# Wage Bargaining, Labor Market Institutions and Business Cycles

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# Abstract

**ZUBEYIR KILINC: Wage Bargaining, Labor Market Institutions and Business Cycles.**
**(Under the direction of Richard Froyen.)**

This dissertation investigates the impacts of labor market institutions on the business cycles. The first chapter builds a DSGE search and match model that can capture the labor market facts and explain the business cycle facts within a tractable framework. The basic framework incorporates labor market frictions to the standard new Keynesian model and extends it with staggered wage contracts. The empirical performance of the model is tested by the impulse responses of the key macroeconomic variables to a monetary policy shock. It shows that incorporating search and matching frictions and a bargaining framework into the baseline new Keynesian model can generate a lower response for inflation and the real wage to a monetary policy shock and can explain some of the labor market facts simultaneously.

The second chapter analyzes the effects of alternative labor market institutions on the responses of key macroeconomic variables. It considers severance payments, unemployment benefits and worker's bargaining power. The impulse responses of key macroeconomic variables are derived under alternative parameterizations for these institutions. It finds that the severance payments do not have an impact on the responses. Increasing the unemployment benefits makes key macroeconomic variables less responsive to disturbances where increasing worker's bargaining power makes output more responsive and inflation less responsive to macroeconomic shocks.

Third chapter explores the differences between the characteristics of the business cycles in the US and the euro area. It first compares the features of the business cycles dated by the NBER and CEPR; then, it looks at the characteristics of the business cycles derived from the real GDP and industrial production. It finds that although the results of a comparison of the features of the business cycles of the US and the euro area differ depending on the time period

considered and data used in the analysis, the business cycles have very different characteristics in the two areas. Thus, the model in the second chapter can be used to answer a question of to what extent the labor market institutions are responsible for the differences in the features of business cycles in different economies.

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## 1.1 Introduction

The new Keynesian dynamic stochastic general equilibrium models have served as a workhorse in explaining the relationships among money, inflation and business cycles for decades. These models incorporate imperfect competition and nominal rigidities into an individually optimizing framework and focus on the demand-side effects of monetary policy.[1] While these models have made significant progress in explaining the relationships mentioned above, they flawed in two ways. First, Walsh (2005) and Trigari (2006) state that the first generation of these models do not replicate the responses of output and inflation to monetary policy shocks as they are observed in the data. The data suggests that following a monetary policy shock response of inflation dies out very slowly and large and persistent response of output follows a hump-shaped path. Moreover, the inflation is also found to respond by a moderate amount.[2] Second, these models do not explain the stylized facts in the labor market such as the existence of unemployment in equilibrium, low volatility in real wage, lower employment volatility compared with unemployment volatility and the Beveridge curve relationship. In the baseline

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[1]Early examples of new Keynesian models are Yun (1996), Rotemberg and Woodford (1997), Goodfriend and King (1997), Gali and Gertler (1999) and McCallum and Nelson (1999).

[2]See Nelson (1998), Christiano, Eichenbaum and Evans (2005) and Bernanke nd Mihov (1998) for the response of inflation and Estrella and Fuhrer (2002) for the response of output.

new Keynesian model, the existence of unemployed workers is ruled out by the assumption of a frictionless perfectly competitive labor market. Moreover, due to the same assumption, the real wage is equated to the marginal rate of substitution between consumption and leisure. This results in a very significant change in hours worked and the real wage in the case of a monetary policy shock.[3]

Researchers tried to overcome these shortcomings by extending the baseline model in several ways such as including structural inflation inertia, adding habit persistence, assuming variable capital utilization, incorporating sticky wages and adding labor market frictions.[4] Recent studies including Gali, Gertler and Lopez-Salido (2001), Smets and Wouters (2003), Christiano, Eichenbaum and Evans (2005) and Levin, Onatski, Williams and Williams (2005) proved that incorporating labor market frictions would improve the understanding of business cycle fluctuations. In other words, by concentrating on labor market frictions, it might be possible to simultaneously generate desired responses of inflation and output to monetary policy shocks while capturing the labor market dynamics.

If one wants to incorporate labor market frictions into the new Keynesian model, search and matching model, along with the works of Mortensen and Pissarides,[5] is going to be a natural candidate.[6] According to Shimer (2005) this model is attractive due to a couple of reasons. First, it is analytically tractable and it describes the way the labor market functions in an appealing way. Second, the model is able to deliver rich and intuitive comparative statics. However, the conventional Mortensen and Pissarides model is blamed for not being able to account for some of the business cycle facts. In particular, under standard calibrations of parameters it cannot account for labor market volatility observed in the data. This idea has recently been discussed by Costain and Reiter (2003), Hall (2005), Shimer (2005), Hagedorn

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[3] See Christiano, Eichenbaum and Evans (1997) and Bernanke and Mihov (1998).

[4] See Gali and Gertler (1999), Erceg, Henderson and Levin (2000), Sbordone (2002), Gali, Gertler and Lopez-Salido (2001), Smets and Wouters (2003), Blanchard and Gali (2005), Christiano, Eichenbaum and Evans (2005) and Levin, Onatski, Williams and Williams (2005) for examples.

[5] See Pissarides (1990), Mortensen and Pissarides (1994, 1999) and Pissarides (2000)

[6] For recent examples of search and matching models, see Gerke and Rubart (2003), Trigari (2004), Christoffel and Linzert (2005), Krause and Lubik (2005), Walsh (2005), Trigari (2006), Gertler and Trigari (2009), and Gertler, Sala and Trigari (2008).

and Manovskii (2008) and Mortensen and Nagypal (2007). For an example to the facts that conventional model falls short of explaining is the low volatility of the real wage as observed in the data. In the conventional model the wages are determined by period-by-period Nash bargaining between firms and workers. This results in a high volatility in real wage. To improve the empirical performance of the conventional model Shimer (2005) and Hall (2005) incorporate an ad hoc real wage stickiness to the models they employ. They find that sticky wages assumption results in a lower volatility in the real wage. In the literature there are some other prominent works that show that introducing sticky wages assumption to the dynamic general equilibrium macroeconomic framework improves the empirical performance of that model. Recent examples are Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005). Gertler and Trigari (2009), however, mention that ad hoc sticky wages assumption must have primitive forces behind it.

Basic purpose of this chapter is to create a model that is able to capture the labor market facts while generating impulse responses of the key macroeconomic variables to a monetary policy shock as observed in the data. Following the findings in the literature above, a search-theoretic model is employed. The basic framework incorporates labor market frictions to the standard new Keynesian model and extends it with staggered wage contracts. Among a lot of alternative search-theoretic models,[7] search and matching model provided by Gertler and Trigari (2009) is chosen. In that study, the authors extend the model of Trigari (2004) with staggered wage contracts in a non-monetary environment.

The agents operating in the economy are households, intermediate firms, retailers, a final good producer and monetary authority. The households maximize their utility and if they are matched with an open vacancy they are hired by the intermediate firms. The search and matching frictions and bargaining over the wage occur at this level of the economy. The unemployed workers search for a job and firms post vacancies. At each period only a fraction of the firms are allowed to bargain over the wage with their existing workforces. There are two side effects of the assumption that only a fraction of the workers are allowed to bargain at

[7] For a detailed analysis on search-theoretic models see Rogerson, Shimer and Wright (2005).

each period: horizon effect and spillover effects. The horizon effect stems from the difference between the time horizons of the bargaining worker and bargaining firm. The source of the spillover effects is the difference between the average market wage and contract wage. There are direct spillover effects and indirect spillover effects. Retailers buy intermediate goods at their marginal cost and produce the retail goods without any other cost. The retailers set their prices as in Calvo (1983). Final good producer buys all the goods produced by retailers in a monopolistically competitive market and produce the final good that is available for consumption. The final good producer employs a Dixit-Stiglitz aggregator.

To test the empirical performance of the model, the model is calibrated according to the US economy and the impulse responses of the key macroeconomic variables to a one standard deviation shock to the nominal interest rate under alternative specifications are derived. First specification is the case of a frictionless economy. In the second case sticky prices assumption is introduced and the impacts of this assumption on the impulse responses are observed. In the final case sticky wages assumption is incorporated to the framework. In the final case, two subcases are considered. In the first subcase the impulse responses when the spillover effects are not in present are derived. In the second subcase, the spillover effects are allowed to operate and the impulse responses are derived accordingly.

The framework of this study differs from that of Gertler and Trigari (2009) such that this model is a monetary model with nominal price rigidities. With price rigidities, the model is capable of capturing the adjustments in the prices and generate responses of inflation to shocks. The differences between this study and Gertler, Sala and Trigari (2008) are: First, this study considers real wage rigidity instead of nominal wage rigidity; second, this study calibrates the model according to the US economy where Gertler et al. (2008) estimate their model with Bayesian techniques; third, the focus of this study is the impulse responses of variables to a monetary policy shock where they focus on productivity shock and investment shock; fourth, this study investigates the impacts of alternative assumptions of sticky prices and sticky wages on the responses of macroeconomic variables where they consider only the full model with all rigidities.

The results of this study are twofold. First, the model is able to generate qualitatively similar responses to a monetary policy shock for nominal interest rate, output, employment, unemployment, the real wage and inflation as in the case of a frictionless economy. However, inflation and real wage are much less volatile once sticky prices assumption and staggered wage setting are introduced to the model. Second, the model is able to explain some of the labor market facts observed in the data. The model generates unemployment in equilibrium and the model displays the Beveridge curve relationship. These results lead to the conclusion that incorporating search and matching frictions and a bargaining framework into the baseline new Keynesian model can generate a lower response for inflation and the real wage to a monetary policy shock and can explain some of the labor market facts at the same time.

The next section discusses the details of the model and provides log-linearized version of the relevant equations for deriving the impulse responses. The calibration of key parameters of the model is discussed in the third section. The impulse responses of the key variables of the model under alternative specifications are derived in the fourth. The last section concludes the paper and gives some ideas for further research.

## 1.2 The Model

The model of this study has five agents: households, intermediate goods producers, retailers, a final good producer and monetary policy authority. The households maximize their lifetime utilities given their budget constraints. The main purpose of the distinction between the intermediate goods producers and the retailers is the desire for distinguishing between labor market frictions and price rigidities.[8] The intermediate goods producers face labor market frictions, namely search and matching frictions. Moreover, at any point in time only a fraction of the intermediate firms are allowed to negotiate their wages. This fraction is assumed to be exogenous to the model. The retailers operate in a monopolistically competitive market and set their prices as in Calvo (1983). If there was only one type of firm, it would face the labor market frictions in the decision process of employment and price rigidities while setting the price of the retail good. This would make the model very complicated. The final good

[8] This type of modeling was first proposed by Bernanke, Gertler and Gilchrist (1999).

producer buys all retail goods, combines them via a Dixit-Stiglitz aggregator and produces the final good. Finally, the monetary authority conducts the monetary policy by a simple Taylor rule using the short-term nominal interest rate as the instrument.

### 1.2.1 The Household

The households are assumed to form a large extended family and distributed on a unit interval, $[0, 1]$. At any point in time, some of the members of the family are employed while the rest are unemployed. Because the income of an individual depends on her employment status, her saving and consumption decisions depend on her employment history. Therefore, there is heterogeneity in income and thus in consumption decisions of households. This creates distributional problems. To avoid these problems, it is assumed that the family pools incomes and then makes its consumption and saving decisions.[9] In other words, each household is perfectly insured by other households.[10] The large extended family is simply referred as the household.

The household chooses consumption, saving in the form of capital, and bond holdings at each period. The household's problem can be formalized as:

$$max \quad E_t \sum_{t=0}^{\infty} \beta^t log(c_t)$$

$$s.t. \quad c_t + k_{t+1} + \frac{B_t}{P_t r_t^n} = w_t n_t + (1 - n_{t-1})b + (z_t + 1 - \delta)k_t + \Pi_t + T_t + \frac{B_{t-1}}{P_t} \tag{1.1}$$

$c_t$, $k_t$, and $B_t$ are consumption of the final good, saving in terms of capital, and per-capita holdings of a nominal one-period bond, respectively. The nominal return of a one-period bond at time $t$ is denoted by $r_t^n$. The household rents the capital she owns to the firms with a rental rate $z_t$. $P_t$ is the aggregate price level, $b$ stands for flow value of unemployment, and $T_t$ is total transfers from the government at time $t$. Thus, total income is sum of wage income

[9]This assumption is very common in the literature. See Merz (1995), Andolfatto (1996), den Haan, Ramey and Watson (2000), Christoffel and Linzert (2005), Trigari (2006), Walsh (2005), and Gertler and Trigari (2009).

[10]The appendix of Andolfatto (1996) provides a simple institutional structure that implements the full-insurance result.

of employed individuals, unemployment benefits of not working households, total rent income from capital remaining after depreciation, total payments of the firms to the laid-off workers in the beginning of the period, the profits distributed by the firms, transfers from the government and return from bonds that the household bought in the previous period.

Assuming that $\lambda_t$ shows the marginal utility of consumption at date $t$ and $\omega_t$ is the Lagrange multiplier, the first order conditions of the problem above can be written as:

$$\omega_t = \frac{1}{c_t} \tag{1.2}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} r_t^n \tag{1.3}$$

$$\lambda_t = \beta E_t \lambda_{t+1} [z_{t+1} + 1 - \delta] \tag{1.4}$$

$$c_t + k_{t+1} + \frac{B_t}{P_t r_t^n} = w_t n_t + (1 - n_{t-1})b + (z_t + 1 - \delta)k_t + \Pi_t + T_t + \frac{B_{t-1}}{P_t} \tag{1.5}$$

The real interest rate is defined as $1 + r_t = E_t \frac{P_t}{P_{t+1}} r_t^n$. With this definition the first order conditions pin down to:

$$\lambda_t = \beta E_t \lambda_{t+1}(1 + r_t) \quad \textit{and} \quad \lambda_t = \beta E_t \lambda_{t+1}[z_{t+1} + 1 - \delta] \tag{1.6}$$

Therefore, in the equilibrium the first order conditions yield $r_t = E_t[z_{t+1} - \delta]$.

### 1.2.2 Matching and production in intermediate market

In the labor market, search and matching model pioneered by Mortensen and Pissarides is employed. The wages are set under Nash bargaining framework. The firms and workers meet on a matching market where firms post vacancies and search for workers from an unemployment pool and workers search for jobs. Therefore, the hiring process is time-consuming and costly for both the firms and the workers. However, in this model it is assumed that the unemployed workers passively search for jobs and do not bear any kind of cost from looking for job. The firm pays the cost of searching for a worker by labor adjustment cost.

Within this model some firms create jobs, some others destroy jobs and some workers generate unemployment while looking for new jobs. This process is summarized by a single matching function. In particular, the matching function is given by:

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{1.7}$$

where $m_t$, $u_t$ and $v_t$ represent the matches within period $t$, the number of workers seeking for a job and the number of vacancies posted at time $t$, respectively. $\sigma_m$ is a scale factor that measures the efficiency of the matching process and $\sigma \in (0, 1)$ is the elasticity of the matching with respect to unemployment. The matching function is increasing in both of its arguments and it displays constant returns to scale. While an alternative assumption for the matching function like increasing returns to scale might be interesting, Diamond (1981, 1982) states that increasing returns can generate multiple equilibria. Moreover, the assumption of constant returns provides a more tractable model.[11]

The sum of the vacancies posted in the economy at time $t$ is given by $v_t = \int_0^1 v_t(i)di$. Similarly, the level of employment is defined as the sum of the workforces of each intermediate firm. Formally, it is given by $n_t = \int_0^1 n_t(i)di$. The unemployment level is determined by the difference between the unity and the number of people working as of the end of the previous period. Here, the distinction between unemployed worker and not in the labor force worker is ignored. Blanchard and Diamond (1989) and Andolfatto (1996) support this idea and the former study states that the flows from unemployed pool and not in the labor force pool to employment are almost the same. For the reverse flows, Clark (1990) finds 0.02 for transition from employment to unemployment and for transition from employment to non-participation he finds 0.033. Like Cole and Rogerson (1999) it is reasonable to consider these not in the labor force as unemployed people searching for jobs at a lower intensity. Therefore, the unemployment level is defined as:

$$u_t = 1 - n_{t-1} \tag{1.8}$$

[11] Blanchard and Diamond (1989) and Pissarides (1990) provide empirical evidence for the assumption of constant returns to scale for the matching function.

It is assumed that all workers in the unemployment pool look for a job and if a worker finds a match, she starts working within that period immediately.

Let $q_t$ denote the probability an open vacancy is going to be matched with a searching worker and $s_t$ represent the probability a worker searching for a job is going to be matched with an open vacancy. Then,

$$q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \quad and \quad s_t = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{1.9}$$

where $\theta_t$ is the ratio of the number of open vacancies to the number of job-seeking workers, $\frac{v_t}{u_t}$. In other words, $\theta_t$ measures the labor market tightness. Keeping the number of unemployed people constant, an increase in the number of vacancies will result in a decrease in the probability of an open vacancy to be filled, $q_t$, and an increase in the probability of an unemployed worker to be matched with a job, $s_t$. This means that the tighter the labor market, the higher the exit rate from unemployment, and the lower the probability that a vacant job will be filled.

The intermediate firms are distributed on the unit interval. They use labor and capital as their inputs and sell their products to retailers. The production of the firm $i$ is denoted by $y_t(i)$ and the inputs used by this firm are expressed by $k_t(i)$ and $n_t(i)$.

In this model adjustment in employment occurs only in the extensive margin. The variations in the intensive margin are ignored for two reasons as mentioned in Sala, Soderstrom and Trigari (2008). First, there are powerful findings in the literature that most of the cyclical variations in the total work hours occur in the extensive margin. According to Hansen (1985) more than sixty percent of the variation in the total work hours occurs in the extensive margin. The second reason is the microeconomic evidence. The earlier estimates prove that the Frisch elasticity is close to zero. For example MaCurdy (1981) finds that a ten percent increase in the wage of a male worker causes a one to five percent change in his working hours. Altonji (1986) uses a similar framework as in MaCurdy (1981) and reports that inter-temporal labor supply elasticity is between 0 and 0.35. More recent studies including Ziliak and Kniesner (1999) and French (2004) report comparable results with those earlier estimates. With the assumption of a Cobb-Douglas type production function, the output produced by firm $i$ can be written as:

$$y_t(i) = k_t(i)^{\alpha} n_t(i)^{1-\alpha} \tag{1.10}$$

In the literature it is common to define the hiring rate of the intermediate firms, which is given by the ratio of new hires of a firm to its existing workforce. The hiring rate of firm $i$ is denoted by $x_t(i)$ and defined as:

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)} \tag{1.11}$$

At each period firm $i$ separates from a fraction of its existing workforce. This fraction is assumed to be exogenous and denoted by $1-\rho$. Then, the workforce of the firm at time $t$ will be the sum of the surviving workers from previous period and new hires at current period. Thus, the evolution of the workforce of firm $i$ is:

$$n_t(i) = \rho n_{t-1}(i) + q_t v_t(i) \tag{1.12}$$

The value of firm $i$ at time $t$ is:

$$F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_{t-1}(i) - z_t k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i) \tag{1.13}$$

Here $w_t(i)$ stands for the ongoing wage rate at firm $i$ and $E_t\Lambda_{t,t+1}$ is the firm's discount factor, where $\Lambda_{t,t+1} = \frac{\lambda_{t+1}}{\lambda_t}$. Finally, $\frac{\kappa}{2} x_t(i)^2$ gives the quadratic labor adjustment cost which is a function of the firm's hiring rate.

The firm takes the existing employment stock, probability of filling a vacancy, the rental rate and current and expected path of the real wage as given and maximizes its value with respect to vacancies and capital stock. If the firm is allowed to negotiate at that period, it will bargain over the wage with its workforce. If it is not allowed, previous period's wage will prevail. Before discussing the details of wage determination process I elaborate on the intermediate firm's hiring and renting decisions.

The firm chooses its employment level at time $t$ by setting the hiring rate. The hiring decision of the firm can be written as:

$$\kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta\frac{\kappa}{2}E_t\Lambda_{t,t+1}x_{t+1}(i)^2 + \rho\beta\kappa E_t\Lambda_{t,t+1}x_{t+1}(i) \tag{1.14}$$

The capital decision is straightforward to derive:

$$z_t = \alpha p_t^w \frac{y_t(i)}{k_t(i)} \tag{1.15}$$

It is assumed that the capital market is perfectly competitive and capital is perfectly mobile. Due to these assumptions the output/capital ratios are equal across the firms. Therefore, $\frac{y_t(i)}{k_t(i)} = \frac{y_t}{k_t}$.

Finally, the value of adding another worker to the labor force for the firm $i$ is:

$$\frac{\partial F_t(i)}{\partial n_t(i)} = J_t(i) = p_t^w f_{nt}(i) - w_t(i) - \beta\frac{\kappa}{2}E_t\Lambda_{t,t+1}x_{t+1}(i)^2 + \beta E_t\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(i)J_t(i) \tag{1.16}$$

Equation 1.16 can also be rewritten by using the fact that $n_{t+1}(i) = \rho n_t(i) + x_{t+1}(i)n_t(i)$ as:

$$J_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta\frac{\kappa}{2}E_t\Lambda_{t,t+1}x_{t+1}(i)^2 + \rho\beta E_t\Lambda_{t,t+1}J_t(i) \tag{1.17}$$

After defining the decision process of the firms, the values of being employed at firm $i$ and being unemployed to a worker are derived. These values are denoted by $V_t(i)$ and $U_t$, respectively. The value of being employed differs across the workers because the wage is different at each firm. The value of being employed at firm $i$ is:

$$V_t(i) = w_t(i) + \beta E_t\Lambda_{t,t+1}[\rho V_{t+1}(i) + (1-\rho)U_{t+1}] \tag{1.18}$$

It is the sum of the wage that the worker is receiving from the firm at time $t$ and discounted value of expected return at time $t+1$. In the next period, the worker will survive at the same firm with probability $\rho$ or will be separated with probability $1-\rho$.

While deriving the value of being unemployed to a worker first the average value of employment, $V_{x,t}$, to the new workers is defined. In particular,

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di \tag{1.19}$$

The equation 1.19 can be rearranged such that $V_{x,t} = \frac{1}{v_{t-1}} \int_0^1 V_t(i) v_{t-1}(i) di$. In other words, the average value of being employed is the average value of all vacancies to the worker. The value of being unemployed on the other hand is the sum of the unemployment benefits received at that period and expected discounted value of the next period's income. In the next period, an unemployed worker will find a successful match with an open vacancy with probability $s_{t+1}$ or will be unemployed again with probability $1 - s_{t+1}$. Formally, the value of being unemployed to a worker can be expressed as:

$$U_t = b + \beta E_t \Lambda_{t,t+1}[s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1}] \tag{1.20}$$

The wage setting process is a Poisson process as in Calvo-type price setting. At any period each intermediate firm re-negotiates its wage with its existing workforce with probability $1 - \varphi_w$. In other words, average duration of the wage at a firm is $\frac{1}{1-\varphi_w}$. Moreover, the probability a firm can re-negotiate its wage is independent from its negotiation history. Once an intermediate firm is allowed to bargain with its existing workforce, they negotiate through Nash bargaining. This process is summarized by:

$$max \quad H_t(r)^\eta J_t(r)^{1-\eta} \tag{1.21}$$

where $J_t(r)$ denotes the value of adding another worker to the bargaining firm and $H_t(r)$ stands for the surplus of an average negotiating worker. To distinguish between the negotiating and not-negotiating firms, the former ones are indexed with $r$.

Let $w_t^*$ stand for the wage of a firm that is allowed to re-negotiate. The bargained wage is the same among all bargaining firms due to the constant returns because all firms and workers face with the same problem. All workers are identical at the margin and the firm bargains with the marginal worker. To be specific, the surplus of a worker working at firm $i$ is:

$$H_t(i) = V_t(i) - U_t \tag{1.22}$$

and the average surplus of a worker hired at time $t$ is:

$$H_{x,t} = V_{x,t} - U_t \tag{1.23}$$

Plugging the equations 1.18 and 1.20 into equation 1.23 and rearranging it results in

$$H_t(i) = w_t(i) - b + \beta E_t \Lambda_{t,t+1}[\rho H_{t+1}(i) - s_{t+1} H_{x,t+1}] \tag{1.24}$$

Due to the assumption of a Poisson process in wage determination, both the workers and firms do not know the exact time of next negotiation. Therefore, both parties have to take the future path of wage into consideration. Before expressing the surpluses of the firm and worker, corresponding cumulative discount factors have to be defined. The firm's cumulative discount factor, $\Sigma_t(r)$, depends on the future employment size of the firm and expected duration of the contract. The worker's cumulative discount factor, $\Delta_t$, depends on the probability the worker will survive in the job and expected duration of the contract. These factors are defined as:

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)(\beta\varphi_w)^s \Lambda_{t,t+s} \tag{1.25}$$

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\beta\varphi_w)^s \Lambda_{t,t+s} \tag{1.26}$$

There is only one difference between these two discount factors. The firm's discount factor depends on the expected employment size of the firm, $\frac{n_{t+s}}{n_t}(r)$, while the worker's discount factor depends on the probability the worker will survive, $\rho^s$. The rest of the terms in the discount factors is identical. On average, expected employment size exceeds the probability a worker will be unemployed in the future. Therefore, the worker places less weight on the future than does the firm. The worker just considers her wage in that firm but the firm has to consider its employment size in the present time and in the future. Having defined the discount factors, the sum of the expected wage revenue of a bargaining worker, $W_t^w(r)$, and expected wage payment of a firm, $W_t^f(r)$, can be expressed as:[12]

$$W_t^w(r) = \Delta_t w_t^* + (1-\varphi_w)E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}^* \tag{1.27}$$

$$W_t^f(r) = \Sigma_t w_t^* + (1-\varphi_w)E_t \sum_{s=1}^{\infty} \frac{n_{t+s}}{t}(r) \beta^s \Lambda_{t,t+s} \Sigma_{t+s}(r) w_{t+s}^* \tag{1.28}$$

As the appendix proves, the surpluses of the bargaining worker and firm can be deduced to:

$$H_t(r) = W_t^w(r) - E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} [b + \beta s_{t+s} \Lambda_{t+s,t+s+1} H_{x,t+s+1}] \tag{1.29}$$

$$J_t(r) = E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} [p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa}{2} x_{t+s+1}^2(r)] - W_t^f(r) \tag{1.30}$$

Then, the solution to the Nash bargaining problem is:

$$\eta \Delta_t J_t(r) = (1-\eta)\Sigma_t(r) H_t(r) \tag{1.31}$$

Rearranging this solution results in:

$$\chi_t J_t(r) = (1-\chi_t(r)) H_t(r) \tag{1.32}$$

with $\chi_t(r) = \frac{\eta}{\eta + (1-\eta)\frac{\Sigma_t(r)}{\Delta_t}}$, which denotes the relative weight in the sharing rule. There is only one difference between the solution to the Nash bargaining in this model and a period-by-period Nash bargaining solution. This difference is named as horizon effect by Gertler and Trigari (2009). If the firms are allowed to bargain every period, the Nash bargaining solution will depend only on the bargaining power of the worker. It will be the case that $\eta J_t(r) = (1-\eta)H_t(r)$. However, in this model the sharing rule depends on the worker's bargaining power and the ratio of the firm's and worker's cumulative discount factors. As discussed above, during the bargaining process the worker cares only about her position at the

[12]The appendix provides the details of the derivations.

firm where the firm considers its existing workforce as well as its future employment size. This results in, on average, $\frac{\Sigma_t(r)}{\Delta_t}$. Therefore, $\chi_t(r) < \eta$. This means that the bargaining power of the firm in this model is higher than that in the conventional model due to the horizon effect.

The bargained wage can be driven by plugging the value of hiring another worker to the firm and value of being employed to a worker into the bargaining problem.

$$\Delta_t w_t^* = w_t^o(r) + \rho\beta\varphi_w E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* \tag{1.33}$$

where $w_t^o(r)$ stands for the target wage and defined as:

$$w_t^o(r) = \chi_t(r)[p_t^w f_{nt} + \beta \frac{\kappa}{2} E_t \Lambda_{t,t+1} x_{t+1}^2(r)] + (1 - \chi_t(r))[b + \beta s_{t+1} E_t \Lambda_{t,t+1} H_{x,t+1}]$$

The target wage is a convex combination of the contribution of a worker to the match and his foregone benefits from being unemployed. Iterating equation 1.33 renders in

$$w_t^* = E_t \sum_{s=0}^{\infty} \phi_{t,t+s} w_{t+s}^o(r) \tag{1.34}$$

where

$$\phi_{t,t+s} = \frac{(\rho\beta\varphi_w)^s \Lambda_{t,t+s}}{E_t \sum_{s=0}^{\infty} (\rho\beta\varphi_w)^s \Lambda_{t,t+s}} \tag{1.35}$$

In the limit case, $\varphi_w = 0$, the bargained wage converges to the target wage, which is the case in period-by-period bargaining solution.

The average wage across all workers in the economy can be written as:

$$w_t = \int_0^1 w_t(i) \frac{n_t(i)}{n_t} di \tag{1.36}$$

It is already assumed that a certain fraction of the firms can re-negotiate and rest of the firms accept the previous period's wage. Then equation 1.36 can be written as:

$$w_t = (1 - \varphi_w) w_t^* + \varphi_w \int_0^1 w_{t-1}(i) \frac{n_t(i)}{n_t} di \tag{1.37}$$

### 1.2.3 Retailers and final goods market

This section presents price setting in the retail goods market, where Rotemberg and Woodford (1997) is followed. There is a continuum of monopolistically competitive retailers distributed on a unit interval. They buy intermediate goods, differentiate them with a technology that transforms one unit of intermediate good to one unit of retail good at no cost, and resell them to the final good producer. Final good producer collects these goods and combines them via a Dixit-Stiglitz aggregator. The profit generated in the retail market is assumed to be distributed to the households, who are the ultimate owners of the firms. The monopoly powers of the retailers provide the nominal price rigidity in the economy. It is assumed that the marginal cost of each retailer is the relative price of each intermediate good. The aggregate output, $y_t$, can be described in terms of each retailer's output, $y_{jt}$, and the firm's own price elasticity of substitution across differentiated retail goods, $\varepsilon$. Formally,

$$y_t = \left[ \int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.38}$$

The aggregate price index can be defined as the aggregate of the nominal sale price of each retailer:

$$P_t = \left[ \int_0^1 P_{jt}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{1.39}$$

The demand curve for good $j$ then is determined by relative price of the good and the aggregate demand. This demand curve is given by:

$$y_{jt} = \left[ \frac{P_{jt}}{P_t} \right]^{-\varepsilon} y_t \tag{1.40}$$

In the price setting a Calvo-type framework is employed. In any given period, with a probability $1-\varphi_p$, each retailer is able to reset its price while the remaining firms are not allowed to change their prices. The probability does not depend on the elapsed time between two adjustment dates. Therefore, assuming that $P_t^*$ is the optimal price for the firms that are able to reset their prices, the price index can be written as:

$$P_t = \left[(1-\varphi_p)(P_t^*)^{1-\varepsilon} + \varphi_p(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{1.41}$$

where the optimal price is the solution to the profit-maximization problem of the firm. The firm sets its price for the expected number of periods that it will not be able to reset its price. The profit maximization problem of the firm can be written as:

$$max \quad E_t \sum_{s=0}^{\infty} (\beta\varphi_p)^s \left[\frac{P_{jt}^*}{P_t} - p_{t+s}^w\right]^{-\varepsilon} y_{jt+s} \tag{1.42}$$

subject to the demand for that good. The optimal price level for the firm that is able to change its price is:

$$p_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} E_t \frac{\sum_{s=0}^{\infty} (\beta\varphi_p)^s P_{t+s}^{\varepsilon} p_{t+s}^w y_{t+s}}{\sum_{s=0}^{\infty} (\beta\varphi_p)^s P_{t+s}^{\varepsilon-1} y_{t+s}} \tag{1.43}$$

### 1.2.4 Monetary athority

The studies that analyze the effects of monetary policy shocks in a new Keynesian framework generally define the monetary policy by a rule for setting the nominal interest rate. This policy rule has either been derived from a specification of the monetary authority's objective function or specified as an arbitrary instrument rule, e.g. Taylor rule (Taylor, 1993).[13] The researchers that derive the rule from a specification of an objective function generally choose to minimize a quadratic loss function that depends on inflation and output gap. Another way to derive the objective function is to derive the policy rule by determining the quadratic loss function from a welfare criterion that is obtained as a log-linear approximation of the utility function.[14] In this study, for simplicity, it is assumed that monetary policymaker uses short-term nominal interest rate as the policy instrument and employs a Taylor-type rule to conduct monetary

[13]For some examples see Clarida, Gali and Gertler (1999), Woodford (1999, 2001), McCallum and Nelson (1999), Svensson and Woodford (2003).

[14]For a detailed analysis of the link between welfare criterion and the typesof quadratic loss functions, see Woodford (2001a).

policy. Defining monetary policy via minimizing a quadratic loss function or deriving it from a welfare criterion is left to further research. The gross nominal interest rate is determined by:

$$r_t^n = \beta^{-(1-\rho_m)}(r_{t-1}^n)^{\rho_m} E_t(\pi_{t+1})^{\gamma_\pi(1-\rho_m)}(y_t^z)^{\gamma_y(1-\rho_m)} e^{\varepsilon_t^m} \tag{1.44}$$

where the degree of interest rate smoothing is captured by the parameter $\rho_m$ and the output gap is denoted by $y_t^z$. The response coefficients of expected inflation and output gap are represented by $\gamma_\pi$ and $\gamma_y$, respectively. The last term on the right hand side is an *i.i.d.* monetary policy shock.

To complete the model, the economy-wide resource constraint is imposed.

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + \frac{\kappa}{2}x_t^2 n_t \tag{1.45}$$

This condition implies that the total output produced at time $t$, $y_t$, must be equal to the sum of the consumption of the household, $c_t$, investment made on capital, $k_{t+1} - (1-\delta)k_t$, and aggregate labor adjustment costs of the intermediate firms, $\frac{\kappa}{2}x_t^2 n_t$.

### 1.2.5 Log-linearizing the model

The model is log-linearized around a zero inflation steady-state. As a general note, the variables with a hat denotes the percentage deviations from their steady state values and the variables without a hat and without a time subscript denote the steady state values of the regarding variables. First, the wage and hiring equations are log-linearized. The percentage deviation of the target wage from its steady state is:

$$\begin{aligned}\hat{w}_t^o(r) = \chi p^w f_n w^{-1}\left(\hat{p}_t^w + \hat{f}_{nt}\right) + \chi w^{-1}\left[p^w f_n + \beta\frac{\kappa}{2}x^2 - b - \frac{s\beta\chi\kappa x}{1-\chi}\right]\hat{\chi}_t(r) + \\ \chi\beta\frac{\kappa}{2}x^2 w^{-1}E_t\hat{\Lambda}_{t,t+1} + \chi\beta\kappa x^2 w^{-1}E_t\hat{x}_{t+1}(r) + s\beta\chi\kappa x w^{-1}E_t\left(\hat{s}_{t+1} + \hat{H}_{t+1} + \hat{\Lambda}_{t,t+1}\right)\end{aligned} \tag{1.46}$$

According to equation 1.46 the variations in the target wage is the sum of the variations in the marginal contribution of a worker to the bargaining firm and foregone benefits of a worker from unemployment. In addition to these, the target wage is also affected by the horizon effect. The horizon effect influences the variations in the target wage in two ways. The first one stems from the steady state value of relative bargaining power, $\chi$. In the case of period-by-period Nash bargaining the steady state value of the relative bargaining power equals the actual bargaining power, $\eta$. When the multi-period wage contracts are introduced, $\chi$ will be less than $\eta$.[15] In other words, the impacts of the factors determining the variations in the target wage are lessened by the introduction of multi-period wage bargaining. The second channel is the deviations in the relative bargaining power, which is captured by $\hat{\chi}_t(r)$. Since the relative bargaining power of the worker depends on the discount factors of the bargaining firm and worker, the movements in these discount factors will impact the target wage too. The percentage deviations of bargaining weight, discount factors of the bargaining firm and worker from their steady state values are:

$$\hat{\chi}_t(r) = -(1-\chi)\left(\hat{\Sigma}_t(r) - \Delta_t\right) \tag{1.47}$$

$$\hat{\Sigma}_t(r) = (1-\rho)\beta\varphi_w E_t \hat{x}_{t+1}(r) + \beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Sigma}_{t+1}(r)\right] \tag{1.48}$$

$$\hat{\Delta}_t = \rho\beta\varphi_w E_t\left(\hat{\Lambda}_{t,t+1} + \hat{\Delta}_{t+1}\right) \tag{1.49}$$

According to these equations, the bargaining weight responds to any variation in the discount factors of both worker and firm. It is also obvious that when the flexible wages areallowed by setting $\varphi_w = 0$, it will be the case that $\chi_t(r) = \eta$. This implies that the variations in the target wage will occur only due to the reasons creating variations in conventional bargaining model. In other words, the horizon effect will be ineffective.

As the appendix proves, equation 1.46 can be simplified to:

$$\hat{w}_t^o(r) = \hat{w}_t^o + \frac{\tau_1}{1-\rho\beta\varphi_w} E_t\left(\hat{w}_{t+1} - \hat{w}_{t+1}^*\right) + \frac{\tau_2}{1-\rho\beta\varphi_w}\left(\hat{w}_t - \hat{w}_t^*\right) \tag{1.50}$$

[15] Under the calibration presented in the next section at the steady state $\chi = 0.44$ when $\eta = 0.5$

where

$$\hat{w}_t^o = \varphi_{f_n}\left(\hat{p}_t^w + \hat{f}_{nt}\right) + \varphi_\chi \hat{\chi}_t + \varphi_s(1-\chi)^{-1}E_t\hat{\chi}_{t+1}$$
$$+(\varphi_x + \varphi_s)E_t\hat{x}_{t+1} + \left(\frac{\varphi_x}{2} + \varphi_s\right)E_t\hat{\Lambda}_{t,t+1} + \varphi_s E_t\hat{s}_{t+1} \tag{1.51}$$

with $\tau_1 = \varphi_s\Gamma(1-\rho\beta\varphi_w)$ and $\tau_2[\varphi_x\varphi_w - \varphi_\chi(1-\chi)(1-\rho)\Psi]\epsilon\Sigma w(1-\rho\beta\varphi_w)$. $\hat{w}_t^o$ stands for the deviations in the spillover-free target wage. There are two spillover effects in this model: direct spillover effects and indirect spillover effects. The significance of these effects for the target wage is captured by $\tau_1$ and $\tau_2$, respectively. The direct spillover effect stems from the difference between the expected average market wage and the expected contract wage. If, for example, the expected average market wage exceeds the expected contract wage, workers will move into employment in the next period. In the other case, the workers will wait for another period and try to find a job. Therefore, there is an impact of the expected average market wage on the wage bargaining process. The hiring rate of the bargaining firm is the source of the indirect spillover effects. The appendix proves that the deviations in the hiring rate of a bargaining firm depends on the difference between the deviations in the average market wage and the contract wage. This relationship is summarized by:

$$\hat{x}_t(r) = \hat{x}_t + w\Sigma\epsilon(\hat{w}_t - \hat{w}_t^*) \tag{1.52}$$

The hiring rate influences the target wage through the bargaining firm's savings in adjustments costs and the bargaining weight. Finally, the percentage deviations of the contract wage and the wage index from their steady state levels are:

$$\hat{w}_t^* = (1-\rho\beta\varphi_w)\hat{w}_t^o(r) + \rho\beta\varphi_w E_t\hat{w}_{t+1}^* \tag{1.53}$$
$$\hat{w}_t = (1-\varphi_w)\hat{w}_t^* + \varphi_w\hat{w}_{t-1} \tag{1.54}$$

After log-linearizing the target wage, log-linearized versions of the rest of the equations in the model are presented as follows. The hiring rate of the firm is

$$\hat{x}_t = p^w f_n \epsilon \left( \hat{p}_t^w + \hat{f}_{nt} \right) - w\epsilon \hat{w}_t + \beta E_t \hat{x}_{t+1} + \frac{\beta}{2}(1+\rho) E_t \hat{\Lambda}_{t,t+1} \tag{1.55}$$

Furthermore, aggregating and log-linearizing the vacancies around its steady state results in

$$\hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_t \tag{1.56}$$

The percentage deviations of production function, marginal product of labor and capital rental rate from their steady state levels are the same across the firms. Therefore,

$$\hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{n}_t \tag{1.57}$$

$$\hat{f}_{nt} = \hat{y}_t - \hat{n}_t \tag{1.58}$$

$$\hat{z}_t = \hat{p}_t^w + \hat{y}_t - \hat{k}_t \tag{1.59}$$

The equations that build the matching process; matching function, transition probabilities and employment and unemployment dynamics are:

$$\hat{m}_t = \sigma \hat{u}_t + (1-\sigma)\hat{v}_t \tag{1.60}$$

$$\hat{q}_t = \hat{m}_t - \hat{v}_t \tag{1.61}$$

$$\hat{s}_t = \hat{m}_t - \hat{u}_t \tag{1.62}$$

$$\hat{n}_t = \rho \hat{n}_{t-1} + (1-\rho)\hat{m}_t \tag{1.63}$$

$$\hat{u}_t = -\frac{n}{u}\hat{n}_t \tag{1.64}$$

The Euler equation of the model, the second of the first order conditions and the marginal utility of consumption can be written as:

$$\hat{\lambda}_t = E_t[\hat{\lambda}_{t+1} + \hat{r}_t] \tag{1.65}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \beta z E_t \hat{z}_{t+1} \tag{1.66}$$

$$\hat{\lambda}_t = -\hat{c}_t \tag{1.67}$$

The real interest rate equation, $r_t^n = \frac{P_{t+1}}{P_t}(1 + r_t)$, therefore;

$$\hat{r}_t^n = E_t\hat{\pi}_{t+1} + (1-\beta)\hat{r}_t \tag{1.68}$$

The model produces a forward-looking Phillips curve relationship:

$$\hat{\pi}_t = \theta \hat{p}_t^w + \beta E_t \hat{\pi}_{t+1} \tag{1.69}$$

where $\theta = \frac{(1-\varphi_p)(1-\beta\varphi_p)}{\varphi_p}$. Finally, the nominal interest rate rule and the resource constraint are:

$$\hat{r}_t^n = \rho_m \hat{r}_{t-1}^n + (1-\rho_m)\gamma_\pi E_t\hat{\pi}_{t+1} + (1-\rho_m)\gamma_y E_t \hat{y}_{t+1} + \varepsilon_t^m \tag{1.70}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \delta\frac{k}{y}\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] + \frac{\kappa x^2 n}{2y}\left[2\hat{x}_t + \hat{n}_t\right] \tag{1.71}$$

## 1.3 Model calibration

The length of the period is taken as a quarter and the time discount factor, $\beta$, is set to 0.99. This yields approximately a four percent annual rate of interest at the steady state. Following Christiano, Eichenbaum and Evans (2005), Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008), the capital depreciation rate, $\delta$, is calibrated as 0.025. This results in a ten percent annual rate of depreciation in capital. In the studies cited above the steady state share of capital income, $\alpha$, ranges from 0.3 and 0.36. Erceg, Henderson and Levin (2005) and Walsh (2005) also report values in the same range. In this study the middle point is chosen and $\alpha$ is set to 0.33.

For the exogenous job separation rate, $1-\rho$, Davis, Haltiwanger and Shuh (1996) and Hall (1995) find values between 0.08 and 0.1. Gertler and Trigari (2009) use the fact that an average job lasts for about two and a half years. This fact results in an exogenous job separation rate of 0.035 for monthly data. Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008) follow the same path and set $\rho$ to 0.895, which is consistent with Shimer (2005). Since the framework employed here is similar to the latter studies, job separation rate is set to 0.105.

In the literature, there is not a consensus for the value of elasticity of matches to unemployment, $\sigma$. There is a wide range of values used in the literature (0.4 to 0.72).[16] This parameter is not a very critical one for the results of this paper; therefore, following the majority of the studies $\sigma$ is set to 0.5.

To calibrate the coefficient of the labor adjustment cost, $\kappa$, the steady state value of the probability that an unemployed worker finds a job or shortly job finding rate, $s$, has to be calibrated. The values used in the literature varies from 0.6 to 0.95.[17] To be able to compare the model in this study with the studies that use a similar framework $s$ is set to 0.95. The bargaining power of the workers, $\eta$ is set to 0.5. This leads to an equally shared surplus between the bargaining worker and the bargaining firm. The frequency of wage renegotiation, $\varphi_w$, is calibrated to 0.718. This implies that an average firm can re-negotiate its wage once in every three and a half quarters.

The parameter $\bar{b}$ is interpreted differently in the literature. Shimer (2005) thinks that this is unemployment insurance and estimates it as 0.4. Hall (2005) interprets more broadly and treats it as the sum of the unemployment benefits and utility from leisure. He sets it to 0.7. Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008) estimate this parameter as 0.72. In this study a value within this range is chosen for $\bar{b}$, 0.5. Finally, the efficiency parameter in matching function, $\sigma_m$ is calibrated as 0.92 to match the average duration of a vacancy. With this calibration the probability a vacancy will be filled is 0.9. The duration of an average vacancy is reported to be under one month by Blanchard and Diamond

[16] Blanchard and Diamond (1989) and Shimer (2005) set the lowest and highest value, respectively.

[17] Walsh (2005) uses 0.2, actually he uses 0.6 for quarterly data under which the average duration of unemployment is 1.67 quarters.

(1989), which requires $q$ to be 1. However, the distinction between the removal time of the advertisement and actual date that the vacancy is filled was pointed out by van Ours and Ridder (1992). They report that in the first two weeks after the vacancy is posted, seventy five percent of the vacancies are filled, however, it takes on the average forty five days to choose the best applicant from the pool.

Calibration of parameters in the final goods market starts with setting the probability a firm cannot change its price, $\varphi_p$. Gali and Gertler (1999) and Sbordone (2002) set $\varphi_p = 0.85$, which implies that only 15 percent of all firms adjust their prices optimally each period. However, Bils and Klenow (2002) provides empirical evidence that the prices are fixed on average for two quarters. This implies $\varphi_p = 0.5$. In the mean time Christiano, Eichenbaum and Evans (2005) sets it to 0.6. Therefore, setting $\varphi_p = 0.6$ is consistent with the literature and closed to the empirical findings. This implies that an average retailer will keep its price unchanged for 2.5 quarters. Moreover, assuming $\varepsilon = 10$, which is suggested by Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008), results in an approximately 11 percent markup of prices on marginal costs.

Finally the parameters in the nominal interest rate rule are calibrated by following Clarida, Gali and Gertler (2000). Interest rate smoothing parameter, $\rho_m$, is set to 0.9, and the response coefficients in the Taylor rule are calibrated as $\gamma_p = 1.5$ and $\gamma_y = 0.5$. Table 1.1 collects the descriptions and values of important parameters of the model.

## 1.4 Alternative specifications and impulse responses to a monetary policy shock

Three alternative cases are considered and the impulse responses of the key macroeconomic variables under these specifications are derived. The first case is a frictionless economy. It is assumed that the prices and wages are fully flexible. Then, in the second case sticky prices assumption to the first case is introduced and impulse responses with those in the first case are compared. Finally, sticky wages assumption is introduced and the impacts of this assumption on the impulse responses are observed. In the last case, two subcases are considered and impulse responses in the cases of first without spillover effects and second with spillover effects are derived. In the frictionless case, the model consists of the following equations.

Case 1: Frictionless Economy ($\varphi_p = 0 \;\; and \;\; \varphi_w = 0$)

$$\hat{\lambda}_t = -\hat{c}_t \tag{1.72}$$

$$\hat{r}_t^n = E_t\hat{\pi}_{t+1} + (1-\beta)\hat{r}_t \tag{1.73}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \beta z E_t\hat{z}_{t+1} \tag{1.74}$$

$$\hat{m}_t = \sigma\hat{u}_t + (1-\sigma)\hat{v}_t \tag{1.75}$$

$$\hat{u}_t = -\frac{n}{u}\hat{n}_t \tag{1.76}$$

$$\hat{q}_t = \hat{m}_t - \hat{v}_t \tag{1.77}$$

$$\hat{s}_t = \hat{m}_t - \hat{u}_t \tag{1.78}$$

$$\hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_t \tag{1.79}$$

$$\hat{n}_t = \rho\hat{n}_{t-1} + (1-\rho)\hat{m}_t \tag{1.80}$$

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \tag{1.81}$$

$$\hat{z}_t = \hat{p}_t^w + \hat{y}_t - \hat{k}_t \tag{1.82}$$

$$\hat{f}_{nt} = \hat{y}_t - \hat{n}_t \tag{1.83}$$

$$E_t\hat{\Lambda}_{t,t+1} = E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t \tag{1.84}$$

$$\hat{x}_t = p^w f_n \epsilon\left(\hat{p}_t^w + \hat{f}_{nt}\right) - w\epsilon\hat{w}_t + \beta E_t\hat{x}_{t+1} + \frac{\beta}{2}(1+\rho)E_t\hat{\Lambda}_{t,t+1} \tag{1.85}$$

$$\hat{w}_t^* = \hat{w}_t^o(r) = \hat{w}_t, \quad \hat{\Sigma}_t = \hat{\Delta}_t = \hat{\chi}_t = 0, \quad \hat{J}_t = \hat{x}_t = \hat{H}_t \tag{1.86}$$

$$\begin{aligned}\hat{w}_t = {} & \frac{\eta p^w f_n}{w}\left(\hat{p}_t^w + \hat{f}_{nt}\right) + \frac{\eta\beta\kappa x^2 + 2\eta\beta\kappa x s}{2w}E_t\hat{\Lambda}_{t,t+1} + \\ & \frac{\eta\beta\kappa x s}{w}E_t\hat{s}_{t+1} + \frac{\eta\beta\kappa x(x+s)}{w}E_t\hat{x}_{t+1}\end{aligned} \tag{1.87}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \delta\frac{k}{y}\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] + \frac{\kappa x^2 n}{2y}\left[2\hat{x}_t + \hat{n}_t\right] \tag{1.88}$$

$$\hat{r}_t^n = \rho_m\hat{r}_{t-1}^n + (1-\rho_m)\gamma_\pi E_t\hat{\pi}_{t+1} + (1-\rho_m)\gamma_y E_t\hat{y}_{t+1} + \varepsilon_t^m \tag{1.89}$$

$$\hat{\pi}_t = \hat{p}_t^w \tag{1.90}$$

The equations from 1.72 to 1.85, 1.88 and 1.89 are shared by all cases. In the frictionless case, the target wage and aggregate wage are the same as stated in equation 1.86. Moreover, the discount factors are constant and equal to one at all times. The effective bargaining power is equal to the actual bargaining power. Then, in this case the aggregate wage is a convex combination of the worker's contribution to the firm's revenues once she is added to the labor force of the firm and the unemployment benefits that she loses once she accepts a job with a firm. Since the effective bargaining power, $\chi_t$, is constant over time, the horizon effect is not present. The last equation shows the relationship between inflation and marginal cost. Since the final goods producers are allowed to change their prices every period, the percentage change in inflation is governed only by the percentage change in the marginal cost.

Figure 1.1 plots the responses of inflation, output, consumption, employment, unemployment, vacancies and the real wage to a one standard deviation increase in the nominal interest rate under the case of no rigidities. An increase in the nominal interest rate causes a decrease in consumption through the optimality condition of the household presented in equation 1.3. This decrease in consumption results in a decrease in output, employment and vacancies in the economy. The decrease in employment renders in an increase in unemployment. The hiring rate decreases as a result of the decrease in vacancies. Then, the wage rate and the marginal cost decrease. Since the firms are allowed to adjust their prices every period, the percentage deviation of inflation from its steady state level is only determined by the percentage deviation of the marginal cost. Therefore, the aggregate price level adjusts immediately and inflation decreases when the shock hits the economy. All of the variables except unemployment respond to the shock contemporaneously. The unemployment responds with a one quarter delay due to the timing assumption in employment. Since there is a high degree of autocorrelation in the nominal interest rate, the shock dies out very slowly in the real economy. Output and consumption slowly return to their original levels. In the labor market the impacts of the shock last for almost three years. However, inflation returns back to the steady state very soon.

The case that has the sticky prices assumption is labeled as the case 2. Under this case the equations 1.72 to 1.89 of the case 1 will be the same. The only impact of the introduction of this assumption is the replacement of equation 1.90 with the following one.[18]

$$\hat{\pi}_t = \theta \hat{p}^w_t + \beta E_t \hat{\pi}_{t+1} \tag{1.91}$$

This assumption does not influence the equations for the wage bargaining process. Equation 1.91 states that, when the prices are sticky, variations in inflation depends not only on the variations in marginal cost but also on the variations in the expected inflation for the next period.

Figure 1.2 shows the impulse responses of inflation, output, consumption, employment, unemployment, vacancies and the real wage to a one standard deviation increase in the nominal interest rate under the assumption of sticky prices. Compared to the first case the response of inflation is much lower in the second case. However, the sticky price assumption does not have an impact on the responses of the other variables. In other words, the real side of the economy is not influenced by that assumption in terms of responses to a monetary policy shock. An increase in the nominal interest rate causes an increase in the real interest rate because a big fraction of the firms are not allowed to adjust their prices. One explanation for these variables not responding more than they do in the first case is the flexible wage assumption. The impact of the change in nominal interest rate, therefore, real interest rate is fully taken by the real wage and thus by marginal cost. It can be concluded that adding the Calvo-type price stickiness to the model only results in a lower response in inflation.

In the case 3, the sticky wages assumption is incorporated into the previous case. As mentioned above two subcases will be considered. To identify the impacts of spillover effects first the economy when the spillover effects are not present is considered and the impulse responses are derived. Then the spillover effects are included and the influences of these effects on the impulse responses of the variables are identified. The relevant equations for the

[18] To save space the equations that are the same as in case 1 are not replicated.

third case will be the equations 1.72 to 1.85 and the equations below.

$$\hat{\chi}_t = -(1-\chi)(\hat{\Sigma}_t - \hat{\Delta}_t) \tag{1.92}$$

$$\hat{\Sigma}_t = (1-\rho)\beta\varphi_w E_t\hat{x}_{t+1} + \beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Sigma}_{t+1}\right] \tag{1.93}$$

$$\hat{\Delta}_t = \rho\beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Delta}_{t+1}\right] \tag{1.94}$$

$$\hat{\Delta}_t + \hat{w}_t^* = (1-\rho\beta\varphi_w)\hat{w}_t^o(r) + \rho\beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Sigma}_{t+1} + \hat{w}_{t+1}^*\right] \tag{1.95}$$

$$\hat{\pi}_t = \frac{(1-\beta\varphi_p)(1-\varphi_p)}{\varphi_p}\hat{p}_t^w + \beta E_t\hat{\pi}_{t+1} \tag{1.96}$$

$$\hat{w}_t = (1-\varphi_w)\hat{w}_t^* + \varphi_w\hat{w}_{t-1} \tag{1.97}$$

$$\begin{aligned}\hat{w}_t^o(r) = \varphi_{f_n}\left(\hat{p}_t^w + \hat{f}_{nt}\right) + \varphi_\chi\hat{\chi}_t + \varphi_x E_t\hat{x}_{t+1} + \left(\frac{\varphi_x}{2} + \varphi_s\right)E_t\hat{\Lambda}_{t,t+1}\\ + \varphi_s E_t\left[\hat{s}_{t+1} + \hat{x}_{t+1} + \frac{1}{1-\chi}\hat{\chi}_{t+1}\right]\end{aligned} \tag{1.98}$$

$$\begin{aligned}\hat{w}_t^o(r) = \varphi_{f_n}\left(\hat{p}_t^w + \hat{f}_{nt}\right) + \varphi_\chi\hat{\chi}_t + \varphi_s(1-\chi)^{-1}\hat{\chi}_{t+1} +\\ (\varphi_x + \varphi_s)E_t\hat{x}_{t+1} + \left(\frac{\varphi_x}{2} + \varphi_s\right)E_t\hat{\Lambda}_{t,t+1} + \varphi_s E_t\hat{s}_{t+1} +\\ \left[\varphi_x\varphi_w - \varphi_\chi(1-\chi)x\Psi\right]\epsilon\Sigma w\left(\hat{w}_t - \hat{w}_t^*\right) + \varphi_s\Gamma E_t\left(\hat{w}_{t+1} - \hat{w}_{t+1}^*\right)\end{aligned} \tag{1.99}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \delta\frac{k}{y}\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] + \frac{\kappa x^2 n}{2y}\left[2\hat{x}_t + \hat{n}_t\right] \tag{1.100}$$

$$\hat{r}_t^n = \rho_m\hat{r}_{t-1}^n + (1-\rho_m\gamma_\pi)E_t\hat{\pi}_{t+1} + (1-\rho_m\gamma_y)E_t\hat{y}_t + \varepsilon_t^m \tag{1.101}$$

The spillover effects impact the model through the target wage equation. Target wage equation for the subcase when spillover effects are not in effect is given by equation 1.98 and for the other subcase it is given by equation 1.99.

When the assumption that at any period only a fraction of the firms are allowed to bargain their wages with their workforces is introduced to the model, the effective bargaining power, $\chi_t$, will not be equal to the actual bargaining power, $\eta$, anymore. As stated by equation 1.92, in the case of sticky wages the variations in the effective bargaining power depends on the variations in the discount factors of the bargaining firm and worker too. These variations are denoted by $\hat{\Sigma}_t$ and $\hat{\Delta}_t$, respectively. This also introduces the horizon effect to the model as discussed in the previous section. Since at the steady state $\chi < \eta$, the variations in the effective bargaining power of the worker is lessened due to the horizon effect.

Figure 1.3 depicts the impulse responses of inflation, output, consumption, employment, unemployment, vacancies and the real wage to a one standard deviation increase in the nominal interest rate when the spillover effects are not present. The first impact of the sticky wages assumption is the drastic decrease in the response of real wage to the monetary policy shock. Since the prices are sticky, the increase in the nominal interest rate causes an increase in the real interest rate. This results in a decrease in the output and employment. The monetary transmission mechanism discussed in the second case is still in effect. However, in the third case the shock to the nominal interest rate is not fully taken by the real wage and marginal cost. Since the wages are sticky, the responses of output, employment, unemployment and vacancies, in absolute values, are higher compared to the previous case. Finally, it is worth to mention that the real wage slowly returns back to its steady state.

Figure 1.4 presents the impulse responses of the same variables in Figure $B.3$ in the case of sticky prices and sticky wages with spillover effects. The first impact of these spillover effects is that the response of real wage to the change in nominal interest rate is higher when these effects are in present. The reason for this higher volatility in the real wage is due to the higher volatility of the bargained wage compared to the aggregate wage. Not reported here but the impulse response of the bargained wage is much higher than that of the aggregate wage. This finding is in the same line with Haefke, Sonntag and Rens (2008).

Finally, the model is able to generate a negative relationship between unemployment and vacancies, so called Beveridge curve relationship, under all specifications.

## 1.5 Conclusion

Through the last decades the new Keynesian dynamic stochastic general equilibrium models have been used to explain the relationships among money, inflation, and business cycles. However, some studies showed that incorporating imperfect competition and price rigidities into an individually optimizing framework do not replicate the responses of output and inflation to monetary policy shocks as they are empirically observed in the data. Moreover, in these models unemployment in equilibrium is ruled out by a perfectly competitive labor market assumption. These shortcomings triggered other lines of research. Researchers focused on alternative specifications such as including structural inflation inertia, adding habit persistence, assuming variable capital utilization, including sticky wages along with other labor market frictions. Prominent studies in the literature showed that labor market frictions improve the understanding of business cycle fluctuations and effects of monetary policy shocks. It is claimed that by concentrating on the labor market frictions, it might be possible to simultaneously generate desired responses of inflation and output to monetary policy shocks found in the data while capturing the labor market dynamics.

The main purpose of this chapter is to build a tractable model that is able to capture the labor market facts and to generate desired responses of inflation and output to a monetary policy shock simultaneously. To be able to do that search-theoretic models are chosen due to their strengths mentioned in the literature. Among a lot of alternatives, the model built in Gertler and Trigari (2009) is employed and a monetary policy and nominal price rigidities are introduced to that model. To evaluate the model the impulse responses of the key macroeconomic variables to a monetary policy shock under alternative specifications are derived.

The results are as follows. The impulse responses of the variables in the case of sticky prices and sticky wages follow the same pattern as they do in a frictionless economy. However, the responses of inflation and real wage are much less volatile. Sticky prices assumption does not influence the variables other than inflation in terms of responses to a monetary policy shock. The introduction of sticky wages assumption causes a higher response in output, employment, unemployment and vacancies in absolute value. Staggered wage contracts bring two spillover effects and their impacts on the impulse responses to a monetary policy shock are also identified. The real wage is more volatile when these effects are in present because the bargained wage is more volatile than the market wage. Finally, the model is able to generate the Beveridge curve relationship. The case with both sticky prices assumption and sticky wages assumption is able to explain some of the business cycle facts can generate some of the labor market facts simultaneously. Therefore, it can be concluded that it is possible to build a tractable model that explains some of the labor market facts and generate some of the desired responses to a monetary policy shock. This can be done by incorporating labor market frictions in the form of Mortensen-Pissarides framework and staggered wage contracts to a new Keynesian framework.

For further research the model built in this chapter can be extended in several ways. First, since the model is able to explain the business cycle facts and some of the labor market facts simultaneously, it can be used to investigate the influences of the labor market institutions on the responses of macroeconomic variables to macroeconomic disturbances. Second, one can try to improve the empirical performance of the model by capturing the facts that it cannot explain. A couple of these facts are the hump-shaped pattern of the response of output to a monetary policy shock and persistence in the response of inflation.

In the second chapter, I follow the first possible extension and analyze the impacts of labor market institutions on business cycles. The model in that chapter uses a very similar framework to the one built in this chapter with sticky prices and and sticky wages assumptions. It extends the model of the third case with endogenous separations and includes severance payments that is made to the involuntarily separated workers. With the severance payments insitution, there are three institutions considered in that chapter. The other two institutions are unemployment benefits and worker's bargaining power, which are already present in the baseline framwework. It derives the impulse responses of key macroeconomic variables to macroeconomic disturbances under alternative parameterizations of these institutions.

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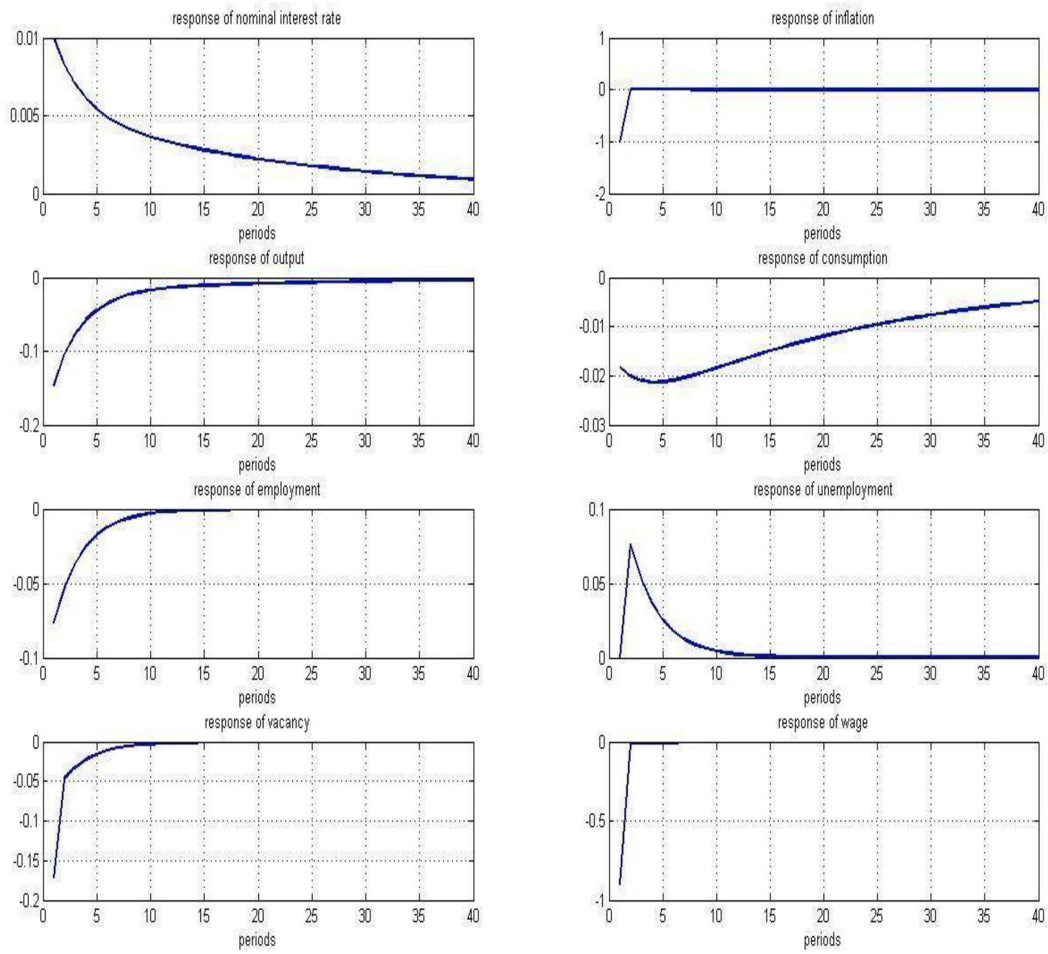
Table 1.1: Parameter Values

| **Parameter** | **Description** | **Value** |
|---|---|---|
| $\beta$ | discount factor | 0.99 |
| $\delta$ | capital depreciation rate | 0.025 |
| $\alpha$ | share of capital income | 0.33 |
| $1-\rho$ | job separation rate | 0.105 |
| $\sigma$ | elasticity of matches to unemployment | 0.5 |
| $s$ | job finding rate at steady-state | 0.95 |
| $\eta$ | bargaining power | 0.5 |
| $\bar{b}$ | unemployment insurance | 0.5 |
| $\varphi_p$ | infrequency of price setting | 0.6 |
| $\varepsilon$ | elasticity of substitution | 10 |
| $\rho_m$ | interest rate smoothing | 0.9 |
| $\gamma_\pi$ | response coefficient of expected inflation | 1.5 |
| $\gamma_y$ | response coefficient of output gap | 0.5 |

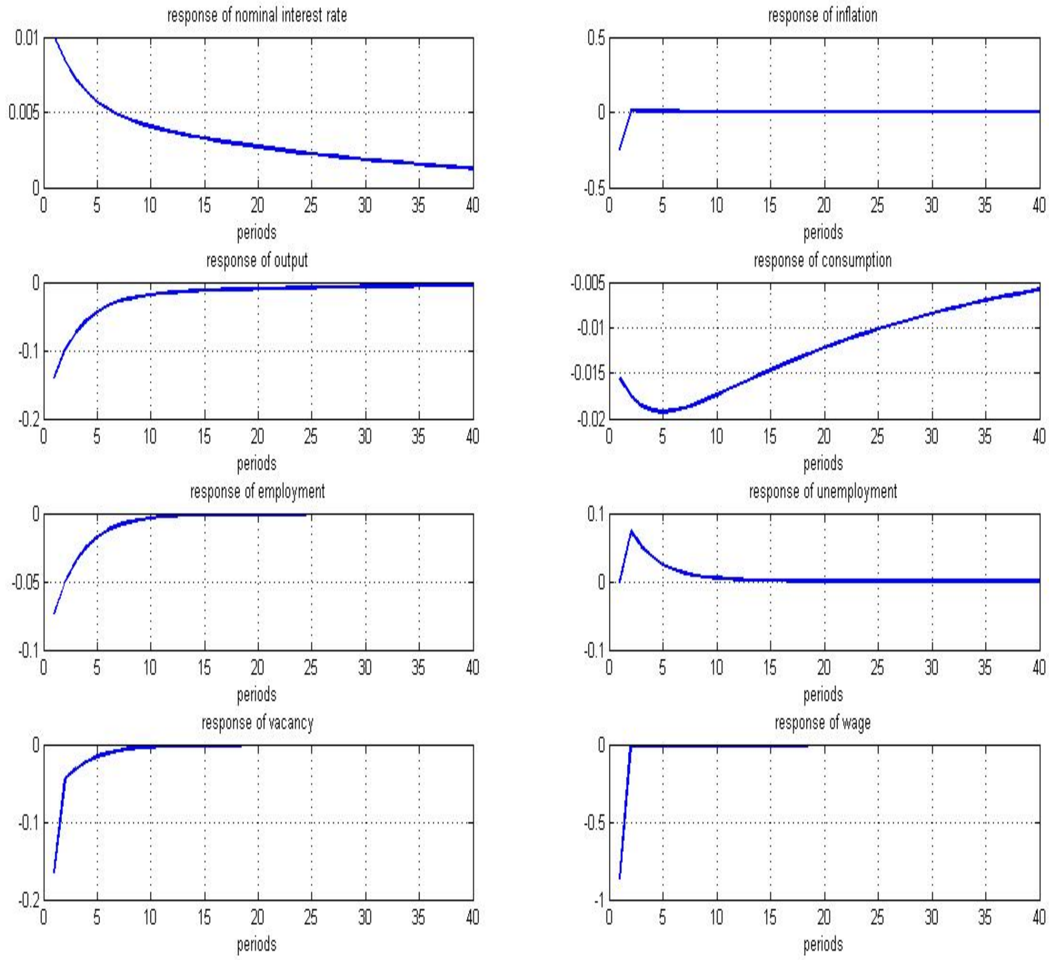
This table collects the descriptions and values of important parameters of the model.

Figure 1.1: Impulse responses in the case of a frictionless economy



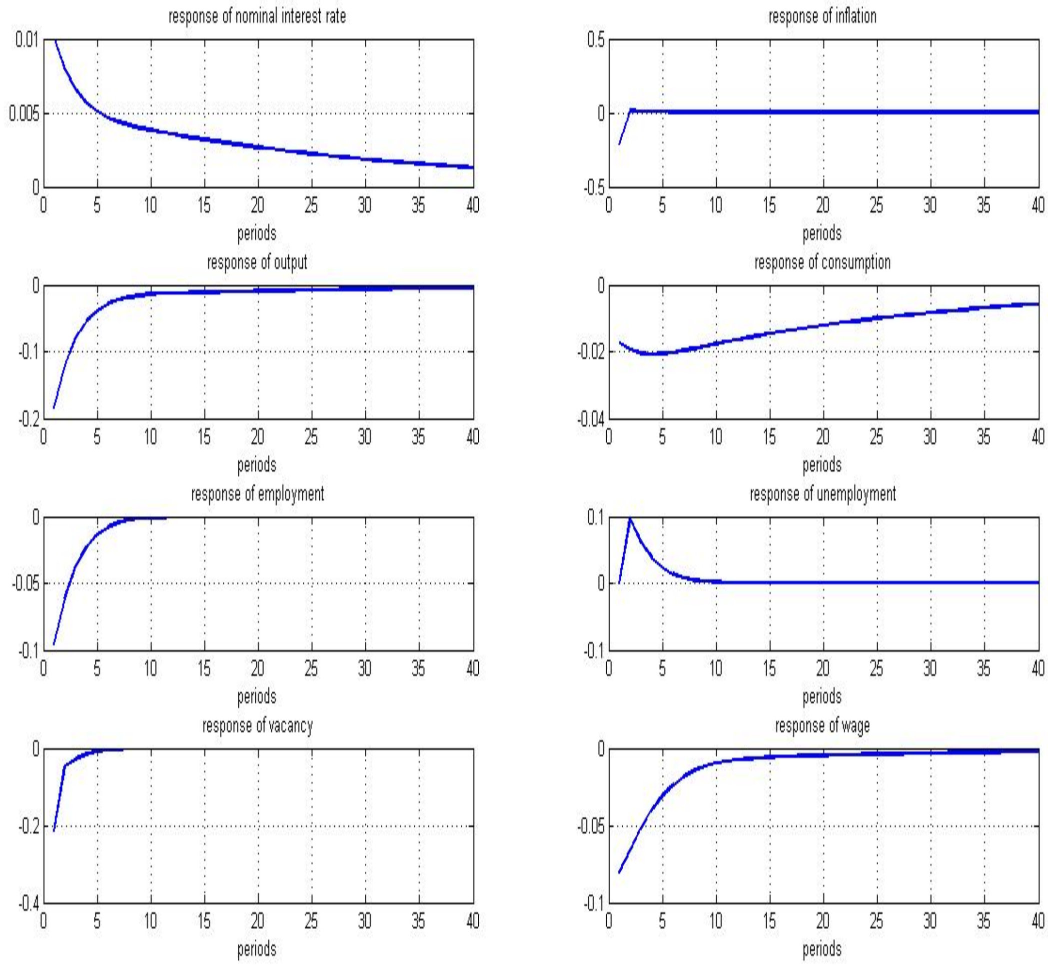

This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock in the case of a frictionless economy, which means $\varphi_p = 0$ and $\varphi_w = 0$.

Figure 1.2: Impulse responses in the case of sticky prices



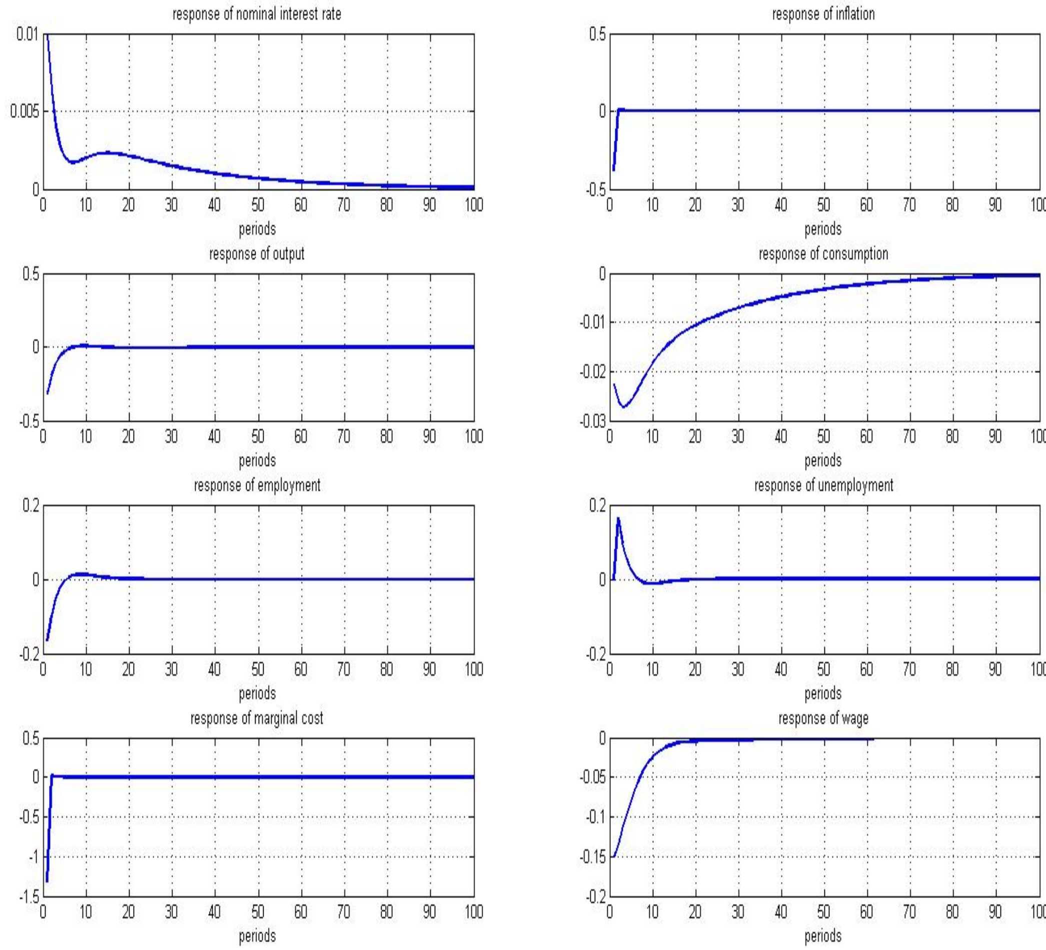

This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock in the case of sticky prices, which means $\varphi_p \neq 0$ and $\varphi_w = 0$.

Figure 1.3: Impulse responses in the case of sticky prices and sticky wages without spillover effects




This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock in the case of sticky prices and sticky wages without spillover effects, which means $\varphi_p \neq 0$, $\varphi_w \neq 0$, $\tau_1 = 0$, and $\tau_2 = 0$.

Figure 1.4: Impulse responses in the case of sticky prices and sticky wages with spillover effects




This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock in the case of sticky prices and sticky wages with spillover effects, which means $\varphi_p \neq 0$, $\varphi_w \neq 0$, $\tau_1 \neq 0$, and $\tau_2 \neq 0$.

# Chapter 2

# Labor market institutions and business cycles

## 2.1 Introduction

Although there has been a significant progress in explaining the impacts of labor market institutions (LMI) on macroeconomic performance, one aspect of this relationship still remains unexplored: their impacts on business cycles. These institutions might have an impact on business cycles due to a number of reasons. First, Calmfors and Driffill (1988) and Rumler and Scharler (2009) claim that the characteristics of the wage bargaining process influence the responses of macroeconomic variables to economic shocks. The behaviors of the bargaining agents will influence the macroeconomic outcomes to the degree that they internalize the macroeconomic consequences of their actions. Second, these institutions play a very important role in determining the worker and job flows at the business cycle frequency. Therefore, LMI might influence the responses of the macroeconomic variables to the disturbances.[1] Third, Veracierto (2008) discusses that some of these institutions are introduced by the policymakers to reduce the magnitude of economic downturns. Finally, Abbritti and Weber (2008) argue that the economies that are characterized by high levels of LMI will experience smoother and more prolonged cycles compared to the economies characterized by low levels of LMI;

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[1] For some examples see Bentolila and Bertola (1990), Bertola (1990), Garibaldi (1998), Blanchard and Portugal (2001) and Pries and Rogerson (2005).

because high levels of LMI amplifies the adjustment via prices and restrict the responses of real variables.

The main goal of this chapter is to investigate the impacts of LMI on business cycles; however, there are various institutions employed in different countries.[2] In the literature these institutions are collected under three main categories - employment protection regulations (e.g. firing costs, severance payments, lifetime employment), unemployment insurance benefits (e.g. duration of benefits, replacement ratio) and collective bargaining (e.g. coverage of collective bargaining, density, rules of bargaining, coordination between unions). In this chapter, I employ representatives of all of these categories. In particular, I look at the impacts of firing costs measured by severance payments, unemployment benefits and bargaining power of workers on business cycles.

In the literature there are a few theoretical studies that analyze the impacts of LMI on business cycles to different extents. Alvarez and Veracierto (2000) build a Lucas-Prescott equilibrium search model and investigate the role played by the labor market policies in explaining the differences in employment across economies. Joseph, Pierrard and Sneessens (2003) focus on the impacts of real wage rigidities as well as employment protection and unemployment benefits within a variation of an RBC model. Pries and Rogerson (2005) develop a matching model to account for the differences in the worker turnovers in the US and in the euro area. Veracierto (2008) employs an RBC model to analyze the effects of firing costs on cyclical fluctuations. All of these studies ignore inflation dynamics and nominal disturbances and some of them do not incorporate labor market frictions in their analyses. Bowdler and Nunziata (2007), Zanetti (2007) and Campolmi and Faia (2009) incorporate these features to their frameworks and show that labor market frictions play an important role in explaining the impacts of LMI on business cycles. Furthermore, Campolmi and Faia (2009) find that alternative institutions will have a role in explaining the differentials in inflation volatilities across economies.

This chapter contributes to this literature by building a dynamic stochastic general equilibrium search and match model by taking these findings into account. In particular, it builds

[2]For a list of these institutions, see Freeman (2008), page 3.

a variation of Mortensen and Pissarides (MP) model.[3] It extends the conventional MP model with staggered wage contracts and nominal price rigidities.

The closest studies to this one in terms of the main question of interest and the methodology are Zanetti (2007) and Abbritti and Weber (2008). The framework of this study differs from Zanetti (2007) in two ways. First, that study considers only nominal price rigidity, however in this study real wage rigidity is incorporated into the model. In the conventional model the wages are determined by period-by-period Nash bargaining between firms and workers. This results in a high volatility in real wage. To improve the empirical performance of the model Shimer (2005) and Hall (2005) include an ad hoc real wage stickiness to the conventional model. They find that sticky wages assumption improves the empirical performance of the model. Moreover, although beyond the scope of this study, the model here is able to test the arguments of Abbritti and Weber (2008) and Joseph et al. (2003). The former study claims that real wage rigidities and labor market rigidities might have opposite effects on business cycle fluctuations. The latter study argues that downward rigidities, rather than LMI, may play a dominant role in explaining the cyclical properties of an economy. Secondly, I consider three institutions where Zanetti (2007) considers only firing costs and unemployment benefits. The difference between Abbritti and Weber (2008) and this study is that they do not focus on the institutions per se. They look at the impacts of real wage rigidities and labor market frictions on the business cycles and measure the frictions with the steady state values of unemployment and job-finding rate. They do not identify the institutions where in this studythese institutions are clearly modeled.

This work also contributes to the literature, pioneered by Trigari (2006) and Gertler and Trigari (2009), which aims to replicate the observed dynamics of unemployment and inflation by incorporating search and match labor market frictions into the standard New Keynesian framework.[4] This line of research has been criticized by not being able to capture the business

[3]Due to Pissarides (1990), Mortensen and Pissarides (1994, 1999) and Pissarides (2000).

[4]Some other examples of this literature are, Walsh (2005), Krause and Lubik (2007), Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008).

cycle facts due to exogenous separation rate.[5] This study incorporates endogenous separations and sets a basis for a further analysis of the impacts of endogenous separations on the empirical performance of this type of model.

To identify the impacts of LMI on business cycles I derive the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of institutions. The results of this study are as follows. A change in the firing cost does not have a significant impact on the impulse responses of the variables. An increase in the unemployment benefits makes the output, employment, unemployment and inflation less responsive to the macroeconomic disturbances. An increase in the bargaining power, however, increase the responses of the real side of the economy where it reduces the response of inflation to a shock.

The next section lays out the main framework, which is very similar to the one built in the previous chapter. Third section calibrates the model. In the fourth section, I present the log-linearized version of the key equations of the model to derive the impulse responses under alternative levels of institutions. Final section concludes the paper.

## 2.2 The Model

The agents in the economy are: households, intermediate goods producers, retailers, a final good producer and a monetary authority. Households maximize their lifetime utility subject to a budget constraint. The intermediate firms produce intermediate goods that are bought by retailers and they face with labor market frictions. They hire workers through a search and matching process and set their wages according to Calvo-type price setting framework. Retailers operate in a monopolistically competitive market and set their prices as in Calvo (1983). They transform one unit of intermediate good to a unit of retial good and sell it to the final good producer. The final good producer collects all goods produced by retailers, combines them via a Dixit-Stiglitz aggregator and produces the consumption good. Finally, the monetary authority employs a Taylor-type monetary policy rule and uses the nominal interest rates as the instrument.

[5] See Ramey (2008) for a detailed discussion of exogenous versus endogenous separations.

### 2.2.1 The Household

Households in the economy are distributed on a unit interval and are assumed to form a large extended family. This family is simply referred as the household.[6] At any point in time some members of the family are employed. Rest of the members is unemployed and search for a job. The household enjoys consumption and maximizes her lifetime utility subject to the budget constraint. Formally, the problem of the household can be written as:

$$max \quad E_t \sum_{t=0}^{\infty} \beta^t log(c_t)$$

$$\begin{aligned} s.t. \quad c_t + k_{t+1} + \frac{B_t}{P_t r_t^n} = {} & w_t n_t + (1 - n_{t-1})b + \rho_t^{in} n_{t-1} \Upsilon + \\ & (z_t + 1 - \delta)k_t + \Pi_t + T_t + \frac{B_{t-1}}{P_t} \end{aligned} \tag{2.1}$$

$c_t$, $k_t$ and $B_t$ are consumption of the final good, saving in terms of capital and per-capita holdings of a nominal one-period bond, respectively. $w_t$ is the aggregate wage rate paid to the working members of the family and $b$ is the unemployment benefits received by the unemployed members. $\Upsilon$ denotes the amount of payment to a worker who is involuntarily separated[7] in the beginning of the period and $\delta$ stands for the depreciation rate of the capital. $r_t^n$ represents the nominal return from a one-period bond and $P_t$ is the aggregate price level. The household rents the capital she owns to the firms with a rental rate $z_t$. Then the total income of the household at time $t$ is the sum of wage income of employed members, $w_t n_t$, unemployment benefits received by unemployed members, $b(1 - n_{t-1})$, total payments to the workers that are separated involuntarily in the beginning of the period, $\rho_t^{in} n_{t-1} \Upsilon$, total rent income from capital remaining after depreciation, $(z_t + 1 - \delta)k_t$, the profits distributed by the firms, $\Pi_t$, total transfers from the government, $T_t$, and real return from bonds that the household bought in the previous period, $\frac{B_{t-1}}{P_t}$. Assuming that $\lambda_t$ shows the marginal utility of consumption at

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[6] To avoid the distributional problems among individuals it is assumed that the members of the family combine their incomes and make the decision of consumption. This is a common assumption in the literature.

[7] The involuntary separations are further discussed below.

date $t$, the first order conditions of the problem above can be written as:

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{P_t}{P_{t+1}} r_t^n \tag{2.2}$$

$$\lambda_t = \beta E_t \lambda_{t+1} [z_{t+1} + 1 - \delta] \tag{2.3}$$

The real interest rate is defined in percentage terms: $1 + r_t = E_t \frac{P_t}{P_{t+1}} r_t^n$.

### 2.2.2 Matching and production in intermediate market

In the intermediate market, the firms and workers meet on a matching market where firms post vacancies and search for workers from an unemployment pool and workers search for jobs. The matching function depends on the vacancies posted by the firms and the number of unemployed workers. It is increasing in both of its arguments and represents constant returns to scale.

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma} \tag{2.4}$$

$m_t$, $u_t$ and $v_t$ denote the matches within period $t$, the number of unemployed workers and the number of vacancies posted at time $t$, respectively. $\sigma_m$ measures the efficiency of the matching process and $\sigma \in (0, 1)$ is the elasticity of the matching with respect to unemployment. The aggregate number of vacancies posted at time $t$ and total employment in the economy are given by $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. The number of unemployed workers at time $t$ will be the difference between the unity and the number of workers employed in the beginning of the period.

$$u_t = 1 - n_{t-1} \tag{2.5}$$

Let $q_t$ denote the probability an open vacancy is going to be matched with a searching worker and $s_t$ denote the probability a worker searching for a job is going to be matched with an open vacancy; then, it is convenient to define

$$q_t = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma} \quad and \quad s_t = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma} \tag{2.6}$$

$\theta_t$ measures the labor market tightness and is defined as the ratio of the number of open vacancies to the number of job-seeking workers, $\theta_t = \frac{v_t}{u_t}$.

The intermediate goods producers are distributed on the unit interval. They have access to a Cobb-Douglas type production function. Intermediate firm $i$ rents capital from households, $k_t(i)$, and hires workers, $n_t(i)$. The total production of the firm, $y_t(i)$, is given by

$$y_t(i) = k_t(i)^{\alpha} n_t(i)^{1-\alpha} \tag{2.7}$$

In the literature it is common to define the hiring rate, $x_t(i)$, as the ratio of new hires to the existing workforce.

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)} \tag{2.8}$$

The workforce of the firm at time $t$ is the sum of the surviving workers from the previous period and current hires. I assume that new workers immediately go to work. Therefore, the total workforce at time $t$ will be

$$n_t(i) = (1 - \rho_t(i))n_{t-1}(i) + q_t v_t(i) \tag{2.9}$$

The separation can occur due to reasons such as migration, death, retirement,...etc, or due to exogenous shocks which results in involuntary separations. The rate of former type of separations is assumed to be constant and denoted by $\rho_x$. The latter separation rate is represented by $\rho_t^{in}$. Therefore, the total separations rate is given by

$$\rho_t = \rho_x + \rho_t^{in} \tag{2.10}$$

I also assume that involuntary separations follow an AR(1) process.

$$\rho_t^{in} = \rho_t^{in} + \rho_\rho \rho_{t-1}^{in} + \varepsilon_t^\rho \quad with \quad \varepsilon_t^\rho \propto NIID(0, \sigma_\rho^2) \tag{2.11}$$

Given the evolution of the workforce and the hiring rate, the value of firm $i$ at time $t$ can be described as:

$$\begin{aligned} F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa}{2} x_t(i)^2 n_{t-1}(i) \\ -\rho_t^{in} \Upsilon n_{t-1}(i) - z_t k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i) \end{aligned} \tag{2.12}$$

The firm's discount factor is denoted by $E_t \Lambda_{t,t+1} = E_t \frac{\lambda_{t+1}}{\lambda_t}$, relative price of the good that the firm produces is given by $p_t^w$ and the ongoing wage rate at firm is represented by $w_t(i)$. $p_t^w$ is also the marginal cost of the retailer who buys this intermediate good. It is assumed that only the workers that are separated due to exogenous shocks are eligible to receive severance payments. Therefore, the total severance payments made by the firm is $\rho_t^{in} \Upsilon n_{t-1}(i)$. The firm also bears labor adjustment cost, which is paid in terms of the final good and is a function of the firm's hiring rate, $\frac{\kappa}{2} x_t(i)^2$.

The wage rate at each intermediate firm is different because at any period only a fraction of the firms are allowed to bargain their wages with their existing workforces. If a firm is allowed to bargain its wage, it will go through the bargaining process over the new wage. If it is not allowed to bargain, the previous period's wage will prevail.

At each period the firm maximizes its value with respect to vacancies and capital stock given its existing employment stock, probability of filling a vacancy, the rental rate and current and expected path of the real wage. The first order conditions are:

$$\kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \left[ (1 - \rho_{t+1}) \kappa x_{t+1} + \frac{\kappa}{2} x_{t+1}(i)^2 - \rho_{t+1}^{in} \Upsilon \right] \tag{2.13}$$

$$z_t = \alpha p_t^w \frac{y_t(i)}{k_t(i)} \tag{2.14}$$

Here, $f_{nt}(i) = (1 - \alpha) \frac{y_t(i)}{n_t(i)}$, is the marginal product of labor. The firm chooses the employment level by choosing the hiring rate. The equation 2.13 represents the hiring decision of the firm. Equation 2.14 gives the capital decision. Due to the assumptions that the capital market is perfectly competitive and capital is free to move, the output/capital ratios are equal across the firms. Therefore, I can drop the indices in equation 2.14 and say $z_t = \alpha p_t^w \frac{y_t}{n_t}$.

It is convenient to derive the marginal contribution of a worker to the firm's value as:

$$\frac{\partial F_t(i)}{\partial n_t(i)} = J_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\partial F_{t+1}(i)}{\partial n_t(i)} = \kappa x_t(i) \tag{2.15}$$

This value will be used in the bargaining process. By using the hiring decision, equation 2.15 can be revised as:

$$J_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \left[ \frac{\kappa}{2} x_{t+1}(i)^2 - \rho_{t+1}^n \Upsilon \right] + \beta E_t \Lambda_{t,t+1} (1 - \rho_{t+1}) J_{t+1}(i) \tag{2.16}$$

Before defining the bargaining process, I will discuss the value of each state to the workers. To derive the value of being unemployed, $U_t$, first the average value of all vacancies posted in that period, $V_{x,t}$, should be derived. Since the unemployed worker does not know which firm she is going to be matched with, she considers $V_{x,t}$ during her job search. This value is given by

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di = \frac{1}{v_{t-1}} \int_0^1 V_t(i) v_{t-1}(i) di \tag{2.17}$$

In the current period the unemployed worker receives unemployment benefits, $b$. In the next period the unemployed worker will find a successful match with probability $s_{t+1}$ and with probability $1 - s_{t+1}$ the worker will be unemployed again.

$$U_t = b + \beta E_t \Lambda_{t,t+1} \left[ s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1} \right] \tag{2.18}$$

The unemployment benefits are calculated as a fraction, $\bar{b}$, of the worker's marginal contribution to the production and firm's savings from labor adjustment cost.

The value of being employed at firm $i$ at time $t$, $V_t(i)$, is the sum of the wage that the worker is receiving from the firm at the current period and expected discounted return in the next period. At time $t+1$ the worker will either work for the same firm or be separated from the firm. The worker will be eligible for unemployment benefits once she is separated. Moreover, if the worker is involuntarily separated from the firm, the worker will receive a severance payment. Therefore,

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1} \left[ (1-\rho_t)V_{t+1}(i) + \rho^{in}_{t+1}(\Upsilon + U_{t+1}) + \rho_x U_{t+1} \right] \tag{2.19}$$

In the wage setting process a Calvo-type price-setting framework is employed. It is assumed that the probability that a firm is allowed to re-negotiate its wage at each period follows a Poisson process. At any point in time an intermediate firm will change its wage with probability $1-\varphi_w$. Therefore, the average duration of the wage at a firm is $\frac{1}{1-\varphi_w}$. The probability that a firm can re-negotiate its wage is independent from its negotiation history.

In the bargaining process, the total surplus generated from the contract is distributed according to the bargaining powers of each party. This process is summarized by:

$$max \quad H_t(r)^{\eta} J_t(r)^{1-\eta} \tag{2.20}$$

$H_t(r)$ stands for the surplus of an average negotiating worker once she accepts a job and $J_t(r)$ denotes the value of adding another worker to the firm. Since only a fraction of the firms are allowed to bargain at time $t$, the firms that are bargaining and that are not bargaining have to be distinguished. This distinction is made by indexing the bargaining ones with $r$. The surplus of the worker from accepting a job with firm $i$, $H_t(i)$, and the average surplus of a worker hired at time $t$, $H_{x,t}$ are defined as:

$$H_t(i) = V_t(i) - U_t H_{x,t} = V_{x,t} - U_t \tag{2.21}$$

Plugging the values of being employed and unemployed to the worker and rearranging the terms result in:

$$H_t(i) = w_t(i) - b - \beta E_t \Lambda_{t,t+1} \left[ s_{t+1} H_{x,t+1} - \rho^{in}_{t+1} \Upsilon \right] + \beta E_t \Lambda_{t,t+1} (1 - \rho_{t+1}) H_{t+1}(i) \quad (2.22)$$

The uncertainty about the time of the next negotiation period results in horizon effect, which is discussed by Gertler and Trigari (2009) in details. Since both the firm and the worker do not know when they will have a chance to re-negotiate, they consider the future path of the wage during the bargaining. However, the worker considers only his tenure at the firm where the firm has to think about its future workforce as well as its existing workforce. In other words, the firm has a longer horizon than the worker. This effect is observed in the difference between the firm's cumulative discount factor, $\Sigma_t(r)$, and the worker's cumulative discount factor, $\Delta_t$, which are given by

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} (\beta \varphi_w)^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \quad (2.23)$$

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\beta \varphi_w)^s \Lambda_{t,t+s} (1 - \rho_{t+s}) \quad (2.24)$$

The discount factors are similar except the fact that the firm's discount factor depends on the employment of the firm at time $t + s$ relative to time $t$, where the worker's discount factor depends on the expected survival rate. On the average $\frac{n_{t+s}}{n_t}(r) > (1 - \rho_{t+s})$, which implies that the firm places relatively more weight on the future than does the worker.

Given these discount factors, the expected wage revenue of a bargaining worker and expected wage payment of a firm to its workers can be written as:[8]

$$\begin{aligned} W^w_t(r) = {} & \Delta_t w^*_t + \beta(1 - \varphi_w) E_t \Lambda_{t,t+1} (1 - \rho_{t+1}) \Delta_{t+1} w^*_{t+1} \\ & \beta^2 (1 - \varphi_w) E_t \Lambda_{t,t+2} (1 - \rho_{t+1})(1 - \rho_{t+2}) \Delta_{t+2} w^*_{t+2} + ... \end{aligned} \quad (2.25)$$

$$W^f_t(r) = \Sigma_t(r) w^*_t + (1 - \varphi_w) E_t \sum_{s=1}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t} \Sigma_{t+s}(r) w^*_{t+s} \quad (2.26)$$

[8]The appendix provides the details.

The first order condition of the bargaining problem is:

$$\eta \Delta_t J_t(r) = (1-\eta)\Sigma_t(r) H_t(r) \quad \Rightarrow \quad \chi_t(r) = J_t(r) = (1-\chi_t(r))H_t(r) \tag{2.27}$$

$\chi_t(r)$ denotes the relative weight in the sharing rule and is derived as $\chi_t(r) = \frac{\eta}{\eta+(1-\eta)\frac{\Sigma_t(r)}{\Delta_t}}$. Due to constant returns assumption, each negotiating firm settles on the same wage rate, $w_t^*$. In other words, the firm bargains with the marginal worker instead of bargaining with its whole existing workforce. Let $w_t^o(r)$ stand for the target wage, then, the following equation defines the optimum bargained wage.

$$\Sigma_t(r) w_t^* = w_t^o(r) + \beta \varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t} \Sigma_{t+1}(r) w_{t+1}^* \tag{2.28}$$

where

$$\begin{aligned} w_t^o(r) = \chi_t(r) \left[ p_t^w f_{nt}(r) - \beta E_t \Lambda_{t,t+1} \left( \frac{\kappa}{2} x_{t+1}(r)^2 + \rho_{t+1}^{in} \Upsilon \right) \right] + \\ (1-\chi_t(r)) \left[ b + \beta E_t \Lambda_{t,t+1} \left( s_{t+1} H_{x,t+1} - \rho_{t+1}^{in} \Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x_{t+1}(r)^2 \right) \right] \end{aligned} \tag{2.29}$$

According to the equation above, the target wage is a convex combination of the contribution of a worker to the match and his foregone benefits from being unemployed.[9] The average wage in the economy can be driven by taking the average wage of all employed workers.

$$w_t = \int_0^1 w_t(i) \frac{n_t(i)}{n_t} di \tag{2.30}$$

Using the law of large numbers and with the fact that only a fraction of the firms can change their wages, the equation above can be rewritten as:

$$w_t = (1-\varphi_w) w_t^* + \varphi_w \int_0^1 w_{t-1}(i) \frac{n_t(i)}{n_t} di \tag{2.31}$$

[9] The target wage in Gertler and Trigari (2009) represents the wage that would arise under period-by-period Nash bargaining, modified to allow for the horizon effect. In our case the target wage is also adjusted for endogenous separations. I represent the bargained wage in this form to make it comparable with that of Gertler and Trigari (2009).

### 2.2.3 Retailers, final good market and monetary authority

The retailers are distributed on the unit interval. The only function of the retailers is to buy intermediate goods, differentiate them with a technology that transforms one unit of intermediate good at no cost, and re-sell it to the final good producer. Each retailer has a monopolistic power on the good it produces and at any point in time only a fraction of the firms are allowed to reset their prices. Marginal cost of buying an intermediate good is its relative price. The final good producer uses a Dixit-Stiglitz aggregator and produces the final good that is directly sold to the consumers. The monetary authority uses the short-term nominal interest rates as the policy instrument and employs a Taylor-type rule.

The total output produced in the economy can be written as:

$$y_t = \left[\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.32}$$

where the aggregate output, $y_t$, is described in terms of each retailer's output, $y_{jt}$, and the firm's own price elasticity of substitution across differentiated retail goods, $\varepsilon$. The monopolistic power of the retailers and Calvo-type price setting results in a nominal price rigidity in the economy. The aggregate price level then is the aggregate of the nominal sale price of each retailer's product. Formally,

$$P_t = \left[\int_0^1 P_{jt}^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \tag{2.33}$$

Solving for the demand for each retail good results in

$$y_{jt} = \left[\frac{P_{jt}}{P_t}\right]^{\varepsilon} y_t \tag{2.34}$$

At any point in time each retailer can reset its price with a probability $1-\varphi_p$. This probability is independent from the firm's price setting history. Therefore, the aggregate price index in the economy is defined as:

$$P_t = \left[(1-\varphi_p)(P_t^*)^{1-\varepsilon} + \varphi_p(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{2.35}$$

where $P_t^*$ is the optimal price for the firms that are able to reset their prices. The firm sets its price for the expected number of periods that it will not be able to reset its price. $P_t^*$ is the solution to the following problem:

$$max \quad E_t \sum_{s=0}^{\infty} (\beta\varphi_p)^s \left[\frac{P_{jt}^*}{P_t} - p_{t+s}^w\right]^{-\varepsilon} \tag{2.36}$$

subject to the demand for that good. The optimal price level for the firm that is able to change its price is:

$$p_{jt}^* = \frac{\varepsilon}{\varepsilon - 1} E_t \frac{\sum_{s=0}^{\infty} (\beta\varphi_p)^s P_{t+s}^{\varepsilon} p_{t+s}^w y_{t+s}}{\sum_{s=0}^{\infty} (\beta\varphi_p)^s P_{t+s}^{\varepsilon-1} y_{t+s}} \tag{2.37}$$

Finally, the model is completed with the monetary policy rule and market clearing condition.

$$r_t^n = \beta^{-(1-\rho_m)} (r_{t-1}^n)^{\rho_m} E_t(\pi_{t+1})^{\gamma_\pi(1-\rho_m)} (y_t^z)^{\gamma_y(1-\rho_m)} e^{\varepsilon_t^m} \tag{2.38}$$

The degree of interest rate smoothing is captured by the parameter $\rho_m$. The monetary authority responds to the output gap, denoted by $y_t^z$, and the expected inflation. The corresponding response coefficients of the output gap and expected inflation are represented by $\gamma_y$ and $\gamma_p$, respectively. The monetary policy shock is represented by $\varepsilon_t^m$, which is an *i.i.d.* process.

The market clearing condition for the final good is given by

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + \frac{\kappa}{2} x_t^2 n_{t-1} \tag{2.39}$$

The equation 2.39 states that the consumption of the household, $c_t$, investment on capital, $k_{t+1} - (1-\delta)k_t$, and aggregate labor adjustment costs of the intermediate firms, $\frac{\kappa}{2} x_t^2 n_{t-1}$, sum up to the amount of final good produced in the economy.

## 2.3 Calibration

To analyze the impacts of LMI, first the model is calibrated and then the steady state of the model with zero inflation is derived. The parameters are calibrated for the US economy. Table 2.1 collects the values that are assigned to the parameters of the model and implied steady state values of some key variables. The length of a period is assumed to be a quarter.

I start with setting the discount factor, $\beta$, to 0.99. This results in an approximately 4 percent annual real rate of interest. $\delta$ is set to 0.025; therefore, annual depreciation rate of capital is around 10 percent. These values are consistent with Christiano, Eichenbaum and Evans (2005), Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008). In the literature, the steady state capital's share of income, $\alpha$, is found to be between 0.3 and 0.36. The midpoint of these values is chosen and it is set to 0.33. The parameter that measures the elasticity of matches to unemployment, $\sigma$, is calibrated as 0.5. This value is within the range of the lowest and highest values assigned to this parameter in the literature, 0.4 and 0.72.[10]

The separation rate at the steady state has two components: $\rho_x$ and $\rho^{in}$. According to Hall (1995) and Davis, Haltiwanger and Shuh (1996) the average separation rate is between 0.08 and 0.1. Moreover, Gertler and Trigari (2009) argues that the average duration of a job in the US is two and a half years. To match this fact, they set monthly separation rate to 0.035. In this study I set the total separation rate to ten percent per quarter. To do that I set $\rho_x = 0.09$ and $\rho^{in} = 0.01$. To get the involuntary separations the other parameters determining that value are set accordingly.

To be able to calibrate $\kappa$, first I need to determine the steady state value of job finding rate, s. In the literature the values vary between 0.6 and 0.95. The studies that use a similar model to the one employed here set $s = 0.95$. Therefore, I use the same value.

In order to match the average duration of a vacancy, the efficiency parameter in the matching function, $\sigma_m$ is calibrated as 0.925. With this calibration the probability a vacancy will be filled at the steady state, $q$, is 0.9. The duration of an average vacancy is reported to be under one month by Blanchard and Diamond (1989), which requires $q$ to be 1. However, the

[10]See Blanchard and Diamond (1989) and Shimer (2005) for the lowest and highest values, respectively.

distinction between the removal time of the advertisement and actual date that the vacancy is filled is pointed out by van Ours and Ridder (1992). In that study it is reported that in the first two weeks after the vacancy is posted, seventy five percent of the vacancies are filled. However, it takes around forty five days to choose the best applicant from the pool.

The probability that an intermediate firm is allowed to reset its wage at any point in time is assumed to be $1 - \varphi_w = 1 - 0.718$. In other words, a firm will be able to re-negotiate its wage on average once in every three quarters.

The probability that a retailer is not allowed to change its price is set as $\varphi_p = 0.6$. This value implies that an average firm will keep its price unchanged for around 2.5 quarters. This is consistent with the literature and close to the empirical findings.[11] Gertler, Sala and Trigari (2008) and Sala, Soderstrom and Trigari (2008) assume that markup rate on marginal cost is around 11 percent. This requires setting $\varepsilon = 10$. While calibrating the parameters of the nominal interest rate rule Clarida, Gali and Gertler (2000) are followed and it is assumed that $\rho_m = 0.9$, $\varphi_p = 1.5$ and $\gamma_y = 0.5$.

Finally, I calibrate the parameters that represent LMI. To save space I discuss only the benchmark calibration of these parameters. In the impulse response analysis I consider alternative values to examine their impacts on responses of variables. In the benchmark case I set $fr = 0.3$, $\bar{b} = 0.5$ and $\eta = 0.5$. The first parameter represents the fraction of the average wage paid to the separated worker as severance payment, $\Upsilon = fr * w$. In the benchmark case, the separated worker will receive 30 percent of the average wage in the economy. The total unemployment benefits paid to a worker is $b = \bar{b}(p^w f_n + \beta \frac{\kappa}{2} x^2)$, which is a fraction of the sum of her possible contribution to the production and firm's saving from labor adjustment cost. Given this calibration the unemployment benefits will be fifty percent of this sum. Finally, setting $\eta = 0.5$ results in an equal bargaining power to each bargaining agents.

Given this calibration, the steady state employment rate is 90 percent and unemployment rate is 10 percent. The unemployment rate is much higher than the average unemployment rate of the US in the last decades. However, in this model the workers do not have an option

[11] See Gali and Gertler (1999), Bils and Klenow (2002), Sbordone (2002) and Christiano, Eichenbaum and Evans (2005)

of leaving the labor force and in the literature there is evidence that shows that the flow from unemployed to employment and out-of-labor-force to employment are almost the same. Therefore, setting unemployment to 10 percent is acceptable in this framework.

## 2.4 Log-linearization and impulse responses

To identify the impacts of the LMI on the impulse responses of variables to a monetary policy shock, the key equations of the model are log-linearized around the steady state. A variable with a hat denotes percentage deviation from its steady state value and a variable without a hat and without a time subscript denotes the steady state value of the regarding variable. The log-linearization process will start with the aggregate wage index.

$$\hat{w}_t = (1-\varphi_w)\hat{w}_t^* + \varphi_w \hat{w}_{t-1} \tag{2.40}$$

The variations in the aggregate wage will be the weighted sum of the variations in the previous period's aggregate wage and in the bargained wage. The weights are determined by the frequency of the wage bargaining in the intermediate market. The variations in the bargained wage will be given by

$$\hat{\Sigma}_t + \hat{w}_t^* = \Sigma^{-1}\hat{w}_t^o(r) + \rho\beta\varphi_w E_t\left[\hat{x}_{t+1} - \hat{\rho}_{t+1}\right] + \beta\varphi_w E_t\left[\Lambda_{t,t+1} + \hat{\Sigma}_{t+1} + \hat{w}_{t+1}^*\right] \tag{2.41}$$

The percentage deviation of the target wage from its steady state is

$$\begin{aligned} \hat{w}_t^o(r) = \chi p^w f_n w^{-1}\left[\hat{p}_t^w + \hat{f}_{nt}\right] + \left[\left(s+\frac{\rho}{2}\right)\beta\chi\kappa x - \beta\rho^{in}\Upsilon\right] w^{-1}E_t\Lambda_{t,t+1} \\ \beta s\chi\kappa x w^{-1}E_t\left[\hat{s}_{t+1} + \hat{H}_{x,t+1}\right] + \left[p^w f_n - b - \beta\kappa x^2 - (s+\rho)\beta H\right]\chi w^{-1}\hat{\chi}_t(r) \\ \beta\frac{\chi}{1-\chi}\kappa x^2 w^{-1}E_t\hat{\chi}_{t+1}(r) + \beta\chi\kappa x^2 w^{-1}E_t\hat{x}_{t+1}(r) - \beta\rho^{in}\Upsilon w^{-1}E_t\hat{\rho}_{t+1}^{in} \end{aligned} \tag{2.42}$$

According to the equation 2.42, the variations in the target wage will be affected by the variations in the marginal worker's contribution to the firm value and the worker's foregone benefit from unemployment. These impacts are captured by the first three terms. Except the differences in the coefficients in front of these terms, these are the factors that affect the variations in the target wage in the case of conventional period-by-period bargaining framework. Once the sticky wages assumption is introduced, the horizon effect takes place. As discussed in Gertler and Trigari (2009) there are two dimensions of the horizon effect. First, in the baseline calibration, although $\eta = 0.5$, the effective bargaining power, $\chi$, is 0.45. In other words, the pure bargaining power will be lessened. Second, the variations in $\chi_t(r)$ will lead to variations in the target wage. This effect is captured by the fourth term in the equation 2.42. The rest of the terms are the results of endogenous separation and severance payment assumptions.

The variations in the separation rate have two effects on the target wage. First effect is captured by the last term in the equation 2.42. The second effect operates through the discount factor of the bargaining firm which in turn affects the expected variations in the effective bargaining power of the worker, $\hat{\chi}_{t+1}(r)$.

$$\hat{\Sigma}_t(r) = \beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \rho(\hat{x}_{t+1}(r) - \hat{\rho}_{t+1}) + \hat{\Sigma}_{t+1}(r)\right] \tag{2.43}$$

$$E_t\hat{\chi}_{t+1}(r) = -(1-\chi)E_t\left[\hat{\Sigma}_{t+1}(r) - \hat{\Delta}_{t+1}\right] \tag{2.44}$$

Although I do not investigate the impacts of spillover impacts, I revise the equation 2.42 as in Gertler and Trigari (2009). As derived in the appendix, the variations in the target wage is

$$\begin{aligned}\hat{w}_t^o(r) = {} & \varphi_{f_n}\left[\hat{p}_t^w + \hat{f}_{nt}\right] + \varphi_\chi\hat{\chi}_t + \varphi_\Lambda E_t\hat{\Lambda}_{t,t+1} + \frac{\varphi_x + \varphi_s}{1-\chi}E_t\hat{\chi}_{t+1} + \left[\varphi_x + \varphi_s\right]E_t\hat{x}_{t+1} \\ & + \varphi_s E_t\hat{s}_{t+1} - \beta\rho^{in}\Upsilon w^{-1}E_t\hat{\rho}_{t+1}^{in} + \tau_1\left(\hat{w}_t - \hat{w}_t^*\right) + \tau_2 E_t\left(\hat{w}_{t+1} - \hat{w}_{t+1}^*\right)\end{aligned} \tag{2.45}$$

where $\tau_2$ measures the direct spillover effect and $\tau_1$ measures the indirect spillover effect. The market wage has a direct spillover effect on the bargained wage because the worker's outside option depends on the wage that she can expect to earn elsewhere. If the expected wage exceeds the expected bargained wage, the outside option of the worker is good and the worker will move to employment in the next period. This induces a spillover effect on the bargained wage. The indirect spillover effect stems from the difference between the hiring rate of the bargaining firm and the average hiring rate. The appendix proves that there is a positive relationship between this difference and the gap between average market wage and contract wage.

$$\hat{x}_{t+1}(r) = \hat{x}_{t+1} + w\Sigma\epsilon\varphi_w \left(\hat{w}_t - \hat{w}_t^*\right) \tag{2.46}$$

To save space rest of the log-linearized versions of the equations are presented in the appendix.

To analyze the impacts of LMI on the business cycles, I derive the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of institutions. However, I do not report the results for the firing costs because a change in the firing cost does not have an impact on the responses of macroeconomic variables. The firing cost appears in the equations for the hiring rate and the bargained wage. Since the expected variation in the involuntary separation is zero, the firing cost does not have an impact on the target wage. Moreover, its impact on the hiring rate decreases to insignificant levels due to the same reason.

The literature does not provide a conclusive answer for the impacts of firing costs on the business cycles. Although Rumler and Scharler (2009), Zanetti (2007) and Veracierto (2008) report a negative relationship, Fonseca, Patureau and Sopraseuth (2007) and Joseph et al. (2009) find almost no relationship. When the results of this study is compared with those of Zanetti (2007), it is observed that the real wage rigidities play an important role. The period-by period bargaining makes the real wage very sensitive to the severance payments. However, if only a fraction of the firms are allowed to bargain at each period, the impacts of a change in the severance payments on the responses decrease to insignificant levels.

Figure 2.1 plots impulse responses of the nominal interest rate, inflation, output, consumption, employment, unemployment, hiring rate and the real wage to a one standard deviation monetary policy shock under alternative levels of unemployment benefits. According to the figure, the most significant impact of a change in unemployment benefits is on the responses of real wage and inflation. A change in the benefits has a negative impact on the target wage, therefore on the aggregate wage. This impact is carried onto inflation through marginal cost.

This leads to the conclusion that as the unemployment benefits increase, the response of inflation decreases. These results are supported by the findings of Campolmi and Faia (2007) and Zanetti (2007), where they find a negative impact of unemployment benefits on inflation volatility. The responses of output, employment and unemployment decreases with an increase in the unemployment benefits but these changes are not very significant.

Figure 2.2 shows the responses of key variables to the monetary policy shock under alternative values for worker's bargaining power. The figure reveals the fact that the impacts of a change in the worker's bargaining power has opposite effects on the responses of the real side of the economy with the impacts of a change in the unemployment benefits. Compared to the influence of a change in unemployment benefits, the bargaining power has more impact on responses of output, employment and unemployment. The figure plots that the higher the bargaining power the more sensitive the output, consumption and employment to the monetary policy shock. In the literature, the impacts of bargaining power on the business cycles are ambiguous. Rumler and Scharler (2009) find a limited impact where Campolmi and Faia (2007) report a positive impact. However, Fonseca et al. (2007) claim that the response of employment is larger for lower bargaining power. The main reason for this ambiguity might be the different aspects of collective bargaining. As discussed in Rumler and Scharler (2009) these different aspects might have different impacts. Although they find that strong unionization has a significantly positive impact on output volatility, the decisions given by the union and coordination between the unions might reduce the impacts of disturbances on the economy. When the change in the reponse of inflation to the monetary policy shock is considered, it is seen that an increase in the bargaining power of the workers has a negative impact.

## 2.5 Conclusion

In this chapter I investigate the impacts of alternative labor market institutions on the business cycles. I consider three institutions which represent the main categorization of a variety of labor market rules and regulations. I use firing cost to represent employment protection institutions, unemployment benefits for unemployment insurance institutions and worker's bargaining power for collective bargaining institutions.

To be able to identify the impacts of these institutions on the business cycles a dynamic stochastic general equilibrium search and match model is employed. The impulse responses of key macroeconomic variables to a monetary policy shock under alternative parameterization of labor market institutions are derived. It is found that a change in the firing cost does not have a significant impact on the impulse responses of the variables. As the unemployment benefits are increased, all of the variables, including output and inflation, become less responsive to the monetary policy shock. An increase in the worker's bargaining power reduces the response of inflation to the same shock and makes the real side of the economy more responsive to the shocks.

This study can be extended in several directions. This study analyzes the impacts of labor market institutions only in terms of the changes in the responses of the macroeconomic variables. A formal statistical analysis is required to measure the exact impacts of these institutions on the behaviors of macroeconomic variables. Another direction would be investigating an optimal monetary policy under alternative levels of institutions since main impacts of changes in the institutions fall on inflation. Furthermore, the framework built here can be used to explore the explanatory powers of the labor market institutions on the differences between the characteristics of the business cycles in alternative economies. In the literature there are a lot of studies showing that these institutions are employed in different economies at different levels. Therefore, these institutions might be responsible for the differences in the features of busness cycles. However, to be able to start working on this direction, the differences between the characteristics of the business cycles in different economies have to be analyzed.

In the next chapter I take upon this task and investigate the features of the business cycles in different economies. I basically compare the characteristics of the US and the euro area business cycles. I first define a business cycle and discuss the methodologies employed by the National Bureau of Economic Research and the Center for Economic and Policy Research, since these two organizations are assumed to be the main source for the dates in the business cycles. Then I discuss the features of the business cycles derived by using the real gross domestic product and industrial production. These two time series are the most commonly used series to derive the business cycles in the empirical literature. It is shown in the next chapter that the US and the euro area countries business cycles exhibit very different characteristics. Thus, it becomes a valid question that to what extent the labor market institutions are responsible for these differences.

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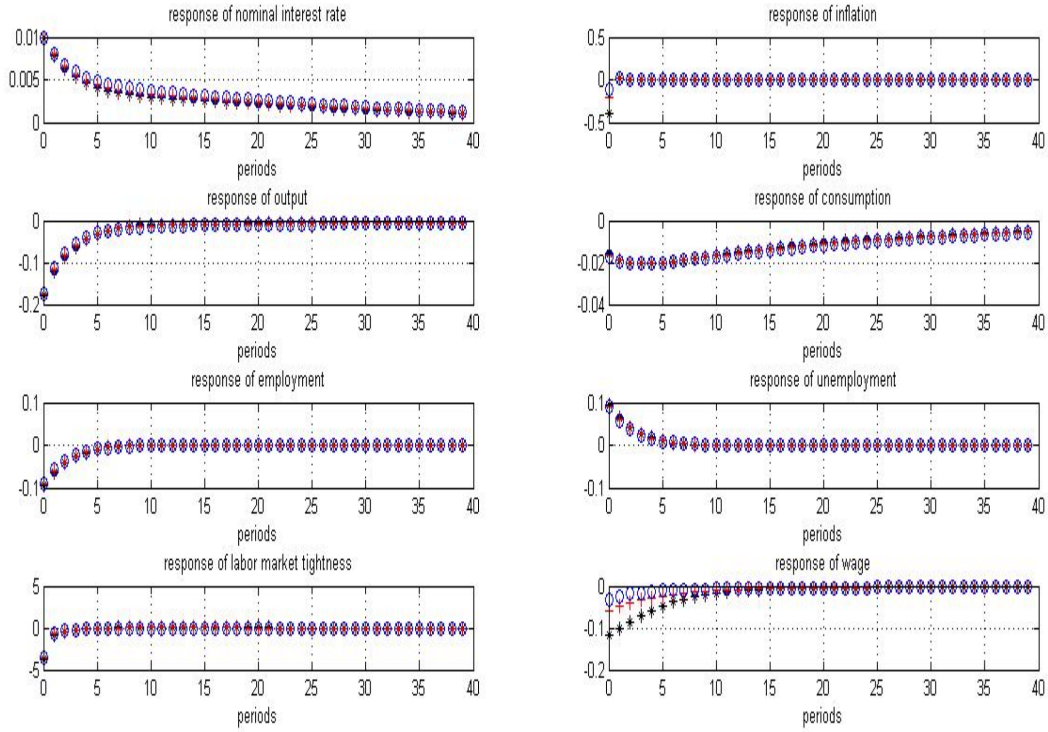
Table 2.1: Benchmark Calibration

| **Parameter** | **Description** | **Value** |
|---|---|---|
| $\beta$ | discount factor | 0.99 |
| $\delta$ | capital depreciation rate | 0.025 |
| $\alpha$ | share of capital income | 0.33 |
| $\rho$ | job separation rate | 0.1 |
| $\sigma$ | elasticity of matches to unemployment | 0.5 |
| $\bar{b}$ | unemployment insurance | 0.5 |
| $\eta$ | bargaining power | 0.5 |
| $\varphi_p$ | infrequency of price setting | 0.6 |
| $\varphi_w$ | infrequency of wage renegotiation | 0.7 |
| $\varepsilon$ | elasticity of substitution | 10 |
| $\rho_m$ | interest rate smoothing | 0.9 |
| $\gamma_\pi$ | response coefficient of expected inflation | 1.5 |
| $\gamma_y$ | response coefficient of output gap | 0.5 |
| $s$ | job finding rate at steady-state | 0.95 |
| $u$ | unemployment rate at the steady state | 0.5 |
| $n$ | employment rate at the steady state | 0.5 |

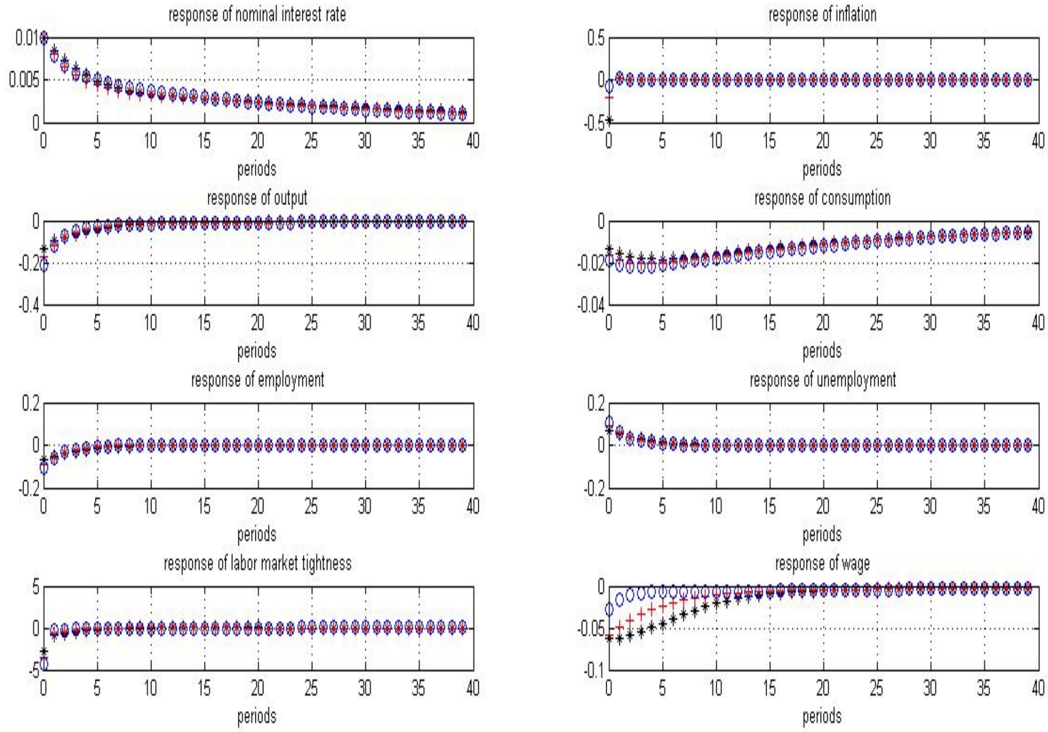
This table collects the values that are assigned to the parameters of the model and implied steady state values of some key variables under the benchmark case. The parameters representing the LMI are assigned different values while deriving the impulse responses. Other values assigned for $\bar{b}$ are 0.1 and 0.8 and for $\eta$ are 0.2 and 0.5.

Figure 2.1: Impulse responses under alternative levels of unemployment benefits




This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of unemployment benefits levels. Black line with stars on it is for $\bar{b} = 0.1$, red line with plus signs on it is for b=0.5, blue line with zeros on it is for $\bar{b} = 0.72$.

Figure 2.2: Impulse responses under alternative levels of worker's bargaining power




This figure shows the impulse responses of key macroeconomic variables to a monetary policy shock under alternative levels of worker's bargaining power. Black line with stars on it is for $\eta = 0.2$, red line with plus signs on it is for $\eta = 0.5$, blue line with zeros on it is for $\eta = 0.8$.

# Chapter 3

# Characteristics of the business cycles in the US and the euro area

## 3.1 Introduction and business cycles

The previous chapter showed that alternative levels of institutions might create different responses for macroeconomic variables at the business cycle frequency. These institutions are employed in many countries at different levels.[1] Therefore, the explanatory power of these institutions on the differences between the characteristics of business cycles in different economies will be a good research path to follow. The main question will be to what extent these institutions are responsible for the differences in the features of business cycles in different economies. However, before attempting to answer this question, the differences between the characteristics of the business cycles among alternative economies have to be explored. This chapter of my dissertation takes upon this task. With this chapter, I do not aim to make an original contribution to the literature but rather try to set a basis for my future research. The comparison will be particularly between the business cycles of the US and the euro area countries.

The modern literature on business cycles dates back to the seminal work of Burns and Mitchell (1946), in which business cycles are defined as: “Business cycles are a type of fluctuation found in aggregate economic activity of nations that organize their work mainly in

[1] For a detailed discussion of the usage of each institution around the world; see Belot and van Ours (2001), Howell, Baker, Glyn and Schmitt (2006) and Freeman (2008).

business enterprises: a cycle consists of expansions occurring at the same time in many economic activities, followed by similarly general recessions, contractions, and revivals that merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten to twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own." (p.3).

Based on the definition above the researchers used two broad definitions of the business cycle: classical cycle and deviation (or growth) cycle. As explained by Artis, Marcelino and Proietti (2004) the difference between these two definitions is the description of the turning points, which are peak and trough. These turning points date that the economy switches from expansion to recession and vice versa. A turning point is called a peak (trough) when a(n) decline (upturn) in real economic activity is observed. The phase starting right after a peak until the next immediate trough is named as contraction[2] and the period between a trough and the next peak is called expansion. In the classical cycle case, selecting the turning points is made on the basis of an absolute decline in the real economic activity. In the deviation cycle case, turning points are defined with respect to the deviations of the rate of growth observed in the variables representing the economic activity from an appropriately defined trend rate of growth. It has been a challenge to find the best method to determine the turning points and there are numerous techniques that have been developed in the literature.[3]

Many researchers assume that the chronologies provided by the National Bureau of Economic Research (NBER) and the Center for Economic and Policy Research (CEPR) are the "official" sources of peak and trough dates in the US and the euro area.[4] The former organization has been doing research on the business cycles of the US for a long time. They provide a business cycle chronology for the US economy. The latter one establishes the chronology

[2]Sometimes these contractions are named as recessions.

[3]Since the evaluation of all of these techniques is beyond the scope of this paper, I will refer the readers to Boldin (1994) for a detailed discussion on this issue.

[4]Both organizations have a Business Cycle Dating Committee to maintain the chronologies of the business cycles in the two areas.

of contractions and expansions of the 11 original euro area member countries plus Greece for 1970 - 1998, and of the euro area as a whole from 1999 onwards.

As discussed in the next section, the committees of the NBER and CEPR consider several different time series while determining the turning points in the economic activity. However, in the empirical literature it is very difficult to replicate or test the validity of the dates declared by these organizations, since they utilize from a variety of time series and the weight of each series in the decision process is not revealed. Therefore, as pointed out by Boldin (1994), for a technique to be qualified as a useful one, a procedure first needs a careful and clear documentation of the data that is examined. In the literature several different series have been used to approximate the business cycles. Among these series, the most commonly used series are: quarterly measured real gross domestic product series and monthly measured industrial production series. The researchers apply the classical business cycle or the deviation business cycle definition to these series.[5]

While applying the definitions above on the time series and analyzing the business cycles, the researchers focus on three features: length, depth and shape. Harding and Pagan (2002) summarizes how these features are measured. For the first feature, they consider the duration of an expansion and a recession. The duration of an expansion (recession) is the time spent between the trough (peak) and the following peak (trough). For the second feature the researchers use amplitude, which is interpreted as the percentage that have been lost (gained) during a(n) recession (expansion). To measure the amplitude they compare the log level of the time series at two consecutive turning points. Finally, to consider the shape of a business cycle they use "excess", which measures the departures of the actual time series path from a hypothetical path if the transition between two consecutive turning points was linear. The main idea is to examine whether the business cycle phase is concave or convex.

The aim of this chapter is to compare the characteristics of the business cycles in the US and the euro area countries. Since the chronologies provided by the NBER and CEPR are assumed to be the main source of the dates of turning points in the two areas, I start the

[5] For some examples see Artis et al. (2004), Bergman (2004), Canova et al (2004), Camacho et al. (2008), Cesaroni et al. (2008) and Giannone et al. (2009).

comparison with the chronologies provided by these two organizations. Then, I look at the business cycles derived from the real GDP and industrial production.

According to the comparison of the business cycles dated by the NBER and CEPR, expansion phases of the business cycles are much longer than the contraction phases in both areas. However, the contraction period of the euro are business cycle is almost two times longer than that in the US. Similarly, the expansion period is much longer in the euro area than in the US.

While comparing the business cycles with GDP and industrial production series, I rely on two recent studies; Cesaroni, Maccini and Malgarini (2009) and Camacho, Perez-Quiros and Saiz (2008). The former study compares the business cycles derived from the real GDP of the US and the euro core countries. The latter study uses stationary bootstrap and model-based clustering methods to analyze the similarities and differences among the business cycles derived from industrial production of a large set of countries.

Cesaroni et al. (2009) finds that the US experiences on the average longer recessions than the euro core countries. The average loss during a recession in the US is higher than that of all of the euro core countries. As for the expansions in the US and in the euro core countries, the average gain during an expansion in the US is higher than the average gain in Germany and Italy, which have almost the same average length of expansion with the US. The average gain in France during an expansion is higher than all other countries but also has a longer average duration of expansion as well.

Camacho et al. (2008) does a similar analysis with the industrial production data. According to their findings the average expansion period in the US is longer than all of the euro area countries except Ireland. The average duration of recession in most of the euro area countries is longer than that in the US. Finally, the average gain during an expansion and the average loss during a recession in the US are lower than those in most of the European countries.

To sum up, the business cycles across Atlantic exhibit different characteristics. However, the results of a comparison will depend on the definition of the turning points and the time series considered. For instance, average duration of an expansion is shorter in the US than in France and Germany when GDP is used where it is longer in the US than in Germany and France when industrial production is considered. Average gain in GDP during an expansion in the US is lower than that in France where average gain during an expansion in industrial production is higher in the US.

The next section first discusses and compares the chronologies of the business cycles provided by NBER and CEPR. Then, it looks at the characteristics of the GDP business cycles in the euro core countries and the US and industrial production business cycles for a larger set of countries. The final section concludes the paper.

## 3.2 Business cycles in the US and in the euro area

The first mean of the comparison will be the chronologies provided by the NBER and CEPR. These two organizations date the peaks and troughs in the business cycles of the US and the euro area, respectively. Although these two organizations employ a similar definition for the turning points in the business cycles, the methodologies they use show a few differences. These differences are discussed below. Then, I compare the characteristics of the real GDP business cycles and industrial production business cycles in the US and the euro area. I consider the findings of two recent studies: Cesaroni et al. (2009) and Camacho et al. (2008). The former study compares the business cycles derived from the real GDP of the US and the euro core countries[6] where the latter study uses stationary bootstrap and model-based clustering methods to analyze the similarities and differences among the business cycles derived from industrial production of a large set of countries.

### 3.2.1 The NBER and CEPR chronologies

The dating committee of the NBER has been maintaining the chronology of the business cycles in the US for a very long time. The first trough that the NBER identified is as early as 1857. The complete list of the dates of the peaks and troughs determined by the NBER can be found in the official website of the organization.[7] However, the CEPR has been providing the same

[6]Euro core countries are France, Germany and Italy. It is very difficult to find a reliable GDP data for most of the European countries, therefore, the analysis here is bounded with only the Euro core countries and the US.

[7]http://www.nber.org/cycles/

chronology starting from only the seventies. Therefore, during the comparison I consider the business cycles identified only after the seventies. But before looking at the dates provided by these organizations, I will discuss the differences between the methods of the NBER and CEPR.[8]

First of all the NBER business cycle dating committee defines a recession as a significant decline in economic activity spread across the economy, lasting more than a few months. In other words, the committee primarily maintains a monthly chronology. However, the CEPR committee maintains only a quarterly chronology due to the lack of a reliable monthly European data. Therefore, the CEPR defines a recession as a significant decline in the level of economic activity spread the economy of the euro area, usually visible in two or more consecutive quarters of negative growth in economic activity.

Second, while identifying the dates in the business cycles, the NBER committee considers several different series at both monthly and quarterly frequencies such as real gross domestic product, real gross domestic income, payroll employment measure, real personal income less transfer payments, real manufacturing and wholesale-retail trade sales, industrial production, and employment estimates based on the household survey. The CEPR committee, however, views the real GDP as the main source of macroeconomic activity.

Third, the CEPR monitors country statistics along with the euro area aggregates to make sure that expansions or recessions are widespread over the countries of the area. The committee does not use a fixed rule by which country information is weighted. Despite the differences in the methodologies used by the NBER and CEPR, it will be very informative to look at the chronologies provided by the NBER and CEPR.

Although the primary role of the NBER committee is to maintain a monthly chronology, it also provides a quarterly chronology. This make it easier to compare the features of the business cycles in the two areas. Table 3.1 provides the peaks and troughs declared by the business cycle committees in the two areas after the seventies. According to that table, on the average, the length of an expansion is much longer than the length of a recession in both areas.

[8] For further discussion on the differences between the methodologies used by the NBER and CEPR visit http://www.cepr.org/data/Dating/cepr-nber.asp.

In other words, both business cycles of the US and the aggregate euro area are characterized by long periods of expansions and short periods of declines in real economic activity. However, the averages of the durations of contractions and expansions in the two areas show that both of the phases of the business cycle are shorter in the US than in the euro area. A contraction period lasts on the average almost two times longer in the euro area than in the US where an expansion period lasts on the average almost four years longer in the euro area than in the US.

### 3.2.2 GDP cycles

As explained in the previous section the NBER and CEPR committees consider several different time series while determining the turning points in the economic activity. This makes it very difficult to use the definitions of these organizations in the empirical studies. Therefore, the researchers use different time series to approximate the business cycle. The two most commonly used time series are the real gross domestic product and industrial production. They apply the classical cycle or deviation cycle definitions on these series and investigate the features of the business cycles.

In this study, I consider two recent studies, Cesaroni et al. (2009) and Camacho et al. (2008), that compare the characteristics of the business cycles in different economies by applying the classical cycle definition on the real GDP and industrial production, respectively. I focus on the classical cycle definition due to two reasons. First, in the literature there is a big debate on finding the most appropriate method to extract a trend from the data while using the deviation cycle definition.[9] Second, the definition used by the NBER and CEPR are closer to the classical cycle definition.

Before going into the details of the studies I want to mention the fact that comparison of the results of these studies might be misleading due to a couple of reasons. First, industrial production data can show different characteristics than gross domestic product data. Within the same quarter there might be two turning points. These points will be ignored in an

[9] See Artis et al. (2004) and thereference therein for further discussion.

analysis that uses GDP. Secondly, the industrial production data is available for much longer time periods for a lot of countries.

Cesaroni et al. (2009) investigates the features of the business cycles in the US and the euro core countries by applying the classical definition on the real GDP. Table 3.2 collects the characteristics of the real GDP business cycles of the countries considered in that study. It shows the averages of duration, amplitude and the value for the measure of excess during an expansion and a recession. Although on average the expansion period lasts longer than contraction period in every country, the average lengths of expansions and recessions vary a lot across countries. The average duration of an expansion swings from 16.22 (Italy) quarters to 33.5 (France) quarters where the maximum and minimum average durations of a recession is 2.67 (France) quarters and 3.20 (the US) quarters, respectively.

According to the amplitudes during the two phases of the business cycles, the average gain during an expansion is very high in France and the US. Average loss during a recession is very high in absolute value in the US compared to the euro core countries. The average gain during an expansion in Germany and Italy is lower compared to the others.

Final feature to compare is the excess of movements observed in the business cycles. A value of excess close to zero indicates that the cyclical fluctuation is almost linear in its behavior. A negative (positive) number for the excess during an expansion implies a concave (convex) expansion, which means the gain in output intensifies (slows down) towards the end of the expansion. A similar description can be made for the contraction period. During a recession a negative (positive) number for the excess implies a concave (convex) recession, which indicates an intensification in the output loss towards (at) the end (beginning) of the recession. Given these definitions and the measures in the table 3.3, all countries experience concave expansions and convex recessions. The highest number for excess in the recession column is for the US business cycle this means that the recessions in the Euro core countries are more linear than recessions in the US.

### 3.2.3 Industrial production cycles

Cesaroni et al. (2009) does the same analysis above with the industrial production data and the first finding is that industrial production fluctuates more than total GDP in all the countries considered. I will, however, look at Camacho et al. (2008) while comparing the features of industrial production business cycles across countries, since it considers a larger set of countries than Cesaroni et al. (2009). In their study, Camacho et al. (2008), applies the classical cycle definition on the industrial production of countries including the euro area countries[10] (except Malta) and the US.[11] The statistics for the euro area countries and the US are collected in table $B.5$.

According to that table, the average duration of a recession varies from 11 months (Slovakia and Ireland) to 24 months (Greece) and the average duration of an expansion varies between 18.5 months (Italy) and 47 months (Ireland). In other words, the average durations of two phases of the business cycles vary a lot across the countries. it is also observed that the recession in the US is shorter and the expansion is longer on the average than those of most of the euro area countries.

When the averages of the amplitudes during an expansion in the industrial production are compared, it is seen that it differs a lot. The gain in the euro core countries seems very limited. A similar result can also be seen in the average loss during a recession. The average loss in the euro core countries during a recession is lower than the average loss in most of the other countries. The most striking result regarding the comparison of the recessions in the US and in the euro area countries is that the average loss in the US industrial production is lower or equal to that in all of the euro area countries. In other words, the US industrial production does not decrease during a recession as much as it does in the euro area countries.

As for the comparison of the excess of the business cycles in the US and in the euro area countries, the expansions and recessions in the industrial production are almost linear

[10] Austria, Belgium, Cyprus, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Slovakia, Slovenia and Spain.

[11] Other countries considered in that study are Canada, the Czech Republic, Denmark, Estonia, Hungary, Japan, Latvia, Lithuania, Norway, Poland, Romania, Sweden, Turkey and the UK.

in the large economies like euro core countries and the US. Most of the economies experience concave expansions and concave recessions. Finally, the US, France and Germany have convex expansions and concave recessions where Italy experiences concave expansions and recessions.

## 3.3 Conclusion

This study compares the characteristics of the business cycles of the US and the euro area countries. Seminal work of Burns and Mitchell (1946) is the study that the modern literature on business cycles has been based on. The researchers following that study came up with two broad definitions for the business cycle: classical cycle and deviation cycle. These two definitions differ basically with respect to the definitions of the turning points. In this study, however, I concentrate on the classical cycle definition since the definitions used by the NBER and CEPR are closer to that definition. These organizations maintain the chronologies of the business cycles by analyzing a variety of time series. Moreover, there is still a debate on the best way of defining an appropriate trend rate of growth.

Since the business cycles determined by the NBER and CEPR are assumed to be the "official" sources of peak and trough dates in the US and the euro area, I first compare the durations of expansions and recessions provided by these two organizations. According to the chronologies provided by the NBER and CEPR, the expansions last much longer than recessions in both areas. However, the contraction period of the euro are business cycle is almost two times longer than that in the US and the expansion period is much longer in the euro area than that in the US.

The NBER and CEPR consider a variety of times series while determining the turning points in the economic activity. Moreover, they do not explicitly declare the weight of the times series considered in the decision process. Several different techniques have been developed and several different time series have been used to approximate the business cycle. The two most common time series used in the literature are the gross domestic product and industrial production. The researchers focused on three features while analyzing the characteristics of the business cycles: length or duration of contractions and expansions, depth or amplitude of the loss or gain during a recession and an expansion and shape of the business cycle.

While comparing the characteristics of the business cycles derived by GDP and industrial production, I consider the findings of two recent studies; Cesaroni et al. (2009) and Camacho et al. (2008), respectively. Former study finds that the US experiences on the average longer recessions than the euro countries. The average loss during a recession is higher in the US than that of all of the euro core countries. The average gain during an expansion in the US is higher than the gain in Germany and Italy, which have almost the same average length of expansion with the US. The average gain in France during an expansion is higher than all other countries but also has a longer average duration of expansion as well.

According to the findings of Camacho et al. (2008), on the average, the expansion period in the US is longer than all of the euro area countries except Ireland. The average duration of a recession in most of the euro area countries is longer than that in the US. Finally, the average gain during an expansion the loss during a recession in the US is lower than that in most of the European countries.

As a conclusion it can be said that the results of a comparison of the features of the business cycles of the euro area and the US differ depending on the time period considered and data used in the analysis. However, the main result still stands, the business cycles have very different characteristics in the two areas.

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Table 3.1: Comparison of the US and the euro area business cycles according to the NBER and CEPR dates

| The US | | | The euro area | |
|---|---|---|---|---|
| Contractions | 1973(IV) - 1975(I) | 5* | 1974(III) - 1975(I) | 2 |
| | 1980(I) - 1980(III) | 2 | 1980(I) - 1982(III) | 10 |
| | 1981(III) - 1982(IV) | 5 | 1992(I) - 1993(III) | 6 |
| | 1990(III) - 1991(I) | 2 | | |
| | 2001(I) - 2001(IV) | 3 | | |
| Average | | 3.4 | | 6 |
| Expansions | 1975(I) - 1980(I) | 20 | 1975(I) - 1980(I) | 30 |
| | 1980(III) - 1981(III) | 4 | 1982(III) - 1992(I) | 28 |
| | 1982(IV) - 1990(III) | 31 | 1993(III) - 2008(I) | 57 |
| | 1991(I) - 2001(I) | 36 | | |
| | 2001(IV) - 2007(IV) | 24 | | |
| Average | | 23 | | 38 |

*All data provided here is in terms of quarters.

This table shows the contractions and expansions identified by the NBER and CEPR in the US and in the euro area. NBER provides both monthly and quarterly chronologies where CEPR is able to provide only a quarterly one due to monthly data availability problem. Therefore, during the comparison will be based on quarterly chronologies. Although the NBER maintains a chronolgy for a very long time, CEPR starts analyzing the data after the seventies. Thus, the comprison between the business cycles in the two areas is made for after the seventies.

Table 3.2: Business cycle characteristics with GDP - Euro core countries and the US

| Countries | Expansion | | | Contraction | | |
|---|---|---|---|---|---|---|
| | Duration | Amplitude | Excess | Duration | Amplitude | Excess |
| the US | 17.50* | 18.07 | -8.00 | 3.20 | -2.04 | 0.38 |
| Germany | 18.29 | 14.61 | -6.50 | 3.00 | -1.44 | 0.24 |
| France | 33.50 | 21.63 | -10.17 | 2.67 | -1.25 | 0.16 |
| Italy | 16.22 | 14.25 | -6.25 | 3.11 | -1.32 | 0.23 |

*All data provided here is in terms of quarters.

This table was taken from Table 1 of Cesaroni et al. (2009), pg. 14. It shows the duration, amplitude and excess values for expansions and contractions of the business cycles identified by applying the classical cycle definition on the real gross domestic product.

Table 3.3: Business cycle characteristics with industrial prodcution - The euro area countries and the US

| Countries | Expansion | | | Contraction | | |
|---|---|---|---|---|---|---|
| | Duration | Amplitude | Excess | Duration | Amplitude | Excess |
| Austria | 35.50* | 0.18 | 0.15 | 13.00 | -0.06 | -0.02 |
| Belgium | 28.00 | 0.12 | 0.03 | 18.75 | -0.08 | 0.04 |
| Cyprus | 23.50 | 0.14 | 0.22 | 22.00 | -0.16 | 0.17 |
| Finland | 33.33 | 0.22 | 0.35 | 14.25 | -0.09 | -0.07 |
| France | 30.67 | 0.08 | 0.04 | 18.50 | -0.04 | -0.05 |
| Germany | 22.75 | 0.08 | 0.04 | 13.17 | -0.06 | -0.02 |
| Greece | 30.33 | 0.12 | 0.31 | 23.67 | -0.09 | 0.08 |
| Ireland | 47.33 | 0.45 | 0.44 | 10.67 | -0.16 | 0.07 |
| Italy | 18.50 | 0.08 | -0.01 | 16.67 | -0.05 | -0.04 |
| Luxemburg | 28.33 | 0.17 | 0.36 | 15.50 | -0.12 | -0.05 |
| Netherlands | 31.33 | 0.10 | -0.18 | 17.67 | -0.07 | -0.12 |
| Portugal | 28.00 | 0.14 | -0.28 | 22.00 | -0.12 | -0.17 |
| Slovakia | 36.33 | 0.21 | 0.18 | 11.00 | -0.09 | 0.05 |
| Slovenia | 27.67 | 0.15 | -0.21 | 16.33 | -0.11 | -0.04 |
| Spain | 32.25 | 0.12 | 0.11 | 14.25 | -0.07 | 0.00 |
| the US | 34.00 | 0.14 | 0.04 | 14.00 | -0.04 | -0.03 |

*All data provided here is in terms of months.

This table is produced from the table 2 of Camacho et al. (2008), page 2178. It shows the duration, amplitude and excess values for expansions and contractions of the business cycles identified by applying the classical cycle definition on the industrial production. The business cycle characteristics are calculated by using the stationary bootstrap method.

# Appendix A

# Dynamics of the model in "Staggered wage contracts and Nash bargaining"

## A.1 Deriving the equilibrium wage rate

• To derive the equilibrium wage, first the sum of the expected wages is derived. Then, the surpluses of the bargaining worker and the bargaining firm are derived. The expected lifetime wage revenues of a worker negotiating at time $t$ depends on whether the relationship will survive or not. It can be written as:

$W_t^w(r) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(r)$

Then, by using the fact that $E_t w_{t+1}(r) = \varphi_w w_t^* + (1-\varphi_w) E_t w_{t+1}^*$ I can rewrite the equation above as

$W_t^w(r) = E_t[1 + \rho\beta\varphi_w \Lambda_{t,t+1} + (\rho\beta\varphi_w)^2 \Lambda_{t,t+2} + (\rho\beta\varphi_w)^3 \Lambda_{t,t+3} + ...] w_t^* + (1-\varphi_w)\rho\varphi_w E_t \Lambda_{t,t+1}[1 + \rho\beta\varphi_w E_t \Lambda_{t+1,t+2} + (\rho\beta\varphi_w)^2 E_t \Lambda_{t+1,t+3} + ...] w_{t+1}^* + ...$

The discount factor for the worker can be defined as $\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\lambda\varphi_w)^s \Lambda_{t,t+s}$. Therefore,

$W_t^w(r) = \Delta_t w_t^* + (1-\varphi_w) E_t \sum_{s=1}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Delta_{t+s} w_{t+s}^*$

The expected wage payments of the firm, $W_t^f$, can be written as:

$W_t^f(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} w_{t+s}(r)$

$\Rightarrow$

$W_t^f(r) = E_t[1 + \frac{n_{t+1}}{n_t}(r)\beta\varphi_w \Lambda_{t,t+1} + \frac{n_{t+2}}{n_t}(r)(\beta\varphi_w)^2 \Lambda_{t,t+2} + ...] w_t^* + (1-\varphi_w) E_t \frac{n_{t+1}}{n_t}(r)\beta\Lambda_{t,t+1}[1 + \frac{n_{t+2}}{n_{t+1}}(r)\beta\varphi_w \Lambda_{t+1,t+2} + ...] w_{t+1}^* + (1-\varphi_w) E_t \frac{n_{t+2}}{n_t}(r)\beta^2 \Lambda_{t,t+2}[1 + \frac{n_{t+3}}{n_{t+2}}(r)\beta\varphi_w \Lambda_{t+2,t+3} + ...] w_{t+2}^* + ...$

Defining the discount factor of the firm as $\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)(\beta\varphi_w)^s \Lambda_{t,t+s}$ yields

$W_t^f(r) = \Sigma_t(r) w_t^* + (1-\varphi_w) E_t \sum_{s=1}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} \Sigma_{t+s}(r) w_{t+s}^*$

• $H_t(r)$ is the surplus of the negotiating worker and given by:

$H_t(r) = w_t(r) - b + \beta E_t \Lambda_{t,t+1}[\rho H_{t+1}(r) - s_{t+1} H_{x,t+1}]$

$\Rightarrow$

$H_t(r) = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[b + s_{t+s}\beta\Lambda_{t+s,t+s+1} H_{x,t+s+1}]$

$\Rightarrow$

$H_t(r) = W_t^w(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[b + s_{t+s}\beta\Lambda_{t+s,t+s+1} H_{x,t+s+1}]$

Finally, substituting $W_t^w(r)$ results in

$H_t(r) = \Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b + \beta\Lambda_{t+s,t+s+1} \left(s_{t+s+1} H_{x,t+s+1} - (1-\varphi_w)\rho\Delta_{t+s+1} w_{t+s+1}^*\right)\right]$

• The surplus of the firm, $J_t(r)$, has to be log-linearized before log-linearizing the bargained wage.

$J_t(r) = p_t^w f_{nt} - w_t(r) - \beta E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}^2 + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) J_{t+1}(r)$

$\Rightarrow$

$J_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} \left[p_{t+s}^w f_{nt+s} - \beta\Lambda_{t+s,t+s+1} \frac{\kappa}{2} x_{t+s+1}^2\right] - W_t^f(r)$

• The bargained wage can be driven by plugging the surpluses into the first order condition of the Nash bargaining problem:

$\chi_t(r)[E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[p_{t+s}^w f_{nt+s} + \beta\Lambda_{t+s,t+s+1}(\frac{\kappa}{2} x_{t+s+1}^2 - (1-\varphi_w)\rho\Delta_{t+s+1} w_{t+s+1}^*)] - \Delta_t w_t^*] =$

$(1-\chi_t(r)) \left[\Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[b + \beta\Lambda_{t+s,t+s+1}(s_{t+s+1} H_{x,t+s+1} - (1-\varphi_w)\rho\Delta_{t+s+1} w_{t+s+1}^*)]\right]$

$\Rightarrow$

$\Delta_t w_t^* = E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s}[\chi_t(r)(p_{t+s}^w f_{nt+s} + \beta\Lambda_{t+s,t+s+1} \frac{\kappa}{2} x_{t+s+1}^2) +$

$(1-\chi_t(r))(b + \beta s_{t+s+1} \Lambda_{t+s,t+s+1} H_{x,t+s+1}) - (1-\varphi_w)\rho\beta\Lambda_{t+s,t+s+1}\Delta_{t+s+1} w_{t+s+1}^*]$

Furthermore, a recursive equation for the equilibrium wage can be derived by rearranging the terms such that

$\Delta_t w_t^* = \chi_t(r) \left[p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}(r)^2\right] + (1-\chi_t(r)) \left[b + \beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1}\right] -$

$(1-\varphi_w)\rho\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^* + \rho\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$

$\Rightarrow$

$\Delta_t w_t^* = w_t^o(r) + \rho\beta\varphi_w E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$

where $w_t^o(r)$ is the target wage and given by

$w_t^o(r) = \chi_t(r) \left[p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}(r)^2\right] + (1-\chi_t(r)) \left[b + \beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1}\right]$

## A.2 Log-linearizing the bargained wage

Before log-linearizing the bargained wage, the percentage deviation of the target wage from its steady state has to be derived.

$w_t^o(r) = \chi_t(r) p_t^w f_{nt} + \chi_t(r)\beta E_t \Lambda_{t,t+1} \frac{\kappa}{2} x_{t+1}(r)^2 + b + \beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1}(1 - \chi_t(r)b -$

$\chi_t(r)\beta E_t \Lambda_{t,t+1} s_{t+1} H_{x,t+1}$

$\Rightarrow$

$w^o(r)(1 + \widehat{w}_t^o(r)) = \chi p^w f_n(1 + \widehat{\chi}_t(r) + \widehat{p}_t^w + \widehat{f}_{nt}) + \chi\beta\frac{\kappa}{2}x^2(1 + \widehat{\chi}_t(r) + \widehat{\Lambda}_{t,t+1} + 2\widehat{x}_{t+1}) + b +$

$s\beta H(1 + \widehat{s}_{t+1} + \widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}) - \chi b(1 + \widehat{\chi}_t(r)) - \chi s\beta H(1 + \widehat{\chi}_t + \widehat{s}_{t+1} + \widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1})$

where at the steady state $w = \chi(r)p^w f_n + \chi(r)\beta\frac{\kappa}{2}x^2 + b - b\chi(r) + s\beta H_x - \chi(r)s\beta H_x$. Getting rid of the constant term renders in

$w\widehat{w}_t^o(r) = \chi p^w f_n(\widehat{p}_t^w + \widehat{f}_{nt}) + \chi(p^w f_n + \beta\frac{\kappa}{2}x^2 - b - s\beta H)\widehat{\chi}_t(r) + \chi\beta\kappa x^2\widehat{x}_{t+1}(r) + (\chi\beta\frac{\kappa}{2}x^2 + (1 -$

$\chi)s\beta H)\widehat{\Lambda}_{t,t+1} + (1 - \chi)s\beta H(\widehat{s}_{t+1} + \widehat{H}_{t+1})$

$\Rightarrow$

$\widehat{w}_t^o(r) = \chi p^w f_n w^{-1}(\widehat{p}_t^w + \widehat{f}_{nt}) + \chi w^{-1}[p^w f_n + \beta\frac{\kappa}{2}x^2 - b - \frac{s\beta\chi\kappa x}{(1-\chi)}]\widehat{\chi}_t(r) + \chi\beta\kappa x^2 w^{-1} E_t\widehat{x}_{t+1}(r) +$

$\chi\beta\frac{\kappa}{2}x^2 w^{-1} E_t\widehat{\Lambda}_{t,t+1} + s\beta\chi\kappa x w^{-1} E_t(\widehat{s}_{t+1} + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1})$

- To find $\widehat{x}_t(r)$, the unconditional average of the hiring rate can be defined as:

$x_t = \int_0^1 x_t(i)\frac{n_t(i)}{n_t}di$

with

$\kappa x_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1}\frac{\kappa}{2}x_{t+1}(i)^2 + \rho\beta\kappa E_t\Lambda_{t,t+1}x_{t+1}(i)]$

Combining these equations and log-linearizing around the steady state yields

$\widehat{x}_t(r) = p^w f_n \epsilon(\widehat{p}_t^w + \widehat{f}_{nt}) - w\epsilon\widehat{w}_t(r) + \beta E_t\widehat{x}_{t+1}(r) + \frac{\beta}{2}(1 + \rho)E_t\widehat{\Lambda}_{t,t+1}$

Then, the difference between the hiring rate of the bargaining firm and the average hiring rate is

$\widehat{x}_t(r) - \widehat{x}_t = w\epsilon(\widehat{w}_t - \widehat{w}_t(r)) + \beta E_t(\widehat{x}_{t+1}(r) - \widehat{x}_{t+1})$

iterating this equation forward provides

$\widehat{x}_t(r) - \widehat{x}_t = w\epsilon(\widehat{w}_t - \widehat{w}_t(r)) + \beta w\epsilon E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)\right) + \beta^2 w\epsilon E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right) +$

$\beta^3 w\epsilon E_t\left(\widehat{w}_{t+3} - \widehat{w}_{t+3}(r)\right) + ...$

This equation can be simplified by using the real wage index.

$\widehat{w}_t = \varphi_w \widehat{w}_{t-1} + (1-\varphi_w)\widehat{w}_t^*$

$\Rightarrow$

$E_t\widehat{w}_{t+1} = \varphi_w \widehat{w}_t + (1-\varphi_w)E_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$E_t\widehat{w}_{t+2} = \varphi_w \widehat{w}_{t+1} + (1-\varphi_w)E_t\widehat{w}_{t+2}^*$

It is also known that expected bargained wage for the next period is the convex combination of the optimum bargained wage at next period and the optimum wage of the current period.

$\widehat{w}_t(r) = \widehat{w}_t^*$

$\Rightarrow$

$E_t\widehat{w}_{t+1}(r) = \varphi_w \widehat{w}_t^* + (1-\varphi_w)E_t\widehat{w}_{t+1}^*$

$\Rightarrow$

$E_t\widehat{w}_{t+2}(r) = \varphi_w \widehat{w}_{t+1}^* + (1-\varphi_w)E_t\widehat{w}_{t+2}^*$

Thus,

$E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)\right) = \varphi_w\left(\widehat{w}_t - \widehat{w}_t^*\right)$

$\Rightarrow$

$E_{t+1}\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right) = \varphi_w\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$

Using the fact that

$E_t\widehat{w}_t(r) = \widehat{w}_t^* \Rightarrow E_{t+1}\widehat{w}_{t+1}(r) = \widehat{w}_{t+1}^*.$

and taking the expectation of both sides:

$E_t\widehat{w}_{t+1}(r) = E_t\widehat{w}_{t+1}^* \Rightarrow \varphi_w\widehat{w}_t^* + (1-\varphi_w)E_t\widehat{w}_{t+1}^* = E_t\widehat{w}_{t+1}^* \Rightarrow \widehat{w}_t^* = E_t\widehat{w}_{t+1}^*.$

Therefore,

$E_{t+1}\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right) = \varphi_w\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$

$\Rightarrow$

$E_t\left[E_{t+1}\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right)\right] = E_t\left[\varphi_w\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)\right]$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right] = \varphi_w\left(E_t\widehat{w}_{t+1} - E_t\widehat{w}_{t+1}^*\right)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right] = \varphi_w(\varphi_w\widehat{w}_t + (1-\varphi_w)E_t\widehat{w}_{t+1}^* - E_t\widehat{w}_{t+1}^*)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}(r)\right]=\varphi_w^2(\widehat{w}_t-E_t\widehat{w}_{t+1}^*)$

$\Rightarrow$

$E_t\left[\widehat{w}_{t+2}-\widehat{w}_{t+2}(r)\right]=\varphi_w^2(\widehat{w}_t-\widehat{w}_t^*)$

Then,

$\widehat{x}_t(r)-\widehat{x}_t=w\epsilon(\widehat{w}_t-\widehat{w}_t(r))+\beta w\epsilon E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}(r)\right)+\beta^2 w\epsilon E_t\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}(r)\right)+...$

$\Rightarrow$

$\widehat{x}_t(r)-\widehat{x}_t=w\epsilon(\widehat{w}_t-\widehat{w}_t(r))+\beta\varphi_w w\epsilon(\widehat{w}_t-\widehat{w}_t^*)+(\beta\varphi_w)^2 w\epsilon(\widehat{w}_t-\widehat{w}_t^*)+(\beta\varphi_w)^3 w\epsilon(\widehat{w}_t-\widehat{w}_t^*)+...$

$\Rightarrow$

$\widehat{x}_t(r)=\widehat{x}_t+w\Sigma\epsilon(\widehat{w}_t-\widehat{w}_t^*)\Rightarrow\widehat{x}_{t+1}(r)=\widehat{x}_{t+1}+w\Sigma\epsilon\varphi_w(\widehat{w}_t-\widehat{w}_t^*)\Rightarrow\widehat{x}_{t+2}(r)=\widehat{x}_{t+2}+w\Sigma\epsilon\varphi_w^2(\widehat{w}_t-\widehat{w}_t^*)$ and so forth.

• Following the same path for the weights in the Nash bargaining first order condition yields

$\widehat{\Sigma}_t(r)=x\beta\varphi_w\widehat{x}_{t+1}(r)+\beta\varphi_w E_t(\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1}(r))$

$\Rightarrow$

$\widehat{\Sigma}_t=x\beta\varphi_w\widehat{x}_{t+1}+\beta\varphi_w E_t(\widehat{\Lambda}_{t,t+1}+\widehat{\Sigma}_{t+1})$

$\Rightarrow$

$\widehat{\Sigma}_t(r)-\widehat{\Sigma}_t=x\beta\varphi_w(\widehat{x}_{t+1}(r)-\widehat{x}_{t+1})+\beta\varphi_w E_t(\widehat{\Sigma}_{t+1}(r)-\widehat{\Sigma}_{t+1})$

$\Rightarrow$

$\widehat{\Sigma}_{t+1}(r)-\widehat{\Sigma}_{t+1}=x\beta\varphi_w(\widehat{x}_{t+2}(r)-\widehat{x}_{t+2})+\beta\varphi_w E_t(\widehat{\Sigma}_{t+2}(r)-\widehat{\Sigma}_{t+2})$

Therefore,

$\widehat{\Sigma}_t(r)-\widehat{\Sigma}_t=x\beta\varphi_w[(\widehat{x}_{t+1}(r)-\widehat{x}_{t+1})+\beta\varphi_w E_t(\widehat{x}_{t+2}(r)-\widehat{x}_{t+2})+(\beta\varphi_w)^2(\widehat{x}_{t+3}(r)-\widehat{x}_{t+3})+...]$

$\Rightarrow$

$\widehat{\Sigma}_t(r)-\widehat{\Sigma}_t=x\Psi\epsilon\Sigma w(\widehat{w}_t-\widehat{w}_t^*)$

where $\Psi=\frac{\beta\varphi_w^2}{1-\beta\varphi_w^2}$, $\epsilon=(\kappa x)^{-1}$ and $\Sigma=\frac{1}{1-\beta\varphi_w}$.

• The first order condition of the Nash bargaining is log-linearized as

$\widehat{\chi}_t(r)=-(1-\chi)(\widehat{\Sigma}_t(r)-\widehat{\Delta}_t)$

Averaging this accross all firms will give

$\widehat{\chi}_t(r)-\widehat{\chi}_t=-(1-\chi)(\widehat{\Sigma}_t(r)-\widehat{\Sigma}_t)$

$\Rightarrow$

$\widehat{\chi}_t(r)=\widehat{\chi}_t-(1-\chi)x\Psi\epsilon\Sigma w(\widehat{w}_t-\widehat{w}_t^*)$

and

$E_t\widehat{\chi}_{t+1}(r) = E_t\widehat{\chi}_{t+1} - (1-\chi)x\Psi\epsilon\Sigma w\varphi_w(\widehat{w}_t - \widehat{w}_t^*)$

• To finish the analysis for the spillover effects, I need to log-linearize the worker's and firm's surpluses. The worker's surplus is given by

$H_t(i) = w_t(i) - b + \beta E_t\Lambda_{t,t+1}\left[\rho H_{t+1}(i) - s_{t+1}H_{x,t+1}\right]$

Then, the percentage deviation of the worker's surplus from its steady state is:

$H(1+\widehat{H}_t(r)) = w(1+\widehat{w}_t(r)) - b + \rho\beta HE_t(1+\widehat{\Lambda}_{t,t+1}+\widehat{H}_{t+1}(r)) - s\beta HE_t(1+\widehat{\Lambda}_{t,t+1}+\widehat{s}_{t+1}+\widehat{H}_{x,t+1})$

Canceling the constant terms and using the fact that $\widehat{H}_t = \widehat{H}_{x,t}$ results in

$H\widehat{H}_t(r) = w\widehat{w}_t(r) + \rho\beta HE_t(\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}(r)) - s\beta HE_t(\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{t+1})$

$\Rightarrow$

$H\widehat{H}_t = w\widehat{w}_t + \rho\beta HE_t(\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}) + s\beta HE_t(\widehat{\Lambda}_{t,t+1} + \widehat{s}_{t+1} + \widehat{H}_{t+1})$

$\Rightarrow$

$H(\widehat{H}_t(r) - \widehat{H}_t) = w(\widehat{w}_t(r) - \widehat{w}_t) + \rho\beta HE_t(\widehat{H}_{t+1}(r) - \widehat{H}_{t+1})$

$\Rightarrow$

$\widehat{H}_t(r) - \widehat{H}_t = wH^{-1}(\widehat{w}_t(r) - \widehat{w}_t) + \rho\beta E_t(\widehat{H}_{t+1}(r) - \widehat{H}_{t+1})$

iterating the equation above forward yields

$\widehat{H}_t(r) = \widehat{H}_t - \frac{wH^{-1}}{1-\rho\beta\varphi_w}(\widehat{w}_t - \widehat{w}_t^*)$

At the steady state $H = \frac{\chi\kappa x}{1-\chi}$ and $\Delta = \frac{1}{1-\rho\beta\varphi_w}$. Then, this equation can be revised as

$\widehat{H}_t = \widehat{H}_t(r) + (1-\chi)\chi^{-1}\epsilon\Delta w(\widehat{w}_t - \widehat{w}_t^*)$

From solution to the Nash bargaining problem it is known that

$\chi_t(r)J_t(r) = (1-\chi_t(r))H_t(r)$

$\Rightarrow$

$\widehat{H}_t(r) = \widehat{J}_t(r) + (1-\chi)^{-1}\widehat{\chi}_t(r)$

Using $\widehat{x}_t(r) = \widehat{x}_t + w\Sigma\epsilon(\widehat{w}_t - \widehat{w}_t^*)$, $\widehat{\chi}_t(r) = \widehat{\chi}_t - (1-\chi)x\Psi\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*)$, and $\widehat{J}_t(r) = \widehat{x}_t(r)$

$\widehat{H}_t = \widehat{H}_t(r) + (1-\chi)\chi^{-1}\epsilon\Delta w(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{J}_t(r) + (1-\chi)^{-1}\widehat{\chi}_t(r) + (1-\chi)\chi^{-1}\epsilon\Delta w(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{x}_t(r) + (1-\chi)^{-1}\widehat{\chi}_t(r) + (1-\chi)\chi^{-1}\epsilon\Delta w(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{x}_t + w\Sigma\epsilon(\widehat{w}_t - \widehat{w}_t^*) + (1-\chi)^{-1}(\widehat{\chi}_t - (1-\chi)x\Psi\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*)) + (1-\chi)\chi^{-1}\epsilon\Delta w(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + \left[\epsilon\Sigma w - x\Psi\epsilon\Sigma w + (1-\chi)\chi^{-1}\epsilon\Delta w\right](\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + \left[\Sigma - x\Psi\Sigma + \frac{(1-\chi)}{\chi}\Delta\right]\epsilon w(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + \left[\eta^{-1} - (1-\rho)\Psi\right]\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*)$

where $\eta = \frac{\chi\Sigma}{(1-\chi)\Delta+\chi\Sigma}$ and $x = (1-\rho)$. The equation above can be rewritten as:

$\widehat{H}_t = \widehat{x}_t + (1-\chi)^{-1}\widehat{\chi}_t + \Gamma(\widehat{w}_t - \widehat{w}_t^*)$

$\Rightarrow$

$\widehat{E_t H}_{t+1} = E_t\widehat{x}_{t+1} + (1-\chi)^{-1}E_t\widehat{\chi}_{t+1} + \Gamma E_t(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*)$

where $\Gamma = (1-\eta x\Psi)\epsilon\Sigma w\eta^{-1}$. Now the target wage is:

$\widehat{w}_t^o(r) = \chi p^w f_n w^{-1}(\widehat{p}_t^w + \widehat{f}_{nt}) + \chi w^{-1}[p^w f_n + \beta\frac{\kappa}{2}x^2 - b - \frac{s\beta\chi\kappa x}{(1-\chi)}]\widehat{\chi}_t(r) + \chi\beta\kappa x^2 w^{-1}E_t\widehat{x}_{t+1}(r) +$
$\chi\beta\frac{\kappa}{2}x^2 w^{-1}E_t\widehat{\Lambda}_{t,t+1} + s\beta\chi\kappa x w^{-1}E_t(\widehat{s}_{t+1} + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1})$

$\Rightarrow$

$\widehat{w}_t^o(r) = \varphi_{f_n}(\widehat{p}_t^w + \widehat{f}_{nt}) + \varphi_\chi\widehat{\chi}_t(r) + \varphi_x E_t\widehat{x}_{t+1}(r) + (\frac{\varphi_x}{2} + \varphi_s)E_t\widehat{\Lambda}_{t,t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_s E_t\widehat{H}_{t+1}$

where $\varphi_{f_n} = \chi p^w f_n w^{-1}$, $\varphi_\chi = \chi w^{-1}[p^w f_n + \beta\frac{\kappa}{2}x^2 - b - \frac{s\beta\chi\kappa x}{(1-\chi)}]$, $\varphi_x = \chi\beta\kappa x^2 w^{-1}$ and $\varphi_s = s\beta\chi\kappa x w^{-1}$.

$\widehat{w}_t^o(r) = \varphi_{f_n}(\widehat{p}_t^w + \widehat{f}_{nt}) + \varphi_\chi[\widehat{\chi}_t - (1-\chi)(1-\rho)\Psi\epsilon\Sigma w(\widehat{w}_t - \widehat{w}_t^*)] + \varphi_x[\widehat{x}_{t+1} + w\Sigma\epsilon\varphi_w(\widehat{w}_t - \widehat{w}_t^*)] +$
$(\frac{\varphi_x}{2} + \varphi_s)E_t\widehat{\Lambda}_{t,t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_s E_t[\widehat{x}_{t+1} + (1-\chi)^{-1}\widehat{\chi}_{t+1} + \Gamma(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*)]$

$\Rightarrow$

$\widehat{w}_t^o(r) = \varphi_{f_n}(\widehat{p}_t^w + \widehat{f}_{nt}) + \varphi_\chi\widehat{\chi}_t + \varphi_s(1-\chi)^{-1}E_t\widehat{\chi}_{t+1} + (\varphi_x + \varphi_s)E_t\widehat{x}_{t+1} + (\frac{\varphi_x}{2} + \varphi_s)E_t\widehat{\Lambda}_{t,t+1} +$
$\varphi_s E_t\widehat{s}_{t+1} + [\varphi_x\epsilon\Sigma\varphi_w w - \varphi_\chi(1-\chi)(1-\rho)\Psi\epsilon\Sigma w](\widehat{w}_t - \widehat{w}_t^*) + \varphi_s\Gamma E_t(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*)$

Thus,

$\widehat{w}_t^o(r) = \widehat{w}_t^o + \frac{\tau_1}{1-\rho\beta\varphi_w}E_t(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*) + \frac{\tau_2}{1-\rho\beta\varphi_w}(\widehat{w}_t - \widehat{w}_t^*)$

where

$\widehat{w}_t^o = \varphi_{fn}(\widehat{p}_t^w + \widehat{f}_{nt}) + \varphi_\chi\widehat{\chi}_t + \varphi_s(1-\chi)^{-1}E_t\widehat{\chi}_{t+1} + (\varphi_x + \varphi_s)E_t\widehat{x}_{t+1} + (\frac{\varphi_x}{2} + \varphi_s)E_t\widehat{\Lambda}_{t,t+1} + \varphi_s E_t\widehat{s}_{t+1}$

with $\tau_1 = \varphi_s\Gamma(1-\rho\beta\varphi_w)$, $\tau_2 = [\varphi_x\epsilon\Sigma\varphi_w w - \varphi_\chi(1-\chi)(1-\rho)\Psi\epsilon\Sigma w](1-\rho\beta\varphi_w)$

Now it is easy to derive the percentage deviation of the bargained wage form its steady state.

$\Delta_t w_t^* = w_t^o(r) + \rho\beta\varphi_w E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$

$\Rightarrow$

$\Delta w(1 + \widehat{\Delta}_t + \widehat{w}_t^*) = w^o(1 + \widehat{w}_t^o(r)) + \rho\beta\Delta\varphi_w w^* E_t(1 + \widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1} + \widehat{w}_{t+1}^*)$

It can easily be shown that at the steady state $w = w^* = w^o(r)$. Therefore,

$\Delta(1 + \widehat{\Delta}_t + \widehat{w}_t^*) = (1 + \widehat{w}_t^o(r)) + \rho\beta\Delta\varphi_w E_t(1 + \widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1} + \widehat{w}_{t+1}^*)$

$\Rightarrow$

$\widehat{\Delta}_t + \widehat{w}_t^* = (1 - \rho\beta\varphi_w)\widehat{w}_t^o(r) + \rho\beta\varphi_w E_t(\widehat{\Lambda}_{t,t+1} + \widehat{\Delta}_{t+1} + \widehat{w}_{t+1}^*)$

# Appendix B

# Dynamics of the model in "Labor market institutions and business cycles"

## B.1 Expected wage revenue of a negotiating worker, $W_t^w(r)$

The expected lifetime wage revenues, which depends on whether the relationship will survive or not, of a worker negotiating at time $t$ can be written as

$W_t^w(r) = w_t(r) + \beta E_t \Lambda_{t,t+1}(1-\rho_{t+1})w_{t+1}(r) + \beta^2 E_t \Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2})w_{t+2}(r) + \beta^3 E_t \Lambda_{t,t+3}(1-\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3})w_{t+3}(r) + ...$

By using the fact that $w_t = (1-\varphi_w)w_t^* + \varphi_w w_{t-1}$ it can be said that $w_t(r) = w_t^*$. Then, in the forthcoming periods

$E_t w_{t+1}(r) = (1-\varphi_w)E_t w_{t+1}^* + \varphi_w w_t^*$

$\Rightarrow$

$E_t w_{t+2}(r) = (1-\varphi_w)E_t w_{t+2}^* + \varphi_w w_{t+1}^* = \varphi_w^2 w_t^* + \varphi_w(1-\varphi_w)E_t w_{t+1}^* + (1-\varphi_w)E_t w_{t+2}^*$

$\Rightarrow$

$E_t w_{t+3}(r) = (1-\varphi_w)E_t w_{t+3}^* + \varphi_w w_{t+2}^*$

$\Rightarrow$

$E_t w_{t+3}(r) = \varphi_w^3 w_t^* + \varphi_w^2(1-\varphi_w)E_t w_{t+1}^* + \varphi_w(1-\varphi_w)E_t w_{t+2}^* + (1-\varphi_w)E_t w_{t+3}^*$

and so on. Therefore,

$W_t^w(r) = w_t^* + \beta E_t \Lambda_{t,t+1}(1-\rho_{t+1})[\varphi_w w_t^* + (1-\varphi_w)w_{t+1}^*] + \beta^2 E_t \Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2})[\varphi_w^2 w_t^* + \varphi_w(1-\varphi_w)w_{t+1}^* + (1-\varphi_w)w_{t+2}^*] + \beta^3 E_t \Lambda_{t,t+3}(1-\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3})[\varphi_w^3 w_t^* + \varphi_w^2(1-\varphi_w)w_{t+1}^* + \varphi_w(1-\varphi_w)w_{t+2}^* + (1-\varphi_w)w_{t+3}^*] + ...$

$\Rightarrow$

$W_t^w(r) = [1 + \beta\varphi_w E_t \Lambda_{t,t+1}(1-\rho_{t+1}) + (\beta\varphi_w)^2 E_t \Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2}) + (\beta\varphi_w)^3 E_t \Lambda_{t,t+3}(1-$

$\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3})+...]w_t^* + [\beta(1-\varphi_w)E_t\Lambda_{t,t+1}(1-\rho_{t+1}) + \beta^2\varphi_w(1-\varphi_w)E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2}) + \beta^3\varphi_w^2(1-\varphi_w)E_t\Lambda_{t,t+3}(1-\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3}) + ...]w_{t+1}^* + [\beta^2(1-\varphi_w)E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2}) + \beta^3\varphi_w(1-\varphi_w)E_t\Lambda_{t,t+3}(1-\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3}) + ...]w_{t+2}^* + ...$

Assuming that

$\Delta_t = 1 + \beta\varphi E_t\Lambda_{t,t+1}(1-\rho_{t+1}) + (\beta\varphi_w)^2E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2}) + (\beta\varphi_w)^3E_t\Lambda_{t,t+3}(1-\rho_{t+1})(1-\rho_{t+2})(1-\rho_{t+3}) + ...$

$\Rightarrow$

$\Delta_t = 1 + \beta\varphi E_t\Lambda_{t,t+1}(1-\rho_{t+1})\Delta_{t+1}$

Then,

$\Rightarrow$

$W_t^w(r) = \Delta_t w_t^* + \beta(1-\varphi_w)E_t\Lambda_{t,t+1}(1-\rho_{t+1})\Delta_{t+1}w_{t+1}^* + \beta^2(1-\varphi_w)E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2})\Delta_{t+2}w_{t+2}^* + ...$

## B.2 Expected surplus of the negotiating worker, $H_t(r)$

The average surplus of a worker hired at time $t$, $H_{x,t}$, is defined as $H_{x,t} = V_{x,t} - U_t$ and the surplus of a worker working at firm $i$ is defined as $H_t(i) = V_t(i) - U_t$. Substituting $V_t(i)$ and $U_t$ will yield

$H_t(i) = w_t(i) - b + \beta E_t\Lambda_{t,t+1}[(1-\rho_{t+1})V_{t+1}(i) + \rho_{t+1}U_{t+1} + \rho_{t+1}^{in}\Upsilon - s_{t+1}V_{x,t+1} - (1-s_{t+1})U_{t+1}]$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t\Lambda_{t,t+1}[s_{t+1}H_{x,t+1} - \rho_{t+1}^{in}\Upsilon] + \beta E_t\Lambda_{t,t+1}(1-\rho_{t+1})H_{t+1}(r)$

$\Rightarrow$

$H_{t+1}(r) = w_{t+1}(r) - b - \beta E_t\Lambda_{t+1,t+2}[s_{t+2}H_{x,t+2} - \rho_{t+2}^{in}\Upsilon] + \beta E_t\Lambda_{t+1,t+2}(1-\rho_{t+2})H_{t+2}(r)$

$\Rightarrow$

$H_{t+2}(r) = w_{t+2}(r) - b - \beta E_t\Lambda_{t+2,t+3}[s_{t+3}H_{x,t+3} - \rho_{t+3}^{in}\Upsilon] + \beta E_t\Lambda_{t+2,t+3}(1-\rho_{t+3})H_{t+3}(r)$

Then,

$\Rightarrow$

$H_t(r) = w_t(r) + \beta E_t\Lambda_{t,t+1}(1-\rho_{t+1})w_{t+1}(r) + \beta^2E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2})w_{t+2}(r) - b - \beta E_t\Lambda_{t,t+1}(1-\rho_{t+1})b - \beta^2E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2})b - \beta E_t\Lambda_{t,t+1}[s_{t+1}H_{x,t+1} - \rho_{t+1}^{in}\Upsilon] -$

$\beta^2 E_t\Lambda_{t,t+2}(1-\rho_{t+1})[s_{t+2}H_{x,t+2}-\rho^{in}_{t+2}\Upsilon]-\beta^3 E_t\Lambda_{t+1,t+3}(1-\rho_{t+1})(1-\rho_{t+2})[s_{t+3}H_{x,t+3}-\rho^{in}_{t+3}\Upsilon]+$
$\beta E_t\Lambda_{t+2,t+3}(1-\rho_{t+3})H_{t+3}(r)$

$\Rightarrow$

$H_t(r) = W^w_t(r) - [1+\beta E_t\Lambda_{t,t+1}(1-\rho_{t+1}) + \beta^2 E_t\Lambda_{t,t+2}(1-\rho_{t+1})(1-\rho_{t+2}) + ...]b$
$-\beta E_t\Lambda_{t,t+1}[s_{t+1}H_{x,t+1}-\rho^{in}_{t+1}\Upsilon]-\beta^2 E_t\Lambda_{t,t+2}(1-\rho_{t+1})[s_{t+2}H_{x,t+2}-\rho^{in}_{t+2}\Upsilon]-\beta^3 E_t\Lambda_{t+1,t+3}(1-$
$\rho_{t+1})(1-\rho_{t+2})[s_{t+3}H_{x,t+3}-\rho^{in}_{t+3}\Upsilon]$

## B.3 Expected surplus of the negotiating worker, $W^f_t(r)$

The expected lifetime wage payments of the bargaining firm at time $t$ can be written as $W^f_t(r) = E_t\sum_{s=0}^{\infty}\beta^s\Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)w_{t+s}(r)$

$\Rightarrow$

$W^f_t(r) = w_t(r) + \beta E_t\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r)w_{t+1}(r) + \beta^2 E_t\Lambda_{t,t+2}\frac{n_{t+2}}{n_t}(r)w_{t+2}(r) + ...$

$\Rightarrow$

$W^f_t(r) = w^*_t + \beta E_t\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r)[\varphi_w w^*_t + (1-\varphi_w)w^*_{t+1}] + \beta^2 E_t\Lambda_{t,t+2}\frac{n_{t+2}}{n_t}(r)[\varphi^2_w w^*_t + \varphi_w(1-$
$\varphi_w)w^*_{t+1} + (1-\varphi_w)w^*_{t+2}] + \beta^3 E_t\Lambda_{t,t+3}\frac{n_{t+3}}{n_t}(r)[\varphi^3_w w^*_t + \varphi^2_w(1-\varphi_w)w^*_{t+1} + \varphi_w(1-\varphi_w)w^*_{t+2} +$
$(1-\varphi_w)w^*_{t+3}] + ...$

$\Rightarrow$

$W^f_t(r) = E_t[1+\beta\varphi_w\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r)+(\beta\varphi_w)^2\Lambda_{t,t+2}\frac{n_{t+2}}{n_t}(r)+...]w^*_t+(1-\varphi_w)E_t\beta\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r)[1+$
$\beta\varphi_w\Lambda_{t+1,t+2}\frac{n_{t+2}}{n_{t+1}}(r)+...]w^*_{t+1}+(1-\varphi_w)E_t\beta^2\Lambda_{t,t+1}\frac{n_{t+2}}{n_t}(r)[1+\beta\varphi_w\Lambda_{t+1,t+2}\frac{n_{t+3}}{n_{t+2}}(r)+...]w^*_{t+2}+...$

If the discount factor for the firm is defined as

$\Sigma_t(r) = E_t\sum_{s=0}^{\infty}(\beta\varphi_w)^s\Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)$

$\Rightarrow$

$\Sigma_t(r) = 1+\beta\varphi_w E_t\Lambda_{t,t+1}\frac{n_{t+1}}{n_t}(r)\Sigma_{t+1}(r)$

Then

$W^f_t(r) = \Sigma_t(r)w^*_t + (1-\varphi_w)E_t\sum_{s=1}^{\infty}\beta^s\Lambda_{t,t+s}\frac{n_{t+s}}{n_t}(r)\Sigma_{t+s}(r)w^*_{t+s}$

## B.4 Expected surplus of the negotiating firm, $J_t(r)$

Having derived the expected wage revenues of a negotiating worker and expected wage payments of a negotiating firm, I can now derive the explicit functions of surpluses of firms and workers.

$J_t(r) = p_t^w f_{nt}(r) - w_t(r) + \beta E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(r)^2 - \rho_{t+1}^{in} \Upsilon\right] + \beta E_t \Lambda_{t,t+1} (1-\rho_{t+1}) J_{t+1}(r)$

Using the fact that $n_{t+1}(r) = (1-\rho_{t+1}) n_t(r) + q_{t+1} v_{t+1}(r) \Rightarrow (1-\rho_{t+1}) = \frac{n_{t+1}}{n_t}(r) - x_{t+1}(r)$ and $J_{t+1}(r) = \kappa x_{t+1}(r)$, I can rewrite the equation above as

$J_t(r) = p_t^w f_{nt}(r) - w_t(r) - \beta E_t \Lambda_{t,t+1} \left[\frac{\kappa}{2} x_{t+1}(r)^2 + \rho_{t+1}^{in} \Upsilon\right] + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) J_{t+1}(r)$

$\Rightarrow$

$J_{t+1}(r) = p_{t+1}^w f_{nt+1}(r) - w_{t+1}(r) - \beta E_t \Lambda_{t+1,t+2} \left[\frac{\kappa}{2} x_{t+2}(r)^2 + \rho_{t+2}^{in} \Upsilon\right] + \beta E_t \Lambda_{t+1,t+2} \frac{n_{t+2}}{n_{t+1}}(r) J_{t+2}(r)$

and so forth. Therefore,

$J_t(r) = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[p_{t+s}^w f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left[\frac{\kappa}{2} x_{t+s+1}^2 + \rho_{t+s+1}^{in} \Upsilon\right]\right]$

$- E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) w_{t+s}(r)$

$\Rightarrow$

$J_t(r) = E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[p_{t+s}^w f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left[\frac{\kappa}{2} x_{t+s+1}^2 + \rho_{t+s+1}^{in} \Upsilon\right]\right] - W_t^f(r)$

## B.5 Deriving the contract wage

The Nash bargaining is summarized by

$max \quad H_t(r)^\eta J_t(r)^{1-\eta}$

The first order condition will result in

$\eta \Delta_t H_t(r)^{\eta-1} J_t(r)^{1-\eta} = (1-\eta) \Sigma_t(r) H_t(r)^\eta J_t(r)^{-\eta}$

$\Rightarrow$

$\eta \Delta_t J_t(r) = (1-\eta) \Sigma_t(r) H_t(r)$

Letting $\chi_t(r) = \frac{\eta \Delta_t}{\eta \Delta_t + (1-\eta)\Sigma_t(r)}$ will provide $\chi_t(r) J_t(r) = (1-\chi_t(r)) H_t(r)$. Then

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1} [s_{t+1} H_{x,t+1} - \rho_{t+1}^{in} \Upsilon] + \beta E_t \Lambda_{t,t+1} (1-\rho_{t+1}) H_{t+1}(r)$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1} \left[s_{t+1} H_{x,t+1} - \rho_{t+1}^{in} \Upsilon\right] + \beta E_t \Lambda_{t,t+1} \left(\frac{n_{t+1}}{n_t}(r) - x_{t+1}(r)\right) H_{t+1}(r)$

$\Rightarrow$

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1} \left[ s_{t+1} H_{x,t+1} - \rho^{in}_{t+1} \Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x_{t+1}(r)^2 \right] +$

$\beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) H_{t+1}(r)$

Then,

$H_{t+1}(r) = w_{t+1}(r) - b - \beta E_t \Lambda_{t+1,t+2} \left[ s_{t+2} H_{x,t+2} - \rho^{in}_{t+2} \Upsilon + \frac{\chi_{t+2}(r)}{1-\chi_{t+2}(r)} \kappa x_{t+2}(r)^2 \right] +$

$\beta E_t \Lambda_{t+1,t+2} \frac{n_{t+2}}{n_{t+1}}(r) H_{t+2}(r)$

ans so forth. Thus,

$H_t(r) = W^f_t(r) -$

$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[ b + \beta \Lambda_{t+s,t+s+1} \left( s_{t+s+1} H_{x,t+s+1} - \rho^{in}_{t+s+1} \Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)} \kappa x_{t+s+1}(r)^2 \right) \right]$

I can derive the contract wage by substituting $H_t(r)$ and $J_t(r)$ into the first order condition of the Nash bargaining.

$\chi_t(r) J_t(r) = (1 - \chi_t(r)) H_t(r)$

$\Rightarrow$

$W^f_t(r) = \chi_t(r) E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r) \left[ p^w_{t+s} f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left( \frac{\kappa}{2} x_{t+s+1}(r)^2 + \rho^{in}_{t+s+1} \Upsilon \right) \right] +$

$[1 - \chi_t(r)] E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r)$

$\left[ b + \beta \Lambda_{t+s,t+s+1} \left( s_{t+s+1} H_{x,t+s+1} - \rho^{in}_{t+s+1} \Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)} \kappa x_{t+s+1}(r)^2 \right) \right]$

$\Rightarrow$

$\Sigma_t(r) w^*_t + (1 - \varphi_w) E_t \sum_{s=1}^{\infty} \beta^s \frac{n_{t+s}}{n_t}(r) \Sigma_{t+s}(r) w^*_{t+s} =$

$\chi_t(r) E_t \sum_{s=0}^{\infty} \beta^s \frac{n_{t+s}}{n_t}(r) \left[ p^w_{t+s} f_{nt+s}(r) - \beta \Lambda_{t+s,t+s+1} \left( \frac{\kappa}{2} x_{t+s+1}(r)^2 + \rho^{in}_{t+s+1} \Upsilon \right) \right]$

$+ [1 - \chi_t(r)] E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \frac{n_{t+s}}{n_t}(r)$

$\left[ b + \beta \Lambda_{t+s,t+s+1} \left( s_{t+s+1} H_{x,t+s+1} - \rho^{in}_{t+s+1} \Upsilon + \frac{\chi_{t+s+1}(r)}{1-\chi_{t+s+1}(r)} \kappa x_{t+s+1}(r)^2 \right) \right]$

$\Rightarrow$

$\Sigma_t(r) w^*_t = \chi_t(r) \left[ p^w_t f_{nt}(r) - \beta E_t \Lambda_{t,t+1} \left( \frac{\kappa}{2} x_{t+1}(r)^2 + \rho^{in}_{t+1} \Upsilon \right) \right]$

$+ (1 - \chi_t(r)) \left[ b + \beta E_t \Lambda_{t,t+1} \left( s_{t+1} H_{x,t+1} - \rho^{in}_{t+1} \Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x_{t+1}(r)^2 \right) \right] -$

$(1 - \varphi_w) \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1} + \beta E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1}$

$\Rightarrow$

$\Sigma_t(r) w^*_t = w^o_t(r) + \beta \varphi_w E_t \Lambda_{t,t+1} \frac{n_{t+1}}{n_t}(r) \Sigma_{t+1}(r) w^*_{t+1}$

where

$w^o_t(r) = \chi_t(r) \left[ p^w_t f_{nt}(r) - \beta E_t \Lambda_{t,t+1} \left( \frac{\kappa}{2} x_{t+1}(r)^2 + \rho^{in}_{t+1} \Upsilon \right) \right] +$

$(1 - \chi_t(r)) \left[ b + \beta E_t \Lambda_{t,t+1} \left( s_{t+1} H_{x,t+1} - \rho^{in}_{t+1} \Upsilon + \frac{\chi_{t+1}(r)}{1-\chi_{t+1}(r)} \kappa x_{t+1}(r)^2 \right) \right]$

## B.6 Dynamics of the intermediate market

The target wage can be log-linearized as the following

$\hat{w}_t^o(r) = \chi p^w f_n w^{-1}\left[\hat{p}_t^w + \hat{f}_{nt}(r)\right] + \left[p^w f_n - \beta\frac{\kappa}{2}x^2 - b - \beta s H_x - \beta\frac{\chi}{1-\chi}\kappa x^2\right]\chi w^{-1}\hat{\chi}_t(r) +$
$\left[(1-\chi)\beta s H_x - \beta\rho^{in}\Upsilon + \frac{\beta\chi\kappa x^2}{2}\right] w^{-1}E_t\hat{\Lambda}_{t,t+1} + \frac{\beta\chi\kappa x^2}{(1-\chi)}E_t\hat{\chi}_{t+1}(r) +$
$\beta\chi\kappa x^2 w^{-1}E_t\hat{x}_{t+1}(r) - \beta\rho_{in}\Upsilon w^{-1}E_t\hat{\rho}_{t+1}^{in} + (1-\chi)\beta s H_x w^{-1}E_t\left[\hat{s}_{t+1} + \hat{H}_{x,t+1}\right]$

Here I used the fact that $f(x_t) \approx f(x) + f'(x)(x_t - x) \approx f(x)(1 + \upsilon\hat{x}_t)$ where $\upsilon = \frac{\partial f(x)}{\partial x}\frac{x}{f(x)}$. Due to constant returns $\hat{f}_{nt}(r) = f_{nt}$ and $\hat{H}_{x,t} = \hat{H}_t$. At the steady state $H_x = H = \frac{\chi J}{1-\chi}$ and $J = \kappa x \Rightarrow H = \frac{\chi\kappa x}{1-\chi}$. Therefore, the equation above can be revised such that.

$\hat{w}_t^o(r) = \chi p^w f_n w^{-1}[\hat{p}_t^w + \hat{f}_{nt}] + \left[p^w f_n - b - \beta\frac{\kappa}{2}x^2 - (s+\rho)\beta H\right]\chi w^{-1}\hat{\chi}_t(r) +$
$\left[\left(s + \frac{\rho}{2}\right)\beta\chi\kappa x - \beta\rho^{in}\Upsilon\right] w^{-1}E_t\hat{\Lambda}_{t,t+1} + \beta\frac{\chi}{1-\chi}\kappa x^2 w^{-1}E_t\hat{\chi}_{t+1}(r) + \beta\chi\kappa x^2 w^{-1}E_t\hat{x}_{t+1}(r) -$
$\beta\rho^{in}\Upsilon w^{-1}E_t\hat{\rho}_{t+1}^{in} + \beta s\chi\kappa x w^{-1}E_t\left[\hat{s}_{t+1} + \hat{H}_{x,t+1}\right]$

To find the percentage deviation of the target wage from its steady state, I will follow the footsteps in the appendix to the first chapter and start with the hiring rate of a bargaining firm. It is given by

$\kappa x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t\Lambda_{t,t+1}\left[(1-\rho_{t+1})\kappa x_{t+1}(i) + \frac{\kappa}{2}x_{t+1}(i)^2 - \rho_{t+1}^{in}\Upsilon\right]$

$\Rightarrow$

$\hat{x}_t(r) = p^w f_n\epsilon\left[\hat{p}_t^w + \hat{f}_{nt}\right] - w\epsilon\hat{w}_t(r) + \left(1 - \frac{\rho}{2} - \rho^{in}\Upsilon\epsilon\right)\beta E_t\hat{\Lambda}_{t,t+1} + \beta E_t\hat{x}_{t+1}(r) - \rho\beta E_t\hat{\rho}_{t+1} -$
$\beta\rho^{in}\Upsilon\epsilon E_t\hat{\rho}_{t+1}^{in}$

where $\epsilon = (\kappa x)^{-1}$. At the steady state $n = (1-\rho)n + qv$ and $x = \frac{qv}{n}$, then $x = \rho$. Given the equation above, the difference between the hiring rate of a bargaining firm and the aggregate hiring rate will be given by

$\hat{x}_t(r) - \hat{x}_t = w\epsilon\left(\hat{w}_t - \hat{w}_t(r)\right) + \beta E_t(\hat{x}_{t+1}(r) - \hat{x}_{t+1})$

iterating this equation forward will provide us

$\hat{x}_t(r) - \hat{x}_t = w\epsilon(\hat{w}_t - \hat{w}_t(r)) + \beta w\epsilon E_t(\hat{w}_{t+1} - \hat{w}_{t+1}(r)) + \beta^2 w\epsilon E_t(\hat{w}_{t+2} - \hat{w}_{t+2}(r)) + ...$

I can simplify this equation by using the real wage index derived in the previous section by

$\hat{x}_t(r) = \hat{x}_t + w\Sigma\epsilon(\hat{w}_t - \hat{w}_t^*)$

where $\Sigma = \frac{1}{1-\beta\varphi_w}$. Then,

$\hat{x}_{t+1}(r) = \hat{x}_{t+1} + w\Sigma\epsilon(\hat{w}_{t+1} - \hat{w}_{t+1}^*)$

I can borrow the percentage deviations of the Nash bargaining weight, employment rate and the effective bargaining power from their steady states. Given these, I can derive the percentage deviaton of the bargaining firm's surplus from hiring another worker. That surplus is given by

$H_t(r) = w_t(r) - b - \beta E_t \Lambda_{t,t+1}[s_{t+1} H_{x,t+1} - \rho^{in}_{t+1} \Upsilon] + \beta E_t \Lambda_{t,t+1}(1 - \rho_{t+1}) H_{t+1}(r)$

Log-linearizing it around the steady state will yield

$\hat{H}_t(r) - \hat{H}_t = wH^{-1} [\hat{w}_t(r) - \hat{w}_t] + \beta(1 - \rho) E_t \left[\hat{H}_{t+1}(r) - \hat{H}_{t+1}\right]$

where $H^{-1} = \frac{1-\chi}{\chi \kappa x}$. Iterating the equation will give

$\hat{H}_t(r) = \hat{H}_t - w\Delta H^{-1} (\hat{w}_t - \hat{w}^*_t)$

By using the log-linearization of the Nash bargaining condition it can be driven that

$\hat{H}_t = \hat{x}_t + (1 - \chi)^{-1} \hat{\chi}_t + \left[(1 - \rho\Psi) w\Sigma\epsilon + w\Delta H^{-1}\right] (\hat{w}_t - \hat{w}^*_t)$

$\Rightarrow$

$E_t \hat{H}_{t+1} = E_t \hat{x}_{t+1} + (1 - \chi)^{-1} E_t \hat{\chi}_{t+1} + \left[(1 - \rho\Psi) w\Sigma\epsilon + w\Delta H^{-1}\right] E_t \left(\hat{w}_{t+1} - \hat{w}^*_{t+1}\right)$

Finally, the percentage deviation of the target wage from its steady state will be

$\hat{w}^o_t(r) = \varphi_{f_n} \left[\hat{p}^w_t + \hat{f}_{nt}\right] + \varphi_\chi \hat{\chi}_t + \varphi_\Lambda E_t \hat{\Lambda}_{t,t+1} + \frac{\varphi_x + \varphi_s}{1-\chi} E_t \hat{\chi}_{t+1} + [\varphi_x + \varphi_s] E_t \hat{x}_{t+1} - \beta \rho^{in} \Upsilon w^{-1} E_t \hat{\rho}^{in}_{t+1} +$
$\varphi_s E_t \hat{s}_{t+1} + \tau_1 (\hat{w}_t - \hat{w}^*_t) + \tau_2 E_t \left(\hat{w}_{t+1} - \hat{w}^*_{t+1}\right)$

where

$\varphi_{f_n} = \chi p^w f_n w^{-1}$, $\varphi_\chi = \left[p^w f_n - b - \beta \frac{\kappa}{2} x^2 - (s + \rho)\beta H\right] \chi w^{-1}$,

$\varphi_\Lambda = \left[\left(s + \frac{\rho}{2}\right) \chi\kappa x - \rho^{in} \Upsilon\right] \beta w^{-1}$, $\varphi_x = \beta\chi\kappa x^2 w^{-1}$, $\varphi_s = \beta s \chi \kappa x w^{-1}$

$\tau_1 = [\varphi_x \varphi_w - (1 - \chi)\varphi_\chi \rho\Psi] w\Sigma\epsilon$ and $\tau_2 = \left[(\varphi_x - \varphi_s) \rho\Psi\Sigma\epsilon + \varphi_s \left(\Sigma\epsilon + \Delta H^{-1}\right)\right] w$.

Given this equation, the percentage deviation of the bargained wage from its steady state will be

$\hat{\Sigma}_t + \hat{w}^*_t = \Sigma^{-1} \hat{w}^o_t(r) + \rho\beta\varphi_w E_t [\hat{x}_{t+1} - \hat{\rho}_{t+1}] + \beta\varphi_w E_t \left[\hat{\Lambda}_{t,t+1} + \hat{\Sigma}_{t+1} + \hat{w}^*_{t+1}\right]$

## B.7 Complete log-linearized model

All of the equations that are used to derive the impulse response functions are collected below.

$\hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t$

$\hat{q}_t = \hat{m}_t - \hat{v}_t$

$\hat{s}_t = \hat{m}_t - \hat{u}_t$

$$\hat{y}_t = \alpha\hat{k}_t - (1-\alpha)\hat{n}_t$$

$$\hat{x}_t = \hat{q}_t + \hat{v}_t - \hat{n}_{t-1}$$

$$\hat{n}_t = (1-\rho)\hat{n}_{t-1} + \rho\left(\hat{q}_t + \hat{v}_t - \hat{\rho}_t\right)$$

$$\hat{\rho}_t = \frac{\rho^{in}}{\rho_\rho}\hat{\rho}_t^{in}$$

$$\hat{\rho}_{t+1}^{in} = \rho_\rho\hat{\rho}_t^{in}$$

$$\hat{z}_t = \hat{p}_t^w + \hat{y}_t - \hat{k}_t$$

$$\hat{x}_t = p^w f_n \epsilon\left[\hat{p}_t^w + \hat{f}_{nt}\right] - w\epsilon\hat{w}_t + \left(1 - \frac{\rho}{2} - \rho^{in}\Upsilon\epsilon\right)\beta E_t\hat{\Lambda}_{t,t+1} + \beta E_t\hat{x}_{t+1} - \rho\beta E_t\hat{\rho}_{t+1} - \beta\rho^{in}\Upsilon\epsilon E_t\hat{\rho}_{t+1}^{in}$$

$$\hat{f}_{nt} = \hat{y}_t - \hat{n}_t$$

$$\hat{u}_t = -\frac{n}{u}\hat{n}_{t-1}$$

$$\hat{\Delta} = (1-\rho)\beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Delta}_{t+1}\right] - \rho\beta\varphi_w E_t\hat{\rho}_{t+1}$$

$$\hat{\Sigma}_t = \beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \rho\left(\hat{x}_{t+1} - \hat{\rho}_{t+1}\right) + \hat{\Sigma}_{t+1}\right]$$

$$\hat{\chi}_t = -(1-\chi)\left[\hat{\Sigma}_t - \hat{\Delta}_t\right]$$

$$\hat{\Sigma}_t + \hat{w}_t^* = \Sigma^{-1}\hat{w}_t^o(r) + \rho\beta\varphi_w E_t\left[\hat{x}_{t+1} - \hat{\rho}_{t+1}\right] + \beta\varphi_w E_t\left[\hat{\Lambda}_{t,t+1} + \hat{\Sigma}_{t+1} + \hat{w}_{t+1}^*\right]$$

$$\hat{w}_t^o(r) = \varphi_{f_n}\left[\hat{p}_t^w + \hat{f}_{nt}\right] + \varphi_\chi\hat{\chi}_t + \varphi_\Lambda E_t\hat{\Lambda}_{t,t+1} + \frac{\varphi_x + \varphi_s}{1-\chi}E_t\hat{\chi}_{t+1} + [\varphi_x + \varphi_s]E_t\hat{x}_{t+1} - \beta\rho^{in}\Upsilon w^{-1}E_t\hat{\rho}_{t+1}^{in} +$$
$$\varphi_s E_t\hat{s}_{t+1} + \tau_1\left(\hat{w}_t - \hat{w}_t^*\right) + \tau_2 E_t\left(\hat{w}_{t+1} - \hat{w}_{t+1}^*\right)$$

$$\hat{w}_t = (1-\varphi_w)\hat{w}_t^* + \varphi_w\hat{w}_{t-1}$$

$$\hat{\lambda}_t = E_t\left(\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\right) + \hat{r}_t^n$$

$$\hat{\lambda}_t = E_t\left(\hat{\lambda}_{t+1} + \beta z\hat{z}_{t+1}\right)$$

$$\hat{\lambda}_t = -\hat{c}_t$$

$$E_t\hat{\Lambda}_{t,t+1} = E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t$$

$$\hat{r}_t^n = E_t\hat{\pi}_{t+1} + (1-\beta)\hat{r}_t$$

$$\hat{r}_t^n = \rho_m\hat{r}_{t-1}^n + \gamma_\pi(1-\rho_m)E_t\hat{\pi}_{t+1} + \gamma_y(1-\rho_m)\hat{y}_t + \varepsilon_{t+1}^m$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{k}{y}\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] + \frac{\kappa x^2 n}{2y}\left[2\hat{x}_t + \hat{n}_t\right]$$

$$\hat{\pi}_t = \theta\hat{p}_t^w + \beta E_t\hat{\pi}_{t+1}$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t$$