Construction and Simulation of a 1 MeV Electron Accelerator for High Precision Beta Decay Studies

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Abstract

Sensitive experiments with low energy neutrons are being used as tests for physics beyond the standard model. A 1 MeV electron accelerator for use as a calibration tool for detectors in high precision neutron beta decay experiments is currently being constructed at Triangle Universities Nuclear Laboratory (TUNL). The adjustable potential allows electrons to be accelerated to energies across the continuous beta decay spectrum, which is an improvement over current calibration methods. In this thesis, the Pelletron charging system employed by the accelerator is described and the results of a high voltage test of the accelerator are reported. Furthermore, magnetic and electric field maps are produced for the accelerator and spectrometer that holds the detectors and electron transport through the accelerator and spectrometer is analyzed using the finite element analysis software COMSOL, showing the validity of the design of the accelerator.
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1. INTRODUCTION

Neutron beta decay is the decay of a neutron into a proton, an electron, and an electron antineutrino. At the fundamental level, neutron decay is caused by the conversion of a down quark in the neutron into an up quark through the emission of a $W^-$ boson, converting the neutron into a proton. The $W^-$ boson subsequently decays into an electron, also known as a beta particle, and an electron antineutrino [1]. This process is shown in the proceeding Feynman diagram.

![Feynman diagram for neutron beta minus decay, the decay of a neutron into a proton, electron, and electron antineutrino.](image)

FIG. 1: Feynman diagram for neutron beta minus decay, the decay of a neutron into a proton, electron, and electron antineutrino.

In 1911, Lise Meitner and Otto Hahn experimentally showed that the energy spectrum of the emitted electron in beta decay is continuous. At the time, beta minus decay was thought to be the decay of a neutron into only a proton and an electron. If this were the case, then the continuous energy spectrum of the electron would imply a loss of energy and momentum. In addition, since neutrons, protons, and electrons are fermions and therefore carry intrinsic spin, this decay could not conserve angular momentum. In order to explain how beta decay could conserve energy, momentum, and angular momentum, Wolfgang Pauli postulated the existence of the particle that eventually became known as the neutrino in 1930. In 1934, Enrico Fermi developed a successful theory of beta decay based on Pauli’s hypothesis. The electron and the electron antineutrino, along with the recoiling proton, share the total decay energy $Q$, accounting for the continuous electron energy spectrum. Below is a plot of the electron energy distribution calculated using Fermi’s theory, which treats the decay-causing interaction as a weak perturbation [2].
Today, sensitive experiments with low energy neutrons are being used as tests for physics beyond the standard model. At kinetic energies of 340 neV and below, neutrons are totally reflected at all angles of incidence by some material surfaces and can be confined by magnetic or gravitational potentials. Neutrons at these energies are known as ultracold neutrons (UCN). Experiments using UCN include measurements of the neutron lifetime, measurements of the neutron electric dipole moment, observations of the gravitational interactions of the neutron, and measurements of the A-correlation of neutron beta decay [4].

2. UCNA EXPERIMENT

At Los Alamos National Laboratory, the Ultra-Cold Neutron Asymmetry (UCNA) collaboration uses polarized UCN to study the correlation between the number and energy of the electrons emitted along and against the spin direction of the decaying neutrons [5]. The A-correlation is the energy dependence of the asymmetry in the electron emission direction relative to the neutron spin polarization. The A-correlation is related to the ratio of the axial-vector to the vector weak coupling constants. The axial-vector weak coupling constant is important in the spin and flavor structure of the nucleon, big bang nucleosynthesis, and the solar neutrino flux [6, 7].

Energy calibration for the detectors in these and similar experiments is typically achieved using conversion electron sources. Internal conversion occurs when an excited nucleus emits a gamma ray that is absorbed by one of its inner-shell electrons causing the electron to be ejected. Conversion electrons have a discrete kinetic energy equal to the energy of the gamma ray minus the binding energy of the inner-shell electron [3]. Conversion electron

![FIG. 2: Electron kinetic energy (T_e) distribution for a Q value of 2.5 MeV [3].](image)
sources used by the UCNA collaboration include cadmium-109, cesium-139, indium-114m, tin-113, strontium-85, and bismuth-207. The conversion sources are placed on mylar foils and inserted into the spectrometer housing the detectors [8]. A figure of the spectrometer is provided below.

![Spectrometer diagram](image)

**FIG. 3:** Spectrometer used in UCNA collaboration experiments [9].

However, the calibration points from conversion electron sources are not evenly distributed over the continuous beta decay energy spectrum. Furthermore, the foil backing produces perturbations in the calibration spectrum that can adversely affect high precision measurements. This can be improved by using an external, tunable electron source coupled by a magnetic field to calibrate the detector.

A 1 MeV electron accelerator is currently under construction at Triangle Universities Nuclear Laboratory (TUNL) by Dr. Albert Young of North Carolina State University to be used for detector calibration in the high precision beta decay experiments performed by the UCNA collaboration. The Pelletron charging system is used to build up voltages of up to 1 MV across the accelerator. This potential difference can be adjusted to accelerate electrons to a desired energy. As a result, detectors can be calibrated at energies across the continuous beta decay spectrum. Furthermore, the magnetic mirroring effect due to the low magnetic field in the accelerator and the high magnetic field in the spectrometer produces a range of pitch angles that can be determined by the spread in arrival time of the electrons at the detector.
3. ELECTRON ACCELERATOR CONSTRUCTION

This section focuses on the theory behind the Pelletron charging system of the 1 MeV electron accelerator and the progress that has been made on the construction and testing of the accelerator.

3.1 Pelletron Charging System

The 1 MeV electron accelerator in construction at TUNL employs a Pelletron charging system. The Pelletron charging system is a chain charging system utilizing a chain of stainless steel pellets connected by insulating nylon links that is rotated by a motor. A diagram illustrating the Pelletron charging system is provided below.

![Diagram of the Pelletron chain charging system](image)

FIG. 4: Diagram of the Pelletron chain charging system [10].

The chain rotates on two wheels and is driven by a motor. Positive charge is induced on the chain as it leaves the grounded end of the column by a negatively charged electrode biased by a high-voltage power supply called the inductor. As the wheel rotates, contact between the pellets and the wheel is broken and the positive charge becomes trapped on the pellets by the insulating nylon links. The rotating chain then transports the positively charged pellets to the terminal. As a charged pellet reaches the terminal it passes through an electrode, which develops a mirror negative charge by induction. If the gap between the electrode and the chain is the same as the gap between the inductor and the chain, the electrode and inductor voltages with respect to ground will be equal. A conductive charge pickoff wheel on the other side of the electrode picks up positive charge as the chain...
passes and applies it to another inductor on the opposite side of the terminal wheel. This positive inductor induces a negative charge on the pellets leaving the terminal. Thus, the charge is carried in both directions and the charging efficiency is doubled. The charged pellets arriving in the terminal contact the conductive rim of the terminal pulley and the trapped charge is transferred to the terminal [10]. The high voltage difference between the grounded end of the column and the terminal can be used to accelerate charged particles. Note that the polarities of the inductors can be switched so that either a large positive or a large negative voltage can be produced between the grounded end of the column and the terminal. An image of the Pelletron charging system employed by the 1 MeV electron accelerator in construction at TUNL is shown below.

![Image of the Pelletron charging system](image_url)

FIG. 5: Image of the 1 MeV electron accelerator under construction at TUNL. The chain of stainless steel pellets connected by insulating nylon links, the wheels, and the driving motor of the Pelletron charging system are visible.

### 3.2 Construction and Testing

A high voltage test of the electron accelerator was performed by biasing the inductors to the maximum voltage possible before dielectric breakdown in air occurred. In order to perform this test, a motor control system was installed. When the Pelletron charging system is running, the chain should not have any bounce over the length of the run so that the pellets do not come into contact with the inductors or the electrodes. The suggested tension is 34 N/m of chain length [10]. A motor tensioning system was designed using Autodesk Inventor, a 3D CAD software, to provide sufficient tension to the chain to perform a high
voltage test of the electron accelerator. The tensioning system consists of an L-shaped piece of aluminum with a pair of springs that connect to a form-fitting aluminum piece placed flush to the motor. In addition, there is an aluminum piece that is bolted to the bottom of the motor where the motor is bolted to a base plate. The L-piece can be slid along the base plate on which the motor is bolted to provide tension to the motor and therefore to the Pelletron chain. Once the chain is sufficiently tensioned, the L-shaped piece and the piece on attached to the bottom of the motor can be bolted into place. The tensioning system is shown below.

(a) CAD Model of the motor tensioning system.   (b) Picture of the motor tensioning system applied to the motor.

FIG. 6: Images of the motor tensioning system.

The tensioning system above was able to provide enough tension to perform a high voltage test of the electron accelerator. A string of 30 resistors, each with a resistance of 1 GΩ, steps down the voltage from the terminal to the grounded end of the column. A current of 7 µA was measured through the terminal before dielectric breakdown occurred and charge jumped between pellets. By Ohm's law, this gives a voltage across the accelerator column of 210 kV. The image below shows the string of resistors.
FIG. 7: Image of the 1 MeV electron accelerator under construction at TUNL. The string of resistors is on the far edge of the Lucite case. There are 30 resistors, each with a resistance of 1 GΩ. The red wires connect the resistors to electrodes on the column of the accelerator.

It is planned to enclose the electron accelerator in a pressure vessel so that higher voltages can be reached without breakdown. In addition, the current motor will be replaced by a smaller one and a new tensioning system will be designed.

4. ELECTRON ACCELERATOR COMSOL MODEL

This section focuses on the modeling of the 1 MeV electron accelerator using COMSOL, a finite element analysis solver and simulation software. Magnetic and electric field maps are produced for the accelerator and spectrometer system. The trajectories of the electrons, including the timing and energy spreads, are analyzed.

4.1 Finite Element Analysis

COMSOL is a simulation software that uses finite element methods for various physics and engineering applications. Finite element methods approximate the solutions to partial differential equations (PDE) with given boundary values and are useful since they can be applied to arbitrarily shaped domains. The first step in finite element analysis is to place a distribution of discretized points inside and along the boundary of the domain of the system being studied. These points are known as nodes. In COMSOL, the nodes are typically
connected to form triangular subdomains in 2D and tetrahedral subdomains in 3D. The subdomains must fill the entire volume of the original domain. In general, the smaller the subdomains, the better the approximation to the solution. An example of the mesh generated for the COMSOL model of the electron accelerator is provided below.

![Free tetrahedral mesh generated for the COMSOL model of the electron accelerator.](image)

The PDE is approximated on each subdomain with a system of algebraic equations for steady state problems or with a system of ordinary differential equations (ODE) for transient problems. The ODE system can be solved using numerical techniques such as Euler’s method or the Runge-Kutta method [11]. For the transient problems described in sections 4.4 and 4.5, COMSOL uses the generalized-α method as the integrator. The generalized-α method is a one step implicit method that achieves high-frequency numerical dissipation while minimizing low-frequency numerical dissipation and avoiding degradation of the second order accuracy. In highly non-linear problems, high-frequency numerical dissipation has been found to improve the convergence of iterative equation solvers [12].

As an example of how the finite element method works in the steady state, consider a one dimensional Poisson equation on the domain $0 \leq x \leq L$ with the following boundary conditions:

$$\frac{d^2 u}{dx^2} - f(x) = 0 \quad u(0) = 0 \quad u(L) = 0$$

(1)

Consider multiplying the differential equation by a weight function $v(x)$ and integrating over
the interval. Then, for arbitrary \( v(x) \):

\[
\int_0^L v(x) \left( \frac{d^2 u}{dx^2} - f(x) \right) \, dx = 0
\]  

(2)

Equivalently:

\[
\int_0^L d^2 u v(x) \, dx = \int_0^L f(x) v(x) \, dx
\]  

(3)

Since \( u(0) = 0 \) and \( u(L) = 0 \), choose the weight function to fulfill \( v(0) = 0 \) and \( v(L) = 0 \).

Integrating the left hand side of the above equation by parts yields:

\[
\int_0^L d^2 u v(x) \, dx = v(x) \frac{d u}{dx} \bigg|_{x=0}^{x=L} - \int_0^L \frac{d u}{dx} \frac{d v}{dx} \, dx = - \int_0^L \frac{d u}{dx} \frac{d v}{dx} \, dx
\]  

(4)

Here, the first term of the first equality is zero since \( v(0) = 0 \) and \( v(L) = 0 \). Therefore, equation 3 can be written as:

\[
- \int_0^L \frac{d u}{dx} \frac{d v}{dx} \, dx = \int_0^L f(x) v(x) \, dx
\]  

(5)

Consider discretizing the domain \( 0 \leq x \leq L \) into subdomains using \( N \) nodes. The value of \( u(x) \) is constant at the nodes and interpolation functions can be used to determine the values of \( u(x) \) inside the subdomains. The interpolation functions are typically polynomials and are chosen such that solutions in neighboring subdomains are continuous [13]. The general form of the approximated solution \( u_h(x) \) to the differential equation can be written as a linear combination:

\[
u_h(x) = \sum_{i=1}^{N} U_i \phi_i(x)
\]  

(6)

Here, \( \phi_i(x) \) are the interpolation functions and are piecewise basis functions that take the value 1 at node \( i \) and take the value 0 at all other nodes. \( U_i \) are unknown scalar coefficients. Define the residual as \( r(x) = \frac{d^2 u_h}{dx^2} - f(x) \). The goal is to construct \( u_h(x) \) so that the integral of the residual will be zero for some choices of the weight function \( v(x) \). This minimizes the estimated error of the solution. In the Galerkin method, the weight functions are chosen to be linear combinations of the basis functions \( \phi_i(x) \) [14]:

\[
v(x) = \sum_{j=1}^{N} V_j \phi_j(x)
\]  

(7)

Here, the index \( j \) is used for clarity. Substituting this expression and the expression for \( u_h(x) \) into equation 5 and rearranging gives:

\[
- \sum_{j=1}^{N} V_j \sum_{i=1}^{N} U_i \int_0^L \frac{d \phi_j}{dx} \frac{d \phi_i}{dx} \, dx = \sum_{j=1}^{N} V_j \int_0^L f(x) \phi_j(x) \, dx
\]  

(8)
This equation must be satisfied for the residual to be zero. It is clear from this equation that the constants $V_j$ can be arbitrarily chosen since they cancel. Performing this cancellation results in:

$$- \sum_{i=1}^{N} U_i \int_{0}^{L} \frac{d\phi_j}{dx} \frac{d\phi_i}{dx} \, dx = \int_{0}^{L} f(x) \phi_j(x) \, dx \quad (9)$$

This can be written as a matrix equation of the form $K_{ji} U_i = F_j$ where $K_{ji} = -\int_{0}^{L} \frac{d\phi_j}{dx} \frac{d\phi_i}{dx} \, dx$ and $F_j = \int_{0}^{L} f(x) \phi_j(x) \, dx$. The values of $U_i$ can be solved for by using techniques from linear algebra, giving the solution $u_h(x)$. COMSOL automatically chooses the best method to solve the linear system. Direct methods used by COMSOL include MUMPS, PARADISO, and SPOOLES, which are based on LU decomposition. A variety of iterative methods are also used by COMSOL [15].

4.2 Electric Field Analysis

The 1 MeV electron accelerator model consists of 30 electrodes. The electrodes are cylindrical with an inner diameter of 1 inch, an outer diameter of 5 inches, and a thickness of 1/16 inch. There is a spacing of 3/4 inch between each electrode. All of the electrodes were modeled to have the physical properties of aluminum. The figure below shows an image of the electrode configuration.

![Configuration of the 30 electrodes in the electron accelerator](image_url)

**FIG. 9:** Configuration of the 30 electrodes in the electron accelerator. The beam line is along the z axis. The units for all position axes are inches.

The first electrode is set to have a potential of -1 MV in COMSOL. The second electrode
is set to have a potential of -28/29 MV, and the third electrode is set to have a potential of
-27/29 MV. Following this pattern, the nth electrode is set to have a potential of \(-(30-n)/29\) MV. Consequently, the final (30th) electrode is at zero potential. Negative potentials were used since electrons are negatively charged and move from low to high potentials. The region before the first electrode was set to have a potential of -1 MV. This simulates the high voltage of the electron source and prevents the electrons from not entering the acceleration column. The region after the final electrode was set to zero potential, simulating the grounded tube attached to the end of the acceleration column. COMSOL’s electrostatics solver was used to generate a map of the electric field and potential. Under static conditions, the electric field \(\vec{E}\) is given by the gradient of the potential \(V\):

\[
\vec{E} = -\vec{\nabla}V
\]  

(10)

COMSOL solves the Poisson equation for the electric field at all points in the model given the assigned conditions on the voltage [15]:

\[
\vec{\nabla} \cdot (\epsilon_0 \epsilon_r \vec{E}) = \rho
\]  

(11)

Here, \(\epsilon_0\) is the vacuum permittivity, \(\epsilon_r\) is the relative permittivity of the material, and \(\rho\) is the volume charge density.

The results of the electric field and potential simulations shown in the figures below.

![Image: Electric Potential map for the beam line of the electron accelerator. The vertical bar farthest to the right corresponds to the electric potential slice and shows the constant drop in potential from -1 MV to 0 along the beam line from the first to the last electrode. The units of the position axes are inches.]
To calibrate the detector, the electron accelerator will be attached to either the left end or the right end of the spectrometer, which is set to be at zero potential, shown in figure 3. The electric field profile for the combined accelerator and spectrometer system is in the following figure.

![Electric field map for the electron accelerator and spectrometer](image)

**FIG. 11:** Electric field map for the electron accelerator and spectrometer. The vertical bar corresponds to the electric potential slice and shows the increase in potential from -1 MV to zero along the beam line. The electric potential surrounding the accelerator and spectrometer are also shown. The region directly to the left of the first electrode is set to -1 MV to simulate the electron source and the region directly to the right of the final electrode is grounded.

A plot of the potential along versus position along the z axis is provided below.
4.3 Magnetic Field Analysis

A current-carrying coil geometry was designed in order to produce a guiding magnetic field of approximately 0.01 to 0.05 T along the beam axis of the electron accelerator. The purpose of this magnetic field is to keep the electrons along the z axis. A total of 10 solenoids were used in the design. All solenoids have a width of 2 inches and are spaced 2 inches apart. The left edge of the first solenoid is 8 inches from the left edge of the first electrode. This solenoid has an inner diameter of 6 inches and an outer diameter of 8 inches. Since the first solenoid has a width of 2 inches and there is a spacing of 2 inches between all solenoids, the left edge of the second solenoid is 4 inches from the left edge of the first electrode. The dimensions of the second solenoid are the same as the first solenoid. The third through eighth solenoids have an inner diameter of 38 inches and an outer diameter of 40 inches. The ninth and tenth solenoids have the same dimensions as the first and second solenoids. The electron accelerator will be enclosed in a pressure vessel that the solenoids will be outside of, necessitating the larger diameters of the middle solenoids. An image of these solenoids is provided in the proceeding figure.
FIG. 13: Configuration of the 10 solenoids that provide a guiding magnetic field along the z axis for the electrons in the accelerator. The units for the position axes are inches.

In COMSOL, all 10 solenoids were set to be made of copper wire with a cross sectional area of 0.01 cm\(^2\). The smaller solenoids were set to have 400 turns and the larger solenoids were set to have 10000 turns. All solenoids were set to have a current of 5 A passing through the copper wire composing them. COMSOL treats the windings of a solenoid as one homogenized domain when performing calculations.

Ampère’s law in the case of magnetostatics is:

\[ \nabla \times \vec{B} = \mu_0 \mu_r \vec{J} \]  

(12)

Here, \(\vec{B}\) is the magnetic field, \(\mu_0\) is the magnetic permeability of a vacuum, \(\mu_r\) is the relative magnetic permeability of the material, and \(\vec{J}\) is current density. In this case, the current density is the number of coil turns multiplied by the current assigned to pass through the copper wire making up the solenoid divided by the cross sectional area of the coil domain.

Consider the magnetic vector potential \(\vec{A}\), which is related to the magnetic field by:

\[ \vec{B} = \nabla \times \vec{A} \]  

(13)

Using the magnetic vector potential, Ampère’s law can be rewritten as:

\[ \nabla \times (\nabla \times \vec{A}) = \mu_0 \mu_r \vec{J} \]  

(14)

This is the equation that COMSOL’s magnetostatics solver uses to find the magnetic field at all points in the model [15].
The result of the magnetic field simulation is shown below.

FIG. 14: Magnetic flux density in T due to the 10 solenoids. As seen in the figure, the field is fairly uniform along the beam line at approximately 0.025 T.

During calibration, the electron accelerator will be attached to either the left end or the right end of the spectrometer shown in figure 1. As seen in figure 1, the spectrometer contains a 1 T superconducting solenoidal magnet. For simulation purposes, this was modeled as a solenoid made of copper wire with a cross sectional area of 0.01 cm$^2$, an inner diameter of 38 inches, an outer diameter of 40 inches, and a width of 120 inches. The solenoid has 100000 turns and a current of 550 A. The spectrometer was placed 12 inches from the end of the last solenoid in the accelerator. The magnetic field for the combined accelerator-spectrometer system is provided in the following figure.
FIG. 15: Magnetic flux density in T for the spectrometer. The magnetic field is approximately 1 T in the interior of the spectrometer and drops at the ends of the spectrometer. This is similar to the magnetic field described of the actual spectrometer in figure 3.

4.4 Electron Trajectory Analysis without Spectrometer

The electric and magnetic fields are solved for by COMSOL independently. The results of the calculations are then used in the charged particle tracing analysis. Since the electrons in this model will be accelerated to energies of 1 MeV, it is necessary to use the relativistic correction. COMSOL does this by computing the particle mass $m_p$ as [16]:

$$m_p = \frac{m_r}{\sqrt{1 - \frac{\vec{v} \cdot \vec{v}}{c^2}}}$$  \hspace{1cm} (15)

In this equation, $m_r$ is the rest mass of the particle, $\vec{v}$ is the particle velocity, and $c$ is the speed of light.

The equation of motion for a particle of charge $q$ in both external electric and magnetic fields is given by:

$$\frac{d}{dt}(m\vec{v}) = q\vec{E} + q(\vec{v} \times \vec{B})$$  \hspace{1cm} (16)

This is Newton’s second law in the case where the total force is given by the Lorentz force. COMSOL uses this equation to calculate the motion of the charged particles.

In order to determine the effectiveness of the designed solenoids in keeping the electrons along the beam line, two simulations of the electron trajectories without the spectrometer
were run. First, an ideal magnetic field of 0.025 T in the z direction was programmed into COMSOL and the electron trajectories were determined. Later, the magnetic field due to the solenoid assembly shown in figures 13 and 14 was used to calculate the trajectories of the electrons.

The electron source was set to be 1 inch to the left of the first electrode and the initial velocities of the electrons were set to be $10^6$ m/s, which corresponds to a kinetic energy of 2.843 eV. Electrons were generated using the release from grid feature and the constant speed, hemispherical setting. The hemisphere was oriented so that electrons have initial velocity vectors with non-negative z components. Electrons are released evenly over the the hemisphere. This gives the electrons a range of initial velocities along the beam line from 0 to $10^6$ m/s. 10000 electrons were released beginning at time zero with a delta function pulse width.

The calculations performed by COMSOL ignore the interactions between the electrons. The electron source produces $10^4$ electrons per second. Thus, very few electrons are expected to be in the accelerator at any given time. As a result, particle-particle interactions between the electrons are negligible.

4.4.1 Electron Trajectories for Ideal Magnetic Field

First, the electron trajectories were calculated using the ideal magnetic field of 0.025 T in the z direction. This magnetic field was only applied in the portion of the domain containing the electrodes. The average electron kinetic energy was calculated as a function of electron position along the beam line and is provided in the following figure. The results were found using a time step size of $10^{-11}$ s. As a check of the robustness of the simulation, the calculations were repeated for a time step size of $5 \times 10^{-12}$ s and for a time step size of $5 \times 10^{-11}$ s. The results found for these three time step sizes were the same.
FIG. 16: Average electron kinetic energy through the accelerator using the ideal magnetic field of 0.025 T. The kinetic energy increases linearly as the electrons pass through the electrodes. The kinetic energy is 1.000 MeV after the electrons have passed through the electrodes, which is the desired value.

The electron trajectories using the ideal magnetic field of 0.025 T are shown in the following figure.

FIG. 17: Electron trajectories through the accelerator and several feet beyond the accelerator. Particle paths are represented by the colored lines. The color gradient represents the electron kinetic energies in units of keV. The maximum kinetic energy is 1000 keV. The axes units are in inches.
The electron trajectories found using the ideal magnetic field are very well focused along the beam line, as seen in the figure below.

![Electron spatial distribution](image)

FIG. 18: Electron spatial distribution 162 inches from the first electrode in the z direction. The black circle has a diameter of 1 inch and is the inner diameter of the electrodes. The electron spread is about 1/3 inch in diameter.

The results above show that the electrodes accelerate the electrons to the desired energy of 1 MeV and the ideal guiding magnetic field is successful in focusing the electrons along the beam line.

4.4.2 Electron Trajectories for Magnetic Field due to Designed Solenoids

Next, the electron trajectories were calculated using the magnetic field due to the designed solenoids. The average electron kinetic energy was calculated as a function of the electron position along the beam line and is provided in the following figures. The results were found using a time step size of $10^{-11}$ s. As a check of the robustness of the simulation, the calculations were repeated for a time step size of $5 \times 10^{-12}$ s and for a time step size of $5 \times 10^{-11}$ s. The results found for these three time step sizes were the same.
FIG. 19: Average electron kinetic energy through the accelerator using the magnetic field due to the designed solenoids. The kinetic energy increases linearly as the electrons pass through the electrodes. The kinetic energy is 1.000 MeV after the electrons have passed through the electrodes, which is the desired value.

The electron trajectories using the magnetic field due to the designed solenoids are shown in the following figure.

FIG. 20: Electron trajectories through the accelerator and several feet beyond the accelerator. Particle paths are represented by the colored lines. The color gradient represents the electron kinetic energies in units of keV. The maximum kinetic energy is 1000 keV. The axes units are in inches.
The electron trajectories found using the magnetic field due to the designed solenoids are well focused along the beam line, as seen in the figure below.

FIG. 21: Electron spatial distribution 162 inches from the first electrode in the z direction. The black circle has a diameter of 1 inch and is the inner diameter of the electrodes. The electron spread is about 1/3 inch in diameter.

The results of the simulations in this section and in section 4.4.1 demonstrate that the design of the electron accelerator is valid. In both models, the electrons are accelerated from a small initial kinetic energy to the desired kinetic energy of 1.000 MeV. Furthermore, the guiding magnetic field of approximately 0.025 T along the z axis produced by the designed solenoids has kept the electrons along the beam line as intended and performs as well as the ideal magnetic field.

4.5 Electron Trajectory Analysis with Spectrometer

The trajectory of an electron in the accelerator-spectrometer system is greatly impacted by the change in magnetic field from the low magnetic field region of the accelerator to the large magnetic field region of the spectrometer. The impact on the electron trajectory is known as the magnetic mirroring effect. In our simulation, we have a total magnetic field that varies spatially but is constant with time. Consider the magnetic moment $\mu$ of the electron, which is given by:

$$\mu = \frac{mv_{\perp}^2}{2B}$$

In this expression, $m$ is the mass of the electron, $v_{\perp}$ is the velocity of the electron perpendicular to the magnetic field, and $B$ is the magnitude of the magnetic field. The magnetic moment $\mu$ has adiabatic invariance, meaning that it does not change if the magnetic field is
slowly varied spatially or temporally [17]. Since the electron moves from a region of lower magnetic field to region of higher magnetic field as it leaves the accelerator and enters the spectrometer, the electron velocity perpendicular to the magnetic field must increase so that \( \mu \) remains constant. Now, consider the total energy \( E \) of the electron. By splitting the velocity into components parallel \((v_\parallel)\) and perpendicular \((v_\perp)\) to the magnetic field, the total energy can be written as:

\[
E = qV + \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2
\]  

(18)

Here, \( q \) is the electron charge and \( V \) is the potential. Note that the total energy of the electron is conserved. Since \( v_\perp \) increases as the electron moves from low to high magnetic field, \( v_\parallel \) must decrease for energy to be conserved. This is the magnetic mirroring effect.

Since the electron velocities perpendicular to the magnetic field lines increase while their velocities parallel to the magnetic field lines decrease as they travel from low to high magnetic field, a range of electron pitch angles from the beam line that depends on the initial velocities is produced. To show this, first write \( v_\perp \) as a function of the pitch angle \( \theta \) from the beam line:

\[
v_\perp = v \sin \theta
\]  

(19)

Next, since \( \mu \) is conserved, for two locations with magnetic field magnitude \( B_0 \) and \( B(z) \) we must have:

\[
\frac{\sin^2 \theta_0}{B_0} = \frac{\sin^2 \theta(z)}{B(z)}
\]  

(20)

This can be rearranged to give:

\[
\sin^2 \theta(z) = \frac{B(z)}{B_0} \sin^2 \theta_0 = \frac{B(z)}{B_0} \left( \frac{v_\perp}{v_0} \right)^2
\]  

(21)

Therefore, the pitch angle as a function of \( z \) is:

\[
\theta(z) = \arcsin \left( \frac{v_\perp}{v_0} \sqrt{\frac{B(z)}{B_0}} \right)
\]  

(22)

5. FUTURE WORK

It is planned to run the simulation for the entire accelerator-spectrometer system using the same method as the simulations of the accelerator without the spectrometer in order to observe the magnetic mirroring of the electron trajectories. The timing spread of the
electrons arriving at the end of the spectrometer will be analyzed. In addition, the effects of different initial conditions on the electron trajectories will be explored. Furthermore, the new motor for the Pelletron charging system of the accelerator utilizes magnetic fluid feedthrough. As a result, a current-carrying coil geometry must be designed to compensate for the effects of the magnetic field due to the magnetic fluid feedthrough along the beam line of the accelerator.

6. SUMMARY

The Ultra-Cold Neutron Asymmetry (UCNA) collaboration uses polarized ultracold neutrons (UCN) to study the correlation between the number and energy of the electrons emitted along and against the spin direction of the decaying neutrons. A 1 MeV electron accelerator is currently being constructed by Dr. Albert Young at Triangle Universities Nuclear Laboratory (TUNL) and will be used as a calibration tool for detectors in high precision neutron beta decay experiments, like those performed by the UCNA collaboration. The adjustable potential allows electrons to be accelerated to energies across the continuous beta decay spectrum, which is an improvement over current calibration methods. A high voltage test of the accelerator was performed and the voltage across the column was found to be 210 kV. Using COMSOL, magnetic and electric field maps were produced for the accelerator and spectrometer that holds the detectors. A guiding magnetic field of about 0.025 T along the beam line was generated by a set of 10 current-carrying coil geometries. The 1 T magnetic field along the axis of the beam in the spectrometer was produced using a current-carrying coil for modeling purposes. In addition, electron transport through the accelerator-spectrometer system was analyzed. The simulations showed that the electrons are accelerated from their small initial kinetic energies to the desired kinetic energy of 1.000 MeV. Furthermore, the magnetic field produced by the current-carrying coil geometries is successful in keeping the electrons along the beam line. These simulation results show the validity of the accelerator design.
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