UNIFICATION IN DESCARTES' PHILOSOPHY OF MATHEMATICS

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ABSTRACT

CATHAY LIU: Unification in Descartes' Philosophy of Mathematics
(Under the direction of Alan Nelson)

I argue that Descartes unified mathematics in a rather striking and strong way. He held that there was a single shared subject matter of all mathematics, that the objects of mathematics are ontologically identical to extended substance.

The most entrenched conception that I argue against in my dissertation is the view that for Descartes mathematical objects—particularly numbers—have some sort of ontological status independent of material extension. It is easier to see that geometrical objects such as triangles depend on geometrical extension or reduce to extension but similar claims about the dependence on extension for numbers or other abstract, arithmetic, or algebraic objects doesn't not seem as immediately plausible.

I argue that the best way to understand Descartes' views about mathematics is to see that Descartes viewed numbers as epistemically and metaphysically dependent on extension. I give three arguments for this view.

The first argument I offer is based on ontological problems with mathematical realism as an interpretation of Descartes. Many commentators have thought Descartes is best understood as having some form of realism about the universals in our mathematical cognitions. Problems for these realist readings arise when the need to ontologically locate

the universals conflicts with central Cartesian metaphysical doctrines. The alternative is a nominalism about mathematical objects.

The second argument I offer is found in Descartes' concept of numbers. There I show that in order for numbers to be conceived at all, a prior idea of extension (specifically its divisibility) must be contained in the idea. Without the idea of the nature of extension, there can be no idea numbers. Knowledge of extension is epistemically prior to and necessary for mathematical knowledge.

The final argument concerns Descartes' unification of algebra and geometry. I show that the best account of Descartes' mathematics is given using this way of distinguishing the metaphysical and epistemic unity and priority between algebra and geometry. Descartes' development of analytical geometry exhibits the dependence of number on extension through the dependence of algebra on geometry and the priority of geometrical magnitudes to discrete algebraic multitudes.

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Introduction To The General Topic

Imagine you have developed mild myopia. An optometrist examines you to determine the degree of refractive error in your vision, i.e. how far away from your retina two parallel rays of light entering your eye are being focused. Then using the relationships between some triangles, and curved lines that she diagrams, she determines the shape of a particular parabola (the anaclastic curve), which she explains is the shape of the refracting surface of the lenses you will need to properly focus those parallel rays of light at a single point on your retina. She expresses this curve as an equation, and gives it to the lens grinder. The lens grinder uses the equation to grind the lenses into the appropriate shapes to produce your glasses.

Consider what needed to take place in the scenario above. The optometrist and lens grinder relied on a special relationship between various physical magnitudes and qualities (distances, shapes of surfaces, etc.) and mathematical facts (geometrical depictions of triangles, and curves, expressed as numerical equations, etc). Further examples of this assumed special relationships between (a) algebra and geometry, and (b) mathematics and physics, are legion. But such associations are crucial; the development of empirical science relied heavily on our ability to characterize its phenomena in mathematical terms. In this sense, the seamless alternation between physical objects like lenses and mathematical objects like triangles, curves, and equations illustrates nothing less than the birth of the modern world.

With our physics so desperately dependent on mathematics today, we myopically see connection between equations, geometrical diagrams, and the physical world as obvious and even un-mysterious. But a historical perspective of the development of mathematics, its applications, and also the development of physics will quickly expose that these assumptions we have today have not always been obvious or given or taken for granted. Physics was hardly mathematical: numbers and equations had nothing to do with final causes, efficient causes, nor any other Aristotelian cause that had ruled the sciences for many centuries previous. Similarly, it was not always the case that there was the ready acceptance of "pure mathematics" or even the unification of algebra and geometry. In the earliest days, geometry was merely, literally, a *geo*-metry: a technique used to measure parcels of land for the purposes of calculating tax burdens and strategic farming along the Nile river. Mathematics was very much a practical, and applied craft, as opposed to a pure, theoretical and abstract science that we practice today.

The turning point for what is no less than the birth of the modern world, as we now understand it, depends vitally on the use of mathematics in our physical sciences. This single handedly opened up the door to the scientific revolution, the development of calculus, advances of leaps and bounds in optics, astronomy, mechanics… etc. None of this would have occurred had not the applied classical geometry come together with what was seen as questionable algebraic methods to produce analytic geometry. What was required was the systematic unification of algebra with geometry, and mathematics with physics.

The above example of the anaclastic line is just one of many examples that is emblematic of the radical changes occurring during the early modern period as the mathematics and sciences surged forward. It embodies the various unifications that set the ground for everything to come in the next two hundred years. And in the center of this exciting change and the surge of our ability to understand the world was a figure who had a hand in it all: Rene Descartes.

Descartes plays a central role in the modern understanding of the relationship between mathematics and empirical science. He made revolutionary contributions to both mathematics (*e.g.*, his analytic geometry which unified geometry and algebra), and to physics and optics (*e.g.*, his conservation laws, the sine law of refraction, and the determination of the anaclastic curve), which, moreover, were often characterized in a mathematical fashion: a mathematical-physics.

Like other philosophers of the early modern period there were important and pressing fundamental questions about the relationship between the mathematical and the empirical that required answers. An interpretation of Descartes' answers to these questions needs to account for his philosophical views about numbers and the relationship between algebra and geometry such that he could unify them in a single mathematics allowing then for the connection between mathematics and physics to be addressed. Much work has been done over the years to examine his physics, and the metaphysics underlying it. But it is to Descartes' philosophy of mathematics, and particularly its metaphysical underpinnings that I will explore here.

The Arguments For My Interpretation: An Overview

This more limited topic of his philosophy of mathematics to which I have confined myself is only a small portion of the more ambitious topic of his mathematical-physics. In what follows, I argue for a particular interpretation of Descartes' philosophy of mathematics. My claim is that Descartes unified mathematics in a rather striking and strong way. He held that there was a single shared subject matter of all mathematics, that the objects of mathematics (e.g. triangles, lines, curves, ratio, proportions, numbers, and etc.) are ontologically identical. The single subject matter of all mathematics was geometrical extension, and thus, for Descartes, material or extended substance. Perhaps the single most entrenched conception that I am arguing against in my dissertation is the view that for Descartes the mathematical objects—particularly numbers—have some sort of ontological status that is independent of material extension. It is easier for people to accept that geometrical objects such as triangles depend on geometrical extension or reduce to extension. But any similar claims about the dependence on extension for numbers or other abstract, arithmetic, or algebraic objects doesn't not seem so acceptable nor plausible. But I argue that the best way to understand Descartes' views about mathematics is to see that Descartes viewed numbers as epistemically and metaphysically dependent on extension.

In order to elaborate on this account of Descartes' philosophy of mathematics in this dissertation, I concentrate on three arguments that speak in favor of understanding the truths of mathematics as consisting in the true and immutable nature of extension: the metaphysical problems with realism about numbers, the conceptual dependence of numbers on extension, and Descartes' account of the relationship between algebra and geometry.

The first argument I offer addresses the problem with a long standing and tempting alternative account of mathematical objects. Many commentators have thought that in order to account for the necessity, certainty and universal applicability of mathematics, Descartes is best understood as having some form of realism about the universals in our mathematical cognitions, e.g. geometrical figures such as triangles and numbers such as *two*. However I argue that these realist readings of Descartes create more problems than they purport to solve. Problems for these realist readings arise because the need to ontologically locate the universals conflicts with central Cartesian metaphysical doctrines. The alternative that emerges is a nominalism about mathematical objects and truths, and the ultimate nature or essence of the ideas named is the nature of extension. As we have an innate idea of extension implanted by God, the necessity, certainty and universal applicability of mathematical ideas comes from the true and immutable nature of extension. The nature or essence of the mathematical truths reduces to the same metaphysical essence of extended substance.

The second argument I offer addresses the dependence of number or counting on extension. There I show that in order for numbers to be conceived at all, a prior idea of extension must be contained in the idea. The conceptual dependence of numbers on extension is due to the need to conceive of numeric quantities that rely on a relationship between parts and wholes; and the only things that can admit of this conception (divisibility) is extension. Thus, without the idea of the nature of extension, there can be no idea numbers. Knowledge of extension is epistemically prior to and necessary for mathematical knowledge.

The final argument I offer is based on Descartes' unification of algebra and geometry. I show that the best account of Descartes' mathematics it is given using this way of distinguishing the metaphysical and epistemic unity and priority between algebra and geometry. In some sense this argument can also be seen an extension to the one previously offered about the part/whole relationships that are necessary to counting. However, rather than talk of part and wholes in our conception of the numeric quantities (multitudes) of arithmetic or algebra, Descartes speaks in terms of their dependence on the notion of a unit of measure. It is the necessity of the unit (the parts by which we are measuring or counting wholes) that makes the magnitudes or continuous quantities of pure geometrical extension necessary and prior to algebra.

In general, I have tried to present an account of the metaphysical roots of Descartes' philosophy of mathematics. This metaphysical root (the first principle, or the simple nature) is the nature of extended substance. And as a clear and certain first principle, its certainty and necessity is what accounts for the truths of mathematics, which we can deduce from the nature of extension. There are many more philosophically relevant issues that I have not been able to address here due to my own lack of time or expertise, but what I hope that I have been able to do is present an interpretation of Descartes' philosophy of mathematics that has better accounted for Descartes' claims about mathematics as well as one that will later better fit with his wider conception of the unified system of human knowledge.

The Broader Project And A Note About The Method

My arguments for these three points are based on Descartes' claims about his method for philosophizing and the unity of philosophical knowledge—neither of which I have the luxury of discussing in any depth at present. Instead, for the purposes of this dissertation, I will very briefly lay out below my understanding of Descartes' method and the unity of philosophical knowledge that I have assumed. My purpose in providing this rough sketch is to give the reader a sense of the interpretative foundations that inform not just this discussion of his philosophy of mathematics but the larger project of which the mathematics is just one part.

The unity of science is inextricably tied to Descartes' new philosophical method of investigation he characterized in an unpublished early work, *Rules for the Direction of the Mind*.[1]

[1] I shall use '*Rules*' to refer to this early work (*Regulae Ad Directionem Ingenii*), which can be found in René Descartes, *Oeuvres de Descartes*, ed. C. Adam and P. Tannery, 12 vols., revised

This unity of science was a significant theme present throughout Descartes' corpus (as I gloss below). But for Descartes, knowledge consisted in understanding how simpler things composed other things, and his method instructs us on how to arrange these component parts serially in chains of thought so we can see the interconnections among the composite things while reducing them to their most fundamental, simple natures. The discerning and ordering of the parts of complexes will explain how, and to what extent, the complexes are related.

To see why this will help us better understand Descartes' philosophy of mathematics we should see how his method for philosophy arises from mathematics in the first place. Then all we need to do is turn the method back on the subject (mathematics) that inspired it.

> Thus the whole of philosophy is like a tree. The roots are metaphysics, the trunk is physics, and the branches emerging from the trunk are all the other sciences…[2]

Descartes uses the analogy of the tree to represent the interconnectedness of the whole of philosophy. This conception of a unified science can be seen in both in his later published work such as the *Principles*, and also in very early unpublished manuscripts such as the *Rules*. In the *Principles*, Descartes says that philosophy, or the study of wisdom, must start with the search for the first causes or principles in order to attain perfect knowledge of all things that mankind is capable of knowing.[3] Descartes believes that only two reasons are

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edition (Paris, 1964-76), X: 359-472 subsequently cited as "AT" followed by volume and page number; René Descartes, *The Philosophical Writings of Descartes*, English translation by J. Cottingham, R. Stoothoff, and D. Murdoch, 2 vols. (Cambridge, 1985), I: 9-78—subsequently cited as "CSM" followed by volume and page number. For the 3rd volume in the series, *The Philosophical Writings of Descartes: The Correspondence* (translated by J. Cottingham, R. Stoothoff, D. Murdoch, & A. Kenny), I cite as "CSMK" followed by page number.

[2] AT IX B: 14; CSM I: 186.

[3] AT IXB: 2; CSM I: 179.

necessary to show that "the true principles, enabling one to reach the highest degree of wisdom which constitutes the supreme good of human life, are the principles which [he has] set down in [*The Principles of Philosophy*]."[4] The first is that his principles are very clear. The second is that all other knowledge depends on these principles: they enable all other things to be deduced from them. This *scientia* or knowledge *is* the whole of human wisdom, and it is unified and interconnected.[5] In the *Rules*, Descartes writes that

> "The sciences as a whole are nothing other than human wisdom, which always remains one and the same, however different the subjects to which it is applied, it being no more altered by them than sunlight is by the variety of the things it shines on. Hence there is no need to impose any restrictions on our mental powers; for the knowledge of one truth does not, like skill in one art, hinder us from discovering another; on the contrary, it helps us."[6]

He also explains that though the arts of harp-playing and farming are two distinct activities requiring distinct skills, and are best mastered when not practiced concurrently, science, i.e., human wisdom or knowledge, requires only a single skill: a good reason (*bona mente*), which Descartes otherwise calls "universal wisdom". Descartes instructs us to direct our studies not to particular scholastic problems but instead to the general end and study of universal wisdom: we should "consider simply how to increase the natural light of [our] reason... in order that [our] intellect should show [our] will what decision it ought to make in each of life's contingencies."[7] This will be far more effective in our investigation of truth

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[4] AT IXB: 9; CSM I: 183.

[5] See *Rule* 1; AT X: 359-361; CSM I 9-10.

[6] AT X: 360; CSM I: 9.

[7] AT X:361; CSM I: 110.

because all the sciences are closely *interconnected and interdependent*. Specializing is thus far less effective.

This systematic and unified body of knowledge based on a set of self-evident first principles is contrasted with the specialization of the Scholastics. Like many philosophers of the early modern period, Descartes was so impressed by the success of mathematics that he was inspired to adopt as a model for all human inquiry the method of mathematics. Descartes observes that there is "hardly any question in the sciences about which clever men had not frequently disagreed," and "just arithmetic and geometry" are "free from any taint of falsity or uncertainty."[8] These two disciplines enjoy the certainty that other disciplines lack for they alone concern themselves with an object that is so pure and simple, there is no room for doubt or uncertainty. When Descartes surveys why this is so, he first considers the various ways we come by our cognitions: "There are only two ways of arriving at a cognition (*cognitionem*) of things--through experience and through deduction."[9] Whereas our experiences are often deceptive and dubitable, a rational intellect can never perform a pure inference or deduction incorrectly. Crucially, arithmetic and geometry arrive at knowledge of their objects through deduction, and hence they make "no assumptions that experience might render uncertain."[10] The "conclusion we should draw from these considerations is ... that in seeking the right path (*iter*) of truth we ought to concern ourselves only with objects which admit of as much certainty as the

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[8] *Rule* 2. AT X: 363-4; CSM I: 11-2.

[9] AT X:364-5; CSM I: 12.

[10] AT X: 365; CSM I: 12.

demonstrations of arithmetic and geometry."[11] Descartes seeks to develop a *scientia* modeled after arithmetic and geometry.

The result of first Descartes' study of arithmetic, and geometry and then a more general investigation of mathematics was his method (his universal mathematics)[12].[13] He resolved in his search for the knowledge of things, "to adhere unswervingly to a definite order, always starting with the simplest and easiest things and never going beyond them till there seems to be nothing further which is worth achieving where they are concerned."[14]

According to Descartes, the contents of our minds can be divided into two types: simple natures, and things that are composed of simple natures.[15] Simple natures are innate ideas[16] that we can clearly and distinctly perceive[17](or "intuit", to use Descartes' term in the

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[11] AT X: 366; CSM I: 12-13.

[12] For discussions about the nature and continuity of Descartes' method and the *mathesis universalis*, see Alexandrescu (2009); Beck (1952); Dear (1998); Flage and Bonnen (1999); Garber (1992), (2000), (2002); Machamer and Maguire (2009); Sasaki (2003); Shuster (1977); Smith, K (2010); and Smith, N (2010).

[13] Cf. *Rule* 4 (AT X: 371-9; CSM I: 15-20).

[14] *Rule* 4; AT X: 379; CSM I: 20.

[15] "… it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another. Indeed, it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others." (*Rule* 12; AT X: 422; CSM I: 46).

[16] Or as he describes them in the *Discourse* (AT VI: 41; CSM I: 131) and *Principles*, the clear and certain principles(*e.g.*, AT IXB: 9; CSM I: 183). See also Nelson (2008).

[17] "… we term 'simple' only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known…all the rest we conceive to be in a sense composed out of these [simples]" (*Rule* 12; AT X, 418; CSM I 44).

Rules).[18] There are not very many of these ideas: Descartes remarks that "there are very few pure and simple natures which we can intuit…"[19] The different simple natures (or different substances, or essences), can form various mixtures or compounds, which can further combine to form yet more complex composites. This compounding of simples in multiple iterations and patterns, according to Descartes, can account for all our other more complex ideas.[20] Given his theory of ideas, "*the whole of human knowledge consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things*."[21] His entire method, as he conceived it, consists in *ordering and arranging our ideas according to their degree of confusion or complexity*.[22]

[18] "... we need take no great pains to discover these simple natures, because they are self evident enough. What requires effort is distinguishing one from another, and intuiting each one separately with steadfast mental gaze." (*Rule* 12; AT X: 425; CSM I: 48).

And also:

"…we should note that there are very few pure and simple natures which we can intuit straight off and per se (independently of any others) either in our sensory experience or by means of a light innate within us. We should, as I said, attend carefully to the simple natures which can be intuited in this way, for these are the ones which in each series we term simple in the highest degree. (*Rule* 6; AT X: 383; CSM I: 22).

[19] *Rule* 6; AT X: 383; CSM I: 22.

[20] "… it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another. Indeed, it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others." (*Rule* 12; AT X: 422; CSM I: 46).

[21] *Rule* 12; AT X: 427; CSM: 49. Emphasis mine.

[22] "The whole method consists entirely in the ordering and arranging of the objects of which we must concentrate our mind's eye if we are to discover some truth. We shall be following this method exactly if we first reduce complicated and obscure propositions step by step to simpler ones, and then, starting with the intuition of the simplest ones of all, try to ascend through the same steps to a knowledge of all the rest." (*Rule* 5; AT X: 379; CSM I: 20).

In this fashion, Descartes' universal mathematics, as given in the *Rules* is not so much a method for mere mathematical computation, but a method for all the sciences. Descartes thinks that mathematics has something important to offer us[23] due to its focus on order and patterns.[24] What Descartes finds valuable about the study of mathematics is the training of our minds in his method, which we can then use for proper scientific enquiry that yields knowledge of the true nature of things.[25] All human knowledge would consist in understanding the necessary order of dependence (or their composition) in terms of the few and most simple natures, which are the metaphysical roots on his tree of knowledge. The unity of human knowledge lies in the reduction of all things to the simple natures.

At present, though, I only argue that all that we consider mathematical reduces to the nature of extension: it is the nature of extension that unifies algebra and geometry, but the

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[23] Though he can otherwise at times be surprisingly disparaging of just doing mathematics, for example, Descartes writes in the *Rules* that he "would not value these Rules so highly if they were good only for solving those pointless problems with which arithmeticians and geometers are inclined to while away their time, for in that case, all I could credit myself with achieving would be to dabble in trifles with greater subtlety than they." *Rule* 4; AT X: 374; CSM I: 18.

And that, "there is really nothing more futile than so busying ourselves with bare numbers and imaginary figures that we seem to rest content in the knowledge of such trifles. *Rule* 4; AT X: 374; CSM I: 18.

[24] "Number-games and any games involving arithmetic, and the like, belong [in the simplest and least exalted arts]. It is surprising how much all these activities exercise our minds, provided of course we discover them for ourselves and not from others. For, since nothing in these activities remains hidden and they are totally adapted to human cognitive capacities, they present us in the most distinct way with innumerable instances of order, each one different from the other, yet all regular. Human discernment consists almost entirely in the proper observance of such order." *Rule* 10; AT X: 404; CSM I: 35.

[25] "… these Rules are so useful in the pursuit of deeper wisdom that I have no hesitation in saying that this part of our method was designed not just for the sake of mathematical problems; our intention was, rather, that the mathematical problems should be studied almost exclusively for the sake of the excellent practice which they give us in the method." *Rule* 14; AT X: 442; CSM I: 59.

nature of extension is also necessary for our mathematical ideas. This interpretation has several virtues. It avoids the metaphysical problems of competing accounts. It accounts for a larger set of texts than the alternative interpretations. It can better account for Descartes' mathematical practices. And it can unify his early works with his later works. My hope is that the virtues of the resulting account of Descartes' philosophy of mathematics will render the underlying interpretive assumptions about Descartes' method and the unity of knowledge that I have employed worthy of discussion as an interpretation for Descartes' philosophical corpus more generally.

Chapter 2: Universals and Descartes' metaphysics of numbers

0. The Metaphysics Of Mathematical Objects

Eugene Wigner begins his famous 1960 publication[26] with a story about former high-school classmates:

> One of them became a statistician and was working on population trends. He showed a reprint to his former classmate. The reprint started, as usual, with the Gaussian distribution and the statistician explained to this former classmate the meaning of the symbols for the actual population, for the average population, and so on. His classmate was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How do you know that?" was his query. "And what is this symbol here?" "Oh said the statistician, "this is pi" "What is that?" "The ratio of the circumference of the circle to its diameter." "Well, now you are pushing your joke too far," said the classmate, "surely the population has nothing to do with the circumference of the circle."

Examples like the one in Wigner's story are legion. As Wigner's title (*The Unreasonable Effectiveness of Mathematics in the Natural Sciences)* aptly observes, the applicability of mathematics is surprising, uncanny and inexplicable. A particularly striking feature of mathematics is the universality of its application in various and sundry particular instances. Consider a mathematical notion such as the irrational number we refer to with 'pi'. Used to express the ratio of a circle's circumference and its diameter, this number holds for all circles, any circle with any diameter. It can also be used to express a third geometric magnitude: that of a line constructed from the circle's circumference and diameter.

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[26] "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" in *Commutations in Pure and Applied Mathematics*, vol. 13, No. 1 (February 1960). New York: John Wiley & Sons, Inc.

Moreover, this number is also used in expressing probability distributions for things like populations. The mathematical notion of this irrational (or transcendental) number does not resemble a ratio of two geometric magnitudes any more than it does a line or population distribution. Nor, for that matter, does the comparison between a diameter and circumference seem to resemble population distributions.

In a philosophy of mathematics some explanation is required for such striking universality of mathematical notions. What explains the possibility of Thales' pioneering practice of formulating properties of geometrical figures as general statements? What explains the systematic deducibility of these properties in the form of geometric propositions as Euclid did in his *Elements*? What explains the number-theoretic investigation of the properties of numbers by the Pythagoreans,[27] or the seeming necessity and universality of '5+7=12'? Accounts of mathematics generally fall into one of two categories: realist accounts or anti-realist accounts. My use of 'realism'[28] here aims to cover the type of views in which "mathematical objects" have some sort of independence from all the particular cases to which they apply while remaining silent about whether mathematical objects are properties, relations, propositions, concepts, or forms. The truths about these mathematical objects are prior to any particular in the sense that they are independent and underived. Thus, a hallmark of these types of realist views is that all of the mathematical objects will be abstract. This indeterminacy allows mathematical objects to apply in a wide variety of particular instances. This indeterminacy also makes mathematical objects independent of particular

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[27] For more, see Heath (1981).

[28] The term 'realism' has come to cover a variety of accounts in the philosophy of mathematics and can apply to positions ranging from the ontology of mathematical objects to the truth of mathematical propositions. It is certainly not my intention to trample the delicate taxonomy of positions offered in contemporary philosophy of mathematics.

occurrences of them in the world, e.g., even if nothing in the world were circular, truths about circles or the concept of circle would not change. "Anti-realism" is used to describe views in which abstract, independent mathematical objects do not exist. Note that the focus of anti-realism is the denial of the prior existence of something that is abstract and independent of any particular instance that can be said to exhibit or instantiate the object.

A common view of Rationalists and Empiricists of the early modern period divides them neatly along party lines into realists and anti-realists, respectively.[29] Rationalists were so impressed by the certainty and aprioricity of mathematics that they accepted the existence of universals and sought to emulate the method of mathematics. They even went so far as to provide axioms and demonstrate from philosophical first principles all of human knowledge.[30] Because Empiricists believed instead that humanity could not reach the Platonic heavens, they sought to explain how it was that mathematics could appear necessary and apply universally without actually appealing to our cognitive grasp of universals.[31]

There is a long tradition of commentators who favor reading Descartes, the father of modern day rationalism, as a mathematical realist[32]. Several passages seem to make inevitable some interpretation of Descartes as a realist. Traditional commentators think that these texts recommend that Descartes was a realist about universals generally, though I will

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[29] Shapiro (2005) p.1-8.

[30] Descartes' *Rules*, Spinoza's *Ethics*, Malebranche's *Search for Truth*.

[31] See Parsons (1983) Essay 1.

[32] For reasons that well become clear below, what I am calling a realist reading includes accounts that have been presented as an alternative to realism (e.g. conceptualism, or nominalism).

focus on numbers in particular for present purposes.[33] I argue that these tricky interpretive issues are not best resolved by a realist understanding of Descartes because doing so creates problems for understanding his philosophy of mathematics and general metaphysics that are more severe than the difficulties they purport to solve. In short, we should resist a realist reading of Descartes and seek an alternative.

Given the problems of realist accounts, I argue that Descartes is best understood as an anti-realist. More specifically, Descartes has a nominalist account of universals that is similar to accounts typically ascribed to Empiricists such as Berkeley or Hume. A reading of Descartes as a mathematical nominalist is not necessarily free from all of the worries typical of nominalism, but it is a reading that will be free from the interpretive problems of realist accounts.

But the nominalist account of universals for Descartes that I offer is also significantly different from the accounts in Berkeley and Hume, despite having some similarities. Numbers and geometrical figures are names of ideas, but to Descartes they are ultimately ideas of extension or of the nature of extension. The nature of extension grounds the necessity and apriority of what we call the mathematical truths. For Descartes, number-ideas depend on extension. This dependence can be understood metaphysically or epistemically. Strictly speaking, the things that are numbered are metaphysically identical to extension. However, epistemologically, ideas of number must conceptually contain[34] ideas

[33] Unless otherwise noted, for the remainder of this dissertation my discussion of universals is limited to only those that Descartes discusses the most: the mathematical objects—by which I mean to refer *both* to *geometrical figures* (e.g. triangles) and to *arithmetical quantities and numbers* (e.g. two or twoness).

[34] Cf. definition 9 from the geometrical proofs in Second Replies (AT VII: 162; CSM II: 114): When we say that something is ***contained in the nature or concept of a thing***, this is the same as saying that it is true of that thing, or that it can be asserted of that thing.

of extension. This is to say that numeric thinking depends on conceptually prior ideas of extension such that if the requisite idea of extension were lacking, an idea of number could never occur.

In what follows, I survey three types of accounts of Cartesian mathematical objects. I divide these accounts into two camps. The first camp attributes to Descartes what I call a "strong realism" (SR) about mathematical objects. I examine two types of SR: one found in Anthony Kenny and the other in Tad Schmaltz. The second camp attributes a "weak realism" (WR) to Descartes; WR was developed to avoid the problems arising from a SR reading of Descartes. Proponents of WR views have presented their interpretations as attributing to Descartes a "conceptualism" about numbers. More importantly for my present purposes, proponents of WR interpretations have presented their accounts as an alternative to resorting to "realism" and thereby avoiding all of the problems associated with reading Descartes as a realist. However, some rather serious problems for these conceptualist readings have gone unnoticed as yet. These problems can be traced to a shared motivation for realism among conceptualist readings. Thus, I am intentionally labeling these interpretations "Weak Realism" in an effort to emphasize their similar realist motivations.

All three interpretations offer an account of how Descartes thought about mathematical objects, but they disagree substantially about the ontological status of mathematical objects. As such, the particular problems arising for each account also differ across accounts. Even so, for all three interpretations it is the realism invoked on Descartes' behalf that is philosophically and interpretively problematic. The exploration of the problems nonetheless suggests a promising alternative: that numbers depend on extension. I argue that this view avoids the problems of the realist interpretations by not running afoul of basic Cartesian tenets.

1. Three Central Cartesian Doctrines.

There are three fundamental doctrines of Cartesian metaphysics that any interpretation seeks to preserve. In general, no interpretation of Descartes ought to run afoul of these core positions, or at least not do so without good cause or explanation. Before we proceed further to examine and evaluate the various interpretations that have been offered, it would be expedient to briefly review these three central claims.

(1). The perfection of God

Descartes' God is infinitely perfect and must have all of the perfections.[35] Simplicity is a perfection; thus, God has divine simplicity.[36] While we sometimes speak as though God wills and understands, Descartes warns that we ought not think that God has distinct faculties; rather, "there is always a single identical and perfectly simple act by means of which he simultaneously understands, wills and accomplishes everything."[37] Necessary existence is

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[35] "The substance which we understand to be supremely perfect, and in which we conceive absolutely nothing that implies any defect or limitations in that perfections, is called *God*." AT VII:162; CSM II:114; emphasis original.

[36] "On the contrary, the unity, the simplicity, or the inseparability of all the attributes of God is one of the most important of all the perfections which I understand him to have." AT VII: 50; CSM II: 34.

[37] *Principles* I: 23; AT VIIIA: 14; CSM I: 201.

also a perfection that belongs to God[38]; that is, God is self-caused and cannot depend on anything else for His existence.

(2). Cartesian Dualism

Of the things that God created, they can be divided into only two types: mind or body. Descartes writes in the *Principles*:

> But I recognize only two ultimate classes of things: first, intellectual or thinking things, i.e. those which pertain to mind or thinking substance; and secondly, material things, i.e. those which pertain to extended substance or body.[39]

(3). Real Distinction between substances

Any two different substances must be really distinct from one another.[40] This distinctness means that God, mind, and body are metaphysically distinct: one is never numerically identical with any of the others.

With these in hand, let us now turn to the various realist interpretations of Descartes that have been offered.

2. Realism In Cartesian Mathematics

Many difficulties surround Descartes' account of the nature and status of mathematical objects. These difficulties leave much room for interpretation, as is evidenced by the wide range of available accounts in the literature. I claim that while scholars have

[38] *Principles* I : 14; AT VIIIA: 10; CSM I: 197-8.

[39] *Principles* I: 48; AT VIIIA: 23; CSM I: 208.

[40] *Principles* I: 60; AT VIIIA: 28-9; CSM I: 213.

mostly appealed to a realism in some form or another to resolve these difficulties, their accounts differ in other significant ways. For many scholars, a major point of tension stems from apparent conflicts between accounts of mathematical objects, especially Descartes' discussion of true and immutable natures in the *Meditations* as compared to his discussion of universals in the *Principles*.

In the *Principles*, Descartes speaks of numbers and geometrical figures such as triangles as "modes of thinking" and not as objects that have some separate, real, mind-independent existence of their own. Here, the universals seem to be mere ideas or modes of our mind. E.g., in the *Principles* I: 58, Descartes writes that, "Number and all universals are simply modes of thinking."[41] Earlier, in the *Principles* I: 55, he explains that we should not regard number as "anything separate from the things which are ordered and numbered, but should think of them simply as modes under which we consider the things in question."[42]

In contrast, Descartes writes in the Fifth Meditations that, "there is still a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented by me or dependent on my mind."[43] This passage seems to suggest that while geometrical objects (and presumably other mathematical objects like numbers) may be conceived of by our minds, they also have some independent existence of their own.

To further complicate matters, mathematical truths and the true immutable natures or essences of the Fifth Meditations are also connected to Descartes' doctrine of the eternal truths. For example, in a letter to Mersenne dated April 15, 1630, Descartes writes, "The

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[41] AT VIII A: 27; CSM I: 212.

[42] AT VIII A: 26; CSM I: 211.

[43] AT VII: 64; CSM II: 45.

mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures."[44] About a month later in another letter to Mersenne, Descartes writes that the essence of created things, "is nothing more than the eternal truths."[45] By connecting mathematical truths and the true, immutable and eternal nature of geometrical objects, a connection can be made from mathematical objects to the eternal truths. This connection also serves to further encourage a reading of Descartes that allows for a realism about abstract mathematical objects. Such readings stress the existing, immutable nature of the mathematical objects. The objects' nature is independent of the way we think of them, which ascribes these readings to the existence of mind-independent real mathematical objects in some form or another. I call this reading a *strong* realism. On the other hand, *weak* realism will focus more on the passages from the *Principles* and not adopt the same mind-independent desiderata.

I will present the various realist interpretations in terms of how they locate the universal. As opposed to merely differentiating the interpretations in terms of the text they emphasize the most, this presentation will allow us to get a better grasp on the differences between what I call a strong realism and a weak realism.

Recall that any realist reading supports the existence of abstract non-particulars that are independent of the things that contain, exhibit or exemplify them. According to Cartesian ontology there are three substances: God (i.e. infinite substance), mind (i.e. thinking substance) and body (i.e. extended substance). If a universal is real, that is, if it

[44] AT I: 145; CSMK: 23.

[45] AT I: 152; CSMK: 25.

exists, it will need to be located somewhere in ontological space. Thus, every realist account faces a trilema for which there are broadly three possibilities.

Mathematical objects must be located either:

(i) in God,
(ii) in one of the created substances, mind or body,
(iii) in some third realm.

The "strength" of the realism in SR readings stems from the elimination of (ii). The SR-universals have a much more robust existence than WR-universals. WR accounts will attempt to locate the universal in the created mind (ii)—exactly what a SR account rules out.

However, I show below that this need to locate the abstract universal in any realist accounts will ultimately be problematic.

2.1 Strong Realism

As I mentioned above, SR interpretations are primarily motivated by two sets of passages in Descartes writings: the passages from the Meditations and those from the correspondence. SR accounts stress two ideas from these passages. First, SR readings call attention to the independence of mathematical objects from the world and from our human minds. Second, SR readings underscore the emphasis this world- and mind-independence then places on the existence of the nature of the mathematical object. For example, in addition to what we saw above, Descartes also writes in the Fifth Meditation:

> When, for example, I imagine a triangle, even if perhaps no such figure exists, or has ever existed, anywhere outside my thought, there is still a determinate nature, or essence, or form of the triangle which is immutable and eternal, and *not invented by me or dependent on my mind.* This is clear from the fact that various properties can be demonstrated of the triangle, for example, that its three angles equal two right angles, that its greatest side subtends its greatest angle, and the like; and since these properties are ones which I now clearly recognize whether I want to or not, even if

> I never thought of them at all when I previously imagined the triangle, it follows that they cannot have been invented by me.[46]

A bit later, he writes:

> It is not necessary for me to imagine any triangle; but whenever I choose to consider a rectilinear figure that has just three angles, I must ascribe to it properties from which it is rightly inferred that its three angles are not greater than two right angles, even if I do not notice this at the time…[47]

Both Anthony Kenny and Tad Schmaltz appeal to passages like the ones above to support the view that Descartes' mathematical objects cannot be identified with created mind or body existing in the actual world. That is, they appeal to the two passages above to rule out (ii).

Universals do not depend on the thoughts of finite minds, as evidenced by our inability to manipulate or alter the natures of universals at will.

Nor do they depend on the existence of material things; for the nature of the triangle is unaffected by whether or not any actual triangles exist. In fact, as Kenny notes, "the supposition that no triangles exist in the world is not merely a part of Descartes' hyperbolical doubt. He believes the supposition to be true of the macroscopic world even after he provides the solution to his methodic doubts"[48].

Kenny and Schmaltz part ways in their SR according to how each interpretation treats Descartes' writings on the eternal truths. After ruling out option (ii), two possibilities remain open. Kenny and Schmaltz each locate universals in a different place. Kenny opts to locate universals in some third realm (iii), while Schmaltz locates them in God (i).

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[46] AT VII: 64; CSM II: 44-45.

[47] AT VII: 67-68; CSM II: 47.

[48] Kenny (1958) 148-149.

Kenny describes Descartes as "the founder of modern Platonism"[49] and thinks "for Descartes the geometer's triangle is *an eternal creature of God*, with its own immutable nature and properties, a real thing lacking only the perfection of actual existence."[50] Kenny thinks that regardless of "whether [a geometrical figure] existed or not, it had a kind of being that was sufficient to distinguish it from nothing, and it had its eternal immutable essence."[51]

Kenny's understanding of mathematical objects as eternal creatures of God is motivated largely by his understanding of Descartes' doctrine of the eternal truths. For Kenny, these mathematical entities have essences of their own that are distinct from God's essence, and they "stand in a causal relationship to Him"[52] as things created by God. This relationship must be the case; otherwise, as Gassendi points out, "it seems very hard to propose that there is any 'immutable and eternal nature' apart from almighty God."[53] And indeed, Descartes is clear that

> "The mathematical truths which you call eternal have been laid down by God and depend on him entirely no less than the rest of his creatures. Indeed to say that these truths are independent of God is to talk of him as if he were Jupiter or Saturn and to subject him to the Styx and the Fates."[54]

Descartes also tells Gassendi that the immutable essences and eternal truths "are themselves immutable and eternal, because God so willed, because he so arranged"[55]

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[49] Kenny (1970) 692-693.

[50] Kenny (1970) 697, emphasis mine.

[51] Kenny (1970) 699.

[52] Kenny (1970) 696.

[53] AT VII:219; CSM II: 221.

[54] AT I: 145; CSMK: 23.

[55] AT VII: 380; CSM II: 261.

Furthermore, Descartes writes in the Sixth *Replies* that the eternal truths depend "on God alone, who as supreme legislator, has instituted them from eternity"[56]. Thus, Kenny understands Descartes' immutable essences or natures as akin to the eternal truths: eternal substances that are distinct from God.

Kenny's account has its strengths. Kenny's interpretation of Descartes also accounts for Descartes' emphasis on the independence of mathematical objects from the physical extended world and from our minds. But despite its strengths, Kenny's account is not without its problems.

Kenny's Descartes endorses the following two things about mathematical objects. First, for Kenny's Descartes, mathematical objects are distinct, non-identical to and independent of both human minds and the actual physical world. Second, on Kenny's account, these mathematical objects have some form of existence and are creatures distinct from God, for they are under the control of God's will.

But by endorsing both of these claims, Kenny is presented with a problem for his interpretation when he has to locate these universals. To which class of created entities does these mathematical objects and truths belong? It is already clear on Kenny's account that as a creature of God, mathematical objects are distinct from and not identical to God. However, Kenny's interpretation of mathematical objects is that they are independent and distinct in nature and essence of both minds and bodies. Thus, his interpretation of mathematical objects seems to require some sort of third realm or type of existence.[57] Such a third realm

[56] AT VII: 436; CSM II: 294.

[57] In addition to strong suggestions to this effect made by Kenny himself when he compares the existence of Cartesian mathematical objects to a Meinongian subsistence (c.f. Kenny (1968) 155-156).

of creatures would conflict with Descartes' dualism stated in the *Principles*[58] in which he claims that created things fall into only two kinds: minds or bodies.

As an SR interpretation, Schmaltz's reading locates the true and immutable natures outside both human minds and material extension. But Schmaltz does not go the way of Kenny's Platonic forms. Instead Schmaltz understands these mathematical objects (i.e. eternal truths, natures, or essences) as divine ideas or volitions.[59] As Schmaltz himself notes:

> Whereas Kenny took [the doctrine of divine creation of the eternal truths] to reveal that Descartes identifies immutable essences with eternal substances distinct from God, I hold instead that it shows that [Descartes] identifies immutable essences with God himself, or more precisely, with laws that are not distinct from divine decrees.[60]

For Schmaltz, Kenny and others were misled by the comparison of eternal truths to other creatures, which seems to suggest that Descartes thinks the eternal truths are themselves creatures. However, Schmaltz argues, "despite the appearance, Descartes holds that these truths are not really distinct from God."[61] The mathematical objects are just like the eternal truths that God creates by his divine decree, which Schmaltz locates in God as part of his divine volition. Numbers and geometrical figures on this view are only *rationally*[62], not *really,* distinct from God, preserving the dualism of created things.

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[58] *Principles*1: 48; AT VIIIA: 23; CSM I: 208.

[59] While this understanding means that mathematical objects are not independent from the mind of God, for my purposes mathematical objects' independence of human minds makes Schmaltz's interpretation fall in the SR category.

[60] Schmaltz (1991) 135.

[61] Schmaltz (1991) 145.

[62] A rational distinction is one that is made merely in our mind by our reason. But things that are merely rationally distinct are still metaphysically, numerically identical, and not in reality distinct.

Although placing mathematical essences and truths in God allows Schmaltz to escape Kenny's problems, Schmaltz runs into serious problems of his own as a result. Descartes' commitment to divine perfection is the source of two such problems.

First, it is unclear how to understand these truths as not also necessary in the same way that Descartes' God is necessary if the truths are identified with God himself. Recall in a letter to Mersenne from above that Descartes claims God was free to not create these truths. So if God is simple, and these truths are identical with God, and the essence of God is that he has necessary existence (which presumably applies to his entire simple self) why are the eternal truths not also necessary to God? Yet according to Descartes, like all other created things the eternal truths are "no more necessarily attached to His essence than other created things."[63]

The second problem for Schmaltz's interpretation is the resulting view that while these truths are supposed to be identical to God, God is also their efficient cause. In the sixth set of replies Descartes writes that:

> There is no need to ask what category of causality is applicable to the dependence of this goodness upon God, or to the dependence on him of other truths, both mathematical and metaphysical. For since the various kinds of causality were enumerated by thinkers who did not, perhaps, attend to this type of causality, it is hardly surprising that they gave no name to it. But in fact they did give it a name, for it can be called efficient causality…[64]

This passage is problematic for Schmaltz's reading because Descartes thinks that God is not his own efficient cause.[65] Thus, on this reading we are claiming that God is both his own

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[63] AT I: 151-154; CSMK 25-26.

[64] AT VII: 436; CSM II: 294.

[65] "There is no need to say that God is the efficient cause of himself, for this might give rise to a verbal dispute." AT VII:110-111; CSM II: 80.

efficient cause and not his own efficient cause. Schmaltz's realistic reading of Descartes fares no better than Kenny's.

While there are other versions of SR accounts of Descartes' mathematical objects, such views[66] all suffer from some version of the problems confronting Kenny and Schmaltz. This is because they all face the same trilemma: Either mathematical objects are located (i) in God, (ii) in the material world or in the mind as ideas, or (iii) in some third type of existence. SR accounts see Meditation 5 as ruling out (ii), which motivates a SR strategy in the first place. Thus, any other SR account will be, like Kenny and Schmaltz, left with only two options: (i) and (iii). As we have seen, these options are deeply problematic.

Our discussion of Kenny's Platonic version of a SR-account revealed that mathematical objects can't have their own third type of existence, like Platonic forms in their own realm, lest we give up Cartesian dualism. And our discussion of Schmaltz's SR-account revealed that mathematical objects cannot be in or identical to God lest we give up divine perfection. What hasn't yet been explored is the possibility the SR accounts ignored: option (ii). This option then leaves us with only two possibilities: the two created natures. Either numbers and other mathematical objects are or exist in bodies or they are or exist in minds. This option is what WR-accounts of Descartes' mathematical objects consider.

2.2 Weak Realism

SR-accounts place much of their emphasis on the texts coming from the *Meditations* about true immutable natures and less emphasis on the passages found in the *Principles* on the nature of universals. However, there are other views that do the opposite. These views

[66] Marleen Rozemond (1998) has an account that is not too different from Schmaltz's. Rather than locating the mathematical objects in God in the form of his divine volitions, Rozemond locates them in God's intellect.

attempt to account for the *Meditations* while putting more emphasis on the conceptualism of the *Principles*. Because such views read Cartesian universals as abstract concepts, they have been traditionally labeled "conceptualist" views and have often been presented as an alternative to realism.[67] But, as I argue below, the conceptualist views still require mathematical objects to be prior in a way that entails a weak realism. WR-style views have been suggested or foreshadowed by commentators such as Martial Gueroult[68], Alan Gewirth[69], Lawrence Nolan[70], and Vere Chappell[71]. As Nolan has the most developed[72] account, I will use his account as the representative for WR accounts in general.

Nolan rejects the idea that universals like numbers or triangles exist in extended things, especially based on passages from the *Principles*. He takes very seriously Descartes' claims about numbers and universals being thoughts or ideas in our minds—even innate ideas[73]. On such an account numbers and other universals are concepts in our minds. According to Nolan these innate ideas should be thought of as dispositions that need to be

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[67] For example, see the discussion in Miller (1950). Also see Nolan (1998) and Chappell (1997)—both being anti-realist, conceptualist.

[68] See Gueroult (1968).

[69] See Gewirth (1970) and (1971).

[70] Nolan (1997a), (1997b) and (1998).

[71] Nolan (1998).

[72] Nolan's account of universals as concepts is a corollary to his account of Descartes' theory of attributes and is supposed to come apiece with his account of Cartesian natures. See Nolan (1997a), (1997b) and (1998).

[73] Cf. Nolan (1998) and Nolan (1997) 183, and ft. 15 on page 189-191.

awakened in our thought by the process of intellectual abstraction when occasioned.[74] The abstractive process effects the discovery of an innate idea, not the creation of the idea.[75]

Before describing Nolan's WR interpretation more fully, let me first emphasize how attractive this view is. WR can account for passages such as those in the Fifth Mediation that motivate others to go towards the problematic SR views. WR can account for the objectivity and immutability[76] of mathematical natures by making them fixed innate ideas in our minds, placed there by God. However, WR avoids those problems of SR by understanding texts such as those in the Fifth Meditation as claiming that mathematical natures are not subject to our will. Our minds are not inventing the ideas or natures of triangles or other mathematical objects; instead, we are thinking of a nature that isn't created

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[74] Nolan (1998).

[75] "Like many figures of the modern period, Descartes conceives abstraction in terms of selective attention. One difference between his conception and the view traditionally associated with Locke is that, in this case at least, we are not abstracting to a new more general idea; rather we are selectively attending to different aspects of a single idea." Nolan (1998) 167.

[76] For example see Nolan (1997a) 183, or Chappell (1997) 125:

> What I now wish to claim is that, for Descartes, God's creation of numbers and figures consists in his creation of minds containing the ideas of numbers and figures. Mathematical objects just are ideas according to the ontology of the *Principles*; according to the doctrine of innate ideas, God creates them by including them within the minds that are the direct products of his creating action. These ideas need not be consciously present to the minds in which they are housed, either from the beginning or at every moment thereafter. They may exist originally or intermittently as unconscious dispositions, as Descartes acknowledges in his *Comments* on Regius's *Programma* (VIIB.357f.,361). But doesn't this position destroy the objectivity of mathematics by making its objects differ from mind to mind? And doesn't it render these objects mutable, since minds grow and wither and the ideas within them change? No, for Descartes holds that God installs the same ideas in every mind that he creates; and no again, since the ideas that God makes to be innate in us are constant and never change. So the position I am attributing to Descartes secures both the objectivity and the immutability of mathematical objects.

by the mind. The nature is immutable, since we cannot change it, for it has been fixed in our mind by God. Furthermore, in creating us with these innate ideas, God can also be said to create these truths, and these truths depend on God just as any of his other creatures depend on him.

Thus, WR asserts a weak realism. The realism of the universal is not one that posits some mysterious universal out there in the world but relies instead on the reality of the concept in our minds. There is a universal; it is an abstract general idea or concept in our minds. This concept is supposed to be fully general and not at all particular, which makes this concept a *real universal* in the weak sense. This is how the idea of *twoness* in general can apply to all the (not innate) instances of things occurring in twos.

The machinery for Nolan's interpretation can be partially found in the Third Meditation. As Nolan reads it, innate ideas are ideas whose content is not determined by our will.[77] We cannot change the content or nature of innate ideas. Nolan also uses the distinction between formal and objective reality in the Third Meditation[78]. The *formal reality* of an idea is idea qua mode of the mind (or the idea insofar as it is a piece of mental stuff). In this sense, all of our ideas are mere modes of the mind. However, an idea's *objective reality* is determined by what sort of thing the idea represents. For example, the objective reality of a particular mode of my mind, say an idea of God, is God. However, for Nolan the objective reality of my idea of an idea of God is not God but instead an idea. Nolan uses this

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[77] Nolan (1997a) 178-181.

[78] AT VII: 40-41; CSM II:28-9.

to explain how we can have WR for universals. Ultimately for Nolan the "Cartesian universals are *ideas taken objectively*"[79].

Nolan uses this distinction to provide an account of how we can go step by step to a WR about universals. Let me fill out Nolan's discussion of *Principles* 1:59 (Descartes' example of the two stones).[80]

We start with the idea of a pair of stones. On Nolan's account the idea has a formal reality: it is a mode of my mind. The objective reality of this idea is that of the stones. Next, Nolan appeals to the "process of abstraction," where we can "focus on certain aspects of that idea [of the pair of stones] while ignoring – but not excluding—other aspects."[81] Doing so will allow us to distinguish the stones from their number in our thought: we just focus "exclusively on their twoness and ignore their extension, duration, existence, etc. But the distinction between the two stones and their twoness is confined to our thought; in reality, there is no such distinction."[82] At this point we have a different mode of thought, where the objective reality of this new thought is not that of the stones but is that of the idea of the twoness of the stones.

Then, Nolan goes on to explain, we can again abstract from this particular idea of the twoness of the stones (or from other ideas of twoness, like the idea of the twoness of

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[79] Nolan (1998) 173, emphasis mine.

[80] Nolan (1998) 173-175.

[81] Nolan (1998) 174.

[82] Nolan (1998) 174.

two birds, or of two trees) to an *even more abstract idea* of twoness in general.[83] This new level of abstraction presumably also is a different mode of thought, the idea of twoness in general, as opposed to the twoness of the stones, which we have abstracted out.

Now, from other things, such as birds, trees or fish, we can abstract to the ideas of the twoness of the birds, the twoness of the trees, and the twoness of the fish. And then once more, we can abstract away the birds, trees or fish from the twoness of the birds, the trees and the fish so that we have just an idea of twoness. Nolan explains, "this requires a further level of abstraction because we are no longer attending to the particular substances (i.e. the stones, the birds, or the trees), as we were in at the first level."[84]

To review:

At **Level 1,** we have an idea of a bird, and another bird.

Then at **Level 2**, we have an idea of the twoness of the birds.

Finally at **Level 3**, we have an idea of the idea of twoness in general: the universal—and its all to this numerically identical universal that we arrive each time. Regardless of what we started with at level 1 (it could be and idea of a bird and another bird, a tree and another tree, a fish and another fish, etc.) in the end, we arrive at the same universal, innate concept of twoness.

Though Nolan uses extended objects in his example of the abstraction that allows for an idea of the universal, Nolan's Descartes is not committed to the view that the necessary abstraction to occasion the innate idea could not come from non-extended things

[83] "First, after producing a number of rational distinctions concerning the twoness of various substances—e.g. the twoness of the stones, the twoness of the birds, the twoness of the trees, etc.—we can abstract one further level to twoness in general." Nolan (1998) 174.

[84] Nolan (1998) 174-175.

as well. According to Nolan, we can "arrive at universal twoness simply by iterating abstractions from a single instance. We can move directly from the twoness of the two stones to twoness in general simply by abstracting the stones."[85] But now it seems that we could also arrive at universal twoness simply by iterating abstractions from a single instance when we move directly from the twoness of two ideas, for example, to twoness in general simply by abstracting the particular ideas being counted.

The main purpose of the abstractive process for Nolan's conceptualist reading serves to bridge the move from particulars to universals, and to explain how the universals are innate—despite the fact that the particular instances where the universals apply are not innate. To this end, the conceptualist account opens up the abstractive process: it can originate from both extended and non-extended objects.

As for passages in which Descartes asserts that number is not "separate from the things which are ordered and numbered,"[86] Nolan appeals to his account of rational distinctions as the way Descartes abstracts (or selectively directs his attention). Here, again, an illustration would be helpful.

Consider my idea of God. The thing (or the substance or nature) conceived of in this idea of God *is* God. Next, consider my idea of an attribute of God such as his omnipotence. On Nolan's account, that thing or nature, which is conceived in my idea of omnipotence, is also God. The two ideas are ideas *of the same thing*. In the first case, I attempt to consider God's entire simple nature. In the second case, I may only attend to one particular aspect of God's nature; but God's omnipotence is only *rationally distinct* from God

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[85] Nolan (1998) 175.

[86] AT VIII A: 26; CSM I: 211.

himself. God is numerically identical with that thing which is omnipotent. The thing in each idea is one and the same. Things that are rationally distinct are metaphysically and numerically identical. In this way, Nolan is able to make sense of passages in which Descartes asserts, "number is not distinct from the things numbered"[87]. There are not two separate things we have in mind when we consider the two trees, and the twoness of the trees. The idea of the twoness of the trees is an idea containing the same content as an idea of the trees. On this account, the twoness of the trees is numerically identical *in re* with the trees themselves much like the omnipotent-thing is numerically identical *in re* with God. In each case, it is the same thing that is being thought about.[88]

WR accounts of Cartesian Universals such as Nolan's are not without their own set of problems. A first problem for Nolan's account concerns the number of innate universals that are held by our finite minds. Given the indefinite number of sides a polygon can have, and the indefinite[89] cardinality of even just the counting numbers, it would seem that our finite minds would have to have an infinite number of innate ideas. But this first problem is not the most serious.

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[87] AT VIII A: 26; CSM I: 211.

[88] "We distinguish the two stones in our thought from their number within out thought by focusing exclusively on heir twoness and ignoring their extension, duration, existence, etc. But the distinction between the two stones and their twoness is confined to our thought; in reality, there is no such distinction." Nolan (1998) 174. See also Nolan (1997b) for more on the numeric identity between things that are merely rationally distinct.

[89] Provided that our finite minds were allowed to inquire into such matters, Descartes thinks that at best we can only claim there to be an indefinite number of counting numbers. Strictly speaking, only God is infinite. All other cases we may be tempted to think of as involving something infinite are cases in which we "merely acknowledge in a negative way that any limits [that may exist] cannot be discovered by us" as opposed to positively understanding there to be a lack of limits as in the case of infinity. See *Principles* I:26-27; AT VIIIA: 14-15; CSM I: 201-202.

The most serious problem arises from Nolan's use of rational distinction in his view about abstraction. What is in Level 1 is merely rationally distinct from what is in Level 2: the two stones are merely rationally distinct from their twoness (or the twoness of the stones). Continuing further, if I again abstracted in the manner suggested by Nolan, what is in Level 2 is merely rationally distinct from what is in Level 3: the twoness of the stones is merely rationally distinct from twoness in general. I've merely abstracted out the stones. Abstracting out simply means that I don't attend to the stones, not that they are not part of the "thing" in the idea.[90] Similarly, the two birds are only rationally distinct from the twoness of the birds, and the twoness of the birds is merely rationally distinct from twoness in general. However, on a WR account, there is only one concept of twoness innate in our minds. It is the same general concept of twoness that we get in all the particular instances of the two-step iterative abstractions discussed above.

This claim to a general concept presents an awkward problem for Nolan's account of universals and mathematical objects that has gone unnoticed. Given that (a) the general concept of twoness is merely rationally distinct from *both* the twoness of the stones as well as

[90] For abstraction doesn't work in the way the conceptualist readings need:

> This intellectual abstraction consists in my turning my thought away from one part of the contents of this richer idea the better to apply it to the other part with greater attention. Thus, when I consider a shape without thinking of the substance or the extension whose shape it is, I make a mental abstraction. I can easily recognize this abstraction afterwards when I look to see whether I have derived this idea of the shape on its own from some other, richer idea which I also have within myself, to which it is joined in such a way that although one can think of the one without paying much attention to the other, it is impossible to deny one of the other when one thinks of both together. For I see clearly that the idea of the shape in question is joined in this way to the idea of the corresponding extension and substance, *since it is impossible to conceive a shape while denying that it has an extension or to conceive an extension while deny that it is the extension of the substance*. (From a letter to Gibieuf dated January 19, 1642. AT III: 175; CSMK 202. Emphasis mine.)

the twoness of the birds; and that (b) *both* the twoness of the stones is merely rationally distinct from the stones, and the twoness of the birds is merely rationally distinct from the birds; then it would seem to follow that *the birds are merely rationally distinct from the stones*!

To make the tension more obvious consider this more problematic case:

At **Level 1**, we have an idea of an idea of my Mind and God.

At **Level 2**, we have an idea of the twoness of the substances in my idea.

Finally at **Level 3**, we have an idea of the idea of twoness in general.

Recall, however, that what is at Level 1 is merely rationally distinct from what is in Level 2, thus, the two substances (God and Mind) are merely rationally distinct from their twoness (or the twoness of the substances, God and Mind). Continuing further, what is in Level 2 is merely rationally distinct from what is in Level 3. The twoness of the substances I'm thinking about (God and Mind) is merely rationally distinct from twoness in general. I've merely abstracted out the idea of the substances. But the universal at Level 3 (the concept of twoness in general) is merely rationally distinct from Level 2, so then the concept of twoness is merely rationally distinct from *both* the twoness of the substances as well as the twoness of the stones. But this would make Mind and God merely rationally distinct from the two stones!

This presents a very troubling problem for WR interpretations because for Descartes both Mind and God are **really distinct** from extension.[91] This distinction was the third Cartesian doctrine. There is a real distinction between extension, minds and God. If Nolan's theory of rational distinction is to be preserved, the abstract idea of twoness in general that I

[91] See for examples: AT VIIIA:13-14; CSM I: 201, ATVII:86, CSM II: 59, AT V 403; CSMK:381.

occasioned from the idea of two [extended] trees cannot then be the same abstract concept of twoness occasioned from another idea of two things.

2.3 How To Countenance Real Universals?

If a consistent account of Cartesian universals is to be given, it must avoid the problems of SR views and not locate them in a third realm or in God. A consistent account must also account for the conceptualism of the *Principles*, as the WR accounts do. Nevertheless, part of the problem for understanding Cartesian universals in a WR way is that it still allows for a realism, albeit a weak one. By giving an account of numbers that is so abstract and general, even if only as innate ideas or concepts, it opens the door to the same problems of SR that WR was attempting to avoid, namely that WR also comes into conflict with one of the three central Cartesian doctrines. The only difference is the exchange of the problems of the realism of platonic forms under SR for the problem of the realism of the concepts under WR. *The need to find a way to countenance the realism for Descartes, strong or weak, creates problems.*

No matter what the ontological status of the number is or where the number is located, all of these readings of Descartes understand the number as abstract and prior in the sense that it can be applied to or instantiated by anything, or that anything can represent its true immutable nature (even non-extended things). Whatever a number is, it is not a particular. The number two is twoness, not two stones, or two trees, or two rocks or two souls. While the disparate accounts may not agree on the status, nature or location of a number, they do agree about its general, abstract and universal nature. Any type of object, extended or unextended, can occasion the concept or express the nature or exhibit the form of twoness. These views all hold that numbers and their natures are not derived from other substances. Numbers and their natures do not depend on other things (with the exception of God)—and the accounts all agree that numbers do not depend on extension. But given the problems with realism for Descartes, an alternative is needed.

I propose that we start with the most promising view above: the WR account Nolan offers. Recall that Nolan successfully avoided SR problems by taking very seriously Descartes' claims in the *Principles* that numbers are modes of thought. The trouble for WR accounts comes in when the ideas of numbers become reified into abstract concepts that are distinct from our other innate ideas. This problem suggests a strategy for avoiding WR's problems: move towards a view in which the numbers reduce to an innate idea we have. There are at least a few options, but given Descartes' commitment to the veracity of our clear and distinct perceptions and the problems of locating numbers in God and Minds (*i.e.*, SR and WR), we should reduce ideas of numbers to ideas of extension.

This reduction would allow us to jettison the weak realism involved in abstract or truly general ideas of number in favor of a nominalism about ideas of number. This kind of nominalism would make all the so-called "universals" not universals but rather particular ideas and names that we make use of in a general, Berkelian sort of way. As for what those particular ideas are like, I think that for Descartes any idea of a number of things qua number reduces to an idea whose object is always extension. That is, ideas of numbers *just are* particular ideas of extension that we use a number-name to signify. As I show below, such a view would avoid the ontological problems that realist accounts face.

3. Nominalism About Universals: A Sketch

A nominalist account of Descartes' universals starts out in much the same way a WR account would, particularly with regard to mathematical objects. The WR accounts have several strengths with respect to how WR interprets many of the texts in question. As mentioned above, the problems with such accounts arise from the abstract or general quality of the idea they claim is the universal. And this quality is a subtle but ultimately important distinction between WR accounts and my account. Briefly explaining the similarities and differences between WR and my nominalist reading will be useful in filling out just what I am suggesting. It will also clarify any remaining confusions caused by both my attributing to WR accounts some form of realism and my contrasting of my reading as one that is opposed to nominalism found in WR accounts.

Indeed, many WR accounts conceive of their conceptualism as opposed to realism, by which they mean of a strong Platonic sort. Moreover, they also describe themselves as giving a nominalist account of Descartes.[92] But proponents of such accounts are quick to contrast their readings with the nominalism of empiricists such as Locke[93]. These proponents cite passages[94] in which Descartes makes it clear that we do not get our idea of mathematical objects empirically through our sense experiences: e.g. our idea of triangle is not an idea that we had to arrive at through triangular-shaped visual perceptions. It is true that my claim about Descartes' nominalism is closely aligned with an idea of nominalism typically held by the empiricists. But the empiricists disagree amongst themselves about our ideas of mathematical objects: the account I have in mind is opposed to Locke's and more

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[92] For more about the realism, conceptualism, and nominalism, see chapters 1-4 of Marilyn McCord Adam's book, *William Ockham*.

[93] Both Nolan and Chappell emphasize that they do not mean to suggest that Descartes thinks that "universal ideas are derived empirically—that the idea of twoness for example can be produced in our mind by abstracting from our adventitious idea of two stones…" (Nolan (1998): 173) nor are we to think that Descartes is like Locke in the sense that when we form an idea of two upon seeing two stones, "we are in effect creating that idea: if first comes to be in our minds by the process of abstraction" (Chappell (1997): 121). Rather the abstractive process effects the discovery of an innate idea, not the creation of the idea. See also Nolan (1998) 167.

[94] See AT VII: 382; CSM II: 262.

closely aligned with that of Berkeley and of Hume. The disagreement between Berkeley and Locke with respect to our ideas of mathematical objects tracks very closely my disagreement with WR about our ideas of universals. Locke thought that we could form or construct a general idea that we call 'two' that was neither of two trees, two triangles, two birds, nor two minds, etc., etc… But Berkeley denied the possibility of any such general idea, and instead insisted that it is the term or name that is general, not the idea. The idea is particular and is merely regarded as general. Berkeley thinks an abuse of language is the source of the problem for philosophers such as Locke who think they can form abstract ideas.[95] This is a view that Descartes shares. Descartes writes in the *Principles* that one of the causes of error

> is that we attach our concepts to words which do not precisely correspond to real things… people often give their assent to words they do not understand, thinking the once understood them, or that he got them from others who did understand them correctly.[96]

Using WR accounts of number, a number term such as 'two' becomes general by being made the sign or name of an abstract, general idea: the concept *twoness*. This idea is the same idea that we abstract to when we consider any of our ideas of two things. This applies indifferently for all particular instances of twoness we can think. For example, whether we start with an idea of two trees, two line segments or two minds, we can abstract away from the particular set of trees, line segments or minds in the three different ideas to a single, numerically identical, general idea of twoness. It is this general idea of twoness that the general term 'two' signifies.

On the nominalist reading, numbers ideas are ways of thinking about particular things. Nominalism will not explain the generality of a term like 'two' by appeal to a general

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[95] See section 6 of the Introduction to Berkeley's *Principles of Human Knowledge*.

[96] *Principles* I: 74; AT VII: 37-8; CSM I: 220-221.

idea or concept. A word becomes general when it is used to sign for a particular idea that we understand to resemble all other particular ideas to which we would apply the term. We call a particular way of thinking 'two' and then use 'two' to name all the other particular ways of thinking. When we mean for some particular idea to represent all the other particulars for which the term 'two' signifies, the particular idea is "regarded" generally and the same term used to name that idea also signs other particular ideas that are similar. For example, when we think of the two trees in a particular way, we may name that way of regarding as the 'two' way. Then when we think of two line segments in a similar way, we may use the same term ('two') to name that idea. There is no single, numerically identical idea named 'two'. The general term 'two' can be used to signify any one of these ideas—with the understanding that we are using that particular idea to represent, or stand in for all the other cases that the term 'two' may apply if we wish.

What separates Descartes from the likes of Berkeley is the nature of the particular ideas that are considered mathematical. For empiricists like Berkeley or Hume there is no true and immutable nature that we come to understand when we think about the "mathematical truths". And they think the ideas we come to have of so-called "mathematical objects" *do* require empirical experience and do not carry any necessity.

But Descartes is able to adopt a nominalism that is similar to Berkeley's and to Hume's while also maintaining that these mathematical objects have true and immutable natures. I agree with WR accounts about universals insofar as they deny that empirical experience with the world is a necessary condition to forming ideas of mathematical truths. Seeing a triangle drawn on a piece of paper or some other such empirical experience is not necessary to produce what Descartes is talking about when he writes of our understanding of the nature of the triangle. But I do not think an abstract innate idea of each mathematical object is necessary either. Our innate idea of extension can provide the true and immutable nature of all our ideas of mathematical objects. Each universal is merely a name used to signify particular ideas of extension. When we regard our innate idea of extension in certain ways, the true and immutable nature (i.e. the essence) of extended substance is the idea that is signed by a general name such as 'triangle' or 'two'.[97] Descartes seems to suggest as much in various passages. For example, in the fifth set of replies of the *Meditations*, Descartes writes:

> Not that there are in the world substances which have length but no breadth, or breadth but no depth; it is rather that the geometrical figures are considered not as substances but as boundaries within which a substance is contained.[98]

But boundary, a surface or line, is not a thing unless it is a way to think of the nature of extension.[99] But as was noted above, Descartes claims in the fifth meditation, whatever is true is something[100]. A true and immutable nature not only needs to be something, it must be a simple thing: an essence. But a general abstract idea cannot be of something that is simple in virtue of its general nature which must apply to different kinds of cases. Consider again what Descartes writes about a boundary or limit:

> We are abstracting, for example when we say that shape is the limit of an extended thing, conceiving by the term 'limit' something more general than shape, since we can talk of the limit of a duration, the limit of a motion, etc… since the term 'limit' is also applied to other things – such as the limit of a duration or a motion, etc., things totally different in kind from shape—it must have been abstracted from these as well. Hence, it is something compounded out of many quite different

[97] See Nelson (n.d.) for an account of how God's attributes receive their names.

[98] AT VII: 381; CSM II: 262.

[99] "This concept [body] is thus very different from that of a surface or a line, which cannot be understood as complete thing unless we attribute to them not just length and breadth but also depth." (AT VII: 228; CSM II: 160)

[100] AT VII: 65; CSM II:25.

> natures, and the term 'limit' does not have a univocal application in all these cases.[101]

Here Descartes describes *limit* as a compound idea: an idea composed of other ideas. An abstracted idea cannot be a simple idea—simple ideas being more clear and less obscure. Obscure ideas are the source of errors[102] as in the case of our confused ideas of 'cold' where we are unclear as to what is the real or positive thing that our idea represents.[103] Just as we can be confused in the case of obscure ideas such as 'cold' with respect to what is actually contained in our idea, we can be similarly confused about what is contained in ideas that we have obscured in our attempt to make them too general.[104] If we don't try to conceive of 'limit' too generally, 'limit' or 'boundary' is just a name for some particular idea of the nature of extension: a thing with length, breadth, and depth. So too, a triangle or the number *two* is a name for some particular idea of the nature of extension. This idea of extension is a simple idea we can conceive clearly and distinctly. It is an innate idea that God has placed in our minds when he created us. An anti-realist, nominalist account of universals is a possible interpretation of Descartes' mathematical objects.

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[101] *Rule* 12; AT X: 418-419; CSM I: 44.

[102] "I shall here briefly list all the simple notions which are the basic components of our thoughts; and in each case I shall distinguish the clear elements from those which are obscure or liable to lead us into error." (*Principles* I: 47; AT VIIIA: 22; CSM I: 208.)

[103] "And since there be there can be no ideas which are not as it were of things, if it is true that cold is nothing but the absence of heat, the idea which represents it to me as something real and positive deserves to be called false; and the same goes for other ideas of this kind." (AT VII: 44; CSM II: 30.)

And also "But the obscurity of the idea is the only thing that leads me to judge that the idea of the sensation of cold represents some object called 'cold' which is located outside me…" (AT VII: 234-5; CSM II:164.)

[104] See *Principles* I:63; AT VIIIA:31; CSM I:215. For a nice discussion of this passage, see Nelson (n.d.).

Chapter 3: Cartesian Counting? Go Figure.

0. The Motivations Behind A Counting Problem

The goal of the previous chapter was to show that the texts in Descartes that have traditionally been thought to recommend some form of realism about mathematical objects need not be read that way. These texts do not require Descartes to be interpreted as a realist. Moreover, a realism results in consequences that are in tension with the most fundamental principles of his philosophy. While the realisms invoked on Descartes' behalf have varied in the ontological grounding of the mathematical objects, these accounts all agree with respect to the abstract and general nature of these universals.

I sought to bring out this primary difference between the realist readings and the nominalist alternative I have sketched out over the non-abstract nature of our ideas of mathematical objects. Furthermore, under a nominalist reading the true immutable nature contained in any of our mathematical ideas is not the innate idea of the universals themselves. Rather the true nature of these ideas is our innate idea of the true and immutable nature of extension. Our mathematical ideas all derive from an idea of extension: there are no numbers that do not depend on our idea of the nature of extension. The idea of extension is prior to our mathematical ideas. Thus, metaphysically speaking, all of our mathematical ideas reduce to an idea of extension: the ultimate object of thought is always extended substance. The corresponding epistemic claim follows: all of our mathematical ideas must conceptually contain an idea of extension. Mathematical thinking depends metaphysically and epistemologically on a prior idea of extension such that if the idea of extension were lacking, the mathematical ideas could never naturally occur.

Such a nominalist position may be easier to defend when the universal or mathematical object in question is a triangle or some other geometrical figure, but the position is far more dubious to defend when the universal is a number such as *two*. While realists may not agree on the status, nature or location of a number, they do agree about its general, abstract and thus universal nature. Whether extended or unextended, any type of object can occasion the concept, express the nature or exhibit the form of twoness. To a realist our ideas of number need not ever involve extension. In slightly different terms, these realist views all hold that numbers and their natures are not derived from other substances. Numbers and their natures do not depend on other things (with the exception of God), so they do not depend on extension. Realists of all stripes can set aside their differences and unite on at least this: nominalist interpretations in which ideas of extension are prior to our ideas of numbers cannot be correct.

If numbers depend on extension, viz. if ideas of numbers contain some idea of extension, absent extension we would be unable to count things. Unextended things, by definition are not extended, and our ideas of them do not contain extension. It seems ideas of numbers would not apply to unextended things on a nominalist account for there is no extension from which the can be derived. But do we not count unextended things? Can we not think of a number of unextended things?

There is no reason to think that numbers do not apply to extended as well as unextended things equally. In fact it is perfectly reasonable to say that we can count or speak about the quantity of things such as value, time, souls, ideas and other unextended things. We do this kind of counting much of the time, so it would seem ludicrous to think that Descartes would deny that we can. And so much the worse for any nominalist interpretation of Descartes that thinks that he is committed to a view where our ideas of numbers are of extension only. Any case in which we count unextended things appears to present a problem for such a view. Common features of realist interpretations, sc. that numbers are not derived from extension, lead the realist to appeal to cases in which we count unextended things to generate for the nominalist what I call a *Counting Problem*. If this problem can be properly formulated, it would be a very compelling counterexample to my view.

Fortunately, there is no formulation of the Counting Problem that can adequately serve as a counterexample to nominalism, and what may seem like potentially problematic cases of counting are actually easily accounted for using the nominalist account. My argument starts by clarifying what is required for a case of counting to not involve extension (§1). The metaphysical status of the things being counted is only one of four different ways extension can be involved in counting. The others concern the modes in which we represent distinct things and the determination of a quantity. The realist needs to formulate a case of counting in which extension is not involved in any of the four ways. I discuss several cases that a realist might make in attempting to satisfy the necessary conditions for the Counting Problem (§2). These cases serve to illustrate the difficulties involved in formulating a Counting Problem and highlights the central hurdle facing the realist: the idea of a quantity requires extension. This hurdle emerges once the four ways extension can be involved in counting is made clear and we consider passages about what is involved in counting and measuring that realist have ignored (§3). In addressing the failure of Counting Problem attempts, I also show why Descartes might have thought that numbers depend on extension, *sc.* things that admit of part/whole relationships. But this view does not mean that Descartes thought we are unable to count unextended (and therefore indivisible) things. The solution I suggest (§4) is that unextended things can be counted by analogy to extension. This solution is based on Descartes' discussion and uses of analogy found in various places of his correspondence. To conclude (§5) I briefly revisit the attempted Counting Problem cases to illustrate how counting by analogy might work. This use of analogy as a way to determine numeric quantities not only accords nicely with the nominalism of Descartes but it is also able to account for Descartes' brief but surprising comments concerning the determination of the ratio of two intensive magnitudes.

1. Conditions For Formulating A Counting Problem

To demonstrate that extension is not required for counting, the realist needs to show that counting is possible even absent extension or an idea of extension. She just needs to provide a case in which we can have (for example) an idea of two things that does not involve extension. But what it is to "have an idea of two things that does not involve extension" is ambiguous. There are different ways to understand how extension is not involved in our idea of two things, and each is satisfied differently so that what might satisfy one understanding of the requirement many not satisfy the other. Let me address four of the most salient interpretations.

(1) We can understand this requirement to be focused one the metaphysical nature of the objects themselves (idea of two things qua nature of things). In this case, all that would be required is for there to be two unextended, metaphysically distinct things in the world that we can think about. We need not think of both of these things at the same time, nor do we have to recognize in our thought that they are not metaphysically identical. They may even be represented in our minds as extended. We merely need there to be two unextended things about which we may form ideas.

(2) We can understand this requirement to be about the objects as they are represented in our idea (idea of the things qua representation of the things). In this case, our representation of metaphysically distinct things may not involve extension. The things in our ideas, as we think of them or represent them, need to be unextended.

(3) We can understand the requirement to be about the idea of things as not numerically identical. To satisfy the requirement in this case would be to recognize numerically distinct things as such without using an idea of extension, or representing the things in an extensive way to facilitate the idea of their mutual distinctness. The things merely need to be recognized as non-identical while not represented with extension.

Finally (4) we can understand this requirement to be about the idea of the quantity (idea of two things qua two). This is less about the nature of the things or the way the things are represented, but more about the way their quantity (that is, their count or measure) is understood. It requires more than recognizing that there are distinct, non-identical things; but that number of distinct, non-identical things is two. This difference between these concepts is more easily recognized in cases where there are larger quantities. Consider a bucket of marbles. We can have an idea of there being many numerically distinct and non-identical marbles but not have an idea of how many marbles there are. In order to satisfy this requirement, we would have to be able to understand that the distinct things are *two* in number without needed to use an idea of extension.

In order for a realist to use a Counting Problem as a counterexample to my nominal interpretation of numbers, she needs to formulate a very particular type of case. To deny the metaphysical and epistemological relationship between our ideas of numbers and our idea of extension found in my account, the counterexample the realist is looking for needs to meet all four interpretations of what is required. These four interpretations can be reformulated into two general conditions, one I will call the **metaphysical condition** because it pertains mostly to objects being counted, the other I will call **epistemological condition** because it pertains to a particular type of idea we need to have about the objects.

The *metaphysical condition* covers the first three interpretations (the metaphysical nature, the representation of, and the non-identity of the object). It requires that the objects we count be unextended, that the objects cannot even be represented to us in our ideas through extension when we count them, nor can we use the extension to distinguish or individuate the objects we are counting from one another.

The *epistemological condition* covers the fourth interpretation (the idea of the numeric quantity of objects) and requires that when we consider the number of things we are counting, we must be able to think of them in terms of their quantity without using extension to represent their number or quantity.

Because the realist aims to deny that our ideas of numbers reduce to an idea of extension, she needs to produce a case in which the number idea cannot possibly be an idea of extension: a case where no extension is necessary at all. If a proposed counterexample fails to meet these two conditions, the example would not yet present the attempted challenge for my interpretation of Descartes. The first and second conditions are importantly different. The first condition requires that the things being counted are not extended and that we are able to think of them without including in our idea an idea of extension. If she fails to meet the first condition, then the idea of the number of (even merely partially) extended things would contain extension. But the realist wants to eliminate the possibility that it is extended parts that are considered when we count those things, so she must not fail to meet the first condition. It is not enough that what we end up counting is not an extended thing. The second condition requires that in counting unextended things, we are not making use of an idea of extension in the counting process, e.g. that we not index the unextended objects to extension when we count them. If she fails to meet the second condition, then the things we are counting are being represented by extension. Again, this idea would not demonstrate that extension is not necessary for number, so likewise she must not fail to meet the second condition.

This difference (the distinction between the metaphysical status of the things counted and the epistemic act involved in counting them) is crucial to my treatment of the Counting Problem. Regardless of whether or not an example meets the metaphysical condition, I argue that the realist will never be able to meet the second epistemic condition. What is most easy to overlook is the difference between any of the first three requirements (that constitutes the metaphysical condition) and the fourth one (the epistemological condition).

Some cases are better than others at satisfying the first condition. Instead of immediately turning to a discussion of what I take to be the best case, I will first consider some of the more problematic attempts. Going through several attempts to formulate the Counting Problem will be instructive and useful. The development of the cases will make clearer the four different ways of interpreting what is required for an adequate counterexample. The subtlety of these four distinctions, and the ease in which they can be confused is why there are far fewer clear-cut cases than the realist initially believes that can satisfy the first condition. And as I show below, *this difficulty of extracting extension from our ideas of the individuated, unextended things supports my explanation of how they could be counted in the first place.*

It is important that the realist not overlook the second condition because every attempted Counting Problem will fail the second condition. Extension is necessarily contained in out conception of things as counted or measured. Ultimately, I think that

Descartes would say an idea of extension is necessary even for thinking of things *as numerically non-identical*. That is: extension must be involved in our idea of two things in the third way of understanding an "idea of two things". This view would mean that even the first condition of a Counting Problem could never be satisfied. However, as I will not be arguing for this stronger claim here, I will allow that a realist can formulate a case for the Counting Problem that will pass the first condition.

2. Attempts To Generate A Counting Problem

2.1 Counting Monetary Value

A promising strategy for the realist may start with cases in which she counts abstract objects like value, measures of health, counts of abstractions (such as the numbers themselves) and so on. These cases seem to count immaterial, abstract objects. I will illustrate the general strategy of response for these types of cases using the example case of counting money.[105] The point of this example will be show that counting or measuring abstractions such as monetary-value do not make as strong and straightforward a counterexample as a realist may suppose. The ontology of the abstractions quantified are themselves questionable, and the nominalist account of Descartes that I am exploring would consider the ontological status of value or money to be as suspect as the ontological status of numbers. At the end of the day, a nominalist-Descartes would be more amenable to an account of that which we call "value" or "money" as reducing to proportions of ratios of commodities, or rates of exchange between various goods and services. The suspicion about the realist's abstraction, value, prevents it from being a clear counterexample that satisfies the metaphysical condition.

[105] I am grateful to Marc Lange for raising the "money-objection" and his helpful comments in our discussions on the topic.

Consider the way we can count money. At first glance, money doesn't seem to be extended, yet we can count money (indeed, many people often do so frequently and with great relish). But to successfully meet the first condition and avoid counting extended things, we must ignore what I will call "hard-currency." Hard currency is anything that we can *physically exchange* in our monetary transactions; this includes both commodity-currency and fiat-currency. Commodity-currency are things that have some intrinsic value like gold or silver bars. Fiat-currency are things that are used to represent an extrinsic value. These things usually do not have much intrinsic value or they are used to designate something of greater value. Examples of fiat-currency include bank- or government-issued notes, coins, cashier checks, bonds, and other legal tender. Fiat currency merely represents money, it is not money itself.

This difference between counting things and counting representations of those things can be illustrated with an (albeit overly simplified) example. Suppose we are back in the mean and harsh economy of the elementary school cafeteria. In this economy, the currency is chocolate pudding cups. Pudding cups get traded for not just hamburgers, but also solutions to homework problems. Everyone knows that Wimpy's mother packs a chocolate pudding cup in his lunch box every day. Having already spent today's pudding cup to bribe the hall monitor, Wimpy notices that Olive has not consumed her hamburger. Wimpy just loves hamburgers, so he offers to give her his pudding cup tomorrow for her hamburger today. She agrees, and Wimpy writes on his napkin a promissory note that transfers over his pudding cup.

The napkin, which is otherwise just a napkin, can be thought of as fiat-currency that represents the actual currency, the pudding cup. As fiat-currency, it represents the pudding cup: the napkin is not itself a pudding cup. Similar to the fiat-currency napkin, a fiat-currency dollar bill is only *called* a dollar in virtue of its representing a measure of money, one dollar. The bill itself is not a dollar, much like the napkin is not a pudding cup. When the realist points to cases in which we count money, she does not mean cases in which we count dollars using dollar-bills.

Bur the realist is not yet done formulating her example. She still has to explain what or how she is thinking when she is counting. It is not enough to declare that she is counting (abstract) dollars. (This could perhaps take care of only one of the three requirements that comprise the metaphysical condition). In order to count them, she needs to be able to think of them and she hasn't yet explained how that can be done without using extension. Even if a sparse Cartesian ontology ruled out the existence of money (that is that is a real thing or substance), Descartes allows that our ideas may at least contain something that is true or is caused by something with reality.[106] Perhaps "money" is similar to "limit" or "cold": ideas where the names we use to signify them have tricked us into thinking that they are things. But the realist doesn't need money to be a real substance; she just needs what is confused in the idea we call money to not be extended. Put another way: she needs to show that we are not representing money using extension (the second of four different interpretations). I do not think that the realist can. I will explain why I am skeptical the realist can rise to this challenge by returning to the school cafeteria.

Suppose they are now more sophisticated and moved from a system of direct bartering to using pudding cups as commodity-currency to facilitate more complex trades. Perhaps everyone had accounts that kept track of their wealth in units of CPCs (chocolate pudding cups). Unfortunately the school has been experiencing some hard-times. Bluto has

[106] See AT VII: 165; CSM II: 116-7.

ousted the hall monitor and inflation is rising. Now, Wimpy has to transfer 6 CPCs for just a bite of Olive's hamburger and homework solutions are billed at 1.8 CPC per line. The Math-letes have been busy managing everyone's accounts, and sending out regular account balances. Today, Wimpy's balance is down to 27.5 CPC.

How can we count how much money Wimpy has? Since their economy is built around the commodity chocolate pudding cups, at the end of the day, their money-units, CPC, is understood in terms of that commodity, *i.e.*, chocolate pudding cups. (Indeed, as Searle and many others have noted, a monetary system depends upon conventions to treat certain *things* as standing in for certain possibly abstract magnitudes, such as work or value.[107]) But counting his money in terms of CPCs is not the only option available to Wimpy. He can count his money in terms of the things he can trade them for, i.e. their purchasing power. How much money does Wimpy have? He has enough money for an essay 15 lines long, or for 4.5 bite of Olive's hamburger. Wimpy takes big bites, so he thinks he has enough money for a whole hamburger. In Wimpy's case we represent money either in terms of the currency used or the things we can trade for with them.

My skepticism about this objection from money is very similar to the cafeteria-economy case. I grant that our economy is much more complex and sophisticated than the school cafeteria's economy, but the basic principle remains the same. Regardless of how abstract or immaterial the objector thinks money might be, any understanding we have of our currency only makes sense of in terms of the extended forms of that currency. I understand the dollars in my account only in terms of the bills that I can get from an ATM and exchange for goods in trades. When I think about converting my dollars into Euros and

[107] Searle (1984).

the exchange rate, I think about how many bills in various denominations have to be traded for various denominations of other kinds of bills.

The realist is after this monetary value that is given in terms of different forms of currencies, so she can point out that it is the *monetary-value* that is quantified and expressed or represented in terms of one currency or another. The objector notes that exchange rates between currencies only make sense because there is some same value that we are talking about. For example, as I write this, one US dollar is worth .83 Euros. Imagine you have a choice between getting one million US dollars and getting 830,000 Euros. Ignoring preferred locales, costs of exchanging currencies, etc., you should be indifferent between these two outcomes because both choices represent the same thing: the value expressed by that money.

The realist wants to claim that this equivalence indicates that there is a quantity of value that is represented by various forms of currencies. Just as we can draw a distinction between money (dollars) and hard currency (dollar bills), she now carefully separates the monetary-value measured from_the form of currency used to measure it._The value of the money itself is distinct from the expression of value in terms of dollars, Euros, or yen (the money itself). And so the realist means to use this *monetary-value* to generate the counterexample. In this sense one might think of temperature or heat as analogous to monetary-value, and the temperature measured in terms of degrees in Kelvin, Celsius, or Fahrenheit as analogous to dollars, Euros, or yen.[108]

But, again, the issue is what is in the idea or understanding of the abstract notion of the value of money. Our understanding of this value exists only in terms of its various

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[108] For more about the measurement of temperature, see Hasok Chang's excellent book (2004), *Inventing Temperature*. The discussion in Chapter 4 about ontological and theoretical uncertainties regarding what was being measured are particularly interesting.

(equivalent) representations in particular monies or in terms of what else they can yield in a trade. Pretend that CPCs did not start as commodities and that CPC units are not pudding cups. Conceived of as just a measure of monetary-value, how do we consider the value? Wimpy's ways of considering the quantity of CPC was to either think of it as represented through the fiat currency (the number of chocolate pudding cups) or to think of it in terms of what they can be traded for, sc. the value was a hamburger, or a 15-line essay.

These responses are in line with a nominalism about abstract objects. What we have in mind when we say we are thinking of some quantity of value is some idea that we regard as representing other things we would consider acceptable trades: *w* dollars, *x* Euros, *y* guns, or *z* lbs. of butter. A nominalist views counting money as a case that does not pass the first condition because the abstract thing the realist wishes to count isn't a thing that we can think without using an extended representation. At the very least, the money-counting case is not as clear-cut as a realist may have initially thought. However, other possibilities still remain which we should consider on her behalf.

2.2 Counting People

Another way the realist may try to construct her counterexample is from counting distinct souls or minds. But if we think more carefully about what is available to Descartes in terms of how different minds may be counted, it will become clear to us why this also cannot be done without appealing to extension. We would fail in terms of representing the thing itself—the mind—thereby failing to satisfy the first condition; but also in terms of how to understand there being a number of minds, thereby failing the second condition. Saving the failure of the second condition for last, when I collect all the various versions of the objection from counting together, let us now turn presently to how formulating the objection from counting minds fails the first condition.

Counting minds is perhaps the most natural version of the objection that the realist considers. Given Descartes' real distinction between mind and body, the immaterial, non-extended mind is naturally one of the first places to look for something that we can count that is not extended. Despite the fact that we only know ourselves as mind-body unions, we have a clear and distinct idea of ourselves as thinking things, minds—and as minds, it is part of our essence that we are not extended. So far, this seems pretty promising for the satisfaction of the first condition. Clearly, the realist means to count the minds, which are not extended by their very nature; she does not mean to count the number of mind-body unions. She is thinking, when she counts people like Peter and Paul, of Peter's *mind* and Paul's *mind*—not the *embodied minds* of Peter and Paul, for she wishes to avoid the extension of their embodiment representing the mind of Peter and Paul. But this presents a problem. In what sense can we know or think about the other mind that we are counting?

We cannot have direct access to other minds—Descartes finds such a thought utterly unintelligible.[109] The problem this presents for formulating an objection that will pass the first condition is to find a way for us to come to know or think of the other minds we wish to count without having to think of them as the minds that are united to the bodies that we use to represent them. It seems the only way I can do this is to think of other minds as also embodied, and then come to know the minds indirectly though the extension that they are intimately joined with. However, the realist may protest that we can have an idea of mind (for example like our own) directly; and a clear and distinct perception of that idea of our mind would include its immaterial, non-extended nature. The mind is what the realist is

[109] In a letter to More, Descartes writes that "sensory nerves so fine that they could be moved by the smallest particles of matter are *no more intelligible to me than a faculty enabling our mind to sense or perceive other minds directly*." (AT V: 341; CSMK: 372; emphasis mine)

interested in counting. What the mind is can, at least in one case in particular, be known entirely without being represented through extension. This strikes the realist as enough for her purposes, at least insofar as her present interest is in passing the first condition. All she needs to show to satisfy the first condition is that the things we are to count can be known without having to be represented through extension first, and she has done this: we can clearly and distinctly perceive (our own) immaterial mind. We can know our mind directly. Once we do, even if we don't know Peter's mind and Paul's mind directly, we can think about their minds being like our own.

Here, again, upon further consideration, things turn out to be more complicated than they seem. In this formulation of the objection, we are interested in counting minds: it could be Peter's mind and Paul's mind, but it also could be Peter's mind and my mind. We are not counting my mind and my mind. I need to be able to have the concept of more than one mind, but without locating them in different regions of extension on pain of failing the first condition. The problem for the realist still remains. I can't think of other minds qua other without appealing to extension. Put in different terms, without being able to appeal to extension, the realist has a problem of individuating minds from one another. Whatever difficulty we may have separating our own mind from Peter's without appealing to extension, it is even more difficult to think that Peter's mind is separate and distinct from Paul's mind without appealing to extension. Without direct knowledge of other minds, in virtue of what could we say that those are two distinct minds?[110]

[110] As an interesting side note, if we allow ourselves the possibility of knowledge of other distinct minds through their embodiment, Descartes thinks the next step is learning how to tell there is an embodied mind there as opposed to a mere animal that has no soul. The crucial difference between a human and a mere animal is the human's ability to use signs or names to signify our ideas. Our ability to use language—to assign names for our purely intellectual thoughts—is distinctive. Descartes does not mean just words that we say and hear or write and read, but also gestured words; but no matter the form, all these words and

2.3 Counting Our Ideas

Based on the two previous cases, the realist may now shift her strategy—realizing that she can avoid those problems and issues by formulating her object by using the ideas of her own mind, to which she does have direct access, and is by its very essence not extended.

If she formulates her counterexample based on counting ideas in her own mind, she will not have to worry about the first condition, for the thing that she intends to count, her ideas, are not extended. However, as before, she will have to avoid counting ideas that contain extended things. Since the nominalist is claiming that ideas of numbers of things are ideas of extension considered in particular ways, she needs to be able to find a case where no idea of extension is involved in order to formulate a successful case. Formulating a counterexample using her own ideas, there are two options open to her. She can either base her count on the duration of thoughts or on the content of thoughts. I will consider each below.

2.3.1 Counting Ideas—With Respect To Time

The nice feature of formulating a counterexample based on counting the time that thought lasts is that the realist can use just a single thought, like the thought, "I am: I exist". This is a promising start for two reasons. One, the content of the thought does not contain any extension. The thought is just a clear and distinct perception of my mind. As

names convey to us that its source can only be another mind. No matter what form the words are conveyed (either we see the writing or gestures, hear the words spoken, or feel the braile or gesturing of the other person's hands) we known them only as they are conveyed to us through our bodily sense modalities. The other mind, when we discover that another mind is there, is known to us only indirectly through representations that necessarily involve extension: both in thinking of it as being distinct from our own mind in that it occupies an other region of extension, and also in our method of thinking of it as another embodied mind as opposed to a mere animal.

mentioned above, minds are essentially, *i.e.*, necessarily, not extended. This leads to the second reason: the thought itself, a mode of the mind, is also not extended. Descartes is very clear that thoughts, as modes of the mind, are not extended.

Despite these promising features, I argue that the case of counting time fails to satisfy the first condition for it is analogous to the case of counting money in the following way. Just as money is understood in terms of currency, which is in turn understood in terms of extension (fiat-currency); so too is duration understood in terms of time, which is in turn understood in terms of extension (its motion). Descartes, in his discussion of time and duration, writes that:

> For example, when time is distinguished from duration taken in the general sense and called the measure of movement, it is simply a mode of thought. For the duration which we understand to be involved in the movement is certainly no different than the duration involved things which do not move. This is clear from the fact that if there are two bodies moving for an hour, one slowly and the other quickly, we do not reckon the amount of time to be greater in the latter case than the former, even though the amount of movement may be greater. But in order to measure the duration of all things, we compare their duration with the duration of the greatest and most regular motions which give rise to years and days, and we call this duration "time". Yet nothing is thereby added to duration, taken in this general sense, except for a mode of thought.[111]

Time is what we call measures of duration, much like currency measures money. And what our idea of times amounts to is an idea of motion. This is significant because motion is an attribute of extension[112], exclusively[113]. Time is measure of duration with respect to a regular motion, like the motion of the sun across, the sky, or even the pendulum of a clock. Regular motions of bodies represent time and duration to us in the way that we

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[111] *Principles* Part I: 57; AT VIIIA: 27; CSM I: 212.

[112] *Principles* I: 48; AT VIIIA: 23; CSM I: 208-9.

[113] *Principles* I: 53; AT VIIIA: 25; CSM I: 210-11.

might think printed bits of paper represent currency or money. Just as we cannot have an idea of money without having an idea of its extended representation, so too we cannot think of time without thinking of the motions of bodies that represent it. Thus, ideas of duration or time, for Descartes, depend on ideas of extension. In a letter to More, Descartes even writes that it "involves a contradiction to conceive of any duration intervening between the destruction of an earlier world and the creation of a new one"[114] –further suggesting that Descartes thought extension to be a necessary part of our conception of duration.

2.3.2 Counting Ideas—With Respect To The Content

The second strategy the realist can adopt to formulate a case is to count ideas based on the content. Again, for the similar reason previously mentioned, counting ideas has the advantage of counting a thing that is, as a mode of the mind, not extended. Burman reports Descartes as saying, "Thought will be extended and divisible with respect to its duration, since duration can be divided into parts. But it is not extended and divisible with respect to its nature, since its nature remains unextended."[115] The only thing the realist needs to be concerned with is finding suitable ideas to count, *i.e.*, ones that do not involve extended things.

One possibility she might try is to use sensory ideas, like ideas of sounds, to serve this end. There are two ways we can use the content of ideas as things we count. She can either count the number of ideas we have and individuate them based on content, or she can count the number of things within the same idea. I will start with the former.

[114] AT V: 343; CSMK: 373.

[115] AT V: 148: CSMK: 335.

An example of how she can count the number of ideas based on sounds is to individuate the ideas based on the different sounds we hear—examples include: the different notes of a song, where each different note or sound is a different idea we count; or perhaps the different sounds of different voices singing in a choir are the different ideas we can count. Each of these cases presents its own challenges. If the realist attempts to count the notes of a song, she will run back into the same problems concerning our conception of time from above. To conceive of the different notes, *i.e.*, to individuate one from another, or even the same note that occurs more than once, would be to notice the changing succession of notes that make up the tune. The idea of succession and time requires the idea of the motion of bodies. It seems the realist is best to avoid a successive series of ideas. The other example given, the many different voices of a choir, each constituting a different idea that one can count, is also problematic. In this case the problem is with the individuation of one voice from another. Perhaps one voice is higher, and the other lower. Would Descartes have to grant that those can be individuated and counted? In the far less distant past, P.F. Strawson[116] has considered a very similar problem regarding whether one could individuate and reidentify sound particulars without appeal to a space.[117] While there are some large differences in their respective projects, I think there are enough similarities that Strawson's position is useful in at least suggesting a possible Cartesian response. The individuation of different sound-ideas relies on making a comparative or relative judgment: one note being higher than the other. Strawson insists that such comparisons seem to rely

[116] Strawson (1959).

[117] I thank Bob Adams for suggestion here.

on a spatial analogy[118], for "relations between elements in respect of the auditory analogue of the spatial dimension cannot be presented simultaneously, all at once… two auditory elements cannot be heard all at once as at a certain auditory distance from one another" and "the momentary states of the sound-patches of the auditory scene do not audibly exhibit the auditory analogue of spatial relations to each other at a moment."[119] Strawson does not think it is possible to individuate sounds that are at the same moment without making an appeal to extension. And I think this is also what Descartes would claim. Perhaps a suggestion of such a view can be found in a very brief comment Descartes makes about comparing intensive magnitudes in Rule 14 where he writes that we cannot compare shades of white without imagining shapes of white patches:

> One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceed the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape.[120]

I will take up this surprising remark about intensive quantities in greater detail elsewhere. Let us move on for though I think Descartes might say that we often measure abstract quantities using physical instantiations even if only imaginary, there is still the more promising option of using the content of a single instance of her ideas open to the realist. Taking the lessons we have just learned from the sound cases, the realist realizes that the content that is to be counted in her single idea (so she can avoid the issues of time and succession) should be as distinct from one another as possible (so she can avoid problems

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[118] Later, I will return to what Strawson calls "spatial analogy" and what I have called "appealing to an idea of extension", and which just is to make something "extended by analogy" as Descartes would say.

[119] Strawson (1959) 79-80.

[120] AT X: 441; CSM I: 58.

that resemble those found in the case of sound) while still being unextended by their very nature. Such a case would be the strongest possible form of her objection.

2.4 The Strongest Case: Counting Substances In An Idea

The strongest version of the realist's argument is to start from an idea that requires no extension to represent the objects at all, one where the objects by their very nature are not extended, but are very clearly distinct from one another. We can have clear and distinct ideas of two distinct non-extended things: our own minds, and God. All the realist has to do is find a case where we think of both of these two things in a single thought. There are many possibilities, like "I'm not god" or "I'm distinct from god". In such a case, there are two different things in the thought: the "I" and God. These two natures are so distinct, there is no question about their individuation. Thus, in this single unextended thought, the realist can count two (necessarily) unextended things. This formulation of her counterexample satisfies the first condition. But as I will argue below, it does not satisfy the second condition that they be counted without any appeal to extension. In fact, no case can ever satisfy the second condition.

3. The Problem For Counting Problems: Part-Whole Relations

There is a crucial set of texts that Realist interpretations have so far ignored. These texts appear in Descartes' earlier manuscript, *Rules for the Direction of the Mind* (*Rules*). In this section I will review what Descartes writes about numbers in the *Rules*, and then explain how it connects number to the essence or principle attribute of extension.

Simply put, for Descartes, ideas of numbers are ideas of parts, and parts are conceptually entwined with divisibility. In Rule 16 of the *Rules*, Descartes reminds us that numbers have "a dual function" which is "sometimes to express order, sometimes measure."[121] Descartes writes in Rule 14 regarding the two that orders differ from measures in that when we are said to be *counting*, we consider the order of the parts in relation to the whole; but when we are said to be *measuring*, we are regarding the whole as being divided up into parts:

> By 'dimension' we mean simply a mode or aspect in respect of which some subject is considered measurable. Thus length, breadth and depth are not the only dimensions of a body: weight too is a dimension--the dimension in terms of which objects are weighed. Speed is a dimension--the dimension of motion, and there are countless other instances of this sort. For example, *division into several equal parts whether it be real or merely intellectual division is, strictly speaking, the dimension in terms of which we count things*. The mode which gives rise to number is strictly speaking a species of dimension, though there are some differences between the meaning of the two terms. If we consider the *order of the parts in relation to the whole, we are then said to be counting*; if on the other hand, we *regard the whole as being divided up into parts, we are measuring it*. For example, we measure centuries in terms of years, days hours, minutes; if on the other hand we count minutes, hours days and year, we end up with centuries.[122]

An intuitive way to grasp Descartes' distinction between count and measure is to consider what will held fixed and what will be allowed to vary. When we are counting, we fix our units of measure and it is the whole (the sum or total) that we allow as a variable quantity. In contrast, when we are measuring, we start with a fixed quantity, and it is the unity with which we use to measure it that we may vary. Descartes' example of a century is helpful for illustrating the subtle distinction.

When we are said to be *counting* something, we are thinking about the how the parts of some total or larger thing relate to the whole thing. Thus, we first start off with the parts, like minutes, hours, days, or years, and we think about its relationship (a relation of order) to some whole that the parts come to comprise, their total, like a century. On the other hand,

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[121] *Rule* 16, AT X: 457; CSM I: 68.

[122] *Rule* 14, AT X: 448; CSM I: 62. Emphasis added.

we would be *measuring* something if we first start off thinking of a thing as a whole and then consider (in a relation of measure) the various ways we can divide it up in to parts. In this case, we could measure a century in various ways, depending on what kind of division of parts we would use, it could be divided into (and thereby measured in terms of) minutes, hours, days, or years. Descartes thinks that having parts is preconditioned on divisibility:

> Now tangibility or impenetrability in body is… not a true and essential differentia such as I claim extension to be. Consequently, just as man is not defined as an animal capable of laughter, but as a rational animal, so body should be defined not by impenetrability but by extension. This is confirmed by the fact that tangibility and impenetrability *involve a reference to parts and presuppose the concept of division or limitation;* where as we can conceive a continuous body of indeterminate size, or an indefinite body in which there is nothing to consider but extension. [123]

The condition that gives rise to a number, be it a count or a measure, is the ability to think about things in terms of wholes and parts; in other words, divisibility.

The essence of extension, what separates it from all other substances, is among other things its divisibility. There are many passages in which he connects divisibility (i.e. having parts within parts) with the essence or nature of extension. Here are just a few examples:

> For example, *the nature of a body includes divisibility* along with extension in space, and since divisibility is an imperfection, it is certain that God is not a body.[124]

> [W]hile to extended substance belong size (that is, extension in length, breadth, and depth), shape, motion, position, *division of component parts* and the like.[125]

> [A]nd *divisibility is contained in the nature of body*, or of an extended thing (for we cannot conceive of any extended thing which is so small that we cannot divide it, at least in our thought).[126]

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[123] AT V: 269; CSMK: 361.

[124] *Principles* I: 23; AT VIIIA: 13; CSM I: 201.

[125] *Principles* I: 48; AT VIIIA: 23; CSM I: 208-9. Emphasis mine.

[126] AT VII: 163; CSM II: 115.

Furthermore, *extension is differentiated from all other substances through its divisibility*. That is, what makes extension uniquely different and unlike other substances is that extension alone is divisible. It is how we know mind and body are really distinct. Again, Descartes makes such claims in numerous texts, below are just a few examples:

> For example, the nature of a body includes divisibility along with extension space, and *since divisibility is a imperfection, it is certain that God is not a body.* [127]

> As for the faculties of willing, of understanding, of sensory perception and so on, these cannot be termed parts of the mind, since it is one and the same mind that wills, and understands and has sensory perceptions. *By contrast, there is no corporal or extended thing that I can think of which in my thought I cannot easily divide into parts; and this very fact makes me understand that it is divisible. This one argument would be enough to show me that the mind is completely different from the body*, even if I did not already know as much from other considerations.[128]

> Remember that nothing has no properties, and that what is commonly called empty space is not nothing, but a real body deprived of all its accidents (i.e. all the things which can be present or absent without their possessor ceasing to be). *Anyone who has fully realized this, and who has observed how each part of this space or body differs from all others and is impenetrable, will easily see that no other thing can have the same divisibility, tangibility, and impenetrability.* [129]

But how is it that numbers express the relations of order and measure among parts and wholes? Let me attempt to give an account of how numbers and extension can be related through the part whole relationship as I've laid it out above. In making this account of numbers or counting clearer, I think that it is important that I not saddle Descartes with a view that would have been implausible even by his own lights, but I also wish to render the view tenable even from our own contemporary point of view. To this end, the following remarks are meant to both explicate how I understand Descartes' views and also respond to

[127] *Principles* I: 23; AT VIIIA: 13; CSM I: 201.

[128] AT VII: 86; CSM II: 59. Emphasis mine.

[129] AT V: 401; CSMK: 381. Emphasis mine.

a contemporary philosopher who is wary of numbers entailing a composition relation as opposed to a mere non-identity relation.[130]

In terms of the connection of number to extension, the central connection comes from the fact that the part/whole relation depends on the concept of divisibility, which is the essence or nature of extension. Here, let me start of by clarifying that I think Descartes means to say that in thinking numerically, we must make use of a *conceptual* whole, with conceptual parts (not that the things we are thinking about numerically actually be a composite whole, metaphysically). For example, when we count all the objects in a room, the whole being assumed is "all the objects in the room". Thus the things being counted, the pluralities, need not actually constitute some metaphysical whole: in fact, they may very well be decidedly not parts of a whole at all. What I think Descartes means is that we conceive of there being some whole, total or sum that we regard as having within it all the divisions or distinctions that are the parts, elements, or members counted.

So let us suppose you set out to count some things. Which things are you going to count? In giving *any* reply at all (e.g. blue cars, sheep, marbles, or *that* plurality), you will have done what Descartes would be claiming is to regard as a whole: you will have picked out a single (whole) group that you wish to enumerate such that anything you do count is only counted in virtue of your thinking that it belongs *in* that group. This is all that is meant by conceptualizing something as a whole. The whole that gets assumed just is all the objects getting counted or measured. Think of this as partitioning the world into two types: the things you are/will count and the things that you will not count. The partition determines all and only the things you are interested in: it individuates what you are interested in from the

[130] I have Laurie Paul to thank for her comments and discussion on this matter.

things you are not. By doing this individuation, you would be thinking of the things you are interested in *as if it were* a single thing or collection. This is why it is so crucial to the concept of numbers: *it is only in virtue of something being a part of that relevant partition that it gets counted.* If you did not think of it as a part of that whole (i.e. the group of things you are interested in) you would not count it.

I think this makes good sense of the passage in Rule 14 on counts and measures. When a number is used to express a count, it functions like an ordinal: it expresses the order of the parts with respect to the whole series, or however far along the series you are at that moment. That is to say, when you count your third marble as you throw it into your bucket, it depends on there also being a first marble and a second marble. Otherwise, it wouldn't be a third marble; it would have been marble, marble, marble. When a number is used to express a measure, it functions like a cardinal: it expresses the whole as divided up into parts. If we were to measure our bucket of marbles, we would treat the total number of marbles as the whole and find that it is divisible into 3 unit-parts. What is most important is that in Rule 14, he connects numbers with thinking about parts and wholes. The condition that gives rise to number, be it a count or a measure, is the conception of things in terms of wholes and parts; *vis*, a conception that requires something divisible. It is this that requires numbers to be connected with extension.

But perhaps, still skeptical of this type of thinking, you think the notion of *difference* is what is really crucial to counting. If you are going to count a plurality, one way you might identify that plurality would be to say, "I'm going to count this thing, and this thing, and this thing." But when you do that, crucially, what you are saying is that this thing is different from this thing, which is also different from this other thing. And so the difference relation is doing most of the work. Perhaps you would be effectively creating (in a non-metaphysically loaded way) the relevant plurality or partition by identifying the various distinct things that you are going to count. Thus the notion of difference is what you are going to count by and it is also the thing that is underlying how you characterize the plurality. So the notion of parts and wholes I was talking about earlier is only playing some kind of secondary role: it's rather otiose, because it is difference that is doing all the work.

I think Descartes would agree that distinction or difference is what gets you numerical distinction (or the semantic condition that there is a multitude or number of things)—in fact I think he says as much in the *Principles* when he discusses real, rational and modal distinction.[131] But I this is not what is at issue. The view about numbers that we are discussing is about the concept of the various numbers we have qua numeric quantity. So while the difference relation is what can get you numerical individuation, for Descartes it is not enough to get you particular numerical concepts like *two*. *Two* is not the same concept as *different*. For example, we would not count, "one, different, three," but instead we count, "one, two, three".

What is at issue is how we must think of things numerically. Even if we grant that is only by using the notion of difference or distinction that we can tell there are multiple things out there in the world, this is not yet the same as being able to think the number of things are there. (The thought that x and y are different things is not yet to have the thought that x and y are two things.) As for the priority of distinction, I think even though you start with only things that are different, when you go to count them you still have to partition them into the relevant group of distinct things that you mean to count which is to still think of them as a whole in the Cartesian sense.

[131] See AT VIIA: 28; CSM I:213.

As no other substance but extension is something that strictly speaking admits of conceiving in a parts/whole way. If the dual function of numbers is to express the relations of order and measure among parts to wholes, then that severely limits numbers to expressions of various relationships that hold of only extension. So strictly speaking ideas of number are ideas of extension, and no adequate counterexample the realist seeks can be generated for Descartes.

But a question still remains, given that our ideas of numbers are ideas of extension how might Descartes explain how we count things that are not extended?

4. Counting Indivisible Things By Analogy.

The discussion the first and second condition of the Counting Problem above already suggests how my reading of Descartes would account for counting unextended things. We count unextended things *by analogy* to extension. That is, when consider unextended things *as if* they were extended. We can do this by indexing them to (or representing them with) some extension. The extension that we use to stand in their place allows us to think of the unextended things as if they had parts. In this sense, any idea of a number still, strictly speaking, involves an idea of extension: either it is the extension of the extended things counted, or it is the extension used to represent the unextended things counted.

Though Descartes does not directly address this counting per se, we can piece together how this can be done from a couple of different letters he wrote to More in which he describes the use of analogy. For example:

> Now just as we can say that health belongs only to humans beings, though by analogy medicine and a temperate climate and many other things also are called

> healthy, *so too I call extended only what is imaginable as having parts within parts, each of determinate size and shape—although other things may be called extended by analogy.*[132]

Strictly speaking only what is imaginable as having parts within parts is what can properly be called extension, other things (unextended things) can be called extended by analogy. What do this mean? Consider what Descartes writes in a letter to More about God's extension. Descartes writes that:

> For my part, in God and angels and in our mind I understand there to be no extension of substance, but only extension of power. An angel can exercise power now on a greater and now a lesser part of corporeal substance; but if there were no bodies, I could not conceive of any space with which an angel or God would be co-extensive.[133]

And then a little bit later he writes that,

> You seem here to make God's infinity consist in his existing everywhere, which is an opinion I cannot agree with. I think that God is everywhere in virtue of his power; yet in virtue of his essence he has no relation to place at all."[134]

So while Descartes is very clear that metaphysically speaking God is not extended, we can think about God's power as being extended by thinking about the parts within parts of the extended word as being places on which God's power can act. This is a way in which we may think of God as extended by analogy. God is not extended: his essence has nothing to do with extension. However, we can think about God in extended ways by thinking of the extension itself as representing, in this case, the power of God. Since "in God there is no distinction between essence and power,"[135] this is to think the essence of God through this extension. In the letter, Descartes is warning More against confusing ideas of God's

[132] AT V: 270; CSMK: 362.

[133] AT V: 342; CSMK: 372.

[134] AT V: 343; CSMK: 373.

[135] AT V: 343; CSMK: 373.

extension of power with the nature of God himself, but for my purposes, it shows nicely how, despite the dangers of confusing people like More, unextended things are thought of as extended by analogy. There are other examples of this type of discussion in Descartes' letters to More[136], but this appeal to analogy also appears in Descartes' discussions with other people. For example in a case reported by Burman in which Descartes makes similar claims about God being extended, thought his time in terms of his duration:

> Thought will be extended and divisible with respect to its duration, since duration can be divided into parts. But it is not extended and divisible with respect to its nature, since its nature remains unextended. It is just the same with God: we can divide his duration into an infinite number of parts, even though God himself is not therefore divisible…[137]

There, Descartes indicates that it is because duration is divisible into parts that allows for thought and God to be extended, though neither are extended by their nature.

These passages show that while something may not be extended with respect to its nature, we can nonetheless think of them as extended by analogy and thereby consider the unextended things as if it were divisible or having parts within parts. By doing so, we can

[136] "Nothing of this kind can be said about God or about our mind; they cannot be apprehended by the imagination, but only by the intellect; nor can they be distinguished into parts, and certainly not into parts which have determinate sizes and shapes. Again we easily understand that the human mind and God and several angels can be at the same time in one and the same place. So we clearly conclude that no incorporeal substances are in any strict sense extended. I conceive them as sorts of powers or forces, which although they can act upon extended things, are not themselves extended..." (AT V: 270; CSMK: 361.)

"I have already replied to what follows by observing that the extension which is attributed to incorporeal things is an extension of power and not of substance. Such a power, being only a mode in the thing to which it is applied, could not be understood to be extended once the extended thing corresponding to it is taken away." (AT V: 343; CSMK: 373.)

"Moreover my mind cannot be more of less extended or concentrated in relation to place, in virtue of its substance, but only in virtue of its power, which it can apply to larger or smaller bodies… " (AT V: 347; CSMK: 375.)

[137] AT V: 148-9: CSMK: 335.

think of non-extended things as having parts and being divisible through its representative. Once we consider the relationship of the parts to the whole, we can understand the non-extended things in terms of its number (*i.e.* count or measure).

With this in hand, now let us revisit the attempt to find an appropriate example that will generate the counting dilemma, and see how this view I have just sketched out can be applied

5. Counting Problems Accounted.

All of the attempted counting problem examples can be handled the same way for it does not matter what it is that anyone is counting, even if the things being counted are not extended. It will be true in all cases that we understand the things numerically though ideas of extension, even in the realist's cases when she attempts to count unextended things. If she wants to understand the idea of a quantity of something, she has to understand it in some sort of part to whole relation. But this relation is just what it means to understand something as divisible: that it can have parts. Extension is the only substance that is divisible. So in all other cases, when the things we are counting are not divisible according to their own nature, if we are to think of them in terms of a count or measure, we must represent them as extended things and use the division of the extended representations to assist us.

In the case of money, what we think of as the value of the money that can be measured in terms of Euros or dollars as the whole (let us represent it as a line). To think of the value in terms of dollars is to think of the line as divided into segments of one size; and to think of the value in terms of Euros is to think of the line as divided up into segments of a different size. Alternatively, we can start with some particular sized segments and think about how many of them have been concatenated together to form the particular line. This would be like asking how many dollars comprise or make up the whole monetary value we are interested in.

The same application of the basic ideas applies to all the other cases. If we want to count the number of minds in the room, we think of how many parts are in the whole, the collection of the parts. And, again, the whole being assumed is all the minds in the room. For example: We can think of the minds in the room as being a jar of marbles, and each mind being a marble that we will need to count. But just like the money case, we can also use the line as the total or whole of all the minds, and some particular lengthed line segments as the parts that represent a mind. In the case of time or duration, again for simplicity's sake, we can think of the whole duration as a line and measure it according to some particular lengthed line segment that represents a unit or part of time. To count sounds, we can consider the whole tune or song a line, and think of each note in the sequence as a segment that is part of the line. There is always part/whole relationship (a relation of order or measure) involved in understanding things in terms of quantities.

So too, when we consider the things we think of in a single idea like "I am distinct from God". We think of the idea as a whole, and when we want to individuate things within it, even though the idea is not extended, we have to think of the idea as extended in order to understand it *as divisible* into parts. Understanding that the idea contains two different substances is to think that some whole, like a line, can be divided up into two parts, such as line segments. One part represents one substance in the idea (God), and the other part represents the other substance in the idea (the thinker). So while the idea itself is not extended and does not really have parts, we represent it as extended, or make it extended by analogy, and think if it as divisible through the divisibility of the extension we use to represent it to think about it as if it had parts. Only then can we make sense of it as standing in a part/whole relation that is necessary for thinking about in terms of a measure or count numbers. So in every idea of a number of something, no matter what that something is, if we think of it numerically, it means we have included the idea of extension in order to mediate the part/whole idea of the non-extended thing to make sense of its number.

6. Further Considerations: The Compendium Of Music

On this nominalist account I have laid out, numbers are names of ideas. These ideas are not abstract, general, innate ideas of universals. Instead the ideas that signified by number-terms are all ideas that contain some idea of extension. These ideas of extension are used to express a relationship between parts and wholes in one form or another: either in the form of a collection of discrete extended objects, or in the form of imaginary divisions in some continuous geometrical magnitude. In either case, the division in to parts is necessary to generate a number because units for comparison are required. Numbers are ideas that necessarily require an idea of extension.

But why think this is a candidate account of Descartes' views on the nature of numbers? Aside from the various complications that would arise from adopting a realist reading of Descartes, consider one of his more striking claims:

> One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceed the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape.[138]

One sound can be said to be more or less sharp than another. But if we are to quantify how much sharper (that is, if we were to quantify the difference or *measure* it and put a number to it), we would have to use an analogy with extension. The question then concerns what he

[138] AT X: 441; CSM I: 58.

means when he speaks of using an analogy to an extension of a body. I submit that the answer to this question can be found in another early treatise, The *Compendium of Music (Compendium)*[139]. There he makes remarks about how differences perceived by the senses are less when there is greater proportion between them, but importantly, this proportion must be arithmetic rather than geometric.[140] The difference between the two types of proportions is then shown using two sets of line segments. The one set of line segments illustrate geometrical comparisons. The other set illustrate arithmetic comparisons. The difference between the two sets of line segments is that in the arithmetic set, the line segments are divided into smaller parts (the divisions are marked out with contain hash marks) that allow one to say "that AB consists of two parts, whereas BC consists of three". It is only in relation to an arithmetic proportion that this can be properly perceived. Otherwise, AB and BC are "incommensurable".[141]

There are many reasons to think that Descartes thought of numbers as ideas of extension, or more specifically, as ideas of the relations that hold between extended objects. In these last two chapters I have presented a few metaphysical and epistemological considerations and offered some texts from the *Rules* and *Compendium* that have been previously neglected by the literature to motivate this position. Next, I will offer some motivations for adopting this reading of Descartes that arise from a philosophical account of the relationship between algebra and geometry based on Descartes' general mathematical views and practices.

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[139] I use Walter Robert's translation of Descartes' *Compendium of Music*, or *Compendium Musicae.* Latin text may be found in AT X.

[140] Compendium, page 12.

[141] Ibid.

Chapter 4 Unification and Priority in Descartes' Algebra & Geometry

0. Features Of Descartes' Mathematical Practices

There is a question about how we ought to understand how Descartes' algebra and geometry fit together. There are two distinctive features of his thinking about mathematics: on the one hand, he thinks that the equations of algebra and the figures of geometry can be translated each into the other—this is what I call the ***unification*** of algebra and geometry. On the other hand, Descartes' mathematical practices are consistently focused on geometry, and it is only in the service of finding geometrical solutions that he makes use of algebraic equations—this is what I call the ***priority*** of geometry.

Two types of interpretations of Descartes' philosophy of mathematics have generally been offered: I'll call them the Traditionalist and Progressivist readings. ***Traditionalist*** readings focus on the priority of geometry and understand Descartes' mathematics in terms of the classical tradition he inherited—one that focused on geometrical constructions. Algebra, on this account, was merely a convenient way to find solutions for his real focus: the classical geometrical problems. ***Progressivist*** readings focus less on the influence of the historical tradition, and focus more on Descartes' contributions towards our current mathematical conceptions. While Traditionalists hold that Descartes thinks geometry is prior to algebra, the Progressivists give an explanation of how Descartes was able to justify the unification of algebra and geometry by denying geometry any real priority. These accounts see Descartes as having taken a large step towards our current, pure mathematics by separating the concepts of multitudes and magnitudes. This is achieved by his adoption of abstract algebraic entities that allow for, but are independent of, geometrical instantiations.

I argue that both these interpretive strategies involve important mistakes. In this paper I offer an alternative account of how Descartes conceived the relationship between algebra and geometry that will better explain both features of his mathematical practices: unification and priority. I argue that his philosophy of mathematics is better interpreted when we take into consideration his fundamental views about metaphysics and epistemology. On the one hand, I argue that Descartes' unification of algebra and geometry should be understood in a strong, metaphysical sense. His unification is based on the *metaphysical unity of the subject matter* of both algebra and geometry. For Descartes, thoughts about algebra and thoughts about geometry are both thoughts about one and the same metaphysical nature, viz. *extension*, the essence of matter. On the other hand, his epistemological views will explain the priority he accorded to geometry. Though Descartes thinks that the subject matter of algebra is the same as the subject matter of geometry, geometry represents that subject matter in a way that is more fundamental. Algebraic conceptions, for Descartes, depend metaphysically and epistemologically on geometrical conceptions. This, I argue, is why there is a *priority* of geometry.

I proceed as follows. In the first half of the paper I explain in more detail the two distinctive features of Descartes' mathematics: unity and priority, (§1); and I then explain the two different interpretations of Descartes' philosophy of mathematics that have been offered, the Traditionalist and Progressivist readings (§2). The Traditionalist has some way of accounting for priority but not for unity; the Progressivist has something to say about unity but little about priority. And so a third reading is needed. In the Second half of the paper I argue for my third reading, and explain both unity and priority using the connections between Descartes' philosophy of mathematics and his fundamental metaphysics and epistemology.

1. Unity And Priority: The Interpretive Challenge

While Descartes was an important figure in the unification of algebra with geometry, his mathematical practices did not resemble what we identify today as analytic geometry. For example, Descartes did not use what we today call *Cartesian* coordinates for he did not understand negative and positive numbers in terms of opposing directions from a designated origin.[142] He thought of negative quantities as "less than nothing," or as "false".[143] And yet, at the same time, he was making innovations in algebraic theory in order to bring those algebraic methods to bear on classical geometrical problems. Descartes was deeply concerned with the nature of the relationship between the numbers and their algebraic expressions[144] (multitudes) and the continuous figures of geometry (magnitudes). His efforts to address this concern led him to formulate some of his most important mathematical innovations. His contributions that led to the development of analytic geometry are found in his re-conceptions of arithmetical operations that allowed for their generalized use for solving traditional geometrical problems. That is, he gave algebraic methods acceptable geometrical interpretations, thereby legitimating their use for solving geometrical

[142] This innovative and essential conceptual leap for mathematics did not occur until 1685 and was made by John Wallis in his work, *Algebra*.

[143] Cf. Book 3 of Descartes' *Geometry*.

[144] In algebra, ranges of values for variables are numbers, or vectors of numbers, and they are not the same as the figures of geometry. Since this is the relevant distinction I am aiming to emphasize, for the purposes of the paper, I will treat algebra as a generalized form of arithmetic, though there is some controversy and little agreement as to whether or not the early modern philosophers conceived of it this way.

problems.[145] Descartes' mathematical practices and philosophy of mathematics, in this respect, are curious; they simultaneously justify the unification of algebra with geometry on philosophical grounds while maintaining a distinctive focus on geometry.

The *priority* of geometry to algebra, and the *unification* of geometry with algebra poses an interpretative challenge for Descartes' philosophy of mathematics. To better understand the challenge, it will help to briefly review a few points from the history of mathematics, and then compare Descartes with two of his contemporaries: Thomas Hobbes and Pierre Fermat.

Mathematics was not always the abstract[146], unified science that it is today.[147] In its earliest stages, before it underwent its many transformations, mathematics was divided into geometry and algebra. Geometry has a long history and tradition, but in its earliest stages it was merely a method or technique for measuring parcels of land: literally a *geo*-metry. Geometry dealt with continuous (geometrical) magnitudes like lines, perimeters, surfaces, and solids, as opposed to discrete multitudes that referred to the number of objects in a collection. For geometers like Euclid, multitudes and magnitudes were different in kind. For instance, Euclid never multiplied two magnitudes together.[148] Instead, to handle

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[145] For more detailed discussion of this, see Bos (2000); Mancosu (1999).

[146] Certainly mathematics that are neither beholden to nor grounded on the actual physical world (*e.g.*, Zermelo-Fraenkel set theory) would have been considered too abstract and unintelligible by many early mathematicians.

[147] Even as late as the 17th century, British surveyors referred to themselves as 'geometers' in the original sense of the word. They measured the Earth by angle as well as distance, intentionally setting themselves apart from mere "mathematicians'. It was thus that mathematicians were thought to have a lower status than that of the geometers, a convention that persisted for two more centuries. Cf. Grattan-Guiness (1997), p. 196.

[148] For example, in Book 1, Proposition 46 of the Elements, when Euclid described the square on the line AB, he proved that the construction yields a defined square, not that the

magnitudes, Euclid used ratios. Again, let me emphasize that a ratio such as 1:3 was, for Euclid, different in kind from the unit fraction 1/3.[149] For Euclid and the geometers that followed him for hundreds of years, numbers and magnitudes were different types of quantity, each with their own distinct discipline: arithmetic dealt with the discrete and geometry with the continuous. While geometers were less conservative as time went on, going so far as to multiply two, even three, magnitudes together (*e.g.*, $a \times a$, or a^2, as a "*square*" of *a,* or $a \times a \times a$, or a^3, as a "*cube*" of *a*) these arithmetic operations on magnitudes always resulted in another magnitude, and multiplying four magnitudes together was considered problematic. Algebra was not considered an acceptable method for solving geometrical problems.

Algebra had its own history and tradition going at least as far back as the ninth century when al-Khwarizmi introduced his method of solving polynomials in his book on equations. While some developed algebra for geometry in the early modern period, [150] others, such as Hobbes, opposed the use of the new methods for solving the geometrical problems from the classical tradition. One reason Hobbes opposed using algebraic techniques was due to the then common belief that the objects and relations of geometry are the true and sole subjects of mathematics. Algebraic expressions and solutions needed to be translated into their geometrical counterparts; but this presented many problems. How are

area is AB × AB, *i.e.*, the square on the side is not the side squared: it is a region, and Euclid was not concerned with its area.

[149] One would never find in Euclid an equation such as $a \times d = b \times c$, but instead the proportion $a:b :: c:d$.

[150] Cf. Molland, (1979). Also see Klein (1968) for a discussion about the influence of François Viète's work on Descartes and Fermat regarding this matter.

we to understand a negative magnitude, much less complex numbers? Furthermore, how are we to make geometrical sense of the sum of a cube and a square (*i.e.*, $x^3 + x^2$)?

Yet Descartes, unlike Hobbes, did not object to the use of algebra with geometry. Descartes did not think discrete multitudes were so different from continuous magnitudes that the latter couldn't also be multiplied. He didn't think multiplying four magnitudes together resulted in dimensionality problems. Instead, he unified algebra and geometry by connecting multitudes with magnitudes. This is what a unification of algebra and geometry required.

On the other hand, many commentators have observed that Descartes lets essentially geometrical problems dictate and constrain his algebraic practices. In fact, Descartes never introduces geometrical objects (such as curves) using their algebraic equations. Curves are always first geometrically defined. Equations are only subsequently derived from the given figures and are representations of relevant geometrical relations. Scholars, thus, have observed that Descartes uses algebraic methods only to find solutions to geometrical problems. As Henk Bos notes, "it appears that Descartes was not interested in algebra for its own sake."[151] Descartes' "creation and adoption of algebraic analysis," is described by Bos to be merely "a tool for geometry"[152]: and is what Ivor Grattan-Guinness describes as a "handmaiden"[153] to Descartes' geometrical pursuits. According to Emily Grosholz, Descartes was "supplying the field of geometry with the computational device of algebra", not "unifying two fields".[154] These are only a few examples.[155] Descartes' development of

[151] Bos (2001) 386-387.

[152] Bos (2001) 259.

[153] Grattan-Guiness (1998) 729.

analytic geometry is often contrasted with that of Fermat's complementary development and emphasis on algebra. Whereas Fermat works with algebraic equations and assumes the geometrical objects corresponding to them exist, Descartes starts, first, with geometrical objects, and seeks their equations second—and only as needed. This is what I am calling the "priority" of geometry over algebra, for Descartes.

Accounting both for unity and for priority is the interpretive challenge that current interpretations of Descartes do not adequately meet, or so I argue. In the next section I characterize both the Traditionalist and Progressivist accounts according to their explanatory focus and explain how they conceive Descartes' unity and priority.

2. Traditionalist And Progressivist Readings Of Descartes

If Descartes' philosophy of mathematics and his mathematical practices are the *explanandum*, the ***Traditionalist*** reading of Descartes appeals to the classical views about magnitude and geometry as the *explanans*. According to ***Traditionalist*** readings, as I characterize them, Descartes' geometrical emphasis results from an acceptance of the Aristotelian tradition of mathematics. This type of appeal can be found in various discussion of Descartes' mathematics. For example, Bos[156] maintains that, "[t]he starting point of

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[154] Grosholz (1980), 159.

[155] Further details can be found in Grattan-Guinness (1998), 225; Grosholz (1980), 160; Bos (2001), 383; and Boyer (2004), 101.

[156] I only aim to broadly characterize a strategy in the literature that shares this general feature of appealing to the traditional cannon of construction. Bos' full account of Descartes includes many subtle details that I do not have the space to explain in detail here. Bos does have much to say about Cartesian mathematical practices in terms of what he identifies as philosophical issues with which Descartes was concerned. But Bos appeals to the classical tradition to explain these issues instead of grounding them in any of Descartes' systematic, philosophical views. I think a better explanation can be found within Descartes' metaphysical and epistemological views.

[Descartes'] interpretation of exactness was classical Greek geometry; geometrical problem solving meant construction by intersection of curves and his classification of problems can be seen as a modification of Pappus' "[157] and "[w]e may therefore characterize Descartes' algebra as subservient to geometry, more precisely to *the canon of construction* that Descartes elaborated in order to solve 'all the problems of geometry'."[158] These accounts of Descartes' mathematical practices seem to suggest that he has no philosophically principled reason for the subservient role he assigns to algebra. Carl Boyer offers a strong view of this sort:

> Some attention has been given to the status of analytic geometry vis-à-vis other branches of mathematics; but the impact of the wider intellectual milieu has been referred to only where it was regarded as of particular significance. It is of interest to note in this connection that the development of coordinate geometry was not to any great extent bound up with general philosophical problems. The discoveries of Descartes and Fermat in particular are relatively free of any metaphysical background. Indeed, *La Géométrie* was in many respects an isolated episode in the career of Descartes – one suggested by a classical problem of Greek geometry.[159]

In short, Boyer denies the very *plausibility* of any such philosophical explanation. This is unexpected: given what Boyer describes as Descartes' otherwise "iconoclastic attitude"[160], why does he not transcend the classical tradition, but instead maintain a devotion to, as Grosholz puts it, "the classical canon of problems and its emphasis on constructability"[161,162]? Such Traditionalist accounts have a difficult time explaining what led

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[157] Bos (2001) 285.

[158] Bos (2001) 387, emphasis added.

[159] Boyer (2004) viii.

[160] Boyer (2004) 84.

[161] Grosholz (1980) 161.

[162] See also Grattan-Guinness (1998) 223; and Boyer (2004) 83,100.

Descartes to use algebraic techniques to solve geometrical problems, and why he regarded himself as justified in so doing.

In other words, while the Traditionalists can offer an explanation for the *priority* of geometry in Descartes' thinking by appealing to an acceptance of the classical tradition, their historical focus does not offer an explanation for why Descartes was able to justify the use of algebra for geometry, *i.e.*, the *unification* of algebra and geometry. The classical tradition to which the Traditionalists appeal is one that would have motivated Descartes to avoid the use of algebraic methods.[163] Explaining the priority of geometry in Descartes in terms of the classical cannon does not explain why he did not, instead, share the views of Hobbes. The difficult task a Traditionalist strategy faces, if it is to be retained, is explaining why Descartes rejects *some* of the traditional conceptions of mathematics *but not others*. But such an explanation most likely will not be based on the Traditionalist's appeal to the classical canon. Traditionalist readings offer an account of priority at the expense of an adequate account of his unification.

In opposition, a ***Progressivist*** reading takes Descartes to have posited the existence of pure, abstract numbers in order to justify the unification of algebra and geometry. This interpretive strategy understands Descartes as taking the first fundamental steps in developing a pure mathematics by allowing for the existence of truly abstract mathematical

[163] Many commentators have explained how Descartes was able to geometrically reinterpret arithmetical operations, and thus explain how Descartes is able to make quantities dimensionally homogenous. [Again, cf. Bos (2000); Mancosu (1999) for thorough explanations.] This, by itself, it not an explanation for why he was justified in unifying the algebraic methods with geometry in the way he does. More conservative thinkers of the time found algebraic methods, in general, to be suspect. There was no denying the usefulness of algebraic techniques, but Descartes was not merely unifying the methods for use by craftsmen where effective approximations would be sufficient. Instead, he makes the unification legitimate as a method for *knowledge*—which must be indubitable (cf. *Rule* 2 of the *Rules*).

objects that are not necessarily physically grounded. Progressivists think Descartes' philosophy about the nature of abstraction allowed him to jettison the need to ground abstract ideas like numbers and quantities in extension. And it is through this "spatial liberation" that Descartes makes great progress in our journey to the pure mathematics we have today.

The Progressivist reading champions how novel Descartes' mathematical approach was with respect to the classical canon; thus, the account doesn't offer a reason for why Descartes would accord geometry any priority so much as it *deemphasizes* any geometrical priority and any need for such a priority on Descartes' part. Various interpretations of Descartes' mathematics focus on this distinction between abstract numbers and continuous, geometric magnitudes.[164] For example, in Descartes' work Mahoney finds that "for the first time there appear new, purely abstract, non-intuitive objects in mathematics, which arise out of structural considerations. Descartes also frees the concept of number from its classical intuitive foundations."[165] Schouls characterizes Descartes' algebraic entities and their relations as "free from the constraints of sense or corporeal imagination."[166] Gaukroger considers Descartes to have inaugurated what he takes to be "the first stage in the development of algebra, namely the freeing of number from spatial intuitions."[167] He argues

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[164] Ernst Cassirer is one of the first Progressivists: cf. chapter 3 of his *Substance and Function*.

[165] Mahoney (1980) 146.

[166] Schouls (2000) 122.

[167] Gaukroger (1992) 112.

that in denying the necessity of a spatial conception of numbers, Descartes had taken a significant step away from the Aristotelian "metric geometry".[168]

According to Progressivists, Descartes maintains that even though ideas like "extension", "body" and "number" are inseparable in our imagination, they have distinct meanings and are understood separately by the intellect.[169] The intellect separates these ideas by abstraction; "number" and "extension" are understood by the intellect as having different content. And "[t]he proper objects of the intellect," along this type of reading, "are completely abstract entities and are free of images or 'bodily representations'."[170] Thus mathematical entities are conceived *independently* of any determinate entities that can represent them. Gaukroger explains that "the objects of [algebra], insofar as these are conceived in the intellect, are indeterminate."[171] By indeterminate, he means that a concept, like "fiveness" is not conceived in terms of any particular instance of five things. The indeterminate content of algebraic objects in the intellect such as "fiveness" is understood by the intellect "as something separate from five objects (or line segments, or points, or whatever)".[172] Only in the imagination is a general magnitude like "fiveness" made specific and determinate by the five objects, whatever they are, we imagine.[173] But in the intellect, we

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[168] Gaukroger (1992) 100-103.

[169] Gaukroger (1992) 109.

[170] Gaukroger (1992) 110,

[171] Gaukroger (1980) 109.

[172] Gaukroger (1992) 110.

[173] Gaukroger (1992) 110.

conceived of fiveness independently of any particular spatial determination, or any spatial determination at all, for that matter.

In this way, Progressivist readings, such as Gaukroger's, take Descartes to have freed numbers from spatial intuitions.[174] The abstract numerical entities of algebra can, by our imagination, "be represented geometrically, i.e. purely in terms of spatial extension"[175]. But insofar as the intellect understands an algebraic abstraction, it isn't as an idea of extension.[176] The indeterminate, abstract entities of algebra—numbers free from the constraints of spatial intuitions—are, for Gaukroger and other Progressivist readings, what so sharply separates Descartes from the Greek tradition.[177] Descartes departs from the classical tradition when he allows for the intellect to understand the meaning of abstract algebraic entities (such as "fiveness") as something separate from the extended, determinate way we imagine five things. *On this account, Descartes doesn't collapse the distinction between multitudes and magnitudes so much as he makes the discrete quantity, the multitude, prior to magnitudes.*[178] Various geometrical

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[174] On this point Gaukroger is not alone, cf. Mahoney, Schouls.

[175] Gaukroger (1992) 110.

[176] Patrick Suppes also takes a Progressivist reading of Descartes in this respect. He writes, "A claimed reduction much closer to the formal spirit promoted here and one of great importance in the history of ideas is *Descartes' reduction of geometry to algebra*." Suppes (2002) page 52: emphasis mine.

[177] "One of the central features of Descartes' algebra is that it deals with magnitudes, or 'proportions', in general. Above everything, this serves to distinguish it from Greek mathematics. In the Aristotelian tradition, there are only numbers of things, geometric magnitudes, periods of time etc., that is, specific kinds of quantity of specific kinds of things." Gaukroger (1980) 103.

[178] Progressivist readings of Descartes require that he subscribed to some version of Realism (either Strong or Weak) about universals such as numbers. See Chapter 2 of this dissertation. I call consider all readings that give some sort of independence or priority to numbers "Realist" readings. Strong Realism differs from Weak Realism in terms of the

magnitudes are just one of many determinations of a more fundamental (and abstract) object, the number.[179]

But just as the Traditionalist readings struggle to account for the difference between Descartes and Hobbes, the Progressivist cannot account for the difference between Descartes and Fermat. While the Progressivists can offer an explanation for Descartes' unification of algebra and geometry by appealing to the indeterminate nature of algebraic abstractions, their focus does not offer an explanation for why Descartes was focused on the geometry: why he would use algebraic techniques only after he was first able to geometrically characterize an object. A Progressivist reading of Descartes would have to deny any real meaningful priority of geometry in Descartes' mathematics. With the existence of abstract numeric entities, multitudes are distinct from magnitudes and there is no reason that Descartes could not have developed his mathematics in the same way his contemporary, Fermat, had done. The difficult task a Progressivist reading faces is explaining why Descartes' mathematics did not in fact resemble Fermat's; and why Descartes focused overwhelmingly on the geometrical aspect of the unification of algebra and geometry. Any such explanation offered by a Progressivist reading will most likely not be based on their appeal to Descartes' creation of algebraic abstractions. Progressivist readings offer an account for unity at the expense of an adequate account of priority.

Neither the Traditionalist nor the Progressivist readings provide the best account of Descartes' mathematical views. Neither adequately accounts for both unity and priority.

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ontological status of numbers. For example, a Platonist about numbers would be considered a Strong Realism, while a conceptualism about numbers would be a Weak Realism.

[179] There are many interpretive problems with Realist readings of Descartes that are also additional problems for Progressivist readings. For arguments against Realist readings, see Chapter 2.

Instead, I offer a better interpretation of Descartes that will explain both unity and priority. My interpretation will also explain why it would be a mistake to take a Traditionalist or Progressivist approach to understanding Descartes philosophy of mathematics. Taking the issues in reverse, I argue, *pace* Progressivist accounts, that Descartes *does not* free numbers from spatial intuitions. Descartes' philosophical views regarding the conception of quantity and the nature of abstract ideas do not countenance the pure, abstract, mathematical objects required for the Progressivist understanding of Descartes. It is not that "algebraic entities *can* be represented geometrically"[180]. Rather, I argue that according to Descartes they *must* be represented geometrically. To show this, I explain what Descartes takes to be the relationship between algebra and geometry. This reading, based largely on the methodological considerations given in an early, unpublished manuscript, the *Regulae ad Directionem Ingenii*, will offer an "internal" and systematic account for his subordinating of algebra to geometry. Hence, *pace* Boyer and the Traditionalist understanding of his philosophy of mathematics, Descartes was *not* merely following in the classical tradition either. On my account, Descartes' mathematical practices are a consequence of his metaphysical and epistemological views concerning the nature of mathematics—ones that are themselves grounded in his methodological considerations. Extension, the shared nature of the objects of both algebra and geometry, accounts for their metaphysical identity—justifying their unification. But the relative ordering and deductive distance from that shared absolute nature accounts for the priority Descartes places on the geometry. Thus, it would be a mistake to think that Descartes' mathematical views are orthogonal to his philosophical views. My reading, which starts with considerations from Descartes' stated Method, can

[180] Gaukroger (1992) 110: emphasis mine.

comfortably account for both the priority of geometry while explaining its unification with algebra.

The remainder of this paper is structured as follows. I briefly explain why our investigation of Descartes' philosophy of mathematics should be viewed through his philosophical Method as presented in the *Rules*. After I present the rudiments of Descartes' Method, I then turn (§4) to his views regarding the general enterprise of mathematics as given in the *Rules*. There we see that there is no real (metaphysical) distinction between the objects or subject matter of geometry and algebra. Such a view rules out any possibility of the "numeric-liberation" espoused by a Progressivist reading, and instead offers a different explanation for how Descartes justified the unification of algebra and geometry. Then (§5) I argue that despite the metaphysical identity between objects of geometry and algebra, geometry is in an epistemic sense more fundamental than algebra. It is this epistemic priority of geometry that grounds the geometrical emphasis in his mathematical practices. This, I suggest, strongly undermines the Traditionalists readings that see Descartes' mathematical views as merely a product of his time. Finally (§6) I summarize and briefly conclude.

3. Through Descartes' Eyes: Using His Method As A Lens

As mentioned above, Descartes' contributions to the development of analytic geometry are both lasting and integral. Given the esteem with which we hold Descartes' mathematical achievements, it may be perplexing that in Descartes' writings he is rather dismissive of mathematics. For example, he writes in the *Rules* that he

> would not value these Rules so highly if they were good only for solving those pointless problems with which arithmeticians and geometers are inclined to while

> away their time, for in that case, all I could credit myself with achieving would be to dabble in trifles with greater subtlety than they.[181]

And that, "there is really nothing more futile than so busying ourselves with bare numbers and imaginary figures that we seem to rest content in the knowledge of such trifles."[182]

The Rules to which Descartes refers above are the rules that explain his Method for the investigation of the truth in all things. Descartes goes on to explain that the Method he developed (his universal mathematics) was the result of first his study of arithmetic, and geometry; and then a more general investigation of mathematics. What he discovered led him to resolve in his search for the knowledge of things, "to adhere unswervingly to a definite order, always starting with the simplest and easiest things and never going beyond them till there seems to be nothing further which is worth achieving where they are concerned."[183]

Before I briefly summarize Descartes' universal mathematics, let me first review some of the relevant machinery central to the Method. According to Descartes, the contents of our minds can be divided into two types: simple natures, and things that are composed of simple natures.[184] Simple natures are innate ideas that we can clearly and distinctly

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[181] *Rule* 4; AT X: 373; CSM I: 17.

[182] *Rule* 4; AT X: 374; CSM I: 18.

[183] *Rule* 4; AT X: 379; CSM I: 20.

[184] "… it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another. Indeed, it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others." (*Rule* 12; AT X: 422; CSM I: 46)

perceive[185] (or "intuit", to use Descartes' term in the *Rules*).[186] There are not very many of these ideas: Descartes remarks that "there are very few pure and simple natures which we can intuit…".[187] The different simple natures (or different substances, or essences) can form various mixtures or compounds that can further combine to form yet more complex composites. This compounding of simples in multiple iterations and patterns, according to Descartes, can account for all our other more complex ideas.[188] Given his theory of ideas, "the whole of human knowledge consists uniquely in our achieving a distinct perception of how all these simple natures contribute to the composition of other things."[189] His entire

[185] "… we term 'simple' only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known…all the rest we conceive to be in a sense composed out of these [simples]" (*Rule* 12; AT X: 418; CSM I: 44).

[186] "... we need take no great pains to discover these simple natures, because they are self evident enough. What requires effort is distinguishing one from another, and intuiting each one separately with steadfast mental gaze." (*Rule* 12; AT X: 425; CSM I: 48).

And also:

"…we should note that there are very few pure and simple natures which we can intuit straight off and per se (independently of any others) either in our sensory experience or by means of a light innate within us. We should, as I said, attend carefully to the simple natures which can be intuited in this way, for these are the ones which in each series we term simple in the highest degree. (*Rule* 6; AT X: 383; CSM I: 22).

[187] *Rule* 6; AT X: 383; CSM I: 22.

[188] "… it is not possible for us ever to understand anything beyond those simple natures and a certain mixture or compounding of one with another. Indeed, it is often easier to attend at once to several mutually conjoined natures than to separate one of them from the others." (*Rule* 12; AT X: 422; CSM I: 46).

[189] *Rule* 12; AT X: 427; CSM I: 49.

Method, as he conceived it, consists in ordering and arranging our ideas according to their degree of confusion or complexity.[190]

In order to do this, we compare two objects to one another in order to determine their relative dependence. The object or nature that is called "absolute" is that one that has the simpler nature of the two, and the other, the "relative" nature, shares (at least partially) that same nature.[191] This comparing and ordering according to dependence and complexity is done repeatedly until a continuous series can be completed that connects the two extremes: the most complex with the most simple (which Descartes sometimes calls "simple in the highest degree"[192], or "absolute in the highest degree"[193]). This will allow us to "observe how all the rest [the more complicated] are more or less, or equally removed from the simplest."[194] But no matter how far removed from the simplest nature, any compound in the continuous series will share in that simple nature. That is to say, it will have that

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[190] "The whole method consists entirely in the ordering and arranging of the objects of which we must concentrate our mind's eye if we are to discover some truth. We shall be following this method exactly if we first reduce complicated and obscure propositions step by step to simpler ones, and then, starting with the intuition of the simplest ones of all, try to ascend through the same steps to a knowledge of all the rest." (*Rule* 5; AT X: 379; CSM I: 20).

[191] "I call 'absolute' whatever has within in the pure and simple nature in question; that is, whatever is viewed as being independent, a cause, simple, …and other qualities of that sort… The 'relative', on the other hand, is what shares the same nature, or at least something of the same nature, in virtue of which we can relate it to the absolute and deduce it from the absolute in a definite series of steps." (*Rule* 6; AT X: 381-382; CSM I: 21)

[192] "We should, as I said, attend carefully to the simple natures which can be intuited in this way, for these are the ones which in each series we term simple in the highest degree." (*Rule* 6; AT X: 383; CSM I: 22).

[193] "The secret of this technique consists entirely in our attentively noting in all things what which is absolute in the highest degree." (*Rule* 6; AT X: 382; CSM I: 22)

[194] *Rule* 6; AT X: 381; CSM I: 21.

nature as a part of its complex, and will depend on that simple nature. This Method of investigation (reducing a complex thing into its simplest nature) provides us with knowledge of what the true nature or essence of the thing is.

Equipped with the understanding of Descartes' Method as I have briefly summarized, we can better understand Descartes' contributions to the development of analytic geometry. *A true unification of the disciplines of algebra and geometry is ultimately grounded on the fact that the objects of algebra and the objects of geometry share an identical metaphysical nature.* My Method-based understanding of Descartes' mathematics will offer an explanation of how he justified the unification of algebra and geometry in a way that was acceptable on his own philosophical grounds. It will also explain why his unification has its distinctive characteristic focus on geometry as by appeal to the dependence relation that the method picks out.

4. Unification Through The Nature Of Mathematics

In this section, I develop an account of Descartes' philosophy of mathematics and his unification of algebra and geometry that is at odds with the Progressivist reading sketched above. In order to bring out this conflict (and, I suggest, Descartes' true view), let me first step back and review a few key points from Descartes' *Rules* insofar as they pertain to some very general views about mathematics. Once we have done this, not only will we be in position to see the problem for the Progressivist account of the unification of algebra and geometry, but we will also have a better explanation for how Descartes was able to justify it.

According to Descartes, all knowledge (mathematical or otherwise) results from "a comparison between two or more things" along, or with respect to, some common feature or nature.[195] Regarding mathematics in particular, Descartes writes that he, "came to see

[195] *Rule* 14; AT X: 440-441; CSM I: 57-58.

that the exclusive concern of mathematics is with questions of *order and measure* and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds, or any other object whatever."[196] As terminology, a *relation of order* relates things directly to each other with respect to some shared "dimension" of comparison. For example, we can know the relation of order between a line segment and a congruent line segment by comparing them side-by-side; or we could tell if a third line segment was longer than the other two put together if we concatenated the first two and compared it against the third. In contrast, *relations of measure* require the mediation of an extra term: the specified unit of measure. We cannot know the count or measure of a magnitude of extension unless we know the unit by which we are to compare them. E.g., we cannot know the number of an object's height until we decide on a scale of measurement that, thereby, fixes the units. The mediating unit of measure allows us to understand magnitudes of extensions either as counts or as measures. There is no real distinction between a count and a measure. The two are merely rationally distinct.[197]

Note here, in connection with the Method, the same two things can be compared or related in both ways: order and measure. We can use a relation of order to compare two objects, *A* and *B*, along a shared nature, like that of length, directly to each other and

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[196] *Rule* 4; AT X: 378; CSM I: 19: emphasis mine.

[197] See *Rule* 14; AT X: 448; CSM I: 62. Counting and measuring are only rationally distinct. Counting consists in considering the order of the parts in relation to the whole. Measuring consists in considering a whole magnitude as something as divided into parts. When something is measured, it is counted when considered in the reverse direction. We can *count* the years in a century; or we can *measure* centuries in terms of years. It is the same magnitude we consider, first by attending to the order of the parts to the whole, then by attending to the measure of the whole in terms of its parts. The important point is that counting and measuring are different ways to view the same relationship between parts and wholes, *i.e.,* different ways in which we can attend to the same thing.

determine that object *A* is longer than object *B*. We can use a relation of measure to compare the same two objects and their lengths according to a shared unit of measure that we specify and determine that object *A* is longer than object *B* by *x* units. In either case, the same two objects are being compared with respect to the same common nature, *i.e.*, the dimension of length; however, the relation of measure is not considered to be as simple or as easily known as the relation of order because the objects are *not directly related* to one another but instead require the extra mediation of the unit of measure. In this sense, the relation of measure is *more complex* relative to the relation of order. Thus following Rule 6,[198] *the relation of order is prior to the relation of measure.*

In sum, mathematical knowledge results from comparisons of the order and measure of things with respect to a *common nature*. Descartes' Method for attaining this knowledge of the shared nature instructs us to reduce all the relevant relations and proportions among the things compared to some *equality* between what is sought and what is known. That is, we find an equation to express what we want to know, based on the relationships of the equalities between what we do know. To give an example, let me adapt a problem from Rule 6[199]: If we are given two extremes, 3 and 48, and were required to find the three missing means to complete the continued proportional (viz. 6, 12 and 24), we would could use the relations we do know (viz. $3 \times 48 = y \times y$; $3 \times y = w \times w$; and $y \times 48 = z \times z$) to produce the solutions (we would just solve for *y, w*, and *z* one by one).

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[198] *Rule* 6; AT X: 381-387; CSM I: 21-24. *Rule* 6 instructs us to arrange things serially according to what can be known on the basis of others, sorting out what can be apprehended only by deducing them from those which are simpler.

[199] *Rule* 6; AT X: 386; CSM I: 24.

But, Descartes says, "nothing can be reduced to such equality except *what admits of differences of degree, and everything covered by the term* '*magnitude*'."[200] He continues on in Rule 14 to explain that our ideas of magnitudes are ideas of the real extensions of bodies: they are *ideas of the same thing*, *i.e.*, the same, shared, identical nature. For "it follows", he writes, "from what we said in Rule Twelve that [magnitude in general] is the real extension of a body considered in abstraction from everything else about it save its having a shape."[201] Descartes then nicely summarizes that our mathematical knowledge consists solely in the discovery of "a certain extension on the basis of a comparison with some other extension which we already know."[202] We need to discern the magnitudes of extension as relations and proportions. Being able to do so would show how the relations and proportions merely result from, and are known on the basis of, the simpler shared nature: the extension of bodies, *i.e.*, geometrical magnitudes. Fortunately, extension has three characteristics that help us do just this: dimension, unity, and figure. I discuss these three characteristics in turn.

By **dimension**, Descartes means "simply a mode or aspect in respect of which some subject is considered to be measurable."[203] This includes length, breadth and depth,[204] but also other dimensions such as weight or speed. Any mode that gives rise to number or measure is a species of dimension. There can be, within the same object, many different

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[200] *Rule* 14; AT X: 440-441; CSM I: 57-58: emphasis mine.

[201] *Rule* 14; AT X: 441; CSM I: 58. (By extension, Descartes means that which has length, breadth, and depth, but he does not mean to make a distinction between a body and a space.)

[202] *Rule* 14; AT X: 447; CSM I: 62.

[203] *Rule* 14; AT X: 447; CSM I: 62.

[204] *Rule* 14; AT X: 449; CSM I: 63. He notes that these are merely nominally different.

dimensions, and each one is not to be considered really distinct from the extended body or, therefore, from each other. Though some geometers mistakenly think the line, plane, and solid are different species of quantity, Descartes reminds us that they are not really distinct from one another. They are only rationally distinguished through the abstractions we make with our reason. "Indeed," he writes, "if they are thought of without respect to anything else, as abstractions of the intellect, then they are no more different species of quantity than 'animal' and 'living' in man are different species of substances."[205] We do not think that there are two separate parts of man, one that is 'animal', and the other that is 'living'. Instead, we view them as different ways in which we can regard a man. Similarly, we can think of an extended object, like a book, and regard it in different ways. If we regard the object *qua* volume, we may attend to the object as a rectangular prism. If we regard the object *qua* surface area of a face, we may attend to the object as a rectangle. Alternatively, if we regard the book *qua* length of an edge, we may attend to the object as a line segment. In each case above we have successively further delimited our thought of the book. But in all cases, we would be thinking of the book: sometimes as an object with volume, other times as an object with area, etc. In each instance, we are thinking of the same extended thing (the same magnitude of extension), though we are, in each instance, attending to it along a different dimension.

The second characteristic of extension, **unity**, is "the *common nature* which, …, all the things which we are comparing must participate in equally."[206] So long as no unity is specified in the problem, we can take any magnitude, and use it as the unit of measure for all

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[205] *Rule* 14; AT X: 448-449; CSM I: 63.

[206] *Rule 14*; AT X: 450; CSM I: 63, emphasis added.

other magnitudes. It just needs to have as many dimensions as the most extreme term we're comparing has. Thus, if the most extreme magnitude sought has two dimensions, our common measure should have two. Note that determining a common unit of measure already assumes that some common dimension(s) has been selected. Only after we specify the dimension(s) of interest can we undertake the task of determining what a unit of that measure will be.

The third characteristic of extension is shape, or **figure**. When we are engaged in mathematics, only two types of figures are useful for comparing relations or proportions of multitudes and magnitudes: figures that represent sets, and figures that illustrate continuous magnitudes.[207] Figures that represent sets or **multitudes** are discrete, like a collection of points or lines. Figures that illustrate continuous **magnitudes** can include polyhedrons, polygons, lines, etc.

These three features, dimension, unity, and figure, are not really distinct from extension, but are the ways that we can attend to the shared nature that allows for comparisons of extension to be made. Even when extensive magnitudes seem at first incomparable, using the three features of extension, we can understand what relations and proportions hold among the different extended magnitudes. As mathematics is exclusively concerned with order and measure, the features of dimension, unity and figure allow us to compare the extensive magnitudes in terms of these two relations.

In Rule 14, Descartes warns against making the same mistakes that mathematicians make when they think that the magnitudes abstracted from objects in their thoughts are really distinct from the objects. Sometimes arithmeticians "think that numbers are

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[207] *Rule 14*; AT X: 450; CSM I: 64.

abstracted from every subject by means of the intellect and that they are even to be really distinguished (*vere distinguendos*) from every subject by means of the imagination."[208] Geometers, Descartes continues, thinking, "lines have no breadth, surfaces no depth," create a surface using the extended quality of the line, not realizing that they are all simply "mode[s] of body.[209] When we are engaged with mathematical problems, we are concerned with an extended object that we think of solely in terms of its extensive magnitudes. But we should not think that magnitudes are really distinct from the things that have the magnitudes (the real extension of a body). It would be a mistake to think the things whose magnitude we are regarding are somehow excluded in our cognition of the magnitudes when considered generally in mathematics, as if they somehow had a separate existence—like in the case of the "animal" and the "living" in the man. In short, Descartes takes pains, repeatedly, to emphasize that the magnitudes—or what admits of differences of degree[210]—are modes of attributes: mere rational distinctions of extended objects. This is important to for him, in part because it is easy for us to be misled by an incorrect understanding of some of the crucial terms of the discussion, much as the geometers have been.

In the same vein, Descartes warns that we should be careful to distinguish what ideas words like "extension" and "body" mean when conveyed to our intellect. His discussion of the sentence "Extension is not body" is particularly informative:

> In this sense there is no specific idea corresponding to [extension] in the imagination. In fact this expression is entirely the work of the pure intellect: it

[208] *Rule* 14; AT X: 446; CSM I: 61.

[209] *Rule* 14; AT X: 446; CSM I: 61-62.

[210] *Rule* 14; AT X: 440; CSM I: 58: "We should note, moreover, that nothing can be reduced to such an equality except what admits of differences of degree, and everything covered by the term 'magnitude'."

> alone has the ability to distinguish between abstract entities of this sort. This is a source of error for many who, not realizing that extension taken in this sense cannot be grasped by the imagination, represent it by means of a real idea. Now such an idea ***necessarily*** involves the concept of body. So if they say that extension so conceived is not body, they are unwittingly ensnared into saying 'The same thing is at once body and not body.'[211]

Descartes, here, clearly warns both against reifying the abstractions of the intellect, and against observing "distinctions" between ideas that cannot be separated in the imagination. It is, Descartes claims, "a source of error" to think that we have an idea of extension that does not include what necessarily is involved in the concept, *i.e.*, its necessary *nature*. Descartes does not stop with just extension. He continues:

> It is important to distinguish utterances in which such terms as 'extension', 'shape', '*number*', 'surface', 'line', 'point', 'unity', etc. are given such a narrow sense that they exclude something which is not really distinct from what they signify, as for example in the statements: 'Extension or shape is not body', '*A number is not the thing numbered*' (*numerus non est res numerata*) … etc.[212]

Just as extension is not really distinct from body, and it would be "a source of error" to think we have an idea of extension that does not include body; so too, number is not really distinct from the things numbered, and it would be a "source of error" to think we can have an idea of number that does not include what is "necessarily involved in the concept", *i.e.*, its necessary nature.

All this leads to our first conclusion. Crucially, in Rule 14, Descartes rules out the possibility of the existence of the Progressivist reading's abstract numeric entities. In fact, *the Progressivist reading makes exactly the mistake Descartes warns against.* Descartes recommends that "the imagination nonetheless ought to form a real idea of the thing" when the intellect employs terms like "number" so as to prevent the intellect from excluding "the other

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[211] *Rule* 14; AT X: 444-445; CSM I: 60: emphasis mine.

[212] *Rule* 14; AT X: 445; CSM I: 60: emphasis mine.

features of the thing which are not conveyed by the term in question, and so that it may never injudiciously take these features to be excluded."[213] Specifically, in the case of number, "we imagine some subject which is measurable in terms of a set of units" and though we can allow the intellect to confine "its attention to this set of units; nevertheless we must see to it that, in doing so, it does not draw a conclusion which implies that the thing numbered has been excluded from our conception."[214] *We cannot have an abstract idea of "fiveness" that excludes any determinate representation of it.* Any attempt to conceive of a number that is not a thing numbered is one in which the term is "given such a narrow sense that [it] exclude[s] something which is not really distinct from what [it] signif[ies]."[215] We would thus be ensnared into saying something like, "The same thing is at once the thing numbered and not the thing numbered." This would be to simultaneously affirm and deny the nature of the thing on which it depends. Numbers *cannot* be free of "images or 'bodily representations' " as a Progressivist reading of Descartes' claims.[216] Numbers like "five" can only signify, or stand in for, some geometric extension. They *cannot* be understood as anything but some extended magnitude.

The view just sketched goes hand-in-hand with Descartes' insistence that dimensions of extension are neither really distinct from each other nor from the extended body. How can a number (which requires the notion of a unit) be distinct from the body if the

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[213] *Rule* 14; AT X: 445; CSM I: 61.

[214] *Rule* 14; AT X: 445; CSM I: 61.

[215] *Rule* 14; AT X: 445; CSM I: 60.

[216] Gaukroger (1992) 110.

dimensions (along which any unit of measure is dependent on) are not really distinct? If dimensions do not exist independently of extension, numbers cannot either.

Even more can be said on this front. A unit of measure allows us to move between multitudes (given by discrete counting numbers) and magnitudes (given by continuous numbers). Once we have established a unit, Descartes tells us that "it is sometimes possible completely to reduce continuous magnitudes to a set and that this can always be done partially at least."[217] This is because we can shift our attention and think about the same thing in different ways. We can use our unit to think of the number of "parts" in a continuous whole (a count), thus yielding a set, or multitude. (If the continuous whole is a whole number in the scale of units, there need be no "rounding" error.[218]) For example, we can take a continuous quantity, like a line segment, and establish the centimeter as our unit. Then we can measure our line segment with our unit, and determine that it is 5 centimeters long. Now, we can think of that same continuous magnitude, the line segment, in terms of a number or multitude like 5 by thinking of the line segment as consisting of a set of five unit-line segments that are concatenated together to form the whole line segment we have measured. We can apprehend the measures of the figure by working with the orders; in this way, a figure can either represent sometimes a continuous magnitude, or sometimes a set, a number, or multitude. Comparing extension with respect to the relation of measure differs from comparing extension with respect to the relation of order only insofar as we are considering the extension as a multitude or a magnitude. Given that the objects of geometry and the objects of algebra have the same nature, it is clear from methodological

[217] *Rule* 14; AT X: 452; CSM I: 65.

[218] Indeed, if the continuous whole is a rational number in this scale, there still needn't be any rounding error, but the units may not be the same as our original unit.

considerations that algebraic entities cannot be different types of things, for otherwise they would have different natures. There is, thus, no metaphysical distinction between the geometrical extension and the numeric quantity. For Descartes, it is this metaphysical identity that justifies the unification of algebra and geometry; they concern the same nature: extension.[219]

5. Descartes' Subordination Of Algebra To Geometry In Focus

The Progressivist reading (unsuccessfully) attempts to explain the unification of algebraic methods and geometrical problems. Moreover, the proposed explanation is at the expense of an adequate explanation for Descartes' geometrical emphasis. We've just seen that Descartes was not the type of mathematical innovator that the Progressivist reading claims, but we have also seen that there is a better way to account for the unification of algebra and geometry. But how are we to understand his view of geometrical priority? Above, we had noted a problem for the Traditionalist readings. In accounting for the geometrical emphasis in Descartes' work (something the Progressivist reading cannot as explain), the Traditionalists are left without an explanation for how Descartes' was able to philosophically ground the use of algebra in geometry.

In contrast, my Method-based reading of Descartes' mathematical views successfully explains his unification of algebra and geometry, and is superior to both the Progressivist and Traditionalist readings in this respect. Furthermore, my Method-based reading is also able to systematically account for the geometrical emphasis—which was the main reason for

[219] My reading presents Descartes' views on human psychology as influencing his epistemology. The way in which we think of things naturally will constrain how we can know the nature of things. On this point, my resolution of the Counting Problem will align Descartes with contemporary, empirical research in cognitive science about the use of spatial schemas in our abstract reasoning. Cf. Dehaene (1997) and also Giaquinto (2007).

favoring a Traditionalist reading over a Progressivist one. We've seen that my Methodological reading uses the identity of the natures of geometrical and algebraic objects to explain the unification. But now I will explain how—based on the ordering of what needs to be deduced from the other (*i.e.*, relative simplicity and complexity)—my reading is able to account for the priority that geometry receives in Descartes' mathematical practices.

Despite the *metaphysical* identity of geometrical magnitudes and numerical quantities of multitudes, geometrical magnitudes are *epistemologically* prior. We have seen that measures of continuous magnitudes can be reduced to discrete multitudes that we can count only through the mediation of a unit. When we regard a geometrical figure as a set or number, a unit of measure is required. But in order to determine a unit of measure, we assume the required dimension is specified. This means that before we can think of a multitude or a quantity of measure along some dimension of the extended body, we need to determine in what respect we are going to regard it. This is selected from the many dimensions or attributes that are available for us to consider. To illustrate this, let's return to the case of the book. We could chose to direct our focus and attention to the cover of the book, and pick out the dimension "surface area", which will pick out the quantity of measure as an area. Alternatively, we could choose to regard just one edge of a face of the book, thereby picking out the dimension of length. In that case we would be attending to the quantity of measure of length.

But in order to determine what dimension of measure we are interested in, we must first determine the object of interest. This requires thinking of the whole extended object, or some aspect of the extended object. In doing so, we are delimiting in our thought the particular way in which we will be thinking of the extension. We could consider the volume of the whole book, in which case the whole rectangular prism is the object of interest; or in focusing on the area of one of its faces, the object of interest is the rectangular cover or face of the book. In each case, we will determine a distinct dimension. Importantly, as we saw above, although we might think of the square as a different object from the cube for the purposes of determining a dimension of measure, they are really one and the same thing.

The subject matter of **geometry** consists in this activity of determining or delimiting figures. Geometry concerns the *relations of the aspects or attributes* that we use to rationally distinguish our figure. Geometry takes as its subject matter the continuous "geometric" magnitudes, *i.e.,* dimensions of measure (size, shape, length, area, etc), and then considers them as relations and proportions of order. I explained earlier how these relations do not require the mediation of a unit of measure. Relations of order hold *directly* between comparisons of continuous magnitudes. We can, for example, use relations of order to discover that the interior angles of a triangle can be joined together to equal two right angles joined together, as we do in Euclid's 32^{nd} proposition.

When we are engaged in the activity or practice of **algebra**, we consider the magnitudes as multitudes and then relate particular measures of these magnitudes or quantities. This is distinguished from geometry because it involves *relations of measures*—which require that a unit of measure be specified. If we are counting (considering the relation of order of the parts to the whole), we need to determine a unit of measure before we can determine how many there are. E.g., it is meaningless to report that an object is 10, without specifying whether our units are feet, acres, tons, etc. And if we were to use relations of measure, and we determined our unit of measure to be a degree, then we could discover upon measuring and summing up the measures of the interior angles of a triangle and finding them to equal 180°, that it was equal to the measure of the degrees in a straight line, or two right angles.

It is precisely because units are a measure along a dimension that has already been specified that geometry is epistemically prior to algebra for Descartes. In terms of the Cartesian Method, arranging things in order from the most simple and absolute to the more complex: geometry (and its relations of order) is prior to algebra (and its relations of measure). Algebra depends on geometry. Geometry is less removed from the simplest nature than algebra. Not only does geometry delimit the extended object, it does so while specifying dimensions of measure that allow for the possibility of numeric quantities once a unit of measure is assumed. Algebra concerns the relations of orders of measures; however, we cannot specify a measure unless we do so in relation to a unit of measure. This unit of measure is given along a particular dimension of measure. The dimensions—along which there can be measures—are the concern of geometry. We engage in geometry when we consider and determine dimensions and their relations of order. Only then we can be in a position to begin considering the particular quantities of measure along those dimensions prior to counting the parts. Since the relations of measures require there first being relations of order, from whence it can be subsequently deduced, the relations of measure depend on the relations of order.

The upshot is that the Traditionalist's interpretation is wrong. Descartes' commitment to the priority of geometry over algebra is not some enculturated whim independent of his philosophy. Rather, it is a straightforward consequence of some very fundamental and basic features of his philosophical and Methodological views. Descartes' general views forced him to ***both*** emphasize the epistemic priority of geometry and to identify the objects of algebra and geometry, thus unifying the two together.

6. Unified Mathematics: A Case Study For Unified Wisdom

In this paper I have explained the unification of algebra and geometry by showing how algebra is concerned with relations of measure and geometry with relations of order, and then by explaining how relations of measure are relations of order. Once this is done, their unification becomes inevitable, as they are most fundamentally concerned with relations of the same metaphysical nature: extension. Geometry and algebra regard the relationship differently, depending on whether the objects are seen as discrete sets or multitudes, or instead as continuous magnitudes. There is no real, metaphysical difference between the two ways of considering the objects, because for Descartes moving between relations of order and relations of measure requires only a specification of some unit of measure, thereby allowing us to move back and forth between continuous magnitudes, and discrete multitudes. In this way, we are also able to move back and forth between geometric and algebraic considerations of the relationship between objects. However, in all cases, we merely regard the same metaphysically identical object, its nature, first as a magnitude and then as a multitude. Since there is no metaphysical distinction between a multitude and a discretely considered magnitude, the unification of geometry and algebra is assured.

The only difference is the epistemic priority of the objects being considered. That is to say, the difference between a multitude and a discretely considered magnitude can be understood in terms of the difference between the complexities of the ideas involved. A multitude is more complex than, and depends on, the magnitude. The multitude is relative to the magnitude; the magnitude is absolute relative to the multitude. Without there being first a magnitude (of any dimension), there could be no specification of the unit of measure required for any relation of measure. It is in this sense that the objects of geometry (*i.e.*, magnitudes) are epistemically prior to the numeric considerations of those objects: that is, the objects of algebra (*i.e.*, multitudes).

The picture of Descartes I have presented offers not only a philosophical, systematic explanation of the unification of algebra and geometry, but also an explanation for why Descartes nonetheless retains a geometrical emphasis. This geometrical emphasis is not merely the result of echoes of the classical geometrical tradition reverberating in his ear—no, it its better explained in terms of the same systematic considerations which lead Descartes to the unification in the first place. The fact that geometry considers the objects more directly makes geometry epistemically more fundamental. Grounding numeric quantities on continuous magnitudes is also consistent with claims Descartes makes in the *Rules* that deny the existence of abstract, numeric quantities. These abstractions are things Descartes' mathematics could not countenance. Moreover, my reading is also in keeping with his claims in the *Rules* regarding his Method for all knowledge. I do not deny that Descartes' mathematical contributions were in fact innovative and revolutionary; I only claim that their brilliance and significance to the development of our mathematics should not be falsely attributed to his development of abstract, indeterminate, algebraic entities whose existence he firmly rejects.

What then are we to make of this picture of Descartes? On the one hand, he does not appear to be innovative in the way that the Progressivist readings have maintained. But on the other hand, we have also seen, against the Traditionalists, that this lack of innovative numeric-liberation and priority of geometry was due to a very systematic and consistent philosophy of mathematics (at least in the present respects). Rather than importing the classical fashions of the day into his philosophy, we see Descartes as taking very seriously his own basic principles and methodological rules. Moreover, in true Cartesian fashion, he lets them lead the way to particular conclusions, whether they are popular, innovative, or anything else. While Traditionalists readings were able to account for priority at the cost of unity, and Progressivists were able to account for unity only at the cost of priority, I am able to account for both unity and priority by using Descartes' Method. This allows for a philosophically systematic understanding of Descartes' mathematical practices and his philosophy of mathematics that avoids the philosophically superficial reading offered by Traditionalists, as well as the textual problems of the Progressivists.

But more than just Descartes' philosophy of mathematics is better understood by his method. Descartes' universal mathematics, as given in the *Rules,* is not so much a Method for mere mathematical computation, but a method for all the sciences. This is not to say that mathematics such as algebra or geometry, trifling though they may be, has no value to Descartes at all. Indeed, Descartes thinks that mathematics has something important to offer us.[220] What Descartes finds valuable about the study of mathematics is the training of our minds in his Method, which we can then use for a proper scientific investigation that yields knowledge of the true natures of things.[221]

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[220] *Rule* 10; AT X: 404; CSM I: 35: Number-games and any games involving arithmetic, and the like, belong [in the simplest and least exalted arts]. It is surprising how much all these activities exercise our minds, provided of course we discover them for ourselves and not from others. For, since nothing in these activities remains hidden and they are totally adapted to human cognitive capacities, they present us in the most distinct way with innumerable instances of order, each one different from the other, yet all regular. Human discernment consists almost entirely in the proper observance of such order

[221] *Rule* 14; AT X: 442; CSM I: 59. Descartes writes that "… these Rules are so useful in the pursuit of deeper wisdom that I have no hesitation in saying that this part of our method was designed not just for the sake of mathematical problems; our intention was, rather, that the mathematical problems should be studied almost exclusively for the sake of the excellent practice which they give us in the method."

While a study of Descartes' Method has informed our understanding of his mathematics, that is only one of the many things that is covered in Descartes' universal wisdom. Descartes plays a central role in the modern understanding of the relationship between mathematics and empirical science. He made revolutionary contributions to both mathematics (*e.g.*, his analytic geometry which unified geometry and algebra), and to physics and optics (*e.g.*, his conservation laws, the sine law of refraction, and the determination of the anaclastic curve), which, moreover, were often characterized in a mathematical fashion: a mathematical-physics. Yet, Descartes saw them all as a part of a single "universal wisdom" or "human wisdom, which always remains one and the same"[222]. For example, he wrote in the *Principles* that, "The only principles which I accept, or require, in physics are those of geometry and pure mathematics; these principles explain all natural phenomena, and enable us to provide quite certain demonstrations regarding them."[223] He also wrote that his "entire physics is nothing but geometry."[224] These passages illuminate how Descartes may have conceived the relationship between mathematics and physics.

But if we are to take Descartes' comments about his Method seriously, we should be able to understand his unity of science while also accounting for his identification of his physics with mathematics in such a way that it explains *why* mathematics is applicable to his physics (*e.g.*, why numbers and equations are relevant to the surface of a lens in the case of the anaclastic curve). The unity of science is inextricably tied to Descartes' philosophical Method of investigation in the *Rules*. Most contemporary scholarship on Descartes'

[222] AT X: 360; CSM I: 9.

[223] AT VIIA: 78; CSM I: 247.

[224] AT V: 268; CSMK: 119.

mathematics and physics has not adequately considered the role that his Method plays in his larger project. Many dismiss this earlier work as irrelevant to his considered views on a metaphysically grounded physics. Instead, his method is taken either to refer merely to his use of algebra, or as one designed solely to solve specific problems in mathematics or physics that he later abandoned.[225]

In contrast, I think we should understand Descartes' Method as he claims we should: as a universal method to be used for all systematic inquiry, a Method he continued to employ throughout his corpus, including his metaphysics. The Method is of a piece with his view about universal wisdom. For Descartes, knowledge consisted in understanding how simpler things composed other things, and his Method instructs us on how to arrange these component parts serially in chains of thought so we can see the interconnections among the composite things while reducing them to their most fundamental, simple natures. The discerning and ordering of the parts of these complexes will explain how, and to what extent, these complexes are related.

All this would suggest that our understanding of Descartes' philosophy, and how he came to see the unity of science, is best seen when viewed though the lenses of the Method with which he used. It will help us take Descartes' seemingly diverse contributions, and appropriately focus them so that we can correctly see how Descartes saw the convergence of his natural philosophy (*e.g.*, physics), mathematics (*e.g.*, geometry, algebra), and metaphysics (*e.g.*, the essence of matter) into *a single, unified science*. This is an attractive position because showing how physical objects and mathematical objects both consist in *the exact same nature*

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[225] For discussions and debates about the nature and continuity of Descartes' method, see Alexandrescu (2009); Beck (1952); Dear (1998); Flage and Bonnen (1999); Garber (1992), (2000), (2002); Machamer and Maguire (2009); Sasaki (2003); Shuster (1977); Smith, N. (2010); and Smith, K. (2010).

can thereby offer an explanation for the applicability and unity of physics and mathematics for Descartes that is based on his fundamental metaphysics and epistemology.

Of course, the larger project of understanding the whole of Descartes' mathematical physics cannot be attempted here, but it does suggest an interesting, larger, interpretive project for further investigation. The larger project of reconstructing Descartes' unification of science and the applicability of mathematics to physics will show the natures of mathematical and physical objects to be identical in the same way that the objects of geometry and algebra are identical. This shared nature, as we have seen above in the case of mathematical objects, is geometrical extension—or what we know from his metaphysics as *extended substance*. Descartes' Method uses the combining of simple natures into complicated and obscure compounds to explain the diversity of our ideas. Here, I showed how this difference in ordering could explain the difference between algebra and geometry, despite their metaphysical identity. I suggest this difference in the complexity of compounded natures is also what will account for the apparent differences between the objects of mathematics, the objects of physics, and the material substance of Descartes' metaphysics. And just as the ordering of magnitudes and multitudes in the case of Descartes' analytic geometry can explain how they all reduce to the same simple nature, Descartes' Method will also show how the objects of mathematics, the objects of physics and material substance all reduce to the same simple nature. I believe the complete unification of mathematics and physics, as Descartes understood it, can be explained in a fashion consistent with and informed by the Method he lays out in the *Rules*. The result is a beautiful, elegant and complex systematic account of the major philosophical issues in the 17th century.

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