Anisotropic microstructure–induced reduction of the Rayleigh instability for liquid crystalline polymers

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Abstract: We analyze a macroscopic 3-D model for flows of liquid crystalline polymers (LCPs), deduced from Doi-type kinetic equations. The Doi model accounts for rigid-rod microstructure, which introduces elastic relaxation and polymer-induced viscosity in addition to a Newtonian solvent viscosity, thus capturing all effects contained in standard isotropic viscoelastic models for Maxwell and Oldroyd B fluids. The rod-like microstructure further introduces *anisotropic effects* in the form of drag on the rods, together with a short-range, Maier-Saupe intermolecular potential, whose critical points vary with LCP concentration and yield stable isotropic (at low density) and nematic (at high density) equilibrium phases. From this single model, we compare various physical mechanisms for reducing the capillary instability of inviscid cylindrical jets: solvent viscosity as studied by Rayleigh and Chandrasekhar; isotropic viscoelasticity, both with and without Newtonian solvent viscosity; anisotropic polymer friction; and finally, the nematic, highly aligned prolate phase at high LCP density. Realistic parameter values for LCPs correspond to a regime in which the LCP capillary number (polymer bulk free energy relative to surface tension) is above an identified critical value; in such regimes, the unstable growth rates of the isotropic and nematic phases are lowered arbitrarily close to zero if the molecular drag is sufficiently anisotropic even in the absence of solvent viscosity. In low capillary number regimes, where surface tension dominates LCP bulk free energy, the LCP growth rates are sandwiched below the inviscid Rayleigh curve and above an explicit positive lower bound.

1. Introduction

The capillary instability of cylindrical columns of inviscid fluids was characterized by Lord Rayleigh [?]: all perturbations along the jet axis of symmetry of wavelength exceeding the jet circumference are unstable. Rayleigh derived an explicit formula for the *inviscid linearized growth rate*, $\nu_{inv}(K)$, (given in nondimensional units, normalized with respect to a capillary timescale t_0 made explicit below), as a function of dimensionless axial wavenumber

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$$K = \phi_0 k, k = 2\pi/\text{wavelength},$$

$$\nu_{inv}^2(K) = \frac{(1 - K^2)K I_1(K)}{W I_0(K)}, \quad (1)$$

where $W = (\rho\phi_0^3)/(\sigma_s t_0^2)$ is the Weber number measuring inertia relative to surface tension, ϕ_0 is the jet radius, σ_s is the surface tension of the fluid/air interface, ρ is the fluid density, and I_1, I_0 are the modified Bessel functions of the first kind of orders 1, 0, respectively. The dimensional instability band is $0 < k < (\phi_0)^{-1}$; the wavenumber cutoff, $(\phi_0)^{-1}$, is purely geometrical, independent of surface tension. Rayleigh and later Chandrasekhar [?] coupled viscous stress and derived a transcendental equation for the *viscous growth rate*, $\nu_{visc}(k)$. They showed the unstable band of wavenumbers is also independent of fluid viscosity, and that viscosity lowers the growthrate of all unstable wavenumbers, $0 < \nu_{visc} < \nu_{inv}$, for all $0 < K < 1$ (as illustrated in Figure 1).

Here we mimic the analysis of Rayleigh and Chandrasekhar by *coupling orientational and viscous stresses* to the inviscid equations. When the LCP is in an isotropic phase, we can carry out the full linearized analysis including viscous stress, while the nematic phase presents difficulties which limit our analysis to an LCP with zero solvent viscosity but nonzero polymer-induced viscosity. In many LCPs, the solvent viscosity is indeed quite low relative to the polymer viscosity, so this limitation is likely to yield a reasonable approximation comparable to using inviscid equations for low-viscosity fluids. In the figures reported below, all material properties are based on literature values for LCPs. *The goal of this paper is to distinguish the order of reduction in the inviscid capillary instability of an axisymmetric liquid column due to: solvent viscosity, polymer viscosity, anisotropy in the polymer drag, and the isotropic versus nematic (ordered) LCP phase.* Each effect influences capillary instability. However, the combination of sufficiently high LCP capillary number and sufficiently high anisotropic polymeric drag lead to a dramatic reduction, both for low density isotropic phases and high density nematic phases.

We adopt a macroscopic Doi-type constitutive law and orientation dynamics derived for nematic liquid crystalline polymers by Bhawe et al. [?]; the microstructure consists of rod-like molecules. The constitutive stress law is (with τ the *total stress tensor*, p the *scalar pressure*, $\mathbf{D}$ the *rate of strain tensor*, η the *solvent viscosity*, and $\mathbf{I}$ the identity tensor):

$$\tau = -p\mathbf{I} + 2\eta\mathbf{D} + \tau_{or}, \quad (2)$$

which generalizes the inviscid relation to include a viscous and an *orientational stress* τ_{or} ,

$$\tau_{or} = 3cBT[(1 - \frac{N}{3})\mathbf{Q} - N(\mathbf{Q} \cdot \mathbf{Q}) + N(\mathbf{Q} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3) + 2\lambda(\nabla \mathbf{v} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3)], \quad (3)$$

where c is the *LCP number density* of rod-like molecules per unit volume, B is the Boltzmann constant, T is absolute temperature, N is the *dimensionless LCP concentration* which governs the shape and critical points of the bulk free energy, λ is the *LCP relaxation time* and $\mathbf{Q}$ is the *nematic orientation tensor* defined by

$$\mathbf{Q} = \langle \mathbf{m} \otimes \mathbf{m} \rangle - \mathbf{I}/3, \quad (4)$$

where $\mathbf{m}$ is the LCP molecular direction and the average $\langle \bullet \rangle$ is with respect to a molecular probability distribution function. For *uniaxial nematic* LCPs, $\mathbf{Q}$ has two equal eigenvalues and admits a representation

$$\mathbf{Q} = s(\mathbf{n} \otimes \mathbf{n} - \mathbf{I}/3), \quad (5)$$

where $\mathbf{n}$ is the distinguished *optical axis* or *uniaxial LCP director*, and s is the *uniaxial order parameter* which relates nonlinear average information about the LCP molecular direction $\mathbf{m}$ relative to the optical axis $\mathbf{n}$,

$$s = \frac{3}{2}(\langle \mathbf{n} \cdot \mathbf{m}^2 \rangle - 1), \mathbf{n} \cdot \mathbf{m}^2 = \cos^2 \theta, \quad (6)$$

where θ is the acute angle between the director and molecular direction. Corresponding to $0 < s \leq 1$, $-\frac{1}{2} \leq s < 0$ and $s = 0$, uniaxial nematic LCPs are said to be in the prolate, oblate and isotropic phase, respectively.

The governing dynamical 3-D equations are (where $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$):

Incompressibility:

$$\nabla \cdot \mathbf{v} = 0. \quad (7)$$

Conservation of linear momentum:

$$\rho \frac{d}{dt} \mathbf{v} = \text{div} \tau. \quad (8)$$

Orientation tensor equation:

$$\frac{D}{Dt} \mathbf{Q} = -\frac{\sigma_d}{\lambda} \left[\left(1 - \frac{N}{3}\right) \mathbf{Q} - N(\mathbf{Q} \cdot \mathbf{Q}) + N(\mathbf{Q} : \mathbf{Q})(\mathbf{Q} + \frac{\mathbf{I}}{3}) \right] + \frac{2}{3} \mathbf{D} - 2(\nabla \mathbf{v} : \mathbf{Q})(\mathbf{Q} + \mathbf{I}/3), \quad (9)$$

where

$$\frac{D}{Dt}(\bullet) = \frac{d}{dt}(\bullet) - (\nabla \mathbf{v})(\bullet) + (\bullet) \nabla \mathbf{v}^T \quad (10)$$

is the upper-convected derivative, σ_d is the dimensionless *anisotropic drag parameter*, $0 \leq \sigma_d \leq 1$, representing the ratio of drag on the rod molecules orthogonal versus parallel to the rod axis; $\sigma_d = 1$ corresponds to isotropic polymer drag, whereas the limit $\sigma_d = 0$ is a hyper-anisotropic drag limit which essentially prohibits motion transverse to the rod axis. (This unphysical limit turns out to be very useful for providing lower bounds on the microstructure-induced capillary growth rates, as we shall exhibit below.)

We use cylindrical polar coordinates (r, θ, z) , with standard basis $\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_z$, where $\mathbf{e}_z$ is the jet axis of symmetry. The free surface is axisymmetric, given by

$$r = \phi(z, t), \quad (11)$$

and is convected with the fluid velocity:

$$\frac{d}{dt}(r - \phi) = 0. \quad (12)$$

There is a normal stress jump at the fluid/air interface proportional to *surface tension* (σ_s) times *mean curvature* (κ):

$$[\tau - p_a \mathbf{I}] \mathbf{n}_{f.s.} = -\sigma_s \kappa \mathbf{n}_{f.s.}, \quad (13)$$

where $\mathbf{n}_{f.s.}$ is the unit outward normal to the free surface, the mean curvature is

$$\kappa = \phi^{-1} [1 + \phi_z^2]^{-1/2} - \phi_{zz} [1 + \phi_z^2]^{-3/2}, \quad (14)$$

and $p_a \mathbf{I}$ corresponds to a passive ambient with constant pressure p_a .

2. Equilibrium Solutions

For a cylindrical LCP jet, flow/microstructure equilibria are [?, ?]:

$$\mathbf{v} = v_0 \mathbf{e}_z, \mathbf{n} = \mathbf{e}_z, p = p_0, \phi = \phi_0, s = s_*, \quad (15)$$

where $p_0 = p_a + \frac{\sigma_s}{\phi_0}$, s_* are the real critical points of the uniaxial bulk free energy $\int U(s) ds$, with

$$U(s) = s[1 - \frac{N}{3}(1-s)(2s+1)], \quad (16)$$

$$s_0 = 0, s_{\pm} = \frac{1}{4}[1 \pm 3\sqrt{1 - 8/(3N)}].$$

Here we focus on the isotropic phase, $s_0 = 0$, at low LCP concentration ($N < 3$), and the highly aligned “prolate” phase, s_+ , which exists only for sufficiently high polymer concentration, $N \geq 8/3$. As shown in [?], these are the only stable LCP equilibrium phases. For $8/3 < N < 3$, the isotropic and nematic phases are bi-stable.

3. Linearized Dispersion Relations

Paralleling the classical Fourier analysis for flows with zero angular velocity [?, ?], we seek solutions of the *linearized* 3-D dimensionless governing equations and boundary conditions in the form: (with $\mathbf{u} = (v_r, v_z, p, \mathbf{Q})$, for which the equilibrium cylindrical jet is $\mathbf{u}_0 = (0, v_0, p_0, s_*(\mathbf{e}_z \otimes \mathbf{e}_z - \mathbf{I}/3))$ and $\phi = \phi_0$)

$$\begin{aligned} \mathbf{u}(\tilde{r}, \tilde{z}, \tilde{t}) &= \mathbf{u}_0 + \mathbf{u}_1(\tilde{r}) e^{iK(\tilde{z} - \tilde{v}_0 \tilde{t}) + \omega(K)\tilde{t}}, \\ \phi(\tilde{z}, \tilde{t}) &= \phi_0 + \phi_1 e^{iK(\tilde{z} - \tilde{v}_0 \tilde{t}) + \omega(K)\tilde{t}}, \end{aligned} \quad (17)$$

where the dimensionless variables $\tilde{r}, \tilde{z}, \tilde{t}$ are scaled with respect to $\phi_0, \phi_0, t_0 = \sqrt{\rho \phi_0^3 / \sigma_s}$, respectively, $\tilde{v}_0 = \frac{v_0 t_0}{\phi_0}$, $\Omega(K) = \omega t_0$ and $\omega - ikv_0$ is the wave speed. We have chosen a *capillary timescale* t_0 so that $W = 1$ throughout the paper and all growth rates can be compared to the Rayleigh inviscid formula. The stability information is captured by the *dimensionless linearized growth rate*, denoted for an LCP jet by $\nu_{lcp}(K)$,

$$Re(\omega(K)) = \nu_{lcp}(K), \quad (18)$$

which indicates *linearized instability* for all $K = \phi_0 k$ such that $\nu_{lcp} > 0$. Of interest is the *unstable wavenumber band*, $\{K | \nu_{lcp}(K) > 0\}$, and the relative order of the growth rate $\nu_{lcp}(K)$ as we compare the inviscid, viscous, isotropic viscoelastic, and anisotropic LCP jets.

Note that the LCP growth rate depends on: (i) the *microstructure phase*, either isotropic ($s_0 = 0$) or prolate ($s_+ \geq \frac{1}{4}$); (ii) the *LCP concentration* ($0 < N < 3$ for the isotropic phase, and $N \geq \frac{8}{3}$ for the prolate nematic phase); (iii) the *Newtonian capillary number*, $Ca = \frac{\eta\phi_0}{\sigma_s t_0}$, and the *LCP capillary number*, $Ca_{lcp} = \frac{3cBT\phi_0}{\sigma_s}$; (iv) the *anisotropic drag parameter*, $0 \leq \sigma_d \leq 1$; and (v) the *Deborah number*, $\Lambda = \frac{\lambda}{t_0}$, which is the ratio of elastic relaxation to capillary timescales. From the Table, we use literature estimates for a LCP, which yields typical values of $Ca = 2.2$, $Ca_{lcp} = 25$, and $\Lambda = 2$. The polymer concentration parameter, N , can be varied either through changing the density of rod-like molecules or by changing temperature. We choose the value $N = 2.9$ first, for which the LCP is “bi-phasic”, that is, the isotropic and prolate equilibrium phases are both stable at this molecular concentration [?]; later we use $N = 4$ for which the prolate phase is the unique stable phase [?]. We now remark on the Doi model predictions as indicated in the Figures.

Inviscid, viscous, elastic, and viscoelastic isotropic fluids with isotropic polymeric drag (Figure 1).

Figure 1 contains in order from top to bottom: the classical Rayleigh inviscid growth rate ν_{inv} ([?]); the classical Rayleigh/Chandrasekhar viscous growth rate, ν_{visc} ([?]); the isotropic ($s_0 = 0$) LCP phase with inviscid solvent ($Ca = 0$); and the isotropic LCP phase with viscous solvent ($Ca = 2.2$). The two lowest curves isolate the viscoelastic (nonzero Deborah and LCP capillary numbers) effects on the capillary instability, with all anisotropic LCP effects suppressed. Referring to Figure 1:

- The two top curves in Figure 1 isolate the *difference due to Newtonian viscosity*, with capillary numbers $Ca = 0, 2.2$, respectively, reviewing classical results of Rayleigh and Chandrasekhar. Viscosity acts to lower the strength of the instability, and to shift the dominant mode of instability toward longer wavelengths (smaller K).
- The top and third highest curves isolate the *effect of isotropic polymer viscosity and elastic relaxation on an approximately inviscid solvent fluid*, which has the same qualitative effect as solvent viscosity, though more significant for these parameter values.
- The second and fourth highest curves isolate the *effect of polymeric viscosity and elastic relaxation when solvent viscosity is present*. There is a clear enhancement of the lowering of the growth rate, and of the shift to longer dominant wavelength of instability.
- The two lowest curves isolate the *effect of solvent viscosity on an isotropic LCP jet*. The added effect of a relatively low solvent viscosity on an isotropic LCP jet is not nearly as significant as the same order viscous contribution to an inviscid jet (the difference in the top two curves versus the bottom two).

These comparisons underscore the significant regularizing effect of isotropic viscoelasticity (polymer viscosity and relaxation), effecting a reduction in the maximum growth rate on

the order of 99.8% if the solvent is presumed inviscid and even higher percentages if solvent viscosity is accounted for.

The effect of anisotropic polymer friction on an isotropic LCP phase, in the supercritical (Figure 2) and subcritical (Figure 3) LCP capillary number regimes.

Beginning from the isotropic LCP phase of Figure 1b, we explore the effect of anisotropic polymer drag. The second lowest curve there, corresponding to an inviscid solvent ($Ca = 0$), is repeated in Figure 2 (where it is the top curve). Note the change in vertical scale. We then make the drag anisotropic by lowering σ_d to values of 0.5 (the middle curve) and 0.1 (the lowest curve).

For every value of LCP concentration $0 < N < 3$, there is a *critical LCP capillary number associated with the isotropic phase*,

$$Ca_{lcp}^{crit} = \frac{3}{2}[3 - N]^{-1}; \quad (19)$$

for all *supercritical LCP capillary numbers*, $Ca_{lcp} > Ca_{lcp}^{crit}$, the capillary growth rate curve vanishes in the limit as σ_d goes to zero [?]. Since the polymer concentration parameter $N = 2.9$ in Figures 1 and 2, we find $Ca_{lcp}^{crit} = 15$, and $Ca_{lcp} = 25$ is clearly supercritical.

In Figure 3, we retain all LCP parameter values of Figure 2 *except we lower the LCP capillary number to subcritical*, $Ca_{lcp} = 10$. In Figure 3a, we vary the anisotropic drag parameter for an inviscid solvent ($Ca = 0$), while in Figure 3b we use the literature estimate for an LCP solvent viscosity $Ca = 2.2$ consistent with Figure 1. Both Figures 3a, 3b illustrate the result [?]: for all *subcritical LCP capillary numbers*, $Ca_{lcp} < Ca_{lcp}^{crit}$, there is an explicit nonzero lower bound, given by the $\sigma_d = 0$ value of $\nu_{lcp}(K)$, and the LCP capillary growth rate of the isotropic phase $s_0 = 0$ for variable drag parameter $0 < \sigma_d \leq 1$ lies between the classical inviscid (Figure 3a) or viscous (Figure 3b) curve and this explicit lower bound. The lower bound curve, however, has a reduced wavenumber cutoff, $K_{cutoff} < 1$, defined by the nonnegative zeros of the equation

$$2Ca_{lcp}(1 - N/3) - 3(K^2 - 1)[K^2 + 1 - \frac{KI_0(K)}{I_1(K)}]^{-1} = 0. \quad (20)$$

The diminishing slopes of the growth rate curves at $K = 1$ as drag anisotropy is enhanced is indicative of the preferential suppression of the instability in the shortest wavelengths; compare this behavior to the longwave regime $K = 0$, where the curves are all tangent.

There are two results from [?] and the Appendix particularly relevant to Figures 1-3.

- The linearized constitutive equation (??, ??) and nematodynamic equation (??) for the isotropic phase is given by:

$$\begin{cases} \tau = -p\mathbf{I} + 2\eta\mathbf{D} + \hat{\tau}, \\ \lambda_{eff}[\frac{d}{dt}\hat{\tau} - (\nabla\mathbf{v} \cdot \hat{\tau} + \hat{\tau} \cdot \nabla\mathbf{v}^T)] + \hat{\tau} = 2\eta_{eff}\mathbf{D}, \end{cases} \quad (21)$$

where $\hat{\tau} = 3cBT(1 - N/3)\mathbf{Q}$ is the extra stress corresponding to the effective polymeric stress, $\eta_{eff} = nBT\lambda/\sigma$ is an *effective polymer viscosity* and $\lambda_{eff} = \frac{\lambda}{\sigma(1 - \frac{1}{3}N)}$ is an *effective polymer relaxation time*.

This identification shows the Doi model at low concentrations ($N < 3$) and in the isotropic phase is linearly equivalent to the upper convected Oldroyd B (Maxwell if $\eta = 0$) model for viscoelastic fluids. This clearly illustrates how the Doi model limits to standard viscoelastic models in low-density isotropic phases and reveals that Figure 1 corresponds to viscoelastic effects on capillary instability.

- The linearized stability calculation for the axisymmetric LCP jet in the isotropic phase can be finessed as follows. Remarkably, upon introducing normal (Fourier) modes, the linearized equations are identical in form to the linearized Navier-Stokes free surface equations, with the usual Newtonian viscosity replaced with an effective microstructure viscosity. We can therefore invoke results from Chandrasekhar [?], as detailed in [?] and the Appendix.

The role of the isotropic-to-nematic phase transition on capillary instability (Figure 4).

Since the isotropic and prolate nematic phases are bi-stable at density $N = 2.9$, we can retain all material parameters from the top growth rate curve of Figure 2, and change only the equilibrium phase from $s_* = 0$ to s_+ . This isolates the stabilizing effect of the ordered nematic phase: the maximum growth rate drops on the order of 40%, and the wavenumber of maximum instability shifts further downward. These calculations assume an inviscid solvent ($Ca = 0$) and isotropic drag ($\sigma_d = 1$). Since the isotropic-phase flow equations are identified above with an isotropic viscoelastic fluid, this figure indicates how the isotropic-nematic phase transition further *suppresses capillary instability*.

Scaling behavior of the maximum growth rate and corresponding most unstable wavenumber for the prolate nematic phase (Figure 5).

Figures 2 and 3 illustrate the effect of anisotropic drag in supercritical and subcritical LCP capillary number regimes for the isotropic, low density phase $s_0 = 0$. In Figure 5 we focus on this scaling behavior for the high density ($N > \frac{8}{3}$) prolate nematic phase; we fix $N = 4$ so that the nematic phase, with $s_+ = \frac{1+\sqrt{3}}{4}$, is the unique stable phase. Instead of discrete growth rate curves, we focus here on the distinguished “peak” of each growth rate curve, which carries the maximum unstable rate and the most unstable wavelength. For this concentration and parameter values, the critical capillary number for the prolate phase is

$$Ca_{lcp}^{crit} = [2(1 - s_+)(2s_+ + 1)U'(s_+)]^{-1} \approx 0.43. \quad (22)$$

For *subcritical LCP capillary numbers* below this value (the top curves in Figure 5a and 5b are given the value $Ca_{lcp} = 0.4$), *the growth rate curve is bounded between two non-zero rate curves*, analogous to Figure 3. The maximum growth rate and corresponding wavenumber of maximum instability decrease as the anisotropic drag parameter approaches zero, but approach non-zero values of K_{max} and ν_{lcp}^{max} . The lower curves correspond to *supercritical LCP capillary number*, $Ca_{lcp} = 25$. In this supercritical regime, both the wavenumber K_{max} of maximum instability and the maximum growth rate ν_{lcp}^{max} approach zero as σ_d approaches zero! This implies that in supercritical LCP capillary regimes, a highly anisotropic polymeric

drag leads to significant weakening of the capillary instability, and the dominant wavelength of instability is extremely long.

The anisotropic effect on capillary instability of the prolate phase s_+ in a subcritical LCP capillary regime (Figure 6).

To illustrate the linearized stability behavior of the prolate phase for subcritical LCP capillary numbers, we fix $N = 4$, $\Lambda = 2$, and $Ca = 0$ as in Figure 5. We pick a subcritical $Ca_{lcp} = 0.4$, and then show the growth rate curves for four values of the anisotropic drag parameter, $\sigma_d = 1, 0.1, 0.01, 0$. The curves in order from top to bottom correspond to $\sigma_d = 1, 0.1, 0.001, 0$, respectively. The lower curve is a lower bound for all LCP capillary numbers with all other parameters held constant. As with the subcritical isotropic phase, Figure 3, the lower bound has a reduced cutoff wavenumber of instability,

$$K_{cutoff} = \sqrt{1 - 2(1 - s_+)(2s_+ + 1)U'(s_+)/Ca_{lcp}} < 1, \quad (23)$$

and all anisotropic drag parameter values lead to a characteristic drop in the slope of the growth rate at $K = 1$.

4. Conclusion

Analysis of the macroscopic free surface equations deduced from Doi-type kinetic theories for LCP flows indicates significant microstructural effects on capillary instability. We have illustrated the analogy between viscous and viscoelastic regularization of capillary instability, using the Doi theory for low density, isotropic phases and with isotropic polymeric drag. However, the Doi model allows one to further isolate the role of the ordered nematic phase, and to isolate the role of anisotropic drag, on capillary instability. Both effects, like viscosity and viscoelasticity, weaken the Rayleigh instability and shift the dominant mode of instability to longer wavelengths. Perhaps the most salient result of this study occurs when LCP-induced surface effects dominate interfacial tension, in what we call a supercritical LCP capillary number regime: in the limit of highly anisotropic drag, the Rayleigh instability can be virtually eliminated. When surface tension dominates LCP surface energy, the subcritical LCP capillary number regime, precise upper and lower bounds are given for the lowering of the LCP capillary growth rates.

Experimentally, these results quantify the suggestion that capillary breakup of low-density isotropic, and high-density nematic, LCP filaments always results in larger drops which grow on longer timescales than a corresponding Newtonian fluid with the same surface tension, geometry, viscosity, and flow conditions.

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APPENDIX

A. Dispersion equation for LCPs

The *dispersion equation for isotropic LCPs* is

$$2K^2(2K^2 + Re_m\Omega)\frac{I_1'(K)}{I_0(K)}\left[1 - \frac{2KY}{K^2+Y^2}\frac{I_1(K)}{I_1(Y)}\frac{I_1'(Y)}{I_1'(K)}\right] + Re_m\Omega(2K^2 + Re_m\Omega) = Re_m^2K(1 - K^2)\frac{I_1(K)}{I_0(K)}. \quad (24)$$

where

$$\frac{1}{Re_m} = Ca + \frac{Ca_{lcp}(1-\frac{N}{3})}{3(\Omega + \frac{\sigma}{\Lambda}(1-\frac{N}{3}))}, Y^2 = K^2 + Re_m \Omega. \quad (25)$$

This dispersion equation reduces to the Newtonian formula when $N = 3$ and further to the inviscid Rayleigh formula when $Ca = 0$.

The *dispersion equation for anisotropic LCPs* is

$$\Omega(G_1 I_0(\nu K) - G_2 G_0^{(2)} \frac{I_1(\nu K)}{\nu K}) = G_0^{(1)} G_0^{(2)} K^2 (1 - K^2) \frac{I_1(\nu K)}{\nu K}, \quad (26)$$

where

$$\begin{aligned} \nu^2 &= \frac{\Omega}{G_1} G_0^{(1)} G_0^{(2)}, G_0^{(1)} = \Omega + \frac{\sigma N}{\Lambda} s_0, G_0^{(2)} = \Omega + \frac{\sigma U'(s_0)}{\Lambda}, \\ G_1 &= \Omega G_0^{(1)} G_0^{(2)} + Ca_{lcp} K^2 [2\Lambda s_0^2 G_0^{(1)} G_0^{(2)} - (U'(s_0)((2s_0^2 - \frac{2}{3}s_0 - \frac{4}{3})G_0^{(1)} + \frac{4\sigma N}{9\Lambda}(1-s_0)^2 s_0) - \frac{4N}{9}(1-s_0)^2 s_0 G_0^{(2)})], \\ G_2 &= \frac{2}{3} Ca_{lcp} K^2 N s_0 (1 - s_0). \end{aligned} \quad (27)$$

B. Physical parameters for the LCP and dimensionless numbers

Liquid density: ρ	1300 kg/m^3
$3cBT$	1000 Pa
Surface tension: σ_s	0.04 N/m
Relaxation time: λ	0.01 $\sim$ 0.1 s
Solvent viscosity: η	0.5 $Pa \cdot s$
Radial length scale: ϕ_0	1.0×10^{-3} m
Time scale t_0 :	0.57×10^{-1} s
<hr/>	
$Ca_{lcp} = \frac{3cBT\phi_0}{\sigma_s}$	0.25×10^2
$Ca = \frac{\eta\phi_0}{\sigma_s t_0}$	0.22×10^1
$\Lambda = \frac{\lambda}{t_0}$	$0.18 \times 10 \sim 0.18 \times 10^2$

Figure 1: Dimensionless *growth rate curves for isotropic fluid jets*, normalized with respect to the capillary timescale $t_0^2 = \rho\phi_0^3/\sigma_s$, so that the Weber number $W = 1$. In order from the strongest instability (top curve) to the weakest (bottom curve): The classical Rayleigh *inviscid fluid growth rate*: ν_{inv}/t_0 uniquely specified by $W = 1$; ν_{visc} , the classical *viscous fluid growth rate*, for $Ca = 2.2$; ν_{lcp} for the *isotropic LCP equilibrium phase with inviscid solvent*, with polymer concentration parameter $N = 2.9$, Deborah number $\Lambda = 2$, solvent capillary number $Ca = 0$, LCP capillary number $Ca_{lcp} = 25$, and $\sigma_d = 1$ (isotropic polymeric drag); and finally, ν_{lcp} for the *isotropic LCP phase with viscous solvent* ($Ca = 2.2$), retaining all other LCP parameters as in the third curve. The dimensionless unstable wave number band is $0 < K < 1$, normalized with respect to ϕ_0 .

Figure 2: *The effect of anisotropic drag on capillary instability of the LCP isotropic phase, $s_0 = 0$, in the supercritical LCP capillary number regime. The top curve is a repeat of the second lowest curve of Figure 1: inviscid solvent $Ca = 0$, Deborah number $\Lambda = 2$, concentration $N = 2.9$, LCP capillary number $Ca_{lcp} = 25$, and isotropic drag parameter $\sigma_d = 1$. The middle curve has $\sigma_d = 0.5$, corresponding to twice the drag transverse to the rod axis as along the axis, while the lowest (nearly zero) growth rate curve corresponds to a highly anisotropic drag coefficient, $\sigma_d = 0.1$. Note this LCP capillary number is supercritical (the critical value for the isotropic phase is $\frac{3}{2}[3 - N]^{-1} = 15$), for which the Doi theory predicts microstructure overcomes capillary instability in the limit $\sigma_d = 0$.*

Figure 3: *The effect of polymer drag anisotropy on the isotropic LCP phase, $s_0 = 0$, in the subcritical LCP capillary number regime, with $Ca_{lcp} = 10$. Figure 3a is for an inviscid solvent ($Ca = 0$), 3b is with solvent viscosity ($Ca = 2.2$); in both figures 3a,3b, the top curve corresponds to isotropic polymer drag, $\sigma_d = 1$, and the curves in descending order correspond to $\sigma_d = .10, 0$, respectively. Otherwise, LCP parameters are consistent with Figures 1,2: $\Lambda = 2$, $N = 2.9$. This subcritical LCP capillary number has capillary growth rate curve bounded between the Rayleigh inviscid (if $Ca = 0$) or viscous (if $Ca \neq 0$) curve and the non-zero curve with $\sigma_d = 0$.*

Figure 4: *The effect of the isotropic-to-nematic phase transition on capillary instability, for inviscid solvents. The top curve is for the isotropic LCP phase, the lower curve for the nematic prolate phase. LCP parameter values coincide with Figures 1 and 2, with polymer concentration $N = 2.9$ in the range for which the isotropic and nematic phases are bi-stable orientation equilibria, with remaining parameters: $Ca = 0$, $\Lambda = 2$, $\sigma_d = 1$, $Ca_{lcp} = 25$.*

Figure 5: *Anisotropic drag and LCP capillary number: scaling of the peak of the growth rate curve for the prolate nematic phase, s_0 : the dominant wavenumber (K_{max} , Figure 5a) and corresponding maximum growth rate (ν_{lcp}^{max} , Figure 5b). We fix the polymer concentration $N = 4$, for which: $s_0 = \frac{1+\sqrt{3}}{4}$, the critical capillary number is $Ca_{lcp}^{crit} = 0.4265$, and the prolate phase is the unique stable orientation equilibrium. The scaled relaxation time is $\Lambda = 2$, and the solvent is inviscid ($Ca = 0$). The top curve is the subcritical LCP capillary number $Ca_{lcp} = 0.4$, and the lower curve is with supercritical $Ca_{lcp} = 25$, depicting how the maximum wave number and growthrate scale with σ_d for all values of the anisotropic drag parameter σ_d .*

Figure 6: *Anisotropic drag* effect on capillary instability of the LCP prolate equilibrium s_+ for subcritical LCP capillary number and an inviscid solvent. The parameter values are $\Lambda = 2$, $Ca = 0$, $Ca_{lcp} = 0.4$, $N = 4$. The curves from top to bottom correspond to $\sigma_d = 1, 0.1, 0.01, 0$, respectively. The curves illustrate the Doi theory prediction in the *subcritical LCP capillary regime*, for which an LCP jet has capillary growth rate bounded between the Rayleigh inviscid rate and the non-zero LCP rate with $\sigma_d = 0$. Note the lowering of the cutoff wavenumber in the limit of zero polymer drag.