# Essays on Applied Information Economics Theory

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## ABSTRACT

**DAVID FRAGOSO GONZALEZ: Essays on Applied Information Economics Theory.**
**(Under the direction of Gary Biglaiser.)**

This dissertation contains two essays on applied microeconomic theory, each addressing situations of asymmetric information between economic agents. The first essay develops a theoretical model to illustrate how a short-termist board with positive inside information can use disclosed executive compensation to credibly signal its optimism to the less informed outsiders pricing the company's stock. The board uses the fact that performance based pay is more valuable to the executive when prospects are good to give her a compensation package that she would not accept if the inside information were bad. Signaling does not always distort compensation packages away from optimal incentive provision; when it does, the distortion is magnified if the moral hazard in the agency relation between board and executive is large, the executive's contribution to company performance is relatively unimportant, the company's operations are relatively risky, and the enforcement of disclosure rules is weak. By outlining some conditions in which compensation is likely to be used as a signal and characterizing the distortions that such use induces in different circumstances, this paper proposes new explanations for the observed heterogeneity in compensation practices.

The second essay develops a new theory of the organization of the financial audit market to explain the observed variation in market concentration across market segments. It provides a micro-founded model of audit demand whereby audit clients enjoy network benefits from having other clients audited by their auditor. Using two versions of the model, and under the assumption of diseconomies of scale in the audit sector, the essay derives two alternative explanations for audit market outcomes. The first is that, because they value the network

benefits from auditing more highly, client companies in which informational asymmetries are more severe retain auditors with more clients. The second is that, because they generate a larger network effect, companies in which accounting errors are more likely to become public are also audited by larger auditors. The hypotheses behind these explanations are that the visibility of a company and the severity of information asymmetries are correlated with size and ownership type.

To my parents, Lídia and Celso.

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# CHAPTER 1: A THEORY OF DISCLOSED EXECUTIVE COMPENSATION AS A SIGNAL OF INSIDE INFORMATION

## 1.1 Introduction

As corporate insiders, the members of a company's board - which typically includes a few of the company's executives - are often better informed about their company's future prospects than the outsiders that finance it and price its stock[1]. As representatives of current stockholders' interests, board members are also often motivated to influence how outsiders perceive those prospects. This is true when the company issues securities - during an I.P.O., a bond issue, or a new stock issue - or when the company's stockholders have a short-term position in the company. When motivated to influence outsiders, board members must solve the "corporate signaling problem" (CSP): how to credibly signal optimism given that purporting it is beneficial even when non-existent.

This paper develops a theoretical model in which publicly disclosed executive compensation is a credible signaling device. The purpose of the model is to help advance the understanding of the following questions: Under what conditions is executive compensation *available* as a signal? How does the signaling intent affect compensation practices (and hence accounts for heterogeneity in them)? Under which circumstances is executive compensation *likely* to be used as a signal? This introduction will explain how the model's assumptions and conclusions help us address these questions. Before, however, it must explain the intuition behind the idea of executive compensation as a signal and provide a basic overview of how that idea is modeled.

In the model, the board engages in two relationships: one of agency with the executive, and one of signaling with outsiders that gives rise to the CSP. The CSP is modeled through

[1]For an overview of some of the evidence supporting the hypothesis of asymmetric access to information between the insiders of a company and the outsiders pricing its securities see Tirole (2006).

the inclusion of two assumptions: first, that boards have soft, or unverifiable, private information that outsiders lack and, crucially, that such information is common knowledge with the executive; second, that the board maximizes short-term stock prices - set in a competitive stock market composed by outsiders - with short-term being some moment *before* the public unraveling of the board's private information.

In the context of the model, the fact that allows executive compensation to solve the CSP is that optimistic executives are more willing to accept packages with less guaranteed pay, which comes in the form of instruments like salary and perks, and more performance-based pay, which comes in the form of bonuses and equity based instruments like stock grants and options, than pessimist ones. An optimistic board can then signal its optimism by offering the executive a package that, given her alternative employment opportunities, she would not be able to accept if she were less optimistic.

The last sentence conveys the basic reasoning as to why executive compensation can serve as signal, but does so somewhat imprecisely. This is why: if a compensation package that a pessimist executive would never accept outright leads outsiders to infer that the company's prospects are good, then a pessimistic board may be tempted to "bribe" its executive into accepting such a package by giving her additional compensation without disclosing so to the public. To credibly signal optimist, the board must therefore account for the possibility of a bribe and make its offer unprofitable. Note that, the more pessimistic the executive is, the more it will cost the board to convince the executive to accept a package heavy in variable pay and light in fixed pay. To put it more technically, the cost of signaling is negatively correlated with the value of the information being signaled. As first observed by Spence (1973) in his seminal work on signaling, this condition of negative correlation between the information being signaled and the cost of signaling it is sufficient to ensure that the proposed signaling device is effective.

With the basic intuition behind this paper's argument as to why executive compensation can solve the CSP now exposed, we can go back to the discussion of how the paper contributes to answering its guiding questions.

**Under what circumstances is executive compensation *available* as a signal?**

The model does not characterize all the circumstances under which executive compensation can be used as a signal, and so cannot give a definitive answer to this question. However, by showing that executive compensation is a viable signal under particular assumptions, it suggests a partial one, based precisely on those assumptions.

It was previously remarked that, in the model, a board signals optimism by connecting the executive's pay to future performance in a way that a pessimistic executive would find unacceptable. This conclusion is made possible by two implicit assumptions. The first assumption concerns executive compensation regulation: for the model to apply it must not be possible to undo, through undisclosed means, the connection between the executive's pay and the company's performance. That is, there must be rules in place to prevent the executive from secretly short-selling the company or engaging in secret trading with the company or a third party and protect herself from the risk associated with her compensation package. In reality, there is a class of companies subject to such rules: those that are publicly listed or in the process of being listed (going through IPOs). In the United States, for example, the Securities and Exchange Commission (SEC) enforces rules limiting secret trading and short-selling by executives. The SEC also enforces compensation disclosure rules, that constraint companies from awarding undisclosed compensation. This need for rules on secret trading and disclosure practices seemingly discards private companies as potential users of executive compensation as a signal.

The second assumption concerns the outsiders' understanding of the market for the executive's labor. For an outsider to determine that an accepted package would be unacceptable to a pessimistic executive, he must have some knowledge about the market value of the executive. That is, for the model to apply, outsiders must have a good understanding of the executive's value. The extent to which they do is likely to vary across firms and industries.

**How does the use of executive compensation for signaling affect compensation practices?**

The model works within the boundaries of the classical principal-agent approach to executive compensation in that the board sets executive pay not just to signal information but also to correct the moral hazard in the executive's actions. As in other works in the classical mold (which started with Jensen and Meckling (1976)), moral hazard is modeled through the assumption that the executive must exert unobservable effort at a personal cost. This means that, absent a signaling motive, we have a textbook moral hazard model in which the board awards performance-based pay to motivate the executive to exert effort.

As already mentioned, to fulfill its signaling goal, an optimistic board must eliminate the temptation that a pessimistic board may have to secretly bribe the executive into accepting a package signaling good news. This need to suppress the board's temptation to bribe holds even though, as suggested in the previous subsection, the companies that can use compensation as a signal are those that are subject to rules requiring all compensation to be disclosed. This is because disclosure rules apply to (or can be more easily enforced when dealing with) standard instruments of compensation, like cash, equity-based compensation, or monetary retirement benefits (and, at least in the U.S., and with particular emphasis since 2006, to perquisites like airplane use as well), not to less evident forms of compensation, like state of the art offices, the ability to recruit friends to work for the company, the sponsoring of the executive's pet projects (like charities and foundations), or non-pecuniary post-retirement benefits[2]. While these forms of compensation may be used for a variety of other reasons, their existence gives boards an instrument with which to secretly pay their executives, even if it is a costly one[3].

[2]Tirole (2006) provides an example of non-pecuniary retirement benefits that were not disclosed to stockholders at the time of its grant: "Unbeknownst to outsiders, [Jack Welsh's] retirement package included continued access to private jets, a luxurious apartment in Manhattan, memberships of exclusive clubs, access to restaurants, and so forth." Tirole also documents instances of nepotism and of attribution of undisclosed perks.

[3]The impossibility to convert most instruments of secret compensation into money means that these instruments ares likely to hold less value to the executive than what it costs the company to provide them.

The analysis of the model shows that there is a signaling equilibrium, and that this equilibrium is unique[4]. In some cases, boards signal optimism "for free" by offering the package that addresses moral hazard optimally. This is the case when moral hazard in the agency relation between board and executive is small the executive's contribution to company performance is important, the company's operations are relatively safe or the executive is not too averse to risk. In other words, free signaling is possible when the optimal moral hazard solving package prescribes that the executive's pay should be largely attached to the company's future performance. Free signaling is also possible when the value of undisclosed compensation to the executive is low relative to what it costs the board to provide it.

When these conditions are not present, however, signaling distorts the provision of incentives. The distortions are magnified as the conditions move in the opposite direction towards: significant moral hazard in the agency relation between board and executive; the executive's contribution to company performance being relatively unimportant, the company's operations being risky, and a high valuation by the executive of undisclosed forms of compensation.

The paper's equilibrium predictions thus propose different sources of heterogeneity for observed compensation: the first source is the nature of inside information (equilibrium packages change according to the insiders' confidence); the other sources are the idiosyncratic features of the company and of the executive discussed above. All of this, however, is contingent on executive compensation being used for signaling, which brings us to the next question.

**Under which circumstances is executive compensation *likely* to be used as a signal?**

Over the years, what I have here labeled as the CSP has received some attention in the finance literature. As a consequence, dividend policies (starting with Bhattacharya (1979)), debt/equity ratios (starting with Ross (1977)), and stock splits (starting with Grinblatt et al. (1984)), the stakes retained by entrepreneurs in their own projects (starting with Leland and Pyle (1977)), and executive compensation (starting with Hayes and Schaefer (2009) and now

[4]The uniqueness of the equilibrium follows from the application of a refinement - the intuitive criterion - that discards "implausible" equilibria.

with the present work) have all been cast as signals of insider information.

On a first take, the plurality of signals would seem to indicate that boards are only likely to use executive compensation as a signal when compensation is the most cost effective in the pool of corporate signaling devices. A closer look, though, reveals that the signals in this pool all seem to share a common trait: they are not systematically available. A company cannot split stock, announce a new dividend policy, or change the way it compensates executive every time new information is generated. Therefore, it is likely that at most points in time, a board wishing to signal information may not have any credible devices with which to do so. Therefore, every time any of the devices in the pool is available to a board facing the CSP, it is probably going to be used. The relevant question is then: in what circumstances is the CSP modeled in the paper likely to arise in reality?

One of the cornerstones of the CSP is the assumption of asymmetric unverifiable information between the board and the executive on one side, and the outsiders on the other. As already mentioned, the existence of asymmetric information is supported empirically. Moreover, it has an intuitive explanation: board members and executives owe their positions to their expertise in the company's activities; this expertise puts them in a position to interpret hard information - that can and, sometimes, by virtue of the disclosure rules affecting public companies and IPO candidates, must be disclosed to the public - in a way that outsiders cannot.

The other cornerstone of the CSP at the heart of this paper is the assumption that the board's goal is to maximize short-term stock prices[5]. In the case of IPO candidates, the short-term motivation is obviously present: a higher initial price allows the company to obtain more capital, enhances the reputations of all parties involved, and allows the initial investors - who are likely to be heavily represented in the board - to cash in on their holdings and move on to new projects.

In the case of already listed companies, the short-term motivation is also likely to be present, at least in some cases. Using data from Froot et al. (1992), Bolton et al. (2006) estimate that "the effective horizon of institutional investors (...) is about 1 year." Because institutional

[5]Short-termism is also the motive assigned to signal-senders in other research on the CSP. Examples include (Ross, 1977), Bhattacharya (1979), and Grinblatt et al. (1984).

investors appoint most of the board members, their short-term orientation is likely to be a significant factor in board members' decisions.

The answer to this subsection's question is thus that, if available as such, executive compensation is likely to be used as signal by any public company where there is substantial generation of inside information and where the board is responsive to the impatience of its stockholders. We thus have that the model predicts that stockholder impatience and influence are sources of heterogeneity in executive compensation practices.

In sum, if this paper's theory has any merit, we should expect IPO candidates and at least some publicly listed companies (those with a short-term inclination) to approach the negotiation over their executives' compensation with a signaling intent. Moreover, given the model's unique equilibrium prediction that the disclosure of compensation is always informative when the company has a signaling intent, we should expect stock prices to react positively to the announcement of packages whereby the executive's pay depends strongly upon performance. Certo et al. (2003) obtain "partial support" for this hypothesis in their study of the case of IPO candidates by using CEO stock options as a measure of performance-based pay. Unfortunately, no other empirical works address it.

The rest of the paper goes on as follows. Section 1.2 places this paper in the two literatures to which it is related. Section 1.3 introduces the multilateral model behind the paper's signaling theory of executive compensation. Section 1.4 analyzes the basic version of that model, whereby the insiders' private information is binary, and section 1.5 extends this analysis to a model where private information is drawn from a continuous interval. The comparative statics are discussed an interpreted in section 1.6. The conclusion outlines ideas for future research.

### 1.2 Literature

This paper stands in the intersection between the executive compensation literature and the corporate signaling literature. This section explains the paper's relation with other papers in that intersection, and also with key contributions to either of the two literatures.

At least two other papers address the signaling function of executive compensation from a theoretical standpoint. Lund (2012) dismisses incentive provision and puts forth the hypothesis that observed compensation practices reflect two signaling goals: to convey governance quality and, as in the present paper, to betray the executive's optimism or pessimism about the company's future. This paper's approach differs from Lund's in three ways. First, it considers incentive provision in tandem with signaling as the forces that shape compensation practices. Second, it acknowledges the incentives of pessimistic boards to mimic the behavior of optimistic ones and the channel through which they can do it: by bribing the executive with undisclosed compensation. And third, it studies the signaling role of executive compensation with a game theoretical model.

Hayes and Schaefer (2009) also employ a game theoretical approach to study executive compensation as signaling, but they do so to address a different question: they ask whether it is possible to explain the documented rise in CEO pay over the last decades with the existence of a "Lake Wobegon Effect", whereby boards do not want to admit to having a below average CEO and hence try to signal her value by awarding her more compensation than what her retention would warrant. In their treatment, the information being signaled concerns only the CEO's match with the firm. Moreover, signaling is performed through fixed compensation, as pay cannot be attached to performance and there is no agency problem warranting the provision of incentives.

A few papers in the corporate signaling literature have considered the CSP created when the signal-sender is driven by short-termist motivations[6]. The devices considered as solutions to the signaling problem include dividend policies (Bhattacharya (1979), Bhattacharya (1980), and Miller and Rock (1985)), the allocation of the company's financing between debt and equity (Ross (1977) and Myers and Majluf (1984)), and stock splits (Grinblatt et al. (1984) and Mcnichols and Dravid (1990)).

While the nature of their signaling problem is different - they consider an entrepreneur looking for lenders who can finance her venture - Leland and Pyle (1977) have one of closest

[6]This section does not intend to be an exhaustive survey of the corporate signaling literature, and therefore focuses only on contributions addressing the same problem or containing similar arguments to the one made here. For a broader review of this literature, see Tirole (2006).

arguments to that presented in this paper. They propose that entrepreneurs signal their confidence in the future future performance of their venture by foregoing a larger diversification of their investment portfolio, thus keeping a larger stake in their venture than optimal risk considerations would recommend.

As already mentioned in the introduction, this paper takes a classical approach to executive compensation, as it assigns to performance based pay the role of diluting the moral hazard in the agency relationship between stockholders and executives. In this sense it follows the lead of works like Jensen and Meckling (1976), Holmstrom and Ricart i Costa (1986), Holmstrom (1999), Holmstrom and Tirole (1993), and Bolton et al. (2006)[7]. Moreover, it borrows a popular modeling tool from some of these papers: the assumption of linear compensation schemes, normally distributed future profits, and mean-variance preferences for the executive. Bolton and Dewatripont (2005) present the textbook version of the principal-agent model with these assumptions, while Holmstrom and Tirole (1993) and Bolton et al. (2006) present applied versions of it.

## 1.3 The Model

### 1.3.1 Description

The model features three types of agent connected by their relation to the same company: a board, an executive, and a mass of homogenous outsiders making up for a perfectly competitive demand for the company's stock.

Acton unfolds according to the timeline in figure 1.1. In keeping with the standard timeline in the corporate signaling literature, the model has three stages. From the perspective of the initial stage, 0, these stages are the present (0), the short-term, (1), and the long-term (2).

In stage 0 board and executive jointly acquire private information about the long-term (stage 2) profits (gross of executive compensation) of the company. The board then offers a compensation package to the executive. If the executive accepts the package, the board

[7] The literature on moral hazard in general agency relationships is extensive. Here, I only mention references focusing exclusively on managerial moral hazard.

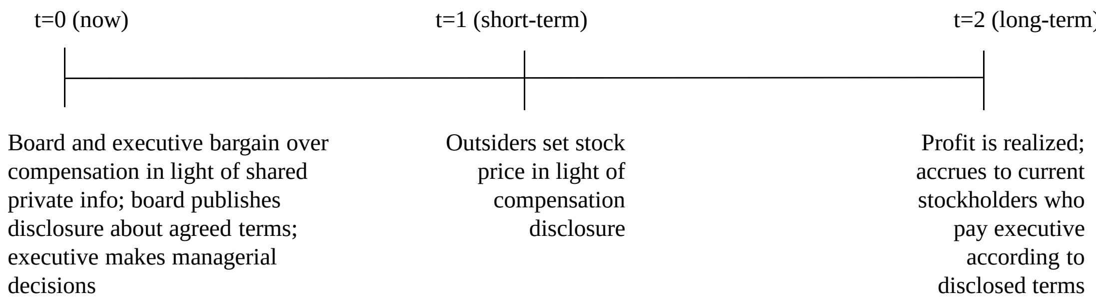



Figure 1.1: The timeline of the model

makes a public disclosure about it and the executive begins her employment, which entails the exertion of managerial effort. In stage 1 the outsiders bid for the company's stock with disclosed compensation as their only information. In stage 2 profits are realized, the executive is paid according to the terms of her package, and net profits accrue to the new stockholders.

The board approaches the setting of executive compensation motivated by its full internalization of the goals of the non-modeled initial stockholders of the company, who wish to sell the company in the short term (stage 1)[8].The board therefore sets executive compensation in stage 0 to maximize the price of the company's at stage 1 net of executive compensation costs incurred until the moment of the sale. The stock's price will be larger the more the outsiders are convinced that the state of the company is good and herein lies the corporate signaling problem at the core of this paper.

The informal description of the model makes clear that it contains two relationships: the first is of agency between the board (the principal) and the executive (the agent); the other of signaling in the market for the company's stock between the board (the sender) and the outsiders (the receivers). The next two subsections introduce the model formally by sequentially addressing each relationship.

#### 1.3.1.1 Agency within the Company

In stage 0 the board and the executive jointly observe the company's state $\theta \in \Theta$ (which affects the company's profit). Outsiders do not observe $\theta$ but hold the correct prior belief $p_\theta$ for all

[8]Stage 1 can be interpreted as the date of an IPO or of a common stock issue, or simply the moment in which the stockholders' short-term interest in the company expires because of liquidity needs or optimal portfolio considerations.

$\theta \in \Theta$. $p_\theta$ is a probability if the state space $\Theta$ is discrete and a probability density function if $\Theta$ is continuous. The initial analysis is based on the two state version of the model, as it allows for a quicker, and less technical, characterization of the signaling role of executive compensation[9].

Let the state space then be $\Theta = \left\{0, \bar{\theta}\right\}$, where $\bar{\theta} > 0$ is the "good" state and 0 the "bad" state, and define $p \equiv p_{\bar{\theta}}$.

After the two insiders observe $\theta$ the board offers the executive a compensation package $C$. If the executive accepts the job, she is charged with a single managerial task: the exertion of costly effort $e$ in stage 0. State and effort affect the stochastic gross profit $\pi(\theta, e)$ that the company yields in stage 2 and which is given by:

$$\pi(\theta, e) = \theta + \mu e + \eta \tag{1.1}$$

where $\eta \sim N\left(0, \sigma^2\right)$ is normally distributed and $\mu$ is the marginal value of managerial effort.

A compensation package $C$ includes three elements. Two are publicly observed: they are the wage $w$ and the share in the company's gross profit $b$. The other is unobservable: the undisclosed fixed compensation $k$[10]. The two observable elements are paid to the executive in stage 2, the unobservable element is paid in stage 0. Formally, compensation package $C$ is a triple[11]

$$C \equiv (w, b, k) \in \Omega_C \equiv \mathbb{R} \times [0,1] \times \mathbb{R}_+.$$

Disclosed compensation is a pair

$$D \equiv (w, b) \in \mathbb{R} \times [0,1] \equiv \Omega_D.$$

[9] Section 1.5 tackles the case in which $\Theta$ is a bounded continuous interval.

[10] As mentioned in the introduction, undisclosed compensation comes in the form of benefits that do not fit standard definitions of compensation, and therefore can be awarded without subjecting the company to charges of non-compliance with insider trading and disclosure rules. The fact that undisclosed compensation must take non-standard forms also implies that it cannot be made contingent on performance, which is why the model only has fixed undisclosed compensation.

While hidden forms of compensation may serve different purposes in reality, here they serve as tool with which companies can bribe their executives into accepting disclosed terms that displease them.

[11] The assumption that $b$'s domain is $[0,1]$ reflects the notion that the executive cannot own more than 100% of the company nor own a short position in it.

In game theoretical terms, a strategy by the board entails the selection of a compensation package in stage 0 in every possible state. Notationally, a strategy by the board is represented as $s_{Bd} \equiv (C_\theta)_{\theta\in\Theta} = (w_\theta, b_\theta, k_\theta)_{\theta\in\Theta}$. In every state, the board selects the package that maximizes the price of the company's stock in stage 1 net of compensation costs expensed in stage 0. That is, if we let $q_1$ denote the sale price of the company in stage 1, the board maximizes $q_1 - k$. This assumption about the board's preferences synthesizes two deeper assumptions: first, that the board internalizes the preferences of unmodeled initial stockholders; second, that these initial stockholders are short-termist risk-neutral agents who want to sell the company in stage 1.

A strategy by the executive entails deciding whether to accept or reject any package $C \in \Omega_C$ in every possible state, as well as selecting a level of effort for every package in every state. Notationally, a strategy by the executive is represented as $s_{Ex} = (\delta_{\theta C}, e_{\theta C})_{\theta\in\Theta, C\in\Omega_C}$, where $\delta_{\theta C} \in \{Y, N\}$ is her decision to accept or reject package $C$ in state $\theta$, and $e_{\theta C}$ is her effort contingent on having accepted package $C$ when in state $\theta$.

If the executive rejects the board's offer, she earns a reservation expected utility of $\bar{u}$ by taking on her next best employment opportunity. If the executive accepts the board's offer of $C$ and decides to exert a level of effort of $e$ when the state of the company is $\theta$, she obtains an expected income given by

$$E\left(I|\theta; C, e\right) = w + \alpha k + bE\left[\pi\left(\theta, e\right)|\theta\right] \tag{1.2}$$

where $\alpha$ (such that $0 \leq \alpha < 1$) represents the value attached by the executive to every dollar spent by the board in undisclosed compensation[12]. Expected income enters the executive's utility linearly: given the state of the company, the executive's preferences are represented by the CARA mean-variance utility function

$$Eu_{Ex-\theta}\left(C, e\right) = E\left(I|\theta; C, e\right) - \frac{\gamma}{2}var(I|C) - \frac{\psi}{2}e^2 \tag{1.3}$$

[12]The fact that a dollar spent by the board in undisclosed compensation is worth less to the executive than a dollar in salary follows from the very nature of undisclosed compensation. Since it must be awarded through non-standard forms that cannot be easily converted to cash, undisclosed compensation is unlikely to be worth as much as cash. Moreover, the value of undisclosed compensation is likely to depend upon the ability of the regulator to monitor the company, and upon the idiosyncrasies of the company.

in which $\gamma, \psi > 0$ measure the executive's aversion to risk and effort, respectively.

The assumption that compensation is linear in the profits of the company is common in effort based models of managerial compensation. (See Bolton and Dewatripont (2005) for a discussion and Holmstrom and Tirole (1993) and Bolton et al. (2006) for examples of its application.) It is used in combination with the mean-variance utility function of the executive and the normal distribution of profits to circumvent the technical difficulties associated with trying to find general optimal compensation schemes, as it allows for closed form solutions. Moreover, linear contracts, with their fixed and variable components, bear some resemblance to typical executive contracts.

#### 1.3.1.2 Signaling to the Stock Market

The asymmetry of information between board and prospective stock buyers means that variables controlled by the board and observable to the buyers will used by the latter as information, making them signaling devices. In this model, the only variable that is set by the board and observed by the buyers is disclosed compensation $D$.

Aware that prospective buyers will use disclosed compensation to make inferences about the company's state, and given its objective of maximizing $q_1 - k$ (stock prices net of compensation costs expensed in period 0 ), the board has an incentive to set $D$ as to induce optimistic beliefs, regardless of the true state of the company.

Upon observing $D$, the prospective buyers update their beliefs about the state of the company and, therefore, their valuation of the company's stock. Formally, the outsiders' updated beliefs after they observe the disclosure of package $D$ are denoted as $\rho(D) \equiv (\rho(\theta|D; s))_{\theta\in\Theta}$ - in which $\rho(\theta|D; s)$ is the probability assigned by the prospective buyers to the company's state being $\theta$ given their observation of $D$ and their belief that a strategy profile $s = (s_{Bd,} s_{Ex})$ is being played by the other players. A fully specified system of beliefs is denoted as $\rho = (\rho(D))_{D\in\Omega_D}$.

The prospective buyers are homogenous, risk-neutral, and numerous (which ensures perfect competition in the market for the company's stock). Combined, these assumptions imply that the equilibrium stock price in period 1, $q_1^\star(D; s)$, must be equal to the outsiders' assessment of the company's expected gross profit net of executive compensation given their observation of package $D$ and their belief that the other players are behaving according to strategy profile $s$. That is, $q_1^\star(D;s)$ must be given by

$$q_1^\star(D;s) = \int_{\theta\in\Theta} \{(\theta + \mu e_{\theta C})(1-b) - w\}\, \rho(\theta|D;s)\, d\theta \tag{1.4}$$

which, in the two state version of the model, boils down to

$$q_1^\star(D;s) = \left[\bar{\theta}\rho\left(\bar{\theta}|D;s\right) + \mu e_{\theta C}\right](1-b) - w \tag{1.5}$$

The expressions reveal the three channels through which disclosed executive compensation affects stock prices: first, the observation of $D$ allows outsiders to observe the executive compensation costs that the final (stage 2) owners of the company will bear; second, it lets the outsiders infer the executive's effort; and third, it informs the outsiders' beliefs.

Given a strategy profile $s$ and a belief system $\rho$, the board's payoff expected payoff from offering package $C$ in state $\theta$ is then given by:

$$Eu_{Bd-\theta}(C;s,\rho) = q_1^\star(D;s) - k \tag{1.6}$$

The expression makes clear that compensation package $C$ affects the board's payoff in two ways: directly trough $k$, and indirectly through the three channels by which $D$ affects the stock price.

With the formal description of the players' preferences and actions complete, only one ingredient is missing from the model: an equilibrium concept. The next subsection provides one.

### 1.3.2 Equilibrium Concept

Equilibrium is defined as a *perfect Bayesian equilibrium* (PBE) that satisfies the intuitive criterion of Cho and Kreps (1987). This refinement discards PBE that contain implausible beliefs in off-the-equilibrium paths. Formally:

**Definition 1.** A profile of strategies and a system of beliefs $(s^\star;\rho^\star)$ is an equilibrium of this game if:

1. it is a PBE:
   (a) Given his/her beliefs and the equilibrium strategies of his/her opponents, no player finds it profitable to deviate from $s^\star$ once any of his/her information sets (on and off the equilibrium path) is reached;
   (b) The system of beliefs $\rho^\star$ of the prospective buyers is derived from strategy profile $s$ through Bayes' rule whenever possible. That is,

$$\rho^\star\left(\theta|D^\star;s^\star\right) \quad = \quad \Pr\left[\theta|D^\star;s^\star\right] = \frac{\Pr\left[D^\star|\theta;s^\star\right]p_\theta}{\int_{\theta'\in\Theta}\Pr\left[D^\star|\theta';s^\star\right]p_{\theta'}d\theta'}$$

2. it passes the intuitive criterion. That is, there cannot be a state $\theta$ in which the board would surely have in $D^{dev}$ a profitable deviation if only the prospective buyers' beliefs assigned zero probability to the state of the company being any $\tilde{\theta}$ such that, for any beliefs after $D^{dev}$, it is true that $Eu_{Bd-\tilde{\theta}}\left(C^{dev};s,\rho\right) = q_1^\star\left(D^{dev};s^\star\right)-k^{dev} < Eu_{Bd-\tilde{\theta}}\left(C^\star;s,\rho\right) = q_1^\star\left(D^\star;s^\star\right)-k^\star$.

The intuitive criterion thus discards PBE in which prospective buyers would hold beliefs that fail to recognize credible signaling attempts by the board off the equilibrium path.

The next section applies this equilibrium concept to the two state model, while the section after does the same to the continuous state model.

### 1.4 Analysis

As customary in dynamic games, I start by analyzing the last strategic decision of the game and then move backwards in time.

The last decision belongs to the prospective buyers who compete in period 1 for the ownership of the company. As first argued in subsection 1.3.1.2, the equilibrium price of the company that results from this competition is, in the two state model, given by:

$$q_1^\star\left(D;s\right) = \left[\bar{\theta}\rho\left(\bar{\theta}|D;s\right) + \mu e_{\theta C}\right](1-b) - w$$

In equilibrium, the buyers correctly guess the strategies being followed by the other players and update beliefs by using the on and off equilibrium path rules provided in definition 1.

The second to last decision in the model belongs to the executive: contingent on having accepted a compensation package, she must decide how much effort to exert. In equilibrium, she selects the amount of effort that maximizes her expected utility given the state of the company and the terms of her compensation. That is, she solves the following maximization problem:

$$\max_{e\in\mathbb{R}_+} Eu_{Ex-\theta}(C,e) \quad = \quad \max_{x\in[0,1]} \left\{ w + \alpha k + b\left(\theta + \mu e\right) - \frac{\gamma}{2}\left(b\sigma\right)^2 - \frac{\psi}{2}e^2 \right\}. \tag{1.7}$$

The first order condition of this problem yields the executive's optimal effort

$$e^{\star}_{\theta C} = e^{\star}_{C} = b\frac{\mu}{\psi}\,\forall\theta\in\Theta \tag{1.8}$$

which does not directly depend upon the state of the company, but may be affected by it indirectly if the executive's stake varies with the state of the company.

The executive has her optimal effort selection rule in mind when she takes the decision that precedes effort exertion: whether to accept or reject the compensation package that the board proposes. In equilibrium, the executive accepts package $C = (w, b, k)$ if it is rational to do so. By exerting the optimal amount of effort $e^{\star}_{\theta C}$ under $C$, she must obtain an expected utility that is no less than the utility yielded by her alternative employment option. This condition of individual rationality is denoted as:

$$IR_{Ex-\theta}:\ Eu_{Ex-\theta}(C,e^{\star}_{C}) = w + \alpha k + b\left(\theta + b\frac{\mu^2}{\psi}\right) - \frac{\gamma}{2}\left(b\sigma\right)^2 - \frac{\psi}{2}\left(b\frac{\mu}{\psi}\right)^2 \geq \bar{u} \tag{1.9}$$

In sum, the equilibrium strategy of the executive is $s^{\star}_{Ex} = (\delta^{\star}_{\theta C}, e^{\star}_{C})_{\theta\in\Theta, C\in\Omega_C}$ where $\delta^{\star}_{\theta C} = Y$ if $Eu_{Ex-\theta}(C,e^{\star}_{C}) \geq \bar{u}$ and $\delta^{\star}_{\theta C} = N$ otherwise.

The derivation of the executive's equilibrium strategy leaves only one decision to be characterized: the board's offer of a package. Subsection 1.4.2 does this characterization. Before we get there, subsection 1.4.1 derives a useful benchmark by solving the model with symmetric information.

### 1.4.1 Symmetric Information

The signaling function of disclosed compensation is lost if the buyers can observe the state of the company. Therefore, the compensation package that maximizes the price of the company in period 1 net of undisclosed compensation is the package that maximizes gross profit net of undisclosed and disclosed compensation costs. The symmetric information model is similar to the textbook model of effort exertion with normally distributed profits, linear compensation packages and mean-variance preferences for which Bolton and Dewatripont (2005) provide a solution. The only difference is that here there are two forms of fixed compensation, $w$ and $k$, instead of one.

The problem that the board solves in each state, with the optimal effort provision rule $e_C^\star = \frac{b\mu}{\psi}$ already plugged in, is:

$$\begin{aligned} & \max_{C\in\Omega_C} \left(\theta + b\tfrac{\mu^2}{\psi}\right)(1-b) - w - k \\ s.t.:\quad & IR_{Ex-\theta} : w + \alpha k + b\left(\theta + b\tfrac{\mu^2}{\psi}\right) - \tfrac{\gamma}{2}\left(b\sigma\right)^2 - \tfrac{\psi}{2}\left(b\tfrac{\mu}{\psi}\right)^2 \geq \bar{u}. \end{aligned} \tag{1.10}$$

Proposition 1 offers the solution to this problem. Throughout the paper, this solution will be referred to as the solution to the symmetric case or the first-best solution[13].

**Proposition 1.** *In the unique equilibrium of the symmetric information case, the board offers compensation package $C_\theta^{SI}$ to the executive when in state $\theta$ such that:*

$$\begin{aligned} b_\theta^{SI} &= \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2} \\ w_\theta^{SI} &= \bar{u} - b^{SI}\left(\theta + b^{SI}\frac{\mu^2}{\psi}\right) + \frac{\gamma}{2}\left(b^{SI}\sigma\right)^2 + \frac{\psi}{2}\left(b^{SI}\frac{\mu}{\psi}\right)^2 \\ k_\theta^{SI} &= 0. \end{aligned}$$

[13] The solution is first-best in the sense that it is not distorted by signaling. It is, however, distorted by moral hazard.

*Given $C_\theta^{SI}$, the executive exerts effort*

$$e^{SI} = \frac{\mu^3}{\psi\mu^2 + \psi^2\gamma\sigma^2}.$$

*Proof.* First observe that the constraint must bind; otherwise the board would find it profitable to decrease $w$. The stake in profits $b_\theta^{SI}$ can then be found by taking the first-order condition of the problem with respect to $b$. Since it is more expensive to reward the executive through $k$ than through $w$, and $k$ cannot be negative, we have that $k_\theta^{SI} = 0$. The expression for $w_\theta^{SI}$ follows from solving the binding constraint with respect to $w$. The executive's equilibrium effort is found by substituting the expression of $b^{SI}$ in the optimal effort exertion rule. □

We thus have that in the symmetric information case, the executive's stake in the firm's profits is independent of the state. The next subsection shows that the same may not be true when information is asymmetric.

### 1.4.2 Asymmetric Information

A pure strategy by the board specifies the compensation package that is offered in each state. A strategy is then an object $s_{Bd} = (C_0, C_{\bar{\theta}})$. To determine which strategies can be sustained in equilibrium, it is convenient to separately consider pooling and separating strategies. The two kinds of strategy differ in terms of their prescriptions for the part of the compensation package that can be observed by outsiders, $D = (w, b)$. A pooling strategy prescribes the same disclosed compensation terms in each state ($D_0 = D_{\bar{\theta}}$), which makes disclosed compensation uninformative about the state of the company. A separating strategy prescribes different disclosed compensation terms for each state ( $D_0 \neq D_{\bar{\theta}}$), which allows outsiders to fully infer the state of the company from their observation of disclosed compensation. For executive compensation to be a solution to the corporate signaling problem outlined in this paper, a separating strategy must be supported in equilibrium.

The following two sub-subsections study whether either kind of strategy can be sustained in equilibrium.

#### 1.4.2.1 Separating Strategies

When the board follows a separating strategy, stock prices fully reflect the value of the company's stock. This is beneficial to the board if the state is good, but detrimental if it is not, and therefore can only be sustained if the optimistic board can prevent the pessimist board from purporting optimism. In other words, for a separating strategy prescribing compensation packages $D_{\bar{\theta}}$ and $D_0$ to be supported in equilibrium, the board must design $D_{\bar{\theta}}$ as to make it sufficiently costly to offer in the bad state.

I begin the search for a separating equilibrium by conjecturing that at least one separating strategy, $s_{Bd}^{sep} = \left(C_0^{sep}, C_{\bar{\theta}}^{sep}\right)$, is indeed supported in equilibrium. By the end of this subsection, I will have shown that the conjecture is true and that $s_{Bd}^{sep}$ is the only separating strategy that is supported in equilibrium. I will also have derived a closed form solution for each of its elements.

Denote by $s^{sep} = \left(s_{Bd}^{sep}, s_{Ex}^{\star}\right)$, where $s_{Ex}^{\star}$ is the previously derived equilibrium strategy of the executive, the equilibrium strategy profile that includes $s_{Bd}^{sep}$. On the path that follows from the implementation of $s^{sep}$, the prospective buyers of the company are able to perfectly infer its state from their observation of the disclosed portion of the compensation package. It follows that the equilibrium package offered in the bad state, $C_0^{sep}$, must be the same as the first-best package $C_0^{SI}$. As the solution to the symmetric information case, $C_0^{SI}$ has already been shown to be the best package under the very worst beliefs. The conclusion that $C_0^{sep} = C_0^{SI}$ is formally stated and proved in lemma 1.

**Lemma 1.** *In any separating equilibrium, the board offers compensation package* $C_0^{sep} = (w_0^{sep}, b_0^{sep}, k_0^{sep}) = C_0^{SI}$ *when the state of the company is bad.*

*Proof.* Suppose instead that a package $\hat{C}_0 = \left(\hat{w}_0, \hat{b}_0, \hat{k}_0\right) \neq C_0^{SI}$ could be supported as part of a proposed equilibrium strategy $\hat{s}_{Bd}$. The observation of the disclosed part $\hat{D}_0 = \left(\hat{w}_0, \hat{b}_0\right)$ would induce the belief $\hat{\rho}\left(\bar{\theta}|D_0^{sep}; s^{sep}\right) = 0$ and the payoff of the board would then be $Eu_{Bd-0}\left(\hat{C}_0; \hat{s}_{Bd}, s_{Ex}^{\star}, \hat{\rho}\right) = q_1^{\star}\left(\hat{D}_0; \hat{s}_{Bd}, s_{Ex}^{\star}\right) - \hat{k}_0$. Now consider a deviation by the board whereby it offers package $C_0^{SI}$ (the package that maximizes the board's payoff in state 0 in the symmetric information case). The deviation cannot induce worse beliefs than those induced by $\hat{C}_0$, which implies that the board's payoff from this package can be no worse than $q_1^\star\left(D_0^{SI};\hat{s}_{Bd},s_{Ex}^\star\right)$. It is then true that:

$$Eu_{Bd-0}\left(C_0^{SI};\hat{s}_{Bd},s_{Ex}^\star,\hat{\rho}\right)\geq q_1^\star\left(D_0^{SI};\hat{s}_{Bd},s_{Ex}^\star\right)>Eu_{Bd-0}\left(\hat{C}_0;\hat{s}_{Bd},s_{Ex}^\star,\hat{\rho}\right)$$

where the second inequality follows from the fact that $C_0^{SI}$ is the unique maximizer of the board's payoff in the case with symmetric information. Combined, the two inequalities imply that $C_0^{SI}$ is a profitable deviation from $\hat{C}_0$, which contradicts the initial supposition. □

Now that we know the package $C_0^{sep}$ offered in the bad state in a separating equilibrium, consider a strategy $s_{Bd}=(C_0^{sep},C_{\bar{\theta}})$ such that $D_{\bar{\theta}}\neq D_0^{sep}$. Package $C_{\bar{\theta}}$ is supported in a separating equilibrium if:

1. when in the good state, the executive can rationally accept it;
2. when in the bad state, the board cannot profitably deviate *to* it;
3. when in the good state, the board cannot deviate *from* it.

The first condition implies that package $C_{\bar{\theta}}$ must satisfy the individually rationality condition in expression (1.9):

$$IR_{Ex-\bar{\theta}}:\ Eu_{Ex-\bar{\theta}}\left(C_{\bar{\theta}},e_{\bar{\theta}C}^\star\right)=w_{\bar{\theta}}+\alpha k_{\bar{\theta}}+b_{\bar{\theta}}\left(\bar{\theta}+b_{\bar{\theta}}\frac{\mu^2}{\psi}\right)-\frac{\gamma}{2}\left(b_{\bar{\theta}}\sigma\right)^2-\frac{\psi}{2}\left(b_{\bar{\theta}}\frac{\mu}{\psi}\right)^2\geq\bar{u}. \quad (1.11)$$

The second condition imposes incentive compatibility: it asserts that, in equilibrium, the benefits accruing to the board from offering the deviation package $C_{\bar{\theta}}$ in the bad state cannot be larger than the costs. Let the prospective buyers believe that the board is following strategy $s_{Bd}$ as defined above and that the executive is following her optimal strategy $s_{Ex}^\star$. The benefit from a deviation in the bad state is equal to the difference between the buyers' valuation of the company after observing $D_{\bar{\theta}}$ and their valuation after observing $D_0^{sep}$. This benefit is defined as:

$$
\begin{aligned}
Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) &\equiv q^{\star}_{1}\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) - q^{\star}_{1}\left(D^{sep}_{0}; s_{Bd}, s^{\star}_{Ex}\right) \\
&= \left[\left(\bar{\theta} + b_{\bar{\theta}}\frac{\mu^{2}}{\psi}\right)\left(1 - b_{\bar{\theta}}\right) - w_{\bar{\theta}}\right] \\
&\quad - \left[b^{sep}_{0}\frac{\mu^{2}}{\psi} - \bar{u} - \frac{\gamma}{2}\left(b^{sep}_{0}\sigma\right)^{2} - \frac{\psi}{2}\left(b^{sep}_{0}\frac{\mu}{\psi}\right)^{2}\right]. \qquad (1.12)
\end{aligned}
$$

In equilibrium, $Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right)$ cannot be negative; otherwise, the board would find it profitable to deviate to $C^{sep}_{0}$ in the good state.

The cost of a deviation is equal to the "bribe" that the board must pay the executive to make her accept the deviation package. The cost is defined as:

$$
F\left(D_{\bar{\theta}}\right) \equiv \max\left\{0, \frac{1}{\alpha}\left[\bar{u} - Eu_{Ex-0}\left(D_{\bar{\theta}}, 0, e^{\star}_{C}\right)\right]\right\} \qquad (1.13)
$$

where $Eu_{Ex-0}\left(D_{\bar{\theta}}, 0, e^{\star}_{C}\right)$ represents the executive's utility from getting a package with observable $D_{\bar{\theta}}$ and no undisclosed compensation. In equilibrium, the cost must be strictly positive, as otherwise the board would be able to deviate for free when in the bad state.

The formal expression for the board's incentive compatibility is then:

$$
IC_{Bd-0}: \; Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) \leq F\left(D_{\bar{\theta}}\right). \qquad (1.14)
$$

which, through algebraic manipulation, can be rewritten as

$$
Eu_{Ex-0}\left(D_{\bar{\theta}}, Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right)\right) \leq \bar{u} \qquad (1.15)
$$

The rewritten condition illustrates how the board achieves separation if the state is good: by offering a package including compensation terms $D_{\bar{\theta}}$ that would not be acceptable to the executive if the state were bad for any bribe that the board might be rationally willing to pay.

Note that every contract that fulfills $IR_{Ex-\bar{\theta}}$ and $IC_{Bd-0}$ induces, by the intuitive criterion, the belief that the firm's state is good regardless of whether such contract is on or off the equilibrium path. In effect, this means that the board gets to choose the best package from among those that satisfy $IR_{Ex-\bar{\theta}}$ and $IC_{Bd-0}$. It follows that the third condition for $C_{\bar{\theta}}$ to be a part of an equilibrium is that it must maximize the board's payoff subject to the constraints given in expressions (1.11) and (1.14):

$$\begin{aligned} & \max_{C_{\bar{\theta}} \in \Omega_C} \left(\bar{\theta} + b_{\bar{\theta}} \tfrac{\mu^2}{\psi}\right)(1 - b_{\bar{\theta}}) - w_{\bar{\theta}} - k_{\bar{\theta}} \\ s.t. : & \\ IR_{Ex-\bar{\theta}} : \quad & Eu_{Ex-\bar{\theta}}\left(C_{\bar{\theta}}, e^{\star}_{C}\right) \geq \bar{u} \\ IC_{Bd-0} : \quad & Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) \leq F\left(D_{\bar{\theta}}\right) \end{aligned}$$

This problem is simplified by lemma 2, which asserts that $IR_{Ex-\bar{\theta}}$ must bind in equilibrium. The proof conveys the intuition behind the result.

**Lemma 2.** *In equilibrium, the executive cannot earn rents above her reservation utility if the company's state is good. That is, $IR_{Ex-\bar{\theta}}$ holds as an equality.*

*Proof.* Suppose instead that a package $C_{\bar{\theta}}$ is supported in equilibrium such that the executive earns rents; that is, such that $Eu_{Ex-\bar{\theta}}\left(C_{\bar{\theta}}, e^{\star}_{C}\right) > \bar{u}$. Now consider a deviation whereby the board offers a smaller wage, equal to $w_{\bar{\theta}} - \epsilon$, where $\epsilon$ is arbitrarily small. The deviation does not violate either constraint: the executive's participation is left unaffected as $IR_{Ex-\bar{\theta}}$ was slack by assumption under $C_{\bar{\theta}}$; and the benefit from deviating $Q\left(D_{\bar{\theta}}; s_{Bd}, s^{\star}_{Ex}\right)$ increases by $\epsilon$ while the cost $F\left(D_{\bar{\theta}}\right)$ increases by $\frac{1}{\alpha}\epsilon$, which, since $\alpha \leq 1$, implies that $IC_{Bd-0}$ still holds. Since incentive compatibility is still respected after the deviation, the intuitive criterion applies and therefore it must be that the buyers still assess the firm's state as being good. This implies that the deviation package credibly signals that the state is good. Because the it reduces compensation costs while leaving the gross profit and the buyers' beliefs intact, the proposed deviation is profitable, which contradicts the initial assertion that the executive can earn rents in equilibrium. $\square$

The lemma makes it possible to re-write constraint $IR_{Ex-\bar{\theta}}$ as to illustrate how the wage adjusts to keep the constraint binding:

$$w_{\bar{\theta}} = \bar{u} - \alpha k_{\bar{\theta}} - b_{\bar{\theta}}\left(\bar{\theta} + b_{\bar{\theta}}\frac{\mu^2}{\psi}\right) + \frac{\gamma}{2}\left(b_{\bar{\theta}}\sigma\right)^2 + \frac{\psi}{2}\left(b_{\bar{\theta}}\frac{\mu}{\psi}\right)^2. \tag{1.16}$$

After substituting $w_{\bar{\theta}}$ in the board's objective function and in the $IC_{Bd-0}$ constraint, we get the following simplified maximization problem

$$\begin{array}{rcl} & \max\limits_{b_{\bar{\theta}},k_{\bar{\theta}}} \bar{\theta} + b_{\bar{\theta}}\frac{\mu^2}{\psi} - \bar{u} - (1-\alpha)\,k_{\bar{\theta}} - \frac{\gamma}{2}\left(b_{\bar{\theta}}\sigma\right)^2 - \frac{\psi}{2}\left(b_{\bar{\theta}}\frac{\mu}{\psi}\right)^2 & \\ s.t.: & & (1.17) \\ IC_{Bd-0}: & Q\left(D_{\bar{\theta}}; s_{Bd}, s_{Ex}^{\star}\right) \leq F\left(D_{\bar{\theta}}\right) & \end{array}$$

where, after some algebra, $Q\left(D_{\bar{\theta}}; s_{Bd}, s_{Ex}^{\star}\right)$ is now given by

$$Q\left(D_{\bar{\theta}}; s_{Bd}, s_{Ex}^{\star}\right) = \bar{\theta} + \frac{\mu^2}{\psi}\left(b_{\bar{\theta}} - b_0^{sep}\right) - \frac{1}{2}\left(\gamma\sigma^2 + \frac{\mu^2}{\psi}\right)\left[\left(b_{\bar{\theta}}\right)^2 - \left(b_0^{sep}\right)^2\right] + \alpha k_{\bar{\theta}}, \quad (1.18)$$

and $F\left(D_{\bar{\theta}}\right)$ by

$$F\left(D_{\bar{\theta}}\right) = \frac{1}{\alpha}\left[b_{\bar{\theta}}\bar{\theta} + \alpha k_{\bar{\theta}}\right]. \quad (1.19)$$

If constraint $IC_{Bd-0}$ holds for the package that solves the symmetric information problem, $C_{\bar{\theta}}^{SI}$, then the solution to the problem in expression (1.17) is trivial: $C_{\bar{\theta}}^{sep}$ must be the same as $C_{\bar{\theta}}^{SI}$. When this is the case, we have that the executive's stake is the same in both states $\left(b_{\bar{\theta}}^{sep} = b_{\bar{\theta}}^{SI} = b_0^{SI} = b_0^{sep}\right)$. The fixed wage, however, is not: because the same stake $b_{\bar{\theta}}^{SI}$ in profit gives the executive $b_{\bar{\theta}}^{SI}\bar{\theta}$ more in expected utility in the good state than it does in the bad state, in the bad state the board must pay a higher fixed wage to ensure the executive's acceptance of the compensation package (that is, $w_0^{sep} = w_0^{SI} > w_{\bar{\theta}}^{SI} = w_0^{sep}$). It follows that, when $IC_{Bd-0}$ holds when evaluated at $C_{\bar{\theta}}^{SI}$, signaling is performed through the fixed wage. Proposition 2 derives the necessary and sufficient condition for $C_{\bar{\theta}}^{sep}$ to be equal to $C_{\bar{\theta}}^{SI}$.

**Proposition 2.** *Compensation package $C_{\bar{\theta}}^{sep} = C_{\bar{\theta}}^{SI}$ is supported in a separating equilibrium if and only if the value of undisclosed compensation to the executive is such that*

$$\alpha \leq b_{\bar{\theta}}^{SI} = \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2}.$$

*Proof.* $C_{\bar{\theta}}^{SI}$ solves (1.17) if and only if $IC_{Bd-0}$ holds when evaluated at $C_{\bar{\theta}}^{SI}$ as given in proposition 1. (This is true because $C_{\bar{\theta}}^{SI}$ maximizes the unconstrained version of problem (1.17).)

Constraint $IC_{Bd-0}$ evaluated at $C_{\bar{\theta}}^{SI}$ comes down to $\alpha \leq b_{\bar{\theta}}^{SI}$. $\square$

Proposition 2 thus asserts that the board can offer package $C_{\bar{\theta}}^{SI}$ in a separating equilibrium if and only if the executive's first-best stake in the company's profit is sufficiently large relative to the valuation that the executive attaches to undisclosed compensation. This follows from the fact that the benefit of a bad state deviation to $C_{\bar{\theta}}^{SI}$ is $\bar{\theta}\left(1 - b_{\bar{\theta}}^{SI}\right) + b_{\bar{\theta}}^{SI}\left(\bar{\theta} - 0\right) = \bar{\theta}$ - where the first term is the signaling gain derived from convincing outsiders that the company's state is good and the second term reflects the savings in wages relative to the equilibrium contract $C_0^{SI}$ - and its cost is $\frac{b_{\bar{\theta}}^{SI}\bar{\theta}}{\alpha}$ - which reflects the facts that the deviation contract falls short by $b_{\bar{\theta}}^{SI}\bar{\theta}$ from achieving the participation of the executive and that it costs the board $\frac{1}{\alpha}$ dollars to increase the executive's income by 1 dollar through undisclosed compensation.

In conclusion, signaling does not distort compensation practices if solving the pure moral hazard problem - that is, if finding the optimal balance between protecting the executive from risk and incentivizing to exert effort - entails offering the executive a sufficiently large stake in the company. This large stake is offered if the executive's effort is valuable (large $\mu$), if she is not too averse to exerting unobservable effort (small $\psi$) or not too risk-averse (small $\gamma$), and if the company's prospects are relatively safe (low $\sigma^2$). The sufficient size of the stake is determined by the value that the executive attaches to undisclosed compensation. The larger $\alpha$ is, the larger the stake must be.

One of the features of the scenario to which proposition 2 applies is that $IC_{Bd-0}$ may not bind. In other words, it is possible that the costs of a deviation are larger than its benefits. In the alternative scenario - $\alpha > b_{\bar{\theta}}^{SI}$ - which will concern us from here on after, $IC_{Bd-0}$ must bind. Lemma 3 establishes the result formally.

**Lemma 3.** *Let the value that the executive attaches to undisclosed compensation be such that $\alpha > \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2}$ holds. Then, in any separating equilibrium, the incentive compatibility constraint of the board, $IC_{Bd-0}$, must bind.*

*Proof.* After plugging expressions (1.18) and (1.19) in for the benefit and cost of a deviation, the $IC_{Bd-0}$ constraint is given by:

$$\bar{\theta} + \frac{\mu^2}{\psi}\left(b_{\bar{\theta}} - b_0^{sep}\right) - \frac{1}{2}\left(\gamma\sigma^2 + \frac{\mu^2}{\psi}\right)\left[\left(b_{\bar{\theta}}\right)^2 - \left(b_0^{sep}\right)^2\right] + \alpha k_{\bar{\theta}} \leq \frac{1}{\alpha}\left[b_{\bar{\theta}}\bar{\theta} + \alpha k_{\bar{\theta}}\right]. \qquad (1.20)$$

Suppose that the inequality holds strictly. Since $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$, proposition 2 implies that $C_{\bar{\theta}}^{sep}$ cannot be equal to $C_{\bar{\theta}}^{SI}$. Combined with lemma 2, this implies that at last two of the elements of $C_{\bar{\theta}}^{sep} = \left(w_{\bar{\theta}}^{sep}, b_{\bar{\theta}}^{sep}, k_{\bar{\theta}}^{sep}\right)$ must differ from the corresponding elements in $C_{\bar{\theta}}^{SI}$.

First suppose that $b_{\bar{\theta}}^{sep}$ is one of the elements that differs. That is, let $b_{\bar{\theta}}^{sep} > b_{\bar{\theta}}^{SI}$. (It cannot be that $b_{\bar{\theta}}^{sep} < b_{\bar{\theta}}^{SI}$; otherwise, since $b_{\bar{\theta}}^{SI} = b_0^{sep}$, a deviation in the bad state would be profitable). Now consider a new package prescribing a stake that is smaller than $b_{\bar{\theta}}^{sep}$ by an arbitrarily small amount, and a wage adjusted as to make the participation of the executive rational. The new package thus satisfies the executive's individual rationality constraint (because of the adjustment in the wage) and, because $IC_{Bd-0}$ is slack at $b_{\bar{\theta}}^{sep}$, it also satisfies the incentive compatibility constraint. With the two constraints still holding, the intuitive criterion applies and therefore outsiders do not revise their beliefs downwards after observing the new package. Since the board's objective function - see expression (1.17) - is concave in $b_{\bar{\theta}}$, and $b_{\bar{\theta}}^{SI}$ (which is smaller than $b_{\bar{\theta}}^{sep}$) is the maximizer of the function, we can conclude that the function takes on a higher value when evaluated at the new package than when evaluated at $b_{\bar{\theta}}^{sep}$, which implies that the new package configures a profitable deviation.

Now suppose that $b_{\bar{\theta}}^{sep} = b_{\bar{\theta}}^{SI}$. It must then be that $w_{\bar{\theta}}^{sep} \neq w_{\bar{\theta}}^{SI}$ and $k_{\bar{\theta}}^{sep} \neq k_{\bar{\theta}}^{SI} \Rightarrow k_{\bar{\theta}}^{sep} > 0$. Consider a new package $C_{\bar{\theta}}^{dev}$ that prescribes less undisclosed compensation: $k_{\bar{\theta}}^{dev} = k_{\bar{\theta}}^{sep} - \epsilon$, where $\epsilon$ is arbitrarily small. To keep the participation of the executive intact, the wage increases - see expression (1.16). The net effect is an increase in the board's payoff - see the objective function in expression (1.17). The new package decreases the benefits of a bad state deviation by $\alpha\epsilon$ and decreases its cost by $\epsilon$, making the deviation relatively more attractive. However, since $IC_{Bd-0}$ is slack for $C_{\bar{\theta}}^{sep}$, if $\epsilon$ is sufficiently small the constraint still holds for $C_{\bar{\theta}}^{dev}$, making it a profitable deviation from $C_{\bar{\theta}}^{sep}$.

The analysis of the two possible cases for $b_{\bar{\theta}}^{sep}$ shows that a profitable deviation must exist when the $IC_{Bd-0}$ constraint is slack and $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$ holds, which is a contradiction of the initial supposition. Therefore, the constraint most hold as an equality. □

The idea behind the proof is simple: if the incentive compatibility constraint does not bind in equilibrium, then the cost of a deviation must be strictly larger than its benefits. This means that the board can let the benefits from deviation increase (say, by decreasing the executive's stake or her undisclosed compensation and increasing her wage as to keep her participation intact) without making the deviation profitable; by doing so, however, it improves its own payoff, as it substitutes the relatively expensive forms of compensation (variable and undisclosed) with the relatively cheap one (the wage).

Solving the binding $IC_{Bd-0}$ with respect to $k_{\bar{\theta}}$ then yields:

$$k_{\bar{\theta}} = \frac{1}{1-\alpha}\left\{\left(1-\frac{b_{\bar{\theta}}}{\alpha}\right)\bar{\theta} + \frac{\mu^2}{\psi}\left(b_{\bar{\theta}} - b_0^{sep}\right) - \frac{1}{2}\left(\gamma\sigma^2 + \frac{\mu^2}{\psi}\right)\left[(b_{\bar{\theta}})^2 - (b_0^{sep})^2\right]\right\}, \tag{1.21}$$

which simplifies the board's problem even further to:

$$\begin{aligned} &\max_{b_{\bar{\theta}}} \frac{b_{\bar{\theta}}}{\alpha}\bar{\theta} + b_0^{sep}\frac{\mu^2}{\psi} - \bar{u} - \frac{1}{2}\left(\gamma\sigma^2 + \frac{\mu^2}{\psi}\right)(b_0^{sep})^2 \\ st: \quad & k_{\bar{\theta}} \geq 0. \end{aligned} \tag{1.22}$$

where $k_{\bar{\theta}}$ is given by expression (1.21).

The simplified objective function is strictly increasing in $b_{\bar{\theta}}$, thus implying that, among all the compensation packages that guarantee separation while making $IC_{Bd-0}$ bind, the one that the board prefers is that which gives the executive the largest feasible stake in the company's profit. Feasibility is determined by the constraint of the simplified problem, which states that undisclosed compensation cannot be negative. Lemma (4) goes one step further: it states that undisclosed compensation must be equal to zero.

**Lemma 4.** *Let $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$. Then, in any separating equilibrium, the board offers no undisclosed compensation to the executive if the company's state is good. That is, $k_{\bar{\theta}}^{sep} = 0$.*

*Proof.* a) For $b_{\bar{\theta}} = b_0^{sep}$, the right hand side of equation (1.21) is strictly positive, which illustrates that the constraint in problem (1.22) holds for values in some subset of the domain of $b_{\bar{\theta}}$[14]. b) The right hand side of the equation is a continuous quadratic function of $b_{\bar{\theta}}$ with a negative quadratic coefficient. Since the board's objective function in optimization problem (1.22) is increasing in $b_{\bar{\theta}}$, this implies that the the right hand side of equation (1.21) must be

[14] Too see why the right hand side of expression (1.21) is positive for $b_{\bar{\theta}} = b_0^{sep}$, first evaluate the expression at $b_0^{sep}$ and simplify. The assertion then follows from $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$, and $b_0^{sep} = \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$.

equal to zero, which implies that $k_{\bar{\theta}}^{sep}$ must be equal to zero. $\square$

It follows from the lemma that the executive's share of the final output in the good state must solve

$$\frac{1}{1-\alpha}\left\{\left(1-\frac{b_{\bar{\theta}}}{\alpha}\right)\bar{\theta}+\frac{\mu^2}{\psi}\left(b_{\bar{\theta}}-b_0^{sep}\right)-\frac{1}{2}\left(\gamma\sigma^2+\frac{\mu^2}{\psi}\right)\left[\left(b_{\bar{\theta}}\right)^2-\left(b_0^{sep}\right)^2\right]\right\}=0 \qquad (1.23)$$

which follows from plugging in $k_{\bar{\theta}}=0$ into equation (1.21).

Proposition 3 presents the solution to this equation and summarizes the conclusions about separating equilibria in this model.

**Proposition 3.** *Let $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$. Then, one, and only one, separating strategy profile is supported in equilibrium. In that equilibrium:*

1. *the board's separating strategy is $s_{Bd}^{sep}=\left(C_0^{sep},C_{\bar{\theta}}^{sep}\right)$ such that: $C_0^{sep}=C_0^{SI}$ and $C_{\bar{\theta}}^{sep}=\left(w_{\bar{\theta}}^{sep},b_{\bar{\theta}}^{sep},k_{\bar{\theta}}^{sep}\right)$, where $k_{\bar{\theta}}^{sep}=0$, $w_{\bar{\theta}}^{sep}$ is given by expression (1.16) and*

$$b_{\bar{\theta}}^{sep} \quad = \quad b_0^{sep}+\left(\frac{\psi}{\mu^2+\psi\gamma\sigma^2}\right)\left[-\frac{\bar{\theta}}{\alpha}+\sqrt{\left(\frac{\bar{\theta}}{\alpha}\right)^2+2\left(\frac{\psi\gamma\sigma^2+\mu^2\left(1-\frac{1}{\alpha}\right)}{\psi}\right)\bar{\theta}}\right].$$

2. *the executive follows strategy $s_{Ex}^{\star}=[\delta_{\theta C}^{\star},e_C^{\star}]_{\theta\in\Theta,C\in\Omega_C}$.*

3. *the prospective buyers have beliefs*

   (a) *$\rho\left(\bar{\theta}|D_{\bar{\theta}};s_{Bd},s_{Ex}^{\star}\right)$, $\rho\left(\bar{\theta}|D_0;s_{Bd},s_{Ex}^{\star}\right)$ given by Bayes' rule on the equilibrium path of any strategy $s_{Bd}\in\Omega_C\times\Omega_C$;*

   (b) *given by $\rho\left(\bar{\theta}|D;s_{Bd},s_{Ex}^{\star}\right)=1$ if*

$$Eu_{Ex-0}\left(D,Q\left(D;s_{Bd},s_{Ex}^{\star}\right),e_C^{\star}\right)<\bar{u}\leq Eu_{Ex-\bar{\theta}}\left(D,0,e_C^{\star}\right)$$

   *and $\rho\left(\bar{\theta}|D;s_{Bd},s_{Ex}^{\star}\right)=0$ otherwise, off the equilibrium path of any strategy $s_{Bd}\in\Omega_C\times\Omega_C$.*

4. *the price of the company's stock is $q_1^{\star}(D;s)$ for all $D$ and $s$.*

*Proof.* The proposition contains two assertions that have not been proved yet. They concern:

1. The equilibrium expression for $b_{\bar{\theta}}^{sep}$.

   It is known that $b_{\bar{\theta}}^{sep}$ must be a solution to equation (1.23), which equates a quadratic function of $b_{\bar{\theta}}$ to zero. The equation has two solutions but, since the board's payoff function is increasing in $b_{\bar{\theta}}$, it must be that $b_{\bar{\theta}}^{sep}$ is the largest of the two. The expression of $b_{\bar{\theta}}^{sep}$ presented in the proposition is that solution (after algebraic simplification).

2. The description of the outsiders beliefs.

   (a) Beliefs on the equilibrium path: on the path induced by any strategy of the board $s_{Bd} \in \Omega_C \times \Omega_C$, beliefs must be derived through Bayes' rule.

   (b) The belief $\rho\left(\bar{\theta}|D; s_{Bd}, s_{Ex}^{\star}\right) = 1$ after a package $D$ such that

   $$Eu_{Ex-0}\left(D, Q\left(D; s_{Bd}, s_{Ex}^{\star}\right), e_C^{\star}\right) < \bar{u} \leq Eu_{Ex-\bar{\theta}}\left(D, 0, e_C^{\star}\right)$$

   is implied by the application of the intuitive criterion to beliefs off the equilibrium path. If the buyers observe a package that the executive cannot rationally accept in the bad state (even when receiving the most undisclosed compensation that the board can rationally offer), but can accept in the good state, they must believe that the company is in the good state.

   (c) The belief $\rho\left(\bar{\theta}|D; s_{Bd}, s_{Ex}^{\star}\right)$ after other packages: while Bayes' rule and the intuitive criterion say nothing about these beliefs, I set them at 0 to ensure that, when the strategy being played is $s_{Bd}^{sep}$, there are no profitable deviations in either state.

□

The proposition states that there is a unique separating strategy profile supported in equilibrium instead of stating that the equilibrium is unique because of the multiplicity of beliefs that support that equilibrium. While the beliefs on the equilibrium path and those that follow deviations to which the intuitive criterion applies are determined by the equilibrium concept presented in definition 1, beliefs after other deviations are unrestricted. The proposition states that those beliefs are that the state is bad for sure, ensuring that there are no profitable deviations from the proposed strategy profile, but other beliefs - those that assign a sufficiently low probability to the state being good - also work. For convenience, however, I will often refer to the equilibrium derived in the proposition as the unique separating equilibrium.

In sum, the model always has a separating equilibrium. If the value of undisclosed compensation to the executive is low separation is achieved at no cost. Otherwise, it entails distortions as it forces the board to give sub-optimal incentives to the executive: as asserted in proposition 3, the executive's stake in this case is given by:

$$b_{\bar{\theta}}^{sep} = b_0^{sep} + \left(\frac{\psi}{\mu^2 + \psi\gamma\sigma^2}\right)\left[-\frac{\bar{\theta}}{\alpha} + \sqrt{\left(\frac{\bar{\theta}}{\alpha}\right)^2 + 2\left(\frac{\psi\gamma\sigma^2 + \mu^2\left(1 - \frac{1}{\alpha}\right)}{\psi}\right)\bar{\theta}}\right]. \tag{1.24}$$

Since $b_0^{sep}$ is the optimal effort inducing stake, the expression allows us to easily assess the size of the distortion induced by signaling over the executive's incentives. The distortion is bigger the more important the asymmetric information problem is, as measured by $\bar{\theta}$ (this is illustrated by figure 1.3 in section 1.6, for $\theta = \bar{\theta}$).

The characterization of the separating equilibrium is the main result of this paper. It provides theoretical support to the hypothesis that executive compensation can be used as a credible signal of inside information. The next sub-subsection confirms that the equilibrium found in this section is the unique equilibrium of the model by showing that there are no pooling strategies.

#### 1.4.2.2 Pooling Strategies

The board is said to be playing a pooling strategy if the observable component of the compensation package is the same in both states. That is, a pooling strategy is an object $s_{Bd}^{pool} = \left(C_0^{pool}, C_{\bar{\theta}}^{pool}\right)$ with $C_\theta^{pool} = \left(w^{pool}, b^{pool}, k_\theta^{pool}\right)$ for $\theta\left\{0, \bar{\theta}\right\}$ and $D^{pool} = \left(w^{pool}, b^{pool}\right)$.

Proposition 4 formally states that the model has no pooling equilibria as the board always has a profitable deviation in the good state from any proposed equilibrium pooling strategy. The deviation entails offering a package that credibly signals the company's state at a sufficiently low cost for the board. By the intuitive criterion, a credible signal gives the buyers the belief that the firm's state is good, and hence induces them to make a bid that fully prices in the value of the state.

**Proposition 4.** *There are no pooling equilibria.*

*Proof.* The proof is by contradiction. Suppose that the pooling strategy $s_{Bd}^{pool}$ is supported in equilibrium. Now consider the set:

$$\Lambda^{dev}\left(D^{pool}\right) \equiv \left\{D^{dev} = \left(w^{dev}, b^{dev}\right) \in \Omega_D : b^{dev} > b^{pool}\, and\, Eu_{Ex-\bar{\theta}}\left(D^{dev}, 0, e^{\star}\right) = \bar{u}\right\}$$

which contains the deviations from $D^{pool}$ that: $a$) give the executive a larger stake in the firm than that prescribed by the pooling package; $b$) if the firm's state is good, give the executive just enough expected utility as to make the package acceptable to her without need for undisclosed compensation.

Suppose that, upon observing a package $D^{dev} \in \Lambda^{dev}\left(D^{pool}\right)$, the prospective buyers become certain that the firm's state is good. That is, suppose that $\rho\left(\bar{\theta}|D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) = 1$. We can compute the benefits and costs of a deviation to $D^{dev}$ from $D^{pool}$ in both states to verify whether the supposition that $\rho\left(\bar{\theta}|D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) = 1$ follows from the intuitive criterion for some $D^{dev} \in \Lambda^{dev}\left(D^{pool}\right)$. If it does, then $D^{dev}$ is a profitable deviation from $D^{pool}$, as the criterion only applies if the board is better off with the deviation in the good state and strictly worse off in the bad state. In other words, the criterion applies if the net benefit from deviating is positive in the good state and strictly negative in the bad state:

$$\left[Q_{\bar{\theta}}\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - 0\right] \geq 0 > \left[Q_0\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - F_0\left(D^{dev}\right)\right] \tag{1.25}$$

Lets start by computing the benefit from a deviation to $D^{dev}$. For all $\theta \in \left\{0, \bar{\theta}\right\}$, the benefit is given by

$$Q_\theta\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) = q_1\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - \left[q_1\left(D^{pool}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - k_\theta^{pool}\right].$$

It is useful to compute the difference in the benefits from a deviation to $D^{dev}$ in state 0 relative to state $\bar{\theta}$:

$$Q_0\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - Q_{\bar{\theta}}\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) = k_0^{pool} - k_{\bar{\theta}}^{pool}. \tag{1.26}$$

Since $s_{Bd}^{pool}$ is, according to the initial supposition, supported in equilibrium, the undisclosed compensation that it prescribes in each state must be exactly what is necessary to induce the participation of the executive:

$$k_{\theta}^{pool} = \max\left\{0, \frac{\bar{u} - Eu_{Ex-\theta}\left(D^{pool}, 0, e^{\star}\right)}{\alpha}\right\}.$$

The difference in the hidden payments awarded in each state is then bounded above:

$$k_0^{pool} - k_{\bar{\theta}}^{pool} \leq \frac{b^{pool}}{\alpha}\bar{\theta} \tag{1.27}$$

The combination of (1.26) and (1.27) yields an upper limit to the difference in benefits from a deviation:

$$Q_0\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - Q_{\bar{\theta}}\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) \leq \frac{b^{pool}}{\alpha}\bar{\theta}. \tag{1.28}$$

Lets now turn to the cost of deviating to package $D^{dev}$. By definition of the set $\Lambda\left(D^{pool}\right)$ to which $D^{dev}$ belong, in the good state the cost is $F_{\bar{\theta}}\left(D^{dev}\right) = 0$. In the bad state, it is $F_0\left(D^{dev}\right) = \frac{b^{dev}}{\alpha}\bar{\theta}$ , which means that the difference in costs is given by:

$$F_0\left(D^{dev}\right) - F_{\bar{\theta}}\left(D^{dev}\right) = \frac{b^{dev}}{\alpha}\bar{\theta}. \tag{1.29}$$

Expressions (1.28) and (1.29), combined with the fact that $b^{dev} > b^{pool}$, imply that the net benefit from a deviation in the good state to a package $D^{dev} \in \Lambda^{dev}\left(D^{pool}\right)$ that prompts belief $\rho\left(\bar{\theta}|D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) = 1$ is greater than the net benefit from a deviation in the bad state to that same package:

$$\begin{aligned} &\left[Q_{\bar{\theta}}\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - 0\right] \\ -&\left[Q_0\left(D^{dev}; s_{Bd}^{pool}, s_{Ex}^{\star}\right) - F_0\left(D^{dev}\right)\right] \quad \geq \quad \left(\frac{b^{dev}}{\alpha} - \frac{b^{pool}}{\alpha}\right)\bar{\theta} > 0. \end{aligned} \tag{1.30}$$

Now note that the net benefit from a deviation in either state is continuous in $b^{dev}$ and its graph is a negative parabola truncated to the left at $b^{pool}$. Because of the change in beliefs that

$D^{dev}$ elicits relative to $D^{pool}$, this net benefit is also positive for values of $b^{dev}$ sufficiently close to $b^{pool}$ and negative for larger values. By the intermediate value theorem, it follows that there is an observable package $\hat{D}^{dev} \in \Lambda^{dev}\left(D^{pool}\right)$ such that $Q_0\left(\hat{D}^{dev}; s^{pool}_{Bd}, s^{\star}_{Ex}\right) - F_0\left(\hat{D}^{dev}\right) = 0$ which, by expression (1.30), implies that $Q_{\bar{\theta}}\left(\hat{D}^{dev}; s^{pool}_{Bd}, s^{\star}_{Ex}\right) > 0$.

The continuity of the net benefit from a deviation in both states and the fact that the net benefit of a deviation in the bad state is monotonically decreasing in $b^{dev} > \hat{b}^{dev}$ then imply that for some $\tilde{D}^{dev}$ such that $\tilde{b}^{dev}$ is in the vicinity of $\hat{b}^{dev}$, we have

$$\left[Q_{\bar{\theta}}\left(\tilde{D}^{dev}; s^{pool}_{Bd}, s^{\star}_{Ex}\right) - 0\right] \geq 0 > \left[Q_0\left(\tilde{D}^{dev}; s^{pool}_{Bd}, s^{\star}_{Ex}\right) - F_0\left(\tilde{D}^{dev}\right)\right]$$

which implies that the intuitive criterion applies, proving the existence of a profitable deviation from $D^{pool}$. $\square$

This proof completes the analysis of the two state model, which has a unique, separating, equilibrium. This analysis can be extended without too much complication to a case with a continuum of states.

### 1.5 Extension: equilibrium with a continuum of states

This section considers a modified model whereby the company's state is no longer binary but continuous with bounded support: $\theta \in \Theta = [0, \theta_M]$.

In the absence of informational asymmetries, the compensation package derived in proposition 1 would still be offered in every state $\theta \in \Theta = [0, \theta_M]$. The rest of this section is dedicated to the case of asymmetric information.

Board and executive are privately informed about $\theta \in \Theta = [0, \theta_M]$, while the buyers' prior beliefs about the company's state are represented by the probability density function $p_\theta$.

I begin by addressing whether a separating strategy $s^{sep}_{Bd} = \left(C^{sep}_\theta\right)_{\theta\in\Theta}$ such that $D^{sep}_\theta \neq D^{sep}_{\theta'}$ for all $\theta \neq \theta'$ can be supported in equilibrium. As before, I start by characterizing some conditions that a separating strategy must fulfill to be supported in equilibrium, and then show that a separating equilibrium exists.

The claims in lemmas 1 and 2 still hold in the continuous state model, and their proofs are similar to those presented in the analysis of the two state case, thus not warranting repetition. As a refresher, lemma 1 established that when the firm is in the lowest state 0, the compensation package of the executive is that of the symmetric information case: $C_0^{sep} = C_0^{SI}$. Lemma 2 established that the the executive cannot earn rents in any state: $Eu_{Ex-\theta}\left(C_\theta^{sep}, e^\star\right) = \bar{u}$ (that is, the lemma states that $IR_{Ex-\theta}$ binds).

The problem that each type $\theta > 0$ must solve is then:

$$\begin{aligned}
&\max_{C_\theta^{sep}\in\Omega_C} \theta + b_\theta \tfrac{\mu^2}{\psi} - \bar{u} - (1-\alpha)\, k_\theta - \tfrac{\gamma}{2}\left(b_\theta \sigma\right)^2 - \tfrac{\psi}{2}\left(b_\theta \tfrac{\mu}{\psi}\right)^2 \\
st: & \\
IC_{Bd-\theta'}^{\theta}: \quad & Q_{\theta'}\left(D_\theta^{sep}; s_{Bd}^{sep}, s_{Ex}^\star\right) \leq F_{\theta'}\left(D_\theta^{sep}\right) \; \forall \theta' \in [0,\theta)
\end{aligned} \tag{1.31}$$

In each state $\theta \in \Theta$, the board now designs the compensation package as to respect a "family" of incentive compatibility conditions - $\left\{IC_{Bd-\theta'}^{\theta}\right\}_{\theta'\in[0,\theta)}$ - that ensure that offering that package is not profitable in any inferior state.

As in the two state case, there are parametrizations of the model for which the first-best package, offered in the symmetric information case, is a feasible, and thus the optimal, solution to the problem in every state. Since we must have $b_\theta^{SI} = b_{\theta'}^{SI}$ for all $\theta, \theta' \in \Theta$, the channel of signaling is the wage.

**Proposition 5.** *Offering compensation package $C_\theta^{sep} = C_\theta^{SI}$ in every state $\theta \in \Theta$ is supported in a separating equilibrium if and only if the value of undisclosed compensation to the executive is such that:*

$$\alpha \leq b_\theta^{SI} = \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2}.$$

*Proof.* $C_\theta^{SI}$ solves the problem in expression (1.31) for every state $\theta$ if and only if $IC_{Bd-\theta'}^{\theta}$ holds when evaluated at $C_\theta^{SI}$ as given in proposition 1. Suppose then that, in every state $\theta \in \Theta$, the board is implementing the first-best package $C_\theta^{SI}$. When in an inferior state $\theta' < \theta$, the board has no incentive to deviate to $C_\theta^{SI}$ if $Q_{\theta'}\left(D_\theta^{SI}; s_{Bd}^{SI}, s_{Ex}^\star\right) \leq F_{\theta'}\left(D_\theta^{SI}\right)$, which simplifies down to $\alpha \leq b_\theta^{SI}$. □

If $\alpha < b_\theta^{SI}$, no incentive compatibility condition binds. If, however, we have that $\alpha > b_\theta^{SI}$, at least one element of the family of constraints $\left\{IC^{\theta}_{Bd-\theta'}\right\}_{\theta'\in[0,\theta)}$ must bind for every $\theta$. This is proved in lemma 5, which is similar in its proof and explanation to lemma 3.

**Lemma 5.** *Let $\alpha > \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$. Then, in every state $\theta \in \Theta \setminus \{0\}$, the board designs the compensation package so that, for at least one $\tilde{\theta}(\theta) \in [0,\theta)$, $IC^{\theta}_{Bd-\tilde{\theta}}$ binds.*

*Proof.* Suppose instead that, in a separating equilibrium, there is some state $\theta \in \Theta \setminus \{0\}$ such that $IC^{\theta}_{Bd-\theta'}$ does not bind for any $\theta' \in [0,\theta)$. Then, the board can decrease $b_\theta$ and increase $w_\theta$ as to keep $IR_{Ex-\theta}$ binding and still respect all $\left\{IC^{\theta}_{Bd-\theta'}\right\}_{\theta'\in[0,\theta)}$, thus maintaining separation (because of the intuitive criterion) and increasing its payoff. (The payoff increases because $b_0^{sep} = b_0^{SI}$ and because of the already asserted fact that the executive's stake must increase with the state in any separating equilibrium of the continuous state case.) This constitutes a contradiction of the assertion that this is an equilibrium, and so it follows that there must be at least one $\tilde{\theta}(\theta) \in [0,\theta)$ such that $IC^{\theta}_{Bd-\tilde{\theta}(\theta)}$ binds. □

The lemma simply asserts that there is at least one state in which the board is indifferent between deviating by offering package $D_\theta$ and staying on path. The lemma does not, however, assert that such state is unique. So let $\tilde{\Theta}(\theta)$ denote the set of states in which the board is indifferent between offering the compensation package prescribed in the separating strategy being played and deviating to $D_\theta$:

$$\tilde{\Theta}(\theta) \equiv \left\{\tilde{\theta} \in [0,\theta) : IC^{\theta}_{Bd-\tilde{\theta}}\, binds\right\}.$$

As in the two state case, it is possible to rewrite, by solving for $k_\theta$, any incentive compatibility constraint that holds as an equality and use the resulting equation to simplify the board's problem in state $\theta$. I do this for state $\tilde{\theta}(\theta) \in \tilde{\Theta}(\theta)$:

$$\begin{aligned} &\max_{b_\theta} \quad \tfrac{b_\theta}{\alpha}\theta + b_{\tilde{\theta}(\theta)}\tfrac{\mu^2}{\psi} - \bar{u} - k_{\tilde{\theta}(\theta)} - \tfrac{\gamma}{2}\left(b_{\tilde{\theta}(\theta)}\sigma\right)^2 - \tfrac{\psi}{2}\left(b_{\tilde{\theta}(\theta)}\tfrac{\mu}{\psi}\right)^2 \\ &st: \\ &IC^{\theta}_{Bd-\tilde{\theta}(\theta)}: \qquad k_\theta = k_{\tilde{\theta}(\theta)} + \tfrac{1}{1-\alpha}H \geq 0 \end{aligned} \tag{1.32}$$

with

$$H=\left(1-\frac{b_{\theta}}{\alpha}\right)\left[\theta-\tilde{\theta}\left(\theta\right)\right]+\left(b_{\theta}-b_{\tilde{\theta}(\theta)}\right)\frac{\mu^{2}}{\psi}-\frac{1}{2}\left(\gamma\sigma^{2}+\frac{\mu^{2}}{\psi}\right)\left[\left(b_{\theta}\right)^{2}-\left(b_{\tilde{\theta}(\theta)}\right)^{2}\right].$$

Again as in the two state case, the simplified objective function is strictly increasing in $b_{\bar{\theta}}$ and subject to the constraint that undisclosed compensation be non-negative. Also as in the two state case, the constraint binds in any separating equilibrium when $\alpha$ is large enough. Lemma (6) states this formally.

**Lemma 6.** *Let* $\alpha>\frac{\mu^{2}}{\mu^{2}+\psi\gamma\sigma^{2}}$. *Then, in any separating equilibrium, the board sets undisclosed compensation at zero in all states:* $k_{\theta}^{sep}=0$ *for all* $\theta\in\Theta$.

*Proof.* Same as that of lemma 4. □

It follows from lemma (6) that the executive's share $b_{\theta}^{sep}$ of the final output in state $\theta\in\Theta$ solves equation $k_{\tilde{\theta}(\theta)}^{sep}+\frac{1}{1-\alpha}H=0$ . In the two state case, the lemma asserting that no undisclosed compensation is paid in equilibrium was the final step before we could find $b_{\theta}^{sep}$ as it was known, by virtue of there being just two states, that $\tilde{\theta}\left(\bar{\theta}\right)=0$. In the continuous state case, it is still necessary to identify at least one state for which incentive compatibility binds for each $\theta\in\Theta$.

To that end, lets again consider the benefits and costs of deviating to a package offered in equilibrium in a superior state. The purpose is now to write benefits and costs in a way that helps shed light over each $\tilde{\theta}\left(\theta\right)\in\tilde{\Theta}\left(\theta\right)$.

The benefit from a deviation in state $\theta'$ to a package $D_{\theta}$ offered on the equilibrium path in state $\theta$ is given by

$$Q_{\theta'}\left(D_{\theta};s_{Bd},s_{Ex}^{\star}\right)=q_{1}^{\star}\left(D_{\theta};s_{Bd},s_{Ex}^{\star}\right)-q_{1}^{\star}\left(D_{\theta'};s_{Bd},s_{Ex}^{\star}\right).\tag{1.33}$$

By adding and subtracting $q_{1}^{\star}\left(D_{\theta''};s_{Bd},s_{Ex}^{\star}\right)$ to this expression for any $\theta''$ such that $\theta''<\theta'<\theta$, we can re-write $Q_{\theta'}\left(D_{\theta};s_{Bd},s_{Ex}^{\star}\right)$ as the difference between the benefits from a deviation in state $\theta''$ to the package offered in state $\theta$ and the benefits from a deviation in that same state $\theta''$ to the package offered in state $\theta'$:

$$Q_{\theta'}\left(D_{\theta}; s_{Bd}, s_{Ex}^{\star}\right) = Q_{\theta''}\left(D_{\theta}; s_{Bd}, s_{Ex}^{\star}\right) - Q_{\theta''}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right). \tag{1.34}$$

The cost from a deviation in state $\theta'$ to package $D_{\theta}$ is, as in the two state case,

$$F_{\theta'}\left(D_{\theta}\right) \equiv \max\left\{0, \frac{1}{\alpha}\left[\bar{u} - Eu_{Ex-\theta'}\left(D_{\theta}, 0, e_{C}^{\star}\right)\right]\right\}$$

which, using the facts that $\theta'$ is smaller than $\theta$ and that, in equilibrium, $IR_{Ex-\theta}$ binds - and so, $Eu_{Ex-\theta}\left(C_{\theta}, e_{C}^{\star}\right) = \bar{u}$ - and $k_{\theta} = 0$, boils down to

$$F_{\theta'}\left(D_{\theta}\right) = \frac{b_{\theta}}{\alpha}\left(\theta - \theta'\right). \tag{1.35}$$

Armed with expressions (1.34) and (1.35), we are in a better position to find $\tilde{\theta}(\theta)$. In fact, lemma 7 asserts that $0 \in \tilde{\Theta}(\theta)$ for all $\theta > 0$.

**Lemma 7.** *Let $\alpha > \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2}$. Then, for all $\theta \in \Theta \setminus \{0\}$, $IC_{Bd-0}^{\theta}$ binds.*

*Proof.* Suppose instead that there is at least one state $\theta'$ such that $IC_{Bd-0}^{\theta}$ does not bind. That is, the benefits from a deviation in state 0 to the package offered in state $\theta'$ are strictly lower than the costs:

$$IC_{Bd-0}^{\theta'}: Q_{0}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right) < \frac{b_{\theta'}}{\alpha}\theta' \tag{1.36}$$

while for every $\tilde{\theta} \in \tilde{\Theta}(\theta')$ (which is non-empty by lemma 7) the following is true:

$$IC_{Bd-\tilde{\theta}}^{\theta'}: Q_{\tilde{\theta}}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right) = \frac{b_{\theta'}}{\alpha}\left(\theta' - \tilde{\theta}\right). \tag{1.37}$$

Using expression (1.34) I re-write $Q_{\tilde{\theta}}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right)$ in terms of the benefits from deviation in some $\hat{\theta} < \underline{\tilde{\theta}} \equiv \inf \tilde{\Theta}(\theta')$ to $\theta'$ and any $\tilde{\theta} \in \tilde{\Theta}(\theta')$:

$$Q_{\tilde{\theta}}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right) = Q_{\hat{\theta}}\left(D_{\theta'}; s_{Bd}, s_{Ex}^{\star}\right) - Q_{\hat{\theta}}\left(D_{\tilde{\theta}}; s_{Bd}, s_{Ex}^{\star}\right) \tag{1.38}$$

Note that $\hat{\theta}$ is well defined because $0 \notin \tilde{\Theta}(\theta')$. Combining expressions (1.37), and (1.38), it follows that:

$$Q_{\hat{\theta}}\left(D_{\theta'}; s_{Bd}, s^{\star}_{Ex}\right) - Q_{\hat{\theta}}\left(D_{\tilde{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) = \frac{b_{\theta'}}{\alpha}\left(\theta' - \tilde{\theta}\right) \tag{1.39}$$

By definition of $\hat{\theta}$, it must also be true that:

$$Q_{\hat{\theta}}\left(D_{\theta'}; s_{Bd}, s^{\star}_{Ex}\right) < \frac{b_{\theta'}}{\alpha}\left(\theta' - \hat{\theta}\right) \tag{1.40}$$

Combining (1.39) and (1.40), it follows that:

$$Q_{\hat{\theta}}\left(D_{\tilde{\theta}}; s_{Bd}, s^{\star}_{Ex}\right) \quad < \quad \frac{b_{\theta'}}{\alpha}\left(\tilde{\theta} - \hat{\theta}\right)$$

for all $\hat{\theta} < \underline{\tilde{\theta}}$. This implies that there is no state for which $IC^{\tilde{\theta}}_{\theta}$ binds, a contradiction to lemma 5. □

It follows from the lemma that, in every state $\theta \in (0, \theta_M]$, the package offered in each state $\theta$ in a separating equilibrium of the continuous state case is exactly the package that would be offered in state $\bar{\theta}$ in the two state case if $\bar{\theta}$ were equal to $\theta$.

Proposition 6 summarizes the conclusions regarding the continuous state model.

**Proposition 6.** *If $\alpha > \frac{\mu^2}{\mu^2 + \psi\gamma\sigma^2}$, one, and only one, separating strategy profile is supported in equilibrium. In such equilibrium:*

1. *the board's separating strategy is $s^{sep}_{Bd} = \left(C^{sep}_{\theta}\right)_{\theta\in\Theta}$ such that $C^{sep}_0 = C^{SI}_0$ and $C^{sep}_{\theta} = C^{sep}_{\bar{\theta}}$, where $C^{sep}_{\bar{\theta}}$ is the package offered to the executive in the good state in the two state model if that good state is equal to $\theta$.*

2. *the executive follows strategy $s^{\star}_{Ex} = [\delta^{\star}_{\theta C}, e^{\star}_{C}]_{\theta\in\Theta, C\in\Omega_C}$.*

3. *the prospective buyers have beliefs*

   *(a) $\rho\left(\theta|D; s_{Bd}, s^{\star}_{Ex}\right)$ given by Bayes' rule on the equilibrium path of strategy $s_{Bd} \in \Omega^{\#\Theta}_{C}$;*

   *(b) $\int_{\theta'}^{\theta_M} \rho\left(\theta|D; s_{Bd}, s^{\star}_{Ex}\right) d\theta = 1$ if*

$$Eu_{Ex-\theta''}\left(D, Q\left(D; s_{Bd}, s^{\star}_{Ex}\right), e^{\star}_{C}\right) < \bar{u} \leq Eu_{Ex-\theta'}\left(D, 0, e^{\star}_{C}\right)$$

for all $\theta'' < \theta'$ *off the equilibrium path of strategy* $s_{Bd} \in \Omega_C^{\#\Theta}$*;*

4. *the price of the company's stock is* $q_1^\star(D; s)$ *for all* $D$ *and* $s$*.*

*Proof.* The only assertion that has not been proved is that contained in clause (b) of the statement on beliefs. The clause follows directly from the application of the intuitive criterion. If, when the buyers believe that board is following strategy $s_{Bd}$, the board can profitably induce the executive into accepting an observable deviation package $D$ (such that $\int_{\theta'}^{\bar{\theta}} \rho(\theta | D; s_{Bd}, s_{Ex}^\star)\, d\theta = 1$) when in state $\theta'$ or better but not when in any inferior state, then the intuitive criterion implies that the buyers cannot believe that there is any probability that the state of the company is worse than $\theta'$. □

The model has no equilibria that include pooling by the board in some states by the same reason employed in the proof of proposition 4: because beliefs are assumed to satisfy the intuitive criterion, when in one of the highest states in which it is supposed to offer a pooling package, the board can always find a profitable deviation.

As was true about the analysis of the two state model, the analysis of the continuous state model establishes that only one strategy by the board is supported in equilibrium, and that is a separating strategy. The next section computes and discusses the comparative statics of this equilibrium.

### 1.6 Discussion and comparative statics

The continuous state space model underpins this section, which discusses how the parameters of the model - measuring risk, the executive's attitudes towards risk and effort, the value of her effort to the company, and the value of undisclosed compensation - affect the size of the distortion to compensations packages created by signaling, and affect economic efficiency.

To start the analysis, it is convenient to recall the executive's stake in the company. By proposition 6, in state $\theta \in \Theta$, that stake is given by the expression

$$b_\theta^{sep} = \begin{cases} b_0^{sep} + \left(\frac{\psi}{\psi\gamma\sigma^2 + \mu^2}\right)\left[-\frac{\theta}{\alpha} + \sqrt{\left(\frac{\theta}{\alpha}\right)^2 + 2\left(\frac{\psi\gamma\sigma^2 + \mu^2\left(1 - \frac{1}{\alpha}\right)}{\psi}\right)\theta}\,\right] & if\, \alpha > b_0^{sep} \\ b_0^{sep} & if\, \alpha \le b_0^{sep} \end{cases} \tag{1.41}$$

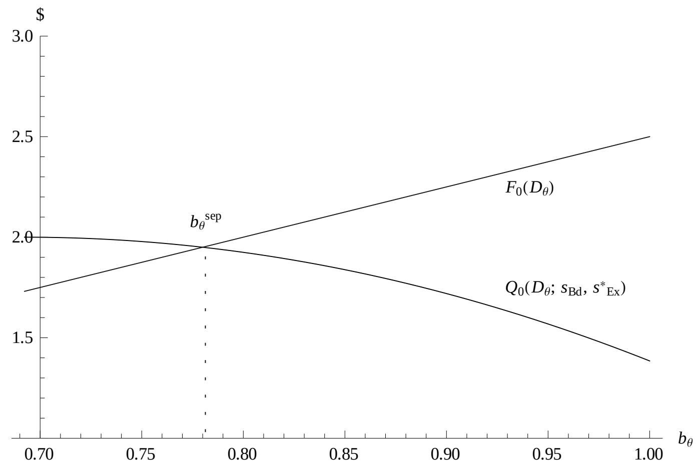



Figure 1.2: Graphical determination of the executive's equilibrium stake in the company.

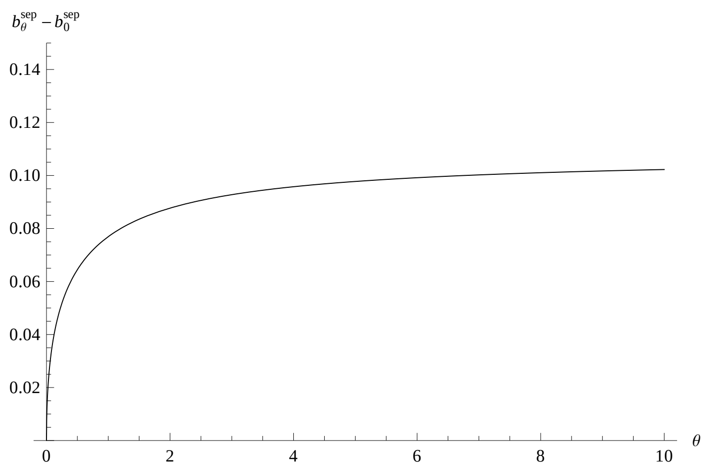



Figure 1.3: Relation between $\theta$ and the distortion to the executive's stake $b_\theta^{sep} - b_0^{sep}$

where $b_0^{sep} = \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$. Recall that the equilibrium stake $b_\theta^{sep}$ was found by equating the benefits of a deviation to its costs. (Figure illustrates this equilibrium determination for a baseline parametrization of $\theta = 2$, $\mu = 3$, $\psi = 1$, $\gamma = 1$, $\sigma^2 = 4$, $\alpha = 0.8$, which also underlies the other figures in this section. Note that the origin for the x-coordinate is $b_0^{sep}$.)

Since stake $b_0^{sep}$ optimally balances the executive's exposure to risk with incentive provision, the more $b_\theta^{sep}$ deviates from it, the larger the distortion introduced by signaling will be. This distortion to performance-based pay, given by $b_\theta^{sep} - b_0^{sep}$, can be easily computed from expression (1.41). For large $\alpha$ ($\alpha > b_0^{sep}$), the executive's stake $b_\theta^{sep}$ increases with $\theta$ and so does the distortion created by signaling. This is illustrated in figure 1.3.

Since the executive never earns rents above $\bar{u}$ and competition in the stock market prevents the buyers from keeping any surplus, changes in the parameters affect welfare only through their impact upon the utility of the board. In state $\theta$, social welfare is given by

$$WF_\theta = q_1^\star \left(D_\theta^{sep}; s^{sep}\right) + \bar{u} = \frac{b_\theta^{sep}}{\alpha}\theta + b_0^{sep}\frac{\mu^2}{\psi} - \frac{\gamma}{2}\left(b_0^{sep}\sigma\right)^2 - \frac{\psi}{2}\left(b_0^{sep}\frac{\mu}{\psi}\right)^2. \tag{1.42}$$

The welfare loss relative to the case with symmetric information is then a function of the distortion to the executive's stake in the company:

$$L_\theta \equiv WF_\theta - WF_\theta^{SI} = \frac{\theta}{\alpha}\left(b_\theta^{sep} - b_0^{sep}\right). \tag{1.43}$$

The following subsections contain the analysis of the comparative statics induced by each parameter. This analysis is underpinned by the derivatives of the distortion to compensation with respect to the parameters. Given their size, the derivatives are omitted. Since some parameters measure related characteristics, some sections contain more than one parameter.

**Informational Asymmetries - The Probability Density $p_\theta$**

The density function $p_\theta$ has a natural interpretation as a measure of the importance of informational asymmetries between the insiders and outsiders of the company. The larger the cumulative probability is when assessed at a state close to zero, the less important the informational asymmetries are.

The executive's stake in the company is not affected by $p_\theta$ in any state. What $p_\theta$ does affect, if $\alpha > b_0^{sep}$, is the average stake in the company's profit handed out to the executive and, consequently, her average total income, both of which are larger when large states are likelier[15]. The model thus predicts that, among companies that use executive compensation as a signaling device, those in which asymmetries are unimportant should give out smaller stakes to their executives and less total pay.

There is some empirical support for the prediction that performance-based pay is more

[15]Because the executive is risk averse, her expected income must increase when her stake in the company increases to ensure that her expected utility remains the same (at $\bar{u}$).

important when asymmetric information is likely to be present. This support is found when two different proxies for asymmetric information are used. First, Bizjak et al. (1993) find that the ratio of salary plus end-of-year bonus - which, in the context of this paper's model, might be seen as a proxy for the wage - to total incentives is decreasing in the the growth opportunities of a company, while Smith and Watts (1992) show that growth firms are likelier to implement equity based plans[16].

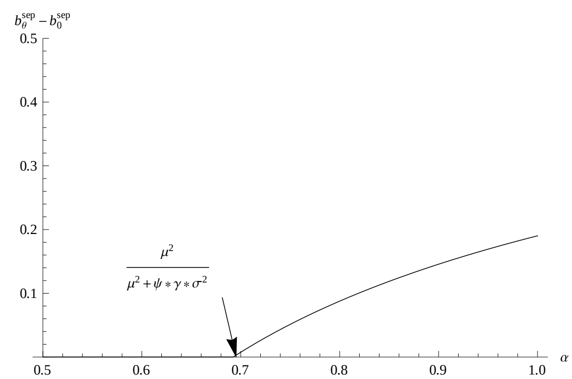
Second, Smith and Watts (1992) and Bizjak et al. (1993) also find that regulated firms are less likely to implement equity based plans. Using two deregulation events as the basis for their work, Cuñat and Guadalupe (2009) find that "deregulations substantially changed the level and structure of compensation: the variable components of pay increased along with performance-pay sensitivities and, at the same time, the fixed component of pay fell."

**The Value of Undisclosed Compensation $\alpha$**

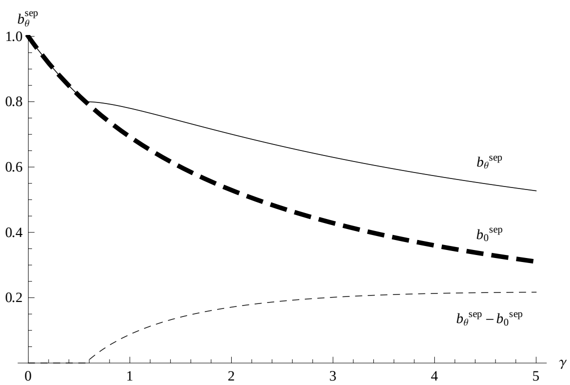
As previously discussed, when $\alpha$ is small the board is able to signal the state of the company without inducing any distortions. When $\alpha$ crosses the threshold $b_{\theta}^{SI} = b_0^{sep} = \frac{\mu^2}{\mu^2+\psi\gamma\sigma^2}$, however, the executive's stake becomes distorted. This distortion, as well as the welfare loss caused by it, increase with the size of $\alpha$, a phenomenon illustrated in figure 1.4a. The intuition behind this dynamic is the following: an increase in $\alpha$ makes undisclosed compensation more valuable to the executive, which reduces the cost that the board must support, in every state but the best, to induce the executive into accepting a contract offered in better states. Then, separation can only be maintained if, in every state but the worst, the executive receives a sufficiently larger stake in the company that again raises the cost of a deviation in any of the states beneath it unaffordable. Graphically, the change in $\alpha$ amounts to a shift to the right of the cost of deviation function $F_0\left(D_{\theta}\right)$ represented in figure 1.2.

As the monetary value attached by the executive to a dollar in undisclosed compensation, $\alpha$ is affected by the opportunities available to the board to offer undisclosed compensation.

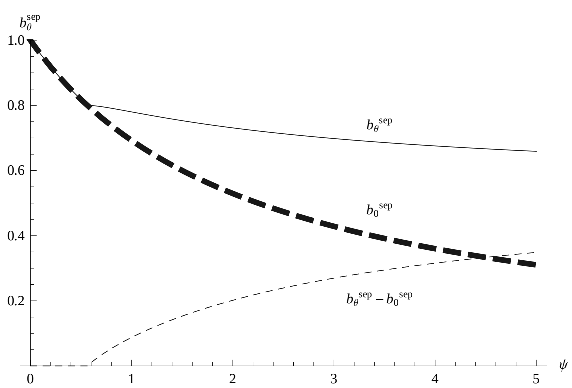
[16]Bizjak et al. (1993) base the hypothesis that growth opportunities measure informational asymmetries on two observations: first, growth is usually derived from new products, about which the information gap between insiders and outsiders is likely to be at its largest; second, growth opportunities can take many years to develop which implies that the information gap takes a long time to be resolved.



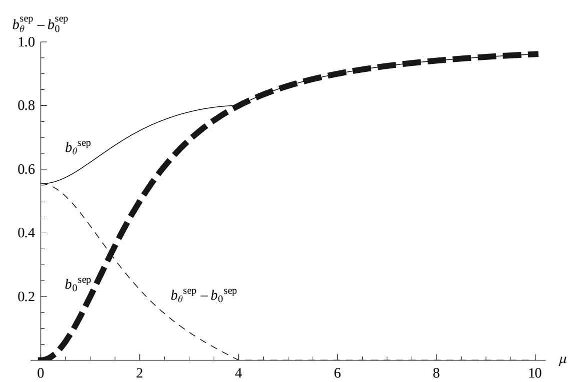

(a) The value of undisclosed compensation $\alpha$.




(b) Risk parameters $\gamma$ and $\sigma^2$ (exemplified for $\gamma$)




(c) The executive's aversion to effort $\psi$.




(d) The value of the executive's effort $\mu$

Figure 1.4: Comparative statics of the size of the distortion to the executive's stake.

These opportunities are likely to be determined by the securities exchange regulations that the company is subject to - in particular, by the scope of inside trading and compensation disclosure rules that apply to the company and by the resources available to the regulator that enforces those rules - as well as by idiosyncratic features of the company and of its large investors and board members. For example, the pool of pet projects that an executive can adopt is likely to be larger, and hence more valuable, when the company has its headquarters in a large city and operates in large markets; and better connected and more powerful investors and board members can exert more influence to favor the executive's friends and family members in their professional endeavors.

**Risk $\sigma^2$ and risk aversion $\gamma$**

As figure 1.4b illustrates, the executive's stake in the company in the lowest state ($b_0^{sep}$) and in any state $\theta$ ($b_\theta^{sep}$) decreases monotonically with risk $\sigma^2$ and with her aversion to risk $\gamma$, as is standard in the executive compensation literature. For low values of either parameter, the stake is the same in all states and therefore decreases at the same rate in all states; eventually, as risk or risk-aversion keep increasing, the first-best stake can no longer be part of a compensation package that credibly signals the company's state, and the executive's stake in the company's profits is distorted. The distortion, measured by $b_\theta^{sep} - b_0^{sep}$, then increases with $\gamma$ and $\sigma^2$, as does the welfare loss that the distortion causes. Two effects account for this result. The first degree effect is that an increase in risk or risk aversion prompts the company to reduce the executive's stake across states to protect her from risk. The second degree effect is that this lower stake reduces the costs of deviating in lower states, which in turn forces the board to increase the distortion in its provision of incentives to the executive.

**The marginal value of effort $\mu$ and the degree of effort aversion $\psi$**

An increase in the marginal value of the executive's effort decreases, and eventually eliminates, the distortions caused by signaling (see figure 1.4d). Two effects account for this result. The first degree effect is that when the executive's effort is more valuable, the company finds it optimal to induce more of it in all states by giving the executive a higher stake in the company. The second degree effect is that these higher stakes across states raise the costs of deviating in lower states, which in turn allows the board to reduce the distortion in its provision of incentives to the executive.

As a measure of the executive's marginal value, $\mu$ is likely to be larger in companies in which executives have a more entrepreneurial and less managerial role.

The executive's stake in the company in the lowest state ($b_0^{sep}$) and in any state $\theta$ ($b_\theta^{sep}$) decreases monotonically with her aversion to effort $\psi$; as for the distortion in the executive's stake, after it appears it increases with $\psi$ (see figure 1.4c). The mechanics are similar to those of the risk parameters, so I will not repeat them.

Parameter $\psi$ might be interpreted as a measure of the agency problem faced by the company. In light of this interpretation, the comparative statics of $\psi$ predict that companies where the board is better able to monitor their executives endure fewer distortions in the setting of executive compensation.

## 1.7 Conclusion

This paper shows how, under some conditions, boards can use disclosed executive compensation to credibly convey information to outsiders, and then characterizes the distortions that this use introduces in compensation practices. This conclusion outlines the empirical and theoretical projects that I intend to pursue as a continuation to the research agenda here initiated.

The theory outlined in this paper would benefit from an empirical study that directly addressed its prediction that when boards have a short-term bias, executive compensation practices may be distorted by signaling. To this end, I plan to use stockholder turnover in public companies as a proxy for the temporal orientation of the board and then observe how stock prices react to the disclosure of new compensation packages. Since companies without a short-term bias do not have a motive to use compensation as a signal and therefore are not (intentionally) attempting to transmit information, their stock prices should exhibit a smaller variance after disclosure than those of companies with a short-term bias. The decision to observe the variance of stock prices after disclosure instead of average power of incentives is due to the fact that companies showing less stockholder turnover may also suffer from fewer agency problems, and therefore might not need as much performance-based pay to induce effort provision.

I also intend to apply and extend the signaling approach to executive compensation proposed in this paper to address equity grants to non-executive workers who cannot affect their company's stock price. The underlying idea is that, if insiders have imperfect signals about future performance, and those signals are not perfectly correlated, offering variable pay to several agents may increase the amount of information available for the statistical inference of a company's state.

# CHAPTER 2: A NETWORK EFFECTS THEORY OF THE FINANCIAL AUDITS MARKET

## 2.1 Introduction

Recent studies indicate that concentration in national markets for financial audits varies significantly across market segments. When segmentation is performed according to company revenue, the data indicate that concentration is higher in segments containing companies with high revenue: figure 2.1 illustrates how the "Big 4" audit firms audited 98% of the companies listed in the U.S. with more than $1 billion dollars in revenue in 2006 but only 22% of those with less than $100 million. When segmentation is performed according to the type of ownership, concentration is higher in the public company segment (Langli and Svanström, 2013 and Velte and Stiglbauer, 2012). The purpose of this paper is to provide a theoretical explanation for these market outcomes.

Existing theories explain audit market outcomes as the result of heterogeneity of audit firms in terms of quality. They argue that the Big 4 auditors perform better, but costlier, audits and thus audit only the clients that value high quality auditing enough. This paper proposes a new economic theory of the organization of financial audit markets whereby the correlation between the size or ownership type of a company and the likelihood that it is audited by a large audit firm can arise even though the expected - or perceived - quality of every audit firm is the same. First, the paper provides a micro-founded model of the demand for financial audits that explains concentration in audit markets. Then, it provides a simple model of the supply of financial audits based on diseconomies of scale. Finally, the two models are brought together to explain the variability of concentration across segments.

The model of audit demand provides the underlying force behind concentration. It proposes that concentration appears in audit markets because it generates a network externality. Investors - whose preferences drive their companies' procurement of an auditor - care about the quality of the audit firms that audit their companies, as this quality determines the accuracy

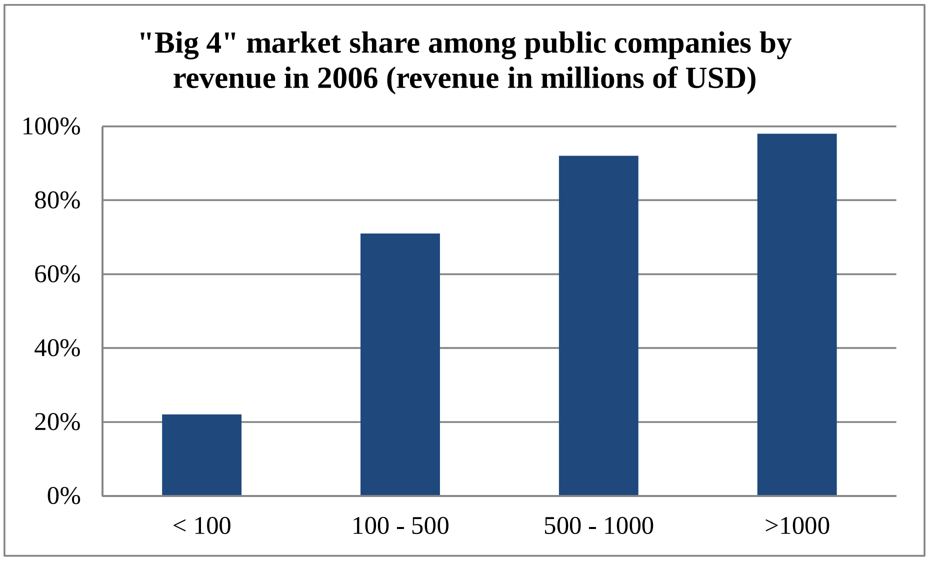



Figure 2.1: "Big 4" market share among public companies by 2006 revenue (revenue in millions of USD). Source: (U.S. Government Accountability Office, 2008)

of the financial statements of those companies. Quality is random and unobservable but, with time, information about it appears in the form of publicly observable audit errors, thus allowing investors to update their beliefs about their audit firm's quality. Audit firms with large client portfolios allow for more observations and, therefore, for more precise beliefs, which improves the decision making of investors.

The model of audit supply creates a limit to concentration and, therefore, helps answer the question of why a financial audit market contains more than one audit firm. The main tenet of the model is that audit firms face diseconomies of scale, caused by rising managerial and bureaucratic costs in the spirit of Coase (1937). The underlying idea is that audit firms see important agency conflicts between the teams conducting field work and the managerial ranks that increase in importance as the firms expand.

The paper builds on the idea of diseconomies of scale and on the micro-founded portrayal of audit firms as economic networks that give their clients positive network effects, to develop two alternative explanations for the variability of market concentration across segments. The two explanations rely on two different parametrizations of the paper's baseline model. The first explanation is driven by heterogeneity in the informational asymmetries within each company. It proposes that when companies differ in terms of how much their investors value auditing, those where investors value auditing the most - that is, those in which the informational asymmetries between management and investors are more severe - gain more from being audited by big audit firms. I show that, in any efficient equilibrium of the heterogeneous informational asymmetries case, companies suffering from more severe informational asymmetries "cluster" around big auditors, while companies with milder asymmetries do the same, but to a less significant extent, around small auditors.

The second explanation is driven by heterogeneity in the "public visibility" of each company, with visibility defined as the probability that a company's audit errors become public. The hypothesis behind the public visibility explanation is that companies have idiosyncratic properties - like the industry they operate in, the brands they own, their corporate culture, or their aforementioned size and legal status - that affect the probability that accounting misstatements, and hence, audit mistakes, will be publicly observed. When companies differ in terms of the likelihood that errors in their audits become public, those that are more visible are more valuable network users from the perspective of the other users than those that are less visible, as their audit outcomes provide more informative observations of their auditors' quality. I show that, in any efficient equilibrium of the heterogeneous visibility case, high visibility clients appoint big auditors.

In both parametrizations of the model, large auditors are able to charge higher audit fees. Also in both parametrizations, auditors are able to earn positive economic profits without creating an incentive for entry in the market.

Both the public visibility and informational asymmetries explanations fit the stylized facts about market concentration in different market segments: informational asymmetries are, on average, more significant in public or large companies than in private or small companies; and big and/or public companies are typically more visible than their small and/or private counterparts.

The network effects theory of audit markets developed here thus seems capable to account for market concentration patterns. But at what level (global, continental, national, regional) of the audit market does the theory work? The paper's explanation for concentration is that clients prefer audit firms with large client portfolios. This preference is due to the fact that an audit firm's audits are uniform in terms of quality, and therefore observing how the audit firm performs in its other audits allows the client to learn about its quality. Depending upon the purpose of the analysis, an audit firm can be the set of legal entities operating under a global "brand" name, a national legal entity, or a local office with regional scope. Local offices are almost fully autonomous entities led by partners who monitor the staff conducting the field work and make all the decisions with material impact in audit reports. While a complete audit may draw upon work from several offices in different geographic locations, one single office typically coordinates this work and makes most of the material contributions, and therefore has a dominant influence over the quality of the audit. At the same time, local offices abide by nation-wide company standards. So, at which level can significant quality uniformity be guaranteed: the national level or the office level?

If company wide standards - and the extent to which they are enforced - play a significant role in audit quality, uniformity might be guaranteed at the national level. If the standards are relatively unimportant, the paper's theory applies to the organization of local geographic markets for financial audits. Either way, while the theory explains the number of audit firms that dominate a local or national market and how such market is segmented, it does not account for the stylized fact that it is the local offices of a few global audit brands that dominate most markets. A possible explanation for this fact is that audit brands serve as a coordination device for clients pursuing network effects.

In summary, this paper's theory of the audit market is as follows. Auditors work as networks that bestow on each client a positive benefit that increases with the number of its other clients. In one version, this benefit varies according to the visibility of those clients. In another version, clients are homogenous in terms of visibility, but heterogeneous in terms of their taste for network effects. Outcomes in local or national markets are determined competitively in light of these network effects and of diseconomies of scale. Global aggregates reflect these outcomes and the fact that brands act as a coordination device.

The rest of the paper unfolds as follows. The next section addresses the related literature. Section 2.3 develops a model of audit demand, while section 2.4 presents a simple model of audit supply, a model of audit market interaction, and the equilibrium analysis of that market.

The conclusion proposes paths for future research.

## 2.2 Discussion of the literature

Existing literature argues that big auditors owe their size to the superior quality of their audits - see, for example, Lennox (1999) or Khurana and Raman (2004). This consensus around the cause of size breaks down when the subject of study is the motive for the large auditors' higher quality. There are two popular explanations: reputation concerns (DeAngelo, 1981) and deep pockets (Dye, 1993). These two theories of quality differentiation assert that the alleged superior quality of large audit firms is in fact a by-product of their size, in the sense that size creates stronger incentives to the provision of quality. This superior quality then reinforces the size of the large audit firms by making their retention even more beneficial to clients. These accounts, however, treat auditor size differences as exogenously given and do not address the motives that made big auditors big in the first place, something that this paper attempts to address.

Perceived auditor quality is likely to be a relevant parameter when a company searches for an auditor. Because of that, audit firms competing for the same client have an incentive to provide the quality that gives that client the most value. In reality, the quality of a financial auditor depends upon two factors: the quality of the field work of collecting and analyzing data; and the quality of the auditing firm's internal monitoring. Both factors are at least partially explained by the features and decisions of the people that carry them out: honesty, technical skill, and effort. An auditing firm's staff, from field work operative to partners, is not a firm specific asset and can be hired in an open labor market in which good talent, while scarce, is not rarefied. It follows that *ex-ante* quality differences alone should not explain why visible clients tend to choose one of the Big 4 auditors. Otherwise, smaller auditors could replicate that quality and capture a larger share of those clients.

This is not, of course, to say that audit firms have the same realized quality. A talented, apparently honest partner may turn out to be corrupt; an apparently competent one may turn out to be incompetent; and internal controls may be designed with the best intentions but contain flaws. Hiring a large auditor gives that client the knowledge that, in the future, she will have a more accurate perspective of that auditor's *true* quality and a better idea of how much weight to put on its reports. This gives clients an incentive to select auditors with more, and more visible, clients.

Having completed the discussion of the determinants of auditor size, I now turn to research containing evidence related to this paper's claims. Chaney and Philipich (2002) and Weber et al. (2008) detail how the stock prices of companies audited by audit firms involved in two different accounting scandals suffer abnormal decreases. Their findings provide support to the hypothesis that the outcomes of other companies' audits contain information about the quality of the audits of companies using the same auditor and inform stockholders in their decision making[1].

The literature also presents corroboration for the ideas that companies with higher agency costs tend to hire larger auditors (Johnson and Lys, 1990) - which is compatible with this paper's informational asymmetries explanation - and that large auditors charge higher audit fees (Chan et al., 1993) - something that the paper also predicts.

### 2.3 A model of audit demand with network effects

This section provides a micro-founded model of audit demand whereby the benefit that an audit client derives from the appointment of a particular auditor depends upon the number of clients in that auditor's portfolio and upon idiosyncratic characteristics of those clients. This model will be used in section 2.4 in conjunction with a model of audit supply to characterize equilibria in the financial audit market.

#### 2.3.1 Model description

The fundamental premise of the model of audit demand is that companies harbor a conflict of interests between managers and investors regarding the communication of financial information: investors want to base their investment decisions on correct information, while managers have

[1] Chaney and Philipich (2002) for example, look at the Enron scandal and report that "(o)n the three days following Andersen's admission that a significant number of documents had been shredded, we find that Andersen's other clients experienced a statistically negative market reaction, suggesting that investors downgraded the quality of the audits performed by Andersen. We also find that audits performed by Andersen's Houston office suffered a more severe decline in abnormal returns on this date."

a vested interest in reporting positive information. Auditing is valuable to investors because it serves as a verification mechanism of the financial reports produced by managers.

The model addresses a market for financial audits in which there are $I$ potential client companies, indexed by $i \in \{1, ..., I\}$, each with one manager and one investor. Each company has one asset in place that needs financing from the investor to reach maturity and yield a return.

If financed, company $i$'s asset yields a return:

$$\pi_i = v_i + L_i$$

where $L_i$ is the certain component of the return and $v_i$ is the uncertain component which may take two values: $v_i \in \{0, \theta_i\}$, with $\theta_i > 0$. Manager and investor both know the value of $L_i$; however, they are asymmetrically informed about $v_i$: the manager knows its value, the investor does not. The investor is the claimant of the return if she does indeed finance the company, and she has an alternative investment opportunity that yields a net return of $\bar{\pi}$. I assume that this alternative opportunity is such that it can only be profitable to finance company $i$ if $v_i = \theta_i$. This is codified in assumption A1:

$$\text{A1: } L_i < \bar{\pi} < \theta_i + L_i.$$

It follows that for the investor to decide to finance the company, she must attach a sufficiently high probability to the value of $v_i$ being $\theta_i$. What follows is the description of how this paper models the initial situation of asymmetric information and of the channels through which auditing helps the investor acquire information with which to decide whether to finance the company. Events unfold over four stages, numbered 1 through 4. In the first two, the investor acquires information; in the third, she makes a decision; and in the fourth, she claims her returns.

**Stage 1 - asymmetric information and information acquisition by the investor: financial reporting and auditing**

The value of the uncertain return is randomly determined in the beginning of stage 1. With probability $q$, $v_i$ is equal to 0, and with probability $1-q$ it is equal to $\theta_i$. The manager observes $v_i$, while the investor has beliefs given by $1-q$ and $q$.

After learning $v_i$, the manager must prepare a financial statement about the value of asset; this statement can be interpreted as a balance sheet. For simplicity of notation, and since the only component of the asset's return over which there is asymmetric information is the uncertain one, the financial statement concerns only $v_i$ and is denoted as $V_i$. I assume that the manager has a vested interest in obtaining financing and therefore always reports $V_i = \theta_i$[2]. The manager is therefore not a strategic agent in this model, and his introduction here merely serves a narrative purpose.

On its own, the financial statement prepared by the manager is uninformative. Now, suppose that company $i$ previously retained an auditor $a(i)$. This auditor examines the financial statement of the company and issues an audit report to the investor expressing its *truthful* opinion about whether $V_i$ is correct or not. In effect, then, the audit report is the auditor's opinion about the value of the uncertain component of the company's asset. I denote the audit report as $\hat{v}_i$ and let it take a value in the set $\{0, \theta_i\}$. The following matrix describes the probability $\Pr[\hat{v}_i|v_i]$ of the auditor's report being $\hat{v}_i$ conditional on the true value of the uncertain component of the return, $v_i$, under the assumption that the auditor always reports its true opinion:

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[2]The manager's motivations are unmodeled here but may be seen as being due to be career concerns (admitting that the asset has a low return may reflect poorly on his managerial ability), job security considerations (he needs the company's asset to be funded to keep his job), or empire building ambitions. There is an implicit assumption that if it is later on determined that the manager's financial reporting was incorrect, he is either shielded from negative consequences (say, because it cannot be proved that the manager purposely "cooked" the books), or those consequences are small enough. In reality, accounting decisions often involve making judgment calls about how to recognize assets or revenue with uncertain value; when this uncertainty is present, there is an interval of valuations that might be plausible to outsiders, giving management the opportunity to report a value in the frontier of that interval. This kind of leway is precisely the reason why auditors play an important role.

We can interpret the manager's report of $V_i = \theta_i$ as standing at the frontier of plausibility (before audit).

| | | | |
|---|---|---|---|
| | | $v_i$ | |
| | | $\theta_i$ | $0$ |
| $\hat{v}_i$ | $\theta_i$ | $1$ | $1-d_{a(i)}$ |
| | $0$ | $0$ | $d_{a(i)}$ |

These probabilities reflect the auditor's technical ability to find an accounting misstatement (which is defined as occurring when $v_i \neq V_i$, and that, given my assumption that $V_i = \theta_i$ for all $v_i \in \{0, \theta_i\}$, occurs whenever $v_i = 0$). While the auditor never forms an incorrect opinion about $v_i$ when $v_i = \theta_i$, it only detects misstatements with probability $d_{a(i)}$.

**Stage 2 - additional information acquisition: private observation of audit errors and public observation of audit errors**

When $d_{a(i)}$ is smaller than 1, the auditor is susceptible to committing an error. When the auditor does commit an error in stage 1, the investor detects it with probability $d$ in stage 2[3]. Notationally, error observation is represented by the variable $e_i$, which takes value $e$ (error is observed) or value $\emptyset$ (no error is observed) and we have that $d \equiv \Pr\left[e_i = \varepsilon | \hat{v}_i = \theta_i, v_i = 0\right]$. Error detection leads the investor to the conclusion that the uncertain return is equal to 0.

Audit errors are initially private. With probability $\sigma_i$, however, an audit error becomes public information visible by all companies in the market. Notationally, public error observation is represented by the variable $e_i^p$, which may either take value $\varepsilon^p$ (error is observed by the public) or value $\emptyset$ (no error is observed). The probability $\sigma_i$ is a measure of company $i$'s public visibility. The $I$ companies in the model differ across this measure: a number $\bar{I} \leq I$ of them have "high visibility" $\bar{\sigma} > 0$, while the remaining $I - \bar{I}$ have "low visibility" 0[4].

It follows from the description above that the public revelation of an error in company $i$'s audit report does not improve the information available to that company's investor. Why, then, does the public disclosure of errors matter? The answer lays in the fact that the investor $i$ does not know her auditor's ability to detect misstatements. Instead, each investor enter the

[3]Investors may detect an error through their own research efforts or through the work of financial analysts, consultants, or additional auditors.

[4]The extreme assumption that a low visibility company's parameter $\sigma_i$ is equal to zero instead of a positive number smaller than $\bar{\sigma}$ makes the analysis simpler without impacting the conclusions of the analysis.

game with the belief that $d_{a(i)}$ is uniformly distributed over $[0,1]$:

$$d_{a(i)} \sim U\left[0,1\right].$$

It follows that, from the investor's stage 1 perspective, the expected quality of each auditor is $E_1\left(d_{a(i)}\right)=\frac{1}{2}$ and the variance of that quality is $var_1\left(d_{a(i)}\right)=\frac{1}{12}$. More importantly, it also follows that, from this *ex-ante* perspective, auditors are similar[5].

We thus have that, because the investor does not know her auditor's quality, the observation of the auditor's errors (or lack thereof) in other companies - if the auditor does indeed have more clients - gives her more information with which update her beliefs about that quality. We will see that, under some conditions, these beliefs affect the investor's decision making.

**Stages 3 and 4 - the investor's decision and return realization**

In stage 3 each investor must decide whether to finance her company or to pursue the alternative investment opportunity that yields an expected return of $\bar{\pi}$. The decision of company $i$'s investor is denoted by $y_i$, which is equal to $i$ if the investor decides to finance company $i$, and to $\bar{y}$ if she selects the alternative opportunity.

The investor selects $y_i$ after observing the financial report $V_i$ (which is completely uninformative, as it is always equal to $\theta_i$), the audit report $\hat{v}_i$, the realization of the private audit error (or lack thereof) $e_i \in \{\emptyset, \varepsilon\}$, and the realization of public errors (or lack thereof) for each company in the market, $\left\{e_j^p\right\}_{j=1,\ldots,I} \in \{\emptyset, \varepsilon^p\}^I$. The only information that matters to her, however, is the audit report, the realization of the private audit error, and the realization of public errors committed by her auditor, which I denote as $\mathcal{P}_{a(i)} \equiv \left\{e_j^p\right\}_{j\in\Upsilon_{a(i)}} \in \{\emptyset, \varepsilon^p\}^{I_{a(i)}}$, where the set $\Upsilon_{a(i)}$ is the client portfolio of company $i$'s auditor:

$$\Upsilon_{a(i)} = \{j \in \{1,...I\} : a\left(j\right) = a\left(i\right)\}$$

and $I_{a(i)} = \#\Upsilon_{a(i)}$ is the number of companies in that set.

[5]Under this premise, the phenomenon that this paper sets out to explain - the structure of the financial audits market observed in reality - cannot be explained with the argument that different companies have different preferences regarding the auditor's quality.

| Stage | Relevant information obtained by $i$ |
|---|---|
| Common knowledge | $\theta_j$, $d$, $\sigma_j$ for all $j \in \{1, ..., I\}$<br>$\Pr[v_j = \theta_j] = (1-q)$ for all $j \in \{1, ..., I\}$<br>$\Upsilon_a = \{j \in \{1, ..., I\} : a(j) = a\}$ for all $a$<br>$d_a \sim U[0,1]$ for all $a$ |
| 1 | Audit report $\hat{v}_i$ from auditor $a(i)$ |
| 2 | Private error observation $e_i \in \{\emptyset, \varepsilon\}$ :<br>$\Pr[e_i = \varepsilon \vert v_i = 0, \hat{v}_i = \theta_i] = d$<br>Public error observation $\mathcal{P}_{a(i)} \equiv \left\{e_j^p\right\}_{j \in \Upsilon_{a(i)}} \in \{\emptyset, \varepsilon^p\}^{I_{a(i)}}$:<br>$\Pr\left[e_j^p = \varepsilon^p \vert e_j = \varepsilon\right] = \sigma_j$ |

Table 2.1: Stockholder $i's$ acquisition of information by stage

An investment selection strategy $\hat{y}_i$ therefore specifies the investor's course of action for any realization of $\left(\hat{v}_i, e_i, \mathcal{P}_{a(i)}\right) \in \{0, \theta_i\} \times \{\emptyset, \varepsilon\} \times \{\emptyset, \varepsilon^p\}^{I_{a(i)}}$. Table 2.1 summarizes the information acquired by stockholder $i$ in each stage and serves as a reference for the rest of the paper.

In stage 4 the investor claims the return from the opportunity that she decided to finance. She is risk-neutral, and therefore her realized utility is given by:

$$u_i(y_i, a(i)) = \begin{cases} \pi_i - F_{i,a} & \text{if } y_i = i; a(i) = a \\ \bar{\pi} - F_{i,a} & \text{if } y_i = \bar{y}; a(i) = a \\ \pi_i & \text{if } y_i = i; a(i) = \emptyset \\ \bar{\pi} & \text{if } y_i = \bar{y}; a(i) = \emptyset \end{cases} \tag{2.1}$$

where $F_{i,a}$ is the audit fee due to the auditor $a$ when $a$ is selected to audit company $i$. Two assumptions are implicit in this utility function. First, the amount of cash initially available to the investor is normalized to zero. Second, the investor bears the cost of paying for the financial audit.

### 2.3.2 The economic value of audits

The model described in subsection 2.3.1 provides a framework with which to study the economic value of auditing and, in particular, to study how an auditor may be the source of network effects. The current subsection begins this study by computing the benefit that the investor derives from auditing.

Let $E_0(\pi_i|\hat{y}_i, a(i))$ denote investor $i$'s expectation regarding the return to the company's asset as assessed before the beginning of stage 1, given the identity of company $i$'s auditor $a(i)$, and an investment selection strategy $\hat{y}_i$. I then define the expected benefit that investor $i$ derives when some auditor $a$ is in place to be:

$$\alpha_{i,a} \equiv E_0(\pi_i|\hat{y}_i, a(i)) - E_0(\pi_i|\hat{y}_i, \emptyset) \tag{2.2}$$

This expression is at the center of this paper. Its analysis will make apparent how, under some assumptions, the expected benefit $\alpha_{i,a}$ from appointing an auditor $a$ has a fixed component that does not vary across auditors and a variable component the value of which depends upon the characteristics of each auditor's clientele. This variable component is, in effect, a positive network effect.

To find the conditions in which a network effect can arise we must first characterize the investor's stage 3 decision. After all, auditing is only valuable if the information that it creates affects that decision. When $\hat{v}_i = 0$ or $e_i = \varepsilon$, the investor knows that the true value of the uncertain component of the return is 0, and the optimal decision for her is to finance the alternative investment project $\bar{y}$.The stage 3 decision faced by investor $i$ is therefore non-trivial only when the audit report confirms the information in the financial statement of the company (that is, $\hat{v}_i = \theta_i$) and the investor does not observe an audit error ($e_i = \emptyset$).

Lets, then, consider the event in which $\hat{v}_i = \theta_i$ and $e_i = \emptyset$. Define

$$\begin{aligned} \gamma_x &\equiv \Pr[v_i = \theta_i | \hat{v}_i = \theta_i, e_i = \emptyset; x] \\ &= \frac{\Pr[\hat{v}_i = \theta_i, e_i = \emptyset | v_i = \theta_i, x] \Pr[v_i = \theta_i]}{\Pr[\hat{v}_i = \theta_i, e_i = \emptyset | x]} \\ &= \frac{(1-q)}{(1-q) + (1-x)(1-d)q}. \end{aligned} \tag{2.3}$$

The newly defined $\gamma_x$ expresses the assessment that investor $i$ would make, upon observing $\hat{v}_i = \theta_i$ and $e_i = \emptyset$, of the likelihood of the uncertain return to the company's asset being $\theta_i$

if she could be sure that the auditor's quality, $d_{a(i)}$, was equal to $x$[6]. In stage 3, however, the investor does not know what $d_{a(i)}$ is equal to. Instead, she has a system of prior beliefs - stipulating that $d_{a(i)}$ is uniformly distributed over $[0, 1]$ - and her observation of the auditor's performance in other companies (when the auditor is employed by companies with visibility).

So, let $\rho\left(x|\mathcal{P}_{a(i)}\right)$ denote investor $i$'s updated belief that the quality of company $i$'s auditor, $d_{a(i)}$, is equal to $x$; also let $\rho|\mathcal{P}_{a(i)} = \left(\rho\left(x|\mathcal{P}_{a(i)}\right)\right)_{x\in[0,1]}$ denote a fully specified system of beliefs. The investor decides to finance company $i$'s asset if she expects the return to that asset - of which she would be the claimant - to be larger than the return to her outside opportunity:

$$\begin{aligned} E_3\left[\pi_i|\hat{v}_i = \theta_i, e_i = \emptyset, \mathcal{P}_{a(i)}\right] &= \\ \int_0^1 \theta_i \Pr\left[v_i = \theta_i|\hat{v}_i = \theta_i, e_i = \emptyset; x\right]\rho\left(x|\mathcal{P}_{a(i)}\right)\partial x + L_i &= \\ \theta_i \int_0^1 \gamma_x \rho\left(x|\mathcal{P}_{a(i)}\right)\partial x + L_i &\geq \bar{\pi}. \end{aligned} \tag{2.4}$$

Expression (2.4) makes clear that the investor's expectation regarding the return to company $i$'s asset depends upon her beliefs about the auditor's ability. For the value derived by the investor from the appointment of a particular auditor to have a variable component that depends upon the characteristics of the auditor's clientele, however, it is not enough that $E_3\left[\pi_i|\hat{v}_i = \theta_i, e_i = \emptyset, \mathcal{P}_{a(i)}\right]$ depend upon $\mathcal{P}_{a(i)}$; it is also necessary that these beliefs be pivotal in the investor's selection of an investment[7]. Otherwise - that is, if the investor always choose the same investment regardless of observed performance $\mathcal{P}_{a(i)}$ - such information would be worthless and only the audit report would matter. Beliefs are pivotal if the following two assumptions hold:

$$\begin{aligned} \text{A2}: \ \theta_i\gamma_0 + L_i &< \bar{\pi} \\ \text{A3}: \theta_i\gamma_1 + L_i &> \bar{\pi} \end{aligned}$$

[6] $\gamma_x$ is found by using Bayes' rule.

[7] That is, there must be some updated beliefs for which the stockholder decides to finance the company, and other beliefs for which she decides to finance her alternative opportunity.

Assumption A2 (A3) states that when the investor is sure that the auditor's ability is 0 (1), she prefers the to finance the company (the alternative opportunity).

Letting $\rho \equiv [\rho(x)]_{x\in[0,1]}$ denote a system of updated beliefs, define the set

$$\Phi = \{\rho : E_3[\pi_i|\hat{v}_i = \theta_i, e_i = \emptyset, \rho] + L_i \geq \bar{\pi}\}.$$

$\Phi$ contains every belief system under which the investor prefers to invest in the company rather than in the outside opportunity. It follows directly from assumption A3 that this set is non-empty, and from assumption A2 that its complement is non-empty as well. The two assumptions also imply that there is a threshold $\delta^\star$ such that, when the investor is sure that the auditor's ability to detect misstatements is equal to $\delta^\star$, she is indifferent between financing the company and the alternative opportunity. Lemma formally states this conclusion and presents an expression for the threshold. The belief system prescribing $\rho\left(\delta^\star|\mathcal{P}_{a(i)}\right) = 1$ is in set $\Phi$ by definition.

**Lemma 8.** *Under assumptions A2 and A3, there is a threshold* $0 < \delta^\star < 1$ *such that if investor* $i$ *knew that* $d_{a(i)} = \delta^\star$*, she would be indifferent between financing* $i$ *and* $\bar{y}$ *when* $\hat{v}_i = \theta_i$ *and* $e_i = \emptyset$*. That threshold is given by expression*

$$\delta^\star = 1 - \frac{1}{q(1-d)}\left[\frac{(1-q)(\theta_i + L_i - \bar{\pi})}{\bar{\pi} - L_i}\right].$$

*Proof.* Because A2 and A3 both hold, the intermediate value theorem then implies that $\delta^\star$ must exist. By definition, $\delta^\star$ solves the equation

$$\begin{aligned} E_3[\pi_i|\hat{v}_i = \theta_i, e_i = \emptyset, \delta^\star] &= \bar{\pi} \\ \Leftrightarrow \frac{(1-q)}{(1-q) + (1-\delta^\star)(1-d)q}\theta_i + L_i &= \bar{\pi}, \end{aligned}$$

which, after solving for $\delta$, yields the expression presented in the lemma. □

Assumption A2 has one final implication of interest, summarized in lemma 9.

**Lemma 9.** *Under assumption A2, if company* $i$ *does not have an auditor* ($a(i) = \emptyset$)*, then the investor will not finance the company. That is, if* $a(i) = \emptyset$*, then* $y_i = \bar{y}$*.*

*Proof.* If company $i$ does not have an auditor, then no information is generated. The investor's expectation of the value of the company's return in the decision stage - stage 3 - is then $E_3(\pi_i) = E_0(\pi_i) = (1-q)\,\theta_i + L_i$; since there is no audit fee, this is the investor's expected utility if she finances company $i$.

Assumption A2 states that $\theta_i \gamma_0 + L_i < \bar{\pi}$. By evaluating expression (2.3) at 0 we find that

$$\gamma_0 = \frac{(1-q)}{(1-q)+(1-d)\,q}$$

which implies that $\gamma_0 > (1-q)$. We then have that

$$E_3(\pi_i) = (1-q)\,\theta_i + L_i < \theta_i \gamma_0 + L_i < \bar{\pi},$$

where the last inequality follows from assumption A2. This inequality shows that the expected return to financing the company is smaller than the return to the alternative opportunity, which implies that the investor is better off pursuing the latter. □

The reason why the investor does not finance the company when there is no auditor is the following: by assumption A2, if the investor knows the auditor to be the worst possible, she does not invest in the company even if she observes a positive audit report and no audit error; however, even that worst possible auditor allows the investor to collect more information than no auditor at all, so it follows that she cannot invest in that case either.

Lemma 9 completes the characterization of the investor's optimal decision rule. The investor finances company $i$ if she observes an audit report $\hat{v}_i = \theta_i$, no audit error $e_i = \emptyset$, and her updated beliefs are optimistic enough ($\rho|\mathcal{P}_{a(i)} \in \Phi$). She does not finance the company if any of these conditions fails or if there is no auditor in place. Denoting by $y_i^\star$ the investor's optimal decision, we have that:

$$y_i^\star = \begin{cases} \bar{y} & \text{if } a(i) = \emptyset \\ \bar{y} & \text{if } \hat{v}_i = 0\text{; or } e_i = \varepsilon\text{; or } \rho|\mathcal{P}_{a(i)} \notin \Phi \\ i & \text{if } \hat{v}_i = \theta_i,\ e_i = \emptyset \text{ and } \rho|\mathcal{P}_{a(i)} \in \Phi \end{cases} \tag{2.5}$$

| $v_i$ | Outcome | $\Pr\left[v_i, \text{outcome}\vert d_a\right]$ | $y_i^\star$ | $\pi_i\vert\hat{y}_i, a\left(i\right)$ |
|---|---|---|---|---|
| $0$ | $\hat{v}_i = 0$ | $qd_a$ | $\bar{y}$ | $\bar{\pi}$ |
| $0$ | $\hat{v}_i = \theta_i, e_i = \varepsilon$ | $q\left(1-d_a\right)d$ | $\bar{y}$ | $\bar{\pi}$ |
| $0$ | $\hat{v}_i = \theta_i, e_i = \emptyset$ $\rho\vert\mathcal{P}_a \notin \Phi$ | $q\left(1-d_a\right)\left(1-d\right)\Pr\left[\rho\vert\mathcal{P}_a \notin \Phi\vert d_a\right]$ | $\bar{y}$ | $\bar{\pi}$ |
| $0$ | $\hat{v}_i = \theta_i, e_i = \emptyset$ $\rho\vert\mathcal{P}_a \in \Phi$ | $q\left(1-d_a\right)\left(1-d\right)\Pr\left[\rho\vert\mathcal{P}_a \in \Phi\vert d_a\right]$ | $i$ | $L_i$ |
| $\theta_i$ | $\hat{v}_i = \theta_i, e_i = \emptyset$ $\rho\vert\mathcal{P}_a \notin \Phi$ | $\left(1-q\right)\Pr\left[\rho\vert\mathcal{P}_a \notin \Phi\vert d_a\right]$ | $\bar{y}$ | $\bar{\pi}$ |
| $\theta_i$ | $\hat{v}_i = \theta_i, e_i = \emptyset$ $\rho\vert\mathcal{P}_a \in \Phi$ | $\left(1-q\right)\Pr\left[\rho\vert\mathcal{P}_a \in \Phi\vert d_a\right]$ | $i$ | $\theta_i + L_i$ |

Table 2.2: Probability of and return to each outcome for a fixed auditor quality

It is now possible to compute the net expected benefit that accrues to investor $i$ when some auditor $a$ is in place in company $i$, as defined in expression (2.2), contingent on the investor following the optimal strategy $y_i^\star$:

$$\alpha_{i,a} = E_0\left(\pi_i|\hat{y}_i, a\left(i\right)\right) - E_0\left(\pi_i|\hat{y}_i, \emptyset\right) = E_0\left(\pi_i|\hat{y}_i, a\left(i\right)\right) - \bar{\pi}.$$

Table 2.2 is of use in the computation of $E_0\left(\pi_i|\hat{y}_i, a\left(i\right)\right)$. It identifies every possible outcome leading up to the investor's decision in stage 3, the probability of that outcome, the decision made by the investor after each outcome, and the return that follows from that decision, all for a fixed auditor ability of $d_a$. The value of $\alpha_{i,a}$ that results from adding the returns weighted by their probability, integrating over the quality of the auditor (which is uniformly distributed over $[0,1]$ and therefore has a density of 1 at all values in its support), and performing some algebraic simplifications is:

$$\begin{aligned}\alpha_{i,a} &= \int_0^{\delta^\star}\left(L_i-\bar{\pi}\right)q\left(1-d_a\right)\left(1-d\right)\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]\partial d_a \\ &+ \int_{\delta^\star}^1\left(L_i-\bar{\pi}\right)q\left(1-d_a\right)\left(1-d\right)\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]\partial d_a \\ &+ \int_0^{\delta^\star}\left(\theta_i+L_i-\bar{\pi}\right)\left(1-q\right)\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]\partial d_a \\ &+ \int_{\delta^\star}^1\left(\theta_i+L_i-\bar{\pi}\right)\left(1-q\right)\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]\partial d_a.\end{aligned} \tag{2.6}$$

Expression (2.6) highlights the crucial insight that, from an ex-ante perspective, the investor gains if she can reduce the probability that she will hold misleading beliefs about the auditor's ability[8]. That is, $\alpha_{i,a}$ is larger the higher $\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]$ for all $d_a > \delta^\star$ is and the smaller $\Pr\left[\rho|\mathcal{P}_a \in \Phi|d_a\right]$ for all $d_a < \delta^\star$. It follows that the largest benefit that an investor may derive from auditing is given by:

$$\begin{aligned}\alpha_{i,a}^{max} &= \int_{\delta^\star}^{1} (L_i - \bar{\pi})\, q\,(1-d_a)\,(1-d)\,\partial d_a + \int_{\delta^\star}^{1} (\theta_i + L_i - \bar{\pi})\,(1-q)\,\partial d_a \\ &= \frac{(\delta^\star - 1)^2}{2} q\,(1-d)\,(L_i - \bar{\pi}) + (1-\delta^\star)\,(1-q)\,(\theta_i + L_i - \bar{\pi})\,.\end{aligned}$$

which, using the value of $\delta^\star$ presented in lemma 8, simplifies to

$$\alpha_{i,a}^{max} = \frac{(1-q)^2\,(\theta_i + L_i - \bar{\pi})^2}{2q\,(1-d)\,(\bar{\pi} - L_i)}$$

If company $i$ has an auditor that has a numerous and visible set of clients - that is, an auditor for which there is a large, informative, sample of performance observations - the likelihood that the executive will have misleading beliefs is small. Herein lays the explanation as to why the value associated to appointing an auditor $a$, given by $\alpha_{i,a}$, embeds a network effect.

Does auditing have a value even if the auditor has no other visible clients that can generate informative observations about the auditor's performance? Lemma 10 provides an answer.

**Lemma 10.** *Auditing has an intrinsic value to the investor in company $i$ if, and only if, the following inequality holds:*

$$\theta_i \geq \frac{(1-q) + \frac{1}{2}(1-d)\,q}{(1-q)}\,(\bar{\pi} - L_i)\,.$$

*Proof.* When the company is audited by an auditor $a'$ that has $\bar{I}_{a,-i} = 0$ clients of high visibility (not including $i$), the investor finances it after observing an audit report $\hat{v}_i = \theta_i$ and no audit

[8]The investor holds misleading beliefs whenever the investor's observation of the auditor's performance in other companies, $\mathcal{P}_a$, leads her to choose the alternative $\bar{y}$ when the auditor's true ability is in fact greater than $\delta^\star$ or to finance $i$ whenever the auditor's true ability is smaller than $\delta^\star$.

error $e_i = \emptyset$ if and only if:

$$\begin{aligned} E_3\left[v_i | \hat{v}_i = \theta_i, e_i = \emptyset\right] + L_i &\geq \bar{\pi} \Leftrightarrow \\ \frac{(1-q)}{(1-q)+\frac{1}{2}(1-d)\,q}\theta_i + L_i &\geq \bar{\pi} \Leftrightarrow \\ \theta_i &\geq \frac{(1-q)+\frac{1}{2}(1-d)\,q}{(1-q)}\left(\bar{\pi} - L_i\right) \end{aligned}$$

□

From hereon after, I will assume that auditing does indeed have an intrinsic value:

$$\text{A4: } \theta_i \geq \frac{(1-q)+\frac{1}{2}(1-d)\,q}{(1-q)}\left(\bar{\pi} - L_i\right), \forall i.$$

With assumption A4, I have fully formalized this paper's model of audit value. Its main tenets are the following. The model begins by assuming that auditing reduces the uncertainty regarding the company's asset. By assumption A4, this reduction in uncertainty is important enough to make auditing by any auditor valuable from an *ex-ante* perspective. Put differently, auditing improves the decision-making of the investor. By assumptions A2 and A3, this decision-making is further improved if the investor can reduce the uncertainty about the auditor's quality. The investor can achieve such a reduction in uncertainty if the company retains an auditor about which there are (in stage 2) many informative observations of performance. That is, if the company retains an auditor with many visible clients. The next subsection explains exactly how the value derived by investor $i$ from appointing some auditor $a$ is affected by the number of clients of the auditor and the visibility of those clients.

### 2.3.3 Belief formation and network effects in the value for financial audits

The purpose of this subsection is to give an analysis of the benefit function $\alpha_{i,a}$ and, in particular, of its dependence upon the composition of auditor $a$'s client portfolio $\Upsilon_a$. This dependence exists because the investor derives value from having precise beliefs about the auditor's ability. It is therefore important to understand how updated beliefs are formed, and how their precision is affected by the characteristics of $\Upsilon_a$.

As mentioned in the previous subsection, investor $i$ updates her beliefs about her auditor, $a(i)$ by observing the auditor's performance in its $\bar{I}_{a(i),-i}$ high visibility clients, that is, all $j \in \Upsilon_{a(i)}$ such that $\sigma_j = \bar{\sigma}$. It follows that, whatever network benefit investor $i$ derives from being audited by auditor $a(i)$ is created by the $\bar{I}_{a(i),-i}$ high visibility clients of that auditor.I reflect this dependence of $\alpha_{i,a}$ upon $\bar{I}_{a,-i}$ by denoting it as $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$.

Given that the probability that an audit error is made public is the same for all high visibility clients, all that matters to investor $i$ is the count in $\mathcal{P}_a$ of the two possible public outcomes: $\emptyset$ and $\varepsilon^p$. As before, let company $i$'s auditor be $a(i) = a$. Then, let $N_{a,-i}$ be the number of clients of that auditor, excluding $i$, for which a public audit error is observed. Formally, we have $N_{a,-i} \equiv \#\{\varepsilon^p \in \mathcal{P}_a\}$. The probability $\Pr[\rho|\mathcal{P}_a \in \Phi|d_a]$ that enters the investor's benefit from auditing is then the probability that $N_{a,-i}$ is low enough relative to $\bar{I}_{a,-i}$ such that the investor chooses to finance the company, given $d_a$. Conditional on $d_a$, $N_{a,-i}$ follows a binomial distribution with $\bar{I}_{a,-i}$ trials and probability $Q \equiv \Pr\left[e_j^p = \varepsilon^p|d_a\right] = q(1-d_a)\,d\bar{\sigma}$ of "success":

$$N_{a,-i}|d_a \sim \mathrm{Bin}\left(\bar{I}_{a,-i}, Q\right).$$

where $Q$ is a random variable uniformly distributed over interval $[0, qd\bar{\sigma}]$, with density $g(Q) = \frac{1}{qd\bar{\sigma}}$, mean $E(Q) = \frac{1}{2}qd\bar{\sigma}$, and variance $Var(Q) = \frac{1}{12}(qd\bar{\sigma})^2$[9]. Lemma 11 explains how the investor updates her beliefs about $Q$ - and, by extension, about $d_a$ - given the observation of the binomially distributed random variable $N_{a,-i}$.

**Lemma 11.** *When auditor $a$ has $\bar{I}_{a,-i}$ high visibility clients and it commits a public audit error in $N_{a,-i}$ of them, investor $i$'s updated beliefs about $Q = q(1-d_a)\,d\bar{\sigma}$ are given, for all $Q \in [0, qd\bar{\sigma}]$, by the probability density function:*

$$\hat{g}\left(Q|N_{a,-i};\bar{I}_{a,-i}\right) = \frac{Q^{N_{a,-i}}(1-Q)^{\bar{I}_{a,-i}-N_{a,-i}}}{\int_0^{qd\bar{\sigma}} x^{N_{a,-i}}(1-x)^{\bar{I}_{a,-i}-N_{a,-i}}\,\partial x}.$$

[9] $d_{a(i)}$ is uniformly distributed over $[0,1]$, which implies that so is $\left(1-d_{a(i)}\right)$. $Q$ is a linear transformation of $\left(1-d_{a(i)}\right)$, and therefore it is uniformly distributed as well.

*The investor's updated beliefs about the auditor's ability $d_a$ are given by*

$$\rho\left(d_a|\mathcal{P}_a\right)=\hat{g}\left(q\left(1-d_a\right)d\bar{\sigma}|N_{a,-i};\bar{I}_{a,-i}\right).$$

*Proof.* Investor $i$ uses her observation of $N_{a,-i}$ to update her beliefs by using Bayes' rule:

$$\hat{g}\left(Q|N_{a,-i};\bar{I}_{a,-i}\right) \quad = \quad \frac{\Pr\left[N_{a,-i}|Q;\bar{I}_{a,-i}\right]g\left(Q\right)}{\Pr\left[N_{a,-i}|\bar{I}_{a,-i}\right]},$$

where the conditional probability function of $N_{a,-i}$ is given by:

$$\Pr\left[N_{a,-i}|Q,\bar{I}_{a,-i}\right]=\binom{\bar{I}_{a,-i}}{N_{a,-i}}Q^{N_{a,-i}}\left(1-Q\right)^{\bar{I}_{a,-i}-N_{a,-i}}.$$

Using the fact that $g\left(Q\right)=\frac{1}{qd\bar{\sigma}}$ we then have:

$$\begin{aligned}\hat{g}\left(Q|N_{a,-i};\bar{I}_{a,-i}\right) &= \frac{\binom{\bar{I}_{a,-i}}{N_{a,-i}}Q^{N_{a,-i}}\left(1-Q\right)^{\bar{I}_{a,-i}-N_{a,-i}}\frac{1}{qd\bar{\sigma}}}{\int_0^{qd\bar{\sigma}}\binom{\bar{I}_{a,-i}}{N_{a,-i}}x^{N_{a,-i}}\left(1-x\right)^{\bar{I}_{a,-i}-N_{a,-i}}\frac{1}{qd\bar{\sigma}}\partial x} \\ &= \frac{Q^{N_{a,-i}}\left(1-Q\right)^{\bar{I}_{a,-i}-N_{a,-i}}}{\int_0^{qd\bar{\sigma}}x^{N_{a,-i}}\left(1-x\right)^{\bar{I}_{a,-i}-N_{a,-i}}\partial x}.\end{aligned}$$

$\rho\left(d_a|\mathcal{P}_a\right)=\hat{g}\left(q\left(1-d_a\right)d\bar{\sigma}|N_{a,-i};\bar{I}_{a,-i}\right)$ follows from the fact that $Q=q\left(1-d_a\right)d\bar{\sigma}$. □

The posterior beliefs presented in the lemma are single peaked. They are also more precise the larger $\bar{I}_{a,-i}$ is. However, precision increases at a decreasing rate, as the addition of observations of the auditor's performance means that each observation has a smaller weight in the investor's formation of updated beliefs. This has an important implication: as $\bar{I}_{a,-i}$ increases, the probability $\Pr\left[\rho|\mathcal{P}_a\in\Phi|d_a\right]$ for $d_a>\delta^\star$ must eventually increase, while the probability $\Pr\left[\rho|\mathcal{P}_a\in\Phi|d_a\right]$ for $d_a<\delta^\star$ must eventually decrease. In other words, the benefit that the investor derives from being audited by auditor $a$ must eventually increase at a decreasing rate with the size of the auditor's visible clientele. The qualifier "eventually" is necessary because, for small $\bar{I}_{a,-i}$, the benefit from being audited by $a$ may be constant. Proposition 7 establishes this formally.

**Proposition 7.** *There is a threshold $T \in \{0, 1, 2, ...\}$ such that investor $i$'s benefit from retaining an auditor $a$, $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$, is:*

1. *a constant $\bar{\alpha}$ if auditor $a$ has any number $\bar{I}_{a,-i} \leq T$ of high visibility clients, not including $i$;*
2. *increasing at a decreasing rate if auditor $a$ has any number $\bar{I}_{a,-i} > T$ of high visibility clients, not including $i$.*

*For some parametrizations of the model, the threshold $T$ is strictly greater than* 0.

*Proof.* The proposition contains two claims:

1. There is a threshold $T \in \{1, 2, ...\}$ such that $\alpha_{i,a}\left(\bar{I}\right) = \bar{\alpha}$ if $\bar{I}_{a,-i} = \bar{I} \leq T$ and $\alpha_{i,a}\left(\bar{I}\right)$ is increasing at a decreasing rate if $\bar{I}_{a,-i} = \bar{I} > T$.

   The novel part of this claim is that for small clienteles, $\alpha_{i,a}\left(\bar{I}\right) = \bar{\alpha}$, which is trivially true for $T = 0$.

2. For some parametrizations of the model, the threshold $T$ is strictly greater than 0.

   By assumption A4, investor $i$ finds it profitable to hire an auditor $a'$ that has no other highly visible clients. That is the case because the parameters of the model are assumed to be such that she finds it profitable to finance the company if a positive audit report $\hat{v}_i = \theta_i$ is not contradicted by the observation of an audit error. It is then possible that if investor $i$ instead had an auditor $a''$ such that $\bar{I}_{a'',-i} = 1$, she might opt to finance the company after observing $\hat{v}_i = \theta_i$ and $e_i = \emptyset$ even if she observes a public audit error in the other client of $a''$. That is, it could be that $\rho|\mathcal{P}_{a''} \in \Phi$ even if $\mathcal{P}_{a''} = \{\varepsilon^p\}$, which is the scenario in which the updated beliefs would be the most pessimistic. In this case, having auditor $a''$ audit the company instead of $a'$brings no additional benefit to the investor which means that $\alpha_{i,a}(1) = \alpha_{i,a}(0) = \bar{\alpha}$ and, thus, that $T > 0$.

$\square$

Figure 2.2 illustrates $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$ for $T = 3$ and$\bar{\alpha} = 1$. The figure captures the essential features of the benefit from auditing: it may be fixed when a new high visibility client is added to the auditor's portfolio if that portfolio contains few highly visible clients; and it increases at a decreasing rate when a new high visibility client is added to the auditor's portfolio if the

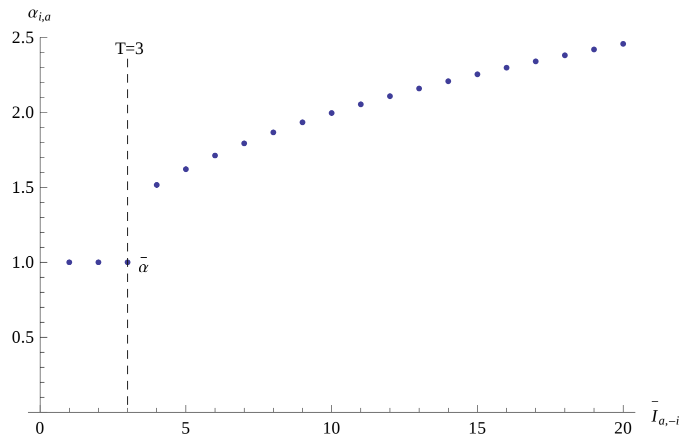


Figure 2.2: A representation of $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$ with $T = 3$ and $\bar{\alpha} = 1$.

portfolio contains sufficiently many highly visible clients.

Proposition 7's characterization of audit value as a function of the auditor's clientele gives us a theory of the demand for audit services. The next section proposes a theory of audit supply and characterizes the audit market equilibria that arise under the proposed theories of demand and supply.

## 2.4 A model of the financial audit market

### 2.4.1 Audit supply: free entry and diseconomies of scale

This paper's model of audit supply has two main tenets. The first is that there are no barriers to entry in the industry and, therefore, that entry is free. The second is that auditors face diseconomies of scale. With $I_a$ denoting the number of clients in auditor $a$'s client portfolio $\Upsilon_a$, the cost function of auditor $a$ is given by

$$C\left(I_a\right) = cI_a^2.$$

Note that the cost function faced by auditors implicitly assumes that client companies are homogenous as far as audit technology goes: what determines the marginal cost of auditing a client company is the number of companies that the auditor has, not any idiosyncratic feature of the client. Diseconomies of scale play a crucial role in this model: they provide a natural -

that is, non-regulatory - boundary to the size of auditors.

The assumption of diseconomies of scale is rooted in the hypothesis that increasing the size of the auditor increases bureaucratic and monitoring costs. In reality, auditing firms may benefit from economies of scale when the number of clients that they serve is low: an audit firm may increase its portfolio of clients moderately without having to expand office space, hire more back office staff or implement complex internal monitoring and control mechanisms. It is only as they keep expanding that audit firms are likely to experience the rising marginal costs that appear in association with the need for increased delegation. For simplicity of analysis, the model assumes that diseconomies of scale start at the lowest auditor size ($I_a = 2$).

Subsection 2.4.3 explains how diseconomies of scale and the network effects in the value of audits shape market equilibria. First, I must explain how market interaction is modeled.

### 2.4.2 The audit market: stage 0

The demand for financial auditing services comes from the $I$ companies in the market. Each company's investor appoints the auditor and is responsible for the payment of the audit fees. The underlying hypothesis is that, even if investors (or stockholders) do not actually directly appoint the external auditor of the company, their preferences are heeded by the board and the audit committee[10]. In the context of the model, each investor cares about only two variables: the size of the auditor's portfolio of visible clients and the audit fee.

The model of audit value presented in section 2.3 unfolded over stages $1-4$ under the assumption that an auditor had been appointed. The auditor is actually appointed when the market meets in stage 0. The timing of stage 0 market interaction, exemplified in figure 2.3 for a market with four clients, is the following: each auditor simultaneously submits *private* audit fee proposals to a subset of its choice of the prospective client set $\{1, 2, ..., I\}$. The fee proposal of auditor $a$ to investor $i$ is the previously introduced $F_{i,a}$, and cannot be made contingent upon the audit outcome. Each investor then appoints at most one auditor from the list of auditors

[10] In the United Kingdom, auditors are appointed at the shareholders' annual general meeting; in the U.S., the audit committee - which draws its members from the board of directors - oversees the hiring of the external auditor; in the European Union, the auditor is recommended by the audit committee - which contains a combination of non-executive members of the board and members appointed by the shareholders' general meeting.

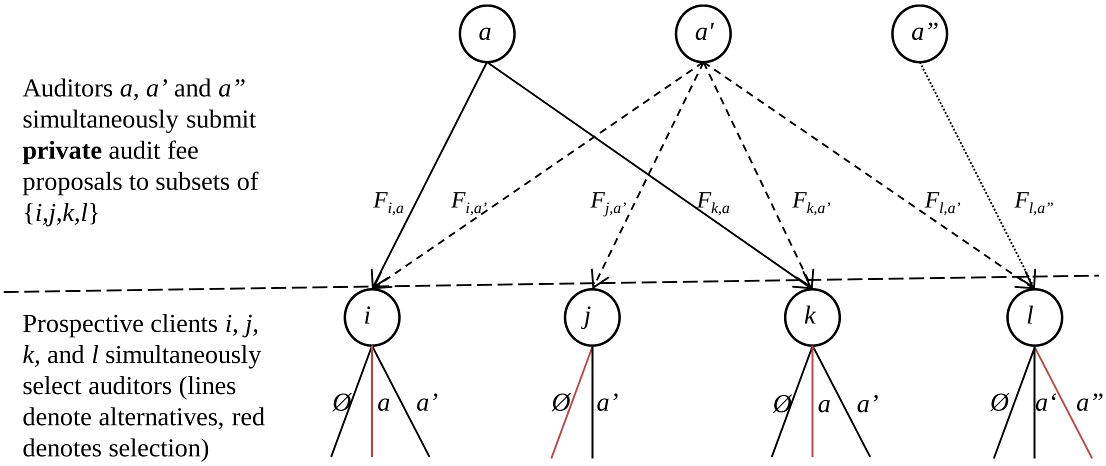


Figure 2.3: Example of market with prospective clients $\{i, j, k, l\}$ and three proposing auditors $\{a, a', a''\}$.

that made a proposal to audit her company. Investors appoint auditors simultaneously. If investor $i$ chooses auditor $a$, she pays the proposed fee $F_{i,a}$. Once investors appoint auditors, every investor observes the client portfolio $\Upsilon_a$ of every auditor $a$.

The model of market interaction reflects the facts that audit firms are free to deny service to clients by not extending offers and, more importantly, that they are free to price discriminate.

### 2.4.3 Equilibrium in the market for financial audits

This subsection brings together the models of audit demand, supply, and market interaction previously developed. The model of audit value developed over 2.3 established that the benefit enjoyed by an investor from appointing an auditor $a$ is a function $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$ that depends upon the number of highly visible clients of the auditor. The benefit is flat up to a critical threshold $T$ and increasing but "concave" after that threshold. The model of audit supply developed in subsection 2.4.1 proposed that the industry is characterized by free entry and diseconomies of scale.

In light of this characterization, stage 0 interaction fits a model of network externalities whereby: i) network benefits increase at a decreasing rate with the number of visible users of the network (those $j$ such that $\sigma_j = \bar{\sigma}$) and do not change with the number of non-visible users; ii) the cost function of the provider of the network good is convex in the total number of users and given by $C\left(I_a\right) = cI_a^2$; iii) the provider is allowed to discriminate while setting prices

- that is, it can charge different fees to different clients.

Stage 0 can be interpreted as a standalone two period dynamic game in which auditors maximize audit fees net of audit costs and each investor $i$ maximizes $\alpha_{i,a}\left(\bar{I}_{a,-i}\right)$: in the first period, auditors simultaneously submit (private) audit fee proposals to the stockholders of client companies - with each auditor being free to submit proposals to as many clients as it wishes; in the second, the investors simultaneously select one auditor each - or none - from among the auditors that made them a proposal.

The baseline solution concept of the 5 stage model $(0-5)$ is Perfect Bayesian equilibrium (PBE), which is consistent with the investors' Bayesian updating of beliefs regarding auditor quality in stage 3. PBE requires sequential rationality in every contingency. For the investors, this means that:

**Eq1.** each investor holds correct beliefs about the equilibrium portfolio of clients $\Upsilon_a$ that each auditor $a$ will end up with.

**Eq2.** in light of those beliefs, each investor chooses the auditor from which she derives the most utility net of the audit fee. That is, given the audit proposals that she receives, investor $i$ accepts a proposal from which she cannot unilaterally deviate with profit.

For auditors, sequential rationality means that:

**Eq3.** each auditor holds correct beliefs about the equilibrium portfolio of clients $\Upsilon_a$ that each auditor $a$ will end up with.

**Eq4.** each auditor $a$ charges each of its clients a fee that makes them indifferent between appointing $a$ and the next best alternative (another auditor or no auditor).

**Eq5.** no auditor can profit by making an audit proposal that "poaches" a client that appoints a different auditor on the equilibrium path. This condition includes a no profitable entry condition, whereby no audit firm without clients on the proposed equilibrium path can profitably poach a client.

**Eq6.** no auditor can profit by not making a proposal that the other players expect it to.

If the cost parameter $c$ is sufficiently large, the model has a unique equilibrium whereby companies do not audit any clients. If, on the other hand, $c$ is small, there can be multiple equilibria.

In particular, there can be "bad" equilibria reflecting dis-coordination between the players when $T \geq 2$. Lemma 12 formalizes this result and its proof conveys the intuition as to why this is the case.

**Lemma 12.** *Consider a parametrization of the model whereby network effects appear only when an auditor has more than two clients ($T = 2$). Let that parametrization be such that it has at least one equilibrium in which at least one auditor has more than one client. Then, the model also has an equilibrium whereby each company is audited by an auditor without any other clients and is charged an audit fee equal to $c$.*

*Proof.* Consider a proposed equilibrium outcome in which each client is indeed audited by an auditor without any other clients and charged an audit fee equal to $c$ (equilibrium cannot be sustained in this case if the fee is greater than $c$ because of free entry). Since investors expect the proposed equilibrium to be played out, they believe that each auditor will end up with exactly one or zero clients, and they will judge deviation offers by any of these auditors accordingly. So consider a proposed deviation by an auditor $a$ that has one client on the equilibrium path. Since every investor expects $a$ to have exactly one client (investor beliefs are formed according to the proposed equilibrium path) and network effects appear only when an auditor has more than two clients ($T = 2$), investors receiving a deviation proposal by $a$ assign the same value to appointing $a$ as they do to appointing their equilibrium path auditor. It follows that the deviation proposal is profitable to the investors receiving it only if the deviation fee is smaller than their equilibrium path fee of $c$. Auditor $a$, however, cannot profitably offer such deviation fee as the marginal cost of auditing the second client is $c2^2 - c1^2 = 3c$.

Now consider an auditor that has no clients on the equilibrium path: it is also unable to create network benefits to investors, and cannot rationally offer a fee strictly lower than $c$. The lack of profitable deviations shows that the proposed equilibrium is an equilibrium indeed. $\square$

The inefficient equilibria characterized in the lemma rely on investors not being able to coordinate their choice of auditor in order to generate the positive network effect of more precise beliefs. To discard these equilibria, I limit the analysis from hereon after to the class of equilibria that are efficient from a social welfare standpoint.

Let an outcome in the audit market be an allocation of clients to auditors $\Upsilon = (\Upsilon_a)_{a:I_a>0}$ -

which is a partition of the client set $\{1,...,I\}$ - and a schedule of audit fees $F = \left[(F_{j,a})_{j\in\Upsilon_a}\right]_{a:I_a>0}$. Definition 2 lays out the conditions that outcome $(\Upsilon, F)$ must fulfill to be supported in an efficient equilibrium.

**Definition 2.** A strategy profile yielding an outcome $(\Upsilon, F)$ is an efficient equilibrium if it maximizes the welfare function

$$W(\Upsilon) = \sum_{a:I_a>0}\left[\sum_{i\in\Upsilon_a}\alpha_{j,a}\left(\bar{I}_{a,-j}\right) - cI_a^2\right]$$

subject to fulfilling equilibrium conditions Eq1-6.

The definition makes clear that, as long as $c$ is not too large, efficiency requires network externalities to be generated and hence discards the "one client per auditor" scenario. Two remarks are now pertinent. The first remark concerns the size that auditors may take. The welfare generated by auditor $a$ is defined as:

$$W_a(\Upsilon_a) \equiv \sum_{i\in\Upsilon_a}\alpha_{i,a}\left(\bar{I}_{a,-i}\right) - cI_a^2. \tag{2.7}$$

Given the characteristics of the benefit function $\alpha_{i,a}(\cdot)$, it must be that the welfare generated by auditor $a$, $W_a(\Upsilon_a)$, becomes decreasing in the number of clients after some number of clients is reached, even if all clients have high visibility: when a client $j$ with high visibility is added to $a$'s portfolio, each of the other clients derives a marginal externality of $\alpha_{i,a}\left(\bar{I}_{a,-i}\right) - \alpha_{i,a}\left(\bar{I}_{a,-i}-1\right)$, whereas $j$ derives a benefit $\alpha_{j,a}\left(\bar{I}_{a,-j}\right)$. The externality is bounded above and goes to zero as the number of high visibility clients $\bar{I}_a$ increases, which means that for high $\bar{I}_a$, the benefit of adding a new client approaches a constant $\alpha_{j,a}\left(\bar{I}_{a,-j}\right)$. The convexity of the cost function then implies that $W_a(\Upsilon_a)$ becomes decreasing at some point. It follows that the size of an auditor's portfolio has a well defined upper boundary in an efficient equilibrium: the biggest size before the welfare generated by $a$ starts decreasing.

The second remark concerns audit fees and is summarized in proposition 8.

**Proposition 8.** *Let company $i$ be audited by auditor $a$. Then, $i$'s audit fee must be such that:*

$$c(2I_a - 1) \leq F_{i,a} \leq \alpha_{i,a}\left(\bar{I}_{a,-i}\right) - \alpha_{i,a}(0) + c \tag{2.8}$$

*Proof.* For equilibrium condition Eq6 to hold, audit fees must be larger than the "marginal" cost of auditing the last client. That is, for every auditor $a$ and every client $i \in \{1, ..., I\}$, it must be that:

$$F_{i,a} \geq cI_a^2 - c\left(I_a - 1\right)^2 = c\left(2I_a - 1\right).$$

For equilibrium condition Eq5 to hold, no auditor without other clients may profit by making an audit proposal to $i$ that she would prefer over the proposal of $a$. An auditor without clients on the equilibrium path profits if she can convince any client $i$ to pay a fee $c + \epsilon$, for an arbitrarily small $\epsilon$ . Under such auditor, $i$ would obtain a benefit $\alpha_{i,a}(0)$ and no network externalities. Therefore, it must be that:

$$\begin{aligned} \alpha_{i,a}\left(\bar{I}_{a,-i}\right) - F_{i,a} &> \alpha_{i,a}(0) - c - \epsilon \Leftrightarrow \\ F_{i,a} &< \alpha_{i,a}\left(\bar{I}_{a,-i}\right) + c + \epsilon. \end{aligned}$$

□

Proposition 8 has an important implication: every auditor with more than one client has a positive economic profit. Moreover, the lower bound for profits is bigger for companies with larger client portfolios. On the other hand, if in equilibrium there are auditors with only one client, they earn a profit of zero.

The two sub-subsections that follow conclude the characterization of market outcomes in efficient equilibria. Each addresses a different parametrization of the model: in the first, client companies have heterogeneous returns $(\theta_i, L_i)$ and homogenous $\sigma_i$; in the second, client companies have homogenous $\theta_i$ and heterogeneous $\sigma_i$. The equilibria of the parametrizations are interpreted in light of the stylized facts that motivate this paper: the patterns of market concentration across segments of the audit market. The sources of heterogeneity are considered in isolation rather than in tandem for tractability.

### 2.4.4 Equilibrium with homogenous visibility and heterogeneous informational asymmetries

This subsection analyzes the case in which companies all have high visibility ($\sigma_i = \bar{\sigma}$ for all $i \in \{1, ..., I\}$, or $\bar{I} = I$), but differ in terms of the parameters $(\theta_i, L_i)$ of their returns. In particular, this section assumes that companies may have one of two alternative parameter pairs, $\left(\bar{\theta}, \bar{L}\right)$ or $(\underline{\theta}, \underline{L})$, such that $\bar{\theta} > \underline{\theta}$ and

$$(1-q)\,\bar{\theta} + \bar{L} = (1-q)\,\underline{\theta} + \underline{L}$$

which combined imply that $\bar{L} < \underline{L}$ . It follows that, while the expected return is the same for all companies, any company $i$ such that $\theta_i = \bar{\theta}$ sees a larger share of its return come from the uncertain component, whereas any company $j$ such that $\theta_i = \underline{\theta}$ sees a larger share of its return come from the certain component. We have, then, that the case under consideration in this section is one in which companies exhibit heterogeneous informational asymmetry between managers and investors. In companies with parameter pair $\left(\bar{\theta}, \bar{L}\right)$, the asymmetry is more severe.

From expression (2.6) it is clear that companies where informational asymmetries are more severe benefit more from auditing and from the network benefits generated by the auditor than companies where those asymmetries are less severe. The interpretation for this positive correlation between information asymmetries and audit value is intuitive: the larger $\theta_i$ is, the more damaging an undetected audit error becomes. Translating the meaning of this positive correlation into the language of models of network externalities, we have that the network users differ in the value that they assign to the network good.

Proposition 9 provides a first condition that an outcome must satisfy to be supported in an efficient equilibrium when the audit clients differ in their valuation of auditing. It asserts that in a audit market defined by the heterogeneity of clients in terms of the information asymmetries, an auditor with a large client portfolio necessarily has more clients with severe informational asymmetries than a smaller auditor.

**Proposition 9.** *In any efficient equilibrium of the case in which companies exhibit heterogeneous asymmetric information between managers and investors, if an auditor $a'$ has more clients than another auditor $a''$, then it must have more clients with severe informational asymmetries. That is,*

$$I_{a'} > I_{a''} \Rightarrow \# \left\{ i \in \Upsilon_{a'} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\} > \# \left\{ i \in \Upsilon_{a''} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\}.$$

*Proof.* Suppose instead that we had $I_{a'} > I_{a''}$ and

$$\# \left\{ i \in \Upsilon_{a'} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\} \leq \# \left\{ i \in \Upsilon_{a''} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\}$$

in an efficient equilibrium. That is, suppose that auditor $a'$ had more clients than $a''$, but that $a''$ had at least as many clients with severe informational asymmetries. Then, a welfare improvement is available. The improvement consists of "swapping" one of the clients of $a''$ with severe asymmetries to $a'$ for one of $a'$'s clients with mild asymmetries. The swap does not change the cost of either company and allocates a client that values the network effects more to an auditor were these effects are stronger. Since the original outcome was supported in equilibrium, the new one must be supported as well: just have the clients that swapped auditors pay the fees that their respective counterparts would have paid. Because the new outcome can be supported in a PBE and improves welfare, we have a contradiction of the assertion that the initial outcome was supported in an efficient equilibrium. □

By a similar argument, auditor $a'$ has at least as many clients as $a''$ if it has more clients with severe informational asymmetries[11]. That is,

$$\# \left\{ i \in \Upsilon_{a'} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\} > \# \left\{ i \in \Upsilon_{a''} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\} \Rightarrow I_{a'} \geq I_{a''}.$$

Proposition 10 provides a second condition that an outcome must satisfy to be supported

[11] The proof is the same as that of proposition 9: it starts by assuming $\# \left\{ i \in \Upsilon_{a'} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\} > \# \left\{ i \in \Upsilon_{a''} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right) \right\}$ and proceeds through a contradiction argument whereby it supposes, for the sake of that argument, that $I_{a'} < I_{a''}$. The swap argument presented in proposition 9 then applies, which results in the demonstration of the desired contradiction.

in an efficient equilibrium. The proposition establishes that if an auditor $a'$ has clients of both types, then it cannot happen that an audit client with severe informational asymmetries has an auditor with fewer clients than $a'$.

**Proposition 10.** *Let auditor $a'$ have clients of both types of information asymmetries in its client portfolio. Then, there cannot exist an auditor $a''$ that has any clients with severe informational asymmetries and that has strictly fewer clients than $a'$. That is, $a'$ is the smallest auditor with clients with severe informational asymmetries.*

*Proof.* Suppose instead that an auditor $a''$ existed such that $I^{a'} > I^{a''}$ and

$$\#\left\{i \in \Upsilon_{a''} : (\theta_i, L_i) = \left(\bar{\theta}, \bar{L}\right)\right\} > 0.$$

Then, a welfare improvement is available. The improvement consists of "swapping" one of the clients of $a''$ with severe asymmetries to $a'$ for one of $a'$'s clients with mild asymmetries. The swap does not change the cost of either company and allocates a client that values the network effects more to an auditor were these effects are stronger. Since the original outcome was supported in equilibrium, the new one must be supported as well: just have the clients that swapped auditors pay the fees that their respective counterparts would have paid. Because the new outcome can be supported in a PBE and improves welfare, we have a contradiction of the assertion that the initial outcome was supported in an efficient equilibrium. □

Combined, the two propositions imply that, in any efficient equilibrium, every company with severe informational asymmetries is audited by one of the largest auditors, which is consistent with the stylized fact that large auditors, namely the Big 4, audit most publicly listed companies and most large companies. Moreover, if we assume that the model's market for auditing has fewer companies with severe informational asymmetries than companies with small agency problems, we get the prediction that market concentration is stronger in the strong informational asymmetry segment. The assumption and the prediction are respectively compatible with reality and the stylized facts that motivate this paper.

The relevant comparative statics in this section concern how changes in the parameters measuring informational asymmetry affect the nature of efficient equilibria. We can use the statics to help answer at least one important question: how are markets where informational asymmetries are stronger across the board structured? If both $\underline{\theta}$ and $\bar{\theta}$ are larger, than the set of efficient equilibria will remain the same or there will be new equilibria calling for fewer auditors with more clients (as auditing is more valuable to everyone). The prediction is then that geographic markets in which informational asymmetries are stronger - which might be those where there are more large companies - are likely to be more concentrated.

### 2.4.5 Equilibrium with heterogeneous visibility and homogenous informational asymmetries

This subsection analyzes the case in which companies have homogenous informational asymmetries but differ in terms of their visibility. That is, the case in which $\bar{I} < I$ and $(\theta_i, L_i) = (\theta, L)$ for all $i \in \{1, ..., I\}$. As a reminder, note that there are $\bar{I}$ with high visibility $\bar{\sigma}$ and $I - \bar{I}$ with no visibility. Visibility determines whether audit errors detected within a company can ever be public knowledge and, therefore, whether a company generates a network externality for the other companies that are audited by that company's auditor.

Translating the new parametrization into the language of models of network externalities, we now have that all network users assign the same value to the network good, but only a few of them (the $\bar{I}$ companies with high visibility) are desirable as co-users of a network. Proposition 11 establishes that in a audit market defined by the heterogeneity of client visibility, larger auditors cannot have fewer highly visible clients. This result, similar to that presented in proposition 9 for the case of heterogeneous informational asymmetries, is again consistent with the stylized fact that large auditors, namely the Big 4, audit most public and large companies.

**Proposition 11.** *In any efficient equilibrium of the case in which companies exhibit heterogeneous visibility, auditor $a'$ has more clients than auditor $a''$ only if it has at least as many high visibility clients. That is,*

$$I_{a'} > I_{a''} \Rightarrow \bar{I}_{a'} \geq \bar{I}_{a''}.$$

*Proof.* Suppose instead that we had $I_{a'} > I_{a''}$ and $\bar{I}_{a'} < \bar{I}_{a''}$. That is, suppose that auditor $a'$ had more clients, but that auditor $a''$ had at least as many highly visible clients, which also implies that auditor $a'$ has at least one client with no visibility. A welfare improvement is available. The improvement consists of moving one of auditor $a'$'s clients with no visibility, $k$, to auditor $a''$. Since $I_{a'} > I_{a''}$, it follows that $\left[c\left(I_{a'}-1\right)^2 - cI_{a'}^2\right] - \left[c\left(I_{a''}+1\right)^2 - cI_{a''}^2\right] \geq 0$. In words, the cost savings of auditor $a'$ are larger than the increase in costs of auditor $a''$. Since $k$ has no visibility, its exit does not reduce the network effects enjoyed by the other clients of $a'$. And, finally, $k$ enjoys a strictly larger network benefit when it joins auditor $a''$. Because the new outcome can be supported in a PBE and improves welfare, we have a contradiction of the assertion that the initial outcome was supported in an efficient equilibrium. □

By a similar argument, it follows that auditor $a'$ has at least as many clients as $a''$ if it has more clients with high visibility. That is:

$$\bar{I}_{a'} > \bar{I}_{a''} \Rightarrow I_{a'} \geq I_{a''}.$$

In contrast to the previous sub-subsection, where the case of heterogeneous informational asymmetries was treated, here it is not possible to establish that if an auditor has both types of clients, it cannot happen that audit clients with high visibility appoint auditors with fewer clients than that auditor. In fact, whereas in a market in which companies differ in the severity of informational asymmetries it cannot be bad, from a social welfare standpoint, to have perfect segregation according to that severity, here it can be bad to have segmentation according to visibility. The reason is that only high visibility clients generate network externalities and, hence, complete segmentation precludes low visibility clients from enjoying those externalities. Complete segmentation might give rise to large auditors in which the marginal network value of the last highly visible clients is very low relative to what their marginal network value would be in a smaller auditor with a few low visibility clients. In sum, unless the diseconomies of scale are very strong, in efficient equilibria big audit firms contain a mix of high visibility and no visibility clients.

Because of diseconomies of scale, auditors that only audit no visibility clients must have only one client. If we assume that the model's market for auditing has relatively fewer highly visible companies, we get the prediction that market concentration is stronger among the high visibility market segment. Once again, the assumption and the prediction are compatible with reality and the stylized facts that motivate this paper.

### 2.5 Conclusion

This paper provides a micro-founded model of auditing demand in which clients benefit from network effects. By using two different parametrizations of the model, the paper then suggests two new explanations for the pattern of market concentration across segments in audit markets. The paper also provides an explanation for the observation that large auditors charge higher audit fees, and suggests that large auditors may enjoy positive economic profits. Naturally, the paper's theory would benefit from empirical study.

The line of research that this paper follows would also benefit from a formal approach to the explanation of why the big auditors are, in almost every local market, autonomous subsidiaries of the Big 4 audit firms.

The network effects approach followed here might also help explain the pattern of mergers that transformed the Big 8 (1980s) into the Big 4 (2000s). Changes in the corporate environment, the increased dispersion of companies' stocks - and the consequent deepening of informational asymmetries - and financial innovation may have enhanced the value of auditing and thus made larger audit firms viable. Mergers may have then been a way to capture the new benefits from the existence of large auditors.

The network effects approach to the audit market might be adapted to answer new questions. The paper assumes throughout that client portfolios are determined simultaneously. A dynamic approach would enable the investigation of the effects of incumbency in the audit market. Another, perhaps less immediately obvious, question concerns market interaction in new audit markets. This question is relevant for the analysis of audit markets in emerging economies, in which the increase in size of companies, the development of financial markets, and the introduction of financial regulations might create a new demand for audit services where the supply is incipient. In these markets, established audit "brands" might have a first mover advantage.

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