ELEMENTARY PRESERVICE TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING: USING SITUATED CASE STUDIES AND EDUCATIVE EXPERIENCES TO EXAMINE AND IMPROVE THE DEVELOPMENT OF MKT IN TEACHER EDUCATION

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ABSTRACT

TRACY L. JOHNSON: Elementary Preservice Teachers' Mathematical Knowledge For Teaching: Using Situated Case Studies And Educative Experiences To Examine And Improve the Development of MKT In Teacher Education
(Under the direction of Olof Steinthorsdottir)

This study examined pre-service teachers' development of mathematical knowledge for teaching (MKT) over their final year in a university based teacher education program. This was done through analyzing written reflections, focus group interviews, individual interviews, teaching observations, and post-observation interviews as well as through the use of a quantitative measure, the MKT test (Learning Mathematics for Teaching, 2005). The study design employed situated case studies, in which tiered participation resulted in extensive data for three focal pre-service teachers as well as a comparison to larger groups of their peers through interviews (n=8), focus groups (n=11), and written reflections and the MKT test (n=35). A new protocol for coding elementary pre-service teachers' mathematics lessons was developed to extend Rowland et al.'s (2009) work on the Knowledge Quartet (KQ) model.

The study investigated pre-service teachers' definitions of MKT, demonstrations of MKT in their teaching, and educative experiences that contributed to their development of MKT. Insights were gained into pre-service teachers' definitions of MKT, the development of which was dynamic, non-linear, individual, and shared similarities to the aggregate definition only at the end of the year. The KQ category of foundation tended to dominate the pre-service teachers' definitions of MKT, the transformation category remained vague, connection was an inconsistent category in their definition, and contingency arose late in the year and at a relatively small proportion.

Insights were also gained into pre-service teachers' demonstration of MKT in their teaching of mathematics. Dimensions of MKT were most often demonstrated at a minimum level, growth on a dimension as indicated by scores that improved over time was extremely rare, scores were more variable than predicted across the four observed lessons, and the connection category was particularly challenging.

This study used the theoretical lenses of cognitive views of learning and Dewey's (1904/1964; 1916/1985) philosophy of educative experiences. These foundations enhanced this study and led to more substantial suggestions by which to improve teacher education in order to better develop pre-service teachers' development of MKT through the methods course, initial field placements, student teaching and content-based discussion groups such that pre-service teachers' can better develop MKT via educative experiences that encourage conceptual rather than procedural teaching knowledge.

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would entail. As I finish this work, Ainsleigh has recently finished kindergarten and I couldn't be prouder of the person she is. To Ripken, thank you for being silly and sweet. To both of you, I hope this work and that which comes after it might lead to better teachers of mathematics for you and all children, because everyone deserves as much. It is for this reason that these pages contain my attempt at not merely conducting and reporting on a study, but at creation: creation of new ideas and new methodologies fueled by an ever present drive to improve that which is and contribute possibilities to that which could be. In the words of G.H. Hardy (1940/2005), a true mathematician:

> (T)hat I have created is undeniable: the question is about its value. The case for my (work), then, …is this: that I have added something to knowledge, and helped others to add more; and that these somethings have a value which differs in degree only, and not in kind, from that of the creations of the great mathematicians, or of any of the other artists, great or small, who have left some kind of memorial behind them. (pp.53-54)

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Elementary Preservice Teachers' Mathematical Knowledge For Teaching: Using Situated Case Studies And Educative Experiences To Examine And Improve The Development Of MKT In Teacher Education

CHAPTER ONE

INTRODUCTION

Shulman's (1986, 1987) work on pedagogical content knowledge (PCK) made a significant contribution to the field of teacher education by re-focusing attention on content knowledge during a time when the primary focus within teacher education research was on pedagogy. Since the construct of PCK was introduced to the field, work has been done to further explicate what PCK entails in various subject matter domains. Within the field of mathematics education, Ball and her colleagues' work at Michigan and Rowland and his colleagues' work at University of Cambridge have made significant contributions to teacher education and mathematics education (Ball, Thames & Phelps, 2008; Rowland et al., 2009). Both projects have yielded models of mathematical knowledge for teaching that explicates what elementary classroom teachers need to know and be able to do during mathematics planning and instruction.

Although it could be argued that there is growing consensus in the field as to the "types" of knowledge required to teach elementary mathematics, to date there has been little work done to describe the emergence and growth of this knowledge with novice or beginning teachers (Leinhardt, 1993). Becoming clearer on expert knowledge is significantly different than becoming aware of how this knowledge comes to be, in the first place. This has resulted in significant gaps in current scholarship surrounding theories about learning to teach elementary mathematics, where such theories tend to be oversimplified and underspecified (Feiman-Nemser & Remillard, 1996; Munby et al., 2001). This lack of knowledge, in turn, is a problematic omission within teacher education because of the relative lack of theories and research on which to build courses, field work, and programs for preservice teachers.

In addition to gaps in the field of teacher development, I perceive helpful yet underspecified parallels between research in mathematics education about the way young children learn mathematics and the way novice teachers learn to teach. Two such parallels are foundational to the philosophies of the researcher and therefore influence the assumptions that prefigure the research questions. The first idea is the distinction between procedural knowledge versus conceptual understanding, and the second is the application of constructivist learning principles to teacher education candidates. The parallels between mathematics education and teacher education will now be explained for each of these ideas.

First, the National Council for Teachers of Mathematics (NCTM) asserted that students' school experiences with mathematics should emphasize conceptual understanding rather than procedural knowledge (NCTM, 2000, 2006; Skemp, 1978). Furthermore, an early emphasis on conceptual understanding in the primary grades lays a foundation for more advanced mathematics in upper elementary school, middle school, and high school. As Kathy Richardson, a well-known researcher and curriculum designer in early-grades learning, said, "If you have a problem in your high school algebra classrooms, you have a problem in your elementary classrooms" (North Carolina Council of Teachers of Mathematics Conference, October 29, 2009).

I believe that a parallel can be made between these reform-based mathematics beliefs and teacher education: Preservice teachers should have experiences with teaching which emphasize conceptual understanding rather than procedural knowledge. Likewise, if there is a problem with the mathematics teaching in elementary classrooms, teacher educators should look back to preservice classrooms and establish a better foundation for the work necessary to enter and continue in the profession. It can be argued that the work of elementary mathematics teaching can be enhanced by expanding teachers' mathematical pedagogical content knowledge (Ball et al., 2008; Rowland et al., 2009; Davis & Simmt, 2006). Therefore, better understanding the development of such knowledge and the experiences which promote its development is important to establish stronger foundations in preservice teacher education.

Secondly, constructivist learning principles, to which I subscribe, claim that students do not write new knowledge on blank slates nor are the empty vessels to be filled with knowledge pre-packaged by the teacher (Freire, 1970). Rather, constructivism indicates that learning is a process unique to each individual because new knowledge is seen through the lens of an individual's prior knowledge, background, and beliefs (Greeno, Collins & Resnick, 1996). Thus, in a classroom of twenty-five students there may be as many different interpretations of a concept presented in a lesson. A parallel can be made to teacher education in this domain, as well. Although teacher education and mathematics education researchers have suggested "best-practices" and many teacher education courses and programs are structured according to this research, teacher educators must acknowledge that their students, too, construct their own knowledge. As Munby, Russell, and Martin (2001) wrote, "constructivism in teacher education seems to apply only to school-age learners and not to the preparation of their teachers" (p.896). Just as it is possible for P-12 students to have multiple interpretations of content presented in the same lesson, so too is it possible for preservice teachers to interpret the content and experiences they have in courses and field placements in multiple ways.

This is complicated even more by what Lorite (1975) deemed the "apprenticeship of observation." Since preservice teachers have typically been in schools and observed teachers for approximately thirteen to fifteen years in advance of entering a teacher education program, constructivist learning theory indicates this impacts how they evaluate new knowledge and how they integrate this new knowledge into their future practices as a teacher. Therefore, considering "what works" in teacher education must take into account constructivist learning principles. Just because courses and field placements *should* result in positive impacts on many domains, including the development of pedagogical content knowledge, does not mean that experiences will "stick" in the intended way or be translated into practice. Therefore, considering how knowledge growth occurs over a preservice teacher education program addresses a significant gap in existing studies.

In addition to addressing gaps in the research of teacher knowledge, it was my intent to design a study with a substantial philosophical foundation. The problematic nature of disconnects in teacher education between course work and field placements have been explained by researchers (ref). I perceive a similar problematic divide in content area research void of substantial philosophical grounding. Therefore, Dewey's (1916/1985; 1938/1963) philosophy of experience and distinctions between experiences that are educative, mis-educative, and non-educative was used as a way to frame content area research in teacher education while enhancing it with a substantial theoretical foundation. Shulman, Ball, Rowland, and Dewey's work was used in conjunction to make sense of the growth and development that occurred with preservice teachers: Shulman, Ball, and Rowland's work was used to examine and explain the knowledge growth that occurred over the final year in a preservice teacher program and Dewey's work was used to identify and describe the preservice teachers' experiences that contributed to the growth of their knowledge.

Lastly, this study also addressed a gap in current qualitative methodologies. The phenomenon of preservice teachers' knowledge development seemed best suited to a case study design, however case studies are intended to offer specific rather than general understanding of the phenomenon at hand (Grossman, 1990). Given the significant gaps in current research I wanted to have a way to situate the specific understanding of a smaller group of preservice teachers I gained against a larger number of the other preservice teachers in the same cohort. Therefore, I developed a new methodology that I termed “situated case studies” in which data is collected in a series of nested tiers, thus offering a broader scope of data in which to situate the focal case studies. Developing this new methodology provided a way to research more participants and make more substantial suggestions for teacher education than would have been possible using a traditional case study design.

Research Questions

This study spanned the fields of mathematics education and teacher education and addressed gaps in content, theory, and methodology. The dual purpose was to a) describe preservice teachers’ growth in mathematical knowledge for teaching and b) investigate what experiences contribute to preservice teachers’ development of mathematical knowledge for teaching. Specifically, the research questions for the study were as follows:

1. To what extent does elementary preservice teachers’ mathematical knowledge for teaching develop over their last year in a teacher education program?
2. What aspects of mathematical knowledge for teaching translate into elementary preservice teachers’ practices?
3. What experiences are educative for elementary preservice teachers’ development of mathematical knowledge for teaching

CHAPTER TWO

LITERATURE REVIEW

This study investigated mathematical knowledge for teaching, which traces its roots to Shulman's work. In the mid-1980s Shulman introduced the field of teacher education to a new construct, "pedagogical content knowledge" (Shulman, 1986, 1987; Wilson, Shulman & Richert, 1987). Pedagogical content knowledge refers to the specialized subject-matter knowledge that is necessary for teaching. To situate the study and the tradition of pedagogical content knowledge upon which it is founded and that it seeks to extend, pedagogical content knowledge (PCK) will first be described. Second, two bodies of work that attempt to explicate PCK within the field of elementary mathematics education and that the study operationalized for data analysis will be explained (Ball, Thames, & Phelps, 2008; Rowland, Turner, Thwaites, & Huckstep, 2009). Third, the literature on teacher development will be reviewed.

Part One: Pedagogical Content Knowledge

Munby et al. (2001) reviewed the literature and revealed a "root tension (about) what counts as professional knowledge and even how to conceptualize knowledge" (p.878). Differences also exist in the ways that questions about teacher knowledge are addressed, with some work situated within theory and philosophy, and other work situated within an epistemology of practice[1]. Research into pedagogical content knowledge has been of particular interest since 1985 (Munby et al., 2001). Lee Shulman's (1986, 1987) work is most-often linked with research on pedagogical content knowledge. His work with the Knowledge Growth in Teaching study was based on observations of effective teachers. Shulman (1987) explained that effective teachers not only manage students but also ideas, and do so in ways that are flexible, adaptive, and complex. At the time of his writing Shulman indicated policymakers and even teacher educators generally considered teaching to be comprised of basic skills, content knowledge, and general pedagogical skills. Shulman contended that this formulation was problematic because "teaching is trivialized, its complexities ignored, and its demands diminished" (p.6). Additionally, a rush to delineate competencies of effectiveness often oversimplifies the complexities of teaching (Shulman, 1987).

Shulman proposed a focus on pedagogical content knowledge at a time when content and pedagogy were viewed separately and pedagogy ruled the day. In the 1980s content was "conspicuously absent" from teacher education programs and research (Shulman, 1986, p.6). Thus, Shulman (1987) identified the absence of subject matter in studying teaching as "the missing paradigm problem" (p.6). Shulman (1987) explained the lack of focus on subject matter as a blind spot that left several important questions

[1] Carter (1990) categorized research into three different approaches to address teacher knowledge. First, information-processing studies of the 1980s, including expert-novice comparison studies, used psychological frameworks and focused on cognitive processes (Borko, Lalik, & Tomchin, 1987; Borko, Livingston, McCaled, & Mauro, 1988; Rohrkemper, 1989). Second, research was done on teachers' practical knowledge in classroom settings and linked knowledge to specific and personal situations (Elbaz, 1983; Lampert, 1985). Third, research on pedagogical content knowledge integrated what teachers know and how that knowledge is enacted in the classroom within a specific domain.

unasked.[2] Attempts in research and teacher education to make general statements about effective teaching often skipped over "critical features" of teaching including the subject-specific decisions a teacher makes (Shulman, 1987).

To address both oversimplified and underspecified conceptions of teachers' knowledge, Shulman (1987) described seven categories of teacher knowledge: 1) content knowledge, 2) general pedagogical knowledge, 3) curriculum knowledge, 4) pedagogical content knowledge, 5) knowledge of learners and their characteristics, 6) knowledge of educational contexts, and 7) knowledge of educational ends, purposes, and values. A definition of each of these categories is found in the following table.

[2] These questions included, "Where do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it and how to deal with the problems of misunderstanding?" (Shulman, 1986, p.8).

Table 1

Categories of Shulman's (1987) Knowledge Base

Knowledge Base Category	Definition
Content knowledge	Adequacy and accuracy of the subject matter ideas presented
General pedagogical knowledge	Broad principles and strategies of classroom management and organization that appear to transcend subject matter
Curriculum knowledge	Grasp of the materials and programs that serve as "tools of the trade" for teachers
Pedagogical content knowledge	Special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding
Knowledge of learners and their characteristics	Physical and psychological characteristics of students
Knowledge of educational contexts	Ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
Knowledge of educational ends	Purposes, values, and their philosophical and historical grounds

Note: adapted from Shulman, L. S. (1987). Knowledge and teaching: Foundation of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Of the seven categories he identified, Shulman (1987) described pedagogical content knowledge as the category of "special interest because it identifies the distinctive bodies of knowledge for teaching" (p.8). He explained that it was the intersection of content and pedagogy that contained the heart of a distinct knowledge base for teaching. For Shulman (1987), the combination of content and pedagogy revealed "the capacity of the teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p.15). Pedagogical content knowledge "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching"* (Shulman, 1986, p.7, emphasis in original). PCK includes knowing how to use representations, analogies, illustrations, examples, explanations, and demonstrations so that the content is understandable to the learner. It also includes understanding what makes learning a topic difficult or easy, the pre-conceptions students are likely to have, and strategies that will most likely address students' (mis)understandings. PCK "places emphasis upon the intellectual basis for teaching performance rather than on behavior alone" (Shulman, 1987, p.20; see also Dewey, 1904/1964).

The aim of Shulman's (1986, 1987) project was to focus the attention of researchers, policymakers, and teacher educators on the types of knowledge necessary for teaching and the need to conceptualize domain-specific knowledge for teaching. His work called attention to the need for subject-specific development of teacher knowledge, rather than continuing to "confine activity to the content-free domains of pedagogy and supervision" (Shulman, 1987, p.20). Since the construct of PCK was introduced, various researchers have taken up Shulman's project in their own content areas, including mathematics, social studies, and English (Munby et al., 2001). One domain in which there has been an attempt to describe the knowledge base for teaching is elementary mathematics.

Part Two: Two Models of Professional Knowledge for Mathematics Teaching

As Munby et al. (2001) noted, "many bridges remain to be built in both directions between practice and knowledge as we seek to understand the nature of teachers' knowledge and its development" (p.885). There have been a handful of attempts to erect bridges between practice and knowledge in the field of elementary mathematics education. Two such projects are Ball, Thames, and Phelps (2008) and Rowland, Turner, Thwaites, and Huckstep (2009).[3] Both models conceptualize knowledge for teaching mathematics differently and therefore link practice and knowledge differently, as well. The two models will next be described.

Mathematical Knowledge for Teaching. Ball, Thames, and Phelps (2008) have worked over the last fifteen years to identify and describe the mathematical knowledge required for teaching (Ball, 1990; Ball, 1993; Ball, Hill, & Bass, 2005; Hill, Rowan & Ball, 2005; Phelps & Schilling, 2004). The aim of their project is to further develop Shulman's (1986, 1987) notion of pedagogical content knowledge[4], which they describe as foundational because of the attention to the role of content in teaching as well as contending that teaching requires unique specialized knowledge. However, Ball et al.

[3] The discussion of MKT is presented throughout this section in the present tense, because both models of MKT are part of a present discussion within the field. The current and ongoing nature of this discussion is obscured by using the past tense.

[4] Shulman was clear that his work was incomplete and provisional, and that the aim was not to list what teachers needed to know but rather "sought to provide a conceptual orientation and a set of analytic distinctions that would focus the attention of the research and policy communities on the nature and types of knowledge needed for teaching a subject" (Ball et al., 2008, p.392).

(2008) believe PCK has remained “thinly developed” and “underspecified,” (p.389) noting that the term is used differently across and even within subject areas. They believe the combination of a mushy definition and lack of empirical basis limit its usefulness (Ball et al., 2008). Overall, Ball et al. (2008) believe “the field has made little progress on Shulman’s initial charge: to develop a coherent theoretical framework for content knowledge for teaching. The ideas remain theoretically scattered, lacking clear definition” (p.394). Still, they acknowledge the significance of Shulman’s work, noting that “the power of the idea…that teaching requires a special kind of content knowledge is worth our collective investment and cultivation” (Ball et al., 2008, p.394).

To describe professional knowledge within mathematics Ball and her colleagues use the term “mathematical knowledge for teaching” (MKT). They define MKT as “the mathematical knowledge needed to carry out the work of teaching mathematics”, where teaching is defined as “everything that teachers must do to support the learning of their students” (Ball et al., 2008, p.395). Ball et al. (2008) offer four domains and two categories that comprise MKT. The four domains were empirically generated through factor analysis, and the two categories build off of the researchers’ experiences related to teaching elementary mathematics (Ball et al., 2008).[5] Their depiction of the model is provided in the following figure.

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[5] Ball, Thames, and Phelps attempted to extend Shulman’s work through their research projects entitled Mathematics Teaching and Learning to Teach Project and Learning Mathematics for Teaching Project. They empirically identified and defined sub-domains of Shulman’s three subject-related categories within elementary mathematics: subject-matter knowledge, pedagogical content knowledge, and curriculum knowledge (Ball et al., 2008). To develop a practice-based theory, Ball et al. (2008) focused on the mathematical demands of the tasks of teaching. That is, they focused on the knowledge needed to “perform the recurrent tasks of teaching mathematics to students” (Ball et al., 2008, p.399). To do this, Ball et al. examined a qualitative NSF-funded data-base that documented the 1989-1990 school year in a third grade U.S. public school classroom for all mathematics teaching, including “videotapes and audiotapes of the classroom lessons, transcripts, copies of students’ written class work, homework, and quizzes as well as the

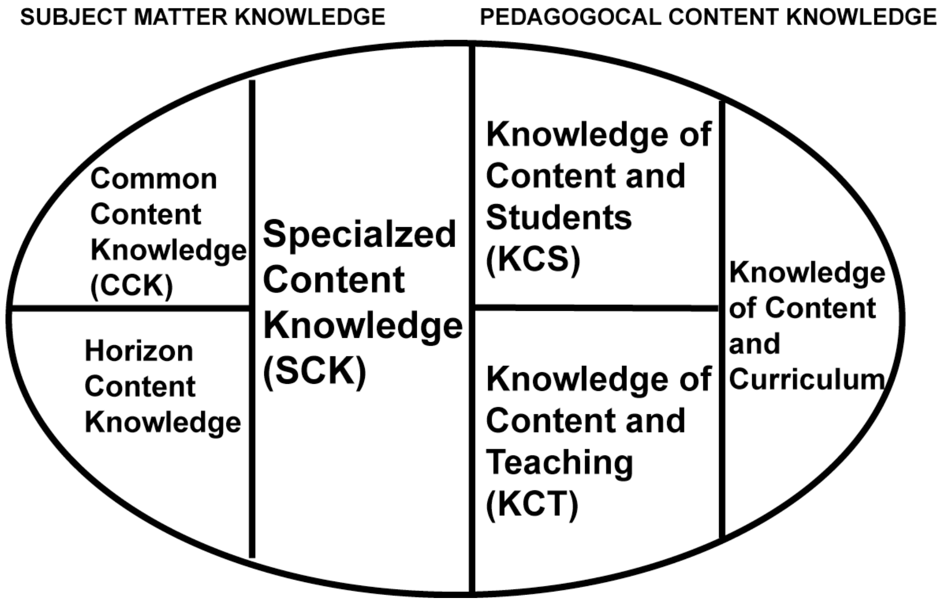

Figure 1. Mathematical Knowledge for Teaching Model: adapted from Ball et al. (2008). This figure shows a visual representation of Ball et al.'s MKT model.

The first domain within MKT is c*ommon content knowledge (CCK)*, which is a sub-domain of Shulman's subject-matter knowledge category. This is defined as "the mathematical knowledge and skill used in settings other than teaching" (Ball et al., 2008, p.399) and involves the ability to perform calculations, solve mathematical problems, give mathematical definitions, and use terms and notation correctly. By calling this knowledge "common" Ball et al. do not mean that everyone has this knowledge, but rather it is knowledge used in many settings and is not exclusive to teaching. They do not intend to trivialize the importance of this knowledge by calling it common, noting that it is essential because "an understanding of the mathematics in the student curriculum plays a critical role in planning and carrying out instruction" (Ball et al., 2008, p.399).

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teacher's plans, notes, and reflections" (p.395). They also developed quantitative survey measures and used factor analysis to determine whether there were sub-domains of pedagogical content knowledge for teaching mathematics (see Phelps & Schilling, 2004).

The second domain of MKT, which is also a sub-domain of Shulman's (1987) subject-matter knowledge category, is s*pecialized content knowledge (SCK)*. This includes the ability to work with mathematical content in a way that is unique to teaching. The SCK domain constitutes knowledge that teachers must have beyond the information they teach to their students. Likewise, these forms of knowledge extend beyond a conceptual understanding of mathematics because they require working with mathematics with "pedagogically strategic intent" (Ball et al., 2008, p.401). Ball et al. (2008) contend that teaching requires using decompressed and unpacked mathematical knowledge "because teaching involves making features of particular content visible to and learned by students" (p.400). Tasks that Ball et al. (2008) identified as both unique to and routine within teaching mathematics and comprise SCK are found in Appendix A.

The third domain of MKT is k*nowledge of content and students (KCS)*, which combines knowing about students and knowing about mathematics. This is a sub-domain of Shulman's (1987) pedagogical content knowledge category. KCS entails the ability to anticipate the way students will interact with mathematics: conceptions and misconceptions, likely errors, things that will be easy, hard, confusing, interesting, or motivating, and the ability to interpret student thinking based on their explanations (Ball et al., 2008). As Ball et al. (2008) explained, "each of these tasks requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (p.401).

The final domain of MKT is k*nowledge of content and teaching (KCT)*, which is a second sub-domain of Shulman's pedagogical content knowledge category and combines knowing about teaching and knowing about mathematics. This domain entails instructional design tasks: how to sequence content, how to select and sequence examples, and the selection of representations, methods, and procedures. KCT also involves the ability to lead a mathematical discussion, which includes making decisions about which student comments to pursue or ignore and when to interject a question or task into the discussion. Each of the tasks within KCT involves the intersection of mathematical knowledge with "pedagogical issues that affect student learning" (Ball et al., 2008, p.401).

Although the four empirically-identified domains receive the most explanation in the literature, two more categories exist within MKT. The first is h*orizon content knowledge*, which is provisionally placed as the third aspect of Shulman's (1987) subject-matter knowledge category. Horizon content knowledge is the awareness of the mathematical trajectory of curriculum. For example, an early grades teacher would know the mathematics of later grades and how the mathematics she teaches relates to that which is to come. Horizon content knowledge entails a "vision useful in seeing connections to much later mathematical ideas" (Ball et al., 2008, p.403).

Ball et al.'s final category of MKT is k*nowledge of content and curriculum*, which is synonymous with Shulman's third category of curricular knowledge. Ball et al. group this under the broader heading of PCK, noting that in later work Shulman's colleagues did likewise (see Grossman, 1990). Ball et al.'s work on MKT provides insights into the work of mathematics teaching and offers a way to identify and discuss teachers' knowledge within elementary mathematics.

Knowledge Quartet. University of Cambridge researchers Rowland, Turner, Thwaites and Huckstep have explored knowledge for teaching mathematics over the past six years (Rowland, Huckstep, & Thwaites, 2005; Rowland & Turner, 2008; Rowland, Turner, Thwaites & Huckstep, 2009; Thwaites, Huckstep & Rowland, 2005). Their aim is to offer a way to build professional knowledge for mathematics teaching. Rowland and his colleagues wanted to build on Shulman's work by considering the specific knowledge required for teaching elementary mathematics. Similar to Ball et al. (2008), Rowland et al. (2005, 2009) focused on Shulman's (1987) three content-specific categories rather than the four categories that are, as they term, generic.

Rowland et al. (2005, 2009) discussed Ball et al.'s (1990, 2005) work and used it to support their comparable position that teachers' mathematical content knowledge is important because of the impact on students' learning and achievement. However, they take a different approach and critique Ball et al.'s development of quantitative measures to ascertain a teacher's MKT (see Phelps & Schilling, 2004). Rowland et al. (2009) cautioned:

> If you were to take such a test, it might give some indication of your pedagogical content knowledge. However, although your responses would give a measure of your *theoretical* pedagogical content knowledge, they might not necessarily reflect how you would act in practice. In order to make such assessments someone would need to actually observe you teaching. (p.24, emphasis added)

Rowland et al. (2009) believe that given the connected nature of PCK and the way a teacher's different knowledge types interact within a teaching scenario, it is problematic to assess PCK void of actual teaching. They ask, "How can you judge whether you will give clear explanations and use the most appropriate resources without knowing the context?" (Rowland et al., 2009, p.25). Therefore, Rowland et al.'s (2009) work is founded on the belief that "mathematical content knowledge for teaching will be most clearly seen in the action of teaching" (p.25). The aim of their project, then, was to "identify different aspects of teacher knowledge that have an impact on teaching from observations of that teaching" (Rowland et al., 2009, p.26). Furthermore, their ultimate goal was to help beginning teachers develop their knowledge for teaching mathematics. They wanted to identify how university supervisors, cooperating teachers, district coaches, and novice teachers "might observe and reflect on mathematics lessons in a way that helped them identify aspects of content knowledge that affected their teaching" (Rowland et al., 2009, p.33). Their model of teacher knowledge is called the Knowledge Quartet (KQ) [6] and is summarized in the following table.

[6] To develop a conception of mathematical content knowledge based in teaching, Rowland et al. (2009) worked with 12 student teachers towards the end of their one-year postgraduate initial teacher training course, which is the typical form of teacher education in the UK. Each student teacher videotaped two mathematics lessons. The 24 lessons were then analyzed to identify portions of the lessons that were informed by the student teacher's mathematical content knowledge or mathematical pedagogical knowledge. These particular moments were then analyzed through grounded theory, which ultimately resulted in 18 codes. Given their goal of practical use in lesson observation and reflection, Rowland et al. (2009) grouped similar codes into four broader categories that function as "big ideas" within elementary mathematics teaching. The four broad categories of teacher's mathematical knowledge comprise the Knowledge Quartet, with each category containing three to seven of the 19 codes (see Appendix B).

Table 2

Rowland et al.'s (2009) Knowledge Quartet Model

Category	Dimension
Foundation	Adheres to textbook
	Awareness of purpose
	Concentration on procedures
	Identifying errors
	Overt subject knowledge
	Theoretical underpinning
	Use of terminology
Transformation	Choice of examples
	Choice of representation
	Demonstration
Connection	Anticipation of complexity
	Decisions about sequencing
	Making connections between procedures
	Making connections between concepts
	Recognition of conceptual appropriateness
Contingency	Deviation from agenda
	Responding to children's ideas
	Use of opportunities

The first category of the Knowledge Quartet is *foundation*. As the name indicates, this knowledge serves as the foundation for the remaining three categories. The three components that comprise a teacher's theoretical background are mathematical content knowledge, knowledge of mathematics pedagogy, and beliefs about how students best learn mathematics. Dimensions of teaching that relate to foundation are the ability to perform calculations and procedures, identify errors, and use terms and notation. This category "has the potential to inform pedagogical choices and strategies in a fundamental way," where fundamental means "a rational, reasoned approach to making decisions about teaching based on something other than imitation or habit" (Rowland et al., 2009, p.30; see also Dewey, 1904/1964). Foundation incorporates two of Shulman's categories: subject-matter knowledge and knowledge of purpose. A theoretical foundation of knowledge about mathematics and its teaching combines within the KQ's first category and serves as a foundation for the three subsequent categories (Rowland et al., 2009).

Transformation is the second category of the Knowledge Quartet. This maps back to Shulman's (1987) category of pedagogical content knowledge, which he described as "the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful" (emphasis added, p.15). Such knowledge, seen in both planning and teaching, takes the form of analogies, illustrations, examples, explanations, and demonstrations (Rowland et al., 2009). Teachers need to be able to use this range of representations at different times and in different ways over the course of teaching, to introduce concepts, demonstrate procedures, and select practice activities.

The third category of the Knowledge Quartet is *connection*, which involves "the coherence of the planning or teaching across an episode, lesson, or series of lessons" (Rowland et al., 2009, p.31). While it does not directly equate with Shulman's (1987) categories, it incorporates aspects of subject-matter knowledge as well as pedagogical content knowledge. Connection includes teachers' knowledge of the connectedness of mathematical content as well as the ability to sequence topics within and between lessons and order tasks, exercises, and activities. Such knowledge is demonstrated both in planning and teaching. Rowland et al. (2009) noted that "the ability to sequence teaching effectively involves making choices based on knowledge of structural connections within mathematics itself and as awareness of the relative cognitive demands of different topics and tasks" (p.31). By considering each of these dimensions, a teacher is more likely to provide students with coherent and continuous mathematical experiences (Rowland, 2009; see also Dewey, 1904/1964, 1938/1963).

Contingency is the final ategory of the Knowledge Quartet, which incorporates all seven of Shulman's (1987) categories. This dimension acknowledges that students' responses cannot be planned the way teacher's actions are planned. Although increased knowledge can improve a teacher's ability to anticipate challenges and likelihoods that may arise in a given lesson, contingency captures the idea that teachers need to be able to "think on their feet." The three main dimensions of this dimension are a teacher's readiness to respond to students' ideas, to deviate from the lesson plan, and to use opportunities that arise in the course of the lesson. Rowland et al.'s (2005, 2009) work on the Knowledge Quartet provides a framework to identify dimensions of content knowledge that affect teaching and can also be used in observations and reflections to develop teachers' mathematical knowledge.

Similarities between MKT and KQ models[7]. Similarities exist between Ball et al.'s (2008) MKT model and Rowland et al.'s (2009) KQ model. One similarity is that both models operate under a positive rather than deficit perspective of teacher knowledge. Their projects consider the types of mathematics that teachers engage in rather than beginning with gaps in teacher knowledge. A second similarity of the two models is the conceptualization that knowledge for teaching mathematics is unique to mathematics teaching. Each research team argues that knowledge required for teaching mathematics is not simply more or deeper mathematical knowledge than that which is expected of students, but rather that teacher knowledge is qualitatively different (Ball et al., 2008; Rowland et al., 2009). A third similarity is that each research team states its model of teacher knowledge is provisional, tentative, and likely to change. Neither team is claiming their model is correct or complete, but rather that it is an attempt to use their experiences and research to date to describe what teachers need to know and be able to do.

In addition to their orientation to teacher knowledge, the MKT (Ball et al., 2008) and KQ (Rowland et al., 2009) models contain largely the same components of teacher knowledge despite differences in arrangement. Rowland et al.'s KQ model captures a few additional aspects of teaching than Ball et al.'s MKT model. To borrow a phrase from Munby et al. (2001) the researcher "assumes a certain compatibility among the different ways of representing knowledge" (p.883) across the two models.

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[7] Ball et al. (2008); Rowland et al. (2009)

Building on the MKT and KQ models. While there are significant areas of overlap, the strengths of each model seem to be useful footholds for investigating the still relatively new domain of mathematical teacher knowledge. In terms of quantitative measurement of mathematical knowledge for teaching, the MKT authors have developed a well-vetted instrument. Hill, Schilling, and Ball (2004) conducted a series of studies to investigate how knowledge for teaching mathematics is organized and defined, with a particular focus on mathematics teaching at the elementary school level. From 1999 to 2004 they developed an assessment that was based on existing, well-known theories in the fields of mathematics and teacher education (e.g., Ball & Bass, 2003; Grossman, 1990; Shulman, 1987; Wilson et al., 1987). The authors constructed "survey-based teaching problems thought to represent components of the knowledge of mathematics needed for teaching" (Hill, Schilling & Ball, 2004, p.12). Data analysis, including factor analysis, indicated there were three dimensions within mathematical knowledge for teaching, including knowledge of content in number concepts and operations; knowledge of content in patterns, functions, and algebra; and knowledge of students and content in number concepts an operations (Schilling, 2002; Hill, Schilling & Ball, 2004). The resulting instrument, the Mathematical Knowledge for Teaching (MKT) instrument, is the primary means by which the field of mathematics education assesses teachers' mathematical knowledge for teaching. Many publications exist detailing the development and validity of the MKT instrument (e.g., Hill, 2007; Hill & Ball, 2004; Hill, Schilling & Ball, 2004; Schilling, Blunk, & Hill, 2007; Schilling & Hill, 2007). A released sample item is found in Appendix C.

Although the quantitative offering of the MKT instrument is substantial, there were advantages to building off the KQ model for the qualitative aspects of this study. To begin, the similar language and terms used by Ball et al. (2008) in describing the four domains and two categories would have made it more difficult to make distinctions between the MKT categories when analyzing data. The MKT authors themselves noted, “It is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions” (Ball et al., 2008, p.403). Alternatively, the four categories of the KQ are arranged into more distinct groupings and therefore its use is preferable for accurate and consistent data analysis.

The KQ model contains an additional aspect of knowledge for teaching that the MKT model does not, found in the KQ “contingency” category. This category acknowledges the real-time decisions and moves teachers must make. The inclusion of this category is especially important for this study, since contingency is an area of known difficulty for preservice teachers (Rowland et al., 2009). Rowland et al. ‘s (2009) KQ model incorporates the big ideas of teaching elementary mathematics in a way that is complicated enough to provide fruitful data analysis while straightforward enough to consistently analyze data through the four distinct KQ categories. The KQ model is also flexible enough to work across a wide range of elementary school curricula.

A potential pitfall to mapping out knowledge for teaching is thinking categories are the literal means by which knowledge is held in teachers’ minds rather than as a heuristic device for helping us think about teacher knowledge (Borko & Putnam, 1996). Teacher knowledge does not exist in discrete categories but is connected across categories and contexts. Despite researchers’ attempts to complexify the teaching of elementary mathematics by identifying categories and proposing heuristics, the act of teaching remains entirely more complicated. In mathematical terms, this is an instance where the whole is greater than the sum of its parts; the act of teaching still exceeds the sum of its heuristical parts even as these parts attempt to complicate our views of teaching. Still, the MKT and KQ models offer a window into teacher knowledge. The ultimate goal of expanding teachers' MKT is to "help to ensure that every child has a qualified teacher" (Hill et al., 2007, p.150). In addition, "Mathematical knowledge for teaching may also act upon…teachers' ability to motivate students to learn, to organize classrooms for productive instruction, and to produce learning goals" (Hill et al., 2007, p.152). Understanding the development of mathematical knowledge of preservice teachers builds upon the literature related to PCK and also teacher development. The teacher development literature will be reviewed next.

Part Three: Teacher Development

The work on PCK within elementary mathematics teaching has focused attention on the subject-specific aspects of teacher knowledge. Sowder (2007) stated that, "We now have at least some knowledge about what a teacher needs to know to be an effective teacher, and this knowledge should guide us in developing programs for teacher education and professional development" (p.166). However, mapping out teacher knowledge should not be conflated with knowing how to teach this knowledge. While research coming out of the tradition of PCK has been instrumental, there remain underdeveloped and undertheorized aspects of PCK. These include a nearly singular focus on improving teaching and learning through the increase of teacher knowledge, as well as a fascination with mapping out the sub-components of this knowledge. To date the map remains void of discernable paths on which to walk en route to the development of the identified knowledge.

The expert teacher, portrayed in studies and teacher education, conveys the image of an autonomous knowledge source. As Britzman (1991) cautioned, “the understanding that all knowledge is a construct and can thus be deconstructed and transformed by the knower is disregarded” (p.230). Leinhardt (1993) argued that “Knowing how experts tend to behave does not help in getting someone to that point, and more importantly, simply copying expertise alone is likely to result in an inappropriate conservatism and lack of innovation” (p. 890; see also Dewey, 1904/1964). Overall, research in teacher education and teacher develop has neglected this crucial aspect. As Munby et al. (2001) explained, “The problem of tracing the path from novice to expert and detailing how expert knowledge is acquired from experience is…a serious omission” (p.890; see also Carter, 1990; Clift, 1989; Munby & Russell, 1994).

While teacher educators often talk about learning to teach, “research insufficiently emphasizes a theory of learning to teach” (Munby et al., 2001, p.893). Here it is helpful to quote Feiman-Nemser and Remillard (1996) at length:

> The phrase learning to teach rolls easily off the tongue, giving the impression that it is a straightforward, easily understood process. In fact, we do not have well-developed theories of learning to teach and the phrase itself covers many conceptual complexities…Knowing what good teachers do, how they think, or what they know is not the same as knowing how teachers learn to think and act in particular ways and what contributes to their learning. (p.63)

Two oversimplified assumptions prevail regarding learning to teach, the first that teaching is learned through trial and error and the second that it is learned through acquiring knowledge and then using it in a practice setting (Feiman-Nemser & Remillard, 1996). Cochran and Jones (1998) examined preservice teachers' knowledge and found that the process of teaching itself increased teacher knowledge and its connectedness, although they noted that "we have very little information on how this occurs." This finding is consistent with constructivist teaching and learning. As Munby et al. (2001) stated, "In the area of science teaching, at least, the sense is that teaching experience makes a substantial contribution to the richness and interconnectedness of teachers' knowledge. This approach merits extension to all subject areas" (p.882). This study extended this work into the development of elementary mathematics teachers. Munby et al. (2001) acknowledged that, despite progress, "the nature and development of (teachers') knowledge is only beginning to be understood by the present generations of researchers in teaching and teacher education" (p.877). This study extended this work.

CHAPTER THREE

PHILOSOPHICAL FRAME

Maxine Greene (1986) asserted the contributions philosophy can make to research. In reference to her work Munby et al. (2001) stated, "philosophic perspectives do not provide guidelines for situation-specific practice. But they can provoke far-reaching questions and more examination of preconceptions and assumptions, opening possibilities for further inquiry and clarification of concepts and terms" (p.879). As Greene (1986) wrote, "If the 'doing' of philosophy moves researchers and teachers to do more thinking about their own thinking, it is justified. If it intensifies the wonder with regard to teaching, enhances awareness of what remains unsolved, philosophers may have accomplished what they have set out to do" (p.499).

Munby et al. (2001) noted that the field has acknowledged the importance of experience for the development of teacher knowledge, but tends to do so in an oversimplified way:

> Without a treatment of the epistemology of experience and of experiential knowledge, the review implies that the having of an experience is enough for the development of the knowledge of teaching…(This assumes) two unproblematic transfers: an unproblematic transfer of experience to propositional knowledge, and an unproblematic transfer of propositional knowledge to teaching actions. (p.883)

For these reasons, John Dewey's philosophy of experience was used to focus and intensify thinking about experiences that take place in teacher education, specifically in regards to the development of elementary teachers of mathematics.

Dewey's Philosophy of Experience

Current readers of Dewey's work must take care to not read too superficially, lest we may misappropriate or oversimplify his philosophies. Particular attention must be given to Dewey's use of terms and language that may sound similar or even the same as present-day language, but may in fact have been used differently by him.[8] Integral concepts including education, experience, continuity, and interaction must be understood in order to understand his philosophy of educative experience. This is necessary because, as Archambault (1964) explained, "(Dewey's) educational principles can only be understood in the context of his total philosophy" (p.xxix). After establishing a foundation with Dewey's integral concepts, the characteristics of educative experiences will be described.

Dewey's (1916/1985) conception of education is that it is a "reconstruction or reorganization of experience which adds to the meaning of experience, and which increases ability to direct the course of subsequent experience" (p.82). Education, therefore, is experience-based and thus one must take care to understand the way in which Dewey conceptualized experience in order to have an appropriate conception of education. Dewey explained this by stating, "There is an intimate and necessary relation between the processes of actual experience and education. If this be true, then a positive and constructive development of its own basic idea depends upon having a correct idea of experience" (Dewey, 1938/1963, p.20). Dewey conceptualized experience as something distinct from the prevalent notion of "everything or anything that happens to a person" (Hook, 1985, p.x). Additionally, he explained that "mere activity does not constitute

[8] Thanks to Lynda Stone and her helpful facilitation of the Dewey reading group in which this idea was discussed.

experience" (Dewey, 1916/1985, p.146). Experiences are present-centered interactions between people, objects, and environments such that there is the formation of or impact on habits, attitudes, preferences, aversions, sensitivities, and/or responsiveness. As a result of experiences, future objective conditions are changed and particular ends are easier or harder to attain (Dewey, 1938/1963). Hook (1985) explained, "As Dewey uses the term, (experience) refers to a pattern of events in which the organism is deliberately or with some awareness attending or acting upon something and undergoing or suffering the consequences of the action" (p.x). Although learning can and does happen unconsciously in a process that begins "almost at birth" (Dewey, 1897/1964, p.427), an experience requires an increased level of consciousness (Dewey, 1916/1985).

It is important to note that although education, growth, and development are interchangeable terms for Dewey, education and experience are not. To be sure education and experience are related concepts, as Dewey (1938/1963) explained that "education is a development within, by, and for experience" (p.28), but they are not synonymous. This is true for two reasons. First, all experiences have an impact, but not all experiences are educational. Dewey (1938/1963) wrote in *Experience and Education*, "The belief that all genuine education comes about through experience does not mean that all experiences are genuinely or equally educative" (p.25). Second, Dewey believed that learning can occur not only from one's direct experience but also from the experiences of others. Therefore there could be learning without a first-hand experience, which would not be captured if one equated education with experience.

Although they are not synonymous, Dewey (1938/1963) believed there is an "organic connection between education and experience" (p.25). The relationship between education and experience is that "education is the process by which on the basis of present experiences we make future experiences more accessible, meaningful, or controllable" (Dewey, 1916/1985, p.x). Education serves as the process by which experiences are reorganized and restructured such that learning, growth, and development can occur (Dewey, 1916/1985, 1938/1963).

Dewey's Philosophy of Educative Experience

Dewey (1938/1963) explained that a theory or philosophy of experience is necessary to conceive of education based on experience:

> If, then, I am asked why I have spent so much time on expounding a rather abstract philosophy, it is because practical attempts to develop schools based upon the idea that education is found in life-experience are bound to exhibit inconsistencies and confusions unless they are guided by some conception of what experience is, and what marks off educative from non-educative and mis-educative experience. (p. 51)

If one rushes to develop practical applications too quickly, "they are at the mercy of every intellectual breeze that happens to blow" (Dewey, 1938/1963, p.51). In order to begin developing a conception of experience Dewey (1938/1963) explained, "I have tried to illustrate the need for such a theory by calling attention to two principles which are fundamental in the constitution of experience: the principles of interaction and of continuity" (p.51). Interaction and continuity are two concepts which function in tandem to integrally support Dewey's conception of educational experiences, which he termed educative experiences (Fishman & McCarthy, 1998). In order to understand Dewey's ideas about experience and subsequently educative experience, one must consider his conception of continuity and interaction.

It is important to note that although continuity and interaction are named separately, this is only due to the convention of language and ought not suggest a separation or hierarchy between the two (Dewey, 1938/1963). Dewey (1938/1963) described continuity and interaction as organic wholes that have an intimate and active union. He wrote, "the two principles of continuity and interaction are not separate from each other. They intercept and unite. They are, so to speak, the longitudinal and lateral aspects of experience" (Dewey, 1938/1963, p.44). Additionally, Dewey (1938/1963) believed that the organic whole of continuity and interaction "provide the measure of the educative significance and value of an experience" (p.45). Therefore, continuity and interaction will first be described as concepts and second discussed as criteria for educative experiences.

The concept of continuity. Just as Dewey (1938/1963) described growth and development as continuous processes, so too are experiences continuous (Fishman & McCarthy, 1998; Talisse, 2000; Webb, 1976). Although an experience is not everything that happens, every experience does contain a measure of continuity. Dewey's (1938/1963) conception of continuity means that "no experience lives or dies to itself…every experience lives on in further experiences" (p.27). There is an inherent projection of past experiences into the present and present experiences into the future, such that experience is continuous and without definitive ends or stopping points (Webb, 1976). As Dewey (1938/1963) explained, "(T)here is some kind of continuity in any case since every experience affects for better or worse the attitudes which help decide the quality of further experiences, by setting up certain preference and aversion, and making it easier or harder to act for this or that end. Moreover, every experience influences in some degree the objective conditions under which further experiences are had" (p.37).

Dewey's conception of the continuity of experience has two implications. First, it amplifies the impact of any given experience, as it will live into the future. Second, the forward-feeding nature of experience also requires one to consider the future in order to discuss a present experience. Continuity of experience means that "every means is a temporary end[9] until we have attainted it. Every end becomes a means of carrying activity further as soon as it is achieved" (Dewey, 1916/1985, p.113). Using Dewey's conception of continuity as a basis, its relationship to educative experiences will now be considered.

Continuity as criteria. Although every experience "has" continuity, the quality of the experience determines the way continuity plays out in that and subsequent experiences (Dewey, 1938/1963; Fishman & McCarthy, 1998). As Dewey (1938/1963) wrote, "Every experience is a moving force. Its value can be judged only on the ground of what it moves toward and into" (p.38). Dewey referred to experiences that move a person towards and into positive ends as educative. Although evaluation of an experience ultimately needs to occur on a case by case basis, there are some broad statements that can be made as to the "types" of experiences that Dewey deemed educative.

Related to the concept of continuity, an educative experience is one that promotes growth in knowledge, mental processes, attitudes, and as a cumulative result, expands the

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[9] Dewey termed the consideration of the fluidity of means and ends as end in view (Webb, 1976). For Dewey (1916/1985), "the doing with the thing, not the thing in isolation, is (the) end" (p.112).

possibilities for the future. The primary criterion for educative experiences is that they contribute to growth. As Dewey (1916/1985) wrote, “the educative process is a continuous process of growth, having as its aim at every stage an added capacity of growth” (p.59). Dewey (1938/1963) explained that “Growth, or growing as developing, not only physically but intellectually and morally, is one exemplification of the principle of continuity” (p.36). He further explained the link between continuity and growth by stating, “Growing is not something which is completed in odd moments; it is a continuous leading into the future” (Dewey, 1916/1985, p.61). Dewey (1938/1963) believed that continuity, growth, and reconstruction of experience meant that every experience should prepare a person for deeper and more expansive experiences in the future. Broadly defined across several areas, growth was Dewey’s primary criterion for educative experiences.

Educative experiences result in growth not only in the accumulation of knowledge but also in the development of positive mental processes. Dewey (1916/1985) made a distinction between training and education, where the former is concerned with answers and outward actions and the latter is focused on the development of metal processes. As Dewey (1938/1963) wrote, “no experience is educative that does not tend both to knowledge of more facts and entertaining of more ideas and to a better, a more orderly, arrangement of them” (p.82). This indicates that not only must one’s factual knowledge increase, but in turn one must be opened up to increased possibilities through the restructuring and reorganizing affect of experience. Merely learning information without also expanding the quality of mental processes is insufficient to be considered educative (Dewey, 1916/1985, 1938/1963). Routine or repeated action may increase skills and even fix habits, but in as much as one lacks awareness of his/her responses the experience remains within the realm of training rather than education (Dewey, 1916/1985). The result is that our habits "posses us, rather than we them" (Dewey, 1916/1985, p.34; see also Kestenbaum, 1977; Webb, 1976). It follows that educative experiences are active experiences that lead to growth in mental processes (i.e., critical thinking and reflection).

Educative experiences also contribute to growth in one's attitude (Webb, 1976). As Dewey (1916/1985) wrote, "the criterion of the value of school education is the extent in which it creates a desire for continued growth and supplies means for making the desire effective in fact" (p.58). The continuity of an educative experience is such that it "arouses curiosity, strengthens initiative, and sets up desires and purposes that are sufficiently intense to carry a person over dead places in the future" (Dewey, 1938/1963, p.38). Educative experiences generate additional positive possibilities because of the way they are carried into the future through one's attitude, which is "fundamentally what counts in the future" (Dewey, 1938/1963, p.48). Fishman and McCarthy (1998) summarized Dewey's ideas on this topic by writing, "For Dewey, continuity is educationally effective when a sequence of experiences, despite occasional cul-de-sacs and detours, is so driven by deeply held purposes that it coheres, develops, and finds fulfillment; for example, a problem is solved, a discussion is played out, a piece of writing reaches consummation" (p.32). The combination of growing in the moment and for the future embodies Dewey's conception of educative experiences.

To contrast the above description of educative experiences as ones centered on growth, mis-educative experiences will be briefly considered, which are experiences with negative ends. Mis-educative experiences are detached, isolated, or sporadic and do not build upon each other or towards a consistent end. This disconnectedness results in the formation of habits that do not help generate self-control or control of future experiences. Instead, these habits have "the effect of arresting or distorting the growth of further experience" (Dewey, 1938/1963, p.25). Mis-educative experiences "engender callousness" and reduce sensitivity and responsiveness (Dewey, 1938/1963, p.25). It is important to note that using enjoyment as a criterion for experience should be avoided, because "an experience may be immediately enjoyable and yet promote the formation of a slack and careless attitude." This is problematic because such an attitude will interfere with future growth (Dewey, 1938/1963, p.26). Experiences also mis-educate when too great an appeal is made to preparation for the future, with the false assumption that the acquisition of skills in the present will translate into future application of these skills under new conditions (Dewey, 1938/1963). In sum, the continuity of mis-educative experiences results in a trajectory or cumulative affect that "narrow(s) the field of further experience" (Dewey, 1938/1963, p.26) and causes a "low plane of development, in a way which limits later capacity for growth" (pp.37-38).

The concept of interaction. Just as every experience contains some form of continuity, so too does every experience contain some degree of interaction (Fishman & McCarthy, 1998; Talisse, 2000). As Dewey (1938/1963) explained, "An experience is always what it is because of a transaction taking place between an individual and what, at the time, constitutes his environment"[10] (p. 43). Dewey did not believe that experiences

[10] The environment includes people, subjects, and/or objects and is any and every condition that "interact(s) with personal needs, desires, purposes, and capacities to create the experience which is at hand" (Dewey, 1938/1963, p.44). Experiences are created by interaction and cannot occur void of interaction, because at the very least an experience always occupies a physical environment and therefore there is interaction between the individual and the surrounding objects. The environment can also extend beyond the visible and external (see Webb, 1976).

occurred exclusively within an individual (Talisse, 2000). As he stated, "It ought not to be necessary to say that experience does not occur in a vacuum" (Dewey, 1938/1963, p.40). Rather, he believed in conjoint experiences in which each person present is both a participant in and impacted by the experience (Dewey, 1916/1985, 1938/1963; Webb, 1976). Dewey (1934/1964) believed social interaction was crucial, because "the material of genuine development is that of human contacts and associations" (p.11). The quality of interaction, and in particular social interactions, contribute to the educative nature of an experience.

Interaction as criteria. Just as the concept of continuity can be used to inform the distinction between experiences which are educative versus those that are mis-educative, so too can the concept of interaction be used in this way. One criterion to consider is whether interactions and experience occur in a thoughtfully planned environment. Dewey (1916/1985) explained that "We never educate directly, but indirectly by means of the environment" (p.23). Environments can be carefully planned to encourage experiences therein to be educative, or the environment can be left haphazard, chaotic, and disintegrated and therefore experiences therein are likely to be the same (Dewey, 1916/1985, 1938/1963). The environment needs to be "deliberately regulated with reference to its educative effect" (Dewey, 1916/1985, p.23). In order to provide an educative experience, the concept of continuity must be considered in terms of the trajectory of any given experience. Furthermore, in order to enhance educative experiences in regards to interaction one must intentionally select materials and methods that will encourage growth, since there is interaction between the learner and the objects and subjects of the environment (Dewey, 1916/1985).

Using Dewey's conception of interaction, a second criterion to consider whether an experience is educative is if there are opportunities for positive social interactions. Similar to the continuity of experience, although every experience "has" interaction the quality of the experience is determined, in part, by the type of interaction that exists. Given Dewey's conception of education as growth and development arising out of social contexts, social interaction must be considered as a criterion for educative experiences. Dewey (1938/1963) wrote, "The principle that development of experience comes about through interaction means that education is essentially a social process" (p.58). This is not to say that every educative experience must entail social interaction, but Dewey encouraged social interactions to occur whenever possible in order to promote growth.

In *Democracy and Education* (1916/1985), Dewey cautioned that books and conversations can be helpful but are over-utilized in schools, whereas social activities are under-utilized: "Schools require for their full efficiency more opportunity for conjoint activities in which those instructed take part, so that they may acquire a *social* sense of their own powers and of the materials and appliances used" (p.45, emphasis in original). Through increased contact and communication with others, more expansive ends are possible (Dewey, 1938/1963). The social environment "is truly educative in its effect in the degree in which an individual shares or participates in some conjoint activity. By doing his share in the associated activity, the individual appropriates the purpose which accentuates it, becomes familiar with its methods and subject matters, acquires needed skill, and is saturated with its emotional spirit" (Dewey, 1916/1985, p.26). Consistent with Dewey's holistic perspective, the reason for social interactions is not only that they promote skills and academic knowledge but also that they encourage broader learning of processes, development in social, emotional and moral dimensions, and expand one's possibilities for the future. Fishman and McCarthy (1998) summarized Dewey's philosophy on this topic by writing: "Interaction is educationally worthy when classroom conditions and student interests are synchronized so that students maximally explore—get the fullest meaning they can from—their school environment" (p.32).

Summary. Educative experiences are those which are present centered and contain continuity and interaction such that they serve as a positive trajectory for future experiences. In regards to continuity, educative experiences promote growth in knowledge and skills, mental processes, and attitudes and outlooks. In regards to interaction, educative experiences provide intentional environments within which interactions take place with objects, subject matter, and people. Of particular importance is the quality of social interactions that an experience promotes. It is the intent that the cumulative impact of these educative experiences would promote growth in mental processes through a focus on intellectual responsibility and initiative rather than imitation.

For Dewey, continuity and interaction function as twin pillars upon which experience is built and by which it can be judged. Fishman and McCarthy (1998) explained educative experiences with the following, which can be applied to teacher education: "Whether a particular experience is educative depends on the *quality* of its continuities and interactions, on students taking a residue of greater rather than lesser curiosity and initiative, sensitivity and responsiveness, desire and purpose from their classrooms" (pp.31-32, emphasis in original). As Dewey (1916/1985) wrote in *Democracy and Education*, "Experience, in short, is not a combination of mind and world, subject and object, method and subject matter, but is a single continuous interaction of a great diversity (literally countless in number) of energies" (p. 174). Experience as continuous interaction offers a more complex way to think about both evaluating and planning experiences using Dewey's criteria for educative experiences as a foundation.

Educative Experiences in Teacher Education

Given his broad conception of growth, development, and education, Dewey's philosophies can be applied beyond the schooling of children. It is the assertion of the researcher that the educative criteria illuminated through considering the continuity and interaction of experience can be used to consider what constitutes educative experiences for teachers. Given that most of Dewey's work on educative experiences was about children learning things for the first time, his writings most directly apply to teachers who are likewise in the initial process of learning how to teach. Additionally, Dewey's piece *The Relation of Theory to Practice in Education* (1904/1964) linked educative experiences and the education of new teachers. For these reasons this study applied Dewey's criteria for educative experiences based on the twin pillars of continuity and interaction to the education of preservice teachers.

Given that Dewey (1938/1963) himself stated "there is no such thing as educational value in the abstract" (p.46), it is helpful to use the previously delineated concepts to consider what would constitute an educative experience for preservice teachers. In *The Relation of Theory to Practice in Education* (1904/1964), Dewey addressed the education of preservice teachers; he called attention to pitfalls of prevalent turn of the century practices and also encouraged more thoughtful possibilities related to learning to teach. Dewey wrote about field experiences, which he termed practice work, including classroom observations, supervision, and lesson planning. These areas will now be explored in terms of continuity and interaction.

In *The Relation of Theory to Practice in Education* (1904/1964) Dewey described two approaches to teacher education, the apprentice approach and the laboratory approach, whose core philosophies are so different that they "radically alter the amount, conditions, and method of practice work" (p.313). Depending on the approach adopted "the amount of time given to practice work, the place at which it is introduced, the method of conducting it, of supervising, criticizing, and correlating it, will differ widely" (Dewey, 1904/1964, p.314). Next, whether each approach can be deemed educative will be considered, based on whether it promotes experiences that contribute to growth in knowledge and skills, mental processes and attitudes.

Dewey (1904/1964) described the approach where preservice teachers gain the skills, techniques, and tools they will need for teaching as one of apprenticeship (Tanner, 1997). Under this method the immediate and long-term aim is one of practicality. This coupled with a short amount of time in the training school results in an overall aim of "independent mastery of practical skill, rather than at turning out at once masters of the craft" (Dewey, 1904/1964, p.315). As a result, practice work takes the form of learning to imitate a teacher, with a heavy emphasis on classroom management (see also Strong & Baron, 2004).

In contrast, Dewey described the laboratory approach as a second approach to teacher education. Dewey (1904/1964) stated the purpose of the laboratory approach was to make theory "real and vital" and to "supply the intellectual method and material of good workmanship" (p.314). These intellectual methods then become a means to the ultimate aim, which is for teachers to be students of teaching. Based on these aims, practice work is focused on "giving the student a better hold upon the educational significance of the subject-matter he is acquiring and of the science, philosophy, and history of education" (Dewey, 1904/1964, p.314). Practice work takes on a very different look and feel than it did in the apprentice approach. The major components of teacher education (i.e., university coursework, lesson planning, supervision, and student teaching) will be described and evaluated in terms of whether or not they meet Dewey's criteria for educative experiences. This will be done so the reader will see that Dewey's criteria for educative experiences can be applied to preservice teachers and also to lay a foundation for the data analysis of this study.

Relation of coursework to field placements. Dewey's first criterion[11] for educative experiences was creating a purposeful environment in which interactions and experiences will occur. To this end, Dewey (1904/1964) called for "convergence and concentration" (p.322) of training school (university) and practice school (field work) experiences. He stated that the conditions in the practice school should "stand in the closest and most organic relation to the instruction in subject-matter which is given by the teachers of the professional school" (p.331). When there are disconnects between the

[11] Upon trying to evaluate the educative capacity of Dewey's (1904/1964) recommendations for pre-service teaching field work based on the core criterion of continuity and interaction, it becomes apparent that it is not possible to separate the two criterion in order to first discuss Dewey's recommendations that only relate to interaction followed by those that only relate to continuity. Dewey's holistic philosophy meant that instead of making recommendations based solely on one concept or the other he made recommendations based upon both concepts, since they are, as he wrote, inseparable (1938/1963). Therefore, Dewey's recommendations for pre-service teaching will be described and then discussed in terms of both continuity and interaction.

training school and practice school, it is often because either the training class is too isolated from classroom application, or because the conditions in the elementary practice school result in trivializing subject matter for the sake of drill rather than thinking. Convergence between the training and practice school would result in increased continuity of experiences for preservice teachers across teacher education programs (Dewey, 1904/1964).

Convergence would likewise improve the educative potential of interactions taking place in both the training and practice school settings by increasing the amount and quality of preservice teachers' interactions with subject matter. The laboratory method elevates the role of liberal arts coursework. Dewey (1904/1964) believed that scholastic knowledge "may itself be the most effective tool for training and turning out good teachers" (p.327). It is important to remember that for Dewey subject-matter and method are not distinct but rather intertwined, lest he be misinterpreted as advocating against pedagogical training. Still, he wrote that increased interactions with subject matter and focus on scholarship may have higher yields than training in pedagogy because of the focus on metal activity. In sum, the convergence of the training and practice school would improve the continuity of preservice teachers' experiences between the two settings and would also increase their interaction with scholastic knowledge and thereby mental activity, all of which increase the educative potential of preservice teaching experiences.

Another way the laboratory approach capitalizes on the continuity of experience is by integrating preservice teachers' P-12 experiences. Given the concept of continuity, teachers' P-12 schooling experiences will live on in their classrooms (Dewey, 1938/1963;

Lortie, 1975; Webb, 1976). Dewey (1904/1964) suggested intentionally using past experiences, both negative and positive, to offer practical examples of incidents of arrested and positive growth and thereby provide a personal link between theory and practice within the training school. When past experiences are not acknowledged in teacher education, it conveys the implicit message that there is a divide between the mental activity that takes place within the classroom and that which takes place in all other settings. Operating under a holistic philosophy, Dewey saw this separation of the classroom from life as artificial and problematic. As Dewey (1904/1964) stated, the preservice teacher "comes to believe in the potency for schoolroom purposes of materials, methods, and devices and it never occurs to him to trust in his experience outside of school" (p.323). The convergence of the training school with preservice teachers' past experiences also improves the transition to the practice teaching classroom because the preservice teacher has already gained experience sifting through, selecting and organizing aspects of her own experience (Dewey, 1904/1964).

Field placements. A second way that interaction operates as a criterion for educative experiences is that the environment should promote educative social interactions. The apprenticeship method maintains a traditional role of the university supervisor. Dewey (1904/1964) explained the impact this has on lesson planning: "The student who prepares a number of more or less set lessons; who then has those lesson plans criticized; who then has his actual teaching criticized from the standpoint of success in carrying out the prearranged plans, is in a totally different attitude from the teacher who has to build up and modify his teaching plans as he goes along from experience gained in contact with the pupils" (p.317). The aim of practice teaching for skill mastery sets up the preservice teacher's focus on the interactions between herself and the supervisor, not between herself and the students. In what amounts to a scathing indictment of traditional approaches to supervision worth quoting at length, Dewey (1904/1964) wrote:

> No greater travesty of real intellectual criticism can be given than to set a student to teaching a brief number of lessons, have him under inspection in practically all the time of every lesson, and then criticize him almost, if not quite, at the end of each lesson, upon the particular way in which that particular lesson has been taught, pointing out elements of failure and success. Such methods of criticism may be adapted to giving a training-teacher command of some of the knacks and tools of the trade, but are not calculated to develop a thoughtful and independent teacher. (p.335)

Due to the environment of practice teaching, preservice teachers are not genuinely interacting with students. The deference to the university supervisor instead of students results in mis-educative experiences because of the distortion of social interactions. Further, Dewey (1904/1964) stated that "the habit of making isolated and independent lesson plans for a few days' or weeks' instruction in a separate grade here or there…is likely to be distinctly detrimental" (p.332).

One of Dewey's criterions for educative experiences is that the role of interaction must be considered, first in regards to creating a purposeful environment in which interactions and experiences will occur. Dewey (1904/1964) believed that the environment of practice teaching in the apprentice method was too controlled and did not approximate the actual conditions of teaching because the most significant features of real schools and classrooms were intentionally eliminated. Dewey (1904/1964) wrote, "As matter of fact, the 'best interests of the children' are so safe-guarded and supervised that the situation approaches learning to swim without going *too near* the water" (p.317, emphasis in original). The stripping of realistic experiences from practice teaching occurs by reducing the number of students with whom the preservice teacher works, not giving the preservice teacher sufficient responsibility for classroom discipline, the presence of the expert cooperating teacher and supervisor who at any point can, and often do, step in to make suggestions or help, and the way lesson planning and observation is conducted (see also Grossman & McDonald, 2008). The result is preservice teachers are not given an adequate opportunity to grow within practice teaching because the conditions of the environment in which they interact are both prescriptive and restrictive. Additionally, there is a lack of continuity from practice teaching into the profession where the safety nets are removed at once, often leaving new teachers overwhelmed with the realities of teaching (Dewey, 1904/1964).

Under the laboratory approach, a key aspect of the practice teaching environment is the intentionality with which practice teaching is planned and sequenced. The interactions of practice work begin with a focus on observing the students rather than imitating the teacher. The aim of initial classroom observations, explained Dewey (1904/1964), is not to memorize methods of the cooperating teacher, but rather to observe interactions of a group of minds learning together. The focus on the psychological rather than the practical enhances the educative potential of initial classroom observations, as growth in preservice teachers' mental processes rather than skills are the focus. As a result, preservice teachers will not only know *what* works, but better understand *how* and *why* they work. Habits are more likely to be fixed with reference to principles from psychology, subject matter, logic, history of education, or philosophy rather than to external behaviors of children (see also Shulman, 1987). Dewey maintained that growth in skills will still occur under this method. He explained, “It would be very strange if practice work in doing what the laboratory does for a student of physics or chemistry in a way of securing a more vital understanding of its principles, should not at the same time insure some skill in the instruction and management of a class” (Dewey, 1904/1964, p.314). The development of mental processes increases continuity with future teaching and shifts the focus of interaction from imitating the expert teacher to learning from a group of students. Thus the focus of initial observations on psychology and reflective thinking increases the educative potential of experiences for preservice teachers.

One aspect of the laboratory approach is field placements that provide interaction with the curriculum in consecutive and longitudinal ways rather than only cross-sectionally. After having time to observe groups of students, Dewey (1904/1964) recommended intermediate field placement experiences that span several grade levels. Dewy (1904/1964) cautioned against the traditional approach of preservice teachers writing lesson plans for a few days or weeks in separate grades, because “what is needed is the habit of viewing the entire curriculum as a continuous growth, reflecting the growth of mind itself” (p.332; see also Ball et al., 2008; Davis & Simmt, 2006; Rowland et al., 2009). Dewey recommended having preservice teachers interact with subject-matter and its development over the entire grade span of the school before adapting subject matter to sets of lessons for particular grades. Such experiences would expand preservice teachers’ schemes for the development of subject matter and offer “some conception of the educational movement of the school as a whole” (Dewey, 1904/1964, p.333). As a result, there would be less of a tendency towards imitation.

Intermediate experiences that provide longitudinal rather than cross-sectional views of children, subject-matter, growth, and classrooms would continue to broaden preservice teachers' conceptions about the continuity of K-5 schooling rather than promoting narrow ideas of growth and development. Such field experiences would also shift the focus from the actions of the teacher to interactions with children, subject matter and growth. The cumulative impact of offering field placement experiences spanning several grade levels would be the promotion of educative experiences because of the continuity and interaction the experiences provide (Dewey, 1904/1964). In her description of Dewey's lab school, Tanner (1997) explained the way teachers worked with curriculum across grade levels, which is important because "a clear picture of the curriculum in its cumulative sense is necessary if we want the curriculum to have a cumulative effect. Indeed, there is something almost perverse about teachers who cannot see beyond their own grades…such people cannot be trusted to guide others; indeed, we fear for their own footing" (p.164).

Following initial observations focused on observing interactions of students and a field placement spanning multiple grades, Dewey (1904/1964) recommended a "more intimate introduction to the lives of the children and work of the school" (p.334). This would occur through preservice teachers operating as helpers of the classroom teacher in ways that were genuinely helpful to the teacher, students, and school. This is important so that the conditions of practice teaching can be consistent with those of the actual classroom, lest they devolve into merely training the preservice teacher. The gradual introduction of the preservice teacher into the classroom maintains the focus on psychological and subject-related principles, without which premature emphasis on mechanics and imitation can result. This enhances the educative nature of practice teaching by providing authentic interactions whose continuity allows for the increase of responsibility and development of maturity as a practice teacher (Dewey, 1904/1964).

After a solid foundation of experiences in subject-matter, educational theory, and observation, Dewey (1904/1964) recommended preservice teachers treat the classroom as a laboratory in which teaching experiments occur. To this end, Dewey (1904/1964) recommended removing the safety nets that often limit practice teaching experiences and instead giving preservice teachers "the maximum amount of liberty possible" (p.334). Under this conception of practice teaching, preservice teachers would not only be permitted but expected to rely on their own intellectual initiative. This kind of re-visioning of practice teaching requires a re-conceived role for the university supervisor. In the spirit of offering more liberty and promoting more initiative, Dewey recommended the supervisor provide the preservice teacher with less supervision and criticism and more emphasis on self-evaluation and reasoning about both success and failure, with an aim of being thoughtful about principles rather than a simplified identification of "good" and "bad" methods (Tanner, 1997; see also Darling-Hammond & Sclan, 1992). In so doing, interactions between the preservice teacher, her students and subject matter become the focus rather than interactions with the supervisor. This offers more educative potential because it approximates the interactions of actual teaching and therefore provides continuity from teacher education to the profession (Dewey, 1904/1964).

Finally, after going through the experiences in all of the previous stages, Dewey (1904/1964) stated that the preservice teacher would be ready to undergo training of the apprenticeship approach. Dewey explained that he was not opposed to training in skills and techniques, so long as this was situated after training in educational theory and history, subject-matter, observation, and laboratory-method practice work. Even when implementing apprentice methods as a final set of experiences, Dewey argued that some of the laboratory methods needed to be sustained. Chief among these is that the preservice teacher's level of initiative and responsibility was to remain as high as possible, with low levels of supervision and criticism (Dewey, 1904/1964). Dewey (1904/1964) acknowledged this final set of apprentice-focused experiences would add to the amount of time teacher education required. Therefore he noted that if there was not sufficient time within the practice teaching school then this final step could be completed independently as a new teacher rather than prolonging the teacher education program beyond what it could bear (Dewey, 1904/1964).

Focus. At its broadest level, Dewey's critique of the apprentice method was against the focus on outward behaviors to characterize teaching. Dewey (1904/1964) believed a preservice teacher's attention was diverted from understanding learning and children while she focused on mastering skills. This can lead to an illusion of teaching, where the preservice teacher has an orderly classroom and therefore appears to be teaching, but in fact lacks the necessary mental processes to thoughtfully engage children with subject matter. Also, a focus on visible skills can result in a preservice teacher abandoning her own thinking in attempt to master teaching through imitation of the cooperating teacher rather than relying on her own initiative (Dewey, 1904/1964).

Based on Dewey's (1938/1963) second criterion of continuity, the apprenticeship method's focus on imitation within practice teaching experiences largely results in mis-educative experiences for preservice teachers. Dewey (1904/1964) acknowledged that under this method teachers would increase their outward skills for teaching, especially within the realm of classroom management. Despite growth in skills, the apprenticeship method results in stifled knowledge growth overall because there is little perceived need for and attention given to scholastic knowledge and intellectual responsibility. Preservice teachers' habits become fixed based on the trial and error of the moment and what more experienced teachers do. This is because preservice teachers in the apprenticeship model tend to use external student behavior as their reference point for changing or maintaining their method of teaching, rather than using the principles from subject matter, psychology, logic, history of education, or philosophy (see also Shulman, 1987). Regarding this tension Dewey (1904/1964) explained that while principles of related fields may seem dominant, "in practice, the moving forces are the devices and methods which are picked up through blind experimentation; through examples which are not rationalized; through precepts which are more or less arbitrary and mechanical; through advice based upon the experience of others" (p.320).

The focus of the laboratory method on developing mental processes of preservice teachers results in an increased opportunity for educative experiences. Beyond promoting knowledge growth more than the apprentice method, the distinct advantage of the laboratory method is that the ultimate goal and therefore focus of practice teaching is much more far-sighted than immediate practical concerns. Dewey's criterion that educative experiences promote growth in mental processes is met through a focus on intellectual responsibility and initiative rather than imitation. Additionally, Dewey's criterion that educative experiences promote growth in attitudes is enhanced by the laboratory method, because preservice teachers are encouraged to develop deeper beliefs of what it means to teach and be a teacher as they become students of teaching. The focus on encouraging present and future initiative through the development of mental processes will increase and improve the resources available to a teacher and the likelihood that she will be able to carry herself over dead places in the future (Dewey, 1938/1963). For these reasons, the laboratory approach offers consistent opportunities for preservice teachers to have educative experiences that are based upon Dewey's philosophy of experience, and in particular the concepts of continuity and interaction.

While teacher education experiences may not fall into two distinct categories as readily as the descriptors of the apprenticeship or laboratory approach, it does seem that a teacher education program may be more closely aligned with one approach or the other in terms of the connection between university coursework and field placements, the way field experiences are sequenced and laid out, the approach to lesson planning, and the role of the university supervisor. Moreover, the lengthy descriptions were provided to demonstrate they way Dewey's philosophy of educative experiences can be used as criteria to evaluate experiences that happen within teacher education. This study is framed by Dewey's philosophy of educative experiences and offers a philosophical foundation and language with which data will be analyzed, which will be described in a subsequent chapter.

CHAPTER FOUR

METHODOLOGY—STUDY DESIGN AND DATA COLLECTION

The purpose of this study was to describe the development of preservice teachers' development of MKT over the course of their final year in a teacher education program. This chapter describes the study design and participants. It also presents a description of the data collection that occurred.

Part One: Situated Case Study Design

The primary task of this study was to make sense of three focal preservice teachers' experiences over the course of their last year in a teacher education program, as it relates to the three research questions. The study employed a methodology that the researcher developed and termed "situated case studies." Because the research questions drove the methodology, the majority of data sources were qualitative in nature. Qualitative data was most helpful to understand the knowledge growth and experiences of preservice teachers as asked in the research questions. Although one quantitative measure was used, given the predominantly qualitative nature of the research questions, it is inappropriate to characterize the study design as mixed methods when so much weight was given to qualitative work.

The situated case study design was developed by the researcher to situate the three focal case studies against a larger backdrop of data that was gathered from other members of the same cohort. This new methodology builds on the strengths of case studies which contribute a holistic examination of a phenomenon that results in a richer understanding of the phenomenon. In addition, the data and results for the three focal participants was situated within a broader set of data, leading the researcher to term this expanded methodology as “situated case studies.” This new methodology directly addresses one of the primary critiques of traditional case studies, that of their lack of generalizeability. By using a situated case study methodology, similarities and differences of the experiences and growth in MKT of the three focal participants could be compared to their peers, which, depending on the data source, ranged from n=8 to n=35 in a tiered data collection design. Although various numbers of levels could be used within a situated case study design, nested tiers of data collection are the primary characteristic of situated case studies. The specfic form used in the study at hand and will now be further explained as one possible way to emply a situated case study design.

Some advantages of quantitative research were incorporated into the study by using a quantitative instrument to collect data from a larger number of participants and thus create a broader context in which to situate the follow-up qualitative work. To expand on the quantitative data collected and further situate the case studies, qualitative methods were employed with larger sample sizes and included analysis of written reflections, focus group interviews, and individual interviews. For the three focal participants mathematics teaching observations and post-observation interviews were also completed. Data collection spanned preservice teachers’ senior year and began in August 2009 and continued until April 2010.

Given the situated case study design, the number of participants was tiered depending on the extent to which the preservice teachers were willing to participate (Tiers One, Two, and Three) and the level of depth required for data collection and analysis (Tier Four). At the broadest level, all 35 preservice teachers agreed to complete the quantitative pre-test, post-test, and two written reflections in the Fall 2009 semester. Thirty-three of these preservice teachers also completed the quantitative measure again in April, 2010[12]. At the second tier, eleven of the 35 preservice teachers volunteered to participate in four focus group interviews over the course of the year (two in Fall 2009 and two in Spring 2010) and submitted their math lessons plans for the year. Besides volunteering to participate, there was no additional selection criteria. At the third tier, eight preservice teachers also volunteered to participate in four individual interviews over the course of the year (two in Fall 2009 and two in Spring 2010). Again there was no additional selection criteria beyond volunteering. In a situated case study design each tier is nested within the former, meaning that the preservice teachers who participated in the individual interviews also participated in the focus groups, written reflections, and quantitative measure.

Sampling. At the highest tier, Tier Four, purposeful sampling was used to select four of the eight preservice teachers who participated in all other forms of data collection to observe and videotape as they taught four mathematics lessons during their Spring 2010 student teaching. The teacher education program in the university at hand places student teachers in districts that use either EnVisions or Math Expressions as their district-wide mathematics curriculum. Of the eight possible preservice teachers who participated in all other aspects of data collection, two were placed in a classroom that

[12] The other two declined to participate

used Math Expressions as the mathematics curriculum in their district. The remaining six preservice teachers were placed in a classroom that used the EnVisions curriculum. Initially, it was of interest to the researcher to use purposeful sampling to select two preservice teachers in each of the two curriculae (resulting in four preservice teachers total) in order to consider whether interaction with a curriculum seemed to result in similarities or differences in the development of MKT over the course of their senior year. Therefore, at the end of the Fall 2009 semester the two preservice teachers placed in Math Expressions classrooms were approached about and volunteered to participate in the observational portion of the study. Wendy[13] was placed in a first grade classroom, and Molly was placed in a fouth grade classroom.

Of the six possible choices from the EnVisions classroom, several were excluded from consideration based on unique circumstances. In the first example, several weeks into the fall semester one preservice teacher changed her field placement classroom due to a significant personality clash with her original cooperating teacher. This student even changed schools and districts. In the second example, two preservice teachers had cooperating teachers who were going out on maternity leave during the spring semester and therefore they would not have a cooperating teacher throughout the entire year. These three students had unique experiences that were not typical of the majority of their peers and therefore they were not selected as a focal preservice teacher in this study given the focus on describing preservice teachers' growth and experiences over the year. There is no claim on the part of the researcher that the absence of such variables equates with the existence of a "typical" experience since each preservice teacher's experiences are

[13] All names are pseudonyms.

unique within the intersection of an extensive number of factors including herself, her cooperating teacher, her students, the curriculum, PLC members, resources, facilities, and policies. However, all three of these preservice teachers were going to change cooperating teachers or go without one, which was atypical. Therefore purposeful sampling was used to select two of the three preservice teachers who would have the same cooperating teacher throughout the entire year.

Of the remaining three preservice teachers, one was placed in a second grade classroom and two were placed in a fourth grade classroom. Therefore, to similarly select a preservice teacher from the lower grades and upper grades as were the two preservice teachers in Math Expressions, the preservice teacher in second grade and one of the two in fourth grade were originally approached about further participating in the observational portion of the study. The determination of which of the two preservice teachers to ask who were placed in fourth grade was again done through purposeful sampling. One of the preservice teachers, Holly, had more experience teaching than her peers because her mother was a teacher at a local private school and Holly substitute taught there many times. Again, this was a unique situation for Holly and so the other two remaining preservice teachers, Alice and McKenzie, were approached. Although both initially agreed to participate in the observations, upon returning from the semester break in January 2010 Alice indicated she was overwhelmed with the upcoming student teaching and would rather not participate in the observational portion of the study.14 Therefore, Holly was approached and volunteered to participate. This resulted in both preservice teachers who were using EnVisions were placed in fourth grade classrooms.

[14] Alice continued to participate in the focus groups and individual interviews throughout the entire study.

Although four preservice teachers were observed during the Spring 2010 semester, only three of their observational data was considered for the present report of the study. This was again the result of purposeful sampling, because the fourth case, (Name), did not talk very much in focus groups, gave short answers in individual interviews, and had relatively brief post-observation interviews. This contrasted with her three peers, Wendy, McKenzie, and Holly, who were all examples of what Patton (1990) refered to as "information-rich cases," which is the essence of purposeful sampling. Patton explained, "the logic and power of purposeful sampling lies in selecting information-rich cases for study in depth. Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research" (p. 169). Since the purpose of this study was to learn about preservice teachers' growth and experiences largely through their comments at various types of interviews, I selected the three participants' for Tier Four who were conversational during the study to report on. Therefore, Wendy, McKenzie, and Holly's data was analyzed and will be reported in this document.

The following figure depicts the nested tiers of data collection in the study, where participants completed all of the data collection in the tier(s) under and including their own (e.g., Tier Four participants completed all of the data collection elements in Tiers One, Two, Three and Four).

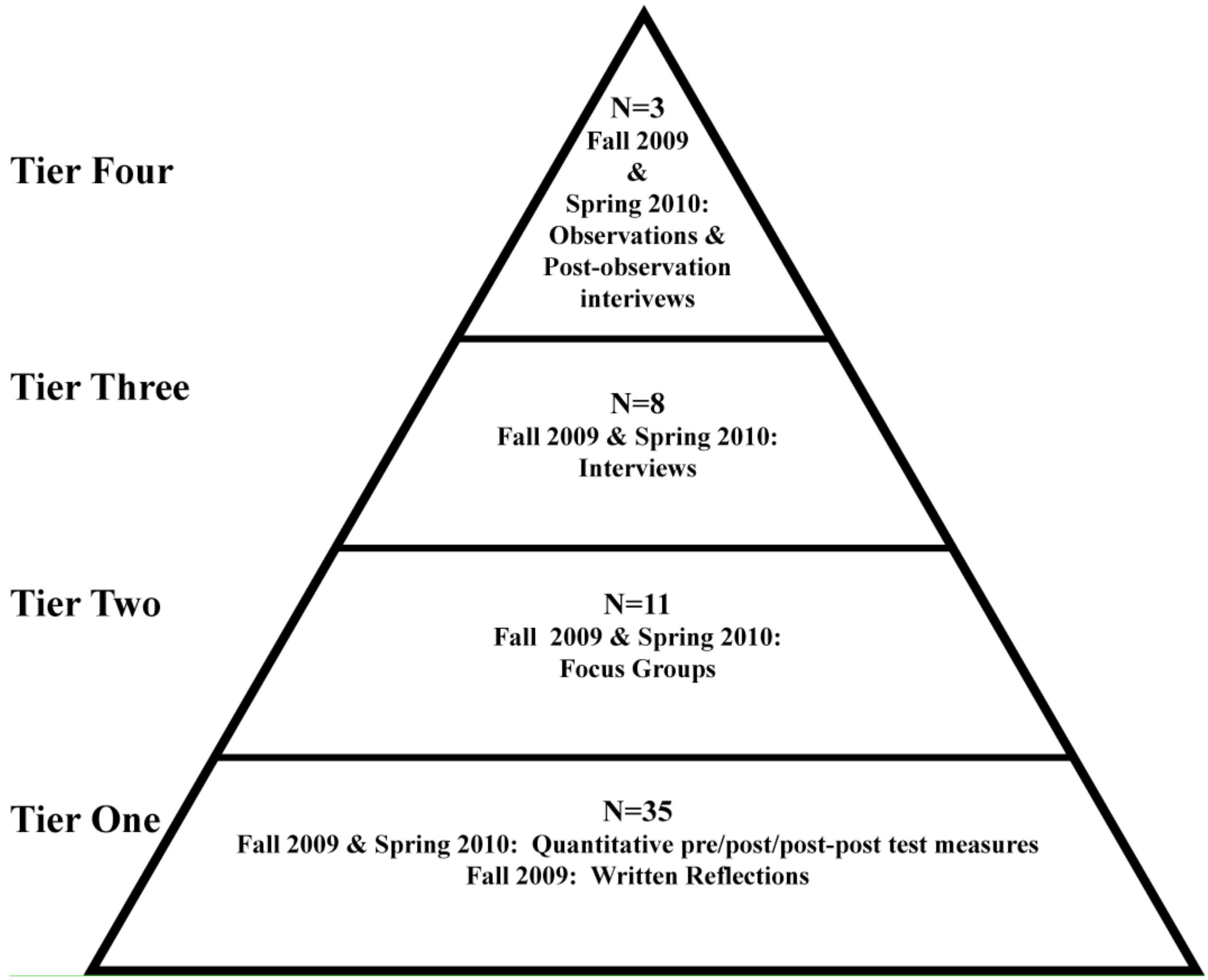

Figure 2. Situated case study design: Nested tiers of data collection. This figure shows a visual depiction of the new methodology for data collection that was used in this study.

Participants. The subjects for this study were 35 preservice teachers, all of whom were senior elementary education majors enrolled in the same section for their required mathematics methods course at a university in the southeastern United States. Thirty-four of the participants were female and one was male. There were three African-Americans, one Latino, and 31 Caucasians. The ethnic and gender breakdown for each tier of participation is found in the following table.

Table 3

Demographics of each participation tier

Tier	n	Ethnicity	Gender
One	35	Latino/a: 1 African American: 3 Caucasian: 31	Females: 34 Male: 1
Two	11	Caucasian: 11	Female: 11
Three	8	Caucasian: 8	Females: 8
Four	3	Caucasian: 3	Female: 3

University context. The elementary education program at the university at hand follows a traditional, four-year design in which students apply to the program in March of their sophomore year and spend their final two years at the university in the School of Education. The degree consists of general college requirements that are completed during the first two years; professional core courses in child development, learning, theory, social foundations of education, teaching methods; and a major academic concentration in one of the following areas: the arts, language and literature, mathematics and science, or social studies. For the cohort in the present study, field work began in the fall of Junior year with a bi-weekly course that was taught at a public school site and included three hours per week of field placement time with a requirement to teach one lesson individually or with a peer. In the spring of their Junior year, the present cohort of students participated in a second bi-weekly course that was taught at a different public school site and included three hours per week of field placement time and a requirement to teach a two or three lesson "unit". In the fall of their senior year, students were enrolled in content area methods courses and participated in a weekly field placement all day on Wednesday. This is the same classroom in which they complete their student teaching, which is the exclusive activity during spring semester of their senior year. Completion of the program results in a Bachelor of Arts in Education and state licensure, pending passing PRAXIS scores. The program is approved by the state department of public instruction and the National Council for the Accreditation of Teacher Education (NCATE).

Fall 2009: Methods course. All 35 participants were enrolled in the same section for all of their methods courses during Fall 2009, including the mathematics methods course which was a four credit semester-long required course for senior elementary education majors. It met once a week for four hours and was a freestanding course that did not have a field placement component, although the preservice teachers attended their student teaching placement each Wednesday for the entire day. The course focused on reform-based mathematics teaching and learning. The syllabus stated the following as the course description: “Students study mathematics content for grades K-6 as well as materials, techniques and teaching aids.” During the Fall 2009 semester there were two sections of the course that were taught by different professors. Participants were recruited from only one section so that they would be exposed to similar experiences in the way they learned to teach mathematics and thereby did not introduce another variable into the study by selecting participants from two different sections. Further, it was predicted that having participants from the same section would promote more discussion in the focus groups since everyone was already familiar with one another. Participants could also build off of each other’s comments since they were more likely to be familiar with anecdotes that were shared by virtue of their shared experiences in their methods courses.

Spring 2010: Student teaching. Some studies have indicated that the most influential aspect of teacher education is the student teaching placement, with the cooperating teacher being the most influential person in terms of the development of practices by the preservice teacher (see Cochran-Smith, 1991; Zeichner & Tabachnick, 1981). If this is an accurate depiction of teacher education, it means that the content specialists who teach methods courses are less influential despite their subject-specific expertise. Therefore, it was important to consider development of mathematical knowledge for teaching that occurred both during the methods course and into the spring semester student teaching placement. Student teachers had much more time observing and teaching mathematics in the spring semester since they attended their field placements every day in Spring 2010 instead of once per week as they did in Fall 2009. It seemed likely that the increased time spent observing their cooperating teacher teach mathematics, interacting with students who were working on mathematics, teaching the mathematics unit they planned in Fall 2009 as the culmination of their methods course, and full-time student teaching would have an impact on their development of mathematical knowledge for teaching.

In Spring 2010, preservice teachers attended their student teaching placement every day throughout the semester. In the beginning of the semester they began "picking up" areas to teach, usually beginning with parts of the day such as morning meeting or read aloud. In February, preservice teachers usually taught their math, science, and literacy units that they wrote in Fall 2009 as part of their methods coursework. In March

2010 they usually taught full-time. The Elementary Education program requires that student teachers teach all day, every day for a minimum of three weeks and a maximum of six weeks. The length of full-time teaching is determined with the cooperating teacher and university supervisor and is dependent on a) schedules and b) the perceived readiness of the student teacher to take on the responsibilities of full-time teaching. In Spring 2010 preservice teachers were formally observed every week by their cooperating teacher and every-other week by their university supervisor. They attend a bi-monthly seminar with their university supervisor and the other preservice teachers who also were assigned to the same supervisor.

Focal Preservice Teachers. In addition to the general description of the program, methods coursework, and student teaching given in the previous section, the three focal preservice teachers will be introduced individually. The following section will explain the mathematics content background of Wendy, McKenzie and Holly in terms of their high school and college mathematics coursework and general disposition towards the field of mathematics, as indicated in focus group and interview comments.

Wendy. Wendy took a mathematics course each of her four years in high school. She took geometry as a freshman, algebra II as a sophomore, pre-calculus as a junior, and AP Statistics as a senior. Although she did not remember her exact math SAT score, she thought it was in the range of 650-700. At the university Wendy did not take any courses that were strictly math content courses. In addition to the math methods course that her entire cohort took in the fall of their senior year, Wendy took two additional courses that were cross-listed with the math department and taught within the School of Education. One of the courses was required and the second was not. The required course is three credits; students usually take it during their sophomore or junior year but sometimes take it the fall of their senior year. The purpose of the course is to develop an in-depth understanding of real numbers and algebra through content focused on problem solving and mathematical reasoning. The optional course investigates ways elementary concepts in mathematics can be developed and applied.

McKenzie. McKenzie took a mathematics course each of her four years in high school. She took geometry as a freshman, algebra I and II as a sophomore, pre-calculus as a junior, and AP Calculus as a senior. She participated in a program called MathCounts that involved team competitions at a local university that she participated in with her classmates. McKenzie scored 700 on the math portion of the SAT. At the university McKenzie took one course that was strictly a math content course about the integration of mathematics with art. Like Wendy, McKenzie took the the required math methods course in the fall of her senior year as well as the two additional courses that were cross-listed with the math department and taught within the School of Education, one of which was required and the second was not.

Holly. Holly took a mathematics course each of her four years in high school. She took geometry as a freshman, honors algebra II/trigonometry as a sophomore, honros calculus as a junior, and AP Calculus (Calculus AB) as a senior. Holly scored 590 on the math portion of the SAT. At the university Holly took one course that was strictly a math content course that she described as "a very low level algebra class." Holly took the the required math methods course in the fall of her senior year and the one required course that was cross-listed with the math department and taught within the School of Education on real numbers and algebra. She did not take the optional course.

Researcher

Background. Since the majority of the work in this study is qualitative, it is important to name my own background, beliefs and biases in regards to preservice teachers and teacher education. Therefore, in this section I will describe my role as a researcher. I am a white female who was born in the North Eastern United States and attended university in a neighboring region of the country. In high school I took advanced mathematics courses including AP Calculus. I began college as an architecture major and after completing my first year of studies decided to change my major to elementary education. For the next two years I participated primarily in school of education courses. At the end of my Junior year, I applied and was selected to participate in a year-long professional development school experience as an alternative to traditional student teaching. Following completion of the program, I was a primary grades elementary school classroom teacher for one year in the mid-atlantic and three years in the south-east. During my time as a classroom teacher I enacted reform-based mathematics practices in my teaching and instructional decisions and excelled in teaching mathematics.

While in graduate school I have had numerous experiences in the elementary education program. First, I was a TA for the required mathematics specialty content course on rational numbers and algebra. During this time I attended all class meetings and taught some of the sessions. The following year I was an instructor of the fall semester senior year math methods course. Therefore I am familiar with the syllabus, content, and assignments of the course. I have also participated in other aspects of the elementary education program, including being an instructor of the fall semester junior year educational psychology course and the associated field placement. In addition to a high level of familiarity with the courses in the elementary education program, I also have been a university supervisor for elementary education student teachers for the past three years. For all of these reasons, I have had significant experiences with the contexts, populations, and material related to this study, which both prompted the development of the research questions and enabled me to collect and analyze relevant and appropriate data and use my expertise to ensure the success of this study.

One experience in particular was pivotal during my time as a student teaching supervisor because it influened my thinking and wondering about student teaching, the role of supervison, and teacher education and was the catalyst for developing the research questions that then developed into the present study. Therefore, it will be related in some length to make my initial thinking at the outset of the study more tranparent to the reader. This story is also included not because it is unique but because it is, I suspect, a story to which most supervisors will be able to relate.

During the 2008-2009 school year I supervised elementary preservice teachers for the first time. Working with preservice teachers was the main reason I left my first grade classroom to go to graduate school so it was with much anticipation that I began interacting with a handful of preservice teachers in this new role. I supervised at the elementary school at which I last taught before beginning graduate school. One of my student teachers was placed in a fifth grade classroom with a former colleague and friend.

Although I had observed this preservice teacher teach one lesson in the fall and several in the spring, it was not until March that I observed her teach a lesson in mathematics. It was a geometry lesson about chords and diameters--and it was a mess.

At various points in the lesson she gave incomplete and incorrect definitions of chords and diameters. The lesson addressed only procedural knowledge with no attempts at conceptual understanding (see Skemp, 1978). Her representations clouded rather than clarified the mathematics she was trying to explain. The examples she selected and their sequence seemed haphazard. I was especially discouraged since the lesson took place after her methods course but she seemed to implement little that was taught during her semester-long exposure to reform-based mathematics education.

Despite all of this, the real shock came when I filled out her evaluation. I cringed as I reached for the form and thought something along the lines of "This is not going to be good." The first category was about planning. Did she have the objective listed? *Check.* Did she have her materials listed? *Check*. The next category was about instruction. Did she have her materials ready? *Check*. Did she use multiple approaches? *Check*. I finished the two-page form and looked back over it. Almost every item was checked. How was this possible? Based on my knowledge of mathematics teaching and learning I was certain the lesson had been a flop at best, and quite possibly detrimental to students given the errors she made in her definitions and explanations. But this was not captured on the evaluation other than in the "suggestions for improvement" box at the end that I crammed with specific comments on her mathematics teaching.[15]

The post-conference was quite awkward. She thought the lesson went well. When asked why, she referenced many of the items on the evaluation form. She was pleased that she followed the lesson plan as written and did not deviate for student comments or questions. She spoke at length about her classroom management. When I

[15] The observation form was changed from the one described beginning in the 2009-2010 school year. Instead of using a checklist it now offers broad categories in which supervisors record evidence and feedback.

talked with her about her mathematical definitions, selection and sequence of examples, and practice work problems she seemed perplexed as to why I was giving her suggestions about things that were not even on the form. I am pretty sure she wrote off my suggestions as overly picky.

This experience caused me to reflect on supervision as well as teaching. First, I wondered what the evaluation and post-conference would have focused on if the supervisor did not have a background in mathematics education; it seems likely that in this case the boxes would have been checked and the post-conference would have been about generic aspects of teaching such as classroom management and pedagogy (Strong & Baron, 2004). Likewise, I wondered if I essentially did this as I observed literacy, science, or social studies lessons; did I lack the content specific knowledge to give feedback with any depth when observing lessons in areas other than mathematics? Additionally, I thought about teaching and how it seemed like there were things unique to teaching mathematics, and therefore likely other subjects, which were not captured by a general description of what elementary teachers do. Finally, although I was not this student's instructor, I had taught a different section of the same math methods course she took in the fall that should have prevented this lesson from being presented in this way. I was curious about preservice teachers' MKT development over their final year in teacher education throughout their required fall math methods course and spring student teaching experiences.

Role of the researcher. The role I played as a researcher in this study was one of an "active learner" (Creswell, 1998, p. 18). Most of the interactions I had with the preservice teachers in this study were held in settings other than their student teaching or university classrooms. When I was present for the observational portion of the study, I was more of an observer than a full participant in the classroom. Although I would sometimes circulate during work time to help students, there was limited time to fully participate in the setting during four classroom visits and the majority of my time in each classroom was spent video recording the preservice teacher's interactions. Therefore, it is more accurate to describe my role in the field as an observer rather than a participant.

In the re-telling of their experiences, I have used the preservice teachers' voices through direct quotations as much as possible to maintain the preservice teachers' ownsership of their stories rather than telling their stories from my perspective. My primary motive for conducting this study was a desire to sincerely understand the preservice teachers' experiences over their final year in a teacher education program. Therefore, I entered this study with openness to their stories, experiences, and challenges. I set aside my preconceptions of the relative strengths and weaknesses of this specific teacher education program as well as traditional teacher education, in general. I asked follow-up questions and practiced re-voicing during interviews and focus groups to confirm or clarify what the preservice teachers said in real-time to best understand their experiences as they wanted to express them. This validated my interpretations as a researcher.

Given the primarily qualitative nature of this study, a critical aspect of this work was understanding and interpreting participants' experiences and doing so in a transparent and trustworthy way. This was done through credibility, transferability, and confirmability, each of which will be described in the subsequent sections.

Credibility. Patton (1990) explained that the credibility of qualitative research is dependant on three essential aspects: "(1) rigorous techniques and methods for collecting data that are carefully analysed, with attention to issues of validity, reliability, and triangulation; (2) credibility of the researcher, which is dependent on training, experience, previous work, status, and presentation of self; and (3) fundamental appreciation of naturalistic inquiry, qualitative methods, inductive analysis, and holistic thinking" (p. 464). I established credibility within this study through prolonged engagement with the participants and triangulation (Creswell, 1998).

The credibility of this study was established, in part, through the prolonged engagement I had with the preservice teachers. This was especially true at Tiers Two, Three, and Four, where I had long and meaningful interactions with the participants through regularly meeting with them for focus groups, interviews, teaching observations and/or post-observation interviews throughout an entire academic year. Over this time, I developed relationships with the preservice teachers through exhibiting sincerity in learning about their experiences as an active listener who sought to understand. The relationships developed between myself and the participants is further evident through many of them voluntarily participating in an extension of the study through continuing to meet for focus groups and teaching observations throughout their first year as classroom teachers, which indicates a level of comfort and relationship that the participants felt with me. Prolonged and meaningful contact contributed to the credibility of the research.

Triangulation was another way that the credibility of this study was established, which was done through "comparing and cross-checking the consistency of information derived at different times and by different means" (Patton, 1990, p. 467). Through the four nested tiers of data that included the quantitative measure, written reflections, lesson plans, focus group interviews, individual interviews, teaching observations, and post-observation interviews, there was an extensive amount and variety of data to compare against itself. Multiple sources of data were used to address the three research questions. Data triangulation across the four tiers of study participation contributed to the credibility of the study.

Part Two: Data Collection

Data collection occurred in the form of one quantitative measure and several qualitative measures in the form of written reflections, focus group interviews, individual interviews, lesson plan analysis, mathematics teaching observations, and post-observation interviews. Data collection methods for the study will be described per each of the three research questions. After detailing the data that was collected, how the data helps answer the research question will be explained.

Research Question One: To what extent does elementary preservice teachers' mathematical knowledge for teaching develop over their last year in a teacher education program?

MKT instrument. Hill, Schilling, and Ball (2004) conducted a series of studies to investigate how knowledge for teaching mathematics is organized and defined, with a particular focus on mathematics teaching at the elementary school level. From 1999 to 2004 they developed an assessment that was based on existing, well-known theories in the fields of mathematics and teacher education (e.g., Ball & Bass, 2003; Grossman, 1990; Shulman, 1987; Wilson et al., 1987). The authors constructed "survey-based teaching problems thought to represent components of the knowledge of mathematics needed for teaching" (Hill, Schilling & Ball, 2004, p.12). Data analysis, including factor analysis, indicated there were three content groupings within mathematical knowledge for teaching: knowledge of content in number concepts and operations; knowledge of content in patterns, functions, and algebra; and knowledge of content in geometry (Schilling, 2002; Hill, Schilling & Ball, 2004). The resulting instrument, the Mathematical Knowledge for Teaching (MKT) instrument[16], is comprised of three separate assessments, one for each content strand of knowledge for teaching. The MKT instrument is the primary means by which the field of mathematics education quantitatively assesses teachers' mathematical knowledge for teaching. Many publications exist detailing the development and validity of the MKT instrument (e.g., Hill, 2007; Hill & Ball, 2004; Hill, Schilling & Ball, 2004; Schilling, Blunk, & Hill, 2007; Schilling & Hill, 2007). A sample item can be found in Appendix C.

Of the three content strands for which there is an MKT instrument, the participants' methods course most directly addressed knowledge in number concepts and operations (NCOP). Therefore, it was within this domain that one would expect the most growth to occur. In order to be able to make a stronger claim that growth in number concepts and operations was the result of methods coursework and student teaching, an internal control was used each time the NCOP instrument was administered. The patterns, functions, and algebra (PFA) assessment was used as an internal control because this domain of knowledge was not specifically addressed in the participants' methods course. Therefore, it was anticipated that scores in number concepts and operations would rise while scores on patterns, functions, and algebra would remain constant over the three administrations. Using the PFA MKT instrument as an internal control was

[16] For the sake of clarity the measure will be referred to throughout this document as "the MKT instrument" to distinguish from the "MKT model" (Ball et al., 2008) or MKT meaning "mathematical knowledge for teaching."

recommended by and discussed with Geoffrey Phelps, one of the primary developers of the MKT instrument (personal communication, August 20, 2009).

The MKT instrument in number concepts and operations as well as patterns, functions, and algebra was given to the 35 participants in August 2009 at the second class meeting of their methods course[17]. This data provided a quantitative baseline for the participants' mathematical knowledge for teaching at the beginning of their senior year in advance of their methods course. A parallel posttest was given to the 35 participants at the conclusion of the course in early December 2009. This data was an indication of participants' mathematical knowledge for teaching at the conclusion of their methods course. It also served as a pre-test or baseline measure of their mathematical knowledge for teaching prior to their full-time student teaching placement in the spring of 2010. Finally, the MKT instrument was given to 33 participants at the end of student teaching in April 2010. This data was an indication of the participants' mathematical knowledge for teaching at the conclusion of student teaching and upon completion of their teacher education program, and likewise served as a pre-test or baseline measure for their mathematical knowledge for teaching prior to their entry to the profession.

The quantitative data from the MKT instrument contributed to answering the first research question, which asked the extent to which the preservice teachers' mathematical knowledge for teaching developed over the year. Data collected from the MKT instrument provided a way to quantitatively measure preservice teachers' mathematical knowledge for teaching, and scores between the time points were compared as an indication of knowledge growth.

[17] This was the earliest it was possible to administer the MKT instrument, because the study was introduced and consent forms were filled out at the first class meeting.

Written reflections. To expand on the initial quantitative data from the MKT instrument, qualitative methods were also employed. The 35 Tier One participants who completed the MKT instrument also contributed qualitative data to the study. Qualitative data in Tier One took the form of the 35 participants' written reflections that were class assignments for their methods course. This allowed documentation of the preservice teachers' thoughts and ideas about mathematical knowledge for teaching. Two prompts were given during the Fall 2009 semester. The first prompt was given in September and asked the students to write one to two pages on the following:

> The purpose of the (MKT instrument) you took last class is to measure a person's "mathematical knowledge for teaching." What do you think *mathematical knowledge for teaching* entails? What experiences have contributed to your mathematical knowledge for teaching?

This prompt was given at its particular time point for three reasons: 1) to collect qualitative baseline data from all 35 participants, 2) to provide a natural starting point for the discussion in the first wave of focus groups, which were held approximately one week later, and 3) so written reflections could be followed-up on during the focus group.

The second prompt was given at the last class of the semester on December 8, 2009 and asked the students to write two to three pages on the following:

> What do you think *mathematical knowledge for teaching* entails? How have your thoughts about *mathematical knowledge for teaching* changed over the course of the semester?
>
> Please describe the two most significant experiences that have contributed to your growth as a teacher of mathematics. First explain the experience and then explain why it was significant.

The prompt was given at this time point for four reasons: 1) to collect qualitative data at the very end of the semester, 2) to allow the collection of qualitative data from all 35 participants and capture growth and related experiences that occurred during the semester, 3) so that the focus group participants had something to begin their second focus group meeting discussing, 4) so written reflections could be followed-up on during the second wave of focus groups.

The written reflections contributed to answering the first research question because both prompts directly asked for the preservice teachers' definitions and thoughts about mathematical knowledge for teaching. The first written reflection provided an indication of preservice teachers' initial conceptions of mathematical knowledge for teaching and what it entailed. The second written reflection provided an indication of changes in their thinking about mathematical knowledge for teaching. The written reflections were one way to capture preservice teachers' thoughts about and growth in mathematical knowledge for teaching.

Focus group interviews. Data collection at Tier Two included the addition of focus groups. Eleven of the 35 participants who completed the MKT instrument and written reflections also participated in focus groups throughout the 2009-2010 school year. This was all of the people who were willing to participate in a focus group; there were no additional selection criteria. The demographic and student teaching placement information on the eleven focus group participants is presented in the following table.

Table 4

Tier Two participants: MKT instrument, written reflections, and focus group interviews (n=11)

Ethnicity	Gender	Student teaching placement (Grade: n)
Caucasian:11	Female: 11	Kindergarten: 1 First: 1 Second: 2 Third: 2 Fourth: 3 Fifth: 2

The eleven participants were divided into two focus groups, one group of five and one group of six, to keep groups of a size where people felt comfortable participating. Assignment to focus group A or B was determined by participants' willingness, at that point in time, to participate in a follow-up individual interview. Those who initially indicated they were not interested in participating in a follow-up individual interview were placed in Focus Group A, which was held first in order to pilot the focus group protocol (Appendix D: non-italicized items). The individuals who were initially interested in participating in a follow-up individual interview were placed in Focus Group B, which met three days after Group A. Minimal additions, in the form of follow-up questions, were made to the protocol in advance of Focus Group B (Appendix D: all items). Following the first focus group, two of the members from Focus Group A volunteered to participate in the individual interviews.

The first focus group was held the last week of September, 2009. This time point was selected for two reasons. First, because it was early enough in the semester to consider growth between this and the second focus group held at the end of the semester.

Second, it was five weeks into the semester to allow the preservice teachers to have a sufficient amount of experiences both in their methods course and in their once per week field placement to ensure there was enough to discuss. The second focus group meeting was held the first week of December, one week from the end of the Fall 2009 semester. This time point was selected because it was as late in the semester as possible while still allowing time for the follow-up individual interviews to be completed.

In the focus groups the participants were asked to discuss their mathematical knowledge for teaching and their experiences. The full Fall 2009 focus group protocol can be found in Appendix D (September 2009) and Appendix E (December 2009).[18] The purpose of the focus groups in the fall semester was to further explore the experiences and perspectives of as many preservice teachers as possible. Questions centered on asking the preservice teachers what they thought mathematical knowledge for teaching was and what elementary teachers needed to know and be able to do to teach mathematics effectively. The preservice teachers also discussed experiences they found helpful to their development as teachers of mathematics. The second focus group had the additional purpose of discussing whether, how, and why the preservice teachers believed they developed as teachers of mathematics during the semester.

The eleven Tier Two preservice teachers continued participating in focus group meetings during the Spring 2010 semester. The preservice teachers continued to meet in the same groups, which met for the third time in early February. This time point was selected for three reasons. First, because it was early enough in the semester to consider

[18] The research questions became more focused following the first wave of focus group interviews that were held (9/29/09 & 10/2/09). Some of the focus group questions asked go beyond the scope of the current research questions for future projects. The protocol for the December 2009 focus groups was updated to reflect the current research questions and can be found in Appendix E.

growth between this time point and the fourth focus group held towards the end of the semester. Second, the field placement component of the elementary education program switched from weekly visits in Fall 2009 to daily visits as of the Spring 2010 semester; the third focus group was held four weeks into the semester to allow the preservice teachers time for a sufficient amount of experiences in their field placements to ensure there are enough new experiences to discuss. Third, the preservice teachers were likely to teach the mathematics unit they wrote in their Fall 2009 methods course around this time. Therefore, having the focus group at this time allowed discussion of another experience that may have contributed to the preservice teachers' growth. It also provided the opportunity to discuss instructional decisions preservice teachers made when teaching their unit, giving insight into which practices carried over from their methods courses, which were adapted, and if any were abandoned.

The fourth and final focus group meeting was held in April two weeks from the end of the Spring 2010 semester. This time point was selected because it was as late in the semester as possible and still allowed time for the follow-up individual interviews to be completed. Having the focus groups late in the semester allowed the preservice teachers to have as many experiences with teaching mathematics as possible in advance of the final focus group. Although it was approximately two weeks before the end of their placement, most of the preservice teachers were done full-time teaching and had finished teaching mathematics by this time. Therefore, it was unlikely participants would have any significant experiences that would impact their growth or MKT after the focus group occurred.

The purpose of the focus group in the spring semester was to continue to follow all eleven participants in a manageable way and continue to discuss their conceptions of MKT, their development in this domain, and the experiences they had which positively contributed to such growth. The focus group script for Spring 2010 can be found in Appendix E. It was slightly updated prior to the fourth focus group in April. Changes for the fourth meeting are italicized while other items were asked at both spring focus groups.

The focus group meetings provided data to address research question one, which asked about the preservice teachers' development of mathematical knowledge for teaching over the year. The focus groups included specific questions that asked participants to describe their understanding of what mathematical knowledge for teaching entailed (see Appendix E). One way growth in mathematical knowledge for teaching was assessed was by comparing responses over time to these questions, because growth in mathematical knowledge for teaching was predicted to impact the preservice teachers' definition and discussion of mathematical knowledge for teaching over the course of the year. Holding four waves of focus groups over the 2009-2010 school year provided data to indicate whether and in what way(s) preservice teachers' mathematical knowledge for teaching developed.

Lesson plans. All of the mathematics lesson plans the eleven Tier Two participants wrote over the course of the 2009-2010 school year were collected. Particular focus was on three lessons: one from the beginning of the year, one from the middle of the year, and one from the end of the year. These lessons were selected by the preservice teachers as an example of a typical lesson from that time of year. This was done to investigate what the preservice teachers included and focused on in their lesson plans, and whether and how this changed over the course of the year. Of particular interest was to what extent mathematics was focused on in each lesson plan. For example, to what extent did the preservice teachers include items related to mathematical content, planning and sequencing of examples and tasks, reinforcing mathematical ideas through class discussion, and the anticipation of student moves? These aspects of lesson planning map onto MKT, as opposed to generic aspects of teaching that in lesson planning include listing materials, describing transitions, and describing the organizational set-up of the lesson. It was expected that growth in MKT would be evident by lesson plans that attended to a broader range and/or depth of key issues related to learning mathematics. It was anticipated that analyzing lesson plans would help answer research question one because they would provide data on the planned enactment of mathematical knowledge for teaching at various time points throughout the year, and therefore to what extent growth occurred.

Individual interviews. Data collection at Tier Three included the addition of individual interviews. Eight of the 35 participants who completed the MKT instrument, written reflections, and focus group interviews also participated in individual interviews throughout the 2009-2010 school year. All of the people who were willing to participate in an interview did so; there were no additional selection criteria. Following the first wave of focus group meetings, three people who had previously indicated no interest in the follow-up individual interviews volunteered to participate. Therefore, two of five participants from Focus Group A and six of six participants from Focus Group B ultimately participated in individual interviews. The following table shows the demographic and student teaching placement information for the eight people who participated in the individual interviews.

Table 5

Tier Three participants: MKT instrument, written reflections, focus group interviews, and individual interviews (n=8)

Ethnicity	Gender	Student teaching placement (Grade: n)
Caucasian: 8	Female: 8	Kindergarten: 1 First: 2 Second: 1 Third: 1 Fourth: 3 Fifth: 0

This study used individual interviews to engage preservice teachers to a greater level of depth than was possible during a focus group. In addition, the interviews functioned as a way to follow-up on the focus group discussions. For this reason the interviews occurred approximately one week after the focus group meeting. This allowed sufficient time for the researcher to review the focus group audio tapes and revise the interview protocol, if needed. It also meant as little time passed as possible so that the preservice teachers were at nearly the same "point" in terms of their development of MKT as they were during the focus group. The first wave of individual interviews was held the first week in October, approximately one week after the first focus group meeting. The second wave of interviews was held December 8-10, 2009, approximately one week after the second focus group meeting and after the last methods class meeting of the semester. The data collected indicated preservice teachers' ideas about MKT at the end of the semester, and indicated growth over as much of the semester as possible.

In the individual interviews the preservice teachers were asked to discuss MKT in general, their own MKT, and their experiences in greater depth than in the focus groups. To begin, participants were asked to expand on their previous answers from the focus group, including their definition of MKT. The majority of the time was then spent discussing the individual's ideas and definition of mathematical knowledge for teaching and the mathematically-related experiences the preservice teacher had to date. Participants were asked to describe helpful experiences and discuss the impact they had on their development as a teacher of mathematics. The full interview protocol can be found in Appendix F.

The purpose of the interview in the fall semester was to further explore the experiences and perspectives of as many preservice teachers as possible. The interviews offered insight into the preservice teachers' MKT and significant experiences that seemed to contribute to their development of MKT. Additionally, if the preservice teacher taught a mathematics lesson in their weekly field placement the lesson planning and teaching were discussed. The second interview had the same purposes with the additional purpose of discussing whether, how, and why participants believed they developed as teachers of mathematics during the semester. Of particular interest was discussion of any lessons they planned and taught, as well as the math unit that was the culmination of their methods course, because discussion of lesson planning and teaching potentially offered additional evidence and insight into their growth of MKT.

The eight preservice teachers continued their participation in individual interviews during the Spring 2010 semester. The interviews were held for the third time in mid-February, approximately one week after the third focus group interview. This time point was selected for three reasons. First, because it was early enough in the semester to consider growth between this time point and the fourth interview held later in the semester. Second, the third interview was approximately five weeks into the semester to allow the preservice teachers to have a sufficient amount of experiences in their field placements to ensure there were enough new experiences to discuss. Third, having the interview after preservice teachers had time to write and teach mathematics lessons allowed for individual follow-up regarding the focus group about instructional planning and implementation. This allowed discussion of practices that carried over from their methods course, those which were adapted, and if any were abandoned in their instructional planning.

The fourth and final interview was held in April, approximately one week from the end of the Spring 2010 semester. This time point was selected because it was as late in the semester as possible given the demands on participants as they finished their student teaching placement. Having the interviews late in the semester allowed the preservice teachers to have as many experiences with teaching mathematics as possible in advance of the interview, which was the final form of data collection. In this way, the interview functioned as a kind of qualitative post-test for the preservice teachers MKT and their reflection over the 2009-2010 academic year and their most salient experiences.

The purposes of the interviews in the spring semester were to continue to follow all eight preservice teachers in a manageable way and continue to discuss their conceptions of mathematical knowledge for teaching, their growth in this domain, and the experiences they had which they perceived as positively contributing to such growth. The spring interview script is found in Appendix F. It was slightly updated prior to the fourth focus group in April. Changes for the fourth meeting are italicized while other items were asked at both spring focus groups.

The individual interviews helped answer research question one by providing data about an individual's ideas and definition of MKT at a depth that was not captured in the focus group setting. The interviews included specific questions that asked preservice teachers to describe their understanding of what mathematical knowledge for teaching entailed (see Appendix F). One way growth in MKT was assessed was by comparing responses over time to these questions and discussion about lesson planning and teaching. This is because development in MKT should result in a more rich definition and discussion of MKT, lesson planning, and teaching. Holding four interviews over the 2009-2010 school year provided a significant amount of data to indicate whether and in what way(s) preservice teachers mathematical knowledge for teaching developed.

Summary of data collection for research question one. The multiple data sources that were used in this study generated a considerable amount of data that enabled the researcher to answer the first research question regarding the extent to which preservice teachers' MKT developed over the course of their last year in a teacher education program. Data collection incorporated both quantitative and qualitative techniques that allowed a more thorough understanding of knowledge growth. The MKT instrument, written reflections, lesson plans, focus groups, and individual interviews each provided data that helped answer research question one.

Part Three: Research Question Two--What aspects of mathematical knowledge for teaching translate into preservice teachers' practices?

Research question two focused on identifying the aspects of mathematical knowledge for teaching that were enacted by the preservice teachers. The data sources that were used to answer this question were lesson plans, teaching observations and post-observation interviews. It will now be explained how each of the three data sources answered research question two.

Lesson plans. Analyzing three mathematics lesson plans for each of the eleven Tier Two preservice teachers was one data source that was anticipated would help answer research question two. Lesson planning indicates the way a teacher plans to sequence and implement a learning experience for her students. A considerable number of decisions are made in lesson planning (e.g., how to address a goal or objective; whether and how to adapt curricular materials; whether students will work on a task as a class, in small groups, or with partners; how to introduce the topic). Therefore, lesson planning seemed a significant step towards and indication of the way a teacher enacts her MKT. Additionally, many aspects related to mathematical knowledge for teaching are decided upon at the planning stage of teaching and therefore are more clearly found in lesson planning than in the actual teaching of a lesson (e.g., anticipation of complexity, recognition of conceptual appropriateness, and decisions about sequencing) (Rowland et al., 2009). Therefore, lesson plan analysis was predicted to provide evidence for the intended enactment of MKT and help answer the second research question.

Teaching observations. Some studies have indicated that the most influential aspect of teacher education is the student teaching placement (see Cochran-Smith, 1991; Zeichner & Tabachnick, 1981). If this is an accurate depiction of teacher education, it means that the content specialists who teach methods courses are less influential despite their subject-specific expertise. Therefore, it was imperative to study knowledge growth throughout student teaching in Spring 2010 and not just during the Fall 2009 semester. In addition, there is “ample and impressive testimony that student teaching tends to be the most practical and useful part of preservice education in the minds of prospective teachers” (Peck & Tucker, 1973, p.967). A better understanding of the growth that occurs during student teaching and how preservice teachers perceive their professional growth is needed. Therefore, this study collected teaching data from three of the preservice teachers’ student teaching placement.

Additionally, Rowland et al. (2009) critiqued measuring mathematical knowledge for teaching by using the MKT instrument without accompanying the measure with observations of actual teaching. Rowland et al. (2009) believe that given the connected nature of PCK and the way a teacher’s different knowledge types interact within a teaching scenario, it is problematic to assess PCK void of actual teaching. They ask, “How can you judge whether you will give clear explanations and use the most appropriate resources without knowing the context?” (Rowland et al., 2009, p.25). For this reason an important element of this study was to observe the three Tier Four participants as they taught mathematics throughout the Spring 2010 semester. By so doing, insight was gained into whether the preservice teachers’ knowledge developed not only in terms of their definitions, ideas, thoughts, and hypothetical projections, but whether that knowledge was actually enacted in their teaching.

Eight preservice teachers participated in Tier Three data collection. As previously explained, the researcher selected two preservice teachers using Math Expressions and two who were using Envisions. All four preservice teachers did their student teaching in different schools; two were in District A and two were in District B. The two in District B had the same university supervisor, and the two District A had different university supervisors, resulting in three university supervisors total. Given the extensive amount of data, for the purposes of this document three of the four preservice teacher's data will be reported upon. Their information is presented in the table below.

Table 6

Tier Four participants: MKT instrument, written reflections, focus group interviews, individual interviews, teaching observations, and post-observation interviews (n=3)

Name[19]	Ethnicity	Gender	Student teaching placement	District	School	Math curriculum
Wendy	Caucasian	Female	First	A	1	Expressions
McKenzie	Caucasian	Female	Fourth	B	2	Envisions
Holly	Caucasian	Female	Fourth	B	3	Envisions

Four lessons of each of the four Tier Four participants were observed over the course of the Spring 2010 semester. This number was arrived at based on the research of the Learning Mathematics for Teaching (LMT, 2006b) work group at University of Michigan. When validating the MKT instrument, researchers analyzed ten videotaped lessons per participant. After correlating their MKT instrument scores with their coded scores from video analysis, it was found that the reliability between observational scores and the MKT instrument score remained high at four observations and was referred to as "a safe bet for making inferences about the mathematical quality of teaching" (LMT, 2006b, p.5). This means that four observations are sufficient to reliably indicate a teacher's mathematical knowledge for teaching. Observing less than four lessons

[19] All names used throughout this document are pseudonyms

produced lower reliabilities, and was “okay” at three observations and “dicey” at two observations (LMT, 2006b, pp.5-6). Although the authors noted that their results may only pertain to the coding system they used, to my knowledge there are no other studies which indicate a specific number of lessons to observe in order to assess a teacher’s MKT. Therefore, I observed each of the three preservice teachers four times in order to meet the four lesson threshold identified by the LMT (2006b) research.

Detailed field notes were taken while observing preservice teachers as they taught mathematics lessons. A descriptive synopsis was written immediately following the lesson. This was a brief (400-500 words) summary of what happened during the lesson (see Rowland et al., 2009). The lessons were video tapped and the preservice teacher carried an additional audio recorder so that the lessons could be more accurately analyzed after the lesson concluded rather than trying to code the lesson as it was taught.

Observing the preservice teachers throughout their Spring 2010 full-time student teaching placement provided data to help answer research question two. While the written reflections, focus groups, and individual interviews captured the development of MKT in regards to preservice teachers’ definitions, thoughts and ideas, the teaching observations provided data in the actual enactment of MKT. The observations were demonstrations of MKT at various time points throughout the semester, and therefore offered insight into the preservice teachers’ development of MKT.

Post-observation interviews. Following each observed lesson the preservice teacher was interviewed. The full post-observation interview protocol is found in Appendix G. The interviews took place immediately after the conclusion of each lesson and lasted for approximately 45 minutes. The purpose of the post-observation interviews was to gain insight into the preservice teacher's moves during the lesson and the reasons for these moves (e.g., asking why a student's comment was or was not elaborated upon or why the lesson was altered from the written lesson plan). The interviews helped clarify the observations so the researcher did not rely on inferences to code the lessons. In this way, the interviews functioned as an initial member check for the observations. The post-observation interviews also helped answer research question two by clarifying the data gathered during the observation so that more accurate analysis resulted. Since the data from the observations was necessary to identify the aspects of mathematical knowledge for teaching that appeared in practice, it was necessary to understand the decisions that the preservice teachers made while teaching. Some of these decisions (e.g., choosing whether and how to respond to a student's question) were not captured in lesson planning or in observing, since the observations demonstrated practices but did not typically reveal the reasons for particular moves because teachers tend to do this internally. Therefore, the post-observation interviews were important and increased the accuracy of observation data analysis and thereby helped more accurately answer research question two.

Summary of data collection for research question two. The multiple data sources that were used in this study generated sufficient data to answer the second research question regarding the aspects of mathematical knowledge for teaching that translated into preservice teachers' practices. Data collection was in the form of qualitative sources so as to be most helpful to capture teaching practices. Lesson plan analysis, teaching observations, and post-observation interviews each provided data that helped answer research question two.

Part Four: Research Question Three--What experiences are educative for elementary preservice teachers' development of mathematical knowledge for teaching?

The third research question focused on what experiences positively contributed to preservice teachers' development of mathematical knowledge for teaching. The data sources used to answer this question were the written reflections, focus group interviews, and individual interviews. An explanation of how each of these helped answer the third research question will now be provided.

Written reflections. Written reflections provided data to answer research question three, which identified educative experiences related to preservice teachers' development of mathematical knowledge for teaching. Both of the reflection prompts asked preservice teachers to identify and describe experiences that related to mathematical knowledge for teaching. The second reflection prompt especially helped identify what preservice teachers considered to be educative experiences, because preservice teachers were asked to identify and describe their two most significant[20] experiences in regards to their development as a teacher of mathematics. Therefore, the written reflections provided data to address research question three.

Focus group interviews. The focus group interviews also provided data to address research question three. Specific questions asked the preservice teachers to identify and describe experiences that contributed to their mathematical knowledge for teaching (see Appendix E). This produced data to identify the experiences the preservice teachers deemed as helpful to their growth as teachers of mathematics and therefore addressed research question three.

[20] The words "significant", "helpful", and "meaningful" are not synonymous with "educative." Data analysis will take this into account in order to evaluate whether or not experiences are truly educative.

Individual interviews. The individual interviews also provided data to address research question three. A significant amount of time in each individual interview was spent addressing this question to produce data at more depth than was possible in the focus group setting. Specific questions asked preservice teachers to identify and describe experiences that contributed to their mathematical knowledge for teaching, with substantial time dedicated for preservice teachers to explain in depth the experience and growth they believe resulted (see Appendix F). The interviews further identified and described experiences the preservice teachers viewed as helpful to their growth as teachers of mathematics and therefore addressed research question three.

Summary of data collection for research question three. The multiple data sources that were used in this study generated a considerable amount of data to answer the third research question regarding the experiences that preservice teachers identified as educative in regards to their development of mathematical knowledge for teaching. Data collection comprised of qualitative techniques to better understand experiences preservice teachers identified as helpful to their development as teachers of mathematics. Written reflections, focus group interviews, and individual interviews each provided data that helped answer research question three.

CHAPTER FIVE

METHODOLOGY—DATA ANALYSIS

This chapter will provide a detailed description of how data were analyzed. Data analysis largely relied upon a coding protocol the researcher developed based on the KQ model (Rowland et al., 2009) of mathematical knowledge for teaching (see Appendix H). As previously explained in the literature review, the KQ model incorporates all of the aspects of knowledge identified in the MKT model (Ball et al, 2008), adds a category about contingency, and is organized in a way that allowed for more consistent coding. However, the authors of the KQ model have yet to define each of the dimensions of MKT. Therefore after reviewing all of the available literature on the KQ, the researcher defined each of the dimensions of the Knowledge Quartet. Once the dimensions were defined, it was possible to identify when a particular dimension appeared in a participant's written reflection, lesson plan, focus group or individual interview comments, or teaching. The final coding protocol will be further described when explaining the data analysis for the observed lessons. Quantitative and qualitative data were analyzed to answer the first research question. Research questions two and three did not use the MKT instrument and therefore relied on strictly qualitative analysis. Data analysis will be described for each research question, followed by a description of what such analysis indicated.

Part One: Research Question One--To what extent does elementary preservice teachers' mathematical knowledge for teaching develop over their last year in a teacher education program?

MKT instrument. Although the results for the MKT instrument will be reported first, this was the last data source that was analyzed. The researcher did not want the preservice teachers' collective or individual instrument scores to influence the analysis of any of the other data, all of which was qualitative. Therefore, data analysis for the MKT instrument was conducted at the very end of the study after all other analysis was complete.

The MKT instrument was statistically analyzed using SPSS in accordance with the instructions from LMT Training Workshop that the researcher attended on November 20, 2009. The MKT test has two separate parallel forms that are designed for pre-test/post-test designs. One of the instrument's designers, Geoffrey Phelps (personal communication, August 20, 2009), indicated that it was appropriate to administer and analyze the MKT in what amounts to a pre-test/post-test/post-post-test design for the beginning of the Fall 2009 semester, the end of the Fall 2009 semester[21], and the end of Spring 2010 semester. Analyzing the data in this way produced quantitative scores which indicated the preservice teachers' initial level of MKT at the beginning of the Fall 2009 semester, level of MKT at the end of the Fall 2009 semester and beginning of the Spring 2010 semester, and level of MKT at the end of the Spring 2010 semester. Furthermore, generating descriptive statistics and paired t-tests allowed for analysis of growth in MKT during the Fall 2009 semester by comparing results from August and December 2009,

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[21] MKT scores from the end of Fall 2009 are assumed to be the same at the beginning of Spring 2010 semester, due to the short time span and absence of teaching that will occur over the semester break . This makes an additional administration of the MKT at the beginning of the Spring 2010 semester unnecessary (Geoffrey Phelps, personal communication, August 20, 2009).

growth in MKT in the Spring 2010 semester by comparing results from December 2009 and April 2010, and growth in MKT over the 2009-2010 year by comparing results from August 2009 to April 2010. Descriptive statistics and paired t-tests were calculated, and scores will be reported in terms of growth in standard deviations as compared to the average teacher as per the parameters of MKT instrument use (LMT Training Workshop; Cambridge, MA; November 20, 2009).

Analysis for pre-test, post-test, and post-post-test scores occurred on both of the MKT instrument content strands that were administered (number concepts and operations, NCOP; patterns, functions, and algebra, PFA). Since the Fall 2009 methods course and Spring 2010 student teaching placement focused heavily on numbers and operation and only minimally on algebra, the PFA score functioned as an internal control for the study. Growth in standard deviations were compared across the two strands at each of the three administrations. It was expected that scores on NCOP would improve over the course of the year, while scores in PFA would remain relatively constant since experiences were not focused on developing this strand of MKT with elementary preservice teachers. By having the PFA score as an internal control with relatively constant scores, it would be possible to attribute growth in NCOP scores to the experiences preservice teachers had over the year rather than to other confounding factors such as aging nine months. A relatively constant score on the internal control would allow for a stronger claim that growth in MKT was because of the experiences preservice teachers had over the course of their last year in teacher education. Therefore, this approach to data analysis provided information to help answer the first research question.

The MKT instrument was also analyzed to determine if the subgroups of participants at Tier Two, Tier Three, and Tier Four are representative of the Tier One class of 35 preservice teachers. The paired t-tests at the three time points as well as growth in standard deviations will be compared between the 35 preservice teachers in the class and each subgroup of those participating in the focus groups (n=11), interviews (n=8), and teaching observations (n=3). If the subgroups mapped onto the class in terms of their MKT instrument scores and growth in scores, stronger claims could be made about the representativeness of the subgroups to the larger class, and therefore the likelihood that their additional qualitative data mapped onto the larger class.

Written reflections. The written reflections, which were completed twice in Fall 2009 by each of the 35 Tier One preservice teachers, were coded to address the first research question. First, all of the text where participants defined mathematical knowledge for teaching was extracted. Next, these excerpts were coded using Atlas.ti in order to more specifically and consistently track the components of the preservice teachers' definitions of mathematical knowledge for teaching. Initial coding used two broad categories that Rowland et. al. (2009) used to describe teaching: generic and specific. Generic aspects of teaching include classroom management and things that are strictly pedagogical. Aspects of participants' written definitions that related to these topics were coded as "generic." In contrast, aspects that were mathematically specific and mapped onto mathematical knowledge for teaching and were coded as specific. In addition to the "generic" and "mathematically specific" codes, a third broad code of "other" was going to be used for any aspect not adequately captured by the initial codes of generic or specific. However, the “other” code was not needed, as all aspects could be coded as generic or specific.

A second wave of coding took place on all excerpts that were coded as “mathematically specific.” The KQ definitions that the researcher developed were used to identify which KQ category each excerpt described (foundation, transformation, connection, contingency) (see Appendix H). Table 5 indicates the categories that were used as codes for the generic and specific categories, as well as dimensions within each category.

Table 7

Data analysis codes: written reflections, focus groups, individual interviews, and post-observation interviews[22]

Code	Dimensions
Generic	Classroom management Pedagogy
Specific	
Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Use of questioning Choice of examples Choice of representation Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between procedures Making connections between concepts Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities
Other	Not adequately captured by any of the above codes

[22] The definitions for each KQ dimension can be found in Appendix F

Coding preservice teachers' definitions of MKT as generic or specific helped identify their individual and collective conceptions of MKT. Coding in such a way increased understanding of changes in the definitions that occurred in "specific" compared to "generic" definitions of MKT, as well as in each of the four KQ categories.

Focus group interviews. In order to address the first research question the eight focus group transcripts were coded. Following transcription, all of the places preservice teachers' defined mathematical knowledge for teaching were extracted. The KQ codes the researcher defined were used in Atlas.ti in order to more specifically attend to particular aspects of the preservice teachers' definitions of MKT. These codes were the same KQ category codes used to analyze the written reflections (e.g., generic; specific foundation, transformation, connection, contingency; other). As with the written reflections, portions coded as "other" were going to be extracted and compared to see if any new code(s) could be developed for comments initially coded as other, however this code was not needed. Data was analyzed in this way at all four time points over the year. By so doing changes in definitions over the year were discernable.

Individual interviews. In order to address the first research question, the interviews from the three Tier Four focal preservice teachers were transcribed and coded in a similar method to the focus group transcripts. Following transcription, all of the places the preservice teachers defined mathematical knowledge for teaching were extracted. The KQ codes were again used in Atlas.ti to more specifically attend to particular aspects of the preservice teachers' definitions of MKT. The codes were the same KQ category codes as those used to analyze the written reflections and focus groups (e.g., generic; specific foundation, transformation, connection, contingency; other). As with the written reflections and focus groups, portions coded as “other” were to be extracted and compared to see if any new code(s) could be developed for comments initially coded as other, but this was not necessary. Data was analyzed this way at all four time points over the year. This allowed changes in depth or focus that occurred over the year to be apparent. Analyzing the interview data in this way provided evidence of growth.

Reading across the data for research question one. Each data source provided a unique way to gain insight into preservice teachers’ development of mathematical knowledge for teaching over their last year in a teacher education program. In addition to having individual reasons for using each data source, the data also collectively answered the first research question. This was done by comparing the extensive qualitative data to the MKT instrument score to see if the qualitative data supported preservice teachers’ quantitative measurement of MKT. In accordance with information provided by the instrument authors (Geoffrey Phelps, personal communication, August 20, 2009), the MKT instrument is not intended to be “officially” analyzed at the individual level but rather is designed to be analyzed at the group level to measure the overall level of MKT this group of preservice teachers had before and after their math methods course, after their student teaching placements and their net growth. However, I also considered whether individual MKT score changes appeared to be a useful indicator when compared to the other qualitative data collected throughout the 2009-2010 school year.

An additional way data was compared across measures was to determine if the subgroups (Tier Two: n=11, Tier Three: n=8, Their Four: n=3) mapped onto the larger group (Tier One: n=35) by comparing MKT instrument scores and written reflections across the groups. If they did, the qualitative data conducted on the subgroups would be more representative of the class as a whole. Also under consideration was whether the written reflections, focus group interviews, and individual interviews produced consistent data, which would indicate more robust data than if different sources told different stories.

Finally, a comparisons between a) written reflections, focus group interviews, and individual interview data from research question one with b) lesson plans, observations, and post-observation interviews from research question two indicated whether growth in MKT occurred at a theoretical level, an enacted level, or if it was consistent across both domains. Analyzing data across sources further described the growth of preservice teachers' development of mathematical knowledge for teaching over the course of the year.

Part Two: Research Question Two--What aspects of mathematical knowledge for teaching translate into preservice teachers' practices?

The data collected to answer research question two were lesson plans, observations of teaching, and post-observation interviews. This section will explain how each of these was coded, with particular explanation given to the development of the coding protocol for the observed lessons.

Lesson plan analysis. In order to address the second research question lesson plans gathered over the 2009-2010 school year were coded. Although all mathematics lesson plans were collected from the eleven Tier Two participants who also completed the MKT instrument and participated in the focus groups, only three of each participants' lesson plans were rigorously analyzed. The codes that were used to analyze the lesson plans were the KQ category codes that have already been described (e.g., generic; specific foundation, transformation, connection, contingency; other).

It was planned to use these codes to answer the second research question because using the KQ categories as codes would identify particular aspects of MKT that the preservice teachers enacted into the planning phase of teaching. This would also identify aspects of MKT that they intended to enact in the teaching phase. While analyzing lesson plans at three time points over the year was expected to provide the opportunity to consider changes in the preservice teachers' lesson planning practices over time, it was not possible to code the lessons with the KQ codes in any depth. At all three time points over the year, the preservice teachers' lesson plans were scant on MKT and mainly included lists of things to do in the lesson. However, these were not described in enough depth to make use of the KQ codes without significant inferences. The researcher attempted to analyze the lesson plans in multiple ways to try to make sense of them, but ultimately they simply did not contain enough information to be useful to the present query. Therefore, the lesson plans were discarded from the study as a data source because they were so thin on evidence of MKT. While this is interesting in and of itself, it is not helpful in answering the current three research questions. Therefore, lesson plan analysis was not pursued.

Teaching observations. In order to address the second research question the observed lessons from the three focal preservice teachers were analyzed and coded for each of their four lessons. Because indicating the presence of a KQ category does not adequately indicate the way in which MKT was used, more specific codes were required than the definitions that were used to code the written reflections, focus group interviews and individual interviews. To better describe the teaching that was observed, the KQ dimensions were individually defined and more specific codes were developed. This was an iterative process in which the protocol was written, used to code lessons, and adjusted several times. This process will now be described.

Protocol Development. The original descriptor codes that were used when first coding the lessons in this study were developed in a video analysis study to further validate the MKT instrument as a quantitative indicator of mathematical knowledge for teaching (Learning Mathematics for Teaching, 2006). The resulting instrument is called the Quality of Mathematics Instruction (QMI) and is an observational rubric designed for use with video analysis. It was not possible to use the full QMI coding protocol in the study, because it contains 83 codes that were evaluated within every five-minute segment of a lesson. This protocol was used by the Learning Mathematics for Teaching group (2006) with a large team of researchers and was not feasible to use for the study at hand.

Another reason why the QMI protocol was not used is this study was designed prior to knowing whether teachers and principals would allow lessons to be videotaped and therefore the coding protocol had to be possible to use in a real-time situation, in case permissions were not granted. The Learning Mathematics for Teaching group (2006) acknowledged the protocol would need significant modifications to be used with live teaching, stating "this instrument must be adapted for use in real-time classroom situations, where coders do not have the luxury of replaying events, consulting transcripts, or studying curriculum materials" (p.20). Therefore, the researcher pursued that use of the KQ dimensions as codes because they were more manageable and mapped onto the MKT model, as previously explained.

Although the full protocol was not used, the original protocol for this study was based on a helpful set of descriptor codes that were developed in the QMI. These codes indicated whether or not a KQ dimension was present and was initially coded as "present" or "not present." Next, each of the KQ dimensions was first coded as "appropriate" or "inappropriate," where appropriate was defined as an "accurate use of mathematical language, the avoidance of mathematical errors or oversights, the provision of mathematical explanations when warranted, the connection of classroom work to important mathematical ideas, and the work of ensuring all students access to mathematics" (LMT, 2006a, p.3). Inappropriate was defined as "significantly hindering or obscuring mathematics, or significantly limiting students' access to mathematics in the moment" (LMT, 2006a, p.13). Given that the two codes are used simultaneously, there were four possible combinations for each KQ dimension, as depicted in the example in Figure 3.

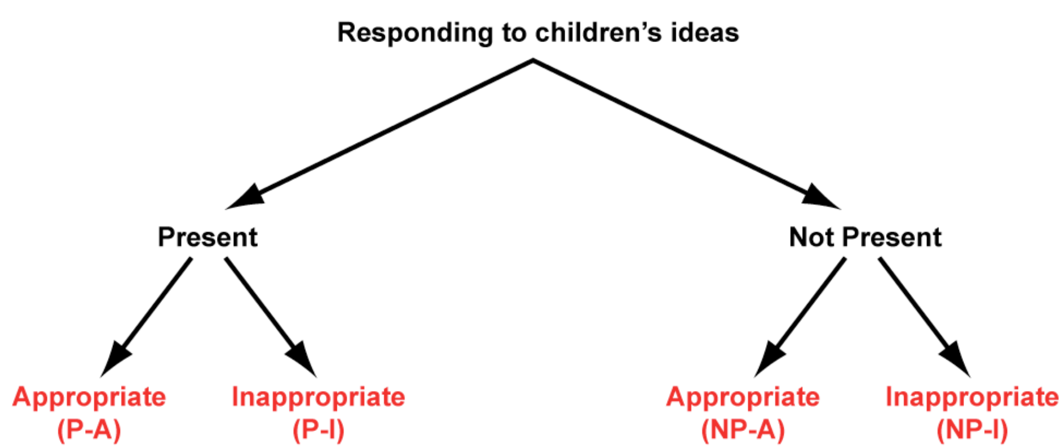

Figure 3. Initial codes for teaching observations. Adapted from LMT (2006a). This figure shows the initial four codes the researcher used in the original coding protocol.

The original protocol for this study integrated the four possible combinations of descriptor codes from the QMI (e.g., present/not present and appropriate/inappropriate) with the nineteen KQ dimensions.

After coding the first few lessons with this protocol, it was evident that most of the time KQ dimensions were coded as “Present: Appropriate.” However, the researcher recognized that within this code there was a large range of how well that dimension was demonstrated by the preservice teacher. Under the original protocol, whether a teacher demonstrated a dimension one time minimally or twenty times exceptionally the dimension was given a score of Present: Appropriate. Therefore, the researcher returned to the protocol to make additional levels within the Present: Appropriate category to better reflect the teaching that occurred.

During the secondary phase of developing the coding protocol, the researcher used codes of “minimum” and “maximum” to further specify the extent to which a dimension was demonstrated. For example, if a preservice teacher responded to children’s ideas (a dimension in the Contingency category) appropriately a few times during the lesson, it was coded as minimum. If she responded to children’s ideas nearly all of the time, it was coded as maximum. This protocol was then used to code all twelve of the lessons. As it turned out, the majority of the time dimensions were coded at “Present: Appropriate--Minimum,” because the participants were demonstrating initial levels of MKT but rarely demonstrating a maximum level. Therefore, the scores continued to “bunch up” at the minimum level.

Not only were the vast majority of dimensions coded at Present: Appropriate—Minimum, but also there was still a noticeable range of the extent to which a given dimension was demonstrated within that one level. In terms of the previous example, a preservice teacher who responded to children's ideas appropriately two or three times was coded the same as a preservice teacher who responded to children appropriately ten times (both at Present: Appropriate—Minimum). To better describe the data, the protocol was refined for a third time by adding a third level to Present: Appropriate, which included minimum, middle, and maximum as possible scores on each dimension. Once this was decided, the protocol had to be re-worked so that each dimension was defined at each of the three levels. A depiction of the final version of the coding protocol, which comprised of six possible coding levels, is seen in the following figure.

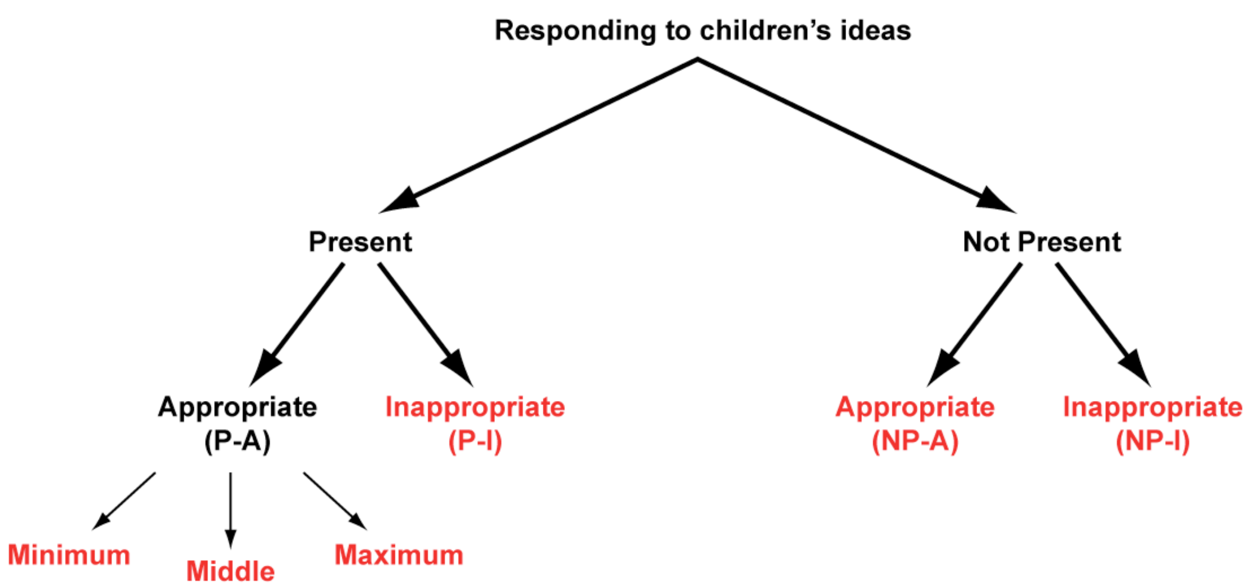

Figure 4. Final codes for teaching observations. This figure shows the final six codes the researcher used in the coding protocol.

Throughout the process of making an increasingly refined definition for each dimension, which ultimately needed to be defined over six possible levels, some areas of overlap began to emerge between a few of the dimensions. This was true with two of the dimensions, recognition of conceptual appropriateness and use of opportunities, which were not definable in unique ways. Recognition of conceptual appropriateness overlapped with anticipation of complexity, another dimension in the connection category. It was not possible to make a definition for recognition of conceptual appropriateness that made a distinction between these two dimensions without then overlapping with other dimensions, including choice of examples, choice of representation, demonstration, and questioning, as these are key ways a teacher demonstrates her recognition of conceptual appropriateness. Therefore, recognition of conceptual appropriateness was not used to code the observed lessons because it was captured by multiple other dimensions that were coded.

The second overlapping dimension was use of opportunities, which is in the contingency category. When developing a definition unique from responding to children's ideas was difficult, all twelve lessons were reviewed for times when use of opportunities applied. After reviewing the notes from the lessons, it was found that every instance where use of opportunities applied was also an example of responding to children's ideas. At least in the current data set, no opportunity arose in a lesson that did not relate to the students' ideas—usually in the form of accurate or inaccurate statements students made, questions they asked, or work they completed. Therefore, responding to children's ideas was used to code observed lessons and use of opportunities was discarded since a unique definition could not be generated.

Since two dimensions were dropped, the final protocol includes seventeen of the nineteen dimensions of the KQ model. Therefore, when the preservice teachers' definitions of MKT are presented for the first research question it is in terms of nineteen dimensions, and when results for research question two are presented it is in terms of seventeen out of the total nineteen KQ dimensions.

The definitions and levels of the protocol tie back to the literature. For example, the awareness of purpose dimension was defined by synthesizing seminal pieces from Simon (2006) on Key Developmental Understandings and Skemp (1978) on relational and procedural knowledge. This protocol was then attempted by coding four lessons, making further adjustments to the definitions of the levels, and then re-coding all twelve lessons with the final protocol. The process of developing the protocol was extremely iterative, stayed close to the literature, and considered the data. Many considerations went into developing the final protocol, including professional standards, mathematics education literature, and ethics. This process was tremendously tedious and ultimately resulted in the lengthy protocol found in Appendix I. Feedback on the protocol was received from Tim Rowland on January 20, 2011 to further clarify the way he thought about the dimensions. During this meeting he also indicated support for attempting to use the KQ model as a coding protocol.

It should also be noted that the protocol was designed specifically for preservice and novice teachers and the levels were defined with that population in mind. This means that the "maximum" score was set as something potentially attainable by preservice teachers. Therefore the ceiling is not so high that one must be "perfect" on a dimension in order to score a maximum. Since the purpose of this study was to consider demonstrated MKT and growth of preservice teachers the scale was intentionally developed to reflect increments of improvement and not be so difficult that higher scores were essentially impossible for a preservice teacher. Therefore, the researcher developed what might be considered a "preservice teacher friendly" scale. The researcher makes no claim that "maximum" on this scale is the best possible demonstration of MKT on a particular dimension; indeed it is not. Such a scale would be more suitable for veteran or master teachers but inappropriate for use with preservice teachers.

Due to its length a thorough explanation of each of the seventeen dimensions, each of which has six levels (present: appropriate—maximum; present: appropriate—middle; present: appropriate—minimum; present: inappropriate; not present: appropriate; not present: inappropriate) cannot be given in this document. Therefore, two of the seventeen dimensions and the corresponding levels will be reviewed to give the reader some examples of the coding protocol used in this study. The levels for additional pertinent dimensions will be described throughout subsequent sections as necessary.

Use of terminology. The first dimension that will be described is Use of Terminology, which is one of seven dimensions in the Foundation category. After reviewing the literature, the researcher defined use of terminology in the following way:

> One aspect of mathematical knowledge for teaching is a teacher's knowledge of mathematically correct terminology and notation (Rowland et al., 2009; Ball, Thames & Phelps, 2008). This includes verbally using mathematically correct language as well as writing correct mathematical expressions. Therefore, this code is used to indicate whether the preservice teacher correctly used verbal and written notation in her lesson.

The three levels of "Present: Appropriate" were defined by the researcher to indicate the number of errors the preservice teacher made in her verbal or written notation. If the preservice teacher made one or two mistakes in her verbal or written notation, she was coded at "Present: Minimum" because usually, although not always, she used correct terminology with her students. If the preservice teacher made zero mistakes and did not engage in any "press" with her students about the mathematical terminology or notation at hand, she was scored "Present: Middle." This level indicated a lack of error on her part, but also a lack of explicit discussion about terminology or notation with her students. At the "Present: Maximum" level, the preservice teacher committed no mistakes in verbal or written notation and did include a press for terminology with her students by providing explanation, definition, discussion, or correction. The difference between a teacher at "medium" and "maximum" was whether or not she made mathematical terminology and/or notation an explicit part of the lesson. Two examples will now be presented to demonstrate the difference between these two levels.

The following example of the maximum level came from McKenzie's second lesson on classification of angles and triangles. During the beginning of the lesson she asked her students to tell her the difference between a regular and an irregular polygon. A student began to answer by saying that a regular polygon is "kind of like a mountain." McKenzie asked the student to come up to the Smartboard to show her and the class what he meant. The student drew an equilateral triangle on the board. Then McKenzie drew an isosceles and scalene triangle on the board and asked her students if they were regular or irregular polygons, and why. This led to a discussion about regular polygons having the same size angles and the same length sides and irregular polygons do not have the same size angles and sides. Next, McKenzie drew a regular hexagon on the board and asked students to identify it, followed by an irregular hexagon for which she asked students if it was regular. Next she said that a regular polygon has congruent sides and congruent angles and asked her students what having congruent sides meant. Finally, she demonstrated how to notate regular polygons by "drawing a little line through all sides to say all these sides are exactly equal."

During this lesson McKenzie made no errors in her verbal or written terminology. What distinguished this as an example of Present: Appropriate--Maximum as opposed to medium was the way she focused students' attention on mathematical terminology, both verbal and written, during this part of the lesson. Had she used terms such as regular, irregular, and congruent without explanation the lesson would have been coded at medium. For example, had the lesson gone as follows with fictitious preservice teacher Julia, a medium code would have resulted.

During the beginning of the lesson Julia drew a regular polygon on the board and asked students if it was regular or irregular. Then Julia drew an isosceles and scalene triangle on the board and asked her students if they were regular or irregular polygons. Next, Julia drew a regular hexagon on the board and asked students to identify it, followed by an irregular hexagon for which she asked students if it was regular. Finally Julia drew a regular and irregular polygon and asked students which one had congruent sides and congruent angles.

In the above example, the preservice teacher made no errors in her terminology but also did not encourage students to focus on, discuss, or explain the terms that were being used (polygon, regular, irregular, congruent). Therefore this lesson would be coded at Present: Appropriate, medium.

The remaining three levels for the use of terminology dimension will now be explained. “Not present: Inappropriate” (NP:I) was used when a preservice teacher used terminology or notation that was mathematically inaccurate more than three times in a lesson. For example, in Holly’s lesson three on coordinate grids she referred to the “points” on the grid as “dots” throughout the entire lesson. Therefore, she was coded at “Not present: Inappropriate” because she used mathematically inaccurate terminology well over three times in her lesson. This is not to say that she never used any mathematically correct terminology during the lesson, but rather her use of inaccurate mathematical terminology was consistent enough to be reflected in her score. A preservice teacher was also coded as NP:I if she did not demonstrate accuracy in her written notation, an example for which would be using the equal sign incorrectly when writing out a long series of steps in a solution.

“Present: Inappropriate” was the one level that frequently was defined as “not possible.” This was true in the case of use of terminology, because there does not seem to be an instance in which appropriate mathematical terminology and vocabulary would be inappropriate. Not present: Appropriate was defined as an intentional decision on the part of the preservice teacher to use different terminology in the present lesson which she would then build upon and introduce mathematically correct language in a subsequent lesson, using today’s vocabulary as a foundation for more complicated terms to come. An example of this is introducing younger children to the commutative property of addition and initially using other terms that the children generate to describe this property, with the intention of providing the proper terminology ("commutative property") in a subsequent lesson. Likewise, a teacher may intend for notation to be formalized in a subsequent lesson.

Choice of examples. The second dimension from the coding protocol that will be explained is choice of examples, one of four dimensions in the transformation category. Again, with support from the literature (Stein et al., 2000; Stein, Remillard & Smith, 2007; Rowland et al., 2009; Ball, Thames & Phelps, 2008), the researcher defined this dimension in the following way:

> One aspect of Mathematical Knowledge for Teaching is a teacher's ability to select or create examples with which her students will interact (Rowland et al., 2009; Ball, Thames & Phelps, 2008). This includes the ability to match a concept, skill, or objective with corresponding examples. This includes selecting high (as opposed to low) cognitive demand tasks (Stein, Smith, Henningsen & Silver, 2000; Stein, Remillard & Smith, 2007). One aspect of selecting or creating tasks is the purposeful use of numbers and operations so that the use of the particular numbers or operations focuses students' attention on the desired concept or strategy.

The three levels within the Present: Appropriate category will now be defined. These levels were based upon the work of Stein, Smith, Henningsen and Silver (2000) who described tasks as either low cognitive demand or high cognitive demand. Low cognitive demand tasks can be completed by retrieving memorized information whereas high cognitive demand tasks require knowledge to be applied in new and often demanding ways. Within each of those two categories, there are two further levels of tasks. Within low cognitive demand tasks the lowest level is Memorization, which is characterized by "reproducing previously learned facts, rules, formulas or definitions or committing these to memory" (Stein et al., 2000). The higher level, although still low cognitive demand, is Procedures Without Connections, which is characterized by a task or solution that is algorithmic and relies upon "the use of a procedure that either is specifically called for or is evident from prior instruction and/or experience" and "little ambiguity about what needs to be done and how to do it" (Stein et al., 2000).

Within high cognitive demand tasks the lower level is Procedures With Connections, which is characterized by "focusing students' attention on the use of procedures for the purpose of developing deeper understanding of mathematical concepts and ideas…can usually be represented in multiple ways." Although these tasks may be solved by using procedures "they cannot be followed mindlessly" (Stein et al., 2000). The highest level of high cognitive demand tasks in the Stein et al. (2000) model is Doing Mathematics, which is characterized by "complex, non-algorithmic thinking" which "requires considerable cognitive effort" as students "explore and understand the nature of mathematical concepts, processes, or relationships" (Stein et al., 2000). Each of the four levels of tasks will now more fully be described.

A preservice teacher was coded at Not present: Inappropriate (NP:I) if all examples in the lesson were low cognitive demand within either the memorization or procedures without connections subcategories, which are the lowest two levels of the Stein et al. (2000) model. An example of a low cognitive demand task that would be coded as NP:I is performing calculations on naked number sentences, such as multiplying two fractions together. This is an example of a low cognitive demand task because its solution only requires an algorithmic use of procedures.

The Present: Appropriate--Minimum code was used when one of a preservice teacher's examples or tasks was high cognitive demand: procedures with connections. An example of this code was in Wendy's lesson one, which was a first grade lesson on dimes and pennies in which she asked students to show more than one way to make 24 cents without nickels. This was an example of procedures with connections because students' solutions could be represented in multiple ways through manipulatives and drawings, which is one of the characteristics of procedures with connections. The Present: Appropriate—Minimum code would still be used in a lesson in which additional examples were low cognitive demand but at least one example was procedures with connections.

The Present: Appropriate--Middle code was used when at least two of a preservice teacher's examples and tasks was high cognitive demand: procedures with connections. For example, Holly's second lesson was a fourth grade lesson on probability. During the lesson she asked students to complete several tasks, two of which were as follows.

> The numbers used are 2, 3, 4, and 5. If we know the probability is ½, how many of the numbers will be 4?
>
> There are 2 orange, 5 yellow and 1 blue t-shirts. What is the probability of picking a blue or an orange shirt?

These were each examples of procedures with connections because students' solutions could use procedures but not in an algorithmic way, because they had not been taught any algorithms by which to calculate the solution.[23] Therefore, they needed to use procedures

[23] Knowledge of the classroom and the way the example was presented in the lesson are important when determining which level is appropriate for an example. The probability examples that are listed above as Present: Appropriate--Middle were high cognitive demand tasks for that particular group of students.

mindfully and with some cognitive effort, both of which are characteristics of procedures with connections. The Present: Appropriate—Medium code would still be used in a lesson in which additional examples were low cognitive demand but at least two examples were procedures with connections.

Finally, the Present: Appropriate--Maximum code was used when at least one of a preservice teacher's examples or tasks was high cognitive demand: Doing mathematics, which is the highest level on the Stein et al. (2000) scale. For example, in McKenzie's fourth lesson, which was a fourth grade lesson, she asked students to complete the following problem:

> Four roller bladers travel at different speeds for a 20-minute workout. How many times will they have simultaneously returned to the spots where they started?

This is an example of a high cognitive demand: doing mathematics task because it required students to engage in complex, non-algorithmic thinking and required considerable effort for fourth graders to solve.

The Present: Inappropriate (P:I) code was used when a preservice teacher used an example that was potentially high cognitive demand, except the example was not an appropriate match for the age group or developmental level of the students for whom it was intended or did not align with the desired concept, skill, or objective. That is to say, the task itself may have been high cognitive demand (HCD), but it was inappropriate to use in the given lesson because it did not align with the goals and objectives of that lesson or for that age group. For example, had the previously listed HCD task from

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Because the examples were presented as problems that needed to be solved rather than as a means to practice a known procedure, in which case they would be considered low cognitive demand tasks. Familiarity with the lesson and the context is important to reduce inferences made during coding.

McKenzie's lesson (i.e., the rollerblading problem) been done in a lesson with first graders, it would have been coded as P:I because first graders, in general, would not have had adequate background knowledge with which to begin working on the problem. For example, they would be highly unlikely to realize that 20 minutes is one-third of an hour, which turns out to be an aspect of the solution strategy. Likewise, had McKenzie presented this same problem in a lesson that was focused on place value, that would also be coded P:I. In both of these examples although the task was HCD, it was inappropriate for the students or lesson at hand.

Choice of examples is another dimension for which the code "Not Present: Appropriate" (NP:A) does not make sense and is therefore not possible. Of the seventeen dimensions, there are only six that have a NP: A code, but even for these dimensions it seems the use of this code will be rare because it is seldom appropriate to omit one of the dimensions unless it is done for a very intentional purpose.

The full coding protocol can be found in Appendix I. A description of two of the seventeen dimensions, use of terminology and choice of examples, was presented to introduce the protocol and demonstrate how the six levels operate. Additional work on the protocol will continue in the future so that it can be shared with a wider audience.

Protocol use. To use the protocol the lessons were transcribed from the videotapes. The supplemental audio recording, made by placing a voice recorder on the preservice teacher during the lesson, was used when necessary. Then, the lesson was divided into sections as seemed to match the flow and topic of the lesson. Most typically the lessons contained two or three sections. This was done for ease of coding so that smaller sections could be focused on at one time. Next, the transcript from one section at a time was sorted per KQ dimension by using a table containing all seventeen dimensions. At this initial stage of coding, all excerpts that applied to a dimension were inserted in the appropriate dimension of the table. Once all of the tables were made for each section of the lesson, all of the excerpts on a given dimension for the entire lesson were considered and the definitions of the six levels in the final protocol found in Appendix H were used to determine the overall lesson code for that dimension. This process was repeated for all seventeen dimensions, resulting in seventeen codes per lesson, or one for each dimension.

In regards to research question two, analyzing the videos of the teaching observations that were conducted over the course of the Spring 2010 semester with the protocol the researcher designed provided the opportunity to consider changes in the preservice teachers' practices. Student teaching is the period of time when preservice teachers were in an environment that approximated "real" teaching as close as it was ever approximated in the teacher education program in which they were enrolled.[24] Therefore, it was an important window in which to view and consider the dimensions of MKT that translate into practice.

Post-observation interviews. Although student teaching is an attempt to approximate the conditions of "real" teaching, there are usually shortcomings (Dewey, 1904/1964; Grossman & McDonald, 2008). For example, student teachers are never fully "in charge" of all aspects of the day-to-day functioning of the classroom. Student teachers must also work within the space they are given by their Cooperating Teacher.

[24] For a critique of the ways in which student teaching fails to approximate actual teaching, see Dewey (1904/1964) and Grossman and McDonald (2008).

While some student teachers may be allowed to develop and teach lessons however they wish, others have constraints including differences in teaching and learning philosophies with their Cooperating Teacher or mandated curriculum set forth by the district, school, or PLC in which they are placed. Given the disconnect that may exist between what the preservice teachers *would* do in their own classroom versus what was observed in their teaching during Spring 2010, the post-observation interviews were important to further explore they ways MKT may translate into lesson planning and teaching practice if participants were in a different setting. Therefore, the additional code of "future" was used when a preservice teachers indicated she would do something differently were she given the opportunity.

The other codes that were used to analyze the post-observation interviews were previously described as the post-observation interview was coded the same way the written reflections, focus group interviews, individual interviews, and lesson plans were coded (e.g., generic; specific foundation, transformation, connection, contingency; other).

Reading across the data for research question two. Each data source provided additional insight into preservice teachers' enacted mathematical knowledge for teaching over their last year in a teacher education program. The teaching observations and post-observation interviews collectively answered the second research question. For this study, the purpose of the post-observation interviews was not to compile information about the post-observation interview itself but rather to more accurately code the observed lessons. Therefore the post-observation interview codes were referred to when determining the code for each KQ dimension on the twelve observed lessons because during the post-observation interview preservice teachers talked about their decisions and reasoning and the researcher asked questions to better evaluate the lesson without having to rely upon inference. Analyzing data across sources further described the preservice teachers' demonstrated MKT.

Part Three: Research Question Three--What experiences are educative for elementary preservice teachers' development of mathematical knowledge for teaching?

To answer research question three, written reflections, focus group interviews, and individual interviews were analyzed in two phases: First, the KQ categories were used to determine what preservice teachers' indicated as helpful experiences, and second, whether these experiences could be considered educative was determined (Dewey, 1904/1964, 1938/1963).

Written reflections, focus group interviews, and individual interviews. To analyze the three data sources that contributed to answering research question three, the text or transcription in which the preservice teachers' wrote or talked about experiences that were helpful towards their growth in MKT and as a teacher of mathematics were extracted. These ideas were then analyzed individually to determine trends in responses. It was intended to use the KQ codes that were used for most other aspects of data analysis to code each helpful or significant experience that was identified (e.g., generic; specific foundation, transformation, connection, contingency; other). This was going to be done to identify the particular categories of MKT the preservice teachers found most helpful in their development as teachers of mathematics.

However, once the data about helpful and significant experiences was extracted and coding with the KQ codes was attempted, it was evident that using the KQ categories as codes was not possible given the data. This was because the majority of the time the preservice teachers identified and described helpful experiences in generic terms that did not map onto particular aspects of MKT. Despite being asked specifically for experiences that helped them develop as teachers of mathematics, the preservice teachers tended to describe helpful experiences in generic terms that were not unique to learning to teach mathematics but rather that applied to teaching, writ large. This was true across all three data sources (written reflections, focus group interviews, and individual interviews).

Since it was not possible to adequately describe the preservice teachers' individual or collective conceptions of helpful experiences through the KQ categories, a second wave of analysis was conducted following an inductive approach for the three focal preservice teachers. First, all of the helpful experiences that Wendy, McKenzie, and Holly identified over the course of the year were compiled from their written reflections, focus group comments, and individual interviews using Atlas.ti. Since these experiences did not have a comparable impact on their growth, each of their experiences were further analyzed to focus on the experiences they thought were most helpful. A new list of the most helpful experiences was created for each preservice teacher, which resulted in identifying between three to five experiences for each of the three focal preservice teachers. Next, inductive analysis was used to reveal similar ways in which the three focal preservice teachers described their most helpful experiences. This second wave analysis resulted in three descriptors of helpful experiences.

While the preservice teachers may identify experiences as "helpful", this does not necessarily indicate an experience was "educative" (Dewey, 1904/1964, 1916/1985, 1938/1963). Therefore, the last wave of analysis considered Dewey's descriptions of educative experience to evaluate the degree to which an experience was educative. Particular attention was given to whether experiences included continuity and interaction, since these were the twin-pillars of Dewey's philosophy on educative experiences.

Analyzing the data in this way provided two groupings of data with which to answer research question three. First, it was possible to describe experiences the preservice teachers found helpful in their development as teachers of mathematics. Second, it was possible to compare these experiences to Dewey's philosophy of educative experiences and consider the extent to which the experiences were actually educative. Therefore, the data analysis helped indicate the experiences that preservice teachers had that were educative in regards to their development as teachers of mathematics.

Reading across the data for research question three. Each data source provided a unique way to gain insight into the experiences preservice teachers identified and described as significant in their development of mathematical knowledge for teaching. In addition to having individual reasons for using each data source, there were ways that the data collectively answered the third research question. Analyzing data across sources further described the experiences the preservice teachers considered helpful in regards to their growth of mathematical knowledge for teaching over the course of the year. It also allowed for a deeper consideration of the degree to which experiences the preservice teachers identified as helpful were educative and contained continuity and interaction.

CHAPTER SIX

FINDINGS—RESEARCH QUESTION ONE

This chapter examines the overall findings for the study based on synthesizing data from multiple sources. Results will be presented per each of the three research questions. Given this study was based largely on qualitative data with small *n* values, descriptive statistics were generated by compiling coded data and will be reported to best describe the data throughout this chapter.

Research Question One: To what extent does elementary preservice teachers' mathematical knowledge for teaching develop over their last year in a teacher education program?

The first area this study explored was preservice teachers' development of mathematical knowledge for teaching over the course of their senior year. In order to answer research question one, the MKT instrument was analyzed as per its coding protocol, and the written reflections, focus group interviews, and individual interviews were analyzed using the KQ categories (foundation, transformation, connection, contingency). First, the MKT instrument results will be reported. Second, the written reflections, focus groups, and interview results will each be reported separately. Third, similarities and differences across the way these data explain the development of MKT across this particular group of preservice teachers will be discussed. Fourth, a detailed description of the individual growth of each of the three focal preservice teachers will be given. Finally, similarities and differences across the three focal preservice teachers' definitions will be discussed.

MKT Instrument

The 35 preservice teachers took the MKT instrument three times over the course of the year: beginning of fall semester (August), end of fall semester (December), and end of spring semester (April). The following section will report the findings of the MKT instrument analysis. First descriptive statistics will be presented for the control and then the actual test. Then correlations and t-tests between the test dates will be reported. Finally a comparison plot of all three test dates will be presented and discussed.

As per the guidelines of MKT instrument administration results will be discussed in terms of standard deviations, not in terms of raw scores or percentages. The following table shows the descriptive statistics for the internal control, which was the MKT instrument on patterns, functions, and algebra (PFA).

Table 8

Descriptive statistics--Control

Test	N	Minimum	Maximum	Mean	Median	Standard Deviation
August 2009	35	-2.005	1.719	0.344	0.361	0.844
December 2009	35	-3.015	1.933	0.383	0.464	0.902
April 2010	33[25]	-1.830	2.019	0.412	0.553	0.867

[25] All participants took the control and test in August and December. Two participants declined to complete the control and test measures in April 2010, resulting in an n of 33 for the final test data.

From the above data it is apparent that there was quite a wide range of scores, based on the minimum and maximum scores. The mean score increased over time, while the median increased more than the means indicating the distribution became increasingly more skewed to the right. The assumption was that the control scores would remain the same over the three test administrations. There proved to be more variation in the control test scores than anticipated.

Although the test score averages increased somewhat over time, the correlations were quite low. The correlations between the control scores are provided in the matrix below. The correlation between scores for the August and December tests was 0.499, the December and April tests was 0.695, and the correlation between scores for the August and April tests was 0.621.

Table 9

Correlation Matrix--Control (n = 33)

	August 2009	December 2009	April 2010
August 2009	1.000		
December 2009	0.499	1.000	
April 2010	0.621	0.695	1.000

The correlations were positive, indicating increases or decreases on one test moved in the same direction as the other tests. The association between scores was moderate between the August and December tests and more strongly correlated between April and the other two tests. However, if scores had been consistent at the three time points as expected, then correlations would have been higher. The slight upward trend of mean scores and relatively low correlations call into question using the PFA test as a control measure for preservice teachers. Therefore, the control data will be set aside and taken up again after presenting the remainder of the test scores.

The following table shows the descriptive statistics for the test measure, which was the MKT test on number concepts and operations.

Table 10

Descriptive statistics-- Test

Test	N	Minimum	Maximum	Mean	Median	Standard Deviation
August 2009	35	-1.840	1.640	0.233	0.483	0.806
December 2009	35	-1.369	2.507	0.454	0.506	0.782
April 2010	33	-1.254	2.201	0.411	0.483	0.795

The data for the August test does not seem surprising, other than perhaps the fact that this group of preservice teachers at the beginning of their senior year did slightly better than an average teacher on the pre-test. The descriptive statistics for the three test dates indicate that the first test from August 2009 had the lowest range and mean of the three tests. This is consistent with expectations since it was the pre-test. Since this test has not been used with preservice teachers, at least in published research, there was little basis for expectations on the range and mean scores. Nevertheless, the fact that participants scored

0.2 standard deviations above the mean without experiencing methods or student teaching suggests a good pre-test score, because the test is normed on practicing teachers.

As indicated by increased minimum and maximum scores in the above table, the December test distribution shifted to the right, had a higher mean and median, and the data was tighter than it had been in August (as indicated by a slightly smaller standard deviation). All of these trends were expected based on administering a different version of the same test four months later and after a semester-long methods course and weekly field placements. However, the amount of increase was lower than anticipated. The average score increased only by about 0.2 standard deviations, resulting in an average score of 0.45 standard deviations above the normed mean. Furthermore, when examining the 35 individual's scores, it was not the case that most people increased slightly from August to December, as mean increase may suggest. Rather, it was not uncommon for large fluctuations in both positive and negative directions. In fact, while 19 people's scores increased from August to December, 16 people's scores decreased. This was very surprising because "at worst" it was expected that scores would remain the same. That is to say, if there was no treatment effect by the intervening experiences of methods coursework and the weekly field placement then scores should have been the same in August and December. The fact that almost half of the participants scored lower in December after participating in a methods course and weekly field placement is perplexing.

Comparing the test scores from December 2009 to April 2010, the lowest score improved slightly but all other descriptive statistics were less favorable for the April scores than for December. The mean decreased slightly by .04 standard deviations and the median dropped by 0.02 standard deviations. This means that when comparing August (pre-test) data to April (post-test) data, there is even less of an increase than there was between the August and December scores. From December to April fifteen scores increased and eighteen decreased.

Comparing August to April the range shifted to the right as indicated by higher minimum and maximum scores and the mean increased by 0.18 standard deviations. The median was identical. Fifteen scores increased and fourteen decreased. Again, there was growth but much less than expected considering the August test was before and the April test was after methods coursework, the fall weekly field placement, and spring student teaching. This is even more surprising because the test given in August and April was not an equated test (as it was in August and December or December and April), but rather it was the exact same version. As previously explained, the same version of the test was administered at these two time points because of the long time in between test administrations which decreased test-retest effects (Geoffrey Phelps, personal communication, November 20, 2009). Therefore to see participants answer multiple questions incorrectly in April that they had correct in August was alarming.

Given the small differences in means, scores were not significantly different between any two administrations. The p values generated from paired t-tests for each comparison are as follows: August-December: $p = 0.463$; December-April: $p = 0.818$; August-April: $p = 0.623$. A nonparametric Kruskal-Wallace rank test of differences between distributions was also conducted, and again no statistically significant results were present.

Data from a kernel smoothed density plot is presented in the following figure. This graph indicates the density of scores for the test.

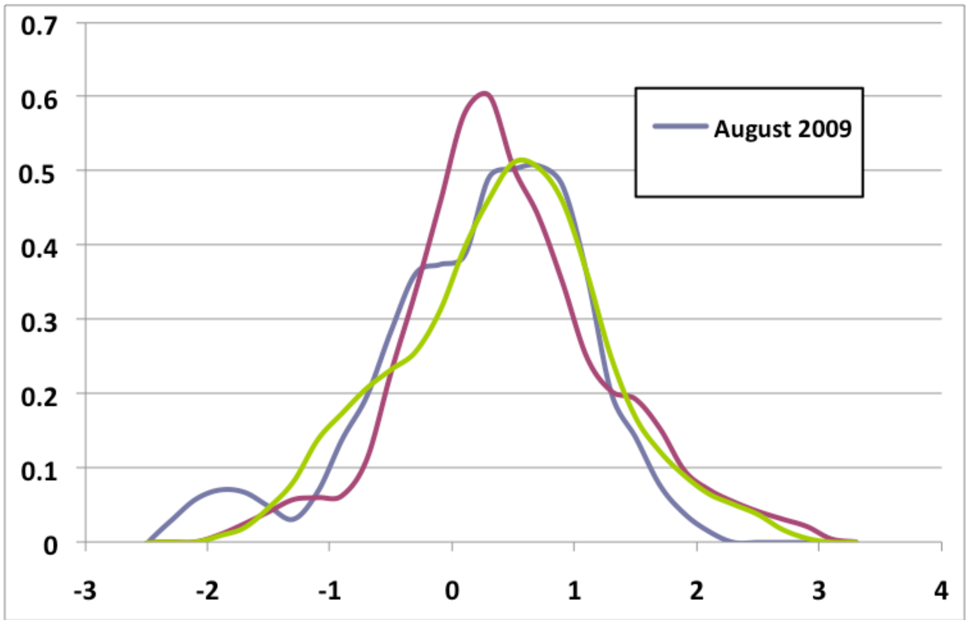


Figure 5. Kernel Smoothed Density Plots (n=33). This figure shows the MKT instrument scores at the three test administrations.

The above figure offers a depiction of the data to go along with the previous description. For all three tests, positively shifted scores and a slight positive skew are visible. The first test in August had a relatively high concentration of scores in the lower tail and dropped off more quickly in the upper tail when compared to the December and April tests.

From about -1.5 through -0.6 the April scores were the most prevalent. From -0.5 to 1.4 the April scores were to the right of the August and December scores. Then in the upper tail from about 1.4 through 3.2 the April scores are to the right of the August scores but to the left of the December scores. This indicates that the middle of the distribution (from -0.5 to 1.4) the April scores were the best, while the December scores were better than the April scores in both tails.

Although the December test had the highest mean, the mode of the distribution was shifted to the left of the August and April tests. Thus, the more dense positive tail resulted in the highest mean in December. While the modes for the August and April distributions are nearly identical, some positive improvement is visible in both tails of the April distribution, which are positively shifted in comparison with the August test. Although differences do exist and there is some improvement, it is surprising the extent to which results from all three tests are similar despite the intervening events of a methods course and student teaching.

It is difficult to explain this result. It could be that students did not try equally hard on all three tests. This may have been particularly true for the test in April, which was administered for some participants in a less formal setting than the tests in August and December, which were administered to all participants during class. However, this would only account for the April 2010 scores and does not explain the lack of growth from August to December 2009 when both tests were administered under standard conditions.

The lack of consistency among the control scores and for the same test form in August and April calls into question the reliability of the measures for use with preservice teachers. The lack of growth could also be attributable to the fact that this test is not an accurate measure of either MKT or growth in MKT for preservice teachers. That is to say, perhaps the scores are not a valid measure of MKT for at least this group of preservice teachers. The relative lack of change in scores (0.18 standard deviations) over the final year of a teacher education program does not seem to make sense. Perhaps the test is not sufficiently sensitive preservice teachers' improvement in MKT. This result will be further considered in the Discussion section.

The final way the MKT test was used in this study was to compare the test scores of the participants at the four tiers of data collection. This comparison helps situate the qualitative work for the study and determine if the sub-groups of preservice teachers who participated in higher tiers of data collection are representative of the larger group on the MKT instrument. The following table shows the mean scores for the three test administrations for each tier of data collection.

Table 11

MKT instrument--Mean scores for each tier of participation

Test	Tier One (n=35)	Tier Two (n=11)	Tier Three (n=8)	Tier Four (n=3)
August 2009	0.233	0.631	0.572	0.695
December 2009	0.454	0.824	0.596	0.364
April 2010	0.411	0.566	0.526	0.568

These results show that, almost always, the participants at the higher tiers of data collection also had higher MKT test scores on the NCOP measure than the larger group of Tier One participants. The only exception to this was in December the three focal preservice teachers had a lower mean by approximately 0.09 standard deviations than the

Tier One participants. On this measure the participants at the higher tiers of data collection were similar to the larger group of preservice teachers at Tier One, and if anything may have had slightly higher levels of MKT, which ranged from 0.09 standard deviations below the Tier One mean to 0.46 standard deviations above the Tier One mean. This suggests that the participants at the higher tiers of data collection may be representative of the larger group of Tier One participants for other data throughout the study.

MKT Definitions

The next several sections will report the preservice teachers' definitions of MKT in terms of the four KQ categories: foundation, transformation, connection, and contingency. Pie charts will be used to show the proportional contribution of each of the four categories to their overall definition of MKT at the given time point. A fifth category, labeled "generic", was sometimes included in the preservice teachers' definitions of MKT. This was used when they included aspects of teaching that were not specific to teaching mathematics in their definition, such as classroom management.

For ease of comparison, the following figure shows how the KQ model proportionally defines MKT. That is to say, if a preservice teacher included each of the nineteen dimensions of MKT in her definition then her overall definition of MKT would be identical to the figure below. This figure shows the foundation category has the most dimensions of the four categories (seven), connection has the second most dimensions (five), transformation has the third largest amount (four), and contingency has the fewest (three).

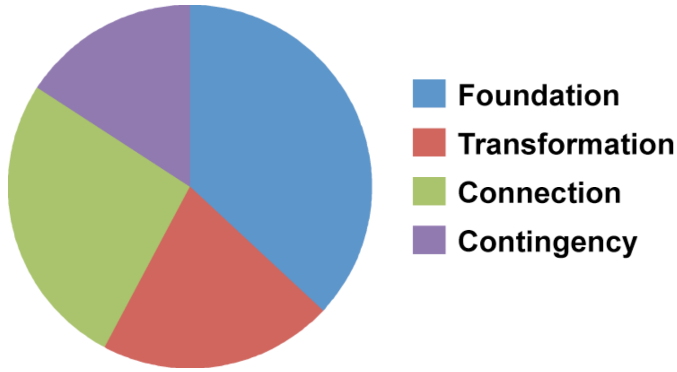


Figure 6. Category proportions of the KQ model. This figure shows the relative proportion of each of the four KQ categories in the model.

Written reflections. The following section will report the data from the written reflections. Some of the pre-test written reflection files were corrupted and not recoverable even with IT support due to an issue with the digital drop box to which they were submitted. The reduced n of 13 for whom there is data at time one map onto, and are nearly identical with, the larger n of 35 in December for which all data was accessible. The following figure shows the comparison of data in December between the reduced n of 13 for which there was paired data from August and the larger n of 35.

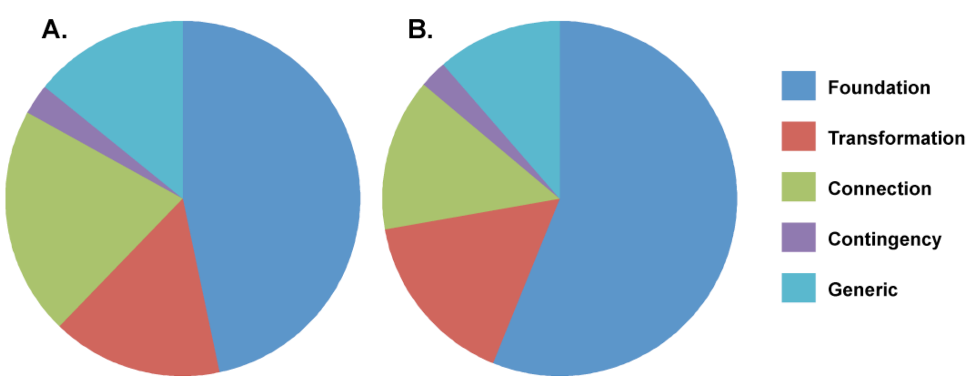


Figure 7. Written reflections—MKT definition, December. A. December 2009, n = 13; B. December 2009, n = 35. This figure shows the MKT definition for the written reflection data from December 2009.

Because of the similarities between the two groups (n = 13 and n = 35) in December, it seems likely that the pre-test data from August for the reduced n of 13 may similarly represent the larger group of 35 at time one on the pre-test. The assumption of the representativeness of the pre-test underlies the analysis of the written reflection data.

The following figure shows how the preservice teachers, as a group, defined MKT in their written definitions at the beginning and end of the fall semester (August and December). This data from the written reflections offers a snapshot of the preservice teachers' definition of MKT before and after their methods course.

A. August 2009, n = 13; *B. December 2009, n = 35*

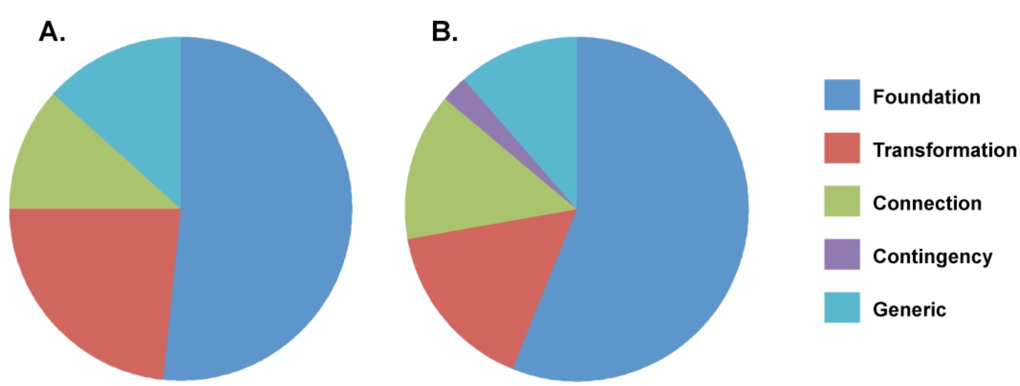


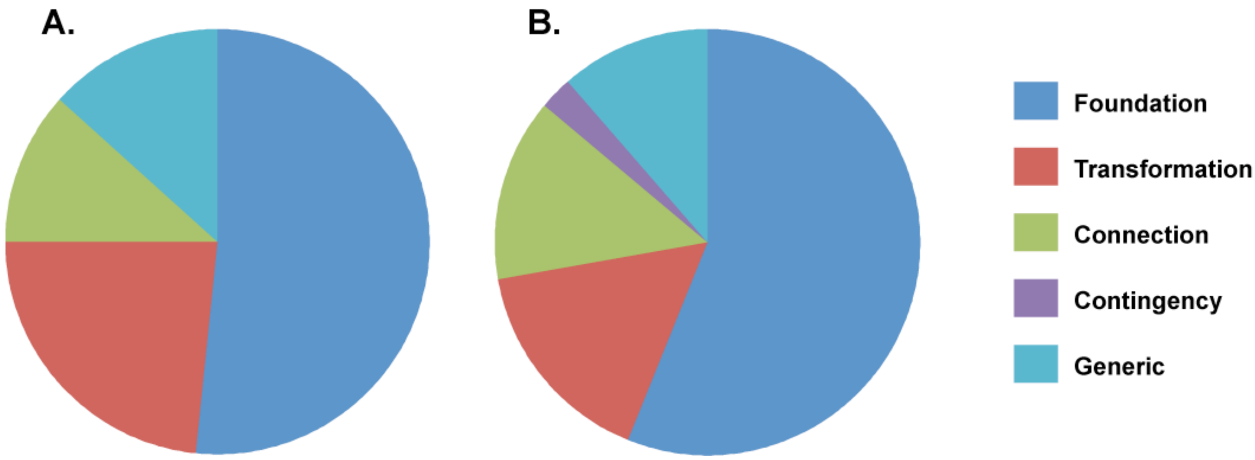
Figure 8. Written reflections—MKT definition, fall semester. A. December 2009, n = 13; B. December 2009, n = 35. This figure shows the MKT definition for the written reflection data in August and December 2009.

A primary result from this data is the preeminence of the foundation category that was evident in their written definitions of MKT, which was 52% in August and 56% in December. The remaining three categories that appeared in both August and December were also fairly consistent across the two time points: connection (August: 12%, December: 14%), transformation (August: 23%, December: 16%), and generic (August: 13%, December: 11%).

One aspect of growth that was identified in the fall semester and is depicted in the previous figure is the emergence of the contingency category. Whereas no preservice teachers mentioned contingency in their written definition of MKT in August, 11% included an aspect of contingency in December. Across the group this growth should not be overstated since it is still a low percentage, but it is a category of MKT that was not discussed at all in August and so its emergence during the fall semester is of interest. More specific discussion of growth within each category will take place after summarizing the group data from the focus groups and interviews.

Focus groups. An overview of the focus group results will now be presented. The focus group data from the two groups (n=5 and n=6) was compiled at each of the four time points. Data from the focus groups (n=11) from the first and second meetings held in the end of August/beginning of September and December provided a comparison for the written reflection data and is presented in the following figure.

Written reflections--A. August 2009, n = 13; *B. December 2009, n = 35*



Focus Groups--A. September 2009, n = 11; *B. December 2009, n = 11*

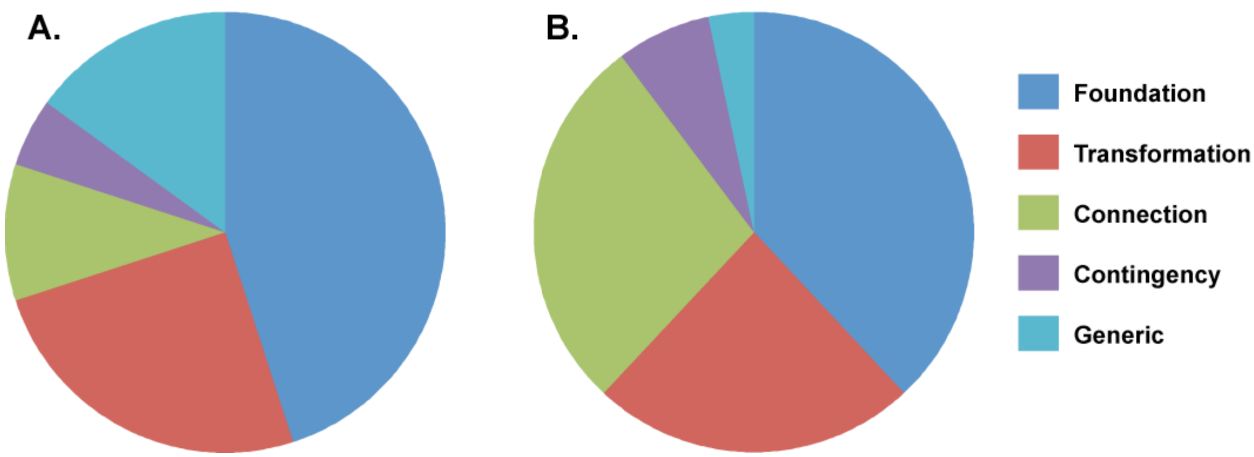


Figure 9. Written reflection and focus group comparison—fall semester. This figure shows the MKT definitions from September and December 2009 for the written reflection and focus group data.

Data in the above figure indicated similarities across data sources in the proportion that each KQ category contributed to the preservice teachers' overall definition of MKT. The additional data from the focus groups seemed to indicate similar ways the preservice teachers defined MKT across the two data types at the beginning and end of the fall semester. For example, the preeminence of the foundation category was seen in both the written and focus group data.

Turning to the focus group data exclusively, the combined data is described in the following figure regarding the definition of MKT over the course of the year for the eleven focus group participants.

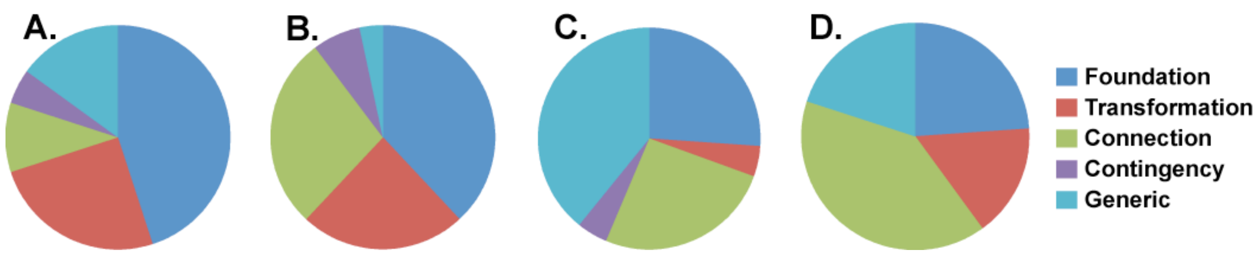


Figure 10. Focus Groups—MKT definitions, full year (n = 11). This figure shows the MKT definitions from the focus group data.

In addition to the overall similarities between the written reflection and focus group results some slight differences were revealed through the focus group data from September and December, including the presence of the contingency category in September (albeit in a very small proportion at 5%), the expansion of the connection category from 10% in September to 28% in December, and the reduction of the generic category from 15% in September to 3% in December.

The focus group data also provided an initial look at participants' definition of MKT in the spring semester (February and April 2010). A notable result in February was an expansion in the generic category from 3% in December to its peak of 39% in February. In February, the foundation category was at its lowest proportion of the focus group data at 24%. Transformation was also at its lowest proportion in February, falling from 24% in December to 4% in February.

In April, the main results for the eleven preservice teachers' definitions of MKT included the expansion of the connection category from 26% in February to its peak at 40%. The generic category decreased to 20%, which was nearly the same level as it was in August 2009 when it was15%. Also in April the transformation category expanded to 16% and the contingency category was absent, which had ranged from 4-7% in September, December, and February. Discussion of specific areas of MKT in which the preservice teachers developed over the year will be included after the interview data is introduced.

Indivdual interviews. In order to give an overview of the interview results, the eight preservice teachers' data was compiled at each of the four time points. The following figure depicts the eight preservice teachers' definition of MKT over the course of the year as indicated through their individual interviews.

A. October, B. December, C. February, D. April

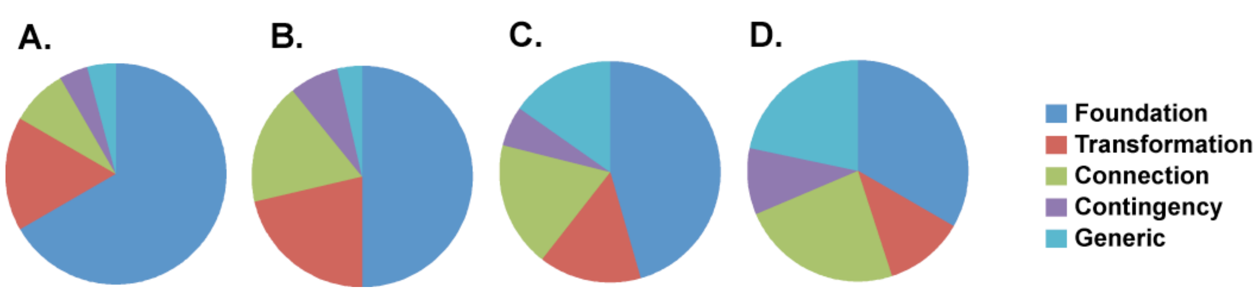
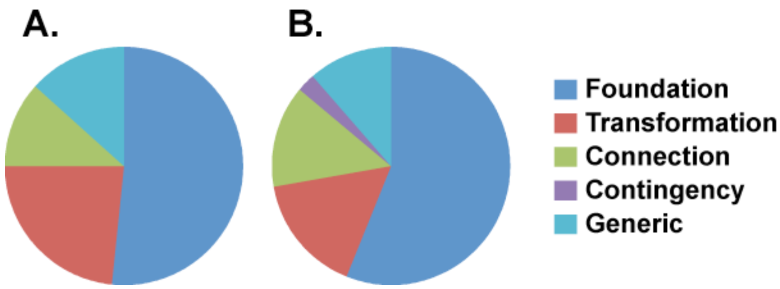


Figure 11. Interviews—MKT definitions, full year (n = 8). This figure shows the MKT definitions from the interview data.

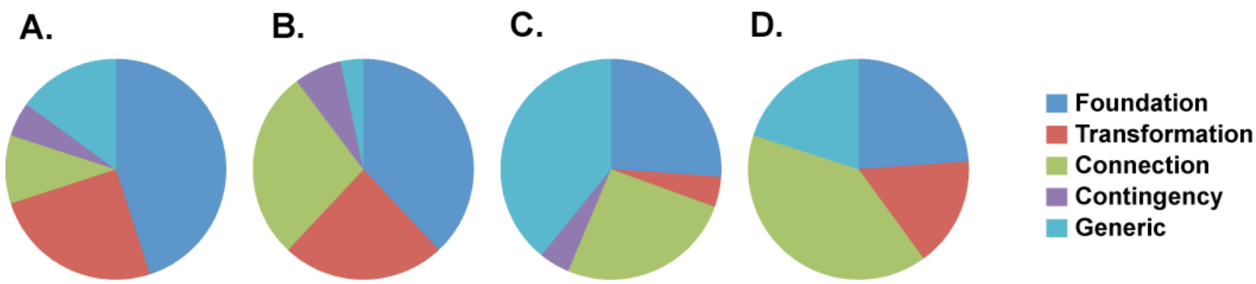
The aggregate interview data again indicated the prominence of the foundation category over the course of the year, which was the most referenced category at all four time points and consistently declined from 64% in September 2009 to 33% in April 2010. The transformation category also tended to decrease over time and ranged from a high of 21% in December 2009 to a low of 13% in April 2010. The remaining three categories expanded over the course of year: connection increased from 8% in September 2009 to 22% in April 2010, contingency expanded slightly from 4% at September 2009 to 10% in April 2010, and the generic category expanded from 4% at September 2009 to 22% in April 2010. This summary of the interview data will next be compared to the previously presented written reflection and focus group data to compare definitions across data sources.

Comparison data across group sources. In order to compare the data from the written reflections, focus groups, and interviews, it was helpful to look at all of the summary charts together. The following figure compiles previous figures to provide an overview of the preservice teachers' definition of MKT over the course of their senior year.

Written Reflections—A. August, n=13; B. December, n = 35



Focus Groups—MKT definitions, full year (n = 11)



Interviews—MKT definitions, full year (n = 8)

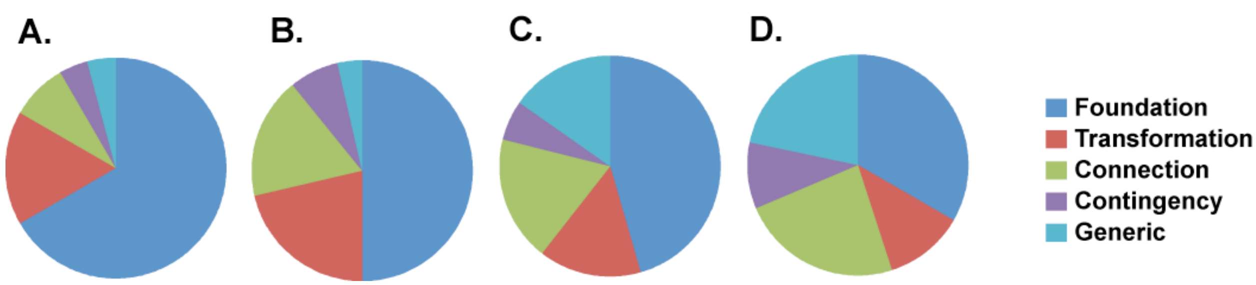


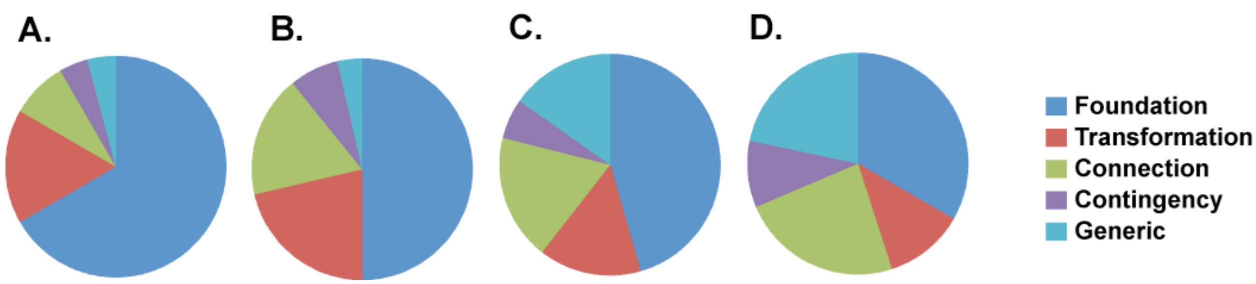
Figure 12. Comparison of written reflection, focus group, and interview MKT definitions. This figure shows the MKT definitions from the written reflection, focus group, and interview data.

Despite some differences across the three data sources, when viewed together the preservice teachers' collective definition of MKT at a given time point tended to look similar across data types. As a group, the preservice teachers tended to have a similar definition of MKT across the two or three data sources at each of the four time points. This seemed to suggest that the smaller group of preservice teachers for whom data existed in the spring semester (n=11 for focus groups and n=8 for interviews) was likely representative of the larger group of 35 preservice teachers who completed the written reflections and MKT instrument in the fall semester.

When looking across all three of the group data sources (written reflections, focus groups, and interviews), a pattern of consistent results was viewable and can be used to describe the preservice teachers' collective growth in MKT over the year. First there was the prominence of the foundation category, which declined over time but usually remained the most referenced category at all time points, with the only two exceptions being the focus groups in February and April 2010. The connection category tended to increase over the year and by December 2009 was the second-largest aspect of MKT preservice teachers referenced in their definition. The transformation category remained fairly steady over the year with a slight decrease. Contingency was the smallest proportional math-related aspect of their definition of MKT at every time point and at times was not mentioned at all (e.g., written reflections in August 2009 and focus group in April 2010). The generic category expanded to its peak in February and was larger in the spring semester (February and April 2010) than it was in the fall semester (September and December 2009).

Situated case studies. Having presented the group data in which to situate the three focal preservice teachers' data, Wendy, McKenzie, and Holly's growth in their definitions of MKT over the course of the year will now be discussed in more detail. First, their interview data will be compared to the larger group of interviewed preservice teachers (n=8). Second, their individual interview data will be presented. The following figure shows the definition of MKT in the interviews for all interviewed preservice teachers (n=8) and for the focal preservice teachers (n=3).

Interviews—MKT definitions (n=8)



Interviews—MKT definitions for Wendy, McKenzie, and Holly (n=3)

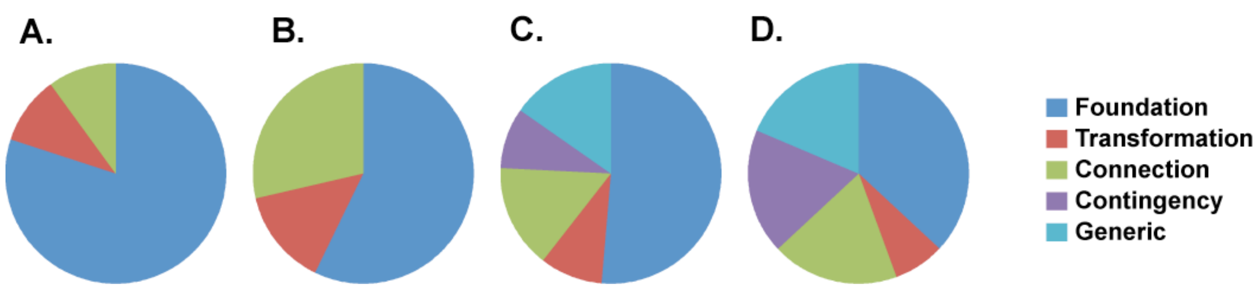
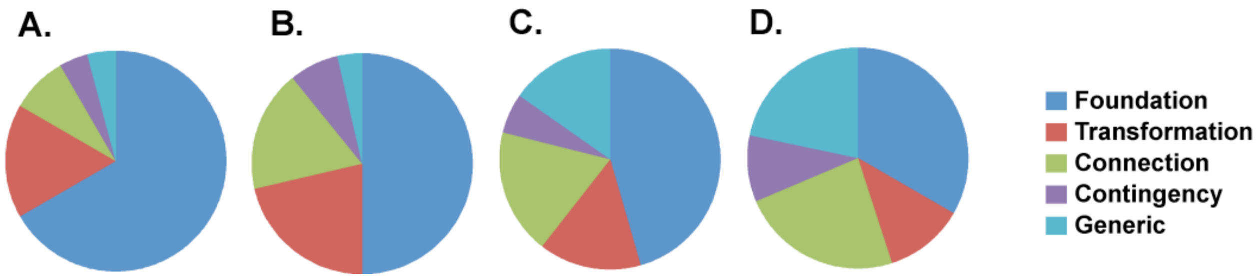


Figure 13. Comparison of interview participants' definitions of MKT. This figure shows the MKT definitions from the interview data for all eight interviewed preservice teachers and for the three focal preservice teachers.

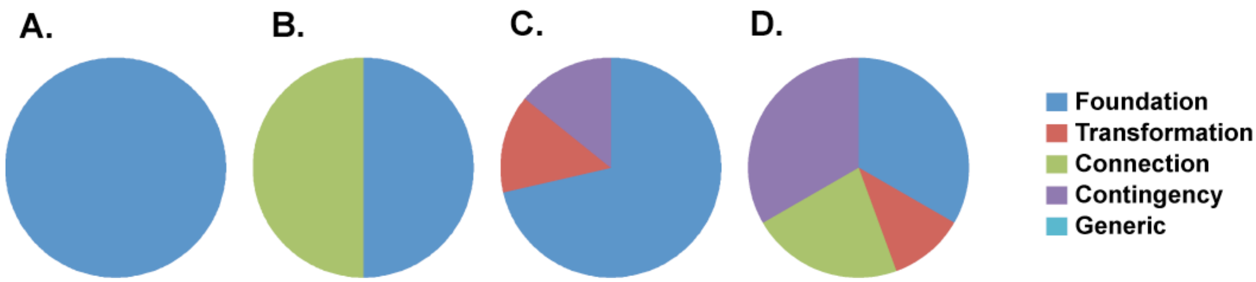
The above figure shows a great deal of similarity between the focal preservice teachers (n=3) and all interview participants (n=8), especially in February and April 2010 where the MKT definitions were nearly identical in their proportional composition across the five categories. Therefore, the three focal preservice teachers' definitions of MKT seemed comparable to the larger group of interviewed preservice teachers (n=8). In turn, as previously discussed, the group of interviewees (n=8) seemed comparable to the larger group of preservice teachers (n=35) because of similarities in data between the written reflections, focus groups, and individual interviews. As a result it seemed likely that the three focal preservice teachers' definitions of MKT represented the larger group. Therefore, it may be helpful to further examine the way their definitions changed over the course of the year to understand the growth in MKT of these three individuals, which may also represent the larger group (n=35) of preservice teachers.

Before reporting the results for the three focal preservice teachers individually, an important result was found in the interview data of the eight preservice teachers' individual definitions compared to the aggregate definition. The following figure shows the aggregate definition of MKT for the eight participants as compared to each of the eight participant's individual definitions of MKT at the four time points.

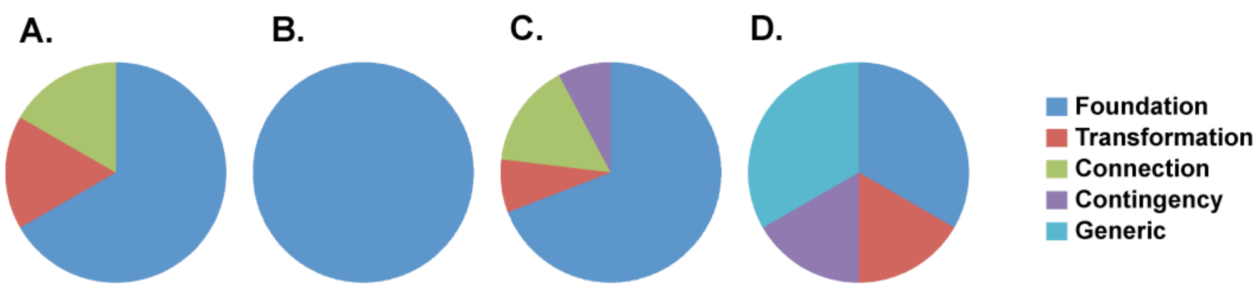
Aggregate Interview data (n=8)



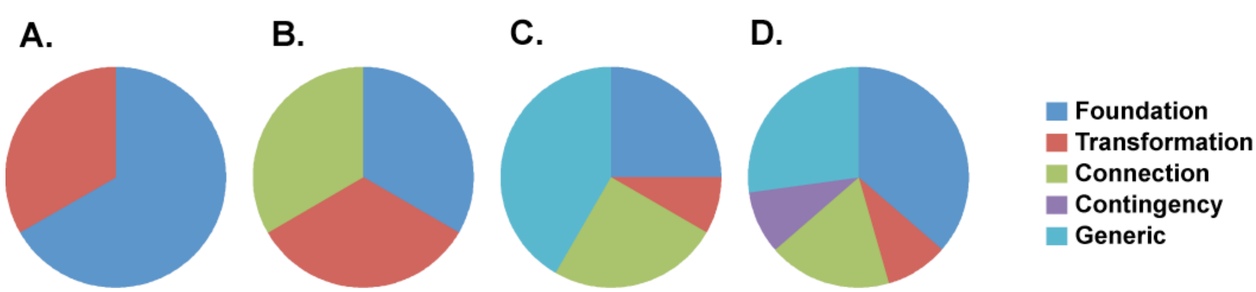
Wendy's definition of MKT



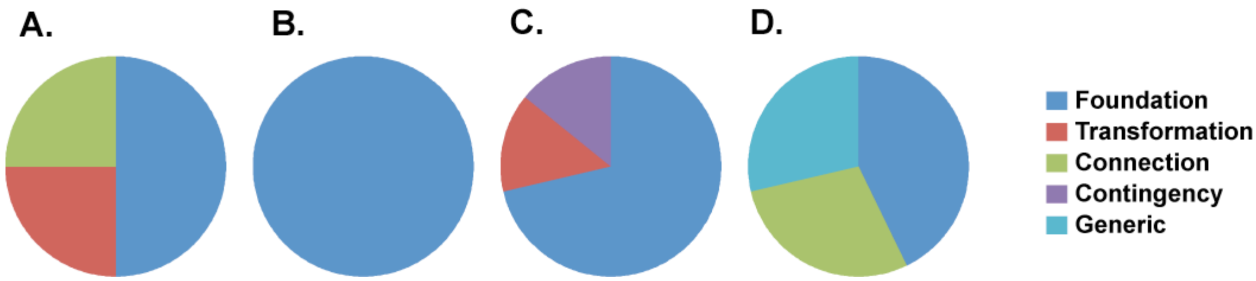
McKenzie's definition of MKT



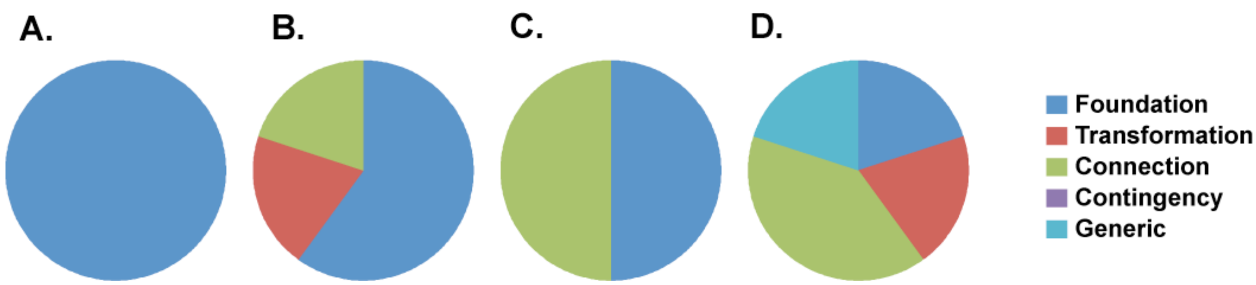
Holly's definition of MKT



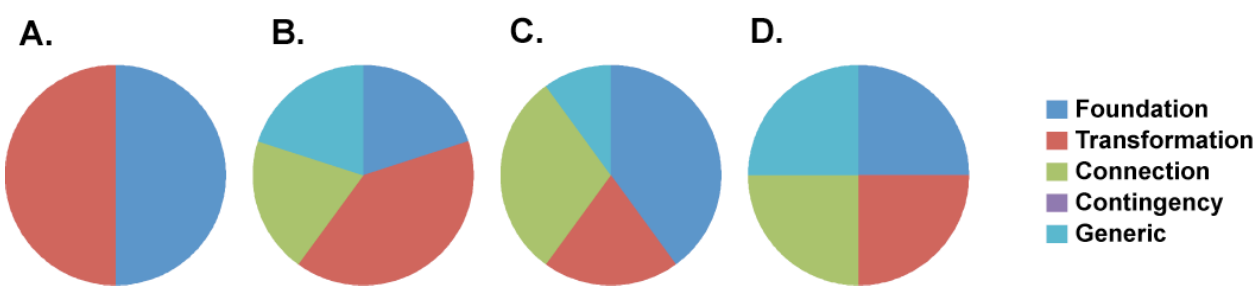
Molly's definition of MKT



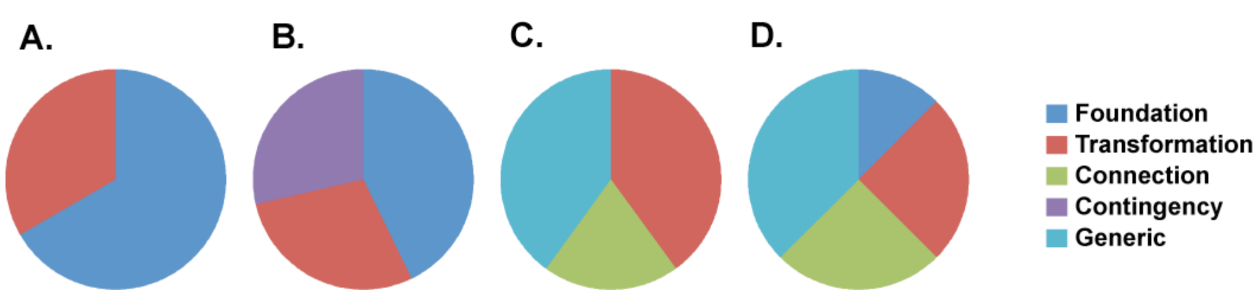
Alice's definition of MKT



Claire's definition of MKT



Kate's definition of MKT



Amanda's definition of MKT

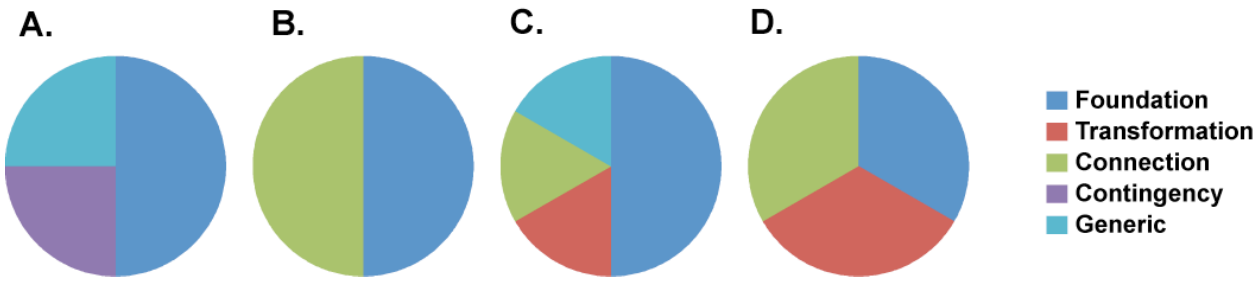


Figure 14. Aggregate and individual MKT definitions. This figure shows the MKT definitions from all interview participants aggregated and individually.

The above depiction of the data indicates that individual's definitions of MKT did not map onto the aggregate definition generated when their eight definitions were combined. That is to say, the compiled definition (n=8) is essentially a mathematical artifact that represents the averaged data, but the compilation does not represent the individual definitions of the preservice teachers over the course of the year. The compiled interview data (n=8) made it seem like each preservice teacher described MKT using all five categories at each of the four time points. However, examining the individual data for each of the eight interviewed preservice teachers revealed that only one of the participants, Holly, defined MKT with all five categories, and this only occurred in February and April 2010. In fact there were four instances where participants used only one category to define MKT, that category always being foundation, and six instances where they only used two categories. The individual data made it clear that a description of MKT growth needs to occur not just at the group level, but also at the individual level. Therefore, the three focal preservice teachers' individual data will be presented next.

By April 2010, the eight preservice teachers used three (n=1), four (n=6), or five (n=1) categories to define MKT. Individually, they either used more categories in April than they had at any point in the year (n=5) or the same number of categories as they had in February (n=3). In April there appeared to be some convergence of the individual and aggregate data, where the preservice teachers' individual definitions of MKT looked closer to the aggregate definition than at any other time point. This was especially true for Holly's definition of MKT, which was proportionally quite similar to the aggregate definition (n=8). Furthermore, the preservice teachers' individual definitions of MKT were more similar to each other in April than they were at any previous time point. There seemed to be some convergence across the group of preservice teachers' definitions of MKT in April. Based on their disparate definitions over the course of the year in which they began with and added categories to their definitions at various times and amounts, it seemed that the preservice teachers' individual growth was varied but eventually more consistent ideas about MKT began to emerge at the end of the year.

Each of the three focal preservice teachers' interview data will now be used to more fully describe their growth in MKT definitions over the year. Based on the similarities of their definitions to larger groups of preservice teachers as indicated through multiple data sources, it is likely that the three focal preservice teachers' growth may also have been similar to other preservice teachers. Again, there is the phenomenon that the aggregate interview data for the three focal preservice teachers does not accurately reveal anyone's individual definition of MKT. Therefore, each of their MKT definitions will now be presented individually.

Wendy. The first participant whose results will be discussed is Wendy, who did her student teaching in a first grade classroom that primarily used Math Expressions. First her definition of MKT will be summarized to provide an overview, and then her definition will be examined in more depth. The following figure shows Wendy's definition of MKT over her senior year.

A. September; B. December; C. February; D. April

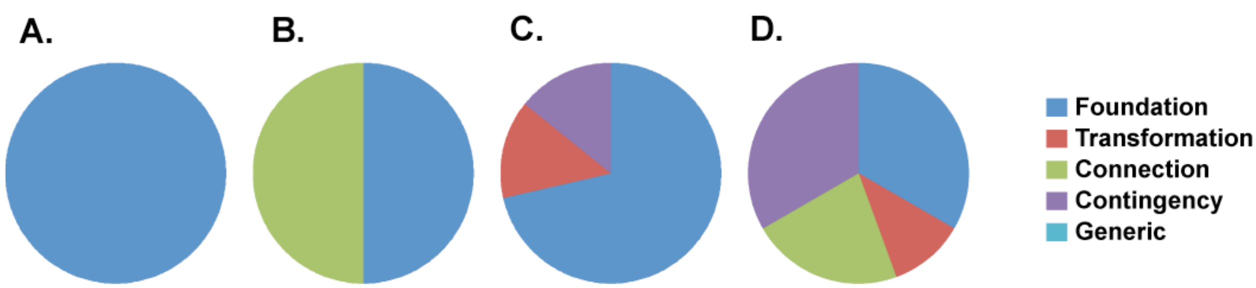


Figure 15. Wendy's MKT definition. This figure shows Wendy's definition of MKT.

In September, Wendy described MKT entirely in terms of foundational knowledge. In December, she defined MKT as half foundation and half connection. At February, she described MKT as 67% foundation and equal parts transformation and contingency at 17% each. In April, Wendy defined MKT as equal parts foundation and contingency at 33% each, 22% connection and 11% transformation.

Based on these results, some statements can be made about Wendy's definition of MKT over the course of the year. In September 2009 Wendy defined MKT with one category, and then she added one or two new categories to her definition at each subsequent time point so that in April 2010 she defined MKT with all four KQ categories. At every time point, foundation was the category Wendy used most or tied for the most to define MKT. The foundation category tended to decline over time and ranged from 33% to 100% of her definition of MKT. Contingency and transformation were not present in the fall semester (0% at September and December 2009) and were added to her definition of MKT in the spring (February and April 2010). Contingency increased from 17% in February to 33% in April 2010. Transformation decreased from 17% in February to 11% in April. Connection appeared in December 2009 at 50% and again in April 2010 at 22% of her definition of MKT. Wendy's definition of MKT never included any generic aspects of teaching, which distinguished her from her interviewed peers, all seven of whom included generic aspects of teaching in their definitions of MKT (see Figure 14).

Wendy's definition of MKT over the course of the year will now be described to add to the above summary. In her first written reflection from August she explained her initial conception of MKT in the following way: "Mathematical knowledge for teaching is a vague and nebulous topic that will hold meaning differently for each person." At the beginning of the year Wendy believed that MKT was not specific or standardized. Furthermore, what little she did say about MKT was entirely about foundational knowledge. In her first interview Wendy defined MKT in the following way:

> Math teachers need to be able to show or demonstrate why their students should be learning this. So, understanding why you need to learn algebra, so that you're not just doing it, but you might use it someday. And I think sometimes a math teacher might know in their head why you need to know a fill-in-the-blank type of math, but they forget to explicitly tell their students, and so it takes a while—or maybe it never happens--that their students realize that they can apply that to some kind of real-world setting. So I think that is super critical. And then definitely understanding the math, just underneath the surface, so that if a kid asks, "What is 0?" you actually have a bigger image, or a concept, of what that means, or why it's important.

In the above August definition Wendy focused on foundational knowledge, and more specifically on awareness of purpose and concentration on understanding. She believed that teachers needed to have a broad enough conception of the content they taught in order to tell their students why they were learning something as well as to conceptually explain mathematical ideas.

In December, Wendy's definition of MKT changed to half foundation and half connection. In regards to foundation, she focused on her growth in mathematical content knowledge:

> What maybe has changed about my understanding [since August] is I am more flexible in the way I can do math. I came in thinking, "Oh yeah, a kid can do fill-in-the-blank strategy or fill-in-the blank strategy, but I would never do that." My understanding of math was pretty rigid, but now I just feel like I understand the math a little bit more so I find myself using different strategies and my own math usage is a little more flexible.

Despite the foundation category being proportionally reduced in her definition from time one to time two, Wendy expressed her sense of growth within this category, specifically regarding overt subject knowledge. It is important to note that a decrease in proportional attention a category received over time is not necessarily indicative of a "worsening level" of knowledge within that category. Although Wendy's definition of MKT changed from 100% foundational in September to 50% foundational in December, within this category she reported growth. A frequent finding of this study was that the preservice teachers generally talked about each of the four KQ categories in deeper and more specific ways over the course of the study. Therefore it is important to interpret the data about their definitions of MKT in terms of both the relative proportion of each of the five categories to the others as well as the depth within each category. As this example with Wendy indicated, some areas of growth are not visible if only considering the frequency counts of aspects of MKT that appeared in each definition.

The second aspect of Wendy's definition of MKT in December was the addition of the connection category. In her second interview Wendy said that part of MKT was "trying to anticipate what questions my kids would have and how to address them." This indicated Wendy's new awareness that anticipation of complexity was an important aspect of teaching mathematics, which was not part of her conception of MKT in August. By comparing her definition of MKT in September and December, growth was evident in both the depth of her definition within the foundation category as well as her expansion to include the connection category.

In February, Wendy included even more aspects of foundation in her definition of MKT. Whereas in September she referenced two aspects of foundation (awareness of purpose and concentration on understanding) and in December she referenced one aspect of foundation (overt subject knowledge), in February she defined MKT with four aspects of foundational knowledge: concentration on understanding, overt subject knowledge, theoretical underpinning, and use of terminology.

Wendy talked about concentration on understanding and theoretical underpinning in her explanation of why overt subject knowledge was important in MKT. This was also a time Wendy included transformation in her definition of MKT:

> I would say that it's definitely about being a facilitator of knowledge and not being a dispenser of it, and I think knowing enough about math to be able to create these inquiry lessons and knowing enough about math that you can create an environment where you can be a facilitator. And not needing to be a dispenser just so if kids ask you questions then you would know the answers. So, knowing

> enough that you can give kids the freedom to learn it on their own and know that you can supplement them regardless of what pitfalls they might encounter.

This revealed a shift for Wendy because previously she thought content knowledge was important to answer students' questions, but in February she thought content knowledge was important to create lessons that focused on understanding instead of rote learning. The above statement also indicated development in Wendy's awareness that mathematics teaching has a theoretical basis that supports children learning in classroom environments in which the teacher is a facilitator rather than a dispenser of knowledge.

In regards to transformation, Wendy stated that a teacher needs to be able to create inquiry lessons as well as an environment in which she facilitated and students learned. Although Wendy was vague and never specified how this happened, Wendy's description matched the transformation category because of its focus on making the teacher's knowledge accessible to students. It is interesting to note that this was the first time Wendy included transformation in her definition of MKT and that it was quite vague.

The final aspect of foundational knowledge that Wendy included in her definition of MKT in February was a teacher's use of terminology. She said that a master mathematics teacher does not use "really bizarre intangible terms that don't make any sense." She went on to explain what she meant by this using the following example:

> I learned regrouping when I was in school, but when I think about my kids now, I'm like, "Why would you ever teach a kid the word 'regrouping?' That doesn't mean anything to a kid." So I think a master mathematics teacher, any kid of master teacher, really connects with kids and says things the way they need to be said.

Although the particular example Wendy gave of regrouping being a “bizarre intangible term” could be debated (especially when compared to other words such as “borrowing”), she communicated her understanding that wording and language matter in mathematics instruction and are an additional aspect of MKT that she had not previously voiced.

Wendy’s February interview was also the first time she included contingency in her definition of MKT. Specifically she explained that MKT was about using opportunities:

> I think that a master mathematics teacher capitalizes on teachable moments. I was just in the bookroom, perusing non-fiction texts, and I found *Going to the Grocery Store*, a non-fiction text, but it has dollars and dimes and nickels. I see that as a really good opportunity to teach kids about money if that’s what you’re doing in math, and so I think that a master mathematics teacher just does those kinds of things naturally.

Although Wendy’s example was more about a planned opportunity, her initial comment about capitalizing on teachable moments was in the realm of contingency situations that occur in teaching and using those as opportunities to further student learning. Compared to December 2009, Wendy’s definition of MKT in February 2010 revealed growth through incorporating two new categories of MKT (contingency and transformation) and including more aspects of foundational knowledge, including the addition of theoretical underpinning and use of terminology. Furthermore, Wendy continued to indicate growth in overt subject knowledge in terms of how she viewed the purpose of her own content knowledge.

In April, Wendy’s definition of MKT comprised of all four of the KQ categories: foundation, connection, contingency, and transformation. Within the foundation category, Wendy discussed concentration on understanding and overt subject knowledge, both of which she included in her definition of MKT in February. Similar to February, Wendy's definition in April was grounded in her understanding of the role of overt subject knowledge. At the time of the fourth interview Wendy had just begun teaching geometry, which she described as "very fun and very practical." She went on to say that geometry "was always my favorite type of math to learn, which I think has put a very strong bearing on it being my favorite math to teach." During the fourth interview she said that, for her, "Having comfort with the subject matter is something new with geometry." Wendy went on to explain her current definition of MKT:

> The newest element of my understanding of mathematical knowledge for teaching is that comfort level and the idea that when you're really comfortable with content matter, then you can deviate from your lesson plans, and you can really supplement it. When I was teaching more abstract concepts like how to subtract two digit numbers using ten sticks and circles, I wasn't frequently walking around being like, "Oh, here's a good application while we're standing in the hall waiting for (physical education) to start." But now I see myself doing that a lot more, when my kids are in the hallway or when we're walking to the bus, "Look at that shape. Look at that rotated version of the same shape," and sort of making those mini-connections for students in my casual conversation.

Wendy talked about contingency more in her April interview than she did at any other time. The above comment again included the use of opportunities, which she also included in February. For the first time Wendy added deviation from agenda to her definition of MKT, which she explained was possible due to a teacher's content knowledge.

Wendy included two ideas about connection in her definition of MKT in April. The first was contained in the previous comment about making what she referred to as "mini-connections" between concepts for students when opportunities arose. The second way Wendy talked about connection more thoroughly was in the following excerpt from her fourth interview, when she described the shift she made from viewing herself as a dispenser to a facilitator. Wendy then went on to explain her new thinking about overt subject knowledge and how it related to understanding and connection:

> At the very beginning of the study in the fall, I thought of (MKT) very much like having a very critical understanding of the content in a way that I could answer any questions that students pose and plan effective lessons. And then we move from that to sort of thinking about being a facilitator of it, so not just being able to stand in front of the class and say, "This is how you do it," and have that be right. But actually, perhaps it requires an even more critical understanding. And being able to step back and say, "If I give children this set of prompts, they'll be able to discover that strategy on their own." So, I move from like dispenser of knowledge to facilitator of understanding, and I still believe in that whole facilitator of understanding philosophy, but I think with geometry has come that understanding of master teachers of mathematics connect it to every day, and they connect it frequently and without trying.

Wendy realized that teaching mathematics with a focus on promoting student understanding required a deeper understanding of the content knowledge than a teacher would use to teach procedurally. When she experienced a deeper understanding of the content she was teaching, Wendy became more aware of the importance of connection within mathematics teaching than she had previously. She also included transformation in her above definition of MKT when she talked about giving students a certain "set of prompts." Although it was difficult to determine if this statement refers more to choice of examples or demonstration, it is an example of Wendy's awareness of the importance of transformational knowledge in teaching mathematics.

Compared to February, in April Wendy's definition of MKT was broader because it included at least one reference to each of the four KQ categories of MKT. Furthermore, connection and contingency both increased from February to April 2010 and contributed to Wendy's definition of MKT at the highest proportional amount.

Transformation was fairly similar to February in that it was mentioned but only peripherally. Foundation continued to be the basis for Wendy's conception of MKT. In particular Wendy's expanded awareness of the way overt subject knowledge was linked to other aspects of MKT was a key way she described her growth from the beginning to the end of the year.

McKenzie. McKenzie did her student teaching in a fourth grade classroom that used EnVisions. First her definition of MKT will be summarized to provide an overview, and then her definition will be examined in more depth. McKenzie's individual data for her definition of MKT at the four time points will now be presented.

A. September; B. December; C. February; D. April

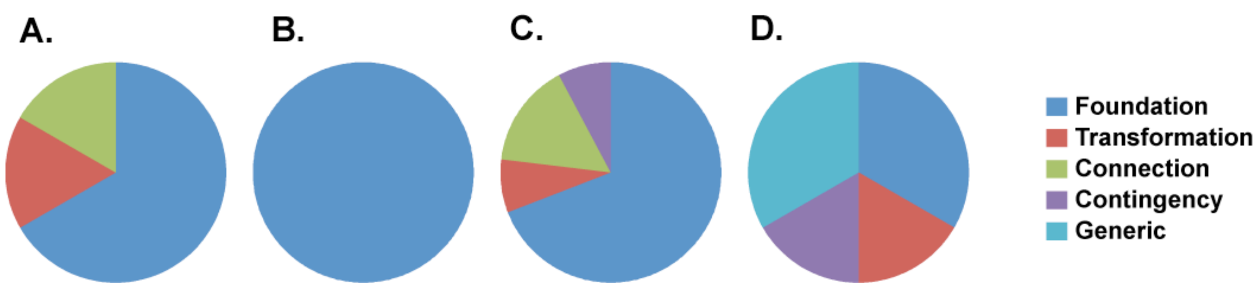


Figure 16. McKenzie's MKT definition. This figure shows McKenzie's definition of MKT over the year.

In September 2009, McKenzie described MKT predominantly in terms of foundational knowledge at 67% with equal mention of connection and transformation at 17% each. In December, she defined MKT entirely in terms of foundational knowledge at 100%. In February 2010, McKenzie described MKT as 69% foundational, 15% connection, and equal parts transformation and contingency at 8% each. In April, McKenzie defined

MKT as equal parts foundation and generic at 33% each, and equal parts transformation and contingency at 17% each.

Based on these results, it is possible to make some statements about McKenzie's definition of MKT over the course of the year. At every time point, the foundation category was the primary way she defined MKT and ranged from 33%-100% of her definition. Her definition of MKT in the fall semester (September and December 2009) was especially dominated by foundational knowledge. Proportionally, foundation declined in the spring semester (February and April 2010) and was at its lowest percentage in April at 33%, although it still ranked first or tied for first at each of the four time points. Transformation appeared in September, February, and April in the range of 8-17% and were equal in September 2009 and April 2010 at 17% each. Contingency appeared in McKenzie's definition of MKT during the spring semester (February and April 2010), and increased from 8% in February to 17% in April. McKenzie included connection in her definition half of the time in nearly the same amount when it was included, which was in September at 17% and February at 15%. The generic, non-mathematical category appeared in her definition of MKT in April where it ranked as tied for the most referenced category at 33%.

McKenzie's definition of MKT over the course of the year will now be described to add to the above summary. In her first written reflection in August, she explained her initial conception of MKT in the following way:

> It seems that having mathematical knowledge *in general* and having mathematical knowledge *for teaching* are two very different concepts. Understanding the difference between the two can lead to very different beliefs and attitudes towards the way in which math is taught in classrooms. The way in which teachers view

> mathematical knowledge can create closed and fearful math environments for students or can produce open and risk-taking environments for students. (emphasis in original)

McKenzie explained her initial conception of MKT was that there was something different or additional within MKT than only overt subject knowledge. Furthermore, she believed that a teacher's knowledge impacted the classroom environment she established for her students.

In September, the primary way McKenzie defined MKT was in terms of the importance of concentrating on understanding. She stated that teaching math included more than content knowledge. McKenzie described what she realized her Junior year when she took a required math content course taught within the School of Education which resulted in her definition of MKT in September:

> I never thought about changing the way I thought about math. I thought, "Why can't we just teach it the way we were taught it? I don't understand. It works." By the end (of the course) I was more open minded about how kids think about math.

McKenzie explained that her conception of MKT in September included overt subject knowledge and concentration on understanding. She stated her ideas about concentration on understanding again later in the interview when she said:

> I cannot get away from asking leading questions…(Students) just need to figure it out for themselves and I just want to help them out so much but that is not helping them understand it for themselves.

McKenzie again expressed her belief in the importance of concentration on understanding rather than procedural learning. She also expressed a vague awareness of transformational knowledge. That is to say, she realized she needed to develop her own transformational knowledge, in particular her use of questions, and included this knowledge in her definition of MKT.

The final aspect of foundational knowledge that McKenzie included in her initial definition of MKT was use of terminology. Related to this, she said, “There are so many times when a teacher asks, ‘How did your teacher explain it last year?’…There’s different language that each teacher uses.” McKenzie included the idea of consistency in terminology in her definition, and also spoke about the benefit of teaching the same students multiple years through the category of connection: “It almost makes you want to loop up with your students from K-5…You could have the same concepts and structure through all of elementary school.” This comment maps onto the KQ dimension making connections between concepts.

In September McKenzie’s primary way of defining MKT was based on the foundational dimension of concentration on understanding. In addition to directly focusing on concentration on understanding in her definition of MKT, she also described the two other categories she included in her definition of MKT, transformation and connection, as important because of the way these aspects of MKT further supported students’ development of mathematical understanding.

In December McKenzie’s definition of MKT consisted of two dimensions of foundational knowledge: curricular knowledge and overt subject knowledge. She defined MKT in the following way:

> Knowing the curriculum, and knowing the content, too, because I feel like I’m not going to know how to create a rubric or a set of strategies I’m looking for if I don’t know the strategies themselves…If I understood the content, it would be more beneficial.

Knowing the curriculum was a new aspect of MKT that appeared only in December. Overt subject knowledge was the one dimension of MKT that McKenzie included in her definition at all four time points. She indicated her belief that content knowledge was important because of its necessity for assessment. From September to December, the definition McKenzie gave for MKT changed from being comprised of foundation, transformation, and connection in September to entirely foundation in December. Within the foundation category McKenzie did not mention concentration on understanding, which had been the foundation of her definition in September.

In time February McKenzie included more aspects of MKT in her definition than she did at any other time during the year. At the category level she included all four KQ categories: foundation, transformation, connection, and contingency. Additionally within foundation and connection she referred to multiple dimensions of MKT. She included the most dimensions of MKT in the foundation category, for which she included five dimensions of MKT: adapting the textbook, identifying errors, awareness of purpose, concentration on understanding, and overt subject knowledge.

Whereas in December McKenzie made a statement about curriculum knowledge, in February described how she enacted this in her lesson planning. During the third interview she explained, “Every lesson that I’ve taught I’ve definitely had to go back into the manual and say, ‘What is this? What are (the students) supposed to know? What are the concepts?’” McKenzie explained she did this in order to make decisions about adapting the textbook during lesson planning. McKenzie’s comments during February interview which mapped onto the foundational dimension adapts textbook had more depth than they did in December, when she used the phrase "knowledge of curriculum" but never talked about this in terms of adapting the curriculum.

McKenzie started teaching math full-time a few weeks prior to her February interview, which she referenced in some of her comments about MKT. For example when asked about her definition of MKT, McKenzie first said, "A lot of it for me in the last couple of weeks, with now taking over math, has been about understanding or being aware of or anticipating their misconceptions." Within the KQ model this dimension of MKT is identifying errors, which McKenzie included for the first time in February 2010.

McKenzie went on to give a longer definition that included multiple dimensions of MKT, in which she focused on concentration on understanding from the foundation category. She also included overt subject knowledge and awareness of purpose (foundation) as well as connections between concepts (connection):

> I definitely think (MKT) means knowing how to teach the concept without just teaching definitions and practice problems. Like knowing how to talk about it, knowing how to pick up on what kids already know, knowing how to pick up on what kids think they know but aren't really understanding, and what are just fake answers, like loaded answers, and what kids are actually understanding what you're talking about. And then part of it is definitely knowing the content, knowing what you're teaching, and why you're teaching, and how it relates to other concepts that you've already taught.

McKenzie continued to define MKT first and foremost in regards to foundational knowledge, and in particular in regards to concentration on understanding, which was also the predominant theme of her definition in September. McKenzie briefly included connection in the above definition in terms of connections between concepts, which she later spoke more about in regards to MKT:

> And knowing what we've already learned and how what we're doing right now is going to build on what we've already learned. So for example we started out with polygons and then we moved to triangles and then quadrilaterals.

The two dimensions within connection that McKenzie included were connections between concepts and sequencing instruction, the latter of which McKenzie included for the first time in her definition of MKT.

The one aspect of transformation McKenzie included in her February MKT definition was similar to the idea she expressed at time one about leading questions. In her third interview she said:

> I'm just really tired of asking leading questions. I don't feel like it's authentic. I mean, sometimes it's good to just ask a leading question and have somebody say the right answer because you need a right answer, but I would rather ask them a question and have them not know than ask them a question and have them say an answer, and they still don't really know, and then convince myself that they do know it because they said the right answers. So I've steered away from there; I've learned which questions to ask.

McKenzie indicated that questioning to develop and assess student learning (transformation) was both a dimension of MKT as well as an area of personal growth.

Another area of growth in her February definition of MKT was that McKenzie included an aspect of contingency for the first time when she said:

> As far as mathematical knowledge for teaching, the last two weeks have been really good for me as far as like picking up on whose comments I should expand on and whose comments I should, not ignore, but be like, "Mmm…That's interesting, but let's not necessarily go there right now."

This contingency dimension is responding to children's ideas. At the time of the third interview in February, McKenzie had recently realized that part of effectively teaching mathematics involved her awareness of which student comments would be helpful to further discuss and which were unlikely to contribute to student learning.

Compared to December 2009, McKenzie included several more aspects of MKT in her definition in February 2010. Dimensions of MKT that emerged for the first time in February were: adapting the textbook, awareness of purpose, identifying errors (foundation), decisions about sequencing (connection), and responding to children's ideas (contingency). Similar to time one, McKenzie's primary focus within her definition of MKT was on foundational knowledge related to concentration on understanding.

During her April interview McKenzie related her impression of MKT and how it changed over the course of the year. She described what her definition of MKT was at the beginning of the fall semester:

> I thought that math teaching was all content, that you just needed to know what your students need to know. Then at the beginning of the (spring) semester [roughly interview three], I think I was a little bit *too* far extreme. Like, "no, it's just how you go about it. It's just like how you're able to talk with your kids and pull it out of them and things like that." And I feel like now I have a better balance of knowing that I need to understand the content *and* knowing that I need to understand my kids and how they learn and how to approach a certain concept and how to structure a lesson. So, I think it's a balance of those two things, between the content and how you structure a lesson. (emphasis in original)

McKenzie reported her initial definition in terms of overt subject knowledge, which she felt subsequently declined in relative importance and reached its lowest point at the beginning of the spring semester. In its place, she included generic aspects of teaching in her definition, including the need to understand students and how they learn. McKenzie went on to explain what she meant by the "structure" of a lesson in terms of transformational knowledge:

> The structure part has been a little bit more what I've focused on this semester, mainly because when you're up in front of a class, you need to know how you're going to do what you're going to do. It's not just like sitting there and lecturing, and so you need to know, "What questions am I going to ask? What activities am I going to be doing? How is this thing going to actually come about?"

The knowledge of how things "actually come about" is a trademark of transformational knowledge. In April, McKenzie said that her current definition of MKT was a balance between foundational knowledge (overt subject knowledge and a brief comment later in the interview about use of terminology), generic knowledge (understanding children and learning) and transformational knowledge (questioning, choice of examples). This was the first time McKenzie included non-mathematical aspects of MKT in her definition. She included additional generic aspects in her definition of MKT when she described another teacher on her PLC who she considered a master mathematics teacher. These included using kid-friendly language and students knowing the teacher's system and expectations.

Examining the remainder of McKenzie's April interview responses showed that McKenzie also included contingency in her definition of MKT. In her description of the PLC member she considered to be a master mathematics teacher, McKenzie highlighted how skillful her team member was with contingency, which was why McKenzie considered her a master mathematics teacher. McKenzie's description of the teacher's facility with contingency was as follows:

> They were going over their morning work at the very beginning of class, and one of the markers that I use (for a master mathematics teacher) was how she was walking around the whole time, and she was talking to a student that's on the other side of the classroom and saying, "Alright, Jeff, what did you get? Can you read number four for us?" And then he's starting to read, but she's already walking around and checking everybody's answers. But he's explaining what he's doing, and she's still picking up on what he's doing. But she's still checking other people, and then she's like, "Well, Jeff, hold on. Sarah got this…" And she's bringing everybody in, which is more a master teacher-type thing in the math classroom.

McKenzie's retelling of what she observed in her PLC member's classroom focused on contingency, especially responding to children's ideas, which was also a dimension of her definition of MKT in February.

A primary way McKenzie's definition of MKT changed from February to April was by adding generic aspects of teaching which were not present at any other time point. Furthermore, although McKenzie made longer comments about MKT in April she did not include any mention of connection. McKenzie's description of foundation was more narrowly defined in April and comprised of two dimensions, as opposed to four dimensions in February. McKenzie's September and February core aspect of MKT of concentration on understanding was not mentioned at all in her definition in April. Compared to February, McKenzie increased her proportional attention to transformation and contingency. McKenzie spoke about contingency in similar ways in February and April, which was in terms of responding to children's ideas. She reported focusing on developing her transformational knowledge during the spring semester.

Holly. Holly did her student teaching in a fourth grade classroom that used EnVisions. Her student teaching placement was in the same district and she had the same university supervisor as McKenzie. First her definition of MKT will be summarized to provide an overview, and then her definition will be examined in more depth. An overview of Holly's definition of MKT throughout her senior year is provided in the following figure:

A. September; B. December; C. February; D. April

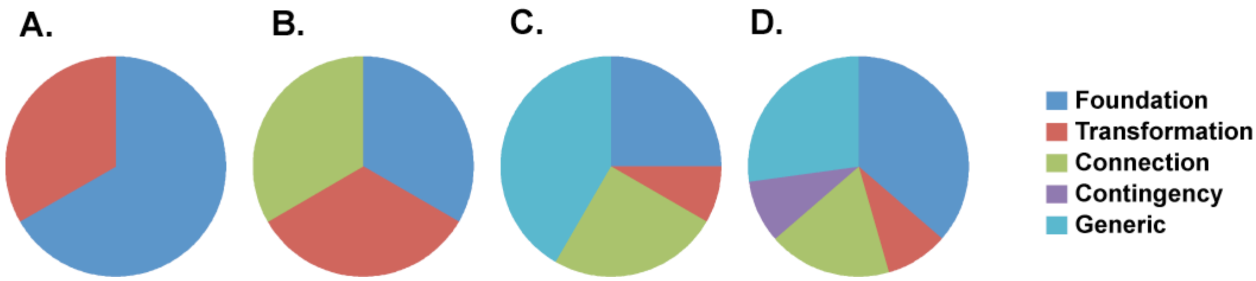


Figure 17. Holly's MKT definition. This figure shows Holly's definition of MKT over the year.

In September, Holly described MKT largely in terms of foundational knowledge at 67%, as well as transformation at 33%. In December she defined MKT as equal parts foundation, transformation, and connection at 33% each. In February Holly described MKT as 45% generic, 27% foundation, 18% connection and 9% transformation. In April she defined MKT as 36% foundation, 27% generic, 18% connection, and equal parts transformation and contingency at 9% each.

Based on these results some statements can be made about Holly's definition of MKT over the course of the year. In September Holly defined MKT with two categories, in December she used three categories, and in February and April she used all five categories. Usually foundation was the category she used most to define MKT, with the only exception being in February. Along with foundation, transformation appeared at all four time points and was higher in the fall (September and December 2009) than in the spring (February and April 2010). Connection first appeared and also peaked in December (33%) and then declined to an equal proportion in February and April (18%). Contingency appeared in April where it comprised the smallest proportional amount of

Holly's definition of MKT at 9%. The generic category only appeared in the spring semester (February and April) where it ranked first and second, respectively, in Holly's definition of MKT and comprised a substantial amount of her definition at 45% in February and 27% in April.

Holly's definition of MKT over the course of the year will now be described to add to the above summary. In her first written reflection in August Holly explained her initial conception of MKT was that it "entails many different things." In September, Holly's primary focus was on foundational knowledge and all of her comments within this category addressed overt subject knowledge. When asked what it meant to be a master mathematics teacher she said, "A master in my mind is like a professor, knowing every single thing there could be to know." She expanded her definition of MKT by stating:

> The phrase MKT is not just content knowledge. That "for teaching" aspect of the phrase really entails that you have to be able to not only solve the problem but explain to the best of your ability why you solved it to be able to talk about it out loud to the class to have a discussion around it and understand the process behind it. If you just borrow and add that's not going to help your students, so really being able to explain to your students why something happens I think is important.

The process of a teacher making her knowledge accessible to students is transformational knowledge, although Holly did not offer much specificity beyond the idea of explanation. In September Holly's definition of MKT consisted of foundational and transformational knowledge, which she said were both necessary to effectively teach mathematics.

Compared to September, in December Holly's definition of MKT was identical in amount for transformation, slightly less for foundation, and connection was included for the first time. Each of these three categories of MKT comprised of one-third of Holly's definition in December. Whereas in September she focused on overt content knowledge within foundation, in December she said that identifying errors was a dimension of MKT: "Misconceptions, that's something to think about that I think we overlook a lot, or we just don't take the time to think about them, but they're important." This was a new aspect of Holly's definition of MKT in December.

Holly also included connection and transformation in her definition of MKT. This was revealed in her statement that MKT is about "how to develop (student) strategies, like knowing where are they going to start, where are they going to end up, and how do you get them there?" The first part of this statement was about connection as it indicates a sequence in instruction and a progression in conceptual difficulty. Holly mentioned transformation at the end of the statement when she talked about knowing "how to get students there." While there was some specificity in how she talked about foundation and connection (identifying errors and decisions about sequencing, respectively) Holly was still vague in how she talked about transformation, as she had been in September. In terms of proportion, Holly's definition of MKT in December was equally balanced across foundation, connection, and transformation.

In February Holly included generic aspects of teaching as a new category, which was nearly half of her MKT definition at 45%. Similar to September, within the foundation category overt subject knowledge was the primary referent. Holly's definition of MKT in February began with a statement explaining MKT as a combination of overt subject knowledge, plus transformational knowledge and generic aspects of teaching:

> I think (MKT) encompasses the math side and then the knowledge of teaching. And you have to break it up, so in math, it's content knowledge, and it's knowing

> strong techniques for teaching math, so the way you structure your classrooms. Knowing how to do things like centers, whole group activities versus small group. It's knowing what are the best methods to reach the kids on any given, you know, content, within that subject. I think there's that piece, and I think there's the basics of teaching behavior management, classroom management. And assessment, that's something I'm realizing is coming into this big time that I hadn't really considered: How to assess kids in math, what to include on assessments, how to prepare them for assessments. In my mind, that's what it encompasses. It encompasses knowing everything you know to teach, and then the content of that.

Holly indicated a conception of MKT in which there was foundational knowledge (overt subject knowledge, "what to include on assessments"), transformational knowledge ("knowing what are the best methods to reach the kids"), and generic knowledge (how to structure a classroom and behavior management, how to prepare students for assessments).

Another way Holly included generic aspects of teaching in her definition was in her example of a student in her classroom who she found behaviorally challenging. She said through this experience she learned "how to juggle individual issues you have to deal with" as she taught the whole class. Her comments centered on behavior management and therefore were coded as generic rather than contingency. Another time Holly talked about MKT in terms of behavior management was when she said, "Now I can teach math and run the class at the same time. I think that's one more step. I see some teachers that teach math great, but their class is off the wall, or they can control their class but not teach math so great." Holly included an emphasis on behavior management in her definition of MKT in February.

Connection was the final aspect of MKT that Holly included in her February definition. She spoke about differentiation at two different times during the interview as it related to MKT:

> Differentiation is another thing I haven't really thought about too much until this semester. I hadn't thought about it too much until I'm in (the classroom) now, and I'm realizing I've got some kids who could go on to middle school curriculum and how some are back in second and third grade and how to meld all that together.

Differentiation incorporates two aspects within the connection category: anticipation of complexity and recognition of conceptual appropriateness. When asked to describe a master mathematics teacher, Holly's response included the categories of foundation and connection:

> I don't think it means knowing all of the math before you're hired as a fourth grade teacher. I think, however, it means being able to look at content, look at curriculum, and implement it in a way that reaches all students in the classroom, that's fully differentiated, that's engaging, that's challenging.

Within the foundation category, Holly included the dimensions of overt subject knowledge and adapts textbook. She also incorporated recognition of conceptual appropriateness, which is a dimension in the connection category. Holly did this through her comments on reaching students through differentiation, engagement, and challenge, all of which require recognition of conceptual appropriateness.

Compared to December 2009, Holly's definition of MKT in February 2010 changed in the addition of a proportionally large focus on generic aspects of teaching, particularly about classroom management. Her sense of transformation continued to remain vague, as it also was in September and December. Although her definition proportionally declined in connection compared to December, Holly included both more and more specific comments about connection in her definition of MKT in February.

In April, Holly included generic aspects of teaching in her definition of MKT less, although this category still comprised 27% of her definition. Looking back over the course of the year, Holly explained that her definition of MKT in April compared to the fall semester was different because of what she referred to as “little things” that she added to her definition. She said that previously her definition of MKT consisted of content knowledge (foundation) and knowledge of teaching (generic), to which she added:

> Understanding students’ misconceptions and how to address them, how to word questions, how to (know) when students are confused or something that doesn’t even matter. Lots of times I’ll have a student raise their hand and say, “Eli’s comment confused me.” Well, that’s because Eli’s comment isn’t relevant. Knowing how to bring the class back and get them focused so that they’re not confused, because then that brings up another whole can of worms.

In April, Holly included identifying errors (foundation) in her definition, which she also included in December. She also mentioned questioning, which is a dimension of MKT within transformation. This was the first time Holly gave an example of transformational knowledge; she included transformation in the three previous interviews but it always remained vague. For the first time Holly also included an aspect of contingency in her definition of MKT, in regards to decisions made about responding to children’s ideas that are raised in the midst of a lesson.

Holly continued her definition of MKT by addressing other areas that she now considered aspects of MKT:

> Knowing how to differentiate homework, knowing how to differentiate class

> work, understanding it's okay to pull a small group while the rest of the class is working, but knowing how to structure that. Knowing how to look at a worksheet and decide, "Okay, is this meaningful work for them, or is this just busy work? Are they going to learn something from it?"

Through this comment Holly indicated the inclusion of connection (differentiating homework and class work), generic (how to pull a small group while the rest of the class works on something else), and foundation (adapting the textbook) in her definition of MKT. Holly went on to say more about connection:

> And then, of course, how do you deal with a class when you've got a low (group of students) and a high (group of students)? How do you teach them all so that you're reaching all the kids, teaching them at their level, challenging them? How do you challenge kids in math? Those kids that are bored which I had all the time, do you give them extra packets, do you sit with them at lunch and teach them separately, what do you do? How do you reach them?

These ideas addressed recognition of conceptual appropriateness and anticipation of complexity. Referring to all of the above "little things" that Holly said she added to her definition of MKT over the year, she said:

> The nitty-gritty stuff I've learned and added to (my) definition this semester, that I just simply didn't even think about before. In the fall it was broad: you have to know how to be a teacher, and you have to know math, which is true. But more than that, it's the little stuff within a lesson that you need to know how to do.

Based on Holly's April interview, this "little stuff" included the use of questions to develop and assess learning (transformation), differentiation (connection), adapting the textbook and identifying errors (foundation), and responding to children's ideas (contingency).

Holly added contingency to her definition of MKT in April, although it remained the lowest proportionally contributing category of her definition. Generic aspects of teaching proportionally decreased in her definition of MKT. In April, Holly gave an example of transformation for the first time. At the conclusion of the year Holly articulated that MKT included content knowledge, generic teaching knowledge, and other mathematically-specific aspects including the use of questions to develop and assess learning (transformation), differentiation (connection), adapting the textbook and identifying errors (foundation), and responding to children's ideas (contingency).

MKT Definitions compared. Looking at the three focal preservice teachers' data individually reveals both similarities and differences that were not initially evident by looking at their aggregate data. Over the course of the year, the three preservice teacher's definitions of MKT were dynamic and changed at each of the four time points. All three of the preservice teachers included substantially more dimensions of MKT in their definitions in the spring semester (February and April) than in the fall (September and December 2009). The number of dimensions of MKT the preservice teachers included in their definitions peaked in February (for McKenzie) or April (for Wendy and Holly). In the fall the preservice teachers included not only fewer categories but also fewer dimensions of MKT within those categories. The following table shows the number of KQ dimensions that each preservice teacher included in her definition of MKT at each of the four time points over the course of her senior year. For comparison, there are nineteen total dimensions in the KQ model.

Table 12

Number of dimensions included in MKT definition

	September 2009	December 2009	February 2010	April 2010
Wendy	2	2	6	9
McKenzie	6	2	14	4
Holly	3	3	8	8

The largest increase of included dimensions happened between December 2009 and February 2010. This could be because of the increased exposure the preservice teachers had to mathematics teaching and learning when they began their daily student teaching placement in early January, whereas in the fall they attended their placements only once per week. This meant that they were likely to have seen substantially more mathematics teaching and learning from January until February than they had in the entire fall semester. While the reason is difficult to definitively identify, an overall progression of including more categories and more dimensions tended to occur over the course of the year.

One similarity for all three focal preservice teachers was the predominance of the foundation category in their definition of MKT. Within this category, overt subject knowledge was the dimension of MKT that was the primary focus. The general decline in the foundation category was less a function of a reduced level of foundational knowledge and more attributable to the preservice teachers' new awareness of other categories and dimensions of MKT, whose inclusion resulted in proportionally less of the definition being about foundation. For Wendy, overt subject knowledge seemed to form the nucleus of her definition of MKT. Wendy believed that increased content knowledge led to more facility not only in terms of foundational knowledge but also in terms of connection and contingency. Overt subject knowledge functioned as the basis for how she explained other aspects of MKT.

Transformation was an area of MKT that remained fairly vague for all three preservice teachers. Wendy, who included transformation in February and April, and Holly, who included it at all four time points, almost never gave a specific examples of transformation beyond phrases like “being a facilitator of knowledge” and queries such as “How is this thing going to actually come about?” The one exception to this was in April Holly specifically included questioning in her definition of MKT. Furthermore, when Holly and McKenzie gave an example of transformation, it was almost always in terms of questioning techniques. Never did any of the three focal preservice teachers include choice of representation in their definition of MKT. Furthermore, only McKenzie included choice of examples and demonstration in her definition, and that was only one time and so vague that it was not possible to determine if she was talking about one or both of these dimensions.

Contingency was a category of MKT that did not present in anyone’s definition until February (Wendy and McKenzie) or April (Holly). Even when it did appear, contingency comprised a fairly small percent of the overall MKT definition for McKenzie and Holly. Once it appeared, Wendy included contingency the most of the three preservice teachers, which was at its maximum at 33% in February.

Connection was a category of MKT that came and went in Wendy's and McKenzie's definitions of MKT, appearing two out of four times but not at sequential time points. Holly included connection in December, February, and April.

None of the three focal preservice teachers included generic aspects of MKT in their definition in the fall semester (September and December 2009), and Wendy never did at any time point. Holly included a substantial amount of generic aspects of MKT in her definition in February and April, when she predominately talked about behavior management. McKenzie included a substantial amount of generic knowledge in her definition in April, which was generally about knowledge of students and generic aspects of teaching. This could be because of their field placement experience where such aspects of teaching were frequent and legitimate concerns, whereas in the fall their methods course focused on mathematical aspects of MKT.

An exception to the idea that an increase or decrease in proportional attention a category received over time was not necessarily indicative of an "improving" or "worsening" conception of knowledge within that category exists in regards to the generic category. This category comprises non-mathematics-related aspects of teaching such as behavior management. Given the scope of the study and the specificity of the questions the preservice teachers were asked, a definition of MKT that contained the generic category was not desirable. This is not to say that generic aspects of teaching are not important even when teaching mathematics, but rather that when asked specifically about mathematical knowledge for teaching it was less desirable for preservice teachers to discuss generic rather than mathematics-specific aspects of teaching in their responses. While aspects that were coded as generic are necessary for effective teaching, when asked specifically about mathematical knowledge for teaching it was hoped that the preservice teacher could respond in mathematically related ways.

Other than the above statements, it is difficult to make other summary statements about the development of MKT across the four time points for these three preservice teachers. This is because although there were some similarities, each one began the study in September with a different definition of MKT and then added to and changed her definition in different ways over the course of the year, as evidenced by the data from December, February, and April. Their development, as indicated by the interviews, was not linear and was not neat. For example, there were times when categories were not present in their definitions despite being present previously. There were also times when a new category emerged for one preservice teacher and that same category was absent from another preservice teacher's definitions at the same time point.

A final result was that in April, the MKT definitions for the three focal preservice teachers were beginning to look like the aggregate data. The following figure shows the aggregate data of MKT definitions for all interviewed preservice teachers (n=8) and each focal preservice teacher (n=3) in April 2010.

A. aggregate (n=8); B. Wendy; C. McKenzie; D. Holly

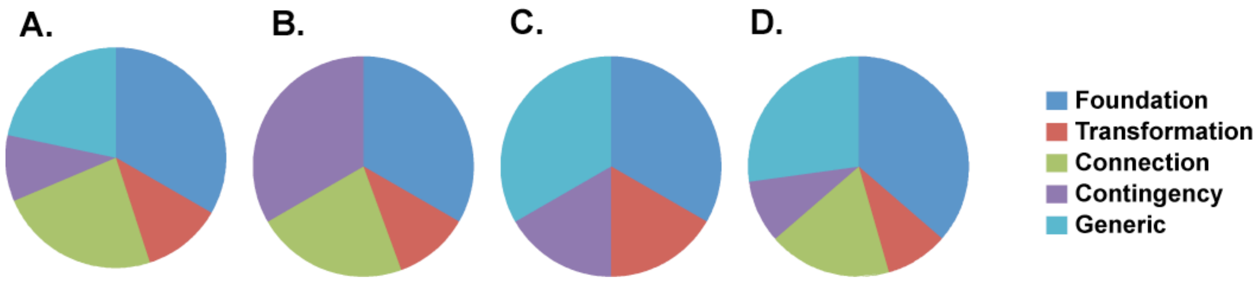


Figure 18. MKT Definitions--April 2010. This figure shows the April MKT definitions of all interviewed preservice teachers and each focal preservice teacher.

Whereas significant discrepancies existed between individual and group data at previous time points as shown in Figure 14 (especially in September and December 2009), in April 2010 the three focal preservice teachers' data looked the most similar to each other and the most similar to the aggregate data (n=8) as it had all year. Holly's definition of MKT, especially, was quite similar to the aggregate definition of MKT from the larger group (n=8). Wendy's definition differed in that she did not include the generic category and her contingency category was larger than the other seven preservice teachers. McKenzie's definition differed because she did not include the connection category. Other than these differences, there were several similarities in the proportional amount the four KQ categories, as well as the generic category, contributed to the focal preservice teachers' definitions and the aggregate definition of MKT. For example, the foundation category was nearly identical, and the ranges for transformation, connection (except McKenzie), contingency and generic categories (except Wendy) were also similar.

In April 2010 similarities were beginning to emerge between the aggregate and individual data regarding preservice teachers' definitions of MKT. This seems to indicate that over the course of the year each preservice teacher developed along a fairly individual trajectory (see Figure 14), but by the end of the year there was some similarity across the group. Said another way, the categories and dimensions seemed to emerge in preservice teachers' definitions of MKT at different times, but eventually similar categories and dimensions did seem to emerge. In summary, the preservice teachers' definitions of MKT over the course of their senior year developed in terms of the number and depth of categories and dimensions that were included. The development of definitions of MKT was also dynamic, non-linear, individual, and shared significant similarities only at the end of the year.

CHAPTER 7

FINDINGS—RESEARCH QUESTION TWO

Results from the twelve observed lessons will be reported to answer the second research question about the dimensions of MKT that translated into preservice teachers' practices. The results will first be presented through the aggregate data from the three focal preservice teachers by reporting trends across the four KQ categories (foundation, transformation, connection, contingency). Second, results will be reported within each of the four KQ categories separately. Third, dimensions of MKT will be discussed through tiered groupings which indicated the consistency with which the dimension was demonstrated by the preservice teachers.

Research Question Two: What aspects of mathematical knowledge for teaching translate into elementary preservice teachers' practices?

Demonstrated MKT. The coding protocol made use of seventeen out of nineteen KQ dimensions, which were each scored at one of six levels of demonstration (Appendix I). The six possible levels for each dimension were Present: Appropriate Maximum; Present: Appropriate Middle; Present: Appropriate Minimum; Present: Inappropriate; Not Present: Appropriate; Not Present: Inappropriate. Within the twelve lessons, Not Present: Appropriate was never scored. Also, Present: Inappropriate and Not Present: Inappropriate occurred in relatively low numbers and it did not seem helpful to maintain a distinction between the two categories. Therefore Present: Inappropriate and Not Present: Inappropriate were combined into one category, entitled "Inappropriate" (In). This category represents inappropriate aspects of teaching, whether they were overt mistakes or mistakes of omission. The lessons will be reported using four categories to make an initial simplification of the original six categories of fairly complicated data. The three Present: Appropriate categories will be used to report distinctions between high levels of demonstration (Maximum, or "Max"), medium levels of demonstration (Middle, or "Mid") or low levels of demonstration (minimum, or "Min"), along with the In category.

An initial overview of the data from the twelve lessons is presented in the following table which reports the frequency of each of the four scores (Max, Mid, Min, In) for each preservice teachers' four observed lessons. For example the table indicates that in lesson one, of the seventeen KQ dimensions, Wendy scored maximum on two dimensions, middle on ten dimensions, minimum on two dimensions, and inappropriate on four dimensions. The following table can be used to estimate the relative strength of demonstrated MKT by comparing the number of "high" scores with the number of "low" scores. For example, such a comparison with the following table indicates that in McKenzie's third lesson she demonstrated MKT at a higher level than in her fourth lesson based on higher "high" scores and lower "low" scores.

Table 13

Score frequencies for each observed lesson

Name	Level	Lesson 1: February	Lesson 2: early March	Lesson 3: late March	Lesson 4: April	Total
Wendy	Max	2	0	1	0	3
	Mid	10	4	3	4	20
	Min	2	11	9	12	34
	In	4	3	5	2	14
McKenzie	Max	3	3	2	0	8
	Mid	4	7	5	3	19
	Min	10	4	9	7	30
	In	1	3	2	8	14
Holly	Max	3	3	0	0	6
	Mid	6	9	1	2	18
	Min	6	5	9	14	34
	In	3	1	8	2	14

The above table shows the frequency at which each of the four levels was demonstrated within each lesson. The most frequently demonstrated level of MKT for the three focal preservice teachers was the minimum level, which they demonstrated 45% of the time. This means that nearly half of the time the preservice teachers demonstrated a minimum level of MKT. The second most demonstrated level was the middle level, which was demonstrated 26% of the time. The third most demonstrated level was inappropriate at 20%. The least occurring level the preservice teachers demonstrated was maximum, which they only demonstrated 9%. These frequencies indicated that usually the preservice teachers positively demonstrated a KQ dimension, and it was most likely to be at a minimum level of proficiency and rarely at the maximum level.

Mathematically computing an average score per lesson is problematic because of the categorical nature of the data. Therefore, the frequency distribution of scores per lesson was used to indicate the relative demonstration of MKT between lessons. Based on the frequency of each score over the course of the four lessons it is evident that lessons did not consistently improve from one lesson to the next, where improvement was indicated by an increase in "high" scores (maximum and medium) and a decrease of "low" scores (minimum and inappropriate). It was predicted that lessons, which were spaced three weeks apart and spanned from February to April, would tend to improve from lower demonstrated levels of MKT in earlier lessons to higher demonstrated levels of MKT during later lessons. However the data from this study was not consistent with this prediction. For example Wendy's lesson one, which was predicted to be her lowest demonstration of MKT, was actually her highest demonstration of MKT. It should further be noted that lesson one was also one of the very first mathematics lessons she ever taught. McKenzie and Holly each demonstrated their highest overall level of MKT in lesson two. Furthermore lesson four, which was predicted to be the highest demonstration of MKT, was actually the lowest demonstration of MKT for McKenzie and Holly. Wendy's lowest demonstration of MKT occurred in lesson two, although her summary scores for lesson four were nearly identical.

A third result conveyed in Table 13 was the relative consistency of demonstrated MKT across the three preservice teachers. Each preservice teacher had nearly the same overall frequency score and a similar range of scores for each of the four levels. No preservice teacher was substantially different than the other two in her overall level of demonstrated MKT. Furthermore, none of the individual lessons were substantially different from other lessons in terms of frequency scores. Although there was greater variation from one lesson to the next than expected, as a group the data seemed coherent across participants and lessons.

Additional results were found when the preservice teachers' scores were reviewed on each of the seventeen KQ dimensions. The following table shows Wendy, McKenzie, and Holly's scores on each of the seventeen KQ dimensions for each lesson. Each of the seventeen dimensions received one score of maximum, middle, minimum, or inappropriate based on the definitions in Appendix I for each lesson. The following table is read such that in the first lesson for the adapts textbook dimension, Wendy scored middle and McKenzie and Holly scored maximum. The dimensions are grouped by KQ category (foundation, transformation, connection, contingency).

Table 14

Demonstrated MKT scores on 17 KQ dimensions

Category		Lesson One	Lesson Two	Lesson Three	Lesson Four
Foundation					
Adapts Textbook	Wendy	Mid	Min	Min	Min
	McKenzie	Max	Max	Max	In
	Holly	Max	Max	In	Min
Awareness of purpose	Wendy	Mid	Min	Min	Min
	McKenzie	Min	Min	Min	In
	Holly	Mid	Mid	Min	Min
Concentration on understanding	Wendy	Min	Min	In	Min
	McKenzie	Min	Min	Mid	In
	Holly	Min	Mid	In	Min
Identifying errors	Wendy	Max	Min	In	Min
	McKenzie	Max	Max	Min	Mid
	Holly	Mid	Mid	Min	Min
Overt subject knowledge	Wendy	In	Mid	Mid	Mid
	McKenzie	Mid	Mid	Mid	Mid
	Holly	Mid	Mid	In	In

Theoretical underpinning	Wendy	Mid	Mid	Min	Min
	McKenzie	Min	Min	Min	Min
	Holly	Min	Min	Min	Min
Use of terminology	Wendy	Mid	Mid	Mid	Mid
	McKenzie	Max	Max	Mid	Min
	Holly	Min	Max	In	Min
Transformation					
Questioning	Wendy	Mid	Min	Min	Min
	McKenzie	Mid	Min	Mid	Mid
	Holly	Max	Max	Min	Min
Choice of examples	Wendy	Min	Min	Mid	Min
	McKenzie	Mid	Mid	Mid	Max
	Holly	Mid	Mid	Mid	Min
Choice of representation	Wendy	Max	Min	Min	Min
	McKenzie	In	In	Min	In
	Holly	In	In	In	Min
Demonstration	Wendy	Mid	Min	Min	Min
	McKenzie	Min	In	Min	Min
	Holly	Min	Min	In	Min

Connection					
Anticipation of complexity	Wendy	In	In	In	Min
	McKenzie	Min	In	In	In
	Holly	In	Min	Min	Mid
Decisions about sequencing	Wendy	Mid	Min	Min	Min
	McKenzie	Max	Mid	Max	Min
	Holly	Max	Mid	Min	Mid
Connections between procedures	Wendy	Mid	Min	Max	In
	McKenzie	Min	In	Min	Min
	Holly	In	Mid	Min	Min
Connections between concepts	Wendy	Max	Min	In	Min
	McKenzie	Mid	Min	Min	In
	Holly	Min	Max	In	Min
Contingency					
Deviation from agenda	Wendy	Mid	Mid	Min	Max
	McKenzie	Min	Mid	Min	Min
	Holly	Mid	Mid	Min	Min
Responding to children's ideas	Wendy	In	In	Min	Min
	McKenzie	Min	Mid	Min	Min
	Holly	Mid	Min	In	In

From these results further statements can be made about the teaching of the three focal preservice teachers. First, it is evident that the seventeen KQ dimensions were variable in their teaching across the four lessons not just in terms of the overall scores that were presented in Table 13, but also in terms of individual dimensions. Over all twelve lessons, there was only one dimension that Wendy demonstrated (use of terminology) and one dimension that McKenzie demonstrated (overt subject knowledge) at the same level in all four lessons. Since these were the only two dimensions of MKT that remained the same over the lessons, this means almost all of the dimensions were variable over the course of the lessons. Although it was not surprising that almost no dimension remained constant across the four lessons given the time span and novice nature of the participants, what was surprising was that the variation in demonstrated level of MKTwas seldom consistently positive in nature. That is to say, improving or maintaining a certain level of demonstrated MKT without "backsliding" to a lower level in a subsequent lesson happened infrequently. There was only one dimension in which McKenzie improved her level across the four lessons. This happened with her use of questioning for which she scored at the minimum level in lessons one and two and at the middle level in lessons three and four. Wendy improved in two dimensions: choice of examples and responding to children's ideas. For choice of examples she scored minimum in lessons one and two and middle in lessons three and four. For responding to children's ideas Wendy scored inappropriate in lessons one and two and minimum in lesson three and four. The only time anyone improved by more than one level was Holly, whose anticipation of complexity improved from inappropriate in lesson one to minimum in lessons two and three and improved again to the middle level in lesson four. This was the only KQ dimension Holly improved upon. It was expected that more growth without a subsequent drop in level would have been demonstrated across the lessons of preservice teachers than actually occurred.

As a result of this initial view of the data, the scores were aggregated over the twelve lessons. This was done because the scores did not follow a pattern over time in which they consistently changed but rather they were highly variable across the four lessons. In addition, the three preservice teachers scores were similar to each other. Therefore as a way to make sense of the data, the demonstrated scores were totaled across the three participants and across the twelve lessons. Next, results across the twelve lessons will be presented for each of the four KQ categories (foundation, transformation, connection, contingency). Following this overview, specific dimensions that translated into the preservice teachers' practices will be identified.

The results in the foundation category will be presented first. Within this category there are seven dimensions. The following figure depicts the frequency of each of the four levels (inappropriate, minimum, middle, maximum) per each of the seven dimensions within the foundation category. The figure is read such that for the "adapts textbook" dimension, over the twelve lessons two of the lessons were scored inappropriate, four were scored minimum, one was scored middle and five were scored maximum. The figure gives a visual depiction of the foundation scores found in Table 14.

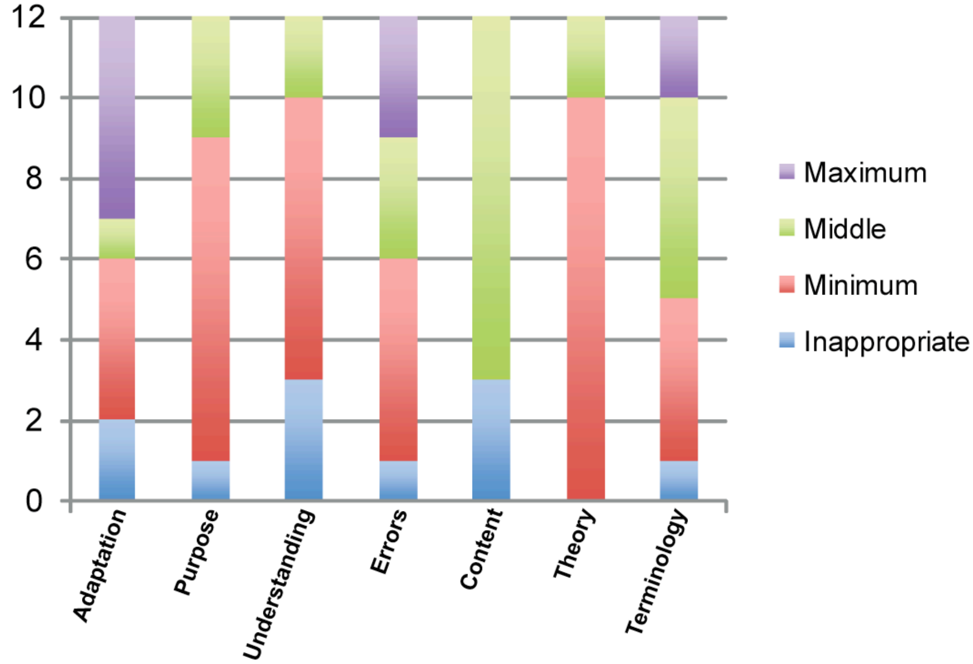


Figure 19. Foundation Category—Demonstrated MKT. This figure shows the demonstrated MKT scores in the foundation category.

Viewing the data this way shows the prevalence of the minimum category, which was closely followed by medium in the foundation category. A score of inappropriate appeared between one to three lessons on six out of the seven dimensions. The majority of the time the preservice teachers demonstrated foundational aspects of MKT at minimum or medium levels. Foundation was the category for which the preservice teachers scored maximum the most number of times; over the twelve lessons they scored maximum eighteen total times, and ten of those maximum scores were in the foundation category. Still, maximum levels of demonstrated MKT were infrequent.

The results in the transformation category will be presented next. Within this category there are four dimensions. The following figure depicts the frequency of each of the four levels (inappropriate, minimum, middle, maximum) per each of the four dimensions within the transformation category.

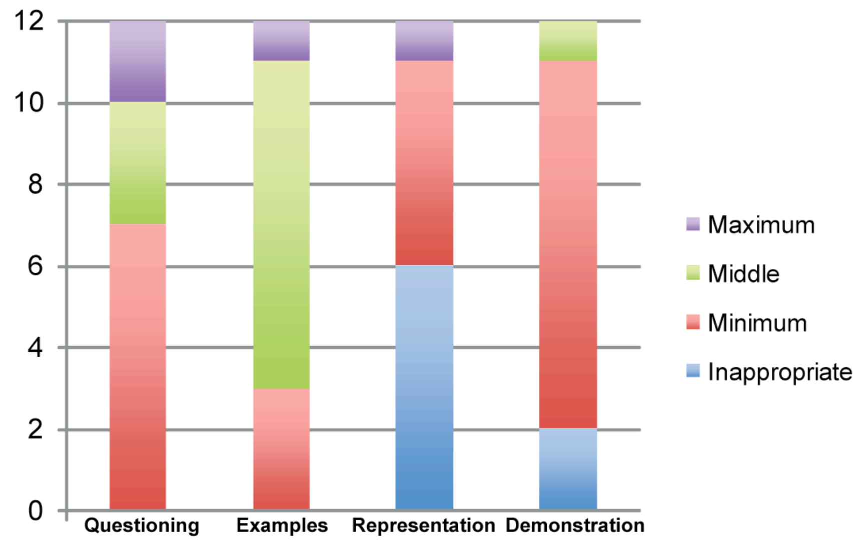


Figure 20. Transformation Category—Demonstrated MKT. This figure shows the demonstrated MKT scores in the transformation category.

Transformation was the category that had the most variation of demonstrated MKT across its dimensions. Whereas the frequency of scores for a given dimension in the other three categories also had other similar-looking dimensions in the same category, the scores for the four transformation dimensions were the most dissimilar. The most prevalent score in transformation was minimum. Other than choice of examples, a medium score did not appear frequently in the transformation category, and maximum occurred even less at only four times total. An inappropriate level of MKT was demonstrated in two of the four dimensions, including half of the lessons on choice of representation; this was the second highest prevalence of an inappropriate score across all seventeen dimensions.

The results in the connection category will next be presented. Within this category there are four dimensions. The following figure depicts the frequency of each of the four levels (inappropriate, minimum, middle, maximum) per each of the four dimensions within the connection category.

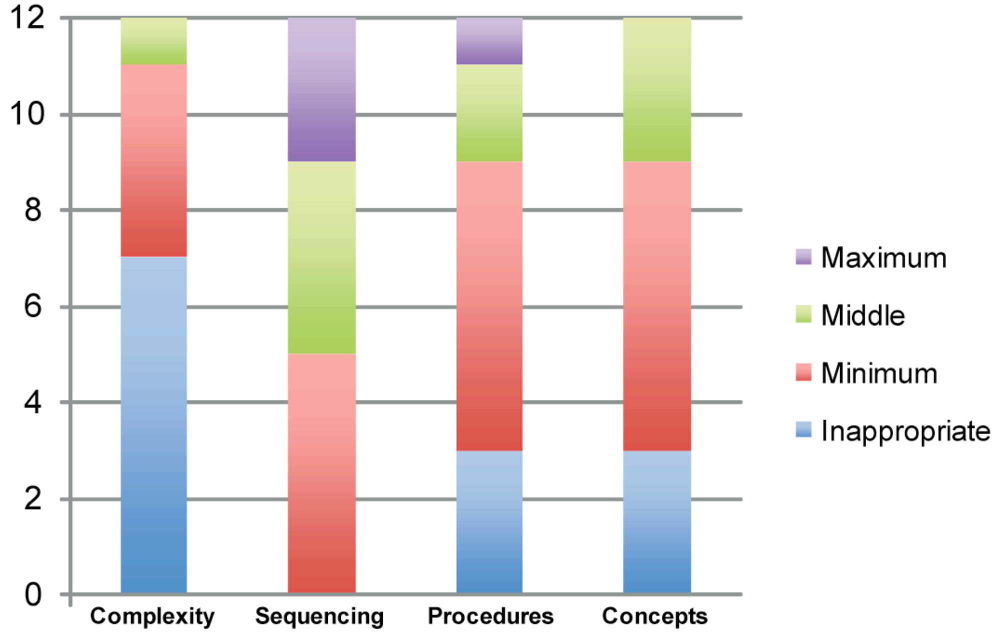


Figure 21. Connection Category—Demonstrated MKT. This figure shows the demonstrated MKT scores in the connection category.

The connection category had the highest level of inappropriate scores, which ranged between three to seven lessons on three of the four dimensions. Medium scores appeared between one to four lessons on each dimension. Maximum scores were rare, only appearing four times across two dimensions. Minimum continued to be the level that contributed the most to three of the dimensions (decisions about sequencing, connections between procedures, and connections between concepts), while inappropriate contributed the most to anticipation of complexity.

The results in the contingency category will be presented next. Within this category there are two dimensions. The following figure depicts the frequency of each of the four levels (inappropriate, minimum, middle, maximum) per each of the two dimensions within the contingency category.

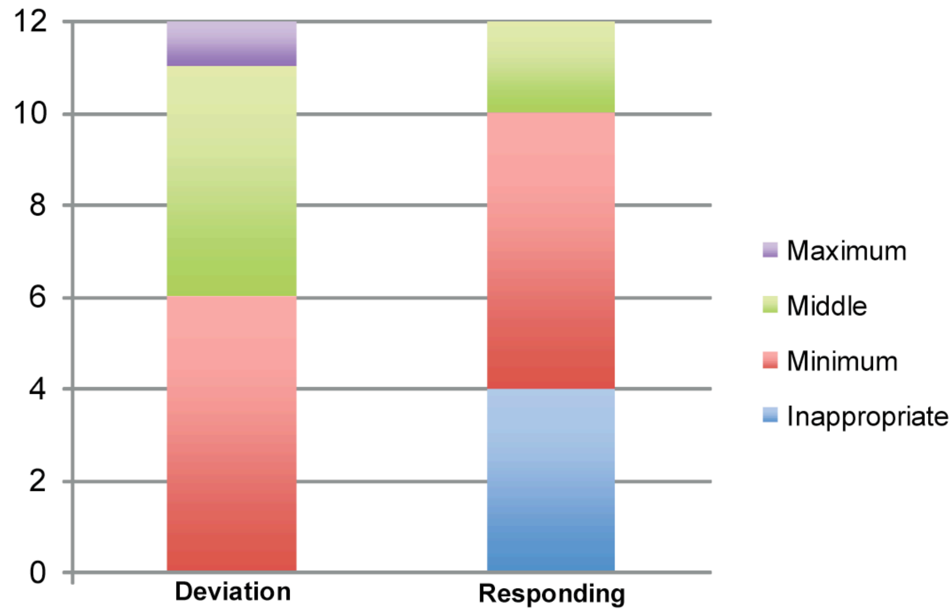


Figure 22. Contingency Category—Demonstrated MKT. This figure shows the demonstrated MKT scores in the contingency category.

It is difficult to see paterns in the contingency category since there are only two dimmensions to compare. An inappropriate level of MKT was demonstrated on one dimension, responding to chidren's ideas, during four of the lessons. The minimum level was the most prevelant contingency score, and medium was the second most frequent. There was one lesson in which a maximum level of contingency was demonstrated and that was on deviation from the agenda. The data indicated that the preservice teachers tended to demonstrate low levels of contingency in their teaching, as they almost never demonstrated a maximum level of MKT.

Translation to practice: Tiers. Having summarized the aggregate data for each of the four KQ categories as an overview, the second research question of what dimensions of MKT translated into the preservice teachers' practices will now be addressed. MKT was demonstrated at various levels and frequencies for each of the seventeen dimmensions of MKT. Given the fluctuation of demonstrated MKT from one lesson to the next, translation to practice will be described as a series of four tiers. The highest tier, Tier One, represents the highest demonstration of translation to practice for these preservice teachers, and the lowest tier, Tier Four, represents the lowest demonstration of translation to practice.

The highest tier, Tier One, is comprised of dimmensions on which the preservice teachers consistently demonstrated MKT. That is to say, dimensions were considered a consistent part of the preservice teacher's practice if she scored any combination of minimum, medium, and/or maximum over her four lessons but never scored inappropriate. What may be critiqued as the most extreme incidence of exclusion will be presented to explain the decision to determine tiers of demonstration in this way. McKenzie scored maximum on adpats textbook in lessons one, two, and three. Then in lesson four she scored inappropriate. Therefore, the researcher is not comfortable claiming that McKenzie will reliably demonstrate adpats textbook in the future. Furthermore, while three maximums and one inappropriate may seem to "average" to an acceptable level of MKT for the adapting textbook dimension, the experience of sitting in the classroom while the lesson unfolded offered a much different view. A score of inappropriate may not seem as problematic when presented in the above tables and figures, but as a lesson taught in a classroom it was highly problematic when a dimension of MKT was inapproriate because it was either absent or enacted incorrectly. No amount of averaging will take away from the fact that on that day the preservice teacher did a poor job on the given dimmension and that her lesson suffered noticeably as a result. Furthermore, the requirements for scoring a minimum were intentionally made low enough to credit preservice teachers for even initial attempts at demonstrating MKT. Again, the rating system was preservice teacher-friendly. For all of these reasons the researcher used a series of tiers to describe the relative translation of practice of each of the seventeen KQ dimensions, which was determined by how consistent a dimension was demonstrated across the three focal preservice teachers.

Each of the four tiers and their corresponding dimensions will now be presented. Tier One will be presented in the most depth since those are the dimensions that all three focal preservice teachers consistently translated into practice. Following a detailed description of Tier One, an overview of the Tier Two, Tier Three, and Tier Four dimensions will be reported.

Translation to practice: Tier One. First the dimensions on Tier One will be presented. These dimensions were consistently part of the preservice teachers' practices, as demonstrated by scores of minimum, middle, or maximum in each of the four lessons. When compared across the group there were five dimensions of MKT that all three preservice teachers' demonstrated at least the minimum level in each lesson. These five dimmensions were theoretical underpinning, questioning, choice of examples, decisions about sequencing, and deviation from agenda. These dimmensions were spread across the four KQ categories of foundation (theoretical underpinning), transformation (questioning and choice of examples), connection (decisions about sequencing), and contingency (deviation from agenda). The figure below shows the scores of each of these five dimensions.

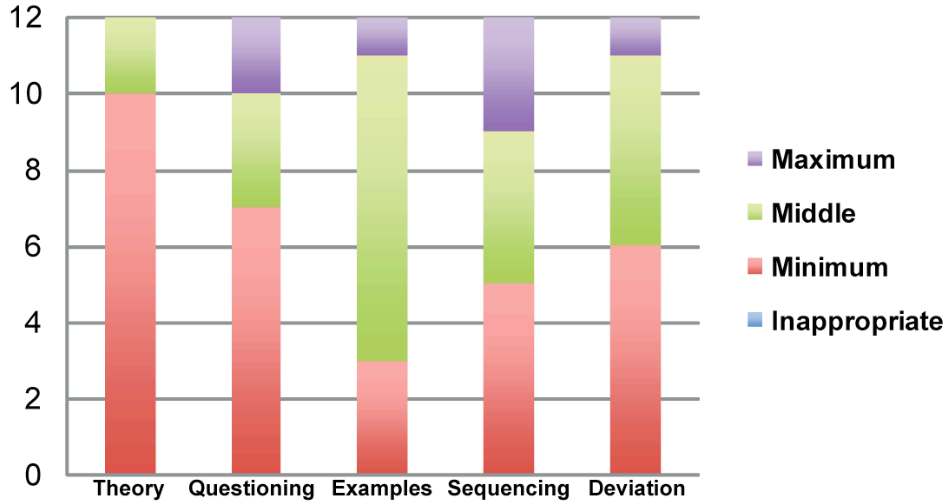


Figure 23. Tier One dimensions. This figure shows the dimensions that were consistently demonstrated by all three focal preservice teachers.

In the majority of lessons preservice teachers demonstrated these dimensions at a minimum level, doing so in half or more of the lessons on three dimensions (theoretical underpinning, questioning, deviation from agenda). The second most prevalent score was the medium category, which the scored between two to five lessons. They scored maximum in zero to five lessons. In sum, the preservice teachers almost always demonstrated a minimum or medium level of MKT on each of these five dimensions. Despite rarely scoring maximum, they never scored inappropriate over the twelve lessons. Based on the data these five dimensions translated to practice for these three preservice teachers. Each of these dimmensions will now be examined individually to describe how each dimension of MKT manifested itself in Wendy, McKenzie, and Holly's teaching.

Questioning. One of the highest four dimensions of MKT the preservice teachers demonstrated as a group was their use of questioning. Differences across the levels pertained to the way in which questions were asked and the percentage of questions that were open-ended ("What shape is this?") as opposed to either/or ("Is this a square or a rhombus?") or yes/no questions ("Is this a square?"). At the minimum level, 25-49% of the questions were open-ended, at the middle level 50-75% of the questions were open-ended and at the maximum level over 76% of questions were open-ended. Seven of the lessons contained minimum levels of questionging, three lessons contained middle levels, and two lessons cpntained maximum level of questioning.

Questioning was the one area in which McKenzie demonstrated consistent growth across the four lessons; in lessons one and two McKenzie's questioning was minimum and in lessons three and four her questioning was medium. Reviewing instances of her questioning over the four lessons revealed noticeable growth. The first difference in McKenzie's teaching practices occurred when the number of questions increased drastically from lesson one to lesson two. From lesson one to lesson two the number of questions McKenzie asked nearly doubled, from 33 questions asked in lesson one to 63 questions asked in lesson two. However the relative quality of questioning did not increase, and in fact slightly decreased from 48% of open-ended questions in lesson one to 40% in lesson two.

A typical series of McKenzie's questioning is included below from her second lesson about similarity and congruence. In this excerpt she began with an open-ended question but before students answered her initial question she quickly moved to asking a series of either/or and yes/no questions.. Students were looking at a picture of two objects on the Smartboard when McKenzie asked the following:

McKenzie: “Why are they not similar or congruent? Are they the same size?”

Students (in unison): “No.”

McKenzie: “Are they the same shape?”

Students (in unison): “No.”

McKenzie: “So are they similar or congruent?”

Students (in unison): “No.”

McKenzie asked fewer open-ended questions during her first two lessons, and when she did she quickly moved away from her initial question to more simplified “funneling” questions, as in the above example.

For lesson three the quality of McKenzie’s questions increased to 68% open-ended. The following excerpt from the beginning of lesson three was a typical example of her more developed questioning technique. In this excerpt she introduced a lesson about equations by referring back to the previous use of a scale as an analogy for how equations work. She had a picture of a scale on the Smartboard and asked,

McKenzie: Who can remind me about this? Who can tell me what this picture represents?

Student: A scale.

McKenzie: A scale. What is this scale doing for us?

Student: Showing to where it's equal.

McKenzie: To where it's equal? What do you mean?

Student: Where it's the same thing as.

McKenzie: To where it's the same thing as. Do the two parts of a scale always have to be equal, though?

Students: some answer "yes", some say "no."

McKenzie: What would happen if they weren't equal?

Student: They wouldn't be equal

McKenzie: What would happen if you had more weight on this side than this side? What would the scale look like, then?

Student: Lopsided.

McKenzie: So if we're looking at an equation, what has to happen on both sides?

Student: Equal.

McKenzie: It has to be equal on both sides.

Although there is room for further growth in the above series of questions, it is an improvement when compared to McKenzie's questioning in lessons one and two. For example, in the above excerpt she only asked a yes/no question once, and she also asked a student to further explain his thinking. These were new aspects of McKenzie's questioning that emerged in lesson three and continued in lesson four. One way McKenzie enacted a greater amount of MKT was in her questioning, both in terms of the number of questions she asked and the quality of those questions.

The best demonstration of questioning was found in Holly's first two lessons, which were the two times a maximum score was achieved on the questioning dimension. In Holly's first lesson, 78% of the questions she asked were open-ended, and in her second lesson 91% were open-ended. The following is an excerpt from Holly's second lesson, which was on probability.

> Holly: If I were to add up all of the numerators, what should I get?
>
> Student: 17
>
> Holly: Why should that be the case? If I add 2 plus 2 plus 4 plus 5 plus 1 plus 3 I should get 17. Why should that be the case?
>
> Student: There are six shapes
>
> Holly: There are six different shapes. Why should I get 17 for my numerator?
>
> Student: That's the total
>
> Holly: We know that our total number of shapes is 17, so if we add all of the numerators, we should have 17 because I asked you to find the probability for each of the shapes listed.

Whereas McKenzie moved away from her initial open-ended question by asking either/or or yes/no questions in lesson two, Holly continued to ask the same question in an open-ended way until students answered it.

Another interesting result is that in lessons three and four Holly's questioning decreased to the minimum level, which coincided with a decrease in her overt subject knowledge, as well. Whereas Holly demonstrated sufficient overt subject knowledge in lessons one and two, in lessons three and four her overt subject knowledge was not always sufficient, as indicated by errors she made in her terminology and explanations. It was interesting that her level of questioning also declined during lessons three and four. Further exploration of the correlation between dimmensions will be mentioned in the discussion as an area for future research.

Wendy scored medium on her questioning for the first lesson and minimum on lessons two through four. Since Wendy did not tend to ask questions in a series the way McKenzie and Holly did, providing an excerpt of a lesson is not helpful for Wendy because she only tended to ask one question at a time. No explanation could be made as to why Wendy's questioning level declined from lesson one to two.

What did stand out in Wendy's questioning was the difference that occured when she worked with the whole class versus when she worked with individual students. During the first three lessons there was a noticeable discrepency between the quality of questions Wendy asked for the beginning portion of the lesson when students were all seated on the carpet and Wendy presented the lesson through problems and examples on the Smartboard versus when students worked at their seats, usually on an EnVisions worksheet. This stood out even before the data was coded.

After this pattern was noticed for Wendy further analysis was done on McKenzie's and Holly's quesitons. McKenzie's class did not engage in hardly any individual student work time, and when they did it was for a very short increment of time in which students completed one problem before sharing their soloution. During this time McKenzie circulated but did not ask students questions. Therefore she had no data to contribute to this query since all of her questions were to the whole class. Likewise during Holly's third lesson there was no individual work time. The following table presents the percentage of open-ended questions Wendy and Holly asked for lessons in which they interacted with students in both a whole class and individual setting.

Table 15

Percentage of open-ended questions asked by setting

		Whole class	Individual student
Wendy			
	Lesson One	67%	46%
	Lesson Two	44%	23%
	Lesson Three	58%	33%
	Lesson Four	28%	41%
Holly			
	Lesson One	88%	65%
	Lesson Three	44%	57%
	Lesson Four	38%	39%

There tended to be a discrepency between the kinds of questions that were asked in whole class versus one-on-one interactions within the same lesson, with the exception of Holly's fourth lesson. In the other six of lessons there was a difference in questioning across settings, although not always in the same direction (Wendy and Holly asked more opened-ended questions during their whole class instruction in four lessons and in their individual student interactions in three lessons). This data suggests that questioning was not always enacted across settings in consistent ways. This result could use further exploration specifically within questioning, and also raises the question as to whether other dimesnions of MKT are demonstrated differently depending on the grouping arrangement of students.

Decisions about sequencing. The highest dimension of MKT the preservice teachers demonstrated as a group was decisions about sequencing, a dimension in the connection category. Differences across the levels considered three aspects of sequencing: sequencing across the unit (whether a given lesson was connected to the previous lesson), sequencing within the lesson (whether different sections of the lesson were purposfully sequenced) and sequencing of examples (whether examples, problems, or tasks in the lesson were purposfully sequenced). When one of these three aspects of sequencing was demonstrated the lesson was scored minimum, when two aspects were demonstrated it was scored middle, and when all three aspects of sequencing were demonstrated the lesson was scored maximum. Five of the lessons were examples of minimum levels of sequencing, four lessons were examples of the middle level, and three lessons were examples of the maximum level of sequencing.

The preservice teachers demonstrated at least one of the three areas of sequencing in every lesson. Over each of their four lessons the scores for sequencing varied and no participant demonstrated consistent growth. Of the three participants, Wendy's level of sequencing tended to be at a more initial level followed by Holly and next by McKenzie, who demonstrated sequencing at the highest level across her four lessons. Each of the preservice teachers' demonstration of sequencing is presented in the following table.

Table 16

Aspect(s) of sequencing demonstrated by each participant

Name	Lesson	Sequencing of the Unit	Sequencing of the Lesson	Sequencing of Examples
Wendy	Lesson One	X	X	
	Lesson Two	X		
	Lesson Three	X		
	Lesson Four	X		
Holly	Lesson One	X	X	X
	Lesson Two	X	X	
	Lesson Three	X		
	Lesson Four	X		X
McKenzie	Lesson One	X	X	X
	Lesson Two	X		X
	Lesson Three	X	X	X
	Lesson Four	X		

The above table indicates that in every lesson all three preservice teachers connected the observed lesson to the previous lesson. This result may indicate that within the sequencing dimension the first aspect of sequencing that translated to practice was the ability to connect a lesson to the previous lesson. This seems to indicate that connection to the previous lesson may happen in advance of a consistent demonstration of sequencing within a lesson or sequencing of examples, which were both demonstrated less frequently. The data do not indicate whether there was a distinction in the relative timing of translation to practice for sequencing within a lesson or sequencing of examples, only that these seem to remain inconsistently demonstrated even once connection to previous lessons is consistent. Decisions about sequencing will next be discussed for each preservice teacher.

Wendy scored middle on leson one and minimum in lessons two, three, and four for decisions about sequncing. In three of the four lessons she began with a brief statement connecting the lesson to a previous lesson. Wendy did this most explicitly in lesson two when she introduced the lesson on subtraction in the following way.

> We have been adding the unknown partner. We have been looking at math problems that look like this (draws on the board 7 + □ = 14). We've been using our strategies to work out what goes in this box. Today, instead of adding the unknown partner we're subtracting the unkown partner. So it will be 7 minus "box" (changes the addition sign to a minus sign).

In one lesson Wendy did not start off with a connection to the previous lesson for her students, although she did talk about connections to the previous lesson in her post-observation interview, indicating she had made decisions about sequencing even though these were not made explicit to her students.

Wendy did not demonstrate sequencing of examples in any of her lessons. Other than when she wrote story problems for the Smartboard ahead of time, she did not decide upon the examples she was going to use until she was teaching that part of the lesson. This was true in lesson one, which was the first lesson in a new unit on money and also one of the very first full math lessons Wendy taught. During the lesson Wendy had the students chorally count dimes and pennies from a large flip chart which totaled 55, 44, and 37 cents, respectively. Later in the lesson at their seats, students worked with a partner to use dimes and pennies to show 63, 24, and 45 cents, respectively. In the post-observation interview the reasearcher asked Wendy whether the numbers for the six examples were from EnVisions or if she decided to use her own numbers. Wendy answered:

> I just made them up. I just kind of fly by the seat of my pants…Every time that I was like, "Let's count (this amount)," I was just making it up. And then when they were in the small groups I was also making up those numbers, and I kept having to remind myself that I had only given them five pennies, and so I'd be like, "Count out 67, oh wait, you can't." Because I hadn't introduced covering up the pennies on the strips.

Wendy created her examples as she taught and did not sequence them in an intentional way based on the goals of her lesson.

In lesson one Wendy sequenced across the lesson, but she did not in subsequent lessons. In lessons two, three, and four, the worksheet or independent practice she asked students to do towards the end of the lesson did not directly connect with what she taught at the beginning of the lesson. Although it was on the same general topic, it was oftern focused on a different instructional goal than the one she had just taught. For example in lesson three she taught a lesson on how to use a graph as a strategy for solving addition and subtraction problems. After demonstrating this for nineteen minutes, students were sent to their seats to work individually on a worksheet in which they were supposed to use a different matching strategy to solve subtraction problems. Although the two parts of the lesson were on the same broad topic, the particular skills were different. Overall, Wendy's demonstration of sequencing was consistent in her connection to the previous lesson. In addition, once she included connections within the lesson. Wendy never demonstrated sequencing of examples.

Like Wendy, McKenzie and Holly also explicitly connected the beginning of the lesson to a previous lesson three out of four times and indicated the connection to the previous lesson during the post-observation interview in the fourth lesson. Of the particpants, Holly was the most explicit with her students when it came to connecting a lesson to the previous lesson. Holly's introduction to lesson two, which was about probability, was as follows:

> Holly: What did we talk about on Friday?
>
> Students: Probability
>
> Holly: We talked about probability. It was our introduction to probability. Probability is the likelihood if something is going to happen. We used a tree diagram. There were examples with a bag. There was a bag with four numbers. And if I said how many outcomes are there possible in the bag, what would you say?
>
> Students: Four
>
> Holly: An outcome is a solution. You'd simply count up the numbers in bag one and that's your number of total outcomes in that bag. A tree diagram was helpful because it helped us figure out how many outcomes when pulling two different

> items from two different bags at the same time. That's when a tree diagram came into play. Today we're going to work on writing probability as a fraction.

Holly's connection to the previous lesson often was a review of the previous lesson followed by a statement about what the students would be working on during the given lesson.

Holly demonstrated sequencing within a lesson and sequencing of examples in two out of four lessons. In lesson one, which was the one time she scored maximum on sequencing, Holly demonstrated both of these. In regards to sequencing within the lesson Holly made an intentional progression which began with reviewing rounding of whole numbers and then rounding to the tenths. Next Holly introduced estimation, then students estimated sums and differences with naked number problems, and finally students estimated sums and differneces through word problems. Holly's examples also followed an intentional sequence in that she purposefully chose numbers that were easier to round before asking students to work with numbers that were likely to give them more difficulty. For example, the mental math problems she included at the very end of the lesson to round to the nearest whole number were presented in the following order, from easier to harder: 1) 1.4 + 1.9; 2) 3.1 + 4.8; 3) 0.18 + 6.2. In the following excerpt from the post-observation interview, Holly explained her sequencing within the lesson and her sequencing of examples:

> The whole thing I had in a certain order, so the first thing was, "What does it mean to estimate? Talk about it with your partner." And then discussing when we estimate using decimals we look to the tenths place, and then to the whole number…And then get into some examples, and the word problems were going to build and become increasingly more difficult. Even the problems I had for them to practice--just with rounding, just the decimals--they increasingly got more difficult, the decimals themselves got more challenging.

Holly demonstrated in her teaching and confirmed in her post-observatiom interview the intentional decisions she made to sequence across the lesson and sequence her examples. In sum, Holly's demonstration of sequencing was consistent in her connection to the previous lesson. In addition, twice she included connections within the lesson and sequencing of examples.

Of the three participants McKenzie demonstrated the highest level of sequencing, scoring maximum twice and scoring middle and minimum once each. McKenzie demonstrated sequencing of examples in three lessons and sequencing within a lesson twice. She demonstrated both of these as well as connection to the previous lesson in her third lesson, resulting in a maximum score on sequencing. To introduce the lesson on equations, McKenzie used the previously described set of questions to connect to the previous lesson by reviewing what a scale was and how it worked. Following this introduction, McKenzie sequenced within the lesson by beginning with the topic of equations and naked number sentences, then had students write equations to match story problems she gave them, and finally had students solve equations by substituting a value for the variable to determine if the equation was true or false.

McKenzie also sequenced the examples within each of those topics. She did this by intentionally selecting the operations students worked with and their order. For example in the first section when she introduced equations, McKenzie first had students work on x+1=3 and later had them work on n/7 = 6. In the third section in which students solved equations by substituting a value for the variable to determine if the equation was true or false McKenzie began with addition (n+32.5=36.5), second did a problem with subtraction and multiplication (24-n=10x2), third did a problem with division and subtraction (n/2=7-n), fourth completed problems that used the same variable in two places (nxn=8x2 and nx1=n+1), and finally introudcued new notation (3n=12). McKenzie's third lesson was a solid example of sequencing and connection to the previous lesson, sequencing within the lesson, and sequencing of examples. In sum, McKenzie's demonstration of sequencing was consistent in her connection to the previous lesson. In addition, McKenzie demostrated connections within the lesson twice and more frequently demonstrated sequencing of examples by doing so in three of her lessons.

The three focal preservice teachers were more successful demonstrating sequencing at higher levels than they were with any other KQ dimmension. One result that the data suggested was that connection to a previous lesson may be the first aspect of sequencing that consistently translates to practice and sequencing within a lesson and of examples may take longer to consistently demonstrate. This may be because connecting a lesson to the previous lesson is a generic tenat of teaching that the participants were exposed to throughout coursework and field experiences, whereas how a lesson mathematically builds across itself and sequencing examples is a task more specific to mathematics teaching. It seemed that Holly and McKenzie demonstrated a more advanced level of sequencing than Wendy, as the former both demonstrated all three aspects of sequencing in at least two lessons. Wendy, on the other hand, only demonstrated sequencing within a lesson in one lesson and never demonstrated sequencing of examples.

Choice of Examples. One of the highest dimensions of MKT that the preservice teachers demonstrated as a group was choice of examples. While the previous discussion of the decisions about sequencing dimension reflected whether the preservice teachers presented examples in an intentional order, this dimension reflected the mathematical quality of the examples or tasks. Scores on this dimension reflected the cognitive demand of the examples and tasks used in the lesson (Stein et al., 2000). A minimum score indicated that one example was High cognitive demand: Procedures with connections. A medium score indicated that two or more examples or tasks in the lesson were High cognitive demand: Procedures with connections. A maximum score indicated one or more examples or tasks were at the highest level, High cognitive demand: Doing mathematics. At each of these levels, other examples or tasks in the lesson may have been lower levels of cognitive demand. The most frequent score on this dimension was medium, which occurred in eight of the twelve lessons, indicating that the three focal preservice teachers most often used two or more examples or tasks that were High cognitive demand: Procedures with connection (Stein et al., 2000).

Wendy demonstrated an improvement in her scores on this dimension, scoring minimum in the first and second lessons and middle in the third and fourth lessons. Her first lesson was when she said she "flew by the seat of her pants" and made up the examples as she was teaching. She scored a minimum on choice of examples for that lesson, indicating she used one High cognitive demand: Procedures with connections task. This was because the Math Expressions worksheet that she used for the practice work portion of the lesson had one such task, in which the students were asked to make 24 cents without nickels. All of the problems she made up as she was teaching were low cognitive demand.

Wendy did not make up problems as she taught in her subsequent lessons. Instead, she used the teacher's manual from which she selected examples. In regards to the teacher's manual, she explained in her post-observation interview after her second lesson, "I actually have one that's highlighted, like marked up, and so I read it and studied it, and just had no intention of teaching every single thing, but I had highlighted the problems I wanted to use and just those kinds of things. So, that is what I had with me (while teaching)." She explained that she could not teach the Math Expressions lessons as they were laid out in the teacher's manual due to her district's pacing guide, which frequently combined lessons so that Wendy had to cover two lessons in one day. Therefore, Wendy selected the examples she was going to use from the teacher's manual prior to the lesson. Wendy's score on this dimension depended on a combination of the quality of tasks in the curriculum and her ability to select ones that were higher cognitive demand.

McKenzie demonstrated the highest levels on the choice of examples dimension, scoring middle in the first three lessons and maximum in the fourth lesson. Unlike Wendy, McKenzie did not closely adhere to the Envisions curriculum her district used. Following the first lesson, she was asked if she used the teacher's manual for the lesson, to which she answered,

> I went through the teacher's guide mainly for the vocabulary, and sometimes they have pretty good visuals, so I'll get a screen capture of the visuals they have on the software and use those…But that's about it as far as using the teacher's guide. I feel like with the AIG group I use the bare minimum basics of what Math Expressions gives me, but then I can step up with them and go a little bit further.

This meant that nearly every one of the examples McKenzie used was one she created. The main way she described how she made her examples during the four post-observation interview was that she tried to make examples to address students' misconceptions, or at least not further contribute to them. One example of this was in her first lesson, which had been on angle measurement. At one point in the lesson she had a series of seven different angles that students needed to measure with a protractor. When asked how she decided on the angles, she explained:

> I wanted to do acute and obtuse and right. So I was trying to hammer the idea that it doesn't matter, because I remember when we measured angles in elementary school, I feel like they were always—the bottom line was always lined up on the line, and it was very easy to measure an angle, because you just set (the protractor) on there, and there's the angle. But I was trying to hammer in the point with them that it doesn't matter what direction the angle is going in, you can still measure it at that angle.

McKenzie's score on this dimension depended on her ability to make high cognitive demand tasks. Unlike Wendy, this score was not an indication of the curriculum tasks or her ability to select tasks, but rather her ability to make her own tasks.

Whereas Wendy generally selected tasks from the curriculum and McKenzie generally made her own, Holly shifted from the former to the latter over the course of the study. In the first post-observation interview when asked where the problems came from that she had the students complete, she explained she "picked four or five from Envisions to use, that I felt were relevant, that showed how you could use this in real life." Holly said there were approximately fifteen problems in the teacher's manual, of which she selected four or five to use based on the numbers and operations used. She explained that one of the last examples she used "had whole numbers in the hundreds, to see if I could throw them off…I wanted to find the trickier problems to see if they could figure those out. Not to trick them, but to see, 'Can you work your way through this type of problem?'" This seemed to indicate a difference in approach to examples than McKenzie had. Whereas McKenzie used examples to dispel or not contribute to misconceptions, Holly used examples as a way for students to apply their mathematical knowledge to a new situation.

Unlike her first lesson, for her second lesson Holly said that only one of her examples came from the book and the others she made herself. She explained during her post-observation interview that she selected one of the tasks from the book, which was a task involving two orange, five blue, and one yellow tee shirts that were used to calculate various probabilities.

> I thought it was good…so I pulled that one, but even in the book it only had one or two questions associated with it, but I wanted to pull more out of it…So I asked a question for each color because I wanted them to see that the total of (each probability) should equal a whole fraction.

Holly went on to explain, "the other question, I just made that one up. They had one kind of like it in the book…but I thought, 'They're going to get this. I'm going to give them one a little more challenging.' And, they got it." Whereas in the first lesson she used examples from the teacher's manual, in the second lesson she only used one example and extended it by developing more questions, and also made her own problem by making an example from the book more challenging.

In Holly's fourth lesson she made all of the examples she used during the lesson portion, and the centers were based on a combination of Envisions and ideas from her cooperating teacher. Holly's score on choice of examples depended on different abilities during the study. At the beginning of the study, Holly's score on choice of examples depended on her ability to select tasks from the curriculum. In the second lesson, her score depended on her ability to select and enhance tasks from the curriculum. In the fourth lesson, Holly's score on this dimension depended on her ability to make high cognitive demand tasks.

The data from the three preservice teachers suggested that the choice of examples dimension involves three sub-domains: the ability to select existing tasks, the ability to enhance existing tasks, and the ability to make tasks. Based on their descriptions, it seemed that during student teaching Wendy primarily practiced selecting existing tasks, McKenzie primarily practiced making her own tasks, and Holly primarily began with a focus on selecting existing tasks, then moved to enhancing existing tasks, and finally made her own tasks.

Theoretical underpinning. Of the five MKT dimmensions that all three preservice consistently demonstrated in their teaching, theoretical underpinning was more frequently demonstrated at the minimum level than the other dimensions. The minimum level was characterized by broad principles of pedagogy such as having students work in groups or the use of discussion. These instructional strategies are generic to all domains of mathematics, whereas the middle level included aspects of pedagogy specific to the content area of the lesson. The only participant who demonstrated content-specific pedagogy was Wendy, who did so during two of her lessons. McKenzie and Holly scored minimum on all four of their lessons as did Wendy on two of hers.

The best demonstration of theoretical underpinning was found in Wendy's second lesson, which was a first grade subtraction lesson. Some of the general aspects of mathematics pedagogy that she included in the lesson were that students used small dry erase boards and markers to work out their own solution strategies to story problems, she encouraged the use of multiple strategies, and she asked students to share their solutions. In addition to these mathematically generic elements of theoretical underpinnning, Wendy incorporated specific elements that demonstrated a higher level of awareness of significant factors for teaching operations in the primary grades. One example of this included intentionally selecting students to share based on their solution strategies, so that the class would be presented with a variety of different, efficient, and reliable strategies. This can be contrasted with a teacher calling on a student without knowing or evlauating his solution stragegy, thus leaving the solution shared with the class to chance which then impacts the subsequent comments and discussion. Wendy took this into account by looking at students' white boards and then selecting the students she wanted to share to demonstrate their strategy at the board or dictate it to her as she wrote it on the board. Also, as strategies were shared she instructed students to pay attention and "ask yourself, is that how I did it? Is her way faster, or easier, or make more sense?" This indicated Wendy's higher level of understanding than a teacher who has students share strategies but does not intentionally select them or overtly discuss why different strategies are being shared. This was the highest demonstration of theroetical underpinning across the twelve lessons.

Holly demonstrated a minimum level of theoretical unerpinning in all four of her lessons. The most frequent ways she demonstrated this was she asked students to explain their answer or thinking, had students work with a partner or in centers for part of the time, or had students complete a mental math exercise at the conclusion of the lesson. Each of these aspects of MKT is generic across content areas and was not specfic to the particular lesson at hand.

McKenzie also demonstrated a minimum level of theoretical unerpinning in all four lessons. McKenzie most frequently demonstrated an initial level on this dimmension through having students regulalry share solution strategies at the board, although she did this without intentionally selecting strategies as Wendy did. Occassionally, McKenzie had students work with a partner or a small group. Each of these aspects of MKT is generic across content areas and was not specfic to the content of the particular lesson at hand.

On the whole the three preservice teachers tended to demonstrate an intial level of theoretical underpinning in their teaching. In all twelve lessons Wendy, Holly, and McKenzie incorporated some aspects of pedagogy shown to be significant in the teaching of mathematics such as having students work in different grouping arrangements, engaging in problem solving, using manipulatives, and participating in mathematical discussions (Rowland et al., 2009). However, rarely did participants demonstrate theoretical knowledge specific to the content area of the given lesson. Although Wendy did in half of her lessons, Holly and McKenzie never demonstrated content-specific theoretical knowledge in their teaching or in their post-observation interview.

Deviation from agenda. The final MKT dimmension that participants consistently demonstrated was part of the contingency category. As such, this was an aspect of teaching that the preservice teachers could not plan for in advance of their lesson, unlike the other four dimmensions on Tier One. Deviation from the agenda was one of the highest dimensions of MKT the participants consistently demonstrated as a group. Half of the time (six lessons) the preservice teachers scored at the minimum level and half of the time they scored at the middle level. The minimum level was characterized by deviation for a non-math specific reason. The two most frequently occuring examples of generic deviation were increasing the pace of the lesson to cover all of the content in the alloted time and deciding what students would do once they had completed the assigned work for the lesson. Generic deviation was demonstrated in half of the lessons: one of Wendy's lessons, two of McKenzie's lessons, and two of Holly's lessons. These were elements of teaching that are necessary though not specific to teaching mathematics and so further exploring examples of generic deviaiton will not be pursued. Instead, attention will be given to the middle and maximum levels of deivation which were mathematical in nature.

The middle level of deviation was characterized by the preservice teacher making a change in her lesson for a mathematical reason that resulted in an increased focus on procedural knowledge, whereas the maximum level was characterized by the preservice teacher doing so to increase focus on conceptual knowledge. An example from each preservice teacher will now be presented to explain how the KQ dimension of deviation from the agenda was enacted in their teaching practices.

Of the three preservice teachers McKenzie demonstrated the most intial level of deviation from the agenda. She did so at the minimum level in three lessons and the medium level in one lesson. McKenzie demonstrated deviation the most during her second lesson which was on similarity and congruence. Students were trying to determine whether two shapes were similar, congruent or neither. Several students did not agree with another student's answer that it was neither. McKenzie then asked the class if the two shapes were congruent, but the class still disagreed. One student called out that it would be neither similar nor congruent. McKenzie then asked the following series of questions:

> McKenzie: Why would it be neither?
>
> Student: (does not know the answer)
>
> McKenzie: Are they the same size?
>
> Students: No
>
> McKenzie: Are they the same shape?
>
> Students: No
>
> McKenzie: So are they congruent, similar, or neither?
>
> Student: Similar?

At this point McKenzie deviated from her intended lesson in the following way.

> McKenzie: Okay. Let's go back.

She went back to the previous Smartboard slide, which was about similarity, and said, "Similar figures have the same shape but not the same size." Then she returned to the slide on congruency and said:

> McKenzie: To be congruent, they have to be exactly the same size and shape. So they are not congruent. Maybe they're similar. Are they the same size?
>
> Students: No
>
> McKenzie: Are they the same shape?

Students: No

McKenzie then dragged the two objects to the part of the board labeled “neither” and moved on to the next part of the lesson, which was determining whether a new pair of objects was congruent, similar, or neither.

The above vignette demonstrated the middle level of deviation. McKenzie deviated for a mathematical reason because her students were unsuccessful at identifying whether two objects were congruent, similar, or neither. McKenzie’s response was procedural in focus. She went back to a previous slide, restated the definitions of similarity and congruence, and again asked students about the size and shape of the objects before telling them it was neither congruent nor similar. Each of these moves is characteristic of a focus on procedural knowledge (Skemp. 1978).

Three aspects of this example seem worth further consideration. First, McKenzie decided to deviate from her lesson at an appropriate time and for an appropriate reason, which was because her students were not successful with the content at hand. A second aspect that is worth noting is the manner in which McKenzie deviated, which was by repeating what she previously presented to her students instead of presenting either new content or a new explanation of the same content. Finally, her deviation was focused on procedural knowledge of the content rather than on conceptual understanding, in this case focusing on the definitions of similarity and congruence. McKenzie demonstrated an initial amount of deviation because she deviated generically in three of her lessons and the one time she deviated for a mathematical reason she repeated previous and procedural aspects of her lesson rather than presenting students with an alternative approach to the content.

Of the three preservice teachers, Holly demonstrated the next highest level of deviation from the agenda. In two of her lessons Holly demonstrated the minimum level of deviation and in the other two she demonstrated the middle level of deviation. Lesson one contained two instances where Holly deviated in a procedurally mathematical way, resulting in a score at the medium level. At the very beginning of lesson one, which was a fourth grade lesson on estimating sums and differences, the Smartboard in Holly's classroom stopped working. Holly planned to use it throughout her lesson and had loaded slides with all of the problems she wanted students to complete throughout the lesson. The first aspect of deviation Holly demonstrated was getting an 18 x 24 inch whiteboard on which she wrote the numbers for the problems and held it up for students to refer to throughout the lesson. Despite limited space and no longer having access to the problems she created, she did her best to continue the lesson as she had originally intended. In the post-observation interview Holly explained her use of the whiteboard as follows:

> The whiteboard was helpful to be able to show them. And yeah, I didn't have the Smartboard, but thank goodness I had the white board. If I didn't have that, I think I probably would have taken a big piece of construction paper and wasted all this paper, but I would have to show them somehow. So at least I had the whiteboard.

Another incidence of deviation occurred at the end of Holly's lesson when students were doing mental math excercises. The last problem was .18 + 6.2, which students were to solve by rounding to the nearest whole number. One student said that his answer was seven, to which Holly replied, "How did you get 7? What does .18 round to?" At this point a student said "18." Then Holly deviated from her lesson by saying and doing the following:

> Holly: Ah. Good point. Does that mean we're dealing with the whole number 18?

As she said this she got the whiteboard back out and wrote on it ".18". Then she continued.

> Holly: It looks like this. What does this number round to?
>
> Students: Zero.
>
> Holly: Remember, this is a common error. When we say "let it rest", it stays the same whole number. The 6 is never going to turn into a 5. I make that mistake, too.

Throughout this lesson Holly certainly worked hard and deviated from her original plan, largely because of the Smartboard becoming inoperable at the beginning of her lesson. Furthermore, Holly recognized when a student did not understand a problem and deviated at an appropriate time to give him additional support. Similar to McKenzie's deviation, Holly's deviation was in the form of procedural knowledge focused on the mechanics of rounding rather than conceptual understanding (which would render turning .18 into 18 absurd). Another similarity to McKenzie's deviation was Holy's repetition of previous instruction ("when we say 'let it rest', it stays the same whole number"). In these three ways McKenzie and Holly's deviation was quite similar, with the main difference being that Holly demonstrated a medium level of MKT in two lessons and McKenzie demonstrated a medium level of MKT in one lesson.

Of the three preservice teachers Wendy most frequently demonstrated the highest level of MKT within the deviaiton from agenda dimmension. She scored minimum on one lesson, medium on two lessons, and maximum on one lesson. This was the only time a maximum was demonstrated, as defined by deviation that had some aspects of conceptual understanding. This happened during Wendy's fourth leson which was on rotating and combining shapes to form other shapes. After Wendy combined two triangles she asked the class:

> Wendy: Do we have a new shape? Raise your hand if you know what new shape we have.
>
> Student A: A diamond.
>
> Student B: Or it is a square?

At this point Wendy deviated from her planned lesson; she drew a square on the Smartboard and asked, "Does everyone agree that this is a square?" The students said yes. Then Wendy rotated the square one-eighth of a turn so that one corner was at the top and one was at the bottom.

> Wendy: Is it still a square?

The students answered both "no" and "yes" in about equal number.

> Wendy: Oh, a little dissent!

Again, she drew the square in its typical orientation.

> Wendy: Is this a square?
>
> Students (in unison): Yes.

Then Wendy rotated the square one-eighth of a turn again.

> Wendy: Is it still a square?

Most of the students said no.

> Wendy: If you think that it is not a square anymore, raise your hand. Jordan, why do you think it's not a square any more?
>
> Jordan: 'Cuz its shape is a diamond.
>
> Wendy: 'Cuz its shape is a diamond? What describes a square? It has four sides, four corners, and all the sides are even, right? When it's shaped like this [referring to the square with the typical orientation], does it have four sides?
>
> Students (in unison): Yes.
>
> Wendy: Four corners?
>
> Students (in unison): Yes.
>
> Wendy: Are all the sides even?
>
> Students (in unison): Yes.
>
> Wendy: When it's shaped like this [referring to the square rotated 1/8 turn], does it have 4 sides?
>
> Students (in unison): Yes.
>
> Wendy: Four corners?
>
> Students (in unison): Yes.
>
> Wendy: Are all the sides even?
>
> Students (in unison): Yes.

Wendy: Interesting. It might be that this is still a square.

Wendy then used the Smartboard to drag a square on top of the two triangles.

Wendy: When we combine these two triangles, they make a square.

This excerpt is offered as an example of deviation from the agenda, not in regards to Wendy's questioning, which stands to be improved. In this example Wendy demonstrated the most focus on conceptual understanding of any of the incidences of deviation. Again, it is not perfect, but one might consider it a nice attempt for a preservice teacher. This example is consistent with the previous two examples from McKenzie and Holly in which deviation occurred at an appropriate time and for an appropriate reason, which was her students did not understand that when a square was rotated it remained a square. Unlike the previous examples of deviation, Wendy's deviation had some inclusion of conceptual understanding and presented information not previously covered in the lesson. In these two ways Wendy demonstrated a higher level of deviation than her peers, who deviated in procedural and repetitious ways.

The data from the three preservice teachers suggests that the first aspect of deviation that translated to practice may be the ability to recognize when deviation would be mathematically beneficial for students. The next aspect of deviation that translated to practice seemed to be deviating for mathematical reasons, and this deviation was characterized by a procedural focus and repetition of previous lesson content. A third phase of deviation that translated to practice seemed to be deviating for mathematical reasons, where this deviation was characterized by a conceptual focus and presentation of the concept in a new way that did not repeat previous lesson content.

Summary. The five dimensions that all three focal preservice teachers consistently demonstrated in their teaching were questioning, decisions about sequencing, choice of examples, theoretical underpinning and deviation from agenda. Closer examination of each dimension indicated some similarities in the way MKT translated to practice across the five dimensions. Chief among these was the result that translation to practice seemed to occur in a way such that even within one dimension, some aspects of that dimension appeared before others, and some continued to be absent even when others were consistently demonstrated.

Translation to practice: Tier Two. Although the focus of this results chapter was the five previously described MKT dimensions that all preservice teachers consistently translated into practice, an overview of the next three tiers will now be reported beginning with Tier Two. In order to be included at Tier Two, a dimension had to be consistently demonstrated by two of the three preservice teachers as indicated by a score of minimum, medium, and/or maximum over the four lessons with no inappropriate score. This meant that one preservice teacher inconsistently demonstrated the dimension, indicated by scoring inappropriate on at least one of the four lessons. The Tier Two dimensions were all in the foundation category: awareness of purpose, identifying errors, and use of terminology. The demonstrated scores for these dimensions are presented in the following figure.

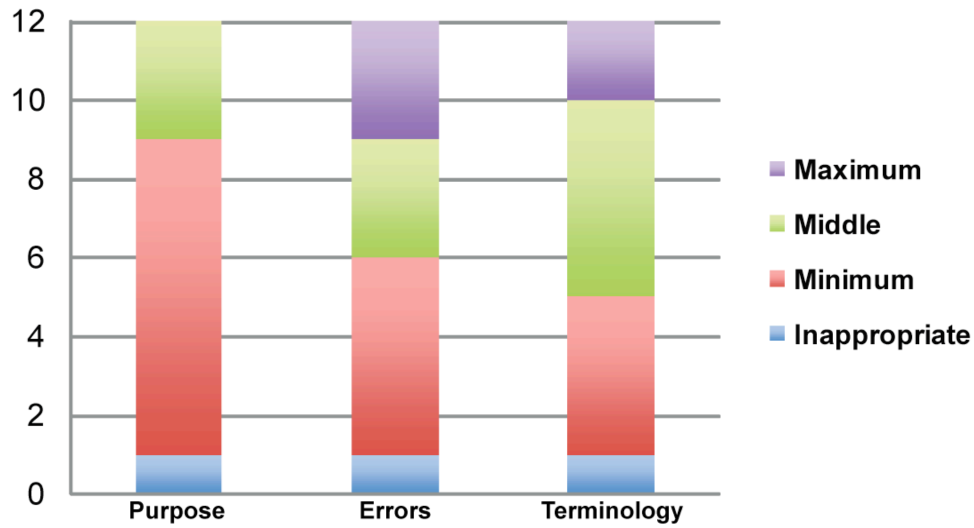


Figure 24. Tier Two dimensions. This figure shows the dimensions that were consistently demonstrated by two of the three focal preservice teachers.

The first result that is apparent when viewing the three MKT dimensions that two of the three participants consistently demonstrated was that the third participant scored inappropriate in only one of the four lessons. As previously explained, the fact that there was even one lesson at an inappropriate level is problematic. An interesting finding was that the inappropriate score always occurred in lesson three. This suggested that even after a substantial amount of practice and successful demonstration, the dimension still had not consistently translated to practice for one of the three preservice teachers.

These three dimensions were consistently demonstrated in practice for two of the preservice teachers and may be the next dimension to consistently translate to practice for the third preservice teacher, based on the fact that the participant demonstrated the dimension in three of her four lessons. By demonstrating the dimension in three of her lessons she showed she was capable of enacting the given dimension, but it had not become a consistent aspect of her teaching.

The demonstration of each of the three dimensions will now be briefly summarized. The dimensions with the best two scores were identifying errors and use of terminology. McKenzie and Holly consistently demonstrated identifying errors, and Wendy did in lessons one, two, and four. Wendy and McKenzie consistently demonstrated use of terminology, and Holly did in lessons one, two, and four. Of the three dimensions on Tier Two, the scores for awareness of purpose ranked third. Wendy and Holly consistently demonstrated awareness of purpose, and McKenzie did in her first three lessons.

Whereas examples of every dimension were given for each of the three focal preservice teachers for all Tier One dimensions, discussion of Tier Two, Three, and Four will concentrate on one dimension of MKT. The one dimension discussed was selected based on the richness of the example(s) and similarities across the preservice teachers. For Tier Two, the dimension that will now be discussed is awareness of purpose.

Awareness of purpose was selected to discuss because of the prevalence of an idea that two of the three focal preservice teachers had regarding instructional decisions. In the post-observation interviews, the preservice teacher was asked about her reasoning for instructional planning and real-time decisions that occurred before and during the lesson. It was hoped that the preservice teacher would respond with a dimension of MKT to explain a given decision, such as one of the nineteen dimensions on the KQ model. Certainly there were many times when this was the case. However, out of the twelve lessons, there were also seven lessons for which the preservice teacher indicated she made an instructional decision because she wanted her students to "have fun." Out of four lessons, this was true one time for Wendy, three times for McKenzie, and three times for Holly. The following excerpts are from post-observation interviews in which the preservice teacher identified "having fun" as a reason for an instructional decision in the lesson that just concluded.

McKenzie ended lesson two by having students play a game in which they stood facing a partner and used their bodies to demonstrate various reflections, rotations, and translations that McKenzie called out. For example, she called out "reflection: arm, leg" and students would quickly demonstrate a reflection by each positioning an arm and a leg in an appropriate orientation. They played this for six minutes and then McKenzie handed out homework and ended math for the day. After the lesson McKenzie explained the inclusion of the game in the following way:

> The translation game is something we did last Thursday, so that's just kind of a good review, and it's fun for them, and some of them are really good at it. Some of them are having a little bit more trouble figuring out rotation, especially, but it's kind of fun and just sort of like a good review.

Later in the post-observation interview McKenzie was asked to name a few things that she thought went well in the lesson, to which she similarly said the following:

> I liked the review game at the end…They're excited about it, they like it, they enjoy it. It just takes maybe five minutes. I don't let it go on for too long, but I think they enjoy it and they kind of get to do something else.

Although McKenzie did say that the game was an opportunity for her to assess her students, her students' excitement and enjoyment seemed to be her primary reason for including this activity in the lesson.

Similarly, Holly often explained portions of her lesson in terms of "fun." Her third lesson on coordinate grids involved a lengthy set of directions followed by working with a partner in which each partner first made a design on a blank coordinate grid and then they took turns reading their points to their partner, who had to duplicate the design without seeing it. In the post-observation interview, Holly said the following:

> My intention initially was for them to get to do something fun to review. I wanted this to be fun, because they're going to be doing a big group project Wednesday and Thursday with this material...So, I thought it would be a good activity…It probably wasn't the best activity to do today with this group, but I wanted them to do something fun. I thought it might be fun for them, but, I don't know.

Similar to McKenzie, when Holly was asked what went well in the lesson one of the things she said was that she was happy that they were "having fun with it. Because I wanted them to have fun with it. And that was one of the goals of the lesson--to enjoy it."

At the end of the same post-observation interview Holly made a telling statement about the connection she viewed between mathematics lessons and "fun." She explained that of all the content areas she was the least comfortable teaching math. She went on to say that, at that time, she didn't "dread" and "hate" teaching math like she did compared to when she was observed in March.

> It's not as terrible as I thought it was. I thought math was the worst thing in the world. I still don't like math, but, I like teaching it. That's the thing—I tried to make it really fun, and they liked a lot of activities that we've done, and that's good….(A student) said to me a week or two ago, "You make math more fun than I've ever had it." They may not be learning as much, but, it's fun. And that's my goal--I want it to be fun for them.

This was a tremendously helpful piece of information to gain from Holly. She spoke openly from the beginning of the study about not liking math in school and feeling like she was not good at mathematics, even though she made As. She also spoke multiple times about wanting mathematics to be fun for her students. In this excerpt it was apparent how these two ideas related to Holly: because she did not like mathematics she wanted to make sure that her students liked mathematics, which to her meant making math lessons fun.

Although the above excerpt helped explain Holly's over-reliance on "fun" as a basis for instructional decisions due to her dislike of mathematics, this did not explain McKenzie's reliance on "fun." McKenzie was quite confident in her mathematical abilities as both a student and a teacher and enjoyed teaching math from the beginning of the study. Even after examining all the data it remained unclear as to why McKenzie used "fun" as a barometer for instructional decisions.

Translation to practice: Tier Three. Having presented results about the five KQ dimensions that all preservice consistently translated into practice (Tier One) and the three dimmensions that two out of the three preservice teachers consistently demonstrated (Tier Two), an overview of the next tier of dimensions will now be reported. Dimensions were included in Tier Three when one of the three preservice teachers consistently demonstrated the dimension in all four lessons and the other two preservice teachers scored inappropriate at least once. This means that for each dimension on Tier Three, one preservice teacher scored minimum, middle, or maximum for all four lessons, while the other two scored inappropriate in at least one lesson. This resulted in five Tier Three dimensions, which are show in the following figure.

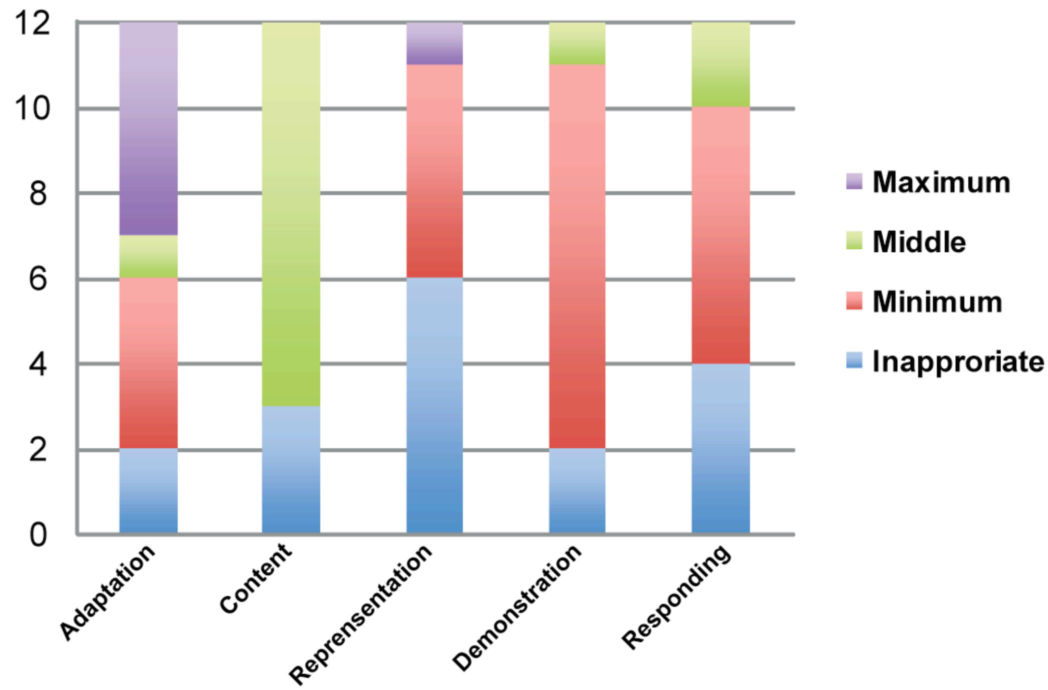


Figure 25. Tier Three dimensions. This figure shows the dimensions that were consistently demonstrated by one of the three focal preservice teachers.

Overall there was a higher prevalence of inappropriate scores for the dimensions on Tier Three than Tier Two, which ranged from two to six lessons on this tier. The most common score continued to be minimum. Medium scores appeared on four of the five dimensions. As a category, maximum scores were rare. The one exception to this was for adapts textbook, for which a maximum score was demonstrated in five of the lessons.

The dimension that will be further discussed for the Tier Three dimensions is choice of representations. As seen in the above figure, choice of representations was demonstrated inappropriately by the preservice teachers in half of the lessons. Of the seventten dimensions, this was the second highest amount of inappropriate scores. The most noteworthy classroom episode that happened with choice of representations occurred in a lesson McKenzie taught in her fourth grade student teaching placement, which was considered an AIG class. This episode, which was scored inappropriate, will first be described and then it will be discussed in terms of choice of representation.

At the beginning of her fourth lesson McKenzie briefly talked with her students about the importance of place value and then told her students, “Yesterday we were using our M&Ms to talk about place value, so we’re going to use our M&Ms to talk about place value for a little bit more today.” Whereas the day before they used real M&Ms, today she used the same representation but did so on the Smartboard. She displayed a slide that showed different colored circles, representing the M&Ms, in a column down the left-hand side. Each color was a different amount, which was indicated next to the corresponding circle. She referred to this as the “key.” For example, a blue M&M represented 1, a red M&M represented 10 and a green M&M represented 100.

To begin, McKenzie showed the students several different colored circles on the Smartboard and asked, “Based on our key over here what number is this?” The number was 141. She then made a second number with different colored circles and asked, “If I was adding these together, how could I use this visual to add these numbers together? Show me how you would use these pieces to find the answers.” A student was called to the board and added the two groups of circles together to get the answer of 483. Next, McKenzie drew two new groups of circles and asked students, “What do my numbers end up added together?” After students answered 524, she asked one student to come to the board “to show me how we would add these pictures together.” Again the student slid the circles around on the Smartboard to arrange them in the standard algorithm format by “stacking” numbers of like place value and adding the columns. The student traded in a group of ten, tens circles for one, one hundred circle as she worked. She left the ten, tens circles off to the side. When the student finished McKenzie said, “You left some over here. You need to add them all together” as a prompt to have the student explain how she accounted for those circles when she traded them for one, one hundred. After the student explained this, McKenzie said, “Oh! So you had ten, tens, and you changed it into one hundred.”

McKenzie showed the students the final problem for the M&M model, which she wrote on the board as 12 x 13. She asked the students, “How am I going to show this? Take about 30 silent seconds to show me how you would show this. Using these representations, not numbers, to show this. If you just click and drag those M&Ms, how would you do it?” She called a student to the board, who again arranged them in the standard algorithm format and wrote the answer. Based on how she arranged the circles, it was clear the student was not using the representation but rather was “translating” the circles to their corresponding numeric value in her head and performing the calculation in numeric form. McKenzie, to her credit, realized this. She asked the student to explain her thinking, which the student did in numeric form while ignoring the M&M model. McKenzie then demonstrated making three groups of 12 with the circles and said, “When I was looking at this problem right here I was looking at it like this: 12, 3 times.” After this demonstration McKenzie said “Good job guys, I just wanted to go over that a little bit more” and moved on to the next part of the lesson.

The above episode is an example of an inappropriate use of representations. The main reason for this is because of the significantly reduced level of mathematics that was done compared to what these students usually did and of which they were capable. A fourth grade AIG class should not be working on problems such as 12 x 3. This level of mathematics was far below their instructional range. This was an example of changing the mathematics to fit the representation rather than using a representation to promote an understanding of mathematics. The representation used was also inefficient, in that it took a noticeable amount of time for the McKenzie and her students to duplicate and rearrange the circles on the Smartboard. More of the time during this portion of the lesson was spent sliding circles around rather than discussing mathematics. This was due to both the choice of representation and McKenzie's use of the representation. Indeed, the only time she discussed place value was once when she asked the student at the board about trading in ten tens for one hundred, although whether even this statement was beneficial to fourth grade AIG students was dubious given the depth at which it was discussed.

This episode when McKenzie used the M&M model as a supposed representation of place value is offered as an example of an inappropriate use of representation. An inappropriate score on this dimension was never a matter of the preservice teacher not using a representation, but rather it was always a lesson in which she used an inappropriate representation for the concept at hand. This indicated that these preservice teachers may not have been clear on the purpose of using representations and/or how to select appropriate representations that help make a mathematical concept clearer. Since choice of representation was demonstrated at an inappropriate level in half of the twelve observed lessons, this may indicate it is a dimension of particular challenge amongst preservice teachers to translate to practice.

Translation to practice: Tier Four. There were five KQ dimensions that all participants consistently translated into practice (Tier One), three dimmensions that two out of the three participants consistently demonstrated (Tier Two), and five dimensions that one of the three participant's consistently demonstrated (Tier Three). Tier Four dimensions were those that were not consistently demonstrated by any of the preservice teachers. This meant that each of the three preservice teachers scored inappropriate for at least one of their lessons on each dimension in Tier Four. These dimensions were concentration on understanding (foundation), anticipation of complexity, connection between concepts, and connection between procedures (connection). The following figure shows the preservice teachers' scores on each of these four dimensions.

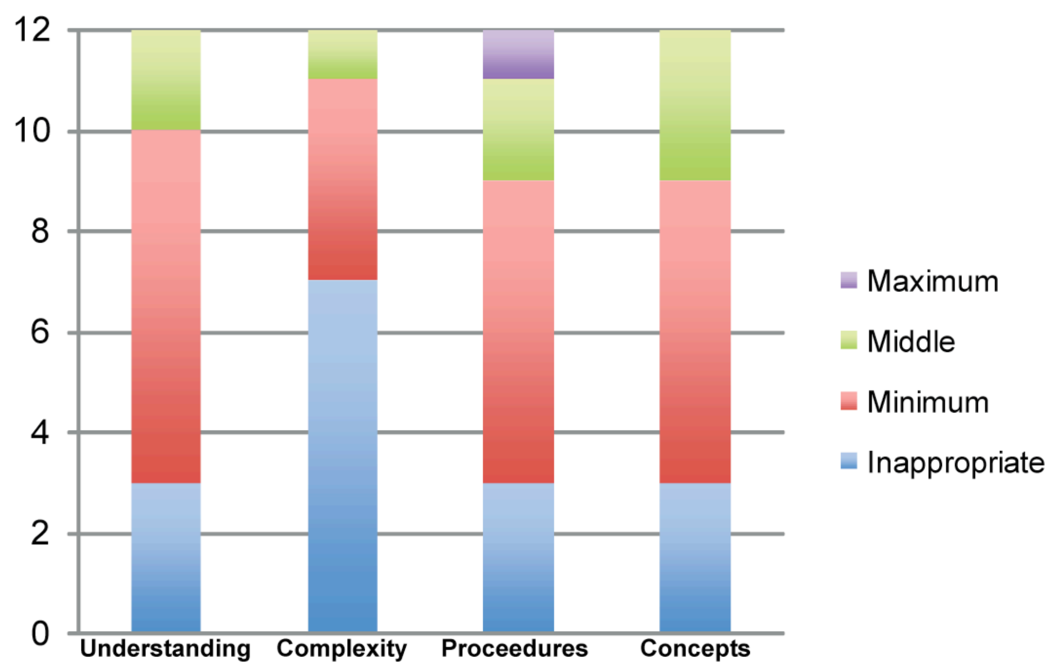


Figure 26. Tier Four dimensions. This figure shows the dimensions that were not consistently demonstrated by any of the three focal preservice teachers.

The number of lessons for which there was an inappropriate score for each of the four dimensions on Tier Four was slightly higher than it was for Tier Three. For Tier Four, an inappropriate score occurred in three to seven lessons per dimensions, whereas for Tier Three the range was two to six lessons per dimension. When preservice teachers did demonstrate MKT on these four dimensions it was usually at the minimum level. Scores were less prevalent at the higher levels, occuring between one to three lessons per dimension at the medium level and only once total at the maximum level. For three of the dimensions (concentration on understanding, connection between concepts, and connection between procedures) each preservice teacher scored inappropriate on one of her four lessons. For anticipation of complexity, participants scored inappropriate in either one (Holly) or three (Wendy and McKenzie) lessons.

Inappropriate scores occurred for all four lessons, including three inappropriate scores for lesson one, three inapprorpiate scores for lesson two, six inappropriate scores for lesson three, and four in appropriate scores for lesson four. Again it was suprising that inappropriate scores did not decline, and in fact increased, in lessons three and four compared to lesson one and two. For the preservice teachers, the group of dimensions that comprise Tier Four did not consistently translate to their teaching.

The dimension that will be further discussed for Tier Four is anticipation of complexity. As seen in the above figure, anticipation of complexity was demonstrated inappropriately by the preservice teachers in seven of the lessons. Of the seventeen dimensions this was the highest amount of inappropriate scores, meaning that the preservice teacher did not make any instructional decisions (i.e., use of a demonstration, example, question, or representation) based on their anticipation of complexity. The most noteworthy classroom episode that happened with anticipation of complexity occurred in a lesson Wendy taught in her first grade student teaching placement. This episode, which was scored inappropriate, will first be described and next be discussed in terms of anticipation of complexity.

Wendy's second lesson introduced a new subtraction strategy to the class. Previously they worked on "adding the unknown partner," and on this day Wendy had them begin subtracting the missing partner, which Math Expressions called the "making ten strategy." To use this strategy, first the students wrote the minuend in expanded notation. Next they they wrote the subtrahend, and then they made the number of "counting dots" it would take to make a ten with the subtrahend. Then students counted on with the counting dots from the expanded notation number to arrive at the difference. After the lesson, Wendy explained that she had not anticipated the complexity of this strategy for use with first graders:

> When I studied it I was just like, "Oh, okay. It seems weird, but okay." Because I just can't know--I still haven't really locked into what they're going to stuggle with, so when I saw them doing it, I was like, "Ohhhh, okay. This is a lot harder than I thought it was going to be." So then I kind of had to rewind.

In her lesson, "rewinding" took the form of having students stop working on their individual whiteboards and telling them what to do in a step-by-step fashion. This happened after several minutes in which students floundered with the strategy and Wendy struggled with her responses. During this time she said she realized "something about what I said or something about this strategy is a little more complex than they know what to do…I didn't realize that was going to be quite so complex for them. But I saw it in their faces when they did it, so, I thought that was a pro." To be sure, although Wendy did not anticipate the complexity of the subtraction strategy and stuggled to respond during the lesson, the fact that she did recognize her students' difficulty was a positive indication that she could recognize the complexity of a concept based on her students' responses and questions. After the lesson when asked if there was anything she would do differently were she to teach the same lesson again, she said:

> I would demonstrate to them additional times. Knowing that they would struggle with it, I would make more problems ahead of time, because, also, when I saw they needed additional reinforcement, I was like, "Oh, well, okay, let me think of a teen problem with a partner." So I would anticipate that, and I would make up more problems ahead of time.

This episode when Wendy struggled to introduce a new subtraction strategy to her class is offered as an example of inappropriate anticipation of complexity. After the majority of lessons during the post-observation interview the preservice teacher described one or more things that she did not anticipate would challenge her students. What was encouraging was that usually they were able to recognize their shortcoming on this dimension and offer some ideas as to how they would try to make adjustments in a subsequent lesson. It seemed like the challenge with this dimension was less about translating knowledge to practice and more about not having acquired the necessary knowledge. That is to say, the preservice teachers did not struggle with finding a way to make an instructional decision based on their anticipation of complexity, but rather lacked an awareness of the complicated nature of what they were about to ask their students to do. Since anticipation of complexity was demonstrated at an inappropriate level in over half of the observed lessons, this may indicate it is dimension of particular challenge amongst preservice teachers.

Summary. Based on the data, results for the second research question were conceptualized as a series of tiers. Distinctions between each tier were based on the number of preservice teachers who consistently demonstrated the given dimension. Tier

One contained dimensions of MKT that were consistently demonstrated by all three preservice teachers, Tier Two dimensions were consistently demonstrated by two preservice teachers, Tier Three dimensions were consistently demonstrated by one preservice teacher, and Tier Four dimensions were not consistnetly demonstrated by any of the preservice teachers. The following table summarizes the four tiers of translation to practice for the three preservice teachers; the seventeen KQ dimensions are sorted by their corresponding category (foundation, transformation, connection, contingency).

Table 17

Tiers of MKT translation to practice

Tier	Foundation	Transformation	Connection	Contingency
Tier One (n=3)[26]	Theoretical underpinning	Questioning, Choice of examples	Sequencing	Deviation
Tier Two (n=2)	Awareness of purpose, Identifying errors, Use of terminology			
Tier Three (n=1)	Adapts textbook, Overt subject knowledge	Choice of representation, Demonstration		Responding to children's ideas
Tier Four (n=0)	Focus on understanding		Anticipation of complexity, Connect procedures, Connect concepts	

[26] n = number of preservice teachers (out of 3) who consistently demonstrated all dimensions on the tier. For example, all three preservice teachers consistently demonstrated all of the dimensions on Tier One.

From this table it is evident that the foundation category tended to translate to practice more consistently than the other three categories, as evidenced by the prevalence of foundation dimensions in Tier One and Tier Two and their tapering amount in Tier Three and Tier Four. The contingency category and transformation category were the next most successful categories for translation to practice, with half of the dimensions at Tier One and half at Tier Three. Finally, the majority of the connection category dimensions were demonstrated at the lowest levels of translation to practice, with one dimension on Tier Three and three dimensions on Tier Four.

Another result based on the above table is that eight of the dimensions were consistently demonstrated by all or most of the preservice teachers (Tier One or Tier Two), nine of the dimensions were consistently demonstrated by one or zero preservice teachers (Tier Three or Tier Four). Wendy consistently translated ten of the seventeen dimensions of MKT to her practice, Holly consistently translated eight of the dimensions to her practice, and McKenzie consistently translated nine of the dimensions to her practice. This means that, as a group, just over half of the seventeen KQ dimensions did not consistently translate to preservice teachers practices. This result is a likely indication that consistent demonstration of these dimensions probably did not occur before the end of their teacher education program, because the final observation was completed only two weeks prior to the end of student teaching. Furthermore, during this time, the student teachers had completed their full time teaching and were no longer teaching mathematics every day. Therefore, it seems unlikely that consistent translation to practice would occur under such circumstances, especially when it hadn’t to date.

In summary, it was seldom possible to identify areas of straightforward development of MKT in the preservice teachers' teaching. Their demonstrations of MKT were variable and approximately half of the dimensions were not consistently demonstrated. Therefore, four tiers were used to indicate the consistency with which this group of preservice teachers demonstrated MKT in their teaching. The preservice teachers demonstrated some dimensions of MKT at higher levels than other dimensions. Based on this data, it can be claimed that dimensions of MKT do not translate to practice at the same time. Furthermore, insights were gained on sub-practices within the Tier One dimensions through the study of preservice teachers' teaching.

CHAPTER 8

FINDINGS—RESEARCH QUESTION THREE

The third area this study explored was preservice teachers' experiences throughout their final year in a teacher education program. In order to answer research question three, data analysis was conducted in a series of waves to analyze experiences the preservice teachers identified as helpful contributions to their growth as teachers of mathematics. The results from this analysis will now be presented. First, preservice teachers' experiences will be introduced. Second, the three most significant experiences will be discussed. Following the description of each of the three sets of experience, it will be compared to Dewey's (1938/1963) criteria of continuity and interaction to consider if it is as educative experience.

Research Question Three: What experiences are educative for elementary preservice teachers' development of mathematical knowledge for teaching?

Initial coding. In the first wave of data analysis, all "helpful experiences" that were indicated by the preservice teachers were extracted from the written reflections (n=35), focus group interviews (n=11), and individual interviews (n=8). Analysis of this data involved coding each experience as per the four KQ categories (foundation, transformation, connection, contingency). However, when this coding was attempted it was not possible to code the majority of reported helpful experiences in terms of the four KQ categories. This was because the majority of the time preservice teachers identified and described helpful experiences in generic terms that did not map onto particular categories of MKT. Despite being asked specifically for experiences that helped them develop as teachers of mathematics, the preservice teachers tended to describe these experiences in generic terms that were not unique to learning to teach mathematics but rather that applied to teaching, writ large. The following focus group excerpt is a typical example of how the preservice teachers tended to describe helpful experiences over the course of the year, in which they identified experiences that were helpful but did not describe the experiences with enough depth to be able to link them back to a particular KQ category. This excerpt is during the third focus group, and is Holly's response to the question, "What experiences have contributed to your mathematical knowledge for teaching?":

> Everyday lessons. Having to plan and then execute it with kids. Everything that goes into that. And PLCs, anything when you are planning with other teachers. And seminars.

Although the preservice teachers had no trouble answering the questions and identifying experiences they thought were helpful, even when asked follow-up questions they seemed generally unaware of what it was about the stated experience that made it helpful. Even when focus group comments were followed-up on the next week in the individual interviews, the preservice teachers still did not describe helpful experiences in term of MKT. For example, during Holly's first interview she answered the same question by saying that a helpful experience was the class discussions in her methods course. The researcher asked her to continue to describe this two more times, and while more details about the experience were elicited, they were not in terms of MKT. After Holly identified class discussions the researcher asked if she could "pick out one that's been the most helpful to describe more." Holly responded:

> The ones that are most beneficial to me are like when we did a class discussion on inquiry-based methods…We did it with our tables and wrote it on chart paper and posted it. That's helpful to me. I like it when we do that, because we can get our ideas together as a group, and then we see all the other groups' ideas. That's been the most beneficial to me, because for math teaching (inquiry) is not something I'm really familiar with, and I'm not seeing it in my placement. I've never really seen it before…So it was really helpful talking through it with other people in my class to see what they got out of the readings—and if they've seen it and things like that. So, the inquiry discussion was good.

At this point Holly was asked a third time about the discussion, and if she remembered the specific prompts or what they talked about. She explained how they used two questions from their book and said, "I think one of them maybe was lesson planning and inquiry." She did not remember the second one, "but basically it was just how do you understand inquiry in general? How can you see it being beneficial in the classroom? That was the general discussion." As these excerpts indicate, although it helpful information was gained, it was not possible to code these comments based on the KQ categories because of their general nature.

Since it was not possible to adequately describe the preservice teachers' individual or collective conceptions of helpful experiences through the KQ categories, a second wave of analysis was conducted following a grounded theory methodology for the three focal preservice teachers. First, all of the helpful experiences that Wendy, McKenzie, and Holly identified over the course of the year were compiled. Since these experiences were not all discussed with the same amount of depth nor as having the same impact on growth, each preservice teacher's experiences were further analyzed to focus on the experiences she thought were most helpful to her. A new list of the three to five most helpful experiences was created for each preservice teacher. Next, a grounded theory analysis revealed similar ways in which the three focal student teachers described their most helpful experiences. This second wave analysis resulted in three descriptors of helpful experiences.

Although these experiences were discussed by the preservice teachers in generic terms that seemed to better address the question, "What was helpful to your growth as a teacher?" rather than the questions they were asked about what was helpful to their growth as a *mathematics* teacher, it still seems worthwhile to consider their responses to better understand what they perceive as helpful experiences. In the next section, these categories will be explained and responses from each of the three focal preservice teachers included to more fully describe each category. The three categories will be discussed in the chronological order in which they emerged over the course of the year, from earliest to latest. Following the three focal preservice teachers' responses, the third wave of analysis will be presented, in which the "helpful" experiences were compared to Dewey's (1916/1985) conception of educative experiences.

Re-teaching. The first descriptor of helpful experiences emerged in the initial interviews held in early October. During this interview, all three of the focal preservice teachers identified re-teaching as a helpful experience. They each had an opportunity in a classroom setting to teach the same lesson more than one time, although this looked different for each of the them: For Wendy this took the form of teaching calendar math weekly in her student teaching placement, McKenzie taught the same center multiple times in a row during in her student teaching placement, and Holly taught the same lesson two times in two different fourth grade classrooms. Additionally, towards the end of the year Wendy had another helpful experience with re-teaching when she re-taught a lesson that, in her words, “bombed” the first time. Each of these experiences will now be described in greater depth.

Re-teaching via calendar math. As previously described, in the fall semester preservice teachers spent each Wednesday in their student teaching placement classroom. They were only required to teach two lessons in the fall, one that was observed by their cooperating teacher and one that was observed by their university supervisor. However, preservice teachers generally also began leading other short activities, including teaching during the calendar portion of the day. Since Wendy was in a first grade classroom, calendar was a daily activity that was completed as students sat on the carpet and watched slides on the Smartboard to review various aspects of the calendar and do a brief series of mental math exercises. This followed the same sequence each time: reviewing the calendar, talking about the repeating patterns on the date cards, counting on from a given number, regrouping units into tens and ones, determining numbers that were one greater or one less than a given number, writing number sentences using the number of the day, and determining whether a number was even or odd. The daily math lesson followed calendar time.

As of her first interview in early October, Wendy had taught calendar math three times and it was already part of her weekly routine in her placement classroom. Between the first and third time she taught calendar two changes were made to the routine, which included having students use small white boards on their laps and having a student, instead of the teacher, write on the Smartboard. In regards to her repeated teaching of calendar Wendy said, “It’s nice to come back to the same thing and be able to tweak it almost experimentally, like, ‘I did this one thing differently from last time, and I got these different results.' So, that's really nice. Things like the white board, to see, 'This is what happened beforehand, and this is what happened after.' That's been really advantageous" Wendy used a repetitive teaching experience as an opportunity to experiment with adjustments, reflect on their results, and adapt or discard changes in subsequent calendar lessons. She reported this was a helpful experience as she learned to teach mathematics.

Re-teaching via centers. In the fall McKenzie's class had 36 students. Due to the large class size, her cooperating teacher taught math using centers only on Wednesdays when McKenzie was in the classroom and could lead one of the centers, which reduced the number of students working independently at any given time. McKenzie taught a small group of students at a center for approximately 15 minutes, then the students rotated and she taught a new group of students at the same center. This meant that in one math block she taught the same lesson four times in a row.

In her first interview in early October McKenzie described her weekly experience teaching the same center four times. She explained that in each of the past two weeks when she taught centers her confidence and teaching improved with each successive group:

> I would say like after group one in both situations, I was heavily discouraged and frustrated that we didn't really accomplish anything. And then, after Group 2, when I realized that I had made a couple conscious and unconscious changes and that it went somewhat better…I felt a little bit, not necessarily confident, but like, "all right, I know what to do this next time now." So, after Group 3, it was kind of like, "I see improvement, like that was good," and then after Group 4, probably both times I felt a lot better about the situation and took away a lot as to "okay, if I was to run this (lesson) again, this is what I would do. And I would know how to handle this information." Because even though I had bad experiences with probably the first one or two groups both (weeks), after the fourth group I was like, "Okay, well there is potential to do well in that activity." I felt really bad

> about those first two groups that I handled so poorly, but if we had a second chance to do it, it would go a lot better.

Similar to Wendy, one reason that re-teaching was helpful to McKenzie was the practice developing a lesson by making adjustments with each successive attempt. She said, "If we were able to do the same lesson two or three times, it would give us a better sense of how we develop it. I guess because if you talk about (the lesson after you teach it), that's fine, that's great, you learn mistakes, but unless you do it again, you don't really experience doing it again."

In addition to not being able to make adjustments and corrections, McKenzie noted another drawback to teaching a lesson only one time as the following: "It does put a lot of pressure on you when you're teaching just one lesson, and if it flops, it flops, and you can talk about it afterwards with your supervisor, but you're not going to do anything about it, really." Reduced pressure, increased confidence, and improved teaching were ways that McKenzie described re-teaching as helpful.

Re-teaching via two classrooms. Holly taught her first mathematics lesson in her placement at the end of September, one-and-a-half weeks before her first interview. The lesson was about making organized lists as a precursor to combinations and lasted just over one hour. Although she had taught a substantial number of mathematics lessons by virtue of working at her mother's school, she had never taught this concept before, had not seen her mother or anyone else teach it before, and her mother did not give her any help or ideas for the lesson. Holly's cooperating teacher formally observed her and told her it was a "very good lesson", and her only suggestion was that Holly not turn her back to the class when writing on the board. All but two students scored 100% on the daily assessment that took place at the end of each lesson. Two days later Holly went to her mother's classroom to teach the same lesson again, having made some changes from the first time. These changes included using an overhead projector so she did not have to turn her back to the class, managing the manipulatives differently, and having students bring their manipulatives to the overhead when they shared solution strategies instead of just verbalizing them.

Similar to Wendy and McKenzie, Holly described her re-teaching experience as an opportunity to try making changes to her instruction. Her mother encouraged her to do this: "(My mother) told me, 'If it doesn't work, I'll teach it again. It's not a big deal. This is your chance to learn and to experiment.'" Holly tried to do this, stating that:

> When I taught it the second time…it was really good to kind of throw other things at (the students) and to look at it on a different level…If I give them more time to do more strategies, is that good or bad? I was very curious, are they going to just tune out if I have this many groups share? And they didn't…But that was good for me to kind of gauge to see how that worked.

In addition to the benefit of experimenting with changes to her teaching, Holly said that re-teaching the same lesson a second time was helpful in the following way:

> It was very beneficial, and it's hard for me to articulate why. I went in more confident. Maybe that's the biggest thing. I went into the second time I taught it much more confident than the first time. And mainly, as I said, I knew the lesson worked—the lesson as a whole should work. It worked once, it should work again.

Having the confidence that the lesson "worked" allowed Holly to focus on skills she wanted to develop. She explained that having taught the lesson successfully once allowed her to be less worried about the lesson itself in terms of activities and materials. As a result, she was able to focus on specific aspects of her teaching:

> One of my biggest goals this year is to learn to be more concise when I teach a lesson and to not harp on the same thing over and over again. And I know when I (taught) it in my (cooperating teacher's) classroom…I probably emphasized things more than I needed to…So, when I went into (my mother's) classroom, I focused on that a lot. And it cut down time. I noticed, too, I was able to get more in, in the same amount of time, and I think a lot of it was because I realized that the first group, they understood it. I realized I didn't need to harp on stuff so much…But it was just a good feeling to be able to do it again more confident, focusing more on the management and how I said things.

Holly considered re-teaching helpful because of the opportunity to correct aspects of her lesson she perceived as mistakes she made the first time, to make and reflect upon changes and their results, and to work on more specific aspects of teaching because of increased confidence in the lesson. Holly indicated that re-teaching the same lesson in a second fourth grade classroom was one of the most helpful experiences during the fall semester as she was learning to teach.

Re-teaching via a second attempt. Previously Wendy's initial experience with re-teaching via teaching the calendar each week was described. Towards the end of student teaching she had another experience with re-teaching that she discussed in both the final focus group as well as her final interview. The lesson to which she refers in the excerpt below was the final lesson recorded for this study. The day after her lesson was the final focus group, at which Wendy explained what happened in her lesson the day before:

> Yesterday I bombed a math lesson. It was a behavior management disaster. It should have been good. It was like, "manipulatives are great for children." And I wanted everyone to know you could combine shapes to make shapes, and it was going to be phenomenal. And then it just wasn't… So not only did the Smartboard break, but I wasn't so sure about their prior knowledge of shape names, and we ran into this problem where they called them "diamonds" instead of "rhombuses." There were two shapes that could be called a diamond, if you didn't have the vocabulary. So there were a lot of misconceptions of theirs and a lot of logistical things that I didn't anticipate. So it turned into this free-for-all

> where they were like "I built a rocket ship." And I was like, "Great, but I wanted you to build a hexagon."

Despite "bombing" the lesson, Wendy described this as a helpful experience because of the process that it initiated:

> It was actually really helpful, because then I really had to sit down and go back and be like, "What about this was developmentally appropriate? What about it was right, or wrong? What seemed good?...What was I doing because I'd seen it modeled before versus what was actually, instructionally appropriate?"

Following this reflection, Wendy decided to re-teach the lesson the next day. The final focus group meeting was later that evening, when she explained:

> So today I sat down and everyone had the same set of shapes and the same set of instructions. And it was much more structured, and much more, "This is what you're going to do. Now your only materials are these four equilateral triangles. What shapes can you assemble?" And they did a lot better with it. And they were able to articulate what they knew about shape combinations. So that was really helpful to me…having that re-do option, to be like, "Alright kids, let's scrap what we learned yesterday," and seeing that progress…because it was more successful today.

Whereas towards the beginning of the year Wendy described re-teaching as beneficial due to making changes and reflecting on their relative success or failure, towards the end of the year she also articulated that it was the added critical reflection that was helpful in regards to re-teaching that particular lesson as well as to her teaching in general.

Summary. The three focal preservice teachers all described re-teaching as one of the most helpful experiences they had for growth in teaching mathematics. Each preservice teacher had a different way of experiencing re-teaching, including teaching the same lesson weekly, teaching the same lesson multiple times in a row, teaching the same lesson in two different settings, and teaching the same lesson on two consecutive days. The real power of the experience seemed to come from preservice teachers going through a process of reflection to identify things to change, re-teach with these changes, evaluate the results as compared to the first lesson, and continue to reflect on possible subsequent changes. The increase in not only perceived quality of teaching but also the preservice teachers' increased confidence is worth consideration. Furthermore, it is interesting to note that of the four different ways of re-teaching these preservice teachers reported as helpful, only one of them--teaching the same lesson weekly--consistently occurs within the program that exists in the present program. Suggestions for including re-teaching experiences more intentionally in teacher education will be made in the Discussion section.

Educative evaluation of re-teaching. Since the study participants were not familiar with Dewey's concepts and language, during the focus groups and interviews they were asked questions about the "helpfulness" of experiences which can be found in Appendices F and E. However, identifying an experience as "helpful" does not mean it is necessarily "educative." Therefore, the three focal preservice teachers' experiences with re-teaching will now be evaluated based on Dewey's (1938/1963) conception of educative experiences. His ideas about continuity and interaction will be used as the two criteria, as these are the twin pillars of educative experiences.

Continuity. Re-teaching seems to embrace the spirit of continuity that Dewey (1938/1963) described for experiences to be educative. The continuity of an experience means that it has a forward feeding nature in which the means are only temporary ends. With re-teaching, this process is exemplified because of the way in which a lesson is not viewed as a finished product--or end--but rather as a means to creating a more fully developed lesson in the subsequent attempt. This idea was perhaps best articulated by

McKenzie in regards to her experiences re-teaching the same center four times in a row and the way she described her thoughts at each stage of the lesson development, in which she learned what to try the next time and continually had a better idea of how to approach the lesson and content with each successive attempt. The way re-teaching builds upon itself and uses temporary ends as means towards the next temporary end is consistent with an educative experience.

Dewey's (1938/1963) idea that educative experiences prepare a person for deeper experiences in the future maps onto re-teaching experiences, in which preservice teachers described moving from a state of concern about whether or not the activity would "work" to giving more attention to specific aspects of their own teaching. The continuity and connectedness of educative experiences helps generate self-control for future experiences. Through re-teaching, the preservice teachers gained more control over themselves as teachers with each successive iteration of their lesson, which then allowed for more control of their subsequent teaching experience.

Dewey's (1938/1963) main criterion for continuity was its relationship to growth. An educative experience is one that promotes growth in knowledge, mental processes, attitudes, and as a cumulative result, expands the possibilities for the future. Re-teaching encouraged the preservice teachers to engage in a process of growth in terms of their development of a lesson, their confidence, and their teaching. Not only were their knowledge and skills developed, but they also had to engage in a process of reflection in order to determine the adjustments they wanted to make to each subsequent attempt.

The way the preservice teachers described their experiences with re-teaching contrasts with Dewey's conception of mis-educative experiences which are detached, isolated, or sporadic and do not build upon each other or towards a consistent end. Re-teaching stands in sharp contrast to such descriptors and seems to embody a larger measure of continuity than does teaching a lesson only one time, which seems to convey a notion of an artificial end. Re-teaching, however, makes it more transparent that a teaching experience functions as a means to the next temporary end. The way re-teaching encourages growth both in the moment (within a specific teaching interaction) and for the future (for the subsequent iteration of the lesson, as well as for growth that will transcend that particular lesson) seems to embody Dewey's conception of educative experiences.

When considering re-teaching in light of Dewey's descriptions of educative experience, one caution must be considered: it is possible to engage in a series of repeated actions in a way that devolves into routine rather than reflective responses. Such an occurrence would confine the experience to the realm of training rather than education. This is a possible concern in regards to re-teaching, where a preservice teacher could teach something two times in a row but do so without awareness and reflection. The risk of perfunctory responses seems to increase with the number of times a lesson is re-taught. Although McKenzie reported that teaching the same center four times in a row was a positive experience and from her description it is evident that she was engaged in reflection over the series of lessons, it is important to acknowledge that without the mental engagement of the preservice teacher the experience may no longer be educative if habits are formed without reflection and growth in mental processes does not occur.

Interaction. Along with continuity, the other major concept to consider regarding the educativeness of an experience is interaction. The two chief criteria for interaction are whether experiences occur in a thoughtfully planned environment and whether there are opportunities for positive social interactions. The criterion of the environment relates to re-teaching that occurred in a classroom setting and is therefore transpiring in an authentic environment, or at least in the most authentic environment in which we ever place student teachers. That is to say, the re-teaching that the three focal student teachers described took place in their student teaching placement, as opposed to in a classroom or school that was visited as essentially a “field trip” for a methods class. This means the student teachers interactions are as potentially meaningful as possible, since they are interacting in a familiar environment with their own students. However, the environment still has some shortcomings in that the student teachers are operating in someone else’s classroom. Still, compared to alternatives of going to work with any other group of students other than those with whom they are most engaged would reduce the amount of positive interaction that would be possible in the given environment. Therefore, we can conclude that re-teaching in the student teaching placement is fairly educative in terms of the environment, so long as we realize that the student teachers do not have full reign of the environment in which they are interacting by virtue of being in someone else’s classroom.

The second aspect of interaction in regards to educative experiences is the degree to which there are opportunities for positive social interactions. It does not seem like a stretch to claim that re-teaching is entirely a social experience because it happens between the student teacher and her students. An additional aspect to consider in terms of social interaction is the role of the cooperating teacher and the university supervisor in regards to re-teaching and whether these people could be used to enhance the social interactions within re-teaching. However, this will be best discussed in light of subsequent findings and therefore will be discussed in the Discussion section. As for the present query into whether re-teaching the way the three focal student teachers experienced it was educative in regards to social interaction, it can be concluded that it was because of the authentic social experience of re-teaching with their students.

Summary. Re-teaching seems to match Dewey's conception of an educative experience based on the criteria of continuity and interaction. Re-teaching exemplifies continuity through its focus on temporary ends as means, reflection, and growth. It also incorporates interaction via transpiring in a meaningful environment and with authentic social interactions. All of these aspects seem to make re-teaching both helpful to the preservice teachers as well as an educative experience.

Teaching alone. The second category the three focal preservice teachers independently identified as one of their most helpful experiences was having the opportunity to teach without being observed, either formally or informally. That is to say, the three preservice teachers reported that one of the most helpful experiences they each had was teaching in a classroom in which they were the only adult present. Discussion of such experiences did not occur at a consistent time point for the three focal preservice teachers but rather was talked about over the course of the entire study. The three focal preservice teachers described similar experiences at various points in time ranging from the first to the last interview (early October until April).

McKenzie described the difficulty of having her cooperating teacher in the classroom when she wanted to try something new in her teaching. As previously mentioned, Holly entered her senior year having substitute taught for approximately 40-50 days, during which time she was not observed. Wendy described a pivotal moment that happened towards the end of student teaching when her cooperating teacher was out of the classroom. Each of these experiences will now be presented in more depth.

It should be noted that none of the student teachers were claiming that their cooperating teacher should usually be out of the classroom when they were teaching. They spoke of positive outcomes of being observed formally as well as informally and the conversations and growth that resulted from these experiences. However, they all indicated that having some time with their students otherwise by themselves was one of the most helpful experiences for their growth. Each of the three focal preservice teachers' experiences teaching without being observed will now be described.

McKenzie: Teaching centers. McKenzie was similar to her peers in the way she described her feelings about and responses to being observed, as the following interview excerpt indicates. She felt she benefited from not being observed in her initial attempts at teaching, which happened by virtue of leading a small group of students at a center while her cooperating teacher was across the room teaching a different group of students:

> I think for myself, I don't know for all student teachers, but I find for myself if somebody is watching me, I even get a little more timid, and I don't necessarily interact…I feel like I wouldn't maybe have said as much or I wouldn't have directed as much, or I would have been scared that I would have been saying the

> wrong thing or giving the wrong instruction as to how to complete the math problem. So, at least maybe for these first couple of times it was good for me to just kind of go in and see what it was like.

McKenzie explained her perception that being able to teach the centers "on her own" allowed her to interact in a more natural way with her students than had her cooperating teacher watched as McKenzie taught. She explained a tension in student teaching between feedback and freedom: "Feedback from (cooperating) teachers and supervisors would be helpful, but too much would be—I mean, there's got to be a balance between feeling like you're being watched all the time and being able to create your own flow and style in the classroom." McKenzie did not ever discuss a specific instance of teaching without her cooperating teacher present as did the other two focal preservice teachers, but she did communicate that she interacted differently with her students when she was being observed. Even teaching in centers while her cooperating teacher was teaching across the room gave her some increased feelings of calm and freedom.

Holly: Substitute teaching. When Holly was asked to identify the most helpful experience for her development as a teacher of mathematics at the third interview, she said that it was her experience substitute teaching at her mother's school. She described why this was so helpful to her growth by explaining:

> You don't have a (cooperating teacher) to watch you. You don't have a supervisor watching you. It's you and the kids…Sometimes I feel like when…I'm being observed by my (cooperating teacher), I'm more hesitant, and I won't necessarily try something on the spot, because I'm observed, and I think, "What if it doesn't work?" Whereas when I'm subbing, I try stuff… More often than not it will work. And, you know, it's just me and the kids, so I don't feel self-conscious that someone's watching me, you know? So that has been such a good experience for me.

Holly explained that sometimes when her cooperating teacher was out of the classroom she would try new things, such as the day when her cooperating teacher was sick. In the following interview excerpt Holly described what she did in the middle of the literacy block that day:

> At one point the kids were exhausted, so I said, "Alright, close your packets, stand up, do what I do." And we did some theater stuff, we just moved for a few minutes, it wasn't long, but we stretched, and we made funny sounds, and it was just that kind of thing, and then they were so focused. And they came back, and we got everything accomplished. And I thought, "I would not have done that if she were watching me," simply because it's not something I see her doing, and I don't think I'd have the guts to just try it.

This is not to say that Holly always tried new things when her cooperating teacher was out of the classroom, as she explained in the following excerpt, which is also from the third interview:

> The other day she walked out of the room to go to the office, and I was like, "Yes!" I didn't change anything, but I felt like I could breathe easier because I wasn't being watched, and that is the difference. When you sub, you don't have someone watching you. You have that freedom to do with those kids what you want to do: you know, to talk to them, you get to try out with behavior management issues, the way that *you* would handle issues best. You don't have that opportunity when you have someone watching you all the time. That has been so beneficial to me. I look back on that all the time.

Holly had a unique opportunity to teach in multiple settings at an early stage in her preservice training as a result of her mother teaching at a local private school. She found the opportunity to interact with the students and without being observed to be the most helpful contribution to her development as a teacher. Even within her student teaching placement, she reported that she was more likely to try new things when her cooperating teacher and supervisor were not present. Such opportunities resulted in the development of responses to situations that she felt represented her knowledge and beliefs about teaching in authentic ways that may not have surfaced or developed when she was watched by her cooperating teacher or supervisor.

Wendy: Math circle. As was the case with all three of the focal preservice teachers, Wendy's cooperating teacher was more traditional in her mathematics instruction than Wendy. During her cooperating teacher's lessons on the carpet, students sat in rows faced towards the teacher and rarely used manipulatives. In her fourth interview in mid-April, Wendy explained,

> I created recently the math circle, and so I can sit with my kids in a circle, and they can use a lot more manipulatives in a way that they couldn't before when they were sitting in rows on the carpet. (We) all sit in a circle and face one another and talk about strategies, which is much different than what they're used to.

As Wendy described the origins of math circle, she explained that a key aspect was being the only adult in her classroom. The following is her response to being asked what she has found helpful in her development as a teacher of mathematics:

> Never give this to my cooperating teacher because she'll cry, but having her spend some time out of the classroom (has been helpful), and I appreciate having her in the classroom often when I'm teaching, I don't mind it in the way that other student teachers do, but she's quick to behavior manage for me in the way that like a teaching assistant would. And, so for a long time I didn't even realize that maybe not having the kids sit in rows at a carpet with a white board in front of them, trying to fit into like their small assigned box, I didn't even realize that the reason that might be a behavior management thing is because there's a better strategy. Because she just really quickly jumped in and would be like, "Put your board down. Stop doodling." And so when she was out of the room, it brought a lot of those behavior things to light a lot more, because I was managing them. And so then that prompted that critical reflection piece of like, "Why are we having these behavior management issues?" I mean, having her out of the class so that I had to see those behavior management things and then continuing to have her out of the class so that when I made a math circle, she wasn't there to behavior manage that or jump in. She's very supportive, but it was kind of nice not to have her opinion on it, so I could just see how it would work.

Having her cooperating teacher out of the classroom resulted in different dynamics and interactions with her students, which then provided Wendy with different information to reflect upon. In turn, this reflection allowed Wendy to develop her own personalized response to the interactions that she observed, rather than attempting to continue her cooperating teacher's system or having her cooperating teacher or supervisor tell her what to do. Further, Wendy identified that it was helpful for both the reflection phase as well as the implementation phase to use the time and space afforded by her teacher's absence to see how her attempt fared with her students.

Summary. Compared to the other two focal preservice teachers, McKenzie spoke the least about specific instances when her cooperating teacher was out of the room and trying something new as a result. This is not to say it never happened or was not true, since she was never asked directly, but she did not indicate that such an experience was one of the most helpful to her, as did her two peers. However, all three focal preservice teachers described how being observed, even informally, had an impact on them beyond affective measures such as being nervous in the way that it impacted their teaching. All three reported interacting with their students differently when they were in a situation in which they were potentially being watched. That is to say, this did not just apply to formal observations, but also from the sheer presence of their cooperating teacher. The physical absence of the cooperating teacher allowed the preservice teacher to interact with her students in an individualized way that best fit herself, rather than trying to maintain or replicate responses that were more consistent with her cooperating teacher.

Educative evaluation of teaching alone. The three focal preservice teachers' experiences with teaching as the only adult in the classroom will now be evaluated based on Dewey's (1938/1963) conception of educative experiences. His ideas about continuity and interaction will be used as the two criteria, since these are the twin pillars of educative experiences.

Continuity. Having the opportunity to teach without always being formally or informally observed seems to be consistent with Dewey's (1938/1963) conception of educative experiences. In regards to continuity, Dewey described how experiences set up preferences or aversions that are then played out in subsequent experiences and make it easier or harder to act for various ends. The preservice teachers all described how they tended to feel and sometimes act differently when their cooperating teacher was present, even if she was not conducting a formal observation. This reduced range of responsiveness on the part of the preservice teachers is likely to be detrimental in future experiences because it may make it harder to act for more natural ends. That is to say, continuing to develop as a teacher in a setting in which one does not feel she has can fully act seems like it may lead to fixing habits rather than promoting reflective thinking. Holly described how this can be an easy habit to slip into, and how at times she found herself imitating her cooperating teacher "even for aspects of her teaching I don't like, and then I find myself doing it." Perhaps having some more time in which preservice teachers can truly experiment with teaching would provide more expansive ends rather than fitting their habits to imitate their cooperating teacher or respond in ways they feel she would approve.

Continuity also incorporates growth in mental processes. Occasionally teaching without another adult in the room seems like it would increase, or at least not diminish, a preservice teacher's mental processes in the form of reflection which may happen in a wider scope than when her cooperating teacher is present. This was the case for Wendy who described her cooperating teacher's absence as leading to "that critical reflection piece" that probably would not have occurred in the same way had her cooperating teacher stayed in the room to manage aspects of the lesson from off to the side. Another way that Dewey (1938/1963) described growth associated with continuity was that it strengthened initiative. This maps onto the experiences of the preservice teachers when they were the only adult present, in that at times they demonstrated a wider range of initiative than they would have had their cooperating teacher or supervisor been present. The fostering of increased initiative is indicative of an educative experience.

The claim that an acquisition of skills in the present will necessarily translate into future application of these skills under new conditions relates to mis-educative experiences in regards to continuity. Although it is not possible to entirely replicate the preservice teachers' future classroom, one of the most significant differences compared to student teaching is the almost constant presence of the cooperating teacher. As Dewey (1904/1964) wrote, educating a teacher in such a setting is akin to "learning to swim without going *too near* the water" (p.317, emphasis in original). One assumption in teacher education is that what a preservice teacher does in someone else's classroom should transfer to a different future setting, namely that of their future classroom. Although the majority of characteristics of their future setting are not known, one thing that is known is that the preservice teacher will be the lead teacher a short amount of time from when the teacher education program ends. Occasionally having a more authentic replication of the future setting may help diminish the assumption that what a preservice teacher can or will do in her own classroom is exhibited when she is being watched in someone else's classroom. This is an additional reason why having occasional opportunities to teach without being watched maps onto an educative experience.

Interaction. Along with continuity, the interaction that takes place, or is likely to take place, within an experience must be considered when discussing the educative potential of an experience. Dewey's (1938/1963) two main criteria for interaction are whether experiences occur in a thoughtfully planned environment and whether there are opportunities for positive social interactions. The criterion regarding the environment relates to teaching in a classroom setting that would be more authentically representative of future conditions by the occasional absence of the cooperating teacher. The classroom setting in the student teaching placement, as opposed to in any other classroom or school, means the preservice teacher's interactions are as potentially meaningful as possible since they are interacting in a familiar environment with their own students. Although it is still not "their" classroom, alternatives of working with any other group of students in any other setting would reduce the amount of positive interaction that would be possible in the given environment. Therefore, occasionally teaching when no other adults are present in the student teaching placement is fairly educative in terms of the environment, so long as it is realized that the preservice teachers do not have full reign of the environment in which they are interacting by virtue of still being in someone else's classroom. However, preservice teachers move somewhat closer to interacting an authentic environment by occasionally being the only adult in the classroom.

The second aspect of interaction in regards to educative experiences is the degree to which there are opportunities for positive social interactions. Similar to the previous analysis of re-teaching, teaching in a room as the only adult present is entirely a social experience because it happens between the preservice teacher and her students. Furthermore, it potentially increases the authenticity of the social interactions. Wendy made an insightful comment about this in her fourth interview, when she was discussing implementing her math circle without the presence of her cooperating teacher:

> My kids have a great relationship with both my (cooperating teacher) and myself, I would say. But, I mean, there's something to be said for the way they respond to me. Like I don't know that my (cooperating teacher) could sit all the kids down in a circle and have a meaningful conversation, just because of her personality and the way they mesh. And that's not the fault of the children or the teacher. But I think there's just that interplay of personalities that you don't fully realize when everybody's in the room, when all the adults are in the room.

It can be concluded that because of the more authentic social interactions that result for the preservice teacher, occasionally teaching when the only adult present is educative in regards to social interaction.

Summary. Occasionally teaching as the only adult present matches Dewey's (1938/1963) conception of an educative experience based on the criteria of continuity and interaction. Occasionally teaching as the only adult present exemplifies continuity through allowing for more expansive ends, increased initiative, and a reduced discrepancy in the training environment to the future environment. It also incorporates interaction because it transpires in a meaningful environment and with authentic social interactions. All of these aspects seem to make occasionally teaching as the only adult present both helpful to the preservice teachers as well as an educative experience.

Teaching experiments. The first two helpful and educative experiences that emerged from the data were re-teaching and occasionally teaching as the only adult in the classroom. These are two separate examples of educative experiences that can also be grouped together under a common heading. These two experiences can be understood as examples of the preservice teachers finding ways to engage in teaching experiments in their student teaching placement. Both re-teaching and teaching as the only adult present allowed the preservice teachers to have experiences closer to Dewey's (1904/1964) ideas of the laboratory approach, in which he believed preservice teachers should treat the classroom as a laboratory in which teaching experiments should occur. Both re-teaching and occasionally teaching as the only adult in the classroom promoted interactions between the preservice teacher, her students and the subject matter as her focus rather than interactions with the cooperating teacher or supervisor. Both resulted in more authentic experiences that promoted the synthesis of experiences with students and subject matter, their own ideas, and reflection and critical thinking. Re-teaching the same lesson and occasionally teaching as the only adult in the room are examples of how teaching experiments can occur in field placements.

Participation in the study. The third category all three focal preservice teachers identified as one of the most helpful experiences for their development as teachers of mathematics, and the most surprising, was their participation in this study. In addition to the three focal preservice teachers identifying study participation as a helpful experience, the other eight preservice teachers who participated in the focus groups also reported participation in the study as one of their most helpful experiences. This section will first report the overall feedback from the eleven preservice teachers who participated in the focus groups regarding the helpfulness of the meetings. Second, a synthesis of the three focal preservice teachers' responses in regards to the focus groups will be presented. Third, the overall feedback from the three focal preservice teachers who participated in the video observations will be reported. Fourth, the three focal preservice teachers' responses regarding the observational aspect of the study will be presented. Last, the focus group and observational aspects of the study will be compared to Dewey's

(1938/1963) conceptualization of educative experiences. Given the amount of data that exists and how strongly the preservice teachers felt about their experiences in the study and offering subsequent experiences to future cohorts, more space will be dedicated to discussing this experience than the previous discussions of re-teaching and teaching as the only adult in the classroom.

All eleven preservice teachers who participated in the focus groups indicated that the four focus group meetings that occurred over the course of the year were one of the most helpful experiences in their development as teachers of mathematics. It should be noted that this was brought up in each of the two focus groups as an answer to a broad question asking what were their most helpful experiences. That is to say, in each of the two focus groups, preservice teachers were not asked specifically about the study but offered these comments on their own. As one preservice teacher said of the focus group meetings, "There's something of value coming together with people from your cohort outside of school." The following list denotes more specifically the aspects of the focus group meetings that the preservice teachers indicated as helpful.

- The opportunity to get out of their self-reported "student teaching bubbles" by hearing about the experiences of their peers and sharing their own experiences with their peers.
- The opportunity to reflect on their own experiences and philosophies about teaching mathematics.
- Having sustained discussions in small groups over the course of the year with the same group of people.

- Engaging in discussion focused on one specific content area in some depth rather than talking about teaching generically across subjects.
- Increased awareness of mathematics teaching across different grade levels, schools, districts, and curricula.
- Framing discussions with a positive tone by focusing on participants' growth and improvement.
- Enjoying the informal atmosphere of talking off campus, without assignments, over a meal.
- Facilitation by a graduate student so that conversations were more open and low-stakes.
- Feeling better supported by and connected to the School of Education.

Preservice teachers identified all of the above aspects of the focus groups as helpful but given the extensive amount of data, the next section will focus on the paramount reason they discussed. Upon analyzing the responses of both the three focal preservice teachers and the additional eight preservice teachers who participated in the focus groups, the main reason both the larger and smaller group of preservice teachers described the focus group meetings as helpful can be summarized by stating it was the feeling of connection that resulted from the meetings. The discussion of connection will be reported through more specific forms of connection including personal connection, connection leading to support, connection in the form of idea sharing, and connection to the university-based program. Each of the four aspects of connection that developed from the focus group meetings will now be discussed through the three focal preservice teachers' focus group and interview responses. Additional comments from the other eight participants will also be used.

Personal connection. One aspect of the focus groups that several of the preservice teachers specifically commented on was the personal nature of the groups. In a program that over the past five years has incorporated more on-line requirements via interfaces like Blackboard, Sakai, and blogs, the preservice teachers in the focus groups appreciated the personal connection of face-to-face discussions. As Julia, who was student teaching in second grade, explained,

> I've been thinking about this a lot…It would have significantly changed my experience to not have done this [participated in the focus groups]. Theoretically articles and blog posts are great, but teaching is such a solitary profession anyway… This [participating in the focus group] is the most meaningful, and if we could get our supervisors and everyone to see that, it would change the outcome, you wouldn't come out of student teaching bogged down by little things.

Claire, who was student teaching in kindergarten, expressed strong feelings about learning from her peers through face-to-face discussions:

> I have no idea what the other student teachers are doing, and I don't see why I shouldn't know what they're doing in other schools, in other districts, it would be interesting to see and to expose us. Just going to visit those schools is not really that helpful. Having discussions is helpful. And being able to sit down and talk about it, not just write on-line.

Both Wendy and Holly, as well as other participants in the larger group, described student teaching as a "bubble." They felt that coming together with their peers at the four focus group meetings was a welcome change to their generally isolating routines in their classrooms and schools. At the fourth focus group Wendy explained,

> It can sometimes be frustrating to be in a bubble and be isolated from your peers. There are 70 of us going through this experience and I see only one or two of them on a regular basis. It's a lucky day when we can interact as student teachers, and come and sit with people that I haven't seen and who are going through a very similar experience, but not an identical experience. So it's helpful to see where everyone is. I just thought those (meetings) were really nice.

Holly also expressed the idea of the focus groups providing a way to overcome the isolation she usually felt in her student teaching placement:

> You're in your own bubble. We're lucky if we have two or three other student teachers at our school, and for me they're not in my grade. So we don't get to talk to anyone and we never see each other…It's good to know I'm not the only one experiencing some of these issues or having these hesitations or freaking out over these little things that I think are little but they're not to all of us. And it's nice to be able to see where everyone is. Because you wonder. It's good to be able to come together and realize we're all at the same place and we're all having the same challenges and experiencing the same thing. And it gives you peace of mind.

Both Wendy, Holly, and the majority of focus group participants expressed that the focus group meetings diminished the isolation of student teaching by coming together with their peers to hear and share experiences. One way they described this as helpful was in regards to awareness of their peers' experiences and the similarities that existed between experiences and challenges that they were going through and could relate to. This awareness helped the preservice teachers feel less isolated.

Connection in the form of support. The peer connection that was generated by the focus group meetings took on an element of not just awareness of their peers, but also became a support structure for the preservice teachers. In her final interview, Holly explained that the focus groups were "hugely helpful" because of the support she gained from participating:

> Because it's helpful to see, not only where everyone is…but to have that support system is really nice. And to be able to come together and those focus groups, I

> felt like I walk out of there like, "Okay, I'm doing things right." You get that feeling which you need in order to get through this semester, because it's rough. There are times when you just want to cry. And so, those have been so important, and I've looked forward to every single one. And my semester couldn't be the same if I hadn't had those groups, I can guarantee it. It wouldn't have been the same.

Holly's student teaching placement was at a school with two other student teachers who were in different grade levels than her. When they did see each other, which happened rarely, she explained that the conversations they had,

> were not about school, so we don't get that support within the school, and we never talk about our lessons or anything like that. So I look forward to be able to go (to the focus group meetings), and I know, "I'm going to see Claire and McKenzie, and we get to talk about how we're doing," And even though it was just a couple of times, I really looked forward to that. And it helped me get through the teaching because I knew that I'm going to hear what they're doing. I get to talk to them, see how their interactions are with their (cooperating teacher), because it was about the math, but it wasn't just about the math at those focus groups. And you learned about the math and our math teaching, but it was also how you interacted with your (cooperating teacher) and how things were going with your students. What methods are you using to fix this…And so it was time to be with friends and talk about your teaching that I didn't have at my school.

For Holly and others, the focus group meetings were helpful not only to see her peers and learn about their experiences and challenges, but also for the sense of peer support that these interactions generated. Holly found this sense of support helpful to face the stresses and challenges associated with student teaching.

Connection in the form of idea sharing. In addition to being more generally aware of each other's experiences within student teaching and feeling supported through similar experiences, challenges, and feelings, the preservice teachers also reported that the focus groups were helpful because of the specific things people discussed. This was true even though the two focus groups were comprised of preservice teachers who were student teaching across grade-levels and ranged from kindergarten to fifth grade in one group and first through fourth grade in the other. As Holly explained,

> It was nice being with teachers from other grade levels because Wendy is in first grade, Amanda is in a lower grade, and then there were a couple of us in third, fourth, fifth. They make me think about my own classroom. The whole time we're sitting there, I always think of my own classroom and how I can use it or have I seen that or how I relate to that.

Holly felt that the time to reflect on her classroom and experiences as compared to what her peers shared was helpful. Wendy offered a more specific example of how her peer's comments were helpful to her reflecting on her own classroom in her final interview:

> When I hear about other people's struggles or successes, it reminds me to think about those components. Because, for example, I became really concerned with assessment, and so now I still consider assessment pretty frequently, but I never considered motivation, which Molly discussed. And so it reminds me to consider motivation, even though it's not an issue for me. It keeps me from getting bogged down or stuck in my own isolation.

The comments shared at the focus group made Wendy aware of not only her peer's experiences, but also specific aspects about teaching that she could then reflect upon in her own teaching. Even more specifically, several participants cited times when a peer's comment offered a new idea for them to think about or try in their student teaching placement. Wendy explained how her peer who was student teaching in fourth grade shared an idea that impacted her interactions with her first graders:

> I can think really vividly about Molly's comments about how her kids were really bored with the work that she gives them. My kids don't seem to ever have a problem or take issue with worksheets out of that math workbook, but hearing that her kids really do, it just reminds me, "Oh, not all kids are my kids, and not all kids are first graders." So hearing about children's reactions was really helpful. That one in particular is really vivid to me, and it reminds me every day to not—I mean, my kids won't advocate for themselves because they're six. So, it reminds me to advocate for them to myself.

The comment Molly made led to reflection for Wendy not just in that moment, but provided a specific consideration to regularly reflect upon as she continued student teaching. Holly had a similar experience when Wendy shared about re-teaching the previously discussed lesson that "bombed." In her final interview, Holly explained what she thought as she listened to Wendy describe the first attempt at the lesson at the focus group the week before:

> Immediately I thought, "Yeah, I've done that." And then when she said she taught it again the next day, "I didn't do that." So, I thought, "You know what? There's no reason why you can't do it again and give it a second chance. If it doesn't work, tweak it, and try it again." And I didn't think about that (before). And I liked that. But you know that's something that I would try if a lesson bombs, and if I'm confident enough that there's something in that lesson that I really think is important, I would absolutely re-work it and try it again. But I would never have thought of that.

In addition to providing ideas to reflect upon or try in the future, sometimes participants did take an idea that one of their peers stated during a focus group and tried to implement it in their student teaching placement. For example, Holly was in a fourth grade classroom and wanted to introduce math centers in her placement classroom, even though she was in an upper grade and centers were generally only done in grades K-2. She explained that her idea to try centers originated with a comment at a focus group from a peer who was student teaching in a third grade classroom that used centers:

> The whole centers thing started because Amanda mentioned centers, and right when she said that, I was like, "Oh, they do centers in math! There's no reason why fourth graders can't." And that was the whole reason why I started this center thing. It's all because of that. I wouldn't have thought of doing that. I had thought about it, but I would have never actually done it or considered doing it.

The preservice teachers indicated the focus groups were helpful in the specific ideas shared that they reflected upon during and after the meetings, some of which led to an idea they implemented in their student teaching placement or something they indicated they may try in their future teaching.

Connection to the university. While Wendy, Holly, and the majority of participants described the connections the focus groups generated in terms of support and idea sharing with their peers, McKenzie and some other participants expressed that the focus groups made them feel more connected to the university elementary education program. As Kate explained,

> I think it's important to have support when you are student teaching from the college level because it is a lot to take on and it is an awkward stage. Conversations like this are so much more meaningful…getting together, supporting each other, learning together, and sharing experiences.

The feeling of connection to the university was a sentiment McKenzie voiced during both the focus group and her interview. She stated,

> It felt like every time we came together in a focus group, I felt like it was another arm of connection to the School of Ed. Obviously this (study) is a separate thing, but still it felt like a connection in that somebody was making me really think about what I was teaching in math. I'm thinking of science—if somebody did this for me for science, my science teaching probably would have been a lot better this semester. And so, whether that's true or not, that's how I felt.

Summary. The most unexpected result of this study was that the study itself functioned as a vehicle for helpful experiences in which the preservice teachers believed they experienced growth as mathematics teachers. All eleven preservice teachers reported that the focus groups were a key experience. The main way they described the helpfulness of the focus groups was through the idea of connection: participants felt the focus groups provided a venue for needed personal connection, connection that lead to peer support, connection that lead to idea sharing, and connection to the university-based program.

Study teaching observations. In addition to the focus groups, four of the eleven preservice teachers completed the video observation portion of the study, including the three focal preservice teachers. The four participants all reported that the video portion of the study was helpful in ways beyond the focus group meetings. As Holly explained,

> Whereas the focus groups were you reflect on your teaching in a group, and you got feedback from other people, the observations are more self-reflection. So I think they're both really important, but in different ways.

The preservice teachers identified multiple ways in which the video observation portion of the study was beneficial. Given the extensive amount of data, the next section will focus on the paramount reason they discussed. Upon analyzing the responses of the three focal preservice teachers, the main reason they described the video observations as helpful surrounded reflection. The discussion of reflection will be reported through increased reflection and math-specific reflection. Each of the two aspects of reflection that was generated by the video observation portion of the study will now be discussed through the three focal preservice teachers' focus group and interview responses.

Increased reflection. The three focal preservice reported that the increased reflection caused by the questions and conversations in the focus group meetings and post-observation interviews was helpful in their development as teachers of mathematics. As McKenzie said, "Thinking about what (the researcher) is actually asking forced me to be more reflective. I can tend to not be reflective sometimes, and it's definitely made me more-so." Specific to the post-observation interviews, Wendy commented that they helped her reflect more on her mathematics lessons than she would have otherwise:

> The thing that I think is really nice, especially in the beginning (of teaching math), I would get through a math lesson, and just be like, "Okay. Well, that's done. Everybody go recess," and I would go to recess, and I would mentally go to

> recess. And then if I reflected on that lesson, great, but I probably wouldn't in a very meaningful way. Maybe on my drive home, I'd be like, "Man, I can't believe so-and-so was flopping around on the carpet. I'll have to be harder on them tomorrow." But in like a really true way, I just probably wouldn't have done it.

Having the post-observation interview immediately following the lesson provided an additional opportunity for the preservice teachers to engage in reflection, which they all said resulted in more reflection than they would have done on their own. The three focal preservice teachers reported that it was helpful to have the focus of post-observation interviews on one specific lesson, and even more specifically, particular aspects of that one lesson. This was an idea that Holly described in the following excerpt from her final interview:

> Because of those (post-observation) interviews, (the researcher) asked a question, and I had to answer it. I had to reflect whether I wanted to or not. And so it forces you to think about one lesson. At the end of the week, you're like, "Okay, how was math this week?" You don't always ask yourself, "How was math Tuesday, how was that one activity?" You don't always ask yourself that. I mean I tend to, but not always, no, not always. And so, when (the researcher) asked us about those *individual* things—how do you think this part of a lesson went, and this part went? That's important, because we can't group math as a week of lessons because it's not. And we may have had four great lessons that week, but if one of them bombed, we need to think about why that bombed, and how we can improve it…it's forced me to think about each lesson as an individual lesson. I don't group math or literacy—I don't group it together anymore. Everyday I think of every lesson and how they can be improved.

One aspect of the video observation portion of the study that the preservice teachers found beneficial was the specificity of the post-observation interview that caused them to reflect upon their mathematics teaching for a specific lesson, as well as specific aspects of planning and enactment within a particular lesson.

Math-specific reflection. In addition to an increased amount of reflection, one area of reflection that the three focal preservice teachers indicated as helpful was through the use of specific questions that focused on teaching mathematics, as opposed to generic aspects of teaching. When asked about helpful experiences, McKenzie responded that the study helped her distinguish between things that are particular to teaching mathematics and others that are not:

> Reflection has been good, like when (the researcher has) been asking us to answer these different questions, distinguishing between classroom management and behavior, and then lesson planning and how it relates to math.

One reason why the observations and post-observation interviews were cited as helpful was although university supervisors conducted weekly observations of each preservice teacher, these were spread across the entire school day which meant preservice teachers were rarely observed teaching mathematics. This was the case with Wendy, whose supervisor only observed her teach one-half of one mathematics lesson all year. In regards to the video observation aspect the study, she said:

> I actually really appreciated (the researcher) observing. (My supervisor) really only ever saw me teach literacy, and so in conversations with her, we only had conversations about literacy, which I feel very comfortable with and don't really have any qualms with and never did. And obviously I can't anticipate what it would have been like if (the researcher) hadn't been there, but I can only think that it was immensely helpful.

Wendy also described how participating in the post-observation interviews helped her reflect not just more, but how it helped her reflect more specifically about her mathematics teaching in a way that was unique from how her university supervisor approached post-observation conferences. Her description was similar to the other preservice teachers' and is worth quoting at length:

> I think this study has been really helpful. It's definitely a different kind of reflective practitioning than I do with my supervisor, because when we sit down with her it's like, "What went well in this lesson, and what didn't" in terms of strategy and theory and the execution of a lesson. So the questions are essentially

> the same for a guided reading lesson, or a math lesson, or a social studies lesson, because to her it doesn't matter. She's coming at it as a "good teaching is good teaching" mindset. The questions (the researcher) asks are different in the sense that (the researcher is) like, "How did shape combinations affect your lesson?" And I'm like, "Ohhhh, there's a theoretical basis for math teaching." And not theoretical in the Vygotsky and Piaget sense. And the questions that (the researcher) asks us, the reflective questions are so different from the ones my supervisor asks, so I often find myself in my conversation with her being like, "I should have had the papers ready. I should have done this. I should have done that." And that's not your goal. So it really challenges me to switch gears and not think about the behavior management. My supervisor is very focused on behavior management. (The researcher) really challenge(s) me to get outside of that and think about it in a more conceptual way, like, "Why did we teach that to kids? How are kids learning that? And what do I think about that?" I think it's very helpful.

The generic way supervisors interacted with preservice teachers void of consideration for the specific content area they observed, with some exceptions for giving specific literacy feedback, was a typical comment that came up repeatedly across the focus groups and interviews. The post-observation conferences were deemed helpful because they engaged the preservice teachers in math-specific reflection that did not happen otherwise. Holly made a similar comment stating the value of the post-observation interview that focused on math-specific reflection:

> I think (the researcher is) the reason why I'm so reflective, because we talk after the lessons. In many ways for me, it's been more beneficial than my supervisor coming in. Because (the researcher) ask(s) questions and we are forced to answer them. When it's just one-on-one with our supervisor, at least for me, my supervisor says, "Great, great, great, this you could work on, but it was great" and then that was it, and I don't really say much. No questions are posed to me, really. I usually try to jump in and say, "Well, I thought this was bad" and tell her stuff, but there's not much discussion. If I'm lucky I have ten minutes to talk to her, if any, usually nothing after an observation. So when (the researcher) observes I actually get to talk out what I just did, and it forces me to think about it, and (the researcher) ask(s) questions that are specific so I have to actually think about what I said and how I said it and how it went, and, at least personally, that's not something I get to do with my supervisor, so for me it's more beneficial having (the researcher) come in and having those observations than coming in and reading (my supervisor's) observations on a piece of paper.

One aspect of the post-observation interviews that all three focal preservice teachers indicated was helpful was the math-specific reflection that they were asked to verbalize in order to answer the interview questions (see Appendix G). Although follow-up questions specific to the content and lesson at hand were asked, it is interesting to note that the interview questions were still fairly broad and general, yet the preservice teachers felt these questions were noticeably more focused on mathematics teaching than what their supervisor asked. This was an idea that McKenzie explained:

> When (my supervisor) came to observe me, she was focusing more on the big picture of teaching, rather than like mathematical teaching. So her comments weren't necessarily, "The way you described this concept was good," or "Maybe you could have changed your wording in this concept." It was more like—"There were a couple boys that weren't paying attention, how can you engage them more in the lesson?" It was more of those types of things than mathematical knowledge. But I feel like when (the researcher) came to observe me, we did talk about those things.

Another important aspect of the post-observation interviews that should be pointed out is that the researcher did not give feedback or make evaluative comments on the lesson afterwards, but rather only asked questions as to what happened during the lesson and why the preservice teacher made various choices in lesson planning and execution. Therefore, although more specific conversations were had about the mathematics teaching and learning that took place in the lesson, it was not done in the form of evaluation or suggestion. Rather, the preservice teacher was asked to explain what happened, what she tried to do, and what she thought about the various aspects of the lesson. As Holly explained,

> This with (the researcher) has been hugely beneficial to me, and I think a lot of it has to do with my experiences with my supervisor haven't been, just because I don't really ever see her. So, really, when you come in is the only time I get to *talk* about my lesson. I talk to my teacher about it, but she doesn't really care to

> listen. And I need to talk things out. So, it's you or my mom. And my mom was tired of hearing about it. So, that sounds so stupid, but it's good for me to be able to talk it out like that, so these (post-observation interviews) have been so important for me.

Preservice teachers reported low amounts of time being observed by and dialoguing about teaching mathematics with their supervisors, ranging from zero to three lessons over the year with a mode of one. Given the relative absence of math-specific reflection that occurred with the university supervisor, as reported across all eleven focus group participants, this was another way in which the present study functioned as a helpful experience.

McKenzie explained how the absence of feedback from anyone else and her own lack of specific reflection left her to duplicate her previous instructional attempts regardless of their effectiveness. Therefore, she thought the way the post-observation interview asked about specific instructional decisions and moments was helpful in her growth as a teacher:

> Because if nobody's telling me what specific parts were *good* or if I'm not going back and reflecting and say that *specific* thing was good in my lesson, I'm just going to keep on doing lessons without really any purpose or direction, not knowing which specific things were effective and which specific things were helpful. You might look at a whole lesson, and say that lesson was good. But maybe there was one thing that wasn't effective, but you thought the whole lesson was good. And so you would still use that thing because you thought the lesson was good, but really that one thing wasn't very effective.

McKenzie reported that the post-observation interview helped her learn to reflect upon her teaching in more specific ways than generally across a whole lesson. That is to say, she identified smaller areas of her teaching to reflect upon as a result of participating in the study.

In addition to being more reflective about practices, Holly also reported that the video observation helped her try new things in her teaching. In addition, she said the combination of being video taped and the specific questions following her lessons helped her become more aware of what she does and says during her lessons throughout the day, not just in mathematics:

> Because of being observed for the study I've been able to see how I've grown over the course of the semester in math, and I've been able to focus on different ways of teaching math, and I don't think I would have been as risky at trying some of the things I've tried if I had not been observed and videotaped. I think it's been helpful because it forces you to think about what you're saying and how you're saying it, and how you're being perceived by these kids, because you're on camera and it makes a difference. And for me that's played over into all of the lessons I teach because now it's like second nature; I'm always constantly aware of what I'm saying and how I'm saying it because I did that when you videotaped. So it's been helpful for me.

Holly described the benefit of the study observations not only in terms of her mathematics teaching, but also in other lessons due to her increased awareness of more specific aspects of teaching. When asked about reflection and whether it carried over to lessons other than the four observed for the study, the three focal preservice teachers indicated that it did, including Holly who explained:

> Oh, yeah. Because the same questions that you would ask, I asked myself. I mean, (the researcher) always says, "Well, how do you think it went?" and then specifics. So, "Introduction, how did that go? How did the centers go?" And now like whenever I teach a lesson, I think of them as separate entities, so that helped me. That helped me grow, because I think about now—I want to improve my centers, I want to improve my introduction, I want to improve how I taught the meat of the lesson. Like I think of them, I don't just think of the whole lesson, I mean obviously, the flow's important and how it fits together is important, but I look at it as different pieces. And in literacy, too, I think about it now in different areas, not just math.

Asking questions that caused the preservice teachers to reflect not just on teaching in general, or mathematics teaching more specifically, but about specific parts of the lesson or moves they made within the lesson was an aspect of the post-conferences that the preservice teachers consistently reported as helpful in their developments as teachers of mathematics. Additionally, being observed resulted in their increased awareness of what they said and did. Participation in the observations and post-observation interview portion of the study helped the preservice teachers improve on their specificity of reflection, which they and then applied to other lessons besides the four that were observed and other content areas besides mathematics.

Educative evaluation of study teaching observations. The eleven preservice teachers' experiences with the focus groups and the three focal preservice teachers' additional experiences with the video observations and post-observation interviews will now be evaluated based on Dewey's (1904/1964) conception of educative experiences. His ideas about continuity and interaction will be used as the two criteria, as these are the twin pillars of educative experiences.

Continuity. Participation in the focus group was consistent with Dewey's (1904/1964) conception of educative experiences. The focus group meetings were, as it turns out, a solid embodiment of the spirit of continuity because the preservice teachers met with the same peer group multiple times over the course of the year. There was a sense of not just answering questions for the study but rather participating in an on-going dialogue with each other about their experiences. The way the focus groups resulted in peer support also is indicative of the continuity of the experience. As Dewey (1904/1964) explained, one outcome of a continuous experience is that it "arouses curiosity, strengthens initiative, and sets up desires and purposes that are sufficiently intense to carry a person over dead places in the future" (p.31). The focus group participants expressed that the peer support was helpful in overcoming some of the "dead" places in student teaching, namely the isolation and stress. The focus groups offered a continuous experience that allows the experience to be considered not only helpful but also educative.

There are additional ways in which both the focus groups and observational portion of the study increased the continuity of the preservice teachers' experiences. Another way Dewey (1904/1964) described continuity was that it functions as a means of carrying an activity further and towards positive ends. The way the preservice teachers reported their individual outcomes from the focus groups in terms of trying new things in their classrooms based on the ideas their peers shared is an example of how they carried mathematics instruction further than they reported they would have on their own. This was also true of the observational portion of the study, in which preservice teachers reported trying new things as a result of their increased focus and reflection on mathematics instruction, which they reported exceeded the amount of reflection and resulting ideas that would have occurred without the observations.

Another key aspect of continuity is the contribution to growth in knowledge and mental processes. Participants explained that two of the ways they benefited from the focus groups included increased awareness of their peer's experiences (growth in knowledge) and increased reflection (growth in mental processes). For the observational portion, preservice teachers benefited from even more growth in metal processes, as they reported ways that they increased the amount of reflection they engaged in and also how they grew in the way they reflected as a result of the study. This coincides with Dewey's (1938/1963) conception of continuity contributing to the restructuring and reorganizing affect of experience, as the three focal preservice teachers all reported how they developed in the way they reflected as a result of participating in the post-observation interviews.

Considering Dewey's (1938/1963) description of mis-educative experiences further illuminates the educative nature of the observational portion of the study. Mis-educative experiences that suffer from low levels of continuity increase skills and fix habits but reduce sensitivity and responsiveness. The preservice teachers indicated much the opposite result from the post-observation interviews, again because they became more aware of their actions and practices. This allowed them to increase their self-control in regards to future experiences with mathematics instruction. Participation in the study resulted in unique experiences that were consistently continuous and educative.

Interaction. Along with continuity, the interaction that takes place, or is likely to take place, within an experience must be considered when discussing the educative potential of that experience. The two main criteria for interaction are whether experiences occur in a thoughtfully planned environment and whether there are opportunities for positive social interactions (Dewey, 1938/1963). The criterion of the environment relates to the focus group in terms of the setting in which the meetings were held. As previously explained, both the individual interviews and focus group meetings were held after school, off campus, in local restaurants. The focus groups were always held over dinner. Although originally this was done as an incentive to encourage participation, it became one of the contributing factors of the meetings. At the last focus group meeting, the preservice teachers reported their appreciation for having the meetings in an informal, comfortable location, including the three focal preservice teachers.

During the last meeting the preservice teachers started talking about how it would be beneficial to continue something similar in the future for the next cohort of preservice teachers, to which Holly said:

> I think it would actually be really worthwhile. I'll leave school and my (cooperating teacher) will say to me, "Oh, you have to go to another one of those interviews?" like she said today and I'm like, "Yeah, but it's a dinner and I'm looking forward to it!" Never *once* have I looked at these or our one-on-one interviews as something that I dread or that I'm not looking forward to. I don't leave school and think, "Ugh, I have to go talk for an hour about math." I've never thought of that. Whereas I left school at 3:30 today and thought, "I have to go sit through seminar." And that, I didn't look forward to. Part of it is this is so informal, and you can sit and have dinner, and talk to your friends about your experiences, and what's working, what isn't working. So if they could form groups of five or six people, and say twice a month you get dinner or once a month, or even meeting for coffee if (the program) can't afford dinner, I think it would be really worthwhile for them.

Following Holly's comment, Wendy and McKenzie commented about the focus group environment:

> Wendy: I like the venue. It's a comfortable place. I like that it's not in a classroom, which is not to say that it wouldn't be beneficial, but I think I'm more open minded because we're not in an institutional setting. And we're sitting in real sized chairs. And when I go to seminar sometimes I'm like, "This is going to be so helpful but I have been sitting in a chair that is six inches off the ground all day today and it's the last thing I want to do right now." So I really like the environment. So in the future, keep them here.
>
> McKenzie: I'll say the same thing. Our seminar is in the school so there are intercoms going off, and teachers trying to pack up their things, and today my brain was like--I was just done, I had been in school all day, and I couldn't get into it. Even right before I came here I was like, "I have to make my brain work," but I don't feel that way now. I don't feel stressed out right now.

From these comments, which were consistent with the other eight preservice teachers' comments about the meeting locations, it was evident that the preservice teachers would concur that the experiences took place in an environment that further promoted social interaction and growth than would have been the case had the meetings been held on campus or at a local school. In regards to the observational portion of the study, the participants' teaching took place in their student teaching placement, as opposed to in any other classroom or school. This means the preservice teachers interactions are as potentially meaningful as possible since they were interacting in an authentic and familiar environment with their own students.

The second aspect of interaction in regards to educative experiences is the degree to which there are opportunities for positive social interactions. The focus group meetings were saturated with positive social interactions. A secondary result that is interesting to note was that at the first focus group meeting, preservice teachers' generally talked only to the researcher. That is to say, the researcher would ask a question and then nearly every participant would answer the question. The preservice teachers did this while speaking to the researcher and rarely acknowledged what the previous person had said before starting their answer. Each of the four times we met, however, the preservice teachers interacted more with each other. They increasingly responded to each other's comments and asked questions of each other. In each meeting the researcher spoke less and the participants engaged in sustained conversation with each other. These meaningful peer interactions certainly embody the spirit of Dewey's (1938/1963) conception of social interaction as a means of growth.

Although the majority of interactions occurred between the preservice teachers themselves, of course it must be acknowledged that the researcher's presence had an impact on the experience. As the preservice teachers talked about how to continue the study for the next cohort of students during the final meeting, they discussed the presence of the researcher at the focus groups. The three focal preservice teachers discussed the researcher's presence in the following way, making these three comments in succession:

> Wendy: There's something to be said for having (the researcher) here, to guide the conversation, and focus it, because if there were a student leader—not to say that we wouldn't be able to handle it—but it *could* become really mechanized. Like, "Alright ladies, let's get through this." And I think, (the researcher's) perspective on it, and knowing what you're looking for and the nuggets of conversations that you want, and you know when to move us along versus if you gave us this list of questions, we would either answer every single one, in good true to form elementary ed fashion, and probably just go around the table, and that would be that. Or we would not answer any of them. So it's nice you are here and it's also nice you are not our supervisor. Because if (my supervisor) was sitting here asking this same set of questions I would be like, "Uhhhhh" (nervously). It's just a different expectation when your actual supervisor has that kind of pull on you. So it's just different with you.
>
> McKenzie: I was just going to say the same thing. I realize (the researcher is) *a* supervisor, but you aren't *my* supervisor. And I feel like this doesn't have anything to do with being a supervisor. This is your study, you're a student, and for some reason I feel like I'm aiding you. I don't know if there is a way to have all the doctoral students leading the focus groups? It has more meaning for me knowing that it's not for a checklist, but that you're going to do something with this."
>
> Holly: I was going to say the same thing about possibly using grad students to do this. Because you're also in school. So you're a student just like we're a student. And you've also been a teacher more recently, too, than our professors, some of whom have been out of the classroom for twenty years, so I feel like you can relate more. And especially if this were like a dinner group, and not just towards math, I feel like you could relate to what we're talking about and give us advice and just talk to us on more on an even keel than our supervisors, whereas there is something at stake there when you're talking with your actual supervisor and you're constantly wondering, " Am I saying the right thing?" And you have to think about it. I hate having to think before I speak, which I should worry about. It's just nice to be able to say whatever I'm thinking and not have to worry about it. So it would be nice if it were with a grad student or someone like that, and someone we met with consistently, throughout the whole school year, starting in the fall.

All of the focus group participants felt the presence of the researcher helped facilitate reflection and growth, and therefore added to the quality of social interactions that took place during the meetings.

The observational portion of the study incorporated positive social interactions in a different way than the peer interactions of the focus group. Here, the social interaction occurred between one participant and the researcher. As previously noted, the researcher did not make evaluative comments about the lesson. McKenzie discussed this at her final interview, stating:

> I would definitely say that a huge part of this being a helpful experience, like the fact that I haven't been stressed out by this study or like dreading this study is that—I mean, I would say (the researcher) has been successful in like staying neutral, staying like laid back and non-forceful; I feel like that's a big part of it. If I felt like there was like pressure or this was high stakes, I probably wouldn't have been as willing to share all this information or be as vulnerable with my answers as I have been. I mean, if I felt like you were going to criticize my lessons, which, I wouldn't say that my lessons are all that stellar, but it's nice to know you won't. And you've helped me self-monitor myself, and I do that now independently of you helping me.

McKenzie expressed that the researcher maintained a neutral stance during the study and that social interactions with the researcher were helpful to her growth. Over the course of the year of the study, the researcher spent approximately twenty hours with each of the focal preservice teachers, which probably contributed to the preservice teachers feeling comfortable with and around the researcher. Furthermore, two of the three[27] focal preservice teachers continued to meet voluntarily as a small group with the researcher over their first year of teaching, and one of these two offered to have the researcher continue to observe her teaching. Preservice teachers' continued involvement in the extension of the study seems to indicate some level of comfort with the researcher and further speaks to the quality of social interaction present throughout the study. Participating in the focus group and observational portions of the study seemed to be

[27] The third focal preservice teacher was in a teaching position where she did not teach mathematics

educative for the preservice teachers in regards to the social interactions that occurred as a result of, and were unique to, the study.

Summary of study participation. Participation in both the focus group meetings and observational portion of the study aligned with Dewey's (1938/1963) conception of educative experiences based on the criteria of continuity and interaction. The focus groups embodied continuity by having the same group of students meet multiple times throughout the year. The resulting peer support, increased focus and reflection on mathematics instruction, and growth in knowledge, mental processes, and self control are all exemplifications of the continuity of the focus group and observational portion of the study. These aspects of the study also incorporated interaction because they took place in environments that were conducive to interaction and growth and were saturated with social interactions between peers as well as with the researcher. All of these aspects contributed to both the focus group meetings and observational portion of the study helpful to the preservice teachers as well as educative.

Summary of educative experiences. The three experiences that emerged from the data as most helpful to the preservice teachers were experiences with re-teaching the same lesson at least two times, teaching as the only adult in the classroom, and participating in the study through focus groups and post-observation interviews. All of their peers who participated also indicated that participating in the focus groups was one of the most helpful experiences they had in their development as teachers of mathematics. All three of these findings will be further considered in the final chapter in regards to suggesting improvements for teacher education.

CHAPTER 9

PROCEDURAL VERSUS CONCEPTUAL TRAJECTORIES OF TEACHING MATHEMATICS

Having addressed the research questions individually, this chapter will wholistically present the trajectory of development experienced by each of the three focal preservice teachers. To do this, statements the three focal preservice teachers made about their development over the year as teachers of mathematics will be presented and integrated with Dewey's philosophies of educative experiences and constructivist views of learning.

A Return to Shared Philosophies

The main reason for developing and using a new research design of situated case studies instead of traditional case studies was to be able to consider the representativeness of the findings from the higher tiers of data collection and analysis and perhaps generalize them to the larger groups of preservice teachers at the lower tiers of the study (see Figure 2). Data analysis at several points indicated that data across tiers (which ranged from n = 3 to n = 35) was consistent. Specifically, the MKT instrument, written reflections, focus group definitions, and individual definitions of MKT were similar (see Figure 12). The three focal preservice teachers' data from their MKT instrument, written reflections, focus groups and individual interviews indicated that their aggregate data was consistent with the aggregate data at the other tiers. Therefore, the findings from the teaching observations are likely to generalize beyond the three focal preservice teachers to other members of their cohort. An important consideration, then, is if their individual development was as follows, what might this mean for the development of the larger group? The following chapter will discuss this question.

One of the main findings of this study was the dominance of a relatively few dimensions of MKT throughout the written reflections, focus groups and individual interviews and corresponding absence of the majority of the KQ dimensions. Similarly, although the three focal preservice teachers demonstrated seven (Holly), eight (McKenzie), or ten (Wendy) dimensions of MKT consistently, this left seven to ten dimensions of MKT that they sometimes demonstrated at an inappropriate level. This is even more troublesome because the scale on which they were rated was designed specifically to be preservice teacher friendly, and yet there were still few instances of the preservice teachers demonstrating MKT at medium or maximum levels. The three focal preservice teachers, and likely the larger group, remained unaware of pertinent aspects of mathematics teaching and learning. That is not to say that they were not taught the dimensions they did not identify or demonstrate. Rather, it may be that some aspects of teaching emerge and solidify before others. Still, the fact that they demonstrated fairly low levels of MKT at the conclusion of their teacher education program leaves significant reflection to be done by the researcher, and likely others, who have taught methods courses and supervised student teachers. Based on my experiences observing preservice teachers as a university supervisor for the past three years, the three focal preservice teachers can be considered above average in their mathematics teaching abilities, which makes their level of demonstrated MKT even more worrisome.

At the beginning of this study constructivist learning and procedural knowledge versus conceptual understanding were introduced as two philosophies that have been widely adopted in their application to educating P-12 students. It was suggested that these philosophies need additional consideration within teacher education in terms of educating preservice teachers (Greeno, Collins & Resnic, 1996). As preservice teachers' knowledge is constructed, an important consideration is if the resulting transformations are leading towards a development of MKT that is a conceptual versus procedural framework for teaching mathematics. This study was also framed through Dewey's philosophy of educative experiences. In Dewey's terms, consideration must be made for the educativeness of an experience and what temporary ends experiences lead a preservice teacher towards and into.

The next section will return to these philosophies to consider how they can be used to describe the results from this study. First, the three focal preservice teachers' development over the year will be summarized. Second, each of the three focal preservice teacher's individual trajectories of development will be presented. Consideration of procedural versus conceptual development will be woven throughout the individual trajectories.

Individual Trajectories

Based on the individual results from the first two research questions, it is evident that this group of preservice teachers developed mathematical knowledge for teaching along individualized paths over the course of the year, both in regards to their definition and demonstration of MKT. Although these paths seemed to converge to an extent in April, they were often quite distinct at earlier times in the year (see Figure 14). As mentioned in the introduction, constructivist learning principles need to be considered for preservice teachers not just the education of young people. One way to consider the differences in the preservice teachers' trajectories of development is through a lens of constructivist learning that took place and yielded multiple interpretations of content and experiences in course work and field placements. This was particularly evident in the fall semester, when preservice teachers' experiences were most similar because of the common methods courses in which they participated, and yet they still developed unique definitions of and conceptions about teaching mathematics during this time (see Figure 14).

Even when the preservice teachers were in the same section for all of their courses and they only had a weekly field placement they defined MKT differently throughout the semester, suggesting that their learning is better described with a constructivist view of learning than by an information processing or behavioral view (Greeno, Collins, & Resnick, 1996). A major tenet of constructivism is that learning is an active process that has a transformative effect rather than being a passive process of assimilating information (Greeno, Collins, & Resnick, 1996). Individual trajectories of development continued in the spring semester during the daily student teaching placement. Each preservice teacher had a different context in which to student teach and there was no claim on the part of the program that they had identical experiences during the spring semester. However it is one of the assumptions of the program as well as teacher education that there are similar outcomes to field placements in general, and student teaching in particular.

In addition to the previously presented descriptions of MKT development that emerged from the data via research questions one and two, the researcher asked the eleven preservice teachers who participated in focus groups and the eight who participated in individual interviews to describe their development of MKT over the course of the year. Wendy, McKenzie, and Holly's descriptions of their development over the course of the year were similar to their peers' descriptions shared through focus groups and interviews and were likely similar to other preservice teachers in their cohort and perhaps in general. The focal preservice teachers' trajectories will be the focus of the next three sections.

Wendy and Holly described their development as a teacher of mathematics in terms of five or six distinct stages. McKenzie described her trajectory in less detail than Wendy and Holly but still identified three phases through which she passed over the course of the year. These individual trajectories, as reported by the three focal preservice teachers, will now be presented in order to give a wholistic picture and first-hand account of their development over the course of the year and to inform ideas to improve preservice teachers' development of MKT in teacher education, which will be presented in the final chapter.

McKenzie. McKenzie described her development of MKT over the course of the year in three phases. Each phase will be presented using excerpts from her interviews and focus groups. McKenzie explained her conception of mathematics teaching at the outset of the year by saying, "Before (the methods course began) I thought that math teaching was all content. It was all--you just need to know what your students need to know." As will be presented shortly, a focus on content is what all three focal preservice teachers identified as the starting point of what they considered important in mathematics teaching at the beginning of the year. By the end of the semester they each recognized this was an over-simplified view of teaching mathematics, but it is interesting to note that all three focal preservice teachers began the year with this conception. This finding will be considered when proposing suggestions for methods coursework in the next chapter.

At the end of the fall semester during the second focus group meeting in December, McKenzie said that one thing she realized over the course of the semester in regards to teaching mathematics was that, "It's almost like the more you know, the more you realize you don't know." She went on to say,

> At the beginning of the semester I was a little more confident and I just assumed that if I realized there were multiple strategies, that as long as I went into the classroom with the perception that all of my students should have different strategies and I could recognize different strategies that it would be alright, for the most part. But now that we've done different (content areas in methods class) and I've seen an actual curriculum, I realize that I don't know as much as I need to know.

Although all eleven preservice teachers who participated in the focus groups believed they experienced growth over the course of the fall semester, just over half indicated they were more confident about teaching mathematics at the beginning of the semester than they were at the end. All eleven voiced or agreed with the sentiment "the more you know, the more you realize you don't know." As the fall semester ended, they viewed their awareness of what is involved in teaching mathematics as expanding, which some of them found discomforting.

McKenzie explained her conception of teaching mathematics at the end of the fall semester and into the beginning of spring semester as follows:

> During methods and at the beginning of the spring semester, I think I was a little bit *too* far extreme. Like, "No, it's just how you go about it. It's just like how you're able to talk with your kids and pull it out of them" and things like that.

McKenzie believed she focused on content less and more on generic ("how you talk with your kids") and transformational knowledge ("how you go about it"). It is interesting that she described this point in her development as “too extreme.” Based on her description, which remained quite vague and did not include any references to specific dimensions of MKT, this may be an accurate statement since it is not clear how “talking with your kids and pull(ing) it out of them” would actually come about. This reflects a different oversimplification of teaching mathematics than McKenzie had at the beginning of the year when her focus was on content knowledge. Whereas previously she had placed all of the onus for teaching mathematics on her own content knowledge, at the end of the fall semester she seemed to place too much of the onus of learning on the students themselves.

Perhaps McKenzie’s “too extreme” view of teaching mathematics stemmed from a misinterpretation of one of the core themes of the methods course, which was that young children can do more advanced mathematics than they are typically asked to do in U.S. classrooms (Carpenter, 1999). However, this does not mean that the teacher only needs to talk with her students and “pull it out of them.” The fact that at the end of the methods course McKenzie thought about teaching mathematics in vague transformational terms does not seem like a solid basis for entering the classroom full time and beginning student teaching.

The third phase McKenzie reported came at the end of the spring semester, when she said that her conception of teaching mathematics shifted again as follows:

> I feel now I have a good balance—or not maybe a good balance, but a better balance--of knowing that I need to understand the content and knowing that I need to understand my kids and how they learn and how to approach a certain concept and how to structure a lesson. So, I think in that way, it’s a balance of those two things.

McKenzie described her year as developing a balance of content with generic and transformational aspects of teaching, which she said she was able to achieve by the end of student teaching. As will be presented shortly, the idea of development for the sake of achieving balance is different than the way Wendy and Holly described their year as a series of discrete stages. McKenzie further described her focus during the spring semester enroute to achieving this balance as follows:

> So a focus or a balance between the content and how you structure a lesson, the structure part has been a little bit more what I've focused on this semester, mainly because when you're up in front of a class, you need to know how you're going to do what you're going to do. It's not just like sitting there and lecturing, and so you need to know, "What questions am I going to ask? What activities am I going to be doing? How is this thing going to actually come about?"

McKenzie indicated that her focus during student teaching was on transformational aspects of teaching, including questioning and activities. She further explained the basis for her decisions regarding activities in two of her post-observation interviews. Following her second and third observed lessons, McKenzie was asked how she developed the lesson. In the third post-observation interview, McKenzie said that she worked with the other AIG teacher on her PLC in the following way:

> We always decide the homework the week before, we just decide the homework based on how quickly we think (students) are going to go through it. And then we kind of plan our lessons based on the homework, what we know is going to have to get done.

Following her second lesson she reported doing the same thing, stating, "So, I'm looking at their homework and seeing their worksheet tonight and see what they're going to need to do tonight and basing it on that." McKenzie explained that deciding "how this would come about" was driven largely by the nightly homework assignment and working backwards from the skills her students needed to know in order to successfully complete the assignment. Although one could see why McKenzie would do this, it seems to be a fairly rudimentary approach to lesson planning to employ at the end of student teaching in that it is reactive instead of proactive. Furthermore, the lack of generalizeable skills this approach left McKenzie with was exposed during her fourth observed lesson, which took place on a day the PLC did not have any homework assigned. This meant McKenzie had to plan a lesson without having a homework activity as a starting point. Of her four lessons, this lesson was McKenzie's lowest demonstration of MKT. Even prior to coding the lesson it stood out as McKenzie's worst lesson. The lesson was split into thirds and three discrete topics were covered: place value, roman numerals, and problem solving. After the lesson McKenzie said, "There wasn't necessarily a theme or a progression in today's lesson, but it was a little bit more of a mishmash."

McKenzie's experience is an example of why Dewey (1938/1963) called for developing mental processes of a sufficient quality that would be able to carry a person over dead places in the future. When McKenzie was presented with a "dead place," in this case not having a homework assignment from which to plan, it exposed a weakness in her approach to instructional planning that focused on the immediate practical concern of her students completing the homework instead of promoting growth in her mental processes related to lesson planning. The fact that McKenzie had a difficult time planning a lesson without a corresponding homework assignment seems to indicate that she would benefit from a more developed conception of MKT from which to work proactively towards lesson planning. Suggestions to this end involving the methods course and student teaching will be presented in the final chapter.

McKenzie explained more about her spring semester experiences that helped her become, to use her words, “better balanced”:

> And, you know, I just had to do it. It was like practice doesn’t make perfect, but it makes permanent, I guess…doing it more and more often and now I feel like, I’m not completely to the point where I’m done with structure and done with knowing how to run a lesson, but I’m to the point where now I can start focusing again on content and assessment and inquiry-based lessons and things like that.

McKenzie’s description is revealing. The first thing that is striking is her comment about “practice doesn’t make perfect, but it makes permanent.” This is a powerful way to state one of the problems with an assumption of field placements in general and student teaching specifically: that repeated practice, along with reflection, helps student teachers improve their teaching. However, if preservice teachers’ reflection is only as deep as McKenzie’s language seems to indicate and focuses on content, questioning, activities, how to structure a lesson, and how a lesson comes about, then the vast majority of MKT dimensions seem to escape her. Said another way, preservice teachers cannot reliably reflect on that which they are not aware of when self-assessing their teaching, which is the case for the majority of lessons they teach since they are formally observed only once or twice each week during the spring semester in the program at hand. The guiding questions provided to preservice teachers to reflect upon following a lesson by the program are likewise generic in nature. It seems that transformations which result from repeated attention to generic aspects of teaching may continue to obscure salient aspects of mathematics teaching and learning.

The idea of practice making permanent was also described by John Dewey. Dewey explained that routine or repeated action may increase skills and even fix habits, but in as much as one lacks awareness of his/her responses, the experience remains within the realm of training rather than education (Dewey, 1916/1985). The result is that our habits "posses us, rather than we them" (Dewey, 1916/1985, p.34; see also Kestenbaum, 1977; Webb, 1976). Habits that are fixed "through blind experimentation; through examples which are not rationalized; through precepts which are more or less arbitrary and mechanical; through advice based upon the experience of others" (p.320) do not help generate self-control or control of future experiences and are formed through miseducative experiences. The resulting habits have the detrimental effect of "arresting or distorting the growth of further experience" (Dewey, 1938/1963).

The other striking aspect of McKenzie's previous description was when she said, "I'm to the point where now I can start focusing again on content and assessment and inquiry-based lessons and things like that." McKenzie indicated that she spent most of her spring semester student teaching focused on generic aspects of teaching, specifically classroom management and how to structure a lesson. Based on her description it seems that these aspects of teaching required all of her attention for most of the semester, and it was only at the end of student teaching when she started thinking about more specific aspects of mathematics teaching including content. Again, this fits in with Dewey's conceptions about focusing attention on the mechanics over substance of teaching.

To further round out their descriptions of development as teachers of mathematics, the preservice teachers were asked how their cooperating teachers contributed to their development of MKT. During the third interview in February McKenzie responded with the following:

> The first half of (fall) semester I was just kind of observing the classroom in general, and then after a while, I was observing, "Oh, this is how she phrases things sometimes," or, "This is how she asks these questions," or, "This is what

> she goes off on a tangent with, and this is not what she goes off on a tangent with." And then even (spring) semester I started picking up on, "Oh, this is the big picture of how she sets up her lessons." And then the next week I'd realize, "Oh, this is how she does homework and how she relates it back." So I started picking up on the details, the logistics of things, without necessarily asking her. And so really just watching her, how she did it last time.

The fact that McKenzie's cooperating teacher contributed mostly through McKenzie "just watching her…without necessarily asking her" seems to have resulted in a heavily procedural contribution to McKenzie's development of MKT. To be sure, almost everything McKenzie listed in the above excerpt is a generic aspect of teaching, which McKenzie summarized as "logistics." As evidenced by the researcher-developed coding protocol, the seventeen KQ dimensions are nearly all observable aspects of teaching. However, McKenzie only specifically mentioned one of these dimensions, "questioning", in her list of things she learned by watching her cooperating teacher.

Based on her interview excerpt it is clear that at different times in the year McKenzie "saw" different things as she observed her cooperating teacher. Again this suggests that using a constructivist lens to describe preservice teachers' learning seems to be an appropriate match for the data at hand because of the complicated way learning occurred relative to the preservice teacher, not just dependant on the demonstration of the cooperating teacher as in an information processing approach (Greeno, Collins, & Resnick, 1996). However, McKenzie largely did not see MKT dimensions as she watched. The only two MKT dimensions McKenzie referenced throughout her explanations of development over the year were overt subject knowledge and questioning. The majority of things she talked about were vague aspects of transformation or generic elements of teaching. This result is further supported by studies such as that conducted by Berliner (1988) and his colleagues in which advanced beginner teachers[28] watched a video of a lesson. The advanced beginner teachers "found the videos confusing and made inconsistent interpretations about what was going on", whereas teachers who were identified as experts "were quickly able to discern, interpret, and make hypothesis" about what they saw in the lesson (Greeno, Collins, & Resnick, 1996, p. 682). This raises the question of how teacher educators can help preservice teachers "see" dimensions of MKT in mathematics lessons they observe. Otherwise when they begin to teach they may narrowly attempt to replicate the peripheral "logistics" of teaching instead of the core aspects of MKT. This seemed to be the case with McKenzie, who planned lessons based on the skills needed to complete the homework rather than based on a conceptual foundation of MKT.

McKenzie's development as a teacher of mathematics involved essentially three phases: first a focus on content knowledge, second a "too extreme" focus on talking to students and eliciting information from them, and third a balance between the two. Her description and language provided helpful insights into how she, and likely other, preservice teachers viewed and experienced her culminating year in a teacher education program. Based on her comments, some areas to improve in teacher education are visible and will be discussed in a following chapter. Next Wendy and Holly's summaries of their year will be presented.

Wendy. Wendy described her development of MKT as a series of stages in which, at a given time point, she focused on one particular skill or aspect of teaching which rose to prominence during that time period. Unlike McKenzie who described her

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[28] Where "advanced beginner" was defined as student teachers and first year teachers who had received excellent evaluations during student teaching

year as achieving a balance between two views of teaching mathematics, Wendy described six stages over the course of the year for her development as a teacher of mathematics. Each of these stages will now be presented using excerpts from Wendy's interviews and focus groups.

Wendy said she began the fall semester with the following vision of teaching mathematics:

> At the very beginning of the study in the fall, I thought of (teaching mathematics) very much as having a very critical understanding of the content in a way that I could answer any questions that students pose and plan effective lessons. And being able to stand in front of the class and say, "This is how you do it," and have that be right.

Similar to McKenzie and Holly, Wendy began the year with a conception that teaching mathematics was entirely about her content knowledge. She also explained her initial conception of how students learned mathematics:

> I was very focused on the product. Not to say that I cared how students reached it, but I cared *that* they reached it. So, as we watched videos (of children working on mathematics during methods class), I was interested in their thinking, but what was most important to me was that the student got the answer.

At the beginning of the year Wendy had two over-simplified conceptions of mathematics: teaching was about the teacher's content knowledge, and learning was about students producing correct answers. During the second focus group meeting in December, Wendy explained her development as a teacher of mathematics over the course of the fall semester in the following way:

> I felt like at the beginning of the semester I was like, "Oh my God I don't even like math!" and so there was no way I was going to be a master mathematics teacher. But now, I feel like I don't have to know as much rotely as opposed to knowing what questions to ask, and I feel a lot more confident in my ability to recognize strategies and push kids to get to that more sophisticated method of problem solving. It's not about being able to execute a perfect lesson, where every kid comes up with the right answer every single time, because I guess that

> was my perception of what a good math teacher accomplishes at the end of the day, is that all her kids can do the math the right way. But now my understanding of how kids learn math is different, so I feel like I'm going to be better at teaching. So, I moved from like dispenser of knowledge to facilitator of understanding.

This description summarizes how Wendy viewed her growth over the fall semester and how her initial singular focus on her content knowledge expanded to include questioning, recognizing multiple solution strategies, encouraging more advanced problem solving, and being a facilitator. She went on to explain that over the fall semester,

> I've really flushed out my role as the teacher. And it's broken my own misconception about the teacher in a one room school house where the teacher does all of the talking and sends kids away with all of this knowledge they never could have had without me. And that's been debunked. And now I have this conception of kids are responsible for their learning and they're coming in with all of the skills and secret knowledge to know this material and all I have to do is give them the keys to unlock that, however cheesy that is to say, and I just have to teach them to access what they already know. I'd say that's been my biggest take away (from fall semester).

This description seems similar to McKenzie's description of where she was at the conclusion of methods coursework when she said she was "too extreme." Wendy's explanation of teaching mathematics as being about teaching children "to access what they already know" likewise seems too extreme, especially when it is not supported by any specific dimensions of MKT to foster such "unlocking." That both Wendy and McKenzie came out of their methods coursework with an oversimplified and underspecified conception of mathematics teaching is problematic, especially when considering their data throughout the study was representative of the larger group of preservice teachers. This finding further supports the results from the preservice teachers' definitions of MKT, which in December were comprised of two of the four categories for Wendy (foundation and connection), and only one category for McKenzie (foundation). It also supports using a constructivist lens to describe the preservice teachers' learning that took place during the fall semester and highlights the challenge that is inherent when learning is viewed as a transformation and reorganization of mental structures rather than rote memorization. Despite such challenges, suggestions about reorganizing the methods course with an aim to improve oversimplified and underspecified conceptions of teaching mathematics will be presented in a subsequent section entitled Improving Teacher Education.

The third stage Wendy described took place at the beginning of the spring semester when she began her daily student teaching placement. She explained her third stage in the following way:

> Moving into the early parts of the spring semester, I was very concerned with saying exactly the right thing the first time and expecting it to be perfect for all students. So, I thought very critically and very carefully about what specific words I was saying. I was very concerned that if I flip-flopped numbers or made a misstep in my speaking, that it would throw these kids off 100 percent, and they would never learn, and that would ruin them… I was very concerned about not saying things like, "Wait a minute. Let me go back. Let me rewind. Sorry I misspoke." I was very concerned about not misspeaking and all of that…And so I thought of my own language as being much more high stakes in the mathematical learning for children.

When asked if she felt this way for other content areas or just for mathematics, Wendy indicated that she thought her language, and teaching in general, was "much more high stakes" specifically in mathematics lessons but not in other content areas. During this stage, Wendy focused on the MKT dimension of terminology along with the transformational language she was using. Wendy explained the origins of her conception that mathematics teaching was high stakes related to language:

> I was never a natural mathematics learner, and so I have to assume that the kids weren't as resilient in math. So, it seems more high stakes to me. Like if

> I tank this math lesson, kids are never going to learn to multiply or divide or do fractions or exponents or graph equations, and it's all going to be my fault. That's an exaggeration, but, it just seems more high stakes to me, and I think of kids being less resilient. So, that's where I was, and now I trust my kids more, I trust in their learning process more.

Whereas Holly's discomfort with math resulted in her focus on students "having fun," Wendy's discomfort with math resulted in a belief that her students were less resilient and therefore the lessons she taught were high stakes. As Wendy said in her third interview about her then current thoughts on teaching mathematics, "I am ready for it in the way that you're ready for a shot: I'm not sure how this is going to go, but I'm just going to do it, and I'm going to survive it, and that will be okay. I just have a very trial-by-fire attitude about it right now." Even after the fall methods course Wendy provided quite an image of bracing for teaching mathematics.

Through the focus groups and interviews, it became clear that the idea that teaching mathematics lessons was high stakes was also an artifact of programmatic requirements to teach two stand-alone lessons in the fall semester. This further contributed to preservice teachers' views of initial attempts at teaching as evaluative events rather than learning experiences. Again, this is an example of a programmatic requirement having unintended consequences and promoting a distorted view of teaching and the purposes of student teaching. This was discussed by the preservice teachers in both focus group meetings and individual interviews throughout the study and will be further discussed when suggestions for teacher education are made.

The fourth stage Wendy described also transpired in the beginning of the spring semester. She explained,

> I had a very difficult time pacing a math lesson early on in the spring, which I wasn't concerned with in the fall. It arose as a concern in the spring early on because I just couldn't know how long to take on a math problem, or how much time to spend on the calendar portion of the math lesson versus the new material portion, or the whole group versus the small group. I found that very difficult, and so that came to light as a concern pretty early on (in the spring semester).

Wendy's identification of pacing is another example of an important but generic aspect of teaching. It makes sense that she focused on this skill during the beginning of the spring semester when she began teaching full math lessons as opposed to only leading the calendar portion of the lesson as she had in the fall. Still, it is worth noting that Wendy was focused on a generic aspect of teaching during this time, as she was during her fifth stage, which she described as follows:

> I became very concerned with assessments somewhere in the middle of the (spring) semester, and so I was really concerned with documenting how many of my students were understanding a particular concept, and so when I look in my assessment folder I have like really heavy-duty assessments for just a few weeks. But I was just very concerned with assessing their progress and my own through that. So, being very concerned with assessment, and then I became more comfortable with my ability to assess students, so I didn't need to keep as careful tallies or such rigid spreadsheets of their success rates, because I could intuit how well they were understanding a particular concept.

There are elements of assessment that require the use of MKT, such as deciding what to include in an assessment being an aspect of foundational knowledge. The description Wendy provided was too vague to determine if she considered assessment in terms of any dimension(s) of MKT or if she considered them in a generic way, such as how to keep tallies and spreadsheets.

The final stage Wendy experienced during the year explained her focus within teaching mathematics at the time of the interview, which was in mid-April near the end of student teaching. Wendy explained,

> I moved from my "assessment phase" to my "learning styles phase" or the "creativity phase." And so that's when I started really looking at, "Here are the essential skills that Math Expressions wants me to teach, but what do I know about my own students and what do I know about learning styles, learning preferences? What do I know about teaching that relates to math teaching?"… (Earlier) in the spring I was like, "Well, I don't know what kids know, so I'm just going to teach this lesson. This (curriculum guide) was made up by thoughtful and well-meaning people, so if I just teach it, then they'll just learn it, and we'll be great." But now I understand my students' learning styles better and can see how they would benefit from using manipulatives that aren't written in the lesson plan or different kinds of things…And so that's when I started to sort of deviate from our Math Expressions day in and day out criteria. And that's where I am now.

In this excerpt Wendy described the MKT dimension of adapting the textbook and beginning to use curriculum resources critically and as a starting point rather than as the sole authority for her lesson planning. Of all of the stages she went through, this final stage was most explicitly about mathematical dimensions of teaching, or at least Wendy described it in a more mathematical way than she did the other five stages. This was the first stage in which Wendy described a synthesis of multiple forms of knowledge. Whereas previously she focused on one skill at a time, such as terminology or assessment, in the above description she explained using an integrated knowledge of students' thinking with curriculum knowledge in order to adapt and supplement materials to best meet her students' needs.

Overall, it is interesting that Wendy could so readily identify six stages she passed through as she learned to teach mathematics. It is also worth noting that the majority of the stages Wendy described with generic terms that did not directly relate to dimensions of MKT, at least in the terms that she used. Although she described six stages, Wendy's summary of her development as a teacher of mathematics over the course of the year included only two of the four KQ categories (foundation and transformation) and referenced three out of nineteen dimensions of MKT (adapts textbook, overt subject knowledge, and use of terminology). Based on her descriptions, it seems that the majority of the time her primary focus was on generic or procedural aspects of teaching. This again raises the question of how can teacher educators help focus preservice teachers' attention on not only generic aspects of teaching, such as facilitating, pacing, and assessing—all of which are important aspects of teaching—but also on content-specific practices that will foster more awareness and higher demonstrated levels of MKT in their teaching.

Wendy's development as a teacher of mathematics involved six stages: first a focus on content knowledge, second a transition to viewing herself as a facilitator, third a high stakes view of terminology, fourth being challenged by pacing, fifth a concentration on assessments, and finally a synthesis of adapting the textbook with knowledge about her students. Her description and language provided helpful insights into how she, and likely other, preservice teachers viewed and experienced her final year in a teacher education program. Based on her comments, it seems that improvements within the methods course and field placements that help her focus on dimensions of MKT in her teaching would be helpful to Wendy as well as McKenzie. Suggestions to this end will be presented in the final chapter.

Holly. Holly explained her development as a teacher of mathematics as a series of stages that were more similar to Wendy's descriptions than McKenzie's in that they each had a particular focus rather than working out a balance between two views of teaching mathematics. Holly described five distinct stages that occurred over the course of the year, which will now be presented using excerpts from her interviews and focus groups.

Holly began the year similar to McKenzie and Wendy with a primary focus on content knowledge. Holly explained,

> When I entered math methods in the fall, I was focused so much on the content. I was so afraid—I've never been strong in math and never enjoyed math at all, so I was so focused on the fact that I don't know math. How am I going to teach it? So, it was more of the content when I started.

Holly and Wendy were similar in their discomfort with mathematics as students and began the year concerned about their ability to teach mathematics to their students. It should be remembered that their MKT pre-test scores in August were slightly above the class average as well as identical to each other and to McKenzie, who considered herself a "math person" and enjoyed teaching math more than any other subject.

Holly explained that the second stage happened at the end of the fall semester, when she felt better about content knowledge but less confident about other aspects of teaching mathematics:

> I felt okay with the content at that point. That fear had kind of subsided because (my methods professor) explained that you will learn the content as you go, and not to worry about that so much. I would say by the end of the fall semester, I was scared that I hadn't learned anything, that I wasn't going to be able to teach a math class. It wasn't so much the content, it was, "How do I teach math to a whole class because I haven't learned that?" I could do math interviews and do them great at the end of the fall semester. I could take small group math and do well with that, but it was the whole class math at that point that I didn't know.

Similar to McKenzie, Holly was another one of the focus group participants who were less confident at the end of the fall semester than at the beginning, although all eleven believed that they had experienced growth as teachers of mathematics during the semester. This stands in contrast to Wendy, who had more confidence with the new facilitator role she adopted during the fall semester.

Following the two stages she experienced during fall semester, Holly's third stage happened at the beginning of the spring semester and her daily student teaching field placement. Holly explained:

> Then like right before I started my student teaching, so January, I had watched my teacher do enough whole class math that I thought, "Okay, well at least I can mimic her," which I could do fine. And so I did, I mimicked her, and that was in January when I started my full-time teaching.

The excerpt from Holly's second stage indicated that as she finished her methods course she felt like she did not know how to teach math to a whole class. Therefore, when she entered the classroom full time at the start of the spring semester she learned to teach math to the whole class by watching and mimicking her cooperating teacher. This is both understandable and also problematic. At the most basic level, it is problematic because the cooperating teacher may or may not be consistently demonstrating all seventeen dimensions of MKT.

Furthermore, the larger problem with Holly's watching and mimicking approach is the same as previously discussed in regards to McKenzie's statement about learning by watching but not asking her cooperating teacher: although it is possible to "see" MKT while observing a teacher, whether specific dimensions--let alone all seventeen dimensions--will be identified by a preservice teacher who is unaware of many of the dimensions that are at play in teaching mathematics is dubious. Said another way, it seems that a preservice teacher would have to know what she was looking at and looking for as she observes her cooperating teacher in order to reliably attend to dimensions of MKT. Otherwise her attention will likely be focused on the most outwardly visible aspects of teaching, which tend to be generic or procedural. This, in turn, will likely result in the preservice teacher mimicking surface level aspects of teaching. These ideas are supported with data from this study, in which even at the end of the year following repeated observation of their cooperating teachers the preservice teachers continued to define and demonstrate MKT on relatively few of the KQ dimensions. These ideas are also supported by studies that showed that beginning teachers struggle to interpret observations of teaching (Berliner, 1988).

Holly's explanation of mimicking her teacher to learn to teach whole class mathematics is also reminiscent of Dewey's (1904/1964) description of the unfavorable apprenticeship model of teacher education. Dewey strongly cautioned against a premature emphasis on imitation and mechanics, which focused preservice teachers' attention on outward behaviors to characterize teaching. As a result of focusing on imitation, attention is diverted from understanding learning and children to instead focus on mastering skills. Thus, mimicking is likely to fix habits in regards to external behaviors rather than adherence to principles from psychology, subject matter, logic, history of education, or philosophy. Dewey also explained that a focus on visible skills could result in a preservice teacher abandoning her own thinking in an attempt to master teaching through imitation of the cooperating teacher rather than reliance on her own initiative.

Holly was not the only preservice teacher who said she mimicked her cooperating teacher. The presence, and perhaps prevalence, of preservice teachers mimicking their cooperating teachers gives further reason to consider providing additional experiences in teacher education to better develop preservice teachers' MKT and discourage generic conceptions of teaching. The fall semester weekly field placement also needs to be evaluated in terms of where the preservice teachers' attention is focused through assignments, expectations, requirements, and interactions, and what the likely ramifications of each of these may be towards directly or indirectly encouraging imitation rather than reliance on principles and initiative.

After mimicking her cooperating teacher for approximately two months, Holly said her fourth stage of development as a teacher of mathematics took place. She explained,

> By March I was like, “Okay, I’m confident in this format, in this structure. I do it fine. My kids learn. I feel confident that they’re going to get the material, and they enjoy the lesson. I can make it engaging.”

Holly indicated that she was confident teaching mathematics within the system her cooperating teacher established after mimicking her for two months. This means that Holly’s attention was focused on mimicking her cooperating teacher and perfecting her imitation for a significant portion of her student teaching experience. Again, this is similar to Dewey’s (1904/1964) apprenticeship model and stands in contrast to the laboratory model in which preservice teachers treat the classroom as a laboratory where teaching experiments occur; preservice teachers were expected to develop and rely upon their own intellectual initiative rather than relying on imitation.

The fifth and final stage Holly reported in her development as a teacher of mathematics was the following:

> And then by April 1, I had grown to not only be able to teach that one structure, but being able to do group projects and centers and small group work and feel confident with that.

This is the most encouraging of the excerpts Holly related about her development as a teacher, because ultimately she was able to engage in some form of teaching experiments towards the end of her student teaching placement. Whereas throughout the year Holly reported in her interviews that she wanted to try teaching in different ways than her cooperating teacher, whenever she asked her cooperating teacher about trying to use centers, for example, her cooperating teacher told her she could not. Then towards the end of her student teaching Holly planned some mathematics lessons that used centers. She explained in her fourth interview, "I planned it first and then showed (my cooperating teacher) when I was doing it. I didn't ask her. I just kind of did it." Holly's cooperating teacher thought the lesson plans were good and agreed to let Holly use them. It was only by doing an "end-run" that Holly was able to gain the approval she needed to try new things in her classroom. Holly's commitment to wanting to try different formats of teaching mathematics and her strategy to gain approval resulted in an opportunity that she would not have had otherwise. As a result, Holly was able to have some experiences that were more akin to a laboratory approach than the time she spent working on mimicking her teacher.

Holly's development as a teacher of mathematics involved five stages: first a focus on content knowledge, second a feeling that she did not know how to teach whole class mathematics, third mimicking her cooperating teacher, fourth becoming confident in her cooperating teacher's structure for mathematics lessons, and fifth trying different ways to structure a mathematics lesson. Her description and language provided helpful insights into how she, and likely other, preservice teachers viewed and experienced her final year in a teacher education program. Based on their comments, it seems that improvements within the methods course and field placements to help focus on dimensions of MKT in teaching would be helpful to Holly, McKenzie, and Wendy and likely other preservice teachers.

CHAPTER 10

ACROSS SITUATED CASE ANALYSIS

Having described the three focal preservice teachers' development as individual trajectories, this chapter will look across the three focal preservice teachers to compare results from the first two research questions in order to further synthesize study results. First, aggregate data from research questions one and two will be compared across the three focal preservice teachers. Second, individual data from research questions one and two will be presented and compared for each of the three focal preservice teachers.

Synthesizing Results

Comparing the results from research questions one and two can further describe preservice teachers' development of MKT over the course of their final year in a teacher education program. One way to consider the results for the first research question is that it appraises growth in MKT at a theoretical level. Preservice teachers were given tests and asked about their definition and description of MKT over the course of the year to better understand their knowledge growth. However as Rowland et al. (2009) discussed this offers a theoretical indication of their MKT, but whether it is an accurate representation of their MKT in a classroom setting is unclear. Therefore, what was learned from research question one through the written reflections, focus group interviews, and individual interviews can be compared with what was learned from research question two through teaching observations and post-observation interviews to indicate if growth in MKT occurred at a theoretical level, an enacted level, or if it was consistent across both. Analyzing data across sources further described preservice teachers' development of mathematical knowledge for teaching over their final year in a teacher education program.

Comparing theoretical and enacted MKT. First, the preservice teachers' definitions of MKT will be collectively and individually compared to their demonstrated levels of MKT. Since the preservice teachers' definitions of MKT were predominantly considered within the four KQ categories (foundation, transformation, connection, and contingency), the three focal preservice teachers' demonstrated MKT scores were averaged within each of the four KQ categories to enable comparison. Doing so indicated the relative strengths and weaknesses of the preservice teachers' enacted MKT at the category level. The following figure depicts the focal preservice teachers' final aggregate proportional definition of MKT on the pie chart and their aggregate averaged demonstrated level of MKT on the column chart. For example, the column chart indicates that, on average, the three focal preservice teachers demonstrated foundational aspects of MKT at a medium or maximum level in just over 40% of their lessons.

MKT definition, April 2010 (n=3) *Average demonstrated MKT (n=3)*

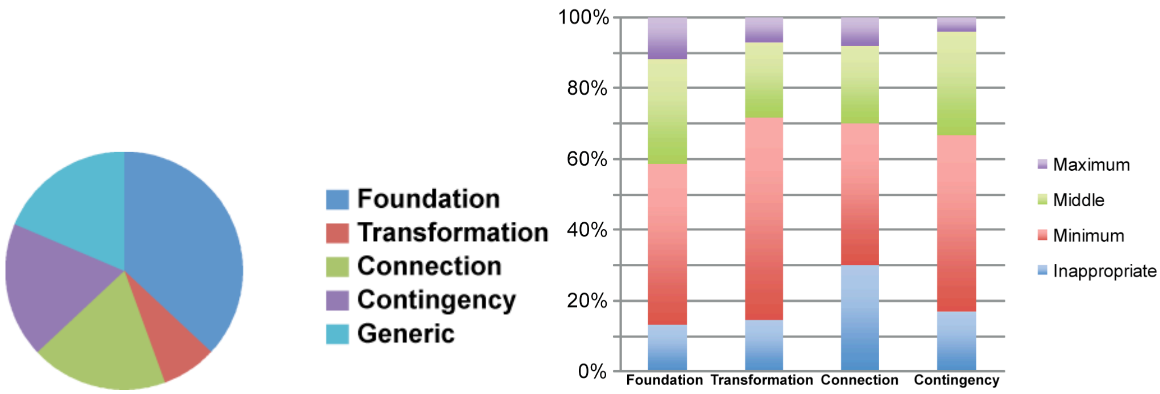

Figure 27. MKT definition and demonstration for the three focal preservice teachers. This figure shows the aggregate April 2010 MKT definition and levels of demonstrated MKT sorted by KQ category.

Two main findings emerged from comparing the preservice teachers' theoretical and enacted MKT. The first finding is the discrepancy that existed between their defined and demonstrated levels of MKT. The second finding is the relative challenge of the connection category. Both of these findings will now be discussed in more detail.

The first finding is the discrepancy that existed between preservice teachers' theoretical and enacted levels of MKT. As a group, the preservice teachers' definition of MKT focused primarily on foundation throughout the year (see Figure 14). It is interesting to note that in their teaching the preservice teachers tended to demonstrate foundation at a higher level than the other three KQ categories, as indicated in the above figure. Scores in the foundation category had fewer "inappropriate" and more "medium" and "maximum" scores than the other three KQ categories. Still, the average scores in the foundation category look fairly similar to the other three KQ categories. Whereas the foundation category dominated the preservice teachers' definitions of MKT over the course of the year, there was much less discrepency in teaching proficiency between foundation and the other three KQ categories when looking at their aggregate data.

A similar finding exists in regards to the transformation and contingency categories. The transformation category tended to remain a peripheral and under-specified aspect of the preservice teachers' definitions of MKT throughout the year. The contingency category appeared in their definitions later in the year, most often beginning at February, and was always the smallest category within their definitions. Both of these categories stand in contrast to the foundation category which was consistently the largest category of the preservice teachers' definitions of MKT. However, the discrepency that existed between the transformation and contingency categories with the foundation category in their definitions of MKT is not nearly as prevalent for the preservice teachers' enacted MKT. These three categories look fairly similar to each other for the preservice teachers' demonstrated MKT.

As was the case when considering the aggregate versus individual definitions of MKT (see Figure 14), there was a discrepency between the averaged and individual levels of demonstrated MKT. Although the foundation, transformation, and contingency categories seem to be demonstrated in similar ways when averaged, as shown in Figure 27, they tend to be more variable for an individual's scores. Therefore the three focal preservice teachers' individual definitions of MKT and demonstraed scores, which were compiled for each of the four KQ categories to show an average lesson score, will now be presented and compared to the aggregate data.

Wendy. Wendy's April definition of MKT and her average lesson score for each category are presented in the following figure.

MKT definition, April 2010 *Average demonstrated MKT*

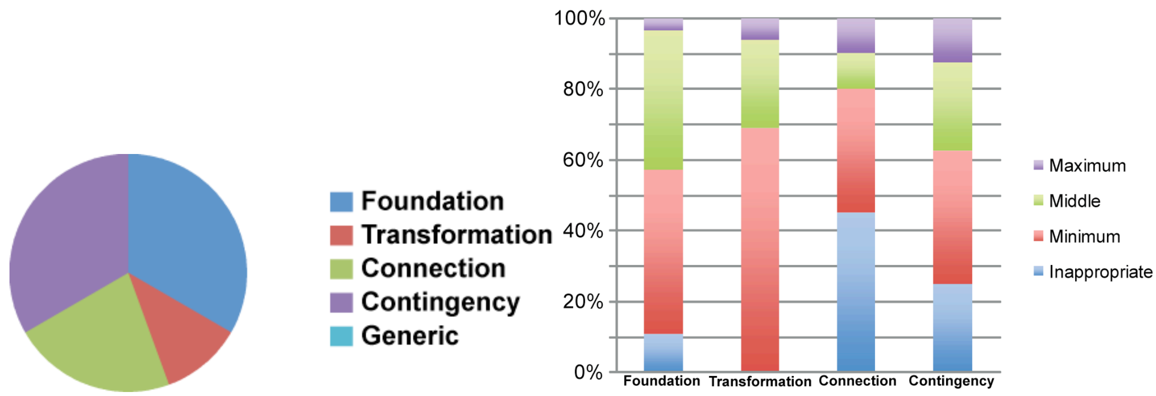

Figure 28. MKT definition and demonstration for Wendy. This figure shows Wendy's April 2010 MKT definition and levels of demonstrated MKT sorted by KQ category.

Wendy's definition of MKT was dominated by a focus on the foundation category throughout the year; she only mentioned the transformation category briefly. However, when comparing her definition to her teaching scores, these two categories were demonstrated at a similar level despite a significant difference in attention within her definition. This finding can be contrasted with the connection and contingency category. Despite including contingency in her definition in an amount that was tied with foundation for the highest amount, she demonstrated contingency at a significantly lower level than foundation, scoring an inappropriate 25% for contingency compared to 11% for foundation. Similarly, transformation was included the least in her definition but it was the one category she consistently demonstrated in her teaching. Based on these comparisons, there did not seem to be a consistent relationship between Wendy's theoretical and enacted MKT.

One finding did emerge from examining the data on Wendy's defined and demonstrated MKT individually. Of the three focal preservice teachers, Wendy's April definition of MKT seemed to be the "best" because she included the most KQ dimensions (nine out of seventeen) and did not include any generic aspects of teaching. However, of the three focal preservice teachers her demonstrated MKT was the most variable at the category level, as evidenced in the figure above by the large discrepancy between the foundation and transformation categories compared to the connection and contingency categories. For example, Wendy's inappropriate scores ranged from 0% on transformation to 44% on connection. Whereas the aggregate data in Figure 25 made the four categories appear more evenly demonstrated, Wendy's individual data revealed marked variability at the category level. Again, this points to the importance of considering individual trajectories, growth, and competencies in teacher education in addition to considering these at the group level. The relative difficulty Wendy had with demonstrating dimensions of MKT in the connection category will be further discussed after McKenzie's and Holly's individual data is presented.

McKenzie. McKenzie's April definition of MKT and her average lesson score for each category are presented in the following figure.

MKT definition, April 2010 *Average demonstrated MKT*

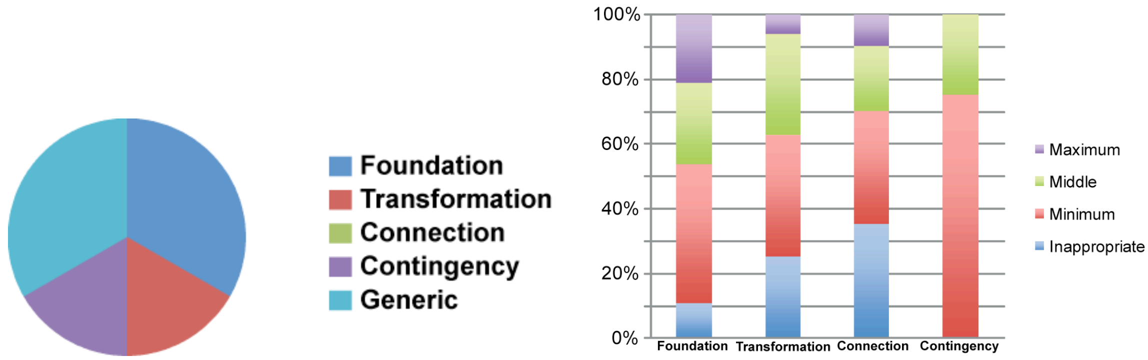

Figure 29. MKT definition and demonstration for McKenzie. This figure shows McKenzie's April 2010 MKT definition and levels of demonstrated MKT sorted by KQ category.

McKenzie's April definition of MKT primarily focused on the foundation and generic categories, and secondarily on transformation and contingency. Although she included the foundation category in her definition twice as much as transformation and contingency, her demonstrated scores indicated she consistently demonstrated contingency across her four lessons whereas she demonstrated foundation and transformation inappropriately 11% and 24% of the time, respectively.

The one place there does seem to be a correlation between McKenzie's defined and demonstrated MKT is in the connection category. McKenzie did not include connection in her definition of MKT in April, and only included it twice during the year in September and December. Connection was also the category McKenzie demonstrated at the lowest level in her teaching, doing so at an inappropriate level 34% of the time. Although her scores on the four KQ categories were not as variable as Wendy's, there was still a wide range of demonstrated level across the four KQ categories, with inappropriate scores ranging from 0 to 34%. Again, the relative difficulty of the connection category will be further discussed after Holly's individual data is presented.

Holly. Holly's April definition of MKT and her average lesson score for each category are presented in the following figure.

MKT definition, April 2010 *Average demonstrated MKT*

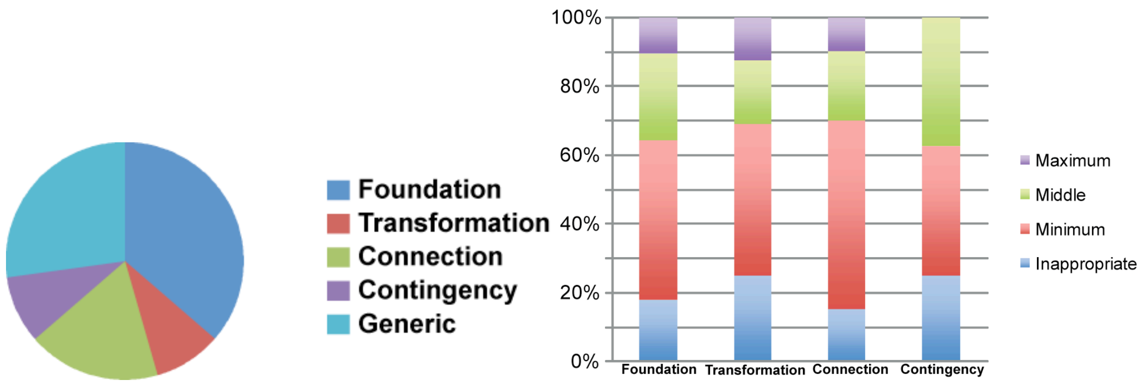

Figure 30. MKT definition and demonstration for Holly. This figure shows Holly's April 2010 MKT definition and levels of demonstrated MKT sorted by KQ category.

Of the four KQ categories, Holly included foundation and connection the most in her April definition of MKT. These were also the two categories she demonstrated at the highest levels in her teaching, as evidenced in the above figure by the two lowest inappropriate scores. April was also the first time Holly included the contingency category in her definition and it was one of the weaker categories of her teaching, as shown in the above figure as tied for the highest level of inappropriate scores. Of the three focal preservice teachers, Holly was the only preservice teacher whose relative amount each category was included in her definition of MKT at the end of the year mapped onto her relative success demonstrating that category in practice. For example, Holly's definition of MKT included the four KQ categories in the following amount from greatest to least: foundation, connection, and a tie between transformation and contingency. This is also the order in which she successfully demonstrated each category of MKT: foundation received the fewest inappropriate scores, connection the second fewest, and transformation and contingency were tied at the highest amount of appropriate scores.

Holly's teaching scores across the four KQ categories were the least variable of the three focal preservice teachers. While the other two focal preservice teachers scored inappropriate between 0-34% (McKenzie) and 0-44% (Wendy), Holly scored inappropriate between 15-25% of the time. Similarly, Wendy scored at the middle and maximum levels 20-45% of the time and McKenzie did 25-45% of the time, whereas Holly did 30-38% of the time. It is interesting that Holly, who had substantially more teaching experience than anyone else in her cohort, did not score "better" than the other two preservice teachers on any category other than connection. However, she enacted MKT more consistently than did Wendy and McKenzie across the four categories. Holly's individual scores were also more similar to the aggregate scores found in Figure 25 than were Wendy's or McKenzie's individual scores.

Summary. Based on the above descriptions, there was a frequent discrepency between the preservice teachers' theoretical and enacted MKT. This was true for both the aggregate data as well as for Wendy's and McKenzie's individual data. There was no discrepency in Holly's individual data at the category level in April, although there was at the previous interview in Febrary. It may be that Holly's additional time spent teaching reduced the variability of her scores across the four KQ categories and also resulted in more similarity with the aggregate data, although it is not possible to determine this based on the present data.

Challenges with the connection category. The second finding from comparing definitions with demonstrated MKT for both the aggregate and individual data was the relative difficulty of the connection category, which was the most variable category in the preservice teachers' definitions of MKT throughout the year (see Figure 12). That is to say, it was not a predictable or stable aspect of the preservice teachers' MKT definitions. At times connection did not appear at all in definitions and when it did it had fairly large fluctuations in amount, for example contibuting to 50% of Wendy's defintion in December followed by 0% in February. The connection category came and went in Wendy's and McKenzie's definitions of MKT, appearing in two out of four interviews but not at sequential time points. Holly included connection in her definition more consistently and included it in December, February, and April.

In the aggregate data for the three focal preservice teachers as well as Wendy's and McKenzie's individual data, the conneciton category was demonstrated at a lower level than the other three KQ categories. This is indicated in Figure 27 by a higher amount of inappropriate scores than any other category at 30% on average, and in Figures 28 and 29 for Wendy's and McKenzie's individual data. This seems to be a high rate of inappropriate scores to occur at the end of a teacher education program and is roughly twice the amount of inappropriate scores as the other three KQ categories. This raises the question of whether there is something about the connection category in general or its four specific dimensions that makes it a more challenging aspect of MKT for preservice teachers to identify and demonstrate. This question will now be considered.

The main reason the connection category had a lower demonstrated average than the other three KQ categories was that it contained the dimension on which the preservice teachers scored the lowest out of all seventeen KQ dimensions. This dimension was anticipation of complexity, for which the three focal preservice teachers scored an inappropriate in seven out of twelve lessons. This was the highest prevalence of an inappropriate score on any dimension of MKT. An inappropriate score on this dimension meant that the preservice teacher could not identify an instructional decision, such as a demonstration, example, intentional sequence, or question, within the lesson that was made as a result of anticipating the complexity of an idea or concept. This dimension of MKT was not mentioned by the three focal preservice teachers in their written reflections, focus groups, or individual interviews. The only time anticipation of complexity was discussed during the entire year was in response to a question from the researcher during the post-observation interviews.

Although the intent of this study was to discuss the preservice teachers' MKT in terms of development over the course of the year, it is worth briefly noting anticipation of complexity and the other dimensions that stood out as the lowest demonstrations of MKT for the three focal preservice teachers. Anticipation of complexity was the lowest level, and the other tier four dimensions were focus on understanding, connections between procedures, and connection between concepts. Three of the four of these dimensions are found within the connection category. It is also worth noting that there were only four dimensions in the connection category and three of them were in tier four, meaning they were inconsistently demonstrated by all three focal preservice teachers. In sum, connection was demonstrated least well of the four KQ categories.

It is difficult to determine why connection seemed to be especially challenging for these preservice teachers. It may be that they viewed mathematics as a series of discrete topics rather than as a connected subject, probably largely based on their own experiences with school mathematics, and therefore remained largely unaware of the presence of this dimension as an aspect of teaching mathematics. The preservice teachers' possible lack of awareness is supported by the low presence of connection in their definitions of MKT over the course of the year. When it was included, the preservice teachers talked about connection in terms such as Wendy's statement that " master teachers of mathematics connect it to every day, and they connect it frequently and without trying." If their conception of the role of connection in teaching mathematics was only as deep as their comments indicated, then it follows that they would have a difficult time enacting such vague notions. This hypothesis raises the question of how did the preservice teachers' experiences in teacher education not make them better aware of the importance of connection within mathematics teaching? Suggestions to address this will be made in the subsequent section entitled Improving Teacher Education.

The other possible way to explain the challenge of the connection category is not aas n issue of awareness but rather as a difficulty with translation to practice. Perhaps the preservice teachers' connected knowledge was variable and vague because it was just coming into focus through their experiences in teacher education and the main difficulty was not in their awareness but rather in enacting the connected knowledge they had. Two of the four connection dimensions, making connections between procedures and making connections between concepts, are dependent on the teacher's content knowledge and an awareness of the importance of connection within mathematics. The anticipation of complexity dimension requires an understanding of content knowledge as well as children's thinking. Perhaps connection is an especially challenging category to enact because its dimensions draw on three realms of knowledge that are in and of themselves challenging: understanding the connected nature of mathematics, having content knowledge that is connected across procedures and concepts, and understanding children's thinking. It was not possible to determine whether the relative difficulty associated with the connection category for these preservice teachers was an issue of awareness or translation to practice. Either way, it seems that these preservice teachers may have especially benefited from additional awareness of and experiences with dimensions in the connection category. Suggestions to this end will be provided in a subsequent section entitled Improving Teacher Education.

Challenging dimensions. The final dimension in Tier Four, and the only one that was not part of the connection category, was focus on understanding. This dimension was also demonstrated by all three focal preservice teachers inconsistently, meaning they each sometimes received an inappropriate score on this dimension. A score of inappropriate indicated that less than 25% of the preservice teachers' interactions in the lesson focused on understanding and over 75% of the interactions focused on procedural aspects of mathematics (see Appendix I; Skemp, 1978). In order to teach for understanding, preservice teachers must also conceptualize mathematics as a field of understanding rather than one of memorized rules and procedures. These may be areas of difficulty for preservice teachers. One of the dominant themes of their methods course was a focus on understanding. The importance of understanding was included by the three focal preservice teachers in their definitions of MKT beginning in their second interviews in December, and by the majority of the eleven preservice teachers who participated in focus groups and the eight who participated in interviews.

The inclusion of a focus on understanding as a dimension of MKT was reported frequently starting in December and throughout the spring semester. However the three focal preservice teachers, all of whom included a focus on understanding in their spring definitions of MKT, did not consistently demonstrate a focus on understanding in their teaching. Whereas there was a question as to whether the preservice teachers were aware of the importance of the three dimensions from the connection category that were also on Tier Four, it was clear from the written reflections, focus groups, and interviews that the preservice teachers were aware of and committed to a focus on understanding at a theoretical level. In this case, the challenge did not seem to be one of awareness but rather one of difficulty with translation to practice.

A comparison of dimensions that were demonstrated at Tier One, Tier Two, Tier Three and Tier Four revealed a possible way to consider the relative success the preservice teachers had demonstrating a given dimension. As was discussed for the four dimensions on tier four, these dimensions required a synthesis of multiple aspects of mathematical knowledge, namely an understanding of the nature of mathematics, content knowledge that is deep and connected, and an understanding of children's thinking. These three requirements were considered for each of the seventeen KQ dimensions. The descriptions and definitions for each of the dimensions as delineated in the coding protocol (Appendix I) were used to evaluate whether a dimension required one, two, or all three of these "kinds" of knowledge for successful demonstration. Doing so indicated that, predictably, all seventeen KQ dimensions required content knowledge to be demonstrated successfully. However, a difference was noticed in the remaining two aspects of knowledge: understanding children's thinking and understanding the nature of mathematics. The following table shows the seventeen KQ dimensions by tier and indicates which of these two requirements are needed for successful demonstration of each dimension. Content knowledge is not included in the following table since it is required for all dimensions.

Table 18

Aspects of understanding required for successful demonstraion of each KQ dimension

Tier	Dimension	Aspect(s) of understanding required[29]	
		Understanding children's thinking	Understanding the nature of mathematics
Tier One	Theoretical underpinning	X	X
	Questioning	X	
	Sequencing	X	
	Deviation	X	
	Choice of examples	X	
Tier Two	Awareness of purpose		X
	Identifying errors	X	
	Use of terminology	(Content knowledge only)	

[29] Note: all dimensions also require content knowledge

Tier Three	Adapts textbook	X	
	Overt subject knowledge	(Content knowledge only)	
	Choice of representation	X	
	Demonstration		X
	Responding to children's ideas	X	
Tier Four	Focus on understanding		X
	Anticipation of complexity	X	
	Connect procedures		X
	Connect concepts		X

The dimensions for which an understanding of the nature of mathematics is required comprised the majority of dimensions at Tier Four, where three of the four dimensions required this kind of knowledge. In contrast, the majority of dimensions at Tiers One, Two, and Three required an understanding of children's thinking. This may indicate that the three focal preservice teachers tended to have more difficulty enacting dimensions of MKT that required an understanding of the nature of mathematics than student thinking.

This raises the question of how teacher educators develop not only preservice teachers' content knowledge and knowledge of student thinking, but also how we are encouraging development of their understanding of the nature of mathematics as one that is connected across topics and based on understanding rather than memorized procedures. This is not to say that a focus on student thinking should not remain a primary focus in teacher education, since ten out of seventeen of the aspects of MKT in the KQ model require an understanding of children's thinking while fewer dimensions, six out of seventeen, require an understanding of the nature of mathematics. However, providing preservice teachers with experiences in which they broaden their conception of the nature of mathematics should be addressed in teacher education in order to help preservice teachers better develop and demonstrate MKT. These experiences could be provided within a methods course or within a content course that is designed specifically for elementary education majors. It is interesting to note that both of these courses exist and are required at the present university. Both the methods course and content coutse have as a primary theme the importance of understanding in mathematics teaching and learning. Connections which exist across procedures and connections within mathematics are not a substantial focus for either course. The findings from this study that the connection category in general, and dimensions which require an understanding of the connected nature of mathematics specifically, seem to be especially difficult for preservice teachers to translate to practice should be reflected upon in regards to the experiences that preservice teachers are provided through coursework and field placements in teacher education. Some suggestions to these ends will be provided in the next chapter.

CHAPTER 11

DISCUSSION—IMPLICATIONS AND RECOMMENDATIONS FOR TEACHER EDUCATION

Based on the extensive amount of data from this study, suggestions for improving teacher education to enhance preservice teachers' development of MKT will now be presented. These ideas integrate the findings from the three research questions, the individual trajectories of the three focal preservice teachers, Dewey's (1938/1963) philosophies on educative experiences, and constructivist views of learning. Specific suggestions for improving teacher education through methods coursework, initial field placements, and student teaching will be made. Finally, the study will be summarized, limitations and contributions noted, and future research questions posed.

Improving Teacher Education

Although they experienced growth in identifiable ways over their final year in a teacher education program, a primary finding from this study was that the preservice teachers did not include a majority of the dimensions of MKT in their definitions nor did they successfully demonstrate all dimensions of MKT consistently in their teaching. Particular difficulty tended to occur with identifying and enacting the connection category. Synthesizing these results along with the three focal preservice teachers' statements about their development as teachers of mathematics indicates several ways in which teacher education could be improved with an aim of focusing preservice teachers' attention on and development of dimensions of MKT. This needs to be considered holistically across the program including the methods course, initial field placements, and student teaching. This chapter will discuss ideas to this end based on viewing the data from this study through a cognitive perspective of learning.

Methods course. One way to help preservice teachers become aware of the dimensions of MKT is to use the KQ model (or another similar model) in methods coursework. This could be done in two ways. One way is to introduce preservice teachers to the model. The KQ model was designed to use with preservice teachers and is organized in a straightforward way around four categories (foundation, transformation, connection, and contingency) so that it is memorable, easy to understand, and user-friendly while still being complicated enough to give a thorough description of effective mathematics teaching. Even if the preservice teachers do not remember all nineteen dimensions, their exposure to them and understanding of the spirit of the four categories seems likely to persist into spring semester student teaching and hopefully beyond their time in teacher education.

Although awareness of a conceptual model is an initial step to help preservice teachers' attend to and develop dimensions of MKT, this should be supported through additional experiences with the dimensions. It is unlikely that telling the preservice teachers about the seventeen dimensions of MKT will have a significant impact on their practices, in a similar way that it is unlikely that telling a student about a mathematical concept will achieve an understanding of that concept without accompanying experiences. Here another parallel can be made between teacher education and the field of mathematics education through Simon's (2006) work on key developmental understandings, or KDUs. A KDU "involves a conceptual advance on the part of students...a change in the students' ability to think about and/or perceive particular mathematical relationships" (p.362). Likewise, using a conceptual model of MKT is suggested not so that preservice teachers' can memorize the dimensions, but rather to foster a conceptual advance and a change in preservice teachers' ability to think about teaching mathematics. One characteristic of a KDU is that "students without the knowledge do not tend to acquire it as the result of an explanation or demonstration. That is, the transition requires a building up of the understanding through students' activity and reflection and usually comes about over multiple experiences" (Simon, 2006, p.362). This is consistent with cognitive views of learning, which contribute to the foundation of this study, and explain why additional experiences with a conceptual model of MKT will be necessary beyond naming its components. Familiarity with the categories and dimensions of MKT will likely expand preservice teachers' awareness of considerations that contribute to mathematics teaching, but additional experiences are necessary for a conceptual advance to occur. Suggestions for additional experiences will now be presented.

In their work on cognition and learning, Greeno, Collins, and Resnick (1996) explained two forms of practical conceptualization that emerge from a cognitive view of learning in terms of formulating curricula. The first is the importance of giving explicit attention to generality. They explained that curriculum "can be organized so that students come to understand the major unifying principles of the domain" (Greeno, Collins, & Resnick, 1996, p.27). Thus, a second and more sophisticated way to use the KQ model in methods coursework and give preservice teachers deeper experiences with dimensions of MKT would be to organize the course around the model. Currently in the program at hand, and typically within teacher education programs, mathematics methods courses are organized around content areas. For example, the methods course within the current program is organized by topic and taught in the following order: principles of number, counting, operations, place value, fractions, geometry, and measurement. Within each of these areas broader aspects of mathematics teaching are discussed, such as the use of questioning or how to design and select tasks. Still, such an organizational framework may focus preservice teachers' attention on content areas rather than broader principles of teaching mathematics. In order to help preservice teachers understand the unifying principles of teaching mathematics, an alternate way to organize a methods course would be to use the four KQ categories as the main units of the course, and then address the dimensions within that category via the content area(s) that seem to best elucidate each given dimension. This may help preservice teachers learn that there are aspects of teaching mathematics broader than and extended across content areas.

Additional suggestions become apparent when integrating findings with the principles of constructivism to methods course design. One of the findings of this study was that all three focal preservice teachers began the year with a focus on content knowledge as the singular dimension of MKT that contributed to teaching. This seems to indicate that many other preservice teachers in the cohort, and perhaps in general, may first be aware of and focus on the KQ dimension of overt subject knowledge. This should be considered when planning methods coursework in order to build on specific experiences in which the role of content knowledge can be discussed. Constructivist learning principles emphasize that "learning must be viewed as transforming significant understanding that (people) already have, rather than as simple acquisitions written on blank slates" (Greeno, Collins, & Resnick, 1996, p 18). Therefore, it is important to consider the conceptions of preservice teachers throughout their time in teacher education. Therefore if preservice teachers are focused on the role of content knowledge during the first weeks of the semester they may need help to "see" how initial experiences in their methods course and field placements relate to MKT in ways beyond overt subject knowledge.

Awareness of preservice teachers' conceptions about teaching mathematics and dimensions that are at the forefront of their thinking at the beginning and during the methods course is important for the professor to recognize not only from a deficit perspective, in which other dimensions of MKT are essentially invisible to the preservice teacher at a given time, but also from a perspective that views their conceptions as assets. One of the tenets of constructivism is that "intuitive understanding provides the basis for new understandings that develop and should be treated as an essential resource in students' learning" (Greeno, Collins, & Resnick, 1996, page 22). In regards to the data from this study, the intuitive understanding of this group of preservice teachers resided primarily in the foundation category; foundation was the category the preservice teachers, both as a group and individually, focused on in their definitions of MKT throughout the entire year. It was also the category that, as a group, the three focal preservice teachers demonstrated at the highest level. If a subsequent cohort had a similar awareness and demonstration of the foundation category of MKT, this could be reflected in the methods course in order to use their knowledge as a resource from which to deepen their understanding of those foundational dimensions of MKT as well as broaden their knowledge to the other categories of MKT, perhaps by exploring how and in what ways foundational knowledge relates or contributes to the other three categories in general, and whether there are specific dimensions that seem especially related to foundational knowledge.

Greeno, Collins, and Resnick (1996) explained a second form of practical conceptualization for formulating curricula from a cognitive view of learning as the importance of sequencing learning activities. A cognitive view of learning indicates that experiences best promote learning when the sequence "proceed(s) from issues and problems that are within the reach of students' initial understanding and reasoning ability to issues and problems that require greater extensions of their intuitive capabilities (and) accomplish conceptual growth by refining and extending their initial understandings" (p.27). In the case of the group of preservice teachers from this study, this would mean beginning the methods course with the foundation category and then moving to the transformation category. This is because transformation was present in their definitions to a lesser amount than foundation but from the beginning of the year. The preservice teachers had an awareness of the importance of transformational knowledge but rarely identified specific dimensions within this category, and failed to identify some of the dimensions at any point during the year (e.g., choice of representation).

Based on the eight preservice teachers' individual definitions of MKT and the aggregate definitions over the course of the year connection would be the third category to discuss, as this was generally the third category of which the preservice teachers became aware. Contingency tended to be the last category that this group of preservice teachers became aware of, and therefore this would be the final category to introduce in the methods course. Having contingency as the final category also makes sense in terms of the KQ model because it is the one category of mathematics teaching that cannot be planned for ahead of time, and for which teachers need to respond in way(s) that connect to dimensions from the other three categories. Therefore, having contingency last when sequencing the four KQ categories makes sense both in terms of the model and the data from this study.

A final suggestion for improving methods coursework is offered for categories or dimensions that the preservice teachers may find particularly challenging. For the preservice teachers in the present study, this was the connection category, as well as the tier three and tier four dimensions that were demonstrated inconsistently in their practices. A way to respond to areas of difficulty is again provided through consideration of a cognitive view of learning, in which the aim of teaching is to help students develop and use cognitive structures. As these structures are formed, initial conceptual understandings, as well as misunderstandings, should be addressed. When a misconception arises, two possible approaches are as follows:

> If one believes that students have an incorrect scientific theory or misconception in the domain, then it is appropriate to elicit their beliefs and confront them with contradictory evidence. On the other hand, if one believes that their understanding is based on intuitions that are valid in some circumstances, then a more exploratory kind of conversation is probably more effective. Then students can develop ways of talking and thinking about phenomena and gradually become more attuned to the ways in which properties in the domain are related (Greeno, Collins, & Resnick, 1996, page 22).

There were several points in the study where the preservice teachers had a misconception regarding the teaching of mathematics. This was usually is the form of an oversimplified view such as content knowledge being the sole dimension of MKT at the beginning of the year. Sometimes they also had an incorrect idea, such as making instructional decisions so that students would “have fun,” in the case of McKenzie and Holly, or planning lessons based on the homework, in the case of McKenzie. Contradictory evidence for incorrect knowledge of MKT may be found in guided discussions of teaching observations that are selected to target and eliminate the particular misconception. Reading assignments in the form of classroom vignettes or research studies could also provide contradictory evidence. Exploratory conversations can be a regular part of the methods class in which the preservice teachers have the opportunity to develop their thinking and talking about dimensions of MKT and may further reveal misconceptions, oversimplifications, or underspecifications to pursue.

All of these suggestions for the methods course are made with a goal of improving the translation to practice for dimensions of MKT. In cognition terms, translation to practice is a form of knowledge transfer. Throughout the literature there are an extensive number of studies whose results indicate that knowledge transfer from one situation to another or one task to the next is incredibly difficult and not in the students' favor (Greeno, Collins, & Resnick, 1996). Under a constructivist view, a student's success on transfer depends on the formation of schema that indicates relations and is "invariant across situations" (Greeno, Collins, & Resnick, 1996, page 23). Use of the KQ model in the ways discussed throughout this chapter will likely focus preservice teachers' attention on the dimensions of MKT, which are features of mathematics teaching whose principles are invariant across situations. Said another way, regardless of the content area, topic of the lesson, or grade-level, a preservice teacher needs to be aware of the core features of MKT that are contained in the KQ model and persist across such superficial differences. Reorganizing the methods course in the suggested ways may help preservice teachers develop a more conceptual basis for teaching mathematics.

Initial field placements. Along with reorganizing the methods course around a conceptual model of teaching mathematics, field placements must also be improved to eliminate practices that result in distorted views of teaching and obscure rather than illuminate dimensions of MKT. Particular to the current program, the first way this could be done with the aim of helping preservice teachers focus on the dimensions of MKT is to re-envision the weekly field placement that takes place during the fall semester. In more general terms, evaluating initial field placements based on what they focus attention on and lead preservice teachers towards and into and applying Dewey's (1938/1963) principles of interaction and continuity may indicate areas to improve.

To begin with the program at hand, the continuity of the weekly fall field placement was called into question by the preservice teachers during focus groups and individual interviews. In the course of discussing their experiences they shared some of the challenges with being in their field placement only once per week. One challenge that was mentioned in the focus groups was that it was difficult to see the progression throughout a unit of instruction by only being present once per week. In focus groups the preservice teachers often said their placement class worked on an entirely different unit from one week to the next. This was especially true for preservice teachers who were placed either in the lower grades or in a classroom that used the EnVisions curriculum, and a majority of the preservice teachers fell into one or both of these groups. McKenzie talked about the challenges she had with the weekly fall field placement, which occurred on Wednesdays, in the fourth focus group when she reported the following in a way that was similar to her peers' comments:

> I taught two math lessons last fall, one kind of at the beginning of the semester and one towards the end of the semester, and I was coming smack-dab in the

> middle of the week. My (cooperating) teacher had told me, "We'll be on this lesson, so you can do this lesson," but who knows if on Tuesday they got up to the point they were supposed to be when I picked up on Wednesday? And she could *tell* me, but I wasn't there to hear the language that they were using, to hear whether or not they had heard some of the words I was saying before.

All eleven of the preservice teachers in the focus groups shared or agreed with statements similar to McKenzie's. These statements fit with Dewey's (1938/1963) philosophy of continuity, which is one of the two main pillars of an educative experience. Although teacher education programs may claim weekly field placements are continuous because preservice teachers are exposed to the progression of a classroom, teaching, and learning with the same group of students over time, the preservice teachers in the present study did not feel that their weekly experiences were continuous. Although changing the weekly aspect of the field placement so that preservice teachers have a more continuous classroom experience may not be possible in the current program, the challenges and limitations inherent in this structure should be considered and discussed with preservice teachers, cooperating teachers, supervisors, and professors so that the preservice teachers are best supported during this time. Conversations between cooperating teachers and preservice teachers that intentionally and specifically discuss aspects of the classroom to broaden the preservice teacher's one-day-per-week snapshot may be one way to promote some sense of continuity within a discontinuous structure.

The requirement to teach two observed lessons in the fall semester in the current program, or the observation of early teaching experiences in general, should be reconsidered. The preservice teachers described their lack of awareness of their students, the classroom, and the curriculum throughout the fall semester as a result of their novice standing and only once per week presence. Therefore, they were left to do something similar to what Wendy did even into the spring semester, saying, “Well, I don’t know what kids know, so I’m just going to teach this lesson. This (curriculum guide) was made up by thoughtful and well-meaning people, so if I just teach it, then they’ll just learn it, and we’ll be great.” In the present context, the push for evaluated teaching was premature and focused preservice teachers’ attention on generic and procedural aspects of teaching rather than giving them an initial experience with dimensions of MKT. Here it is worth quoting Dewey’s (1904/1964) statement again to further indicate the miseducative impact early evaluations of teaching can have:

> No greater travesty of real intellectual criticism can be given than to set a student to teaching a brief number of lessons, have him under inspection in practically all the time of every lesson, and then criticize him almost, if not quite, at the end of each lesson, upon the particular way in which that particular lesson has been taught, pointing out elements of failure and success. Such methods of criticism may be adapted to giving a training-teacher command of some of the knacks and tools of the trade, but are not calculated to develop a thoughtful and independent teacher. (p.335)

At the very least, eliminating the evaluative component of being observed by the university supervisor and cooperating teacher during the fall semester could encourage using initial teaching experiences as opportunities to interact with children and dimensions of MKT in more of an exploratory way. Instead of a rush to teaching and evaluation, a better use of the fall semester in this program, or initial field placements in general, would be to support preservice teachers by engaging in guided practice of initial observations. As previously indicated through results from this study as well as other studies that show the confusion beginning teachers have when observing another teacher (Berliner, 1988), it is challenging for preservice teachers to “see” dimensions of MKT as they observe their cooperating teacher. Therefore, initial field experiences could provide an opportunity and support so that as preservice teachers observe mathematics lessons they focus on dimensions of MKT.

This could be done in conjunction with the methods course, so that whichever of the four KQ categories is the focus in the methods course at a given time could, in turn, be the primary focus of the preservice teachers as they attend their fall field placement. This would also build off of their intuitions of mathematics teaching, an important aspect of successful constructivist learning, since that would be the basis for determining the sequence of KQ categories in the methods course. Based on the data from this study, at the beginning of the semester this would mean focusing on the foundation category. Initial observations could take the form of preservice teachers identifying and reflecting upon the incidents where foundational knowledge was demonstrated by their Cooperating Teacher in an observed lesson. Conversations with cooperating teachers and in the methods course could further explore what was observed, the decisions that went into those dimensions of teaching, important considerations within the dimensions they may be missing or misunderstanding, and possible improvements. This kind of observation would focus preservice teachers' attention on dimensions of MKT as well and form conceptions around aspects of teaching that persist and are "invariant across situations" (Greeno, Collins, & Resnick, 1996, page 23).

When teaching does begin, initial approximations of practice could be designed so that, again, preservice teachers are focused on specific categories or dimensions of MKT rather than being expected to enact all seventeen dimensions at once. For example, the preservice teachers could work on their connected knowledge by engaging in a series of experiences of individual, small group, and whole class interactions in which their focus is on the four dimensions within the connection category. Additional individualized approximations of practice could also occur with the categories and/or dimensions that each preservice teacher finds particularly challenging. For example, Wendy would have benefited from focused practice on responding to children's ideas and Holly would have benefited from practice on choice of representation. These approximations of practice should be designed and evaluated based on whether they lead the preservice teachers towards a conceptual or procedural view of teaching mathematics, clarify rather than distort their conception of MKT, and foster rather than impede growth.

As previously presented in the results to research question three, one of the most helpful experiences the three focal preservice teachers discussed in their development as teachers of mathematics was the opportunity to repeatedly teach the same lesson. Teaching the same lesson at least twice could be one of the initial experiences preservice teachers have with teaching a full lesson. Following multiple experiences with guided observations and initial teaching experiences focused on specific categories and dimensions of MKT, preservice teachers could teach a lesson at centers or to an otherwise small group of students in which they try to enact multiple and eventually all dimensions of MKT. Following the first attempt, the preservice teacher could have a short amount of time to reflect upon the lesson through the KQ dimensions and then teach the same lesson, now slightly refined, to at least a second group of students. Doing so would emphasize both dimensions of MKT and development of mental processes (i.e., reflection and critical thinking), and stands in contrast to Dewey's (1904/1964) critique of traditional lesson planning:

> The student who prepares a number of more or less set lessons; who then has those lesson plans criticized; who then has his actual teaching criticized from the standpoint of success in carrying out the prearranged plans, is in a totally different attitude from the teacher who has to build up and modify his teaching plans as he goes along from experience gained in contact with the pupils. (p.317)

Developing teaching plans based on dimensions of MKT and refining them based on interactions with students would likely provide educative experiences for preservice teachers as they begin teaching full lessons.

All of the above improvements would result in many intentional and educative experiences with dimensions of MKT through observing and beginning to teach so that a conceptual basis for teaching mathematics would be established in advance of full-time student teaching. To this end, a significant contribution can be found in work within the situative/pragmatist-sociohistoric perspective that considers the system in which an individual learns. Lave and Wenger (1991) examined multiple studies of learning through a system that initiated individuals to a community through apprenticeship, which maps onto traditional arrangements of student teaching. In such systems, beginners are initially involved peripherally in the activities of the community, and their participation becomes increasingly central as they become more experienced. However, Lave and Wenger (1991) explained that an apprentice system could be unproductive for learning. They identified that a crucial factor in the success of learning in such a system is that the beginners must be "afforded legitimate peripheral participation, which involves access to the practices that they are expected to learn and genuine participation in the activities and concerns of the group" (Greeno, Collins, & Resnick, 1996, page 23). Within teacher education, this points to the importance of initial peripheral practices in which preservice teachers are engaged through field placements. Lave and Wenger (1991) continued, "For an environment of apprenticeship to be a productive environment of learning, learners need to have opportunities to observe and practice activities in which their abilities will become stronger in ways that correspond to progress toward more central participation" (cited in Greeno, Collins, & Resnick, 1996, page 23). Developing approximations of practice that are grounded in and focused on dimensions of MKT would help preservice teachers become involved in legitimate aspects of teaching so that as they move to the center of classroom activities, they do so with a conceptual foundation.

Student teaching. In addition to improving initial field experiences, student teaching must also be examined in terms of the educative nature of experiences and whether these contribute to a procedural or conceptual development of mathematics teaching. Despite the essence of each methods course conveying that there are unique aspects of a given discipline that are not common across teaching--otherwise separate courses for each content area would not exist--most of what transpired during student teaching for the preservice teachers in this study glossed over these distinctions and instead reinforced a generic and procedural view of teaching. One way this was done in the current program was by using generic prompts upon which the preservice teachers reflected following a lesson they taught. Therefore, once the preservice teachers begin teaching another improvement is to use content-specific reflection prompts following a mathematics teaching experience as opposed to the generic ones which are currently used in the program. This suggestion is made because repeated attention to generic aspects of teaching may continue to obscure or distort salient aspects of mathematics teaching and learning. Therefore, using reflection prompts following mathematics teaching that are aligned with the KQ model would further focus preservice teachers' attention on dimensions of MKT. Additionally, using reflection prompts which align with the methods course would promote the "convergence and concentration" (p.322) of coursework and field experiences, another emphasis within Dewey's (1904/1964) writings on improving teacher education. Using subject-specific reflection prompts is one way to promote convergence between the training and practice school and would result in increased continuity of experiences for preservice teachers across teacher education programs (Dewey, 1904/1964).

Another way the current student teaching practices focus on generic and procedural aspects of teaching and could be improved is in the use of a generic observation form that the university supervisor and cooperating teacher complete when observing a lesson. The same observation form is used across content areas whether it is a mathematics, guided reading, or science lesson, and therefore the form only contains elements common across teaching. As mentioned previously this means that unless the supervisor or cooperating teacher hold additional knowledge for the discipline that she is observing, the preservice teacher is likely to only receive generic and procedural feedback about her mathematics teaching. This sets up a cycle in which preservice teachers receive generic feedback and then improve their teaching based on the generic comments their supervisors and cooperating teachers tend to make. Based on focus group and individual interview data, generic feedback was especially prevalent from university supervisors, who never gave any content-specific feedback in mathematics, and only did occasionally in literacy. The preservice teachers reported that their cooperating teachers were more likely to give mathematics teaching-specific feedback, but this was still not done regularly. The prevalence of generic feedback across content areas, with the rare exception of literacy-specific feedback, was reported by all eleven preservice teachers who participated in the focus groups and was further discussed by all eight participants in their individual interviews. McKenzie explained this cycle in her third interview when she explained how her cooperating teacher helped her development of MKT in the following excerpt:

> It's been really good to notice the things that (my cooperating teacher) comments on about my lessons. And just by her saying a little something like, "Oh, I really liked how you blah-blah-blah," that helps me to know, "Oh, that was a good aspect of my lesson. I should do that again. And I should maybe drop this other part of the lesson that wasn't necessary."

This seems to set up a feedback loop in which if the preservice teachers receive generic feedback from their supervisors and cooperating teachers then it follows that they will place more of their attention in subsequent lessons on things like classroom management instead of dimensions of MKT. Furthermore, one of the consistent comments during focus groups and interviews was that supervisors rarely observed math lessons, ranging from zero to three times per year with a mode of one, and when they did they never gave any math-specific feedback but rather gave generic feedback about things like classroom management, who the preservice teacher called on, and general poise. Therefore, encouraging both reflection and feedback that is specific to teaching mathematics would likely help preservice teachers reflect upon and try to improve dimensions of MKT in their teaching.

However, changing the forms used during student teaching will likely be ineffective without also training cooperating teachers and supervisors on the model, the spirit of the four categories, and the nineteen dimensions of MKT. To this end, models of teacher education that include on-going professional development for cooperating teachers and university supervisors should be pursued. One way to do this would be to have the monthly university supervisor meetings shift from a focus on paperwork and procedures to more of a seminar format with accompanying readings and discussion, including pieces on subject-specific knowledge and methods. This would help university supervisors develop their pedagogical content knowledge in areas beyond their primary field. This would also increase consistency with the methods course and thereby promote the continuity of experiences for preservice teachers. Similarly, cooperating teachers could participate in their own seminars that would also involve reading about and discussing pertinent topics to mentoring, including subject-specific dimensions of teaching. In return for their additional participation in the program, college credits towards graduate degrees could be awarded. Developing and implementing models that integrate supervisors and cooperating teachers with the content of methods coursework should be pursued to form more continuous programs and experiences for preservice teachers (Johnson, Willox, Brown & Persson, February 2010).

A broad final suggestion for student teaching is for it to be re-envisioned so that experiences can more closely align with Dewey's (1904/1964) laboratory approach to teacher education. To this end, a specific suggestion arose through this study from the preservice teachers' discussions of helpful experiences. As previously presented in the results to research question three, one of the most helpful experiences the three focal preservice teachers discussed was having an opportunity to teach as the only adult in the room. All three focal preservice teachers indicated they interacted with their students differently when there was another adult in the room, even if they were not being formally observed. The absence of the cooperating teacher allowed the preservice teacher to interact with her students in ways she found more natural, rather than trying to replicate responses consistent with her cooperating teacher. One of Dewey's (1904/1964) critiques of the apprentice method of teacher education was that the experiences are too controlled and do not approximate the actual conditions of teaching. Dewey (1904/1964) wrote, "As matter of fact, the 'best interests of the children' are so safe-guarded and supervised that the situation approaches learning to swim without going *too near* the water" (p.317, emphasis in original). One way this occurs in student teaching is through the presence of the expert cooperating teacher or supervisor who at any point can step in to make suggestions and whose presence is interpreted by the preservice teacher as evaluative. One way to reduce this restrictiveness is for the preservice teacher to engage in some experiences in which she is the only adult in the classroom so she can treat the classroom as more of a laboratory in which teaching experiments occur, such as Wendy setting up a math circle. Even if no new routines or structures resulted from such experiences, having time to further develop and reflect upon her own teaching would be beneficial and would increase the continuity from student teaching into the profession.

Other approaches to encourage teaching experiments by means other than having the cooperating teacher leave the room should be considered, such as re-visioning the cooperating teacher/student teacher relationship and making clear the purposes and expectations of student teaching to everyone involved: preservice teachers, cooperating teachers, and university supervisors. To this end, before the placement begins reading[30] and discussing with all of these groups of people a vision for what teaching experiments may look like is recommended.

[30] The researcher has used and recommends Dewey's 1904/1964 piece on the apprentice versus laboratory methods with preservice teachers entering their first field placement and with preservice teachers entering their student teaching placement.

Many suggestions for the methods course and field placements have been offered. I envision these as more intentional and conceptual as well as more fluid and individualized experiences than presently exist in the current program and in teacher education, in general. The suggestions should be enacted for preservice teachers in the order presented, but not in a rigid way that sets dates by which each step needs to be accomplished. The sequence of suggestions, including guided observations, initial experiences with specific categories of MKT, and experiences with re-teaching, should happen in the proposed order and by the end of the teacher education program. However, individual time-lines should be allowed to vary lest a push for programmatic requirements again results in miseducative experiences because they are not aligned with the preservice teacher's conceptual readiness at a given time. Instead, the idea is for a preservice teacher to have time to conceptualize the specific aspects of teaching within a set of experiences before proceeding to a more centralized set of experiences. This means that preservice teachers will proceed at their own rate and may at times be focused on dimensions of MKT that are distinct from their peers. Again this is an opportunity to apply tenets of differentiated instruction not only to the education of children but also to the education of preservice teachers.

Discussion groups. One of the primary findings from this study was that the preservice teachers reported participating in the study contributed to their development as teachers of mathematics. As a result of the overwhelming positive feedback on their experiences participating in the focus groups, which all of the eleven participants indicated as one of their most significant experiences, an additional suggestion for teacher education programs to consider is offering preservice teachers the opportunity to participate in a similar peer community as the focus groups for this study.

Within this study, eleven preservice teachers participated in the two groups, which met four times over the year. There were multiple aspects of the groups that the preservice teachers consistently reported valuing, both individually in their interviews and collectively in the focus groups. Some important characteristics of the groups included meeting with a small number of their peers across grade levels and schools, meeting throughout the year with the same group of people, meeting in a comfortable setting outside of university and elementary schools, and having the meetings facilitated by someone other than their university supervisor. As previously discussed in the results for research question three, the preservice teachers valued the opportunity to get out of their self-reported "student teaching bubbles" by hearing about the experiences of their peers, as well as the opportunity to reflect on their own experiences and philosophies about teaching mathematics and sharing their own experiences with their peers. They also valued engaging in discussion focused on one specific content area in some depth rather than talking about teaching generically across subjects. Therefore, it is suggested that one way to improve teacher education is to offer something akin to mathematics teaching discussion groups, as well as discussion groups in other content areas.

For such discussion groups to be successful, replicating the general ethos of the groups from this study will likely be more important than duplicating the characteristics of the groups, because it was the spirit pf the groups that the preservice teachers found most compelling. One of the main elements to this end was framing discussions using a positive tone by focusing on participants' growth and improvement (see Appendices D and E). The preservice teachers spoke about how often in other seminar-style meetings with their peers there was a lot of complaining and venting, and they appreciated the contrast of the positive tone of the discussion groups. As McKenzie said, "I feel like this has also remained pretty positive. Sometimes it's a gripe session at our seminar. It's burdening, it's heavy to handle, and it makes it difficult to get through." It should be emphasized that the purpose of the discussion groups is not to teach the preservice teachers anything in particular, but rather to create an experience in which they can engage in meaningful social interactions and reflection and thereby collectively further their own professional growth. The idea is for the facilitator to hear about the preservice teachers' conceptions about MKT, development, and experiences.

Having preservice teachers participate voluntarily was a key aspect of the efficacy of the discussion groups in this study. Therefore in terms of recruitment an invitation to participate can be offered at the beginning of the fall semester to all seniors in the program. Students hopefully would volunteer to participate in one content area discussion group of their choosing over the course of the school year. Perhaps some students will sign up for a content area in which they feel particularly strong, while others will sign up for a content area in which they feel they need more support; the current study drew preservice teachers for both reasons. It is important for the preservice teachers to voluntarily participate as well as be able to select which content area discussion group they join. It is also important for them to participate in the same group over the course of the year rather than fluctuate between different groups of people or different content areas. The nature of the discussion groups is to provide some depth to one content area rather than "skipping around" to multiple topics.

The two primary reasons for suggesting content-area discussion groups are first to better support the development of preservice teachers in content area teaching, and second to engage in collaborative research that may further inform work within a specific teacher education program. As was discussed throughout previous constructivist-based suggestions for the methods course, it is important that the intuitions and misconceptions of the preservice teachers are known if they are to be built upon or disassembled in coursework and field placements. Holding mathematics teaching discussion groups would be a way to do this, while providing a rich and educative experience for the preservice teachers.

Although organizing and leading discussion groups may seem like a substantial addition to a teacher education program where no corollary currently exists, once the groups are organized and the protocol is established (see Appendix E for the one used in the present study) there is no "prep work" that has to be done for the meetings since the same guiding questions are used throughout the year. Also, the chance to engage in collaborative research would hopefully provide an incentive for doctoral students and/or faculty members to facilitate a group. Of all of the suggestions that have been made in this section, participation in discussion groups is one that has already been done and all participating preservice teachers valued the interactions therein.

Educative experiences. A final, albeit less tangible, suggestion for teacher education programs is to evaluate and enhance preservice teachers' experiences in terms of educative and mis-educative experiences (Dewey, 1938/1963). As demonstrated by the data from this study, the foremost experiences that the preservice teachers identified as helpful and instrumental to their growth were also educative because they integrated continuity and interaction. This is not to say that every experience identified as helpful was educative, but perhaps there is some correlation between what preservice teachers perceive to be significantly helpful experiences and the characteristics of educative experiences. The philosophy of educative experiences provides a framework to evaluate and enhance experiences in teacher education. In general, teacher education would benefit from a wider theoretical stance with which to frame programmatic decisions, course assignments, and field placement experiences. This could be done through a constructivist lens as well as through John Dewey's (1938/1963) work on educative experiences.

Many suggestions for teacher education have been made throughout this section based on integrating the results from this study with a cognitive view of learning and John Dewey's (1904/1964) philosophies of educative experiences and the laboratory method of teacher education. Suggestions have been made to improve the methods course, initial field placements, and student teaching. An additional suggestion of offering content-area discussion groups was made based on its success in this study and alignment with the principles of educative experiences.

Conclusion

Summary. This study examined preservice teachers' development of MKT over their culminating year in a university-based teacher education program. This was done through analyzing written reflections, focus group interviews, individual interviews, teaching observations, and post-observation interviews as well as through the use of a quantitative measure, the MKT instrument. A new study design of situated case studies was introduced, in which tiered participation resulted in extensive data for three focal preservice teachers as well as comparison to larger groups of their peers through interviews (n=8), focus groups (n=11), and written reflections and the MKT instrument (n=35). In this way, data collection and analysis was more substantial than is typical for traditional case studies, hence the naming of this methodology as situated case studies.

Insights were gained into preservice teachers' definitions of MKT and revealed that they developed conceptions of MKT in distinct ways from each other and from the aggregate definition, although there was some convergence at the end of the year. The KQ category of foundation tended to dominate the preservice teachers' definitions of MKT, the transformation category remained vague, connection was an inconsistent category in their definition, and the contingency category arose late in the year and at a relatively small proportion.

Insights were also gained into preservice teachers' demonstration of MKT in their teaching of mathematics. Dimensions of MKT were most often demonstrated at a minimum level, growth on a dimension as indicated by scores that improved over time was extremely rare, and scores were variable across the four observed lessons. Particular dimensions were enacted consistently by all three focal preservice teachers (theoretical underpinning, questioning, choice of examples, sequencing, deviation from agenda), some dimensions were enacted consistently by two of the three focal preservice teachers (awareness of purpose, identifying errors, use of terminology), some dimensions were enacted consistently by only one of the three focal preservice teachers (adapts textbook, overt subject knowledge, choice of representation, demonstration, responding to children's ideas), and some dimensions were inconsistently enacted by all three focal preservice teachers (focus on understanding, anticipation of complexity, making connections between procedures, making connections between concepts).

The final realm of exploration was into the preservice teachers' experiences throughout the year. Written reflections, focus group interviews, and individual interviews revealed experiences the preservice teachers considered to be helpful. Dewey's (1938/1963) philosophies of educative experiences were applied to the preservice teachers' ideas about helpful experiences to evaluate them on the basis of continuity and interaction. Similarities between helpful experiences were apparent and indicated that the preservice teachers benefited from re-teaching the same lesson, teaching as the only adult in the room, and participating in this study through peer discussion groups and post-teaching reflection.

Limitations of study. One limitation of this study was the overall small sample size. Although significantly more data was collected than with a traditional case study design, for the purposes of this document only three preservice teachers' data was examined in depth, resulting in limitations on the generalizeability of the findings. One other limitation is the loss of some of the written reflection data from the beginning of the study due to a technical problem with the digital drop-box to which the preservice teachers uploaded their written reflections, although the remaining data seemed representative of the larger group (n=35).

Although the data for the three focal preservice teachers was consistent with the data from the larger group (n=35), it is still not possible to claim that these three preservice teachers are an accurate and complete representation of their peers. While I am comfortable claiming that their three stories seem to capture elements of their peer's development based on data from interviews and focus groups with larger groups of preservice teachers, there may be other trajectories of development that were not captured through the study of these three preservice teachers.

Another caution should be noted in regards to the analysis and reporting of the educative experiences of the three focal preservice teachers. Dewey's conception of social interaction goes beyond simply being present in a classroom. That is to say, it is possible to be in a classroom (or in any setting where there is a group of people) and not, in fact, be in a social environment or have genuine social interaction. Particular to the study at hand, the social interaction in the focus groups and the proposed content area discussion groups is a truer exemplification of Dewey's philosophy of social interaction than the examples of re-teaching and teaching as the only adult in the classroom, which contained educative interactions with students and subject matter but did not embody the same kind of peer interaction that was present in the focs groups and in Dewey's conception of social interaction.

Areas for further research. In particular, teaching observations of more preservice teachers would be beneficial to further explore the tiered nature of the dimensions of MKT in demonstration and better map out how the dimensions translate to practice. A primary question this study raised is whether a typical trajectory of the development of MKT exists for preservice teachers and can be established with additional research. To this end, additional research with larger numbers of preservice teachers should be conducted to continue to investigate and explore how the dimensions of MKT translate to practice and if it is possible to determine which dimensions seem more intuitive versus more challenging for preservice teachers. Additional teaching observations of a larger sample size would also make it possible to consider whether some of the dimensions of MKT correlate with each other, which may indicate a way to simplify the model by combining dimensions with high correlations or give further insight into methods course dedsign to teach dimensions in correlated groups.

Framing the work of MKT development in a similar way to Simon's (1996) work on KDUs may be a helpful way forward. In mathematics learning, a key developmental understanding "identifie(s) critical transitions that are essential for students' mathematical development" (p.360). Determining critical transitions that exist in preservice teachers' development of MKT would be of significant importance to the field of teacher education.

Another question arose based on Wendy's and Holly's demonstrated MKT on the questioning dimension is whether MKT is demonstrated differently in different settings. For example, Wendy and Holly exhibited differences in their questioning when they were in a whole class setting versus interacting individually with students. Whether some dimensions tend to be demonstrated differently according to the setting by preservice teachers as MKT is developed is another question for future research.

Additional research needs to be conducted using the MKT instrument with preservice teachers to verify the test is appropriate to use with this population. Especially given the current political climate, test reliability and validity needs to be determined for preservice teachers should decisions, certification, or funding ever be linked to scores on this or similar measures. Test items may need to be dropped or added to make it a valid instrument to use with preservice teachers. Scores obtained from this sample suggest that use with preservice teachers is discouraged for purposes other than research and possibly providing a learning experience for preservice teachers.

In addition to conducting similar research with larger sample sizes, other methodologies may provide additional insight into present and future data. For example, using discourse analysis to analyze focus groups and interviews for growth and change that occurred over the course of the year is of interest. A future study could also unpack areas of overlap and areas of divergence between the two theoretical traditions used in this study: philosophical (e.g., Dewey's philosophy of experience) and pshycological (e.g., constructivist learning principles).

Finally, this study touched upon many sub-topics that are worth considering individually, both with the current data set and with larger sample sizes. For example, analyzing the data from all of the focus group participants (n=11) to further explore the preservice teachers' interactions and relationships with their supervisors and cooperating teachers is of interest. Additionally, data analysis that focuses entirely on continuity and interaction could result in more substantial insights into current practices and possibilities in teacher education. Lastly, implementing any of the suggestions from the previous section within a teacher education program or with a sub-set of preservice teachers to investigate the impact the change(s) seems to have on preservice teachers' development of MKT is an area for future research.

Contributions. Although it was not the intent at the outset, the primary contributions of this study are likely to be methodological ones. The most immediate contribution of this study is the development of the coding protocol by which to evaluate preservice and beginning teachers' enactment of MKT in their teaching (see Appendix I). The development of a protocol that is tied to the literature will make a substantial contribution to the field because currently such a protocol does not exist for the KQ model and requests for such a protocol are increasing (Rowland, personal communication, January 2011). Additionally, few protocols for any theoretical model of MKT have been published about, and the most well-known one from the QMI project is so tedious to use that its authors acknowledged significant revisions would be required to use it for anything other than rigorous video studies (Learning Mathematics for Teaching, 2006a). Furthermore, there are no known protocols that are designed specifically for use with preservice and beginning elementary education teachers in mathematics. Therefore, publishing this protocol will make a substantial contribution to mathematics and teacher education. In this way the development of a rigorous MKT protocol addresses Ball et al.'s (2008) critique that, to date, "ideas have remained theoretically scattered, lacking clear definition" (p.394). Furthermore, the data from this study will be used to write a series of cases for each dimension of MKT. These vignettes from teaching episodes will accompany the coding protocol to provide an image of a given dimension in a classroom situation.

An additional methodological contribution of the study is the development of the new methodology of situated case studies to incorporate strengths of qualitative work while enabling comparisons to a larger sample size in order to discover the potential applicability of findings. The third methodological contribution of this study is the finding that the aggregate data from the preservice teachers' definitions and demonstrations of MKT were substantially different from their individual definitions and demonstrations until the end of the year. This is an important realization for future research design.

Another contribution of this study is that the MKT instrument results provide the first look at preservice teachers' scores on this measure and call into question the validity of the instrument with this population and whether it is an accurate depiction of their development of MKT over time.

The final contribution of this study is demonstrating the power and usefulness of theoretical lenses within teacher education, particularly cognitive views of learning and Dewey's (1938/1963) philosophy of educative experiences. These foundations enhanced this study and led to more substantial suggestions to improve teacher education and preservice teachers' development of MKT. Currently teacher education, and especially field placements, suffer from a scant theoretical base (Munby, Russell, Martin, 2001). As Greeno, Collins, and Resnick (1996) stated, it is important to use the strengths of multiple perspectives pluralistically when attempting to address educational problems. This study has made such an attempt and offered numerous suggestions for improvements in teacher education for the methods course, initial field placements, student teaching and content-based discussion groups such that preservice teachers' can better develop MKT via educative experiences that encourage conceptual rather than procedural knowledge.

Appendix A

Ball, Thames, and Phelps'[31] mathematical tasks of teaching

Presenting mathematical ideas

Responding to students "why" questions

Finding an example to make a specific mathematical point

Recognizing what is involved in using a particular representation

Linking representations to underlying ideas and to other representations

Connecting a topic being taught to topics from prior or future years

Explaining mathematical goals and purposes to parents

Appraising and adapting the mathematical content of textbooks

Modifying tasks to be either easier or harder

Evaluating the plausibility of students' claims (often quickly)

Giving or evaluating mathematical explanations

Choosing and developing useable definitions

Using mathematical notation and language and critiquing its use

Asking productive mathematical questions

Selecting representations for particular purposes

Inspecting equivalencies

[31] (Ball et al., 2008, p.400)

Appendix B

Rowland, Turner, Thwaites, and Huckstep's[32] Knowledge Quartet codes

Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Choice of examples Choice of representation Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between procedures Making connections between concepts Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities

[32] (Rowland et al., 2009, p.29)

Appendix C

MKT Instrument Sample Item[33]

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought. Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

[33] Hill, H.C., Schilling, S.G., & Ball, D.L. (2004)

Appendix D

Focus Group Protocol

September 2009

(Items in italics were added as a result of Focus Group A and asked at Focus Group B)

Part I: Questions regarding the Survey[34]

1. What are your thoughts about the Survey?
    a. What were your thoughts and/or feelings as you took the Survey?
    b. What were your thoughts and/or feelings after you took the Survey?
    c. *Did the survey impact the scope of what you considered a teacher had to do in teaching mathematics?*

2. Do you think the questions on the Survey are appropriate?
    a. Follow-up: Why/why not?
    b. What do you think about using a Survey that does/doesn't include "I'm not sure" as a choice?
    c. Do you think this is a good instrument to use with preservice teachers at the beginning of the semester?
    d. *Discuss reactions to the "as a teacher…" questions*
    e. *Do you feel like you should know the answers to these questions?*

3. Can you give an example of something you felt confident about answering on the Survey?
    a. Follow-up: Can you trace your confidence on that item back to a specific experience, or several experiences? *Where did your confidence come from?*
        i. If "no": How did you know the answer if it was not from an experience you had?

4. Can you give an example of something you were *not* confident about answering on the Survey?
    a. Follow-up: Why do you think you were unconfident?
        i. Do you think it had anything to do with the experiences you have had or not had? Explain.
        ii. *Lack confidence in relation to "you are a teacher" questions?*

[34] Survey" means the MKT instrument. It was called a survey in attempt to reduce test anxiety.

5. At this point, PI reads a selected question from the Survey back to the focus group to allow participants to discuss the above questions with a specific example.
    i. What do you think about this question?
    ii. Do you think it is appropriate? Why/why not?
    iii. What about this question did you feel confident answering?
    iv. What about this question did you *not* feel confident answering?
    v. How was your answer on this question influenced by your prior experiences?
6. The purpose of the Survey is to measure a person's "mathematical knowledge for teaching." What do you think mathematical knowledge for teaching entails?
    a. What does an elementary teacher need to know to be able to teach mathematics?
    b. Do you think it's important to master all of these areas as a preservice teacher? *Is that your goal, to master all of these things?*
        i. Do you think some areas are more important?
        ii. As you consider the scope of MKT, what are your goals for the semester and year?

7. After the survey I asked about whether you consider yourself a "master mathematics teacher."
    a. What does being a "master mathematics teacher" mean?
    b. What is involved with being a master mathematics teacher?
    c. *Is it possible to define master teacher of mathematics?*
    d. What is the relationship between content and being a master teacher?
    e. *Would you have responded differently had it said "master mathematical teacher for elementary school?" What's the difference?*
    f. What do you think about yourself and why? *Why did you rank yourself as you did?*
    g. *Surprising that peers put 4 or 5?*

8. *Survey Part 2*
    How did this make you look at yourself?
    Did this have an impact at how you looked at yourself?
    Impact of putting the survey at the beginning or end?

9. *Do you think using this Survey as part of a course would be helpful? How?*
    a. *Instructor's use*
    b. *Was it helpful to you personally?*

Part II: Questions regarding experiences

10. Think about your current mathematical knowledge for teaching. What experiences have you had that have contributed to your current mathematical knowledge for teaching?

    a. Ask about various time-frames
        i. Before enrolling in the School of Education?
            1. What have you learned from mathematics content courses (at UNC or in high school) that have contributed to your current MKT?
        ii. Since enrolling in the School of Education?
            1. What did you learn from Math 307 that has contributed to your current MKT?
        iii. Since the beginning of the semester?
            1. What have you learned from EDUC 512 that has contributed to your current MKT?
            2. What have you learned from your Wednesday field placement that has contributed to your current MKT?

    b. Ask about various "kinds" of experiences
        i. What formal experiences have you had?
        ii. What informal experiences have you had?

    c. In what settings have these experiences occurred?
        i. What experiences outside of those coordinated by the School of Education have you had that have contributed to your current mathematical knowledge for teaching?

    d. What has been the most helpful to your development of MKT? *If we sent you out to teach a lesson today, what would you draw from the most?*
        i. Coursework, class discussions, class readings, class assignments, field placements, other
        ii. What do you anticipate being the most helpful to your growth in MKT this semester? Spring semester?

11. How and in what way have you grown as a teacher of mathematics since the time you took the Survey?
    a. What experiences have you had that have contributed to your growth in mathematical knowledge for teaching since the beginning of the semester?
    b. Describe the one specific experience that, to date, has resulted in the most growth for you in terms of your mathematical knowledge for teaching.

Appendix E
Focus Group Protocol

Fall 2009- Spring 2010

December 2009, February 2010, April 2010

Part I: Questions regarding the Survey[35]

1. What are your thoughts about the Survey?
    a. What were your thoughts and/or feelings as you took the Survey?
    b. What were your thoughts and/or feelings after you took the Survey?
    c. Did the survey impact the scope of what you considered a teacher had to do in teaching mathematics?

2. Do you think the questions on the Survey are appropriate?
    a. Follow-up: Why/why not?
    b. Do you think this is a good instrument to use with preservice teachers at the end of the semester?
    c. Discuss reactions to the "as a teacher…" questions
    d. Do you feel like you should know the answers to these questions?

3. Do you think using this survey as part of a course would be helpful? How?
    a. How might it be helpful for the Instructor?
    b. Is it helpful to you personally? How?

4. Did taking the survey at the end of the semester make you think about or reflect on areas you have improved this semester? areas you want to focus on/improve next semester?

Part II: Questions regarding Mathematical Knowledge for Teaching

5. The purpose of the Survey is to measure a person's "mathematical knowledge for teaching." What do you think mathematical knowledge for teaching entails?
    a. What does an elementary teacher need to know to be able to teach mathematics effectively?
    b. Do you think it's important to master all of these areas as a preservice teacher? Is that your goal, to master all of these things?
        i. Do you think some areas are more important?
        ii. As you consider the scope of MKT, what are your goals for the spring semester?

6. After the Survey you indicated whether you consider yourself a "master mathematics teacher."

[35] "Survey" means the MKT instrument. It was called a survey in attempt to reduce test anxiety.

a. Is it possible to define master teacher of mathematics?
b. What does being a "master mathematics teacher" mean?
c. What is involved with being a master mathematics teacher?
d. What is the relationship between content and being a master teacher?
e. Would you have responded differently had it said "master mathematical teacher for elementary school?" What's the difference?
f. What do you think about yourself and why? Why did you rank yourself as you did?
g. Talk about your growth in this area (compare your scores to earlier this semester).
    i. In what ways have you developed as a teacher of mathematics this semster?
    ii. What do you know now that you didn't know at the beginning of the semester in regards to teaching mathematics?
h. Do you think you will learn as much, the same, or less in spring semester when you are done with your methods course and in your full-time student teaching placement?

Part III: Questions regarding experiences

7. What experiences have you had that have contributed to your <u>current</u> mathematical knowledge for teaching?
    a. What experiences have helped you develop as a teacher of mathematics?
        i. Identify and describe the experience
        ii. What was it about this experience that made it so helpful?

    b. What have you learned from EDUC 512 that has contributed to your current MKT?
    c. What have you learned from your Wednesday field placement that has contributed to your current MKT?
    d. Have you had any informal experiences, experiences outside of those coordinated by the School of Education, that have contributed to your MKT?
    e. What has been the most helpful to your development of MKT? If you were to teach a lesson today, what would you draw from the most?
        i. Coursework, class discussions, class readings, class assignments, field placements, other
        ii. What do you anticipate being the most helpful to your growth in MKT this semester? Spring semester?

Part IV: Questions regarding growth

8. How and in what way have you grown as a teacher of mathematics since the beginning of the semester?
    a. Describe your growth as a teacher of mathematics since the beginning of the semester
    b. How have you grown as a teacher of mathematics since the beginning of the semester?
    c. Why have you grown as a teacher of mathematics since the beginning of the semester?
        i. What experiences have you had that have contributed to your growth in mathematical knowledge for teaching since the beginning of the semester?
        ii. Describe the one specific experience that, to date, has resulted in the most growth for you in terms of your mathematical knowledge for teaching.

Appendix F

Individual Interview Protocol

September 2009, December 2009, February 2010, April 2010

Part I: Follow-up on Focus Group Discussion

1. PI first provides brief review of focus group discussion.

2. Ask follow-up questions regarding comments this participant made, if necessary.

3. Of the things that were said during the focus group, what did you agree with most strongly?

4. Was there anything said during the focus group with which you disagreed?

5. Are there additional aspects regarding these topics (list) that you would like to discuss? Was there anything that the focus group discussion did not sufficiently address?

Part II: Mathematical knowledge for teaching

6. What do you think mathematical knowledge for teaching entails?
    a. What does an elementary teacher need to know to be able to teach mathematics effectively?
    b. Is it possible to define master teacher of mathematics?
    c. What does being a "master mathematics teacher" mean?
    d. What is involved with being a master mathematics teacher?
    e. Do you think it's important to master all of these areas as a preservice teacher? Is that your goal, to master all of these things?
        i. Do you think some areas are more important?
        ii. As you consider the scope of MKT, what are your goals for the spring semester?
    f. Talk about your growth in this area.
        i. In what ways have you developed as a teacher of mathematics this semester?
        ii. What do you know now that you didn't know at the beginning of the semester in regards to teaching mathematics?
    g. Do you think you will learn as much, the same, or less in spring semester when you are done with your methods course and in your full-time student teaching placement?

Part III: Significant experiences

7. What experiences have you had over the course of this semester that have been helpful in your development as a teacher of mathematics? Begin with describing the experience that, to date, has resulted in the most growth for you in terms of your mathematical knowledge for teaching. Please describe the experience, and then discuss the impact of the experience on your development as a teacher of mathematics.

8. You shared several examples of experiences that have positively contributed to your development as a teacher of mathematics. Have you had any experiences that had a negative impact on you or your practices?

9. Have you taught any mathematics lessons this semester? Please describe at length. What aspects of the lesson were you pleased with? What would you do differently were you to teach the same lesson again?

10. You have had many experiences in regards to teaching mathematics this semester. Are there any experiences you feel like you are missing? Are there any experiences you would have liked to have, or are hoping to have at some point before entering the classroom as a teacher?

Part IV: Growth

11. How and in what way have you grown as a teacher of mathematics since the beginning of the semester?

a. Describe your growth as a teacher of mathematics since the beginning of the semester
b. How have you grown as a teacher of mathematics since the beginning of the semester?
c. Why have you grown as a teacher of mathematics since the beginning of the semester?

Appendix G

Post-observation Interview Protocol

January 2010 - April 2010

Part I: Lesson Planning

1. Tell me about your lesson plan

2. What were the source(s) of the activities?
   Curriculum
   Cooperating Teacher
   Methods
   Other

3. What was your goal for the lesson?

Part II: Teaching

4. Tell me about your actual lesson

5. Discussion will focus on
   a. Aspects of the lesson that were different from or not included in the lesson plan.
   b. The Preservice teacher's responses to students' comments and questions
      i. Why did you call on (student name)?
      ii. Why didn't you call on (student name)?
      iii. Why did you expand on (student name)'s comment/question?
      iv. Why didn't you expand on (student name)'s comment/question

Part III: Intended vs. Enacted

6. Were there any differences between your lesson plan and the lesson as it was taught?
   a. If not, why did you not make any changes?
   b. If so, why did you make changes?

Part IV: Reflections

7. What things do you think went well?

8. What things did not go as planned?

9. As you were teaching this lesson, were you surprised by anything?

10. As you were teaching this lesson, were you uncertain about anything?

11. If you were teaching in your own classroom, would you have done anything differently in your lesson planning or your teaching?

12. Did you learn anything from teaching this lesson?
    a. What did you learn from teaching this lesson?

Appendix H

Coding protocol

Research question one: written reflections, focus group interviews, individual interviews, and post-observation interviews

Research question two: post-observation interviews

Code 1	Indicators
Generic	Classroom management Pedagogy
Specific	
Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Choice of examples Choice of representation Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between procedures Making connections between concepts Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities
Other	Not adequately captured by any of the above codes

Appendix I

The format for this protocol is as follows:

CATEGORY: KQ Dimension[1]: Each dimension is introduced with a brief description, which reflects current ideas and research about best practice in the field.

Coding Scheme. Each dimension can be coded one of the following 6 ways[2,3]:

Present: Appropriate—Maximum

The teacher demonstrated the dimension at a high, although not perfect, level.[4] Depending on the dimension, this may mean demonstrating the dimension several times or demonstrating it at a high level.

Present: Appropriate—Middle

The teacher demonstrated the dimension at a moderate level. Depending on the dimension, this may mean demonstrating the dimension two or more times time or demonstrating it at a moderate level.

Present: Appropriate—Minimum

The teacher demonstrated the dimension at an initial level. Depending on the dimension, this may mean demonstrating the dimension one time or demonstrating it at a low level.

Present: Inappropriate

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[1] This protocol is based on Rowland et al.'s (2009) Knowledge Quartet. Each dimension is included in the KQ model. Two dimensions, Connection: Recognition of conceptual appropriateness and Contingency: Use of opportunities, were eliminated from this protocol because they were redundant with other dimensions.

[2] Evidence for each dimension may be drawn from observed teaching and/or the post-observation interview.

[3] The one dimension that was not coded at six levels was Foundation: Overt subject knowledge, which was coded at two levels

The teacher demonstrated a dimension but based on the goals of the lesson or the developmental level of the students it was inappropriate for the given lesson.

Not Present: Appropriate:

In general this is not possible, because the premise is these descriptors of mathematics teaching are so fundamental to teaching and learning mathematics that it is never appropriate, or at least highly questionable, to omit them. One exception to this is if the teacher is thinking ahead to a subsequent lesson in which she will cover a concept in depth and therefore intentionally decides to not attend to that item/concept in the given lesson. Unless this is done intentionally, it is unlikely that this code will be used.

Not Present: Inappropriate

The teacher did not demonstrate a dimension in her lesson and this was detrimental to the lesson. That is to say, the lesson would have been improved by the presence of the dimension. When a dimension is not demonstrated, it will almost always be coded with this code, since the premise is these descriptors of mathematics teaching are so fundamental to teaching and learning mathematics that it is never appropriate, or at least highly questionable, to omit them.

FOUNDATION: **Adapts textbook:** This dimension was re-phrased from the original KQ dimension of "adheres to textbook" so that reverse scoring would not need to be done for this code when summarizing. Although the name has been changed, its definition remains the same as the original dimension.

Curricular materials serve as a starting point and a guide for teaching, but rigid adherence to given materials may indicate a lack of specialized content knowledge on the part of the teacher (Stein, Remillard & Smith, 2007). One aspect of this specialized knowledge is the ability to critically use curriculum, which is described by analyzing given or mandated curricular materials and modifying them in order to meet the needs of students (Rowland et al., 2009; Ball, Thames & Phelps, 2008). Therefore, this dimension considers whether the preservice teacher critically used the curriculum rather than strictly adhering to the given curriculum. Critical use entails careful selection of tasks rather than doing every problem provided and/or following the sequence exactly as written in the teacher's manual. It may also mean supplementing with additional resources and teaching strategies to improve on what is provided.

Appropriate: Present--Maximum

The preservice teacher taught a lesson that met the same goals and objectives as delineated in the curriculum or pacing guide and critically used the curriculum by carefully selecting tasks from the existing curriculum. Over 50% of the materials, examples, questions, or demonstrations were supplemental materials the preservice teacher added to the curriculum.

Appropriate: Present--Middle

The preservice teacher taught a lesson that met the same goals and objectives as delineated in the curriculum or pacing guide and critically used the curriculum by carefully selecting tasks from the existing curriculum. Less than 50% of the materials, examples, questions, or demonstrations were supplemental materials the preservice teacher added to the curriculum.

Present: Appropriate--Minimum

The preservice teacher taught a lesson that met the same goals and objectives as delineated in the curriculum or pacing guide and critically used the curriculum but no supplemental materials were added. The preservice teacher carefully selected materials, examples, questions, or demonstrations from the curriculum.

Present: Inappropriate

Either of the two following scenarios will be coded as Present: Inappropriate:

A) The preservice teacher taught a lesson that did not meet the same goals and objectives as delineated in the curriculum or pacing guide.

B) The preservice teacher taught a lesson that met the same goals and objectives as delineated in the curriculum or pacing guide. The preservice teacher used materials, examples, questions, or demonstrations to supplement the curriculum, however these resources were less effective than the ones she replaced from the curriculum; using the curriculum suggestions would have been more instructionally advantageous.

Not present: Appropriate

(Not possible)

Not present: Inappropriate

The preservice teacher demonstrated a lack of critical use of the curriculum. For example, the preservice teacher may use every problem given in the curriculum and/or do so in the same order as the manual suggests and does so without reflection.

FOUNDATION: Concentration on understanding: This dimension was re-phrased from the original KQ dimension of "concentration on procedures" so that reverse scoring would not need to be done for this dimension when summarizing. Although the name has been changed, its definition remains the same as the original dimension.

Current research indicates that an important element of effective mathematics teaching is a teacher's focus on developing the conceptual understanding rather than procedural knowledge of her students (Simon, 2006). Therefore, this dimension considers whether the teacher uses instructional practices to develop relational rather than instrumental understanding (Skemp, 1978). This is done by evaluating whether the preservice teacher's practices concentrate on the development of such understanding or focus on procedures. This focus is demonstrated through interactions with students including direct instruction, explanation, demonstration, and/or questioning. Each incident is tallied as either a focus on understanding or procedures.

Appropriate: Present--Maximum

The preservice teacher spends over 75% of the lesson focused on understanding. While explanations of procedures may be included, they are not the primary emphasis of the lesson.

Appropriate: Present—Middle

The preservice teacher spends 50-74% of the lesson focused on understanding. While explanations of procedures may be included, they are not the primary emphasis of the lesson.

Present: Appropriate--Minimum

The preservice teacher spends 25-49% of the lesson focused on understanding. The primary emphasis of the lesson is on procedural knowledge not conceptual understanding.

Present: Inappropriate

The preservice teacher spends the majority of the lesson focused on understanding of her students. However, the material at hand should have been addressed primarily procedurally. The preservice teacher's approach seems to obscure rather than clarify the content at hand.

Not Present: Appropriate

The preservice teacher spends the majority of the lesson focused on procedures rather than understanding. However, the material at hand lends itself to being addressed procedurally. The preservice teacher's approach seems to clarify rather than obscure the content at hand.

Not Present: Inappropriate

The preservice teacher spends less than 25% of the lesson focused on understanding. However, the material at hand does not lend itself to being addressed procedurally. The preservice teacher's approach seems to obscure rather than clarify the content at hand.

FOUNDATION: Identifying errors: One aspect of mathematical knowledge for teaching is a teacher's knowledge of common errors and misconceptions that are typical for students of a particular age to have regarding mathematical concepts and procedures (Rowland et al., 2009; Ball, Thames & Phelps, 2008; Munby & Russell, 2001). Therefore, this dimension is used to indicate whether the preservice teacher demonstrated this knowledge in her planning and delivery of the lesson and whether she took steps to address or avoid these errors or misconceptions.

Appropriate: Present--Maximum

The preservice teacher made two or more intentional instructional moves based on her knowledge of common errors and misconceptions. The preservice teacher attempted to counter-act these common errors and misconceptions with her students by intentionally using situations or tasks to address them. The preservice teacher also used discussion, explanation, demonstration, or questioning to explicitly highlight the misconceptions or errors.

Appropriate: Present--Middle

The preservice teacher made one intentional instructional move based on her knowledge of common errors and misconceptions. The preservice teacher attempted to counter-act this common error or misconception with her students by intentionally using a situation or task to address it. The preservice teacher also used discussion, explanation, demonstration, or questioning to explicitly highlight the misconception or error.

Present: Appropriate--Minimum

The preservice teacher made one intentional instructional move based on her knowledge of common errors and misconceptions. The preservice teacher attempted to counter-act this common error or misconception with her students by intentionally using a situation or task to address it. However, the preservice teacher did not include an explicit discussion, explanation, demonstration, or questioning to explicitly highlight the misconception or error.

Present: Inappropriate

The preservice teacher attempted to address or counter-act one or more common errors and misconceptions, but a) this was tangential to the goals and objectives of the lesson at hand or b) used an inappropriate example or task that she attempted to use to address the error or misconception.

Not present: Appropriate

The preservice teacher indicated that when planning the lesson, she thought about potential errors and misconceptions her students may have. Upon reflection, she realized that either a) addressing them in the given lesson would interrupt the flow or go beyond the scope of the given lesson, and so she decided to address it in a later lesson or b) her students were sufficiently well versed in these errors and misconceptions and therefore she did not need to spend time in the given lesson addressing them.

Not present: Inappropriate

The preservice teacher made no intentional instructional moves regarding common errors and misconceptions. The preservice teacher may indicate that she was unaware of any common errors or misconceptions related to the content at hand. Additionally, she may have further contributed to these errors and misconceptions.

FOUNDATION: Use of terminology: One aspect of mathematical knowledge for teaching is a teacher's knowledge of mathematically correct terminology and notation (Rowland et al., 2009; Ball, Thames & Phelps, 2008). This includes verbally using mathematically correct language as well as writing correct mathematical expressions. Therefore, this dimension is used to indicate whether the preservice teacher correctly used verbal and written notation in her lesson.

Appropriate: Present--Maximum

The preservice teacher made zero mistakes in her verbal or written notation during the lesson. The preservice teacher did engage in "press" with her students about the mathematical terminology or notation at hand. This level indicates a lack of error and an explicit discussion about terminology or notation with students by providing explanation, definition, discussion, or correction.

Appropriate: Present--Middle

The preservice teacher made zero mistakes in her verbal or written notation during the lesson. The preservice teacher did not engage in any "press" with her students about the mathematical terminology or notation at hand. This level indicates a lack of error and also a lack of explicit discussion about terminology or notation with her students.

Present: Appropriate--Minimum

The preservice teacher made one or two mistakes in her verbal or written notation during the lesson.

Present: Inappropriate

(Not possible)

Not present: Appropriate

The preservice teacher indicated that when planning the lesson, she thought about the terminology she would use with her students during the lesson. Upon reflection, she thought her instruction would be more effective if she used different terminology in the lesson which she would build upon and use mathematically correct language in subsequent lessons, using today's vocabulary as a foundation for more complicated terms to come. An example of this is introducing younger children to the commutative property and using other terms that the children generate to describe it, with the intention of providing the proper terminology in a subsequent lesson. Likewise, a teacher may intend for notation to be formalized in a subsequent lesson.

Not present: Inappropriate

The preservice teacher used terminology or notation that is mathematically inaccurate more than three times in a lesson either verbally or in her written notation. For example, instead of referring to the hypotenuse of a triangle as such, she called it "slanty." The preservice teacher is also coded as NP:I if she did not demonstrate accuracy in her written notation, for example using the equals sign incorrectly when writing out a long series of steps in a solution.

FOUNDATION: Overt Subject Knowledge: One aspect of mathematical knowledge for teaching is the teacher's content knowledge of the topic they are teaching; clearly a teacher cannot teach what she does not know (Shulman, 1987; Rowland et al., 2009; Ball, Thames & Phelps, 2008). Therefore, this dimension indicates the teacher's subject knowledge as displayed in her teaching. Indicators of such knowledge include demonstrating an accurate understanding of mathematical ideas or concepts (e.g., adding a 0 is not helpful when multiplying by 10), demonstrating a good understanding of the processes involved in operations, and demonstrating a knowledge of quick mental methods (Rowland et al., 2009). These components of subject matter knowledge may be demonstrated in explanations of mathematical concepts and processes.

One additional note regarding this dimension is that it is difficult to ascertain a teacher's content knowledge based on her teaching. Just because she does not make any mistakes in a lesson does not mean she has "maximum" content knowledge. The intention of this code is not to determine the high end of the scale but rather to note when content knowledge was obviously lacking and detracted from the lesson. Because of the difficulty and inferences needed to assess content knowledge based on teaching, this dimension was evaluated at only two of the six levels to indicate whether content knowledge was adequate or inadequate.

Present: Appropriate

This code was used when the preservice teacher made no content-related mistakes in her lesson. In her demonstration and explanation throughout the lesson she always presented content accurately. This code operates independently of pedagogy; that is to say, even if she did not present the information clearly or in the "best" way she was still coded at this level if she made no content-related errors. If she made a mistake that she corrected (for example making a computational error) she would still be coded at this level.

Not Present: Inappropriate

This code was used when the preservice teacher made one or more content-related mistakes in her lesson. These may have occurred in her demonstration or explanation throughout the lesson. This code operates independently of pedagogy; that is to say, even if she present information clearly or in the "best" way she was still coded at this level if she made content-related errors.

FOUNDATION: Awareness of purpose: One aspect of a teacher's mathematical knowledge for teaching is her ability to consider the purpose of a lesson as it relates to previous and subsequent lessons, the given concept, related concepts, and broader notions about how the idea relates to understanding mathematics (Ball, Lubienski & Mewborn, 2001; Rowland et al., 2009). Therefore, this dimension will be defined by the level of broadness that the preservice teacher used to describe the purpose of the given lesson.

Appropriate: Present--Maximum

The preservice teacher broadly described the purpose of the lesson across mathematics. She explained how the lesson related to other domains within mathematics, and discussed the purpose of the lesson in relation to at least one additional domain or mathematical idea within K-8 mathematics. The purpose of the lesson was described in both conceptual and instrumental terms (Skemp, 1978).

Appropriate: Present--Middle

The preservice teacher described the purpose of the lesson across the given concept. She explained the purpose of the lesson in relation to at least one additional idea within the same concept. For example, for a lesson on fractions, she described one Key Developmental Understanding (Simons, 2006) within fractions to which the lesson related (e.g., understanding of equivalence or part-whole relationships). The purpose of the lesson was described in both conceptual and instrumental terms (Skemp, 1978).

Present: Appropriate-Minimum

The preservice teacher described the purpose of the lesson in relation to the specific concept being addressed in the lesson. The preservice teacher described the purpose of the lesson in terms of preparing for the next lesson within the same topic or in regards to developing instrumental knowledge (e.g., preparing to add and subtract fractions) (Skemp, 1978).

Present-Inappropriate

The preservice teacher described the purpose of the lesson across the given concept. However, she did so in a way that was mathematically inaccurate regarding her perception of the relationships between mathematical concepts or domains.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The preservice teacher was unable to provide an explanation for the purpose of the lesson at hand.

FOUNDATION: Theoretical Underpinning: One aspect of Mathematical Knowledge for Teaching is a teacher's use of a theoretical foundation to guide instructional decisions, rather than relying on imitation of another teacher or trial and error. Therefore, it is important for preservice teachers to know factors that are significant in the teaching and learning of mathematics (Rowland et al., 2009; Ball, Lubienski & Mewborn, 2001). One way preservice teachers demonstrate such knowledge is by referring to the writings of mathematics educators. Therefore, the levels on this dimension evaluate whether preservice teachers made any pedagogical decisions related to the particular topic of the lesson (e.g., principles of counting), whether they made any pedagogical decisions related to broad notions of mathematics teaching and learning (e.g., problem-solving, construction of knowledge, discussion of mathematical ideas), or whether pedagogical decisions were not specific to mathematics but rather were generic to teaching (e.g, behavior management concerns).

Appropriate: Present--Maximum

The preservice teacher made pedagogical decisions based on broad notions of mathematics teaching and learning (e.g., having students work in groups, the use of discussion) as well as the particular topic of the lesson (e.g., principles of counting), which was demonstrated by two or more topic-related pedagogical moves in the lesson.

Appropriate: Present--Middle

The preservice teacher made pedagogical decisions based on broad notions of mathematics teaching and learning (e.g., having students work in groups, the use of discussion) as well as the particular topic of the lesson (e.g., principles of counting), which was demonstrated by one topic-related pedagogical move in the lesson.

Present: Appropriate--Minimum

The preservice teacher made pedagogical decisions based on broad notions of mathematics teaching and learning, such as having students work in groups or the use of discussion, but not based on the particular topic of the lesson. These instructional strategies are generic to all domains of mathematics.

Present: Inappropriate

The preservice teacher made pedagogical decisions based on principles of the given content area (e.g., principles of counting). However, the preservice teacher did so in a way that was mathematically inaccurate.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The preservice teacher made pedagogical decisions based on generic aspects of teaching, such as classroom management concerns, but were not particular to mathematics.

TRANSFORMATION: Choice of examples: One aspect of Mathematical Knowledge for Teaching is a teacher's ability to select or create examples with which her students will interact (Rowland et al., 2009; Ball, Thames & Phelps, 2008). This includes the ability to match a concept, skill, or objective with corresponding examples. This includes selecting high (as opposed to low) cognitive demand tasks (Stein, Smith, Henningsen & Silver, 2000; Stein, 2007). One aspect of selecting or creating tasks is the purposeful use of numbers and operations so that the use of the particular numbers or operations focuses students' attention on the desired concept or strategy.

Appropriate: Present--Maximum

One or more examples or tasks in the lesson are High cognitive demand: Doing mathematics. Other examples or tasks may be lower levels of cognitive demand.

Appropriate: Present--Middle

Two or more examples or tasks in the lesson are High cognitive demand: Procedures with connections. Other examples or tasks may be lower levels of cognitive demand.

Present: Appropriate--Minimum

One example or task in the lesson is High cognitive demand: Procedures with connections. Other examples or tasks may be lower levels of cognitive demand.

Not Present: Appropriate

(Not possible)

Present: Inappropriate

An example is provided that is potentially High Cognitive Demand, except it is either not an appropriate match for the age group or developmental level of the students for whom it is intended or it fails to match the goals and objectives of the given lesson.

Not Present: Inappropriate

All examples or tasks in the lesson are Low cognitive demand: Memorization or Procedures without connections.

TRANSFORMATION: Choice of representation: One aspect of mathematical knowledge for teaching is a teacher's ability to select representations in order to transform her students' understanding of a concept (Rowland et al., 2009; Ball, Thames & Phelps, 2008). Therefore, it is important that a teacher can select a representation that reflects the concepts and/or procedures the teacher wants her students to learn and use the selected representation correctly to explain the given mathematical concepts or procedures. Examples of representations include a number line, place value grid, or manipulatives.

Appropriate: Present--Maximum

The preservice teacher selected two or more representations that matched the concepts and/or procedures that were the focus of the lesson. The teacher also used the representations to focus students' attention on the salient features of the mathematics at hand.

Appropriate: Present--Middle

The preservice teacher selected one representation that matched the concepts and/or procedures that were the focus of the lesson. The teacher also used the representation to focus students' attention on the salient features of the mathematics at hand.

Present: Appropriate--Minimum

The preservice teacher selected at least one representation that matched the concepts and/or procedures that were the focus of the lesson. However, the teacher did not use the representation(s) to focus students' attention on the salient features of the mathematics at hand.

Present: Inappropriate

The preservice teacher selected representation(s) that either did not reflect the concepts and/or procedures pertinent to the given lesson, or used the selected representation incorrectly when trying to explain the given mathematical concepts or procedures.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The preservice teacher did not select or use any representations during a lesson.

TRANSFORMATION: Demonstration: Another aspect of Mathematical Knowledge for Teaching is the teacher's ability to demonstrate mathematics to her students. This means demonstrating clearly and accurately how to carry out procedures as well as giving clear explanations of mathematical ideas or concepts (Rowland et al., 2009; Ball, Thames & Phelps, 2008).

Appropriate: Present--Maximum

The preservice teacher demonstrated procedures accurately throughout the lesson. The preservice teacher's demonstrations focused on a conceptual understanding of mathematics 75% or more of the time (Skemp, 1978).

Appropriate: Present--Middle

The preservice teacher demonstrated procedures accurately throughout the lesson. The preservice teacher's demonstrations focused on a conceptual understanding of mathematics 50-74% of the time (Skemp, 1978).

Present: Appropriate--Minimum

The preservice teacher demonstrated procedures accurately throughout the lesson. The preservice teacher's demonstrations focused on a conceptual understanding of mathematics 25-49% of the time (Skemp, 1978).

Present: Inappropriate

The preservice teacher demonstrated procedures inaccurately or gave mathematically inaccurate explanations of ideas or concepts or the preservice teacher's demonstrations focused on a conceptual understanding of mathematics less than 25% of the time (Skemp, 1978).

Not Present: Appropriate

Depending on the topic/concept, at times it may be appropriate to intentionally *not* demonstrate the steps of a procedure. For example, when encouraging students to generate their own solution strategies, it would be appropriate for the teacher to not demonstrate the steps in the procedure but rather to leave it open-ended in order to encourage student thinking. When a demonstration is not included in a lesson, the post-observation interview will be used to determine the intentionality of the preservice teacher in this regard.

Not Present: Inappropriate

The preservice teacher did not attempt to demonstrate procedures or give explanations of ideas or concepts even though it seems that this would have been helpful to students.

TRANSFORMATION: **Questioning to assess and develop knowledge:** Another aspect of Mathematical Knowledge for Teaching is a teacher's ability to ask questions, the answers to which provide her with information to assess students' knowledge or further develop their knowledge (Rowland et al., 2009; Ball, Thames & Phelps, 2008).). Not all questions are equally effective. Open-ended questions ("What shape is this?") are more likely to assess and develop knowledge than are either/or questions ("Is it a square or a rectangle?") or yes/no questions ("Is this a square?") because of their relative cognitive demand.

Appropriate: Present--Maximum

75% or more of the questions asked by the preservice teacher were open-ended and 25% or less of the questions were either/or or yes/no questions.

Appropriate: Present--Middle

50-74% of the questions asked by the preservice teacher were open-ended and 26-50% of the questions were either/or or yes/no questions.

Present: Appropriate--Minimum

25-49% of the questions asked by the preservice teacher were open-ended and 51-75% of the questions were either/or or yes/no questions.

Present: Inappropriate

The preservice teacher asked questions that focused on tangential aspects of mathematics, did not match the current level of learning for the student(s) to whom the questions were directed or took an inordinate amount of time within the lesson.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

Less than 25% of the questions asked by the preservice teacher were open-ended and over 75% of the questions were either/or or yes/no questions.

CONNECTION: Anticipation of complexity: Mathematical Knowledge for Teaching includes awareness on the part of the teacher that children view and experience mathematics differently than adults (Carpenter et al., 1999). A teacher must also be aware of the different levels of difficulty that exist within a topic or concept. Finally, a teacher must use this knowledge to anticipate the complexity of an idea and break it down into steps that can be understood by her students (Rowland et al., 2009).

Appropriate: Present--Maximum

The preservice teacher anticipated the complexity of an idea(s) or concept(s) and made three or more instructional decision as a result. This may be evident in her demonstrations, examples, sequencing, and/or questioning, and was verified in her post-observation interview.

Appropriate: Present--Middle

The preservice teacher anticipated the complexity of an idea(s) or concept(s) and made two instructional decisions as a result. This may be evident in her demonstrations, examples, sequencing, and/or questioning, and was verified in her post-observation interview.

Present: Appropriate--Minimum

The preservice teacher anticipated the complexity of an idea or concept and made one instructional decision as a result. This may be evident in her demonstrations, examples, sequencing, and/or questioning, and was verified in her post-observation interview.

Present: Inappropriate

(Not possible)

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The preservice teacher did not anticipate the complexity of any ideas or concepts in the lesson and made no instructional decision as a result. This was verified in her post-observation interview.

CONNECTION: Decisions about sequencing: Another aspect of Mathematical Knowledge for Teaching is a teacher's ability to sequence instructional activities in an intentional way. Ideas and strategies need to be introduced in a progressive order (Rowland et al., 2009). Therefore, there are three aspects of sequencing instruction: sequencing across a unit (linking the given lesson to the previous lesson), sequencing across a lesson (ordering the sections of the given lesson and topics therein) and sequencing of examples (using examples in a progressive and intentional way).

Appropriate: Present--Maximum

The preservice teacher demonstrated all of the following: sequencing across the unit, sequencing across the lesson, or sequencing of examples. This was verified in the post-observation interview.

Appropriate: Present--Middle

The preservice teacher demonstrated two of the following: sequencing across the unit, sequencing across the lesson, or sequencing of examples. This was verified in the post-observation interview.

Present: Appropriate--Minimum

The preservice teacher demonstrated one of the following: sequencing across the unit, sequencing across the lesson, or sequencing of examples. This was verified in the post-observation interview.

Present: Inappropriate

(Not possible)

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The preservice teacher demonstrated none of the following: sequencing across the unit , sequencing across the lesson, or sequencing of examples. This was verified in the post-observation interview.

CONNECTION: Making connections between procedures: One aspect of Mathematical Knowledge for Teaching is a teacher's ability to focus students' attention on the connections that exist between mathematical procedures (Rowland et al., 2009; Ball, Thames & Phelps, 2008). Examples of this include making connections between operations of fractions and operations of whole numbers, between repeated addition and multiplication, and between different solution strategies for the same problem.

Appropriate: Present--Maximum

During the lesson the teacher identified a connection between procedures. The connection is stated and explained and/or discussed. Similarities and/or differences between the two procedures are explained and/or discussed.

Appropriate: Present--Middle

During the lesson the teacher identified a connection between procedures. The connection is stated and briefly explained and/or discussed. Similarities and/or differences between the two procedures are not explained and/or discussed.

Present: Appropriate--Minimum

During the lesson the teacher identified a connection between procedures. The connection is stated but not explained. The connection is mentioned but not discussed.

Present: Inappropriate

The teacher identified a connection between procedures in a way that was mathematically inaccurate.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The teacher identified no connections between procedures throughout the entire lesson.

CONNECTION: Making connections between/within concepts: One aspect of Mathematical Knowledge for Teaching is a teacher's ability to make conceptual connections within the subject matter (Rowland et al., 2009; Ball, Thames & Phelps, 2008). For example, when teaching a lesson on fractions of shapes, a teacher may refer back to previously discussed concepts regarding fractions of numbers (Rowland et al., 2009). In this way students come to see mathematics as a connected body of knowledge rather than a series of discrete topics.

Appropriate: Present--Maximum

During the lesson the teacher identified a connection between concepts. The connection is stated and explained and/or discussed. Similarities and/or differences between the two concepts are explained and/or discussed.

Appropriate: Present--Middle

During the lesson the teacher identified a connection between concepts. The connection is stated and briefly explained and/or discussed. Similarities and/or differences between the two concepts are not explained and/or discussed.

Present: Appropriate--Minimum

During the lesson the teacher identified a connection between concepts. The connection is stated but not explained. The connection is mentioned but not discussed.

Present: Inappropriate

The teacher identified a connection between concepts in a way that was mathematically inaccurate.

Not Present: Appropriate

(Not possible)

Not Present: Inappropriate

The teacher identified no connections between concepts throughout the entire lesson.

CONTINGENCY: Deviation from agenda: Although a teacher can intentionally plan many aspects of her lesson ahead of time and anticipate likely student responses, she cannot fully plan how students will respond. Therefore, another aspect of Mathematical Knowledge for Teaching is a teacher's ability to deviate from her lesson plan. Even with an intentionally planned lesson, it may be necessary or beneficial to alter the lesson as it is enacted to better respond to the students (Rowland et al., 2009).

The coding levels for Deviation are different from other dimensions because the amount of deviation is not the most helpful descriptor for this dimension. That is to say, more deviation is not necessarily better than less deviation. Therefore, the purpose of coding this dimension was to denote the "type" rather than "amount" of deviation.

Appropriate: Present--Maximum

The preservice teacher deviated from her lesson plan at least one time for a mathematical reason. When she deviated for a mathematical reason she did so in a way that focused on conceptual understanding. The preservice teacher may have also deviated at other times in a mathematically procedural or generic way. These changes were verified during the post-observation interview.

Appropriate: Present—Middle

The preservice teacher deviated from her lesson plan at least one time for a mathematical reason. When she deviated for a mathematical reason she did so in a way that focused on procedural knowledge. The preservice teacher may have also deviated at other times in a generic way, for example in response to adjusting the pacing of the lesson by going faster, slower, or skipping an example. These changes were verified during the post-observation interview.

Present: Appropriate--Minimum

The ST deviated from her lesson plan at least one time. When she deviated from her lesson plan she always did so in a generic way, for example in response to adjusting the pacing of the lesson by going faster, slower, or skipping an example. This change was verified during the post-observation interview.

Present: Inappropriate

The preservice teacher deviated from her lesson plan in a way that omitted an example or demonstration that would have been helpful to retain in the lesson. This change was verified during the post-observation interview.

Not Present: Appropriate

The preservice teacher did not deviate from her lesson plan. There were no instances in the lesson where it would have been beneficial for the ST to adjust the lesson as it was being taught.

Not Present: Inappropriate

The preservice teacher did not deviate from her lesson plan. There was at least one instance in the lesson where it would have been beneficial for the ST to adjust the lesson as it was being taught, but she did not.

CONTINGENCY: **Responding to children's ideas**: One aspect of mathematical knowledge for teaching is a teacher's ability to respond to the interactions that take place during a lesson. These include verbal comments, questions, and correct and incorrect answers and statements made by students (Rowland et al., 2009; Ball, Thames & Phelps, 2008; Franke, Kazemi & Battey, 2007). A teacher must also address students' responses to the activities in the lesson. Each response is an opportunity for the teacher to focus on conceptual or procedural components of mathematics (Skemp, 1978).

Present: Appropriate--Maximum

The preservice teacher accurately responds to students' comments, questions, answers and statements. The preservice teacher's responses, which include answers, questions, explanations, demonstrations, and/or deviation, focused on conceptual understanding 75% or more of the time.

Present: Appropriate--Middle

The preservice teacher accurately responds to students' comments, questions, answers and statements. The preservice teacher's responses, which include answers, questions, explanations, demonstrations, and/or deviation, focused on conceptual understanding 50-74% of the time.

Present: Appropriate--Minimum

The preservice teacher accurately responds to students' comments, questions, answers and statements. The preservice teacher's responses, which include answers, questions, explanations, demonstrations, and/or deviation, focused on conceptual understanding 25-49% of the time.

Present: Inappropriate

The preservice teacher responds to students' comments, questions, answers and/or statements in a way that is mathematically inaccurate.

Not Present: Appropriate

At times a teacher may intentionally choose not to respond to a student's comment or question at the time it is presented. This may be done so that the teacher can come back to the idea later in the lesson or in a future lesson. During the post-observation interview the preservice teacher explained why she deferred her response in such terms.

Not Present: Inappropriate

The preservice teacher failed to respond to a student's comment or question.

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