

KIRBY N. SMITH: Computer Modeling of Contaminant Jet Flow into Local Exhaust Hoods. (Under the direction of Assistant Professor, Michael R. Flynn, Sc.D.).

A computer model was developed and coded in BASIC to predict the streamline that a jet of gaseous sulfur hexafluoride will follow in the flow field of a flanged circular exhaust hood (FCH). This approximate solution is based on the vector addition of a modified potential flow solution for the FCH, and a jet flow solution. The assumptions underlying the equations describing jet flow are those of the Prandtl mixing length hypothesis. The computer program generates streamlines for the combined flow by means of iterative vector addition. The interactive program prompts the user for the hood and jet diameters and flows, and the distance from the hood at which the jet is placed. A graphic plot of the predicted streamline followed by the gas jet is displayed.

The program is used to predict the critical distance  $[Z/D]_{50}$ , the distance along the hood centerline ( $Z$ ), as a fraction of the hood diameter ( $D$ ), where the jet can be placed such that 50% of the jet contaminant flow is captured. A series of such  $[Z/D]_{50}$  values was generated for twenty-one hood and jet flow combinations.

The program was validated in the laboratory. A probe was placed in the duct of a flanged circular exhaust hood and was connected to an electron-capture gas chromatograph, to determine the concentration of  $\text{SF}_6$  in the hood. Capture efficiencies (ratios of "captured" gas concentrations at various jet-hood distances to concentrations in the duct when the jet flow is fully captured) were determined for jet positions at intervals along the hood centerline. Five replicate measurements were collected per position, for all combinations of jet and hood flow.

Results indicate that the model is quite accurate when crossdrafts are accounted for, except for predicted  $[Z/D]_{50}$  values of less than 0.7, which occur quite close to the hood face. The approximate model errs in this region because it neglects the effects on the jet of the static pressure gradient created by the flow of the exhaust hood, and the shear turbulence of the interacting streamlines of jet and hood flow.

The model may be expanded in the future to include definitive crossdraft variations, other jet locations or directions, hoods of other shapes, or heat and gas buoyancy effects.

Key Words: Critical distance, flanged circular exhaust hood, capture efficiency, ventilation.

# TABLE OF CONTENTS

iii

|                                                                                     |                   |
|-------------------------------------------------------------------------------------|-------------------|
| LIST OF FIGURES                                                                     | <u>Page</u><br>iv |
| LIST OF TABLES                                                                      | v                 |
| LIST OF APPENDICES                                                                  | vi                |
| ACKNOWLEDGEMENTS                                                                    | vii               |
| I. INTRODUCTION                                                                     | 1                 |
| II. LOCAL EXHAUST VENTILATION DESIGN                                                |                   |
| A. CAPTURE VELOCITY CONCEPTS                                                        | 5                 |
| B. CAPTURE EFFICIENCY CONCEPTS                                                      | 18                |
| III. BACKGROUND TO THE COMPUTER MODEL                                               |                   |
| A. ELEMENTS OF POTENTIAL THEORY                                                     | 27                |
| B. POTENTIAL FLOW SOLUTIONS FOR<br>FLANGED CIRCULAR HOODS                           | 31                |
| C. CIRCULAR JET FLOW AND<br>THE SCHLICHTING EQUATIONS                               | 36                |
| IV. PURPOSE AND OBJECTIVES                                                          | 49                |
| V. METHODS                                                                          |                   |
| A. COMPUTER MODEL                                                                   | 52                |
| B. LABORATORY VALIDATION                                                            | 58                |
| C. STATISTICAL EVALUATION                                                           | 62                |
| VI. RESULTS AND DISCUSSION                                                          |                   |
| A. LOGIT CAPTURE EFFICIENCY REGRESSION<br>ON $[Z/D]$ RATIOS                         | 64                |
| B. PREDICTED VS. EXPERIMENTAL $[Z/D]_{50}$<br>CRITICAL DISTANCES                    | 67                |
| C. HOOD TO JET FLOW RATIOS VS. $[Z/D]_{50}$ :<br>PREDICTIONS AND CROSSDRAFT EFFECTS | 73                |
| D. THE ROLE OF $\epsilon_0$ , THE APPARENT<br>KINEMATIC (EDDY) VISCOSITY            | 80                |
| VII. CONCLUSIONS AND RECOMMENDATIONS                                                | 82                |
| VIII. REFERENCES                                                                    | 86                |
| APPENDICES                                                                          |                   |

## LIST OF FIGURES

|                                                                                         | <u>Page</u> |
|-----------------------------------------------------------------------------------------|-------------|
| 1. EXHAUST AIR SYSTEM                                                                   | 4           |
| 2. HOOD DESIGN TYPES                                                                    | 6           |
| 3. EQUAL VELOCITY CONTOURS                                                              | 9           |
| 4. HOOD FLOW NOMOGRAM                                                                   | 13          |
| 5. POTENTIAL LINES AND STREAMLINES FOR AN FCH,<br>DISTURBED BY A CROSSDRAFT             | 22          |
| 6. IDEALIZED CONTAMINANT STREAMLINES                                                    | 24          |
| 7. STREAMLINES IN A FREE CIRCULAR TURBULENT JET                                         | 37          |
| 8. VELOCITY PROFILES FOR AN AXISYMMETRIC JET                                            | 38          |
| 9. AXISYMMETRIC JET DIAGRAM                                                             | 40          |
| 10. THE MIXING LENGTH CONCEPT                                                           | 42          |
| 11. VELOCITY DISTRIBUTION IN A CIRCULAR TURBULENT JET                                   | 48          |
| 12. EXPERIMENTAL DESIGN                                                                 | 57          |
| 13. EXPERIMENTAL vs. PREDICTED $[Z/D]_{50}'s$                                           | 68          |
| 14. EXPERIMENTAL AND PREDICTED $[Z/D]_{50}'s$ vs. $[Q_h/Q_j]$ ,<br>at 25 fpm CROSSDRAFT | 74          |
| 15. EXPERIMENTAL AND PREDICTED $[Z/D]_{50}'s$ vs. $[Q_h/Q_j]$ ,<br>at no CROSSDRAFT     | 75          |
| 16. EXPERIMENTAL AND PREDICTED $[Z/D]_{50}'s$ vs. $[Q_h/Q_j]$ ,<br>at 5 fpm CROSSDRAFT  | 76          |

## LIST OF TABLES

|                                                                                                                                               | <u>Page</u> |
|-----------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| 1. CAPTURE VELOCITIES                                                                                                                         | 7           |
| 2. EMPIRICAL DESIGN DATA FOR NON-DIMENSIONAL<br>CENTERLINE VELOCITY GRADIENTS                                                                 | 16          |
| 3. SUMMARY OF INITIAL RESULTS                                                                                                                 | 65          |
| 4. VALUES OF $\epsilon_0$ , AND OF CROSSDRAFT, REQUIRED TO<br>MATCH PREDICTED $[Z/D]_{50}$ WITH EXPERIMENTAL<br>(MODIFIED LOGIT) $[Z/D]_{50}$ | 79          |

## LIST OF APPENDICES

- A. RESEARCH COMPUTER PROGRAM
- B. COMPUTER PREDICTIONS OF  $[Z/D]_{50}$ 's AT CROSSDRAFT=25fpm  
(3 EXAMPLES)
- C. EXPERIMENTAL DATA, CALIBRATION THEREOF, AND  
EFFICIENCY CALCULATIONS
- D. DATA FOR REGRESSION OF LOGIT-TRANSFORMED EFFICIENCY  
ON  $[Z/D]$ ; CALCULATION OF  $\omega$  AND  $\mu$
- E. LOGIT CAPTURE EFFICIENCY REGRESSION ON  $[Z/D]$
- F. EXPERIMENTAL DATA: RESULTS SUMMARY
- G. "EPSILON" MODIFICATION OF RESEARCH COMPUTER PROGRAM

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## I. INTRODUCTION

Industrial hygienists typically use a variety of control measures to abate the danger to workers of inhaling toxic materials. These may include engineering or administrative controls, and possibly the use of personal protective equipment. Because inhaled toxic materials may give rise to a variety of deleterious health effects, it is important to minimize such exposures.

Engineering controls are easily the more desirable of protective measures because they ensure that the worker is actually exposed to the toxin or otherwise hazardous material as little as possible. Engineering controls in general do not require active participation on the part of the worker to be effective (controls are "designed in"), and are therefore recommended over measures requiring considerable training and, especially, supervision, such as personal protective equipment or even administrative rotation [1].

Ventilation is a desirable and useful engineering control. Dilution ventilation reduces the air concentration of toxin in the entire work area by bringing in uncontaminated air with which it is diluted. Dilution ventilation is useful when the contaminant concentration or toxicity is fairly low, if the contaminant is released reasonably uniformly in the workroom, and if the worker(s) are somewhat removed in location from the process.

Otherwise, dilution ventilation is insufficient. Local exhaust ventilation is particularly necessary for close work with concentrated toxic materials.

Local exhaust ventilation (LEV) is most usefully designed so the contaminant does not have a chance to escape in quantity into the room air. When LEV is properly designed, other forms of protection, such as masks and/or respirators may not be necessary. The basic elements of LEV consist of a hood or hoods, ductwork, fan(s) and an air cleaning system [2].

Hoods preferably are designed to be enclosures encompassing the exhaust from the entire process. When this is not possible, the hood may be placed to receive or capture the bulk of the air flow from a process, and should be placed as close to the process as possible. Receiving hoods are placed so that the contaminant material will flow into them. Grinding wheel hoods and canopies over hot processes are receiving hoods. "Capture" hoods on the other hand must be designed so the ventilation system creates a strong enough flow field to entrain and capture the contaminant. LEV hood design will be reviewed in the next section.

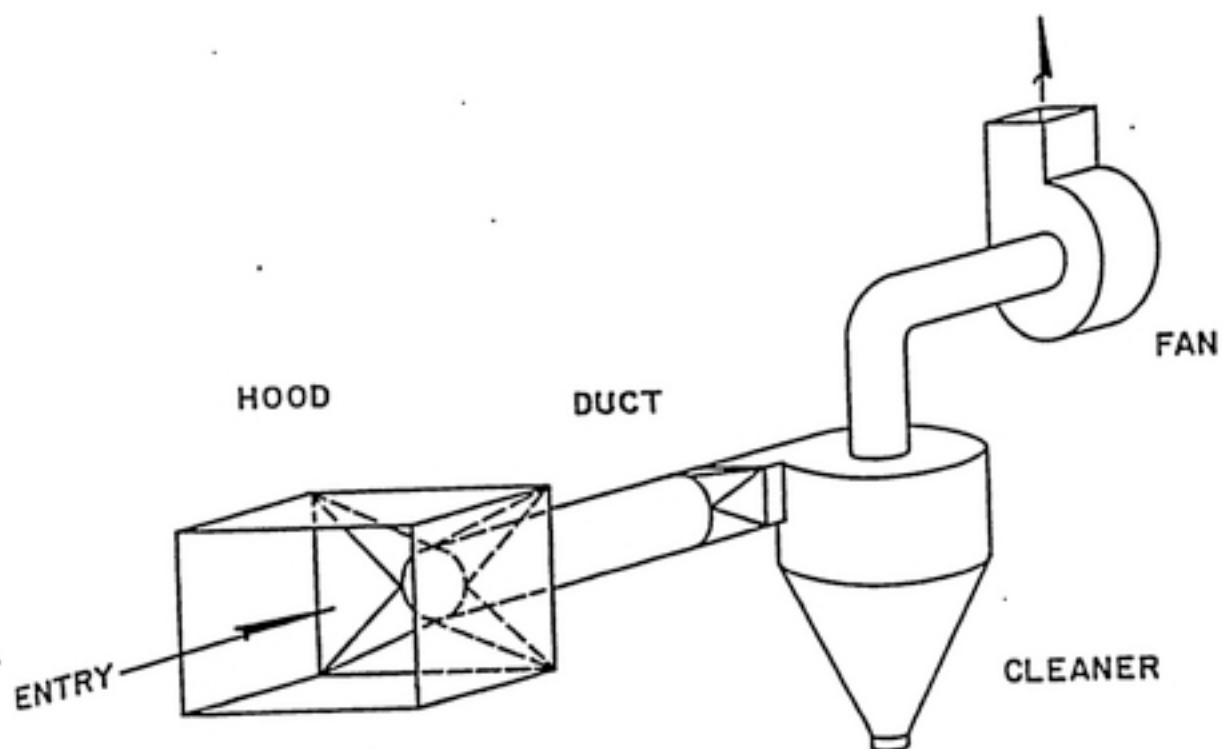
The contaminant in air is removed through the ductwork to the air cleaning device by the fan. By creating in the ductwork a static pressure differential negative to the atmosphere, the specifically-chosen fan moves a quantity of air with a certain velocity. Ductwork and air cleaner

design depend on the process employed, its temperature, the particle size and density of the material expelled, the toxicity thereof, and the cleaning efficiency required [Figure 1].

The air cleaning device removes the contaminant from the airstream brought to it by the ventilation system described above. Air is usually exhausted to the outside atmosphere through an exhaust stack once the particulates and toxins have been largely removed. Under certain circumstances, e.g., where atmospheric air would have to be excessively heated or otherwise conditioned, some proportion of exhaust air may be recirculated.

Designs of the LEV systems have remained fairly stagnant since World War II, partially because the older methods were seen as "adequate." Until the 1980s a relative lack of theoretical work was available which would affect system design concepts. The goal of such theoretical work is not only to understand better the fundamentals, but is also to provide workers with better protection for the engineering dollar spent.

FIGURE 1.  
EXHAUST AIR SYSTEM



SOURCE: REFERENCE 3

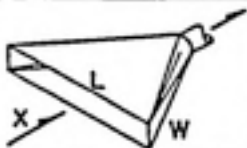
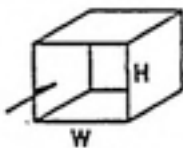
## II. LOCAL EXHAUST VENTILATION DESIGN

### A. CAPTURE VELOCITY CONCEPTS

Many different configurations of hood designs are possible for control of the exhaust of every conceivable industrial process. Nonetheless there are a few standard designs that are used routinely, and which have been tested widely. Slots, rectangular and round openings are the most common; cabinets and booths are used to enclose whole processes, and canopies are placed over evaporative processes [Figure 2]. Traditionally, local exhaust ventilation designs relied on a single unifying concept, that of "capture velocity." The design equations developed by Dalla Valle and Silverman in the 1930s all rely on this design parameter, and it is the primary focus of designs still promulgated by the ACGIH, in their Industrial Ventilation Manual [3].

Velocity must be sufficient to entrain the contaminant in the airflow toward the hood so it does not disperse or settle out before being "captured" by the exhaust system. Particular processes generate contaminants of different characteristics (gaseous vs. particulate; light vs. heavy or dense particles; contaminants released with low or very high initial velocity). Each characteristic should contribute to the evaluation of the capture velocity necessary [Table 1]. Then the volumetric flow ( $Q$ ) necessary may be calculated

FIGURE 2.  
HOOD DESIGN TYPES

| HOOD TYPE                                                                                            | DESCRIPTION     | ASPECT RATIO $\frac{W}{L}$  |
|------------------------------------------------------------------------------------------------------|-----------------|-----------------------------|
|                     | SLOT            | 0.2 OR LESS                 |
|                     | FLANGED SLOT    | 0.2 OR LESS                 |
| <br>$A=WL$ (sq.ft.) | PLAIN OPENING   | 0.2 OR GREATER<br>AND ROUND |
|                    | FLANGED OPENING | 0.2 OR GREATER<br>AND ROUND |
|                   | BOOTH           | TO SUIT WORK                |
|                   | CANOPY          | TO SUIT WORK                |

SOURCE: REFERENCE 3

TABLE 1.  
CAPTURE VELOCITIES

| Condition of Dispersion<br>of Contaminant                                | Examples                                                                                                     | Capture Velocity, fpm |
|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|-----------------------|
| Released with practically no<br>velocity into quiet air.                 | Evaporation from tanks; degreasing,<br>etc.                                                                  | 50-100                |
| Released at low velocity into<br>moderately still air.                   | Spray booths; intermittent container<br>filling; low speed conveyor transfers;<br>welding; plating; pickling | 100-200               |
| Active generation into zone of<br>rapid air motion                       | Spray painting in shallow booths;<br>barrel filling; conveyor loading;<br>crushers                           | 200-500               |
| Released at high initial velocity<br>into zone of very rapid air motion. | Grinding; abrasive blasting, tumbling                                                                        | 500-2000              |

In each category above, a range of capture velocity is shown. The proper choice of values depends on several factors:

| <u>Lower End of Range</u>                                  | <u>Upper End of Range</u>         |
|------------------------------------------------------------|-----------------------------------|
| 1. Room air currents minimal or favorable to capture.      | 1. Disturbing room air currents.  |
| 2. Contaminants of low toxicity or of nuisance value only. | 2. Contaminants of high toxicity. |
| 3. Intermittent, low production.                           | 3. High production, heavy use.    |
| 4. Large hood—large air mass in motion.                    | 4. Small hood—local control only. |

SOURCE: REFERENCE 3

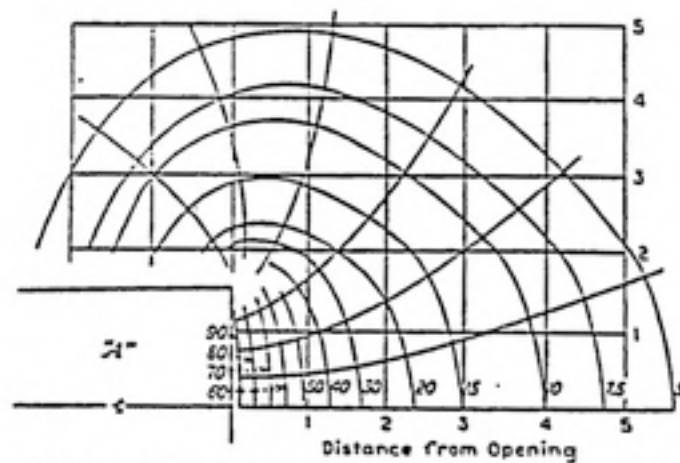
easily, in conjunction with the use of a "VS Print" (a plan of a ventilation system typical for the process) taken from or adapted from the ACGIH Manual.

For standard hood configurations such as round or rectangular, flanged or unflanged hoods, Dalla Valle [4] developed the original "rule-of-thumb" equations. He mathematically related several variables he found to be characteristic of hood velocity values he measured at various locations in front of LEV hoods. In general Dalla Valle established the concept of the centerline velocity gradient as a function of distance from the hood ( $X$ ), volume airflow ( $Q$ ), and hood shape and flanging. He showed that the surfaces of equal velocity into an exhaust hood were of the same shape and relative position for all similarly shaped hoods. While he mistakenly equated equal velocity contours with equipotential surfaces, in alluding to potential theory as a possible basis for description of streamlines of airflow, he not only formed the basis for the capture velocity concept, but also paved the way for the theory which superceded it.

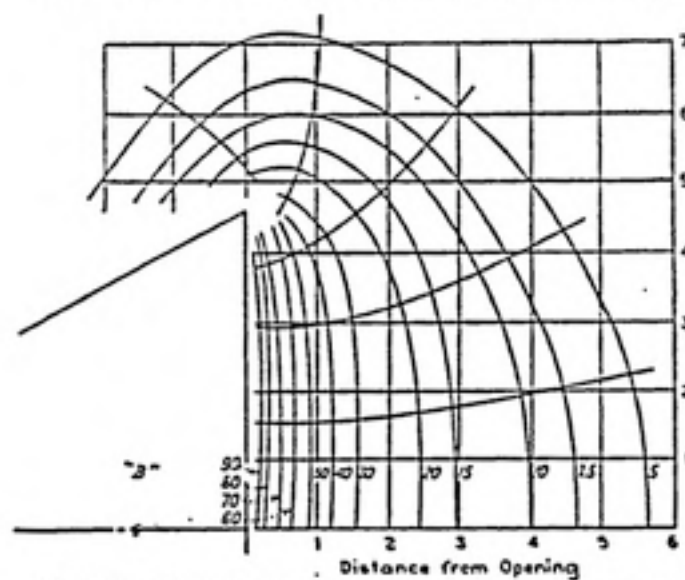
The use of a modified Pitot tube allowed Dalla Valle to map the equal velocity contours of various exhaust hoods [Figure 3]. A general equation was the result of his studies:

$$f(Y) = m/(X^n), \quad (1)$$

where:  $n = \text{a constant: } -1.91;$



(A, Contours in center-plane, perpendicular to long side of opening.)



(B, Contours in center-plane, perpendicular to short side of opening.)

FIG. 3 VELOCITY CONTOURS FOR 3-INCH BY 9-INCH OPENING

SOURCE: REFERENCE 4

$X$  = the horizontal distance from the hood  
along its centerline;

$$m = bA^k,$$

where:  $b$  depends on the aspect ratio of a  
rectangular hood, or = 0.0825 for  
round hoods;

$A$  = the hood face area;

$k$  = a constant: 1.04; and

$f(Y)$  = the point velocity at  $X$ , as fraction  
of  $Y$  = the average face velocity.

Dalla Valle later simplified this model for round  
hoods, or rectangular hoods with aspect ratio ( $AR$  =  
width/length) greater than 0.2. This simplification has  
been rearranged in the Ventilation Manual as:

$$V = Q/(10X^2 + A), \quad (2)$$

where:  $X \leq 3/2 D$ ;

$D$  = the hood diameter or side length;

$V$  = the air velocity in feet per minute (fpm);

$Q$  = volume flow in cubic feet per minute (cfm).

Dalle Valle believed that flanges reduce the volume  
flow required by about 33% for the same required capture  
velocity, so the simplified (ACGIH) equation for flanged  
hoods became:

$$V = Q/[\cdot 75(10X^2 + A)], \quad (3)$$

which Garrison [10] says is good for the region beyond about .4D away from the hood face.

Several years after Dalla Valle's work Silverman continued his investigations [5]. While he was unable to improve upon Dalla Valle's simple equations for round hoods and rectangular hoods of aspect ratios of  $>0.2$ , Silverman was able to provide handy equations for slots (defined as having  $AR \leq 0.2$ ). His empirical solutions were:

$$\text{for unflanged:} \quad V = 23.8 Q[(W+1)/W]/XL; \text{ and} \quad (4)$$

$$\text{for flanged:} \quad V = 55.4 Q/XL, \quad (5)$$

where:  $L$  = the length of the slot hood;  
 $W$  = the width of the slot hood; and  
 $X$  is defined as in the Dalla Valle equations.

These equations have been reduced and corrected in the ACGIH Ventilation Manual to the following:

$$\text{for unflanged:} \quad V = Q/(3.7LX); \text{ and} \quad (6)$$

$$\text{for flanged:} \quad V = Q/(2.6LX). \quad (7)$$

A much more extensive investigation of the effect of aspect ratio on the centerline velocity gradient was conducted by Fletcher [6]. For fixed volume flows and hood areas, the velocity at any given point  $X$  on the hood

centerline increased as the AR decreased (became more slot-like). Fletcher developed an equation for unflanged slot hoods of AR's from 1:1 to 1:16 relating these variables, and then constructed a convenient nomogram [Figure 4].

Fletcher's equation is:

$$V/V_0 = 1/(0.93 + 8.58a^2), \quad (8)$$

where:  $a = [X/A^{.5}] [W/L]^{-B}$ ; and  
 $B = 0.2[X/A^{.5}]^{-1/3}$ ; and  
 $V_0$  = hood face velocity; and  
 other variables are defined as before.

The effects of flanging on the centerline velocity were studied subsequently by Fletcher [7]. Because flanges cut down significantly on the volume flow necessary to produce a given centerline velocity, they increase the efficiency and decrease the cost of ventilation systems which use them [8]. He was able to demonstrate that the optimum flange width equalled the square root of the hood opening area, and the effect of the flange increases as the aspect ratio decreases (becomes more slot-like). An adjacent surface [9] likewise increases the centerline velocity of an exhaust hood by cutting down the air volume from which flow is drawn into the hood. Equal centerline velocities may be obtained in either case with the use of lower total volume flows.

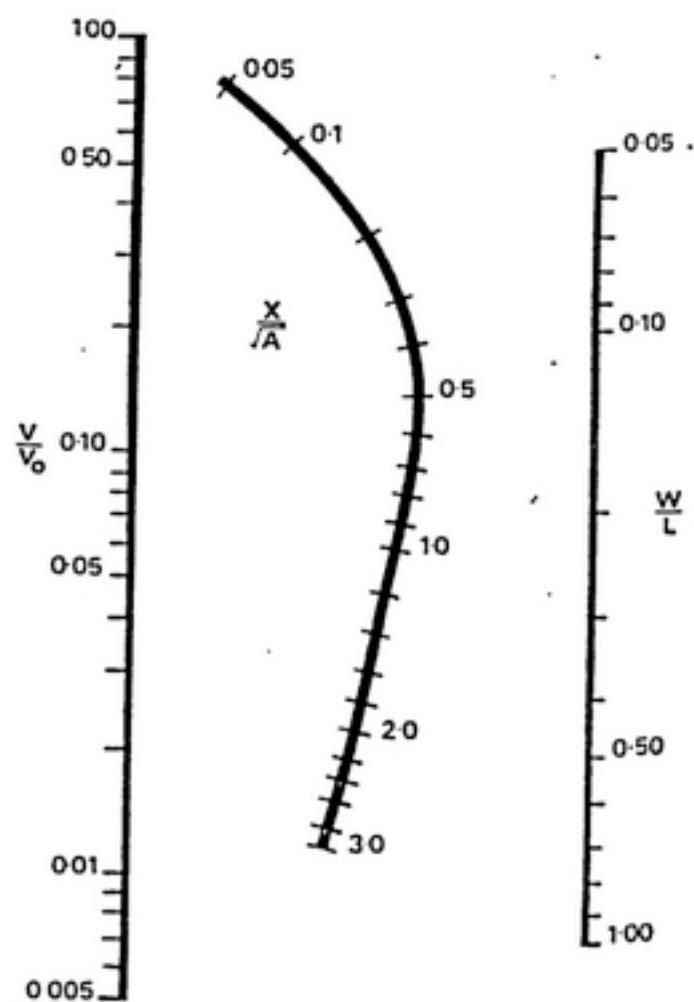


FIGURE 4.  
HOOD FLOW NOMOGRAM  
(FLETCHER)  
SOURCE: REFERENCE 6

More recently, Garrison [10] has studied high velocity-low volume (HVLV) systems and compared the results to the work of Dalla Valle and followers. Generally, he found that the ACGIH Ventilation Manual equations were suitable, but disagreed that flanging added 33% to centerline velocity gradient values. He suggested that the actual increase is probably between 10 to 30%. Silverman's equations cannot be used very near the hood face, because as  $X$  approaches zero,  $V$  at  $X$  becomes indeterminate; Garrison suggests that a limit of accuracy of Silverman's equations (or their simplifications in the ACGIH Manual) is reached when centerline distance  $X$  to hood diameter or width ratios  $X/D$  or  $X/W = 0.4$ .

Garrison subsequently [11] conducted analyses of the relationship of non-dimensional velocity ratios to non-dimensional distance ratios for circular, rectangular and slot hoods, for flanged and unflanged cases, and for various aspect ratios.  $V$ , the centerline velocity at any given  $X$  distance, may be related in a ratio to the hood face velocity  $V_0$ :  $Y = V/V_0$ . Likewise, the centerline distance  $X$  may be related in a ratio to hood diameter  $D$ , rectangular hood width  $W$ , or slot hood length  $L$ :  $X_{DW} = X/D$  or  $X/W$  or  $X/L$ . Then non-dimensional ratios  $Y$  and  $X_{DW}$  may be related to one another through empirically derived equations:

$$Y(\text{"near"}) = a(b)^{X_{DW}}; \text{ and} \quad (9)$$

$$Y(\text{"far"}) = a(X_{DW})^b, \quad (10)$$

where:           a and b are empirical constants, which vary  
                  depending on hood characteristics.

Later, Garrison expanded his explorations to include other practice design concepts using various graphical techniques for a number of "real-world" situations [12]. Obstacles and surfaces frequently block ideal airflow streamlines, and methods such as sketching, conformal mapping, and velocity vector addition may assist in the evaluation of two-dimensional velocity gradients on the hood centerline [Table 2].

A great deal of work has been done, summarized briefly above, using capture velocity as the core theoretical concept upon which practical design of exhaust hoods, and analysis of exhaust hood flow, has been built. However, there are significant deficiencies therein.

Recently, a number of investigators have criticized the capture velocity concept. Heinson and Choi [13] have provided a good summary of the problems associated with this design method. It is as follows:

- 1) Contaminant concentration in the vicinity of the source cannot be predicted;
- 2) The effect of changes in design (such as system dimensions or volumetric flow) on the performance of a system cannot be estimated;

TABLE 2.  
Empirical Design Data for Nondimensional Centerline Velocity Gradients

| Engineered Design Data for Non-dimensional Centerline Velocity Gradients |                            |                                    |      |                             |      |                                             |      |                     |      |                                              |     |     |
|--------------------------------------------------------------------------|----------------------------|------------------------------------|------|-----------------------------|------|---------------------------------------------|------|---------------------|------|----------------------------------------------|-----|-----|
| Nozzle<br>End<br>Shape                                                   | Nozzle<br>Profile<br>Shape | Y = a(b) <sup>X<sub>ow</sub></sup> |      |                             |      | Y = a(X <sub>ow</sub> ) <sup>b</sup>        |      |                     |      | Specific<br>Y Values<br>at X <sub>ow</sub> = |     |     |
|                                                                          |                            | 0 ≤ X <sub>ow</sub> < 0.5          |      | 0.5 ≤ X <sub>ow</sub> < 1.0 |      | 1.0 ≤ X <sub>ow</sub> ≤ X <sub>ow,max</sub> |      | X <sub>ow,max</sub> |      |                                              |     |     |
|                                                                          |                            | a                                  | b    | a                           | b    | a                                           | b    | a                   | b    | X <sub>ow,max</sub>                          | 0.5 | 1.0 |
| Circular                                                                 | Plain                      | 110                                | 0.06 | --                          | --   | 8                                           | -1.7 | 8                   | -1.7 | 1.5                                          | 26  | 8   |
|                                                                          | Flanged                    | 110                                | 0.07 | --                          | --   | 10                                          | -1.6 | 10                  | -1.6 | 1.5                                          | 30  | 10  |
|                                                                          | Flared                     | 90                                 | 0.20 | 90                          | 0.20 | --                                          | --   | 18                  | -1.7 | 2.0                                          | 40  | 18  |
|                                                                          | Rounded                    | 98                                 | 0.50 | 145                         | 0.23 | --                                          | --   | 33                  | -2.2 | 2.5                                          | 69  | 33  |
| Square<br>(WLR=1.0)                                                      | Plain                      | 107                                | 0.09 | --                          | --   | 10                                          | -1.7 | 10                  | -1.7 | 1.5                                          | 32  | 10  |
|                                                                          | Flanged                    | 107                                | 0.11 | --                          | --   | 12                                          | -1.6 | 12                  | -1.6 | 1.5                                          | 36  | 12  |
| Rectangular<br>(WLR=0.50)                                                | Plain                      | 107                                | 0.14 | --                          | --   | 18                                          | -1.2 | 18                  | -1.7 | 2.0                                          | 41  | 18  |
|                                                                          | Flanged                    | 107                                | 0.17 | --                          | --   | 21                                          | -1.1 | 21                  | -1.6 | 2.0                                          | 45  | 21  |
| Rectangular<br>(WLR=0.25)                                                | Plain                      | 107                                | 0.18 | --                          | --   | 23                                          | -1.0 | 23                  | -1.5 | 2.5                                          | 46  | 23  |
|                                                                          | Flanged                    | 107                                | 0.22 | --                          | --   | 27                                          | -0.9 | 27                  | -1.4 | 3.0                                          | 50  | 27  |
| Narrow slot<br>(WLR=0.10)                                                | Plain                      | 107                                | 0.19 | --                          | --   | 24                                          | -1.0 | 24                  | -1.2 | 3.5                                          | 48  | 24  |
|                                                                          | Flanged                    | 107                                | 0.22 | --                          | --   | 29                                          | -0.8 | 29                  | -1.1 | 4.0                                          | 50  | 29  |

SOURCE: REFERENCE 12

3) Even though the performance of a particular system is known, the effect of geometrically scaling it up or down is unpredictable;

4) An engineer designing a system for a new process (one for which an LEV design does not appear in published literature) is left to design basically from scratch with little knowledge of the effectiveness of the resulting system;

5) The idea of providing a certain velocity to capture contaminants is inconsistent with the laws of fluid mechanics.

For example, Fletcher and Johnson [14] show that traditional design methods are adequate for gases and micron-sized particles released on the centerline of an LEV hood at low velocities. But, especially if the direction of release is away from the hood, if the release velocity is higher than a certain low amount (0.21 m/sec in a certain set of cases), higher "capture velocities" are required. Moreover, as Ellenbecker et al. [16] point out, crossdrafts and other air disturbances cannot be accounted for, energy expenditure optimization is difficult, and there are significant uncertainties in shaping the hood to distribute velocity contours for efficient capture in three dimensions. Only qualitative predictions of hood performance can be obtained using capture velocity concepts as the theoretical foundation.

## B. CAPTURE EFFICIENCY CONCEPTS

Capture efficiency is a notion which may be used to evaluate hood performance comprehensively. It is useful because hood and system designs of all types may be compared effectively to one another, and the effects of changes in any design parameter may be evaluated along a single scale.

Dalla Valle was quite aware of the inadequacies of the theoretical approach in use at the time he was doing his original work. He states [4]: "Without attempting to minimize the importance of experience in engineering design, it seems proper to point out that most of the past experience in the design of local exhaust hoods has not been associated with quantitative studies of the actual efficiency of dust removal."

The first study using capture efficiency as the central concept for the evaluation of hoods was conducted by Burgess and Murrow [15]. Field conditions of contaminant generation from machining operations were modeled in the laboratory, and hood shape was demonstrated by the authors to be a primary factor in the efficiency of contaminant control.

Once the central concept underlying hood design changed, a new era in ventilation research began. However, a careful definition of the new parameter was required.

Capture efficiency,  $\eta$ , is defined by Ellenbecker, et al. [16] as "the fraction of the airborne contaminants

generated by a source that is captured by the LEV system controlling it," or mathematically as:

$$\eta = G'/G \quad (11)$$

where:  $G'$  = the exhaust contaminant capture rate in grams per second (g/s), and  
 $G$  = the contaminant generation rate, g/s.

The capture efficiency is a function of at least five variables:

- $Q$ , the volume flow of the hood;
- $A$ , the hood face area;
- $X$ , the centerline distance of the hood to the source;
- $V_c$ , the crossdraft velocity; and
- $T$ , the temperature of the source.

When the temperature variable can be ignored, the others may be analysed more easily. It is found by application of the Buckingham  $\pi$  Theorem (see the relevant discussion later in this section) that the capture efficiency is related to a function of two (dimensionless) ratios: the crossdraft velocity to hood face velocity; and the centerline distance of source to hood divided by the square root of the hood area:

$$\eta = K(V_c/V_o)^a (X/\sqrt{A})^b \quad (12)$$

The specific functional variable (K) and exponents (a, b) defining the relationship are determined by experiment.

The limiting conditions which apply are:

$$\eta = 0 \text{ when } X \rightarrow \infty; \quad (13)$$

$$\eta = 0 \text{ when } V_o = 0; \quad (14)$$

$$\eta = 0 \text{ when } V_c \rightarrow \infty; \quad (15)$$

$$\eta = 1 \text{ when } X = 0; \quad (16)$$

$$\eta = 1 \text{ when } V_c = 0. \quad (17)$$

Actual measurement of capture efficiency in the laboratory entails direct measurement of contaminant concentrations in the duct of the exhaust hood. One must assure good mixing within the hood's duct. Direct measurement is made in the duct of the exhaust hood for the contaminant concentration. The source is placed just within the hood itself, to obtain the "100%" value. Then, the source is placed at various distances X away from the hood. The latter contaminant concentration values are compared at every time interval measured with the 100% value, and the ratio of the two is capture efficiency.

A subsequent paper by Flynn and Ellenbecker [17] offered an analytically detailed approach to capture efficiency, specifically to flanged circular exhaust hoods (FCH). Their approach was based on the intuitive idea that capture efficiency depends upon the interaction of three

flow fields: 1) that generated by the hood; 2) the flow field generated by the contaminant source; and 3) the flow field due to perturbing crossdrafts [Figure 5]. It is the interactions of these flow fields that ultimately determine whether a contaminant enters the exhaust hood. Velocity vector average values were determined for each field by mathematical functions; in addition they accounted for some degree of variability about these averages due to turbulence.

In their model, Flynn and Ellenbecker calculated by vector addition the path of streamlines of a contaminant issuing in all directions from a point source, as they were affected by the flow fields of the hood, and by a crossdraft. They based their model on the modified potential flow solution for airflow into flanged circular hoods [18].

The cylindrical coordinate system is assumed in this model such that the FCH centerline is the Z-axis. The crossdraft is assumed to blow at velocity  $V_c$  perpendicular to the Z-axis, from the  $\theta = 180^\circ$  to the  $\theta = 0^\circ$  half-planes. The model assumes irrotational incompressible air flow,  $Q$ . A series of point sources of isothermal nonbouyant gas release at flow volume  $Q_s$ , at some point (at distance  $Z$ ) from the FCH. Flynn and Ellenbecker developed a computer model for the IBM XT personal computer [17] which maps the streamlines for the contaminant flow. It displays a visual plot of a semicircle of point-source streamlines, as they

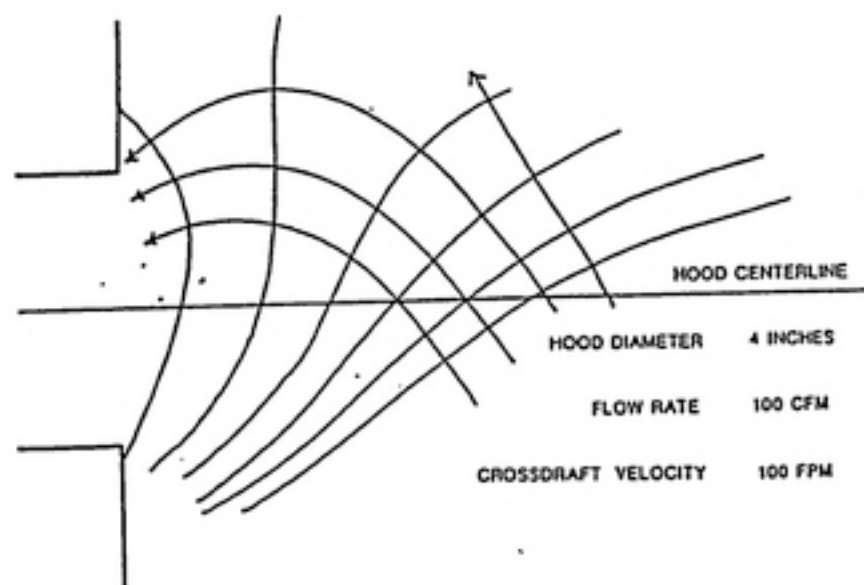


Figure 5 — Theoretical potential lines and streamlines for a flanged circular hood operating in the presence of a crossdraft perpendicular to the hood centerline.

SOURCE: REFERENCE 18

exist in the plane of the  $Z$  axis, and shows whether or not, when under the combined influence of the hood flow and crossdraft flow, they enter the FCH [Figure 6].

Previously developed similar models include Fialkovskaya's simplified point-sink model in which he described equations for the streamline which would just enter a hood in the presence of a cross-draft [19], and Strauss' modified  $n$ -sinks model allowing iterative processing [20]. Empirical studies have validated Flynn and Ellenbecker's "Final Model" [21]; their work recently has been extended to mathematical analysis and quantitative evaluation of potential flow modeling for hoods of other configurations [22].

To calculate capture efficiencies in such systems, one must apply the Buckingham  $\pi$  theorem. The  $\pi$  stands for the Product of variables. Each  $\pi$  is a dimensionless group of variables formed by application of the theorem. The theorem assures that for a process depending on  $n$  dimensional variables, then a reduction to  $k$  dimensionless variables is possible, where  $n - k = j$ , where  $j$  is the maximum number of variables which do not form a  $\pi$  among themselves. The reduction number  $j$  is always less than or equal to the number of dimensions (time, length, mass, temperature),  $m$ , in the  $n$  descriptive variables. The choice of the  $n$  dimensional variables is critical; if one is inadvertently omitted, then the analysis will be incorrect.

D= 4 IN.  
Q= 104 CFM  
U= 114 FPM

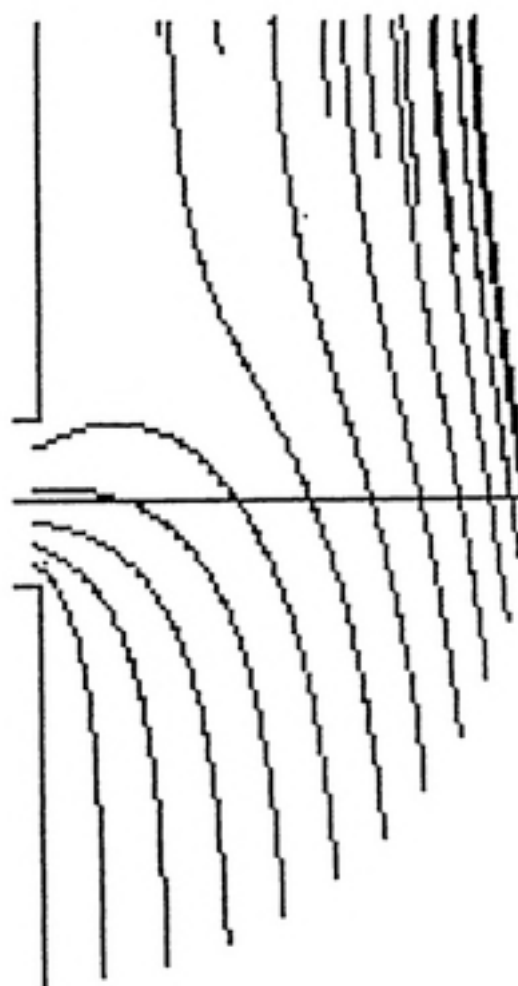


FIGURE 6.

IDEALIZED CONTAMINANT STREAMLINES

SOURCE: REFERENCE 21

In the capture efficiency analysis, each of the following variables:

$D$  = diameter of the hood;

$Z$  = distance to the point of origin;

$Q$  = the volume flow of the hood in cfm; and

$V_C$  = the velocity of the crossdraft,

must be considered. Application of the Buckingham  $\pi$  theorem suggests that one dimensionless group will be  $[Z/D]$ , and the second will be  $[V_f/V_C]$ , where the hood face velocity is extracted from the hood flow variable. A third dimensionless group,  $[Q_s/Q]$  will appear when the contaminant source flow  $[Q_s]$  is considered with the other variables.

However, the functional relationship between the  $\pi$ 's cannot be specified explicitly without experiment. The  $[Z/D]$  is the ratio of the hood-source distance to the diameter of the hood. It will have a profound effect on capture efficiency. Near the hood, most of the source of flow will be captured by the hood's flow field; if the source is far away the hood's field is weak. The second group  $[V_f/V_C]$  is the ratio of face velocity to that of the crossdraft. The weaker the crossdraft, the less distorted are the effects of the hood and its flow field. Similarly with  $[Q_s/Q]$ , the hood flow field will have predominance over a contaminant source with a low flow rate.

The use of plotted streamlines from the source to the hood is important; each streamline either does or does not enter the hood. Thus whatever proportion of multiple

streamlines enter the hood defines the capture efficiency for that particular set of conditions [Figure 6].

Alternatively, when a single streamline is calculated from a point source, it may be seen statistically as the first moment of distribution of the flow from that source; turbulence and dispersion are assumed to be equally distributed around such a streamline. When such a streamline hits the edge of the hood, half the flow is assumed captured and half is not. The distance  $Z$  from the hood to the contaminant source then forms a dimensionless ratio with the hood diameter  $D$ ; and at the point of probable 50% capture, is designated  $[Z/D]_{50}$ , the "critical distance." It is assumed that turbulence is primarily accounted for by the  $[V_f/V_c]$  ratio; it is used as a predictor for the effects of turbulent diffusion on contaminant dispersion around the streamline. The computer model can be used iteratively to obtain the  $[Z/D]_{50}$  for any given combination of other variables. Then one determines the regression between the  $\pi$  groups.

### III. BACKGROUND TO THE COMPUTER MODEL

#### A. ELEMENTS OF POTENTIAL THEORY

Since Flynn and Ellenbecker's model [17, 21] is based on the potential flow solution [18], they are assuming that the airflow into the exhaust hood is incompressible and irrotational. Moreover, in potential flow, frictional forces are negligible, so that inviscid flow is assumed. Laplace's equation:

$$\nabla^2 \phi = 0 \quad (18)$$

is used to describe such a flow field.

Laplace's equation is derived from the continuity equation:

$$[\partial \rho / \partial t] + \nabla \cdot (\rho \vec{v}) = 0 \quad (19)$$

where:  $\rho$  = the fluid density;  
 $t$  = the elapsed time;  
 $\nabla$  = del, the gradient operator; and  
 $\vec{v}$  = the velocity vector.

The continuity equation is the summary of conservation of mass requirements in fluid mechanics. Continuity is said to exist wherever the volume flow,  $Q$ , equals the area of any hypothetical velocity contour surface times the velocity magnitude through that surface.

Incompressibility of a fluid means that density changes are negligible, so the first term drops out, and the continuity equation becomes:

$$\nabla \cdot \vec{v} = 0, \quad (20)$$

and it is said that the "divergence" of the velocity field is zero. "Divergence" is a measure comparing flow into and out of a defined differentially small control volume in space. When it is zero, all fluid flowing into such a volume leaves at the same rate. The velocity field then is neither converging (volume shrinking with increasing density) nor diverging (getting larger with decreasing density). About 330 ft/sec is the upper velocity limit for incompressible flow of standard air.

The gradient operator,  $\nabla$ , can be written out as:

$$\nabla( ) = [\partial( ) / \partial x] \vec{i} + [\partial( ) / \partial y] \vec{j} + [\partial( ) / \partial z] \vec{k} \quad (21)$$

in a three dimensional (x, y, z) coordinate system. The gradient operator converts a scalar to a vector function, and when solved gives the direction and maximum rate of increase of the function.

An irrotational fluid flow has no vorticity or "curl." In the mathematical description of an irrotational fluid, the cross-product of the gradient operator and the velocity vector function must always equal zero:

$$\nabla \times \vec{v} = 0 \quad (22)$$

because the angular momentum of an irrotational flow is zero.

From this it follows directly (partly by definition) that the velocity vector function is the gradient of a "potential" function:

$$\vec{v} = \nabla \phi \quad (23)$$

where  $\phi$  is the (scalar) potential function. Substituting equation (23) into equation (20) yields Laplace's equation (18).

The "potential" function,  $\phi$ , is defined for every point in space ( $x, y, z$ ) as "the sum of the potential of the extraneous impulsive forces by which the actual motion at any instant could be produced instantaneously from rest" [27]. The potential function may be analysed as the product of time and force, divided by area and density:  $tF/A\rho$ ; simplified, the units are usually  $\text{cm}^2/\text{sec}$ .

Viscous forces are negligible in potential flow. Inviscid flow occurs where no solid surfaces exist over which boundary layers would form. It is assumed in the strict potential flow model for FCH's that all hood flow is potential flow. This simplifying assumption yields results which are inaccurate only at points close to the hood face.

Using the assumptions of potential flow, and within certain boundary conditions, one can use Laplace's equation

to define the velocity flow field. Boundary conditions are discussed in the next section.

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## B. POTENTIAL FLOW SOLUTIONS FOR FLANGED CIRCULAR HOODS

The potential flow solution is described in detail in Flynn and Ellenbecker's original papers [18, 21]. The potential flow model for the FCH was developed because it is amenable to practical application, in contrast to the more accurate, but difficult, constant velocity analytic solutions of Lamb [27] and Drkal [28].

In contrast to centerline velocity gradient studies, potential flow solutions describe the velocity field of airflow into the hood in three dimensions. This is particularly useful and important where sources are not on the centerline, where there is significant dispersion, where the direction of contaminant generation is not directly toward the hood face, or where there is a crossdraft.

Boundary conditions and simplifying assumptions for the potential flow model for the FCH are:

- 1) an infinite flange;
- 2) no flow through the flange:  $\partial\phi/\partial z = 0$ , for the conditions  $z = 0$ ,  $r > a$ , where  $a$  = the hood radius;
- 3) constant potential,  $\phi$ , at the hood face; and
- 4)  $\phi \rightarrow 0$ , as  $X \rightarrow \infty$ .

The strict potential flow solution however, is not entirely adequate. The assumptions of inviscid, irrotational flow begin to break down in the region near the flange and the hood face, because of the increasing

importance of shear stress due to turbulence of the boundary layer, and vena contracta formation. Real centerline velocities at the hood face are about twice the predicted value. Additionally, the strict potential flow solution predicts infinite velocity along the edge of the hood. However, turbulence there considerably reduces actual air velocity.

To address these anomalies in the theory, Flynn and Ellenbecker noted that Dalla Valle's equal velocity contours are elliptical. They make the assumption that the velocity vector field is uniform everywhere along each confocal ellipsoid equipotential surface provided by the theory. (In reality, the velocity field is the gradient of the potential.) For their modified potential flow solutions, a set of conditions, similar to the boundary conditions for the strict solution, apply, with the exception that in addition the hood face velocity is constant. The derived expression for velocity at every point in the field is then reasonably consistent with experiment. Additionally, because Laplace's equation is linear, other potential flows may be added vectorially at any given point. Thus, crossdraft effects and source vectors can be added to affect the velocity vector field of the hood.

In the validation of their solution [21], Flynn and Ellenbecker considered three versions of their model. The first was the strict potential flow solution. The second was the modification just discussed. They distinguish

between an "inconsistent" model and one which contains a singularity. While "an inconsistent model is one that is not exact mathematically, a singularity refers to a region where unrealistic fluid behavior occurs." Thus while Model 1 is a consistent model, it is also a singular one. Model 2, however, is inconsistent due to the "inexact" approximation made to obtain it. The modified velocity field equation cannot be integrated to give the true  $Q$ ; for example, the theoretical (Model 2) average face velocity is 87% of the true value.

Flynn and Ellenbecker's Final Model employed a radial correction factor  $C_r$ , a strong function of the eccentricity, where:  $C_r = 2.6 \epsilon^{18} + 0.853$ . The eccentricity,  $\epsilon$ , is the ratio of the hood diameter to the sum of the distances from the edges of the hood opening to the point in question:

$$\epsilon = \frac{2 a}{(\Gamma_1 + \Gamma_2)} . \quad (24)$$

Here,  $\Gamma_1 = \sqrt{[z^2 + (a^2 + r^2)]}$ , and  $\Gamma_2 = \sqrt{[z^2 + (a^2 - r^2)]}$ . The radial correction was necessary because the radial velocity as measured increased more rapidly than predicted as the eccentricity approached 1 (i.e. near the hood face).

Additionally, the theoretical axial velocity calculations were also adjusted, by a factor of 0.9, based on the graphical results of the validation experiments. These empirical corrections were an attempt to overcome the mathematical inconsistency previously discussed.

The modified potential flow solution calculates the velocity at any point in the hood coordinate system  $(R, \theta', Z)$  as:

$$V_{T2} = \frac{\sqrt{3} Q \epsilon^2}{2 \pi a^2 \sqrt{(3-2\epsilon^2)}} \quad (25)$$

The Final Model component velocity vectors are:

$$V_R = -C_r V_{T2} (\sin \beta); \text{ and} \quad (26)$$

$$V_Z = -0.9 V_{T2} (\cos \beta); \quad (27)$$

where:  $\beta = \tan^{-1}(V_{R1}/V_{Z1}), \text{ and} \quad (28)$

where  $V_{R1}$  and  $V_{Z1}$  are calculated as defined in both papers [18, 21].

The Final Model was incorporated into an interactive BASIC program, which required the input of three variables:

$D$  = the hood diameter in inches;

$Q$  = the hood flow in cubic feet per minute, cfm;

$V_c$  = the crossdraft velocity, feet per minute, fpm.

The output for one of the possible combinations of these variables is seen in [Figure 6]. Through this program it is possible to "define the regions under control of the hood, and those that are dominated by the crossdraft." Some level of control will be exerted over contaminant processes

located in those regions from which the streamlines are drawn into the hood. The program just described forms the basis of both the experiments and program modifications for this thesis; the contaminant source will be a jet.

## C. CIRCULAR JET FLOW AND THE SCHLICHTING EQUATIONS

An ideal circular jet of fluid, flowing into a still medium, maintains constant static pressure throughout itself. However, the flow,  $Q$ , the area,  $A$ , and the jet width,  $b$ , are not at all constant; they are continually increasing with the entrainment of the surrounding air [Figure 7]. Its energy losses are likewise proportional to jet length, almost entirely in kinetic energy (i.e. in velocity) [Figure 8]. The momenta of external forces on a jet entering still air sum to zero, so the momentum of the mass flow of air (kinematic momentum) throughout such a jet remains constant [19].

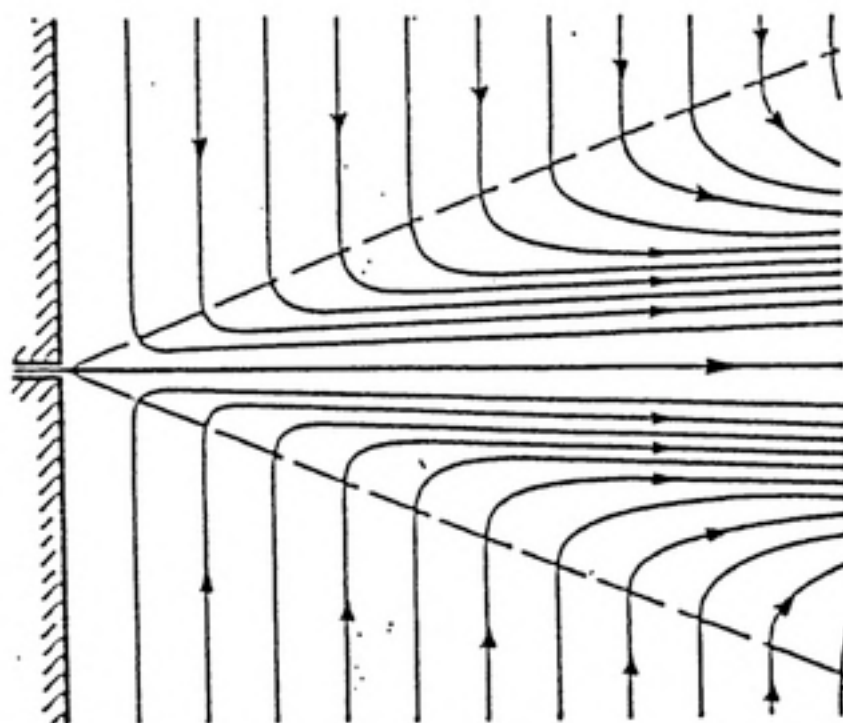
The kinematic momentum,  $K$ , can be calculated for a jet of known flow. Since, in cylindrical coordinates

$$K = 2\pi \int_0^{\infty} V_z^2 r \, dr, \quad (29)$$

where:  $V_z$  = velocity in the axial direction of the jet, and  
 $r$  = the radius of the jet flow at  $z$ ; and  
 since at  $z = 0$ ,  $V_z$  is not a function of  $r$ , then simple integration will yield:

$$K = \pi r^2 V_z^2 = QV_z, \quad (30)$$

Fig. 7. Pattern of streamlines  
in a circular, turbulent free jet



SOURCE: REFERENCE 23

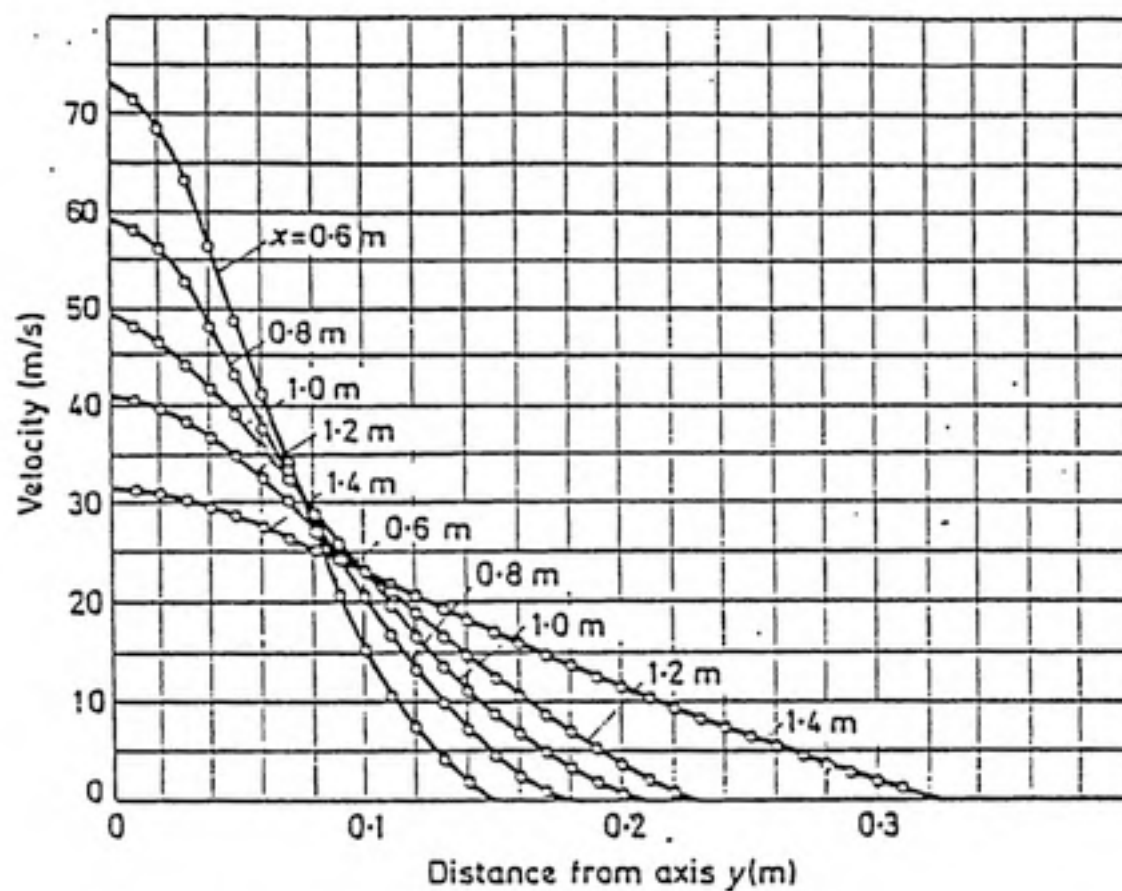


FIG. 8. Velocity profiles for axisymmetric jet.

SOURCE: REFERENCE 23

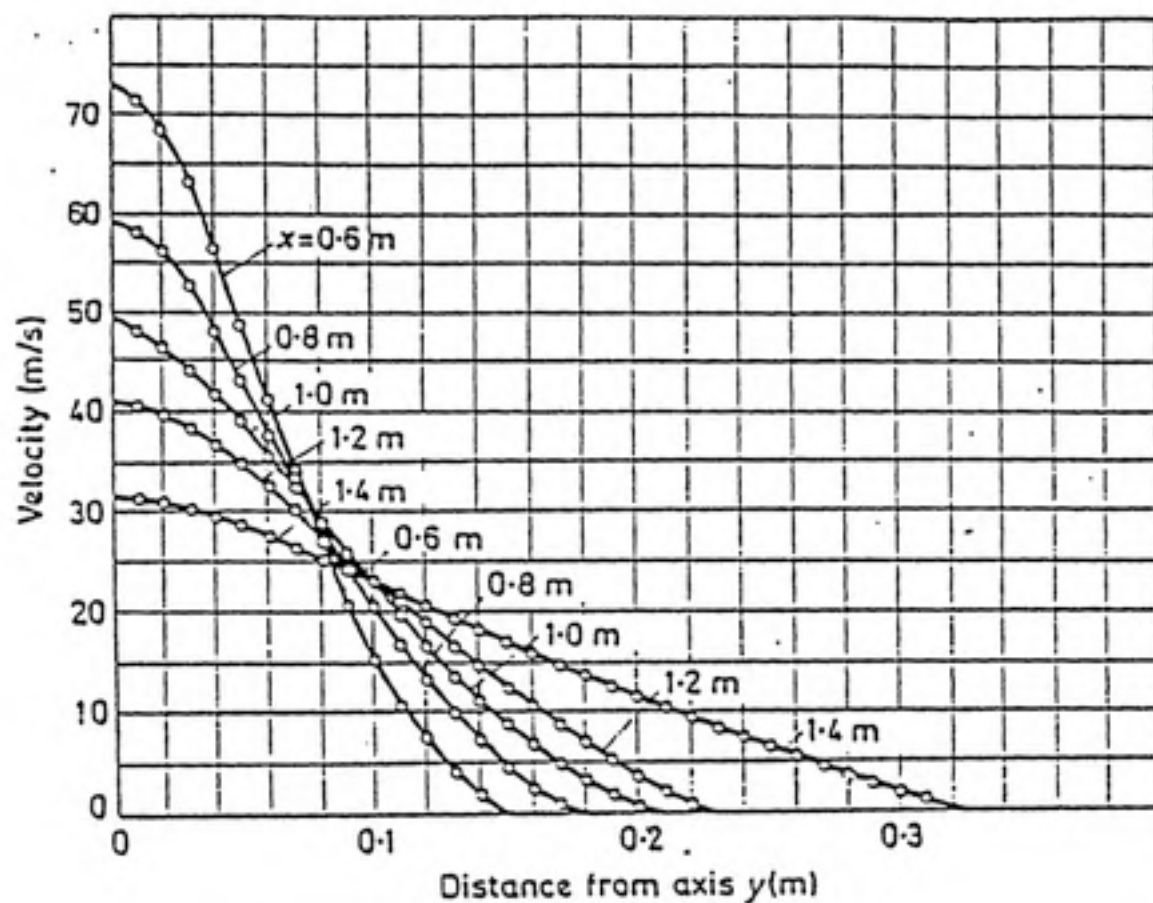


FIG. 8. Velocity profiles for axisymmetric jet.

SOURCE: REFERENCE 23

since flow equals the product of area and velocity. For any given jet flow,  $Q$ , and starting velocity,  $V$ , we can thus calculate  $K$ .

The spread of a circular jet can be described [23] as beginning at a single point ("pole" or "virtual point") [Figure 9]. Experimentally, it has been discovered that for a cylindrical jet the virtual point is located 1.86 times the jet opening diameter, inside its opening [19]. It is there that the flow calculations must begin. See the program, located in the Appendix.

Lines drawn from the virtual point through the orifice edges then extend outward such that they form the boundary of the mixing zone. With increasing distance from the origin, the material in the jet core becomes diffused by mixture with the surrounding air. In the core (in the "initial section"), the velocity profile remains square, and the temperature and concentration remain constant. The core tapers. In the "main section," the velocity profile widens and flattens. Throughout, the velocity profiles are symmetric, and similar.

Turbulent jet flow is characterized by a cross-transfer of vortices, and as these move beyond the limits of the jet, impart their momentum to surrounding layers of air. Successive cone-shaped layers of air are entrained in the jet motion. This incorporation retards the boundary layer. The thickness of the turbulent boundary zone increases with the increasing distance from the jet source, until from the

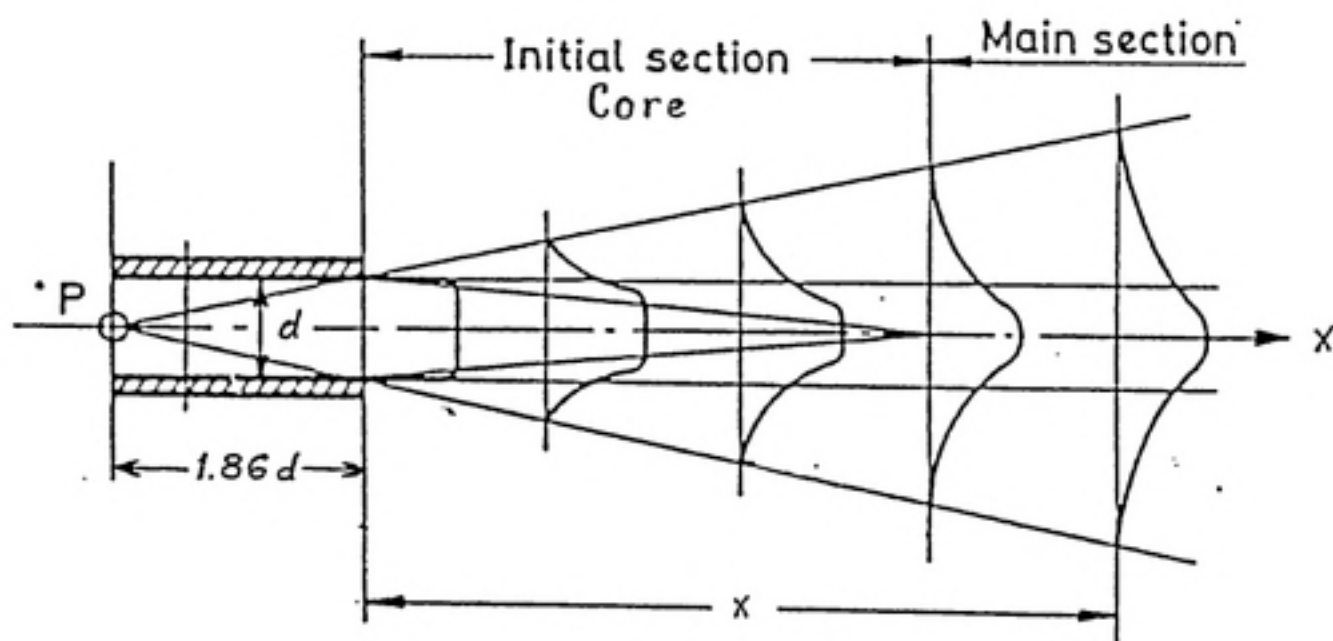


FIG. 9. Axisymmetric jet.

SOURCE: REFERENCE 19

entire periphery it "meets itself" on the axis of the jet [Figure 9]. In the main section of the jet, the entire flow is turbulent.

In a circular jet, the amount of turbulent shear stress associated with the boundary layer can be analyzed by Prandtl's modified mixing length theory. To visualize a physical interpretation of "mixing length" one must use a simple model of turbulent flow of a jet along a wall [Figure 10]. This is the simplest case of parallel flow, in which velocity is assumed to vary only from streamline to streamline. As the flow progresses and turbulent mixing zones move longitudinally, they also may move transversely, while retaining their momentum.

Prandtl's mixing length,  $l$ , is defined as "that (transverse) distance which must be covered by an agglomeration of fluid particles, travelling with its original velocity, in order to make the difference between its velocity and the velocity in the new lamina equal to the mean transverse fluctuation in turbulent flow" [23].

The overall variations in the velocity contours of the jet are controlled by this transverse movement of turbulence eddies. The thickness and rate of motion of the mixing layers is a critical determinant in the calculation of the magnitude and direction of the velocity vectors at any given point in the jet.

*The difference in forward velocity, between the laminae defining lateral movement, is related quantitatively to the*

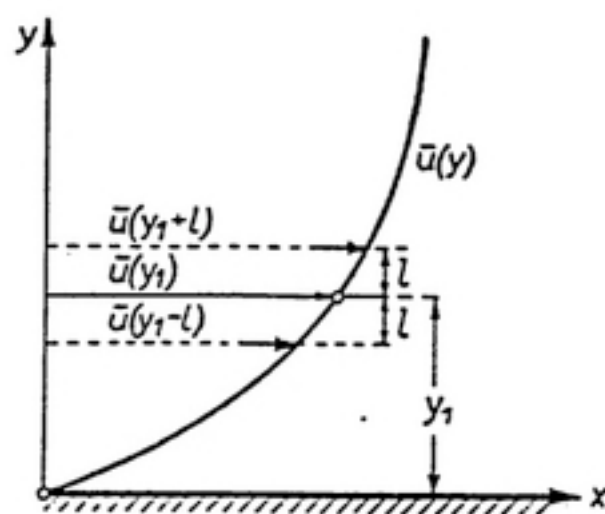


Fig. 10. Explanation of the mixing-length concept

SOURCE: REFERENCE 23

extent of lateral movement. Prandtl's mixing length hypothesis combines the equation describing this relation, with the equation for the shearing stress of the turbulent flow, and obtains an equation hypothetically describing the turbulent shear stress,  $\tau_t$ , in terms of:

$\rho$  = the density of the flowing medium;

$l$  = the thickness of the laminae defining lateral movement; and

$du/dy$  = the rate of change of mean velocity between laminae:

$$\tau_t = \rho l^2 |du/dy| du/dy, \quad (31)$$

where the absolute value operator is to ensure the proper sign of the result. Equation 31 is the formal definition of Prandtl's mixing length hypothesis.

Turbulent flow contains both time-average (mean) motions, and fluctuating (eddy) motions, in all three directions. Over a sufficient length of time, the time-average of all the eddy motions sum to zero. However, these fluctuations influence the mean motion such that the mean motion exhibits an apparent increase in resistance to deformation: the apparent (or virtual) viscosity, or "eddy viscosity." A mixing coefficient has been introduced in the fluid dynamics literature,  $A_r$ , for this Reynolds stress of turbulent flow. It is analogous to the Stokes coefficient of viscosity for laminar flow,  $\mu_1$ , and it likewise relates the (turbulent) shear stress to the velocity gradient:

$\tau_t = \Lambda_T d\bar{u}/dy$ . It is not, however, a fluid property like the coefficient of viscosity, and its value depends on the mean fluid velocity. The apparent kinematic viscosity,  $\epsilon_T$ , is likewise analogous to the derivation of kinematic viscosity of laminar flow,  $\nu_1$ , and is defined as the mixing coefficient divided by the fluid density.

In order to cure a theoretical defect in the calculation of the apparent kinematic viscosity,  $\epsilon_T$ , based on Equation 31 and its assumptions, Prandtl modified its derivation. The modification is valid only in free turbulent flow, and is derived from extensive experimental data. The original hypothesis had assumed that the volumes of fluid moving transversely during turbulent mixing had diameters very small compared to the transverse dimensions of the movement. The modified hypothesis [23] assumes the diameters of the transversely-moving volumes of fluid are of the same order of magnitude as that of the mixing zone. "The virtual kinematic (eddy) viscosity,  $\epsilon_T$ , is now formed by multiplying the maximum difference in the time-mean flow velocity with a length which is assumed to be proportional to the width,  $b$ , of the mixing zone":

$$\epsilon_T = x_1 b (\bar{u}_{\max} - \bar{u}_{\min}), \quad (32)$$

where  $x_1$  is a dimensionless experimentally-derived constant; with this treatment,  $\epsilon_T$  remains constant throughout the width of every cross-section of flow. Due to the direct

proportionality of length and width of the jet, and the simple inverse proportionality thereof to velocity, the virtual kinematic viscosity of turbulent flow,  $\epsilon_t$ , becomes a constant,  $\epsilon_0$ , over the entire length of the jet.

As a result, the velocity distribution differential equations become formally similar to those of laminar jets; only the term therein for kinematic viscosity of laminar flow ( $\nu_1$ ) needs to be replaced by that for the virtual kinematic viscosity ( $\epsilon_0$ ) of turbulent flow.

To calculate the vector equations, one must know how to calculate  $\epsilon_0$ . According to measurements by Reichardt [referenced in 23], the half-width,  $b_{1/2}$ , of a circular turbulent jet at the point where  $V_z$  = one-half the maximum centerline velocity, is given by:

$$b_{1/2} = 0.0848 z, \quad (33)$$

where:  $z$  = the distance from the nozzle.

Reichardt's measurements also yielded an equation:

$$b_{1/2} = (5.27 z \epsilon_0) / \sqrt{K} \quad (34)$$

that can be used in conjunction with the previous one, such that for any given value of  $z$ , and with  $K$  determined as previously discussed,  $\epsilon_0$ , the virtual or apparent kinematic ("eddy") viscosity can be directly calculated.

In summary, given a few fairly reasonable simplifying assumptions, it is possible to calculate two critical characteristics of the flow of a circular turbulent jet. The kinematic momentum,  $K$ , and the eddy viscosity,  $\epsilon_0$ , are calculated by knowing: 1) the flow,  $Q$ , and initial velocity,  $V$ ; and 2) the axial distance,  $z$ , of any particular point in the jet.

As alluded to earlier, there is formal similarity of the equations for the velocity vectors of turbulent flow with those of laminar flow. For a turbulent jet,  $V_z$  is the magnitude of the velocity vector in the direction of the jet axis ( $z$ ):

$$V_z = \frac{3K}{8 \pi \epsilon_0 z [1+.25\eta^2]^2}, \text{ and} \quad (35)$$

$V_r$  is the magnitude of the velocity vector in the radial direction ( $r$ ):

$$V_r = \frac{\sqrt{3K} [\eta-.25\eta^3]}{4 \sqrt{\pi} z [1+.25\eta^2]^2}, \quad (36)$$

where in either case:

$$\eta = \frac{r \sqrt{3K}}{4 \sqrt{\pi} \epsilon_0 z}. \quad (37)$$

Reichardt evaluated this model by comparing the predicted velocity distribution of a circular turbulent jet

with the distribution of experimentally-determined velocity values, for three different axial distances. [Figure 11]. The axes of Figure 11 are in dimensionless ratios. There is impressive correspondence between the experimental findings and the model predictions.

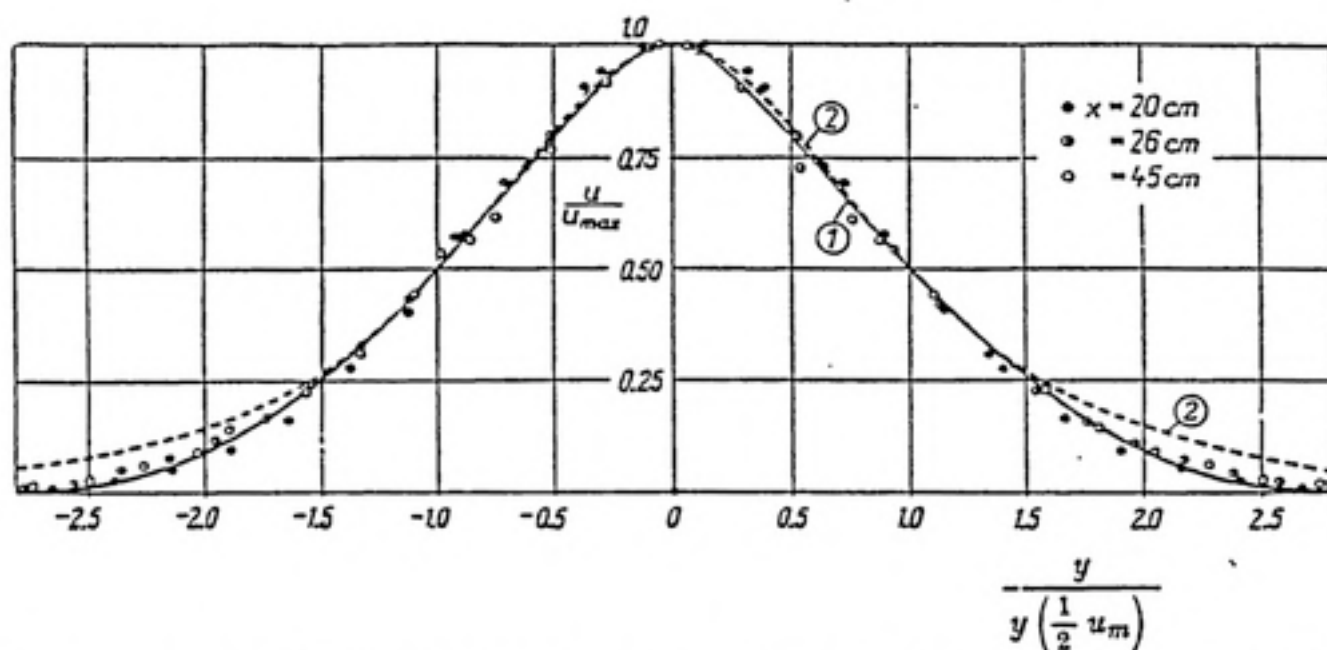


Fig. 11. Velocity distribution in a circular, turbulent jet. Measurements due to Reichardt

SOURCE: REFERENCE 23

#### IV. PURPOSE AND OBJECTIVES

The purpose of this work is to validate a computer model that predicts the streamline that a jet of gaseous contaminant will follow in the flow field of a flanged circular hood. These studies will assist in developing reliable estimates of breathing zone concentrations of gaseous or other jets of workplace contaminants.

The objectives of this research are:

1. To write a new interactive computer program to describe the flow of a circular turbulent contaminant jet within the flow field of a flanged circular exhaust hood. This is accomplished by combining a modified BASIC computer program from Flynn and Ellenbecker [21], for the validated potential flow solution for airflow into a flanged circular hood, with the appropriate expressions for the velocity vectors of the flow of a circular jet;
2. To run the program for a matrix of hood and jet flows, and distance values of the jet from the hood face, to create predictions of the specific hood centerline locations of the jet,  $[Z/D]_{50}$ , at which half of the jet flow would be captured by the exhaust hood; and
3. To perform replicate laboratory experiments for each combination of hood and jet flows and distances, to determine actual  $[Z/D]_{50}$  values for each, and to compare the results statistically with the predictions.

A basic premise of potential theory is that of a free field, in which there is unbounded, unobstructed flow. Inviscid flow may be assumed, and this assumption allows the neglect of friction. The use of strict potential theory in the description of hood flow yields an analysis in which the gradient of potential (the magnitude of velocity vectors) varies strongly along the confocal ellipsoids of equal potential. For the modified potential flow solution, a simplifying assumption is made [18], equivalent to Dalle Valle's original error. It is that the equal velocity contours found in experimental work are equivalent to the equipotential confocal ellipsoidal surfaces described in potential flow field theory. This simplification, with appropriate correction factors [21], yields a quite accurate descriptive model of an unobstructed FCH flow field.

When plumes of jet contaminant are introduced, an appropriate jet-flow theory must be used. The Prandtl mixing-length hypothesis for turbulent jet flow, which assumes a constant virtual kinematic viscosity, and yields a constant kinematic momentum, seems to be applicable; viscosity is an important consideration in its derivation. The Schlichting equations calculate the velocity vectors of any given point in the flow field of the jet. It is assumed that each of these vector components can be added to those velocity vector

quantities calculated for each corresponding point along streamlines of the flow field of the hood. Vector additions are made iteratively at desired increments, to obtain the entire combined streamline.

This model of combined flow is validated experimentally. Computer predictions are made of the specific locations along the hood centerline of the jet, such that a 50% capture efficiency is achieved by the hood. This is necessary to determine if the velocity vectors of the two parts of this model, one (for the hood flow) which ignores viscosity, and the other (for the jet) which assumes its significance, can be added together to predict jetstream trajectory while in the flow field of the hood. If so, then the entire field of points of actual jet location can be mapped such that the capture efficiency of the hood is at least 50%.

## V. METHODS

### A. COMPUTER MODEL

The computer model is composed of the union of two parts, with accompanying reminders, explanatory notes, and instructions for graphic display and printouts. The two parts of the computer model are: 1) those that describe and calculate the flow field of the flanged circular exhaust hood; and 2) those that describe and calculate the flow of a free turbulent gas jet. Each of these parts of the overall program calculates the vector magnitude and direction of velocity in its own cylindrical coordinate system. These are denoted as  $(r, \theta, z)$  for the jet, and  $(R, \theta', Z)$  for the hood.

The jet is arranged in relation to the hood such that its tip is in front of the hood on the hood centerline, and the jet centerline ( $z$ ) axis is perpendicular to the centerline ( $Z$ ) axis of the hood. The "base plane," in which all calculations are done, is the plane of the two (hood and jet) centerlines.

Vector transformations are contingent upon the original orientation of the jet to the hood. The hood's  $R$  directional axis for calculation purposes was in the half-plane of the base plane in the direction of original jet flow. Additionally, only the  $r$ -vector of the jet in the base plane was considered for calculation purposes. Due to

the specific arrangement of jet to hood axes chosen, there was no  $\theta$  component (rotation out of the base plane) to be considered for either jet or hood. In the base plane, r- and z-direction vector magnitudes of the jet were transformed into the coordinate system of the hood, prior to the calculation of their combined magnitude. They were also calculated to account for the distance of the virtual point, within the tip of the jet tube, from the tip of the jet nozzle.

In order to account for the effects of the flange of the hood on the flow of the jet, use is made in the computer program of an image jet located "behind" the flange, the vector calculations for which are assumed to be equal and opposite to its real counterpart. It is necessary for the proper calculation of the velocity vectors of jet flow. Any given velocity vector for the real jet equals the scalar sum of the corresponding velocity vectors of both the real and imaginary jets. Thus, when combined, jet velocity vectors will be calculated to yield streamlines which follow a path which "sees" the barrier the flange presents.

The velocity vectors of the hood and jet flows are iteratively calculated and added (once transformed to the same coordinate system) for the entire length of the centerline flow of the jet within the flow field of the hood. The program directs the display of the jet's calculated centerline in relation to a cross section of the hood, and the hood centerline. Each time the program is

run, it may be used to calculate the jet trajectory for any given hood flow ( $Q_h$ ), jet flow ( $Q_j$ ), jet-to-hood distance ( $z$ ), and crossdraft velocity ( $V_c$ ) parallel to the axis of the jet.

The program can be run, using the given assumptions, with the jet pointing along any quadrant line. The jet could be placed pointing away from the hood ---0° to the hood Z axis--- or toward the hood (180°) along its axis. In contrast, the arrangement tested for the experiments reported in this thesis is placement of the jet axis perpendicular to the hood axis. Note that, without a crossdraft, a 270° placement is equivalent to 90°.

The program displays, for each run, the following variables:

- a)  $Q_h/Q_j$  = Ratio of hood to jet flows;
- b)  $V_h/V_j$  = Ratio of hood to jet velocities;
- c)  $D_h/D_j$  = Ratio of hood to jet diameter;
- d)  $Z/D$  = Ratio of the distance of the jet from the hood face, to the hood diameter;
- e)  $Q_h$  = Hood flow, cfm;
- f)  $V_h$  = Hood face velocity, fpm;
- g)  $R_h$  = Hood radius, ft.;
- h)  $Q_j$  = Jet flow, cfm;
- i)  $V_j$  = Jet face velocity, fpm;
- j) "Jet X" (Hood Z) = Distance along hood centerline of jet tip, in.;

- k) "Jet Y" (Hood R) = Hood radial distance of the jet tip from the centerline, in.;
- l) Ang = Angle of jet from hood centerline;
- m) Jet Orig = Distance inside jet tip from which spread of jet begins, in.
- n) Jet D = Jet diameter, in.;
- o) Xdrft = Crossdraft velocity (in the base plane of the jet and hood centerlines), fpm;
- p) Xdf Ang = Angle of the crossdraft from the hood axis.

The hood-to-jet distance, such that the jet trajectory loops over to just reach the edge of the hood opening, is called the "critical distance." See the computer program printouts in the Appendix.

It is assumed that the spread of the jet is symmetrical around its centerline. Therefore, when the jet begins at the critical distance, it is predicted that 50% of the jet flow is captured, while 50% of the jet flow escapes capture. The escaping contaminant potentially endangers nearby workers by entering their breathing zones.

Computer-predicted critical distances,  $[Z/D]_{50}$ 's, for any given (operator-entered) set of hood and jet flows can be determined by use of the program. This was done for a set of 21 combinations of hood and jet flow. The matrix of experimental design conditions, all  $[Z/D]$  distance ratios

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tested for each of the 21  $[Q_h/Q_j]$  values comprising the experiment, is shown [Figure 12].

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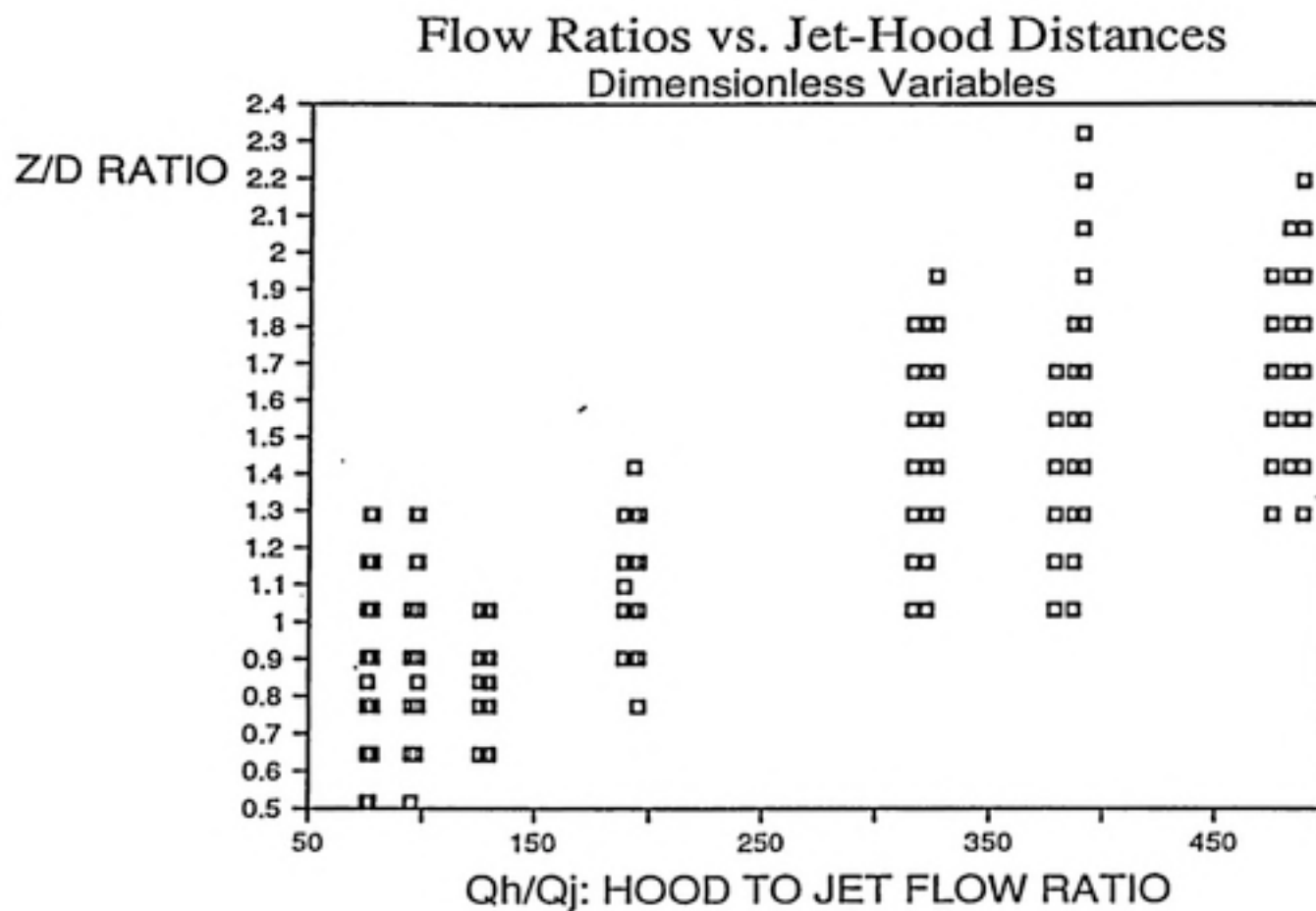


FIGURE 12. EXPERIMENTAL DESIGN

## B. LABORATORY VALIDATION

The experimental set-up consisted of the jet and gas source, the FCH and fan, devices to control flow through each, and the calibration apparatus.

The jet was made of a machined steel cylinder 0.25 inches diameter, and eight inches long, connected to tubing, rotameters, and a laboratory air pump. Sulfur hexafluoride ( $\text{SF}_6$ ) tracer gas at 900 ppm was drawn into the jet by the flow-induced vacuum created by the force of a laboratory air source. The individual flows were monitored carefully. Sulfur hexafluoride ( $\text{SF}_6$ ) was used as the tracer primarily because of its low toxicity and low flammability and its relative ease of detection.

The jet axis was placed perpendicular to the hood centerline, and so that its tip was on the hood centerline. This created a geometrical plane, the "base plane," in which all flow calculations were made.

A machined flanged circular exhaust hood, 3.875 inches in inside diameter, with a 4 inch wide flange, was used as the primary opening. It was connected to a flexible duct, through which air was drawn at various flow rates by an industrial fan. Flows were measured by means of a manometer which measured the pressure drop across a Venturi constriction in the intake pipe of the fan.

The detector probe consisted of a steel cylinder six inches long, 0.125" in diameter, with a handle through which the passage continued, connected to a flexible tube. The probe was placed well behind the hood face opening, through and into the hood's flexible duct, so that its tip reached the duct centerline. The probe was placed about 15 hood diameters back of the hood face, two 90° bends away, so there was complete mixing of the captured contaminant jet gases with the hood flow. The probe drew samples of duct air to an ITI gas chromatograph (GC), to be analyzed for the concentration of indicator gas,  $\text{SF}_6$ , being drawn into the exhaust hood. Peak heights were displayed on a chart recorder.

The GC generated a current proportional to the concentration of tracer gas being used. Quantitation was possible by calibrating concentration vs. peak height, which, over suitable ranges, is linear. The calibration equipment consisted of the exponential dilution flask, a gas-tight syringe, the  $\text{SF}_6$  gas source, a chart recorder, and a stopwatch or its equivalent. The exponential dilution flask had a volume of 3.7 liters, and was stirred with paddle blades to achieve to achieve gas concentrations of

$$C_t = C_0 \exp[-Qt/V] \quad (38)$$

where:  $t$  = the elapsed time;

$C_0$  = the initial concentration;

$Q$  = the flow through the flask; and

$V$  = the flask volume.

The initial flask concentration can be determined from the concentration of the  $SF_6$  source gas (900 ppm), the flask volume, and the amount injected.

Capture efficiency is a relative measurement and therefore actual concentrations are not as important as the relative changes in peak height. The calibrations were least squares regressions of peak height with concentration. Calibrations were performed before every run of measurements.

Capture efficiency measurements were made with the GC. It was calculated as the relative concentration of  $SF_6$  in the hood duct when the jet source was located at some hood centerline position  $Z$ , to the concentration in the hood duct when the jet source was located at the hood face.

$$\eta = C_z/C_f \quad (39)$$

The experimental design consisted of twenty-one combinations (ratios) of hood to jet flows. Three hood flows were used (220, 145 and 75 cfm), by setting the fan volume flows with a damper, and reading the calibrated manometer settings.

The actual laboratory source air jet flow was carefully regulated. First, the known hood flow was divided by the desired experimental ratio, and the desired total jet flow

was obtained. From this was subtracted the value in cfm of the  $\text{SF}_6$  flow necessary to obtain a preliminary 100% reading in the calibration procedure. The  $\text{SF}_6$  and laboratory air source flows both were set by carefully calibrated rotameters, so that the total jet flow just equaled the desired value.

A large laboratory exhaust hood was located above the experimental set-up, and was used primarily to draw off any excess  $\text{SF}_6$  escaping into the room. The essentially vertical crossdraft periodically was measured, but was so apparently low (average 25 fpm) that it was not expected originally to have an effect on the results.

For each of the the twenty-one hood to jet flow ratios tested, the jet was moved incrementally out along the hood centerline away from the hood. At each position, five or more measurements of  $\text{SF}_6$  concentration in the hood duct were taken. The capture efficiencies for each were calculated; then for every jet location, for each hood-to-jet flow combination  $[Q_h/Q_j]$ , the capture efficiency ( $\eta$ ) averages and standard deviations were calculated.

### C. STATISTICAL EVALUATION

At each incremental position of the jet along the hood centerline, for 21 experimental hood-to-jet flow  $[Q_h/Q_j]$  ratios, the average of five or more capture efficiencies was calculated. It has been found previously [17] that logit transformation is a useful treatment of capture efficiency data. Each of the calculated average capture efficiency values,  $\eta$ , was treated with the logistic transform:

$$y = \ln [\eta / 1 - \eta], \quad (40)$$

and

$$x = Z/D \quad (41)$$

where  $y$  is the natural logarithm of the odds of being captured, and  $x$  is the dimensionless centerline distance.

These values may be related to one another by simple linear (least squares) regression procedures, the form of which is:

$$y = \alpha x + \beta. \quad (42)$$

A consistent strong relationship would suggest that capture efficiency is described by a cumulative logistic function with the form:

$$\eta = 1 / (1 + \exp[(x - \mu)/w]), \quad (43)$$

where:  $w = -1/\alpha$ ; and

$$\mu = w\beta.$$

The values of  $\alpha$  and  $\beta$  are taken from the slope and intercept, respectively, of the regression of  $y$  on  $x$ .

For the logistic model, the parameter  $w$  is analogous to a standard deviation in a normal distribution. However, the distribution of the logistic model is much narrower (leptokurtic, or more peaked) than a normal distribution; 92.4% of the logistic distribution lies between  $-w$  to  $+w$ . The probability distribution function is, of course, readily obtainable from the cumulative distribution.

The experimentally estimated "true"  $[Z/D]_{50} = \mu$ . In the logistic function,  $\mu = w\beta$ . Restated,

$$\mu = -\beta/\alpha. \quad (44)$$

## VI. RESULTS AND DISCUSSION

### A. LOGIT CAPTURE EFFICIENCY, $y$ , REGRESSION ON JET DISTANCE TO HOOD DIAMETER RATIO, $x$

#### 1. Summary of Equations:

$$a) \quad y = \ln [\eta/(1-\eta)], \quad (40)$$

where:  $\eta$  = capture efficiency.

$$b) \quad x = Z/D, \quad (41)$$

where:  $D$  = hood diameter; and

$Z$  = jet to hood distance.

$$c) \quad \text{Regression: } y = \alpha x + \beta, \quad (42)$$

where:  $\alpha$  is the regression slope; and

$\beta$  is the  $y$ -intercept value.

$$d) \quad \text{Predicted } [Z/D]_{50}: \mu = -\beta/\alpha. \quad (44)$$

#### 2. Results and Analysis

The raw data tables and graphs, including regressions, are in the Appendix.

The summary results of the regressions are tabulated [Table 3] for each of the  $[Q_H/Q_j]$  ratios employed experimentally. Logistic function estimates,  $\mu$ , of each  $[Z/D]_{50}$  may or may not fall within the range between the two neighboring hood axial distances actually experimentally determined. Therefore,  $\mu$  can be modified to do so, and is

-----TABLE 3-----

## -----SUMMARY INITIAL RESULTS-----

| Qh/Qj<br>ratio | Logit C. E.<br>vs. (Z/D)               |                          |  | Experimental<br>(Z/D) 50<br>modlogit | Vc=0 fpm:<br>Predicted<br>(Z/D) 50 | Vc=25fpm:<br>Predicted<br>(Z/D) 50 |
|----------------|----------------------------------------|--------------------------|--|--------------------------------------|------------------------------------|------------------------------------|
|                | Regression<br>y=ax+b<br>R <sup>2</sup> | p-<br>value<br>CorrCoeff |  |                                      |                                    |                                    |
| 76             | 0.932                                  | 0.000                    |  | 0.87                                 | 0.548                              | 0.484                              |
| 77             | 0.925                                  | 0.001                    |  | 1.11                                 | 0.548                              | 0.484                              |
| 78             | 0.892                                  | 0.005                    |  | 1.03                                 | 0.548                              | 0.484                              |
| 95             | 0.859                                  | 0.023                    |  | 0.75                                 | 0.677                              | 0.548                              |
| 97             | 0.918                                  | 0.003                    |  | 0.94                                 | 0.677                              | 0.613                              |
| 98             | 0.860                                  | 0.023                    |  | 0.92                                 | 0.677                              | 0.677                              |
| 126            | 1.000                                  | 0.000                    |  | 0.86                                 | 0.937                              | 0.742                              |
| 129            | 0.953                                  | 0.024                    |  | 0.82                                 | 0.939                              | 0.806                              |
| 130            | 0.905                                  | 0.013                    |  | 0.84                                 | 0.948                              | 0.871                              |
| 189            | 0.948                                  | 0.005                    |  | 1.14                                 | 1.388                              | 0.935                              |
| 193            | 0.928                                  | 0.037                    |  | 1.15                                 | 1.413                              | 1.129                              |
| 195            | 0.919                                  | 0.010                    |  | 1.03                                 | 1.452                              | 1.194                              |
| 316            | 0.830                                  | 0.004                    |  | 1.41                                 | 2.199                              | 1.258                              |
| 322            | 0.765                                  | 0.023                    |  | 1.41                                 | 2.243                              | 1.516                              |
| 326            | 0.960                                  | 0.001                    |  | 1.69                                 | 2.252                              | 1.710                              |
| 379            | 0.973                                  | 0.000                    |  | 1.43                                 | 2.529                              | 1.323                              |
| 387            | 0.945                                  | 0.000                    |  | 1.59                                 | 2.568                              | 1.645                              |
| 391            | 0.882                                  | 0.000                    |  | 2.07                                 | 2.613                              | 1.903                              |
| 475            | 0.775                                  | 0.021                    |  | 1.52                                 | 2.987                              | 1.387                              |
| 483            | 0.972                                  | 0.000                    |  | 1.76                                 | 3.000                              | 1.839                              |
| 489            | 0.939                                  | 0.000                    |  | 1.98                                 | 3.065                              | 2.097                              |

denoted  $\mu_{\text{mod}}$ . The predicted  $[Z/D]_{50}$ , determined by this modified logit transformation of the capture efficiency data, may be referred to as "modified logit" or even "modlogit." Computer program predictions of the  $[Z/D]_{50}$ 's are also displayed, for comparison, at two crossdraft velocity values.

Regressions of the true logit of capture efficiency,  $y$ , on the jet distance to hood diameter ratios,  $x$ , yield  $R^2$  values of which two-thirds are greater than 0.9, and none is less than 0.75. In addition,  $p$ -values for the significance of the correlation coefficient in the regression equations are uniformly less than 0.05, and two-thirds are equal to or less than 0.01.

These results suggest that the logit transformation is a useful treatment, and that estimates of critical distances,  $[Z/D]_{50}$ 's, can be made with validity from capture efficiency data with this statistical procedure.

B. PREDICTED VS. EXPERIMENTAL  $[Z/D]_{50}$  CRITICAL DISTANCES

Actual conditions may differ from those specified or assumed in the model. They do so appreciably near the hood face. There, the hood flow creates a strong static pressure gradient; there is shear turbulence between the flows of jet and hood; and frictional forces in the hood boundary layer become important. Predictions made for a jet entering a uniform flow field may not be borne out in this region.

The experimental findings near the hood face are consistent with this evaluation. See the graph of experimental vs. predicted  $[Z/D]_{50}$  [Figure 13], and the tabulated values [Table 3]. The strong static pressure gradient, shear turbulence, and the turbulence (frictional) effects of viscous flow near the hood face create conditions of increased capture efficiency (longer  $[Z/D]_{50}$ 's) there compared to predicted values. This is true where the predicted critical distance of the jet from the hood is less than about seven-tenths of a hood diameter, i.e. where the predicted hood-to-jet flow ratios,  $[Q_h/Q_j]$ , necessary to capture 50% or more of the jet flow are less than 100.

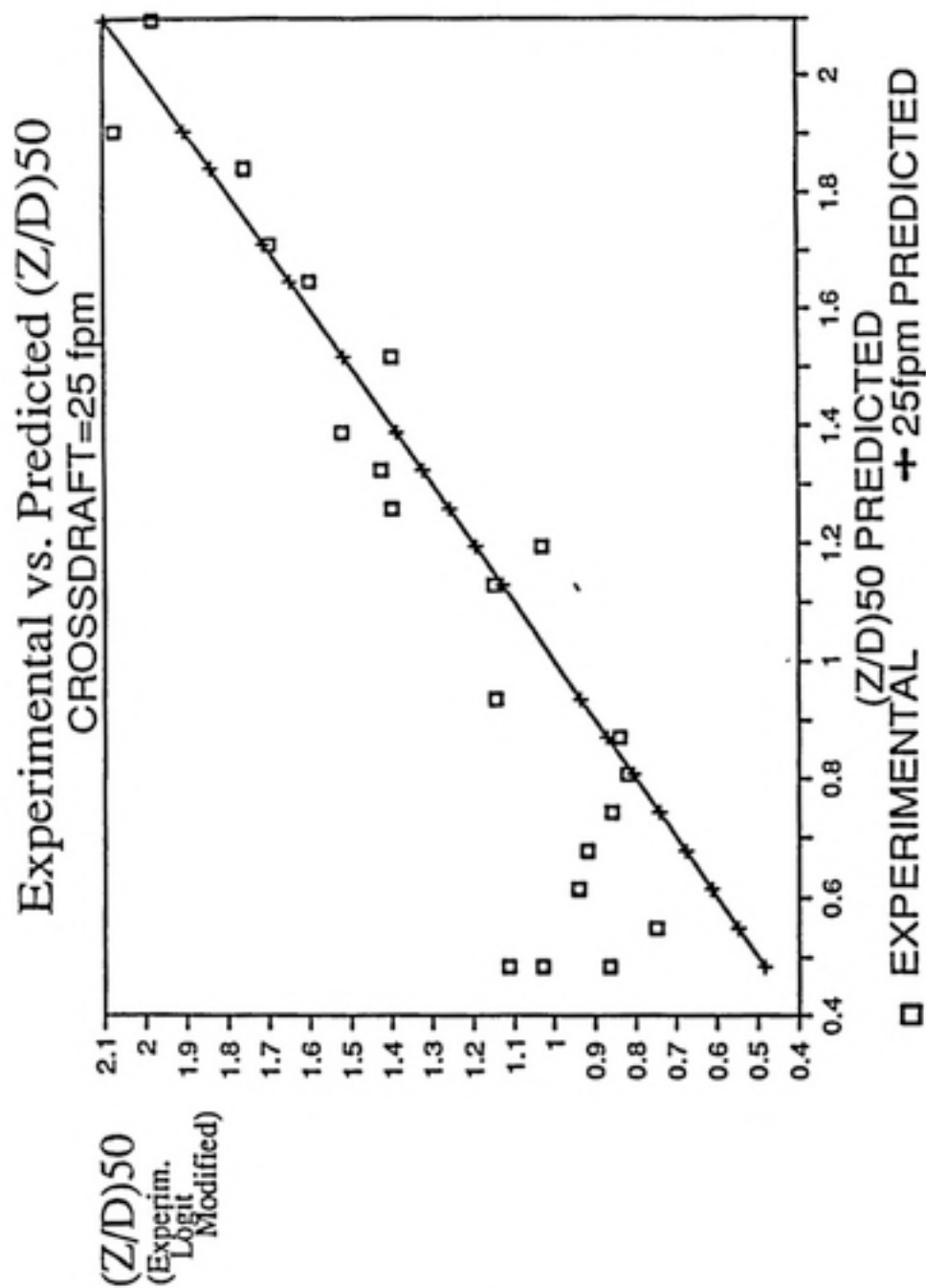


FIGURE 13.

Turbulence may be classified in a number of ways. A useful set of distinctions is between shear turbulence (turbulence generated in free space between interacting streams of fluid); and wall turbulence (generated by fluid interactions with solid surfaces and characterized by boundary layers).

Both types of turbulence are occurring in this case; shear turbulence between a) the jet streams with each other; b) jet and hood streams; c) hood flow streams with each other; and d) each of the above with any crossdraft streams. Wall turbulence is occurring near the hood face along the entire reach of the flanges, and at the corners of the flanges with the duct.

Flynn and Ellenbecker [17] could assume that the characteristics of turbulence were primarily determined by the interaction of hood flow with crossdraft flow. They had no velocity component from their point contaminant source(s), let alone a jet, with which to contend. See [Figure 6], a depiction of the trajectories of single idealized contaminant streamlines (ICS) in a flow field of interacting hood and crossdraft effects.

Therefore, in their analysis, the  $[V_f/V_c]$  ratio (hood face velocity to crossdraft velocity) serves them as a suitably predictive dimensionless variable for the effects of turbulent diffusion around each ICS. Thus Flynn and Ellenbecker rationalize the use of  $\omega$ , the spread parameter in the logistic function, to describe in probabilistic terms

the likely distribution of the turbulent diffusion of single streamlines. They therefore can relate mathematically predictions of the spread parameter,  $w$ , to the dimensionless velocity ratio  $[V_f/V_c]$ . From this they can simplify their model so that capture efficiency,  $\eta$ , may be predicted directly as a function of the actual,  $x$ , and experimentally estimated,  $\mu$ , ratios of centerline distances of the source from the hood face to the hood diameter.

However, a spreading turbulent jet creates a flow field very different indeed from that of Flynn and Ellenbecker's idealized contaminant streamlines (ICS), primarily by imparting significant turbulence to the hood flow field, and thus creating strong velocity gradients within it [Figure 10]. They acknowledged [17] that the velocity field into the hood (without a jet) already shows considerable gradients both in the direction of mean flow, as well as perpendicular to the streamlines, and that this indicates a non-homogenous turbulent field.

There is evidence [24] that the turbulence intensity in shear flows is quite large, and that the bulk of the transport by turbulent diffusion occurs both quickly and near the source. This suggests [17] that "the effects of turbulent diffusion may be largely determined by conditions at the source."

One might therefore suspect that the turbulence intensity near a jet source may be sufficient to cause contaminant gas dispersion into the hood flow field near the

jet source. Thus, when the jet is within a short distance of the hood face, these effects are combined with those of a strong static pressure gradient across the jet created by the hood flow, so that more than predicted amounts of contaminant are captured, even at hood jet flow ratios of less than 100.

Where the predicted critical distance for the jet is farther than 0.7 hood diameters from the hood face (where the required hood-to-jet flow ratios are higher than 100), the predicted  $[Z/D]_{50}$  and the experimentally estimated (modified logit procedure)  $\mu_{\text{mod}} = [Z/D]_{50}$  are very close. If the crossdraft is nominally assumed to be 25 fpm (about the average of laboratory measurements), regression of all  $\mu_{\text{mod}}$  on all predicted  $[Z/D]_{50}$  values yields:

$$\mu_{\text{mod}} = 0.713 [Z/D]_{50} + 0.451 \quad (45)$$

with an  $R^2 = 0.852$ .

For those values of the predicted  $[Z/D]_{50} > 0.7$  (i.e. where  $[Q_h/Q_j] > 100$ ), under the same conditions, this regression yields:

$$\mu_{\text{mod}} = 0.915 [Z/D]_{50} + 0.139, \quad (46)$$

with an  $R^2 = 0.924$ .

This is excellent agreement in the outer region between predicted and experimental critical distances. There is,

however, no discernible relationship between  $w$  with the hood face to assumed average crossdraft velocity ratio,  $[V_f/V_C]$ , probably because, while the statistical concept of  $w$ , the spread parameter, was useful for a single ICS from a point source, it is irrelevant in turbulent spreading jet flow.

C. HOOD TO JET FLOW RATIOS VS.  $[Z/D]_{50}$ 's:  
PREDICTIONS AND CROSSDRAFT EFFECTS

The effects of the crossdraft were apparent in the relationships of the dimensionless predictor variables  $[Z/D]$  and  $[Q_h/Q_j]$ . Regressions of  $\mu_{\text{mod}}$  (modified logit estimated experimental  $[Z/D]_{50}$ ) on the computer program prediction of  $[Z/D]_{50}$  are more significant at  $V_c = 25$  fpm than they are, for example, at 0 fpm (no) crossdraft.

Moreover, in the graphical representation [Figure 14] of  $\mu_{\text{mod}}$  vs.  $[Q_h/Q_j]$ , and predicted  $[Z/D]_{50}$  also vs.  $[Q_h/Q_j]$ , there is complete overlap (except for a single data point) of the modified logit-transformed experimental  $[Z/D]_{50}$  with the computer program-predicted values, for all predicted  $[Z/D]_{50}$ 's  $> 0.7$ , when the crossdraft is set at 25 fpm. (When the program crossdraft value was fixed at 0 fpm, or at 5 fpm, however, there is no meaningful overlap [Figures 15 and 16].) There is a significant spreading of the predictions of  $[Z/D]_{50}$  in proportion to the  $[Q_h/Q_j]$  ratio in the presence of a crossdraft. As the programmed crossdraft gets larger, that predicted spread becomes wider until, at 25 fpm, the experimental findings are nearly completely encompassed.

The three lines in Figures 14 and 16 represent the predictions of the  $[Z/D]_{50}$  for each  $[Q_h/Q_j]$ , for each of three  $[V_f/V_c]$  ratios tested. There are three  $[V_f/V_c]$  ratios

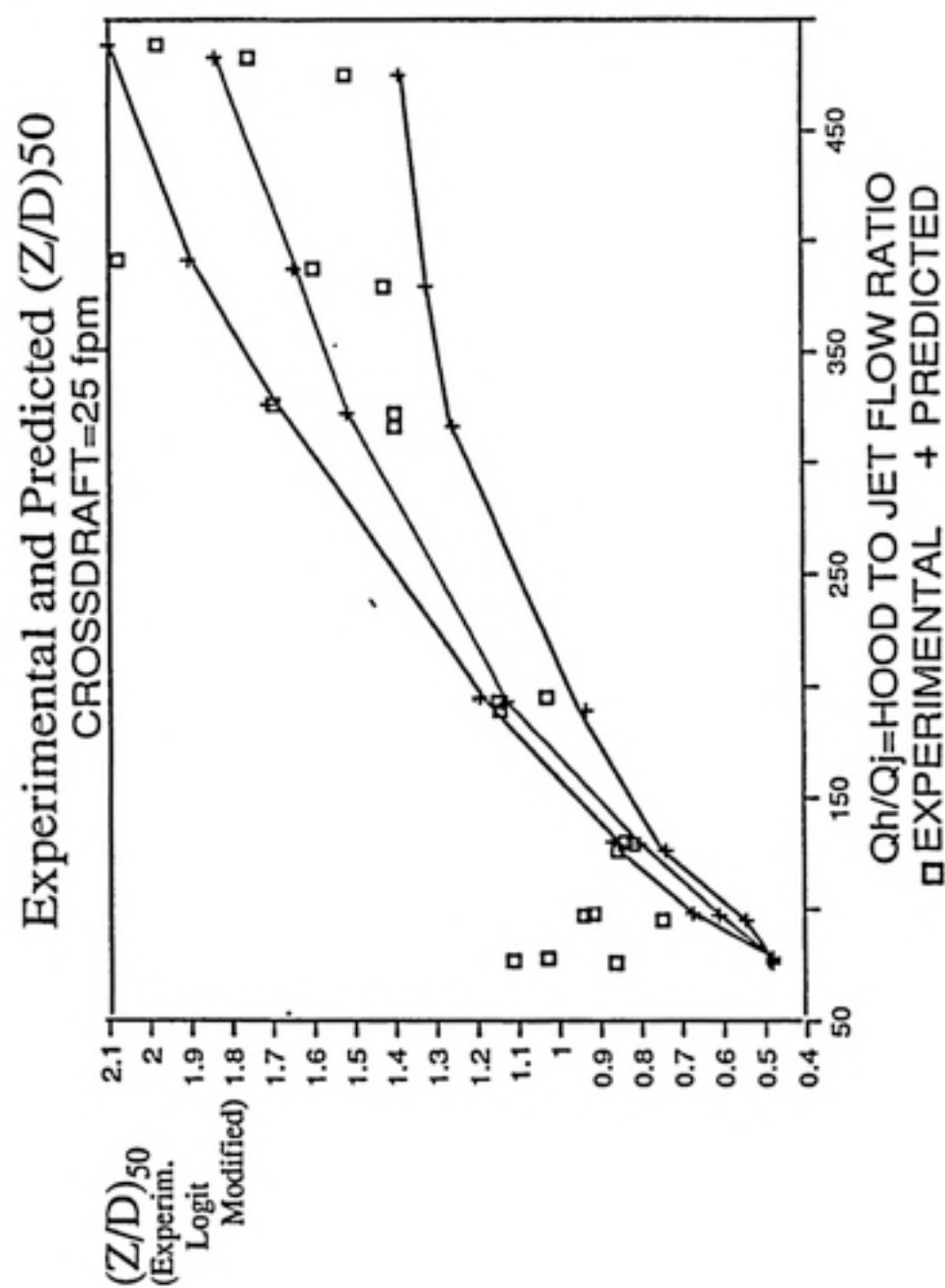


FIGURE 14

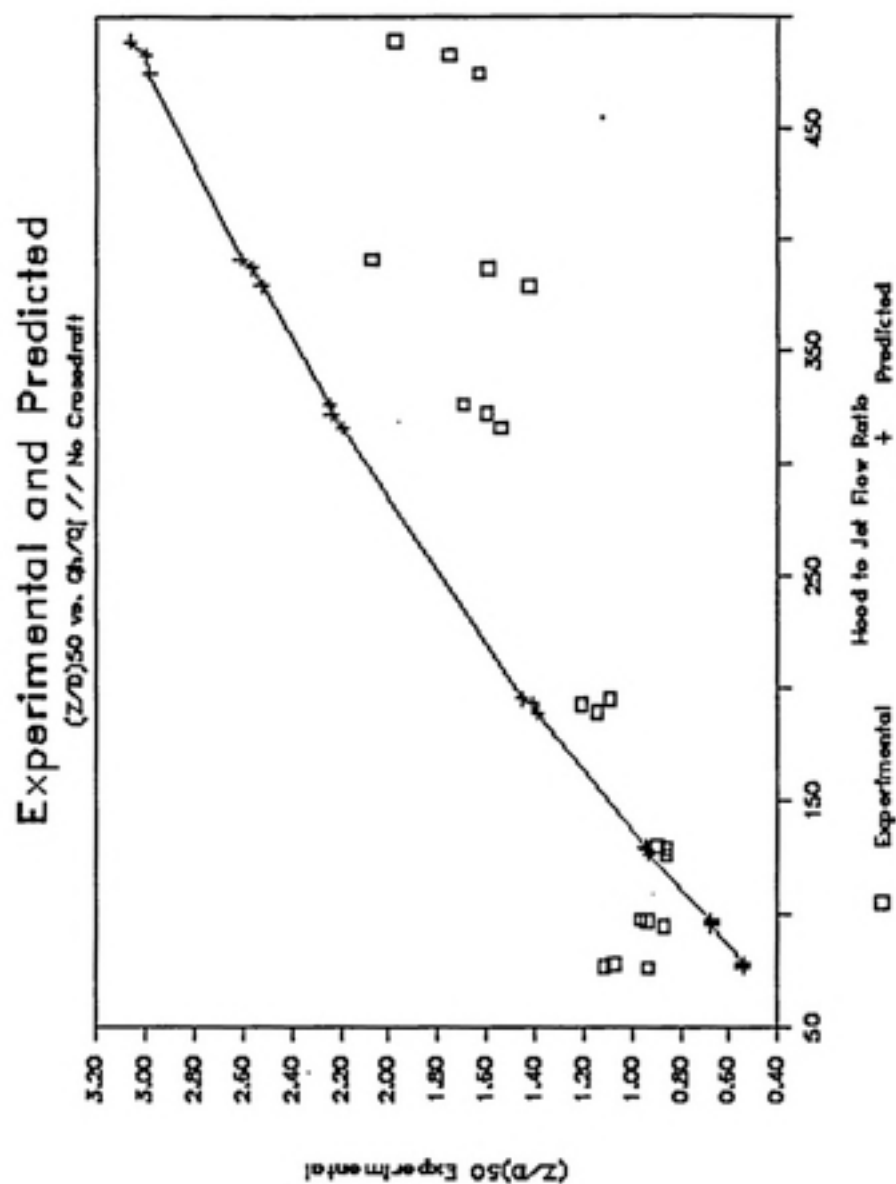


FIGURE 15

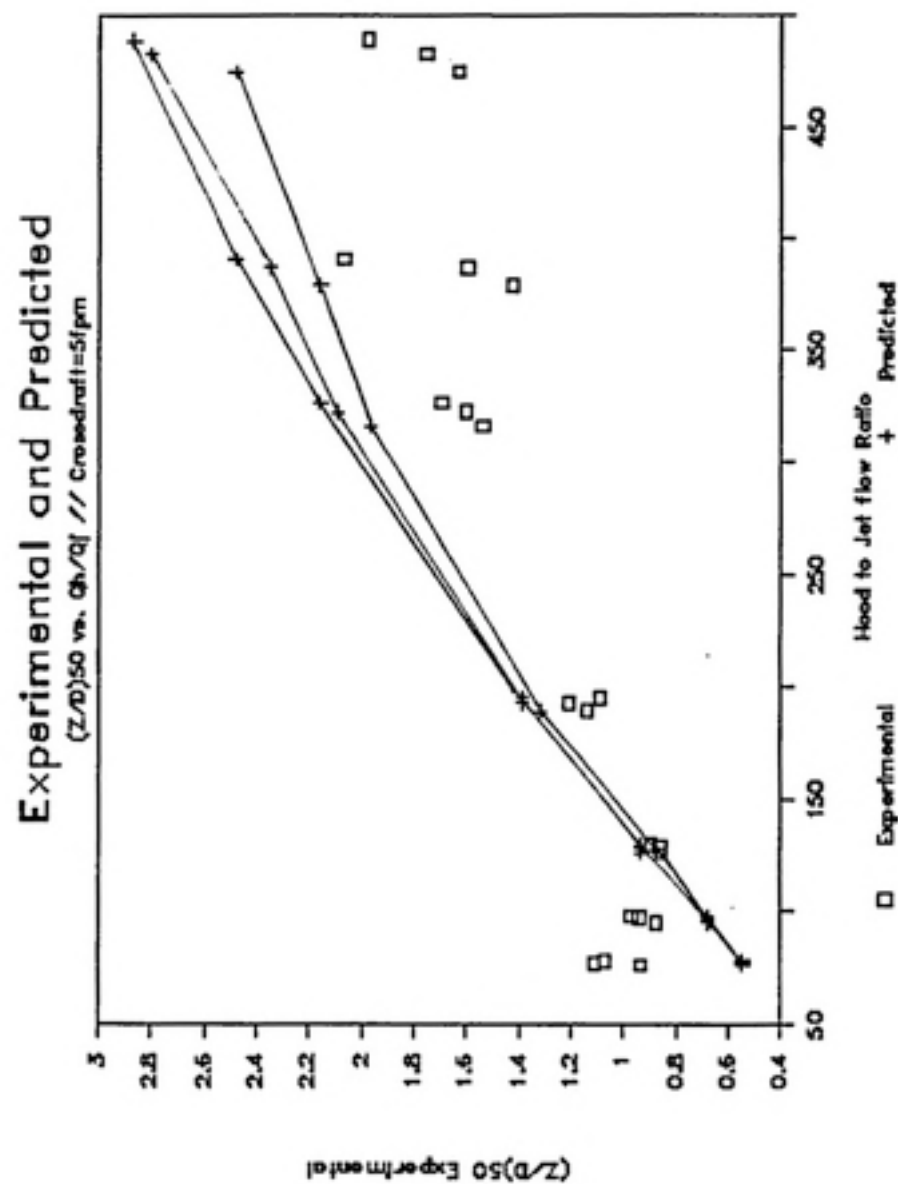


FIGURE 16

because there were three hood face velocities in the experiments (three  $Q_h$ 's tested). It is assumed that a non-zero crossdraft is constant. In Figure 15, for no (0 fpm) crossdraft, these three prediction lines simply overlap.

With the crossdraft fixed at 25 fpm, power regression analysis was conducted for values of  $\mu_{\text{mod}}$ , modified experimental estimates of  $[Z/D]_{50}$ , upon values of the corresponding  $[Q_h/Q_j]$  ratios, for each of the  $[V_f/V_C]$  ratios. The results are:

For  $[V_f/V_C] = 36.64$ :

$$\mu_{\text{mod}} = .144 [Q_h/Q_j]^{0.387}, \quad (47)$$

with  $R^2 = 0.922$ .

For  $[V_f/V_C] = 70.82$ :

$$\mu_{\text{mod}} = .211 [Q_h/Q_j]^{0.331}, \quad (48)$$

with  $R^2 = 0.729$ .

For  $[V_f/V_C] = 107.44$ :

$$\mu_{\text{mod}} = .107 [Q_h/Q_j]^{0.471}, \quad (49)$$

with  $R^2 = 0.804$ .

To confirm the validity of the program when incorporating the average measured value of the crossdraft

velocity (25 fpm), the computer program was re-run. This time, the variables were set such that for every  $[Q_h/Q_j]$  ratio tested, the program  $[Z/D]$  (jet location) ratio was set at the best statistically evaluated laboratory value for each experiment. Then the program crossdraft velocity,  $V_c$ , was sequentially altered until the predicted jet trajectory within the hood flow curved until it just hit the edge of the computer-displayed hood opening, i.e. until that particular  $[Z/D]$  became the critical distance, the  $[Z/D]_{50}$ . For all  $[Q_h/Q_j]$  ratios over 100, the average value of the crossdrafts necessary in the computer program to cause the predicted  $[Z/D]_{50}$  to match the real one was 24.33 fpm [Table 4].

## VALUES REQUIRED TO MATCH COMPUTER

-----MODEL PREDICTIONS OF [Z/D]50 WITH-----  
 EXPERIMENTAL (mod. logit) [Z/D]50

| ---EXPERIMENTAL CRITERIA--- |           |          | Eddy Viscosity,<br>epsilon-sub-0,<br>Required: |          | Crossdraft<br>fpm<br>Required: |
|-----------------------------|-----------|----------|------------------------------------------------|----------|--------------------------------|
| Qh/Qj                       | modlogit  | modlogit | at                                             | at       |                                |
|                             | [Z]50,in. | [Z/D]50  | Vc=0 fpm                                       | Vc=25fpm |                                |
| =====                       |           |          |                                                |          |                                |
| 76                          | 3.35      | 0.87     | 4.3E+00                                        | 6.9E+00  | N/A                            |
| 77                          | 4.32      | 1.11     | 1.2E+01                                        | 2.1E+01  | N/A                            |
| 78                          | 3.99      | 1.03     | 1.7E+01                                        | 1.9E+01  | N/A                            |
| 95                          | 2.91      | 0.75     | 2.1E+00                                        | 2.7E+00  | N/A                            |
| 97                          | 3.65      | 0.94     | 3.9E+00                                        | 6.5E+00  | N/A                            |
| 98                          | 3.57      | 0.92     | 5.9E+00                                        | 7.8E+00  | N/A                            |
| 126                         | 3.33      | 0.86     | 5.0E-01                                        | 1.6E+00  | 5                              |
| 129                         | 3.18      | 0.82     | 7.0E-01                                        | 2.0E+00  | 24                             |
| 130                         | 3.26      | 0.84     | 1.0E+00                                        | 1.3E+00  | 32                             |
| 189                         | 4.43      | 1.14     | 2.0E-01                                        | 1.4E+00  | 10                             |
| 193                         | 4.46      | 1.15     | 2.0E-01                                        | 1.3E+00  | 21                             |
| 195                         | 3.99      | 1.03     | 5.0E-05                                        | 1.0E-02  | 59                             |
| 316                         | 5.43      | 1.40     | 1.0E-06                                        | 1.0E+00  | 16                             |
| 322                         | 5.43      | 1.40     | 2.0E-06                                        | 4.0E-03  | 33                             |
| 326                         | 6.58      | 1.70     | 2.0E-05                                        | 5.0E-01  | 25                             |
| 379                         | 5.52      | 1.43     | 5.0E-07                                        | 6.9E-01  | 19                             |
| 387                         | 6.19      | 1.60     | 3.0E-06                                        | 1.3E-01  | 28                             |
| 391                         | 8.04      | 2.07     | 1.0E-05                                        | 1.5E+00  | 17                             |
| 475                         | 5.89      | 1.52     | 4.0E-07                                        | 5.5E-01  | 20                             |
| 483                         | 6.81      | 1.76     | 2.0E-06                                        | 1.1E-01  | 27                             |
| 489                         | 7.67      | 1.98     | 3.0E-06                                        | 2.0E-02  | 29                             |

-----  
 Avg. =  
 24.333fpm

D. THE ROLE OF  $\epsilon_0$ , THE VIRTUAL OR APPARENT KINEMATIC ("EDDY") VISCOSITY

To clarify a little the events occurring when the jet location is set within one hood diameter's distance of the hood face, and additional effects of the crossdraft on the system, a new program was written, EPSILON. This program is a modification of the original, constructed to change iteratively the value of  $\epsilon_0$ , the eddy (apparent kinematic) viscosity of the jet flow.

Viscosity is a natural property of fluids and is a measure of resistance to shear forces. Kinematic viscosity is calculated as the viscosity divided by the fluid density. The apparent kinematic viscosity of a fluid is an analytic concept useful in helping account for the retarding effects on flow due to the excess frictional forces of turbulence.

The program was written to assist in the determination of the approximate value of  $\epsilon_0$  such that when inserted into the original program, the computer program would actually predict the best statistically evaluated  $[Z/D]_{50}$  values determined experimentally for all sets of trial conditions. The incorporation of a nominal crossdraft was critical in this determination, and had its most significant effect on the value of the necessary  $\epsilon_0$  at the higher hood to jet flow  $[Q_h/Q_j]$  ratios, where the  $[Z/D]_{50}$  distances are large, i.e. where real crossdrafts would have the most relative effect [Table 4].

Moreover, where the  $[Q_h/Q_j]$  ratios are less than 100, the crossdraft seems to have no relative effect on the  $\epsilon_o$  values required at all; this is expected because the jet is located within one hood diameter of the hood opening. In this region, the value of the required eddy viscosity value,  $\epsilon_o$ , of the jet flow is generally much greater than it is with the jet further away from the hood face. Since viscous forces in turbulent flow are approximately proportional to the square of the mean velocity, the fact that the eddy viscosity rises when the jet is near the hood face is expected.

## VII. CONCLUSIONS AND RECOMMENDATIONS

The object of local exhaust ventilation is to reduce of workers' breathing zone concentrations of toxic materials. Industrial hygiene has developed rapidly in recent years in the analysis of the mechanisms of operation of local exhaust outlets, after a fallow period in the 1950's and 1960's. From the core concept of centerline velocity gradients as the basis of evaluation (and installation - it is still used by the ACGIH), the profession has recently moved to analytic and numerical solutions based in fluid mechanics theory, and to the concept of capture efficiency. These analyses have yielded promising results, and may be even more productive than the past "practical" approach. This thesis is a piece of a larger body of ongoing research work in the field. Analytic evaluations of capture efficiency concepts, hood flows in various configurations, and experimental validations of applied potential flow theory have formed some of the background to this paper. Although frictional forces are ignored in the potential flow solution, in its modified form it has been found to be highly predictive in evaluating flows into both flanged round and rectangular hoods, both with and without crossdrafts.

Jets of contaminant gases are commonly found in industrial settings. They may include gas jets of

various kinds, intentionally occurring and accidental; spray paint jets; jets of welding fume, and many other examples. It is important to remove these dangerous materials as efficiently as possible from the industrial workers' environment; where they contaminate the breathing zone of a stationary worker, local exhaust ventilation is a critical need.

This thesis describes a computer program written to address this problem and act as a first step in its solution. Schlichting's derivation of velocity vectors for a circular free jet was used as the basis of the jet flow in the program. The assumptions used were those inherent in the mixing length hypothesis of Prandtl. Viscous forces are of importance in the theoretical derivations. (In contrast, viscous forces are neglected in the potential theory model of hood flow.) The program adds iteratively the vectors of hood and jet flow at every point along the jet's hypothetical centerline. The computer program calculates the velocity vector values for any diameter flanged circular hood of any specified flow, if the jet tip is perpendicular to and on the centerline of the hood. A crossdraft parallel to the jet (either direction) can be programmed in.

The program, by predicting the critical distance of the jet from the hood, where 50% of the contaminant will be captured by the hood, forms a basis for future work. Outside of the zone within about seven-tenths of one hood

diameter of the hood face, the program quite accurately predicts the critical distance. Within that short region, the critical distance is found experimentally to be longer than expected (the error is in a hypothetical worker's favor). The discrepancy near the hood face is due to: 1) the steep static pressure gradient created by the hood flow upon a jet placed within it; 2) turbulent eddies propagating from the shear between interacting streamlines of hood and jet flow; and 3) turbulence effects near the flange not accounted for in the potential flow solution. Values of the apparent or virtual kinematic (eddy) viscosity of the jetstream are high when the jet is located near the hood face.

The jet is circular, and is flowing freely in a conical dispersion pattern. This, and the real turbulence effects described above, are the reason that  $w$ , the spread parameter of a single idealized streamline, when analyzed in a logistic function, does not describe either statistically or analytically the distribution of jet flow in the hood velocity field.

Even very weak crossdrafts have strong effects outside about one hood diameter away from the hood face. It is recommended that systematic experimental variations be made of the crossdraft velocity in a wind tunnel to confirm the strong probable effect demonstrated here of the average crossdraft velocity on the  $[Z/D]_{50}$  and, thus

on the capture efficiency of flanged circular exhaust hoods for a jet of contaminant.

The described computer model adds the vectors for the potential flow field into a local exhaust hood with those of a free turbulent jet model in which viscous forces are important. The program does not attempt to solve Navier-Stokes or any of the energy equations of accurate fluid mechanics models. Numerical simulations of such problems are very expensive in time, computer power and code development. Recently, investigators have evaluated the interactions of crossdraft and jets using such methods [25, 26]. The program is, however, useful as an approximate model, for building upon in the design of effective local exhaust ventilation systems, and for further research work into the interactions of hood, jet and crossdraft flow fields.

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APPENDIX A

RESEARCH COMPUTER PROGRAM

```

10 CLS
20 DIM P(1600,2)
30 PI=3.1415927#
40 PRINT "WELCOME"
50 'Author: This program was prepared by Kirby N. Smith.
60 'Date: This program was finalized in October, 1987.
70 'Purpose: To describe the contaminant jet flow near a flanged circular
80 '          hood, from a jet pointing in any axial or radial direction
              (0,90,180,270,360 degrees) in the base plane of the hood.
90 'Method: The flow field into the hood is described with the equations
100 '         of potential theory, which assumes inviscid, irrotational and in-
              compressible flow. The equations of the Prandtl mixing length hypo-
              thesis, which takes viscosity into account, describe the jet flow.
110 '        These two types of equations are added vectorially at every point
              in the base (R-Z) plane of the hood. The jet flow is portrayed
              graphically in the printout.
120 '        The line of 50% Capture Efficiency can be determined with a function
              of the dimensionless variables (Qh/Qj) and (Z/D)50, and e-o.
130 'Initial Variables: Variables containing "J" are for the jet. Jet direction
              is in hood coordinates. Initial inquiries are self-explanatory.
140 '        Constants in the program and/or input variables, or even whether a
              certain value should be a constant or an input variable, may be
              altered at the user's careful discretion.

150 SCREEN 0
160 PRINT "THE HOOD DIAMETER IS 3 7/8 IN."
170 DH=3.875
180 RH=DH/2
190 A=RH/12          'Note: A is the hood radius in feet.
200 INPUT "WHAT IS THE HOOD FLOW IN CFM";Q
210 VH=Q/(PI*A^2)
220 PRINT "THE JET DIAMETER JD=.25 INCHES."
230 JD=.25
240 DJ=JD/12
250 RJ=DJ/2
260 AJ=PI*(RJ)^2
270 INPUT "WHAT IS THE JET FLOW IN CFM";QJ
280 VJ=QJ/AJ
290 INPUT "WHERE IS THE JET FACE IN HOOD AXIAL COORD. (0to12in.)";XJFIN
300 PRINT "THE RADIAL COORD. (-12to+12 IN) OF JET FACE, YJFIN=0."
310 YJFIN=0
320 XJF=XJFIN/12
330 YJF=YJFIN/12
340 'Note: (+Z) is to the right. Negative Z is into the hood. Positive R
              is up on the screen. The R-Z plane is the base plane.
350 PRINT "JET DIRECTION DIR=90 DEG. FROM THE HOOD(+Z) AXIS"
360 DIR=90
370 RADIANS=PI*DIR/180
380 PRINT "THE CROSSDRAFT ANGLE XDFA=90 DEG."
390 XDFA=90
400 VC=25
410 'INPUT "WHAT IS THE CROSSDRAFT VELOCITY";VC
420 PRINT "THE CROSSDRAFT VELOCITY IS";VC"FPM."
430 RDXDF=PI*XDFA/180
440 PRINT "THE INCREMENT TO RECALCULATION IS .03 FT."
450 INC=.03
460 'Note: The virtual point in the Prandtl mixing length hypothesis of jet
              flow is taken as the actual origin of the jet.
470 PRINT "THE JET ORIGIN IS"1.86*JD"IN. INSIDE THE JET TIP."
480 PRINT "THE JET VELOCITY IS";VJ;"FPM."
490 PRINT "THE HOOD VELOCITY IS";VH;"FPM."
500 INPUT "ENTRIES OK(Y/N)";A#

```

```

510 IF A$="N" OR A$="n" THEN GOTO 150 ELSE 520
520 CLS
530 COLOR 5
540 PRINT "PLEASE STAND BY."
550 'Note: The following three equations set the angle of the image jet.
560 IF RADIANS=0 THEN RADIM=PI
570 IF 0<RADIANS(=PI THEN RADIM=PI-RADIANS
580 IF PI<RADIANS(2*PI THEN RADIM=(3*PI)-RADIANS
590 ' Note:The following equations set the angle of the image crossdraft.
600 IF RDXDF=0 THEN XDFIM=PI
610 IF 0<RDXDF(=PI THEN XDFIM=PI-RDXDF
620 IF PI<RDXDF(2*PI THEN XDFIM=(3*PI)-RDXDF
630 'Note: The following equations calculate the vectors for each increment
      of hood flow.
640 'Note: Hood coordinates Z and R are named in hood flow calculations
      as X and Y, respectively.
650 X=XJF
660 Y=YJF
670 FOR I=1 TO 1600
680 GAMMA1=SQR(X^2+(A+Y)^2)
690 GAMMA2=SQR(X^2+(A-Y)^2)
700 ECC=(2*A)/(GAMMA1+GAMMA2)
710 ECC2=ECC^2
720 T1=A+Y
730 T2=Y-A
740 T3=GAMMA1+GAMMA2
750 T4=GAMMA1*GAMMA2
760 T5=4*A^2
770 T6=SQR(T3^2-T5)
780 T7=-Q/PI
790 T8=(T1*GAMMA2)+(T2*GAMMA1)
800 T9=T3*T4*T6
810 VR1=(T8/T9)*T7
820 VZ1=(T7*X)/(T4*T6)
830 V=SQR(VR1^2+VZ1^2)
840 VTF=(Q*ECC2*SQR(3))/(2*PI*A^2*SQR(3-2*ECC2))
850 VR2=(VR1/V)*VTF
860 VZ2=(VZ1/V)*VTF
870 VRC=(2.6*ECC^18+.853)*VR2
880 VZC=.9*VZ2
890 'Note: The following equations transform the coordinates of the jet and
      its image to the coordinates of the hood.
900 IF DIR=0 OR DIR=360 THEN GOTO 910 ELSE 960
910 Z=X-XJF+(1.86*DJ)
920 R=YJF-Y
930 ZI=2*((1.86*DJ)-(2*XJF))-Z
940 RI=-R
950 GOTO 1140
960 IF DIR=90 THEN GOTO 970 ELSE GOTO 1020
970 Z=Y-YJF+(1.86*DJ)
980 R=X-XJF
990 ZI=Z
1000 RI=(2*XJF)+R
1010 GOTO 1140
1020 IF DIR=180 THEN GOTO 1030 ELSE GOTO 1080
1030 Z=XJF-X+(1.86*DJ)
1040 R=Y-YJF
1050 ZI=2*((1.86*DJ)+XJF)-Z
1060 RI=-R
1070 GOTO 1140
1080 IF DIR=270 THEN GOTO 1090 ELSE GOTO 350
1090 Z=YJF-Y+(1.86*DJ)
1100 R=XJF-X
1110 ZI=Z
1120 RI=R-(2*XJF)
1130 'Note: The following equations calculate the vectors for any given R and

```

Z points of the jet flow.

```
1140 K=AJ*(VJ)^2
1150 EO=.0161*SQR(K)
1160 ETA=(R*SQR(3*K))/(4*(EO)*Z*SQR(PI))
1170 ETA1=ETA-((ETA^3)/4)
1180 ETA2=(1+((ETA^2)/4))^2
1190 IF -.00001(ETA AND ETA(.00001 OR -2.00001(ETA AND ETA(-1.99999 OR
1.99999(ETA AND ETA(2.00001 THEN GOTO 1200 ELSE 1220
1200 VRJ=0
1210 GOTO 1230
1220 VRJ=(SQR(3*K)*ETA1)/(4*(ETA2)*Z*SQR(PI))
1230 VZJ=3*K/(8*PI*(EO)*Z*ETA2)
1240 IF VZJ<0 THEN VZJ=0 AND VRJ=0
1250 'Note: The following calculations are for the effect of the image jet
field on the flow of the actual jet into the hood. Note that
K and epsilon-sub-zero have already been calculated for the jet.
1260 ETA1=(R1*SQR(3*K))/(4*EO*Z1*SQR(PI))
1270 ETA11=ETA1-((ETA1^3)/4)
1280 ETA12=(1+((ETA1^2)/4))^2
1290 IF -.00001(ETA1 AND ETA1(.00001 OR -2.00001(ETA1 AND ETA1(-1.99999 OR
1.99999(ETA1 AND ETA1(2.00001 THEN GOTO 1300 ELSE 1320
1300 VRIM=0
1310 GOTO 1330
1320 VRIM=(SQR(3*K)*ETA11)/(4*ETA12*Z1*SQR(PI))
1330 VZIM=3*K/(8*PI*EO*Z1*ETA12)
1340 IF VZIM<0 THEN VZIM=0 AND VRIM=0
1350 'Note: The following equations add the vectors and then determine
the next point for vector calculation based on the combined
values and the desired increment of calculation.
1360 VRTOT=VRC+(COS(RADIANS)*VRJ)+(COS(RADIM)*VRIM)+(SIN(RADIANS)*VZJ)+
(SIN(RADIM)*VZIM)+(SIN(RDXDF)*VC)+(SIN(XDFIM)*VC)
1370 VZTOT=VZC+(COS(RADIANS)*VZJ)+(COS(RADIM)*VZIM)+(SIN(RADIANS)*VRJ)+
(SIN(RADIM)*VRIM)+(COS(RDXDF)*VC)+(COS(XDFIM)*VC)
1380 IF VZTOT<=-.00001 THEN GOTO 1390 ELSE GOTO 1410
1390 ANGLE=ATN(VRTOT/VZTOT)+PI
1400 GOTO 1490
1410 IF VZTOT>=.00001 THEN GOTO 1420 ELSE GOTO 1440
1420 ANGLE=ATN(VRTOT/VZTOT)
1430 GOTO 1490
1440 IF -.00001(VZTOT AND VZTOT(.00001 AND VRTOT<0 THEN GOTO 1450 ELSE 1470
1450 ANGLE=3*PI/2
1460 GOTO 1490
1470 IF -.00001(VZTOT AND VZTOT(.00001 AND VRTOT)=0 THEN GOTO 1480
1480 ANGLE=PI/2
1490 X=X+(COS(ANGLE)*INC)
1500 Y=Y+(SIN(ANGLE)*INC)
1510 P(I,1)=X
1520 P(I,2)=Y
1530 IF X<=0 OR X>1 OR Y<=-1 OR Y>1 THEN I=1600 ELSE COUNT=I+1
1540 'Note: A repetitive cycle is programmed to calculate the new
magnitude and direction of flow at each subsequent increment.
1550 NEXT I
1560 'Note: The following section is the instructions to the computer
to draw the hood, and to display the flow into it.
1570 CLS
1580 SCREEN 1
1590 COLOR 16,0
1600 WINDOW (-1.6,-1.1)-(1.6,1.1)
1610 LINE (0,A)-(0,1),2
1620 LINE (0,A)-(-.05,A),2
1630 LINE (-.05,0)-(1,0),3
1640 LINE (0,-A)-(0,-1),2
1650 LINE (0,-A)-(-.05,-A),2
1660 FOR J=1 TO COUNT
1670 G=P(J,1)
1680 H=P(J,2)
```

```

1690 PSET (G, H), 1
1700 NEXT J
1710 KEY OFF
1720 LOCATE 4, 4:PRINT "QH/QJ=";Q/QJ
1730 LOCATE 5, 4:PRINT "VH/VJ=";VH/VJ
1740 LOCATE 6, 4:PRINT "DH/DJ=";2*A/DJ
1750 LOCATE 7, 4:PRINT "(Z/D)=";XJF/(2*A)
1760 LOCATE 9, 4:PRINT "QH=";Q"CFM"
1770 LOCATE 10, 4:PRINT "VH=";VH"FPM"
1780 LOCATE 11, 4:PRINT "RH=";A"FT"
1790 LOCATE 13, 4:PRINT "QJ=";QJ"CFM"
1800 LOCATE 14, 4:PRINT "VJ=";VJ"FPM"
1810 LOCATE 15, 4:PRINT "JETX=";XJFIN"IN"
1820 LOCATE 16, 4:PRINT "JETY=";YJFIN"IN"
1830 LOCATE 17, 4:PRINT "ANG=";DIR"DEG"
1840 LOCATE 18, 4:PRINT "JET ORIG=";-1.86*JD"IN"
1850 LOCATE 19, 4:PRINT "JETD=";JD"IN"
1860 LOCATE 21, 4:PRINT "XDRFT=";VC"FPM"
1870 LOCATE 22, 4:PRINT "XDFANG=";XDFA"DEG"
1880 LOCATE 24, 1:PRINT "PRINT"
1890 INPUT "ANOTHER(Y/N)";ANN$
1900 IF ANN$="Y" OR ANN$="y" THEN GOTO 1910 ELSE GOTO 1930
1910 CLS
1920 GOTO 150
1930 CLS
1940 PRINT "END OF JET-HOOD FLOW ANALYSIS RUNS"
1950 SYSTEM

```

Ok

APPENDIX B

COMPUTER PREDICTIONS OF  $[Z/D]_{50}$ 's AT CROSSDRAFT=25 fpm  
(3 EXAMPLES)

QH/QJ= 77.33334  
UH/UJ= .321887  
DH/DJ= 15.5  
(Z/D)= .483871

QH= 145 CFM  
UH= 1770.505 FPM  
RH= .1614583 FT

QJ= 1.875 CFM  
UJ= 5500.394 FPM  
JETX= 1.875 IN  
JETY= 0 IN  
ANG= 90 DEG  
JET ORIG= -.465 IN  
JETD= .25 IN

XDRFT= 25 FPM  
XDFANG= 90 DEG

PRINT

● OTHER(Y/N)? ■

QH/QJ= 193.3333  
UH/UJ= .8047175  
DH/DJ= 15.5  
(Z/D)= 1.109678

QH= 145 CFM  
UH= 1770.505 FPM  
RH= .1614583 FT

QJ= .75 CFM  
UJ= 2200.158 FPM  
JETX= 4.3 IN  
JETY= 0 IN  
ANG= 90 DEG  
JET ORIG=-.465 IN  
JETD= .25 IN

XDRFT= 25 FPM  
XDFANG= 90 DEG

PRINT

NOTHER(Y/N)? ■

QH/QJ= 322.2222  
UH/UJ= 1.341196  
DH/DJ= 15.5  
(Z/D)= 1.516129

QH= 145 CFM  
UH= 1770.505 FPM  
RH= .1614503 FT

QJ= .45 CFM  
UJ= 1320.095 FPM  
JETX= 5.875 IN  
JETY= 0 IN  
ANG= 90 DEG  
JET ORIG= -.465 IN  
JETD= .25 IN

XDRFT= 25 FPM  
XDFANG= 90 DEG

PRINT  
OTHER(Y/N)?

APPENDIX C

EXPERIMENTAL DATA, CALIBRATION THEREOF, AND  
EFFICIENCY CALCULATIONS

# Calibration of Experimental Data & Efficiency Calculation

| z   | d     | regress<br>y | m       | exp.peak<br>x | b       | theoret.<br>max | ef=y/max | avg.eff. | stdev    | eff | run |
|-----|-------|--------------|---------|---------------|---------|-----------------|----------|----------|----------|-----|-----|
| 2.5 | 3.875 | 241.3756     | 2.47635 | 98            | -1.3067 | 246.33          | 97.98871 | 95.97811 | 1.421705 |     | 78a |
|     |       | 233.9465     |         | 95            |         |                 | 94.97282 |          |          |     |     |
|     |       | 231.4702     |         | 94            |         |                 | 93.96752 |          |          |     |     |
|     |       | 238.8992     |         | 97            |         |                 | 96.98341 |          |          |     |     |
|     |       | 236.4229     |         | 96            |         |                 | 95.97811 |          |          |     |     |
| 3   |       | 211.6594     |         | 86            |         |                 | 85.92514 | 79.29017 | 4.477808 |     | 78b |
|     |       | 204.2303     |         | 83            |         |                 | 82.90924 |          |          |     |     |
|     |       | 191.8486     |         | 78            |         |                 | 77.88275 |          |          |     |     |
|     |       | 181.9432     |         | 74            |         |                 | 73.86156 |          |          |     |     |
|     |       | 186.8959     |         | 76            |         |                 | 75.87216 |          |          |     |     |
| 3.5 |       | 157.1797     |         | 64            |         |                 | 63.80859 | 60.99375 | 11.39849 |     | 78c |
|     |       | 110.1290     |         | 45            |         |                 | 44.70793 |          |          |     |     |
|     |       | 139.8452     |         | 57            |         |                 | 56.77150 |          |          |     |     |
|     |       | 147.2743     |         | 60            |         |                 | 59.78739 |          |          |     |     |
|     |       | 196.8013     |         | 80            |         |                 | 79.89335 |          |          |     |     |
| 4   |       | 139.8452     |         | 57            |         |                 | 56.77150 | 49.93548 | 4.689474 |     | 78d |
|     |       | 117.5581     |         | 48            |         |                 | 47.72382 |          |          |     |     |
|     |       | 117.5581     |         | 48            |         |                 | 47.72382 |          |          |     |     |
|     |       | 107.6527     |         | 44            |         |                 | 43.70263 |          |          |     |     |
|     |       | 132.4162     |         | 54            |         |                 | 53.75561 |          |          |     |     |
| 4.5 |       | 120.0344     |         | 49            |         |                 | 48.72912 | 41.69203 | 11.67184 |     | 78e |
|     |       | 97.7473      |         | 40            |         |                 | 39.68144 |          |          |     |     |
|     |       | 97.7473      |         | 40            |         |                 | 39.68144 |          |          |     |     |
|     |       | 142.3216     |         | 58            |         |                 | 57.77680 |          |          |     |     |
|     |       | 55.64935     |         | 23            |         |                 | 22.59138 |          |          |     |     |
| 5   |       | 97.7473      |         | 40            |         |                 | 39.68144 | 29.22634 | 5.600871 |     | 78f |
|     |       | 72.9838      |         | 30            |         |                 | 29.62846 |          |          |     |     |
|     |       | 68.0311      |         | 28            |         |                 | 27.61787 |          |          |     |     |
|     |       | 63.0784      |         | 26            |         |                 | 25.60727 |          |          |     |     |
|     |       | 58.1257      |         | 24            |         |                 | 23.59667 |          |          |     |     |
| 2   |       | 336.4977     | 3.58133 | 93            | 3.43405 | 361.567         | 93.06649 | 98.01900 | 2.582914 |     | 77a |
|     |       | 361.5670     |         | 100           |         |                 | 100.0000 |          |          |     |     |
|     |       | 361.5670     |         | 100           |         |                 | 100.0000 |          |          |     |     |
|     |       | 357.9857     |         | 99            |         |                 | 99.00951 |          |          |     |     |
|     |       | 354.4043     |         | 98            |         |                 | 98.01900 |          |          |     |     |
| 2.5 |       | 336.4977     |         | 93            |         |                 | 93.06649 | 92.07599 | 3.373529 |     | 77b |
|     |       | 311.4284     |         | 86            |         |                 | 86.13297 |          |          |     |     |
|     |       | 340.0790     |         | 94            |         |                 | 94.05699 |          |          |     |     |
|     |       | 329.3350     |         | 91            |         |                 | 91.08549 |          |          |     |     |
|     |       | 347.2417     |         | 96            |         |                 | 96.03800 |          |          |     |     |
| 3   |       | 282.7777     |         | 78            |         |                 | 78.20895 | 78.01085 | 3.094428 |     | 77c |
|     |       | 282.7777     |         | 78            |         |                 | 78.20895 |          |          |     |     |
|     |       | 261.2898     |         | 72            |         |                 | 72.26594 |          |          |     |     |
|     |       | 293.5217     |         | 81            |         |                 | 81.18046 |          |          |     |     |
|     |       | 289.9404     |         | 80            |         |                 | 80.18996 |          |          |     |     |
| 3.5 |       | 279.1964     |         | 77            |         |                 | 77.21845 | 67.51153 | 5.651942 |     | 77d |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z    | d | regress<br>y | exp.peak<br>x | theoret.<br>max | ef=y/max | avg.eff. | stdev.eff | run      |     |
|------|---|--------------|---------------|-----------------|----------|----------|-----------|----------|-----|
|      |   | 221.8951     | 61            |                 | 61.37041 |          |           |          |     |
|      |   | 254.1271     | 70            |                 | 70.28494 |          |           |          |     |
|      |   | 232.6391     | 64            |                 | 64.34192 |          |           |          |     |
|      |   | 232.6391     | 64            |                 | 64.34192 |          |           |          |     |
| 4    |   | 232.6391     | 64            |                 | 64.34192 | 59.98371 | 5.962791  | 77e      |     |
|      |   | 203.9885     | 56            |                 | 56.41790 |          |           |          |     |
|      |   | 243.3831     | 67            |                 | 67.31343 |          |           |          |     |
|      |   | 221.8951     | 61            |                 | 61.37041 |          |           |          |     |
|      |   | 182.5005     | 50            |                 | 50.47489 |          |           |          |     |
| 4.5  |   | 146.6872     | 40            |                 | 40.56986 | 45.52237 | 5.241246  | 77f      |     |
|      |   | 153.8499     | 42            |                 | 42.55087 |          |           |          |     |
|      |   | 150.2685     | 41            |                 | 41.56036 |          |           |          |     |
|      |   | 196.8258     | 54            |                 | 54.43690 |          |           |          |     |
|      |   | 175.3378     | 48            |                 | 48.49388 |          |           |          |     |
| 5    |   | 175.3378     | 48            |                 | 48.49388 | 36.21165 | 8.230115  | 77g      |     |
|      |   | 157.4312     | 43            |                 | 43.54137 |          |           |          |     |
|      |   | 114.4552     | 31            |                 | 31.65534 |          |           |          |     |
|      |   | 103.7112     | 28            |                 | 28.68383 |          |           |          |     |
|      |   | 103.7112     | 28            |                 | 28.68383 |          |           |          |     |
| 2    |   | 163.1203     | 93            | 1.651           | 175.274  | 93.06593 | 92.86782  | 0.741282 | 76a |
|      |   | 164.8566     | 94            |                 | 94.05651 |          |           |          |     |
|      |   | 163.1203     | 93            |                 | 93.06593 |          |           |          |     |
|      |   | 161.3841     | 92            |                 | 92.07535 |          |           |          |     |
|      |   | 161.3841     | 92            |                 | 92.07535 |          |           |          |     |
| 2.5  |   | 157.9117     | 90            |                 | 90.09419 | 89.89607 | 4.491602  | 76b      |     |
|      |   | 163.1203     | 93            |                 | 93.06593 |          |           |          |     |
|      |   | 168.3290     | 96            |                 | 96.03767 |          |           |          |     |
|      |   | 145.7580     | 83            |                 | 83.16013 |          |           |          |     |
|      |   | 152.7030     | 87            |                 | 87.12245 |          |           |          |     |
| 3    |   | 119.7146     | 68            |                 | 68.30142 | 66.32026 | 5.352802  | 76c      |     |
|      |   | 119.7146     | 68            |                 | 68.30142 |          |           |          |     |
|      |   | 104.0885     | 59            |                 | 59.38620 |          |           |          |     |
|      |   | 130.1320     | 74            |                 | 74.24490 |          |           |          |     |
|      |   | 107.5610     | 61            |                 | 61.36736 |          |           |          |     |
| 3.25 |   | 123.1871     | 70            |                 | 70.28258 | 58.98996 | 7.159642  | 76d      |     |
|      |   | 90.19873     | 51            |                 | 51.46155 |          |           |          |     |
|      |   | 112.7697     | 64            |                 | 64.33910 |          |           |          |     |
|      |   | 97.14365     | 55            |                 | 55.42387 |          |           |          |     |
|      |   | 93.67119     | 53            |                 | 53.44271 |          |           |          |     |
| 3.5  |   | 69.36397     | 39            |                 | 39.57459 | 41.75386 | 5.582516  | 76e      |     |
|      |   | 79.78135     | 45            |                 | 45.51807 |          |           |          |     |
|      |   | 67.62774     | 38            |                 | 38.58401 |          |           |          |     |
|      |   | 88.4625      | 50            |                 | 50.47097 |          |           |          |     |
|      |   | 60.68282     | 34            |                 | 34.62168 |          |           |          |     |
| 4    |   | 46.79298     | 26            |                 | 26.69704 | 39.57459 | 18.37253  | 76f      |     |
|      |   | 114.5059     | 65            |                 | 65.32968 |          |           |          |     |
|      |   | 43.32052     | 24            |                 | 24.71588 |          |           |          |     |
|      |   | 39.84806     | 22            |                 | 22.73472 |          |           |          |     |
|      |   | 102.3523     | 58            |                 | 58.39562 |          |           |          |     |
| 4.5  |   | 38.11183     | 21            |                 | 21.74414 | 26.69704 | 9.186265  | 76g      |     |
|      |   | 24.22199     | 13            |                 | 13.81949 |          |           |          |     |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z    | d | regress<br>y | a        | exp.peak<br>x | b       | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run |
|------|---|--------------|----------|---------------|---------|-----------------|----------|----------|-----------|-----|
|      |   | 71.1002      |          | 40            |         | 40.56517        |          |          |           |     |
|      |   | 57.21036     |          | 32            |         | 32.64052        |          |          |           |     |
|      |   | 43.32052     |          | 24            |         | 24.71588        |          |          |           |     |
| 3    |   | 144.3033     | 1.432617 | 100           | 1.04164 | 144.304         | 99.99954 | 99.99954 | 0         | 98a |
|      |   | 144.3033     |          | 100           |         |                 | 99.99954 |          |           |     |
|      |   | 144.3033     |          | 100           |         |                 | 99.99954 |          |           |     |
|      |   | 144.3033     |          | 100           |         |                 | 99.99954 |          |           |     |
|      |   | 144.3033     |          | 100           |         |                 | 99.99954 |          |           |     |
| 3.25 |   | 107.0552     |          | 74            |         | 74.18733        | 72.99600 | 4.322857 |           | 98b |
|      |   | 115.651      |          | 80            |         | 80.14400        |          |          |           |     |
|      |   | 101.3248     |          | 70            |         | 70.21623        |          |          |           |     |
|      |   | 105.6226     |          | 73            |         | 73.19456        |          |          |           |     |
|      |   | 97.02697     |          | 67            |         | 67.23789        |          |          |           |     |
| 3.5  |   | 78.40295     |          | 54            |         | 54.33179        | 51.35346 | 9.206635 |           | 98c |
|      |   | 49.75061     |          | 34            |         | 34.47625        |          |          |           |     |
|      |   | 85.56604     |          | 59            |         | 59.29568        |          |          |           |     |
|      |   | 85.56604     |          | 59            |         | 59.29568        |          |          |           |     |
|      |   | 71.23987     |          | 49            |         | 49.36791        |          |          |           |     |
| 4    |   | 65.50940     |          | 45            |         | 45.39680        | 32.88781 | 9.939676 |           | 98d |
|      |   | 62.64417     |          | 43            |         | 43.41125        |          |          |           |     |
|      |   | 29.69398     |          | 20            |         | 20.57737        |          |          |           |     |
|      |   | 44.02015     |          | 30            |         | 30.50514        |          |          |           |     |
|      |   | 35.42444     |          | 24            |         | 24.54848        |          |          |           |     |
| 4.5  |   | 75.53772     |          | 52            |         | 52.34624        | 32.09359 | 11.10136 |           | 98e |
|      |   | 48.31800     |          | 33            |         | 33.48348        |          |          |           |     |
|      |   | 32.55921     |          | 22            |         | 22.56293        |          |          |           |     |
|      |   | 44.02015     |          | 30            |         | 30.50514        |          |          |           |     |
|      |   | 31.12659     |          | 21            |         | 21.57015        |          |          |           |     |
| 5    |   | 32.55921     |          | 22            |         | 22.56293        | 21.37159 | 9.042449 |           | 98f |
|      |   | 48.31800     |          | 33            |         | 33.48348        |          |          |           |     |
|      |   | 41.15491     |          | 28            |         | 28.51959        |          |          |           |     |
|      |   | 15.36781     |          | 10            |         | 10.64960        |          |          |           |     |
|      |   | 16.80042     |          | 11            |         | 11.64238        |          |          |           |     |
| 2.5  |   | 143.4793     | 1.79844  | 78            | 3.201   | 183.064         | 78.37658 | 83.87808 | 4.241465  | 97a |
|      |   | 159.6652     |          | 87            |         | 87.21828        |          |          |           |     |
|      |   | 148.8746     |          | 81            |         | 81.32382        |          |          |           |     |
|      |   | 165.0606     |          | 90            |         | 90.16551        |          |          |           |     |
|      |   | 150.6730     |          | 82            |         | 82.30623        |          |          |           |     |
| 3    |   | 111.1074     |          | 60            |         | 60.69320        | 59.51430 | 8.391419 |           | 97b |
|      |   | 125.4949     |          | 68            |         | 68.55248        |          |          |           |     |
|      |   | 93.123       |          | 50            |         | 50.86909        |          |          |           |     |
|      |   | 89.52612     |          | 48            |         | 48.90427        |          |          |           |     |
|      |   | 125.4949     |          | 68            |         | 68.55248        |          |          |           |     |
| 3.5  |   | 138.084      |          | 75            |         | 75.42935        | 51.19656 | 16.14929 |           | 97c |
|      |   | 91.32456     |          | 49            |         | 49.88668        |          |          |           | (6) |
|      |   | 53.55732     |          | 28            |         | 29.25606        |          |          |           |     |
|      |   | 64.34796     |          | 34            |         | 35.15052        |          |          |           |     |
|      |   | 93.123       |          | 50            |         | 50.86909        |          |          |           |     |
|      |   | 121.8980     |          | 66            |         | 66.58766        |          |          |           |     |
| 4    |   | 30.1776      |          | 15            |         | 16.48472        | 33.06290 | 10.99946 |           | 97d |
|      |   | 39.1698      |          | 20            |         | 21.39677        |          |          |           | (8) |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | m        | exp.peak<br>x | b        | theoret.<br>max | ef=y/max | avg.eff. | stdevff  | run  |
|-----|---|--------------|----------|---------------|----------|-----------------|----------|----------|----------|------|
|     |   | 69.74328     |          | 37            |          | 38.09775        |          |          |          |      |
|     |   | 67.94484     |          | 36            |          | 37.11534        |          |          |          |      |
|     |   | 78.73548     |          | 42            |          | 43.00981        |          |          |          |      |
|     |   | 46.36356     |          | 24            |          | 25.32642        |          |          |          |      |
|     |   | 57.1542      |          | 30            |          | 31.22088        |          |          |          |      |
|     |   | 94.92144     |          | 51            |          | 51.85150        |          |          |          |      |
| 4.5 |   | 75.1386      |          | 40            |          | 41.04498        | 34.65932 | 10.95068 |          | 97e  |
|     |   | 37.37136     |          | 19            |          | 20.41436        |          |          |          | (6)  |
|     |   | 60.75108     |          | 32            |          | 33.18570        |          |          |          |      |
|     |   | 98.51832     |          | 53            |          | 53.81632        |          |          |          |      |
|     |   | 44.56512     |          | 23            |          | 24.34401        |          |          |          |      |
|     |   | 64.34796     |          | 34            |          | 35.15052        |          |          |          |      |
| 5   |   | 31.97604     |          | 16            |          | 17.46713        | 19.62844 | 8.086855 |          | 97f  |
|     |   | 46.36356     |          | 24            |          | 25.32642        |          |          |          |      |
|     |   | 58.95264     |          | 31            |          | 32.20329        |          |          |          |      |
|     |   | 22.98384     |          | 11            |          | 12.55508        |          |          |          |      |
|     |   | 19.38696     |          | 9             |          | 10.59026        |          |          |          |      |
| 2   |   | 176.7998     | 1.7298   | 100           | 3.81985  | 176.8           | 99.99991 | 99.60855 | 0.782714 | 95a  |
|     |   | 176.7998     |          | 100           |          |                 | 99.99991 |          |          |      |
|     |   | 176.7998     |          | 100           |          |                 | 99.99991 |          |          |      |
|     |   | 176.7998     |          | 100           |          |                 | 99.99991 |          |          |      |
|     |   | 173.3402     |          | 98            |          | 98.04312        |          |          |          |      |
| 2.5 |   | 138.7442     |          | 78            |          | 78.47525        | 83.56290 | 3.239059 |          | 95b  |
|     |   | 143.9336     |          | 81            |          | 81.41043        |          |          |          |      |
|     |   | 152.5826     |          | 86            |          | 86.30240        |          |          |          |      |
|     |   | 154.3124     |          | 87            |          | 87.28079        |          |          |          |      |
|     |   | 149.1230     |          | 84            |          | 84.34561        |          |          |          |      |
| 3   |   | 71.28205     |          | 39            |          | 40.31790        | 46.77529 | 5.272443 |          | 95c  |
|     |   | 76.47145     |          | 42            |          | 43.25308        |          |          |          |      |
|     |   | 83.39065     |          | 46            |          | 47.16665        |          |          |          |      |
|     |   | 83.39065     |          | 46            |          | 47.16665        |          |          |          |      |
|     |   | 98.95885     |          | 55            |          | 55.97220        |          |          |          |      |
| 3.5 |   | 57.44365     |          | 31            |          | 32.49075        | 30.53396 | 2.143553 |          | 95d  |
|     |   | 52.25425     |          | 28            |          | 29.55557        |          |          |          |      |
|     |   | 52.25425     |          | 28            |          | 29.55557        |          |          |          |      |
|     |   | 59.17345     |          | 32            |          | 33.46914        |          |          |          |      |
|     |   | 48.79465     |          | 26            |          | 27.59878        |          |          |          |      |
| 4   |   | 26.30725     |          | 13            |          | 14.87966        | 18.01052 | 4.436343 |          | 95e  |
|     |   | 28.03705     |          | 14            |          | 15.85805        |          |          |          |      |
|     |   | 38.41585     |          | 20            |          | 21.72842        |          |          |          |      |
|     |   | 22.84765     |          | 11            |          | 12.92287        |          |          |          |      |
|     |   | 43.60525     |          | 23            |          | 24.66360        |          |          |          |      |
| 2.5 |   | 179.1342     | 1.794434 | 100           | -0.30916 | 179.134         | 100.0001 | 97.79633 | 2.717617 | 130a |
|     |   | 179.1342     |          | 100           |          |                 | 100.0001 |          |          |      |
|     |   | 168.3676     |          | 94            |          | 93.98976        |          |          |          |      |
|     |   | 179.1342     |          | 100           |          | 100.0001        |          |          |          |      |
|     |   | 170.1620     |          | 95            |          | 94.99149        |          |          |          |      |
| 3   |   | 114.5346     |          | 64            |          | 63.93795        | 76.76005 | 6.805849 |          | 130b |
|     |   | 146.8344     |          | 82            |          | 81.96904        |          |          |          |      |
|     |   | 137.8622     |          | 77            |          | 76.96040        |          |          |          |      |
|     |   | 148.6288     |          | 83            |          | 82.97076        |          |          |          |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z    | d | regress<br>y | m        | exp.peak<br>x | b        | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run  |
|------|---|--------------|----------|---------------|----------|-----------------|----------|----------|-----------|------|
| 3.25 |   | 139.6566     |          | 78            |          | 77.96213        |          |          |           |      |
|      |   | 96.59026     |          | 54            |          | 53.92067        | 50.51480 | 7.748992 |           | 130c |
|      |   | 64.29045     |          | 36            |          | 35.88958        |          |          |           |      |
|      |   | 103.7680     |          | 58            |          | 57.92758        |          |          |           |      |
|      |   | 98.38470     |          | 55            |          | 54.92240        |          |          |           |      |
|      |   | 89.41253     |          | 50            |          | 49.91377        |          |          |           |      |
| 3.5  |   | 78.64592     |          | 44            |          | 43.90340        | 42.50098 | 8.584515 |           | 130d |
|      |   | 100.1791     |          | 56            |          | 55.92413        |          |          |           |      |
|      |   | 80.44036     |          | 45            |          | 44.90513        |          |          |           |      |
|      |   | 67.87932     |          | 38            |          | 37.89304        |          |          |           |      |
|      |   | 53.52385     |          | 30            |          | 29.87922        |          |          |           |      |
| 4    |   | 39.16838     |          | 22            |          | 21.86540        | 24.46989 | 4.972443 |           | 130e |
|      |   | 44.55168     |          | 25            |          | 24.87059        |          |          |           |      |
|      |   | 35.57951     |          | 20            |          | 19.86195        |          |          |           |      |
|      |   | 39.16838     |          | 22            |          | 21.86540        |          |          |           |      |
|      |   | 60.70158     |          | 34            |          | 33.88613        |          |          |           |      |
| 2.5  |   | 161.8823     | 1.59208  | 100           | 2.67434  | 161.884         | 99.99897 | 99.99897 | 0         | 129a |
|      |   | 161.8823     |          | 100           |          |                 | 99.99897 |          |           |      |
|      |   | 161.8823     |          | 100           |          |                 | 99.99897 |          |           |      |
|      |   | 161.8823     |          | 100           |          |                 | 99.99897 |          |           |      |
|      |   | 161.8823     |          | 100           |          |                 | 99.99897 |          |           |      |
| 3    |   | 99.79122     |          | 61            |          | 61.64365        | 66.95439 | 9.668061 |           | 129b |
|      |   | 99.79122     |          | 61            |          | 61.64365        |          |          |           |      |
|      |   | 102.9753     |          | 63            |          | 63.61059        |          |          |           |      |
|      |   | 99.79122     |          | 61            |          | 61.64365        |          |          |           |      |
|      |   | 139.5932     |          | 86            |          | 86.23039        |          |          |           |      |
| 3.25 |   | 123.6724     |          | 76            |          | 76.39570        | 49.25194 | 13.99985 |           | 129c |
|      |   | 58.39714     |          | 35            |          | 36.07344        |          |          |           |      |
|      |   | 74.31794     |          | 45            |          | 45.90814        |          |          |           |      |
|      |   | 69.5417      |          | 42            |          | 42.95773        |          |          |           |      |
|      |   | 72.72586     |          | 44            |          | 44.92467        |          |          |           |      |
| 3.5  |   | 52.02882     |          | 31            |          | 32.13956        | 48.07177 | 16.42600 |           | 129d |
|      |   | 39.29218     |          | 23            |          | 24.27181        |          |          |           |      |
|      |   | 98.19914     |          | 60            |          | 60.66018        |          |          |           |      |
|      |   | 98.19914     |          | 60            |          | 60.66018        |          |          |           |      |
|      |   | 101.3833     |          | 62            |          | 62.62712        |          |          |           |      |
| 4    |   | 44.06842     |          | 26            |          | 27.22222        | 20.33793 | 5.241072 |           | 129e |
|      |   | 26.55554     |          | 15            |          | 16.40405        |          |          |           |      |
|      |   | 24.96346     |          | 14            |          | 15.42058        |          |          |           |      |
|      |   | 42.47634     |          | 25            |          | 26.23875        |          |          |           |      |
|      |   | 26.55554     |          | 15            |          | 16.40405        |          |          |           |      |
| 2.5  |   | 130.2020     | 1.293731 | 100           | 0.828975 | 130.2           | 100.0015 | 100.0015 | 0         | 126a |
|      |   | 130.2020     |          | 100           |          |                 | 100.0015 |          |           |      |
|      |   | 130.2020     |          | 100           |          |                 | 100.0015 |          |           |      |
|      |   | 130.2020     |          | 100           |          |                 | 100.0015 |          |           |      |
|      |   | 130.2020     |          | 100           |          |                 | 100.0015 |          |           |      |
| 3    |   | 70.69044     |          | 54            |          | 54.29373        | 72.17942 | 19.07184 |           | 126b |
|      |   | 130.2020     |          | 100           |          | 100.0015        |          |          |           |      |
|      |   | 117.2647     |          | 90            |          | 90.06510        |          |          |           |      |
|      |   | 79.74656     |          | 61            |          | 61.24928        |          |          |           |      |
|      |   | 71.98418     |          | 55            |          | 55.28738        |          |          |           |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z    | d | regress<br>y | m        | exp.peak<br>x | b        | theoret.<br>max | ef=y/max | avg.eff. | stdevff  | run  |
|------|---|--------------|----------|---------------|----------|-----------------|----------|----------|----------|------|
| 3.25 |   | 47.40329     |          | 36            |          | 36.40805        | 54.88992 | 13.61842 |          | 126c |
|      |   | 82.33402     |          | 63            |          | 63.23658        |          |          |          |      |
|      |   | 86.21522     |          | 66            |          | 66.21752        |          |          |          |      |
|      |   | 52.57821     |          | 40            |          | 40.38265        |          |          |          |      |
|      |   | 88.80268     |          | 68            |          | 68.20482        |          |          |          |      |
| 3.5  |   | 91.39014     |          | 70            |          | 70.19212        | 37.79916 | 18.12258 |          | 126d |
|      |   | 44.81582     |          | 34            |          | 34.42075        |          |          |          |      |
|      |   | 18.94120     |          | 14            |          | 14.54777        |          |          |          |      |
|      |   | 40.93463     |          | 31            |          | 31.43981        |          |          |          |      |
|      |   | 49.99075     |          | 38            |          | 38.39535        |          |          |          |      |
| 4    |   | 13.76628     |          | 10            |          | 10.57318        | 12.56048 | 2.810463 |          | 126e |
|      |   | 21.52867     |          | 16            |          | 16.53507        |          |          |          |      |
|      |   | 16.35374     |          | 12            |          | 12.56048        |          |          |          |      |
|      |   | 11.17882     |          | 8             |          | 8.585885        |          |          |          |      |
|      |   | 18.94120     |          | 14            |          | 14.54777        |          |          |          |      |
| 3    |   | 203.1016     | 2.009064 | 100           | 2.195281 | 203.1           | 100.0008 | 93.86779 | 4.781016 | 195a |
|      |   | 199.0835     |          | 98            |          | 98.02242        |          |          |          |      |
|      |   | 176.9838     |          | 87            |          | 87.14123        |          |          |          |      |
|      |   | 191.0472     |          | 94            |          | 94.06563        |          |          |          |      |
|      |   | 183.0110     |          | 90            |          | 90.10883        |          |          |          |      |
| 3.5  |   | 160.9113     |          | 79            |          | 79.22764        | 82.98659 | 6.603230 |          | 195b |
|      |   | 158.9022     |          | 78            |          | 78.23844        |          |          |          |      |
|      |   | 195.0654     |          | 96            |          | 96.04403        |          |          |          |      |
|      |   | 162.9204     |          | 80            |          | 80.21683        |          |          |          |      |
|      |   | 164.9294     |          | 81            |          | 81.20603        |          |          |          |      |
| 4    |   | 90.59409     |          | 44            |          | 44.60566        | 44.20998 | 9.582478 |          | 195c |
|      |   | 96.62128     |          | 47            |          | 47.57325        |          |          |          |      |
|      |   | 102.6484     |          | 50            |          | 50.54085        |          |          |          |      |
|      |   | 52.42188     |          | 25            |          | 25.81087        |          |          |          |      |
|      |   | 106.6666     |          | 52            |          | 52.51925        |          |          |          |      |
| 4.5  |   | 96.62128     |          | 47            |          | 47.57325        | 33.52662 | 9.959032 |          | 195d |
|      |   | 58.44907     |          | 28            |          | 28.77847        |          |          |          |      |
|      |   | 42.37656     |          | 20            |          | 20.86487        |          |          |          |      |
|      |   | 86.57596     |          | 42            |          | 42.62726        |          |          |          |      |
|      |   | 56.44000     |          | 27            |          | 27.78927        |          |          |          |      |
| 5    |   | 78.53971     |          | 38            |          | 38.67046        | 26.40439 | 11.74775 |          | 195e |
|      |   | 52.42188     |          | 25            |          | 25.81087        |          |          |          |      |
|      |   | 28.31311     |          | 13            |          | 13.94047        |          |          |          |      |
|      |   | 82.55784     |          | 40            |          | 40.64886        |          |          |          |      |
|      |   | 26.30404     |          | 12            |          | 12.95128        |          |          |          |      |
| 3.5  |   | 155.4185     | 1.52847  | 100           | 2.571589 | 155.42          | 99.99909 | 99.99909 | 0        | 193a |
|      |   | 155.4185     |          | 100           |          |                 | 99.99909 |          |          |      |
|      |   | 155.4185     |          | 100           |          |                 | 99.99909 |          |          |      |
|      |   | 155.4185     |          | 100           |          |                 | 99.99909 |          |          |      |
|      |   | 155.4185     |          | 100           |          |                 | 99.99909 |          |          |      |
| 4    |   | 130.9630     |          | 84            |          | 84.26397        | 77.37986 | 12.74690 |          | 193b |
|      |   | 94.27978     |          | 60            |          | 60.66129        |          |          |          |      |
|      |   | 112.6214     |          | 72            |          | 72.46263        |          |          |          |      |
|      |   | 111.0929     |          | 71            |          | 71.47919        |          |          |          |      |
|      |   | 152.3616     |          | 98            |          | 98.03220        |          |          |          |      |
| 4.5  |   | 82.05202     |          | 52            |          | 52.79373        | 47.28644 | 14.40805 |          | 193c |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z    | d | regress<br>y | exp.peak<br>x | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run  |
|------|---|--------------|---------------|-----------------|----------|----------|-----------|------|
|      |   | 65.23885     | 41            | 41.97584        |          |          |           |      |
|      |   | 39.25486     | 24            | 25.25728        |          |          |           |      |
|      |   | 108.0360     | 69            | 69.51230        |          |          |           |      |
|      |   | 72.88120     | 46            | 46.89306        |          |          |           |      |
| 5    |   | 48.42568     | 30            | 31.15795        | 41.38577 | 9.153964 |           | 193d |
|      |   | 54.53956     | 34            | 35.09173        |          |          |           | (10) |
|      |   | 71.35273     | 45            | 45.90962        |          |          |           |      |
|      |   | 63.71038     | 40            | 40.99240        |          |          |           |      |
|      |   | 65.23885     | 41            | 41.97584        |          |          |           |      |
|      |   | 69.82426     | 44            | 44.92618        |          |          |           |      |
|      |   | 59.12497     | 37            | 38.04206        |          |          |           |      |
|      |   | 83.58049     | 53            | 53.77718        |          |          |           |      |
|      |   | 88.16590     | 56            | 56.72751        |          |          |           |      |
|      |   | 39.25486     | 24            | 25.25728        |          |          |           |      |
| 5.5  |   | 23.97016     | 14            | 15.42283        | 24.07714 | 12.35231 |           | 193e |
|      |   | 63.71038     | 40            | 40.99240        |          |          |           |      |
|      |   | 11.74240     | 6             | 7.555275        |          |          |           |      |
|      |   | 33.14098     | 20            | 21.32350        |          |          |           |      |
|      |   | 54.53956     | 34            | 35.09173        |          |          |           |      |
| 3.5  |   | 101.8050     | 100           | -0.79960        | 101.8050 | 99.99999 | 96.77486  | 189a |
|      |   | 101.8050     | 100           |                 |          | 99.99999 |           |      |
|      |   | 101.8050     | 100           |                 |          | 99.99999 |           |      |
|      |   | 85.38830     | 84            | 83.87432        |          |          |           |      |
|      |   | 101.8050     | 100           | 99.99999        |          |          |           |      |
| 4    |   | 89.49248     | 88            | 87.90574        | 84.07589 | 8.702633 |           | 189b |
|      |   | 73.07574     | 72            | 71.78007        |          |          |           |      |
|      |   | 95.64876     | 94            | 93.95287        |          |          |           |      |
|      |   | 92.57062     | 91            | 90.92930        |          |          |           |      |
|      |   | 77.17993     | 76            | 75.81149        |          |          |           |      |
| 4.25 |   | 60.76318     | 60            | 59.68582        | 55.65441 | 19.04817 |           | 189c |
|      |   | 34.08597     | 34            | 33.48161        |          |          |           |      |
|      |   | 34.08597     | 34            | 33.48161        |          |          |           |      |
|      |   | 78.20597     | 77            | 76.81934        |          |          |           |      |
|      |   | 76.15388     | 75            | 74.80364        |          |          |           |      |
| 4.5  |   | 56.65900     | 56            | 55.65441        | 32.07062 | 15.86658 |           | 189d |
|      |   | 17.66923     | 18            | 17.35594        |          |          |           |      |
|      |   | 19.72132     | 20            | 19.37165        |          |          |           |      |
|      |   | 47.42458     | 47            | 46.58372        |          |          |           |      |
|      |   | 21.77341     | 22            | 21.38736        |          |          |           |      |
| 5    |   | 14.59109     | 15            | 14.33238        | 19.57322 | 7.278912 |           | 189e |
|      |   | 19.72132     | 20            | 19.37165        |          |          |           |      |
|      |   | 13.56504     | 14            | 13.32453        |          |          |           |      |
|      |   | 17.66923     | 18            | 17.35594        |          |          |           |      |
|      |   | 34.08597     | 34            | 33.48161        |          |          |           |      |
| 5    |   | 116.2535     | 96            | -0.42082        | 121.115  | 95.98604 | 96.18674  | 326a |
|      |   | 112.6074     | 93            | 92.97562        |          |          |           |      |
|      |   | 116.2535     | 96            | 95.98604        |          |          |           |      |
|      |   | 121.1149     | 100           | 99.99994        |          |          |           |      |
|      |   | 116.2535     | 96            | 95.98604        |          |          |           |      |
| 5.5  |   | 107.7459     | 89            | 88.96172        | 81.53602 | 10.93003 |           | 326b |
|      |   | 106.5306     | 88            | 87.95825        |          |          |           |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | n        | exp.peak<br>x | b      | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run  |
|-----|---|--------------|----------|---------------|--------|-----------------|----------|----------|-----------|------|
|     |   | 77.36206     |          | 64            |        | 63.87487        |          |          |           |      |
|     |   | 89.51563     |          | 74            |        | 73.90961        |          |          |           |      |
|     |   | 112.6074     |          | 93            |        | 92.97562        |          |          |           |      |
| 6   |   | 59.13169     |          | 49            |        | 48.82276        | 66.68460 | 12.68352 |           | 326c |
|     |   | 72.50063     |          | 60            |        | 59.86098        |          |          |           |      |
|     |   | 94.37706     |          | 78            |        | 77.92351        |          |          |           |      |
|     |   | 101.6692     |          | 84            |        | 83.94435        |          |          |           |      |
|     |   | 76.14670     |          | 63            |        | 62.87140        |          |          |           |      |
| 6.5 |   | 84.65420     |          | 70            |        | 69.89572        | 53.03736 | 13.15125 |           | 326d |
|     |   | 48.19348     |          | 40            |        | 39.79150        |          |          |           |      |
|     |   | 81.00813     |          | 67            |        | 66.88530        |          |          |           |      |
|     |   | 46.97812     |          | 39            |        | 38.78802        |          |          |           |      |
|     |   | 60.34705     |          | 50            |        | 49.82624        |          |          |           |      |
| 7   |   | 31.17847     |          | 26            |        | 25.74286        | 31.42922 | 5.330914 |           | 326e |
|     |   | 31.17847     |          | 26            |        | 25.74286        |          |          |           | (6)  |
|     |   | 33.60919     |          | 28            |        | 27.74981        |          |          |           |      |
|     |   | 48.19348     |          | 40            |        | 39.79150        |          |          |           |      |
|     |   | 42.11669     |          | 35            |        | 34.77413        |          |          |           |      |
|     |   | 42.11669     |          | 35            |        | 34.77413        |          |          |           |      |
| 7.5 |   | 23.88633     |          | 20            |        | 19.72202        | 20.47463 | 2.172584 |           | 326f |
|     |   | 28.74776     |          | 24            |        | 23.73592        |          |          |           | (4)  |
|     |   | 25.10168     |          | 21            |        | 20.72549        |          |          |           |      |
|     |   | 21.45561     |          | 18            |        | 17.71507        |          |          |           |      |
| 4   |   | 276.0373     | 2.684206 | 100           | 7.6167 | 275.59          | 100.1623 | 99.99999 | 0.000000  | 322a |
|     |   | 276.0373     |          | 100           |        |                 | 100.1623 |          |           |      |
|     |   | 276.0373     |          | 100           |        |                 | 100.1623 |          |           |      |
|     |   | 276.0373     |          | 100           |        |                 | 100.1623 |          |           |      |
|     |   | 276.0373     |          | 100           |        |                 | 100.1623 |          |           |      |
| 4.5 |   | 276.0373     |          | 100           |        |                 | 100.1623 | 98.60392 | 3.116752  | 322b |
|     |   | 254.5636     |          | 92            |        | 92.37042        |          |          |           |      |
|     |   | 276.0373     |          | 100           |        | 100.1623        |          |          |           |      |
|     |   | 276.0373     |          | 100           |        | 100.1623        |          |          |           |      |
|     |   | 276.0373     |          | 100           |        | 100.1623        |          |          |           |      |
| 5   |   | 265.3004     |          | 96            |        | 96.26636        | 98.21433 | 1.742317 |           | 322c |
|     |   | 270.6688     |          | 98            |        | 98.21433        |          |          |           |      |
|     |   | 276.0373     |          | 100           |        | 100.1623        |          |          |           |      |
|     |   | 265.3004     |          | 96            |        | 96.26636        |          |          |           |      |
|     |   | 276.0373     |          | 100           |        | 100.1623        |          |          |           |      |
| 5.5 |   | 144.5112     |          | 51            |        | 52.43702        | 45.37563 | 11.11314 |           | 322d |
|     |   | 157.9322     |          | 56            |        | 57.30695        |          |          |           | (4)  |
|     |   | 120.3533     |          | 42            |        | 43.67116        |          |          |           |      |
|     |   | 77.40605     |          | 26            |        | 28.08739        |          |          |           |      |
| 6   |   | 74.72185     |          | 25            |        | 27.11341        | 38.47657 | 11.17138 |           | 322e |
|     |   | 80.09026     |          | 27            |        | 29.06138        |          |          |           | (6)  |
|     |   | 168.6690     |          | 60            |        | 61.20289        |          |          |           |      |
|     |   | 106.9323     |          | 37            |        | 38.80123        |          |          |           |      |
|     |   | 109.6165     |          | 38            |        | 39.77521        |          |          |           |      |
|     |   | 96.19549     |          | 33            |        | 34.90529        |          |          |           |      |
| 6.5 |   | 88.14288     |          | 30            |        | 31.98333        | 37.24285 | 8.811202 |           | 322f |
|     |   | 106.9323     |          | 37            |        | 38.80123        |          |          |           |      |
|     |   | 139.1427     |          | 49            |        | 50.48905        |          |          |           |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | exp.peak<br>x | theoret.<br>max | ef=y/max | avg.eff. | stdevff  | run      |      |
|-----|---|--------------|---------------|-----------------|----------|----------|----------|----------|------|
|     |   | 66.66923     | 22            |                 | 24.19145 |          |          |          |      |
|     |   | 112.3007     | 39            |                 | 40.74920 |          |          |          |      |
| 7   |   | 74.72185     | 25            |                 | 27.11341 | 30.27886 | 12.37536 | 322g     |      |
|     |   | 50.56399     | 16            |                 | 18.34754 |          |          |          |      |
|     |   | 61.30082     | 20            |                 | 22.24348 |          |          |          |      |
|     |   | 147.1954     | 52            |                 | 53.41101 |          |          |          |      |
|     |   | 72.03764     | 24            |                 | 26.13942 |          |          |          |      |
| 4   |   | 101.1575     | 98            | 5.881978        | 103.102  | 98.11407 | 99.62279 | 0.754359 | 316a |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
| 4.5 |   | 103.1019     | 100           |                 | 99.99997 | 96.22817 | 7.543597 | 316b     |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 103.1019     | 100           |                 | 99.99997 |          |          |          |      |
|     |   | 83.65797     | 80            |                 | 81.14098 |          |          |          |      |
| 5   |   | 49.63097     | 45            |                 | 48.13774 | 63.97930 | 9.227426 | 316c     |      |
|     |   | 72.96377     | 69            |                 | 70.76853 |          |          |          |      |
|     |   | 61.29737     | 57            |                 | 59.45314 |          |          |          |      |
|     |   | 70.04717     | 66            |                 | 67.93968 |          |          |          |      |
|     |   | 75.88037     | 72            |                 | 73.59738 |          |          |          |      |
| 5.5 |   | 54.49197     | 50            |                 | 52.85249 | 48.84495 | 6.613264 | 316d     |      |
|     |   | 51.57537     | 47            |                 | 50.02364 |          |          | (4)      |      |
|     |   | 56.43637     | 52            |                 | 54.73839 |          |          |          |      |
|     |   | 38.93677     | 34            |                 | 37.76529 |          |          |          |      |
| 6   |   | 62.26957     | 58            |                 | 60.39609 | 42.66863 | 17.19584 | 316e     |      |
|     |   | 38.93677     | 34            |                 | 37.76529 |          |          |          |      |
|     |   | 65.18617     | 61            |                 | 63.22494 |          |          |          |      |
|     |   | 17.54837     | 12            |                 | 17.02040 |          |          |          |      |
|     |   | 36.02017     | 31            |                 | 34.93644 |          |          |          |      |
| 6.5 |   | 59.35297     | 55            |                 | 57.56724 | 30.22170 | 15.60859 | 316f     |      |
|     |   | 15.60397     | 10            |                 | 15.13450 |          |          |          |      |
|     |   | 15.60397     | 10            |                 | 15.13450 |          |          |          |      |
|     |   | 30.18697     | 25            |                 | 29.27875 |          |          |          |      |
|     |   | 35.04797     | 30            |                 | 33.99349 |          |          |          |      |
| 7   |   | 22.40937     | 17            |                 | 21.73515 | 22.86669 | 7.599963 | 316g     |      |
|     |   | 17.54837     | 12            |                 | 17.02040 |          |          |          |      |
|     |   | 38.93677     | 34            |                 | 37.76529 |          |          |          |      |
|     |   | 19.49277     | 14            |                 | 18.90630 |          |          |          |      |
|     |   | 19.49277     | 14            |                 | 18.90630 |          |          |          |      |
| 5   |   | 112.5296     | 100           | 2.53289         | 115.8375 | 97.14437 | 93.72589 | 2.855057 | 391a |
|     |   | 112.5296     | 100           |                 | 97.14437 |          |          |          |      |
|     |   | 104.8298     | 93            |                 | 90.49733 |          |          |          |      |
|     |   | 105.9298     | 94            |                 | 91.44691 |          |          |          |      |
|     |   | 107.0297     | 95            |                 | 92.39648 |          |          |          |      |
| 5.5 |   | 112.5296     | 100           |                 | 97.14437 | 95.81496 | 1.761204 | 391b     |      |
|     |   | 112.5296     | 100           |                 | 97.14437 |          |          |          |      |
|     |   | 107.0297     | 95            |                 | 92.39648 |          |          |          |      |
|     |   | 111.4296     | 99            |                 | 96.19480 |          |          |          |      |
|     |   | 111.4296     | 99            |                 | 96.19480 |          |          |          |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | exp.peak<br>x | theoret.<br>max | ef=y/max | avg.eff. | stdevff  | run      |          |      |
|-----|---|--------------|---------------|-----------------|----------|----------|----------|----------|----------|------|
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
| 5.5 |   | 116.6323     | 100           |                 | 99.99769 | 97.93095 | 2.614247 | 489b     |          |      |
|     |   | 109.4006     | 94            |                 | 93.79746 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 111.8112     | 96            |                 | 95.86420 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
| 6   |   | 116.6323     | 100           |                 | 99.99769 | 97.72427 | 4.546837 | 489c     |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 103.3743     | 89            |                 | 88.63060 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
| 6.5 |   | 81.67937     | 71            |                 | 70.02990 | 79.33025 | 11.14893 | 489d     |          |      |
|     |   | 81.67937     | 71            |                 | 70.02990 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 87.70574     | 76            |                 | 75.19676 |          |          |          |          |      |
|     |   | 94.93738     | 82            |                 | 81.39699 |          |          |          |          |      |
| 7   |   | 63.60027     | 56            |                 | 54.52932 | 69.20320 | 21.61107 | 489e     |          |      |
|     |   | 88.91102     | 77            |                 | 76.23013 |          |          |          |          |      |
|     |   | 116.6323     | 100           |                 | 99.99769 |          |          |          |          |      |
|     |   | 91.32156     | 79            |                 | 78.29688 |          |          |          |          |      |
|     |   | 43.11062     | 39            |                 | 36.96199 |          |          |          |          |      |
| 7.5 |   | 66.01082     | 58            |                 | 56.59606 | 60.10953 | 14.60824 | 489f     |          |      |
|     |   | 70.83191     | 62            |                 | 60.72955 |          |          |          |          |      |
|     |   | 67.21609     | 59            |                 | 57.62943 |          |          |          |          |      |
|     |   | 99.75848     | 86            |                 | 85.53048 |          |          |          |          |      |
|     |   | 46.72644     | 42            |                 | 40.06211 |          |          |          |          |      |
| 8   |   | 46.72644     | 42            |                 | 40.06211 | 33.65520 | 8.068248 | 489g     |          |      |
|     |   | 25.03151     | 24            |                 | 21.46141 |          |          |          |          |      |
|     |   | 51.54753     | 46            |                 | 44.19559 |          |          |          |          |      |
|     |   | 33.46843     | 31            |                 | 28.69501 |          |          |          |          |      |
|     |   | 39.49480     | 36            |                 | 33.86187 |          |          |          |          |      |
| 8.5 |   | 26.23679     | 25            |                 | 22.49478 | 23.94150 | 6.423553 | 489h     |          |      |
|     |   | 29.85261     | 28            |                 | 25.59490 |          |          |          |          |      |
|     |   | 40.70007     | 37            |                 | 34.89525 |          |          |          |          |      |
|     |   | 17.79987     | 18            |                 | 15.26117 |          |          |          |          |      |
|     |   | 25.03151     | 24            |                 | 21.46141 |          |          |          |          |      |
| 5.5 |   | 94.94        | 1.05          | 92              | -1.66    | 103.34   | 91.87149 | 91.46506 | 7.833579 | 483a |
|     |   | 83.39        |               | 81              |          |          | 80.69479 |          |          |      |
|     |   | 103.34       |               | 100             |          |          | 100      |          |          |      |
|     |   | 87.59        |               | 85              |          |          | 84.75904 |          |          |      |
|     |   | 103.34       |               | 100             |          |          | 100      |          |          |      |
| 6   |   | 63.44        |               | 62              |          |          | 61.38958 | 73.78556 | 12.85872 | 483b |
|     |   | 63.44        |               | 62              |          |          | 61.38958 |          |          |      |
|     |   | 99.14        |               | 96              |          |          | 95.93574 |          |          |      |
|     |   | 81.29        |               | 79              |          |          | 78.66266 |          |          |      |
|     |   | 73.94        |               | 72              |          |          | 71.55022 |          |          |      |
| 6.5 |   | 52.94        |               | 52              |          |          | 51.22895 | 59.35746 | 8.051942 | 483c |
|     |   | 76.04        |               | 74              |          |          | 73.58234 |          |          |      |
|     |   | 55.04        |               | 54              |          |          | 53.26107 |          |          |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | a        | exp.peak<br>x | b        | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run  |
|-----|---|--------------|----------|---------------|----------|-----------------|----------|----------|-----------|------|
|     |   | 135.4095     |          | 70            |          | 70.01420        |          |          |           |      |
|     |   | 150.8745     |          | 78            |          | 78.01043        |          |          |           |      |
|     |   | 152.8076     |          | 79            |          | 79.00996        |          |          |           |      |
|     |   | 154.7407     |          | 80            |          | 80.00949        |          |          |           |      |
| 6   |   | 147.0082     |          | 76            |          | 76.01137        | 59.41920 | 9.042280 |           | 387e |
|     |   | 94.81410     |          | 49            |          | 49.02411        |          |          |           |      |
|     |   | 106.4128     |          | 55            |          | 55.02128        |          |          |           |      |
|     |   | 116.0784     |          | 60            |          | 60.01892        |          |          |           |      |
|     |   | 110.2790     |          | 57            |          | 57.02033        |          |          |           |      |
| 6.5 |   | 50.35238     |          | 26            |          | 26.03495        | 36.23014 | 9.616274 |           | 387f |
|     |   | 98.68033     |          | 51            |          | 51.02316        |          |          |           |      |
|     |   | 60.01797     |          | 31            |          | 31.03259        |          |          |           |      |
|     |   | 85.14851     |          | 44            |          | 44.02646        |          |          |           |      |
|     |   | 56.15174     |          | 29            |          | 29.03354        |          |          |           |      |
| 7   |   | 15.55626     |          | 8             |          | 8.043445        | 20.63750 | 6.796793 |           | 387g |
|     |   | 46.48615     |          | 24            |          | 24.03590        |          |          |           |      |
|     |   | 54.21862     |          | 28            |          | 28.03401        |          |          |           |      |
|     |   | 44.55303     |          | 23            |          | 23.03637        |          |          |           |      |
|     |   | 38.75368     |          | 20            |          | 20.03778        |          |          |           |      |
| 4   |   | 100.7442     | 1.069988 | 91            | 3.3753   | 110.373         | 91.27613 | 91.66390 | 2.572186  | 379a |
|     |   | 101.8141     |          | 92            |          | 92.24556        |          |          |           |      |
|     |   | 106.0941     |          | 96            |          | 96.12328        |          |          |           |      |
|     |   | 99.67422     |          | 90            |          | 90.30670        |          |          |           |      |
|     |   | 97.53424     |          | 88            |          | 88.36784        |          |          |           |      |
| 4.5 |   | 95.39426     |          | 86            |          | 86.42898        | 86.23510 | 6.019834 |           | 379b |
|     |   | 107.1641     |          | 97            |          | 97.09270        |          |          |           |      |
|     |   | 95.39426     |          | 86            |          | 86.42898        |          |          |           |      |
|     |   | 88.97434     |          | 80            |          | 80.61241        |          |          |           |      |
|     |   | 88.97434     |          | 80            |          | 80.61241        |          |          |           |      |
| 5   |   | 94.32428     |          | 85            |          | 85.45955        | 75.57138 | 12.28385 |           | 379c |
|     |   | 63.29462     |          | 56            |          | 57.34611        |          |          |           |      |
|     |   | 88.97434     |          | 80            |          | 80.61241        |          |          |           |      |
|     |   | 98.60423     |          | 89            |          | 89.33727        |          |          |           |      |
|     |   | 71.85453     |          | 64            |          | 65.10154        |          |          |           |      |
| 5.5 |   | 29.05501     |          | 24            |          | 26.32438        | 39.70250 | 16.27255 |           | 379d |
|     |   | 71.85453     |          | 64            |          | 65.10154        |          |          |           |      |
|     |   | 46.17482     |          | 40            |          | 41.83524        |          |          |           |      |
|     |   | 51.52476     |          | 45            |          | 46.68239        |          |          |           |      |
|     |   | 20.49510     |          | 16            |          | 18.56895        |          |          |           |      |
| 6   |   | 24.77506     |          | 20            |          | 22.44666        | 33.88593 | 10.51998 |           | 379e |
|     |   | 46.17482     |          | 40            |          | 41.83524        |          |          |           |      |
|     |   | 41.89486     |          | 36            |          | 37.95753        |          |          |           |      |
|     |   | 51.52476     |          | 45            |          | 46.68239        |          |          |           |      |
|     |   | 22.63508     |          | 18            |          | 20.50780        |          |          |           |      |
| 6.5 |   | 10.86521     |          | 7             |          | 9.844088        | 17.40563 | 6.643242 |           | 379f |
|     |   | 10.86521     |          | 7             |          | 9.844088        |          |          |           |      |
|     |   | 29.05501     |          | 24            |          | 26.32438        |          |          |           |      |
|     |   | 20.49510     |          | 16            |          | 18.56895        |          |          |           |      |
|     |   | 24.77506     |          | 20            |          | 22.44666        |          |          |           |      |
| 5   |   | 116.6323     | 1.205273 | 100           | -3.89505 | 116.635         | 99.99769 | 99.99769 | 0         | 489a |
|     |   | 116.6323     |          | 100           |          | 99.99769        |          |          |           |      |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | exp.peak<br>x | b   | theoret.<br>max | ef=y/max | avg.eff. | stdev eff | run      |      |
|-----|---|--------------|---------------|-----|-----------------|----------|----------|-----------|----------|------|
| 6   |   | 112.5296     | 100           |     | 97.14437        | 89.35784 | 6.423527 |           | 391c     |      |
|     |   | 99.33001     | 88            |     | 85.74944        |          |          |           |          |      |
|     |   | 112.5296     | 100           |     | 97.14437        |          |          |           |          |      |
|     |   | 96.03011     | 85            |     | 82.90071        |          |          |           |          |      |
|     |   | 97.13007     | 86            |     | 83.85028        |          |          |           |          |      |
| 6.5 |   | 96.03011     | 85            |     | 82.90071        | 64.09906 | 9.746944 |           | 391d     |      |
|     |   | 70.73086     | 62            |     | 61.06041        |          |          |           |          |      |
|     |   | 70.73086     | 62            |     | 61.06041        |          |          |           |          |      |
|     |   | 70.73086     | 62            |     | 61.06041        |          |          |           |          |      |
|     |   | 63.03109     | 55            |     | 54.41337        |          |          |           |          |      |
| 7   |   | 66.33099     | 58            |     | 57.26210        | 68.27721 | 9.430986 |           | 391e     |      |
|     |   | 75.13073     | 66            |     | 64.85873        |          |          |           |          |      |
|     |   | 76.23069     | 67            |     | 65.80830        |          |          |           |          |      |
|     |   | 99.33001     | 88            |     | 85.74944        |          |          |           |          |      |
|     |   | 78.43063     | 69            |     | 67.70746        |          |          |           |          |      |
| 7.5 |   | 66.33099     | 58            |     | 57.26210        | 52.51421 | 3.343808 |           | 391f     |      |
|     |   | 61.93112     | 54            |     | 53.46379        |          |          |           |          |      |
|     |   | 60.83115     | 53            |     | 52.51421        |          |          |           |          |      |
|     |   | 60.83115     | 53            |     | 52.51421        |          |          |           |          |      |
|     |   | 54.23135     | 47            |     | 46.81675        |          |          |           |          |      |
| 8   |   | 68.53092     | 60            |     | 59.16126        | 51.37472 | 9.503372 |           | 391g     |      |
|     |   | 55.33132     | 48            |     | 47.76632        |          |          |           |          |      |
|     |   | 64.13105     | 56            |     | 55.36295        |          |          |           |          |      |
|     |   | 39.93177     | 34            |     | 34.47223        |          |          |           |          |      |
|     |   | 69.63089     | 61            |     | 60.11084        |          |          |           |          |      |
| 8.5 |   | 48.73151     | 42            |     | 42.06886        | 46.24700 | 9.080268 |           | 391h     |      |
|     |   | 35.53190     | 30            |     | 30.67392        |          |          |           |          |      |
|     |   | 57.53125     | 50            |     | 49.66548        |          |          |           |          |      |
|     |   | 65.23102     | 57            |     | 56.31252        |          |          |           |          |      |
|     |   | 60.83115     | 53            |     | 52.51421        |          |          |           |          |      |
| 9   |   | 41.03174     | 35            |     | 35.42181        | 34.28232 | 3.769716 |           | 391i     |      |
|     |   | 36.63187     | 31            |     | 31.62350        |          |          |           |          |      |
|     |   | 35.53190     | 30            |     | 30.67392        |          |          |           |          |      |
|     |   | 47.63154     | 41            |     | 41.11928        |          |          |           |          |      |
|     |   | 37.73184     | 32            |     | 32.57308        |          |          |           |          |      |
| 4   |   | 172.1388     | 1.933118      | 89  | 0.09132         | 193.403  | 89.00524 | 92.60355  | 2.331280 | 387a |
|     |   | 177.9381     |               | 92  |                 |          | 92.00383 |           |          |      |
|     |   | 185.6706     |               | 96  |                 |          | 96.00194 |           |          |      |
|     |   | 177.9381     |               | 92  |                 |          | 92.00383 |           |          |      |
|     |   | 181.8044     |               | 94  |                 |          | 94.00289 |           |          |      |
| 4.5 |   | 189.5368     |               | 98  |                 |          | 98.00100 | 95.00241  | 4.193257 | 387b |
|     |   | 185.6706     |               | 96  |                 |          | 96.00194 |           |          |      |
|     |   | 193.4031     |               | 100 |                 |          | 100.0000 |           |          |      |
|     |   | 170.2057     |               | 88  |                 |          | 88.00572 |           |          |      |
|     |   | 179.8712     |               | 93  |                 |          | 93.00336 |           |          |      |
| 5   |   | 135.4095     |               | 70  |                 |          | 70.01420 | 84.20751  | 7.436921 | 387c |
|     |   | 176.0050     |               | 91  |                 |          | 91.00430 |           |          |      |
|     |   | 170.2057     |               | 88  |                 |          | 88.00572 |           |          |      |
|     |   | 170.2057     |               | 88  |                 |          | 88.00572 |           |          |      |
|     |   | 162.4732     |               | 84  |                 |          | 84.00760 |           |          |      |
| 5.5 |   | 170.2057     |               | 88  |                 |          | 88.00572 | 79.00996  | 5.724427 | 387d |

## Calibration of Experimental Data &amp; Efficiency Calculation

| z   | d | regress<br>y | a      | exp.peak<br>x | b      | theoret.<br>max | ef=y/max | avg.eff. | stdevff  | run    |
|-----|---|--------------|--------|---------------|--------|-----------------|----------|----------|----------|--------|
|     |   | 64.49        |        | 63            |        | 62.40565        |          |          |          |        |
|     |   | 58.19        |        | 57            |        | 56.30927        |          |          |          |        |
| 7   |   | 36.14        |        | 36            |        | 34.97193        | 42.49080 | 14.67640 |          | 483d   |
|     |   | 43.49        |        | 43            |        | 42.08438        |          |          |          |        |
|     |   | 69.74        |        | 68            |        | 67.48596        |          |          |          |        |
|     |   | 23.54        |        | 24            |        | 22.77917        |          |          |          |        |
|     |   | 46.64        |        | 46            |        | 45.13257        |          |          |          |        |
| 7.5 |   | 13.04        |        | 14            |        | 12.61854        | 20.74704 | 5.299138 |          | 483e   |
|     |   | 23.54        |        | 24            |        | 22.77917        |          |          |          |        |
|     |   | 25.64        |        | 26            |        | 24.81130        |          |          |          |        |
|     |   | 27.74        |        | 28            |        | 26.84342        |          |          |          |        |
|     |   | 17.24        |        | 18            |        | 16.68279        |          |          |          |        |
| 8   |   | 22.49        |        | 23            |        | 21.76311        | 16.88600 | 4.329908 |          | 483f   |
|     |   | 22.49        |        | 23            |        | 21.76311        |          |          |          |        |
|     |   | 16.19        |        | 17            |        | 15.66673        |          |          |          |        |
|     |   | 10.94        |        | 12            |        | 10.58641        |          |          |          |        |
|     |   | 15.14        |        | 16            |        | 14.65066        |          |          |          |        |
| 5   |   | 92.8178      | 0.9986 | 98            | -5.045 | 94.815          | 97.89358 | 99.29786 | 0.992974 | 475aII |
|     |   | 94.815       |        | 100           |        | 100             |          |          |          | (3)    |
|     |   | 94.815       |        | 100           |        | 100             |          |          |          |        |
| 5.5 |   | 52.8738      |        | 58            |        | 55.76522        | 67.35052 | 12.95632 |          | 475bII |
|     |   | 49.878       |        | 55            |        | 52.60560        |          |          |          | (6)    |
|     |   | 84.829       |        | 90            |        | 89.46791        |          |          |          |        |
|     |   | 61.8612      |        | 67            |        | 65.24410        |          |          |          |        |
|     |   | 58.8654      |        | 64            |        | 62.08448        |          |          |          |        |
|     |   | 74.843       |        | 80            |        | 78.93582        |          |          |          |        |
| 6   |   | 28.9074      |        | 34            |        | 30.48821        | 47.10550 | 14.87622 |          | 475cII |
|     |   | 44.885       |        | 50            |        | 47.33955        |          |          |          | (9)    |
|     |   | 74.843       |        | 80            |        | 78.93582        |          |          |          |        |
|     |   | 26.9102      |        | 32            |        | 28.38179        |          |          |          |        |
|     |   | 54.871       |        | 60            |        | 57.87164        |          |          |          |        |
|     |   | 30.9046      |        | 36            |        | 32.59463        |          |          |          |        |
|     |   | 48.8794      |        | 54            |        | 51.55239        |          |          |          |        |
|     |   | 44.885       |        | 50            |        | 47.33955        |          |          |          |        |
|     |   | 46.8822      |        | 52            |        | 49.44597        |          |          |          |        |
| 6.5 |   | 21.9172      |        | 27            |        | 23.11575        | 23.95831 | 12.76774 |          | 475dII |
|     |   | 28.9074      |        | 34            |        | 30.48821        |          |          |          |        |
|     |   | 6.9382       |        | 12            |        | 7.317618        |          |          |          |        |
|     |   | 13.9284      |        | 19            |        | 14.69008        |          |          |          |        |
|     |   | 41.8892      |        | 47            |        | 44.17992        |          |          |          |        |
| 7   |   | 7.9368       |        | 13            |        | 8.370827        | 19.42952 | 9.034502 |          | 475eII |
|     |   | 17.9228      |        | 23            |        | 18.90291        |          |          |          | (6)    |
|     |   | 10.9326      |        | 16            |        | 11.53045        |          |          |          |        |
|     |   | 34.899       |        | 40            |        | 36.80746        |          |          |          |        |
|     |   | 18.9214      |        | 24            |        | 19.95612        |          |          |          |        |
|     |   | 19.92        |        | 25            |        | 21.00933        |          |          |          |        |
| 7.5 |   | 17.9228      |        | 23            |        | 18.90291        | 13.90017 | 4.732120 |          | 475fII |
|     |   | 6.9382       |        | 12            |        | 7.317618        |          |          |          | (4)    |
|     |   | 16.9242      |        | 22            |        | 17.84970        |          |          |          |        |
|     |   | 10.9326      |        | 16            |        | 11.53045        |          |          |          |        |

APPENDIX D

DATA FOR REGRESSION OF LOGIT-TRANSFORMED EFFICIENCY  
ON  $[Z/D]$ ; CALCULATION OF  $\mu$  AND  $\sigma$

NOV 1965

43

HANDBOOK FOR ROAD

DESIGN

Data for Regression of Logit Efficiency on  $Z/D$ 

|          | y         |         | x     | m        | b          | y         | $mu = -b/m$ | $w = 1/m$ |
|----------|-----------|---------|-------|----------|------------|-----------|-------------|-----------|
| efficien | logit eff | $Qh/Dj$ | $Z/D$ | logiteff | regression | predicted | $(Z/D)50$   | spread    |
| 0.929    | 2.567     | 76      | 0.516 | -5.919   | 5.539      | 2.484     | 0.935799    | 0.168947  |
| 0.899    | 2.186     | 76      | 0.645 | -5.919   | 5.539      | 1.720     |             |           |
| 0.663    | 0.678     | 76      | 0.774 | -5.919   | 5.539      | 0.957     |             |           |
| 0.590    | 0.364     | 76      | 0.839 | -5.919   | 5.539      | 0.575     |             |           |
| 0.418    | -0.333    | 76      | 0.903 | -5.919   | 5.539      | 0.193     |             |           |
| 0.396    | -0.423    | 76      | 1.032 | -5.919   | 5.539      | -0.571    |             |           |
| 0.267    | -1.010    | 76      | 1.161 | -5.919   | 5.539      | -1.335    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 0.980    | 3.902     | 77      | 0.516 | -5.407   | 6.027      | 3.236     | 1.114666    | 0.184945  |
| 0.921    | 2.453     | 77      | 0.645 | -5.407   | 6.027      | 2.539     |             |           |
| 0.780    | 1.266     | 77      | 0.774 | -5.407   | 6.027      | 1.841     |             |           |
| 0.675    | 0.731     | 77      | 0.903 | -5.407   | 6.027      | 1.143     |             |           |
| 0.600    | 0.405     | 77      | 1.032 | -5.407   | 6.027      | 0.446     |             |           |
| 0.455    | -0.180    | 77      | 1.161 | -5.407   | 6.027      | -0.252    |             |           |
| 0.362    | -0.566    | 77      | 1.290 | -5.407   | 6.027      | -0.950    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 0.960    | 3.172     | 78      | 0.645 | -5.706   | 6.144      | 2.463     | 1.076761    | 0.175254  |
| 0.793    | 1.343     | 78      | 0.774 | -5.706   | 6.144      | 1.726     |             |           |
| 0.610    | 0.447     | 78      | 0.903 | -5.706   | 6.144      | 0.990     |             |           |
| 0.499    | -0.003    | 78      | 1.032 | -5.706   | 6.144      | 0.254     |             |           |
| 0.417    | -0.335    | 78      | 1.161 | -5.706   | 6.144      | -0.482    |             |           |
| 0.292    | -0.884    | 78      | 1.290 | -5.706   | 6.144      | -1.219    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 0.996    | 5.539     | 95      | 0.516 | -12.836  | 10.874     | 4.249     | 0.847148    | 0.077905  |
| 0.836    | 1.626     | 95      | 0.645 | -12.836  | 10.874     | 2.593     |             |           |
| 0.468    | -0.129    | 95      | 0.774 | -12.836  | 10.874     | 0.936     |             |           |
| 0.305    | -0.822    | 95      | 0.903 | -12.836  | 10.874     | -0.720    |             |           |
| 0.180    | -1.516    | 95      | 1.032 | -12.836  | 10.874     | -2.376    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 0.839    | 1.649     | 97      | 0.645 | -4.231   | 3.983      | 1.253     | 0.941385    | 0.236350  |
| 0.595    | 0.385     | 97      | 0.774 | -4.231   | 3.983      | 0.707     |             |           |
| 0.512    | 0.048     | 97      | 0.903 | -4.231   | 3.983      | 0.161     |             |           |
| 0.331    | -0.705    | 97      | 1.032 | -4.231   | 3.983      | -0.384    |             |           |
| 0.347    | -0.634    | 97      | 1.161 | -4.231   | 3.983      | -0.930    |             |           |
| 0.196    | -1.410    | 97      | 1.290 | -4.231   | 3.983      | -1.476    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 1.000    | 12.295    | 98      | 0.774 |          |            | 0.000     |             |           |
| 0.730    | 0.994     | 98      | 0.839 | -4.472   | 4.33       | 0.579     | 0.968246    | 0.223613  |
| 0.514    | 0.054     | 98      | 0.903 | -4.472   | 4.33       | 0.291     |             |           |
| 0.329    | -0.713    | 98      | 1.032 | -4.472   | 4.33       | -0.286    |             |           |
| 0.321    | -0.749    | 98      | 1.161 | -4.472   | 4.33       | -0.863    |             |           |
| 0.214    | -1.303    | 98      | 1.290 | -4.472   | 4.33       | -1.440    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 1.000    | 16.237    | 126     | 0.645 |          |            | 0.000     |             |           |
| 0.722    | 0.953     | 126     | 0.774 | -11.178  | 9.593      | 0.939     | 0.858203    | 0.089461  |
| 0.549    | 0.196     | 126     | 0.839 | -11.178  | 9.593      | 0.215     |             |           |
| 0.378    | -0.498    | 126     | 0.903 | -11.178  | 9.593      | -0.503    |             |           |
| 0.126    | -1.940    | 126     | 1.032 | -11.178  | 9.593      | -1.946    |             |           |
|          |           |         |       |          |            | 0.000     |             |           |
| 1.000    | 11.488    | 129     | 0.645 |          |            | 0.000     |             |           |
| 0.670    | 0.706     | 129     | 0.774 | -7.633   | 6.579      | 0.670     | 0.861915    | 0.131010  |
| 0.493    | -0.030    | 129     | 0.839 | -7.633   | 6.579      | 0.177     |             |           |
| 0.481    | -0.077    | 129     | 0.903 | -7.633   | 6.579      | -0.315    |             |           |

Data for Regression of Logit Efficiency on  $[Z/D]$ 

|       |        |     |       |         |        |        |          |          |
|-------|--------|-----|-------|---------|--------|--------|----------|----------|
| 0.203 | -1.365 | 129 | 1.032 | -7.633  | 6.579  | -1.300 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.978 | 3.793  | 130 | 0.645 | -12.606 | 11.288 | 3.155  | 0.895446 | 0.079327 |
| 0.768 | 1.195  | 130 | 0.774 | -12.606 | 11.288 | 1.529  |          |          |
| 0.505 | 0.021  | 130 | 0.839 | -12.606 | 11.288 | 0.715  |          |          |
| 0.425 | -0.302 | 130 | 0.903 | -12.606 | 11.288 | -0.098 |          |          |
| 0.245 | -1.127 | 130 | 1.032 | -12.606 | 11.288 | -1.725 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.968 | 3.401  | 189 | 0.903 | -13.07  | 14.958 | 3.153  | 1.144452 | 0.076511 |
| 0.841 | 1.664  | 189 | 1.032 | -13.07  | 14.958 | 1.466  |          |          |
| 0.557 | 0.227  | 189 | 1.097 | -13.07  | 14.958 | 0.623  |          |          |
| 0.321 | -0.751 | 189 | 1.161 | -13.07  | 14.958 | -0.220 |          |          |
| 0.196 | -1.413 | 189 | 1.290 | -13.07  | 14.958 | -1.907 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 1.000 | 11.610 | 193 | 0.903 |         |        | 0.000  |          |          |
| 0.774 | 1.230  | 193 | 1.032 | -5.716  | 6.911  | 1.011  | 1.209062 | 0.174947 |
| 0.473 | -0.109 | 193 | 1.161 | -5.716  | 6.911  | 0.273  |          |          |
| 0.414 | -0.348 | 193 | 1.290 | -5.716  | 6.911  | -0.464 |          |          |
| 0.241 | -1.148 | 193 | 1.419 | -5.716  | 6.911  | -1.202 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.939 | 2.728  | 195 | 0.774 | -7.578  | 8.294  | 2.427  | 1.094484 | 0.131960 |
| 0.830 | 1.585  | 195 | 0.903 | -7.578  | 8.294  | 1.449  |          |          |
| 0.442 | -0.233 | 195 | 1.032 | -7.578  | 8.294  | 0.472  |          |          |
| 0.335 | -0.684 | 195 | 1.161 | -7.578  | 8.294  | -0.506 |          |          |
| 0.264 | -1.025 | 195 | 1.290 | -7.578  | 8.294  | -1.484 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.996 | 5.576  | 316 | 1.032 | -8.139  | 12.548 | 4.146  | 1.541712 | 0.122865 |
| 0.962 | 3.239  | 316 | 1.161 | -8.139  | 12.548 | 3.096  |          |          |
| 0.640 | 0.574  | 316 | 1.290 | -8.139  | 12.548 | 2.046  |          |          |
| 0.488 | -0.046 | 316 | 1.419 | -8.139  | 12.548 | 0.996  |          |          |
| 0.427 | -0.295 | 316 | 1.548 | -8.139  | 12.548 | -0.054 |          |          |
| 0.302 | -0.837 | 316 | 1.677 | -8.139  | 12.548 | -1.105 |          |          |
| 0.229 | -1.216 | 316 | 1.806 | -8.139  | 12.548 | -2.155 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 1.000 | 16.118 | 322 | 1.032 |         |        | 0.000  |          |          |
| 0.986 | 4.257  | 322 | 1.161 | -8.71   | 13.963 | 3.848  | 1.603099 | 0.114810 |
| 0.982 | 4.007  | 322 | 1.290 | -8.71   | 13.963 | 2.724  |          |          |
| 0.454 | -0.186 | 322 | 1.419 | -8.71   | 13.963 | 1.600  |          |          |
| 0.385 | -0.469 | 322 | 1.548 | -8.71   | 13.963 | 0.477  |          |          |
| 0.372 | -0.522 | 322 | 1.677 | -8.71   | 13.963 | -0.647 |          |          |
| 0.303 | -0.834 | 322 | 1.806 | -8.71   | 13.963 | -1.771 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.962 | 3.228  | 326 | 1.290 | -6.709  | 11.384 | 2.727  | 1.696825 | 0.149053 |
| 0.815 | 1.485  | 326 | 1.419 | -6.709  | 11.384 | 1.862  |          |          |
| 0.667 | 0.694  | 326 | 1.548 | -6.709  | 11.384 | 0.996  |          |          |
| 0.530 | 0.122  | 326 | 1.677 | -6.709  | 11.384 | 0.130  |          |          |
| 0.314 | -0.780 | 326 | 1.806 | -6.709  | 11.384 | -0.735 |          |          |
| 0.205 | -1.357 | 326 | 1.935 | -6.709  | 11.384 | -1.601 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |
| 0.917 | 2.398  | 379 | 1.032 | -6.386  | 9.102  | 2.510  | 1.425305 | 0.156592 |
| 0.862 | 1.835  | 379 | 1.161 | -6.386  | 9.102  | 1.686  |          |          |
| 0.756 | 1.129  | 379 | 1.290 | -6.386  | 9.102  | 0.862  |          |          |
| 0.397 | -0.418 | 379 | 1.419 | -6.386  | 9.102  | 0.038  |          |          |
| 0.339 | -0.668 | 379 | 1.548 | -6.386  | 9.102  | -0.786 |          |          |
| 0.174 | -1.557 | 379 | 1.677 | -6.386  | 9.102  | -1.610 |          |          |
|       |        |     |       |         |        | 0.000  |          |          |

Data for Regression of Logit Efficiency on [I/D]

|       |        |     |       |        |        |        |          |          |
|-------|--------|-----|-------|--------|--------|--------|----------|----------|
| 0.926 | 2.527  | 387 | 1.032 | -5.519 | 8.823  | 3.126  | 1.598659 | 0.181192 |
| 0.950 | 2.945  | 387 | 1.161 | -5.519 | 8.823  | 2.414  |          |          |
| 0.842 | 1.674  | 387 | 1.290 | -5.519 | 8.823  | 1.702  |          |          |
| 0.790 | 1.326  | 387 | 1.419 | -5.519 | 8.823  | 0.990  |          |          |
| 0.594 | 0.381  | 387 | 1.548 | -5.519 | 8.823  | 0.277  |          |          |
| 0.362 | -0.565 | 387 | 1.677 | -5.519 | 8.823  | -0.435 |          |          |
| 0.206 | -1.347 | 387 | 1.806 | -5.519 | 8.823  | -1.147 |          |          |
|       |        |     |       |        |        | 0.000  |          |          |
| 0.937 | 2.704  | 391 | 1.290 | -3.598 | 7.463  | 2.820  | 2.074207 | 0.277932 |
| 0.958 | 3.131  | 391 | 1.419 | -3.598 | 7.463  | 2.356  |          |          |
| 0.894 | 2.128  | 391 | 1.548 | -3.598 | 7.463  | 1.892  |          |          |
| 0.641 | 0.580  | 391 | 1.677 | -3.598 | 7.463  | 1.428  |          |          |
| 0.683 | 0.767  | 391 | 1.806 | -3.598 | 7.463  | 0.963  |          |          |
| 0.525 | 0.101  | 391 | 1.935 | -3.598 | 7.463  | 0.499  |          |          |
| 0.514 | 0.055  | 391 | 2.065 | -3.598 | 7.463  | 0.035  |          |          |
| 0.462 | -0.150 | 391 | 2.194 | -3.598 | 7.463  | -0.429 |          |          |
| 0.343 | -0.651 | 391 | 2.323 | -3.598 | 7.463  | -0.894 |          |          |
|       |        |     |       |        |        | 0.000  |          |          |
| 0.993 | 4.952  | 475 | 1.290 | -9.16  | 14.964 | 3.145  | 1.633624 | 0.109170 |
| 0.674 | 0.724  | 475 | 1.419 | -9.16  | 14.964 | 1.963  |          |          |
| 0.471 | -0.116 | 475 | 1.548 | -9.16  | 14.964 | 0.781  |          |          |
| 0.240 | -1.155 | 475 | 1.677 | -9.16  | 14.964 | -0.401 |          |          |
| 0.194 | -1.422 | 475 | 1.806 | -9.16  | 14.964 | -1.583 |          |          |
| 0.139 | -1.824 | 475 | 1.935 | -9.16  | 14.964 | -2.765 |          |          |
|       |        |     |       |        |        | 0.000  |          |          |
| 0.915 | 2.372  | 483 | 1.419 | -6.113 | 10.739 | 2.062  | 1.756747 | 0.163585 |
| 0.738 | 1.035  | 483 | 1.548 | -6.113 | 10.739 | 1.274  |          |          |
| 0.594 | 0.379  | 483 | 1.677 | -6.113 | 10.739 | 0.485  |          |          |
| 0.425 | -0.303 | 483 | 1.806 | -6.113 | 10.739 | -0.304 |          |          |
| 0.207 | -1.340 | 483 | 1.935 | -6.113 | 10.739 | -1.093 |          |          |
| 0.169 | -1.594 | 483 | 2.065 | -6.113 | 10.739 | -1.881 |          |          |
|       |        |     |       |        |        | 0.000  |          |          |
| 1.000 | 10.679 | 489 | 1.290 |        |        | 0.000  |          |          |
| 0.979 | 3.857  | 489 | 1.419 | -6.87  | 13.602 | 3.851  | 1.979912 | 0.145560 |
| 0.977 | 3.760  | 489 | 1.548 | -6.87  | 13.602 | 2.965  |          |          |
| 0.793 | 1.345  | 489 | 1.677 | -6.87  | 13.602 | 2.078  |          |          |
| 0.692 | 0.810  | 489 | 1.806 | -6.87  | 13.602 | 1.192  |          |          |
| 0.601 | 0.410  | 489 | 1.935 | -6.87  | 13.602 | 0.305  |          |          |
| 0.337 | -0.679 | 489 | 2.065 | -6.87  | 13.602 | -0.581 |          |          |
| 0.239 | -1.156 | 489 | 2.194 | -6.87  | 13.602 | -1.468 |          |          |

APPENDIX E

LOGIT CAPTURE EFFICIENCY REGRESSION ON (3/D)

FOX E. 1988

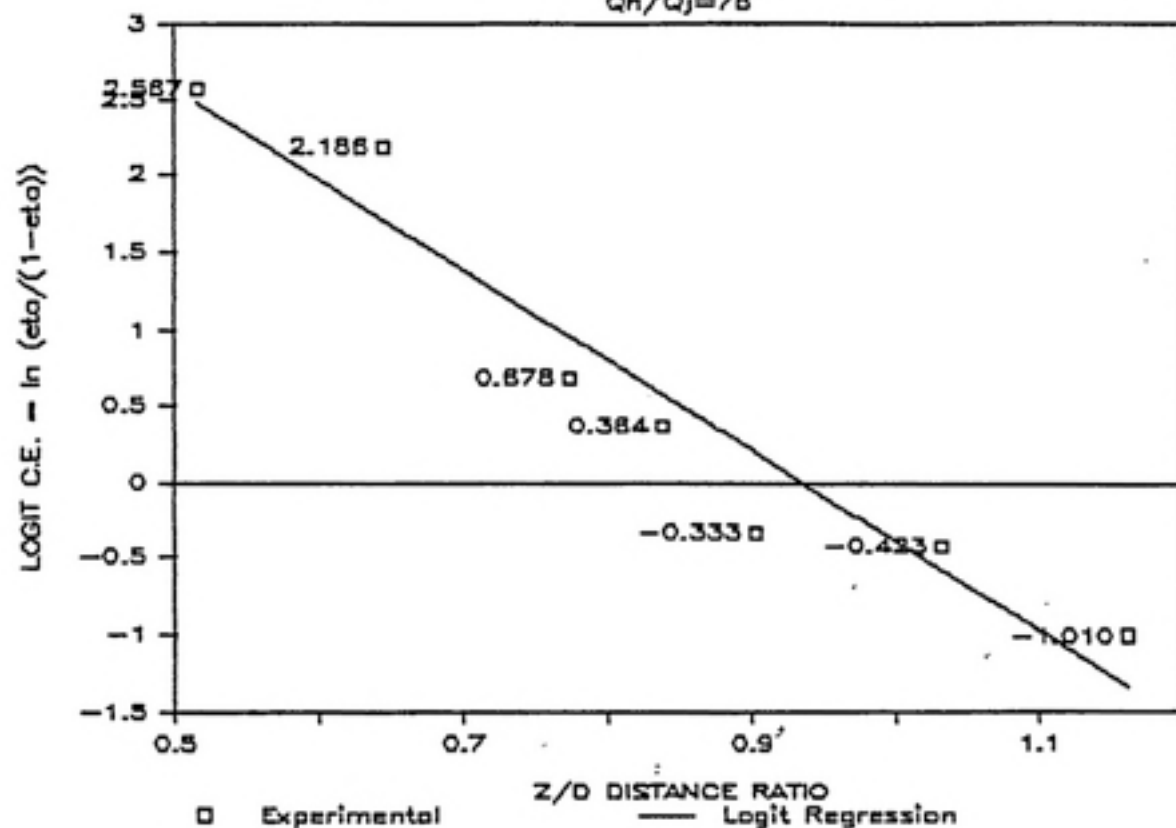
1988

MINNESOTA FISH

100% CATCH

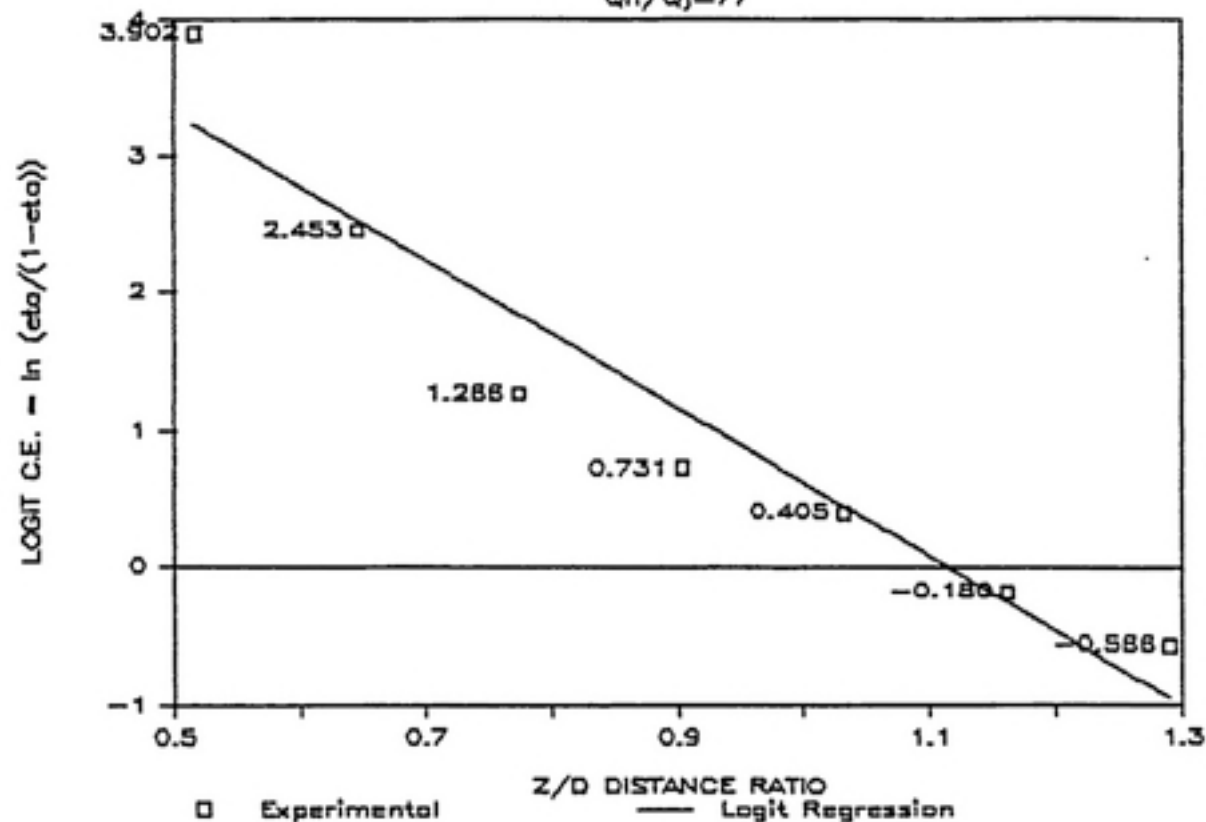
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=78$



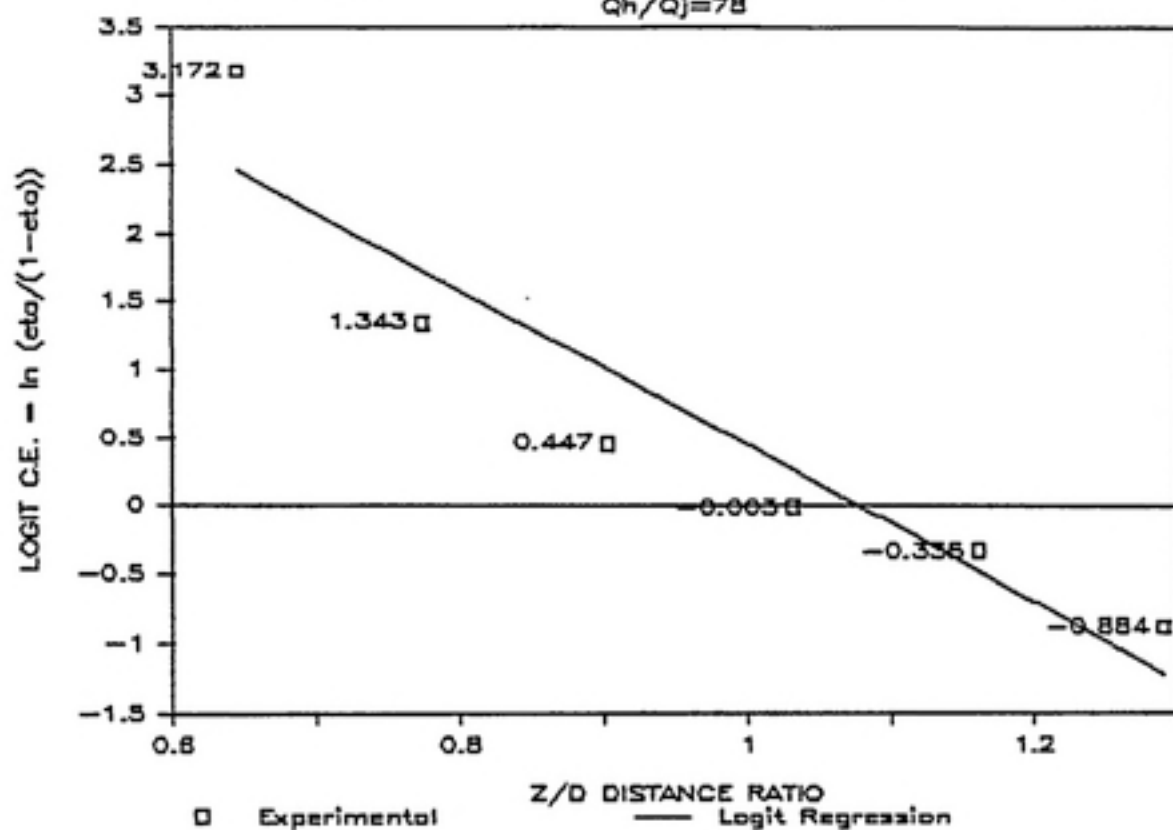
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=77$



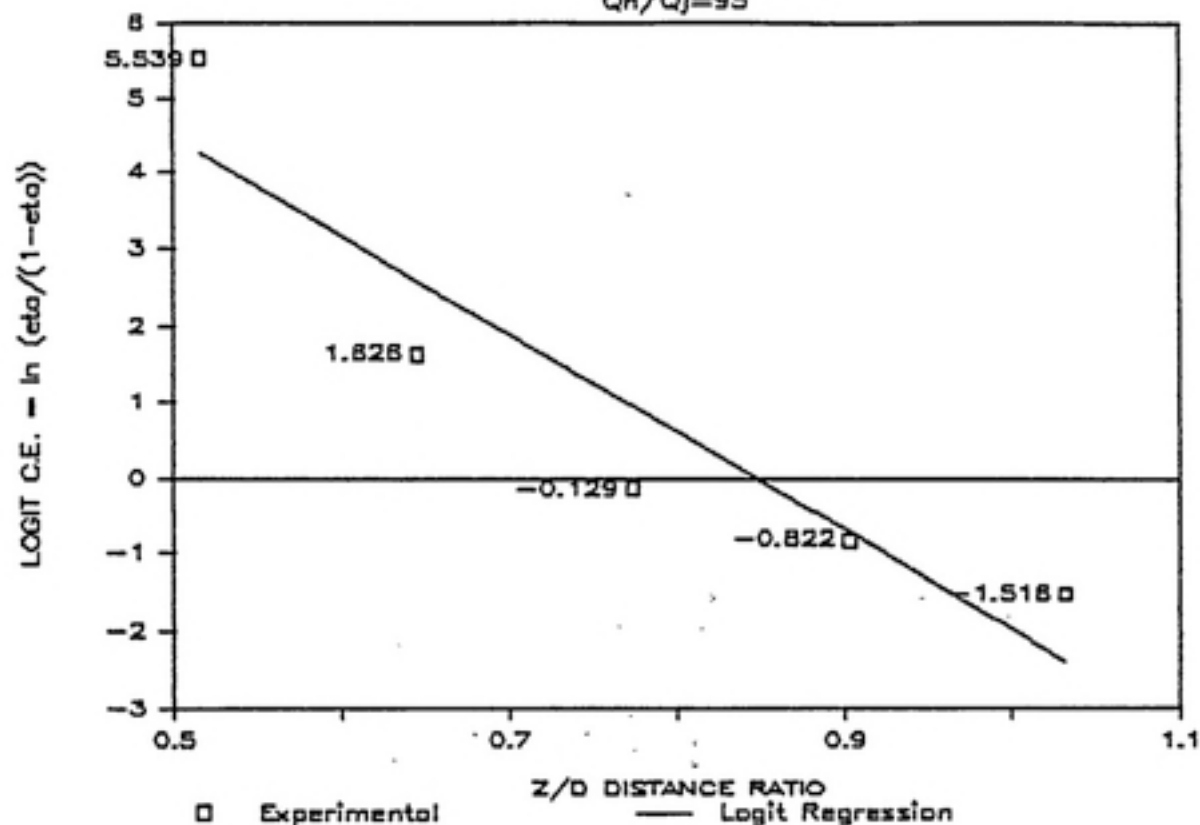
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=78$



# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=95$



12X 1/2

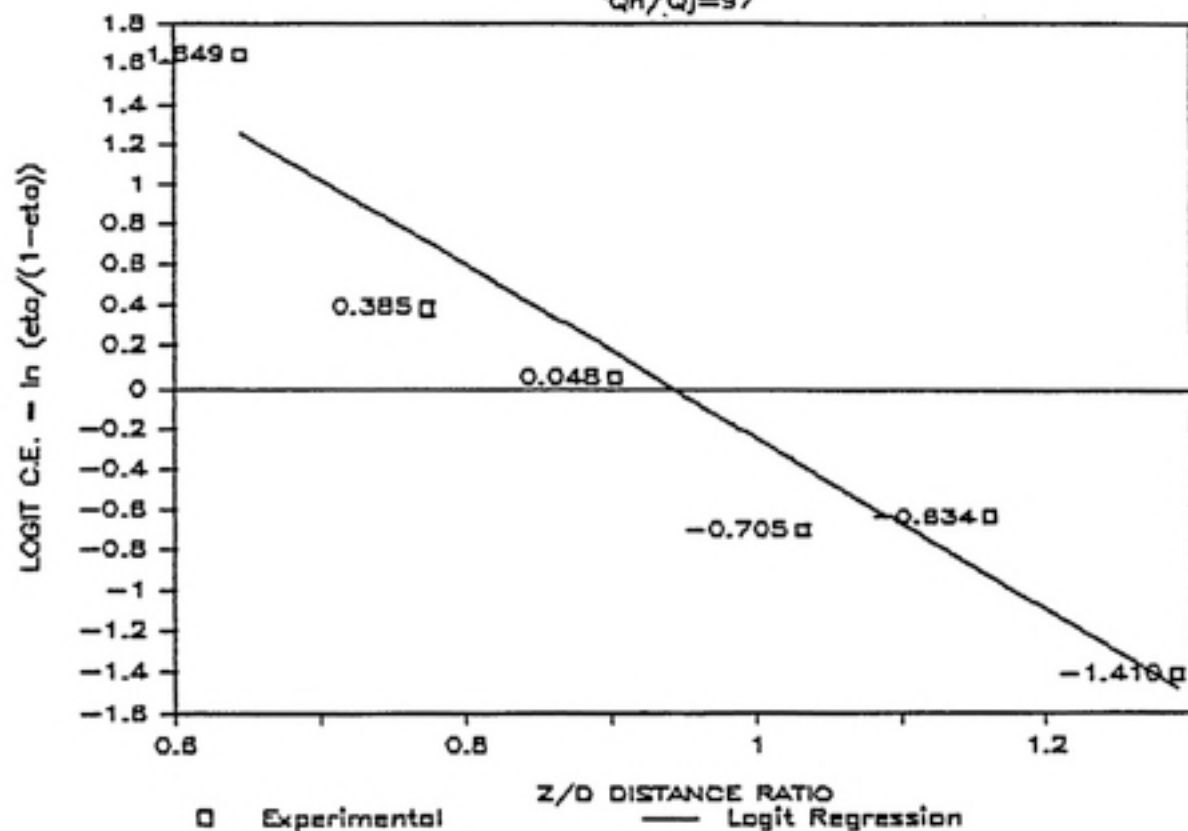
ON

WINNER 2000

100% COTTON

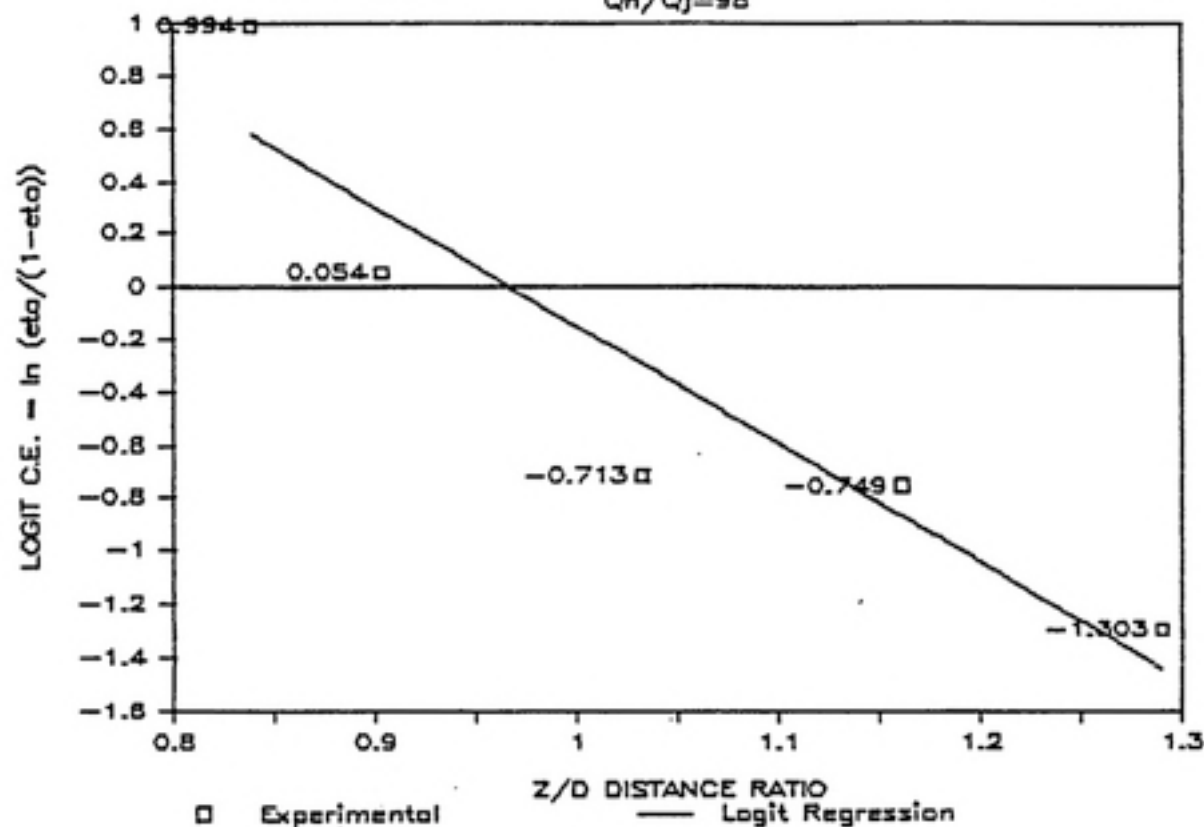
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=97$



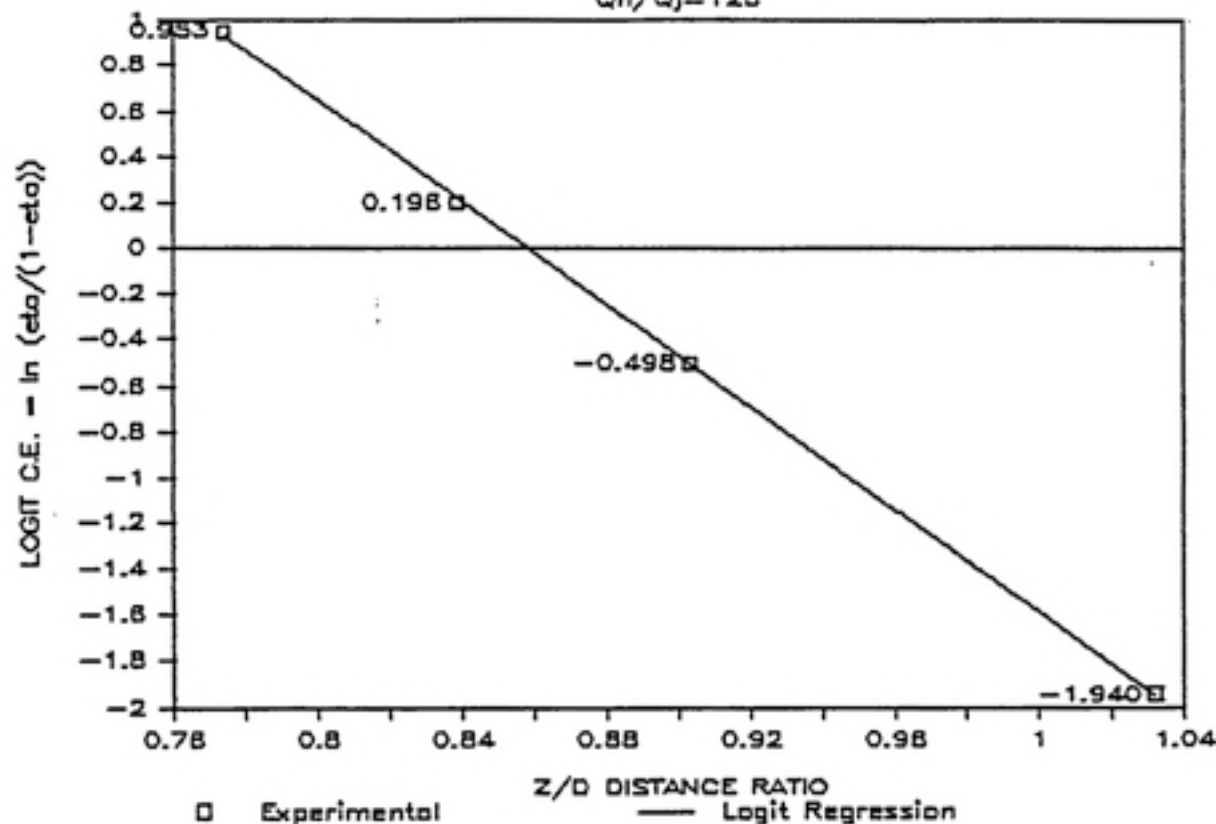
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=98$



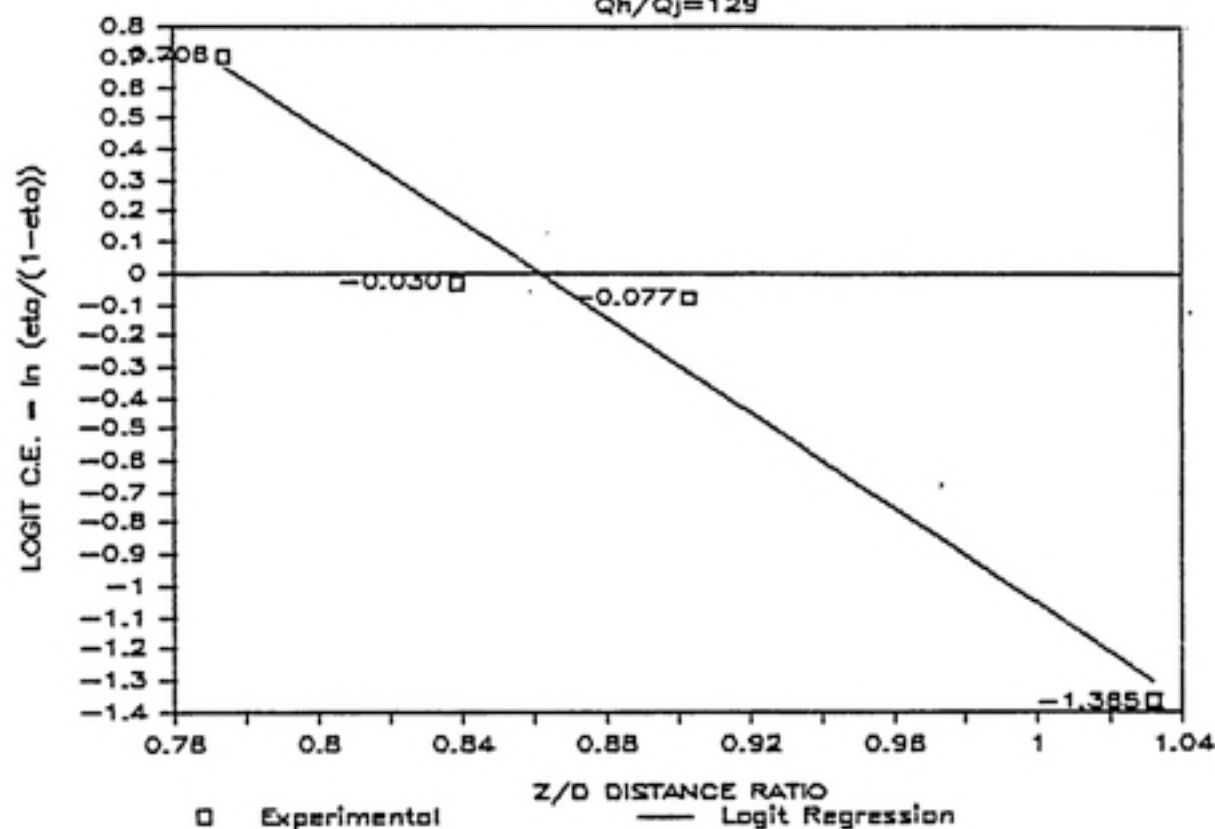
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=128$



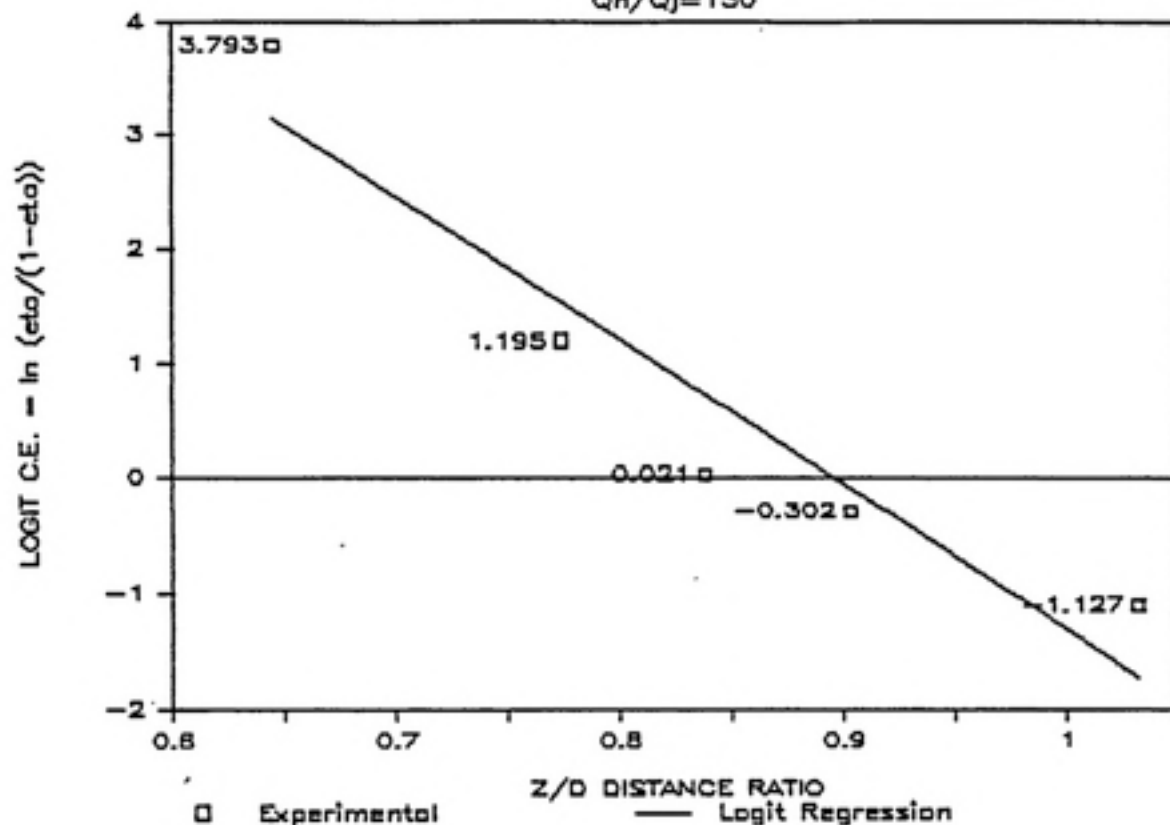
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=129$



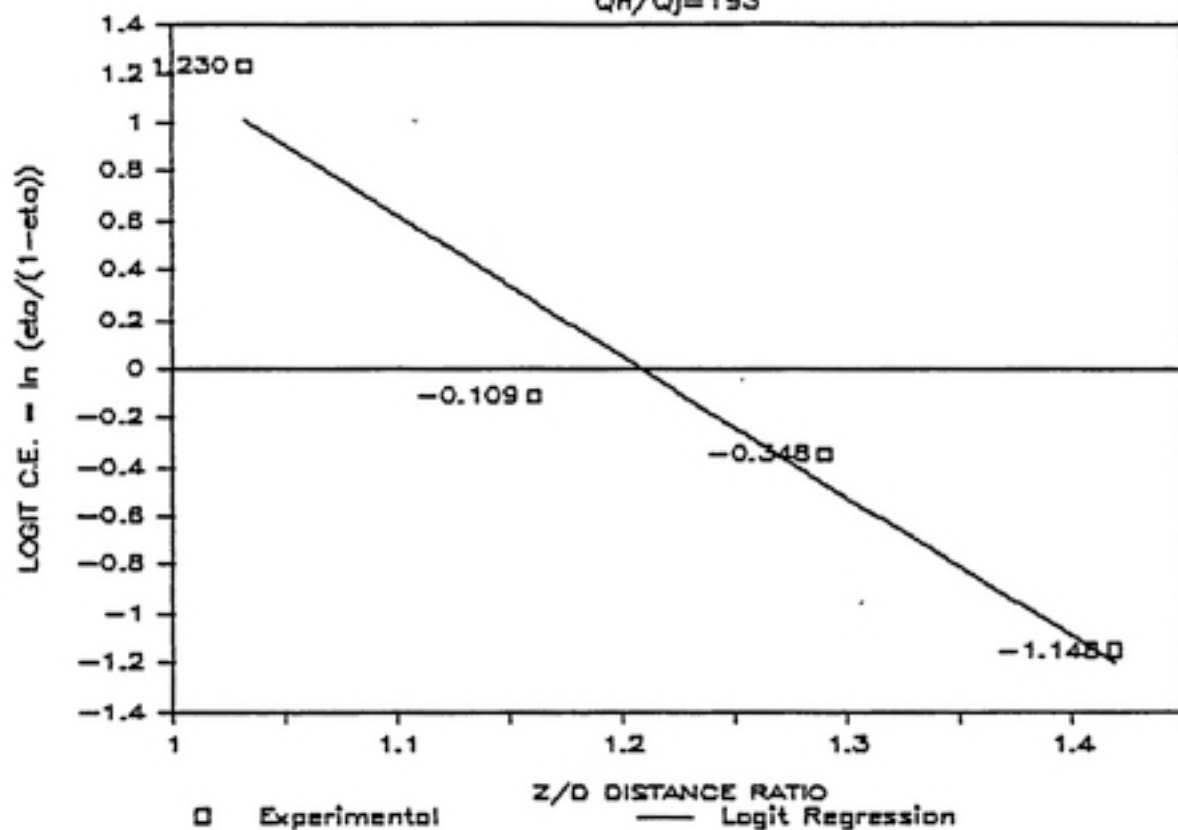
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=130$



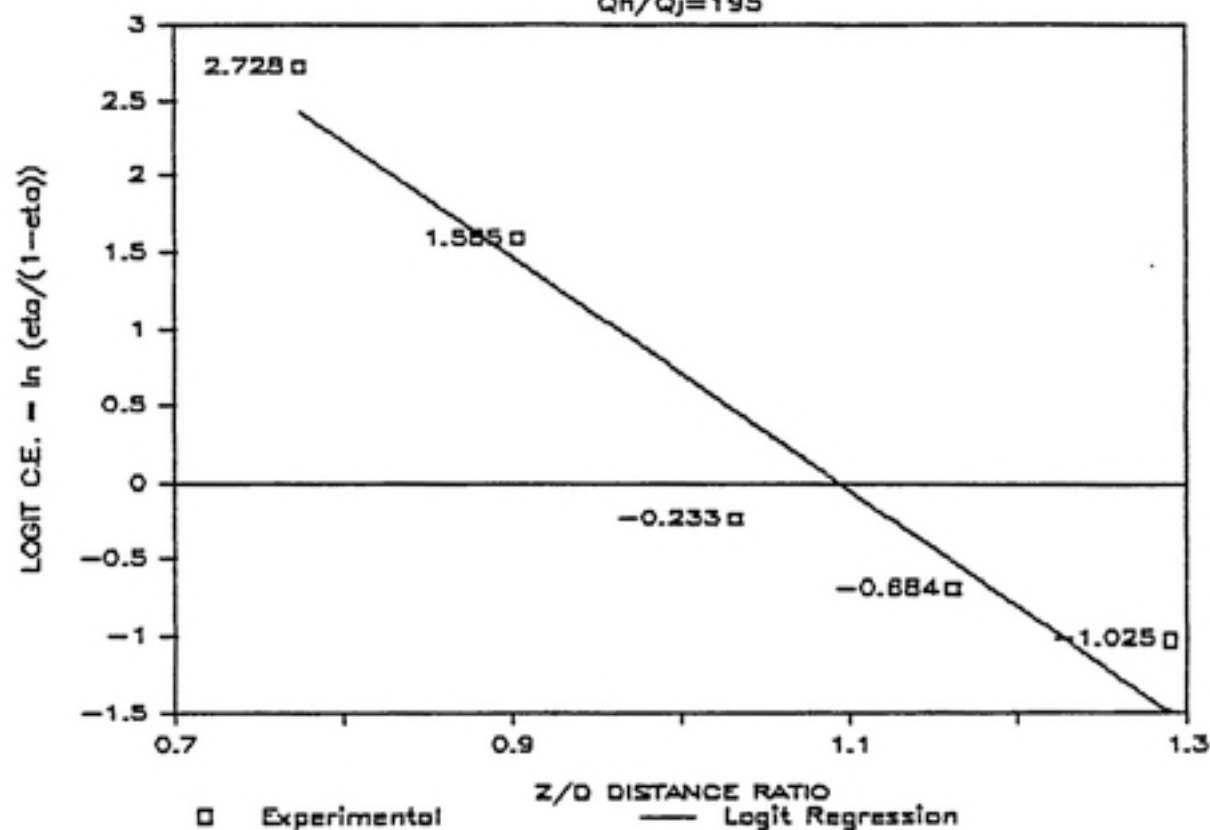
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=193$



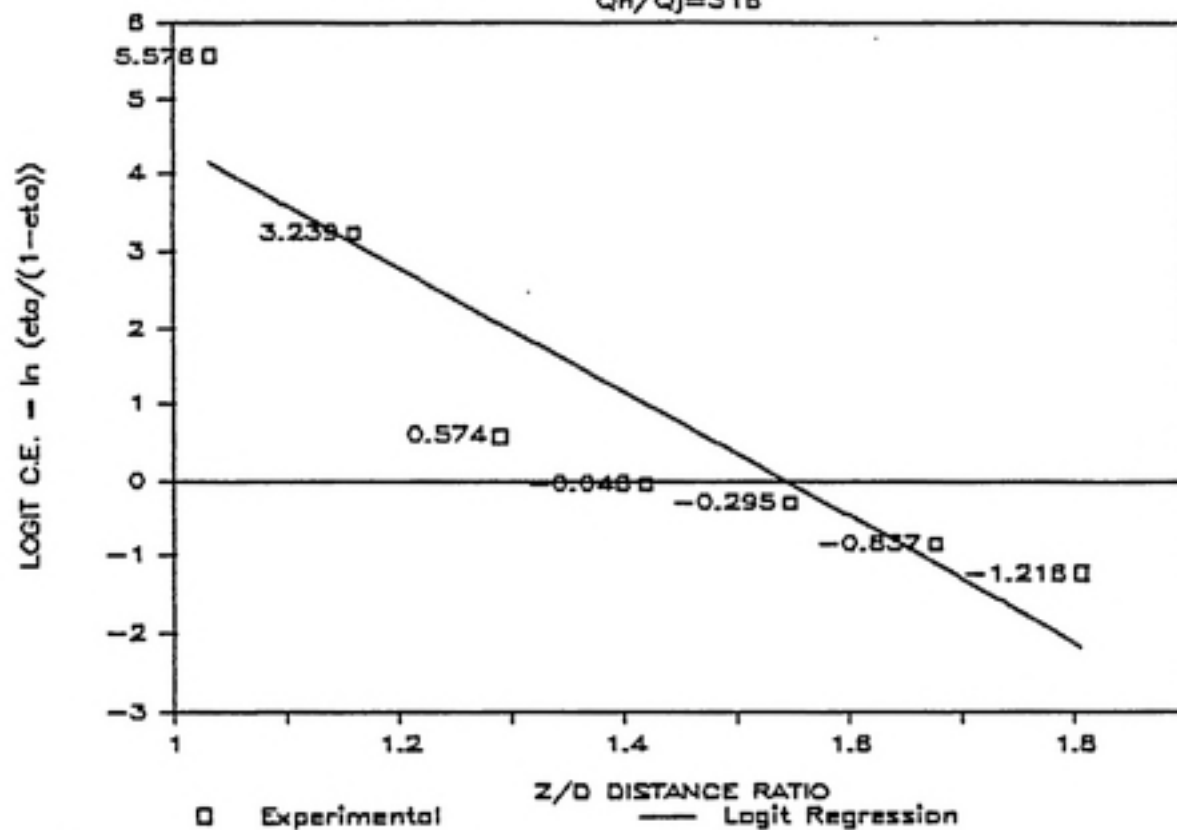
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=195$



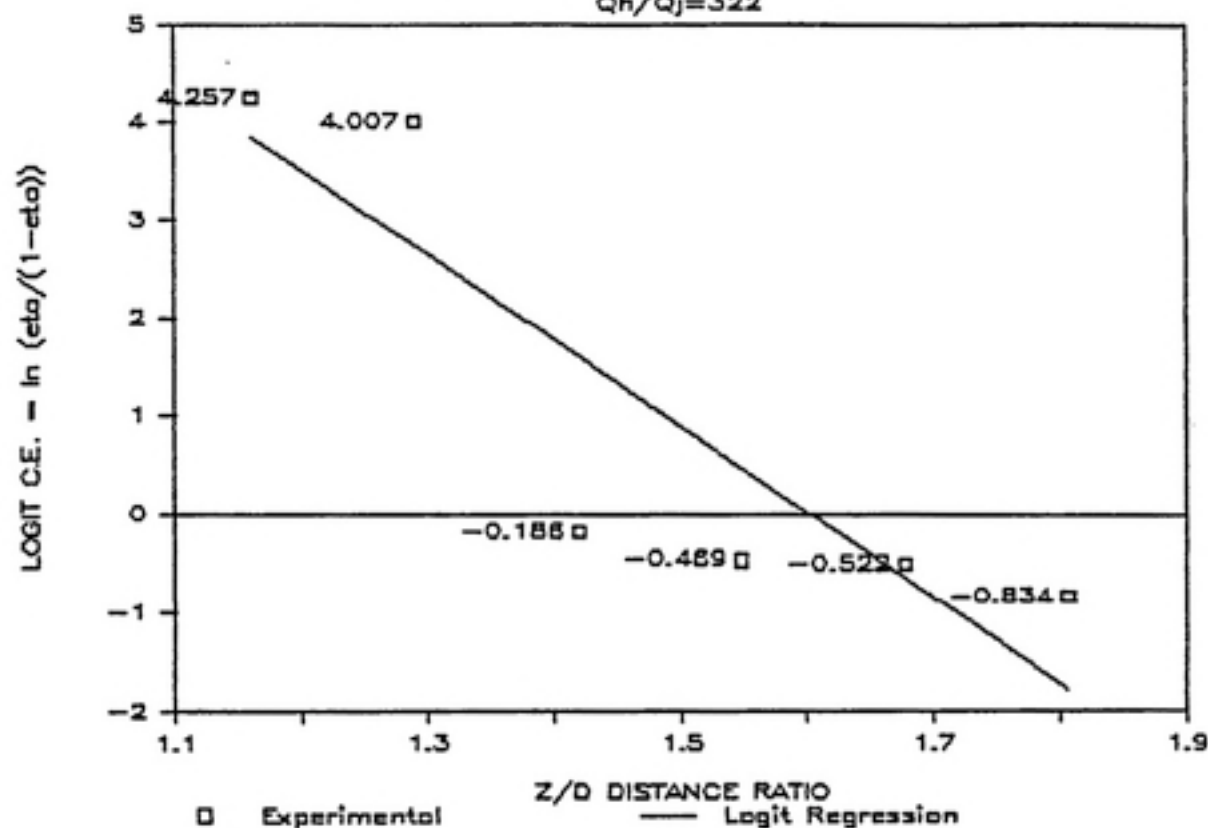
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j = 3.18$



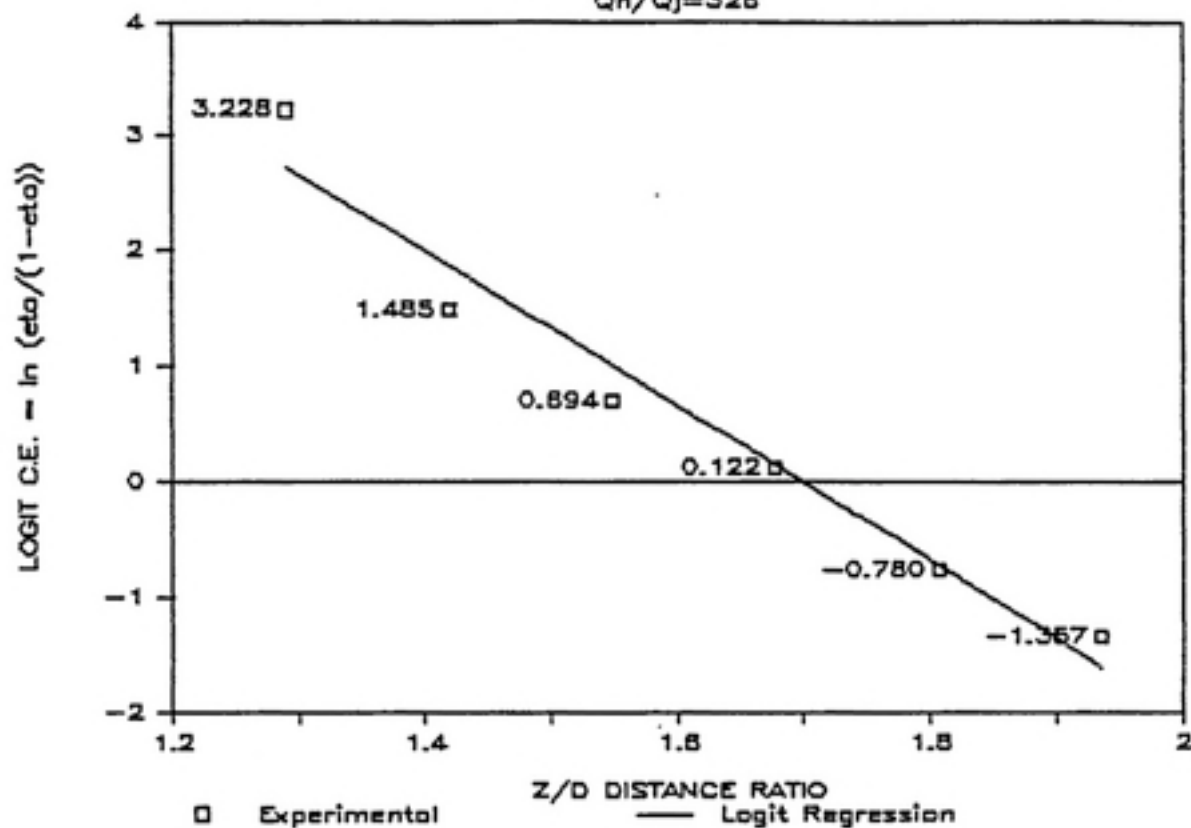
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=322$



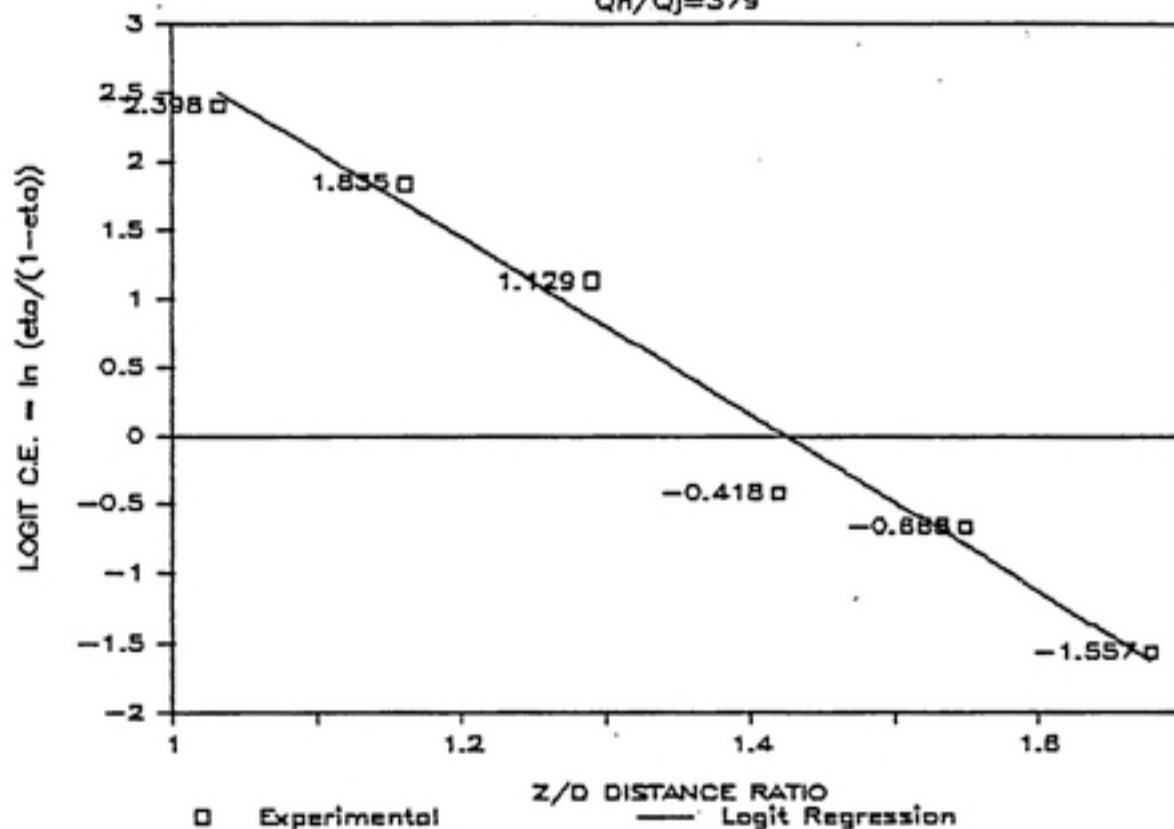
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=328$



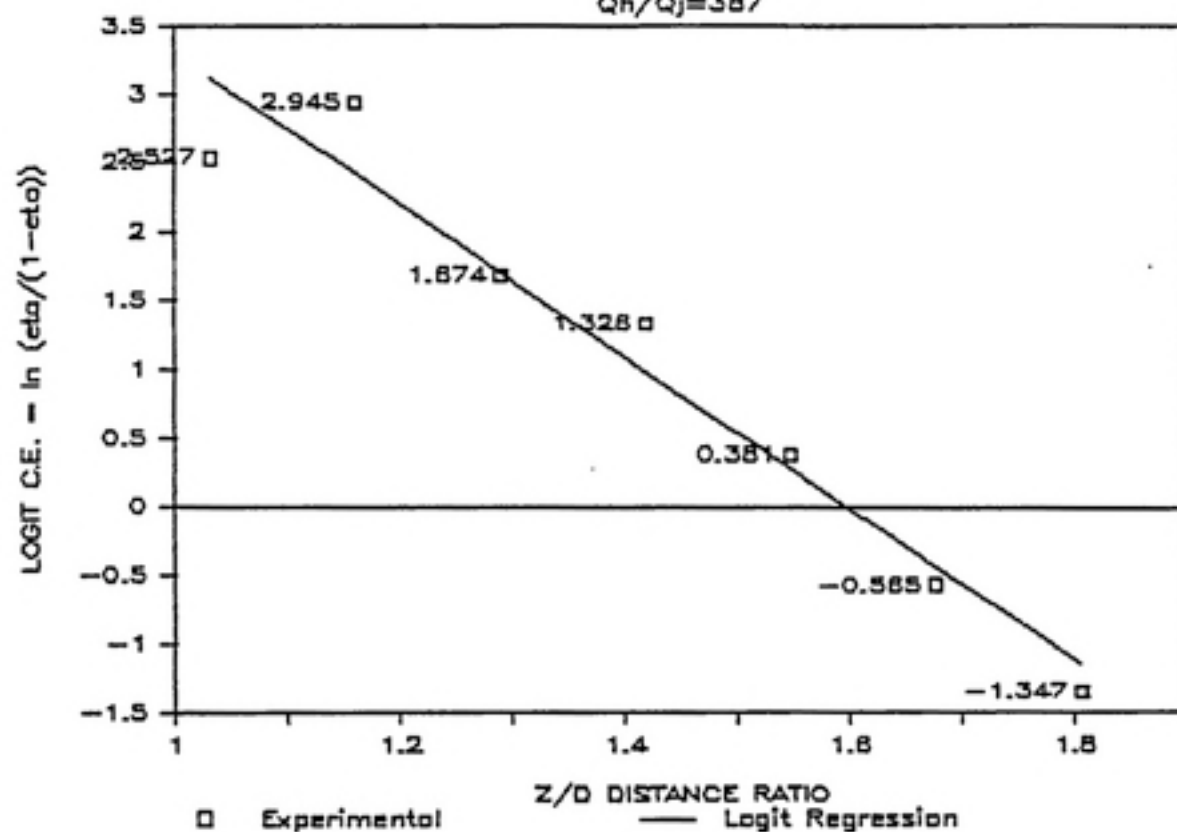
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=379$



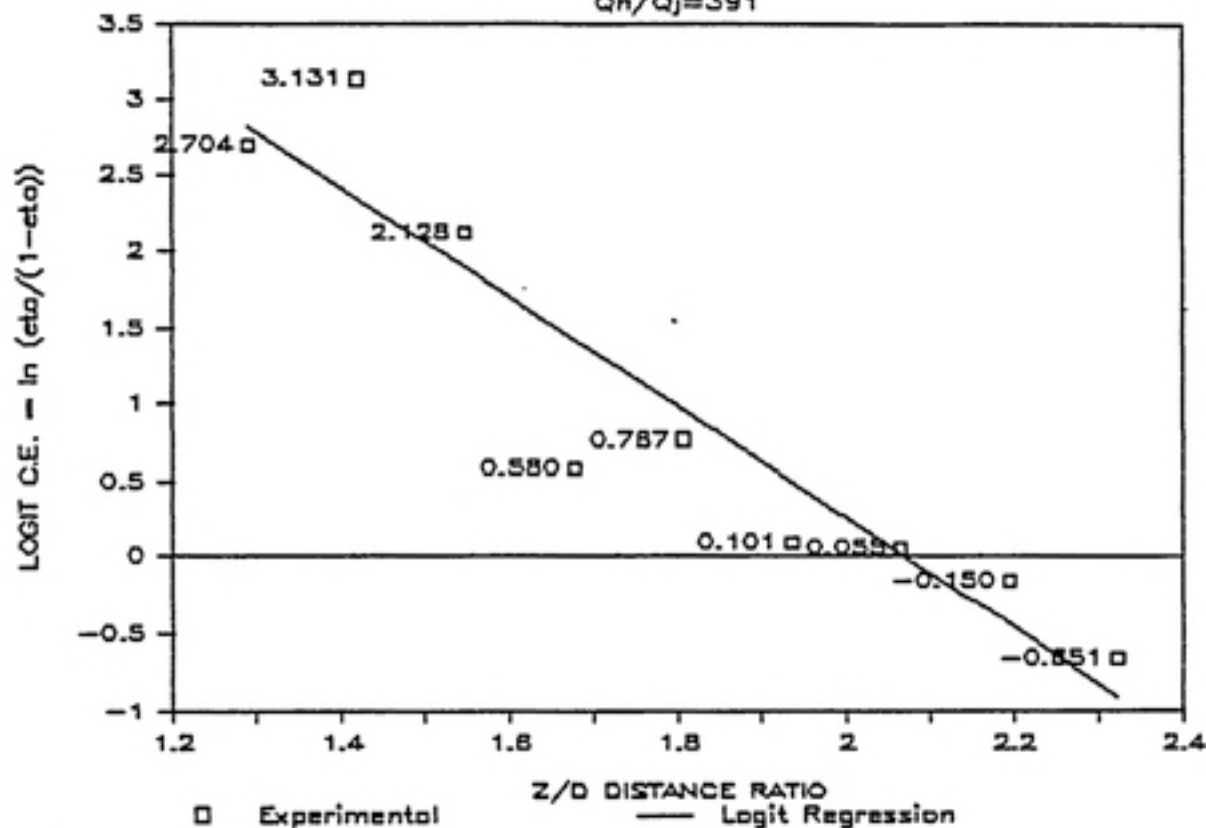
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=387$



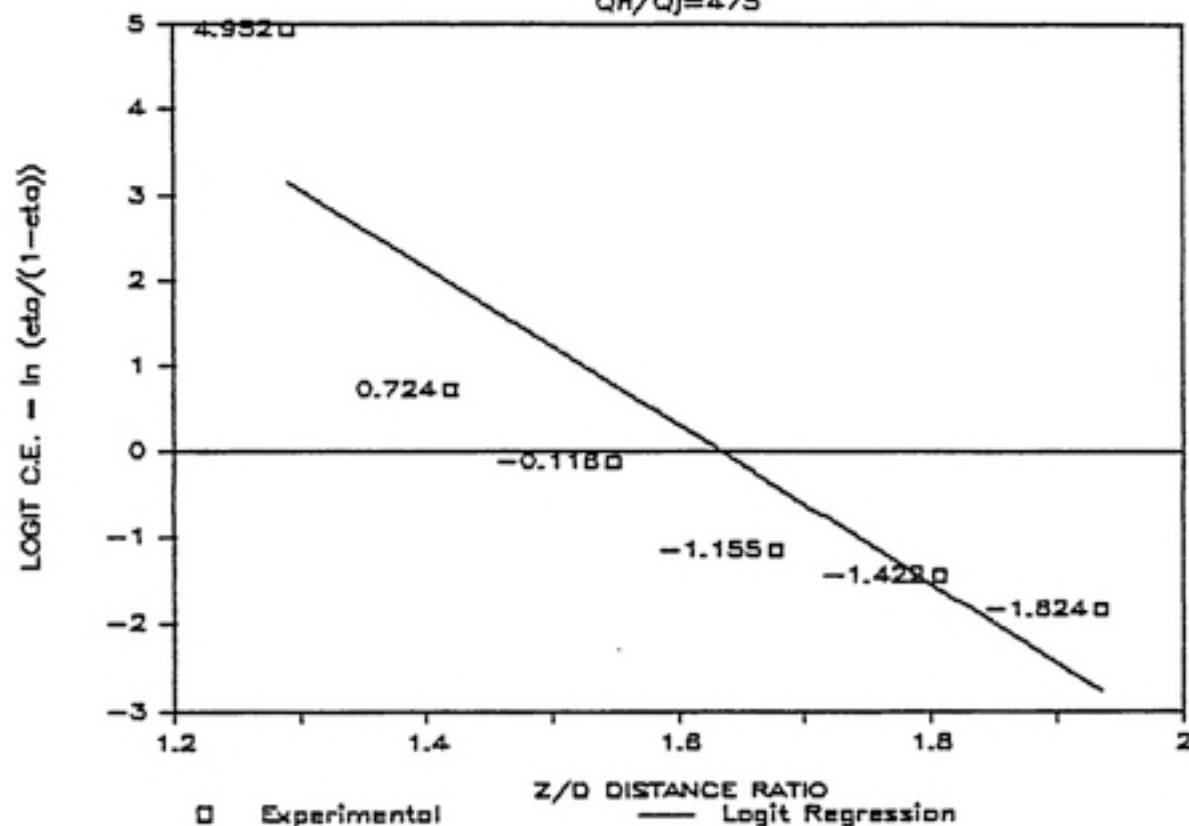
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=3.91$



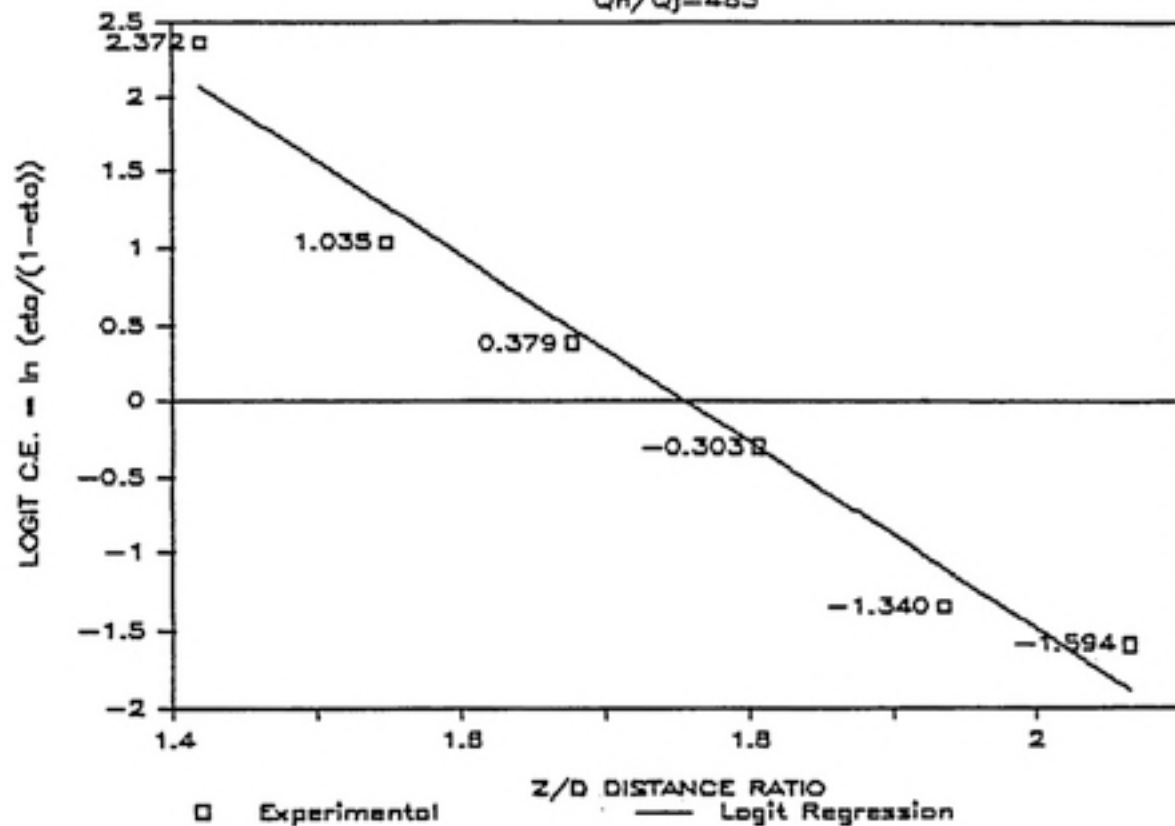
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=475$



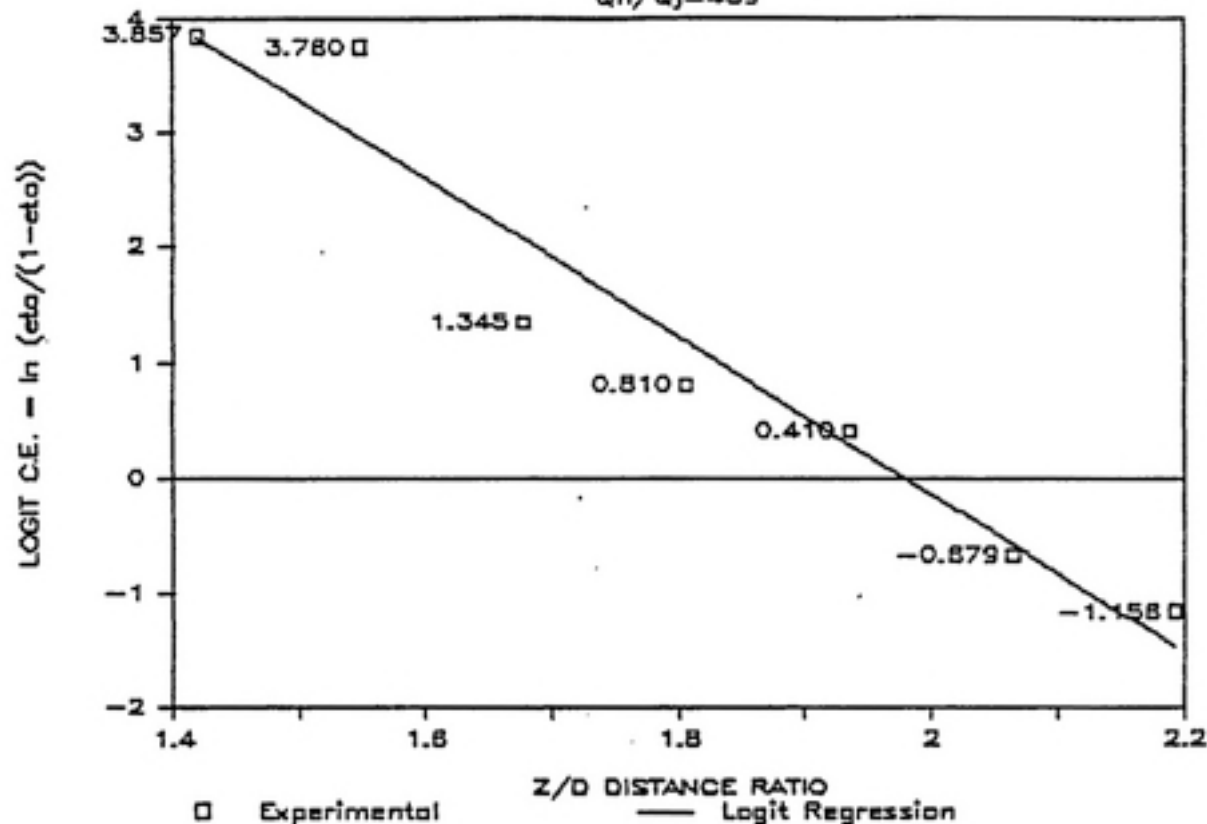
# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=483$



# Logit Capt.Effic. vs. Jet-Hood Distance

$Q_h/Q_j=489$



LOA BASH

THE AMERICAN BOMB

100% COLLECT

APPENDIX F

EXPERIMENTAL DATA: RESULTS SUMMARY

LOA BASH

# Experimental Results Data Summary

| Qh/Qj<br>ratio | Qh<br>cfm | Vh<br>fpm | Vh/25<br>Projected<br>Crossdra | Vh/ideal<br>or/25 | (Z/D)50<br>logit mu | (Z)50,<br>inches |       |
|----------------|-----------|-----------|--------------------------------|-------------------|---------------------|------------------|-------|
| 76             | 75        | 916       | 36.64                          | 25                | 36.64               | 0.9358           | 3.626 |
| 77             | 145       | 1770.5    | 70.82                          | 25                | 70.82               | 1.1147           | 4.319 |
| 78             | 220       | 2686      | 107.44                         | 25                | 107.44              | 1.0768           | 4.173 |
| 95             | 75        | 916       | 36.64                          | 25                | 36.64               | 0.8742           | 3.388 |
| 97             | 145       | 1770.5    | 70.82                          | 25                | 70.82               | 0.9414           | 3.648 |
| 98             | 220       | 2686      | 107.44                         | 25                | 107.44              | 0.9683           | 3.752 |
| 126            | 75        | 916       | 36.64                          | 5                 | 183.2               | 0.8582           | 3.326 |
| 129            | 145       | 1770.5    | 70.82                          | 24                | 73.77083            | 0.8619           | 3.340 |
| 130            | 220       | 2686      | 107.44                         | 32                | 83.9375             | 0.8955           | 3.470 |
| 189            | 75        | 916       | 36.64                          | 10                | 91.6                | 1.1445           | 4.435 |
| 193            | 145       | 1770.5    | 70.82                          | 21                | 84.30952            | 1.2091           | 4.685 |
| 195            | 220       | 2686      | 107.44                         | 59                | 45.52542            | 1.0945           | 4.241 |
| 316            | 75        | 916       | 36.64                          | 16                | 57.25               | 1.5417           | 5.974 |
| 322            | 145       | 1770.5    | 70.82                          | 33                | 53.65151            | 1.6031           | 6.212 |
| 326            | 220       | 2686      | 107.44                         | 25                | 107.44              | 1.6968           | 6.575 |
| 379            | 75        | 916       | 36.64                          | 19                | 48.21052            | 1.4253           | 5.523 |
| 387            | 145       | 1770.5    | 70.82                          | 28                | 63.23214            | 1.5987           | 6.195 |
| 391            | 220       | 2686      | 107.44                         | 17                | 158                 | 2.0742           | 8.038 |
| 475            | 75        | 916       | 36.64                          | 20                | 45.8                | 1.6336           | 6.330 |
| 483            | 145       | 1770.5    | 70.82                          | 27                | 65.57407            | 1.7568           | 6.808 |
| 489            | 220       | 2686      | 107.44                         | 29                | 92.62068            | 1.9799           | 7.672 |

# Experimental Results Data Summary

| w=-1/m<br>spread | (Z/D)50<br>modlogit | (Z)50,<br>modlogit" | 25fpmPred<br>inches | 25fpmPred<br>(Z/D)50 | 5fpmPred<br>Z,inches | 5fpmPred<br>(Z/D)50 | Qh/Qj<br>ratio |
|------------------|---------------------|---------------------|---------------------|----------------------|----------------------|---------------------|----------------|
| 0.168947         | 0.865               | 3.352               | 1.875               | 0.484                | 2.125                | 0.548               | 76             |
| 0.184945         | 1.1147              | 4.319               | 1.875               | 0.484                | 2.125                | 0.548               | 77             |
| 0.175254         | 1.03                | 3.991               | 1.875               | 0.484                | 2.125                | 0.548               | 78             |
| 0.077905         | 0.75                | 2.906               | 2.125               | 0.548                | 2.625                | 0.677               | 95             |
| 0.23635          | 0.9414              | 3.648               | 2.375               | 0.613                | 2.625                | 0.677               | 97             |
| 0.223613         | 0.92                | 3.565               | 2.625               | 0.677                | 2.625                | 0.677               | 98             |
| 0.089461         | 0.8582              | 3.326               | 2.875               | 0.742                | 3.625                | 0.935               | 126            |
| 0.13101          | 0.82                | 3.178               | 3.125               | 0.806                | 3.625                | 0.935               | 129            |
| 0.079327         | 0.84                | 3.255               | 3.375               | 0.871                | 3.625                | 0.935               | 130            |
| 0.076511         | 1.1445              | 4.435               | 3.625               | 0.935                | 5.125                | 1.323               | 189            |
| 0.174947         | 1.15                | 4.456               | 4.375               | 1.129                | 5.375                | 1.387               | 193            |
| 0.13196          | 1.03                | 3.991               | 4.625               | 1.194                | 5.375                | 1.387               | 195            |
| 0.122865         | 1.4                 | 5.425               | 4.875               | 1.258                | 7.625                | 1.968               | 316            |
| 0.11481          | 1.4                 | 5.425               | 5.875               | 1.516                | 8.125                | 2.097               | 322            |
| 0.149053         | 1.6968              | 6.575               | 6.625               | 1.710                | 8.375                | 2.161               | 326            |
| 0.156592         | 1.4253              | 5.523               | 5.125               | 1.323                | 8.375                | 2.161               | 379            |
| 0.181192         | 1.5987              | 6.195               | 6.375               | 1.645                | 9.125                | 2.355               | 387            |
| 0.277932         | 2.0742              | 8.038               | 7.375               | 1.903                | 9.625                | 2.484               | 391            |
| 0.10917          | 1.52                | 5.890               | 5.375               | 1.387                | 9.625                | 2.484               | 475            |
| 0.163585         | 1.7568              | 6.808               | 7.125               | 1.839                | 10.875               | 2.806               | 483            |
| 0.14556          | 1.9799              | 7.672               | 8.125               | 2.097                | 11.125               | 2.871               | 489            |

# Experimental Results Data Summary

| OfpmPred<br>Z, inches | OfpmPred<br>(Z/D)50 | Projected<br>Crossdraft | Vh/Vc<br>Projected | Z"for<br>PredXdrft | (Z/D)50<br>fromXdrft | Qh/Qj<br>ratio |
|-----------------------|---------------------|-------------------------|--------------------|--------------------|----------------------|----------------|
| 2.125                 | 0.548               | 25                      | 36.640             | 1.875              | 0.484                | 76             |
| 2.125                 | 0.548               | 25                      | 70.820             | 1.875              | 0.484                | 77             |
| 2.125                 | 0.548               | 25                      | 107.440            | 1.875              | 0.484                | 78             |
| 2.625                 | 0.677               | 25                      | 36.640             | 2.125              | 0.548                | 95             |
| 2.625                 | 0.677               | 25                      | 70.820             | 2.375              | 0.613                | 97             |
| 2.625                 | 0.677               | 25                      | 107.440            | 2.625              | 0.677                | 98             |
| 3.63                  | 0.937               | 5                       | 183.200            | 3.325              | 0.858                | 126            |
| 3.64                  | 0.939               | 24                      | 73.771             | 3.18               | 0.821                | 129            |
| 3.675                 | 0.948               | 32                      | 83.938             | 3.255              | 0.840                | 130            |
| 5.38                  | 1.388               | 10                      | 91.600             | 4.43               | 1.143                | 189            |
| 5.475                 | 1.413               | 21                      | 84.310             | 4.46               | 1.151                | 193            |
| 5.625                 | 1.452               | 59                      | 45.525             | 3.991              | 1.030                | 195            |
| 8.52                  | 2.199               | 16                      | 57.250             | 5.425              | 1.400                | 316            |
| 8.69                  | 2.243               | 33                      | 53.652             | 5.425              | 1.400                | 322            |
| 8.725                 | 2.252               | 25                      | 107.440            | 6.575              | 1.697                | 326            |
| 9.8                   | 2.529               | 19                      | 48.211             | 5.52               | 1.425                | 379            |
| 9.95                  | 2.568               | 28                      | 63.232             | 6.195              | 1.599                | 387            |
| 10.125                | 2.613               | 17                      | 158.000            | 8.04               | 2.075                | 391            |
| 11.575                | 2.987               | 20                      | 45.800             | 5.89               | 1.520                | 475            |
| 11.625                | 3.000               | 27                      | 65.574             | 6.81               | 1.757                | 483            |
| 11.875                | 3.065               | 29                      | 92.621             | 7.67               | 1.979                | 489            |

APPENDIX G

"EPSILON" MODIFICATION OF COMPUTER PROGRAM

10X BITE

12

UNCLASSIFIED

CONFIDENTIAL

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10 'Title: EPSILON
20 'Author: This program was prepared by Kirby N. Smith.
30 'Date: This program was finalized in March, 1988.
40 'Purpose: To evaluate the effect of sequential changes in epsilon sub-zero
              on the calculation of jet flow.
50 'Note: The equations of the Prandtl mixing length hypothesis,
              which takes viscosity into account, describe the jet flow.
60 'Note: The line of 50% Capture Efficiency can be determined as a function
              of the dimensionless variables (Qh/Qj) and (Z/D)50, and of  $\alpha$ -o.

70 CLS
80 SCREEN 0
90 COLOR 5
100 PRINT "EPSILON RUNNING--PLEASE STAND BY."
110 DIM P(600,2)
120 PI=3.1415927#
130 DH=3.875          ''''Hood diameter in inches.
140 RH=DH/2
150 A=RH/12           ''''A is the hood radius in feet.
160 AH=PI*(A^2)
170 JD=.25           ''''Jet diameter in inches.
180 DJ=JD/12
190 RJ=DJ/2
200 AJ=PI*(RJ)^2
210 FOR MM=1 TO 21
220 IF MM=1 THEN Q=220
230 IF MM=2 THEN Q=145
240 IF MM=3 THEN Q=75
250 IF MM=4 THEN Q=220
260 IF MM=5 THEN Q=145
270 IF MM=6 THEN Q=75
280 IF MM=7 THEN Q=220
290 IF MM=8 THEN Q=145
300 IF MM=9 THEN Q=75
310 IF MM=10 THEN Q=220
320 IF MM=11 THEN Q=145
330 IF MM=12 THEN Q=75
340 IF MM=13 THEN Q=220
350 IF MM=14 THEN Q=145
360 IF MM=15 THEN Q=75
370 IF MM=16 THEN Q=220
380 IF MM=17 THEN Q=145
390 IF MM=18 THEN Q=75
400 IF MM=19 THEN Q=220
410 IF MM=20 THEN Q=145
420 IF MM=21 THEN Q=75
430 IF MM=1 THEN QJ=Q/489
440 IF MM=2 THEN QJ=Q/483
450 IF MM=3 THEN QJ=Q/475
460 IF MM=4 THEN QJ=Q/391
470 IF MM=5 THEN QJ=Q/387
480 IF MM=6 THEN QJ=Q/379
490 IF MM=7 THEN QJ=Q/326
500 IF MM=8 THEN QJ=Q/322
510 IF MM=9 THEN QJ=Q/316
520 IF MM=10 THEN QJ=Q/195
530 IF MM=11 THEN QJ=Q/193
540 IF MM=12 THEN QJ=Q/189
550 IF MM=13 THEN QJ=Q/130
560 IF MM=14 THEN QJ=Q/129
570 IF MM=15 THEN QJ=Q/126
580 IF MM=16 THEN QJ=Q/98
590 IF MM=17 THEN QJ=Q/97
600 IF MM=18 THEN QJ=Q/95
610 IF MM=19 THEN QJ=Q/78
620 IF MM=20 THEN QJ=Q/77

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630 IF MM=21 THEN QJ=Q/76
640 IF MM=1 THEN XJFIN=7.67
650 IF MM=2 THEN XJFIN=6.81
660 IF MM=3 THEN XJFIN=5.89
670 IF MM=4 THEN XJFIN=8.04
680 IF MM=5 THEN XJFIN=6.195
690 IF MM=6 THEN XJFIN=5.52
700 IF MM=7 THEN XJFIN=6.575
710 IF MM=8 THEN XJFIN=5.425
720 IF MM=9 THEN XJFIN=5.425
730 IF MM=10 THEN XJFIN=3.99
740 IF MM=11 THEN XJFIN=4.46
750 IF MM=12 THEN XJFIN=4.43
760 IF MM=13 THEN XJFIN=3.255
770 IF MM=14 THEN XJFIN=3.18
780 IF MM=15 THEN XJFIN=3.325
790 IF MM=16 THEN XJFIN=3.57
800 IF MM=17 THEN XJFIN=3.65
810 IF MM=18 THEN XJFIN=2.91
820 IF MM=19 THEN XJFIN=4
830 IF MM=20 THEN XJFIN=4.32
840 IF MM=21 THEN XJFIN=3.35
850 IF MM=1 THEN GG=-2
860 IF MM=2 THEN GG=-1
871 IF MM=3 THEN GG=0
880 IF MM=4 THEN GG=0
890 IF MM=5 THEN GG=-1
900 IF MM=6 THEN GG=0
910 IF MM=7 THEN GG=-1
919 IF MM=8 THEN GG=-2
930 IF MM=9 THEN GG=0
939 IF MM=10 THEN GG=-2
950 IF MM=11 THEN GG=0
960 IF MM=12 THEN GG=0
970 IF MM=13 THEN GG=-1
979 IF MM=14 THEN GG=0
989 IF MM=15 THEN GG=0
999 IF MM=16 THEN GG=0
1009 IF MM=17 THEN GG=0
1019 IF MM=18 THEN GG=0
1029 IF MM=19 THEN GG=0
1040 IF MM=20 THEN GG=1
1041 IF MM=21 THEN GG=0
1042 IF MM=1 THEN FF=4
1043 IF MM=2 THEN FF=4
1044 IF MM=3 THEN FF=4
1045 IF MM=4 THEN FF=3
1046 IF MM=5 THEN FF=4
1047 IF MM=6 THEN FF=4
1048 IF MM=7 THEN FF=9
1049 IF MM=8 THEN FF=1
1050 IF MM=9 THEN FF=5
1051 IF MM=10 THEN FF=1
1052 IF MM=11 THEN FF=2
1053 IF MM=12 THEN FF=4
1054 IF MM=13 THEN FF=9
1055 IF MM=14 THEN FF=2
1056 IF MM=15 THEN FF=3
1057 IF MM=16 THEN FF=4
1058 IF MM=17 THEN FF=5
1059 IF MM=18 THEN FF=4
1060 IF MM=19 THEN FF=8
1061 IF MM=20 THEN FF=2
1062 IF MM=21 THEN FF=8
1064 FOR DD=.1 TO .9 STEP .2
1070 VH=Q/AH

```

```

1080 VJ=DJ/AJ
1090 RATIO=D/DJ
1100 YJFIN=0
1110 YJF=YJFIN/12
1120 XJF=XJFIN/12
1130 DIR=90
1140 RADIANS=PI*DIR/180
1150 XDFA=90
1160 RDXDF=PI*XDFA/180
1171 VC=25
1180 INC=.03
1190 'Note: The following three equations set the angle of the image jet.
1200 IF RADIANS=0 THEN RADIM=PI
1210 IF 0<RADIANS<PI THEN RADIM=PI-RADIANS.
1220 IF PI<RADIANS<2*PI THEN RADIM=(3*PI)-RADIANS
1230 ' Note: The following equations set the angle of the image crossdraft.
1240 IF RDXDF=0 THEN XDFIM=PI
1250 IF 0<RDXDF<PI THEN XDFIM=PI-RDXDF
1260 IF PI<RDXDF<2*PI THEN XDFIM=(3*PI)-RDXDF
1270 'Note: The following equations calculate the vectors for each increment
      of hood flow.
1280 'Note: Hood coordinates Z and R are named in hood flow calculations
      as X and Y, respectively.
1290 X=XJF
1300 Y=YJF
1310 FOR I1=1 TO 600
1320 GAMMA1=SQR(X^2+(A+Y)^2)
1330 GAMMA2=SQR(X^2+(A-Y)^2)
1340 ECC=(2*A)/(GAMMA1+GAMMA2)
1350 ECC2=ECC^2
1360 T1=A+Y
1370 T2=Y-A
1380 T3=GAMMA1+GAMMA2
1390 T4=GAMMA1*GAMMA2
1400 T5=4*A^2
1410 T6=SQR(T3^2-T5)
1420 T7=-Q/PI
1430 T8=(T1*GAMMA2)+(T2*GAMMA1)
1440 T9=T3*T4*T6
1450 VR1=(T8/T9)*T7
1460 VZ1=(T7*X)/(T4*T6)
1470 V=SQR(VR1^2+VZ1^2)
1480 VTF=(Q*ECC2*SQR(3))/(2*PI*A^2*SQR(3-2*ECC2))
1490 VR2=(VR1/V)*VTF
1500 VZ2=(VZ1/V)*VTF
1510 VRC=(2.6*ECC^18+.853)*VR2
1520 VZC=.9*VZ2
1530 'Note: The following equations transform the coordinates of the jet and
      its image to the coordinates of the hood.
1540 'Note: The virtual point in the Prandtl mixing length hypothesis of jet
      flow is taken as the actual origin of the jet.
1550 IF DIR=0 OR DIR=360 THEN GOTO 1560 ELSE 1610
1560 Z=X-XJF+(1.86*DJ)
1570 R=YJF-Y
1580 ZI=2*(1.86*DJ)-(2*XJF)-Z
1590 RI=-R
1600 GOTO 1790
1610 IF DIR=90 THEN GOTO 1620 ELSE GOTO 1670
1620 Z=Y-YJF+(1.86*DJ)
1630 R=X-XJF
1640 ZI=Z
1650 RI=(2*XJF)+R
1660 GOTO 1790
1670 IF DIR=180 THEN GOTO 1680 ELSE GOTO 1730
1680 Z=XJF-X+(1.86*DJ)
1690 R=Y-YJF

```

```

1700 Z1=2*((1.86*DJ)+XJF)-Z
1710 RI=-R
1720 GOTO 1790
1730 IF DIR=270 THEN GOTO 1740 ELSE PRINT "DIRECTION"
1740 Z=YJF-Y+(1.86*DJ)
1750 R=XJF-X
1760 Z1=Z
1770 RI=R-(2*XJF)
1780 'Note: The following equations calculate the vectors for any given R and
      Z points of the jet flow.
1790 K=AJ*(VJ)^2
1800 EO=.0161*SQR(K)*(FF-1+DD)*10^GG
1810 F=(FF-1+DD)*10^GG
1820 ETA=(R*SQR(3*K))/(4*(EO)*Z*SQR(PI))
1830 ETA1=ETA-((ETA^3)/4)
1840 ETA2=(1+((ETA^2)/4))^2
1850 IF -.00001<ETA AND ETA<.00001 OR -2.00001<ETA AND ETA<-1.99999 OR
      1.99999<ETA AND ETA<2.00001 THEN GOTO 1860 ELSE 1880
1860 VRJ=0
1870 GOTO 1890
1880 VRJ=(SQR(3*K)*ETA1)/(4*(ETA2)*Z*SQR(PI))
1890 VZJ=3*K/(8*PI*(EO)*Z*ETA2)
1900 IF VZJ<0 THEN VZJ=0 AND VRJ=0
1910 'Note: The following calculations are for the effect of the image jet.
      field on the flow of the actual jet into the hood. Note that
      K and epsilon-sub-zero have already been calculated for the jet.
1920 ETAI=(RI*SQR(3*K))/(4*EO*Z1*SQR(PI))
1930 ETAI1=ETAI-((ETAI^3)/4)
1940 ETAI2=(1+((ETAI^2)/4))^2
1950 IF -.00001<ETAI AND ETAI<.00001 OR -2.00001<ETAI AND ETAI<-1.99999 OR
      1.99999<ETAI AND ETAI<2.00001 THEN GOTO 1960 ELSE 1980
1960 VRIM=0
1970 GOTO 1990
1980 VRIM=(SQR(3*K)*ETAI1)/(4*ETAI2*Z1*SQR(PI))
1990 VZIM=3*K/(8*PI*EO*Z1*ETAI2)
2000 IF VZIM<0 THEN VZIM=0 AND VRIM=0
2010 'Note: The following equations add the vectors and then determine
      the next point for vector calculation based on the combined
      values and the desired increment of calculation.
2020 VRTOT=VRC+(COS(RADIANS)*VRJ)+(COS(RADIM)*VRIM)+(SIN(RADIANS)*VZJ)+
      (SIN(RADIM)*VZIM)+(SIN(RDXDF)*VC)+(SIN(XDFIM)*VC)
2030 VZTOT=VZC+(COS(RADIANS)*VZJ)+(COS(RADIM)*VZIM)+(SIN(RADIANS)*VRJ)+
      (SIN(RADIM)*VRIM)+(COS(RDXDF)*VC)+(COS(XDFIM)*VC)
2040 IF VZTOT<=-.00001 THEN GOTO 2050 ELSE GOTO 2070
2050 ANGLE=ATN(VRTOT/VZTOT)+PI
2060 GOTO 2150
2070 IF VZTOT>=.00001 THEN GOTO 2080 ELSE GOTO 2100
2080 ANGLE=ATN(VRTOT/VZTOT)
2090 GOTO 2150
2100 IF -.00001<VZTOT AND VZTOT<.00001 AND VRTOT<0 THEN GOTO 2110 ELSE 2130
2110 ANGLE=3*PI/2
2120 GOTO 2150
2130 IF -.00001<VZTOT AND VZTOT<.00001 AND VRTOT=0 THEN GOTO 2140
2140 ANGLE=PI/2
2150 X=X+(COS(ANGLE)*INC)
2160 Y=Y+(SIN(ANGLE)*INC)
2170 P(II,1)=X
2180 P(II,2)=Y
2190 IF X<=.01 OR X=1 OR Y<=-1 OR Y=1 THEN II=600 ELSE COUNT=II+1
2200 'Note: A repetitive cycle is programmed to calculate the new
      magnitude and direction of flow at each subsequent increment.
2210 NEXT II
2220 'Note: The following section is the instructions to the computer
      to draw the hood, and to display the flow into it.
2230 CLS
2240 SCREEN 1

```

```

2250 COLOR 16,0
2260 WINDOW (-1.6,-1.1)-(1.6,1.1)
2270 LINE (0,A)-(0,1),2
2280 LINE (0,A)-(-.05,A),2
2290 LINE (-.05,0)-(1,0),3
2300 LINE (0,-A)-(0,-1),2
2310 LINE (0,-A)-(-.05,-A),2
2320 FOR JJ=1 TO COUNT
2330 G=P(JJ,1)
2340 H=P(JJ,2)
2350 PSET (G,H),1
2360 NEXT JJ
2370 KEY OFF
2380 LOCATE 8,4:PRINT "QH/QJ=";RATIO
2390 LOCATE 9,4:PRINT "QH=";Q"CFM"
2400 LOCATE 10,4:PRINT "JETX=";XJFIN"IN"
2410 LOCATE 17,4:PRINT "VZJ=";VZJ"FPM"
2420 LOCATE 16,4:PRINT "VJ=";VJ"FPM"
2430 LOCATE 15,4:PRINT "(Z/D)=";XJF/(2*A)
2440 LOCATE 18,4:PRINT "epsilon-0=";EO
2450 LOCATE 19,4:PRINT "F=";F
2460 PRINT
2470 PRINT
2480 INPUT "BREAKPT?(Y/N)";ANS
2490 IF ANS="Y" OR ANS="y" THEN GOTO 2510 ELSE GOTO 2630
2500 PRINT "BREAKPT FILE"
2510 OPEN "EPSICALC.DAT" FOR APPEND AS #1
2520 WRITE #1, RATIO, Q, VJ, VZJ, XJFIN, XJF/(2*A), EO, F
2530 CLOSE #1
2540 LPRINT "QH/QJ=";RATIO
2550 LPRINT "QH=";Q"CFM"
2560 LPRINT "VJ=";VJ"FPM"
2570 LPRINT "VZJ=";VZJ"FPM"
2580 LPRINT "JETX=";XJFIN"IN"
2590 LPRINT "(Z/D)S0=";XJF/(2*A)
2600 LPRINT "epsilon-0=";EO
2610 LPRINT "eoFACTOR=";F
2620 LPRINT "-----"
2630 CLS
2640 " " " " " " SCREEN 0
2650 " " " " " " COLOR 4
2660 " " " " " " INPUT "ANOTHER?(Y/N)";ANN$
2670 " " " " " " IF ANN$="Y" OR ANN$="y" THEN GOTO 1977 ELSE GOTO 1970
2680 " " " " " " PRINT "END OF PROGRAM"
2690 " " " " " " SYSTEM
2700 " " " " " " END
2710 SCREEN 0
2720 COLOR 5
2730 PRINT "EPSILON LOOPS RUNNING--PLEASE STAND BY."
2741 NEXT DD
2750 NEXT MM
2760 CLS
2770 COLOR 3
2780 PRINT "END OF EPSILON LOOPS."
2790 OPEN "EPSICALC.DAT" FOR INPUT AS #2
2800 LPRINT" EPSICALC DATA DATE"
2810 LPRINT" "
2820 LPRINT" "
2830 LPRINT "QH/QJ", "QH", "VJ", "VZJ", "XJFIN", "ZD50", "E-0", "eoFACTOR"
2840 LPRINT"-----"
2850 IF EOF(2) THEN CLOSE #2:SYSTEM
2860 INPUT #2, RATIO, Q, VJ, VZJ, XJFIN, XJF/(2*A), EO, F
2870 LPRINT RATIO, Q, VJ, VZJ, XJFIN, XJF/(2*A), EO, F
2880 GOTO 2850
Ok
system

```