Abstract

Evaluating Levels of Abstraction in Modeling:
The Durham Mixed Integer Program

Gregory John Tallmadge
Master of Science in Environmental Engineering
University of North Carolina at Chapel Hill
Professor Donald T. Lauria, Advisor

The City of Durham’s Water Resources Department operates a complicated water supply system consisting of multiple reservoirs, treatment plants, and pumping modes. The annual electrical operating cost for this system is approximately $600,000. Almost all of this cost is attributable to electric pumping. The Water Resources Department would like to minimize these costs, however, the complexity of the system and the electric tariff make it difficult to identify least-cost pumping patterns. The Durham Mixed Integer Program (MIP) was developed to provide the Water Resources Department with guidance in identifying these pumping patterns.

A fundamental part of developing the MIP program was determining the appropriate levels of abstraction to use in representing the system elements. The principle of parsimony suggests that a simple model containing only the most essential elements of a system is preferable to a more complicated model which would include more detail, but fail to provide additional useful information. The key to applying this principle is
determining which elements of the system are the most essential. The determination of these elements depends on the intended use of the model.

This report describes the process whereby the appropriate levels of abstraction were determined for key elements of the Durham water supply system. These levels of abstraction were then evaluated to determine whether additional detail would improve the model's ability to identify least-cost operating strategies. The results of the analyses conducted indicated that the chosen levels of abstraction were, in general, sufficient for the intended use of the model. In addition, the analyses indicated the relative importance of finished water operations in determining total operating costs.
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Chapter 1. Introduction

1.1 Motivation

The City of Durham’s water supply system is managed by the city’s Water Resources Department (WRD). It currently meets an average demand of 25 million gallons per day (MGD). The raw water storage of the system consists of Lake Michie and Little River reservoirs. The WRD has comparatively little experience operating the Little River reservoir because it is a relatively new addition to the system. The reservoirs are connected by transmission mains to terminal reservoirs for the two water treatment plants (WTPs), the Brown plant and the Williams plant. The system schematic is shown in Figure 1.1.

The City of Durham pays approximately $600,000 per year for the electrical power required to operate the water supply system. This cost is due almost entirely to pumping. Raw water pumping costs can be reduced by the use of hydraulically driven pumps (hydropumps) located at both reservoirs. These pumps operate by using the potential energy of the water contained in the reservoirs to drive the pump turbines, allowing water to be pumped without incurring electrical costs. However, this mode of pumping wastes
Figure 1.1. Durham water supply system
the water used to drive the turbines, resulting in the rapid drawdown of the reservoir
during the dry season (June - November). The drawdown cycle begins in March at which
time, historically, both reservoirs have always been full. The WRD has been cautious
about using the hydropumps late in the drawdown cycle because of the risk of drawing
down the reservoirs too much and thus, being unable to meet demand.

Electric pumping costs can also be reduced by delivering water from Little River
reservoir to the Brown terminal reservoir by gravity flow when sufficient head
differential exists. However, the raw water pumping modes and pathways shown in
Figure 1.1 are mutually exclusive at each reservoir. Thus, water from Little River
reservoir cannot simultaneously be transferred by hydropumps to Williams and by gravity
flow to Brown. The relative cost-efficiency of tradeoffs between these options depends
on the difference in costs associated with finished water pumping at each treatment plant.

The Williams treatment plant is located closer to the distribution network than the Brown
plant, resulting in lower energy requirements for pumping water to the network.
However, the Williams plant has a lower treatment capacity and older equipment than the
Brown plant, resulting in higher chemical treatment costs. The complicated electric tariff
makes it difficult to determine the relative costs of using the Brown and Williams plants.

The Water Resources Department believes that it may be possible to operate the water
supply system more cost effectively; however, it is uncertain about the levels of risk
associated with using the hydropumps late in the drawdown cycle. The tradeoffs between
raw water pumping options also complicate the decision of how to transfer raw water cost effectively. These factors, combined with the complicated electric tariff, make it difficult for the WRD to evaluate alternative pumping patterns to identify least-cost strategies.

To aid the Water Resources Department, a mathematical optimization model was formulated to determine least-cost system operation. An important consideration in the development of the model was determining the appropriate level of abstraction. Model development seeks to include only the minimal level of abstraction which will provide useful solutions. The principle of parsimony suggests that a simple model containing only the most essential elements of a system is preferable to a more complicated model which would include more detail, but fail to provide additional useful information. The key to applying this principle is determining which elements of the modeled system are most essential. The determination of these elements depends primarily on the intended use of the model. This report focuses on the correspondence between the intended use of a model and its appropriate level of abstraction.

1.2 Tasks

This report includes four principle tasks, as follows:

- The first is to describe Durham’s water supply system.
• The second is to describe the operational decisions the Water Resources Department faces and the guidance they desire for making these decisions. The WRD's primary goal of cost minimization is discussed.

• The third is to explain how the WRD's goal can be met using a mathematical optimization model. The purpose and intended use of the model are discussed. The levels of abstraction and formulation for key model elements are presented and explained. A discussion of the preliminary results of the model is also included.

• The final task is to evaluate the level of the abstraction in the original model formulation (i.e. the Basic Model). This is accomplished by comparing the results from the Basic Model to those from Expanded Models which contain additional levels of detail.

1.3 Organization

The Durham water supply system is described in Chapter 2. Section 2.1 describes the reservoirs and the modes of pumping for transferring water to the treatment plants. Section 2.2 describes the treatment plants and the pumping hardware for supplying finished water to the distribution network. Section 2.3 describes the electric tariff under which the system operates.
Chapter 3 describes the operational problems the Water Resources Department faces and the proposed modeling approach for guiding their decisions. Section 3.1 describes the operational decisions the WRD faces and its goal of cost minimization. Section 3.2 explains how the proposed model will be used to indicate optimal (least-cost) decisions. Section 3.3 describes the levels of abstraction used in formulating the Basic Model. Details about model verification are provided in Section 3.4. Section 3.5 compares model recommendations with the WRD's actual operation for 1994. Section 3.6 contains general observations about the relative potential for savings in raw and finished water operations and the implications for model abstraction.

Chapter 4 contains evaluations of the levels of abstraction for selected model elements. Section 4.1 describes which model elements will be analyzed and how the levels of abstraction will be evaluated. These evaluations are described and presented in Sections 4.2, 4.3, 4.4, and 4.5.

Chapter 5 contains a summary of the analyses. Section 5.1 discusses the conclusions which can be drawn from the analyses.
Chapter 2. Durham Water Supply System

2.1 Raw Water Storage and Transmission

Raw water storage consists of Lake Michie and Little River reservoirs. The Lake Michie reservoir has been in operation since 1925. Its inflow is from the Flat River which has a drainage area of 160 square miles (mi$^2$) at its entrance into the reservoir. The reservoir has a total volume of 3.8 billion gallons (BG), with a water elevation of 341 feet when full, and dead storage of 1.2 BG at 319 feet. The maximum working volume of the reservoir is 2.6 BG. The Little River reservoir was completed in 1988. It is fed by the Little River which has a drainage area of 90 mi$^2$ at its entrance into the reservoir. It has a total volume of 4.9 BG at elevation 355 feet, with dead storage of 1.7 BG at elevation 330 feet. The maximum working volume of the reservoir is 3.2 BG.

Both reservoirs have several electrical pumps to transfer water to the terminal reservoirs at the treatment plants. In addition to these pumps, water can be transferred from Little River to the Brown terminal reservoir by gravity due to a water elevation difference of 7 feet when both reservoirs are full. As the Little River reservoir is drawn down, the flow decreases from its maximum rate of 45 MGD. Gravity can be used to transfer water from
Little River reservoir to the Brown terminal reservoir as long as the head differential is at least 1 foot.

Both Lake Michie and Little River are equipped with hydraulically driven pumps which use water in the reservoirs to drive the pump turbines. The water used for this purpose is wasted downstream. As the reservoir levels are drawn down, the amount of wasted water per unit of pumped water increases, while the flows from the hydropumps decrease. The hydropump at Little River reservoir can only transfer water to the Williams plant, but the hydropumps at Lake Michie can transfer water to either the Williams or the Brown plant.

Historically, the drawdown cycle for the reservoir system begins at the end of March at which time both reservoirs have always been full. The dry period of the cycle generally occurs from June to November. The reservoirs typically reach their lowest point in November or December.

2.2 Finished Water Storage and Transmission

The Brown water treatment plant is supplied by a terminal reservoir (90 MG capacity) which receives water from Lake Michie and Little River. The terminal reservoir is equipped with electric pumps which transfer water to the treatment plant. An additional electric pump located at the Brown terminal reservoir can transfer water to the Williams terminal reservoir. The treatment plant has a capacity of 30 MGD. Downstream of the treatment plant are two clearwells with a combined storage capacity of 10 MG. The layout is shown in Figure 2.1. The clearwell pumping station has 3 variable speed pumps
Figure 2.1 Brown water treatment plant layout.
and a backup engine-driven pump. The variable speed pumps operate at rates between 12 and 23 MGD; the engine-driven pump's capacity is 18.5 MGD. Generally, only one pump is used to supply the distribution network at a time. Due to the head characteristics of the network, a minimum pumping rate of at least 12 MGD must be maintained at all times.

The Williams treatment plant has a capacity of 22 MGD and is fed by gravity from a 45 MG terminal reservoir. The majority of the chemical handling equipment at the Williams plant is older and more costly to operate than that of the Brown plant. The additional chemical costs of treating water at the Williams plant have been estimated as $10 per MG.

The treatment plant has two clearwells with 2 MG of combined finished water storage. Water can be pumped to the distribution network by 6 pumps with capacities between 8 and 21 MGD. Only one pump can be used to supply the network at a time. A minimum rate of 8 MGD must be pumped from the clearwells to the network at all times. The layout is shown in Figure 2.2.

2.3 Electrical Tariff

The City of Durham pays about $600,000 each year in electrical operating costs for its water system. These costs are almost entirely attributable to pumping. The electric tariff is a complex arrangement consisting of basic facilities charges, demand charges, energy
Figure 2.2 Williams water treatment plant layout.
charges, and contract demand charges. All major electrical connections in the Durham water supply system incur these charges.

The basic facilities charge is a fixed “cost of service” charge assessed each month regardless of the amount of energy used. The demand charge is based on the maximum kilowatt (kW) draw incurred during the billing period and is expressed as dollars per kW. The energy charge is based on the kilowatt-hours (kWh) used during the billing period and is expressed as dollars per kWh. The contract demand charge represents the cost of providing the hardware necessary to meet demand at any time. It is fixed based on maximum demand estimates and is expressed as dollars per kW.

Because the demand for electrical power is influenced by the time of year, the tariff has different rates for summer and winter. The summer pricing schedule is applicable between June and September. The tariff also distinguishes between on-peak and off-peak use of electricity. The on-peak hours are between 1:00 pm and 9:00 pm during the summer months and between 6:00 am and 1:00 pm during the winter months. The energy charge and the demand charge are subject to daily time of use. The on-peak component of the demand charge is termed the on-peak demand charge, and the off-peak component is termed the economy demand charge. The demand charge is also subject to seasonal time of use. Table 2.1 shows the seasonal and on/off-peak pricing for these charges.
Table 2.1 Electric tariff charges (Duke Power\(^1\)).

I. Basic Facilities Charge  $36.07

II. Demand Charge
A. On-Peak Demand Charge  Summer Months  $12.98 per kW  Winter Months  $7.64 per kW
B. Economy Demand Charge  $1.03 per kW

III. Energy Charge
A. On-Peak Energy  4.2 cents per kWh  4.2 cents per kWh
B. Off-Peak Energy  2.1 cents per kWh  2.1 cents per kWh

The demand charges associated with using large finished water pumps during on-peak hours can be considerable. For example, the demand charge for using a variable speed pump at the Brown Plant to pump 18 MGD during on-peak hours is $14,000 (for each month of use) during the summer months.

\(^1\) The Duke Power electric tariff is given in Appendix A.
Chapter 3. The Basic Model

3.1 Operational decisions

The Water Resources Department wants to minimize pumping costs. The complexity of the system and the tariff make decision making difficult with respect to this goal. Following are examples of the operational decisions faced by the WRD which affect pumping costs:

What percentage of the water supply should be obtained from Lake Michie?

What percentage of the water supply should be obtained from Little River?

How far down can the reservoirs be drawn down in September before the risk of failing to meet demands exceeds 5%?

How should water be transferred to the Williams treatment plant?
   Electric pumping from Lake Michie?
   Hydropumping from Lake Michie?
   Hydropumping from Little River?

How should water be transferred to the Brown treatment plant?
   Electric pumping from Little River?
   Hydropumping from Lake Michie?
   Gravity flow from Little River?

How do changes in inflows and reservoir levels affect these decisions?
What percentage of finished water flows should come from the Williams treatment plant during summer on-peak hours?

What percentage of finished water flows should come from the Brown treatment plant during winter off-peak hours?

These are the kinds of questions the WRD must address to achieve their goal of cost minimization. While they are not seeking instruction for which pumps to select on a day by day basis, the WRD does desire general guidelines for operating the system at minimal cost under a variety of inflow conditions.

In general, the decisions faced by the WRD are: what volumes of water should be transferred by each pump and pathway to meet demand at minimal cost? Guidance for these decisions requires information such as: the optimal amount of water to be transferred from Lake Michie to Williams by hydropumps during a specific time period. This type of information is needed for all pumping options. The time steps chosen should allow for seasonal and on/off-peak variations. Such guidance must also account for interactions between raw water operations and finished water operations and the effects of changing inflows.

3.2 The Basic Model

The objective of cost minimization can, in principle, be met using a mathematical optimization model, such as a linear program (LP). However, LPs cannot model the demand charges in the tariff because they do not behave as continuous functions, rather, they are incurred in discrete quantities based on the kW of the largest pump used during
the billing period. Mixed Integer Programming\(^2\) models (MIPs) are identical to linear programming models except that one or more of the variables are constrained to values of 0 or 1. Using these variables, MIP models can be formulated to represent discrete functions. However, the algorithms used to solve MIP models cause solution times to increase significantly as 0,1 variables are added. Thus, it is desirable to use as few as possible.

An MIP model was formulated to provide the WRD with information about optimal (least-cost) pumping options which would satisfy demand. The original formulation of this model is referred to in this report as the “Basic Model”. The model consists of linear equations describing the physical constraints and requirements of the system\(^3\).

Information about the costs associated with electrical pumping is laid on top of this framework.

3.3 Purpose and Intended Use

The primary purpose of the Basic Model is to determine the optimal operation of the system (i.e. the volumes of water which should be transferred by each pump and pathway to satisfy demand at the lowest cost). These results will be expressed in time steps which capture seasonal variations. However, we expect that the optimal operation of the system will change with changing inflows. Therefore, the intended use of the Basic Model is to analyze the changes in optimal flows over a range of inflow values. This analysis will

\(^2\) For a general discussion of MIP models, see Appendix B.

\(^3\) As indicated in Appendices C and D
seek to identify optimal pumping patterns for each water transfer option as a function of inflow values.

Those pumping patterns which do not change with varying inflows can be considered "robust" and can be implemented for most inflow conditions. Those patterns which change with varying inflow values will be analyzed to determine general "rules of thumb" for least-cost operation for comparatively wet or dry inflows. This will provide the WRD with operating information such as; the optimal amount of water to be pumped from Lake Michie to the Brown treatment plant using hydropumps during the latter part of a dry year.

The model will also be used to analyze changes in optimal pumping options in response to changes in other parameters, such as consumer demand and maximum allowable working volumes for the reservoirs.

3.4 Levels of Abstraction

An important step in developing the MIP model was determining the appropriate level of abstraction. Model development seeks to determine the minimal level of abstraction which will provide useful results for its intended application. The key to efficient model formulation is determining the essential details of the system which should be represented. This determination is made by considering the intended use of the model. As an example, a map can be considered as a model which may contain various levels of abstraction. The level of abstraction required depends on the intended use of the map. If
it is to be used to reach a distant city, details about interstate routes will be essential.

However, if it is to be used to reach a street address, additional details, such as street
names, will be required. In either case, the essential details which should be represented
depend on the use which will be made of the map.

The most fundamental decisions pertaining to the level of abstraction for the Basic Model
involved selecting system boundaries and time steps. Other important decisions
regarding the appropriate levels of abstraction involved representing pumping options,
the electric tariff, and non-linear parameters.

3.4.1 Model Boundaries

The model boundaries were chosen so as to include the pumping stations. This was
essential since pumping is the primary determinant of the electrical operating cost. Thus,
the system was modeled from the raw water reservoirs to the clearwells⁴. Figure 3.1 on
the following page shows the boundary limits and system components that were used in
the model formulation.

3.4.2 Time Step

The model was designed for a calendar year period of analysis using monthly time steps.
The period of analysis was chosen to simulate a full drawdown cycle. Monthly time
steps were chosen since they are the tariff billing period for the WRD. This is important

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⁴ The terminal reservoirs, clearwells, and pumping stations were considered together with the treatment
plants.
Figure 3.1 MIP model boundaries
since the demand charges are based on the maximum kW draw incurred during the billing month.

3.4.3 Tariff Structure
The tariff was divided into two types of charges: marginal and fixed. The marginal costs refer to the amount charged per kWh of energy used (energy charge). The fixed costs refer to demand charges, basic facility charges, and contract demand charges, which are independent of the amount of energy used. The basic facility and contract demand charges are constant for each billing period, however, the energy and demand charges vary considerably based on daily (on/off-peak) time of use (see Table 2.1). This indicated that it would be necessary to include on/off-peak pricing to describe the energy and demand charges.

A peak demand was determined which represented the amount of water the WRD needed to deliver during on-peak hours. The treatment plant operators indicated that, typically, the treatment plants pumped a combined total of 20 MGD during on-peak hours in the winter and 26 MGD during on-peak hours in the summer. These values were used in the model formulation.
The model was formulated to reflect on/off-peak pumping by using two variables to represent each option. For example, the following variables describe the option of electric pumping between Lake Michie and the Williams treatment plant.

\[ \text{rmwp}_t = \text{the amount of water transferred by electric pumping from Lake Michie to the Williams WTP on-peak in month } t \text{ (MG)} \]
\[ \text{rmwop}_t = \text{the amount of water transferred by electric pumping from Lake Michie to the Williams WTP off-peak in month } t \text{ (MG)} \]

Thus, each pumping option was formulated with on/off-peak components. The marginal cost (energy charge) was represented by including these variables in the objective function with coefficients representing the corresponding on/off-peak marginal cost of transferring water by electric pump from Lake Michie to the Williams treatment plant.

The fixed charges for the finished water pumps were modeled using 0,1 variables. Since the basic facilities and contract demand charges are constant, they were modeled by including a 0,1 variable and setting its minimum value at 1. This variable was included in the objective function with a coefficient equal to the sum of both charges. The demand charges were also modeled using 0,1 variables where a value of 1 indicated that a specific pump had been used on-peak during the specified month. These variables were

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5 The variable naming convention used is as follows:
\[ \begin{align*}
  r & = \text{electric pumping (r in first position)} \\
  m & = \text{Lake Michie} \\
  w & = \text{Williams treatment plant} \\
  p & = \text{on-peak} \\
  t & = \text{month (t: 1-12)} \\
  q & = \text{hydropumping} \\
  r & = \text{Little River (r in second position)} \\
  b & = \text{Brown treatment plant} \\
  op & = \text{off-peak} \\
  d & = \text{demand}
\end{align*} \]

See Appendix D for the complete variable naming convention.

6 See Appendix C.

7 The technique used to model the demand charges is shown in Appendix C. The actual equations used in the formulation are given in Appendix D.
This representation was chosen since the larger pumps at both reservoirs are used infrequently. In addition, the difference in cost of operation between the larger and smaller pumps is negligible. Thus, only the small electric pump was represented in this formulation. Also, it was unnecessary to represent demand charges since raw water electric pumps are never used on-peak.

This formulation also represents the option of using a single hydropump between the Lake Michie reservoir and each treatment plant. In reality, two hydropumps can be used simultaneously to pump water from Lake Michie to either treatment plant. However, this option is not often used since it significantly decreases the efficiency of the hydropumps.

3.4.4.2 Finished Water Pumping Options

The decision about how to represent finished water pumps was constrained by the need to account for demand charges. The formulation used to account for these charges required the extensive use of 0,1 variables if more than two pumps were to be modeled. Therefore, at each water treatment plant the finished water pumping options were represented by modeling two constant capacity pumps: high capacity and low capacity. The capacities for these pumps were selected based on the pumps typically used by the operators. For example, the Brown WTP uses variable speed pumps to supply the distribution network. The commonly used 78% and 88% speed settings (corresponding to capacities of 12 and 18 MGD) were modeled as separate pumps. The corresponding variables used to describe on/off-peak pumping at the Brown WTP are given below:
\( r \text{lowbdp}_t = \) the amount of water transferred by low capacity pump (12 MGD) from the Brown WTP to demand on-peak
\( r \text{lowbdop}_t = \) the amount of water transferred by low capacity pump from the Brown WTP to demand off-peak
\( r \text{highbdp}_t = \) the amount of water transferred by high capacity pump (18 MGD) from the Brown WTP to demand on-peak
\( r \text{highbdop}_t = \) the amount of water transferred by high capacity pump from the Brown WTP to demand off-peak

Finished water pumping options at the Williams plant were represented by modeling the 8 MGD and 12 MGD pumps. This formulation was not intended to indicate exactly which pumps the WRD should use, rather, it allows the WRD to observe the optimal amounts of water to pump from each treatment plant. When that distribution is known, the operators can select the appropriate pumps accordingly.

3.4.5 Non-Linearities

The flows of the hydropumps and the gravity flow line decrease non-linearly as the reservoir levels are drawn down. Thus, these flows cannot be represented explicitly in the model.

3.4.5.1 Gravity Flow

The gravity flow line between the Little River reservoir and the Brown terminal reservoir operates using the 7 feet of head differential that exists when both reservoirs are full. As the reservoir is drawn down, the head differential decreases. The flow in the pipeline can be described by the Hazen-Williams equation\(^8\).

\(^8\) \( Q = 0.279 \times C_{hw} \times D^{2.63} \times (H/L)^{0.54} \)
Although the diameter of the pipeline is known, the exact length is not. Using the observed flow (45 MGD) at full reservoir capacity (H = 7 ft) and an estimated Hazen-Williams coefficient of 100, a pipe length of approximately 1300 feet was calculated. Using this length Table 3.1 shows the gravity flow at different head differentials.

Table 3.1 Gravity flow as a function of head differential.

<table>
<thead>
<tr>
<th>Head Differential (ft)</th>
<th>Gravity Flow (MGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>45</td>
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<tr>
<td>5</td>
<td>38</td>
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</tbody>
</table>

The relationship is non-linear and thus cannot be included in the model explicitly. Use of an approximation based on linear regression implicitly constrained the reservoir levels, and thus, could not be used. Although the non-linear relationship could be modeled using 0,1 variables, it was decided that gravity flow be formulated as a pump with a constant capacity of 20 MGD. This decision was, in part, to minimize the number of 0,1 variables in the model. Also, since 23 MGD can be transferred when only 2 ft of head differential remains, the model will be prone to underestimate the amount of water that can be transferred by gravity flow. However, the Brown WTP treated an average of only 17 MGD during 1994. Unless the model indicates that the optimal amount of water to be

Where:

- \( Q \) = maximum pipeline flow possible under the given head conditions (MGD)
- \( C_{HW} \) = Hazen-Williams Coefficient
- \( D \) = pipe diameter (ft)
- \( H \) = head difference between Little River and the Brown terminal reservoir (ft)
- \( L \) = length of the pipeline (ft)
treated at the Brown WTP is substantially larger, the flow limitation of 20 MGD should adequately represent the option of flowing water by gravity.

3.4.5.2 Hydropumps

The hydropump flows (hydroflows) also decrease as the reservoir levels are drawn down. Table 3.2 shows these reductions in flow for hydropumping between Lake Michie and the Williams terminal reservoir. The estimated reductions are based on pump performance curves.

Table 3.2 Lake Michie hydropump flow reduction as a function of reservoir drawdown.

<table>
<thead>
<tr>
<th>Reservoir Drawdown (ft)</th>
<th>Hydropump Flow (MGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>

As shown, the relationship between hydroflow and drawdown is non-linear. The hydropumps at Lake Michie and Little River were modeled as constant flow pumps with flows equal to those observed when the reservoirs are full. This decision was based on the observation that the decrease in hydroflow between Lake Michie and Williams did not appear to be significant until drawdowns in excess of 15 ft. Currently, hydropumps are not used when drawdowns exceed 8 ft. In addition, the modeling of hydroflow as a non-linear function would have required the use of 0,1 variables.
The wastage ratio (gallons of water wasted per gallon of water pumped) increases non-linearly as the reservoir is drawn down. Hydropump performance curves were used to estimate wastage ratio as a function of reservoir drawdown. The estimated wastage ratios for hydropumping between Lake Michie and the Williams terminal reservoir are shown in Table 3.3.

Table 3.3  Lake Michie hydropump wastage ratio as a function of reservoir drawdown.

<table>
<thead>
<tr>
<th>Reservoir Drawdown (ft)</th>
<th>Wastage Ratio (gallons wasted/gallon pumped)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.7</td>
</tr>
<tr>
<td>5</td>
<td>1.7</td>
</tr>
<tr>
<td>10</td>
<td>1.8</td>
</tr>
<tr>
<td>15</td>
<td>1.9</td>
</tr>
<tr>
<td>20</td>
<td>2.2</td>
</tr>
</tbody>
</table>

A statistical analysis was also conducted to empirically determine the wastage ratio for hydropumps at Lake Michie when the reservoir was full. This analysis used hydropumping data for 1993 and 1994. Days were selected for which the only streamflow below the dam was due to wastage from the hydropumps. The streamflow was obtained from the USGS gauge located just below the Lake Michie dam. A regression model was fitted to the data to estimate the wastage ratio. The data and subsequent analysis are given in Appendix F. This analysis yielded an estimate of 2.2 gallons wasted per 1 gallon pumped. The reservoir drawdowns observed during the period of analysis were between 1-3 feet. To be conservative and to avoid the use of

* See Appendix E for calculations.
additional 0.1 variables, the wastage ratio was represented as a constant of 2.5 gallons/gallon.

3.4 Model Verification

Model verification was conducted by constraining the Basic Model to simulate WRD operation for 1994. This was accomplished by making the following adjustments:

- 1994 inflows and demands were used as inputs.
- Maximum allowable working volumes were set to 1660 and 2330 MG at Lake Michie and Little River, respectively.
- The amounts of water to be treated at each treatment plant were specified.
- Hydropumping between Little River and Williams was not permitted.
- The use of the Brown to Williams pumping option was only permitted during June and September.

Since the model is only an abstraction of the water supply system, it was not possible to constrain it to perform exactly as the WRD. The actual WRD operation was compared with the verification model output to evaluate the abstractions used in the model. The WRD pumped flows are compared with the model output in Table 3.4.

---

10 The data provided by the WRD did not indicate whether the hydropumping was directed to the Williams or Brown plant.
As shown in Table 3.4 on the following page, the verification model output agrees well with the actual WRD operation. The major differences occur during the months of January and August where the verification model flows approximately 650 MG less by gravity and 650 MG more by electric pumping when compared to WRD operation. Other than this notable exception, the month by month operations are in close agreement. This result is significant since it indicates that the Basic Model can closely approximate month by month operation with gravity flow and hydroflow represented as constant flow pumps. Table 3.5 shows the cost comparison between the WRD and the verification model.

<table>
<thead>
<tr>
<th></th>
<th>Actual operating costs</th>
<th>Operating costs for MIP verification model</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Raw water operations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake Michie</td>
<td>$20,300</td>
<td>$20,300</td>
<td>$0</td>
</tr>
<tr>
<td>Little River</td>
<td>$19,000</td>
<td>$18,500</td>
<td>-$500</td>
</tr>
<tr>
<td>Total</td>
<td>$39,300</td>
<td>$38,800</td>
<td>-$500</td>
</tr>
<tr>
<td><strong>Finished water operations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Williams</td>
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<td>$144,700</td>
<td>-$800</td>
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<tr>
<td>Brown</td>
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</tr>
<tr>
<td>Total</td>
<td>$558,500</td>
<td>$557,200</td>
<td>-$1,300</td>
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<tr>
<td>Total</td>
<td>$597,800</td>
<td>$596,000</td>
<td>-$1,800</td>
</tr>
<tr>
<td>Raw Water From LM</td>
<td>January WRD</td>
<td>January MIP</td>
<td>February WRD</td>
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<td>-------------</td>
<td>-------------</td>
<td>--------------</td>
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<tr>
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</tr>
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</tr>
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</tr>
<tr>
<td>Reservoir Drawdown</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LR</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Raw Water From BR</td>
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<td></td>
</tr>
<tr>
<td>to WR</td>
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</tr>
<tr>
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<td>0</td>
</tr>
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</tr>
<tr>
<td>Gas</td>
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<td>0</td>
</tr>
</tbody>
</table>

¹ Hydros from Lake Michie may be run to the Williams or the Brown terminal reservoir

² Hydros from Little River may only be run to the Williams terminal reservoir
Table 3.4 Comparison of WRD 1994 operation and MIP verification model operation (all values given in MG)

<table>
<thead>
<tr>
<th></th>
<th>August</th>
<th>September</th>
<th>October</th>
<th>November</th>
<th>December</th>
<th>Total</th>
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<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>309</td>
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<td>575</td>
<td>534</td>
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<td>0</td>
</tr>
<tr>
<td>Raw Water From LR</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>534</td>
<td>262</td>
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<td>12</td>
<td>0</td>
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<td>416</td>
<td>1050</td>
<td>1239</td>
</tr>
<tr>
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<td>670</td>
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<td>522</td>
<td>1710</td>
<td>940</td>
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<tr>
<td>Raw Water From BR to WR</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0</td>
<td>140</td>
<td>219</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>Finished Water From BR</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
<td>560</td>
<td>560</td>
<td>532</td>
<td>537</td>
<td>510</td>
<td>511</td>
</tr>
<tr>
<td>Diesel</td>
<td>0.06</td>
<td>0</td>
<td>4.7</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
</tr>
<tr>
<td>Finished Water From WR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
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<td>292</td>
<td>267</td>
<td>269</td>
<td>262</td>
<td>262</td>
</tr>
<tr>
<td>Gas</td>
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<td>1.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

1 Hydros from Lake Michie may be run to the Williams or the Brown terminal reservoir
2 Hydros from Little River may only be run to the Williams terminal reservoir
The agreement in costs and flows for finished water operations indicates that the abstraction of providing only two pumps at each plant suitably represents the actual system. The total electrical operating cost returned by the model is only 0.3% lower than the actual operating cost. This result suggests that the abstractions used to model the electric tariff accurately\textsuperscript{11} describe the electrical operating costs associated with the pumped flows. This is particularly important for ensuring that the model is useful for identifying least-cost pumping options.

Based on the agreement for operational cost and month by month pumped flows, the levels of abstraction chosen for the Basic Model can accurately simulate the operation of the actual system.

3.5 Basic Model Results for 1994

The Basic Model was used to determine optimal flows for 1994. The optimal flows were then compared to the actual operation of the water supply system for 1994. These results are included here to illustrate those system components with potential for cost reduction. A more complete comparison between the model results and actual WRD operation is given by Fragapane (1996).

A cost comparison between the MIP recommended strategy and that used by the WRD in 1994 projected that a savings of $53,500 would have been realized if the optimal pumped

\textsuperscript{11} The model did not account for demand charges which the WRD incurred in response to spikes in on-peak demand. These charges amounted to $15,600 for 1994.
flows had been used. As shown in Table 3.6, the bulk of that savings would have been due to decreased finished water operating costs.

Table 3.6 Comparison of electric operating costs (1994).

<table>
<thead>
<tr>
<th></th>
<th>Actual operating costs</th>
<th>Projected costs for optimal flows</th>
<th>Projected savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw water operations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake Michie</td>
<td>$20,300</td>
<td>$18,500</td>
<td>$1,800</td>
</tr>
<tr>
<td>Little River</td>
<td>$19,000</td>
<td>$14,600</td>
<td>$4,400</td>
</tr>
<tr>
<td>Total</td>
<td>$39,300</td>
<td>$33,100</td>
<td>$6,200</td>
</tr>
<tr>
<td>Finished water operations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Williams</td>
<td>$145,500</td>
<td>$187,100</td>
<td>-$41,600</td>
</tr>
<tr>
<td>Brown</td>
<td>$413,000</td>
<td>$324,100</td>
<td>$88,900</td>
</tr>
<tr>
<td>Total</td>
<td>$558,500</td>
<td>$511,200</td>
<td>$47,300</td>
</tr>
<tr>
<td>Total</td>
<td>$597,800</td>
<td>$537,500</td>
<td>$53,300</td>
</tr>
</tbody>
</table>

Since the recommendations for finished water operation were found to be more important in obtaining cost savings, they are presented first.

3.5.1 Finished Water Operations

The finished water operations analysis considered the model results for pumping from the water treatment plants. The model indicates that the use of the optimal finished water flows would have saved\(^\text{12}\) approximately $47,300 compared to those used by the WRD in 1994. Table 3.7 shows the comparison between 1994 WRD operation and MIP recommended operation for raw and finished water pumping.

---

\(^{12}\) The model actually predicts a savings of $62,100 for finished water electrical pumping costs, but due to higher chemical treatment costs at the Williams plant, the actual savings would be $53,700.
<table>
<thead>
<tr>
<th></th>
<th>January WRD</th>
<th>January MIP</th>
<th>February WRD</th>
<th>February MIP</th>
<th>March WRD</th>
<th>March MIP</th>
<th>April WRD</th>
<th>April MIP</th>
<th>May WRD</th>
<th>May MIP</th>
<th>June WRD</th>
<th>June MIP</th>
<th>July WRD</th>
<th>July MIP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Raw Water From LM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydros</td>
<td>484</td>
<td>444</td>
<td>200</td>
<td>360</td>
<td>210</td>
<td>183</td>
<td>249</td>
<td>80</td>
<td>258</td>
<td>221</td>
<td>143</td>
<td>411</td>
<td>409</td>
<td>410</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>592</td>
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<td>447</td>
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</tr>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>LM</td>
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<td>0</td>
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<td>830</td>
<td>437</td>
<td>510</td>
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<td></td>
<td></td>
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<td></td>
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<td>1.4</td>
<td>0</td>
<td>2.7</td>
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</tr>
<tr>
<td><strong>Finished Water From WR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
<td>250</td>
<td>403</td>
<td>203</td>
<td>284</td>
<td>237</td>
<td>351</td>
<td>248</td>
<td>382</td>
<td>271</td>
<td>456</td>
<td>327</td>
<td>459</td>
<td>287</td>
<td>443</td>
</tr>
<tr>
<td>Gas</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0.9</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2.3</td>
<td>0</td>
<td>1.6</td>
<td>0</td>
</tr>
</tbody>
</table>

1 Hydros from Lake Michie may be run to the Williams or the Brown terminal reservoir
2 Hydros from Little River may only be run to the Williams terminal reservoir
Table 3.7 Comparison of WRD 1994 operation and MIP optimal flows (all values given in MG)

<table>
<thead>
<tr>
<th></th>
<th>August</th>
<th></th>
<th>September</th>
<th></th>
<th>October</th>
<th></th>
<th>November</th>
<th></th>
<th>December</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>WRD</td>
<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
<td>WRD</td>
<td>MIP</td>
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<td>MIP</td>
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<td>Raw Water From LM</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydros¹</td>
<td>288</td>
<td>403</td>
<td>130</td>
<td>153</td>
<td>309</td>
<td>290</td>
<td>287</td>
<td>126</td>
<td>0</td>
<td>100</td>
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<td>3181</td>
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<tr>
<td>Electric</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>52</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>251</td>
<td>0</td>
<td>256</td>
<td>148</td>
</tr>
<tr>
<td>Raw Water From LR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Hydros²</td>
<td>0</td>
<td>46</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>219</td>
<td>39</td>
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<td>Electric</td>
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<td>0</td>
<td>438</td>
<td>0</td>
<td>483</td>
<td>0</td>
<td>421</td>
<td>0</td>
<td>437</td>
<td>360</td>
<td>1872</td>
<td>360</td>
</tr>
<tr>
<td>Gravity</td>
<td>575</td>
<td>402</td>
<td>262</td>
<td>598</td>
<td>12</td>
<td>476</td>
<td>0</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>4154</td>
<td>4166</td>
</tr>
<tr>
<td>Reservoir Drawdown</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>LM</td>
<td>120</td>
<td>0</td>
<td>360</td>
<td>416</td>
<td>1050</td>
<td>1239</td>
<td>1660</td>
<td>1471</td>
<td>1520</td>
<td>1660</td>
<td>1660</td>
<td>1660</td>
</tr>
<tr>
<td>LR</td>
<td>670</td>
<td>0</td>
<td>830</td>
<td>522</td>
<td>1710</td>
<td>940</td>
<td>2070</td>
<td>1408</td>
<td>2330</td>
<td>2330</td>
<td>2330</td>
<td>2330</td>
</tr>
<tr>
<td>Raw Water From BR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>to WR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
<td>0</td>
<td>0</td>
<td>140</td>
<td>196</td>
<td>31</td>
<td>116</td>
<td>0</td>
<td>238</td>
<td>0</td>
<td>0</td>
<td>383</td>
<td>827</td>
</tr>
<tr>
<td>Finished Water From BR</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
<td>560</td>
<td>402</td>
<td>532</td>
<td>402</td>
<td>510</td>
<td>360</td>
<td>485</td>
<td>360</td>
<td>452</td>
<td>360</td>
<td>6075</td>
<td>4488</td>
</tr>
<tr>
<td>Diesel</td>
<td>0.06</td>
<td>0</td>
<td>4.7</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>1.4</td>
<td>0</td>
<td>31.06</td>
<td>0</td>
</tr>
<tr>
<td>Finished Water From WR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electric</td>
<td>290</td>
<td>449</td>
<td>267</td>
<td>403</td>
<td>262</td>
<td>409</td>
<td>253</td>
<td>366</td>
<td>246</td>
<td>319</td>
<td>3141</td>
<td>4724</td>
</tr>
<tr>
<td>Gas</td>
<td>2.4</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.7</td>
<td>0</td>
<td>1.5</td>
<td>0</td>
<td>15.2</td>
<td>0</td>
</tr>
</tbody>
</table>

¹ Hydros from Lake Michie may be run to the Williams or the Brown terminal reservoir
² Hydros from Little River may only be run to the Williams terminal reservoir
The optimal pumped flows for 1994 would result in 51% of the water being treated by the Williams plant and 49% by the Brown plant. This is compared to the actual distribution used in 1994 where 35% of the water was treated by the Williams plant and 65% by the Brown plant.

From Table 3.7 we observe that the optimal operation used the minimum\textsuperscript{13} pumping rate for the Brown plant under all conditions except on-peak pumping during summer months. The optimal operation for the Williams plant used the minimum pumping rate on-peak and an off-peak pumping rate\textsuperscript{14} to supply whatever demand was left unmet by the Brown plant.

3.5.2 Raw Water Operations

The raw water operations analysis considered the model results for pumping modes, transmission routes, and reservoir levels. A comparison between the optimal raw water flows and reservoir levels generated by the MIP model and the actual flows used was given in Table 3.7. A summary comparison of raw water operations is given in Table 3.8.

\textsuperscript{13} Recall that the Williams and Brown treatment plants have minimum required pumping rates of 8 MGD (56 MGM) and 12 MGD (84 MGM), respectively.

\textsuperscript{14} The varying off-peak values indicated various combinations of the two off-peak pumps which were provided.
Table 3.8 Comparison of yearly summary statistics.

<table>
<thead>
<tr>
<th>Raw Water Flow Distribution</th>
<th>Source Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WRD</td>
</tr>
<tr>
<td>Hydros</td>
<td>32%</td>
</tr>
<tr>
<td>Electric</td>
<td>23%</td>
</tr>
<tr>
<td>Gravity</td>
<td>45%</td>
</tr>
</tbody>
</table>

Brown to Williams Transport (MG)

<table>
<thead>
<tr>
<th>WRD</th>
<th>MIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>383</td>
<td>476</td>
</tr>
</tbody>
</table>

Although there are substantial differences between the raw water flows, Table 3.6 indicates that only $6,200 in raw water savings could have been realized by using the optimal flows. These results indicate that there is little potential for cost savings in manipulating raw water pumping modes. However, the raw water pumping pathways are essential in determining finished water flows.

3.5.3 Summary of Model Results

The model results for 1994 indicate the relative importance of finished water operations in identifying potential cost savings. This suggests that those details associated with finished water flows and costs (e.g. finished water pumping options, demand charges) and raw water pathways are essential elements and should be represented accurately in the model. This agrees with our decision to use 0,1 variables to model demand charges rather than hydroflow and gravity flow.
3.6 General Considerations

Based on the model verification, the levels of abstraction chosen for the Basic Model accurately simulate the system operation and the associated pumping costs. The model results for 1994 indicate that there is significant potential for savings in using optimal finished water flows. This suggests that accurately representing finished water pumping, costs, and raw water pathways is important to the intended use of the model.
Chapter 4. Evaluation of Basic Model

Abstraction

4.1 Concerns about Basic Model Abstraction

Some of the levels of abstraction chosen for the Basic Model were significantly different from the observed physical operation, e.g. gravity flow was treated as a constant when, in reality, it varies between 45 MGD and 16 MGD. This was particularly true for those parameters that vary non-linearly with reservoir level, i.e. gravity flow and hydroflow. It was also noted that no accounting of evaporation and precipitation had been included in the Basic Model. Finally, there was concern that the provision of only two pumping options at the finished water treatment plants would limit the model’s ability to model future demand scenarios. The decision was made to explore additional levels of detail for these elements.

The evaluation of additional detail in the model focuses on the effect the additional detail has on model results. It is important to consider the intended use of the model when making these judgments. In this case the intended model use is to conduct sensitivity
analyses over a variety of inflow conditions to determine least-cost pumping patterns. This requires that we evaluate additional levels of detail not only by considering changes in optimal pump flows, but also the effect on overall cost. It is possible that additional levels of detail may significantly alter the optimal pump flows without changing the cost of operation. This would suggest that the overall cost is not sensitive to those flows. Under these conditions it might reasonably be concluded that the additional level of detail does not result in useful information for cost minimization and thus, does not contribute to the purpose of the model.

The evaluations were conducted by comparing results from the Basic Model to those from Expanded Models that contained additional details. Results were generated and compared using five years of inflow data. Since preliminary analyses showed that changes in model solutions were most significant for extreme weather conditions, inflow years were selected to represent the wettest and driest years on record (1925 - 1995). The inflow years selected were: 1974, 1973, 1977, 1976, 1933. These inflow data were used for each sensitivity analysis.

4.2 Gravity Flow

In the Basic Model gravity flow was modeled as a pump with constant capacity which could only operate between reservoir elevations of 349 and 355 feet. It was decided that an alternative formulation that accounted for variation in gravity flow with reservoir level should be evaluated. The effect of allowing gravity flow to vary according to reservoir level was explored using two methods of analysis:
1) Increasing gravity flow
2) Providing two level gravity flow

In the first analysis gravity flow was increased to determine whether the model would make additional use of it. In the second analysis gravity flow was split into two levels, high and low, and the model was allowed to choose between them based on reservoir levels. These analyses were used to determine the desirability of increasing the detail in the description of the gravity flow line. In this evaluation, the changes in optimal gravity flow and other pumped flows were analyzed as well as changes in the objective function value.

4.2.1 Increased Gravity Flow

The Basic Model assumed a flow of 20 MGD for the gravity line. In the Expanded Models constant flows of 30 and 40 MGD were assumed. The Expanded Models did not represent gravity flow as a function of reservoir level, rather, they were used to indicate whether the cost of operating the system could be reduced by increased use of gravity flow. Table 4.1 shows the model runs made for the analysis.

<table>
<thead>
<tr>
<th>Model Run</th>
<th>Inflow Year</th>
<th>Gravity Flow</th>
<th>Model Run</th>
<th>Inflow Year</th>
<th>Gravity Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1974</td>
<td>20</td>
<td>10</td>
<td>1976</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>1974</td>
<td>30</td>
<td>11</td>
<td>1976</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>1974</td>
<td>40</td>
<td>12</td>
<td>1976</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>1973</td>
<td>20</td>
<td>13</td>
<td>1933</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>1973</td>
<td>30</td>
<td>14</td>
<td>1933</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>1973</td>
<td>40</td>
<td>15</td>
<td>1933</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>1977</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>8</td>
<td>1977</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td>1977</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The results of these model runs were evaluated based on the differences observed in the objective functions and in the optimal pumped flows. The largest differences between the Basic and Expanded Models were observed when 1977 inflow data were used. Table 4.2 shows optimal raw water transfer for the Basic and Expanded Models for 1977 inflows. Since the solutions for both Expanded Models were identical, they are presented here as a single model.

Table 4.2  Comparison of Basic and Expanded Models (increased gravity flow capacity).

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th>Obj. Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>595</td>
<td>598</td>
<td>447</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,569</td>
<td>$531,900</td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>588</td>
<td>629</td>
<td>423</td>
<td>845</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,815</td>
<td>$531,500</td>
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</tbody>
</table>

Water transferred by hydropumps (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>412</td>
<td>172</td>
<td>6</td>
<td>398</td>
<td>253</td>
<td>403</td>
<td>409</td>
<td>484</td>
<td>679</td>
<td>5,334</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>412</td>
<td>169</td>
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<td>6</td>
<td>403</td>
<td>409</td>
<td>484</td>
<td>679</td>
<td>5,334</td>
<td></td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Lake Michie and Little River) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48</td>
<td>257</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>1,309</td>
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<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>1,063</td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Brown to Williams) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
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<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>235</td>
<td>196</td>
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<td>196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>672</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>228</td>
<td>227</td>
<td>21</td>
<td>443</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>919</td>
<td></td>
</tr>
</tbody>
</table>
The Expanded Model uses the higher flow to transfer more water by gravity only in the months of June and August. In these months a total of 31 MG and 247 MG, respectively, of additional flow was transferred by the Expanded Model compared to the Basic Model. In the months of May and July the Basic Model actually transferred more water by gravity than did the Expanded Model.

Table 4.2 shows that the increased gravity flow caused slight shifts in the amounts of water transferred by electric, hydro, and cross-connection pumping between the two model formulations. Although the model formulations differed in the amounts of water transferred by electric pumps located at the reservoirs and those pumps located at the Brown terminal reservoir, the overall amount of water transferred by electric pumps was equal. The Expanded Model used the Brown to Williams pumping option to transfer the additional gravity flow to the Williams plant, thus resulting in identical finished water flows between the models. The finished water pumped flows for both models were identical to those presented in Section 3.5.1.

The 20 MGD capacity chosen for the Basic Model represents the flow with only 1.6 ft of head differential between Little River and the Brown terminal reservoir (as estimated by the Hazen-Williams equation). Thus, we expect that representing gravity flow as a function of reservoir elevation would increase the amount of water that could be transferred by gravity. From Table 4.2 we observe that increasing the gravity flow only improves the objective function by $400 (approximately 0.07% of the annual electrical
operating cost). This suggests that the level of abstraction contained in the Basic Model is satisfactory for our purpose of identifying least-cost pump flows.

4.2.2 Two Level Gravity Flow

To further test the assumption that the gravity line could be modeled as a pump with constant flow, a second experiment was designed. This experiment used an Expanded Model which would allow the option of choosing between two flow levels based on reservoir elevations\(^{15}\). The formulation treated the two flow levels as separate, mutually exclusive pipelines. A 0,1 variable was used to allow the Expanded Model to choose the higher flow option when the reservoir level was high. Thus, gravity flow was represented as a piecewise linear function. To accomplish this, the following variables and constraints were added to the Basic Model:

Variables:

\[
\begin{align*}
\text{Grblop}_t & = \text{Water transferred on-peak by gravity flow using the lower level (20 MGD) in month } t \\
\text{Grbloop}_t & = \text{Water transferred off-peak by gravity flow using the lower level in month } t \\
\text{Grbhip}_t & = \text{Water transferred on-peak by gravity flow using the higher level (33, 41 MGD) in month } t \\
\text{Grbhiop}_t & = \text{Water transferred off-peak by gravity flow using the higher capacity in month } t \\
\text{[GHI]}_t & = 0,1 \text{ variable such that} \\
& \quad \text{when } = 1, \text{ higher capacity can be used in month } t \\
& \quad = 0, \text{ higher capacity cannot be used in month } t
\end{align*}
\]

\(^{15}\) Working volumes were used as a surrogate for reservoir elevation.
Constraints:

GHIon,: \[ \text{GHI}_t - 1 \leq -\alpha[(S^M_t - C_s) - (S'_{t-1} + S'_t)/2] \]

Gravflo,: \[ .001(\text{GrbHlp}_t + \text{GrbHlop}_t) \leq \text{GHI}_t \]

Where: \[ S^M_t - C_s = \] The critical storage value specified for month t (MG)
\[ S'_{t-1} = \] The working volume at Little River at the end of month t

The GHIon, and Gravflo, constraints allowed the model to choose between the higher and lower gravity flow levels only when the average of the reservoir working volumes for months t-1 and t exceeded the minimum critical value. Between that critical level and the gravity flow cutoff level\(^\text{16}\) the Expanded Model was only allowed to use 20 MGD as the gravity flow. The high gravity flow level and its corresponding minimum critical storage value were chosen based on typical reservoir levels at different periods during the drawdown cycle. These values are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Month</th>
<th>January, February, June - December</th>
<th>March - May</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Gravity Flow Level (MGD)</td>
<td>33</td>
<td>41</td>
</tr>
<tr>
<td>Critical Working Volume Range (MG)</td>
<td>2737 to 3250</td>
<td>3075 to 3250</td>
</tr>
</tbody>
</table>

Results were generated and compared for the Basic and the Expanded Models using the five years of inflow data. The results of these model runs were evaluated based on the differences observed in the objective functions and in the optimal pumped flows. The

\(^{16}\) Gravity flow cannot occur below a working volume of 2260 MG.
largest differences between the Basic and Expanded Models were observed when 1977 inflow data were used. These results are shown in Table 4.4.

Table 4.4  Comparison of Basic and Expanded Models (two level gravity flow).

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th>Obj. Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>595</td>
<td>598</td>
<td>447</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,569</td>
<td>$531,900</td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>448</td>
<td>790</td>
<td>444</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,619</td>
<td>$531,800</td>
</tr>
</tbody>
</table>

Water transferred by hydropumps (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>412</td>
<td>172</td>
<td>6</td>
<td>398</td>
<td>253</td>
<td>403</td>
<td>409</td>
<td>484</td>
<td>679</td>
<td>5,334</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>420</td>
<td>113</td>
<td>70</td>
<td>393</td>
<td>253</td>
<td>403</td>
<td>410</td>
<td>484</td>
<td>679</td>
<td>5,342</td>
<td></td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Lake Michie and Little River) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48</td>
<td>257</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>1,309</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>254</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>1,259</td>
<td></td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Brown to Williams) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>235</td>
<td>196</td>
<td>45</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>672</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>88</td>
<td>388</td>
<td>50</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>722</td>
<td></td>
</tr>
</tbody>
</table>

The differences in the optimal pumped flows between the Basic and the Expanded Model formulations are appreciable only in the months of May and June. The differing amounts of water transferred by gravity flow for these months are counterbalanced by
hydropumping and electrical pumping to ensure the same finished water distribution\textsuperscript{17}. This is shown in Table 4.5.

<table>
<thead>
<tr>
<th></th>
<th>May</th>
<th></th>
<th>June</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic</td>
<td>Expanded</td>
<td>Basic</td>
<td>Expanded</td>
</tr>
<tr>
<td>Lake Michie to Williams</td>
<td>170</td>
<td>111</td>
<td>4</td>
<td>68</td>
</tr>
<tr>
<td>Little River to Williams</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Electrical pumps</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake Michie to Williams</td>
<td>48</td>
<td>254</td>
<td>257</td>
<td>0</td>
</tr>
<tr>
<td>Brown to Williams</td>
<td>235</td>
<td>88</td>
<td>196</td>
<td>388</td>
</tr>
<tr>
<td>Gravity Flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Little River to Brown</td>
<td>595</td>
<td>448</td>
<td>598</td>
<td>790</td>
</tr>
<tr>
<td>Total Water Treated at Brown</td>
<td>360</td>
<td>360</td>
<td>459</td>
<td>458</td>
</tr>
<tr>
<td>Total Water Treated at Williams</td>
<td>455</td>
<td>455</td>
<td>402</td>
<td>402</td>
</tr>
</tbody>
</table>

The piecewise linear representation of gravity flow resulted in a predicted savings of only $100 when compared to the operating costs obtained from the Basic Model. This agrees with our assumption that increased description of gravity flow will not significantly improve the model’s ability to identify least-cost pump flows. Thus, we conclude that the original level of abstraction contained in the Basic Model is sufficient for the intended use.

4.3 Hydroflow

In the Basic Model hydropumps are represented as constant capacity pumps with a fixed wastage ratio. The capacity used for the Basic Model was based on the observed flow

\textsuperscript{17} The finished water pumping strategy for both models remained identical to that presented in Section 3.5.1.
when the reservoirs were full. This assumption was evaluated by comparing results from the Basic Model to those from an Expanded Model in which hydroflow capacity was reduced to 75% of full capacity for months\(^\text{18}\) where the reservoirs might be drawn down below 2-3 ft. In addition, the wastage ratio used in the Expanded Model for these months was increased from 2.5 to 4 gallons wasted per gallon pumped. Results were generated and compared for the Basic and the Expanded Model using the five years of inflow data. Table 4.6 shows the model runs made for the analysis.

<table>
<thead>
<tr>
<th>Model Run</th>
<th>Inflow Year</th>
<th>Hydro Flow</th>
<th>Model Run</th>
<th>Inflow Year</th>
<th>Hydro Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>1974</td>
<td>100%</td>
<td>30</td>
<td>1976</td>
<td>100%</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>75%</td>
<td>31</td>
<td></td>
<td>75%</td>
</tr>
<tr>
<td>24</td>
<td>1973</td>
<td>100%</td>
<td>33</td>
<td>1933</td>
<td>100%</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>75%</td>
<td>34</td>
<td></td>
<td>75%</td>
</tr>
<tr>
<td>27</td>
<td>1977</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
<td>75%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of these model runs were evaluated based on the differences observed in the objective functions and in the optimal pump flows. The largest differences between the Basic and Expanded Models were observed when 1977 inflow data were used. These results are shown in Table 4.7.

As shown in Table 4.7 the differences in optimal pumped flows between the Basic Model and the Expanded Model are significant for hydro and electric pumping. The Expanded Model pumps approximately 750 MG less by hydropumps than does the Basic Model.

\(^{18}\) Based on historical reservoir levels, we can assume that the reservoirs would not be drawn down significantly in the months of March, April, and May. Thus, the hydroflow capacity was not adjusted for these months.
Table 4.7  Comparison of Basic and Expanded Models (reduced hydroflow).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Pathway</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th>Overall Total</th>
<th>Obj. Function Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>LMWR</td>
<td>84</td>
<td>84</td>
<td>84</td>
<td>382</td>
<td>170</td>
<td>4</td>
<td>396</td>
<td>250</td>
<td>28</td>
<td>320</td>
<td>193</td>
<td>84</td>
<td>2078</td>
<td>5334</td>
<td>$531,900</td>
</tr>
<tr>
<td></td>
<td>LRWR</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1587</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>LMWR</td>
<td>186</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>315</td>
<td>138</td>
<td>115</td>
<td>315</td>
<td>251</td>
<td>122</td>
<td>226</td>
<td>310</td>
<td>315</td>
<td>2293</td>
<td></td>
</tr>
<tr>
<td>(75%)</td>
<td>LRWR</td>
<td>217</td>
<td>284</td>
<td>351</td>
<td>67</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>282</td>
<td>183</td>
<td>56</td>
<td>4</td>
<td>1454</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic</td>
<td>LMWR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48</td>
<td>257</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1982</td>
</tr>
<tr>
<td></td>
<td>LRBR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2725</td>
</tr>
<tr>
<td>Expanded</td>
<td>LMWR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>160</td>
<td>145</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(75%)</td>
<td>LRBR</td>
<td>222</td>
<td>23</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>BRWR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>156</td>
<td>195</td>
<td>126</td>
<td>195</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Model</td>
<td>Pathway</td>
<td>Jan</td>
<td>Feb</td>
<td>Mar</td>
<td>Apr</td>
<td>May</td>
<td>Jun</td>
<td>Jul</td>
<td>Aug</td>
<td>Sep</td>
<td>Oct</td>
<td>Nov</td>
<td>Dec</td>
<td>Total</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic</td>
<td>LRBR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>595</td>
<td>598</td>
<td>447</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2569</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>LRBR</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>360</td>
<td>516</td>
<td>597</td>
<td>528</td>
<td>597</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2598</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

LMWR = Lake Michie to Williams Reservoir
LMBR = Lake Michie to Brown Reservoir
LRWR = Little River to Williams Reservoir
LRBR = Little River to Brown Reservoir
BRWR = Brown Reservoir to Williams Reservoir (cross-connection)
The Expanded Model maintains the same finished water pumped flows as shown in Section 3.5.1 by pumping almost 750 MG more using electric pumps. Virtually all of this additional pumping occurs between Little River and the Brown treatment plant. This causes the raw water costs of the Expanded Model to be $1,100 greater than those of the Basic Model. Thus, although there are significant differences in hydroflow and electric pumping, the result is an increase of only 0.2% in overall cost. In addition, the differences between the Basic Model and the Expanded Models were less pronounced for the other inflows analyzed. Table 4.8 shows a comparison of the objective values for the inflow years used (inflow years are arranged from wet to dry).

Table 4.8 Comparison of objective functions for Basic and Expanded Models (decreased hydroflow).

<table>
<thead>
<tr>
<th>Model</th>
<th>Inflow Year</th>
<th>Obj. Function</th>
<th>Difference from Basic Model for the same year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>1974</td>
<td>$520,600</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>1973</td>
<td>$520,700</td>
<td>$100</td>
</tr>
<tr>
<td>Basic</td>
<td>1977</td>
<td>$520,700</td>
<td></td>
</tr>
<tr>
<td>Expanded</td>
<td>1977</td>
<td>$531,900</td>
<td>$100</td>
</tr>
<tr>
<td>Basic</td>
<td>1976</td>
<td>$533,000</td>
<td>$1100</td>
</tr>
<tr>
<td>Expanded</td>
<td>1976</td>
<td>$534,500</td>
<td></td>
</tr>
<tr>
<td>Basic</td>
<td>1933</td>
<td>$535,200</td>
<td>$700</td>
</tr>
<tr>
<td>Expanded</td>
<td>1933</td>
<td>$535,500</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$535,600</td>
<td>$100</td>
</tr>
</tbody>
</table>

The objective function values for Basic and Expanded Models are not substantially different for the range of inflow years used. This suggests that the Basic Model's representation of hydropumps as having constant flows and wastage ratios is sufficient for the intended use of the model.
4.4 Evaporation and Precipitation Effects

Evaporation and precipitation considerations were not included in the Basic Model formulation. However, since these effects had been accounted for in similar models (Speight, 1994) it was decided to investigate the effect they might have on the MIP model output. An Expanded Model used the method employed by Speight to account for average evaporation and precipitation effects. These calculations used a net evaporation factor (Gt) to describe the average water removal (evaporation - precipitation) from the reservoirs. These factors are given in Table 4.9.

Table 4.9 Evaporation/Precipitation data and associated Gt factors (Speight, 1994).

<table>
<thead>
<tr>
<th>Month</th>
<th>Evaporation</th>
<th>Rainfall inches per month</th>
<th>(Gt)(^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.89</td>
<td>3.59</td>
<td>-1.80</td>
</tr>
<tr>
<td>February</td>
<td>3.12</td>
<td>3.96</td>
<td>-0.84</td>
</tr>
<tr>
<td>March</td>
<td>3.75</td>
<td>4.14</td>
<td>-0.39</td>
</tr>
<tr>
<td>April</td>
<td>5.25</td>
<td>3.60</td>
<td>+1.65</td>
</tr>
<tr>
<td>May</td>
<td>5.85</td>
<td>3.95</td>
<td>+1.90</td>
</tr>
<tr>
<td>June</td>
<td>6.39</td>
<td>4.21</td>
<td>+2.18</td>
</tr>
<tr>
<td>July</td>
<td>6.45</td>
<td>4.90</td>
<td>+1.55</td>
</tr>
<tr>
<td>August</td>
<td>5.70</td>
<td>4.97</td>
<td>+0.73</td>
</tr>
<tr>
<td>September</td>
<td>4.56</td>
<td>3.57</td>
<td>+0.99</td>
</tr>
<tr>
<td>October</td>
<td>3.21</td>
<td>3.18</td>
<td>+0.03</td>
</tr>
<tr>
<td>November</td>
<td>2.19</td>
<td>3.02</td>
<td>-0.83</td>
</tr>
<tr>
<td>December</td>
<td>1.59</td>
<td>3.44</td>
<td>-1.85</td>
</tr>
<tr>
<td>Total</td>
<td>49.95</td>
<td>46.63</td>
<td>+3.32</td>
</tr>
</tbody>
</table>

Linear relationships were derived for both reservoirs relating the working volume to total surface area. These relationships\(^{20}\) are given as:

Lake Michie:

\[ H_{LM} = 0.13s_{LM} + 192 \text{ for } s_{LM} \leq 2570 \]

\(^{19}\) Evaporation - Rainfall

\(^{20}\) The correlation coefficients for each linear regression were greater than 0.98.
Little River:

\[ H_{t}^{LR} = 0.085sr_{t-1} + 277 \text{ for } sr_{t-1} \leq 3250 \]

Where:
- \( H_{t}^{LM} \) = Surface area at Lake Michie in month \( t \) (acres)
- \( H_{t}^{LR} \) = Surface area at Little River in month \( t \) (acres)
- \( sm_{t-1} \) = Working volume at Lake Michie in month \( t-1 \) (MG)
- \( sr_{t-1} \) = Working volume at Little River in month \( t-1 \) (MG)

Using this information and the Gt factors derived by Speight (1994), equations were derived to account for evaporation and precipitation effects as a function of reservoir working volume. These equations were added to the Expanded Model as constraints and are shown below.

NELM:

\[ Gt(0.027)(0.13)sm_{t-1} + nelm_{t} = -Gt(0.027)(192) \]

NELR:

\[ Gt(0.027)(0.085)sr_{t-1} + nelr_{t} = -Gt(0.027)(277) \]

Where
- \( nelm_{t} \) = Net evaporation from Lake Michie in month \( t \) (MG)
- \( nelr_{t} \) = Net evaporation from Little River in month \( t \) (MG)
- 0.027 = Conversion factor (MG/acre-in)

The net evaporation variables were then entered as withdrawals or additions (depending on the sign of the Gt factor) in the continuity equations for the two reservoirs. Results were generated and compared for the Basic and the Expanded Models using the five years of inflow data. The results of these model runs were evaluated based on the differences observed in the objective functions and in the optimal pump flows. The largest differences between the Basic and Expanded Models were observed when 1977 inflow data were used. These results are shown in Table 4.10.
Table 4.10  Comparison of Basic and Expanded Models (evaporation/precipitation included).

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expanded</td>
<td>35</td>
<td>21</td>
<td>10</td>
<td>-47</td>
<td>-56</td>
<td>-58</td>
<td>-36</td>
<td>-14</td>
<td>-16</td>
<td>0</td>
<td>11</td>
<td>24</td>
<td>-127</td>
</tr>
</tbody>
</table>

Water transferred by gravity flow (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
<th>Obj. Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>330</td>
<td>595</td>
<td>598</td>
<td>447</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,569</td>
<td>$531,900</td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>323</td>
<td>413</td>
<td>598</td>
<td>598</td>
<td>598</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2,530</td>
<td>$532,700</td>
</tr>
</tbody>
</table>

Water transferred by hydropumps (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>412</td>
<td>172</td>
<td>6</td>
<td>398</td>
<td>253</td>
<td>403</td>
<td>409</td>
<td>484</td>
<td>679</td>
<td>5,334</td>
</tr>
<tr>
<td>Expanded</td>
<td>763</td>
<td>644</td>
<td>711</td>
<td>419</td>
<td>403</td>
<td>263</td>
<td>247</td>
<td>100</td>
<td>403</td>
<td>153</td>
<td>488</td>
<td>679</td>
<td>5,273</td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Lake Michie and Little River) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>48</td>
<td>257</td>
<td>0</td>
<td>0</td>
<td>402</td>
<td>360</td>
<td>243</td>
<td>0</td>
<td>1,309</td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>152</td>
<td>402</td>
<td>616</td>
<td>239</td>
<td>0</td>
<td>1,409</td>
</tr>
</tbody>
</table>

Water transferred by electric pumps (Brown to Williams) (MGM)

<table>
<thead>
<tr>
<th>Model</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>235</td>
<td>196</td>
<td>45</td>
<td>196</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>672</td>
</tr>
<tr>
<td>Expanded</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>53</td>
<td>196</td>
<td>196</td>
<td>196</td>
<td>0</td>
<td>256</td>
<td>0</td>
<td>0</td>
<td>897</td>
</tr>
</tbody>
</table>

When evaporation and precipitation effects are included, the model projects that an additional 127 MG would be withdrawn from the system. This withdrawal results in slightly less water being transferred by gravity flow and hydropumps for the Expanded Model. The Expanded Model uses additional electric pumping from the reservoirs to make up the difference (about 100 MG). This causes the objective function of the
Expanded Model to be approximately $800 higher than that of the Basic Model. The finished water flows for both models remained identical to those presented in Section 3.5.

The difference in objective function values between the Basic and the Expanded Models indicates that the additional detail does not contribute to the purpose of identifying least-cost pumping options. Therefore, we conclude that evaporation and precipitation effects can be neglected, as in the Basic Model.

4.5 On-Peak Pumping Options at the Water Treatment Plants

The Basic Model formulation included only two electric pumps at each WTP. These pumps are shown in Table 4.11.

Table 4.11  Pumps used in Basic Model formulation for on-peak finished water pumping.

<table>
<thead>
<tr>
<th>Treatment Plant</th>
<th>Pump HP</th>
<th>Pump Capacity (MGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams</td>
<td>400</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>600</td>
<td>14</td>
</tr>
<tr>
<td>Brown</td>
<td>Variable Speed (78%)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Variable Speed (88%)</td>
<td>18</td>
</tr>
</tbody>
</table>

The on-peak demand to be satisfied was represented in the Basic Model as 20 and 26 MGD for the winter and summer months, respectively.

To provide the WRD with guidance for future operation of their water system it will be necessary to evaluate optimal operation for increased on-peak demands. However, when on-peak summer demand exceeds 26 MGD, only one pumping combination is feasible, i.e. use of the 12 and 18 MGD pumps at Williams and Brown, respectively. If the
summer on-peak demand were 28 MGD, this combination would use larger pumps, and thus incur larger demand charges, than necessary. As indicated in Table 2.1, these additional charges could be substantial. Based on this observation, it was determined that additional pumping options should be provided at the water treatment plants.

An alternative formulation which included 3 on-peak pumps at each WTP proved to be computationally undesirable due to the additional 0,1 variables required\(^2\). Subsequently, an Expanded Model was created which included only two pumps at each WTP, but allowed the user to select the pumps according to the on-peak demand to be satisfied. This would permit the evaluation of multiple pump combinations. The utility of this modification is shown in the following example.

The summer on-peak demand for both the Basic and Expanded Models was increased to 28 MGD. The Expanded Model used a modified pumping combination to identify the optimal strategy. This combination is shown in Table 4.12.

Table 4.12 Pumps used in Expanded Model formulation for summer on-peak finished water pumping.

<table>
<thead>
<tr>
<th>WTP</th>
<th>Pump HP</th>
<th>Pump Capacity (MGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams</td>
<td>600</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>16</td>
</tr>
<tr>
<td>Brown</td>
<td>Variable Speed (78%)</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Variable Speed (86%)</td>
<td>17</td>
</tr>
</tbody>
</table>

\(^2\) The formulation required the addition of 36 new 0,1 variables. This increase in binary variables required solve times greater than two hours for reasons discussed in Appendix B. This was deemed unacceptable.
This formulation allowed the Expanded Model to choose between two pumping options as shown below in Table 4.13.

<table>
<thead>
<tr>
<th></th>
<th>Williams WTP (MGD)</th>
<th>Brown WTP Pump (MGD)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Option 1</strong></td>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td><strong>Option 2</strong></td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Results were generated and compared for the Basic and the Expanded Models using the five years of inflow data. The results of these model runs were evaluated based on the differences observed in the objective functions and in the optimal summer on-peak pump flows. These results are shown for 1977 inflow data\textsuperscript{22} in Table 4.14.

<table>
<thead>
<tr>
<th>Model</th>
<th>Williams WTP (MGD)</th>
<th>Brown WTP (MGD)</th>
<th>Obj. Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>12</td>
<td>18</td>
<td>$548,500</td>
</tr>
<tr>
<td>Expanded</td>
<td>16</td>
<td>12</td>
<td>$539,000</td>
</tr>
</tbody>
</table>

As shown, the Expanded Model made use of Option 2, whereas the Basic Model was constrained to use Option 1. The difference between the objective functions is $9,500. However, no evaluation was made for changes in winter on-peak demands. It is probable that winter on-peak demands will also face similar scenarios. In addition, a new 42-inch transmission main from the Brown plant to the distribution network will be completed in the winter of 1997. This may significantly alter the hydraulics, and thus, the energy to capacity ratios of the pumps at the Brown plant. The identification of the new optimal

\textsuperscript{22} Identical optimal finished water pumping strategies were observed for all inflow years.
pumping patterns will require considerable flexibility in evaluating pumping combinations. Given the impact of the modification on the objective function, and its importance in evaluating uncertain future scenarios, it was decided that the level of abstraction contained in the Basic Model was inadequate and additional detail was required.
Chapter 5 Conclusions

5.1 Model Analyses and Conclusions

The MIP model was formulated to aid the WRD in achieving its goal of minimizing pumping costs for the water supply system. Its intended use is to provide information about optimal operation by conducting sensitivity analyses using a range of inflow conditions and observing the changes in optimal pump flows. These changes would be analyzed to identify general least-cost pumping patterns.

The initial formulation of the MIP model was referred to as the Basic Model. Several abstractions were made to represent the system elements in the Basic Model formulation. The task of this report was to determine whether the levels of abstraction contained in the Basic Model were sufficient for its intended use. This was accomplished by two methods:

1) Constraining the model to simulate WRD operation and comparing the pumped flows and resulting costs (model verification).
2) Comparing Basic Model results to those from Expanded Models containing additional detail for specific model elements.
The results of the model verification led to the following conclusions:

- The Basic Model can be constrained to reasonably approximate actual operation.
- The Basic Model accurately describes the costs associated with those pumping options.

The specific model elements for which additional detail was explored were:

1) Gravity flow
2) Hydropumping
3) Evaporation and precipitation
4) Finished water pumping options

The results of these analyses lead to the following conclusions:

- The cost of operating the water system is sensitive to raw water pumping pathways and insensitive to raw water pumping modes.
- The representations used in the Basic Model for raw water pumping options (gravity flow, hydropump, and electric pumps) are sufficient for the intended use of the model.
- The representation used in the Basic Model for finished water pumping options may be insufficient to identify least-cost pump flows for uncertain future demands.

In addition to these conclusions, the optimal finished water flows for each analysis were identical. This suggests that these flows constitute an optimal pumping pattern that is insensitive to inflows. Based on comparison with WRD 1994 operating costs, the use of this optimal pumping pattern could result in significant savings.
References


Appendix A
Duke Power Schedule
SCHEDULE OPT (NC)
OPTIONAL POWER SERVICE, TIME-OF-USE

AVAILABILITY (North Carolina Only)
Available to the individual customer:
Service under this Schedule shall be used solely by the contracting Customer in a single enterprise, located entirely on a single, contiguous premises.
This Schedule is not available to the individual customer who qualifies for a residential schedule, nor for auxiliary or breakdown service. Power delivered under this schedule shall not be used for resale or exchange or in parallel with other electric power or as a substitute for power contracted for or which may be contracted for, under any other schedule of the Company, except at the option of the Company, under special terms and conditions expressed in writing in the contract with the Customer.
The obligations of the Company in regard to supplying power are dependent upon its securing and retaining all necessary rights-of-way, privileges, franchises and permits, for the delivery of such power. The Company shall not be liable to any customer or applicant for power in the event it is delayed in, or is prevented from furnishing the power by its failure to secure and retain such rights-of-way, rights, privileges, franchises and permits.

TYPE OF SERVICE
The Company will furnish 60 Hertz service through one meter at one delivery point, at one of the following approximate voltages, where available:
- Single-phase, 120/240 volts, or
- 3-phase, 208Y/120 volts, 240Y/138 volts, 480Y/277 volts; or
- 3-phase, 3-wire, 240, 480, 573, or 2300 volts, or
- 3-phase, 4160Y/2400, 12470Y/7200, or 24940Y/14400 volts; or
- 3-phase voltages other than those listed above may be at the Company's option if the size of the Customer's contract warrants a substation solely to serve the Customer and if the Customer furnishes suitable outdoor space on the premises to accommodate a ground-type transformer installation, or substation, or a transformer vault built in accordance with the Company’s specifications.
The type of service supplied will depend upon the voltage available. Prospective customers should determine the available voltage by contacting the nearest office of the Company before purchasing equipment.
Motors of less than 5 H.P. may be single-phase. All motors of more than 5 H.P. must be equipped with starting compensators. The Company reserves the right, when in its opinion the installation would not be detrimental to the service of the Company, to permit other types of motors.

RATE:
I. Basic Facilities Charge $56.07
II. Demand Charge
A. On-Peak Demand Charge
For the first 2000 KW of Billing Demand per month
For the next 3000 KW of Billing Demand per month
For all over 5000 KW of Billing Demand per month
B. Economy Demand Charge
III. Energy Charge
A. All On-Peak Energy per month 4.201 cents per Kwh
B. All Off-Peak Energy per month 2.0647 cents per Kwh

DETERMINATION OF ON-PEAK AND OFF-PEAK HOURS

<table>
<thead>
<tr>
<th>Summer Months</th>
<th>Winter Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>June - September</td>
<td>October - May</td>
</tr>
<tr>
<td>1:00 p.m. - 9:00 p.m.</td>
<td>6:00 a.m. - 12:00 p.m.</td>
</tr>
<tr>
<td>Monday - Friday</td>
<td>Monday - Friday</td>
</tr>
</tbody>
</table>

On-Peak Period Hours: All other weekday hours and all Saturday and Sunday hours.
All hours for the following holidays shall be considered as Off-Peak: New Year's Day, Memorial Day, Good Friday, Independence Day, Labor Day, Thanksgiving Day, Day after Thanksgiving, and Christmas Day.
DEFINITION OF "MONTH"
The term "month" as used in this Schedule means the period intervening between meter readings for the purpose of monthly billings. Readings are taken each month in intervals of approximately thirty (30) days.

Summer and winter on-peak hour rates are applied to the monthly bills as follows:

For customers who receive end of month bills and for customers in the later billing cycles 13-20 and 33-40, summer months rates apply to the billing months of June through September. Winter months rates apply to the billing months of October through May.

For customers in billing cycles 1-12 and 21-34, summer months rates apply to the billing months of July through October. Winter months rates apply to the billing months of November through June.

CONTRACT DEMAND
The Company will require contracts to specify the maximum demand to be delivered to the Customer which shall be the Contract Demand.

Where the Customer can restrict on-peak demand to levels considerably below that of the Contract Demand, the Company may also contract for a limited On-Peak Contract Demand in addition to the Contract Demand.

DETERMINATION OF BILLING DEMAND
A. The On-Peak Billing Demand each month shall be the largest of the following:
   1. The maximum integrated thirty-minute demand during the on-peak period during the month for which the bill is rendered
   2. Fifty percent (50%) of the Contract Demand (or 50% of the On-Peak Contract Demand if such is specified in the contract)
   3. 15 kilowatts (KW)

B. Economy Demand
   To determine the Economy Demand, the larger of
   1. The maximum integrated thirty-minute demand during the month for which the bill is rendered; or
   2. 50% of the Contract Demand
   shall be compared to the On-Peak Billing Demand as determined in A. above. If the demand determined by the larger of B.1. and B.2. above exceeds the On-Peak Billing Demand, the difference shall be the Economy Demand.

MINIMUM BILL
The minimum bill shall be the bill calculated on the Rate above including the Basic Facility Charge, Demand Charge, and Energy Charge, but the sum of the On-Peak Demand Charge and the Economy Demand Charge shall not be less than $1.64 per month per KW of Contract Demand. If the Customer's measured demand exceeds the Contract Demand, the Company may at any time establish the minimum based on the maximum integrated demand in the previous twelve months including the month for which the bill is rendered.

POWER FACTOR CORRECTION
When the average monthly power factor of the Customer's power requirements is less than 85 percent, the Company may correct the integrated demand in kilowatts for that month by multiplying by 85 percent and dividing by the average power factor in percent for that month.

PAYMENT
Bills under this Schedule are due and payable on the date of the bill at the office of the Company. Bills are past due and delinquent on the fifteenth day after the date of the bill. If any bill is not so paid, the Company has the right to suspend service. In addition, all bills not paid by the twenty-fifth day after the date of the bill shall be subject to a one percent (1%) late payment charge on the unpaid amount. This late payment charge shall be rendered on the following month's bill and it shall become part of and be due and payable with the bill on which it is rendered.

CONTRACT PERIOD
Each customer shall enter into a contract to purchase electricity from the Company for a minimum original term of one (1) year and thereafter from year to year upon the condition that either party can terminate the contract at the end of the original term, or at any time thereafter by giving at least sixty (60) days' previous notice of such termination in writing; but the Company may require a contract for a longer original term of years where the requirement is justified by the circumstances.
FUEL COST ADJUSTMENT RIDER

APPLICABILITY (North Carolina Only)
Service supplied under the Company's rate schedules are subject to approved fuel charge adjustments, if any, over or under the rate set forth in the approved rate schedules. Adjustments are made pursuant to North Carolina General Statute 52-133.2 as ordered by the North Carolina Utilities Commission.

APPROVED FUEL CHARGE ADJUSTMENTS
The Commission has ordered, effective for service rendered on and after July 1, 1995 that such an adjustment shall be included in the Company's rate schedules and an appropriate fuel cost adjustment rider be filed together with the rate schedules to implement the changes to the fuel component.
Effective July 1, 1995 the Approved Fuel Cost was ordered decreased from the 1.1032e/kwh Base Fuel Cost established November 12, 1991 in the general rate case Docket No. E-7, Sub 487 to 1.0400e/kwh.

APPROVED FUEL COST
The Approved Fuel Cost of 1.0400e/kwh will remain in effect until a new cost of fuel is established by the Commission.

EXPERIENCE MODIFICATION FACTOR
An Experience Modification Factor (EMF) decrement of -.0923e/kwh and an EMF interest decrement of -.0138e/kwh are effective for service on and after July 1, 1995 and will continue until June 30, 1996.

The following shows the calculation of the Approved Fuel Charge Adjustments:

<table>
<thead>
<tr>
<th>Fuel Cost Effective</th>
<th>Fuel Cost Effective</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 1, 1994, E-7, Sub 540</td>
<td>July 1, 1995, E-7, Sub 559</td>
</tr>
<tr>
<td>Base Fuel Cost Effective November 12, 1991</td>
<td>1.1032e/kwh</td>
</tr>
<tr>
<td>Docket No. E-7, Sub 487</td>
<td></td>
</tr>
<tr>
<td>Adjustment to Base Fuel Cost</td>
<td>-.0307e/kwh</td>
</tr>
<tr>
<td>Approved Fuel Cost</td>
<td>1.0725e/kwh</td>
</tr>
<tr>
<td>Experience Modification Factor</td>
<td>-.0681e/kwh</td>
</tr>
<tr>
<td>Experience Modification Factor Interest</td>
<td>-.0102e/kwh</td>
</tr>
<tr>
<td>Net Fuel Factor</td>
<td>.9942e/kwh</td>
</tr>
<tr>
<td>Change in Net Fuel Factor</td>
<td></td>
</tr>
<tr>
<td>Gross Receipts Tax Multiplier</td>
<td></td>
</tr>
<tr>
<td>Change in Rates</td>
<td></td>
</tr>
</tbody>
</table>

EFFECT ON RATES
As a result of the Commission's June 7, 1995 order in Docket No. E-7, Sub 559, the Approved Fuel Cost, Experience Modification Factor, and EMF interest are included in the current rate schedules effective for service on and after July 1, 1995. The effect of the Commission's order including its impact on the Company's gross receipts tax expense is a decrease in all rate schedules of .0623e/kwh as compared to the rates in effect immediately prior to July 1, 1995.

USE OF RIDER
Since adjustments are already included in the Rates of the Company's current rate schedules which are effective for service on and after July 1, 1995, this Rider should not be used in addition to such rate schedules for bill calculations.
Appendix B
MIP Models
MIP models include decision variables, parameters, constraints and an objective function. The decision variables are associated with the components of the system over which the user has control. For the Durham MIP most of these variables will represent the amount of water pumped from one location to another using a specific pump, during a specific time period. Variables representing reservoir storage will also be included. These are the values for which the model will solve.

System parameters are the exogenous variables specified by the user. For the Durham system, they include such things as monthly inflows, pump capacities, costs, maximum working volumes, etc.

The system constraints consist of those conditions which the system must meet. The Durham MIP contains six basic types of constraints:

1) Continuity constraints
2) Demand constraints
3) Capacity constraints
4) Gravity flow constraints
5) Fixed cost constraints
6) Minimum flow constraints
7) Mutual exclusivity constraints
8) Hydrowaste definition constraints

The objective function of the Durham MIP model represents the costs or benefits associated with each unit increase in the decision variables. It is this function which the model will attempt to minimize or maximize. In the case of the Durham MIP model the objective function contains the costs associated with transferring a unit of water from one location to another by specific pathways using specific pumping modes. This function is
of primary importance since all decision variable values will be determined based on their contribution to the objective function value.

The use of 0,1 variables distinguishes an MIP model from an LP model. These variables are used when it is desirable to constrain a variable to have an "either/or" value (e.g. on/off, yes/no). Typically constraints are written such that the 0,1 variable must take on a specific value for a given set of conditions. Thus, an MIP model can represent the scenario where a system component must be turned off when other system components reach threshold values. This makes MIP models very powerful for applications where these types of system interactions exist. However, unlike LP models, the time required for the solution of an MIP model increases exponentially with an increasing number of 0,1 variables. This is due to the "branch and bound" algorithm which is used to solve MIP problems. This algorithm first solves the problem allowing all variables to be continuous. This solution provides an upper (or lower) bound for the optimal objective value. The algorithm then solves a set of sub-problems created by allowing the 0,1 variables to take on integer values. Since the variables will only take on integer values one at a time, the number of sub-problems increases exponentially. Thus, it is clearly optimal, in terms of solution time, to include as few 0,1 variables as possible.
Appendix C
Durham MIP Model Formulation:
General Form
Decision Variables

A total of 452 decision variables were used in the formulation of the model. These variables represent decisions that the WRD can make when operating their water supply system (e.g. how much water to send from Lake Michie to the Williams treatment plant by hydropumps in the month of January). These are the variables for which the model will solve.

System Inputs

The system inputs are the exogenous variables specified by the user. In this formulation the monthly inflows, demands, and maximum working volume are the primary system inputs. Values for these inputs may be chosen as the user desires.

Objective Function

The objective function contains the electrical costs associated with operating the Durham water supply system. These costs are broken down into two primary categories; fixed costs and marginal costs. The marginal costs are based on the actual cost of electricity for transferring a unit of water from one location to another by a specific pathway. The fixed costs are based on the kW draw of the pumps used. There are also fixed costs associated with contract demands. These costs, as well as other background electrical costs, are relatively constant from year to year. Thus, they were included in the objective function as required costs of operation.
Constraints

The Durham MIP contains six basic types of constraints:

1) Continuity constraints
2) Demand constraints
3) Capacity constraints
4) Gravity flow constraints
5) Fixed cost constraints
6) Minimum flow constraints
7) Mutual exclusivity constraints
8) Hydrowaste definition constraints

Since monthly time steps were used in the formulation, each system constraint is written 12 times. The following discussion will give the general form of these constraints.

Continuity Constraints

Continuity constraints were written for each node\(^2\) of the Durham water supply system. These constraints were written as mass balances between storage, inflow, and outflow. The constraints took the general form of:

\[
\text{Storage in month } t = \text{Storage in month } t-1 + \text{Inflow in month } t - \text{Outflow in month } t
\]

The storage values indicate the amount of water in storage at the end of the time period shown (e.g. Storage\(_1\) is the amount of water in storage at the end of January). Since the terminal reservoirs for the Williams and Brown treatment plants were not included in the model, there is no storage at these nodes and the constraints simplify to:

\(\text{Storage in month } t = \text{Storage in month } t-1\)

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\(^1\) Examples include lighting and heating for the personnel offices.

\(^2\) The nodes include Lake Michie reservoir, Little River reservoir, Williams treatment plant, and Brown treatment plant.
Inflow in month \( t \) = Outflow in month \( t \)

Demand Constraints

Two types of demand constraints were included: overall monthly demands and monthly on peak demands. The overall monthly demand constraints ensures that the amount of finished water leaving both treatment plants is at least equal to the total monthly demand (MG). These constraints are written as:

\[
\text{Total demand in month } t = \text{Amount of water pumped from Williams WTP in month } t + \text{Amount of water pumped from the Brown WTP in month } t
\]

Since the demand represents an overall value, the constraints include the water pumped on and off peak from the water treatment plants.

The monthly on peak demand constraints require that the model solution provide a minimum specified amount of water during on peak hours. As discussed previously, there are approximately 7 days worth of on peak hours during any month. Thus, the model must meet a specified demand for those 7 days. This demand is also expressed in MG. The constraint is written the same as the overall monthly demand constraint, however, it only includes the finished water pumping which occurs on peak.

\[
\text{On peak demand in month } t = \text{Amount of water pumped from Williams on peak in month } t + \text{Amount of water pumped from Brown on peak in month } t
\]
Capacity Constraints

The capacity constraints represent the capacity limitation of system components. Treatment capacity limitations exist at the Williams and Brown treatment plants. The Williams treatment plant can treat at an approximate rate of 22 MGD. Thus the constraints for the Williams plant are written as:

\[
\text{Amount of water pumped from the Williams plant in month } t \leq 660 \text{ MG (22 MGD x 30 days)}
\]

Similar constraints were included for the Brown plant which can treat water at a rate of 30 MGD.

Gravity Flow Constraints

Water can be transferred by gravity flow at the Little River reservoir when the reservoir elevation maintains a hydraulic head of at least 1 foot relative to the terminal reservoir at the Brown treatment plant. This elevation translates to a drawdown of 990 MG. Thus, constraints were needed to ensure that the model did not allow gravity flow below this level. These constraints used a 0,1 variable to model the can flow/can’t flow condition. The primary gravity flow constraint is given below.

\[
\alpha (sr^{M_{t-1}} - 990) \leq \alpha sr_{t-1} + X_t \leq 1 + \alpha (sr^{M_{t-1}} - 990)
\]
Where:  
\[ \alpha \quad = \quad 0.0001 \]
\[ sr_{t-1}^{M} \quad = \quad \text{Maximum working volume for the Little River reservoir in month } t-1 \]
\[ sr_{t-1} \quad = \quad \text{Storage in the Little River reservoir at the end of month } t-1 \]
\[ X_{t} \quad = \quad 0,1 \text{ variable such that: } \begin{cases} X_{t} = 0, \text{ gravity flow can be used} \\ X_{t} = 1, \text{ gravity flow cannot be used} \end{cases} \]

This constraint connects the ability to use gravity flow with the storage value\(^3\) observed at the end of the previous month. If a maximum working volume of 3250 MG\(^4\) is chosen for the reservoir, the critical level, below which gravity flow cannot occur, becomes 2260 MG. The \(\alpha\) value chosen for the formulation was sufficiently small to force \(X_{t}\) to be 1 when the storage value at the end of the previous month is less than 2260. Suppose that the storage value at the end of month \(t-1\) were 2500. The constraint would be:

\[ \alpha(2260) \leq \alpha 2500 + X_{t} \leq 1 + \alpha(2260) \]

If we let \(\alpha = 0.0001\) we have:

\[ 0.226 \leq 0.25 + X_{t} \leq 1.226 \]

Under these conditions the value of \(X_{t}\) can only be 0, i.e. gravity flow can be used. This is accurate since the storage value at the end of month \(t-1\) exceeds the critical storage level of 2260 MG. Suppose that the storage value at the end of month \(t-1\) were 2000 MG. The constraint would then be:

\[ 0.226 \leq 0.2 + X_{t} \leq 1.226 \]

\(^3\) Working volumes are used as a surrogate for reservoir elevations.

\(^4\) This value represents the maximum working volume that can be chosen for the Little River reservoir.
Under these conditions the value of \( X_t \) can only be 1, i.e. gravity flow cannot be used. The 0.1 variable, \( X_t \), is connected to the actual on peak gravity flow variables by the following constraint:

\[
Grbp_t + 1500X_t \leq 1500
\]

Where: \( Grbp_t \) = The amount of water transferred by gravity flow on peak in month \( t \)

Thus, when \( X_t = 1 \), \( Grbp_t \) must be equal to 0, i.e. no water can be transferred by gravity flow. When \( X_t = 0 \), up to 1500 MG\(^5\) of water can be transferred by gravity flow. A similar constraint was written for the off peak condition.

Fixed Cost Constraints

Fixed cost constraints use 0.1 variables to model the fixed cost pricing schedule under which the WRD operates. These fixed costs are assessed based on whether or not a specific pump is used at any time during the billing period. The magnitude of the cost is based on the kW draw of the pump. Therefore, these constraints allow the model to assign a fixed cost based on the kW draw of the specific pumps at the water treatment plants\(^6\). Only two pumping options were included at each treatment plant. The available pumps for both treatment plants consist of a small capacity pump and a large capacity pump. The fixed cost for the small capacity pump has already been included as an unavoidable cost since no smaller pump can be used. Thus, the fixed costs apply only to

---

\(^5\) Due to other constraints in the model, the gravity line will never be able to transfer 1500 MG of water. Any number in excess of the maximum gravity flow capacity (1350 MGM) could have been used.

\(^6\) The fixed costs associated with the electric raw water reservoir pumps were not included since they are not used in practice.
the larger pumps. The fixed cost constraint for turning on the large pump on peak at the Williams treatment plant are shown below.

\[-\alpha rwhidp_t + Fwp_t \geq 0\]

Where:  
\(\alpha = 0.001\)  
\(rwhidp_t\) = The amount of water transferred from Williams to demand on peak with the large pump  
\(Fwp_t\) = 0,1 variable where:  
\(Fwp_t = 0\), large pump is not used on peak  
\(Fwp_t = 1\), large pump is used on peak

Thus, when any amount of water is pumped using the larger pump at Williams (\(Fwp_t = 1\)) the cost is assessed. The 0,1 variable is also included in the objective function with a coefficient equal to the actual fixed cost of using the specific pump. This ensures that when the pump is used, the appropriate fixed cost will be assessed. Similar constraints have been written for on/off peak conditions at the Brown plant.

Minimum Flow Constraints

The Lake Michie and Little River reservoirs have minimum instream flow requirements. They must discharge at least 9 MGM and 6 MGM, respectively. Constraints were written which account for the fact that these flow requirements may be met by spillage or wastage from the hydropumps. The constraints take the form of:

\[
\text{Spillage in month } t + \text{Hydrowaste in month } t \geq \text{Minimum flow requirement}
\]

Mutual Exclusivity Constraints
The pumping stations at the reservoirs and treatment plants are generally\(^7\) confined to using only one pump and one pathway at a time. Thus, constraints were needed to represent this condition. The mutual exclusivity constraints are based on the amount of time, in days, for which water is transferred by a specific pump along a specific pathway. Separate constraints for on/off peak pumping were required. The general constraint for on peak pumping for a specific pathway during any month is given below.

\[
\text{Amount of time pump A is used} + \text{Amount of time pump B is used} + \text{Amount of time pump C is used} \leq 7 \text{ days (on peak time)}
\]

In the actual constraints the amount of time a pump is used is obtained by dividing the variable which accounts for the amount of water transferred along a specific pathway by its capacity. The constraint shown ensures that the model will not produce a solution that requires on peak pumping combinations that exceed the capacity of the physical system. Similar constraints were written to account for the on/off peak conditions at both reservoirs and treatment plants.

Besides the problem of ensuring that the model accurately reflects the upper bound of pumping capacities, the Williams and Brown treatment plants also have a lower bound which must be represented. The pumping stations adjacent to the Williams and Brown treatment plants must maintain minimum flows due to the pressure characteristics of the distribution network. Since we have modeled the pumps as if they were attached to the treatment plants we may say that the treatment plants themselves must treat at some

\(^7\) The Lake Michie pumping station can actually pump along a single pathway using two hydropumps.
minimum rate. Therefore, the Williams plant must maintain a flow of at least 8 MGD and the Brown plant must maintain a flow of at least 12 MGD at all times. To ensure this condition mutual exclusivity constraints were used. Both on and off peak constraints were needed. The general form for the on peak constraints is shown below

\[
\frac{\text{Amount of time pump } A}{\text{is used}} + \frac{\text{Amount of time pump } B}{\text{is used}} \geq 7 \text{ days (on peak time)}
\]

This constraint, combined with the previous mutual exclusivity constraints, ensures that the treatment plants will maintain a minimum flow at least equal to the minimum capacity of the two pumps. Since we have included the 8 MGD and the 12 MGD pumps as the smallest available pumps at Williams and Brown, respectively, the minimum flow conditions are satisfied.

Hydrowaste Definition Constraints

These constraints were used to account for the water used to drive the hydropump turbines. The constraints define the magnitude of the hydrowaste based on the water transferred by the hydropumps using the wastage ratio previously determined. The general form of the constraints is shown below.

\[
2.5 \times \frac{\text{Amount of water transferred by hydro pumps on peak}}{} + 2.5 \times \frac{\text{Amount of water transferred by hydropumps off peak}}{} = \frac{\text{Total amount of water wasted by hydropumps}}{}
\]

Constraints were written for the both Lake Michie and Little River.

However, since this application is uncommon, it was not modeled.
Appendix D
Durham MIP Model Formulation:
Variables and Equations
Definition of Variables:

All variables denoting transfer of water have units of MG and indicate the amount of water transferred in month t.

\[ q_{mwp_t} = \text{water transferred by hydropump from Lake Michie to the Williams WTP on peak} \]
\[ q_{mwop_t} = \text{water transferred by hydropump from Lake Michie to the Williams WTP off peak} \]
\[ q_{mbp_t} = \text{water transferred by hydropump from Lake Michie to the Brown WTP on peak} \]
\[ q_{mbop_t} = \text{water transferred by hydropump from Lake Michie to the Brown WTP off peak} \]
\[ q_{rwp_t} = \text{water transferred by hydropump from Little River to the Williams WTP on peak} \]
\[ q_{rwop_t} = \text{water transferred by hydropump from Little River to the Williams WTP off peak} \]
\[ r_{mwp_t} = \text{water transferred by electric pumping from Lake Michie to the Williams WTP on peak} \]
\[ r_{mwop_t} = \text{water transferred by electric pumping from Lake Michie to the Williams WTP off peak} \]
\[ r_{mbp_t} = \text{water transferred by electric pumping from Lake Michie to the Brown WTP on peak} \]
\[ r_{mbop_t} = \text{water transferred by electric pumping from Lake Michie to the Brown WTP off peak} \]
\[ r_{rbp_t} = \text{water transferred by electric pumping from Little River to the Brown WTP on peak} \]
\[ r_{rbop_t} = \text{water transferred by electric pumping from Little River to the Brown WTP off peak} \]
\[ r_{lowwdp_t} = \text{water transferred by low capacity pump from the Williams WTP to demand on peak} \]
\[ r_{lowwdop_t} = \text{water transferred by low capacity pump from the Williams WTP to demand off peak} \]
\[ r_{highwdp_t} = \text{water transferred by high capacity pump from the Williams WTP to demand on peak} \]
\[ r_{highwdop_t} = \text{water transferred by high capacity pump from the Williams WTP to demand off peak} \]
\[ r_{lowbdp_t} = \text{water transferred by low capacity pump from the Brown WTP to demand on peak} \]
\[ r_{lowbdop_t} = \text{water transferred by low capacity pump from the Brown WTP to demand off peak} \]
\[ r_{highbdp_t} = \text{water transferred by high capacity pump from the Brown WTP to demand on peak} \]
\[ r_{highbdop_t} = \text{water transferred by high capacity pump from the Brown WTP to demand off peak} \]
$\text{gaswdp}_t$ = water transferred by gas pump from the Williams WTP to demand on peak
$\text{gaswdop}_t$ = water transferred by gas pump from the Williams WTP to demand off peak
$\text{diesbdp}_t$ = water transferred by diesel pump from the Brown WTP to demand on peak
$\text{diesbdop}_t$ = water transferred by diesel pump from the Brown WTP to demand off peak
$\text{rbwp}_t$ = water transferred by electric pumping from the Brown WTP to the Williams WTP on peak (never use this)
$\text{rbwop}_t$ = water transferred by electric pumping from the Brown WTP to the Williams WTP off peak
$\text{Grbhip}_t$ = water transferred by high capacity gravity flow from Little River to the Brown WTP on peak
$\text{Grbhiop}_t$ = water transferred by high capacity gravity flow from Little River to the Brown WTP off peak
$\text{Grblop}_t$ = water transferred by low capacity gravity flow from Little River to the Brown WTP on peak
$\text{Grbloop}_t$ = water transferred by low capacity gravity flow from Little River to the Brown WTP off peak
$\text{sm}_t$ = storage (working volume) in Lake Michie at the end of month $t$
$\text{sr}_t$ = storage (working volume) in Little River at the end of month $t$
$\text{Em}_t$ = spillage from Lake Michie during month $t$
$\text{Er}_t$ = spillage from Little River during month $t$
$\text{wm}_t$ = hydro waste flow from Lake Michie in month $t$
$\text{wr}_t$ = hydro waste flow from Little River in month $t$

$[\text{Ghi}_t]$ = 0,1 variable such that:
when $= 1$, high capacity gravity flow can be used
$= 0$, high capacity gravity flow cannot be used

$[\text{X}_t]$ = 0,1 variable such that:
when $= 1$, gravity flow can be used
$= 0$, gravity flow cannot be used

$[\text{Fbp}_t]$ = 0,1 variable such that:
when $= 1$, high capacity pump at Brown WTP is used on peak
$= 0$, high capacity pump at Brown WTP is not used on peak

$[\text{Fbop}_t]$ = 0,1 variable such that:
when $= 1$, high capacity pump at Brown WTP is used off peak
$= 0$, high capacity pump at Brown WTP is not used off peak
\[[Fwp_t]\] = 0,1 variable such that:

- when \( = 1\), high capacity pump at Williams WTP is used on peak
- \( = 0\), high capacity pump at Williams WTP is not used on peak

- net evaporation at Lake Michie occurring in month \(t\)
- net evaporation at Little River occurring in month \(t\)
- 0,1 variable forced \( = 1\) to incur a base fixed cost of operation

System Parameters:

- \(D_t\) = Consumer demand in month \(t\)
- \(S_M^t\) = Maximum allowable working volume for Lake Michie in month \(t\)
- \(S_R^t\) = Maximum allowable working volume for Little River in month \(t\)
- \(I_M^t\) = Inflow into Lake Michie in month \(t\)
- \(I_R^t\) = Inflow into Little River in month \(t\)

Objective Function:

\[
\text{Fixed} + 700 \sum_{t=1}^{12} F_{bop} + 3100 \sum_{t=1}^{5} F_{bp} + 5600 \sum_{t=6}^{9} F_{bp} + 3100 \sum_{t=10}^{12} F_{wp} + 1000 \sum_{t=1}^{5} F_{wp} + 1800 \sum_{t=6}^{9} F_{wp} + 1000 \sum_{t=10}^{12} F_{wp} \\
+ 22 \sum_{t=1}^{12} m_{wp} + 11 \sum_{t=1}^{12} m_{wp} + 13 \sum_{t=1}^{12} m_{bp} + 6 \sum_{t=1}^{12} m_{bp} + 3 \sum_{t=1}^{12} r_{bp} + 2 \sum_{t=1}^{12} r_{bp} + 19 \sum_{t=1}^{12} r_{bw} + 10 \sum_{t=1}^{12} r_{bw} \\
+ 38 \sum_{t=1}^{12} r_{lowd} + 38 \sum_{t=1}^{12} r_{lowd} + 19 \sum_{t=1}^{12} r_{hiwp} + 18 \sum_{t=1}^{12} r_{hiwp} + 49 \sum_{t=1}^{12} r_{obdp} + 57 \sum_{t=1}^{12} r_{obdp} \\
+ 24 \sum_{t=1}^{12} r_{obdp} + 58 \sum_{t=1}^{12} r_{obdp} + 500 \sum_{t=1}^{12} g_{sw}\text{dpt} + 500 \sum_{t=1}^{12} g_{sw}\text{dpt} - 0.0001 \sum_{t=1}^{12} sm - 0.0001 \sum_{t=1}^{12} sw
\]

Constraints

Continuity Constraints

Continuity at Lake Michie:

\(q_{wp_t} + q_{mwop_t} + q_{mbop_t} + r_{wp_t} + r_{mwop_t} + r_{mbop_t} + r_{mbop_t} + n_{elm_t} + s_{m_t} - s_{m_{t-1}} + E_{m_t} + w_{m_t} = I_{M_t}^t\) for all \(t\)

Continuity at Little River:

\(q_{rp_t} + q_{rwop_t} + r_{rbp_t} + r_{rbop_t} + G_{rhp_t} + G_{rhop_t} + G_{rboop_t} + G_{rboop_t} + n_{elr_t} + s_{r_t} - s_{r_{t-1}} + E_{r_t} + w_{r_t} = I_{R_t}^t\) for all \(t\)
Continuity at Williams Treatment Plant:

\[ r\text{lowdp}_t + r\text{hiwdp}_t + r\text{lowdp}_t + r\text{hiwdp}_t + \text{gaswdp}_t + \text{gaswdp}_t - q\text{mwp}_t - q\text{mwop}_t - q\text{rwp}_t - q\text{rwp}_t - r\text{bwop}_t - r\text{bwop}_t - r\text{mwp}_t - r\text{mwop}_t = 0 \text{ for all } t \]

Continuity at Brown Treatment Plant:

\[ r\text{bwp}_t + r\text{bwop}_t + r\text{lobdp}_t + r\text{hibdp}_t + r\text{lobdp}_t + r\text{hibdp}_t + \text{diesbdp}_t + \text{diesbdp}_t - q\text{mbp}_t - q\text{mbp}_t - r\text{mbp}_t - r\text{mbp}_t - r\text{rbp}_t - r\text{rbp}_t - \text{Grbhip}_t - \text{Grbhiop}_t - \text{Grblop}_t - \text{Grbloop}_t = 0 \text{ for all } t \]

**Demand Constraints**

**Monthly Demands**

\[ r\text{lowdp}_t + r\text{hiwdp}_t + \text{gasdp}_t + r\text{lowdp}_t + r\text{hiwdp}_t + \text{gaswdp}_t + r\text{lobdp}_t + r\text{hibdp}_t + \text{diesbdp}_t + r\text{lobdp}_t + r\text{hibdp}_t + \text{diesbdp}_t \geq D_t \]

**On-Peak Demands**

\[ r\text{hiwdp}_t + r\text{hiwdp}_t + \text{gasdp}_t + r\text{hibdp}_t + r\text{hibdp}_t + \text{diesdp}_t \geq D_{pw} \text{ for } t = 1-5 \text{ and } t = 10-12 \]

\[ r\text{hiwdp}_t + r\text{hiwdp}_t + \text{gasdp}_t + r\text{hibdp}_t + r\text{hibdp}_t + \text{diesbdp}_t \geq D_{ps} \text{ for } t = 6-9 \]

**Capacity Constraints**

**Treatment Capacity at Williams Treatment Plant:**

**On Peak**

\[ r\text{lowdp}_t + r\text{hiwdp}_t + \text{gasdp}_t \leq 154 \text{ MG for all } t \]

**Off Peak**

\[ r\text{lowdp}_t + r\text{hiwdp}_t + \text{gasdp}_t \leq 506 \text{ MG for all } t \]

**Treatment Capacity at Brown Treatment Plant:**

**On Peak**

\[ r\text{lobdp}_t + r\text{hibdp}_t + \text{diesbdp}_t \leq 210 \text{ MG for all } t \]
Off Peak
rlobdop_t + rhbdep_t + diesdop_t ≤ 690 MG for all t

Gravity Flow Constraints

Critical Level at Little River
0.0001sr^M - 0.990 ≤ 0.0001sr_{t-1} + X_t ≤ 1 + 0.0001sr^M - 0.990 for all t

Critical Level for High Capacity Gravity Flow at Little River
-1 +0.001(sr^M_{t-541}) ≤ .0005sr_t + .0005sr_{t-1} - GHI_t

Activation of High Capacity Gravity Flow at Little River
0.0001Grbhipt + 0.0001Grbhiop_t ≤ GHI_t for all t
GrbHIpt + GrbHIop_t + GrbLOpt + GrbLOop_t + 1700[X_t] ≤ 1700 for all t

Fixed Cost Constraints

Fixed Costs at Williams Treatment Plant (on peak only):
Fwp_t ≥ 0.001rhiwdpt for all t

Fixed Costs at Brown Treatment Plant:

On Peak
Fbp_t ≥ 0.001rhbdpt for all t

Off Peak
Fbop_t ≥ 0.001 rhbdpt for all t

Minimum Flow Constraints

Minimum Flow Below Lake Michie
Em_t + wm_t ≥ 9 MG for all t
Minimum Flow Below Little River

$E_{r_t} + w_{r_t} \geq 6 \text{ MG} \text{ for all } t$

Minimum Flow at Williams Treatment Plant:

On Peak

$(1/C_{rlowdp})rlowdp_t + (1/C_{rhiwdp})rhiwdp_t + 1/(C_{gaswdp})gaswdp_t \geq 7 \text{ days for all } t$

Off Peak

$(1/C_{rlowdop})rlowdop_t + (1/C_{rhidwop})rhiwdop_t + 1/(C_{gaswdop})gaswdop_t \geq 23 \text{ days for all } t$

Minimum Flow at Brown Treatment Plant:

On Peak

$(1/C_{rlodbp})rlodbp_t + (1/C_{rhibdp})rhibdp_t + 1/(C_{diesbdp})diesbdp_t \geq 7 \text{ days for all } t$

Off Peak

$(1/C_{rlodbop})rlodbop_t + (1/C_{rhbobp})rhbdop_t + 1/(C_{diesbdop})diesbdop_t \geq 23 \text{ days for all } t$

Mutual Exclusivity Constraints

Mutual Exclusivity at Lake Michie:

On Peak

$(1/C_{qmwp})qmwp_t + (1/C_{qmbp})qmbp_t + (1/C_{rmwp})rmwp_t + (1/C_{rmbp})rmbp_t \geq 7 \text{ days for all } t$

Off Peak

$(1/C_{qmwp})qmwp_t + (1/C_{qmbp})qmbp_t + (1/C_{rmwp})rmwp_t + (1/C_{rmbp})rmbp_t \geq 23 \text{ days for all } t$

Mutual Exclusivity at Little River:

On Peak

$(1/C_{qrwp})qrwp_t + (1/C_{rrbp})rrbp_t + (1/C_{Grbhp})Grbhp_t + (1/C_{Grblop})Grblop_t \leq 7 \text{ days for all } t$
Off Peak

\[ (1/C_{qrwp_t}) + (1/C_{rbwp_t}) + (1/C_{Grbhiop_t}) + (1/C_{Grbloop_t}) \leq 23 \text{ days for all } t \]

Mutual Exclusivity at Williams Treatment Plant:

On Peak

\[ (1/C_{rlowdp_t}) + (1/C_{rhiwp_t}) + (1/C_{gaswdp_t}) \leq 7 \text{ days for all } t \]

Off Peak

\[ (1/C_{rlowdp_t}) + (1/C_{rhiwp_t}) + (1/C_{gaswdp_t}) \leq 23 \text{ days for all } t \]

Mutual Exclusivity at Brown Treatment Plant:

On Peak

\[ (1/C_{rlobdp_t}) + (1/C_{rhibdp_t}) + (1/C_{diesbdp_t}) \leq 7 \text{ days for all } t \]

Off Peak

\[ (1/C_{rlobdp_t}) + (1/C_{rhibdp_t}) + (1/C_{diesbdp_t}) \leq 23 \text{ days for all } t \]

Hydrowaste Definition Constraints

Hydrowaste Definition at Lake Michie

\[ \frac{wm_i}{R_m} - qmwp_t - qmwop_t - qmbp_t - qmbop_t = 0 \text{ for all } t \]

Hydrowaste Definition at Little River

\[ \frac{wrt}{R_t} - qrwp_t - qrwop_t = 0 \text{ for all } t \]

Evaporation Constraints

Evaporation at Lake Michie:

\[ 0.00353G_t + nelm_t = -4.176G_t \text{ for all } t \]

Evaporation at Little River:

\[ 0.00231G_t + nelr_t = -7.52G_t \text{ for all } t \]
Appendix E
Hydropump Flow and
Wastage Ratio Analysis
The hydropump flows for different reservoir drawdowns were estimated using the attached pump performance curve for a hydropump located at Lake Michie. The hydropump is known to pump 14 MGD to Williams when the reservoir is full. Using this information, the Total Dynamic Head (TDH) at 0 feet of drawdown (120 feet) was determined from the pump performance curve. Drawdown increments of 5 feet were added to the TDH at full reservoir capacity and the pump flows were read from the performance curve. The results are shown in Table 3.2.

The wastage ratio was estimated using the relation:

\[ P = HQ \]

Where:

- \( P \) = HP of the pump
- \( H \) = Head under which the pump is operating
- \( Q \) = Flow

Since the pump was known to produce 260 HP when the reservoir was at capacity (\( H = 70 \) ft), the required flow was calculated as 21.2 MGD. This is the amount of water theoretically required to pump 14 MGD to the Williams treatment plant. The pump efficiencies can be taken from the performance curve to calculate the actual amount of water required. Table 3.3 shows the calculated wastage ratios associated with different reservoir drawdowns.
I certify that this test correctly represents the performance of
16 LNC-23 PUMP SERIAL NO. 1597749

[Signature]

WORTHINGTON CORPORATION

MARIPOSA, CA

DATE 12-10-63

E192406

ORDER NO: H 40358

APPR: AD

SUBJECT: 16 LNC-23 PUMP CITY OF DURHAM, N.C.
Appendix F
Empirical Hydrowastage Analysis
A regression analysis was used to empirically relate the amount of water wasted per
gallon of water pumped for the hydropumps at Lake Michie. Daily hydropumping and
streamflow data were compared for days when the reservoir was full, elevation 341 ft.
This ensured that the measured streamflow was due entirely to water wasted from the
hydropumps.

The streamflow data was obtained from USGS station # 02085500. The hydropumping
data was obtained from WRD records for 1993-1994. During this period there were only
59 days when hydropumping occurred while the reservoir elevation was less than 341 ft.
A regression analysis was performed using the hydroflow data as the independent
variable. Thus, the hydroflow coefficient represents the hydrowastage ratio.

The first regression analysis indicated that the intercept value was not significant (t-score
= 0.89). The second regression analysis was conducted forcing the intercept to 0,
representing that no streamflow should be observed when hydropumps are not in use.
The data and regression analysis are given in Table F.1.

The analysis determined that approximately 2.2 gallons were wasted per gallon
transferred by hydropump. However, the observed $r^2$ value was only 0.69. Given the
apparatus used at the USGS guage station, it is likely that the low correlation was due to
inaccuracies in streamflow measurement. Since no additional data were available, a
conservative hydrowastage ratio of 2.5 was used for the model. Subsequent sensitivity
analyses indicated that the model was insensitive to changes in this ratio (i.e. increased wastage ratios did not significantly impact the solutions, see Section 4.3).
analyses indicated that the model was insensitive to changes in this ratio (i.e. increased wastage ratios did not significantly impact the solutions, see Section 4.3).

Table F.1 Regression analysis for all low elevation days (< 340 ft) where hydroflow occurs

<table>
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Regression Statistics

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