Direct numerical simulation of turbulent channel flows with boundary roughened with virtual sandpaper

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A method to simulate the effects of a roughened surface on a turbulent boundary layer is introduced. The method is easy to implement, does not increase the numerical overhead of the code, and affects the mean velocity in an a priori predictable way. A single parameter \( k \) is sufficient to fully characterize the roughness. The procedure has been tested in turbulent channel flows at \( \text{Re}_x = 1000 \), with roughness heights \( k^+ \) spanning the transitional regime. The properties of the rough flow agree well with experimental data. © 2006 American Institute of Physics.

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Hydrodynamically rough boundary layers are found in many industrial applications and are usually the norm in geophysical flows. Yet, to date, our knowledge of the physics of these flows is nowhere close to the one we have for smooth boundary layers. One of the few properties for rough boundary layers that is well established is that roughness shifts the mean velocity profile downward in a Clauser plot. In fact, the magnitude of the shift \( \Delta U \) is often used to classify the roughness by assigning an equivalent sand roughness, defined as the diameter of sand particles that would create the same shift. However, it is often difficult to estimate \( \Delta U \) a priori this quantity. Conversely, the effects of roughness outside the roughness layer on Reynolds stresses and dissipation are still not quite understood; the question being tied to the larger problem of how the inner and outer layers interact within a boundary layer. Given the present state of affairs, direct numerical simulation (DNS) of the Navier-Stokes equations has the potential to increase the level of understanding of the problem. However, roughness in a boundary layer introduces a new set of variables, such as the height \( k \) of the roughness elements, their aspect ratio \( \sigma \), one or more “wavelengths” \( \lambda_i \), and possibly other quantities related to the spatial distribution of roughness. This proliferation of parameters works against DNS, a computationally expensive and time-consuming technique, not very well suited for exploring wide swaths of the parameter space. We surmise that the paucity of DNS studies published so far is partly due to this problem (note that similar considerations apply to theoretical works as well). There is also a class of flows where roughness is found, with different characteristics, at different scales. For example, consider the flow of water over sand ripples (other flows of this kind include atmospheric flows over vegetated hills, urban flows, and so on). Both ripples and individual sand grains over them constitute roughness. At the ripple scales, a DNS or (LES) is possible. While other methods, such as lattice Boltzmann equation, exist to calculate flows over complex boundaries, it would be desirable to include the effects of the small-scale roughness into standard Navier-Stokes-based solvers. Such roughness should, at the minimum, satisfy the following requirements: (i) be characterized by a small (ideally, one) set of parameters; (ii) be easy to implement numerically; (iii) the effects on the bulk properties of the flow (velocity defect, friction coefficient, . . . ) be known a priori; and (iv) the boundary layer has properties that match observations. In this Letter, we show that it is indeed possible to achieve these goals. The idea is to consider a set of self-similar surfaces, for which a single parameter characterizes the hydrodynamical properties. Since the surface mimics real sandpaper we call it “virtual sandpaper.” Here we apply the method to a channel with otherwise flat walls, but it can be easily used to make surfaces of more general large-scale shape rough. Miyake et al. used a somewhat similar approach in their pioneering work, but their roughness still depended on several parameters, and the methodology was tested only at \( \text{Re}_x=400 \), where low-Reynolds number effects still affect the flow. Here we consider \( \text{Re}_x=1000 \).

The Navier-Stokes equations

\[
\frac{Du}{Dt} = -\nabla p + \nu \nabla^2 u + g, \quad \nabla \cdot u = 0, \tag{1}
\]

are solved on a Cartesian staggered grid. The advective and diffusive terms are treated explicitly, since for this combination of grid and forcing the time step is determined by the Courant-Friedrich-Lewy condition. In the streamwise and spanwise directions (\( x \) and \( z \), respectively) we use periodic boundary conditions. At the top boundary (\( y=H \)) we impose free-slip conditions (“half-channel” approximation), while at the bottom the usual no-slip condition is enforced. The channel dimensions are \( 6H \times 2H \times H \). The choice of free-slip conditions at the surface is dictated mainly by the need to achieve a reasonably high Reynolds number, while maintaining the computational load within reasonable limits. The domain is discretized with \( 386 \times 386 \times 256 \) points in the streamwise, spanwise, and vertical directions, with uniform grid spacing in the horizontal (\( \Delta x^+ = 15.5 \) and \( \Delta z^+ = 5.2 \)). In the vertical, the grid is stretched to have a higher resolution near the lower wall (20 points within \( y^+ < 10 \)). The flow is
forced by a steady and uniform body force $G$ acting in the streamwise direction, which defines unambiguously the friction velocity $u^* = \sqrt{GH}$ set so that $Re_u = u^* H / \nu = 1000$. After reaching steady state, statistics are accumulated over 20,000 wall time units (w.t.u.) for each case considered.

To roughen the wall, we start by partitioning the bottom surface into $N_x \times N_z$ square cells of side $2k$. Within each cell, an ellipsoid with semiaxes $k, 1.4k$ and $2k$ is placed, randomly oriented, with its center located at $z_0 = -0.5k$. The geometry is inspired by sand grains, which are usually not spherical. We overlay the assemblage with the DNS Cartesian grid and for each cell we calculate the fraction $v_f$ of its volume not occupied by the roughness elements (i.e., $v_f = 0$ for a fully occupied cell, $v_f = 1$ for a cell completely free). Neighboring ellipsoids are allowed to intersect. The result is a self-similar family of surfaces characterized by a single parameter, $k$. When averaged over horizontal planes, $v_f$ plotted against $y/k$ collapses on a single curve equal to 0.5 at $y/k = 1$. Above $y/k = 1.5$ the fluid is free of obstacles. We take $k$ as the roughness height. We stress that the volume fraction is calculated once for a given Cartesian grid and roughness height $k$. $v_f$ is then used to include the effect of the rough boundary at each time step based on the following procedure: (i) Compute the intermediate velocity field which includes the effects of advection and diffusion; (ii) multiply the intermediate field by $v_f$ to include the effect of roughness; (iii) calculate the pressure field necessary to project the corrected intermediate field onto the space of divergenceless fields; (iv) apply the pressure correction to obtain the velocity field at the end of the time step. The appeal of this method is that it is extremely simple to implement numerically. Of course, it is possible to use an immersed boundary method to achieve the same result with higher accuracy. However, the computational cost would be higher, without a clear benefit, since we are interested in boundaries (sand grains, trees, etc.) whose surface is not known if not approximatively. In this Letter, we consider three values of $k^+ = 10, 20$, and 40, spanning the region of smooth and transitional roughness. Higher values of $k^+$ require a larger horizontal domain (to have a sufficiently large number of elements) and are not considered here. We also calculate a smooth flow for comparison.

The smooth case ($k^+ = 0$) is first run for 25,500 w.t.u. to achieve steady state, and then run for an additional 20,000 w.t.u. to accumulate statistics. The final field is then used as an initial condition for $k^+ = 10$, and so on. Profiles of velocity and Reynolds stress components are collected every 12.5 w.t.u., while full pressure and velocity fields are recorded every 125 w.t.u. Statistical convergence is gauged by measuring the deviation of the total stress (turbulent and viscous) from a linear profile. Convergence is deemed satisfactory when the discrepancy is less than 0.5%.

The use of the half-channel approximation has been used in the past to simulate flows with free surfaces at low values of Froude and Weber numbers. It is known that turbulent structures generated near the wall can be advected near the surface and interact with it, hence extra care needs to be taken to resolve the scales of this interaction. Here, the free-slip approximation is chosen as an expedient to keep the size of the problem within manageable limits. In fact, the rather coarse resolution near the upper boundary (where $\Delta y^+ = 10$) means that the upper boundary is not very well resolved. It is thus necessary to compare to what extent the artificial upper boundary condition affects the flow underneath. For purpose of comparison, we consider the channel flow data of Ref. 10 at $Re_u = 1000$. The DNS data and the experimental results agree very well up to $y^+ = 900$ (Fig. 2) at which point the vertical velocity fluctuations decay rapidly to zero, because of the $y = 0$ boundary conditions. The peak in $u_{rms}$ appears slightly shifted toward the wall. On the other hand, the development of the plateau is well reproduced. The skin friction coefficient $C_f = 2(Re_u / Re_b)^{1/2}$ calculated from the DNS data is within 0.5% of the value obtained using Dean’s correlation. Thus, the upper boundary condition does not alter in an appreciable way the near-wall dynamics.

The standard approach to analyze velocity profiles in boundary layers over rough surfaces is to assume that in the overlap region $C_f = \frac{1}{\kappa} \log(y^+ - d) + B - \Delta U^*(k^+)$, where $\kappa$ is the Von-Karman constant, $5 < B < 5.5$ a universal constant, $\Delta U^*$ a roughness-dependent velocity defect, and $d$...
the location of the virtual wall. Usually, experimental data points are fitted to Eq. (2) to determine appropriate values of \( B, d, \Delta U^+, \) and \( u^+ \) \((\text{Ref. 13}). It is known that this method produces results that are often inaccurate.\(^{14}\) Here we adopt a different approach which relies on the fact that \( u^+ \) is determined by the forcing and which does not make an a priori assumption on the law of the wall. To calculate \( d \), we define 
\[
d \approx \int_0^s F(y) dy / \int_0^r F(y) dy \quad \text{(Ref. 16)},
\]
where \( F(y) dy \) is the horizontally averaged drag force exerted by the roughness elements on the flow comprised between \( y \) and \( y+d \), and \( h \) is the maximum height of the roughness elements \((1.5k \text{ in our experiments})\). In other words, \( d \) is the equivalent arm of the moment exerted by the ellipsoids on the flow. \( F(y) \) can be easily calculated from the time-averaged momentum balance
\[
0 = \int_0^y F(s) ds + (u^+ u_+^+) + \frac{\partial U^+}{\partial y^+} + \frac{y^+}{H^+}.
\]
(3)

Because of the self-similar nature of the roughness, \( F(y) \) for the different cases considered collapse reasonably on a single curve. The ratio \( d/k \) is about 0.8 in all three cases, close to values reported in the literature for sand-grain-type roughness.\(^{16}\) Once \( d \) is known, \( \Delta U^+ \) is calculated by averaging \( U_r^+ - U_s^+ \) \((\text{the subscripts } s \text{ and } r \text{ refer to smooth and rough, respectively})\) over the range \( 100 < y^- < 300 \). The calculated values are plotted in Fig. 1 together with the experimental values of Ref. 8.

The interesting result is that \( k \) as defined above coincides with the “equivalent” sand roughness. Only the velocity defect is taken into account, the profiles for \( U^+ \) collapse very well on each other from about \( y^+ = 30 \) to the upper wall, with the exception of \( k^+ = 40 \), where the agreement degrades above \( y^+ = 500 \) (Fig. 1). The friction coefficient in the rough case agrees well with Colebrook and White’s \(^{12}\) correlation (Table 1).

Figure 2 shows the rms fluctuations of the three components of velocity as a function of distance from the virtual wall. Only data above 1.5\(k\) are shown, as the interpretation of turbulence within the roughness is beyond the scope of this Letter. All profiles collapse reasonably well sufficiently far away from the wall \((y^+ > 3k)\), marking the extent of the roughness layer. In the roughness layer, \( w_{\text{rms}} \) is essentially the same whether the boundary is rough or smooth. Vertical fluctuations are slightly enhanced, whereas streamwise fluctuations are damped. Experiments with closely packed spheres\(^{17}\) show a similar drop in the intensity of \( u_{\text{rms}} \) in the roughness layer with increasing equivalent roughness height.

Another aspect of rough boundary layers for which experiments give different answers is the balance between energy production and dissipation. Adding to the uncertainty, there is the fact that measuring directly dissipation in experiments relies of a number of assumptions about the small-scale flow that may not be verified for rough flows. In our experiments, the ratio of production to dissipation \( P/\epsilon \) collapses on a single curve above the roughness layer [Fig. 3(a)]. Moreover, in the region \( 30 < y^+ - d < 400 \) the ratio is close to unity. In the roughness layer we distinguish two cases: for small \( k^+ \), \( P/\epsilon \) falls below the smooth case, while for higher values of \( k^+ \) the ratio is larger. It must be noted, however, that for small \( k^+ \) the roughness layer is contained in the region around \( y^+ = 15 \), where production peaks. In any case, the deviation from unity in the region \( 30 < y^+ - d < 400 \) is small, and this is good news for the geophysical community, since in the field, where conditions are almost always rough, dissipation is frequently estimated by measuring production and assuming local balance. In a smooth boundary layer, the statistical distribution of \( \epsilon \) in the near-wall region (where dissipation peaks), departs from Gaussian, the hallmark of a strongly intermittent signal. In fact, the probability density function of \( \epsilon \) is well represented by an exponential; that is PDF(\( \epsilon^+ \)) \( \sim \exp(-a\epsilon^+) \) [Fig. 3(b)]. When we consider the PDF of dissipation just above the tallest roughness elements, there is an apparent increase in the slope.
Experiments show differences in streamwise spectra at strongly affected by the discretization employed. Available considered with caution, since high-wave numbers are small. Spanwise spectra are virtually unaffected, while streamwise spectra show a slight increase in energy at the small wave numbers. The difference could be the result of the tail. However, if we consider the distribution of $\epsilon/\sigma$, where $\sigma$ is the standard deviation, we see that the distributions for the renormalized dissipation are very similar [Fig. 3(c)]. There is a tendency for the slope to decrease with increasing roughness height (from $-0.3$ to $-0.22$), which would imply that roughness tends to increase intermittency in dissipation. However, this result needs to be verified with larger data sets to weed out a possible bias due to poor statistical convergence.

The problem of intermittency in wall bounded flows is considerably more complex than for isotropic and homogeneous turbulence, because of the existence of extra scales related to coherent structures. However, we know that dissipation remains strongly intermittent in the vicinity of the wall. What appears from our analysis is that whatever the mechanism is, our virtual sandpaper essentially maintains it. The effect of roughness on near-wall spectra ($y^+ \sim 30$) is small. Spanwise spectra are virtually unaffected, while streamwise spectra show a slight increase in energy at the smallest wave numbers. These results, however, need to be considered with caution, since high-wave numbers are strongly affected by the discretization employed. Available experiments show differences in streamwise spectra at small wave numbers. The difference could be the result of the large-scale nature of the roughness in their experiments (mesh) versus the present work. A more detailed analysis of this problem, as well as of the related problem of higher-order statistics and coherent structures, is outside the scope of this Letter.

In summary, the results presented here show that a surface roughened with the procedure outlined above modifies the turbulent flow in a way that is consistent with experimental results. The method is not meant to precisely reproduce the flow near the roughness but to account for its effects on the flow above. It is very easy to implement, requires virtually no computational overhead, and introduces only one extra parameter, with a clear physical interpretation. It does not require the modification of the original computational grid, and can be easily applied to any numerical scheme that discretizes the equations in physical space.

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