A CASE STUDY OF PRE-SERVICE TEACHERS' EXPERIENCES
IN A REFORM GEOMETRY COURSE

By

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Abstract

Chadd W. McGlone. A Case Study of Pre-Service Teachers' Experiences in a Reform Geometry Course (Under the direction of Carol E. Malloy).

Abstract

This study documented 25 pre-service elementary and middle school teachers' experiences in an inductive geometry course. It utilized a qualitative case study design in order to gain insight into the participants' reactions. Data were collected through in-depth student interviews that elicited information about students' previous mathematics courses; their reaction to this particular mathematics course; their view of themselves as mathematicians; their beliefs about mathematics teaching and learning; and their perspective about the role that this particular mathematics class played in their pre-service teacher training. Additional data came from student reflections on a summary essay question; their responses to an attitude about mathematics assessment; and their answers on a geometry knowledge assessment. Data were also collected in the form of frequent classroom observations.

Overall, Students' beliefs about teaching and learning were transformed during this semester.   They also gained pedagogical skills on which to draw when they become teachers and learned how to create a constructivist classroom environment complete with supportive tools and resources. Students developed an appreciation for (1) the process of obtaining an answer, (2) multiple solutions to mathematical problems, (3) learning for mathematical understanding, (4) the value of cooperative learning in the classroom, (5) the impact of meaningful, high-demand mathematics on understanding,

and (6) the role of classroom tools, like manipulatives and technology, in the learning process.

Students came to realize that mathematical knowledge originates from within students in the classroom, not just the teacher and textbook. Out of this expectation grew a commitment to the efficacy of cooperative learning; consequently, many students reported that their mathematics class will look different than they believed it would at the beginning of the semester.

Other findings were: (a) when discussing memories of previous mathematics classes, students described traits unique to traditional instruction; (b) participants describe the student-centered lessons in this particular geometry course as being entirely different from previous courses; (c) students became knowledgeable, confident mathematicians as a result of their exposure to the instruction in this course; and (d) students reported that this reform-based geometry course played an important role in their pre-service teacher training.

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Table of Contents

List of Tables

List of Figures

Chapter One

Introduction

At a large state university in the Southeastern United States, pre-service elementary and middle school teachers currently take a standards-based mathematics course that is different other courses they have taken in college In this course, they develop confidence from the mathematical understanding that occurs when they grapple with the mathematics in order to complete challenging tasks. In addition to gaining this understanding and confidence in the content knowledge, the students' experience in this course seem to impact their learning of and beliefs about mathematics instruction.   The course offered by this university focuses on geometry as part of a requirement for an education degree. In this dissertation, I will endeavor to determine the ways in which students benefit from their experiences in this course. If they do report increased understanding of and confidence with mathematics and changing views of mathematics teaching, I will attempt to determine why those changes occurred.

In this introduction, I will take the first steps in that exploration by briefly exploring how a standards-based geometry class is developed and describing how such a class might impact pre-service teachers' beliefs about teaching mathematics. I will then explain why this dissertation is an important study and what it will contribute to the literature.

Standards-Based Mathematics Classrooms

Constructivist and sociocultural theorists, such as Piaget (1972) and Vygotsky (1989), hypothesize that individuals build their knowledge through meaningful experiences. When people experience something that does not fit into their existing mental framework, disequilibrium occurs, causing them to adapt by either modifying their framework or building entirely new ones. A distinction in the meaning of knowledge and knowing exists between these two perspectives. In the mid 1990's, various theorists identified and reconcile that distinction (Cobb and Yackel, 1995; Driver, Asoko, Leach, Mortimer, & Scott, 1994).

In education, constructivist learning theory suggests a shift in the roles of the student and teacher in the classroom. In these classrooms, students become responsible for engaging in the learning process and building new knowledge. The teacher, on the other hand, is no longer seen as the keeper of knowledge. Rather, he or she becomes a guide for students as they work independently or in groups. As a facilitator, the teacher moves from the role of director in the front of the classroom to the role of conductor, nudging students toward understanding as needed.

In 1989, 1991, and 2001, National Council of Teachers of Mathematics (NCTM) published documents that provided recommendations to help mathematics teachers create constructivist classrooms. In these classrooms, students build mathematical knowledge by actively participating in the instruction, solving problems through logic, conjecture, and mathematical reasoning. The teaching described in the NCTM documents is often called standards-based or reform mathematics instruction. Reform classrooms provide students with opportunities to complete the mental tasks that

encourage mathematical understanding (Carpenter & Lehrer, 1999) through high cognitive demand tasks (Stein, Smith, Henningsen, & Silver, 2000). This instruction occurs in classrooms that have a high press for learning (Kazami & Stipek, 2001) and establish sociomathematical norms (Yakel & Cobb, 1996) that facilitate student learning. A classroom discourse in which all students participate in learning through conversations about mathematics (Reinhart, 2000) as well as through cooperative learning groups (Slavin, 1995) is an important component of a reform mathematics classroom.

Tall (2004) characterized the mathematical thinking that occurs at the post-secondary level as passing through three worlds that represent the sophistication of student mathematical thinking and communication. Thus, students who think mathematically in the first world, called the Embodied World, communicate and perceive mathematics through physical experiences. Students in the Procept World utilize symbolic manipulations and calculations to communicate about mathematics. Advanced students in the Formal World express mathematical objects according to formally deduced definitions and theorems. Robert, Dorier, Robinet, & Rogalski, (2000) emphasized how important it is for instructors to match their communication to a student's world. Students struggle when professors and teaching assistants jump from their world to a more advanced world before the students are ready. Dorier (2000) demonstrated that beginning linear algebra students work in the Embodied World as they were introduced to this new type of mathematics, even though they might have functioned in the Procept or Formal Worlds in different courses.

What role does a standards-based geometry course have in an undergraduate teacher preparation program? Borko, Underhill, Brown, Jones, and Agard (1992) suggest that prospective teachers must experience university coursework that allows them to strengthen their core content knowledge. This strengthening occurs in mathematics courses that provide students opportunities to explore the material in ways that stimulate understanding. In these classrooms, future teachers develop models of teaching when they experience instruction that demonstrates the strategies needed to support mathematical understanding (Wilson & Ball, 1996). Likewise, university coursework for pre-service teachers should challenge their beliefs about teaching and learning mathematics (Borko et al., 1992). Finally, exposure to discourse can change student beliefs about its role in their classrooms (Blanton, 2002). In sum, experiences in these courses will allow future teachers to strengthen their pedagogical content knowledge.

Importance of Research and Contribution to the Literature

What is the role of standards-based geometry instruction in a teacher training programs? This question is important to ask at a time when NCTM is calling for teachers to establish reform-style mathematics classrooms, and teacher training programs are being called to produce large numbers of mathematics teachers. In order to meet this need to produce teachers, these programs are forced to streamline certification programs in order to get teachers out into the workforce. Consequently, some pre-service teachers learn about constructivist mathematics classroom in their methods class and begin teaching without ever wrestling with mathematics themselves. Moreover, the entirety of their mathematics training may occur in a university

mathematics department without input from pre-service teaching programs (Grover & Connor, 2000). Do these classes challenge pre-service teachers' beliefs about teaching and learning as Borko et al. (1992) suggest? Does the instruction model the types of teaching that teachers are expected to exhibit in their own classrooms (Wilson & Ball, 1996)? The results of this project will help guide pre-service teacher programs as they design content coursework for their students.

In this study, I wanted to explore students' experiences in a standards-based geometry class. I determined the impact of those experiences on the students' beliefs about geometry learning and instruction.

Four general questions guiding this study are as follows:

1. To what extent do students perceive that they have improved as mathematicians based on their experiences in a standards-based geometry course?

2. What do students' report that they learned about teaching and learning in mathematics during the semester?

3. Did students beliefs about teaching and learning in mathematics change as a result of their experiences in this course?

4. Do students believe that this standards-based geometry course played an important role in preparing them for subsequent teaching methods coursework?

Chapter Two

Review of the Literature

In this project, I documented student reactions to a standards-based geometry course that was taught to pre-service teacher. In the syllabus for this course, the instructor informs that students that it will be taught from a "constructivist and sociocultural perspective" and that they will "move from concrete to abstract reasoning using the van Hiele model of Geometric Thought." She also notifies students that they will connect geometry to real world situations, complete inquiry-based activities, and utilize technological models to build a personal understanding of geometry. This course provided the foundation for the theoretical framework that guides this study and the literature that generates my research questions, which I present in this chapter. This framework is woven throughout the literature review.

I will open this chapter with an outline of the constructivist and sociocultural perspectives of learning and their role in education. These learning theories form a portion of the theoretical foundation of this research. The first section of this chapter discusses the constructivist theories of Piaget and the sociocultural theories of Vygotsky and outlines their contribution to education research. I will next establish a distinction between these two perspectives and identify ways that theorists (Cobb and Yackel, 1995; Driver, Asoko, Leach, Mortimer, & Scott, 1994) have attempted to bridge the gap.

The manifestation of constructivism in the mathematics classroom forms a key component of the theoretical framework guiding this research. The next section explains how the NCTM *Principles and Standards for School Mathematics* applied the constructivist and socioculturalist perspectives to mathematics teaching and learning by endorsing reform mathematics instruction. I will review the literature to outline key components of standards-based mathematics instruction, also called reform mathematics instruction, and describe how it is effectively implemented in the classroom.

Piaget's and van Hiele's theories of geometric proof and geometric thought, respectively, comprise the third part of the theoretical framework guiding this study. Rooted in the constructivist theories, Piaget proposed stages through which students pass as they gain an understanding of proof in geometry. Likewise, van Hiele described a sequential level model that describes increasingly complex geometric thinking. In addition, van Hiele presented four phases of instruction to support teachers as they guide their students from one stage to the next. The above mentioned theories will be presented in the third section of this chapter.

The fourth section of this chapter will describe how reform mathematics instruction looks in a geometry classroom. In it, I will illustrate how the guidelines proposed by NCTM's *Principles and Standards for School Mathematics* facilitate mathematical learning in a geometry classroom. In addition, the theories and tools teachers of standards-based geometry courses utilize to support instruction will be presented.

The fifth section will discuss the research regarding reform mathematics instruction and mathematical thinking in post-secondary courses. It contains a review of a successful attempt to implement reform mathematics instruction in college calculus courses. Next, I will describe the research about mathematical thinking that grew out of a desire to articulate the progression mathematics majors make during their post-secondary education.

In the final section of this chapter, I will consider the role of standards-based mathematics courses in teacher preparation programs. This section contains a description of the unique knowledge that teachers of mathematics must possess in order to meet the needs of their students. In addition, a review of the literature regarding mathematical content knowledge and teacher preparation can be found in this chapter.

Little research exists that specifically explores the impact of a standards-based geometry course on pre-service teachers' mathematical learning and perceptions of mathematics instruction. Therefore, I will primarily look at the research regarding reform instruction about general mathematics.

Theories of Learning

The ability to grasp progressive abstract concepts is a critical tool in the development of mathematical thinking. The source of that ability is derived from maturing cognitive structures. Over the past century, many theories of cognitive development and learning have been proposed in the field of psychology. Jean Piaget and Lev Vygotsky have proposed theories of cognitive development that are still driving research in child development and educational theory today (Huitt & Hummel, 2003; Byrnes, 2001).

Constructivist Perspective - Piaget

In the early 1950s, Jean Piaget was one of the first scholars to articulate the constructivist perspective of learning, proclaiming that children are not "empty vessels" waiting to be filled with knowledge (Piaget, 1954). Piaget described human knowledge as consisting of a set of frameworks that have been constructed as a result of adaptations to previous experiences (Berk, 1992). When an individual is confronted with an environmental situation that does not fit into one of these frameworks, disequilibrium occurs. To restore order, the person must equilibrate his or her frameworks through the process of assimilating that information into an existing framework or accommodating it by constructing entirely new frameworks (Piaget, 1985).

For Piaget, learning is predicated on the biological maturity of the individual. Piaget (1954) believed that individuals matured cognitively in response to their environment; however, general biological growth provides the framework for that cognitive maturation. He believed that the mind builds mental structures that allow it to achieve progressively better adaptations to its environment. The child's mind selects, interprets, and reorganizes information with regard to its existing structure. If the environment does not fit the existing model, the mind adapts to account for the disequilibrium created by the misfit information. In other words, cognitive development is an account of how an individual experiences and adapts to his or her environment (Piaget, 1974).

Piaget (1952) described stages of cognitive development as occurring in a sequential and invariant stepwise progression from simple thinking to sophisticated

abstract reflection. These stages emerge in a fixed order, are non-reversible, cannot be skipped, and are unaffected by temporary environmental variations (Berk, 1986). In other words, an individual will employ the same cognitive structures, or schemes, regardless of the situation.

An individual's progression through the stages is based both on physical maturity and environmental stimuli (Brainard, 1978). As a child's mind develops, it builds schemes to help it understand its world. These structures are constantly being modified as the child attempts to make sense of the environment. Piaget describes this equilibration as the continuous movement between phases of cognitive equilibrium and disequilibrium (Piaget, 1985). Equilibration occurs when an individual assimilates familiar information and accommodates unfamiliar stimuli.

Piaget's theories have had a major impact on children's education. Educators have relied on principles rooted in his theories to develop education programs (Van Glaserfeld, 1989). For example, Piaget places greater emphasis on the process of an individual's thinking than the product of that thinking. This principle has led teachers to ask more questions to gain insight into the processes a child uses when completing classroom tasks. Likewise, Piaget recognized the importance of a child's self-initiated active participation in learning activities (Van Glaserfeld, 1989). The Piagetian classroom limits the amount of direct instruction of ready-made knowledge activities. Rather, teachers design activities that encourage students to discover the knowledge for themselves.

A criticism of Piaget's theory is that he viewed cognitive development as a special case of an individual's biological development (Berk, 1992). For him,

knowledge being organized by the individual knower is unique to that person and inaccessible to others (Smith, 1995). Consequently, he de-emphasized the importance of language and individuals' social histories in knowledge construction. Relevant to this discussion is Vygotsky (1978) theory of cognitive development which incorporates social structures and communication with the constructivist notion of experientially-based cognitive maturity.

Sociocultural Perspective - Vygotsky

Since its translation into English, Vygotsky's research in cognitive processes has gained great status in the fields of child psychology and education. Vygotsky described knowledge as occurring in two forms: concepts and functions. A concept is a class of things that has a label and is defined according to a set of criteria. For example, the class "right triangles" would be a concept because it has a label and is defined based on a set of standards: one right angle, etc. A function is the process of applying the knowledge of those concepts to succeed on problem solving and memory tasks (Byrnes, 2001).

Vygotsky described two specific types of concepts dependent on an individual's understanding: spontaneous and scientific. Children who understand spontaneous concepts can correctly label something but cannot provide the specific classifying criterion for that item (Vygotsky, 1989). For example, a child might correctly describe a shape as being square but remain unable to provide some defining characteristics of that square. Vygotsky found that pre-adolescent children were, in general, only capable of this lower level spontaneous conceptual understanding.

11

As children mature toward adolescence, they develop the ability to provide several defining characteristics for a specific item (Byrnes, 2001). This mature understanding of the criterion for classifying an item was described as scientific conceptual understanding. According to Vygotsky (1989), as children mature they develop the knowledge that the label for an object is arbitrary. Therefore, they place greater emphasis on the defining characteristics of the object. This emphasis on definitions was viewed by Vygotsky as the individual demonstrating an understanding of scientific concepts.

In addition to concepts, Vygotsky proposed that the ability to integrate five main cognitive functions is a key skill for memory and problem solving tasks (Byrnes, 2001). He loosely defined these functions (language, thinking, perception, attention, and memory) with regard to how they might be utilized and integrated in order to succeed on a problem solving or memory task. Vygotsky believed that all higher mental functioning is rooted in social interactions (Berk, 1992); consequently, he placed the greatest importance on the integration of language function (Vygotsky, 1978).

How do individuals acquire the knowledge and skills to integrate these five cognitive functions? First, when trying to solve a problem, children communicate with themselves through private speech (Berk, & Garvin, 1984; Kohlberg, Yaeger, & Hjertholm, 1968). As children mature, private speech becomes more internalized with clear verbal utterances being replaced by soft whispers and silent lip movements (Berk, 1986; Frauenglass & Diaz, 1985). Vygotsky believed that all higher order mental functioning has social origins (Berk, 1986); however, the content of this verbal speech

is more than an imitation of teachers, parents, and friends. The sophistication of this speech is predicated on the individual's current developmental level (Byrnes, 2001).

Second, individuals acquire knowledge and skill through concept development. Through social interactions with teachers, peers, and parents, children's spontaneous concepts develop into scientific concepts (Berk, 1992: Byrnes, 2001). During these social experiences, children experience concept criteria that do not fit their current model. Through speech with themselves and others, they develop a greater set of guidelines for the label. Over time, these spontaneous concepts grow into scientific concepts.

Third, knowledge and intellectual skills are acquired and mastered in a progressive nature. Consequently, information and new skills must be presented to children with regard to their current mastery level. This social communication within a zone of proximal development plays an important role in cognitive development. Tasks that fall within a child's zone of proximal development are too difficult to be done alone but can be accomplished through cooperative communication with more advanced individuals. To be effective, this cooperative dialogue must offer a support system, called a scaffold, which helps the child master a task. According to Vygotsky, peers, parents, and teachers provide the resources to help students build a scaffold and master new and "higher" skills.

Vygotsky's theory of cognitive development has had a great influence in the classroom. Students in either a Piagetian or Vygotskian classroom have opportunities for active participation, with an emphasis on the process of thinking rather than the product, as well as acceptance of individual differences (Berk, 1986). However, in the

Vygotskian environment, the teacher guides the child's learning with verbal prompts, explanations, and demonstrations (Tharp & Gillmore, 1988). Slowly, the teacher pulls away to allow the children to assist themselves in discovering a solution. Finally, in Vygotsky's classroom, the teacher acts in the learning process by providing cooperative learning experiences in the form of group work.

As with all theories of cognitive development, challenges to Vygotsky's theory exist. One specific challenge is that his theory of instruction might be culturally specific. For example, Rogoff (1990) observed young Micronesian children's education in sailing a canoe and found that they learned more from direct practice and observation than they did with verbal guidance by adults. Without regard to culture, Carpenter, Fennema, Franke, Levi, and Empson (1999) proposed that teachers allow young children to discover their own methods of solving math problems, rather than guide them toward a solution, as Vygotsky would suggest. Children come to the classroom with a certain knowledge base. By using that knowledge to solve a problem, students gain a greater understanding of the mathematics than if they had been led to the solution by their teacher.

Emergent Perspective

In the mid 1990's, theorists began to write about a perceived limitation regarding both the constructivism and sociocultural perspectives of learning (Smith, 1995). Specifically, they identified the gap that exists between knowing and knowledge in the two perspectives. Constructivists identify knowledge as the internal mental processes that individuals perform to organize information; consequently, it is unique to the individual knower and cannot be made part of the community of knowledge (Cobb,

1995). In contrast, the sociocultural perspective emphasizes interactions between individuals and groups of individuals and the social constructs and language that grow out of those interactions.

Cobb and Yackel's (1995) emergent perspective is an attempt to bridge the gap that exists between the constructivist and sociocultural perspectives. The emergent perspective accepts that knowledge for the individual learner is organized through independent processes and is unique for that individual. This organization occurs in reaction to the individual's participation in and reactions to social practices of the community. Therefore, a child's understanding of a particular concept might be unique to the child, but it is influenced by that child's interactions with others in the community. Moreover, the way that the child organizes his/her understanding of the concept will influence the way he/she communicates it to others in the community.

In contrast, Driver et al. (1994) affirms the gap that exists between the constructivist and sociocultural perspective. They propose that these two orientations are not mutually exclusive; rather, the distinction between constructivism and socioculturalism forms a continuum. They establish their position on this continuum as neither exclusive constructivism nor socioculturalism (Driver & Scott, 1995). In fact, these authors claim that as educators "it is particularly important to adopt a perspective that embraces both perspectives" (p. 28) because educators are concerned with the interactions that occur between a student's personal knowledge and knowledge as a social construct.

In her syllabus, the instructor claims that the course under examination in this project will be taught from a "constructivist and sociocultural perspective."

Consequently, I will locate my theoretical perspective in this paper in a manner that is similar to Driver et al. (1994): somewhere along the constructivism/socioculturalism continuum, exclusively in neither orientation. In the next section, I will outline the impact that these two perspectives have had on education.

Constructivist and Socioculturalist Influence in the Classroom

While constructivist and socioculturalist theories do not specifically suggest one particular pedagogy, their fundamental precept, that individuals learn through doing, has had a tremendous impact in the classroom. Specifically, constructivism addresses the roles and responsibilities of the learner and teacher in the learning process.

Constructivism and sociocultural theories place the responsibility for learning on the learner. The learner is actively involved in the learning process, looking for regularity and order in classroom events in order to build knowledge (Von Glasserfeld, 1989). The level and source of motivation is another crucial assumption regarding the nature of the learner. Confident learners have a sustained motivation to learn (Von Glasserfeld, 1989). The confidence that develops from multiple first-hand experiences mastering problems is much more powerful than external acknowledgment and motivation that derives from the teacher (Prawat & Floden, 1994). For Vygotsky, successful mastery of these problems is predicated on the teacher presenting problems that occur within a student's zone of proximal development and problems for which appropriate scaffolding has occurred (Vygotsky, 1989).

According to the constructivist and socioculturalist approaches, instructors should be learning facilitators who guide student learning rather than knowledge keepers who supply knowledge to students (Bauersfeld, 1995). Constructivist teachers

help students gain understanding of content by providing appropriate learning scenarios about which students create order. Consequently, the emphasis turns from the instructor and content toward the knowledge-building learner (Gamoran, Secada, & Marrett, 1998). In constructivist classrooms, teachers, serving as facilitators, ask students questions to guide discovery, provide flexible environments for students to build understanding, continuously communicate with learners through dialogue, and steer learning experiences in order to make them meaningful to the learner (Rhodes & Bellamy, 1999; Brownstein, 2001).

In mathematics education, the constructivist and socioculturalist approaches represent a dramatic shift from the traditional teacher as teller and student as listener model that was so prevalent in the 20th century. In a constructivist mathematics classroom, understanding must be derived by the student, not given by the teacher. In the following section, I will discuss specific characteristics of the constructivist mathematics classroom.

General Mathematics Classroom

In 1989 and 1991, the National Council of Teachers of Mathematics published two sets of recommendations regarding the teaching and learning of mathematics (NCTM, 1989; NCTM 1991). The *Professional Teaching Document* recommended a shift from teacher as lecturer and student as passive listener to the concept of a classroom community (Malloy, 2003). In these classrooms, students would actively participate in the instruction by using logic and mathematical facts to solve problems, and they would employ mathematical reasoning and conjecture in the application of those problems (NCTM, 1991). In 2000, NCTM published a revised set of standards,

17

called the *Principles and Standards for School Mathematics,* to address criticisms of the initial version; these revised standards are more strongly grounded in the cognitive development literature. This revised document provides specific suggestions for mathematics instruction. Specifically, mathematics instruction should emphasize both procedural and conceptual understanding. Individuals should learn not only *how* to solve the mathematics problems, but *why* their solution strategy works. This notion of a mathematics classroom is a dramatic shift from the traditional one in which students memorize formulas to solve the problems.

Stimulating mathematical understanding, writing appropriate tasks, and creating an appropriate classroom environment are some of the important components of a standards-based classroom. In the next three sections, I will discuss how each of these components impacts the constructivist mathematics classroom described in the NCTM documents.

Mathematical Understanding

In the discussion section of his review of the Euclidean style geometry classroom, Schoenfeld (1988) provides the following general guidelines for a constructivist approach in mathematics education that stimulates mathematical understanding:

1. Thinking mathematically is a major goal for mathematics instruction.
2. Mathematics is a complex and highly structured discipline.
3. Making connections to previously learned facts and procedures is an important component of thinking mathematically.
4. Thinking mathematically involves meaningfully and flexibly applying formal knowledge to mathematical situations.
5. Students interpret and make sense of their world by building mathematical frameworks.
6. Those frameworks shape the way they experience future mathematics.

Schoenfeld's recommendations predate the NCTM documents and describe guidelines that, when followed, can lead students to gain mathematical understanding of the topics being presented.

More recently, Carpenter and Lehrer (1999) defined five mental processes that stimulate mathematical understanding. Students can gain mathematical understanding when they construct relationships by making meaning from the way current concepts are related to other mathematical concepts or previous mathematical experiences. Mathematical understanding also occurs when students extend and apply mathematical knowledge by creating rich, integrated knowledge structures for mathematical concepts. Moreover, individuals who reflect on their mathematical experiences regarding the concepts are able to apply that knowledge to solve unfamiliar problems and gain mathematical understanding. Fourth, when working in a mathematics community, students can gain mathematical understanding by articulating what they know about the mathematical concept in their own words. Finally, when students construct knowledge of the concepts, they gain understanding because they make mathematical knowledge their own. Individuals do not perceive that knowledge as something told or explained to them. Instead, they adopt a stance that knowledge is their own and has evolved based on their experiences through discovery.

Carpenter and Lehrer (1999) propose that the occurrence of one or more these mental processes is a necessary, but not a sufficient, step toward gaining mathematical understanding. In order for understanding to occur, students must also complete meaningful mathematical tasks in a cooperative learning classroom environment.

Task Design

The authors of the 1991 NCTM Standards document communicated the importance of tasks by choosing as the first standard "Worthwhile Mathematical Tasks" (NCTM, 1991). This standard provides teachers with guidelines on which to base tasks. For instance, tasks in a standards-based classroom should engage students intellectually, develop mathematical understanding and skills, stimulate students to make connections to previous mathematical experiences, require that students engage in problem solving in order to solve the task, stimulate the students to communicate about the task, and represent mathematics as an ongoing human activity by placing the task in a relevant context (NCTM, 1991).

The Standards document also produced a set of considerations for teachers to follow when writing tasks. Teachers should provide tasks that are based on sound mathematics. They are also called to generate tasks that are based on an understanding of the students for whom the tasks are written (NCTM, 1991). While this document provides suggestions for the use of tasks in the classroom, it does not attempt to classify them.

In a report of their work for the QUASAR project, Stein, Smith, Henningsen, and Silver (2000) emphasized the importance of task design in stimulating mathematical understanding in their students. They classified two broad categories of tasks based on the cognitive demand required to complete them: low demand tasks and high demand tasks.

These authors classified "memorization tasks" and "mathematics without connection tasks" as low cognitive demand tasks. These tasks require low cognitive

demand to complete and are appropriate when the goal is to promote speedy and accurate reproduction of simple mathematical skills. The exclusive use of low demand tasks can lead to limited understanding of the mathematical concepts being taught. Unfortunately, low demand tasks are a common element in a direct instruction lesson (Stein et al., 2000).

In contrast to low demand tasks, high cognitive demand tasks are presented to help students gain a greater understanding of the mathematics underlying the task. These tasks, subdivided into "mathematics tasks with connection" and "doing math tasks" are often presented in student-centered classrooms in which the students construct their mathematical knowledge and understanding through meaningful experiences (Stein et al., 2000).

In order to maximize the effectiveness of mathematical tasks, teachers must consider various student factors. Teachers help students gain mathematical understanding by writing tasks that match their students' readiness and provide a proper scaffold for learning. Carpenter and Lehrer (1999) recommend that teachers begin slowly when presenting high demand tasks. During these tasks, teachers must observe students and be flexible to adjust for any difficulties (NCTM, 1991).

The best tasks can be rendered ineffective if teachers forget their facilitator role in the classroom and provide specific solution strategies, thereby stifling the students' ability to explore the tasks (Stein et al., 2000). However, when tasks are presented in an environment that supports cooperative learning, encourages classroom discourse, and values appropriate sociomathematical and sociocultural norms, high-cognitive demand

21

tasks can lead to mathematical understanding. In the next section, I will discuss the classroom environments that support standards-based instruction.

Classroom Environment and Discourse

Numerous mathematics researchers have placed an emphasis on nurturing classroom discourse that contains specific properties (Blanton, 2002; Elliott & Kenney, 1996; NCTM, 2000; Sherin, 2002). During this discourse, students explain previously stated ideas that have been generated in response to comments heard from other classmates. The teacher's role during this discourse is to guide these conversations to elicit student ideas that further the exploration of the topic.

In his article titled *Never Say Anything a Kid Can Say,* Reinhart (2000) provides more specific guidance for teachers as they facilitate classroom discourse. In this paper, he describes the types of classroom discussions that lead students toward mathematical understanding. With regard to questioning strategies, Reinhart suggests that teachers should employ patience and, as the title suggests, let the kids do the talking. In an effort to maximize the amount of material covered during instruction, teachers often bail their students out by answering questions for them. Herbel-Eisenmann and Breyfogle (2005) proposed questioning styles such as funneling and focusing to help guide students toward understanding.

Reinhart (2000) also recommends general communication strategies for success in the classroom. For example, he suggests that teachers share with students their reasons for asking the question. By learning the ways teachers link each question to the mathematics, students can develop their own connections. The author also recommends that teachers do not answer the question, "Is this correct?" Students might obtain an

answer to that question through conversations with other classmates. Likewise, teachers should not repeat student answers. Rather, they should allow students to work through the conversations themselves by repeating the question for each other to gain clarity. Finally, Reinhart recommends that teachers require everyone to participate. Participation may take many forms, but it requires the student to engage in the learning process.

Heaton (2000) would say that Reinhart provides recommendations that facilitate a classroom discourse and nurture learning. She describes this discourse as a dance in which the teacher serves as the choreographer, the dancer, the stage manager, and the set designer. Reinhart (2000) would likely say that the teacher is also the audience. To establish a healthy classroom discourse, Reinhart requires students in his classes to pose questions to each other if they cannot contribute to the discussion and to ask the class for help when it is needed. He also is careful never to carry a pencil, instead requiring individuals in the class to answer his questions. In this way, students build knowledge by talking to each other about mathematics, while the teacher, who is no longer the source of knowledge, functions as a facilitator.

In general, Reinhart promotes a classroom environment that Kazami and Stipek (2001) would describe as having a "high press for learning." These classrooms place value on students' learning and understanding of mathematics. They emphasize the importance of the students' efforts toward obtaining understanding. Reasons for solutions are more highly valued than the solutions themselves. In these classrooms, students have a sense of autonomy and responsibility for their learning.

Likewise, Yakow and Cobb (1996) describe a set of sociomathematical norms that facilitate mathematical understanding in the classroom. Classrooms that view errors as a method of furthering understanding, regard mathematics as a collaborative effort, provide explanations that are rooted in mathematical reasoning, and employ mathematical thinking that is based on mathematical understanding have a set of values that will stimulate lifelong learning.

Students in classrooms that contain a high press for learning and the sociomathematical norms described above view the teacher as a guide and a resource to help them gain understanding, not as a source of knowledge. Likewise, teachers in these classrooms relinquish their front-of-the-room control of the mathematical knowledge by providing meaningful, high cognitive demand tasks that foster classroom discourse.

Another component of the classroom environment that must be considered is the context of the instruction. Students do not come to the classroom as empty vessels ready to be filled (Piaget, 1954). Rather, they bring with them a rich history of experiences with mathematics, both inside and outside of school. Teachers setting mathematics in context can encourage discoveries that promote confidence in students (Boaler, 1993). Fasheh (1999) claims that providing a context for mathematics makes it meaningful. By seeing this human side of the discipline, students begin to see the role that they can play in the mathematics universe, thereby gaining confidence in themselves.

The mathematical learning that occurs in the above described environment takes place when students work as a cooperative community of learners to gain

understanding. In the following section, I will briefly review the literature regarding cooperative learning groups.

Cooperative Learning

Johnson and Johnson (1999) described cooperative learning as existing "when students work together to accomplish shared learning goals" (p. 1). The impact of cooperative learning research on education has been tremendous. In fact, Slavin, Hurley, and Chamberlain (2003) report that "research on cooperative learning is one of the greatest success stories in the history of educational research" (p. 177).

A substantial body of research exists to support the effectiveness of cooperative learning when assessed with both student achievement and non-achievement factors. Higher student achievement in cooperative learning has been demonstrated when compared with a variety of control methods on a wide range of outcome measures (Slavin, 1995; Stevens & Slavin, 1995; Slavin et al., 2003; Springer, Stanne, & Donovan, 1999). This achievement effect has been reported in studies of cooperative learning at all grade levels in many educational settings (Slavin et al. 2003). In addition, more recent research has demonstrated the positive, non-achievement impact of cooperative learning. For example, students who work in cooperative learning groups demonstrated more willingness to take on difficult tasks, greater intrinsic motivation to complete tasks, and increased ability to generalize concepts across content areas (Slavin et al., 2003).

While researchers agree on the outcome of cooperative learning, some confusion about how and why cooperative methods affect learning can be found in the literature. Slavin et al. (2003) defined the motivational, social cohesion, cognitive-development,

and cognitive-elaborative to be four mutually inclusive theoretical perspectives on the consequences of cooperative learning.

The first two perspectives, the motivational and social cohesion perspectives, presume that motivation to complete a task impacts the student learning process in cooperative learning. The motivational perspective focuses on the extrinsic and intrinsic motivation that students experience in their drive to learn. Motivationalist scholars believe that students achieve their own goals to learn by helping others in their group (Slavin, 1995). On the other hand, the social cohesion perspective proposes that performance in cooperative learning groups is dependent on the cohesiveness of the group. Therefore, students who have developed self-identification from being a part of the group are motivated to help their peers because they care about them (Johnson & Johnson, 1999).

The two cognitive perspectives, the developmental and cognitive elaboration perspectives, hold that the mental processing of information that occurs during interactions among students will increase achievement. Drawing on Piaget's and Vygotsky's theories, the developmental perspective suggests that learning occurs in the interactions among students around developmentally appropriate tasks. It emphasizes the role more advanced students play in guiding learning of their peers through interactions occurring in their zone of proximal development (Slavin et al., 2003). In contrast to the developmental perspective, the cognitive elaboration perspective emphasizes how the interactions that occur in cooperative learning benefit both students receiving support and the providers of the support. Learning occurs during the

elaborations that occur as students explain their understanding of concepts to group-mates.

While these four perspectives are unique, they compliment one another and may all positively affect learning. For example, both individual and group goals can lead to an increase in personal motivation to learn as well as commitment to ensure that all group-mates succeed. In the effort to achieve these goals, students gain mathematical understanding when participating in developmentally appropriate interactions among peers. These interactions come in the forms of assessment and corrections through peer modeling and elaborative explanations that enhance learning.

Webb (1991) reports that when implemented properly, cooperative learning can be almost as effective as one-to-one instruction from a teacher and can be stronger than teacher-led instruction. Significant research examines the conditions under which cooperative learning positively influences student understanding. Slavin (1995) found significant positive effects of group work when interventions are designed to reward learning of all group members. Webb (1991) reported that group work benefited students when feedback from peers comprised elaborative explanations rather than answers or procedural information. Others reported that establishing group goals and individual accountability (Fantuzzo, King, & Heller, 1992; Fantuzzo, Davis, & Ginsburg, 1995) promoted cooperative group activity that enhanced learning.

Meaningful tasks play an important role in the success of cooperative learning groups (Cohen, 1994). Cohen proposes that instructors present students with open-ended tasks that emphasize higher order thinking and require input from all members of the group. Tasks that do not offer solution strategies allow diverse thinkers in the group

27

to present original strategies to group-mates. Cohen also suggests that more members

of the group make connections to the mathematical concepts when they are presented

multiple tasks that relate to a central intellectual theme. Diverse learners experience

tasks differently and thus make unique contributions to group work because of their

varied perspectives.

Thus, students who work cooperatively in groups can develop mathematical

understanding as they explore meaningful tasks. To facilitate these explorations,

mathematics teachers have at their disposal a variety of tools. In the following section,

I will briefly discuss the literature regarding the use of tools such as manipulatives and

technology in the classroom.

Tools that Support Learning

In constructivist mathematics classrooms, teachers utilize a variety of tools to

facilitate student learning. Manipulatives, like pattern blocks, counting cubes, and

pictures, stimulate physical activity and can provide a means to explore mathematics in

a task (Clements & Battista, 1992). Carpenter et al. (1999) suggest that elementary

school-aged students utilize pictures, counting manipulatives, number lines, or some

other tool to solve meaningful mathematics problems. By exploring the CGI problem

with concrete tools and discussing solution strategies with classmates, children

eventually gain a conceptual understanding of the mathematics. This understanding

eventually allows the students to employ internal strategies, such as derived facts, to

solve similar CGI problems.

Physical tools, such as manipulatives and pictures, are most effective when

students utilize them to discuss high-demand cognitive tasks. Specifically, students will

not gain understanding if the teacher tells the students exactly how to use a particular

manipulative to solve the problem (Stein et al., 2000). Likewise, pictures can become

non-concrete if they are used ineffectively (Sowell, 1989), but pictures that vary and/or

are utilized in conjunction with other tools can be effective in the classroom (Clements,

2003).

The 2001 NCTM *Principles and Standards* call for an increased use of

technology in the classroom to enhance student learning. Significant research exists

regarding the efficacy of technology in the classroom. For instance, Battista (2007)

found that draggable geometric figures that are constructed based on a set of geometric

principles help students explore that figure to develop a notion of proof. Likewise,

Garfield and Ben-Zvi (2006) report that the automation of complicated calculations

allows students to more deeply explore certain statistical concepts. Finally, calculators

can provide students the ability to investigate aspects of functions and gain a deeper

understanding of them (Fey, 1992).

Heibert (2003) reports that a simple answer to the complex question regarding

the effectiveness of the NCTM Standards Document is that it is "consistent with the

best and most recent evidence on teaching and learning in mathematics (p. 5)." This

document, in conjunction with other theories of geometry learning can inform

instruction in geometry classrooms. In this next section, I will present those theories

and how they have been implemented in standards-based geometry classrooms.

<center>*Theories of Geometry Instruction*</center>

In addition to general learning, Jean Piaget was interested in the way individuals

justify mathematical results, particularly with regard to proof (Battista & Clements,

<center>29</center>

1995). He described stages through which individuals pass that entail the development of logical reasoning without regard to specific content. Piaget believed that an individual's ability to provide theoretical justification for a mathematical phenomenon progresses through three stages that vary in their level of sophistication. He believed that individuals pass from one stage to the next through the debates that develop in the mathematical discourse occurring in collaborative classrooms.

Piaget's Stages for the Development of Proof

Using his theory of cognitive development as a framework, Piaget proposed three stages of thinking with regard to proof. As with his theory of cognitive development, Piaget's stages describe the development of logic without regard to a specific concept (Battista & Clements, 1995). In other words, Piaget described a general complexity of mathematical thought rather than specific understanding of individual concepts. Piaget believed that an individual's progression through these stages was sequential and dependent on that person's cognitive maturity.

At stage one, children think about mathematical problems unsystematically, illogically and unreflectively (Clements & Battista, 1992). At this stage, they fail to integrate all of the information about the problem into a coherent whole. They work toward solutions randomly, without a plan. Consequently, their conclusions may be contradictory. Battista and Clements (1995) illustrate this thinking by presenting children a "proof" that the sum of angles inside a triangle is 180 degrees. They presented children a single triangle and instructed them to cut off its corners. Then, the children were asked to put the corners together and discuss their findings. While many stage one students recognized that the corners of their triangle made a straight line, they

30

failed to predict that the corners of another triangle would also make a straight line. Moreover, some children failed to recognize that the angles of their particular triangle would still make a straight line if the order of the angles was changed. Their lack of systematic and logical thinking about the problem led to contradictory conclusions about subsequent triangles.

Children begin to use empirical results to make and justify their conclusions at stage two. However, their systematic search for information and logical, reflective thinking about the problem is limited to issues in which they believe. Thus, even after these students have determined that the angles of the triangle make a straight line, they conduct an analysis of each new triangle. They struggle to establish the relationship of the three angles in the new triangle, often being misled by the shape of the triangles. Gradually, they begin to believe the generality of their findings and are able to make logical predictions about new triangles. Similar to concrete operational thinkers, these children require concrete evidence of a fact before they begin to believe a general theorem.

Students become sophisticated thinkers about problems at stage three. These students are capable of using formal deductive reasoning based on assumptions to make logical conclusions. They use their abstract thinking ability to operate explicitly in mathematical systems. When these students are presented the triangle problem, they believe the generality of the outcome. They employ an axiomatic system of theories based on existing knowledge to generate a logical proof of this notion. Furthermore, they are able to make related deductions based on existing knowledge. For example,

when investigating the sum of a triangle's interior angles, these students can deduce that three angles, whose sum is greater than 180 degrees, cannot form a triangle.

Piaget believed that children progress from one stage to the next as a result of contact with others (Clements & Battista, 1992). The debates that arise out of peer group work stimulate the individual to seek verification of statements. Through argument with others, individuals begin to become aware of their own thoughts and definitions about a topic. They develop the ability for introspections and begin to take the perspective of others. Finally, with the achievement of formal operational thought, students are able to mentally test ideas to produce logically constructed and reflective proofs (Piaget, 1928).

The Van Hiele Model of Geometric Thought

In response to the great difficulty they saw students having in geometry, Pierre and Dina van Hiele began to investigate how their students understood geometry and the complexity of their thinking about geometry. This investigation led to the creation of a stage model that describes the level of students' geometric aptitude. Beginning with a rudimentary understanding of geometric shapes and figures, aptitude progresses to an internalized and integrated understanding of geometric systems. Additionally, the van Hieles developed a five phase classification of instruction to help educators teach students to be more sophisticated thinkers about geometry.

An emphasis of this model is that both the learner and educator play a fundamental role in geometry teaching. A teacher's awareness of the hierarchy of students' geometric thinking helps guide them through the instructional phases that the van Hieles present. For students, learning is a stepwise progression through five

necessary and sequential levels of geometric thinking. Conceptual understanding that is

typical of a lower level will be fully understood by individuals thinking at a more

advanced level. Unlike in Piaget's theory, movement to a more advanced van Hiele

level is topic specific (Battista & Clements, 1995). Therefore, a person might have a

level II understanding of polygons, yet possess level I understanding for polyhedra.

Progression through the levels is dependent upon the student's exposure to cognitively

appropriate geometry experiences. According to Crowley (1987), the levels are:

Level 0 – Visual/recognition. Students focus on the appearance of geometric

constructs. They make identification of and conduct operations on those shapes

based on concrete information gained through physical appearance and visual

transformations of objects. Students at this level can recognize shapes, but will

not provide explanations of the properties of the shapes. Rather, they base their

observations on concrete examples from the past. For example, a student might

say, "that shape is a square because it looks like checker board." If the teacher

cuts the square in half, the child might classify the two shapes as a rectangle,

"like one of my books at home." No mention would be made of the properties

of squares and rectangles.

Level I – Description/analytic. Establishing rules for shapes via

experimentation is a characteristic of students at this level. They use

measurements, illustrations, observations, and models to establish properties that

they use to classify, identify, and describe geometric constructs. Students at this

level would identify the square because they can see that the four sides have

equal measurements. They would then classify the rectangles appropriately

because the opposite sides have equal measurements. These students would remember the rules about squares and rectangles based on previous experience. One distinction that they would not be able to make is the general classification of these shapes. Thus, even though they would know that both a rectangle and square are four-sided figures with opposite sides congruent, they would not be able to classify them jointly as rectangles. The problem would lie in their focus on the fact that all four sides of the square are congruent and their neglect of the fact that this observation would mean that the object can also be classified as a rectangle. Senk (1989) reports that over 70 percent of students entering a high school geometry course are at this level.

Level II - Abstract/relational. Students use logical reasoning to form abstract definitions, to distinguish between necessary and sufficient conditions for a concept, and to understand and present logical arguments. This ability allows them to create a hierarchical classification of geometric properties. De Villiers (1987, in Battista & Clements, 1995) suggests that this is the stage where deductive reasoning for proof occurs. Senk (1989) claims that this deductive reasoning is a necessary prerequisite for success in a proof-oriented geometry course. Even so, these students do not fully understand the importance of axioms, and they do not make deductions based on these theorems. Rather, they reason based on experimentally obtained properties of geometric constructs. Their arguments are experimentally based rather than axiomatically based. With regard to the square and rectangle example, these students would identify the squares and rectangles appropriately because of properties of those shapes.

Likewise, they would correctly classify them as rectangles. They would also be able to generalize the results of this experiment to state that all squares that are similarly cut in half will result in two rectangles.

Level III - Formal deduction. At this stage, logically interpreted geometric statements are utilized to provide formal definitions, postulates, definitions, and theorems. Sutherland, Trzinki-Beker, and Tsering (2002) report that advanced high school students think geometrically at this level. They understand the necessity of the axiomatic system to establish geometric relationships and classifications. Similarly, they can use deductive reasoning and understanding of rigorous mathematics to make reason based conclusions. When presented the square problem, these students might attempt to prove that all squares that are cut in this specific way will necessarily result in two rectangles. They would base their proof on the theories and axioms that they already know.

Level IV - Rigor/mathematical. Students focus on formal reasons about mathematical systems rather than postulates within them. These students can compare and formulate different theorems, axioms, and postulates by precisely dealing with the fundamental relationships between structures. They can reason formally by manipulating geometric statements to establish and compare axiomatic systems of geometry.

Advancement from one level to the next is experientially based. A student advances from one level to the next through experiences and instruction that lead to the mastery of a particular level. Consequently, the teacher plays a vital role in a student's advancement from one level to the next. By using appropriately leveled language and

examples, the teacher can provide the experiences that allow the individual to gain

mastery of the material. The van Hieles have provided instruction guidelines to help

teachers choose the appropriate geometric exercises and classroom activities to assist

students in moving from one phase to the next. Van Hiele's five phases of instruction

are the following (Clements & Batista, 1992):

> *Phase 1 - Information*. Students become engaged with the object of study
>
> through conversation with each other and the teacher. In order to introduce the
>
> new concept, the instructor makes observations, introduces terms, asks relevant
>
> questions, and encourages independent work. During this time, the teacher is
>
> collecting data regarding the students' way of thinking in order to generate
>
> activities that lead to useful and purposeful understanding.
>
> *Phase 2 – Guided Practice*. Students are aware of key facts about the objects or
>
> concepts being studied. In this stage, they deepen that understanding by
>
> exploring that material through carefully sequenced activities. As the students
>
> explore the geometry, the teacher emphasizes different relationships involving
>
> the topic through classroom discussion and activities.
>
> *Phase 3 - Explication*. As students become more aware of the material, they try
>
> to express key concepts and relationships in their own language. The teacher's
>
> role is to guide this expression by clarifying previously introduced terms,
>
> introducing new terms, and providing activities that motivate the students to
>
> employ their new understanding.
>
> *Phase 4 – Free Orientation*. Through activities and problem-solving tasks,
>
> students learn to use these newly acquired skills to solve relevant tasks in many

different ways. To facilitate this learning, teachers select appropriately

challenging geometric problems. They guide the students toward different

solutions through instruction and discourse. During discussions, the students

provide comments and descriptions of their problem solving strategies.

Phase 5 - Integration. Students are able to internalize and integrate the new

concepts, relationships and skills into an existing knowledge base. The

instructor's role in this phase is to provide activities and lead discussions that

encourage students to summarize and employ all that they have learned about

the topic. Teachers can assess this learning by providing a survey of what they

have learned.

After completing the fifth phase of instruction, students attain a new, more

sophisticated, level of thinking about the geometric concepts. As mentioned earlier,

students must have a complete understanding of the geometric topic before they move

from one level to another (Sutherland et al., 2002). Consequently, a student with a level

I understanding of a particular topic will be unable to comprehend material that is

geared to students with a level II understanding. In addition, students must be provided

with all five phases of instruction before they can optimally move from one level of

understanding to another.

Similar to Piaget's stages of proof, advancement from one level to another is

experientially based. While the achievement of abstract thought is a necessary

condition for progress to the most advanced levels, it is not a sufficient condition.

Rather, the primary method for an individual to advance from one level to another is

through student centered instruction, experiences, and reflection. With the assistance of

teachers who provide appropriate geometric exercises and allow sufficient debate within the classroom, students can develop the language and logic skills necessary to advance through the stages.

In general, the efficacy of the van Hiele model is well accepted (Battista, 2007). However, some limitations do exist. For instance, two studies (Gutiereze, Jamie, & Fortuny, 1991; Lehrer, Jenkins, & Osana, 1998) observed that students develop the same concept at more than one level at a time. Likewise, Pegg and Davie (1998) noted that progression through the stages is more continuous than the van Hieles proposed.

Various authors have addressed these concerns in the literature. For example, Pegg and Davie (1998) have adopted Biggs' Structure of Observed Learning Outcomes or SOLO taxonomy (Biggs, 1999). According to this model, students pass through various modes of thinking as they progress from one stage to another. Clements and Battista (2001), in contrast, describe the students' progress through the stages of reasoning as occurring in continuous waves of acquisition. Both of these alternative models work within van Hiele's well-regarded framework (Battista, 2007).

What role do Piaget's and van Hiele's models of proof and geometric reasoning, respectively, play in geometry classrooms? In this next section, I will explore two types of geometry instruction presented in the literature, one of which is grounded in the recommendations that grow out of the aforementioned models.

Geometry Instruction

How do the geometry classrooms of the past 25 years look in comparison to what the Piagetian and van Hiele models suggest? When discussing what should be covered in a geometry course, Ball (1993) suggests that the geometry curriculum

replicate what geometers do in the field. In order to examine the feasibility of her

proposal, we must first examine the ways in which geometry is taught.

Goldenberg and Couco (1998) propose a continuum of geometry instruction that

progresses from traditional to reform. The most traditional instruction falls in line with

the Euclidean model, utilizing definitions of indefinable objects such as points, lines,

and planes to build an axiomatically-based model of geometry. In these classrooms,

called replicating Euclid classrooms, teachers present knowledge that strictly adheres to

proof based on objects with which students have little experience.

Until the 1980s, the Euclidean model of geometry instruction was highly

regarded (Chazan & Yerushalmy, 1998). Good teachers presented clear arguments and

had "10 different ways" to say the same thing (Shoenfeld, 1988). The teacher, as the

source of knowledge, helps students memorize key concepts and "proofs" by providing

specific examples for the students to follow.

Schoenfeld (1988) reviewed a typical geometry classroom being taught by a

well-regarded teacher whose students regularly performed well on end-of-course

assessments. He described a classroom which emphasized correct answers to the

problems presented, sometimes at the expense of mathematical understanding. Students

in this classroom seemed to believe that all mathematics tasks can be solved in five

minutes or fewer by employing a pre-formed strategy presented by the teacher.

Consequently, simple modifications in problems, such as turning a triangle on its point

or creating longer tasks that required critical thinking, caused students to struggle and

eventually surrender.

Goldenberg and Couco (1998) labeled geometry classes at the reform end of the continuum as inductive. Contrary to the traditional Euclidean-style geometry classrooms, students in these student-centered classrooms complete mathematical tasks that help them reason from specific to general. Teachers utilize the instructional strategies proposed by Piaget and van Hiele in order to support the students' construction of geometric understanding.

Inductive Geometry Classroom

Inductive geometry classrooms are student-centered, with teachers facilitating student discovery through discourse with other students and the teacher. These geometry classrooms fit the guidelines set forth by the NCTM *Principles and Standards* document (NCTM, 2001), in which students build mathematical knowledge under the guidance of a teacher, who facilitates their discovery through relevant questions and tasks. Inductive geometry classrooms provide students the opportunity to perform the mental functions that develop mathematical understanding (Carpenter & Lehrer, 1999) through high cognitive demand tasks (Stein et al., 2000). This instruction occurs in classrooms that have established sociomathematical norms that stimulate mathematical understanding (Yakel & Cobb, 1996) and have a high press for learning (Kazami & Stipek, 2001). A discourse in which all students participate by asking questions of each other for clarity (Reinhart, 2000) is an important component of an inductive geometry classroom. In sum, inductive geometry instruction is a special type of standards-based mathematics instruction.

While discovery plays an important role in the inductive classroom, Klausmier (1992) suggests instructors should strive to include both discovery and expository

methods in each lesson. The expository method involves utilizing verbal cues to aid

learning and to provide feedback toward mathematical understanding. For example, if

students were working to discover number patterns in geometry and connect them to

algebra, then an instructor would move about a classroom watching and listening to the

group-work. If the teacher noticed that a group's progress had stagnated, she/he would

provide a verbal or visual prompt to guide it along. This combination of discovery task

with expository feedback stimulates the groups' movement toward mathematical

understanding.

Manipulatives and pictures play an important role in geometry classes and are

frequently used to provide a rich learning experience (Battista & Clements, 1992).

Battista (2007) suggests that utilizing manipulatives in the classroom stimulates higher

geometric thinking by allowing students to construct sound geometric representations of

specific concepts. Nevertheless, Fuey (1992) cautions against relying so heavily on the

use of manipulatives that instruction suffers.

Pictures can also be employed in the classroom to facilitate mathematical

understanding because they can give an immediate physical representation of the

geometric concept (Clements, 2003). However, pictures can also be too abstract unless

utilized in conjunction with other tools (Sowell, 1989).

While manipulatives and pictures can be useful tools toward developing

geometric reasoning, they are only marginally useful in developing an understanding of

proof. However, understanding proof is an important component of any geometry

classroom. In the next section, I will discuss the role of proof in inductive geometry

classrooms.

Geometric Proof

Proof is critically important in mathematics classes because it allows students to see the validity and reasoning of specific mathematics concepts (Battista & Clements, 1995). Students who work to justify the facts involved with a geometric construct gain a deeper understanding of that construct (Goldenberg & Cocco, 1998). Proof allows students to visualize, illuminate, and classify geometric concepts (Ball, 1976).

Nonetheless, students at all levels have great difficulty with proof, thus leading to significant misunderstanding. Clements and Battista (2000) report that students struggle to develop proof and cannot establish the mathematical truth of statements. Similarly, McCrone and Martin (2004) found that geometry students demonstrated poor performance on nearly all proof-constructions. Students do not seem to understand the power of proof, believing that proofs are irrelevant (Healy & Hoyles, 1998). Further, Chazan (1993) highlighted students' belief that proof merely provided evidence that a statement is true, not proven.

The literature is unclear about why students struggle so much with proof (Battista, 2007). Still, proof plays an important role in gaining mathematical understanding. Fortunately, teachers have modern tools at their disposal to help students gain an understanding of proof. In this next section, I will review the role of technology, such as dynamic graphing environment (DGE) software, in the development of proof.

Dynamic Graphing Environment

One strategy for assisting students in the development of proof is the use of dynamic graphing software. Dynamic graphic software programs such as Geometers

Sketch Pad (GSP) provide a framework in which students can explore geometric constructs. These programs allow users to construct objects according to specific restrictions. Students can explore those objects by modifying their unrestricted characteristics

This "draggability" of objects is an important feature of DGEs. Battista (2007) reports that students gain a greater conceptual understanding when they can manipulate geometric objects that have been constructed according to specific geometric rules. Similarly, students can gain a deeper understanding of the relationship between specific classes of objects in these environments (Jones, 2000).

Clearly, when used within the NCTM framework, DGEs play an important role in helping students gain an understanding of proof. Students can describe the exact properties of specific geometric objects and gain the ability to formally discuss those properties (Battista, 2007). Jones (2000) claims that the conjectures and tests that occur in DGEs help students bridge the gap between justification and proof. In general, the use of DGEs can enhance geometric thinking (Yerushalmy, 1993; Yerushalmy & Chazan, 1993; Clements & Battista, 1992).

Mathematical Instruction in the Post-Secondary Classroom.

Earlier in this chapter, a model for general reform mathematics instruction for geometry students in secondary classrooms is constructed from the literature; however, this project focuses on an inductive geometry course intended for pre-service college teachers. In this section, I will consider the impact of reform mathematics instruction at the post-secondary level through a review of the calculus reform movement. I will also review the literature regarding mathematical thinking at the post-secondary level.

Calculus Reform Movement

In the late 1970s and early 1980s, mathematics faculty began to consider changes to help students succeed in mathematics. In this section, I will briefly discuss the history, implementation, and evaluation of that movement. A first step to reform occurred when Ralston proposed substituting discrete math for the typical third calculus course. This proposition met significant opposition that became more pronounced when Ralston received a large grant to explore his theories. Douglas was a leader of this opposition (Tucker & Leitzel, 1994).

In 1984, Douglas organized the Tulane Conference, which was called *Toward a Lean and Lively Calculus* and was organized to deal with five current problems in calculus. Mathematics faculty felt that (1) too few students were successfully completing calculus courses; (2) the coursework relied too heavily on algorithms; (3) too little technology was being incorporated into the classroom; (4) the workload needed to pull students through with a passing grade was too great; and (5) the material was being diluted to get non-majors though the coursework. The conference consisted of workshops and working groups to discuss these issues (Tucker & Leitzel, 1994).

Individuals at the conference produced a blueprint for calculus reform. The foundation of that blueprint was to change instruction fundamentally to include technology and to increase focus on conceptual understanding. The participants in the conference pledged to alter calculus instruction by focusing on conceptual understanding, modifying the mode of instruction to include more exploration, and fostering an inclusive spirit for all students.

The calculus reform movement proponents obtained significant funding from NSF and proceeded to transform their calculus curricula (Tucker & Leitzel, 1994). Initial evaluations of the reform movement were a resounding success. Tucker and Leitzel (1994) report that more than 60% of undergraduate institutions have made at least a partial movement to the reform curriculum. In addition, the types of instruction proposed by the reform movement were influencing pre- and post-calculus instruction. Reform curriculum materials containing open-ended questions that encouraged students to be active participants in the material were offered to replace tedious computations. Students worked together on course projects, discussed written assignments, and utilized technology to gain a deeper understanding of the material (Tucker & Leitzel, 1994).

By the late 1990s, NSF funding for the calculus reform movement began to diminish. The few evaluations of the movement found that students in the reform classes demonstrated greater proficiency than students who did not take reform classes (Tucker & Leitzel, 1994); nevertheless, opposition to the movement arose from concerns about the amount of preparation time required, the potential decrease in algorithm skills, and complaints from traditionally successful students. Consequently, the reform calculus movement has stagnated, or in some cases, vanished.

Advanced Mathematical Thinking (AMT)

Based on the success of the calculus reform movement and in response to instructor requests to present challenging material to mathematics majors, researchers asked, "What transformation in a person's thinking must occur for him or her to move through a post-secondary mathematics curriculum?" The research regarding

mathematical thinking would indicate that individuals generally move to a more advanced, formal form of mathematical thinking during their undergraduate experiences.

In the early 1990s, Dreyfus (1991) attempted to articulate the type of thinking required at the graduate level of mathematics by describing the mental processes that generally occur in mathematics. He labeled those processes (1) representing, (2) visualizating, (3) generalizing, (4) classifying, (5) conjecturing, (6) inducing, (7) analyzing, (8) synthesizing, (9) abstracting, and (10) formalizing. For example, induction, conjecture, and generalization are considered to be mathematical processes because an individual might observe a mathematical phenomenon and induce a possible reason for it. That person then might make a conjecture about a general rule that follows from the initial observation. Dreyfus proposed that these processes constitute advanced mathematical thinking (AMT).

Edwards, Dubinski, and McDonald (2005) describe the nature of AMT as being the thinking that permits deductive and rigorous reasoning about mathematics occurring beyond our five senses. Thus, if a class explored a mathematical concept with manipulatives, then AMT would not occur unless the students more deeply explored the mathematics beyond the manipulatives.

Dubinski (1991) elaborated on the process of moving to reasoning without the senses. He describes this process as Action – Process – Objects – Schemes (APOS). In thinking about mathematics, an individual manipulates previously constructed frameworks by taking physical *action* on the mathematics. Those actions are internalized to form *processes*. The processes are restructured into new frameworks to

46

support mathematical *objects*. Finally, the objects are incorporated into the individual's overall mathematical *scheme*. Research on Dubinski's theory supports progression through action, process, and objects, but Dubinski's notion of schemes was found to be poorly defined (Artigue, Batanero, & Kent, 2007).

Recent research has seen a movement away from the APOS model to one that includes pathways through three mathematical worlds. Tall (2004) depicted these pathways as moving from simple to sophisticated and describing how individuals express mathematical objects through communication. As an individual gains experience with a particular concept or field of mathematics, that person will move from the basic world to the more complicated world.

Tall describes the first world, or the Embodied World, as originating from perceptions of physical experiences. Thus, a student might conduct a scientific experience using marbles and a ramp to explore acceleration in order to understand a specific concept of calculus. In the second world, called the Procept World, students explore and describe mathematical concepts through symbol manipulation and calculations. Typical calculus instruction might fit into this category. For example, students might consider several equations that determine acceleration through symbolic manipulation and calculations of specific numbers. Finally, Tall describes the most advanced world, the Formal World as the one in which objects are expressed according to formally and deductively derived definitions. In this world, analytical students might describe the mathematical constructs that build the formulas typically used in calculus classrooms. These individuals would be able to formally derive these formulas through mathematical proof.

The objective of an undergraduate curriculum that prepares students for graduate programs in mathematics is to move them to the formal world in all relevant mathematical fields. University mathematics faculty and teaching assistants customarily function in the formal world, which creates difficulty for them when they are required to communicate entirely in the other two worlds for their students' sake. Therefore, undergraduate students struggle because instructors jump from one world to the next before the students are ready (Robert, Dorier, Robinet, & Rogalski, 2000). Most undergraduates have limited exposure to calculus and are unable to communicate exclusively in the Procept and Formal Worlds (Artigue et al., 2007).

Some research documents beginning linear algebra students' movement through these worlds (Dorier, 2000; Robert et al., 2000). Linear algebra was chosen because undergraduates had very little prior experience with matrices. Dorier (2000) found that new linear algebra students communicate almost exclusively in the Embodied World early in the course. During this time, students perform arithmetic operations on matrices. As the semester progresses, students move into the Procept World, performing more sophisticated matrix calculations and operations. Predictably, many students struggle when the course materials venture into the Formal World with applications of eigen values, linear independence, etc. In subsequent courses, students were able to move into the Formal World. Houndement and Kuzniak (1999, in Artigue, Batanero, & Kent, 2007) have also documented a similar pattern of progression through college geometry courses.

Have the calculus reform movement and more recent work regarding AMT had an impact on mathematics instruction in today's college classroom? A review of the

literature seems to indicate that the results of the reform calculus movement do not appear to have been explicitly absorbed into the mathematics teacher preparation literature. Nonetheless, examples of successful attempts to teach reform mathematics to pre-service teachers do exist. In the next section, I will report the current thinking about a mathematics curriculum for all students at the post-secondary level and the implications of that thinking for content coursework found in pre-service teacher training programs.

Mathematics Instruction in Today's University Classroom

In the wake of the diminishing calculus reform movement, many programs have returned to traditional instruction. Others have developed a hybrid type of instruction that blends technology, class projects, and more traditional methods.

While undergraduate mathematics instruction is oriented toward future mathematics graduate students, enrollment data show that most undergraduate mathematics majors do not progress to graduate programs (Tucker, 1999). Consequently, Tucker (1999) and Wu (1999) suggest that the undergraduate mathematics curriculum be divided three ways, according to the students' future intentions regarding mathematics. Distinct curricula could be offered for students intending to progress to graduate school or professions in mathematics, for students intending to move to graduate school or professional work in another field, and for future mathematics educators.

Regarding the latter, proponents of providing unique coursework for mathematics educators can turn to recent work in mathematics education. For example, Usiskin, Peressini, Marchisotto, and Stanley (2003) offer a textbook designed for the

special content knowledge needs of high school mathematics teachers. This textbook covers a variety of concepts that teachers will be expected to know in the classroom. True to the reform-based philosophy, the concepts build on previously acquired knowledge. Likewise, assessment problems are project-like, in that they are constructed so that they build a deeper understanding of the material.

Roth-McDuffie, McGinnis, and Watanabe (1996) have also proposed that pre-service teachers be actively involved in learning experiences that follow the pedagogical model they expect to employ. In their study, pre-service teachers participated in a reform mathematics course that emphasized conceptual understanding constructed by the students. In the course, the instructor provided a learning environment in which students explored concepts through collaborative experiences; consequently, students were engaged in active learning, meaningfully solving problems. In addition, students acquired mathematical understanding and a new perspective on teaching. Most importantly, after the course, students wanted to teach mathematics in their future classrooms the same way that their instructor did in this course.

The pre-service teachers' experiences in the above study seemed to impact their view of themselves as mathematicians and as mathematics teachers. In this next section, I will explore the role of these reform-based mathematics classes in the preparation of pre-service teachers.

Teacher Education Training Theory

Design of effective pre-service teacher education training programs should be based on sound theories of adult learning, utilizing the constructivist approach to teacher training (Mewborn, 2001). In accordance with the constructivist model,

students should experience key components of teaching in the context of that environment. A standards-based mathematics classroom in which students wrestle with relevant tasks to gain mathematical understanding is an essential component of that teaching environment.

Grover and Connor (2000) propose that content knowledge is a key characteristic of effective teacher preparation courses. The interaction between teaching and subject content knowledge has been identified as an essential teacher characteristic for student success in the mathematics classroom (Darling-Hammond, 1999; Ball, 1991; Shulman, 1987). Different from a typical mathematics student, pre-service mathematics teachers must possess a unique understanding of mathematics in order make sense of their students' solutions (Hill, Sleep, Lewis, & Ball, 2007). Consequently, teachers who obtain greater mathematical knowledge are more capable of the conceptual teaching than their counterparts, who utilize rules-based instruction (Brown & Baird, 1993).

The profound understanding of fundamental mathematical concepts teachers must possess grows out of mathematics coursework that provides pre-service teachers with personally meaningful models of mathematics learning (Ma, 1999). Sowder (2007) calls for a revamping of undergraduate mathematics education to allow coursework in which students "grapple" with their learning of mathematics. In order to make mathematics meaningful for the students, pre-service teachers must be provided the opportunity to utilize mathematical concepts and language to make connections between representations and applications, algorithms and procedures (Sowder, 2007).

Unfortunately, substantial evidence suggests that many mathematics teachers know the rules and procedures required to do mathematics but lack knowledge of

concepts and reasoning skills required to teach it (Borko & Putnam, 1995). Many pre-service teachers have not been provided the chance to strengthen their subject knowledge, because a majority of the students in mathematics classes have different needs than future mathematics teachers. (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992). For pre-service elementary school teachers, in particular, the negative experiences that they have in their undergraduate mathematics coursework can lead to mathematical anxiety as teachers. This anxiety is exacerbated by the teachers' superficial understanding of the mathematics they teach (Sowder, 2007).

Some hope does exist in the literature for teachers who experience this mathematical anxiety. Training programs that provide mathematical experiences and allow teachers to work together to explore mathematics can help them gain confidence in their abilities to develop understanding (Nelson & Hammerman, 1996; Sowder, Phillips, Armstrong, & Schappelle, 1998). These experiences can empower teachers to escape the anxiety that they have associated with mathematics (Hargreaves, 1995).

Sowder and Schappelle (1995) demonstrated the changes that can occur in teachers' confidence with mathematics and attitudes about instruction when they are provided the opportunity to explore mathematical concepts in a meaningful way. In this study, in-service teachers met to investigate the relationship between mathematical understanding and instructional practices. During these meetings, they explored mathematical concepts in a reform instruction setting. After one year of wrestling with the mathematics and discovering the importance of this subject, teachers' confidence about mathematics grew. Moreover, the teachers' instructional practices began to improve because they changed their expectations of the students, probed for

understanding, and encouraged classroom discourse among their students. Consequently, the researchers found that student learning in these teachers' classrooms was enhanced.

Why did the teachers' instructional practices change along with their understanding? Before teachers of mathematics became teachers, they were students of mathematics. Consequently, they obtained "incidental pedagogies" (Blanton, 2002, p 118) of teaching through their experiences as mathematics students (Lortie, 1975). As pre-service or practicing teachers, their strategies in the classroom are likely to reflect recent experiences in mathematics courses (Grossman, 1990). In addition, Darling-Hammond (1999) proposes that knowledge of teaching and learning, teacher behaviors, and best practices are important components that affect student achievement. Simon (1997) emphasizes that, in addition to knowledge of the mathematics, teachers must have a personally meaningful model of mathematics learning. The teachers in Sowder and Schappelle's (1995) study gained a new, incidental pedagogy by experiencing instruction requiring them to grapple with mathematics to develop new, personally meaningful models of teaching and learning.

Like Roth-McDuffie et al. (1996), Blanton (2002) found support for the proposition that pre-service teachers change beliefs about instructional strategies as a result of their experiences in mathematics classes. In her study, Blanton examined students' beliefs about and understanding of classroom discourse as they progressed through a reform-based undergraduate geometry course. She found that students in the course developed the ability to participate in mathematical discourse and came to believe that such discourse was an active process in which students built mathematical

understanding through interactions with their peers. More importantly, the results of her study indicated that students began to analyze their own habits of discourse as teachers and intended to incorporate them in their future practice as mathematics teachers.

Does the shift in pedagogy that is described by Blanton transfer to the classroom? Some research supports the notion that teachers' experiences as students of mathematics in a reform-style classroom establish their ability to create reform-style teaching and learning environments of their own. Shifter and Fostnot (1993) presented a summer workshop for elementary school teachers about implementing reform instruction in their classrooms. They concluded that teachers must experience mathematics for understanding as learners before they can be expected to implement it.

In conclusion, reform mathematics instruction seems to influence pre-service and in-service teachers in a variety of ways. First, when students experience this instruction, they gain a deeper understanding of the mathematical concepts presented. Second, student experiences in these courses seem to affect the teachers' beliefs about mathematics instruction.

Summary

Little research exists that explores pre-service and in-service teachers' experiences in reform-based mathematics classes. Moreover, a majority of that research focuses on students' experiences with reform instruction in general mathematics. Little research to date has examined students' overall experiences in an inductive geometry course.

The theoretical framework that guides this research is rooted in the constructivist learning theories. Theorists, such as Piaget (1972) and Vygotsky (1989), propose that individuals build knowledge through meaningful experiences. This perspective manifests itself in the classroom by redefining the roles of teachers and students. Specifically, in constructivist classrooms, students become responsible for their knowledge as active participants in the learning process. Meanwhile, the teachers' role becomes more passive, guiding students toward understanding by providing a lattice to support learning. Teachers provide support for student learning by creating an environment that facilitates discovery (von Glasserfeld, 1989).

In mathematics education, NCTM developed documents to guide teachers in building a constructivist classroom (NCTM, 2000). Students in these classrooms work in cooperative learning groups (Slavin et al., 2003) on meaningful high-demand tasks (Stein et al., 2000) to develop mathematical understanding (Carpenter & Lehrer, 1999). In their groups and as a whole class, students solidify their understanding through meaningful discourse (Elliott & Kenney, 1996). Finally, teachers are encouraged to guide discussion and provide learning support tools in the form of manipulatives and technological resources.

Piaget's theory of the development of geometric proof and the van Heile model of Geometric Thinking lay the foundation for inductive geometry instruction (Battista & Clements, 1995). Much like the constructivist classroom, teachers in inductive geometry classrooms facilitate student discovery through discourse (Schoenfeld, 1988). In these classrooms, students, who are required to justify their work, utilized learning aids such as manipulatives and DGEs to establish mathematical truths.

Few studies specifically connect the calculus reform movement and AMT to content area coursework in teacher-training programs. Nonetheless, a connection does exist. For example, the calculus reform moment can provide a model of successfully implemented reform instruction at the college level. In addition, Tall (2004) provides a model of student thinking that, when followed, can help instructors of pre-service teacher's mathematical content courses communicate with their students in meaningful ways.

In recent years, authors have proposed that mathematics teachers must possess a unique understanding of the content they are teaching to be effective (Hill et al., 2007). Consequently, they call for a specific content curriculum to be offered for future mathematics educators (Wu, 1999). An objective of the coursework in these curricula would be to develop the rich content knowledge that teachers must possess. Moreover, these classes would implement the recommendations set forth in the NCTM Standards document. An early study that investigated the impact of such a course demonstrated that pre-service teachers acquired mathematical understanding and a new perspective on teaching (Roth-McDuffie et al., 1996).

The literature regarding student experiences in the above mentioned courses is limited. Roth et al. (1996) found that after taking a reform mathematics course, students expressed the desire to incorporate the teaching strategies that they witnessed in that class in their future practices. Similarly, Blanton (2002) reported that students' view of classroom discourse changed as a result of experiencing it effectively implemented in a course that she was teaching. Initial findings seem to indicate that in addition to increased mathematical understanding, pre-service teachers obtain an

"incidental pedagogy" (Blanton, 2002, p. 118) from exposure to reform-based mathematics instruction.

This study is an attempt to erase the gaps in the literature regarding pre-service teachers' experiences in inductive geometry courses. Some of the questions that were considered in this project are the following: how do students' experiences in an inductive geometry course influence their understanding of and confidence with the material being presented? What do students learn about teaching and learning in mathematics during this course? How will students' participation in this course alter their beliefs about mathematics instruction? Finally, do students believe that this course was an important component of their pre-service teacher training program?

Chapter Three

Methodology

Introduction

In this chapter, I will present the methodology that I employed to conduct this research. First, I will outline my history as a mathematician, mathematics educator, and researcher in order to inform the reader the perspective from which I analyzed the data. Second, I will present the results of a pilot study conduct in the previous year. Third, I will present the four research questions that guide this project. Forth, I will present the design of this study and support it with the literature. Next, I will describe the participants of this project and details of the classroom in which they learned. Sixth, I will describe the procedures I used to collect the data. Finally, I will present my method for analyzing the data.

History of Researcher

Qualitative data must filter through the researcher's mind during the analysis process; consequently, qualitative researchers must guard against prejudices or preconceptions about the data that may creep into the process (Bogden & Biklen, 2007). While qualitative researchers attempt to objectively study the observable states of their subject, the personal history of the researcher, by necessity, plays a role in the analysis (Coffey & Atkinson, 1996). In this section, I will describe my personal history as a

mathematician, as a teacher of mathematics, and as a student and teacher of the geometry course examined in this study.

Being a Mathematician

Some of my fondest memories as a child occurred during the weekends that I spent with my grandparents, when I would spend late evenings with my grandfather working mathematics problems. Despite leaving school after the eighth grade, my grandfather was able to talk with me about mathematical principles to the extent that I created a sufficient understanding.

As I progressed through school, I maintained my love for mathematics. While I often did not perform well during the teacher-directed lectures, I thrived when the teachers gave me responsibility for my own learning. Geometry was my favorite class because the homework often reminded me of the Saturday evenings I spent with my grandfather. Like his problem-solving games, my geometry homework consisted of mathematical activities nested in critical thinking tasks. As my knowledge grew through these and other experiences, I developed an approach of "seeing behind the math." This ability to learn mathematics on my own served me well in university-level statistics and mathematics courses, where the professors expected students to work outside of the classroom to learn the material.

The ability to look "behind the mathematics" was also useful when I helped my college and graduate school classmates with their coursework. The thrill that I felt when I saw they understood my thinking led me to become a mathematics teacher.

Becoming a Teacher

Directly after I completed my course work in my statistics graduate program, I became a middle school lateral-entry mathematics teacher. As a confident mathematician, I spent the majority of my lessons showing my students what was going on behind the mathematics rather than the trick that would help them succeed on the upcoming test.

Despite observing my students make adequate progress in learning mathematics, I felt that I could provide a better environment for them; thus, I began my coursework at the University of North Carolina to obtain my teaching certificate. During my first year, I was required to take a geometry course, which was designed to teach elementary and middle school pre-service teachers some of the mathematics behind the concepts they would be covering as teachers. I found that this course encouraged my self-directed learning style.

My perspective underwent a dramatic change during one memorable class session when I was completing the generalization of a task as other members of my group were struggling. The instructor came over to our group and asked a leading question to nudge the others forward. With great enthusiasm, I interrupted her saying, "I can show them how to do it." The instructor replied, "Well, then you are *showing* them. I want them to *discover* it."

As a result, I learned that, in order for everyone in the group to gain understanding, I needed to guide them toward solutions, not give them the answers. Moreover, I learned that, if I was patient, other members of the group might provide insight into the tasks that I had not considered. I believe that my experiences in this

class helped me discover the efficacy of a reform mathematics classroom. Since then, I have had the opportunity to teach this same geometry course, where I observed the value of reform strategies from my perspective as a teacher.

A Pilot Study

Midway through the spring semester of 2007, a pilot study was conducted with a cohort of pre-service elementary and middle school teachers enrolled in a geometry course to meet graduation and licensure requirements. In that study, I observed classroom group work and communication, taught two classes, and conducted interviews with six students at the end of the semester.

The data from these classroom observations indicated that the students appeared engaged in the mathematics and regularly participated in the cooperative learning tasks. For example, I saw groups of students exchanging ideas while working together to solve classroom tasks. Often, groups would begin by sub-dividing into pairs or threesomes in order to explore the activities. Next, they would discuss their findings with the group as a whole. During these group discussions, someone might propose a conjecture that would be debated by other members of the group. This discourse helped members of the group arrive at the generalizations that were an objective of the tasks.

I found that many of the students I interviewed had limited experience in a reform mathematics classroom. Moreover, the time that they spent in a traditional classroom appeared to have negatively affected their confidence as mathematicians. Consequently, some individuals reported feeling that they were, as one participant stated, "not good mathematicians," because they could not follow the teacher's lectures. One student reported that she stopped feeling skilled at mathematics midway through

middle school, as her teachers shifted from student-centered reform instruction to traditional instruction. However, during this geometry class, she began to regain her confidence. She reported that her fear of failing in the course subsided after she experienced success on the daily classroom cooperative learning tasks.

An examination of the pilot study data also led me to conclude that students had a favorable impression of the reform mathematics classes they were taking. For example, students commented that cooperative group work helped them to learn the concepts being taught in the class. During interviews, students reported that they were initially apprehensive about the group work that was required. Students remarked that in previous classes, they were often the student in the group who did all of the work. They reported that the group work in the reform mathematics class was different. These cooperative learning tasks were designed so that progress toward the solution was more important than the solution itself. This focus on the process required that all group members engage with the task. One student said that she really liked the group work in "this class because [she] could rely on her group to help [her] when [she] was lost."

As a result of these findings, I predicted that the students' experiences in this class would have an impact on the learning environment that they intended to establish in their future classrooms. One student described the way that she would teach mathematics before she experienced this reform classroom. During class, she would offer knowledge to students by presenting lectures and examples based on sound mathematics. She would then provide homework assignments that covered the material provided in class and that the students would complete independently. She would recognize that her students understood the material when they were able to answer

homework and test problems correctly. In short, she described herself as a traditional didactic mathematics instructor.

By the end of the semester, however, this student articulated a very different classroom. For her, the optimal environment would be comprised of cooperative group work to solve problems that would promote understanding. During class, the teacher would observe the group's progress and provide guidance only when needed. Students would have the opportunity to consult classmates on homework assignments, for which grades were assigned in consideration of effort toward the answer, not just the answer itself. Most importantly, this student wanted to establish a learning environment similar to the one she experienced in this class.

This pilot study formed a foundation for my dissertation. I believe students gained a deeper understanding of geometry and their view of mathematics instruction changed as a result of their experience in this standards-based geometry class. Furthermore, I explore this observation in a larger scale qualitative study. In the following section, I will describe the formal qualitative design I conducted to investigate these questions.

Research Questions

This case study was conducted over a period of four months during a spring semester standards-based geometry course for pre-service teachers. It documented students' general experiences in this mathematics course and investigated three general issues. First, this research explored what the participants learned about mathematics as well as teaching and learning in mathematics during the semester that they took this course. Second, this project documented students' attitudes about mathematics

instruction and how those attitudes changed during the semester. Third, this study

explored this course's perceived pedagogical value to the participants in this study. The

four specific research questions guiding this study were:

1.   To what extent do students perceive they have improved as

         mathematicians based on their experiences in a standards-based

         geometry course?

2.   What do students report they learned about teaching and learning in

         mathematics during the semester?

3.   Did students' beliefs about teaching and learning in mathematics

         change as a result of their experiences in this course?

4.   Do students believe this standards-based geometry course played an

         important role in preparing them for subsequent teaching methods

         coursework?

Design

In order to gain insight into the individuals' experiences in and reactions to this

particular reform-based geometry classroom, this study utilized a qualitative case study

design (Stake, 1988). A case study provides insight into the accounts of individuals or

groups in the context of a natural setting (Glesne, 2006). Case studies illuminate issues

by allowing the researcher to become a participant observer while conducting an in-

depth analysis of a system utilizing a variety of data (Yin, 1994; Glesne, 2006). They

allow researchers to answer "how and why" questions (Yin, 1994) in the investigation

of phenomena, populations, and generalizations (Glesne, 2006). In summary, the case

study utilized data from a variety of sources to conduct a comprehensive examination of a bounded system in context.

The case study is a good fit for this research for several reasons. First, this research occurred in the context of a bounded integrated system (Yin, 1994), formed by the geometry course, with various individual, interrelated elements that constitute an organized whole (Johnson & Christensen, 2000). It explored students' experiences of and reactions to that closed system. Second, the case study is the optimal design to answer "how and why" research questions. Specifically, this design allowed me to investigate how students experienced this particular course and why their attitudes changed during the semester. In this project, all aspects of this particular stanrdards-based geometry course will form the single case being studied.

The case study methodology provides a rich variety of elements and tools to examine a bounded system in its context. In the next section, I will describe the procedures I used to collect the data.

Context

This geometry course officially titled "Selected Topics in Mathematics" was offered through the Department of Mathematics but was instructed by a professor of mathematics education. The stated curricular perspective of this course on the syllabus (see Appendix A for the course syllabus) was to "allow students to move from concrete to abstract reasoning using the van Hiele model of Geometric Thought.

According to the instructional perspective stated on the course syllabus, this course was taught from a "constructivist and socio-cultural perspective." In it, students constructed "personal understandings of geometry" through the use of inquiry-based

activities. The students' investigations were supported with "investigations and technology."

The syllabus stated that a major objective of the course was to "provide students with a mathematical foundation and cognitive support for the teaching of elementary and middle school geometry." It also listed the specific goals that "students will:

a. connect geometric concepts to real world situations;

b. understand properties and relationships of shape, size, and symmetry in two-and three-dimensional space;

c. understand systems of measurement and use systems to perform measurements in realistic settings;

d. understand concepts of transformations in two- and three-dimensional space through the investigations of rotations, reflections, and translations and apply these concepts to congruence and similarity;

e. study geometric reasoning, conjecturing, and proof in geometry-both written and oral; and

f. represent and solve geometric concepts, problems, and solutions using technology and models."

The course was initially designed as a requirement for all students seeking a license in middle grades mathematics education. At a later time, the course was opened to future elementary school teachers. Currently, all middle and elementary school pre-service teachers specializing in mathematics education are required to take this course. It is an elective course for all other students. Middle grade mathematics education

majors are also required to take a subsequent proof-based geometry course taught by a Department of Mathematics professor.

Most lessons were taught in a classroom in the School of Education building, in close proximity to the instructor's office. The classroom was designated for mathematics and science education courses and contains substantial resources in the form of manipulatives and investigation tools. Students, who spent the entire semester working in groups, sat with their four other group members at one of five tables in the classroom. A table sat in the center of the classroom that contained relevant handouts and classroom resources for the day. One wall contained a dry-erase board, orienting instruction toward that pre-designated front of the classroom. However, during instruction, the professor often stood in the middle of the classroom to speak and visited group tables during cooperative learning activities. During observations, I sat at a desk that was placed in the back of the room, opposite the dry-erase board.

Participants

The population consisted of 25 total students, 23 pre-service elementary and middle school teachers and two students who were not formally participating in a teacher preparation program. The seven pre-service elementary school teachers were taking this course as an elective in the teacher preparation program at a large public university in the Southeastern region of the United States of America.

The population included three sophomores, 17 juniors, three seniors, and two non-degree students, all of whom had not taken any prior teaching methods courses. While three students were ranked as seniors, they had one year of coursework remaining. Some of the students in the class had previously taken another standards-

based mathematics course about real numbers. The three non-degree students already possessed an undergraduate degree and were taking the course as part of a requirement for a lateral entry license (see Table 1).

Table 1

All Student Major, Class Rank, and Program of Study

Student Number	Major	Class Rank	Teacher Education Program
1	Education	Senior	Elementary
2	License	Graduate	Middle Grades
3	Education	Senior	Middle Grades
4	Undeclared	Sophomore	Undecided
5	Education	Junior	Elementary
6	Education	Junior	Middle Grades
7	Education	Junior	Middle Grades
8	Education	Senior	Middle Grades
9	Education	Junior	Middle Grades
10	Education	Junior	Middle Grades
11	Education	Junior	Elementary
12	Education	Junior	Middle Grades
13	Education	Junior	Middle Grades
14	Education	Junior	Elementary
15	License	Graduate	Middle Grades
16	Education	Junior	Elementary
17	Education	Junior	Middle Grades
18	Education	Junior	Middle Grades
19	Education	Junior	Middle Grades
20	Education	Junior	Middle Grades
21	Education	Junior	Middle Grades
22	Education	Junior	Middle Grades
23	Education	Sophomore	Elementary
24	Education	Junior	Elementary

I conducted summary interviews with the eight students who volunteered to participate in them. Six of the interview participants were women and the remaining two were men. Two of the participants were elementary education majors while the

remaining six students were in the middle grades teacher preparation program (see

Table 2).

Table 2

Name, Gender, Class Standing, and Major of Interview Participants

Name[1]	Gender	Class Standing	Major
1. Emily	Female	Sophmore	Elementary Ed.
2. Isabella	Female	Junior	Middle Grade Ed.
3. Jacob	Male	Junior	Middle Grade Ed.
4. Emma	Female	Junior	Middle Grade Ed.
5. Ava	Female	Junior	Middle Grade Ed.
6. Michael	Male	Junior	Middle Grade Ed.
7. Madison	Female	Junior	Middle Grade Ed.
8. Sophia	Female	Junior	Elementary Ed.

[1] All names are pseudonyms.

The female instructor designed the course and was teaching it for the eighth time

in ten years. She is a reform mathematics teacher who has written extensively about

teaching and learning in the field of mathematics education.

Procedure

In their qualitative methods book, Gay, Mills, and Airasian (2006) suggest that

researchers triangulate their data by gathering them from a variety of sources.

Triangulation allows researchers to gain multiple perspectives of a phenomenon. In this

study, I collected data about students' experiences as well as their beliefs about

mathematics teaching and learning through several processes. I triangulated my

investigation by observing components of the classroom as a passive observer and

interviewing students at the end of the study. In addition, I reviewed results of a

mathematics attitude assessment presented during the first and last week of the semester

and student responses to a summary essay question that was completed by all students in the class.

Interviews

The central goal of an interview is to understand how the person being interviewed thinks (Bogdan & Biklin, 2007). Consequently, interviews are conducted to enable researchers to glean information that cannot be observed and to gain a deeper understanding of what is observable (Glesne, 2006). Researchers can gain insight into how the interviewee thinks by means of the unexpected turns the questioning takes during the interview (Glesne, 2006).

In this study, eight students were interviewed during the last week of the semester. During the in-depth student interviews, I asked questions designed to elicit information about their experiences in previous mathematics courses; their reaction to this particular mathematics course; their view of themselves as mathematicians before and after their experience in this geometry course; their beliefs about mathematics teaching and learning; and their perspective about the role that this particular mathematics class played in their pre-service teacher training. In addition, I asked questions that sought their impressions about specific components of this class, such as group work or computer-based geometry lessons (see Appendix B to read student interview questions).

The instructor's reflections were collected during brief weekly meetings. During these check-ins, I inquired about her perspective on how well the class and lessons in general were progressing. A more complete interview was conducted approximately one month after the semester concluded. During the post-semester

interview, I asked the instructor questions about the history of the class and her orientation as an educator. I also asked her specific questions about trends that I observed during the data collection (see Appendix C for instructor interview questions).

Observation

Qualitative researchers utilize observations to gain an understanding of the natural environment as experienced by the participants (Gay et al., 2006). To varying degrees (Bogden & Biklen, 2007), observers become a part of the environment, either acting in the situation or passively filling space. By observing events that occur in the environment, the researcher can gain a deeper insight into the participants' experiences in the system being studied (Gay et al., 2006).

In this project, I observed a total of eighteen 75-minute class periods. Of the 18 class periods I observed, one was led by a guest who was another mathematics education faculty member, one consisted of independent group work, and one was dedicated to student-led presentations. Two of the 18 observations took place in the computer lab, while the remaining observations occurred in the classrooms. During computer lab observations, I sat at a table in the back of the room. A small number (up to eight) of students were working on computers directly in front of me and two were working on each side of me. I took observational notes on my laptop computer. Overwhelmingly, students did not look at or communicate with me, and on the rare occasions that they did initiate a conversation, I quickly ended it to avoid unduly influencing the results.

In addition to making observations during the class period, I also gathered data outside of the classroom. For example, I recorded my observations of events that

occurred in the instructor's office during our frequent check-ins. I also watched student interactions and discussions in the hallway before class.

Document Review

Document analysis and review comprised the third source of data in this study. The artifacts from the classroom can form an authentic representation of events that contribute to a researcher's understanding of a particular environment (Gay et al., 2006). Unlike interviews or observations, which occur in the presence of the researcher, artifacts are typically produced independent of the researcher's influence.

In this study, I examined a variety of documents in order to obtain deeper understanding of the students' experiences. First, I examined student responses to a summary essay question that was presented by the instructor at the end of the semester. The question was written as follows: "State one of the most important things that you believe you have learned in this course. Explain why this knowledge is important to you personally or professionally." Every student in the class provided a summary essay.

Second, I reviewed student responses on two versions of the Geometry and Measurement Assessment (GMA), one presented at the beginning and one presented at the end of the semester (Bush, 2007). The GMA is designed to provide instructors insight into the breadth and depth of pre-service middle school teachers' geometry content knowledge by describing their strengths and weaknesses in geometry knowledge. These assessments were distributed by the teacher with instructions to voluntarily complete at home and to return them to her. Nineteen students returned the assessment that was provided at the beginning of the semester (see Appendix D for

Bush's Geometry and Measurement Assessment-Version I), and only nine students completed the end-of-year version (see Appendix E for Bush's Geometry and Measurement Assessment-Version 5). Many of the end-of-year assessments were partially finished. In all, seven students completed both versions of the assessment.

Finally, I reviewed instructional material, homework assignments, and tests as well as lesson plans from previous semesters. The instructor provided me with a copy of each document she handed out to the students and her lesson plans.

In conclusion, I collected data from a variety of sources in order to triangulate my research findings. These different methods of collecting data allowed me to explore the impact of the students' experiences along several notable dimensions.

Data Analysis

Qualitative researchers utilize inductive analysis to reveal outcomes about which they have not hypothesized. They conduct qualitative analysis in order to organize the data into manageable units so they can synthesize it and search for patterns (Bogden & Biklen, 2007). In this study, I analyzed data in order to search for meaning and understanding (Borg & Gall, 1989).

Bogden and Biklen (2007) suggest that beginning researchers do not have the theoretical experience to identify themes and other issues while simultaneously collecting data in the field. Consequently, they advise beginning qualitative researchers to reserve their formal analysis until data gathering in the field is completed. In light of these recommendations, I focused my efforts on gathering sufficient and accurate information while conducting only a partial investigation of the data during the experience.

Bogden and Biklen (2007) offer suggestions to aid in the data analysis that occurs during the collection phase. Following their counsel, I developed a research focus based on early observations that narrowed the scope of the study. While collecting data, I wrote substantial "observer comments" about ideas that I generated. These ideas became a resource for critical thinking when I analyzed the data at a later time. Similarly, throughout the process, I wrote memos to myself about what I was learning. These one- to two-page summaries about my experiences provided reflections on the issues that arose during the fieldwork. Finally, I utilized visual devises, such as charts, outlines, and graphs, to supply a concrete representation of my observations. These visual aids helped form a blueprint of the themes and ideas that developed during my fieldwork. In sum, my data analysis was an ongoing process.

After completing my fieldwork, I conducted an in-depth analysis of the data. I began by searching for patterns in the data and was thus able to derive a coding system (Gay et al., 2005). I narrowed the types of coding that I used as my data analysis progressed. Finally, I developed a list of coding categories and sorted the data mechanically (Bogden & Bilken, 2007).

The sorting of the data phase preceded the identification of various themes phase of research. Qualitative research must be prepared for the data to take unexpected turns (Stake, 1988). While identifying themes, this research project led me places that I did not expect to go at the beginning of the project. For example, while I had experienced the cooperative learning component of the course as a student and instructor, I did not expect for it to impact the students as dramatically as it did.

Nonetheless, this theme analysis helped me recognize general patterns in the data (Gay et al., 2005) and addressed my research questions. One category of themes emerged while investigating the first research question, "To what extent do students perceive they have improved as mathematicians based on their experiences in a standards-based geometry course?" This category directly addresses the research question and includes the themes, "mathematical knowledge gain" and "increased confidence in mathematical abilities."

Likewise, a category of themes emerged to directly address research question #2, which states, "What do students report they learned about teaching and learning in mathematics during the semester?" The first theme in this category identifies what the students learned through observing and experiencing standards-based instruction. The second theme describes student learning about mathematics teaching and learning that occurred through specific instruction from the professor.

Two categories of themes emerged while investigating the third research question, "Did students' beliefs about teaching and learning in mathematics change as a result of their experiences in this course?" The first category of themes directly refers to the above questions and includes the three following themes: change in beliefs about epistemology, cooperative learning, and intentions for teaching mathematics as professionals. The second category of themes refers to student experiences in mathematics courses before their enrollment in any School of Education mathematics course. Experiencing direct instruction, poorly administered cooperative learning, and generally negative feelings about mathematics are themes in this category. These themes helped to establish the students' initial beliefs.

One category of themes, titled "classroom components that influenced learning," emerged in response to an investigation of all of the three above mentioned research questions. Themes in this category include instructor caring for the students, students experiencing reform instruction, and students utilizing classroom resources. These themes address the first two research questions, because components of this particular classroom directly impact the students' views of themselves as mathematicians and what they learned about teaching mathematics during the semester. These themes provide insight into the third research question because they provided elucidation how student beliefs about mathematics education changed.

The final theme emerged as a direct investigation of the final research question, "Do students believe this standards-based geometry course played an important role in preparing them for subsequent teaching methods coursework?" It describes students' beliefs about the place of this standards-based geometry course in their overall training to become teachers.

In this project, data was gathered from a variety of sources and coded and categorized according to identified themes. These themes where then grouped into broader categories that document the student's experiences in the course and address the research question that guided this study. In the following table, I identify themes, the source of data analyzed to obtain the theme, and the research question each theme addresses (See Table 3).

Table 3

Theme Identification: Analysis of Research Questions

I. Experiences in Mathematics Courses		
Theme	Source of Data	Research Question
1. Direct instruction	Interviews	
2. Cooperative learning	Interviews Reflection essay	Question #1 Question #3
3. General unhappiness	Interviews Reflection essay	

II. Classroom Components that Influenced Learning		
Theme	Source of Data	Research Question
1. Instructor caring for students	Interviews Reflection essay	
2. Reform instruction	Interviews Reflection essay Observations	Question #1 Question #2 Question #3
3. Classroom resources	Interviews Reflection essay Observations	

III. Perception of Self as Mathematician		
Theme	Source of Data	Research Question
1. Knowledge gain	Interviews Reflection essay GMA	Question #1
2. Increased confidence	Interviews Reflection essay	

IV. Learning About Teaching Mathematics		
Theme	Source of Data	Research Question
1. Learning through experiencing mathematics instruction	Interviews Reflection essay Observations	Question #2
2. Learning through direct instruction and course assignments	Interviews Observations	

V. Changed Beliefs		
Theme	Source of Data	Research Question
1. Epistemology	Interviews	
2. Cooperative Learning	Interviews Reflection essay	Question #3
3. How they would teach mathematics	Interviews GMA	

VI. Necessity of this Course		
Theme	Source of Data	Research Question
1. Necessity of this course	Interviews Reflection essay	Question #4

In the following chapter, I present the category and themes in an order that I believe best describes the students' progression through the semester. First, I document their experiences in previous mathematics courses to establish a picture of the students at the beginning of the semester. Second, I describe components of the classroom that influenced the students. Third, I evaluate how students' perceptions of themselves as mathematicians improved during the semester. Fourth, I chronicle what students discovered about mathematics education as a result of their experiences in this course. Fifth, I identify how students' beliefs about teaching and learning mathematics changed during this course. Finally, I document the role that students perceive this class played in their pre-service teacher training program. The following table identifies categories and themes, sources of data, and research questions the themes address.

Summary of Methodology

In summary, this study was designed as a qualitative case study in order to observe pre-service teachers' experiences during and reactions to a standards-based geometry course. In this project, all aspects of this particular geometry course for the single case being studied. One of the strengths of qualitative case study research is that it produces rich data, because the object is studied in its normal setting. The case study is used to answer "how and why" questions; it is also useful when there is no control over the situation or behavior of the individual to be studied (Yin, 1994). Several limitations to the case study methodology are presented in the literature. For example, bias can occur in collecting and interpreting the data. This bias could in turn influence the conclusions or emphasize a particular viewpoint. Researchers also question the ability to generalize from a single case (Yin, 1994).

In the literature, triangulation is referred to as the use of multiple data collection methods and data sources as corroborative evidence for the validity of the research findings (Gall, Borg, & Gall, 1996). Multiple data collection methods in this study included student interviews, classroom observations, and document reviews. The qualitative data in this project were analyzed via an ongoing process in accordance with the guidelines proposed by Bogden and Biklen (2007). During this process, the data were coded and categorized according to identified themes. At all times, I was aware that my history with this course posed a risk of potential bias and attempted to guard against it.

Chapter Four

Results

The data collection occurred during one semester at a mid-size public university in the Southeastern United States. During this semester-long geometry class, I conducted eight interviews with students and one with the instructor, observed several periods of classroom instruction, and reviewed a variety of documents. A large portion of the data reported in this section was derived from personal interviews, classroom observations, and student responses to the summary essay. While I guided the conversation during the interviews with my questions, the classroom observations and summary essays were completely unscripted and open-ended. In particular, student responses to the summary essay covered a variety of topics. Consequently, when results are presented in this chapter, I do not calculate the proportion of students who responded as such. The fact that someone chooses to write that feeling confident about mathematics was an important thing they learned during the semester does not imply that that person did not also learn how to effectively implement cooperative learning.

For many of the students, this course was their first experience in a mathematics course in the School of Education. However, some of the students had taken another mathematics course in the School of Education the previous semester. In this paper, my discussion of previous experiences will refer to the students before their enrollment in any School of Education mathematics course.

This study was conducted to investigate the following four research questions:

1.   To what extent do students perceive that they have improved as mathematicians based on their experiences in a standards-based geometry course?

2.   What do students report that they learned about teaching and learning in mathematics during the semester?

3.   Did students' beliefs about teaching and learning in mathematics change as a result of their experiences in this course?

4.   Do students believe that this standards-based geometry course played an important role in preparing them for subsequent teaching methods coursework?

A review of the data revealed themes that fit into six general categories that address the above questions as outline in the previous chapter. A review of the data revealed themes that fit into six general categories that address the above questions as outline in the previous chapter. The first two categories of themes provide insight into the students' experiences with mathematics and inform the first three research questions: (1) students' experiences in previous mathematics courses and (2) influential components of the students' experiences in this particular course. The remaining four categories of themes directly address the four research questions: (3) the ways in which the students improved their perception of self-as-mathematicians; (4) what the students learned about teaching and learning mathematics; (5) changes in beliefs about mathematics learning and teaching that occurred as a result of student experiences in this course; and (6) the necessity of completing this standards-based mathematics

course before a teaching methods class. These themes will directly address the research questions.

<center>*Experiences in Mathematics Courses*</center>

Whereas students entered this course with a wide range of mathematical abilities, they consistently described experiences in prior mathematics classes that were characteristic of traditional instruction. Specifically, they described the instruction they received as directed by the teacher. The students' role, in contrast, was to follow along. They infrequently worked in cooperative learning groups and expressed a dislike of group work. In this section, I will present themes associated with previous experiences in mathematics classes.

Students described their history of participating minimally in mathematics classrooms. For example, Jacob remarked, "In previous math classes, we went over homework, then we'd be introduced to a new topic, then we'd go start [on new homework], or take it home." Emily stated that "in a typical math class, you would have lecture, writing on the board with examples, then homework." During the lecture portion of the class, students often took notes to keep track of what the teacher was saying. Emma expressed her frustration with these notes by commenting that "in math class, you copied notes, and then you took notes, and finished class with more notes." Ava summarized her school mathematics experience as follows:

> In previous classes, you take notes for the first part of class and do problems for the second part. Notes were given by the teacher and consisted of terms, definitions, and examples. The teacher might work some examples on the overhead and show some tricks like FOIL. The teacher would go to the overhead, going through problems that we would write down. Students had no interactions in the class, we just took notes.

<center>82</center>

Because of their teachers' delivery, students would "simply memorize what they are supposed to do without even really understanding the material." In general, students described experiencing very traditional instruction that started "by checking homework, then introducing a new topic, and finishing with new homework" in prior mathematics courses.

Other students reported unhappiness with their overall experience in previous mathematics classes. For example, Madison remarked, "Math courses throughout elementary and middle school did not provide enough different learning spaces for me to be able to connect certain topics." Sophia stated that she "didn't have much interaction in math classes and no group work." In these classes, the mathematical knowledge came from the teacher. Jacob said, "It's like the teacher would say, 'Here's how you do the problem, now you go practice the exact way I showed you.'" Consequently, many students reported that they would fall behind because information was "thrown at you at such a fast pace that it made for a bad experience." In the end, they reported that they "retained nothing or would mindless[ly] learn the material."

All but one of the students who talked about group work reported negative experiences with it in previous classes. The typical goal of group work had been to complete an assignment. Ava summarized two common scenarios that occurred when students were presented group work in the following statement:

> You get the smartest person in the class to be in your group so that they can do all of the work for you and you get an A. Usually I did all the work. If you found someone who was even with you, then you would divide the work. 'You do the evens and I'll do the odd ones.' Then you get the work done fast and goof off the rest of class.

As a result, the majority of students who mentioned group work "hated it, because [they were] always selected to be the one to do all of the work."

While students were excited to learn about teaching mathematics in this class, a majority expected the mathematics instruction to be similar to what they had received. Therefore, students reported a variety of emotions at the beginning of the semester. Those students who were strong in mathematics were either excited to be taking another mathematics course or were glad to be able to check off another requirement for graduation. Students who thought of themselves as poor mathematicians were very nervous about taking a college level course in a subject that had caused them so much trouble in the past. All of the students reported feeling concerned when they learned the amount of time they would be spending in groups during the semester. As Jacob stated, "I was a little hesitant about the group work because she [the instructor] paired me with four…girls who were very different from me."

Nonetheless, the data indicated that dramatic shifts occurred in students' knowledge and attitudes about mathematics during the semester. In this next section, I will discuss the classroom components that facilitated this change.

Classroom Components that Influenced Learning

Students in this course experienced mathematics instruction that was fundamentally different from their prior classes. These differences helped them gain a deeper understanding of the geometric concepts presented. In this section, I will describe themes that emerged from student interviews and classroom observations regarding their experiences in this course. The data can be separated into six themes: (1) the instructor's caring for students and their learning; (2) classroom components that

nurtured mathematical understanding; (3) elements of the environment that encouraged learning; (4) the role of cooperative learning in the classroom; (5) the implementation of high cognitive demand tasks; and (6) the manipulatives and technology that were employed to support learning.

Instructor's Caring for Students

A variety of students reported they felt as if the instructor "genuinely cared about [them and their] success." The instructor's concerned attitude motivated her to find ways to connect to the students so that "they get it." For students, receiving kindness enabled them to take risks with the material and to seek help when they needed it.

I occasionally met with the instructor before class to prepare for upcoming lessons. During some of these check-ins, I would find her reflecting on past classes and making plans to eliminate perceived gaps in student understanding. On one occasion, when I walked into her office, she reported that she did not think the students truly understood the material from a previous class. This belief seemed to upset her, and she considered altering the lesson plan. On another occasion, a student stopped by the instructor's office after receiving a low grade on a test. The student told the instructor that she was struggling with the material because she had not had geometry in high school. In response, the instructor let her borrow a supplemental book and offered to meet with her once a week for extra instruction. By the end of the semester, this student said, "I can't believe that I am saying this, but I think that I can be good at geometry because of this class."

Other students reported that the caring attitude of the instructor encouraged their learning. When asked if there was anything else that she would like to add to her interview, Madison said,

> I just wanted to say that I think [the instructor] really cared about us. Simple things such as having a smile…and overall concern for all students' academic success makes me want to go to class and…[helped me] develop a deep love or appreciation of the subject.

Emma commented on how the instructor's willingness to work with her helped her succeed:

> If I had difficulty getting the homework done, I would go to her and she would give me extra time. Other teachers would have just told me that I have a bad grade and forget about it. I think that [the instructor] gave me extra time because she really wanted me to know the material. It gave me the opportunity to do the individual study that I need along with the group work. She really cares about us knowing the material and has a heart for us.

Students such as the ones quoted above could see the instructor was concerned about their learning the material, because she was willing to make adjustments in her schedule.

Reform Instruction

Mathematical Understanding. Instruction in this course incorporated activities requiring the five forms of mental processes identified by Carpenter and Lehrer (1999). First, the instructor provided activities that necessitated students connecting new concepts with prior knowledge and experiences. Isabella articulated this process as follows:

> Sometimes finding patterns was easy, but you had to connect it to something else. You have to start with some problems that you know the answer to and then do some difficult problems to make connections to harder stuff. Then you see the whole big picture.

Second, during group work, students would articulate their knowledge of the concepts to other members. Isabella summarized this process by stating, "Just helping someone else understand what you know helps you learn it better." Third, students took ownership of the mathematical discoveries they made. This ownership was especially evident during group work when students debated particular solutions. Fourth, students extended their knowledge to other situations. During class work, students were encouraged to develop a generalized statement that would summarize their findings. Finally, students were encouraged to reflect on their efforts in class. Often, the reflection took place while solving problem sets, during which students were required to formally write up their work (see Appendix G for Steps in Writing about Mathematics). Part of the write-up process included explanation of the strategies used to derive solutions.

In general, the instructor accomplished the above by providing activities that nurtured mathematical understanding. She also promoted understanding by pushing students to utilize the aforementioned five mental processes. Emily described the problem-solving process in this course as follows:

> When you get to a stopping point in other classes, it was like, "Let's raise our hand and see what the teacher is going to tell us or what hint she'll give." If someone gets the correct answer, then it's like we all do the same thing as that person. [This instructor] is like, "Oh, we have to keep working" even if you are stuck. We have to figure it out on our own and with our group. We just break it down and build each idea back up. In this class, we have to figure it out by working together. We just have to talk to each other until we have it figured out.

This observation highlights the instructor's insistence on focusing on the *process* of obtaining the answer rather than the answer itself. In the following section, I will discuss more aspects of the classroom environment that facilitated learning.

Classroom Environment. Students quickly discovered the classroom environment was different from their prior mathematics classes. Early in the semester, the instructor established sociocultural norms in the classroom that facilitate learning. For example, during each lesson, students worked in collaborative groups on mathematical tasks that were not graded. Instead, they were designed to emphasize reasons for solutions rather than the solution itself. Emily remarked, "To some extent, just the fact that we thought about the process and how we got there…not just the answer helped us learn." Consequently, students were motivated to make an effort to complete the work, because learning and understanding the mathematical concepts underlying the tasks was the focus. With this goal, students took responsibility for the learning, because they began to realize that they, not the teacher, were the source of mathematics knowledge. For example, during an interview, Isabella said, "Now, I don't have a problem if I have come up with an answer that is different than [the instructor's]," because, "I might have a different understanding than her." This approach also emerged during classroom discussion, where students initiated conversations with other classmates and the instructor about the discoveries they were making.

The sociomathematical norms that were established in this class created an environment that nurtured understanding. Students reported that they "truly learned the math" because the instructor would ask them to explain their reasoning "and she

wouldn't take just any answer." Students were required to justify their work with sound mathematical statements. In addition, students spent the entire semester working in collaborative groups to complete tasks that were designed to build relationships between mathematical concepts. One student wrote, "Even though I have always been good at math, sometimes it would be a group-mate explaining something in a way that helped me to understand better." Finally, the frequent errors that occurred as students worked to solve problems were utilized to enrich understanding.

One student wrote that this math class "really opened my eyes because so much progress can result when a teacher develops that great classroom environment." The classroom environment this student endorsed incorporated sociocultural and sociomathematical norms that promote learning.

Cooperative Learning/Group Work. Students consistently praised their experience with cooperative learning in this class. In fact, students believed they learned the mathematics better because of their learning groups. During classroom observations, I recorded many examples of effective cooperative learning. In this section, I will briefly describe some aspects of the group work that students mentioned as particularly helpful. However, I will discuss group work in much greater detail in subsequent sections of this chapter.

Students reported that the focus of cooperative learning group efforts in this course was to understand mathematics, not to get a grade. For Ava, this goal "helped a lot. It doesn't matter if I contribute grade wise, but it matters a lot if I am getting it. You feel a responsibility to the group because it's what you have to do." Consequently, the members of the groups began to rely on each other for support and differing

perspectives. For example, Michael commented, "Sometimes, someone did not understand so well and we would all go back and kind of cover it again. We would work together and use everything that we had done to arrive at an understanding."

This commitment to helping other members of the group understand the mathematics led three students to say, "I don't think anyone in our group ever left class totally lost." The group setting caused many students to develop a special bond with other members and to depend on them for assistance with difficult problems. Isabella exclaimed, "I love my green group because we all have different ideas and we all put them together. I love group work now. Go green!"

Nonetheless, during classroom observations, one of the five groups did not seem to work as effectively as the others. One member of the group, who had particularly strong mathematics skills, assumed a dominant leadership role in the group. He would quickly obtain an answer to the group tasks and spend the remainder of the time talking to other members, thereby removing them from the elaboration process. On one occasion, group members were grappling with triangular numbers. Within minutes, the group leader discovered the general formula and turned to the remainder of the group, saying, "Here's how it works…let me show you." He proceeded to tell the group how he solved the task as they took notes (see Appendix H for Triangular Number Activity).

While class observations revealed the imbalance in this group's discussions, subsequent data did not consign this group to any negative effects. No students wrote disparaging comments about their cooperative learning experience on the end-of-semester reflection statements, and no one from that particular group volunteered to participate in the personal interviews.

Mathematical Tasks. During the semester, students completed meaningful high demand mathematical tasks. These tasks rarely offered a specific solution strategy and often necessitated connecting previous mathematical knowledge (See Appendix I for the First Day's Tasks). In addition, students found that the tasks offered in this class challenged them to think more extensively about mathematics.

During interviews, students described tasks that they "did not immediately know how to solve." In order to reach solutions, students reported that they had to think in different ways and push a little further. One student wrote that, "All of our [problems] challenged me to push my thinking of math to a new level."

Not only did the mathematical tasks students experienced during this semester enhance student learning, they also reinforced the importance of wrestling with the mathematics through challenging problems. One student summarized this thought by saying, "This class was less about math and more about working through problems and going deep into them…We took easier problems to a higher level with this class."

Classroom Resources

Student learning in this course was supported by manipulatives and technology. The instructor supplied manipulatives nearly every lesson to help students build a concrete representation of the concepts being presented. For example, during one lesson, students were presented cut-out paper polygons, pattern blocks, and tracing paper to thoroughly investigate tessellations (see Appendix J for this Tessellation Lesson). Moreover, the instructor encouraged students to utilize any other resource they could find to further explore the concept. In addition, students utilized a dynamic graphing environment (DGE) in the form of Geometers Sketchpad (GSP) to more

deeply investigate geometric concepts. They utilized it to complete work during at least four class periods, on numerous homework assignments, and on tests and exams.

Observations revealed that student work on tasks in the computer lab was not effective in comparison to their work on tasks in the classroom. This finding was especially evident when the instructor was demonstrating to the students how to utilize the GSP program and leading activities with it. Occasionally, at the beginning of lab, some of the more advanced students would open Internet browser windows with their email and Facebook accounts. Then, when the instructor assigned them computer tasks to accomplish, they would quickly complete the activity and switch to these extracurricular interests. On other occasions, students would complete the task presented by the instructor and then proceed to create sophisticated geometric figures on GSP, utilizing the motion function to explore the movement of their creations. Often, these figures did not connect to the lesson being presented and involved more sophisticated geometric operations than the students had experienced during the year (see Appendix K for the Computer Lab, GSP Lesson).

Additionally, observations of student endeavors in the computer lab revealed that they spent significant time working individually on tasks. Students would arrive early for class, choose a computer toward the back of the lab, and begin to pursue non-class related activities. They frequently sat next to people who were not in their group, and they did not engage in discussion. When tasks were assigned during class, students worked independently until they completed the task or reached a sticking point. When students needed help, they often sought it from the instructor or a neighbor. Rarely did

they work in groups of more than two people. As stated in the above paragraph, those students who did complete the task often filled time with some off-task pursuit.

Despite these observations, probably because students are accustomed to working independently at a computer, interview data were positive. Students reported that their experiences with the manipulatives and technology motivated them to learn and made the lessons and other computer-based activities fun (see Appendix L for the Take Home Test 3, which contains sample GSP activities). Michael summarized the importance of manipulatives and technology with the following statement:

> One aspect of this course that truly aided with my motivation and with my enjoyment of the homework and class time was the hands-on nature of the problems. I do not recall ever being so actively engaged in a math classroom…The material used allowed for much more interactions with the math than before.

In addition, students utilized the manipulatives to enrich the mathematical learning that took place during this semester.

A variety of classroom constructs served to enrich the students' learning experience during this semester. A caring instructor provided the meaningful experiences required to facilitate mathematical understanding. Students worked in a cooperative learning environment that contained the sociomathematical and the sociocultural norms supportive of learning. Additionally, students used manipulatives and technological tools to complete high cognitive demand mathematical tasks. In this next section, I will describe the ways that this class helped students learn and become more confident about mathematics.

Perception of Self as Mathematician

Two broad themes emerged to suggest that students became better mathematicians as a result of their experiences in this geometry course. The first way in which students became better mathematicians was through increased mathematical knowledge. In addition, students revealed more positive feelings about mathematics.

Knowledge Gain

Students became better mathematicians during the semester because of the increased understanding of geometry that resulted from their efforts in this course. Evidence of this understanding was found in student reports during interviews and on the summary essay question. Also, they appeared to recognize the connectedness of geometry to other topics. Finally, their performance on a geometry assessment presented at the beginning and end of the semester improved.

During interviews and on the end-of-semester essay question, many students claimed that they believed their knowledge of mathematics increased dramatically during the semester. Some participants made general comments such as "I learned a great deal (about geometry) in this course" or "I learned a deep understanding of geometry in this course." Other students referred to the quality of the work they were able to complete as evidence of their knowledge gain. For example, Isabella remarked, "I learned a whole lot of math in this course…That's pretty obvious by the way I do my work." Likewise, Jacob expressed amazement at the content he was able to master by the end of the semester.

> If someone had shown me at the beginning of the semester some of the work that I had done, I would be like, 'no way I could do that work!' After doing

this class, I can do some crazy things. Now that I have done some of those things, it's a piece of cake.

Every student interviewed after the semester expressed the belief that she or he had a much more profound understanding of geometry after the class then the student did before the semester. Many essay responses concurred with this finding. For example, one advanced student wrote, "I thought that I knew geometry before I came into this class, but know I *really* know it."

For many of the students in this class, the extent to which topics were explored led them to believe that they had gained a deeper understanding of the concepts. One student suggested that the class "helped [her] to be more rigorous about certain things that I had only guessed at or understood intuitively, before." The instructor's efforts to push for generalities helped one student to "learn to look for exceptions to the patterns that I think that I have found. There were times during the year that I tricked myself into believing something that was wrong when I look at it more." These investigations helped many students develop knowledge of concepts that they had only partially grasped in previous classes. Emma summarized this belief with the following response:

> I think that I learned math very well in this course. It is obviously topics that we had learned before, but I feel like we took a different approach at it. [We covered] a lot of topics in more depth. A lot of the ways that we covered them will help it stick really well… better than in the past.

The above statements support the finding that students believed their skill in geometry increased by the end of the semester.

Many students reported that connecting geometry to other subjects in mathematics and to life experiences helped them gain a more meaningful understanding

of geometry. Some students reported that they were surprised when they discovered that "geometry was more than what they learned in high school…it's everywhere." For one person, "the most important thing I learned in this class is that geometry isn't just all theorems." Many of the students in this course learned that geometry is connected to other subjects in mathematics, and that making these connections will lead to an enhanced understanding. Jacob summarized this discovery by stating, "Before entering this class, I thought that I knew a lot about math. After completing this class, I [knew] what math can do…All math has meaning and makes so many connections it is crazy!" Not only did students discover that concepts in geometry relate to other types of mathematics, they also discovered that geometry can be found outside the mathematics classroom, where they discover it themselves. Students learned to "look at the world through the eyes of a geometer," seeing "geometry in every building, classroom, and organism of nature." In this class, they developed the ability to search these discoveries out and "dig deeper to learn more."

While many students who completed the end-of-semester version of this test reported that they rushed through it because of a busy exam schedule, they still demonstrated a greater geometric understanding with their responses on this test. Frequently, students were able to complete problems on the follow-up test that they did not even attempt on the first one. For example, one student utilized skills that she learned in this course to solve the following triangle equality problem at the end of the semester that she had left blank on the prior test (See Figure 1).
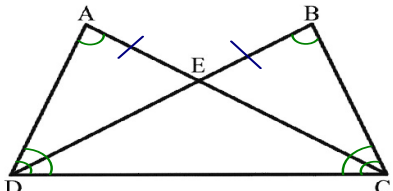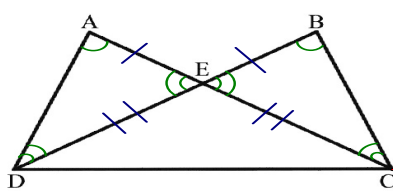
First Assessment	
In the figure below, segment AE is congruent to segment BE and angle A is congruent to angle B. Use deductive reasoning to prove that angles ADC and BCD are congruent. A B E D C	**? – I don't know.**
Follow-up assessment	
In the figure below, segment AE is congruent to segment BE and angle A is congruent to angle B. Use deductive reasoning to prove that triangle DEC is isosceles. A B E D C	∠AED is congruent to ∠BEC because DB and AC are intersectiong lines. If 2 angles in each triange are congruent, then the third angle has to be as well, thus ∠ADE is congruent to ∠BCE. Since all angles in △ AED aand △ BEC are congruent, the triangles <u>must</u> be proportional to each other, and since side AE is the same length as side BE, <u>all</u> sides of the two triangles are congruent. Thus, DE is congruent to EC and △ DEC is an iscosceles triangle.

Figure 1: STUDENT RESPONSE - GEOMETRY AND MEASUREMENT ASSESSMENT (PROBLEM 11)

On other occasions, students answered problems on both tests correctly, but demonstrated a greater ability to employ mathematical knowledge and provide clear answers. For example, one student utilized rules about transformation that she/he had learned during this semester to provide a detailed solution to a double reflection problem. While her/his response to each version of the question is accurate, the following response on the follow-up assessment demonstrates a greater focus on the process of solving the problem than the response to the first test (See Figure 2).
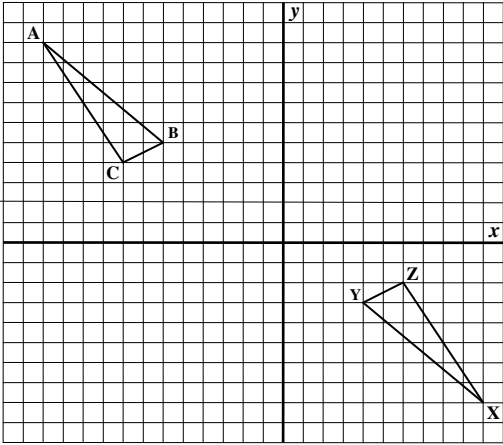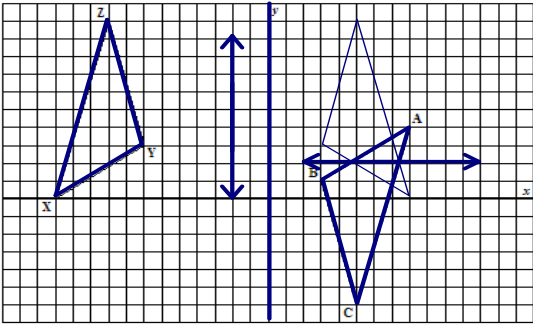
First Assessment	
  Two reflections on triangle ABC map A to X, B to Y, and C to Z. (Please note that each line of reflection is parallel either to the x-axis or the y-axis). a. Identify each line of reflection by writing its equation. b. Justify your solutions.	a. $y = 1$ & $x = -1$ b. from each of these lines all pairs are equal distances apart.

Follow-up assessment	
  Two reflections on triangle ABC map A to X, B to Y, and C to Z. (Please note that each line of reflection is parallel either to the x-axis or the y-axis). a. Identify each line of reflection by writing its equation. b. Justify your solutions.	a. $x = -2$ & $y = 2$ b. The reflection of line $x = -2$ brings the shape above its final rotation with the wrong orientation. The second reflection about the line $y = 2$ allows the final image to be seen.

Figure 2: STUDENT RESPONSE: GEOMETRY AND MEASUREMENT ASSESSMENT (PROBLEM 14)

Thus, even though students rushed to complete the end-of-semester assessment, they demonstrated a greater knowledge of geometric concepts.

Many students reported that their experiences in this course will help them become better mathematicians, which in turn will help them succeed in subsequent mathematics classes. Isabella summarized her learning in this course with the comment, "The way that we have been taught to explore mathematics with hands-on activities and critical thinking are crucial …to [the ability that I] developed to deeply and critically think about the subject matter." The well-formulated thinking about mathematics that students demonstrated during this course enhanced their confidence and generated a more positive attitude toward mathematics.

Increased Confidence

After completing this course, many students reported that they had generally positive feelings about mathematics. Evidence of this effect was found in student responses on the summary essay and student statements of increased confidence in their mathematical abilities during interviews.

Students who claimed to have been afraid or uncomfortable with geometry before the semester reported that their experience in this course augmented their confidence in all mathematics, not just geometry. Isabella, who early in the year told a visiting instructor that she had always "been awful at geometry" and "hated it," articulated the transformation that occurred for her during the semester:

> "I did not feel comfortable with geometry at the beginning of the semester, but now I am not scared of it. Like before, I was scared of 3-D stuff, but we needed to solve 3-D problems and I could do it with the toys [manipulatives]…I would say that I like geometry now, I wouldn't say that I love it…(chuckling) Yeah, I do love it. I like it a lot."

99

Like many of her classmates, Isabella was able to use the tools provided by the instructor to solve difficult geometric problems and gain confidence about mathematics.

Successfully completing the mathematical tasks and building a sense of understanding led to improved confidence and a generally improved impression of mathematics for many students. When asked to describe the most important thing they learned during the semester, one student wrote, "The most important thing that I learned is that I CAN DO GEOMETRY!" This student started the semester afraid that he/she would fail because of previous experiences in mathematics classes; however, as time progressed, this student acquired more confidence. In fact, he/she wrote that in regard to teaching, improving "this confidence is as important as learning any specific geometry." The self-assurance allowed the student to set aside his/her fears to focus on learning the mathematics.

Many students rely on this new confidence when they approach future mathematical problems. They have learned to depend on themselves and to trust their thinking when faced with new problems. Students leaned that they can do well in geometry without relying on others to explain it to them. In addition, they believed that they can critically think about mathematics to develop knowledge. One student wrote that "this class has made me aware that math does not have to be just one way, because you can look at it a whole lot of ways and get the same answer."

Experiences in this course have given many students the ability to tackle large problems in the future. One student wrote, "When I see a tough problem, I just sit down and do it because I know that I can." Some students have the self-assurance to solve difficult problems even when they involve unfamiliar mathematics. The following

quote is from Madison, who claimed to "be awful at geometry" at the beginning of the

semester:

> I am not scared to death of geometry anymore. Before, I have left math
> classes and felt like I know something, but not necessarily had
> confidence…Now I feel strong in math. Not only do I get it, but I feel strong
> and confident. This is largely due to (the instructor) asking why did you get
> this or how does that happen? I mean, "Can I write a formal proof?" At the
> beginning of class, I probably couldn't do it, but now I can!

While formal proofs had not been presented as a requirement of the course, this student

was enthusiastically anticipating an advanced geometry course to try out what she

learned in this class.

The qualitative data revealed that the mathematical knowledge and confidence

these students gained during the semester had an impact on their view of themselves as

mathematicians. As better mathematicians, some students reported "geometry can be

fun!" One student illustrated this transition by writing that "not only did I truly learn

about geometry this semester, but I can say I don't dislike it anymore…in fact, I now

think that it's really fun." This positive attitude about mathematics coupled with a

greater understanding of key geometric concepts was a major objective of this course.

However, a secondary purpose of this class was to provide students with experiences to

help them learn how to teach mathematics. For some of the students, this learning was

as or more important than the mathematical discoveries they made. In the next section,

I will illuminate some themes associated with the ways in which students learned the

pedagogy of mathematics instruction in this course.

Learning about Teaching Mathematics

Students gained valuable experiences to help them begin to build an understanding of teaching and learning in mathematics. Many students reported that they "really learned a lot about teaching" from observing the instruction they received during the semester. Ava stated, "A lot that I realized about teaching and how to get through to people came from being in this class and having the experiences that I had this semester." In addition, students learned about mathematics pedagogy through specific instruction and assignments from the instructor. In this section, I will discuss themes associated with the learning about teaching mathematics that occurred for the students in this course.

Learning Through Experiencing Mathematics Instruction

Many of the students' responses indicated that they were able to learn about teaching because the instructor "modeled good teaching." While building mathematical knowledge themselves, students learned to value the discovery process, to create an effective learning environment, and to provide tools to facilitate mathematical understanding.

Focus on process. As a result of their experiences in this course, many students learned to focus on the process of obtaining answers rather than the answer itself. They realized that people learn while struggling to solve the problems because "the important thing is not the answer to a math problem…but all the work it took to get it." One student stated that her encounter with take-home problem sets and exams taught her "how important it is to persevere. When you struggle to get the answers, you really learn the math behind the problem." When students wrestled with challenging

problems, they learned to have "patience, because there are times that you have to set aside the problem to work on it later. I did not know how to do that before this class." In addition, attending to the process of completing problems helps students learn "that it is important to know the math behind the shortcuts that are taken…because each student will evaluate solutions using different methods."

Multiple solutions. The realization that students solve mathematical problems and learn mathematics through a variety of methods is another important discovery that participants made during this semester. Isabella articulated this idea as follows:

> I learned that there is more than one way to solve a problem and all of them are worthwhile…Personally, I had a hard time in geometry and now realize that I don't have to solve the problem a genius way, I just have to solve it, to look at it from different angles. I think that this is how a lot of people feel.

Some of the more advanced students even learned to value the different approaches of others, because when they "knew a way to get the correct answer, others had other effective and valid ways of finding the same solutions." Students ascertained that when they present material in a classroom setting, they must account for diverse abilities and thinking. This knowledge will lead them to account for "different learning methods and styles…when break[ing] down concepts." Therefore, they discovered how important it is to "present things in a lot of different ways, [because] the more ways [there are] to reach the students, the better the understanding will be."

Mathematical understanding. During this semester, students developed an appreciation for mathematical understanding as opposed to memorization of mathematical facts, rules, or algorithms. They learned that, whereas they might be proficient enough to answer basic mathematical questions, they begin to truly

understand the mathematics behind the questions by discovering and constructing their own knowledge. One student wrote, "I have seen in this class the importance of discovering things by working through them instead of just being taught." During an interview, Ava exclaimed, "I have discovered a new way to teach geometry…Students can learn geometry and theorems without having to memorize them!" Lessons, classroom tasks, and homework assignments were designed to facilitate understanding by making connections to existing knowledge. Students learned the importance of connecting new concepts to previous mathematical learning. Michael summarized this discovery as follows:

> To learn, you have to have some prior knowledge. As a teacher, you need to find that knowledge and go from there. Value what they bring to the classroom and connect that to the class, especially in geometry. This is what I learned in this class."

This student has articulated a discovery that was repeated on numerous occasions in the data.

Cooperative learning. Every interviewee and many who completed the end of the semester essay reported that they learned the importance of cooperative learning based on their positive experiences with it in this class. At least two people wrote that the efficacy of cooperative learning was "the most important thing that I have learned." Students realized that if the purpose of group work is "just to discover math, then everyone in the group will actually talk…[and] all members will contribute." They discerned that learning in groups allows people to "talk about the math," "learn how to explain things better," and to be "reassure[d] that you are on the right track."

In addition, group-mates in this class provided a community of support when members were struggling with concepts. One student wrote:

> In previous classes, I felt like I was on my own. In here, I had my group to help when I did not understand or when I thought that the question that I had was too embarrassing to present to the teacher.

Having their group-mates for assistance prompted three interviewees to state that neither they nor anyone in their group ever "left class feeling lost."

The effectiveness of the cooperative learning that occurred during this course taught students that people other than the instructor have valuable contributions to make to their learning experience and vice versa. One student explained, "I think that this class has taught me that students can sometimes come up with important ideas even before the teacher has to tell them." In addition to facilitating mathematical understanding, the instructor's efforts to provide students the opportunity to discover mathematics helped some students to "learn to allow [people] to think about problems and strategies for a while [because] it only takes a student thinking about it for a long time to get an epiphany."

Students' experiences with effective cooperative learning groups in this class prompted many of them to indicate that they will utilize group work in their classes. One student predicted that "when I teach, I will use group work as a tool to let students explore on their own and piece things together with the help of their peers." Their exposure to cooperative learning in this course has served as a "great model to reference while [they are] teaching." This reference gives the students the confidence to incorporate cooperative learning in their practices as teachers, because they "know how to make it effective and beneficial for all involved." During an interview, Isabella

declared, "When I teach, I will definitely put [my students] into groups…That's important. By letting them work in groups, they can come up with it before you even have to tell them."

Mathematical tasks. Participants also learned the supportive role that meaningful mathematical tasks play in the discovery process. They realized that one benefit of these tasks will be to help their students focus on understanding the concepts rather than memorizing a procedure. According to Jacob, "Ultimately, I have learned that math treated as an investigation is *always* more effective than math treated as memorization." To illustrate how much this student's opinion changed during this semester, he had remarked at the outset of the study that he would insist that his students repeat his procedures in "exactly the same way as me."

These examples show how participants express a desire to "foster minds that think like a mathematician instead of relying on rote memorization." Students learned that mathematical tasks requiring effort and building on previous mathematical experiences and knowledge will foster understanding.

Participants expressed a desire to implement meaningful mathematical tasks in their practice as teachers. First, they intend to utilize classroom activities that require effort to generate a "solution that is not always fast or immediate." During an interview, Sophia said that she would provide tasks that require "more talking than before" because such tasks are more challenging. She commented, "Patience is going to be an important thing. Part of being patient is giving my students all the help and information I can give, then letting them *think* about it." Second, nearly all participants said they will develop activities that build on previous mathematical knowledge. One

student wrote, "In my future profession of education, I will incorporate activity-based learning and encourage my students to find meaningful relationships among what they learn." Based on their exposure to this course, students learned the importance of developing high cognitive demand tasks that make connections to previous experiences.

Technological tools and manipulatives. After utilizing classroom manipulatives and dynamic graphing environments (DGEs), like Geometers SketchPad (GSP), students learned that classroom tools can help people at all levels. While most students believed that manipulatives in elementary school were important, they learned that "even at the upper levels they are important. There are things that I think that we would have never learned as well without them." Another student wrote, "One of the most important things that I learned in this course was how valuable hands-on activities can be to the learning experience." Likewise, students learned that DGEs can enrich student learning. During an interview, Michael described what he learned about DGEs as follows:

> I learned…the usefulness of computers for interpreting, understanding, and manipulating geometry. Figures that would be tedious to draw and manipulate on paper can be easily drawn with GSP. By moving points and lines, you can easily see the relationships between them.

Their work with the manipulatives and DGEs in this course taught students to use them to "picture relationships that [they could not see] in their head" and to develop a "better understanding of the geometry."

All interviewees reported that that they will provide manipulatives as a classroom resource to support their students' learning. For example, Jacob stated that he "will definitely hand [his] students some toys [manipulatives] to learn something."

107

Ava described her belief that manipulatives will help her meet the needs of diverse

learners:

> In my class, I would definitely try to include resources and manipulatives because students are different learners. I sort of knew that it was helpful for other students to experience it differently before this class, but it helped a lot to see it in this class. I remember days when we worked with two or three different types of manipulatives. I saw it really clicked differently for other people in my group.

Moreover, students intended to support learning by utilizing computer

technology. One student wrote, "There is a vast amount of math technology out there.

Technology makes things fun, so by incorporating it into my curriculum, I'll be able to

accomplish things that I wasn't able to accomplish without it." Some pre-service

middle school teachers who utilized GSP during this semester "hope to use it in [their]

classroom[s] one day." One student reported that she "will use GSP in my future

classroom so that my students can continue to learn and explore geometry." After

completing this course, students learned the value of manipulatives and technology and

believed that they will be a valuable resource for their classrooms.

The data revealed that students learned "about teaching because [they] watched

a great teacher teach math" by building an "overall environment for learning." They

learned to "encourage [their] students to find the math themselves, not from the teacher"

by making mathematical connections and employing group work. They also learned

how to provide tools to help others "learn through concrete activities instead of

memorization." In the next section, I will discuss some additional discoveries students

made regarding course assignments and direct instruction.

Learning Though Direct Instruction and Course Assignments

When this geometry course became open to pre-service elementary school teachers, the instructor made an effort to incorporate more instruction about pedagogy into the syllabus. A review of the data revealed that some of the ways the instructor met this goal were through direct instruction and two course assignments.

During the semester, the instructor occasionally provided direct guidance about how students might work in a classroom setting. For example, when a student was working at the board with his back turned to the class, her instruction helped him "learn that when working at the board, I need to stand at a certain place where I am facing the class." On another occasion, a student was working at the board without including the class. The instructor interrupted him and encouraged him to allow the class to provide assistance. During this interaction, she was encouraging him to stop giving the answers to the class and allow them to take responsibility for their own learning. During interviews, Sophia reported that she developed an awareness of the diversity in all classrooms when "she [the instructor] told us about the diversity that she saw in our classroom."

The data revealed that students learned about the role of technology in the classroom from the work they did on an assigned research project. For this project, students were instructed to conduct research, write a short paper, and give a class presentation on some technology to use in the classroom. Michael stated that he "really found it helpful to know about all of the websites that are available to math teachers." During the presentations, students were actively taking notes about the discoveries

made by their classmates. Some students even pulled out their laptop computers to check and mark websites during the presentations and after class.

In addition to the technology project, students wrote a paper about the van Hiele model of geometric thinking and instruction. Learning the details of the model helped students "understand how important it is to [know] and meet the students at their level" and to "realize that language is really important" when communicating with people at different levels. Writing and thinking about van Hiele's phases of instruction have reinforced the importance of providing concrete representations of key concepts during geometry instruction. One student wrote that knowing the "van Hiele model for learning will benefit my future students because I have learned the value of physically interacting with shapes."

The students' efforts on the van Hiele paper and subsequent reflection and discussions about the model seemed to have an impact on their intention to recognize the uniqueness of their students and modify instruction to account for the different levels of understanding their students possess. For example, one student wrote, "I have to keep the van Hiele levels in mind when I teach because all of the students in my class will not be on the same level." In general, students in this class realized that individual students are unique and stated an intention to adjust their instruction accordingly. One way that students intend to accommodate their students' disparate stages of content mastery is to incorporate a range of instruction strategies.

For many students, their work with the instructor during class and their work on assignments outside of class helped them build an understanding of what it means to teach mathematics for understanding. Moreover, this knowledge provided students with

a foundation of good mathematics instruction. Jacob described how he has applied his knowledge of mathematics teaching to make sense of some observations he made during a school visit:

> I watched a teacher in math during a school visit. The teacher put homework on the board and told the kids to check their work. She lectured a bit and then she gave them homework and told them to work. Only about four kids did their work and shared their answers with the rest of the class. Knowing what I know from this class [that I am taking at UNC], I realized how these kids were not doing their work because they did not care. They were not given a chance to discover the math and interact with it with their group mates.

Earlier in the interview, Jacob reported that if asked to lead a mathematics class before this semester, it would look very similar to the class he had just described. However, during this semester, his view of how mathematics should be taught shifted significantly. During this next section, I will elaborate on this and other changes in student beliefs about mathematics instruction that occurred.

Changed Beliefs

During the course of the semester, a variety of changes occurred in students' beliefs about mathematics education. In this section, I will elaborate on the three most prominent changes that were uncovered by the data. First, students' beliefs about the source of mathematical knowledge altered during this course. Second, students' beliefs about the role of cooperative learning in the classroom changed. Finally, participants in this class developed different beliefs about the way that that they would teach mathematics.

Epistemology

The data revealed that as the semester progressed students began to see themselves and the classroom community rather than the instructor as the source of mathematical knowledge. During interviews, students indicated that they initially believed knowledge originated with the instructor and the textbook. However, by the end of the course, every participant stated that the role of the instructor and book were to guide students toward finding the knowledge within themselves. For example, Madison commented:

> [Before, I believed that] the book and the teacher hold the knowledge and they impart their wisdom by showing the steps, methods, and examples. The teacher shares the knowledge. [Now], I would say that a lot of knowledge comes from the students and students share it with the rest of the class. But the book and teacher have knowledge, too. As a student, you need to share how you learned and what you bring to the classroom.

Isabella reiterated the view that knowledge can be self-generated in the following comment:

> In this class, [the instructor] pulls the knowledge from us. Everyone has different amounts, but a lot of times it is her fishing and us pulling it out of each other. I think it comes from everywhere. I may not understand the topic, but the questions that I ask can help.

Further evidence of the students' belief that they on whole are the source of mathematical knowledge was observed in the classroom. At the beginning of the semester, students were inclined to wait for the instructor's prompts before participating in the conversation. For example, during a class meeting early in the semester, the instructor asked students to work in their groups on an activity that was intended to discover a proof for Thale's Theorem. After providing groups some time to explore the theorem, she asked students to supply a generalization by asking, "Can you give me a

112

statement [to generalize what you found]?" After a period of silence, she asked, "What would Thale's Theorem say?" Finally, after another 15 to 30 seconds of silence, she pulled the class together and began to lead a class discussion (see Appendix M for the Thale's Theorem Lesson). Later in the semester, students in the class took responsibility for their learning during an exercise designed to discover dilations and scale factor. As before, the instructor provided students some time to explore the task. However, this time, Isabella prompted class discussion by saying, "The scale factor is 1:4." Then she joked, "That's ¼." Then, to the whole class, not just the instructor, she asked, "Can you say it that way?" After a bit of a discussion, the instructor pointed out that another person had a different approach to the task. The student moved to the board and presented a geometric illustration of scale factor. During this entire exchange, the instructor's role was to serve as a guide while the students shared their newly-discovered knowledge with each other (see Appendix N for the Dilations Lesson). During an interview, Emily articulated this change in roles with the following statement:

> [Before the semester], I [didn't] know how you get knowledge, just by doing,
> that's how I used to do math. [The teacher] taught you the steps and you
> practiced. [Now], I would say that people learn through discovery, or better
> understanding comes from discovery. You can have someone tell you things
> all day long, but it might make sense to you. You can have someone show
> you the steps to do a problem and you might even be able to do it, but still
> have no idea why you are doing it. Now teaching for me would be a lot more
> about leading a discovery as opposed to telling them how to do it.

Classroom experiences such as the one above regarding dilations and scale factor activity have helped students learn that mathematical discoveries come from someone other than the instructor. If the students believe that the source of knowledge

comes from themselves and their classmates, with the teacher guiding students to discoveries, then students might have more confidence working in groups. I will discuss how this confidence led to changed views about cooperative learning in the next section.

Cooperative Learning

Some of the most striking changes occurred regarding students' opinions about cooperative learning. At the beginning of the semester, all of the students who were interviewed were unhappy to learn that they would be doing so much work in groups. Their experience in previous classes had taught them that group work was a time of "goofing off" where "everyone gets the same grade, no matter how unfair it is." Over the semester, every participant's beliefs about group work changed. Ava stated;

> I am just amazed with how group work works. Any other time I have been in a group work situation, I am just stressed and exhausted, and I hate it. I am actually pretty sad that my group is not going to work on future problems. I would like to, as a teacher, try to use more of it. I think it is a good thing for everyone.

Students began to value cooperative learning for a variety of reasons during this course. One reason was that they recognized that "group-mates helped [them] learn." For some students, a small group setting produced the assurance to attempt difficult mathematical tasks. Others learned to appreciate different approaches to solving the problems. Isabella described the change that occurred for her:

> Before this class, I hated group work…Instead of 'lets all learn,' it was 'how about you do all the work so that we can have a good grade.' Usually, I was the one who did all the work. Now, I love my group because we all have different ideas and we all put them together.

The different abilities, experiences, and perspectives that people bring to cooperative

learning can, under the right conditions, lead to a richer educational experience.

During interviews, some participants suggested that one factor accounting for

the success of cooperative learning was the focus of the work. Emma made the

following statement:

> [Before this class], I had mixed feelings about group work. A lot of my
> group experiences have been frustrating. You all get one grade, so if they do
> not do their work, then you hurt. There is not a good answer for it. In this
> class, there were more repercussions if you did not do your work. Like, if
> you did not do your work, yeah, you could get a good grade, but there was so
> much more that you would miss if you did not do your part. If you do not do
> your work, you won't learn.

Ava went a step further with her statement. After reporting dissatisfaction with prior

group work, she commented,

> [Now] thinking in groups and talking in groups is different than completing
> an assignment in groups. When there is no time limit and no grade, it is just
> kind of a discussion in groups…The purpose of group work is to see how
> you think and to see how other people think…to see if the person next to you
> has a different idea. The cool thing about math is there is more than one way
> to get an answer. You see that a lot with group work

Both of these students had come to the realization that when the focus of group work is

to learn rather than turn in work for a grade, groups can function effectively.

Participants who were interviewed felt strongly about the role of collaborative

learning in mathematics. In fact, when answering the question, "How would you have

taught a math class before this semester?" Isabella hesitated and quietly stated that her

"students would work alone." When asked why she hesitated, she said,

> I would change my answer now because I think that group work is really
> important. I have always thought that group work might work. I just did not
> know how to work it. I really like collaboration! ...Now that I have seen the
> importance of group work, I think that management of the classroom might

be more difficult [when students do group work], but the benefits really pay off.

Similar to this student, many participants now realize the benefits of working with others and will try to implement cooperative learning when they become teachers. In general, students describe altered ideas about how they expect to teach. In this next section, I will elaborate on some of these changes.

How They Would Teach Mathematics

Interviews with students revealed that their beliefs about how they would teach a mathematics class changed during the semester. All interviewees reported that they would have presented a traditional lesson if they were asked to teach at the beginning of the semester; however, by the end of the semester, each insisted that she/he would present very different lessons based on experiences in this class. For example, Sophia, who had a nebulous perspective on classroom instruction early in the semester, stated the following:

> [Before taking this class], it would have been very hard to tell you how a class that I teach would look. I would probably spend some time teaching on the board and then give some time for students to work on their own. [Now] I would add how important it is for students to work in groups...Instead of going up there and teaching the topic, you can draw it out of the class.

Other students, who had a well-defined idea of how their mathematics instruction would look before taking this class, changed their view of mathematics teaching during the semester. For example, Jacob stated,

> Before this semester, students in my math class would probably see me work math on the overhead, do worksheets, and follow my steps EXACTLY. But now, it would be more interactive. I like the whole 'explore as a group' sort of thing and using manipulatives is really fun. Actually holding the objects in your hand was good. I would use these things in my class, now.

Participants' responses to Bush's (2007) Geometry and Measurement

Assessment revealed a similar modification in the way they believe that they will teach

mathematics as professionals. Some questions on this assessment asked respondents to

find the conceptual error in a student's response to a problem and propose a way to help

the student "correct his/her thinking." Some participants found the error in the students'

thinking on both assessments, but offered more open-ended, discovery-based activities

as a strategy to help the students. For example, to correct the students' thinking on the

pre-test (see below), one participant wrote that she/he would provide a specific example

and "explain that a quadrilateral is any 4-sided figure then show him with some

examples." Her/his response on the post-test demonstrates a commitment to utilize

reform-style instruction. On the follow-up test, she/he said, "I would have a group of

quadrilaterals and go through each one to see if there was symmetry…She would test it

first and then I would check and explain if need be." This participant has demonstrated

a desire to allow the student to uncover her mistake with a variety of examples.

Moreover, she/he is willing to offer an explanation only if it is needed (See Figure 3).
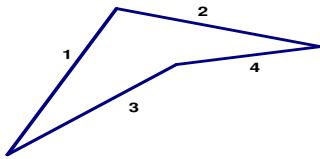
First Assessment	
A student was asked to investigate the rotational symmetry of quadrilaterals. After investigating rectangles and squares, the student concluded, "These shapes have rotational symmetry so all quadrilaterals have rotational symmetry." a. What is incorrect about this student's conclusion? b. Explain how you would help the student correct his thinking.	All quadrilaterals do not fit into the rectangle or square category and therefore upon rotation may not have symmetry. I would then give an example *[hand-drawn quadrilateral with vertices labeled 1, 2, 3, 4]* Then after explaining that a quadrilateral is any 4-sided figure then show him with some examples.
Follow-up assessment	
A student was asked to investigate the lines of symmetry of quadrilaterals. After investigating two rectangles, the student concluded, "These rectangles have lines of symmetry, so all quadrilaterals have lines of symmetry." a. What is incorrect about this student's conclusion? b. Explain how you would help the student correct her thinking.	She did not take different types of quadrilaterals to test her theory like squares, trapezoids, parallelograms, etc. I would have a group of quadrilaterals and go through each [one] to see if there was symmetry along with her. She would test it first and then I would check and explain if need be.

Figure 3: STUDENT RESPONCE – GEOMETRY AND MEASUREMENT ASSESSMENT (PROBLEM 19)

This geometry course provided a model for mathematics instruction for the students who did not have a clear notion of what their teaching would look like. Additionally, experiences in this course helped to reassure students who believed that they understood how to be a good mathematics teacher. In both cases, students left the

course with the desire to utilize a reform-based, inductive style of mathematics instruction when they become professionals. The data revealed that students have such powerful expectations about the role of these activities in their classrooms because they actually experienced their effectiveness. In this next section, I will discuss the importance of this particular class in the development of these participants' pedagogy.

Necessity of this Course

Many participants expressed the belief that their exposure to the mathematics instruction in this course will allow them to become more invested in teaching methods they learn in subsequent courses. For example, on the topic of various instructional strategies, Jacob stated, "I sort of knew that it was helpful for other students to experience [mathematics] differently before this class, but it helped a lot to see it in this class." Whereas this student had some knowledge of different learning styles, his direct experience made that knowledge more salient. In this section, I will report themes associated with the necessity of taking this course before other methods classes.

Participants indicated that seeing how to teach mathematics for themselves has provided concrete examples of specific instruction that they will receive in subsequent mathematics education methods course. For example, one student indicated that "when the [van Hiele] model was first introduced, I was a bit confused by the whole concept. But, after experiencing it as a student, I can certainly see the benefits...and hope to use it in my classroom." Regarding cooperative learning, one student indicated that "the knowledge that I gained about groups is…something that is only truly understood from experiencing, not from reading about it."

Other students expressed how experiences in this course have produced a

general change in their beliefs about mathematics instruction, changes that they feel

were necessary to make before they take their methods course. During interviews, I

specifically asked students, "Before taking this class, if someone told you in your

methods class that people can learn mathematics the way you learned it in this class,

would you believe them?" All participants indicated that the exposure to reform

mathematics instruction in this course would help them trust what they learn in their

methods course. Eva indicated that she would have been a skeptical believer. She said:

> I would have agreed [with the methods instructor] but having these
> experiences as a student, I realize how important discovery is. I would
> [sarcastically] say, 'Oh yeah, that really works." If I had not had this class, it
> would be coming from nowhere.

Participating in this course gave her a framework on which to build new ideas about

mathematics teaching that will be presented in her methods course. She continued:

> I would not have been able to say anything about it [the teaching
> methods]. Having these experiences doing math in this "how to
> teach math" environment helps me believe that it works. Otherwise,
> I would be having this stuff thrown at me without having any
> experiences seeing what it is like. It would have no relevance and
> you would not need to believe it because it comes out of nowhere.

Her willingness to believe that teaching mathematics in a reform classroom is

beneficial came from work in this class. She concluded as follows:

> Now, you can know it works! In methods, I will be able to draw on
> experiences in this course that have affected my learning and my thinking
> about how to teach math. I think that this is a really valuable experience as I
> head to methods. It makes me excited to become a teacher.

Sophia said:

> I think that I would have been unsure if I trusted it. I'd think that the
> students still need to be told the answers, but this class has made me realize
> that there is a lot more that [students] can do on their own than I had
> thought."

Instead of being incredulous learners in their teaching methods class, these students

have become eager participants who are excited to learn more about innovative

mathematics instruction.

Isabella indicated that her experience in this class was crucial, because now she

will understand how to use the tools she will learn in her methods class. She said:

> If my methods teacher had told me about teaching math this way and I had
> not had [this instructor's] class, I would have probably said that she does not
> know what middle schoolers are like. I would probably be open minded, but
> I would need examples…I would be skeptical and would not know how to
> use the tools that she was talking about. Like, if you had handed me some
> block to teach a lesson, I would have said, 'What am I going to use these
> blocks for?'

While Jacob endorsed collaborative work in other subjects, he did not think it would

work in mathematics. He explained his transformed attitude as follows:

> I think that if I had been told about a different way to teach math in my
> methods class, I would say that, "Yeah, in theory, it's great." Having seen it
> work has helped me believe. I have always felt better about working with
> others, but I have not done it in a math class before.

Exposure to the reform-based mathematics instruction in this class taught

students how to use the tools and strategies presented in their methods course.

Moreover, after reflecting on their personal experiences, students now believe in

reform-based instruction.

This foundation of understanding allows the theories presented in methods

courses to take hold. As one student wrote, "Without having these experiences [in this

class], I do not think that I could put together how to teach math." Many students commented that they were more likely to accept what their methods teacher was telling them due to having witnessed its worth.

Summary of Results

The study documented students' experience in a reform-based geometry course and the impact of those experiences on participants. Students' experiences in previous mathematics courses and influential components of their experiences in this particular class provided the first two broad categories of themes. Regarding the impact, the data analysis reveals categories of themes that addressed (1) how students' perceptions of themselves as mathematicians changed, (2) what students learned about teaching and learning in mathematics, (3) how students' beliefs about teaching and learning in mathematics shifted, and (4) whether students believed that this course played an important role in their preparation as future teachers.

Experiences in Mathematics Courses

The data revealed that when discussing memories of previous mathematics classes, student describe traits that are unique to traditional instruction. During these lessons, teachers directed instruction, parsing out relevant knowledge to students who passively reviewed previous homework, took notes, and worked on current homework. While cooperative learning rarely occurred during mathematics instruction, students consistently reported recollections of unfair divisions of labor and poor group discourse. These imbalances led all of them believe that group work did not have a place in mathematics classrooms.

Classroom Components that Influenced Learning

Students describe the student-centered lessons in this particular geometry course as being entirely different from previous courses. In this class, students worked in cooperative learning groups to solve meaningful mathematical tasks. They were provided tools, like manipulatives and technology, to explore these tasks. Their caring instructor developed a learning environment that would facilitate mathematical understanding by placing the students at the center of instruction.

Improved Mathematician Skills

The data revealed that students became knowledgeable, confident mathematicians as a result of their exposure to the instruction in this course. During the semester, students demonstrated improved mathematical skills on assessments, and they reported a belief that they expanded their knowledge. Moreover, students reported that they could utilize the skills that they learned in this course to gain understanding about entirely new mathematical concepts. This belief was a demonstration of the growth in their confidence in both geometry and general mathematics.

Learning About Teaching

In addition to learning about mathematics, the data revealed that students gathered pedagogical skills on which to draw when they become teachers. By experiencing a reform mathematics classroom, students learned how to create a constructivist learning environment complete with tools and resources that support learning. Students learned to appreciate (1) the process of obtaining an answer, (2) multiple solutions to mathematical problems, (3) learning for mathematical

understanding, (4) the value of cooperative learning in the classroom, (5) the impact of meaningful, high-demand mathematics on understanding, and (6) the role of classroom tools, like manipulatives and technology, in the learning process. Additionally, specific course assignments and instruction taught students theories of geometry learning and strategies to support lesson planning and classroom instructions.

Changed Beliefs

Many participants demonstrated that their beliefs about teaching and learning were transformed during this semester. Most importantly, students came to realize that mathematical knowledge originates from within students in the classroom, not just the teacher and textbook. Out of this expectation grew a commitment to the efficacy of cooperative learning; consequently, many students reported that their mathematics class will look different than they believed it would at the beginning of the semester.

Necessity of this Course

Finally, many students reported that this reform-based geometry course played an important role in their pre-service teacher training. They stated that their concrete experiences in this class will make them more likely to accept the instruction of their methods instructor.

Chapter Five

Interpretations and Implications

Discussion of Results Regarding the Research Questions

The general purpose of this study was to document student experiences in a reform-based geometry course and the impact of those experiences on participants in the course. Regarding the latter, this study specifically examined (1) how students' perceptions of themselves as mathematicians changed, (2) what students learned about teaching and learning in mathematics, (3) how students' beliefs about teaching and learning in mathematics shifted, and (4) whether students believed that this course played an important role in their preparation as future teachers.

Question 1: To what extent do students perceive that they have improved as mathematicians based on their experiences in a standards-based geometry course?

The first research question was designed to explore focused on the degree to which students in this geometry course began to see themselves as mathematicians. Specifically, did students report that they gained a deeper understanding of geometry as well as confidence in mathematics? The research indicates that content knowledge is a key characteristic of effective teacher preparation courses (Grover & Conner, 2000) and that teachers must possess a unique understanding of mathematics to teach it effectively (Hill et al., 2007). Unfortunately, teachers who lack that mathematical understanding also lack confidence when teaching the subject (Sowder, 2007). Two major objectives

for this geometry course were (1) to provide pre-service mathematics education students an environment in which to develop a rich understanding of the geometric concepts being taught, and (2) to create learning experiences that will provide these future teachers with the skills and confidence to learn new mathematical concepts as professionals.

The results indicated that students did believe that they gained an understanding of geometry resulting from their work in this course. One reason students reported this increase in understanding was because of the connections they made to previous mathematical knowledge. This finding is consistent with Carpenter and Lehrer's (1999) document. In addition, students indicated that they gained a deeper insight through the interactions with peers that occurred while solving meaningful mathematical tasks. The high-demand mathematical tasks (Stein et al., 2000) that students solved in their cooperative learning groups (Slavin, 1995) formed the foundation for the richer understanding the students reported.

Students also reported feeling more confident in their abilities in mathematics as a result of their experiences in this course. Not only did they believe that they were capable of solving the challenging geometry problems they saw in class, but they expressed enthusiasm about solving original geometry tasks. Moreover, students reported feeling convinced of their ability to derive meaning from other areas of mathematics they will see in their future as professionals. Expanding from experiences in this course, students have become empowered to escape their fear of mathematics, as Hargreaves (1995) describes.

Question 2: What do students report that they learned about teaching and learning in mathematics during the semester?

      The second research question investigated what students reported they learned about teaching and learning in mathematics as a result of their experiences in this class. During the investigation of this question, two components emerged. First, do students gain pedagogy incidentally by experiencing an NCTM Standards-based mathematics environment and observing a reform mathematics instructor? The results indicated that students learned about teaching mathematics from their experiences in this class. For example, by utilizing mathematical tools to solve challenging tasks, students recognized the powerful role these resources can play in the classroom. In addition, they learned how to build a classroom environment that establishes necessary sociomathematical norms (Yakel & Cobb, 1996). In fact, students' beliefs about classroom environments that stimulate learning in mathematics is a theme that runs through the results. Consequently, I will discuss the classroom environment in much greater detail in the next section.

      The second component that emerged while investigating research question #2 was what students learned about mathematics teaching and learning through direct instruction. While this course was formally offered through the Department of Mathematics, students understood that it was taught by an education professor and was intended for education majors; consequently, they began the semester expecting to study teaching. Students reported that their research projects about van Hiele levels stimulated them to think about various models of instruction. Additionally, many students stated that they remembered the strategies the instructor provided them during

class discussions and group presentations.   The above finding seems to be consistent with Sowder and Schappelle's (1995) and Shifter and Fostnot's (1993) finding that teachers learn about teaching mathematics when they participate in reform-style instruction.

An investigation of the language students used to answer interview questions and the summary essay was an indicator of how students in this class internalized what they studied. Specifically, students employed terminology that sounded as if it came from the reform mathematics literature. For example, they would talk about making connections or discoveries, making generalizations, finding solutions as groups, and solving meaningful mathematical tasks. They used these descriptors even though they were never required to read about reform instruction, and the instructor did not present it during class time. However, one caveat about this conclusion is that I was unable to interview these students at the beginning of the semester. Students might be drawing on language that they have learned in other education classes to express the pedagogy they derived from this one.

Question 3: Did students' beliefs about teaching and learning in mathematics change as a result of their experiences in this course?

The third research question sought to determine the extent to which student beliefs about teaching and learning in mathematics changed as a result of experiences in this class. The results revealed that a dramatic shift occurred during the semester. Most importantly, participants who supposed that mathematical knowledge originated with the teacher and book at the beginning of the semester came to believe that mathematical knowledge originates in both students and teacher. This view of the student as an active

participant in the learning process adheres more closely to the constructivist perspective on which reform instruction is based.

A change in belief about the learning environment grew out of the above transformation. At the beginning of the semester, all of the students reported that they wanted to create a traditional classroom environment in which the teacher presented information in the front of the room while students gathered it through notes and homework problems. By the end of the semester, every interview participant reported that he or she intended to develop a reform classroom in his or her teaching practice. In those classrooms, students would gain mathematical understanding by making connections to previous knowledge and conducting conversations with peers while solving meaningful tasks.

These findings seem to be supported in the literature. Blanton (2002) reported that student beliefs about discourse changed after they participated in geometry instruction that modeled the effective use of discourse. Likewise, Shiftner and Fostnot (1993) reported that teachers implemented new mathematics teaching pedagogies rooted in the principles of constructivism after experiencing it in an in-service teacher education program.

Question 4: Do students believe that this standards-based geometry course played an important role in preparing them for subsequent teaching methods coursework?

The fourth research question evaluated whether students believed that their experiences in this geometry course played an important role in their pre-service teaching program. Specifically, are students more likely to believe the theories about

reform instruction that will be presented in their methods class, having experienced it in this course?

To varying degrees, seven of the eight interview students reported that they would more likely believe in the efficacy of reform instruction because they have witnessed it. This course appears to have given students concrete examples on which to draw when they are exposed in the future to unfamiliar methods that are characteristic of reform instruction. Likewise, when learning about teaching strategies in a reform classroom, students will know how they look from direct experience. Free of skepticism, students will become more open to new ideas in subsequent teaching methods classes.

Participants in this project mimic similar propositions in the literature. Ma (1999) proposes that students must experience mathematics instruction in an environment that is similar to what they are expected to create as professionals. Pre-service teachers begin to develop what Blanton (2002) describes as "incidental pedagogies" (p. 118) from their experiences as students of mathematics. These pedagogies, which are likely to model what students experience in recent mathematics courses (Grossman, 1990), should be built from knowledge of teaching and learning, teacher behaviors and best practices (Darling-Hammond, 1999).

Discussion of General Results

The participants in this study demonstrated an understanding of teaching and learning in mathematics across research questions. In this section, I will discuss three general findings that seem to be evident throughout the results. First, participants learned how to build a classroom environment that will help their students find the

mathematics within themselves through the discoveries they make. Second, students

came to believe that cooperative learning is an important component of that

environment, and they learned how to implement it. Finally, students compiled their

experiences and observations during this semester to develop a teaching toolbox of

strategies and resources they will use as professionals.

Classroom Environment

Most of the participants in this study reported that they experienced traditional

instruction in prior mathematics classes; consequently, many of them expressed anxiety

about the mathematics that they would study during this course. In contrast to

traditional methods, this class encouraged active participation in the learning process.

Students made connections to previous mathematical knowledge through interactions

with peers under the guidance of the instructor. The data revealed that by the end of the

semester, students had developed a conceptual model of a classroom environment that

facilitated mathematical understanding.

The results indicated that participants wish to establish a classroom that contains

sociomathematical (Yakow & Cobb, 1996) and sociocultural norms (Kazami & Stipek,

2001) that nurture student learning. During interviews and on the essay, some

participants in this study made the following commitments in their practice as teachers:

1. To emphasize the process that their students took to obtain an answer, not the answer itself.

2. To examine and find meaning in the errors that their students made.

3.   To require their students to utilize sound mathematical arguments and reasoning to justify conjectures.

4.   To encourage the class or smaller groups to collaborate while finding solutions to classroom tasks.

Students expressed a strong desire to implement these norms.

Believing that they gained a deeper understanding of geometry in this course, students committed themselves to finding ways to connect mathematics to their students' knowledge and experiences outside the classroom. Participants reported feeling surprised when they realized how many ways they use geometry outside of the classroom. Many of them enthusiastically endorsed a desire to make mathematics come alive by helping their students discover that mathematics is everywhere. The results suggested that participants believe their students must utilize existing mathematical knowledge to create new concepts. Therefore, they will search for means to introduce these mathematical connections in their practice as teachers. In other words, much like the students in other studies (Blanton, 2002; Roth-McDuffie et al., 1996; Schifter & Fosnot, 1993), participants in this course gained an incidental pedagogy from their experiences in this class.

During classroom work, students often were combined into groups to solve challenging mathematical tasks that Stein, Smith, Henningsen, and Silver (2000) would describe as high demand. They discovered that tasks lacking an obvious solution strategy stimulated diverse thinking about the mathematical concepts presented. The conjectures and verifications occurring during group and class-level discourse kindled the mathematical understanding described by Carpenter and Lehrer (1999). Indeed,

after these conversations, some students remarked that they did not know a person could comprehend mathematics in such a profound sense. Consequently, while most students reported that they were uncertain how to create high demand tasks, many of them left this course willing to utilize them as professionals.

In conclusion, this study exposed students to an experience of mathematics different from what they had experienced in previous classes. Nonetheless, by the end of the semester, most students reported that they gained a deep understanding of the concepts presented, gained confidence with regard to all mathematics, and intended to build a similar classroom environment in their employment as teachers. Moreover, not one student reported a desire to establish a traditional classroom environment as a professional.

Cooperative Learning

Students spent a portion of every non-computer lab class period working in cooperative groups to solve mathematical tasks, to examine homework, and to explore new concepts. Such group work left a positive impression on all interview participants, who vowed to make it an integral component of their classrooms. This uniformly affirming response reveals the value of cooperative learning in the classroom and the impact that it can have when effectively implemented.

As the semester progressed, students began to work more closely with other members of the group and developed the social commitment to one another that is described by Slavin et al. (2003). Some participants reported they became friends with their group-mates while others commented that they learned to value the different perspectives in the group. By the end of the semester, I observed groups staying after

class to work on problem sets or to help each other solidify understanding. I also observed group members arranging study sessions.

The students' affinity for group work is surprising, at first glance, given their negative history with it. During interviews, seven of eight students reported unsuccessful experiences with prior group work. Some students even declared that they "hated" group work and were unhappy to see it included in the course. Nevertheless, those opinions had changed so radically by the end of the semester that all eight interview students vowed to use cooperative learning in their classrooms.

Why did students' attitudes towards cooperative learning shift over the semester? I believe they experienced it implemented correctly for the first time. The cooperative learning literature supports this finding. Antil, Jenkins, Wayne, and Vadasy (1998) report that while 93% of teachers say they incorporate collaborative learning strategies in their classroom, follow-up interviews reveale that only 4% of them employ it in the most effective manner. Cohen (1994) proposes that cooperative learning is most effective when teachers provide open-ended tasks that emphasize critical thinking, when group tasks require participation of all members, and when a variety of tasks related to a central intellectual theme are presented. Further, Webb (1991) found that cooperative learning succeeds when the purpose of the tasks is to gain understanding, not complete an assignment. The results of this project indicate that, in the classroom, the students' participation in effectively implemented cooperative learning influenced their beliefs about group work.

Whereas students reported their experiences with Geometers Sketch Pad (GSP) on homework and tests helped them gain fuller comprehension of the reasons behind

the truth of certain geometric statements, many spent a substantial amount of class time in the computer lab off-task. They would quickly complete the GSP task and begin work on the Internet or their Facebook pages. I believe that this finding is related to the students' experiences with cooperative learning in the classroom. In the computer lab, students worked on their own to complete relatively simple mathematical tasks. They did not sit with their groups, because the environment did not support cooperative learning.

Students did not work in groups in the computer lab for two reasons. First, the computer lab tasks were designed primarily to provide operational practice with GSP, not to develop a mathematical concept as Cohen (1994) suggests. Consequently, students who were more familiar with the computer program quickly completed the task and had no reason to think of alternative strategies. Once they were finished, they moved on to other activities, such as editing their Facebook pages. In addition, solutions to most computer lab tasks were straightforward procedures to complete a function. Students who helped others often simply demonstrated the steps required to complete the activity. No discussion about the activity developed.

A second reason that group work did not occur in the computer lab was because technological limitations inhibited group discourse. In the computer lab, students' efforts focused on a single computer directly in front of them. If they wanted to demonstrate a proposition to other members of the group, everyone had to leave his/her terminal to crowd around another one. Furthermore, follow-up by another group member was almost prohibited, in that he/she would have to recreate the original object

on another computer. This cumbersome process slowed interactions by preventing group-mates from freely interacting with each other.

This data regarding group work in the laboratory points to the big issue regarding cooperative learning and technology in the classroom. Often, when using technology, people work individually or in pairs. For example, when incorporating calculators in the classroom, many teachers encourage students to explore concepts autonomously; consequently, students work alone on their piece of technology. They cannot explore their problems with peers or share their solutions with others in their group. I believe that further exploration of the role of cooperative learning in a technological environment must be conducted.

In general, participants underwent a transformation regarding their cooperative learning experiences in this class. As mentioned before, not one student reported a negative attitude toward group work, despite initial antipathy. On the contrary, many of the participants promised to implement cooperative learning frequently.

Teaching Toolbox

A major finding of this project was that participants emerged from the semester with precise ideas about how they would like to teach when they enter the profession. As noted above, they reported that they plan to build a standards-based learning environment that incorporates cooperative learning. Additionally, by the end of the semester, students stated they had compiled a set of teaching strategies and resources from their experiences in this class that I call a *teaching toolbox*.

When the participants in this course become teachers, they plan to recognize the uniqueness of their students and modify instruction to account for differences. The data

136

revealed that participants will attempt to accommodate the variety of learning styles of the students in their classrooms by using different methods of instruction and explanation so that they can connect with all students. In addition, participants are aware that their students have assorted levels of understanding and intend to explain concepts in a range of ways to fit the needs of every student. For example, their knowledge of the van Hiele model led many participants to report that they would rely on it during geometry instruction. In general, the students in this class became aware of the uniqueness of individual students and stated an intention to adjust their instruction accordingly.

Participants also stated their intention to utilize the technology and manipulatives included in this class. During the semester, students operated dynamic graphing environment technology to solve complicated homework and exam tasks. They also conducted research to find teaching support technology on the Internet, and they generated solutions to various tasks with support from manipulatives. Some students reported they would not have known how to incorporate these resources meaningfully into lessons without experiencing them in this course.

These findings seem to be supported in the literature. For example, Hill et al. (2007) suggest that pre-service mathematics teachers must possess a rich understanding of the concepts they will be teaching as professionals. This understanding will allow them to make sense of the variety of solutions they will see from students and to meaningfully connect those solutions to concepts being taught. Likewise, Roth-McDuffie et al. (1996) report that pre-service teachers in the classes they observed

ended the semester with a desire to teach their future classes in a manner that is similar to what they experienced.

Limitations and Weaknesses

At least three limitations or weaknesses have been identified with regard to this project. All three of these limitations were practical in nature. The first two limitations involved restrictions in the scope of the study introduced by the Institutional Review Boards review process. The third limitation involved a lack of long-term follow-up data.

Limited Interview Data

An intent of this study was to compare student beliefs about mathematics teaching and learning before their participation in this course with their beliefs after it. Unfortunately, I was only permitted to interview students at the end of the semester. I compensated for this limitation by asking students to respond to questions as if they had never taken this or any other reform mathematics course at this university. While students appeared to answer these questions as honestly as possible, I believe they might have been influenced by their experiences during the semester.

Unrepresentative Sample

This study attempted to document the learning experiences of the entire population of students taking the geometry course. Therefore, I attempted to review summary essays written by every student and record the behavior of all students during classroom observations. I also tried to evaluate student responses to the Geometry and Measurement and Assessment and the Modified Fennema-Sherman Attitude Scale

presented at the beginning and end of the semester. Finally, I intended to conduct interviews with a representative sample drawn randomly from the population.

The only true representation of the entire population that I obtained was in the form of the summary essays. During classroom observations, my records about students who sat near my desk are more detailed than those about the rest of the class. Unfortunately, the same two or three groups typically sat in my vicinity during the semester. Likewise, only about thirty percent of the population returned either of the assessments at the end of the semester. As an evaluator, I cannot be certain that students who returned the assessments are not different from the entire population in some way. This caveat also applies to my interview data, as I included all eight students who volunteered to participate; consequently, I am unable to ensure that the interviewees formed a representative sample.

Long-Term Impact

The results indicated that significant changes occurred in students' beliefs about teaching and learning in mathematics. Because of these changes, many students expressed a desire to implement reform-like instruction in their practice as teachers. Unfortunately, I cannot say for certain that the changes documented in this paper will actually translate into future behavior.

<div align="center">*Implications*</div>

The transformations that occurred within many of the students in this project underscore the potential value of reform mathematics courses in teacher training programs. This study documented the individual components of a standards-based geometry course that can have a dramatic impact on pre-service teachers. In addition,

this project identified ways in which a general reform mathematics course can benefit

teachers-in-training. Finally, the results of this study draw attention to the need to

include mathematical content in professional development presentations and in-service

training.

A Model for Geometry Instruction for Pre-Service Teachers

This project identified components of a standards-based geometry course that

can facilitate a deeper understanding of geometry and encouraged reform-minded views

of teaching and learning for pre-service teachers. Two recommendations can be derived

from the data. First, geometry courses that are developed with the objective of

preparing pre-service teachers for the classroom should include components of an

inductive geometry course. Second, such a course should be developed with

recognition of the incidental and deliberate pedagogy that students will gain.

Mathematics teachers must possess a profound understanding of fundamental

concepts in mathematics and geometry in their practice (Ma, 1999). This study

demonstrated that the participants believed they held such an understanding. Some

components of this particular geometry course that might have facilitated the learning

process and should be included in future courses are as follows:

- Students were presented high cognitive demand tasks for class work and
  homework assignments. These tasks provided opportunities for students to
  utilize previous knowledge and personal strengths to solve problems in
  unconventional ways.

- Students worked in cooperative groups to solve these tasks with the objective of acquiring geometric understanding. The discussions that occurred during the group work allowed students to articulate and reflect on the concepts presented.

- Students participated in group and whole class discussions during which they proposed and defended conjectures about the geometric concepts presented during the activity.

- Students utilized numerous resources and learning tools to explore and discover geometry concepts. Technological resources, such as GSP, were important aids for students during problems sets and tests. Students used numerous geometry manipulatives, including, but not limited to, pattern blocks, geo-boards, and mirrors, to explore challenging tasks.

- Student errors were treated by the classmates and the instructor as an opportunity to learn. Focusing on the process of obtaining the solution rather than the answer itself demonstrated to the students the value of the reasoning one makes to obtain a solution.

- Students were required to use sound mathematical arguments grounded in logic and facts to justify their work.

Geometry courses that are designed for pre-service teachers must acknowledge that students are attending the class expecting to gain knowledge about teaching and learning in mathematics. Therefore, students in these classes might be especially attentive to the pedagogy of the instructor. Teachers who include the above-mentioned components will model a successful learning environment for their students. This

project has demonstrated that the student will, in turn, incorporate some of these components into his or her own pedagogy.

Additionally, designers of a standards-based geometry course would be advised to deliberately encourage students to think about teaching and learning in geometry as well as geometric concepts. Components of the course in this project that stimulated participants to think about teaching and learning are as follows:

- Students researched and wrote a paper about the van Hiele Model of Geometric Thought. The reflections students undertook during this assignment prompted many of them to say they would modify instruction to meet the needs of their students.

- Students presented to the class the results of their research on the technological resources available to teachers. Their research and observations of their colleagues' research helped to illustrate the numerous resources available to teachers through the Internet and other outlets. These presentations may have prompted many students to remark that they would implement technology in their lesson plans.

- Students completed a group project about a geometric concept and presented it to their peers. These presentations provided students with practice explaining new mathematical concepts in a safe environment under the guidance of an experienced teacher.

Some students in this project were able to reflect extensively on their learning about instruction. By volunteering as interview participants, eight students gained the opportunity to contemplate how their experiences in this class might influence their

142

pedagogy. During these interviews, I saw some of the students refine their

understanding about teaching and learning in mathematics as they spoke. For instance,

I asked one student, "If I said at the beginning of the semester that people could learn

mathematics the way you learned it in this class, would you have believed me?" The

student provided the following answer:

> I think that I would have been unsure if I trusted it. I would say maybe
> some students could. They still need to be told the answer. I still think
> that they need to be told, but this class has made me realize that there is a
> lot more that they can do on their own than I did before this class.

I responded by asking, "From where does mathematical knowledge come, the

instructor or you?" The student replied;

> I think that [the instructor] pulls it from us. Everyone has different
> amounts, but a lot of time is her fishing and us pulling it out of each other.
> I think it comes from everywhere. Even people who do not understand it
> as well can help a lot. I may not understand the topic, but the questions
> that I ask can help someone who understands it better. Or, the question
> could challenge their understanding. I guess we all pull it [mathematical
> knowledge] from each other.

Within a few seconds and with no prompting, this student moved from reporting that

people still need to be told mathematics to claiming that "we all pull it from each

other." I believe that the reflection necessary for answering the interview questions

helped this particular student gain a better understanding about teaching and learning

mathematics. Future instructors might want to embrace the pedagogical learning

component of this class by encouraging students to contemplate their experiences in a

manner similar to this study's interviews and attitude assessment.

A Case for Reform Mathematics Instruction in All Teacher Training Programs

More generally, this study identified aspects of a standards-based mathematics course for pre-service teachers that will supply them with necessary conceptual knowledge and will influence their beliefs about mathematics instruction. Consistent with results reported by Roth-McDuffie, McGinnis, and Graeber (2000), this project suggests that mathematics courses including a generalized version of the above-mentioned components will facilitate mathematical understanding. In addition, such courses have the potential to shape students' thinking about mathematics learning, motivating them to create a classroom environment in their practice that imitates what they experienced as students.

A Model for Professional Development for Practicing Teachers

Some research evinces similar transformations in experienced teachers. For example, Schifter and Fosnot (1993) document the development of constructivist pedagogical beliefs resulting from practicing teachers' participation in a summer in-service program. The results of this study can guide professional development presenters as they conduct training in standards-based mathematics instruction. The data indicated that some students are more likely to believe in the efficacy of reform instruction having experienced it in this course. This finding should motivate professional development consultants to include meaningful mathematical tasks in their presentations. The findings in this study suggest that teachers may be more likely to believe the teaching methods proposed by the presenter when they have experienced the effectiveness of those methods directly.

144

The results of this project have implications for training programs for pre- and in-service teachers. For pre-service teacher training, these findings provide guidance for designers of the mathematics courses that the students will take. In general, the findings encourage instructors to allow participants to discover the efficacy of reform mathematics instruction in the manner that students learn mathematics in a reform classroom: by discovering it through meaningful experiences.

Recommendations for Future Research

The results of this project prompt several directions for future research, often in response to an identified limitation or weakness of this study. In this section, I will present potential research projects that might address those weaknesses.

Replication with Different Instructor and Content

Because this research evaluated students' experiences in one particular reform-based geometry course, characteristics specific to this course might explain the results. For example, the instructor might have influenced the participants in a way that another instructor could not reproduce. Likewise, the particular content of this course might have affected students' understanding and beliefs.

The participants identified the instructor as an exceptional, model teacher. For example, classroom and office observations and several student interviews identified the instructor's caring attitude toward the students as an important component in their learning. Some students reported they felt safe to take risks with mathematical conjectures during class because of the trust they had for her. Another student remarked that he could focus on learning the material rather than producing work because he believed the teacher was concerned about the process.

145

A substantial body of literature exists with regard to the influence of teacher caring on student performance. For example, teacher support has been found to positively relate to students' mastery of goal orientations (Wentzel, 1997). Other studies have suggested that a perceived caring instructor may be an important motivational element in the classroom context (Brigham, Scruggs, & Mastropieri, 1992). Finally, Turner and her colleagues (2002) reported that teachers of high-mastery classrooms express more positive affect and support, and made fewer negative comments during lessons. Therefore, the transformations that were documented in this project might have been due to caring behaviors rather than the components that I identified in an earlier section.

In addition, some students in this project identified this instructor as a prominent person in mathematics education. During the semester, some students recognized her as an author of some of the resources that they were using in other classes. Moreover, students learned that the instructor was giving an important speech at the National Council of Teachers of Mathematics annual conference during the semester. Consequently, students might have been more inclined to emulate the instruction of such a respected professor.

Clearly, replicating this study with a different instructor and, if possible, with different content would increase further the generalizability of the findings reported in this paper. Previous research (Blanton, 2002) as well as personal experience lead me to predict that a follow-up study would result in similar outcomes.

Replication on a Grander Scale

Practical restrictions and limitations introduced by the Institutional Review Board's review process lessened the scope of this study. For example, scheduling requirements prevented the inclusion of a control group of students who did not participate in a reform mathematics classroom. There were also restrictions on the extent to which I had contact with and the documentation that I received from the students during the semester.

Comparison group. An important finding of this project was that most students believed the reform mathematics course played an important role in their teacher preparation program. They reported they would have been less likely to subscribe to the reform teaching methods presented in subsequent courses if they had not experienced them in this one. One way to examine this finding further would be to compare interviews of students who participated in this class with interviews of students who did not take a reform mathematics class after they had taken their methods course. This comparison would help document whether participation in a mathematics teaching methods course is sufficient to produce the changes in opinions about teaching and learning documented in this study.

More representative data. A weakness of this study was that the data may not have been truly representative of the entire population of students in this class. A solution to this weakness would be to provide student incentives to participate in the study and vary the seating arrangement of the groups during class. The incentives would increase the sample size by motivating more students to participate in the more

time-consuming components of the study. Also, varying the group placement would ensure that the observer collects data from the interactions of every group.

Long-Term Impact

The results revealed that changes occurred in the students who participated in this course. Students began to view themselves as mathematicians, gained an understanding about teaching and learning in mathematics, and developed a passion to teach mathematics in their classroom the way they learned it in this course, through standards-based instruction. Unfortunately, no data exists to determine whether these changes will have a long-term impact in their journey as professionals.

The logical solution to this gap in the data would be to conduct various follow-up interviews and observations with the participants in this study. This additional contact would provide data about their experiences as pre-service teachers in a methods course, as beginning teachers experiencing the classroom for the first time, and as advanced teachers with a history in the classroom.

Conclusion

On the first day of class, 25 students brought with them a variety of histories with mathematics and diverse expectations about the coming semester. Some of the students were frightened about the prospect of taking a college-level course in a topic that had caused them so much difficulty in high school, and they expected to scrape by with a near-failing grade. A much smaller group of students, who had historically succeeded in mathematics courses without much effort, expected this course to be an easy "A." Most of the students expected to participate in class as passive listeners while

the teacher lectured to them about key geometric concepts. Nearly all of the students

expected to have negative experiences completing course work in groups.

During the semester, students experienced a different type of mathematics class.

They found themselves actively participating in the instruction, using logic and

mathematical facts to solve meaningful tasks. Under the guidance of a caring

instructor, students utilized reasoning and conjecture in group work to gain

mathematical understanding, not simply to complete an assignment. Those students

who had previously struggled in mathematics courses contributed to the discussions

occurring at their tables. Meanwhile, advanced students gained insight by looking at

the problems in different ways.

As students grasped the geometric concepts being presented in the assigned

tasks, they also learned about a new way to teach mathematics. They learned that

mathematical knowledge resides within themselves, and, thus, within their future

students. They discovered that by creating a learning environment that nurtures

mathematical understanding, they can help their students succeed as mathematicians.

Most importantly, many of the students in this project became passionate about building

a learning environment like the one they saw in this course.

This passion might provide them the strength to hold onto their new pedagogical

beliefs in the face of the stubborn challenges they might receive from colleagues and

students' parents. Recently, a local school district made the decision to drop a reform

textbook from its lineup of mathematics books (Keung Hui, 2009). One parent, serving

on the textbook selection committee, called the book series an "idiotic, myopic exercise

in futility (p. B1)." This committee member was repeating complaints from numerous

parents about reform instruction textbooks that look so different from what the parents used in school and that contain elementary school level problems that they cannot solve. I believe students who participated in this geometry course can zealously defend these types of reform textbooks because they can draw on their positive experiences with reform instruction.

In a recent conversation, Amy Roth-McDuffie (2009) told me a great challenge for new teachers is to hold onto the pedagogy they obtain in their training when they become professionals. When they arrive as new teachers, they might be mentored by an experienced teacher who has spent many years successfully guiding students toward graduation with traditional instructional strategies. Moreover, new teachers face time constraints to produce multiple lesson plans each day derived from traditional textbooks. The time crunch is exacerbated by the multitude of non-teaching duties. Finally, teachers feel tremendous pressure to conform to the school teaching culture in the face of high-stakes exams.

The participants in this project might draw on their strong conviction for reform mathematics instruction to overcome these challenges. In this course, students experienced a classroom environment that stimulated mathematical understanding. For many of them, this classroom environment was instrumental in helping them gain a rich understanding of mathematics. I believe these experiences will provide the pre-service teachers in this study with the confidence to implement reform mathematics instruction in a high-stakes environment.

If the future teachers in this project withstand parental complaints, school culture, and time pressures, they will help the students in their classrooms discover the

mathematical knowledge held within them. In their classrooms, the teacher will not be

the keeper of knowledge, distributing it at appropriate times in the curriculum. Rather,

they will serve as guides for all students on the journey toward learning mathematical

concepts. These teachers will facilitate learning by providing meaningful high-demand

mathematical tasks that connect new material to prior experiences both inside and

outside the classroom. They will encourage students to construct their mathematical

knowledge socially, through meaningful interactions with peers and, occasionally, their

teacher. Consequently, the students who leave their classrooms will be lifelong

learners, capable of solving the challenging mathematical problems they face in

everyday life.

MATH 411
Selected Topics in Mathematics
Geometry
Tuesay and Thursday 2:00 – 3:15 AM
Room 310 ▓▓▓▓▓▓▓

Professor: ▓▓▓▓▓▓▓▓▓▓▓▓▓

Office: ▓▓▓▓▓▓▓▓▓▓▓

Phone: ▓▓▓▓▓▓ *(office)*   ▓▓▓▓▓▓
(home)
Email: ▓▓▓▓▓▓▓▓▓▓▓▓▓▓

Office Hours: M. 1-3 or by appointment

**Book: O'Daffer, P. G., and Clemens, S. R. (1992). <u>Geometry: An Investigative Approach</u>.
      Reading, MA: Addison-Wesley Publishing Company.**

**Technology: The Geometer's SketchPad, Key Curriculum Press, can be purchased
      online from Key Curriculum Press.**

Course Perspectives
<u>Goals</u>
This course will provide students with a mathematical foundation and cognitive support for the
teaching of elementary and middle school geometry. Specific goals address the structure of
school geometry. Students will:
  a. connect geometric concepts to real world situations;
  b. understand properties and relationships of shape, size, and symmetry in two- and three-
      dimensional space;
  c. understand systems of measurements and use systems to perform measurements in
      realistic settings;
  d. understand concept of transformations in two-and three-dimensional space through the
      investigations of rotations, reflections, and translations and apply these concepts to
      congruence and similarity;
  e. study geometric reasoning, conjecturing, and proof in geometry--both written and oral;
      and
  f. represent and solve geometric concepts, problems, and solutions using technology and
      models.

<u>Instructional Perspective</u>
The course will be taught using a constructivist and socio-cultural perspective. Students will
use inquiry-based activities using manipulatives and technology to construct their personal
understanding of geometry.

Curricular Perspective
The curriculum is developed to allow students to move from concrete to abstract reasoning using the van Hiele Model of Geometric Thought.

van Hiele Levels

Level 0 - Visual, judges shapes by their appearance
Level 1 - Analysis, sees figures in terms of their components and discovers properties of a class of shapes
Level 2 - Informal deduction, logically interrelates previously discovered properties
Level 3 - Deduction, proves theorems deductively
Level 4 - Rigor, established theorems in different postulational systems

Course Content
Four units of study, listed below, provide a loose frame for topics. The Euclidean Geometry topics of axiomatic systems, congruency, similarity, polygons, transformations, and circles will be used as a basis for course content.

Unit 1   Two- and Three-Dimensional Geometry Concepts, Chapters 1 - 3
Unit 2   Spatial Relationships / Properties of Shapes and Angles, Chapters 4 - 6
Unit 3   Transformational Geometry, Chapters 7 - 8
Unit 4   Special Topics, Chapters 9 - 10

Course Requirements
1. **Discussions**. Members of the class bring a rich diversity of backgrounds, interests, and experiences to our discussions. A part of learning is listening to other's ideas, questioning them, and sharing your ideas. Your participation in classroom discussions is expected.

2. **Readings**. You are required to complete readings and be prepared for class discussions related to the readings.

3. **Problem Sets** [Individual and Group]. **Selected exercises from problem sets will be collected and graded.** Answers for problems not graded will be provided for student checking. Problem sets are designed to provoke concrete and abstract thinking about topics and generate class discussions about geometry. You are permitted to discuss the **individual problem sets** with a partner, but you are not expected to submit the same work as your partner because your solutions are unique to your way of thinking and solving problems.

    One group problem set, Set 5, will be completed in groups, presented to the class by groups, and graded as groups. In your presentations, you will be teaching specific topics to your classmates. The rubric below should guide the detail of your solutions and presentations.

    Group Problem Set and Presentation Rubric (12 minutes per group)

Content	Points	Grade
1. Explanation of Problem	10	
2. Strategies used to solve	10	
3. Soution	15	
4. Explanation of new and interesting knowledge gained by group members	15	
Total	**50**	

<u>Curricular Perspective</u>
The curriculum is developed to allow students to move from concrete to abstract reasoning using the van Hiele Model of Geometric Thought.

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Level 1 - Analysis, sees figures in terms of their components and discovers properties of a class of shapes
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2. Strategies used to solve	10	
3. Soution	15	
4. Explanation of new and interesting knowledge gained by group members	15	
Total	**50**	

4.   **Paper.** You are required to complete one paper at least four pages in length on the Van Hiele Levels for Geometry Learning. This paper should include the rationale for the van Hiele Model for teaching Geometry, elements of the model, and your opinions on the use of this model in geometry classrooms. Your paper should have at least three references that are **published articles**, should be typed, double-spaced with 12-point font. Margins should be no more than 1.25 inches and no less than one inch. *(Due date, February 26th)* The paper will be graded on content (cohesiveness of ideas and correctness), grammar, and form. One grade will be given the paper (project) and will understandably be subjective.

Paper Rubric

Content	Points	Grade
1. Rationale	10	
2. Description	10	
3. Application	20	
4. Your evaluation—critique	10	
Total	**50**	

5. **Report**. This brief report (and presentation) on technology use in geometry should present the use any phase of technology in learning geometry such as computer software, webpages, or information from internet websites. Your selection of technology should be easily used by elementary or middle grades students and/or teachers and should make use of tools that go beyond the capabilities of a written text. You should include a description of the technology, how it is used in geometry, and your evaluation of the use of the technology including limitations.   *(Due date, April 3rd)*

Presentation Rubric

Content	Points	Grade
1. Selection of technology	10	
2. Description	10	
3. Use in geometry	10	
4. Your evaluation	20	
Total	**50**	

Evaluation
Your course grade will be a combination of thoughtful class participation, solutions to problem sets, examinations, and papers weighted as follows:

Class Participation   10%
Problem Sets   25%
Papers/Reports   10%
Unit Examinations   40%
Final Examination   15%

Problem Sets and Unit Examinations have allocated points for all work. Correct answers with correct thinking and processes are given full credit. Partial credit will be given on occasion for answers that may not be exact, but use correct thinking and processes.

Attendance Policy
Adopted by School of Education Faculty March 1999

You are enrolled in a professional school, the School of Education, and are beginning [or continuing] the process of your own professional development. Members of the education profession have special responsibilities since so many other people depend on them. Among these responsibilities are meeting all obligations on time and being thoroughly prepared. With this in mind, the following attendance policy has been adopted for all classes in the School of Education.

1. Attendance and punctuality are required. The Undergraduate Bulletin of the University describes regular class attendance as "a student obligation" and reminds us that "no right or privilege exists that permits a student to be absent from a given number of class meetings."

2. On rare occasions, it may be necessary to request that an absence be excused, e.g., for illness, death of an immediate family member, or other emergencies. The appearance of a student's name on the Infirmary List constitutes an excused absence for the days in which the student was in the Infirmary. Also, according to legislation adopted by the Faculty Council, students who are members of regularly organized and authorized University activities are to be excused when out of town taking part in a scheduled event. It is the student's obligation to give prior notification of such absences. Last of all, although the University calendar does not recognize religious holidays, instructors are encouraged to make reasonable accommodations for students requesting to miss class due to the observance of religious holidays.

    Students should make every effort to attend class. Students who do not attend class should call the instructor immediately to explain the absence and discuss ways to make up missed work. An unexplained absence is automatically an unexcused absence.

3. Any unexcused absence or tardiness will result in a lower course grade, provided in both cases that advance notice is given. Instructors also have the right to limit the number of excused absences.

The Faculty Council gives each instructor the authority to prescribe attendance regulations for his or her class, at the beginning of the class. **You will loose one point of your grade for every unexcused absence and one point for every two tardies.**

Tentative Topic, Readings, and Assignment Calendar

Reading, problem set, and paper due dates are listed on this calendar. The information from the readings is assumed content knowledge for our class work. Reading assignments **must be completed** on the date indicated below so that our discussions in class concentrate on the mathematics rather than the readings. If you have questions about the readings, we certainly will discuss them in class.

Tuesday	Thursday
	January 10 Geometry in the Physical World
January 15 Chap. 1: Axiomatic Structure and van Hiele Model Preface pp. v-ix, Chap. 1, pp. 1-21	January 17 <u>PS 1 due in my office</u> Chap. 2: Points, lines, and planes Chap. 2, pp. 24-34
January 22 **LAB** (SketchPad) Chap. 2: Lines, figures, relationships Chap. 2, pp. 38-49.	January 24 <u>PS 2 due</u> **LAB** (Sketch Pad) Chap. 3: Polygons Chap. 3, pp. 56-62
January 29 Chap. 3: Regular polygons Chap. 3, pp. 65-71	Jan 31 Chap. 3: Star polygons Chap. 3, pp. 74-81.
February 5 <u>PS 3 due</u> Chap. 4: Tessellations -regular polygons **Exam 1-3 (Take Home)** Chap. 4, pp. 86-115.	February 7 I will be absent. ***Exam due in my office. I will pick it up on 2/9***
February 12 <u>PS 4 due</u> Chap. 5: Polyhedra Chap. 5, pp. 118-127	February 14 **LAB** (Sketch Pad) Triangles, circles, and polygons Chap. 5, pp. 132-137.
February 19 Chapter 5 Group Presentations	February 21 Chap. 6: Measurement-length Chap. 6, pp. 158-168.
February 26 ***van Hiele paper due*** Chap. 6 Measurement-area Chap. 6, pp. .172-176.	February 28 <u>PS 6 due</u> Measurement-volume Chap. 6, pp. 180-191.
March 4 **Exam Chapters 4-6 (In class)**	March 6 Chap. 7: Translations and rotations Chap. 7, pp. 196-205.

March 11
 	
            S p r i n g
	March 13
	
	             B r e a k

March 18 **Lab** Chap. 7: Translations and rotations Chap. 7, pp. 196-205	March 20 Chap. 7: Reflections Chap. 7, pp. 213-217
March 25 PS 7 due Chap. 7: Combinations of motions Chap. 7, pp. 222-231.	March 27 Chap. 8: Magnification and similarity Chap. 8, pp. 234-244.
April 1 **Lab** PS 8 due Chap. 8: Similarity Chap. 8, pp. 248-250.	April 3 ***Tech Report Presentations*** Review for Exam **Exam 7-8 (Take Home)**
April 8 NCTM	April 10 NCTM
April 15 ***Exam due*** Chap. 9: Topology\Networks & Jordan Curve Chap. 9, pp. 256-279	April 17 PS 9 due Chap. 10: Patterns-points Chap. 10, pp. 282-288
April 22 Chap. 10: Patterns-points	April 24 PS 10 due Chap. 10: Patterns-lines and cubes Chap. 10, pp. 294-298 Review for Final
	Final

Interview Questions for Students

1. Before this semester, what was your history in mathematics?

2. What was your group color?

3. At the beginning of the semester, how did you feel about taking this geometry course?
    - Now that the semester is finally over, how do you now feel about taking this geometry course?

4. Describe a typical math class that you have taken in the past (instruction, tasks).

    a. What was your favorite? Why?
    b. What was you least favorite? Why?

5. At the beginning of the semester, how did you think people learn math?
    - How has that opinion changed during this semester?

6. If I asked you at the beginning of the semester how a typical lesson that you will teach would look, what would you say?
    - Now how would you answer that question?

7. At the beginning of the semester, what was your view of group work? What do you like about working in groups? What do you dislike?
    - Now what do you think?

8. How well did you learn math in this class? Did you learn it in any way that was different than other math classes?

9. Do you think that your experience in this class will impact the way that you teach math in the future? How?

10. Warm-down
    a. Do you like math class?
    b. What is your favorite mathematics subject?

11. Is there anything else that you want to talk about in relation to you experiences in this math class that I have not asked you?

Interview Questions for Instructor

1. Why is the School of Education offering this particular course to pre-service teachers?

2. Did you help design this course?

3. Describe the typical student in this class. What is your general impression of this particular group of students?

4. What are the general objectives of this course? Do you think that you met them this semester?

5. How do you think people learn math?

6. When you prepare a lesson, what are the types of things that you think about with regard to…

   a. content knowledge?
   b. pedagogy?

7. What did you expect to see happen for the students in this class regarding
   c. Content knowledge?
   d. pedagogy?
      • Did you observe those changes?

8. Do you think that changes occurred in student attitudes regarding the following questions?
   e. Describe a typical math class that you have taken in the past (instruction, tasks).
   f. How do you think people learn math? If you taught a math class, how would it look?
   g. Describe a typical math teacher?
   h. What is your view of group work?
   i. Do you like math class?

9. What was the most important knowledge that you students take from their experience in this class?

Is there anything else that you want to talk about in relation to you experiences in this math class that I have not asked you?

Geometry and Measurement Assessment– Version 1

Diagnostic Teacher Assessments in   Date _____
Mathematics and Science
Middle School Mathematics

Please provide the following information about yourself:	Gender: M ☐ F **Testing # _____**

Number of *college* math courses:	0-3 ☐	4-6 ☐	7-9 ☐	Licensure Program	Elem. ☐	M.S. ☐	H.S. ☐	Spec. ☐
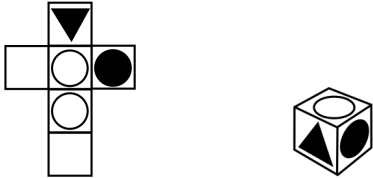
Directions for completing items:
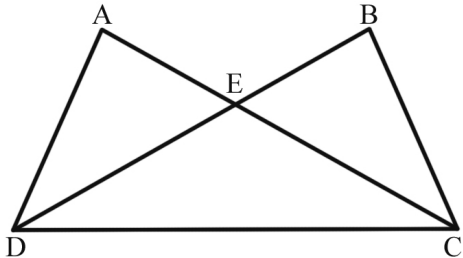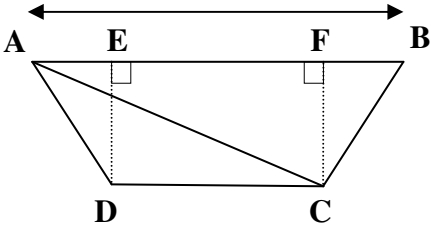
Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page.
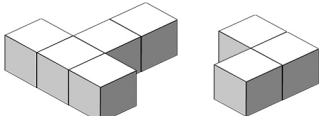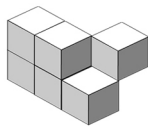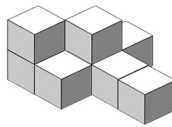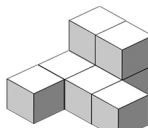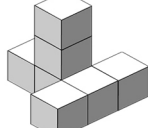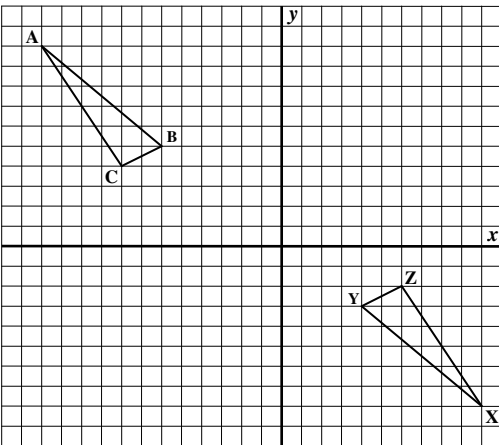
Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items.

Remember, your answers will not be reviewed until after the semester.
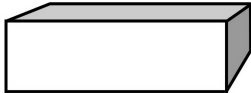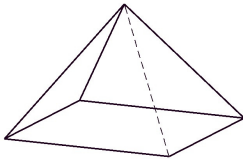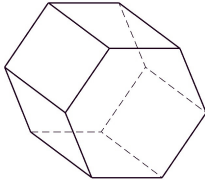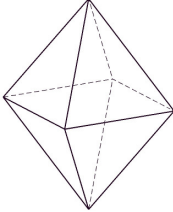
#	Item	Answer
1	A(n) _____ is the union of two rays with a common endpoint. a. line segment c. point b. line d. angle	
2	How many edges does a rectangular prism have? a. 4 b. 6 c. 8 d. 12	
3	Put the following units in order from shortest to longest: meter, centimeter, millimeter, kilometer, decimeter a. kilometer, decimeter, meter, centimeter, millimeter b. meter, decimeter, centimeter, millimeter, kilometer c. millimeter, centimeter, decimeter, meter, kilometer d. millimeter, centimeter, meter, decimeter, kilometer	
4	Which expression below can be used to find area? a. 2l + 2w b. bh c. 4s d. 6s	
5	For which unit of measure below can a thimble serve as an estimation benchmark? a. one millimeter c. one kiloliter b. one centiliter d. one liter	
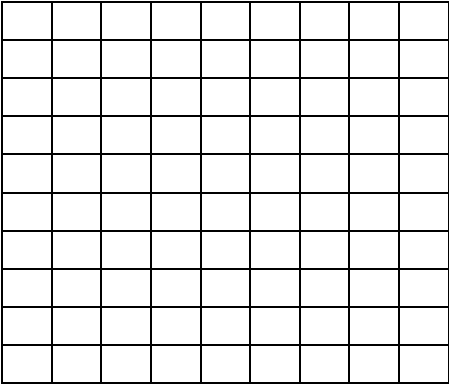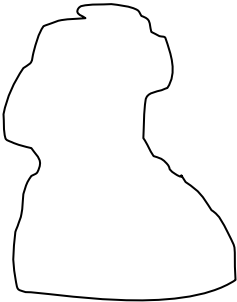
6	Which shape below meets all of these properties: the diagonals are always equal, always bisect each other, but are not necessarily perpendicular? a. rectangle c. kite b. rhombus d. parallelogram	
7	In order to rotate (turn) a geometric shape to another position which of the following information is needed? a. line of rotation c. distance of rotation b. angle of rotation d. direction of rotation	
8	Can the net on the left be folded to make the cube on the right? a. Yes b. No c. Not enough info	
9	The transformations that were performed on Square A to get Square B were: A translation followed by a: a. reflection about a horizontal line b. reflection about a vertical line c. clockwise rotation of $180°$ about the center d. clockwise rotation of $270°$ about the center	
10	Which shape below always has both reflective and rotational symmetry? a. equilateral triangle c. quadrilateral b. scalene triangle d. trapezoid	

11	In the figure below, segment AE is congruent to segment BE and angle A is congruent to angle B. Use deductive reasoning to prove that angles ADC and BCD are congruent. 	
12	In isosceles trapezoid ABCD find the length of diagonal AC by using the Pythagorean Theorem. You are given these lengths: AB = 42; AD = 20; CD = 18. 	

13	a. Given the cube combinations below, select the building at the bottom that can be formed from the cube combinations.	
	b. Justify your selection.	
	Cube Combinations:	
	Buildings:	
	A B	
	C D	
1		
4	Two reflections on triangle ABC map A to X, B to Y, and C to Z. (Please note that each line of reflection is parallel either to the x-axis or the y-axis.	
	a. Identify each line of reflection by writing its equation.	
	b. Justify your solutions.	

15	Companies that make containers are concerned about the "cost efficiency" of their containers. Cost efficient containers have low surface-area-to-volume ratios. That is, the amount of material needed is low compared to the capacity to hold liquid.	
	a. How do the containers below compare with respect to cost efficiency?	
	b. Explain your answer. (Assume the containers have tops.)	
	6 cm 10 cm 10 cm 6 cm 6 cm	
16	A student claims that all squares are congruent to each other because they all have four right angles. a. Why is this claim incorrect? b. Explain how you would help the student understand the error in her thinking.	

17	As an assignment, you give students a picture of a rectangular prism like the one illustrated on the right and ask them to determine as many different types of cross sections as possible. One student draws the three shapes below: a. Identify the student's limited thinking. b. Describe how you would help this student understand that there are other different cross-sections.	
18	Consider the following task and student response: Task: Find the number of edges, faces, and vertices of the three shapes below: A B C Student response: The table below shows the student's response: a. What misconception did the student have about the relationship among edges, faces and vertices? b. Explain how you would help this student correct this misconception.	

Table for Question 18:

Shape	# edges	# faces	# vertices
A	8	5	13
B	18	8	26
C	12	8	20

19	A student was asked to investigate the rotational symmetry of quadrilaterals. After investigating rectangles and squares, the student concluded, "These shapes have rotational symmetry so all quadrilaterals have rotational symmetry." a. What is incorrect about this student's conclusion? b. Explain how you would help the student correct his thinking.	
20	 Figure A A group of seventh graders estimated the area of Figure A by placing a string around the perimeter of the shape and cutting the string so that its length was the same as the perimeter of the shape. They made a rectangle with the new string, placed it on centimeter grid paper, and counted the squares inside. Describe an instructional activity that you would use to address this misconception.	

Geometry and Measurement Assessment– Version 5

Diagnostic Teacher Assessments in
Mathematics and Science   Date _____
Middle School Mathematics

Please provide the following information about yourself:	Gender: M ☐ F	Test # _____

Number of *college* math courses:	0-3 ☐ 4-6 ☐ 7-9 ☐	Licensure Program	Elem. ☐ M.S. ☐ H.S. ☐ Spec. ☐
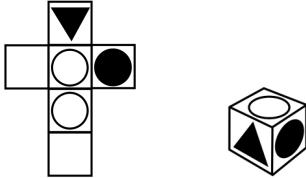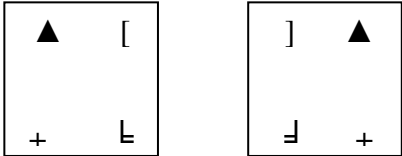
Directions for completing items:
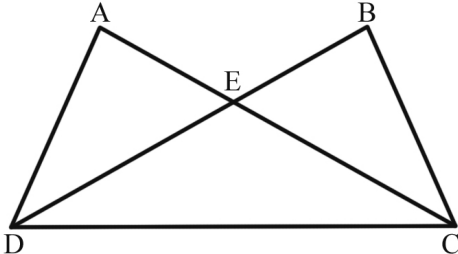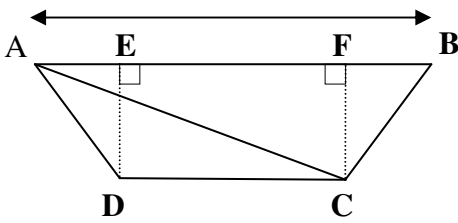
Please record date and starting and finishing times in the spaces in the upper right-hand corner of this page.
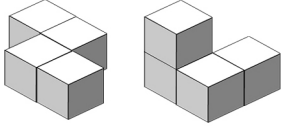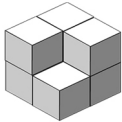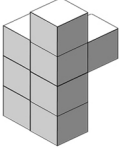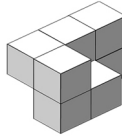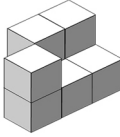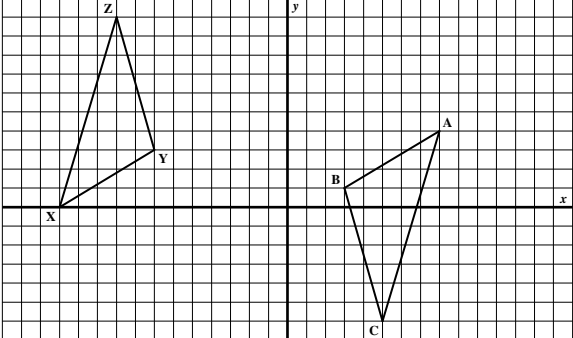
Please answer all questions as completely as possible. Show all work in responding to items and briefly explain your thinking on all items.
Remember, your answers will not be reviewed until after the semester.
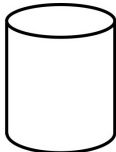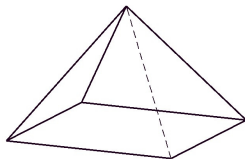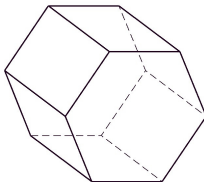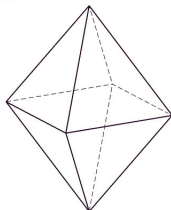
#	Item	Answer
1	An angle is the union of two _____ in a plane that have a common endpoint. a. lines c. rays b. line segments d. points	
2	How many faces does a triangular prism have? a. 3 b. 4 c. 5 d. 9	
3	Put the following units in order from most to least in volume: liter, centiliter, milliliter, kiloliter, deciliter a. kiloliter, deciliter, meter, centiliter, milliliter b. kiloliter, liter, deciliter, centiliter, milliliter, c. milliliter, centiliter, deciliter, liter, kiloliter d. milliliter, centiliter, meter, deciliter, kiloliter	
4	Which expression below can be used to find the surface area of a rectangular prism with length *l,* width *w,* and height *h*? a. *lwh* c. $2lw + 2wh + 2lh$ b. $2l + 2w + 2h$ d. $lw + wh + lh$	

167
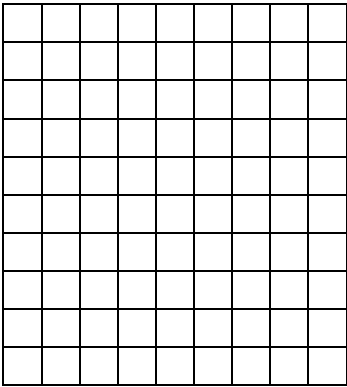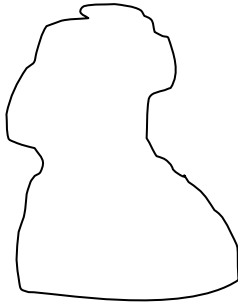
5	For which unit of measure below does the length of the average high school running track serve as an estimation benchmark? a. one centimeter c. one decimeter b. one meter d. one kilometer	
6	Which shape below meets these properties: the diagonals are not always equal, are always perpendicular, and always bisect each other? a. parallelogram c. kite b. rhombus d. rectangle	
7	In order to translate (slide) a geometric shape to another position which of the following information is needed? a. distance of translation only b. direction of translation only c. angle of translation only d. both direction and distance of translation	
8	Can the net on the left be folded to make the cube on the right? a. Yes b. No c. Not enough info	
9	The transformations that were performed on Square A to get Square B were: A translation followed by a: a. reflection about a horizontal line b. reflection about a vertical line c. clockwise rotation of $180°$ about the center d. clockwise rotation of $270°$ about the center	
10	Which shape below always has exactly two lines of symmetry? a. triangle c. rectangle b. square d. parallelogram	

11	In the figure below, segment AE is congruent to segment BE and angle A is congruent to angle B. Use deductive reasoning to prove that triangle DEC is isosceles.	
12	In trapezoid ABCD find the length of diagonal AC by using the Pythagorean Theorem. You are given these lengths: DC = AD = BC =13; FB = 5.	

13	a. Given the cube combinations below, select the building at the bottom that can be formed from the cube combinations. b. Justify your selection. Cube Combinations: Buildings: A B C D	
14	 Two reflections on triangle ABC map A to X, B to Y, and C to Z. (Please note that each line of reflection is parallel either to the x-axis or the y-axis. a. Identify each line of reflection by writing its equation. b. Justify your solutions.	

15	a. How does doubling the dimensions (length, width, and height) of a cube affect its volume? b. Justify your answer.	
16	A student claims that all equilateral triangles are congruent to each other because they all have three 60° angles. a. Why is this claim incorrect? b. Explain how you would help the student understand the error in his thinking.	

17	As an assignment, you give students a picture of a cylinder like the one illustrated below and ask them to determine as many different types of cross sections as possible. One student draws the two shapes below: a. Identify the student's limited thinking. b. Describe how you would help this student understand that there are other different cross-sections.	
18	Consider the following task and student response: <u>Task:</u> Find the number of edges, faces, and vertices of the three shapes below: A B C <u>Student response:</u> The table below shows the student's response: | Shape	# edges	# faces	# vertices
---	---	---	---
A	4	5	5
B	9	8	12
C	6	8	6 | c. What misconception did the student have about the relationship among edges, faces and vertices? d. Explain how you would help this student correct this misconception.	

19	A student was asked to investigate the lines of symmetry of quadrilaterals. After investigating two rectangles, the student concluded, "These rectangles have lines of symmetry, so all quadrilaterals have lines of symmetry." a. What is incorrect about this student's conclusion? b. Explain how you would help the student correct her thinking.	
20	 Figure A A group of seventh graders estimated the area of Figure A by placing a string around the perimeter of the shape and cutting the string so that its length was the same as the perimeter of the shape. They used the new string to make a rectangle with a width of 4 cm, placed it on centimeter grid paper, and counted the squares inside. Describe an instructional activity that you would use to address this misconception.	

1. State the problem

   Describe the problem clearly enough that someone reading your paper will understand exactly what you were asked to do.

2. Discuss your approach

   Describe how you went about solving the problem using these questions:

   - How did you get started?
   - What strategies did you use to solve the problem?
   - What strategies did you try that did not help you solve the problems?
   - What did you do when you got stuck?
   - Did you talk to anyone about the problem? Did this help?
   - Did you notice any patterns?
   - Did anything else help you?

   Include any lists, charts, or pictures you used in your description

3. State solution and use reflection

   State your answer and solution to the problem.

   Explain what makes you think that your answer is reasonable and what you learned about mathematics.

Figurate Numbers

The triangular numbers are 1, 3, 6, 10, 15, ...; the square numbers are 1, 4, 9, 16, 25, ...; the pentagonal numbers are 1, 5, 12, 22, 35, ... The geometrical language is justified by the following diagrams:

Triangular numbers:

Square numbers:
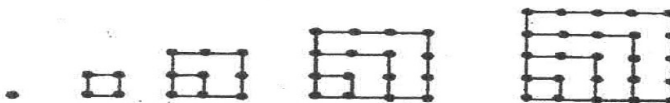
Pentagonal numbers:

1. What are the first five hexagonal numbers?

2. What are the first five septagonal numbers?
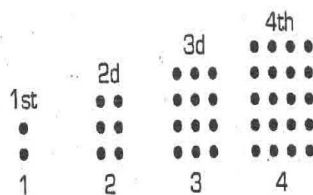
3. What are the first five r-gonal numbers?

FIGURATE (POLYGONAL) NUMBERS

Examine the patterns and complete each table below.

1. Oblong numbers:
   G = number of dots

	1st	2d	3d	4th	5	6 ··· 10 ··· n
	1	2	3	4	5	6 ··· 10 ··· n
G	2	6	12	20	?	? ? ?

How can you predict the next oblong number? What patterns can you find? Describe.

2. Triangular numbers:
   T = number of dots
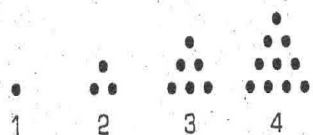
	1	2	3	4	5	6 ··· 10 ··· n
T	1	3	6	10	?	? ? ?

How can you predict the next triangular number? What patterns can you find? Describe.

3. Square numbers:
   S = number of dots
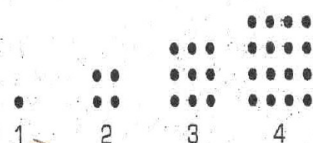
	1	2	3	4	5	6 ··· 10 ··· n
S	1	4	9	16	?	? ? ?

How can you predict the next square number? What patterns can you find? Describe.

4. Can you find a relationship between the oblong and the triangular numbers? Describe it. How would you explain the relationship by using only the dot representations?

5. What relationships can you find between two consecutive triangular numbers and a square number? Describe. How would you explain these relationships by only using the dot representations?

Golden Ratio Activity
January 15, 2008

1. Draw several rectangles that are pleasing to you on a sheet of paper. **Measure the lengths and widths in millimeters and find the ratio of length to width for each**.

2. Determine *your* ratio by measuring the length from your navel to chin, length of head, navel to ground, and navel to top-of-head. What ratios did you find for your *navel height : total height* and *navel to top of head : navel height*?

3. Use the chart below to find our class ratios. Consider the reciprocals of your ratios.

RATIOS

Class member	Navel height	Navel to top of head	Total height	Navel height / Total height	Navel of top of head / Navel height

Golden Ratio: The Greeks used the Golden Ratio in sculptures of men and women and in their design plans for buildings. Mathematics is filled with golden relationships! Consider golden relationships in the Fibonacci sequence.

A. The Fibonacci Sequence begins 1,1,2,3,5,8, ... Expand the sequence to at least 12 terms. Find the ratio of consecutive pairs of numbers in the sequence. How are they related to the Golden Ratio?

B. What conjecture can you make about the limit of these pairs of Fibonacci Numbers and the Golden Ratio?

C. Do the **Problem Solving: Developing Skills and Strategies** on the bottom of p. 7. (Write up using the procedure on the Steps for Writing about Mathematics handout.)

Problem Solving: Skills and Strategies *

- -

"The line segment in Figure 1.18 is divided so that $x - y = 1$ and $xy = 1$. This division of a segment was of special interest to early Greek geometers. Use a calculator to find two decimals, to the nearest thousandths, for x and y. What did you discover about this pair of numbers?

To solve this problem with the guess-and-revise strategy, simply guess a value for x, see how close it is, revise your guess, and try again. Continue this process until the correct pair of numbers has been found. Try it.

Can you think of another way to solve the problems? Explain." (O'Daffer & Clemens, 1992, p. 7).
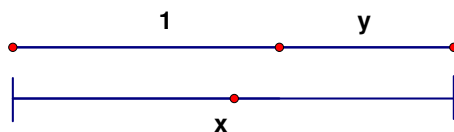
- -

Figure 1.18

* O'Daffer, P.G., & Clemens, S.R. (1992). *Geometry: An Investigative Approach.* Reading, MA; Addison-Wesley Publishing Company.

Investigation – Tessellations
2/05/08

Square regions can be arranged into a repeating pattern that completely covers the plane, like the ceiling. There are no "holes" and no "overlapping" areas. We say that the squares "tile" or "tessellate" the plane.

   A. Consider the following polygons, use tracing paper to determine which ones tessellate.
   B. Can you state some general conclusions about which polygons will tessellate the plane?

Investigation – Regular Tessellations
2/05/08

A Tessellation is a Regular tessellation if it is constructed from regular convex polygons of one size and shape such that each vertex figure is a regular polygon.

1. Use the pattern blocks or pattern cut-outs to determine which regular polygons will tessellate the plane. Draw them in the following table.

2. Complete the following table regarding which polygons form regular tessellations.

Polygon	Measure of vertex angle	Number at each vertex

3. Use your answers to problems one and two to other regular tessellations exist other than the ones that you found. Why?

APPENDIX J: COMPUTER LAB, GSP LESSON
Classwork, January 22, 2008

Task: Use **Geometer's Sketchpad** to draw a triangle and find the sum of the measure of the interior angles of the triangle.

Change defaults on display

Tool	Menu	Selection	Action
None	Display	Preferences	select point, change degree precision to tenths

Draw a Triangle

Tool	Menu	Selection	Action
Point. Click on three different points A, B, C Select all points by holding down the shift key and clicking on points.	Construct	Segment	Draws triangle

Measure an Angle--Repeat for three angles

Tool	Menu	Selection	Action
Arrow. Select three points in the order of the angle. (Hold down shift to select)	Measure	Angle	Measure appears on screen

Find the sum of angles

Tool	Menu	Selection	Action
Arrow. Select three angles and measures.	Measure	Calculate	Displays calculator

Use value bar and "+" to add the measures of the angles. Select okay when all three measures are in the display. Your display should look something like this:

m∠ABC= 21.1 °
m∠ACB= 93.3 °
m∠BAC= 65.6 °
m∠ABC+ m∠ACB m∠BAC = 180.0 °
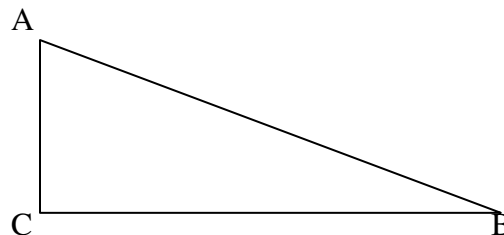
Select the Arrow tool and click on a vertex. Slide the vertex to a different place. What happens to the angle measures? Explore other polygons.

Task: Use **Geometer's Sketchpad** to draw a triangle and find the sum of the measure of the exterior angles of the triangle.

Use the same directions to draw the triangle as in the interior angles.

Change the segments in the triangle to rays.

Tool	Menu	Selection	Action
Point. Select two points in the direction of your ray.			
Ray.	Construct	Ray	Draws a ray.

Place another point on the ray by changing the tool to **point** and clicking on the extended part of the ray (outside of the triangle).

Use the same method as before to find the measure of the angles and to calculate the sum.

Change the figure and see what happens. Select the Arrow tool and click on a vertex. Slide the vertex to a different place. What happens to the angle

GSP

Test 3

Two each (16 points) **[Do not use GSP]**
1. Complete each of the following problems. In exercises *a-f* identify the coordinates of the image resulting from the
    a. reflection of △ ABC in the y-axis. _____

    b. reflection of △ CDE over OP _____

    c. translation of AB using slide RR' _____

    d. 90° counter clockwise rotation of △ ABC with center P _____

    e. dilation of △ABC using center O and scale factor ½ _____

    f. dilation of △CDE using center O and scale factor 3 _____

    g. Give the ratio of the pre-image to the image in *e* and *f*? _____ ,
        _____
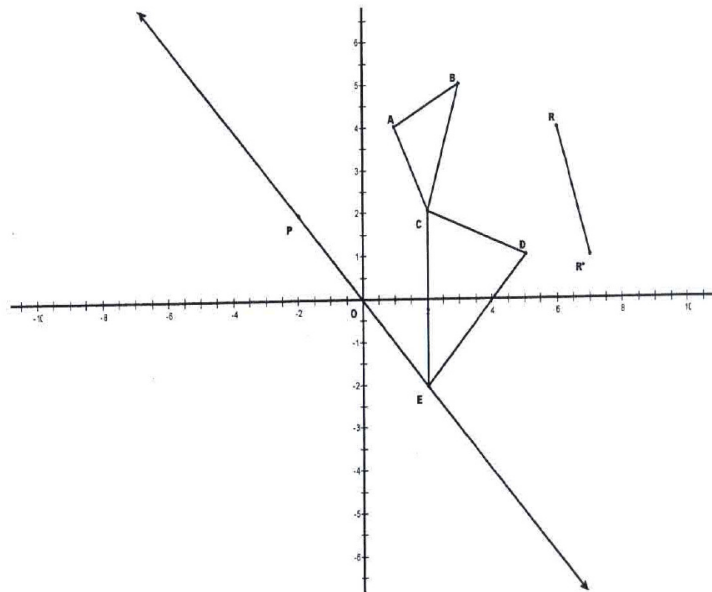
Activities on Exam

10 points

2. Find the center and the scale factor or preimage to image of dilation for triangles ABC and A'B'C'
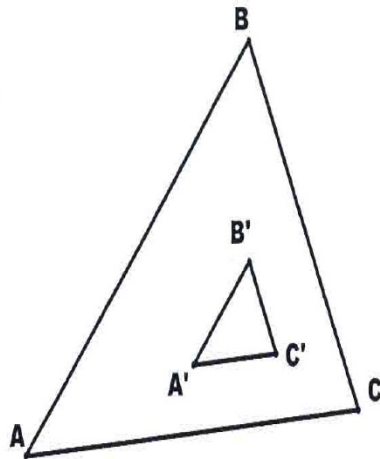
14 Points

3. Use the definitions and preservations of translation, reflection, rotation, and dilation to **compare and contrast** congruence and similarity.

184

20 Points
4. Does a reflection in a line followed by a clockwise rotation result in the same image as a clockwise rotation with the same center followed by a reflection in the same line? Explore this question first using the figure below. Test for special cases on GSP.
Given: Pre-image R, and line of reflection. You place the center of rotation.

a. Explain your conclusions. (10)

R

b. Does the measure of the angle of rotation make a difference? Explain. (5)

c. Does the placement of the center of rotation make a difference? Explain. (5)

185

10 points

5.

Use the enlarged internal dilation of below and your knowledge of dilations to explain that the sum of the geometric series

$$\frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \ldots = \frac{1}{3} \text{ is true.}$$
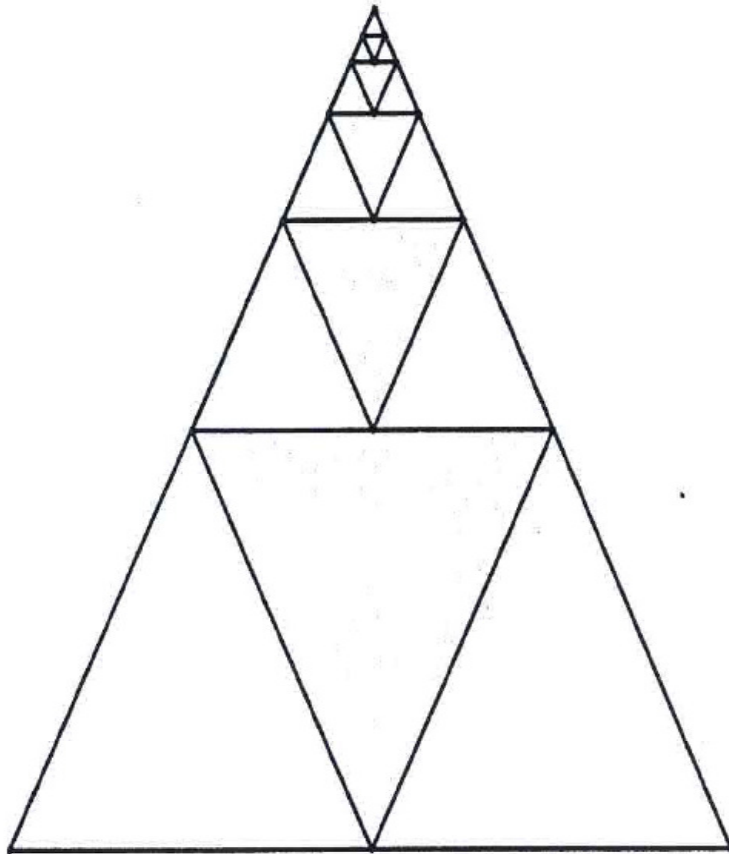
30 points

6. In this string of quadrilaterals determine the motions that transformed the shapes. **Construct** all lines, centers, angles, and support your conclusions with short explanations. Please use hand tools and show your work.
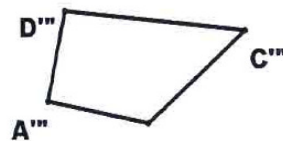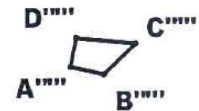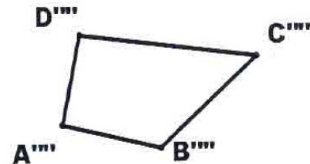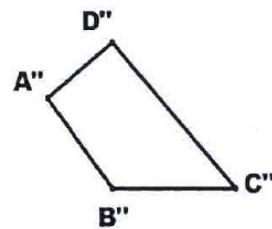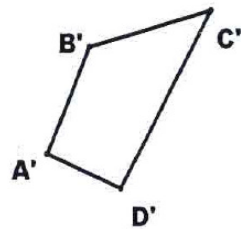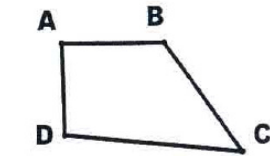
APPENDIX L: THALES THEOREM ACTIVITY

Discovery with Thales' Theorem

Use the drawing below of circle O, for this activity.

Select any point on the circle (not A or B)
and label that point C. Draw segments AC
and BC. Use your protractor to find the
measure of ∠s A, B, and C. Within your
group, complete the chart below listing the
data you have found. Compile your data
with data from the other groups.

Angle Measures

∠A	∠B	∠C

O
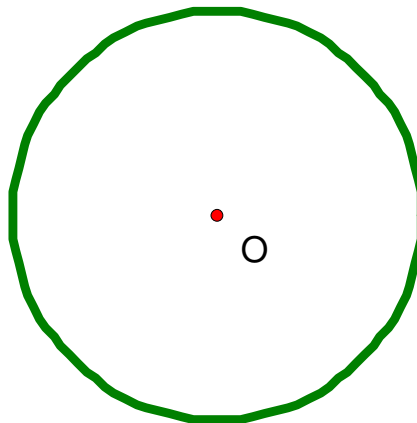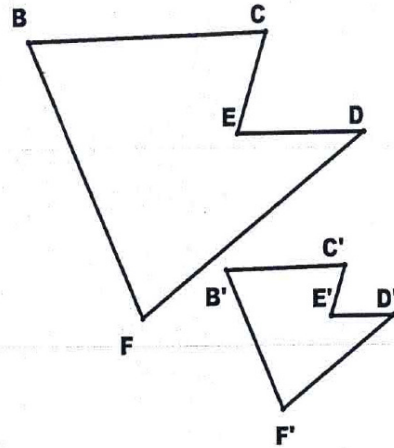
1.   What conclusions could you draw from the data you have listed?

2.   State your conjecture about the triangle that you formed.

Dilations Lab

What is the scale factor of the image in relationship to the pre-image?

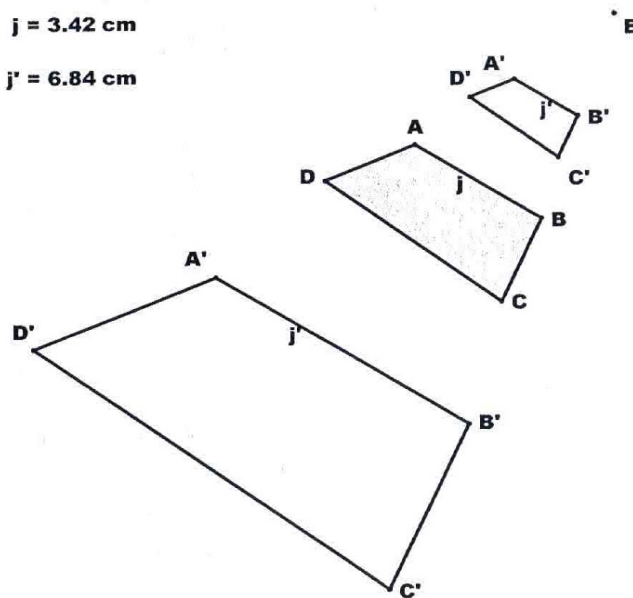A. Describe how these dilations where constructed. Include scale factors used on GSP.

j' = 1.71 cm

j = 3.42 cm

j' = 6.84 cm

B. Draw any polygon on GSP. Dilate this polygon with a scale factor of 1/3, ½, -3, 2, and 5.

1. Explain the dirferences between and among these images resulting from these dilations.

2. Determine the relationship between the perimeter and the area of a preimage and image in a dilation.

3. Generalize the relationship you established in #2 and develop a proof of this generalization.

4. Make a conjecture about the relationship between perimeter and volume.

C. Describe what was done to obtain the image B'C'D''E'F'? Find the intermediate image.
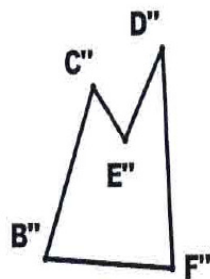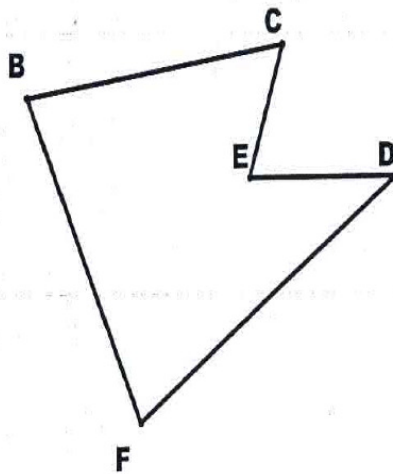
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