Rationalizing Size, Value, and Momentum Effects with an Asymmetric CAPM

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Abstract

ISACCO PICCIONI: Rationalizing Size, Value, and Momentum Effects with an Asymmetric CAPM.
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This work shows that an Asymmetric Capital Asset Pricing Model (A-CAPM) that is based on recent decision theory models can rationalize size, value, and momentum anomalies. The A-CAPM is derived by approximating the utility function of the representative agent with asymmetric polynomial models that allow for different attitudes toward risk in the domains of gains and losses. Results show that the A-CAPM rationalizes the cross-section of stock returns, significantly improving with respect to several existing asset pricing models. Furthermore, size, value, and momentum factors do not load when they are tested on the A-CAPM because the model already captures the sources of risk that drive these anomalies: (i) preference for security/potential, (ii) loss aversion, and (iii) goal achievement.

To my family in Italy.

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1. Introduction

The traditional mean-variance Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965) has long served as the backbone of academic finance, and it has been used in numerous important applications. However, several studies indicate that the cross-section of stock returns cannot be explained by market beta alone as predicted by the CAPM. In particular, the mean-variance CAPM fares poorly in explaining the high risk premiums of portfolios formed by small cap, high book-to-market securities (Fama and French, 1992), and momentum winners (Jagadeesh and Titman, 1993). In response, researchers have examined the performance of alternative models of asset prices. For example, Harvey and Siddique (2000) and Dittmar (2002) studied the three-moment CAPM, which takes into account skewness in addition to mean and variance. The literature on downside risk, in contrast, considers semivariance, rather than variance, as the primary source of risk (e.g. Ang, Chen, and Xing, 2006). These alternative asset pricing models fit stock return data better than the standard mean-variance CAPM, but size, value, and momentum effects remain.

A fundamental limitation of the existing asset pricing models is the poor descriptive ability of the utility functions from which they are derived. The literature on decision making under risk points out that models displaying asymmetries in the domains of gains and losses can explain risky choice behavior significantly better than standard utility functions (e.g. Levy and Levy, 2002; Diecidue and Van de Ven, 2008; Piccioni, 2011). For example, Piccioni develops a utility function that is (i) concave for losses and convex for gains, (ii) steeper for losses, and (iii) with a positive shift at the reference point.[1] Piccioni shows that such a utility function can capture the aspects that are crucial to explain how people face risk: (i) preference for security/potential, i.e. downside risk aversion and preference for upside potential; (ii) loss aversion, i.e. the empirical observation that losses loom larger than gains; and (iii) goal achievement, i.e. the importance of satisfying relevant aspiration levels.

In this paper I examine whether asset pricing models based on asymmetric utility functions can shed light on the sources of risk that drive size, value, and momentum anomalies. In order to do so, I develop and test a new Asymmetric Capital Asset Pricing Model (A-CAPM). The A-CAPM is derived by approximating the utility function of the representative agent with asymmetric polynomial models.[2] In this way, I can estimate several models of the empirical decision making literature, which share the common trait of displaying asymmetries for gains and losses. Furthermore, the A-CAPM nests several existing asset pricing models (mean-variance CAPM, three-moment CAPM, and downside risk models), allowing for model comparison tests.

Testing the model on the cross-section of stock returns, I find that the A-CAPM rationalizes size, value, and momentum portfolio returns and it yields a significant improvement with respect to the mean-variance CAPM, the three-moment CAPM, and downside risk models. As shown in Figure 1, I find that the cross-section of stock returns is rationalized by an asymmetric quadratic utility function that is (i) concave for losses and mildly convex for gains (consistent with preference for security/potential), (ii) steeper in the

[1]See Appendix A for more details.

[2]Asymmetric polynomial models are polynomial models with different parameters in the positive and negative domains. In other words, the utility function is approximated with separate polynomial expansions in the domains of gains and losses.

negative domain (consistent with loss aversion), and (iii) with a positive shift at the target return[3] (consistent with goal achievement).[4] The corresponding pricing kernel is (i) decreasing for losses and slightly increasing for gains, (ii) with a higher level in the negative domain, and (iii) a bump at the target return. It follows that the A-CAPM implies aversion—therefore higher risk premiums—for securities delivering: (i) *higher* covariance with market returns during market downturns and *lower* covariance with market returns during rising markets (higher loadings on preference for security/potential), (ii) *lower* average returns during market downturns (higher loadings on loss aversion), and (iii) *lower* average returns when market returns are close to the target return (higher loadings on goal achievement).

The three main characteristics of the model allow the A-CAPM to rationalize size, value, and momentum effects. In fact, small cap, high book-to-market securities, and momentum winners deliver higher loadings on the three factors that are priced by the A-CAPM (the risky choice factors[5]). In other words, these securities are riskier according to the A-CAPM; therefore, the model can rationalize their higher risk premiums. Furthermore, I find that size, value, and momentum factors do not load when they are tested on the A-CAPM because the model already captures the sources of risk that drive these anomalies.

Running a wide range of model comparison tests, I find that only the A-CAPM that includes in the analysis all three risky choice factors can rationalize size, value, and momentum anomalies. Consistent with the results of Piccioni (2011) related to the empirical decision making literature, we need all three main characteristics of the utility function

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[3]The estimated target return is equal to the risk free rate.

[4]These results are consistent with Piccioni (2011). It's important to note that in the present work I do not take a stand on the precise functional form of the utility function. In fact, utility is approximated with asymmetric polynomial models that nest a wide range of utility functions.

[5]Preference for security/potential, loss aversion, and goal achievement are defined as the three "risky choice factors", since they are related to the fundamental aspects that drive risky choice behavior.

(concavity for losses and convexity for gains, utility steeper in the domain of losses, and positive shift at the target return) in order to explain the cross-section of stock returns.

The first main feature of the A-CAPM utility function is to display concavity in the domain of losses and mild convexity in the domain of gains. In fact, approximating the utility function with an asymmetric quadratic model, I find a negative parameter for the quadratic term in the negative domain and a slightly positive parameter for the quadratic term in the positive domain. This implies preference for security/potential. Preference for security/potential, in turn, implies preference for positive asymmetry, a factor already included in the three-moment CAPM, which is derived from cubic utility functions. Running model comparison tests, I find that asymmetric quadratic utility functions (A-CAPM) can capture preference for positive asymmetry better that cubic utility functions (three-moment CAPM). Since stocks are more correlated during market downturns, the market portfolio yields a relatively small reduction in downside risk at the cost of a relatively large reduction in upside potential. The three-moment CAPM cannot rationalize the efficiency of the market portfolio because it cannot break preference for positive asymmetry into strong downside risk aversion and mild preference for upside potential.[6] The A-CAPM, instead, has such flexibility.[7]

Besides finding asymmetries in the parameters of the quadratic terms of the utility function, I also find significant asymmetries in the parameters of the linear terms. Consistent with loss aversion, I find that the parameter of the linear term in the negative domain is

[6]As pointed out by Post, Vliet, and Levy (2008), with cubic utility, high preference for positive skewness necessary implies high preference for upside potential relative to downside risk aversion.

[7]The model can imply strong downside risk aversion and mild preference for upside potential because of the asymmetries in the quadratic terms of the utility function.

significantly higher.[8] It follows that the utility function has a higher slope (and the pricing kernel has a higher level) in the negative domain. This implies higher risk premiums for securities delivering *lower* average returns during market downturns. It follows that the A-CAPM keeps track of securities paying off less when investors need it the most. This is a clearly desirable property that is not included in models such as the mean-variance CAPM, the three-moment CAPM, or downside risk models. Running model comparison tests, I find that loss aversion significantly contributes to the improvement of the A-CAPM with respect to the existing asset pricing models.

Finally, I include a positive constant in the upside component of the utility function, in order to take into account the importance that investors assign to the overall probability of achieving the target return (goal achievement).[9] Since I use *continuous* asymmetric polynomials to approximate utility, the inclusion of the positive constant produces a positive shift, rather than a discrete jump, at the target return (as emphasized in Figure 2). It follows that the pricing kernel displays a bump around the target return. Since marginal utility is higher around the target return, the A-CAPM implies higher risk premiums for securities paying off *less* when market returns are close to the target return. The importance of goal achievement is highlighted in the empirical analysis. In particular, when goal attainment is included in the analysis in addition to preference for security/potential and loss aversion, the A-CAPM yields an almost perfect fit (as shown in Figure 3) and drives out the size, value, and moment factors, even when the model is tested on size, value, and moment portfolios.

The remainder of this work is organized as follows. In section 2 I examine the theory behind the asset pricing models tested in this study. Section 3 outlines the empirical analysis,

[8]Both linear terms in the positive and negative domains are positive, consistent with monotonicity.

[9]See Diecidue and Van de Ven (2008).

including the estimation methods and model comparison tests. Section 4 reports the estimation results, and section 5 concludes.

2. Theory

2.1 The Investment Problem

In this paper I study asset pricing models based on a wide range of utility functions. To focus my attention on the role of preferences, I adhere to the remaining assumptions of the CAPM. I consider a single period, portfolio-based, representative investor model of a frictionless and competitive capital market that satisfies the following assumptions:

1. The investment universe consists of N risky assets with return $r \in \mathbb{R}^N$, and a risk free asset with return $r \in \mathbb{R}_+$.
2. The returns $r \in \mathbb{R}^N$ are treated as random variables with continuous joint cumulative distribution function $G: \mathbb{R}^N \to [0,1]$.
3. The representative investor constructs a portfolio by choosing portfolio weights $w \in \mathbb{R}^N$, so as to maximize the expectation of a utility function $u: \mathbb{R} \to \mathbb{R}$, differentiable and strictly increasing. The weight assigned to the risk free asset is$1 - w'i$.
4. The utility function is defined over portfolio returns in excess of the risk free rate r_F and a constant spread τ.

These assumptions are common to several works like, for example, Post and Levy (2005) and Post, Vliet, and Levy (2008). The only difference is that I consider a target return that doesn't have to be fixed to a pre-specified level.

The investment problem is:

$$\max_{w \in \mathbb{R}^N} E\left(u(r^{\tau}_{w,t+1})\right) \tag{1}$$

with utility defined over:

$$r^{\tau}_{w,t+1} = w' r_{t+1} + (1 - w'i)\, r_{F,t} - \left(r_{F,t} - \tau\right) = w' r^{e}_{t+1} - \tau \tag{2}$$

With $r^{e}_{t+1} = r_{t+1} - i\, r_{F,t}$. The value-weighted market portfolio $w_m \in \mathbb{R}^N$ must represent the optimal solution to this problem. The well-known Euler equation gives the First Order Conditions (FOC) for optimality:

$$E(m_{t+1} r^{e}_{t+1}) = 0 \tag{3}$$

with $m_{t+1} = u'(r^{\tau}_{w_m,t+1})$ Stochastic Discount Factor,[10] or pricing kernel, that prices all risky payoffs under the law of one price and is non-negative under the condition of no arbitrage.

Assumption 3 imposes only differentiability and monotonicity of the utility function. It follows that global concavity is not imposed and the FOC are no longer sufficient for optimality. In fact, we may wrongly classify a minimum or a local maximum (which also satisfy the FOC) as the global maximum. Therefore, for each model I verify if the Second Order Conditions (SOC) are satisfied, by checking that the following matrix is negative definite

$$E\left(u''\left(r^{\tau}_{w_m,t+1}\right) r^{e}_{t+1}\, {r^{e}_{t+1}}'\right) \tag{4}$$

Checking the SOC represents an improvement with respect to many works of the asset pricing literature. In fact, several works estimate models that could imply local convexity of the utility function, but fail to verify either local risk seeking or the SOC for optimality. On the contrary, in this paper, I analyze both local convexity of the utility function and the SOC, since the most recent decision theory models explicitly display local risk seeking.

[10]Appendix B provides details about: efficiency of the market portfolio; the connection of the pricing kernel defined in this paper and the Stochastic Discount Factor defined over consumption; unconditional and conditional expectations models.

2.2 Continuously Asymmetric Utility Functions

The purpose of this paper is to verify that the cross-section of stock returns can be rationalized by asset pricing models based on utility functions that are able to explain risky choice behavior. The first contribution of the paper is to show that continuously asymmetric quadratic utility functions can assess all the relevant risky choice factors (i.e. the aspects that are crucial to explain how people face risk), while models based on higher-order polynomials or lower partial moments are not able to do so. Continuously asymmetric utility functions can be defined as follows:

$$u(r_m) = \tilde{u}_-(r_m)\,(1 - F_m) + \tilde{u}_+(r_m)\,F_m \tag{5}$$

With $r_m = r^{\tau}_{w_m,t+1}$ to simplify notation. The utility function is the weighted sum of two continuous functions: $\tilde{u}_-(r_m)$ reflects the risk attitude in the domain of losses, while $\tilde{u}_+(r_m)$ captures the risk attitude in the domain of gains. The weighting function F_m is a strictly increasing and continuous function, with values between zero and one. For example, we can choose F_m to be the cumulative distribution function, evaluated at r_m, of a normal random variable with mean zero and variance σ^2. If σ^2 is low enough, F_m is similar to an indicator function, but the continuity of the utility function on its entire support is preserved.

The above utility function, given by the weighted sum of upside and downside components, is equivalent to a model given by the sum of symmetric and upside components:

$$u(r_m) = u_S(r_m) + u_+(r_m)\,F_m \tag{6}$$

with $u_S(r_m) = \tilde{u}_-(r_m)$ applied to the entire domain, and $u_+(r_m) = \tilde{u}_+(r_m) - \tilde{u}_-(r_m)$ applied to the domain of gains and weighted by F_m. This formulation has the advantage of nesting standard symmetric utility functions—by setting $u_+(r_m) = 0$—and it doesn't impose any restriction on the shape of the positive portion of the utility function.

Model (6) can be used to test several utility functions studied in the decision theory literature. For example, Piccioni (2011) shows that the aspects that are crucial to explain how people face risk are: (i) preference for security/potential, i.e. downside risk aversion and preference for upside potential; (ii) loss aversion, i.e. the empirical observation that losses loom larger than gains; and (iii) goal achievement, i.e. the importance of satisfying relevant aspiration levels. To consider these factors, Piccioni develops a utility function that is (i) concave in the domain of losses and convex in the domain of gains (consistent with preference for security/potential), (ii) steeper for losses (consistent with loss aversion), and (iii) with a positive shift at the target return (consistent with goal achievement).[11]

The A-CAPM is derived from a continuous asymmetric quadratic model that is able to approximate several models of the empirical decision making literature (e.g. Levy and Levy, 2002; Diecidue and Van de Ven, 2008; Piccioni, 2011) and assess the importance of all factors that are important to explain how people face risk:

$$\begin{gathered} u_S(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 \\ u_+(r_m) = \theta_0^+ + \theta_1^+\, r_m + \theta_2^+\, r_m^2 \end{gathered} \tag{7}$$

Preference theory implies the following sign restrictions:

$$\theta_1 > 0, \quad \theta_2 < 0, \quad \theta_0^+ \geq 0, \quad \theta_1^+ \leq 0, \quad \theta_2^+ \geq 0 \tag{8}$$

Standard sign restrictions are applied to the symmetric component: positive linear term—$\theta_1 > 0$—for monotonicity,[12] and negative quadratic term—$\theta_2 < 0$—for risk aversion.

Regarding the upside component, θ_0^+ is assumed to be positive, consistent with goal seeking behavior.[13] In fact, as shown by Diecidue and Van de Ven (2008), if we include in

[11]See Appendix A for more details.

[12]Imposing $\theta_0^+ \geq 0$ alone is not a sufficient condition for monotonicity. If θ_2 or θ_1^+ are sufficiently strong, the utility function may be decreasing in some intervals. Any violation of monotonicity will be investigated in the empirical analysis.

the utility function a positive constant multiplied by an indicator function equal to one if portfolio returns are greater than the target return, when we compute expected utility we get:

$$E(\theta_0^+ \ I_m^+) = \theta_0^+ \ E(\ I_m^+) = \theta_0^+ \ P(r_m > 0) \tag{9}$$

Therefore, θ_0^+ quantifies the importance of the overall probability of achieving the target return: $P(r_m > 0)$. It follows that θ_0^+ measures what can be defined as goal achievement, goal seeking behavior.

With continuous asymmetric utility we get the same results. In fact, since $F_m \approx \ I_m^+$, we have:

$$E(F_m) \approx E(\ I_m^+) = P(r_m > 0) \tag{10}$$

The ability of quantifying the importance of the overall probability of achieving the target return represents the main innovation of the present work. Note that, in order to assess the effect of goal seeking behavior on asset prices, we need to estimate continuous utility functions with a positive shift (highlighted in Figure 2), rather than discontinuous utility functions with discrete jumps. Continuity is required because each model is estimated on moment conditions defined on its first derivative. This technical aspect allows the assessment of goal achievement in asset pricing models without compromising the original intuition of the decision theory models.

Moving to the next term of the polynomial approximation, the upside linear term is negative, $\theta_1^+ \leq 0$, consistent with a utility function that has a lower slope in the domain of gains (therefore the utility function is steeper for losses). In this way we can comply with loss aversion (e.g. Kahneman and Tversky, 1979), the empirical observation that losses loom larger than gains.

[13]Parameter θ_0^+ has to be positive. Both intuition and the results of the empirical decision making literature point toward this direction. Furthermore, besides contradicting goal seeking behavior, a negative θ_0^+ would lead to violations of monotonicity.

Finally, the upside quadratic term is imposed to be positive: $\theta_2^+ \geq 0$. If θ_2^+ is large enough—$\theta_2^+ > |\theta_2|$—the utility function is convex in the domain of gains,[14] consistent with preference for upside potential (e.g. Levy and Levy, 2002). If instead $\theta_2^+ < |\theta_2|$, the utility function is concave in both domains of gains and losses, but the concavity is stronger in the negative domain. This is consistent with the intuition that investors are, at the least, more averse to downside variance than upside variance.[15]

The marginal utility of (7) is given by:

$$m = \theta_1 + 2\,\theta_2\, r_m + \theta_0^+\, f_m + \theta_1^+ (F_m + r_m\, f_m) + \theta_2^+ (2\, r_m\, F_m + r_m^2\, f_m) \tag{11}$$

As shown in Appendix C, we can derive the expected excess return of each asset from the First Order Conditions:

$$E(r_i) = \frac{-cov(m, r_i)}{E(m)} = \frac{cov\left(-2\,\theta_2\, r_m - \theta_0^+ r_{0,m}^+ - \theta_1^+ r_{1,m}^+ - \theta_2^+ r_{2,m}^+, r_i\right)}{E(m)} \tag{12}$$

with:

$$r_{0,m}^+ = f_m\,, \quad r_{1,m}^+ = F_m + r_m\, f_m\,, \quad r_{2,m}^+ = 2\, r_m\, F_m + r_m^2\, f_m \tag{13}$$

The estimated utility and pricing kernel of model (7) are shown in Figure 1. The upside component $u_+(r_m)$ has a significant impact on the utility function. In fact, the estimated utility function is (i) concave in the negative domain and mildly convex in the positive domain, (ii) it is steeper in the domain of losses, and (iii) it has a positive shift at the target return. The corresponding pricing kernel is (i) decreasing in the domain of losses and slightly increasing for gains, (ii) with a higher level in the domain of losses, and (iii) a bump around

[14]But if θ_2^+ is too large, the SOC will be violated. This condition is verified in the empirical analysis.

[15]Note that Prospect Theory (Kahneman and Tversky, 1979) would imply opposite sign restrictions on the quadratic terms of the utility function. As shown in section 4.1 a model with Prospect Theory sign restrictions is rejected. More details are provided in Appendix A.

the target return. Applying equation (12), these characteristics are consistent with higher risk premiums for securities delivering:

(i) *Higher* covariance with market returns during market downturns and *lower* covariance with market returns during rising markets (higher loadings on the first risky choice factor: preference for security/potential). In fact:

$$cov\left(-2\,\theta_2\, r_m - \theta_2^+\, r_{2,m}^+\,, r_i\right) \approx$$

$$\approx -2\,\theta_2\; cov(r_m\, I_m^-\,, r_i) - 2\,(\theta_2 + \theta_2^+)\; cov(r_m\, I_m^+\,, r_i) \tag{14}$$

With

$$I_m^+ = \begin{cases} 1 & if \quad r_m > 0 \\ 0 & if \quad r_m \le 0 \end{cases} \tag{15}$$

and $I_m^- = 1 - I_m^+$;

(ii) *Lower* average returns during market downturns (higher loadings on the second risky choice factor: loss aversion). In fact:

$$cov\left(-\theta_1^+\, r_{1,m}^+\,, r_i\right) \approx \theta_1^+ E(I_m^-\, r_m) \tag{16}$$

(iii) *Lower* average returns when market returns are close to the target return (higher loadings on the third risky choice factor: goal achievement). In fact:

$$cov\left(-\theta_0^+\, r_{0,m}^+\,, r_i\right) \approx \theta_0^+ E(f_m\, r_m) \tag{17}$$

As shown in section 4, only the A-CAPM that includes all three risky choice factors can rationalize size, value, and momentum anomalies and significantly improve with respect to existing asset pricing models. Consistent with Piccioni (2011) findings related to the empirical decision making literature, we need all three main characteristics of the utility function (concavity for losses and convexity for gains, utility steeper in the domain of losses, and positive shift at the target return) in order to explain the cross-section of stock returns.

2.3 Standard Polynomial Models

Besides allowing to approximate decision theory models, the continuously asymmetric utility function defined in (6) is also able to nest several models studied in the asset pricing literature. For example, standard polynomial models are obtained by setting $u_+(r_m) = 0$ and

$$u_S(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 \tag{18}$$

Or

$$u_S(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 + \theta_3\, r_m^3 \tag{19}$$

In this way, we get the standard quadratic and cubic utility functions, implying the mean-variance and three-moment CAPM, respectively. It's immediate to verify that standard polynomial models are not able to capture any risky choice factor. In fact, preference for security/potential, loss aversion, and goal achievement cannot be assessed if $u_+(r_m) = 0$.

The cubic model is consistent with preference for skewness ($\theta_3 > 0$), which is related to preference for security/potential. However, preference for security/potential and preference for positive skewness are two different concepts, and the results of this study emphasize the importance of breaking preference for positive asymmetry into strong downside risk aversion and mild preference for upside potential. In fact, several woks find evidence that models based on higher-order polynomials are not flexible enough to imply strong preference for positive asymmetry without violating the necessary SOC for optimality. For example, Dittmar (2002) finds that admissible pricing kernels display values of θ_3 high enough (i.e. preference for positive asymmetry strong enough) to imply a reverse S-shaped utility function, with concavity for losses and convexity for gains. However, the convexity of the utility function in the positive domain is too strong to satisfy the SOC. If global concavity is imposed, instead, Dittmar finds that models based on higher-order polynomials are no longer admissible for the cross-section of stock returns. In other words, if global concavity is imposed, the cubic model (three-moment CAPM) is not able to improve with respect to the standard quadratic utility function (mean-variance CAPM). This result is consistent with Tsiang (1972), who demonstrates that a quadratic function is likely to give a good approximation for any *concave* utility function over the typical sample range, and that higher-order polynomials are unlikely to improve the fit.

The optimality issues of higher-order polynomials are made clear by Post, Vliet, and Levy (2008), who find that the market portfolio represents the global minimum, rather than the global maximum, for investors with cubic utility functions. Since stocks are more correlated during market downturns, the market portfolio yields a relatively small reduction in downside risk at the cost of a relatively large reduction in upside potential. With cubic utility functions, investors with high preference for skewness assign a relatively high weight to upside potential. Therefore, the small reduction in downside risk investing in the market portfolio does not sufficiently compensate them for the large reduction in upside potential. It follows that, in order to explain asset returns, we need a utility function that implies strong downside risk aversion and mild preference for upside potential at the same time.

It's clear that models based on higher-order polynomials are not flexible enough to break preference for positive asymmetry into strong downside risk aversion and mild preference for upside potential. On the other hand, such flexibility can be achieved with asymmetric quadratic utility functions.

2.4 Non-Continuous Asymmetric Utility Functions

Asymmetric quadratic utility functions can be defined as:

$$u(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 + (\theta_1^+\, r_m + \theta_2^+\, r_m^2)\, I_m^+ \tag{20}$$

As pointed out above, defining asymmetric models as the sum of a symmetric and an upside component allows the model to nest standard polynomial models, and it doesn't impose any shape restriction in the positive portion of the utility function. However, to get more intuition about the implications of model (20), we can consider the following formulation:

$$u(r_m) = \left(\tilde{\theta}_1^-\, r_m + \tilde{\theta}_2^-\, r_m^2\right) I_m^- + \left(\tilde{\theta}_1^+\, r_m + \tilde{\theta}_2^+\, r_m^2\right) I_m^+ \tag{21}$$

Models (20) and (21) are equivalent if

$$\theta_1 = \tilde{\theta}_1^-, \quad \theta_2 = \tilde{\theta}_2^-, \quad \theta_1^+ = \tilde{\theta}_1^+ - \tilde{\theta}_1^-, \quad \theta_2^+ = \tilde{\theta}_2^+ - \tilde{\theta}_2^- \tag{22}$$

Note that the utility function is non-continuous, since it has a discontinuity point at zero. Since each model is estimated on its first derivative, the kink of the utility function doesn't allow estimating the shift at the target return essential to assess the effect of goal achievement. Therefore, to quantify the effect of goal achievement we need to develop *continuously* asymmetric utility functions, as in (6)−(7).

Nevertheless, the non-continuous asymmetric quadratic model can consider two other important risky choice factors: preference for security/potential, and loss aversion. Loss aversion can be captured by imposing the linear term in the negative domain to be higher than the linear term in the positive domain ($\tilde{\theta}_1^- > \tilde{\theta}_1^+ > 0$), resulting in a utility function that is steeper for losses. Regarding preference for security/potential, the model has enough f\mbox{}lexibility to imply strong downside risk aversion and mild preference for upside potential ($\tilde{\theta}_2^- < 0$ and $0 < \tilde{\theta}_2^+ < \left|\tilde{\theta}_2^-\right|$). In this way the model can imply strong preference for positive asymmetry without violating the necessary SOC for optimality.

The expected excess return of each asset is:

$$E(r_i) = \frac{-\tilde{\theta}_1^- cov(\, I_m^-, r_i) - 2\, \tilde{\theta}_2^- cov(r_m\, I_m^-, r_i) - \tilde{\theta}_1^+ cov(\, I_m^+, r_i) - 2\, \tilde{\theta}_2^+ cov(r_m\, I_m^+, r_i)}{E(m)} \tag{23}$$

First of all, the model implies higher risk premiums for securities delivering lower average returns during market downturns—since $\tilde{\theta}_1^- > \tilde{\theta}_1^+$. This is a clearly desirable property (not included in models such as the mean-variance CAPM, the three-moment CAPM, or the downside risk models) whose importance is stressed in section 4.1.

Furthermore, expected returns depend explicitly upon downside risk—covariance with the market return during falling markets: $cov(\, I_m^-, r_i)$—and upside potential---covariance with the market return during rising markets: $cov(\, I_m^+, r_i)$.

The non-continuous asymmetric quadratic model in (21) is strictly connected with the extensive literature on downside risk. For example, Ang, Chen, and Xing (2006) run Fama-MacBeth regressions using all stocks in the cross-section, finding a positive premium (higher excess returns) for stocks delivering higher downside risk and lower upside potential. These results are consistent with an asymmetric quadratic utility function with concavity for losses and convexity for gains. However, when estimating a Stochastic Discount Factor model able to capture downside risk, Ang, Cheng, and Xing (2006, p.1219) find parameters estimates implying a pricing kernel that is increasing in the negative domain and decreasing in the positive domain (see Figure 5). Such a pricing kernel is consistent with preference for *negative* asymmetry, and it contradicts Ang, Chen, and Xing previous findings of a positive premium for downside risk and negative premium for upside potential. Furthermore Ang, Chen, and Xing pricing kernel violates monotonicity for the typical sample range, since it displays a reverse V-shape with negative values for market returns below a negative threshold or above a positive one, implying decreasing utility and violations of the no-arbitrage condition in asset pricing.

In this paper, asymmetric quadratic models are estimated imposing sign restrictions that are consistent with both intuition and the results of the empirical decision theory literature, in order to avoid paradoxical results like preference for negative asymmetry or violations of monotonicity. Furthermore, section 4.1 shows that the asymmetric quadratic model with Ang, Chen, and Xing signs is rejected.

3 Empirical Analysis

3.1 Estimation Methods

This paper compares asset pricing models defined over a broad set of utility functions, listed in Table 1. The four models on which I focus my attention are:

1. CAPM: standard mean-variance CAPM.
2. CAPM(SK): three-moment CAPM, which considers skewness in addition to mean and variance.
3. NCA-CAPM(SP,LA): Non-Continuous-Asymmetric CAPM, which considers preference for security/potential and loss aversion.[16]
4. A-CAPM(SP,LA,GA): Asymmetric-CAPM, which considers all risky choice factors (preference for security/potential, loss aversion, and goal achievement).[17]

The utility functions of these four models are defined as:

$$CAPM: \quad u(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 \tag{24a}$$

$$CAPM(SK): \quad u(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 + \theta_3\, r_m^3 \tag{24b}$$

$$NCA-CAPM(SP,LA): \quad u(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 + (\theta_1^+\, r_m + \theta_2^+\, r_m^2)\, I_m^+ \tag{24c}$$

$$A-CAPM(SP,LA,GA): \quad u(r_m) = \theta_1\, r_m + \theta_2\, r_m^2 + (\theta_0^+ + \theta_1^+\, r_m + \theta_2^+\, r_m^2) F_m \tag{24d}$$

where I_m^+ is the indicator function equal to one for positive excess returns of the market, while F_m is the weighting function defined in Section 2.2. The signs restrictions are the same for all models and are applied depending on each model's extension:

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[16]When not specified, the notation "NCA-CAPM" is used for NCA-CAPM(SP,LA).

[17]When not specified, the notation "A-CAPM" is used for A-CAPM(SP,LA,GA).

$$\theta_1 > 0, \quad \theta_2 < 0, \quad \theta_3 > 0, \quad \theta_0^+ \geq 0, \quad \theta_1^+ \leq 0, \quad \theta_2^+ \geq 0 \tag{25}$$

The symmetrical linear term—θ_1—is always assumed to be positive (to allow for positive monotonicity); the symmetrical quadratic term—θ_2—is negative (risk aversion for symmetrical models, and downside risk aversion for asymmetrical models); the symmetric cubic term—θ_3—is positive (preference for positive skewness); the upside constant—θ_0^+—is positive (goal achievement); the upside linear term—θ_1^+—is negative (loss aversion); and the upside quadratic term—θ_2^+—is positive (preference for upside potential). These restrictions, derived from preference theory, deliver greater economic and statistical power to the tests of the model.

Regarding A-CAPM(SP,LA,GA), the weighting function F_m requires the estimation of the weighting parameter σ, which will be constrained to the value of 0.01. This constraint is imposed to avoid too much smoothing of the weighting function, which may compromise the distinction between the two components of the utility function.

The pricing kernels corresponding to the four utility functions in (24) are:

$$CAPM: \qquad m = \theta_1 + 2\,\theta_2\, r_m \tag{26a}$$

$$CAPM(SK): \qquad m = \theta_1 + 2\,\theta_2\, r_m + 3\,\theta_3\, r_m^2 \tag{26b}$$

$$NCA-CAPM(SP, LA): \quad m = \theta_1 + 2\,\theta_2\, r_m + \theta_1^+ I_m^+ + \theta_2^+ r_m I_m^+ \tag{26c}$$

$$A-CAPM(SP, LA, GA): \quad m = \theta_1 + 2\,\theta_2\, r_m + \theta_0^+ r_{0,m}^+ + \theta_1^+ r_{1,m}^+ + \theta_2^+ r_{2,m}^+ \tag{26d}$$

For each utility specification it’s possible to directly test the CAPM (derived in Appendix C). The betas relative to the four utility functions in (24) are given by

$$CAPM: \qquad \beta_i = \frac{cov(2\,\theta_2\, r_m\,, r_i)}{cov(2\,\theta_2\, r_m\,, r_m)} \tag{27a}$$

$$CAPM(SK): \qquad \beta_i = \frac{cov(2\,\theta_2\, r_m + 3\,\theta_3\, r_m^2\,, r_i)}{cov(2\,\theta_2\, r_m + 3\,\theta_3\, r_m^2\,, r_m)} \tag{27b}$$

$$NCA-CAPM(SP,LA): \quad \beta_i = \frac{cov(2\,\theta_2\,r_m + \theta_1^+ I_m^+ + \theta_2^+ r_m I_m^+\,, r_i)}{cov(2\,\theta_2\,r_m + \theta_1^+ I_m^+ + \theta_2^+ r_m I_m^+\,, r_m)} \qquad (27c)$$

$$A-CAPM(SP,LA,GA): \quad \beta_i = \frac{cov\left(2\,\theta_2\,r_m + \theta_0^+ r_{0,m}^+ + \theta_1^+ r_{1,m}^+ + \theta_2^+ r_{2,m}^+\,, r_i\right)}{cov\left(2\,\theta_2\,r_m + \theta_0^+ r_{0,m}^+ + \theta_1^+ r_{1,m}^+ + \theta_2^+ r_{2,m}^+\,, r_i\right)} \qquad (27d)$$

Note that the model, based on unconditional expectations, implies constant betas. Furthermore, the solution to the problem of the identification of all parameters is discussed in Section 3.2.

3.2 Generalized Method of Moments estimation

For each utility specification, I study the following moment conditions:

$$E\begin{pmatrix} r-\beta\,r_m \\ m\,R_F-1 \end{pmatrix} = 0 \qquad (28)$$

with r vector of the excess returns of the N risky assets; β is the vector of betas of the risky assets, defined in (27); r_m is the excess return of the market portfolio; m is the pricing kernel, defined in (26); and R_F is the gross risk free rate, $R_F = 1 + r_F$. The moment condition relative to the risk free asset is added to identify all parameters. Furthermore, Dahlquist and Soderlind (1999) and Farnsworth et al. (1999) find that imposing this restriction on the pricing kernel is important in the context of performance valuation. The risk free rate moment condition can be derived from the First Order Conditions with an approach based on risk neutral probabilities, as shown in Appendix D.

Equation (28) forms a set of moment conditions that can be used to test the asset pricing models via Hansen’s (1982) Generalized Method of Moments (GMM). The sample version of (28) is:

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} u_t = 0 \tag{29}$$

Where T represents the number of observations, θ is the vector of parameters, and u_t is

$$u_t = \begin{pmatrix} r_t - \beta\, r_{m,t} \\ m_t\, R_{F,t} - 1 \end{pmatrix} \tag{30}$$

Equation (30) is a system of $N + 1$ equations, with K parameters. Hansen (1982) shows that a test of model specification can be obtained by minimizing the following quadratic form:

$$J(\theta) = g_T(\theta)'\, W_T(\theta)\, g_T(\theta) \tag{31}$$

where $W_T(\theta)$ is the GMM weighting matrix. Hansen shows that the efficient weighting matrix is the inverse of the covariance matrix of the moment conditions:

$$W_T(\theta) = Var\big(g_T(\theta)\big)^{-1}$$

The sample orthogonality conditions (29) can be interpreted as the pricing errors that are obtained using the approximate utility function. If the model provides a good description of the data, then these pricing errors should be close to zero and the minimized value of equation (31) will be small. Hansen (1982) shows that $J_T = T \cdot J\big(\hat{\theta}_{GMM}\big)$ has a χ^2 distribution with degrees of freedom equal to the number of moment conditions minus the number of parameters (in our case, $N + 1 - K$). This statistic is commonly referred to as the "J test" or as the test of the model's "over identifying restrictions". The J_T statistic tests the magnitude of the weighted average of the pricing errors. However, using the efficient weighting matrix, there are two ways to get a small value of the J_T statistic: first, generate small pricing errors with a high degree of precision; second, generate large pricing errors with even larger standard deviations. In fact, a model can achieve a low J_T by simply blowing up the covariance matrix of the moment conditions, rather than improving the moment conditions. It follows that when evaluating the descriptive ability of a model, it's important to consider the magnitude of the average pricing errors. To this end I check, for each model, the Mean Absolute Average Error (MAAE):

$$MAAE(\hat{\theta}_{GMM}) = \frac{1}{N}\sum_{n=1}^{N}\left|g_{T,n}(\hat{\theta}_{GMM})\right| \tag{33}$$

This metric directly checks the pricing ability of each model and is particularly appealing in the CAPM context, since it gives a measure of the average distance between predicted and actual returns. Another important metric is represented by the Root Mean Squared Error (RMSE), which is the square root of the Mean Squared Error:

$$RMSE(\hat{\theta}_{GMM}) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}u_t\left(\hat{\theta}_{GMM}\right)^2} \tag{34}$$

The RMSE is an absolute metrics not taking into account the volatility of the assets under analysis. To make sure that any improvement in the J_T statistic is not due to volatile pricing errors, we need to impose restrictions based on relative metrics that consider the dispersion of the pricing errors relative to the assets under investigation. To this end I check the R^2 of each moment condition, defined in the following way:

$$R_i^2 = 1 - \frac{MSE_i}{TSS_i} \tag{35}$$

Where TSS_i is the Total Sum of Squares of asset i returns, while MSE_i is the Mean Squared Error of asset i. To guarantee that the estimation process doesn't lead to regions of the parameter space that simply blow up the errors volatility (rather than lowering the average errors g_T), I impose the non-negativity of all the R_i^2 as one of the admissibility conditions that all models have to satisfy.[18]

The four admissibility conditions that are verified during the estimation process are the following:

1. Monotonicity: each utility function has to be globally increasing, with pricing kernel strictly positive. This is a minimum condition that each model has to satisfy. Any violation would imply that individuals prefer less to more and would lead to violations of the no-arbitrage condition in asset pricing.
2. Second Order Conditions: since global concavity is not imposed, the SOC for optimality defined in equation (4) have to be verified.
3. Hansen and Jagannathan bounds: the variance of the pricing kernel has to be sufficiently high, in order to satisfy the Hansen and Jagannathan bounds.
4. Non-negative R_i^2: the non-negativity of all the R_i^2 guards against parameter estimates that blow up errors volatility rather than improving pricing ability.

3.3 Model Comparison Tests

Since the models studied in this paper are nested with each other, model comparison tests can be obtained by checking the significance of single or groups of parameters. For example, A-CAPM(SP,LA,GA) nests NCA-CAPM(SP,LA) if we set to zero the upside constant (goal achievement), and the weighting parameter (continuity); NCA-CAPM(SP,LA) nests the standard CAPM by setting the upside linear (loss aversion) and quadratic (preference for

[18]The maximum value that R_i^2 can achieve is 1 (if $MSE_i = 0$). A value of R_i^2 lower than zero implies that residuals are more volatile than the asset under analysis: $MSE_i > TSS_i$.

security/potential) terms to zero; while CAPM(SK) nests the standard CAPM by setting the cubic term (skewness) to zero.

Studying a wide range of models—going from the standard CAPM to the most developed model with all factors, A-CAPM(SK;SP,LA,GA)—we can check the contribution of each factor or group of factors with the following χ^2 difference test:

$$T\ g_T(\theta_R)'\ W_T(\theta)\ g_T(\theta_R) - T\ g_T(\theta)'\ W_T(\theta)\ g_T(\theta) \sim \chi_p^2 \tag{36}$$

where θ_R is the set of optimal parameters of the restricted model. Note that the restricted optimization is obtained using the weighting function of the unrestricted model. A low value of the difference test implies that the parameters set to zero are statistically not significant (since the J test of the restricted model doesn't rise much). The test statistic is distributed according to a chi-square distribution with p degrees of freedom (p is the number of restrictions).

3.4 Data

I analyze investor preferences by testing if asset pricing models defined on different utility functions can rationalize the market portfolio. To this end, we need a proxy for the market portfolio and proxies for the individual assets in the investment universe. I approximate the market portfolio by using the CRSP all-share index, which is the value-weighted average of all common stocks listed on the NYSE, AMEX, and NASDAQ. Furthermore, I use the one-month US Treasury bill as the riskless asset.

The main results of the paper are obtained testing all models on the 30 portfolios given by the aggregation of size, book-to-market and momentum portfolio deciles. I focus on these assets because they represent the most studied phenomena in the literature (Fama and

French, 1992; Jagadeesh and Titman, 1993). Robustness results are obtained using portfolios based on different classifications of size, book-to-market and momentum: 6 double sorted size and book-to-market portfolios; 6 double sorted size and momentum portfolios; and 25 double sorted size and book-to-market portfolios. Furthermore, I also use the 48 Industry portfolios (based on four digit SIC code classification) to check the results obtained using portfolios built on formation criteria independent of size, value, and momentum effects. For all securities under analysis, I use data at monthly frequency from July 1963 to June 2010 (564 observations) obtained from the data library on the homepage of Kenneth French.[19] All portfolios are value weighted.

[19]The data set starts in 1963 because the COMPUSTAT data used to construct the benchmark portfolios are biased towards big historically successful firms for the earlier years (Fama and French (1992)).

4 Results

4.1 Main Results

Table 2 shows the main results of the paper, which are related to the estimation of the four models defined in equation (24): standard CAPM (quadratic utility function), CAPM(SK) (cubic utility function), NCA-CAPM(SP,LA) (non-continuous asymmetric quadratic utility), and A-CAPM(SP,LA,GA) (continuously asymmetric quadratic utility with positive shift at zero). The table shows the J test of the "over identifying restrictions", parameters values (with p-values in parenthesis), and goodness to fit (MAAE) of each model. The test assets are the 30 value-weighted portfolios given by the aggregation of size, book-to-market, and momentum portfolio deciles.

Results are consistent with the intuition that the cross-section of stock returns can be rationalized by asset pricing models based on utility functions that are able to explain risky choice behavior. The two standard polynomial models (quadratic and cubic utility functions, implying the standard CAPM and CAPM(SK)) are both rejected, while the two asymmetric quadratic models (NCA-CAPM(SP,LA) and A-CAPM(SP,LA,GA)) are admissible. In particular, the asymmetric models significantly improve the fit of the CAPM. In fact, the MAAE (the Mean Absolute Average Error, defined in equation (33)) decreases from the 20 basis points[20] of the mean-variance CAPM, to the 2.73 basis points of the Asymmetric-CAPM (A-CAPM(SP,LA,GA)).

[20]One basis point is equal to 0.0001.

4.1.1 Standard Mean-Variance CAPM

The first column of Table 2 shows the estimation results of the standard mean-variance CAPM, derived from a quadratic utility function. The model, with a J test equal to 44.40, is rejected at the 5% level. Figure 3 shows the predicted returns versus the actual returns of the 30 test assets. The graph clearly shows that the model fails to capture the risk-return profile of the securities under analysis. These results are consistent with the extensive literature documenting the failure of the standard CAPM to explain the risk-return trade off of size, value, and momentum portfolios.

4.1.2 Three-moment CAPM: CAPM(SK)

The second column of Table 2 shows the estimation results of the model able to consider preference for positive skewness (CAPM(SK)), which is derived from a cubic utility function. The model is rejected and there's no improvement in the fit. Note that the cubic parameter is low and not significant, implying a utility function concave over the entire sample range, and a pricing kernel decreasing and almost linear. In fact, if the Second Order Conditions for optimality are imposed, the cubic parameter cannot be too high (with a high value of the cubic parameter, the utility function would be convex in the positive domain, and the SOC would be violated). Not reported results, relative to the estimation of the cubic model without imposing the SOC for optimality, confirm the findings of Post, Vliet, and Levy (2008) and Dittmar (2002) (when they don't impose global concavity of the utility function): the cubic parameter is high and significant, the utility function is convex in the positive domain, and CAPM(SK) is not rejected. These results confirm that the cubic model doesn't have enough flexibility to imply strong preference for positive asymmetry, necessary to explain the cross-section of stock returns, and satisfy the SOC at the same time.

4.1.3 Non-Continuous-Asymmetric CAPM: NCA-CAPM(SP,LA)

The third column of Table 2 shows the estimation results of NCA-CAPM(SP,LA), derived from a non-continuous asymmetric quadratic utility function that allows considering the first two risky choice factors: preference for security/potential, and loss aversion. The model is admissible (the J tests is 30.67 with p-value equal to 24%) and is able to significantly improve the pricing ability of the CAPM. In fact, the MAAE is 11 basis points, and the improvement of the fit (more than 44% with respect to the mean-variance CAPM) is apparent in Figure 3.

As shown in Figure 4, the NCA-CAPM(SP,LA) utility function is concave in the negative domain, mildly convex in the positive domain (preference for security/potential), and steeper in the domain of losses (loss aversion). The corresponding pricing kernel is decreasing in the negative domain, slightly increasing for gains, and has a higher level in the domain of losses. Even though the utility function is convex over a range, the Second Order Conditions for optimality are satisfied, since the mild convexity in the domain of gains is balanced by the strong concavity in the domain of losses. The point is that the model can imply strong preference for positive asymmetry, without violating the SOC, because of its flexibility in disentangling preference for positive asymmetry in strong downside risk aversion and mild desire for upside potential. This result represents one of the main improvements obtained by considering asymmetric quadratic models rather than higher-order polynomials.

Table 3 shows the estimation results of the non-continuous asymmetric models with preference for security/potential alone (NCA-CAPM(SP)) and loss aversion alone (NCA-CAPM(LA)). The table highlights that neither preference for security/potential, nor loss aversion is sufficient to explain the cross-section of stock returns.

The model with preference for security/potential alone (NCA-CAPM(SP)) is the model typically studied in the downside risk literature[21] (for example, Ang, Chen, and Xing, 2006).[22] The estimation results shown in the first column of Table 3 highlight that preference for security/potential alone is not able to explain the cross-section of stock returns. In fact, NCA-CAPM(SP) is rejected and its fit is even worse than that of the mean-variance CAPM.

The second column of Table 3 shows the estimation of the model with loss aversion alone: NCA-CAPM(LA). The results of Table 3 clearly show that loss aversion alone is not sufficient to explain the cross-section of stock returns. In fact, NCA-CAPM(LA) is rejected and the fit of the model is only slightly better than the mean-variance CAPM (MAAE of 19 basis points).

The χ^2 comparison tests results of Table 4 highlight that both preference for security/potential and loss aversion are required to explain the cross-section of stock returns. In fact, model NCA-CAPM(SP,LA) is significantly better than models excluding either loss aversion or preference for security/potential (highly significant χ^2 difference tests of 48.62

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[21]Several works of the downside risk literature just replace variance with semivariance: $u(r_m) = \theta_1 r_m + \theta_2^- r_m^2 I_m^-$. This utility function (concave for losses and linear for gains) can also be achieved by neutralizing the upside quadratic term in NCA-CAPM(SP), i.e. by setting $\theta_2^+ = |\theta_2|$. Non-reported results reject the model: J test equal to 48.45 with p-value of 0.0096, and MAAE of 21 basis points.

[22]The results obtained in Table 3 are different from those of Ang, Chen, and Xing (2006, p.1219) because of the imposition of the sign restrictions defined in (25), as well as admissibility conditions defined in Section 3.1 (in particular, positive monotonicity). Section 2.4 highlighted the problems of Ang, Chen, and Xing SDF estimation (violation of monotonicity and preference for negative asymmetry). Furthermore, not reported estimation results clearly reject the asymmetric quadratic model obtained imposing Ang, Chen, and Xing SDF sign restrictions. For example, regarding NCA-CAPM(SP,LA), the J test is equal to 50.31 with p-value of 0.0042, and the MAAE is 19 basis points.

and 33.71, when NCA-CAPM(SP,LA) is compared with NCA-CAPM(SP) and NCA-CAPM(LA), respectively), or both risky chioce factors (highly significant χ^2 difference test of 48.62, when NCA-CAPM(SP,LA) is compared with the standard mean-variance CAPM). Finally, neither the model with loss aversion alone nor the one with preference for security/potential alone are able to improve with respect to the standard CAPM (non significant χ^2 difference tests of 0.01 and 0.00, respectively, when they are compared with the mean-variance CAPM).

4.1.4 Asymmetric-CAPM: A-CAPM(SP,LA,GA)

The fourth column of Table 2 shows the estimation results of the Asymmetric-CAPM, A-CAPM(SP,LA,GA), derived from a continuously asymmetric quadratic model that allows considering all the risky choice factors: preference for security/potential, loss aversion, and goal achievement. The ability of A-CAPM(SP,LA,GA) at capturing all relevant risky choice factors allows the model to significantly improve the performance of the CAPM. The high significance of the model (J test equal to 7.75, with p-value of 99%) is due to the considerable improvement in the fit; in fact, the MAAE is only 2.73 basis points, compared to 20 basis points of the standard CAPM (improvement of more than 86%), and 11 basis points of NCA-CAPM(SP,LA) (improvement of more than 75%). The improvement of the fit is apparent in Figure 3, showing that predicted returns are aligned with the average returns of the test assets.

Utility function and pricing kernel of A-CAPM(SP,LA,GA) are shown in Figure 1. The estimated utility function is in line with the results of Piccioni (2011). In fact, the utility function is concave in the negative domain and convex in the positive domain (preference for security and potential), steeper in the domain of losses (loss aversion), and has a positive shift at the target return (goal achievement). The corresponding pricing kernel is decreasing in the negative domain and increasing in the positive domain, has a higher level in the domain of losses, and displays a bump at the target return.

Model comparison tests in Table 4 highlight the significance of each risky choice factor (highly significant χ^2 difference tests when each risky choice factor is removed from A-CAPM(SP,LA,GA)) and the joint significance of all risky choice factors (highly significant χ^2 difference tests when all risky choice factors are removed from A-CAPM(SP,LA,GA)).

The third column of Table 3 shows that, like preference for security/potential and loss aversion, goal achievement alone is not sufficient to rationalize the cross-section of stock returns. In fact, A-CAPM(GA) is rejected and the upside constant term is not significant.

The fourth column of Table 3 and comparison tests of Table 4, instead, show that we can't get any improvement by simply adding continuity to NCA-CAPM(SP,LA). In fact, Table 4 shows that A-CAPM(SP,LA) doesn't improve with respect to NCA-CAPM(SP,LA) (non-significant χ^2 difference test of 2.13). However, A-CAPM(SP,LA,GA) is significantly better than A-CAPM(SP,LA) (highly significant χ^2 difference test of 23.34). This means that the improvement obtained moving from NCA-CAPM(SP,LA) to A-CAPM(SP,LA,GA) is not obtained simply by adding continuity to NCA-CAPM(SP,LA), but the inclusion of the upside constant term (goal achievement) is crucial to improve the performance of the model.

4.2 Asymmetrical Cubic Models

The asymmetric quadratic models studied in Tables 2 and 3 don't nest the cubic polynomial model (CAPM(SK)). In order to test whether preference for positive asymmetry is better captured by asymmetric quadratic models (implying preference for security/potential) or cubic polynomials (implying preference for skewness), we have to extend the asymmetric quadratic models by including a cubic term. In this way, the resulting models are able to nest both asymmetric quadratic models and cubic polynomials, allowing the required comparison tests.

The last two columns of Table 5 show the results obtained by including preference for skewness in NCA-CAPM(SP,LA) and A-CAPM(SP,LA,GA). For both models, it's clear that preference for security/potential drives out preference for skewness. In fact, the cubic term is highly non-significant and close to zero for both models, while the upside quadratic term is significant and the estimation results are very similar to those obtained with NCA-CAPM(SP,LA) and A-CAPM(SP,LA,GA).

The first two columns of Table 5, instead, show the results obtained by replacing preference for security/potential with preference for skewness. Note that model A-CAPM(SK;LA,GA) is admissible and has a performance similar to A-CAPM(SP,LA,GA). This result shows that models that are able to capture preference for positive asymmetry, loss aversion, and goal achievement are able rationalize the risk-return tradeoff of size, value, and momentum portfolios. However, model comparison tests clearly show the importance of breaking preference for positive asymmetry into strong downside risk aversion and mild preference for upside potential. In fact, the χ^2 difference test for the A-CAPM(SK;SP,LA,GA) and A-CAPM(SK;LA,GA) comparison is 49.84 (preference for security/potential is statistically significant even after controlling for preference for skewness), while the χ^2 difference test for the A-CAPM(SK;SP,LA,GA) and A-CAPM(SP,LA,GA) comparison is very close to zero (preference for skewness is not statistically significant after controlling for preference for security/potential).[23]

4.3 Size, Value, and Momentum factors

Table 6 shows the results obtained by including size, value, and momentum factors in the four models of Table 2. The inclusion is obtained by adding three terms to the original utility functions: $\theta_{smb}\ SMB + \theta_{hml}\ HML + \theta_{mom}\ MOM$[24] In this way, the covariance between the three additional factors and each asset enters in the CAPM. The methodology used to introduce these factors is somehow arbitrary, but it allows assessing the contribution of size, value, and momentum factors. This exercise is justified by the popularity of these factors, and by the fact that the present work focuses on the risk-return tradeoff of size, value, and momentum portfolios.

Note that size, value, and momentum factors parameters are negative, consistent with a positive premium for securities having higher loadings (higher covariance) with these factors.

The first two columns of Table 6 clearly show that both mean-variance and three-moment CAPM are significantly improved by including size, value, and momentum factors: the two models become admissible and there's a considerable improvement in the fit (MAAE of 7 basis points for each model, instead of 20 basis points obtained without including the

[23]Similar results are obtained for non-continuous models. The χ^2 difference test for the NCA-CAPM(SK;SP,LA) and NCA-CAPM(SK;LA) comparison is 41.35, while the χ^2 difference test for the NCA-CAPM(SK;SP,LA) and NCA-CAPM(SP,LA) comparison is very close to.

[24]Where *SMB*, *HML*, *MOM* are the "Small Minus Big", "High Minus Low", and "Momentum" factors obtained from the data library on the homepage of Kenneth French.

additional factors). Furthermore, each additional factor is highly significant, and the tests of their joint significance clearly reject the null hypothesis that size, value, and momentum factors' parameters are jointly equal to zero (highly significant χ^2 difference tests of 35.42 and 30.40). These results are consistent with the extensive literature documenting the importance of size, value, and momentum factors at improving the performance of existing asset pricing models.

The last two columns of Table 6 show the results obtained by including size, value, and momentum to the asymmetrical quadratic models. Regarding the A-CAPM (A-CAPM(SP,LA,GA)), size, value, and momentum factors parameters drop in value and completely lose their significance. Note that size, value, and momentum factors are jointly significant for NCA-CAPM(SP,LA). This is further evidence of the importance of including *all* risky choice factors in the analysis.

The results of Table 6 show that when we consider the CAPM built on a utility function that is able to explain risky choice behavior there's nothing left to explain for size, value, and momentum factors, even when the model is tested on size, value, and momentum portfolios.

4.4 Robustness Tests

Tables 7, 8, and 9 show several robustness tests, obtained by using different sets of test assets: 12 portfolios given by the aggregation of 6 double sorted Size and Book-to-Market portfolios and 6 double sorted Size and Momentum portfolios, for Table 7; 25 double sorted Size and Book-to-Market portfolios for Table 8; and 48 Industry portfolios for Table 9. Test assets of Table 8 have been chosen to check the results obtained using different aggregations of size, value, and momentum portfolios. Test assets of Table 8 have been chosen to assess the performance of the models with the most studied Size-Value classification: the 25 Fama-French portfolios. Test assets of Table 9 have been chosen to check the results obtained using portfolio formation criteria independent of size, value, and momentum effects. Lo and MacKinlay (1990) and Conrad, Cooper, and Kaul (2003) argue that many of the empirical regularities observed studying portfolios built on characteristics such as Size and Value may be overstated due to data snooping. Then, it's important to check that results are robust to the choice of portfolio formation strategy.

Results of Table 2 are robust to changes in tested assets. In particular, results relative to the A-CAPM are always in line with the message coming from decision theory literature (the shape of the utility function is always the same: concave for losses and convex for gains, steeper for losses, and with a positive shift at the target return). There are only two relevant differences with respect to Table 2. First, results of Table 7 show that only A-CAPM(SP,LA,GA) is not rejected and significantly able to improve the fit. Second, regarding Table 9, the J test doesn't have enough power to reject any model, but the asymmetric quadratic models are able to significantly improve with respect to symmetric models (in particular, MAAE decreases from 18 basis points to 6 basis points).

5 Conclusions

This paper shows that the same utility function that can explain risky choice behavior is also able to rationalize the risk-return tradeoff of size, value, momentum portfolios. The Asymmetric-CAPM is derived from asymmetric quadratic utility functions that are in line with Piccioni (2011), who points out that the risky choice factors, i.e. the fundamental aspects necessary to explain risky choice behavior, are: (i) preference for security/potential (downside risk aversion and preference for upside potential), (ii) loss aversion (losses loom larger than gains), and (iii) goal achievement (importance of satisfying relevant aspiration levels). These factors can be captured by a utility function (i) concave in the domain of losses and convex in the domain of gains, (ii) steeper for losses, and (iii) with a positive shift at the target return.

The first risky choice factor is preference for security/potential, which is related to preference for positive asymmetry. This study shows that asymmetric quadratic utility functions (implying preference for security/potential) are better than cubic utility functions (implying preference for skewness) at capturing preference for positive asymmetry. In fact, asymmetric quadratic models can imply strong aversion toward downside risk and mild preference for upside potential, therefore strong preference for positive asymmetry (necessary to explain the cross-section of stock returns) without violating the SOC for optimality. Cubic models, instead, don’t have such flexibility. In fact, cubic models either imply strong preference for positive asymmetry (at the cost of violating the SOC for optimality), or satisfy the SOC for optimality (at the cost of not being able to explain the cross-section of stock returns). Comparing a wide range of models with symmetrical cubic and asymmetric quadratic terms, this paper finds clear evidence that preference for positive asymmetry is better captured by asymmetric quadratic models rather that cubic polynomials.

The second risky choice factor is loss aversion, which is captured with the asymmetry in the linear terms of the utility function. Because of loss aversion, the A-CAPM predicts higher risk premiums for securities delivering lower returns during market downturns. In other words, the model keeps track of securities paying off less when investors need it the most. The inclusion of loss aversion significantly contributes to improving with respect to the existing asset pricing models.

The third risky choice factor is goal achievement. Goal achievement (the importance of the overall probability of achieving the target return) can be effectively incorporated in the model by introducing a positive shift of the utility function in proximity of the target return. The theory section of the paper shows that only continuously asymmetrical models are able to quantify the importance of this factor in the CAPM context, and the empirical analysis section shows the crucial importance of goal achievement at improving the performance of the CAPM (in particular, by achieving an almost perfect fit of the model).

Finally, this paper shows clear evidence that size, value, and moment factors are completely driven out when all risky choice factors are taken into account. When we consider the CAPM built on utility functions able to explain risky choice behavior (A-CAPM), there’s nothing left to explain for size, value, and moment factors, even when the model is tested on size, value, and moment portfolios.

Running a wide range of model comparison tests, I find that only the A-CAPM that includes all three risky choice factors can rationalize size, value, and momentum anomalies and significantly improve with respect to the existing asset pricing models. Consistent with Piccioni (2011) findings in the empirical decision making literature, we need all three main characteristics of the utility function (concavity for losses and convexity for gains, utility steeper in the domain of losses, and positive shift at the target return) in order to explain the cross-section of stock returns.

This paper can be extended in several directions. First of all, further research is needed to develop conditional models that are able to incorporate time varying preferences. Furthermore, a deeper analysis is needed to distinguish between the factors analyzed in this paper and other cross-sectional effects. For example, liquidity effects and downside risk are interconnected, because liquidity typically dries up when the largest losses occur and, in turn, liquidity dry–up may cause or amplify these losses. It follows that an approach able to disentangle liquidity and downside risk is required.

Table 1: Models under analysis

	Symmetric component			Asymmetric component			
	linear	quadratic	cubic	constant	linear	quadratic	weighting
A-CAPM(SP,LA,GA)	✓	✓		✓	✓	✓	✓
A-CAPM(SP,LA)	✓	✓			✓	✓	✓
A-CAPM(LA,GA)	✓	✓		✓	✓		✓
A-CAPM(SP,GA)	✓	✓		✓		✓	✓
A-CAPM(GA)	✓	✓		✓			✓
A-CAPM(LA)	✓	✓			✓		✓
A-CAPM(SP)	✓	✓				✓	✓
NCA-CAPM(SP,LA)	✓	✓			✓	✓	
NCA-CAPM(LA)	✓	✓			✓		
NCA-CAPM(SP)	✓	✓				✓	
CAPM	✓	✓					
A-CAPM(SK;SP,LA,GA)	✓	✓	✓	✓	✓	✓	✓
A-CAPM(SK;LA,GA)	✓	✓	✓	✓	✓		✓
NCA-CAPM(SK;SP,LA)	✓	✓	✓		✓	✓	
NCA-CAPM(SK;LA)	✓	✓	✓		✓		
CAPM(SK)	✓	✓	✓				

Table 1 provides a list of the models studied in this paper. The notation used to identify each model complies with the following logic:

- "A" identifies asset pricing models derived by approximating utility with \emph{continuous} Asymmetric polynomials.
- "NCA" identifies asset pricing models derived from Non-Continuous Asymmetric polynomials.
- "SP" identifies models capturing preference for Security/Potential.

- "LA" identifies models capturing Loss Aversion.
- "GA" identifies models capturing Goal Achievement.
- "SK" identifies models capturing preference for positive Skewness.

The Table specifies which terms of the polynomial approximation of the utility function are included in each model. Note that asymmetric polynomial models are defined as the sum of a symmetric component (applied to both domains of gains and losses) and an upside component (applied to the domain of gains only). This formulation is equivalent to the sum of upside and downside components, but allows asymmetric polynomial models to nest symmetric ones, as shown in Section 2.2.
Models extended to include size, value, and momentum factors are shown in Table 6 and discussed in Section 4.3.

Table 2: Testing the CAPM on Size, Value, and Momentum Portfolios

	CAPM	CAPM SK	NCA-CAPM SP,LA	A-CAPM SP,LA,GA
J test	44.40	46.59	31.67	7.75
	(0.03)	(0.01)	(0.20)	(0.99)
MAAE ($\times 10^{-4}$)	19.79	19.82	11.06	2.73
# parameters	3	4	5	7
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	1.01	0.99	1.16	0.27
	(0.00)	(0.00)	(0.00)	(0.00)
Quadratic: θ_2	-2.93	-5.51	-13.60	-14.36
	(0.00)	(0.00)	(0.00)	(0.00)
Cubic: θ_3		8.68		
		(0.99)		
Upside Constant: θ_0^+				0.04
				(0.00)
Upside Linear: θ_1^+			-0.94	-0.18
			(0.00)	(0.00)
Upside Quadratic: θ_2^+			15.54	16.53
			(0.00)	(0.00)
Weighting: σ				0.01
				(0.00)

The table shows the results of the GMM estimation of the four models defined in Section 3.1: CAPM (standard mean-variance CAPM), CAPM(SK) (three-moment CAPM, which considers skewness in addition to mean and variance), NCA-CAPM(SP,LA) (Non-Continuous-Asymmetric CAPM, which considers preference for security/potential and loss aversion), and A-CAPM(SP,LA,GA) (Asymmetric-CAPM, which considers all risky choice factors: preference for security/potential, loss aversion, and goal achievement).

Each model is tested on 30 value-weighted portfolios given by the aggregation of size, book-to-market, and momentum portfolio deciles. Data are at monthly frequency over the 1963.07−2010.06 period (564 observations).

The table shows the J test of the models' over identifying restrictions, as well as the parameter estimates (p-values are in parenthesis). Moment conditions and econometric methodology are defined in Section 3.2. The GMM weighting matrix is the efficient weighting matrix, given by the inverse of the covariance matrix of the moment conditions. The covariance matrix of the residuals is obtained with Heteroskedasticity and Autocorrelatation Consistent (HAC) estimation. Furthermore, the optimization process is run with the Continuously Updated GMM procedure. Finally, the table shows the measure of each model's fit, the Mean Absolute Average Error, defined in equation (33) in Section3.2.

Admissibility conditions, defined in Section 3.2, of monotonicity, Second Order Conditions for optimality, non-negative R^2, and Hansen-Jagannathan bounds, have been verified. All models satisfy these conditions, except the standard CAPM that doesn't satisfy the Hansen-Jagannathan bounds.

Table 3: Testing Alternative Asymmetric Models

	NCA-CAPM SP	NCA-CAPM LA	A-CAPM GA	A-CAPM SP,LA
J test	46.45	46.44	31.67	7.75
	(0.01)	(0.01)	(0.20)	(0.99)
MAAE ($\times 10^{-4}$)	20.80	18.67	11.06	2.73
# parameters	4	4	5	7
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	0.56	1.21	0.86	1.11
	(0.00)	(0.00)	(0.21)	(0.00)
Quadratic: θ_2	-16.90	-3.65	-2.41	-13.58
	(0.99)	(0.00)	(0.00)	(0.00)
Cubic: θ_3				
Upside Constant: θ_0^+			0.01	
			(0.99)	
Upside Linear: θ_1^+		-0.36		-0.88
		(0.93)		(0.00)
Upside Quadratic: θ_2^+	18.48			16.08
	(0.04)			(0.00)
Weighting: σ			0.01	0.01
			(0.99)	(0.14)

The table shows the results of the GMM estimation of moment conditions defined in (28). Test assets and econometric methodology are the same as in Table 2. The models estimated are alternative specifications of asymmetrical utility functions, studied to highlight the contribution of each risky choice factor.

All models satisfy the admissibility conditions, defined in Section 3.2, of monotonicity, Second Order Conditions for optimality, non-negative R^2, and Hansen-Jagannathan bounds (except A-CAPM(GA) that doesn't satisfy the Hansen-Jagannathan bounds).

Table 4: Model Comparison Tests

	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)	A-CAPM(SP,LA,GA)
A-CAPM(SP,LA,GA)	-									
A-CAPM(SP,LA)	23.34 (0.00)	-								
A-CAPM(LA,GA)	41.24 (0.00)	-	-							
A-CAPM(SP,GA)	41.24 (0.00)	-	-	-						
A-CAPM(GA)	41.24 (0.00)	-	0.00 (0.99)	11.78 (0.00)	-					
A-CAPM(LA)	41.24 (0.00)	38.39 (0.00)	0.00 (0.99)	-	-	-				
A-CAPM(SP)	41.24 (0.00)	38.39 (0.00)	-	14.96 (0.00)	-	-	-			
NCA-CAPM(SP,LA)	25.32 (0.00)	2.13 (0.14)	-	-	-	-	-	-		
NCA-CAPM(LA)	41.24 (0.00)	38.39 (0.00)	0.00 (0.99)	-	-	0.14 (0.71)	-	33.71 (0.00)	-	
NCA-CAPM(SP)	41.24 (0.00)	38.39 (0.00)	-	14.96 (0.00)	-	-	0.01 (0.91)	48.62 (0.00)	-	-
CAPM	41.24 (0.00)	38.39 (0.00)	0.00 (0.99)	14.96 (0.01)	0.00 (0.99)	0.15 (0.93)	0.01 (0.99)	48.62 (0.00)	0.01 (0.93)	0.00 (0.99)

The table shows the χ^2 difference tests for model comparison (with p-values in parenthesis) defined in equation (36) in Section 3.3. The test is given by the difference between the J test of each restricted model (rows) and each unrestricted model (columns). Note that the comparison test can be obtained only when the unrestricted model (column) nests the restricted one (rows).

Table 5: Testing Asymmetric Cubic Models

	NCA-CAPM SK;LA	NCA-CAPM SK;LA,GA	NCA-CAPM SK;SP,GA	A-CAPM SK;SP,LA,GA
J test	44.19	12.16	31.72	7.75
	(0.01)	(0.98)	(0.17)	(0.99)
MAAE ($\times 10^{-4}$)	15.72	3.31	11.77	2.73
# parameters	5	7	6	8
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	1.51	1.09	1.30	0.18
	(0.00)	(0.00)	(0.07)	(0.00)
Quadratic: θ_2	-4.42	-2.33	-9.55	-15.31
	(0.99)	(0.00)	(0.00)	(0.00)
Cubic: θ_3	37.26	24.50	0.00	0.00
	(0.00)	(0.00)	(0.99)	(0.99)
Upside Constant: θ_0^+		0.02		0.04
		(0.00)		(0.00)
Upside Linear: θ_1^+	-1.09	-0.72	-0.98	-0.12
	(0.00)	(0.00)	(0.00)	(0.00)
Upside Quadratic: θ_2^+			10.91	17.63
			(0.00)	(0.00)
Weighting: σ		0.01		0.01
		(0.99)		(0.00)

The table shows the results of the GMM estimation of moment conditions defined in (28). Test assets and econometric methodology are the same as in Table 2. The Table shows four asymmetric models extended to include a cubic term. These models are studied to test whether preference for positive asymmetry is better captured by including a symmetric cubic term (preference for skewness) or by allowing for asymmetry in quadratic terms (preference for security/potential).

All models satisfy the admissibility conditions, defined in Section 3.2, of monotonicity, Second Order Conditions for optimality, non-negative R^2, and Hansen-Jagannathan bounds.

Table 6: Testing the CAPM including Size, Value, and Momentum factors

	CAPM	CAPM SK	NCA-CAPM SP,LA	A-CAPM SP,LA,GA
J test	34.37	35.81	28.76	7.75
	(0.10)	(0.06)	(0.19)	(0.99)
MAAE ($\times 10^{-4}$)	7.64	7.06	6.12	2.73
# parameters	6	7	8	7
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	1.07	0.89	1.51	0.34
	(0.00)	(0.00)	(0.01)	(0.01)
Quadratic: θ_2	-1.31	-9.04	-4.70	-12.96
	(0.00)	(0.00)	(0.00)	(0.00)
Cubic: θ_3		69.80		
		(0.99)		
Upside Constant: θ_0^+				0.03
				(0.00)
Upside Linear: θ_1^+			-1.08	-0.21
			(0.00)	(0.00)
Upside Quadratic: θ_2^+			5.36	15.88
			(0.00)	(0.00)
Weighting: σ				0.01
				(0.00)
Size	-2.39	-2.21	-1.89	-0.87
	(0.01)	(0.00)	(0.09)	(0.41)
Value	-7.20	-7.09	-2.97	-0.00
	(0.00)	(0.00)	(0.00)	(0.99)
Momentum	-2.56	-3.12	-1.36	-0.00
	(0.00)	(0.01)	(0.13)	(0.99)
	Joint Significance of Size, Value, and Momentum factors			
χ^2 difference test	35.42	30.40	10.27	0.09
	(0.00)	(0.01)	(0.02)	(0.99)

The table shows the results of the GMM estimation of moment conditions defined in (28). Test assets and econometric methodology are the same as in Table 2. The estimated models are extended to include size, value, and momentum factors.

Since size, value, and momentum factors are assumed to be sources of risk, their parameters are imposed to be negative. These sign restrictions are consistent with a positive premium for securities having a positive covariance with these factors.
All models satisfy the admissibility conditions, defined in Section 3.2, of monotonicity, Second Order Conditions for optimality, non-negative R^2, and Hansen-Jagannathan bounds (except the standard CAPM that doesn't satisfy the Hansen-Jagannathan bounds).
Furthermore, the table shows the results of the χ^2 difference test, defined in equation (36), for the joint significance of size, value, and momentum factors (with p-values in parenthesis). A low value of the difference test implies that the parameters set to zero are statistically not significant. The test has three degrees of freedom (p = 3).

Table 7: Robustness, Testing the CAPM on 12 Portfolios (6 Size and Book-to-Market, 6 Size and Momentum)

	CAPM	CAPM SK	NCA-CAPM SP,LA	A-CAPM SP,LA,GA
J test	47.35	55.42	31.48	6.81
	(0.00)	(0.00)	(0.00)	(0.34)
MAAE ($\times 10^{-4}$)	28.03	28.20	14.49	3.07
# parameters	3	4	5	7
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	1.00	0.83	0.84	0.25
	(0.00)	(0.00)	(0.00)	(0.00)
Quadratic: θ_2	-1.79	-9.33	-20.98	-13.84
	(0.00)	(0.00)	(0.00)	(0.00)
Cubic: θ_3		62.57		
		(0.99)		
Upside Constant: θ_0^+				0.03
				(0.00)
Upside Linear: θ_1^+			-0.76	-0.16
			(0.00)	(0.00)
Upside Quadratic: θ_2^+			24.84	17.83
			(0.00)	(0.00)
Weighting: σ				0.01
				(0.00)

The table shows the results of the GMM estimation of moment conditions defined in (28). Econometric methodology and models estimated are the same as in Table 2. Test assets are defined over the 6 double sorted size and book-to-market (value-weighted) portfolios, plus the 6 double sorted size and momentum (value-weighted) portfolios. Data are at monthly frequency over the 1963.07−2010.06 period (564 observations). All models satisfy the admissibility conditions defined in Section 3.2 of monotonicity, Second Order Conditions for optimality, non–negative R^2, and Hansen-Jagannathan bounds (except the standard CAPM that doesn't satisfy the Hansen-Jagannathan bounds).

Table 8: Robustness, Testing the CAPM on 25 Size and Book-to-Market Portfolios

	CAPM	CAPM SK	NCA-CAPM SP,LA	A-CAPM SP,LA,GA
J test	44.27 (0.00)	46.93 (0.00)	32.64 (0.05)	7.96 (0.99)
MAAE ($\times 10^{-4}$)	29.16	29.26	16.13	3.69
# parameters	3	4	5	7
Target spread: τ	0.00 (0.99)	0.00 (0.99)	0.00 (0.99)	0.00 (0.99)
Linear: θ_1	1.02 (0.00)	0.99 (0.00)	0.36 (0.00)	0.63 (0.00)
Quadratic: θ_2	-2.32 (0.00)	-5.90 (0.00)	-28.83 (0.00)	-9.40 (0.00)
Cubic: θ_3		8.30 (0.99)		
Upside Constant: θ_0^+				0.02 (0.00)
Upside Linear: θ_1^+			-0.29 (0.00)	-0.38 (0.00)
Upside Quadratic: θ_2^+			33.72 (0.00)	12.42 (0.00)
Weighting: σ				0.01 (0.00)

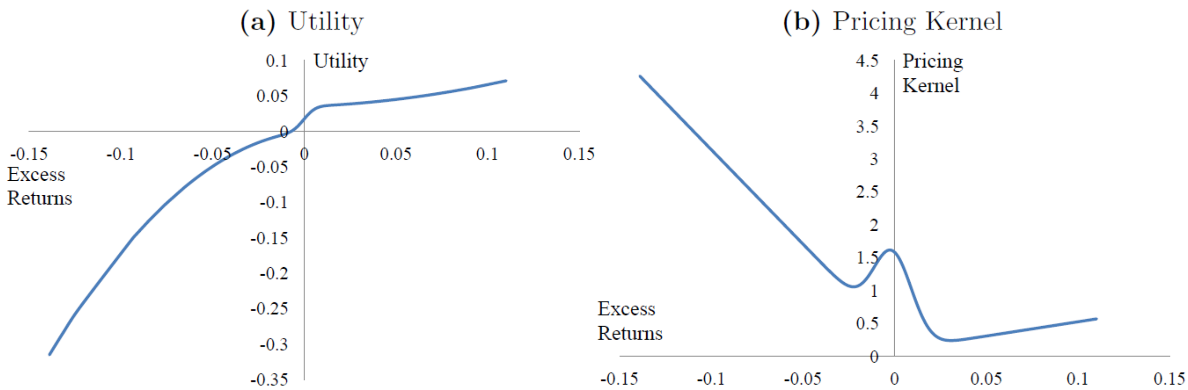
The table shows the results of the GMM estimation of moment conditions defined in (28). Econometric methodology and models estimated are the same as in Table 2. Test assets are defined over the 25 double sorted size and book-to-market (value-weighted) portfolios. Data are at monthly frequency over the 1963.07−2010.06 period (564 observations). All models satisfy the admissibility conditions defined in Section 3.2 of monotonicity, Second Order Conditions for optimality, non–negative R^2, and Hansen-Jagannathan bounds (except the standard CAPM that doesn't satisfy the Hansen-Jagannathan bounds).

Table 9: Robustness, Testing the CAPM on 48 Industry Portfolios

	CAPM	CAPM SK	NCA-CAPM SP,LA	A-CAPM SP,LA,GA
J test	5015	51.27	41.16	15.47
	(0.31)	(0.24)	(0.59)	(0.99)
MAAE ($\times 10^{-4}$)	18.19	18.42	11.81	6.43
# parameters	3	4	5	7
Target spread: τ	0.00	0.00	0.00	0.00
	(0.99)	(0.99)	(0.99)	(0.99)
Linear: θ_1	1.04	0.99	1.59	1.03
	(0.00)	(0.00)	(0.00)	(0.00)
Quadratic: θ_2	-4.08	-6.74	-5.98	-4.19
	(0.00)	(0.00)	(0.00)	(0.00)
Cubic: θ_3		15.57		
		(0.99)		
Upside Constant: θ_0^+				0.01
				(0.00)
Upside Linear: θ_1^+			-1.27	-0.53
			(0.00)	(0.00)
Upside Quadratic: θ_2^+			6.78	6.08
			(0.00)	(0.00)
Weighting: σ				0.01
				(0.00)

The table shows the results of the GMM estimation of moment conditions defined in (28). Econometric methodology and models estimated are the same as in Table 2. Test assets are defined over 48 industry (value-weighted) portfolios based on four-digit SIC code classification. Data are at monthly frequency over the 1963.07−2010.06 period (564 observations). All models satisfy the admissibility conditions defined in Section 3.2 of monotonicity, Second Order Conditions for optimality, non–negative R^2, and Hansen-Jagannathan bounds (except the standard CAPM that doesn't satisfy the Hansen-Jagannathan bounds).

Figure 1: Asymmetric CAPM, A-CAPM(SP,LA,GA)



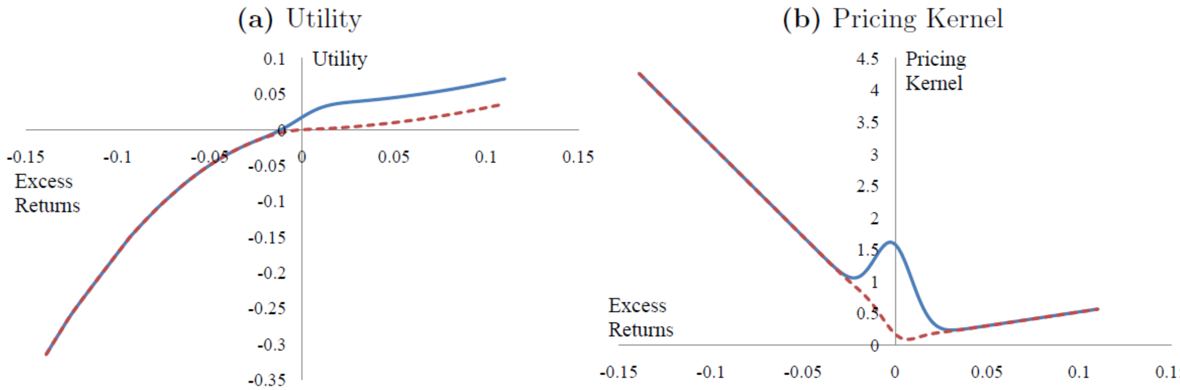
The graph shows the utility function and the pricing kernel of the Asymmetric-CAPM. The utility function (defined on returns of the market portfolio in excess of the risk free rate) is approximated with a continuously asymmetric quadratic polynomial model, which allows for asymmetries in linear and quadratic terms, and a positive shift at zero.

The utility function, defined in Section 2.2 and estimated in Table 2, is (i) concave for losses and slightly convex for gains, (ii) steeper for losses, and (iii) with a positive shift at zero. The corresponding pricing kernel is (i) decreasing for losses and slightly increasing for gains, (ii) with a higher level for losses, and (iii) a bump at zero.

These characteristics imply:

1. Preference for Security/Potential (downside risk aversion and preference for upside potential), implying higher risk premiums for securities delivering *higher* covariance with market returns during market downturns and *lower* covariance with market returns during rising markets;
2. Loss Aversion (losses loom larger than gains), implying higher risk premiums for securities delivering *lower* average returns conditional on market downturns;
3. Goal Achievement (importance of satisfying relevant aspiration levels), implying higher risk premiums for securities delivering *lower* average returns when market excess returns are low.

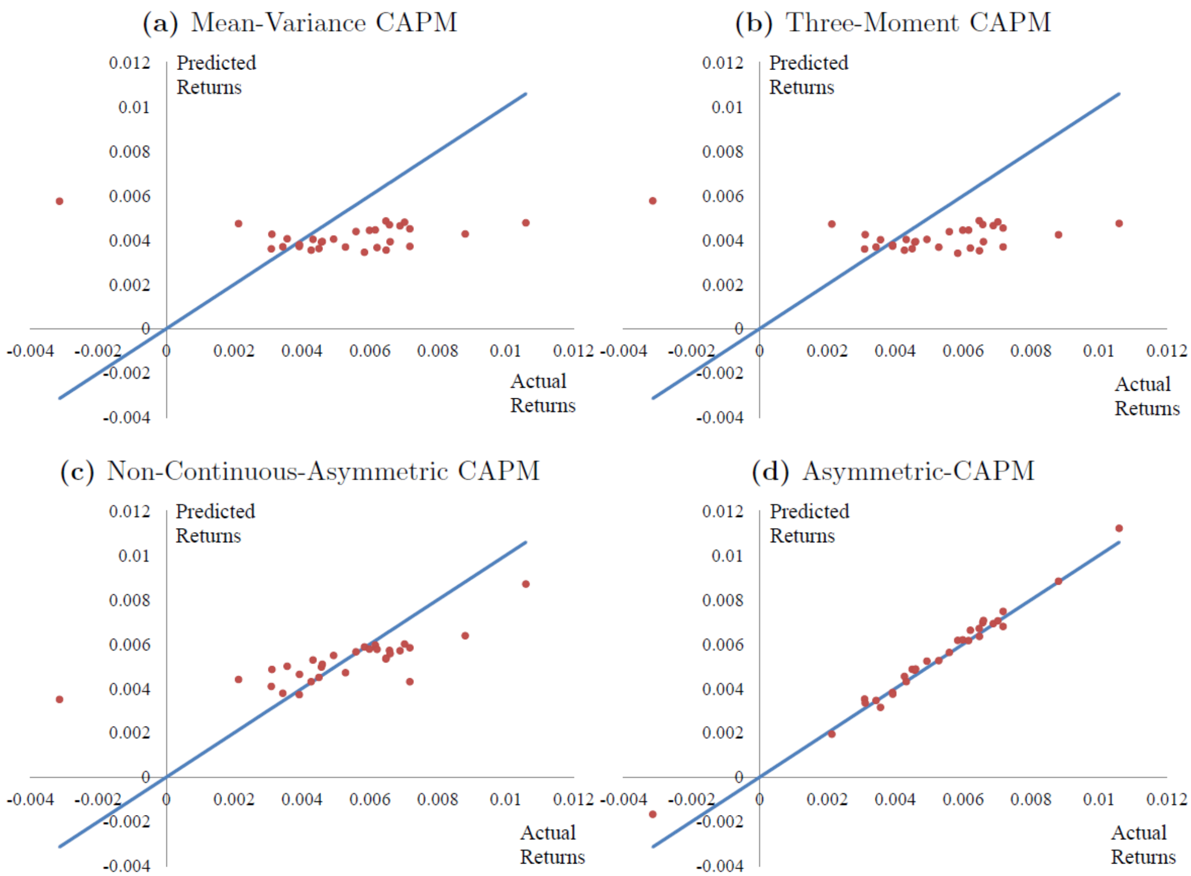
Figure 2: Effect of Goal Achievement on A-CAPM(SP,LA,GA)



The graph highlights the effect of goal achievement on the Asymmetric-CAPM (A-CAPM(SP,LA,GA)). The solid lines show the utility function and the pricing kernel, with parameters estimated in Table 2. The dotted lines show utility function and pricing kernel obtained by removing the upside constant term (necessary to assess goal achievement) from the polynomial approximation in (7).

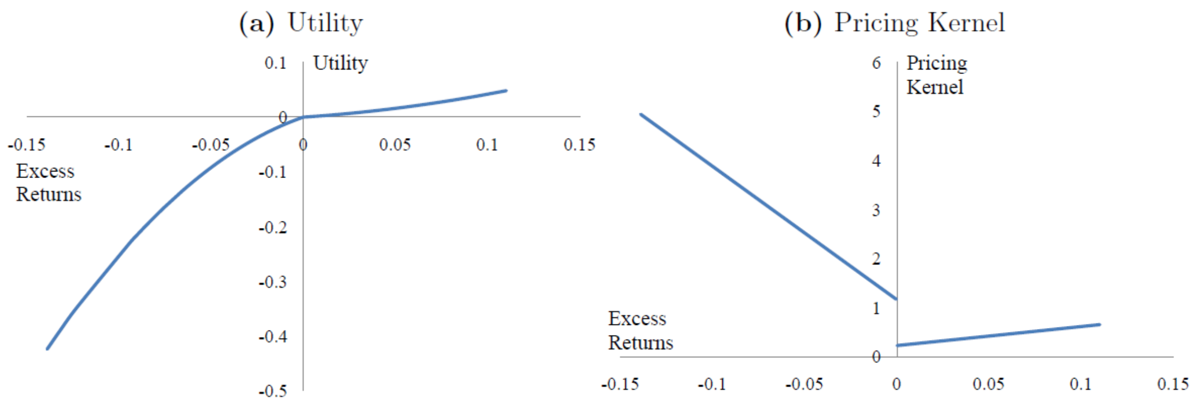
The utility function has a positive shift in the domain of gains (solid line), produced by $\theta_0^+ F_m$. The weighting function F_m, used to distinguish between positive and negative domains, is the c.d.f. of a normal distribution. The pricing kernel has a bump around the target return (solid line), produced by $\theta_0^+ f_m$. The function f_m is the p.d.f. of a normal distribution.

Figure 3: Predicted and Realized Returns



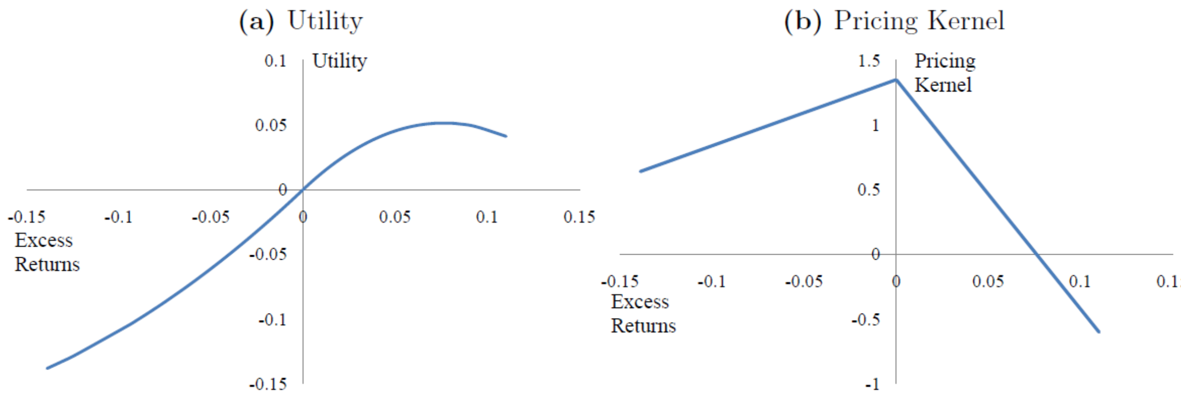
The graph shows the realized excess returns (solid line) versus the predicted excess returns (red dots) of the four models estimated in Table 2. The measure of fit, the Mean Absolute Average Error in equation (33), is equal to 19.79 basis points for the standard mean-variance CAPM, 19.82 basis points for the three-moment CAPM (CAPM(SK), with mean, variance, and skewness), 11.06 basis points for the Non-Continuous-Asymmetric CAPM (NCA-CAPM(SP,LA), with preference for security/potential, and loss aversion), and 2.73 basis points for Asymmetric-CAPM (A-CAPM(SP,LA,GA), with preference for security/potential, loss aversion, and goal achievement).

Figure 4: Non-Continuous-Asymmetric CAPM, NCA-CAPM(SP,LA)



The graph shows the utility function and pricing kernel of the non-continuous asymmetric model NCA-CAPM(SP,LA), estimated in Table 2. The parameter estimates imply a utility function concave in the domain of losses (the pricing kernel is decreasing in the negative domain), convex in the domain of gains (the pricing kernel is increasing in the positive domain), and steeper for losses (the level of the pricing kernel is higher for losses). These characteristics are consistent with the two risky choice factors captured by NCA-CAPM(SP,LA): preference for security/potential, and loss aversion.

Figure 5: Ang, Cheng, and Xing (2006)



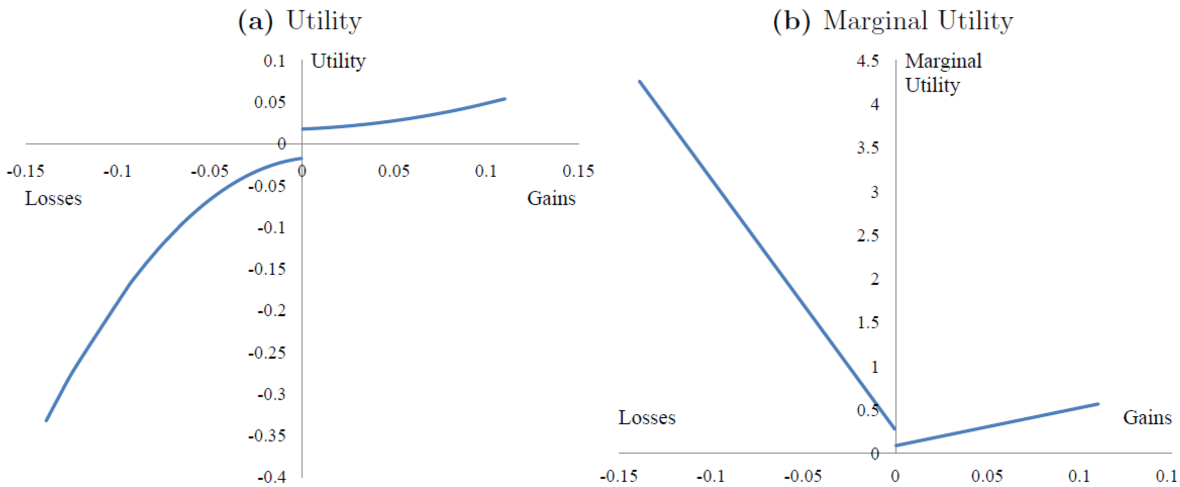
The graph shows the pricing kernel (and implied utility function) estimated by Ang, Chen, and Xing (2006, p.1219), described in Section 2.4. The pricing kernel is equal to $m = a + b_m\, r_m + b_m^-\, r_m\, I_m^-$, with $a = 1.35$, $b_m = 17.73$, and $b_m^- = 22.84$.

The pricing kernel, being increasing in the negative domain and decreasing in the positive domain, implies convexity of the utility function in the domain of losses (preference for downside risk) and concavity in the domain of gains (upside potential aversion), leading to the counterintuitive result of preference for negative asymmetry.

Furthermore, the pricing kernel is negative for returns above a positive threshold (7.61%) and below a negative one (-26.42%), implying violations of monotonicity (decreasing utility function) and the no-arbitrage condition in asset pricing.

Note that Ang, Cheng, and Xing pricing kernel parameters' signs are partially consistent with Prospect Theory. The reasons why Prospect Theory is not tested in the present work are discussed in Appendix A.

Figure 6: Target Utility Theory (Piccioni, 2011)



The graph shows the utility function (and marginal utility) of Target Utility Theory (Piccioni, 2011), described in Appendix A. The three main characteristics of the utility function---concavity in the domain of losses and convexity in the domain of gains, utility steeper for losses, and the jump of the utility function at the reference point---are consistent with preference for security/potential, loss aversion, and goal seeking behavior. Note that the discontinuity at zero of the marginal utility is due to the different inclination of the utility function in the positive and negative domains, and not to the jump in utility.

In this paper each model is tested on moment conditions defined on marginal utility. It follows that the discontinuity of the utility function at zero doesn't allow estimating the jump of the utility function. If, instead, we guarantee the continuity of the utility function at zero, as in model A-CAPM(SP,LA,GA) (Asymmetric-CAPM) in Figure 1, we can assess the importance of goal achievement without compromising the original intuition of the model.

A. Appendix: Decision Theory

Piccioni (2011) develops a model for risky choice behavior—called Target Utility Theory (TUT)—that rationalizes several puzzles of the empirical decision theory literature. According to TUT, the expected utility of lottery X, when it faces lottery Y, is given by:

$$E\big(u(X,Y)\big) = \sum_{x_i \le 0} P(x_i)\big(v_-(x_i) - \varphi_+(Y)\big) + \sum_{x_i \ge 0} P(x_i)\big(v_+(x_i) - \varphi_-(Y)\big) \qquad (37)$$

With

$$\begin{aligned} \varphi_+(Y) &= \textstyle\sum_{y_i \ge 0} P(y_i)\, r_+(y_i) \ge 0 \\ \varphi_-(Y) &= \textstyle\sum_{y_i \le 0} P(y_i)\, r_-(y_i) \le 0 \end{aligned} \qquad (38)$$

In the negative domain, the utility function is given by the sum of a (concave) value function $v_-(x_i)$—capturing the pure utility associated with each outcome—and a (negative) constant equal to $-\varphi_+(Y)$—capturing the expected regret associated with each negative outcome. Regret is the negative feeling associated with the ex-post knowledge that a different past decision would have given a positive payoff, instead of the negative payoff obtained with the chosen prospect. The anticipated regret associated with choosing prospect X, instead of prospect Y, is equal to the expected value of a (convex) function $r_+(y_i)$ computed on the positive payoffs of Y. Because of expected regret, the utility function of each $x_i \le 0$ suffers a negative shift.

Regarding the positive domain, instead, the utility function is given by the sum of a (convex) value function $v_+(x_i)$ and a (positive) constant equal to $-\varphi_-(Y)$, with the latter equal to the expected rejoice associated with each positive outcome. Rejoice is the positive feeling associated with the avoidance of a loss. Because of expected rejoice, the utility function of each $x_i \geq 0$ gets a positive shift, which is equal to the expected value of a (concave) function $r_-(y_i)$ computed on the negative payoffs of the alternative gamble Y.

As shown in Figure 6, TUT utility function is consistent with preference for security and potential because of its concavity in the domain of losses and convexity in the domain of gains. Loss aversion, instead, is captured by the higher slope of the utility function in the negative domain. Finally, the importance of goal achievement is assessed by the jump of the utility function at the reference point.

To compare the performance of TUT with several models of the decision making literature, Piccioni (2011) runs a Maximum Likelihood estimation using a Logit model defined on a wide range of results of the empirical decision theory literature. The estimation results clearly show that TUT represent a significant improvement with respect to: Expected Utility Theory, Original Prospect Theory (Kahneman and Tversky, 1979), Cumulative Prospect Theory (Tversky and Kahneman, 1992), SP/A theory (Lopes, 1987), Regret Theory (Loomes and Sudgen, 1982), Disappointment Aversion (Gul, 1991), and Expected Utility Theory with jumps (Diecidue and Van de Ven, 2008). Furthermore, several comparison tests show that each aspect of TUT (concave utility function for losses, convex utility function for gains, and anticipatory feeling component creating the jump at the reference point) is important to rationalize the phenomena of the empirical literature.

Regarding the connection with the present work, TUT can be approximated with the continuously asymmetrical quadratic model in (7). Note that the continuity of the utility function is necessary to assess the shift of the utility function. In fact, models with discrete jumps cannot be estimated in the present work, since each model is estimated on its first derivative.[25] Furthermore, TUT displays a negative shift of the utility function in the negative domain and a positive shift in the domain of gains. However, in the context of asset pricing models estimation, only the difference in levels can be assessed, and not the precise extent of positive and negative shifts. This doesn't create any problem in the present work, since the only thing that matters to determine preferences is the overall difference in levels in the positive and negative domains, and not the precise extent of each component. In fact, preferences are not affected by positive or negative shifts affecting the utility function as a whole. It follows that θ_0^+ contains enough information for the problem at hand.[26] In the context of decision making models, instead, the way in which (and the reason why) each shift is determined is important.

Finally, a few points have to be made regarding Prospect Theory, the most studied model in the behavioral literature. Prospect Theory could be tested with the asymmetric quadratic utility function defined in equation (20). However, Prospect Theory utility function is convex for losses and concave for gains, requiring different sign restrictions, with respect to (8), for the quadratic terms: $\theta_2 > 0$, $\theta_2^+ < 0$, and $\theta_2 + \theta_2^+ < 0$. As shown in the present work (Section 4.1), a quadratic model with such sign restrictions is rejected. Furthermore, Prospect Theory sign restrictions imply preference for negative asymmetry, inconsistent with the intuition (and the empirical results) that investors are downside risk averse and prefer upside potential. Finally, Prospect Theory sign restrictions would lead to a pricing kernel with a reverse V-shaped, increasing in the negative domain and decreasing in the positive

[25]Piccioni (2011), instead, estimates the utility function with Maximum Likelihood, using a Logit model defined over differences in expected utility.

[26]Note that we can only approximate the avoided regret/rejoice by assessing the magnitude of the shift of the utility function at the target return (θ_0^+). In fact, the regret/rejoice component of the utility function cannot be measured precisely, since the reference portfolio (with respect to which any alternative portfolio is compared) is not known.

domain, which would likely violate monotonicity (and the no-arbitrage condition in asset pricing), implying a negative pricing kernel (and decreasing utility function) for returns below negative and above positive thresholds.

However, it's important to note that a fundamental component of Kahneman and Tversky's model is the probability weighting function, according to which individuals overestimate low probabilities and underestimate higher ones. Some authors argue that the joint effect of Prospect Theory value and probability weighting functions is such that individuals are downside risk averse and upside potential seeking (see, for example, Post and Levy, 2005). However, because of the distortion in expectations produced by the probability weighting function, the full version of Prospect Theory cannot be tested within the rational expectation framework.

B. Appendix: The Optimization Process

B.1 Efficiency of the Market Portfolio

The purpose of this work is to find a utility function that rationalizes the efficiency of the market portfolio. In order to do so, I study a wide range of utility functions, allowing for combinations of risk aversion and risk seeking. In fact, the only conditions imposed on the utility function are differentiability and monotonicity. The use of non-globally concave utility functions raises doubts about the efficiency of the market portfolio. However, Khanna and Kurdorff (1999) found that the mutual fund theorem (guaranteeing the efficiency of the market portfolio) "holds for all investors, regardless of their attitude toward risk, as long as they do not prefer less to more [i.e. monotonicity]. This includes investors who are risk seekers as well as those with a combination of risk seeking and risk averse preferences".

Nevertheless, as pointed out by Post and Levy (2005), given the large number of investors who appear to hold the market portfolio in the form of passive mutual funds and exchange traded funds that track broad value-weighted equity indexes, it is interesting to ask what kind of utility functions could rationalize such behavior, in the face of attractive premiums offered by size, book-to-market and momentum portfolios—the so called *revealed preferences* approach.

B.2 Stochastic Discount Factor definition

Even though the pricing kernel defined in equation (3) is the first derivative of a utility function defined over portfolio returns in excess of a target, the Stochastic Discount Factor

(SDF) studied in this paper is equivalent to the SDF of utility functions defined over consumption, which is equal to the marginal rate of substitution: $U'(C_{t+1})/U'(C_t)$. Brown and Gibbons (1985) show the conditions under which consumption and wealth are equivalent, and the marginal rate of substitution can be expressed as a function of aggregate wealth: $U'(W_{t+1})/U'(W_t)$. Several works, like Dittmar (2002) and Harvey and Siddique (2000), run Taylor approximations of $U'(W_{t+1})$ around W_t, deriving a pricing kernel that is a polynomial function of the market portfolio return. The approximation of the SDF in these works is equivalent to the expression of the pricing kernel defined in Section 2.1.

B.3 Conditional and Unconditional Expectations

The model defined in (1)−(3) is based on unconditional expectations. However, there exists a wealth of evidence that the risk/return characteristics of securities show structural and cyclical variation, justifying the use of conditional models. The problem with conditional models is that they entail a large risk of specification error, as they have to specify how each aspect of investor preferences depends on the state of the world. As Ghysels (1998) points out, “if the beta risk [in capital asset pricing models] is inherently misspecified, there is a real possibility that we commit serious pricing errors, potentially larger than with a constant traditional beta model”. Ghysels finds that pricing errors of the unconditional CAPM are smaller than those of the conditional CAPM. In addition, Post and Vliet (2006) point out that the problem of imposing the regularity conditions is very severe, as we have to make sure that the utility function is well-behaved for all possible states of the world. The development of conditional models with time varying preferences is beyond the scope of the present work, and it is left for future research.

C. Appendix: CAPM Derivation

The derivation of the Capital Asset Pricing Model from the First Order Conditions in (3) is quite straightforward. First, we can derive the expected value of the excess returns of the risky assets using the well known fact that the expected value of the product of two random variables is equal to the product of their expected values plus their covariance:

$$E(m)\, E(r) + cov(m\,, r) = 0 \tag{39}$$

rearranging:

$$E(r) = \frac{-cov(m\,, r)}{E(m)} \tag{40}$$

Equation (40) holds for all securities, including the market. Then, for each asset we can derive:

$$E(r_i) = \beta_i\, E(r_m) \tag{41}$$

with:

$$\beta_i = \frac{cov(m\,, r_i)}{cov(m\,, r_m)} \tag{42}$$

and r_m representing the excess return of the value-weighted portfolio of risky assets. The beta relative to each security is equal to the covariance between the pricing kernel and the excess returns of that particular security, divided by the covariance between the pricing kernel and the market portfolio.

D. Appendix: Risk-Neutral Probabilities and the Risk-Free Rate Moment Condition

Besides pricing all securities through the Euler equation (3), the pricing kernel also serves as change of measure, transforming the physical (also known as true or objective) probability density function into the risk-neutral measure. In fact, the no-arbitrage condition in asset pricing implies that the risk-neutral and physical probability measures are related by the equation:

$$m(s)\,p(s) = \frac{\pi(s)}{R_F} \tag{43}$$

With $m(s)$ Stochastic Discount Factor in state s, $p(s)$ physical probability and $\pi(s)$ risk-neutral probability. From (43) we can get:

$$E(m_{t+1}) = \frac{1}{R_F} \tag{44}$$

Then,

$$E(m_{t+1}\,R_F) = E(m_{t+1})\,R_F = 1 \tag{45}$$

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