

BANK CAPITAL REGULATION AND SYSTEMIC RISK IN THE PRESENCE OF ENDOGENOUS FIRE SALES

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ABSTRACT

SAMUEL ROSEN: Bank Capital Regulation and Systemic Risk in the Presence of Endogenous Fire Sales.

(Under the direction of Mariano M. Croce)

In a model with heterogeneous banks and endogenous fire sales, the tightening of bank capital regulation can aggravate fire sales, leading to larger bank losses and higher systemic risk. When calibrated to the data, the least costly policies to mitigate systemic risk raise both ex ante capital requirements and ex post shortfall penalties. These policies also assign relatively higher capital requirements to banks that can better offset price declines during a fire sale, consistent with the recently implemented capital surcharge for global systemically important banks (G-SIBs). My findings provide further support for leading-edge macroprudential tools, including stress tests and countercyclical capital buffers.

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CHAPTER 1

INTRODUCTION

In my dissertation, I investigate the channels through which tighter bank capital regulation affects the real costs of a financial crisis when fire sales are determined endogenously. The motivation for this topic is twofold. First, regulators responded to the financial crisis of 2007–2009 with the “most significant reregulation of the banking industry since Glass-Steagall.”¹ These reforms are intended to reduce the severity of the next financial crisis, and many involve tighter regulation of bank capital. Second, many papers document and argue that fire sales are an important contributor to the severity of a financial crisis through their ability to propagate losses throughout the financial system (Brunnermeier 2009; Hanson, Kashyap, and Stein 2011; Shleifer and Vishny 2011; Duarte and Eisenbach 2015). However, no paper to my knowledge has modeled the interplay between capital regulation and fire sales. The main contribution of this paper is to show the significant impact of this connection for policy analysis.

In the first step of my analysis, I develop a three-period model with heterogeneous banks and a regulator. Banks choose their asset portfolios anticipating a potential crisis. Conditional upon the crisis state, banks may choose to sell assets. These selling decisions can cause market prices to decline because of limited potential buyers, and I refer to this outcome as a fire sale. The severity of the fire sale is endogenous because it depends on the volume sold. Banks are heterogeneous in their ability to liquidate assets during a crisis, which affects both optimal portfolio and selling decisions. As a result, the cross section of banks is a key input in determining equilibrium outcomes.

The regulator can influence bank behavior through two capital-based policy tools: *ex ante*

¹A description of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 from a February 13, 2012, letter written by the Chief Investment Officer of the California Public Employees’ Retirement System (CalPERS) to all regulatory agencies.

capital requirements and an ex post penalty. The ex post penalty is increasing function of the amount by which a bank violates its requirement, commonly known as its capital “shortfall.” The regulator’s objective is to reduce systemic risk, which is a measure that indirectly captures the real costs of a crisis. I define systemic risk according to the level of aggregate equity capital in the banking sector during a crisis state.² Through its capital-based tools, the regulator can ensure that aggregate bank capital remains high during a crisis and therefore reduce systemic risk.

In the second step, I calibrate the model using data from regulatory filings and perform quantitative policy analysis. As part of this process, I also document novel stylized facts about banks’ selling behavior during the financial crisis. Using the calibrated model, I assess the effects from tightening regulatory parameters and solve for the least costly policies to mitigate systemic risk.

The key mechanism behind my results is found in the optimal selling decision of a bank. When a bank sells an asset in a fire sale, it incurs a *realized* loss. This loss occurs because the bank is selling while the price declines, so it effectively sells at a weighted average price between the asset’s initial and fire sale values. If a bank does not sell an asset that experiences a price decline, it incurs an *unrealized* loss from marking down the value on its balance sheet (i.e., mark-to-market accounting). There is a benefit to selling while the asset price is declining, because a realized loss is necessarily smaller than an unrealized loss. A bank optimally decides to sell if this opportunity to minimize the loss is large enough. Many factors affect the size of this benefit, including regulation and the specific weighted average price at which the bank can sell.

My analysis delivers three main results. First, I show the existence of an endogenous fire sale channel through which tighter capital regulation can unintentionally lead to higher systemic risk. Undercapitalized banks sell securities to improve their capital ratio in order to reduce ex post shortfall penalties. Tighter regulation effectively increases this benefit and therefore makes the decision to sell more attractive, all else held equal. If more banks sell, the fire sale externality sharpens, and system-wide losses increase. For this endogenous fire sale channel to materialize,

²Researchers have proposed many alternative definitions and measures for systemic risk, and currently there is no consensus definition (cf. Allen and Carletti 2013; Bisias, Flood, Lo, and Valavanis 2012; Hansen 2012). The measure I have chosen to use is based on a specific definition of systemic risk from the literature; I elaborate on the underlying rationale for this measure in in section 2.3.

the probability of a crisis must be sufficiently small so that banks do not hold a capital buffer to avoid selling assets.

Second, in my quantitative analysis, I show that the least costly policies to mitigate systemic risk raise both ex ante capital requirements and ex post shortfall penalties. This result highlights that these regulatory tools provide systemic risk reduction with different cost-benefit tradeoffs. Capital requirements are costly to banks because they are applied ex ante and force a bank to hold additional capital in both noncrisis and crisis states of the world. Ex post penalties, on the other hand, are generally less costly to banks because the penalty is only applied in the crisis state (i.e., the penalty is state contingent). However, banks find it increasingly expensive to adjust their portfolios in response to larger penalties. As a result, raising capital requirements eventually becomes the less costly option for achieving further systemic risk reduction.

Third, these least costly risk-mitigation policies assign relatively higher capital requirements to banks that can better offset price declines during a fire sale. This “capital surcharge,” which is 6% to 8% depending on parameter assumptions, is the result of bank heterogeneity and the regulator’s objective. I assume banks fundamentally differ in the effective price they receive when selling during a fire sale, an assumption motivated by the data and empirical evidence from the literature. Banks that can sell at a lower discount can generally be described as better able to offset price declines. A simpler and more specific interpretation is that some banks can sell faster during periods of rapid price decline. “Slow” banks incur larger realized losses when selling. As a result, a higher ex post shortfall penalty is more costly to these banks. Therefore, in order to make up for the disparity in costliness from increasing the ex post penalty, the regulator optimally raises capital requirements only for “fast” banks. Intuitively, this capital surcharge compensates for the fact that, through their selling behavior, “fast” banks impose losses on other banks by causing larger price declines while incurring smaller realized losses themselves.

My findings have two broad implications for current and future regulatory policy. First, my quantitative results are consistent with the recently implemented enhanced prudential standards for institutions deemed “systemically important” and, in particular, the capital surcharge for the global systemically important banks (G-SIBs). The G-SIBs correspond to the “fast” banks in my

calibration, which are assigned higher capital requirements under the least costly policies. Therefore, my results provide a fire-sale-based justification for the current regulatory regime. Second, my findings provide further support for leading-edge macroprudential tools, including stress tests and countercyclical capital buffers (CCyBs). Their key potential advantage is to reduce systemic risk without triggering the endogenous fire sale channel.

The model in this paper is closely related to the recent literature that seeks to measure systemic risk generated through fire sales (sometimes denoted “indirect contagion”). Greenwood, Landier, and Thesmar (2015) (henceforth GLT) describe a framework in which the effects from an initial shock can be traced throughout the banking system in terms of selling that both generates and amplifies losses. Cont and Schaanning (2017) modify the GLT framework to feature asymmetric selling. They argue that fire sales arise only when one-sided portfolio constraints are breached as a result of large portfolio losses, which implies an asymmetric reaction of banks to gains and losses. These models, however, differ from mine along two relevant dimensions: they assume that asset portfolios are exogenous and also that banks follow prespecified rules in making their selling decisions. Building upon a similar fire sale framework, I endogenize both the ex ante portfolio choice and distressed selling decision. After doing so, the connection between policy and systemic risk becomes clear.

My paper is also related to the theoretical literature that analyzes endogenous fire sales in the banking sector (e.g., Allen and Gale 2004; Acharya, Shin, and Yorulmazer 2011). As a particularly relevant example, Diamond and Rajan (2011) develop a model in which fire sales in a future period are exacerbated because of the risk-shifting incentives at troubled banks in the initial period. My endogenous fire sale channel result is similar in that the private incentives and decisions of banks lead to a negative externality via larger fire sale losses. However, my result differs in that selling decisions do not rely on banks’ limited liability and risk shifting. Instead, the banks sell according to a cost-benefit analysis influenced by regulatory parameters. Also, unlike Diamond and Rajan (2011), my model features endogenous portfolio choice and ex ante capital regulation.

Although my paper is the first to jointly *model* capital regulation and fire sales, there is a growing literature that empirically investigates the connection. These studies face the dual challenge

of both identifying regulation as the cause of selling and demonstrating a fire sale. Boyson, Helwege, and Jindra (2014) do not find evidence that commercial banks sell affected assets at fire sale prices during crisis periods. This lack of evidence, however, may be due to data limitations. Other studies focus on the insurance sector for which the data quality is high and institutional details can be exploited to make causal inference (Ellul, Jotikasthira, and Lundblad 2011; Merrill, Nadauld, Sherlund, and Stulz 2014). These papers convincingly demonstrate that the violation of regulatory constraints causes selling that can generate significant price distortions.

More broadly, the present study contributes to the literature on the role of bank capital regulation (see Thakor (2014) for a survey). The basic idea is that bank capital affects both risk management and the ability to withstand economic shocks. However, forcing banks to hold more regulatory capital may also create costs due to reductions in valuable maturity transformation, the supply of deposits as safe liquid assets (Gorton and Pennacchi 1990), and/or monitoring from debt holders (Diamond and Rajan 2001). The optimal capital structure will in theory trade off these benefits and costs. My model, in which the losses from fire sales are a function of the cross section of banks, provides a richer setting for policy analysis that seeks to determine sufficient levels of bank capital to mitigate negative real externalities during a financial crisis (i.e., systemic risk). A full welfare analysis, however, is beyond the scope of this paper, given my reduced-form representation of agents outside of the banking sector.

The policy analysis in this study contributes to the broader literature analyzing regulatory policy to address systemic risk (e.g., Allen and Gale 1998; Davila and Korinek 2017; Diamond and Dybvig 1983; Farhi and Tirole 2012; Lorenzoni 2008; Stein 2012). My results are consistent with those of Acharya, Pedersen, Philippon, and Richardson (2017), who find that institutions should be taxed according to the systemic risk they generate. The contribution of this study is to identify a specific mechanism through which banks actually create systemic risk, thereby delivering an implementation strategy based on available regulatory tools. Further, I specify the characteristic that leads these banks to generate more risk in the first place: their greater ability to offset price impact.

Finally, this study is related to the literature that explores why profit-maximizing banks choose to hold correlated downside risk. Farhi and Tirole (2012) develop a model in which expectations

of central bank policy and strategic complementarities lead banks to optimally choose correlated portfolios. Their model, however, does not offer a specific mechanism explaining how banks are able to coordinate failure in the same states of the world. Allen, Babus, and Carletti (2012) motivate asset commonality in a network model as a means to reduce the costs of debt, and Acharya and Yorulmazer (2008) suggest that “herding” is a way to minimize the information spillover from bad news about other banks. My model provides a distinct channel through which downside risk materializes (i.e., the fire sale channel) that is consistent with profit-maximizing behavior.

The remainder of my dissertation is organized as follows. Chapter 2 describes the model and the main theoretical results. Chapter 3 presents the quantitative policy analysis. Chapter 4 relates my findings and analysis to current regulatory policy. Finally, chapter 5 concludes.

CHAPTER 2

A MODEL OF THE CROSS SECTION OF BANKS

I consider a stylized model of the banking sector in order to highlight the interaction between portfolio choice, capital regulation, fire sales, and systemic risk. After describing the setup, I characterize the equilibrium and select optimality conditions. I then relate these results to equilibrium systemic risk, focusing on the endogenous fire sale channel described in the introduction.

2.1 Setup

The model features three periods ($t = 0, 1, 2$) and an arbitrary number of banks (B). At $t = 0$, each bank chooses a portfolio from among three risky assets subject to a regulatory capital constraint. At $t = 1$, there is a systematic asset shock and the banks may choose to sell assets. These selling decisions can lead to market price declines, which create additional losses for banks holding the affected assets. The equity capital of each bank is affected by the shock, the bank's own selling decisions, and losses due to market price declines. As a result, the bank's capital ratio may fall below the regulatory minimum. In such a case, the bank is considered “undercapitalized” and, equivalently, it has a positive “capital shortfall.” At $t = 2$, the stochastic payoffs from the risky assets are realized and the bank pays a penalty according to its level of capital shortfall at the end of the previous period. This setup is summarized in a timeline in figure 2.1.

Three Risky Assets. Banks can choose from among three risky assets for their portfolios. These assets broadly represent the types of risky financial assets that comprise the majority of bank balance sheets in the data.

1. Asset 1 is an illiquid two-period investment (e.g., bank loans) subject to a systematic shock at $t = 1$, which is described further below.
2. Asset 2 is perfectly liquid marketable security (e.g., US Treasuries).

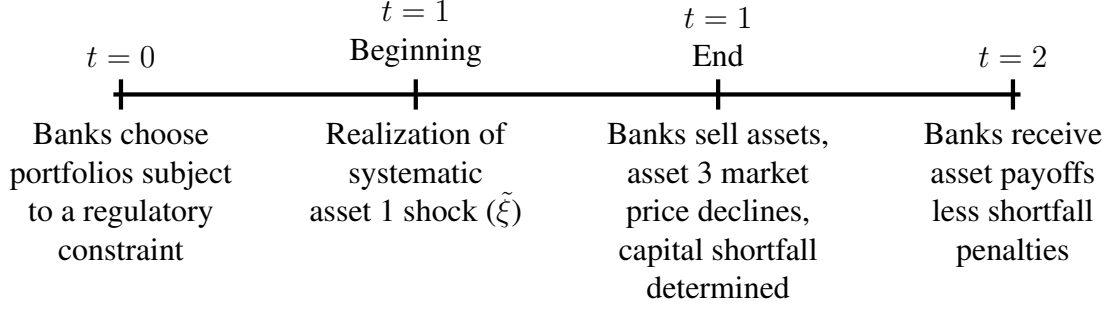


Figure 2.1: Model Timeline

3. Asset 3 is a marketable security with downside fire sale risk (e.g., asset-backed securities).

The downside fire sale risk refers to a potential market price decline at $t = 1$ that occurs due to aggregate selling.

The asset payoffs at $t = 2$ are determined by a stochastic gross return vector $\tilde{R}_A \sim (\mu, \Sigma)$. The elements of the mean return vector μ and covariance matrix Σ represent the fundamental (i.e., primitive) riskiness of the assets.

Bank Portfolio Problem. At $t = 0$, bank b is endowed with a fixed amount of equity capital (E_0^b). Given a cost of debt (R_D) and a regulatory framework $(\kappa_{reg}^b, \phi, w)$, the bank chooses a vector of asset holdings (A_0^b), a capital buffer (β^b), and an amount of borrowing (D_0^b) to maximize a mean-variance objective function. Formally, the bank solves the following problem:

$$\max_{A_0^b, \beta^b, D_0^b} \mathbb{E}_0 [\tilde{R}_E^b] - \frac{\gamma}{2} Var_0 [\tilde{R}_E^b] \quad (2.1)$$

subject to

$$\frac{E_0^b}{w' A_0^b} = \kappa_{reg}^b (1 + \beta^b) \quad (2.2)$$

$$D_0^b = 1' A_0^b - E_0^b \quad (2.3)$$

as well as non-negativity constraints for the choice variables $\{A_0^b, \beta^b, D_0^b\}$.

Mean-variance optimization is often used in the literature for the formulation for the portfolio

problem of a financial institution choosing among multiple assets (e.g., Hart and Jaffee 1974; Kim and Santomero 1988; Rochet 1992; Calomiris 2009; Glasserman and Kang 2014; Haddad and Sraer 2015). These preferences capture, in reduced form, a bank's concern for risk unrelated to regulation. The mean-variance setup also has intuitive appeal because it is essentially the same as imposing a value-at-risk (VaR) constraint on the bank's portfolio. VaR is a common tool used in practice both internally and by regulators for monitoring the riskiness of bank portfolios.

Equation (2.2) represents the regulatory capital requirement constraint. Given equity capital (E_0^b), a bank's portfolio and borrowing choices must result in a capital ratio that is greater than or equal to the minimum level specified by the regulator (κ_{reg}^b). This required level can be bank specific. The vector w contains risk weights for each asset that are used to compute risk-weighted assets ($w' A_0^b$), the denominator in the bank's capital ratio. In practice, risk weights are intended to align a bank's capital cushion to the riskiness of its assets, and the specific weight values are determined by the regulator. The constraint in (2.2) holds with equality because of the non-negativity constraint for the bank's capital buffer choice (β^b).

The level of borrowing in (2.3) is determined by the bank's portfolio and buffer choices (A_0^b and β^b). The debt is to be repaid at $t = 2$ and the cost of debt (R_D) is assumed constant. This modeling choice has two implications. First, it assumes that debt is not priced according to asset risk, which is a clear violation of the Modigliani and Miller (1958) conditions. Second, it implies that all bank debt is long-term, which both is counterfactual and neglects the fundamental aspect of maturity transformation in banking (Diamond and Dybvig 1983).

I justify the first implication by a combination of deposit insurance and collateralization, both of which serve to make bank debt information insensitive. Although these elements are not explicitly included in the model, the literature has long acknowledged these features as common to and necessary for bank debt.¹ Moreover, banking theory often views bank debt as information

¹The long-standing justification for deposit insurance is a means to prevent panic-based bank runs (Diamond and Dybvig 1983). Repurchase agreements ("repo") are another common form of non-deposit bank debt. Repos are short-term and collateralized and therefore viewed as information insensitive during normal times (Krishnamurthy, Nagel, and Orlov 2014). Repos, like uninsured deposits, are vulnerable to runs as observed during the financial crisis of 2007-2009 (Gorton and Metrick 2012).

insensitive by design (Diamond 1984).

Despite its long-term appearance, the model still captures the costs associated with short-term bank debt. The bank pays a penalty at $t = 2$ based on its financial distress during $t = 1$. In reduced form, this cost represents in part the various costs that a distressed institution may pay when rolling over short-term debt. This penalty and its interpretation are discussed in greater detail later in this paper. To summarize, the modeling choice for debt is a simplification that allows me to more easily focus the analysis on portfolio choice and fire sales but still captures key aspects of bank debt.

The stochastic return on the bank's equity $\left(\tilde{R}_E^b\right)$ is equal to the gross return on assets still held at the end of $t = 1$ less the cost of remaining debt and penalty costs

$$\tilde{R}_E^b E_0^b \equiv \tilde{R}_A' \tilde{A}_1^b - R_D \tilde{D}_1^b - \tilde{\Phi}^b \quad (2.4)$$

The quantity values for assets and debt $\left(\tilde{A}_1^b\right.$ and $\left.\tilde{D}_1^b\right)$ are stochastic because they depend on the outcome of an asset shock at $t = 1$. The same is true for the penalty $\left(\tilde{\Phi}^b\right)$.

Bank Heterogeneity. The key heterogeneity across banks is their ability to offset price impact during a fire sale. Specifically, the bank-specific parameter α^b determines the price that bank b receives if it sells during $t = 1$. As a result, banks can differ in their optimal selling decision even while under the same regulatory framework. Additional consequences are that banks choose different optimal portfolios at $t = 0$ and changes to regulatory parameters affect bank decisions differently. I discuss the interpretation of this heterogeneity later in this chapter after describing losses due to fire sales.

Banks may also differ in initial equity capital $\left(E_0^b\right)$ and minimum regulatory requirement $\left(\kappa_{reg}^b\right)$. The former effectively determines the cross-sectional distribution of α^b in the banking sector. The latter is set by the regulator and therefore is not a fundamental characteristic. However, the requirement level is a significant input into the bank's optimal decisions.

Systematic Shock to Asset 1 $\left(\tilde{\xi}\right)$. At the beginning of $t = 1$, there is a permanent shock to the value of asset 1. This shock is perfectly correlated across banks and thus can be interpreted as a

systematic crisis shock. The distribution of this shock is

$$\tilde{\xi} = \begin{cases} 0 & \text{with probability } 1 - q \\ \xi & \text{with probability } q \end{cases} \quad (2.5)$$

where $\xi \in (0, 1)$ and $q \in (0, 1)$. The shock is in percentage units and therefore the value of asset 1 holdings for bank b at the end of $t = 1$ is

$$\tilde{A}_{1,1}^b = (1 - \tilde{\xi}) A_{0,1}^b \quad (2.6)$$

Equivalently, the bank suffers a proportional loss $(A_{0,1}^b \xi)$ when the crisis state is realized $(\tilde{\xi} = \xi)$.

Selling Assets (s). If the crisis state is realized at $t = 1$, the bank may want to sell its marketable assets. The motivation for doing so is to reduce a penalty that is paid at $t = 2$. Each bank b chooses to sell fractions $s_2^b \in [0, 1]$ and $s_3^b \in [0, 1]$ of its asset 2 and asset 3 holdings, respectively. As a result, its holdings of the marketable assets become

$$\begin{aligned} A_{1,2}^b &= (1 - s_2^b) A_{0,2}^b \\ A_{1,3}^b &= (1 - s_3^b) A_{0,3}^b \end{aligned}$$

Given a realization of the crisis state $(\tilde{\xi} = \xi)$, each bank b optimally chooses $\{s_2^b, s_3^b\}$ to solve

$$\max_{s_2^b \in [0,1], s_3^b \in [0,1]} \mathbb{E}_0 \left[\tilde{R}_E^b \mid \tilde{\xi} = \xi \right] - \frac{\gamma}{2} Var_0 \left[\tilde{R}_E^b \mid \tilde{\xi} = \xi \right] \quad (2.7)$$

For simplicity, I assume that the bank does not sell in the good state of the world.²

²Technically, a bank would optimally re-optimize its portfolio given the noncrisis state $(\tilde{\xi} = 0)$. Given the illiquidity of asset 1, this re-optimization would consist only of a reallocation from asset 2 to asset 3. Accounting for this term in the solution, while easy to identify, would only serve to add notational clutter without providing meaningful economic substance.

Price Impact from Selling. The volume of selling of asset 3 has an impact on its market price. I denote this price decline in percentage units as $\Psi_3 \in (0, 1)$. The level of the price decline is determined according to a price impact function in which the only input argument is the aggregate quantity of asset 3 sold:

$$\Psi_3 \equiv \Psi \left(\sum_{b=1}^B s_3^b A_{0,3}^b \right) \quad (2.8)$$

This function satisfies the following general properties:

1. Bounded range: $\Psi : \mathbb{R}^+ \rightarrow [0, 1]$
2. Positive price decline requires positive selling: $\Psi(0) = 0$
3. Monotonicity: $\Psi'(\cdot) > 0$

The form of this price impact follows from the theoretical literature on fire sales, which characterizes fire sales as events in which many potential buyers of an asset are concurrently distressed (Shleifer and Vishny 2011). As a result, any selling that takes place occurs at a price below the fundamental value of the asset. The price impact function above represents in reduced form an outside investor who provides liquidity to the banking sector during a fire sale. As long as this investor has limited wealth and other investment options, the investor's demand will be downward sloping. Equivalently, the offered price is declining in the volume being sold. This exact type of specification is similarly used in other fire sale frameworks (Greenwood, Landier, and Thesmar 2015; Cont and Schaanning 2017).

Losses Due to Asset 3 Price Decline. Given selling decision $s_3^b \in [0, 1]$ and price decline $\Psi_3 \in (0, 1)$, bank b will incur two types of losses related to asset 3: unrealized and realized. The market value of the remaining asset 3 holdings for bank b is

$$(1 - \Psi_3) (1 - s_3^b) A_{0,3}^b \quad (2.9)$$

This expression implies an *unrealized* loss equal to $\Psi_3 (1 - s_3^b) A_{0,3}^b$. In other words, bank b marks down the value of these assets on its balance sheet and the corresponding losses are reflected in

equity capital.³

For the share of asset 3 holdings it sells, bank b receives

$$(1 - \alpha^b \Psi_3) s_3^b A_{0,3}^b \quad (2.10)$$

This expression implies a *realized* loss equal to $\alpha^b \Psi_3 s_3^b A_{0,3}^b$. In other words, the bank receives less than the amount it paid for the holdings that it sells. Here we see the exact role that the bank-specific parameter $\alpha^b \in (0, 1)$ plays within the model framework. When selling, a bank receives a weighted average price between the initial and fire sale levels with weights $(1 - \alpha^b, \alpha^b)$. If α^b is close to zero, bank b receives a high price. If α^b is close to one, bank b receives a low price. This price received directly affects the size of the realized loss. As long as $\alpha^b < 1$, the realized loss per unit sold will always be less than the unrealized loss per unit of asset 3 held.

I follow Cont and Schaanning (2017) in incorporating realized losses and in the way that I model them (i.e., the α parameter). The authors note that previous studies do not account for the fact that banks liquidate assets at a discount to the current market price, which is known as “implementation shortfall” in the literature on optimal trade execution (e.g., Almgren and Chriss 2000). Greenwood, Landier, and Thesmar (2015), for example, assume that banks sell at the current market price *and then* the market price declines according to a price impact function, which would imply that $\alpha^b = 0$ for all banks. The inclusion of a realized loss, however, is critical when the decision to sell is endogenous, as in my setup.

A novel feature of my model relative to that of Cont and Schaanning (2017) is that I also allow

³I assume mark-to-market accounting (also known as fair value accounting or FVA) in my setup because of its empirical relevance. On average, 93% of security holdings in my dataset are accounted for using fair value. There is an open debate in the literature as to whether FVA should be used in the banking sector (Acharya and Ryan 2016; Laux and Leuz 2010). The main argument against FVA is that it can force banks to sell off assets during a crisis resulting in costly fire sales (Allen and Carletti 2008; Plantin, Sapra, and Shin 2008). Although fire sales in my model are indeed driven heavily by FVA, an assessment on the optimality of FVA is beyond the scope of this paper. My model does not account for the potential downsides of historical cost accounting (Bleck and Liu 2007; Ellul, Jotikasthira, Lundblad, and Wang 2015), which is the alternative accounting treatment. My model does capture, however, an important purported *benefit* from FVA: banks internalize the probability of fire sales and choose safer ex ante portfolios. In fact, this portfolio reallocation channel is a crucial aspect of the regulatory policies I analyze in my quantitative analysis. Ellul, Jotikasthira, Lundblad, and Wang (2014) provide empirical evidence for this benefit in the insurance sector.

banks to be heterogeneous in the α parameter. This type of heterogeneity allows banks to make different optimal selling decisions, all else held equal. I do not explicitly model the underlying source of this heterogeneity, but there are two broad interpretations. First, some banks may fundamentally be able to trade faster, and therefore they receive higher volume-weighted average prices when selling during periods of price decline. This higher speed may arise from a larger set of established counterparties or from a larger trading operation within the firm (e.g., a larger number of traders employed). Second, some banks may have superior information about the direction of financial markets. Knowing that a large price decline is coming, these banks can begin trading sooner and therefore will receive a higher volume-weighted average price. These interpretations are not mutually exclusive, and clearly they imply the same end result. In the quantitative analysis in chapter 3, I present empirical evidence that banks differ along this dimension.

Summing unrealized and realized, the total losses incurred by bank b due to the asset 3 price decline is

$$L^b = (1 - (1 - \alpha^b) s_3^b) \Psi_3 A_{0,3}^b \quad (2.11)$$

Capital Shortfall Penalty (Φ). A bank pays a penalty at $t = 2$ if its capital ratio is lower than the required minimum at the end of $t = 1$. Specifically, this penalty is an increasing function of the amount by which a bank's equity capital is below its minimum, which is commonly known as "capital shortfall." Capital shortfall at the end of $t = 1$ (CS_1) is defined implicitly

$$\frac{E_1 + CS_1}{w' A_1} = \kappa_{reg} \quad (2.12)$$

where E_1 and $w' A_1$ are the equity capital and risk-weighted asset values, respectively, at the end of $t = 1$ after all selling activity has concluded. Capital shortfall can be positive or negative. If it is positive, the bank's capital is below its minimum required level, and the bank is considered "undercapitalized."

The penalty that the bank pays at $t = 2$ is a linear function of its positive capital shortfall at the end of $t = 1$:

$$\Phi \equiv \phi \times \max \{CS_1, 0\} \quad (2.13)$$

At $t = 0$, this quantity is stochastic because it depends on the realization of the asset shock at $t = 1$.

This penalty represents all of the costs that a bank's equity holders should expect to incur if its capital ratio falls too low. Broadly, these costs come from the regulator, debt holders, and the issuing of equity. A clear distinction between the costs by source is not important as long as we assume that the regulator effectively controls the penalty coefficient ϕ . In chapter 3, I conduct the quantitative policy analysis under this assumption.

The regulatory aspect of this penalty is based on the prompt corrective action (PCA) framework addressing financial deterioration in banks. This framework describes a sequence of increasingly stringent interventions dependent primarily upon a bank's capital ratios. These interventions range from increased oversight to prohibiting capital distributions to closing and liquidating the bank.⁴ The goal of PCA is to rehabilitate a bank in order to avoid losses in the FDIC insurance fund, so these measures are technically not to be viewed as a "penalty" in the punishment sense. However, from the perspective of equity holders of a bank, PCA measures are effectively a punishment for becoming distressed. Accordingly, PCA measures were utilized heavily during and after the financial crisis of 2007–2009.⁵ Thus PCA resembles quite well the ex post penalty in the model.

The capital shortfall penalty also captures in reduced form the costs that arise from debt holders in times of distress. Debt-related financial distress costs have been studied extensively in the literature; these include fundamental-based bank runs (e.g., Allen and Gale 1998), difficulties in rolling over short-term debt (e.g., Acharya, Gale, and Yorulmazer 2011), and overhang (Admati, DeMarzo, Hellwig, and Pfleiderer 2011). A common feature of these costs is that they are an increasing function of a bank's perceived distress level.

Finally, the capital shortfall penalty captures in reduced form the costs of issuing equity (Gomes

⁴For more details on PCA, see, e.g., the Government Accountability Office report "Bank Regulation: Modified Prompt Corrective Action Framework Would Improve Effectiveness," GAO-11-612, Jun 23, 2011.

⁵Between 2006 and 2010, 569 banks underwent the PCA process and 295 ultimately failed. PCA was implemented prior to almost all bank failures. During this period, only 25 banks failed that did not first undergo the PCA process. See Government Accountability Office, "Bank Regulation: Modified Prompt Corrective Action Framework Would Improve Effectiveness," GAO-11-612, June 23, 2011.

2001). These costs can be viewed as voluntary if the bank chooses to issue equity during or after the distress period. The costs can also be imposed through formal or informal pressure by the regulator. As with debt-related costs, any equity-issuance costs would be increasing the level of bank distress.

Parameter Assumptions. I make the following assumptions about the parameter space to either simplify the analysis or focus on specific outcomes.

Assumption 1 *The probability of a crisis (q) is sufficiently small such that no bank holds an excess capital buffer in order to buy fire sale assets.*

I describe the “small” condition for q in appendix A. The equilibrium implications from this assumption are that banks will either sell asset 3 ($s_3^* > 0$) or not sell at all ($s_3^* = 0$) and the outside investor represented by the price impact function is the only buyer. Other papers in the literature use the expected profit from purchasing assets at fire sale prices as a primary driver in a bank’s portfolio choice (Allen and Gale 2005; Acharya, Shin, and Yorulmazer 2011), but this type of story is not the focus of this paper. Instead, by focusing on a crisis for which banks choose not hold excess liquidity beforehand, my analysis is in the spirit of papers that model liquidity coming from outside the banking sector (Allen and Gale 1998; Stein 2012).⁶

Assumption 2 *The asset return covariance matrix (Σ) is diagonal.*

This assumption greatly simplifies the analytical expressions yet leaves the economics of the problem unchanged.⁷

⁶Allen and Gale (1998) include a large number of wealthy, risk-neutral speculators who hold cash in order to purchase assets when bank sell off assets cheaply to obtain liquidity. Stein (2012) defines a subset of “patient” investors that both invest in new projects and absorb assets being sold in a crisis state.

⁷If the true covariance matrix of the asset returns S is not diagonal, we can perform a Cholesky decomposition (S is positive definite because it is a covariance matrix) to find $S = LL'$ where L is a lower triangular matrix and Σ is diagonal matrix. The bank problem can be rewritten so that the bank chooses a vector of exposure levels \hat{A}_0 to orthogonal factors with returns summarized by (μ, Σ) . To recover asset holdings, one can simply multiply the exposure levels by the triangular matrix from the Cholesky decomposition ($A_0 = L'\hat{A}_0$).

Assumption 3 *The fundamental expected asset return parameters satisfy*

$$\mu_1 > \mu_3 > \mu_2 > R_D$$

This assumption has a few implications. First, in a world with no capital regulation or systematic shock, a bank would want to hold a positive amount of each asset. Second, it implies that asset 1 earns the highest risk premium and asset 2 earns the smallest risk premium, which is consistent with the types of real-world assets they represent (bank loans and Treasuries). Third, it guarantees that banks sell their holdings of asset 3 before selling those of asset 2, which simplifies the presentation of the optimal selling decision by removing a few special cases (appendix B). This outcome was also a stylized feature of the data at the height of the financial crisis as shown in chapter 3.

2.2 Equilibrium and Optimality Conditions

Before analyzing the optimal decisions, let us first define an equilibrium.

Definition 1 *Equilibrium.* *An equilibrium is a vector of asset holdings (A_0^b) , a capital buffer (β^b) , a level of debt (D_0^b) , and a set of distressed selling decisions (s_2^b, s_3^b) for each bank b as well as a price decline for asset 3 (Ψ_3) such that*

1. **Bank optimality:** $\{A_0^b, \beta^b, D_0^b\}$ solves the $t = 0$ bank b problem in (2.1)–(2.3) and (s_2^b, s_3^b) solves the $t = 1$ bank b problem in (2.7) given

- Asset 3 price decline (Ψ_3)
- Regulatory framework (κ_{reg}^b, ϕ)
- The other model parameters

2. **Market clearing:** Asset 3 price decline (Ψ_3) satisfies

$$\Psi_3 = \Psi \left(\sum_{b=1}^B s_3^b A_{0,3}^b \right)$$

given the price impact function $\Psi(\cdot)$

Compared to a pure portfolio choice problem (i.e., $q = 0$), solving this model is significantly more difficult. The asset 3 price decline (Ψ_3) is taken as given by each bank at $t = 0$ but also must be consistent with the portfolio choice and selling behavior of all banks at $t = 1$. However, we gain richness in the model insights because the equilibrium asset 3 price decline (Ψ_3) incorporates the selling behavior of the entire cross section of banks.

The following proposition formalizes the key insight regarding the optimal bank selling decision:

Proposition 1 *Optimal Asset 3 Selling.* *Ceteris paribus, bank b optimal selling for asset 3 (s_3^b) is weakly increasing in*

- *The ex post penalty parameter (ϕ)*
- *The minimum capital requirement (κ_{reg}^b)*
- *The fire sale price decline (Ψ_3)*
- *The bank's ability to offset price impact ($-\alpha^b$)*

as long as $\alpha^b < (1 - \kappa_{reg}^b w_3)$.

Proof See appendix A.

From a policy perspective, there are two important implications from proposition 1. First, tightening bank regulation can cause bank b to sell asset 3. This finding is the foundation of the endogenous fire sale channel. Second, banks that can sell at a higher price (i.e., banks that have a low α) choose to sell at lower threshold values than other banks, all else held equal. This implication may seem obvious, but it highlights the reason that the cross section of banks is an important input to both equilibrium outcomes and the choice of the best regulatory policies.

To better understand the optimal selling decision and proposition 1, it is helpful to consider a numerical example. In figure 2.2, shows the policy function for selling asset 3 (s_3^b) using specific parameter values. Each panel represents the same policy as a function of a different input. The

policy function is weakly increasing in each panel. The policy functions shown represent a partial equilibrium outcome in which all other inputs (including the equilibrium price Ψ_3) are held constant.

The intuition for this outcome can be understood by comparing the extreme cases in which the bank sells nothing or everything. If a bank does not sell asset 3 at all ($s_3 = 0$), its payoff at $t = 2$ from asset 3 is a mean-variance payoff less the penalty from the unrealized loss due to Ψ_3 :

$$\underbrace{(\mu_3 - R_D) A_{0,3} - \frac{1}{E_0} \frac{\gamma}{2} A_{0,3}^2 \sigma_3^2}_{\text{Mean-variance payoff}} - \underbrace{\phi (1 - \kappa_{reg} w_3) \Psi_3 A_{0,3}}_{\text{Penalty unrealized loss}}$$

On the other hand, if a bank sells its entire holding of asset 3 ($s_3 = 1$), its payoff at $t = 2$ from asset 3 is purely a realized loss due to Ψ_3

$$- \underbrace{\phi (\alpha \Psi_3 - \kappa_{reg} w_3) A_{0,3}}_{\text{Penalty realized loss}}$$

Combining the expressions yields the net payoff at $t = 2$ from selling its entire holdings:

$$\underbrace{\phi \left[\underbrace{(1 - \Psi_3) \kappa_{reg} w_3}_{\text{From reducing assets}} + \underbrace{(1 - \alpha) \Psi_3}_{\text{From } \alpha < 1} \right] A_{0,3}}_{\text{Benefit from reduced penalty}} - \underbrace{\left[(\mu_3 - R_D) A_{0,3} - \frac{1}{E_0} \frac{\gamma}{2} A_{0,3}^2 \sigma_3^2 \right]}_{\text{Lost mean-variance payoff}}$$

In this expression, we can see that the benefit term is increasing in all of the inputs listed in proposition 1. This benefit comes from two sources. First, selling assets reduces capital shortfall by reducing assets (the denominator of the capital ratio). As the minimum capital ratio increases ($\kappa_{reg} \uparrow$), the reduction in assets has a larger effect in reducing capital shortfall because of the lower leverage. Second, realized losses are smaller than unrealized losses for a given Ψ_3 given that $\alpha < 1$. As the fire sale worsens ($\Psi_3 \uparrow$), this benefit becomes larger. This benefit is also larger for banks that can better offset price impact (i.e., that have a lower α). Finally, if the bank expects to be penalized more severely ($\phi \uparrow$), both sources of benefit are more valuable.

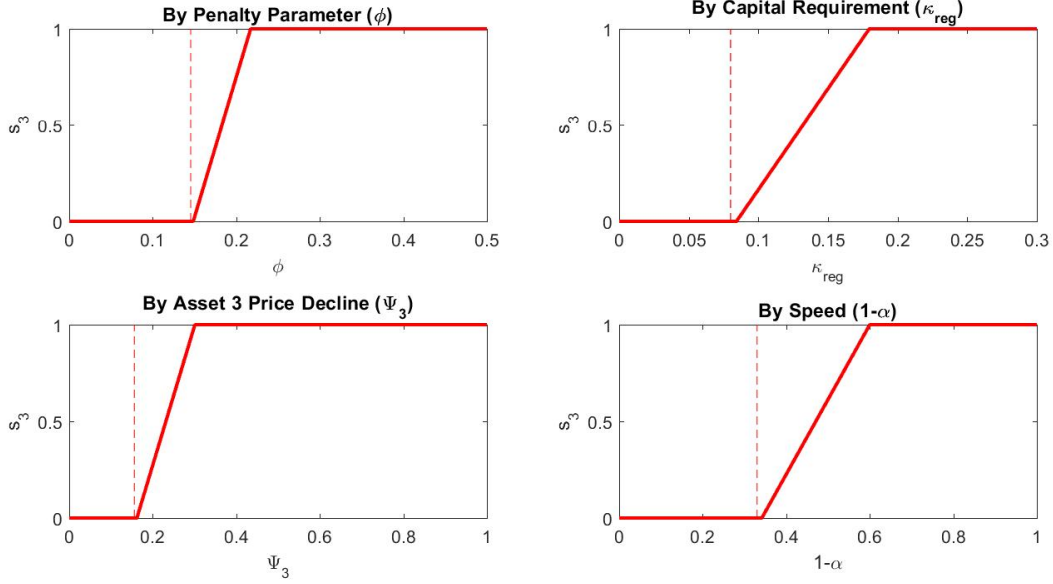


Figure 2.2: Example Optimal Asset 3 Selling Function

The underlying parameters are from the calibration in chapter 3 ($\mu_3 = 1.023$, $R_D = 1.0045$, $w_3 = 0.5$, $\alpha = 0.67$, $\phi = 0.1454$, $\kappa_{reg} = 0.08$, and $\Psi_3 = 0.1558$). The dashed lines in each plot indicate the underlying parameter or equilibrium values, which show that the bank in this example does not sell its asset 3 holdings.

Proposition 2 Capital Buffer. *A bank holds zero additional capital buffer if the probability of a crisis (q) is sufficiently small.⁸*

Proof See appendix A.

Absent the ex post shortfall penalty costs, a bank only holds a capital buffer if the asset return fundamentals are not sufficiently attractive for the capital requirement to bind. Including these costs, the bank may want to hold an additional capital buffer to reduce the penalties or amount of selling in the crisis state. Holding such a buffer is costly, however, because the bank gives up profits in the noncrisis state. The larger the probability of the crisis state (q), the larger the expected benefit from holding a capital buffer. This is the intuition that is formalized in proposition 2.

2.3 Systemic Risk

Following Acharya et al. (2017) and others, I define systemic risk (SR^{agg}) using a measure of aggregate capital shortfall. This measure captures the relative capitalization of the banking sector

⁸Whether or not this condition also holds given assumption 1 is difficult to show analytically. Therefore I simply present it as its own condition on q , assuming that banks do not consider a strategy to buy fire sale assets at a discount during the crisis, which is the implication from assumption 1.

as a whole. The underlying idea is that aggregate undercapitalization creates negative externalities in the real economy (e.g., reduced intermediation). Aggregate capital shortfall can be considered a sufficient statistic for these costs that is relatively easy to compute in practice. As a result, it has become popular as a measure for systemic risk.

Definition 2 Systemic Risk. *Systemic risk is the non-negative amount of aggregate capital shortfall relative to a fixed capital ratio level*

$$SR^{agg} \equiv \max \left\{ \sum_{b=1}^B SR^b, 0 \right\} \quad (2.14)$$

where each bank's contribution is defined by

$$\frac{E_1^b + SR^b}{w' A_1^b} = \zeta \quad (2.15)$$

and ζ is the fixed capital ratio level.

Note that, compared to the definition of capital shortfall in (2.12), a bank's systemic risk contribution is measured relative to a fixed capital ratio level ζ instead of its regulatory minimum κ_{reg}^b . For reference, I set $\zeta = 0.08$ in the calibration in the quantitative analysis (chapter 3), which is the value used in the *SRISK* measure of Brownlees and Engle (2017). This aspect of measuring systemic risk also follows from the literature and explains why increasing capital requirements can lower systemic risk. Similar to capital shortfall, we can decompose the systemic risk contribution from any given bank b into the following three components:

$$SR^b \approx \underbrace{\xi A_{0,1}^b}_{\text{Asset 1 Loss}} - \underbrace{\left(\frac{E_0^b / (w' A_0^b) - \zeta}{E_0^b / (w' A_0^b)} \right) E_0^b}_{\text{Initial Capital Ratio Above SR Threshold}} + \underbrace{L(s_2^b, s_3^b; A_0^b, \Psi_3)}_{\text{Net Fire Sale Loss}} \quad (2.16)$$

This decomposition will help us understand the impact from changing policies discussed in chapter 3.

2.4 Endogenous Fire Sale Channel

Using the key results from the model and the definition of systemic risk, I now ready describe the endogenous fire sale channel. This channel describes a sequence of endogenous effects, starting with tightening regulation that leads to higher systemic risk. Specifically, tighter regulation can lead to higher systemic risk through the following sequence:

$$\{\kappa_{reg}^b, \phi\} \uparrow \implies s_3^b \uparrow \implies \Psi_3 \uparrow \implies \underbrace{L(s_2^b, s_3^b; A_0^b, \Psi_3)}_{\text{Net Fire Sale Loss}} \uparrow \implies SR^b \uparrow \implies SR^{agg} \uparrow \quad (2.17)$$

In the first link, tighter regulation leads bank b to sell asset 3. This effect follows from proposition 1 and requires that bank b was not already selling the asset. In the next link, more selling leads to a larger fire sale price decline, which is a direct result from the properties of the price impact function in (2.8). In the third link, the larger fire sale price decline leads to larger fire sale losses. This outcome, which is formally described in the following proposition, occurs as long as the probability of a crisis (q) is sufficiently small.

Proposition 3 *Fire Sale Losses Increasing in Asset 3 Price Decline.* *Ceteris paribus and assuming a positive asset 3 holding ($A_{0,3}^* > 0$), a worsening fire sale ($\Psi_3 \uparrow$) creates larger bank b fire sale losses if*

$$\min \left\{ \frac{\mu_3 - R_D}{2q\phi(1 - \kappa_{reg}^b w_3)}, \frac{(1 - q)(\mu_3 - R_D) + \kappa_{reg} w_3 2q\phi}{2q\phi\alpha^b} \right\} > \Psi_3$$

Proof See appendix A.

The intuition for proposition 3 is simple: bank b does not shift its ex ante holdings of asset 3 much if the probability of the crisis is low.⁹ The condition in the proposition guarantees that the elasticity of a bank's holding of asset 3 with respect to the fire sale price decline (Ψ_3) is less than one, which implies that total losses (price decline times holding) go up.

⁹The expression in proposition 3 can alternatively be characterized as holding for a sufficiently small ex post penalty parameter (ϕ) value or for a sufficiently large expected return on asset 3 (μ_3). These characterizations provide a similar intuition as to why the bank's fire sale losses in a crisis state increase despite a larger anticipated fire sale price decline (Ψ_3).

In the final two links of (2.17), larger fire sale losses lead to larger systemic risk, as defined above. To summarize, the tightening of bank regulation can lead to more selling, larger fire sale price declines and losses, and higher systemic risk.

The sequence of effects in (2.17), however, does not describe all of the effects that occur in equilibrium as the result of tighter capital regulation. There are several other endogenous effects that occur concurrently, many of which mitigate increases in systemic risk. For example, an increase in capital requirements also forces banks to hold a larger initial capital ratio level. In the decomposition of systemic risk in (2.16), this effect directly reduces systemic risk.

The ultimate impact on equilibrium systemic risk from changing regulatory parameters depends on the quantitative magnitudes from all effects describe above, and the direction is difficult to establish analytically. In the next chapter, I explore the equilibrium outcomes from policy changes numerically in a calibrated model. I also characterize the least costly policies to mitigate systemic risk that account for the impact of the endogenous fire sale channel.

CHAPTER 3

QUANTITATIVE POLICY ANALYSIS

In this chapter, I investigate the quantitative impact from tightening capital regulation, given parameter values that represent the US banking sector. First, I calibrate the model using asset holdings data disclosed in regulatory filings. As part of this process, I document novel stylized facts about asset selling by banks during the financial crisis. Using the calibrated model, I then assess regulatory policies that mitigate systemic risk under different parameter assumptions and potential regulatory objective functions.

3.1 Stylized Facts and Calibration

I calibrate the model to both the period before the financial crisis and the observed outcomes during the financial crisis. The financial crisis offers a relevant empirical example for the model developed in chapter 2. In particular, the fourth quarter of 2008 resembles the crisis state and resulting fire sale. During this quarter, large and sharp price declines occurred across many types of securities, as shown in figure 3.1. The left panel shows return indices for “risky” security types that experienced steep declines from the beginning of the quarter, while the right panel shows return indices for “safe” security types that experienced no price declines.

In aggregate, banks were net sellers of risky securities during this period (see table 3.1). Banks sold 10.6% of their holdings in “risky” securities, while they increased their holdings of “safe” securities by 7.6% (overall sales of 2.7% of their total securities holdings). A more detailed breakdown of the underlying asset types for the risky and safe subtotals is reported in table 3.1.

Figure 3.2 shows the share of asset-backed securities and private mortgage-backed securities sold, by bank asset size groups. I focus on these two types of securities because of their notoriety during the crisis (Acharya, Cooley, Richardson, and Walter 2009) and because they accounted for a large volume of the selling that did occur (table 3.1). The top panels of figure 3.2 show that

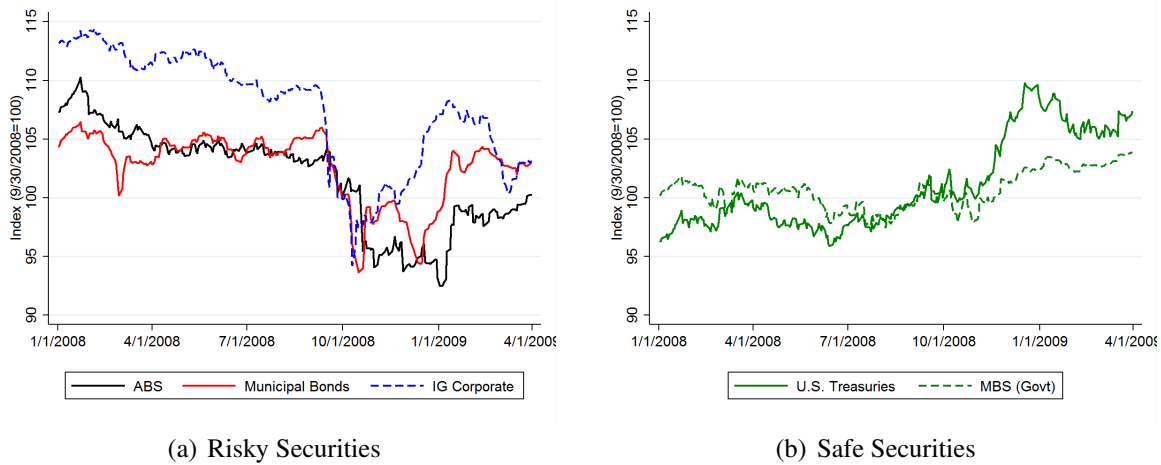


Figure 3.1: Return Indices for Various Security Types 2008Q1-2009Q1

All indices are computed to have a value of 100 on September 30, 2008. The ABS index is computed from Bloomberg Barclays US Agg ABS Total Return Value Unhedged USD, downloaded from Bloomberg (LUABTRUU Index). The Municipal Bond index is computed from the S&P Municipal Bond Index. IG Corporate is computed from the FINRA Investment Grade Corporate Bond Index. U.S. Treasury values are computed from ICE U.S. Treasury Core Bond TR Index, downloaded from Bloomberg (IDCOTCTR Index). MBS (Govt) is computed from the net asset value of the iShares MBS ETF, which tracks an index composed of investment-grade mortgage-backed pass-through securities issued and/or guaranteed by U.S. government agencies.

the largest banks are the only significant net sellers of asset-backed and private mortgage-backed securities.

The model suggests a few potential explanations for the cross-sectional difference in selling behavior. These explanations include differences in (1) the ability to offset price impact (α^b), (2) the level of distress (CS_1^b), (3) capital requirements (κ_{reg}^b), and (4) the ex post penalty parameter (ϕ). Let us consider each of these alternative explanations.

According to market-based capitalization measures, the largest banks faced similar levels of distress compared to the other size groups. In figure 3.2, I show the relative share of banks and assets in distress by bank size group. The bottom panels support the conclusion that the largest banks, as a group, were equivalently undercapitalized according to market-based capital ratios. I use the market-based measures because they have two advantages relative to the accounting-based regulatory measures: they are available within the quarter, and they capture changes to the market value of all assets. In the model, banks have an incentive to sell assets at the beginning of a fire sale if they expect to incur penalties as a result of being further undercapitalized. Figure 3.2 suggests

Category	Percentage of Starting Portfolio	Percentage Sold during Quarter
Asset-Backed (Total)	12.0	41.4
Equities	10.0	0.0
MBS (Private)	9.4	6.8
Other Debt	19.4	6.3
US State and Muni	6.2	-2.1
Risky Subtotal	57.0	10.6
MBS (Govt.-backed)	32.1	-12.1
US Govt. Agency	6.2	-9.8
US Treasury	4.6	23.1
Safe Subtotal	43.0	-7.6
Total	100.0	2.7

Table 3.1: Aggregate Banking Sector Securities Portfolio in 2008Q4

Data are from FR Y-9C. Figures used to compute percentage of portfolio are fair value. The classifications into “Risky” and “Safe” are subjective judgments based upon observed price declines during the quarter (see Figure 3.1). Subtotals for percent sold are computed as weighted averages of the share sold for underlying types and cannot be . Wells Fargo and Wachovia are excluded from the computations because of the data issues created by the merger during the quarter. For data details see Appendix C.

that the largest banks did not have a particularly greater incentive to sell during the fourth quarter of 2008.

The largest banks faced the same regulatory framework (κ_{reg}^b and ϕ) as other banks leading up to the crisis. Even if there were informal differences (e.g., in enforcement), these differences would not account for the largest banks selling. According to proposition 1, the optimal selling decision is an increasing function of a bank’s ex ante capital requirement (κ_{reg}^b) and the ex post penalty parameter (ϕ). Therefore, the largest banks would be the only group selling only if they had *higher* values for either parameter. The precrisis evidence does not support this claim. In appendix D.3, I show that the largest banks had lower capital ratios than most banks. Also, the popular narrative that large banks were “too big to fail” (Sorkin 2010) implies that, if anything, the largest banks had a lower value for the ex post penalty parameter (ϕ). To summarize, tighter capital regulation is not a consistent explanation for the largest banks selling more of their assets.

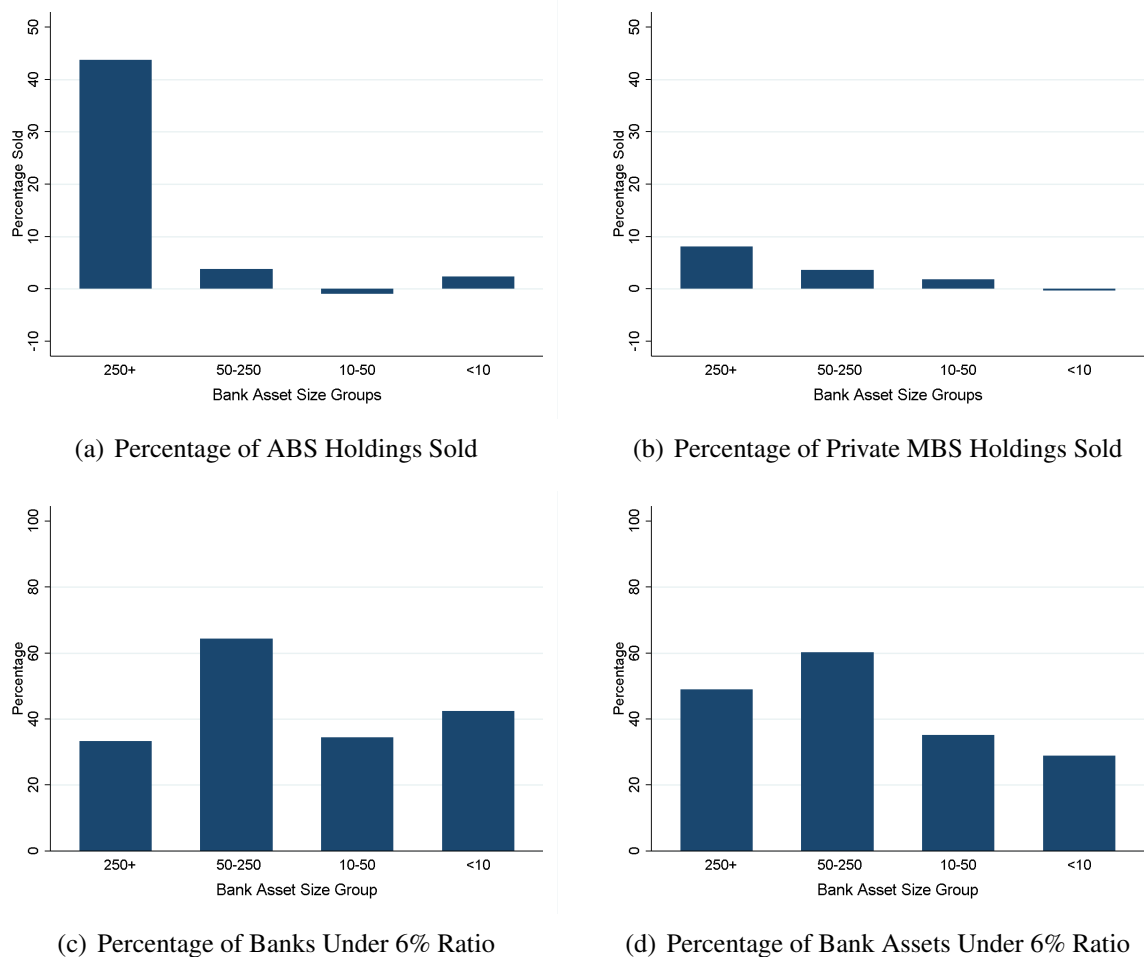


Figure 3.2: Risky Security Selling and Market-Based Capital Ratios during 2008Q4 by BHC Asset Size

Asset size is total assets in billions of dollars at the beginning of 2008Q4. Wells Fargo and Wachovia are excluded when computing the figures for share sold because of the data issues created by their merger during the quarter. The market-based capital ratio used in the bottom panels is computed as the minimum equity valuation during the quarter divided by risk-weighted assets. The bottom left panel shows the number of banks with ratios below 6% divided by the total number of banks within each asset-size group. The right panel shows the sum of the assets for banks with ratios below 6% divided by the total sum of assets within each asset-size group. For data details, see appendix C.

There is a logical argument based on empirical evidence that may explain why the largest banks can better offset price impact. These banks have significant broker-dealer subsidiaries and tend to hold a large percentage of their securities as trading assets¹, and these trading businesses provide advantages during periods of market turmoil. In the context of the corporate bond market,

¹The largest banks held 6% to 8% of their securities as trading assets between 2001–2006 compared to approximately 1% for all other banks. The largest banks also held 45% to 65% of their risky securities (as defined in table 3.1) as trading assets between 2001–2006 compared to 10% to 20% for all other banks.

Di Maggio, Kermani, and Song (2017) find empirically that being a central dealer was valuable between September 2008 and July 2009 in two ways.² These dealers charged higher prices to both peripheral dealers and clients, and they also shrank significantly their holdings of bonds that their clients were selling aggressively. The first finding directly supports a lower value for α^b . The second suggests that these dealers can and do act on private information about market activity from order flow. This idea is also supported by Barbon, Di Maggio, Franzoni, and Landier (2017), who find evidence that brokers leak order flow information to their best clients in order to predate on large liquidations by the other clients. In sum, the advantages from having large dealer subsidiaries are consistent with better ability to offset price impact (i.e., lower α^b).

Proposition 1 tells us that a bank that is better able to offset price impact (i.e., one with low α^b) can choose to sell while an otherwise identical bank with a higher α^b does not. Therefore, if we interpret the largest banks as having a low α^b , we can find an equilibrium in which only the largest banks are selling.

All together, the stylized facts in the data discussed above provide the following implications for the benchmark model calibration:

1. Set $B = 2$ where bank 1 represents banks over \$250 billion in assets as of the beginning of 2008Q4 and bank 2 represents all other banks.
2. The initial equilibrium under pre-2008 regulation must feature only bank 1 selling.
3. Bank 1 can better offset price impact ($\alpha^1 < \alpha^2$).

Table 3.2 shows the parameters that I can directly measure in the data. These parameters include, for example, the return parameters for asset 2, which represents perfectly liquid assets that do not incur any fire sale discount in the crisis state. Given my distinction between banks 1 and 2, I can also easily measure the relative size of these banks during the sample period.

²Other over-the-counter markets follow a similar core-periphery structure. Li and Schurhoff (2014) show core dealers trade faster in the municipal bond market. Hollifield, Neklyudov, and Spatt (2014) describe a similar network structure in the securitization market.

Asset-Specific Fundamentals	Symbol	Value	Source
Mean Return, Asset 2	μ_2	1.09%	Ibbotson Associates
Volatility of Return, Asset 2	σ_2	2.40%	Ibbotson Associates
Probability of Asset 1 Shock	q	4.00%	Laeven and Valencia (2012)
Asset 1 Percentage Decline Given Shock	ξ	4.98%	FR Y-9C
Initial Equilibrium Asset 3 Price Decline	Ψ_3	15.58%	FINRA
Cost of Debt	$R_D - 1$	0.45%	FR Y-9C
<hr/>			
Bank-Specific Fundamentals			
<hr/>			
Share of Aggregate Equity Capital			
Bank 1	$E^1 / (E^1 + E^2)$	0.5	FR Y-9C
Bank 2	$E^2 / (E^1 + E^2)$	0.5	FR Y-9C
<hr/>			
Regulatory			
<hr/>			
Minimum Capital Ratio			
Bank 1	κ_{reg}^1	0.08	FR Y-9C
Bank 2	κ_{reg}^2	0.08	FR Y-9C

Table 3.2: Parameters Directly Measured in the Data

See appendix D for details.

I set the regulatory risk weights (w_1, w_2, w_3) according to the following formula:

$$w_i = \frac{\mu_i - R_D}{\mu_1 - R_D} \quad (3.1)$$

so that the risk weight values do not distort portfolio outcomes (Kim and Santomero 1988; Glasserman and Kang 2014).³

There are several parameters that are either difficult or impossible to measure directly. I choose these parameters in order to match the observed portfolio holdings and selling decisions (see table 3.3). For banks' ability to offset price impact (α^1 and α^2), I solve the model over a range of input values as a form of sensitivity analysis. Given the crucial nature of these parameters, it is important to understand how equilibrium outcomes change with cross-sectional heterogeneity.

For the final element of the calibration, I follow Greenwood, Landier, and Thesmar (2015) and Duarte and Eisenbach (2015) in choosing a linear price impact function for the quantitative

³Other papers find similar results. Rochet (1992) proposes setting risk weights proportional to systematic risk. Calomiris (2009) proposes tying capital requirements (effectively risk weights) to loan interest rates under the assumption that higher rates imply higher risk.

Panel A: Parameters		
Asset-Specific Fundamentals	Symbol	Value
Mean Return		
Asset 1	μ_1	4.30%
Asset 3	μ_3	2.30%
Volatility of Return		
Asset 1	σ_1	3.40%
Asset 3	σ_3	5.50%
Bank-Specific Fundamentals		
Risk Aversion	γ	1.0
Ability to Offset Price Impact		
Bank 1	α^1	[0.20,0.39]
Bank 2	α^2	[0.66,0.95]
Regulatory		
Shortfall Penalty Coefficient	ϕ	0.145

Panel B: Moments				
	Bank 1		Bank 2	
	Data	Model	Data	Model
Ability to Offset Price Impact (α^b)	[0.20,0.39]		[0.66,0.95]	
Portfolio Shares				
Asset 1	0.631	0.687	0.740	0.692
Asset 2	0.154	0.184	0.191	0.185
Asset 3	0.191	0.129	0.069	0.122
Distressed Capital Ratio	0.047	0.031	0.043	0.013

Table 3.3: Parameters Chosen to Match the Data

For α^1 and α^2 , I consider a range of values such that $\alpha^1 < \alpha^2$. Data values for portfolio shares are average observed holdings during 2002–2006. Model portfolio shares are averages over the $\{\alpha^1, \alpha^2\}$ space, but the values do not change much (at most by 0.002). Distressed capital ratios are computed using the minimum market-based capital ratio during 2008Q4. For data details, see appendix C. For calibration details, see appendix D.

exercise:

$$\Psi \left(\sum_{b=1}^B s_3^b A_{0,3}^b \right) \equiv \max \left\{ \min \left\{ \psi_3 \times \left(\sum_{b=1}^B s_3^b A_{0,3}^b \right), 1 \right\}, 0 \right\} \quad (3.2)$$

The max and min functions ensure that the range of the function is bounded between zero and one. Otherwise, the basic linear function satisfies all of the price impact function properties described in chapter 2. In the ensuing quantitative analysis, the equilibrium price decline is never equal to one. The coefficient ψ_3 is determined such that the benchmark calibration delivers an initial equilibrium

price decline (Ψ_3) equal to the value specified in table 3.2.⁴

In the bottom panel of table 3.3, I compare benchmark calibration model outcomes against the data. I am able to qualitatively match the data values insofar as they differ across banks. For example, the model results show that bank 1 holds a smaller percentage of its portfolio in asset 1 and experiences less market-based capital shortfall during the fire sale period.

3.2 Increasing Ex Post Penalties and Capital Requirements

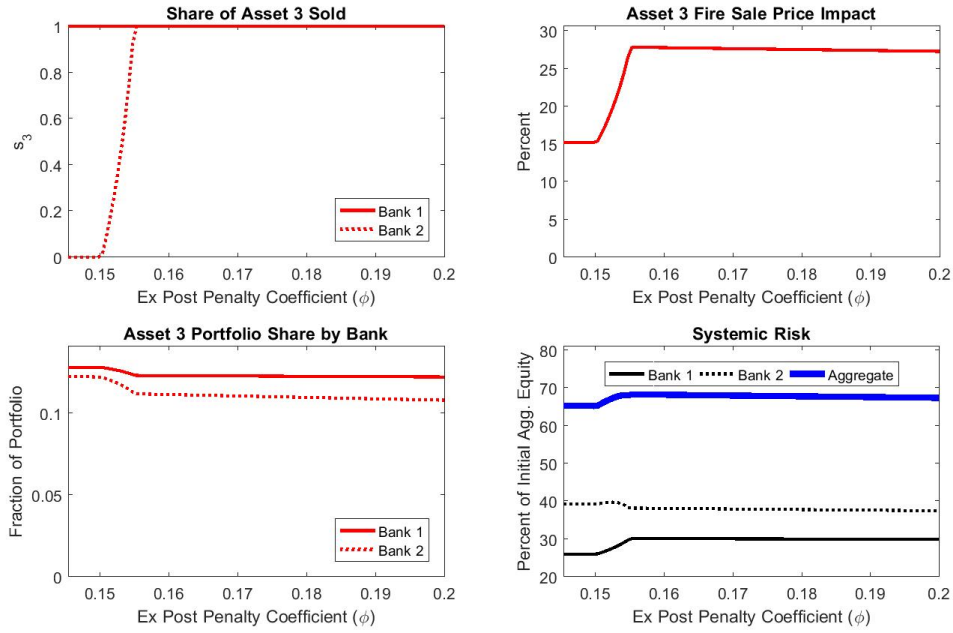
Given a calibration, we can begin our steps toward policy analysis. In this section, I investigate the effect of tightening each policy tool used in capital regulation, ex post shortfall penalties and ex ante requirements, one at a time. The goal is to understand the impact from each type of policy tool and how they may differ. These insights will be helpful in understanding the policies that combine both.

Figure 3.3(a) shows the effects from relatively small increases in the ex post penalty parameter (ϕ). Consistent with proposition 1, bank 2 joins the fire sale. As a result, there is a jump in the asset 3 fire sale price decline, and both banks shift their portfolios away from asset 3. Since the probability of a crisis is only 4%, the portfolio rebalance is modest. Importantly, systemic risk actually increases over this interval for the penalty parameter because the endogenous fire sale channel described in section 2.4 dominates.

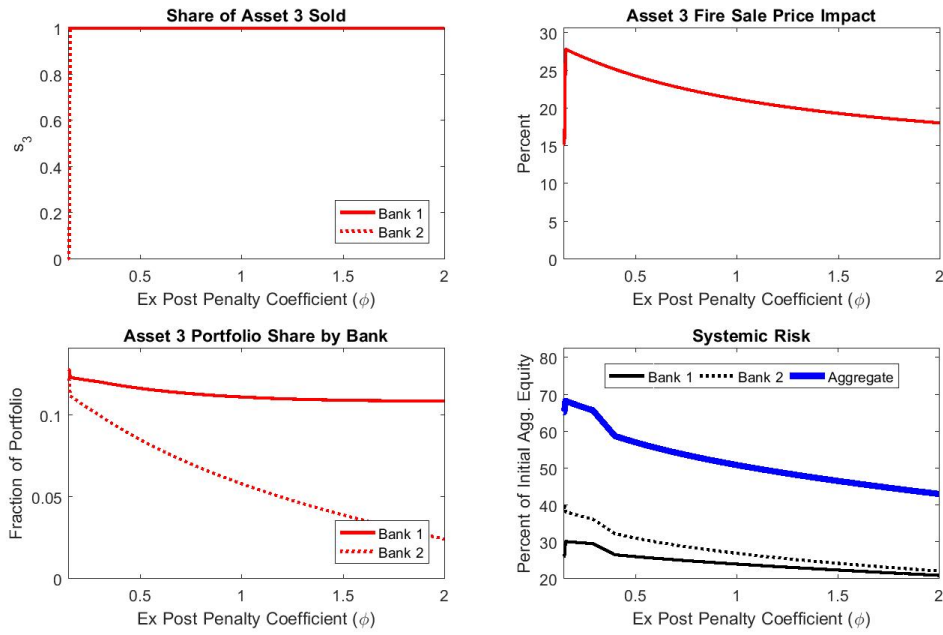
Figure 3.3(b) highlights the positive effects from increasing the ex post penalty parameter. Further increases in this penalty parameter steadily lower the fire sale price decline, as both banks shift their portfolios away from asset 3. Bank 2 shifts its holdings more dramatically because this bank suffers larger realized losses during the fire sale episode given its lesser ability to offset the price impact (high α). As a result, bank 2 is more affected by the increasing ex post penalty parameter. Systemic risk also steadily declines following the initial increase. Based on this outcome, a regulator with the goal of reducing systemic risk should either leave the ex post penalty unchanged or increase it significantly to take advantage of its positive effect.

Before discussing increasing capital requirements (κ_{reg}^b), I must clarify an important aspect of

⁴The expression is $\psi_3 = \Psi_3 / \left(\sum_{b=1}^B s_3^{*b} A_{0,3}^{*b} \right)$, where s_3^{*b} and $A_{0,3}^{*b}$ denote the optimal selling and asset holding decisions given all of the other parameter values in tables 3.2 and 3.3.



(a) Small Increases in ϕ



(b) Large Increases in ϕ

Figure 3.3: Effect of Increasing the Ex Post Penalty Parameter (ϕ)

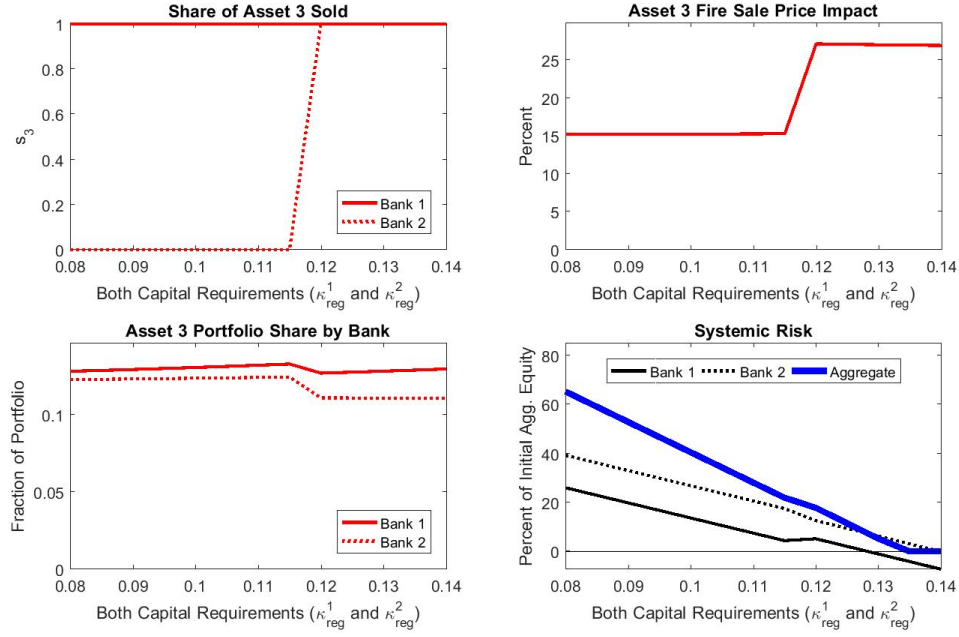
The underlying calibration uses parameter values from tables 3.2 and table 3.3 and the specific combination $\alpha_1 = 0.33$ and $\alpha_2 = 0.67$. Further calibration details are discussed in appendix D.

this exercise. Technically, banks can meet a higher capital requirement in two ways: by raising more equity capital (E_0^b) or by reducing risk-weighted assets ($w' A_0^b$). In order to preserve the size distribution of the banks and the size of the banking sector, I adjust equity capital (E_0^b) in step with capital requirements (κ_{reg}^b). A benefit to this approach is that systemic risk does not mechanically shrink, because banks become smaller with rising capital requirements. See appendix D.3 for implementation details and recent empirical evidence.

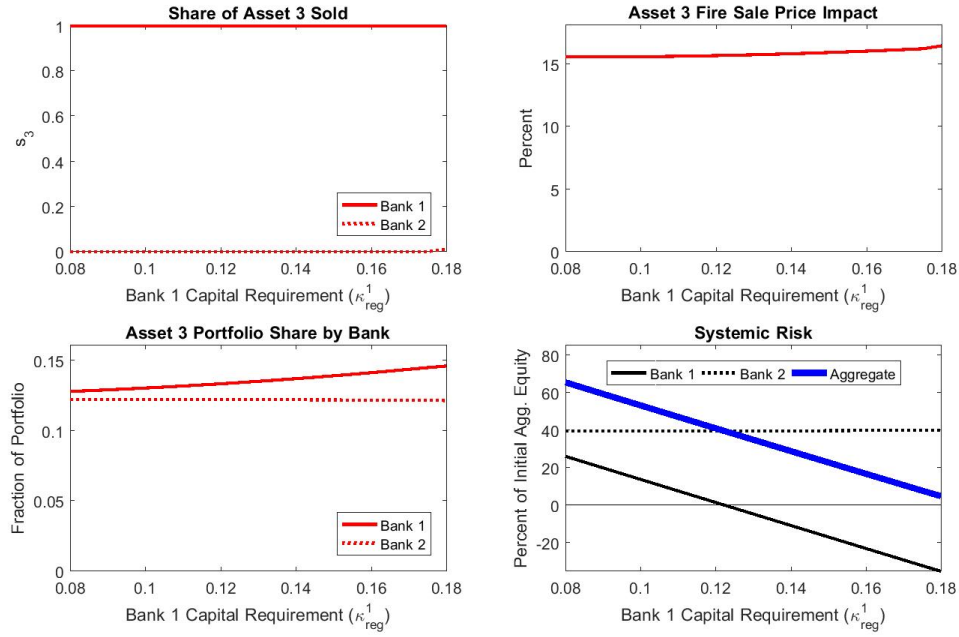
In figure 3.4(a), I show the effect of increasing capital requirements for both banks ($\kappa_{reg}^1, \kappa_{reg}^2$). In the top left panel, we see that bank 2 switches to selling asset 3 after its capital requirement is raised above 11%, which is an expected outcome based on proposition 1. As a result, the fire sale price decline of asset 3 jumps up and both banks shift their portfolios away from asset 3. In the bottom right panel, we see that, for each bank, contributions to systemic risk are monotonically decreasing, except for the slight uptick to bank 1 when bank 2 switches to selling. Note, however, that systemic risk still declines in aggregate, so the endogenous fire sale channel does not dominate in this instance.

The key difference in the effects from increasing capital requirements versus increasing the ex post penalty parameter can be explained through the decomposition of a bank's contribution to systemic risk in (2.16). Increasing the capital requirement effectively forces a bank to hold a larger initial capital ratio level relative to the systemic risk threshold (ζ). Therefore the bank's systemic risk contribution mechanically declines, all else held equal. Systemic risk contributions can even turn negative, which means that a bank's distressed capital ratio level is above the systemic risk threshold (ζ) at the end of the fire sale period. This beneficial effect differs from the portfolio reallocation caused by increasing the ex post penalty parameter (figure 3.3(b)), as the reallocation effect lowers the net fire sale loss component of a bank's systemic risk contribution.

If the regulator only increases the capital requirement for bank 1 (κ_{reg}^1), bank 2 does not switch to selling asset 3. This result is shown in figure 3.4(b). This outcome should be expected, as proposition 1 tells us that a bank's optimal selling function is increasing in its own capital requirement, not the requirements of other banks. As a related outcome, the bottom right panel shows that the systemic risk contribution for bank 2 remains constant while the contribution of bank 1 declines.



(a) Increasing Both κ_{reg}^1 and κ_{reg}^2



(b) Increasing Only κ_{reg}^1

Figure 3.4: Effect of Increasing Capital Requirements (κ_{reg}^1 and κ_{reg}^2)

The underlying calibration uses parameter values from tables 3.2 and table 3.3 and the specific combination $\alpha_1 = 0.33$ and $\alpha_2 = 0.67$. Further calibration details are discussed in appendix D.

Systemic risk therefore declines at a slower pace. This result highlights a tradeoff to the regulator: increasing capital requirements for bank 1 helps to avoid larger fire sale losses but will require a larger increase to achieve similar levels of systemic risk reduction compared to raising both capital requirements.

3.3 Policies To Mitigate Systemic Risk

In this section, I assess policies that mitigate systemic risk within the model framework. The goal is to find an optimal set of policy parameter values for each given set of non-regulatory model parameters. Aggregate capital shortfall, however, only relates to the real costs of a financial crisis. Therefore, without a mapping from aggregate capital shortfall to the level of real costs, it is not possible to precisely identify policies that maximize aggregate output or welfare. Thus, assessing the optimality of the regulatory tools from a welfare perspective is beyond the scope of this paper.

As an alternative approach, I define optimal policies as those that mitigate systemic risk while imposing the smallest cost on banks. Specifically, I solve for the policy parameters $\{\kappa_{reg}^1, \kappa_{reg}^2, \phi\}$ that maximize the average bank objective function value conditional upon zero systemic risk ($SR^{agg} = 0$). There are two reasons for specifying minimum cost as the regulator's secondary objective. First, one could view the value of a bank as a representation of the value it creates through financial intermediation and services. Second, higher costs imposed on banks through regulation may lead to unintended real costs from banks trying to minimize their regulatory burden.⁵ Therefore, minimizing the cost of regulation is consistent with a welfare-maximizing regulatory goals.

In the model, I define individual bank value as the objective function value as

$$V^b \equiv \max_{A_0^b, \beta, D_0^b} \mathbb{E}_0 \left[\tilde{R}_E^b \right] - \frac{\gamma}{2} Var_0 \left[\tilde{R}_E^b \right] \quad (3.3)$$

⁵Numerous papers have documented examples of how banks have worked around regulation to maximize profits (see, e.g., Acharya, Schnabl, and Suarez 2013; Duchin and Sosyura 2014; Houston, Lin, and Ma 2012). Acharya et al. (2009) describe how banks “manufactured” tail risk prior to the financial crisis of 2007-2009. Much of this behavior was motivated by favorable regulatory treatment and corresponding increases in profit. As a direct result, the financial crisis and corresponding recession were more severe than they would have been otherwise (Thakor 2015). While some may argue that such actions were specific to the precrisis period, others such as Kane (1981; 2014) argue that “loophole-mining” is actually a pervasive feature of the American financial system.

Therefore the average bank value is

$$V^{Avg} \equiv \frac{1}{2} (V^1 + V^2) \quad (3.4)$$

In table 3.4, I summarize the optimal policy and resulting equilibrium outcomes for the benchmark calibration.⁶ There are two key features of the optimal policy. First, it raises the capital requirement of only bank 1 (κ_{reg}^1) from 8.0% to 15.2% and leaves the capital requirement of bank 2 (κ_{reg}^2) unchanged at 8.0%. In other words, the optimal policy implies a “capital surcharge” for bank 1 relative to bank 2 (κ_{reg}^1 minus κ_{reg}^2) equal to 7.2%. Second, it increases the ex post penalty parameter significantly, from 0.145 to 1.368. In sum, the optimal policy is a capital surcharge for bank 1 and a higher ex post penalty parameter.

Both regulatory tools are tightened as part of the optimal policy because they provide systemic risk reduction with different cost-benefit tradeoffs. Capital requirements are costly to banks because they are applied ex ante. A bank is forced to hold additional capital in both noncrisis and crisis states of the world, which reduces the bank’s return on equity because it is less levered. Ex post penalties, on the other hand, are generally less costly to banks because the penalty is only applied in the crisis state (i.e., the penalty is state contingent). To understand why, note that banks optimally respond to a higher ex post penalty parameter by shifting their ex ante portfolios away from assets that generate capital shortfall. As the penalty parameter increases, banks further shift their ex ante portfolios and the corresponding cost to the bank of this distortion grows. In other words, banks find it increasingly expensive to shift their portfolios further away from the portfolio that would be optimal without the threat of ex post penalties. Intuitively, a bank only chooses to shift its portfolio further given a higher expected ex post penalty. Because the marginal return to increases in the ex post penalty parameter is decreasing, there is a level of the penalty parameter at

⁶For explanatory purposes, I use the same cross-sectional distribution for the bank-specific speed parameters $\{\alpha^1, \alpha^2\}$ as used in figures 3.3 and 3.4. Later in this section, I show how the optimal policy parameter values vary with the cross section. Although the specific values change, the optimal policy remains qualitatively similar in all cases.

	Symbol	Initial	Optimal	Restricted Optimal $\kappa_{reg}^1 = \kappa_{reg}^2$	Only Optimal Penalty
<i>Policy Parameters</i>					
Capital Requirement					
Bank 1	κ_{reg}^1	0.080	0.152	0.109	0.080
Bank 2	κ_{reg}^2	0.080	0.080	0.109	0.080
Ex Post Penalty Coeff.	ϕ	0.145	1.368	2.240	1.368
<i>Equilibrium Outcomes</i>					
Systemic Risk					
Aggregate	SR^{agg}	0.653	0.000	0.000	0.476
Bank 1	SR^1	0.260	-0.242	0.001	0.228
Bank 2	SR^2	0.393	0.242	-0.001	0.248
Objective Function					
Average	V^{avg}	1.376	1.280	1.268	1.350
Bank 1	V^1	1.376	1.212	1.270	1.352
Bank 2	V^2	1.376	1.347	1.265	1.347
Asset 3 Port. Fraction					
Bank 1	$A_{0,3}^1/1'A_0^1$	0.128	0.127	0.120	0.110
Bank 2	$A_{0,3}^2/1'A_0^2$	0.123	0.027	0.001	0.044
Bank 2 Selling Asset 3		No	Yes	Yes	Yes

Table 3.4: Optimal Policy to Mitigate Systemic Risk

The bottom section of the table shows select equilibrium outcomes using the policy parameter values listed in the top section; non-policy parameter values from tables 3.2 and 3.3; and the specific combination $\alpha_1 = 0.33$ and $\alpha_2 = 0.67$. The “Initial” column is the calibrated policy from table 3.2 and table 3.3. The “Optimal” column is the policy that maximizes (3.4) conditional upon zero systemic risk ($SR^{agg} = 0$). The “Restricted Optimal ($\kappa_{reg}^1 = \kappa_{reg}^2$)” column is the optimal policy with the additional restriction that bank capital requirements must be equal. The “Only Optimal Penalty” column is the policy that increases the ex post penalty parameter according to the optimal policy but keeps the initial capital requirements.

which the regulator finds it more cost effective to use capital requirements to achieve any remaining systemic risk reduction.⁷ This outcome is indeed the case for the benchmark calibration, and hence the optimal policy in table 3.4 tightens both capital requirements and the ex post penalty.

The optimality of the capital surcharge for bank 1 is due to bank heterogeneity and the regulator’s objective. The only difference between the two banks in the calibration is their speed parameter (α^b). The faster speed of selling (lower α^b value) for bank 1 means that the bank incurs

⁷Interestingly, the marginal return to raising capital requirements is actually increasing in the level of the capital requirement. This outcome occurs because there is a positive feedback effect in terms of reducing capital shortfall from lower leverage.

a smaller realized loss due to the fire sale. As a result, the ex ante portfolio choice of bank 1 is significantly less affected by the ex post penalty. We can see this outcome in the “Only Optimal Penalty” column in table 3.4, which shows the equilibrium outcomes if only the optimal ex post penalty is implemented relative to the initial calibration. In this case, bank 1 only reduces its relative portfolio holding of asset 3 from 12.8% to 11.0%; meanwhile bank 2 reduces its relative holding from 12.3% to 4.4%. This larger decrease in ex ante holdings of asset 3 reflects the fact that bank 2 is more penalized by the higher ex post penalty parameter. This interpretation is confirmed by the larger decrease in the value of bank 2’s objective function (from 1.376 to 1.347) compared to bank 1 (from 1.376 to 1.352). Given an objective to maximize the *average* bank value, the regulator makes up for this disparity by increasing capital requirements for bank 1 only (i.e., the capital surcharge).

As further evidence for the optimality of the capital surcharge, the average bank value (V^{avg}) under the optimal policy would be smaller if the regulator restricted itself to policies with equal capital requirements (i.e., no capital surcharge). This outcome is shown in the “Restricted Optimal ($\kappa_{reg}^1 = \kappa_{reg}^2$)” column of table 3.4. The restricted optimal policy is to increase capital requirements for both banks to 10.9% and to significantly increase the ex post penalty parameter. The average objective function value is 0.1268, which is lower than the value of 0.1280 for the unrestricted optimal policy. Although it is not surprising that restricting the regulator’s choice set yields a worse objective outcome, it is interesting to see that bank 1 is better off than bank 2 (i.e., has a higher objective function value) as a result of the restricted optimal policy ($V^1 = 0.1270 > 1.265 = V^2$). We observe the opposite outcome under the unrestricted optimal policy ($V^1 = 0.1212 < 1.347 = V^2$) due to the large capital surcharge for bank 1.

The result that bank 1 is made relatively worse off compared to bank 2 by the optimal policy can be explained as follows. Bank 1, through its speed advantage, effectively creates an externality on bank 2 in the form of a larger capital shortfall due to additional fire sale losses. In the initial equilibrium under the benchmark calibration, only bank 1 is selling asset 3 and triggering a price impact (i.e., a higher Ψ_3), which creates an unrealized loss for bank 2. As the regulator increases the ex post penalty parameter, bank 2 also begins to sell asset 3 and incurs a realized loss instead.

However, bank 1 makes this realized loss for bank 2 larger by maintaining a large holding of asset 3 (12.7% of its portfolio) that has significant price impact when sold. So not only is bank 1 less affected by the ex post penalty parameter, but its lessened response to higher penalties creates larger losses for bank 2. Thus it makes sense that bank 1 is effectively more penalized than bank 2 in the optimal policy. In fact, we see in table 3.4 that bank 1 *negatively* contributes to systemic risk under the optimal policy.⁸

As a final step, I investigate how the optimal policies change depending on the assumed cross-sectional distribution of the bank-specific “speed” parameter $\{\alpha^1, \alpha^2\}$. In figure 3.5, I show the optimal policies and select equilibrium outcomes for different combinations of α^1 and α^2 . The optimal bank 2 capital requirement (κ_{reg}^2) remains at 8% and therefore is not shown in the figure. The top left panel shows that the optimal capital surcharge for bank 1 (κ_{reg}^1 minus κ_{reg}^2) is 6% to 8%. The top right panel shows that the optimal ex post penalty parameter is always significantly higher than the benchmark calibrated value (the dotted line) and is generally increasing in both α_1 and α_2 , with one exception (high α_1 and $\alpha_2 = 0.70$). Consistent with the large capital surcharge, the bottom left panel shows that bank 1 always has a negative systemic risk contribution. Finally, we see that both banks end up selling asset 3 during the fire sale, resulting in a higher fire sale price impact (bottom right panel; dotted line shows the initial value). In sum, we see the optimal policies are qualitatively similar to the benchmark optimal policy in table 3.4.

⁸A negative contribution to systemic risk occurs when the bank’s distressed capital ratio at the end of $t = 1$ is above the fixed systemic risk threshold (ζ), which does not depend on regulation.

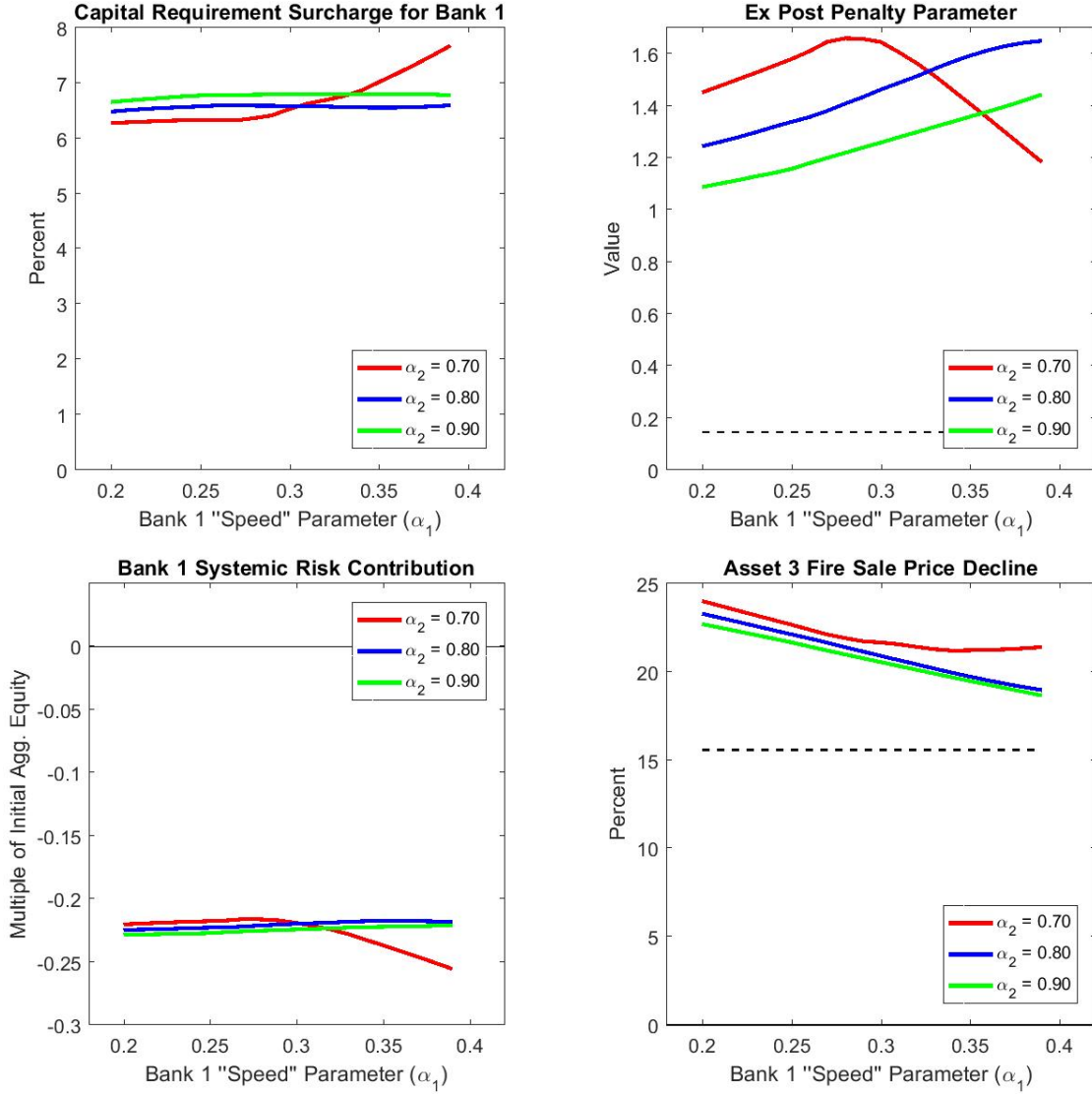


Figure 3.5: Optimal Policies for Different Cross Sections

Non-policy parameter values from table 3.2 and table 3.3. The optimal capital requirement for bank 2 is not shown above and is $\kappa_{reg}^2 = 0.08$. The capital requirement surcharge for bank 1 is the additional minimum requirement for bank 1 compared to bank 2 ($\kappa_{reg}^1 - \kappa_{reg}^2$). The dotted lines in the top right and bottom right panels are the benchmark calibration values. Further calibration details are discussed in appendix D.

CHAPTER 4

RELATION TO CURRENT REGULATORY POLICY

In this chapter, I discuss how the model insights and quantitative results relate to the new regulatory framework for banks in the US. I find that, in general, my results for the least costly policies to mitigate systemic risk are consistent with the steps that have been implemented since 2010. I also identify how some of the new macroprudential regulatory tools can potentially address the endogenous fire sale channel directly. This discussion is not exhaustive of the many changes to the regulatory landscape over the past decade, but rather it is meant to highlight the key areas in which the analysis in this study is most applicable. I do not discuss the many proposals for tools or schemes that have not yet gained significant traction in the US, although they might address the concerns raised in my analysis.¹

4.1 Capital Requirements

The first significant post-crisis change to the regulatory framework came in 2010. Lawmakers passed the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 (“Dodd-Frank”), a piece of legislation described by some as the “most significant reregulation of the banking industry since Glass-Steagall.”² Dodd-Frank led to two particularly notable changes: (1) an increase in the minimum capital requirement ratio of 2% for all banks,³ and (2) the establishment of enhanced

¹An example of a proposed capital tool is contingent convertible bonds (CoCos), which is debt that is converted to equity in response to a triggering event. These hybrid securities offer a way to reduce risk shifting and “bail in” creditors, but there are concerns about their benefits in practice (see the discussion in Thakor 2014). Initial evidence from CoCos issued in Europe suggests that uncertainty about regulatory discretion is an additional concern in practice (Gleason, Bright, Martinez, and Taylor 2017). Other proposals include two kinds of capital requirements (Acharya, Mehran, and Thakor 2016), required preemptive rights offerings when capital too low (Duffie 2011), minimum capital levels based on the prices of credit default swaps (Hart and Zingales, 2011), and many more.

²From a February 13, 2012 letter written by the Chief Investment Officer of the California Public Employees’ Retirement System (CalPERS) to all regulatory agencies.

³Federal Register, Vol. 78, No. 198 (Friday, October 11, 2013).

prudential standards for institutions deemed “systemically important.” This latter change includes a capital surcharge for global systemically important banks (G-SIBs).⁴

My quantitative policy results regarding optimal capital requirements are broadly consistent with the changes to capital requirements described above. In section 3.3, I found that the least costly policies to mitigate systemic risk involve a capital surcharge 6% to 8% on the banks that can best offset price impact. These banks correspond to the largest banks in the data, specifically the G-SIBs. In the finalized 2015 rule, regulators decided that these banks would be subject to a capital surcharge of 1% to 4.5% depending on regulatory assessment.⁵ The stated rationale for the G-SIB surcharge is (1) to create incentives for the G-SIBs to shrink their systemic footprint and (2) to combat the funding advantage that G-SIBs enjoy from being perceived as “too big to fail.” Although similar, the rationale for the bank 1 capital surcharge in my setting differs in the source of the benefit. Specifically, the surcharge compensates for bank 1’s speed advantage and role in creating fire sale losses for other banks. Thus, my findings offer a fire-sale-based justification for the G-SIB surcharge.

4.2 Ex Post Penalty

The ex post shortfall penalty parameter in the model (ϕ) represents the costs that a bank’s equity holders expect to pay as the result of being undercapitalized during a crisis. Although captured in a single parameter, these penalties have two fundamentally different sources: distress and regulatory. For the purpose of policy analysis, I considered ϕ to be a regulatory parameter under the assumption that the regulator has both the ability to offset distress costs and to increase regulatory penalties. Importantly, regulatory penalties are assumed to take the form of restrictions or forced actions that lower the market value of equity, as opposed to penalties assessed to the bank that would directly weaken its capital level. This distinction is important because it means that reducing anticipated assistance in offsetting distress costs is functionally equivalent to increasing the regulatory parameter.

⁴See Federal Register, Vol. 80, No. 157, (Friday, August 14, 2015).

⁵See “Calibrating the GSIB Surcharge,” Federal Reserve Board, July 20, 2015 available at <https://www.federalreserve.gov/publications/2015-calibrating-thegsib-surcharge-introduction.htm>.

Both regulatory statements and actions signal that distressed banks are now more likely to be allowed to fail relative to before the crisis. The full title of Dodd-Frank explicitly mentions that the act is intended to end “too big to fail,” which is the idea that large banks would expect to be bailed out by regulators during a crisis. This expectation, along with the corresponding moral hazard, was believed to have been a significant contributing factor to the severity of the financial crisis (Thakor 2015). Dodd-Frank also requires large banks to submit “living wills” (also known as “resolution plans”) each year that specify how the bank is to be wound down in time of distress.⁶ Moreover, Dodd-Frank created the Orderly Liquidation Authority (OLA), allowing the federal government to take over and wind down a failing financial institution even if it is not a commercial bank.⁷

In sum, equity holders should expect the value of their claims to decline more in the event of a crisis, which translates into a higher ϕ through the lens of the model. Recent policy changes are therefore consistent with the optimal policies found in section 3.3. In line with model predictions, Ignatowski and Korte (2014) provide empirical evidence that US banks now subject to the OLA have responded by reducing their riskiness. Duffie (ming) notes that creditors seem convinced that banks are more likely to be allowed to fail according to credit default swap spreads and unsecured borrowing rates.

4.3 Regulatory Stress Tests

Annual stress tests have been a significant addition to the regulatory toolkit since 2012. This annual exercise is formally known as the Comprehensive Capital Analysis and Review (CCAR). The purpose is to ensure that the largest banks have “sufficient capital to continue operations throughout times of economic and financial stress.”⁸ In the exercise, banks submit their plans to make capital distributions (dividend payments or stock repurchases), and the Federal Reserve evaluates

⁶These resolution plans are also published on the Federal Reserve website:
<https://www.federalreserve.gov/supervisionreg/resolution-plans.htm>.

⁷Prior to the crisis, the FDIC’s authority was restricted to commercial banks. See Aaron Klein, “A primer on Dodd-Frank’s Orderly Liquidation Authority,” *Brookings Up Front*, June 5, 2017.

⁸<https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm>

the banks' hypothetical capital ratios under multiple adverse economic scenarios. If a bank is projected to be undercapitalized in any of these scenarios, the bank fails the stress test. As a result, its proposed capital distribution plan is likely rejected and the bank will also likely not be allowed to pay dividends.⁹

Stress tests impact a bank's portfolio choice in the same way as raising the ex post penalty parameter in the model. By disallowing dividend payments, a bank is effectively penalized today for a capital shortfall in a future distressed state of the world. In the model, banks internalize the impact from expected penalties in the crisis state at $t = 1$ into the portfolio choice at $t = 0$ (see, e.g., figure 3.3(b)). As a result, increasing the penalty parameter ϕ causes banks to shift their portfolio at $t = 0$. This ex ante shift represents the present value of the impact from a higher ex post penalty. From this perspective, increasing the ex post penalty can be interpreted as making the stress test scenarios more severe.

Stress tests offer a potential antidote to the endogenous fire sale channel, but this promise may not hold in practice. Despite acting like an increase in the ex post penalty parameter, increasing the severity of the stress tests does not actually require imposing a larger ex post penalty. Therefore banks are not more incentivized to sell as proposition 1 predicts, and the endogenous fire sale channel should not be a concern. This conclusion has two problems. First, banks may learn to game the stress tests over time, rendering them ineffective. Second, stress tests are expensive for both regulators and the bank. As a result, there have been recent calls to conduct stress tests less frequently and also to exclude more banks from full participation.¹⁰

⁹The punishment is determined at the regulator's discretion. According to Hirtle and Lehnert (2014): "If the CCAR qualitative assessment reveals significant weaknesses ... the bank holding company may make only those dividend payments and share repurchases approved by the Federal Reserve and must resubmit its capital plan after addressing the concerns raised in the initial review. Depending on the nature and extent of the concerns about a bank holding company's capital plan and current capital position, the Federal Reserve could require the company to stop dividend payments and share repurchases entirely or could permit these actions within certain bounds."

¹⁰In 2017, only 13 of the 34 banks included in CCAR were required to participate in the full stress testing exercise (quantitative and qualitative). Reducing "significant burden on these firms" is the stated reason for exempting so many banks from the qualitative assessment. See <https://www.federalreserve.gov/newsevents/pressreleases/bcreg20170130a.htm>

4.4 Countercyclical Capital Buffer

The countercyclical capital buffer (CCyB) is a new addition to the Federal Reserve's toolkit. Introduced in the 2010 Basel III Accord,¹¹, the CCyB is meant to protect the banking sector from periods of excess credit growth and leverage buildup.¹² It works by allowing the Federal Reserve to increase minimum capital requirements by up to 2.5% when systemic risk is perceived to be high.

In the model, a CCyB could be represented as a conditional buffer added to the bank's minimum capital requirements that has a value of zero in the crisis state. Formally, a CCyB of η would imply that capital shortfall at $t = 1$ is computed as

$$\frac{E_1 + CS_1}{w' A_1} = \kappa_{reg} + \eta \quad (4.1)$$

This definition is identical to (2.12) when $\eta = 0$. This implementation, however, is not quite the same as the CCyB is intended to function. A more accurate and complete representation would provide a signal to the regulator about the shock at $t = 0$, and the regulator would implement the CCyB based on this signal.

Including a CCyB would appear to allow the regulator to avoid the endogenous fire sale channel. When banks are deciding whether to sell during $t = 1$, the relevant capital ratio is only κ_{reg} . Therefore, by proposition 1, the bank has less incentive to sell compared to the case when the relevant capital ratio is $\kappa_{reg} + \eta$. The regulator, however, still benefits from forcing banks to hold more capital initially.

In practice, however, the CCyB faces many challenges. First of all, the regulator needs to

¹¹Basel III is a comprehensive set of reform measures developed by the Basel Committee on Banking Supervision, which is an international committee of banking supervisory authorities. The goal of the framework is to strengthen the regulation, supervision, and risk management of the banking sector. See <http://www.bis.org/bcbs/basel3.htm>.

¹²In addition to being part of the US Basel III implementation, the CCyB also addresses the provision in Dodd-Frank that the regulatory agencies "shall seek to make such [capital] requirements countercyclical, so that the amount of capital required to be maintained by a company increases in times of economic expansion and decreases in times of economic contraction, consistent with the safety and soundness of the company." See 12 U.S.C. 1467a; 12 U.S.C. 1844; 12 U.S.C. 3907 (as amended by section 616 of the Dodd-Frank).

enact it *before* a crisis occurs and with sufficient lag so that banks can raise the needed capital. Such foresight is understandably difficult to expect (Adrian, de Fontnouvelle, Yang, and Zlate 2017). Second, regulators have little experience with CCyBs. The Federal Reserve only finalized its framework for planned future implementation of a CCyB in October 2016.¹³ The literature has only just begun to address the issue (see, e.g., Davydiuk 2017).

4.5 Liquidity Requirements

In addition to raising capital requirements, regulators have also introduced new liquidity requirements since the crisis. These requirements effectively limit the extent to which banks can hold long-term illiquid portfolios given short-term debt. The belief is that banks engaged in “excessive” maturity transformation prior to the crisis, which made solvent institutions vulnerable to runs and created additional distress during the height of the crisis (e.g., Yellen 2014). Introduced as part of Basel III, liquidity requirements take the form of a liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR). The LCR requires that a bank has enough high-quality liquid assets to meet withdrawals during a 30-day run scenario. The NSFR requires that a bank has sufficient stable (i.e., long-term) funding to withstand an extended period of market distress (up to one year). In the US, banks have been required to comply with a LCR since 2015 while an NSFR is still in development.¹⁴

If taken literally, the LCR and NSFR would not have any impact if introduced within my model framework because the assets and bank debt are both long-term with the same maturity ($t = 2$). However, suppose that we interpret all bank debt in the model as one-period and that the ex post shortfall penalty (ϕ) captures all of the distress costs from rolling over this debt during the crisis. In this case, I can represent a LCR as a required minimum relative holding for asset 2

$$\frac{A_{t,2}^b}{1'A_t^b} \geq \ell$$

¹³See Federal Reserve Systemic Docket No. R-1529 RIN 7100 AE-43, 2016-21970. See also 12 CFR 217, Appendix A.

¹⁴For the LCR rule, see Federal Register, Vol. 79, No. 197, (Friday, October 10, 2014). The June 2016 NSFR proposal can be found at <https://www.gpo.gov/fdsys/pkg/FR-2016-06-01/pdf/2016-11505.pdf>.

where ℓ is specified by the regulator. Note that this constraint must bind in every period, which is a feature of the LCR in practice. The above constraint nests my model as described in chapter 2 in which $\ell = 0$.

The LCR described above offers a potentially more cost-effective way to reduce systemic risk compared to the ex post penalty. In chapter 3, we saw that increasing the ex post penalty parameter (ϕ) reduces systemic risk by causing a bank to shift its portfolio towards asset 2. Using the LCR, the regulator could achieve similar systemic risk reduction by setting the liquidity requirement parameter (ℓ) equal to the asset 2 portfolio share that would be observed under the larger ex post penalty parameter. This approach offers the added benefit that it will not induce additional selling during a fire sale because banks' ex post selling decisions are unaffected. However, the LCR also generates a distinct cost relative to the ex post penalty in terms of excess liquidity. The LCR prevents banks from deploying excess liquidity even during a crisis, which implies a potentially large social cost (Diamond and Kashyap 2016).

CHAPTER 5

CONCLUSION

To study the impact of bank capital regulation on systemic risk, I have developed a model with heterogeneous banks and endogenous fire sales. The decisions of individual banks affect the losses of others through a fire sale externality that can occur in a crisis state, and a regulator can influence bank behavior (and therefore systemic risk) through two capital-based policy tools: ex ante capital requirements and ex post shortfall penalties.

My analysis delivers three key results. First, I show the existence of an endogenous fire sale channel through which tighter capital regulation can unintentionally lead to higher systemic risk. Second, I show in a calibrated version of the model that the least costly policies to mitigate systemic risk raise both ex ante capital requirements and ex post shortfall penalties. This result highlights that these regulatory tools provide systemic risk reduction with different cost-benefit tradeoffs. Third, these least costly risk-mitigation policies assign relatively higher capital requirements to banks that can better offset price declines during a fire sale. This capital surcharge is the result of bank heterogeneity and the regulator's objective.

My findings have two broad implications for current and future regulatory policy. First, my quantitative results are consistent with the recently implemented enhanced prudential standards for institutions deemed "systemically important" and, in particular, the capital surcharge for the global systemically important banks (G-SIBs). Therefore, my results provide a fire-sale-based justification for the current regulatory regime. Second, my findings provide further support for leading-edge macroprudential tools, including stress tests and countercyclical capital buffers (CCyBs). Their key potential advantage is to reduce systemic risk without triggering the endogenous fire sale channel.

Further research should expand this framework to include other sectors of the economy. For

example, I currently model in reduced form outside investors who provide liquidity during a fire sale. It would be interesting to consider how changes in banking regulation indirectly affect this group of investors and consequently their role in fire sales. These investors likely play in large role in the recovery following a financial crisis, particularly in the provision of capital. Addressing these types of questions will help to advance the broader agenda of a welfare analysis of the current and hypothetical regulatory regimes.

APPENDIX A

PROOFS

Proof Proposition 1 From lemma B.0.1, the expression for the optimal asset 3 selling choice of bank b ignoring the lower and upper bounds is

$$1 - \frac{(\mu_3 - R_D) - \phi [(1 - \alpha^b - \kappa_{reg}^b w_3) \Psi_3 + \kappa_{reg}^b w_3]}{\gamma \sigma_3^2 A_{0,3}^b / E_0^b} \quad (\text{A.1})$$

This expression is increasing in ϕ because

$$(1 - \alpha^b - \kappa_{reg}^b w_3) \Psi_3 + \kappa_{reg}^b w_3 = (1 - \alpha^b) \Psi_3 + (1 - \Psi_3) \kappa_{reg}^b w_3 > 0$$

given that $\alpha^b \in (0, 1)$ and $\Psi_3 \in [0, 1]$. This expression is also increasing in κ_{reg}^b because $(1 - \Psi_3) \in [0, 1]$ as well.

The expression in (A.1) is increasing in Ψ_3 only if $(1 - \alpha^b - \kappa_{reg}^b w_3) > 0$, which is equivalent to

$$\alpha^b < 1 - \kappa_{reg}^b w_3$$

Finally, the expression in (A.1) is weakly decreasing in α^b because $\Psi_3 \in [0, 1]$. If $\Psi_3 > 0$, the expression is strictly decreasing in α^b . The consequence of increasing α^b is that the bank receives a lower price when selling during a fire sale. Therefore, in words, the bank's optimal selling decision is weakly increasing in its ability to offset price impact.

In lemma B.0.1, the optimal selling choice (s_3^*) is bounded between 0 and 1. If the optimal choice is strictly below or above these boundaries ($s_3^* < 0$ or $s_3^* > 1$), all partial derivatives are zero. Therefore, as a general statement, the optimal selling choice can only be described as weakly increasing in $\{\phi, \kappa_{reg}^b, \Psi_3\}$ and weakly decreasing in $\{\alpha^b\}$.

Proof Proposition 3 For an optimally chosen level of selling (s_3^*) and portfolio choice (A_0^*), the

net capital shortfall generated from asset 3 (CS_3) is

$$CS_3 \equiv \{[(1 - \kappa_{reg} w_3) (1 - s_3^*) + \alpha s_3^*] \Psi_3 - s_3^* \kappa_{reg} w_3\} A_{0,3}^* \quad (\text{A.2})$$

Take the partial derivative with respect to Ψ_3

$$\begin{aligned} \frac{\partial CS_3}{\partial \Psi_3} &= [(1 - \kappa_{reg} w_3) (1 - s_3^*) + \alpha s_3^*] A_{0,3}^* \\ &\quad + \{[(1 - \kappa_{reg} w_3) (1 - s_3^*) + \alpha s_3^*] \Psi_3 - s_3^* \kappa_{reg} w_3\} \frac{\partial A_{0,3}^*}{\partial \Psi_3} \end{aligned} \quad (\text{A.3})$$

This partial derivative is positive if

$$-\frac{\frac{\partial A_{0,3}^*}{\partial \Psi_3}}{A_{0,3}^*} < \frac{(1 - \kappa_{reg} w_3) (1 - s_3^*) + \alpha s_3^*}{[(1 - \kappa_{reg} w_3) (1 - s_3^*) + \alpha s_3^*] \Psi_3 - s_3^* \kappa_{reg} w_3} \quad (\text{A.4})$$

In words, the percent decline in asset 3 holding must not be too large. We will see below in (A.6) that $\frac{\partial A_{0,3}^*}{\partial \Psi_3} < 0$ with certainty and so the left-hand side represents always the percent decline in holding. The right-hand side is greater than or equal to 1. If $s_3 > 0$ then it is greater than 1 with certainty.

From lemma B.0.2, the expression for the optimal asset 3 holding is

$$\begin{aligned} \frac{\gamma (1 - q + q (1 - s_3)^2) \sigma_3^2}{E_0} A_{0,3}^* &= (1 - q s_3) (\mu_3 - R_D) \\ &\quad - q \phi ([(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3) \\ &\quad - \left(\frac{w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c}) - \frac{\gamma}{\kappa_{reg} (1 + \beta)}}{w' \bar{\Sigma}^{-1} w} w_3 \right) + Z_3 \end{aligned} \quad (\text{A.5})$$

where we have removed the non-negativity multiplier ($\Gamma_{x,3} = 0$) because we are assuming that the asset 3 holding is positive. In the remainder of the proof, we will assume that either $s_3 = 0$ or $s_3 = 1$. If a set of parameters delivers our outcome of interest at either of these extremes then we know the outcome will also hold for values of s_3 in between. Because we assume $s_3 = 0$ or $s_3 = 1$, we can set $Z_3 = 0$.

Using expressions as needed from lemma B.0.2 and assuming $Z_3 = 0$, take the partial derivative of (A.5)

$$\frac{\partial x_{0,3}^*}{\partial \Psi_3} = \frac{E_0}{\gamma (1-q + q(1-s_3)^2) \sigma_3^2} \left(-q\phi [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \left(1 - \frac{\frac{w_3^2}{(1-q+q(1-s_3)^2)\sigma_3^2}}{w' \bar{\Sigma}^{-1} w} \right) \right) \quad (\text{A.6})$$

Combine (A.5) and (A.6) to form an expression for the percent decline in holding

$$\frac{-\frac{\partial A_{0,3}^*}{\partial \Psi_3}}{A_{0,3}^*} = \frac{q\phi_1 [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \left(1 - \frac{\frac{w_3^2}{(1-q+q(1-s_3)^2)\sigma_3^2}}{w' \bar{\Sigma}^{-1} w} \right)}{(1 - qs_3) (\mu_3 - R_D) - q\phi_1 [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 - \frac{w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c}) - \frac{\gamma}{\kappa_{reg}(1+\beta)}}{w' \bar{\Sigma}^{-1} w} w_3}$$

If $s_3 = 0$, the condition for $\frac{\partial CS_3}{\partial \Psi_3} > 0$ is

$$\frac{q\phi_1 (1 - \kappa_{reg} w_3) \left(1 - \frac{\frac{w_3^2}{\sigma_3^2}}{w' \bar{\Sigma}^{-1} w} \right)}{\mu_3 - R_D - q\phi_1 (1 - \kappa_{reg} w_3) \Psi_3 - \frac{w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c}) - \frac{\gamma}{\kappa_{reg}(1+\beta)}}{w' \bar{\Sigma}^{-1} w} w_3} < \frac{1}{\Psi_3}$$

Given that the Lagrange multiplier term must be non-negative, we know that

$$\frac{q\phi_1 (1 - \kappa_{reg} w_3) \left(1 - \frac{\frac{w_3^2}{\sigma_3^2}}{w' \bar{\Sigma}^{-1} w} \right)}{\mu_3 - R_D - q\phi_1 (1 - \kappa_{reg} w_3) \Psi_3 - \frac{w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c}) - \frac{\gamma}{\kappa_{reg}(1+\beta)}}{w' \bar{\Sigma}^{-1} w} w_3} < \frac{q\phi_1 (1 - \kappa_{reg} w_3)}{\mu_3 - R_D - q\phi_1 (1 - \kappa_{reg} w_3) \Psi_3}$$

Therefore a sufficient condition for our outcome of interest is

$$\frac{q\phi_1 (1 - \kappa_{reg} w_3)}{\mu_3 - R_D - q\phi_1 (1 - \kappa_{reg} w_3) \Psi_3} < \frac{1}{\Psi_3}$$

Take the inverse of both sides to find

$$\frac{\mu_3 - R_D}{2q\phi_1 (1 - \kappa_{reg} w_3)} > \Psi_3 \quad (\text{A.7})$$

If $s_3 = 1$ then the condition for capital shortfall due to asset 3 increasing $\left(\frac{\partial CS_3}{\partial \Psi_3} > 0\right)$ is

$$\frac{q\phi_1 \left(1 - \frac{\frac{w_3^2}{(1-q)\sigma_3^2}}{w'\bar{\Sigma}^{-1}w}\right)}{(1-q)(\mu_3 - R_D) - q\phi_1(\alpha\Psi_3 - \kappa_{reg}w_3) - \frac{w'\bar{\Sigma}^{-1}(\bar{\mu}-\bar{c}) - \frac{\gamma}{\kappa_{reg}(1+\beta)}}{w'\bar{\Sigma}^{-1}w}w_3} < \frac{1}{\alpha\Psi_3 - \kappa_{reg}w_3}$$

Follow a similar process to find

$$\frac{(1-q)(\mu_3 - R_D)}{2q\phi_1} + \kappa_{reg}w_3 > \alpha\Psi_3 \quad (\text{A.8})$$

In conclusion, we can combine the sufficient conditions in (A.7) and (A.8) to find the following single sufficient condition such that capital shortfall due to asset 3 is increasing in Ψ_3

$$\min \left\{ \frac{\mu_3 - R_D}{2q\phi(1 - \kappa_{reg}^b w_3)}, \frac{(1-q)(\mu_3 - R_D) + \kappa_{reg}w_3 2q\phi}{2q\phi\alpha^b} \right\} > \Psi_3 \quad (\text{A.9})$$

Assumption 1: Probability of Crisis (q) Sufficiently Small. Suppose that a bank optimally chooses portfolio (A_0^*) and capital buffer ($\beta^* \geq 0$) given an equilibrium asset 3 price decline (Ψ_3) under the assumption that it cannot purchase asset 3 during a fire sale. Suppose further that the bank is weakly undercapitalized in the crisis state ($CS_1^* \geq 0$) as a result of these optimal decisions. If it could purchase asset 3 during a fire sale, the bank may want to hold a portfolio (A_0^{**}) and capital buffer ($\beta^{**} > \beta^*$) that ensures it is overcapitalized in the crisis state ($CS_1^{**} < 0$) so it can purchase asset 3 cheaply. The expected profit from this trade is the probability of a crisis (q) multiplied by the price return from purchasing asset 3 at a discount ($1/(1 - \Psi_3)$) multiplied by the amount of asset 3 it can purchase. A bank is limited in its ability to purchase asset 3 by its available holding of asset 2, which it sells in order to purchase asset 3 cheaply.

If $q = 0$, this strategy has zero expected profit and therefore a bank will optimally choose $\{A_0^*, \beta^*\}$ if given the choice. As q increases, the expected profit from the trade increases and this additional benefit can eventually outweigh the objective function value loss from choosing $\{A_0^{**}, \beta^{**}\}$ instead of $\{A_0^*, \beta^*\}$. This threshold value for q , if it exists, is an upper bound for the

bank-specific “sufficiently small” value to avoid following the purchase strategy. Let us denote this value \bar{q}^b .

If the bank does switch to the “purchase” strategy, the equilibrium asset 3 price decline (Ψ_3) will become smaller for two reasons. First, the bank may have been selling under its old strategy and therefore no longer contributes to the aggregate quantity of asset 3 sold. Second, the bank will be purchasing asset 3 and therefore further reducing the aggregate quantity sold. Both effects reduce the aggregate quantity of asset 3 sold, which is the input to the price impact function. As a result, the asset 3 price decline (Ψ_3) would necessarily become smaller. Therefore \bar{q}^b is potentially much larger than the threshold value of q for which the bank would switch strategies *given* Ψ_3 , and it may even be larger than 1.

The final “sufficiently small” value for q is the minimum value of \bar{q}^b among all banks, which will guarantee that no bank follows the strategy to hold additional capital in order to purchase asset 3 at a fire sale price.

APPENDIX B

OPTIMALITY CONDITIONS

Lemma B.0.1 *Optimal Selling Decisions.* *If $\tilde{\xi} = 0$, optimal selling is zero for both assets by assumption ($s_2^* = 0$ and $s_3^* = 0$). See footnote 2 for the explanation.*

If $\tilde{\xi} = \xi$, optimal selling decisions are

$$s_3^* = \min \{s_3^{**}, \bar{s}_3\} \quad (\text{B.1})$$

$$s_2^* = \min \{s_2^{**}, \bar{s}_2\} \quad (\text{B.2})$$

where

$$s_3^{**} = \begin{cases} 0 & \phi \leq \underline{\phi} \\ 1 - \frac{(\mu_3 - R_D) - \phi[(1 - \alpha - \kappa_{reg} w_3) \Psi_3 + \kappa_{reg} w_3]}{\gamma \sigma_3^2 A_{0,3} / E_0} & \phi \in (\underline{\phi}, \bar{\phi}) \\ 1 & \phi \geq \bar{\phi} \end{cases} \quad (\text{B.3})$$

$$\underline{\phi} \equiv \frac{\mu_3 - R_D - \gamma \sigma_3^2 A_{0,3} / E_0}{(1 - \alpha - \kappa_{reg} w_3) \Psi_3 + \kappa_{reg} w_3} \quad (\text{B.4})$$

$$\bar{\phi} \equiv \frac{\mu_3 - R_D}{(1 - \alpha - \kappa_{reg} w_3) \Psi_3 + \kappa_{reg} w_3} \quad (\text{B.5})$$

and

$$\bar{s}_3 = \frac{\max \left\{ (1 - \kappa_{reg} w_1) \xi A_{0,1} + (1 - \kappa_{reg} w_3) \Psi_3 A_{0,3} - \left(1 - \frac{1}{1+\beta}\right) E_0, 0 \right\}}{[(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3] A_{0,3}} \quad (\text{B.6})$$

and

$$s_2^{**} = \begin{cases} 0 & \phi \leq \frac{\mu_2 - R_D - \gamma \sigma_2^2 A_{0,2} / E_0}{\kappa_{reg} w_2} \\ 1 - \frac{(\mu_2 - R_D) - \phi \kappa_{reg} w_2}{\gamma \sigma_2^2 A_{0,2} / E_0} & \frac{\mu_2 - R_D - \gamma \sigma_2^2 A_{0,2} / E_0}{\kappa_{reg} w_2} < \phi < \frac{\mu_2 - R_D}{\kappa_{reg} w_2} \\ 1 & \phi \geq \frac{\mu_2 - R_D}{\kappa_{reg} w_2} \end{cases} \quad (\text{B.7})$$

and

$$\overline{s_2} = \frac{\max \left\{ (1 - \kappa_{reg} w_1) \xi A_{0,1} - \left(1 - \frac{1}{1+\beta} \right) + \{ \alpha \Psi_i - \kappa_{reg} w_3 \} A_{0,3}, 0 \right\}}{\kappa_{reg} w_2 A_{0,2}} \quad (\text{B.8})$$

Proof Suppose that $\tilde{\xi} = \xi$. A bank chooses optimal selling for asset 2 and 3 (s_2 and s_3) to solve the following problem

$$\max_{s_2 \in [0,1], s_3 \in [0,1]} \mathbb{E}_0 \left[\tilde{R}_E \mid \tilde{\xi} = \xi \right] - \frac{\gamma}{2} \text{Var}_0 \left[\tilde{R}_E \mid \tilde{\xi} = \xi \right] \quad (\text{B.9})$$

Plug in the expressions for \tilde{R}_E , A_1 , D_1 , and Φ and $\tilde{\xi} = \xi$ and then take expectations. The rewritten problem is

$$\max_{s_2 \in [0,1], s_3 \in [0,1]} \hat{\mu}(\xi)' A_0 + R_D - \mathbb{I}_{CS_1 \geq 0} \phi CS_1 - \frac{\gamma}{2E_0} A_0' \hat{\Sigma}(\xi) A_0 + \Gamma_0' s + \Gamma_1' (1 - s) \quad (\text{B.10})$$

where

$$\begin{aligned} \frac{CS_1}{E_0} &= (1 - \kappa_{reg} w_1) \xi A_{0,1} - \left(1 - \frac{1}{1+\beta} \right) E_0 - s_2 \kappa_{reg} w_2 A_{0,2} \\ &\quad + \{ [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 \} A_{0,3} \end{aligned} \quad (\text{B.11})$$

and I have defined

$$\hat{\mu}(\xi) \equiv \begin{bmatrix} \mu_1 (1 - \xi) - R_D \\ (1 - s_2) (\mu_2 - R_D) \\ (1 - s_3) (\mu_3 - R_D) \end{bmatrix} \quad (\text{B.12})$$

$$\hat{\Sigma}(\xi) \equiv \begin{bmatrix} (1 - \xi)^2 \sigma_1^2 & 0 & 0 \\ 0 & (1 - s_2)^2 \sigma_2^2 & 0 \\ 0 & 0 & (1 - s_3)^2 \sigma_3^2 \end{bmatrix} \quad (\text{B.13})$$

and Γ_0 and Γ_1 are vectors of Lagrange multipliers for the lower and upper bounds of s_2 and s_3 . The indicator function $\mathbb{I}_{CS_1 \geq 0}$ captures that the penalty is only paid if capital shortfall is non-negative.

The first order conditions for s_2 and s_3 are

$$0 = -(\mu_2 - R_D) A_{0,2} + \mathbb{I}_{CS_1 \geq 0} \phi \kappa_{reg} w_2 A_{0,2} \quad (\text{B.14})$$

$$- \frac{\gamma}{2E_0} A'_0 \left[\frac{\partial}{\partial s_2} \hat{\Sigma}(\xi) \right] A_0 + \Gamma_{0,2} - \Gamma_{1,2}$$

$$0 = -(\mu_3 - R_D) A_{0,3} + \mathbb{I}_{CS_1 \geq 0} \phi [(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3] A_{0,3} \quad (\text{B.15})$$

$$- \frac{\gamma}{2E_0} A'_0 \left[\frac{\partial}{\partial s_3} \hat{\Sigma}(\xi) \right] A_0 + \Gamma_{0,3} - \Gamma_{1,3}$$

Given our assumption that Σ is diagonal (assumption 2), the expressions for the partial derivatives of $\hat{\Sigma}(\xi)$ are

$$\frac{\gamma}{2E_0} A'_0 \left[\frac{\partial}{\partial s_2} \hat{\Sigma}(\xi) \right] A_0 = -\frac{\gamma}{2E_0} 2(1 - s_2^{**}) (-1) \sigma_2^2 x_{0,2}^2 \quad (\text{B.16})$$

$$\frac{\gamma}{2E_0} A'_0 \left[\frac{\partial}{\partial s_3} \hat{\Sigma}(\xi) \right] A_0 = -\frac{\gamma}{2E_0} 2(1 - s_3^{**}) (-1) \sigma_3^2 x_{0,3}^2 \quad (\text{B.17})$$

so therefore we have

$$0 = -(\mu_2 - R_D) x_{0,2} + \mathbb{I}_{CS_1 \geq 0} \phi \kappa_{reg} w_2 x_{0,2} + \gamma (1 - s_2^{**}) \sigma_2^2 x_{0,2}^2 + \Gamma_{0,2} - \Gamma_{1,2} \quad (\text{B.18})$$

$$0 = -(\mu_3 - R_D) x_{0,3} + \mathbb{I}_{CS_1 \geq 0} \phi [(1 - \alpha) \Psi_3 + (1 - \Psi_3) \kappa_{reg} w_3] x_{0,3} \quad (\text{B.19})$$

$$+ \gamma (1 - s_3^{**}) \sigma_3^2 x_{0,3}^2 + \Gamma_{0,3} - \Gamma_{1,3}$$

Note that each s_i^{**} for $i \in \{2, 3\}$ corresponds to the maximum because that the second derivative is

$$- \frac{\gamma}{E_0} \sigma_i^2 x_{0,i}^2 < 0 \quad (\text{B.20})$$

Rearrange and simplify to find

$$s_2^{**} = 1 - \frac{(\mu_2 - R_D) - \mathbb{I}_{CS_1 \geq 0} \phi \kappa_{reg} w_2}{\gamma \sigma_2^2 A_{0,2} / E_0} + \frac{\Gamma_{0,2} - \Gamma_{1,2}}{\gamma \sigma_2^2 A_{0,2} / E_0} \quad (\text{B.21})$$

$$s_3^{**} = 1 - \frac{(\mu_3 - R_D) - \mathbb{I}_{CS_1 \geq 0} \phi [(1 - \alpha) \Psi_3 + (1 - \Psi_3) \kappa_{reg} w_3]}{\gamma \sigma_3^2 A_{0,3} / E_0} + \frac{\Gamma_{0,3} - \Gamma_{1,3}}{\gamma \sigma_3^2 A_{0,3} / E_0} \quad (\text{B.22})$$

Both expressions are decreasing in ϕ . Therefore these expressions can be equivalently written as cases according to the value of ϕ without the Lagrange multipliers for the lower and upper bounds as in (B.3) and (B.7).

Suppose that $CS_1 \geq 0$ given s_2^{**} and s_3^{**} assuming that $\mathbb{I}_{CS_1 \geq 0} = 1$. In this case, the optimal selling decisions are exactly s_2^{**} and s_3^{**} . If instead we find $CS_1 < 0$ given s_2^{**} and s_3^{**} then we have a corner solution. A bank will never optimally choose to sell beyond the amount that achieves $CS_1 = 0$ because there is no more benefit from reducing the penalty beyond this point. Given a corner solution, we must figure out which asset the bank sells first. Even if $\Psi_3 = 0$, we see that the bank would sell its entire holding of asset 3 before asset 2 if

$$w_3 > \frac{\mu_3 - R_D}{\mu_2 - R_D} w_2 \quad (\text{B.23})$$

This condition is guaranteed by assumption 3 and my calibration for risk weights in (3.1). As a result, we can define the expression for the largest share of asset 3 holdings that the bank will sell

$$\bar{s}_3 = \frac{\max \left\{ (1 - \kappa_{reg} w_1) \xi A_{0,1} + (1 - \kappa_{reg} w_3) \Psi_3 A_{0,3} - \left(1 - \frac{1}{1+\beta} \right) E_0, 0 \right\}}{[(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3] A_{0,3}} \quad (\text{B.24})$$

This quantity \bar{s}_3 represents the amount of asset 3 selling in order for $CS_1 = 0$. The max operator signifies that the bank will sell nothing if capital shortfall is already negative ($CS_1 < 0$). If $\bar{s}_3 \leq 1$, it is an upper bound to the optimal selling value and hence we conclude

$$s_3^* = \min \{ s_3^{**}, \bar{s}_3 \} \quad (\text{B.25})$$

If $\bar{s}_3 > 1$, we may still be at a corner solution if the bank can achieve $CS_1 = 0$ by selling asset 2.

The expression for the largest share of asset 2 holdings that the bank will sell

$$\bar{s}_2 = \frac{\max \left\{ (1 - \kappa_{reg} w_1) \xi A_{0,1} - \left(1 - \frac{1}{1+\beta} \right) + \{ \alpha \Psi_i - \kappa_{reg} w_3 \} A_{0,3}, 0 \right\}}{\kappa_{reg} w_2 A_{0,2}} \quad (\text{B.26})$$

This quantity represents the amount of asset 2 selling in order for $CS_1 = 0$ given that $s_3 = 1$. Like

\bar{s}_3, \bar{s}_2 is an upper bound to the optimal selling value and hence we conclude

$$s_2^* = \min \{s_2^{**}, \bar{s}_2\} \quad (\text{B.27})$$

Lemma B.0.2 Optimal Portfolio. *The optimal 3x1 portfolio vector for bank b is*

$$A_0^{b*} = \frac{E_0^b}{\gamma} \bar{\Sigma}^{-1} \left(\bar{\mu} - \bar{c} + \Gamma_x - \frac{\max \left\{ w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} + \Gamma_x) - \frac{\gamma}{\kappa_{reg}(1+\beta^b)}, 0 \right\}}{w' \bar{\Sigma}^{-1} w} w \right) + Z^b \quad (\text{B.28})$$

where

$$\bar{\mu} \equiv \begin{bmatrix} (1 - q\xi) \mu_1 - R_D \\ (1 - qs_2^b) (\mu_2 - R_D) \\ (1 - qs_3^b) (\mu_3 - R_D) \end{bmatrix} \quad (\text{B.29})$$

$$\bar{c} \equiv q\phi \times \mathbb{I}_{CS_1^b \geq 0} \times \begin{bmatrix} (1 - \kappa_{reg}^b w_1) \xi \\ -s_2^b \kappa_{reg}^b w_2 \\ [(1 - \kappa_{reg}^b w_3) (1 - s_3^b) + \alpha^b s_3^b] \Psi_3 - s_3^b \kappa_{reg}^b w_3 \end{bmatrix} \quad (\text{B.30})$$

$$\bar{\Sigma} \equiv (1 - q)\Sigma + q \times \begin{bmatrix} (1 - \xi)^2 \sigma_1^2 & 0 & 0 \\ 0 & (1 - s_2^b)^2 \sigma_2^2 & 0 \\ 0 & 0 & (1 - s_3^b)^2 \sigma_3^2 \end{bmatrix} \quad (\text{B.31})$$

and Γ_x is the 3x1 vector of Kuhn-Tucker multipliers on the non-negativity constraints and Z^b is a 3x1 vector of terms that capture the effect of portfolio holdings on the selling decision at $t = 1$. If $s_i^b \in \{0, 1\}$ for $i \in \{2, 3\}$ then $Z^b = [0, 0, 0]'$.

Proof In the following analysis, I suppress the bank-specific superscript to reduce notational clutter. Plug in the expressions for $\{\tilde{R}_E, A_1, D_1, \Phi\}$ and the distribution for $\tilde{\xi}$ to rewrite the bank problem at $t = 0$ as

$$\begin{aligned} \max_{x_0^b, \beta^b} (1-q) \times \left(\mu' x_0 + R_D - \frac{\gamma}{2} x_0' \Sigma x_0 \right) \\ + q \times \left(\hat{\mu}(\xi)' x_0 + R_D - \frac{\gamma}{2} x_0' \hat{\Sigma}(\xi) x_0 - \frac{1}{E_0^b} \Phi \right) \end{aligned} \quad (\text{B.32})$$

subject to

$$\frac{1}{w' x_0} = \kappa_{reg} (1 + \beta) \quad (\text{B.33})$$

$$x_0 \geq 0 \quad (\text{B.34})$$

$$\beta \geq 0 \quad (\text{B.35})$$

where

$$x_0 \equiv \frac{1}{E_0} A_0 \quad (\text{B.36})$$

$$\frac{1}{E_0} \Phi = \phi \times \max \left\{ \frac{C S_1}{E_0}, 0 \right\} \quad (\text{B.37})$$

$$\frac{C S_1}{E_0} = \left(\frac{1}{1 + \beta} - 1 \right) + (1 - \kappa_{reg} w_1) \xi x_{0,1} - s_2 \kappa_{reg} w_2 x_{0,2} \quad (\text{B.38})$$

$$\begin{aligned} & + \{ [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 \} x_{0,3} \\ \hat{\mu}(\xi) & \equiv \begin{bmatrix} \mu_1 (1 - \xi) - R_D \\ (1 - s_2) (\mu_2 - R_D) \\ (1 - s_3) (\mu_3 - R_D) \end{bmatrix} \end{aligned} \quad (\text{B.39})$$

$$\hat{\Sigma}(\xi) \equiv \begin{bmatrix} (1 - \xi)^2 \sigma_1^2 & 0 & 0 \\ 0 & (1 - s_2)^2 \sigma_2^2 & 0 \\ 0 & 0 & (1 - s_3)^2 \sigma_3^2 \end{bmatrix} \quad (\text{B.40})$$

We normalize the holdings by E_0 to avoid carrying around the E_0 in the remaining analysis. We have also incorporated the assumption that Σ is diagonal (assumption 2).

The Lagrangian is

$$L(\Lambda, \Gamma_x, \Gamma_\beta) = (1 - q) \times \left(\mu' x_0 + R_D - \frac{\gamma}{2} x_0' \Sigma x_0 \right) \quad (\text{B.41})$$

$$+ q \times \left(\hat{\mu}(\xi)' x_0 + R_D - \frac{1}{E_0} \Phi - \frac{\gamma}{2} x_0' \hat{\Sigma}(\xi) x_0 \right) \quad (\text{B.42})$$

$$+ \Lambda (1 - \kappa_{reg} (1 + \beta) w' x_0) + \Gamma_x' x_0 + \Gamma_\beta \beta$$

Take the partial derivative with respect to x_0

$$0 = (1 - q) \times (\mu - \gamma \Sigma x_0) \quad (\text{B.43})$$

$$+ q \times \left[\frac{\partial \hat{\mu}(\xi)' x_0}{\partial x_0} - \frac{1}{E_0} \frac{\partial \Phi}{\partial x_0} - \gamma \left(\frac{\partial}{\partial x_0} \left(\frac{1}{2} x_0' \hat{\Sigma}(\xi) x_0 \right) \right) \right]$$

$$- \Lambda \kappa_{reg} (1 + \beta) w + \Gamma_x$$

The above matrices partial derivatives are

$$\frac{\partial \hat{\mu}(\xi)' x_0}{\partial x_0} = \hat{\mu}(\xi) - \begin{bmatrix} \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,1}} (\mu_i - R_D) x_{0,i} \\ \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,2}} (\mu_i - R_D) x_{0,i} \\ \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,3}} (\mu_i - R_D) x_{0,i} \end{bmatrix} \quad (\text{B.44})$$

and

$$\frac{\partial}{\partial x_0} \left(\frac{1}{2} x_0' \hat{\Sigma}(\xi) x_0 \right) = \hat{\Sigma}(\xi) x_0 - z(\xi) \quad (\text{B.45})$$

where

$$z(\xi) \equiv \begin{bmatrix} 0 \\ (1 - s_2) \sigma_2^2 x_{0,2}^2 \frac{\partial s_2}{\partial x_{0,2}} \\ (1 - s_3) \sigma_3^2 x_{0,3}^2 \frac{\partial s_3}{\partial x_{0,3}} \end{bmatrix} \quad (\text{B.46})$$

The partial derivative of Φ with respect to x_0 is

$$\frac{1}{E_0} \frac{\partial \Phi}{\partial x_0} = \phi \times \max \left\{ \frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial x_0}, 0 \right\}$$

For $i = 1$, the partial derivative is

$$\frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial x_{0,1}} = (1 - \kappa_{reg} w_1) \xi - \left[\frac{\partial s_2}{\partial x_{0,1}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,1}} \{ (1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3 \} x_{0,3} \right]$$

For $i = 2$, the partial derivative is

$$\frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial x_{0,2}} = -s_2 \kappa_{reg} w_2 - \left[\frac{\partial s_2}{\partial x_{0,2}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,2}} \{ (1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3 \} x_{0,3} \right]$$

For $i = 3$, the partial derivative is

$$\begin{aligned} \frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial x_{0,3}} &= [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 \\ &\quad - \left[\frac{\partial s_2}{\partial x_{0,3}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,3}} \{ (1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3 \} x_{0,3} \right] \end{aligned}$$

Putting it all together, the optimality condition is

$$0 = \bar{\mu} - \gamma \bar{\Sigma} x_0^* - \gamma \bar{z} - \bar{c} - \Lambda \kappa_{reg} (1 + \beta) w + \Gamma_x \quad (\text{B.47})$$

where

$$\begin{aligned} \bar{\mu} &\equiv \begin{bmatrix} (1 - q\xi) \mu_1 - R_D - q \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,1}} (\mu_i - R_D) x_{0,i} \\ (1 - qs_2) (\mu_2 - R_D) - q \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,2}} (\mu_i - R_D) x_{0,i} \\ (1 - qs_2) (\mu_3 - R_D) - q \sum_{i=2}^3 \frac{\partial s_i}{\partial x_{0,3}} (\mu_i - R_D) x_{0,i} \end{bmatrix} \\ \bar{\Sigma} &\equiv \begin{bmatrix} (1 - q + q(1 - \xi)^2) \sigma_1^2 & 0 & 0 \\ 0 & (1 - q + q(1 - s_2)^2) \sigma_2^2 & 0 \\ 0 & 0 & (1 - q + q(1 - s_3)^2) \sigma_3^2 \end{bmatrix} \\ \bar{z} &\equiv q \begin{bmatrix} 0 \\ (1 - s_2) \sigma_2^2 x_{0,2}^2 \frac{\partial s_2}{\partial x_{0,2}} \\ (1 - s_3) \sigma_3^2 x_{0,3}^2 \frac{\partial s_3}{\partial x_{0,3}} \end{bmatrix} \end{aligned}$$

and

$$\bar{c} \equiv q\phi_{CS_1 \geq 0} \begin{bmatrix} (1 - \kappa_{reg} w_1) \xi - \left[\frac{\partial s_2}{\partial x_{0,1}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,1}} \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \right] \\ -s_2 \kappa_{reg} w_2 - \left[\frac{\partial s_2}{\partial x_{0,2}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,2}} \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \right] \\ [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 - \left[\frac{\partial s_2}{\partial x_{0,3}} \kappa_{reg} w_2 x_{0,2} + \frac{\partial s_3}{\partial x_{0,3}} \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \right] \end{bmatrix}$$

Rearrange to find

$$x_0^* = \frac{1}{\gamma} \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} - \Lambda \kappa_{reg} (1 + \beta) w + \Gamma_x)$$

If $\Lambda > 0$ then we know $1 = \kappa_{reg} (1 + \beta) w' x_0$. Multiply both sides of the above equation by $\kappa_{reg} (1 + \beta) w'$ and plug the above expression for x_0^* to find

$$1 = \frac{1}{\gamma} \kappa_{reg} (1 + \beta) w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} - \Lambda \kappa_{reg} (1 + \beta) w + \Gamma_x)$$

Rearrange further to find

$$\Lambda = \frac{\max \left\{ w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} + \Gamma_x) - \frac{\gamma}{\kappa_{reg} (1 + \beta)}, 0 \right\}}{\kappa_{reg} (1 + \beta) w' \bar{\Sigma}^{-1} w} \quad (\text{B.48})$$

where the max operator incorporates the fact that $\Lambda \geq 0$. Plug this expression back into the optimality condition to get

$$x_0^* = \frac{1}{\gamma} \bar{\Sigma}^{-1} \left(\bar{\mu} - \bar{c} - \gamma \bar{z} + \Gamma_x - \frac{\max \left\{ w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} + \Gamma_x) - \frac{\gamma}{\kappa_{reg} (1 + \beta)}, 0 \right\}}{w' \bar{\Sigma}^{-1} w} w \right) \quad (\text{B.49})$$

noting that the $\kappa_{reg} (1 + \beta)$ term in the denominator of Λ cancels out with the $\kappa_{reg} (1 + \beta)$ in $\Lambda \kappa_{reg} (1 + \beta) w$.

The expressions for $\frac{\partial s_2}{\partial x_{0,i}}$ and $\frac{\partial s_3}{\partial x_{0,i}}$ are messy (see lemma B.0.1 for the optimal expressions for s_2 and s_3). However, we can simplify things immensely by noting that these expressions are zero when $s_i^* = 0$ or $s_i^* = 1$ for $i \in \{2, 3\}$. From this perspective, we write the optimal portfolio as

$$x_0^* = \frac{1}{\gamma} \bar{\Sigma}^{-1} \left(\bar{\mu} - \bar{c} + \Gamma_x - \frac{\max \left\{ w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} + \Gamma_x) - \frac{\gamma}{\kappa_{reg}(1+\beta)}, 0 \right\}}{w' \bar{\Sigma}^{-1} w} w \right) + Z$$

where

$$\begin{aligned} \bar{\mu} &\equiv \begin{bmatrix} (1 - q\xi) \mu_1 - R_D \\ (1 - qs_2) (\mu_2 - R_D) \\ (1 - qs_3) (\mu_3 - R_D) \end{bmatrix} \\ \bar{c} &\equiv q\phi \times \mathbb{I}_{CS_1 \geq 0} \times \begin{bmatrix} (1 - \kappa_{reg} w_1) \xi \\ -s_2^b \kappa_{reg}^b w_2 \\ [(1 - \kappa_{reg} w_3) (1 - s_3) + \alpha s_3] \Psi_3 - s_3 \kappa_{reg} w_3 \end{bmatrix} \\ \bar{\Sigma} &\equiv (1 - q) \Sigma + q \times \begin{bmatrix} (1 - \xi)^2 \sigma_1^2 & 0 & 0 \\ 0 & (1 - s_2)^2 \sigma_2^2 & 0 \\ 0 & 0 & (1 - s_3)^2 \sigma_3^2 \end{bmatrix} \end{aligned}$$

and Z is a 3x1 vector that captures all of the terms related to $\frac{\partial s_2}{\partial x_{0,i}}$ and $\frac{\partial s_3}{\partial x_{0,i}}$.

Lemma B.0.3 Optimal Capital Buffer (β). *The optimal capital buffer for bank b satisfies*

$$\begin{aligned} 0 &= \beta^* \times q \times \left[\phi \left[\frac{1}{(1 + \beta)^2} + \kappa_{reg} w_2 x_{0,2} \frac{\partial s_2}{\partial \beta} + \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \frac{\partial s_3}{\partial \beta} \right] \times \mathbb{I}_{CS_1 \geq 0} \right] \\ &+ \beta^* \times q \times \left[\sum_{i=2}^3 \left[(\gamma (1 - s_i) \sigma_i^2 x_{0,i}^2 - (\mu_i - R_D) x_{0,i}) \left(\frac{\partial s_i}{\partial \beta} \right) \right] \right] \\ &- \Lambda \frac{\beta^*}{1 + \beta^*} \end{aligned}$$

where Λ is the Lagrange multiplier on the capital requirement constraint as shown in (B.48), Γ_β is the Kuhn-Tucker multiplier on the non-negativity constraint, and the optimal selling expressions are from lemma B.0.1.

Proof Start by taking the partial derivative of the Lagrangian as seen in (B.42) with respect to β

$$0 = q \times \left[\frac{\partial \hat{\mu}(\xi)' x_0}{\partial \beta} - \frac{1}{E_0} \frac{\partial \Phi}{\partial \beta} - \gamma \left(\frac{\partial}{\partial \beta} \left(\frac{1}{2} x_0' \hat{\Sigma}(\xi) x_0 \right) \right) \right] - \Lambda \kappa_{reg} w' x_0 + \Gamma_\beta$$

The above matrices partial derivatives are

$$\frac{\partial \hat{\mu}(\xi)' x_0}{\partial \beta} = - \sum_{i=2}^3 \frac{\partial s_i}{\partial \beta} (\mu_i - R_D) x_{0,i}$$

and

$$\frac{\partial}{\partial \beta} \left(\frac{1}{2} x_0' \hat{\Sigma}(\xi) x_0 \right) = \sum_{i=2}^N (1 - s_i) \sigma_i^2 x_{0,i}^2 \left(-\frac{\partial s_i}{\partial \beta} \right)$$

The partial derivative of Φ with respect to x_0 is

$$\frac{1}{E_0} \frac{\partial \Phi}{\partial \beta} = \phi \times \max \left\{ \frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial \beta}, 0 \right\}$$

The partial derivative within the max operator is

$$\frac{\partial \left(\frac{CS_1}{E_0} \right)}{\partial \beta} = \frac{-1}{(1 + \beta)^2} - \kappa_{reg} w_2 x_{0,2} \frac{\partial s_2}{\partial \beta} - \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \frac{\partial s_3}{\partial \beta}$$

Putting it all together, the optimality condition is

$$\begin{aligned} 0 = & q \times \left[\phi \left[\frac{1}{(1 + \beta)^2} + \kappa_{reg} w_2 x_{0,2} \frac{\partial s_2}{\partial \beta} + \{(1 - \kappa_{reg} w_3 - \alpha) \Psi_3 + \kappa_{reg} w_3\} x_{0,3} \frac{\partial s_3}{\partial \beta} \right] \times \mathbb{I}_{CS_1 \geq 0} \right] \\ & + q \times \left[\sum_{i=2}^3 \left[(\gamma (1 - s_i) \sigma_i^2 x_{0,i}^2 - (\mu_i - R_D) x_{0,i}) \left(\frac{\partial s_i}{\partial \beta} \right) \right] \right] \\ & - \Lambda \frac{1}{1 + \beta} + \Gamma_\beta \end{aligned}$$

where we have substituted in the binding capital requirement constraint equation

$$\frac{1}{w'x_0} = \kappa_{reg}(1 + \beta)$$

and where Λ is the Lagrange multiplier on the capital requirement constraint as shown in (B.48).

The three terms can be interpreted as follows:

- The first term is negative and shows that increasing β is costly because the bank sells less and therefore reduces the penalty less
- The second term is positive and shows that increasing β is beneficial because it means the bank needs to sell less
- The third term is positive and shows that increasing β is costly if it forces the bank to hold a smaller portfolio that it otherwise would optimally hold

To help understand the optimal β value that satisfies the optimality condition, let us consider the following cases.

Case 1: $q = 0$

In this case, $\beta^* = 0$ if $\Lambda_{\beta=0} > 0$. Otherwise, $\beta^* = \frac{E_0}{\kappa_{reg}w'A_0} - 1 > 0$ meaning that the bank prefers to hold a capital buffer purely because the assets returns are not sufficiently attractive to level up to the point where leverage yields a capital ratio below κ_{reg} . The optimal portfolio is the solution to the portfolio problem where the capital requirement effectively does not matter.

Case 2: $CS_1 \leq 0$ without any selling required

In this world $\frac{\partial s_3}{\partial \beta} = 0$ and $\frac{\partial s_2}{\partial \beta} = 0$ and also $\mathbb{I}_{CS_1 \geq 0} = 0$ by the case assumption. In order for this outcome to occur, we must have $\beta^* > 0$ given that $\xi > 0$. This means $\beta^* = \frac{E_0}{\kappa_{reg}w'A_0} - 1 > 0$ as in Case 1 and the interpretation is the same: the assets are not sufficiently attractive to hold high leverage.

Case 3: $CS_1 = 0$ with some positive selling

In this case, we have $\frac{\partial s_3}{\partial \beta} \neq 0$ or $\frac{\partial s_2}{\partial \beta} \neq 0$ because the bank has to be selling one of the assets according to either $s_2 = \bar{s}_2$ as in (B.8) or $s_3 = \bar{s}_3$ as in (B.6). This is how the bank ends

at $CS_1 = 0$. We cannot simplify the optimality condition any further in this case and the optimal $\beta^* \geq 0$ is the value that satisfies the equation. We can think of this case, however, as corresponding to a sufficiently small ξ value. If the loss from the systematic shock ($\xi x_{0,1}$) is sufficiently small, then the bank can sell.

Case 4: $CS_1 > 0$ given optimal selling decisions

The only way for this outcome to occur is for $s_3 = s_3^{**}$ and $s_2 = s_2^{**}$ as in (B.3) or (B.7). Otherwise $CS_1 = 0$. Therefore the optimal selling decisions are not a function of β and $\frac{\partial s_3}{\partial \beta} = 0$ and $\frac{\partial s_2}{\partial \beta} = 0$. The optimality condition is reduced to

$$0 = q\phi \left[\frac{1}{(1+\beta)^2} \right] - \Lambda \frac{1}{1+\beta} + \Gamma_\beta$$

Because the first term is positive and $\Gamma_\beta \geq 0$, this means that we must have $\Lambda > 0$. Plug in the expression for Λ from (B.48). Multiply both sides by $(1+\beta)$, utilize $\Gamma_\beta \beta = 0$, and rearrange to find

$$\beta^* = \frac{\gamma}{\kappa_{reg}} \left[w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} + \Gamma_x) - (q\phi + \Gamma_\beta) \kappa_{reg} w' \bar{\Sigma}^{-1} w \right]^{-1} - 1$$

or

$$\beta^* = \max \left\{ \frac{\gamma}{\kappa_{reg}} \left[w' \bar{\Sigma}^{-1} (\bar{\mu} - \bar{c} - \gamma \bar{z} + \Gamma_x) - q\phi \kappa_{reg} w' \bar{\Sigma}^{-1} w \right]^{-1} - 1, 0 \right\}$$

The expression inside the max operator effectively tells us that $\beta^* = 0$ if

$$\Lambda_{\beta=0} - q\phi > 0$$

This result makes sense that either q or ϕ must be larger enough to induce the bank to hold a capital buffer if the bank would otherwise want to hold zero buffer based on the asset return parameters.

APPENDIX C

DATA DETAILS

C.1 Bank Sample Construction

For the primary data source, I utilize bank holding company (BHC) data collected by the Federal Reserve through the *Consolidated Financial Statements for Holding Companies* (FR Y-9C). Raw data are downloaded from the Federal Reserve of Chicago website (<https://chicagofed.org/banking/financial-institution-reports/bhc-data>). Throughout the description of the dataset, I use the terms BHCs and banks interchangeably to refer to the entities in this dataset.

The FR Y-9C data broadly provides balance sheet and income statement information on a quarterly basis. Of particular use in this study, it provides a detailed breakdown of securities holdings both in the banking book and trading book (Schedules HC-B and HC-D). I am also able to see contributions of these assets to regulatory ratios (Schedule HC-R).

Onto the FR Y-9C dataset, I merge equity returns, prices, and shares outstanding from CRSP using the FRBNY CRSP-FRB Link dataset. This dataset created and maintained by Federal Reserve Bank of New York links PERMCOs from CRSP to RSSD identifiers (https://www.newyorkfed.org/research/banking_research/datasets.html). My final sample covers 2001Q1 through 2016Q4.

Banks that meet any of the following criteria are dropped from the dataset:

- Non-typical BHCs (AIG, American Express, Discover, Goldman Sachs, Metlife, Morgan Stanley)
- Foreign owned BHCs , which are BHCs with a non-missing Financial High Holder ID (RSSD9364)
- Savings and Loans Holding Companies, which are BHCs for which the entity type (RSSD9346) is “SLHC”
- Banks that drop out of the FR Y-9C dataset in 2006Q1 because they are too small (under \$500 million in assets). Prior to 2006Q1, the minimum size threshold was \$150 million in assets.

C.2 Key Variables Within FR Y-9C Dataset

Total Assets: BHCK2170.

Risk-Weighted Assets: BHCKA223 before 2014, BHCAA223 thereafter.

Tier 1 Capital: BHCK8274 before 2014, BHCA8274 thereafter.

Tier 1 Capital Ratio: BHCKA223 before 2014, BHCAA223 thereafter.

C.3 Constructing Time Series of Holdings for Security Types

The security types shown in Table 3.1 and utilized as part of the calibration are constructed by summing together numerous individual series in the FR Y-9C. This construction of these series is complicated by three factors:

1. Banks separately report securities as Held to Maturity (HTM), Available for Sale (AFS), or Trading Assets (TA). The first two accounting categories are reported in Schedule HC-B (Securites) and the last category is reported in Schedule HC-D (Trading Assets and Liabilities).
2. For HTM or AFS securities, banks report both the amortized cost (AC) and fair value (FV). These values can be thought of as book and market values.
3. The reporting form changes over the years and hence the names of the variables in the FR Y-9C dataset change as well.

Table C.1 reports the definitions for each security type in terms of the FR Y-9C variable names and related information. The fair value for a bank's entire holdings of a given security type is the sum of the fair value of its holdings reported across all schedules.

Security Type	Description	Schedule	Value	Period	Formula Using FR Y-9C Variable Names
Asset-backed (Total)	Includes the following reference assets: Credit card receivables, Home equity lines, Automobile loans, Other consumer loans, Commercial and industrial loans, Other	HC-B	AC	Full	BHCKB838 + BHCKB840 + BHCKB842 + BHCKB844 + BHCKB846 + BHCKB848 + BHCKB850 + BHCKB852 + BHCKB854 + BHCKB856 + BHCKB858 + BHCKB860
		HC-B	FV	Full	BHCKB839 + BHCKB841 + BHCKB843 + BHCKB845 + BHCKB847 + BHCKB849 + BHCKB851 + BHCKB853 + BHCKB855 + BHCKB857 + BHCKB859 + BHCKB861
		HC-D	FV	Full	BHCKF643 + BHCKF644 + BHCKF645 + BHCKF646 + BHCKF647 + BHCKF648
Equities	Investments in mutual funds and other equity securities with readily determinable fair values	HC-B	AC	Full	BHCKA510
		HC-B	FV	Full	BHCKA511
		HC-D	FV	Full	BHCKF652 + BHCKF653
Loans	Total Loans and Lease Financing Receivables on the balance sheet (HC-C) and held as Trading Assets (HC-D)	HC-C	AC	Full	BHCK2122
		HC-D	FV	Before 2011	BHCKF610 + BHCKF614 + BHCKF615 + BHCKF616 + BHCKF199 + BHCKF617 + BHCKF618
				2011 - End	BHCKF610 + BHCKF614 + BHCKF615 + BHCKF616 + BHCKF199 + BHCKF210 + BHCKF618
MBS (Govt-backed)	Includes mortgage-backed securities categorized as (1) Pass-through securities issued or guaranteed by FNMA, FHLMC, or GNMA and (2) Other mortgage-backed securities issued or guaranteed by FNMA, FHLMC, or GNMA (include CMOs, REMICs, and stripped MBS)	HC-B	AC	Before 2009	BHCK1699 + BHCK1701 + BHCK1703 + BHCK1706 + BHCK1714 + BHCK1716 + BHCK1718 + BHCK1731
				2009 - 2011	BHCKG300 + BHCKG302 + BHCKG304 + BHCKG306 + BHCKG312 + BHCKG314 + BHCKG316 + BHCKG318 + BHCKG324 + BHCKG326
				2011 - End	BHCKG300 + BHCKG302 + BHCKG304 + BHCKG306 + BHCKG312 + BHCKG314 + BHCKG316 + BHCKG318 + BHCKK142 + BHCKK144 + BHCKK150 + BHCKK152
		HC-B	FV	Before 2009	BHCK1700 + BHCK1702 + BHCK1705 + BHCK1707 + BHCK1715 + BHCK1717 + BHCK1719 + BHCK1732
				2009 - 2011	BHCKG301 + BHCKG303 + BHCKG305 + BHCKG307 + BHCKG313 + BHCKG315 + BHCKG317 + BHCKG319 + BHCKG325 + BHCKG327
				2011 - End	BHCKG301 + BHCKG303 + BHCKG305 + BHCKG307 + BHCKG313 + BHCKG315 + BHCKG317 + BHCKG319 + BHCKK143 + BHCKK145 + BHCKK151 + BHCKK153
		HC-D	FV	Before 2008	BHCK3534 + BHCK3535
				2008 - 2009	BHCM3534 + BHCM3535
				2010 - End	BHCKG379 + BHCKG380 + BHCKK197
MBS (Private)	Includes mortgage-backed securities categorized as (1) Other pass-through securities and (2) All other mortgage-backed securities	HC-B	AC	Before 2009	BHCK1709 + BHCK1711 + BHCK1733 + BHCK1735
				2009 - 2011	BHCKG308 + BHCKG310 + BHCKG320 + BHCKG322 + BHCKG328 + BHCKG330
				2011 - End	BHCKG308 + BHCKG310 + BHCKG320 + BHCKG322 + BHCKK146 + BHCKK148 + BHCKK154 + BHCKK156
		HC-B	FV	Before 2009	BHCK1710 + BHCK1713 + BHCK1734 + BHCK1736
				2009 - 2011	BHCKG309 + BHCKG311 + BHCKG321 + BHCKG323 + BHCKG329 + BHCKG331
				2011 - End	BHCKG309 + BHCKG311 + BHCKG321 + BHCKG323 + BHCKK147 + BHCKK149 + BHCKK155 + BHCKK157
		HC-D	FV	Before 2008	BHCK3536
Other Debt	Includes "Other domestic debt securities" and "Foreign debt securities"	HC-B	AC	Full	BHCK1737 + BHCK1739 + BHCK1742 + BHCK1744
		HC-B	FV	Full	BHCK1738 + BHCK1741 + BHCK1743 + BHCK1746
		HC-D	FV	Before 2008	BHCK3537
				2008 - 2009	BHCM3537
				2010 - End	BHCKG383 + BHCKG384 + BHCKG385 + BHCKG386
U.S. Govt Agency	U.S. government agency obligations (exclude mortgage-backed securities)	HC-B	AC	Full	BHCK1289 + BHCK1291 + BHCK1294 + BHCK1297
		HC-B	FV	Full	BHCK1290 + BHCK1293 + BHCK1295 + BHCK1298
		HC-D	FV	Before 2008	BHCK3532
U.S. State and Muni	Securities issued by states and political subdivisions in the U.S.	HC-B	AC	Full	BHCK8496 + BHCK8498
		HC-B	FV	Full	BHCK8497 + BHCK8499
		HC-D	FV	Before 2008	BHCK3533
				2008 - End	BHCM3533
U.S. Treasury	U.S. Treasury securities	HC-B	AC	Full	BHCK0211 + BHCK1286
		HC-B	FV	Full	BHCK0213 + BHCK1287
		HC-D	FV	Before 2008	BHCK3531
				2008 - End	BHCM3531

Table C.1: Constructing Securities Holdings from FR Y-9C Data

Notes: All information in the table is from the FR Y-9C reporting form and instructions. "AC" is amortized cost, "FV" is fair value.

C.4 Computing Share Sold and Unrealized Losses

The computations in this section can be applied to an individual bank's holdings or the aggregated holdings across many banks.

For a security type i , the transition equations for the fair value (FV_i) and book value (AC_i) of a bank's holdings are

$$FV_{i,t} = (1 - s_{i,t})(1 - \Psi_{i,t})FV_{i,t-1} \quad (C.1)$$

$$AC_{i,t} = (1 - s_{i,t})AC_{i,t-1} \quad (C.2)$$

where $\Psi_{i,t}$ is percent decline in the market value of the holdings over the quarter and $s_{i,t}$ is the share of the holdings sold during the quarter. In the data, we only observe FV and AC as described in Table C.1. We can back out the values for s and Ψ using the beginning and ending values for FV and AC .

The expression for share sold ($s_{i,t}$) is

$$s_{i,t} = \frac{AC_{i,t-1} - AC_{i,t}}{AC_{i,t-1}} \quad (C.3)$$

Using the above expression, the expression for percent value decline ($\Psi_{i,t}$) is

$$\Psi_{i,t} = 1 - \frac{FV_{i,t}}{(1 - s_{i,t})FV_{i,t-1}} \quad (C.4)$$

Using both of these expressions, the unrealized loss over the quarter is

$$\left(\frac{\Psi_{i,t}}{1 - \Psi_{i,t}} \right) FV_{i,t} \quad (C.5)$$

This figure in dollars can be easily summed across banks or security types.

In Table C.1, we see that book value is only reported for the security type holdings in the securities book (HC-B). Only fair value is reported for the security type holdings in the trading book (HC-D). I estimate the share of the holdings in the trading book sold ($s_{i,t}^{ta}$) using the following

expression

$$s_{i,t}^{ta} = 1 - \frac{FV_{i,t}^{ta}}{FV_{t-1}^{ta}(1 - \Psi_{i,t}^{agg})} \quad (C.6)$$

where $\Psi_{i,t}^{agg}$ is the market price decline computed according to (C.4) using the aggregate banking book holdings of the security type.

To compute the share sold across all holdings (securities and trading), I compute the weighted average

$$s_{i,t}^{agg} = s_{i,t}^{sec} \frac{FV_{i,t-1}^{sec}}{FV_{i,t-1}^{ta} + FV_{i,t-1}^{sec}} + s_{i,t}^{ta} \frac{FV_{i,t-1}^{ta}}{FV_{i,t-1}^{ta} + FV_{i,t-1}^{sec}} \quad (C.7)$$

To compute the share sold for a subtotal category k of trading assets (e.g., “Risky”), I take the weighted average of all of the underlying J types

$$s_{k,t}^{ta} = \sum_{j=1}^J s_{j,t}^{ta} \frac{FV_{j,t-1}^{ta}}{\sum_{j=1}^J FV_{j,t-1}^{ta}} \quad (C.8)$$

APPENDIX D

CALIBRATION DETAILS

I set $B = 2$ where bank 1 represents banks over \$250 billion in assets as of the beginning of 2008Q4 and bank 2 represents all other banks. Specifically, bank 1 is comprised of the following BHCs:

1. JPMORGAN CHASE & CO (1039502)
2. BANK ONE CORP (1068294)
3. WACHOVIA CORP (1073551)
4. BANK OF AMER CORP (1073757)
5. STATE STREET CORP (1111435)
6. WELLS FARGO & CO (1120754)
7. CITIGROUP (1951350)

The name for each institution is the last reported entity short name (RSSD9010) and the number in parentheses is the RSSD identifier in the FR Y-9C dataset. Bank One was acquired by J.P. Morgan Chase on July 1, 2004. I include this bank as part of Bank 1 despite not meeting the strict definition because it was over \$250 billion in assets when purchased. Wachovia was purchased by Wells Fargo on December 31, 2008.

D.1 Parameters Directly Measured in the Data

Asset 2 Return Parameters (μ_2 and σ_2). The sample mean and volatility of the return on intermediate-term government bonds over the period 2002-2006. Underlying monthly data are from Ibbotson Associates. The monthly mean and volatility are converted to quarterly return units. Intermediate-term return series (ITGOVBD) corresponds to government bonds with maturity of 5 years. Banks do not report maturity structure of specific security types on the FR Y-9C, but they do report a rough maturity structure for their entire securities holdings. On average, portfolios

include roughly 60-65% with maturity greater than 5 years, 20-30% with maturity between 1-5 years, and 10-20% with maturity less than 1 year. Thus a weighted average maturity of 5 years seems reasonable.

Probability of Asset 1 Shock (q). The advanced economy average probability of a crisis based on historical observations. Laeven and Valencia (2012) compile a database that includes all systemic banking, currency, and sovereign debt crises during the period 1970-2011. The advanced economy average probability of crisis in their database is 4.0 percent.

Asset 1 Percent Decline Given Shock (ξ). The cumulative excess net charge-off rate for the aggregate loan portfolio over the period 2008-2010. In Figure D.1, we see that the net charge-off rate started to increase in 2008. The average annual rate between 2002-2006 is 0.77%. Subtract this average rate from the values in 2008-2010 to compute excess rates and sum over the period 2008-2010 to find 4.98%. I use the cumulative excess net charge-off rate including 2009 and 2010 even though the crisis outcome in the calibration is for 2008Q4 because the market and bank managers generally anticipated these future losses.

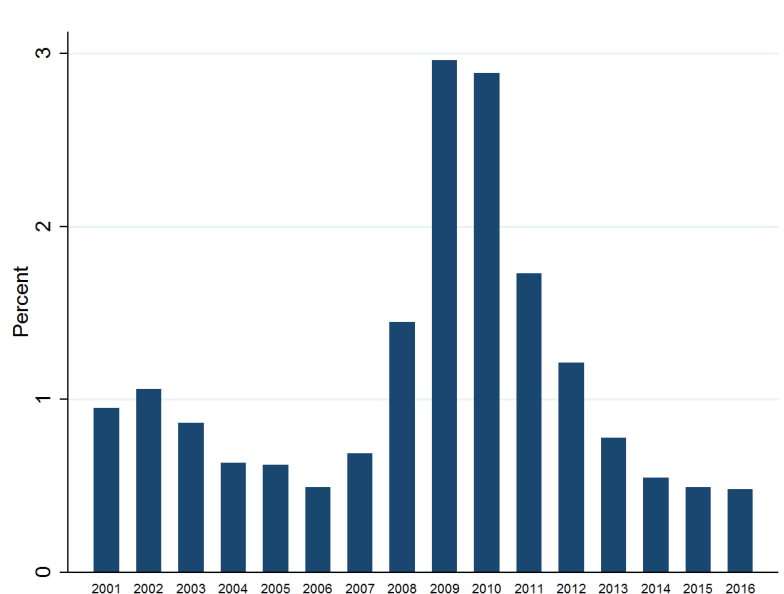


Figure D.1: Aggregate Net Charge-Off Rates

Net charge-offs are charge-offs (BHCK4635) less recoveries (BHCK4605). For data details see Appendix C.

Initial Equilibrium Asset 3 Price Decline (Ψ_3). The percent decline in the FINRA-Bloomberg Price Index for investment grade bonds between September 10, 2008 and October 10, 2008. The index is constructed using all bonds in the following credit categories as defined by NASD Rule 6200 Series as “Investment Grade”: AAA, AA, A, BBB. See time series plot in Figure 3.1. The coefficient for the linear price impact function in (3.2), ψ_3 , is determined such that the benchmark calibration delivers an initial equilibrium price decline equal to this value.

Cost of debt ($R_D - 1$). The average cost of funds across all banks over the period 2002-2006. Cost of funds for a given quarter is computed as the sum of total interest expense on deposits (BHCKA517 + BHCKA518 + BHCK6761 + BHCK4172) and total expense on federal funds purchased and securities sold under agreements to repurchase (BHCK4180) divided by the average balance of the sum of total deposits (BHDM6631 + BHDM6636 + BHFN6631 + BHFN6636) and federal funds purchased and securities sold under agreements to repurchase (BHDMB993 + BHCKB995).

Shares of Aggregate Equity Capital ($E^b/(E^1 + E^2)$). The relative share of aggregate tier 1 capital for each group of banks over the period 2002-2006. The sum of the shares must equal one. The share for the largest banks (bank 1) is 0.503. See time series in Figure D.3

Minimum Capital Ratios (κ_{reg}^b). The approximate average tier 1 capital ratio for the largest banks over the period 2002-2006. Before 2013, all banks formally had the same minimum tier 1 capital ratio of 6% to be considered “well capitalized” (i.e., avoid PCA). In practice, however, all banks held a consistent buffer over this minimum (Figure D.3). Given that the model only includes a crisis with relatively low probability, it will not generate a similar consistent capital buffer. As a compromise, I set the minimum capital ratio as if it was equal to the average capital ratio held by the largest banks, which is approximately 8%. This value has the additional benefit that it is the same as the calibrated ζ value used in the systemic risk measure (Definition 2). Therefore the benchmark calibration implies the smallest possible minimum capital ratios to avoid systemic risk during non-crisis times, which is intuitively consistent with the belief that regulation was insufficient prior to the financial crisis of 2007-2009.

D.2 Parameters Chosen to Match Initial Portfolio Shares and Selling Decisions

There are several parameters that are either difficult or impossible to measure directly. The values for these parameters are chosen to match two types of empirical observations: portfolio shares and selling decisions. For portfolio shares, there are six data points (three shares for two banks) corresponding to the average relative portfolio share over the period 2002-2006. See Table D.2. For the selling decisions, the outcome must be that bank 1 is sells asset 3 in equilibrium and bank 2 does not.

	Assets as of Sept. 30, 2008	
	Over \$250 billion	Under \$250 billion
Asset 1	0.631	0.740
Asset 2	0.154	0.191
Asset 3	0.191	0.069

Table D.1: Portfolio Shares in the Data

Portfolio shares are average values over the period 2002-2006. Portfolio shares are also fractions relative to the sum of assets 1-3, not balance sheet assets. Asset 1 corresponds to all “Loans” as defined in Table C.1. Asset 2 corresponds to the sum of the fair value of the safe securities as shown in Table 3.1. Asset 3 corresponds to the sum of the fair value of the securities with fire sale risk as shown in Table 3.1. The definitions for these security types in the data can be found in Table C.1. For data details see Appendix C.

Despite seeming to have seven parameters $(\mu_1, \sigma_1, \mu_3, \sigma_3, \gamma, \alpha^1, \alpha^2, \phi)$ and exactly seven target values, it is not possible to exactly match the portfolio shares. The issue is that heterogeneity in α^b is not flexible enough to generate the significantly different portfolio shares observed. Therefore I choose the set of parameters that minimizes the following objective function

$$\sum_{b=1}^2 \sum_{i=1}^3 \left(\frac{A_{0,i}^{*b}}{1'A_0^{*b}} - f_i^b \right)^2 \quad (\text{D.1})$$

where A_0^{*b} denote optimal bank portfolio holdings and f_i^b represents the empirical data averages shown in Table D.2. In Table 3.3 in the main text, I present the empirical portfolio shares alongside the shares implied by the model.

D.3 Increasing Capital Requirements and Equity Capital

Assuming $\beta^* = 0$ and suppressing the b superscripts, the capital requirement constraint is

$$\frac{E_0}{w' A_0} = \kappa_{reg}$$

In this form, the bank can only satisfy a higher value for κ_{reg} by either increasing E_0 or decreasing $w' (A_0)$ or some combination of the two. I formally represent this choice with a parameter $g \in [0, 1]$ that is an input to the following equation that determines a bank's updated equity capital

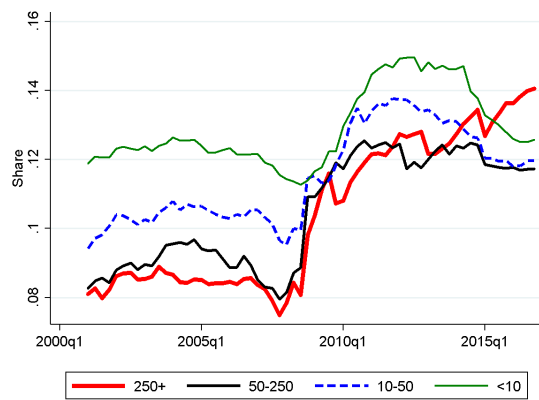
$$E_0^{New} = E_0^{Initial} \times \left(1 + g \times \left(\frac{\kappa_{reg}^{New}}{\kappa_{reg}^{Initial}} - 1 \right) \right) \quad (D.2)$$

If $g = 0$, the bank achieves a higher minimum capital requirement entirely through shrinking its risk-weighted assets. If $g = 1$, the bank does so entirely through increasing its initial equity capital.

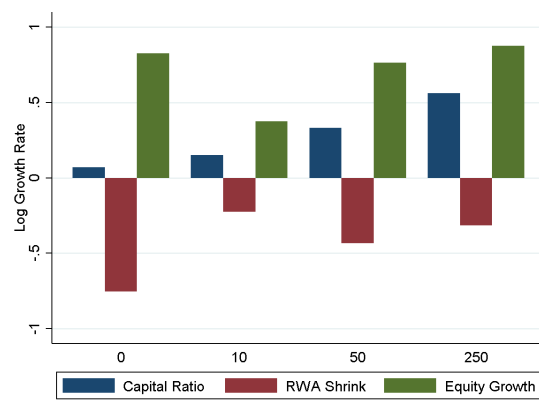
For guidance, we can use the observed increase in bank capital ratios since the financial crisis. Following the financial crisis, regulators put pressure on banks to increase their capital ratios. In 2013 and 2015, regulators finalized rules that formalized increases in minimum capital requirements.¹ In the left panel of Figure D.3, we can see that banks have indeed increased their tier 1 capital ratios to comply with the new minimums. In the right panel, I decompose these observed changes into the underlying changes from risk-weighted assets and equity capital levels. The fact that all of the red bars are negative means that all banks *increased* risk-weighted assets over the period from 2006 to 2016. The positive green bars reiterate this finding by displaying the large and positive growth in equity capital over the same period. In conclusion, it appears that banks met the higher capital requirements entirely through increasing equity capital ($g = 1$).

For an alternative perspective, I show the shares of equity capital and assets for each BHC size group over the same time period. In Figure D.3, we see that the relative size of the largest group has increased slightly in terms of equity capital from 2006 to 2016. However, the relative size of

¹See Federal Register, Vol. 78, No. 198 (Friday, October 11, 2013) and Federal Register, Vol. 80, No. 157, (Friday, August 14, 2015).



(a) Tier 1 Capital Ratios



(b) Decomposition of Change, 2006Q4 to 2016Q4

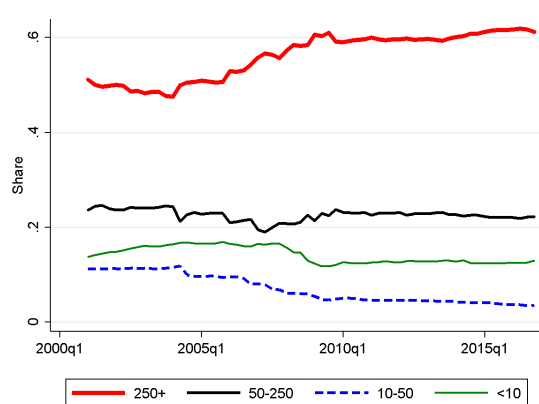
Figure D.2: Tier 1 Capital Ratios by BHC Size Group

The values for the bars in the right panel are found by using the following decomposition formula for the ending capital ratio (2016Q4) as a function of the initial capital ratio (2006Q4) and the growth rates of the components:

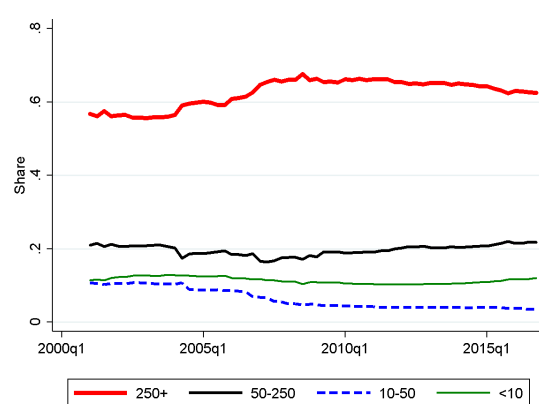
$$\frac{E_{end}}{RWA_{end}} = \frac{E_{end}}{E_{start}} \times \frac{RWA_{start}}{RWA_{end}} \times \frac{E_{start}}{RWA_{start}}$$

For data details see Appendix C.

assets for the same group has remained very similar over the same period. Given that this group also increased its capital ratio by the largest amount, it supports the conclusion that banks met higher capital requirements entirely through equity capital.



(a) Tier 1 Equity Capital



(b) Tangible Assets

Figure D.3: Relative Size of Equity Capital And Assets by BHC Size Group

For data details see Appendix C.

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