0.11	0.11	1.00
24	500	6	High	0.05	0.13	0.14	1.02	0.14	0.14	1.00	0.13	0.13	1.00

Table 17: RT Model with known identifiers – PQL Simulation Fixed Effect Standard Error Recovery. *Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$			$\beta_2 = 0$		
					SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD
1	100	2	Low	-3	0.36	0.33	0.93	0.45	0.42	0.95	0.31	0.31	1.02
2	100	2	Mixed	-3	0.35	0.33	0.94	0.44	0.42	0.97	0.33	0.32	0.98
3	100	2	High	-3	0.34	0.33	0.97	0.41	0.42	1.04	0.34	0.32	0.94
4	100	2	Low	0.05	0.27	0.25	0.91	0.35	0.35	0.98	0.28	0.26	0.95
5	100	2	Mixed	0.05	0.28	0.28	0.98	0.36	0.37	1.04	0.30	0.29	0.97
6	100	2	High	0.05	0.29	0.30	1.02	0.38	0.39	1.03	0.32	0.31	0.96
7	100	6	Low	-3	0.18	0.19	1.09	0.24	0.23	0.98	0.19	0.18	0.97
8	100	6	Mixed	-3	0.20	0.21	1.04	0.23	0.24	1.03	0.20	0.20	1.01
9	100	6	High	-3	0.22	0.22	1.00	0.23	0.24	1.06	0.22	0.21	0.99
10	100	6	Low	0.05	0.15	0.15	0.97	0.20	0.19	0.99	0.15	0.16	1.04
11	100	6	Mixed	0.05	0.18	0.18	1.00	0.21	0.21	1.00	0.18	0.18	0.99
12	100	6	High	0.05	0.20	0.20	0.99	0.22	0.22	1.00	0.21	0.21	0.99
13	500	2	Low	-3	0.14	0.14	1.00	0.18	0.18	0.99	0.13	0.13	1.03
14	500	2	Mixed	-3	0.14	0.14	0.98	0.18	0.18	1.00	0.14	0.13	0.98
15	500	2	High	-3	0.15	0.14	0.97	0.18	0.18	1.03	0.13	0.14	1.04
16	500	2	Low	0.05	0.12	0.11	0.91	0.15	0.15	1.02	0.11	0.11	1.00
17	500	2	Mixed	0.05	0.12	0.12	0.97	0.16	0.16	1.00	0.12	0.12	1.01
18	500	2	High	0.05	0.12	0.13	1.04	0.17	0.17	1.02	0.13	0.13	0.98
19	500	6	Low	-3	0.08	0.08	1.02	0.10	0.10	1.03	0.08	0.08	0.97
20	500	6	Mixed	-3	0.09	0.09	1.00	0.10	0.11	1.07	0.09	0.09	1.00
21	500	6	High	-3	0.10	0.10	0.99	0.10	0.11	1.07	0.09	0.09	1.01
22	500	6	Low	0.05	0.07	0.07	0.99	0.08	0.09	1.03	0.07	0.07	1.00
23	500	6	Mixed	0.05	0.08	0.08	0.99	0.09	0.09	1.09	0.08	0.08	1.00
24	500	6	High	0.05	0.09	0.09	1.02	0.09	0.10	1.06	0.09	0.09	1.00

Table 18: RT Model with known identifiers – MCMC Simulation Random Effects Bias.  
*Factors italicized and conditions with relative bias above or below 25% of parameter bolded.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect			Reciprocator Random Effect		
					TRUE	Bias	Relative Bias	TRUE	Bias	Relative Bias
1	100	2	Low	Low	0.35	0.89	<b>2.53</b>	0.35	0.32	<b>0.92</b>
2	100	2	Low	Mixed	0.35	0.82	<b>2.33</b>	1.40	0.92	<b>0.66</b>
3	100	2	Low	High	1.40	1.50	<b>1.07</b>	1.40	0.93	<b>0.67</b>
4	100	2	High	Low	0.35	0.32	<b>0.90</b>	0.35	0.09	<b>0.25</b>
5	100	2	High	Mixed	0.35	0.40	<b>1.14</b>	1.40	0.71	<b>0.51</b>
6	100	2	High	High	1.40	1.11	<b>0.79</b>	1.40	0.77	<b>0.55</b>
7	100	6	Low	Low	0.35	0.06	0.17	0.35	-0.03	-0.07
8	100	6	Low	Mixed	0.35	0.09	<b>0.27</b>	1.40	0.43	<b>0.31</b>
9	100	6	Low	High	1.40	0.62	<b>0.45</b>	1.40	0.47	<b>0.34</b>
10	100	6	High	Low	0.35	0.10	<b>0.29</b>	0.35	0.08	0.23
11	100	6	High	Mixed	0.35	0.12	<b>0.33</b>	1.40	0.66	<b>0.47</b>
12	100	6	High	High	1.40	0.70	<b>0.50</b>	1.40	0.62	<b>0.44</b>
13	500	2	Low	Low	0.35	0.01	0.04	0.35	-0.05	-0.13
14	500	2	Low	Mixed	0.35	0.06	0.17	1.40	0.39	<b>0.28</b>
15	500	2	Low	High	1.40	0.45	<b>0.32</b>	1.40	0.33	0.24
16	500	2	High	Low	0.35	0.14	<b>0.39</b>	0.35	0.08	0.23
17	500	2	High	Mixed	0.35	0.06	0.17	1.40	0.67	<b>0.48</b>
18	500	2	High	High	1.40	0.62	<b>0.44</b>	1.40	0.59	<b>0.42</b>
19	500	6	Low	Low	0.35	0.05	0.13	0.35	0.04	0.12
20	500	6	Low	Mixed	0.35	0.06	0.16	1.40	0.33	0.24
21	500	6	Low	High	1.40	0.41	<b>0.29</b>	1.40	0.39	<b>0.28</b>
22	500	6	High	Low	0.35	0.15	<b>0.43</b>	0.35	0.14	<b>0.41</b>
23	500	6	High	Mixed	0.35	0.13	<b>0.36</b>	1.40	0.56	<b>0.40</b>
24	500	6	High	High	1.40	0.57	<b>0.40</b>	1.40	0.56	<b>0.40</b>

Table 19: RT Model with known identifiers – PQL Simulation Random Effects Bias.  
*Factors italicized and conditions with relative bias above or below 25% of parameter bolded.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect			Reciprocator Random Effect			PD*
					TRUE	Bias	Relative Bias	TRUE	Bias	Relative Bias	
1	100	2	Low	Low	0.35	-0.03	-0.10	0.35	-0.05	-0.15	0.40
2	100	2	Low	Mixed	0.35	-0.11	<b>-0.33</b>	1.40	-0.63	<b>-0.45</b>	0.49
3	100	2	Low	High	1.40	-0.77	<b>-0.55</b>	1.40	-0.80	<b>-0.57</b>	0.73
4	100	2	High	Low	0.35	-0.09	<b>-0.25</b>	0.35	-0.09	<b>-0.27</b>	0.45
5	100	2	High	Mixed	0.35	-0.13	<b>-0.37</b>	1.40	-0.60	<b>-0.43</b>	0.60
6	100	2	High	High	1.40	-0.72	<b>-0.52</b>	1.40	-0.70	<b>-0.50</b>	0.85
7	100	6	Low	Low	0.35	-0.06	-0.18	0.35	-0.08	-0.22	0.86
8	100	6	Low	Mixed	0.35	-0.15	<b>-0.44</b>	1.40	-0.48	<b>-0.34</b>	0.84
9	100	6	Low	High	1.40	-0.64	<b>-0.46</b>	1.40	-0.65	<b>-0.46</b>	1.00
10	100	6	High	Low	0.35	-0.10	<b>-0.30</b>	0.35	-0.10	<b>-0.29</b>	0.92
11	100	6	High	Mixed	0.35	-0.16	<b>-0.44</b>	1.40	-0.45	<b>-0.32</b>	0.89
12	100	6	High	High	1.40	-0.60	<b>-0.43</b>	1.40	-0.61	<b>-0.43</b>	1.00
13	500	2	Low	Low	0.35	-0.14	<b>-0.40</b>	0.35	-0.15	<b>-0.42</b>	0.63
14	500	2	Low	Mixed	0.35	-0.25	<b>-0.71</b>	1.40	-0.68	<b>-0.49</b>	0.60
15	500	2	Low	High	1.40	-0.87	<b>-0.62</b>	1.40	-0.89	<b>-0.64</b>	0.99
16	500	2	High	Low	0.35	-0.12	<b>-0.36</b>	0.35	-0.13	<b>-0.38</b>	0.86
17	500	2	High	Mixed	0.35	-0.22	<b>-0.63</b>	1.40	-0.58	<b>-0.42</b>	0.78
18	500	2	High	High	1.40	-0.77	<b>-0.55</b>	1.40	-0.76	<b>-0.54</b>	1.00
19	500	6	Low	Low	0.35	-0.10	<b>-0.29</b>	0.35	-0.10	<b>-0.29</b>	1.00
20	500	6	Low	Mixed	0.35	-0.18	<b>-0.52</b>	1.40	-0.51	<b>-0.37</b>	0.98
21	500	6	Low	High	1.40	-0.67	<b>-0.48</b>	1.40	-0.67	<b>-0.48</b>	1.00
22	500	6	High	Low	0.35	-0.10	<b>-0.30</b>	0.35	-0.10	<b>-0.29</b>	1.00
23	500	6	High	Mixed	0.35	-0.17	<b>-0.49</b>	1.40	-0.48	<b>-0.34</b>	1.00
24	500	6	High	High	1.40	-0.63	<b>-0.45</b>	1.40	-0.62	<b>-0.44</b>	1.00

\*Proportion of replications with a positive definite random effect covariance matrix.



Table 20: RT Model with known identifiers – MCMC Simulation Random Effect Standard Error Recovery.  
Standard deviation of parameter estimates compared to mean standard error across replication (*factors italicized*)  
with conditions **bolded** when the ratio is off by more than 25%.

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect				Reciprocator Random Effect			
					TRUE	SD	M(SE)	M(SE) / SD	TRUE	SD	M(SE)	M(SE) / SD
1	100	2	Low	Low	0.35	5.01	0.64	<b>0.13</b>	0.35	2.35	1.83	<b>0.78</b>
2	100	2	Low	Mixed	0.35	4.35	0.78	<b>0.18</b>	1.40	2.63	3.42	<b>1.30</b>
3	100	2	Low	High	1.40	5.94	1.53	<b>0.26</b>	1.40	3.76	3.74	1.00
4	100	2	High	Low	0.35	1.12	0.27	<b>0.24</b>	0.35	0.64	0.82	<b>1.28</b>
5	100	2	High	Mixed	0.35	1.45	0.38	<b>0.26</b>	1.40	1.63	2.02	1.24
6	100	2	High	High	1.40	2.95	0.82	<b>0.28</b>	1.40	2.26	2.45	1.09
7	100	6	Low	Low	0.35	0.51	0.14	<b>0.27</b>	0.35	0.35	0.41	1.16
8	100	6	Low	Mixed	0.35	0.59	0.17	<b>0.29</b>	1.40	0.79	1.03	<b>1.30</b>
9	100	6	Low	High	1.40	1.35	0.48	<b>0.35</b>	1.40	0.90	1.14	<b>1.26</b>
10	100	6	High	Low	0.35	0.41	0.12	<b>0.28</b>	0.35	0.33	0.37	1.14
11	100	6	High	Mixed	0.35	0.50	0.13	<b>0.26</b>	1.40	0.67	0.90	<b>1.33</b>
12	100	6	High	High	1.40	1.11	0.31	<b>0.28</b>	1.40	0.78	0.99	<b>1.28</b>
13	500	2	Low	Low	0.35	0.49	0.13	<b>0.27</b>	0.35	0.36	0.40	1.12
14	500	2	Low	Mixed	0.35	0.56	0.17	<b>0.30</b>	1.40	0.72	0.91	<b>1.26</b>
15	500	2	Low	High	1.40	1.14	0.36	<b>0.31</b>	1.40	0.78	0.99	<b>1.28</b>
16	500	2	High	Low	0.35	0.44	0.16	<b>0.37</b>	0.35	0.34	0.35	1.04
17	500	2	High	Mixed	0.35	0.45	0.14	<b>0.32</b>	1.40	0.53	0.72	<b>1.35</b>
18	500	2	High	High	1.40	0.92	0.26	<b>0.28</b>	1.40	0.65	0.84	<b>1.29</b>
19	500	6	Low	Low	0.35	0.23	0.06	<b>0.28</b>	0.35	0.17	0.20	1.17
20	500	6	Low	Mixed	0.35	0.24	0.08	<b>0.32</b>	1.40	0.30	0.40	<b>1.34</b>
21	500	6	Low	High	1.40	0.48	0.17	<b>0.36</b>	1.40	0.35	0.44	<b>1.27</b>
22	500	6	High	Low	0.35	0.18	0.05	<b>0.30</b>	0.35	0.13	0.17	<b>1.33</b>
23	500	6	High	Mixed	0.35	0.21	0.06	<b>0.28</b>	1.40	0.28	0.36	<b>1.31</b>
24	500	6	High	High	1.40	0.42	0.12	<b>0.29</b>	1.40	0.30	0.40	<b>1.33</b>

Table 21: RT Model with known identifiers – PQL Simulation Random Effect Standard Error Recovery.  
*Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect				Reciprocator Random Effect			
					TRUE	SD	M(SE)	M(SE) / SD	TRUE	SD	M(SE)	M(SE) / SD
1	100	2	Low	Low	0.35	0.36	0.38	1.07	0.35	0.35	0.38	1.10
2	100	2	Low	Mixed	0.35	0.32	0.31	0.97	1.40	0.50	0.57	1.14
3	100	2	Low	High	1.40	0.49	0.53	1.08	1.40	0.49	0.52	1.05
4	100	2	High	Low	0.35	0.27	0.27	1.00	0.35	0.26	0.27	1.01
5	100	2	High	Mixed	0.35	0.27	0.27	1.00	1.40	0.44	0.47	1.08
6	100	2	High	High	1.40	0.49	0.48	0.99	1.40	0.49	0.49	1.01
7	100	6	Low	Low	0.35	0.19	0.20	1.06	0.35	0.19	0.20	1.05
8	100	6	Low	Mixed	0.35	0.17	0.18	1.03	1.40	0.29	0.32	1.09
9	100	6	Low	High	1.40	0.27	0.30	1.10	1.40	0.27	0.30	1.09
10	100	6	High	Low	0.35	0.15	0.15	1.02	0.35	0.15	0.15	1.00
11	100	6	High	Mixed	0.35	0.15	0.14	0.99	1.40	0.27	0.28	1.05
12	100	6	High	High	1.40	0.26	0.27	1.04	1.40	0.26	0.27	1.05
13	500	2	Low	Low	0.35	0.17	0.18	1.04	0.35	0.17	0.17	1.02
14	500	2	Low	Mixed	0.35	0.13	0.13	0.99	1.40	0.21	0.25	1.15
15	500	2	Low	High	1.40	0.21	0.24	1.15	1.40	0.21	0.24	1.16
16	500	2	High	Low	0.35	0.14	0.14	1.05	0.35	0.14	0.14	0.99
17	500	2	High	Mixed	0.35	0.12	0.13	1.05	1.40	0.19	0.21	1.12
18	500	2	High	High	1.40	0.21	0.22	1.03	1.40	0.21	0.22	1.05
19	500	6	Low	Low	0.35	0.09	0.09	1.06	0.35	0.09	0.09	1.07
20	500	6	Low	Mixed	0.35	0.08	0.09	1.07	1.40	0.12	0.14	1.10
21	500	6	Low	High	1.40	0.12	0.13	1.06	1.40	0.12	0.13	1.07
22	500	6	High	Low	0.35	0.06	0.07	1.05	0.35	0.06	0.07	1.07
23	500	6	High	Mixed	0.35	0.06	0.07	1.05	1.40	0.12	0.12	1.05
24	500	6	High	High	1.40	0.11	0.12	1.06	1.40	0.11	0.12	1.08

### A.3: RT MODEL – WMM APPROACH

Table 22: RT Model with WMM approach – PQL Simulation Fixed Effects Bias.  
Factors italicized and conditions with relative bias above or below 25% of parameter bolded.

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$		$\beta_1 = 0.5$		$\beta_2 = 0$	
					Bias	Relative Bias	Bias	Power ( $\alpha=0.05$ )	Bias	Type I ( $\alpha=0.05$ )
1	100	2	Low	-3.00	0.16	-0.05	0.00	0.21	0.00	0.03
2	100	2	Mixed	-3.00	0.35	-0.12	-0.07	0.15	0.00	0.04
3	100	2	High	-3.00	0.49	-0.16	-0.10	0.13	0.00	0.04
4	100	2	Low	0.05	-0.06	<b>-1.24</b>	-0.03	0.25	0.00	0.05
5	100	2	Mixed	0.05	-0.15	<b>-2.97</b>	-0.10	0.17	0.01	0.05
6	100	2	High	0.05	-0.25	<b>-4.98</b>	-0.10	0.14	0.00	0.05
7	100	6	Low	-3.00	0.13	-0.04	-0.01	0.53	-0.01	0.05
8	100	6	Mixed	-3.00	0.26	-0.09	-0.05	0.47	0.01	0.04
9	100	6	High	-3.00	0.34	-0.11	-0.10	0.36	0.01	0.03
10	100	6	Low	0.05	-0.18	<b>-3.53</b>	-0.05	0.62	0.00	0.03
11	100	6	Mixed	0.05	-0.38	<b>-7.66</b>	-0.07	0.50	-0.01	0.03
12	100	6	High	0.05	-0.50	<b>-10.01</b>	-0.10	0.42	0.01	0.04
13	500	2	Low	-3.00	0.21	-0.07	-0.04	0.72	0.00	0.05
14	500	2	Mixed	-3.00	0.40	-0.13	-0.08	0.63	0.00	0.05
15	500	2	High	-3.00	0.53	-0.18	-0.11	0.57	0.00	0.05
16	500	2	Low	0.05	-0.09	<b>-1.84</b>	-0.05	0.85	0.00	0.05
17	500	2	Mixed	0.05	-0.20	<b>-4.04</b>	-0.08	0.73	0.00	0.05
18	500	2	High	0.05	-0.28	<b>-5.57</b>	-0.12	0.58	0.00	0.04
19	500	6	Low	-3.00	0.18	-0.06	-0.04	1.00	0.00	0.04
20	500	6	Mixed	-3.00	0.28	-0.09	-0.07	0.99	0.00	0.04
21	500	6	High	-3.00	0.36	-0.12	-0.11	0.96	0.00	0.04
22	500	6	Low	0.05	-0.19	<b>-3.74</b>	-0.05	1.00	0.00	0.04
23	500	6	Mixed	0.05	-0.38	<b>-7.69</b>	-0.08	1.00	-0.01	0.04
24	500	6	High	0.05	-0.51	<b>-10.27</b>	-0.10	0.98	0.00	0.04

Table 23: RT Model with WMM approach – PQL Simulation Fixed Effect Standard Error Recovery.  
*Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$			$\beta_2 = 0$		
					SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD
1	100	2	Low	-3.00	0.35	0.33	0.96	0.45	0.43	0.96	0.31	0.32	1.03
2	100	2	Mixed	-3.00	0.35	0.33	0.96	0.43	0.43	1.00	0.32	0.33	1.02
3	100	2	High	-3.00	0.34	0.33	0.98	0.40	0.43	1.07	0.34	0.33	0.97
4	100	2	Low	0.05	0.27	0.26	0.95	0.35	0.36	1.02	0.28	0.27	0.98
5	100	2	Mixed	0.05	0.28	0.29	1.03	0.36	0.38	1.06	0.30	0.30	1.01
6	100	2	High	0.05	0.29	0.31	1.07	0.38	0.40	1.06	0.32	0.33	1.02
7	100	6	Low	-3.00	0.19	0.19	1.02	0.23	0.24	1.03	0.19	0.19	0.98
8	100	6	Mixed	-3.00	0.20	0.21	1.03	0.23	0.24	1.04	0.19	0.20	1.05
9	100	6	High	-3.00	0.21	0.22	1.03	0.23	0.24	1.06	0.20	0.21	1.07
10	100	6	Low	0.05	0.15	0.16	1.07	0.20	0.20	1.00	0.15	0.17	1.13
11	100	6	Mixed	0.05	0.19	0.20	1.03	0.21	0.21	1.02	0.18	0.20	1.11
12	100	6	High	0.05	0.21	0.22	1.04	0.22	0.23	1.03	0.22	0.23	1.04
13	500	2	Low	-3.00	0.14	0.14	0.99	0.18	0.18	1.00	0.13	0.13	1.00
14	500	2	Mixed	-3.00	0.14	0.14	1.00	0.18	0.18	1.00	0.13	0.13	1.03
15	500	2	High	-3.00	0.14	0.14	1.01	0.18	0.18	1.01	0.13	0.14	1.05
16	500	2	Low	0.05	0.11	0.11	0.99	0.15	0.15	1.01	0.11	0.11	1.03
17	500	2	Mixed	0.05	0.12	0.12	1.02	0.16	0.16	1.02	0.12	0.13	1.05
18	500	2	High	0.05	0.12	0.13	1.10	0.17	0.17	1.01	0.14	0.14	0.97
19	500	6	Low	-3.00	0.08	0.08	1.06	0.10	0.10	1.04	0.08	0.08	0.99
20	500	6	Mixed	-3.00	0.09	0.09	1.00	0.10	0.10	1.05	0.08	0.09	1.08
21	500	6	High	-3.00	0.09	0.09	1.05	0.10	0.11	1.07	0.09	0.09	1.03
22	500	6	Low	0.05	0.07	0.07	1.01	0.08	0.09	1.10	0.07	0.07	1.05
23	500	6	Mixed	0.05	0.08	0.09	1.07	0.09	0.09	1.04	0.08	0.09	1.07
24	500	6	High	0.05	0.09	0.10	1.07	0.09	0.10	1.11	0.09	0.10	1.09

Table 24: RT Model with WMM approach – PQL Simulation Random Effects Bias.  
*Factors italicized and conditions with relative bias above or below 25% of parameter bolded.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect		Reciprocator Random Effect		PD*
					TRUE	Bias	TRUE	Bias	
1	100	2	Low	Low	0.35	-0.05	0.35	-0.06	0.38
2	100	2	Low	Mixed	0.35	-0.10	1.40	-0.72	0.51
3	100	2	Low	High	1.40	-0.83	1.40	-0.82	0.71
4	100	2	High	Low	0.35	-0.07	0.35	-0.08	0.45
5	100	2	High	Mixed	0.35	-0.08	1.40	-0.63	0.60
6	100	2	High	High	1.40	-0.69	1.40	-0.70	0.82
7	100	6	Low	Low	0.35	-0.09	0.35	-0.10	0.81
8	100	6	Low	Mixed	0.35	-0.14	1.40	-0.57	0.85
9	100	6	Low	High	1.40	-0.72	1.40	-0.72	1.00
10	100	6	High	Low	0.35	-0.06	0.35	-0.06	0.93
11	100	6	High	Mixed	0.35	-0.10	1.40	-0.35	0.91
12	100	6	High	High	1.40	-0.50	1.40	-0.51	1.00
13	500	2	Low	Low	0.35	-0.15	0.35	-0.15	0.66
14	500	2	Low	Mixed	0.35	-0.22	1.40	-0.77	0.68
15	500	2	Low	High	1.40	-0.91	1.40	-0.92	0.99
16	500	2	High	Low	0.35	-0.12	0.35	-0.12	0.85
17	500	2	High	Mixed	0.35	-0.17	1.40	-0.63	0.86
18	500	2	High	High	1.40	-0.76	1.40	-0.75	1.00
19	500	6	Low	Low	0.35	-0.12	0.35	-0.12	1.00
20	500	6	Low	Mixed	0.35	-0.17	1.40	-0.60	0.98
21	500	6	Low	High	1.40	-0.74	1.40	-0.74	1.00
22	500	6	High	Low	0.35	-0.07	0.35	-0.06	1.00
23	500	6	High	Mixed	0.35	-0.12	1.40	-0.40	1.00
24	500	6	High	High	1.40	-0.54	1.40	-0.54	1.00

\*Proportion of simulations with a positive definite random effect covariance matrix.

Table 25: RT Model with WMM approach – PQL Simulation Random Effect Standard Error Recovery. *Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect			Reciprocator Random Effect		
					TRUE	SD	M(SE) / SD	TRUE	SD	M(SE) / SD
1	100	2	Low	Low	0.35	0.35	0.59	0.35	0.34	0.59
2	100	2	Low	Mixed	0.35	0.33	0.58	1.40	0.48	0.63
3	100	2	Low	High	1.40	0.47	0.61	1.40	0.49	0.62
4	100	2	High	Low	0.35	0.29	0.43	0.35	0.28	0.43
5	100	2	High	Mixed	0.35	0.33	0.48	1.40	0.46	0.54
6	100	2	High	High	1.40	0.50	0.57	1.40	0.50	0.57
7	100	6	Low	Low	0.35	0.19	0.22	0.35	0.19	0.22
8	100	6	Low	Mixed	0.35	0.17	0.21	1.40	0.29	0.31
9	100	6	Low	High	1.40	0.28	0.30	1.40	0.27	0.30
10	100	6	High	Low	0.35	0.18	0.18	0.35	0.18	0.19
11	100	6	High	Mixed	0.35	0.18	0.20	1.40	0.33	0.35
12	100	6	High	High	1.40	0.33	0.34	1.40	0.31	0.34
13	500	2	Low	Low	0.35	0.17	0.22	0.35	0.16	0.22
14	500	2	Low	Mixed	0.35	0.14	0.22	1.40	0.21	0.25
15	500	2	Low	High	1.40	0.20	0.24	1.40	0.20	0.24
16	500	2	High	Low	0.35	0.14	0.16	0.35	0.15	0.16
17	500	2	High	Mixed	0.35	0.14	0.18	1.40	0.20	0.22
18	500	2	High	High	1.40	0.22	0.23	1.40	0.22	0.23
19	500	6	Low	Low	0.35	0.09	0.09	0.35	0.08	0.09
20	500	6	Low	Mixed	0.35	0.08	0.09	1.40	0.12	0.13
21	500	6	Low	High	1.40	0.12	0.13	1.40	0.12	0.13
22	500	6	High	Low	0.35	0.08	0.08	0.35	0.07	0.08
23	500	6	High	Mixed	0.35	0.08	0.08	1.40	0.14	0.15
24	500	6	High	High	1.40	0.13	0.15	1.40	0.14	0.14

## A.4: RT MODEL – AIP APPROACH

Table 26: RT Model with AIP approach – PQL Simulation Fixed Effects Bias.  
Factors italicized and conditions with relative bias above or below 25% of parameter bolded.

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$		$\beta_1 = 0.5$		$\beta_2 = 0$		
					Bias	Relative Bias	Bias	Relative Bias	Power ( $\alpha=0.05$ )	Bias Type I ( $\alpha=0.05$ )	
1	100	2	Low	-3.00	-0.12	0.04	0.09	0.17	0.25	-0.02	0.05
2	100	2	Mixed	-3.00	-0.04	0.01	0.13	<b>0.26</b>	0.30	0.00	0.06
3	100	2	High	-3.00	0.26	-0.09	-0.05	-0.09	0.16	0.05	0.04
4	100	2	Low	0.05	0.01	<b>0.29</b>	0.02	0.05	0.31	-0.06	0.05
5	100	2	Mixed	0.05	-0.11	<b>-2.24</b>	0.02	0.03	0.24	0.03	0.11
6	100	2	High	0.05	-0.18	<b>-3.57</b>	-0.06	-0.11	0.22	-0.04	0.12
7	100	6	Low	-3.00	0.00	0.00	-0.02	-0.04	0.51	0.00	0.08
8	100	6	Mixed	-3.00	0.10	-0.03	-0.01	-0.03	0.53	0.03	0.09
9	100	6	High	-3.00	0.20	-0.07	-0.07	-0.13	0.44	0.03	0.11
10	100	6	Low	0.05	-0.07	<b>-1.35</b>	-0.01	-0.02	0.64	-0.01	0.07
11	100	6	Mixed	0.05	-0.18	<b>-3.69</b>	-0.06	-0.13	0.57	-0.03	0.11
12	100	6	High	0.05	-0.18	<b>-3.50</b>	-0.10	-0.20	0.44	0.02	0.13
13	500	2	Low	-3.00	0.06	-0.02	0.01	0.02	0.78	0.01	0.04
14	500	2	Mixed	-3.00	0.25	-0.08	-0.09	-0.18	0.69	-0.01	0.11
15	500	2	High	-3.00	0.34	-0.11	-0.07	-0.15	0.67	0.02	0.05
16	500	2	Low	0.05	-0.03	<b>-0.70</b>	-0.02	-0.04	0.86	0.00	0.07
17	500	2	Mixed	0.05	-0.16	<b>-3.16</b>	-0.05	-0.11	0.75	0.00	0.10
18	500	2	High	0.05	-0.21	<b>-4.25</b>	-0.04	-0.08	0.76	0.00	0.12
19	500	6	Low	-3.00	0.09	-0.03	-0.03	-0.06	1.00	-0.02	0.06
20	500	6	Mixed	-3.00	0.19	-0.06	-0.06	-0.11	0.99	-0.01	0.13
21	500	6	High	-3.00	0.25	-0.08	-0.09	-0.18	0.98	0.00	0.13
22	500	6	Low	0.05	-0.08	<b>-1.57</b>	-0.01	-0.02	1.00	0.00	0.07
23	500	6	Mixed	0.05	-0.23	<b>-4.53</b>	-0.04	-0.08	0.99	0.00	0.17
24	500	6	High	0.05	-0.24	<b>-4.72</b>	-0.06	-0.12	0.97	-0.01	0.09

Table 27: RT Model with AIP approach – PQL Simulation Fixed Effect Standard Error Recovery.  
Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized)  
with conditions bolded when the ratio is off by more than 25%.

Cond.	Nodes	Links	RE. Variance	TRUE	$\beta_0$			$\beta_1 = 0.5$			$\beta_2 = 0$		
					SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD	SD	M(SE)	M(SE) / SD
1	100	2	Low	-3.00	0.42	0.34	0.82	0.47	0.44	0.94	0.35	0.33	0.94
2	100	2	Mixed	-3.00	0.41	0.35	0.86	0.50	0.44	0.89	0.34	0.33	0.96
3	100	2	High	-3.00	0.41	0.33	0.82	0.47	0.43	0.92	0.32	0.32	0.99
4	100	2	Low	0.05	0.31	0.25	0.81	0.41	0.36	0.88	0.28	0.27	0.96
5	100	2	Mixed	0.05	0.32	0.28	0.86	0.37	0.38	1.04	0.35	0.29	0.83
6	100	2	High	0.05	0.29	0.29	1.00	0.40	0.39	0.99	0.38	0.30	0.79
7	100	6	Low	-3.00	0.23	0.19	0.82	0.24	0.24	0.97	0.21	0.18	0.85
8	100	6	Mixed	-3.00	0.23	0.19	0.85	0.24	0.24	0.98	0.23	0.18	0.81
9	100	6	High	-3.00	0.24	0.20	0.85	0.23	0.24	1.05	0.23	0.19	0.81
10	100	6	Low	0.05	0.17	0.14	0.85	0.26	0.19	<b>0.75</b>	0.16	0.15	0.92
11	100	6	Mixed	0.05	0.20	0.17	0.84	0.22	0.20	0.94	0.20	0.17	0.82
12	100	6	High	0.05	0.23	0.19	0.83	0.27	0.22	0.82	0.26	0.19	<b>0.71</b>
13	500	2	Low	-3.00	0.15	0.14	0.91	0.19	0.18	0.97	0.13	0.13	1.01
14	500	2	Mixed	-3.00	0.14	0.14	0.97	0.17	0.18	1.03	0.16	0.13	0.85
15	500	2	High	-3.00	0.17	0.14	0.83	0.18	0.18	1.03	0.13	0.13	1.02
16	500	2	Low	0.05	0.12	0.10	0.90	0.18	0.15	0.84	0.12	0.11	0.91
17	500	2	Mixed	0.05	0.15	0.12	0.76	0.17	0.16	0.92	0.14	0.12	0.87
18	500	2	High	0.05	0.13	0.13	0.93	0.18	0.17	0.96	0.15	0.13	0.82
19	500	6	Low	-3.00	0.09	0.08	0.93	0.10	0.10	0.98	0.07	0.08	1.05
20	500	6	Mixed	-3.00	0.10	0.08	0.82	0.12	0.10	0.89	0.10	0.08	0.82
21	500	6	High	-3.00	0.10	0.09	0.88	0.10	0.10	1.06	0.11	0.08	0.77
22	500	6	Low	0.05	0.07	0.06	0.87	0.11	0.09	0.78	0.07	0.07	0.97
23	500	6	Mixed	0.05	0.09	0.07	0.82	0.10	0.09	0.90	0.10	0.07	<b>0.72</b>
24	500	6	High	0.05	0.11	0.08	<b>0.73</b>	0.12	0.10	0.82	0.09	0.08	0.83



Table 28: RT Model with AIP approach – PQL Simulation Random Effects Bias.  
*Factors italicized and conditions with relative bias above or below 25% of parameter bolded.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect			Reciprocator Random Effect			PD*
					TRUE	Bias	Relative Bias	TRUE	Bias	Relative Bias	
1	100	2	Low	Low	0.35	0.23	<b>0.66</b>	0.35	0.25	<b>0.71</b>	0.97
2	100	2	Low	Mixed	0.35	0.34	<b>0.97</b>	1.40	-0.48	<b>-0.34</b>	0.98
3	100	2	Low	High	1.40	-0.42	<b>-0.30</b>	1.40	-0.44	<b>-0.31</b>	0.98
4	100	2	High	Low	0.35	0.06	0.17	0.35	0.07	0.20	0.98
5	100	2	High	Mixed	0.35	0.37	<b>1.05</b>	1.40	-0.47	<b>-0.34</b>	0.99
6	100	2	High	High	1.40	-0.32	-0.23	1.40	-0.30	-0.21	0.99
7	100	6	Low	Low	0.35	0.02	0.06	0.35	0.03	0.09	0.99
8	100	6	Low	Mixed	0.35	0.09	<b>0.27</b>	1.40	-0.53	<b>-0.38</b>	1.00
9	100	6	Low	High	1.40	-0.51	<b>-0.37</b>	1.40	-0.49	<b>-0.35</b>	1.00
10	100	6	High	Low	0.35	0.00	0.01	0.35	0.00	0.01	1.00
11	100	6	High	Mixed	0.35	0.27	<b>0.77</b>	1.40	-0.53	<b>-0.38</b>	1.00
12	100	6	High	High	1.40	-0.11	-0.08	1.40	-0.12	-0.09	1.00
13	500	2	Low	Low	0.35	-0.01	-0.04	0.35	0.00	0.01	0.99
14	500	2	Low	Mixed	0.35	0.04	0.13	1.40	-0.75	<b>-0.54</b>	1.00
15	500	2	Low	High	1.40	-0.68	<b>-0.49</b>	1.40	-0.66	<b>-0.47</b>	1.00
16	500	2	High	Low	0.35	-0.03	-0.08	0.35	-0.03	-0.09	1.00
17	500	2	High	Mixed	0.35	0.22	<b>0.64</b>	1.40	-0.69	<b>-0.49</b>	1.00
18	500	2	High	High	1.40	-0.46	<b>-0.33</b>	1.40	-0.41	<b>-0.29</b>	1.00
19	500	6	Low	Low	0.35	-0.08	-0.22	0.35	-0.07	-0.20	1.00
20	500	6	Low	Mixed	0.35	0.02	0.06	1.40	-0.67	<b>-0.48</b>	1.00
21	500	6	Low	High	1.40	-0.63	<b>-0.45</b>	1.40	-0.61	<b>-0.44</b>	1.00
22	500	6	High	Low	0.35	-0.02	-0.07	0.35	-0.01	-0.04	1.00
23	500	6	High	Mixed	0.35	0.22	<b>0.62</b>	1.40	-0.65	<b>-0.46</b>	1.00
24	500	6	High	High	1.40	-0.25	-0.18	1.40	-0.22	-0.16	1.00

\*Proportions of iterations with random effect variances > 0.

Table 29: RT Model with AIP approach – PQL Simulation Random Effect Standard Error Recovery.  
*Standard deviation of parameter estimates compared to mean standard error across replication (factors italicized) with conditions bolded when the ratio is off by more than 25%.*

Cond.	Nodes	Links	Hazard	RE. Variance	Nominator Random Effect				Reciprocator Random Effect			
					TRUE	SD	M(SE)	M(SE) / SD	TRUE	SD	M(SE)	M(SE) / SD
1	100	2	Low	Low	0.35	0.30	0.33	1.10	0.35	0.27	0.34	<b>1.25</b>
2	100	2	Low	Mixed	0.35	0.30	0.35	1.16	1.40	0.36	0.42	1.18
3	100	2	Low	High	1.40	0.41	0.41	1.00	1.40	0.35	0.41	1.18
4	100	2	High	Low	0.35	0.24	0.12	<b>0.50</b>	0.35	0.22	0.12	<b>0.54</b>
5	100	2	High	Mixed	0.35	0.33	0.20	<b>0.59</b>	1.40	0.35	0.23	<b>0.66</b>
6	100	2	High	High	1.40	0.50	0.29	<b>0.59</b>	1.40	0.43	0.29	<b>0.68</b>
7	100	6	Low	Low	0.35	0.19	0.06	<b>0.30</b>	0.35	0.17	0.06	<b>0.33</b>
8	100	6	Low	Mixed	0.35	0.21	0.06	<b>0.29</b>	1.40	0.26	0.10	<b>0.40</b>
9	100	6	Low	High	1.40	0.29	0.11	<b>0.38</b>	1.40	0.26	0.11	<b>0.41</b>
10	100	6	High	Low	0.35	0.15	0.03	<b>0.20</b>	0.35	0.14	0.03	<b>0.21</b>
11	100	6	High	Mixed	0.35	0.25	0.05	<b>0.22</b>	1.40	0.26	0.07	<b>0.29</b>
12	100	6	High	High	1.40	0.39	0.14	<b>0.35</b>	1.40	0.36	0.13	<b>0.37</b>
13	500	2	Low	Low	0.35	0.11	0.05	<b>0.42</b>	0.35	0.14	0.05	<b>0.35</b>
14	500	2	Low	Mixed	0.35	0.14	0.05	<b>0.33</b>	1.40	0.14	0.06	<b>0.39</b>
15	500	2	Low	High	1.40	0.18	0.06	<b>0.33</b>	1.40	0.17	0.06	<b>0.37</b>
16	500	2	High	Low	0.35	0.10	0.02	<b>0.17</b>	0.35	0.10	0.02	<b>0.17</b>
17	500	2	High	Mixed	0.35	0.12	0.03	<b>0.23</b>	1.40	0.12	0.03	<b>0.25</b>
18	500	2	High	High	1.40	0.19	0.04	<b>0.23</b>	1.40	0.19	0.05	<b>0.24</b>
19	500	6	Low	Low	0.35	0.08	0.01	<b>0.11</b>	0.35	0.07	0.01	<b>0.12</b>
20	500	6	Low	Mixed	0.35	0.08	0.01	<b>0.12</b>	1.40	0.11	0.02	<b>0.15</b>
21	500	6	Low	High	1.40	0.12	0.02	<b>0.13</b>	1.40	0.12	0.02	<b>0.14</b>
22	500	6	High	Low	0.35	0.06	0.00	<b>0.09</b>	0.35	0.05	0.01	<b>0.10</b>
23	500	6	High	Mixed	0.35	0.08	0.01	<b>0.11</b>	1.40	0.09	0.01	<b>0.13</b>
24	500	6	High	High	1.40	0.17	0.02	<b>0.13</b>	1.40	0.15	0.02	<b>0.15</b>

## APPENDIX B: FIGURES

Figure 23: RT Model – Standard Deviation of Nominators Random Effect Recovery, Low Nominator Random Effect.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

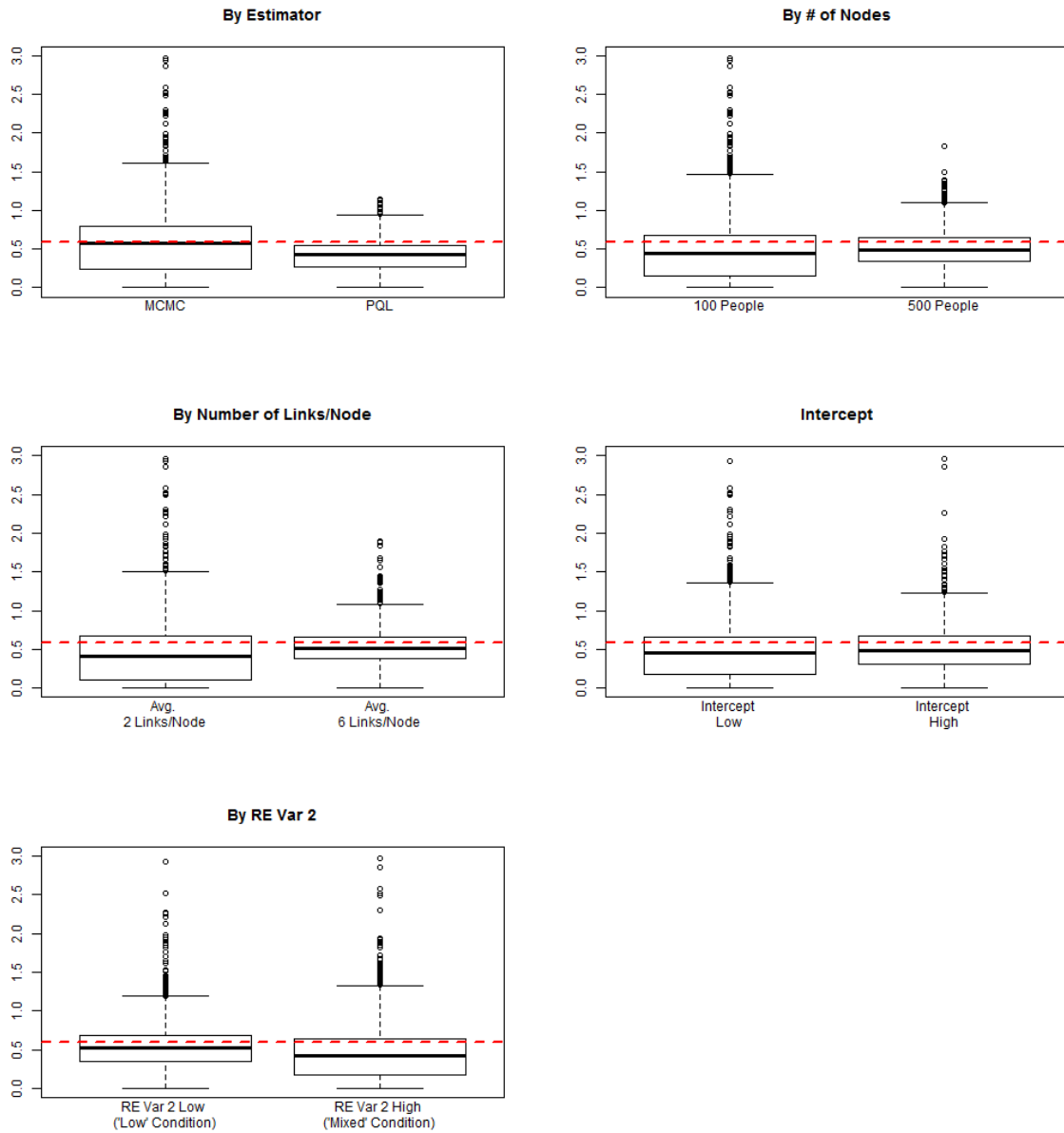


Figure 24: RT Model – Standard Deviation of Nominators Random Effect Recovery, High Nominator Random Effect.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

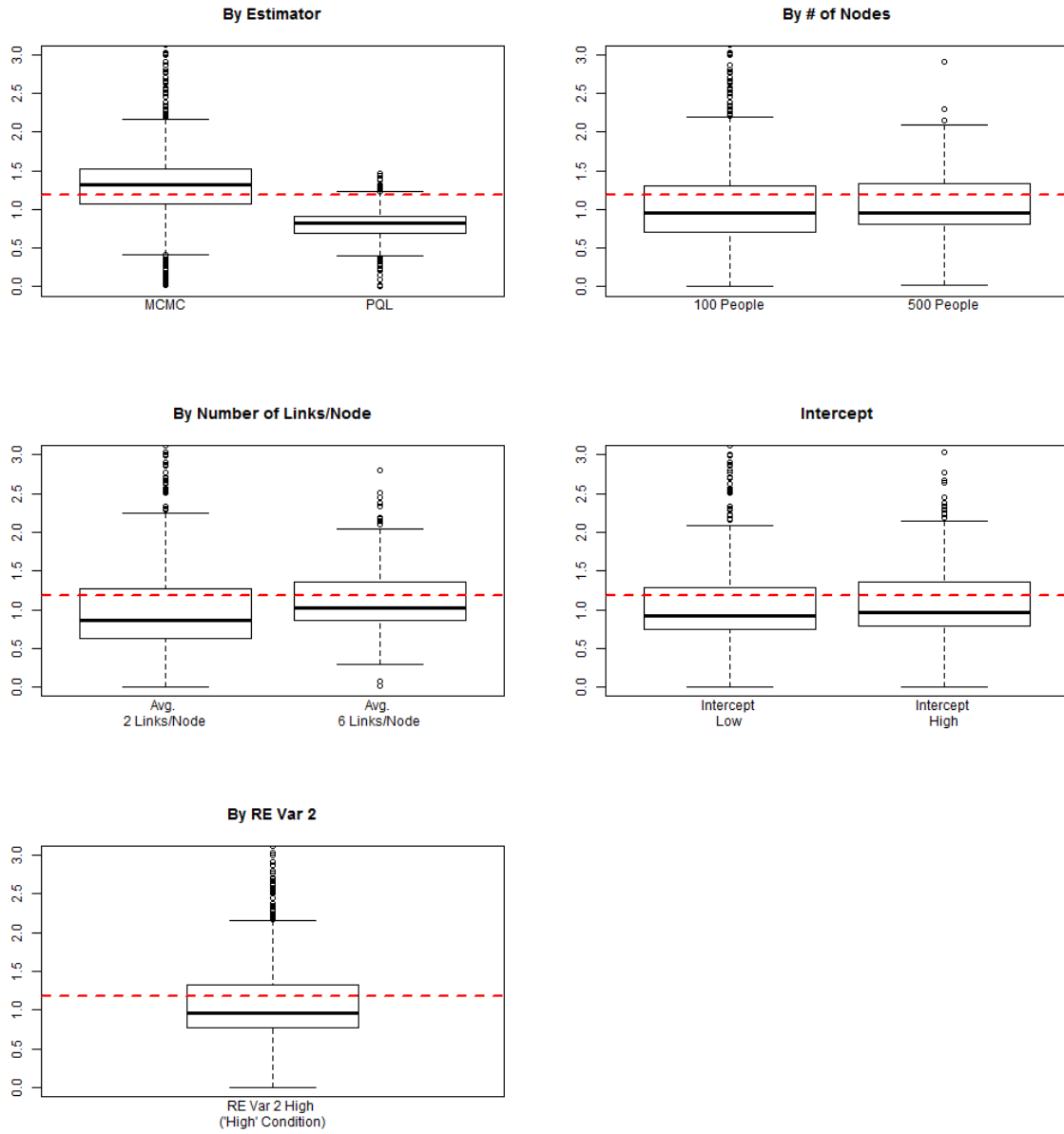


Figure 25: RT Model – Standard Deviation of Reciprocators Random Effect Recovery, Low Reciprocator Random Effect.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

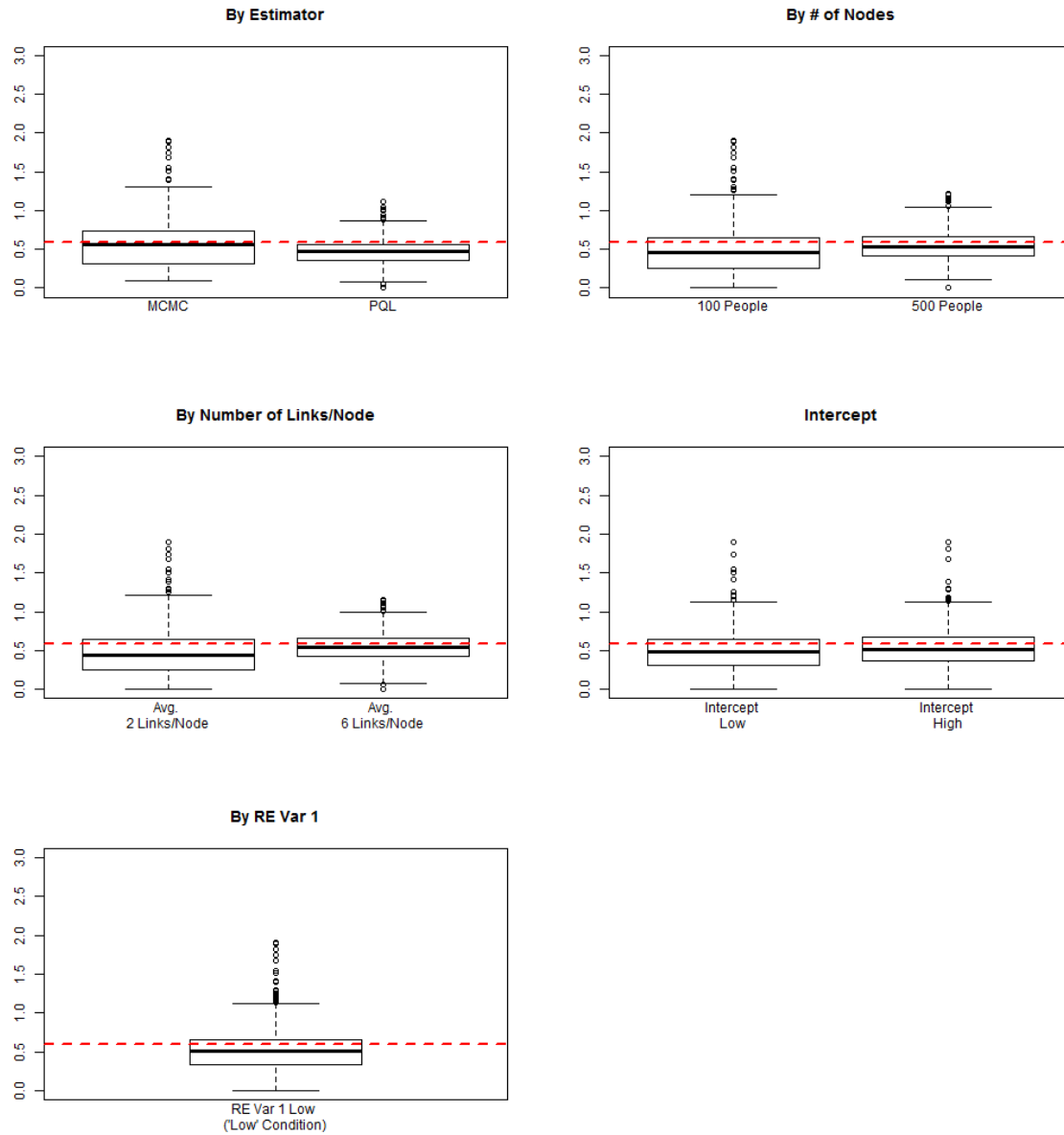


Figure 26: RT Model – Standard Deviation of Reciprocators Random Effect Recovery, High Reciprocator Random Effect.

*Boxplot of parameter estimates across simulations, aggregated by each factor*

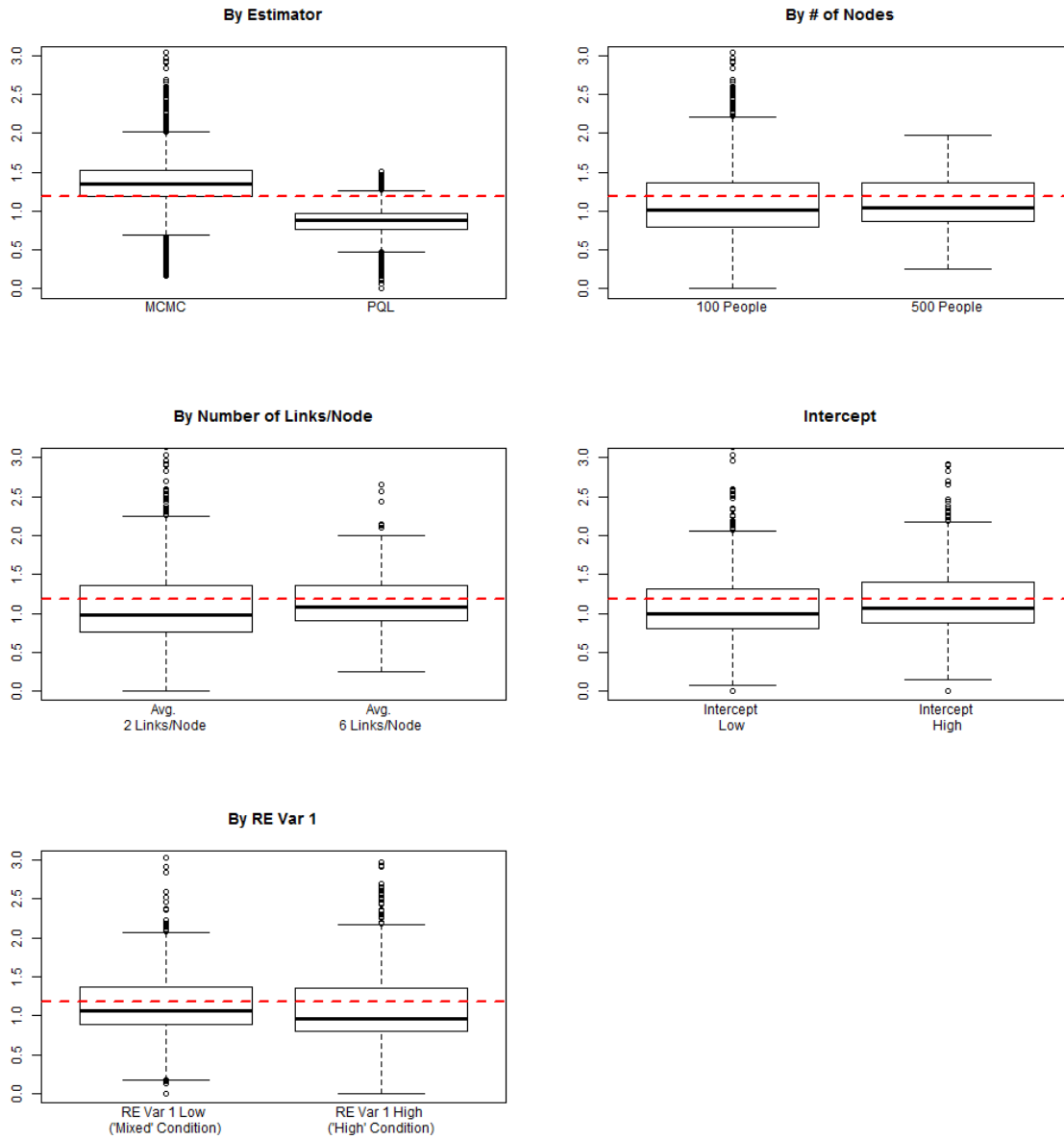
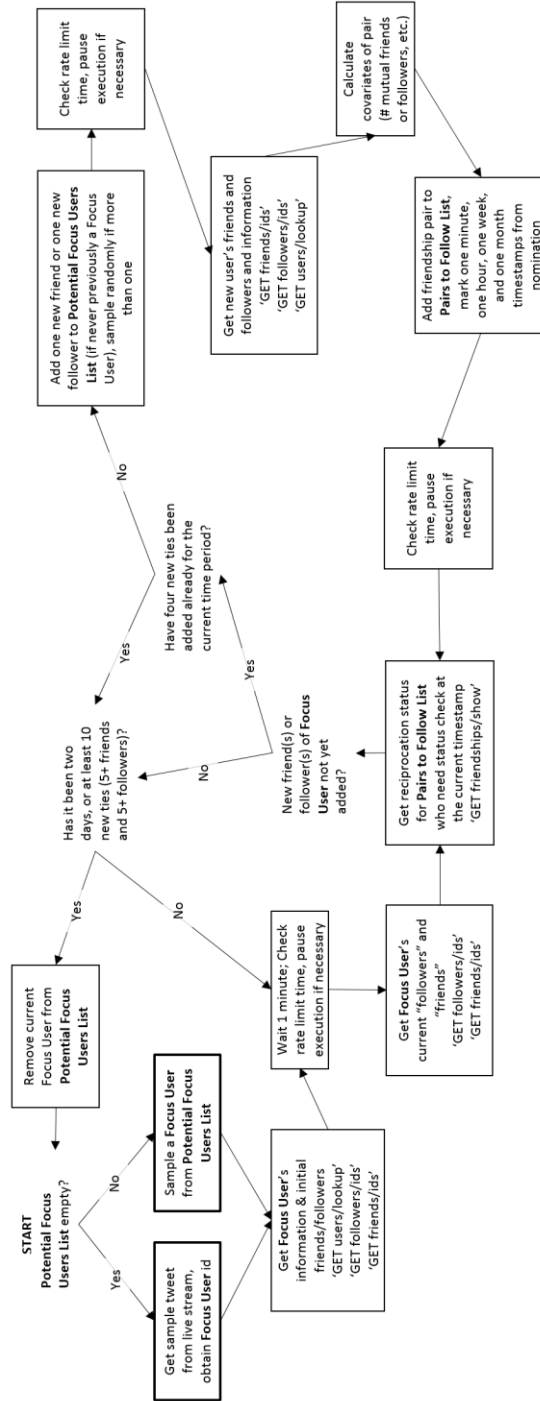


Figure 27: Gathering data from Twitter.

Flow diagram of data collection code, to collect a sample of users without hitting the Twitter rate limit.



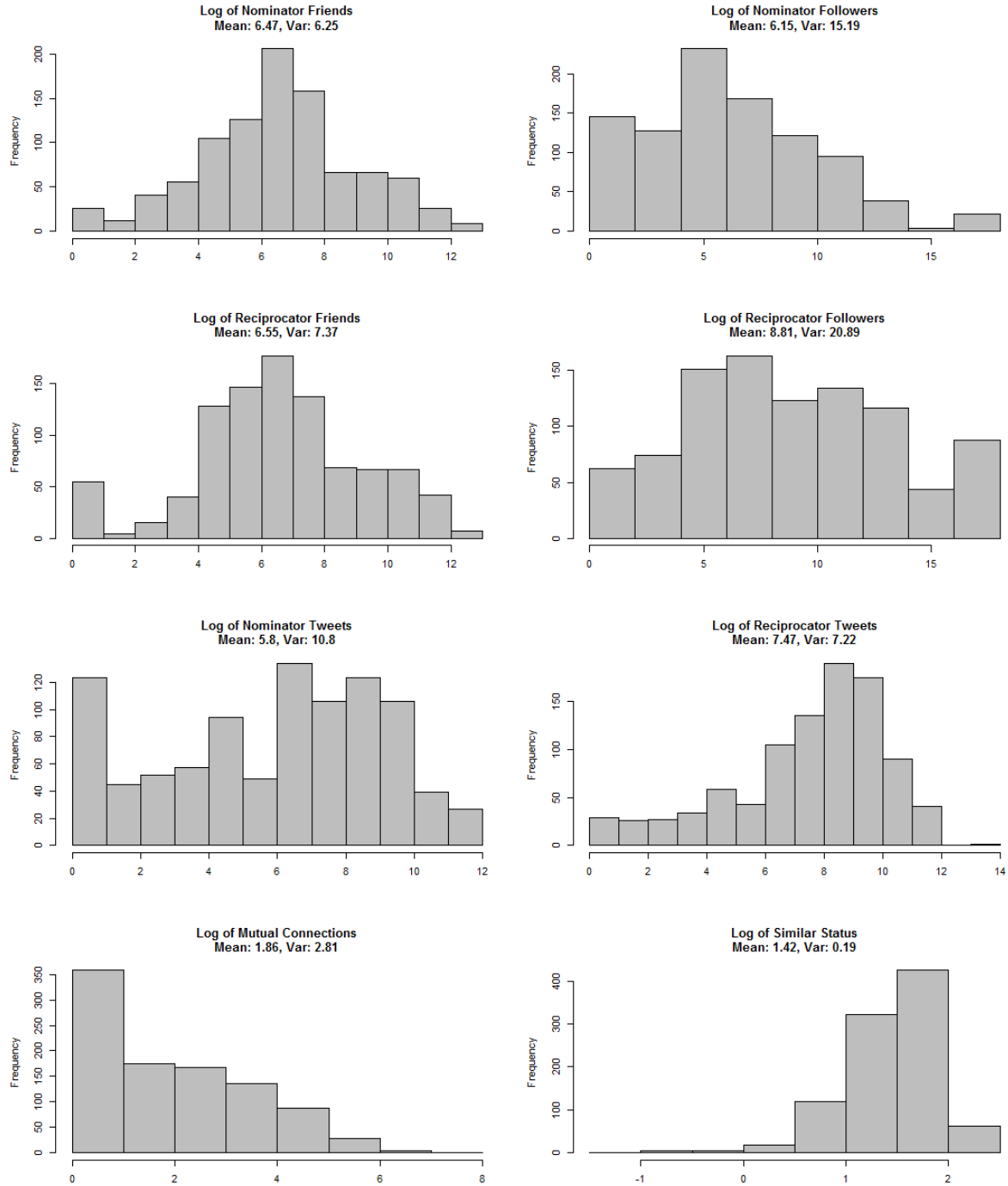
"Focus User" is the user currently being examined over time to see the addition of friends/followers.

"Potential Focus Users List" is the list of users who added links to previous focus users (e.g. new friends/followers) but have not yet been Focus Users.

"Pairs to Follow List" is the list of pairs of users who had new links between them, status checked again to see if reciprocation happened by one hour, one day, one week, and one month after nomination.

Figure 28: RT Model Application – Distribution of Predictors.

*The predictors are logged (as displayed below) and centered before entering into the model.*



Nominators with a protected account are 3% of the sample, and reciprocators with a protected account are also 3% of the sample.



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