ESSAYS ON SELLING TO STRATEGIC CUSTOMERS  
WITH A SUPPLY CHAIN PERSPECTIVE

Mustafa Onur Kabul

A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Statistics and Operations Research.

Chapel Hill  
2017

Approved by:  
Ali K. Parlaktürk  
Nilay Tanık Argon  
Bin Hu  
Vidyadhar G. Kulkarni  
Serhan Ziya
ABSTRACT

MUSTAFA ONUR KABUL: Essays on Selling to Strategic Customers with a Supply Chain Perspective (Under the direction of Ali K. Parlaktürk and Nilay Tanık Argon)

We consider a decentralized supply chain consisting of a retailer and a supplier that serves forward-looking consumers in two periods. In each period, the supplier and the retailer dynamically set the wholesale and retail price to maximize their own profits. The consumers are heterogeneous in their evaluations of the product and are strategic in deciding whether and when to buy the product, choosing the option that maximizes their utility, including waiting for a price mark-down. We derive the equilibrium and study the value of price and quantity commitments from both the retailer’s and the supplier’s perspective. We consider not only unilateral commitments but also simultaneous and concurrent commitments. We find that, while a centralized system always benefits from making price and quantity commitments, this is not true for a firm in a decentralized supply chain due to how the other firm responds to these commitments. We show that the retailer suffers from making a price or quantity commitment and that, similarly, the supplier does not benefit from making a price commitment. In these cases, commitments can harm not only the firm itself but also profitability of the other firm in the supply chain, thereby disadvantaging the entire supply chain. This happens because such commitments aggravate double-marginalization inefficiency in the supply chain. We strengthen our results with extensive robustness checks by altering key modeling assumptions. We find that majority of our results remain the same. In addition, we show eliminating this inefficiency through a coordinating contract (e.g., a two part tariff or quantity discount) makes commitments beneficial.

Furthermore, we extend our model by introducing demand uncertainty to explore the effects of uncertainty to firm profits, prices and quantities in a decentralized setting when selling to forward looking customers. We find that demand uncertainty intensifies the strategic behavior. In addition we find that supplier’s loss due the demand uncertainty is bigger than that of retailer’s and retailer’s
loss due to customers’ strategic behavior is bigger than that of supplier’s. Moreover we show that while in a centralized system demand uncertainty decreases the strategic loss it has a bi-model affect on the decentralized supply chain’s loss such that when volatility in demand is smaller than a threshold value it increases the loss beyond that level it decreases it.
To my wife Ilknur Kaynar Kabul and my family. Joys of my life ...
ACKNOWLEDGEMENTS

I would like to express my gratitude to my advisor Ali K. Parlaktürk. Without his guidance and patience I could not have accomplished this. I would like to thank my committee members Nilay Tanık Argon, Vidyadhar G. Kulkarni, Serhan Ziya and Bin Hu for their valuable feedback and time. I would like to especially thank Prof. Argon for keeping me on track in the final years and making the end possible. I would like to thank my supervisors Radhika Kulkarni, Manoj Chari and Michelle Opp at SAS Institute for their support, flexibility and the academic environment they provided.

I am very very grateful to my wife İlknur Kaynar Kabul who has always supported me, made me believe that I could accomplish this and spent countless weekends with our little one in my absence. Without her support, advice, encouragement, inspiration and sacrifices I could not have done this. She was always there when I had hard times and never given up on me. I would like to thank my daughters Nilufer and Bahar for all the joy and love they brought to our family.

I would like to thank my parents Fahriye and Kazım Kabul for their love, prayers and support.

I would like to thank the UNC Board of Governors, the Department of Statistics and Operations Research at UNC, and SAS Institute for providing me with employment and learning opportunities.

I would like to thank all the faculty and staff who contributed to my education, especially the professors and staff in the UNC STOR department, for their hard work and dedication.

Finally, I would like to thank all my friends who have inspired and enriched my life.

Thank you all!
# TABLE OF CONTENTS

LIST OF TABLES ................................................................................................................. x

LIST OF FIGURES ................................................................................................................. xi

1 Introduction ......................................................................................................................... 1
   1.1 Motivation ....................................................................................................................... 1
   1.2 Literature Review .......................................................................................................... 6

2 The Model ........................................................................................................................... 9
   2.1 Equilibrium ..................................................................................................................... 12
   2.2 Myopic Consumers Benchmark .................................................................................. 16
   2.3 Centralized System Benchmark .................................................................................. 18

3 Quantity Commitments ...................................................................................................... 19
   3.1 Centralized System Quantity Commitment Benchmark ........................................... 20
   3.2 Retailer’s Quantity Commitment ................................................................................. 20
   3.3 Retailer’s Period Specific Quantity Commitment ...................................................... 25
   3.4 Supplier’s Quantity Commitment ................................................................................. 25
   3.5 Supplier’s Period-specific Quantity Commitment ....................................................... 30
   3.6 Simultaneous Quantity Commitment .......................................................................... 34
   3.7 Concluding Remarks .................................................................................................... 35

4 Price Commitments ............................................................................................................ 36
   4.1 Centralized System Price Commitment Benchmark .................................................. 37
   4.2 Retailer’s Price Commitment ....................................................................................... 37
   4.3 Supplier’s Price Commitment ....................................................................................... 39
   4.4 Simultaneous Price Commitment ................................................................................ 42
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.2 The Model</td>
<td>72</td>
</tr>
<tr>
<td>8.3 Equilibrium</td>
<td>74</td>
</tr>
<tr>
<td>8.4 Supply Chain Performance</td>
<td>77</td>
</tr>
<tr>
<td>8.5 Centralized System Benchmark</td>
<td>84</td>
</tr>
<tr>
<td>8.6 Centralized vs. Decentralized System Performance</td>
<td>85</td>
</tr>
<tr>
<td>8.7 Concluding Remarks</td>
<td>91</td>
</tr>
<tr>
<td>9 Conclusion and Future Research</td>
<td>93</td>
</tr>
<tr>
<td>Appendix A Explicit Statement of Period 2 Equilibrium Values</td>
<td>97</td>
</tr>
<tr>
<td>Appendix B Proofs</td>
<td>98</td>
</tr>
<tr>
<td>B.1 Proofs of Chapter 2</td>
<td>98</td>
</tr>
<tr>
<td>B.2 Proofs of Chapter 3</td>
<td>100</td>
</tr>
<tr>
<td>B.3 Proofs of Chapter 4</td>
<td>118</td>
</tr>
<tr>
<td>B.4 Proofs of Chapter 5</td>
<td>123</td>
</tr>
<tr>
<td>B.5 Proofs of Chapter 6</td>
<td>126</td>
</tr>
<tr>
<td>B.6 Proofs of Chapter 7</td>
<td>132</td>
</tr>
<tr>
<td>B.7 Proofs of Chapter 8</td>
<td>134</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>139</td>
</tr>
</tbody>
</table>
LIST OF TABLES

A.1 Summary of the period 2 equilibrium values ........................................ 97
## LIST OF FIGURES

2.1 Sequence of events. ................................................................. 10
2.2 Equilibrium Regions in Proposition 1. ................................................ 15
3.1 Sequence of events for retailer’s aggregate quantity commitment. ................. 20
3.2 Equilibrium Regions in Proposition 5. .................................................. 23
3.3 Sequence of events for supplier’s aggregate quantity commitment. ................. 26
3.4 Sequence of events for supplier’s period specific quantity commitment. ............. 31
4.1 Sequence of events when retailer commits to prices. ..................................... 38
4.2 Sequence of events when supplier commits to prices. ..................................... 39
5.1 Retailer profits comparison under every commitment scenario. ....................... 45
5.2 Supplier profits comparison under every commitment scenario. ....................... 47
8.1 Sequence of events with demand uncertainty. ........................................... 72
8.2 Relative importance regions for the loss factors when $h = 1/4$. ....................... 80
8.3 Relative importance regions for the loss factors for a centralized firm and the decentralized retailer and supplier, when $h = 1/4$. ....................... 87
8.4 Fractional loss due to uncertainty for centralized firm and decentralized supply chain. . . 88
CHAPTER 1
Introduction

1.1 Motivation

Firms are dealing with increasingly sophisticated and better informed consumers who are becoming more prudent in their purchasing decisions. In one example of consumer savvy, many consumers strategically postpone their purchases, anticipating price mark-downs. Indeed recent reports show that a larger portion of sales are made at discount prices in each succeeding year (Rozhon 2004, O’Donnell 2006).

Existing work has shown that making commitments can be effective in dealing with strategic customer behavior (cf. Tirole 1988, Bulow 1982, Butz 1990). These studies show that committing to future prices (e.g., no mark-downs) or availability (e.g., no more than a limited quantity) can deter customers from waiting for price mark-downs. However, in spite of their assumed benefits, such commitments are not too prevalent in practice.

In this dissertation, we start with studying the value of commitments to all parties in a decentralized supply chain. We find that supply chain relations constitute one reason why firms may shy away from making these commitments. It is well known that supply chains suffer from a lack of coordination (Cachon 2003). Wholesale price contracts, commonly used on account of their simplicity, are especially vulnerable to double-marginalization inefficiency (Arrow 1985, Lariviere and Porteus 2001, Cachon 2003, Iyer and Villas-Boas 2003). We show that commitments can further exacerbate the inefficiency of wholesale price contracts and hurt firm profitability, illustrating how commitments can be undesirable.

Our work demonstrates that the inefficiencies that result from a lack of coordination and of strategic customer behavior are related. When double marginalization inefficiency is not addressed, commitments, which aim to address strategic customer behavior, generally backfire and intensify double marginalization inefficiency. However, when double marginalization inefficiency is eliminated
through a more sophisticated contract (e.g., a two-part tariff), commitments can reduce the loss due to strategic customer behavior and improve supply chain profits.

We explicitly model the interaction between the retailer and its supplier in addition to the relation between consumers and the retailer. Our model is closest to that of Besanko and Winston (1990). They consider a single seller; we extend their model to a supply chain. Specifically, we consider a supply chain that consists of a supplier and a retailer selling a product to consumers over two periods. In each period, first, the supplier sets its wholesale price; the retailer then decides its procurement quantity and sets its retail price; finally, the consumers make their purchasing decisions. While the supplier and the retailer set wholesale and retail prices dynamically to maximize their individual profits given the remaining consumers in the market, each consumer decides whether and when to buy the product to maximize her own utility based on her expectation of future prices. Customers are heterogeneous in their valuations of the product and they are willing to pay more to purchase a product in the first period, rather than wait to buy it in the second period. In particular, the product’s value decreases over time. This may result, for example, from declining popularity (common for technology and fashion products) or seasonality (common for apparel). In addition to our base (no-commitment) model, we study alternative models in which the retailer or the supplier can make price or quantity commitments. We also consider the vertically integrated centralized system as a benchmark. Comparing these models allows us to tease out the value of commitments to both the supplier and the retailer; we also characterize the impact of these commitments on the other supply chain partner, as well as on the whole supply chain.

As expected, the centralized system benefits from its ability to make price and quantity commitments. Using these commitments, the centralized system limits the sale of its product to a single period, which in turn eliminates the opportunity for strategic customer behavior. In contrast, we show that, in a decentralized supply chain, the impact of commitments is more intricate because of how they affect the interactions between the retailer and the supplier. In particular, ignoring the relations within a supply chain can lead to overestimating the benefits of commitments.

We find that the retailer becomes worse off from its ability to make a quantity commitment in a decentralized supply chain. This happens because of how the quantity commitment affect the supplier’s pricing. For example, had the supplier’s wholesale price stayed the same, the retailer would have benefited from making a quantity commitment at the expense of the supplier by limiting
sales to only a single period, as in the centralized system. However, the supplier responds to this threat by increasing its period 1 wholesale price to force the retailer to sell the product in both periods, which in turn harms the retailer’s profitability.

In our model the power lies with the supplier, as it moves first and sets a take-it-or-leave-it wholesale price. Therefore, one would expect the supplier to benefit from its commitments. Indeed, the supplier benefits from its ability to make a quantity commitment. Intuitively, the supplier can always nullify its commitment by choosing an arbitrarily large commitment quantity, ensuring that it never becomes worse off. We also consider period specific quantity commitment models. We show that the supplier can do better by committing only to period 2 quantity rather than to the total quantity. Intuitively, limiting period 2 sales is more aligned with the supplier’s objective of pushing sales to period 1, which enables setting a higher wholesale price. Furthermore, when both the supplier and the retailer can make simultaneous but independent commitments, their profitability continues to suffer, showing that the problem is due not to unilateral nature of commitments, but rather to a lack coordination.

Similar, to the quantity commitment case retailer becomes worse off when it can make a price commitment. This time the supplier sets a higher period 1 wholesale price, which hurts the retailer. In contrast to quantity commitment, we find that the supplier, can become worse off by making a price commitment because of how this commitment affects the retailer’s ordering policy. When the supplier does not commit to a period 2 wholesale price, the retailer has an incentive to buy more in period 1, as it results in a lower period 2 wholesale price. However, price commitment eliminates this incentive.

Interestingly, the retailer’s commitments hurt not only itself but also the supplier, thus leaving the whole supply chain worse off. We find that the supplier’s price commitment hurts the retailer and the whole supply chain as well.

In these examples, commitments hurt profitability as they amplify existing double marginalization inefficiency. However, we show that eliminating this inefficiency through a two-part tariff or a quantity discount contract can make the commitments beneficial.

Some previous studies have also observed that making commitments can hurt profitability (e.g., Aviv and Pazgal 2008, Cachon and Swinney 2009). These papers, however, do not consider the relation between the retailer and its supplier; instead, demand uncertainty is the driver of
their results as making commitments impairs a firm’s ability to deal with demand uncertainty. In contrast, demand is deterministic in our model, and the inefficiency caused by the vertical relation between the retailer and the supplier drives our results.

After an extensive investigation about commitments we shift our focus to the implications of uncertainty. We extend our base model by introducing demand variability. Specifically, we consider two demand scenarios, either high or low and we capture the uncertainty by varying the total market size. This allows us to explicitly formulate the equilibrium. We analyze the implications of uncertainty on the firm profits by considering different values of model parameters. In particular we show how uncertainty affects the fraction of customers that delay their purchase strategically. In contrast to current work on strategic customers that considers a stochastic demand model we find that demand volatility increases the fraction of such customers.

Dealing with uncertainty in a decentralized supply chain is already a hard problem, strategic consumers add more complexity to this. By explicitly modeling the interactions between supply chain parties, the level of demand variability and the level of customer patience we provide insights such that when the level of customer patience and the volatility in demand has more impact on the firms’ profits. In addition to this decentralized supply chain with demand uncertainty we analyze the vertically integrated system as a benchmark. We find that while decentralized supply chain has a smaller loss than the centralized system its loss increases up until a threshold when demand volatility increases, however centralized firm’s loss decreases as the demand becomes more variable.

The rest of the dissertation is organized as follows: In the next Section we review the relevant literature, in Chapter 2 we introduce our main model and provide the baseline equilibrium. In Chapter 3 we investigate the value of various quantity commitments made by retailer and supplier unilaterally or simultaneously. Similarly, in Chapter 4 we analyze the value of price commitments. Next in Chapter 5, we provide comparison for those commitments and further extend with concurrent commitment models. Chapter 6 is devoted for evaluating the robustness of our results in terms of the changes in underlying modelling assumptions. We provide alternatives to two key modelling choices and show that our main results about the value of commitments continue to hold. To complete our investigation about the commitments in Chapter 7, we introduce a coordinating contract into our decentralized supply chain design and show that under such an arrangement commitments
are beneficial. Next in Chapter 8, we extend our main model to investigate the implications of demand uncertainty in a decentralized supply chain when selling to strategic customers. We conclude and comment on the future work in Chapter 9.
1.2 Literature Review

A growing body of research dating back to Coase (1972) examines the effects of the forward-looking behavior of consumers. Coase (1972) points out that a durable goods monopolist loses monopoly power and ends up pricing at marginal cost when faced with strategic customers. Coase (1972) notes that the monopolist can avoid this problem if it can make some contractual agreements that include price and quantity commitments. Bulow (1982) notes that the ability to make binding quantity commitments alleviates the loss to the monopolist caused by strategic customer behavior. Likewise, Stokey (1979) finds that a firm benefits from its ability to make quantity commitments. In this case, the firm gives up the opportunity to exercise inter-temporal price discrimination and limits the sale of its product to a single period to eliminate the opportunity for strategic customers to await for mark-downs.

Butz (1990) shows that making price commitments in the form of best-price provisions increases a firm’s profitability. With a best-price provision, the firm promises to refund the price difference between the original price and the sale price, if ever the firm decreases the price of its product. Similarly, Besanko and Winston (1990) and Aviv and Pazgal (2008) show that a firm facing strategic consumers can benefit from a commitment to a price path, as opposed to setting prices dynamically. Liu and Zhang (2013) find that committing to a price path has even greater benefits in a competitive environment.

In contrast to the findings of the above papers, we show that firms may not necessarily benefit from their ability to make quantity and price commitments; we find that, on the contrary, such commitments can hurt firms’ profitability. The key difference between these papers and ours is that, while these papers consider only the interactions between consumers and a single seller, ours, in addition, considers the vertical relationship between supplier and retailer.

We should note that Aviv and Pazgal (2008), Cachon and Swinney (2009) and Dasu and Tong (2010) also observed that commitments can hurt a firm’s profitability. However, the drivers of their results are entirely different. Inability to respond to demand variability (or supply-demand imbalance) is the key driver in these papers; they do not consider the relation between retailer and supplier. In contrast, we consider deterministic demand and the vertical relation between retailer and supplier drives our result.
Strategic consumer behavior has recently attracted increasing interest in operations management literature. The various mechanisms studied in the context of strategic customers include capacity rationing (Liu and van Ryzin 2008, Zhang and Cooper 2008, Lai et al. 2010); quick response (Cachon and Swinney 2009, Swinney 2011); inventory display format (Yin et al. 2009); product improvement (Kim and Swinney 2009); resale (Su 2009); product variety (Parlakturk 2009); availability guarantees (Su and Zhang 2009); reservations (Alexandrov and Lariviere 2012); and price and quantity commitments (Elmaghraby et al. 2008, Aviv and Pazgal 2008, Lai et al. 2009). Shen and Su (2007) survey and Netessine and Tang (2009) book provide many other examples. While the above papers study the optimal policy of a single seller facing strategic consumers, our work differs in that we consider a decentralized supply chain consisting of a supplier and a retailer, and we explicitly model the interactions between them. This approach allows us to address our key research question—the value of commitments to both the retailer and the supplier.

Su and Zhang (2008), Arya and Mittendorf (2006) and Desai et al. (2004) find that a decentralized supply chain can do better than a centralized supply chain due to strategic consumers. Su and Zhang (2008) show that in a decentralized supply chain some contracts can mimic the price and quantity commitments of a centralized supply chain. Our work complements this result in that we find actual price and quantity commitments in a decentralized supply chain may not be beneficial with a wholesale price contract. Our model differs from Su and Zhang (2008) in important ways. In Su and Zhang (2008), wholesale price and marked-down retail price are exogenously set. Supply-demand mismatch due to demand uncertainty—which, in turn, creates availability risk at marked-down price—generates the strategic customer behavior. When demand uncertainty is removed, the strategic behavior vanishes because the retailer never has available units at marked-down price. In contrast, demand is deterministic, and endogenously chosen wholesale and retail prices are critical in our model, because how commitments affect these prices drive our finding. These modeling differences can lead to opposite results: While we show that price and quantity commitments may harm profitability, such commitments are always beneficial in Su and Zhang (2008). Furthermore, customers have the same valuation of the product in Su and Zhang (2008); therefore, they all either buy at full price or wait for mark-down. In contrast, heterogeneity in customers’ valuations in our model may cause them to choose different actions (buy now or wait).
Both Arya and Mittendorf (2006) and Desai et al. (2004) consider durable goods with second-hand markets. They show that decentralization through the addition of a retailer in a distribution channel can increase a manufacturer’s profit if the product is sufficiently durable, i.e., lasting more than two periods (Arya and Mittendorf 2006) or if the manufacturer can commit to future wholesale prices (Desai et al. 2004). In other words, they show that a decentralized system can outperform a centralized system. These papers, however, do not address our key research question about the value of commitments in a decentralized supply chain. Furthermore, in Desai et al. (2004), a more durable product that retains more of its value in the next period makes purchasing it sooner relatively more attractive. In our paper, when the product retains more of its value (i.e., customers are more patient), waiting for period 2 becomes relatively more attractive.

Finally, Jerath et al. (2009) is also relevant for our paper. They show that, when facing strategic customers, selling through opaque intermediaries that hide the exact product specifications (e.g., departure times for airlines) can enhance profitability.
CHAPTER 2

The Model

In the following, we first introduce the supply side of our model; we then discuss how customers make their buying decisions resulting in strategic customer behavior. We finally describe the structure of the equilibrium. We consider a supply chain consisting of a supplier and a retailer selling a product over two periods. The supplier and the retailer set their prices dynamically to maximize their individual profits over two periods. Specifically, before the start of each period $i$, the supplier sets its wholesale price $c_i$, and the retailer decides its procurement quantity $Q_i$, which is delivered at the beginning of the period. The retailer then sets the retail price $p_i$, and consumers decide whether to buy the product in that period. The sequence of events is summarized in Figure 2.1. We assume that the supplier has unlimited capacity and its production costs are normalized to zero. Furthermore, the retailer does not incur any cost for holding inventory; however, the salvage value of any unsold unit at the end of period 2 is zero.

All consumers are present at time zero and they leave the market when they make a purchase. Each consumer buys at most one unit of inventory. Dynamic retail prices provide an opportunity for strategic consumers. They decide whether to buy the product and when to buy it, choosing the option that maximizes their utility. Thus, in period 1 each consumer decides whether to buy a product in that period or wait for period 2. The consumers do not observe the retailer’s procurement quantities $Q_i$, thus they rely on their beliefs $\hat{Q}_i$ when making their decisions. In most settings, consumers indeed do not observe a firm’s order quantities. Note, however, that all of our key insights carry over when consumers can observe order quantities.

The consumer evaluations decrease over time, that is, everything else being equal, each customer prefers buying the product sooner rather than later. Specifically, the value of the product decreases by $1 - \delta$ in period 2. Thus $1 - \delta$ indicates perishability of the product value for consumers and is a measure of customer impatience. Hence, its complement $\delta$ shows the degree of customer patience. The assumption that valuations decline over time seems to be common in the sales of fashion,
technology, and seasonal products (Aviv and Pazgal 2008, Cachon and Swinney 2009, Lai et al. 2009, Desiraju and Shugan 1999). Throughout the paper, we assume that $0 < \delta \leq 4/5$, that is, the value of a product decreases by at least 20% in period 2. This approach preserves the concavity of profit functions and helps us focus on more interesting cases. Our analysis carries over to $4/5 < \delta \leq 1$ as well; however, this extension requires considering many additional equilibrium regions that would needlessly complicate our exposition without contributing to our insights.

The customers differ in their willingness to pay. Specifically, when type $\theta$ customer buys the product at price $p_i$ in period $i$, her utility is equal to

$$U_i(\theta, p_i) = \delta^{i-1} \theta - p_i, \; i : 1, 2. \quad (2.1)$$

Thus, the value of the product for type $\theta$ customer is $\theta$ and $\delta \theta$ in periods 1 and 2 respectively. Not buying a product yields zero utility. An alternative discounting assumption is discussed in Section 6.1, where the discount factor $\delta$ applies to both the product value and the payment.

Customer types $\theta$ are uniformly distributed on the unit interval $[0, 1]$ and have a total mass of 1. The distribution of customer types is common knowledge. However, the retailer does not know the type of any particular customer, hence perfect price discrimination is not feasible. Note that as is common in models of strategic customer behavior (e.g. Besanko and Winston 1990, Su 2007, Liu and van Ryzin 2008, Elmaghraby et al. 2008, Liu and Zhang 2013), the total market size is deterministic in our model, however its allocation to no purchase option and demand in each period depends on the retailer’s pricing policy.

We analyze the above game between the supplier, the retailer, and consumers by looking for a subgame perfect Nash equilibrium (SPNE) (Selten 1975). We restrict our analysis to only pure

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Supplier sets $c_1$</th>
<th>Retailer orders $Q_1$ sets $p_1$</th>
<th>Consumers decide whether to buy or wait</th>
<th>Supplier sets $c_2$</th>
<th>Retailer orders $Q_2$ sets $p_2$</th>
<th>Remaining consumers decide whether to buy or wait</th>
</tr>
</thead>
</table>

**Figure 2.1:** Sequence of events.
strategies. To explicitly define the equilibrium, we introduce additional notation. Let \( \bar{\theta} \) show the type of marginal consumer who is indifferent between buying in periods 1 and 2. We make use of the fact that if type-\( \bar{\theta} \) consumer finds it attractive to buy in period 1 then all consumers with higher valuations also find it attractive to buy in period 1.

Even though the demand is deterministic in our model, the retailer may choose to carry inventory because of its impact on the supplier’s period 2 wholesale price \( c_2 \). This is called strategic inventory in Anand et al. (2008). An alternative model in which the retailer cannot carry over inventory is considered in Section 6.2.

An SPNE in our model is defined by the solutions of the following equations. Note that \( ^* \) denotes SPNE strategies.

\[
(p_2^*, Q_2^*) = \arg \max_{(p_2, Q_2) \in S_2} [p_2(\bar{\theta} - p_2/\delta) - c_2 Q_2], \quad (2.2)
\]
\[
c_2^* = \arg \max_{c_2} c_2 Q_2^*, \quad (2.3)
\]
\[
\bar{\theta}^* = \inf \{ \theta : \theta - p_1 \geq \delta \theta - p_2^*(\hat{Q}_1) \}, \quad (2.4)
\]
\[
(p_1^*, Q_1^*) = \arg \max_{(p_1, Q_1) \in S_1} [p_1(1 - \bar{\theta}^*) - c_1 Q_1 + p_2^*(\bar{\theta}^* - p_2^*/\delta) - c_2^* Q_2^*], \quad (2.5)
\]
\[
\hat{Q}_1 = Q_1^*, \quad (2.6)
\]
\[
c_1^* = \arg \max_{c_1} [c_1 Q_1^* + c_2^* Q_2^*]. \quad (2.7)
\]

where the retailer’s feasible strategy sets \( S_i \) in each period are given by

\[
S_1 = \{ (p_1, Q_1) : Q_1 \geq 1 - \bar{\theta}^*(p_1) \geq 0 \}, \quad (2.8)
\]
\[
S_2 = \{ (p_2, Q_2) : Q_1 + Q_2 \geq 1 - p_2/\delta \}. \quad (2.9)
\]

Basically, (2.2) states that the retailer chooses its order quantity \( Q_2 \) and price \( p_2 \) in period 2 to maximize its profit in that period, given the remaining consumer segment \( [0, \bar{\theta}] \), the wholesale price \( c_2 \), and its carry-over inventory. The constraint \( (p_2, Q_2) \in S_2 \) ensures that the retailer does not sell more than it has on-hand. Because the demand is deterministic, the retailer always sells all of its remaining inventory in period 2 in equilibrium. The supplier chooses its wholesale price \( c_2 \) to maximize its profit in period 2 in (2.3). The optimal \( c_2^* \) takes into account its impact on the retailer’s optimal order quantity \( Q_2^* \). Each consumer chooses her best option in period 1, given her
conjecture of period 2 price, $p_2^*(\hat{Q}_1)$, which depends on her belief $\hat{Q}_1$ about how many units the retailer procured in period 1. Because the difference between period 1 and 2 utilities is monotone in $\theta$, a threshold-type purchasing policy emerges in equilibrium. In particular, (2.4) states that it is optimal to wait for period 2 for all consumers with lower valuations than the marginal customer $\bar{\theta}$. As a result of monotonicity, (2.4) also implies that it is optimal to buy in period 1 for all consumers with higher valuations than the marginal customer $\bar{\theta}$. Therefore, the marginal customer $\bar{\theta}$ shows indifference between buying in period 1 and waiting for period 2.

Furthermore, (2.5) shows that the retailer chooses its period 1 order quantity $Q_1$ and price $p_1$ to maximize its total profit over two periods given the wholesale price $c_1$ in period 1. The condition $(p_1, Q_1) \in S_1$ states that the retailer’s sales quantity in period 1 cannot exceed $Q_1$ units, its on-hand inventory.

The equilibrium requires the consumers to share the same belief on retailer period 1 order quantity and this belief to be consistent with the actual outcome as stated in (2.6). Note that consumers’ beliefs on period 2 order quantity is not relevant since period 2 is the last period. Finally, (2.7) states that the supplier chooses its period 1 wholesale price, $c_1$, to maximize its total profit, taking into account its impact on the retailer’s period 1 and 2 actions.

2.1 Equilibrium

We solve for the equilibrium using backward induction. Essentially, we follow the order in equations (2.2-2.7). First, we find the retailer’s optimal price in period 2, then we derive the supplier’s optimal wholesale price in that period. Next, we characterize the consumers’ optimal choice in period 1 (buy in period 1 vs. wait for period 2). Then, we solve for the retailer’s optimal price in period 1. Finally, we determine the supplier’s optimal wholesale price in period 1, which fully characterizes the equilibrium. Note that all proofs are in Appendix B of the online supplement.

In period 2, the supplier sets its wholesale price $c_2$ to extract maximum profit from the retailer while the retailer sets its price $p_2$ to extract maximum profit from the remaining consumers $[0, \bar{\theta})$. Because period 2 is the last period, the consumers do not have the strategic option of delaying their purchases further. They decide whether or not to buy the product. Specifically, there exist $\theta_2 \leq \bar{\theta}$,
such that consumers in $[\theta_2, \bar{\theta})$ buy the product and the remaining consumers do not buy it. The following Lemma describes the retailer’s and supplier’s optimal policy in period 2.

**Lemma 1.** Suppose the retailer orders $Q_1$ units in period 1, and consumer segment $[0, \bar{\theta})$ remains in the market in period 2.

(i) For any given wholesale price $c_2$, the retailer orders $Q_2^* = \max(\frac{\delta(2-2Q_1-\bar{\theta})}{2\delta} - c_2, 0)$ units and sets $p_2^* = \delta(1 - Q_1 - Q_2^*)$ in period 2.

(ii) The supplier sets $c_2^* = \frac{\delta(2-2Q_1-\bar{\theta})}{2}$ in period 2.

This yields profits $\Pi_{R,2} = \frac{\delta(2-2Q_1-\bar{\theta})}{16} + \delta(1 - Q_1)(Q_1 - 1 + \bar{\theta})$ and $\Pi_{S,2} = \frac{\delta(2-2Q_1-\bar{\theta})}{8}$ for the retailer and supplier respectively.

Next, we consider the consumers’ choices in period 1. The consumers conjecture period 2 price, given their beliefs about the retailer’s inventory level $\hat{Q}_1$. Specifically, following Lemma 1, they expect $p_2^*(\hat{Q}_1) = \frac{\delta(2-2\hat{Q}_1+\bar{\theta})}{4}$. The marginal consumer $\bar{\theta}$, who is indifferent about whether to buy in period 1 or wait for period 2, is given by the solution of $\bar{\theta} - p_1 = \delta \bar{\theta} - p_2^*(\hat{Q}_1)$, where the left and right hand sides correspond to the utility of buying in periods 1 and 2, respectively. This leads to

$$\bar{\theta} = \frac{4p_1 - 2\delta(1 - \hat{Q}_1)}{4 - 3\delta}.$$  \hspace{1cm} (2.10)

Thus, consumers in $[\bar{\theta}, 1]$ buy the product in period 1, and the remaining consumers wait for period 2.

Now, consider the retailer’s period 1 problem. To determine the retailer’s optimal price, we can equivalently solve for the optimal consumer segment that it should entice in period 1. Following (2.10), in order to induce consumers in $[\bar{\theta}, 1]$ to buy the product in period 1, the retailer needs to set the following price:

$$p_1(\bar{\theta}, \hat{Q}_1) = \bar{\theta} - \frac{\delta(2(\hat{Q}_1 - (1 - \bar{\theta})) + \bar{\theta})}{4}.$$  \hspace{1cm} (2.11)

Setting price $p_1$ equal to $\bar{\theta}$ would be sufficient to attract consumer segment $[\bar{\theta}, 1]$ if the consumers were myopic, i.e., if they were not considering a future option in their decision making. The terms in parentheses are non-negative in (2.11) since $\hat{Q}_1 - (1 - \bar{\theta})$ corresponds to the consumers’ belief about the retailer’s inventory level at the end of period 1. Therefore, the retailer suffers a margin
loss from price \( \bar{\theta} \) that results from strategic customer behavior. Observe that this loss increases in \( \hat{Q}_1 \), that is, when customers believe that the retailer has a higher inventory level, the retailer needs to set a lower price to convince the customer to buy in period 1 rather than to wait for sales in period 2. Because the actual order quantity \( Q_1 \) is unobservable, it does not have a direct impact on this loss. By definition, the retailer can conjecture the equilibrium choices. Therefore, it predicts \( \hat{Q}_1 \), which is equal to the equilibrium order quantity in period 1.

We can now reformulate the retailer’s total profit in terms of its order quantity \( Q_1 \) and target consumer segment \([\bar{\theta}, 1]\) in period 1:

\[
\Pi_R(Q_1, \bar{\theta}, \hat{Q}_1) = [1 - \bar{\theta}]p_1(\bar{\theta}, \hat{Q}_1) - Q_1c_1 + \Pi_{R,2}(Q_1, \bar{\theta}).
\]

(2.12)

Note that \( \Pi_{R,2}(Q_1, \bar{\theta}) \) is given in Lemma 1. The retailer’s optimal policy in equilibrium then satisfies the following:

\[
(Q_1^*, \bar{\theta}^*) = \arg \max_{(Q_1, \bar{\theta}) \in S'_1} \Pi_R(Q_1, \bar{\theta}, \hat{Q}_1)
\]

(2.13)

\[
\hat{Q}_1 = Q_1^*.
\]

(2.14)

where

\[
S'_1 = \{(Q_1, \bar{\theta}) : Q_1 \geq 1 - \bar{\theta} \geq 0\}.
\]

(2.15)

Here, the restriction to set \( S'_1 \) ensures that the retailer does not sell a negative quantity, or more than its on-hand inventory in period 1. The solution of (2.13-2.14) gives the retailer’s optimal order quantity and price in period 1, which are stated in the following Proposition.

**Proposition 1.** The SPNE order quantity \( Q_1^* \) and retail price \( p_1^* \) and the corresponding marginal customer \( \bar{\theta}^* \) in period 1 are as follows.

(i) When \( c_1 < \frac{3\delta(4 - 3\delta)}{2(16 - 15\delta)} \), the retailer sells the product in both periods, and it carries inventory between periods, where \( Q_1^* = \frac{11}{14} - \frac{c_1(112 - 9\delta) + 6\delta}{21\delta(8 - 7\delta)} \), \( p_1^* = \frac{1}{2} - \frac{4(5 - 6\delta)c_1 - 9\delta(1 - \delta)}{6(8 - 7\delta)} \), and \( \bar{\theta}^* = \frac{3(4 - 3\delta) + 2c_1}{3(8 - 7\delta)} \).

(ii) When \( \frac{3\delta(4 - 3\delta)}{2(16 - 15\delta)} \leq c_1 < \frac{8 - 3\delta}{8} \), the retailer sells the product in both periods and it does not carry inventory between periods, where \( Q_1^* = \frac{8(1 - c_1) - 3\delta}{16 - 9\delta} \), \( p_1^* = \frac{(4 - \delta)(4 + 4c_1 - 3\delta)}{2(16 - 9\delta)} \), and \( \bar{\theta}^* = \frac{8(1 + c_1) - 6\delta}{16 - 9\delta} \).
(iii) When \( c_1 \geq \frac{8-3\delta}{8} \), the retailer sells the product only in period 2, and it does not carry inventory between periods, where \( Q_1^* = 0 \), and \( \bar{\theta}^* = 1 \).

In part (i), \( Q_1^* > 1 - \bar{\theta}^* \), that is, the retailer sells less than it orders in period 1, thus it carries inventory into the next period. In contrast, in parts (ii) and (iii), \( Q_1^* = 1 - \bar{\theta}^* \), so the upper bound in (2.15) binds, and the retailer sells exactly what it orders in period 1, and, therefore, does not have any remaining inventory at the end of period 1. Furthermore, in part (iii), \( \bar{\theta}^* = 1 \), thus, the lower bound in (2.15) also binds, and the retailer does not sell any units in period 1. The regions characterized in parts (i)-(iii) are depicted in Figure 2.2.

Finally, let us consider the supplier’s pricing problem in period 1. The supplier chooses \( c_1 \) to maximize its total profit:

\[
\Pi_S = c_1 Q_1^* + \Pi_{S,2}(c_1, Q_1^*, \bar{\theta}^*),
\]  

(2.16)

where \( Q_1^* \) and \( \bar{\theta}^* \) are given by Proposition 1 and \( \Pi_{S,2} \) follows from Lemma 1. The following Proposition describes the supplier’s optimal policy.

**Proposition 2.** The SPNE wholesale price \( c_1 \) in period 1 is as follows.

(i) When \( \delta \leq \frac{16}{21} \), \( c_1^* = \frac{16-3\delta}{32} \).

(ii) When \( \frac{16}{21} < \delta \), \( c_1^* = \frac{3(4-3\delta)}{2(16-15\delta)} \).

Propositions 1 and 2 state the firms’ optimal policy in period 1. Recall that Lemma 1 describes their optimal policy in period 2 and (2.10) identifies the marginal customer. Therefore,
the equilibrium of the game between the supplier, retailer and consumers is fully characterized by Propositions 1 and 2, Lemma 1 and (2.10). Note, because our analysis follows backward induction, period 2 equilibrium decisions are stated in terms of period 1 outcomes in all scenarios. They can be stated explicitly by plugging in equilibrium values of these period 1 actions. These expressions are provided in Appendix A.

Propositions 1 and 2 show that the supplier’s pricing in period 1 makes carrying inventory into the next period unattractive for the retailer. In other words, the supplier sets its period 1 wholesale price $c_1$ sufficiently high so that the retailer can not use inventories to get a better period 2 wholesale price $c_2$. In part (ii) of Proposition 2, the solution is at the boundary; any lower wholesale price $c_1$ will result in the retailer carrying inventory into period 2.

It is well known that strategic customer behavior leads to profit loss (cf. Besanko and Winston 1990, Cachon and Swinney 2009, Su and Zhang 2008). Indeed, in our setup, strategic customer behavior hurts the profits of both the supplier and retailer; that is, their profits would be higher if the customers were myopic, i.e., not considering their future pay-offs. We formally state this result in the following where we provide the myopic consumers benchmark.

### 2.2 Myopic Consumers Benchmark

When customers are myopic they buy the product whenever their value exceeds its retail price. Specifically, they do not consider second period price when they make their purchasing decision in period 1. Note that since myopic consumers do not consider waiting for the second period they do not need to make beliefs about the second period quantity. The SPNE is then defined by the following equations.

\[
(p_2^*, Q_2^*) = \arg \max_{(p_2, Q_2) \in S_2} \left[ p_2(p_1 - p_2/\delta) - c_2Q_2 \right],
\]

\[
c_2^* = \arg \max_{c_2} c_2Q_2^*
\]

\[
(p_1^*, Q_1^*) = \arg \max_{(p_1, Q_1) \in S_1^1} \left[ p_1(1 - p_1) - c_1Q_1 + p_2^*(p_1 - p_2^*/\delta) - c_2^*Q_2^* \right],
\]

\[
c_1^* = \arg \max_{c_1} \left[ c_1Q_1^* + c_2^*Q_2^* \right].
\]
where the retailer’s feasible strategy sets $S_i$ in each period are given by

$$
S^M_1 = \{(p_1, Q_1) : Q_1 \geq 1 - p_1^* \geq 0\}, \tag{2.19}
$$

$$
S_2 = \{(p_2, Q_2) : Q_1 + Q_2 \geq 1 - p_2/\delta\}.
$$

The following lemma then describes the supplier’s and retailer’s second period optimal policy.

**Lemma 2.** Suppose the retailer orders $Q_1$ units and charges $p_1$ in period 1.

(i) For any given wholesale price $c_2$, the retailer orders $Q_2^* = \max(\delta(2(1-Q_1)-p_1-c_2)/2\delta, 0)$ units and sets $p_2^* = \delta(1-Q_1-Q_2^*)$ in period 2.

(ii) The supplier sets $c_2^* = \delta/2(2(1-Q_1)-p_1)$ in period 2.

This yields profits $\Pi_{R,2} = \frac{\delta}{16}(p_1^2 + 12p_1(1-Q_1) - 12(1-Q_1)^2)$ and $\Pi_{S,2} = \frac{\delta}{8}(2(1-Q_1)-p_1)^2$ for the retailer and supplier respectively.

Since customers are myopic the consumer segment in $[1-p_1]$ stays in the market and we can solve for retailer’s optimization problem by plugging the values in Lemma 2 into equation (2.18).

The following proposition describes retailer’s optimal policy in period 1.

**Proposition 3.** The SPNE order quantity $Q_1^*$ and retail price $p_1^*$ in period 1 are as follows.

(i) When $c_1 < \frac{6\delta}{16-7\delta}$ the retailer sells the product in both periods, and in carries inventory between periods, where $Q_1^* = 1 - \frac{2+c_1}{2(1-\delta)}$ and $p_1^* = \frac{2+c_1}{4-\delta}$.

(ii) When $\frac{6\delta}{16-7\delta} \leq c_1 \leq 1 - \frac{\delta}{8}$ the retailer sells the product in both periods and it does not carry inventory between periods where $Q_1^* = 1 - \frac{8(1+c_1)}{16-\delta}$, $p_1^* = \frac{8(1+c_1)}{16-\delta}$.

When $c_1 > 1 - \frac{\delta}{8}$ retailer does not sell the product in period 1.

To fully characterize the equilibrium we solve the supplier’s optimization problem in period 1, which is stated by the following proposition.

**Proposition 4.** The SPNE wholesale price $c_1$ is $c_1^* = \frac{4}{8-\delta} - \frac{\delta}{32}$.

Wholesale price in Proposition 4 corresponds to part (ii) of Proposition 3. Hence in equilibrium the retailer serves the market in both periods and does not carry inventory.
It is a well established result that strategic customers cause firms to lose profit. The following corollary formally states this result. Let $NC^M$ denote the no commitment scenario when customers are myopic.

**Corollary 1.** (i) $\Pi_{S}^{NC^M} > \Pi_{S}^{NC}$.

(ii) $\Pi_{S}^{NC^M} + \Pi_{R}^{NC^M} > \Pi_{S}^{NC} + \Pi_{R}^{NC}$.

(iii) $\Pi_{R}^{NC^M} > \Pi_{R}^{NC}$.

Recall that we assume $\delta > 0$. However, at the extreme, when $\delta = 0$, strategic and myopic models become identical as strategic customers do not benefit from waiting at all since the product’s value in period 2 becomes 0. As $\delta$ increases the loss due to strategic customers increases.

### 2.3 Centralized System Benchmark

In this section, we consider a centralized supply chain as a benchmark. Similar to our main model, the centralized system sets the retail price $p_i$ in each period $i : 1, 2$ and consumers decide whether to buy the product in that period. The resulting equilibrium is given by the following Lemma.

**Lemma 3.** Consider a centralized supply chain. The SPNE is as follows. The centralized firm sells the product in both periods, setting $p_1^* = \frac{(2-\delta)^2}{2(4-3\delta)}$ in period 1 and $p_2^* = \frac{(2-\delta)\delta}{2(4-3\delta)}$ in period 2, and the marginal customer is given by $\bar{\theta}^* = \frac{2-\delta}{4-3\delta}$.

For completeness let’s provide the centralized system with myopic consumers.

**Lemma 4.** Consider a centralized supply chain with myopic customers. The SPNE is as follows. The centralized firm sells the product in both periods, setting $p_1^* = \frac{2}{4-\delta}$ in period 1 and $p_2^* = \frac{\delta}{4-\delta}$ in period 2.

The following corollary formally states the result that the strategic customers hurt the profit of the centralized supply chain as they do in the decentralized version.

**Corollary 2.** $\Pi_{C}^{NC^M} > \Pi_{C}^{NC}$
In this chapter we discuss quantity commitment policies implemented by retailer and supplier unilaterally or simultaneously by both of the supply chain parties and the impact these commitments on firm profits. Quantity commitment (keeping quantities low) is one of the policies that are recommended in supply chain literature to induce customers to buy early i.e. scare away from strategic waiting. This can be in the form of limited editions of various consumer goods or occasional stock-outs to create scarcity. For example whenever Apple introduces a newer version of iPhone or iPad it always is out of stock and customers have to pre-order. It is also common that popular toys or electronics to go out of stock during the holiday season (Wingfield and Guth 2005).

Throughout this chapter we assume that the supply chain member who is making the quantity commitment has in its disposition some sort of a commitment device, which convinces customers that no more than the committed quantity will be available in the relevant periods. This could be through a long term relationship of the seller and buyers or through the reputation of the seller. However, the specific form and the details of the commitment device is beyond the scope of this study. Our model captures the relationship between the supplier and retailer explicitly. Although quantity commitments are assumed to be beneficial when customers are strategic we provide one explanation to why they are not widely adopted, by demonstrating the inefficiencies that result from lack of coordination and the strategic customer behavior in a decentralized supply chain.

In the following sections we first introduce the vertically integrated firm’s quantity commitment benchmark, then we provide retailer’s and supplier’s maximum and period specific quantity commitment scenarios. We also discuss the simultaneous quantity commitment model. We address our main research question whether the quantity commitments are valuable when selling to strategic customers in a decentralized supply chain.
3.1 Centralized System Quantity Commitment Benchmark

Suppose that the vertically integrated firm that we introduced in Section 2.3 can make quantity commitments. When the firm commits to sell a limited quantity, it does not sell more than the committed quantity even when doing so would increase its profit.

Lemma 5. Suppose a centralized firm can make quantity commitment to sell no more than \( Q_{\text{max}} \) units in two periods. In equilibrium, the firm sells its product only in period 1 at price \( p_1^* = \frac{1}{2} \) and sets \( Q_{\text{max}}^* = \frac{1}{2} \).

When the firm can make a quantity commitment, it limits its sale to a single period to eliminate the strategic behavior. Let \( \Pi^C_{j : NC, QC} \) show the centralized system’s profit when it cannot make any commitments (NC) and when it can make quantity (QC) commitments. The following Theorem compares the centralized firm’s profit in these scenarios.

Theorem 1. \( \Pi^C_{NC} < \Pi^C_{QC} \).

The Theorem shows that quantity commitments increase the centralized system’s profit. In the rest of this chapter, we study the impact of such commitments for a decentralized system.

3.2 Retailer’s Quantity Commitment

Suppose that the retailer can credibly convince customers that it will not sell more units than its committed quantity. The order of events is similar to our no-commitment model and summarized in Figure 3.1.

When determining its period 1 procurement quantity \( Q_1 \), the retailer commits not to sell more than a total of \( Q_{\text{max}} \) units in two periods. Therefore, the retailer’s period 2 procurement quantity
$Q_2$ needs to satisfy $Q_2 \leq Q_{max} - Q_1$. Otherwise, the order of events is the same as in our no-commitment model shown in Figure 2.1. Note we show period-specific quantity commitments rather than an aggregate commitment by the retailer lead to the same outcome (see Section 3.3). We exploit the following Lemma for deriving the equilibrium.

**Lemma 6.** For any SPNE in which retailer’s quantity commitment does not bind, i.e., $Q_1^* + Q_2^* < Q_{max}$, there is an equivalent SPNE with the same outcome (wholesale and retail prices, quantities and supplier and retailer profits) in which the retailer’s quantity commitment binds, i.e., $Q_1^* + Q_2^* = Q_{max}$.

Therefore, we can assume that the retailer’s quantity commitment binds in equilibrium without loss of generality. This is because the retailer can always match the profit of a non-binding commitment with a binding commitment by simply setting $Q_{max}' = Q_1^* + Q_2^*$.

Let us define the critical period 2 wholesale price $\bar{c}_2$ below which the retailer finds it attractive to procure all of its committed quantity. Thus, $\bar{c}_2$ solves $Q_2^*(c_2) = Q_{max} - Q_1$. In order for the retailer’s quantity commitment to bind, the following condition needs to be satisfied

$$\frac{d\Pi_{R,2}}{dQ_2} \bigg|_{Q_2=Q_{max} - Q_1} \geq 0, \text{ and } \frac{d\Pi_{S,2}}{dc_2} \bigg|_{c_2=\bar{c}_2} \leq 0. \quad (3.1)$$

Basically, (3.1) ensures that it is optimal for the retailer to procure all of its committed quantity given the supplier’s period 2 wholesale price, and it is not optimal for the supplier to increase this price. Note that decreasing its wholesale price does not benefit the supplier in this case, as the retailer cannot procure more than its committed quantity. The solution of (3.1) leads to the following Lemma, which characterizes the firms’ equilibrium strategies in period 2.

**Lemma 7.** When the retailer’s quantity commitment binds, $Q_2^* = Q_{max} - Q_1$, $p_2^* = \delta(1 - Q_{max})$ and $c_2^* = \delta(2(1 - Q_{max}) - \bar{\theta})$ in a SPNE in period 2. This yields $\Pi_{R,2} = p_2^*(Q_{max} - (1 - \bar{\theta})) - c_2^*Q_2^*$ and $\Pi_{S,2} = c_2^*Q_2^*$. Furthermore, the retailer quantity commitment binds if and only if $Q_{max} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}$.

The Lemma shows that for the supplier to set a low enough wholesale price to persuade the retailer to procure all of its remaining committed quantities in period 2, the retailer’s period 1
price, quantity and commitment choices should satisfy $Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\theta}{4}$. Otherwise, given the supplier’s optimal wholesale price, the retailer’s quantity commitment does not bind.

When the retailer’s commitment binds, it essentially dictates period 2 retail price $p_2^* = \delta(1 - Q_{\text{max}})$; therefore, customers do not need to form beliefs about the retailer’s order quantities to conjecture its period 2 price $p_2$. The marginal customer $\bar{\theta}$, who is indifferent between buying in period 1 and waiting for period 2 then solves $\bar{\theta} - p_1 = \delta \bar{\theta} - p_2^*$ leading to

$$p_1(\bar{\theta}, Q_{\text{max}}) = \bar{\theta} - \delta(Q_{\text{max}} - (1 - \bar{\theta})).$$ (3.2)

The retailer’s margin loss resulting from strategic customer behavior, which is given by $\delta(Q_{\text{max}} - (1 - \bar{\theta}))$, increases in the number of units the firm will sell in the second period, that is, $Q_{\text{max}} - (1 - \bar{\theta})$.

Following Lemmas 6 and 7 and (3.2), we can now state the retailer’s period 1 problem in terms of its order quantity $Q_1$, commitment $Q_{\text{max}}$ and target consumer segment $[\bar{\theta}, 1]$ in period 1:

$$\max_{Q_1, Q_{\text{max}}, \bar{\theta}} \quad [1 - \bar{\theta}] \quad p_1(\bar{\theta}, Q_{\text{max}}) - Q_1 c_1 + \Pi_{R,2}(Q_1, Q_{\text{max}}, \bar{\theta})$$

s.t.  

$$Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4};$$ (3.4)

$$Q_1 \leq Q_{\text{max}},$$ (3.5)

$$(Q_1, \bar{\theta}) \in S_1'.$$ (3.6)

Note that $S_1'$ is defined in (2.15). Lemmas 6 and 7 leads to condition (3.4) which ensures that the quantity commitment binds in period 2. Condition (3.5) ensures that the quantity commitment is honored in period 1. Similarly, (3.6) ensures that the sales quantity in period 1 is non-negative, and it cannot be more than the retailer’s period 1 order quantity $Q_1$. The solution of the retailer’s problem in (3.3-3.6) leads to the following Proposition.

**Proposition 5.** Suppose the retailer can make a quantity commitment. Its SPNE order quantity $Q_1^*$, quantity commitment $Q_{\text{max}}^*$, retail price $p_1^*$ and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.

(i) When $c_1 < \frac{\sqrt{16 - 5\delta} - 1}{\delta}$, the retailer sells the product only in period 1 and the quantity commitment binds strictly, where $Q_1^* = \frac{1 - c_1}{2}$, $Q_{\text{max}}^* = \frac{1 - c_1}{2}$, $p_1^* = \frac{1 + c_1}{2}$ and $\bar{\theta}^* = \frac{1 + \epsilon_1}{2}$.  

22
(ii) When \( \frac{\sqrt{16-5\delta}-1}{5} \leq c_1 < \frac{8-3\delta}{8} \), the retailer sells the product in both periods, it does not carry inventory between periods and the quantity commitment binds weakly, where \( Q_1^* = \frac{8(1-c_1) - 3\delta}{16-5\delta} \), \( Q_{\text{max}}^* = \frac{2(10-6c_1)-7\delta}{2(16-5\delta)} \), \( p_1^* = \frac{(4(1+c_1)-\delta)(4-\delta)}{2(16-5\delta)} \), and \( \bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta} \).

(iii) When \( c_1 \geq \frac{8-3\delta}{8} \), the retailer sells the product only in period 2, it does not carry inventory between periods and the quantity commitment binds weakly, where \( Q_1^* = 0 \), \( Q_{\text{max}}^* = \frac{1}{4} \), and \( \bar{\theta}^* = 1 \).

We say that the quantity commitment binds strictly (weakly) when the first inequality in (3.1) holds strictly (weakly). In other words, when the commitment binds strictly, ordering more units than its committed quantity would increase the retailer’s profit. In contrast, when the commitment binds weakly, the retailer does not want to exceed its committed quantity. Figure 3.2 shows the regions characterized by parts (i)-(iii). When period 1 wholesale price \( c_1 \) is small enough as in part (i), the quantity commitment enables the retailer to limit sales of the product to only period 1 similar to the centralized benchmark. However, when \( c_1 \) is sufficiently high, the retailer sells the product in period 2 as well and the quantity commitment does not bind strictly.

Finally, the supplier chooses \( c_1 \) to maximize its total profit:

\[
\Pi_S = c_1 Q_1^* + \Pi_{S,2}(c_1, Q_1^*, Q_{\text{max}}^*, \bar{\theta}^*),
\]
where $Q^*_1$, $Q^*_{max}$ and $\bar{\theta}^*$ are given by Proposition 5 and $\Pi_{S,2}$ follows from Lemma 7. The following Proposition describes the supplier’s optimal policy and completes the characterization of the equilibrium.

**Proposition 6.** Suppose the retailer can make a quantity commitment. The supplier’s SPNE wholesale price $c_1$ in period 1 is as follows. There exists $\bar{\delta}_1 \approx 0.249$ such that

(i) when $\delta \leq \bar{\delta}_1$, $c_1^* = \frac{1}{2}$, and

(ii) when $\delta > \bar{\delta}_1$, $c_1 = \frac{\sqrt{16-5\delta}-1}{5}$.

Note that $\bar{\delta}_1$ is explicitly characterized in the proof of the proposition. Parts (i) and (ii) of Proposition 6 map to parts (i) and (ii) of Proposition 5 respectively. Specifically, when $\delta$ is small, the retailer’s quantity commitment binds strictly and the product is sold only in period 1, while when $\delta$ is big, the commitment binds weakly and the product is sold in both periods.

These findings prepare us to address our key research question: How does the commitment affect the retailer, the supplier and the whole supply chain? The following theorem answers this question. Note that superscripts $NC$ and $RQC$ stand for no commitment and retailer’s quantity commitment respectively.

**Theorem 2.** There exists $\bar{\delta}_2 \approx 0.799$ such that

(i) $\Pi^NC_S > \Pi^RQC_S$,

(ii) $\Pi^NC_S + \Pi^NC_R > \Pi^RQC_S + \Pi^RQC_R$,

(iii) $\Pi^NC_R > \Pi^RQC_R$ when $\delta \leq \bar{\delta}_2$.

Here, $\bar{\delta}_2$ is explicitly characterized in the proof. The Theorem shows that, unlike the centralized benchmark, the retailer can become worse off when it makes a quantity commitment. Furthermore, this hurts the supplier as well as the whole supply chain. Similar to the centralized system benchmark, the retailer would have benefited from its quantity commitment if the supplier’s wholesale prices were given exogenously, that is, if the supplier’s wholesale price stayed the same after the retailer’s commitment. However, the retailer would benefit at the expense of the supplier in this case as it will decrease its order quantity.
Comparisons of Propositions 2 and 6 show that the supplier increases its period 1 wholesale price when the retailer can make a quantity commitment, which exacerbates double marginalization hurting both the retailer and supplier. In this case, if the supplier sets a relatively low price, the retailer would restrict its sales to only period 1 using the quantity commitment and not buy any units from the supplier in period 2. Because of this threat, the supplier sets a high price in period 1 to force the retailer to sell the product in both periods. As a result, both the supplier and the retailer lose compared to no-commitment scenario. A high wholesale price in period 1 means that the retailer can serve only the higher end of the market making the remaining customer segment in period 2 attractive, which in turn, prevents the retailer from limiting its sale to only period 1.

Note, when \( \delta > \bar{\delta}_2 \), the retailer can benefit from its quantity commitment. Everything else being equal, a high \( \delta \) encourages customers to wait for period 2. Therefore, commitment can become more attractive in that case as it discourages customers from waiting for a price mark-down.

### 3.3 Retailer’s Period Specific Quantity Commitment

In Section 3.2 retailer commits to \( Q_{\text{max}} \), which limits the aggregate quantity that can be sold over two periods. Here, we allow period-specific quantity commitments. Specifically, the retailer can commit to \( Q_{1\text{max}} \) and \( Q_{2\text{max}} \), which limit the quantities that can be sold in periods 1 and 2.

However, it is straightforward to show that aggregate and period-specific quantity commitments by the retailer lead to the same outcome. Simply, any equilibrium outcome with an aggregate quantity commitment \( Q_{\text{max}} \) can be replicated by period-specific quantity commitments (and vice versa) by setting \( Q_{1\text{max}} = Q_1 \) and \( Q_{2\text{max}} = Q_{\text{max}} - Q_1 \). This happens because both \( Q_1 \) and \( Q_{1\text{max}} \) are chosen by the retailer. Although period specific quantity commitment for retailer is the same for retailer this can lead to different outcomes for supplier, which we will provide in the coming sections.

### 3.4 Supplier’s Quantity Commitment

We now consider the supplier’s quantity commitment. In Section 3.2, we have seen that the retailer’s quantity commitment always hurts the supplier. The next question becomes whether the supplier can benefit from its own quantity commitment in a decentralized supply chain. Note that
Figure 3.3: Sequence of events for supplier’s aggregate quantity commitment.

the supplier moves first and sets a take-it-or-leave-it wholesale price; in other words the power lies with the supplier in our model. One would expect the supplier to benefit, therefore, from its quantity commitment. Indeed, here, we show that the supplier always benefits from a quantity commitment.

Suppose that the supplier can commit to not selling more than a committed quantity. Specifically, before setting its wholesale price in period 1, the supplier announces that it will not sell more than $Q_{\text{max}}$ units in two periods. The order of events is shown in Figure 3.3.

Without loss of generality, we can restrict our analysis to $Q_{\text{max}} \in [1/4, 1/2]$, as is formally stated in the following Lemma.

**Lemma 8.** Suppose the supplier can commit to sell not more than $Q_{\text{max}}$ units in two periods. $Q_{\text{max}} \geq 1/4$ in all equilibria. Furthermore, for any equilibrium with $Q_{\text{max}} > 1/2$, there is an equivalent equilibrium with $Q_{\text{max}} = 1/2$, which yields the same payoffs to all parties (supplier, retailer and customers).

We follow backward induction to characterize the equilibrium starting with period 2. Following Lemma 7, the quantity commitment binds; that is, the retailer buys all of the supplier’s committed quantity in period 2, if and only if

$$Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}. \quad (3.7)$$

Thus, when (3.7) holds, period 2 equilibrium is given by Lemma 7. On the other hand, when (3.7) does not hold, the commitment is ineffective, and period 2 equilibrium is the same as in a no-commitment scenario, which is given by Lemma 1.
Proposition 7. Suppose the supplier commits to sell not more than $Q_{\text{max}} \in [1/4, 1/2]$ units in two periods. The retailer’s SPNE policy is characterized as follows.

(i) The quantity commitment binds strictly, the retailer sells the product in both periods and it carries inventory between periods with $Q'_1 = \frac{1-c_1-3\delta Q_{\text{max}}}{2(1-2\delta)}$, $p'_1 = \frac{1+\delta(1-c_1)+\delta(Q_{\text{max}}(1+\delta)-3)}{2(1-2\delta)}$ and $\bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{\text{max}})}{2(1-2\delta)}$, when $\delta < 1/2$, $\frac{1-c_1}{2-\delta} < Q_{\text{max}} < \frac{5-3c_1-4\delta}{8-7\delta}$.

(ii) The quantity commitment binds strictly, the retailer sells the product only in period 1 with $Q'_1 = Q_{\text{max}}$ and $p'_1 = \bar{\theta}^* = 1 - Q_{\text{max}}$, when

a) $\delta < 1/2$, $Q_{\text{max}} \leq \frac{1-c_1}{2-\delta}$,

b.1) $\delta \geq 1/2$, $c_1 \leq \frac{2(1-\delta)(5\delta-1)}{2-25\delta}$, $Q_{\text{max}} < K_1$,

b.2) $\delta \geq 1/2$, $c_1 > \frac{2(1-\delta)(5\delta-1)}{2-25\delta}$, $Q_{\text{max}} \leq \frac{4-3c_1-2\delta}{7-5\delta}$.

(iii) The quantity commitment binds weakly, the retailer sells the product in both periods and it does not carry inventory between periods with $Q'_1 = \frac{1}{3}(4Q_{\text{max}}-1)$, $p'_1 = \frac{1}{3}(4-\delta)(1-Q_{\text{max}})$, and $\bar{\theta}^* = \frac{4}{3}(1-Q_{\text{max}})$, when

a) $\delta < 1/2$, $\frac{5-3c_1-4\delta}{8-7\delta} \leq Q_{\text{max}} \leq \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$,

b.1) $\delta \geq 1/2$, $c_1 \leq K_c$, $\frac{4-3c_1-2\delta}{7-5\delta} < Q_{\text{max}} < \frac{10+3c_1-14\delta}{4(1-5\delta)}$,

b.2) $\delta \geq 1/2$, $K_c < c_1 \leq \frac{3\delta(4-3\delta)}{2(16-15\delta)}$, $\frac{4-3c_1-2\delta}{7-5\delta} < Q_{\text{max}} < K_2$,

b.3) $\delta \geq 1/2$, $c_1 > \frac{3\delta(4-3\delta)}{2(16-15\delta)}$, $\frac{4-3c_1-2\delta}{7-5\delta} < Q_{\text{max}} \leq \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}$.
(iv) The quantity commitment binds weakly, the retailer sells the product in both periods and it carries inventory between periods with \( Q_1^* = \frac{4Q_{max}(4-3\delta)-6(1-\delta)}{4(2-\delta)}, \) \( p_1^* = \frac{2+(1-\delta)(4\delta-c_1)-2\delta(4-3\delta)Q_{max}}{2(2-\delta)} \) and \( \bar{\theta} = \frac{2-c_1+2\delta(1-2Q_{max})}{2(2-\delta)}, \) when \( \max(K_1, \frac{10+3c_1-14\delta}{4(4-\delta)}) \leq Q_{max} \leq K_3. \)

(v) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory between periods with \( Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{210(8-7\delta)}, \) \( p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}, \) and \( \bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{9(8-7\delta)}, \) when

\[
a) \quad c_1 \leq K_c, Q_{max} \geq K_3,
\]
\[
b) \quad K_c < c_1 \leq \frac{3\delta(4-3\delta)}{2(16-15\delta)}, \; Q_{max} \geq K_2.
\]

(vi) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory between periods with \( Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}, \) \( p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}, \) and \( \bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}, \) when

\[
a) \quad \delta < 1/2, \; c_1 \leq \frac{8-3\delta}{8}, \; Q_{max} \geq \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta},
\]
\[
b) \quad \delta \geq 1/2, \; \frac{3\delta(4-3\delta)}{2(16-15\delta)} < c_1 \leq \frac{8-3\delta}{8}, \; Q_{max} \geq \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}.
\]

When \( c_1 \geq \frac{8-3\delta}{8}, \) the retailer does not procure any units in period 1. The thresholds \( K_1, K_2, K_3 \) and \( K_c \) are defined as follows.

\[
K_1 = \frac{4+4c_1-14\delta}{4(2-5\delta)} + \frac{\sqrt{(2-\delta)\left(\frac{3c_1^2}{2(1-\delta)} + \delta - 2c_1\right)}}{2(2-5\delta)} \quad (3.8)
\]

\[
K_2 = \frac{20 - 12c_1 - 7\delta}{2(16-5\delta)} + \frac{\sqrt{2\delta(K_2a - K_2b)}}{2\delta(16-5\delta)(8-7\delta)} \quad (3.9)
\]

\[
K_{2a} = 3c_1\delta(512 - 992\delta + 496\delta^2 - 13\delta^3) + 9\delta^2(32 - 88\delta + 68\delta^2 - 13\delta^3)
\]

\[
K_{2b} = 2c_1^2(3072 - 8576\delta + 7980\delta^2 - 2479\delta^3)
\]

28
Next, we study the supplier’s optimal wholesale price and commitment quantity in period 1, which completes the analysis of the equilibrium. The next proposition describes the supplier’s optimal policy. Let $\Pi_S(Q_{\text{max}}, c_1)$ show the supplier’s total profit when it commits to $Q_{\text{max}}$ and sets the wholesale price $c_1$ in period 1.

**Proposition 8.** The supplier’s SPNE wholesale price $c_1^*$ and quantity commitment $Q_{\text{max}}^*$ are as follows.

(i) When $\delta < 1/2$, $Q_{\text{max}}^* = \frac{1}{2(2-\delta)}$ and $c_1^* = 1/2$.

(ii) When $1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$, $Q_{\text{max}}^* = \frac{2-\delta}{1-5\delta}$ and $c_1^* = \frac{2-\delta}{3}$.

(iii) When $\frac{199-3\sqrt{137}}{218} \leq \delta < \delta_3$, $Q_{\text{max}}^* = \frac{7}{16}$ and $c_1^* = \frac{16-3\delta}{32}$.

(iv) When $\delta_3 \leq \delta$, $Q_{\text{max}}^* = K_2$ and $c_1^* = \arg\max_{c_1} \Pi_S(K_2, c_1)$.

where $\delta_3 \approx 0.754$ and it is given by the solution of (B.26) in Appendix B.

Parts (i) and (ii) of Proposition 8 map to part (ii) of Proposition 7; in this case, quantity commitment binds strictly. Similarly, parts (iii) and (iv) of Proposition 8 map to part (iii) of Proposition 7; in this case, quantity commitment binds weakly. Thus, the supplier always finds it attractive to choose a binding (weakly or strictly) quantity commitment. Note that the supplier can always match the outcome of a no-commitment scenario by committing to non-binding arbitrarily large quantity. Thus, the supplier should always benefit from its ability to make a quantity commitment. This is formally stated in the following Theorem. Let superscripts $\text{NC}$ and $\text{SQC}$ denote no commitment and supplier’s quantity commitment respectively.
Theorem 3.  

(i) \( \Pi_{NC}^S \leq \Pi_{SQC}^S \).

(ii) \( \Pi_{NC}^S + \Pi_{NC}^R \leq \Pi_{SQC}^S + \Pi_{SQC}^R \).

(iii) \( \Pi_{NC}^R \geq \Pi_{SQC}^R \) when \( \delta < \bar{\delta}_4 \).

Note \( \bar{\delta}_4 \approx 0.674 \) and it is explicitly characterized in the proof of this theorem. Because the supplier moves first, it can always match the outcome of a no-commitment scenario by committing to a non-binding arbitrarily large quantity. Thus, the supplier always benefits from its ability to make a quantity commitment. Furthermore, when the quantity commitment binds strictly, the supplier’s profit increases strictly. In this case, the product is sold only in period 1. However, the supplier’s commitment can hurt the retailer. This happens because the supplier commits to a smaller quantity than that of the centralized benchmark in order to keep a high wholesale price.

Because quantity commitment discourages customers from waiting for a price mark-down, this benefit would be more valuable when they are more inclined to wait, i.e., when \( \delta \) is high. Therefore, similar to Theorem 2, supplier’s commitment can also help the retailer when \( \delta \) is sufficiently high.

While the supplier’s quantity commitment improves the total supply chain profit, it does not eliminate the loss due to forward-looking customers: The myopic consumers benchmark in Section 2.2 results in a higher profit. Similarly, double marginalization inefficiency is not eliminated; the centralized benchmark also results in a higher profit.

3.5 Supplier’s Period-specific Quantity Commitment

In Section 3.4 the supplier makes a maximum quantity commitment where it does not sell more than a pre-announced quantity. In this section we analyze the policy where supplier is allowed to make one quantity commitment for each period. The sequence of events is shown in Figure 3.4. Recall that when retailer implements this policy it is not different than its maximum quantity commitment.

Lemma 9. Suppose the supplier can make period-specific quantity commitments \( Q_{1\max} \) and \( Q_{2\max} \). It is never optimal for the supplier to make a strictly binding commitment in period 1.
<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier sets $c_1$, commits to $Q_{1\text{max}}$, and $Q_{2\text{max}}$.</td>
<td>Supplier sets $c_2$.</td>
</tr>
<tr>
<td>Retailer sets $p_1$, orders $Q_1$, subject to $Q_{1\text{max}}$.</td>
<td>Retailer orders $Q_2$ subject to $Q_{2\text{max}}$ and sets $p_2$.</td>
</tr>
<tr>
<td>Consumers decide whether to buy or wait.</td>
<td>Remaining consumers decide whether to buy or wait.</td>
</tr>
</tbody>
</table>

**Figure 3.4:** Sequence of events for supplier’s period specific quantity commitment.

Essentially, it is more profitable for the supplier to sell in period 1, thus it does not want to limit the sales in that period. On the other hand, the sales quantity in period 2 is critical for consumers’ tendency to wait for mark-down, therefore limiting the sales in period 2 can be beneficial.

Following Lemma 9 supplier makes a binding quantity commitment only in period 2. Similar to (3.1) let us define the critical period 2 whole sale price $\bar{c}_2$ below which retailer finds it attractive to procure all of this quantity. Thus, $\bar{c}_2$ solves $Q^*_2(c_2) = Q_{2\text{max}}$. In order for supplier’s second period quantity commitment to bind, the following conditions need to be satisfied.

\[
\left. \frac{d\Pi_{R,2}}{dQ_2} \right|_{Q_2 = Q_{2\text{max}}} \geq 0, \quad \text{and} \quad \left. \frac{d\Pi_{S,2}}{dc_2} \right|_{c_2 = \bar{c}_2} \leq 0. \tag{3.12}
\]

Basically, (3.12) ensures that retailer procures all of the commitment quantity given supplier’s second period wholesale price and it is not optimal for supplier to increase this price. The solution of (3.12) leads to the following Lemma, which characterizes the firms’ equilibrium strategies in period 2.

**Lemma 10.** When supplier’s period 2 quantity commitment binds, $Q^*_2 = Q_{2\text{max}}$, $p^*_2 = \delta(1 - Q_1 - Q_{2\text{max}})$ and $c^*_2 = \delta(2(1 - Q_1 - Q_{2\text{max}}) - \bar{\theta})$ in a SPNE in period 2. This yields $\Pi_{R,2} = p^*_2(Q_1 + Q_{2\text{max}} - (1 - \bar{\theta})) - c^*_2 Q^*_2$ and $\Pi_{S,2} = c^*_2 Q^*_2$. Furthermore, the supplier’s second period quantity commitment binds if and only if $Q_{2\text{max}} \leq \frac{1}{4}(2(1 - Q_1) - \bar{\theta})$.

Following lemma 10 retailer buys all of supplier’s commitment quantity in period 2, if and only if

\[
Q_{2\text{max}} \leq \frac{1}{4}(2(1 - Q_1) - \bar{\theta}). \tag{3.13}
\]
Thus, when (3.13) holds, period 2 equilibrium is given by lemma 10. On the other hand, when (3.13) does not hold the commitment is ineffective, and period 2 equilibrium is the same as the no-commitment scenario, which is given by lemma 1. Retailer’s period 1 choices determine whether the commitment in period 2 will bind. When the commitment binds i.e. (3.13) holds the retailer’s objective function is given by the equation below.

\[ [1 - \theta]p_1(\tilde{\theta}, \hat{Q}_1, Q_{2\max}) - Q_1c_1 + \Pi_{R,2}(\tilde{\theta}, Q_1, Q_{2\max}) \]

(3.14)

On the other hand, when the commitment does not bind, i.e., (3.13) does not hold, the retailer’s profit is given by (2.12). We determine the retailer’s optimal policy in period 1 by solving for both of the cases, which is presented in the following proposition.

**Proposition 9.** Suppose the supplier commits to sell not more than \( Q_{2\max} \) units in period 2. The retailer’s SPNE policy is characterized as follows.

(i) The quantity commitment binds, the retailer sells the product in both periods and it carries inventory in between periods with

\[ Q_1^* = \frac{1}{4} \left( 3 - \frac{2c_1}{\delta} - \frac{4Q_{2\max}}{1 - \delta} \right), \quad p_1^* = \frac{1}{4} \left( 2 + 2c_1 - \delta - \frac{(2 - 3\delta)Q_{2\max}}{1 - \delta} \right) \text{ and } \tilde{\theta}^* = \frac{1 - \delta(1 - Q_{2\max})}{2(1 - \delta)} \text{ when } \frac{(2c_1 - \delta)(1 - \delta)}{\delta^2} < Q_{2\max} < \frac{c_1}{\delta^3}. \]

(ii) The quantity commitment binds, the retailer sells the product in both periods and it does not carry inventory in between periods with

\[ Q_1^* = \frac{1 - c_1 - \delta Q_{2\max}}{2 - \delta}, \quad p_1^* = \frac{c_1 + (1 - \delta)(1 - \delta)Q_{2\max}}{2 - \delta} \text{ and } \tilde{\theta}^* = \frac{1 + c_1 - (1 - Q_{2\max})}{2 - \delta} \text{ when } Q_{2\max} \leq \min \left( \frac{(2c_1 - \delta)(1 - \delta)}{\delta^2}, \frac{1 + c_1 - \delta}{\delta}, \frac{1 - c_1}{\delta} \right). \]

(iii) The quantity commitment binds, the retailer sells the product only in period 2 and it does not procure any units in period 1 with

\[ Q_1^* = 0 \text{ and } \tilde{\theta}^* = 1 \text{ when } \frac{1 - c_1}{\delta} < Q_{2\max}. \]

(iv) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory in between periods with

\[ Q_1^* = \frac{6 - c_1 - \delta(6 - 18Q_{2\max}) - 16Q_{2\max}}{8(1 - \delta)}, \quad p_1^* = \frac{(2 + c_1 - 2\delta)(2 - \delta) + 2(2 - 3\delta)Q_{2\max}}{8(1 - \delta)} \text{ and } \tilde{\theta}^* = \frac{2(1 - \delta)(1 + Q_{2\max}) + c_1}{4(1 - \delta)} \text{ when } \frac{c_1}{\delta^3} \leq Q_{2\max} < \min \left( \frac{c_1}{\delta^3}, \frac{2(1 - \delta) + c_1}{2(8 - 7\delta)} \right). \]

(v) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory in between periods with

\[ Q_1^* = 1 - 4Q_{2\max}, \quad p_1^* = (4 - \delta)Q_{2\max} \text{ and } \tilde{\theta}^* = 4Q_{2\max} \text{ when } \max \left( \frac{2(1 - \delta) + c_1}{2(8 - 7\delta)}, \frac{1 + c_1 - \delta}{8 - 5\delta} \right) \leq Q_{2\max} < \min \left( \frac{c_1}{\delta^3}, \frac{4(1 + c_1 - 3\delta)}{2(16 - 9\delta)} \right). \]
(vi) The quantity commitment does not bind, the retailer sells the product in both periods and it carries inventory between periods with
\[ Q_1^* = \frac{11}{14} - \frac{c_1(112-91\delta)+6\delta}{210(8-7\delta)}, \quad p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-7\delta)}, \]
and \( \bar{\theta}^* = \frac{3(4-3\delta)+2c_1}{3(8-7\delta)}, \) when \( c_1 < \frac{3\delta(4-3\delta)}{2(16-15\delta)} \) and \( Q_{2\text{max}} \geq \frac{c_1}{3\delta}. \)

(vii) The quantity commitment does not bind, the retailer sells the product in both periods and it does not carry inventory between periods with
\[ Q_1^* = \frac{8(1-c_1)}{16-9\delta}, \quad p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}, \] and \( \bar{\theta}^* = \frac{8(1+c_1)-6\delta}{16-9\delta}, \) when \( \frac{3\delta(4-3\delta)}{2(16-15\delta)} \leq c_1 \) and \( Q_{2\text{max}} \geq \frac{4(1+c_1)-3\delta}{2(16-9\delta)}. \)

Next we solve the supplier’s optimization problem to completely characterize the equilibrium. The supplier maximizes
\[ c_1Q_1^* + c_2Q_{2\text{max}}, \] where \( Q_1^* \) and \( c_2^* \) are given by Proposition 9 and Lemma 10. The following proposition gives the solution to this problem.

**Proposition 10.** Suppose the supplier commits to sell not more than \( Q_{2\text{max}} \) units in period 2. The equilibrium whole sale price
\[ c_1^* = \frac{1}{2}, \] and the commitment quantity
\[ Q_{2\text{max}}^* = \frac{1-\delta}{2(4-3\delta)}, \] which yields
\[ c_2^* = \frac{\delta}{2}, \quad p_2^* = \frac{\delta(5-3\delta)}{2(4-3\delta)} \] and \( Q_2^* = \frac{1-\delta}{2(4-3\delta)}. \)

The equilibrium solution corresponds to part (ii) of Proposition 9, hence in equilibrium supplier sets a first period wholesale price and commits to a second period quantity such that retailer buys and sells in both periods but does not carry inventory. Next we discuss the impact of this policy on profits. Let \( \Pi_{SQC2}^S \) show the supplier’s profit when it commits to not selling more than \( Q_{2\text{max}} \) units in period 2.

**Theorem 4.** (i) \( \Pi_{SNC}^S < \Pi_{SQC2}^S. \)

(ii) \( \Pi_{SNC}^S + \Pi_{RNC}^S < \Pi_{SQC2}^S + \Pi_{RQC2}^S. \)

(iii) \( \Pi_{RNC}^S > \Pi_{RQC2}^S. \)

Similar to maximum quantity commitment, the supplier’s period specific quantity commitment hurts the retailer while benefiting itself (see Theorem 3). The only difference is that the retailer always suffers from the supplier’s period-specific quantity commitment whereas the retailer may benefit from the supplier’s aggregate quantity commitments when discount factor \( \delta \) is sufficiently high. This happens because when the supplier commits to the aggregate quantity, the retailer has the flexibility to allocate this quantity over the two periods and it can manipulate period 2 wholesale price through carrying inventory. In contrast the supplier’s period-specific quantity commitment
takes away the retailer’s flexibility and reduces its ability to manipulate wholesale price. Because of this benefit, the supplier achieves a higher profit by committing only to a period 2 quantity instead of making an aggregate quantity commitment as formally stated in the following Corollary.

**Corollary 3.** $\Pi_{SQC}^{2} > \Pi_{SQC}^{S}$. 

Interestingly, the Corollary shows that the supplier can do better by limiting only its last period sales rather than trying to control its total sales. Intuitively, limiting period 2 sales is more aligned with the supplier’s objective of pushing sales to period 1, which enables setting a higher wholesale price.

### 3.6 Simultaneous Quantity Commitment

In Sections 3.2, 3.3, 3.4 and 3.5, the supplier and retailer make unilateral quantity quantity commitments. Here, we consider their simultaneous commitments. In particular, we illustrate that they continue to suffer from commitments showing that the problem is due to lack of coordination and is not resolved by concurrent commitments.

Suppose both the supplier and the retailer can make independent quantity commitments following the same sequence of events as in Sections 3.2 and 3.4. Let $\Pi_{i}^{sQC}$, $i : S, R$ show the supplier’s and retailer’s profit under this scenario. In equilibrium, either the supplier’s or the retailer’s quantity commitment would dominate. For example, if the supplier sets a more stringent quantity commitment, the retailer’s quantity commitment becomes irrelevant and vice versa. The following theorem shows the impact of these commitments on their profitability.

**Theorem 5.** There exists $\bar{\delta}_1 \approx 0.249$ and $\bar{\delta}_4 \approx 0.674$ such that

(i) $\Pi_{S}^{NC} > \Pi_{S}^{sQC}$ when $\delta \leq \bar{\delta}_1$.

(ii) $\Pi_{S}^{NC} + \Pi_{R}^{NC} > \Pi_{S}^{sQC} + \Pi_{R}^{sQC}$ when $\delta \leq \bar{\delta}_1$.

(iii) $\Pi_{R}^{sNC} \geq \Pi_{R}^{sQC}$ when $\delta < \bar{\delta}_4$.

Note $\bar{\delta}_1$ and $\bar{\delta}_4$ are given by the solutions of (B.16) and (B.28) in Appendix B.

The Theorem illustrates that the supplier and the retailer as well as the entire supply chain can get worse off due to simultaneous quantity commitments. Recall that the supplier always benefits
from its own unilateral quantity commitments. However, simultaneous quantity commitment can hurt its profitability when $\delta < \tilde{\delta}_1$, because the retailer’s quantity commitment dominates in this case.

### 3.7 Concluding Remarks

In this chapter we provided analysis of several quantity commitment polices in a supply chain selling to strategic customers. We first presented the vertically integrated supply chain and showed that a quantity commitment strategy works well to prevent forward looking customers to wait for a price markdown. Motivated by this result we implemented the same policy on a decentralized system, which makes quantity commitments either by the retailer or the supplier unilaterally as well as simultaneously. We find that contrary to the centralized system result, in a decentralized supply chain unilateral or the simultaneous quantity commitments exacerbate the coordination problem even further. When compared to the no-commitment scenario retailer’s quantity commitments not only hurts itself but also the supplier and the supply chain. For the supplier, however, since it has the first mover advantage in our model, it can always match the no-commitment outcome with a quantity commitment. Therefore, its quantity commitment profit is expected to be at least as good as the no-commitment scenario. Indeed we show that this is the case. However, we show that this happens in the expense of retailer as retailer’s profit gets worse off when supplier makes either an aggregate or period specific quantity commitments. We also show that supplier would benefit more from a period specific quantity commitment rather than an aggregate version since this policy gives better control on how to allocate the commitment quantity to each period. As expected though this increase in supplier’s profit is again in the expense of the retailer’s profit. We conclude the chapter by analyzing the simultaneous commitments and show that the decentralized system still suffers from coordination problem and the supplier, although in its own quantity commitment always benefits can get worse off this time when the retailer’s quantity commitment dominates in equilibrium.
CHAPTER 4
Price Commitments

In this chapter we discuss price commitment policies implemented by retailer, supplier or both of the supply chain parties and the impact of these policies on firms’ profits. Strategic customer behavior has been recognized in the retail industry for the last decade. For example Best Buy’s former CEO has labeled some customers as “devils” since they were constantly searching for clearance prices. This segment of the customers are not profitable and he said in an interview that these customers could wreak enormous economic havoc (McWilliams 2004). Markdown optimization is one of the tools retailers have widely adopted to prevent this kind of profit loss. They help dynamically adjusting prices in an hope for eliminating deep price cuts. When customers are forward looking price commitments are the recommended practice in the literature that might prove useful. Price commitment can eliminate the markdown opportunity completely or restrict the degree of price drop. However, although the assumed benefits, price commitments are not widely seen in the industry. Firms dynamically adjust their prices but they usually do not pre-announce their price path through the selling period. In this chapter we model the price commitment policies in a decentralized supply chain by explicitly showing the interactions between the supplier and retailer. We show that when the coordination problem in a supply chain is not addressed price commitments alone do not provide the benefits that they are promised.

In the following sections we first introduce the vertically integrated firm’s price commitment benchmark, then we provide retailer’s and supplier’s unilateral price commitment models. We also analyze the simultaneous price commitment. We address our main research question whether the price commitments are valuable when selling to strategic customers in a decentralized supply chain.
4.1 Centralized System Price Commitment Benchmark

Suppose that the vertically integrated firm that we introduced in Section 2.3 can make price commitments. When the firm credibly commits to a future price $p_2$, which can be different than $p_1$, it does not change its committed price in period 2 even when this deviation would increase its profit. When the centralized firm makes a price commitment it restricts the sales to the first period. Therefore, the equilibrium with the price commitment is actually not different than the equilibrium with the quantity commitment. For completeness we repeat the price commitment version of the results provided in Section 3.1.

**Lemma 11.** Suppose a centralized firm can commit to future price. In equilibrium, the firm sells its product only in period 1 at price $p_1^* = \frac{1}{2}$.

When the firm can make a price commitment, it limits its sale to a single period to eliminate the strategic behavior. Let $\Pi^j_C, j : NC, PC$ show the centralized system’s profit when it cannot make any commitments (NC) and when it can make price (PC) commitments. The following Theorem compares the centralized firm’s profit in these scenarios.

**Theorem 6.** $\Pi^NC_C < \Pi^PC_C$.

The Theorem shows that price commitments increase the centralized system’s profit. In the rest of this chapter, we study the impact of such commitments for a decentralized system.

4.2 Retailer’s Price Commitment

Suppose that the retailer can credibly commit to a future retail price. This means that when setting the retail price $p_1$ in period 1, the retailer also commits to period 2 retail price $p_2$. The order of events is shown in Figure 4.1. We derive the equilibrium using backward induction. The following Lemma characterizes the equilibrium in period 2.

**Lemma 12.** When the retailer commits to a future retail price, it does not procure any units in period 2 in equilibrium, that is, $Q_2^* = 0$.

Because the retailer already commits to period 2 price $p_2$ before the supplier sets its period 2 wholesale price $c_2$, the supplier has no incentive to set a wholesale price $c_2$ smaller than $p_2$. Because
Supplier sets $c_1$. Retailer orders $Q_1$ sets $p_1$ and commits to $p_2$. Consumers decide whether to buy or wait.

Supplier sets $c_2$. Retailer orders $Q_2$. Remaining consumers decide whether to buy or wait.

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier sets $c_1$.</td>
<td>Supplier sets $c_2$.</td>
</tr>
<tr>
<td>Retailer orders</td>
<td>Retailer orders</td>
</tr>
<tr>
<td>$Q_1$ sets $p_1$ and commits to $p_2$.</td>
<td>$Q_2$.</td>
</tr>
<tr>
<td>Consumers decide whether to buy or wait.</td>
<td>Remaining consumers decide whether to buy or wait.</td>
</tr>
</tbody>
</table>

Figure 4.1: Sequence of events when retailer commits to prices.

the retailer’s sales quantity in period 2 and its order quantity $Q_2$ do not depend on $c_2$ as long as $c_2 \leq p_2$. Anticipating this response, the retailer procures all of its needed units in period 1.

Following Lemma 12, similar to the centralized system benchmark, the retailer limits its sales to only period 1 by committing that period 2 retail price will not be attractive. The next proposition characterizes the resulting equilibrium.

Proposition 11. Suppose the retailer can commit to a future retail price. The SPNE is as follows.

(i) Given the wholesale price $c_1$, the retailer orders $Q_1^* = \frac{1-c_1}{2}$, sets $p_1^* = \frac{1+c_1}{2}$ in period 1 and it does not sell the product in period 2 by committing to a sufficiently high $p_2$ in period 1.

(ii) The supplier chooses $c_2^* = \frac{1}{2}$.

It is straightforward to show that the retailer would benefit from limiting its sales to only period 1, which eliminates the strategic customer behavior, if the supplier’s wholesale prices stay the same as in the no-commitment scenario. How do the changes in supplier’s wholesale prices impact the value of commitment? This is addressed by the next Theorem. Note that superscripts $NC$ and $RPC$ stand for no commitment and retailer’s price commitment respectively.

Theorem 7. (i) $\Pi^S_{NC} > \Pi^S_{RPC}$.

(ii) $\Pi^S_{NC} + \Pi^R_{NC} > \Pi^S_{RPC} + \Pi^R_{RPC}$.

(iii) $\Pi^R_{NC} > \Pi^R_{RPC}$.

Similar to quantity commitment, committing to a future retail price hurts the retailer’s profitability due to how it affects supplier’s pricing. This also makes the supplier as well as the whole
supply chain worse off. The retailer’s price commitment amplifies double marginalization inefficiency as a result of two factors. First, absent any commitments the retailer procures units in both periods where period 2 wholesale price is always cheaper than that of period 1. In contrast, with price commitment, the retailer procures units only in period 1 at a single wholesale price. When the supplier sells the product at two different prices, double marginalization is alleviated. This benefit vanishes with retailer’s price commitment. Second, a comparison of Propositions 2 and 11 shows that the supplier sets a higher wholesale price \( c_1 \) when the retailer can commit to prices. The supplier sets a higher \( c_1 \) anticipating that the retailer is going to limit its sales to only period 1.

Overall, Theorems 2 and 7 show that retailer’s price and quantity commitments can hurt profitability of both itself and its supplier. The vertical relation between the retailer and the supplier, which leads to double-marginalization, is the key driver of this result. In particular, endogenous wholesale price is critical. Commitments would be beneficial if the wholesale price was exogenous (so the supplier does not react to retailer’s commitments) or if the model did not consider the supplier at all as in the centralized benchmark (see Theorem 1).

### 4.3 Supplier’s Price Commitment

In this section, we discuss what happens when the supplier commits to future wholesale prices. In particular, while setting period 1 wholesale price \( c_1 \) before the beginning of period 1, the supplier also commits to period 2 wholesale price \( c_2 \). The order of events is shown in Figure 4.2.

We derive the equilibrium following backward induction. The following Lemma describes the equilibrium in period 2.
Lemma 13. Suppose that the supplier commits to wholesale prices $c_1$ and $c_2$, the retailer orders $Q_1$ in period 1 and consumer segment $[0, \bar{\theta})$ remain in the market in period 2. The retailer then orders $Q_2^* = \max \left( \frac{\delta(2Q_1 - 2\hat{\theta}) - c_2}{2}, 0 \right)$ units and sets $p_2^* = \delta (1 - Q_1 - Q_2^*)$ in period 2.

The marginal customer $\hat{\theta}$, who is indifferent about whether to buy in period 1 or to wait for period 2, solves the following equation

$$\hat{\theta} - p_1 = \delta \hat{\theta} - p_2^*,$$

where the left and right hand sides correspond to the utility of buying in period 1 and 2 respectively. Following Lemma 13, this leads to

$$\hat{\theta} = \frac{2p_1 - c_2}{2 - \delta}. \quad (4.1)$$

Note that (4.1) is independent of the retailer’s period 1 order quantity $Q_1$; in other words, in their purchasing decisions in period 1, customers do not need to rely on their beliefs about $Q_1$. Intuitively, when the supplier already commits to a period 2 wholesale price in period 1, the retailer has no incentive to carry inventory into period 2. Recall that, in our model, the retailer carries inventory only to get a better wholesale price in period 2. Therefore, to conjecture period 2 retail price in period 1, customers do not need to know $Q_1$. Rearranging (4.1), we get

$$p_1(\hat{\theta}) = \frac{(2 - \delta)\hat{\theta} + c_2}{2}. \quad (4.2)$$

The retailer then in period 1 solves the following problem to maximize his total profit in two periods

$$\max_{(Q_1,\hat{\theta})\in S'_1} \left[ (1 - \hat{\theta})p_1(\hat{\theta}) - Q_1c_1 + Q_2^*(p_2^* - c_2) \right], \quad (4.3)$$

where $Q_2^*$ and $p_2^*$ are given in Lemma 13 and $S'_1$ is defined in (2.15), which states that the retailer’s sales quantity in period 1 should be non-negative and cannot exceed $Q_1$. The solution of (4.3) leads to the following Proposition.

Proposition 12. Suppose the supplier commits to wholesale prices $c_1$ and $c_2$. The retailer’s SPNE order quantity $Q_1^*$, and retail price $p_1^*$ in period 1 are as follows.
(i) When $c_2 \leq c_1 + \delta - 1$, the retailer sells the product only in period 2, where $Q_1^* = 0$ and $p_1^* = 1 + \frac{c_2 - \delta}{2}$.

(ii) When $c_1 + \delta - 1 < c_2 < \frac{\delta(2 + 2c_1 - \delta)}{4 - \delta}$, the retailer sells the product in both periods, where $Q_1^* = \frac{2(1 - c_1 + c_2 - \delta)}{4 - 3\delta}$ and $p_1^* = \frac{4 + 2c_1(2 - \delta) - (4 + c_2 - \delta)\delta}{2(4 - 3\delta)}$.

(iii) When $\frac{\delta(2 + 2c_1 - \delta)}{4 - \delta} \leq c_2 < \frac{\delta(1 + c_1)}{2}$, the retailer sells the product only in period 1, where $Q_1^* = 1 - \frac{c_2}{\delta}$ and $p_1^* = \frac{c_2}{\delta}$.

(iv) When $c_2 \geq \frac{\delta(1 + c_1)}{2}$, the retailer sells the product only in period 1, where $Q_1^* = \frac{1 - c_1}{2}$ and $p_1^* = \frac{1 + c_1}{2}$.

The retailer does not carry inventory between periods in all cases, thus the marginal customer is given by $\bar{\theta}^* = 1 - Q_1^*$.

Note that $c_1 - c_2$ difference decreases going from part (i) to (iv), which, as expected, shifts the retailer’s sales from the second period to the first period.

Finally, we need to solve the supplier’s problem to completely characterize the equilibrium. The supplier maximizes $c_1 Q_1^* + c_2 Q_2^*$ where $Q_1^*$ and $Q_2^*$ are given by Proposition 12 and Lemma 13. The solution of this problem is given by the next Proposition.

**Proposition 13.** Suppose the supplier can commit to future wholesale prices. The equilibrium wholesale prices $c_1$ and $c_2$ are as follows.

(i) When $\delta \leq 1 - \frac{1}{\sqrt{17}}$, $c_1^* = \frac{16 - (14 - \delta)\delta}{2(16 - 13\delta)}$ and $c_2^* = \frac{2(5 - 4\delta)}{16 - 13\delta}$.

(ii) When $\delta > 1 - \frac{1}{\sqrt{17}}$, $c_1^* = c_2^* = \frac{\delta(5 - 4\delta)}{2(4 - 3\delta)}$.

Both parts (i) and (ii) of Proposition 13 correspond to part (ii) of Proposition 12; that is, in equilibrium the supplier commits to wholesale prices that induces the retailer to buy and sell the product in both periods.

The next theorem addresses our key research question. What is the value of the supplier’s commitment for the supplier itself, for the retailer and for the whole supply chain? Note that superscripts $NC$ and $SPC$ stand for no commitment and supplier’s price commitment respectively.

**Theorem 8.** (i) $\Pi_S^{NC} > \Pi_S^{SPC}$. 

41
\begin{align*}
(ii) \quad & \Pi^{NC}_S + \Pi^{NC}_R > \Pi^{SPC}_S + \Pi^{SPC}_R. \\
(iii) \quad & \Pi^{NC}_R > \Pi^{SPC}_R.
\end{align*}

The Theorem shows that similar to the retailer’s commitments, the supplier need not benefit from its ability to make price commitments. Furthermore, the supplier’s price commitment not only hurts its own profitability but also that of the retailer, making the whole supply chain worse off. This happens because the supplier’s price commitment increases the inefficiency due to decentralization. When the supplier does not make price commitments, its period 2 wholesale price $c_2$ decreases in the retailer’s period 1 procurement quantity $Q_1$, as seen in Lemma 1.ii. In other words, buying a larger quantity in period 1 allows the retailer to receive a lower wholesale price in period 2. This mimics a quantity discount mechanism and alleviates the double marginalization inefficiency. However, when the supplier already commits to a period 2 wholesale price in period 1, the retailer’s incentive to buy more to pay less later vanishes. Knowing this, the supplier sets a higher period 1 wholesale price (see Propositions 2 and 13) which increases the double marginalization inefficiency. Note that the supplier’s quantity commitment, on the other hand, maintains the retailer’s incentive by preserving the relation between period 1 order quantity and period 2 wholesale price.

The impact of supplier’s price commitment on retailer’s ordering policy is the key driver of Theorem 8. Dynamic wholesale pricing in our base model is critical for this result because dynamic wholesale prices encourages the retailer to act strategically by buying a larger quantity in period 1 in order to receive a lower wholesale price in period 2. As explained above, wholesale price commitment hurts the supply chain because it eliminates the retailer’s incentive to buy more early on.

4.4 Simultaneous Price Commitment

In Sections 4.2 and 4.3, the supplier and retailer make unilateral price commitments. Here, we consider their simultaneous commitments. In particular, we illustrate that they continue to suffer from price commitments unless the coordination problem is resolved.

Suppose both the supplier and retailer can make price commitments following the same sequence of events as in Sections 4.2 and 4.3. The following proposition shows that the retailer’s price commitment always dominates in this case.
Proposition 14. When both the retailer and the supplier can make price commitments, the resulting equilibrium is same as that of the retailer’s unilateral price commitment.

Essentially, the retailer restricts its sales to only period 1 in equilibrium, therefore, the supplier’s price commitment (to period 2 wholesale price) becomes irrelevant. Because the resulting equilibrium is identical to the retailer’s unilateral price commitment, both the supplier and the retailer get worse off compared to no commitment scenario as shown in Theorem 7.

4.5 Concluding Remarks

In this chapter we looked into unilateral and simultaneous price commitments implemented by the retailer and supplier in a decentralized supply chain with forward looking customers. We first provided the result that price commitments when made by a vertically integrated firm increase the profit. We then analyze price commitment policy on the decentralized system by explicitly modeling the interactions between the supply chain members. We find that price commitments cause the double marginalization problem to become even worse when either the retailer or the supplier makes the price commitment. In addition the whole supply chain gets worse off. Note that in Chapter 3 we saw that supplier’s quantity commitment helps itself. However, here we showed that it is not the case with price commitment. We also provided the simultaneous price commitment scenario and showed that the result would not be different than retailer’s price commitment since in equilibrium retailer restricts the sale to only period 1.
CHAPTER 5
Quantity vs. Price Commitments: A Comparison

In this chapter we compare the equilibrium profits of retailer and supplier under different commitment strategies. This provides insights about different commitment scenarios and how they perform for various levels of strategic behavior of customers.

5.1 Retailer’s Profit Under Different Commitment Scenarios

We present the retailer profit comparison in figure 5.1. Since retailer sells only in first period when \( \delta \leq \bar{\delta}_1 \) price and quantity commitments result in the same profit for retailer. For medium \( \delta \) range retailer prefers committing to price rather than quantity, however for very large \( \delta \) it prefers committing to quantity. Note that as \( \delta \) increases the level of strategic behavior of customers as well as the value of the product in second period increases. When the retailer commits to a maximum quantity, the supplier can still manipulate the total sales quantity and its allocation to two periods through its choice of wholesale prices. In contrast, the retailer’s price commitments directly determine the sales quantity in each period. When retailer commits to quantity and sells the product in both periods, ie. in medium \( \delta \) range supplier’s wholesale price choices hurts retailer. However, when product is sufficiently valuable in second period, quantity commitment performs better than price commitment due to the additional profit retailer can get from second period and still can sustain a sufficiently high price.

Retailer prefers committing to price itself rather than supplier for every \( \delta \) value. Note that the supplier’s and retailer’s dynamic pricing have different effects. The retailer’s dynamic pricing encourages consumers to wait for a mark-down, thereby shifting consumer demand to period 2. In contrast, the supplier’s dynamic pricing encourages the retailer to buy more early on, thereby shifting retailer demand to period 1. When supplier commits to wholesale prices it keeps the second period price sufficiently low that retailer always replenishes, which increases the total quantity sold.
However, this helps supplier to extract more of retailer’s profit therefore supplier’s price commitment hurts retailer more than its own price commitment. For high $\delta$, specifically when $\delta > 0.58$ supplier’s aggregate quantity commitment is the best commitment scenario for retailer. However, supplier’s period specific quantity commitment hurts the retailer the most.

Next we study the policy when retailer can make not only a quantity but also a price commitment concurrently. This will answer the question whether retailer would benefit from committing to price when it can also commit to quantity.

### 5.2 Retailer’s Concurrent Price and Quantity Commitment

In this section we provide the equilibrium for the retailer’s concurrent price and quantity commitments. In this case when retailer tells the first period retail price, he not only tells the second period price but also commits to a maximum quantity to sell over two periods. The following lemma describes the second period values.

**Lemma 14.** When retailer commits to both price and quantity then $Q^*_2 = Q_{\text{max}} - Q_1$, $p^*_2 = \delta(1 - Q_{\text{max}})$ and $c^*_2 = p^*_2$.

Note that similar to retailer’s price commitment in Section 4.2, since retailer tells the second period price before supplier announces the second period wholesale price supplier has no incentive
to set $c_2$ lower than $p_2$, which drives the above result. In the following proposition we provide the equilibrium values in period 1.

**Proposition 15.** Suppose the retailer can commit to both future price and a maximum quantity. The SPNE is as follows. Given the wholesale price $c_1$, the retailer orders $Q_{max}^* = Q_1^* = \frac{1-c_1}{2}$ and sets $p_1^* = \frac{1+c_1}{2}$ in period 1. The supplier chooses $c_1^* = \frac{1}{2}$.

In equilibrium retailer restricts the sale only to the first period. Note that this is possible only during $\delta \leq \bar{\delta}_1$ when retailer commits to quantity alone. Committing to prices also makes this strategy optimal in equilibrium for every $\delta$ value. However, this takes away the benefit of making sales in second period when the product value is high with high $\delta$ values, where retailer’s quantity commitments performs better than its price commitment.

Note that equilibrium is the same as retailer’s price commitment therefore Theorem 7 follows. Although quantity commitment provides more flexibility for retailer the benefits are offset by supplier pricing. When retailer commits to quantity it chooses a less aggressive strategy to prevent strategic waiting than it price commitment; where it completely takes away the strategic waiting possibility, which does not leave any room for supplier reaction. Hence, for majority of delta values price commitment performs better for retailer than quantity commitment. Therefore, committing to both price and quantity results in the same equilibrium as its price commitment.

### 5.3 Supplier’s Profit Under Different Commitment Scenarios

We present the supplier profit comparison in figure 5.2. Supplier profits show a nice ordering such that it prefers making a commitment itself rather than retailer making a commitment; either price or quantity. In addition supplier prefers a quantity commitment over a price commitment. As we stated earlier supplier has a first mover advantage in our model. Therefore it is expected that it would benefit from its commitments more than retailer’s commitments. When supplier makes a commitment it not only manipulates the retail prices through wholesale prices but also it controls the availability in each period maximizing its own interest. However, when retailer makes a commitment in order to counter act the potential profit loss by limited sales it has to increase the first period price, which in turn amplifies the double marginalization inefficiency.
5.4 Supplier’s Concurrent Price and Quantity Commitment

Similar to retailer’s concurrent commitment in this section we look into supplier’s concurrent commitments. Our main research question in this section is whether supplier would benefit from committing to prices when it can also make a quantity commitment.

We first consider supplier’s aggregate quantity commitment. The following Theorem formally answers our research question.

**Theorem 9.** When supplier commits to a maximum quantity $Q_{max}$ to sell over two periods, it does not benefit from committing to wholesale prices $c_1$ and $c_2$.

Here instead of fully characterizing the equilibrium we derive the result by showing that any equilibrium profit where all of the commitments bind is dominated by the aggregate quantity commitment profit presented in Section 3.4.

The following Theorem formally answers our research question for the supplier’s period specific quantity commitment policy.

**Theorem 10.** When supplier commits to a maximum second period quantity $Q_{2max}$, it does not benefit from committing to wholesale prices $c_1$ and $c_2$.

Again to show this result instead of fully solving for the equilibrium we first find the retailer’s response when all the commitments bind. Then we find supplier’s best response to these retailer
strategies. We show that this solution may not be the equilibrium strategy, however any equilibrium strategy is not better than the solution presented. Finally we drive the result by comparing supplier’s corresponding profit to its period specific quantity commitment equilibrium, which was presented in Section 3.5.

For supplier, quantity commitment is a more flexible tool than its price commitment in terms of how it reacts with retailer pricing and strategic waiting of the customers. Rather than indirectly determining the availability in second period through retailer prices as in price commitment; quantity commitment sends a stronger signal to customers to encourage them to buy in first period. However, this happens in the expense of retailer as we showed in Sections 3.4 and 3.5. Therefore, supplier does not benefit from committing to wholesale prices when it can also commit to quantity either a maximum or period specific.
CHAPTER 6
Alternative Modeling Assumptions

In this chapter, we consider two alternative modeling assumptions and discuss the robustness of our results.

6.1 Utility Discounting

In our main model, product value diminishes by $1 - \delta$ in period 2, thus, $\delta$ captures the degree of customer patience. The payment term in period 2 is not discounted as seen in (2.1). This approach assumes the loss in customer’s utility due to delayed product use is significantly higher than the benefit of a delayed payment. In contrast, in this section we assume the product value and the payment are discounted at the same rate in customer utility. Specifically, the following replaces the customer utility function in (2.1):

$$U_i(\theta, p_i) = \delta^{i-1}(\theta - p_i), \ i : 1, 2.$$  \hspace{1cm} (6.1)

Note that with this assumption total demand increases because the set $S_2$ in equation (2.9) is replaced with

$$\{(p_2, Q_2) : Q_1 + Q_2 \geq 1 - p_2\}. \hspace{1cm} (6.2)$$

In addition customers’ willingness to pay in period 2 increases, which is reflected in the marginal customer’s decision given by the following equation.

$$\theta^* = \inf\{\theta : \bar{\theta} - p_1 \geq \delta(\bar{\theta} - p_2^*(\hat{Q}_1))\}. \hspace{1cm} (6.3)$$

The following Lemma describes the supplier’s and retailer’s optimal policy in period 2.
Lemma 15. Suppose the retailer orders $Q_1$ units in period 1, and consumer segment $[0, \bar{\theta})$ remains in the market in period 2.

(i) For any given wholesale price $c_2$, the retailer orders $Q_2^* = \max(\frac{1}{2}(2 - 2Q_1 - \bar{\theta} - c_2), 0)$ units and sets $p_2^* = 1 - Q_1 - Q_2^*$ in period 2.

(ii) The supplier sets $c_2^* = 1 - Q_1 - \bar{\theta}$ in period 2.

This yields profits $\Pi_{R,2} = \frac{1}{10}(\bar{\theta}^2 + 12(1 - Q_1)(Q_1 - 1 + \bar{\theta}))$ and $\Pi_{S,2} = \frac{(2 - 2Q_1 - \bar{\theta})^2}{8}$ for the retailer and supplier respectively.

Customers conjecture the second period price and they expect $p_2^*(\hat{Q}_1) = \frac{2 - 2\hat{Q}_1 + \bar{\theta}}{4}$. Note that both $Q_2^*$, $c_2^*$ and $p_2^*$ increase in comparison to the base model. The marginal customer $\bar{\theta}$ in equation (6.3) leads to $\bar{\theta} = \frac{4p_1 - 2\delta(1 - \hat{Q}_1)}{4 + \delta}$, which is the same as equation (2.10) in the base model.

After reformulating the retailer’s total profit in terms of the order quantity $Q_1$ and target consumer segment $[\hat{\theta}, 1]$ in period 1 we can solve for the retailer’s optimal policy.

Proposition 16. The SPNE order quantity $Q_1^*$ and retail price $p_1^*$ and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.

(i) When $c_1 < \min(\frac{3(4 - 3\delta)}{2(9 - 8\delta)}, \frac{3(1 - \delta)}{3 - 2\delta})$ the retailer sells the product in both periods, and it carries inventory between periods, where $Q_1^* = \frac{8 - 7\delta - 2c_1(5 - 4\delta)}{2(6 - 5\delta)}$, $p_1^* = \frac{3(2 - \delta)(4 - 3\delta) - 2c_1(3\delta^2 + \delta - 6)}{6(6 - 5\delta)}$, and $\bar{\theta}^* = \frac{3(4 - 3\delta) + 2c_1(3 - 2\delta)}{3(6 - 5\delta)}$.

(ii) When $\frac{3(1 - \delta)}{3 - 2\delta} \leq c_1 \leq \frac{3}{4}$ the retailer sells the product only second period and carries inventory, where $Q_1^* = \frac{1}{6}(3 - 4c_1)$, $p_1^* = 1 - \frac{\delta}{6}(3 - 2c_1)$, and $\bar{\theta}^* = 1$.

(iii) When $\frac{3(4 - 3\delta)}{2(9 - 8\delta)} \leq c_1 \leq \frac{1}{8}(7 - 2\delta)$ the retailer sells the product in both periods, and it does not carry inventory between periods, where $Q_1^* = \frac{7 - 8c_1 - 2\delta}{15 - 8\delta}$, $p_1^* = \frac{(4 - \delta)(4 + c_1) - 3\delta}{2(15 - 8\delta)}$, and $\bar{\theta}^* = \frac{2(4 + 4c_1 - 3\delta)}{15 - 8\delta}$.

When $c_1 > \max(\frac{1}{8}(7 - 2\delta), \frac{3}{4})$ the retailer does not buy the product in first period.

Note that the region in which the retailer carries inventory increases in comparison to the base model, which is due to the fact that the second period demand increases when customers discount not only their valuations but also the retail price.
To find the equilibrium whole sale price we need to solve the supplier’s optimization problem. The following Proposition describes the solution.

**Proposition 17.** The SPNE whole sale price \( c_1 \) in period 1 is as follows.

(i) When \( \delta \leq \frac{1}{132}(127 - \sqrt{817}) \), \( c_1^* = \frac{9(8 - 7\delta)}{4(33 - 26\delta)} \).

(ii) When \( \delta > \frac{1}{132}(127 - \sqrt{817}) \), \( c_1^* = \frac{9}{16} \).

Parts (i), (ii) of Proposition 17 map to the parts (i), (ii) of Proposition 16. Hence, the supplier allows the retailer to carry inventory in between periods. In addition, \( c_1^* \) also increases compared to the base model.

### 6.1.1 Retailer’s Quantity Commitment when Customers Discount Net Utility

In this section we analyze the retailer’s quantity commitment when customer utility function is given by (6.1). Analysis is similar to that of Section 3.2, specifically Lemma 6 continues to hold. Following equation (3.1), we obtain the following Lemma that describes the optimal policies in the second period.

**Lemma 16.** When the retailer’s quantity commitment binds, \( Q_2^* = Q_{\text{max}} - Q_1 \), \( p_2^* = 1 - Q_{\text{max}} \) and \( c_2^* = 2(1 - Q_{\text{max}}) - \bar{\theta} \) in a SPNE in period 2. This yields \( \Pi_{R,2} = p_2^*(Q_{\text{max}} - (1 - \bar{\theta})) - c_2^*Q_2^* \) and \( \Pi_{S,2} = c_2^*Q_2^* \). Furthermore, the retailer quantity commitment binds if and only if \( Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4} \).

The marginal customer \( \bar{\theta} \), who is indifferent between buying in period 1 or waiting for period 2 then solves \( \bar{\theta} - p_1 = \delta(\bar{\theta} - p_2^*) \) which gives \( p_1(\theta, Q_{\text{max}}) = \bar{\theta} - \delta(Q_{\text{max}} - (1 - \bar{\theta})) \). Following Lemma 16 and the expression for marginal customer we can state retailer’s period 1 problem similar to equations (3.3 - 3.6), which leads to the following proposition.

**Proposition 18.** Suppose the retailer can make a quantity commitment. Its SPNE order quantity \( Q_1^* \), quantity commitment \( Q_{\text{max}}^* \), retail price \( p_1^* \) and the corresponding marginal customer \( \bar{\theta}^* \) in period 1 are as follows.

(i) When \( c_1 < K_{c_2} \), the retailer sells the product in both periods, it carries inventory in between periods and the quantity commitment binds strictly, where \( Q_1^* = \frac{2(1 - c_1)}{3 + \delta} \), \( Q_{\text{max}}^* = \frac{2(1 - c_1)}{3 + \delta} \), \( p_1^* = \frac{2 + c_1(1 + \delta)}{3 + \delta} \) and \( \bar{\theta}^* = \frac{2 + c_1 + \delta}{3 + \delta} \).
(ii) When \( K_{c_2} \leq c_1 < \min\left(\frac{(3-\delta)(2-\delta)}{9-8\delta}, \frac{3(1-\delta)}{4-2\delta}\right) \), the retailer sells the product in both periods, it carries inventory in between periods and the quantity commitment binds weakly, where
\[
Q_1^* = \frac{12 - (11-\delta)\delta - 3c_1(5-4\delta)}{2(9-\delta(6+\delta))}, \quad Q_{\text{max}}^* = \frac{3(4-3c_1) - 10\delta + 7c_1\delta}{2(9-\delta(6+\delta))}, \quad p_1^* = \frac{2(6-(6-\delta)\delta - c_1(3\delta^2 + \delta - 6))}{2(9-\delta(6+\delta))} \quad \text{and} \quad \bar{\theta}^* = \frac{6 - \delta(3+\delta) + c_1(3-2\delta)}{9 - \delta(6+\delta)}.
\]

(iii) When \( \frac{(3-\delta)(2-\delta)}{9-8\delta} \leq c_1 < \frac{3}{4} \), the retailer sells the product only in period 2, it carries inventory between periods and the quantity commitment binds weakly, where \( Q_1^* = \frac{1}{6}(3 - 4c_1) \), \( Q_{\text{max}}^* = \frac{1}{6}(3 - 2c_1) \), and \( \bar{\theta}^* = 1 \).

(iv) When \( \frac{(3-\delta)(2-\delta)}{9-8\delta} \leq c_1 \leq \frac{1}{8}(7 - 2\delta) \), the retailer sells the product in both periods, it does not carry inventory between periods and the quantity commitment binds weakly, where \( Q_1^* = \frac{7c_1 - 4\delta}{15 - 4\delta} \), \( Q_{\text{max}}^* = \frac{6(3-2c_1) - 5\delta}{2(15 - 4\delta)} \), \( p_1^* = \frac{(4-\delta)(4(1+c_1) - \delta)}{2(15 - 4\delta)} \), and \( \bar{\theta}^* = \frac{8(1+c_1) - 2\delta}{15 - 4\delta} \).

(v) When \( c_1 \geq \max(\frac{1}{8}(7 - 2\delta), \frac{3}{4}) \), the retailer does not procure the product in first period commitment binds weakly where, \( Q_1^* = 0 \), \( Q_{\text{max}}^* = \frac{1}{4} \), and \( \bar{\theta}^* = 1 \).

The threshold \( K_{c_2} \) is defined as follows.

\[
K_{c_2} = \frac{(3 - \delta)(1 - \delta)\delta + \sqrt{\delta^2(27 - 36\delta + 8\delta^3 + \delta^4)}}{9 + (3 - 8\delta)\delta} \quad (6.4)
\]

Recall that weak and strict commitments are defined in section 3.2. Finally following Lemma 16 and Proposition 18, we can characterize the equilibrium.

**Proposition 19.** Suppose the retailer can make a quantity commitment, the supplier’s SPNE wholesale price \( c_1 \) in period 1 is as follows. There exists \( \tilde{\delta}_5 \approx 0.735 \) such that

(i) When \( \delta \leq \tilde{\delta}_5 \), \( c_1^* = \frac{108 - 6(159 - \delta(41 + (15-\delta)\delta))}{108 - 48(69 - 20(8+3\delta))} \).

(ii) When \( \delta > \tilde{\delta}_5 \), \( c_1^* = \frac{9}{16} \).

Here \( \tilde{\delta}_5 \) is explicitly characterized in the proof of this proposition. Parts (i) and (ii) of Proposition 19 map to parts (ii) and (iii) of Proposition 18. Specifically the retailer’s quantity commitment always binds weakly and the retailer always carries inventory.

6.1.2 Supplier’s Quantity Commitment when Customers Discount Net Utility

Our results for this section are based on numerical examples and they are stated in Section 6.1.5.
6.1.3 Retailer’s Price Commitment when Customers Discount Net Utility

Suppose the retailer can credibly commit to future prices then similar to the reasoning in Section 4.2 retailer procures all of its needed quantity in period 1. Hence, although customers discount the second period price as well as their valuation of the product, the equilibrium does not change since retailer does not sell in period 2. Therefore, Lemma 12 and Proposition 11 continue to hold.

6.1.4 Supplier’s Price Commitment when Customers Discount Net Utility

Similar to Section 4.3, the following Lemma describes the equilibrium in period 2.

**Lemma 17.** Suppose that the supplier commits to wholesale prices $c_1$ and $c_2$, the retailer orders $Q_1$ in period 1 and consumer segment $[0, \bar{\theta})$ remain in the market in period 2. The retailer then orders $Q_2^* = \max(\frac{1}{2}(2 - 2Q_1 - \bar{\theta} - c_2), 0)$ units and sets $p_2^* = 1 - Q_1 - Q_2^*$ in period 2.

The marginal customer $\bar{\theta}$ solves the following equation $\bar{\theta} - p_1 = \delta(\bar{\theta} - p_2^*)$, which leads to $\bar{\theta} = \frac{2p_1 - \delta c_2}{2 - \delta}$. Note that similar to the base model the marginal customer in this case is also independent from $Q_1$. Then we can formulate the retailer’s optimization problem in period 1 using equation (4.3), which leads to the following Proposition.

**Proposition 20.** Suppose the supplier commits to wholesale prices $c_1$ and $c_2$. The retailer’s SPNE order quantity $Q_1^*$ and retail price $p_1^*$ in period 1 are as follows.

(i) When $c_2 \leq \frac{2c_1 + \delta - 1}{1 + \delta}$, the retailer sells the product only in period 2, where $Q_1^* = 0$.

(ii) When $\frac{2c_1 + \delta - 1}{1 + \delta} < c_2$, the retailer sells the product in both periods, where $Q_1^* = \frac{1 - 2c_1 + c_2 - (1 - c_2)\delta}{3 - 2\delta}$ and $p_1^* = \frac{2c_1(2-\delta)+(2-\delta)^2-c_2(2-(2-\delta)\delta)}{2(3-2\delta)}$.

The retailer does not carry inventory between periods, thus the marginal customer is given by $\bar{\theta}^* = 1 - Q_1^*$, in addition without loss of generality $c_2 \leq c_1$ in both of the cases.

To fully characterize the equilibrium, we solve the supplier’s problem.

**Proposition 21.** Suppose the supplier can commit to future wholesale prices. The equilibrium wholesale prices $c_1$ and $c_2$ are as follows. $c_1^* = c_2^* = \frac{1}{2}$. 
Proposition 21 corresponds to part (ii) of Proposition 20, hence in equilibrium the supplier commits to wholesale prices such that the retailer buys and sells in both periods.

6.1.5 Value of Commitments when Customers Discount Net Utility

The following Theorem shows that the retailer’s commitments can continue to hurt the profitability of both the retailer and the supplier.

**Theorem 11.** Suppose customer utility function is as in (6.1).

(i) \( \Pi^S_{NC} \geq \Pi^S_{RQC} \); \( \Pi^R_{NC} + \Pi^R_{NC} \geq \Pi^R_{RQC} + \Pi^R_{RQC} \); \( \Pi^S_{NC} \geq \Pi^S_{RQC} \).

(ii) \( \Pi^S_{NC} > \Pi^S_{RPC} \); \( \Pi^S_{NC} + \Pi^R_{NC} > \Pi^S_{RPC} + \Pi^R_{RPC} \); \( \Pi^R_{NC} > \Pi^R_{RPC} \).

Inequalities in part (i) are strict when \( \delta < \frac{1}{132} (127 - \sqrt{817}) \approx 0.745 \). Similarly, the supplier’s price commitments can make the retailer and the supplier worse off.

**Theorem 12.** Suppose customer utility function is as in (6.1). There exists \( \tilde{\delta}_6 \approx 0.659 \) such that \( \Pi^S_{NC} > \Pi^S_{SPC} \) when \( \delta < \tilde{\delta}_6 \); \( \Pi^R_{NC} > \Pi^R_{SPC} \); \( \Pi^S_{NC} + \Pi^R_{NC} > \Pi^S_{SPC} + \Pi^R_{SPC} \).

Note that \( \tilde{\delta}_6 \) is explicitly characterized in the proof of this Theorem.

Finally, it is straightforward to show that the supplier never gets worse off from its quantity commitment as in Theorem 3, since it can always nullify this commitment by choosing an arbitrarily large quantity. However, the retailer may not suffer from the supplier’s quantity commitment with the alternative customer utility function in (6.1). We find that \( \Pi^R_{SPC} \leq \Pi^S_{SPC} \) when \( \delta \in \{0.2, 0.4, 0.6, 0.8\} \). The equilibrium of supplier’s quantity commitment involves several regions similar to our main model (see Section 3.4), thus instead of characterizing equilibrium for all scenarios, we resort to numerical examples in this extension.

It is worthwhile to highlight the differences between the results of our main model and this extension. While in our main model the supplier’s price commitment always hurts its profitability (Theorem 8), the supplier can benefit from its price commitment in this extension when \( \delta \) is sufficiently high (Theorem 12). Intuitively, when the discount factor applies to both product value and retail price in period 2 as in (6.1) (as opposed to only product value as in (2.1)), consumers are willing to pay higher prices in period 2, which enables the retailer and the supplier to set higher prices in period 2. Thus, the supplier can commit to a higher price in period 2, which allows setting
a higher price in period 1. Similarly, the supplier’s quantity commitment can be more beneficial to the retailer with the alternative utility function in (6.1): While the retailer in our main model may suffer from the supplier’s quantity commitment, our numerical examples in this extension show that the supplier’s quantity commitment does not hurt the retailer. Because the supplier can set a higher wholesale price in period 2, it does not excessively limit its quantity, which in turn helps the retailer.

6.2 No-Inventory Carryover

Our main model assumes the retailer does not incur inventory holding cost. Here, we consider the other extreme assuming that carrying inventory is prohibitively costly or infeasible so the retailer cannot carry over inventory.

Since retailer cannot carry inventory, it sells all the quantity that it buys in the first period, hence $Q_1$ is always $1 - \bar{\theta}$. The following Lemma describes the supplier’s and retailer’s optimal policy in period 2.

**Lemma 18.** Suppose consumer segment $[0, \bar{\theta})$ remains in the market in period 2.

(i) For any given wholesale price $c_2$, the retailer orders $Q^*_2 = \max(\frac{1}{2}(\bar{\theta} - \frac{c_2}{\delta}), 0)$ units and sets $p^*_2 = \delta (1 - \bar{\theta} - Q^*_2)$ in period 2.

(ii) The supplier sets $c^*_2 = \frac{\delta \bar{\theta}}{2}$ in period 2.

This yields profits $\Pi_{R,2} = \frac{\delta \bar{\theta}^2}{16}$ and $\Pi_{S,2} = \frac{\delta \bar{\theta}^2}{8}$ for the retailer and supplier respectively.

Following Lemma 18, the marginal customer is given by $\bar{\theta} = \frac{4p_1}{4 - \delta}$. Rearranging this equality, we get $p_1(\bar{\theta}) = (1 - \frac{\delta}{4})\bar{\theta}$. We can now solve for retailer’s optimization problem in period 1, which is given in the following Proposition.

**Proposition 22.** The SPNE retail price $p^*_1$ and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.

(i) When $c_1 < \frac{8 - 3\delta}{8}$ the retailer sells the product in both periods, where $p^*_1 = \frac{(4 - \delta)(4(1 + c_1) - \delta)}{2(16 - 5\delta)}$, and $\bar{\theta}^* = \frac{8(1 + c_1) - 2\delta}{16 - 5\delta}$.

(ii) When $c_1 \geq \frac{8 - 3\delta}{8}$ the retailer does not buy the product in first period.
Observe that parts (i) and (ii) of above proposition is the same as the parts (ii) and (iii) of Proposition 5. Hence by not being able to carry inventory retailer is actually making a weak quantity commitment. The equilibrium is described by the following Proposition.

**Proposition 23.** Supplier sets the SPNE wholesale price as $c_1^* = \frac{4(3-\delta)}{24-9\delta} - \frac{11\delta}{96}$.

In equilibrium supplier sets the price so that retailer always sells the product in period 1.

### 6.2.1 Retailer’s Quantity Commitment with No-Inventory Carry Over

When retailer makes a quantity commitment under the condition that it cannot carry inventory the Lemma 6 continues to hold. In addition Lemma 7 also continues to hold since it is for every $Q_1$, but now $Q_1 = 1 - \bar{\theta}$. Moreover, note that the optimal strategies and the prices for retailer’s first period optimization problem given in Proposition 5 do not involve carrying inventory. Hence, when retailer is not allowed to carry inventory it can replicate the best response given in Proposition 5. Therefore Propositions 5 and 6 also continue to hold.

### 6.2.2 Supplier’s Quantity Commitment with No-Inventory Carry Over

When supplier makes a quantity commitment under the condition that retailer cannot carry inventory the Lemma 8 in Section 3.4 continue to hold. In addition, following the discussion in Section 6.2.1, the commitment condition that follows from the Lemma 7 also continues to hold. Similar to the analysis for supplier’s quantity commitment in Section 3.4, the retailer’s profit function exhibits a piecewise structure where it is given by (3.3) when the commitment binds; on the other hand, when the commitment does not bind, it is given by the no commitment objective that is studied in Section 6.2. Since overall retailer objective function is not quasiconcave we solve for each part and compare in order to find the optimal retail price, order quantity and the marginal customer in period 1, which is given by the following proposition.

**Proposition 24.** Suppose the supplier commits to sell not more than $Q_{\text{max}} \in [1/4, 1/2]$ units in two periods. The retailer’s SPNE policy is characterized as follows.

(i) The quantity commitment binds strictly and the retailer sells the product in both periods with $p_1^* = \frac{1+(1-\delta)c_1-\delta(3-\delta)Q_{\text{max}}}{2(1-2\delta)}$ and $\bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{\text{max}})}{2(1-2\delta)}$, when $\delta < 1/2$, $\frac{1-c_1}{2-\delta} < Q_{\text{max}} < \frac{5-3c_1-\delta}{8-7\delta}$.
The quantity commitment binds strictly, the retailer sells the product only in period 1 with
\[ p_1^* = \bar{\theta}^* = 1 - Q_{\text{max}}, \]
when

a) \( \delta < 1/2, \quad Q_{\text{max}} \leq \frac{1-c_1}{2-\delta}, \)

b.1) \( \delta \geq 1/2, \quad c_1 \leq \frac{5(3-\delta)\delta-4}{2(2+5\delta)}, \quad Q_{\text{max}} \leq K_4, \)

b.2) \( \delta \geq 1/2, \quad c_1 > \frac{5(3-\delta)\delta-4}{2(2+5\delta)}, \quad Q_{\text{max}} < \frac{4-3c_1-2\delta}{2-\delta}. \)

The quantity commitment binds weakly and the retailer sells the product in both periods with
\[ p_1^* = \frac{1}{3}(4-\delta)(1 - Q_{\text{max}}) \quad \text{and} \quad \bar{\theta}^* = \frac{4}{3}(1 - Q_{\text{max}}) \quad \text{when} \quad \max \left( \frac{5-3c_1-4\delta}{8-\delta}, \frac{4-3c_1-2\delta}{2-\delta} \right) \leq Q_{\text{max}} \leq \frac{4(5-3c_1)-7\delta}{2(16-5\delta)}. \]

The quantity commitment does not bind and the retailer sells the product in both periods with
\[ p_1^* = \frac{(4-\delta)(4(1+c_1)-\delta)}{2(16-5\delta)}, \quad \text{and} \quad \bar{\theta}^* = \frac{8(1+c_1)-2\delta}{16-5\delta}, \]
when

a) \( \delta < 1/2, \quad c_1 < 1 - \frac{3\delta}{8}, \quad Q_{\text{max}} > \frac{4(5-3c_1)-7\delta}{2(16-5\delta)} \)

b.1) \( \delta \geq 1/2, \quad c_1 < \frac{5(3-\delta)\delta-4}{2(2+5\delta)}, \quad K_4 < Q_{\text{max}} \)

b.2) \( \delta \geq 1/2, \quad \frac{5(3-\delta)\delta-4}{2(2+5\delta)} \leq c_1 < 1 - \frac{3\delta}{8}, \quad Q_{\text{max}} > \frac{4(5-3c_1)-7\delta}{2(16-5\delta)} \)

When \( c_1 \geq 1 - \frac{3\delta}{8}, \) the retailer does not procure any units in period 1. Note that in all the cases above optimal quantity \( Q_1^* = 1 - \bar{\theta}^* \). The threshold is given by \( K_4 = \frac{1}{2} \left( 1 - c_1 + \sqrt{\frac{\delta(3-c_1(2+5c_1)-\delta)}{16-5\delta}} \right) \)
and is the point when the profits from the cases (ii) and (iv) are equal.

Given above optimal retailer strategies, supplier maximizes its first period profit. However, again the overall supplier objective function is not quasiconcave and we find the optimal strategies by considering each of the above cases separately, which is given in the following proposition.

**Proposition 25.** The supplier’s SPNE wholesale price \( c_1^* \) and quantity commitment \( Q_{\text{max}}^* \) are as follows.

(i) When \( \delta < 1/2, \quad c_1^* = 1/2, \quad Q_{\text{max}}^* = \frac{1}{2(2-\delta)}. \)

(ii) When \( \delta \geq 1/2, \quad c_1^* = \frac{2-\delta}{3}, \quad Q_{\text{max}}^* = \frac{2-\delta}{3-5\delta}. \)

Both of the above optimal supplier strategies correspond to the case (ii) of Proposition 24.
6.2.3 Retailer’s Price Commitment with No-Inventory Carry Over

When retailer commits to future prices it commits not to sell in period 2, which does not change whether it can carry inventory or not. Hence, the analysis in section 4.2 and the Proposition 11 continue to hold.

6.2.4 Supplier’s Price Commitment with No-Inventory Carry Over

When supplier commits to wholesale prices retailer has no incentive to carry inventory since carrying inventory does not change \( c_2 \). Therefore when inventory carrying is not allowed the retailer’s actions does not change and the analysis in section 4.3 holds. Specifically the Propositions 12 and 13 continue to hold.

6.2.5 Value of Commitments with No-Inventory Carry Over

The following Theorem shows that the retailer’s and the supplier’s commitments can continue to hurt either themselves or their supply chain partners.

**Theorem 13.** Suppose the retailer cannot carry over inventory.

(i) \( \Pi_{S}^{NC} > \Pi_{S}^{RQC} \); \( \Pi_{R}^{NC} > \Pi_{R}^{RQC} \).

(ii) \( \Pi_{S}^{NC} > \Pi_{S}^{RPC} \); \( \Pi_{R}^{NC} > \Pi_{R}^{RPC} \).

(iii) \( \Pi_{S}^{NC} < \Pi_{S}^{SQC} \); \( \Pi_{S}^{NC} + \Pi_{R}^{NC} < \Pi_{S}^{SQC} + \Pi_{R}^{SQC} \); \( \Pi_{R}^{NC} \geq \Pi_{R}^{SQC} \) when \( \delta \leq \delta_7 \).

(iv) \( \Pi_{S}^{NC} < \Pi_{S}^{SPC} \); \( \Pi_{S}^{NC} + \Pi_{R}^{NC} \geq \Pi_{S}^{SPC} + \Pi_{R}^{SPC} \) when \( \delta \leq \delta_8 \); \( \Pi_{R}^{NC} > \Pi_{R}^{SPC} \).

Note \( \delta_7 \approx 0.694 \) and \( \delta_8 \approx 0.592 \), and they are explicitly characterized in the proof of this theorem.

Different from our main model, the supplier benefits from its price commitment when the retailer cannot carry over inventory. In our main model, the supplier’s price commitment hurts its profitability because it eliminates the retailer’s incentive to buy more in period 1 to pay less in period 2. However, this incentive is dampened in the no-commitment scenario when the retailer cannot carry over inventory. In other words, the supplier’s price commitment does not decrease
its profitability mainly because its no-commitment profit is lower when the retailer cannot hold inventory.
CHAPTER 7
Commitments under a Coordinating Contract

We have shown that commitments can aggravate existing coordination inefficiency under a wholesale price contract making the supplier and/or the retailer worse off. In contrast, now we will show that under a coordinating contract, commitments never hurt the supplier and the retailer and always improve the profitability of a supply chain by eliminating strategic customer behavior. Specifically, we show that a two-part tariff achieves the profit of the centralized benchmark and commitments are always beneficial with a two-part tariff. When the supplier offers a two-part tariff to the retailer, in each period the supplier charges a lump-sum fee $F_i$ in addition to the wholesale price $c_i$ per unit of product. Then the SPNE can be written as:

\[
(p_2^*, Q_2^*) = \arg \max_{(p_2, Q_2) \in S_2} \left[ p_2(\bar{\theta} - p_2/\delta) - c_2Q_2 - F_2 \right],
\]

(7.1)

\[
(c_2^*, F_2^*) = \arg \max_{c_2, F_2} c_2Q_2^* + F_2,
\]

(7.2)

\[
\bar{\theta}^* = \inf \{ \bar{\theta} - p_1 \geq \delta \bar{\theta} - p_2^*(\hat{Q}_1) \},
\]

\[
(p_1^*, Q_1^*) = \arg \max_{(p_1, Q_1) \in S_1} \left[ p_1(1 - \bar{\theta}) - c_1Q_1 - F_1 + p_2^*(\bar{\theta}^* - p_2^*/\delta) - c_2Q_2^* - F_2^* \right],
\]

(7.3)

\[
\hat{Q}_1 = Q_1^*.
\]

\[
(c_1^*, F_1^*) = \arg \max_{c_1, F_1} [c_1Q_1^* + F_1 + c_2Q_2^* + F_2^*].
\]

(7.4)

where the retailer’s feasible strategy sets $S_1$ and $S_2$ are given by

\[
S_1 = \{(p_1, Q_1) : Q_1 \geq 1 - \bar{\theta}^*(p_1) \geq 0 \},
\]

\[
S_2 = \{(p_2, Q_2) : Q_1 + Q_2 \geq 1 - p_2/\delta \}.
\]

The following Lemma describes the supplier’s and retailer’s optimal policy in period 2.
Lemma 19. Suppose the retailer orders $Q_1$ units in period 1, and consumer segment $[0, \bar{\theta})$ remains in the market in period 2.

(i) For any given wholesale price $c_2$, the retailer orders $Q_2^* = \max(\frac{\delta(2(2Q_1-\bar{\theta})-c_2)}{2\delta}, 0)$ units and sets $p_2^* = \delta(1 - Q_1 - Q_2^*)$ in period 2.

(ii) The supplier sets $c_2^* = 0$ and $F_2^* = \frac{\delta \bar{\theta}^2}{4}$ in period 2.

This yields profits $\Pi_{R,2} = 0$ and $\Pi_{S,2} = \delta \bar{\theta}^2$ for the retailer and supplier respectively.

Consumers conjecture second period price and they expect $p_2^* = \frac{\delta \bar{\theta}}{2}$. Note that second period price no longer depends on the customers’ belief about first period quantity since supplier offers a zero second period wholesale price and there is no incentive for retailer to carry inventory.

The marginal consumer $\bar{\theta}$, who is indifferent about whether to buy in period 1 or wait for period 2, is given by the solution of

$$\bar{\theta} - p_1 = \delta \bar{\theta} - p_2^*, \tag{2.10}$$

where the left and right hand sides correspond to the utility of buying in periods 1 and 2, respectively. This leads to

$$\bar{\theta} = \frac{2p_1}{2 - \delta}. \tag{2.11}$$

Now let’s consider retailer’s first period problem. Following (2.10), in order to induce consumers in $[\bar{\theta}, 1]$ to buy the product in period 1, the retailer needs to set the following price:

$$p_1(\bar{\theta}) = \bar{\theta} \left(1 - \frac{\delta}{2}\right). \tag{2.12}$$

Note that retailer’s margin loss from the price $\bar{\theta}$ increases as the value of product increases in second period due to strategic customer behavior.

After reformulating the retailer’s total profit in terms of the order quantity $Q_1$ and target consumer segment $[\bar{\theta}, 1]$ in period 1 we can solve for the retailer’s optimal policy, which is presented in the following proposition.

Proposition 26. The SPNE order quantity $Q_1^*$ and retail price $p_1^*$ and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.
(i) When \( c_1 \leq 1 - \frac{\delta}{2} \), the retailer sells the product in both periods and does not carry inventory between periods, where \( Q_1^* = \frac{1}{2} - \frac{c_1}{2 - \delta} \), \( p_1^* = \frac{1}{4}(2(1 + c_1) - \delta) \) and \( \bar{\theta}^* = \frac{1}{2} + \frac{c_1}{2 - \delta} \).

(ii) When \( c_1 > 1 - \frac{\delta}{2} \), the retailer sells the product only in period 2, and it does not carry inventory between periods, where \( Q_1^* = 0 \) and \( \bar{\theta}^* = 1 \).

Note that unlike the wholesale price contract retailer never carries inventory to second period, due the fact that second period wholesale price is zero.

To find the equilibrium whole sale price we need to solve the suppliers optimization problem. The following Proposition describes that.

**Proposition 27.** The SPNE unit price \( c_1 \) and the entrance fee \( F_1 \) in period 1 are as follows.

\[
c_1^* = \frac{\delta(2-\delta)}{2(1-\delta)}, \quad F_1^* = \frac{2(2-\delta)(1-\delta)^2}{4(1-\delta)^2}.
\]

Supplier’s equilibrium unit price corresponds to part (i) of Proposition 26. Hence in equilibrium retailer serves the market in both periods. In addition equilibrium supplier profit is \( \Pi_S = \frac{(2-\delta)^2}{4(1-\delta)} \), which is the same as the centralized profit given in section 2.3 and equilibrium retailer profit is 0. Therefore, with two part tariff contract supplier can extract all retailer profit and coordinate the supply chain.

### 7.1 Retailer’s Quantity Commitment with Two-Part Tariff Contract

In this section we analyze the scenario when retailer can make an aggregate quantity commitment when supplier offers a two-part tariff contract. Analysis is similar to section 3.2, specifically Lemma 6 holds. Following equation (3.1) we have the following Lemma that describes the optimal values in second period.

**Lemma 20.** When the retailer’s quantity commitment binds, \( Q_2^* = Q_{\text{max}} - Q_1 \), \( p_2^* = 1 - Q_{\text{max}} \), \( c_2^* = 2(1 - Q_{\text{max}}) - \bar{\theta} \) and \( F_2^* = \delta (Q_{\text{max}}^2 + \bar{\theta} + Q_1(2(1 - Q_{\text{max}}) - \bar{\theta}) - 1) \) in a SPNE in period 2. This yields \( \Pi_{R,2} = 0 \) and \( \Pi_{S,2} = c_2^* Q_2^* + F_2^* \). Furthermore, the retailer quantity commitment binds if and only if \( Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\delta}{4} \).
Following Lemma 20 and the expression for marginal customer we can state retailer’s period 1 problem as follows.

\[
\max_{Q_1, Q_{max}, \theta} \quad [1 - \theta] \quad p_1(\bar{\theta}, Q_{max}) - Q_1 c_1 - F_1 + \Pi_{R,2}(Q_1, Q_{max}, \bar{\theta})
\]

\[
\text{s.t.} \quad Q_{max} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4},
\]

\[
Q_1 \leq Q_{max},
\]

\[
(Q_1, \bar{\theta}) \in S'_1.
\]

Note that \(S'_1\) is defined in (2.15). This leads to the following proposition.

**Proposition 28.** Suppose the retailer can make a quantity commitment. Its SPNE order quantity \(Q_1^*\), quantity commitment \(Q_{max}^*\), retail price \(p_1^*\) and the corresponding marginal customer \(\bar{\theta}^*\) in period 1 are as follows. The retailer sells the product only in period 1 and the quantity commitment binds strictly, where \(Q_1^* = \frac{1-c_1}{2}, Q_{max}^* = \frac{1-c_1}{2}, p_1^* = \frac{1+c_1}{2}, \bar{\theta}^* = \frac{1+c_1}{2} \).

Note that unlike other models we presented so far commitment always binds strictly when retailer makes a quantity commitment under two-part tariff contract.

Following Lemma 20 and Proposition 28 we can characterize the equilibrium.

**Proposition 29.** Suppose the retailer can make a quantity commitment, the supplier’s SPNE unit price \(c_1^*\) and the entrance fee \(F_1^*\) in period 1 are as follows. \(c_1^* = 0, F_1^* = \frac{1}{4} \).

At equilibrium supplier offers a zero unit price but extracts all retailer profit with the lump sum fee. Next theorem describes the value of the commitment. Let \(\Pi_{S}^{NC}\) and \(\Pi_{S}^{RQC}\) be the supplier’s equilibrium profit with no commitment and retailer’s quantity commitment under two-part tariff contract respectively.

**Theorem 14.** \(\Pi_{S}^{RQC} > \Pi_{S}^{NC}\).

Theorem states that supplier benefits from retailer’s quantity commitment. Note that supplier’s equilibrium profit is \(\frac{1}{4}\), which is equal to the centralized commitment profit that is presented in section 2.3. It extracts all the retailer profit hence in equilibrium retailer’s profit is zero. Therefore, retailer’s quantity commitment coordinates the supply chain.
7.2 Supplier’s Quantity Commitment with Two-Part Tariff Contract

Similar to Section 3.4, when (3.7) holds, period 2 equilibrium is given by Lemma 20. On the other hand, when (3.7) does not hold, the commitment is ineffective, and period 2 equilibrium is the same as in a no-commitment scenario, which is given by Lemma 19.

When the commitment binds, i.e., (3.7) holds, the retailer’s profit is given by (7.5). On the other hand, when the commitment does not bind, i.e., (3.7) does not hold, the retailer’s profit is given by (7.3). Solving the retailer’s maximum profit for both cases and comparing them, we determine the retailer’s optimal policy in period 1, which is given in the following proposition.

Proposition 30. Suppose supplier commits to sell not more than $Q_{\text{max}} \leq \frac{1}{2}$ units in two periods. The retailer’s SPNE policy is characterized as follows.

(i) The quantity commitment binds strictly the retailer sells the product only in period 1 with $Q^*_1 = Q_{\text{max}}$ and $p^*_1 = \bar{\theta}^* = 1 - Q_{\text{max}}$ when $Q_{\text{max}} \leq \frac{1-c_1}{2-\delta}$. 

(ii) The quantity commitment binds strictly the retailer sells the product in both periods and it does not carry inventory between periods with $Q^*_1 = \frac{1-c_1-\delta Q_{\text{max}}}{2(1-\delta)}$, $p^*_1 = \frac{1}{2}(1 + c_1 - \delta Q_{\text{max}})$ and $\bar{\theta}^* = \frac{1+c_1-\delta(2-Q_{\text{max}})}{2(1-\delta)}$ when, $\frac{1-c_1}{2-\delta} < Q_{\text{max}} < \min\left(\frac{5-3c_1-2\delta}{8-5\delta}, \frac{1-c_1}{2}\right)$.

(iii) The quantity commitment binds strictly and the retailer does not procure any products in period 1 with $Q^*_1 = 0$, $p^*_1 = \bar{\theta}^* = 1$ when $\frac{1-c_1}{2-\delta} \leq Q_{\text{max}} < \frac{1}{4}$.

(iv) The quantity commitment binds weekly the retailer sells the product in both periods and it does not carry inventory between periods with $Q^*_1 = \frac{1}{3}(4Q_{\text{max}} - 1)$, $p^*_1 = \frac{1}{3}(4 - \delta)(1 - Q_{\text{max}})$ and $\bar{\theta}^* = \frac{4}{3}(1 - Q_{\text{max}})$, when

(a) $c_1 \leq 1 - \frac{\delta}{2}$, $\frac{5-3c_1-2\delta}{8-5\delta} \leq Q_{\text{max}} \leq K_5$

(b) $c_1 > 1 - \frac{\delta}{2}$, $\max\left(\frac{5-3c_1-2\delta}{8-5\delta}, \frac{1}{4}\right) \leq Q_{\text{max}}$.

(v) The quantity commitment does not bind the retailer sells the product in both periods and it does not carry inventory between periods with $Q^*_1 = \frac{1}{2} - \frac{c_1}{2-\delta}$, $p^*_1 = \frac{1}{4}(2(1 + c_1) - \delta)$ and $\bar{\theta}^* = \frac{1}{2} + \frac{c_1}{2-\delta}$, when $c_1 \leq 1 - \frac{\delta}{2}$, $K_5 < Q_{\text{max}}$.

The threshold $K_5$ is defined as $K_5 = \frac{5(4 - \delta) - 3\left(4c_1 - \sqrt{3\left(s(1-c_1^2)-\delta(6-\delta)\right)}\right)}{8(4-\delta)}$.  

64
To characterize the equilibrium we solve for the supplier’s period 1 problem, which is presented in the following Proposition.

**Proposition 31.** Suppose the supplier can make a quantity commitment, the supplier’s SPNE unit price \( c_1 \), commitment quantity \( Q_{\text{max}} \) and the entrance fee \( F_1 \) in period 1 are as follows. \( c_1^* = \frac{\delta}{2}, Q_{\text{max}}^* = \frac{1}{2} \) and \( F_1^* = \frac{1-\delta}{4} \).

The equilibrium strategy comes from the boundary between parts (i) and (ii) of Proposition 30, hence at equilibrium supplier sets the price fee and the commitment quantity such that retailer sells the product only in period 1. Next theorem gives the value of commitment.

**Theorem 15.** \( \Pi_{S}^{\text{SQC}} > \Pi_{S}^{\text{NC}} \).

Similar to retailer’s commitments with two-part tariff contract supplier’s quantity commitment also helps itself. In addition it coordinates the supply chain by allocating all retailer profit to supplier and replicating the centralized benchmark.

### 7.3 Retailer’s Price Commitment with Two-Part Tariff Contract

Suppose retailer can credibly commit to future prices then similar to the reasoning in the section 4.2 retailer procure all of its needed quantity in period 1. Therefore Lemma 12 follows. The following proposition describes the equilibrium in period 1.

**Proposition 32.** Suppose the retailer can commit to future prices under two-part tariff contract. The SPNE is as follows.

(i) Given the unit price \( c_1 \), the retailer orders \( Q_1^* = \frac{1-\epsilon_1}{2} \), and sets \( p_1^* = \frac{1+\epsilon_1}{2} \) in period 1 and it does not sell the product in period 2 by committing to a sufficiently high \( p_2 \) in period 1.

(ii) The supplier chooses \( c_1^* = 0 \) and \( F_1^* = \frac{1}{4} \), \( \Pi_S^* = \frac{1}{4} \).

Next we look at the value of commitment.

**Theorem 16.** \( \Pi_{S}^{\text{RPC}} > \Pi_{S}^{\text{NC}} \).

Similar to retailer’s quantity commitment under two-part tariff contract, retailer’s price commitment also helps supplier. Specifically it can extract all retailer profit by letting it ending up with zero profit and it earns the commitment benchmark profit, which is equal to \( \frac{1}{4} \).
7.4 Supplier’s Price Commitment with Two-Part Tariff Contract

When supplier commits to future prices it commits to period 2 unit price and the entrance fee when it sets the period 1 unit price and entrance fee. Given \( c_1, F_1, c_2 \) and \( F_2 \) the retailer’s actions and strategies are the same as in the case of wholesale price contract described in section 4.3. Specifically, Lemma 13 and Proposition 12 follows.

To find the equilibrium we need to solve for supplier’s optimization problem. Supplier maximizes

\[
 c_1 Q_1^* + F_1 + c_2 Q_2^* + F_2,
\]

where \( Q_1^* \) and \( Q_2^* \) are given by Proposition 12 and Lemma 13. The following proposition describes the equilibrium.

**Proposition 33.** Suppose supplier can commit to future prices and fees then equilibrium unit prices and the fees are as follows. \( c_1^* = 0, F_1^* = \frac{1}{4}, c_2^* = 0 \) and \( F_2^* = 0 \).

Proposition 33 corresponds to part \( (iv) \) of Proposition 12. So in equilibrium supplier sets the prices and fees such that she induces retailer to sell only in period 1.

Next we look at the value of commitment.

**Theorem 17.** \( \Pi_{SPC} > \Pi_{NC} \).

Like all the commitment scenarios under two-part tariff contract we presented so far supplier’s price commitment also helps supplier. In addition it coordinates the supply chain by allocating all retailer profit to supplier and attaining the centralized benchmark commitment profit.

7.5 Concluding Remarks

In both the supplier’s and retailer’s price and quantity commitments, the supplier attains the profit of the centralized system leaving no profit to the retailer. Furthermore, the sales are restricted to only period 1 in these examples similar to the commitments of the centralized system.

Note that the supplier can replicate any outcome of a two-part tariff by a quantity discount contract. This can be achieved by simply including the lump-sum fee \( F_i \) in the wholesale price.
of the first unit in the quantity discount contract. Therefore, with a quantity discount, the supplier can extract the entire surplus achieving profit of the centralized system and ensure that the commitments are always beneficial as in Theorems in previous sections.
CHAPTER 8
Implications of Uncertainty when Selling to Strategic Customers

8.1 Introduction

Demand uncertainty is one of the main problems firms face. It makes the already hard planning problem more complicated. Variability in demand causing shortages or excessive production can be a source of profit loss. To mitigate variability in demand and better match with the supply, companies need to implement sophisticated policies. However, such techniques can be quite costly; involving investments in information technology, customer relationship management and advanced analytical capabilities for accurately forecasting the demand. In addition, with the recent developments in on-line retailing and reduced search costs consumers are more price aware. Rather than buying the products on the spot, they strategically time their purchase decisions to maximize their own utility. It is a well established result that forward looking customers, ie. customers that wait for future price markdowns harm the firm profits. This adds another layer of complexity to the operating environment for firms because the interactions between the fluctuations in demand and the customer patience may not be obvious. When demand is highly volatile one may expect customers being forward looking may not be an important issue, since the uncertainty in the late season availability might prevent them from waiting strategically. However, customers may also expect a higher left over quantity since they may believe the demand might be low and may choose to wait longer. Similarly, when customers are highly patient initial observations of sales quantity may not give an accurate picture about the actual demand level. Therefore, it is important to investigate the effects of the forward looking customer behavior together with the volatility in the demand so that we can have a better understanding about the interplay between these two factors.

Given the limited resources it is important for decision makers to assess the relative severity of the demand variability and the strategic customers. When firms make decisions about policies to reverse the implications of these factors they need to be able to evaluate the relative importance of
each of these elements. For example, an extra replenishment within the selling season may help to alleviate potential loss sales, however it may also signal forward looking customers a higher level of inventory availability later in the season with a reduced price. Hence, encouraging customers to delay purchases. In addition, commitments such as fixed price path or limited quantities may benefit firms in terms of inducing customers to buy early with high price, however such policies may also restrict the flexibility against the changes in the demand levels and may leave the firm vulnerable to fluctuations; causing loss sales or overstocking. Moreover, depending on how closely firms can observe these factors and which decisions they may control may have an affect on how they might want to react. For example a manufacturer selling directly to customers can observe both the changes in demand levels and the customer behavior directly. Hence, its priorities in mitigating the loss due to these elements may vary greatly than a retailer in a decentralized supply chain which also can observe both of the factors directly but has a more restricted set of decisions that it can control. Similarly, a supplier selling through a retailer may not be able to observe the customer patience directly but is affected by it through the order decisions made by its retailer. Hence, it still needs to take into account the relative importance of such customer behavior when making decisions about the policies to reduce the affects of the uncertainty in the order quantities. In addition, the interactions between the volatility in demand and the customer patience adds another layer of complexity to the interplay between the up and downstream members in a decentralized supply chain. One might argue that both might intensify the coordination problem the supply chain already experiences. However, it might also be the case that actions taken by one of the supply chain members might have a decreasing affect on one of the losses due either to the volatility in demand or the strategic behavior for the other member. In order to be able to better understand these interactions we need a model that captures the necessary aspects of this problem so that we can draw relevant insights.

In this chapter of the thesis we consider a decentralized supply chain selling to strategic customers with an uncertain demand for the product. We consider two demand scenarios; either high or low and capture the uncertainty by varying the total market size. We determine the level of strategic behavior by the customer patience parameter, which affects the utility a customer will receive if it waits for a price drop. By driving demand uncertainty and the customer patience independently we are able to draw insights about the relative effects of the two on the profit per-
formance. In addition we explicitly model the interactions between the supplier and retailer, which give us the opportunity to compare the implications of uncertainty and customer patience on the up and down stream of the supply chain. Specifically, we introduce metrics such as loss due to uncertainty and loss due to strategic behavior.

We show that demand variability intensifies the strategic behavior in a decentralized supply chain. This is interesting since increasing levels of uncertainty also creates availability or price risks for the customers. However, we find that the price and quantity choices made by the supply chain members causes customers to delay their purchase.

We show that both the uncertainty and the strategic customer behavior are damaging for the supply chain members’ profit performance. More interestingly we find that the severity of these factors in terms of how they impact the profit differ when considered from supplier’s or the retailer’s point of view. We observe that unless there is very little variability in demand or the customers are significantly patient, uncertainty has more impact on the supplier’s total loss than the strategic behavior. In contrast, unless there is very little customer patience or the demand is very volatile strategic behavior is more detrimental to retailer’s profit than the uncertainty. In addition, we quantify the impacts of these factors on the supply chain members’ profits and compare their performance. We find that uncertainty generates a bigger loss for supplier than the retailer, however customer patience hurts retailer more than it damages the supplier. These observations form the evidence that supply chain members may have different priorities for loss mitigating strategies. For example while supplier might want to install policies to control the fluctuations in order quantities; retailer might want to improve on measures that would induce customers to buy earlier with higher prices, which in turn might aggravate coordination problem in the decentralized supply chain.

We also investigate the centralized benchmark and contrast against the decentralized system to better our insights about the decentralized supply chain’s performance. We find that similar to the decentralized supply chain, demand variability intensifies the strategic waiting behavior in the centralized system. Moreover this is even stronger than the decentralized supply chain. This suggests that the interaction between the decentralized supply chain members in terms of how one’s price and quantity decisions affect the other one’s choices, discourages some of the customers from delaying their purchases, which does not exist in a centralized system.
In addition, centralized firm and the retailer are similar when we compare the impact of the strategic behavior and the demand volatility on each one’s total loss such that forward looking customer behavior has a bigger impact than the demand uncertainty. However, recall that demand uncertainty has more impact on the supplier’s loss than the customer patience. This suggests that the effect of customer behavior is dumped on the demand volatility by the retailer through the changes in order quantities. In other words, since supplier is unable to observe the strategic behavior directly, the repercussion of such behavior appears as fluctuations in order quantities, which intensifies the impact of demand uncertainty on the supplier’s profit loss.

We also compare centralized system’s price and quantity decisions against the decentralized supply chain to shed light on the driving factors of the specific choices made by each of its members. We find that different dynamics are at play when firms are making optimal decisions, which lead to different characteristics for the resulting profit performances. We show that demand uncertainty decreases centralized system’s loss due to strategic customers. When demand is highly volatile the centralized firm keeps the availability low early in the selling season to limit the exposure to uncertainty risk and provides a bigger service level later in the season when it has a better understanding of the demand level. In addition when the customers are sufficiently patient they want to delay the purchase, hoping for a reduced price. This alignment in the firm’s and the customers’ preferences provides the necessary ingredient for such a result. In contrast, demand uncertainty has a bimodal effect on the decentralized supply chain’s loss in the presence of strategic customers. As the volatility in the demand increases up to a certain level, the decentralized supply chain’s loss due to customer patience increases. However, beyond that threshold any further increase in variability actually decreases the decentralized supply chain’s loss due to strategic customer behavior. This result is due to how supplier pricing manipulates the retailer quantity decisions in a decentralized supply chain selling to strategic customers. When supplier makes pricing decisions it prevents retailer to take advantage of the potential alignment in its preferences with the customers’ choices, which in turn, amplifies the coordination problem that increases the loss due to strategic customers. However, when uncertainty is significantly high supplier’s leverage on the retailer quantities decreases, which causes the loss to decrease as well. These results provide important insights about the implications of the demand volatility especially when designing mechanisms to mitigate the adverse affects of these factors. For example policies to reduce the profit loss due to
forward looking customers such as commitments might be effective in a centralized system even when there is demand uncertainty, although the value of the commitment is likely to decrease with higher levels of uncertainty. However, it might be a better idea to deal with the uncertainty in a decentralized supply chain even with low levels of variability when customers are strategic since it is likely that a bigger strategic loss is being observed due to volatility in the demand.

The rest of the chapter is organized as follows; in Section 8.2 we introduce our model and in 8.3 the decentralized equilibrium. Using the equilibrium we evaluate the supply chain performance in Section 8.4. Next we provide the decentralized equilibrium in 8.5 and in Section 8.6 we contrast the two equilibriums to tease out the affects of decentralization. We conclude with Section 8.7.

8.2 The Model

The sequence of events is similar to the no-commitment model described in Section 2. Specifically, before the start of each period $i$, the supplier sets its wholesale price $c_i$, and the retailer decides its procurement quantity $Q_i$, which is delivered at the beginning of the period. When retailer orders $Q_i$ it faces a demand risk because of the uncertainty. The retailer then sets the retail price $p_i$, and consumers decide whether to buy the product in that period. Retailer may carry the unsold inventory to second period. It incurs a holding cost $h \in [1/4, 3/8]$ per unit inventory carried over. Note that holding cost is critical in this model in order to avoid unrealistic strategies in equilibrium.

We assume that after all the decisions made for period 1 uncertainty resolves before the start of period 2. The sequence of events is summarized in Figure 8.1.
We model the demand uncertainty as a discrete distribution. Let $X$ be the market size. Then the following defines the total mass of the customers.

$$X = \begin{cases} 
1 + \alpha & \text{with probability } \beta \\
1 - \alpha & \text{with probability } 1 - \beta
\end{cases} \quad (8.1)$$

Note that in our model with deterministic demand the total mass of the customers is 1. To keep the mean demand normalized at 1 we assume $\beta = 1/2$. In addition we assume that demand variability is no bigger than half the mean demand so $\alpha \in [0, 1/2]$. Throughout this chapter we adopt the net utility discount as in Section 6.1, which is defined in Equation 6.1.

An SPNE in our model with demand uncertainty is defined by the solutions of the following equations. We assume that retailer will order at least to satisfy the low market demand in period 1. Note that * denotes SPNE strategies, $\wedge$ denotes the minimum and $\vee$ denotes the maximum operator.

$$(p_2^*, Q_2^*) = \arg\max_{(p_2, Q_2) \in \tilde{S}_2} E[p_2 X(\bar{\theta} - p_2) - c_2 Q_2], \quad (8.2)$$

$$c_2^* = \arg\max_{c_2} E[c_2 Q_2^*], \quad (8.3)$$

$$\bar{\theta}^* = \inf \left\{ \theta : \theta - p_1 \geq \delta \left( \theta - E[p_2^*(\hat{Q}_1)] \right) \right\}, \quad (8.4)$$

$$(p_1^*, Q_1^*) = \arg\max_{(p_1, Q_1) \in \hat{S}_1} E[p_1 (Q_1 \wedge X(1 - \bar{\theta}^*)) - c_1 Q_1 - h((Q_1 - X(1 - \bar{\theta}^*) \vee 0)) + \Pi_{R,2}^*], \quad (8.5)$$

$$\Pi_{R,2}^* = p_2^* X(\bar{\theta}^* - p_2^*) - c_2^* Q_2^*, \quad (8.6)$$

$$\hat{Q}_1 = Q_1^*, \quad (8.7)$$

$$c_1^* = \arg\max_{c_1} E[c_1 Q_1^* + c_2^* Q_2^*]. \quad (8.8)$$
where the retailer’s feasible strategy sets \( \tilde{S}_i \) in each period are given by

\[
\tilde{S}_1 = \{(p_1, Q_1) : Q_1 \geq (1 - \alpha) \left( 1 - \hat{\theta}^*(p_1) \right) \geq 0 \},
\]

\[
\tilde{S}_2 = \{(p_2, Q_2) : Q_1 + Q_2 \geq X(1 - p_2) \}.
\]

### 8.3 Equilibrium

We solve for equilibrium using backward induction following the order in equations (8.2-8.8). In period 2 uncertainty has resolved and the market size is realized. Retailer orders \( Q_2 \) and sets \( p_2 \) to extract maximum profit from customers. Note that if the demand is low retailer might have ordered too much in period 1 therefore it may choose \( p_2 \) in such a way that it does not sell all the remaining inventory. Let \( \chi \) be the market size realized, which is \( 1 - \alpha \) according to Equation 8.1.

**Lemma 21.** Suppose retailer orders \( Q_1 \) in period 1 and the consumer segment \([0, \bar{\theta}]\) remains in the market in period 2.

(i) For any given wholesale price \( c_2 \), the retailer orders \( Q_2^* = \chi \left( 1 - \frac{Q_1}{\chi} - \frac{\bar{\theta} + c_2}{2} \right) \) and sets \( p_2^* = 1 - \frac{Q_1 + Q_2^*}{\chi} \), when \( Q_1 \leq \chi \left( 1 - \frac{\bar{\theta}}{2} \right) \). Otherwise it does not order and sets \( p_2^* = \bar{\theta} \).

(ii) The supplier sets \( c_2^* = 1 - \frac{Q_1}{\chi} - \frac{\bar{\theta}}{2} \) when \( Q_2^* > 0 \).

This yields profits \( \Pi_{R,2} = \frac{\chi}{16} \bar{\theta}^2 + \frac{3(\chi - Q_1) \bar{\theta}}{4} - \frac{3(\chi - Q_1)^2}{4\chi} \) and \( \Pi_{S,2} = \frac{(2Q_1 - (2 - \bar{\theta})\chi)^2}{8\chi} \), when \( Q_1 \leq \chi \left( 1 - \frac{\bar{\theta}}{2} \right) \). Otherwise \( \Pi_{R,2} = \frac{\bar{\theta}^2 \chi}{4} \) and \( \Pi_{S,2} = 0 \).

Next we analyze the consumers’ decision in period 1. The customers conjecture period 2 expected price, given their beliefs about the retailer’s inventory level \( \hat{Q}_1 \). Specifically, following Lemma 21 they expect

\[
E \left[ p_2^*(\hat{Q}_1) \right] = \begin{cases} 
\frac{1}{2} \left( 1 - \frac{\hat{Q}_1}{1 - \alpha} \right) + \frac{\bar{\theta}}{2} & \text{when } \hat{Q}_1 \leq (1 - \alpha) \left( 1 - \frac{\bar{\theta}}{2} \right) \\
\frac{1 + \alpha - \hat{Q}_1}{4(1 + \alpha)} + \frac{3\bar{\theta}}{8} & \text{otherwise}
\end{cases}
\]

(8.11)
The marginal customer $\bar{\theta}$, who is indifferent between buying in period 1 or waiting for period 2 is given by the following, using Equations 8.4 and 8.11.

$$\bar{\theta} = \begin{cases} \frac{2((1-\alpha^2)(2p_1-\delta)+\delta \hat{Q}_1)}{(1-\alpha^2)(4-3\delta)} & \text{when } \hat{Q}_1 \leq (1-\alpha) \left(1 - \frac{\bar{\theta}}{2}\right) \\ \frac{2((1+\alpha)(4p_1-\delta)+\delta \hat{Q}_1)}{(1+\alpha)(8-3\delta)} & \text{otherwise.} \end{cases} \quad (8.12)$$

Now, we consider retailer’s period 1 problem. We can determine retailer’s optimal price by solving for the optimal consumer segment that it should target in period 1. Following Equation 8.12 the retailer needs to set the following price in order for the customers in $[\bar{\theta}, 1]$ to buy in period 1.

$$p_1(\bar{\theta}, \hat{Q}_1) = \begin{cases} \left(1 - \frac{3\delta}{4}\right) \bar{\theta} + \frac{\delta}{2} \left(1 - \frac{\hat{Q}_1}{1-\alpha}\right) & \text{when } \hat{Q}_1 \leq (1-\alpha) \left(1 - \frac{\bar{\theta}}{2}\right) \\ \left(1 - \frac{5\delta}{8}\right) \bar{\theta} + \frac{\delta}{4} \left(1 - \frac{\hat{Q}_1}{1+\alpha}\right) & \text{otherwise.} \end{cases} \quad (8.13)$$

Note that in both cases of the Equation 8.13 retailer has a margin loss from the marginal customer’s valuation and this loss increases as the customers believe that there are more units purchased in period 1.

We can now formulate retailer’s total expected profit in terms of first period quantity $Q_1$ and target consumer segment $[\bar{\theta}, 1]$. Note that retailer’s expected profit function constitutes a piecewise nature. Feasible decision set is divided into four regions depending on whether first period order quantity is enough to serve the high market first period demand and whether retailer replenishes in second period. The following Definition formally describes these strategies and corresponding regions.

**Definition 1.** Depending on the first period order quantity and the market size retailer’s strategies and the corresponding feasible regions are as follows:

(i) The retailer orders enough only to serve the low market demand in period 1 and replenishes in period 2 both in a high or low market, when

$$(1-\alpha)(1-\bar{\theta}) \leq Q_1 \leq \left((1-\alpha) \left(1 - \frac{\bar{\theta}}{2}\right) \wedge (1+\alpha)(1-\bar{\theta})\right).$$

(ii) The retailer orders enough only to serve the low market demand in period 1 and replenishes in period 2 if the demand is high however does not replenish if the demand is low, when

$$(1-\alpha) \left(1 - \frac{\bar{\theta}}{2}\right) \leq Q_1 \leq (1+\alpha)(1-\bar{\theta}).$$
(iii) The retailer orders enough to serve the high market demand in period 1 and replenishes in period 2 both in a high or low market, when \((1 + \alpha)(1 - \tilde{\theta}) \leq Q_1 \leq (1 - \alpha)\left(1 - \frac{\tilde{\theta}}{2}\right)\).

(iv) The retailer orders enough to serve the high market in period 1 and replenishes in period 2 if the demand is high however does not replenish if the demand is low, when \((1 - \alpha)\left(1 - \frac{\tilde{\theta}}{2}\right) \vee (1 + \alpha)(1 - \tilde{\theta}) \leq Q_1\).

Note that in the first two cases retailer runs out of product in the first period when the demand is high. In addition in cases (ii) and (iv) it has a leftover quantity in second period if the demand is low. For every case we write the expected total profit for retailer in terms of \(Q_1\) and \(\tilde{\theta}\) using the Equation 8.13 and Lemma 21. Then we optimize subject to the feasible regions in Definition 1 and belief consistency in Equation 8.7. We omit the details for clarity in here and characterize the optimization problem in the proof of the next proposition, which describes the retailer’s first period optimal actions.

**Proposition 34.** When \(c_1 \geq 3/10\) the SPNE order quantity \(Q_1^*\) and retail price \(p_1^*\) and the corresponding marginal customer \(\tilde{\theta}^*\) in period 1 are as follows.

(i) When \(c_1 \leq \frac{7 - 2\delta + 2\alpha(3 - 4h)}{8(1 + \alpha)}\), the retailer sells the product in both periods, and it carries inventory in between periods if the demand is low, where \(Q_1^* = \frac{(1 - \alpha^2)(7 - 2\delta + 2\alpha(3 - 4h) - 8(1 + \alpha)c_1)}{15 - 8\delta - 3\alpha(1 - 4(\alpha + \delta))}\), \(p_1^* = \frac{4(1 - \alpha)(4(1 - \alpha) - \delta(1 - 3\alpha))c_1 - (4(1 - \Delta - 3\delta)h - 3(12 - 5\delta))\alpha^2 + \alpha(4h(4 - \delta) - 4 + 14\delta - 3\delta^2) + (4 - \delta)(4 - 3\delta)}{2(12\alpha^2 - 3(1 - 4\delta)\alpha + (15 - 8\delta))}\) and \(\tilde{\theta}^* = \frac{2(4(1 - \alpha^2)c_1 + (9 - 4h)\alpha^2 - (1 - 5\delta - 4h)\alpha + (4 - 3\delta))}{12\alpha^2 - 3(1 - 4\delta)\alpha + (15 - 8\delta)}\).

(ii) When \(c_1 > \frac{7 - 2\delta + 2\alpha(3 - 4h)}{8(1 + \alpha)}\), the retailer does not procure in period 1 and sells only in period 2, where \(Q_1^* = 0\), \(p_1^* = 1 - \frac{\delta}{4}\) and \(\tilde{\theta} = 1\).

Note that retailer’s best response strategy comes from the case (iii) in Definition 1. In part (i), \((1 - \alpha)(1 - \tilde{\theta}^*) < Q_1^* = (1 + \alpha)(1 - \tilde{\theta}^*)\), where the lower bound of the feasible region is binding, that is retailer sells less than it orders if the demand is low, thus it carries inventory into the next period. However, in part (ii) first period wholesale price is too high that it becomes non-profitable to purchase and sell in period 1. Note that we provide the best response strategies for retailer only for \(c_1 \geq 3/10\). The strategies when \(c_1 < 3/10\) needlessly complicate the exposition since they are off the equilibrium path. However, in the proof of this proposition we show that there does not exist any equilibrium solution where \(c_1^* < 3/10\).
Finally, we consider the supplier’s period 1 problem. The supplier chooses $c_1$ to maximize its total expected profit:

$$
\Pi_S = c_1 Q_1^* + E[\Pi_{S,2}(c_1, Q_1^*, \bar{\theta}^*)],
$$

(8.14)

where $Q_1^*$ and $\bar{\theta}^*$ are given by Proposition 34 and $\Pi_{S,2}$ follows from Lemma 34. The following Proposition describes the supplier’s optimal policy.

**Proposition 35.** The SPNE wholesale price in period 1 is given by

$$
c_1 = \frac{8(9-4h)\alpha^3 + (16(7-6h)\delta - 2(7-36h))\alpha^2 - (24\delta^2 - 2(23+32h)\delta - 69-104h))\alpha + (16\delta^2 - 98\delta + 121)}{32(4\alpha^3 + (1+6\delta)\alpha^2 + (2+6)\delta + (7-4h))}.
$$

Firms’ optimal policies in period 1 are described by Propositions 34 and 35. Lemma 21 defines the second period optimal decisions and (8.12) gives the marginal customer. Therefore, the equilibrium of the game between the supplier, retailer and consumers is fully characterized by Propositions 34 and 35, Lemma 21 and (8.12).

Propositions 34 and 35 show that supplier set the first period wholesale price sufficiently low that retailer always buys in both periods. However, first period wholesale price is not as low as that it is never feasible to carry inventory to second period when the demand is high. Indeed the equilibrium first period wholesale price is not lower than $3/10$.

Now that we have fully characterized the equilibrium we can analyze the optimal decisions and the corresponding expected profit values for supplier and retailer to figure out the implications of demand uncertainty and the customer patience. Specifically, we want to understand how the strategic customer behavior changes, how the supply chain parties’ optimal decisions are made and what the implications are on the supply chain performance as the demand uncertainty changes when the supply chain faces forward looking customers.

### 8.4 Supply Chain Performance

It is worthwhile to investigate the optimal decisions to better understand the inner workings of the equilibrium. We are especially interested in how the changes in the demand uncertainty changes the equilibrium values. From customers’ point of view while in a low demand market there is an opportunity for deep cuts in retailer price, in a high demand market there is a risk of not being able to afford the product. On one hand we might think that high demand variability
should prevent customers from waiting but on the other hand there could be a good chance of receiving a much lower price in period two. Therefore, it is not trivial to determine how customer patience affects the equilibrium decisions when there is demand uncertainty. Specifically we would like to find out how the market segments in equilibrium and how the portion of customers that time their purchase changes when there is variability in demand. We need the following Definition of equilibrium market composition to answer this question.

**Definition 2.** At equilibrium a customer with valuation $\theta$ belongs to the market segment:

(i) First period buyers when $\theta > \bar{\theta}^*$,

(ii) Strategic waiters when $\bar{\theta}^* > \theta > p_1^*$

(iii) Second period buyers when $p_1^* > \theta > p_2^*$

When uncertainty resolves if the demand is high some of the strategic waiters defined in case (ii) above are actually rationed out since the realized second period optimal retail price is bigger than first period price, which only happens when $\delta$ is sufficiently high. Note that this is not contrary to the rational expectations equilibrium because the expected $p_2$ is not higher than $p_1$, hence these customers would not deviate from their decisions.

**Theorem 18.** Let $\mu = \int_{p_1^*}^{p_2^*} \frac{d\theta}{\int_{p_2^*}^{p_1^*} d\theta}$ be the fraction of customers that wait strategically for a price drop in period two, then $E[\mu]$ increases as the demand becomes more volatile.

Theorem 18 states that in a supply chain selling to forward looking customers demand uncertainty intensifies the strategic behavior. This suggests that equilibrium optimal values formed in such a way that supply chain members are affected more from the variations in demand rather than the customers. Specifically, we find that optimal choices by the supply chain members leave them prone to customers’ strategic actions such that as the demand variability increases the market composition moves to a more forward looking mix. Next we investigate the implications of these on the supply chain performance.

So far we showed that demand uncertainty intensifies the strategic customer behavior. In the following we investigate the implications of this on retailer and supplier profits separately. We are particularly interested in how supplier and retailer experiences this change in market dynamics and
how the supply chain profit is allocated. The following Corollary describes the affects on the firm profits.

**Corollary 4.** Both supplier’s and retailer’s as well as the supply chain’s profit decreases when demand becomes more volatile and/or customers become acting more strategically.

Corollary 4 tells us that both the demand uncertainty and the strategic customer behavior have negative affects on the supply chain members’ profits. Although this result is expected we need it to use as a basis for our main investigation about the relative severity of these two perils. The following definition gives us a good measure that can be used for this purpose.

**Definition 3.** Let \( \Pi^i(\alpha, \delta) \), \( i \in \{S, R\} \) be supplier’s and retailer’s profit respectively. Then

\[
\mathcal{L}^i_\delta = \frac{(\Pi^i(\alpha, 0) - \Pi^i(\alpha, \delta)) + (\Pi^i(0, 0) - \Pi^i(0, \delta))}{2(\Pi^i(0, 0) - \Pi^i(\alpha, \delta))}, \quad i \in \{S, R\}
\]

defines the relative loss due to strategic behavior for supplier and retailer. Similarly, \( \mathcal{L}^i_\alpha = \frac{(\Pi^i(0, \delta) - \Pi^i(\alpha, \delta)) + (\Pi^i(0, 0) - \Pi^i(0, \alpha))}{2(\Pi^i(0, 0) - \Pi^i(\alpha, \delta))}, \quad i \in \{S, R\} \) defines the relative loss due to demand uncertainty for supplier and retailer.

Above measures of loss give us the fraction of the total decline in profit due to either of the two factors, namely demand uncertainty and the strategic customer behavior. Note that \( \Pi \) also depends on the holding cost \( h \) but we omit that dependence here for clarity of exposition. Following Corollary 4 both of the relative loss measures are monotone, continuous and have values in \((0, 1)\) provided \( \{\alpha, \delta\} > 0 \). By comparing \( \mathcal{L}_\delta \) and \( \mathcal{L}_\alpha \) for supplier and retailer we can find out the regions where either the demand uncertainty or the strategic behavior has a dominant effect on the supply chain member’s profit loss. The following Lemma describes which of the factors has a dominant impact on each of the supply chain members’ profit loss within our parameter space.

**Lemma 22.** For the relative loss measures defined in Definition 3

\[
(i) \quad \int 1_{\{\mathcal{L}^S_\delta > \mathcal{L}^S_\alpha\}} d\delta d\alpha < \int 1_{\{\mathcal{L}^S_\delta < \mathcal{L}^S_\alpha\}} d\delta d\alpha \\
(ii) \quad \int 1_{\{\mathcal{L}^R_\delta > \mathcal{L}^R_\alpha\}} d\delta d\alpha > \int 1_{\{\mathcal{L}^R_\delta < \mathcal{L}^R_\alpha\}} d\delta d\alpha
\]

Lemma 22 says that while strategic customers have a bigger impact on the retailer’s profit loss, demand uncertainty plays a bigger role on the supplier’s profit decline. This suggests that supplier would not be affected by strategic customers as it would be affected from the demand uncertainty. This is expected since the supplier does not observe the customers’ decisions directly but only
through retailer’s order quantity, which are passed on to supplier as variability in demand. On the other hand retailer not only deals with forward looking behavior directly but also has to have a solution for demand uncertainty. Clearly, when demand is predictable but customers are forward looking, strategic behavior will have more affect on the profit loss. Similarly, when demand is highly uncertain but customers are acting myopically, demand variability will have a bigger impact. The interesting case is when both of these factors are in the middle range where it is hard to determine intuitively how to attribute the loss. In this case, our relative loss measures give us an important insight as we formally state next.

**Theorem 19.** Given relative loss measures in Definition 3 then, \( \int 1_{\{L_S^\delta < L_{\alpha}^S \cap L_R^\delta > L_{\alpha}^R\}} d\delta d\alpha > 0. \)

Theorem 19 states that there exists a region where the relative loss priorities for the supply chain firms differ from each other. We plot this region in Figure 8.2. In region (i) of Figure 8.2 relative loss due to uncertainty is bigger than relative loss for strategic customer behavior for both retailer and the supplier, ie. \( L_{\alpha}^S > L_S^\delta \) and \( L_{\alpha}^R > L_R^\delta \). Similarly in region (iii) \( L_{\alpha}^S < L_S^\delta \) and \( L_{\alpha}^R < L_R^\delta \). Both of these are expected as we described above. However, in region (ii) when there is moderate to high demand uncertainty and customer patience, while customer foresight has a bigger impact on the retailer’s profit loss, demand uncertainty has a bigger affect on the supplier’s

![Figure 8.2: Relative importance regions for the loss factors when \( h = 1/4 \).](image-url)
profit performance. This finding suggests that in a supply chain, companies would have different incentives in dealing with these factors. We can see how this difference in relative importance of the factors impacting the supply chain members’ profits plays a role in moving the equilibrium as the underlying parameters change. This brings us to one of our main research questions. Do supply chain parties alter their key decisions based on the level of demand uncertainty when the customers are strategically timing their purchase decisions?

**Corollary 5.** For any value of $\alpha$ that determines the demand variability, as the customer patience increases;

(i) $c_1^*$ and $p_1^*$ decrease monotonically,

(ii) $Q_1^*$ increases, while $\bar{\theta}^*$, $c_2^*$, $p_2^*$ and $Q_2^*$ decrease if and only if $\alpha < \bar{\alpha}_1 = \frac{\sqrt{2(32h^2 - 4h + 23) - 4(2h + 1)}}{2(5 - 12h)}$.

Corollary 5 describes the relationship between the uncertainty parameter and the holding cost where the equilibrium is formed in such a way that the optimal decisions increase the number of customers that buy in period one. Specifically, when $\alpha < \bar{\alpha}_1$ supplier’s optimal wholesale price choices induce the retailer to move the sale towards first period by increasing the period one quantity and decreasing the second period quantity. This makes more customers to buy in period one instead of waiting for the second period. However, retailer’s margin loss also increases, which means that retailer has to offer a much less price to make those customers buy in period one. In addition retailer is able to react like this because demand uncertainty is still relatively low and holding cost does not constitute a big issue. In contrast when $\alpha > \bar{\alpha}_1$ uncertainty becomes too much of a risk for the retailer and it no longer can afford the same service level in the first period. Instead, it decreases period one quantity and sells more in period two, which also increases the marginal customer’s valuation.

In addition, following Corollary 5 we observe that when $\alpha < \bar{\alpha}_1$, supplier’s and retailer’s relative loss priorities align in such a way that as the customer patience increases equilibrium decisions cause less customers to delay purchase in period one. One can attribute these choices to the supplier’s understanding of the main factor that drives its loss, ie. uncertainty. Since increasing the sale in the first period is consistent with the goal towards decreasing loss due to uncertainty; it makes sense for supplier to choose the equilibrium first period wholesale price that way. In addition more
customers buying in period one instead of waiting for a reduced price in period two is a good outcome for dealing with strategic customer loss. Therefore retailer’s optimal first period choice is consistent with its incentive. However, the mismatch of the priorities in the decision choices fails to prevent the loss for the supply chain. Moreover it exacerbates the coordination problem the supply chain already is experiencing.

Up until now we tried to show that supplier and retailer have different understandings about the main profit loss driver when operating in an environment when both demand uncertainty and the strategic customer behavior are effective. Next we try to quantify the impact of these factors on the supply chain profits by introducing the fractional loss measures.

**Definition 4.** Given $\Pi^i(\alpha, \delta)$, $i \in \{S, R\}$ supplier’s and the retailer’s profit. The fractional loss due to strategic customer behavior is defined as $\Omega^i_\delta(\alpha, \delta) = 1 - \frac{\Pi^i(\alpha, \delta)}{\Pi^i(\alpha, 0)}$, $i \in \{S, R\}$ and the fractional loss due to demand uncertainty is defined as $\Omega^i_\alpha(\alpha, \delta) = 1 - \frac{\Pi^i(\alpha, \delta)}{\Pi^i(0, \delta)}$, $i \in \{S, R\}$.

Fractional loss measures $\Omega_\alpha$ and $\Omega_\delta$ take into account only the difference in objective along the change in one of the factors that impact the profit by keeping the other constant. Note we showed in Corollary 4 that profits decrease as variability in demand ($\alpha$) or the customer patience ($\delta$) increases. Therefore, $\Omega$ is non-negative. In addition since it is normalized we can use it to compare the impact of uncertainty and the strategic customers on the supplier and retailer profits, which we present in the following Theorem.

**Theorem 20.** Let $\Omega^i_j$, $i \in \{S, R\}$ and $j \in \{\alpha, \delta\}$ be the fractional loss due to uncertainty and the fractional loss due to strategic behavior for the supplier and the retailer respectively. Then for any $\alpha$ and $\delta$,

(i) $\Omega^S_\alpha > \Omega^R_\alpha$.

(ii) $\Omega^S_\delta < \Omega^R_\delta$.

Theorem 20 states that when demand variability increases while customer patience stays the same, supplier’s percent decline in its profit would be bigger than that of retailer’s profit. Similarly when customers’ strategic behavior intensifies while the demand uncertainty remains the same, retailer’s percent loss in its profit will be higher than that of supplier’s profit. Note that this result complements the insight we gathered through the relative loss measures and the Lemma 22, such
that from supplier’s point of view demand uncertainty is not only a more important problem than
the strategic customer behavior but also it has a bigger impact on the supplier’s profit loss than
that of retailer’s. Similarly, having customers waiting for a price decrease constitutes a bigger issue
for retailer rather than the changes in the market size, in addition the impact of more patient
customers on the retailer’s profit is bigger than the supplier’s profit. Next we investigate how the
fractional loss due to uncertainty changes as the customer patience increases.

**Corollary 6.** For any \( \alpha \) that drives the demand uncertainty as the customer patience increases

(i) \( \Omega^S_\alpha \) increases,

(ii) there exists \( \bar{\delta}_9 \) such that \( \Omega^R_\alpha \) increases if and only if \( \delta < \bar{\delta}_9 \).

Corollary 6 says that supplier’s fractional loss increases as the customers become more pa-
tient. This is in line with our observation through the relative loss measures we provided in
Definition 3, such that the effects of the forward looking customers is passed on to the sup-
plier through the fluctuations in order quantities by the retailer, which increases the supplier’s
loss due to uncertainty. Moreover the increase in supplier’s uncertainty loss is bigger than re-
tailer’s loss; in fact when \( \delta > \bar{\delta}_9 \) retailer’s uncertainty loss decreases as the level of strategic
behavior increases. Note that \( \bar{\delta}_9 \) is around the middle range; for example when \( h = 1/4 \) and
\( \alpha = \{1/20, 1/10, 3/20, 1/5, 1/4, 3/10, 7/20, 2/5, 9/20, 1/2\} \) then
\( \bar{\delta}_9 \approx \{0.59, 0.58, 0.57, 0.56, 0.55, 0.53, 0.51, 0.48, 0.45, 0.40\} \)

Corollary 6 together with Theorem 20 provides further evidence that the supply chain’s coor-
dination problem intensifies with demand uncertainty when facing forward looking customers. If
either wants to mitigate its loss it will have a different priority in designing a policy. Therefore any
attempt to alleviate the affects of these factors without coordination might cause harm to the other
party. Furthermore any coordinating policy has to take into account both the demand uncertainty
and the affects of customer patience.

So far we focused on individual supply chain members to get an understanding of the impli-
cations of demand uncertainty separately. Next we introduce the centralized system to use as a
benchmark to further our insights about the supply chain as a whole.
8.5 Centralized System Benchmark

In this section we consider centralized supply chain as a benchmark. For simplicity and to be consistent with the decentralized system we normalize the production costs to zero. However, centralized manufacturer also incurs a holding cost $h$ per the units that are not sold in period 1. Similar to the decentralized model centralized system sets the retail prices $p_i$ in each period $i : 1, 2$ and consumers decide whether to buy the product in that period. Next Lemma describes the second period decisions. Similar to Section 8.3 let $\chi$ be the market size realized, which is $1 \mp \alpha$ according to Equation 8.1.

Lemma 23. Suppose centralized manufacturer produces $Q_1$ in period 1 and the consumer segment $[0, \bar{\theta})$ remains in the market in period 2. Then the firm produces $Q_2^* = \chi \left(1 - \frac{Q_1}{\chi} - \frac{\theta}{\chi}\right)$ and sets $p_2^* = 1 - \frac{Q_1 + Q_2^*}{\chi}$, when $Q_1 \leq \chi \left(1 - \frac{\theta}{\chi}\right)$. Otherwise it does not produce and sets $p_2^* = \frac{\bar{\theta}}{\chi}$.

Following the same analysis in Section 8.3 we can formulate centralized firm’s expected first period profit in terms of first period quantity $Q_1$ and the target consumer segment. Manufacturer’s profit is also a piecewise function. The following Definition explains the regions and the corresponding decisions.

Definition 5. Based on $Q_1$ and the market size, centralized firm’s strategies and the corresponding feasible regions are as follows:

(i) The firm produces enough to serve the low market demand in period 1 and produces more to match the demand in period 2 both in a high or low market, when $(1 - \alpha)(1 - \bar{\theta}) \leq Q_1 \leq (1 + \alpha)(1 - \bar{\theta})$.

(ii) The firm produces enough to serve the high market demand in period 1 and produces more to match the demand in period 2 both in a high or a low market, when $(1 + \alpha)(1 - \bar{\theta}) \leq Q_1 \leq (1 - \alpha) \left(1 - \frac{\bar{\theta}}{\chi}\right)$.

Note that in the first case above the firm may run out of product if the demand is high and in second case it can carry inventory if the demand is low. Next Proposition defines the equilibrium.

Proposition 36. The SPNE order quantity $Q_1^*$ and retail price $p_1^*$ and the corresponding marginal customer $\bar{\theta}^*$ in period 1 are as follows.
(i) When \( \delta \leq 1 - 2h\alpha \) the firm sells the product in both periods, and it carries inventory in between periods if the demand is low, where \( Q_1^* = \frac{(1+\alpha)(1-2h\alpha-\delta)}{2-2\delta} \), \( p_1^* = \frac{(2-\delta)(1+h\alpha-\delta/2)}{2-2\delta} \) and \( \bar{\theta}^* = \frac{2(1+h\alpha-\delta/2)}{2-2\delta} \).

(ii) When \( \delta > 1 - 2h\alpha \) the firm sells the product only in period 2, where \( Q_1^* = 0 \) and \( \bar{\theta}^* = 1 \).

Both of the cases in Proposition 36 comes from case (ii) of Definition 5, and the lower bound on the \( Q_1 \) constraint binds. Note that when the holding cost and / or the demand uncertainty is unrealistically high centralized firm sell the product only in period 2. Although this case is part of the equilibrium it is unlikely to happen in a real life scenario. Proposition 36 and Lemma 23 fully characterize the centralized system equilibrium. Next we use the performance of the centralized system as a benchmark to better understand the implications of decentralization when the system is selling to forward looking customers and it suffers from demand variability.

8.6 Centralized vs. Decentralized System Performance

Recall that we discussed the customers’ trade off of buying in period one or waiting for a sale through the variability in demand point of view in Section 8.4. Although same situation applies to the customers in a centralized system; different dynamics are at play in terms of how the equilibrium is attained. Therefore, it is important to analyze the equilibrium market composition and how it changes with the uncertainty as well as contrasting against the centralized equilibrium market mix to tease out how differently both systems react to strategic customer behavior. Following the customer segments in Definition 2 the following Theorem answers this question.

**Theorem 21.** Let \( \mu^C \) be the fraction of customers that wait strategically for a price drop in period two in a centralized system, which is defined the same way as in Theorem 18 then,

(i) \( E[\mu^C] \) increases as the demand becomes more variable,

(ii) there exists \( \bar{\delta}_{10}(h,\alpha) \) such that \( E[\mu] < E[\mu^C] \) if and only if \( \delta < \bar{\delta}_{10} \).

Theorem 21 shows that, similar to the decentralized supply chain, in a centralized system demand uncertainty intensifies the strategic customer behavior. Moreover, expected number of customers that wait strategically at equilibrium is more in a centralized system than that of decentralized supply chain provided customers are sufficiently patient. This suggests that although
the decentralized supply chain suffers from double marginalization and coordinating against the forward looking customers it still is less prone to strategic behavior than the centralized system. The following Corollary shows the affects on the centralized system’s profit.

**Corollary 7.** *Unless the centralized firm sells the product only in period two in equilibrium, its profit decreases when the demand becomes more volatile (a larger $\alpha$) or the customers become more patient (a larger $\delta$).*

Similar to decentralized supply chain, centralized system is also adversely affected from both the strategic behavior and the demand uncertainty. Next we utilize the relative loss measures given in Definition 3 to give us an insight about the centralized manufacturer’s understanding of the loss severity.

**Lemma 24.** Let $L^*_C$, $j \in \alpha, \delta$ the relative loss measures for the centralized system defined the same way as in Definition 3 then

\[
\int 1_{\{L^*_C \delta > L^*_C \alpha\}} d\delta d\alpha > \int 1_{\{L^*_R \delta > L^*_R \alpha\}} d\delta d\alpha.
\]

Lemma 24 states that similar to decentralized retailer, strategic customers have a bigger impact on the centralized profit than the demand uncertainty. This is expected according to our findings from Theorem 21, where we show that volatility in demand increases the strategic waiting behavior. The following theorem provides further insights about how the decentralization plays a role on the relative loss due to customer patience by comparing decentralized supply chain’s and the centralized manufacturer’s loss priority.

**Theorem 22.** Given $L^*_i$, $i \in \{R, C, S\}$ and $j \in \{\alpha, \delta\}$ defined the same way as in Definition 3 then,

(i) $\int 1_{\{L^*_C \delta > L^*_C \alpha\}} d\delta d\alpha > \int 1_{\{L^*_R \delta > L^*_R \alpha\}} d\delta d\alpha,$

(ii) $\int 1_{\{L^*_C \delta > L^*_C \alpha\}} d\delta d\alpha > \int 1_{\{L^*_S \delta > L^*_S \alpha\}} d\delta d\alpha.$

Theorem 22 shows that the region where the centralized manufacturer experiences a bigger loss from strategic behavior, ie. region where $L^*_\delta > L^*_\alpha$, is bigger than that of retailer and supplier, which can be seen in Figure 8.3. For example, in region (i) of Figure 8.3a both $L^*_i \delta < L^*_i \alpha$, $i \in \{R, C\}$. Similarly in region (iii), $L^*_i \delta > L^*_i \alpha$, $i \in \{R, C\}$. However, in region (ii), $L^*_R \delta < L^*_R \alpha$ but $L^*_C \delta > L^*_C \alpha$. Regions in Figure 8.3b have the same interpretation except they correspond to centralized firm versus the decentralized supplier. These observations suggest that decentralization and particularly
Figure 8.3: Relative importance regions for the loss factors for a centralized firm and the decentralized retailer and supplier, when $h = 1/4$.

the decisions at equilibrium protects supply chain members from further exposure to the adverse affects of strategic customer behavior. This result shows further indication that decentralization may provide some prevention from the disadvantages of selling to forward looking customers. We can see that the centralized system suffers more when selling to strategic customers. Next we investigate whether this is due to the way how centralized firm makes the key decisions.

**Corollary 8.** Given any $\alpha$ that determines the demand variability, then at equilibrium as the customer patience increases centralized firm decreases $p_1$, $Q_1$ and increases $p_2$, $Q_2$. This increases marginal customer’s valuation $\bar{\theta}$.

Corollary 8 shows that centralized system’s optimal decisions are monotone with respect to the strategic customer behavior. In particular as the customers become more eager to wait for second period the firm decreases the service level in period one and increases in period two. Recall we show in Corollary 5 that in decentralized equilibrium when demand variability is low retailer actually increases the period one quantity. Here, Corollary 8 shows that absent supplier pricing centralized firm does not employ this strategy. Shifting sales from first period to second decreases the risk of uncertainty. As $\delta$ increases, customers’ willingness to wait for second period and the firm’s incentive
to sell in period two align. By decreasing period one quantity centralized firm decreases the risk due to uncertainty, however it fails to address the strategic customer behavior as we show next.

**Theorem 23.** Let $\Omega^SC_\delta$, $\Omega^C_\delta$ be the fractional loss due to strategic behavior for the decentralized supply chain and the centralized system respectively. Unless the centralized firm sells the product only in period two in equilibrium, $\Omega^SC_\delta < \Omega^C_\delta$.

Theorem 23 shows that although the decentralized supply chain suffers from lack of coordination, it is more efficient in terms of dealing with strategic customers. Note that decentralized supply chain still suffers from double marginalization such that $\Pi^SC < \Pi^C$. However, information disparity and the difference between the retailer and supplier in the decisions that they can control creates the opportunity to make equilibrium decisions in such a way that the fractional loss due to strategic behavior to become less than that of centralized system. From the retailer’s point of view not only demand uncertainty but also the strategic customer behavior creates a profit risk. However, retailer can only manipulate the demand by the retailer prices but cannot control the availability directly because of the supplier pricing. So it does not have full power to act upon the uncertainty risk. On the other hand centralized firm can control both the prices and the availability. Centralized system makes decisions such that it reduces the cost due to demand variability by carefully controlling the availability, however it fails to address the strategic customer behavior. This can better be depicted by the Figure 8.4.

![Figure 8.4](image_url)

**Figure 8.4:** Fractional loss due to uncertainty for centralized firm and decentralized supply chain.

We see that when there is little uncertainty in the demand and the customers are not patient centralized firm’s uncertainty loss is bigger than the decentralized supply chain. However, as the
uncertainty increases customers need to be very impatient for that to happen. When demand is sufficiently volatile then centralized system always has less fractional uncertainty loss than the decentralized supply chain.

Recall that in Section 8.4 we commented on the supply chain members’ decision choices given in Corollary 5 that the supplier’s and retailer’s optimal decisions align with their understanding of relative loss severity when \( \alpha < \tilde{\alpha}_1 \). Here, centralized manufacturer’s decisions are not in line with the loss priority we get through the relative loss measures. However, note that centralized system lacks any leverage to make decisions as such. Since the only strategy that might decrease the loss due to strategic customers can be either price or quantity commitments, which the centralized system cannot implement without a credible commitment device. On the other hand decentralized system achieves such an equilibrium because supplier moves first and determines the first period wholesale price by which it can always manipulate the retailer’s first period quantity choice. Therefore, it is not because supplier and retailer better coordinate when selling to strategic customers but because the supplier’s pricing decisions provide means for retailer to act upon and better allocate the quantities to both periods so that some of the strategic loss maybe prevented. Next we show that while decentralized supply chain equilibrium has less fractional loss due to strategic behavior than the centralized system, it is more susceptible to demand uncertainty.

**Theorem 24.** Let \( \Omega_{SC}^C \) the fractional loss due to strategic customers for the supply chain defined the same way as in Definition 4 using the total profit, ie. \( \Pi^S + \Pi^R \). For any \( \delta \) that determines the customer patience, as the variability in demand increases,

(i) \( \Omega_{SC}^C \) decreases,

(ii) there exists an \( \tilde{\alpha}_2(h, \delta) \) such that \( \Omega_{SC}^C \) increases if and only if \( \alpha < \tilde{\alpha}_2 \).

Note that \( \tilde{\alpha}_2 \) is closer to the upper bound. When \( h = 1/4 \) and \( \delta = \{0, 1/4, 1/2, 3/4, 1\} \) then \( \tilde{\alpha}_2 \approx \{0.46, 0.43, 0.40, 0.37, 0.32\} \).

Theorem 24 provides us the chance to shed light on the interesting dynamics that are at play when forming the decentralized and the centralized equilibriums. Below we first explain the centralized result to form the basis for our explanation on the decentralized equilibrium. We then introduce Corollary 9 to help us explain the second part of the Theorem.
First part of Theorem 24 shows that a higher demand variability results in a lower fractional loss due to strategic customers for the centralized system. To better understand why, it is helpful to look into how the underlying fractional loss is given in Definition 4. Note that following Corollary 7 both the numerator and the denominator decreases as $\alpha$ increases. Here, the decline in the myopic profit, ie. $\Pi(\alpha, 0)$ is faster than the profit with strategic behavior, ie. $\Pi(\alpha, \delta)$. To understand why the myopic profit decreases faster than the strategic profit the result in Corollary 8 is helpful. Recall that when we investigate the centralized system’s equilibrium decisions in Corollary 8 we observe that the firm’s and the customers’ incentives to move the sale towards the second period align when customers are forward looking. However, when $\delta = 0$, ie. customers are myopic this alignment vanishes. Therefore, increasing demand variability has a higher impact on the equilibrium with the myopic customers than that with strategic customers.

In contrast, second part of Theorem 24 says that unless the demand variability is sufficiently high supply chain’s fractional loss due to strategic behavior increases. Following Corollary 4 both the numerator and the denominator term in the fractional loss measure for supply chain decrease. In this case when $\alpha < \bar{\alpha}_2$ and as the demand becomes more volatile strategic profit, $\Pi(\alpha, \delta)$, decreases faster than the myopic profit, $\Pi(\alpha, 0)$. To understand why there is this contrast between centralized firm and the decentralized supply chain we need to look into how supplier’s first period wholesale price decision affects retailer’s quantity choices. Clearly, if supplier increases $c_1$ retailer will decrease the first period quantity and increase the second period quantity. However, the level of demand volatility determines how big of an impact supplier pricing will have on the retailer’s quantity decisions. The following Corollary describes this relationship.

**Corollary 9.** Let $Q_1^h$, $Q_2^h$ and $Q_2^l$ are the retailer’s first period, second period high and low market best response quantities given in Proposition 34. Then as $\alpha$ increases,

(i) there exists $\bar{\alpha}_3(\delta)$ such that $\left| \frac{dQ_1}{dc_1} \right|$ and $\frac{dQ_2^h}{dc_1}$ increases if and only if $\alpha < \bar{\alpha}_3$,

(ii) there exists $\bar{\alpha}_4(\delta) > \bar{\alpha}_3$ such that $\frac{dQ_2^l}{dc_1}$ increases if and only if $\alpha < \bar{\alpha}_4$.

Corollary 9 shows that the impact of supplier’s first period wholesale price choice has an increasing effect on the retailer’s quantity choices unless the uncertainty is sufficiently high. Otherwise this impact weakens.
Let’s first consider the case of low demand variability. When customers are myopic ($\delta = 0$) they want to buy in first period, which aligns with supplier’s preference of how the retailer allocates quantity to both periods. In other words supplier wants retailer to sell in period one because of the demand uncertainty. However, when customers are strategic ($\delta > 0$) they want to wait for second period, which does not align with the way how supplier manipulates the retailer quantities through $c_1$ choice. As $\alpha$ increases the supplier’s impact on the quantity choices intensifies, which amplifies the misalignment in strategic case. Therefore the decline in the supply chain profit for the strategic case is faster than the myopic case.

Now, when there is high volatility in demand as we showed in Corollary 9 supplier’s impact on the retailer’s quantity choices weakens. Although the misalignment in the strategic case still exists between the supplier’s and the customers’ choices, retailer’s tendency to sell in second period becomes more effective similar to the centralized case. Therefore, the decline in $\Pi(\alpha, 0)$ becomes bigger than the decline in $\Pi(\alpha, \delta)$, which decreases the supply chain’s fractional loss due to strategic behavior.

### 8.7 Concluding Remarks

There have been many work on the strategic customers recently answering questions about dynamic pricing policies, commitments, supply chain performance with different contracts etc., showing that the forward looking behavior can be very harmful to the firm profits. In this research we extend this line of work by considering, the selling to strategic customers problem in a supply chain setting with demand uncertainty. Our careful modeling choices enable us to drive strategic behavior independently from the demand uncertainty, ie. without customers facing a rationing risk, which provides us mechanisms to quantify and compare both of these loss factors separately. We find that supply chain members are not in agreement about the main causes of the loss they observe in their profits. While this misalignment exacerbates the coordination problem, it provides an equilibrium such that the expected customers strategically delaying the purchase is smaller than that of the centralized system. In addition we show that the way uncertainty affects the firms’ equilibrium decisions and the loss metrics we provide is different within the decentralized supply chain and across the decentralized and centralized systems.
These insights have important implications. First, strategic customer behavior may not always be the highest priority for a firm to deal with depending on how much variability in the demand it observes and the supply chain structure. This has to be taken into account when making investment decisions such as advanced analytical products. While it maybe more important for the supplier to generate better forecast it maybe beneficial for the retailer to invest in customer relationship management to mitigate forward looking behavior.

Second, our results show that when designing a coordinating contract for the decentralized supply chain one has to take into account both the demand variability and the intensity of the customer patience as they have different implications on the supplier and retailer.

Third, our findings show that centralized and decentralized equilibriums are driven by much different dynamics when there is variability in demand and the system is facing strategic customers. This provides evidence that policies that mitigate for example the adverse effects of demand variability does not always apply to the decentralized system. This shows that there is need for designing such policies that are special for the decentralized supply chain.
CHAPTER 9
Conclusion and Future Research

In this dissertation we investigate the selling to strategic customers problem in a supply chain setting composed of a supplier and a retailer. There have been some work where the supply chain perspective is pursued as well, however our work differs from them in some significant ways. We consider endogenous retailer and supplier pricing. This provides us the required richness in the model to capture the interactions in a supply chain. We consider a two-period selling season and a second replenishment opportunity for the retailer before period two. This provides further flexibility to our model, which plays an important role in driving certain insights.

In our base model, in period one, the supplier sets the wholesale price, the retailer places its order from the supplier and sets its retail price, and then forward-looking strategic customers decide whether to buy the product in that period. Retailer can carry any of the unsold inventory to period two. Before the second selling season supplier announces the second period wholesale price and retailer decides the order quantity and the second period retail price considering its inventory level and the remaining demand in the market. Then those customers who can afford the price purchase the product. Here we consider the total market size as deterministic, which provides a baseline for analytically tractable models that we investigate later. We provide the sub-game perfect equilibrium of this model where in each period every decision is made optimally by each player in the game given the global information and their beliefs about the unobserved parameters.

Next we focus our attention to various commitment scenarios. Previous studies have shown that a firm’s ability to make commitments is effective in dealing with strategic customer behavior. However, those studies consider only the interactions between a retailer and consumers. In this dissertation, we study the value of commitments in a decentralized supply chain by also taking into account how commitments affect the interactions between the retailer and its supplier. We start with various quantity commitment scenarios where supplier and retailer declare decisions about the quantities unilaterally or simultaneously and stick with them throughout the entire selling season.
We next investigate many price commitment settings where either the supplier or the retailer announces a predefined price path unilaterally or simultaneously. For each of the commitment models we show the equilibrium optimal values, compare them to the baseline performance and we provide the value of such commitments.

We find that while commitments always increase profitability of a centralized system, in a decentralized supply chain, the retailer's and supplier's commitments do not necessarily improve their profitability. Furthermore, they can hurt the other partner in the supply chain, thereby making the whole supply chain worse off. This is because such commitments can exacerbate double-marginalization inefficiency in a supply chain. Without addressing the coordination problem, then, commitments cannot address the strategic customer problem in a decentralized supply chain.

Having shown that quantity and price commitments are not always profitable in a decentralized supply chain in the next chapter we compare the performance of these models for the supplier and retailer. In addition we analyze the scenarios where a supply chain member commits to both quantity and price concurrently. We find that in general quantity commitment is a more flexible tool when compared to price commitment in terms of how it provides responses to the counterpart in the supply chain and how it helps in dealing with the strategic customers. However, committing both the quantity and price takes way that flexibility.

After doing an extensive investigation about the value of several commitment scenarios in a supply chain setting in order to evaluate the robustness of our results we alter two key modelling choices we made initially and reevaluate our findings. We first provide the model with an alternative utility discounting assumption where customers discount the net utility they gain in period two as oppose to discounting only their valuation. With this change we reestablish the baseline equilibrium and the corresponding quantity and price commitment equilibriums for the supplier and retailer. Next we provide the model where it is not feasible for retailer to carry over the unsold inventory to period two and the rest of the main commitment models with this assumption. We find that in general, our main result about the commitments that they may not necessarily be valuable in a decentralized supply chain continues to hold even when these key modelling assumptions are altered. This provides the evidence that our results are not due to our modelling choices but rather due to the interactions in a supply chain.
Recall that the findings we have discussed so far are driven with a wholesale price contract in a decentralized supply chain. A natural direction to analyze further is then a more sophisticated mechanism that coordinates the supply chain members. Therefore we investigate the value of commitments with the two part tariff contract. We show that under this contract commitments never hurt the supplier and the retailer and always benefits the supply chain.

Note that the total market size is deterministic in the models that we analyze the commitments, which enables analytical tractability. Earlier studies (e.g., Aviv and Pazgal 2008, Cachon and Swinney 2009) show that demand uncertainty can make commitments less valuable in a centralized system. This is because making commitments impairs a firm’s ability to respond to demand variability. However, in a decentralized supply chain, the effect of demand uncertainty may be more intricate because of the vertical interactions between the retailer and the supplier. As the final investigation in this dissertation we extend our base model by introducing the demand variability. We drive the volatility in the demand and the customer patience separately, which enables us to evaluate the implications of uncertainty on the strategic customer behavior. We first provide the equilibrium of the decentralized system. Using the optimal values at equilibrium we investigate the effects of demand uncertainty on the equilibrium market composition, supply chain members’ profit performance, relative importance of the demand volatility and the customer patience in terms of how they affect the total loss of each of the supply chain members as well as the percent loss they observe in the presence of forward looking customers. These comparisons within the decentralized supply chain enables us to drive certain insights about the supply chain members’ loss mitigation preferences.

In addition we analyze the centralized benchmark and compare the performance of the centralized and decentralized systems when selling to strategic customers with demand uncertainty. We find that although decentralized system suffers from double marginalization inefficiency the fraction of customers that delay their purchases in equilibrium and the corresponding loss in the presence of strategic customers is smaller than those of the centralized system. After establishing these results we then analyze the implications of demand uncertainty on the profit loss for the two systems and show how the vertical interaction in the decentralized system is effecting the changes in profit loss. In particular we explain the intricate relationship between the supplier pricing and the retailer’s
order quantities and how the changing levels of demand volatility impact the decentralized system’s loss due to strategic customers.

There are many possible directions that this research can evolve towards in the future. As we mentioned earlier the opportunity to replenish before the second period provides flexibility to the retailer. In contrast carrying inventory also provides leverage for the retailer on the second period wholesale price. One interesting extension would be to investigate the value of this second period replenishment when retailer incurs a holding cost for the unsold inventory in period one and selling to a mixture of customers that some are myopic and some are forward looking. One natural extension to the model with demand uncertainty would be to investigate the existence and the performance of the coordinating contracts for the underlying supply chain that we considered.
APPENDIX A
Explicit Statement of Period 2 Equilibrium Values

In the main text, equilibrium 2 values are stated in terms of period 1 quantities. This is because those expressions are obtained by backward induction. By plugging in equilibrium values of period 1 variables into those expressions, we can derive explicit expressions for period 2 equilibrium outcomes. These are stated in the following table.

**Table A.1:** Summary of the period 2 equilibrium values

<table>
<thead>
<tr>
<th>Region</th>
<th>( \bar{\theta} )</th>
<th>( c_2 )</th>
<th>( p_2 )</th>
<th>( Q_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>( \frac{16}{27} \leq \delta &lt; \frac{16}{21} )</td>
<td>( \frac{3}{7} )</td>
<td>( \frac{35}{8} )</td>
<td>( \frac{19}{16} )</td>
</tr>
<tr>
<td>NC</td>
<td>( \frac{16}{21} &lt; \delta )</td>
<td>( \frac{2(1-3\delta)}{16-15\delta} )</td>
<td>( \delta(1-3\delta) )</td>
<td>( \frac{36(4-3\delta)}{21(16-15\delta)} )</td>
</tr>
</tbody>
</table>
| RQC    | \( \delta \leq 0.249 \) | \( \frac{3}{4} \) | \( \frac{\delta(4(16-5\delta+4)-5\delta)}{6(16-5\delta)} \) | \( \frac{36(4(16-5\delta+4)-5\delta)}{10(16-5\delta)} \) | \( 0 \)
| RQC    | \( \delta > 0.249 \) | \( \frac{8}{5\sqrt{16-5\delta}} + \frac{2}{5} \) | \( \delta(4(\sqrt{16-5\delta}+4)-5\delta) \) | \( \frac{4(4(\sqrt{16-5\delta}+4)-5\delta)}{5(16-5\delta)} \) | \( \frac{2}{5\sqrt{16-5\delta}} + \frac{1}{10} \)
| RPC    | \( \delta \leq 1- \frac{1}{3} \) | \( \frac{3}{4} \) | \( \frac{35}{8} \) | \( \frac{92}{16} \) | \( \frac{3}{16} \)
| SQC    | \( \frac{1}{2} \leq \delta < \frac{199-3\sqrt{137}}{218} \) | \( 1 - \frac{2(2-3\delta)}{5\sqrt{16-5\delta}} \) | \( \frac{5-6\delta}{7-5\delta} \) | \( \frac{3}{7} \) | \( 0 \)
| SQC    | \( \frac{199-3\sqrt{137}}{218} \leq \delta < 0.754 \) | \( \frac{3}{4} \) | \( \frac{35}{8} \) | \( \delta(1-K_2) \) | \( \frac{1-K_2}{3} \)
| SQC    | \( 0.754 \leq \delta \) | \( \frac{4}{3}(1-K_2) \) | \( \frac{2}{3}\delta(1-K_2) \) | \( \delta(1-K_2) \) | \( 0 \)
| SPC    | \( \delta \leq 1 - \frac{1}{\sqrt{17}} \) | \( \frac{2(16-5\delta)}{16-13\delta} \) | \( \frac{26(5-4\delta)}{16-13\delta} \) | \( \delta(11-9\delta) \) | \( \frac{1-\delta}{16-13\delta} \)
| SPC    | \( \delta > 1 - \frac{1}{\sqrt{17}} \) | \( \frac{2-\delta}{4-3\delta} \) | \( \delta(6-5\delta) \) | \( \delta(10-7\delta) \) | \( \frac{2-3\delta}{4(4-3\delta)} \)
APPENDIX B
Proofs

The proofs are presented in the order their corresponding results appear in the paper.

B.1 Proofs of Chapter 2

Proof of Lemma 1. Since $c_2 > 0$, the constraint in (2.9) binds. So we can write $p_2$ in terms of $Q_2$, where $p_2 = \delta(1 - Q_1 - Q_2)$. Plugging this result into the optimization problem in (2.2) leads to

$$Q_2^* = \arg \max_{Q_2} -\delta Q_2^2 - (c_2 - (2(1 - Q_1) - \bar{\theta})\delta)Q_2 + \delta(1 - Q_1)(Q_1 + \bar{\theta} - 1)$$  \hspace{1cm} (B.1)

which is a concave function of $Q_2$. First-order condition yields optimal $(p_2, Q_2)$ in part (i) of Lemma 1.

Optimization problem in (2.3) then yields the following.

$$c_2^* = \arg \max_{c_2} \frac{1}{2\delta} \left(-c_2^2 + \delta(2(1 - Q_1) - \bar{\theta})c_2\right)$$  \hspace{1cm} (B.2)

First-order condition gives the optimal $c_2$ in part (ii) of the lemma.

\[\square\]

Proof of Proposition 1. The retailer’s total profit in equation (2.12) can be written as:

$$\Pi_R(Q_1, \bar{\theta}, \hat{Q}_1) = -\frac{3\delta}{4} Q_1^2 - \left(1 - \frac{13\delta}{16}\right) \bar{\theta}^2 - \frac{3\delta}{4} \bar{\theta} Q_1$$  \hspace{1cm} (B.3)

$$- \left(c_1 - \frac{3\delta}{2}\right) Q_1 - \frac{1}{2} \delta(1 - \bar{\theta})\hat{Q}_1 + \frac{1}{4} (2(2 - \delta)\bar{\theta} - \delta).$$

Hessian of equation (B.3) is

$$\begin{pmatrix}
-\frac{3\delta}{2} & -\frac{3\delta}{4} \\
-\frac{3\delta}{4} & -2 + \frac{13\delta}{8}
\end{pmatrix}, \text{ and the eigenvalues of the Hessian are}
$$

$$-\frac{1}{16} \left(16 - \delta + \sqrt{256 - \delta(800 - 769\delta)}\right)$$
and 
\[-\frac{1}{16} \left( 16 - \delta - \sqrt{256 - \delta (800 - 769\delta)} \right).\]

Since both of the eigenvalues are non-positive, equation (B.3) is jointly concave in \((Q_1, \bar{\theta})\). Hence, KKT conditions and equation (2.14) fully characterize the solution, which yield the regions and optimal values in Proposition 1.

**Proof of Proposition 2.** To find the supplier’s best response to the retailer’s optimal strategies we need to analyze all three cases in Proposition 1.

For case (i) in Proposition 1, \(\Pi_S = \frac{1}{16(8-\delta)} \left( -2(32 - 25\delta)c_1^2 - 9\delta(12 - 11\delta)c_1 \right)\), which is a concave function of \(c_1\). 
\[
\frac{d\Pi_S}{dc_1} \Bigg|_{c_1 = \frac{3\delta(4 - 3\delta)}{2(16 - 15\delta)}} = \frac{23}{42} + \frac{2}{56 - 49\delta} - \frac{8}{48 - 45\delta} > 0 \text{ for } \delta < 4/5. \text{ Hence, optimal } c_1 \text{ is not in the interior of this region.}
\]

For case (ii), in Proposition 1, \(\Pi_S = \frac{1}{2(16 - 9\delta)^2} \left( -32c_1^2(8 - 5\delta) + 2c_1(8 - 5\delta)(16 - 3\delta) + (4 - 3\delta)^2\delta \right)\), which is a concave function of \(c_1\).
\[
\frac{d\Pi_S}{dc_1} \Bigg|_{c_1 = \frac{8 - 3\delta}{8}} = -\frac{8 - 5\delta}{16 - 9\delta} < 0 \text{ and } \frac{d\Pi_S}{dc_1} \Bigg|_{c_1 = \frac{3\delta(4 - 3\delta)}{2(16 - 15\delta)}} = \frac{1}{5} \left( 7 - \frac{16}{16 - 9\delta} - \frac{24}{16 - 15\delta} \right), \text{ which is greater than 0 for } \delta \leq \frac{16}{21}, \text{ and less than 0 for } \delta > \frac{16}{21}. \text{ Thus the solutions in two cases of Proposition 2 follow from this region.}
\]

\(\Pi_S\) is continuous at the boundaries and is equal to \(\frac{\delta}{8}\) for case (iii) in Proposition 1, thus the above solutions are globally optimal.

**Proof of Lemma 2.** Similar to proof of lemma 1 both of the profit functions are concave and first order conditions yield the results.

**Proof of Proposition 3.** Retailer’s objective function is jointly concave in \(Q_1, p_1\) and KKT conditions yield the optimal values and the regions.

**Proof of Proposition 4.** Supplier’s objective function is concave in each part and quasiconcave overall, hence the first order condition yield the result.

**Proof of Corollary 1.** Following propositions 2 and 4 there is one cutoff point for \(\delta\) hence we have two cases to consider. Below we present the proof for the result for supplier’s profit. Proof for the supply chain and the retailer profits are similar. When \(\delta \leq \frac{16}{21}\) the difference \(\Pi_S^{NCM} - \Pi_S^{NC} = \frac{(8 + \delta)^2}{64(8 - \delta)} - \frac{8 + 3\delta}{64} = \frac{d^2}{16(8 - d)}\), which is positive. When \(\delta > \frac{16}{21}\),
\[
\frac{d(\Pi_S^{NCM} - \Pi_S^{NC})}{d\delta} = \frac{2028}{25(32 - 25d)^2} + \frac{4}{(8 - d)^2} - \frac{8 + 3\delta}{64} = \frac{23}{42} + \frac{2}{56 - 49\delta} - \frac{8}{48 - 45\delta} > 0 \text{ for } \delta < 4/5. \text{ Hence, optimal } c_1 \text{ is not in the interior of this region.}
\]

For case (ii), in Proposition 1, \(\Pi_S = \frac{1}{2(16 - 9\delta)^2} \left( -32c_1^2(8 - 5\delta) + 2c_1(8 - 5\delta)(16 - 3\delta) + (4 - 3\delta)^2\delta \right)\), which is a concave function of \(c_1\).
\[
\frac{d\Pi_S}{dc_1} \Bigg|_{c_1 = \frac{8 - 3\delta}{8}} = -\frac{8 - 5\delta}{16 - 9\delta} < 0 \text{ and } \frac{d\Pi_S}{dc_1} \Bigg|_{c_1 = \frac{3\delta(4 - 3\delta)}{2(16 - 15\delta)}} = \frac{1}{5} \left( 7 - \frac{16}{16 - 9\delta} - \frac{24}{16 - 15\delta} \right), \text{ which is greater than 0 for } \delta \leq \frac{16}{21}, \text{ and less than 0 for } \delta > \frac{16}{21}. \text{ Thus the solutions in two cases of Proposition 2 follow from this region.}
\]

\(\Pi_S\) is continuous at the boundaries and is equal to \(\frac{\delta}{8}\) for case (iii) in Proposition 1, thus the above solutions are globally optimal.

**Proof of Lemma 2.** Similar to proof of lemma 1 both of the profit functions are concave and first order conditions yield the results.

**Proof of Proposition 3.** Retailer’s objective function is jointly concave in \(Q_1, p_1\) and KKT conditions yield the optimal values and the regions.

**Proof of Proposition 4.** Supplier’s objective function is concave in each part and quasiconcave overall, hence the first order condition yield the result.

**Proof of Corollary 1.** Following propositions 2 and 4 there is one cutoff point for \(\delta\) hence we have two cases to consider. Below we present the proof for the result for supplier’s profit. Proof for the supply chain and the retailer profits are similar. When \(\delta \leq \frac{16}{21}\) the difference \(\Pi_S^{NCM} - \Pi_S^{NC} = \frac{(8 + \delta)^2}{64(8 - \delta)} - \frac{8 + 3\delta}{64} = \frac{d^2}{16(8 - d)}\), which is positive. When \(\delta > \frac{16}{21}\),
\[
\frac{d(\Pi_S^{NCM} - \Pi_S^{NC})}{d\delta} = \frac{2028}{25(32 - 25d)^2} + \frac{4}{(8 - d)^2} - \frac{8 + 3\delta}{64} = \frac{23}{42} + \frac{2}{56 - 49\delta} - \frac{8}{48 - 45\delta} > 0 \text{ for } \delta < 4/5. \text{ Hence, optimal } c_1 \text{ is not in the interior of this region.}
\]
\[
\frac{3}{7(8-7\delta)^2} - \frac{4531}{11200} > 0, \text{ so the difference function is increasing and } \left( \Pi_S^{NCM} - \Pi_S^{NC} \right)_{\delta = \frac{16}{21}} = \frac{2}{399} > 0, \text{ hence } \Pi_S^{NCM} > \Pi_S^{NC}.
\]

Proof of Lemma 3. For the centralized system, we can write (2.2) as

\[
p_2^* = \arg\max_{p_2 \in S_2} [p_2(\bar{\theta} - p_2/\delta)].
\]

First order condition gives \(p_2^* = \frac{\delta}{2} \bar{\theta} \).

Correspondingly, \(Q_1^* = 1 - \bar{\theta}^*\) and following (2.4), \(p_1^* = (1 - \delta)\bar{\theta}^* + \frac{\bar{\theta}}{2}\). Now we can write the seller’s pricing problem in terms of the target consumer segments, similarly to (2.12) as

\[
\bar{\theta}^* = \arg\max_{\bar{\theta}} (-1 + \frac{3\delta}{4}) \bar{\theta}^2 + (1 - \frac{\delta}{2}) \bar{\theta}.
\]

First-order condition leads to \(\bar{\theta}^* = \frac{2 - \delta}{4 - 3\delta}\). Plugging this result into the above expressions for \(p_1\) and \(p_2\) yields the optimal prices in Lemma 3.

Proof of Lemma 4. We can write (2.17) as

\[
p_2^* = \arg\max_{p_2 \in S_2} [p_2(p_1 - p_2/\delta)].
\]

First order condition gives \(p_2^* = \frac{p_1}{2}\). Now we can write the seller’s pricing problem as

\[
p_1^* = \arg\max_{p_1} (-4 + \frac{\delta}{4}) p_1^2 + p_1.
\]

First-order condition yields the result.

Proof of Corollary 2. \(\Pi_C^{NC} - \Pi_C^{NCM} = -\frac{\delta(8 - (8-\delta)\delta)}{4(4 - \delta)(4 - 3\delta)} < 0\). Hence the result follows.

B.2 Proofs of Chapter 3

Proof of Lemma 5. If the seller commits to a maximum quantity of \(Q_{\text{max}}\) to sell in 2 periods then following (2.9) \(p_2^* = \delta(1 - Q_{\text{max}})\) and \(Q_2^* = Q_{\text{max}} - Q_1\). As is similar to the proof of Lemma 3, \(Q_1^* = 1 - \bar{\theta}^*\). With these optimal values and by (2.4), we can write the seller’s first period problem
as
\[
(Q_{\text{max}}^*, \bar{\theta}^*) = \arg \max_{(Q_{\text{max}}, \bar{\theta}) \in Q_{\text{max}} \geq 1 - \bar{\theta} \geq 0} (-\delta Q_{\text{max}}^2 - (1 - \delta)\bar{\theta}^2 + \delta (Q_{\text{max}} - \bar{\theta} + \bar{\theta}),
\]
which is jointly concave in \((Q_{\text{max}}, \bar{\theta})\). First-order condition results in \(\frac{1}{2}\) for both of the decision variables, which gives the optimal \(p_1\) in Lemma 5.

**Proof of Theorem 1.** \(\Pi_{NC}^C = \frac{(2 - \delta)^2}{4(4 - 3\delta)}\) and \(\Pi_{QC}^C = \Pi_{PC}^C = \frac{1}{4}\). Thus, the value of commitment is \(\frac{\delta(1 - \delta)}{4(4 - \delta)}\), which is always positive.

**Proof of Lemma 6.** The proof follows from the discussion following the Lemma 6 in the text.

**Proof of Lemma 7.** Similar to the proof of Lemma 1, we have the following objective function for the retailer in second period.

\[
\Pi_{R,2} = -\delta Q_2^2 - (c_2 - (2(1 - Q_1) - \bar{\theta})\delta)Q_2 + \delta(1 - Q_1)(Q_1 + \bar{\theta} - 1) \quad (B.4)
\]

Then first part of (3.1) gives \(\frac{d\Pi_{R,2}}{dc_2} \bigg|_{Q_2 = Q_{\text{max}} - Q_1} \geq 0\), which is the same as
\[
\delta(2(1 - Q_{\text{max}}) - \bar{\theta})) - c_2 \geq 0. \quad (B.5)
\]
Because commitment binds, \(p_2^* = \delta(1 - Q_{\text{max}})\) and \(Q_2^* = Q_{\text{max}} - Q_1\), thereby the inequality in (B.5) binds, which gives \(\bar{c}_2 = c_2^* = \delta(2(1 - Q_{\text{max}}) - \bar{\theta}))\). To have the retailer’s second period actions to be consistent at this price, the second part of (3.1) needs to hold with the following supplier objective in the second period.

\[
\Pi_{S,2} = \frac{1}{2\delta} \left(-c_2^2 + \delta(2(1 - Q_1) - \bar{\theta})c_2\right). \quad (B.6)
\]
Therefore, \(\frac{d\Pi_{S,2}}{dc_2} \bigg|_{c_2 = \bar{c}_2} \leq 0\), yields
\[
Q_{\text{max}} \leq \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}. \quad (B.7)
\]

\(\square\)
**Proof of Proposition 5.** The retailer’s objective function in (3.3) can be written as follows.

\[ \Pi_{R}(Q_{\max}, Q_1, \bar{\theta}) = \delta Q_{\max}^2 - (1 - \delta)\bar{\theta}^2 + (\delta(2(1 - Q_{\max}) - \bar{\theta}) - c_1)Q_1 + (1 - \delta)\bar{\theta} - \delta(1 - \bar{\theta})Q_{\max}. \]  
(B.8)

Observe that the objective function above is a quadratic concave function of \( Q_{\max} \). So the optimal \( Q_{\max} \) must be at one of two end points of the feasible region. Thus, one can separate the optimization problem defined in (3.3-3.4) into two cases; where in the first case \( Q_{\max} = Q_1 \) and in the second case \( Q_{\max} = Q_1 + \frac{1}{2} - \frac{\bar{\theta}}{4} \). The global optimal solution can be found by comparing the solutions to these two cases of the problem. Hence, in the following we present the solutions of the two cases explicitly.

**Case A:** When \( Q_{\max} = Q_1 \), the objective function reduces to the following.

\[ \Pi_{R,A}(Q_1, \bar{\theta}) = -(1 - \delta)\bar{\theta}^2 - \delta Q_1^2 + (1 - \delta)\bar{\theta} - (c_1 - \delta)Q_1. \]  
(B.9)

Hessian of (B.9) is
\[
\begin{pmatrix}
-2(1 - \delta) & 0 \\ 0 & -2\delta
\end{pmatrix}
\]
with the eigenvalues \(-2(1 - \delta)\) and \(-2\delta\), which are non-positive. Hence, (B.9) is jointly concave in \((Q_1, \bar{\theta})\). KKT conditions characterize the solution for every \( c_1 \) and \( \delta \), which gives \( Q_1^* = \frac{1 - c_1}{2}, \bar{\theta}^* = \frac{1 + c_1}{2} \).

**Case B:** When \( Q_{\max} = Q_1 + \frac{1}{2} - \frac{\bar{\theta}}{4} \), the objective function in (B.8) reduces to the following.

\[ \Pi_{R,B}(Q_1, \bar{\theta}) = (-1 + \frac{13}{16})\bar{\theta}^2 - \frac{\delta}{4}Q_1 - \frac{3\delta}{4}Q_1^2 + (1 - \frac{\delta}{2})\bar{\theta} + (\delta - c_1)Q_1 - \frac{\delta}{4}. \]  
(B.10)

Hessian of (B.10) is
\[
\begin{pmatrix}
-(2 - \frac{13\delta}{8}) & -\frac{\delta}{4} \\ -\frac{\delta}{4} & -\frac{3\delta}{2}
\end{pmatrix}
\]
with the eigenvalues
\[
-\frac{1}{16} \left( 16 - \delta + \sqrt{256 - \delta(800 - 641\delta)} \right)
\]
and
\[
-\frac{1}{16} \left( 16 - \delta - \sqrt{256 - \delta(800 - 641\delta)} \right)
\]
, which are non-positive. Hence, (B.10) is jointly concave in \((Q_1, \bar{\theta})\). KKT conditions characterize the solution for every \(c_1\) and \(\delta\). So, for \(c_1 < \frac{2\delta}{16 - 15\delta}\), \(Q_1^* = \frac{1}{60} \left(36 - \frac{40c_1}{3} - \frac{6 + 5c_1}{6 - 5\delta}\right)\), \(\bar{\theta}^* = \frac{c_1 - 4\delta + 6}{2(6 - 5\delta)}\); for \(\frac{2\delta}{16 - 15\delta} \leq c_1 < \frac{8 - 3\delta}{8}\), \(Q_1^* = \frac{8(1 - c_1) - 3\delta}{16 - 5\delta}\), \(\bar{\theta}^* = \frac{8(1 + c_1) - 2\delta}{16 - 5\delta}\); and for \(c_1 \geq \frac{8 - 3\delta}{8}\), \(Q_1^* = 0\), \(\bar{\theta}^* = 1\). Here, the first solution is in the interior of the feasible region \(S'_1\), which is given in (2.15); the second is the one when the upperbound in \(S'_1\) binds and the last one follows when both the upper and lowerbound in \(S'_1\) bind.

Now to find the global optimal solution we need to compare the solution in case A to the three solutions in case B. Define \(\Pi^*_R,Bj\), \(j = 1, 2, 3\) as the objective values of the case B problem corresponding to the three solutions described above respectively.

\[
\Pi^*_R,A - \Pi^*_R,B1 = \frac{1}{8\delta(6 - 5\delta)} \left(- (16 - 5\delta(5 - 2\delta))c_1^2 + 4\delta(1 - \delta)c_1 + 2(1 - \delta)\delta^2\right), \quad (B.11)
\]

which is a concave function of \(c_1\). When \(c_1 = 0\), it is greater than zero. The only positive \(c_1\) root to this function is \(\bar{r}_1 = \frac{(2(1-\delta) + \sqrt{2}\sqrt{(6-5\delta)(3-2\delta)(1-\delta)})\delta}{16 - 5(5 - 2\delta)\delta}\). To determine whether the first solution in case B is the global solution we need to check if the \(c_1\) cutoff point for that solution is greater than the \(c_1\) root of the difference function. However, \(\bar{r}_1 > \frac{2\delta}{16 - 15\delta}\) since \(\delta < \frac{1}{90}(95 - \sqrt{385})\), so the first solution in case B is not the global solution.

Now consider the second solution in case B,

\[
\Pi^*_R,A - \Pi^*_R,B2 = \frac{\delta}{4(16 - 5\delta)}(-c_1(2 + 5c_1) + 3 - \delta), \quad (B.12)
\]

which is a concave function of \(c_1\). It is greater than 0 for \(c_1 = 0\) and the only positive \(c_1\) root to this function is \(\bar{r}_2 = \frac{\sqrt{16 - 5\delta} - 1}{3}\). However, \(\bar{r}_2 < \frac{8 - 3\delta}{8}\) since \(\delta < \frac{16}{9}\). Hence, for \(c_1 < \frac{\sqrt{16 - 5\delta} - 1}{3}\) the global solution follows from part A and for \(\frac{\sqrt{16 - 5\delta} - 1}{3} \leq c_1 < \frac{8 - 3\delta}{8}\) the global solution is given by the second solution of case B. This leads to the first two parts of Proposition 5.

Now consider the third solution in case B,

\[
\Pi^*_R,A - \Pi^*_R,B3 = \frac{1}{4}(1 - c_1)^2 - \frac{\delta}{16}, \quad (B.13)
\]

which is a convex function of \(c_1\). It is greater than 0 for \(c_1 = 0\). The \(c_1\) root smaller than 1 is \(\bar{r}_3 = 1 - \frac{\sqrt{3}}{2}\). However, \(\frac{8 - 3\delta}{8} > \bar{r}_3\), since \(\delta < \frac{16}{9}\). Hence, the global solution for \(c_1 > \frac{8 - 3\delta}{8}\), is given by
the third solution in case B. This yields the solution in part (iii) of Proposition 5, which completes the proof. □

**Proof of Proposition 6.** To find the best response of supplier to optimal retailer strategies we have to analyze the supplier problem for each case of Proposition 5.

For the first case in Proposition 5, \( \Pi_S = \frac{1}{2}(1 - c_1)c_1 \) and first-order condition yields optimal \( c_1 = \frac{1}{2} \) for \( \delta < \frac{3}{4} \). For \( \delta \geq \frac{3}{4} \), the solution is at the boundary.

For the second case in Proposition 5, \( \Pi_S = \frac{(4 - \delta)^2 - 2c_1(8 - 3\delta))(16c_1 + \delta)}{2(16 - 5\delta)^2} \), which is a concave function of \( c_1 \).

\[
\frac{d\Pi_S}{dc_1} \bigg|_{c_1 = \sqrt{\frac{16 - 5\delta - 1}{8}}} = \frac{1}{(16 - 5\delta)^2} \left( 128 + \frac{32}{5} \left( 1 - \sqrt{16 - 5\delta} \right)(8 - 3\delta) - \delta(72 - 11\delta) \right) < 0.
\]

Hence optimal \( c_1 \) for this case is \( \sqrt{\frac{16 - 5\delta - 1}{8}} \).

For the third case in Proposition 5, \( \Pi_S = \frac{4}{5} \).

Now, as the overall \( \Pi_S \) function is not quasiconcave, we need to compare the solutions in the three cases above to find the global optimal. First, note that the third case is always dominated. Hence, we need to compare only the first two cases. The difference between the profits of the first two cases is as follows:

\[
16 \left( 113 - 28\sqrt{16 - 5\delta} \right) + \delta \left( 4 \left( 5\delta + 22\sqrt{16 - 5\delta} \right) - 693 \right)
\]

\[
200(16 - 5\delta),
\]

which is equal to \( \frac{1}{200} \) and \( \frac{1583 - 944\sqrt{3}}{3000} < 0 \) for \( \delta = 0 \) and \( \delta = \frac{4}{5} \) respectively. Hence, there exists at least one \( \delta \) value, which makes the difference equal to 0. To show that there is at most one such \( \delta \), we need to show that this difference is decreasing. The derivative of (B.14) is

\[
\frac{(152 - 5\delta(248 - 55\delta)) - \sqrt{16 - 5\delta}(512 - 5\delta(32 - 5\delta))}{50(16 - 5\delta)^{5/2}},
\]

(B.15)
where the denominator is positive. Define $N(\delta)$ as the numerator of (B.15) then;

\[
N(\delta) = (152 - 5\delta(248 - 55\delta)) - \sqrt{16 - 5\delta(512 - 5\delta(32 - 5\delta))} \\
< (152 - 5\delta(248 - 55\delta)) - 3(512 - 5\delta(32 - 5\delta)) \\
= -8(173 + 5\delta(19 - 5\delta)) \\
< 0
\]

Thus, (B.14) is decreasing and setting it equal to zero leads to below equation, which explicitly characterizes $\bar{\delta}_1$.

\[
\delta = \frac{16 (113 - 28\sqrt{16 - 5\delta})}{(693 - 4 (5\delta + 22\sqrt{16 - 5\delta}))}. \quad (B.16)
\]

Following the above discussion there exist a $\tilde{\delta}_1 \in [0, \frac{4}{5}]$ that solves (B.16). Thus, when $\delta \leq \bar{\delta}_1$ global optimal comes from case 1 and for $\delta > \bar{\delta}_1$ global optimal follows from case 2.

\[\Box\]

**Proof of Theorem 2.** Following Propositions 2 and 6, we have two cutoff points for $\delta$ hence, there are three cases to consider. Below we present the proof for each case separately.

- When $\delta \leq \bar{\delta}_1$, $\Pi^{NC}_S = \frac{8 + 3\delta}{64}$ and $\Pi^{RQC}_S = \frac{1}{8}$. $\Pi^{NC}_S \geq \Pi^{RQC}_S$ since $\delta > 0$. $\Pi^{NC}_R = \frac{16 + 3\delta}{256}$ and $\Pi^{RQC}_R = \frac{1}{16}$. $\Pi^{NC}_R \geq \Pi^{RQC}_R$ since $\delta > 0$. The result for total supply chain profit follows immediately.

- When $\bar{\delta}_1 < \delta \leq \frac{16}{27}$, $\Pi^{NC}_S = \frac{8 + 3\delta}{64}$, $\Pi^{RQC}_S = \frac{1}{16} (16 - 5\delta)(142 - 5\delta - 352 + 2(56 - 11\delta)\sqrt{16 - 5\delta})$. Thus,

\[
\Pi^{NC}_S - \Pi^{RQC}_S = \frac{14464 - (\delta(215\delta + 4344) + (3584 - 704\delta)\sqrt{16 - 5\delta})}{1600(16 - 5\delta)} \quad (B.17)
\]

The denominator of (B.17) is greater than 0. To show that the numerator is also positive, let $N_1(\delta)$ be the numerator of (B.17). Now, $\frac{d^2 N_1(\delta)}{d\delta^2} = \frac{600(55\delta - 72)}{(16 - 5\delta)^{5/2}}$, so the second derivative is decreasing. In addition $\frac{d^2 N_1(\delta)}{d\delta^2} \bigg|_{\delta = \bar{\delta}_1} = -970.47$ and $\frac{d^2 N_1(\delta)}{d\delta^2} \bigg|_{\delta = \frac{16}{27}} = -430 - \frac{3015\sqrt{21}}{32}$. Hence, $N_1(\delta)$ is a concave function of $\delta$ for $\delta \in (\bar{\delta}_1, \frac{16}{27}]$. Minimum of $N_1(\delta)$ is then attained at one
of the boundary points: $N_1(\bar{\delta}_1) = 275.30$ and 
\[ N_1(\frac{16}{21}) = \frac{256000}{441} (19 - 4\sqrt{21}) \]. So $N_1(\delta) > 0$, which shows that (B.17) is positive for $\delta \in (\bar{\delta}_1, \frac{16}{21})$.

For the retailer’s profit in this case, $\Pi^{NC}_R = \frac{16+3\delta}{256}$ and $\Pi^{RQC}_R = \frac{1}{100} (52 - 5\delta + 12\sqrt{16 - 5\delta})$. Thus,
\[ \Pi^{NC}_R - \Pi^{RQC}_R = \frac{395\delta + 768\sqrt{16 - 5\delta} - 2928}{6400} . \] (B.18)

Let $N_2(\delta)$ be the numerator of (B.18). Then $\frac{d^2 N_2(\delta)}{d\delta^2} = -\frac{4800}{(16 - 5\delta)^{3/2}}$. So $N_2$ is a concave function of $\delta$ and it is positive at both of the boundary points. Hence, (B.18) is positive.

The result for total profit follows from the above analysis.

- When $\frac{16}{21} < \delta \leq \frac{4}{5}$, then
  \[ \Pi^{NC}_S = \frac{\delta(56 - 3(34 - 15\delta)\delta)}{(16 - 15\delta)^2} \]

and
  \[ \Pi^{RQC}_S = \frac{1}{50(16 - 5\delta)} \left( (142 - 5\delta)\delta - 352 + 2(56 - 11\delta)\sqrt{16 - 5\delta} \right) . \]

Thus,
\[ \Pi^{NC}_S - \Pi^{RQC}_S = \] (B.19)
\[ -\frac{2(56 - 11\delta)\sqrt{16 - 5\delta}(16 - 15\delta)^2 + 3\delta(5\delta(5(362 - 135\delta)\delta + 3536) - 53504) + 90112}{50(16 - 15\delta)^2(16 - 5\delta)} \]

The denominator of (B.19) is positive. Let us define $N_3(\delta)$ as its numerator. Then,
\[ \frac{d^4 N_3}{d\delta^4} = \frac{375}{8(16 - 5\delta)^{7/2}} (-(\gamma_1 + \gamma_2) + \delta(\gamma_3 + \gamma_4)) \] (B.20)
where parameters $\gamma_1, \gamma_2, \gamma_3$ and $\gamma_4$ are defined below.

\[
\begin{align*}
\gamma_1 &= 2048 \left( 7201 + 10368\sqrt{16 - 5\delta} \right) , \\
\gamma_2 &= 600 \left( 7175 + 10368\sqrt{16 - 5\delta} \right) \delta^2 , \\
\gamma_3 &= 15360 \left( 919 + 1296\sqrt{16 - 5\delta} \right) , \\
\gamma_4 &= 1125 \left( 385 + 576\sqrt{16 - 5\delta} \right) \delta^2 .
\end{align*}
\]

It is clear that $\gamma_1 > \gamma_3$ and $\gamma_2 > \gamma_4$. Hence, (B.20) is negative. This means that the third derivative of $N_3$ is decreasing, and $\left. \frac{d^3 N_3}{d\delta^3} \right|_{\delta = \frac{4}{5}} = \frac{2990525}{6\sqrt{3}} - 31500 > 0$. So, third derivative of $N_3$ is always positive, which means second derivative of the function is always increasing. Now, $\left. \frac{d^2 N_3}{d\delta^2} \right|_{\delta = \frac{4}{5}} = 158640 - \frac{946100}{3\sqrt{3}} < 0$. Hence, $N_3$ is a concave function $\delta$. However, at both of the boundary points $N_3$ is positive, hence the numerator of (??) is positive; the result thus follows.

For the retailer’s profit in this case, $\Pi_{NC}^R = \frac{(4 - 3\delta)(64 - 3(44 - 23\delta)\delta)}{4(16 - 15\delta)^2}$ and $\Pi_{RQC}^R$ is the same as in case two above. Thus,

\[
\Pi_{NC}^R - \Pi_{RQC}^R = \frac{1}{100} \left( \frac{25(4 - 3\delta)(64 - 3\delta(44 - 23\delta))}{(16 - 15\delta)^2} + 12\sqrt{16 - 5\delta} + 5\delta - 52 \right) \tag{B.21}
\]

Note that $\left. \frac{d(\Pi_{NC}^R - \Pi_{RQC}^R)}{d\delta} \right| = -\frac{1}{50} \left( 9 + \frac{88}{(16 - 15\delta)^2} + \frac{15}{\sqrt{16 - 5\delta}} - \frac{512}{(16 - 15\delta)^2} \right) < 0$. Hence, the difference function is decreasing in $\delta$. The values at the boundary points are $\frac{96\sqrt{21} - 431}{1050}$ and $\frac{2(15\sqrt{3} - 26)}{125}$, which are positive and negative respectively. Hence, the function crosses 0 once. The value of $\delta$ that makes the difference in (B.21) equal to 0 is given by the following, which is approximately equal to 0.799.

\[
\delta = \frac{6(576 - (16 - 15\delta)^2\sqrt{16 - 5\delta})}{5(824 + 135(2 - 3\delta)\delta)} \tag{B.22}
\]

Here, equation (B.22) explicitly characterizes $\delta_2$. 

107
For the supply chain profit in this case the following gives the difference between the profit of the two scenarios.

\[
\frac{4}{375} \left( 20 + 3\sqrt{16 - 5\delta} - \frac{128 - 39(16 - 15\delta)}{(16 - 15\delta)^2} - \frac{96 + 78\sqrt{16 - 5\delta}}{16 - 5\delta} \right) \tag{B.23}
\]

The second derivative of (B.23) is \(-\frac{1}{5} \left( \frac{9216}{(16 - 15\delta)^4} - \frac{936}{(16 - 15\delta)^3} + \frac{78}{(16 - 5\delta)^{3/2}} + \frac{1}{(16 - 5\delta)^{5/2}} + \frac{256}{(16 - 5\delta)^2} \right)\). The second term in parentheses is less than the first term, as \(\delta > \frac{16}{39}\); hence the second derivative is negative. So, the difference function is a concave function of \(\delta\); however, it is positive at both of the boundary points, thus, the result follows.

**Proof of Lemma 8.** We first show that optimal \(Q_{max}\) is never less than \(1/4\).

When \(Q_{max} < \frac{1}{4}\), (3.7) holds and the retailer’s profit is given by (B.8). The decision variables for retailer’s period 1 problem are \(Q_1\) and \(\tilde{\theta}\) and the feasible region is defined by (3.6) and (3.5). Observe that (B.8) is linear with respect to \(Q_1\), so in the optimal solution \(Q_1\) must either be equal to \(1 - \tilde{\theta}\) or \(Q_{max}\). Similar to the proof of Proposition 5, we can separate the problem into two parts, find the solution for each part and compare them to find the global solution. Hence, the following is the solution to the retailer’s optimization problem.

(i) When \(1 - (2 - \delta)Q_{max} < c_1 < 1 - 3\delta Q_{max}\), retailer action and optimal values are the same as part (i) in Proposition 7.

(ii) When \(c_1 \leq \min(1 - (2 - \delta)Q_{max}, 1 - 3\delta Q_{max})\), retailer action and optimal values are the same as part ii in Proposition 7.

(iii) When \(c_1 \geq \max(1 - (2 - \delta)Q_{max}, 1 - 3\delta Q_{max})\), retailer sells the product only in period 2 and it does not carry inventory between periods, where \(Q_1^* = 0, p_t^* = 1 - \delta Q_{max}\) and \(\tilde{\theta}^* = 1\).

Now, let’s consider the supplier’s profit in response to the above retailer actions to see if there is an equilibrium with \(Q_{max} < \frac{1}{4}\). Supplier profit for case (i) above is the following.

\[
\Pi_S = \frac{-c_1^2(2 - 3\delta) - (1 - Q_{max}(2 - \delta))\delta(3 - 4Q_{max} - (4 - 5Q_{max})\delta) + 2c_1(1 - 6Q_{max}\delta - (2 - 9Q_{max})\delta^2)}{4(1 - 2\delta)^2}. \tag{B.24}
\]
Equation (B.24) is jointly concave in \((c_1, Q_{\text{max}})\), first-order condition gives \(\left\{ \frac{4-\delta(6-\delta)}{3(2-3\delta)}, 1/4 \right\} \) for \(c_1\) and \(Q_{\text{max}}\) respectively. Hence, optimal \(Q_{\text{max}}\) is not less than \(1/4\) for this case.

Supplier profit function for case (ii) above is \(c_1 Q_{\text{max}}\); the solution is straightforward where \(Q_{\text{max}}\) values that maximize this function in the given range are \(\left\{ \frac{1}{2(2-\delta)}, \frac{1}{2(1+\delta)} \right\} \) for \(\delta < 1/2\) and \(\delta \geq 1/2\) respectively. Clearly, these values are not less than \(1/4\) for the given \(\delta\) ranges.

Finally, supplier profit function for case (iii) above is \(\delta Q_c(1 - 2Q_c)\) and first-order condition gives \(1/4\) for \(Q_{\text{max}}\). Thus, equilibrium commitment quantity is never less than \(1/4\).

Next, we show that equilibrium commitment quantity never exceeds \(1/2\). Assume there exists an equilibrium with \(Q_{\text{max}} = \frac{1}{2} + \epsilon_1\) where \(\epsilon_1 > 0\) and (3.7) holds, thus commitment binds. From (3.6), \(Q_1 \geq \frac{1}{3}\). Assume at equilibrium \(Q_1 = \frac{1}{3} + \epsilon_2\), where \(\epsilon_2 \geq 0\). Then \(Q_2 = \frac{1}{6} + \epsilon_1 - \epsilon_2\). In addition, following Lemma 7, \(p_2 = \delta(\frac{1}{2} - \epsilon_1)\) and \(c_2 = \delta((1 - 2\epsilon_1) - \bar{\theta})\). Then following (B.8), we can write retailer’s profit function as follows.

\[
\tilde{\Pi}_R = -(1 - \delta)\bar{\theta}^2 + \left(1 - \delta \left(\frac{5}{6} - \epsilon_1 + \epsilon_2\right)\right)\bar{\theta} + \frac{1}{12}(\delta(1 - 2\epsilon_1)(1 - 6(\epsilon_1 - 2\epsilon_2)) - 4c_1(1 + 3\epsilon_2))
\]

(B.25)

\[
\frac{d\tilde{\Pi}_R}{d\epsilon_2} \bigg|_{\epsilon_2=0,\bar{\theta} = \frac{2}{3} - 4\epsilon_1} = -2\delta\epsilon_1, \quad \text{so } \tilde{\Pi}_R \text{ is decreasing in } \epsilon_1, \text{ hence in order to maximize profit, } \epsilon_2 \text{ must be equal to } 0.
\]

Then from (3.6) and (3.7), \(\bar{\theta} = \frac{2}{3} - 4\epsilon_1\). Thus, \(\frac{d\tilde{\Pi}_R}{d\epsilon_1} \bigg|_{\epsilon_2=0,\bar{\theta} = \frac{2}{3} - 4\epsilon_1} = -2\delta\epsilon_1\), so \(\tilde{\Pi}_R\) is decreasing in \(\epsilon_1\), hence in order to maximize profit \(\epsilon_1\) must be 0 as well. However, these yield \(\tilde{\Pi}_R = \frac{4(2 - 3\epsilon_1) - \delta}{36}\) and by setting \(c_1 = \frac{8 - \delta}{12}\) supplier can always extract retailer’s surplus. Retailer can get a better profit by deviating from this strategy thus, when \(Q_{\text{max}} \geq \frac{1}{2}\) commitment does not bind.

When commitment does not bind retailer’s profit is given by (2.12) and the equilibrium is presented in Propositions 1 and 2. The same equilibrium holds if total quantity is equal to \(\frac{1}{2}\). Equilibrium total quantity in a no-commitment model is \(\frac{7}{16}\) for \(\delta \leq \frac{16}{27}\) and \(\frac{20 - 21\delta}{32 - 30\delta}\) for \(\delta > \frac{16}{27}\), which is less than \(\frac{1}{2}\). Thus, when supplier commits to quantity for any equilibrium with \(Q_{\text{max}} > \frac{1}{2}\) there is an equivalent equilibrium with \(Q_{\text{max}} = \frac{1}{2}\), which yields the same payoffs to all parties (supplier, retailer and customers).

\[\blacksquare\]
Proof of Proposition 7. When commitment binds, i.e., (3.7) holds, the retailer’s profit is given by (B.8). The decision variables for retailer’s period 1 problem are \( Q_1 \) and \( \theta \) and the feasible region is defined by (3.6), (3.5) and (3.7). Observe that (B.8) is linear with respect to \( Q_1 \), so in the optimal solution \( Q_1 \) must either be equal to \( 1 - \theta \), \( 2Q_{\text{max}} - 1 + \frac{\theta}{2} \) or \( Q_{\text{max}} \). Similar to the proof of Lemma 8, the optimization problem can be divided into cases according to above \( Q_1 \) values and the global solution can be found by comparing each individual solution. Below, we summarize the solution to retailer’s optimization problem when commitment binds.

(i) The quantity commitment binds strictly, the retailer sells the product in both periods and it carries inventory between periods with \( Q_1^* = \frac{1-c_1-3\delta Q_{\text{max}}}{2(1-2\delta)} \), \( p_1^* = \frac{1+\delta(1-c_1)+\delta(Q_{\text{max}}(1+\delta)-3)}{2(1-2\delta)} \) and \( \bar{\theta}^* = \frac{1+c_1-\delta(4-3Q_{\text{max}})}{2(1-2\delta)} \), when \( \delta < 1/2, \frac{1-c_1}{2-\delta} < Q_{\text{max}} < \frac{5-3c_1-4\delta}{8-7\delta} \).

(ii) The quantity commitment binds strictly, the retailer sells the product only in period 1 with \( Q_1^* = Q_{\text{max}} \) and \( p_1^* = \bar{\theta}^* = 1 - Q_{\text{max}} \), when

a) \( \delta < 1/2, Q_{\text{max}} \leq \frac{1-c_1}{2-\delta} \),
b1) \( \delta \geq 1/2, c_1 \leq \frac{2(1-\delta)(5\delta-1)}{23-25\delta}, Q_{\text{max}} < K_1 \),
b2) \( \delta \geq 1/2, c_1 > \frac{2(1-\delta)(5\delta-1)}{23-25\delta}, Q_{\text{max}} < \frac{4-3c_1-2\delta}{7-5\delta} \).

(iii) The quantity commitment binds weakly, the retailer sells the product in both periods and it does not carry inventory between periods with \( Q_1^* = \frac{1}{3}(4Q_{\text{max}} - 1) \), \( p_1^* = \frac{4}{3}(4 - \delta - Q_{\text{max}}) \), and \( \bar{\theta}^* = \frac{4}{3}(1 - Q_{\text{max}}) \), when max \( \left( \frac{4-3c_1-2\delta}{7-5\delta}, \frac{5-3c_1-4\delta}{8-7\delta} \right) \leq Q_{\text{max}} \leq \frac{10+3c_1-14\delta}{4(4-5\delta)} \).

(iv) The quantity commitment binds weakly, the retailer sells the product in both periods and it carries inventory between periods with \( Q_1^* = \frac{4Q_{\text{max}}(4-3\delta)-6(1-\delta)}{4(2-\delta)} \), \( p_1^* = \frac{2+(1-\delta)(4\delta-c_1)-2\delta(4-3\delta)Q_{\text{max}}}{2(2-\delta)} \) and \( \bar{\theta}^* = \frac{2-c_1+2\delta(1-2Q_{\text{max}})}{2(2-\delta)} \), when max \( (K_1, \frac{10+3c_1-14\delta}{4(4-5\delta)}) < Q_{\text{max}} \).

When the commitment does not bind i.e., (3.7) does not hold the retailer’s profit is given by (B.3). The solution for this optimization problem is given in Proposition 1. From the feasible regions listed there, we need to find where (3.7) is violated. Below, we summarize the solution to the retailer’s optimization problem when commitment does not bind.
(i) Retailer sells the product in both periods and it carries inventory between periods with
\[ Q_1^* = \frac{11}{14} - \frac{c_1(112-9\delta)+6\delta}{218(8-\delta^2)}, \quad p_1^* = \frac{1}{2} - \frac{4(5-4\delta)c_1-9\delta(1-\delta)}{6(8-\delta^2)}, \quad \text{and} \quad \theta^* = \frac{3(4-3\delta)+2c_1}{3(8-\delta^2)}, \]
when \( c_1 \leq \frac{3\delta(4-3\delta)}{2(16-15\delta)} \);
\[ Q_{max} \geq \frac{1}{42} \left(33 - \frac{14c_1}{8} - \frac{2(6+7c_1)}{8-7\delta^2}\right). \]

(ii) Retailer sells the product in both periods and it does not carry inventory between periods
with \( Q_1^* = \frac{8(1-c_1)-3\delta}{16-9\delta}, \quad p_1^* = \frac{(4-\delta)(4+4c_1-3\delta)}{2(16-9\delta)}, \quad \text{and} \quad \theta^* = \frac{8(1+c_1)-6\delta}{16-9\delta}, \]
when \( \frac{3\delta(4-3\delta)}{2(16-15\delta)} < c_1 \leq \frac{8-3\delta}{8} \);
\[ Q_{max} \geq \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}. \]

(iii) Retailer sells the product only in period 2 and it does not carry inventory between periods,
where \( Q_1^* = 0, \quad \text{and} \quad \theta^* = 1, \quad \text{when} \quad c_1 \geq \frac{8-3\delta}{8}. \]

To find the global optimal to the retailer’s first period problem, we need to compare the
solutions to the subproblems presented above. This gives the regions and the optimal values in
the Proposition 7. Note that the first two cases in the solution where commitment binds and last
two cases in the solution where commitment does not bind remain the same in the global solution.
The only comparisons need to be done are between the cases (iii), (iv) of commitment binding
solution and case (i) of commitment non-binding solution where we find the thresholds \( K_2 \) and \( K_3 \)
respectively.

**Proof of Proposition 8.** One has to analyze \( \Pi_S \) in each region to find the optimal \((c_1, Q_{max})\).

For the first case in Proposition 7, supplier profit is given in (B.24), which is jointly concave in
\((c_1, Q_{max})\) for \( \delta < 1/2 \). However, following first order condition there is no interior solution in this
region. In addition \( \Pi_S \) is continuous on the boundary points, hence the optimal solution cannot be
in this region.

For the second case in Proposition 7, \( \Pi_S = c_1 Q_{max} \). One can find optimal \((c_1, Q_{max})\) by
first solving for \( Q_{max} \) then for \( c_1 \). Clearly optimal \( Q_{max} \) must be at the upper bound. For \( \delta \leq 1/2 \), \( Q_{max} = \frac{1-c_1}{2-\delta} \) yields \( c_1 = 1/2 \), and for \( \delta > 1/2 \), \( Q_{max} = \frac{4-3c_1-2\delta}{7-5\delta} \) yields \( c_1 = \frac{2-\delta}{3} \). No
solution comes from \( Q_{max} = K_1 \) because, when upper bound is \( K_1 \) then \( (2(2-\delta)(1-\delta))^{1/2} -
2(1-\delta) < c_1 < \frac{2(6\delta-5\delta^2-1)}{23-25\delta} \) and \( \delta > 5/7 \). \( \Pi_S(Q_{max} = K_1) \) is a concave function of \( c_1 \)
and \( \frac{d\Pi_S(Q_{max}=K_1)}{dc_1} \bigg|_{c_1=(2(2-\delta)(1-\delta))^{1/2}-2(1-\delta)} > 0 \), also \( \frac{d\Pi_S(Q_{max}=K_1)}{dc_1} \bigg|_{c_1=\frac{2(6\delta-5\delta^2-1)}{23-25\delta}} = \frac{2(4-5\delta)(13-11\delta)}{(7-5\delta)(25\delta-23)} > 0 \).

For the third case in Proposition 7, \( \Pi_S = \frac{25Q_{max}^2}{9} + \frac{4(3c_1-\delta)Q_{max}}{9} + \frac{2\delta-3c_1}{3}, \) which is a convex
function of \( Q_{max} \). Hence, optimal \( Q_{max} \) must be at one of the boundary points. We analyzed
lower bound in case 2. When \( Q_{\text{max}} = \frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta} \) for \( \delta \leq \frac{16}{27} \), \( c_1 = \frac{16-3\delta}{32} \). If \( Q_{\text{max}} = K_2 \) then for \( \delta > \frac{71-\sqrt{241}}{75} \), \( c_1 = \arg \max c_1 \Pi_S(K_2, c_1) \). When \( Q_{\text{max}} = \frac{10+3c_1-14\delta}{4(4-5\delta)} \), \( \Pi_S \) is a quadratic convex function of \( c_1 \) and both of the boundary points are analyzed above. However, the two \( \delta \) regions given above intersect. Hence, to find the solution of this case one needs to compare the solutions at each of the \( Q_{\text{max}} \) boundary points. Define \( \bar{c}_1 = \arg \max c_1 \Pi_S(K_2, c_1) \). Then the value of \( \delta \) that solves the following explicitly characterizes \( \bar{\delta}_3 \).

\[
\Pi_S(K_2, \bar{c}_1) - \Pi_S\left(\frac{1}{2} + \frac{2(1-3c_1)}{16-9\delta}, \frac{16-3\delta}{32}\right) = 0 \\
\Pi_S(K_2, \bar{c}_1) - \frac{8 + 3\delta}{64} = 0 
\] (B.26)

For the fourth case in Proposition 7, \( \Pi_S = \frac{8(1-\delta)^2Q_{\text{max}}^2}{(2-\delta)^2} - \frac{(4-3\delta)c_1^2}{8(2-\delta)^2} + \frac{(8-12\delta+5\delta^2)Q_{\text{max}}c_1 - 3(1-\delta)^2(2c_1-\delta(3-8Q_{\text{max}}))}{2(2-\delta)^2} \), which is a convex function of \( Q_{\text{max}} \), hence optimal \( Q_{\text{max}} \) must be at one of the boundary points. The only boundary left for analysis is when \( Q_{\text{max}} = K_3 \), which is a concave function of \( c_1 \). However, \( \frac{d\Pi_S(Q_{\text{max}}=K_3)}{dc_1} \big|_{c_1=C} > 0 \) hence no solution follows from this case.

Solutions for the last two cases in Proposition 7 are same as in Proposition 2, which are superseded by the solution from case 3. Hence, no solution comes from the last two cases.

Thus, to find the optimal \((c_1, Q_{\text{max}})\) one needs to compare the solutions given in cases 2 and 3 above, which gives the result in Proposition 8.

**Proof of Lemma 9.** There may be two scenarios where supplier makes a commitment in period 1. The first scenario is when it makes a commitment only in period 1 and the second is when it makes separate quantity commitments in both periods. Proving that supplier does not benefit from the former scenario is easy, we skip that and present the proof for the latter.

The analysis for the scenario when supplier makes a commitment only in period 2 is presented in Section 3.5. However, in this case since supplier makes a commitment in both of the periods Lemma 10 holds. In addition since we are only interested in the commitment binding solutions it is sufficient to consider only the cases where (3.13) is satisfied. Moreover, customers’ belief about the first period quantity ie. \( \hat{Q}_1 \) is \( Q_{1_{\text{max}}} \) and in order there would be an equilibrium this should also be the optimal first period quantity chosen by the retailer. Hence the following is the retailer’s
first period optimization problem.

\[
\begin{align*}
\max_{Q_1, \theta} & \quad [1 - \bar{\theta}]p_1(\bar{\theta}, Q_{1\max}, Q_{2\max}) - Q_1 c_1 + \Pi_{R,2}(\bar{\theta}, Q_1, Q_{2\max}) \\
\text{s.t.} & \quad Q_{2\max} < \frac{1}{4}(2(1 - Q_1) - \bar{\theta}), \\
& \quad Q_1 \leq Q_{1\max} \\
& \quad (Q_1, \bar{\theta}) \in S'_1.
\end{align*}
\]

Note that retailer’s objective function is jointly concave in \(Q_1\) and \(\bar{\theta}\), so KKT conditions are necessary and sufficient. The following are retailer’s equilibrium strategies.

(i) When \(c_1 \leq \frac{\delta(3-4Q_{1\max}-\delta(3-4Q_{1\max}+Q_{2\max}))}{2(1-\delta)}\) and \(\frac{(1-\delta)(1-2Q_{1\max})}{\delta} \leq Q_{2\max} \leq \frac{(1-\delta)(3-4Q_{1\max})}{8-7\delta}\), the retailer sells the product in both periods and it carries inventory between periods where \(Q_1^* = Q_{1\max}, p_1^* = \frac{1}{2} (1 + \delta(1 - 2Q_{1\max} - Q_{2\max}))\) and \(\bar{\theta}^* = \frac{1-\delta(1-Q_{2\max})}{2(1-\delta)}\).

(ii) When \(c_1 \leq 1 - (2 - \delta)Q_{1\max} - \delta Q_{2\max}\) and \(Q_{2\max} \leq \max\left\{\frac{1-Q_{1\max}}{4}, \frac{(1-\delta)(1-2Q_{1\max})}{\delta}\right\}\), the retailer sells the product in both periods and it does not carry inventory between periods where \(Q_1^* = Q_{1\max}, p_1^* = 1 - Q_{1\max} - \delta Q_{2\max}\) and \(\bar{\theta}^* = 1 - Q_{1\max}\).

Following Lemma 10 and the above regions and optimal values for the retailer we can solve for supplier’s optimization problem in terms of \(c_1, Q_{1\max}\) and \(Q_{2\max}\). Note that supplier’s objective function is linear and increasing in \(c_1\), hence the optimal \(c_1\) is at the boundary. This simplifies the solution and the optimal comes from the second region given above where \(c_1^* = \frac{1}{2}, Q_{1\max}^* = \frac{2-\delta}{2(4-3\delta)}\) and \(Q_{2\max}^* = \frac{1-\delta}{2(4-3\delta)}\), which is the same optimal solution when supplier makes a quantity commitment only in period 2, which is presented in Section 3.5. Therefore, supplier does not benefit from making a quantity commitment in first period when it makes a quantity commitment in second period.

Proof of Theorem 3. The result for supplier profit follows from the fact that supplier can always match the no-commitment scenario profit by committing to an arbitrary large non-binding quantity. Hence, we only need to prove the results for retailer and supply chain profits. Following Propositions 2 and 8, we have four cutoff points for \(\delta\), so there are five regions. Below we present the proof separately for each region.
When $\delta < 1/2$, $\Pi_S^{SQC} = \frac{1}{4(2-\delta)}$. Then, $\Pi_S^{SQC} - \Pi_S^{NC} = \frac{\delta(2+3\delta)}{64(2-\delta)}$, which is positive. $\Pi_R^{NC} = \frac{16+3\delta}{256}$ and $\Pi_R^{SQC} = \frac{1-\delta}{4(2-\delta)^2}$. Therefore, $\Pi_R^{NC} - \Pi_R^{SQC} = \frac{\delta(12+6(4+3\delta))}{256(2-\delta)^2}$, which is also positive.

For the supply chain profit in this case (\(\Pi_S^{SQC} + \Pi_R^{SQC} \)) - (\(\Pi_S^{NC} + \Pi_R^{NC} \)) = \frac{\delta(4+3(4-5\delta))}{256(2-\delta)^2}$, which is positive because $\delta < 4/5$.

When $1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$, $\Pi_S^{SQC} = \frac{(2-\delta)^2}{3(7-5\delta)}$. $\Pi_R^{NC}$ is the same as the previous case and $\Pi_R^{SQC} = \frac{2+\delta(9-5\delta(3-\delta))}{3(7-5\delta)^2}$. Thus,

$$\Pi_R^{SQC} - \Pi_R^{NC} = \frac{\delta(6-5\delta(3-\delta)) - 14}{768(7-5\delta)^2} \quad (B.27)$$

The denominator of (B.27) is clearly positive. Define $N_1(\delta)$ as the numerator of (B.27). Then, $\frac{dN_1}{d\delta} = 6 - 15\delta(2-\delta) > 0$ for $1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$. So, $N_1(\delta)$ is increasing. However, $N_1(\frac{1}{2}) < 0$ and $N_1(\frac{199-3\sqrt{137}}{218}) > 0$. Therefore, (B.27) crosses zero only once for the $\delta$ range of this case and is given by the solution to the following equation, which is the value $\delta_4$ in Theorem 3.

$$\delta = \frac{14}{6-5\delta(3-\delta)} \quad (B.28)$$

For the supply chain profit in this case,

$$(\Pi_S^{SQC} + \Pi_R^{SQC} ) - (\Pi_S^{NC} + \Pi_R^{NC} ) = \frac{\delta(6-5\delta(3-\delta)) - 14}{768(7-5\delta)^2} \quad (B.29)$$

The denominator of (B.29) is clearly positive. Define $N_2(\delta)$ as the numerator of (B.29). Then, $\frac{d^2N_2}{d\delta^2} = 2(874 - 1125\delta) > 0$ since $\delta < \frac{874}{1125}$. So, first derivative of $N_2(\delta)$ is increasing but $\frac{dN_2}{d\delta}|_{\delta=\frac{199-3\sqrt{137}}{218}} < 0$. So, $N_2$ is decreasing. Because, $N_2(\frac{199-3\sqrt{137}}{218}) > 0$, the result follows.

When $\frac{199-3\sqrt{137}}{218} \leq \delta < \bar{\delta}_3$, $\Pi_R^{SQC} = \Pi_R^{NC}$, hence the results immediately follow.

When $\bar{\delta}_3 \leq \delta < \frac{16}{27}$, we show that for every feasible $c_1$ and optimal $Q_{max}$, $\Pi_R^{SQC} > \Pi_R^{NC}$. Define the difference function $\Pi_R^{SQC} - \Pi_R^{NC} \big|_{Q_{max}=K_2} = \Delta_1 \Pi_R(c_1)$ then,

$$\Delta_1 \Pi_R(c_1) = \frac{256c_1^2(192 - \delta(332 - 143\delta)) - 384c_1\delta(224 - \delta(400 - 179\delta)) + 9\delta(3072 - \delta(6592 - \delta(4672 - 1107\delta)))}{2304(8 - 7\delta)^2\delta} \quad (B.30)$$

114
which is a convex function of $c_1$, and from case (iii).b.2 of Proposition 7, $K_e < c_1 < \frac{3\delta(4-3\delta)}{2(16-15\delta)}$. The smaller $c_1$ root for $\Delta_1\Pi_R = 0$ is $c_1 = \frac{3\delta(4-3\delta)}{2(16-15\delta)}$, which is greater than $\frac{3\delta(4-3\delta)}{2(16-15\delta)}$ for $\delta_3 \leq \delta < \frac{16}{21}$. Since $c_1^*$ is less than this boundary, $\Delta\Pi_S > 0$ for optimal $c_1$.

The result for supply chain profit for this case follows from the above.

When $\delta \geq \frac{16}{21}$, similar to the previous case, we show that for every feasible $c_1$ and optimal $Q_{max}$, $\Pi_{R}^{SC} > \Pi_{R}^{NC}$. Define the difference function $\Pi_{R}^{SC} - \Pi_{R}^{NC}$, then,

$$\Delta_2\Pi_R(c_1) = \frac{(3(4 - 3\delta)\delta - 2c_1(16 - 15\delta))(24(1 - \delta)\delta(176 - 3\delta(105 - 47\delta)) - c_1(12 - 11\delta)(16 - 13\delta)(16 - 15\delta))}{18(16 - 15\delta)^2(8 - 7\delta)^2\delta},$$

which is a convex function of $c_1$. The smaller $c_1$ root for $\Delta_2\Pi_R = 0$ is $\frac{3\delta(4-3\delta)}{2(16-15\delta)}$, however, optimal $c_1$ is less than this value. Hence $\Delta\Pi_S > 0$ for optimal $c_1$.

The result for supply chain profit for this case follows from the above.

\[\square\]

**Proof of Lemma 10.** This proof is similar to proof of Lemma 7. First part of Equation (3.12) gives $\delta (2(1 - Q_1 - Q_{2max}) - \bar{\theta}) - c_2 \geq 0$. Note that $\Pi_{R,2}$ is given by the equation (B.4) in the proof of Lemma 7. Since commitment binds the inequality above binds. Hence, $c_2 = c_2^* = \delta (2(1 - Q_1 - Q_{2max}) - \bar{\theta})$. Plugging this into the second part of Equation (3.12) with $\Pi_{S,2}$ in equation (B.5) gives $Q_{2max} \leq \frac{1}{2} (2(1 - Q_1) - \bar{\theta})$.

\[\square\]

**Proof of Proposition 9.** When $Q_{2max} = \frac{1}{2} (2(1 - Q_1) - \bar{\theta})$ the objective function given in (3.14) is the same as the no commitment objective given by (2.12), which is explicitly stated by equation (B.3) in the proof of Proposition 1. Therefore, when finding the solution of retailer’s objective in the region when commitment binds we can restrict the feasible space to where the inequality in (3.13) is strictly satisfied.
Following Lemma 10 we can formulate retailer’s period 1 problem in terms of order quantity $Q_1$, commitment $Q_{2\text{max}}$ and target consumer segment $[\bar{\theta}, 1]$ in period 1 as in the following:

$$\max_{Q_1, \bar{\theta}} \quad p_1(\bar{\theta}, \hat{Q}_1, Q_{2\text{max}})(1 - \bar{\theta}) - Q_1 c_1 + \Pi_{R,2}(\bar{\theta}, Q_1, Q_{2\text{max}})$$

s.t. $Q_{2\text{max}} < \frac{1}{4}(2(1 - Q_1) - \bar{\theta})$,

$$\hat{Q}_1 = Q_1^*$$

$$(Q_1, \bar{\theta}) \in S_1'.$$

Note that $S_1'$ is defined in (2.15). The retailer’s total profit in equation (3.14) can be written as the following.

$$\Pi_R(Q_1, \bar{\theta}, \hat{Q}_1) = \delta(Q_{2\text{max}}^2 + \bar{\theta} - Q_1(Q_1 + \bar{\theta} - 2) - 1) - (1 - \bar{\theta})(\delta(\hat{Q}_1 + Q_{2\text{max}} + \bar{\theta} - 1) - \bar{\theta}) - c_1 Q_1.$$  

(B.32)

Hessian of (B.32) is

$$\begin{pmatrix}
-2\delta & -\delta \\
-\delta & -2(1 - \delta)
\end{pmatrix},$$

where the eigenvalues of the Hessian are $-1 - \sqrt{1 - \delta(4 - 5\delta)}$ and $-1 + \sqrt{1 - \delta(4 - 5\delta)}$. Both of the eigenvalues are non-positive for $\delta \leq 4/5$, hence (B.32) is jointly concave in $(Q_1, \bar{\theta})$.

We showed that retailer’s objective function in the no-commitment scenario is jointly concave in the Proof of Proposition 1. Given the two functions are continuous at the boundary and both jointly concave, and the overall problem is quasiconcave; KKT conditions together with the belief consistency equation fully characterizes the solution, which yield the regions and the optimal values in Proposition 9.

Proof of Proposition 10. $\Pi_S$ defined by all the parts of the proposition 9 is not quasiconcave. Therefore, to find the global optimal we need to analyze each part separately.

For the first part of the proposition 9, $\Pi_S = \frac{1}{4} \left( c_1 \left( 3 - \frac{2c_1}{\delta} \right) + c_1(4 - 5\delta) Q_{2\text{max}} - 8\delta Q_{2\text{max}}^2 \right)$, which is a jointly concave function of $(c_1, Q_{2\text{max}})$. First order condition gives $\frac{48(1-\delta)^2\delta}{48-\delta(88-39\delta)}$, $\frac{3(1-\delta)(4-5\delta)}{48-\delta(88-39\delta)}$ for $c_1$ and $Q_{2\text{max}}$ respectively. However, these values do not satisfy the feasible region defined in first
part of the proposition 9. In addition, $\Pi_S$ is continuous in both of the boundaries of the feasible region therefore global optimal solution does not come from this part.

For the second part of proposition 9, $\Pi_S = \frac{(1-c_1)c_1+\delta Q_{2_{max}}(1-\delta-(4-3\delta)Q_{2_{max}})}{2-\delta}$, which is also a jointly concave function of $(c_1, Q_{2_{max}})$. First order condition gives $c_1^* = \frac{1}{2}$ and $Q_{2_{max}}^* = \frac{1-\delta}{2(4-3\delta)}$. These values satisfy the feasible region defined in second part of the proposition 9. Hence, unless any other solution from the remaining parts gives a better objective value this is the global optimal point.

For the third part of the proposition 9, $\Pi_S = \delta(1-2Q_{2_{max}})Q_{2_{max}}$, which is a function of $Q_{2_{max}}$. Hence the optimal solution has to be at the boundary but $\Pi_S$ is continuous at this boundary between parts (ii) and (iii) of proposition 9 and part (ii) has an interior solution. Therefore, the global solution cannot come from part three.

For the fourth part of the proposition 9, $\Pi_S = \frac{c_1(6(1-\delta)-c_1)-2c_1(8-9\delta)Q_{2_{max}}+16(1-\delta)\delta Q_{2_{max}}^2}{8(1-\delta)}$, which is a quadratic convex function of $Q_{2_{max}}$. Therefore, the solution has to be at one of the boundaries. In addition, both of the boundaries are continuous. First boundary is the one with part (i), which is dominated. We analyze the second boundary in the next part.

For the fifth part of the proposition 9, $\Pi_S = (1-4Q_{2_{max}})c_1 + 2\delta Q_{2_{max}}^2$, which is a quadratic convex function of $Q_{2_{max}}$. Hence, the solution has to be on the boundary. The boundary with part (ii) of the proposition 9 is continuous. Since there is an interior solution in that part no solution can come from this boundary. The boundary with the fourth part of proposition 9 is also continuous, however never produces a solution.

The sixth and seventh parts of the proposition 9 are functions of $c_1$ so we can consider only the solutions at the $Q_{2_{max}}$ boundaries, however both are dominated at those regions.

Therefore the solution in Proposition 10 follows.

Proof of Theorem 4. The result about the supplier profit follows from the Corollary 3. We only need to prove the results about the retailer and supply chain profits.

Following Propositions 2 and 10 there are two cutoff points for $\delta$ below we present the proof for each case separately.

When $\delta \leq \frac{16}{21}$, $\Pi_R^{QC2} - \Pi_R^{NC} = -\frac{(28-9\delta)\delta}{256(4-3\delta)}$, which is clearly negative. $(\Pi_S^{QC2} + \Pi_R^{QC2}) - (\Pi_S^{NC} + \Pi_R^{NC}) = \frac{\delta(20-19\delta)}{256(4-3\delta)}$, which is clearly positive.
When $\delta > \frac{16}{21}$, $d(\Pi_S^{QC2} - \Pi_S^{QC}) = \frac{1}{100} \left( 23 + \frac{176}{(16-15\delta)^2} - \frac{25}{(4-3\delta)^2} - \frac{1024}{(16-15\delta)^3} \right)$, which is greater than 0, hence the difference function is increasing. However, the difference is negative at both end points of the feasible region hence the result follows. The proof for the supply chain result is similar.

**Proof Corollary 3.** Following Propositions 8 and 10 we have four cutoff points for $\delta$. Below we present the proof separately for each region. For every region $\Pi_S^{QC2} = \frac{2-\delta^2}{4(4-3\delta)}$.

When $\delta < \frac{1}{2}$, $\Pi_S^{QC} = \frac{1}{4(2-\delta)}$ and $\Pi_S^{QC2} - \Pi_S^{QC} = \frac{(1-\delta)^2\delta}{4(2-\delta)(4-3\delta)}$, which is clearly positive.

When $1/2 \leq \delta < \frac{199-3\sqrt{137}}{218}$, $\Pi_S^{QC2} - \Pi_S^{QC} = \frac{6(82-\delta^2)(276)-22}{12(7-5\delta)(4-3\delta)}$, which is an increasing function.

In addition it is positive at both end points of the feasible region.

When $\frac{199-3\sqrt{137}}{218} \leq \delta \leq \frac{1}{2}$, $\Pi_S^{QC} = \frac{1}{2(2-\delta)}$ and $\Pi_S^{QC2} = \frac{(1-\delta)^2\delta}{4(2-\delta)(4-3\delta)}$, which is clearly positive.

When $\frac{1}{2} \leq \delta < \bar{\delta}$, $\Pi_S^{QC2} - \Pi_S^{QC} = \frac{6(82-\delta^2)(276)-22}{12(7-5\delta)(4-3\delta)}$, which is clearly positive.

When $\bar{\delta} \leq \delta$, $\Pi_S^{QC2} - \frac{1}{5} = \frac{6(12-5\delta)-6}{20(4-3\delta)} > 0$ since $\delta > \frac{1}{5} (6 - \sqrt{6})$. For every $Q_{max}$ and for every $c_1$ in the region defined by the case (iii).b.2 of Proposition 7 ie. $K_c < c_1 \leq \frac{36(4-3\delta)}{2(16-15\delta)}$ in order for $\Pi_S^{QC} - \frac{1}{5} > 0$ we need $\delta > \frac{1}{210} (265 - \sqrt{9745})$ and $Q_{max} > 1 - \frac{3c_1}{5} + \frac{3\sqrt{\frac{100\delta^2+\delta-5c_1\delta}{4\delta^2}}}{\sqrt{10}} > \frac{9}{20}$. However $K_2 < \frac{9}{20}$ in that region. Therefore $\Pi_S^{QC} < \frac{1}{5}$ and the result follows.

**Proof of Theorem 5.** To find the equilibrium in simultaneous quantity commitment scenario, it is sufficient to compare the equilibriums of individual quantity commitment scenarios and identify the case when retailer would deviate from supplier’s quantity commitment equilibrium. In other words retailer would deviate from supplier’s quantity commitment equilibrium when restricting the equilibrium commitment quantity further and when this action increases its profit. This happens when $\delta \leq \bar{\delta}$ where equilibrium commitment quantity in retailer’s quantity commitment is $\frac{1}{4}$ which is less than corresponding equilibrium commitment quantity in supplier’s quantity commitment, which is $\frac{1}{2(2-\delta)}$. In addition, in this range $\Pi_R^{QC} > \Pi_R^{QC}$ because $\frac{1}{10} < \frac{1-\delta}{4(2-\delta)}$. Therefore, the Theorem 5 follows from Propositions 6 and 8.

**B.3 Proofs of Chapter 4**

**Proof of Lemma 11.** See the proof of Lemma 5.

**Proof of Theorem 6.** See the proof of Theorem 1.
**Proof of Lemma 12.** Supplier sets \( c_2^* = p_2 \). Then, as in the proof of Lemma 1, the optimization problem in (2.2) can be expressed as the following.

\[
Q_2^* = \arg \max_{Q_2} -\delta (Q_1 - (1 - \bar{\theta})) (Q_2 + Q_1 - 1)
\]  
(B.33)

which is a convex and decreasing function of \( Q_2 \). Hence, \( Q_2^* = 0 \). \( \square \)

**Proof of Proposition 11.** Since the retailer commits to \( p_1 \) and \( p_2 \), the marginal customer, which is given by (2.4), can be written as \( \bar{\theta}^* = \frac{p_1 - p_2}{1 - \delta} \). In addition following Lemma 12 and the fact that \( c_1 > 0 \), the constraint in (2.9) binds. Hence, we can write \( Q_1^* = 1 - \frac{p_2}{\delta} \). Plugging these into the optimization problem in (2.5) leads to

\[
(p_1^*, p_2^*) = \arg \max_{(p_1, p_2) \in S_1} [p_1 - \frac{(p_1 - p_2)^2}{1 - \delta} + \frac{(c_1 - p_2)p_2}{\delta} - c_1],
\]  
(B.34)

where \( S_1 \) becomes \( \{(p_1, p_2) : \delta p_1 \geq p_2\} \). However, \( \frac{\partial \Pi_{R, 1}}{\partial p_2} \bigg|_{p_2 = \delta p_1} = \frac{c_1}{\delta} > 0 \), so optimal \( p_2 \) is equal to \( \delta p_1 \), which means that the retailer sells only in period one. Then the pricing problem reduces to

\[
p_1^* = \arg \max_{p_1} (p_1 - c_1)(1 - p_1).
\]  
(B.35)

First-order condition gives \( p_1^* = \frac{1 + c_1}{2} \) and \( Q_1 = \frac{1 - c_1}{2} \).

The supplier’s pricing problem in the first period is the following:

\[
\max_{c_1} c_1 \left( \frac{1 - c_1}{2} \right)
\]  
(B.36)

First-order condition gives \( c_1^* = \frac{1}{2} \). \( \square \)

**Proof of Theorem 7.** Following Propositions 2 and 11, we have one cutoff point for \( \delta \), so there are two different cases to consider. Below we present the proof for each case.

When \( \delta \leq \frac{16}{21} \), \( \Pi_{NS}^{NC} = \frac{1}{8} + \frac{3\delta}{64} \) and \( \Pi_{R}^{RFC} = \frac{1}{8} \), hence the result follows because \( \delta \geq 0 \).

For the retailer’s profit in this case, \( \Pi_{NC}^{R} = \frac{1}{16} + \frac{3\delta}{16} \) and \( \Pi_{R}^{RFC} = \frac{1}{16} \), hence the result follows because \( \delta \geq 0 \).

The result for supply chain profit follows from the above.
When $\delta > \frac{16}{21}$, $\Pi_{NS}^{NC} = \frac{\delta(56 - 3(34 - 15\delta)\delta)}{(16 - 15\delta)^2}$ and $\Pi_{S}^{RPC} = \frac{1}{8}$.

$$\Pi_{S}^{NS} - \Pi_{S}^{RPC} = \frac{\delta(928 - \delta(347 - 120\delta)) - 256}{(16 - 15\delta)^2} \tag{B.37}$$

The denominator of (B.37) is positive. The numerator is a concave function of $\delta$ because the second derivative is $6(360\delta - 347) < 0$ as $\delta \leq \frac{4}{5}$. However, the numerator is positive at the boundaries, hence (B.37) is also positive.

For retailer profits in this case, $\Pi_{R}^{NC} = \frac{(4 - 3\delta)(64 - 3(44 - 23\delta)\delta)}{4(16 - 15\delta)^2}$ and $\Pi_{R}^{RPC} = \frac{1}{16}$.

$$\Pi_{R}^{NC} - \Pi_{R}^{RPC} = \frac{768 - 3(800 - \delta(821 - 276\delta))}{16(16 - 15\delta)^2} \tag{B.38}$$

The denominator of (B.38) is positive. The numerator is a concave function of $\delta$ because the second derivative is $4926 - 4968\delta < 0$. However, the numerator is positive at the boundaries, hence (B.38) is positive.

The result for supply chain profit follows from the above. \[\square\]

**Proof of Lemma 13.** The proof of this result is similar to the proof of Lemma 1 and it is omitted. \[\square\]

**Proof of Proposition 12.** The regions differ, depending on the value of $Q_2^*$ (which is a function of $Q_1$ and $\bar{\theta}$).

When $Q_2^* > 0$, the retailer profit in (4.3) and Lemma 13 lead to the following,

$$\Pi_R = -(1 - \frac{3\delta}{4})\bar{\theta}^2 - (c_1 - c_2)Q_1 - \frac{1}{4}(2c_2 - 2\bar{\theta}(2 - \delta) - \frac{c_2^2}{\delta}). \tag{B.39}$$

When $Q_2^* = 0$, the retailer profit in (4.3) reduces to $(1 - \bar{\theta})p_1(\bar{\theta}) - c_1Q_1$. Because no units are sold in period 2 in this case, $p_1^*(\bar{\theta}) = \bar{\theta}$. These observations lead to the following retailer objective function

$$\Pi_R = (1 - \bar{\theta})\bar{\theta} - c_1Q_1. \tag{B.40}$$
In both of the regions $\Pi_R$ is jointly concave in $(\bar{\theta}, Q_1)$ and $\Pi_R$ is quasiconcave across these two regions. Hence, KKT conditions are sufficient to characterize the optimal solutions in these regions which yield the stated parts in the Proposition 12.

**Proof of Proposition 13.** To find the supplier’s optimal policy, we need to analyze the supplier’s profits in each of the cases in Proposition 12, where we follow the order (i), (iv), (iii) and (ii) on account of the ease of proving the result.

For case (i), $\Pi_S = -\frac{c_2(c_2 - \delta)}{2\delta}$, which is a function of $c_2$. So, the optimal $c_2$ is at the upper bound of the region, which is also in case (ii). Hence, there is no solution in case (i).

For case (iv), $\Pi_S = \frac{(1-c_1)c_1}{2}$, which is a function of $c_1$. So, the optimal $c_1$ is at the upper bound of the region, which is also in case (iii). Hence, there is no solution in case (iv).

For case (iii), $\Pi_S = c_1 - \frac{c_1^2}{\delta}$, which is a convex and decreasing function of $c_2$. So the optimal $c_2$ must be at the lower bound, which is also in case (ii). In addition $\Pi_S$ is continuous at the boundary between the regions of cases (iii) and (iv) so the global solution cannot be in case (iv).

For case (ii), the following is the supplier profit

$$\Pi_S = \frac{-c_2^2(4 - \delta) + c_2\delta(2 + 6c_1 - \delta) + 4c_1\delta(1 - c_1 - \delta)}{2\delta(4 - 3\delta)}, \quad (B.41)$$

which is jointly concave in $(c_1, c_2)$. In addition $\Pi_S$ is continuous at both of the boundaries of the regions between cases (i) and (iii). Thus, KKT conditions for case (ii) are sufficient to characterize the global optimal solution, which leads to the part (i) and (ii) of the Proposition 13.

**Proof of Theorem 8.** Following Propositions 2 and 13, we have two cutoff points for $\delta$. Thus, there are three cases to consider. Below we analyze the result for each of the three cases.

When $\delta \leq 1 - \frac{1}{\sqrt{17}}$, $\Pi^{NC}_S = \frac{8 + 3\delta}{64}$ and $\Pi^{SPC}_S = \frac{4 - \delta(2 + \delta)}{2(16 - 13\delta)}$. Therefore,

$$\Pi^{NC}_S - \Pi^{SPC}_S = \frac{\delta(8 - 7\delta)}{64(16 - 13\delta)}, \quad (B.42)$$

which is always greater than 0.

For retailer’s profit in this case, $\Pi^{NC}_R = \frac{16 + 3\delta}{256}$ and $\Pi^{SPC}_R = \frac{32 - \delta(54 - \delta(24 - \delta))}{2(16 - 13\delta)^2}$, therefore,

$$\Pi^{NC}_R - \Pi^{SPC}_R = \frac{\delta(1024 - \delta(1616 - 635\delta))}{256(16 - 13\delta)^2}. \quad (B.43)$$
Here, the denominator of (B.43) is positive. The term in parentheses in the numerator is also positive because its derivative is $2(635\delta - 808) < 0$, so it is decreasing but its value at $\delta = 1 - \frac{1}{\sqrt{17}}$ is $\frac{2}{17} (683 + 173\sqrt{17})$. Hence, (B.43) is positive.

Result for the supply chain profit follows from above.

When $1 - \frac{1}{\sqrt{17}} < \delta \leq \frac{16}{27}$, $\Pi_{SC}^{NC}$ is the same as the previous case and $\Pi_{S}^{SPC} = \frac{\delta(6-5\delta)^2}{8(4-3\delta)^2}$, thus,

$$\Pi_{S}^{NC} - \Pi_{S}^{SPC} = \frac{128 + \delta((480 - 173\delta)\delta - 432)}{64(4 - 3\delta)^2}.$$  \hfill (B.44)

The denominator of (B.44) is positive. Let us define $N_1$ as the numerator. Then $N_1$ is a decreasing function of $\delta$, because $\frac{dN_1}{d\delta} = -432 + 960\delta - 519\delta^2$, which is concave where the maximum occurs at $\delta = \frac{16}{27}$ with a value of $-\frac{272}{147}$. However, $N_1(\frac{16}{27}) = \frac{9088}{9261}$. Hence, $N_1$ is positive, so is the (B.44).

For the retailer’s profit in this case, $\Pi_{R}^{NC}$ is the same as the previous case and $\Pi_{R}^{SPC} = \frac{64 + \delta((156 - 47\delta)\delta - 172)}{16(4 - 3\delta)^2}$, therefore

$$\Pi_{R}^{NC} - \Pi_{R}^{SPC} = \frac{\delta(2416 - \delta(2424 - 779\delta)) - 768}{256(4 - 3\delta)^2}.$$  \hfill (B.45)

The second derivative of (B.45) is $-\frac{3(8-5\delta)^2}{2(4-3\delta)^2}$, so it is a concave function of $\delta$, however it is positive at both of the boundary points, $\delta = 1 - \frac{1}{\sqrt{17}}$ and $\delta = \frac{16}{27}$. Hence, (B.45) is positive.

The result for supply chain profit follows from the above.

When $\delta > \frac{16}{27}$, $\Pi_{S}^{NC} = \frac{\delta(56 - 3(34-15\delta))}{(16-15\delta)^2}$ and $\Pi_{S}^{SPC}$ is the same as the previous case, therefore

$$\Pi_{S}^{NC} - \Pi_{S}^{SPC} = \frac{\delta(2 - 3\delta)}{8(64 - 9\delta(12 - 5\delta))} (\delta(2880 + \delta(-2642 + 795\delta)) - 1024).$$  \hfill (B.46)

The fraction in (B.46) is negative because $\delta > \frac{2}{7}$. Let us define $N_2(\delta)$ as the second term in (B.46). $\frac{d^2N_2}{d\delta^2} = -5284 + 4770\delta$, so $N_2$ is concave. $\frac{dN_2}{d\delta} \big|_{\delta = \frac{4}{5}} = \frac{896}{5}$, so maximum of $N_2$ occurs at the upper bound of $\delta$, but $N_2(\frac{4}{5}) = -\frac{96}{25}$. So, $N_2$ is negative and therefore (B.46) is positive.
For the retailer’s profit in this case, \( \Pi_{NC_R}^{NC} = \frac{(4-3\delta)(64-3(44-23\delta)\delta)}{4(16-15\delta)^2} \) and \( \Pi_{SPC_R}^{SPC} \) is the same as in the previous case. Thus,

\[
\Pi_{NC_R}^{NC} - \Pi_{SPC_R}^{SPC} = \frac{\delta(4096 + \delta(21932 - 3\delta(4532 - 1041\delta)) - 15552)}{16(64 - 9\delta(12 - 5\delta))^2}
\]  
(B.47)

We claim that (B.47) is positive. Its denominator is positive. It is straightforward to show that its numerator decreases in \( \delta \) and at \( \delta = 4/5 \) it is positive.

The result for supply chain profit follows from the above. \( \square \)

**Proof of Proposition 14.** Since, supplier commits to \( c_2 \) before retailer orders the first period quantity retailer has no incentive to carry inventory. Therefore, \( Q_1^* = 1 - \bar{\theta}^* \). Also, since retailer commits to \( p_1 \) and \( p_2 \) at the beginning of period 1 the marginal customer, \( \bar{\theta} \) becomes \( \frac{p_1-p_2}{\delta} \). Plugging these into retailer’s objective function, it becomes

\[
\delta p_1(1-c_2-\delta-p_1)-c_1\delta(1-\delta-p_1+p_2)+(c_2+2\delta p_1)p_2-p_2^2
\]

which is jointly concave in \( p_1 \) and \( p_2 \). KKT conditions yield the following strategies for the retailer.

(i) When \( c_2 \leq c_1 + \delta - 1 \), the retailer sells only in period 2 where \( p_1^* = \frac{2+c_2-\delta}{2} \) and \( p_2^* = \frac{c_2+\delta}{2} \).

(ii) When \( c_1 + \delta - 1 < c_2 < \delta c_1 \), the retailer sells in both periods where \( p_1^* = \frac{1+c_1}{2} \) and \( p_2^* = \frac{c_2+\delta}{2} \).

To characterize the equilibrium we need to solve the supplier’s problem. Supplier maximizes

\[
c_1(1-\bar{\theta}^*) + c_2(\bar{\theta}^* - \frac{p_2^*}{\delta})
\]

where \( p_1^* \), \( p_2^* \) and \( \bar{\theta}^* \) are given above. This gives the optimal solution \( c_1^* = \frac{1}{2} \) and \( c_2^* = \frac{\delta}{2} \), which forces retailer to sell only in period 1 by making second period price too high. Therefore the equilibrium becomes the same as the one in Section 4.2. \( \square \)

**B.4 Proofs of Chapter 5**

**Proof of Lemma 14.** Since retailer sets \( p_2 \) before supplier sets second period wholesale price, supplier has no incentive to set \( c_2 \) smaller than \( p_2 \). This results \( \Pi_{R,2} = \delta(1 - Q_{max})(Q_1 + \theta - 1) \) and \( \Pi_{S,2} = \delta(1 - Q_{max})(Q_{max} - Q_1) \). \( \square \)

**Proof of Proposition 15.** Following marginal customer equation and Lemma 14 retailer’s first period objective is \( \Pi_{R,1} = Q_1(\delta(1 - Q_{max}) - c_1) + (1 - \delta)\theta - (1 - \delta)\theta^2 \), which is a linear decreasing
function with respect to $Q_{\text{max}}$. Hence, $Q_{\text{max}}$ should be at the lower bound. Substituting $Q_{\text{max}}$ with $Q_1$; $\Pi_{1,R}$ becomes jointly concave with respect to $Q_1$ and $\theta$, and first order condition yields the result.

Following retailer’s best response $\Pi_{S,1} = \frac{1}{2}(1 - c_1)c_1$, where first order condition yields $c_1^* = \frac{1}{2}$.

**Proof of Theorem 9.** The following characterizes the optimal values when supplier commits to both a maximum total quantity and the wholesale prices: $Q_{\text{max}}^* = \frac{5 - 4\delta}{16 - 13\delta}$, $c_1^* = \frac{16 - \delta(14 - \delta)}{2(16 - 13\delta)}$, $c_2^* = \frac{2\delta(5 - 4\delta)}{16 - 13\delta}$, and $\Pi_{S,SPQC} = \frac{4 - \delta(2 + \delta)}{2(16 - 13\delta)}$.

First let’s show that this profit is not better than what supplier gets from a quantity commitment only. We do this by showing $\Pi_{S,SPQC} < \Pi_{S,NC}$, since by Theorem 3 $\Pi_{S,NC} \leq \Pi_{S,SPQC}$. Following Proposition 2 when $\delta \leq \frac{16}{21}$; $\Pi_{S,NC} - \Pi_{S,SPQC} = \frac{\delta(8 - 7\delta)}{64(16 - 13\delta)}$, which is clearly positive. When $\delta > \frac{16}{21}$; $\Pi_{S,NC} = \frac{\delta(4 - 3\delta)(14 - 15\delta)}{(16 - 15\delta)^2}$ where second derivative is $\frac{48(32 - 75\delta)}{(16 - 15\delta)^4}$, which is negative since $\delta > \frac{32}{75}$. Hence, $\Pi_{S,NC}$ is concave in this region. In addition it is equal to $\frac{9}{56}$ and $\frac{4}{25}$ at the boundaries respectively. Therefore $\Pi_{S,NC} \geq \frac{4}{25}$ but $\Pi_{S,SPQC} < \frac{4}{25}$ since $\delta < \frac{1}{25}(27 - \sqrt{29})$.

Second let’s show how we determined above optimal values. Without loss of generality we can consider supplier makes an overall quantity commitment on top of it’s price commitment. Instead of characterizing the full equilibrium we determine when quantity commitment binds. Following Lemma 13 in Section 4.3 this happens when $\frac{\delta(2 - 2Q_1 - \bar{\theta}) - c_2}{2\delta} = Q_{\text{max}} - Q_1$. Using retailer objective function given by Equation (B.39) in the proof of Proposition 12 we find the corresponding retailer optimal prices and the region: $\bar{\theta} = 2(1 - Q_{\text{max}}) - \frac{c_2}{\delta}$, $Q_1 = 1 - \bar{\theta}$ when $c_2 \leq \delta$, $\frac{1}{2}(1 - \frac{c_2}{\delta}) \leq Q_{\text{max}} \leq 1 - \frac{c_2}{\delta}$ and $c_1 \leq 3 - \frac{5\delta}{2} + \left(\frac{5}{2} - \frac{2}{3}\right) c_2 - (4 - 3\delta)Q_{\text{max}}$.

Substituting these in the supplier objective and maximizing in the given region yields the values presented above.

**Proof of Theorem 10.** Following the analysis in Section 3.5, in order for the supplier’s second period quantity commitment to bind first part of the (3.12) need to be satisfied. Hence when $Q_{2,\text{max}} \leq \frac{\delta(2 - 2Q_1 - \bar{\theta}) - c_2}{2\delta}$, $Q_2^* = Q_{2,\text{max}}$, $p_2^* = \delta(1 - Q_1 - Q_{2,\text{max}})$ and

$$\Pi_{R,2} = \delta(1 - Q_1 - Q_{2,\text{max}})(Q_1 - (1 - \bar{\theta})) - c_2Q_{2,\text{max}}.$$
When $Q_{2\max} > \frac{\delta(2-2Q_1-\bar{\theta})-c_2}{2\delta}$, commitment is ineffective and the period 2 equilibrium is the same as the supplier’s price commitment scenario. Retailers period 1 choices determine whether the commitment in period 2 will bind. Let’s characterize retailer’s response assuming that the commitment always binds. Note that this is not the best response of the retailer and for some values of the wholesale prices this solution is actually dominated by the commitment does not bind case.

Assuming commitment binds then $\Pi_{R,1} = (\bar{\theta} - \delta Q_{2\max} - c_1)(1 - \bar{\theta}) + (\delta(\bar{\theta} - Q_{2\max}) - c_2)Q_{2\max}$. Note that this uses the fact that $Q_1 = 1 - \bar{\theta}$ since $c_1$ and $c_2$ are set before retailer orders $Q_1$. $\Pi_{R,1}$ is quadratic concave in $\bar{\theta}$ and the first order condition yields the following optimal values and strategies.

(i) Quantity commitment strictly binds, retailer sells the product in both periods and $\bar{\theta}^* = \frac{1}{2}(1 + c_1 + 2\delta Q_{2\max})$ when,

a) $c_2 \leq c_1 + \delta - 1$ and $Q_{2\max} \leq \frac{1-c_1}{2\delta}$,

b) $c_2 > c_1 + \delta - 1$ and $Q_{2\max} \leq \frac{(c_1+1)\delta-2c_2}{20(2-d)}$.

(ii) Quantity commitment weakly binds retailer sells the product in both periods and $\bar{\theta}^* = \frac{c_2+2\delta Q_{\max}}{3}$ when, $c_2 > c_1 + \delta - 1$ and $\frac{(c_1+1)\delta-2c_2}{20(2-d)} < Q_{2\max} \leq \frac{\delta-c_2}{2\delta}$.

(iii) Quantity commitment strictly binds and the retailer sells the product only in period 2 and $\bar{\theta} = 1$ when, $c_2 \leq c_1 + \delta - 1$ and $\frac{1-c_2}{2\delta} \leq Q_{2\max} \leq \frac{\delta-c_2}{2\delta}$.

Assuming above is the retailer’s response let’s characterize the supplier’s optimal solution. Since commitment binds $\Pi_{S,1} = c_1(1 - \bar{\theta}^*) + c_2Q_{2\max}$, which is linear increasing in $Q_{2\max}$. Therefore in optimal solution $Q_{2\max}$ has to be at the upper bound. This simplifies the optimization problem and the solution is as follows, which correspond to case (i.b) of above retailer response.

(i) $c_1^* = \frac{8-5\delta}{16-9\delta}$, $c_2^* = \frac{2\delta(5-3\delta)}{16-9\delta}$, when $\delta \leq \frac{15-\sqrt{33}}{12}$.

(ii) $c_1^* = c_2^* = \frac{1}{2} - \frac{1}{2\delta} + \delta$, when $\delta > \frac{15-\sqrt{33}}{12}$.

Note that this supplier optimal solution does not constitute the true equilibrium, because when characterizing retailer’s response we assumed that the solution always comes from the commitment
binding region. Therefore, the equilibrium supplier objective value is dominated by this solution. To complete the proof we need to show that optimal supplier objective corresponding to above is not better than the suppliers objective from the case when supplier commits only to second period quantity, ie. SQC2, which is given by Proposition 10 in Section 3.5; \( \Pi_S^{SQC2} = \frac{2-\delta^2}{4(4-3\delta)} \).

Following Proposition 10 and above supplier response there are two cases. For case (i) above \( \Pi_S^{SQC2} - \Pi_S^{SPQC2} = \frac{\delta(2(2-\delta)(28-9\delta))}{4(4-3\delta)(4-9\delta)} > 0 \), since \( \delta \leq 4/5 \). For case (ii) above \( \Pi_S^{SQC2} - \Pi_S^{SPQC2} = \frac{8(2-\delta)^2(2-\delta^2)-(3-2\delta)^2\delta(4-3\delta)}{32(4-3\delta)(2-\delta)^2} \). Clearly the denominator is positive. The numerator is also positive since;

\[
8(2-\delta)^2 (2-\delta^2) > 4\delta(3-2\delta)^2(4-3\delta) \\
8(3-2\delta)^2 (2-\delta^2) > 4\delta(3-2\delta)^2(4-3\delta) \\
8(3-2\delta)^2(4-3\delta) > 4\delta(3-2\delta)^2(4-3\delta) \\
8 > 4\delta
\]

where in above we make left hand side smaller in each step, which completes the proof.

\[ \square \]

### B.5 Proofs of Chapter 6

**Proof of Lemma 15.** This proof is very similar to the proof of Lemma 1. The constraint given by the Set (6.2) binds thus \( p_2 = 1 - Q_1 - Q_2 \). With this \( p_2 \), the following becomes a concave optimization problem of \( Q_2 \).

\[
(p_2^*, Q_2^*) = \arg \max_{(p_2, Q_2) \in \{(p_2, Q_2): Q_1 + Q_2 \geq 1 - p_2\}} [p_2(\bar{\theta} - p_2) - c_2 Q_2]
\]

First order condition yields the optimal values in part (i) of the Lemma 15.

Similarly with above optimal values the following becomes a concave optimization problem in \( c_2 \).

\[
c_2^* = \arg \max_{c_2} c_2 Q_2^*,
\]

where the first order condition gives the optimal \( c_2 \) in part (ii) of the Lemma 15. \[ \square \]
**Proof of Proposition 16.** This proof is very similar to the proof of Proposition 1. Following Lemma 15 retailer’s profit function can be written in terms of $Q_1$ and $\bar{\theta}$, which has an Hessian equal to \[ \begin{pmatrix} -\frac{3}{8} (5 - 4d) & -\frac{3}{4} \\ -\frac{3}{4} & -\frac{3}{2} \end{pmatrix} \]. Therefore, the objective function is jointly concave in $(Q_1, \bar{\theta})$ and KKT conditions fully characterize the equilibrium presented in the Proposition. \qed

**Proof of Proposition 17.** Each part of the supplier’s optimization problem corresponding to the cases presented in Proposition 16 are concave with respect to $c_1$. Since each of these subproblems has one variable and one parameter, $\delta$, defining the optimal solution is straightforward. However, the overall objective function is not quasiconcave. Therefore one needs to compare the solutions from each case, where the result follows from comparing the cases (i) and (ii). \qed

**Proof of Lemma 16.** See proof of Lemma 7. \qed

**Proof of Proposition 18.** This proof is very similar to the proof of Proposition 5. Retailer’s profit function can be written as the following

\[(Q_{\text{max}} - 1 + \bar{\theta})(Q_{\text{max}} + (1 - \delta)(1 - \bar{\theta})) - Q_1(\bar{\theta} + c_1 + 2(Q_{\text{max}} - 1)),\]

which is a quadratic convex function of $Q_{\text{max}}$. Hence the optimal $Q_{\text{max}}$ must be at one of the two end points of the objective function. Following the proof of Proposition 5 we separate the problem into two case where Case A refers to the case when $Q_{\text{max}}$ is at lower bound ie. $Q_{\text{max}} = Q_1$ and Case B refers to the case when $Q_{\text{max}}$ is at upper bound ie. $Q_{\text{max}} = \frac{Q_1 + 1}{2} - \frac{\bar{\theta}}{4}$. In both cases the resulting functions are jointly concave in $(Q_1, \bar{\theta})$ so KKT conditions characterize the local solutions. To find the global solution we compare the local solutions, where the threshold $K_{c_2}$ is the positive root of the difference function of the local interior solutions of the Cases A and B. \qed

**Proof of Proposition 19.** This proof is very similar to the proof of Proposition 6. We follow the same approach. Since overall supplier profit function corresponding to the strategies presented in Proposition 18 is not quasiconcave we need to analyze each case separately. To find the global solution we compare the solutions from each case. Note that in each case supplier profit function is concave in $c_1$, which makes the subproblems easy to characterize. We skip the details and present only the result. Solutions from cases (ii) and (iii) of Proposition 18 dominate the rest. Specifically
when delta is smaller than the first root of the following equation, which characterizes $\bar{\delta}_5$,

$$261 - 2\delta(307 - 2\delta(98 - \delta(15 - 2\delta))) = 0,$$

(B.48)

optimal solution comes from the second case of Proposition 18.

Proof of Lemma 17. The proof of this result is similar to the proof of Lemma 1 and it is omitted. □

Proof of Proposition 20. This proof is very similar to the Proof of Proposition 12. There are two strategies for the retailer depending on the value of $Q_2^*$, which is a function of $Q_1$ and $\bar{\theta}$.

When $Q_2^* > 0$, the retailer profit in (4.3) and Lemma 17 lead to the following,

$$\Pi_R = -\frac{1}{4}(3 - 2\delta)\bar{\theta}^2 + \left(\frac{1}{2}(2 + c_2 - (1 + c_2)\delta)\right)\bar{\theta} + \frac{1}{4}(c_2(c_2 + 2\delta - 4(1 - Q_1)) - 4c_1Q_1).$$

(B.49)

When $Q_2^* = 0$, the retailer profit in (4.3) is the same as equation (B.40), which is given in the Proof of Proposition 12.

In both of the regions $\Pi_R$ is jointly concave in $(\bar{\theta}, Q_1)$ and $\Pi_R$ is quasiconcave across these two regions. However, no solution comes from the second part and KKT conditions on the first part gives the regions presented in the Proposition 20. □

Proof of Proposition 21. To find the supplier’s optimal policy we need to analyze the supplier function in each case of the Proposition 20. Supplier function from the first case is a function of $c_2$ since $\bar{\theta}^* = 1$ in that case. Supplier function from the second case is jointly concave in $c_1$ and $c_2$ therefore overall optimization problem is quasiconcave. KKT conditions give the result in Proposition 21. □

Proof of Theorem 11. The proof of this theorem is very similar to the proof of Theorem 2. Specifically we need to compare two piecewise functions of $\delta$ and determine whether the difference is positive or negative.

First case of the Theorem 11 follows from Propositions 17 and 19. We will show the proof of the result for the supplier profit, the proof for the results for retailer profit and the supply chain
profits are similar. Following Propositions 17 and 19 we have two cutoff points for $\delta$ so three regions to consider.

When $\delta < \bar{\delta}_5$ the difference $\Pi_{S}^{NC} - \Pi_{S}^{RQC}$ can be written as the following.

$$\Pi_{S}^{NC} - \Pi_{S}^{RQC} = \frac{24\delta(1680 - 6283\delta) + 32\delta^2(6578 - 4157\delta) + 16\delta^3(2187 - 130\delta)}{128(6 - 5\delta)(33 - 26\delta)(3(33 - 46\delta) + 4\delta^2(8 + 3\delta))} \quad (B.50)$$

Observe that all parts of the denominator of (B.50) are positive except $33 - 46\delta$ when $\delta > \frac{33}{46}$. However, $4\delta^2(8 + 3\delta) > 3(46\delta - 33)$ in that range, hence the denominator is positive. Similarly, all parts of the numerator of (B.50) are positive except $1680 - 6283\delta$ when $\delta > \frac{1680}{6283}$. However, in that region the numerator is decreasing and positive at both end points of the region. Therefore, $\Pi_{S}^{NC} > \Pi_{S}^{RQC}$ in this region.

When $\bar{\delta}_5 \leq \delta < \frac{1}{132}(127 - \sqrt{817})$ the difference function can be written as the following.

$$\Pi_{S}^{NC} - \Pi_{S}^{RQC} = \frac{9}{64} \left( \frac{58 - 127\delta + 66\delta^2}{(6 - 5\delta)(33 - 26\delta)} \right) \quad (B.51)$$

The denominator of (B.51) is clearly positive and the numerator is decreasing and has one root which is the boundary point $\frac{1}{132}(127 - \sqrt{817})$. Therefore, $\Pi_{S}^{NC} > \Pi_{S}^{RQC}$ in this region.

When $\delta \geq \frac{1}{132}(127 - \sqrt{817})$ the two profit functions are equal. Therefore, $\Pi_{S}^{NC} = \Pi_{S}^{RQC}$ in this region.

Second case of the Theorem 11 follows from Propositions 17 and 11, which is very similar to above second $\delta$ region hence they are omitted.

**Proof of Theorem 12.** Proof of this theorem is similar to the proof of Theorem 8. Following Proposition 17 and 21 we have two cutoff points for $\delta$. Below we present the proof for the result for supplier profit. Proofs for retailer and the supply chain profit are similar and they are omitted.

When $\delta < \frac{1}{132}(127 - \sqrt{817})$ the following is the difference between the supplier profits.

$$\Pi_{S}^{NC} - \Pi_{S}^{SPC} = \frac{96(12 - 35\delta) + 8\delta^2(373 - 102\delta)}{128(6 - 5\delta)(33 - 26\delta)(3 - 2\delta)} \quad (B.52)$$

Clearly the denominator of (B.52) is positive. The numerator of (B.52) is positive when $\delta < \bar{\delta}_6 \approx 0.659$ which is given by the root of the following equation. Hence equation (B.53) explicitly
characterizes $\delta_6$.

\[ \delta^2 = \frac{12(35\delta - 12)}{373 - 102\delta} \]

When $\delta \geq \frac{1}{132}(127 - \sqrt{817})$, $\Pi_{SC}^{NC} = \frac{9}{64} < \Pi_{SC}^{SPC} = \frac{4 - 3\delta}{8(3 - 2\delta)}$, since $\delta < \frac{4}{5}$.

\[ \Box \]

**Proof of Lemma 18.** This proof is the same as the proof of Lemma 1 given $Q_1 = 1 - \bar{\theta}$.

**Proof of Proposition 22.** Following Lemma 18 and the fact that $Q_1 = 1 - \bar{\theta}$, retailer’s profit function is concave in $\bar{\theta}$ and the KKT conditions yield the result.

**Proof of Proposition 23.** Overall supplier’s profit function is quasiconcave corresponding to both cases of Proposition 22 is quasiconcave. Hence KKT conditions yield the result.

**Proof of Proposition 24.** This proof is similar to the proof of Proposition 7, we suffice to give a sketch in the main text.

**Proof of Proposition 25.** This proof is similar to the proof of Proposition 8, we suffice to give a sketch in the main text.

**Proof of Theorem 13.** Below we present the proof for each case separately.

The proof for retailer’s quantity commitment result: Following Propositions 6 and 23 we have two cutoff points for $\delta$. When $\delta \leq \delta_1$ the difference $\Pi_{SC}^{NC} - \Pi_{SC}^{RQC} = \frac{\delta(8 + \delta)}{64(8 - 3\delta)}$, which is clearly positive. The difference $\Pi_{SC}^{NC} - \Pi_{SC}^{RQC} = \frac{27\delta(192 - \delta(112 - 19\delta))}{6012(8 - 3\delta)^2}$ is also positive since $192 > \delta(112 - 19\delta) \forall \delta$. When $\delta > \delta_1$ the difference $\Pi_{SC}^{NC} - \Pi_{SC}^{RQC} = \frac{896 - 256\sqrt{16 - 3\delta} + 8(27(8 - \delta)) \frac{6716}{11520} - \frac{4991}{10800}}{1600(16 - 3\delta)(8 - 3\delta)}$, the second derivative is $\frac{16}{(8 - 3\delta)^3} - \frac{3}{4(16 - 3\delta)^{3/2}} < 0$. So the difference function is concave but positive at both end points therefore the difference is positive, which completes the proof for case (i) of the Theorem 13.

The proof for retailer’s price commitment result: Following Propositions 11 and 23 we have the differences $\Pi_{SC}^{NC} - \Pi_{SC}^{RPC}$ and $\Pi_{SC}^{NC} - \Pi_{SC}^{RQC}$, the same as the first part of the above proof hence positive and completes the proof for case (ii) of the Theorem 13.

The proof for supplier’s quantity commitment result: To complete the proof we need to compare the profits. Following Proposition 23 and Proposition 25; when $\delta < 1/2$ the difference for supplier profits $\Pi_{SC}^{NC} - \Pi_{SC}^{SCC} = \frac{4(48 - (18 - \delta)\delta)}{64(2 - \delta)(8 - 3\delta)} < 0$, because denominator is clearly positive and the numerator
is also positive since $\delta \leq 4/5$. For retailer profits $\frac{d(\Pi_{NC}^{R} - \Pi_{SQC}^{R})}{d\delta} = 19\frac{2304}{2(2-\delta)^2} + \frac{16}{9(8-3\delta)^2} - \frac{3}{4(1-\delta)^2} > 0$, so the difference is increasing but $\left(\Pi_{NC}^{R} - \Pi_{SQC}^{R}\right)_{\delta=0} = 0$, hence $\Pi_{NC}^{R} > \Pi_{SQC}^{R}$.

The result for the supply chain profit when $\delta < 1/2$ follows from these. When $\delta \geq 1/2$, for the supplier profits $\frac{d(\Pi_{NC}^{S} - \Pi_{SQC}^{S})}{d\delta} = \frac{59}{9} + \frac{4}{3(8-3\delta)^2} - \frac{3}{5(7-5\delta)^2} > 0$, so the difference is increasing, however $\left(\Pi_{NC}^{S} - \Pi_{SQC}^{S}\right)_{\delta=1/2} = -\frac{157}{4992}$ and $\left(\Pi_{NC}^{S} - \Pi_{SQC}^{S}\right)_{\delta=4/5} = -\frac{43}{2800}$, so $\Pi_{NC}^{S} < \Pi_{SQC}^{S}$. For the retailer profit, $\frac{d(\Pi_{NC}^{R} - \Pi_{SQC}^{R})}{d\delta} = -\frac{673}{1152} - \frac{6}{5(7-5\delta)^2} + \frac{16}{9(8-3\delta)^2} + \frac{18}{5(7-5\delta)^2} < 0$, so the difference is decreasing. Therefore $\Pi_{NC}^{R} + \Pi_{NC}^{R} < \Pi_{SQC}^{R} + \Pi_{SQC}^{R}$, which completes the proof for case (iii) of the Theorem 13.

The proof for supplier’s price commitment result: To complete the proof we need to compare the profits. Following Propositions 12 and 23 when $\delta \leq 1 - \frac{1}{\sqrt{17}}$ then difference $\Pi_{NC}^{S} - \Pi_{SQC}^{S} = \frac{\delta(288 - 1098\delta - 192)}{64(8-3\delta)(16-13\delta)} < 0$, because the denominator is clearly positive and the term in parenthesis in the numerator is a concave increasing function with derivative $288 - 218\delta$ but the value at upper bound is $-\frac{10}{17}(33 + 7\sqrt{17})$. For the retailer profit in this delta range $\frac{d(\Pi_{NC}^{R} - \Pi_{SQC}^{R})}{d\delta} = \frac{4363}{389376} + \frac{45}{169(16-13\delta)^2} - \frac{16}{9(8-3\delta)^2} + \frac{64}{169(16-16\delta)^2} > 0$, so the difference function is increasing but $\left(\Pi_{NC}^{R} - \Pi_{SQC}^{R}\right)_{\delta=0} = 0$, thus $\Pi_{NC}^{R} > \Pi_{SQC}^{R}$. For the supply chain profit in the same delta range, $\frac{d^2(\Pi_{NC}^{S} + \Pi_{NC}^{R}) - (\Pi_{SQC}^{S} + \Pi_{SQC}^{R})}{d\delta^2} = \frac{2}{13(16-13\delta)^2} \left(19 - \frac{96}{16-13\delta} + \frac{8}{(8-3\delta)^6} \left(1 + \frac{2}{8-3\delta}\right)\right) > 0$, so the difference function is convex.

$$\left(\Pi_{NC}^{S} + \Pi_{NC}^{R} - (\Pi_{SQC}^{S} + \Pi_{SQC}^{R})\right)_{\delta = \delta_0, \delta = \delta_1} = \left\{0, \frac{50961\sqrt{17} - 241757}{735488}, \frac{11153}{8467200}, \frac{50961\sqrt{17} - 241757}{735488}\right\}$$
So the difference function becomes 0 once. The delta value when the difference function is 0 is given by the solution to the following equation, which explicitly characterizes $\bar{\delta}_8$.

$$64\delta(512 - 3\delta(624 - 775\delta)) = (74912 - 12641\delta)\delta^4 \quad (B.55)$$

Therefore, $\Pi^{NC}_S + \Pi^{NC}_R \geq \Pi^{SPC}_S + \Pi^{SPC}_R$, when $\delta \leq \bar{\delta}_8$. The proof for the results when $\delta > 1 - \frac{1}{\sqrt{17}}$ is similar and omitted, which completes the proof for the case (iv) of the Theorem 13.

### B.6 Proofs of Chapter 7

**Proof of Lemma 19.** The proof of the first part of Lemma 19 is the same as the proof of the first part of Lemma 1.

For the second part, observe that supplier’s profit is linear in $F_2$ thus it can set $F_2$ as much as retailer’s profit is nonnegative. Hence it sets $F_2 = \frac{c_2^2 + \delta^2 + 2c_2\delta(2(1-Q_1) - \bar{\theta})}{4\delta}$. Given this value then supplier’s profit is a concave function of $c_2$ and first order condition yields the result.

**Proof of Proposition 26.** Retailer’s profit function is jointly concave in $Q_1$ and $\bar{\theta}$ and KKT conditions yield the result.

**Proof of Proposition 27.** Similar to the proof of Lemma 19 supplier’s profit is linear in $F_1$. So it sets $F_1 = \frac{(2(1-c_1) - \delta)^2}{8(2-\delta)}$. Given this supplier’s profit is concave in $c_1$ and first order condition gives the result.

**Proof of Proposition 27.** Supplier’s profit is linear in $F_1$, also. So it sets $F_1 = \frac{(2(1-c_1) - \delta)^2}{8(2-\delta)}$. Given this, supplier’s profit is concave in $c_1$ and first order condition yields the result.

**Proof of Lemma 20.** Proof of optimal second period wholesale price and commitment condition is the same as in the proof of Lemma 7. Then $F^*_2$ follows from plugging these values into the retailer objective function.

**Proof of Proposition 28.** The retailer’s objective function in (3.3) can be written as follows.

$$\Pi_R(Q_{max}, Q_1, \bar{\theta}) = (1 - \bar{\theta})(\bar{\theta} - \delta(Q_{max} + \bar{\theta} - 1)) - c_1Q_1,$$
which is a jointly concave function of $Q_{max}$, $Q_1$ and $\bar{\theta}$. KKT conditions yield the result.

**Proof of Proposition 29.** Supplier’s profit function is linear in $F_1$ so she sets $F_1 = \frac{1}{4}(1 - c_1)^2$. Given this supplier’s profit is concave in $c_1$ and first order condition yields the result.

**Proof of Theorem 14.** Both of the supplier profits are the same as the centralized profits therefore the proof is the same as the proof of Theorem 1.

**Proof of Proposition 30.** The fact that $Q_{max} \leq \frac{1}{2}$ due to the same reasoning as described in Lemma 8 and the proof is along the same lines.

Both parts of the retailer’s objective function are jointly concave in $Q_1$ and $\bar{\theta}$ so KKT conditions characterize the solutions for each part. Comparing each solution yields the regions in Proposition 30. Specifically $K_5$ is the root of the difference function of the objective values of parts (iv) and (v).

**Proof of Proposition 31.** Supplier’s objective function is linear and increasing with respect to $F_1$. Hence we can set it to the upper bound for each of the five cases. Then case 2 is jointly concave with respect to $c_1$ and $Q_{max}$ in addition continuous at each boundary. All of the objective functions other than second case reduce to a function of one variable. Hence KKT conditions yield the result.

**Proof of Theorem 15.** Both of the supplier profits are the same as the centralized profits therefore the proof is the same as the proof of Theorem 1.

**Proof of Proposition 32.** First part of the proof is the same as the first part of the proof of Proposition 11. Second part follows from the Proof of Proposition 29.

**Proof of Theorem 16.** Both of the supplier profits are the same as the centralized profits therefore the proof is the same as the proof of Theorem 1.

**Proof of Proposition 33.** We need to analyze the supplier’s optimization problem in each case of Proposition 12. In every case supplier’s profit is linear and increasing with respect to $F_i$ so it sets those to the upper bound.
For case (ii), supplier sets \( F_1 = \frac{(1-\delta-c_1+c_2)(4-4c_1(1-\delta)-(4+c_2)\delta+\delta^2)}{(4-3\delta)^2} \) and \( F_2 = \frac{((2+2c_1-\delta-c_2(4-\delta))^2)}{4(4-3\delta)^2} \) that gives objective function \( \frac{(2-\delta)^2-4c_1^2)}{4(4-3\delta)^2}, \) which is jointly concave in \( c_1 \) and \( c_2. \)

Observe that after substituting the corresponding \( F_i \) values; all of the other cases results in a univariate function of either \( c_1 \) or \( c_2. \) Therefore, the overall objective function is quasiconcave. In addition each function is continuous at the corresponding boundaries. Hence KKT conditions yield the result.

**Proof of Theorem 17.** Both of the supplier profits are the same as the centralized profits therefore the proof is the same as the proof of Theorem 1.

\[ \square \]

**B.7 Proofs of Chapter 8**

**Proof of Lemma 21.** This proof is very similar to the Proof of Lemma 1. Since Equation 8.10 binds we can write \( p_2 \) in terms of \( Q_2 \) in optimization problem (8.2). Then first order condition gives optimal \( Q_2 \) and corresponding \( p_2. \) Using these with the optimization problem in (8.3) and first order condition gives the results.

\[ \square \]

**Proof of Proposition 34.** Following Definition 1 the retailer’s optimization problem given by (8.5) constitutes a piecewise nature, where each piece is a quadratic function of \( Q_1 \) and \( \theta \) with parameters \( h, \alpha, \delta \) and \( c_1. \) In addition due to Equation 8.7 it is not continuous on the boundaries defined by each of the cases. To find the optimal solution we treat each case as an individual constrained optimization problem and compare with each other to find the global optimal solution. Below we present the local optimal values and how they are driven implicitly in terms of the inequalities rather than \( c_1 \) to keep the exposition short.

**Case (i) Objective function, which is given below is not quasi concave,**

\[
-\frac{3}{4(1-\alpha^2)}Q_2^2 - \frac{2(\alpha^2 - 1)(4c_1 - \delta + 2h - 6) + (\alpha^2 - 1)(3\delta + 2)\bar{\theta} - 2\delta Q_1}{8(\alpha^2 - 1)} Q_1 + \frac{1}{16}(\alpha(8 - 6\delta) + 6\delta - 7)\bar{\theta}^2 + \frac{\alpha(\alpha(5\delta + 4h - 4) + 6) + 2\delta Q_1 - 5\delta - 4h + 10}{8(\alpha + 1)} \bar{\theta} - \frac{\alpha^2(\delta + 2h) + 3\alpha + \delta(Q_1 - 1) - 2h + 3}{4(\alpha + 1)}
\]

134
KKT conditions are not sufficient to define the optimal solution for this case but are necessary.

Optimal solution is as follows:

(i.a) \[ Q_1 = \frac{(\alpha^2-1)(\alpha(8c_1+8h-6)+8c_1+2\delta-7)}{3\alpha(4(\alpha+\delta-1)-8\delta+15)}, \quad \bar{\theta} = \frac{2(\alpha(\alpha(-4c_1-4h+9)+5\delta+4h-1)+4c_1-3\delta+4)}{3\alpha(4(\alpha+\delta-1)-8\delta+15)}, \text{ when} \]
\[ \frac{3\alpha(\alpha-3\delta+5)+3\delta-4\alpha(3\alpha+1)h-4}{4(\alpha+1)(3\alpha+1)} \leq c_1 \leq \frac{6a-2\delta-8\alpha h+7}{8\alpha+8} \]

(i.b) \[ Q_1 = 0, \quad \bar{\theta} = 1, \text{ when } \frac{8a+c+2\delta+7}{8\alpha+8} < c_1 \leq 1 \]

(i.c) \[ Q_1 = \frac{1-\alpha^2}{3\alpha+1}, \quad \bar{\theta} = \frac{4\alpha}{1+3\alpha}, \text{ when } c_1 \leq \min \left( \frac{-9\alpha+1+4\alpha+3\delta-4}{12\alpha^2+16\alpha+4}, -\frac{-12a^2+3\alpha^2-9\alpha+4\alpha h+15\alpha+3\delta-4}{12\alpha^2+16\alpha+4} \right) \]

**Case (ii) Objective function, which is given below is not quasi concave.**

\[ \frac{-3Q_1^2}{8(\alpha+1)} + \frac{Q_1 \left( -\alpha^2+10\alpha+1 \right)(5\delta-2)\bar{\theta} - 2(\alpha^2+1) \left( 8c_1 - \delta + 4h - 6 - 2\delta \hat{Q}_1 \right)}{16(\alpha+1)} + \]
\[ \frac{1}{32}\bar{\theta}^2(-10(\alpha-1)\delta + 13\alpha - 11) + \frac{\bar{\theta} \left( (\alpha+1)(-7\delta + \alpha(7\delta + 8h - 2) - 8h + 14) - 2(\alpha-1)\delta \hat{Q}_1 \right)}{16(\alpha+1)} + \]
\[ \frac{\delta - \alpha(\alpha\delta + 4h + 3) + 6 + 4h + (\alpha-1)\delta \hat{Q}_1 - 3}{8(\alpha+1)} \]

KKT conditions are not sufficient to define the optimal solution for this case but are necessary.

Optimal solution is as follows:

(ii.a) \[ Q_1 = \frac{(\alpha+1)(\alpha(16c_1+16h-3)+16c_1+6\delta-11)}{3\alpha-16\delta+27}, \quad \bar{\theta} = \frac{2(8(\alpha+1)c_1+8\alpha h-5\delta+8)}{3\alpha-16\delta+27}, \text{ when} \]
\[ c_1 \leq \min \left( \frac{5\delta+\alpha(6a-17\delta-8(3\alpha+1)h+60)-8}{8(\alpha+1)(3\alpha+1)}, \frac{15\delta^2-9\delta-4h(\alpha(25-6\delta)+16\delta-27)+112}{8(3\alpha\delta-11\alpha-13\alpha+13)} \right) \]

(ii.b) \[ Q_1 = \frac{1-\alpha^2}{3\alpha+1}, \quad \bar{\theta} = \frac{4\alpha}{1+3\alpha}, \text{ when } -\frac{17\alpha+28\alpha+5\delta-8}{24\alpha^2+32\alpha+8} \leq c_1 \]

**Case (iii) Objective function, which is given below is jointly concave.**

\[ \frac{-3Q_1^2}{4(1-\alpha^2)} - \frac{1}{4}Q_1 (3\bar{\theta} + 4c_1 + 4h - 6) + \frac{3}{16} (4\delta - 5)\bar{\theta}^2 + \frac{\bar{\theta} \left( - (\alpha^2-1) (5\delta + 4h - 7) - 2\delta \hat{Q}_1 \right)}{4(\alpha^2-1)} + \]
\[ h + \frac{1}{2}\delta \left( \frac{\hat{Q}_1}{\alpha^2-1} + 1 \right) - \frac{3}{4} \]

Optimal solution is as follows:

(iii.a) \[ Q_1 = \frac{(\alpha^2-1)(2c_1(\delta^5-5)+\delta(8h-7)-6h+8)}{3\alpha^2-108+12}, \quad \bar{\theta} = -\frac{2(\alpha^2(6c_1+6h-9)+4\delta c_1+h-6c_1+9\delta+6(h-2))}{9\alpha^2-30\delta+36}, \text{ when} \]
\[ \frac{3\alpha(8-7\delta)-2h(3\alpha(-\alpha-4\delta)+3-10\delta+12)}{6(\alpha+1)(\alpha+4)-4(6\alpha+5)\delta} \leq c_1 \leq \frac{3(\alpha(3\alpha+7\delta-8)-3\delta+4)-2h(3\alpha(2\alpha+4\delta-3)-8\delta+15)}{2(3\alpha(2\alpha+5)-8\delta+9)} \]
(iii.b) \[ Q_1 = \frac{(1-\alpha^2)(7-2\delta+2\alpha(3-4h)-8(1+\alpha)c_1)}{15-8\delta-3\alpha(1-4(\alpha+\theta))}, \quad \bar{\theta} = \frac{2(4(1-\alpha^2)c_1+(9-4h)\alpha^2-(1-5\delta-4h)\alpha+(4-3\delta))}{12\alpha^2-3(1-4\delta)\alpha+(15-8\delta)}, \] when
\[
\max \left( \frac{3(\alpha(3\alpha+7\delta-8)-3\delta+4h(3\alpha(2\alpha+4\delta-3)-8\delta+15)}{2(3\alpha(2\alpha+4\delta-5)-8\delta+9)}, \frac{3\alpha(\alpha-3\delta+5)+3\delta-4\alpha(3\alpha+1)h-4}{4(\alpha+1)(3\alpha+1)} \right) \leq c_1 \leq \frac{7-2\delta+2\alpha(3-4h)}{8(1+\alpha)}
\]

(iii.c) \[ Q_1 = \frac{1-\alpha^2}{3\alpha+1}, \quad \bar{\theta} = \frac{4\alpha}{3\alpha+1}, \text{ when } c_1 \leq \frac{3\alpha(\alpha-3\delta+5)+3\delta-4\alpha(3\alpha+1)h-4}{4(\alpha+1)(3\alpha+1)}
\]

(iii.d) \[ Q_1 = 0, \quad \bar{\theta} = 1, \text{ when } c_1 > \frac{7-2\delta+2\alpha(3-4h)}{8(1+\alpha)}.
\]

**Case (iv)** Objective function, which is given below is jointly concave.

\[
-\frac{3Q^2}{8(\alpha+1)} - \frac{1}{8} Q_1 (3\bar{\theta} + 8c_1 + 8h - 6) - \frac{1}{32} \bar{\theta}^2 (3\alpha - 20\delta+27) + \frac{\bar{\theta} (\alpha+1)(3\alpha-7\delta-8h+11) + 2\delta Q_1}{8(\alpha+1)} + \frac{1}{8} \left(-3\alpha + 8h + \delta \left(2 - \frac{2Q_1}{\alpha+1}\right) - 3\right)
\]

Optimal solution is as follows:

(iv.a) \[ Q_1 = \frac{(a-1)(4(a-1)c_1-13\delta+4(ah+h+4))}{12(a+2)+2-2(11a+9)\delta}, \quad \bar{\theta} = \frac{-\alpha(4a(c_1+h-3)+9\delta+8h-8)+4c_1-5\delta-4h+8}{6(a+2)+2-(11a+9)\delta}, \]
\[
\max \left( \frac{\alpha(48-39\delta)-4h(3\alpha-11\delta+9)-9\delta+12}{12(a+1)(\alpha+4)-4(11a+9)\delta}, \frac{-5\delta+\alpha(12a+17\delta-4(3\alpha+4)h-24)-4h+8}{4(\alpha+1)(3\alpha+1)} \right) \leq c_1 \leq \frac{3\alpha^2+a\delta-2\alpha+2\delta-2(a+1)^2h-2}{2(a^2-1)}
\]

(iv.b) \[ Q_1 = \frac{(a+1)(a(16c_1+16h-3)+16c_1+6\delta-11)}{3a-16\delta+27}, \quad \bar{\theta} = \frac{2(8(a+1)c_1-5\delta+8ah+h)}{3a-16\delta+27}, \text{ when }
\]
\[
c_1 \leq \min \left( \frac{5\delta+\alpha(6a-17\delta-8(3\alpha+1)h+30)-8}{3a-6\delta-16a+11}, \frac{3a-6\delta-16a+11}{3a-16\delta+27} \right)
\]

(iv.c) \[ Q_1 = \frac{1-a}{2}, \quad \bar{\theta} = 1, \text{ when } c_1 \leq \frac{3\alpha^2+a\delta-2\alpha+2\delta-2(a+1)^2h-2}{2(a^2-1)}
\]

(iv.d) \[ Q_1 = \frac{1-a^2}{3a+1}, \quad \bar{\theta} = \frac{4\alpha}{3a+1}, \text{ when }
\]
\[
\frac{5\delta+\alpha(6a-17\delta-8(3\alpha+1)h+30)-8}{8(a+1)(3\alpha+1)} \leq c_1 \leq \frac{-5\delta+\alpha(12a+17\delta-4(3\alpha+4)h-24)-4h+8}{4(\alpha+1)(3\alpha+1)}.
\]

Now that we have found all the local optimal solutions we need to compare and figure out the global solution and corresponding \( c_1 \) region, which is a very tedious task. However, if \( c_1 > 3/10 \) number of comparisons decreases significantly. Given this comparing Case (iii) and (iv) yields the result.

**Proof of Proposition 35.** Following Proposition 34 supplier’s optimization problem is a concave function of \( c_1 \) and first order condition yields the result. Note that \( c_1^* \) is indeed greater than \( 3/10 \).
However, to complete the proof that this is the equilibrium we need to show that there is not any other better option for the supplier if \( c_1 < 3/10 \) and retailer chooses its best response from any of the local optimal solutions provided in the Proof of Proposition 34. We show this without finding the global solution for the case when \( c_1 < 3/10 \) for the Retailer’s response. For all the possible cases we write the corresponding supplier problem find its optimal solution subject to the \( c_1 \) constraint defined by the retailer’s action. However, there is not such solution when \( c_1 < 3/10 \), which completes the proof.

**Proof of Theorem 18.** Follows from segments in Definition 2 and the partial derivative with respect to \( \alpha \).

**Proof of Corollary 4.** Follows from partial derivatives of optimal profit values with respect to \( \alpha \) and \( \delta \).

**Proof of Lemma 22.** \( \mathcal{L}_S^i > \mathcal{L}_C^i \Leftrightarrow \Pi^i(\alpha, 0) > \Pi^i(0, \delta) \). For part (i) \( \Pi^S(\alpha, 0) \) decreases with \( h \). Solve \( \Pi^S(\alpha, 0)|_{h=1/4} - \Pi^S(0, \delta) = 0 \) in terms of \( \delta \). Resulting \( \alpha \) expression always smaller than \( 1/4 \), hence the result follows. Similar argument works for case (ii).

**Proof of Theorem 19.** Follows from the proof of Lemma 22.

**Proof of Corollary 5.** Follows from partial derivatives of the equilibrium values with respect to \( \delta \).

**Proof of Theorem 20.** Follows from comparing fractional loss measures for retailer and supplier.

**Proof of Corollary 6.** Follows from the partial derivatives of the fractional loss measures for supplier and retailer with respect to \( \delta \).

**Proof of Lemma 23.** Similar to the Proof Lemma 21 therefore omitted.

**Proof of Proposition 36.** Following Definition 5 we write the corresponding objective function for the firm and first order condition yields the result.

**Proof of Theorem 21.** First part follows from partial derivative of \( \mu \) with respect to \( \alpha \). Second part follows from comparing \( \mu \) and \( \mu_C \).
Proof of Corollary $??$. Follows from partial derivatives with respect to $\alpha$ and $\delta$.

Proof of Lemma 24. Write $\Pi^C(\alpha, 0) > \Pi^C(0, \delta)$, where $\Pi^C(\alpha, 0)$ decreasing with respect to $h$. When $h$ is $1/4$ show that $\delta < 1/2$.

Proof of Theorem 22. Follows from Lemmas 22 and 24.

Proof of Corollary 8. Follows from partial derivatives of decisions with respect to $\delta$.

Proof of Theorem 23. Follows from comparing the relative loss measures.

Proof of Theorem 24. Follows from partial derivatives of $\Omega_\delta$ measures with respect to $\alpha$.

Proof of Corollary 9. Follows from partial derivatives of $Q_1$, which is given in Proposition 34 and second period $Q_2$ values given by Lemma 21.
BIBLIOGRAPHY


139


