AFFIRMING DIFFERENCE, GENERATING PROBLEMS, AND BECOMING-DEMOCRATIC IN MATHEMATICS EDUCATION

Kathleen Evelyn Rands

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Approved by:

Lynda Stone

George Noblit

Marta Civil

Kurt Stemhagen

Elizabeth St. Pierre

ABSTRACT

KATHLEEN EVELYN RANDS: Affirming Difference, Generating Problems, and Becoming-Democratic in Mathematics Education
(Under the direction of Lynda Stone)

The purpose of this dissertation is twofold. First, the dissertation aims to critique “images of thought” in mathematics education which have ontological underpinnings that maintain oppressive practices and which prevent transformation. The second aim of the dissertation is to launch the transformation to a *post*critical mathematics education through the creation of new concepts and strategies. Following Deleuze and Guattari’s (1980/1987) approach, the dissertation is organized into three “plateaus,” nonlinear writing nodes which catalyze movement in thought from taken-for-granted notions to new ways of thinking. The first plateau engages with notions of equity in mathematics and moves to the Deleuzian concept of *affirming difference*. The second plateau enters through notions of “problems” and “problem-solving” and moves to a Deleuzian concept of *problem-posing*. The third plateau begins with notions about democratic mathematics education and moves to the Deleuzian concept of *becoming-democratic* in the mathematics classrooms. Thinking about difference, problems, and democracy differently opens up launching sites for new ways of becoming anti-oppressive mathematics educators.

dedicated to my mother, Valerie Rands

my father, William J. Rands

and my son, Alexander (Mung Yang) Rands Tang

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Introduction: An Orientation

Introduction

Mathematics education is a relatively young field of academic inquiry which emerged originally from conversations between cognitive psychologists interested in mathematics as a domain of cognition and educators with a special interest in mathematics (de Corte, Greer, & Verschaffel, 1996). Mathematics and mathematics education have traditionally been seen as neutral and outside the social realm. For example, Stemhagen (2007) quoted the mathematician Bertrand Russell as describing mathematics as a “beautiful world; it. . .is eternal, cold and passionless. . . [and has] an immense dignity derived from the fact that its world is exempt from change and time” (p. 92). Stemhagen (2006) has also noted that many math teachers see social justice issues as “out of their hands” (p. 1), as outside the domain of the mathematics classroom. However, in the past quarter-century, a growing number of mathematics educators have reframed mathematics and mathematics education within the social realm. Valero and Zevenbergen (2004) have identified two versions of a “social-turn” in mathematics education. The first version turns to social constructivism and asserts that mathematical knowledge is socially constituted within the social milieu of a classroom culture. Research and theory in this first version of the social turn resonate with the work of Vygotsky and are often referred to “sociocultural” perspectives. The second version of the “social turn” is rooted in sociology and critical theory. In this case, mathematics education is assumed to be a social and political practice, which is "historically constituted in complex systems of action and meaning in the intermesh of multiple contexts such as the classroom, the school, the community, the nation and even the globalized world" (p. 2). This tradition addresses issues of power and raises questions about the ways in which mathematics can be and has been oppressive. Gutiérrez (2002) defined "dominant" mathematics as "mathematics that reflects the status quo in society" (p. 150) and "critical mathematics" as "mathematics that squarely acknowledges students are members of a society rife with issues of power and domination" (p. 151). Yet, certain aspects of extant ways of thinking about diversity in mathematics education, even in critical approaches, maintain ontological underpinnings which block transformations that would escape oppressive practices. The philosophical works of Gilles Deleuze and Félix Guattari offer processes and lenses which can be used to locate such blockages and open new paths for ways of thinking about difference in mathematics education. According to Deleuze (1968/1994), "the conditions of a true critique and a true creation are the same: the destruction of an image of thought which presupposes itself and the genesis of the act of thinking in thought itself" (pp. 136-140). Thinking differently about difference requires both critique of the presuppositions that maintain extant ways of thinking, as well as the creation of different ways of thinking. This dissertation critiques three "images of thought" in mathematics education, each captured by a key concept: 1) equity, 2) problem-solving, and 3) democratic mathematics education. Through the processes and lenses in the work of Deleuze and Guattari, the dissertation then creates three new, transformed concepts: 1) affirming difference, 2) generating problems, and 3) becoming-democratic in mathematics education. Collectively, this critique-creation presses critical mathematics education itself to undergo a process of becoming, to become something different-than-it-was—to become *post*critical mathematics education.

Because an important aim of my dissertation is to create new ways of thinking in mathematics education, my dissertation differs from most dissertations in the field of education and mathematics education in two interrelated ways: 1) the lens through which it is focused, and 2) its approach and form. Because these differences might be disorienting to readers expecting a more traditional dissertation, I have included this introductory section as a way to orient readers. I will begin by introducing the purpose of the dissertation and my research questions. Next, I will trace the context in which Deleuze and Guattari created the processes and lenses used in this dissertation. I will then provide an orientation to the ways in which my dissertation differs from most educational dissertations. Finally, I will situate the dissertation in the broader context of educational scholarship.

Purpose and Research Questions

The purpose of this dissertation is twofold. First, the dissertation aims to critique "images of thought" in mathematics education which have ontological underpinnings that maintain oppressive practices and which prevent transformation. The second aim of the dissertation is to launch the transformation to a Deleuzoguattarian *post*critical mathematics education through the creation of new concepts and strategies. The research questions fall into the following three clusters:

1) What are the current ontological underpinnings of the notion of "equity" in mathematics education and in what ways do these ontological underpinnngs allow or support oppressive practices? What alternatives does Deleuze's concept of *affirming difference* offer?
2) What are the current ontological underpinnings of the notion of "problems" in mathematics education and in what ways do these ontological underpinnngs allow or support oppressive practices? What alternatives does Deleuze's concept of *problem* offer?
3) What are the current ontological underpinnings of the notion of "democracy" in mathematics education and in what ways do these ontological underpinnngs allow or support oppressive practices? What alternatives does Deleuze's concept of *becoming democratic* offer?

Tracing and Mapping

A commonsensical dissertation would, as the King of Hearts said gravely to the White Rabbit in *Alice's Adventures in Wonderland*, "begin at the beginning, and go on until you come to the end: then stop" (Carroll, 1865/2004).[1] If, however, as Kevin Kumashiro (2004) asserted, "teaching and learning toward

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[1] In *The Logic of Sense*, Deleuze (1969/1990) examined time from a Bergsonian perspective, drawing extensively on Lewis Carrol's books about Alice. Although Deleuze did not specifically refer to this quote, it relates to his notions of time.

social justice” entails working “against common sense,” perhaps a noncommonsensical dissertation which dwells instead in “the middle” and follows “lines of flight” might be useful.

The concepts of “the middle” and “lines of flight” are just two of the many novel or transformed concepts that have emerged in the philosophy of the French thinkers Gilles Deleuze and Félix Guattari. To use the concepts of “the middle,” “lines of flight,” or other Deleuzoguattarian concepts, it may help to examine the context out of which such concepts surfaced. This section will *trace* this context. The distinction between *tracing* and *mapping* in the work of Deleuze and Guattari is not obvious in the terms themselves, but is crucial to understanding their projects as well as this dissertation. The English term “tracing” is a translation of the French word *calque*. As a noun, *calque* means a tracing, or more figuratively, a “carbon copy” or a “spitting image.” It can also mean “layer.” *Calque* could also be the present tense or subjunctive form of the verb *calquer*, having the sense of “is tracing” or “would be tracing” respectively. In the verb form, *calque* has the sense of tracing a drawing, or more figuratively, copying something exactly, or even more loosely, imitating a model of behavior (Javamex, 2011). The term “map” is a translation of the French word *carte*, which is often used in the sense of a geographical map. For Deleuze and Guattari (1980/1987), marking the map is laying everything out on the same “plane,” where the “plane” is an ever changing multidimensional “flat” space[2]. A tracing is

[2] Further explanation of Deleuze and Guattari’s use of the term “plane” can be found below, in the section entitled “A Different Approach and Form.”

an attempt to represent the world as is; marking a map creates something new, becomes something different. Deleuze and Guattari (1980/1987) warned against making tracing from a map because in trying to imitate or reproduce the map, one arrives at something that was already presupposed from the beginning, always resulting in the same "image." Deleuze and Guattari (1980/1987) gave examples of psychoanalytic interpretations of behavior, in which every story is the story of Oedipus, always the same. Instead, Deleuze and Guattari (1980/1987) argued for the reverse process: "Plug the tracings back into the map" (p. 14). A still image—that same old story—is only a starting place, not an end in itself. As Deleuze and Guattari (1980/1987) said, one can "find a foothold in formations that Oedipal or paranoid or even worse, rigidified territorialities that open the way for other transformational operations" (pp. 14-15).

Tracing the context in which Deleuze and Guattari wrote might begin with examining the term "poststructuralist," which has often been used to describe the thought of a group of French thinkers in the second half of the twentieth century, Deleuze and Guattari included. However, determining exactly what is meant by the term "poststructuralist" is by no means an easy task. Defining "postructuralism" is difficult for several reasons. First, as James Marshall (2004) has noted, the idea of *post*structuralism is obviously etymologically related to the term "structuralism;" yet, structuralism does not itself refer to a homogenous group of ideas, but rather to the diverse ideas of the linguist Lévi Strauss, Marxist Althusser, the early ideas of psychoanalyst Lacan, among others. Similarly, a second reason poststructualism is difficult to define is that the thought of those labeled "poststructuralist" varies so widely that is difficult to give a substantial reason for the overarching category. Mark Poster (1989; quoted in Marshall, 2004) argued that "poststructuralism" was a term coined by North American academics to categorize the diverse thought of French philosophers of a particular time period. The exogenous origin of the term helps to explain the fact that some "poststructuralist" philosophers may not have considered themselves to be "poststructuralist"(e.g. Foucault, 1977, 1983).[3]

Despite these difficulties, the term "poststructuralist" is still useful because it ties together thinkers through three connections: 1) time and place, 2) against whom they wrote, and 3) centrality of "difference." The first connection among poststructuralist thinkers is simply time and place: poststructuralist thought is that of prominent French academics writing after 1968. The year 1968 specifically is important because of one particular event which had a profound effect on these philosophies. In May of 1968, a student and worker revolution swept across France essentially bringing France "to a standstill" (Marshall, 2004). President Charles de Gualle ended the revolution through a "skillfully orchestrated. . .reassertion of state power" (Marshall, 2004, p. xviii). This event significantly shaped the thinking of poststruturalist philosophers. In fact, poststructuralist thought has even been called "68 philosophy" (Marshall, 2004), since poststructuralist thinkers interpreted and responded to these events in various ways in their works.

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[3] This reaction is even more pronounced in response to the term "postmodern." See the St. Pierre (2000, note 4) for a comparison of these two terms.

Like other poststructuralist thinkers, the work of Gilles Deleuze and Félix Guattari was affected by the events of May 1968. Deleuze, whose works prior to 1968 "read like a Who's Who of philosophical giants" (Massumi, 1987, p. ix) was in the midst of writing his first two major works "in his own voice" (Massumi, 1987, p. ix), *Difference and Repetition* (1968/1994) and *Logic of Sense* (1969/1990) at the time of the student and worker revolution. Shortly after the revolution, Deleuze began his collaboration with Félix Guattari. Guattari was a radical psychoanalyst and political activist before and after the revolution. Beginning in the mid-1950s he was involved at an experimental psychiatric clinic called *La Borde*, which aimed at abolishing "the hierarchy between doctor and patient in favor of an interactive group dynamic that would bring the experiences of both to full expression in such a way as to produce a collective critique of the power relations in society as a whole" (Massumi, 1987, p. x). Given his anti-hierarchical ideas prior to May 1968, it is not surprising that Guattari was involved in the movements that grew out of the student and worker revolution. In 1969 a mutual friend, Jean-Pierre Muyard, arranged for a meeting between Guattari and Deleuze. The encounter between Deleuze's ideas in *Difference and Repetition* and *Logic of Sense* and Gauttari's anti-hierarchical ideas about psychiatry resulted in three collaboratively written books: *Anti-Oedipus* (1972/1977) and *A Thousand Plateaus* (1980/1987), which share the subtitle *Capitalism and Schizophrenia*, and *What is Philosophy* (1991/1994). *Anti-Oedipus* (Deleuze & Guattari, 1972/1977) critiqued both state/party-based versions of Marxism and "school-building strains" of psychoanalysis (Massumi, 1987, p. xi). *A Thousand*

Plateaus (Deleuze & Guattari, 1981/1987) attempted to go beyond mere critique to the creative, positive production of the kind of thought proposed in *Anti-Oedipus*. Their third collaborative work, *What is Philosophy* (1991/1994), laid out their view of the project of philosophy as the creation of concepts such as "the middle" and "lines of flight." The central role played by May 1968 in the encounter between Deleuze and Guattari connects their work to the work of others who fall into the poststruturalist category.

Yet time and place are not enough to distinguish poststructuralist thinkers from non-poststructuralist thinkers, since there were other French thinkers writing in the same time period who are not considered poststructuralist such as Jean Paul Sartre, Paul Riceour, and Emmanuel Levinas. Another distinguishing feature that separates poststrucuralist thinkers from those on this list and others writing in France after May 1968 is against whom the thinkers wrote. Rather than writing against structuralism, Marshall (2004) suggested that it is more accurate to say that poststructuralists were writing against humanism and Marxism (although structuralists also critiqued humanism). Poststructural critiques of humanism followed Nietzsche via Heidegger, who observed that while humanism "purported to liberate human beings, [it] had. . .only oppressed them (Marshall, 2004). Deleuze's (1962) influential book, *Nietzsche and Philosophy*, falls within this strain of thought, a strain which some commentators consider to be poststructuralist (Marshall, 2004). This line of thought critiques the "promises of development, improvement and emancipation" (Marshall, 2004) rooted in the conception of humans as individual, autonomous individuals. Poststructuralists also critiqued Marxism, but this critique was often in some ways a reformulation of Marxism rather than a wholesale rejection. For example, Marshall (2004) described Peters' chapter in the same book as demonstrating that despite Lyotard's break with Marxism, Lyotard maintains certain notions that can be considered Marxist. Similarly, Deleuze and Guattari's (1972/1977, 1980/1987) books *Anti-Oedipus* and *A Thousand Plateaus* are critques, but also reformulations, of Marxism.

While the heterogeneity of "poststructuralist" thought makes defining the term difficult, this heterogeneity can also be seen as its second key aspect. Marshall (2004) forwent "any broad, encompassing definition" of poststructuralism because an implicit theme of the book was the diversity among poststructuralist thinkers' ideas. Rather than similarity, it is difference which ties the work of these thinkers together. The collaboration between Deleuze and Guattari epitomizes this connection across difference. The two men inhabited different worlds: Deleuze, the world of a recognized academic; Guattari, the world of a militant psychoanalyst and activist. The distance between their worlds was symbolized by the fact that they used the formal French word for "you", *vous,* with one another throughout their interactions (Dosse, 2007/2010). Guattari described the collaboration between himself and Deleuze as follows:

> This collaboration is not the result of a simple meeting of two people. . . .At the outset, it was less a matter of sharing a common understanding than sharing the sum of our uncertainties and even a certain discomfort or confusion with respect to the way that May 1968 had turned out. (Lapoujade, 2002/2003, p. 301; quoted in Dosse, 2007/2010, p. 8)

It was connecting across difference that allowed Deleuze and Guattari to produce novel concepts that contributed significantly to philosophy.

The centrality of difference has played an additional role in postructuralist thought. Many poststructuralist thinkers (e.g. Judith Butler, Jacque Derrida, Julia Kristeva) have addressed the concept of difference in their work, albeit in fittingly different ways. Following this pattern, Deleuze developed an ontology of becoming in which difference played a important role. The seeds of these ideas can be found as early as Deleuze's (1962/1983) reference to Heraclitus' doctrine of universal flux in *Nietzsche and Philosophy*. Heraclitus, Deleuze (1962/1983) claimed, "has two thoughts that are like ciphers: according to one there is no being, everything is becoming; according to the other, being is the being of becoming as such" (p. 23). Deleuze's (1968/1994) book *Difference and Repetition* was an elaboration on these two "ciphers" toward an ontology of "an affirmation of becoming" (Deleuze, 1962/1983, p. 23) found in the thinking of Heraclitus and later Nietzsche. A further influence along these lines was "Heidegger's ontological intuition" (Deleuze, 1968/1994, p. 117) that "difference must relate the differing terms to one another. . .[D]ifference must be articulation and connection in itself; it must relate different to different without any mediation whatsoever" (Deleuze, 1968/1994, p. 117). *Difference and Repetition* (Deleuze, 1968/1994) was in large part a critique of ontologies that conceptualized sameness as the basis for existence—that is, ontologies that mediated difference through identity, resemblance, analogy, or opposition. Such ontologies conceptualized *being* as that which remained the same over time; instead, in an ontology of difference, being involves *becoming different* rather than remaining the same. It is the differences--the changes--that matter, not any sort of essential characteristics that remain the same over time.

Deleuze and Guattari (1980/1987) continued developing this ontology of difference or ontology of becoming in *A Thousand Plateaus* in three interconnected ways: 1) in the ideas they expressed, 2) in the approach they took to expressing and creating the ideas, and 3) in the arrangement of the book. In addition to introducing numerous new or transformed concepts such as *assemblages*, *machines,* and *the Body without Organs,* Deleuze and Guattari (1980/1987) also developed and used a new approach to creating such concepts. In line with the idea of an ontology of becoming in which what matters is difference or change, the approach itself constantly changed throughout *A Thousand Plateaus.* Such an approach-in-flight is a strategic response to the context in which Deleuze and Guattari were writing: if the revolutionary thrusts of Marxism can be captured by the State and converted to authoritarian policies and if psychoanalysis is just another form of domination in which a doctor assigns meaning to a patient’s experience based on predetermined and authoritative interpretations, then it seems there is always the potential for revolutionary movement to stagnate, to devolve into some form of domination. An approach-in-flight never remains the same long enough for this conversion into a dominant system of thought to take place. Like a rabbit that follows a zig-zag line of flight from a predator, an approach-in-flight evades “capture” by dominant systems (whether the dominant system is the State, dominant forms of psychoanalysis, or some other form of domination) through constant change and movement. In this spirit, Deleuze and Guattari's approach to critique and creating concepts moved through many variations with different names: rhizomatics, stratoanalysis, pragmatics, micropolitics, nomadology. Each variation was a way to explore, within a given situation, points of stagnation and points of movement, locations of domination or hierarchy and directions in which hierarchies and forms of dominations were being dissolved. The novel concepts and new approach of *A Thousand Plateaus* were supported by its distinct arrangement. The authors composed the book in interconnected "plateaus"; sections of the book were "plateaus" instead of a "chapters" (see the "Different Approach and Form" section for a more complete explanation of "plateaus"). Each plateau title combined a date with a topic from which to launch a line of flight. Eschewing a chronological approach to time, the sequence of the dates zig-zagged through time, sometimes pointing to a single date, sometimes a year, sometimes more than 500 years. The topics were as various as linguistics, literature, music, politics, and psychoanalysis. A superficial scanning of these dates and topics gives the impression of randomness; however, with more careful engagement, certain notions surface and resurface in different forms, connecting back to previous ideas, constantly creating new connections among plateaus, on a "*plane of consistency* of multiplicities" which adds dimensions as it adds nodes of intensity. It is as if their thinking refused to compose ideas only along the two dimensions of the sheets of paper in the book, but instead sprouted new stems that connected nodes along new dimensions.

Orientation

Like Deleuze and Guattari's (1980/1987) *A Thousand Plateaus*, but specifically in relation to mathematics education, this dissertation develops an ontology of becoming through novel or redeployed concepts, a Deleuzoguattarian approach, and its arrangement into plateaus. This section provides an orientation to this different lens, approach, and form.

A Different Lens

A first way in which this dissertation differs from most dissertations in the field of education relates to the lens used. While most dissertations involve an empirical lens, I used instead a philosophical lens. To be more specific, the philosophical lens I used is an ontological one. To be more specific still, the ontological lens I have used is based in the normative ontology of Gilles Deleuze and Félix Guattari.

Noddings (1998) pointed out that although empirical approaches can show whether educational choices result in predicted consequences, "we still need philosophical argumentation to persuade others that the consequences we seek should be valued" (p. 5). Certain educational questions, then, cannot be decided by empirical methods, but rather require philosophical approaches. The questions in the philosophy of education are "philosophical in that they require philosophical methods for their investigation" (Noddings, 1998, p. 4). Hytten (2008) argued that philosophical approaches in education make unique contributions because they "involve asking fundamental questions, uncovering assumptions, making arguments, and exploring alternatives" (p. 189).

Philosophical inquiry, according to Hytten (2008), entails “learning to look at things from different perspectives, notably, from a distance and from alternative angles” (p. 190). Within the broad project of critical mathematics education, looking at things from alternative angles is a part of seeking out and proposing alternatives to the status quo.

The particular philosophical lens used in this dissertation is one that is ontological. Epistemological concerns, that is, questions about knowledge, have dominated scholarship in the field of philosophy of education. Yet, understanding this epistemological domination requires ontological inquiry. Etymologically, ontology is the study of *□ντος (ontos),* “that which is.” Ontology raises questions about what exists, but also about existence itself. Ontology asks: How does “that which is” exist, and why. Ontology also asks questions about boundaries between those things that exist. Traditionally ontological questions about boundaries have been framed in terms of delineating the boundaries of categories. While ontology asks questions about “what is,” ethics asks questions about “what should be.” Ethical questions relate to values. Ontology asks, “What *is being* in the world?”; ethics asks “How *should* we *be* in the world*?*” Normative ontology adds an ethical dimension to ontology, considering not only what is “being,” but also asking questions about how we should “be” in the world. The domination of epistemological concerns in educational scholarship indicates particular underlying assumptions about both ontology and ethics. Ontologically, it is assumed that *knowing* is an (or the) essential aspect of *being* in educational situations. In terms of ethics, it is assumed that what is important or valued in educational situations is knowledge. Ontological and ethical investigations can uncover taken-for-granted assumptions and expand inquiry beyond epistemology. An ontological and ethical lens allows inquiry that fulfills Hytten's (2008) call for posing "fundamental questions, uncovering assumptions, making arguments, and exploring alternatives" (p. 189), even about the nature of education itself.

Most specifically, the normative ontological lens used in the dissertation is one based in the ontology of Gilles Deleuze and Félix Guattari. As traced above, Deleuze, and later Deleuze and Guattari, developed an ontology of difference, one that shifted the focus from "being" to "becoming." An ontology of "being" searches for what endures, what stays the same over time, the "identity" of a thing or person. In Deleuze and Guattari's ontology, what matters or "makes a difference" is not enduring sameness, but rather difference itself. Important within this ontology are the concepts of *difference, problems,* and *becoming.* These concepts not only have ontological implications, but also raise ethical questions. Within the context of this dissertation, the concepts raise questions about *affirming difference, generating problems,* and *becoming democratic* in mathematics classrooms.

A Different Approach and Form

The second way in which this dissertation differs from most others interconnects with the first way. In conjunction with a Deleuzian or Deleuzoguattarian lens, a Deleuzoguattarian approach offers processes for critiquing "images of thought" in mathematics education and creating new ways of thinking. In combination, these differences in lens and approach have resulted in a dissertation whose form is different from most on both the macro level and micro level.

On a macro level, this dissertation has an atypical form. Most dissertations include sections for a literature review, methodology, findings, discussion, and conclusion; this dissertation has a Deleuzoguattarian form modeled on Deleuze and Guattari's collaborative work *A Thousand Plateaus*, the work in which they most thoroughly deployed schizoanalysis to move beyond mere critique to critique-creation. Accordingly, the most important sections of this dissertation are three "plateaus," each of which addresses one of the research questions. Rather than a linear sequence, these plateaus formed intensive interconnected nodes of thinking. Deleuze and Guattari (1980/1987) took the term "plateau" from Gregory Bateson, who originally used the term in reference to sexual practices that maintained and sustained sexual intensity rather than culminating in climax. In the context of philosophy, writing using plateaus differs from writing using chapters in that chapters are organized in a linear fashion whereas plateaus can be thought of as organized on a plane. Instead of a sequence of chapters that build upon one another, "each plateau can be read starting anywhere and can be related to any other plateau" (Deleuze & Guattari, 1980/1987, p. 22). In this way, a plateau is always "in the middle" because it can always be thought of as connected to or between at least two other plateaus. Therefore, where ever one begins in reading or writing plateaus, one begins in the middle. The ideal for a book according to Deleuze and Guattari

(1980/1987) would be to “lay everything out on a plane. . .on a single page, the same sheet” (p. 9). The interconnectedness of plateaus does not allow them to line up into a neat sequence. Instead, a plane allows plateaus to connect through multiple lines in multiple directions in a diagram. Although Deleuze and Guattari (1980/ 1987) use the term “plane” for the backdrop for connections between nodes, they clarify that each node actually creates a new dimension, so the “plane” does not necessarily have only two dimensions: “We will. . .speak of a *plane of consistency* of multiplicities, even though the dimensions of this ‘plane’ increase with the number of connections that are made on it” (p. 9). Here, “multiplicities” is an English translation of the French word *multiplicités*, which, in turn, is a French translation for the German word *Mannigfaltigkeiten.* To complete the circle of translation, *Mannigfaltigkeit* can be translated into English as “manifold” or “manifoldness” as well as via the French as “multiplicity.” In contemporary mathematics, an *n*-manifold is a space which locally appears to be *n*-dimensional Euclidian space $\mathbb{R}^n$, irrespective of its global curvature[4]. Euclidean spaces of any number of dimensions have zero curvature. One dimensional Euclidean space is the “straight” line (called the real line because its points are real numbers). Two-dimensional Euclidean space is the “flat” plane (called the real plane because its points are ordered pairs of real numbers). Three-

[4] A space is a set organized in a particular way. For example, a metric space combines a set with some sort of notion of distance. The elements of the set are organized in a way that allows one to say how far any two elements are from each other. The term “*n*-dimensional” means that the space has n dimensions. The symbol $\mathbb{R}$ means the set of real numbers. An “*n*-dimensional Euclidean space $\mathbb{R}^n$,” then, is a space in which the set is the real numbers, which are organized along n dimensions such that parallel lines, parallel planes, and their higher-dimensional equivalents do not intersect.

dimensional Euclidean space is a “flat” space of three-dimensions, in the same sense. Euclidean space of any dimension can be called “flat” in this sense. People tend to think of a circle as two-dimensional because its points must lie in a two-dimensional plane (that is, the circle is embedded in the plane). However, it is a 1-dimensional manifold because when smaller and smaller sections of it are taken, the pieces approximate more and more closely a line (one-dimensional Euclidean space). Similarly, a sphere is embedded in three-dimensional Euclidean space, but on a micro-level approximates a plane—it is a two-dimensional manifold. When Deleuze and Guattari (1980/1987) remarked that they wanted to lay everything out on a “plane” of consistency of multiplicities, the “plane” is like a multidimensional “flat” Euclidean space in which is embedded globally curved but locally flat manifolds. In this Deleuzian multidimensional space, the manifolds or multiplicities are “plateaus” which interconnect and link with one another. When Deleuze and Guattari (1980/1987) wrote that “the dimensions of this ‘plane’ increase with the number of connections that are made on it” (p. 9), they meant that the way in which new manifolds are “glued” or attached to the existing manifolds creates a new figure that cannot be embedded in the space in which the original manifolds were embedded, but instead must be embedded in a space of a higher dimension. These connections that require the embedding space to change dimension, in Deleuzoguattarian terms, are “lines of flight.”

Embarking on “lines of flight” has resulted in a dissertation whose form differs from most on a micro-level as well as on a macro-level. Most dissertations proceed in a linear fashion on a micro-level as well as on a macro-level. Each paragraph typically includes a topic sentence, supporting details, and a conclusion sentence. In contrast, in order to make the multitudinous connections between plateaus called for in *A Thousand Plateaus*, this dissertation moves by way of “lines of flight,” lines that can intersect and connect in many ways. It is lines of flight that connect the “everything”—“lived events, historical determinations, concepts, individuals, groups, social formations” (p. 9)-- that Deleuze and Guattari (1980/ 1987) would like to lay out on a single plane (or within an *n*-dimensional manifold). Josh Lerner (n.d.) explains the concept of “lines of flight” as follows:

> Lines of flight are creative and liberatory escapes from the standardization, oppression, and stratification of society. Lines of flight, big or small, are available to us at any time and can lead in any direction. They are instances of thinking and acting ‘outside of the box’, with a greater understanding of what the box is, how it works, and how we can break it open and perhaps transform it for the better. (para. 1).

The term Deleuze and Guattari (1980/ 1987) used for this “box” is a “stratum.” According to Deleuze and Guattari (1980/ 1987), the way to follow a line of flight is to “lodge yourself on a stratum, experiment with the opportunities it offers . . .find potential movements. . .possible lines of flight, experience them, produce flow conjunctions here and there. . . .It is through a meticulous relation with the strata that one succeeds in freeing lines of flight. . .” (p. 161). Embarking on lines of flight begins by outlining the contours of what has come to be, but it does not stop with extant situations. Instead, lines of flight move from what has come to be to creating new ways of being.

Each of the plateaus in this dissertation embarks on lines of flight by examining a particular image of thought that has come to exist in mathematics education, then moves to a new way of being through an encounter with an important concept in Deleuze's and Guattari's ontology of becoming. The first plateau begins by examining the image of thought about equity in the National Council of Teachers of Mathematics' (2000) *Principles and Standards* and moves to Deleuze's (1968/ 1994) concept of affirming difference. The second plateau begins by examining the image of thought about problems in the history of mathematics education and moves to Deleuze's (1968/1994) concept of problems. The third plateau begins by examining the image of thought about liberal-democratic mathematics education and moves to Deleuze's (1968/1994) concept of becoming and, more specifically, Deleuze and Guattari's (1991/1994) concept of becoming-democratic. The reader can begin with any of the three plateaus—each serves as a middle from which to begin. Each plateau can also stand on its own, just as a rhizotomous plant can survive if its rhizomes are severed from those to which it connects. However, the concepts in each of the plateaus are also intimately interconnected. The conclusion highlights the "stems" that connect across plateaus.

Situating the Dissertation in the Broader Context of Educational Scholarship

To date, Deleuzoguattarian perspectives are almost completely absent in mathematical education scholarship, with a few notable exceptions. Rivera (1998) used a postructuralist rhizomatic framework in an ethnography with secondary mathematics teachers to explore the meaning of teaching high school mathematics in the late nineties and the extent to which pedagogical practices subjectified students into ways of thinking about/acting/doing mathematics in classrooms. Fleener (2004) drew on "the decentering process of Deleuzian poststructuralism to note the conundrums of mathematics and mathematics education in current educational contexts and offer insights into possibilities of rethinking our curricular futures" (p. 201). Building on Roy's (2003) deployment of Deleuze and Guattari's concepts of smoothness, multiplicity, in-betweenness, becoming, and rhizomes, she interrogated the boundaries and limitations of the standard mathematics curriculum. White-Fredette (2009) referred to Deleuze's assertion that philosophy entails the creation of concepts and interpreted teachers' changing philosophies of mathematics during a mathematics education course to indicate that they were beginning to view both mathematics and philosophy as becomings rather than something static. In using a Deleuzoguattarian philosophical lens and approach, this dissertation makes a significant contribution to this emergent literature.

More broadly, the dissertation is both a critique and transformation of critical mathematics education. Its purpose is to press critical mathematics education to move beyond certain ontological underpinnings that prevent transformation and to launch this transformation through the creation of new concepts and strategies. In this way, the dissertation contributes to the body of work in critical mathematics education which includes work by authors such as Eric Gutstein, Arthur Powell, Marilyn Frankenstein, Olof Steinsthordottir, Bharath

Sriraman, Renuka Vithal, Ole Skovsmose, Gelsa Knijnik, among many others.[5] In the move toward *post*critical mathematics education, the dissertation also contributes to the smaller body of work by mathematics education authors who use poststructural lenses. According to Walshaw (2004), postmodernism and mathematics are "two lines of inquiry that have rarely addressed each other" (p. 1). Those who have used poststructural lenses in their work include, for example, Ernest (2004), Gutiérrez (2008), Mendick (2006), and Walkerdine (1989).[6]

The questions addressed in this dissertation also relate beyond postcritical and critical mathematics education, to fields including reform mathematics education, empirical mathematics educational research, multicultural education, educational philosophy, and Deleuze studies. While reform mathematics education documents such as the National Council for Teachers of Mathematics' (NCTM) (2000) *Principles and Standards for Teaching Mathematics* emphasize the importance of equity, Gutiérrez (2002) pointed out three obstacles to attaining equity: 1) an underlying belief that not all students can learn mathematics, 2) a deficit theory that tends to be applied to students who have been marginalized in mathematics, and 3) a poorly articulated research agenda around issues of equity in mathematics education. This dissertation addresses the first two obstacles by examining the ways in which the ontological underpinnings of extant

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[5] For more thorough overviews of the state of critical mathematics education, see Alrø, Ravn, & Valero (2010), Gutstein (2006), and Tuluk, Bondy, and Adams (2011).

[6] See Walshaw (2004) for a treatment of mathematics and postmodern perspectives.

ways of thinking about equity, problems, and democracy in mathematics classrooms maintain oppressive practices, thereby addressing the third obstacle by laying the groundwork for a more nuanced empirical research agenda.

Beyond mathematics education, the dissertation addresses broader issues in multicultural education in pressing for a transformation of critical multicultural education to postcritical multicultural education. A few education scholars have used the term “postcritical,” mainly in literacy studies and educational research using ethnographic methods. Iyer (2007) used Deleuze and Guattari’s (1980/1987) work along with other works to move from critical literacy to postcritical literacy. Selber (2005) called for a postcritical stance to literacy technologies. Noblit (1999, 2004) critiqued critical ethnography and pushed for the move to postcritical ethnography, a call which has been answered by numerous researchers in recent years (e.g. Murillo, 1999; Childers, 2011; Anders & Lester, 2011). While postcritical multicultural education in other content areas such as social studies, science, writing, art, and others will necessitate that educators situate analysis and strategies within the historical moment of those disciplines, this dissertation situated within mathematics education can serve as one node in an expanding postcritical multicultural education rhizome that allows for differences among disciplines and fields.

This dissertation also contributes to the field of educational philosophy through the elaboration of schizoanalysis as a philosophical approach. “Analysis” has had many different uses and meanings throughout the history of philosophy. Analysis in Aristotle’s *Prior Analytics* addressed syllogisms.

According to Groarke (2011), for Aristotle, “syllogisms can be construed as a vehicle for identifying the deep, immutable natures that make things what they are” (“The Syllogism,” para. 5). Analysis, for Aristotle, meant identifying the role which the middle term of various syllogisms played in order to inventory valid syllogisms. Kant (1764/1992) described analysis in the following way:

> the concept of a thing is always given, albeit confusedly or in an insufficiently determinate fashion. The concept has to be analyzed: the characteristic marks which have been separated out and the concept which has been given have to be compared with each other in all kinds of contexts. (p. 276).

More recently, concept analysis played a strong role in analytic philosophy in the twentieth-century. According to Preston (2006), analytic philosophy has undergone numerous phases, beginning with Russell’s and Moore’s turn away from British Idealism. One instantiation of analytic philosophy, logical atomism, used the method of ideal language analysis, in which propositions were stated using propositional logic notation. An alternative instantiation, initiated by Moore and revived after critiques of logical atomism and its successor logical positivism, used *ordinary* language analysis rather than *ideal* language analysis as a method. Interestingly, Wittgenstein’s philosophy stimulated both the logical positivist movement (through his *Tractatus Logico-Philosophicus)* and, after radically rethinking his initial work, the turn toward ordinary language analysis. According to the later Wittgenstein (1953), “by looking into the workings of our language, and that in such a way as to make us recognize its workings. . .the problems are solved, not by giving new information, but by arranging what we have always known” (p. 109). Ordinary language, then, serves as the medium through which to clarify existing concepts, which are constituted through our everyday use of them. Educational philosophical studies which make use of ordinary language analysis typically address questions such as, "What is teaching? What is learning? What is equity?" For example, in my (Rands, 2003) master's thesis, I combined empirical qualitative methods with philosophical concept analysis to explore the concepts of *dependence, independence,* and *interdependence*. The main tools of ordinary language analysis, according to Miller (1996), include necessary and sufficient conditions, "if-then" conditional thinking, and linking, schematizing, and diagraming concepts.

The most important way that schizoanalysis differs from these previous forms of "analysis" from Aristotle through ordinary language concept analysis is that schizoanalysis is not merely the discovery or clarification of already-existing concepts, but also the creation of *new* concepts, which, for Deleuze and Guattari (1991/1994) is the purpose of philosophy. While the form of "analysis" to which schizoanalysis most directly responds (as a critique) is "psychoanalysis," concept analysis in the analytic philosophical tradition has the same limitation in Deleuze and Guattari's view—both are attempts to *represent* or *reflect* the world as is. In schizoanalysis, the world-as-is is only a beginning point. Schizoanalysis begins by locating structures that maintain the world-as-is, but then uses those sites to begin creative experimentation toward something different. Given the critical acknowledgement that many aspects of the world-as-is are oppressive and the aim to work toward less oppression, an approach that can break open structurations that are oppressive and create something new is much needed.

Moreover, schizoanalysis provides a process for locating and breaking open ontological underpinnings that have come to be taken as common sense and would otherwise be overlooked. Deleuze and Guattari did not first envision schizoanalysis, then apply it to situations. Instead, they created schizoanalysis through “experimentation in contact with the real” (Deleuze & Guattari, 1980/1987, p. 12). That is, they created the process simultaneously with creating new concepts (Buchanan, 2010). One of the most challenging aspects of this dissertation has been the necessity of creating process and concepts simultaneously. It was not possible, as is expected in writing a dissertation proposal, to state clearly in advance the methodology I planned to use. Instead, the process of schizoanalysis emerged in writing the plateaus. Writing the dissertation literally began “in the middle,” with the “Affirming Difference” plateau, which constantly put me in the position of needing to have already explained the process I was using, which I necessarily had to explain later, since it was through writing that the process was developed. Deleuze and Guattari (1980/1987) wrote,

> The map has to do with performance, whereas the tracing always involves an alleged ‘competence.’ Unlike psychoanalysis, psychoanalytic competence (which. . .makes infinite, monotonous tracings. . .), schizoanalysis rejects any idea of pretraced destiny, whatever name is given to it. (p. 13)

Mapping, using schizoanalysis, rendered me “incompetent”, but it was this becoming-incompetent that allowed the dissertation to *perform*, to *do* something—to create something new. Along with other authors who are finding ways to deploy schizoanalysis in educational philosophy (e.g. Evans, Cook, &

Griffiths, 2008; Ringrose, 2011), this dissertation contributes to educational philosophy through the elaboration of schizoanalytic approaches.

More broadly, the dissertation is situated in the growing field of Deleuze studies in education, which touches not only educational philosophy, but also educational qualitative research, literacy studies, teacher education, among other subfields of educational scholarship. A number of influential collections of works related to Deleuze and education have been published in recent years. *Nomadic Education*, edited by Inna Semestky in 2008 brought "innovative educational theory into constructive dialoguq with the intellectual work of French postructuralist philosopher Gilles Deleuze whose conceptualizations strongly resonate with contemporary discourse in education" (p. vii). Contributors included, among others, Ronald Bogue, Noel Gough, Kaustuv Roy, and Elizabeth St. Pierre. Diana Masny and David Cole (2009) edited a collection entitled *Multiple Literacies Theory: A Deleuzian Perspective,* which brought Deleuze's concepts to bear on literacy studies. Several special issues of journals have also addressed Deleuze studies in education. A 2004 special issue of *Educational Philosophy and Theory* addressed the theme "Deleuze and Education," overlapping to some extent with *Nomadic Education* (Semetsky, 2008). A 2011 special issue of *Policy Futures in Education* edited by Inna Semetsky and Diana Masny addressed the theme "Deleuze, Pedagogy, and *Bildung*," Several single-authored books in education have used a Deleuzian perspective. . Roy (2003) wove strands of Deleuze's philosophy into a case study of teacher induction in an urban school. Semetsky's (2006) book *Deleuze, Education, and Becoming* juxtaposed Deleuze's ideas with those of John Dewey. Allan (2008) used Deleuzian notions to think about "inclusivity" in education. Olsson (2009) used a Deleuzian perspective in early childhood pedagogy. Wallin (2011) used a Deleuze and Guattari's ideas to think about curriculum through the concept of *currere.* Although situated within the emergent field of Deleuze studies in education, this dissertation differs from other works in the field. A main way it differs is that it addresses mathematics education, which few Deleuzian works have. The dissertation differs from the few other works that have addressed mathematics education in its philosophical/ontological approach as well as the way in which it takes on a form similar to that of *A Thousand Plateaus*. In sum, the dissertation contributes to the field of mathematics education and more directly critical mathematics education as well as multicultural education, educational philosophy, and Deleuze studies in education by using a Deleuzoguattarian schizoanalytic approach to press critical mathematics education to move beyond ontological underpinnings that maintain oppressive practices and transform into postcritical mathematics education.

Approach

Schizoanalysis

This dissertation enters the encounter between Deleuzian philosophy and teaching mathematics for social justice through three rhizomatic nodes or plateaus. The first plateau, “Affirming Diversity,” stages a collision between the “equity principle” of the National Council for Teachers of Mathematics (2000) *Principles and Standards* and the Deleuzian (1968/1994) concepts of difference and repetition explored in their aptly named book, *Difference and Repetition*. The second plateau, “Generating Problems,” maps shifts in the images of thought about problems in the history of mathematics education and how these shifts relate to teaching mathematics for social justice. The final plateau, “Becoming-Democratic Mathematics Education,” rethinks democratic mathematics education through Deleuze’s and Guattari’s ontology of becoming. The point of entry for each of these plateaus is the approach Deleuze and Guattari (1980/1987) developed in *A Thousand Plateaus* for mapping locations of stagnation and movement within a terrain. This approach goes by many names (schizoanalysis, rhizomatics, stratoanalysis, nomadology, micropolitics, pragmatics), each of which emphasizes a different aspect of the methodology. I will delineate key aspects of the approach with a focus on Deleuze and Guattari's concepts rhizome/ rhizomatics, strata/ stratoanalysis, pragmatics, and schizoanalysis.

Deleuze and Guattari (1980/1987) called for a rhizomatic approach to writing and thinking in contrast to what they called “arborescent” thought. Arborescent thought is based on the image of a tree deeply rooted in one location, roots and branches each a series of dichotomous bifurcations emanating from a central trunk. This is the archetypal image of the tree of knowledge captured in a book, “the classic book, as noble [and] signifying” (Deleuze & Guattari, 1980/1987, p. 5). The task of such a book based in arborescent thought is to represent or reflect the world, to describe or trace a state of affairs. This aim is misguided, according to Deleuze (1969/1995) because it relies on representation, an attempt to reproduce a static reflection of what exists. What is reproduced through such a process is only the aspects that are already taken as common sense forming an image of thought that is stagnant and fixed in place. Representation tries to imitate the world, but “mimicry is a very bad concept, since it relies on binary logic to describe phenomena of an entirely different nature” (Deleuze & Guattari, 1980/1987, p. 11). Like tree branches and roots, arborescent thought bifurcates dichotomously, following the binary logic of negation—“this” and “not this”—reducing every-thing to what it-is-and-is-not. Negation, this binary logic central to representation, is a mediated version of difference, unlike the difference-in-itself that Deleuze argued fuels the existence in the world not as static being, but rather as becoming. “The tree and the root inspire a sad image of thought that is forever imitating the multiple on the basis of a centered or segmented higher unity” (p. 16). The tree/root image of thought is static, an image of thought that has become stagnant and unproductive, locking thought into *doxa*, or commonsensical notions which have come to be taken for granted:

> Such an orientation is a hindrance to philosophy. The supposed three levels—a naturally upright thought, an in principle natural common sense, and a transcendental model of recognition—can constitute only an ideal orthodoxy. Philosophy is left without means to realize its project of breaking with *doxa*. No doubt philosophy refuses every particular *doxa.* (Deleuze, 1968/ 1994, p. 134)

Rhizomatics serve the purpose of breaking out of this stagnant image of thought. A rhizome, unlike the thick vertical trunk of a tree, is an underground horizontal stem, which in turn sends out roots and above-ground stems, creating a decentralized system of new plants expanding in every direction. No longer static, rhizomatic thought engenders movement in thought, constantly creating new nodes and once again sprouting in a new direction. "Make rhizomes, not roots," Deleuze and Guattari (1980/1987, p. 25) wrote. St. Pierre (1997) described rhizomatic writing as a form of inquiry "in which I am able to deterritorialize spaces in which to travel in the thinking the writing produces" (p. 408). Rhizomatic writing enables thinking to change. The distinction between rhizomatic thinking and arborescent thought relate to the two important concepts introduced in the introduction: mapping and tracing. Rhizomatic thinking and writing "mapping" rather than merely "tracing" points of stagnation, those which have come to be taken for granted. Mapping goes beyond representing what exists to engendering movement in thought, to creating something new through experimentation. A map "fosters connections between fields, the removal of blockages. . ., the maximum opening. . ." (p. 12). This openness allows the rhizome, the map, to have multiple entryways and to be maximally connectable: "any point of a rhizome can be connected to anything other, and must be" (p. 17).

This process of connection, a process of creation, finds and puts pressure on points of structuration, locations of stagnation. Deleuze and Guattari (1980/1987) called these points of structuration "assemblages" and explained that each point or assemblage links two heterogeneous elements. The first element is linguistic (in a broad sense) in nature and taking on a "form of expression." The second element is material (in a broad sense) in nature and takes on a "form of content." Rhizomatics poses these two questions in relation to whatever field is being mapped: "Which forms of content have come to be linked with which forms of expression? How can pressure be applied to this link to open it up, to force it to become something other than it is?" In the case of this dissertation, a rhizomatic approach poses these questions in relation to teaching mathematics for social justice: "Which forms of content have come to be linked with which forms of expression in the field of social justice mathematics education? How can pressure be applied to these links to open them up, to force them to become something other than they are?"

In addition to the image of the rhizome, Deleuze and Guattari's (1980/1987) methodology interconnects images of geological strata, geographical territories, organization of organs within bodies, linguistic pragmatics, and a reworking of psychoanalysis. As "stratoanalysis," using a geological image, the methodology identifies locations of stratification and destratification. Deleuze and Guattari (1980/1987) explained stratification thus:

> Strata are Layers, Belts. They consist of giving form to matters, of imprisoning intensities or locking singularities into systems of resonance and redundancy, of producing upon the body of the

earth molecules large and small and organizing them into molar aggregates. Strata are acts of capture. (p. 40)

In stratification, then, a particular organization of reality is created that does not, at least for the time being, allow for other organizations of reality. It "captures" a particular formulation, excluding all other possibilities, locking a particular reality in place. Destratification, on the other hand, is the process of unlocking a particular organization of reality, of unfixing the current organization, of introducing movement into the organization. This dissertation asks, "What stratifications and destratifications have occurred in teaching mathematics for social justice?"

The concept of an assemblage was noted in relation to stratoanalysis. This concept links the geological image of strata to linguistic pragmatics through the work of the Danish linguist Hjelmslev.[7] Hjelmslev (1963/1970) critiqued and reformulated Aristotle's hylomorphism. In *Physics* (2008 version), Aristotle conceptualizes all entities that are susceptible to change (e.g. physical objects) as entailing both "matter" (*hulê, as in the "hylo" in hylomorphism)* and "form" (*morphê* as in the "morph" in hymomorphism) (Shields, 2007; Aristotle, *Physics*). Said another way, that which changes is a compound consisting of substance (matter) and form. Instead of a distinction between form and substance, Deleuze and Guattari (1980/1987) read Hjemslev as saying that *content* and *expression* each have a relation to *form*, such that that-which-changes entails a *form of*

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[77] The term "assemblage" is the term typically used for the Deleuze and Guattari's (1980/1987) French term *agencement.* See Phillips (2006) for an exploration of how this came to be the case and how this translation choice has shaped the use of Deleuze and Guattari's concept.

content and a *form of expression*. In this reformulation, what is distinguished is content and expression rather than substance and form since both content and expression take on forms and have substance. Based on this reformulation, Deleuze and Guattari (1980/1987) developed the concept of double articulation as double, linked processes in the ongoing formation of reality:

> *The first articulation concerns content, the second expression.* The distinction between the two articulations is not between forms and substances but between content and expression, expression having just as much substance as content and content just as much form as expression. . . .Content and expression are two variables of a function of stratification. (p. 44)

Double articulation is the process of constituting a stratum. The stratum consists of both a form of content and a form of expression; these two forms are linked, but their elements are not identical: “The important thing is the principle of simultaneous unity and variety of the stratum: isomorphism of forms but no correspondence; identity of elements or components, but no identity of compound substances[8]” (p. 46). Deleuze and Guattari (1980/1987) called a specific instantiation of a form of content a “machinic assemblage” and a specific instantiation of a form of expression an “assemblage of enunciation” (p. 504). Neither of these (a machinic assemblage nor an assemblage of enunciation) exist on their own, but are two linked sides to a larger assemblage. Deleuze and Guattari (1980/1987) said that every assemblage is “simultaneously and inseparably a machinic assemblage and an assemblage of enunciation” (p. 504).

[8] In mathematics, an isomorphism does involve correspondence, specifically one-to-one correspondence (bijection). Taken in context, I read Deleuze and Guattari’s statement here to mean that the form of content and the form of expression have corresponding structures, organization, or relationships among elements although the elements are not identical.

Through this double articulation, each assemblage relates a form of content and a form of expression in a particular way. An example Deleuze and Guattari (1980/1987; also see Deleuze, 1986/1988) noted from Foucault's work connected the prison as a form of content with “delinquency” as a form of expression:

> Let us follow Foucault in his exemplary analysis, which, though it seems not to be, is eminently concerned with linguistics. Take a thing like the prison: the prison is a form, the ‘prison-form’; it is a form of content on a stratum and is related to other forms of content (school, barracks, hospital, factory). This thing or form does not refer back to the word ‘prison’ but to entirely different words and concepts, such as ‘delinquent’ and ‘delinquency,’ which express a new way of classifying, stating, translating and even committing criminal acts. ‘Delinquency’ is the form of expression in reciprocal presupposition with the form of content ‘prison.’ (p. 66)

In this example, delinquency and the prison are linked in a way that links the form of what is being expressed (delinquency) with a material form (the prison). The linking of these two “sides” of an assemblage is “double articulation:

> .Moreover, the form of expression is reducible not to words but to a set of statements arising in the social field considered as a stratum (that is what a regime of signs is). The form of content is reducible not to a thing, but to a complex state of things as a formation of power (architecture, regimentation, etc.). . . . .Fitting the two types of forms together, segments of content and segments of expression, requires a whole double-pincered, or rather, double-headed, concrete assemblage taking their real distinction into account. . . .We are never signifier or signified. We are stratified. (Deleuze & Guattari, 1980/1987, pp. 66-67)

The view of “delinquency” as a form of expression, as an “assemblage of enunciation,” is a conceptualization of language that differs from common notions of language as consisting of statements made by individuals. Building on the work of Foucault and Austin, Deleuze and Guattari (1980/1987) stated that “there is no individual enunciation. There is not even a subject of enunciation. Yet relatively few linguists have analyzed the necessarily social character of enunciation” (p. 80).

One theorist who did take the social character of enunciation into account was Austin. In his book *How to Do Things with Words*, Austin (1962) began by distinguishing between two types of statements: constatives, which indicate or describe a state of affairs (e.g. “The weather is warm today”), and performatives, which perform an action in their statement (e.g. “I swear” is the act of swearing). Through an investigation of the characteristics of performatives, Austin finally concluded that the constative/ performative distinction is only a special case of a more general framework in which all statements are some type of act. Deleuze and Guattari (1980/1987) identify three consequences of Austin’s work:

> 1) It has made it impossible. . .to conceive of speech as the communication of information: to order, question, promise, or affirm is not to inform someone about a command, doubt, engagement, or assertion but to effectuate these specific, immanent, and necessarily implicit acts.
> 2) It has made it impossible to define semantics, syntactics, or even phonematics as scientific zones of language independent of *pragmatics*. . . .pragmatics becomes the presupposition behind all of the other dimensions and insinuates itself into everything.
> 3) It makes it impossible to maintain the distinction between language and speech because speech can no longer be defined simply as the extrinsic and individual use of a primary signification, or the variable application of a preexisting syntax. Quite the opposite, the meaning and syntax of language can no longer be defined independently of the speech acts they presuppose. (pp. 77-78)

Communication of information is not the sole (or even main) function of language. Rather, speaking, like writing, *does* something. If the function of language is not so much to inform as to act, to do something, then the form and content of language as found in semantics (the study of word- and sentence-level meanings), syntactics (the study of sentence-level structure), and phonematics (the study of word-level sounds) cannot be separated from pragmatics (the study of the use and meaning of language in context). If language is doing, then the ways in which words and sounds function and take on meaning within sentences depend on relations among the people who are using the words and sounds. The social dimensions of language are not an extra component added onto words-as-building-blocks; rather, words always already function within social and power relations. Pragmatics, then, is the presupposition behind all other linguistic components. If pragmatics becomes the presupposition behind all other dimensions of language rather than a “trash heap” (Deleuze & Guattari, 1980/1987, p. 78) into which tangential ideas are thrown, then “stylistics” cannot be separated from “linguistics” as has traditionally been done (Deleuze & Guattari, 1980/1987, p. 97). “The reason for this,” wrote Deleuze and Guattari (1980/1987) “is that. . .when one introduces an internal pragmatics into language, one is necessarily led to treat nonlinguistic elements such as gestures and instruments in the same fashion” (p. 98). Deleuze (1993/1997) examined stylistic choices authors use to indicate character’s intonation. One option is to use dialogic markers such as “she murmured” or “he yelled.” Alternatively, the author can provide a context through characterization, for example, to indicate a character’s intonation, or write the speech itself in a way that indicates intonation. A different possibility which allows pragmatics as presupposition to erupt and shake open other linguistic components , to press further the idea that "*saying is doing"* (Deleuze, 1993/1997, p. 107), is to make language stutter rather than indicate that a character is stuttering: "This is what happens when the stuttering no longer affects preexisting words, but itself introduces the words it affects; these words no longer exist independently of the stutter, which selects and links them together through itself. It is no longer the character who stutters in speech; it is the writer who becomes *a stutterer in language*" (p. 107). Such stylistic effects, according to Deleuze (1993/ 1997), put language into a "state of *boom*, close to a *crash*" (p. 109), and constitute "a cutting edge of deterritorialization of language," a concept which is explained below (Deleuze & Guattari, 1980/1987, p. 99)

Stratification is the creation of a doubly articulated assemblage within a stratum; destratification is the breaking up of such an assemblage. Deleuze and Guattari (1980/1987) also used the term deterritorialization for destratification and reterritorialization for restratification. In a spatial description of assemblages, Deleuze and Guattari (1980/1987) said that the machinic assemblage and assemblage of enunciation are on a horizontal axis; along the vertical axis, "the assemblage has both *territorial sides*, or reterritorialized sides, which stabilize it, and *cutting edges of deterritorialization*, which carry it away" (p. 88). Deterritorialization takes place through the creation of "lines of flight." Put another way, deterritorialization is "the movement by which 'one' leaves the territory. It is the operation of the line of flight" (Deleuze & Guattari, 1980/1987, p. 508).

If every assemblage is the double articulation of a form of content (a machinic assemblage) and a form of expression (a collective assemblage of enunciation), and these assemblages undergo the processes of deterritorialization and reterritorialization, it is *machines* that enact these movements. "As a general rule," wrote Deleuze and Guattari (1980/1987), "a machine plugs into the territorial assemblage. . .and opens it to other assemblages" (p. 333). The distinction between an assemblage and a machine is that "a machine is like a set of cutting edges that insert themselves into the assemblage undergoing deterritorialization" (p. 333). It is a machine that makes new connections among assemblages, or reassembles components of deterritorialized assemblages through reterritorialization.

Stratification, according to Deleuze and Guattari, can also be thought of as "the problem of the organism—*how to 'make' the body an organism"* (p. 41). In this case, the organization of organs within the body is what makes the body an organism rather than simply a body. Deterritorializing the organism is the process of making a "Body without Organs" (BwO). Despite the name, the BwO is a body without organization of organs rather than a body without the organs themselves. Deleuze and Guattari (1980/1987) posed the following questions:

> What is your body without organs? What are your lines? What map are you in the process of making or rearranging? What abstract line will you draw, and at what price, for yourself and for others? What is your line of flight? What is your BwO, merged with that line. . . .Are you deterritorializing? Which lines are you severing, and which are you extending or resuming? (p. 203)

This dissertation asks, “What organs have been constituted within the body of mathematics education? What are its lines of flight? Is it deterritorializing? If so, how? Which lines should we severe, and which should we extend or resume?”

The Body without Organs as a body whose organs are not organized or stratified is a biological image. However, the concept of the Body without Organs is also connected to Deleuze and Guattari's critique and reworking of psychoanalysis, that is, schizoanalysis. In the concept of the Body without Organs, Deleuze and Guattari reapplied a term used by Antonin Artaud (1947/1976) to notions from Melanie Klein's psychoanalytic object-relations theory (Buchanan, 2010). Deleuze and Guattari (1980/1987) critiqued psychoanalysis: “it subjects the unconscious to arborescent structures, hierarchical graphs. . .the phallus tree. . . .Psychoanalysis's margin of maneuverability is therefore very limited” (p. 17). In contrast, schizoanalysis, they argued, “treats the unconscious as an acentered system. . .as a machinic network of finite automata (a rhizome). . . .The issue is to *produce the unconscious* and with it new statements, different desires: the rhizome is precisely this production of the unconscious” (p. 17-18). Schizoanalysis (rhizomatics, stratoanalysis, pragmatics. . .), then, is also an analysis of desire. In fact, according to Deleuze and Guattari, “The BwO is desire; it is that which one desires and by which one desires” (p. 165). In investigating the bodies with and without organs in mathematics education, this dissertation is also an analysis of desire: “What flows of desire circulate in the field of teaching mathematics for social justice? What flows of desire should we open?” Schizoanalysis is not merely an analysis of current flows of desire, but is a way direct flows of desire in new directions. As Deleuze and Guattari (1980/1987) asserted:

> Schizoanalysis, as the analysis of desire, is immediately practical and political, whether it is a question of an individual, group, or society. For politics precedes being. . . .Schizoanalysis is like the art of the new. Or rather, there is no problem of application: the lines it brings out could equally be the lines of a life, a work of literature or art, or a society, depending on which system of coordinates is chosen." (203-204).

It is fitting that Deleuze and Guattari's (1980/1987) methodology is multivocal, shifting names and images throughout *A Thousand Plateaus*. It is a methodology that is constantly deterritorializing and then reterritorializing on a new image. It is a methodology in which what eternally returns is difference[9]. It is a methodology of becoming[10]. It is a methodology that provides an entry point into each of the nodes or plateaus of the encounter between Deleuzian philosophy and teaching mathematics for social justice.

[9] Deleuze's conceptualization of the eternal return of difference is explained and explored in the plateau entitled "Affirming Difference."

[10] Deleuze's (and Guattari's) ontology of becoming is explained and explored in the plateau entitled "Becoming-Democratic Mathematics Education."

Plateaus

1. Affirming Difference

In the National Council for Teachers of Mathematics' *Principles and Standards for School Mathematics* (NCTM, 2000), the “equity principle” stated that “excellence in mathematics education requires equity—high expectations and strong support for all students” (p. 12). The *Principles and Standards* strongly emphasized that “mathematics can and must be learned by *all* students” (p. 13). In fact, within the approximately two pages devoted to the equity principle, the phrase “all students” (or a similar phrase) was repeated 20 times. Such forceful emphasis raises questions about current notions of equity in mathematics education. Why did NCTM put such strong emphasis on “all students”? How has this come to be? This plateau stages an encounter between this notion of equity and Deleuze’s concept of *affirming difference*. Following Deleuze and Guattari’s call to embark on a line of flight by “lodg[ing] yourself on a stratum. . .find potential movements. . .possible lines of flight” (p. 161), this plateau lodges in a stratum constituted by the notion of equity as found in the *Principles and Standards* of the National Council of Teachers of Mathematics (2000). According to Deleuze and Guattari (1980/1987), it is “through meticulous relation with the strata that one succeeds in freeing lines of flight” (p. 161). The equity stratum provides a launching site for freeing lines of flight that will lead to Deleuze and Guattari’s (1968/1994) concept of *affirming* *difference.* This line of flight moves toward (re)conceptualizing the notion of difference through a Deleuzoguattarian lens in a way that moves toward an anti-oppressive ontology and ethics of mathematics education.

1.1 The Talent-Competition Machine

What is it that the forceful repetition of “all” in the “Equity Principle” pushes against and moves toward? The *Principles and Standards* themselves pointed out an image of thought in mathematics education against which the equity principle pushes: “The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics” (p. 12). This image of thought captured, in the terms Deleuze and Guattari use in *A Thousand Plateaus*, a double articulation connecting a form of expression (a collective assemblage of enunciation) with a form of content (a machinic assemblage). In the United States and Britain, the form of expression consists of a stratum such as the following, which might be called the “talent statum”: mathematician—talent—male—masculine—White—suburban—affluent/ middle class—English-speaking. . . . The form of content consists of a stratum such as this one: student/teacher interactions—tracking—scheduling—honors/ advanced placement courses—timed tests—independent practice—decontextualized problems—competitions. . . .This stratum might be called the “competition stratum.” These two strata, or sides of a stratum, are linked through double articulation to form a talent-competition machine at work in a particular image of thought in mathematics education. Walkerdine (1989) examined the gendered aspect of this double articulation in Britain on a micropolitical level. She found that “girls are never unproblematically allowed to enter the categories teachers consistently castigate them for not belonging to: they might be admonished for not 'breaking set' or 'having flair,' but in fact teachers make it very difficult for them to do so” (p. 49). In order to be seen as talented or “having flair” by teachers, students had to be seen as active and willing to break the rules (“breaking set”). Girls found themselves in a double bind: those who “followed the rules,” for example, by showing their work, could not be seen as having talent since “having flair” required breaking the rules; on the other hand, those girls who did fit the profile of “having flair” by challenging the teacher's assertions or skipping steps in their work were “met with resistance” from teachers that boys did not typically face. The micropolitical movements of student/teacher interaction within the competition assemblage prevented girls from entering into the talent assemblage.

On a slightly broader level, school space in the United States often becomes stratified in ways that exclude English Language Learners from honors and advanced-placement level mathematics classes or instruction through scheduling. According to Walqui (2000),

> Typically what happens is that sheltered courses (which are supposed to cover mainstream curricula using texts with special pedagogical accommodations for English learners) and subject matter courses taught in students' native languages are considered watered-down versions of the demanding regular courses. In addition, classes are often offered without the support of appropriate materials [and] classes are crowded (for example, a class might start with 32 students, but every new immigrant student that arrives throughout the year is added to it). (p. 58-59)

Even students placed in mainstream mathematics courses may not have access to honors or advance placement courses if scheduling decisions do not consider scheduling conflicts between ESL language-focused courses and honors/ advanced-placement mathematics courses. Harklau (1994) pointed out that those in charge of making course placement decisions for newcomers faced a dilemma:

> Because there was no truly appropriate place for [newcomer students] in tracked mainstream classes, teachers and counselors were faced with a choice between the lesser of two evils in placement decisions. They could, on the one hand, put students in low track classes, which were linguistically undemanding, but also of poor quality linguistically and academically. Alternatively, they could put students in higher tracks, which provided better learning opportunities, but that could also be very arduous for students who were still learning the language of instruction. In either case, their decisions necessarily confounded perceptions of students' language proficiency and of academic ability. (p. 234)

This course placement process often results in multilingual high school students not fulfilling the requirements to apply for college. For example, it was discovered upon examining the transcript of one otherwise strong candidate for college, Chuy, that although he had taken eight semesters of math, “none of them counted for university admission because all of them were different versions of basic, non-college-preparatory math under different names” (Walqui, 2000, p. 63). Far from being an exceptional case, Walqui (2000) noted that Chuy's experience forms a “pattern common among immigrant students learning English in California high schools” (p. 63). Highlighting the way in which class placement links into the competition-assemblage, Rosenbaum (1976) has compared the course placement process to a tournament in which “students are gradually eliminated from the competition for the highest status placements throughout the process of schooling and once eliminated, may not advance” (Harklau, 1994, p. 232).

The talent-competition machine linking the talent-stratum and the competition-stratum is perhaps best exemplified in the tradition of mathematics competitions, the most “prestigious and important” (Kenderov, 2006) of which is the International Mathematical Olympiad (IMO), a global competition for high-school students. In a milieu influenced by the founding of national mathematical societies, the inception of the International Congress of Mathematicians, the revival of the Olympic Games, and the spread of regional and national mathematics competitions throughout Eastern Europe by the early twentieth century, the first IMO took place in Romania in 1956 (Kenderov, 2006). In 2010, the IMO involved 517 competitors from 97 countries (Mathematical Association of America, n.d.). One of the main stated purposes of this type of competition, as stated on the website of the United States of America Mathematical Olympiad, is to “indicate the talent of those who may become leaders in the mathematical sciences of the next generation” (Mathematical Association of America, para. 1). Here the “talent-stratum” and the “competition-stratum” were explicitly linked in a talent-competition assemblage. On the global level of the IMO as on the micropolitical level examined by Walkerdine (1989), the “talent-stratum” and “competition-stratum” are linked in a way that makes it difficult for girls to enter the “talent-stratum”: of the 517 competitors of the 2010 IMO, 470 of them were boys and 47 of them were girls. A full 10 times as many boys as girls “were talented enough” to compete.

The format of mathematics competitions has become frozen into a particular image of mathematics that emphasizes certain terms of the competition-assemblage: such *competitions* consist of *timed tests* in which students *work individually* on *decontextualized problems.* The contemporary IMO competition shows remarkable resemblance in format to its predecessors from one hundred years earlier. Kenderov (2006) described an 1894 competition in Hungary which is "widely credited as the forerunner of contemporary mathematics. . .competitions for secondary students" (p. 1586). In this competition, secondary students were allotted four hours to solve three problems individually, that is, without interaction with other students or teachers. The set of tasks consisted of decontextualized problems designed to require creativity, mathematical thinking, and often proofs rather than simple technical skills. A similar structure is in place for the IMO, which takes place over the course of two consecutive days. On each day, four and a half hours are allotted for students to solve three problems. Again, problems are decontextualized and require "a significant degree of inventive ingenuity and creativity" (Kenderov, 2006)[11]. Like its predecessor in 1894, the IMO is "a competition for *individuals"* (Kenderov, 2006, p. 1586), although competition among nations is stimulated through unofficial rankings of countries based on the number of medals awarded. This format taps into a particular image of thought in mathematics that has been constituted as *the only* image of mathematics (along with the ancillary notion that

[11] Hence, in this double articulation of the talent and competition components of the talent-competition assemblage, only those previously considered "talented" (that is, for example, ten times as many boys as girls) are presented with problems requiring "a significant degree of inventive ingenuity and creativity" rather than simply technical skills. Sriraman and Steinthorsdottir (2007) argue that the first step toward a synthesis of "equity" and "excellence" is to "give equal opportunity" (p. 99) in mathematics education. Sriraman (2008) actualized "giving equal opportunity" in a research study in which both students identified and not identified as "gifted" kept math journals in which they attempted to solve a set of challenging, non-routine problems.

only certain *people* belong in the image). An ethnomathematics perspective points out the way in which this image of thought of mathematics has come to be seen as common sense:

> The discipline known as mathematics is an ethnomathematics that originated and developed in Europe, having received some contributions from Indian and Islamic civilizations, and that arrived at its current form in the 16^{th} and 17^{th} Centuries, from which point it began to be carried throughout and imposed upon the rest of the world. Today, this mathematics acquires a character of universality, above all due to the predominance of science and modern technologies which were developed beginning in the 17^{th} Century Europe, and which serve to support current economic theories. (D'Ambrosio, 2001, p. 56)

The universalization of this particular image of thought in mathematics connects with the way the “story of mathematics” is told:

> The great heroes of mathematics, that is, those who are historically pointed to as being responsible for the advancement and consolidation of this science, are identified in Ancient Greece, and later in the Modern Age, in the countries of central Europe, above all England, France, Italy, and Germany. The names most often remembered are Pythagoras, Euclides [*sic*], Descartes, Galileu [*sic*], Newton, Leibniz, Hilbert, Einstein, and Hawkings. They are ideas and men who originated north of the Mediterranean. (D'Ambrosio, 2001, p. 57-58)

D'Ambrosio (2001) noted similar processes in the universalization of “jeans” as a form of clothing replacing traditional clothing world-wide and the spread of Coca-Cola products. However, he observed, the universalization of this particular image of mathematics differs in that

> mathematics has a connotation of infallibility, rigor, and precision and of being an essential and powerful instrument of the modern world, so that its presence excludes other ways of thinking. (p. 58)

1.2 Liberal Multicultural Education and the All-Regardless Assemblage

Given such an efficient and effective machine working on every level from the micropolitical to the global in order to link the talent-assemblage and the competition-assemblage in such a way that only certain people can be seen as having mathematical talent, as the future "leaders in the mathematical sciences of the next generation," the claim in the *Principles and Standards* that "mathematics can and must be learned by *all* students" (p. 13) is in fact momentous. The stylistic effect of twenty repetitions of "all" (all—all—all—all—all—all—all—all—all—all—all—all—all—all—all—all—all—all—all—all), then, introduces a stutter into the language of mathematics education, a stutter which "carves out a nonpreexistent foreign language within" (Deleuze, 1993/1997, p. 110) the language of mathematics education, puts the language of mathematics education "into a state of *boom*, close to a *crash*" (p. 109), creates a line of flight away from the smoothly running talent-competition machine. Yet, if the echoing boom of *all-all-all* forms the cutting edge of deterritorialization, what remains stratified within the equity principle?

The resonating boom of *all,* while creating a line of flight away from the talent-competition machine, remains linked to the key word *regardless*: "*All* students, *regardless* [emphasis added] of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics" (NCTM, 2000, p. 12). This statement resonates closely with liberal multicultural education which, according to Kubota (2004), "endorses the idea that *all* individuals, *regardless* [emphasis added] of their background, can socially and economically succeed as long as they work hard" (p. 32). This "all-regardless-assemblage" constitutes a "formation thrown up by the corpus in question," (Deleuze, 1986/1988, p. 18) by liberal multicultural education infused with the "anonymous murmur" (Deleuze, 1986/ 1988, p. 18)[12] of liberal philosophy[13]. The central concept in liberal philosophy is the idea of "liberty" or "freedom," which in liberal theory is accepted as inherently good (Gaus & Courtland, 2003). Because "liberty" is considered inherently good, liberal theorists argue that if anyone wants to restrict "liberty" in any way, that person must prove that there is a good enough reason for the restriction (the Fundamental Liberal Principle) (Gaus, 1996, p. 162-166; Mill, 1859/1991, p. 472). One of the main tasks of liberal theory is to conceptualize "liberty" or "freedom." Positive liberty has been conceptualized as "freedom to" do something, e.g. freedom to go to school, whereas negative liberty has been conceptualized as "freedom from" a constraint, e.g. freedom from being coerced to go to school (Berlin, 1969). The negative concept of liberty implies that people are free if they can "pursue their aims *not* [emphasis added] constrained by others" (Stone, personal communication, 2004). An important idea in liberal thinking is the idea that in order to be free, people must be autonomous, meaning that they choose their own actions (Gaus & Courtland, 2003). Inherent in this idea is a focus on individuals rather than groups. "Power is in the hands of individuals" (Stone, personal communication, 2004) rather than existing at a group level. A related

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[12] In *Foucault*, Deleuze (1986/1988) reads Foucault's work as suggesting that knowledge is not the domain of an individual knower justified in his or her belief that such-and-such is true (as in a liberal conception of knowledge), but rather consists of the anonymous murmur at any given time, of "formations thrown up" in the disjunctions between subjects and between objects, of the "language [which] coagulates around a corpus only in order to facilitate the distribution or dispersion of statements (p. 18).

13 The following descriptions of liberal philosophy and critical theory, and how they are infused into liberal multicultural education and critical multicultural education, respectively, draw on an unpublished paper I wrote with Mina Kim and Lei Zhang entitled "A Multicultural Education Framework for Providing Equitable Education for English Language Learners."

central concept is that of “individual rights.' A “right” is something that is due to a person by law, tradition, or nature. An “individual right” is a right belonging to a single person. In order for an individual to be free, according to liberal philosophy, his or her individual “rights” must not be infringed upon. One of the government's basic tasks, then, is to protect the individual rights, and thereby the freedom, of individuals (Gaus & Courtland, 2003). A main means to do this is through democratic representation (Stone, personal communication, 2004). Another idea stemming from a focus on the individual is the pluralist idea that any society is composed of individuals, each of whom has her or his own “aims, interests, and conception of the good” (Sandel, 1982, p. 1-7). If individual freedom is not to be restricted, then people must be free to pursue different goals and live different ways without others imposing certain aims and concepts of the good upon them (Gaus & Courtland, 2003). Society, then, “being composed of a plurality of persons. . .is best arranged when it is governed by principles that do not themselves presuppose any particular conception of the good” (Sandel, 1982, p. 1-7). (The exception to this rule in liberal theory, of course, is the principle that 'liberty' is itself inherently good). In summary, liberal philosophy attempts to define “liberty” or “freedom” and proposes that “liberty” is inherently good, emphasizes the individual over groups, focuses on individual rights, views the government's role as protecting individual rights, and supports pluralism.

The Liberal Multicultural Education Machine links the ideas of liberal philosophy within the context of education to construct the “all-regardless assemblage,” asserting that *all* students, that is, each individual student, *regardless* of his or her background, must be free. A positive definition of liberty implies that students should be free to take chosen courses of actions regardless of their background. A negative definition of freedom implies that students should be free from restrictions on their activities (to the degree possible), regardless of their background. The implication of these applications of positive and negative definitions of freedom to multicultural education is that students should be treated equally (the same) regardless of their background. The “all-regardless” assemblage, then, in Deleuzian terms, institutes a “generality [which] expresses a point of view according to which one term may be exchanged or substituted for another” (Deleuze, 1968/1994, p. 2). Conceptualizing freedom in this way leads to “difference-blindness,” or the idea that we should not notice (regard) or care about the various ways in which we differ from each other, such as race, culture, class, gender, sexuality, or language(s) spoken. As Kubota (2004) pointed out, “A liberal view of multiculturalism often emphasizes common humanity and natural equality across racial, cultural, class, and gender differences. . . .In the school context, this logic is played out as universal, neutral, and difference-blind institutionalism” (p. 32).

While liberal multicultural education tends to lead to “difference-blindness,” Kubota (2004) pointed out a paradox that emerges when liberal theory is applied to multicultural education: “Along with the focus on commonality and universal humanity, liberal multiculturalism paradoxically also has a tendency to emphasize cultural differences and culturally unique characteristics” (p. 34). This pluralistic aspect of liberal multicultural education tends to

> celebrate. . .superficial aspects of culture, such as artifacts, festivals, and customs, and they are treated in decontextualized and trivialized manners divorced from the everyday life of people

> and the political struggle to define cultural identity. . . .In focusing on only the customs and traditions of different peoples, the culture of the Other is often exoticized and reduced to neutral objects for one to respect and appreciate. At the same time, the Other people and cultures are essentialized as something homogenous, traditional, and authentic. (Kubota, 2004, p. 35)

This pluralist thrust to liberal multicultural education leads to an attempt to represent a diversity of cultures in the curriculum. “Cannot difference,” liberal educators ask with Deleuze (1968/1994), “become a harmonious organism and relate determination to other determinations within a form—that is to say, within the coherent medium of an organic representation” (p. 29)? If the great “Holidays (or historic events) and Heroes” of mathematics have been represented in mathematics curricula as those from Ancient Greece and modern Europe (D’Ambrosio, 2001) can’t we just toss in a few “Brown Holidays and Heroes” (Banks, 1991; Nieto, 1995)? If mathematics education has overlooked queer people and issues, can’t we just “add queers and stir” (Rands, 2009)? Here it is supposed that expanding the boundaries of who is included and represented in the curriculum will in itself establish educational freedom through representation. Thus, liberal multicultural education succumbs to three of what Deleuze (1968/1994) termed the four “shackles” of mediation connected with representation: identity, analogy, and resemblance (p. 29). The pluralist liberal idea of “representing a diversity of cultures in the curriculum” conceives of cultural difference as categorical or generic difference; one’s cultural identity is determined based on inclusion within a particular cultural category. This formulation follows the Aristotelian “method of division” which begins with the largest categories and then divides each category into subcategories, eventually leading to a subcategory including a single individual whose identity is the summation of predicates "said of" all of the broader categories (Aristotle, Metaphysics; Aristotle, Categories). This process of division, according to Deleuze's reading of Aristotle, relies on judgment (distributing or partitioning concepts and establishing a hierarchy of categories and subcategories), which ties specific difference to generic difference and identity to analogy:

> Analogy is itself the analogue of identity within judgment. Analogy is the essence of judgment, but the analogy within judgment is the analogy of the identity of concepts. That is why we cannot expect that generic or categorical difference, any more than specific difference, will deliver us a proper concept of difference. Whereas specific difference is content to inscribe difference in the identity of the indeterminate concept in general, generic (distributive and hierarchical) difference is content in turn to inscribe difference in the quasi-identity of the most general determinable concepts; that is, in the analogy within judgment itself. The entire Aristotelian philosophy of difference is contained in this complementary double inscription, both grounded in the same postulate and together drawing the arbitrary boundaries of the propitious moment[14] (1968/1994, 33-34).

While the "large units" or categories are "determined according to relations of analogy," in Deleuze's reading of Aristotle, "the small units. . .are determined by a direct perception of *resemblances*, which suppose a continuity of sensible intuition in the concrete representation" (p. 34). Hence, the "method of division" ties together analogy, resemblance, and identity (and opposition, which will be addressed in the context of critical multicultural education shortly) in the name of

[14] The original French for "propitious moment" is "*l'heureux moment*" (1968, p. 51). *Heureux* can mean "happy," "fortunate," "good," or "excellent." Deleuze uses this term throughout *Difference and Repetition* in an ironic way to refer to the moment in Greek philosophy in which representational thought emerged. According to Gasché (2007), "However paradoxical as it may sound, for Deleuze, the propitious Greek moment, rather than a suspension, is a triumph of *doxa*" (p. 260).

representation. This mediation of difference through representation, according to Deleuze, is the "mistake of the philosophy of difference from Aristotle to Hegel via Leibniz, [which] lay in confusing the concept of difference with a merely conceptual difference, in remaining content to inscribe difference in the concept in general" (p. 27).

In contrast to the "method of division," Deleuze (1968/1994), drawing on Nietzsche's reworking of the idea of the "eternal return," argued that "difference in itself" is prior to generic and specific difference:

> The eternal return does not appear second or come after, but is already present in every metamorphosis, contemporaneous with that which it causes to return. Eternal return relates to a world of differences implicated one in the other, to a complicated, properly chaotic world *without identity.* (p. 57)

Putting "difference in itself" first

> can be satisfied only at the price of a more general categorical reversal according to which being is said of becoming. . . .That identity not be first, that it exist as a principle but as a second principle, as a principle *become*; that it revolve around the Different: such would be the nature of a Copernican revolution which opens up the possibility of difference having its own concept, rather than being maintained under the domination of a concept in general already understood as identical. (p. 41)

If it is difference which is first, then, "returning is being, but only the being of becoming" (p. 41). An ontology of difference is an ontology of becoming and an ontology of becoming is an ontology of repetition:

> The eternal return does not bring back "the same", but returning constitutes the only Same of that which becomes. Returning is the becoming-identical of becoming itself. Returning is thus the only identity, but identity as a secondary power; the identity of difference, the identical which belongs to the different, or turns around the different. Such an identity, produced by difference, is determined as 'repetition'. Repetition in the eternal return, therefore, consists in conceiving the same on the basis of the different. (Deleuze,

1968/1994, p. 41)

Analogy works in another way in liberal multicultural education. The pluralist vision sees each culture as analogous to every other culture. The Liberal Multicultural Education Machine with its “all-regardless assemblage” is once again at work instituting a “generality” in which “one term [or culture] may be exchanged or substituted for another” (Deleuze1968/1994, p. 2). Here, those who subscribe to the liberal multicultural education perspective often fall into the trap of believing that if each culture is analogous to all others, then individuals from different cultural backgrounds *are indeed* treated equally, (except, perhaps, for certain individual instances of discrimination). Therefore, the liberal multicultural education approach overlooks the “social and economic inequalities and institutional racism that actually exist in schools and society” (Kubota, 2004, p. 33). Frankenberg (1993) suggested the concept of “power evasion” to describe this tendency to ignore issues related to differences in power among groups.

“Power evasion” also has another source within the philosophy of liberalism. Liberal theorists view power as being in the hands of individuals rather than groups. In the school setting, this means that individual students, teachers, and administrators possess power. Individual prejudice leads to discrimination, which is a misuse of power that restricts the freedom of other individuals. Individual prejudiced teachers and administrators make unfair decisions that restrict the freedom of individual students with a particular characteristic, e.g. a particular cultural or linguistic background. According to the liberal perspective, the way to combat this problem is to address prejudice at an individual level, or as stated in the Equity Principle of the *Principles and Standards*, “teachers also need to understand and confront their own beliefs and biases” (p. 14). This focus on power only at the level of the individual (along with difference-blindness) overlooks issues of power and privilege at the group level. Specifically, Kubota (2004) noted that “[l]iberal multiculturalism, influenced by the dominant ideology of individualism and liberal humanism, tends to obscure issues of power and privilege attached to the white middle class” (p. 35). When the unequal power relations between groups that lead to political, economic, educational, and other inequalities in society are ignored, the resulting inequalities are perpetuated.

1.3 Avoiding Difference-Blindness: Critical Multicultural Education

Critical multicultural education, rooted in critical theory, has developed as an alternative to the Liberal Multicultural Education Machine which more viably addresses issues of privilege and oppression in schools (e.g. Kubota, 2004; Frankenstein, 1990; Skovsmose, 1994; Nieto, 1995; Ladson-Billings & Tate, 1995; Gutstein, 2006). Critical theory began as the “[t]heoretical approach of the Frankfurt school of social philosophers,” (Kemerling., 2002, para. 24) a group that originated in Germany in the 1930’s and soon moved to Columbia University in New York to escape the Nazis (Antonio, 1983; also see Lowenthal, 1981; Jay, 1973). Although notably marked by heterogeneity (Antonio, 1983; Löwenthal, 1981; Kellner & Roderick, 1981; Jay, 1973; Leiss, 1974; Buck-Morss, 1977), critical theory is consistently “based on meta-assumptions that derive from Hegel’s dialectics, modified by Marxist materialist critique (Hegel, 1807/1967; Marx, 1844/1964; Schroyer, 1975; Antonio, 1981)” (Antonio, 1983, p. 345). A key idea from the work of Hegel (1807/1967)) and Marx (1867/1976) is the idea that history evolves dialectically. The concept of a dialectical process involves a progression from a thesis (an idea or event) to an antithesis (a reactionary idea or event) and finally to a synthesis (the resolution of the tensions between the thesis and antithesis to form a new state). The synthesis then serves as a new thesis and the process repeats itself. The three moments of dialectical progression (thesis, antithesis, and synthesis) do not necessarily designate distinct points in time and can occur simultaneously (Hegel, 1900). As Antonio (1983) noted,

> Hegel's philosophy, which stresses immanent principles of contradiction, change, and movement, constitutes an alternative to the formal and static nature of Kantianism . . . .For Hegel, the nature of being is characterized by the subject continuously creating, negating, and recreating itself and its object world. . . .In Hegel's thought, emancipatory values are given an historical, rather than a transcendental, foundation. (p. 343-344)

While Hegel (1900) saw this dialectical process as occurring at the level of the mind or “reason,” Marx believed that this dialectical process was rooted in the material world, specifically in economic processes. For Marx (1867/1887), the moments in the dialectical process were economic classes struggling against one another. In the words of Antonio (1983), “Marxian dialectics is Hegelian thought stripped of its phenomenological idiom and re-formulated in a materialist framework” (p. 344). This focus on classes or groups of people rather than individuals has continued in critical theory and contrasts with liberal theory’s focus on the individual. Critical theory “begins with the institution of the state out of which. . .groups. . .acquire power relative to other groups” (Stone, personal communication, 2004). This power differential between groups leads to oppression. Though critical theory is historical, its ultimate aim is not historical description, but rather the “location of contradictions and conditions that contain emancipatory possibilities” (Antonio, 1983, p. 344). Critical theorists see this struggle against oppression as taking place within a dialectic between structure and agency (Stone, 2004; Giroux, 1983). The structure consists of the material constraints in the world. Agency is a person’s or group’s capacity to make choices in order to act. While critical theorists believe that people have the capacity to make choices and to act, they believe that this capacity is limited (though also, some believe, enabled) through the existing conditions of the world. In other words, “the values that define possible social structures belong to an immanent contradiction between ideology and social reality, and any consideration of ‘what can be’ is mediated by a detailed analysis of existing material, cultural, social, and political constraints” (Antonio, 1983, p. 344).

1.4 Critical Multicultural Education: A Way Around the Four Shackles?

Does a critical-theory based critical multicultural education circumvent the four shackles of mediation identified by Deleuze? Can a focus on privilege and oppression at a group level combined with Hegelian and/or Marxian dialectics “save difference from its maledictory state” (Deleuze, 1968/1994, p. 29)? Noblit (2004) noted that critics of the way in which critical theory has played out in ethnography have found that critical theory has been “in itself a form of hegemony—patriarchal, Eurocentric, individualistic, and White” (p. 192; see also, e.g., Bennett & LeCompte, 1990; Delamont, 1989; Lather, 1986; Ellsworth, 1989). Moreover, critical ethnographers’ response to such criticism has been to construct a coherent story based on the parallelist position that each form of domination was equivalent and functioned in basically the same way as did class (Morrow & Torres, 1998; Noblit, 2004). This resulted in “a theory that could never be wrong” (Ladwig, 1996, p. 40, as cited in Noblit, 2004); in short, “critical ethnography has ironically come to be a form of ideological practice” (Noblit, 2004, 192; also see Wexler, 1987). This is an example of the way in which desire “is never an undifferentiated instinctual energy” as Deleuze and Guattari (1980/1987) explained in *A Thousand Plateaus*, “but itself results from a highly developed, engineered setup rich in interactions: a whole supple segmentarity that processes molecular energies and potentially gives desire a fascist determination” (p. 215). Deleuze and Guattari (1980/1987) warned, “Leftist organizations will not be the last to secrete microfascisms” (p. 215), for “it’s too easy to be an anti-fascist on the molar level, and not even see the fascist inside you, the fascist you yourself sustain and nourish and cherish with molecules both personal and collective” (p. 215). A line of flight can be a great site of creation, or it can turn into a terrible “line of destruction” (p. 423). The plan(e) of consistency can be one that “constitutes itself, even piece by piece” or it can turn into “a plan(e) of organization and domination” (p. 423). Again, a warning: “there is communication between these two lines or two planes. . .each takes nourishment from the other, borrows from the other” (p. 423).

Critical theory, whether infused within critical ethnography or critical multicultural education, tends toward hegemony, ideological practice, even (micro)fascism in its maintenance of a reliance on the four shackles of mediation connected with representation. Despite avoiding difference-blindness and power evasion through acknowledging privilege and oppression at a collective level, a critical-theory based Critical Multicultural Education Machine maintains the Aristotelian “method of division” in which one’s identity is determined based on inclusion within a particular class. Critical theory, then, faces the same ontological issues with identity, analogy, and resemblance as the pluralist Liberal Multicultural Education Machine.

1.5 Deleuzian Dialectics: Repetition, not Opposition

The fourth shackle, opposition, however, takes on special significance as the foundation of Hegelian and Marxian dialectics. Deleuze (1968/1994) raised the question of opposition through the example of size: “The question arises, therefore, how far the difference can and must extend—how large? how small?—in order to remain within the limits of the concept” (p. 29). Aristotle (1942) distinguished four types of “opposites”: “correlatives” (e.g. double/ half), “contraries” (e.g. bad/ good), “privatives/ positives” (e.g. blindness/ sight), and “affirmatives/ negatives” (e.g. he sits/ he does not sit). According to Deleuze’s (1968/1994) reading of Aristotle, the “most perfect, the most complete” form of opposition of these four is contrariety, which “alone expresses the capacity of the subject to bear opposites while remaining substantially the same” (p. 30), that is, within a category or “genus.” “In short,” Deleuze read Aristotle as concluding, “contrariety in the genus is perfect and maximal difference, and contrariety in the genus is specific difference” (p. 31). In Aristotle’s conceptualization of difference, specific difference, that is, difference at the level of “species,” is an intermediate between generic difference (difference at the level of the genus) and individual difference[15]. Deleuze noted that "difference" above and below specific difference "tends to become simply otherness" (p. 30), that is, generic difference is too large of a difference and individual difference is too small of a difference to constitute "opposites." However, Deleuze contested Aristotle's conception of specific differences as the perfect form of difference: "It is. . .evident that specific difference is the greatest only in an entirely relative sense. Absolutely speaking, contradiction is greater than contrariety—and above all, generic difference is greater than specific" (p. 31). It is Hegel who saw contradiction as a perfect form of opposition, yet, as Deleuze noted, "It seems that, according to Hegel, 'contradiction' poses very few problems. It serves a quite different purpose: contradiction resolves itself and, in resolving itself, resolves difference by relating it to a ground. Difference is the only problem" (p. 44). Deleuze, (via Nietzsche and Kierkegaard) objected to Hegel in that "he does not go beyond false movement—in other words, the abstract logical movement of 'mediation'" (p. 8). Deleuze along with Nietzsche and Kierkegaard

> want to put metaphysics in motion, in action. They want to make it act, and make it carry out immediate acts. It is not enough, therefore, for them to propose a new representation of movement; representation is already mediation. Rather, it is a question of producing within the work a movement capable of affecting the mind outside of all representation; it is a question of making movement itself work, without interposition, of substituting direct signs for mediate representation. (p. 8)

Unmediated movement, Deleuze concludes, is "*not opposition. . .*but repetition" (p. 10). Hence, the heart of a Deleuzian dialectic consists of the positivities of

[15] The field of biology later adopted the terms "genus" and "species," but this biology-specific connotation is lacking in Aristotle's work and Deleuze's reading of Aristotle.

difference and repetition rather than negation in the form of opposition or contradiction.

1.6 Rhythmic Repetition and Affirming Difference

In fact, it is repetition that has moved critical theory beyond its Frankfurt School conceptualization. The insertion of “race” into critical theory to form Critical Race Theory (CRT) is not merely the parallelist move Morrow and Torres (1998) described. Rather what has returned in CRT is not the *same old* critical theory with race tossed in for good measure, but rather something *different*. A critique of “private property” returns as something *different* in the context of the United States, in which the legal system was based on the idea that those of European ancestry had the right to enslave those of African ancestry as property (e.g. Ladson-Billings & Tate, 1995; Delgado & Stefanic, 2001; Leonard, 2008). Difference returns again and again in TribalCrit’s emphasis on colonization (Brayboy, 2005) and LatCrit’s emphasis on language, immigration, and ethnicity (e.g. Solórzano & Bernal, 2001). In this rhythmic repetition (Deleuze, 1968/1994), “there is no representative concept” (p. 20). Although “Critical” seems to constitute a “Same” that returns, that is, the “identity of the nominal concept which explains the repetition of a word” (p. 21), this cadence-repetition (Deleuze, 1968/1994, p. 21) is “only the outward appearance or the abstract effect of the rhythmic repetition” (p. 21). Repetition of the same “appears only in the sense that another repetition is disguised within it, constituting it and constituting itself in disguising itself” (p. 21), that is, the rhythmic repetition of the different. In this proliferation of simulacra, the simulacrum “is the instance which includes a difference within itself” (p. 69). Unlike Plato’s simulacra as imperfect copies of a perfect original, for Deleuze, “There is no first term which is repeated. . . .Simulacra are the letter of repetition itself. Difference is included in repetition by way of disguise. . . .” (p. 17). Peeling off the disguise, the mask, does not reveal an identity of what/who is beneath, but instead links disguise with disguise, mask with mask, simulacrum with simulacrum in assemblage: “Repetition is truly that which disguises itself in constituting itself, that which constitutes itself only by disguising itself. It is not underneath the masks, but is formed from one mask to another. . .The masks do not hide anything except other masks” (p. 17). This movement from Critical Theory to Critical Race Theory to TribalCrit and LatCrit, then, is not a matter of “dissolving tensions in the identical” (p. 50) in a Hegelian/ Marxian dialectic. It is not a matter of negation, for “negation, like the ripples in pond, is the effect of an affirmation which is too strong or too different” (p. 54). Just as identity comes second, as an effect of difference and the cadence repetition of the Same is only an envelope produced by the rhythmic repetition of the different, negation is secondary in a Deleuzian dialectic, an effect of an affirmation of difference. It is through affirmation of difference that unmediated movement makes metaphysics take action. It is affirmation of difference which sets revolution in motion, as “revolution never proceeds by way of the negative” (p. 208). It is through affirmation of difference that practical struggle stirs, as “practical struggle never proceeds by way of the negative but by way of difference and its power of affirmation” (p. 208). While the “sociopolitical context of [liberal and critical] multicultural education” calls for “affirming diversity” (Nieto, 1992) and, as stated in the “Equity Principle,” “accommodating differences” (NCTM, 2000), for Deleuze, it is “affirming difference" that must and does occur prior to either of these. Deleuze says that "difference is not diversity. Diversity is given, but difference is that by which the given is given, that by which the given is given as diverse" (p. 222). Reassembling multicultural education in a way that moves beyond the Liberal Multicultural Education Machine and the Critical Multicultural Education Machine entails affirming difference, for "history progresses not by negation and the negation of negation, but by. . .affirming differences" (p. 268). But, Deleuze followed these seemingly utopian words with this warning: *"It is no less bloody and cruel as a result"* (p. 268). Affirming difference requires a bigger boom and a louder crash than the Equity Principle stutter of *all-all-all* or a critical theory-based version of critical multicultural education. There is hope in the question, what is it that multicultural education is becoming, can become? Some of the aspects Kubota (2004) describes as part of critical multicultural education return a different rhythm than that of a critical-theory-based critical multicultural education and acknowledge that the cadence repetition of culture is only an envelope of the rhythmic repetition of a cultural becoming. Kubota (2004) called for a problematization of difference. This entails a rejection of a view that difference is neutral and stable. Instead, she argued for an exploration of "why inequality among different groups exists and how various kinds of difference are produced, legitimated, or eliminated within unequal relations of power" (p. 38). Further, Kubota (2004) critiqued a view of culture as homogenous, traditional, and static and instead calls for a view of culture as "diverse, dynamic, and. . .discursively constructed" (p. 38). In other (Deleuzian) words, through ongoing rhythmic repetition, cultures are in the process of becoming. Finally, Kubota (2004) called into question the representations of cultures: “images of a certain culture or language are neither neutral nor objective, rather, they are discursively constructed” (p. 38). Following Noblit’s use of the term “postcritical” in “postcritical ethnography,” which “directly challenges the epistemology of critical ethnography” (Noblit, 2004; Adkins & Gunzenhauser, 1999) and instead understands knowledge as “the product of a moment of mutual construction that at once converges divergent perspectives and preserves the divergence” (Adkins & Gunzenhauser, 1999, p. 71, as cited in Noblit, 2004, p. 194), this reassembled multicultural education might be called “postcritical multicultural education.” Kumashiro (2002, 2004) has conceptualized “anti-oppressive education” along similar lines. Postcritical multicultural education or anti-oppressive education views all knowledge as necessarily partial including its own constructed knowledge. Hence, Kumashiro (2004) wrote, “the field of anti-oppressive education refuses to say that it has found the ‘best’ approach or even an unproblematic approach to teaching toward social justice” (p. xxvi). Kumashiro (2002) conceptualized oppression in terms of repetition: “oppression itself can be seen as repetition, throughout many levels of society, of harmful citational practices” (p. 51). For Kumashiro (2002), drawing on Butler (1990), unlike for Deleuze, “repetition” was conceptualized by default as repetition of the *same*: “oppression is produced by discourse, and in particular, is produced when certain discourses (especially ways of thinking that privilege certain identities and marginalize others) are cited over and over” (Kumashiro, 2002, p. 50). Butler (1990) identified transformative possibilities in the *failure* to repeat: “The possibilities of gender transformation are to be found precisely in the arbitrary relation between. . .acts, in the possibility of a failure to repeat" (p. 179). Here, the failure to repeat is transformative because it is a failure to repeat the *same* gender performance. In the same sentence, however, Butler identifies another source of transformation in repeating *differently*, through "parodic repetition that exposes the phantasmatic effect of abiding identity as a politically tenuous construction" (p. 179). Later, Butler reformulated repetition as necessary: "To enter into the repetitive practices. . .of signification is not a choice, for the 'I' that might enter is always already inside. . . .The task is not whether to repeat, but how to repeat, or, indeed, to repeat, and, through a radical proliferation of gender, *to displace* the very gender on which we might construct a politics" (p. 189). Parodic repetition, repetition which has the effect of displacing, in Deleuzian terms, is rhythmic repetition, in which what returns is difference. Failure to repeat the *same* is to refuse the envelopment of rhythmic repetition within a particular cadence. Kumishiro (2002), although conceptualizing repetition as repetition of the same, saw transformative possibilities in linking repetition to "supplementation" (p. 52): "Of course, the meaning and effects of stereotypes do change in different contexts and over time. . . .What is helpful in this discussion is another poststructural concept: *supplementation*, which means to cite, but also add something new in the process" (p. 52). The way in which Kumashiro's (2002) and Butler's (1990) conceptions of repetition differed from that of Deleuze (1968/1994) was that Butler and Kumashiro conceptualize the repetition of the same as a primary basis upon which to make changes whereas

Deleuze saw repetition of the same as secondary, arising out of rhythmic repetition of the different[16]. It is because repetition necessarily returns the different that affirming difference is a necessary condition of being, that being is becoming.

In addition to maintaining a reflexive critique of anti-oppressive education, the acknowledgement that knowledge is partial is central to anti-oppressive education in another way. Kumashiro (2004) argued that all knowledge is necessarily partial and that often it is partial in ways that reinforce oppression. Moreover, challenging oppression is not simply a matter of filling in gaps in knowledge, of "raising awareness of the more progressive perspectives on the world" (p. 25). The issue is not just that we do not know enough, but "also that we often do not *want* to know more about oppression" (p. 25). Kumashiro (2004) argued that "it is not our lack of knowledge, but our resistance to knowledge and our desire for ignorance that often prevent us from changing the oppressive status quo" (p. 25). Similarly, Deleuze (1968/1994) wrote,

> Questions and problems are not speculative acts, and as such completely provisional and indicative of the momentary ignorance of an empirical subject. On the contrary, they are the living acts of the unconscious, investing special objectivities and destined to survive the provisional and partial state characteristic of answers and solutions. The questions or sources of problems correspond to

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[16] Butler (2004) writes in response to Braidotti's (1995) critique of *Gender Trouble*, "Although Braidotti. . .claim[s] that I reject Deleuze, she needs to know that every year I receive several essays and comments from people who insist that I am Deleuzian. I think this may be a terrible thought for her. . . .Like her, I am in favor of a deinstitutionalized philosophy (a 'minority' philosophy), and. . .I am also looking for the new, for possibilities that emerge from failed dialectics and that exceed the dialectic itself. . . .One reason I have opposed Deleuze is that I find no registration of the negative in his work, and I feared that he was proposing a manic defense against negativity" (p. 198).

> the displacement of the virtual object which causes the series to develop. (p. 106)

Affirming difference, then, rather than the simplistic utopian notion of "celebrating differences," requires "learning through crisis" (Kumashiro, 2004, p. 27), learning through the crisis instigated in the return of difference. That is, as Deleuze (1968/1994) concluded, "Learning takes place not in the relation between a representation and an action (reproduction of the Same) but in the relation between a sign and a response (encounter with the Other)" (p. 22). Such an encounter interrupts presuppositions of the form "Everybody knows. . ." (Deleuze, 1968/ 1994, p. 129), that is, reconceptualized *"givens" as "takens"* (Semetsky, 2006, p. 82) and worked against common sense (Deleuze, 1968/1994; Kumashiro, 2004). For, as Kumashiro (2004) points out, what comes to be seen as common sense "may be comforting for its familiarity and for providing a sense of normalcy, [but] it is also quite oppressive" (p. xxiii). "Many people," Deleuze (1968/1994) argued, "have an interest in saying that everybody knows 'this', that everybody recognizes this, or that nobody can deny it" (p. 131). Converting "givens" to "takens" requires someone, "if only one—with the necessary modesty not managing to know what everybody knows, and modestly denying what everybody is supposed to recognize" (Deleuze, 1968/1994, p. 130). This one person, who neither allows her- or himself to be represented, nor wishes to represent anything (Deleuze, 1968/1994), introduces a crisis into a "dogmatic, orthodox or moral" image of thought. Working *toward* social justice entails working *against* common sense. A postcritical multicultural mathematics education, an anti-oppressive mathematics education, requires working against the commonsensical hum of the Talent-Competition Machine ("Everybody knows only masculine White middle-class English-speaking boys have 'flair' and will inherit the world of mathematics"), the All-Regardless assemblage of the Liberal Multicultural Education Machine ("Everybody knows that if we just celebrate our differences, all of us will succeed regardless of our backgrounds and the material conditions of the world"), and the Contradiction-Dialectic assemblage of the Critical Multicultural Machine ("Everybody knows differences can be resolved through contradiction"). "Common sense," Kumashiro (2004) asserted, "is not what should shape educational reform or curriculum design; it is what needs to be examined and challenged" (p. xxiv). Yet, if the forgoing is taken as the answer, as a solution to the problem of equity in mathematics education, then we have missed the point:

> The anti-oppressive teacher. . .is something we strive for and transitionally become in our practices but never fully are. And the moment that we fix our identities and begin repeating only certain practices and knowledge and relations that we believe are anti-oppressive, we stop doing the necessary work of problematizing how any approach to teaching is partial. (Kumashiro, 2004, p. 15)

Rather than landing squarely in the midst of an answer, that is, moving from the "hypothetical to the apodictic" (Deleuze, 1968/1994, p. 197), we move from the "problematical to the question" (Deleuze, 1968/1994, p. 197). As it turns out, affirming difference entails generating problems.

2. Generating Problems

What is it that, as Deleuze (1968/1994) asked in *Difference and Repetition, "Everybody knows, no one can deny"* (p. 129-130) about mathematical problems in math class[17]? What has become fixed as strata, the organization of organs within the body of mathematics education? Where have there been movements of destratification; what lines of flight have escaped? What lines of flight might we select to press further? Importantly, how do we speed along these lines of flight without "throw[ing] the strata into demented or suicidal collapse" (p. 161)?

2.1 The Problems-Practice Assemblage

Everybody knows that math class is about solving problems. In fact, Stanic and Kilpatrick (1988) claim that "problems in the [mathematics] curriculum go back as far as the ancient Egyptians, Chinese, and Greeks" (p. 1). Everybody recognizes this math classroom: "The teacher [stands] at the blackboard and demonstrate[s] for students the proper procedure to reach a correct solution. This [is]. . .followed with practice problems for students to complete at their seats,

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[17] Some of the ideas in this plateau were presented in a paper entitled "Transdisciplinary Encounters: Deleuzian Philosophy and Mathematics Education" at the 3rd Annual Deleuze Studies Conference: Connect, Continue, Create, July 12-14, 2010.

without discourse with fellow classmates" (McVarish, 2008, 4). This basic math lesson format became common sense in the United States and Europe by the end of the 19th century[18]. The preface of an arithmetic textbook from 1895 entices prospective readers and educators with the following claim: "The number of examples provided for practice and drill is unusually large. . . .There appears to be a demand for an abundance of such material, which this book aims to satisfy" (Seaver & Walton, 1895, p. 3). D'Ambrosio (2003) notes that "educators in the mid- to late 1800s believed that effective teaching involved showing students mathematical procedures, followed by students' application (i.e. use of the procedure to solve word problems) and practice of those procedures" (p. 37). In the "math classrooms of the past" as described by O'Connell (2007), "problem solving was an add-on" (p. 1). Solving problems was "what we did at the end of a chapter that had focused primarily on computations" (p. 1). This image of thought in mathematics education captures a problems-practice assemblage. In this assemblage, "problems" are the form of content or machinic assemblage doubly articulated with "practice" as the form of expression or the collective assemblage of enunciation. Within this assemblage, problems are seen by both teachers and students as completely separate from the rest of the mathematics curriculum.

How does this problems-practice assemblage function in mathematics

[18] This "traditional" mathematics lesson format that came to be taken as common sense in the U.S. and Europe contrasts with the "traditional" problem-centered mathematics lesson format that has been seen as common sense in Japan (e.g. Shimizu, 1999; Becker et al., 1990; Stigler, Fernandez, & Yoshida, 1996; Stigler et al., 1999).

classrooms? Lester and Garofalo (1982) examined the ways in which third- and fifth-grade students whose teachers followed the common sense lesson format described above approached the following problem:

> Tom and Sue visited a farm and noticed there were chickens and pigs. Tom said, ‘There are 18 animals.’ Sue said, ‘Yes, and they have 52 legs in all.’ How many of each kind of animal were there?

Lester and Garofalo (1982) found that third-graders typically added 18 and 52 because “it asks ‘how many *in all*’ (Lester, 1985, p. 41) and fifth-graders typically divided 52 by 18 because “the key words are ‘how many of each’” (Lester, 1985, p. 41). Lester (1985) identified a number of principles which seem to guide how students’ ensconced in the problems-practice assemblage approach problems, two of which are most notable: 1) “All mathematical problems can be solved by direct application of one or more arithmetical operations”, and 2) “Which operations to use is determined by the key words in the problem (these key words usually appear in the last sentence or question” (p. 42). Some students apply these guidelines even in “absurd” problems such as the following: “There are 26 sheep and 10 goats in a ship. How old is the captain?” Some students add the number of sheep and the number of goats and conclude that the captain is 36 years old (Vershaffel & De Corte, 1997; quoted in Cai, 2003, p. 248). Even more telling is the fact that the percentage of students who approached the problem in this way rose from 10 percent in Kindergarten to 60 percent in third and fourth grade (Vershaffel & De Corte, 1997; Cai, 2003). Boaler (2000) has shown that students learn unintended lessons when they participate in the well-ingrained lesson format that accompanies the problems-practice assemblage.

Students learn that the problems at the end of the lesson will require them to use the procedure that was presented during the beginning portion of the lesson or has been a recent focus in the curriculum, eliminating the need to actually think through or comprehend the information in the problem situation. In Lester's (1982) study, third-graders added the numbers partly because addition was a main operation in the third grade curriculum; fifth-graders divided because division was a main operation in the fifth-grade curriculum. Boaler (2000) found that the way students approach problems depends on the context in which they encounter the problem; students approach problems presented in math class very differently from the way in which they approach them outside of school. In school, students use "cue-based" (Schoenfeld, 1987; Boaler, 2000) behaviors in order to make choices of mathematical methods. According to Boaler (2000),

> The students became proficient at finding and interpreting different cues within their mathematics textbooks; these cues helped them to proceed through exercises. Such cue-based practices were specific to the mathematics classroom, yet they were, in many ways, the antithesis of mathematical thought. (p. 114)

Such "cue-based" behaviors proved to be very effective in response to the problems-practice assemblage captured in the common sense lesson format. However, students did not even attempt to use the same behaviors in non-school situations:

> Using school mathematics. . .meant employing the classroom practices that had structured their learning. Thus, students reported that they did not even attempt to make use of school-learned methods in the real world, not because of the form or structure of the mathematical problems they encountered but because the environments of the classroom and their everyday lives were too disparate. The students believed that adopting classroom practices in the real world was inappropriate, so they did not attempt to draw upon school mathematics. (p. 115)

Boaler (2000) suggests that a perspective that takes this observation into account, that is, a situated perspective, shifts the "locus of blame and responsibility in mathematics classrooms away from students" and focuses it instead on "the environments educators provide for students and the practices they encourage" (p. 118). Deleuze and Guattari (1980/1987) describe a similar scenario in developing their concept of the order word:

> When the schoolmistress instructs her students on a rule of grammar or arithmetic, she is not informing them, any more than she is informing herself when she questions a student. She does not so much instruct as 'insign,' give orders or commands. A teacher's commands are not external or additional to what he or she teaches us. . . .The compulsory education machine does not communicate information; it imposes upon the child semiotic coordinates possessing all of the dual foundations of grammar. . . .The elementary unit of language—the statement—is the order-word. (p. 76).

Within the problems-practice assemblage, students learn to expect and obey certain cues embedded in the context of school mathematics; as Deleuze and Guattari (1980/1987) explain, "a pragmatics (semiotic or political)" defines "the effectuation of the *conditions of possibility"* (p. 85). In this case, the pragmatic landscape of the mathematics classroom shapes the conditions of possibility for approaching problems. Hence, in Lester and Garofalo's (1982) study, "even when [students] were made aware that their answers were incorrect, they were unable to devise an alternative procedure for attacking the problem" (Lester, 1985, p. 42) and the students in Boaler's (2000) study saw neither using "school math" outside of school or "real-world math" inside of school as possibilities. The knowledge of school mathematics within the problems-practice assemblage, then, limits rather than enhances students' ability to respond to problems in any meaningful or useful way except in the circumscribed context of practicing school math in school. Schizoanalysis raises the following question: Within the body of mathematics education, what organs of desire form within this problems-practice assemblage? Despite the ostensible objectives of providing opportunities for students to apply knowledge and practice skills in response to problems, it is evident that the flows of desire within math classrooms stratified by the problems-practice assemblage coalesce into nodes of obedience and powerlessness. Boaler (2000) suggests that situated perspectives that take into consideration (and transform) the pragmatic landscape of mathematics classrooms may be helpful in order "to move mathematics away from the discriminatory practices that produce more failures than successes toward something considerably more equitable and supportive of social justice" (p. 118).

2.2 The Pólya Machine

According to Deleuze and Guattari (1980/1987), an assemblage "has both *territorial sides,* or reterritorialized sides, which stabilize it, and *cutting edges of deterritorialization,* which carry it away" (p. 88). What are the cutting edges of deterritorialization along the boundary of the problems-practice assemblage? What machine chews at the borders and makes new connections, unfixing the image of thought? This machine might be called the Pólya Machine. In his 1945 book *How to Solve It*, Pólya described the heuristic processes which established mathematical problem-solvers used in approaching nonroutine problems (those in which a solution method is not initially obvious). In this way, Pólya made the problem solving process a topic to examine in itself rather than an "add-on" or merely a means for practice and application. As Kirschner and Whitson (2000) point out, “what was so powerful about Polya’s *[sic]* exposition was his understanding of heuristics as methods in motion” (p. 382). In the two decades after *How to Solve It* was published, the line of flight opened by the Pólya Machine destratefied the image of thought in mathematics education in the United States when the National Council of Teachers of Mathematics (NCTM) (1980) in *An Agenda for Action* said that “problem solving must be the focus of school mathematics” (p. 1). The *Agenda* went further than simply arguing that problem solving is an important aspect of mathematics to stating that “the mathematics curriculum should be *organized around* problem solving” (Recommended Action 1.1.). In an even stronger statement, NCTM (1989) stated in the *Curriculum and Evaluation Standards:*

> Mathematical problem solving, in its broadest sense is nearly synonymous with doing mathematics. . . . .Problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics. . .is both constructed and reinforced. (p. 137)

Far from being merely supplemental tasks added on to the end of a lesson or unit, this statement places problems at the center of mathematics education. A decade later in the *Principles and Standards,* NCTM (2000) asserted that “problem-solving is an integral part of all mathematics learning, and it should not be an isolated part of the mathematics program” (p. 52). In contrast to the “cue-based” approach students in Boaler’s (2000) study took to problems, the *Principles and Standards* (NCTM, 2000) state that “by learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom" (p. 52). Just as problems should not be separated from the rest of the curriculum, school mathematics and mathematics outside of school should not be such disparate contexts that students cannot conceive of using similar strategies in both settings. Finally, the *Principles and Standards* place problem solving at the heart of mathematics education as both goal and vehicle: "Problem-solving is not only a goal of learning mathematics but also a major means of doing so" (p. 52).

As the problems-practice assemblage was deterritorialized, problems shifted from the end of the lesson to the beginning. As O'Connell (2007) explains, "problem-centered instruction uses problems to launch math lessons" (p. 3). It is through solving problems that students develop understanding of mathematical concepts. Lambdin (2003) describes this process in the following way:

> One can think about a model of learning mathematics in which understanding is represented by an increasingly connected and complex web of mathematical knowledge. . . .Learning through problem solving develops understanding. Students' mental webs of ideas grow more complex and more robust when the students solve problems that force them to think deeply and to connect, extend, and elaborate on their prior knowledge. (p. 7; also see Brownell, 1947; Hiebert & Carpenter, 1992; Van de Walle, 2001).

Frontloading lessons with problems shifts the responsibility for thinking deeply about mathematics from teachers-as-presenters to students-as-problem-solvers.

Within an assemblage, the form of content and form of expression are always linked; if a line of flight opens in one, the other is affected as well (Deleuze & Guattari, 1980/1987). The line of flight that shifts problems (as a form of content) to the center of mathematics education also exerts force on the form of “practice” (as a form of expression) in the problems-practice assemblage. Van de Walle (2003) argues that within problem-based mathematics education, the meaning of “practice” needs to be reconsidered: “If we begin to think of practice as returning regularly to the same basic ideas but through new problem-based experiences, we begin to open up opportunities for all children in our classrooms” (p. 75). Van de Walle (2003) gives an example of what this new form of practice might look like and the way it may open up opportunities for students who develop conceptual understandings at different rates. In Van de Walle’s (2003) example, students encounter the idea of equivalent fractions, that “the same quantity can be represented by different fractions” through a task in which they “use fraction pieces to find names for [a particular] region” (p. 75-76). Students are asked to find as many names as possible. A number of other tasks provide additional “practice” with the idea of equivalence: “This same concept should also be approached with counters. . ., with paper folding, with symbolic tasks, and with variations on each of these” (p. 76). Van de Walle anticipates that some students will continue to engage with the basic idea of equivalence on a concrete level, while others will begin to take note of patterns in conventional numeric representations that move beyond concrete representations. Van de Walle (2003) points out that focusing on a procedural skill or rule too quickly after beginning conceptual exploration results in poor conceptual understanding, and that this danger is even greater for students who continue to engage with the concept at a more basic or concrete level for a longer period of time.

Similarly, in investigating mathematics teaching in three cities in China (Hong Kong, Macau, and Shanghai), Huang, Mok, and Leung (2006; also see Huang, 2002; Huang & Leung, 2004) develop a similar theoretical framework for conceptualizing two different forms of practice: explicit variation and implicit variation. They distinguish between these two forms of varying problems in the mathematics classroom in the following way:

> If the changes from the prototype of problems in which the learnt knowledge can be applied directly by learners to their variations are identified visually and concretely such as variations in number, positions of figures, etc., and the conditions for applying the relevant knowledge are still explicit and direct, then this kind of variation is still explicit. On the other hand, if the changes from the origins to their variations have to be discerned by abstract or logical analysis by learners such as variations in parameters, subtle changes or omissions [of] certain conditions, or changes of contexts, or reckoning on certain strategies etc., so that the conditions or strategies for applying relevant knowledge are implicit and not obvious, then this kind of variation is characterized as implicit. (p. 265).

Huang, Mok, and Leung (2006) examined how teachers in each of the three cities used explicit and implicit variation in problems posed to students involving the method of elimination for solving simultaneous linear equations. Explicit variation involved using the method of elimination with systems of equations all of which retained the form "ax + by = c" whereas "the equations with the implicit variation need a transformation which requires a deeper understanding of the meaning of unknowns in equations" (p. 266). In the highest level of implicit variation, students needed to redefine an abstract expression with new unknowns in order to transform the equations into the form "ax + by = c" before using the method of elimination. For example, in the equation "1/a + 2/b = -2," students needed to first redefine 1/a as one unknown and 1/b as another unknown. The authors conclude that explicit variation is designed for mastering basic skills whereas implicit variation is designed for “enhancing the integration of knowledge and developing advanced thinking” (p. 271). Restructuring “practice” through implicit variation into a form that is problem-based and concept-focused may open up opportunities for students at different current levels of understanding of a concept to expand their “complex web of mathematical knowledge” (Lambdin, 2003, p. 7) rather than encouraging students to approach problems based on the guidelines Lester (1985) identified and come to absurd “solutions” as in the ship captain problem. In Deleuzian (1968/1994) terms, it is encountering difference through repetition with variation that forces one to think and results in learning.

The Pólya Machine reorganized the image of thought in mathematics education to place problem-solving as methods in motion at the center. It also created openings for teachers and students to approach the problem of problems in new ways. However, the flows created by this deterritorialization have often been blocked and restratified as textbooks have converted Pólya’s heuristics from “methods in motion” (Kirschner & Whitson, 2000) into new procedures to be taught (Roy, 2003). In the words of Kirschner and Whitson (2000), “heuristics were subverted from problem-solving process into curricular commodity” (p. 382). In this reterritorialization, problem solving has itself become the skill to be presented and then practiced. One example is Herr and Johnson’s (2001) *Problem solving strategies: Crossing the river with dogs and other mathematical* *adventures*, a book on problem solving for high school students. Once again separating problem solving from the rest of the curriculum, the publisher's website states, "In content-crowded mathematics classes, few students get the practice necessary to fully develop problem-solving skills" (Key Curriculum Press, 2010, "overview," para. 2). Despite the claim to emphasize that "any problem can be solved in many ways" (Key Curriculum Press, 2010, "in-depth," para. 1), the book follows the familiar presentation-then-practice sequence of the problems-practice assemblage in which "each chapter covers a single strategy" so that "students are given time to practice" (Key Curriculum Press, 2010, "in-depth," para. 2). For example, the first chapter entitled "Draw a Diagram" provides an overview of the utility of drawing a diagram in solving problems and presents a number of problems in which the students are instructed to draw a diagram. Here, the textbook authors have transformed "drawing a diagram" into another procedure to be practiced. Pólya's method in motion has been absorbed and stratified back into the original problems-practice assemblage. When problem solving heuristics are absorbed into the problems-practice assemblage as in the "Draw a Diagram" chapter, students are once again encouraged to rely on "cue-based" approaches as described by Boaler (2000). From simply reading the title of the chapter, students know that for each problem, the "answer" is to draw a diagram, removing the need to consider alternative ways to approach the situation[19]. As Kirschner and Whitson (2000) observe, "textbooks are designed

[19] This example raises the following question: Is assigning problems sufficient in order to guide students to become capable problem solvers or is something further needed? O'Connell

to systematically instruct Polya's heuristic methods, using problems specially contrived to illustrate each one" (p. 382). Examples of the reterritorialization of Pólya's heuristics into the problems-practice assemblage are not limited to student textbooks, but can be found in textbooks for prospective teachers as well. For example, *A Problem-Solving Approach to Mathematics for Elementary School Teachers* (Billstein et al., 2007) provides an introductory chapter matching Kirschner and Whitson's description, providing an overview of Pólya's method and heuristics with accompanying example problems, plus a set of practice problems at the end of the chapter. Adding an additional layer to the commodification of Pólya's work, the chapter includes four "sample school book pages" from elementary school textbooks following the same pattern of fixing Pólya's "methods in motion" into the presentation of a procedure to be practiced. One of the sample pages, for example, lists three steps to follow in order to use the strategy "solve a simpler problem" (Scott Foresman-Addison Wesley, Grade 5, 2005, p. 352; reproduced in Billstein et al., 2007, p. 9).

2.3 Problem-Posing Lines of Flight

The Pólya Machine problematized the "problems" side of problem solving; yet the "solving" side remained untouched. The Pólya Machine left intact problem solving as the center of the image of thought in mathematics education.

(2007) argues that "assigning problems is not teaching problem solving" (p. 3). However, rather than following a line of reterritorialization back into the problems-practice assemblage, O'Connell (2007) argues that "when we teach problem solving strategies, we are not implying a drill-and-practice approach to learning about these skills" (p. 3-4). Rather, "the teaching of problem solving involves a variety of approaches so that students can see ideas modeled, experience situations, discuss insights and observations, and process learning through talking and writing. . . .Teaching problem-solving strategies is not about telling students which strategies to use" (pp. 3-4).

Everybody *still* knows that math class is about solving problems. Both the reterritorialized problems-practice assemblage evident in many textbooks and the line of flight that has centered problems and begun to shift the meaning of "practice" remain within the same "problem-solving stratum." What are the cutting edges of deterritorialization along the edges of this problems-practice-assemblage-in-motion, the lines of flight moving away from the "problem-solving" stratum? What machine gnaws at the borders and makes new connections, unfixing this image of thought? The concept of "problem-*posing*" creates new lines of flight by deterritoralizing the "*solving*" side of the problem solving. Rather than a single line of flight, two separate "problem-posing" lines of flight can be followed in the recent history of mathematics education. Along the first problem-posing line of flight moves what might be called the Freire Machine; along the second problem-posing line of flight moves the Brown-and-Walter Machine. Brown (2001) acknowledges the historical juxtaposition of these two lines of flight along with distinguishing their respective emphases:

> Though [Marion Walter and] I first developed [our] thinking about problem posing in the late 1960s—at about the same time that Paulo Freire chose it as a central concept in his educational reform for the oppressed—[our] agenda was more set on ways of motivating students to do mathematical inquiry and less politically rooted than Freire's program. (p. xiv)

Freire, whose ideas were later taken up by mathematics educators in the United States and elsewhere, developed his ideas about problem-posing in the context of teaching literacy with peasants in Brazil; Brown and Walter developed their ideas about problem-posing in the context of team-teaching courses at the Harvard Graduate School of Education in Massachusetts. Despite these vastly different contexts, both lines of flight question where and how problems originate, and who can pose problems, questions which resonate with Deleuze's (1968/1994) critique in *Difference and Repetition:*

> We are led to believe that problems are given ready-made, and that they disappear in the responses or the solution. . . .We are led to believe that the activity of thinking, along with truth and falsehood in relation to that activity, begins only with the search for solutions, that both of these concern only solutions. . . .According to this infantile prejudice, the master sets a problem, our task is to solve it. It is also a social prejudice with the visible interest of maintaining us in an infantile state, which calls upon us to solve problems that come from elsewhere, consoling or distracting us by telling us that we have won simply by being able to respond: the problem as obstacle and the respondent as Hercules. (p. 158)

Within the traditional problems-practice assemblage, the pragmatic landscape of mathematics classrooms directs the flow of desire into nodes of obedience and powerlessness, as indicated by the cue-based approach to problems taken by students in Lester's (1982, 1985) studies and Boaler's (2000) study. The line of flight that deterritorialized the problems-practice assemblage, set problems as the center of mathematics education, and began to change the meaning of "practice" to a more problem-based and concept-oriented perspective also shifted the teacher's role from procedure-presenter to problem-poser. Yet, the pragmatic landscape and the flows of desire have not changed as much as it might seem. The teacher is still "the master" who sets before the students problems which come from elsewhere, praising students for the "Herculean" feat of "simply being able to respond" (Deleuze, 1968/1994, p. 158). The problem-posing lines of flight deterritorialize this dogmatic image of thought about the origins of problems.

In *Pedagogy of the Oppressed*, Freire (1970/ 2000) developed the idea of "problem-posing" pedagogy in contrast to what he called the "banking concept of education" (p. 72), in which education "becomes an act of depositing in which the students are the depositories and the teacher is the depositor" (p. 72). In this model of education, it is the teacher's role to "regulate the way the world 'enters into' the students" (p. 76). As in the problems-practice assemblage, teachers in the banking model present information and students are expected to " 'receive' the world as passive entities" (p. 76). Freire (1970/ 2000) conceptualizes the teacher-student relationship inherent in the banking model of education as a Hegelian/ Marxist dialectical contradiction that must be resolved through dialogue: "The practice of problem-posing education entails at the outset that the teacher-student contradiction. . .be resolved. Dialogical relations—indispensible to the cognitive actors to cooperate in perceiving the same cognizable object—are otherwise impossible" (pp. 79-80). According to Freire (1970/2000), the process of dialogue can dissolve teachers as teachers-of-the-students and students as students-of-the-teacher leaving instead a more equal relationship between teacher-student and students-teachers. The task of the teacher-student, then, is to pose problems *with* students-teachers. For Freire (1970/2000), this process begins before teacher-students and students-teachers first meet: "The dialogical character of education as freedom does not begin when the teacher-student meets with the students-teachers in a pedagogical situation, but rather when the former first asks herself or himself *what* she or he will dialogue with the latter *about"* (p. 93). Making decisions about the content of dialogue consists of “organized, systematized, and developed ‘re-presentation’ to individuals of the things about which they want to know more” (p. 93). Through listening to students-teachers, teacher-students identify generative themes and codify these themes into representations that then become the object of continued dialogue between teacher-students and students-teachers. Hence, “the thematics which have come from the people return to them—not as contents to be deposited, but as problems to be solved” (p. 123). For Freire (1970/2000), problems originate neither solely from teachers nor solely from students, but rather in the dialogical relationship between the two, and the process of posing and grappling with such problems transforms the relationship itself.

Although Freire originally developed the idea of problem-posing pedagogy in the context of literacy education, a number of mathematics educators have applied the ideas to mathematics education (e.g. Gerdes, 1975, 1982; Frankenstein, 1987, 1990, 1995, 2005; Frankenstein & Powell, 1994; Skovsmose, 1994; Lesser & Blake, 2007; Mellin-Olson, 1986; Ferreira, 1990; Gutstein, 2006). One of the most extensive frameworks in mathematics education based on Freire’s ideas is Gutstein’s (2006) book, *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. For Gutstein (2006), problem-posing pedagogy in mathematics education requires a “reconceptualization of the purpose of mathematics education. . .one that includes envisioning mathematical literacy as critical literacy for the purpose of transforming society, in its entirety, from the bottom up toward equity and justice, for all students whether from dominant or oppressed groups” (p. 11). Gutstein

(2006) sees mathematics and mathematics education as vehicles through which society can be transformed. This reconceptualization of the purpose of mathematics education goes further than simply establishing equity within math classrooms. Gutstein (2006) argues that the "goal of increasing equity within mathematics education does not explicitly position teachers and students as having transformative power to rectify fundamental structural inequalities" (p. 13). What is needed, according to Gutstein (2006), is a pedagogy that allows students not only to "read the word," but also to "read the world" with mathematics. Freire (Freire & Macedo, 1987) linked the concept of textual literacy, or "reading the word," with the broader goal of learning to "read the world," or coming to understand the social, political, cultural, and historical conditions of one's life. Bringing this idea into mathematics education, Gutstein (2003) conceptualizes reading the world *with mathematics* as using

> mathematics to understand relations of power, resource inequities, and disparate opportunities between different social groups to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (p. 45)

In addition to reading the world, problem-posing pedagogy in mathematics education entails *writing* the world with mathematics. Gutstein (2006) states that "writing the world with mathematics means using mathematics to change the world" (p. 27). Gutstein (2006) read and wrote the world with middle school students, most of whom identified as Mexican or Mexican American, in the Chicago area through "real-world projects and related conversations" (p. 41). An example of one of the projects with a seventh-grade class was entitled "Mortgage loans—Is racism a factor?" In deciding on the content of dialogue, Gutstein (2006) chose this topic because "the issue of home ownership was real for my students" (p. 60). Gutstein (2006) introduced the project by "discussing whose families owned homes (often in extended family relationships) and whose families wanted to (everyone else)" (p. 60). In the discussion, many students talked about the challenges their families had faced in securing mortgages. The codification of the theme that Gutstein (2006) used was an article in the *Chicago Tribune* that presented data on mortgage rejection rates for people of different races in the local area and nationally. Following Freire's (1970/ 2000) guideline that the "thematic nucleus [of the codification] should be neither overly explicit nor overly enigmatic" (p. 114), this article presented contrasting claims about whether the data indicated institutional racism and "the article was confusing, with many numbers and multiple comparisons, and introduced a *disparity ratio,* the ratio of the rejection rates for different races" (p. 57). The disparity ratio was a complex representation of the data in that it was a ratio between percentages, and therefore, a ratio of ratios. Gutstein (2006) posed the following problem to students based on the codification:

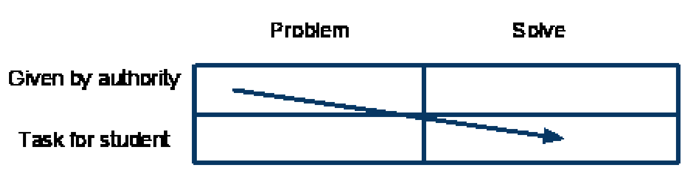
> Write a good essay answering the following question (you **must** use data from the article or the quote above [included in original project] to make your argument): Is racism a factor in getting mortgages in the Chicago area? (p. 57)

Throughout the three weeks in which the class engaged with this project, the class had discussions "in which we mathematically dissected the issues" (p. 60). Gutstein (2006) continually pushed students to justify their arguments using mathematics, questioned their assumptions, and required them to rewrite their essays. Gutstein (2006) asserts that these practices “created conditions for students to grapple genuinely for understanding” (p. 61). In this process, although more time would have allowed for developing a deeper understanding of the historical context of the issues, Gutstein (2006) observed growth in both students’ understanding of mathematics and their “collective sense of justice” (p. 61). In this project, the problem of institutional racism in mortgage rates originated through the dialogic relationship between Gustein and the students, following Freire’s conceptualization of problem-posing pedagogy.

While the Freire-Machine originated in the context of literacy education and then was brought into mathematics education, the second problem-posing line of flight originated within mathematics education itself. Like the Freirean problem-posing line of flight, this line of flight also raises questions about the origin of problems and who should be involved in posing problems. In their book *The Art of Problem Posing,* Brown and Walter (1983) begin with a statement similar to Deleuze’s critique:

> Where do problems come from, and what do we do with them once we have them? The impression we get in much of schooling is that they come from textbooks or from teachers, and that the obvious task of the student is to solve them. (p. 1)

This verbal laying out of common sense is followed by this almost-Deleuzian diagram (p. 1):



The diagram captures the problem-solving stratum in the image of thought of mathematics education. In this image, problems are given by authority (usually the teacher or textbook) to students, whose (Herculean) task is to solve it.

Brown (2001) identifies a tendency among mathematics teachers to "reduce 'problem' to 'problem solving'" (p. 15). He describes an activity he used in a class with a group of experienced mathematics educators. Previously, when giving a talk to a different group of mathematics educators, he had begun by asking members of the audience to respond to a question printed at the top of a sheet of paper. Audience members each received one of the following questions: 1) What are some good reasons for including problems in the school curriculum, or 2) What are some good reasons for including problem solving in the school curriculum. The only difference between the two questions is that one says "problems" while the other says "problem *solving*." After the talk, Brown recorded the answers to the two questions in lists without reference to the questions themselves and asked the class of experienced math teachers to try to determine the questions that had generated the two sets of responses. The suggested questions for both sets of responses were essentially identical, both phrased in terms of problem *solving*, despite the fact that one of the original questions did not mention solving or solutions in the least. The problem-solving stratum in the image of thought in mathematics education is so ingrained that "problem" is automatically interpreted as "problem-solving." When Brown and Walter team taught graduate level courses on problem solving and problem posing, they also noticed that when presented with a situation or an artifact, especially in an academic setting, people often assume that the situation or artifact itself presents a particular problem to be solved. For example, Brown and Walter (1983/2005) begin the second chapter of *The Art of Problem Posing* by asking the reader to consider $x^2 + y^2 = z^2$. When presented with this statement almost everyone begins to list number triples such as 3,4,5 and 5,12,13, or possibly 2, 3, √13 and *i*, 1, 0. Brown and Walter (1983/ 2005) point out, however, that "$x^2 + y^2 = z^2$" is not in itself a question at all. If anything, it begs you to *ask a question* or to *pose a problem* rather than answer a question" (p. 13). The authors point out that typically the only questions students are encouraged to ask in math class are whether or not they have correctly understood the concepts or procedures the teacher has presented. This practice maintains the "infantile prejudice," as Deleuze and Guattari (1968/1994) call it, that problems come from elsewhere and students are empowered simply by responding.

To break out of this practice, Brown and Walter (1983/ 2005) suggest two phases of a problem posing process, first, an "accepting" phase and, second, what they call the "what-if-not" strategy. The accepting phase opens new alternatives despite the acceptance of "the given" of the situation or artifact. Questions that might be posed related to $x^2 + y^2 = z^2$ after beginning a table of number triples include, "Can you get a triple for any value of x you choose?" and "For a fixed x, are y and z always unique?" In each of these questions, the equation is taken as given, but the "problem" is no longer assumed to be simply finding number triples that make the statement true in Euclidean space. The second phase moves outside of “the given” of the situation or artifact. Using the “what-if-not strategy” begins with identifying attributes of the situation or artifact, followed by considering cases in which those attributes do *not* hold. $x^2 + y^2 = z^2$, some examples of attributes people have identified include, “The statement is a theorem,” “The variables are related by an equal sign,” and “There are three exponents all of which are the same” (p. 36). The second step entails asking, “What if not” in relation to each attribute, for example, “What if the variables were *not* related by an equal sign?[20]” This opens up alternatives to consider such as “$x^2 + y^2 < z^2$” or “$x^2 + y^2$ and z^2 are relatively prime” (p. 50). New questions can then be posed related to these new alternatives. For example, if “=” is replaced by “<,” some of the questions that could be posed include the following:

- Does $x^2 + y^2 = z^2$have any geometrical significance?
- For what numbers is the inequality true?
- How many instances are there for which $x^2 + y^2$ differs from z^2 by a particular constant?
- What is the graph of the inequality? (rephrased from Brown & Walter, 1983/ 2005, p. 51)

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[20] In considering triangles, this is basically the question Bolyai, Lobachevsky, and Gauss finally posed after at least a thousand years of attempts to prove Euclid’s fifth postulate, which is equivalent to the “Pythagorean Theorem.” Finally considering the possibility that the Euclid’s fifth postulate about parallel lines (and thus the “Pythagorean Theorem” might *not* hold under all conditions lead to the discovery/ invention of non-Euclidean geometry (Grattan-Guiness, 2005). It is also important to note that the name “Pythagorean Theorem” follows the eurocentric policy of attributing findings to Greek mathematicians when others such as the Egyptians and Chinese had already discovered the relationship (Joseph, 1997).

The "what if not" strategy does not have to begin with a theorem or an equation; other starting points suggested by Brown and Walter (1983/ 2005) include a "real life" situation, a geometric figure such as a picture of an isosceles triangle, a concrete material such as a geoboard, a sequence such as the Fibonacci sequence, a construction such as a regular hexagon construction using a straightedge and compass, a net for a cubical box, and a set such as the set of prime numbers. The process also does not have to end with posing new questions; after brainstorming questions, students can choose a question to further and investigate. During further investigations, often more new questions arise, which can then be investigated. This process of "cycling," argue Brown and Walter (1983/2005), is a "process of varying one attribute followed by varying another [which] suggests a systematic technique we could employ for brainstorming new problems" (p. 60).

2.4 Problem-Posing and "The Real World"

Both problem-posing lines of flight deterritorialize the "solving" side of problem solving and critique the idea that teachers and textbooks should be the source of problems with which students engage. In different ways, these lines of flight opened up by the Freire Machine and the Brown-and-Walter machine also deterritorialize the idea of real-world applications of math in mathematics education. In Gutstein's (2006) Freirean work with middle school students in Chicago, the problem-posing projects and related conversations comprised about 15% to 20% of students' time in class; the rest of the time (80% to 85%) of the time, Gutstein used a reform-based curriculum called Mathematics in Context

(MiC). Gutstein (2006) notes that reform curriculum such as the Mathematics in Context curriculum "are often situated in generic real-world settings about daily life, such as shopping, traveling, working, and building" (p. 31). Gutstein (2006) found that the stories in the Mathematics in Context curriculum were rarely relevant to the students in his classes. When Gustein (2006) surveyed 79 students across three classes using the curriculum at his school, about half of the students said they liked the stories, but only 17 out of 79 said that they could relate to the stories. When Gutstein (2006) discussed the relevance of the stories with students in class, students responded by explaining that the characters in the stories "do things we don't do," "they don't deal with things most people do," and "it's not us" (p. 105). Some of the examples students gave included "we don't go on canoe trips," "they have a friend who went to England for a piano recital," and "We don't have family and friends in Africa, we don't go in hot air balloons. . .we don't go downtown and count cars" (p. 105). These responses raise the question, "mathematics in *whose* context?" (p. 105). Gutstein identified three types of real-world contexts that might be used in curricula: 1) nonrelevant ones such as those used in the Mathematics in Context curriculum, 2) relevant ones unrelated to social justice, such as finding distances between school and home, and 3) political ones involving social justice such as the projects Gutstein (2006) designed based on issues in his students' lives. Freiren problem-posing pedagogy with its goal of social transformation requires the inclusion of the third type of real-world context. Gutstein (2006) writes, "The change in orientation I seek is not just that students believe mathematics is utilitarian, but that they also view it as a tool with which to read the world" (p. 31). Therefore, a curriculum that uses nonrelevant or apolitical relevant contexts does not "*by itself*. . .prepare students to read or write their worlds with mathematics" since such a curriculum "does not challenge students to analyze injustice or see themselves as social change agents" (p. 104). It is problem-posing pedagogies which use the third type of real-world context that are "inherently dangerous to the status quo because they prepare students to ask fundamental questions stemming from the concrete analysis of their lives and begin to 'unveil reality'" (p. 31). It seems that some "real-world" contexts are "more real" than others. While the first two types of "real-world" contexts may exist in the world, only the third has the capacity to enable students to "unveil reality." Here "reality" signifies something more or other than simply that which exists.

The second problem-posing line of flight sets the meaning of "reality" in motion as well. In *Reconstructing school mathematics: Problems with problems and the real world,* Brown (2001) observes that typically what is meant by "application of mathematics to the real world" is that "mathematical models are created that can be used to explain and predict solutions to real-world problems" (p. 139). A clear implication of this formulation is that mathematics is not a part of the "real-world," but rather something separate and in some way unreal: "The justification for using real-world problems is frequently made on the grounds that they provide motivation for mathematical inquiry that might otherwise be considered as isolated from experiences in the world" (p. 139). Brown (2001) also notes that in this way of thinking about real-world applications, "what we seek is some sort of correspondence between elements of the real world and associated mathematical operations. The scheme suggests that we both begin and end with some 'real-world' phenomenon and that we invoke the use of mathematics along the way" (p. 140). Here, the idea is that the world "is always a bit 'messier' than the mathematics that models it" and therefore "we need to figure out what is 'relevant' and what is 'noise'" (p. 141). Once "noisy" elements are identified, they can be put out of mind. As an example, Brown (2001) cites this problem form the NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics:*

> Suppose Anne tells you that under her old method of shooting free throws in basketball, her average was 60%. Using a new method of shooting, she scored 9 out of her first 10 throws. Should she conclude that her new method really is better than her old method? (p. 172; cited in Brown, 2001, p. 8)

In responding to this problem, some of the information that is irrelevant in designing a mathematical model includes the type of sport Anne is playing (she may as well be shooting on the goal in soccer), that the person's name is Anne (it may as well be Kim or Peter), and that the person is a girl (the model would remain the same for a boy). Brown (2001) contrasts this '"real-world" problem with another:

> A close relative of yours has been hit by an automobile. He has been unconscious for one month. The doctors have told you that unless he is operated upon, he will live but will most likely be comatose for the rest of his life. They can perform an operation which, if successful, would restore his consciousness. They have performed ten such operations in the past and have been successful in only two cases. In the other eight, the patient died within a week. What counsel would you give the doctors? (p. 142)

While both problems deal with probability and involve a need to come to a conclusion, the second problem includes an ethical dimension not present in the first; important considerations in the second problem involve not only those related to probability, but also what counts as a life worth living. Brown (2001) argues, however, that “it is not only that the issue we confront is an ethical one. . . .More generally, in order to make sense out of a real-world problem, mathematical consideration is one important dimension, but it is part of a larger cloth” (p. 142). In the second problem, “we cannot come to an intelligent conclusion about what to do if we whittle away at the ‘irrelevancies’ as we did in the example with Anne” (p. 143). In fact, rather than less information, Brown (2001) argues, we would probably want to seek out more information about the comatose relative. As in the case of using the what-if-not strategy, encountering this second problem generates new questions.

The problem of the comatose relative also raises questions about the way in which mathematics relates to the “real-world.” Brown (2001) suggests moving to a view of this relationship that does not conceptualize “mathematics” and “the real-world” as two mutually exclusive fields: “A substantially different point of view is revealed as soon as we begin to relinquish a hold on mathematics that is rooted in a desire to see the field as totally different from other experiences in the world. . .” (p. 145). In considering ways of thinking, experiencing, and feeling that mathematics may share with other ways of experiencing the world, Brown (2001) concludes that

> There is a sense in which mathematical thinking connects mathematical ideas in a metaphorical way to those that everyone experiences in the context of daily living. It is not the pale coin of *applying mathematics to the real world*. Such a view assumes that

> the real world and mathematical thinking are in fact separate entities and that there is an occasional opportunity to seek their linkage. What I have in mind is that mathematical language offers metaphors for understanding the most fundamental qualities of human existence. . . .Thus we arrive at an even more robust view of what it might mean to apply mathematics to the real world. It is already 'in' the real world, and it is only by making believe that it is severed from the world that we arrive at some artificial notion of 'application.' (p. 152-153)

Brown (2001) tells a personal anecdote of the way in which he used a metaphor in approaching a mathematical problem. He began listing multiplication facts beginning with 1X3 and then adding 1 to each factor in each subsequent fact (i.e. 2X4, 3X5, 4X6. . .). As he began looking for patterns in the list of facts, he says, "I saw them not as what they actually *were* but rather as what they seemed to be *trying to become.* With just a small amount of squinting, these numbers were all almost something else—the furthermore they all missed being that something else by the same amount" (p. 149). He discovered that each product was almost a perfect square, but missed by one unit. Using the metaphor of "numbers 'striving' to become a square" allowed Brown (2001) to further investigate this pattern. Brown (2001) claims that although metaphor "is a concept we normally associate with poetry or other forms of literature in the 'real world'. . . metaphor is so deeply implicated in all our thinking that we engage in variations of metaphorical thinking, even when we are not aware that we are doing so" (p. 148). Brown (2001) quotes Keyser (1916) in identifying other mathematical metaphors that we use to understand reality:

> The mathematical concept of constant and variable are represented familiarly in life by the notions of fixedness and change. . . .What is known in mathematics under the name of limit is everywhere present in life in the guise of some ideal, some excellence high-dwelling among the rocks. . . .The supreme concept of functionality

> finds its correlate in life in the all-pervasive sense of interdependence of mutual determination among the elements of the world. . . . (Keyser, 1916, p. 78; quoted in Brown, 2001, p. 153)

The “real-world,” then, or at least our experience of it, is always already mathematical. From this perspective, the drastic separation of school math and real-world math which occurred in Boaler’s (2001) study takes on an uncanny irony. “Real-world” problems tacked onto the end of a mathematics unit are supposed to motivate students and help them apply math to their lives outside of school when the flows of desire throughout the entire unit up to that point have been forcing an artificial separation of these two “worlds.” Simply relinquishing “a hold on mathematics that is rooted in a desire to see the field as totally different from other experiences in the world” (Brown, 2001, p. 145), may allow the “real-world” to come rushing into the mathematics classroom.

2.5 Problem Posing and the Logic of Sense

Both problem-posing lines of flight, though originating in different contexts, deterritorialize not only the “solving” side of problem-solving, but also cut deep lines of deterritorialization into common sense notions of “the real world” and how it relates to mathematics. Both Gutstein (2006) and Brown (2001) call for “real-world” contexts that involve an ethical dimension. Both conceputalize “real” as something more than or other than simply that-which-exists. Brown (2001) claims that literary characters and works of art that are “more real than others” are not those who actually existed, but rather those who “are more vivid, concentrated, focused, delineated, integrated, inwardly beautiful" (p. 190).

In thinking about how this relates to mathematics, Brown (2001) says

> If we can speak of what is 'real' in a more vibrant sense than what 'exists' or what we can 'touch' and 'see,' then we not only legitimize more interesting connections between mathematics and the real world. . ., but we also suppress the need to seek real-world connections as a salve against an otherwise 'unreal' world of mathematics. (p. 191)

Problem-posing in mathematics education ultimately requires a reconceptualization of our relationship with mathematics. As Deleuze (1969/1990) writes in *The Logic of Sense,*

> The relation between mathematics and man [*sic*] may thus be conceived in a new way: the question is not that of quantifying or measuring human properties, but rather, on the one hand, that of problematizing human events, and, on the other, developing as various human events the conditions of the problem. (p. 55)

Pressing further the problem-posing line of flight which deterritorilizes the "solving" side of problem solving, Deleuze (1969/1990) argues that we need to think differently about the relationship between problems and solutions. He sees problems and solutions as different in kind:

> We must then break with the long habit of thought which forces us to consider the problematic as a subjective category of our knowledge or as an empirical moment which would indicate only the imperfection of our method and the unhappy necessity for us not to know ahead of time—a necessity which would disappear as we acquire knowledge. Even if the problem is concealed by its solution, it subsists nonetheless in the Idea which relates it to its conditions and organizes the genesis of the solutions. Without this Idea, the solutions would have *no sense."* (p. 54)

Problems and solutions are different in kind and also exist at different levels. Typically, people think of "truth" and "falsehood" as pertaining to solutions or propositions. For example, most people would agree that x = 9 is a solution to x

+ 2 =11 because substituting 9 for x makes the statement true. While “true” and “false” are the logical values pertaining to such propositions, Deleuze (1969/ 1990) argues that the relation between the proposition and these logical values could not exist without a separate dimension that provides the form of possibility of the proposition or solution. In the example above, letting x = 4 resulting in the statement, 4 + 2 = 11, would be a false, yet interpretable or sensible, solution. It “makes sense,” it is just wrong[21]. On the other hand, x = bird is an absurd response to “x + 2 = 4” because only numbers, not animals, conform to the form of possible solutions. “Bird” makes no sense as a solution. In *The Logic of Sense*, Deleuze (1969/ 1990) examines this notion of sense. He begins with three commonly agreed upon “distinct relations within the proposition” (p. 12). The first relation is denotation or indication, which is the relation between the proposition and an external state of affairs. According to Deleuze (1969/ 1990), “denotation functions through the association of the words themselves with *particular* images *which ought to* ‘represent’ the state of affairs” (p. 12). In the denotation relation, the logical value “true” “signifies that a denotation is effectively filled by the state of affairs” whereas “ ‘false’ signifies that the denotation is not filled” (p. 13). The second relation of the proposition is

[21] The statement 4 + 2 = 11 is just wrong, that is, as long as one of the conditions of the problem is that we are considering the decimal number system. If we change this condition so that we are considering a base 5 number system, then the statement 4 + 2 = 11 is true, and 4 is indeed a solution for x + 2 = 11. This example illustrates the way in which “every solution presupposes a problem--in other words, the constitution of a unitary and systematic field which orientates and subsumes the researches or investigations in such a manner that the answers, in turn, form precisely the cases of solution” (Deleuze, 1968/1994, p. 168). Deleuze (1968/ 1994), therefore, concludes that “a solution always has the truth it deserves according to the problem to which it is a response” (p. 159).

“manifestation,” and is “presented as a statement of desires and beliefs which correspond to the proposition” (p. 13). Manifestation is what makes a state of affairs personal; it constitutes “the domain of the *personal*, which functions as the principle of all possible denotation” (p. 13). Deleuze (1969/1990) suggests that “from denotation to manifestation, a displacement of logical values occurs. . .[so that they are] no longer the true and the false but veracity and illusion” (p. 15). The third relation is that of “signification.” The relation of signification moves beyond the particular of denotation and the personal of manifestation to a “relation of the word to *universal or general* concepts” (p. 14). The logical value of signification is “no longer the truth. . .but rather the *conditions of truth*, the aggregate of conditions under which the proposition ‘would be’ true” (p. 14). Signification as the condition of truth is “not opposed to the false, but to the absurd” (pp. 14-15). Thus, the question “There are 26 sheep and 10 goats in a ship. How old is the captain?” is absurd rather than false, since the conditions of the proposition do not provide conditions of truth. Students who give the answer of “36” fail to recognize that the conditions of truth are missing from the proposition. Yet even absurdities have sense: “the propositions which designate contradictory objects themselves have a sense. . .the two notions of absurdity and nonsense must not be confused” (Deleuze, 1968/1990). According to Deleuze (1968/ 1990), impossible objects such as “square circles, matter without extension” may be absurd but are not nonsense; they are objects “outside of being, but they have a precise and distinct position within this outside: they are of ‘extra-being’—pure, ideational events, unable to be realized in a state of affairs”

(p. 35). Deleuze (1968/ 1990; 1969/ 1995) therefore suggests that the three dimensions of denotation/ indication, manifestation, and signification are not sufficient; a fourth dimension, that of sense itself, is needed:

> Indeed, we must distinguish sense and signification in the following manner: signification refers only to concepts and the manner in which they relate to the objects conditioned by a given field of representation; whereas sense is like the Idea which is developed in the sub-representative determinations. It is not surprising that it should be easier to say what sense is not than to say what it is. In effect, we can never formulate simultaneously both a proposition and its sense; we can never say what is the sense of what we say. (Deleuze, 1969/1995, p. 155)

When Gutstein (2006) and Brown (2001) followed the problem-posing line of flight to the notion that “the real” is something more than or other than simply that-which-exists, what they have arrived at is “sense.” It is the dimension of sense that allows problem-posing pedagogies using Gutstein’s (2006) third type of real-world context to “prepare students to ask fundamental questions stemming from the concrete analysis of their lives and begin to ‘unveil reality” (p. 31). It is the dimension of sense that leads this line of flight to a “profound link. . .[to] ethics” (Deleuze, 1968/1990, p. 31). It is sense or the sensible that Brown (2001) alludes to when he calls us to “speak of what is ‘real’ in a more vibrant sense than what ‘exists’ or what we can ‘touch’ and ‘see’” allowing us to “suppress the need to seek real-world connections as a salve against an otherwise ‘unreal’ world of mathematics” (p. 191). This characteristic that can only be sensed is in fact the object of Deleuze’s (1969/1995) fundamental encounter: “Something in the world forces us to think. This something is an object not of recognition but of a fundamental *encounter*. . . .its primary characteristic is that it can only be sensed" (p. 139). This object of the encounter "is not a quality but a sign. It is not a sensible being but the being *of* the sensible. It is not the given, but that by which the given is given" (p. 140); that is, the object of the encounter is difference (p. 222). Encountering difference, that which can only be sensed, "moves the soul, 'perplexes' it—in other words, forces it to pose a problem" (p. 140).

2.6 Deleuzian Problem-Posing: Generating Problems

Since problem posing is an encounter with difference, and "difference-in-itself" is the center of a Deleuzian dialectic, "*Problems are always dialectical:* the dialectic has no other sense, nor do problems have any other sense" (Deleuze, 169/1995, p. 164). Hence, mathematical problems are actually dialectical problems. "What is mathematical" says Deleuze (1969/1995), "are the solutions" as well as "the expression of problems relative to the field of their solvability which they define, and define by virtue of their very dialectical order" (p. 179). Mathematical problems, then, always participate "in a dialectic which points beyond [them]—in other words, in meta-mathematical and extra-propositional power" (p. 164).

While Deleuze (1969/1995) emphasizes that problems are always dialectical, a Hegelian/ Marxist dialectic based on contradiction overlooks "difference-in-itself" and misinterprets the encounter with the being of the sensible:

> Whenever the dialectic 'forgets' its intimate relation with Ideas in the form of problems, whenever it is content to trace problems from propositions, it loses its true power and falls under the sway of the

> negative, necessarily substituting for the ideal objecticity of the *problematic* a simple confrontation between opposing, contrary or contradictory, propositions. This long perversion begins with the dialectic itself, and attains its extreme form in Hegelianism. (p. 164)

A Deleuzian problem-posing pedagogy rejects a Hegelian dialectic that views difference as the only problem, one that can be resolved through contradiction in favor of one that affirms difference, an anti-oppressive problem-posing pedagogy that involves learning through the crisis instigated by the return of difference (Kumashiro, 2004), as described in the “Affirming Difference” plateau. It is only this form of problem posing that brings real movement to thought, and for this movement to occur, a distinction between knowledge and learning is required. In order to deterritorialize the dogmatic image of thought which has “from Plato to the post-Kantians” defined “the movement of thought as a certain type of passage from the hypothetical to the apodictic” (p. 196), which “maximally betrays and distorts this movement” (p. 197), we must instead conceptualize the movement of thought as going “from the problematical to the question” (p. 197). This distinction is crucial because “the assimilation of the problem and the hypothesis is already a betrayal of the problem or Idea, involving illegitimate reduction of the latter to propositions of consciousness and to representations of knowledge” (p. 197). While knowledge refers to “only the generality of concepts or the calm possession of a rule enabling solutions” (p. 164), problem posing as an encounter with “difference in itself” enables the “subjective acts carried out when one is confronted with the objecticity of a problem (Idea),” that is, the “process of learning” (p. 164). Learning, then, necessitates problem posing, and problem posing is the multivocity of being, the affirmation of difference, the eternal return of difference in repetition, the movement from the problematical to the question. "What are. . .these questions which are the beginning of the world?" asks Deleuze (1969/1995). His answer, fittingly, ends in a question: "The fact is that every thing has its beginning in a question, but one cannot say that the question itself begins. Might the question. . .have no other origin than *repetition?"*

We return to Huang, Mok, and Leung's (2006) chapter about explicit and implicit variation, whose title, "Repetition or variation: Practicing in the mathematics classrooms of China" resonates with the title of Deleuze's (1969/1995) book *Difference and Repetition.* Deleuze's (1969/1995) distinction between cadence repetition (repetition of the Same) and rhythmic repetition (repetition of difference) as explored in the "Affirming Difference" plateau can further the exploration of this resonance. Explicit variation is cadence repetition in that students encounter problems of the same form and repeat the same procedure as was presented by the teacher. Implicit variation, on the other hand, is like rhythmic repetition, repetition with a difference. However, when the source of implicit variation remains solely the teacher, it remains within the problem-solving stratum rather than following a problem-posing line of flight. Implicit variation in a problem-posing pedagogy must originate in the encounter between teachers and students. Brown and Walter's (1983/ 2005) "what if not" strategy locates the process of implicit variation in the encounter between teachers and students, as teachers and students generate questions and problems from a given situation or artifact. Gutstein's (2006) projects were problems that stemmed from conversations between himself and the students, problems that required not only transformation of the mathematics they had been learning, but transformation of their social consciousness as well.

Deleuzian problem-posing, however, presses the problem-posing line of flight yet further than either the Freire Machine or the Brown-and-Walter Machine. Freire's (1970/2003) process of praxis combines "reflection and action upon the world in order to transform it" (p. 51). This takes place through codifications which serve as representations of generative themes which are given back to the people as problems to solve (p. 123). Through reflection and action centered on the representation, teacher-students and students-teachers can overcome false consciousness. A Deleuzian problem-posing pedagogy, one that affirms difference, must break out of the four shackles of that mediate difference through representation: identity, analogy, opposition, and resemblance. For Deleuze, all "consciousness," which remains shackled, is false; it is problems that escape this false consciousness by affirming "difference-in-itself," and allow true movement of thought which deterritorializes the dogmatic image of thought, thought stratified by common sense. Deleuze (1969/1995) writes,

> While it is the nature of consciousness to be false, problems by their nature escape consciousness. The natural object of social consciousness or common sense with regard to the recognition of value is the fetish. Social problems can be grasped only by means of a 'rectification' which occurs when the faculty of sociability is raised to its transcendent exercise and breaks the unity of fetishistic common sense. The transcendent object of the faculty of sociability is revolution. In this sense, revolution is the social power of difference, the paradox of society. (p. 208)

While Freire (1970/2000) wrote that "the important thing is to detect the starting point at which the people visualize 'the given'" (p. 107), Deleuzian problem posing is an encounter not with "the given," which is diversity, but with that-by-which-the-given-is-given, that is, difference (Deleuze, 1969/1995, p. 222). This requires that the encounter take place not only at the level of denotation/ indication, but at the level of sense, at the level of problems themselves, rather than following the "natural illusion (which involves tracing problems from propositions)" (p. 159). This is why a Deleuzian problem -posing pedagogy, as a part of postcritical multicultural education or anti-oppressive education (Kumashiro, 2004), is

> a disarming process that allows students to escape the uncritical, complacent repetition of their prior knowledge and actions. Learning is a disorienting process that raises questions about what was already learned and what has yet to be learned. Learning involves *looking beyond* what students already know, what teachers already know, and what we both are only now coming to know, not by rejecting such knowledge, but by treating it paradoxically, that is, by learning what matters in society. . .while asking why it matters (and how it can reinforce and challenge an oppressive status quo). (Kumashiro, 2004, p. 30)

That is why a Deleuzian problem-posing pedagogy involves teaching and learning against common sense (Kumashiro, 2004). For Deleuze (1968/1994),

> To learn is to enter into the universal of the relations which constitutes the Idea. . .To learn to swim is to conjugate the distinctive points of our bodies with the singular points of the objective Idea in order to form a problematic field. This conjugation determines for us a threshold of consciousness at which our real acts are adjusted to our perceptions of the real relations, thereby providing a solution to the problem" (p. 165)

Yet, the problem does not disappear in the solution, but returns in the process of moving "from the problematical to the question," which is itself repetition, the eternal return of difference. The "generating" in the title of this plateau, then is both adjective and verb: problem posing is the process of generating problems, yet the problems themselves are genetic or generating. Problem posing—that is, learning--is *becoming.*

3. Becoming-Democratic Mathematics Education

On June 1, 2009, the National Governor's Association and the Council of Chief State School Officers in the United States announced that 49 states and territories had joined the Common Core Standards Initiative (National Governors' Association & the Council of Chief State School Officers, n.d.). One year later, standards for English Language Arts and Mathematics were released (Common Core State Standards Initiative, n.d.a, n.d.b.). By June of 2011, 44 states and territories had formally adopted the standards (Common Core State Standards Initiative, n.d.c). The mathematics standards describe in detail over the course of about 90 pages *what* students in Kindergarten through high school should learn. However, *why* students should learn this content and these skills is answered in a single statement repeated in various forms many times throughout the auxiliary documents: "The standards developed . . . must ensure ***all*** American students are prepared for the global economic workplace" (National Governors' Association & the Council of Chief State School Officers, n.d., p. 1). In other words, the purpose of schooling is to prepare students to be workers under global capitalism.

The brief, singular, and repeated purpose of mathematics education, stated without elaboration or further discussion, serves as the stratum from which to embark on a line of flight in this plateau. This line of flight entails a reworking of previous Marxist-influenced ideas through Deleuzoguattarian concepts such as the apparatus of capture, the subject of enunciation and the subject of the statement, machinic enslavement and social subjection. Pressing the line of flight further, through Deleuze and Guattari's ontology of becoming, creates the concept of *becoming-democratic mathematics education.*

3.1 Marxist-Influenced Analyses of Schooling: Two Conflicting Impulses in "Liberal-Democratic" Society

Although the United States is often described as a "liberal-democratic society," as if "liberal-democratic" were one homogenous attribute, Marxist-influenced educational theorists[22] have noted that "liberal-democratic" actually captures two conflicting impulses. The first impulse is the liberal economic impulse towards the capitalist market. The second impulse is the democratic impulse toward egalitarian democratic politics. For Ryoo and McLaren (2010), the United States "is not, and never has been, a democracy" (p. 100), since democracy is impossible under the contemporary capitalist system. "Capitalism," they argued, "through systematic exploitation of human labor power, spawns asymmetrical systems of power and privilege that deny people true direct, participatory or 'protagonistic' democracy and opportunity to unite against conditions of oppression" (p. 101). Ryoo and McLaren (2010) contrasted the current conditions of the United States--such as the simultaneous exploitation of

[22] A thorough summary of the works of "Marxist-influenced educational theorists" is beyond the scope of this section. The works addressed here were chosen because the ideas they include can provide a starting point for a Deleuzoguattarian reformulation. For further investigation see Price (1986), Small (2005), Allman (2010), and Anyon (2011).

immigrants in factories and farms and denial of health care and severe poverty among many people of color, despite being one of the highest level of national wealth--with the democratic ideal of a "government in which the supreme power is vested in the people and exercised by them directly or indirectly. . . .[and] the absence of hereditary or arbitrary class distinctions or privileges" (p. 101). Yet, it is in schools, Ryoo and McLaren (2010) observed, that "learning to believe in the ideals of democracy, while living amidst the anti-democratic ideals of capitalism" begins (p. 102).

Bowles and Gintis (1976) examined the ways in which this tension between the liberal-economic and democratic impulses plays out in schools in ways that reproduce social and economic inequities. Bowles and Gintis (1976) described the system in place in the United States as one consisting of a formally totalitarian economic system embedded in a formally democratic political system:

> The U.S. economy is a formally totalitarian system in which the actions of the vast majority (workers) are controlled by a small minority (owners and managers). Yet this totalitarian system is embedded in a formally democratic political system which promotes the norms—if not the practice—of equality, justice, and reciprocity. (p. 54)

The authors further delineated the ways in which the U.S. educational system reproduces the social and economic inequities in place in the broader society through a structural correspondence between schooling social relations and those of production. According the authors, the organization of social relations within schools replicates the hierarchical division of labor outside of schools. Schools prepare students to take their place in the social and economic hierarchy outside schools not only by "Inur[ing] the student to the discipline of the workplace, but [also by developing] the types of personal demeanor, modes of self-presentation, self-image, and social class identifications which are the crucial ingredients of job adequacy" (p. 131). The hierarchical relations among administrators, teachers, and students replicate the hierarchical relations of the workplace. Just as workers experience alienation from their labor power as it is separated from them in the form of commodities,

> alienated labor is reflected in the student's lack of control over his or her education, the alienation of the student from the curriculum content, and the motivation of school work through a system of grades and other external rewards rather than the student's integration with either the process (learning) or the outcome (knowledge) of the educational 'production process' (p. 131).

Bowles and Gintis (1976) also noted differences in socialization corresponding to different "heights" on the educational ladder. For example, secondary schools closely channel and monitor the activities of students whereas community colleges provide somewhat less supervision and more independence. At the top of the hierarchy, elite four-year colleges "emphasize social relationships conformable with the higher levels of the production hierarchy" (p. 131). Since access to secondary schools, community colleges, and elite four-year colleges differs along class-lines, those from different class backgrounds tend to be "trained" for different sorts of jobs. In Brewster's translation of Althusser (1970/1971), this process is described as schools "ejecting" masses of students at different points along the educational pathway, each mass having been provided with an ideology to match the role it is to fulfill in society[23]. Anyon

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[23] Althusser's original phrasing was a bit less dramatic: "*une énorme masse d'enfants tombe*

(1980; 1981) further investigated this phenomenon and found that students with different class backgrounds experience different schooling even at the same educational level. For example, the schooling of working-class elementary students, middle-class students, affluent professional class students, and executive elite students differs from one another. She found this to be true even when state standards and adopted curricular materials were the same. The ways in which teachers used the curricular materials and the parts they selected to include and exclude differed to such an extent that a different "dominant theme" emerged from each of the social class categories (Anyon, 1981). The dominant theme in working-class schools was *resistance*. Knowledge was seen as consisting of fragmented facts and rule-governed behaviors. Students resisted the irrelevance and meaninglessness of this form of knowledge; teachers responded to student resistance by concluding that students did not care about anything. In the middle class school, the dominant theme was *possibility*. School knowledge was seen as a source of real value when accumulated in large quantities. In the affluent professional school, Anyon (1981) described the dominant theme as *extreme narcissism* or *extreme individualism.* Across all content areas, students were encouraged to learn through discovery and be creative. Finally, the dominant theme in the executive elite school was *excellence*. Knowledge was seen as understanding the internal structure of disciplines and concepts. Teachers strove to teach more content and more

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'dans la production'" A more literal translation would be, "a large mass of children fall into the production [process]."

difficult content to students. Students experienced a great deal of pressure to excel and outperform others in order to get into the “best” schools. The schools in Anyon’s (1981) study served to reproduce the inequitable hierarchical social and economic structure outside of schools in just the way Bowles and Gintis (1976) suggested.

Building on the work of Bowles and Gintis (1976), Labaree (1997) also noted the two conflicting impulses in U.S. “liberal-democratic society”:

> Unfettered economic freedom leads to a highly unequal distribution of wealth and power, which in turn undercuts the possibility for democratic control; but at the same time, restricting such economic freedom in the name of equality infringes on individual liberty, without which democracy can turn into the dictatorship of the majority. (p. 41)

The tension between these two conflicting impulses was evident in the tension between Thomas Jefferson’s political idealism and Alexander Hamilton’s economic realism (Labaree, 1997). Within the context of this tension, three goals or functions for schools have developed: 1) democratic equality, 2) social efficiency, and 3) social mobility. Labaree (1997) argued that the amount of emphasis on each of the three goals has shifted over the course of the history of U.S. schooling. Proponents of the democratic equality goal argue that a democratic society necessitates an education system that prepares all children to become competent and responsible citizens. Since each citizen theoretically has an equal voice in the collective decisions of the society, it is in everyone’s best interest that all citizens gain the knowledge necessary to make informed decisions. Moreover, political equality can be undermined under conditions of too much social inequity. Therefore, the purpose of schools is to promote effective citizenship and social equality. The second goal is that of social efficiency. Proponents of this perspective argue that the economic well-being of society requires that schools effectively and efficiently prepare children for the existing economic roles in society. Such preparation will ensure that society will benefit from a productive workforce. From the perspective of the social mobility goal, the purpose of schools is to provide individual students with a competitive advantage in the race for desirable economic and social positions in society. Labaree (1997) asserted that each of these goals comes from the perspective of a different social role. Democratic equality is the perspective of the citizen; social efficiency is the perspective of the taxpayer and employer; social mobility is the perspective of the individual educational consumer. Democratic equality expresses the politics of citizenship; social efficiency expresses the politics of human capital; social mobility expresses the politics of individual opportunity.

Labaree (1997) suggested that all three of these goals have shaped U.S. schools, but that the relative emphasis on each of the three goals has shifted over time. The goal of democratic equality has entailed a focus on citizenship training, equal treatment, and equal access. The strongest motivation behind the common school movement was the goal of training students for citizenship. The liberal arts tradition was also intended to allow students to gain a broad understanding of many disciplines which they could then bring to bear on societal decisions. Citizenship training as a way to preserve the U.S. as a republic has continued throughout the history of U.S. schooling. Equal treatment and equal access have also been seen as necessary for the preservation of the republic.

The common school movement entailed “universal enrollment, uniform curriculum, and a shared educational experience for all students” (Labaree, 1997, p. 45). The establishment of comprehensive high schools, school desegregation, movements for bilingual education have all focused on providing more equitable access to a broader range of students.

The goal of social efficiency, Labaree (1997) claimed, was operationalized through vocationalism and educational stratification. Vocationalism was a push for schools to shift their curricula from a focus on traditional academic subjects to a focus on training students for particular occupational roles. According to Labaree (1997), vocationalism had a much stronger impact on schooling than simply the establishment of vocational schools, which were always in the minority of schools. Labaree (1997) argued that vocationalism led to a philosophical shift in the general aims of education around the turn of the twentieth century as captured in the following quote: “For a long time all boys were trained to be President. . .Now we are training them to get jobs” (Lynd & Lynd, 1929, p. 194; quoted in Labaree, 1997, p. 47). In addition to vocationalism, educational stratification served the goal of social efficiency. According to Labaree (1997),

> One form [this stratification] has taken is in the emerging hierarchy of educational levels, leading from elementary school to high school to college and then graduate school. The upward expansion of enrollment in this hierarchy over time, while increasing the average years of schooling for the population as a whole, has also provided access to higher levels of education at which individuals can be distinguished from the herd, with the key division being between those who persist in education and those who drop out at an earlier level. . . .[F]rom the perspective of social efficiency, the vertical distribution of educational attainment is quite desirable, since it reflects the vertical structure of the job market and therefore helps allocate individuals to particular locations in the workforce, as

> students move horizontally from a given level in the educational hierarchy to a corresponding level in the occupational hierarchy. (p. 49)

The patterns identified by Bowles and Gintis (1976) and Anyon (1980; 1981) demonstrate the way schools have been set up to be “a mechanism for adapting students to the requirements of a hierarchical social structure and the demands of the occupational marketplace” (Labaree, 1997, p. 46).

Although all three goals have threaded their way through the history of U.S. schooling, Labaree (1997) argued that the goal of social mobility has become the dominant goal in schools. Like social efficiency, social mobility relates to the role of a stratified educational system to prepare students for the stratified economic and social structure in society. However, these two goals view these stratified systems from different vantage points. As representatives of the social efficiency goal, the taxpayer and employer view the structure from the top; as representative of the social mobility goal, the consumer views the structure from the bottom, with the goal being to climb as far toward the top as possible. A particular student’s pathway through the structure and the degree to which the student’s landing spot in the system is desirable economically and socially is irrelevant to the taxpayer and employer, as long as all students are well-prepared for the position they ultimately fill. In contrast, the landing spot in the system matters very much to the student as consumer. The consumer’s goal is to accumulate more and better educational commodities to gain a competitive advantage over others and ultimately “win” by acquiring a desirable job. Rather than desiring equal opportunity, consumers want schools to provide them with more opportunity than others. Therefore, the social mobility goal reinforces the inequitable, stratified educational system described by Bowles and Gintis (1976) and Anyon (1980; 1981). According to Labaree (1997), consumers demand a graded hierarchy, “which requires students to climb upward through a sequence of grade levels and graded institutions and to face an increasing risk of elimination. . . .[A]s students move into an atmosphere that is increasingly rarefied. . .the chance for gaining competitive advantage grows correspondingly stronger” (p. 52). As it turns out, the employer and the consumer demand an educational system with similar characteristics. In addition to a graded hierarchy, both demand qualitative differences between institutions at each level (e.g. some schools are considered better than others), and a stratified structure of opportunities within each institution (e.g. tracking in high school, leveled reading groups, etc.). These distinctions at each level would have little meaning without corresponding signaling systems that communicate the distinctions among consumers (students) to employers. Such signaling systems take the form of program labels (e.g. “Advanced Placement” courses, gifted and talented program, “sheltered” mathematics, magnet programs), standardized testing, differentiated diplomas, college rankings, and the use of grade point averages. Because of the way social efficiency and social mobility hook the consumer along with the employer into the a stratified educational system, consumers protest as loudly (or even more so) when educators “propose the elimination of some form of within-school distinction or another, such as by promoting multi-ability reading groups, ending curriculum tracking, or dropping the gifted and talented program

(Cusick, 1992; Wells & Sema, 1996)" (Labaree, 1997, p. 53).

The Common Core Mathematics Standards documents in their brief, singular, and repeated stated purpose--"The standards developed . . . must ensure ***all*** American students are prepared for the global economic workplace" (National Governors' Association & the Council of Chief State School Officers, n.d., p. 1).--directly invoke the social efficiency goal. The purpose of mathematics education, from this view, is to prepare students to be workers. It seems at first glance, especially with the heavy emphasis on the word "all," that the purpose of the Common Core Standards also encompasses the democratic equality goal. However, given the reproductive model suggested by Bowles and Gintis (1976) and elaborated upon by Labaree (1997), preparing "all" American students for the global workforce does not necessarily entail preparing all students for equally desirable locations in the economic and social hierarchy. In fact, as Labaree (1997) pointed out, "the notion of educational equality is at best irrelevant to the expansion of GNP, and it is counterproductive in a capitalist economy, where the pursuit of competitive advantage is the driving force behind economic behavior" (p. 48). The emphasis on "all" is a linguistic allusion to the democratic equality goal while taking on a distinct form of expression within global capitalism: efficiently preparing ***all*** students for the global workforce must entail differentially preparing students to take social and economic positions in an inequitable, stratified, hierarchical system. The complementary top and bottom views of this system from the perspective of the taxpayer/ employer (in the name of social efficiency) and consumer (in the name of social mobility) ensure consumer “buy-in” as well, since the social mobility goal holds out hope for consumers that they may be able to accumulate enough educational commodities (credentials) in order to land in a favorable position within the collective “***all***.” To facilitate communication between consumers and employers about who has indeed won the race to accumulate educational credentials, the Common Core Standards for Mathematics embeds a signaling system in which a “+” indicates “additional mathematics that students should learn in order to take advanced courses” (Common Core State StandardsInitiative, n.d.b, p. 57).

3.2 A Deleuzoguattarian Perspective on the Two Impulses: Capitalism and the State

Deleuze (1995) explained that “Félix Guattari and I have remained Marxists, in our two different ways, perhaps, but both of us. You see, political philosophy must turn on the analysis of capitalism and the ways it has developed” (p. 171). However, Deleuze approached Marx’s work as he did the work of all previous theorists: as a starting place from which to develop concepts. Deleuze’s work, while remaining profoundly Marxist, was also a critique and reformulation of Marx’s ideas. One point of agreement between Marx (1867/1976) and Deleuze and Guattari (1980/1987; Deleuze, 1968/1994) was the centrality of the “economic instance” (also see Althusser, 1965/1969, 1970/1971; Althusser & Balibar, 1966/1970; Holland, 2009). However, Deleuze and Marx differed on the conceptual formulation of “the economic.” Marx viewed the economic instance as a fundamental contradiction between social classes. Deleuze (1968/1994), on the other hand, formulated the economic as “never given properly speaking, but rather designating a differential virtuality to be interpreted, always covered over by its forms of actualization; a theme or 'problematic' always covered over by its cases of solution" (p. 186). The economic, then, is the presupposition underlying any actual solution in the form of social relations. For Deleuze, problems are always virtual; solutions are always actual. Hence, Deleuze (1968/1994) stated that "The problems of society, as they are determined in the infrastructure in the form of so-called 'abstract' labor, receive their solution from the process of actualization or differenciation (the concrete division of labor)" (p. 207). While what Marx was fascinated by the way in which capitalism entailed the socialization of production relations and the development of human productive forces, it was the process of differenciation that most fascinated Deleuze (Holland, 2009). As a "difference-engine" (Holland, 2009), capitalism accelerates other already existing processes of differenciation such as organic differenciation in biological evolution and linguistic differenciation in the proliferation of discourses and sign systems. Deleuze's ontology is one in which difference-in-itself, rather than contradiction, is primary. Accordingly, in contrast to more Hegelian dialectical readings of Marx, Deleuze (1968/1994) read Marx's *Capital* as indicating that "the category of differenciation (the differenciation at the heart of a social multiplicity: the division of labor) is substituted for the Hegelian concepts of opposition, contradiction and alienation" (p. 207). As a difference-engine, capitalism is an extremely creative, but also dangerous process—a major force of deterritorialization in the contemporary world. Capitalism is both innovative and destructive (Glezos, 2010). Thus, like Bowles and Gintis (1976) and Labaree (1997), Deleuze and Guattari (1980/1987) view capitalism (or "liberal economics") as one of the major forces at work in shaping contemporary society, not only in the U.S., but throughout the world. For Labaree (1997), the force opposed to the liberal economic impulse is the democratic impulse; a Deleuzoguattarian perspective rejects the Hegelian dialectics inherent in a formulation of these two forces as contradictory and is therefore more complicated. Rather than characterizing capitalism as a force opposing the democratic impulse, Deleuze and Guattari (1980/1987) said that capitalism has become an "axiomatic[24]" (p. 436). In mathematics, an axiom is a proposition that is assumed to be true. Undefined terms are words and expressions that are taken as accepted without definition. Axioms and undefined terms serve as building blocks for building mathematical systems or theories. An axiomatic system can be built by using logic, undefined terms, and the original axioms to reason that additional propositions (theorems) must be true. The axioms in an axiomatic system are independent if one does not logically follow from another or a combination other axioms. (If a proposition does follow from other axioms, then it is a theorem rather than an axiom.) An axiomatic system is consistent if it is impossible to derive a contradiction (that is, to derive both a statement and its negation) from its axioms. Axiomatizing a mathematical domain involves beginning with a small number of axioms,

[24] In mathematics "axiomatic" is typically used as an adjective, as in the terms axiomatic system, axiomatic geometry, and axiomatic method. Deleuze and Guattari (1980/1987) use "axiomatic" as a noun, possibly as a shortened form of the phrase"axiomatic system." I follow their usage when discussing capitalism, but follow the more typical usage when discussing axiomatic systems in mathematics.

systematically deriving logically consistent theorems from the axioms, and demonstrating that the axioms are consistent and independent of one another. There are usually multiple ways to axiomatize a given mathematical domain. A model of an axiomatic system is a set of objects in a particular configuration which interprets the meanings of the undefined terms of the system such that all of the axioms of the system are true in the configuration. Two models of an axiomatic system are isomorphic if there is a one-to-one correspondence between the elements of the first model and the elements in the second model such that the relations among elements are preserved. The concept of an axiomatic system was useful for Deleuze and Guattari for two reasons. The first reason was that an axiomatic system is "actualized" in a model of the axiomatic system in the same way that a virtual problem is actualized in the particular economic solutions enacted in societies. For Deleuze and Guattari (1980/1987), the historical emergence of capitalism was not predetermined; rather, it was a historical accident. The authors claimed that it would have been possible for capitalism to emerge at a different time or place, or not emerge at all. However, once capitalism did emerge, a "new threshold of deterritorialization" was surpassed and history became universal--there was no going back. At that point, the rules and properties of capitalism became "given," became "axioms" that regulated the "models of realization" of social relations worldwide. The second reason Deleuze and Guattari (1980/1987) found the conceptual combination of an axiomatic (system) and models of "realization" (models of the axiomatic system) useful was that it allowed them to examine the ways in which different economic "solutions" relate to one another and to capitalism. A model of an axiomatic system is an instantiation that preserves relationships among elements. Deleuze and Guattari (1980/1987) claimed that all modern states—democratic, totalitarian, liberal, and tyrannical alike—are isomorphic in relation to the capitalist axiomatic. "Even the so-called social States are isomorphic," the authors argued, "to the extent that there is *only one world market*, the capitalist one[25]" (p. 455). No State, no matter what political form or manner of social relations it adopts, is outside of the capitalist axiomatic. While it would be easy to conclude that this totalizing capitalist axiomatic must have a global homogenizing effect, Deleuze and Guattari (1980/1987) warned that such a conclusion would be mistaken. On the contrary, the isomorphic relationship among models "allows, and even incites, a great heterogeneity among States (democratic, totalitarian, and especially, 'socialist' States are not facades)" (Deleuze & Guattari, 1980/1987, p. 436). Moreover, the worldwide capitalist axiomatic requires a "certain peripheral polymorphy." (p. 437). On the national scale, the flows of capital in a capital system result in an inequitable distribution of wealth and inequitable social relations; the same phenomenon occurs on the global

[25] In using the term "isomorphic," Deleuze and Guattari (1980/1987) emphasized that all States of varying forms have a similar relationship to capitalism, and in this way are interchangeable. However, they did not establish that relations among elements within the different state forms correspond to relations among elements in another model, which would be necessary for their use of "isomorphic" to be "literal" as they claim (p. 465). Instead, configurations of elements within different states—e.g. the particular ways in which political systems are configured, the ways in which various processes function—do *not* have one-to-one correspondence, which is why the collection of models is heterogenous. When discussing "models of realization" of the capitalist axiomatic system , I follow Deleuze and Guattari's usage of "isomorphic" to indicate that the all of the models are in some ways interchangeable in their relationship with global capitalism.

scale under the global capitalist axiomatic. Inequitable economic and social relations develop not only among individuals but also among States on a worldwide scale. Under the capitalist axiomatic, the “developed” world needs a “developing world.”

The point of agreement, then, among Marx, Bowles and Gintis, Labaree, and Deleuze and Guattari is that capitalism is a powerful force that shapes social relations. While the capitalist (liberal economic) impulse and the democratic impulse are conflicting or opposing forces for Labaree (1997), the second force for Deleuze and Guattari (1980/1987), following Althusser, is the State apparatus. The State’s relation to capitalism is not simply an opposing force; instead, Deleuze and Guattari (1980/1987) reformulated Marx’s concept of the (repressive) State apparatus via Althusser’s(1970/1971) concept of ideological State apparatuses into “State apparatuses of capture.” For Deleuze and Guattari (1980/1987), the State-form was both necessary for the emergence of capitalism and also serves to capture what is deterritorialized through capitalism. In the emergence of capitalism, both towns and the State played a role. However, it was “through the State-form and not the town-form that capitalism triumphed; this occurred when the Western States became models of realization for an axiomatic of decoded flows, and in that way resubjugated the towns” (p. 434). The towns did not create capitalism—instead they anticipated it and warded it off at the same time. This complex relationship between towns and States allows towns to develop which no longer had connection to their own land, but functioned to assure trade between States. With this assurance of trade, the State gives capitalism its modes of realization; through a reciprocal relationship, “what is thus realized is an independent, worldwide axiomatic that is like a single City, megapolis, or ‘megamachine’ of which the States are parts, or neighborhoods” (Deleuze & Guattari, 1980/1987, p. 435). The megamachine coordinates many different types of machinic processes (e.g. polarization of urban societies, anticipation-prevention of hunter-gatherer societies; capture by State apparatuses) which correspond to different forms of social relations; Deleuze and Guattari (1980/1987) defined social formations by these machinic processes rather than by modes of production because the modes of production depend on the machinic processes. The imperial State proceeded by two apparatuses of capture—tribute/ taxation and ground rent; capitalism added a third—capital (Deleuze & Guattari, 1980/1987; Holland, 2009). Capture is “the machinic process whereby a direct comparison (of land, labor, or goods) enables a monopolistic appropriation (of rent, profit, or tribute), the first moment presupposing an established stock constituted by the second moment” (Holland, 2009, p. 156). In ordering the two moments in this way, Deleuze and Guattari (1980/1987) reverse the direction of the process put forth by Marx: Marx argued that surplus arises from production; Deleuze and Guattari (1980/1987; see also Holland, 2009) suggested that production arises from surplus. In the imperial State it is the despot, who is “at once eminent landowner, entrepreneur of large-scale projects, and master of taxes and prices,” who sits at the point of convergence between all three forms of capture. Deleuze and Guattari (1980/1987), drawing on Nietzsche, viewed debt rather than production to be primary in social formations. The social formation of the imperial state is organized around infinite debt owed to the despot (Deleuze & Guattari, 1980/1987; Holland, 2009). This formulation "presupposes nothing 'private'" (p. 448), as all forms of capture flow to the despot, the "sole and transcendent public-property owner" (p. 428). However, even as the despot captures the various forms of surplus, flows also escape along new lines of flight. Large scale public works create independent labor which escapes from the despot's control; taxation creates new forms of commerce and banking; adjacent to public property, private property "is constituted on the margins" (p. 449). In contemporary society this line of flight has accelerated to the point to which "capitalism forms *a general axiomatic of decoded flows*" (p. 453), all of the States are coordinated in one global "megamachine," and the functions of the public and private spheres has shifted. The public sphere "no longer characterizes the objective nature of property but is instead the shared means for a now private appropriation; this yields the public-private mixes constitutive of the modern world" (p. 451). This new social formation still entails an infinite debt, but now the infinite debt is owed to capital itself.

Deleuze and Guattari (1980/1987), following Mumford (1966), defined a machine as "a combination of resistant parts, each specialized in function, operating under human control to transmit motion or perform work" (p. 457). This definition allows for a broader conception of machines than is typically considered in contemporary society. Machines are not limited to motorized or electronic machines, but also include any combination of "parts" that fit the above definition. The ways in which humans are caught up in interactions with motorized and electronic machines as well as serving themselves as parts of more machines more broadly defined results in two processes: machinic enslavement and social subjection. According to Deleuze and Guattari (1980/1987), “there is [machinic] enslavement when human beings themselves are constituent pieces of a machine that they compose among themselves and with other things (animals, tools), under the control and direction of a higher unity” (pp. 456-457). In the imperial state, people served as “pieces of a machine” that allowed the despot to profit, for example, from large-scale public works. Within industrial capitalism, workers were no longer enslaved by the machine, but instead “subjected to” the machine. With global capitalism,

> Capital acts as the point of subjectification that constitutes all human beings as subjects; but some, the ‘capitalists,’ are subjects of enunciation that form the private subjectivity of capital, while others, the ‘proletarians’, are subjects of the statement, subjected to the technical machines in which constant capital is effectuated. (p. 457)

The distinction between the two types of subjects—subject of enunciation and subject of the statement—is crucial in Deleuze and Guattari’s philosophy. It is characteristic of their philosophy to ignore taken-for-granted boundaries and to make connections among different aspects of existence that are not typically thought of as connected, such as connections between linguistics and political and social philosophy. Deleuze and Guattari (1975/1986) first developed the distinction between the two types of subjects in *Kafka: A Minor Literature*, in examining letters Kafka had written to others in his life. Deleuze and Guattari (1975/1986) posed the question:

> But how do the letters function? Without a doubt, because of their genre, they maintain the duality of the two subjects: for the moment, let us distinguish a subject of enunciation as the form of expression that writes the letter, and a subject of the statement that is the form of content that the letter is speaking about (even if *I* speak about *me*). (p. 30)

Take, for example, a letter in which the author writes, “I am planning a trip this coming month.” The subject of the written sentence is “I,” the author of the letter. The letter-writer and the subject of the statement in the letter are one and the same. In contrast, the author of the letter might write, “You will surely enjoy our upcoming trip.” Grammatically, the subject of the sentence or statement written in the letter is “you,” the addressee. However, the addressee is not actually expressing or enunciating anything; it is the author of the letter who is expressing something about the addressee. The two subjects differ in relation to control of the expression. Within global capitalism, everyone “becomes a subject” or is constituted as an individual in relation to capitalism. However, the meaning of “being a subject” within the capitalist system is different depending on the person’s location within the system. “Capitalists’” location provides them control over expression within the system, to be “subjects of enunciation.” The location of “proletarians,” on the other hand, subjects them *to* the system as “subjects of the statement.”

While industrial capitalism induced a shift from machinic enslavement to social subjection, contemporary capitalism has maintained social subjection and reinstated a new form of machinic enslavement: “The axiomatic itself, of which the States are models of realization, restores or reinvents, in new and now technical forms, an entire system of machinic enslavement” (Deleuze & Guattari, 1980/1987, p. 458). This new form of machinic enslavement is catalyzed by cybernetic and informational machines. The relation between humans and machines has transformed from usage or action to one based on communication, with a crucial effect on surplus value:

> In the organic composition of capital, variable capital defines a regime of subjection of the worker (human surplus value), the principal framework of which is the business or factory. But with automation comes a progressive increase in the proportion of constant capital; we see a new kind of enslavement: at the same time the work regime changes, surplus value becomes machinic, and the framework expands to all of society. (p. 458)

Hence, in contemporary global capitalism, the processes of both machinic enslavement and social subjection operate, often simultaneously. Deleuze and Guattari (1980/1987) illustrated this point with the example of television. On the one hand, TV viewers are subjected to TV insofar as they watch or consume it. TV viewing is an excellent example of the processes of social subjection because it illustrates a situation in which “the subject of the statement. . .more or less mistakes itself for a subject of enunciation (‘you, dear television views, who make TV what it is is. . .’)” (p. 458). The process of subjectification leaves the impression that TV viewers create and produce the content of what they view, that they “have some say” in the matter. However, subjectification in global capitalism constitutes subjects as ‘capitalists’ (subjects of enunciation) or ‘proletarians’ (subjects of the statement). In the case of television, TV viewers have the impression that they are “producing” TV programs when, in fact, they are not. On the other hand, TV viewers become enslaved by television “insofar as the television viewers are no longer consumers or users, nor even subjects who supposedly ‘make’ it, but intrinsic component pieces, ‘input’ and ‘output,’ feedback or recurrences that are no longer connected to the machine in such a way as to produce or use it” (p. 458). Holland (2009) gives a second example of the combination of machinic enslavement and social subjection: commercial or political marketing campaigns. Consumers and voters are subjected to opinion polls in which they are asked to give their opinions of products or candidates. Once again, subjects of the statement are mistaken for subjects of enunciation. Opinion polls give the impression that consumers and voters are agents, when in fact, their “voices” are simply being *used* by marketers and candidates. Machinic enslavement is exemplified by the use of galvanic skin response or pupil dilation to extract consumers’ and voters’ ‘opinions’ biologically. Here, consumers and voters become merely suppliers of inputs and outputs.

Building on Foucault’s (1978) analysis of power, Deleuze and Guattari (1980/1987) distinguished three forms of power: sovereign, disciplinary, and control (Holland, 2009). The sovereign power of the despot consists of the right to tax and to take life. As capitalism emerged, a new form of power developed—disciplinary power. This new form of power involved the disciplining of life in order to maximize productivity and reproductivity. It involved subjectification in ways that constituted certain types of individuals. Factories allowed the capture of profit; other institutions supported this capture by producing subjects prepared for their roles as workers and consumers. The most recent type of power—control—corresponds to contemporary capitalism, and involves the complete subsumption of society by capital. Capital has come to completely saturate social life, accelerating processes of deterritorialization and reterritorialization. As the dominant form of power has shifted from fairly fixed models of discipline to a form that consists of continual modulation, the constant pressure of deterritorialization and reterritorialization necessitates constant fluctuations of standards of value as a way to ward off overproduction and the falling or stagnating of profit rates. Holland (2009) summarized these processes as follows:

> All activity is now merely a moment in the circulation of capital; profit can be captured anywhere and everywhere, not just in the factory. Furthermore, the turnover rate of capital (in its increasingly frenetic drive to forestall the falling rate of profit) increases dramatically, becoming so rapid that disciplinary institutions cannot possibly keep pace. Control power operates not by fixed models of discipline, but via continual modulation. The values of currencies, labor-power, fashion styles, brands, musical trends, and so on are allowed to float, because the computer-powered cybernetic apparatus of capture is fast enough to appropriate surplus value without fixed values. (p. 159).

The two major forces in contemporary society are the State and the capitalist axiomatic. These two forces interact in a complex way. Contemporary capitalism has become a totalizing global force that has in some ways exceeded the State. Yet, the role of States has shifted to organizing the flow of capital on a global scale. In this role, the State is still an apparatus of capture operating through social subjection and machinic enslavement, which form its two poles. Like Labaree (1997), Deleuze and Guattari view capitalism or the “liberal economic” impulse as one major force in contemporary society. The democratic impulse, however, is more difficult to categorize within Deleuze and Guattari’s (1980/1987) conceptual landscape. The democratic State is one “model of realization” among many isomorphic models within the global capitalist axiomatic. The combination of "isomorphy" and "heterogeneity" means that very different State-forms all coordinate in the megamachine corresponding to the global capitalist axiomatic. As one model of realization "isomorphic" to others, the democratic State-form involves both poles of the State, that is, processes of machinic enslavement and social subjection, which allow the democratic State to function as an apparatus of capture. However, the particular ways in which these processes function may differ from the way they function under a different State-form.

3.3 Rethinking the Three Goals of Education and the Purpose of Mathematics Education

Deleuze and Guattari's reformulation of Marx's ideas provides a rich conceptual landscape for rethinking the three goals of education and the purpose of mathematics education as stated in the Common Core Standards for Mathematics and auxiliary documents. Like all other aspects of contemporary social life, mathematics education takes place within the interaction between the global capitalist axiomatic and machinic processes such as the apparatus of capture enacted by the State-form. From this view, Labaree's (1997) three goals can be seen as two assemblages. The first assemblage connects the goals of social efficiency and social mobility. It was clear from Labaree's (1997) description of the social efficiency goal and the social mobility goal that both goals connect to a stratified educational system that serves to reproduce the inequities in the stratified economic and social structure outside of schools. In Deleuzoguattarian terms, these linked goals constitute a single assemblage.

Every assemblage contains both an assemblage of enunciation, which is the linguistic aspect of the assemblage or the form of expression, and a machinic assemblage, which is the material aspect of the assemblage or the form of content. In this case, the assemblage of enunciation consists of the goals themselves. The machinic assemblage consists of the stratified educational system (including the hierarchy of grade levels, differential prestige of schools, college rankings, grade point averages, honors and Advanced Placement classes, etc.). When this machinic assemblage is viewed from the top from the perspective of the taxpayer and employer, the assemblage of enunciation sounds something like the following: "Schools exist to prepare workers for their positions within the stratified economic and social structure." From the bottom of the machinic assemblage, from the perspective of the consumer, this same assemblage of enunciation sounds something like the following: "Schools exist to prepare (me) as a worker for my (hopefully more socially and economically desirable) position (than others') in the stratified economic social structure." In an assemblage of enunciation, Deleuze and Guattari (1980/1987) distinguish between the subject of enunciation and the subject of the statement. Given the capitalist axiomatic, "capital acts as the point of subjectification that constitutes all human beings as subjects" (Deleuze & Guattari, 1980/1987, p.457). However, some people (the "capitalists") are constituted as "subjects of enunciation" while others (the "proletarians") are constituted as "subjects of the statement." Within this social efficiency-mobility assemblage, it is those who express the goal of social efficiency, the taxpayer and employer, who are subjects of enunciation and who will ultimately profit through the apparatus of capture. Like the TV viewers in the Deleuze and Guattari's (1980/1987) example, those who "express" the social mobility goal (the consumers) mistake themselves for subjects of enunciation when, in fact, they are merely subjects of the statement. Holland's (2009) example of opinion polling provides an even closer parallel. Voters have the mistaken sense of being subjects whose expressed opinions create the political system through opinion polls, when in fact they are subjected to the opinion polls by a political system in which they have very little "say." As consumers of educational commodities, parents and students have the same sort of mistaken sense that they are creating the educational system through their opinions, that their voice "counts," when in reality they are subjected to an inequitable stratified education system that functions to reproduce a similarly inequitable stratified economic and social structure.

The remaining goal, democratic equality, forms the second assemblage. Once again, since capitalism has become axiomatic, capital acts as the point of subjectification. In this case, what comes to the fore is the interaction between the deterritorializing tendency of the capitalist market and the reterritorializing tendency of the democratic State as one model of realization of the capitalist axiomatic. As Holland (2009) stated, "state reterritorialization produces citizen-subjects through the process of social subjection" (p. 158). Social subjection involves a subject of enunciation and a subject of the statement. The capitalist axiomatic has become the dominant apparatus of capture, with the various State-forms, such as the democratic State, serving to coordinate decoded flows in order to maximize the capture of profit for the capitalist axiomatic. In other words, as Holland (2009) stated, “capitalist profit has superseded ground rent and despotic tribute as the dominant apparatus of capture, with the state serving as point of subjectification and compensatory reterritorialization for the superior deterritorializing power of capitalist axiomatization” (p. 158). Once again, we have a case of a mistaken subject. The citizen who seems to be the subject of enunciation expressing a call for democratic equality turns out be subjected to capital to the same extent as the educational consumer. The effect on the democratic equality assemblage is to redirect statements that allude to democratic equality into the service of social efficiency. This is evident in the Common Core Standards’ apparent allusion to democratic equality in its emphasis on “all” in its call to “ensure ***all*** American students are prepared for the global economic workplace” (National Governors’ Association & the Council of Chief State School Officers, n.d., p. 1). Given the inequitable stratification of the global economic workplace for which all students are being prepared, “preparing all students” entails reproducing the inequitable stratification within schools and classrooms. While the democratic State continues to “produce citizen-subjects through the process of social subjection” (Holland 2009, p. 158), it turns out that citizens are subjects of the statement, subjected to capital, produced for the purpose of maximizing the capture of profit for the capital axiomatic. Efficiently preparing ***all*** students for the global workforce must entail differentially preparing students to take social and economic positions in an inequitable, stratified, hierarchical system. The complementary top and bottom expressions of this system from the perspective of the taxpayer/ employer (in the name of social efficiency) and consumer (in the name of social mobility) ensure consumer "buy-in" as well, since the social mobility goal holds out hope for consumers that they may be able to accumulate enough educational commodities (credentials) in to land in a favorable position within the collective "***all***." Capital has come to be the singular subject of enunciation in schooling, coordinating various subjects of the statement (consumer, citizen) in order to maximize profit[26]. In actuality, both the social efficiency-mobility assemblage and the democratic equality assemblage doubly articulate the very same assemblage of enunciation and machinic assemblage—that is, both link capital as subject of enunciation with the stratified educational system as its machinic assemblage. In the process, students are subjected to capital in ways that hook them into the system by encouraging them to mistake themselves for subjects of enunciation instead of subjects of the statement.

3.4 The State of Democracy in Mathematics Classrooms

Stemhagen and Smith (2008) summarized the state of democracy in

[26] This is true in educational research as well. St. Pierre (2004) described the way in which a new governmental institution, Institute for Educational Sciences was "rapidly putting into place a structure that attempts, in every respect, to discipline and control the field of educational research. . . .This domination begins with training researchers and extends to the dissemination of research findings [through certain databases] to those whom. . .the new Director of IES refers to as our 'customers'—practitioners, school superintendents, and legislators" (p. 285). St. Pierre (2004) used Deleuze and Guattari's concept of "state science" to examine the way in which a certain image of thought about educational research is "turning education into a business" (Deleuze 1990/1995, p. 182). In other words, the capitalist axiomatic serves as a point of subjectification for educational researchers as well. Educational researchers can become "subjects of the statement." The subject of enunciation of educational research under State science is capital.

mathematics classrooms as follows: “While the impact of democratic education proponents on American schooling can be debated, it is interesting to note that any influence the movement has had on schooling in general has been even less evident within the realm of mathematics education” (p. 25). Nevertheless, Mukhopadhyay (2009) examined three forms of democratic education in mathematics classrooms: competitive, participatory, and discourse democracy. Which form is enacted in classrooms impacts the extent to which mathematics education functions as an “allocation machine” serving to reproduce inequitable conditions in the classroom and society.

3.4.1 Competitive Democracy

Mukhopadhyay (2009) argued that characteristics of competitive democracies map directly on to the value systems in current U.S. mathematics classrooms.

Competitive democracy invokes the social efficiency-mobility assemblage of enunciation through the discourse of meritocracy. Mukhopadhyay (2009) posited that a student’s success and position of power within a competitive democracy comes from the student’s ability to quickly access information. In such classrooms, according to Mukhopadhyay (2009), mathematics is typically presented as a collection of facts and procedures—that is, the form the mathematics classroom takes on is the problems-practice assemblage (see the “Generating Problems” plateau). Mathematics knowledge is conceived of as the ability to memorize the facts and procedures and access this information in order to apply it to solve problems. Mukhopadhyay (2009) claimed that this formulation of democracy impacts students differentially, empowering some and marginalizing others. Within this context, the problems-practice assemblage becomes an allocation machine, enacting differential social subjection for different students. Those who are able to quickly access information (memorized facts and procedures) are constituted as “good math students” and those who do not access such information as quickly are constituted as “bad math students.” This is an example of disciplinary power, in which the process of subjectification produces different types of individuals. Mukhopadhyay (2009) included excerpts from a transcript of an interview with two students that demonstrated the way in which competitive democratic politics enact such differential subjectification. One student, Raoul, was constituted as a student who was “bad at math” while another student, Aaron, was constituted as a student who was “good at math.” Both Raoul and Aaron said that math was “hard” because students were expected to “memorize answers to equations.” While both found memorization to be the most challenging aspect of math class, Aaron added, “but I memorize things quicker than other people. . .usually.” In a later interview the students discussed the form math lessons typically took in their classroom. Aaron explained that the teacher typically “gives us the easy problems and then the harder ones. . .and that’s when he goes harder and harder” (Mukhopadhyay, 2009, p. 46). Raoul responded, “That’s why I don’t have friends. . . .they all ask for harder problems” (Mukhopadhyay, 2009, p. 46). With the repetition of daily lessons, certain children were repeatedly “ejected” (to use the term used in Brewster’s translation of Althusser, 1970/1971) prematurely, before getting to the “hard” problems, a process that reproduced the process described by Bowles and Gintis (1976) and Labaree (1997) on a much smaller timescale.

The trend in assessment to establish yearly standardized testing reinforces the value placed on information accessing speed. Gulliksen (1950) distinguished between "speed tests" and "power tests." A speed test consists of items that students would be able to answer easily and correctly without a time limit. However, a time limit is imposed and students are evaluated based on the number of items correctly completed. On a power test, on the other hand, students are not constrained by time; the test is assumed to evaluate students' full "power" to perform. Lu and Sireci (2007) pointed out that standardized tests are rarely "pure power tests" because they typically involve a time limit, partly for practical reasons. Also, researchers have found that when tests have generous time limits, many examinees have extra time, which can have a "troubling" effect on students who are still working. (p. 29; College Entrance Examination Board, 1984). In practice, most standardized tests include a mixture of "speed" and "power" components (Lu & Sireci, 2007; Rindler, 1979), a mixture that is likely to differentially affect students like Raoul and Aaron in ways that amplify the reproduction of the stratified schooling and social/ economic systems.

A second assessment trend is likely to differentially affect students like Raoul and Aaron: the push for value-added assessment systems. This trend illustrates the way in which the subsumption of society to capital reconnects the linguistic aspect of the democratic equality assemblage to the machinic assemblages associated with the social efficiency-mobility assemblage, interlinking the goals of democratic equality, social efficiency, and social mobility in a way that prepares “all” students for the inequitable economic/ social system in place . Sanders (2000) began by invoking the social efficiency assemblage of enunciation as a justification for developing value-added assessment systems such as the Tennessee Value Added Assessment System (TVAAS) and the Education Value-Added Assessment System (EVAAS):

> Huge sums of money have been delivered to the educational community by federal, state and local governments and private philanthropic foundations to support numerous initiatives and programs whose purpose was to improve student academic achievement. The results from these investments have been mixed at best. (p. 330)

Here, the taxpayer and employer are represented by “federal, state and local governments and private philanthropic foundations.” The taxpayer and employer have invested “huge sums of money” in students via teachers, whose “variability in. . .effectiveness is huge” (p. 335). The economic return on these investments in students is mediated by the teacher. Hence, according to Sanders (2000), it is crucial to reduce “the likelihood that students will be assigned to relatively ineffective teachers” (p. 335). Implicit in this justification of value-added assessment is the assumption that academic achievement as measured by standardized tests will ensure employers that “by selecting candidates with the best credentials. . .they are obtaining employees who have acquired the highest level of productive skills” (Labaree, 1997, p. 54; Berg, 1971). Value-added assessment is a refinement of the standards movement, which Sanders (2000) notes is the result of the same sorts of demands from taxpayers and employers (those “outside the traditional educational community” (p. 330)). Before value-added assessment, Sanders (2000) argued, the standards movement was operationalized as a

> step approach to curricula . . .[and] a testing regime that purport[ed] to measure the percentage of students within grades who are at master, proficient, basic, non-mastery (or whatever language is dangled beside the test results). Inevitably when the results of these tests [were] presented, it [became] obvious that differences in results among schools and districts [were] strongly related to socio-economic measure of the demographics of the student population of a school or district" (p. 330).

If a typical standards-based assessment system is analogous to a staircase, Sanders (2000) suggested that the analogue of a value-added assessment system is a ramp: "if the desire is for each student to move up the same ramp. . .and if it is further recognized that all students will not be at the same place at the same time in the same grade, then many problems in assessment and measurement can be mitigated" (p. 331). According to Sanders (2000), this form of accountability "will hold people accountable for things over which they have control, rather than for things they do not. For instance, teachers in the fall have no control over the achievement level of their incoming students" (p. 331). Although the justification for value-added assessment models is based on the social efficiency assemblage of enunciation, Sanders (2000) alludes to the democratic equality assemblage of enunciation by claiming that such a system is more equitable than the typical staircase model. In the previous model, according to Sanders (2000), "Especially in inner city schools, too often it is observed that the previously lower scoring students are being given the opportunity to make progress, but within the same school the earlier higher achieving students are being held to the same pace and place as their lower achieving peers" (p. 337). On the other hand, Sanders (2000) claimed, armed with data about students' predicted growth, teachers can ensure equal growth for all. Sanders (2000) concluded: "[N]ow we are using data more responsibly. . . .With this tool, we can build the education of the future—individualized, equitable, and full of promise for all our kids" (p. 338). However, given the current inequitable and hierarchical economic and social structure that Sanders (2000) himself pointed out in critiquing the staircase model of assessment, an "equal growth for all" approach maintains this same hierarchical structure. If students start at different locations on the ramp, then move up the ramp at an equal rate of growth, then although all students have progressed up the ramp, they have maintained the exact same relative location in reference to one another as was in place at the beginning of the ramp. Schools that promise "equal growth for all" have unlinked the democratic equality assemblage of enunciation from its corresponding material instantiations, and instead, linked it to the machinic assemblage which reproduces the existing hierarchical and inequitable economic and social structure. From the consumer standpoint, "equal growth for all" is alluring only for those beginning toward the top of the pyramid. However, just as the consumer in the social mobility assemblage views "equal opportunity" as the possibility of "more opportunity for me than for others," the consumer can also view the promise of "equal growth for all" as the possibility of "a higher rate of growth for me than for others." In the push for value-added assessment, the subsumption of society by capital (Deleuze & Guattari,1972/1977; Holland, 1999, 2009) has linked all three goals (democratic equality, social efficiency, social mobility) together as Borromean rings. The taxpayer and employer are assured of students who will have grown just enough to be well prepared for the pyramidal job structure. The consumer looks to the possibility of “more growth for me.” And the linguistic allusion to democratic equality quells the citizen’s queasy sense that schools are not performing their role as “the great equalizer” (Cremin, 1951). Meanwhile, schools embody Freire’s “banking” concept of education: students become variable-rate time deposits. The teacher scrambles to deposit as much knowledge as possible into each student, students clamber to get a good bargain on the best credentials, all for the purpose of producing surplus value for future employers in an attempt to repay their infinite debt to capital. The experience of this time in schools--what actually transpires, whether students and teachers are happy, whether students grow in their capacity to care for one another, whether the experience of schooling counts as living in the eyes of students—does not matter except to the extent that it adds value for future employers. After all, as Sanders (2000) asserts, “What is important is NOT the achievement level of third-graders, for instance. . . .What is most important is the achievement level of 11th and 12th graders” (p. 337). When the state of democracy in schools is an embodiment of the values of competitive democracy, what matters is what students can exchange to employers went they, as variable-rate time deposits, mature. As time-deposits, students have come to be information processing component in the broader capital-producing “megamachine.” They compose themselves with other information processing components—for example, electronic devices. Interestingly, over the last several years as I supervised student teachers in a number of school districts , I noticed that the schools have acquired “mobile computer labs” in the form of carts with a class set of computers. The only function I have witnessed for these computers is for students to take practice tests in preparation for the End-of-Grade standardized test. This case is an example of machinic enslavement, which Deleuze and Guattari (1980/1987) described as follows: “In machinic enslavement, there is nothing but transformations and exchanges of information, some of which are mechanical, others human” (p. 458). The flow of information between students and laptops have become an assemblage in students are “intrinsic component pieces, ‘input’ and ‘output,’ feedback or recurrences that are no longer connected to the machine in such a way as to produce or use it” (p. 458). One might protest that students are, in fact, using the computer as a way to gain feedback on their progress in mathematics. However, as Deleuze (1968/1994) said of our image of thought about problems in general (and as previously noted in the discussion of the problems-practice assemblage), “the master sets a problem, our task is to solve it. It is a. . .social prejudice with the visible interest of maintaining us in an infantile state, which calls upon us to solve problems that come from elsewhere” (p. 158) Such a ritual involves “telling us that we have won simply by being able to respond: the problem as obstacle and the respondant as Hercules” (p. 158). Students’ visible anomie during the repetition of this ritual hints that they recognize that outputting prefabricated answers to prefabricated “problems” on a laptop is not a Herculean act, that they are “producing” nothing for their own use, merely biding their time until “maturity.”

3.4.2 Participatory Democracy

The second form of democracy in mathematics classrooms Mukhopadhyay (2009) addressed was participatory democracy. In the participatory form of democracy, value is attached to participation rather than the ability to quickly access information (Hagen, 1992; Mukhopadhyay, 2009). In the participatory approach, the inequitable distribution of resources that arise in the competitive version of democracy under the banner of meritocracy is viewed as unjust. Instead, it is assumed that fair representation necessitates political participation. Mukhopadhayay (2009), following Hagen (1992), noted that democratic participation has often been operationalized through a distributive justice perspective resulting in a narrow focus on voting. Young (2000) has conceptualized this model of democracy as the “aggregative model.” In the aggregative model, democracy is considered to be a process of “aggregating” the preferences of citizens in order to make decisions. In the classroom, a similar phenomenon can be an emphasis on taking turns. One example comes from research in two fourth-grade science classrooms; it is possible the same phenomenon occurs during math class. Carlone, Haun-Frank, and Webb (2011) examined two fourth-grade science classrooms, both of which implemented Reform Based Science following generally accepted instructional processes. The researchers were surprised to find that although students in the two classes performed similarly on achievement measures and expressed positive attitudes about learning science, in one of the classrooms (Mrs. Sparrow’s class) a group of African American and Latina girls in one of the classrooms expressed outright disaffiliation with the constituted meanings of “smart science person.” After 2 years of iterative analysis, the researchers were able to come to the interpretation that a key reason for the students’ disaffiliation was the way in which the notion of “sharing scientific ideas” was constituted differently in the two classrooms. In Mrs. Sparrow’s class, the form for “sharing scientific ideas” consisted of turn-taking. Both the teacher and students emphasized taking turns. In one 27 minute transcript, students mentioned individually trying out their ideas 40 times, using phrases such as “mine,” “can I try,” “my turn,” etc. Scientific investigation came to mean trying out one’s own ideas. Although students were working in small groups, each student I “owned” his or her individual ideas. The investigations became an arena in which students competed to occupy the scientific space through their individual intellectual property. Often, the bids for the floor by the African American and Latina girls were rebuked or ignored. According to Carlone et al. (2011), “the meaning of ‘science person’ (i.e. someone who is assertive in trying out their own ideas with tools) was not equally accessible to all students; not everyone got a fair opportunity to be scientific” (p. 459). Participatory democracy is an attempt to shift the classroom dynamics embedded in competitive democratic classrooms to more equitable ones through a focus on participation. However, a simplistic conception of participation as casting votes or taking turns fails to unlink the Borromean rings tying democratic equality to social efficiency and social mobility in a way that creates an allocation machine which reproduces the stratified educational and social/ economic structures in place.

3.4.3 Discourse Democracy

The third form of classroom democracy Mukhopadhyay (2009) addressed was dialogue or discourse democracy. Young (2000) called this form of democracy the "deliberative model[27]." According to Mukhopadhyay (2009), "the core value of this form of democracy is embedded in the idea that citizens must have an opportunity to participate in dialogue with each other about issues" (p. 47). In the classroom, this means that teachers and students attend not only to frequency or amount of participation, but also to the characteristics of the students' discourse as well as the power relations connected with discourse patterns. Civil and Planas (2004) distinguished among three perspectives on "participation." From the psychological or individual perspective, participation involves an individual's actions and statements. From a social perspective,

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[27] Theoretical distinctions could be made between discourse democracy, dialogue democracy, and deliberative democracy based on the lineages they "cite." The term "dialogue" has associations with Freire's work. In common usage, "dialogue" is similar to conversation; Freire's version of "dialogue" goes beyond mere conversation and involves shifts in power dynamics. The term "deliberative" was used by Young (2000) in contrast with the "aggregative model" mentioned previously in connection with Mukhopadhyay's (2009) "participatory" democracy. The term "deliberative" implies that participants are not simply discussing issues, but carefully considering different options. Young (2000) argued that "reasonableness" should be a criterion for evaluating options put forth. While authors addressing deliberative or dialogue democracy often also address discourse, I am unaware of sources that use the term "discourse democracy." I use it here to "cite" the literature in mathematics education related to discourse. Ryve (2011) analyzed 108 international mathematics education articles that addressed discourse in mathematics education. He identified three topic areas related to discourse: 1) discourse as social interaction, 2) minds, selves and sense making, and 3) cultural and social relations focusing on macro processes of social and institutional actions. I use the term "discourse democracy" in association with all three of these senses. Specifically, the form of democracy following from connections between research on participation, dialogue, and democracy *in mathematics education* connects interaction among students with social norms and sociocultural norms. This form of democracy emphasizes understanding mathematics through discourse in addition to making decisions. A detailed elaboration of the concept of "discourse democracy" in mathematics education is beyond the scope of this dissertation, and is left for future scholarship.

participation is socialization into mathematical practices. Civil and Planas (2004) adopted a third perspective more in line with discourse democracy: the sociocultural perspective participation centers the notion of participation on "the use of discourse and some of its contents (norms, values, valorizations) as crucial mediating tools in order to interpret the mathematical learner in context" (p. 7). This perspective provides insight into why distributive approaches to participation do not jam the inequitable allocation machine found in competitive and participatory democratic classrooms. Civil and Planas (2004) investigated the role of social and organizational structures in shaping participation in two distant settings (Tucson, Arizona in the United States and Barcelona, Spain). The authors found that acquiring concepts and skills is not enough for allow students to become "mathematical learners;" rather, students' "active participation in the reconstruction of a specific discourse" is necessary (Civil & Planas, 2004, p. 7). In a fifth-grade classroom in Tucson, two groups of students found fewer barriers to their active participation in mathematical discussions. First, students in the class were generally more likely to listen to students whose achievements in sports established their popularity. Secondly, students who were in the Gifted and Talented Education (GATE) program dominated discussions that directly focused on mathematics. Civil and Planas (2004) observed that not all GATE students were well liked, but when they spoke, students tended to side with them" (p. 8). Moreover, students were aware of the way racial, ethnic, and economic privilege correlated with access to the GATE program. One White girl who was in the GATE program noted, "GATE tends to be upper class white people, I've noticed, it's kind of a corrupt system" (p. 9). Similar phenomena occurred in Barcelona. In this case, immigrant students who were learning the dominant languages (Catalan or Spanish) or whose behavior was not considered acceptable, irrespective of cognitive factors or mathematical achievement, were placed in a "special needs" mathematical classroom four out of five days a week. On the fifth day, students joined a mainstream class. Both "mainstream" and "special needs" students viewed a passive role as appropriate for the "special needs" students on the fifth day. Students classified as "special needs" a passive role "so as not to confuse the other students. . .or because the co-operative model did not match their reality outside school. . .or because they [did] not feel ready yet for the mainstream group" (p. 109). The "mainstream students" viewed the discourse practices of the "special needs" students as inappropriate and as demonstrating deficiencies in mathematical understanding. The students considered "special needs" students often incorporated connections to their personal lives as they discussed mathematics. Carmen, a "mainstream student," interpreted this discourse practice to indicate that "they want the teachers to teach easier mathematics, but mathematics is very difficult, you cannot always find out what everything means" (p. 11). Ironically, the "special needs" students meaning-making was cast as a deficit while the acceptance of meaninglessness was valorized. In both settings—Tucson and Barcelona—social and organizational aspects of schooling and society made "active participation" easier for some students than others. One organizational feature that had this effect in both settings was the decision to offer different programs for different students in the name of “meeting the needs of all students.” Civil and Planas (2004) noted that “the effectiveness of schooling seems to be associated with the classification of students into certain groups, and the mathematical discourses seem to be developed depending on this classification” and that “the fact is that in both cases the students who seem most negatively affected by these decisions are usually members of certain ethnic and language groups and are economically underprivileged” (p. 12). A discourse democracy perspective requires teachers and students to “anticipate barriers to classroom discourse, such as traditional value systems that students bring with them and a conscious commitment by the classroom teacher to develop a shared value system with students with an emphasis on discourse (Mukhopadhyay, 2009, p. 49).

In Carlone et al.’s (2011) study, the teacher in the second classroom, Mrs. Wolfe, seemed to have viewed classroom participation from the perspective of discourse democracy. She explicitly excluded turn taking in her definition of what it meant to “do science”: “We don’t do ‘turns’ in science. . .I don’t want them taking turns. I don’t want them trying ideas by themselves. . . .[W]hen you’re working together, you need to be able to tell me what question you’re thinking about, what you’re doing, and why you’re doing it” (p. 470). In this classroom, the meaning of participation was constituted as trying out one another’s ideas together rather than taking turns. The different meanings of “sharing” in the two classrooms resulted in different conceptions of what it meant to be a “science person.” In Mrs. Sparrow’s class, a “science person” was someone who had all of the answers; in Mrs. Wolfe's classroom, a "science person" was someone who asked questions, had good reasons, paid attention to the task at hand and others' comments, and always thought about different experiments to do (Carlone et al., 2011, p. 478). Establishing norms for discourse with students which allow all students to identify rather than disidentify with content areas is one step toward shifting the inequitable machinic assemblages currently in place to more ones more in line with the democratic equality goal of schooling. In the mathematics classroom, these norms entail both social norms and sociomathematical norms (Yackel & Cobb, 1996; Cobb et al., 2001). Social norms are general expectations whereas sociomathematical norms pertain specifically to the discipline of mathematics. For example, the expectations to "take turns" or "listen to your partner" are social norms. Taken-for shared understandings of what counts as an acceptable mathematical explanation or justification (e.g. whether personal connections can be included), a sophisticated mathematical solution, or a mathematical solution that is different from those already shared are sociomathematical norms (Cobb et al., 2001). Negotiating and establishing a new version of "normal" discourse patterns sets the scene for "becoming-democratic" mathematical education.

3.5 Deleuze and Guattari, Democracy, and Becoming[28]

Deleuze and Guattari have been considered both opponents (Mengue,

[28] I presented some of the ideas in this section in a paper entitled "Becoming-Democractic Pedagogies: A Deleuzoguattarian Perpective on Democracy in the Classroom" at the Southeast Philosophy of Education Society Conference, September 25-26, 2009.

2003) and proponents (Patton, 2005) of democracy. Philippe Mengue (2003) has argued that Deleuze and Guattari are hostile toward democracy. Mengue suggested that three of Deleuze and Guattari's values contributed to a hostile attitude toward democracy: immanance, the minor or minority, and becoming. First, Mengue argued that Deleuze and Guattari opposed democracy because they valued immanence over transcendence. Democracy is based on notions of human rights. Human rights function as eternal and abstract, and are therefore transcendent rather than immanent. Mengue (2003) suggested that Deleuze and Guattari considered the transcendental quality of notions of human rights inherent in democracies to stop creative movement. Second, Mengue (2003) argued that Deleuze and Guattari were hostile to democracy because they defined democracy as the reign of public opinion or consensus. There is no public opinion today other than a fabrication of the media, which allows the state to serve as an instrument of domination and normalization. This state-invested fabricated public opinion opposes the power of minorities, who are by their nature inventive. Only what is “minor” or “minority” is creative. Therefore, for Mengue's (2003) Deleuze and Guattari, not only is democracy itself not new, but it also flattens and breaks down novelty. Finally, Mengue (2003) argued that Deleuze and Guattari were hostile to democracy because they valued becoming. According to Deleuze and Guattari,(1980/1987) there is no “becoming-majority,” but only “becoming-minority.” While the minor and minorities are creative, the majority is a stoppage, an obstacle to becoming. Since democracies are in principle the rule of the majority, they are obstacles to becoming, to creativity, to novelty. Based on this analysis, Mengue (2003) concluded that Deleuze and Guattari' ideas actually renew the ancient Platonic idea of an intellectual aristocracy of philosophers in which the only worthwhile communication is from one thinker to another or one elite to another in the eviction of the vulgar, multitude, plebe, demos, and dialectic.

Paul Patton (2005) has contested this reading of Deleuze and Guattari's work. Patton (2005) gave broad and narrow definitions of democracy. In the broad sense, democracy refers to

> an association of equals in which there is neither justification for the exclusion of individuals or groups from the widest possible system of basic civil and political liberties, nor any justification for the arbitrary exclusion of particular individuals or groups from the benefits of social and political cooperation. (p. 53)

In a narrower sense, democracy refers to "a form of government in which the governed exercise control over governments and their policies, typically through regular and fair elections" (p. 53). Patton argued that although Deleuze is

> not a theorist of democracy in the narrow sense of the term. . .it does not follow. . .that Deleuze is hostile to democratic governments, or that his political philosophy implies a rejection of democracy in either the broad or narrow sense. In fact there is no shortage of evidence to suggest that, in his political practice as well as in his theoretical views, he is committed to democracy in both senses of the term (p. 54)

Patton notes that Deleuze and Guattari's work focuses on micropolitics rather than addressing the "standard problems of liberal political philosophy, such as the elaboration of principles of justice or freedom or the definition of democracy" (p. 50). Patton points out that Deleuze and Guattari's "neglect of the public sphere" does not, in itself, indicate a hostility toward democracy: "incompleteness is not antipathy and there is no reason to suppose that Deleuzian theory proposes an alternative rather than a supplement to democratic political theory" (p. 55). Patton concludes that "Deleuze and Guattari's micropolitical theory. . .supplements liberal democratic conceptions of decision-making and challenges these to take into account such micropolitical processes" (p. 56).

In contrast to Mengue's view of Deleuze and Guattari as hostile to democracy and Patton's view of their philosophy of democracy as incomplete or supplementary to a liberal conception of democracy, I would contend that Deleuze and Guattari offer a *reconceptualization* of democracy. Deleuze and Guattari are indeed critical of democratic *states.* A consideration of the double meaning of *state,* or in French, *état* in relation to Deleuzoguatarian works is illuminative. A *state* or *état* is both a "territorial or political unit" and a "way or form of being." Deleuze and Guattari's work was a critique of "state" in both of these senses. As we have seen, Deleuze and Guattari (1980/1987) conceptualized States as models of realization in the capital axiomatic which function to "capture" decoded flows in a way that leads maximizes profit and maintains inequitable distributions of wealth within nation-states as well as among nation-states on a global scale. "The State," Deleuze and Guattari (19801987) wrote, "is assuredly not a locus of liberty" (p. 460). This political philosophical critique of States is interconnected with an ontological critique of stasis or "being" in favor of an ontology of becoming. This philosophical connection between politics and ontology is evident in Deleuze and Guattari's (1980/1987) conceptualization of "majority" and "becoming-minoritarian": "When we say majority, we are referring not to a greater relative quantity but to the determination of a state or standard in relation to which larger quantities, as well as the smallest, can be said to be minoritarian" (1980/1987, p. 291). A majority (no matter what size) is invested in an ontology of being, in maintaining its dominant standards or norms. Becoming-minoritarian, on the other hand, is invested in *process* (1980/1987, p. 291) rather than stasis. This is why, as Mengue(2003) pointed out, Deleuze and Guattari (1980/1987) argued that majorities are stoppages since they are invested in stasis, while becoming-minority is a creative process, a process of invention, a stimulus for novelty. Liberal democratic states, as Tocqueville (1835/2007) observed, are majoritarian, and as such, are invested in stasis, in maintaining dominant standards or norms. Deleuze and Guattari (1991/1994) wrote, "Democracies are majorities, but a becoming is by its nature that which always eludes the majority" (p. 108). Instead of supporting liberal democratic states as Patton suggested, Deleuze and Guattari (1991/1994) call for a "becoming-democratic that is not the same as rights-based States" (p. 113; translation modified). "Becoming-democratic" can elude the majority and allow for a creative "resistance to the present" (p. 108). "Becoming-democratic," then, is not a hostility toward democracy nor a supplement to liberal democratic states, but rather a reconceptualization of democracy.

In *A Thousand Plateaus*, Deleuze and Guattari (1980/1987) discussed many different "becomings": becoming-molecular, becoming-woman, becoming-animal, becoming-minoritarian, becoming-imperceptible. They clarified,

> Becoming is not to imitate or identify with something or someone. Nor is it to proportion formal relations. Neither of these two figures of analogy is applicable to becoming: neither the imitation of the subject nor the proportionality of a form.

This is why the actualization of the capital axiomatic in the models of realization (e.g. States) is not “becoming.” An axiomatic system consists of formal relations which are interpreted in a model of the system. In Deleuze and Guattari’s (1980/1987) conception, the capitalist axiomatic is “actualized” in a model but does not “become” the model. Deleuze and Guattari (1980/1987) elaborated on the process of becoming as follows:

> Starting from the forms one has, the subject one is, the organs one has, or the functions one fulfills, becoming is to extract particles between which one establishes the relations of movement and rest, speed and slowness that are *closest* to what one is becoming, and through which one becomes. This is the sense in which becoming is the process of desire (p. 272).

Becoming is a process different from the process of imitating. Imitation is a form of representation. When one imitates someone or something, he or she tries to act *the same as* that which he or she imitates. Imitation is based on resemblance. Becoming relates to difference-in-itself. Deleuze and Guattari (1980/1987) gave this example of becoming-animal:

> Do not imitate a dog, but make your organism enter into composition with something else in such a way that the particles emitted from the aggregate thus composed will be canine as a function of the relation of movement and rest, or of molecular proximity, into which they enter. Clearly, this something else can be quite varied, and be more or less directly related to the animal in question: it can be the animal's natural food (dirt and worm), or its exterior relations with other animals (you can become-dog with cats, or become-monkey with a horse), or an apparatus or prosthesis to which a person subjects the animal (muzzle and reindeer, etc), or something that does not even have a localizable relation to the animal in question. For this last case.. .[a particular person] bases his attempt to become-dog on the idea of tying shoes to his hands using his mouth-muzzle (p. 274).

Becoming-dog is not the same as *acting* like or *looking* like a dog. In fact, becoming-dog in Deleuze and Guattari's sense is not the same as becoming *a* dog. Deleuze and Guattari (1980/1987) redefined two concepts from chemistry to distinguish between two levels of organization, the molecular and the molar: "You become animal only molecularly. You do not become a barking molar dog, but by barking, if it is done with enough feeling, with enough necessity and composition, you emit a molecular dog" (p. 275). In chemistry, a molecule consists of two or more atoms held together in a particular way (that is, through covalent bonds). In working with chemical reactions, it is often necessary to know how much of each reactant is involved. However, molecules are so small that they are difficult to count. Instead of counting molecules, chemists (and others using chemistry) often measure the amount of substances using a more manageable unit of measurement called a "mole," which relates the measured mass of the substance to the number of atoms or molecules contained within that mass. Molar is the adjectival form of "mole." For Deleuze and Guattari (1980/1987), a "molar dog" is what we typically think of when we think of a dog—the whole schnauzer or Dalmatian or Chihuahua. People do not typically think of a dog as an aggregate of atoms or molecules. Imitation entails watching the whole-dog-as-single-entity and attempting to resemble that entity. Becoming-dog, in contrast, entails relating to "dog" not as a single entity, but as a multiplicity, an aggregate of molecules composed into "dog." Similarly,

> there is a becoming-woman, a becoming-child, that do not resemble the woman or the child as clearly distinct molar entities. . . .What we term a molar entity is, for example, the woman as defined by her form, endowed with organs and functions and assigned as a

> subject. Becoming-woman is not imitating this entity or even transforming oneself into it. . .not imitating or assuming the female form, but emitting particles that enter the relation of movement and rest, or the zone of proximity, of a microfemininity, in other words, that produce in us a molecular woman, create the molecular woman (p. 275).

Becoming-democratic in a Deleuzoguattarian sense, then, does not mean imitating "molar" democratic States, nor does it mean coming to identify oneself as a "democratic citizen." Becoming-democratic in classrooms does not entail setting up student government and holding elections for student body president or class representative. It does not mean choosing what to serve at the next special event by taking a vote and serving what the majority of the students choose. It does not mean reading the U.S. Constitution and having class discussions about why democracy is a wonderful form of government. Instead, becoming-democratic entails "starting from the forms one has, the subject one is, the organs one has, or the functions one fulfills" (p. 272) and then extracting "particles between which one establishes the relations of movement and rest, speed and slowness that are closest to" the democratic (p. 272). It means setting up democratic relations on a molecular level rather than on a molar level. It means finding the speeds and slownesses of democracy and finding ways to move in democratic ways. It means making the organism of the class enter into "composition with something else in such a way that the particles emitted from the aggregate thus composed" will be democratic.

3.6 Becoming-Democratic Mathematics Education

All three forms of democracy in the mathematics classroom described by Mukhopadhyay (2009) can be seen as "states" of classroom democracy or

democratic classroom States in that they are intended to exist in the form of stasis. Each one entails its own expectations, social norms, sociomathematical norms—ways of *being* (not *becoming)* in the classroom that repeat like a cadence day-in and day-out. This is not to say that each of these states/States is equally desirable. Certainly, establishing social norms in which students work together collaboratively and listen to one another is preferable to establishing norms in which students just taking turns or, worse yet, make fun of each other. Attention sociomathematical norms for what counts as an acceptable justification, what it means to "do math," and what constitutes a sophisticated strategy is certainly important. As Deleuze and Guattari (1980/1987) stated, "it would be absurd to think that. . .all States are equivalent and homogenous" (p. 466).
A discourse democratic approach involving negotiating and establishing a new version of what "normal" discourse looks and sounds like can set the scene for "becoming-democratic" mathematical education.

However, none of these forms of "establishing" certain "states" in the classroom involve becoming. One might ask: If establishing discourse democracy in the mathematics classroom results in students working collaboratively, following the social and sociomathematical norms we desire for them to follow, and developing understandings of mathematical concepts and the ability to flexibly solve mathematical problems[29]—all on a day-in day-out basis—why should we worry about whether or not this counts as "becoming-

[29] The claim that establishing discourse democracy in the mathematics classroom does in fact have *all* of these effects would need further investigation.

democratic"? There are two reasons something beyond a state/State of discourse democracy is needed. First, the democratic state/State of the classroom is situated within the larger context of a global capitalist axiomatic as exemplified by the singular social efficiency goal for mathematics education stated in the Common Core Standards documents. Even if discourse democracy is established in a mathematics classroom, the democratic state/State of the classroom still serves as an apparatus of capture whose purpose is the ultimate capture of capital for future employers; the school is still a site of machinic enslavement through assessment systems designed to measure the amount of "capital" which has been deposited into them as time-deposits (although the experience of serving as a time-deposit may have been more pleasureable); the school is still a site of social subjection in which students are subjects of the statement expressed by capital. Molecular movements, becomings, are necessary because "molecular movements do not complement but rather thwart and break through the great worldwide organization" (p. 216). It is through lines of flight, becomings, molecular movements that allow escape from totalization.

The second reason becoming-democratic is necessary is that the state/State "itself has always been in a relation with an outside and is inconceivable independent of that relationship" (Deleuze & Guattari, 1980/1987, p. 360). Further, the "law of the State is . . .the law. . .of interior and exterior" (p. 360). The establishment of social and sociomathemtatical norms, even "good" ones, always creates an inside and an outside and, at the same time, through subjectification, Insiders and Outsiders. Biesta (2010) noted two assumptions present in discussions of deliberative (discourse) democracy. The first assumption is that democracy can become a “normal” situation. The question then becomes a practical one: “How can we make our democratic practices even more inclusive. . .and how can we include even more people into the sphere of democratic deliberation” (p. 118). The assumption is that if we continue to increase the bounds of our inclusivity, we will eventually reach a state/State in which democracy is “normal.” In this case, the inside has expanded so much that there is no longer an outside; hence, there are no longer any Outsiders. The second assumption is that inclusion is a process by which Insiders make room on the inside and bring the Outsiders inside—that inclusion is “a process that happens ‘from the inside out’” (p. 119). In this case, only those already considered “democratic” can bring Outsiders inside. In terms of social subjection, Insiders are subjects of enunciation and Outsiders are subjects of the statement—it is the insiders who get to express whether or not the Outsiders are ‘democratic’ enough to enter the inside and determine the terms on which they can do so.

Following Rancière (1999), Biesta (2010) suggested that rather than “normal,” perhaps democracy is something that is sporadic. Rancière (1999) defined democratic politics as something that “happens” from time to time against a background of a police order. This differs from the typical conception of politics as continuous relating to ongoing actions of representatives in government. According to Rancière (1999), “the distribution of places and roles that defines a police regime stems as much from the assumed spontaneity of social relations of social relations as from the rigidity of state functions" (p. 29; quoted in Biesta, 2010, p. 120). The police regime is all-inclusive in the sense that everyone has a position within it, albeit not equally desirable positions. Rancière (1999) conceptualized politics as "the disruption of the police order in the name of equality" (Biesta, 2010). Rancière (1991) reserved the term politics for specific actions that directly break with "the tangible configuration whereby parties and parts or lack of them are defined by a presupposition that, by definition, has no place in that configuration" (pp. 30-31). The act of disrupting the police order in this way also creates new political identities for those who have crossed the line to the inside. The process of inclusion, in this case, comes from the Outsiders rather than by invitation from the Insiders. Democracy-as-disruption, or "democratization," Beista (2010) argued, provides a different type of inclusion—'"the inclusion of what cannot be known to be excluded in terms of the existing order" (p. 125). In Deleuzoguattarian terms, this process is the process of deterritorialization. The Outsiders create a line of flight that escapes from the police state. The cutting edge of deterritorialization enacts shifts in the configuration of the previous assemblage.

Becoming-democratic is deterritorialization. It molecularizes the aggregate (the math class, in the case of mathematics education) in order to change its nature. The outside or the Outsiders suddenly include itself/ themselves. Becoming-democratic is a molecular rather than molar process. Deleuze and Guattari (1980/1987) distinguished two types of multiplicities: mass multiplicities, which are molar multiplicities, and pack multiplicities, which are molecular multiplicities. The characteristics of mass multiplicities include "large quantity, divisibility and equality of the members, concentration, sociability of the aggregate as a whole, one-way hierarchy, organization of territoriality or territorialization, and emission of signs" (p. 33). These are the characteristics of democratic states/States. The democracy of the democratic state/State is proceeds by "rule of the masses." In the mathematics classroom, the democratic state/State can take on any of the forms described by Mukhopadhyay (2009), although the mass multiplicity characteristics take on different senses. In both the competitive and participatory democratic state/State, the notion of meritocracy assumes formal equality among members in the form of imagined *equal opportunity*. Everyone has his or her chance, yet the chances serve to divide students into concentrated categories of "good students" and "bad students." Sociability in the competitive democratic state/State consists of relations in which individuals accumulate or aggregate individual knowledge, which they "own" through turn taking or vote casting. One-way hierarchy installs the teacher as authority dealing out chances, ensuring everyone has a turn, and regulating the accumulation of individual knowledge. Mathematical knowledge is territory to conquer, a mountainous territory with uncanny semblance to a pyramid. Everyone gets a chance, but some will fall (or be knocked) off earlier than others. Ultimately, the goal is the emission of signs: pass-fail; Level I-II-III-IV; A-B-C-D-F; "gifted"-"at grade level"-"special needs." Competitive and participatory democracy maintain the stasis of the pyramid.

The discourse democratic state/State, although preferable to the competitive and participatory forms, has the same characteristics but organized in a different way. Instead of equal opportunity, members are now equal in that they formally have an *equal voice*. The concentration of authority is no longer in the teacher, but the majority. Sociability of the aggregate consists of relations in which majority knowledge is determined through deliberation and regulated by social and sociomathematical norms. The emission of signs takes on a more qualitative and nuanced nature: good explanation, stronger justification, more efficient strategy. Discourse democracy maintains the stasis of normal discourse.

Becoming-democratic disrupts stasis. Whereas the democratic state/State proceeds by “rule of the masses,” becoming-democratic proceeds by the molecular un-ruling of the pack. Deleuze and Guattari (19801987) wrote, “Among the characteristics of the pack are small or restricted numbers, dispersion, nondecomposable variable distances, qualitative metamorphoses, inequalities as remainders or crossings, impossibility of a fixed totalization or hierarchization, and projection of particles” (p. 33). The pack enters from the outside, dispersing among lines of flight through the territory, disrupting the state of the State. The pack does not arrive with proof of invitation, but rather includes itself on its own terms. The pack does not demand equal rights, does not enter a bid to be considered exchangeable. Instead, the pack affirms difference by entering on its own terms. The pack does not demand an equal opportunity to climb and fall from the pyramid, but instead deterritorializes it so that it undergoes a qualitative metamorphosis, changing its shape all together. Nor does the pack demand an equal voice in the normal discourse, but instead introduces a question into the discourse that de-normalizes it.

Deleuze and Guattari (1980/1987) emphasized that the “pack” nor the “mass” ever exist in pure form in actual situations. As a form of deterritorialization, becoming-democratic taken to its abstract limit would result in annihilation. On the other hand, there is no way to perpetually *be* in the *state* of democracy because difference eternally returns. Irrespective of whoever and whatever is brought into the “inside,” a new outside will come into being simply because things, people, situations change.

Scholarship about mathematics education is rarely addressed almost exclusively to educators and rarely addressed to students. Yet, the concept of becoming-democratic involves a disruption from the “outside,” in which the excluded include themselves on their own terms. Assuming the teacher is on the inside (which may not always be the case), and becoming-democratic comes from the outside, what role can teachers play? It is useful to return to Deleuze and Guattari’s (1980/1987) “directions” for how to embark on a line of flight: “lodge yourself on a stratum, experiment with the opportunities it offers . . .find potential movements. . .possible lines of flight, experience them, produce flow conjunctions here and there. . . .It is through a meticulous relation with the strata that one succeeds in freeing lines of flight. . .” (p. 161). Teachers cannot embark on lines of flight alone. However, teachers can provide “launching sites” from which students might embark on lines of flight, and some states/States of democracy make it easier to provide launching sites than others. While discourse democracy involves establishing social norms and sociomathematical norms, teachers can leave cracks in the all of this “establishing” which allow for the process of becoming democratic.

3.7 Becoming-Democratic Through Mathematical Inqu[ee]ry

In my previous work, I (Rands, 2009) used queer theory to develop the concept of mathematical inqu[ee]ry. Mathematical inqu[ee]ry illustrates becoming-democratic mathematics education in two ways. First, the creation of the concept of mathematical inqu[ee]ry is an example of becoming-democratic scholarship in education. Second, mathematical inqu[ee]ry can be used in mathematics classrooms to provide launching sites from which students can embark on lines of flight.

Mathematical inqu[ee]ry un-rules two states in educational scholarship by entering two domains without invitation. On the one hand, I found that some subjects were more “queerable” than others. While numerous educational scholars have explored ways to “queer” reading, writing, science, social studies, and music, math has remained the subject that “dare not speak its name.” Perhaps most illustrative is the journal article title, “Reading, Writing, and Rita Mae Brown” (Boutillier, 1994), in which an author’s name replaced “’rithmetic” in the “three R’s” of elementary education. On the other hand, mathematics educators have overwhelmingly excluded “queer” in mathematics education scholarship. Hence, mathematics is a line of flight into queer scholarship and “queer” is a line of flight into mathematics education.

Mathematical inqu[ee]ry is one of two possible approaches to combining "queer" and mathematics education. The two approaches follow two different meanings for "queer." One way the term "queer" is used as an umbrella term for "lesbian, gay, bisexual, transgender. . ." (LGBT) This use is founded on the notion of identity as fixed and part of an essential self. Here, queer is defined in opposition to "straight." The second use of "queer" is in the sense of queer theory. In this sense, identity is viewed as unfixed, contingent, and discursively produced. Here, queer is not opposed to "straight," but rather against normalcy or normativity. An approach to queering mathematics education based on the first sense involves finding ways to include LGBT people and issues in mathematics class. I called this approach, "Add-Queers-and-Stir" mathematics education. An approach to queering mathematics education through queer theory involves following Nelson's (1999) call to move from inclusion to inquiry. Mathematical inqu[ee]ry involves questioning what comes to be taken-for-granted in mathematics education: tasks, strategies, ways of thinking and doing mathematics, the way mathematics is used to interpret the world. I gave several examples for each of the approaches. In geometry, for example, a teacher taking the "Add-Queers-and-Stir" approach might have students explore the relation between perimeter and area of equilateral triangles in conjunction with learning about the history and symbolism of the pink triangle. A mathematical inqu[ee]ry approach to geometry might involve questioning the ways in which manipulative sets and other curriculum materials normalize certain shapes (e.g. regular polygons over irregular) and the way that influences the way we think and interact with geometrical shapes as well as how certain “shapes” of families are normalized in our society. Mathematical inqu[ee]ries such as this can provide a launching site for students to embark on lines of flight. Students may deterritorialize stagnant images of thought about geometry and families. They may also create new (at least new to them) geometrical shapes or new ways to classify shapes as well as create new ways of thinking about family.

Conclusion

Conclusion

This dissertation has attempted through mapping to “lay everything out on a single plane.” In doing so, each plateau lodged itself on a stratum, and through engagement with the current assemblages on the stratum, moved to something new. A reader already familiar with Deleuzian and Deleuzoguattarian concepts--such as assemblages, strata, lines of flight, subjects of enunciation, subjects of the statement, deterritorialization, the capitalist axiomatic, rhizomes—could read the plateaus in any order. Each plateau is a “middle” and is between each of the others. Yet, each plateau sends out rhizomatic stems that connect to each of the others plateaus. The conclusion highlights two types of rhizomatic interconnections across plateaus: 1) connections among assemblages across plateaus, and 2) lines of flight that provide possibilities for cracking open the strata. Throughout the conclusion, parenthetical numbers refer to sections in the plateaus. For example, “(1.5)” refers to the fifth section in the first plateau, or the section entitled “Deleuzian Dialectics: Repetition, not Opposition.” Parenthetical numbering is intended to allow the reader to revisit the plateaus to follow the rhizomatic stems connecting the plateaus.

Strata in Mathematics Education

The first type of connections across plateaus link assemblages embedded in each stratum of each plateau to other assemblages embedded in strata in the other plateaus. Such linkages form larger strata stretching across plateaus. Strata, as Deleuze and Guattari (1980/1987) can remind us, “consist of giving form to matters, of imprisoning intensities or locking singularities into systems of resonance and redundancy” (p. 40). When read in juxtaposition, the plateaus encompass numerous resonances or redundancies that can be described as “acts of capture” forming three large strata. The first large stratum connects the talent-competition assemblage from the first plateau, the problems-practice assemblage from the second plateau, and the competitive democratic state/State of the third plateau. The second stratum connects the all-regardless assemblage from the first plateau, the problem-solving assemblage from the second plateau, and the participatory democratic state/State from the third plateau. The third stratum connects the contradiction-dialectic assemblage from the first plateau, the problem-posing lines of flight from the second plateau, and the discourse democratic state/State from the third plateau. The three purposes of education that form Borromean rings in the third plateau interconnect in more complicated ways across the three plateaus.

Mathematics Education Stratum I

Schizoanalysis of the concept of equity captured in NCTM’s (2000) equity principle of the *Principles and Standards for Teaching Mathematics* found that the concept of equity pushes against the pervasive belief in North America that mathematical talent is a scarce commodity possessed by only a limited number of individuals. The machine that produces the desire to maintain this belief is the talent-competition machine, which works to doubly articulate a form of expression (mathematician—talent—male—masculine—White—suburban—affluent/ middle class—English-speaking. . .) with a form of content (student/teacher interactions—tracking—scheduling—honors/ advanced placement courses—timed tests—independent practice—decontextualized problems—competitions. . .) to form a talent-competition assemblage, which works to include certain people and exclude others in those who can be seen as having mathematical talent (1.1). The meritocracy machine functions to maintain a competitive democratic state/State in mathematics classrooms, in which student success and position of power comes from the student's ability to quickly access information (3.4.1). The competitive democratic state/State invokes the meritocracy machine, which links the social efficiency assemblage of enunciation and the social mobility assemblage of enunciation into one single assemblage of enunciation with capital as the site of subjectification This single assemblage of enunciation is linked to the machinic assemblage of the stratified educational system (including the hierarchy of grade levels, differential prestige of schools, college rankings, grade point averages, honors and Advanced Placement classes, etc.) (3.3). At the level of the mathematics classroom, the problems-practice assemblage functions to maintain the state/State of competitive democracy. In the problems-practice assemblage, the teacher presents new concepts or procedures, demonstrates how to use them, and then assigns "problems" for students to use to "practice" what they have just been shown (2.1). This problems-practice assemblage easily plugs into the competitive democratic state/State: student success and power come from being able to quickly access and use concept or procedure that was just presented (3.4.1). The problems-practice assemblage becomes an allocation machine (3.4.1). Moreover, the talent-competition assemblage differentially allows students to be seen as talented if they not only access the procedures or concepts quickly, but also “break the rules.” In other words, based on the flows of power in the mathematics classroom, certain students (those who are male, White, English-speaking. . .) may be seen as especially talented if they skip steps or refuse to “show their work” whereas this may not be the case for students not already included in the talent stratum (1.1). Ultimately these flows of power coalesce into credentials (grades, class/ track placements, degrees, etc.) which serve as signaling systems that communicate distinctions among consumers (students) to employers about which match applicants to match with which existing jobs (3.1). Despite the micropolitical flows of power that maintain stratification, the linking of the social efficiency assemblage of enunciation and the social mobility assemblage of enunciation into one single assemblage hooks employers and consumers (students) into this one pyramidal machinic assemblage. From the perspective of the taxpayer and employer, the assemblage of enunciation sounds something like, “Schools exist to prepare workers for their positions within the stratified economic and social structure”. From the perspective of the consumer (student), the assemblage of enunciation sounds something like, “Schools exist to prepare (me) as a worker for my (hopefully more socially and economically desirable) position (than others’) in the stratified economic social structure.” The combined social efficiency-mobility assemblage of enunciation socially subjects both taxpayers/employers and consumers/students at the point of the capital axiomatic; however, taxpayers/employers are subjects of enunciation while consumers/students are subjects of the statement. The social mobility version of the assemblage of enunciation holds out hope to consumers/students that if they accumulate enough credentials, they may be matched to more desirable social and economic positions. Micropolitical flows of power captured in the talent-competition assemblage (1.1) and through the problems-practice assemblage (2.1) enacted in competitive democratic states/States (3.4) create "striated" spaces in mathematics classrooms and broader mathematical education contexts (e.g. school-wide math program) which differentially distribute "talent" along lines of inclusion and exclusion. Striated spaces are the organized spaces under the State as an apparatus of capture (Deleuze & Guattari, 1980/1987)—in this case, the competitive democratic state/State.

Mathematics Education Stratum II

The second large stratum that crosses all three plateaus links the all-regardless assemblage from the first plateau, the problem-solving assemblage of the second plateau, and the participatory democratic state/State of the third plateau. The Liberal Multicultural Education machine deterritorializes the talent-competition assemblage by challenging the idea that talent is a scarce commodity which only a few students can possess. As the talent-competition assemblage is deterritorialized, the Liberal Multicultural Education machine reterritorilizes on the all-regardless assemblage, as seen in the stylistic repetition of the word “all” twenty times in the equity principle of NCTM’s (2000) *Principles and Standards.* The all-regardless assemblage captures the idea that *all* students *regardless* of their (racial, gender, economic, sexual. . .) backgrounds or identities can successfully learn mathematics. Liberal multicultural education is rooted in liberal theory, which attempts to define “liberty” or “freedom” and proposes that “liberty” is inherently good, emphasizes the individual over groups, focuses on individual rights, views the government’s role as protecting individual rights, and supports pluralism. The combining of positive and negative conceptions of freedom to construct the all-regardless assemblage implies that all students should be treated equally (the same) regardless of their background. In Deleuzian (1968/1994) terms, the all-regardless assemblage institutes a “generality [which] expresses a point of view according to which one term may be exchanged or substituted for another,” (p. 2) which, in the context of mathematics education, leads to “difference blindness”—the idea that we should not notice or care about the various ways in which we differ from one another. The Liberal Multicultural Education machine addresses difference by attempting to erase it (1.2). The Liberal Multicultural Education Machine’s reliance on identity, analogy, and resemblance prevent the affirmation of difference, instead both erasing difference through difference-blindness and glorifying a superficial version of diversity through a pluralistic fetish that turns cultural differences into static objects. In mathematics classrooms, this attempt to institute a generality in which one term may be exchanged or substituted for another plays out in the development of a participatory democratic state/State, in which each person supposedly has one “vote” or everyone gets his or her “turn” (3.4). The emphasis on individuals over groups in liberal theory and liberal multicultural education leads to an individualistic view of participation with an emphasis on each individual’s actions and statements. The move from competitive democracy to participatory democracy links with the deterritorialization of the problems-practice assemblage and reterritorialization onto the problem-solving assemblage. In a competitive democratic state/State, the teacher presents and demonstrates procedures or concepts, which students then “practice” in the following “problems.” A new focus on student participation and each student having a voice shifts the teacher’s role to problem-presenter and students’ role to problem-solver. The problems-practice assemblage deterritorializes and a new problem-solving assemblage is constituted (2.2). Yet, this focus on individual participation in participatory democratic states/States fails, as Carlone et al. (2011) found in science classrooms, to escape the talent-competition assemblage. Under this individualistic participatory democratic state/State, students compete to have “my turn” or share “my idea” (3.4). Although students may be working with partners or in groups to solve problems for which procedures have not been demonstrated, each individual “owns” his or her individual ideas. The classroom becomes an arena in which students compete to occupy the classroom space through their individual intellectual property. Micropolitical flows of power regulate whose bids for the floor are acknowledged and the same lines of inclusion and exclusion which striate the talent-competition assemblage (1.1) continue to striate the space of the mathematics classroom. The simplistic conception of participation as casting votes or taking turns fails to unlock the Borromean rings linking democratic equality, social efficiency, and social mobility.

The Liberal Multicultural Education Machine is at work deploying the All-Regardless-Assemblage (1.2) to connect the democratic equality goal to the pyramidal machinic assemblage in place in schools and the social and economic structure under the capitalist axiomatic (3.3), as exemplified in the "equal growth for all" idea of value-added assessment (3.4).

Mathematics Education Stratum III

The third large stratum that stretches across all three plateaus links the contradiction-dialectic assemblage of the first plateau, the problem-posing lines of flight of the second plateau, and the discourse democratic state/State of the third plateau. Critical multicultural education, rooted in critical theory, addresses the difference-blindness and power evasion of liberal multicultural education through a focus on classes or groups rather than individuals. Power is seen not as in the hands of individuals who may or may not be biased, but rather as a dialectical relation between groups or classes, leading to the privileging of certain groups and the oppression of other groups. Furthermore, the struggle against oppression takes place within a dialectic between structure and agency—individuals and groups have the capacity to act in the world, but the material conditions of the world constrain the capacity to act (1.3). Yet, critical-theory based critical multicultural education does not circumvent the four shackles of mediation—identity, analogy, resemblance, and opposition. Despite avoiding difference-blindness and power-evasion through an acknowledgement of privilege/oppression at a group level, critical-theory based multicultural education maintains the Aristotelian “method of division” in which one’s identity is determined based on the inclusion with a particular class or group. Identify, analogy, and resemblance still form the ontological underpinnings of critical multicultural education. The fourth shackle of mediation, opposition, takes on special significance as the foundation of Hegelian and Marxian dialectics. Unlike Aristotle who saw specific (difference at the level of “species”) as the perfect form of difference, in Hegelian and Marxist dialectics, contradiction (difference at the level of “genus”) is seen as the perfect form of difference. However, contradiction, in Hegelian and Marxist dialectics, posed few problems, but instead resolves itself and thereby resolves difference by relating it to a ground. In this way, difference is still assumed to be something to be resolved through contradiction, a new version of difference erasure (1.4). In the discourse democratic state/State in mathematics classrooms, this resolving of differences takes place through deliberation and dialogue. Unlike in a participatory democratic state/State in which teachers and students mainly attended to the frequency or amount of participation and ensured that each person got his or her “turn,:” in a discourse democratic state/State, teachers and students also attend to the characteristics of students’ discourse as well as the power relations connected with discourse patterns. Rather than an individualistic view of participation, the perspective on participation in a discourse democratic state/State is a sociocultural perspective centered on interconnections between discourse and sociomathematical norms. Moreover, in a discourse democratic state/State, teachers and students acknowledge the ways in which organizational aspects of schooling and society make active participation easier for some students than others (3.4.3). Addressing the micropolitical flows of power within the mathematics classroom as well as the ways in which these flows continue outside of the classroom engenders the deterritorialization of the solving side of the problem-solving assemblage and launches problem posing lines of flight. Freirian mathematical problem-posing pedagogy involves resolving the contradiction between teachers and students to create teachers-students and students-teachers who read the world with mathematics in order to transform society. Brown-and-Walter problem posing similarly resolves the contradiction between teachers and students by repositioning students, not only teachers, as posers of mathematical problems (2.3). Brown and Walter's version of problem posing less directed toward transforming the world outside of the classroom, although Brown urges mathematics educators to view mathematical considerations as only one important dimension of real-world problems, which often are more fundamentally ethical problems (2.4). This move to problem-posing and the acknowledgement that mathematical problems are really ethical or political problems resonates with Deleuze's (1968/1994) assertion that "*problems are always dialectical*. . . .What is mathematical. . .are the solutions" (p. 179). However, Deleuzian dialectics reject opposition as the basis of the dialectical and instead move to repetition. The heart of a Deleuzian dialectic consists of the positivities of difference and repetition rather than negation in the form of opposition or contradiction. Rather than ignoring difference altogether through the difference-blindness of liberal multicultural education or assuming that differences can be resolved through Hegelian or Marxist contradiction, Deleuzian dialectics affirm difference (1.5). Affirming difference requires a form of generative form of problem-posing that allows the eternal return of difference in implicit variation arising from encounters between teachers and students (2.6). The discourse democratic state/State can lay the groundwork for such encounters (3.4.3), but since its purpose is to mediate and resolve differences, it does not go far enough to engender Deleuzian problem-posing. The discourse democratic state/State, as a state/State, still serves as an apparatus of capture. Lines of inclusion and exclusion are still determined from the inside. Affirming difference, Deleuzian problem-posing, becoming-democratic in the mathematics classroom, all require encountering unmediated difference from the Outside (1.6, 2.6, 3.6).

Lines of Flight in Postcritical Mathematics Education

Interwoven throughout the plateaus are strategies for embarking on lines of flight, providing possible ways of cracking open the strata to allow encounters with unmediated difference, to generate learning. As a "normative" ontology, Deleuze and Guattari's ontology of becoming is a call for transformative action. This section revisits the interwoven strategies and examines some nascent ideas for ways to use them in mathematics education.

Cultural becoming (1.6)

Often mathematics educators take a multicultural approach by incorporating mathematics games from around the world or from historical cultures of the past. These often become static images of how people of "other" cultures use or have used mathematics. In contrast, Kubota (2004) called for a view of culture as "diverse, dynamic, and. . .discursively constructed" (p. 38). Through ongoing rhythmic repetition, cultures are in the process of becoming. This is true also of mathematics as a cultural practice. Cultural becoming in mathematics class entails using a dynamic ethnomathematics approach that acknowledges that

> identities are not homogeneous and eternal, but rather correspond to an area of tension between permanence and alteration, where--within given contexts--room is left for psycho-social growth processes. A similar dynamic interpretation of culture fully links up with D'Ambrosio's plea for educational reform: "More attention should be paid to students and teachers as human beings, and we have to realize that mathematics--the same is true with respect to other disciplines--are epistemological systems in their socio-cultural and historical perspective and not finished and static entities of results and rules" (D'Ambrosio, 1990, p. 374). (François & Kerkhove, 2010, p. 127).

Cultural becoming in mathematics education also involves recognizing that mathematics is always already cultural—not just when examining the way mathematics is used in cultures other than one's own. Cultural becoming in mathematics education means addressing the becoming-cultural of one's own mathematical practices. "Becoming-cultural" is not intended to imply that one's own mathematical practices were at one time not cultural and are now becoming so; instead, "becoming-cultural" is an eternally returning event, the being-cultural as the cultural “being of becoming as such” (Deleuze, 1962/1983, p. 23). One’s mathematical practices are always already cultural; becoming-cultural in mathematics classes involves asking, “How do my mathematical practices function as cultural practices? What are my mathematical practices *doing* culturally?”

Question how differences are produced, legitimated, eliminated (1.6)

Problematize difference. Do not allow the “problem” of difference to be resolved through mediation, deliberation, erasure. Explore “why inequality among different groups exists and how various kinds of difference are produced, legitimated, or eliminated within unequal relations of power” (Kubota, 2004, p. 38). In mathematics classes, this means acknowledging and addressing the micropolitical flows of power and desire in the classroom and the ways in which they connect outside of the classroom. It means examining interactions in the mathematics classroom and asking questions such as the following: “Have we established a competitive democratic state/State? A participatory democratic state/State? Are we caught up in the problems-practice assemblage? The problem-solving assemblage? In what ways do our interactions plug into the allocation machine? What micropoltical flows of power and desire are taking place? How do these connect with power dynamics outside of the classroom?”

Parodic repetition (displace) (1.6)

Parodic repetition—repetition which has the effect of displacing--in Deleuzian terms is rhythmic repetition, in which what returns is difference.

Failure to repeat the *same* is to refuse the envelopment of rhythmic repetition within a particular cadence. According to Butler (1990), parodic repetition "exposes the phantasmatic effect of abiding identity as a politically tenuous construction" (p. 179). How might parodic repetition serve as a strategy to displace the identity of "math person" as currently constituted in a particular classroom context? James Williams's (2011) performance during the North Carolina Council of Teachers of Mathematics Leadership Seminar, in my reading, incorporated parodic repetition of the problems-practice assemblage to displace the identity of "good mathematics teacher." Without prelude, he began a "lesson" on the "FOIL method[30]" as if the attendees of the leadership seminar (math teachers, math coaches, math teacher educators) were middle school students. The jarring familiarity of the steps of the lesson (following the problems-practice assemblage) and the false earnestness in his tone of voice displaced the familiar image of "what math teachers do" to reveal the way this image of teaching functioned to prevent learning. Might mathematics teachers be able to have a similar effect through parodic repetition of the "good student" who always has the correct answer first? Might such a performance displace the identity of "good math student" in a way that opens up other ways of being—or becoming—a "good math student"? Could parodic repetition be used in this way

[30] In teaching students about the distributive property of multiplication over addition, many teachers teach students to use the mnemonic device "FOIL." Consider an expression of the form $(a + b)(c + d)$. The letters in FOIL stand for First, Outside, Inside, Last, where *a* and *c* are the "first" terms, *a* and *d* are the "outside" terms, *b* and *c* are the "inside" terms, and *b* and *d* are the "last" terms. As a mnemonic device, FOIL is intended to help students remember that $(a + b)(c + d) = ac + ad + bc + bd$. Often this mnemonic device is taught as a method *in place of* a conceptual understanding of the distributive property of multiplication over addition.

without harming certain groups of students?

Refuse to be represented/ to represent (1.6)

This strategy might involve refusing to represent the "token" person in a particular category, refusing to speak "as" a woman/person of color/transgender mathematician or math teacher or "good math student." It might mean challenging the contexts used in a program such as Mathematics in Context (MiC), stating, "That's not *my* context." A teacher might also use this strategy by refusing to be the one to represent the students in the class in problems or contexts—and instead allowing the contexts and problems to come from the students.

Refuse to know what "everybody" knows" (1.6)

"Many people," Deleuze (1968/1994) argued, "have an interest in saying that everybody knows 'this', that everybody recognizes this, or that nobody can deny it" (p. 131). Converting "givens" to "takens" requires someone, "if only one—with the necessary modesty not managing to know what everybody knows, and modestly denying what everybody is supposed to recognize" (Deleuze, 1968/1994, p. 130). This strategy might involve refusing the knowledge that "boys are just better at math than girls," or "you shouldn't see race," or "math is neutral and universal." It might mean refusing the familiar problems-practice assemblage or the refusal to know that mathematics problems come from teachers and textbooks. It might mean refusing to know that "math is hard."

Question cultural representations (1.6)

Call into question the representations of cultures: “images of a certain culture or language are neither neutral nor objective, rather, they are discursively constructed” (Kubota, 2004, p. 38). Question the representations of cultures in mathematics textbooks, curricula, and other mathematics resources. Question “math manipulatives” as cultural objects. Math manipulatives such as pattern blocks, snap cubes, and algebra tiles are often seen as neutral, non-cultural objects; yet, these objects have been created within cultural contexts for particular cultural purposes. What do snap cubes say about mathematics as a cultural practice in schools? What assumptions about how mathematics functions as a cultural practice are captured in base-ten blocks?

Learn through crisis (1.6)

Affirming difference requires learning through the crisis (Kumashiro, 2004, p. 27) instigated in the return of difference. That is, as Deleuze (1968/1994) concluded, “Learning takes place not in the relation between a representation and an action (reproduction of the Same) but in the relation between a sign and a response (encounter with the Other)” (p. 22). Learning through crisis involves acknowledging and addressing desire, and working through the realization that what we desire can be productive in oppressive ways. Mazzei (2011) examined the ways in which White women pre-service teachers’ desire to “carry on as before” (p.660), that is, to maintain White privilege, produced a “desiring silence” (p.660) which continued to “perpetuate a racially inhabited silence that limits, if not negates, an open dialogue regarding race and culture" (p. 661). Recently I witnessed the force with which desiring silences can operate. At a local teacher education conference, I attended a session led by a man who taught multicultural education courses for pre-service teachers. He explained that he had taught the course many times and had found that students were reluctant to talk about issues of power. He shared his new theoretical framework for teaching multicultural education courses which was designed to make the courses more comfortable for students by avoiding overtly political issues. In this case, students' desiring silences won out, maintaining a "safe" space in which privilege could be maintained (and hence, oppression perpetuated).

Mazzei (2011) encourages teacher educators not to overlook or ignore desiring silences, but instead to engage them:

> what is possible is that as teacher educators, we provide opportunities that encourage a continual search for the potential movements of deterritorialization or possible lines of flight that may, over time, produce not a desiring silence, but the production of a desiring pedagogy. If, as teacher educators, we fail to recognize how desire functions with white preservice teachers by failing to attend to a desiring silence, then students can resist and reassert their power. If, on the other hand, we engage the silence, connect our desires with those of our students, then students may still resist, but they may also begin to destratify in ways that produce the possibility of deterritorialization, the possibility of a desiring pedagogy. (pp. 666-667)

Another example of the production of a desiring-silence became evident in a discussion I had with a group of fourth- and fifth-grade students from a variety of schools in central North Carolina. In discussing teachers' responses to the use of the term "gay" in schools, it became evident that in response to "gay" being used as an insult or in the phrase "that's so gay," many teachers were simply telling students not to use the word gay. While teachers' responses may have functioned to deter homophobic bullying, the responses also functioned to produce a desiring-silence about gay lives altogether, which maintained straight privilege in schools (and its correlate, lgbt oppression). Desiring-pedagogy required a pragmatics that engaged this silence and examined how language and silence were functioning to keep "gay" on the outside of schools.

The rhetoric of mathematics and mathematics education as neutral is one way that desiring silences are produced in mathematics education. Math is often the content area to which teachers allude as an example of when and why *not* to use social justice-focused teaching. Many times, I have heard current and future teachers say something like the following in response to the idea of teaching for social justice: "I'm not going to just stop in the middle of my math lesson to talk about racism [or sexism or heterosexism. . .]" The emphasis and tone embedded in the statements take on the ring of what "everybody knows," of common sense: everyone knows that math has nothing to do with social justice; it is just common sense that a teacher would have to *stop* the lesson in order to *completely change the subject* to social justice. Desiring math functions here as desiring silence about social justice. As Mazzei (2011) pointed out, engaging silence requires desire to desire its own transformation. Teaching math for social justice perspectives engage this silence by changing the image of thought that produces "math" and "social justice" as mutually exclusive concepts and instead produce a new concept in which desiring math and desiring social justice recombine into a new assemblage. Desiring-pedagogy (Mazzei, 2011) involves learning through the crisis (Kumashiro, 2004) of transforming desire.

“Conjugate” (2.6)

The problems-practice assemblage assumes that students learn math through imitation, by replicating procedures that *resemble* those the teacher just demonstrated. Learning, Deleuze (1968/1994) claimed, is not imitation. Instead, it is *conjugation*:

> To learn is to enter into the universal of the relations which constitutes the Idea. . .To learn to swim is to conjugate the distinctive points of our bodies with the singular points of the objective Idea in order to form a problematic field. This conjugation determines for us a threshold of consciousness at which our real acts are adjusted to our perceptions of the real relations, thereby providing a solution to the problem.” (p. 165)

In the problem-solving assemblage in which problems are placed at the center of instruction, students are no longer simply imitating the teacher by “applying” a demonstrated procedure to “exercises”; instead students must respond the a problem for which an obvious solution is not available. Yet, most problem solving in problem-centered classrooms still is not “conjugation” because the problems still come from the elsewhere (i.e., from the teacher) and are given to the students as “ready-made” problems. In Deleuzian problem-posing pedagogy, teachers and students encounter one another within a dialectical problematic field in which the political and ethical are not considered irrelevant aspects to be crossed out and ignored. We can replace “swim” with “math” in the quote above:

> To learn *math* is to conjugate the distinctive points of our bodies with the objective Idea in order to form a problematic field. This conjugation determines for us a threshold of consciousness at which our real acts are adjusted to our perceptions of the real relations, thereby providing a solution to the problem.

Conjugation—learning--is the actualization of a virtual Idea through action in the world. Protevi (2010) gave an example of how such actualization occurs according to a Deleuzian perspective. He examined the way in which the Idea of "football games" is actualized under different conditions leading to variations or multiple solutions to the problematic field containing the elements of players, a playing field, and a ball:

> What is the Idea that conditioned the genesis of American football? Well, it would be a multiplicity of differential elements, differential relations, and singularities. . . .But American football is only one actualization of this Idea. Changes in the elements, relations, and singularities will change the game. Forbid the forward pass and blocking and you have rugby. . . .Make it a completely savage festival and you have either Gaelic or Australian rules football. Restrict the handling of the ball to the goalkeeper, change the shape of the goal and the field, install a penalty around the goal and you have association football or soccer. (p. 42)

What this actualization is *not*, Protevi (2010) pointed out, is recognition of the "essence" of football: "It's important to see first of all that we have *not* established a finite set of necessary and sufficient conditions for membership in a class" (p.43). Instead, "we have gone from an actualization to its conditions of genesis in a multiplicity ('vice-diction'), and then experimented with the singularities of the Idea" (p. 43). An actual problem, one that is not a "nonproblem" with an obvious solution, is one whose solutions relate at the level of the "being of the sensible" (the level of sense), not just the level of signification (the level of representation).

Treat knowledge paradoxically (2.6)

Deleuzian problem -posing pedagogy, as a form of anti-oppressive education (Kumashiro, 2004), is

> a disarming process that allows students to escape the uncritical, complacent repetition of their prior knowledge and actions. Learning is a disorienting process that raises questions about what was already learned and what has yet to be learned. Learning involves *looking beyond* what students already know, what teachers already know, and what we both are only now coming to know, not by rejecting such knowledge, but by treating it paradoxically, that is, by learning what matters in society. . .while asking why it matters (and how it can reinforce and challenge an oppressive status quo). (Kumashiro, 2004, p. 30)

Paradox, for Deleuze, is what "both bring[s] language and the world together and keep[s] them separate" (May, 2005, p. 107). It is paradox that enables linguistic meaning to be produced. Paradox is the nexus of sense and the non-sense that allows sense to exist. Encounters with generating problems that require "conjugation"—that is learning—produce knowledge through actualization of solutions. Yet the solutions to dialectical problems do not erase the problems, but move to questions (Deleuze, 1968/1994). Generating problems are generative of questions. As teachers and students construct mathematical knowledge, what images of thought about mathematics are coalescing? What actions and ways of being and becoming do these images of thought about mathematics enable and preclude? How do certain assemblages in the images of thought function to direct flows of desire within our mathematics classrooms?

Disrupt the state/State (3.6)

Deleuze and Guattari are critical of *states*, both in the sense of *stasis* and in the sense of political *States.* As we have seen, Deleuze and Guattari (1980/1987) conceptualized States as models of realization in the capital axiomatic which function to "capture" decoded flows in a way that maximizes profit and maintains inequitable distributions of wealth. “The State,” Deleuze and Guattari (19801987) wrote, “is assuredly not a locus of liberty” (p. 460). This political philosophical critique of States is interconnected with an ontological critique of stasis or “being” in favor of an ontology of becoming.

Disrupting the state/State in mathematics classrooms stimulates the creation of new political identities for those who have crossed the line to the inside. In the process of disrupting the state/State, those on the Outside include themselves rather than being included *by* those on the Inside. In Deleuzoguattarian terms, this process is the process of deterritorialization. The Outsiders create a line of flight that escapes from what Rancière (1999) called the police state. The cutting edge of deterritorialization enacts shifts in the configuration of the previous assemblage. Gutiérrez (2002) has pointed out that although “[m]ost researchers and educators have moved beyond thinking that it is mainly the fault of students themselves, their families, or their cultures as to why they do not perform well in mathematics” (p. 147), it is still the case that “proponents of equity issues tend to frame their arguments in ways that suggest that benefits move from mathematics to persons and not the other way around” (p. 147). This subtle deficit perspective frames inclusion as a process of invitation from those on the Inside: “Hey, you there[31], on the Outside! Come on in and bask in the wonders of mathematics as we see it! We will fill you with

[31] In Althusser’s (1970/1971) explanation of the notion of interpellation, he used the example of the “most commonplace everyday police. . .hailing, ‘Hey, you there.’” Althusser (1970/1971) suggested “that ideology ‘acts’ or ‘functions’ in such a way that it ‘recruits’ subjects among the individuals. . .or ‘transforms’ the subjects into subjects” (p. 86).

mathematical knowledge (like a time deposit)!" The Inside forms a body with organs which desires stasis. It is assumed that the Outsiders will change, not the Inside. Outsiders are invited to take their place on the Inside according to the regulations set forth by those policing the Inside. On the other hand, disrupting the state/State requires that the Inside become a body with*out* organs (BwO)—stasis cannot be maintained. Following Gutiérrez's point, when the state/State is disrupted, benefits flow in the other direction, from person (Outsiders) to mathematics. It is *mathematics* which changes, which is enriched by the uninvited arrival of Outsiders. In studying the "popularization of mathematics" in New Zealand, Knight (1990) found that invitational inclusion that maintained stasis did not result in mathematics becoming more "popular" among Maori people:

> The aim of the popularization of mathematics is to influence the perception which people have of the subject. . . .Blanket attempts at popularization based on the perceptions which mathematicians have of their subject are unlikely to succeed. . .Maori people have been culturally alienated from mathematics and. . .attempts to overcome this must go beyond the superficial introduction of elements of Maori culture into traditional presentation of mathematics. Initiatives, by the Maori themselves, firmly based on their own cultures have much more potential. (p. 136)

Knight (1990) found that non-Maori teachers' attempts to take a Maori perspective (taha Maori) to teaching mathematics promoted cultural awareness among non-Maori students, but were often rejected by Maori students, some of whom "regard[ed] the approach as positively dangerous since it salves the conscience of the pakeha [non-Maori] without confronting real issues" (pp. 140-141). Maori initiatives, on the other hand, were based on the "fundamental principle. . .that instead of starting with the mathematics and introducing a Maori perspective, [one] must begin with Maori culture and introduce a mathematical perspective" (p. 140). In this case, it is mathematics that must change rather than Maori people; indeed, Knight (1990) found that this approach that allowed mathematics to be accepted as "Maori knowledge" transformed mathematics by "blur[ing] the boundaries between subject areas [in ways] very much in line with traditional Maori ways of learning" (p. 142). While Knight (1990) emphasized the importance of non-Maori mathematicians staying in a supporting rather than dominant role, the focus of Knight's (1990) article remained on finding a way to "popularize" mathematics among Maori people without directly addressing the way in which a Maori approach enriched the field of mathematics.

Allow pack infiltration/ Provide launching sites for lines of flight (3.6)

It is the pack that disrupts the state/State. Deleuze and Guattari (19801987) wrote, "Among the characteristics of the pack are small or restricted numbers, dispersion, nondecomposable variable distances, qualitative metamorphoses, inequalities as remainders or crossings, impossibility of a fixed totalization or hierarchization, and projection of particles" (p. 33). The pack enters from the outside, dispersing along lines of flight through the territory, disrupting the state of the State. The pack does not arrive with proof of invitation, but rather includes itself on its own terms. The pack does not demand equal rights, does not enter a bid to be considered exchangeable. Instead, the pack affirms difference by entering on its own terms. The concept of becoming-democratic involves a disruption from the "outside," in which the excluded include themselves on their own terms. When the teacher is on the Inside, he or she cannot disrupt the state/State. However, the teacher can *allow* pack infiltration. Deleuze and Guattari's (1980/1987) "directions" for how to embark on a line of flight were as follows : "lodge yourself on a stratum, experiment with the opportunities it offers . . .find potential movements. . .possible lines of flight, experience them, produce flow conjunctions here and there. . . .It is through a meticulous relation with the strata that one succeeds in freeing lines of flight. . ." (p. 161). Teachers cannot embark on lines of flight alone. Yet, teachers can provide "launching sites" from which students might embark on lines of flight, and some states/States of democracy make it easier to provide launching sites than others. Teachers can "establish" deliberative or discourse democracies while also leaving or opening cracks in the established state/State. Gutstein's projects are an example of ways of opening cracks in the established state/State in the mathematics classroom and beyond the classroom. The projects provided launching sites for lines of flight that could transform stasis by using real-world contexts which were "inherently dangerous to the status quo because they prepare students to ask fundamental questions stemming from the concrete analysis of their lives and begin to 'unveil reality'" (p. 31).

Inqu[ee]r mathematically (3.7)

Another way to provide launching sites for embarking on lines of flight is through mathematical inqu[ee]ry (Rands, 2009). An "Add-Queers-and-Stir" approach includes lgbt people and themes to mathematics projects and problems while maintaining the state/State of the mathematics classroom. This approach attempts to “represent” lgbt people in the mathematics curriculum, falling into the trap of Deleuze’s (1968/1994) four shackles of mediation. Mathematical Inqu[ee]ry, on the other hand, allows pack infiltration, provides launching sites for lines of flight, has the potential to transform the classroom into a Body without Organs and prompt transformation. Mathematical Inqu[ee]ry treats mathematical knowledge paradoxically by raising questions about the nature of that emerging knowledge. Mathematical inqu[ee]ry means “questioning the tasks, the strategies, the very ways of thinking and doing mathematics, as well as the way mathematics is used to interpret and act in the world” (Rands, 2009, p. 186). Mathematical inqu[ee]ry entails “interrogating the ‘regimes of the normal’ (Warner, 1993)” in mathematics and mathematics education. When the queer enters mathematics education without regard to invitation, mathematics and mathematics education must undergo transformation. Gutiérrez (2008, 2011) used Anzaldúa’s (1987, 2002) term “Nepantla” to describe a “liminal space where multiple realities are viewed” (p. 24). In Nepantla, “new forbidden knowledges develop that disrupt previous categories” (p. 24). Mathematical Inqu[ee]ry has the potential to transform that state/State of the mathematic classroom into a smooth Nepantla space . If Gutiérrez (2002) is correct that benefits flow from people to mathematics, the problem of queering mathematics and mathematics education generates the questions: What benefits will flow from queer people to mathematics? How will queering mathematics and mathematics education enrich these fields? What transformations will occur? The liminal space of Nepantla has the potential to provide a space in which “desire desire[s] its own transformation" (Mazzei, 2011, p. 668), a place of affirming difference, generating problems, and becoming-democratic mathematics education.

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