Constraints from Precision Electroweak Data on Leptoquarks and Bileptons

Paul H. Frampton and Masayasu Harada

Department of Physics and Astronomy, University of North Carolina,
Chapel Hill, NC 27599-3255.

Abstract

Explicit expressions are derived for the oblique parameters $S$ and $T$ in certain extensions of the standard model. In particular, we consider leptoquarks and bileptons, and find phenomenological constraints on their allowed masses. Leptoquarks suggested by the neutral and charged current anomalies at HERA can give improved agreement with both $S$ and $T$. If bileptons are the only new states, the singly-charged one must be heavier than the directly-established lower limit. Finally, we study $SU(15)$ grand unification and show that there are regions of parameter space where the theory is compatible with experimental data.
I. INTRODUCTION

Model building for particle theory at and below the TeV energy scale is not made redundant by the standard model but is certainly very constrained by it. There can be little that we do not already know much below the W mass. In the mass region up to the TeV regime one expects the Higgs boson and perhaps supersymmetric partners. But otherwise one must tread softly to avoid upsetting the delicate agreement between the standard model and precision experimental data.

In the present paper we take a fresh look at how certain additional bosons can co-exist with one another and with the successful standard model. Of special interest are particles which can contribute negatively to the very useful parameters $S$ and $T$ which measure compatibility with precision electroweak data. This is because these parameters are generally positive especially for the charged fermionic particles which one might add.

We shall focus on scalar leptoquarks and bileptonic gauge bosons. Scalar leptoquarks are of special interest because of the experimental anomalies see at HERA (see Section II below). Bileptons occur in theoretical frameworks which extend the Standard Model (see Section III below).

The outline of the paper is as follows: In Section II we treat scalar leptoquarks; in Section III we analyse bileptons. In Section IV, we examine whether SU(15) grand unification can survive its additional mirror fermions. Finally, in Section V there are our conclusions.
II. SCALAR LEPTOQUARKS

The H1 [1] and ZEUS [2] collaborations reported a possible excess of neutral current (NC) events in $e^+p$ collisions. This excess can be explained by the existence of the leptoquark of mass around 200 GeV [3]. In addition to the NC channel, an excess in the charged current (CC) was also reported [4].

Recently, D0 [5] and CDF [6] gave the 95% C.L. lower limit on the masses of 225 GeV and 213 GeV with assuming unit branching fraction into a first-generation charged lepton plus jet. For compatibility with the HERA anomaly, therefore, the scalar leptoquark must have a significant branching ratio to other decay channels.

Concerning the contribution from leptoquarks to the weak charge of a nucleus, measured by atomic parity violation experiments: as shown in Ref. [7], there is a relatively large contribution coming from tree level diagrams of $\Delta Q_{LQ}^W$ between $-0.1$ and $-0.8$ which increases the discrepancy of measurements from the Standard Model prediction; however, there is non-negligible contribution coming from the oblique corrections, especially $S$: $\Delta Q_{W}^{\text{oblique}} \simeq -0.79S - 0.01T$ [8] which could improve agreement for $S$ sufficiently negative.

In Ref. [7] a mixing between two scalar leptoquark multiplets carrying different weak hypercharges was introduced to simultaneously explain the NC and CC anomaly at HERA. Since the lightest leptoquark couples to both $e + j$ and $\nu + j$, the CDF/D0 limits are weakened. They also studied the contributions to $\rho$ parameter [8], and showed that $\Delta \rho$ from leptoquark could be negative. Since a relatively large mixing is needed, another electroweak precision parameter $S$ [8] should be also studied. [The parameter $T$ is equivalent to $\Delta \rho$.]

The contributions from single SU(2)$_L$ doublet of leptoquarks to $S$ and $T$ are studied in Ref. [11]. It is shown that the contribution to $S$ can be negative while that to $T$ is positive semidefinite. However, in the present case, the situation is different due to the existence of relatively large mixing between two doublets.

In this section we study the contributions to the oblique parameters $S$ and $T$ from two
doublets of leptoquarks. We show that both the contributions to $S$ and $T$ can be negative.

The NC and CC anomaly at HERA can be simultaneously explained by introducing two SU(2)$_L$ doublets of leptoquarks with weak hypercharges 7/6 and 1/6 [7]. Let us write these doublets as $\Phi_{7/6}$ ($Y = 7/6$) and $\Phi_{1/6}$ ($Y = 1/6$), both of which belong to a 3 representation of SU(3)$_C$. The electric charges are $\Phi_{7/6}(Q = 5/3, 2/3)$ and $\Phi_{1/6}(Q = 2/3, -1/3)$. The SU(3)$_C \times$SU(2)$_L \times$U(1)$_Y$ invariant mass terms are given by

$$\mathcal{L}_M = -M^2\Phi_{7/6}^\dagger \Phi_{7/6} - M'^2\Phi_{1/6}^\dagger \Phi_{1/6}.$$  \hspace{1cm} (2.1)

The interactions to the standard Higgs field $H$ are given by

$$\mathcal{L}_H = -\lambda_1 |H^\dagger \Phi_{7/6}|^2 - \lambda_2 |H^\dagger \Phi_{1/6}|^2$$
$$-\tilde{\lambda}_1 |\tilde{H}^\dagger \Phi_{7/6}|^2 - \tilde{\lambda}_2 |\tilde{H}^\dagger \Phi_{1/6}|^2$$
$$-\lambda_3 \left[(\Phi_{7/6})^\dagger H \tilde{H}^\dagger \Phi_{1/6} + \text{h.c.}\right],$$  \hspace{1cm} (2.2)

where $\tilde{H} = (i\tau_2 H)^T$. After electroweak symmetry breaking by the vacuum expectation value of $H$, the $\lambda_1$ and $\tilde{\lambda}_1$ terms give mass splittings between $Q = 5/3$ and $Q = 2/3$ components ($\Phi_{7/6}^{5/3}, \Phi_{7/6}^{2/3}$) of $\Phi_{7/6}$, and $\lambda_2$ and $\tilde{\lambda}_2$ terms make mass difference between $\Phi_{1/6}^{2/3}$ and $\Phi_{1/6}^{1/3}$. On the other hand, the $\lambda_3$ term gives mixing between two $Q = 2/3$ leptoquarks of $\Phi_{7/6}$ and $\Phi_{1/6}$. Let $\varphi$ denote this mixing angle:

$$\begin{pmatrix} \Phi_{7/6}^{2/3} \\ \Phi_{1/6}^{2/3} \end{pmatrix} = \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} \Phi_{7/6}^{2/3} \\ \Phi_{1/6}^{2/3} \end{pmatrix},$$  \hspace{1cm} (2.3)

where $\Phi_{7/6}^{2/3}$ and $\Phi_{1/6}^{2/3}$ denote mass eigenstates. [We use the convention where the mass of $\Phi_{7/6}^{2/3}$ which we identify with the putative HERA state is lighter than that of $\Phi_{1/6}^{2/3}$.] We do not write explicit form of masses ($m_{5/3}, m_{-1/3}, m_{2/3b}, m_{2/3u}$) and the mixing angle $\varphi$ in terms of original parameters in Eqs. (2.1) and (2.2), since all of them are independent parameters.

---

1 There are other terms like $|H^\dagger H|\Phi_{7/6}^\dagger \Phi_{7/6}$. But their contributions are absorbed into the redefinitions of $M$ and $M'$ in Eq. (2.3).
The D0 [5] and CDF [6] data imply that the three heavier leptoquark states should be above 225 GeV if the BR to $e^+q$ is one; with a smaller BR they can be lighter. However, it should be noticed that for the consistency of the model any differences among all the masses are less than a few 100 GeV [7]. Note also that the leptoquark $\Phi_{h}^{2/3}$ couples to $e^+q$ proportional to $\sin \varphi$ and that as $\varphi$ is increased the branching ratio $\Phi_{t}^{2/3} \to e^+q$ falls below one.

The contributions to $S$ and $T$ from these doublets are given by [11]

$$T = \frac{3\sqrt{2}G_F}{16\pi^2\alpha} \left[ \cos^2 \varphi \left\{ G_1(m_{5/3}, m_{2/3}) + G_2(m_{2/3}, m_{-1/3}) \right\} + \sin^2 \varphi \left\{ G_1(m_{5/3}, m_{2/3}) + G_2(m_{2/3}, m_{-1/3}) \right\} - 4 \sin^2 \varphi \cos^2 \varphi G_1(m_{2/3}, m_{2/3h}) \right],$$

$$S = 3 \left[ \frac{7}{36} \ln \frac{m_{1/3}^2}{m_{2/3}^2} - \frac{1}{6} \ln \frac{m_{2/3}^2}{m_{2/3h}^2} \right] + \frac{7}{6} \left\{ G_2(m_{Z}^2, m_{-1/3}, m_{-1/3}) - G_2(m_{Z}^2, m_{5/3}, m_{5/3}) \right\} - \cos 2\varphi \left\{ G_2(m_{Z}^2, m_{2/3}, m_{2/3}) - G_2(m_{Z}^2, m_{2/3}, m_{2/3}) \right\} - \sin^2 \varphi \left\{ G_2(m_{Z}^2, m_{-1/3}, m_{-1/3}) - G_2(m_{Z}^2, m_{2/3}, m_{2/3}) \right\} - \cos^2 \varphi \left\{ G_2(m_{Z}^2, m_{-1/3}, m_{-1/3}) - G_2(m_{Z}^2, m_{2/3}, m_{2/3}) \right\} + 2 \sin^2 \varphi \cos^2 \varphi \left\{ 2G_2(m_{Z}^2, m_{2/3}, m_{2/3}) - G_2(m_{Z}^2, m_{2/3}, m_{2/3}) \right\} - G_2(m_{Z}^2, m_{2/3}, m_{2/3}) \right\} \right],$$

where $\theta_W$ is the weak mixing angle and

$$G_1(m_1, m_2) = m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2},$$

$$G_2(s, M, m) = \bar{F}_3(s, M, m) - \frac{F_4(s, M, m) - F_4(0, M, m)}{s}$$

with $\bar{F}_3$ and $\bar{F}_4$ given in Appendix [A].

In Figs. [12][1] we show several examples of values of $S$ and $T$ with $m_{2/3l} = 200$ GeV and $\varphi = \pi/6, \pi/4$ and $\pi/3$. For reference, we also show the experimental fit [12] after subtracting the Standard Model contribution with Higgs mass of 300 GeV. The value of $T$ is more sensitive to the parameter choice than $S$. Both $S$ and $T$ can be negative for a wide
range of parameters. To compare with experimental data we may need to include vertex corrections. However, the Yukawa coupling of leptoquarks to fermions is small \[3\], so we expect that vertex and box corrections are small compared with the contributions through \( S \) and \( T \). In such a case leptoquarks can improve the agreement with the data.

![Diagram](image)

**FIG. 1.** Three points indicated by (A), (B) and (C) correspond to the parameter choices \((m_{2/3}, m_{-1/3}, m_{5/3}) = (350, 345, 350), (250, 250, 250)\) and \((275, 300, 250)\), respectively. \( (m_{2/3} = 200\) GeV and \( \varphi = \pi/4 \)\) The contours here (and in Figs. 2 and 3) are for 39%, 90%, 99% confidence levels.
FIG. 2. The points indicated by (A) and (B) correspond to the parameter choices 
\( (m_{2/3}, m_{-1/3}, m_{5/3}) = (310, 250, 340) \) and \( (250, 250, 250) \), respectively. \( (m_{2/3} = 200 \text{ GeV and } \varphi = \pi/3) \)

FIG. 3. The points indicated by (A) and (B) correspond to the parameter choices 
\( (m_{2/3}, m_{-1/3}, m_{5/3}) = (300, 280, 280) \) and \( (250, 250, 250) \), respectively. \( (m_{2/3} = 200 \text{ GeV and } \varphi = \pi/6) \)
III. BILEPTONS

In certain extensions of the standard model, there occur bileptonic gauge bosons \[13–15\] which typically occur in \(SU(2)\) doublets \((Y^{--}, Y^-)\) and their conjugates \((Y^{++}, Y^+)\). The experimental data currently constrain the masses of bileptons differently for the singly-charged and doubly-charged varieties. (For a recent review, see, e.g., Ref. [16].)

The best lower limit on the singly-charged bilepton is presently given by the precision data on the decay of polarized muons. If the normal decay \(\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu\) is contaminated by the bilepton-mediated \(\mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu\) Fierz rearrangement means that the latter contributes proportionally to \(V + A\) rather than \(V - A\). The limit on the Michel parameter \(\xi\) in the coupling \(V - \xi A\) of \(\xi > 0.997\) found at the PSI experiment [17] gives the limit \(M(Y^-) > 230\text{GeV}\).

For the doubly-charged bilepton \(Y^{--}\) a tighter lower bound has been found recently from muonium - antimuonium conversion limits, also at PSI [18]. The data require that \(M(Y^{--}) > 360\text{GeV}\).

In the models which predict bileptonic gauge bosons \(SU(2)_L \times U(1)_Y\) is part of a gauged \(SU(3)_I\), and the bileptonic gauge bosons become massive when the \(SU(3)_I\) is broken. There are generally two types of models for this kind of bileptons: (1) bilepton gauge bosons couple to only leptons as in \(SU(15)\) [13,14]; (2) bilepton gauge bosons couple to quarks as well as leptons as in 3-3-1 model [15].

In the case (1) the usual \(SU(2)_L\) gauge bosons are certain linear combinations of gauge bosons of the unbroken \(SU(2)\) subgroup of \(SU(3)_I\) and other gauge bosons coupling to quarks.

The generators of \(SU(2)_L\) and \(U(1)_Y\), \(I^a\) and \(Y\), are embedded as

\[
I^a = T_I^a + \cdots, \quad (a = 1, 2, 3), \quad Y = -\sqrt{3}T_I^8 + \cdots, \tag{3.1}
\]

where \(T_I^a\) denote the generators of \(SU(3)_I\), and dots stand for other contributions. The relations of the gauge bosons \(W^a_\mu\) and \(B_\mu\) of the standard \(SU(2)_L \times U(1)_Y\) to the \(A_I^a\) of \(SU(3)_I\) are given by
\[ g_t A^a_\mu = g W^a_\mu + \cdots , \quad (a = 1, 2, 3) \]
\[ g_t A^8_\mu = -\sqrt{3}g' B_\mu + \cdots , \] (3.2)

where \( g, g' \) and \( g_t \) denote the corresponding gauge coupling constants. The bileptonic gauge bosons are expressed as

\[ Y^{\pm\pm}_\mu = \frac{1}{\sqrt{2}} \left( A^4_\mu \mp A^5_\mu \right), \quad Y^{\pm}_\mu = \frac{1}{\sqrt{2}} \left( A^6_\mu \mp A^7_\mu \right). \tag{3.3} \]

In the models of type (2) the unbroken SU(2) subgroup of SU(3)_l is nothing but the electroweak SU(2)_L. Then the first relations of Eqs. (3.1) and (3.2) become

\[ T^a = T^a_t , \quad g_t A^a_\mu = g W^a_\mu , \quad (a = 1, 2, 3) , \] (3.4)

with \( g_t = g \). The dots parts in the second relations of Eqs. (3.1) and (3.2) are modified. The definitions of bileptonic gauge fields in Eq. (3.3) remain intact.

In both types of models the bileptonic gauge bosons of SU(3)_l makes SU(2)_L doublet with hypercharge 3/2 and its conjugate. It is convenient to use SU(2)_L doublet notation \( Y_\mu \equiv (Y^{++}_\mu, Y^+_\mu) \). The effective Lagrangian for bileptonic gauge bosons at the scale below SU(3)_l breaking scale can be written as

\[
\begin{align*}
\mathcal{L}_0 &= -\frac{1}{2} (Y^{\mu\nu})^\dagger Y^{\mu\nu} + (D_\mu \Phi - iMY_\mu)^\dagger (D^\mu \Phi - iMY^\mu) \\
&\quad - igY^{\mu\nu} F^{\mu\nu}(W) Y^\mu + i\frac{3}{2} g' Y^{\mu\nu} F^{\mu\nu}(B) Y^\mu ,
\end{align*}
\] (3.5)

where \( \Phi \) are the would-be Nambu-Goldstone bosons eaten by bileptonic gauge bosons: \( \Phi = (\Phi_{++}, \Phi_+) \). The \( Y^{\mu\nu} \) and \( D_\mu \Phi \) are given by

\[ Y_{\mu\nu} = D_\mu Y_\nu - D_\nu Y_\mu , \quad D_\mu Y_\nu = \left[ \partial_\mu - igW_\mu + i\frac{3}{2} g' B_\mu \right] Y_\nu , \]
\[ D_\mu \Phi = \left[ \partial_\mu - igW_\mu + i\frac{3}{2} g' B_\mu \right] \Phi . \tag{3.6} \]

In the simplest case the SU(2)_L doublet Higgs field is introduced as a part of SU(3)_l triplet (or anti-triplet). The other component field generally carries lepton number two, and SU(2)_L singlet with hypercharge one or two:
\[
\phi = \begin{pmatrix} H_1 \\ \phi_- \end{pmatrix}, \quad \phi' = \begin{pmatrix} H_2 \\ \phi_- \end{pmatrix}, \quad (3.7)
\]

where \(H_1 \ (H_2)\) is a SU(2)_L doublet field with hypercharge \(1/2 \ (-1/2)\), and \(\phi_- \ (\phi_-)\) is a SU(2)_L singlet field with hypercharge \(-1 \ (-2)\).

Both the VEVs of these Higgs fields \(H_1\) and \(H_2\) give masses to \(W\) and \(Z\) bosons, and the standard electroweak SU(2)_L \(\times\) U(1)_Y is broken. The VEV of \(H_1\) gives a mass correction to \(Y^-\), while the VEV of \(H_2\) gives a mass correction to \(Y^{--}\). If only one Higgs doublet had VEV, the mass difference of bileptons would be related to the mass of \(W\) boson. But in realistic models several Higgs fields are needed to have VEVs. In such a case both the masses of bileptons and \(W\) boson are independent with each other. In the following we regard the bilepton masses as independent quantities. Moreover, the actual would-be Nambu-Goldstone bosons eaten by bileptonic gauge bosons are certain linear combinations of \(\Phi\) in Eq. \(3.5\) with \(\phi_-\) or \(\phi_{--}\). We assume that the contributions to \(S\) and \(T\) due to these mixings are small compared with the bilepton contributions. Thus we use the following effective Lagrangian for the kinetic term of would-be Nambu-Goldstone bosons and bilepton masses after SU(2)_L \(\times\) U(1)_Y is broken:

\[
\mathcal{L}_{NG} = (D_\mu \Phi - i\hat{M}Y_\mu)^\dagger (D^\mu \Phi - i\hat{M}Y^\mu), \quad (3.8)
\]

where \(\hat{M}\) is \(2 \times 2\) matrix given by

\[
\hat{M} = \begin{pmatrix} M_{++} & 0 \\ 0 & M_+ \end{pmatrix}. \quad (3.9)
\]

The calculations below were done in 't Hooft-Feynman gauge.

The contributions to \(S\), \(T\) and \(U\) from bileptonic gauge bosons coming through the conventional transverse self-energies are given by [19]

\[
S = -16\pi \text{Re} \frac{\Pi^{3Y}(m_Z^2) - \Pi^{3Y}(0)}{m_Z^4} 
= \frac{9}{4\pi} \ln \frac{M^2_{++}}{M^2_+} + \frac{2}{m_Z^2} \left( M^2_{++}\tilde{F}_0(m_Z^2, M_{++}, M_{++}) - M^2_+\tilde{F}_0(m_Z^2, M_+, M_+) \right)
\]

10
\[ T = \frac{4\sqrt{2} G_F}{\alpha} \left( \Pi^{11}(0) - \Pi^{33}(0) \right) \]

\[ U = 16\pi \left[ \frac{\Pi^{11}(m_W^2) - \Pi^{11}(0)}{m_W^2} - \frac{\Pi^{33}(m_Z^2) - \Pi^{33}(0)}{m_Z^2} \right] \]

\[ = \frac{1}{2\pi} \left[ 2 \left( \bar{F}_0(m_Z^2, M_{++}, M_{++}) + \bar{F}_0(m_Z^2, M_{++}, M_{+}) - 2\bar{F}_0(m_W^2, M_{++}, M_{+}) \right) \right. \]

\[ - 3 \left( \frac{M_{++}^2 + M_{+}^2}{m_W^2} \bar{F}_0(m_Z^2, M_{++}, M_{++}) + \frac{M_{+}^2}{m_Z^2} \bar{F}_0(m_Z^2, M_{++}, M_{+}) \right. \]

\[ + \left. \frac{M_{++}^2 + M_{+}^2}{m_W^2} \bar{F}_0(m_Z^2, M_{++}, M_{+}) \right) \]

\[ - \frac{3}{m_W^2} \left( (M_{++}^2 - M_{+}^2) \left( \bar{F}_A(m_W^2, M_{++}, M_{+}) - \bar{F}_A(0, M_{++}, M_{+}) \right) \right. \]

\[ - (M_{++}^2 + M_{+}^2) \bar{F}_0(0, M_{++}, M_{+}) \]

\[ \left. - 3 \left( \bar{F}_3(m_Z^2, M_{++}, M_{++}) + \bar{F}_3(m_Z^2, M_{++}, M_{+}) - 2\bar{F}_3(m_W^2, M_{++}, M_{+}) \right) \right] \] (3.10)

where functions \( \bar{F}_0 \), \( \bar{F}_A \) and \( \bar{F}_3 \) are given in Appendix A.

In the standard model the gauge boson (\( W, Z \) and photon) contributions to the \( S, T \) and \( U \) parameters, which are defined in terms of conventional self-energies, are gauge dependent. To make these parameters gauge invariant we need to add pinch parts arising from vertex and box diagrams [20]. In the present case the contributions of bilepton gauge bosons to conventional self-energies are gauge dependent, so that we need to add pinch parts.

In 't Hooft-Feynman gauge this pinch term arises from a vertex correction which includes a triple vertex of gauge bosons as shown in Fig. 4. We specify here these pinch parts by using current correlation functions. Generally the interactions between fermionic currents and bileptonic gauge bosons can be expressed as

\[ \mathcal{L} = g_l \left[ Y^{++} J^{\mu}_{++} + Y^+ J^\mu_+ \right] + \text{h.c.} \] (3.11)

The \( W \)-fermion-antifermion vertex correction of Fig. 4 can be written as
\[
\Gamma^\mu_W(q) = \frac{gg^2}{\sqrt{2}} \int \frac{d^n k}{(2\pi)^2} \left[ (-2k - q)_\mu g_{\alpha\beta} + (k + 2q)_\alpha g_{\beta\mu} + (k - q)_\beta g_{\mu\alpha} \right] \\
D_+(k)D_{++}(k + q) \int d^n x e^{ikx} \langle f | T J^\alpha_-(x) J^\beta_+(0) | i \rangle,
\]

(3.12)

where \( D_i(k) \) denotes a bilepton propagator denominator:

\[
D_i(k) = \frac{1}{k^2 - M_i^2}.
\]

(3.13)

The pinch parts are triggered by contractions with the four-momentum present in the WYY vertex. By using the equal time commutator

\[
\delta(x_0 - y_0) \left[ J^0_{+-}(x), J^\mu_+(y) \right] = \frac{1}{2} (J^\mu_1(x) + iJ^\mu_2(x)) \delta^n(x - y),
\]

(3.14)

the pinch part of Eq. (3.12) is expressable as

\[
\Gamma^\mu_W(q)|_P = gg^2 B_0(q^2, M_+, M_{++}) \frac{1}{\sqrt{2}} \langle f | (J^\mu_1(0) + iJ^\mu_2(0)) | i \rangle.
\]

(3.15)

where \( B_0 \) is defined by

\[
B_0(q^2, M_1, M_2) = \int \frac{d^n k}{i(2\pi)^n} \frac{1}{[M_1^2 - k^2][M_2^2 - (k + q)^2]}.
\]

(3.16)

By using the relation (3.2), the current associated with the standard SU(2)L gauge boson W is related to the above \( J_{11} + iJ_{12} \) as
\[
\frac{1}{\sqrt{2}} (J_{11}^\mu + i J_{12}^\mu) = g^2 \frac{g_1^2}{g_2^2} J_\nu^{\mu_W} + \cdots. \tag{3.17}
\]

Similarly the pinch part of a Z vertex is given by
\[
\Gamma^\mu_Z (q) |_P = g^3 \left( \frac{1 - 4 \sin^2 \theta_W}{\cos \theta_W} \right) B_0 (q^2, M_{++}, M_{++}) \\
\times \left[ 2 \sin^2 \theta_W \langle f | J_{Q_{\mu}} | i \rangle + \frac{1 - 4 \sin^2 \theta_W}{2 \cos^2 \theta_W} \langle f | J_{Z_{\mu}}^\varphi | i \rangle \right] \\
+ g^3 \left( \frac{1 + 2 \sin^2 \theta_W}{\cos \theta_W} \right) B_0 (q^2, M_+, M_+) \\
\times \left[ \sin^2 \theta_W \langle f | J_{Q_{\mu}} | i \rangle - \frac{1 + 2 \sin^2 \theta_W}{2 \cos^2 \theta_W} \langle f | J_{Z_{\mu}}^\varphi | i \rangle \right] + \cdots, \tag{3.18}
\]

and that of a photon vertex by:
\[
\Gamma^\mu_Q (q) |_P = 4 e g^2 B_0 (q^2, M_{++}, M_{++}) \left[ 2 \sin^2 \theta_W \langle f | J_{Q_{\mu}} | i \rangle + \frac{1 - 4 \sin^2 \theta_W}{2 \cos^2 \theta_W} \langle f | J_{Z_{\mu}}^\varphi | i \rangle \right] \\
+ 2 e g^2 B_0 (q^2, M_+, M_+) \left[ \sin^2 \theta_W \langle f | J_{Q_{\mu}} | i \rangle - \frac{1 + 2 \sin^2 \theta_W}{2 \cos^2 \theta_W} \langle f | J_{Z_{\mu}}^\varphi | i \rangle \right] + \cdots. \tag{3.19}
\]

The self-energies of electroweak gauge bosons are modified by pinch parts which can be expressed as
\[
\Pi_{ZZ} (q^2) |_P = \langle q^2 - m_Z^2 \rangle \left[ (1 - 4 \sin^2 \theta_W)^2 B_0 (q^2, M_{++}, M_{++}) \\
+ (1 + 2 \sin^2 \theta_W)^2 B_0 (q^2, M_+, M_+) \right], \\
\Pi_{ZQ} (q^2) |_P = 2 (q^2 - m_Z^2) (1 - 4 \sin^2 \theta_W) B_0 (q^2, M_{++}, M_{++}) \\
- (q^2 - m_Z^2) (1 + 2 \sin^2 \theta_W) B_0 (q^2, M_+, M_+) , \\
\Pi_{QQ} (q^2) |_P = 4 q^2 \left[ 4 B_0 (q^2, M_{++}, M_{++}) + B_0 (q^2, M_+, M_+) \right], \\
\Pi_{WW} (q^2) |_P = 2 (q^2 - m_W^2) B_0 (q^2, M_+, M_{++}). \tag{3.20}
\]

The corrections to S and T parameters arising from the above pinch parts are therefore given by
\[
S |_P = \frac{1}{\pi} \left[ 3 \ln \frac{M_{++}^2}{M_+^2} + 2 (1 + 2 \sin^2 \theta_W) \bar{F}_0 (m_Z^2, M_{++}, M_{++}) \\
- (1 - 4 \sin^2 \theta_W) \bar{F}_0 (m_Z^2, M_+, M_+) \right],
\]

13
\[ T|_P = \frac{1}{4\pi \sin^2 \theta_W} \left[ \frac{M_{++}^2 + M_{+}^2}{M_{++}^2 - M_{+}^2} \ln \frac{M_{++}^2}{M_{+}^2} - 2 + 3\tan^2 \theta_W \ln \frac{M_{++}^2}{M_{+}^2} \right], \]

\[ U|_P = \frac{1}{\pi} \left[ 4 \sin^2 \theta_W \tilde{F}_0(m_Z^2, M_{++}, M_{++}) - 2 \sin^2 \theta_W \tilde{F}_0(m_Z^2, M_{+}, M_{+}) \right]. \] (3.21)

The expressions \( S|_P, T|_P \) and \( U|_P \) of Eq.(3.21) must be added to \( S, T \) and \( U \) respectively of Eq.(3.10).

In the models of type (2) such as the 3-3-1 model the additional part in Eq. (3.17) represented by \ldots does not exist. So all the pinch part, Eq. (3.15), is already included in \( \Pi_{WW}(q^2)|_P \) of Eq. (3.20). The dots parts in Eqs. (3.18) and (3.19) are proportional to the extra neutral gauge boson \( (Z') \) current, so are related to \( Z-Z' \) and \( \gamma-Z' \) mixings. Similarly in models of type (1) such as SU(15) the \ldots parts in Eqs. (3.17)–(3.19) are related to mixings among the electroweak gauge bosons and certain extra gauge bosons. These mixings are constrained to be very small. Then, in the analysis below, instead of introducing new parameters for these mixings we will neglect the \ldots parts in Eqs. (3.17)–(3.19).

To compare with the electroweak precision data, we need to include the vertex and box corrections as well as a tree level bilepton contribution to muon decay. As shown in Ref. [21][16], the singly-charged bilepton leads to muon decay into \( e^- \bar{\nu}_\mu \nu_\mu \). This decay mode does not interfere with the standard decay (to \( e^- \nu_\mu \bar{\nu}_e \)), so the correction to the measured Fermi constant is given by

\[ G_F^{\text{measured}} = G_F^{\text{SM}} (1 + \delta_Y) \] (3.22)

with

\[ \delta_Y = \frac{1}{2} \left( \frac{g_e^2 m_W^2}{g^2 M_W^2} \right)^2. \] (3.23)

Here, at the tree level of the SM, \( G_F^{\text{SM}} \) is related to the mass of \( W \) by

\[ \delta_Y = \frac{1}{2} \left( \frac{g_e^2 m_W^2}{g^2 M_W^2} \right)^2. \] (3.23)

Here, at the tree level of the SM, \( G_F^{\text{SM}} \) is related to the mass of \( W \) by

---

\[ 2 \text{ If bileptons have flavor changing couplings to leptons there are other decays, but such processes are strongly constrained [16]. In this paper we assume that bileptons have no flavor-changing couplings.} \]
\[ G_F^{\text{SM}} \big|_{\text{tree}} = \frac{g^2}{4\sqrt{2}m_W^2}. \] (3.24)

It should be noticed that the one-loop correction to the above relation is estimated as about 0.55% in the SM \[ \text{[12]} \], while the correction from singly-charged bileptons is about \( \delta_Y \approx 0.005 \), 0.0003 for \( m(Y^+) = 250, 500 \) GeV, respectively.

In models of type (2) bileptons couples to quarks as well as leptons. However, such coupling is related to the heavy \( \text{SU(2)}_L \times \text{U(1)}_Y \) singlet quarks with lepton number two. In the present analysis we consider the case where only the bileptons are light enough to affect the electroweak precision data. So we use an effective theory where bileptons couple to only leptons for both types of models. The non-pinch part of the \( Zl\bar{l} \) vertex correction is then given by

\[ \Gamma_{Zl\bar{l}}(p^\mu) = \frac{g}{\cos \theta_W} \left[ \Gamma_{Z,L}^{\mu}(p^2)\gamma^{\mu} \frac{1 - \gamma_5}{2} + \Gamma_{Z,R}^{\mu}(p^2)\gamma^{\mu} \frac{1 + \gamma_5}{2} \right], \]

\[ \Gamma_{Z,L}^{\mu}(p^2) = \frac{g^2}{(4\pi)^2} \left[ 1 - 4 \sin^2 \theta_W \frac{G_1}{4} \left( \frac{p^2}{M_{++}} \right) + \sin^2 \theta_W \frac{G_2}{2} \left( \frac{p^2}{M_{++}} \right) \right], \]

\[ \Gamma_{Z,R}^{\mu}(p^2) = \frac{g^2}{(4\pi)^2} \left[ 1 - 4 \sin^2 \theta_W \frac{G_1}{4} \left( \frac{p^2}{M_{++}} \right) - \sin^2 \theta_W \frac{G_2}{2} \left( \frac{p^2}{M_{++}} \right) \right. \]

\[ \left. - \frac{1 + 2 \sin^2 \theta_W}{4} \frac{G_1}{G_1} \left( \frac{p^2}{M_{+}} \right) + \frac{1}{4} \frac{G_2}{G_2} \left( \frac{p^2}{M_{+}} \right) \right], \] (3.25)

where we have neglected the lepton masses. The functions \( G_1 \) and \( G_2 \) are given in Appendix A. Similarly the non-pinch part of the \( Z\nu\bar{\nu} \) vertex correction is given by

\[ \Gamma_{Z\nu\bar{\nu}}(p^\mu) = \frac{g}{\cos \theta_W} \Gamma_{Z,L}^{\nu}(p^2)\gamma^{\mu} \frac{1 - \gamma_5}{2}, \]

\[ \Gamma_{Z,L}^{\nu}(p^2) = \frac{g^2}{(4\pi)^2} \left[ -1 + 2 \sin^2 \theta_W \frac{G_1}{4} \left( \frac{p^2}{M_{+}} \right) + \sin^2 \theta_W \frac{G_2}{2} \left( \frac{p^2}{M_{+}} \right) \right]. \] (3.26)

The non-pinch part of the \( W\nu\bar{\nu} \) vertex correction at low-energy limit is given by

\[ \Gamma_{W\nu\bar{\nu}}(p^\mu) = \frac{g}{\sqrt{2}} \Gamma_{W,L}(p^2)\gamma^{\mu} \frac{1 - \gamma_5}{2}, \]

\[ \Gamma_{W,L}(0) = -\frac{g^2}{2(4\pi)^2} \left[ \frac{M_{++}^2 + M_{++}^2}{M_{++}^2 - M_{+}^2} \ln \frac{M_{++}}{M_{+}} - 1 \right]. \] (3.27)

Finally the box diagram correction to the above relation (3.22) is given by
\[ \delta Y_{\text{box}} = \frac{1}{4\sqrt{2}G_F(4\pi)^2} \left[ \frac{1}{M_{++}^2 - M_+^2} \ln \frac{M_{++}}{M_+} \right]. \] (3.28)

Combining all the above formulas we have compared the bilepton models with 15 pieces of data taken from Ref. [22,12] shown in Table I for convenience; aside from the $W$ mass, the 14 data are all $Z$-pole quantities. This implies that the box diagram corrections are needed only for the charged current processes.

<table>
<thead>
<tr>
<th>data</th>
<th>SM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_Z) [GeV]</td>
<td>91.1863 ± 0.0020</td>
</tr>
<tr>
<td>(\Gamma_Z) [GeV]</td>
<td>2.4946 ± 0.0027</td>
</tr>
<tr>
<td>(\sigma_h^0) [nb]</td>
<td>41.508 ± 0.056</td>
</tr>
<tr>
<td>(R_\ell)</td>
<td>20.778 ± 0.029</td>
</tr>
<tr>
<td>(A_{FB}^{0,\ell})</td>
<td>0.0174 ± 0.0010</td>
</tr>
<tr>
<td>(A_r)</td>
<td>0.1401 ± 0.0067</td>
</tr>
<tr>
<td>(A_c)</td>
<td>0.1382 ± 0.0076</td>
</tr>
<tr>
<td>(R_b^0)</td>
<td>0.2178 ± 0.0011</td>
</tr>
<tr>
<td>(R_c^0)</td>
<td>0.1715 ± 0.0056</td>
</tr>
<tr>
<td>(A_{FB}^{0,b})</td>
<td>0.0979 ± 0.0023</td>
</tr>
<tr>
<td>(A_{FB}^{0,c})</td>
<td>0.0735 ± 0.0048</td>
</tr>
<tr>
<td>(A_b)</td>
<td>0.863 ± 0.049</td>
</tr>
<tr>
<td>(A_c)</td>
<td>0.625 ± 0.084</td>
</tr>
<tr>
<td>(A_{LR}) (SLD)</td>
<td>0.1542 ± 0.0037</td>
</tr>
<tr>
<td>(\sin^2 \theta_{\text{eff}}^{\text{lept}} (\langle Q_{FB}\rangle))</td>
<td>0.2320 ± 0.0010</td>
</tr>
<tr>
<td>(m_W) [GeV]</td>
<td>80.356 ± 0.125</td>
</tr>
</tbody>
</table>

TABLE I. Electroweak precision data used in the present fit. The column indicated by SM shows the predictions of the standard model with \(m_H = 300\) GeV.

Two lower bounds on \(m(Y^{++})\) exist in the literature: 360 GeV from Ref. [18], and very recently 850 GeV from Ref. [23]. As well as the best values for \(M(Y^+)\) and \(M(Y^{++})\) we
therefore quote the bounds on \( M(Y^+) \) for each of the lower limits on \( M(Y^{++}) \).

The results for \( m_H = 100 \text{ GeV}, 300 \text{ GeV} \) and \( 500 \text{ GeV} \) are respectively \((M(Y^+), M(Y^{++})) = (566 \text{ GeV}, 574 \text{ GeV}), (1391 \text{ GeV}, 1417 \text{ GeV}) \) and \((1647 \text{ GeV}, 1602 \text{ GeV}) \). (See Figs. 5–7.) For the fixed values \( M(Y^{++}) = 360 \text{ GeV}, 850 \text{ GeV} \) the (lower, upper) limits on \( m_1 \) (GeV) are respectively shown in Table II. In all cases, the lower bound on \( M(Y^+) \) is an improvement on the single-charge bilepton empirical limit (230 GeV) found in Ref. [21].

The best overall fit is for masses \( m(Y^+) = 566 \text{ GeV} \) and \( m(Y^{++}) = 574 \text{ GeV} \) with \( \chi^2/\text{d.o.f} = 17.6/(15 - 2) \). The constraint for \( m(Y^+) \) with fixed \( m(Y^{++}) = 360 \text{ GeV} \) is \( 367 < m(Y^+) < 384 \text{ (GeV)} \); that with \( m(Y^{++}) = 850 \text{ GeV} \) is \( 790 < m(Y^+) < 864 \text{ (GeV)} \).
FIG. 6. Allowed region (90%CL) for $m_H = 300$ GeV. ($m_1 = m(Y^+)$ and $m_2 = m(Y^{++})$)

The best overall fit is for masses $m(Y^+) = 1417$ GeV and $m(Y^{++}) = 1391$ GeV with

$\chi^2$/d.o.f = 17.7/(15 − 2). The constraint for $m(Y^+)$ with fixed $m(Y^{++}) = 360$ GeV is

$387 < m(Y^+) < 397$ (GeV); that with $m(Y^{++}) = 850$ GeV is $837 < m(Y^+) < 894$ (GeV).

FIG. 7. Allowed region (90%CL) for $m_H = 500$ GeV. ($m_1 = m(Y^+)$ and $m_2 = m(Y^{++})$)

The best overall fit is for masses $m(Y^+) = 1647$ GeV and $m(Y^{++}) = 1602$ GeV with

$\chi^2$/d.o.f = 17.7/(15 − 2). $m(Y^{++}) = 360$ GeV is excluded. The constraint for $m(Y^+)$ with

fixed $m(Y^{++}) = 850$ GeV is $865 < m(Y^+) < 905$ (GeV).
| $m_H$ [GeV] | $M(Y^{++})|_{\text{min}}$ [GeV] | allowed $M(Y^+)$ [GeV] |
|------------|-----------------|-----------------|
| 100        | 360             | $367 < M(Y^+) < 384$ |
|            | 850             | $790 < M(Y^+) < 864$ |
| 300        | 360             | $387 < M(Y^+) < 397$ |
|            | 850             | $837 < M(Y^+) < 894$ |
| 500        | 360             | excluded         |
|            | 850             | $865 < M(Y^+) < 905$ |

TABLE II. Bounds for $M(Y^+)$ for fixed values of $M(Y^{++})$.

IV. SU(15) GRAND UNIFICATION

In a grand unified model based on SU(15) [13,14] each generation of quarks and leptons is represented by a fundamental 15. To cancel anomalies of three generations, three generations of mirror fermions are needed. Since these mirror fermions obtain their masses from the VEV which breaks standard SU(2)$_L \times$ U(1)$_Y$ symmetry they necessarily are close to the weak scale in mass and give significant contributions to $S$ and $T$. Even if we assume that members of the same SU(2)$_L$ doublet have degenerate masses, and hence the mirror fermions give no contribution to $T$, they do give a very large contribution to $S$ parameter; $S_{\text{mirror}} = 2/\pi$. Then one might think that this model is already excluded by the precision electroweak analysis? However, there are many extra particles including bileptons and leptoquarks in the model. These extra particles could give non-negligible contributions $S$ and $T$ as discussed in the previous sections. As is easily read from Eqs. (3.10) and (3.21), there is a negative contribution to $S$ coming from bileptons if the singly-charged bilepton ($Y^-$) is heavier than the doubly-charged one ($Y^{--}$). This negative contribution can cancel the large positive contribution coming from mirror fermions. On the other hand, such a mass difference of bileptons gives a large positive contribution to the $T$ parameter. But this could, in turn, be canceled by a negative contribution of leptoquarks without affecting the $S$ parameter.

The vertex and box corrections are quite negligible in this SU(15) case where the $Y^{++}$-
$Y^+$ mass difference is large. We have explicitly checked that there exists an extended region of parameter space where $S$ and $T$ are acceptable: $SU(15)$ grand unification is not yet excluded by experiment!
V. CONCLUSIONS

The continued robustness of the standard model with respect to more and more accurate experimental data gives tight constraints on any attempt at ornamentation of the theory by additional "light" physics.

The parameters $S$ and $T$ provide a very convenient measure of compatibility with the precision electroweak data. Additional states give, in general, positive contributions to $S$ and $T$ and hence rapidly lead to a possible inconsistency. It is therefore of special interest to define what architecture can contribute negatively to $S$ and $T$.

Here we have discussed two examples: bileptons and leptoquarks. For bileptons we have derived a lower bound of 367GeV for the singly-charged bileptonic gauge boson, assuming that bileptons are the only states additional to the standard model; this improves considerably on the mass bound available from direct measurement.

If we identify the putative leptoquark at HERA with a mixture of scalar doublets of SU(2) having different hypercharge, this can also improve agreement with data.

Finally we have addressed the exaggerated reports of the death of SU(15) grand unification due to the large positive $S$ value from its three generations of mirror fermions. Because of the simultaneous presence of both bileptons and leptoquarks in SU(15), there is an extended neighborhood in parameter space where $S$ (and $T$) can be acceptably small in magnitude.

ACKNOWLEDGEMENT

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG05-85ER-40219.
Here we give several functions used in sections II and III.

\[
F_0(s, M, m) = \int_0^1 dx \ln \left( (1 - x)M^2 + xM^2 - x(1 - x)s \right) - \ln Mm
\]

\[
\begin{cases}
-\frac{2}{s} \sqrt{(M + m)^2 - s} \sqrt{(M - m)^2 - s} \ln \frac{\sqrt{(M+m)^2-s}+\sqrt{(M-m)^2-s}}{2\sqrt{Mm}} \\
+ \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } s < (M - m)^2, \\
\frac{2}{s} \sqrt{(M + m)^2 - s} \sqrt{s - (M - m)^2} \tan \frac{\sqrt{s-(M-m)^2}}{(M+m)^2-s} \\
+ \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } (M - m)^2 < s < (M + m)^2, \\
\frac{2}{s} \sqrt{s - (M + m)^2} \sqrt{s - (M - m)^2} \ln \frac{\sqrt{s-(M+m)^2}+\sqrt{s-(M-m)^2}}{2\sqrt{Mm}} - i\pi \\
+ \frac{M^2 - m^2}{s} \ln \frac{M}{m} - 2, & \text{for } (M + m)^2 < s,
\end{cases}
\]

\[
\bar{F}_A(s, M, m) = \int_0^1 dx (1 - 2x) \ln \left( (1 - x)M^2 + xM^2 - x(1 - x)s \right)
\]

\[
= -\frac{M^2 - m^2}{s} \left[ \bar{F}_0(s, M, m) - \bar{F}_0(0, M, m) \right],
\]

\[
\bar{F}_3(s, M, m) = \int_0^1 dx x(1 - x) \ln \left( (1 - x)M^2 + xM^2 - x(1 - x)s \right) - \frac{1}{6} \ln Mm
\]

\[
= \frac{1}{6} \left[ 1 + \frac{M^2 + m^2}{s} - \frac{2(M^2 - m^2)}{s^2} \right] \bar{F}_0(s, M, m)
- \frac{1}{6} \left( 1 - \frac{2(M^2 + m^2)}{s} \right) \frac{M^2 - m^2}{s} \ln \frac{M}{m} + \frac{1}{18} - \frac{(M^2 - m^2)^2}{3s^2}.
\]

\[
\bar{F}_4(s, M, m) = \int_0^1 dx \left[ (1 - x)M^2 + xM^2 \right] \ln \left( (1 - x)M^2 + xM^2 - x(1 - x)s \right)
- \frac{M^2 + m^2}{2} \ln Mm
\]

\[
= \frac{M^2 + m^2}{2} \bar{F}_0(s, M, m) + \frac{M^2 - m^2}{2} \bar{F}_A(s, M, m).
\] (A1)

The functions used in the vertex corrections for bileptons are given by

\[
\bar{G}_1(a) = \frac{2(1 + 2a)}{a(4 - a)} \left[ (J(a))^2 + aJ(a) \right] + J(a) + \frac{9a}{2(4 - a)};
\]

\[
\bar{G}_2(a) = \frac{7}{2} + \frac{2}{a} - \left( 3 + \frac{2}{a} \right) \ln(-a) + 2 \left( 1 + \frac{1}{a} \right)^2 \left[ \text{Sp}(-a) + \ln(-a) \ln(1 + a) \right],
\] (A2)

where \( J(a) = \bar{F}_0(a, 1, 1) \) and \( \text{Sp}(x) \) is the Spence’s function defined by

\[
\text{Sp}(x) = -\int_0^x \frac{dt}{t} \ln(1 - t).
\] (A3)
REFERENCES


[22] The LEP Collaborations ALEPH, DELPHI, L3 OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavour Group, Report No. CERN-PPE/96-183 (unpub-
lished).