
Eric S. Money

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Approved by:
Marc L. Serre
Gregory W. Characklis
Kenneth H. Reckhow
Martin W. Doyle
Lawrence E. Band
D. Derek Aday
The Clean Water Act requires that state and local agencies assess all river miles for potential impairments. However, due to the large number of river miles to be assessed, as well as budget and resource limitations, many states cannot feasibly meet this requirement. Therefore, there is a need for a framework that can accurately assess water quality at un-monitored locations, using limited data resources. Many researchers employ geostatistical techniques such as kriging and Bayesian Maximum Entropy (BME) to interpolate values in areas where no data exist. These techniques rely on the spatial and/or temporal autocorrelation between existing data points to estimate at un-monitored locations. This autocorrelation is traditionally a function of the Euclidean distance between those data points; however, a Euclidean distance does not take into account that many water quality variables may be spatially correlated due to the hydrogeography of the system.

The focus of this work is the development of a space/time geostatistical framework for estimating and mapping water quality along river networks by using river distances instead of the traditional Euclidean distance. The Bayesian Maximum Entropy method of modern space/time geostatistics is modified and extended to incorporate the use of river distances to improve the estimation of basin-wide water quality. This new framework, termed river-BME, uses geostatistical
models that integrate the use of permissible covariance functions with secondary information along with river distance. Factors, such as network complexity, are explored to determine the efficacy of using river-BME for water quality estimation. Additionally, simulation experiments and three real world case studies provide a broad application of this framework for a variety of basins and water quality parameters, including dissolved oxygen, *Escherichia coli*, and fish tissue mercury. Results show that the use of river-BME produces significantly more accurate estimates of water quality at un-monitored locations than traditional Euclidean based methods by more than 30%. Overall, this work provides a new tool for applying modern space/time geostatistics using river distances. It has the potential to aid not only future researchers but can ultimately provide environmental managers with the information necessary to better allocate resources and protect ecological and human health.
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Table of Contents

List of Tables ......................................................................................................................... xi

List of Figures ......................................................................................................................... xii

Chapter

I. Introduction ............................................................................................................................ 1


2.1. Introduction ......................................................................................................................... 6

2.2. The Bayesian Maximum Entropy Framework ................................................................... 6

2.2.1. The Stages of BME ...................................................................................................... 7

2.2.2. The General Knowledge Base ...................................................................................... 9

2.2.3. The Site-Specific Knowledge Base .............................................................................. 10

2.2.4. Bayesian Conditionalization ...................................................................................... 12

2.3. Distance Metrics ............................................................................................................... 13

2.4. Covariance Models Using River Distances .................................................................. 15

2.4.1. Isotropic River Covariance Models ........................................................................... 15

2.4.2. Flow-Weighted River Covariance Models .................................................................. 20

2.4.3. River Covariance Model Selection .......................................................................... 21

2.5. River Estimation and Mapping ...................................................................................... 23

2.5.1. Estimation Neighborhood Selection .......................................................................... 23
4.2.3. Space/Time Covariance Modeling Using River Distance ...... 56
4.2.4. Estimation of Dissolved Oxygen .............................................. 57
4.2.5. Assessment of Impaired River Miles ........................................ 59
4.3. Results and Discussion ................................................................. 60
4.3.1. Covariance of DO in New Jersey ............................................. 60
4.3.2. Euclidean vs. River Estimation .............................................. 63
4.3.3. River-BME Estimation of DO .................................................... 67
4.3.4. Impaired River Miles in the Raritan and Lower Delaware Basins ................................................................................. 68

V. Modern Space/Time Geostatistics Using River Distances: A Case Study of Turbidity and Escherichia coli ................................................................. 72

5.1. Introduction ................................................................................. 72
5.1.1. Fecal Indicator Bacteria in River Systems................................. 72
5.1.2. Autocorrelation in E.coli.............................................................. 74
5.1.3. Turbidity and E.coli ................................................................. 75
5.2. Materials and Methods ................................................................. 75
5.2.1. Data and Study Area ................................................................. 75
5.2.2. Generation of Soft Data ............................................................ 77
5.2.3. Integrating Hard and Soft Data .................................................. 78
5.2.4. Space/Time Covariance Modeling Using River Distance ....... 79
5.2.5. Comparing Euclidean and River Estimation ............................ 79
5.2.6. Estimation of E.coli ................................................................. 80
5.2.7. Assessment of Impaired River Miles ........................................ 81
5.3. Results and Discussion ................................................................. 82
5.3.1. Covariance Analysis ................................................................. 82
5.3.2. Cross-validation Analysis ...................................................... 84
5.3.3. Assessment of Fecal Contamination in the Raritan Basin ...... 85

VI. Modern Space/Time Geostatistics Using River Distances: A Case Study of Fish Tissue Mercury .......................................................... 89
6.1. Introduction .............................................................................. 89
    6.1.1. Mercury in the Environment ............................................. 89
    6.1.2. Autocorrelation of Fish Tissue Mercury ......................... 91
    6.1.3. Factors Influencing the Bioaccumulation of Mercury ....... 92

6.2. Materials and Methods .............................................................. 93
    6.2.1. Data and Study Area ......................................................... 93
    6.2.2. Generation of Soft Data from Multiple Sources ............. 95
    6.2.3. Integrating Hard and Soft Data ....................................... 97
    6.2.4. Space/Time Covariance Models that Use River Distances ... 98
    6.2.5. Comparing Euclidean and River Estimations ................. 99
    6.2.6. Estimation of Fish Tissue Hg ......................................... 100
    6.2.7. Assessment of Impaired River Miles ............................. 100

6.3. Results and Discussion ............................................................. 101
    6.3.1. Covariance Analysis ....................................................... 101
    6.3.2. Cross-validation Analysis .............................................. 103
    6.3.3. Assessment of Fish Tissue Mercury ............................ 105

VII. Concluding Remarks ............................................................. 110

Appendices .................................................................................. 114
Appendix A:  Proof of Permissibility for Exponential Covariance Models
Using River Distance.......................................................... 114

Appendix B:  Mathematical Summary of Flow-weighted Covariance
Models ................................................................................. 117

Appendix C:  Flow-additive Functions.................................................. 122

Appendix D:  Movies Depicting Water Quality Trends Using river-BME ..... 126

Works Cited .................................................................................. 128
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.</td>
<td>Summary of new and modified functions integrated into the river-BME Framework</td>
<td>36</td>
</tr>
<tr>
<td>4.1.</td>
<td>Water quality estimation studies using river covariance models</td>
<td>52</td>
</tr>
<tr>
<td>4.2.</td>
<td>Basic Statistics for monitored DO data (raw-mg/L) for the period January, 1990 – August, 2005 for the Raritan and Lower Delaware River basins in New Jersey</td>
<td>56</td>
</tr>
<tr>
<td>4.3.</td>
<td>Space/time covariance parameters for DO using a river metric</td>
<td>61</td>
</tr>
<tr>
<td>4.4.</td>
<td>Change in cross validation mean square error (MSE) for each basin. A negative change indicates a reduction in overall MSE (i.e. improvement) when using a river metric</td>
<td>64</td>
</tr>
<tr>
<td>4.5.</td>
<td>Seasonal Average Variation in Fraction (%) of River Miles More Likely than Not (MLTN) in Non-Attainment (probability of Violation &gt; 50%) for 2002</td>
<td>69</td>
</tr>
<tr>
<td>4.6.</td>
<td>Summer Fraction (%) of River Miles More Likely than Not (MLTN) in Non-Attainment (probability of Violation &gt; 50%) for the period 2000-2005 (Summer = Jul- Sep)</td>
<td>69</td>
</tr>
<tr>
<td>5.1.</td>
<td>E.coli space/time covariance model parameters</td>
<td>83</td>
</tr>
<tr>
<td>6.1.</td>
<td>Data summary for mercury and pH in the Cape Fear and Lumber Basins, 1990-2004</td>
<td>95</td>
</tr>
<tr>
<td>6.2.</td>
<td>Cross-validation scenarios for fish tissue Hg estimates using river-BME and Euclidean-BME</td>
<td>99</td>
</tr>
<tr>
<td>6.3.</td>
<td>FishHg space/time covariance model parameters</td>
<td>102</td>
</tr>
<tr>
<td>7.1.</td>
<td>Summary of river-BME estimation studies</td>
<td>111</td>
</tr>
</tbody>
</table>
List of Figures

Figure

2.1. The stages of Bayesian Maximum Entropy for space/time geostatistics

2.2. Euclidean distance (A) and river distance (B)

2.3. (Left) Directed tree river network with 5 stream reaches (numbered in circles), and showing point \( (l,i) \) on reach 4, and point \( (l',i') \) on reach 3. (Right) Range of the exponential-power river covariance parameters \( (\alpha,\beta) \) for which the covariance matrix constructed using 20 neighboring points in the Raritan river in New Jersey has a positive lowest eigenvalue, i.e. \( \min(\lambda) > 0 \)

2.4. Estimation neighborhood (squares) for an estimation location (circle) using Euclidean (left) and isotropic river (right) distances

2.5. Estimation grid in Euclidean-BME (left) and river-BME (right)

2.6. Mapping grid in Euclidean-BME (left) and river-BME (right)

3.1. river-BME algorithm for calculating isotropic river distance between pairs of points

3.2. Example of branching level designation for a river network

3.3. The Raritan Network in New Jersey. This network is used in the branching level, meandering ratio, and simulation tests that follow

3.4. Average Efficacy (Eq. 3.2) as a function of branching level in the Raritan basin, New Jersey, with positive standard deviations. Efficacy is defined as the % change in mapping accuracy

3.5. Efficacy (Eq. 3.2) as a function of individual reach meandering ratio in the Raritan Basin, New Jersey

3.6. Simulated data set (row A), estimated using Euclidean-BME (row B) and river-BME (row C). Panel 1 and 2 highlight two areas of distinction between estimates described in the text

3.7. Simulated ‘True’ values vs. estimated values using Euclidean-BME (A) and river-BME (B) on a synthetic stream reach
3.8. Simulated ‘true’ values vs. estimated values using Euclidean-BME (A) and river-BME (B) on a real river network configuration ....................... 49

4.1. Lower Delaware Basin (left) and Raritan Basin(right), with corresponding locations of monitoring stations with at least one measured DO value (circles) ........................................................................ 54

4.2. Space/time covariance of mean-trend removed DO in New Jersey’s Raritan and Lower Delaware River Basins shown as a function of distance \( r \) along the river network for a temporal lag of \( \tau = 0 \) (top plot) and as a function of \( \tau \) for \( r = 0 \) (bottom plot) with squares representing experimental covariance values and plain lines representing the covariance ........................................................................................................................................ 61

4.3. Zonal (a) and Parallel Reach effect (b) on the BME Estimation of DO Residual in the Upper & Lower Branch Raritan Basin on Dec 16, 2002 using a Euclidean metric (left) or a river metric (right). Squares are locations of monitoring stations for this time period and the solid lines indicate the WMA boundary........................................................................................................................................ 66

4.4. River-BME Estimation of dissolved oxygen on July 12, 2002 in the Lower Delaware Basin (left) and Raritan Basin (right). The circle indicates the basin outlet, and squares are locations of actual monitoring data available on July 12, 2002 ........................................................................................................................................ 68

5.1. Locations of at least one E.coli (large circle) and turbidity (small circle) measurement between 2000-2007 in the Raritan Basin, New Jersey ........................................................................................................................................ 76

5.2. Spatial (top) and temporal (bottom) covariance of log-E.coli in the Raritan Basin, New Jersey ........................................................................................................................................ 82

5.3. River-BME estimation of E.coli in the Raritan Basin, New Jersey on 8/16/02 (A), 2/14/03 (B), 5/14/03 (C), and 8/16/05 (D) ........................................................................................................................................ 85

5.4. Percentage of river miles in the Raritan Basin that have a 90% probability of violating the NJDEP standard for primary contact recreation over a 300 day period ........................................................................................................................................ 87

6.1. Lumber (Left) and Cape Fear (Right) basins in North Carolina with locations for fish tissue mercury (circles), pH (squares), and surface water mercury (triangles) ........................................................................................................................................ 94

6.2. Spatial (top) and temporal (bottom) covariance of log-FishHg in the Cape Fear and Lumber Basins, North Carolina ........................................................................................................................................ 101
6.3. river-BME Fish Tissue Mercury estimates (ppm) in the Cape Fear and Lumber Basins on July 23, 1995 (A); July 2, 1999 (B); June 26, 2000 (C); and May 13, 2004 (D). Squares indicate locations of actual fish tissue measurements ................................. 106

6.4. Percentage of river miles with fish tissue mercury median estimate exceeding Mercury Action Levels set by the FDA (top; 1.0ppm), North Carolina (middle; 0.4ppm), and the EPA (bottom; 0.3ppm) ..................... 107

C.1. Example of a river with 5 reaches, indicating for each reach $i$ the contributing watershed area $a_i$, within reach $i$, the total watershed area $A_i$ at the downstream end of reach $i$, and the corresponding flow additive function $\omega(i)$ ............................................................. 124
Chapter I: Introduction

The primary focus of this work is the application of river distances to the geostatistical estimation of water quality along river networks. A substantial portion of this research consists of the development of a river metric that can be incorporated into the Bayesian Maximum Entropy methodology for the spatiotemporal estimation and mapping of water quality. This is the first known attempt to fully implement a river metric into the spatiotemporal estimation of water quality for a series of parameters and across multiple basins. The overall hypothesis is that by accounting for the river connectedness between data points, the space/time estimation and mapping accuracy of basin-wide water quality can be improved significantly.

There have been several studies that attempt to characterize surface water quality using geostatistics. Many of these studies involve traditional kriging techniques, or other interpolation and regression based methods with a Euclidean distance (Rasmussen et al., 2005; Tortorelli and Pickup, 2006; Cressie et al., 2005; Peterson and Urquhart, 2006). Cressie et al. (2005) and Peterson and Urquhart (2006) consider the use of river distance but ultimately perform estimations using a Euclidean approach. These studies raise additional questions about the effect of using a river distance for water quality estimation. Therefore this research extends
previous work to compare geostatistical estimation of water quality using river and Euclidean distances in a space/time framework.

Recent developments in geostatistics have begun to address both the spatial and temporal variability as well (Stein 1986, Christakos 1992, Cressie 1993, Bogaert 1996, Kyriakidis and Journel 1999, Fuentes 2004, Kolovos et al. 2004, Akita et al. 2007). In the case of many water quality parameters, temporal variability plays a key role in understanding the overall impact on a basin-wide system. Spatiotemporal methods aim at rigorously modeling the spatial and temporal variability inherent in data so as to produce more accurate estimates and significantly reduce overall estimation error for a variety of environmental parameters at unmonitored space/time locations using the generally sparse monitoring data available.

One such method is the spatiotemporal Bayesian Maximum Entropy (BME) method (Christakos 1990, 2000; Serre et al. 1998, Serre and Christakos, 1999). This method has been successfully applied to a variety of environmental issues, including air quality (Christakos and Serre, 2000; Christakos et al. 2004; Wilson and Serre 2007), and epidemiology (Law et al. 2004, 2006). There have also been several interesting studies that involve the BME estimation of water quality (Serre et al. 2004, LoBuglio et al., 2007; Akita et al. 2007, Couillette et al., 2008). These studies have shown that by using space/time BME we can produce more accurate maps of water quality than those produced using a purely spatial analysis. In addition, the BME method can rigorously process both actual measurements (hard data) and measurements with some associated error (soft data), leading to more
accurate estimates than typical kriging methods that do not account for soft
information, as shown in several studies (Christakos and Serre, 2000; Lee, 2005;
Savelieva et al., 2005; Serre and Lee, 2006). These spatiotemporal studies use a
Euclidean metric because the water quality parameters considered so far had a
spatial distribution largely driven by processes (overland non point source pollution
and subsurface contamination, respectively) that are adequately described using
distances calculated across land. Akita et al. (2007) suggests, however, that for
other water quality parameters one should investigate whether a river metric is more
appropriate than the classical Euclidean measure.

There have been several recent studies regarding the use of non-Euclidean
distances and stream flow in water quality estimation, and the development of
the corresponding permissible covariance models (Ver Hoef, 2006; Cressie et al., 2006;
Peterson and Urquhart, 2006; Curriero, 2006; Bailly et al., 2006; Bernard-Michel and
Fouquet, 2006; Peterson et al., 2007). Ver Hoef (2006), Cressie et al. (2006), and
Peterson et al. (2006) demonstrate the use of flow-weighted covariance models
using nitrates, change in DO, and dissolved organic carbon (DOC), respectively.
What these studies share, is their restriction to the spatial domain and absence of
soft information. Cressie et al. (2006) and Peterson and Urquhart (2006) also
compared Euclidean and flow-weighted covariance models, and found that the
Euclidean model performed better. Ver Hoef et al. (2006) found a flow-weighted
covariance model was more accurate. Various types of covariance functions (i.e.
spherical, Mariah) were examined as well; however, as Cressie et al. (2006) and Ver
Hoef et al. (2006) point out, only exponential covariance models were assumed
permissible when using river distance based on eigenvalue calculations of the covariance matrix. However, an explicit mathematical proof of permissibility using river distance for any covariance function is not reflected in these studies. Therefore, this research provides a novel computational implementation of a space/time estimation framework that uses demonstrably permissible river distance and covariance for water quality applications involving both hard and soft information. In addition, three case studies are presented that are the first implementations of their kind using a river distance for space/time estimation of water quality. Hence, this research is an examination of modern space/time geostatistics using river distances.

The research is organized around three main themes. The first is the description of the space/time geostatistical framework for estimation of water quality parameters using river distances. The second is the numerical implementation of river based functions within this framework. Third is the application of this framework for real world water quality estimation and mapping.

Theme 1 is addressed in Chapter 2 and describes the Bayesian Maximum Entropy framework in detail and the methodology used in this study to conduct a comprehensive estimation and mapping of water quality using river distances. This chapter provides a review of potential covariance models that use river distances, with details regarding covariance permissibility and the ultimate selection of the river covariance models that will be used throughout the remainder of this work. Chapter 3 addresses theme 2, and describes the numerical implementation of river based functions into the BME framework that will provide a useful geostatistical
library for future researchers interested in river based water quality estimations, which will be referred to as river-BME throughout this work. This includes the creation of an efficient river distance algorithm, new functions, as well as modifications to existing functions which all have an effect on the way geostatistical calculations are performed. In addition the efficacy of using river distances is examined by determining the relationship between efficacy and network complexity, parameter choice, and data density. Finally, a series of simulation experiments are performed to test the numerical implementation of river-BME.

Chapters 4, 5, and 6 describe the implementation of this framework for real world water quality applications, looking at a wide variety of parameters across several types of basins. Chapter 4 examines dissolved oxygen in the Raritan and Lower Delaware basins in New Jersey, where all data are treated as hard data. Chapter 5 builds upon the work in Chapter 4, and is a study of fecal contamination in the Raritan basin, New Jersey using not only hard data for Eschericia coli (E.coli), but also incorporating secondary soft information in the form of turbidity measurements. Chapter 6 uses river-BME for the estimation of fish tissue mercury in the Cape Fear and Lumber basins in North Carolina, again looking at a combination of hard and soft data and the effect this has on estimation accuracy.

This work concludes in Chapter 7, with a major summary of the results of the three major application studies and a discussion of how the development and implementation of river-BME will provide a new and efficient framework for more accurately assessing water quality trends along river networks.
Chapter II: Modern Space/Time Geostatistics Using River Distances: The Conceptual Framework

2.1. Introduction

The field of geostatistics centers on the concept that points that are closer together in space (or time) exhibit more similar physical/chemical/biological characteristics than points that are farther apart. This concept, referred to as autocorrelation, is a central component to the overall methodology employed in this work. Because these autocorrelation functions are based on distance (both in space and time), the choice of distance measure, particularly in a spatial context, becomes extremely important. This chapter will introduce the major methodological and conceptual underpinnings of the river-BME framework, including a discussion on the traditional BME framework, types of distance metrics, covariance model selection and permissibility, and finally the estimation and mapping concepts central to working with river networks.

2.2. The Bayesian Maximum Entropy Framework

2.2.1. The Stages of BME
The BME method provides a rigorous mathematical framework to process a wide variety of knowledge bases. These Knowledge Bases characterize the space/time distribution and uncertainty in monitoring data available for various water quality parameters, and are used to obtain a complete stochastic description of these parameters at any unmonitored space/time point in terms of its posterior Probability Density Distribution (PDF).

The theory of space/time random fields (S/TRF) provides a powerful construct to represent the space/time variability and uncertainty associated with a water quality parameter. Let us consider a modeling approach where the water quality parameter can be modeled as (or transformed to) a homogeneous/stationary S/TRF $X(p)$, where $p=(s,t)$ denote a space/time point at spatial location $s=(s_1,s_2)$ and time $t$. The BME framework is used to process the general and site-specific knowledge bases about the S/TRF $X(p)$ and estimate its value at un-sampled locations. The general knowledge base characterizing the S/TRF $X(p)$ includes its constant mean and the homogeneous/stationary covariance between space/time points $p$ and $p'$, which can be expressed in terms of the spatial distance $d(s,s')$ between spatial locations $s$ and $s'$, and the time difference $\tau = |t-t'|$. This dissertation, as a result, explores the use of a river metric to obtain the distance $d(s,s')$. The mean of the BME posterior PDF is generally selected as our estimator of water quality at some estimation point, with the corresponding posterior variance describing the associated estimation uncertainty.

The framework used throughout this analysis is based on the BME framework as implemented in version 2.0b of the BMElib numerical library written using the
MATLAB R2000a programming platform and modified for river estimation (see Chapter III). The distribution of water quality across space and time is generally modeled as the sum of a non-random function \( m(p) \), and a homogeneous/stationary residual space/time random field (S/TRF) \( X(p) \) modeling the space/time variability and uncertainty associated with the difference between water quality parameter and the non-random function \( m(p) \). The non-random function \( m(p) \) may provide, for example, a model for the known spatial and temporal trends often seen in water quality variables. The site-specific knowledge includes both hard data (e.g. monitoring data measured without error) and soft data (i.e. data with associated measurement error). By way of summary, BME uses the maximization of a Shannon measure of information entropy and an operational Bayesian updating rule to process the general and site specific knowledge bases, and obtain the posterior PDF describing water quality concentration at any un-sampled point of the river network. The BME method for modern space/time geostatistics was introduced by Christakos (1990), and a detailed description of the conceptual underpinnings of the BME framework follows, while it's BMElib numerical implementation is described in Serre et al. (1998), Serre and Christakos (1999) and Christakos et al (2002).

In the special case where only hard data are considered (e.g. when measurement errors are small enough that they can be neglected), then the BME method yields the best estimators of linear geostatistics known as the simple, ordinary and universal kriging methods. The \textit{BMElib} package implements concepts of composite space/time analysis (i.e. composite space/time metrics and neighborhood search, non separable space/time covariance models, etc.) that result
in better geostatistical functions for linear space/time kriging than those provided by classical geostatistics software where time is included as merely another spatial dimension (Christakos et al., 2002; BMElib, 2008). Figure 2.1 summarizes the components of the traditional BME methodology. A more in depth look at each of these steps is presented in the following sections.

**Fig. 2.1: The Stages of Bayesian Maximum Entropy for Space/Time Geostatistics**

It should be noted that there are criticisms related to the use of BME in science applications. Many of these criticisms stem from the use of Bayes theorem in the integration step of the BME methodology. However, as described in Christakos (1990) and Christakos et al. (2002) the integration stage is a generalized use of Bayesian conditionalization and when using only hard data, results in kriging estimates similar to estimates in non-Bayesian approaches. The reader is referred to Christakos et al. (2002) for further information about these approaches and how BME allows for a comparison of these approaches.

### 2.2.2. The General Knowledge Base
As noted earlier, the BME framework is dependent upon the integration of knowledge bases (KB) to develop an accurate representation of the natural system under investigation. The general knowledge base, \( G-KB \), can be expressed in terms of general stochastic equations:

\[
\bar{h}_\alpha(p_{map}) = g_\alpha(x_{map}) \\
g_\alpha(x_{map}) = \int d\chi_{map} g_\alpha(\chi_{map}) f_G(\chi_{map})
\]

(2.1)

where \( g_\alpha \) and \( h_\alpha \) \((\alpha = 0,1,\ldots,N)\) are sets of known functions of \( \chi_{map} \) (values) and \( p_{map} \) (coordinates), and \( N \) is the number of moment equations considered. \( f_G \) refers to the pdf associated with the general knowledge with the left side of the equation representing the stochastic expectations of the fields involved. The \( g_\alpha \)'s are chosen such that the expectations, \( h_\alpha \), can be calculated from field data or other types of general knowledge (Christakos et al., 2002).

There are a variety of general knowledge bases that can be considered in the BME framework. These include statistical correlation functions (means, covariances, variograms, multiple-point moments, non-linear statistics, etc.) as well as scientific models (physical laws, biological theories, etc.). For this work, we derive our general knowledge base from statistical correlation functions, including the mean trend and covariance, as well as information gained from empirical relationships. These are described in detail in Chapters 4-6. The stage of the BME
analysis concerned with processing the G-KB is known as the prior (structural) stage, which will be described later.

2.2.3. The Site-Specific Knowledge Base

Unlike the general KB, the site-specific knowledge base, S-KB, consists of values measured at a specific location in space and time, and can be either hard or soft data. Hard data represent measurements obtained using methodologies or instrumentation that are considered accurate with an error that is either very small or can reasonably be ignored for the mapping analysis. The hard data available at a set of \( n \) points can be expressed as follows:

\[
\chi_{\text{hard}} = (\chi_1, \ldots, \chi_n)
\]  

(2.2)

Soft data, on the other hand, denote data that has been obtained from uncertain observations that can be expressed in terms of interval values, probabilistic statements, etc. As will be shown in Chapters 5 and 6, incorporating soft data can significantly increase the mapping accuracy of water quality at un-monitored locations when combined with existing hard data. Christakos and Serre (2000a,b) utilized both types of data when examining mortality and temperature, as well as particulate matter and showed improved maps when accounting for hard and soft data. With respect to water quality, LoBuglio et al. (2006) showed that using model predictions as soft data can improve the estimation of water quality. There are two types of soft data employed in this work. The first is interval soft data where
a value, $x_{soft}$, is known be between some upper and lower bound, $u_i$ and $l_i$, respectively (Eq. 2.3).

$$\text{Prob}[l_i \leq x_{soft} \leq u_i] = 1$$  \hspace{1cm} (2.3)

In addition to the interval type, probabilistic soft data can also be used to incorporate information provided by secondary variables used as proxies for the primary variable of interest (see Chapters 5, 6). This type of information can be expressed in terms of the probability that the random variable $x_{soft}$ representing water quality at some soft data point is less than a cutoff value $\chi_{soft}$. This results in a cumulative distribution function, $F_s$ constructed on the basis of the site-specific knowledge (Eq. 2.3).

$$F_s(\chi_{soft}) = \text{Prob}[x_{soft} \leq \chi_{soft}]$$  \hspace{1cm} (2.4)

This stage of the BME analysis concerned with organizing the site-specific knowledge into hard and soft data is referred to as the meta-prior (specificatory) stage.

### 2.2.4 Bayesian Conditionalization

The final stage of the BME analysis is referred to as the posterior or integration stage. During this stage of the analysis, given the site-specific knowledge available, the general knowledge based pdf, $f_G$, is updated by means of
a Bayesian conditionalization rule that leads to the BME posterior pdf for any mapping location, \( p_k \) as follows:

\[
f_K(p_k) = \frac{f_s(\chi_{soft}) f_G(\chi_{map})}{A}
\]  

(2.5)

where \( K \) is the total knowledge considered \((G-KB \cup S-KB)\), \( A = \int_\chi^\infty d\chi f_s(\chi_{soft}) f_G(\chi_{data}) \) is the normalization constant, and \( f_s \) is the pdf of site-specific data, dependent on the type used (see Eq. 2.2-2.4). As Christakos et al. (2002) notes, the BME approach offers a substantial improvement – compared to classical Bayesian conditionalization methods – by making sure that a physical connection has been taken into consideration at the \( G-KB \) stage. For a complete description of the Bayesian conditionalization approach in light of BME, as well as other approaches, the reader is referred to Christakos (2000) and Christakos et al. (2002). Once \( f_K \) is calculated, estimation maps are derived based typically on the mode or the mean of the posterior pdf.

### 2.3. Distance Metrics

As noted earlier, distance calculations are an essential component to a river-based geostatistical framework used to estimate water quality at un-monitored points. The way we calculate distances affects correlation functions (such as the covariance), as well as the selection of an estimation neighborhood. The term
A metric is used to describe a distance that meets the following criteria for spatial points $s, s', \text{and} \ s''$

$$d(s, s') \geq 0 \quad \text{(non-negativity)}$$  \hspace{2cm} (2.6)

$$d(s, s') = 0 \quad \text{if and only if} \quad s = s' \quad \text{(identity)}$$

$$d(s, s') = d(s', s) \quad \text{(symmetry)}$$

$$d(s, s') \leq d(s, s'') + d(s'', s') \quad \text{(triangle inequality)}$$

Euclidean distance and isotropic river distance (as described below) both meet the qualifications of a metric, therefore the term ‘metric’ and ‘distance’ are used interchangeably throughout this work.

There are a variety of distance measures to consider when dealing with water quality parameters along river networks. Figure 2.2 describes the types examined in this work.

![Fig. 2.2: Euclidean Distance (A) and River Distance (B)]
The first to consider is the Euclidean metric (Fig. 2.2a). This is the traditional distance metric used within the BME framework and other common geostatistical techniques. A Euclidean distance is best defined ‘as the crow flies’ or straight-line distance in any direction. The other distance to consider is the river distance (Fig. 2.2b), which corresponds to the shortest distance along the river between the two points of interests. We consider these two distances in the next section and examine the impact each may have on the existing space/time framework. One cannot simply substitute a non-Euclidean distance in the calculations of correlation functions, as this can lead to non-permissible covariance functions. Therefore, before river-BME can be established and tested, potential covariance models must be examined for permissibility using the river distance. Section 2.4 provides this examination as well as the details regarding the ultimate covariance function selection used in the application of river-BME to water quality estimation.

2.4. Covariance Models Using River Distances

2.4.1. Isotropic River Covariance Models

Consider the case of a river network that can be represented by a directed tree of river reaches with zero width. This representation is highly adequate for downstream combining stream networks with somewhat narrow reaches; however it is not highly adequate for wider water bodies such as connected estuaries or lakes (Curriero, 2006). The river network is made up of reaches connected at confluence nodes. Each river reach is identified by a unique index $i$ (Fig. 3), and we let $V$ be the
set of all river reach indexes; \( V = \{1, 2, \ldots, n\} \), where \( n \) is the total number of individual reaches. An \( i = 1 \) will denote by convention the downstream-most river reach. The downstream end of the downstream-most reach is the outlet of the river network. The longitudinal coordinate \( l \) of a point on the river network is defined as the length of the continuous line connecting the outlet to that point along the river network (by convention, negative \( l \) values represent fictitious locations downstream of the outlet).

A point \( r = (s, l, i) \) on the river network is uniquely identified by either its spatial coordinate \( s \); or its river coordinate \((l, i)\) identifying the longitudinal coordinate \( l \) and the reach index \( i \) where the point is located (see Fig. 2.3).

Fig. 2.3: (Left) Directed tree river network with 5 stream reaches (numbered in circles), and showing point \((l, i = 4)\) on reach 4, and point \((l', i' = 3)\) on reach 3. (Right) Range of the exponential-power river covariance parameters \((\alpha, \beta)\) for which the covariance matrix constructed using 20 neighboring points in the Raritan river in New Jersey has a positive lowest eigenvalue, i.e. \( \min(\lambda) > 0 \).

A non-negative real-valued function \( d(r, r') \) is a metric if it verifies the properties of a metric (Eq. 2.6) for all \( r, r', r'' \). We denote \( d_E(r, r') \) and \( d_R(r, r') \) as the Euclidean distance and river distance, respectively, as defined in the previous section. It can be easily shown that both the Euclidean and river distances verify the properties of a metric.
We let $X(r)$ be a random field representing the value taken by a water quality parameter $X$ at location $r$. The covariance between $X(r)$ and $X(r')$ is a real-valued function of $r$ and $r'$ that we denote as $\text{cov}(r,r')$. By isotropic river covariance models we refer to the class of permissible models that can be expressed as a function of the distance between the points $r$ and $r'$, i.e. $\text{cov}(r,r')=c(d(r,r'))$. It is well known (Christakos, 1992; Cressie, 1993, Stein, 1999) that permissible covariance functions must verify the positive definiteness condition, which for isotropic river covariance models can be expressed as

$$\sum_{k=1}^{n} \sum_{k'=1}^{n} q_k q_{k'} c(d(r_k, r_{k'})) \geq 0$$  \hspace{1cm} (2.7)

for all choices of $n$ river points $r_k$ and real numbers $q_k$, $k=1,\ldots,n$ (the above condition comes from the fact that $\text{var}(\sum_{k=1}^{n} q_k X(r_k)) = \sum_{k=1}^{n} \sum_{k'=1}^{n} q_k q_{k'} \text{cov}(r_k, r_{k'}) \geq 0$). Some covariance functions are known to be permissible when using the Euclidean distance, such as the following exponential power model (Stein, 1999; Curriero, 2006)

$$\text{cov}(r,r')=\exp\left(-\left(d_E(r,r')/a_r\right)^\beta\right), \quad 0<\beta\leq 2$$  \hspace{1cm} (2.8)

where $a_r$ is the covariance range. This model corresponds to the usual exponential and Gaussian models when $\beta=1$ and $\beta=2$, respectively. Other models (spherical, etc.) are also permissible using the Euclidean metric. However, as demonstrated in Curriero (2006), permissibility of a covariance function with the Euclidean distance
does not ensure permissibility with other distances, even if such distances verify the properties of a metric, therefore caution should be used when using covariance functions with the river distance.

Ver Hoef et al. (2006) propose an appealing method to construct permissible covariance functions for river networks. Using their approach, we define the random variable $X(l,i)$ at longitudinal coordinate $l$ along reach $i$ as the moving-average of a white noise random process $W(u,j)$ defined at longitudinal coordinate $u<l$ along reach $j$ downstream of reach $i$. Let $V_i(u)$ be the set of reaches at longitudinal coordinate $u$ that are flow-connected to reach $i$. By convention, if $u=+\infty$ we let $V_i(u)$ be the set of leaf reaches upstream of reach $i$, and if $u=-\infty$ we let $V_i(u)$ be the outlet reach. Note that if $u>l$ where $l$ is the longitudinal coordinate of a point on reach $i$, then $V_i(u)$ may contain more than one reach index. However, if $u<l$, then $V_i(u)=\{j\}$ is a singleton containing the index of the unique reach at longitudinal coordinate $u$ downstream of $i$. Using this notation $X(l,i)$ can be written as

$$X(l,i) = \int_{-\infty}^{l} du \ g(u-l) \ W(u,V_i(u))$$

(2.9)

where $g(u-l)$ is a moving average function defined on $R^1$. As indicated in Ver Hoef et al. (2006), by choosing a moving average function that is exponentially decaying away from 0, i.e. $g(h)=\sqrt{2} \exp(-|h|)$, the moving average construction leads to a valid covariance function of exponential type that is a function of the river distance, i.e.
\begin{equation}
\text{cov}(r,r') = \exp(-d_R(r,r'))
\end{equation}

An overview of how to obtain this result has already been provided by Ver Hoef et al. (2006) and Ver Hoef and Peterson (2008), therefore we only provide the detailed proof of this result in Appendix A. We note that while the exponential power model is valid for $0<\beta \leq 2$ for the Euclidean distance, that model has only be shown to be valid for the river distance when $\beta=1$.

The most appropriate distance for a given water quality parameter may be a combination of the Euclidean and river distances. We may therefore define a composite Euclidean-river distance as

\begin{equation}
d_\alpha(r,r') = \alpha d_R(r,r') + (\alpha-1)d_E(r,r') , \quad 0 \leq \alpha \leq 1
\end{equation}

which can easily be shown to verify the properties of a metric. Using $d_\alpha(r,r')$, we then propose the following isotropic exponential-power river covariance model

\begin{equation}
\text{cov}(r,r') = \exp\left(-\left(d_\alpha(r,r') / a_r\right)\right) , \quad 0 \leq \alpha \leq 1 \text{ and } 0<\beta \leq 2
\end{equation}

which has not been proposed in this form in earlier works. This covariance model is permissible for any directed tree river network for $(\alpha=0, \beta \in ]0,2])$ and $(\alpha=1, \beta=1)$. Additionally, for a particular river of interest, this covariance model may be valid for other values of $\alpha \in [0,1]$ and $\beta \in ]0,2]$, which can be verified numerically by checking that the lowest eigenvalue $\lambda$ of any covariance matrix used in the estimation of water
quality is non-negative. Fig. 3 depicts the range of \((\alpha, \beta)\) values for which the lowest
eigenvalue is positive, i.e. \(\min(\lambda)>0\), for 20 points randomly selected along an actual
river network. As can be seen from this figure, there is a large range of permissible
\((\alpha, \beta)\) values.

Hence a composite Euclidean-river distance has been developed that can be
used for a variety of water quality parameters. Using an isotropic exponential-power
river covariance model, it is shown that this model is permissible for any directed
tree river network for \((\alpha=0, \beta \in \mathbb{R})\) and \((\alpha=1, \beta=1)\), and provides a river-specific
numerical test to check whether the model is permissible using other choices of
\(\alpha \in [0,1]\) and \(\beta \in ]0,2]\).

2.4.2. Flow-weighted River Covariance Models

Another important class of permissible covariance models for directed tree
river networks are covariance functions that use flow and river distance (Ver Hoef et al., 2006; Cressie et al. 2006; Peterson et al., 2006; Peterson et al., 2007; Bernard-Michel and Fouquet, 2006, see Appendix B for mathematical details of their work
using a unified mathematical notation), which we refer to as flow-weighted
covariance models, and which can be written as

\[
\text{cov}(\mathbf{r}, \mathbf{r}') = \sqrt{\Omega(i, i')} \ c_1(d_R(\mathbf{r}, \mathbf{r}'))
\]  

where the real valued function \(c_1(.)\) can be any permissible covariance function in \(R^1\)
(e.g. such that it is the Fourier transform of a non-negative bounded function in \(R^1\),
Christakos, 1992), and $\Omega(i,i')$ is a real number between 0 and 1 expressing the amount of flow connection between reach $i$ and $i'$ such that $\Omega(i,i')=0$ if they are not flow-connected, $\Omega(i,i')=1$ if they are on the same reach, and $\sum_{i \in V_i(u)} \Omega(i,i') = 1$ for $u>l$.

The above flow-connected covariance model was first derived by Ver Hoef et al. (2006). Cressie et al. (2006) subsequently proposed that the flow connection between reach $i$ and an upstream reach $i'$ can be defined as $\Omega(i,i')=\Omega(i')/\Omega(i)$ where $\Omega(i)$ is a function that increases in the direction of flow. In that case, the property $\sum_{i \in V_i(u)} \Omega(i,i') = 1 \forall u>l$ is verified if and only if $\Omega(i)$ is a flow additive function, i.e. such that if two reaches $i'$ and $i''$ combine into reach $i$, then $\Omega(i') + \Omega(i'') = \Omega(i)$. As shown in Appendix C various additive functions can be used to obtain $\Omega(i)$, including flow discharges if these are available, watershed areas (Ver Hoef et al. 2006; Peterson and Urquhart, 2006; Peterson et al., 2007; Bernard-Michel and Fouquet, 2006), or simply an additive stream-order number (Cressie et al., 2006).

### 2.4.3. River Covariance Model Selection

Flow-weighted covariance models do not belong to the class of isotropic river covariance models because the flow connection term cannot be reduced to a function of the distance between points. Their obvious advantage is that they incorporate flow-connectivity in the model of autocorrelation. However, as noted by Peterson and Urquhart (2006), setting the covariance to zero when points are not flow-connected may be a hindrance if very few monitoring sites are flow-connected, because in that case the number of data points in the estimation neighborhood is
drastically reduced, leading to less informed estimation maps than those produced using an isotropic river covariance model. In addition, by assuming correlation between some points to be zero, the underlying assumption is that there is no across-land influence acting on the system. In the case of water quality, variables such as land use and precipitation may act on a river system in a more uniform manner, meaning that even though some points may not be connected by flow within a river network, they may still be jointly influenced by other basin wide variables. Hence purely flow-connected covariance models may not be appropriate. However; these models should be used when a large fraction of the monitoring samples are flow-connected, and when other across-land influences can be deemed negligible. Recent exciting work by Bailly et al. (2006) may allow us to extend the class of flow-connected covariance models to include models allowing some autocorrelation between points that are not flow-connected (with conditional independence to common downstream points). Therefore, the river-BME framework applied in this work is limited to exponential isotropic covariance models. This provides us with a computationally efficient methodology to compare river-BME with existing geostatistical methodologies using Euclidean distance. Equation 2.14 is an example space/time covariance function.

\[
c(r, \tau) = c_1 \delta(r) \delta(\tau) + c_2 \exp\left(-\frac{3r}{a_2}\right)\exp\left(-\frac{3\tau}{a_2}\right) + c_3 \exp\left(-\frac{3r}{a_3}\right)\exp\left(-\frac{3\tau}{a_3}\right)
\] (2.14)
where \( r \) is chosen to be either the Euclidean or river distance. This model consists of 3 structures where \( c_1 \ldots c_3 \) are calculated portions of the total variance and correspond to the coefficients of each structure (i.e. \( c_1 \) for structure 1, \( c_2 \) for structure 2...). The first term of each structure is the spatial component, while the second term relates to the temporal component of the covariance. The variables \( a_r \) and \( a_t \) are the spatial and temporal ranges for each structure. Other than the initial nugget, the spatial component of the remaining structures is exponential, which as shown above is permissible for any directed tree river network for the Euclidean and river distances, and therefore the overall model is permissible because it corresponds to nested space/time separable permissible covariance functions (Kolovos et al., 2004). The final component of the framework to develop consists of the estimation and mapping concepts for water quality variables across space and time and along river networks.

2.5. River Estimation and Mapping

2.5.1. Estimation Neighborhood Selection

As shown in the previous section, the way distance is measured influences the correlation models that serve as the basis for estimation of water quality along river networks. These models provide us with the information to estimate variables at un-monitored locations. In order to estimate at these locations, an estimation neighborhood must be established. It is from this neighborhood that site-specific knowledge is integrated with our general knowledge about the correlation between data points. Therefore, the selection of this neighborhood is very important to the
accuracy of the estimation at un-monitored locations. Figure 2.4 depicts the differences in neighborhood selection determined by the way distance is calculated. The left side of the figure shows how a Euclidean distance establishes the estimation neighborhood in Euclidean-BME by searching for data points within a specified distance along radii in all directions, with the center (circle) being the location where we would like to estimate a value. This leads to a circular neighborhood containing data points 2, 4, and 5. However, in the river-BME framework we use the river distance. The estimation neighborhood is restricted to the river network and the resulting data points used for the estimation at the un-monitored location are points 1 and 2. Based on the values of these data points, the estimation at the un-monitored location could be very different depending on the distance (Euclidean versus river) used.

![Fig. 2.4: Estimation neighborhood (squares) for an estimation location (circle) using Euclidean (left) and isotropic river (right) distances.](image)

The estimation locations within the S/TRF are generally established using a square grid of estimation points covering the study area of interest (Fig. 2.5 left). In the case of river networks, however, the estimation grid must consist of points that are associated with the river network itself (Fig. 2.5 right).
2.5.2. Mapping River Estimates

Once the estimates have been calculated, they are mapped to a surface depicting the spatial trends. In Euclidean-BME, this requires establishing a mapping grid consisting of equidistant points, generally at a finer resolution than the estimation grid and interpolating a colored surface across the spatial dimension (Fig. 2.6 left). In river-BME, a mapping grid must be established using points along the river network, along with a few outlying points (Fig. 2.6 right).
These outlying grid points are given the same value as the closest point on the river network and are used to establish the color gradient. Since the river network is generally represented by a line feature made up of individual points, establishing a color gradient is not feasible; however, by creating a small buffer around each individual reach, we can then ‘fill in’ this buffer with the appropriate value in order to visualize the river-BME estimates in their original context (See Chapters 4, 5, and 6 for example figures).

2.6. Summary

This chapter has been devoted to establishing the geostatistical concepts used in the application of river distances for the estimation and mapping of water quality. Figure 2.1 summarizes the components of the Euclidean-BME methodology. With the establishment of river-BME, these components do not change; however many of the conceptual underpinnings of this methodology, including the covariance modeling, the estimation, and the mapping procedures, have been refined to incorporate river distances. Now that the conceptual framework for river-BME has been developed, the next step is to numerically integrate these concepts into the existing BME framework, resulting in a new tool for researchers to compare traditional geostatistical methods using Euclidean distances, with those using a river distance. Chapter 3 discusses the numerical implementation of these river-based functions and how they are integrated into the traditional BME framework.
Chapter III: Modern Space/Time Geostatistics Using River Distances: Numerical Implementation

3.1. Introduction

This chapter describes, in detail, the numerical implementation of river based functions to create a new library of geostatistical tools for estimating water quality along river networks. This new library, river-BME, is an extension of the traditional BME framework outlined in the previous chapters. River-BME consists of all of the traditional BME methodologies plus new river based functions outlined below. A series of simulation tests are performed to examine the functionality and efficacy of using river-BME in the geostatistical estimation of water quality. These results provide the basis for using river-BME in the real world applications that follow in Chapters 4, 5, and 6.

3.2. The River Algorithm

There are a variety of tools that can calculate the distance between two points along a network. Hydrology tools exist within commercially available geographic information systems (GIS) such as ArcGIS and the Geographic Resources Analysis...
Support System (GRASS). However, the BME framework is currently implemented in the MATLAB programming language, containing specific functions that must work seamlessly with any new river based functions; therefore, to increase computational efficiency an algorithm was developed to calculate river distances between pairs of points within the BME library of functions. This algorithm, shown in Figure 3.1, relies on a measure of river complexity termed the branching level (BL) (see § 3.4) which is defined as a number starting at 1 for the most downstream reach of the network, and increasing by 1 going upstream each time a reach is divided into two upstream reaches.

**Fig. 3.1: river-BME algorithm for calculating isotropic river distances between pairs of points**
Because we are working with actual networks, obtaining a completely connected network of reaches is an important first step. These files are usually line shapefiles created for GIS platforms and can come from a variety of sources, including state and local government or the National Hydrography Dataset (NHD, 2008). The NHD can provide the user with a tremendous amount of information related to each river reach, including basin contributions, flow characteristics, etc. Pre-processing of any given river network occurs within ArcGIS, where these line shapefiles are converted to interchange files (.e00) for input into the BME framework within MATLAB.

Once in MATLAB, river reach segments are checked for downstream → upstream connectivity relative to the basin outlet, which is defined as the most downstream point of reach 1. All lines are re-organized into river reaches, which are defined as single continuous polylines connecting river reach segments that approximately delineate the centerline of a stream between its upstream and downstream confluence nodes. Each river reach is identified by a unique ID. These re-organized sets of unique river reaches are then saved as the “organized” networks to continue with the analysis.

The organized networks are then used to obtain the river topology for each basin using the branching level convention defined earlier. The topology file describing the reaches making up a river basin consists of four columns, which are (1) the unique reach ID; (2) the reach branching level; (3) the downstream reach ID; and (4) the reach length (i.e. the linear length of the reach).

Using this information each hard or soft data point is associated with the underlying river network by ‘snapping’ the data’s geographical location to the closest polyline
point making up one of the river network reaches. The resulting ‘river’ space/time coordinate of each point is stored in a file consisting of 5 columns, which are (1) the x spatial coordinate (e.g. Easting or longitude) of the point; (2) its y spatial coordinate (e.g. Northing or latitude); (3) the unique ID of the reach onto which the point was snapped; (4) the linear distance from the data point on that reach to its downstream node, and; (5) the time of measurement. Then following the steps of the algorithm presented in figure 3.1, a river distance is calculated between any set of points. Many of these steps required the development of new functions within the BME framework, as well as the modification of some existing functions to accept other types of distance measures. It is these functions that make up the new river-BME framework.

3.3. The river-BME Framework

3.3.1. Development of New River based Functions

The traditional BME functions reside in a numerical library referred to as BMElib. A complete description of BMElib and associated space/time functions are described in Christakos et al. (2002). River-BME, as noted, is an extension of this library to include a variety of new and modified functions for use with river distances. The riverlib directory within BMElib contains all of the new river based functions, while existing functions that were modified retain their same name, but include additional input parameters to specify the type of distance to be used in the analysis.

The first new function introduced takes as input an un-organized river network made up of (not necessarily connected) river segments, and re-organizes that
network into a set of connected stream reaches with an associated river topology.

The MATLAB syntax for this function, named `getRiverTopology.m`, is as follows:

```matlab
[riverReaches,riverTopology,infoval,infoMsg]=
getRiverTopology(riverReachesRaw,sRiverOutlet,distanceTolerance)
```

The first input (`riverReachesRaw`) consists of a cell array describing the un-organized river network, where each cell consists of the geographic coordinates that make up an individual river segment. This input has generally been pre-processed in ArcGIS and converted to an interchange file format (.e00); however the river segments may not correspond to whole stream reaches defined as the river reach between two stream confluence nodes. These are typically recorded in latitude/longitude decimal degrees. Therefore the number of cells is equal to the number of total un-organized river segments that make up the un-organized network. The second input (`sRiverOutlet`) is the geographic location of the river outlet (i.e. the most downstream point of the network). Finally, an optional distance tolerance (`distanceTolerance`) can be specified which will search the specified distance around reach endpoints to determine any issues with connectivity between reaches. If two endpoints are within the specified tolerance, then those points will be matched to connect the two reaches. This parameter is used to capture any small breaks in the continuity of the network that typically occurs in digitized line shapefiles. Caution should be used here since a large distance tolerance may connect two distant reaches that in reality are not connected. The output of this function should be plotted and matched against the original shapefile to spot any irregularities. The
outputs of this function include the set of organized river reaches (riverReaches) consisting of stream reaches made up of points oriented upstream→downstream, and such that river segments consisting of broken fragments between two confluence nodes are merged into a single stream reach, and the river topology (riverTopology) described in the previous section.

The next riverlib function converts the space/time coordinates of any set of points into river coordinates. The MATLAB syntax for this function, named ‘cartesian2riverProj.m’, is as follows:

```
[c1]=cartesian2riverProj(riverReaches,ch,pTolerance)
```

It should be noted here, that a space/time coordinate consists of the geographic location of a point and its time. This could be the time of a measurement for hard/soft data points, or the time of estimation for the estimation points making up a mapping grid. Points that need to be converted include any hard and soft data points, and the estimation points. The inputs for this function include the organized ‘riverReaches’ output from the ‘getRiverTopology.m’ function, the set ‘ch’ of space/time coordinates for the points to be converted, and a distance tolerance ‘pTolerance’. The distance specified is the maximum distance between seed points along the river network. Seed points are defined as equidistant points added to the organized river reaches in order to ‘snap’ points that might not be directly on the network. Locational mismatch is a common occurrence when trying to align points to line features. Therefore, the geographical location of the points (i.e. their longitude/latitude) are modified to equal the geographic location of the closest seed...
point on the river network, if the original data point does not initially fall on the given network and is known to be a part of that network. The output of this function is a set of river coordinates \(c_1\) with the same length as the original input set of points. As described in the previous section, these river coordinates contain the space/time location, reach ID, and length from each point to its respective downstream node.

Another function developed for the river-BME framework uses the outputs from the previous two functions to calculate the river distance between any pairs of points using the algorithm described in figure 3.1. The MATLAB syntax of this function, named ‘coord2distRiver.m’, is as follows:

\[
[rD] = \text{coord2distRiver}(c1, \text{riverTopology})
\]

The specific inputs to this function include the set of river coordinates ‘\(c_1\)’ from the ‘cartesian2riverProj.m’ function, and the ‘riverTopology’ obtained from the ‘getRiverTopology.m’ function. The output ‘\(rD\)’ is a river distance matrix with the distance calculated for all combinations of points.

Another function that uses the outputs from ‘getRiverTopology.m’ is the ‘getRiverStats.m’ function. The MATLAB syntax for this function, named ‘getRiverStats.m’ is as follows:

\[
[rS] = \text{getRiverStats}(\text{riverReaches}, \text{riverTopology}, \text{basinfile})
\]

The inputs include the organized river reaches (\text{riverReaches}) and river topology (\text{riverTopology}) variables generated by the topology function (\text{getRiverTopology.m}).
as well as a boundary file (*basinfile*) for the basin of interest. The output is a vector of five relevant statistics pertaining to the river network under investigation. These include the total number of organized reaches, total river network length, average meandering ratio (MR), maximum branching level (BL), and river ratio. MR, BL, and river ratio can all be considered measures of network complexity, and these concepts are discussed in detail in § 3.4.1.

The final group of new functions deals with the visualization aspect of riverBME. The ‘plotRiverNetwork.m’ function uses the organized river reaches and river topology as input and produces a map of the river network depicting the river topology depending on the scheme chosen by the user. A simple map of the river network can be created, or each reach can be labeled with its unique reach ID (scheme = 1), length (scheme = 2), downstream reach ID (scheme = 3), or branching level (scheme = 4). The syntax for this function is:

```
plotRiverNetwork(riverReaches, riverTopology, plotscheme)
```

The second visualization function, and final function in the *riverlib* directory, discretizes the river network into equidistant grid points for use as a set of estimation points. The inputs include the organized river reaches from ‘getRiverTopology.m’ and a user-specified separation distance which determines the distance between each estimation point along the river network. The result is a river grid which must then be inputted into the ‘cartesian2riverProj.m’ function to assign each river estimation point an appropriate set of river coordinates for use in the subsequent
estimation procedure. The syntax for this function, named
‘discretizeRiverNetwork.m’, is as follows:

\[ \text{[riverGrid]} = \text{discretizeRiverNetwork} (\text{riverReaches}, \text{discreteDistance}) \]

### 3.3.2. Modification of Existing BME Functions

In addition to the new *riverlib* functions, several existing BME *lib* functions were modified to incorporate different types of distance calculations. Functions that calculate the covariance, BME estimates, and neighborhood selection were all modified. They were generalized by adding a single input allowing the user to specify different types of distance algorithms. The current choices include the typical Euclidean distance (‘coord2dist’) and the new isotropic river distance (‘coord2distRiver’). A complete description of all existing BME *lib* functions can be found in Christakos et al. (2002). Table 3.1 summarizes both the new and modified functions that are now a part of river-BME.
Table 3.1: Summary of new and modified functions integrated into the river-BME framework

<table>
<thead>
<tr>
<th><strong>riverlib PROCESSING Functions</strong></th>
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<tbody>
<tr>
<td>getRiverTopology</td>
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<td>getRiverStats</td>
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<tr>
<th><strong>riverlib DISTANCE Functions</strong></th>
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<tbody>
<tr>
<td>coord2distRiver</td>
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<tr>
<td>cartesian2riverProj</td>
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<table>
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<tr>
<th><strong>riverlib VISUALIZATION Functions</strong></th>
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<tbody>
<tr>
<td>plotRiverNetwork</td>
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<tr>
<td>discretizeRiverNetwork</td>
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<table>
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<tr>
<th><strong>BMElib MODIFIED Functions</strong></th>
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<tbody>
<tr>
<td>pairsindex</td>
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<tr>
<td>coord2K</td>
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<td>coord2Kinterface</td>
</tr>
<tr>
<td>simuchol</td>
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<tr>
<td>stcov</td>
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<tr>
<td>neighbors</td>
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<tr>
<td>probaneighbors</td>
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<tr>
<td>BMEprobaMomentsXvalidation</td>
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<tr>
<td>BMEprobaMoments</td>
</tr>
</tbody>
</table>

### 3.4. Efficacy of river-BME

#### 3.4.1. Parameter Choice

There are a variety of factors that can influence whether the use of a river distance is appropriate when estimating water quality. These range from the type of water quality variable, the density of data points, to the complexity of the network. The first thing to consider when deciding between using a river distance as opposed
to more traditional Euclidean distance, is the actual variable under investigation. Water quality is inherently influenced by both in-stream processes and overland processes that affect the spatial and temporal distribution of these variables on a basin-wide scale. If it is determined through empirical evidence that a particular variable is primarily influenced by factors outside the constraints of a river network, then using a river distance may not be an accurate representation of the physical and chemical processes in the true system. For example, tetrachloroethylene (PCE) is a widely detected volatile organic compound in water systems. However, according to several studies, PCE contamination in surface waters is heavily influenced by leaching from groundwater, storm runoff, and other non-point sources (Lopes and Bender, 1998; Moran et al., 2002; Akita et al., 2008). These mechanisms are adequately characterized using Euclidean distances and may therefore not be constrained by the river network. In this case, a river distance may not improve the accuracy of water quality estimation and mapping. On the other hand, variables such as mercury in fish tissue are primarily restricted to the water column and therefore autocorrelation between fish samples is dependent on the network configuration. In this case, the use of a river distance may significantly improve overall estimation and mapping accuracy. In general, however, the physical, chemical, and biological factors that contribute to the true spatiotemporal trends of any given water quality parameter are complex. The system is affected by numerous inputs, both in-stream and overland, therefore a generalized framework that allows for multiple distance measures is an important contribution of this work.
3.4.2. Data Density

Another factor to consider when using river distances is the spatial versus temporal density of hard and soft data points available in the study area. If a particular study area has very few data points arranged at large distances from one another in space, then the autocorrelation functions and estimation maps may not be affected significantly by the choice of a distance measure. In addition, if those same points are temporally abundant (i.e. lots of measurements taken over time at few sparse spatial locations), the spatiotemporal estimation neighborhood will be more informed by (and therefore be biased towards) temporal neighbors rather than spatial neighbors, causing any spatial distance metric to be less influential in the BME estimation, and leading to maps that are not significantly different from one another. However, even if the distance measure has little effect in these situations, the use of the river-BME framework may still lead to overall better estimates in low data density situations. A study by Chaplot et al. (2005) examined the effects of spatial data density on the accuracy of various interpolation techniques on point height data for digital elevation model (DEM) generation and found that kriging estimates were more accurate for low data density areas. Other comparison studies have shown similar results (Dirks et al., 1998; Kravchenko, 2003). Kriging is considered a linear limiting case of the more general BME methodology (Christakos and Li, 1998).
3.4.3. Measures of Network Complexity

In addition to parameter choice and data density, another important consideration is the geographical complexity of the river network itself. There are numerous ways to classify basin complexity, including order, meandering ratio, and drainage density (Strahler, 1952; Julien, 2002; Gordon et al., 2004; Reis, 2006). This section focuses on a modification of order number we term branching level, and meandering ratio. A series of tests were performed examining the relationship between these measures of complexity and the efficacy of using the river distance developed in this work. For this purpose, the efficacy, \( E \), is defined as the percentage change in mean square error (MSE) between river-BME (MSE\(_r\)) and Euclidean-BME (MSE\(_e\)).

\[
E = \left(1 - \frac{MSE_r}{MSE_e}\right) \times 100 \quad \text{where} \quad (3.1)
\]

\[
MSE_{r/e} = \frac{1}{n} \sum_{i=1}^{n} (Y_{i,\text{true}} - Y_{i,\text{observed}})^2 \quad (3.2)
\]

where \( n \) is the total number of points, and \( Y_{i,\text{true}} - Y_{i,\text{observed}} \) is the difference between estimated values and measured values, or the estimation error. Therefore a positive value for \( E \) is equal to the same percentage increase in mapping accuracy. For example, if when comparing river estimates to Euclidean estimates, the MSE\(_r\) is 1.0 and the MSE\(_e\) is 2.0, then the MSE for river-BME is 50% smaller than that of Euclidean-BME, which we say is resulting in an increase in efficacy of 50%, or
stated another way, the estimation accuracy increased by 50% when using a river distance.

3.4.3.1. Branching Level

Branching level (BL) is similar to the ‘order number’ concept, with the modification that the most downstream reach is assigned a BL = 1. Each time a reach at a particular branching level divides into two reaches, the branching level increases by 1. Figure 3.2 shows an example branching level classification. Generally speaking, the higher the branching level, the more complex the network.

![Branching Level Diagram]

Figure 3.2: Example of branching level designation for a river network.

In order to test how branching level affects the efficacy of using a river distance over a Euclidean distance, an actual river network was obtained and simulated values were re-estimated using either river-BME or traditional BME. The network configuration for the Raritan Basin in New Jersey was obtained from the New Jersey
Department of Environmental Protection (NJDEP) to serve as an example (Figure 3.3).

![Figure 3.3: The Raritan Network in New Jersey. This network is used in the branching level, meandering ratio, and simulation tests that follow.](image)

The network was divided into various sub-networks with differing branching levels. There were a total of 32 different sub-networks, with BL = {1, 2, ..., 32}. Values were simulated using river distances on each sub-network individually and a cross-validation performed using a Euclidean distance, to determine a MSE for each. The cross-validation was repeated for the same simulated data using the river distance. In addition each simulation procedure was repeated 14 times, with the resulting average efficacy shown as a function of branching level in figure 3.4. The bars represent the standard deviation among the 14 repeated measurements at each branching level. The lower bounds are connected to provide a conservative estimate of efficacy.
Figure 3.4: Average Efficacy (Eq. 3.2) as a function of branching level in the Raritan basin, New Jersey, with positive standard deviations. Efficacy is defined as the % change in mapping accuracy.

Results of this cross-validation analysis suggest that at extremely low branching levels, one can expect little change in mapping accuracy when using a river metric. This is understandable since at very low branching levels the river network corresponds essentially to a single reach, and as a result there are little differences between the Euclidean distance and river distance between points. However, as the branching level increases, the efficacy tends to increase until it reaches a fairly constant plateau above a branching level of about 5. This cross validation analysis indicates that using river distances should result in estimates that are 10 to 45% more accurate than estimates obtained using Euclidean distances for water quality parameters dominated by in-river processes on river networks with a branching level greater than 5.
3.4.3.2. Meandering Ratio

Another way to examine network complexity is to calculate a meandering ratio (MR). MR is defined as the ratio of river length, $l_r$, to linear length, $l_s$.

$$MR = \frac{l_r}{l_s}$$

This measure can be taken at the individual reach level, but an average MR can also be calculated for a basin as an average of all MR’s for individual reaches, $i$, that make up the total network of $n$ reaches (Equation 3.4).

$$MR_{basin} = \frac{1}{n} \sum_{i=1}^{n} MR_i$$

An MR = 1 corresponds to a straight reach, with no meander while a ratio greater than 1 is considered sinuous or meandering (Gordon et al., 2004).

As with the branching level test, the Raritan Basin serves as our example network. A meandering ratio was calculated for each individual reach (Eq. 3.3). Values were simulated using river distances along the reach and cross-validation estimates were calculated. There were a total of 105 individual stream reaches in the Raritan Basin, with MR’s ranging from 1.0 up to 1.8. Each bar in figure 3.5 represents one of these reaches sorted by meandering ratio. As can be seen, a clear pattern emerges. As the meandering ratio increases, the efficacy of using the river distance also increases. Reaches with an MR less than ~ 1.10 show little to no
difference in estimation accuracy when using a river distance. Like with branching level, this can be expected because reaches with a low MR are essentially straight lines, therefore any distance calculation along the river would be equal to a Euclidean distance. However, as the MR increases, the mapping accuracy increases significantly, by as much as 62% in reaches with extremely high MRs.

![Graph showing efficacy as a function of meandering ratio](image)

**Figure 3.5: Efficacy (Eq. 3.2) as a function of individual reach meandering ratio in the Raritan Basin, New Jersey.**

From this analysis it is evident that many factors influence the efficacy of using a river metric in the geostatistical estimation of water quality. Parameter choice, data density, and network complexity all play a role in determining how well river-BME will perform over the traditional space/time framework. Each of these things should be considered when applying non-traditional distance metrics to estimation along river networks. The last phase of the numerical implementation of river-BME is to test its functionality in a series of simulation tests. The final section
in this chapter details the results of these tests and sets the stage for the application of river-BME to real world applications.

### 3.5. Using river-BME in Simulated Case Studies

#### 3.5.1. Case study using data simulated on a synthetic stream reach

In order to validate the use of a river metric within the river-BME framework, several simulation exercises were performed to gauge the effect a river distance may have on the estimation and mapping of water quality. The simulation experiments involved two scenarios. The first scenario involves simulated (i.e. geostatistically generated) data on a synthetic stream reach. The synthetic stream reach consists of an idealized sinusoidal curve with a high MR, which maximizes the potential effects of river distances. Given this synthetic reach, the data are then simulated using a geostatistical simulation algorithm that generates data having a prescribed (known) covariance function using river distances. This leads to the creation of a dataset of simulated (true) values, which can be separated into a training set (used as monitoring data) and a validation set (where the estimation will be performed using the training set). The monitoring data from the training set can be used with river-BME to model the covariance using both distance metrics, as well as for estimation at the points corresponding to the validation sets (where re-estimated values obtained using the training set can be compared with the “true” values in the validation set). Finally, cross-validation or validation statistics are calculated to compare mean square error results obtained using both river and Euclidean distance metrics.
Figure 3.6 depicts the estimation maps obtained using this simulation scenario. Row A shows the data that were geostatistically generated and represents the ‘truth’. The validation set selected from these simulated data consists of two rows of points, one along the top of the reach, the other along the bottom (within the horizontal bars in figure 3.6). The validation set values were then re-estimated using the remaining (training set) data points. The re-estimation obtained using Euclidean distances is shown in row B, while the re-estimation obtained using river distances is shown in row C.

![Figure 3.6: Simulated data set (row A), estimated using Euclidean-BME (row B) and river-BME (row C). Panel 1 and 2 highlight two areas of distinction between estimates described in the text.](image)

As can be seen in figure 3.6, the Euclidean estimation (row B) creates clouds of similar values that do not follow the pattern of the ‘true’ data shown on row A,
whereas the river (row C) estimated values follow more closely that pattern for this synthetic stream reach. Panels 1 and 2 in figure 3.6 highlight areas where using the river based estimation drastically outperforms the Euclidean-based estimation in reproducing the ‘true’ dataset.

Figure 3.7 shows a scatter plot of simulated (‘true’) values versus estimated values obtained using Euclidean -BME (panel A) or using river-BME (panel B). An apparent reduction in scatter is visible when comparing panel A to panel B, leading one to conclude that the river-BME framework produced more accurate estimates. This is verified by calculating the MSE using both approaches. The $\text{MSE}_e (\text{Euclidean}) = 0.4866$ while the $\text{MSE}_r = 0.2334$. One way to quantify the effect of using river distances versus Euclidean distances is to calculate the efficacy defined in Eq. (3.1). The efficacy calculated here indicates that the use of river-BME resulted in a 52% improvement in estimation accuracy over the classical Euclidean-BME approach.
3.5.2. Case study using data simulated on a real river network

The second simulation scenario examines the efficacy of river-BME using a simulated dataset generated on a real river network (instead of an idealized stream reach). The real river network used in this case study consists, again, in the New Jersey Raritan river network (figure 3.3). Simulated values were generated with a geostatistical simulation method using river distances. Then these simulated values were used in a cross validation analysis to obtain a MSE for both river-BME and Euclidean-BME. As explained earlier, the MSE is calculated based on cross validation errors consisting in the difference between each true value and the value re-estimated based on its neighboring data. Figure 3.8 depicts the relative scatter between true values and re-estimated values obtained using Euclidean-BME (panel A) and river-BME (panel B). The MSE obtained using the data simulated on the Raritan river basin were similar to those obtained in the previous section, with an
MSE\textsubscript{e} = 0.3030 and MSE\textsubscript{r} = 0.1624. This corresponds to a 46% improvement in estimation accuracy (Eq. 3.1) when using river distances instead of Euclidean distances.

Figure 3.8: Simulated ‘true’ values vs. estimated values using Euclidean-BME (A) and river-BME (B) on a real river network configuration.

Overall these results suggest that the use of a river distance can markedly improve the estimation accuracy of water quality parameters that are governed by in-stream processes. However, there are a number of factors that affect the efficacy of using river-BME instead of Euclidean-BME, and simulated data and synthetic river networks can only provide a partial examination of the efficacy of river-BME. Therefore the following chapters provide three real world case studies that examine the river-BME methodology as it relates to various water quality parameters and in several real-world river networks. These case studies will provide a more complete understanding of the differences between river-BME and
Euclidean-BME, and will establish the methodology for use in other real world situations.
4.1. Introduction

Understanding surface water quality is a critical step towards protecting human health and ecological stability. Because of resource deficiencies and the large number of river miles needing assessment, there is a need for a methodology that can accurately depict river water quality where data do not exist. The objective of this research is to implement such a methodology that incorporates a river metric into the space/time analysis of dissolved oxygen data for two impaired river basins using the river-BME framework describes in the previous chapters. We find that using a river distance in a space/time context leads to an appreciable 10% reduction in the overall estimation error, and results in maps of DO that are more realistic than those obtained using an Euclidean distance. As a result river distance is used in the subsequent non-attainment assessment of DO for two impaired river basins in New Jersey.

The identification of impaired river segments is a significant requirement of the federally implemented Clean Water Act (CWA) of 1972. The CWA requires states to assess water quality and identify and report those segments that are impaired for particular uses. Dissolved Oxygen (DO) content is one of the easiest and most basic water quality parameters to measure and is a good indicator of
overall stream health. Because of resource deficiencies, budget constraints, and the sheer number of river miles to be assessed, there is a need for cost efficient and effective methods that can estimate DO for a large number of river miles using limited monitoring data.

As mentioned previously, there have been several recent studies regarding the use of non-Euclidean distances and stream flow in water quality estimation, and the development of corresponding permissible covariance models (Ver Hoef, 2006; Cressie et al., 2006; Peterson and Urquhart, 2006; Curriero, 2006; Bailly et al., 2006; Bernard-Michel and Fouquet, 2006; Peterson et al., 2007). Ver Hoef (2006), Cressie et al. (2006), and Peterson et al. (2006) demonstrate the use of flow-weighted covariance models using nitrates, change in DO, and dissolved organic carbon (DOC), respectively.

A summary of the most recent studies that compare Euclidean and river covariance models is presented in Table 4.1.

<table>
<thead>
<tr>
<th>Study</th>
<th>Water Quality Parameter</th>
<th>Comparison</th>
<th>% Change in MSE¹</th>
<th>Model used in Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cressie et al. (2006)</td>
<td>Change in DO</td>
<td>Euclidean vs. flow-weighted</td>
<td>Not Reported</td>
<td>Euclidean</td>
</tr>
<tr>
<td>Peterson &amp; Urquhart (2006)</td>
<td>DOC</td>
<td>Euclidean vs. flow-weighted</td>
<td>Exponential Cov.: + 31.3%</td>
<td>Euclidean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Spherical Cov.: - 9.0%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mariah Cov.: + 32.4%</td>
<td></td>
</tr>
<tr>
<td>Ver Hoef et al. (2006)</td>
<td>Sulfate</td>
<td>Isotropic river vs. flow-weighted</td>
<td>Exponential Cov with Constant Mean.: - 5.5%</td>
<td>Flow-Weighted</td>
</tr>
</tbody>
</table>

¹ % change in Mean Square Error (MSE). A negative value means that using a flow-weighted covariance model reduces prediction error.

The methods proposed in this work are based on geostatistical principle, most notably the spatial autocorrelation between data points. They are not meant to take
the place of mechanistic and process-based models such as the traditional Streeter-Phelps or the Qual2 models developed by EPA. Geostatistical models can complement these existing methods by taking the outputs of these models and using them as inputs into a geostatistical framework to create larger spatial and temporal coverages of the parameter of interest, possibly leading to more accurate maps (LoBuglio et al. 2007). This study attempts to look at only geostatistical models in order to gain an understanding of the influences that distance measures have on our ability to assess rivers for DO impairments. Future work will examine the use of these models in combination with other mechanistic modeling approaches.

While the majority of studies have focused on purely spatial estimation methods, this research will examine the use of a river metric in a composite space/time analysis. Since very few studies have used a river metric to examine DO in a spatial context, and even fewer have done such analysis in a space/time context, the two objectives of this study are (1) to determine whether the use of a river metric provides a better model for estimation of DO along a river network in a space/time context, and (2) to apply the most appropriate space/time model to estimate DO non-attainment for two impaired river basins based.

4.2. Materials and Methods

4.2.1. Study Area

The two study watersheds are shown in Fig. 4.1 together with the locations of stations that monitored DO at least once during the study period.
Both areas are high priority basins for the state and have impairments related to nutrients, sediments, micro-organisms, and DO. The state of New Jersey is divided into 20 watershed management areas (WMA). The Raritan consists of three WMAs, the North and South Branch, Millstone, and Lower Raritan. The land uses in both basins are primarily urban or agricultural. Overall, the Raritan is 36% urban, 19% agriculture, with the remaining divided between forest, wetland, and water. The Lower Delaware is 46% urban and 21% agricultural. These classifications are based on the 1995/97 Land Use/Land Cover designations by the State of New Jersey.

New Jersey has a generally moderate climate with cold winters and warm, humid summers. These fluctuations in temperature play an important role in determining the amount of available DO found in these basins. Additionally, both the Lower Delaware and Raritan basins are geologically structured such that highlands situated to the west (Raritan) and east (Lower Delaware) feed into flat, highly developed
areas near the basin outlets, where impervious surfaces exceed 50% in many places (NJDEP, 2002). Urban development taking place in both of these basins over the last two decades coupled with relatively little change in agricultural uses produces a wide array of point and non-point sources in both regions. This leads to increased nutrient levels from waste water discharge, urban runoff, and agricultural runoff, and the potential for higher biological oxygen demand (BOD) and reduced DO levels. According to the 2006 integrated water quality report, only 5% of statewide impairments were due to dissolved oxygen, however, greater than 30% of river miles went un-assessed due to insufficient data (NJDEP, 2006b). This is where methods such as the one employed in this study become increasingly important.

4.2.2. Dissolved Oxygen Data

DO data were obtained from two sources for the period beginning January 1, 1990 through August 1, 2005. The first source is the U.S. Geological Survey (USGS) National Water Information System (NWIS). The second source is the USEPA storage and retrieval (STORET) database. Often times these databases report values with clarifying symbols accompanying them to signify uncertainty in the measurement. Therefore, in order to use these values in the analysis, any value reported as ‘less than’ a particular value (i.e. containing a ‘<’ in the database) were treated as equal to 50% of that value, and values reported as estimated (i.e. containing an ‘E’ in the database) were treated as actual values. A summary of the data are given in Table 4.2.
Table 4.2: Basic Statistics for monitored DO data (raw-mg/L) for the period January, 1990 – August, 2005 for the Raritan and Lower Delaware river basins in New Jersey

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Raritan Basin</th>
<th>Lower Delaware Basin</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Space/Time Data Points</td>
<td>1755</td>
<td>1855</td>
</tr>
<tr>
<td># of Monitoring Stations</td>
<td>65</td>
<td>47</td>
</tr>
<tr>
<td>Mean (mg/L)</td>
<td>10.471</td>
<td>7.859</td>
</tr>
<tr>
<td>Variance (mg/L)</td>
<td>7.061</td>
<td>5.359</td>
</tr>
<tr>
<td>Skewness Coefficient</td>
<td>-0.006</td>
<td>0.306</td>
</tr>
<tr>
<td>Kurtosis Coefficient</td>
<td>2.462</td>
<td>2.437</td>
</tr>
</tbody>
</table>

4.2.3. Space/Time Covariance Modeling Using River Distance

As noted in § 2.4 the exponential power model for covariances using isotropic river distances has been proven permissible. Therefore, for this study and the subsequent case studies, we are restricted to this class of covariance functions. The flow-weighted covariance function is not considered here because very few points during a given day of the study period are flow-connected, and as Peterson and Urquhart (2006) suggest, using a flow-weighted covariance in this situation can lead to less informative maps. This analysis uses a space/time random field (S/TRF) \( X(\mathbf{p}) \), where \( \mathbf{p}=(r,t) \) is a space/time point, \( r \) is the spatial river coordinate and \( t \) is time. The covariance \( c_x(\mathbf{p},\mathbf{p}') \) of \( X(\mathbf{p}) \) is said to be spatially isotropic/temporally homogeneous if it can be expressed in terms of the spatial distance \( r=d(r,r') \) and the time difference \( \tau=|t-t'| \). Experimental values of the covariance for a spatial distance \( r \) and temporal lag \( \tau \) are obtained using a covariance statistical estimator on pairs of \( X \) measurements approximately separated by the spatial distance \( r \), and temporal lag \( \tau \). The parameters of a covariance model are then adjusted until a best fit is found between the model and experimental covariance values. The covariance model used in this analysis is given by:
\begin{equation}
    c(r, \tau) = c_1 \delta(r) \delta(\tau) + c_2 \exp\left(\frac{-3r}{a_{r_2}}\right) \exp\left(\frac{-3\tau}{a_{\tau_2}}\right) + \ldots
    \\
    + c_3 \exp\left(\frac{-3r}{a_{r_3}}\right) \cos\left(\frac{2\pi \tau}{a_{\tau_3}}\right) + c_4 \exp\left(\frac{-3r}{a_{r_4}}\right) \exp\left(\frac{-3\tau}{a_{\tau_4}}\right)
\end{equation}

where \( r \) is chosen to be either the Euclidean or river distance and \( \delta \) is the nugget coefficient in space or time. This model consists of 4 structures where \( c_1 \ldots c_4 \) are calculated portions of the total variance and correspond to the coefficients of each structure (i.e. \( c_1 \) for structure 1, \( c_2 \) for structure 2...). The first term of each structure is the spatial component, while the second term relates to the temporal component of the covariance. The variables \( a_r \) and \( a_t \) are the spatial and temporal ranges for each structure. Other than the initial nugget, the spatial component of the remaining structures is exponential, which as shown above is permissible for any directed tree river network for the Euclidean and river distances, and therefore the overall model is permissible because it corresponds to nested space/time separable permissible covariance functions (Kolovos et al., 2004). The temporal component is exponential for structures 2 and 4, while structure 3 is a cosinusoidal function related to the seasonal fluctuations often associated with DO. Further covariance details are found in section 4.3.1.

### 4.2.4. Estimation of Dissolved Oxygen

The river-BME and Euclidean-BME methods were used to estimate DO at unsampled river locations. BME provides a rigorous mathematical framework to
process a wide variety of knowledge bases characterizing the space/time distribution and monitoring data available for DO, and obtain a complete stochastic description of DO at any unmonitored space/time point in terms of its posterior Probability Distribution Function (PDF), as discussed in Chapter 2.

The distribution of DO across space and time is modeled as the sum of a non-random function $m_{DO}(p)$ and an isotropic/stationary residual S/TRF $X(p)$. The spatial and temporal components of $m_{DO}(p)$ were obtained by exponential smoothing of the time-averaged and spatially-averaged data, respectively. The non-random function $m_{DO}(p)$ describes the modeled spatial and temporal trends of DO, while the S/TRF $X(p)$ captures the residual space/time variability and uncertainties.

The site specific knowledge includes both hard data (e.g. measured value) and soft data (i.e. data with associated measurement error). By way of summary, BME uses the maximization of a Shannon measure of information entropy and an operational Bayesian updating rule to process the general and site specific knowledge bases, and obtain the posterior PDF describing the DO concentration at any un-sampled point of the river network (Christakos et al., 2002).

This research uses the special case where only hard data are considered (i.e. the measurement errors are small or unidentified). In this case the BME method yields the estimators of linear geostatistics known as the simple, ordinary and universal kriging methods. This research, therefore, is based on a form of space/time linear kriging.

In order to determine which of the Euclidean or river metrics was more accurate for the assessment of DO in the study basins, a cross-validation procedure
was used. Each data point was removed sequentially and re-estimated using the remaining space/time data points. The Mean Square Error (MSE) is calculated as the sum of the squared differences between re-estimated and measured values. The method with the lowest MSE is then used in the assessment of DO along unmonitored rivers.

Using the selected distance metric within the river-BME framework we estimate DO at equidistant estimation points (i.e. distributed at a fixed interval of 0.1 miles) along the Raritan and Lower Delaware river networks. Monitoring data are treated as hard data because all measurements met the USGS or EPA data quality standards. For each estimation point the hard data situated in its local space/time neighborhood is selected, and the corresponding BME posterior PDF is calculated to describe DO at that estimation point. The BME posterior PDF obtained at equidistant points along the river network are then used to obtain estimated DO values, which are used to produce maps of DO concentration, and delineate river miles that may be impaired.

4.2.5. Assessment of Impaired River Miles

In order to better understand the seasonal pattern of DO impairment and better quantify the probability of these impairments, a criterion-based space/time assessment framework is employed to categorize the fraction of river miles meeting certain probability thresholds, as discussed in Akita et al. (2007). These thresholds give us the ability to classify the probability of violation of a standard for any space/time estimation point based on its BME posterior PDF. The standard for DO
concentration was set at 7 mg/L, which is the standard used by NJDEP for FW-TP streams (NJDEP, 2006a). Using this standard, the probability of violation at space/time point \( p \) is then defined as the probability that the BME mean estimate is > 7 mg/L, i.e.

\[
\text{Prob}[\text{Violation}, p] = \text{Prob}[\text{DO}(p) > 7.0 \text{mg/L}] \quad (4.2)
\]

The fraction of river miles impaired during any given time period is calculated using the fraction of equidistant estimation points for which the probability of violation is in excess of some pre-selected probability threshold.

4.3. Results & Discussion

4.3.1. Covariance of DO in New Jersey

Fig. 4.2 shows the experimental covariance values (squares) obtained using the mean trend removed DO data for the Raritan and Lower Delaware River Basins. These estimates were then used to fit the non separable space/time covariance model (Eq. 4.1). The sills \( c_1, ..., c_4 \), spatial ranges \( a_{r2}, a_{r3}, a_{r4} \), and temporal ranges \( a_{t2}, a_{t3}, a_{t4} \) obtained are listed in Table 4.3, and the resulting model is shown as a solid line in Fig. 4.2.
Figure 4.2: Space/time covariance of mean-trend removed DO in New Jersey’s Raritan and Lower Delaware River Basins shown as a function of distance $r$ along the river network for a temporal lag of $\tau=0$ (top plot) and as a function of $\tau$ for $r=0$ (bottom plot) with squares representing experimental covariance values and plain lines representing the covariance.

Table 4.3: Space/time covariance parameters for DO using a river metric.

<table>
<thead>
<tr>
<th>Covariance structure</th>
<th>Sill $c$ (mg/L)$^2$</th>
<th>Spatial Range $a_r$ (km)</th>
<th>Temporal Range $a_t$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4385</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>2</td>
<td>0.8770</td>
<td>5.0</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>0.0877</td>
<td>2.2</td>
<td>365</td>
</tr>
<tr>
<td>4</td>
<td>0.3508</td>
<td>88.9</td>
<td>10,000</td>
</tr>
</tbody>
</table>

The variance of the first structure of the covariance model, or the nugget effect, is about 25% of the overall variance. The nugget effect typically consists of the variance due to inherent variability of DO over very short distances plus the measurement-error variance. In our case, we assessed that for our dataset, the measurement-error variance for DO contributes at most 20% of the total variation in
the data, which is within the upper bound indicated by the nugget effect. The second structure of the covariance model contains a short range exponential spatial component and a short range exponential temporal component. Fluctuations of DO over this combination of short spatial ranges (5 km) and short temporal ranges (25 days) may be due to local sources of pollution acting over short spatial distances (such as point pollution discharges leading to local increase of BOD and subsequent reductions in DO over short distances) that either have intermittent pollution discharge loading lasting just a few days, or are persistent but have an effect that is altered intermittently by meteorology events lasting from a few days to a month (e.g. rainfall events, or changes in temperature which significantly effects the oxygen saturation of water). This accounts for nearly 50% of the overall variation. The third structure of the covariance also contains a very short range exponential spatial component but coupled with a medium range cosinusoidal hole temporal component with a periodicity corresponding exactly to a calendar year. This covariance structure contributes approximately 5% of the total variation in DO and corresponds to processes acting seasonally. These processes are very localized geographically as they act over distances of about 2.2 km, which may again include localized spikes in BOD and subsequent DO depletion, as well as the natural variability in river morphology and processes acting on DO over distances ranging from of 1 to 3 km. The final covariance structure consists of a long range exponential component in both space and time. The long spatial range of 88.9 km can be attributed to characteristics and impacts from non-point source pollution from suburban development and agricultural runoff that can affect long stretches of rivers at once.
What is interesting to note is that these fluctuations have a temporal range of about 10,000 days or 27.4 calendar years, which captures time scales corresponding to long term effects of human activities and impact on the environment, as well as climatic changes that may alter the air/water interface and oxygen equilibrium. It should be recognized that there is a wider confidence interval for this temporal range than for any of the other spatial or temporal ranges of our covariance model because this temporal range of 27.4 years exceeds the duration of the time period for which data are available (15 years). Nonetheless it is interesting to note that it is a very large temporal range, which suggests that non-point source pollution over large geographical areas may have an impact on DO that is lasting much longer than the impact of point source pollutions. This may have the serious policy implication that, while pollution prevention strategies may have quick responses in abating the effect of point sources pollution, these strategies may face a much greater challenge in abating rapidly the effect of non-point source pollution on the DO in the surface waters of New Jersey.

4.3.2. Euclidean vs. River Estimation

A cross-validation was performed to examine the differences in estimation of DO using a Euclidean versus a river distance. Table 4.4 summarizes the cross-validation MSE obtained for each river basin using both distances.
Table 4.4: Change in cross validation mean square error (MSE) for each basin. A negative change indicates a reduction in overall MSE (i.e. improvement) when using a river metric.

<table>
<thead>
<tr>
<th>Basin</th>
<th>Euclidean MSE</th>
<th>River MSE</th>
<th>% Change in MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raritan</td>
<td>1.7381</td>
<td>1.5416</td>
<td>-11.3%</td>
</tr>
<tr>
<td>Lower Delaware</td>
<td>1.3193</td>
<td>1.1836</td>
<td>-10.3%</td>
</tr>
</tbody>
</table>

The use of a river metric resulted in 11.3% (Raritan) and 10.3% (Lower Delaware) decrease in MSE. We note that the cross-validation points were at a distance from their neighboring training data points corresponding to several times the average spatial and temporal ranges. In this situation there isn’t as much contrast between the Euclidean and river metrics as would be the case if the points were closer across space. Hence, it is possible that the true gain in mapping accuracy is higher than the 10%-11% found. This is supported by other cross-validation analysis we conducted using synthetic datasets (results not shown here). The approximately 10% reduction in estimation error is appreciable because previous studies using river distance in an estimation context found little difference between a Euclidean and river based model and in some cases found a river distance to increase the prediction error (Ver Hoef et al., 2006; Cressie et al. 2006; Peterson et al., 2006; Peterson et al., 2007).

The improvement in mapping accuracy is supported by our covariance analysis. The variance weighted average of the Euclidean and river spatial ranges were 9.7km and 20.4km, respectively. This means that DO levels are correlated over much longer distances along the river network than across land. This is due in part to the fact that a river meanders, so that the distance along two points is longer along the river reach than across land. The ratio of river distance to straight-line distance between two reach endpoints is known as the meandering ratio (MR).
However, even when we account for the meandering of the network, the range of correlation between points along a river network is significantly higher than when using a Euclidean metric. For example, the average (MR) for both the Lower Delaware and Raritan basins is approximately 1.2. Factoring out this effect by dividing the ratio of river range to Euclidean range (2.1) by the average MR (1.2) gives us an adjusted ratio of river vs. Euclidean range of 1.8. This means that, in practice, even after adjusting for meandering, the correlation along the river is still 1.8 times longer than across land.

While use of the river metric produces maps of DO that are more accurate than those obtained using a Euclidean metric, one might ask whether these maps are visually different. The visual difference can best be shown by comparing the DO estimated in two areas of the Raritan Basin, as shown in Fig. 4.3. The maps obtained using a Euclidean metric are shown on the left, while the maps obtained using the river metric are shown on the right. The subfigure contains the zoomed in portion of the northwestern Raritan basin corresponding to the North and South Branch WMA to highlight two major differences when comparing metrics. Fig. 4.3(a) depicts the zonal differences while Fig. 4.3(b) depicts the parallel reach effect.
From Fig. 4.3(a) the differences in zonal influence that points have when using a Euclidean vs. a river metric are apparent. This is directly connected to the differences in covariance ranges. The river covariance has a longer variance weighted spatial range, resulting in a larger zone of influence of data points along the river. For the Euclidean metric this zone is circular in nature with a smaller range than the zone of influence observed with the river metric, as can be seen by comparing the right and left maps of Fig. 4.3(a). Fig. 4.3(b) depicts another phenomenon along parallel reaches. When estimating the DO level at a point along an unmonitored reach, a higher relative weight is assigned to a sample collected at a point that is at a short distance along the river network, than at a point that is at a
short distance across land. So when considering the case shown in Fig. 4.3(b) where two clearly different river branches are running in parallel of one another, we see that the Euclidean map on the left tends to propagate information from the monitoring data point across land, while the river map on the right constrains the propagation of that information to the river branch where the sample was collected, leading to a more realistic map where parallel branches have distinct water quality. Given the monitoring data available in this study, the results support our hypothesis that the river metric provides more accurate and realistic maps of DO across a river network than maps obtained using a Euclidean metric. Based on this conclusion, river distance was incorporated into the estimation of DO in the Raritan and Lower Delaware River Basins for a subset of the study period (2000-2005) to improve our assessment of the fraction of river miles not attaining the FW-TP standard for DO in New Jersey.

4.3.3. River-BME Estimation of DO

Using the river metric the BME posterior PDF was calculated describing DO at estimation points distributed uniformly along all river miles of in the Raritan and Lower Delaware Basins. Fig. 4.4 depicts the BME mean estimate of DO on June 12, 2002. This date is representative of a typical summer month where DO is at its lowest in both basins. The darker areas highlight river miles where DO has fallen below the New Jersey FW-TP standard of 7 mg/L. The inset highlights an area in the southeast quadrant of the Raritan Basin, corresponding to the Millstone WMA where a majority of river miles are impaired for DO during this time period.
Additional movies provided in Appendix D show DO for every 30 days of the 2000 through August 2005 time period, for both the Lower Delaware and Raritan Basins. These maps are used to calculate the fraction of river miles impaired.

![River Maps](image)

**Figure 4.4**: River-BME Estimation of dissolved oxygen on July 12, 2002 in the Lower Delaware Basin (left) and Raritan Basin (right). The circle indicates the basin outlet, and squares are locations of actual monitoring data available on July 12, 2002.

### 4.3.4. Impaired River Miles in the Raritan and Lower Delaware Basins

For illustration purpose we use the New Jersey FW-TP standard of 7 mg/L for waters designated for freshwater trout-production because the Lower Delaware and Raritan Basins contain a significant number of trout producing and trout maintaining streams. The data were examined to see if a temporal or seasonal trend existed as the temporal covariance would suggest.

The average fraction of river miles not meeting the assessment criteria for more likely than not (MLTN) in non-attainment (i.e. probability of violation > 50%, Akita et al.; 2007) increases from 0.00% to 6.61% between winter and spring of 2002 in the Raritan Basin, and from 0% to 19.70% of river miles in the Lower Delaware Basin (Table 4.5).
In the summer of 2002, this fraction of impaired river miles increases even further, to about 23% in the Raritan and about 58% in the Lower Delaware. Much of this phenomenon can be related to temperature changes depending on season. In the warmer months, water temperature is at its highest and therefore does not hold as much oxygen as the colder water in the winter months. Because New Jersey, and particularly the Raritan sit at higher latitudes, the water temperature drops drastically in winter, leading to near 0% of river miles being impaired. Alternatively, the warmer waters of the summer and spring can foster biological growth that consumes large amounts of DO. This increase in BOD compounds the effects of higher temperatures and leads to more river miles in non-attainment.

From 2000 to 2005, between about 8% to 58% of river miles were found to be MLTN in non-attainment in the warmer summer months (Table 4.6).

<table>
<thead>
<tr>
<th>Season</th>
<th>Fraction of Raritan Impaired (% river miles)</th>
<th>Fraction of Lower Delaware Impaired (% river miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter (Jan-Mar)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Spring (Apr-Jun)</td>
<td>6.61</td>
<td>19.70</td>
</tr>
<tr>
<td>Summer (Jul-Sep)</td>
<td>23.47</td>
<td>57.92</td>
</tr>
<tr>
<td>Fall (Oct-Dec)</td>
<td>0.00</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 4.6: Average Summer Fraction (%) of River Miles More Likely than Not (MLTN) in Non-Attainment (probability of Violation > 50%) for the period 2000-2005 (Summer = Jul-Sep).
The fraction of impaired river miles was highest in 2002, that fraction decreased in 2003 and 2004, but it increased again in the last year of our study period, 2005, indicating that low DO may be an on-going problem in these basins. 2005 was also the warmest summer on record in New Jersey and coupled with a drought, could have contributed to the increase in impaired river miles. An analysis was also conducted to determine the fraction of river miles highly likely in non-attainment (i.e. probability of violation > 90%, Akita et al.; 2007). Based on this criterion, we found that the Lower Delaware had a much higher fraction of river miles ascertained as impaired than can be found in the Raritan. Over the study period, the Lower Delaware had as much as 19% of river miles highly likely in non-attainment, while the Raritan remained around 1.8%. DO itself is affected by a number of environmental factors, including temperature, salinity, nutrient levels and biological oxygen demand. One reason for explaining the larger percentage of impaired miles in the Lower Delaware is the fact that not only is the percentage of urban area larger, but the overall amount of agricultural land is also larger than that in the Raritan, possibly contributing non-point source nutrient loading into adjacent streams. Land cover and land use have shown to greatly impact the quality of streams and rivers, especially in areas undergoing rapid conversion to more urban development patterns (King et al., 2005; Chang, H. 2008).

One of the limitations of this approach is the exclusion of other parameters that can be used to predict DO levels. DO is affected by the geochemistry of the water, and therefore process-based models may provide additional information to refine our geostatistical models. Additionally, this approach looks at only one class
of potential covariance functions, the exponential power model, other functions need to be tested for permissibility when using river distance (isotropic or flow-weighted).

Finally, we use a partial cross-validation procedure to examine the predictive capability of our model. Full model validation using measured DO will be useful for further model refinement in future work.

These results suggest that continued DO monitoring is particularly critical in the Lower Delaware basin to evaluate future trends in DO during the summer months. There is more work needed, however, to identify the specific causes of low DO. The DO maps generated provide a general basis to help begin the process of identifying these causes.

Several conclusions can be drawn from the results of this study. First, our implementation of a river distance metric in the BMElib package provides an efficient and flexible tool for the space/time analysis of DO along river networks. Second, application of river-BME to analyze DO in two river basins in New Jersey lead to maps that are about 10% more accurate and appreciably more realistic than maps obtained using the classical Euclidean distance. In addition it was found that after adjusting for river meandering, the correlation of DO along the river is about 1.8 times longer than across land and DO non-attainment was worse in the Lower Delaware, over more river miles and over a longer period of time than in the Raritan. Finally, additional parameters, such as BOD, temperature, salinity, and nutrients should be factored in to improve estimation accuracy at unmonitored locations.
Chapter V: Modern Space/Time Geostatistics Using River Distances: A Case Study of Turbidity and *Escherichia coli*

5.1. Introduction

This chapter is again an investigation into the use of river-BME for real world water quality applications. It builds upon the previous case study by incorporating soft data into the analysis. This study will be the first use of river distances along with the integration of hard and soft data for water quality estimation.

5.1.1. Fecal Indicator Bacteria in River Systems

*Escherichia coli* (*E.coli*) is a widely used indicator of fecal contamination in water bodies. External contact and subsequent ingestion of bacteria coming from fecal contamination can lead to harmful health effects. Since *E.coli* data are sometimes limited, the objective of this study is to use secondary information in the form of turbidity to improve the assessment of *E.coli* at un-monitored locations. We obtained all *E.coli* and turbidity monitoring data available from existing monitoring networks for the 2000 – 2006 time period for the Raritan River Basin, New Jersey. Using collocated measurements we developed a predictive model of *E.coli* from turbidity data. Using this model, soft data are constructed for *E.coli* given turbidity measurements at 739 space/time locations where only turbidity was measured. Finally, the Bayesian Maximum Entropy (BME) method of modern space/time
geostatistics was used for the data integration of monitored and predicted \textit{E.coli} data to produce maps showing \textit{E.coli} concentration estimated daily across the river basin. The addition of soft data in conjunction with the use of river distances reduced estimation error by about 30\%. Furthermore, based on these maps, up to 35\% of river miles in the Raritan Basin had a probability of \textit{E.coli} impairment greater than 90\% on the most polluted day of the study period.

Fecal indicator bacteria (FIB) provide important health and ecological information for many river basins. Although FIB’s themselves are not harmful, their presence in streams suggests that pathogenic microorganisms might also be present, leading to possible human health risks. Diseases and illnesses that can be contracted in water with high fecal contamination include typhoid fever, hepatitis, gastroenteritis, and dysentery (Mallin \textit{et al.}, 2000). The most commonly tested FIBs are total coliforms, fecal coliforms, \textit{Escherichia coli} (\textit{E.coli}), and enterococci. \textit{E.coli} is a species of fecal coliform that is specific to fecal material from humans and other warm blooded animals. Based on studies conducted by the Environmental Protection Agency (EPA), \textit{E.coli} is the best indicator of health risk from water contact in recreational waters (USEPA, 2000). Therefore many states are now measuring \textit{E.coli} instead of total coliforms to assess streams for fecal contamination. However, due to the limited scope of existing monitoring networks, budget limitations, and manpower constraints, it is difficult to assess all river miles. The purpose of this study is to examine the use of a modern spatiotemporal geostatistics technique, known as Bayesian Maximum Entropy (BME), to statistically assess \textit{E.coli}’s presence in both monitored and un-monitored streams using not only existing \textit{E.coli
data but also integrating secondary information in the form of turbidity measurements to further improve the mapping of basin-scale fecal indicators.

5.1.2. Autocorrelation in *E. coli*

Geostatistical techniques such as kriging rely on the fact that many natural phenomenon exhibit spatial autocorrelation. Monitoring stations along the same stream, for example, tend to report similar physical and chemical characteristics. Kriging methods construct a regional model of correlation to estimate variables, such as *E. coli*, at un-sampled locations based on data from sampled locations (Delhomme, 1978; Cressie, 1990; Stein, 1999). Cokriging, subsequently, uses not only the spatial correlation of a single variable, but also the correlations associated with other environmental variables. There have been numerous examples of cokriging for environmental variable estimation ranging from soil salinity, suspended sediment, and rainfall, to regional stream quality (Darwish *et al.*, 2007; Li *et al.*, 2006; Seo *et al.*, 1990; Jager *et al.*, 1990). It is most beneficial where the primary variable is under-sampled with respect to the secondary variable, as is the case for this study when examining *E. coli* and turbidity as secondary information. Generally the inclusion of secondary information results in more accurate local predictions than when considering a single variable alone (Darwish *et al.*, 2007; Goovaerts, 1997). A more general approach, and the approach used in this study, to estimating at un-sampled locations is the BME method of modern space/time geostatistics (Christakos and Li, 1998). As described in Chapter 2, this method accounts for both spatial and temporal correlations between data points. The major component of this
study is to determine whether the use of river-BME along with turbidity as a secondary variable, improves our estimation of *E.coli* for un-monitored stream reaches.

### 5.1.3. Turbidity and *E.coli*

Turbidity is the measure of light attenuation in a water column. It is related to *E.coli* concentration in that research has shown that FIBs are oftentimes associated with particulate matter in the water column and transport of fecal bacteria via suspended sediments is an important aquatic mechanism (Mallin *et al.*, 2000; Sayler *et al.*, 1975). Often-times bacteria will settle out of the water column into the sediment. The sediment can become entrained and allow the release of particulate matter and the associated bacteria. Additionally, as bacteria concentration increases, the amount of light absorbance in water also changes. Numerous studies have examined the relationship between turbidity and *E.coli* and found significant correlation between both parameters (Adams *et al.*, 2007; Dorner *et al.*, 2007; Vidon *et al.*, 2008; Reeves *et al.*, 2004). Our study area contained a larger number of measured turbidity values relative to *E.coli*, therefore turbidity was chosen as a secondary variable.

### 5.2. Materials and Methods

#### 5.2.1. Data and Study Area

Like the previous case study, the area under investigation is the Raritan River Basin in north-central New Jersey (Figure 5.1). The basin is 1100 square miles and
The basin consists of 36% urban, 19% agriculture, 27% forest, and approximately 17% wetland/water land uses. Approximately 1.2 million people live within this basin and both fecal coliforms and turbidity have been cited as major resource concerns (NJDEP, 2002). Water quality data for the Raritan Basin was obtained through the National Water Information System (NWIS), maintained by the United States Geological Survey (USGS) for the period January 1, 2000 – December 30th, 2007. A total of 44 monitoring stations provided 579 space/time data points for measured *E. coli* while 118 monitoring stations yielded 739 measurements of turbidity for the study period. *E. coli* data were log-normally distributed with a mean of 5.4 log-colony forming units (cfu)/100 mL. Figure 5.1 summarizes the locations of *E. coli* and turbidity measurements during the study period.

![Figure 5.1: Locations of at least one *E. coli* (large circle) and turbidity (small circle) measurement between 2000-2007 in the Raritan Basin, New Jersey.](image)
5.2.2. Generation of Soft Data

One of the primary goals of this research is to introduce a secondary variable, in the form of turbidity, to predict *E.coli* concentrations in areas where there are no direct *E.coli* measurements. These predicted values are referred to as ‘soft’ data because of the uncertainty associated with the predicted values. There are two types of soft data employed in this study, probabilistic and interval. To construct the probabilistic soft data, we used a total of 27 locations where both turbidity and *E.coli* were measured. Using these collocated points a simple linear regression was performed using log-transformed data to determine an initial correlation (r-squared = 0.54) which was consistent with other studies relating turbidity to *E.coli* or fecal coliform concentration (Adams et al., 2007; Dorner et al., 2007; Vidon et al., 2008; Reeves et al., 2004). Because of the limited number of collocated points and relatively low values of turbidity represented, the final least squares predictive model for *E.coli* is a continuous piecewise function containing the linear relationship along with a polynomial model of order 2 to reduce overestimation of *E.coli* at extremely high turbidity values:

\[
\text{Log-}E.coli = \begin{cases} 
2.07z^2 - 0.02z + 2.08 & z < 0.6 \\
2.05z + 1.57 & z \geq 0.6 
\end{cases}
\]
where log-\textit{E.coli} is expressed in log-cfu/100mL, and log-turbidity (\(z\)) is expressed in log-NTU. Using this relationship the log-\textit{E.coli} prediction error variance was calculated using the mean of the squared differences between predicted and measured log-\textit{E.coli} for a series of given windows of log-turbidity values. Finally, for every space/time point where log-turbidity (but not necessarily log-\textit{E.coli}) was measured, a Gaussian probability distribution function (PDF) was constructed for log-\textit{E.coli} with a mean given by (1) and a variance corresponding to the prediction error variance at the measured log-turbidity. This resulted in soft log-\textit{E.coli} data of Gaussian probabilistic type at 739 space/time points.

The uncertainty associated with the direct measurements of low levels of \textit{E.coli} was also accounted for. The data downloaded from the USGS uses the membrane filtration (m-Tec) method for bacteria enumeration and several intercalibration studies suggest \(\pm 0.5\) log as a working point to account for measurement error (Noble et al., 2003; Griffith et al., 2003). Therefore, for any measured log-\textit{E.coli} < 2 log-cfu/100mL in this study, an interval soft datum was introduced in the general form of equation (5.2). This resulted in an additional 15 soft data points.

\[
Prob[a < \text{log-\textit{E.coli}} < b] = 1
\]  

\(5.2\)

5.2.3. Integrating Hard and Soft Data

To integrate the soft data with the measured log-\textit{E.coli} values and then estimate at un-monitored locations, the BME method of modern space/time
geostatistics is used (see Chapter 2). As described in previous sections, both the river-BME and Euclidean-BME procedure consists of defining the general knowledge (i.e. covariance), site specific knowledge (i.e. monitoring data), and integrating the two to calculate a posterior PDF (Eq. 2.5). In this case study, site specific knowledge includes both hard data (e.g. measured values) and soft data (i.e. log-

\[ E.coli \] predictions based on turbidity).

5.2.4. Space/Time Covariance Modeling Using River Distance

As with the previous case study, a covariance model is selected that uses river distances and we restrict our model choice to the exponential power model since it has been shown to be permissible when using river distances (see Chapter 2, § 2.4, and Appendix A). In the case of log-\( E.coli \) in the Raritan Basin, considering a spatial range equal to the area of the basin itself, on average only 1.6 data points were flow-connected. Therefore an isotropic covariance model was chosen to estimate log-\( E.coli \) in the Raritan Basin. The final model used in this study for the space/time covariance of log-\( E.coli \) between space/time points \( p=(r,t) \) and \( p'=(r',t') \) is

\[
\text{cov}(p,p') = c_1 \exp\left(\frac{-3h}{a_{r1}}\right) \exp\left(\frac{-3\tau}{a_{t1}}\right) + c_2 \exp\left(\frac{-3h}{a_{r2}}\right) \exp\left(\frac{-3\tau}{a_{t2}}\right) + c_3 \exp\left(\frac{-3h}{a_{r3}}\right) \exp\left(\frac{-3\tau}{a_{t3}}\right) \tag{5.3}
\]

where \( t \) and \( t' \) are times, and \( h=d_\alpha(r,r') \) and \( \tau=|t-t'| \) are the spatial and temporal lags, respectively. In this study we used either \( \alpha=0 \) (Euclidean distance) or \( \alpha=1 \) (river distance).
5.2.5. Comparing Euclidean and River Estimations

A comparison was made between estimations using river distance, as described above, and estimation using the typical Euclidean distance, alongside the incorporation of soft data from measured turbidity. Cross-validation tests were performed on three different scenarios to determine the best model for estimating basin-wide log-\(E. coli\). Scenario 1 used the measured log-\(E. coli\) data (i.e. the 15 interval soft data points and all the hard data) with the Euclidean distance. Scenario 2 contained the same data as scenario 1 except the river distance was used. Scenario 3 built upon scenario 2 by adding in the turbidity data (incorporated as the soft Gaussian data constructed using Eq. 1). The method with the lowest MSE was then used in the assessment and estimation of \(E. coli\) for the entire Raritan Basin.

5.2.6. Estimation of \(E. coli\)

Using the selected distance metric within the BME framework we estimate \(E. coli\) at equidistant estimation points (i.e. distributed at a fixed interval of 0.1 km) along the Raritan River Basin network. For each estimation point we select the hard and soft log-\(E. coli\) data situated in its local space/time neighborhood, and calculate the corresponding BME posterior PDF describing log-\(E. coli\) at that estimation point. The variance of the BME posterior PDF provides an assessment of the estimation uncertainty, while the back-log transform of the mean of the BME posterior PDF is used as an approximation of the median estimator for \(E. coli\) concentrations. This is
then used to produce chloropleth maps of estimated \( E.\text{coli} \) concentration, and delineate river miles that are more-likely-than-not impaired.

### 5.2.7. Assessment of Impaired River Miles

In order to better understand the pattern of fecal contamination impairment and better quantify the probability of these impairments, a criterion-based space/time assessment framework is used to categorize the fraction of river miles meeting certain probability thresholds, as discussed in Akita \textit{et al.} (2007). These thresholds give us the ability to classify the probability of violation of a standard for any space/time estimation point based on the BME posterior PDF of log-\( E.\text{coli} \). The standard for \( E.\text{coli} \) concentration was set at 235\( \text{cfu}/100\text{mL} \), which is the standard set by NJDEP for primary contact recreation. Using this standard, the probability of violation at space/time point \( p \) is defined as the probability that \( E.\text{coli} > 235\text{cfu}/100\text{mL} \), i.e.

\[
\text{Prob.}[\text{Violation}, p] = \text{Prob.}[E.\text{coli}(p) > 235\text{cfu}/100\text{mL}] \tag{5.4}
\]

The fraction of river miles impaired on any given day of the study period is then obtained by calculating the fraction of equidistant estimation points for which the probability of violation (Eq. 5.4) is in excess of some pre-selected probability threshold (e.g. 90%).
5.3. Results and Discussion

5.3.1. Covariance Analysis

Figure 5.2 shows the covariance $c_X(h, \tau)$ of log-$E. coli$ obtained for the Raritan Basin. The top panel displays $c_X(h, \tau=0)$ which shows how the covariance varies as a function of spatial lag $h$ for a temporal lag $\tau$ equal to 0, while the bottom panel displays $c_X(h=0, \tau)$ which shows how the covariance varies as a function of temporal lag for a zero spatial lag.

![Covariance Analysis Diagram](image)

**Figure 5.2**: Spatial (top) and temporal (bottom) covariance of log-$E. coli$ in the Raritan Basin, New Jersey.

Experimental covariance values estimated from data are shown with markers, while the covariance models obtained by fitting Eq. 5.3 to the markers are shown with lines. The covariance was calculated and modeled using both a Euclidean distance
(dashed line) and river distance (plain line). The covariance model parameters obtained with the Euclidean and river distances are summarized in table 5.1.

**Table 5.1: E.coli Space/Time Covariance Model Parameters**

<table>
<thead>
<tr>
<th></th>
<th>$c_1$ (*)</th>
<th>$a_1$ (km)</th>
<th>$a_1$ (days)</th>
<th>$c_2$ (*)</th>
<th>$a_2$ (km)</th>
<th>$a_2$ (days)</th>
<th>$c_3$ (*)</th>
<th>$a_3$ (km)</th>
<th>$a_3$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euclidean</td>
<td>1.35</td>
<td>30</td>
<td>80</td>
<td>1.08</td>
<td>100</td>
<td>200</td>
<td>0.27</td>
<td>200</td>
<td>500</td>
</tr>
<tr>
<td>River</td>
<td>1.35</td>
<td>40</td>
<td>80</td>
<td>1.08</td>
<td>300</td>
<td>200</td>
<td>0.27</td>
<td>400</td>
<td>500</td>
</tr>
</tbody>
</table>

(*) $c_1$, $c_2$, $c_3$ are expressed in $(\text{log-cfu/100 mL})^2$.

The first structure of the covariance model (with parameters $c_{01}$, $a_1$ and $a_{11}$) is similar for both Euclidean and river distance-based models, with 50% of the total variability of log-\(E.coli\) being characterized by a fairly short range of 30-40\(km\) in space and 80 \textit{days} in time. This is not inconsistent with variability we would expect from point-like sources of \(E.coli\) pollution that are not constant and therefore may dissipate over a few months. The second and third structures of both Euclidean and river covariance models indicate that the remaining 50% of variability in log-\(E.coli\) levels is autocorrelated over longer distances and durations. As noted before, \(E.coli\), and fecal bacteria in general, is oftentimes associated with suspended sediment in the water column. This sediment can travel longer distances along a river network and it is hypothesized that \(E.coli\) associated with suspended sediment remains in the water at high levels for a longer period of time than free bacteria. This phenomenon is captured in the longer spatial and temporal ranges of the covariance models. In the Euclidean based model, the longer range was between 100-200\(km\) in space and 200-500 \textit{days} in time. Interestingly, for the river based-model, the spatial ranges were anywhere from 1.5 to 2 times longer (300-400\(km\)), suggesting that by
accounting for the river connections between points, *E.coli* concentrations may remain correlated over much longer distances than previously considered.

### 5.3.2. Cross-validation Analysis

The cross-validation analysis outlined above resulted in mean square errors of \( \text{MSE}_1 = 2.87 (\text{log-cfu/100 mL})^2 \) for scenario 1, \( \text{MSE}_2 = 2.57 (\text{log-cfu/100 mL})^2 \) for scenario 2, and \( \text{MSE}_3 = 1.99 (\text{log-cfu/100 mL})^2 \) for scenario 3. Comparing scenario 1 to scenario 2 we see that by using river distances instead of Euclidean distances we reduce the estimation error by about 10%, which is similar to the reduction found in a previous study examining dissolved oxygen in the Raritan Basin (Money et al., 2008). If we then add in soft log-*E.coli* data derived from measured turbidity (scenario 3), there is an additional 24% decrease in estimation error. Therefore by incorporating river distances along with soft data from turbidity, the estimation error was reduced by 31% when compared to log-*E.coli* estimation using the typical Euclidean distance and no secondary information. This is one of the first instances in a space/time context that river distances and secondary information have been combined to significantly reduce estimation error. As a result, the river-based covariance model was deemed to be the most accurate representation of *E.coli* in the Raritan Basin, and was used in the subsequent basin-wide estimation and mapping of fecal contamination.
5.3.3. Assessment of Fecal Contamination in the Raritan Basin

Median estimates of *E. coli* concentration were calculated for every day of the study period between 2000-2007. A movie showing changes in these estimated concentrations over time and space can be viewed in Appendix D. Figure 5.3 depicts the *E. coli* concentration for 4 different days of and is representative of many of the days in this study. The squares indicate locations of monitoring stations with measured *E. coli* values and the chloropleth map shows areas where the concentration exceeds the single sample standard of 235cfu/100mL.

![Figure 5.3: river-BME estimation of *E. coli* in the Raritan Basin, New Jersey on 8/16/02 (A), 2/14/03 (B), 5/14/03 (C), and 8/16/05 (D).]
One can see from these maps and the animation that extremely high \textit{E.coli} concentrations (> 600cfu/100mL) can be found along the eastern side of the basin in the North and South Branch and Lower Raritan watershed management areas (WMA). Over the last half of the study period the Lower Raritan WMA remained consistently contaminated with \textit{E.coli} well above the state standard for contact recreation. In addition, several hot spots could be identified in the upper Millstone WMA that would appear and then dissipate, suggesting the occurrence of acute point source contamination in those areas. It should also be noted that high \textit{E.coli} concentrations were estimated in many areas where no monitoring stations existed. In these areas, additional monitoring strategies may be needed to capture potential harmful levels of \textit{E.coli}.

It is also important to assess the confidence in these estimations and describe the probability that a particular river mile is impaired for \textit{E.coli}. This information is important for decision-makers and environmental managers when deciding how to allocate resources and devise public warnings of fecal contamination. Using the log-\textit{E.coli} posterior PDF calculated at regularly spaced estimation points along the Raritan, we calculated for each day of the study period the percentage of river miles with a probability of impairment (Eq. 5.4) greater than 90%. Figure 5.4 depicts these results for a 300 day window of the study period.
Figure 5.4: Percentage of river miles in the Raritan Basin that have a 90% probability of violating the NJDEP standard for primary contact recreation over a 300 day period.

The x-axis is the day of estimation and the y-axis is the percentage of river miles in the Raritan Basin that exceeded the standard with 90% confidence. The fraction of river miles having a >90% probability of being impaired was highly variable from one day to another, and reached a maximum of 35% on the most polluted day of this time period.

Overall this study provides a unique spatiotemporal framework for incorporating river distances and secondary information into the basin-wide assessment of water quality. Accuracy has been improved by over 30% when combining river distances and turbidity as an indicator of *E.coli* concentration. By constructing our model in this way, we are better able to estimate *E.coli* along un-monitored stream segments, thereby increasing the overall number of river miles assessed and providing environmental managers with accurate maps that not only show the spatial and temporal distribution of *E.coli* but that can also highlight areas
of concern, which can be useful when evaluating future monitoring strategies and allocating resources.
6.1. Introduction

This study continues to build on the previous applications of river-BME for water quality estimation. As with the previous case studies, this chapter examines the differences in estimation using either river-BME or Euclidean-BME. It extends the analysis presented in Chapter 5 by incorporating multiple sources of soft data (pH and surface water mercury) for the estimation and mapping of fish tissue mercury in the Cape Fear and Lumber basins in North Carolina.

6.1.1. Mercury in the Environment

Mercury (Hg) is a naturally occurring substance that is present in air, water, and sediments as a result of both anthropogenic and natural sources. Due to the reactive chemical nature of mercury, it can change forms depending on the media it comes into contact with. The mercury cycle has been developed to organize the various avenues with which mercury can enter the environment (Morel et al., 1998). Mercury is naturally present at ambient levels in the atmosphere. Additional atmospheric mercury is due to emissions from volcanoes and fossil fuel burning. This mercury undergoes photochemical oxidation and is deposited through
precipitation where it is then allowed to runoff into lakes and streams. Once in the water, inorganic mercury will settle into the sediments, while some mercury remains in the water column and is converted to methylmercury by bacteria present in the water. Methylmercury is a highly toxic substance that is readily bioaccumulated by aquatic organisms (Porcella, 1995). It is the bioaccumulated methylmercury that poses the most risk to human health. The majority of mercury enters the human body through fish consumption (USEPA, 1997a; 1997b). This poses a significant risk because methylmercury can penetrate mammalian cells and alter cell division, putting children and pregnant women at a much larger risk than the general population. Methylmercury can affect the nervous system and in high doses can affect the kidneys and cardiovascular system (Watras et al., 1998; National Research Council, 2000). There have been numerous studies on mercury distribution in ecosystems, including air (Mason et al., 1997; Fulkerson and Nnadi, 2006), surface water (Balogh et al., 1997; Sullivan and Mason, 1998), and fish tissue (Kannan et al., 1998; Peterson and Sickle, 2007).

Many states and local agencies monitor fish tissue mercury and use this information to issue consumption advisories for particular areas and species of fish. Assessing the spatiotemporal trends of fish tissue mercury on a larger scale, based on monitoring data is a difficult task. Inter-species variability, such as trophic level, and intra-species variability such as size or age have an impact on the amount of bioaccumulated mercury present in a system (Huckabee et al., 1979; MacCrimmon et al., 1983; Cope et al., 1990; Wiener and Spry, 1996). In 2006, in the United
States alone, 38% of total lake acreage and 26% of all river miles had fish consumption advisories (USEPA, 2007).

6.1.2. Autocorrelation of Fish Tissue Mercury

Geostatistical techniques such as kriging rely on the fact that many natural phenomenon exhibit spatial autocorrelation. Kriging methods construct a regional model of correlation to estimate variables at un-sampled locations based on data from sampled locations (Delhomme, 1978; Cressie, 1990; Stein, 1999). Cokriging, subsequently, uses not only the spatial correlation of a single variable, but also the correlations associated with other environmental variables. There have been numerous examples of cokriging for environmental variable estimation ranging from soil salinity, suspended sediment, and rainfall, to regional stream quality (Darwish et al., 2007; Li et al., 2006; Seo et al., 1990; Jager et al., 1990). It is most beneficial where the primary variable is under-sampled with respect to the secondary variable. Generally speaking, the inclusion of secondary information results in more accurate local predictions than when considering a single variable alone (Darwish et al., 2007; Goovaerts, 1997).

A more general approach, and the approach used in this study, to estimating at un-sampled locations is the BME method of modern space/time geostatistics (Christakos and Li, 1998). The BME method provides a rigorous mathematical framework to process a wide variety of knowledge bases. These Knowledge Bases characterize the space/time distribution and uncertainty in monitoring data available for various water quality parameters, and are used to obtain a complete stochastic
description of these parameters at any unmonitored space/time point in terms of its posterior Probability Density Distribution (PDF). The major component of this study is to determine whether the use of river-BME along with several types of soft data can improve our estimation of fish tissue mercury at un-sampled locations.

As noted before fish tissue mercury can vary significantly due to inter and intra-species variability. Therefore, distinguishing between the natural spatiotemporal trends and species specific trends can be difficult if the monitoring data are heterogeneous (i.e. contains many different species). Wente (2004) describes an interesting approach to this problem by developing a statistical model for distinguishing trends in fish tissue mercury concentration using a combination of covariance and multiple linear regression. This model serves as the basis for the Environmental Mercury Mapping, Modeling, and Analysis (EMMMA) program (Hearn et al., 2006). Although it is not the focus of this work, EMMMA results could be integrated into the river-BME analysis, potentially leading to more accurate maps of fish tissue mercury distribution. For our purposes, however, we are focusing on the inclusion of river distances for the geostatistical estimation of fish tissue mercury to understand its distribution on a basin scale and across several species. (see § 6.2).

6.1.3. Factors Influencing the Bioaccumulation of Mercury

A major contribution of this work is the incorporation of secondary variables in the form of soft data that effect the concentration of mercury in fish tissue. A number of studies have examined a variety of water quality variables and their relationship to fish tissue mercury. Many found that one of the best predictors of fish tissue
mercury is pH (Grieb et al., 1990; Rose et al., 1999; Song et al., 2001; Sackett et al., 2008). Low pH values have been shown to increase the amount of methylmercury released from sediments, increasing its bioavailability (Ullrich et al., 2001; Miller and Akagi, 1979). Oftentimes measurements such as pH are more readily available than fish tissue samples, therefore it is a good candidate for generating soft data for fish tissue mercury in areas where fish samples do not exist.

The majority of mercury enters from outside the water body, and ultimately a large majority becomes associated with sediments; however, some mercury remains in its elemental form in the water column, where it is directly transformed into methylmercury by bacteria. Surface water measurements for mercury are typically very scarce and difficult to measure, resulting in large datasets with values below the detectable limit of the procedure being used. The USEPA (2001) suggests the use of bioaccumulation factors (BAF), or the ratio of fish tissue mercury to water column mercury ($WCHg$), to determine the appropriate levels of aqueous mercury that would result in compliant fish tissue mercury concentrations. Minute concentrations of aqueous mercury are capable of generating methylmercury at rates significant enough to warrant consumption advisories (Southworth et al., 2004). Therefore, this study incorporates additional soft data for fish tissue mercury derived directly from surface water mercury measurements, rather than a BAF, which are more reliable if site-specific.

### 6.2. Materials and Methods

#### 6.2.1. Data and Study Area
The area under investigation is the Cape Fear and Lumber river basins in eastern North Carolina. Both of these basins have ongoing fish consumption advisories, and the entire Lumber Basin was listed as impaired in the state’s 303(d) list of impaired waters as required by the Clean Water Act (NCDENR, 2006). The lumber basin is approximately 3300 square miles and is primarily forested (60%) or agricultural (30%). The Cape Fear Basin, on the other hand, is 3 times larger than the Lumber. At 9300 square miles, it is the largest basin in the state and contains close to 20% of the total population, or around 2 million people (NCDENR, 2007; 2004). Figure 1 shows the study area along with the locations of fish tissue and secondary variable measurements.

Figure 6.1: Lumber (Left) and Cape Fear (Right) basins in North Carolina with locations for fish tissue mercury (circles), pH (squares), and surface water mercury (triangles).

Fish tissue mercury data were obtained from the North Carolina Department of Environment and Natural Resources (NCDENR), through the North Carolina Division
of Water Quality (NCDWQ) Fish Tissue Assessment Program. The database of fish tissue mercury and secondary variables was assembled by researchers at North Carolina State University, and a complete description of this database can be found in Sackett et al. (2008). Only those data within the Cape Fear and Lumber Basins were used. Collocated pH data and fish tissue mercury were obtained from this database and additional pH measurements were downloaded from the National Water Information System through the United States Geological Survey (NWIS-USGS). Surface water total mercury data included data collected by NCDWQ as part of the Eastern Regional Mercury Study (NCDWQ, 2003, 2006), as well as additional data from the NWIS. Any duplicate measurements were averaged to a single value. Table 1 summarizes the data used in this study.

<table>
<thead>
<tr>
<th>Data Type</th>
<th># of Locations</th>
<th># of space/time data points</th>
<th># collocated with Fish Hg*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fish Hg</td>
<td>75</td>
<td>1663</td>
<td>-</td>
</tr>
<tr>
<td>pH</td>
<td>33</td>
<td>356</td>
<td>143</td>
</tr>
<tr>
<td>Surface Water Hg**</td>
<td>7</td>
<td>80</td>
<td>35</td>
</tr>
</tbody>
</table>

*Independent from space/time data locations in column 3; **starts in 1995

6.2.2. Generation of Soft Data from Multiple Sources

The availability of secondary variables provides an opportunity to incorporate additional soft data points for fish tissue mercury. Money et al. (2008) described the general framework for generating soft data from secondary water quality variables by creating *E. coli* soft data from turbidity measurements. However, that study focused on only one source of soft data. In this study we incorporate two secondary variables, pH and surface water mercury to generate probabilistic soft data and combine them into one analysis.
There were a total of 143 points where both fish tissue mercury ($FishHg$) and pH were measured. Using these collocated points, a simple regression analysis was performed using log-transformed data to create a relationship of predicted $FishHg$ given pH (Eq. 6.1).

$$\log_{-10}FishHg = -3.5 \log_{-10}pH + 5.7$$  \hspace{1cm} (6.1)

where $log_{-10}FishHg$ is expressed in log-mg/kg (or ppm) and log-pH is in log-standard units. Using this relationship the $log_{-10}FishHg$ prediction error variance was calculated using the mean of the squared differences between predicted and measured $log_{-10}FishHg$ for a series of given windows of log-pH values. Finally, for every space/time point where log-pH (but not necessarily log-$FishHg$) was measured, a Gaussian probability distribution function (PDF) was constructed for log-$FishHg$ with a mean given by (1) and a variance corresponding to the prediction error variance at the measured log-turbidity. This resulted in soft log-$FishHg$ data of Gaussian probabilistic type at 356 space/time points.

There were a total of 35 points where both fish tissue Hg and water column mercury ($WCHg$) were measured during the study period. Again, using these collocated points, we constructed a simple relationship to predict fish tissue Hg using log-transformed data.

$$\log_{-10}FishHg = 0.25 \log_{-10}Hg_{sw} - 3.40$$  \hspace{1cm} (6.2)
where log-$FishHg$ is as above expressed in log-mg/kg (or ppm) and log-$WCHg$ is expressed in log-ng/L. For every space/time location where surface water mercury was measured, a Gaussian probability distribution function was constructed for log-$FishHg$ with a mean given by (2) and variance corresponding to the prediction error variance at the measured log-$WCHg$. This resulted in soft log-$FishHg$ data of Gaussian type at an additional 80 space/time locations.

The models shown in equations 6.1 and 6.2 are simplified expressions of a complex system that has many potential secondary variables. As discussed earlier, a variety of factors effect the concentration of mercury in fish tissue, however the intent of this study is to examine how the addition of soft data in conjunction with a river distance affects the estimation accuracy of fish tissue mercury, therefore a simplified model was most appropriate. Future research will examine more complex models for predicting fish tissue and examine ways to incorporate those model predictions as soft information.

**6.2.3. Integrating Hard and Soft Data**

The site specific knowledge for log-$FishHg$ comes from direct measurements of fish tissue mercury (treated as hard data) and from secondary variables, pH and surface water total mercury, which are used to construct soft data. The BME method of modern space/time geostatistics (see Chapter 2-5) is used to integrate these hard and soft data and obtain statistical estimates of log-$FishHg$ at un-monitored locations. As described in previous sections, both the Euclidean-BME and river-BME procedures consist in defining the general knowledge (i.e. covariance) and site
specific knowledge (i.e. monitoring data), and integrating these two knowledge bases to obtain a posterior PDF (Eq. 2.5) characterizing log-$\text{FishHg}$ at any point on the river.

### 6.2.4. Space/Time Covariance Models That Use River Distances

As with previous water quality studies using river-BME (Money et al., 2008a, 2008b), a covariance model is selected that uses either a Euclidean or river distance. We restrict our model choice to the isotropic exponential power covariance model since it has been shown to be permissible when using river distances (Ver Hoef et al., 2006, Money et al., 2008a). Using this model, the covariance of log-$\text{FishHg}$ between space/time points $\mathbf{p}=(r,t)$ and $\mathbf{p}'=(r',t')$ is expressed as

$$
\text{cov}(\mathbf{p}, \mathbf{p}') = c_1 \exp\left(\frac{-3h}{a_{r1}}\right) \exp\left(\frac{-3\tau}{a_{t1}}\right) + c_2 \exp\left(\frac{-3h}{a_{r2}}\right) \exp\left(\frac{-3\tau}{a_{t2}}\right)
$$

(6.3)

where $t$ and $t'$ are times, $h=d_d(r,r')$ and $\tau=|t-t'|$ are the spatial and temporal lags, respectively, and $d_d(r,r')=\alpha d_R(r,r')+(\alpha-1)d_E(r,r')$ (Eq. 2.11) is an $\alpha$-weighted average of the Euclidean distance $d_E(r,r')$ and the river distance $d_R(r,r')$. In this study we used either $\alpha=0$ (Euclidean distance) or $\alpha=1$ (river distance). For each value of $\alpha$, the parameters $(c_1, a_{r1}, a_{t1}, c_2, a_{r2}, a_{t2})$ of the covariance model (6.3) are obtained using a least square fitting between the covariance function and experimental covariance values calculated from the hard log-$\text{FishHg}$ data. An interpretation of these parameters is provided in § 6.3.
6.2.5. Comparing Euclidean and River Estimations

A comparison was made between estimations using river distance, as described above, and estimation using the typical Euclidean distance, alongside the incorporation of soft data from measured pH and measured WCHg. A cross-validation analysis was performed on five different scenarios to determine the best model for estimating basin-wide log-$\text{FishHg}$. Scenario 1 used the measured log-$\text{FishHg}$ data with Euclidean-BME. Scenario 2 contained the same data as scenario 1 except river-BME was used. Scenario 3 built upon scenario 2 by adding in the pH data (incorporated as the soft Gaussian data constructed using Eq. 1). Scenario 4 built upon scenario 2 by adding in Gaussian soft data from WCHg using equation 2. Finally, scenario 5 combined the soft data from scenarios 3 and 4 into one analysis using river-BME. The method with the lowest MSE was then used in the assessment and estimation of FishHg for the Cape Fear and Lumber basins.

Table 6.2 summarizes each scenario.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Metric Used</th>
<th>Hard Data Used</th>
<th>Soft Data Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Euclidean</td>
<td>Measured log-FishHg</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>River</td>
<td>Measured log-FishHg</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>River</td>
<td>Measured log-FishHg</td>
<td>Gaussian from log-pH</td>
</tr>
<tr>
<td>IV</td>
<td>River</td>
<td>Measured log-FishHg</td>
<td>Gaussian from log-Hg$_{sw}$</td>
</tr>
<tr>
<td>V</td>
<td>River</td>
<td>Measured log-FishHg</td>
<td>Gaussian from log-Hg$_{sw}$ + Gaussian from log-pH</td>
</tr>
</tbody>
</table>
6.2.6. Estimation of Fish Tissue Hg

Using the selected scenario within the BME framework we estimate log-$FishHg$ at equidistant estimation points (i.e. distributed at a fixed interval of 0.1 $km$) along the combined Cape Fear and Lumber network. For each estimation point we select the hard and soft log-$FishHg$ data situated in its local space/time neighborhood, and calculate the corresponding BME posterior PDF describing log-$FishHg$ at that estimation point. The variance of the BME posterior PDF provides an assessment of the estimation uncertainty, while the back-log transform of the mean of the BME posterior PDF is used as an approximation of the median estimator for $FishHg$ concentrations. This is then used to produce choropleth maps of estimated $FishHg$ concentration, and calculate the fraction of river miles that exceed specified action levels.

6.2.7. Assessment of Impaired River Miles

The fraction of river miles impaired at any given time is calculated by determining the fraction of equidistant estimation points that exceed a given action level for fish tissue mercury. There are currently three different action levels for fish tissue mercury. The Food and Drug Administration (FDA) has determined a consumer action level of 1.0 ppm (or mg/kg) (FDA, 2001). The state of North Carolina has declared a more stringent action level of 0.4 ppm (Williams, 2006). In addition, the USEPA has set the most stringent mercury action level at 0.3 ppm (USEPA, 2001). For the Cape Fear and Lumber basins, the fraction of total river
miles exceeding each of these threshold concentration values was calculated independently.

6.3. Results and Discussion

6.3.1. Covariance Analysis

Figure 6.2 shows the covariance $c_X(h, \tau)$ of log-$FishHg$ obtained for the Raritan Basin. The top panel displays $c_X(h, \tau = 0)$, which shows how the covariance varies as a function of the spatial lag $h$ for a temporal lag $\tau$ equal to zero, while the bottom panel displays $c_X(h=0, \tau)$, which shows how the covariance varies as a function of temporal lag for a zero spatial lag.

Figure 6.2: Spatial (top) and temporal (bottom) covariance of log-$FishHg$ in the Cape Fear and Lumber Basins, North Carolina.
Experimental covariance values estimated from data are shown with markers, while the covariance models obtained by fitting Eq. 6.3 to the markers are shown with lines. The covariance was calculated and modeled using both a Euclidean distance (dashed line) and river distance (plain line). The covariance model parameters obtained with the Euclidean and river distances are summarized in table 6.3.

**Table 6.3: FishHg Space/Time Covariance Model Parameters**

<table>
<thead>
<tr>
<th></th>
<th>$c_1$ (*)</th>
<th>$a_{r1}$ (km)</th>
<th>$a_{t1}$ (days)</th>
<th>$c_2$ (*)</th>
<th>$a_{r2}$ (km)</th>
<th>$a_{t2}$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euclidean (α=0)</strong></td>
<td>0.38</td>
<td>58</td>
<td>685</td>
<td>0.0001</td>
<td>0.0001</td>
<td>6.16</td>
</tr>
<tr>
<td><strong>River (α=1)</strong></td>
<td>0.30</td>
<td>111</td>
<td>1070</td>
<td>0.10</td>
<td>114</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*c$_1$ and c$_2$ are expressed in log-ppm$^2$

The first structure of the covariance model (with parameters c$_1$, a$_{r1}$ and a$_{t1}$) explains approximately 75% of the overall variability in FishHg using river distance, and nearly 100% of the variability when using Euclidean distance. The corresponding spatial range of the first structure using river-BME is nearly double the spatial range of the Euclidean-BME model. This suggests that by accounting for river distance, fish tissue mercury is spatially more highly correlated than if the constraints of the river network are not taken into account. Physically this can be explained because fish are inherently restricted to following pathways that mimic the river network configuration. As one would expect, if a Euclidean distance is used, the correlation between fish can be lost over a short distance because fish do not generally travel across land. Indeed, the spatial covariance range (a$_{r1}$) indicates that 95% of the correlation in FishHg is lost after about 58km, compared to 111km when
taking river distance into account. Temporally, fish tissue mercury remains highly correlated for a period of about 2-3 years. This is understandable given that bioaccumulated mercury will change gradually over time, depending upon the age and size structure of the fish community.

6.3.2. Cross-validation Analysis

The cross-validation analysis outlined in table 2 resulted in mean square errors of $\text{MSE}_1 = 0.3050 \text{(log-ppm)}^2$ for scenario 1, $\text{MSE}_2 = 0.2556 \text{(log-ppm)}^2$ for scenario 2, $\text{MSE}_3 = 0.2480 \text{(log-ppm)}^2$ for scenario 3, $\text{MSE}_4 = 0.2508 \text{(log-ppm)}^2$ for scenario 4, and $\text{MSE}_5 = 0.2487 \text{(log-ppm)}^2$ for scenario 5. Using river-BME over Euclidean-BME with only hard data reduced estimation error by $(\text{MSE}_1 - \text{MSE}_2)/\text{MSE}_1 = 16.2\%$. This is appreciably higher than what was obtained in previous studies that examined river-BME for DO and E.coli. Those studies resulted in 10%-11% decrease in error. Also when accounting for soft data from either pH or WCHg independent of one another, an additional 2-3% reduction in estimation error was found. Even though this is appreciably smaller than the reduction seen from soft data in the E.coli study by Money et al. (2008), it is significant because relatively few soft data points were available in this study. In the E.coli/turbidity study there were over 700 additional soft data points added to the analysis, and covering a much smaller land area. In this study, there were only ~300 additional soft data derived from pH and only ~30 additional soft data points derived from WCHg. The reduction in estimation error may be even further reduced if more data points can be included for these secondary variables. The basin under investigation is also very large.
The land area of the Cape Fear and Lumber Basins combined is 4 times the size of basins used in previous studies. Overall, when using river-BME with one source of soft data (either pH or $WCHg$), there was an 18.7% reduction in estimation error when compared to the Euclidean-BME without secondary data. This suggests that accounting for the hydrogeography of the system as well as variables that affect the bioaccumulation of mercury, will result in more accurate estimations of fish tissue mercury at un-sampled locations.

One interesting finding is that when combining the two sources of soft data (pH and $WCHg$) into a single estimation, there was no significant decrease in error when compared to estimations calculated using the sources independently. Again, one reason for this result is the number of soft data relative to the number of hard data. There were over 1600 hard data space/time locations and approximately 400 soft data points used in the combined analysis, meaning that for any given estimation point, the estimation neighborhood is more likely to contain hard data points with more weight than any soft data point. Additionally, with the distribution of $WCHg$ measurements concentrated in a small area of the Northeastern Cape Fear Basin (see figure 1), the estimation neighborhood will likely contain either soft data from pH, or soft data from $WCHg$, but rarely would it contain both. This means that more often than not, any decrease in estimation error can primarily be attributed to the use of river distance, followed by the inclusion of either source of soft data, but not both sources of soft data at the same time. Lange et al. (1993) suggested that acidity may increase the methylation process, and generally speaking water column total mercury increases the availability of mercury for methylation and subsequent
bioaccumulation in fish, which can explain why pH and WCHg, when considered separately, lead to more accurate maps of FishHg. But it is also possible that pH and surface water mercury may work together in the uptake of mercury into fish tissue.

For example, Chen et al. (2001) noted that the bioavailable speciation of Hg and factors that bind with Hg depend on pH. Another possibility is that water column total mercury is a source of methylmercury in the sediment, and pH may aid the methylation of mercury at the sediment-water interface (Winfrey and Rudd, 1990). However, the detailed mechanisms of how the combined effect of water column mercury and pH affects methylmercury formation and bioaccumulation are still largely unknown, and our spatially discontiguous datasets for pH and WCHg may not capture their combined effect. This could help explain why the combination of soft data from these two sources does not result in any additional decrease in estimation error in our study.

6.3.4. Assessment of Fish Tissue Mercury

Median estimates of FishHg concentration were calculated every 180 days over the study period between 1990-2004. A movie showing changes in fish tissue mercury over space and time can be viewed in supplementary material. Figure 6.3 depicts the FishHg concentration for 4 sample days during the study period.
These estimates show that over the course of the study period a large proportion of both basins had the potential for fish tissue mercury levels to be above both the EPA and North Carolina action levels (0.3 and 0.4 ppm respectively). Only a small fraction exceeded the FDA action level of 1.0ppm. Generally the Lumber basin contained higher potential for contaminate fish over more of the basin and for a longer period of time. This is reflected by the fact that the entire Lumber basin has been listed as impaired for fish tissue mercury in the 2006 North Carolina Integrated Water Quality report (NCDENR, 2007). The area of Jordan Lake in the northeastern
Cape Fear also contained high estimates of fish tissue mercury during several time periods, however, very few actual samples were taken in this area during the study. One can combine estimation maps and maps of error variance to provide state and local agencies with a tool for designing future monitoring strategies and better pinpointing fish consumption advisories in areas like Jordan Lake. Figure 6.4 summarizes the fraction of river miles impaired during the study period.

![Figure 6.4: Percentage of river miles with fish tissue mercury median estimate exceeding Mercury Action Levels set by the FDA (top; 1.0ppm), North Carolina (middle; 0.4ppm), and the EPA (bottom; 0.3ppm).](image)

The median estimate of fish tissue mercury exceeded the most stringent action level of 0.3ppm in more than 90% of river miles for a majority of the study period. There were more fluctuations in the percent of impaired river miles when the action level was increased to the current North Carolina level of 0.4ppm; however, over 50% of river miles had median estimates of fish tissue mercury exceeding 0.4ppm for almost the entire study period. In addition, during the years 1990-1994,
between 1-4% of river miles had a median estimate of fish tissue mercury above even the most lenient action level of 1.0ppm set by the FDA. Another small peak is seen in 1998; however, at least with respect to the FDA, the majority of waters remained below this action level, suggesting that measures to mediate mercury inputs into streams and lakes may be taking effect. No river miles had a median estimate of fish tissue mercury exceeding the FDA action level since 1999 according to these results.

Overall this study examines a combination of knowledge sources that may improve the estimation of fish tissue mercury concentrations in two large river basins in North Carolina. Estimation maps were produced that were on average 18% more accurate when accounting for river distance and the secondary variables pH and water column total mercury. Both secondary variables contributed to an overall decrease in estimation error, albeit small due to the limited amount of data points available for these secondary variables. The use of river-BME in this study contributed a majority of the reduction in estimation error and provides a good framework for further decreases in estimation error with the addition of other secondary factors in future work. Soft data from pH contributed as much error reduction as soft data from surface water mercury. Generally pH data are much more reliable and easier to measure, therefore state and local agencies may consider using pH measurements to aid in the assessment of fish tissue mercury in areas where samples may be scarce. However, this work demonstrates that water column total mercury data, when available, can provide a valuable alternate source of information to estimate fish tissue mercury levels. Overall, the framework
developed in this work can aid environmental managers in identifying important bioaccumulation factors and areas where sampling and advisory resources can be targeted.
Chapter VII: Concluding Remarks

The primary goal of this work was to establish a new framework for estimating water quality along river networks, using modern space/time geostatistics with a river distance. In order to achieve this goal the existing Bayesian Maximum Entropy Framework was extended with new river-based functions and modified functions to create the new river-BME framework. It was determined that there are several factors that affect the efficacy of using river-BME for water quality assessments. These included parameter choice, data density, and network complexity. Network complexity plays a significant role in determining whether or not a river-based approach is most appropriate. Both branching level and meandering ratio were shown to impact the efficacy of using river-BME. Generally speaking, as network complexity increases, the efficacy of river-BME increases over that of the classical Euclidean-BME approach. In addition, simulation exercises confirmed that the numerical implementation of river-BME was successful and established the framework for use in real world applications.

The real world case studies provide a broad range of applications for using river-BME. A variety of parameters and network configurations were examined to illustrate the potential for river-BME to improve estimation accuracy over the Euclidean based approach. It was hypothesized that by accounting for the hydrogeography of a system, a noticeable decrease in estimation error would occur.
for water quality parameters that are influenced by in-stream physical, chemical, and biological processes. The results of the three case studies confirm this hypothesis, as all three resulted in improvements in estimation accuracy. Table 7.1 contains a summary of the simulation and real world case studies using river-BME. There was a reduction of estimation error between 10%-31% in the real world studies. The range of improvements is due to a number of factors including data density, network complexity, parameter choice, and soft data source and abundance. The highest reduction (31%) was obtained when there was an abundance of soft information relative to the primary variable of interest, and both were distributed across an entire complex basin. If we examine just the effect of using river distance over Euclidean distance, without soft data, error reduction (or efficacy) ranges from 10% - 16% in the real world case studies. However, the simulation studies suggest that under the right conditions (high complexity, large dataset) estimation error could by reduced by up to 50%.

Table 7.1: Summary of river-BME Estimation Studies

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Network</th>
<th>Parameter</th>
<th>River-BME Error Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation I</td>
<td>Simulated Data</td>
<td>52.4%</td>
<td></td>
</tr>
<tr>
<td>Simulation II</td>
<td>Simulated Data</td>
<td>46.7%</td>
<td></td>
</tr>
<tr>
<td>Hard Data</td>
<td>Dissolved Oxygen</td>
<td>11.3%</td>
<td></td>
</tr>
<tr>
<td>Hard Data</td>
<td>Dissolved Oxygen</td>
<td>10.3%</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>-------</td>
<td></td>
</tr>
<tr>
<td>Hard &amp; Soft Data</td>
<td><em>E.coli</em> with soft data from Turbidity</td>
<td>31.2%</td>
<td></td>
</tr>
<tr>
<td>Hard &amp; Soft Data</td>
<td>Fish Tissue Mercury with soft data from pH &amp; WCHg</td>
<td>18.7%</td>
<td></td>
</tr>
<tr>
<td>Hard &amp; Soft Data</td>
<td>Fish Tissue Mercury with soft data from pH &amp; WCHg</td>
<td>18.7%</td>
<td></td>
</tr>
</tbody>
</table>

Overall river-BME was able to significantly reduce water quality estimation error along a variety of river networks and for a variety of parameters. There are limitations, however, to this approach. First, the algorithm developed for this work calculated isotropic river distance and does not take into account flow connectivity between data points. Although the models presented can be generalized to include a combination of Euclidean and river distance, and flow-weighted covariance models are available and should be incorporated into future work. Second, the covariance functions used in the analysis were restricted to the spatial exponential power model for $\alpha=0$ (Euclidean distance) and $\alpha=1$ (river distance), which has been proven...
permissible for any river networks (see Appendix B). There are a variety of other possible covariance functions that need to be examined for permissibility before they can be used in a river-based geostatistical framework. In addition, more complex models of environmental parameters (i.e. Qual2, EMMMA, CMAQ) can be used to generate soft data, which may lead to even further improvements in estimation and mapping of water quality.

Future research directions should examine the use of other types of models to generate soft data in conjunction with river-BME. Also, more water quality variables and additional networks with varying complexity need to be investigated for the efficacy of using river-BME. This will help to establish a general library of network and variable types and their associated efficacy, which will be an invaluable tool to future researchers. Other potential extensions for river-BME include areas where restricted network distances may play a role in the autocorrelation between points (i.e. water distribution systems). The river-BME framework developed here is a general tool that sets the stage for a multitude of research regarding spatiotemporal trends in water quality along river networks. It will provide local, state, and federal environmental managers with a framework for better targeting resources, advising stake-holders, and informing the public of water quality trends and impairments that may affect future ecological and human health.
Appendix A: Proof of Permissibility for Exponential Covariance Models Using River Distance

This appendix contains a proof that the exponential covariance model is permissible for river distances on directed trees, using the powerful procedure outlined in the works of Ver Hoef et al. (2006) and Ver Hoef and Peterson (2008). We have defined $X(l,i)$ at longitudinal coordinate $l$ along reach $i$ as the moving-average of a white noise random process $W(u,j)$ downstream of point $(l,i)$ using the following equation:

$$X(l,i) = \int_{-\infty}^{l} du \ g(u-l) \ W(u,V_i(u))$$

where $g(u-l)$ is a moving average function defined on $\mathbb{R}^1$, $V_i(u)$ designates the reach at longitudinal coordinate $u$ downstream of reach $i$, and $W(u,j)$ is a white noise process with mean zero, i.e. $E[W(u,j)]=0$ where $E[.]$ is the expectation operator, and with covariance $\text{cov}[W(u,j), W(u',j')] = \delta_{j,j} \delta(u-u')$, where $\delta_{j,j'}$ is the kronecker function ($\delta_{j,j'}=1$ if $j=j'$, and $\delta_{j,j'}=0$ otherwise), and $\delta(u-u')$ is the Dirac function (with property $\int_{-\infty}^{\infty} f(u') \delta(u-u') = f(u)$ for sufficiently smooth functions $f(u)$ defined on $\mathbb{R}^1$).

We now restrict ourselves to the case of the exponential moving average function $g(h) = \sqrt{2} \exp(-|h|)$. Without loss of generality we assume that $l' \geq l$. The covariance between two points $r=(s,l,i)$ and $r'=(s',l',i')$ is then given by

$$\text{cov}(r,r') = \text{cov}(X(l,i), X(l',i'))$$

$$= E \left[ \int_{-\infty}^{l} du \ g(u-l) \ W(u,V_i(u)) \int_{-\infty}^{l'} du' \ g(u'-l') \ W(u',V_{i'}(u')) \right]$$
\[
= 2 \int_{-\infty}^{l} du \exp(u - l) \int_{-\infty}^{l'} du' \exp(u' - l') E[W(u, V_i(u)) W(u', V_i(u'))]
\]
\[
= 2 \int_{-\infty}^{l} du \exp(u - l) \int_{-\infty}^{l'} du' \exp(u' - l') \delta_{V_i(u), V_i(u')} \delta(u - u')
\]
\[
= 2 \int_{-\infty}^{l} du \exp(u - l) \exp(u' - l') \delta_{V_i(u), V_i(u')}
\]

where \( l' \geq l \) was used in the last line to obtain the upper bound of the integral.

First consider the case where \( i \) and \( i' \) are flow-connected, i.e. \( i' \) is upstream of \( i \), or equivalently \( V_i(l) = \{i\} \). Then \( \delta_{V_i(u), V_i(u') = 1} \) for all \( u \leq l \) and therefore in that case

\[
\text{cov}(r, r') = 2 \int_{-\infty}^{l} du \exp(u - l) \exp(u - l') = 2 \int_{-\infty}^{0} dv \exp(v) \exp(v - (l' - l))
\]
\[
= \exp(-(l' - l)) 2 \int_{-\infty}^{0} dv \exp(2v) = \exp(-(l' - l))
\]
\[
= \exp(-d_R(r, r'))
\]

where \( d_R(r, r') = l' - l \) was used to obtain the last line since \( i \) and \( i' \) are flow-connected.

Next consider the case where the points \( r \) and \( r' \) are not flow-connected, i.e. \( i \) and \( i' \) are on different branches of the river network. In that case the confluence node of these two branches is at a longitudinal coordinate \( l'' \) such that \( l'' \leq l \) and \( l'' \leq l' \), and the river distance between \( r \) and \( r' \) is \( d_R(r, r') = (l - l'') + (l' - l'') \). It follows that \( \delta_{V_i(u), V_i(u') = 1} \) for \( u \leq l'' \) and \( \delta_{V_i(u), V_i(u') = 0} \) for \( l'' < u \leq l \). Therefore in that case we have

\[
\text{cov}(r, r') = 2 \int_{-\infty}^{l} du \exp(u - l) \exp(u - l')
\]
\[
= 2 \int_{-\infty}^{l''} du \exp(u - l'' + l'' - l) \exp(u - l'') \exp(l'' - l')
\]
\[
\begin{align*}
&= \exp(-(l - l') - (l'' - l'')) \int_{-\infty}^{l''} du \exp(u - l'') \exp(u - l''') \\
&= \exp(-d_R(r, r'))
\end{align*}
\]
Hence we have shown that whether or not two points are flow-connected, their covariance is \(\exp(-d_R(r, r'))\). Since the covariance of the moving-average of a white noise random process is a permissible covariance model (Ver Hoef et al., 2006), then the exponential covariance is permissible with the river metric.
Appendix B: Mathematical Summary of Flow-weighted Covariance Models

The framework proposed in Ver Hoef et al (2006), and further used by Cressie et al. (2006) and Ver Hoef and Peterson (2008) was a break-through to obtain a large class of flow-weighted covariance models based on moving average constructions. We refer the reader to their papers for an in-depth presentation of the framework, but we provide here some mathematical steps using a slightly modified notation that can then be compared with the alternate framework proposed by Bernard-Michel and Fouquet (2006). Let’s define $X(l,i)$ at longitudinal coordinate $l$ along reach $i$ as the moving-average of a white noise random process $W(u,j)$ on the reaches upstream of point $(l,i)$ using the following equation

$$X(l,i) = \int_0^\infty \sum_{j \in V_i(u)} \Omega(i,j) g(u-l) W(u,j)$$

where $V_i(u)$ is the set of river reaches at longitudinal coordinate $u$ upstream of reach $i$, $g(u-l)$ is a moving average function defined on $R^1$, $W(u,j)$ is a white noise process with mean zero, and $\Omega(i,j)$ is real number between 0 and 1 expressing the amount of flow connection between reach $i$ and $j$ such that $\sum_{j \in V_i(u)} \Omega(i,j) = 1$ for $u > l$. The flow connection between reach $i$ and an upstream reach $i'$ can be defined as the ratio $\Omega(i,i') = \Omega(i')/\Omega(i)$ where $\Omega(i)$ is function that increases in the direction of flow. In that case the property $\sum_{j \in V_i(u)} \Omega(i,j) = 1 \forall u > l$ is verified if and only if $\Omega(i)$ is a flow additive function, i.e. such that if two reaches $i'$ and $i''$ combine into reach $i$, then
\(\Omega(i') + \Omega(i'') = \Omega(i)\). Flow discharges or watershed areas are physically meaningful variables that can be used to obtain \(\Omega(i)\) (see next section). The covariance between two points \(r=(s,l,i)\) and \(r'=(s',l',i')\) is then given by

\[
cov(r, r') = \text{cov}(X(l, i), X(l', i'))
\]

\[
= \mathbb{E}\left[ \int_{l} \sum_{j \in V_{l}(u)} \sqrt{\Omega(i, j)} \ g(u-l') \ W(u, j) \right] \int_{l'} \sum_{j' \in V_{l'}(u')} \sqrt{\Omega(i', j')} \ g(u'-l') \ W(u', j') \]

\[
= \int_{l} \int_{l'} g(u-l) g(u'-l') \sum_{j \in V_{l}(u)} \sum_{j' \in V_{l'}(u')} \sqrt{\Omega(i, j) \Omega(i', j')} \ E[W(u, j)W(u', j')]
\]

\[
= \int_{l} \int_{l'} g(u-l) g(u'-l') \sum_{j \in V_{l}(u)} \sum_{j' \in V_{l'}(u')} \sqrt{\Omega(i, j) \Omega(i', j')} \ \delta_{j,j'} \ \delta(u-u')
\]

If \(r\) and \(r'\) are not flow-connected, then \(V_{l}(u) \cap V_{l'}(u') = \emptyset\ \forall \ u \geq l \) and \(u' \geq l'\), as a result \(\delta_{j,j'}=0\ \forall \ j \in V_{l}(u)\) and \(j' \in V_{l'}(u')\), so that the double summation is zero and consequently \(\text{cov}(r, r')=0\). If \(r\) and \(r'\) are flow-connected let us assume without loss of generality that \(r\) is upstream of \(r'\), i.e. \(l \geq l'\). Then using the property of the Dirac function \(\int_{-\infty}^{\infty} f(u') \ \delta(u-u') = f(u)\) for sufficiently smooth functions \(f(u)\) defined on \(R^{1}\) we obtain

\[
\text{cov}(r, r') = \int_{l} \int_{l'} g(u-l) g(u'-l') \sum_{j \in V_{l}(u)} \sum_{j' \in V_{l'}(u')} \sqrt{\Omega(i, j) \Omega(i', j')} \ \delta_{j,j'}
\]

where \(l \geq l'\) was used in obtaining the lower bound of the integral. Since \(r\) and \(r'\) are flow-connected with \(r\) upstream of \(r'\), it follows that \(V_{l}(u) \subset V_{l'}(u) \neq \emptyset\ \forall \ u \geq l\), so that the double summation reduces to a single summation as follow
\[ \text{cov}(r, r') = \int_1^\infty du \ g(u - l)g(u - l') \sum_{j \in V_i(u)} \sqrt{\Omega(i, j)\Omega(i', j)} \]

Recall that \( \Omega(i', j) = \Omega(j)/\Omega(i'), \Omega(i, j) = \Omega(j)/\Omega(i) \) and \( \sum_{j \in V_i(u)} \Omega(i, j) = 1 \) for \( u > l \), from which we also get \( \sum_{j \in V_i(u)} \Omega(j) = \Omega(i) \) for \( u > l \). Using these relationships it follows that

\[ \sum_{j \in V_i(u)} \sqrt{\Omega(i, j)\Omega(i', j)} = \sum_{j \in V_i(u)} \sqrt{\Omega(i)} = \sqrt{\Omega(i)}/\sqrt{\Omega(i)} = \sqrt{\Omega(i, i')} \],

which when substituted in the equation above leads to

\[ \text{cov}(r, r') = \sqrt{\Omega(i, i')} \int_0^\infty du \ g(v)g(v + l - l') \]

where the change of variable \( v = u - l \) was used in the integral. Noting that \( d_R(r, r') = l - l' \) when \( r \) and \( r' \) are flow-connected and \( l \geq l' \), and noting that \( \Omega(i, i') = 0 \) when \( r \) and \( r' \) are not flow-connected, then a permissible model for the covariance between \( r \) and \( r' \) (whether they are flow-connected or not) is given by

\[ \text{cov}(r, r') = \sqrt{\Omega(i, i')} \ C_1( \ d_R(r, r') \ ) \]

where \( d_R(r, r') = |l - l'| \) is the river distance between \( r \) and \( r' \), and \( C_1( . ) \) is the class of permissible covariance functions in \( R^1 \) defined by \( C_1(h) = \int_0^\infty du \ g(v)g(v + h) \) for any suitable moving average functions \( g(v) \). This class of permissible covariance functions includes for example the strikingly beautiful Mariah (Ver Hoef, 2006) model.

De Fouquet and Bernard-Michel (2006) proposed a framework that can be used to expand the class of permissible flow-weighted covariance models. Along the
lines of the framework they proposed, let us here define $X(l,i)$ at longitudinal coordinate $l$ along reach $i$ as

$$X(l,i) = \sum_{j \in V_i(\infty)} \sqrt{\Omega(i,j)} Y_j(l)$$

where $V_i(\infty)$ is the set of flow-connected leaf reaches (i.e. sources) upstream of reach $i$, $\Omega(i,j) \in [0,1]$ quantifies the flow connection between reach $i$ and its source $j$ such that $\sum_{j \in V_i(\infty)} \Omega(i,j) = 1$, and $Y_j(l)$ are independent zero mean random processes on $R^1$ with covariance $\text{cov}(Y_i(l), Y_i(l')) = c_1(h) = |l-l'|$, where $c_1(h)$ may be any permissible covariance function in $R^1$ (i.e. such that it is the Fourier transform of a non-negative bounded function in $R^1$). The covariance between two points $r=(s,l,i)$ and $r'=(s',l',i')$ is then given by

$$\text{cov}(r, r') = \sum_{j \in V_i(\infty)} \sum_{j' \in V_i(\infty)} \sqrt{\Omega(i,j)} \sqrt{\Omega(i',j')} \text{cov}(Y_j(l), Y_{j'}(l'))$$

If $r$ and $r'$ are not flow-connected, then $V_i(\infty) \cap V_{i'}(\infty) = \emptyset$, as a result $\text{cov}(Y_j(l), Y_{j'}(l')) = 0 \ \forall j \in V_i(\infty) \text{ and } j' \in V_{i'}(\infty)$, so that $\text{cov}(r, r') = 0$. If $r$ and $r'$ are flow-connected, assuming without loss of generality that $r$ is upstream of $r'$, i.e. $l \geq l'$, we have

$$\text{cov}(r, r') = \sum_{j \in V_i(\infty)} \sum_{j' \in V_i(\infty)} \sqrt{\Omega(i,j)} \sqrt{\Omega(i',j')} \delta_{j,j'} c_1(|l-l'|)$$

$$= \sum_{j \in V_i(\infty)} \sqrt{\Omega(i,j)\Omega(i',j')} c_1(|l-l'|)$$

$$= \sqrt{\Omega(i,i')} c_1(|l-l'|)$$
Noting that $\Omega(i,i')=0$ when $r$ and $r'$ are not flow-connected, we obtain that a permissible model for the covariance between $r$ and $r'$ (whether they are flow-connected or not) is given by

$$\text{cov}(r,r') = \sqrt{\Omega(i,i')} c_1( d_R(r,r') )$$

where $c_1(\cdot)$ can be any one dimensional permissible covariance function, which includes the functions $C_1(\cdot)$ obtained with the moving average construction.
Appendix C: Flow-additive Functions

As described in Appendix B, the flow connection between reach $i$ and an upstream reach $i'$ can be defined as the ratio $\Omega(i,i') = \Omega(i')/\Omega(i)$ where $\Omega(i)$ is a flow additive function, i.e. such that if two reaches $i'$ and $i''$ combine into reach $i$, then $\Omega(i') + \Omega(i'') = \Omega(i)$. We show here that the framework developed by Ver Hoef et al. (2006) to quantify flow-connection provides a flexible method to construct the flow additive function $\Omega(i)$. Using their approach, let’s define the flow weight of a reach $i'$ as $\omega(i')$ such that if reaches $i'$ and $i''$ combine at a river junction, then $\omega(i') + \omega(i'') = 1$. Using this variable, Ver Hoef et al. (2006) originally defined the flow connection between reach $i$ and an upstream reach $i'$ as $\Omega(i,i') = \prod_{j \in B_{i,i'}} \omega_j$, where $B_{i,i'}$ is the set of reaches in the flow-path between reaches $i$ and $i'$, exclusive of the downstream reach $i$ and inclusive of the upstream reach $i'$. This construction is equivalent to defining the flow additive function as

$$\Omega(i) = \prod_{j \in B_{1,i}} \omega_j,$$

where $B_{1,i}$ is the set of reaches in the flow-path between the river outlet (on a river reach numbered 1 by convention) and reach $i$. Then it follows immediately that

$$\Omega(i,i') = \frac{\prod_{j \in B_{1,i'}} \omega_j}{\prod_{j \in B_{1,i}} \omega_j} = \frac{\prod_{j \in B_{1,i}} \omega_j \prod_{j \in B_{i,i'}} \omega_j}{\prod_{j \in B_{1,i'}} \omega_j} = \prod_{j \in B_{i,i'}} \omega_j,$$

which, after taking its square-root, leads to the multiplier $\sqrt{\Omega(i,i')} = \prod_{j \in B_{i,i'}} \sqrt{\omega_j}$ defined in Ver Hoef et al. (2006) equation for flow-connected covariance models.
The weight of each combining reach can be calculated using a physically meaningful parameter that increases in the direction of flow. Examples include watershed area, discharge, cumulated river length, precipitation, pollution loading, etc. Let us denote the value of this parameter at the downstream end of any reach \( i \) as \( A_i \), and let \( a_i \) be the contribution within reach \( i \), such that if reaches \( i' \) and \( i'' \) combine into reach \( i \), then \( A_i = A_{i'} + A_{i''} + a_i \). Using this parameter we can define the weights of combining reaches \( i' \) and \( i'' \) as 

\[
\omega_{i'} = \frac{A_{i'}}{A_{i'} + A_{i''}} \quad \text{and} \quad \omega_{i''} = \frac{A_{i''}}{A_{i'} + A_{i''}},
\]

respectively, which satisfies \( \omega_{i'} + \omega_{i''} = 1 \). The resulting flow additive function is then given by

\[
\Omega(i) = \prod_{j \in B_i} \omega_j = \prod_{j \in B_i} \frac{A_j}{A_j + A_{C(j)}} = \prod_{j \in B_i} \frac{A_j}{A_{D(j)} - a_{D(j)}}
\]

where \( C(j) \) is the reach combining with reach \( j \), and \( D(j) \) is the reach immediately downstream of reaches \( j \) and \( C(j) \); so that \( A_{D(j)} = A_j + A_{C(j)} + a_{D(j)} \). As can be seen in the example of Fig. C1, the construction shown allows us to account for the contribution of watershed area (or discharge, cumulated river length, etc.) within each reach.
Figure C1: Example of a river with 5 reaches, indicating for each reach $i$ the contributing watershed area $a_i$ within reach $i$, the total watershed area $A_i$ at the downstream end of reach $i$, and the corresponding flow additive function $\Omega(i)$.

A simplified construction might consist in setting $a_i=1$ for each leaf reaches, and setting the contribution of non-leaf reaches to zero, i.e. $a_D(i)=0$ $\forall$ $i>1$. In this case $A_i$ corresponds to the additive stream-order number used in Cressie et al. (2006), and the flow additive function simplifies to $\Omega(i)= A_i/A_1$. For illustration purposes, this corresponds to setting $a_5=a_4=a_3=1$ and $a_2=a_1=0$ in the example of Fig. C1, resulting in the stream-order numbers $A_5=A_4=A_3=1$, $A_2=2$ and $A_1=3$, and in the flow additive function values $\Omega(5)=1/3$, $\Omega(4)=1/3$, $\Omega(3)=1/3$, $\Omega(2)=2/3$, $\Omega(1)=1$. This construction provides a convenient way to obtain flow-connectivity if no information is available.
about the contribution of watershed area (or discharge, cumulated river length, etc.) within each reach.

Another simplification consists in weighting each reach equally, i.e. $\omega(i) = 1/2 \forall i > 1$, which leads to $\Omega(i) = \prod_{j \in B_{1,i}} 1/2$. This would correspond to using $\Omega(5) = 1/4$, $\Omega(4) = 1/4$, $\Omega(3) = 1/2$, $\Omega(2) = 1/2$, $\Omega(1) = 1$ in the example of Fig. C1, which is a slightly different representation of flow-connectivity than that of based on stream-order number.
Appendix D: Movies Depicting Water Quality Trends Using river-BME

This appendix contains links to the animations of water quality for the case studies in Chapters 4, 5, and 6. Each link will take you to a webpage where you can view the movies in any web browser.

Chapter 4 Animations: Dissolved Oxygen

Movie 4.1 depicts monthly space/time trends of Dissolved Oxygen in the Lower Delaware Basin, New Jersey: 2000-2005


Movie 4.2 depicts monthly space/time trends of Dissolved Oxygen in the Raritan Basin, New Jersey: 2000-2005


Chapter 5 Animations: Escherichia Coli

Movie 5.1 depicts daily space/time trends of E.coli in the Raritan Basin, New Jersey for April and May, 2003.

http://www.unc.edu/depts/case/BMElab/studies/EC_NJ/Raritan_Ecoli_AprilMay2003.GIF
Chapter 6 Animations: Fish Tissue Mercury

Movie 6.1 depicts the biannual concentration of fish tissue mercury in the Cape Fear and Lumber river basins, North Carolina: 1990-2004

http://www.unc.edu/depts/case/BMElab/studies/HgFish_NC/CapefearLumber_HgFish_2months.GIF
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