Essays on Antitrust Issues

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ABSTRACT

WONCHUL HWANG: Essays on Antitrust Issues.
(Under the direction of Gary Biglaiser.)

This dissertation consists of three essays on antitrust issues. In the first paper, I propose two models based on quantity setting game to analyze the profitability of merger, its welfare effect and price effect. In the second paper, I deal with the issue how uncertainty on other firms' discount rates affects the competitive behavior in oligopoly market. In the third paper, I analyze two antitrust policy issues for effective cartel deterrence : leniency program and crackdown policy.

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I also owe a huge debt of gratitude to my parents, Jaeseok Hwang and Sookja Ahn, brother, Wonjin Hwang, and my wife, Eunjeong. Their wholehearted support and patience made this journey possible. Finally, I hope that this dissertation can be a good present to my two sons, Junsang and Yoonsang.

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CHAPTER 1

Introduction

This document presents the three essays that form my dissertation in accordance with the Graduate School and Economics Department at UNC at Chapel Hill.

Chapter 2, titled "The Incentive of Horizontal Merger and Its Welfare Effect", reexamines the result of Salant, Switzer, and Reynolds (1983) on horizontal merger's private profitability, its welfare effect and price effect after I generalize their assumptions which include Cournot competition pre- and post-merger, identical constant marginal costs among firms and no entry-exit condition.

On one hand, collusion becomes easier post-merger if firms have constant and identical marginal costs. If such collusive effect exists, a merger becomes profitable but socially more injurious. On the other hand, cost saving from reallocation or synergies can make a merger profitable even under Cournot competition pre- and post-merger. A merger gets more profitable as insiders are more asymmetric, and outsiders are "less" efficient, while its welfare effect improves as insiders are smaller or more asymmetric, and outsiders are "more" efficient. Synergy-creating mergers are not always profitable or welfare-increasing. Consumer surplus (CS) increasing mergers are profitable, and CS-decreasing merger improves another profitable CS-decreasing merger's profitability. Entry-inducing mergers are not always unprofitable while exit-inducing mergers are always profitable. Entry reduces a merger's price effect or may even change the direction of it.

Chapter 3, titled "Collusion under Asymmetric Information on Discount Rates", studies collusion agreement and its sustainability when each firm's discount factor is private information. In order to analyze this issue, I construct a model where firms may have different discount rates and each firm does not know the other firms' discount rate, and also build up an equivalent counterfactual model with perfect information. Then, I solve the model by using Bayesian Nash equilibrium or perfect Bayesian equilibrium concept and compare the features of equilibrium outcome with those under counterfactual model.

Under the private information, cartel could be agreed on even when each firm's incentive constraint for cartel sustainability is not satisfied, and hence cartel agreement contains the possibility that cartel members produce more than cartel output from the beginning. If firms are allowed to agree on the payoff less than monopoly profit, there might exist a continuum of collusion equilibria where firms choose payoff below the monopoly payoff at the beginning. But the output in the first period plays the role of signaling that reveals each firm's discount rate, so if both firms abide by the agreed output, perfect cartel output is produced from period 2 and on. When firms are allowed to agree on the uneven split of monopoly profit after communicating each other's discount factor, truth-telling equilibrium does not exist under money transfer and may or may not exist depending on parameter values under output quota.

Chapter 4, titled "Antitrust Policy Issues for Effective Cartel Deterrence", deals with two antitrust policy issues : leniency program and crackdown policy. To examine the effectiveness of leniency program, I introduce cartel duration model and explain why self-reports are mostly made from "dying cartels" and applied simultaneously by multiple cartel members. These facts are the outcome path of stationary Markov perfect equilibrium in this model and the increase in the number of discovered cartels does not necessarily imply that the introduced leniency program is effective. Optimal law enforcement with leniency program, full exemption to deviator with no reduction to simultaneous leniency applicants, would increase deterrence to cartel if fines are sufficiently high or firm's strategy against deviation is severe enough.

Crackdown policy, on the other hand, means that antitrust authority spends all the resources for target industry's cartel conviction at a period, moves its focus to another in the next period, and so on. The efficacy of this policy depends on antitrust authority's conviction technology, and it is more likely to be effective when conviction technology is less concave.

CHAPTER 2

The Incentive of Horizontal Merger and Its Welfare Effect

2.1 Introduction

Since Stigler (1950) pointed out "free rider's problem" in horizontal merger meaning that firms who do not participate in a merger may get greater benefit than the constituent firms, the incentive of horizontal merger has been an important research topic in oligopoly market theory. Salant, Switzer, and Reynolds (1983) ("S-S-R" henceforth) provided one landmark paper on this issue. Employing a symmetric Cournot model with linear demand and identical constant marginal cost, S-S-R derived two surprising results on private profitability and social desirability of merger: (1) an exogenous merger may reduce the joint profits of the firms that are assumed to merge, and (2) a merger that provides efficiency gains may be socially beneficial even if it is privately injurious to the merging parties. On the first result, which took the name "*The Merger Paradox*", they showed that it is sufficient for a merger to be unprofitable that less than 80% of the firms merge if there is no efficiency gains from the merger.

Although there have been a variety of researches in an attempt to resolve the paradox thus far, this paper's approach is to start from the same setting that S-S-R constructed and then to combine the results of merger's profitability analysis with its welfare effect and price effect. Given that mergers are so common in the real economy, an important question is basically which mergers are profitable, and what will be the welfare effects and price effects of profitable mergers.

To start with, I consider the possibility that firms are engaging in tacit collusion post-merger. This extension shows that tacit collusion becomes easier after merger under the S-S-R model. If such collusive effect exists, the merger becomes profitable whereas it hurts consumers and worsens social welfare.

Another extension of the S-S-R model is to generalize the cost function. As Perry and Porter (1985) pointed out, mergers are not well-defined conceptually in the S-S-R model because the merged entity does not differ from the others in their setting. In order to fix this shortcoming of the S-S-R model, I adopt and slightly modify the cost function that Perry and Porter (1985) proposed. This adjustment of the cost function enables me to capture the cost savings not only from reallocation among the facilities of constituent firms but also from synergies that a merger may create. In contrast, I maintain Cournot assumption of the S-S-R in this exercise because a merger does not necessarily reduce competition under this setting. The linear property of demand and marginal cost allows me to perform the direct analysis on the incentive of merger and its welfare effect. This model shows that a merger without synergies can be both privately profitable and socially desirable only when merging parties have quite different efficiency levels and outsider is sufficiently efficient. While the synergy effect improves both merger's profitability and desirability, a merger with big synergies does not always bring higher welfare or consumer surplus. This extension also shows that a CS-increasing merger is always profitable. On the other hand, CS-decreasing merger may trigger another CS-decreasing merger, which can become profitable after the former CS-decreasing merger takes place.

Finally, I consider an environment where costless entry and exit is possible. Not surprisingly, entry (exit) may occur in a merger that would have been CS-decreasing (CS-increasing) under no entry-and-exit condition. Firms are less likely to merge under free entry condition, because entry harms merger's profitability. But entry-inducing mergers are not always unprofitable whereas exit-inducing mergers are always profitable. The presence of entry or exit reduces a merger's impact on price and may even change the direction of price effect.

This paper is organized as follows. In Section 2.2, I briefly reviewed related literatures. I set up the identical constant marginal cost model and analyze the incentive and welfare

Table 2.1: Key Feature of Each Model

	Marginal Cost	Competition Behavior	Entry-exit Condition
Section 2.3	identical + constant	Cournot/collusion	no entry-exit
Section 2.4	asymmetric + increasing	Cournot	no entry-exit
Section 2.5.1	identical + constant	Cournot/collusion	free entry-exit
Section 2.5.2	asymmetric + increasing	Cournot	free entry-exit

effect of merger in this environment in Section 2.3. In Section 2.4, I construct the asymmetric increasing marginal cost model and perform the analysis on the incentive and welfare effect of merger in this setting. Section 2.5 extends these models in a direction where free entry or exit is allowed after merger. Concluding remarks are following in Section 2.6.

2.2 Literature Review

After S-S-R's research, there have been many attempts to check the robustness of their results in various directions. Some researchers resolve the paradox by relaxing the assumption of linear demand [Cheung (1992), Fauli-Oller (1997), Hennessy (2000)], and others find the incentive of merger from product differentiation [Deneckere and Davidson (1985)], or earning and strengthening the position of Stackelberg leader [Perry and Porter (1985), Mallela and Nahata (1989)].

Although there is no literature, to the author's knowledge, that addresses higher chance of collusion explicitly in order to explain merger's profitability, some papers study the relation between the number of firms and the chance of collusion. For example, Ivaldi, Jullien, Ray, Seabright, and Tirole (2003) considers a simple price competition model in an environment where firms produce the homogenous good with the same unit variable cost. Assuming Nash reversion strategy against unilateral deviation from collusion agreement, they illustrate that collusion is less sustainable as there are more competitors. Kuhn (2008) also points out that collusion gets harder with more firms "when we replicating the assets of firms as we increase the number of firms in an industry." This paper considers a quantity competition model under the S-S-R setting and shows that the result of Ivaldi, Jullien, Ray, Seabright, and Tirole (2003) may hold in strategies other than trigger strategy.

When this paper deals with the collusive effect, it follows the “Folk Theorem” approach based on an infinitely repeated game setting. [Friedman (1971), Abreu (1986), Fudenberg and Maskin (1986) and etc.] In this approach, collusion is understood as a subgame perfect equilibrium in the repeated game. In more detail, firms interacting repeatedly may be able to maintain higher prices by agreeing that any deviation from the collusive path would trigger some retaliation. For the agreement to be sustainable, such retaliation must be sufficiently likely and costly to outweigh the short-term benefits from “cheating” on the collusive path. This paper illustrates that collusive effect may exist under the various retaliation strategies.

Another approach closely related to this research is to modify the S-S-R’s cost function. One attempt along this line is to use asymmetric but “constant” marginal cost functions across firms. Using this cost function and the linear demand, Fauli-Oller (2002) found that a merger can only be profitable if it involves firms that are asymmetric enough. This research confirms Fauli-Oller’s point that cost asymmetry is an important source of merger’s profitability, but it also shows that the degree of asymmetry necessary for a merger to be profitable depends on the outsider’s size or efficiency if asymmetric “increasing” marginal cost functions are assumed.

Another modification of cost function dates back to the Perry-Porter model. The quadratic cost function proposed by Perry-Porter ensures that the newly merged firm retains all of its constituents’ capital. While Perry-Porter modified the S-S-R’s assumption of Cournot competition as well, the following researches in this vein show that the introduction of convex cost function alone may reduce the critical degree of concentration required for a merger to be profitable. For example, Heywood and McGinty (2007) showed that any size of merger in an industry with any number of identical firms can be profitable if cost function is sufficiently convex. Although the cost function of the asymmetric increasing marginal cost model in this paper is heavily indebted to the Perry-Porter model, it has two critical differences from the one in the previous literatures. First, the cost function of each firm may vary due to both capital stock and technology. So, two firms have different cost functions if they have either different capital stock or different technologies with the same capital stock. The other distinctive feature is that it can consider the cost synergies of merger explicitly. These two modifications enlarge the scope of merger analysis. In particular, I can check the profitability, welfare effect and price effect of a merger with synergies.

This research is also closely related to the literatures on the welfare implications of mergers. Assuming Cournot competition pre- and post-merger as in the S-S-R model, Farrell and Shapiro (1990) provided the necessary and sufficient condition that a merger improves consumer surplus (Proposition 1), and the sufficient condition that a profitable CS-decreasing merger increases aggregate welfare (Proposition 5). As a special case where firms have asymmetric constant marginal costs, Froeb and Werden (1998) derives the condition on marginal cost reduction that restores the pre-merger price. The welfare analysis in this paper is another exercise of the Farrell-Shapiro model where demand is linear and cost function is linear or quadratic. Under these functional forms, I can derive a more specific and exact condition for welfare-increasing merger than Farrell-Shapiro's Proposition 5. In addition, combining the welfare analysis of merger with the profitability analysis, I can get the properties of a merger that is privately profitable and socially desirable. In contrast, their Proposition 1 plays an essential role in my paper when I look at the profitability and welfare effect of a CS-increasing merger.

McAfee and Williams (1992), on the other hand, examined the welfare implications of horizontal mergers using linear demand and quadratic cost function with Cournot competition pre- and post-merger. They suggested a necessary condition for a merger to increase welfare, and showed that a merger reduces welfare if it creates a new largest firm or increases the size of the largest firm under moderately elastic demand. The asymmetric increasing marginal cost model has some common features with the McAfee-Williams model because both models assume linear demand and quadratic cost function. But this paper extends their research in that I include merger's profitability and synergy effect into the scope of merger analysis. In particular, this paper demonstrates that their elasticity condition - the prerequisite to apply the McAfee-Williams' condition - is too binding because profitable mergers fail to satisfy this condition.

Finally, literatures on mergers with free entry also have close relation with this research. Using Cournot model with linear demand and identical increasing marginal cost, Werden and

Froeb (1998) showed that significant mergers are normally unprofitable or not so profitable to induce entry. Their result depends on two critical assumptions: the symmetry of cost function and no synergies. I illustrate that if these assumptions are relaxed, Cournot mergers can induce entry or even exit. Based on Bertrand setting, Cabral (2003) showed that cost efficiencies decreases the likelihood of entry, and thus benefit consumers less than under no entry condition. I confirm this result under Cournot setting and add the potential possibility that a marginal incumbent may exit post-merger under strong synergies. I also detail the price effect of entry-inducing or exit-inducing mergers.

Davidson and Muhkerjee (2007), on the other hand, demonstrated that any merger is profitable for any degree of cost synergy with free entry. Their result critically relies on the identical constant marginal cost function. This research shows that mergers creating synergies can be unprofitable even under no entry condition if the assumption on cost function is relaxed.

Spector (2003) studied the price effect of a merger under free entry condition, and showed that any profitable Cournot merger failing to generate synergies raise price even if entry is possible. His result is an extension of Farrell-Shapiro's Proposition 2 which proves the same argument under no entry condition. This research, in contrast, shows that if entry condition is relaxed when there is relatively "small" degree of synergies in that the merger with that amount of synergies would increase price under no entry condition, the merger's price effect becomes not decisive.

2.3 Identical Constant Marginal Cost Model

In this section, I build up the identical constant marginal cost model following the S-S-R model. After I show that a merger in this model increases the scope of collusion, I classify all possible merger cases according to firms' competition pattern post-merger. The situation that S-S-R assumed - firms compete *à la* Cournot pre- and post-merger - becomes just one possible case in this model. Then, I perform case analysis on a merger's profitability and its welfare effect.

2.3.1 Model

The model of this section considers an exogenous merger as in the S-S-R model, but the repeated game is constructed in order to see that the remaining firms makes collusion decision post-merger unlike the S-S-R model.

■ **Demand and Supply** : I assume linear demand and constant marginal cost. Specifically, demand curve is normalized as $P = 1 - Q$ and marginal cost as $MC = 0$.[1] The demand and marginal cost do not change in every period $t \in \{0, 1, 2, \cdots\}$. There are N identical firms in the industry at period 0 that produce homogenous goods. Each firm discounts future profit at $\delta \in (0, 1)$, which is the same and common knowledge across firms. Entry or exit does not take place in this economy.

■ **Game Structure** : Merger is one-time exogenous event under this model in the sense that all the remaining firms post-merger believe that there is no more merger. The timing of the game is as follows.

1. Firms compete *à la* Cournot pre-merger at period 0.
2. $(M + 1)$ firms merge at the end of period 0, where $M \in \{0, 1, \cdots, N - 1\}$.
3. Remaining firms make a collusion decision at the beginning of period 1.
4. Each firm chooses its output in every period $t \geq 1$.

Each firm earns $\frac{1}{(N+1)^2}$ at period 0 since the industry compete *à la* Cournot. M is exogenous and represents the size of merger: no merger if $M = 0$; merger to monopoly if $M = N - 1$. As in the S-S-R model, merger does not affect the marginal cost of merging firms. Efficiency gains take the form of saved fixed cost, F, if exists, which is assumed to be the same across all firms in the industry.

■ **Stage Game Payoff Post-Merger** : $(N - M)$ firms remain in the industry post-merger and interact infinitely from period 1 and on. In each period post-merger, every firm would

[1]While the S-S-R's original model assumed $P = \beta - Q$ and $MC = \alpha$, I normalize $\beta = 1$ and $\alpha = 0$ for computational convenience. But this normalization does not affect the result and inference of this paper.

get $\frac{1}{4(N-M)}$ under perfect collusion with symmetric payoff, $\frac{1}{(N-M+1)^2}$ under Cournot-Nash equilibrium. If a firm deviates with best response output when all other firms produce perfect collusive output ($\sum_{j\neq i} q_j^c = \frac{N-M-1}{2(N-M)}$), its stage payoff would be $\left(\frac{N-M+1}{4(N-M)}\right)^2$.

2.3.2 The Effect of Merger on Firm's Competitive Behavior

The merged entity is identical to the other remaining firms under this model. Let me denote the number of firms post-merger by $L = (N - M)$ in this subsection, which may take a value from 1 to N. Then, the discounted payoff of each firm from perfect collusion is given by $\pi^C(L,\delta) = \frac{1}{4L(1-\delta)}$ if L firms split monopoly profit evenly post-merger. In contrast, each firm's discounted payoff becomes $\pi^D(L,\delta) = \left(\frac{L+1}{4L}\right)^2 + r(\delta, L)$ when it deviates and selects best deviation output. Here, $r(\delta, L)$ represents the discounted continuation payoff achieved under subgame perfect equilibrium in the punishment phase. It depends on firms' strategy after unilateral defection. For example, $r(\delta, L)$ is $\frac{\delta}{(1-\delta)(L+1)^2}$ in Nash reversion strategy [Friedman (1971)]. Then, collusion can be supported as subgame perfect equilibrium if and only if

$$\begin{aligned} \pi^C(L,\delta) &\geq \pi^D(L,\delta) \\ &\Leftrightarrow \delta \geq \left(\frac{L-1}{L+1}\right)^2 + \left(\frac{4L}{L+1}\right)^2 (1-\delta) * r(\delta, L) \end{aligned} \tag{2.1}$$

Let δ^* be the threshold discount rate such that $\pi^C(L,\delta^*) = \pi^D(L,\delta^*)$. By solving this equation, I can define δ^* as a function of the number of firms ($\delta^* = f(L)$). If $\pi^C(L,\delta) > \pi^D(L,\delta)$ for all $\delta \in (0,1)$, then $f(L) \equiv 0$, which means that collusion is agreed for any discount rate. If $\pi^C(L,\delta) < \pi^D(L,\delta)$ for all $\delta \in (0,1)$, then $f(L) \equiv 1$, which means that collusion is never agreed. I assume that firms compete *à la* Cournot if $\delta \leq \delta^*$ and collude with monopoly output if $\delta > \delta^*$. So I do not allow partial collusion if monopoly output cannot be supported as subgame perfect equilibrium outcome.

Note that a smaller L represents a larger size of merger when N is given. So, a larger size of merger increases the scope of collusion if condition (2.1) is more easily satisfied for smaller L. In the appendix, I show that Nash reversion strategy and optimal punishment strategy [Abreu (1986)] satisfy this property. Put it differently, collusion gets more easily incentive compatible under these strategies as the number of firms decreases in the identical constant marginal cost setting. Given this result, I can assume that the threshold discount rate is strictly increasing in the number of firms ($f'(L) > 0$).

2.3.3 Merger's Profitability and Its Welfare Effect

Merger analysis of the identical constant marginal cost model also follows the S-S-R's framework. But I now consider the possibility that a merger changes firms' competitive behavior, so I need to analyze a merger's profitability and its welfare effect in all possible cases.

Framework of Merger Analysis

I will introduce some notations similar to the S-S-R for profitability analysis of merger. $\Pi^{pre}(N, M)$ denotes insiders' pre-merger joint profits each period when the insiders consist of $M + 1$ firms in an industry with N firms. $\Pi^{post}(N, M)$ represents each period's profits of the merged firm if the merger takes place among $M+1$ constituent firms. The incentive of merger function, denoted by $g(N, M)$, is defined as the increase in joint profits each period when a merger takes place among $M + 1$ insiders. So, by definition,

$$g(N, M) = \Pi^{post}(N, M) - \Pi^{pre}(N, M)$$

Then if each firm's per-period profit is given by $\Pi(x)$ in an x-firm equilibrium, I get

$$\begin{aligned} \Pi^{pre}(N, M) &= (M + 1) * \Pi(N) \\ \Pi^{post}(N, M) &= \Pi(N - M), \end{aligned}$$

so $g(N, M)$ comes to

$$g(N, M) = \Pi(N - M) - (M + 1) * \Pi(N) \tag{2.2}$$

Given the assumptions on demand, marginal cost and firms' competitive behavior, $\Pi(x) = \frac{1}{4x}$ if $\delta > f(x)$ while $\Pi(x) = \frac{1}{(x+1)^2}$ if $\delta \leq f(x)$.

In contrast, welfare effect of merger function, denoted by $S(N, M)$, is introduced for the welfare analysis of merger. $S(N, M)$ is defined as per-period increase in total surplus when $M + 1$ firms merge in an industry with N firms.

$$S(N, M) = TS(N, M) - TS(N) \tag{2.3}$$

Here, $TS(N, M)$ and $TS(N)$ represent post-merger total surplus and pre-merger total surplus in each period, respectively. So, each term is defined by

$$\begin{aligned} TS(N, M) &= CS(N, M) + (N - M) * \Pi(N - M) \\ TS(N) &= CS(N) + N * \Pi(N), \end{aligned}$$

where $CS(N, M)$ and $CS(N)$ represent post-merger and pre-merger consumer surplus in each period, respectively.

In terms of firms' competitive behavior post-merger, there are 3 possible cases depending on the number of firms before merger (N), the size of merger (M) and firms' discount rate (δ); (Case 1) firms compete Cournot post-merger for every $M \neq N - 1$, (Case 2) firms collude post-merger for every $M \neq 0$, and (Case 3) firms compete Cournot post-merger if merger size is less than threshold size ($M < M^*$) whereas they collude post-merger if merger size is equal to or larger than threshold size ($M \geq M^*$). In fact, (Case 2) is a special case of (Case 3) such that $M^* = 1$. For example, suppose $N = 10$ and Nash reversion strategy is used against the unilateral defection. (Case 1) is applied for $\delta \leq \frac{9}{17}$, (Case 2) for $\frac{25}{34} < \delta \leq \frac{121}{161}$, and (Case 3) for $\frac{9}{17} < \delta \leq \frac{25}{34}$. In (Case 3), M^* depends on the value of δ.[2]

Even though I consider the infinitely repeated game situation, $g(N, M)$ exactly reflects the private profitability of a merger to $M + 1$ insiders because $\frac{g(N,M)}{1-\delta}$ will be insiders' increase in discounted profit from the merger, which is proportional to $g(N, M)$. Then, it is enough to proceed the analysis with $g(N, M)$. The same holds for $S(N, M)$ in welfare analysis of merger.

Case 1. ***(S-S-R Model) :*** $\delta \leq f(2)$

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[2]If $\delta > \frac{121}{161}$ under our setting, firms are expected to collude pre-merger contrary to the assumption in this paper. In that case, it is easy to show that any merger is not profitable and does not affect social welfare.

This case happens when firms' common discount factor is lower than threshold discount rate under duopoly ($\delta \leq f(2)$). Since firms compete *à la* Cournot pre- and post-merger under any size of merger except the one to monopoly, this is the case where all the results of the S-S-R model can be applicable.

■ **Incentive of Merger** : $g_1(N,M)$, the incentive of merger function in (Case 1), yields

$$g_1(N,M) = \frac{1}{(N-M+1)^2} - \frac{M+1}{(N+1)^2} \tag{2.4}$$

To compare with other cases, the results of S-S-R are summarized[3]:

Claim 1. *[S-S-R] (a) A merger to form monopoly is profitable.*

(b) For any N, it is sufficient for a merger to be unprofitable that less than 80% of the firms merge.

A merger in (Case 1) is profitable if the concentration ratio of the merger $\alpha = \frac{M+1}{N}$ is greater than the threshold concentration ratio $\hat{\alpha}(N) \equiv \frac{\hat{M}+1}{N} \in [0.8, 1)$. Put differently, every merger combining $\hat{M}+1$ firms or more is profitable in (Case 1). I will call $\hat{M}$ as the threshold merger size under the S-S-R.

■ **Welfare Effect of Merger** : $S_1(N,M)$, the welfare effect function in (Case 1), becomes[4]

$$S_1(N,M) = \frac{N-M}{(N-M+1)} - \frac{N}{N+1} - \frac{1}{2}\left(\frac{N-M}{N-M+1}\right)^2 + \frac{1}{2}\left(\frac{N}{N+1}\right)^2 \tag{2.5}$$

Using this function, we can get the following result.

Claim 2. *[S-S-R] (a) Every merger decreases welfare. ($S_1(N,M) < 0$ for all $M \neq 0$)*

(b) Welfare decreases at a slower rate than the incentive of merger does around $M = 0$. $\left(\left|\frac{\partial g_1(N,0)}{\partial M}\right| \geq \left|\frac{\partial S_1(N,0)}{\partial M}\right|\right)$

A merger in the S-S-R model decreases welfare because it does not bring any cost saving with increasing the remaining firms' market power. But part (b) of this Claim implies that

[3]Refer to Salant, Switzer, and Reynolds (1983) p.191 $\sim$ p.195 for Claim 1 and Claim 2.

[4]Equation 2.5 is the same with the one in p.195 of Salant, Switzer, and Reynolds (1983).

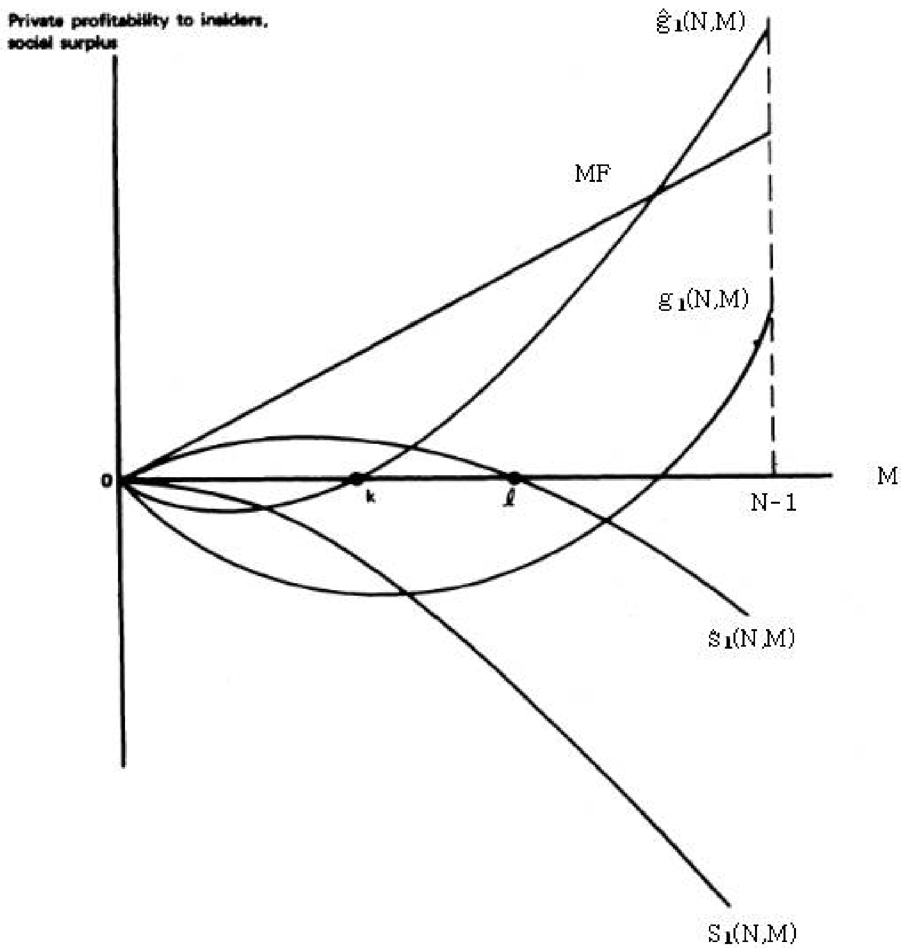

Figure 2.1: Merger's Profitability and Welfare Effect : Case 1 (S-S-R)

social loss is smaller than private loss when the size of merger is small enough. This is because outsider's profit gain outweighs consumers' loss when a merger is sufficiently small.

■ **Existence of Efficiency Gains** : Efficiency gains turns the incentive of merger function into $\hat{g}_1(N, M) = g_1(N, M) + M * F$. So, a merger that creates efficiency gains from elimination of fixed cost duplication may still cause losses depending on the parameter values F and M. Social gain from a merger, on the other hand, becomes $\hat{S}_1(N, M) = S_1(N, M) + M * F$. Since $\frac{\partial g_1(N,0)}{\partial M} < 0$, $\frac{\partial S_1(N,0)}{\partial M} < 0$, and $\left|\frac{\partial g_1(N,0)}{\partial M}\right| \geq \left|\frac{\partial S_1(N,0)}{\partial M}\right|$ for all $N \geq 2$, it is possible to select F so that $\hat{S}_1(N, M) > 0 > \hat{g}_1(N, M)$ for some M.[5] Figure 2.1, which is quoted from Salant, Switzer,

[5]Refer to the discussion in p.195 of Salant, Switzer, and Reynolds (1983).

and Reynolds (1983) (Figure 4. in pp.196), illustrates that a merger is privately unprofitable but socially desirable when the size of a merger (M) is less than k.

Case 2. $f(N-1) < \delta \leq f(N)$

This case happens when firms' common discount factor is lower than threshold discount rate pre-merger, but becomes higher post-merger even under 2-firm merger ($f(N-1) < \delta \leq f(N)$). So any size of merger turns firms' behavior into collusion in this case.

■ **Incentive of Merger** : $g_2(N, M)$, the incentive of merger function in (Case 2), yields

$$g_2(N, M) = \begin{cases} \frac{1}{4(N-M)} - \frac{M+1}{(N+1)^2} & if \quad M > 0 \\ 0 & if \quad M = 0 \end{cases} \tag{2.6}$$

Since $\lim\limits_{M \to 0^+} \frac{\partial g_2(N,M)}{\partial M} < 0$ and $g_2(N, M)$ is convex,[6] I can obtain the following results:

Claim 3. *(a) Every merger is profitable except $M = \frac{N-1}{2}$ while a merger is just break-even if $M = \frac{N-1}{2}$ ($g_2(N, M) > 0$ for all $M \neq \frac{N-1}{2}$, and $g_2(N, \frac{N-1}{2}) = 0$).*

(b) A merger to form monopoly is strictly more profitable than a merger between two firms. ($g_2(N, N-1) > g_2(N, 1)$ for all $N \geqslant 3$)

The proof of every result in this paper is provided in the appendix. Claim 3 (a) shows that any size of merger is at least break-even to the insiders and strictly profitable if $M \neq \frac{N-1}{2}$. So, the incentive of a merger dramatically increases when the merger is expected to change firms' competitive behavior from Cournot competition to collusion. Claim 3 (b) implies that a merger to monopoly is most profitable because $g_2(N, M)$ is convex and $g_2(N, \frac{N-1}{2}) = 0$.

■ **Welfare Effect of Merger** : The equilibrium price and quantity under Cournot competition with N firms are given by $P(N) = \frac{1}{N+1}$, $Q(N) = \frac{N}{N+1}$. So, consumer surplus amounts to $CS_2(N) = \frac{1}{2}\left(\frac{N}{N+1}\right)^2$, and total profit of the industry is $N * \Pi_2(N) = \frac{N}{(N+1)^2}$. Hence, $TS_2(N) = \frac{1}{2}\left(\frac{N}{N+1}\right)^2 + \frac{N}{(N+1)^2}$. Since $(N-M)$ firms collude post-merger in this case, the equilibrium price and quantity are given by $P(N-M) = \frac{1}{2}$, $Q(N-M) = \frac{1}{2}$. So, the post-merger

[6] $\lim\limits_{M \to 0^+} \frac{\partial g_2(N,M)}{\partial M} = \frac{1}{4N^2} - \frac{1}{(N+1)^2} < 0$ and $\frac{\partial^2 g_2}{\partial M^2} = \frac{1}{8(N-M)^3} > 0$

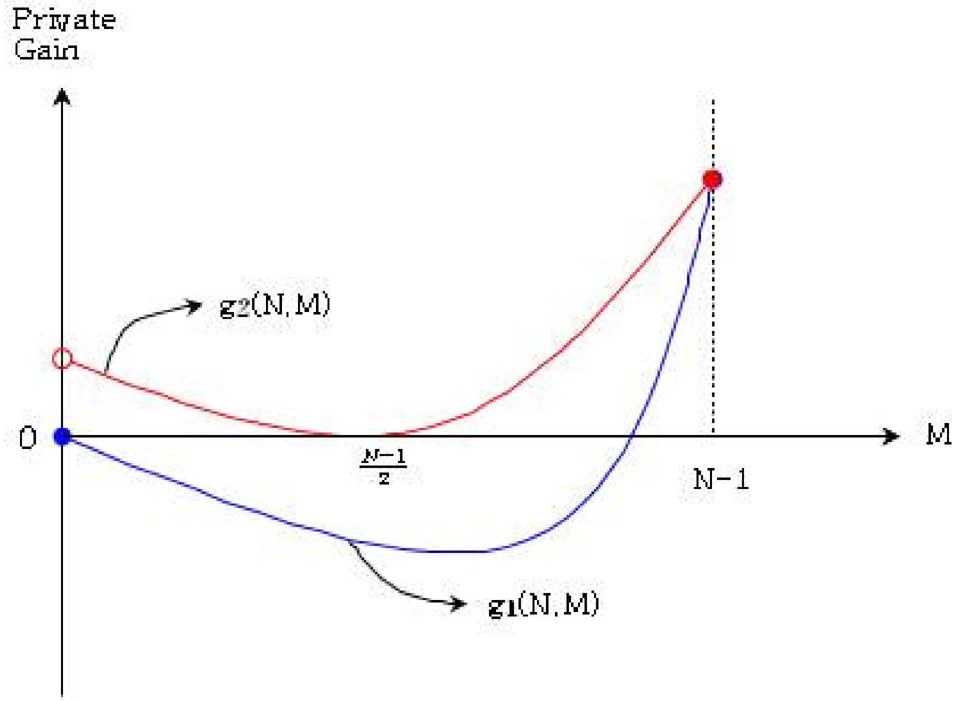

Figure 2.2: Merger's Profitability : Comparison

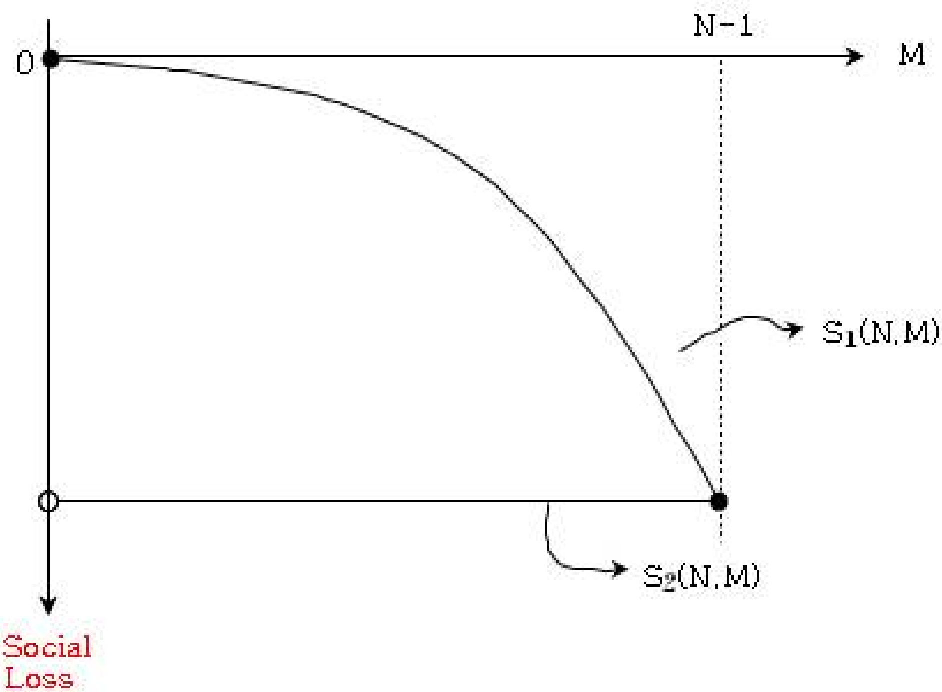

Figure 2.3: Merger's Welfare Effect : Comparison

total surplus, $TS_2(N,M)$, comes to $TS_2(N,M) = CS_2(N,M) + (N-M) * \Pi_2(N-M) = \frac{3}{8}$. Hence, $S_2(N,M)$, the welfare effect function in (Case 2), yields

$$S_2(N,M) = \begin{cases} 0 & if\ M = 0 \\ \frac{4-(N+1)^2}{8(N+1)^2} & if\ M \neq 0 \end{cases} \tag{2.7}$$

It is easy to see that $S_2(N,M) = \frac{4-(N+1)^2}{8(N+1)^2} < 0$ for all $N \geq 2$. So every size of merger causes social loss, and the amount of welfare loss does not depend on the size of merger. This is because the remaining firms collude for every M in this case. Comparing the incentive of merger and its welfare effect between (Case 1) and (Case 2), I can obtain the following result:

Corollary 1. *Given the number of firms and the size of a merger not forming monopoly (N,M),*
(a) a merger in (Case 2) is more profitable than one in (Case 1) $(g_1(N,M) < g_2(N,M))$;
(b) a merger in (Case 2) is socially more injurious than one in (Case 1) $(S_1(N,M) > S_2(N,M))$.

As expected, the private incentive to merge becomes higher and social welfare gets worse if Cournot competition turns to collusion after merger compared with the case that firms compete Cournot pre- and post-merger.

■ **Existence of Efficiency Gains** : If each firm has a fixed cost, the incentive of merger function comes to $\hat{g}_2(N,M) = g_2(N,M) + M * F$. So every size of merger is profitable because $g_2(N,M) \geq 0$ and $M * F > 0$ for all $M > 0$. The welfare effect function turns to $\hat{S}_2(N,M) = S_2(N,M) + M * F$. Note that $\hat{S}_2(N,M)$ and $\hat{g}_2(N,M)$ are largest when $M = (N-1)$. So every size of merger is socially injurious if $F \leq \frac{(N+1)^2-4}{8(N-1)(N+1)^2}$, whereas a merger to monopoly is socially most desirable if $F > \frac{(N+1)^2-4}{8(N-1)(N+1)^2}$.

Case 3. $f(N-M^*) < \delta \leq f(N-M^*+1)$ ***for some*** $M^* \geq 2$

This case happens when there is a threshold size of merger $M^* \geq 2$ such that firms collude post-merger if merger size is greater than or equal to M^*. Given Cournot competition pre-merger, firms' post-merger competitive behavior depends on the discount rate (δ), the size of merger (M), and the pre-merger number of firms in the industry (N).

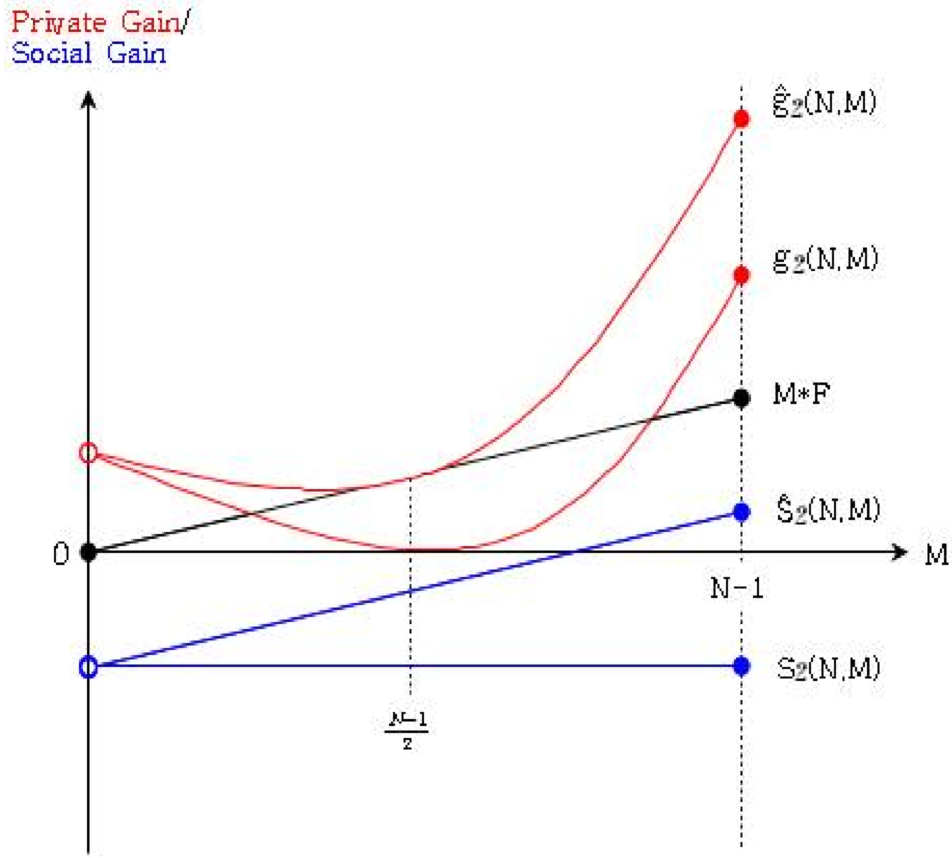

Figure 2.4: Merger's Profitability and Welfare Effect : Case 2

■ **Incentive of Merger** : Without loss, suppose $2 \leq M^* \leq N-2$.[7] For $M < M^*$, the post-merger competition is still Cournot. So, the gains of merging firms are $g_1(N,M)$ in this area. For $M \geq M^*$, firms collude post-merger. So, in this area, the gains of merging firms becomes $g_2(N,M)$. Hence, $g_3(N,M)$, the incentive of merger function in (Case 3), comes to

$$g_3(N,M) = \begin{cases} g_1(N,M) & if \quad M < M^* \\ g_2(N,M) & if \quad M \geq M^* \end{cases} \tag{2.8}$$

Note that $g_3(N,M)$ is convex both in $[0, M^*)$ and $[M^*, N-1]$. A merger turns from privately unprofitable to profitable to the merging firms at M^* if $g_1(N,M^*) < 0$, and the merger becomes more profitable at M^* if $g_1(N,M^*) \geq 0$ due to the change in competitive behavior. Formally, define M^{**} be such that $g_3(N,M^{**}) \geq 0$ and $g_3(N,M) < 0$ for all $M \in (0, M^{**})$. Similar to (Case 1), M^{**} can be called as the threshold merger size in (Case 3). I know that $M^{**} > 1$ because $g_3(N,1) = g_1(N,1) < 0$ for all $N \geq 4$ in this case. Then, I obtain the following result:

Proposition 1. *Threshold merger size in (Case 3) is not greater than that in the S-S-R.* $\left(M^{**} \leq \hat{M}\right)$

This result shows that collusive effect reduces the minimum size of a profitable merger. Hence, if $\alpha^* \equiv \frac{M^*+1}{N}$ is less than 80%, Claim 3 and Proposition 1 imply that the minimum concentration ratio also falls below 80%.

■ **Welfare Effect of Merger** : For the same reason, social gains from a merger are $S_1(N,M)$ if $M < M^*$, and $S_2(N,M)$ if $M \geqslant M^*$. So $S_3(N,M)$, the welfare effect function in (Case 3), can be derived as

$$S_3(N,M) = \begin{cases} S_1(N,M) & if \quad M < M^* \\ S_2(N,M) & if \quad M \geqslant M^* \end{cases} \tag{2.9}$$

So, social welfare always worsens for any size of merger ($S_3(N,M) < 0$ for all $M > 0$), and it decreases discontinuously at M^*. The social loss from a merger is the same in collusion regime regardless of the size of merger.

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[7](Case 2) can be applied if $M^* = 1$, while (Case 1) is applied if $M^* = N-1$. This case also requires that N is greater than or equal to 4.

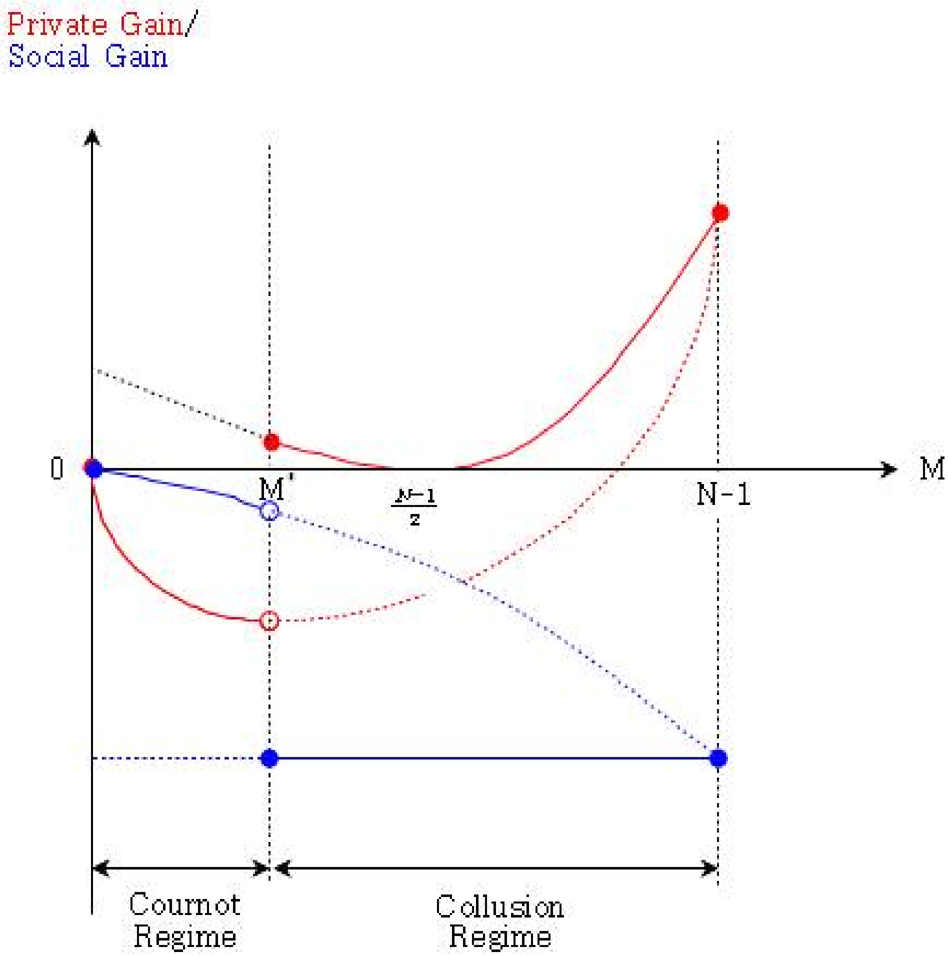


Figure 2.5: Merger's Profitability and Welfare Effect : Case 3 (No Efficiency Gain)

The regime change at M^* is depicted in Figure 2.5. Social gains from a merger are drawn by a blue line while its private profitability is drawn by a red line. They follows (Case 1) for $M < M^*$, (Case 2) for $M \geqslant M^*$, and there is a break at M^*. So a merger to monopoly is the best for insiders, but socially most harmful.

■ **Existence of Efficiency Gains** : When there exist efficiency gains from merger, private gain of merging firms turns to $\hat{g}_3(N,M) = g_3(N,M) + M*F$ and social surplus from a merger is provided by $\hat{S}_3(N,M) = S_3(N,M) + M*F$. Note that $\hat{S}_3(N,M)$ is monotonically increasing in M for collusion area ($M \geq M^*$). Since $M^* \geq 2$, I know that $\hat{g}_3(N,1) = \frac{1}{N^2} - \frac{2}{(N+1)^2} + F$ and $\hat{S}_3(N,1) = \frac{N-1}{N} - \frac{N}{N+1} - \frac{1}{2}\left(\frac{N-1}{N}\right)^2 + \frac{1}{2}\left(\frac{N}{N+1}\right)^2 + F$. Hence, depending on the F value, there can be three possible cases in terms of the signs of $\hat{g}_3(N,1)$ and $\hat{S}_3(N,1)$.

(1) $\hat{g}_3(N,1) \geq 0$, $\hat{S}_3(N,1) > 0$ if $F \geq \frac{2}{(N+1)^2} - \frac{1}{N^2}$
(2) $\hat{g}_3(N,1) < 0$, $\hat{S}_3(N,1) \geq 0$ if $-\frac{N-1}{N} + \frac{N}{N+1} + \frac{1}{2}\left(\frac{N-1}{N}\right)^2 - \frac{1}{2}\left(\frac{N}{N+1}\right)^2 \leq F < \frac{2}{(N+1)^2} - \frac{1}{N^2}$
(3) $\hat{g}_3(N,1) < 0$, $\hat{S}_3(N,1) < 0$ if $F < -\frac{N-1}{N} + \frac{N}{N+1} + \frac{1}{2}\left(\frac{N-1}{N}\right)^2 - \frac{1}{2}\left(\frac{N}{N+1}\right)^2$

In the appendix, I show the following: the merger of optimal size is welfare increasing and profitable if efficiency gain is large ($F \geq \frac{2}{(N+1)^2} - \frac{1}{N^2}$); every profitable merger is socially injurious if efficiency gain is small ($F < -\frac{N-1}{N} + \frac{N}{N+1} + \frac{1}{2}\left(\frac{N-1}{N}\right)^2 - \frac{1}{2}\left(\frac{N}{N+1}\right)^2$); the optimal size of merger may be welfare increasing but unprofitable if efficiency gain is intermediate.

Summary of Merger Analysis

Table 2.2 summarizes the merger analysis in the identical constant marginal cost model. This table is made on the assumption that there is no fixed cost. If a merger does not change firms competitive behavior as in (Case 1), the merger is usually not profitable. But the setting of the paper makes it easier for firms to collude after merger. If this collusive effect works in a merger as in (Case 2) and (Case 3), the merger's profitability improves dramatically. It happens because the merged entity can obtain monopoly profit divided by the number of post-merger firms under any merger having collusive effect. In terms of welfare effect, every merger harms social welfare under this model because it reduces the aggregate output without any efficiency gains. This negative effect on social welfare is larger when firms collude post-merger.

Table 2.2: Profitability and Welfare Effect of Merger

Type of Merger	Profitability	Welfare Effect	Note
Case 1	loss if $\frac{M+1}{N} < 0.8$	negative	S-S-R model
Case 2	benefit	negative*	worse than S-S-R in welfare
Case 3	benefit if $M \geq M^*$	negative*	worse than S-S-R if $M \geq M^*$

2.4 Asymmetric Increasing Marginal Cost Model

As I discussed in the introduction, the merger concept of the S-S-R and the identical constant marginal cost model is not realistic. In this section, I build up the asymmetric increasing marginal cost model where a merged entity has bigger size or better technology than its constituent firms. Having this model, I perform the merger analysis similar to Section 2.3. Since a merger in this model may create cost saving from synergies, merger analysis includes its price effect as well.

2.4.1 Model

This model also basically follows the S-S-R model except that the cost function is modified.

■ **Demand and Supply** : I keep using linear demand curve $P = 1 - Q$. There are N firms before merger. In order to deal with cost asymmetry, I will introduce cost function similar to the one used by Perry and Porter (1985), McAfee and Williams (1992), Rothschild (1999) and etc. :

$$C^i(q_i) = \frac{q_i^2}{2e_i},\ e_i > 0 \tag{2.10}$$

So the marginal cost of firm i is a linear function $MC^i(q_i) = \frac{q_i}{e_i}$, and increases as output increases in this setting. Here, e_i can be seen as efficiency coefficient or technology-adjusted capital stock, and the S-S-R setting is a specific case that e_i is infinity. In order to focus on cost saving effect from reallocation or synergy effect of merger, I assume that there is no fixed cost in this model.

This cost function is slightly different from the Perry-Porter model in that it uses efficiency coefficient instead of capital stock. So it is possible that two firms with the same capital stock may have different cost functions under this setting. I made this modification in order to analyze the incentive and welfare effect of a merger that creates synergy effect.

■ **Merger Scenario** : I will consider a 2-firm merger between firm 1 and firm 2 in this section. Two types of cost saving are analyzed : rationalization and synergy effect. Following the definition of Farrell and Shapiro (1990), rationalization model deals with the case that "the combined entity can better allocate outputs across facilities but its production possibilities are no different from those of the insiders (jointly) before the merger". In fact, the merger concept in the Perry-Porter or McAfee-Williams model exactly coincides with rationalization. On the other hand, synergy effect model looks at the situation where the merged firm's production possibilities are better than those of the constituent firms. Economies of scale or learning effect can be sources of synergy effect.

■ **Firms' Competitive Behavior** : In this model, I assume firms compete *à la* Cournot pre- and post-merger like the S-S-R model. So I rule out the collusive effect as a motive of merger. There is no entry nor exit in this section.

I can derive Cournot-Nash equilibrium when there are N firms. Each firm i solves the following profit maximization problem:

$$\pi^i(q) = (1 - \sum_{k=1}^{N} q_k)q_i - \frac{q_i^2}{2e_i}, \ where \ q = (q_1, \cdots, q_N) \tag{2.11}$$

Let me denote $\lambda_k = \frac{e_k}{1+e_k}$. Then, Cournot-Nash equilibrium output and price are given by

$$Q_N^* = \frac{\sum_{k=1}^N \lambda_k}{1 + \sum_{k=1}^N \lambda_k} \tag{2.12}$$

$$P_N^* = \frac{1}{1 + \sum_{k=1}^N \lambda_k} \tag{2.13}$$

and each firm i's output and market share amount to

$$q_i^* = \frac{\lambda_i}{1 + \sum_{k=1}^N \lambda_k} \tag{2.14}$$

$$s_i^* = \frac{\lambda_i}{\sum_{k=1}^N \lambda_k} \tag{2.15}$$

As e_i gets bigger, firm i's market share becomes higher. Then, each firm's profit yields

$$\pi_i^* = \frac{\lambda_i(1+\lambda_i)}{2(1+\sum_{k=1}^N \lambda_k)^2} \tag{2.16}$$

2.4.2 Analysis of Merger with Rationalization Effect

Having the equilibrium output, price and profit of each firm, I can analyze the incentive and welfare effect of a merger with rationalization. To this end, I need to derive cost function of a merged entity. In order to minimize its cost, the merged firm solves

$$\min_{q_1,q_2} \frac{q_1^2}{2e_1} + \frac{q_2^2}{2e_2} \quad \textit{subject to } q_M = q_1 + q_2$$

So, its cost function becomes $C^{1+2}(q_M) = \frac{q_M^2}{2(e_1+e_2)}$. Note that the merged firm's efficiency is equal to the sum of its constituent firms' efficiency when a merger brings cost saving only from rationalization.

Incentive of Merger

Suppose that firm 1 and 2 merge, then a merged firm's profit at post-merger Cournot-Nash equilibrium, denoted by π_M^{1+2}, would be

$$\pi_M^{1+2} = \frac{\lambda_{1+2}(1+\lambda_{1+2})}{2(1+\lambda_{1+2}+\sum_{k=3}^N \lambda_k)^2}, \text{ where } \lambda_{1+2} = \frac{e_1+e_2}{1+e_1+e_2} \tag{2.17}$$

Then, the incentive of merger function, $g_R^{1+2}(e)$, is defined by using equation (2.16) and (2.17).

$$\begin{aligned} g_R^{1+2}(e) &= \pi_M^{1+2}(e) - (\pi_1^*(e) + \pi_2^*(e)), \ \textit{where } e = (e_1, \cdots, e_N) \\ &= \frac{\lambda_{1+2}(1+\lambda_{1+2})}{2(1+\lambda_{1+2}+\sum_{k=3}^N \lambda_k)^2} - \frac{\lambda_1(1+\lambda_1)+\lambda_2(1+\lambda_2)}{2(1+\sum_{k=1}^N \lambda_k)^2} \end{aligned} \tag{2.18}$$

So merger incentive depends not only on the merger participants' efficiency (e_1, e_2) but also on outsider's efficiency $(e_3, \cdots, e_N)$.

To see the detail of merger's profitability, let me consider a simplest case where there are

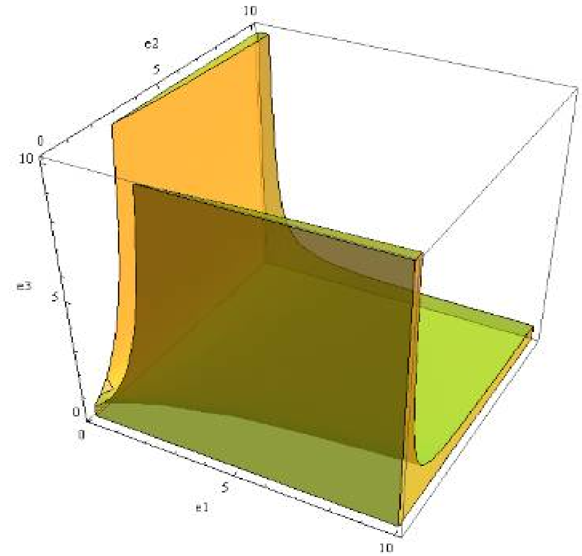


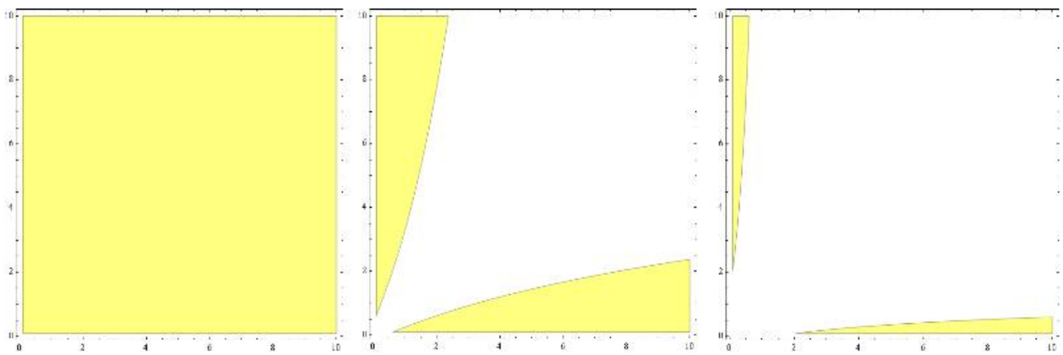
Figure 2.6: Incentive of Merger - Rationalization

3 firms pre-merger ($N = 3$), then $g_R^{1+2}(e)$ yields

$$g_R^{1+2}(e_1, e_2, e_3) = \pi_M^{1+2}(e_1, e_2, e_3) - (\pi_1^*(e_1, e_2, e_3) + \pi_2^*(e_1, e_2, e_3)) \tag{2.19}$$

Figure 2.6 shows the space where a merger is profitable $\left(g_R^{1+2}(e_1, e_2, e_3) > 0\right)$ when each firm's efficiency takes a value between 0.1 and 10 $\left((e_1, e_2, e_3) \in [0.1, 10]^3\right)$. Recall that any merger between 2 firms in triopoly market is not profitable under the S-S-R model because $g_1(3, 1) = \frac{1}{9} - \frac{1}{8} = -\frac{1}{72}$ from equation (2.4). So, this example shows that "merger paradox" does not always hold when cost saving from reallocation is possible.

Given that a merger may be profitable, the important question is how outsider's or insiders' efficiency affects the incentive of merger. We can get some intuitions from Figure 2.7. Panel (A) shows that any merger with rationalization is profitable when outsider's efficiency level is low ($e_3 = .3$). If outsider is efficient enough as in panel (B) and (C), however, there should be an asymmetry in efficiency coefficients between constituent firms so that a merger is profitable ($e_3 = 1$ or $e_3 = 5$). Panel (B) and (C) also implies that the required asymmetry in efficiency level gets bigger as outsider is more efficient. Claim 4 provides the formal relation between outsider's efficiency and a merger's profitability.



(A) $e_3 = 0.3$ (B) $e_3 = 1$ (C) $e_3 = 5$

Figure 2.7: Outsider's Efficiency and Incentive of Merger

Claim 4. *Suppose that a merger is at least break-even at $e = (e_1, \cdots, e_N)$. If any outsider gets more efficient, the merger becomes less profitable or unprofitable. (If $g_R^{1+2}(e) \geq 0$, then $\frac{\partial}{\partial e_j} g_R^{1+2}(e) < 0$ for $j \geq 3$.)*[8]

It is useful to look at outsider's response to a merger in order to understand this result. Using equation (2.13), (2.14) and (2.16), I can get the post-merger equilibrium price, outsider's output and profit and can compare them with the equivalent pre-merger values.

$$
\begin{aligned}
P_N^{1+2} &= \frac{1}{1 + \lambda_{1+2} + \sum_{k=3}^{N} \lambda_k} > \frac{1}{1 + \lambda_1 + \lambda_2 + \sum_{k=3}^{N} \lambda_k} = P_N^* \\
q_o^{1+2} &= \frac{\lambda_o}{1 + \lambda_{1+2} + \sum_{k=3}^{N} \lambda_k} > \frac{\lambda_o}{1 + \sum_{k=1}^{N} \lambda_k} = q_o^* \\
\pi_o^{1+2} &= \frac{\lambda_o (1 + \lambda_o)}{2(1 + \lambda_{1+2} + \sum_{k=3}^{N} \lambda_k)^2} > \frac{\lambda_o (1 + \lambda_o)}{2(1 + \lambda_1 + \lambda_2 + \sum_{k=3}^{N} \lambda_k)^2} = \pi_o^*
\end{aligned}
$$

The inequalities comes from $\lambda_1 + \lambda_2 > \lambda_{1+2}$. Since the equilibrium price and outsider's output increase after a merger, the merged firm's output q_M^{1+2} should be less than the sum of its constituent firms' pre-merger output $q_1^* + q_2^*$. It depends on outsider's efficiency how much the merged firm's output decreases. To see that, note that $\lambda_i = -\frac{dq_i}{dQ}$ from the first order condition of (2.11). So, λ_i represents firm i's responsiveness with respect to the change in

[8] A sufficient condition ($g_R^{1+2}(e) \geq 0$) is required to establish this result analytically, but numerically I could check that this condition is not necessary to get the result when $N = 3$. See the appendix.

market equilibrium output. Therefore, the more efficient an outsider is, the more it increases its output after merger because higher e_i is equivalent to higher λ_i. Big reaction of an efficient outsider, in turn, harms the merged firm's profitability.

Next, look at the relationship between cost asymmetry of constituent firms and the merger's profitability. Note first that marginal costs are different among firms. From equation (2.14) and cost function, firm i's marginal cost is given by $MC^i(q_i^*) = \left[(1+e_i)\left(1+\sum_{k=1}^N \lambda_k\right)\right]^{-1}$. So more efficient firm produces at lower marginal cost in Nash equilibrium. When 2 firms are combined by merger, the merged firm makes the marginal costs of these two facilities equal through reallocation of output in order to minimize its cost. Larger difference in merging parties' efficiencies is equivalent to larger difference in their marginal costs at pre-merger Nash equilibrium. So the merged entity can save more cost through reallocation. Formally, fix $e_1 + e_2 = e_s$ and let $e_1 = (1-\nu)e_s$ and $e_2 = \nu e_s$ for $\nu \in [0.5, 1)$. Then asymmetry between firm 1 and firm 2 comes to $\frac{e_2}{e_1} = \frac{\nu}{1-\nu}$, so larger ν is equivalent to bigger asymmetry. Using (e_s, ν), I can redefine the incentive of merger function as $g_R^{1+2}(e_s, e_3, \cdots, e_N, \nu)$. Then, I could show the following numerical result in the appendix: if a merger under $N = 3$ is not profitable, increase in asymmetry between insiders improves the incentive of merger $\left(if\ g_R^{1+2}(e_s, e_3, \nu) \leq 0,\ \frac{\partial}{\partial \nu} g_R^{1+2}(e_s, e_3, \nu) > 0\ for\ all\ (e_s, e_3) \in R_+^2\right)$.

Welfare Effect of a Merger

As shown in previous subsection, a merger with rationalization increases price and outsiders' output, while the merged firm's output is lower than the sum of its constituent firms' pre-merger output. So, this type of merger increases outsider's profit and decreases consumer surplus. But merger's profitability is not decisive. The merger's overall effect on aggregate welfare depends on the relative magnitude of these 3 effects.

Since pre- and post-merger price and each firm's profit have analytical solutions thanks to linear demand and marginal cost, welfare analysis can be performed directly. Given the characterization of pre- and post-merger Nash equilibrium, welfare effect of a merger yields

$$w_R^{1+2}(e) = g_R^{1+2}(e) + \sum_{k=3}^N (\pi_k^{1+2}(e) - \pi_k^*(e)) - \int_{P_N^*(e)}^{P_N^{1+2}(e)} (1-P)dP \tag{2.20}$$

So the welfare effect of a merger depends on both the merger participants' efficiency (e_1, e_2) and outsiders' efficiency $(e_3, \cdots, e_N)$ as merger's profitability does. It is not surprising because firms' interaction affects consumer surplus and each firm's profit in oligopoly market. With equation (2.20), I can check whether a merger is welfare-enhancing or not when $e = (e_1, \cdots, e_N)$ is given. Moreover, e can be identified from $\frac{e_i}{1+e_i} = \frac{q_i^*}{P_N^*}$ even when it is unobservable. Using equation (2.13), (2.14), (2.20) with cost function, we can obtain the necessary and sufficient condition for welfare-increasing merger[9]

$$\sum_{k=1}^{N} q_k^* MC^k(q_k^*) - q_M^{1+2} MC^{1+2}(q_M^{1+2}) - \sum_{k=3}^{N} q_k^{1+2} MC^k(q_k^{1+2}) > (P_N^{1+2})^2 - (P_N^*)^2 \quad (2.21)$$

This condition says that the decrease in output-weighted marginal cost outweighs the increase in square of price in welfare-increasing merger. Because of the linearity of demand and marginal cost, half of left-hand side in condition (2.21) is equal to the decrease in total cost of the industry from the merger whereas half of right-hand side represents decrease in total revenue and consumer surplus. So, this condition requires that the industry's profit increase from output rationalization outweighs decrease in consumer surplus under welfare-increasing merger.

In order to see when this condition is satisfied in more detail, revisit the example where $N = 3$. Then, I can check the welfare effect of merger in the cube $e \in [0.1, 10]^3$ using equation (2.20). Figure 2.8 shows the space where a merger increases welfare $(w_R^{1+2}(e_1, e_2, e_3) > 0)$. This picture illustrates that a merger is more likely to increase social welfare if joint market share of merger participants is small and the outsider has the largest market share. Higher social welfare mainly comes from output reallocation between insiders and outsider in this case. Firm 3's output increases after merger whereas aggregate output of firm 1 and 2 decreases. Since firm i's marginal cost is equal to $\frac{1}{(1+e_i)(1+\lambda_1+\lambda_2+\lambda_3)}$ in Nash equilibrium before merger, firm 3's marginal cost is lowest if e_3 is largest. Moreover, since larger e_3 implies larger λ_3, firm 3 responds more to the decrease in aggregate output from merger. Therefore, a merger between small firms enables efficient outsider to produce more and relatively inefficient merged

[9] See the appendix for the derivation of condition (2.21).

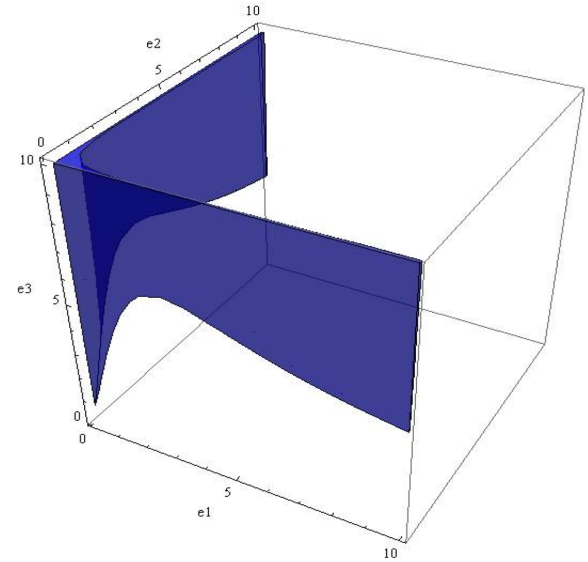


Figure 2.8: Welfare Effect of Merger - Rationalization

entity to produce less, which makes it possible for social welfare to increase.[10]

On the other hand, Figure 2.8 shows that there is another type of welfare-enhancing merger, which combines a big firm and a small firm given the presence of a sufficiently efficient outsider. Output reallocation between insiders can be an additional source of higher welfare in this case. It happens because a merged firm can save its cost by increasing the output of the efficient participant and decreasing the output of the inefficient participant. So, the merger might enhance social welfare even when the efficient participant's initial market share is larger than the outsider's market share.

As shown in panel (A) of Figure 2.9, a merger between firms with small market share is rarely privately profitable although this merger may increase social welfare. So the second feature of the S-S-R model may occur in a merger with rationalization. Since this kind of merger cannot benefit from cost asymmetry between participants by much, cost saving effect from

[10]This is a different observation from the argument that a merger is more likely to be welfare enhancing if the non-merging firms are more concentrated. [Farrell and Shapiro (1990, 1991), Werden (1991), and McAfee and Williams (1992)] In this example, the rest of the industry is completely concentrated for all parameter value of e_3 because there is only one firm out of merger.

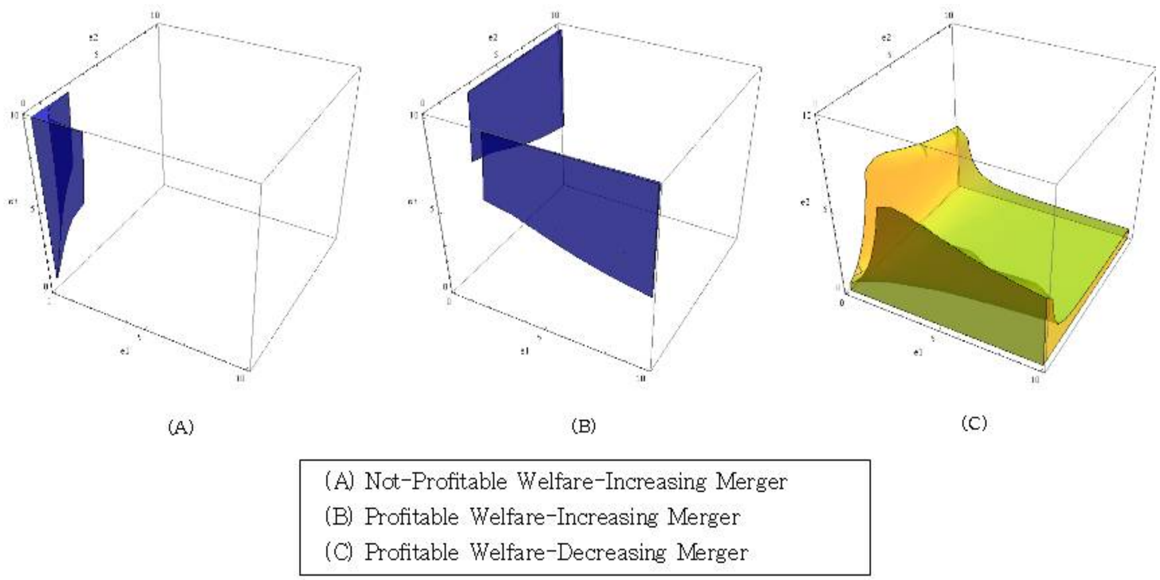


Figure 2.9: Profitability and Welfare Effect of Merger with Rationalization

reallocation is restricted for the merged entity. Moreover, the output response of efficient outsider is big, which harms the merged firm's profitability. So it is unlikely for a merger between small inefficient firms to occur in the market. In contrast, a merger is privately profitable and welfare enhancing when there is a sufficiently efficient outsider and a big asymmetry in cost efficiency between participants as is illustrated in panel (B). Asymmetry of insiders improves both profitability and welfare effect of merger while outsider's response improves welfare but hurts merger's profitability. Overall, the cost saving effect from big asymmetry outweighs profit loss due to outsider's output increase in this case. Panel (C) in Figure 2.9 shows the space where a merger is profitable but welfare-decreasing. This is the type of merger which antitrust policy has to concern about because it is likely to occur but socially undesirable.

The fact that the welfare effect of a merger depends on outsider's efficiency level and insiders' asymmetry has some policy implication as well. A merger with the same efficiency combination (e_1, e_2) of participants might be either socially beneficial or harmful depending on outsider's efficiency level. Similarly, a merger with the same post-merger efficiency $e_s = e_1 + e_2$ might be either socially beneficial or harmful depending on asymmetry level and e_3.

2.4.3 Analysis of Merger with Synergy Effect

Merger models that I analyzed so far have adverse effects on consumer surplus because the equilibrium price increases after merger. It is quite a natural result from Proposition 2 of Farrell-Shapiro, which proves that a merger without synergy effect causes the equilibrium price to rise. Now I will move the scope of analysis to the type of merger creating synergy effect. For that purpose, I let the merged firm's cost function as $C^{1+2}(q_M) = \frac{q_M^2}{2e_M}$ such that $e_M > e_1 + e_2$ holds. Here, e_M represents the merged firm's efficiency level.

Incentive of Merger

Given the merged firm's cost function, its profit at post-merger Cournot-Nash equilibrium would be

$$\pi_M^{1+2} = \frac{\lambda_M(1+\lambda_M)}{2(1+\lambda_M+\sum_{k=3}^{N}\lambda_k)^2}, \text{ where } \lambda_M = \frac{e_M}{1+e_M} \tag{2.22}$$

Then, using arguments similar to those in the rationalization case, the incentive of merger function, denoted by $g_S^{1+2}(e, e_M)$, becomes

$$\begin{aligned} g_S^{1+2}(e,e_M) &= \pi_M^{1+2}(e_M, e_3, \cdots, e_N) - (\pi_1^*(e) + \pi_2^*(e)), \;\; where\; e = (e_1, \cdots, e_N) \\ &= \frac{\lambda_M(1+\lambda_M)}{2(1+\lambda_M+\sum_{k=3}^{N}\lambda_k)^2} - \frac{\lambda_1(1+\lambda_1)+\lambda_2(1+\lambda_2)}{2(1+\sum_{k=1}^{N}\lambda_k)^2} \end{aligned} \tag{2.23}$$

Now the incentive of merger depends on the magnitude of synergy effect besides the insiders' and outsider's efficiency level. Then, the relationship between the magnitude of synergy effect and the incentive of merger can be derived.

Claim 5. *A merger's profitability improves as synergies get stronger.*

$g_S^{1+2}(e, e_M) > g_R^{1+2}(e)$ is immediate from this Claim. I can obtain some economic rationale of this result from the comparison between a merged firm's output in a rationalization merger, denoted by q_M^{1+2}, and that in the equivalent merger with synergies, denoted by $q_M^{1+2}(e_M)$. From equation (2.14), q_M^{1+2} and $q_M^{1+2}(e_M)$ are given by

$$q_M^{1+2} = \frac{\lambda_{1+2}}{1+\lambda_{1+2}+\sum_{k=3}^{N}\lambda_k} < \frac{\lambda_M}{1+\lambda_M+\sum_{k=3}^{N}\lambda_k} = q_M^{1+2}(e_M)$$

The inequality comes from $\lambda_M > \lambda_{1+2}$. The equilibrium output of the merged firm becomes larger as synergy effect gets stronger. In contrast, outsider's output in a rationalization merger is higher than that in an equivalent synergy effect merger $\left(q_o^{1+2} > q_o^{1+2}(e_M)\right)$. Hence, the presence of synergy effect restricts the amount of outsider's output response after merger, which in turn improves the merger's profitability. For the same reason, outsider's post-merger profit, or equivalently "free rider's problem" proposed by J.Stigler, gets smaller with the stronger synergy.

$$\pi_o^{1+2} = \frac{\lambda_o(1+\lambda_o)}{2(1+\lambda_{1+2}+\sum_{k=3}^N \lambda_k)^2} > \frac{\lambda_o(1+\lambda_o)}{2(1+\lambda_M+\sum_{k=3}^N \lambda_k)^2} = \pi_o^{1+2}(e_M) \tag{2.24}$$

Welfare Effect of Merger

The post-merger equilibrium price is given by

$$P_N^{1+2}(e_M) = \frac{1}{1+\lambda_M+\sum_{k=3}^N \lambda_k} \tag{2.25}$$

While the effect of merger on consumer and outsider is not decisive, equation (2.24) and (2.25) imply that higher synergy lowers post-merger equilibrium price and outsider's profit. Given these equations, welfare effect of a merger with synergy effect yields

$$w_S^{1+2}(e,e_M) = g_S^{1+2}(e,e_M) + \sum_{k=3}^N (\pi_k^{1+2}(e_M) - \pi_k^*(e)) - \int_{P_N^*(e)}^{P_N^{1+2}(e_M)} (1-P)dP \tag{2.26}$$

As in the incentive of merger, the welfare effect depends on the magnitude of synergy effect in addition to the insiders' and outsider's efficiency level. Using equation (2.13), (2.14) and cost function, I can transform equation (2.26) into the necessary sufficient condition for welfare-increasing merger ($w_S^{1+2}(e,e_M) > 0$).

$$\begin{aligned}\sum_{k=1}^N q_k^* MC^k(q_k^*) - q_M^{1+2}(e_M)MC^{1+2}(q_M^{1+2}(e_M)) - \sum_{k=3}^N q_k^{1+2}(e_M)MC^k(q_k^{1+2}(e_M)) \\ > (P_N^{1+2}(e_M))^2 - (P_N^*)^2 \end{aligned} \tag{2.27}$$

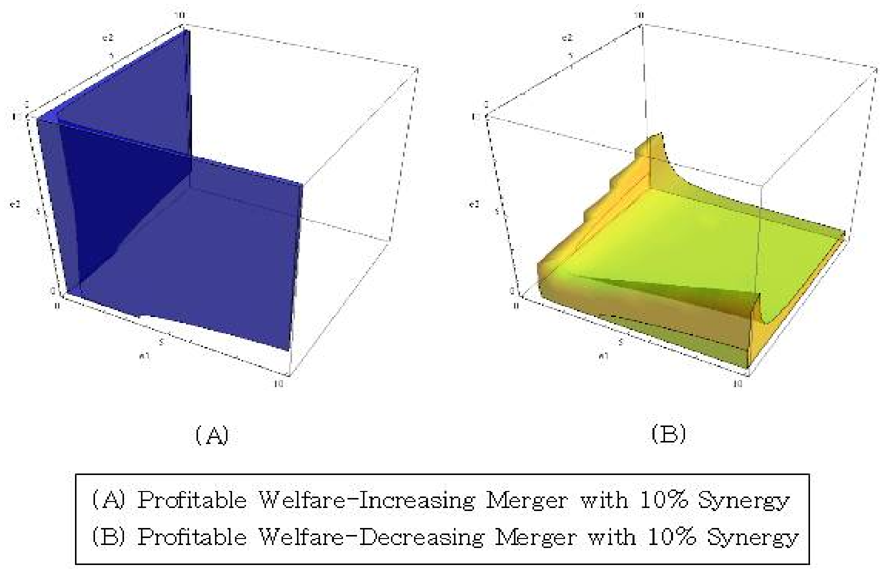


Figure 2.10: Welfare Effect of Profitable Merger - Synergy

Condition (2.27) is similar to condition (2.21), but each term ($q_M^{1+2}(e_M)$, $MC^{1+2}(q_M^{1+2}(e_M))$, $q_k^{1+2}(e_M)$, $MC^k(q_k^{1+2}(e_M))$ and $P_N^{1+2}(e_M)$) has a different value from rationalization case and varies depending on the level of synergy. Using equation (2.26), we can derive the relationship between the magnitude of synergy effect and the welfare effect of merger.

Claim 6. *A merger's welfare effect improves as synergies get stronger if and only if e_M satisfies*

$$e_M > \frac{-1 - 2\sum_{k=3}^{N} \frac{\lambda_k^2}{e_k} + \sum_{k=3}^{N} \lambda_k}{4 + 2\sum_{k=3}^{N} \frac{\lambda_k^2}{e_k} + \sum_{k=3}^{N} \lambda_k} \tag{2.28}$$

Claim 6 shows that stronger synergy always improves merger's welfare effect for N = 3, and it does for N $\geq$ 4 unless e_M is too small. In order to compare with rationalization type merger, let me consider the case of $N = 3$ again. Since synergy effect improves both a merger's profitability and welfare effect in this case, it expands the scope for privately profitable and socially desirable mergers.

Suppose 10% of synergy effect (i.e. $e_M = 1.1 * (e_1 + e_2)$). The shaded space in panel (A) of Figure 2.10 plots the space where a merger with this amount of synergy effect is profitable and welfare-enhancing for $e \in [0.1, 10]^3$. It encompasses all the relevant space in panel (B) of Figure

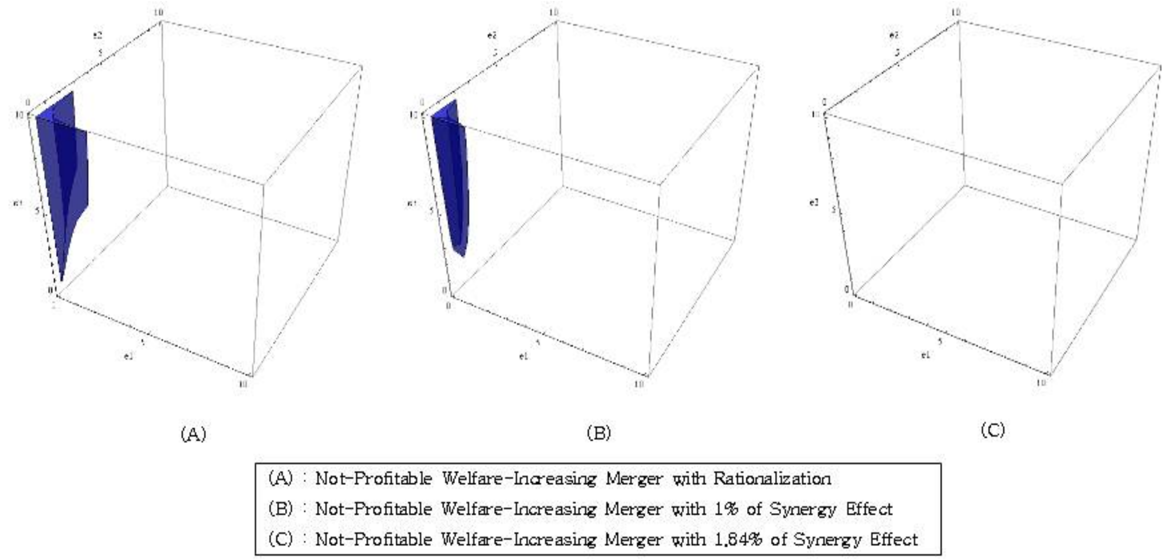


Figure 2.11: Synergy Effect and Profitability of Welfare-Increasing Merger

2.9, so mergers with synergy are profitable and welfare-enhancing in wider range. In fact, the blue shape includes all the unprofitable welfare-increasing mergers without synergy (panel (A) of Figure 2.9) as well. The profitability of a merger with synergy improves not only because the merged entity can save its cost from both reallocation and synergy, but also because the enhanced productivity of the merged firm restricts outsider's output increase post-merger.

It is also worthwhile to note that there still exist profitable welfare-decreasing mergers with 10% synergy from panel (B) in Figure 2.10. This yellow shape do not disappear even with stronger synergy effect, say 100%. So the presence of strong synergy effect is not sufficient for welfare to increase after merger.

Another interesting question is how much synergy is required for a welfare-increasing merger to be profitable for merging parties. Figure 2.11 answers this question in my example. Panel (A) in Figure 2.11 plots the space of unprofitable welfare-increasing merger when there is no synergy. This shape gets smaller in panel (B) with 1% of synergy, and eventually disappears with about 1.84% synergy effect or above, as shown in panel (C). In other words, a welfare-increasing merger with 1.84% synergy or above is always privately profitable. But the inverse is not true; even a profitable merger with 10% synergy may be welfare-decreasing as panel (B) of Figure 2.10 implies.

Synergy Effect of Merger and Consumer Surplus

Farrell-Shapiro's Proposition 1 provides a necessary and sufficient condition for a merger to improve consumer surplus. When two firms merge, the condition comes to $MC^1(q_1^*) - MC^{1+2}(q_1^* + q_2^*) > P_N^* - MC^2(q_2^*)$. Given the functional form of demand and cost function, this condition boils down to the following result.

Claim 7. *[Farrell-Shapiro] A merger improves consumer surplus if and only if*

$$\lambda_1 + \lambda_2 < \lambda_M \tag{2.29}$$

There are two things to note on this Claim. First, it only depends on insiders' efficiency and the magnitude of synergy whether a merger is CS-increasing or not, which is in contrast with welfare effect of merger. More surprisingly, if the constituent firms of a merger are efficient enough in the sense that $\lambda_1 + \lambda_2 > 1$, then the merger cannot improve consumer surplus irrespective of the magnitude of its synergy effect. Using Claim 7, I can check the profitability of any CS-increasing merger.

Claim 8. *Any CS-increasing merger is profitable for merging firms.*[11]

Claim 8 is also in contrast with a welfare-increasing merger, which is not always profitable. This result holds because sufficient synergy effect is required for a merger to improve consumer surplus. But the inverse is not true because a profitable merger is not necessarily CS-increasing.

This model confirms the well-documented fact that a merger increases consumer surplus if and only if outsider's profit decrease. [Stillman (1983), Farrell and Shapiro (1990), Duso, Neven, and Roller (2007), etc.] To see this, using equation (2.16) yields

$$\pi_o^{1+2}(e_M) = \frac{\lambda_o(1+\lambda_o)}{2(1+\lambda_M+\sum_{k=3}^N \lambda_k)^2} < \frac{\lambda_o(1+\lambda_o)}{2(1+\lambda_1+\lambda_2+\sum_{k=3}^N \lambda_k)^2} = \pi_o^*$$

[11] Claim 8 holds in a very general setting. This result only requires the following two conditions, which Farrell and Shapiro (1990) assumes. See the appendix.

Condition 3 : $p'(Q) + q_i p''(Q) < 0$, $i = 1, \cdots, N$

Condition 4 : $\frac{d^2}{dq_i^2} C^i(q_i) > p'(Q)$, $i = 1, \cdots, N$

Table 2.3: The Welfare Effect of Horizontal Merger

Type of Merger	Insiders	Outsiders	Consumer	Net Effect
1. CS-decreasing	non-definite	benefit	loss	non-definite
2. CS-neutral	benefit	neutral	neutral	positive
3. CS-increasing	benefit	loss	benefit	positive

Here, $\pi_o^{1+2}(e_M)$ represents the outsider o's post-merger profit whereas π_o^* denotes its pre-merger profit, and the inequality comes from $\lambda_1 + \lambda_2 < \lambda_M$. So, "free rider's problem" completely disappears in a CS-neutral and CS-increasing merger. Further, a CS-increasing merger reduces outsider's output and its market share as well. Denote the outsider's pre-merger output and market share by q_o^* and s_o^*, and its post-merger output and market share by $q_o^{1+2}(e_M)$ and $s_o^{1+2}(e_M)$. From equation (2.14) and (2.15), I can obtain

$$\begin{aligned} q_o^{1+2}(e_M) &= \frac{\lambda_o}{1+\lambda_M+\sum_{k=3}^N \lambda_k} < \frac{\lambda_o}{1+\sum_{k=1}^N \lambda_k} = q_o^* \\ s_o^{1+2}(e_M) &= \frac{\lambda_o}{\lambda_M+\sum_{k=3}^N \lambda_k} < \frac{\lambda_o}{\sum_{k=1}^N \lambda_k} = s_o^* \end{aligned}$$

Again, the inequality comes from $\lambda_1 + \lambda_2 < \lambda_M$. Therefore, the merged firm's market share gets larger than the joint pre-merger market share of its constituent firms in a CS-increasing merger.

Using these results enables me to check the welfare effect of a CS-neutral merger. Any CS-neutral merger does not affect consumer surplus by definition, nor the outsiders' profit. So the welfare effect of CS-neutral merger is simplified into $w_S^{1+2}(e, e_M) = g_S^{1+2}(e, e_M)$. The proof of Claim 8 shows that a CS-neutral merger is profitable, so it is welfare increasing.

Table 2.3 summarizes the discussion so far on the profitability and welfare effect of a merger. The profitability of CS-decreasing merger is not decisive but improves as insiders are more asymmetric, outsiders are "less" efficient and the merger creates bigger synergies while CS-neutral or CS-increasing merger is profitable.

There is no merger that all the economic agents benefit from it. Given that only profitable mergers are proposed, the one who gets hurt from a merger would be either outsider or consumer but not both, and the dividing line is the merger's price effect. For example, a merger in the S-S-R model or rationalization type merger is also always CS-decreasing, so outsiders benefit from both kinds of merger.

The welfare effect of CS-decreasing merger is not decisive but improves as insiders are "smaller" or more asymmetric, outsiders are "more" efficient and the merger creates bigger synergies whereas CS-neutral or CS-increasing merger is welfare-increasing in general.[12]

2.4.4 One Merger's Effect on Another Merger's Profitability

The comparative statics in this subsection is how one merger affects another merger's profitability. If a merger enhances the profitability of another merger(s), then mergers are more likely to occur simultaneously or in chain, which is called "merger wave".

To deal with this issue, it is useful to rewrite $g_S^{1+2}(e, e_M)$ in (2.23) using $y \equiv \sum_{k=3}^{N} \lambda_k$. Then, the incentive of merger function becomes

$$g^{1+2}(e_1, e_2, e_M, y) = \frac{\lambda_M(1+\lambda_M)}{2(1+\lambda_M+y)^2} - \frac{\lambda_1(1+\lambda_1)+\lambda_2(1+\lambda_2)}{2(1+\lambda_1+\lambda_2+y)^2}$$

Besides a merger between firm 1 and 2 (merger A), let me consider another merger between firm 3 and 4 (merger B) without loss. After merger B, y comes to $y' = \lambda'_M + \sum_{k=5}^{N} \lambda_k$, where $\lambda'_M = \frac{e'_M}{1+e'_M}$ and e'_M represents the efficiency level of the merged firm coming from merger B. Claim 7 implies that y decreases (increases) if and only if merger B is CS-decreasing. (CS-increasing, resp.) Taking a partial of $g^{1+2}(e_1, e_2, e_M, y)$ with respect to y, I can obtain

$$\begin{aligned} \frac{\partial}{\partial y} g^{1+2}(e_1, e_2, e_M, y) &= \frac{\lambda_M(1+\lambda_M)}{(1+\lambda_M+y)^2}\left[\frac{1}{1+\lambda_1+\lambda_2+y} - \frac{1}{1+\lambda_M+y}\right] \\ &\quad - \frac{2g^{1+2}(e_1, e_2, e_M, y)}{1+\lambda_1+\lambda_2+y} \end{aligned} \tag{2.30}$$

Equation (2.30) brings me the following result.

Claim 9. *Suppose that merger A is break-even without merger B. Another CS-decreasing merger B may trigger the occurrence of merger A.*

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[12] If condition (2.28) holds, Claim 6 and Claim 8 implies that a CS-increasing merger is welfare-increasing. But it does not hold in all environment. Cheung (1992) shows one example where a CS-increasing merger may be welfare-decreasing under linear demand and constant marginal cost setting.

This Claim provides a sufficient condition where a merger becomes more profitable after the occurrence of another CS-decreasing merger: $g^{1+2}(e_1, e_2, e_M, y) \geq 0$ and $\lambda_1 + \lambda_2 \geq \lambda_M$. Since this is a sufficient condition, even unprofitable mergers may turn profitable after the occurrence of another CS-decreasing merger. To see this, suppose that merger A is unprofitable before merger B happens. Then $\lambda_1 + \lambda_2 > \lambda_M$ and $g^{1+2}(e_1, e_2, e_M, y) < 0$ hold. If the loss of merger A is small enough, $\frac{\partial}{\partial y} g^{1+2}(e_1, e_2, e_M, y) < 0$ holds from equation (2.30). So it can be the case that $g^{1+2}(e_1, e_2, e_M, y') > 0$.

The reason why this Claim holds is again related to the output response of outsiders. Without merger B, merger A causes firm 3 and 4 to adjust their output individually. But if firm 3 and 4 are combined by merger B, this merged entity will best respond to the change in aggregate output caused by merger A. Since merger B is CS-decreasing, $\lambda_3 + \lambda_4 > \lambda'_M$ holds. Recall that λ_i is firm i's responsiveness with respect to the change in market equilibrium output. So, the merged entity's output response is smaller than the joint output response of firm 3 and 4 for a given aggregate output change. Hence, the presence of a CS-decreasing merger reduces the output response of outsider(s), which improves the profitability of a merger between firm 1 and 2. Claim 9 and this discussion partly explains why mergers are apt to occur simultaneously or in chain.

2.4.5 General Linear Demand and Merger Analysis

In order to analyze the effect of demand side on profitability and welfare effect of merger, I will assume $P = a - bQ$ in this subsection, where $a > 0$ and $b > 0$. So, a is related to the market size while b is related to elasticity. If I do the same exercise with this demand function, the incentive of merger function, denoted by $g^{1+2}(e, e_M, a, b)$, comes to

$$g^{1+2}(e, e_M, a, b) = \frac{a^2}{2b}\left[\frac{\mu_M(1+\mu_M)}{(1+\mu_M+\sum_{k=3}^{N}\mu_k)^2} - \frac{\mu_1(1+\mu_1)+\mu_2(1+\mu_2)}{(1+\sum_{k=1}^{N}\mu_k)^2}\right], \tag{2.31}$$
$$\text{where} \quad \mu_M = \tfrac{be_M}{1+be_M} \text{ and } \mu_k = \tfrac{be_k}{1+be_k}$$

Note that if $e_M = e_1 + e_2$ and $a = b = 1$, $g^{1+2}(e, e_M, a, b) = g^{1+2}(e, e_1 + e_2, 1, 1) = g_R^{1+2}(e)$ in equation (2.18) and if $e_M > e_1 + e_2$ and $a = b = 1$, $g^{1+2}(e, e_M, a, b) = g^{1+2}(e, e_M, 1, 1) =$

$g_S^{1+2}(e, e_M)$ in equation (2.23). The change in outsider o's profit from the merger yields

$$\pi_o^{1+2} - \pi_o^* = \frac{a^2}{2b}\left[\frac{\mu_o(1+\mu_o)}{(1+\mu_M+\sum_{k=3}^N \mu_k)^2} - \frac{\mu_o(1+\mu_o)}{(1+\sum_{k=1}^N \mu_k)^2}\right]$$

Since pre-merger and post-merger Nash equilibrium price is given by

$$P_N^* = \frac{a}{1+\sum_{k=1}^N \mu_k},\ P_N^{1+2} = \frac{a}{1+\mu_M+\sum_{k=3}^N \mu_k}, \tag{2.32}$$

the welfare effect of merger, denoted by $w^{1+2}(e, e_M, a, b)$, comes to

$$w^{1+2}(e, e_M, a, b) = g^{1+2}(e, e_M, a, b) + \sum_{k=3}^{N} (\pi_k^{1+2} - \pi_o^*) - \frac{1}{b}\int_{P_N^*}^{P_N^{1+2}} (a-P)dP \tag{2.33}$$

Similarly, if $e_M = e_1 + e_2$ and $a = b = 1$, $w^{1+2}(e, e_M, a, b) = w^{1+2}(e, e_1 + e_2, 1, 1) = w_R^{1+2}(e)$ in equation (2.20) and if $e_M > e_1 + e_2$ and $a = b = 1$, $w^{1+2}(e, e_M, a, b) = w^{1+2}(e, e_M, 1, 1) = w_S^{1+2}(e, e_M)$ in equation (2.26). Equation (2.31), (2.32), and (2.33) give the following result, immediately.

Claim 10. *(a)* $g^{1+2}(e, e_M, a, b) = \frac{a^2}{b} g^{1+2}(be, be_M)$

(b) $w^{1+2}(e, e_M, a, b) = \frac{a^2}{b} w^{1+2}(be, be_M)$

(c) A merger improves consumer surplus if and only if $\mu_1 + \mu_2 < \mu_M$.

So, $g^{1+2}(e, e_M, a, b) > 0$ is equivalent to $g^{1+2}(be, be_M) > 0$, as is $w^{1+2}(e, e_M, a, b) > 0$ equivalent to $w^{1+2}(be, be_M) > 0$. Then, while market size variable a affects the magnitude of merger's profitability and welfare effect, it relies only on (e, e_M, b) whether a merger is profitable or welfare-increasing. In addition, it only depends on (e_1, e_2, e_M, b) and not on market size variable a whether a merger is CS-increasing. Hence, Claim 10 shows that qualitative merger analysis can be done with demand $P = 1 - Q$ if I substitute (e, e_M) with (be, be_M) even when the real demand is $P = a - bQ$.

Note that (e, e_M, b) is all the information requirement for the qualitative welfare analysis of a merger whereas (e_1, e_2, e_M, b) is required for the qualitative price effect analysis of a merger. Moreover, if one parameter of demand function (either a or b) is identified, it is possible to derive (e, b) from the observable variables at pre-merger Nash equilibrium. So, we can evaluate the welfare effect of rationalization type merger ($e_M = e_1 + e_2$). If synergy effect ($e_M > e_1 + e_2$) is observable, we can also evaluate the welfare effect and price effect of a merger with synergy.

2.4.6 Evaluation of Merger Review Criteria in the Literature

Farrell-Shapiro's Sufficient Condition

Proposition 5 of Farrell and Shapiro (1990) provides a sufficient condition for merger to increase welfare :

> PROPOSITION 5 (Farrell-Shapiro): Consider a proposed merger among firms $i \in I$, and suppose that their initial (joint) market share s_I does not exceed $\sum_{k \in O} \lambda_k s_k$. Suppose further that P'', P''', and $\frac{d^2}{dq_i^2} C^i$ are all nonnegative and $\frac{d^3}{dq_i^3} C^i$ is nonpositive in the relevant ranges and for all nonparticipant firms i. Then, if the merger is profitable and would raise the market price, it would also raise welfare.

In the model setting of this section, $P'' = P''' = \frac{d^3}{dq_i^3} C^i = 0$ and $\frac{d^2}{dq_i^2} C^i = \frac{1}{e_i} > 0$ hold. On the other hand, the initial market share condition in a merger between firm 1 and 2 becomes[13]

$$\lambda_1 + \lambda_2 < \sum_{k=3}^{N} \lambda_k^2 \tag{2.34}$$

Hence, a profitable CS-decreasing merger would raise welfare if it satisfies condition (2.34). There are two observations on this condition. First, there is a risk that a welfare-enhancing merger does not satisfy this condition because condition (2.34) is a sufficient condition. Second, this condition does not depend on the magnitude of synergy effect, so it gives the same condition whether a merger only has a rationalization or even synergy effect.

In order to see the property of condition (2.34) in more detail, revisit the example of $N = 3$. Then this condition comes to $\lambda_1 + \lambda_2 < \lambda_3^2$. Compare the Farrell-Shapiro's condition with $w_R^{1+2}(e) > 0$ for $e \in [0.1, 10]^3$. Panel (A) in Figure 2.12 plots the space where the Farrell-Shapiro's condition is satisfied whereas panel (B) shows welfare-enhancing mergers where this condition cannot capture. So, there exists the risk of type-2 error in which a welfare-enhancing merger fails to satisfy Farrell-Shapiro's condition.

---

[13]This condition is equivalent with equation (17) in Farrell and Shapiro (1990) or equation (6) in Werden (1991) because $s_i = \frac{\lambda_i}{\sum_{k=1}^{N} \lambda_k}$ and $\varepsilon = \frac{1}{\sum_{k=1}^{N} \lambda_k}$ under the our model.

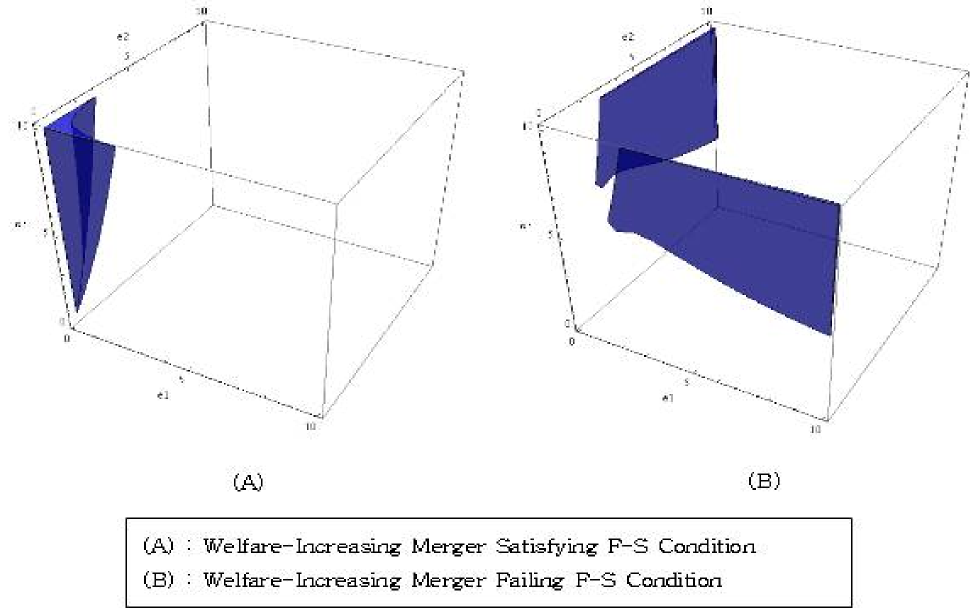


Figure 2.12: Farrell-Shapiro's Condition and Welfare-Increasing Merger

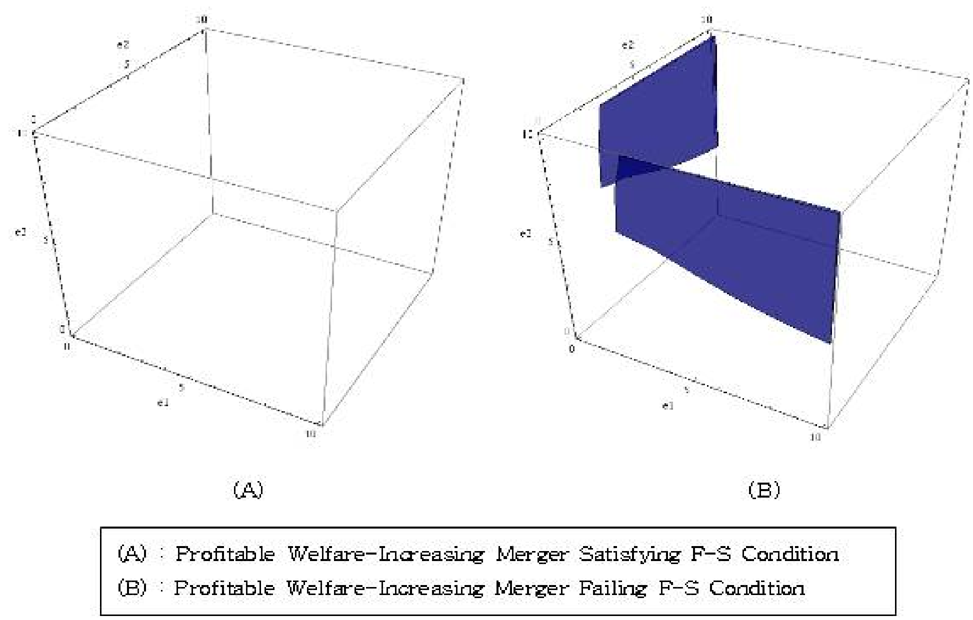


Figure 2.13: Farrell-Shapiro's Condition and Profitable Welfare-Increasing Merger

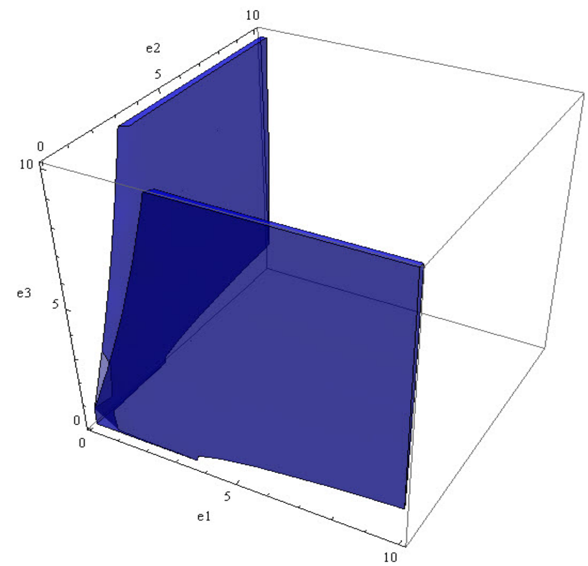


Figure 2.14: Welfare-Increasing Merger Failing Farrell-Shapiro (10% Synergy)

Figure 2.13 illustrates a more negative feature of this condition in a rationalization type merger. Panel (A) in Figure 2.13 shows that no profitable merger satisfies the Farrell-Shapiro's condition, so this condition deals with the merger that is not likely to occur in this specific case. This is because a merger can satisfy the Farrell-Shapiro's condition when it combines two small firms. So, cost saving from reallocation is restricted whereas the merged entity's output decreases a lot because of the efficient outsider's large output increase. In contrast, panel (B) in Figure 2.13 plots the space where merger is profitable and welfare-enhancing but fails to satisfy the Farrell-Shapiro's condition. Hence, the merger, which is socially desirable and more likely to occur, would be disapproved under the Farrell-Shapiro's condition.

Next, move to the kind of merger with synergy effect. I will assume that synergy effect $e_M = 1.1 * (e_1 + e_2)$, and compare the Farrell-Shapiro's condition with $w_S^{1+2}(e, e_M) > 0$ for $e \in [0.1, 10]^3$. Since the presence of synergy effect does not change this condition, the area where the Farrell-Shapiro's condition is satisfied is again given by panel (A) of Figure 2.12. But then, the space of welfare-enhancing mergers which fail to satisfy this condition, which is shown in Figure 2.14, becomes bigger than the case of rationalization merger. So, the probability that the Farrell-Shapiro's condition causes type-2 error gets higher as synergy effect gets stronger. While some profitable mergers with synergy satisfy this condition, profitable and welfare-enhancing mergers more frequently fail to satisfy the Farrell-Shapiro's condition in this case.

In sum, this exercise illustrates the possibility that the Farrell-Shapiro's condition may not be useful as a merger review criterion. Farrell and Shapiro (1990) also noticed that a policy of allowing only mergers satisfying this condition is too restrictive even though they did not analyze the magnitude of this potential type-2 error in detail.

McAfee-Williams' Necessary Condition

McAfee and Williams (1992), on the other hand, provides a condition for welfare-enhancing merger under the same setting with the rationalization model:

$$E \geq \frac{2}{3} \Rightarrow s^*(h_c, E, z) \leq \frac{h_c}{h_c+1}\ , where$$

$$E\ :\ \text{elasticity of demand}$$

$$h_c = \textstyle\sum_{k=3}^{N} \frac{s_k^2}{(1-s)^2},\ where\ s = s_1 + s_2\ and\ h_c \in [0,1]$$

$$z = \tfrac{s_1 s_2}{s^2},\ where\ z \in [0, \tfrac{1}{4}]$$

Here, s is the joint pre-merger market share of firm 1 and 2, $s^*(h_c, E, z)$ is the critical value on s for a merger to improve welfare, and h_c represents the Herfindahl of the non-merging firms. Since a merger improves welfare if and only if $s \leq s^*(h_c, E, z)$, the implied necessary condition for welfare-enhancing merger comes to $s \leq \frac{h_c}{h_c+1}$ when the demand is at least moderately elastic ($E \geq \frac{2}{3}$). Using this condition, they proved that no merger should be allowed that will either create a new largest firm or increase the market share of the largest firm.

There are a few remarks on this condition. First, this condition cannot be used when the demand is not elastic ($E < \frac{2}{3}$). Second, there might be a type-1 error where a welfare-decreasing merger satisfies this condition because it is a necessary condition. Finally, the McAfee-Williams model only consider the rationalization effect of merger, so their condition is not applicable to a merger with synergy effect.

To see the property of this condition in more detail, revisit the example that $N = 3$. Unlike

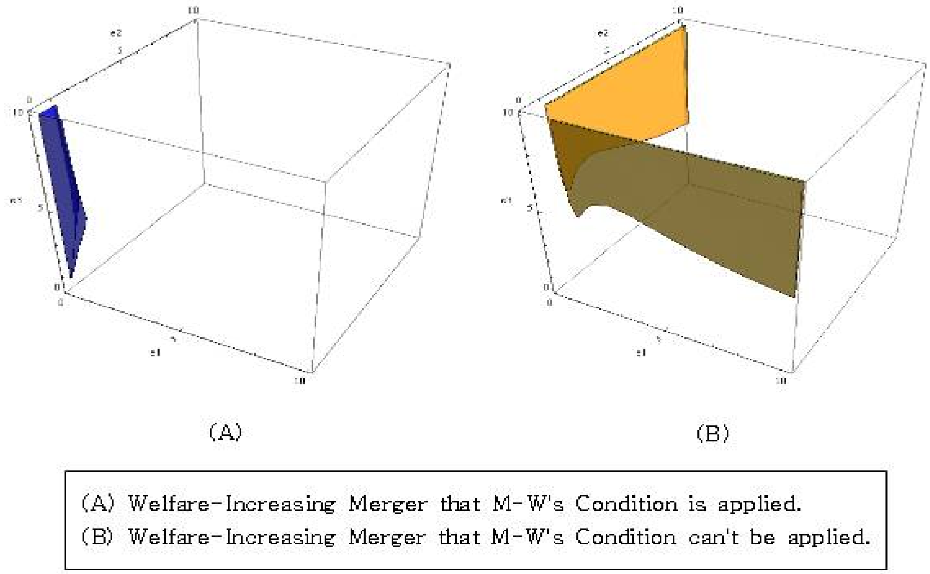


Figure 2.15: McAfee-Williams' condition and Welfare-Increasing Merger

the Farrell-Shapiro's condition, I only need to check a merger with rationalization. Since I have only firm 3 out of merger, h_c is equal to 1 in this case. So the McAfee-Williams' condition comes to $s_1 + s_2 \leq \frac{1}{2}$ if $E \geq \frac{2}{3}$ holds.

Figure 2.15 and Figure 2.16 illustrate the feature of elasticity condition ($E \geq \frac{2}{3}$). Panel (A) in Figure 2.15 plots the space of welfare-improving mergers satisfying $E \geq \frac{2}{3}$, so the McAfee-Williams' condition is applicable to the merger in this shape. Panel (B) in Figure 2.15 shows the welfare-improving mergers that the McAfee-Williams' condition cannot be used because elasticity condition fails. So, this condition is not applied to the welfare-increasing merger with big cost asymmetry between merging firms.

Panel (A) in Figure 2.16 shows that none of welfare-increasing mergers with $E \geq \frac{2}{3}$ is profitable for merging parties. So, the McAfee-Williams' condition only deals with mergers that are not likely to occur for the same reason with the Farrell-Shapiro's condition. Panel (B) in Figure 2.16 plots the space of profitable welfare-increasing mergers but the McAfee-Williams' condition cannot be applied. Hence, this condition cannot tell the welfare effect of this socially desirable merger that is more likely to occur. Finally, panel (C) in Figure 2.16 demonstrates that there may exist a profitable welfare-increasing merger which increases the

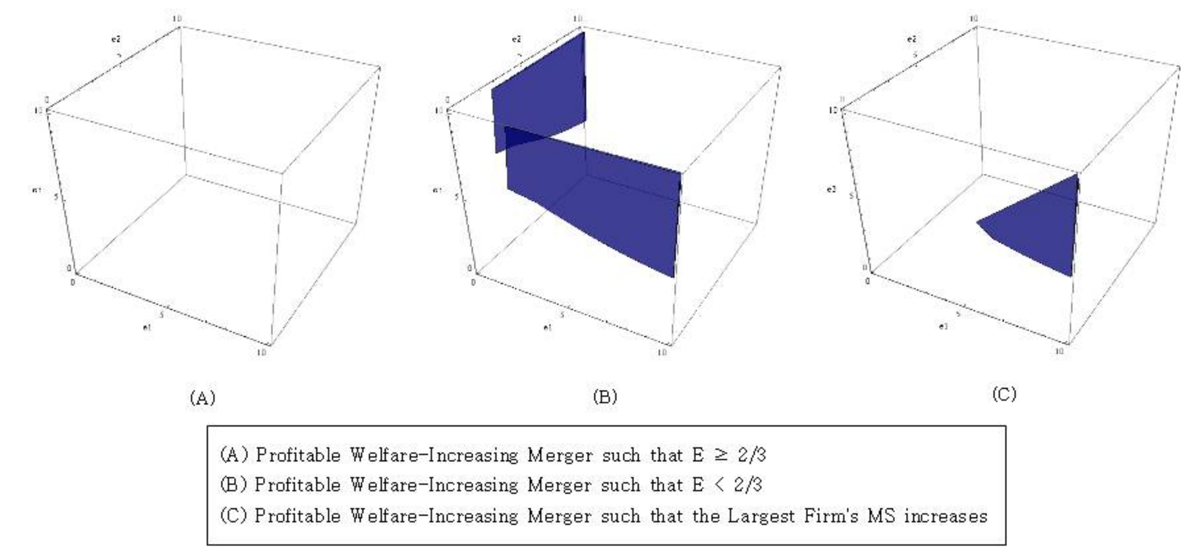


Figure 2.16: McAfee-Williams' condition and Profitable Welfare-Increasing Merger

largest firm's market share. This may happen when the largest firm take over a small firm although the elasticity condition does not hold in this case.

Regarding to the magnitude of a type-1 error in this condition, I could check that some welfare-decreasing mergers satisfy the McAfee-Williams' condition, but none of them is profitable. Hence, type-1 error is not an important problem of this condition.

2.5 Merger Analysis under Free Entry-Exit

Hitherto, this research assumes no entry-exit. The meaning of this assumption was explained by footnote 8 in Farrell and Shapiro (1990): "Our analysis can easily accommodate entry by, or the existence of price-taking fringe firms, if we reinterpret the demand curve as the residual demand curve facing the oligopolists that we model. What we are ruling out is entry by additional large firm that behave oligopolistically." The analysis up to the previous section can be interpreted in the same way. In this extension, I will consider the other extreme setting where free entry and exit is possible.

2.5.1 Identical Constant Marginal Cost Model under Free Entry-Exit

I assume free entry and exit but the presence of fixed cost F as in Davidson and Muhkerjee (2007). At pre-merger equilibrium, firms compete Nash in Cournot fashion as in Section 2.3. In this section, I let π_n be the net operating profit (before F is deducted) accruing to firm n when there are n firms in the industry. Then the (pre-merger) number of firms at free-entry equilibrium, N^*, satisfies

$$\pi_{N^*} > F > \pi_{N^*+1} \tag{2.35}$$

I will assume $N^* \geq 3$ for merger analysis. Given the assumption of linear demand and constant marginal cost, $\pi_{N^*} = \frac{1}{(N^*+1)^2}$ under Cournot competition. So, $\pi_l > \pi_n$ is equivalent to $l < n$ if firms' competitive behavior is the same at l and n. For the same n, on the other hand, π_n from collusion is higher than π_n from Cournot competition.

Consider an exogenous merger combining $(M+1)$ firms. Note that the post-merger number of firms $(N^* - M)$ fails to satisfy condition (2.35) regardless of whether the industry with $(N^* - M)$ firms competes Cournot or collude. This condition is recovered when M new entries occur after merger. Hence, if I assume that the duplication of fixed cost is eliminated after merger, the incentive of merger yields

$$g(N^*, M) = (\pi_{N^*} - F) - (M+1)(\pi_{N^*} - F) = -M(\pi_{N^*} - F) < 0$$

So any size of merger is not profitable. Because the number of firms is the same at pre- and post-merger, the market structure and firms' competitive behavior do not change. As a result, the presence of free entry removes any effect of merger on outsider's profit, welfare, or consumer surplus. Hence, "merger paradox" is reestablished, but "free rider's problem" disappears.

2.5.2 Asymmetric Increasing Marginal Cost Model under Free Entry-Exit

Model Modification

I build up a simple dynamic game under the asymmetric increasing marginal cost model setting in order to analyze how free entry and exit affects the incentive of merger, post-merger price and welfare. The game structure is basically similar to Spector (2003), but my analysis includes mergers with or without synergies under a specific functional form while Spector (2003) deals with rationalization type mergers alone using a general functional form.

■ **Game Structure** : The timing of the game is as the follows:

1. Nature picks a decreasing sequence $\langle e_n \rangle_{n=1}^{\infty}$ for each potential entrant n.

2. Initial entry decision is made.

3. Merger occurs between incumbent i and j.

4. Entry or exit decision is made by incumbents or potential entrants.

■ **Entry/Exit** : I assume free entry and exit but there is a fixed cost F. So firm $i's$ cost function comes to $C^i(q_i) = \frac{q_i^2}{2e_i} + F$. Sequential entry or exit is assumed in (Step 2) and (Step 4), which means that entry or exit occurs according to the efficiency order. So, the least efficient incumbent is more efficient than the most efficient potential entrant.

Market Structure at Pre- and Post-Merger

Let N^* be the number of firms at free-entry equilibrium in (Step 2). Then, firm N^* is the least efficient incumbent from the sequential entry assumption, and its net operating profit amounts to

$$\pi_{N^*} = \frac{\lambda_{N^*}(1+\lambda_{N^*})}{2(1+\sum_{k=1}^{N^*}\lambda_k)^2} \tag{2.36}$$

Note that $\pi_l > \pi_n$ holds for every (l, n) such that $l < n$. π_l (π_n) represents the operating profit of firm l (firm n) when there are the most efficient l (n) firms in the industry. Then N^* satisfies $\pi_{N^*} > F > \pi_{N^*+1}$. I will assume $N^* \geq 3$ for merger analysis.

In (Step 3), a merger occurs between incumbent i and j such that $i < j \leq N^*$. Let firm $N^\dagger \notin \{i, j\}$ be the least efficient post-merger incumbent - except the merged entity if it is the least efficient firm -, then the net operating profit of firm $N^\dagger$ comes to

$$\pi_{N^\dagger}^{i+j} = \frac{\lambda_{N^\dagger}(1+\lambda_{N^\dagger})}{2(1+\lambda_M+\sum_{k\neq i,j}^{N^\dagger}\lambda_k)^2} \tag{2.37}$$

Similarly, $\pi_p^{i+j} > \pi_q^{i+j}$ holds for every $(p,q) \notin \{i,j\}$ such that $p < q$. π_p^{i+j} (π_q^{i+j}) represents the operating profit of firm p (firm q) when firm p (firm q) is the least efficient post-merger incumbent (except the merged firm if necessary). In addition, free-entry-and-exit equilibrium implies that $N^\dagger$ has to satisfy[14]

$$\pi_{N^\dagger}^{i+j} > F > \pi_{N^\dagger+1}^{i+j} \tag{2.38}$$

The post-merger number of firms at the free-entry-and-exit equilibrium is $\big(N^\dagger + 1\big)$ if $N^\dagger < i$, $N^\dagger$ if $i < N^\dagger < j$, and $\big(N^\dagger - 1\big)$ if $N^\dagger > j$.

Case Analysis of Merger under Free Entry-Exit

Sequential entry and exit assumption implies that firm $(N^* + 1)$ enters post-merger if and only if a merger induces entry. Similarly, an exit-inducing merger always causes the least efficient pre-merger incumbent except insiders to exit. If I denote this least efficient incumbent by firm L, then $L = N^*$ for $j \neq N^*$, $L = (N^* - 1)$ for $i \neq (N^* - 1)$ and $j = N^*$, and $L = (N^* - 2)$ for $i = (N^* - 1)$ and $j = N^*$.

$\pi_{N^*+1}^{i+j} > \pi_{N^*+1}$ is a necessary condition so that a merger induces entry, and equation (2.36) and (2.37) imply that $\pi_{N^*+1}^{i+j} > \pi_{N^*+1}$ is equivalent to $\lambda_M < \lambda_i + \lambda_j$. In contrast, the operating profit of firm L should decrease so that a merger may induce exit. Let me denote $\pi_L(N^*)$ be the pre-merger operating profit of firm L, then $\pi_L(N^*)$ is given by

$$\pi_L(N^*) = \frac{\lambda_L(1+\lambda_L)}{2(1+\sum_{k=1}^{N^*}\lambda_k)^2} \tag{2.39}$$

Then, $\pi_L^{i+j} < \pi_L(N^*)$ is a necessary condition that a merger induces exit, and equation (2.37) and (2.39) imply that $\pi_L^{i+j} < \pi_L(N^*)$ is equivalent to $\lambda_M > \lambda_i + \lambda_j$. From this observation, I can divide a merger scenario into 3 cases: (Case 1) $\lambda_M = \lambda_i + \lambda_j$, (Case 2) $\lambda_M < \lambda_i + \lambda_j$, and (Case 3) $\lambda_M > \lambda_i + \lambda_j$.

---

[14]More precisely, condition (2.38) is satisfied if and only if $\big(N^\dagger + 1\big) \notin \{i,j\}$. $\pi_{N^\dagger}^{i+j} > F > \pi_{N^\dagger+2}^{i+j}$ holds if $\big(N^\dagger + 1\big) \in \{i,j\}$ and $\big(N^\dagger + 2\big) \notin \{i,j\}$, and $\pi_{N^\dagger}^{i+j} > F > \pi_{N^\dagger+3}^{i+j}$ holds if $\big(N^\dagger + 1\big) = i$ and $\big(N^\dagger + 2\big) = j$. So there is a slight abuse of notation in $\pi_{N^\dagger+1}^{i+j}$.

(Case 1 : $\lambda_M = \lambda_i + \lambda_j$) In this case, equation (2.36), (2.37) and (2.39) show that $\pi_L(N^*) = \pi_L^{i+j}$ and $\pi_{N^*+1} = \pi_{N^*+1}^{i+j}$ hold. So, there is no entry from $F > \pi_{N^*+1} = \pi_{N^*+1}^{i+j}$, nor exit from $F < \pi_{N^*} \leq \pi_L(N^*) = \pi_L^{i+j}$ post-merger. The incentive of merger function becomes[15]

$$
\begin{aligned}
g_{N^*}^{i+j}(e, e_M) &= \frac{\lambda_M(1+\lambda_M)}{2(1+\lambda_M+\sum_{k\neq i,j}^{N^*}\lambda_k)^2} - \frac{\lambda_i(1+\lambda_i)+\lambda_j(1+\lambda_j)}{2(1+\sum_{k=1}^{N^*}\lambda_k)^2} \qquad (2.40)\\
&= \frac{\lambda_i\lambda_j}{(1+\sum_{k=1}^{N^*}\lambda_k)^2}
\end{aligned}
$$

The last equality holds from $\lambda_M = \lambda_i + \lambda_j$. Then welfare effect function yields

$$
\begin{aligned}
w_{N^*}^{i+j}(e, e_M) &= g_{N^*}^{i+j}(e, e_M) + \sum\nolimits_{k\neq i,j}^{N^*}(\pi_k^{i+j}(L) - \pi_k(N^*)) - \int_{P_{N^*}^*}^{P_{N^*}^{i+j}}(1-P)dP,\\
&\qquad where \;\; \pi_k^{i+j}(L) = \tfrac{\lambda_k(1+\lambda_k)}{2(1+\lambda_M+\sum_{l\neq i,j}^{N^*}\lambda_l)^2},\\
&\qquad\qquad \pi_k(N^*) = \tfrac{\lambda_k(1+\lambda_k)}{2(1+\sum_{l=1}^{N^*}\lambda_l)^2}\\
&= g_{N^*}^{i+j}(e, e_M) \qquad (2.41)
\end{aligned}
$$

Here, $\pi_k(N^*)$ denotes the pre-merger operating profit of firm $k \in \{1, \cdots, N^*\}\backslash\{i, j\}$ when N^* is the least efficient pre-merger incumbent, whereas $\pi_k^{i+j}(L)$ denotes its post-merger operating profit when L is the least efficient post-merger incumbent (except the merged firm if necessary). The second equality comes from $\pi_k(N^*) = \pi_k^{1+2}(L)$ for all k and $P_{N^*}^* = P_{N^*}^{1+2}$. Then, Claim 7 and 8 imply that a merger satisfying $\lambda_M = \lambda_i + \lambda_j$ is profitable, welfare-increasing and CS-neutral irrespective of entry or exit condition.

(Case 2 : $\lambda_M < \lambda_i + \lambda_j$) Given $\pi_{N^*+1}^{i+j} > \pi_{N^*+1}$ in this case, merger induces entry if and only if $\pi_{N^*+1}^{i+j} > F > \pi_{N^*+1}$. But exit cannot occur because $\pi_L^{i+j} > \pi_L(N^*) \geq \pi_{N^*} > F$. $\big($i.e. $N^\dagger \geq L\big)$ Because $\pi_{N^*+1}^{i+j}$ gets larger as e_M becomes smaller, entry is more likely to occur in a merger with no or weak synergies. Due to the potential possibility of entry, the incentive

[15] I do not assume the elimination of fixed cost duplication in this model where the merging firms' facilities are not shut down in general. If I assume some fixed cost can be saved, αF can be added to the incentive of merger function for $\alpha \in (0, 1]$. But the presence of this term does not matter in the following analysis by much.

of merger function comes to

$$g_{N^\dagger}^{i+j}(e, e_M) = \frac{\lambda_M(1+\lambda_M)}{2(1+\lambda_M+\sum_{k\neq i,j}^{N^\dagger}\lambda_k)^2} - \frac{\lambda_i(1+\lambda_i)+\lambda_j(1+\lambda_j)}{2(1+\sum_{k=1}^{N^*}\lambda_k)^2} \tag{2.42}$$

Note that the summation is taken over $k \in \{1, \cdots, N^\dagger\}\backslash\{i, j\}$ for $\lambda_k's$ in the denominator of the first term. Equation (2.42) shows that the incentive of merger gets weaker ($g_{N^\dagger}^{i+j}(e, e_M) < g_{N^*}^{i+j}(e, e_M)$) if any merger-induced entry takes place. Here are two examples to show this aspect.

Example 2.1 (Case that Entry Prevents a Welfare-Increasing Merger) *Let the sequence* $\langle e_n\rangle_{n=1}^{\infty}$ *be* $\langle e_1, e_2, e_3, e_4, e_5, \cdots\rangle = \langle 0.5, 0.4, 0.3, 0.15, 0, \cdots\rangle$, $F = 0.019$, *and consider a merger between firm 2 and firm 3. Then,* $\pi_3 > F > \pi_4$ ($\Leftrightarrow 0.0415 > 0.019 > 0.0188$), *so* $N^* = 3$ *and* $P_{N^*}^* = P_3^* = 0.5406$ in (Step 2). *Suppose* $e_M = 0.8$, *then* $e_M > e_2 + e_3$ *and* $\lambda_M < \lambda_2 + \lambda_3$ *hold. Then,* $\pi_4^{2+3} > F > \pi_5^{2+3}$ ($\Leftrightarrow 0.02 > 0.019 > 0$), *so firm 4 enters the market post-merger. The incentive of merger comes to* $g_{N^*}^{2+3}(e, e_M) = 0.006$ *under no entry, and* $g_{N^\dagger}^{2+3}(e, e_M) = -0.007 < 0$ *under free-entry. In addition, the welfare effect of this merger is given by* $w_{N^*}^{2+3}(e, e_M) = 0.0019$ *under no entry condition.*

Example 2.2 (Case that Entry Prevents a Welfare-Decreasing Merger) *Consider the same sequence and fixed cost with [Example 2.1], but a merger between firm 1 and firm 2. Now suppose* $e_M = 0.9$, *then* $e_M = e_1 + e_2$ *and* $\lambda_M < \lambda_1 + \lambda_2$ *hold. Since* $\pi_4^{1+2} > F > \pi_5^{1+2}$ ($\Leftrightarrow 0.022 > 0.019 > 0$), *firm 4 enters the market post-merger. The incentive of merger comes to* $g_{N^*}^{1+2}(e, e_M) = 0.0015$ *under no entry, and* $g_{N^\dagger}^{1+2}(e, e_M) = -0.015 < 0$ *under free-entry. In contrast, the welfare effect of this merger is given by* $w_{N^*}^{1+2}(e, e_M) = -0.0112$ *under no entry condition.*

These two examples shows that significant entry can make otherwise profitable mergers unprofitable, and firms are less likely to merge under free entry condition. As these examples show, the merger blocked by the presence of free entry may be either welfare-increasing or welfare-decreasing under no entry condition.

Further, because a merger with synergies is not necessarily profitable even under no entry-exit environment, synergy-creating merger may be unprofitable under the free entry condition as well. [Example 2.1] shows that even profitable synergy-creating merger under no entry condition may become unprofitable under the free entry. This is in contrast to Proposition 1 of Davidson and Muhkerjee (2007) that any merger is profitable for any degree of cost synergy. This difference shows that their result relies on the assumption of the identical constant marginal cost function. On the other hand, a merger's profitability improves as synergies get stronger as in no entry condition. There are two reasons why Claim 5 can be extended to a merger under the free entry condition: if entry consequence of a merger is the same under two different synergy levels, the one with stronger synergy is more profitable; in addition, the likelihood of entry is smaller in a merger with stronger synergies.

Next look at the price effect. If a merger does not induce entry $\left(F > \pi^{i+j}_{N^*+1} > \pi_{N^*+1}\right)$, Claim 7 implies that the post-merger equilibrium price increases under free-entry condition as well. If a merger induces entry $\left(\pi^{i+j}_{N^*+1} > F > \pi_{N^*+1}\right)$, however, the equilibrium price may decrease post-merger even if $\lambda_M < \lambda_i + \lambda_j$ holds, which cannot happen under no entry condition. Here is one example to illustrate this possibility.

Example 2.3 (Case that Entry-Inducing Merger is Profitable and CS-increasing) *Again, let the sequence* $\langle e_n \rangle_{n=1}^{\infty}$ *be* $\langle e_1, e_2, e_3, e_4, e_5, \cdots \rangle = \langle 0.5, 0.4, 0.3, 0.15, 0, \cdots \rangle$ *and fixed cost is* $F = 0.019$. *Now consider a merger between firm 2 and firm 3. Suppose* $e_M = 1$, *then* $e_M > e_2 + e_3$ $(\Leftrightarrow 1 > 0.7)$ *and* $\lambda_M < \lambda_2 + \lambda_3$ $(\Leftrightarrow 0.5 < 0.5164)$ *hold. So this merger creates synergy but would increase price post-merger under no entry condition. Then,* $\pi^{2+3}_4 > F > \pi^{2+3}_5$ $(\Leftrightarrow 0.0191 > 0.019 > 0)$, *so firm 4 enters the market post-merger.* $P^{2+3}_{N^\dagger} = P^{2+3}_4 = 0.5092$ *in (Step 4). Hence, this merger decreases the equilibrium price. Furthermore, this merger is profitable because* $g^{2+3}_{N^\dagger}(e, e_M) = 0.0021$.

This example shows a scenario where an entry-inducing merger decreases price post-merger. This example also illustrates that an entry-inducing merger might be profitable. Although a merger satisfying $\lambda_M < \lambda_i + \lambda_j$ may decrease the price as in [Example 2.3], there is a lower bound where the post-merger price can go down.

Claim 11. *If $\lambda_M < \lambda_i + \lambda_j$, post-merger price is higher than the price that would have been formed if firm $(N^* + 1)$ had been an incumbent pre-merger.* $\left(P^*_{N^*+1} < P^{i+j}_{N^\dagger}\right)$

Claim 11 says that a merger either increases price post-merger $\left(P^*_{N^*+1} < P^*_{N^*} < P^{i+j}_{N^\dagger}\right)$, or would increase if firm $(N^* + 1)$ had been an incumbent pre-merger $\left(P^*_{N^*+1} < P^{i+j}_{N^\dagger} \leq P^*_{N^*}\right)$. In [Example 2.3], if firm 4 had been an incumbent before merger, the pre-merger price would have been $P^*_{N^*+1} = P^*_4 = 0.5049$. So $P^*_{N^*+1} < P^{i+j}_{N^\dagger}$ holds, as expected. The reason of this result is simple. If firm $(N^* + 1)$ had entered pre-merger, the price would have been $P^*_{N^*+1}$. We know that firm $(N^* + 1)$ could not have earned the operating profit greater than fixed cost under this price. Hence, its post-merger operating profit would also be less than fixed cost for any post-merger price equal to or lower than $P^*_{N^*+1}$. But then, firm $(N^* + 1)$ would not enter the market, which contradicts that it is an incumbent after merger.

It is worth contrasting this result with Farrell-Shapiro's Proposition 1·2 and Spector's Proposition 1. Proposition 2 of Farrell-Shapiro proves that any merger failing to create cost synergies increases price under no entry assumption. Under the asymmetric increasing marginal cost model, Claim 7 confirms Farrell-Shapiro's result because $\lambda_M = \lambda_{i+j} < \lambda_i + \lambda_j$ holds in a merger without synergies. Spector (2003) extended Farrell-Shapiro's Proposition 2 by showing that the post-merger price increases irrespective of entry condition when a profitable merger does not create synergies. In contrast, Claim 11 and [Example 2.3] imply that the price effect of a profitable merger becomes non-decisive when it creates synergies within the degree such that $\lambda_M < \lambda_i + \lambda_j$. So Farrell-Shapiro's Proposition 1 is not valid if no entry assumption is relaxed and Spector's result cannot be extended up to synergy-creating profitable mergers. More precisely, if I relax no synergy condition in Farrell-Shapiro's Proposition 2 up to the extent with which a merger would increase price under no entry condition, the equivalent merger with the same synergies may decrease price under free entry condition. Even when it happens, however, the post-merger price is at least higher than $P^*_{N^*+1}$ due to the degree of synergies.

Finally, welfare effect function of this case comes to

$$
\begin{aligned}
w_{N^\dagger}^{i+j}(e,e_M) &= g_{N^\dagger}^{i+j}(e,e_M) + \sum\nolimits_{k\neq i,j}^{N^*}(\pi_k^{i+j}(N^\dagger) - \pi_k(N^*)) \\
&\quad + \sum\nolimits_{k=N^*+1}^{N^\dagger}(\pi_k^{i+j}(N^\dagger) - F) - \int_{P_{N^*}^*}^{P_{N^\dagger}^{i+j}}(1-P)dP \\
&\quad where,\ \pi_k^{i+j}(N^\dagger) = \tfrac{\lambda_k(1+\lambda_k)}{2(1+\lambda_M+\sum_{l\neq i,j}^{N^\dagger}\lambda_l)^2}
\end{aligned}
\tag{2.43}
$$

Here, $\pi_k^{i+j}(N^\dagger)$ denotes firm $k's$ post-merger operating profit given that $N^\dagger$ is the least efficient firm at post-merger equilibrium. If entry does not occur ($L = N^\dagger$), $\sum_{k=N^*+1}^{N^\dagger}(\pi_k^{1+2}(N^\dagger) - F)$ is defined by zero. If entry occurs ($L < N^\dagger$), the first two terms in the right-hand side of equation (2.43) are smaller than those under no entry condition. This negative effect of entry on welfare, which is called "business stealing effect" by Mankiw and Whinston (1986), takes place because entrant takes away some of the incumbents' output. In contrast, the other two terms affect positively on welfare effect because the entrant earns positive profit and price does not increase as much as it does under no entry. Entry's influence on welfare effect of a merger relies on the relative magnitude of these two conflicting effects. In the case of [Example 2.3], the welfare effect of this merger is given by $w_{N^\dagger}^{2+3}(e,e_M) = 0.0098$. Since $w_{N^*}^{2+3}(e,e_M) = 0.0153$ under no entry condition, this example shows the possibility that entry makes the welfare effect of merger worse.

(Case 3 : $\lambda_M > \lambda_i + \lambda_j$) The merger analysis of this case is almost symmetric with (Case 2). Given $\pi_L^{i+j} < \pi_L(N^*)$ in this case, exit arises if and only if $\pi_L(N^*) > F > \pi_L^{i+j}$. But merger does not induce entry because $\pi_{N^*+1}^{i+j} < \pi_{N^*+1} < F$. The incentive of merger function is again given by equation (2.42), but $L \geq N^\dagger$ holds in this case due to the possibility of exit. Clearly, the incentive of a merger improves when it induces an exit of a marginal incumbent (*i.e.* $g_{N^\dagger}^{i+j}(e,e_M) > g_{N^*}^{i+j}(e,e_M)$ if $L > N^\dagger$). Combined with Claim 8, this result implies that any merger satisfying $\lambda_M > \lambda_i + \lambda_j$ is profitable regardless of entry-exit condition. A merger's profitability improves as synergies get stronger as in (Case 2) for similar reasons: if exit consequence of a merger is the same under two different synergy level, the one with stronger synergy is more profitable; in addition, the likelihood of exit becomes higher in a merger with stronger synergies because π_L^{i+j} is smaller under this merger.

Next consider the price effect of merger. If a merger does not induce exit ($\pi_L(N^*) > \pi_L^{i+j} > F$), Claim 7 implies that the post-merger equilibrium price decreases under free-exit condition as well. If a merger induces exit ($\pi_L(N^*) > F > \pi_L^{i+j}$), however, the equilibrium price may increase post-merger even if $\lambda_M > \lambda_i + \lambda_j$ holds. But there is a ceiling on the post-merger price even if it rises.

Claim 12. *If $\lambda_M > \lambda_i + \lambda_j$, post-merger price (a) either decreases $\left(P_{N^\dagger}^{i+j} < P_{N^*}^*\right)$ or (b) would decrease if the merger induced one less exit $\left(P_{N^\dagger+1}^{i+j} < P_{N^*}^* \le P_{N^\dagger}^{i+j}\right)$.*

Claim 12 (b) shows that no more exit arises as soon as an exit raises the post-merger price above the pre-merger price. This holds because if an exit raises price above the pre-merger level, every remaining incumbent will make its operating profit greater than fixed cost at the price level. Otherwise, it could not have earned the operating profit greater than fixed cost at the pre-merger price either, which contradicts that it was an incumbent before merger.

Finally, consider the welfare effect function, which is given by

$$\begin{aligned} w_{N^\dagger}^{i+j}(e, e_M) &= g_{N^\dagger}^{i+j}(e, e_M) + \sum_{k \neq i,j}^{N^\dagger} (\pi_k^{i+j}(N^\dagger) - \pi_k(N^*)) \\ &\quad - \sum\nolimits_{k \in \{N^\dagger+1, \cdots, N^*\} \setminus \{i,j\}} (\pi_k(N^*) - F) - \int_{P_{N^*}^*}^{P_{N^\dagger}^{i+j}} (1-P)dP \end{aligned} \tag{2.44}$$

If exit does not occur ($L = N^\dagger$), $\sum_{k \in \{N^\dagger+1, \cdots, N^*\} \setminus \{i,j\}} (\pi_k(N^*) - F)$ is defined by zero. Else if exit occurs ($N^\dagger < L$), the first two terms in the right-hand side of equation (2.44) get bigger than those under no entry condition. Contrary to (Case 2), exit brings "business recovering effect" to the remaining incumbents. But the other two terms affect negatively on welfare effect because the exiting incumbent's profit disappears and price does not decrease as much as it does under no exit. Exit's influence on welfare effect of a merger relies on the relative magnitude of these two effects.

Table 2.4: Merger's Profitability and Price Effect under Free Entry-Exit Condition

Type of Merger	$\lambda_M = \lambda_i + \lambda_j$	$\lambda_M < \lambda_i + \lambda_j$	$\lambda_M > \lambda_i + \lambda_j$
Entry/Exit	none	entry only if any	exit only if any
Profitability	profitable	indefinite	profitable
- Comparison	same with no entry	not better than no entry	not worse than no exit
Price Effect	CS-neutral	indefinite w/ bottom	indefinite w/ ceiling
- Comparison	same with no entry	not worse than no entry	not better than no exit

Summary of Merger Analysis under Free Entry-Exit

This case analysis brings a few results on the incentive of merger, its price effect and welfare effect under the free-entry-and-exit environment.

First, condition (2.29) - the necessary sufficient condition for CS-increasing merger under no entry-exit assumption - becomes a necessary condition that exit occurs after merger. More generally, Farrell-Shapiro's condition for CS-increasing merger (Proposition 1) becomes a necessary condition for exit to occur when free entry and exit is allowed in their model.

Second, free entry-exit condition affects merger's profitability as follows: an exit-inducing merger is always profitable; an entry-inducing merger is not necessarily unprofitable although its profitability gets harmed due to entry; as in no entry-exit condition, stronger synergies improve a merger's profitability and a synergy-creating merger is not necessarily profitable.

Third, entry or exit, if it occurs, reduces the extent of a merger's price effect and its effect on the remaining outsider(s). Note that exit might occur under free entry-exit model in a situation where the merger decreases price under no entry-exit model. Hence exit reduces the amount of price decrease, or may even increase the price. A symmetric inference is possible for entry. Since exit and entry reduces price effect, there is smaller change in aggregate output. But then, outsider's output response gets smaller as well.

Finally, this case analysis could not determine how free entry-exit affects merger's welfare effect. But, it can be the case that merger-induced entry worsens the merger's welfare effect whereas merger-induced exit improves it.

2.6 Concluding Remarks

I studied a merger's profitability, its welfare effect and price effect under two different settings in this paper. One model deals with identical and constant marginal cost whereas the other model assumes asymmetric and increasing marginal costs across firms.

The identical constant marginal cost model highlights on how the change in firms' competitive behavior affects a merger's profitability and its welfare effect. In contrast, the asymmetric increasing marginal cost model shows the importance of cost savings from reallocation or synergies on a merger's profitability, its welfare effect and price effect. These two models illustrate that "merger paradox" in the S-S-R model requires a very specific environment in which marginal cost is constant and identical and firms' competitive behavior does not change. Their model setting eliminates not only the possibility of cost saving from a merger but also the potential benefit from reduced competition. Clearly, these are the important sources that firms try to achieve through a horizontal merger.

Regarding to the second result of S-S-R, the asymmetric increasing marginal cost model agrees that a welfare-increasing merger may be unprofitable as shown in a merger without synergies between small firms. But this model also shows that welfare-increasing merger becomes profitable if it creates strong enough synergies. In particular, CS-increasing mergers are always profitable in general setting. So, the S-S-R's point that socially desirable mergers may not be privately enforceable only holds in a merger where its synergies are relatively "weak" under more general setting, and disappears under consumer surplus criterion.

Rather, more realistic risk in horizontal mergers seems that socially undesirable mergers are more likely to take place. The identical constant marginal cost model shows that a merger with collusive effect is more profitable, but worsens social welfare. The asymmetric increasing marginal cost model implies that the presence of a bigger outsider helps social welfare but harms a merger's profitability. So each firm may have an incentive to find a bigger merger partner, and a socially undesirable merger between "big" firms is more likely to occur than others. Moreover, CS-decreasing mergers may occur simultaneously or in chain.

Hence, the conflict between merger incentive and social welfare should be the target issue which antitrust policy has to concern about. Proposition 1 of Farrell-Shapiro provides the exact criteria for a CS-increasing Cournot merger under no entry condition. Two models in this paper adds merger review criteria to measure the welfare effect when linear approximation is possible for demand and marginal cost. Clearly, they are not sufficient to our goal. There can be many cases that linearity assumption is not maintained for demand or marginal cost. In addition, as this research showed, the price effect or welfare effect of a synergy-creating merger depends on entry-exit condition as well.

CHAPTER 3

Collusion under Asymmetric Information on Discount Rates

3.1 Introduction

The goal of this research is to see how uncertainty on other firms' discount rates affects the competitive behavior in oligopoly market. To this end, this paper considers the situation where each firm in oligopoly market may have a different discount factor and no firm can observe other firms' discount rates.

To my knowledge, Harrington (1989) is the only literature that analyzed the cartel formation issue in an environment where firms may have different discount rates. He pointed out that incomplete capital market or agency problem between shareholders and manager may be the sources of different discount factors. Harrington (1989) analyzed the problem under perfect information using Bertrand model, whereas I constructed Cournot model under the incomplete information on other firms' discount rate in order to see the environment where price competition is not important but there is a private information on each firm's patience.

Asymmetric information issue among cartel members has been analyzed in quite many literatures. Abreu, Pearce, and Stacchetti (1986, 1990) are the representative ones in that regards. These papers focus on the situation where each cartel member cannot monitor competitors' behavior perfectly, and derived the Bang-Bang result as an optimal cartel equilibrium outcome path. My research looks at different situation from Abreu, Pearce, and Stacchetti (1986, 1990) in that each firm can monitor competitors' behavior perfectly but is uncertain about their discount factors.

Athey, Bagwell, and Sanchirico (2004) studied the optimal cartel scheme under the situation where there is a asymmetric information on each firm's cost and firms exchange cost information each period before cartel chooses price. My research is similar to Athey, Bagwell, and Sanchirico (2004) in that uncertainty is about competitors' type instead of their behavior, but two researches take different approach to treat this asymmetric information. Athey, Bagwell, and Sanchirico (2004) explicitly considered the process to exchange the cost information on each firm who receives a privately observed, i.i.d. cost shock in each period. In contrast, I do not consider the information exchange process explicitly (baseline model), or analyze it with cheap talk game setting (extended model). Firm's output serves as a signal which may reveal each firm's type in my model.

Sobel (1985) is also related to one extension in this paper, which studies the possibility that a firm with low discount rate may not deviate from cartel agreement for some periods. The key distinction between two researches is that Sobel (1985) studies sender-receiver model while this paper looks at the situation where both firms are a sender and receiver simultaneously. This paper, however, focuses only on the pure strategy equilibrium because I do not consider the trust building process.

The rest of the paper is organized as follows. Section 3.2 introduces perfect information model as a counterfactual, and then Bayesian model is constructed in order to look at the main topic of this paper in Section 3.3. After finding the equilibrium, I compare the properties of equilibrium outcome in each model. I perform some comparative statics, such as entry, exit or merger in both settings. The following two sections extend the baseline model; Section 3.4 deals with the environment where firms can agree on a collusion with less than monopoly profit until the uncertainty on the other firm's discount rate is resolved, and Section 3.5 introduces the possibility that firms communicate each other on their discount rate in a cheap talk pregame and may agree on uneven split of monopoly profit. Conclusion follows in Section 3.6.

3.2 Perfect Information Model : Counterfactual

Assume an oligopoly industry with perfect information on each other's discount rate across firms. I consider a problem whether there is a collusion equilibrium in this environment. A general way to deal with collusion is to use the infinitely repeated game. But the same problem can be analyzed by using a strategic game form where payoff is given by the sum of current payoff and continuation payoff. In this section I will characterize the equilibrium after I represent a dynamic game of complete information with an equivalent strategic game. The same analysis can be done by subgame perfect equilibrium concept, but the main result does not change from the following analysis.

3.2.1 Duopoly Collusion Game

As a simplest counterfactual, I will start from a perfect information duopoly game. There are two firms in the industry, and $i \in \{1, 2\}$ denotes each firm. Nature draws firm $i's$ discount rate $\delta_i \in \Delta \equiv \{\delta^L, \delta^H \mid \delta^L < \delta^H\}$ with $\Pr(\delta^H) = \gamma$, and each firm knows not only its own discount rate δ_i, but also the other firm's discount rate δ_{-i} in this section. Put it differently, there is uncertainty in the ex-ante sense, but the uncertainty is removed in the interim sense. $\pi_i(q_i, q_{-i})$ represents firm i's profit when it produces q_i while the other produces q_{-i}. $\pi_i(q_i, q_{-i})$ is strictly concave in q_i, decreasing in q_{-i}, continuous, and continuously differentiable. Each firm chooses whether to join cartel or not. If any firm chooses not to join, cartel cannot be agreed on and each firm earns Cournot-Nash equilibrium profit $\pi^n = \pi_1(q^n, q^n) = \pi_2(q^n, q^n)$ in each period. I assume that there exists a unique stage game Nash equilibrium. If both firms choose to join, cartel is agreed upon. In this case, they can also choose their output. If each firm chooses cartel output q^c, cartel is sustained and each firm gets cartel payoff $\pi^c = \pi_1(q^c, q^c) = \pi_2(q^c, q^c)$ in each period.[1] If at least one firm chooses output q_i different from q^c, cartel breaks down and $(1 - \delta_i)\pi_i(q_i, q_{-i}) + \delta_i\pi^r$ would be each firm's average profit per period. This payoff captures the punishment phase after any one firm's deviation from

[1] In this paper, I assume symmetric payoff for simplicity and comparative statics discussed in Section 3.2.3 and Section 3.3.3

Table 3.1: Payoff Matrix of Duopoly Game

F 1 \ F 2	Join, q^c	Join, $q_2 \backslash q^c$	Not Join
Join, q^c	$[\pi^c, \pi^c]$	$[(1-\delta_1)\pi_1(q^c, q_2) + \delta_1\pi^r,$ $(1-\delta_2)\pi_2(q_2, q^c) + \delta_2\pi^r]$	$[\pi^n, \pi^n]$
Join, $q_1 \backslash q^c$	$[(1-\delta_1)\pi_1(q_1, q^c) + \delta_1\pi^r,$ $(1-\delta_2)\pi_2(q^c, q_1) + \delta_2\pi^r]$	$[(1-\delta_1)\pi_1(q_1, q_2) + \delta_1\pi^r,$ $(1-\delta_2)\pi_2(q_2, q_1) + \delta_2\pi^r]$	$[\pi^n, \pi^n]$
Not Join	$[\pi^n, \pi^n]$	$[\pi^n, \pi^n]$	$[\pi^n, \pi^n]$

cartel agreement. Here, π^r is the per-period profit of firm i in the punishment phase after at least one firm deviates. Note that I am using per-period profit instead of discounted profit, but this adjustment does not affect the result. Under this setting, the strategic game can be represented by

$$
\begin{aligned}
G_D^{FI} \quad &= \quad (N, A, \pi \mid \Delta, \gamma)\,,\ where \\
&N = \{1, 2\} \text{ is a set of firms} \\
&A = \Pi_{i=1}^2 A_i \text{ is action space, where} \\
&\quad A_i = \{(Join, q_i), (Not\ Join) \mid q_i \in R_+\} \\
&\pi = \{\pi_1, \pi_2\} \text{ is a payoff vector, where } \pi_i : A \mapsto R\ \ s.t \\
&\quad \pi_i(s_i, s_{-i} \mid \delta_i) = \begin{cases} \pi^c & if\ s_i = (Join, q^c)\ for\ \forall\ i \\ \pi^n & if\ s_i = (Not\ Join)\ for\ \exists\ i \\ (1-\delta_i)\pi_i(q_i, q_{-i}) + \delta_i\pi^r & otherwise \end{cases}
\end{aligned}
$$

Suppose that $\infty > \pi^{d+} > \pi^c > \pi^n > \pi^{d-} > 0$, where $\pi^{d+} = \pi(q(q^c), q^c)$ and $\pi^{d-} = \pi(q^c, q(q^c))$. Here, $q(q^c)$ is one firm's best response output to the other firm's cartel output. Given the strategy profile $s = (s_1, s_2) \in A$, the payoff matrix of the game $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$ is given by Table 3.1.

Let the threshold discount factor δ^* be the smallest $\delta \in (0, 1)$ which satisfies $\pi^c \geq (1-\delta)\pi^{d+} + \delta\pi^r$ for all $\delta \in [\delta^*, 1)$. If $\delta^* \leq \delta^L$, it is easy to see that $s^* = \Pi_{i=1}^2(Join, q^c)_i$ is a unique efficient Nash equilibrium. If $\delta^H \leq \delta^*$, on the other hand, $s^* = \Pi_{i=1}^2(Not\ Join)_i$ is a unique Nash equilibrium. Put it in extensive form terminology, cartel can be supported as subgame perfect equilibrium if the discount rates of all types are greater than δ^*, while cartel cannot be supported as subgame perfect equilibrium if all of them are less than δ^*.

The most interesting case is when $\delta^L < \delta^* < \delta^H$. In the perfect information setting, however, there is no uncertainty on δ_{-i} when firms move, so it is common knowledge which is the case that they face between (1) $\delta_i = \delta^H > \delta^*$ for every firm $i \in N$ and (2) $\delta_i = \delta^L < \delta^*$ for some firm $i \in N$. Clearly, collusion can be supported as a unique efficient Nash equilibrium only in case (1) while $s^* = \Pi_{i=1}^{2}(Not\ Join)_i$ is a unique Nash equilibrium in case (2).[2] From the ex-ante point of view, if $\delta^L < \delta^* < \delta^H$ holds, the probability that cartel is supported as an efficient Nash equilibrium is given by γ^2 for every $\gamma \in [0,1]$. It is also easy to see that cartel can be sustained forever if it is supported as an efficient Nash equilibrium. Claim 13 summarizes the argument so far.

Claim 13. *Given $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$ and $\delta^* \in (0,1)$, the ex-ante equilibrium outcome in the duopoly market is given as follows.*

(a) firms collude for sure if $\delta^* \leq \delta^L$,

(b) firms do not collude for sure if $\delta^* \geq \delta^H$, and

(c) firms collude with probability γ^2 and do not with probability $(1-\gamma^2)$ if $\delta^L < \delta^* < \delta^H$. Moreover, if they collude in the equilibrium outcome, the cartel is sustained forever.

3.2.2 Oligopoly Collusion Game

I now consider an oligopoly market where there are n firms. As before, nature draws each firm $i's$ discount rate $\delta_i \in \Delta$ with $\Pr(\delta^H) = \gamma$, and the uncertainty on discount rates is removed when firms move. So each firm knows both its discount rate δ_i and all other firms' discount rate vector $\delta_{-i} = (\delta_1, \cdots, \delta_{i-1}, \delta_{i+1}, \cdots, \delta_n)$.

[2] I do not allow the possibility that cartel members split monopoly profit unevenly in this baseline model. This possibility is introduced in Section 3.5.

Then, the strategic game can be represented as follows.

$$
\begin{aligned}
G_O^{FI} \;\; = \;\; & (N, A, \pi \mid \Delta, \gamma)\,, where \\
& N = \{1, 2, \cdots, n\} \\
& A = \Pi_{i=1}^{n} A_i \;\; s.t \;\; A_i = \{(Join, q_i), (Not\ Join) \mid q_i \in R_+\} \\
& \pi = \{\pi_1, \pi_2, \cdots, \pi_n\}, \text{ where } \pi_i : A \mapsto R \;\; s.t \\
& \qquad \pi_i(s_i, s_{-i} \mid \delta_i) = \begin{cases} \pi^c & if\ s_i = (Join, q^c)\ for\ \forall\ i \\ \pi^n & if\ s_i = (Not\ Join)\ for\ \exists\ i \\ (1-\delta_i)\pi_i(q_i, q_{-i}) + \delta_i \pi^r & otherwise \end{cases}
\end{aligned}
$$

Similar to duopoly case, I assume that $\infty > \pi^{d+} > \pi^c > \pi^n > 0$, where $\pi^{d+} = \pi_i(q(q^c_{-i}), q^c_{-i})$. Here, $q(q^c_{-i})$ represents each firm's best response output when all other (n-1) firms produce cartel output q^c. Let $\delta^* = f(n)$ be the smallest δ which satisfies $\pi^c \geq (1-\delta)\pi^{d+} + \delta\pi^r$ for all $\delta \in [\delta^*, 1)$. The threshold discount rate δ^* depends on the number of firms. Then, $s^* = \Pi_{i=1}^{n}(Join, q^c)_i$ is a unique efficient Nash equilibrium for $\delta^* \leq \delta^L$, whereas $s^* = \Pi_{i=1}^{n}(Not\ Join)_i$ is a unique Nash equilibrium for $\delta^H \leq \delta^*$. Suppose $\delta^* \in (\delta^L, \delta^H)$. Since there is no uncertainty on δ_{-i} when firms move, it is common knowledge which is the case that they face between (1) $\delta_i = \delta^H$ for every firm $i \in N$ and (2) $\delta_i = \delta^L < \delta^*$ for some firm $i \in N$. Using the same argument in duopoly case, I can obtain the result similar to Claim 13.

Claim 14. *Given $G_O^{FI} = (N, A, \pi \mid \Delta, \gamma)$ and $\delta^* \in (0, 1)$, the ex-ante equilibrium outcome in the oligopoly market is given as follows.*

(a) firms collude for sure if $\delta^* \leq \delta^L$,

(b) firms do not collude for sure if $\delta^* \geq \delta^H$, and

(c) firms collude with probability γ^n and do not with probability $(1-\gamma^n)$ if $\delta^L < \delta^* < \delta^H$. Moreover, if they collude in the equilibrium outcome, then the cartel is sustained forever.

3.2.3 Comparative Statics under Perfect Information

Given this characterization of equilibrium outcome in $G_O^{FI} = (N, A, \pi \mid \Delta, \gamma)$, it is possible to see how entry, exit or merger affects the competitive behavior of firms in the industry. I assume $f'(n) > 0$ for this comparative statics.[3]

Entry

An entrant is assumed to have $\delta_E \in \Delta$ with $\Pr(\delta_E = \delta^H) = \gamma$ like the incumbents. Denote $\gamma_u(n) \equiv 1_{\delta^H > f(n)} * \gamma + 1_{\delta^L > f(n)} * (1-\gamma)$, where $1_{\delta^x > f(n)}$ is an indicator function. In the ex-ante sense, the industry would be colluding with probability $(\gamma_u(n))^n$ before this entry. If a firm enters the industry, the threshold discount rate δ^* increases from $f(n)$ to $f(n+1)$. Hence, $\gamma_u(n+1) \leq \gamma_u(n)$ holds. But then, it is clear that $(\gamma_u(n+1))^{n+1} \leq (\gamma_u(n))^n$, where equality holds if and only if $\gamma_u(n+1) = \gamma_u(n) = 1$ ($\Leftrightarrow \delta^L > f(n+1) > f(n)$) or $\gamma_u(n+1) = \gamma_u(n) = 0$ ($\Leftrightarrow f(n+1) > f(n) \geq \delta^H$). Therefore, entry decreases the probability that an oligopoly industry colludes in the ex-ante sense.

For the interim analysis, suppose that an industry was colluding before entry. Then, this cartel would collapse either if there exists an incumbent $i \in N$ whose discount rate $\delta_i \in [f(n), f(n+1))$ or if $\delta_E < f(n+1)$. If the industry was not colluding before entry, there exists at least one firm i among incumbents such that $\delta_i < f(n)$. Since $\delta_i < f(n) < f(n+1)$ holds for the firm i, this industry does not collude after entry either.

Exit and Merger

An exit from an oligopoly market has exactly the opposite effect compared with entry. Since $(\gamma_u(n))^n \leq (\gamma_u(n-1))^{n-1}$, exit increases the probability that an oligopoly industry colludes in the ex-ante sense. For the interim analysis, suppose an industry was not colluding before exit. The industry would collude after exit if $\delta_i > f(n-1)$ holds for all the remaining firms and not collude if there is at least one firm i among the remaining firms such that $\delta_i \leq f(n-1)$. If the industry was colluding before exit, it clearly colludes after exit as well.

In order to see the effect of merger on firms' competitive behavior, I need to assume the merged firm's discount rate δ_M. Suppose that a merger is implemented between two firms indexed by $(n-1)$ and n, and that δ_M is arbitrarily picked between δ_{n-1} and δ_n. Then, all

[3] If we have linear demand and identical constant marginal cost like Salant, Switzer, and Reynolds (1983), the appendix of Chapter 2 shows that this assumption holds for Nash reversion and optimal punishment strategy.

results in the case of exit can be equally applied to merger case.

One important characteristic in these comparative statics is that the ex-ante probability of collusion strictly increases after exit or merger as long as either $f(n) \in (\delta^L, \delta^H)$ or $f(n-1) \in (\delta^L, \delta^H)$ holds under this perfect information model.

3.3 Asymmetric Information Model on Discount rate

I now introduce asymmetric information about each other's discount rate. In order to analyze collusion formation and sustainability under this setting, I construct a Bayesian game. I perform the analysis after I represent a dynamic game of incomplete information with an equivalent Bayesian game form. The same analysis can be done by using perfect Bayesian equilibrium concept, but the main result does not change from the following analysis.

3.3.1 Duopoly Bayesian Collusion Game

All the settings of the duopoly Bayesian game are exactly the same with $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$ in Section 3.2 except the presence of asymmetric information on the other's discount rate. So each firm i knows its own discount rate δ_i, but does not know the other firm's type δ_{-i} except γ. Under this setting, the Bayesian game $G_D^B = (N, A, \Theta, \pi, p)$ is given by

$$
\begin{aligned}
G_D^B \;=\; & (N, A, \Theta, \pi, p)\,,\ where \\
& N = \{1, 2\} \\
& A = \Pi_{i=1}^2 A_i \ \ s.t \ \ A_i = \{(Join, q_i), (Not\ Join) \mid q_i \in R_+\} \\
& \Theta = \Pi_{i=1}^2 \Theta_i \text{ is type space, where } \ \Theta_i = \Delta \\
& \pi = \{\pi_1, \pi_2\}, \text{ where } \pi_i : A\ X\ \Theta_i \mapsto R \ \ s.t \\
& \qquad \pi_i(s_i, s_{-i}|\ \delta_i) = \begin{cases} \pi^c & if\ s_i = (Join, q^c)\ for\ \forall\ i \\ \pi^n & if\ s_i = (Not\ Join)\ for\ \exists\ i \\ (1-\delta_i)\pi_i(q_i, q_{-i}) + \delta_i \pi^r & otherwise \end{cases} \\
& p \in \Delta(\Theta) \text{ is the prior type distribution such that } \delta_1 \perp \delta_2 \text{ and } \Pr(\delta^H) = \gamma
\end{aligned}
$$

Suppose again $\infty > \pi^{d+} > \pi^c > \pi^n > \pi^{d-} > 0$. A firm $i's$ pure strategy is a map $S_i : \Theta_i \mapsto A_i$, and a pure strategy Bayesian Nash equilibrium (PBNE) of this game is defined as a strategy profile $s^* = \{s_1^*, s_2^*\}$ such that

$$\begin{aligned} &\gamma\pi_i(s_i^*(\delta_i), s_{-i}^*(\delta^H); \delta_i) + (1-\gamma)\pi_i(s_i^*(\delta_i), s_{-i}^*(\delta^L); \delta_i) \\ &\qquad \geq \ \gamma\pi_i(a_i, s_{-i}^*(\delta^H); \delta_i) + (1-\gamma)\pi_i(a_i, s_{-i}^*(\delta^L); \delta_i) \\ &\qquad for\ every\ i \in N,\ \delta_i \in \Theta_i,\ and\ a_i \in A_i. \end{aligned} \tag{3.1}$$

Similar to the perfect information game, if the threshold discount rate $\delta^* \leq \delta^L$, $s_i^*(\delta_i) = (Join, q^c)$ is an efficient PBNE for each firm $i \in N$. Likewise, if $\delta^H \leq \delta^*$, $s_i^*(\delta_i) = (Not\ Join)$ is a unique PBNE for each firm $i \in N$. In other words, cartel is supported as PBNE if the discount rate of every type is greater than δ^*, while cartel is not supported as PBNE if the discount rate of every type is less than δ^*.

The most interesting case is when $\delta^L < \delta^* < \delta^H$. One trivial PBNE is $s_i^*(\delta_i) = (Not\ Join)$ for all firm $i \in N$ as in other cases. Let me consider a pure strategy in which both type of firms choose to join and a firm with $\delta_i = \delta^H$ produces cartel output q^c. Since $\delta^L < \delta^*$, any firm with $\delta_i = \delta^L$ has an incentive to deviate. But there is a possibility $(1-\gamma)$ that the other firm also has low discount factor. So when the low type firm chooses its output, it has to consider the possibility that the other firm may also deviate. Formally, suppose the other firm produces q^c when $\delta_{-i} = \delta^H$ and a given output q_{-i} when $\delta_{-i} = \delta^L$. Then, in order to maximize the expected profit of current period, the low type firm i has to solve

$$\max_{q_i} \ \gamma\pi_i(q_i, q^c) + (1-\gamma)\pi_i(q_i, q_{-i})$$

This optimization problem gives me each firm's best-response function $q_i = q^d(q_{-i}, \gamma)$. Hence, I can get $q^d = q^d(\gamma)$ as a fixed point. Then I can derive a useful result for the following analysis.

Lemma 1. *If $p'(Q) + q_i p''(Q) < 0$, then $\frac{dq^d}{d\gamma} > 0$ holds.*

Proof. See the appendix. □

$p'(Q) + q_i p''(Q) < 0$ is a standard and weak assumption in quantity setting game. [Dixit (1986), Farrell and Shapiro (1990), etc.] Lemma 1 says that higher belief on $\delta_{-i} = \delta^H$ induces low type firm to produce more when it deviates. Here is one example that illustrates the argument so far.

Example 3.1. *Assume that $P = 1 - Q$ and $MC_i = 0$ for $i = 1, 2$. Then, $q^c = \frac{1}{4}$ for each firm. In order to get $q^d(\gamma)$, solve*

$$\max_{q_i} \ \gamma(1 - q_i - \frac{1}{4})q_i + (1-\gamma)(1 - q_i - q_{-i})q_i$$

Then, the best-response function is given by $q_i = \frac{3}{8}\gamma + \frac{1}{2}(1-\gamma)(1-q_{-i})$. So, the fixed point comes to

$$q^d(\gamma) = (\frac{4-\gamma}{12-4\gamma}, \frac{4-\gamma}{12-4\gamma})$$

Hence, $\frac{dq^d}{d\gamma} = \frac{1}{4(3-\gamma)^2} > 0$ holds, as expected.

Using the fixed point output $q^d(\gamma)$, I can construct one pure strategy $\bar{s}_i(\delta_i)$:

$$\bar{s}_i(\delta_i) = \begin{cases} (Join, q^c) & if \quad \delta_i = \delta^H \\ (Join, q^d(\gamma)) & if \quad \delta_i = \delta^L \end{cases} \tag{3.2}$$

This strategy shows that both type of firms join collusion but low type firm deviates by choosing the output reflecting its belief on $\delta_{-i} = \delta^H$. Under this strategy, the interim payoff of firm i with δ^H is given by

$$\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) = \gamma\pi^c + (1-\gamma)\left[(1-\delta^H)\pi_i(q^c, q^d(\gamma)) + \delta^H\pi^r\right] \tag{3.3}$$

The other firm has δ^H with probability γ, and the payoff of firm i comes to π^c in this case because both firms stick to cartel agreement. But the other firm has δ^L with probability $(1-\gamma)$, and it deviates with the fixed point output $q^d(\gamma)$. So $\pi_i(q^c, q^d(\gamma))$ is the current payoff of firm i and π^r is the continuation payoff in this case.

Similarly, the interim payoff of firm i with δ^L yields

$$\begin{aligned}\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) &= \gamma \left[(1-\delta^L)\pi_i(q^d(\gamma), q^c) + \delta^L \pi^r\right] \\ &\quad +(1-\gamma)\left[(1-\delta^L)\pi_i(q^d(\gamma), q^d(\gamma)) + \delta^L \pi^r\right] \qquad (3.4)\end{aligned}$$

Then I get the fundamental result of this research.

Proposition 2. *Suppose $\delta^L < \delta^* < \delta^H$. Then, there exists $\gamma^* \in (0,1)$ such that a strategy profile $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2))$ in equation (3.2) can be supported as a PBNE for all $\gamma \in (\gamma^*, 1)$ in the Bayesian game $G_D^B = (N, A, \Theta, \pi, p)$.*

Proof. See the appendix. □

To see why this result holds, suppose that the belief γ is greater than the threshold belief γ^*. Then, a firm with high discount factor can expect higher payoff when it sticks to cartel agreement because it believes that cartel is sustained with sufficiently high probability. On the other hand, a firm with low discount rate also believes that the chance of $\delta_{-i} = \delta^H$ is high enough so that it benefits from joining and deviating the cartel. PBNE $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2))$, if exists, is clearly more efficient than another PBNE $s^* = ((Not\ Join), (Not\ Join))$. The equilibrium outcome of PBNE $\bar{s}$ is determined by the combination of two firms' discount rates.

Corollary 2. *Suppose that $\delta^L < \delta^* < \delta^H$ and $\gamma \in (\gamma^*, 1)$. Then, the equilibrium outcome of a PBNE $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2))$ is given as follows.*

(a) if $(\delta_1, \delta_2) = (\delta^H, \delta^H)$, then cartel is agreed and sustained.

(b) if $(\delta_1, \delta_2) \neq (\delta^H, \delta^H)$, then cartel is agreed but is not sustained.

The probability of each outcome equals to γ^2 and $\left(1-\gamma^2\right)$, respectively.

So if there is asymmetric information about the other firm's discount rate, it is possible that cartel is agreed on even when one or both firms' incentive constraint for sustaining cartel in subsequent periods is not satisfied. Hence, cartel under this Bayesian environment may collapse with probability $\left(1-\gamma^2\right)$. Put it differently, punishment phase can be an equilibrium outcome path in the Bayesian game, which never happens under the perfect information.

Another interesting feature occurs when $\gamma \leq \gamma^*$ and both firms have high discount rates. This case may happen with probability γ^2. In that case, even though all firms satisfy the incentive constraint for collusion under perfect information, the lack of belief on $\delta_{-i} = \delta^H$ prevents them from agreeing on collusion. It happens because each firm puts the possibility of its partner's deviation too high to collude. If I compare these results with the perfect information game $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$, I obtain the following result.

Corollary 3. *Suppose that $\delta^L < \delta^* < \delta^H$.*

(a) While firms collude with (ex-ante) probability γ^2 and do not with (ex-ante) probability $(1-\gamma^2)$ for every $\gamma \in (0,1)$ in $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$, firms agree on collusion with probability 1 for any $\gamma \in (\gamma^*, 1)$ in $G_D^B = (N, A, \Theta, \pi, p)$.

(b) Moreover, while cartel is sustained forever if it is agreed on in $G_D^{FI} = (N, A, \pi \mid \Delta, \gamma)$, the cartel agreement in $G_D^B = (N, A, \Theta, \pi, p)$ has the probability $\left(1-\gamma^2\right)$ of deviation.

3.3.2 Oligopoly Bayesian Collusion Game

I can extend the duopoly Bayesian game into the environment where there are n firms and 2 types of discount rates. Then the Bayesian game comes to

$$
\begin{aligned}
G_O^B \;&=\; (N, A, \Theta, \pi, p)\,,\ where \\
&N = \{1, 2, \cdots, n\} \\
&A = \Pi_{i=1}^n A_i \ \text{ s.t } \ A_i = \{(Join, q_i), (Not\ Join) \mid q_i \in R_+\} \\
&\Theta = \Pi_{i=1}^n \Theta_i \ \text{ s.t } \ \Theta_i = \Delta \\
&\pi = \{\pi_1, \pi_2, \cdots, \pi_n\}, \text{ where } \pi_i : A\ X\ \Theta_i \mapsto R \ \ s.t \\
&\qquad \pi_i(s_i, s_{-i} \mid \delta_i) = \begin{cases} \pi^c & if\ \ s_i = (Join, q_i^c)\ for\ \forall\ i \\ \pi^n & if\ \ s_i = (Not\ Join)\ for\ \exists\ i \\ (1-\delta_i)\pi_i(q_i, q_{-i}) + \delta_i \pi^r & otherwise \end{cases} \\
&p \in \Delta(\Theta) \text{ s.t } \ \delta_i \sim i.i.d \text{ with } \Pr(\delta_i = \delta^H) = \gamma
\end{aligned}
$$

Each firm knows its type $\delta_i \in \Theta_i$, but does not know the other firm's type vector δ_{-i} except γ. A firm $i's$ pure strategy is a map $S_i : \Theta_i \mapsto A_i$, and a PBNE of the game is a strategy profile

$s^* = (s_1^*, s_2^*, \cdots, s_n^*)$ such that

$$\begin{aligned} &\sum_{j=0}^{n-1} {}_{n-1}C_j \ \gamma^{n-1-j}(1-\gamma)^j \pi_i(s_i^*(\delta_i), s_{-i}^*(\delta_{-i}(j)); \delta_i) \\ &\quad \geq \sum_{j=0}^{n-1} {}_{n-1}C_j \ \gamma^{n-1-j}(1-\gamma)^j \pi_i(a_i, s_{-i}^*(\delta_{-i}(j)); \delta_i) \\ &\quad\quad for\ every\ i \in N,\ \delta_i \in \Theta_i\ and\ a_i \in A_i. \end{aligned} \tag{3.5}$$

Here, $\delta_{-i}(j) = (\delta_1, \cdots, \delta_{i-1}, \delta_{i+1}, \cdots, \delta_n)$ is a vector representing (n-1) other firms' discount rate profiles in which j many firms have type δ^L and (n-j-1) many firms have type δ^H. Like the perfect information game, $s^* = \Pi_{i=1}^n (Join, q^c)_i$ is a unique efficient PBNE if $\delta^* \leq \delta^L$ while $s^* = \Pi_{i=1}^n (Not\ Join)_i$ is a unique PBNE if $\delta^H \leq \delta^*$.

Suppose that $\delta^* \in (\delta^L, \delta^H)$ similar to the duopoly case. One trivial PBNE is $s_i^*(\delta_i) = (Not\ Join)$ for each firm $i \in N$. Again, consider a pure strategy in which both type of firms choose to join the cartel and a firm with $\delta_i = \delta^H$ produces cartel output q^c. While any firm with $\delta_i = \delta^L$ has an incentive to deviate, there is a possibility of $(1 - \gamma^{n-1})$ that one or more other firms also have low discount rate δ^L. When it deviates, it has to maximize the expected profit given (q_{-i}, γ), where $q_{-i} = (q_1, \cdots, q_{i-1}, q_{i+1}, \cdots q_n)$ is other firms' given output profile in which firm 1 is assumed to choose q_1 when $\delta_1 = \delta^L$, firm 2 choose q_2 when $\delta_2 = \delta^L$ and so on. Formally, given (q_{-i}, γ), firm i solves

$$\max_{q_i} \sum_{j=0}^{n-1} {}_{n-1}C_j \ \gamma^{n-1-j}(1-\gamma)^j \pi_i(q_i, q_{-i}^j), \text{ where } q_{-i}^j = q_{-i}^c(n-1-j) + q_{-i}(j)$$

Here, $q_{-i}^c(n-1-j)$ represents $(n-1)$ dimensional vector that has $(n-1-j)$ many cartel output q^c and j many $0's$, and $q_{-i}(j)$ denotes $(n-1)$ dimensional vector that has $(n-1-j)$ many $0's$ in the coordinate where $q_{-i}^c(n-1-j)$ has q^c as its element and has j many given q_k for $k \in \{1, \cdots, i-1, i+1, \cdots, n\}$ in the coordinate where $q_{-i}^c(n-1-j)$ has 0 as its element. Note that there are ${}_{n-1}C_j$ cases with $(n-1-j)$ many cartel output q^c and j many given q_k. $\pi_i(q_i, q_{-i}^j)$ is defined by the average profit of all those cases given q_{-i}. If $n = 2$, then the above problem is exactly the same with the duopoly case. This optimization problem gives me the best-response function $q_i = q^d(q_{-i}, \gamma)$ for each firm. Again, I can construct

$q^d = (q^d(\gamma), q^d(\gamma), \cdots, q^d(\gamma))$ as a fixed point. Now, I can define the pure strategy for each firm $i \in N$, which is equivalent to the duopoly case.

$$\bar{s}_i(\delta_i) = \begin{cases} (Join, q^c) & if \quad \delta_i = \delta^H \\ (Join, q^d(\gamma)) & if \quad \delta_i = \delta^L \end{cases} \tag{3.6}$$

I will denote firm i's average payoff when firm i chooses q_i given that j many other firms have low discount factor by $\pi_i^{j-i}(q_i) = \pi_i(q_i, q_{-i}^d(j))$, where $q_{-i}^d(j) = (q_1, q_2, \cdots, q_{i-1}, q_{i+1}, \cdots, q_n)$ with j many $q^d(\gamma)$ and $(n-1-j)$ many q^c. Under the strategy $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2), \cdots, \bar{s}_n(\delta_n))$, the interim payoff of firm i with type δ^H is given by

$$\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) = \gamma^{n-1}\pi^c + \sum_{j=1}^{n-1} {}_{n-1}C_j \ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^H)\pi_i^{j-i}(q^c) + \delta^H \pi^r] \tag{3.7}$$

while the interim payoff of firm i with type δ^L yields

$$\begin{aligned} \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) \quad = \quad & \gamma^{n-1}[(1-\delta^L)\pi_i(q^d(\gamma), q_{-i}^c) + \delta^L \pi^r] + \\ & \sum_{j=1}^{n-1} {}_{n-1}C_j \ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^L)\pi_i^{j-i}(q^d(\gamma)) + \delta^L \pi^r] \end{aligned} \tag{3.8}$$

Using the same technique, I can get an equivalent result with Proposition 2.

Proposition 3. *Suppose $\delta^L < \delta^* < \delta^H$. Then, there exists $\gamma^* \in (0,1)$ such that a strategy profile $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2), \cdots, \bar{s}_n(\delta_n))$ in equation* (3.6) *can be supported as a PBNE for all $\gamma \in (\gamma^*, 1)$ in the Bayesian game $G_O^B = (N, A, \Theta, \pi, p)$.*

Proof. See the appendix. □

The result holds for the same reason with the duopoly case. If γ is higher than the threshold belief γ^*, a high type firm expects higher payoff when it sticks to cartel agreement because it believes that cartel is sustained with sufficiently high probability. On the other hand, a low type firm also believes that the chance of $\delta_{-i} = (\delta^H, \cdots, \delta^H)$ is high and that it benefits from cartel agreement and unilateral deviation. This belief makes it possible for cartel to be agreed on when one or more firms fail to satisfy the incentive constraint. As in the duopoly, a PBNE

$\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2), \cdots, \bar{s}_n(\delta_n))$ is more efficient than another PBNE $s^* = \Pi_{i=1}^{n}(Not\ Join)_i$. The equilibrium outcome of PBNE $\bar{s}$ depends on the combination of n firms' discount factors.

Corollary 4. *Suppose that $\gamma \in (\gamma^*, 1)$. Then, the equilibrium outcome of a PBNE $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2), \cdots, \bar{s}_n(\delta_n))$ is given as follows.*

(a) if $\delta_i = \delta^H$ for all firm i, then cartel is agreed and sustained.

(b) if $\delta_i = \delta^L$ for a firm i, then cartel is agreed but is not sustained.

The probability of each outcome equals to γ^n and $(1 - \gamma^n)$, respectively.

The implication of this result is also the same with the duopoly case. Collusion in this Bayesian environment may collapse from the beginning with probability $(1 - \gamma^n)$. Again, as in the duopoly case, if $\gamma < \gamma^*$ holds, collusion cannot be agreed on even when all firms satisfy the incentive constraint ($i.e$ $\delta_i = \delta^H$ for all firm i). Comparison with the perfect information game $G_O^{FI} = (N, A, \pi \mid \Delta, \gamma)$ gives the following result.

Corollary 5. *Suppose $\delta^L < \delta^* < \delta^H$.*

(a) While firms collude with (ex-ante) probability γ^n and do not collude with (ex-ante) probability $(1 - \gamma^n)$ for every $\gamma \in (0, 1)$ in $G_O^{FI} = (N, A, \pi \mid \Delta, \gamma)$, firms agree on collusion with probability 1 for any $\gamma \in (\gamma^*, 1)$ in $G_O^B = (N, A, \Theta, \pi, p)$.

(b) Moreover, while cartel is sustained forever if it is agreed on in $G_O^{FI} = (N, A, \pi \mid \Delta, \gamma)$, the cartel agreement in $G_O^B = (N, A, \Theta, \pi, p)$ has the probability $(1 - \gamma^n)$ of deviation from the beginning.

3.3.3 Implication to comparative statistics

Given the above characterization of equilibrium outcome in $G_O^B = (N, A, \Theta, \pi, p)$, I revisit the problem how entry, exit or merger affects the competitive behavior of firms in the industry. I maintain the assumption that $\delta^* = f(n)$ is an increasing function with respect to n.

Entry

As in the perfect information setting, an entrant is assumed to have $\delta_E \in \Delta = \{\delta^H, \delta^L\}$ with $\Pr(\delta_E = \delta^H) = \gamma$. Before entry, the industry would be in one situation among 3 possible cases : (1) $\gamma \in (0, \gamma^*(n))$, (2) $\gamma \in (\gamma^*(n), 1)$ and $\delta_i = \delta^L < f(n)$ for at least one firm $i \in N$,

(3) $\gamma \in (\gamma^*(n), 1)$ and $\delta_i \geq f(n)$ for all firm $i \in N$. Here, $\gamma^*(n)$ represents the threshold belief γ^* when there are n firms in the industry. Then, $\gamma^*(n) = 1$ and cartel agreement is impossible if $f(n) \geq \delta^H$, whereas $\gamma^*(n) = 0$ and cartel is agreed for sure if $f(n) \leq \delta^L$.

If $\gamma \in (0, \gamma^*(n))$ holds, firms could not agree on collusion pre-entry. So the belief γ is not updated. On the other hand, entry changes each firm's payoff ($\pi^{d+}, \pi^c, \pi^n, \pi^{j_{-i}}, \pi^r$ *etc.*) and increases the number of firms in the industry. Hence, $(n+1)$ firms including entrant play a similar Bayesian game $G_O^B = (N, A, \Theta, \pi, p)$. If $\gamma^*(n+1)$ in the new Bayesian game is still greater than γ, firms do not agree on cartel post-entry either.

Next suppose that $\gamma \in (\gamma^*(n), 1)$ and $\delta_i = \delta^L < f(n)$ for at least one firm $i \in N$ pre-entry. Then, the industry would be in the punishment phase before entry because firm i deviates after cartel agreement. Moreover, $\delta_i = \delta^L$ is common knowledge for all incumbents and the entrant. Hence, the industry does not collude after entry either because all firms know that firm i will deviate again due to $\delta_i = \delta^L < f(n) < f(n+1)$.

Finally, consider the case that $\gamma \in (\gamma^*(n), 1)$ and $\delta_i \geq f(n)$ for all firm $i \in N$. In this case, firms collude before entry and it is common knowledge that $\delta_i \geq f(n)$ holds for all incumbents. Clearly, cartel is impossible after entry if $f(n+1) \geq \delta^H$ while cartel is always possible after entry if $f(n) < f(n+1) \leq \delta^L$. If $\delta^L < f(n+1) < \delta^H$, two different kinds of Bayesian game can be played.

One is the subcase that $f(n) \leq \delta^L < f(n+1) < \delta^H$ holds. In this case, there is no update on the incumbents' discount rates. So the incumbents and the entrant play a Bayesian oligopoly game $G_O^B = (N, A, \Theta, \pi, p)$ similar to case 1. For the same reason, firms will not agree on collusion after entry if $\gamma^*(n+1) > \gamma$ holds, while they continue to agree on cartel if $\gamma^*(n+1) < \gamma$ holds. In the latter case, cartel may collapse with probability $(1 - \gamma^{n+1})$.

The other is the subcase that $\delta^L < f(n) < f(n+1) < \delta^H$ holds. Since cartel was sustained before entry, it is common knowledge that $\delta_i = \delta^H > f(n+1) > f(n)$ for all incumbents. So the only uncertainty is about the entrant's discount rate δ_E. Because every incumbent's type is revealed as δ^H, its strategy is just to pick $s_i \in A_i = \{(Join, q_i), (Not\ Join) \mid q_i \in R_+\}$. On the other hand, the entrant's strategy is a function $\Delta \rightarrow A_E = \{(Join, q_E), (Not\ Join) \mid q_E \in R_+\}$. In order to support cartel agreement as equilibrium outcome, each incumbent i has to choose $\bar{s}_i = (Join, q^c)$ and the entrant has to choose

$$\bar{s}_E(\delta_E) = \begin{cases} (Join, q^c) & if \quad \delta_E = \delta^H \\ (Join, q(nq^c)) & if \quad \delta_E = \delta^L \end{cases}$$

In words, the entrant joins the cartel and chooses cartel output when its type is δ^H while it chooses to join but deviates with best response output against sum of incumbents' cartel output. Then, each incumbent's expected payoff under the strategy profile $\bar{s} = (\bar{s}_1, \cdots, \bar{s}_n, \bar{s}_E(\delta_E))$ $\equiv (\bar{s}_{In}, \bar{s}_E(\delta_E))$ is given by

$$\Pi_i(\bar{s}_i, \bar{s}_{-i}, \bar{s}_E(\delta_E); \delta^H) = \gamma\pi^c + (1-\gamma)[(1-\delta^H)\pi_i(q^c_{In}, q(nq^c)) + \delta^H\pi^r]$$

and the entrant's payoff would depend on its type.

$$\begin{aligned} \Pi_E(\bar{s}_E(\delta_E), \bar{s}_{In}; \delta^H) &= \pi^c \\ \Pi_E(\bar{s}_E(\delta_E), \bar{s}_{In}; \delta^L) &= (1-\delta^L)\pi_E(q(nq^c), q^c_{In}) + \delta^L\pi^r \end{aligned}$$

Then, similar arguments with Proposition 2 and 3 imply that there exists $\gamma^{**} \in (0,1)$ such that $\bar{s} = (\bar{s}_{In}, \bar{s}_E(\delta_E))$ can be supported as PBNE for all $\gamma \in (\gamma^{**}, 1)$. Since I already know that $\gamma \in (\gamma^*, 1)$, I have the following result.

Claim 15. *Suppose that $\delta^L < f(n) < f(n+1) < \delta^H$ holds and the industry was colluding before entry without deviation. Then,*

(a) if $\gamma \geq \max\{\gamma^*, \gamma^{**}\}$ holds, the industry still agrees on collusion after entry. In this case, the collusion will be sustained with probability γ after entry while it breaks down with probability $(1-\gamma)$.

(b) If $\gamma^{**} > \gamma > \gamma^*$, the industry does not agree on collusion after entry any longer.

Exit

As in the previous subsection, the industry would be in one situation among 3 possible cases before exit : (1) $\gamma \in (0, \gamma^*(n))$, (2) $\gamma \in (\gamma^*(n), 1)$ and $\delta_i = \delta^L < f(n)$ for some firm $i \in N$, (3) $\gamma \in (\gamma^*(n), 1)$ and $\delta_i \geq f(n)$ for all firm $i \in N$.

If $\gamma \in (0, \gamma^*(n))$ holds, firms could not agree on collusion pre-exit. So the belief γ is not updated. On the other hand, exit changes each firm's payoff $(\pi^{d+}, \pi^c, \pi^n, \pi^{j_{-i}}, \pi^r \ etc.)$ and decreases the number of firms in the industry. Hence, the remaining $(n-1)$ firms play a similar Bayesian game $G_O^B = (N, A, \Theta, \pi, p)$. If the threshold belief γ^* decreases below γ $(\gamma^*(n-1) < \gamma < \gamma^*(n))$, firms will agree on cartel post-exit. In this case, cartel can break down with probability $\left(1-\gamma^{n-1}\right)$. Else if $\gamma < \gamma^*(n-1)$ holds, firms do not agree on collusion post-exit either.

Next suppose that $\gamma \in (\gamma^*(n), 1)$ and $\delta_i = \delta^L < f(n)$ for some firm $i \in N$ before exit. Then, the industry is in the punishment phase before exit because the firm i deviates. Moreover, $\delta_i = \delta^L$ is common knowledge for all firms. If $f(n-1) < \delta^L < f(n)$ holds, the industry will collude for sure after exit. Suppose instead that $\delta^L < f(n-1) < f(n)$ holds. Then the industry will collude after exit only if the firm who exits is a unique deviator. If they collude after exit in any case, the cartel is sustained because $f(n-1) < \delta^L$ in the first case and $\delta_i = \delta^H > f(n-1)$ for all the remaining firms in the other case.

Finally, consider the case where $\gamma \in (\gamma^*(n), 1)$ and $\delta_i \geq f(n)$ for every firm $i \in N$. If $\delta^L \geq f(n) > f(n-1)$, firms continue to collude after exit. If $f(n) > \delta^L$, then $\delta_i = \delta^H > f(n)$ is common knowledge for every firm $i \in N$. So the industry will keep colluding after exit because $\delta_i = \delta^H > f(n) > f(n-1)$ holds for all the remaining firms.

Merger

We can analyze merger's effect on firms' competitive behavior similarly. Suppose that there are n firms competing *à la* Cournot before merger, and that $(m+1)$ firms merge. So there remain $(n-m)$ firms post-merger. Without loss, $I = \{n-m, n-m+1, \cdots, n\}$, a subset of N, is the set of insiders in the merger. If you recall $\gamma_u(n) \equiv 1_{\delta^H > f(n)} * \gamma + 1_{\delta^L > f(n)} * (1-\gamma)$, $\gamma_u(n) \leq \gamma^*(n)$ is implied from the fact that n firms competes *à la* Cournot pre-merger. For simplicity, assume that merger does not change the support of discount rates $\Delta = \{\delta^L, \delta^H\}$ and the merged firm picks its discount rate δ_M arbitrarily among the set of insiders' discount rates $\Delta_{IN} = \{\delta_{n-m}, \delta_{n-m+1}, \cdots, \delta_n\}$.

Then, the analysis is the same with exit case. Merger affects the remaining firms' competitive behavior in two different ways. First, merger decreases $\delta^* = f(n)$ due to $f^{'} > 0$. This effect weakly increases γ_u given the distribution of δ. Second, merger also affects the threshold belief γ^* because it changes each firm's payoff $(\pi^{d+}, \pi^c, \pi^n, \pi^{j_{-i}}, \pi^r etc.)$ and decreases the number of firms in the industry. As a result, it can be the case that a pure strategy $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2), \cdots, \bar{s}_{n-m-1}(\delta_{n-m-1}), \bar{s}_M(\delta_M))$ is supported as a PBNE after merger if $\gamma_u(n-m) > \gamma^*(n-m)$ is satisfied.

Note that the change in firms' competitive behavior takes a cutoff property. Even when merger increases γ_u and decreases γ^*, firms still compete *à la* Cournot until $\gamma_u < \gamma^*$ holds. In contrast, if $\gamma_u > \gamma^*$ is satisfied post-merger, then firms choose a collusive PBNE.

In sum, 3 different scenarios may arise post-merger. If $\gamma_u = 1$ or equivalently $\delta^* < \delta^L$ post-merger, then cartel is agreed and sustained for sure. If $\gamma_u \in (\gamma^*, 1)$, then cartel is agreed on and sustained with probability γ^{n-m} while cartel is agreed on but not sustained with probability $(1 - \gamma^{n-m})$. Finally, if $\gamma_u < \gamma^*$, the remaining firms after merger continue to compete *à la* Cournot irrespective of their real discount factors.

3.4 Collusion with Less than Monopoly Profit

Thus far, firms are only allowed to agree on perfect cartel. In this section I extend the model in a direction where cartel agreement might include the payoff less than monopoly profit until the uncertainty on the other firm's discount rate is not removed.

3.4.1 Dynamic Bayesian Game Structure

Here, I explicitly consider a dynamic Bayesian game which deals with cartel formation and its sustainability in a duopoly market where there is asymmetric information on the other firm's discount rate. Before game starts, nature picks each firm's discount rate $\delta_i \in \Delta$ with $\Pr(\delta^H) = \gamma_0$ and γ_0 is a common knowledge. At the beginning of the game, each firm chooses whether to join the cartel or not, and then decides its output in every period $t \geq 1$. Given firm i's output profile $q_i = (q_{i1}, q_{i2}, \cdots, q_{it}, \cdots)$, its (averaged) payoff is given by

$\Pi_i(q_i, q_{-i}) = (1-\delta_i)\sum_{t=1}^{\infty}\delta_i^{t-1}\pi_i(q_{it}, q_{-it})$, where $\pi_i(q_{it}, q_{-it})$ is firm i's profit at period t when each firm chooses its output as (q_{it}, q_{-it}). I assume $\delta^* \in (\delta^L, \delta^H)$ like the baseline model. So the dynamic Bayesian game $G_D^{DB} = (N, A, \Theta, \Pi, p)$ comes to

$$
\begin{aligned}
G_D^{DB} \quad = \quad & (N, A, \Theta, \Pi, p)\,,\ where \\
& N = \{1, 2\} \\
& A = \Pi_{i=1}^{2} A_i, \text{ where} \\
& \quad A_i = (A_{it})_{t=0}^{\infty} \text{ s.t } A_{i0} = \{(Join), (Not\ Join)\}, A_{it} = \{\ q_{it} \in R_+\} \text{ for } t \geq 1 \\
& \Theta = \Pi_{i=1}^{2}\Theta_i \text{ s.t } \ \Theta_i = \Delta \\
& \Pi = \{\Pi_1, \Pi_2\}, \text{ where} \\
& \quad \Pi_i : A\ X\ \Theta_i \mapsto R\ \ s.t\ \ \Pi_i(q_i, q_{-i}; \delta_i) = (1-\delta_i)\textstyle\sum_{t=1}^{\infty}\delta_i^{t-1}\pi_i(q_{it}, q_{-it}) \\
& p \in \Delta(\Theta) \text{ s.t } \delta_1 \perp \delta_2 \text{ and } \Pr(\delta^H) = \gamma_0
\end{aligned}
$$

Perfect monitoring is assumed. Let γ_{it} be firm i's updated belief at period $t \geq 1$ on $\delta_{-i} = \delta^H$. Define history $H_t = A^{t-1}$ and $H = \cup_{t=1}^{\infty} H_t$, where $A^t = (A_{1s}, A_{2s})_{s=0}^{t}$ for $t \geq 0$. Firm $i's$ pure strategy $s_i = (s_{it})_{t=0}^{\infty}$ is a map $S_i : H\ X\ \Theta_i \mapsto A_i$, so the pure strategy perfect Bayesian equilibrium of this game (PBE) is defined as a pair (s^*, γ^*), where a strategy profile $s^* = \{s_1^*, s_2^*\}$ and a system of beliefs $\gamma^* = (\gamma_{1t}^*, \gamma_{2t}^*)_{t=0}^{\infty}$ with the initial $\gamma_0 = \gamma_{10}^* = \gamma_{20}^*$ such that

(1) (s^*, γ^*) is sequentially rational, or equivalently

$$
\begin{aligned}
& \Pi_{it}(s_i^*|h_t, s_{-i}^*|h_t\ ; \delta_i, \gamma_{it}^*) \geq \ \Pi_{it}(s_i|h_t, s_{-i}^*|h_t\ ; \delta_i, \gamma_{it}^*) \\
& \qquad\qquad for\ every\ i \in N,\ \delta_i \in \Theta_i,\ h_t \in H_t\ and\ s_i|h_t \in (A_{is})_{s=t}^{\infty}
\end{aligned} \tag{3.9}
$$

(2) γ^* is obtained from s^* using Bayes rule whenever it is applicable,

where $\Pi_{it}(\ \cdot\ ; \delta_i, \gamma_{it}^*)$ represents firm i's continuation payoff at period t and $s_i|h_t$ represents firm i's continuation strategy at history $h_t \in H_t$.

3.4.2 Construction of Belief and Strategy

A System of Belief

Given the initial belief γ_0, each firm's belief is updated by Bayes rule in every period whenever it is applicable. If $a_{-i0} = Join$ and $q_{-i} = (q_1^c, \cdots, q_t^c, \cdots)$, where q_t^c is determined by the strategy $\bar{s}_{-it}$ that I construct in the next subsection, then firm i's belief is updated as following.

$$\begin{aligned}
\gamma_0 &= \gamma_{10} = \gamma_{20} : \ initial\ belief\ (Given) \\
\gamma_{i1} &= \frac{\gamma_0 \Pr(Join|\ \delta^H)}{\gamma_0 \Pr(Join|\ \delta^H) + (1-\gamma_0)\Pr(Join|\ \delta^L)} \\
\gamma_{it} &= \frac{\gamma_{it-1} \Pr(q_{t-1}^c|\ \delta^H)}{\gamma_{it-1} \Pr(q_{t-1}^c|\ \delta^H) + (1-\gamma_{it-1})\Pr(q_{t-1}^c|\ \delta^L)} \quad for\ t \geq 2
\end{aligned}$$

Suppose that both types of firms choose ($Join$) and a firm with δ^H always produces q_t^c. Given that $\Pr(Join|\ \delta^H) = \Pr(Join|\ \delta^L) = \Pr(q_{t-1}^c|\ \delta^H) = 1$, I get

$$\gamma_0 = \gamma_{i1},\ \gamma_{it} = \frac{\gamma_{it-1}}{\gamma_{it-1} + (1-\gamma_{it-1})\Pr(q_{t-1}^c|\ \delta^L)} \tag{3.10}$$

Hence, $\gamma_{it-1} = \gamma_{it}$ if $\Pr(q_{t-1}^c|\ \delta^L) = 1$, and $\gamma_{it} = 1$ if $\Pr(q_{t-1}^c|\ \delta^L) = 0$.

If a firm i observes $a_{-i0} = (Not\ Join)$, its belief is updated as $\gamma_{i1} = 0$. If h_t is such that $h_1 = (Join, Join)$ coupled with either (1) $q_{-i1} \neq q_1^c$ or (2) $q_i^{t-2} = q_{-i}^{t-2} = (q_1^c, \cdots, q_{t-2}^c)$ and $q_{-it-1} \neq q_{t-1}^c$ for $t \geq 3$, firm i's belief is also updated as $\gamma_{it} = 0$. If $\gamma_{it} = 0$ at h_t, then $\gamma_{is} = 0$ at all the history h_s following h_t.

Construction of Strategy

If duopolists do not agree on collusion, each firm gets stage Nash payoff π^n every period. In order to find a collusion equilibrium in this game, suppose that high type firm chooses symmetric collusion output $q_t^c \in [q^c, q^n)$ in every period t insofar as its partner chose collusion output q_s^c for $s \in \{1, \cdots, t-1\}$. Next I need to look at when the low type firm deviates. Let t^* denote the optimal deviation period of low type firm. Then, it coordinates with output q_t^c until period (t^*-1) but deviates for sure at period t^*, which means that $\Pr(q_1^c|\ \delta^L) = \cdots = \Pr(q_{t^*-1}^c|$

$\delta^L) = 1$ and $\Pr(q^c_{t^*}|\ \delta^L) = 0$. So firms agree on not only collusion but also each period's output level of collusion in this setting. After any one firm deviates at period t^*, each firm will get (average) punishment payoff π^r from period (t^*+1) and on. Equation (3.10) yields $\gamma_{it} = \gamma_0$ for all $i \in N$ and $t = 1, ..., t^*$ while $\gamma_{it^*+1} = 1$ if $q_{-it} = q^c_t$ and $\gamma_{it^*+1} = 0$ if $q_{-it} \neq q^c_t$. Since there is no update in γ, it is a natural assumption that each firm's collusion profit is the same in every period until period (t^*-1). Let me denote each firm's collusion profit and output until period (t^*-1) by $\bar{\pi}^c$ and $\bar{q}^c$, respectively. If both firms choose $q_{it^*} = \bar{q}^c$ at period t^*, then each firm's collusion profit π^c_t can be updated into π^c from period (t^*+1) and on because γ_{it^*+1} is updated into 1 for each $i \in \{1, 2\}$. Given that $\gamma_{it^*} = \gamma_0$ and $\Pr(\bar{q}^c|\ \delta^L) = 0$ for $t = t^*$, low type firm chooses its output by maximizing the following optimization problem.

$$\max_{q_i}\ \gamma_0 \pi_i(q_i, \bar{q}^c) + (1-\gamma_0)\pi_i(q_i, q_{-i})$$

From this problem, I can get the best response function $q_{it^*} = q^d(\gamma_0, \bar{q}^c, q_{-i})$ and fixed point output $q_{t^*} = q^d(\gamma_0, \bar{q}^c)$.

Example 3.2 *Assume again that $P = 1-Q$ and $MC_i = 0$ for $i = 1, 2$. Now, two firms can agree on $\bar{q}^c$ between $[\frac{1}{4}, \frac{1}{3})$. In order to get fixed point output $q^d = (q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c))$, first I have to solve*

$$\max_{q_i}\ \gamma_0(1-q_i-\bar{q}^c)q_i + (1-\gamma_0)(1-q_i-q_{-i})q_i$$

Then, the best-response function is given by $q_i = \frac{1}{2}\{\gamma_0(1-\bar{q}^c) + (1-\gamma_0)(1-q_{-i})\}$. So, the fixed point output is given by

$$q^d(\gamma_0, \bar{q}^c) = \frac{1-\gamma_0\bar{q}^c}{3-\gamma_0}$$

If I let $\bar{q}^c = q^c = \frac{1}{4}$, then $q^d(\gamma_0, \bar{q}^c) = \frac{4-\gamma_0}{12-4\gamma_0}$, which is the same with what I obtained in [Example 3.1.]

When low-type firm deviates at period t^* with fixed point output $q^d(\gamma_0, \bar{q}^c)$, its expected payoff amounts to

$$\begin{aligned}
&\Pi_i(t^*; \delta^L, \gamma_0) \\
= \; & \gamma_0[(1-(\delta^L)^{t^*-1})\bar{\pi}^c + (1-\delta^L)(\delta^L)^{t^*-1}\pi(q^d(\gamma_0,\bar{q}^c),\bar{q}^c) + (\delta^L)^{t^*}\pi^r] \\
& +(1-\gamma_0)[(1-(\delta^L)^{t^*-1})\bar{\pi}^c + (1-\delta^L)(\delta^L)^{t^*-1}\pi(q^d(\gamma_0,\bar{q}^c),q^d(\gamma_0,\bar{q}^c)) + (\delta^L)^{t^*}\pi^r] \\
= \; & \bar{\pi}^c + (\delta^L)^{t^*-1}[\gamma_0\pi(q^d(\gamma_0,\bar{q}^c),\bar{q}^c) + (1-\gamma_0)\pi(q^d(\gamma_0,\bar{q}^c),q^d(\gamma_0,\bar{q}^c)) - \bar{\pi}^c] \\
& +(\delta^L)^{t^*}[\pi^r - \gamma_0\pi(q^d(\gamma_0,\bar{q}^c),\bar{q}^c) - (1-\gamma_0)\pi(q^d(\gamma_0,\bar{q}^c),q^d(\gamma_0,\bar{q}^c))]
\end{aligned}$$

Taking a derivative with respect to t^* in order to get the optimal deviation period, I obtain

$$\begin{aligned}
\frac{d\Pi_i(t^*)}{dt^*} = \; & (\delta^L)^{t^*-1}\log\delta^L * \\
& [(1-\delta^L)\{\gamma_0\pi(q^d(\gamma_0,\bar{q}^c),\bar{q}^c) + (1-\gamma_0)\pi(q^d(\gamma_0,\bar{q}^c),q^d(\gamma_0,\bar{q}^c))\} + \delta^L\pi^r - \bar{\pi}^c]
\end{aligned} \tag{3.11}$$

Using equation (3.11), I can define $f: X \longmapsto R$ such that

$$\begin{aligned}
X &= [0,1]\times[q^c, q^n] \\
f(\gamma,q) &= (1-\delta^L)(\gamma\pi(q^d(\gamma,q),q) + (1-\gamma)\pi(q^d(\gamma,q),q^d(\gamma,q))) + \delta^L\pi^r - \pi(q,q)
\end{aligned}$$

Then, $f(\gamma,q)$ is continuous and $X \subset R^2$ is a compact set. Let me define an upper contour set $U(0) = \{(\gamma,q)\in X \mid f(\gamma,q) > 0\}$ and a lower contour set $L(0) = \{(\gamma,q)\in X \mid f(\gamma,q) < 0\}$. $U(0)$ is non-empty because $f(1,q^c) = (1-\delta^L)\pi^{d+} + \delta^L\pi^r - \pi^c > 0$. If $\pi^r < \pi^n$, $L(0)$ is also non-empty because $f(0,q^n) = \delta^L(\pi^r - \pi^n) < 0$. $U(0)$ and $L(0)$ are open because $f(\gamma,q)$ is continuous. Hence, there is $\varepsilon_1 > 0$ such that $B_{\varepsilon_1}(1,q^c) \subset U(0)$ and $\varepsilon_2 > 0$ such that $B_{\varepsilon_2}(0,q^n) \subset L(0)$. So, $U(0)$ and $L(0)$ have interior points. Since $sign(\frac{d\Pi_i(t^*)}{dt^*})$ is equal to $-sign(f(\gamma_0,\bar{q}^c))$ due to $\log\delta^L < 0$, $\Pi_i(t^*;\delta^L,\gamma_0)$ is monotonically increasing or decreasing depending on $sign(f(\gamma_0,\bar{q}^c))$. Then, it is most profitable that a low type firm deviates at period 1 if $(\gamma,q)\in U(0)$, and that it does not deviate forever if $(\gamma,q)\in L(0)$.

Based on these observations, I construct the strategy for each type according to the sign of $f(\gamma_0, \bar{q}^c)$. If $f(\gamma_0, \bar{q}^c) > 0$ holds, the (continuation) strategy of a high type firm is given by

$$\bar{s}_i|h_t(\delta^H;\ f(\gamma_0,\bar{q}^c) > 0) = \begin{cases} Join & if \quad t = 0 \\ q^n & if \quad t \geq 1,\ h_1 \neq (Join, Join) \\ \bar{q}^c & if \quad t = 1,\ h_1 = (Join, Join) \\ q^c & if \quad \begin{cases} Given\ h_1 = (Join, Join), \\ t = 2,\ q_i^1 = q_{-i}^1 = \bar{q}^c \text{ or} \\ t \geq 3,\ q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, q^c, \cdots, q^c) \end{cases} \\ q^r & if \quad otherwise \end{cases} \tag{3.12}$$

Here, q_i^{t-1} and q_{-i}^{t-1} are $(t-1)$ dimensional vectors which represent history of each firm's output from period 1 to period (t-1). In words, this strategy is constructed in the following way: high type firm joins cartel at the beginning of the game; if collusion was not agreed on, it chooses Cournot-Nash output every period; if collusion was agreed on at period 1, it chooses restricted cartel output at period 1 and perfect cartel output at period $t \geq 2$ insofar as collusion has been sustained; if collusion had been agreed on but one firm deviated at some period, it chooses output in the punishment phase.

When $f(\gamma_0, \bar{q}^c) > 0$ holds, the (continuation) strategy of a low type firm is given by

$$\bar{s}_i|h_t(\delta^L;\ f(\gamma_0,\bar{q}^c) > 0) = \begin{cases} Join & if \quad t = 0 \\ q^n & if \quad t \geq 1,\ h_1 \neq (Join, Join) \\ q^d(\gamma_0, \bar{q}^c) & if \quad t = 1,\ h_1 = (Join, Join) \\ q^{d+} & if \quad \begin{cases} Given\ h_1 = (Join, Join), \\ t = 2,\ q_i^1 = q_{-i}^1 = \bar{q}^c \text{ or} \\ t \geq 3,\ q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, q^c, \cdots, q^c) \end{cases} \\ q^r & if \quad otherwise \end{cases} \tag{3.13}$$

Low type firm's strategy is different from high type firm's strategy only at the history where cartel was agreed on and has not collapsed: low type firm chooses fixed point output at period

1 and deviation output under perfect information at period $t \geq 2$ at this class of history.

If $f(\gamma_0, \bar{q}^c) < 0$ holds instead, the strategy of both types is the same as follows.

$$\begin{aligned} \bar{s}_i|h_t(\delta^H;\ f(\gamma_0,\bar{q}^c) &< 0) = \bar{s}_i|h_t(\delta^L;\ f(\gamma_0,\bar{q}^c) < 0) \\ &= \begin{cases} Join & if \quad t = 0 \\ q^n & if \quad t \geq 1,\ h_1 \neq (Join, Join) \\ \bar{q}^c & if \quad \begin{cases} Given\ h_1 = (Join, Join), \\ t = 1 \text{ or} \\ t \geq 2,\ q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, \cdots, \bar{q}^c) \end{cases} \\ q^r & if \quad otherwise \end{cases} \end{aligned} \tag{3.14}$$

Each firm's strategy in this case is different from high type firm's strategy in (3.12) only at the history where cartel was agreed on and has not collapsed: each firm chooses restricted cartel output at this class of history. This construction reflects that low type firm does not deviate forever when $f(\gamma_0, \bar{q}^c) < 0$ holds.

3.4.3 Characterization of PBE and Its Outcome

Case 1 : $f(\gamma_0, \bar{q}^c) > 0$

Given with the constructed strategy and system of belief $(\bar{s}, \gamma)$, I can obtain the expected payoff or the expected continuation payoffs. The high type firm's expected payoff of the game amounts to

$$\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^H, \gamma_0) = \gamma_0[(1-\delta^H)\bar{\pi}^c + \delta^H \pi^c] + (1-\gamma_0)[(1-\delta^H)\pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c)) + \delta^H \pi^r]$$

With $(\bar{s}, \gamma)$, $h_1 = (Join, Join)$ is on the outcome path and $\gamma_{i1} = \gamma_0$. So $\Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1\ ; \delta^H, \gamma_{i1}) = \Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^H, \gamma_0)$ is immediate. On the other hand, the expected payoff of a firm with δ^L under $(\bar{s}, \gamma)$ is given by

$$\begin{aligned} \Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^L, \gamma_0) \quad &= \quad \gamma_0[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) + \delta^L \pi^r] \\ &\quad +(1-\gamma_0)[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) + \delta^L \pi^r] \end{aligned}$$

As in the high type, I have $\Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1 \ ;\delta^L, \gamma_{i1}) = \Pi_i(\bar{s}_i, \bar{s}_{-i} \ ;\delta^L, \gamma_0)$. If $h_t \in H_t$ is either $q_i^1 = q_{-i}^1 = \bar{q}^c$ for $t = 2$ or $q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, q^c, \cdots, q^c)$ for $t \geq 3$ given with $h_1 = (Join, Join)$, the continuation payoff of a firm with δ^H is given by

$$\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t \ ;\delta^H, \gamma_{it} = 1) = \pi^c$$

and the continuation payoff of a low-type firm yields

$$\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t \ ;\delta^L, \gamma_{it} = 1) = (1 - \delta^L)\pi^{d+} + \delta^L \pi^r$$

If $h_t \in H_t$ is either (1) $q_i^1 \neq \bar{q}^c$ or $q_{-i}^1 \neq \bar{q}^c$ for $t = 2$, or (2) $q_i^{t-1} \neq (\bar{q}^c, q^c, \cdots, q^c)$ or $q_{-i}^{t-1} \neq (\bar{q}^c, q^c, \cdots, q^c)$ for $t \geq 3$ given with $h_1 = (Join, Join)$, the continuation payoff of a firm with any type is given by

$$\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t \ ;\delta_i, \gamma_{it}) = \pi^r, \text{ where } \gamma_{it} \in \{0, 1\} \text{ and } \gamma_{it} = 0 \text{ for some } i, \ \delta_i \in \Delta$$

and if $h_t \in H_t$ is any history such that $h_1 \neq (Join, Join)$, the continuation payoff of a firm with any type is given by

$$\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t \ ;\delta_i, \gamma_{it}) = \pi^n, \text{ where } \gamma_{it} = 0 \text{ for some } i, \ \delta_i \in \Delta$$

Now, I can derive a result similar to Proposition 2 in the baseline model.

Proposition 4. *Suppose $\delta^L < \delta^* < \delta^H$. Then, there exists $\epsilon^* > 0$ such that $(\bar{s}, \gamma)$, constructed in subsection 3.4.2, can be supported as PBE in the dynamic Bayesian game $G_D^{DB} = (N, A, \Theta, \Pi, p)$ for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon^*}(1, q^c)$.*

Proof. See the appendix. □

Proposition 4 shows that there are a continuum of equilibria in which each firm in duopoly market agrees on restricted initial collusion payoff $\bar{\pi}^c < \pi^c$ if the initial belief γ_0 is close to 1. Given this characterization of PBE, the equilibrium outcome can be derived.

Corollary 6. *Suppose that $\delta^L < \delta^* < \delta^H$ and $(\gamma_0, \bar{q}^c) \in B_{\epsilon^*}(1, q^c)$. Then, the equilibrium outcome of PBE $(\bar{s}, \gamma)$ is given as follows.*

(a) if $(\delta_1, \delta_2) = (\delta^H, \delta^H)$, cartel is agreed and sustained. Each firm's output path is given by $q = (\bar{q}^c, q^c, q^c, \cdots)$ in this case.

(b) if $(\delta_1, \delta_2) \neq (\delta^H, \delta^H)$, cartel is agreed but is not sustained. In this case, high type firm's output path is given by $q = (\bar{q}^c, q^r, q^r, \cdots)$ while low type firm's output path is given by $q = (q^d(\gamma_0, \bar{q}^c), q^r, q^r, \cdots)$.
The ex-ante probability of each outcome path equals to γ_0^2 and $1 - \gamma_0^2$, respectively.

Like the baseline model, this extended model also shows that cartel can be agreed on even when each firm's incentive constraint is not satisfied and the punishment phase can be an equilibrium outcome path. One interesting feature of this equilibrium outcome, which is in contrast with the baseline model, is that when both firms have high type cartel may start from the payoff below full collusion payoff π^c but can achieve π^c from period 2 and on. The reason why this happens is that uncertainty on the other firm's discount rate might prevent duopolists from agreeing on full cartel output q^c from the beginning, but they reach the perfect cartel from period 2 and on because uncertainty is removed after observing the other firm's output at period 1.

Case 2 : $f(\gamma_0, \bar{q}^c) < 0$

In this case, the outcome path from $(\bar{s}, \gamma)$ would be $h_1 = (Join, Join)$ and h_t is such that $h_1 = (Join, Join)$ and $q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ for $t \geq 2$ while the belief is $\gamma_0 = \gamma_{it}$ for all $i \in N$ and $t \geq 0$. So the expected payoff and the expected continuation payoff of this game on the outcome path from $(\bar{s}, \gamma)$ would be

$$\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta_i, \gamma_0) = \Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t; \delta_i, \gamma_{it} = \gamma_0) = \bar{\pi}^c$$

for all $i \in N$, $\delta_i \in \Theta_i$ and $t \geq 1$. For any history h_t such that $h_1 \neq (Join, Join)$, the continuation payoff of this game comes to $\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t; \delta_i, \gamma_{it}) = \pi^n$. If $h_t \in H_t$ is such that $q_i^{t-1} \neq (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ or $q_{-i}^{t-1} \neq (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ for $t \geq 2$ given with $h_1 = (Join, Join)$, the continuation payoff of a firm with any type is given by $\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t\ ; \delta_i, \gamma_{it}) = \pi^r$.

Proposition 5. *Suppose that $\delta^* > \delta^L$ and $\pi^r < \pi^n$. The strategy profile and system of belief $(\bar{s}, \gamma)$, constructed in subsection 3.4.2, can be supported as PBE in the dynamic Bayesian game $G_D^{DB} = (N, A, \Theta, \Pi, p)$ as $\bar{q}^c$ approaches to q^n.*

Proof. See the appendix. $\square$

One thing to note in Proposition 5 is that PBE of this case only requires $\delta^L < \delta^*$, so this low payoff cartel equilibrium is possible even when $\delta^* \geq \delta^H$. In fact, it can be shown that this low payoff cartel equilibrium exists when two firms have common discount rate δ or there is no asymmetric information on the other firm's discount rate. To see this, construct a function $f(q) = \pi(q,q) - (1-\delta_i)\pi(q^d(\gamma, q), q) - \delta_i \pi^r$, then I get $\lim_{q \to q^n} f(q) = \delta_i(\pi^n - \pi^r)$. This implies that $\bar{\pi}^c > (1-\delta_i)\pi(q^d(\bar{q}^c), \bar{q}^c) + \delta_i \pi^r$ if $\bar{q}^c$ is close enough to q^n and $\pi^n > \pi^r$. Hence, the important assumption in Proposition 5 is about firms' strategy against deviation because this low payoff cartel equilibrium is possible only when each firm retaliates more severely than stage Nash payoff against deviation. Given with PBE in Proposition 5, the equilibrium outcome path is given by $q_1 = q_2 = (\bar{q}^c, \bar{q}^c, \bar{q}^c, \cdots)$ irrespective of each firm's type.

3.5 Collusion with Uneven Split of Monopoly Profit

This section considers the situation where duopolists exchange the information on their discount factor and may agree on collusion with uneven split of monopoly profit based on their updated belief on the other firm's discount rate. In order to deal with this extension, I introduce cheap talk game before each firm makes cartel and output decision.

3.5.1 Characteristics of the Extension

Duopolists can split the monopoly profit unevenly in two different ways: one is to set a different output quota, and the other is to use money transfer after two firms produce the same output. Let me introduce one simple example under perfect information.

Example 3.3 *Assume P = 1 - Q and MC = 0 as before. Then, $\pi^d = \frac{9}{64}$, $\pi^c = \frac{1}{8}$, $\pi^n = \frac{1}{9}$, and monopoly profit $\pi^m = \frac{1}{4}$. Consider first even split and Nash reversion strategy against* *deviation, then I have* $\pi^r = \frac{1}{9}$ *and the threshold discount factor* $\delta^* = \frac{9}{17}$*. I also get* $q^c = \frac{1}{4}$, $q^d = \frac{3}{8}$, *and* $q^n = q^r = \frac{1}{3}$*. Let* $(\delta^L, \delta^H) = (\frac{7}{17}, \frac{15}{17})$, *so* $\delta^L < \delta^* < \delta^H$ *holds. Suppose that duopolists with different discount rates set their output quota* $(q^c_L, q^c_H) = (\frac{19}{72}, \frac{17}{72})$*. For any given* (q^c_L, q^c_H), *the cartel payoff is given by* $(\pi(q^c_L, q^c_H), \pi(q^c_H, q^c_L)) = (\frac{q^c_L}{2}, \frac{q^c_H}{2})$ *whereas the deviation payoff is equal to* $\pi^d(q^c_L) \equiv \pi(q^d(q^c_L), q^c_L) = \frac{(1-q^c_L)^2}{4}$ *and* $\pi^d(q^c_H) \equiv \pi(q^d(q^c_H), q^c_H) = \frac{(1-q^c_H)^2}{4}$*. So for* $(q^c_L, q^c_H) = (\frac{19}{72}, \frac{17}{72})$, *each firm's cartel and deviation profit come to* $(\pi(q^c_L, q^c_H), \pi(q^c_H, q^c_L)) = (\frac{19}{144}, \frac{17}{144})$ *and* $\left(\pi^d(q^c_L), \pi^d(q^c_H)\right) = (0.1354, 0.1458)$, *respectively. Then,* $\pi(q^c_H, q^c_L) = \frac{17}{144} > 0.1139 = (1-\delta^H)\pi^d(q^c_L) + \delta^H \pi^r$ *and* $\pi(q^c_L, q^c_H) = \frac{19}{144} > 0.1315 = (1-\delta^L)\pi^d(q^c_H) + \delta^L \pi^r$*. Hence, cartel can be sustained under this uneven split of output quota.*

Now consider uneven split of monopoly profit through money transfer. Each firm produces cartel output $q^c = \frac{1}{4}$ *under the given demand and cost function. Suppose that money transfer in each period is* $m = \frac{1}{144}$*. In other words, the high type firm gives the low type firm* $\frac{1}{144}$ *from its profit every period. So, each firm's per-period profit after this transfer comes to* $(\pi^c_L, \pi^c_H) \equiv (\pi^c + m, \pi^c - m) = (\frac{19}{144}, \frac{17}{144})$*. Then, cartel can be sustained under this profit sharing because* $(\delta^*_L, \delta^*_H) = (\frac{5}{17}, \frac{13}{17})$ *and* $\delta^*_L < \delta^L < \delta^*_H < \delta^H$ *holds.*

This example shows a few characteristics of this extension. First of all, for a given $\delta^H > \delta^*$, there is a lower bound of threshold discount rate, $\hat{\delta}^*_L$, such that there is an uneven split of monopoly profit satisfying each type's incentive constraint for any $\delta^L > \hat{\delta}^*_L$. I can let $\hat{\delta}^*_L = f(\delta^H)$ such that $f^{'} < 0$, which reflects that a high type firm can yield more portion of monopoly profit to a low type firm as it is more patient. Note that if $\delta^L < \hat{\delta}^*_L$ holds, there is no way to satisfy incentive constraint for the low type firm with any feasible uneven split. I make the following assumption in order to exclude this case:

Assumption 1. $\hat{\delta}^*_L < \delta^L < \delta^*$

Second, if there is one profit split satisfying incentive constraint for each type, then there is a continuum of ways to split monopoly profit that satisfy incentive constraint of each type. In the above example, every profit division $(\frac{19}{144} + \varepsilon, \frac{17}{144} - \varepsilon)$ also satisfies incentive constraints if $|\varepsilon|$ is sufficiently small.

Third, compared with even split of monopoly profit, uneven split with money transfer does not change each firm's market share nor deviation payoff whereas uneven split with output quota changes both. So one uneven profit split satisfying incentive constraint of each type in one method does not necessarily satisfy incentive constraint in the other. Given these observations, I will examine the possibility of collusion with uneven profit under Bayesian environment both by money transfer and by output quota.

3.5.2 Counterfactual Model : Perfect Information

Given Assumption 1, cartel can be agreed on and sustained unless both firms have δ^L. Since cartel cannot be agreed on with even split when at least one firm has δ^L, uneven split enlarges the scope of collusion under perfect information. To see this, let $\Pr(\delta^H) = \gamma$ for each firm. Then, cartel can be agreed on with (ex-ante) probability $\left(2\gamma - \gamma^2\right)$, which is greater than γ^2, the (ex-ante) probability of cartel agreement when uneven split is not allowed. If both firms have δ^H, they agree on collusion with even split π^c whereas if one firm has δ^L and the other δ^H, they agree on collusion in which low type firm's profit is greater. The uneven split is not unique because there are infinitely many ways to achieve incentive constraint for each type of firm.

3.5.3 Bayesian Model with Money Transfer

As in the previous models, nature picks each firm's discount rate $\delta_i \in \Delta$ with $\Pr(\delta^H) = \gamma$, which is a common initial belief on $\delta_{-i} = \delta^H$. In order to allow the possibility that firms agree on uneven split collusion under Bayesian setting, I introduce a cheap-talk procedure before firms make cartel or output decision. In this subsection, uneven split of monopoly profit is done by money transfer.

After each firm learns its type, it says its discount rate to the other simultaneously. I will let $\Pr(\delta^H \mid \delta^H) = p \in [0,1]$ and $\Pr(\delta^L \mid \delta^L) = q \in [0,1]$ as the strategy of each type of firm in the cheap-talk game. The discount rate in front of bar $(\delta^X \mid \cdot\,)$ represents the revealed discount rate of a firm, and that behind bar $(\,\cdot \mid \delta^Y)$ does the real discount rate of the firm. Then, p (q) is the probability that a high (low) type firm reveals its real type truthfully in the cheap talk game. Given the strategy profile (p, q) and the initial belief γ, the belief on $\delta_{-i} = \delta^H$ is updated by Bayes' rule.

$$\gamma' = \frac{\gamma p}{\gamma p + (1-\gamma)(1-q)} \tag{3.15}$$

So γ' would be 1 under the truth-telling strategy ($p = q = 1$) if the competitor's revealed type is δ^H. I will check whether there exists a truth-telling equilibrium in the cheap talk game which has the following features in the subgame after cheap talk :

1. if (HH) is the outcome of cheap talk, cartel is agreed on with the same payoff π^c,
2. if (HL) is the outcome, cartel is agreed on with uneven payoff (π^c_H, π^c_L) where $\pi^c_H + \pi^c_L = 2\pi^c$, $\pi^c_H < \pi^c_L$ and each type's incentive constraint holds ($\pi^c_H > (1-\delta^H)\pi^d + \delta^H\pi^r$ and $\pi^c_L > (1-\delta^L)\pi^d + \delta^L\pi^r$),
3. finally, if (LL) is the outcome, cartel is not agreed on.

In other words, firms' cartel agreement decision sticks to the outcome of cheap talk game. Formally, the continuation strategy after the cheap talk is given as follows.

$$\begin{aligned}
\bar{s}_i(\delta_i \quad | \quad HH) &= \begin{cases} (Join, q^c) \quad if \quad \delta_i = \delta^H \\ (Join, q^d) \quad if \quad \delta_i = \delta^L \end{cases} \\
\bar{s}_i(\delta_i \quad | \quad HL) &= \begin{cases} (Join, q^c) \quad if \quad \delta_i = \delta^H \ or \ \delta_i = \delta^c_i = \delta^L \\ (Join, q^d) \quad if \quad \delta_i = \delta^L \ and \ \delta^c_i = \delta^H \end{cases} \\
\bar{s}_i(\delta_i \quad | \quad LL) &= (Not\ Join,\ q^n)
\end{aligned} \tag{3.16}$$

Here, δ_i represents firm i's real type while δ^c_i does its revealed type in the cheap talk. Given the truth-telling strategy, the updated belief γ' on $\delta_{-i} = \delta^H$ is either 0 or 1 for each firm in every subgame after the cheap-talk. Put differently, no subgame after cheap talk involves uncertainty on the other firm's discount factor. Then the continuation strategy $\bar{s}_i(\delta_i \mid \cdot\ )$ is sequentially rational in every history after the cheap-talk.

Now I want to check whether there exists a PBE that consists of truth-telling strategy in the cheap talk stage and the continuation strategy (3.16) in the post cheap talk stage. If there exists a truth-telling strategy, it must be the case that there is an uneven profit share (π^c_H, π^c_L)

such that the expected payoff from truth-telling is greater than that from lying for every type of firm.

First, consider a firm with δ^H. Given the truth-telling strategy of the other firm, the outcome of cheap talk game would be (HH) with probability γ and (HL) with probability $(1-\gamma)$. From continuation strategy (3.16), the expected payoff of the high-type firm from truth-telling comes to

$$E\Pi_{HH} = \gamma\pi^c + (1-\gamma)\pi^c_H$$

The first subscript in $E\Pi_{XY}$ represents firm's real type and the second its revealed type. So $X = Y$ implies truth-telling, and $X \neq Y$ implies lying. In a similar way, if the high type firm lies about its discount rate, its expected payoff would be

$$E\Pi_{HL} = \gamma\pi^c_L + (1-\gamma)\pi^n$$

Hence, truth-telling would be more profitable for the high-type firm if and only if $\pi^c_H - \pi^n \geq \frac{\gamma}{1-\gamma}(\pi^c_L - \pi^c)$. If I use $\pi^c_L = 2\pi^c - \pi^c_H$, this condition yields

$$\pi^c_H \geq \gamma\pi^c + (1-\gamma)\pi^n \tag{3.17}$$

Next consider a firm with δ^L. Given the truth-telling strategy of the other firm and continuation strategy (3.16), the expected payoff of the low type firm from truth-telling becomes

$$E\Pi_{LL} = \gamma\pi^c_L + (1-\gamma)\pi^n$$

If the low type firm lies, then its expected payoff would be

$$\begin{aligned} E\Pi_{LH} &= \gamma\left[(1-\delta^L)\pi^d + \delta^L\pi^r\right] + (1-\gamma)\left[(1-\delta^L)\pi^d + \delta^L\pi^r\right] \\ &= (1-\delta^L)\pi^d + \delta^L\pi^r \end{aligned}$$

Note that the deviation payoff in even split cartel is the same with that in uneven split cartel because firms use money transfer. Hence, truth-telling would be profitable for the low-type firm if and only if $\gamma\pi_L^c + (1-\gamma)\pi^n \geq (1-\delta^L)\pi^d + \delta^L\pi^r$, which is equivalent to

$$\pi_H^c \leq 2\pi^c + \frac{1-\gamma}{\gamma}\pi^n - \frac{1}{\gamma}\Big((1-\delta^L)\pi^d + \delta^L\pi^r\Big) \tag{3.18}$$

Let D_1 and D_2 be the right-hand side of (3.17) and (3.18), respectively. Then, I can obtain the following result.

Claim 16. *There is no π_H^c that satisfies both* (3.17) *and* (3.18)*.*

Proof. Define $f(\gamma) \equiv D_2 - D_1$, then

$$\begin{aligned} f(\gamma) &= (2-\gamma)\pi^c + \frac{(1-\gamma)^2}{\gamma}\pi^n - \frac{1}{\gamma}\Big((1-\delta^L)\pi^d + \delta^L\pi^r\Big) \\ &= \frac{1}{\gamma}\Big(\gamma(2-\gamma)\pi^c + (1-\gamma)^2\pi^n - \Big((1-\delta^L)\pi^d + \delta^L\pi^r\Big)\Big) \\ &= \frac{1}{\gamma}\Big(\pi^c - \Big((1-\delta^L)\pi^d + \delta^L\pi^r\Big) - (\pi^c - \pi^n)(1-\gamma)^2\Big) \end{aligned}$$

Since $\pi^c < (1-\delta^L)\pi^d + \delta^L\pi^r$ and $\gamma \in (0,1)$, $f(\gamma) < 0$ holds for all $\gamma \in (0,1)$. □

This result shows that at least one type of firm has an incentive to lie about its type for any feasible uneven profit split (π_H^c, π_L^c). Hence, truth-telling equilibrium for both types does not exist under money transfer.

3.5.4 Bayesian Model with Output Quota

Similar to money transfer, I will check whether there exists a truth-telling equilibrium in the cheap talk game which has the following features :

1. if (HH) is the outcome of cheap talk, cartel is agreed on with the same cartel payoff π^c,

2. if (HL) is the outcome, cartel is agreed on with output quota (q_H^c, q_L^c) where $q_L^c > q_H^c$, $\pi(q_H^c, q_L^c) + \pi(q_L^c, q_H^c) = 2\pi^c$ and each type's incentive constraint holds ($\pi(q_H^c, q_L^c) > (1-\delta^H)\pi(q^d(q_L^c), q_L^c) + \delta^H\pi^r$ and $\pi(q_L^c, q_H^c) > (1-\delta^L)\pi(q^d(q_H^c), q_H^c) + \delta^L\pi^r$),

3. finally, if (LL) is the outcome, cartel is not agreed on.

As in money transfer, firms' cartel agreement decision follows the outcome of cheap talk game. Then, the continuation strategy is given as follows.

$$\bar{s}_i(\delta_i \quad | \quad HH) = \begin{cases} (Join, q^c) & if \quad \delta_i = \delta^H \\ (Join, q^d) & if \quad \delta_i = \delta^L \end{cases}$$

$$\bar{s}_i(\delta_i \quad | \quad HL) = \begin{cases} (Join, q^c_H) & if \quad \delta_i = \delta^c_i = \delta^H \\ (Join, q^c_L) & if \quad \delta^c_i = \delta^L \\ (Join, q^d(q^c_L)) & if \quad \delta_i = \delta^L \ and \ \delta^c_i = \delta^H \end{cases} \tag{3.19}$$

$$\bar{s}_i(\delta_i \quad | \quad LL) = (Not\ Join,\ q^n)$$

The continuation strategy (3.19) is different from (3.16) in money transfer when (HL) is the outcome of cheap-talk; a high type firm chooses q^c_H if it revealed its type truthfully and q^c_L if it lied, whereas a low type firm chooses q^c_L if it revealed truthfully and $q^d(q^c_L)$ if it lied in the cheap talk. Note that the deviation output $q^d(q^c_L)$ depends on the other firm's output quota.

First, the expected payoff of a high type firm yields

$$E\Pi_{HH} = \gamma\pi^c + (1-\gamma)\pi(q^c_H, q^c_L)$$

Note that the high type firm earns $\pi(q^c_H, q^c_L)$ from the continuation strategy (3.19) when $\delta_i = \delta^c_i = \delta^H$ and $\delta_{-i} = \delta^c_{-i} = \delta^L$. If the high-type firm lies about its discount rate, its expected payoff would be

$$E\Pi_{HL} = \gamma\pi(q^c_L, q^c_H) + (1-\gamma)\pi^n$$

Similarly, the high type firm earns $\pi(q^c_L, q^c_H)$ when $\delta^c_i = \delta^L$ and $\delta_{-i} = \delta^c_{-i} = \delta^H$. Hence, truth-telling would be better for the high-type firm if and only if

$$\pi(q^c_H, q^c_L) \geq \gamma\pi^c + (1-\gamma)\pi^n \tag{3.20}$$

Next consider a firm with δ^L. Given the truth-telling strategy of the other firm and continuation strategy (3.19), the expected payoff from truth-telling becomes

$$E\Pi_{LL} = \gamma\pi(q_L^c, q_H^c) + (1-\gamma)\pi^n$$

When this firm lies, its expected payoff would be

$$E\Pi_{LH} = \gamma\left((1-\delta^L)\pi^d + \delta^L\pi^r\right) + (1-\gamma)\left((1-\delta^L)\pi(q^d(q_L^c), q_L^c) + \delta^L\pi^r\right)$$

$E\Pi_{LH}$ in output quota is less than that in money transfer; deviation payoff gets smaller because of higher output of the other firm ($q_L^c > q^c$) when (HL) is the outcome of the cheap talk. Hence, truth-telling would be profitable for the low-type firm if and only if

$$\pi(q_H^c, q_L^c) \leq 2\pi^c + \frac{1-\gamma}{\gamma}\pi^n - \frac{1}{\gamma}\left((1-\delta^L)\left(\gamma\pi^d + (1-\gamma)\pi(q^d(q_L^c), q_L^c)\right) + \delta^L\pi^r\right) \qquad (3.21)$$

Let D_3 and D_4 be the right-hand side of (3.20) and (3.21), respectively. If I define $f(\delta^L, q_L^c, \gamma) \equiv D_4 - D_3$, then

$$\begin{aligned} f(\delta^L, q_L^c, \gamma) &= (2-\gamma)\pi^c + \frac{(1-\gamma)^2}{\gamma}\pi^n - \frac{1}{\gamma}\left((1-\delta^L)\left[\gamma\pi^d + (1-\gamma)\pi(q^d(q_L^c), q_L^c)\right] + \delta^L\pi^r\right) \\ &= \frac{1}{\gamma}\left(\gamma(2-\gamma)\pi^c + (1-\gamma)^2\pi^n - \left[(1-\delta^L)\left(\gamma\pi^d + (1-\gamma)\pi(q^d(q_L^c), q_L^c)\right) + \delta^L\pi^r\right]\right) \\ &= \frac{1}{\gamma}\left(\pi^c - \left[(1-\delta^L)\left(\gamma\pi^d + (1-\gamma)\pi(q^d(q_L^c), q_L^c)\right) + \delta^L\pi^r\right] - (\pi^c - \pi^n)(1-\gamma)^2\right) \end{aligned}$$

Since $\pi(q^d(q_L^c), q_L^c) < \pi^d$ due to $q_L^c > q^c$, $\pi^c > (1-\delta^L)\left(\gamma\pi^d + (1-\gamma)\pi(q^d(q_L^c), q_L^c)\right) + \delta^L\pi^r$ may hold if q_L^c is sufficiently large and δ^L is close enough to δ^*. Then, it can be the case that $f(\delta^L, q_L^c, \gamma) > 0$ for some $(\delta^L, q_L^c, \gamma)$, or equivalently there may exist $\pi(q_H^c, q_L^c)$ satisfying both (3.20) and (3.21) for the $(\delta^L, q_L^c, \gamma)$. In that case, truth-telling equilibrium may exist if incentive constraint is satisfied for the high type as well ($\pi(q_H^c, q_L^c) \geq (1-\delta^H)\pi(q^d(q_L^c), q_L^c) + \delta^H\pi^r$).

It is crucial for the potential existence of truth-telling equilibrium that the deviation payoff gets smaller in the output quota, which reduces low-type firm's incentive to deviate. [Example 3.3] illustrates the potential existence of truth-telling equilibrium.

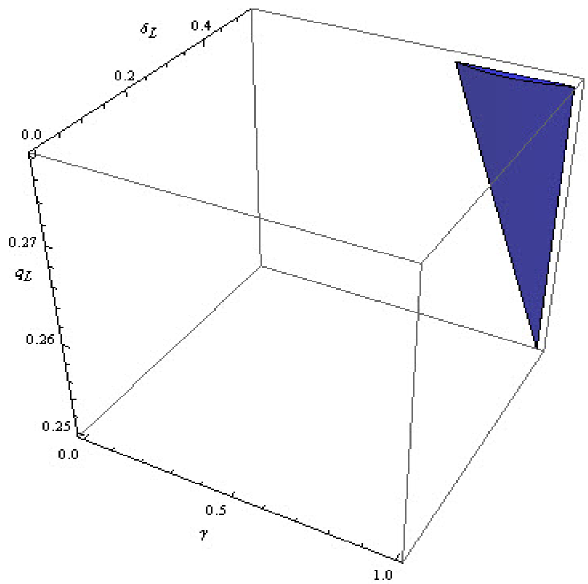


Figure 3.1: Example of Truth-telling Equilibrium

Example 3.3 (Continued) *In this example, $f(\delta^L, q_L^c, \gamma)$ is given by*

$$f(\delta^L, q_L^c, \gamma) = \frac{1}{\gamma}\left(\frac{1}{8} - \left[(1-\delta^L)\left(\gamma\frac{9}{64} + (1-\gamma)\frac{(1-q_L^c)^2}{4}\right) + \frac{\delta^L}{9}\right] - \frac{1}{72}(1-\gamma)^2\right)$$

Here, I only need to consider $q_L^c \in (\frac{1}{4}, \frac{5}{18})$ because both $q_L^c > q^c$ and $\pi(q_H^c, q_L^c) > \pi^n$ should hold. Figure 3.1 plots the space of $(\delta^L, q_L^c, \gamma)$ such that $f(\delta^L, q_L^c, \gamma) > 0$. So truth-telling equilibrium exists for any $(\delta^L, q_L^c, \gamma)$ in the blue shape, if δ^H satisfies $\pi(q_H^c, q_L^c) \geq (1-\delta^H)\pi(q^d(q_L^c), q_L^c) + \delta^H\pi^r \Leftrightarrow \frac{1-2q_L^c}{4} \geq \frac{(1-\delta^H)(1-q_L^c)^2}{4} + \frac{\delta^H}{9}$. If δ^H is close enough to 1, this condition can be satisfied. This picture shows that truth-telling equilibrium may exist when δ^L is close to $\delta^* = \frac{9}{17}$, q_L^c is larger than $q^c = \frac{1}{4}$ and γ is close to 1.

3.6 Conclusion

In this research, I studied cartel formation and its outcome path when there is uncertainty on other firms' discount rates. I could get a few results originated from this asymmetric information, which are in contrast with perfect information model.

First, cartel agreement becomes more difficult under the uncertainty on other firms discount rates, but it is possible even when each firm's incentive constraint for cartel sustainability is not satisfied. This result happens when each firm believes with sufficiently high probability that its cartel partner has a high discount rate.

Second, cartel agreement in this environment contains the possibility that firms fall into price war from the beginning. It may arise because low type firm can have an incentive to agree on collusion and deviate at equilibrium. It is a unique feature of cartel agreement in this Bayesian setting that punishment phase may be an equilibrium outcome path.

Third, if firms are allowed to agree on restricted payoff cartel, there may exist a continuum of collusion equilibria where firms choose the payoff below the perfect cartel payoff at the first period after they agree on collusion. It may happen because concern on the deviation of its partner prevents duopolists from agreeing on full cartel output from the beginning. But the output of the first period plays the role of signaling on each firm's type, so cartel can achieve perfect cartel output from period 2 and on if both firms abide by the agreed output in the first period. Another interesting feature of this extension is the potential presence of low payoff cartel. Although it requires a behavioral assumption that firms punish more severely than Nash reversion, this low payoff cartel may exist in various settings.

Fourth, if I allow uneven split of monopoly profit between duopolists, I can find quite different characteristics between the perfect information setting and the Bayesian setting. The uneven division of monopoly profit enlarges the scope of cartel agreement with the perfect information whereas it is not the case with the Bayesian environment. The pregame communication under the Bayesian model quite often disturb cartel agreement rather than help it because it fails to induce true information on the other firm's discount factor.

Finally, this model is applicable to comparative statics. The equilibrium outcome before entry, exit or merger serves as a signaling on the incumbents' discount rates. So, the event either turns the game into perfect information game or into different Bayesian game.

CHAPTER 4

Antitrust Policy Issues for Effective Cartel Deterrence

4.1 Introduction

It is an important issue in antitrust policy to deter collusion effectively. Towards this end, legislators have introduced some renovative law enforcement schemes, and antitrust authorities put its highest priority on blocking cartels in many countries. Motivated by these efforts, this paper examines two policies for effective cartel deterrence: corporate leniency program and crackdown policy.

Leniency program is "the policy that reduces sanctions against colluding firms who report information on their cartel to antitrust authority and cooperate with it along the prosecution phase." [Spagnolo (2008)] In the U.S., leniency program was introduced in 1978 and revised in 1993. The 1993 version's success in the detection of cartels led many other countries to install similar programs. This paper focuses on three distinctive features that are commonly observed in the countries having leniency program: discovered cartels increase steeply at least in the early periods after the introduction of leniency program; most of leniency cases are cartels that are just or already collapsed, so called "dying cartels" [Harrington and Chang (2009a,b)][1]; finally there is a simultaneous "rush to report" among cartel members. As is shown in Figure 4.1 quoted from Bloom (2006), this aspect is conspicuous in the E.U. where the second or subsequent applicants can get reduction in fine. This research examines what

[1] According to Harrington and Chang (2009b), O.Guersent, a European Commission official, mentioned at the 11^{th} Annual EU Competition Law and Policy Workshop that many leniency applicants are from dying cartels.

brings these facts on leniency program and whether they have any relation with leniency program's deterrent effect.

To this end, I assume firms consider that cartel may collapse due to an unexpected shock or detection by antitrust authority when they agree on a cartel. Empirical researches report about 5 to 9 years as the estimates of average cartel duration depending on their data set. [Posner (1970), Eckbo (1976), Griffin (1989), Zimmerman and Connor (2005), Suslow (2005), Levenstein and Suslow (2010), etc.] There can be many reasons why cartel collapses. Rotemberg and Saloner (1986) suggested a model where cartels are fragile during booms. Empirical researches add various sources of cartel instability: antitrust policy, change in firms' patience, demand shock, entry, buyer/seller concentration and etc. [Zimmerman and Connor (2005), Levenstein and Suslow (2006, 2010), Oindrila (2009), etc.]

From this perspective, firms take the presence of law enforcement and leniency program into account when they form a cartel. So each firm considers not only the expected duration of cartel but also the expected penalty due to the conviction by antitrust authority when it makes cartel decision, and hence cartel is formed only when this adjusted discounted payoff is greater than its unilateral deviation payoff. Using the stationary Markov perfect equilibrium concept based on the recursive game structure, I show that every cartel member applies leniency when cartel is just collapsed and no firm applies when cartel is active. A surge of leniency applications from dying cartels comes from this characterization of equilibrium on leniency decision. Because this application pattern arises in a wide range of leniency program, the fact that antitrust authority discovers more cartels with leniency program does not guarantee that there are fewer cartels under law enforcement with leniency program.

Optimal leniency program, which maximizes deterrent effect to cartel, is one that gives full exemption to deviator irrespective of its leniency application and no reduction to simultaneous leniency applicants; full benefit to deviator maximizes the incentive to deviate from cartel agreement while no benefit to simultaneous leniency applicants maintains the expected fine under the law enforcement with leniency program at the highest level. For the same reason, optimal leniency program does not necessarily give full exemption to the first applicant in my model. Optimal leniency program increases the effectiveness of law enforcement if fines are

Figure 4.1: EC Leniency Notice Cases

Year	Decision	Reduction of fine and number of companies
1998	Stainless Steel	40%: 2 companies, 10% : 4
1998	British Sugar	50%: 1, 10%: 3
1998	Preinsulated Pipes	30%: 5, 20%: 3
1998	Greek Ferries	45%: 1, 20%: 6
1999	Seamless Steel Tubes	40%: 1, 20%: 1
2000	Lysine	50%: 2, 30%: 2, 10%: 1
2001	Graphite Electrodes	70%: 1, 40%: 1, 30%: 1, 20%: 2, 10%: 3
2001	Sodium Gluconate	80%: 1, 40%: 2, 20%: 3
2001	Vitamins	100%: 1, 50%: 2, 35%: 3, 30%: 1, 15%:1, 10%: 1
2001	Brasseries Luxembourg	100%: 1, 20%: 3
2001	Brasseries Belges	50%:1, 30%: 1, 10%: 4
2001	Citric Acid	90%: 1, 50%:1, 40%:1, 30%:1, 20%: 1
2001	Zinc Phosphate	50%: 1, 40%: 1, 10%: 4
2001	Carbonless Paper	100%: 1, 50%: 1, 35%: 1, 20%: 1, 10%: 3
2002	Methionine	100%: 1, 50%: 1, 25%: 1
2002	Industrial & Medical Gases	25%: 2, 15%: 2, 10%: 2
2002	Fine Art Auction Houses	100%: 1, 40%: 1
2002	Plasterboard	40%: 1, 30%: 1
2002	Methylglucamine	100%: 1, 40%: 2
2002	Food Flavour Enhancers	100%: 1, 50%: 1, 40%: 1, 30%: 1
2002	Rond à Beton	20%: 1
2002	Speciality Graphite	100%: 1, 35%: 6, 19%: 1
2003	Sorbates	100%: 1, 50%: 1, 40%: 1, 30%: 1, 25%: 1
2003	Carbon & Graphite Products	100%: 1, 40%: 1, 30%: 2, 20%: 1
2003	Organic Peroxides	100%: 1, 50%: 1, 25%: 1, 15%: 1
2003	Industrial Copper Tubes	50%: 1, 30%: 1, 20%: 1
2004	Copper Plumbing Tubes	100%: 1, 50%: 1, 35%: 2, 15%: 1, 10%: 2
2004	Spanish Raw Tobacco	40%: 1, 25%: 2, 20%: 1, 10%: 1
2004	Needles etc	100%: 1
2004	Choline Chloride	30%: 2, 20%: 1
2005	Monochloroacetic Acid	100%: 1, 40%: 1, 25%: 1
2005	Industrial Thread	Some reductions
2005	Italian Raw Tobacco[12]	50%: 1, 30%: 1
2005	Plastic Industrial Bags	30%: 1, 25%: 2, 10%: 2
2005	Rubber Chemicals	100%: 1, 50%: 1, 20%: 1, 10%: 1
2006	Bleaching Chemicals	100%: 1, 40%: 1, 30%: 1, 10%: 1
2006	Acrylic Glass	100%: 1, 40%: 1, 30%: 1

sufficiently high or firms retaliate severely enough against deviation. The feature of optimal leniency program implies that leniency program should restrict benefits to the first reporting firm in order to minimize its negative effect on the expected fine.

Crackdown policy, on the other hand, means that antitrust authority spends all its resources on the target industry at a given period, then moves its focus to another in the next period and so on. Less extreme form is selective law enforcement where antitrust authority spends more on the target industry's cartel conviction at a given period. Assuming antitrust authority's resource is fixed, I examine whether crackdown policy or selective law enforcement is more effective to deter cartel than non-selective one.

Using two-industry model under the standard repeated game setting, I find that there exists selective law enforcement more effective than non-selective law enforcement given any increasing conviction technology function, which relates antitrust authority's resources to conviction probability. The efficacy of crackdown policy depends on the curvature of conviction technology function. Crackdown policy is more likely to be effective than non-selective law enforcement as this function is less concave. For example, crackdown policy is optimal if conviction probability is linearly related to the amount of budget.

The rest of this paper is organized as follows. Section 4.2 develops a cartel duration model and then analyze the effect of law enforcement. Section 4.3 introduces leniency program into the model and characterizes firms' leniency decision in every possible state. Section 4.4 characterizes optimal leniency program and discusses some policy issues on leniency program and the relation of this paper with literatures. Section 4.5 discusses the effect of selective law enforcement or crackdown policy on cartel deterrence. Conclusion follows in Section 4.6.

4.2 Cartel Duration Model

This section develops a model where cartels may collapse due to the presence of a stochastic shock after they are formed. Then, the model is extended into an environment with law enforcement.

4.2.1 A Representative Industry of Economy

A representative industry of the economy consists of $n \geq 2$ risk-neutral symmetric firms interacting repeatedly in the infinite, discrete time $t = 1, 2, \cdots$, and discounting future profit with the common discount rate $\delta \in (0, 1)$.

I assume that a stage game has a unique symmetric Nash equilibrium, and each firm gets payoff π^n at Nash equilibrium. Let π^c denote cartel payoff of each firm and π^d the static payoff from unilaterally deviating and choosing the static best response. Finally, π^r (π^p) will denote the payoff that a defector (non-defector) would get in the subsequent periods after defection until shock occurs. I assume that $\pi^d > \pi^c > \max(\pi^n, \pi^r, \pi^p) > 0$ holds.

There is a shock with probability $p \in (0, 1)$ in this industry at the beginning of each period. The occurrence of shock is independent across periods. Shock causes cartel to break down immediately and the industry to move to Nash equilibrium with expected stage payoff π^{ns}.

4.2.2 Information

Collusion agreements need to be administered and monitored, and induce members to communicate regularly, to exchange documents, and to produce other kind of hard evidence on the cartel that exposes them to the risk of conviction. So I assume that a piece of "hard" evidence is generated each period while a cartel is formulated and it is active. I also assume that each cartel member possesses a copy of hard evidence produced by the cartel and can costlessly transmit it to a third party if it wishes. For simplicity, I also assume that there is no decay of this hard evidence, so it lasts forever.

4.2.3 Structure of Game

I now consider the simplest environment where there is no antitrust law enforcement. In this environment, firms make cartel decision at the beginning of period 1 when there is no shock without loss. Each firm chooses its output in every period $t \geq 1$.

3 factors affect a firm's payoff at period t in this game: (1) cartel decision at period 1, (2) the presence of deviation action at period t or before and (3) the presence of shock until

period t. For example, each firm earns π^c at period 1 if the industry decides to collude and no firms deviate, while it gets π^n at period 1 if the industry decides not to collude. If a firm deviates unilaterally from cartel agreement at period $t \geq 1$ when there is no shock, it would get π^d at period t and π^r from period (t+1) to one period before shock occurs. When a shock occurs at period $t \geq 2$, each firm gets π^{ns} at period t and on regardless of cartel decision at period 1 and the presence of deviation until period (t-1).

4.2.4 Baseline Model

I will denote each firm's expected discounted payoff from collusion by $E\Pi^C$. Then $E\Pi^C$ is given by

$$\begin{aligned} E\Pi^C(p) &= \pi^c + \delta((1-p)\pi^c + p\pi^{ns}) + \delta^2((1-p)^2\pi^c + (p + p(1-p))\pi^{ns}) + \cdots \\ &= \frac{\pi^c}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \qquad (4.1) \end{aligned}$$

On the other hand, if one firm deviates unilaterally, its expected discounted payoff, denoted by $E\Pi^D$, will be

$$\begin{aligned} E\Pi^D(p) &= \pi^d + \delta((1-p)\pi^r + p\pi^{ns}) + \delta^2((1-p)^2\pi^r + (p + p(1-p))\pi^{ns}) + \cdots \\ &= \pi^d + \frac{\delta(1-p)\pi^r}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \qquad (4.2) \end{aligned}$$

Note that the punishment payoff π^r is obtained until one period before shock occurs. Firm $i's$ expected discounted cartel payoff and deviation payoff can be seen as a function of shock occurrence probability p in equation (4.1) and (4.2). Having these equations, cartel can be supported as subgame perfect equilibrium if and only if $E\Pi^C(p) > E\Pi^D(p)$, which is equivalent to

$$\frac{\pi^c}{1-\delta(1-p)} > \pi^d + \frac{\delta(1-p)\pi^r}{1-\delta(1-p)} \qquad (4.3)$$

In order to make the problem more interesting, I assume the following:

Assumption 2. $\frac{\pi^c}{1-\delta} > \pi^d + \frac{\delta}{1-\delta}\pi^r$

Assumption 2 means that the industry forms cartel if shock never occurs ($p = 0$). Note that the industry does not collude if shock occurs for sure at period 2 ($p = 1$) because $\pi^d > \pi^c$. Then I can derive the threshold shock occurrence probability, p^*, such that cartel is supported as subgame perfect equilibrium for all $p < p^*$.

Claim 17. *$p^* = 1 - \frac{\pi^d - \pi^c}{\delta(\pi^d - \pi^r)} \in (0, 1)$ and $E\Pi^C > E\Pi^D$ for all $p \in [0, p^*)$.*

Proof. Define $f(p) \equiv E\Pi^C(p) - E\Pi^D(p)$. Then I get

$$f(p) = \pi^c - \pi^d + \frac{\delta(1-p)(\pi^c - \pi^r)}{1 - \delta(1-p)}$$

If I let $f(p) = 0$, I can obtain a unique $p^* = 1 - \frac{\pi^d - \pi^c}{\delta(\pi^d - \pi^r)} = \frac{\pi^c - (1-\delta)\pi^d - \delta\pi^r}{\delta(\pi^d - \pi^r)}$. Assumption 2 implies $\pi^c - (1-\delta)\pi^d - \delta\pi^r > 0$, so $p^* > 0$. Since $\frac{\pi^d - \pi^c}{\delta(\pi^d - \pi^r)} > 0$, $p^* < 1$. Moreover,

$$\frac{df}{dp} = \frac{\delta(\pi^r - \pi^c)}{(1 - \delta(1-p))^2} < 0 \tag{4.4}$$

for all $p \in [0, 1]$ from $\pi^r < \pi^c$. So $f(p) > 0$ for all $p \in [0, p^*)$. □

This result shows that cartel is supported as subgame perfect equilibrium only when the expected cartel duration is longer than the threshold duration $D^* = \frac{1}{p^*} = \frac{\delta(\pi^d - \pi^r)}{\pi^c - (1-\delta)\pi^d - \delta\pi^r}$. It is easy to see that D^* is negatively related to π^c or δ, and positively related to π^d or π^r. Hence, an industry's cartelization depends on each firm's cartel or deviation payoff (π^c, π^d), strategy against deviation (π^r), the industry's patience (δ), and the shock occurrence possibility (p): higher π^c or δ increases the likelihood of cartelization, but higher π^d, π^r, or p decreases it.

4.2.5 Introduction of Law Enforcement

I now introduce law enforcement into the baseline model. The analysis of this step serves as a benchmark that I evaluate the effect of leniency program on cartel deterrence after I construct the full model in the next section. Following literatures on antitrust policy [Motta and Polo (2003), Spagnolo (2004), Harrington (2008), etc.], I assume that antitrust authority sets and commits policy parameters and then firms interact in the oligopolistic supergame.

Model Modification

The baseline model is modified in order to examine how law enforcement affects the firms' incentive to collude.

■ **Law enforcement** : Antitrust authority can set and execute a policy vector (F, R, α):

1. A monetary fine $F > 0$ that a colluding firm has to pay when the cartel is convicted,

2. A reduced fine $R \in [0, F]$ that a cartel member can pay if it deviates,

3. Conviction probability $\alpha \in (0, 1)$ with which cartel is discovered and convicted in each period $t \geq 1$ provided that it was not until period $(t - 1)$.

So the probability that cartel is convicted at period t is given by $(1 - \alpha)^{t-1}\alpha$. I assume that antitrust authority detects and convicts cartel at the end of each period, fines are charged at period (t+1), and the industry turns into Nash equilibrium from period (t+1) and on if cartel is convicted. In addition, cartel can be detected and convicted after it collapses. Since there does not exist leniency program yet, a monetary fine of each cartel member does not depend on whether it reveals the hard evidence to antitrust authority or not.

■ **Timing of the game** : With law enforcement, the sequence of the game is as follows.

1. Antitrust authority commit the policy vector (F, R, α) at period 0.

2. The industry decides whether it colludes or not at the beginning of period 1.

3. Each firm chooses its output in every period $t \geq 1$.

Figure 4.2 shows the game tree of this game until period 2.

■ **Factors affecting each period's payoff :** 3 factors in the baseline model affect a firm's payoff in each period of this game as well. Now, the presence of law enforcement also affects a firm's payoff at period t based on cartel conviction at period (t-1) or before. For example, suppose that a firm deviates unilaterally from cartel agreement and cartel is detected and convicted at period $t \geq 1$. Suppose also that shock happens at $(t + 1)$. Then, the deviator earns π^d at period t, $(\pi^{ns} - R)$ at period (t+1) and π^{ns} at period (t+2) and on.

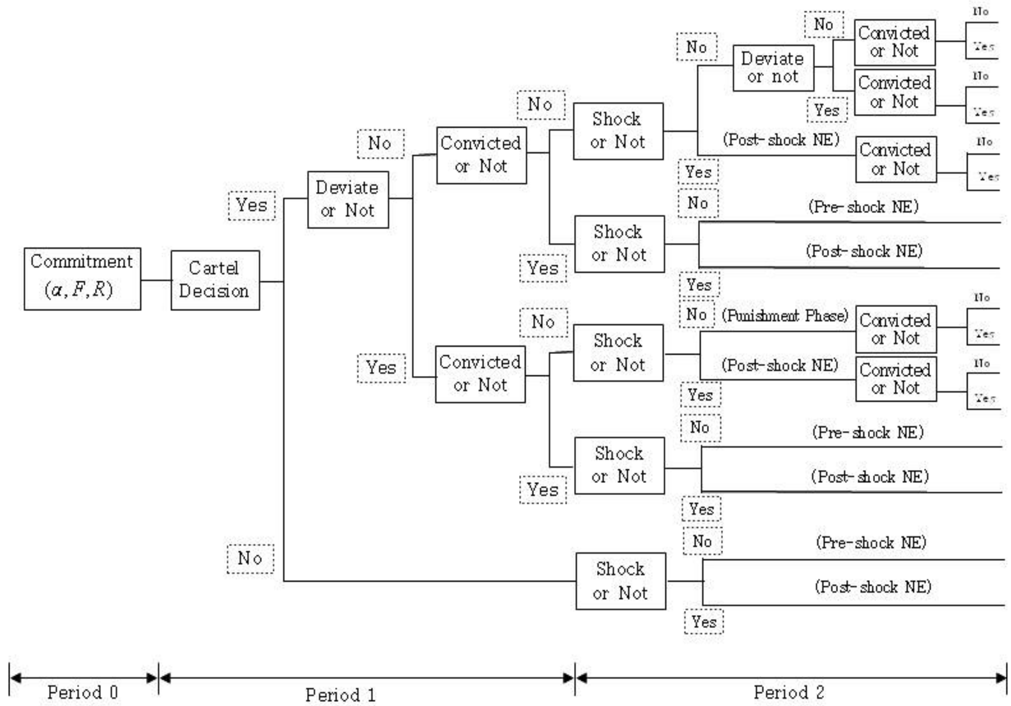


Figure 4.2: Game Tree without Leniency Program

The Effect of Law Enforcement

I need to examine the incentive compatibility constraint of each firm for cartel formation under the law enforcement (F, R, α) in order to see the effect of law enforcement. $E\Pi^{CL}$ denotes each firm's expected discounted cartel payoff under the law enforcement. Then $E\Pi^{CL}$ is equal to the following infinite sum.

$$
\begin{aligned}
E\Pi^{CL} \;=\;& \pi^c + \delta[(1-\alpha)(1-p)\pi^c + \alpha(1-p)(\pi^n - F) + (1-\alpha)p\pi^{ns} + \alpha p(\pi^{ns} - F)] \\
& + \delta^2[(1-\alpha)^2(1-p)^2\pi^c + \alpha(1-\alpha)(1-p)^2(\pi^n - F) \\
& \quad + (1-\alpha)^2 p(1-p)\pi^{ns} + \alpha(1-\alpha)p(1-p)(\pi^{ns} - F) + \alpha(1-p)^2\pi^n \\
& \quad + \alpha(1-p)p\pi^{ns} + \alpha(1-\alpha)p(\pi^{ns} - F) + (1-\alpha)^2 p\pi^{ns} + \alpha p\pi^{ns}] + \cdots
\end{aligned}
$$

At period 1, payoff from cartel is equal to π^c. But the payoff at period 2 depends on both whether the cartel is convicted at period 1 and whether shock takes place at period 2. With probability $(1-\alpha)(1-p)$, there is no conviction at period 1 nor shock at period 2. π^c would be the payoff at period 2 in this case. $\alpha(1-p)$ is the probability that cartel is convicted at period 1 but there is no shock at period 2. $(\pi^n - F)$ would be the payoff at period 2 in this case. With probability $(1-\alpha)p$, cartel is not convicted at period 1 but a shock occurs at period 2. Each firm would earn π^{ns} at period 2 in this case. Finally, αp is the probability that cartel is convicted at period 1 and shock occurs at period 2. $(\pi^{ns} - F)$ would be the payoff at period 2 in this case. So the expected sum of these four terms amounts to the expected payoff at period 2, and I can proceed in this way in order to get the payoff in the following periods.

With some algebra and notation $E\Pi^C(p)$ in equation (4.1), $E\Pi^{CL}$ is simplified into[2]

$$E\Pi^{CL} = E\Pi^C(\alpha + p - \alpha p) - \frac{\alpha\delta F}{1-\delta(1-\alpha)} + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \tag{4.5}$$

Equation (4.5) shows that the expected discounted payoff of cartel consists of three parts under the law enforcement (F, R, α). The first term, $E\Pi^C(\alpha + p - \alpha p)$, is the expected discounted payoff of cartel with expected duration $\frac{1}{\alpha+p-\alpha p}$, which is shortened due to law enforcement. This effect decreases the expected discounted payoff of cartel compared with the baseline model. The second term is the expected discounted fine from detection and conviction, which clearly decreases the cartel payoff. The last term reflects that Nash equilibrium payoff changes pre- and post-shock. If a cartel is convicted, each firm obtains π^n from one period after conviction to one period before shock occurs. This effect is not decisive because it depends on which is greater between π^n and π^{ns}. In aggregate, $E\Pi^{CL}$ is smaller than $E\Pi^C(p)$ in equation (4.1).

On the other hand, since a unilateral defector pays reduced fine R when cartel is convicted by antitrust authority, the expected discounted payoff from deviation comes to the following infinite sum.

$$\begin{aligned}
E\Pi^{DL} \;=\; & \pi^d + \delta[(1-\alpha)(1-p)\pi^r + \alpha(1-p)(\pi^n - R) + (1-\alpha)p\pi^{ns} + \alpha p(\pi^{ns} - R)] \\
& + \delta^2[(1-\alpha)^2(1-p)^2\pi^r + \alpha(1-\alpha)(1-p)^2(\pi^n - R) \\
& \quad + (1-\alpha)^2 p(1-p)\pi^{ns} + \alpha(1-\alpha)p(1-p)(\pi^{ns} - R) + \alpha(1-p)^2\pi^n \\
& \quad + \alpha(1-p)p\pi^{ns} + \alpha(1-\alpha)p(\pi^{ns} - R) + (1-\alpha)^2 p\pi^{ns} + \alpha p\pi^{ns}] + \cdots
\end{aligned}$$

[2] See the appendix for the derivation of equation (4.5), (4.8), (4.10), (4.11), (4.12), (4.16) and (4.24).

Note that π^r is obtained until the period when cartel is convicted or one period before a shock occurs. With notation $E\Pi^D(p)$ in equation (4.2), $E\Pi^{DL}$ becomes

$$E\Pi^{DL} = E\Pi^D(\alpha + p - \alpha p) - \frac{\alpha\delta R}{1-\delta(1-\alpha)} + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \tag{4.6}$$

Hence, under the law enforcement (F, R, α), cartel is supported as subgame perfect equilibrium if and only if $E\Pi^{CL} > E\Pi^{DL}$, or equivalently

$$E\Pi^C(\alpha + p - \alpha p) > E\Pi^D(\alpha + p - \alpha p) + \frac{\alpha\delta(F-R)}{1-\delta(1-\alpha)} \tag{4.7}$$

Similar to the baseline model, I can derive the threshold shock occurrence probability under law enforcement, p^L, such that cartel is supported as subgame perfect equilibrium for all $p < p^L$.

Claim 18. *(a) $p^L < p^*$ holds for any law enforcement (F, R, α).*

(b) The optimal reduced fine is given by $R^* = 0$.

Proof. **[Part (a)]** If I define $f_L(p) = E\Pi^C(\alpha + p - \alpha p) - E\Pi^D(\alpha + p - \alpha p) - \frac{\alpha\delta(F-R)}{1-\delta(1-\alpha)}$ and let $f_L(p) = 0$, I can obtain

$$p^L = 1 - \frac{1}{1-\alpha} * \frac{\pi^d + \frac{\alpha\delta(F-R)}{1-\delta(1-\alpha)} - \pi^c}{\delta(\pi^d + \frac{\alpha\delta(F-R)}{1-\delta(1-\alpha)} - \pi^r)}$$

Since $\alpha \in (0,1)$ and $\frac{\alpha\delta(F-R)}{1-\delta(1-\alpha)} \geq 0$, $p^L < p^* = 1 - \frac{\pi^d - \pi^c}{\delta(\pi^d - \pi^r)}$ holds for any (F, R, α).

[Part (b)] R^* has to make it most difficult that condition (4.7) is satisfied. Hence, $R^* = 0$ is immediate. $\square$

Claim 18 implies that any industry with $p \in [p^L, p^*)$ cannot be cartelized any longer after the introduction of law enforcement (F, R, α). The deterrence of law enforcement (F, R, α) comes from two different effects. The first is a duration effect, which occurs because law enforcement makes the expected duration of cartel shorter from $\frac{1}{p}$ to $\frac{1}{\alpha + p - \alpha p}$. This shortened duration reduces the incentive for firms to collude. The other is fine difference effect, which happens because a unilateral defector pays less fine than non-defector. This effect is maximized when the defector pays nothing.

4.3 Leniency Decision under Law Enforcement

4.3.1 Model Modification

Now I am ready to analyze a firm's leniency decision and the effectiveness of leniency program on cartel deterrence. To this end, I need to introduce leniency program into the model.

■ **Leniency Program** : Leniency program in this paper is defined as (R_1^L, R_2^L) under the law enforcement (F, R, α):

1. A reduced fine $R_1^L \in [0, F)$ that a leniency applicant has to pay if it is a unique applicant when it transmits the hard evidence of cartel to antitrust authority,

2. A reduced fine $R_2^L \in [R_1, F]$ that a leniency applicant has to pay if it is not a unique applicant when it transmits the hard evidence of cartel to antitrust authority[3].

Leniency program is defined as a reduced fine R_j^L, $where \quad j \in \{1, 2\}$, so the amount of reduction is $(F - R_j^L)$. R_j^L may be negative conceptually, but I restrict R_j^L as non-negative because antitrust law in most countries set this value in this range and in any country's legal system it is not easy to reward a cartel member just because it noticed its illegal action. If a cartel member is a unilateral deviator and applies leniency, then it only need to pay $\min(R_j^L, R)$ as fine. A firm, who does not apply leniency when there is a leniency application by another cartel member, will be fined by F for non-deviator or R for deviator. If one or more cartel members report the hard evidence to antitrust authority in a period, the other cartel members are convicted in the period for sure. Firms have to pay fines at the same period when leniency application is made. The industry turns to Nash equilibrium from the period and on when leniency notice is made.

[3]This paper proves the existence of simultaneous leniency application equilibrium in some states. In reality, however, even when the equilibrium is a simultaneous application of all cartel members, there can be a slight time gap among leniency applicants; one of them can be the first applicant, another firm can be the second, and so on. So R_2^L can be interpreted as the expected reduced fine when there are multiple leniency applicants. For example, suppose that antitrust law gives 100% reduction for the first applicant, 50% for the second, and 30% for the third. Also suppose that 3 cartel members apply leniency simultaneously in equilibrium. Then, $R_2^L = \frac{1}{3}(0.0 + 0.5 + 0.7)F = 0.4F$.

■ **Timing of the game** : Introduction of leniency program changes timing of the game as follows.

1. Antitrust authority commit the policy vector $\left(F, R, R_1^L, R_2^L, \alpha\right)$ at period 0.

2. The industry decides whether it colludes or not at the beginning of period 1 and each firm chooses its output at period 1.

3. If leniency is applicable, firms make leniency application decision at the beginning of each period $t \geq 2$ when they learn the existence of shock.

4. Each firm chooses its output at every period $t \geq 2$.

There are three conditions that Step 3 can be played at period t in this game: first, cartel should be formed in Step 2; second, cartel was not convicted until period $(t-1)$; third, leniency was not applied by any firm until period $(t-1)$. If leniency was applied by at least one firm before period t, there is no chance to apply leniency at period t because cartel was already convicted due to leniency notice. If one of these conditions fails at period $t \geq 1$, Step 3 is not played from period t and on.

■ **Factors affecting each period's payoff :** In addition to the factors under law enforcement, a firm's or competitors' leniency decision at period t or before affects its payoff at period t. For example, suppose that a firm deviates unilaterally from cartel agreement and cartel is not detected and convicted at period t. Suppose also that there is no shock until period t, but shock takes place at $(t+1)$ and the firm applies leniency unilaterally at $(t+1)$. Then, it would get π^d at period t, $(\pi^{ns} - \min(R_1^L, R))$ at period (t+1) and π^{ns} at period (t+2) and on. But its competitors would get $\pi(q^c, q^d)$ at period t, $(\pi^{ns} - F)$ at period (t+1) and π^{ns} at period (t+2) and on.

Figure 4.3 shows the game tree of this game until the leniency decision stage in period 2. I will proceed backwards in order to characterize the equilibrium of this game. So leniency decision game is analyzed first and then cartel agreement decision and optimal policy parameters are characterized.

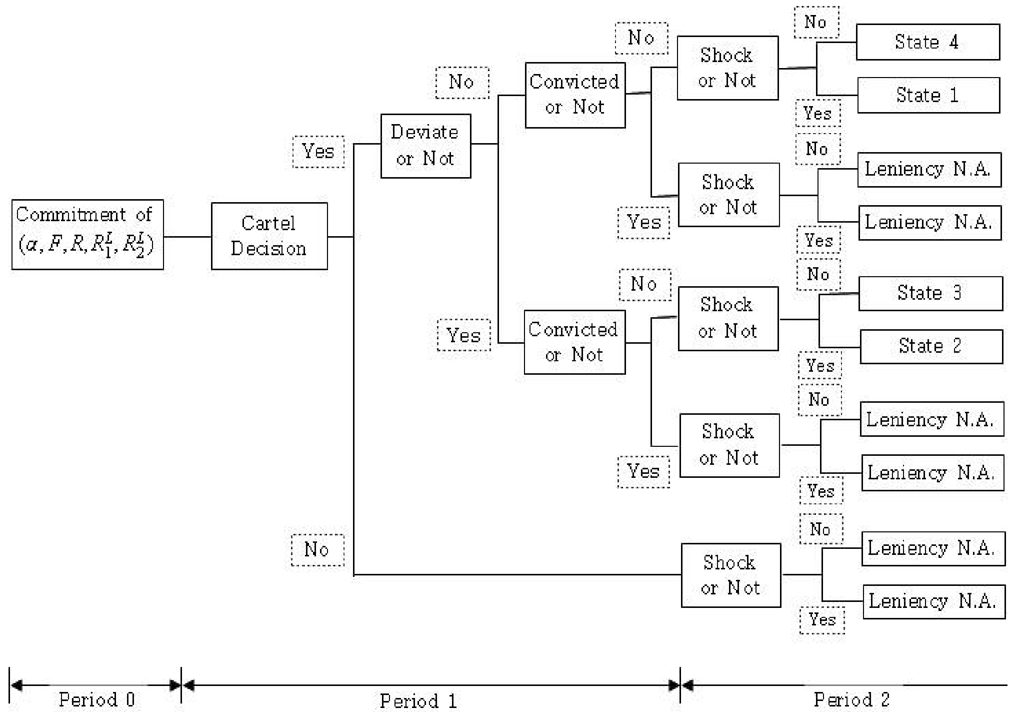


Figure 4.3: Game Tree with Leniency Program

4.3.2 Leniency Decision

In this subsection, I will characterize leniency application decision under law enforcement with leniency program, $\left(F, R, R_1^L, R_2^L, \alpha\right)$. Given three common conditions that leniency decision can be made, there are 4 different states in which firms make leniency decision at period t as shown in Figure 4.3:

1. ($State$ 1) a shock at period t or before, but no deviation until $(t-1)$,

2. ($State$ 2) a shock at period t or before, and a deviation by a firm at $(t-1)$ or before,

3. ($State$ 3) no shock until period t, but a deviation by a firm at $(t-1)$ or before,

4. ($State$ 4) no shock until period t, nor deviation until $(t-1)$.

I assume that each firm plays stationary Markov strategy in the sense that its strategy only depends on the payoff-relevant state whenever it moves. Then, leniency application decision game in each state can be represented by the following strategic game form.

$$
\begin{aligned}
G_s(M) \quad = \quad & (N, \Sigma, E\Pi^s)\,, where \\
& s \in \{State\ 1, State\ 2, State\ 3, State\ 4\} \\
& N = \{1, 2, \cdots, n\} \text{ is a set of firms} \\
& \Sigma = \Pi_{i=1}^{n} \Delta S_i \text{ is action space, where} \\
& \qquad \Delta S_i = [0,1] \text{ is firm i's strategy space,} \\
& \qquad \beta_i = \Pr(not\ report) \in \Delta S_i \text{ is a strategy chosen by firm i} \\
& \qquad \beta = (\beta_1, \beta_2, \cdots, \beta_n) \in \Sigma \text{ is a strategy profile} \\
& E\Pi^s = \{E\Pi_1^s, E\Pi_2^s, \cdots, E\Pi_n^s\} \text{ is an expected payoff vector, where} \\
& \qquad E\Pi_i^s : \Sigma \mapsto R \text{ is firm i's payoff function}
\end{aligned}
$$

Here the expected payoff vector $E\Pi^s$ varies according to the states. Before I characterize leniency decision, I introduce one assumption:

Assumption 3. $R_1^L < \frac{\alpha\delta F}{1-\delta(1-\alpha)}$

Assumption 3 requires that the reduced fine from unilateral leniency application is lower than the expected discounted fine when no firms ever apply for leniency. If this condition fails, the minimum fine of a leniency applicant is greater than the expected discounted fine of each firm without leniency program. Given δ, this assumption holds more easily when R_1^L is lower, and α or F is higher. For example, if R_1^L is equal to 0, this condition trivially holds.

Leniency Decision in $(State\ 1 : Shock + No\ Deviation)$ The characterization of leniency decision in $(State\ 1)$ is provided in the following result.

Claim 19. *The dominant strategy equilibrium in* $G_1(M) = \left(N, \Sigma, E\Pi^1\right)$ *is that all firms apply leniency (i.e.* $\beta_*^1 = 0$*).*

Proof. Consider each firm i's expected discounted continuation payoff in $(State\ 1)$ denoted by $E\Pi_i^1$. The set of each firm's pure strategies is given by $S_i = \{s_i^1, s_i^2\}$, where s_i^1 is to report and s_i^2 is not to report. Since there happened a shock and firm i is not a defector, $E\Pi_i^1(s_i, s_{-i})$ can be derived as follows.

$$E\Pi_i^1(s_i^1, s_{-i}) = \begin{cases} \frac{\pi^{ns}}{1-\delta} - R_1^L & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ \frac{\pi^{ns}}{1-\delta} - R_2^L & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

$$E\Pi_i^1(s_i^2, s_{-i}) = \begin{cases} \frac{\pi^{ns}}{1-\delta} - \frac{\alpha\delta F}{1-\delta(1-\alpha)} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ \frac{\pi^{ns}}{1-\delta} - F & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

Here, $s_{-i} = \Pi_{j\neq i} s_j^2$ represents that none of other firms choose to report while $s_{-i} \neq \Pi_{j\neq i} s_j^2$ means that there is at least one firm who reports. Hence, firm $i's$ best response is to report regardless of other firms' strategy because

$$\begin{aligned} \frac{\pi^{ns}}{1-\delta} - R_1^L &> \frac{\pi^{ns}}{1-\delta} - \frac{\alpha\delta F}{1-\delta(1-\alpha)} \quad \text{if } s_{-i} = \Pi_{j\neq i} s_j^2 \\ \frac{\pi^{ns}}{1-\delta} - R_2^L &\geq \frac{\pi^{ns}}{1-\delta} - F \quad \text{if } s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{aligned}$$

The first inequality comes from Assumption 3. □

This result holds because leniency application reduces the expected fine irrespective of other firms' strategy profile. So all cartel members apply leniency in (*State* 1).

Leniency Decision in $(State\ 2 : Shock{+}Deviation)$ The characterization of firms' leniency decision in (*State* 2) is quite similar to that in (*State* 1).

Claim 20. *The equilibrium after iterated elimination of weakly dominated strategies in* $G_2(M) = \left(N, \Sigma, E\Pi^2\right)$ *is that all firms (or all non-defectors) make leniency application.* (*i.e.* $\beta_*^2 = 0$)

Proof. See the appendix. □

Note that firms' leniency decision is not affected by the presence of deviation when there is a shock. Claim 19 and 20 guarantee that leniency application is made whenever there is shock.

Leniency Decision in $(State\ 3 : No\ Shock + Deviation)$ Since there happened a deviation at period $s < t$ and shock has not occurred until period t in (*State* 3), the industry would be in the punishment phase at period t. Like leniency decision in (*State* 1) and (*State* 2), I need to derive the expected continuation payoff of each firm from all possible combinations of pure strategies in order to characterize leniency decision.

It is not difficult to get the payoff when there is a leniency application. Since leniency application turns the industry into Nash equilibrium and shock did not occur in this state, leniency applicant's expected discounted continuation payoff when it is a defector (non-defector) comes to

$$
\begin{aligned}
E\Pi_{i=d}^{L_j} &= \pi^n - \min(R, R_j^L) + \delta((1-p)\pi^n + p\pi^{ns}) + \delta^2((1-p)^2\pi^n + (1-(1-p)^2)\pi^{ns}) + \cdots \\
&= \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \min(R, R_j^L) \\
E\Pi_{i=n}^{L_j} &= \pi^n - R_j^L + \delta((1-p)\pi^n + p\pi^{ns}) + \delta^2((1-p)^2\pi^n + (1-(1-p)^2)\pi^{ns}) + \cdots \\
&= \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - R_j^L
\end{aligned}
$$

Here, subscript d (n) represents a deviator (non-defector), $R_j^L = R_1^L$ if $s_{-i} = \Pi_{j\neq i} s_j^2$, and $R_j^L = R_2^L$ if $s_{-i} \neq \Pi_{j\neq i} s_j^2$. Similarly, when a firm does not apply leniency while at least one other firm self-reports, its expected discounted continuation payoff yields the following.

$$
\begin{aligned}
E\Pi_{i=d}^{L_{-i}} &= \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - R \\
E\Pi_{i=n}^{L_{-i}} &= \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - F
\end{aligned}
$$

In contrast, it is not trivial to get the expected discounted continuation payoff when no firms apply leniency. Let me denote the expected discounted continuation payoff of a defector by $E\Pi_{i=d}^{NL}$. Then, $E\Pi_{i=d}^{NL}$ can be obtained from the following sum.

$$
\begin{aligned}
E\Pi_{i=d}^{NL} = \; & \pi^r + \delta[(1-\alpha)(1-p)\pi^r + \alpha(1-p)(\pi^n - R) \\
& \quad + (1-\alpha)p(\pi^{ns} - \min\{R, R_2^L\}) + \alpha p(\pi^{ns} - R)] \\
& + \delta^2[(1-\alpha)^2(1-p)^2\pi^r + \alpha(1-\alpha)(1-p)^2(\pi^n - R) \\
& \quad + (1-\alpha)^2 p(1-p)(\pi^{ns} - \min\{R, R_2^L\}) + \alpha(1-\alpha)p(1-p)(\pi^{ns} - R) \\
& \quad + \alpha(1-p)^2\pi^n + \alpha(1-p)p\pi^{ns} + (1-\alpha)p\pi^{ns} + \alpha p \pi^{ns}] + \cdots
\end{aligned}
$$

A deviator earns π^r at period t because the industry is in the punishment phase. The payoff at period $(t+1)$ depends on cartel conviction at period t and shock occurrence at period $(t+1)$. With probability $(1-\alpha)(1-p)$, there is no conviction at period t nor shock at period $(t+1)$. π^r would be the payoff at period $(t+1)$ in this case. With probability $\alpha(1-p)$, cartel is convicted at period t but there is no shock at period $(t+1)$. Then, $(\pi^n - R)$ would be the payoff at period $(t+1)$. With probability $(1-\alpha)p$, cartel is not convicted at period t but there happens a shock at period $(t+1)$. Since the industry is in (*State* 2) at period $(t+1)$ in this case, all firms including the deviator would apply for leniency from Claim 20. Hence, $(\pi^{ns} - \min\{R, R_2^L\})$ would be the payoff at period $(t+1)$. Finally, αp is the probability that cartel is convicted at period t and there happens a shock at period $(t+1)$, so $(\pi^{ns} - R)$ would be the payoff at period $(t+1)$ in this case. As a result, the expected sum of payoffs in these four cases amounts to the expected payoff at period $(t+1)$. I can proceed in this way in order to get the expected payoff in the following periods. Note that Claim 20 is used at all the subsequent histories which end up with (*State* 2) when $E\Pi_{i=d}^{NL}$ is calculated. With some algebra, $E\Pi_{i=d}^{NL}$ is simplified into

$$\begin{aligned} E\Pi_{i=d}^{NL} &= \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\ &\quad - \frac{\pi^n - \pi^r + \delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)} \end{aligned} \tag{4.8}$$

Similarly, the expected discounted continuation payoff of non-defector when no firms apply leniency in (*State* 3), denoted by $E\Pi_{i=n}^{NL}$, comes to the following infinite sum.

$$\begin{aligned} E\Pi_{i=n}^{NL} &= \pi^p + \delta[(1-\alpha)(1-p)\pi^p + \alpha(1-p)(\pi^n - F) \\ &\quad + (1-\alpha)p(\pi^{ns} - R_2^L) + \alpha p(\pi^{ns} - F)] \\ &\quad + \delta^2[(1-\alpha)^2(1-p)^2\pi^p + \alpha(1-\alpha)(1-p)^2(\pi^n - F) \\ &\quad + (1-\alpha)^2 p(1-p)(\pi^{ns} - R_2^L) + \alpha(1-\alpha)p(1-p)(\pi^{ns} - F) \\ &\quad + \alpha(1-p)^2\pi^n + \alpha(1-p)p\pi^{ns} + (1-\alpha)p\pi^{ns} + \alpha p \pi^{ns}] + \cdots \end{aligned}$$

Again, we use Claim 20 at all the subsequent histories which end up with (*State* 2). Then,

$E\Pi_{i=n}^{NL}$ is given by

$$E\Pi_{i=n}^{NL} = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))} \tag{4.9}$$

Given $E\Pi_{i=d}^{L_j}$, $E\Pi_{i=n}^{L_j}$, $E\Pi_{i=d}^{L-i}$, $E\Pi_{i=n}^{L-i}$, $E\Pi_{i=d}^{NL}$ and $E\Pi_{i=n}^{NL}$, the characterization of leniency decision in ($State$ 3) is similar to ($State$ 1) or ($State$ 2).

Claim 21. *(a) If* $R_1^L < \frac{\pi^n-\pi^p+\delta(\alpha F+(1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ *or* $\min(R, R_1^L) < \frac{\pi^n-\pi^r+\delta(\alpha R+(1-\alpha)p\min(R,R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}$ *holds, the equilibrium after iterated elimination of weakly dominated strategies in* $G_3(M) = \left(N, \Sigma, E\Pi^3\right)$ *is that all firms make leniency application.* (*i.e.* $\beta_*^3 = 0$)

(b) If $R_1^L > \frac{\pi^n-\pi^p+\delta(\alpha F+(1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ *and* $\min(R, R_1^L) > \frac{\pi^n-\pi^r+\delta(\alpha R+(1-\alpha)p\min(R,R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}$ *hold, the unique efficient Nash equilibrium is that no firm applies leniency.* (*i.e.* $\beta_*^3 = 1$)

Proof. See the appendix. $\square$

Note that each part of Claim 21 requires one additional condition besides Assumption 3:

Condition 1. $R_1^L < \frac{\pi^n-\pi^p+\delta(\alpha F+(1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ *or* $\min(R, R_1^L) < \frac{\pi^n-\pi^r+\delta(\alpha R+(1-\alpha)p\min(R,R_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$

Condition 2. $R_1^L > \frac{\pi^n-\pi^p+\delta(\alpha F+(1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ *and* $\min(R, R_1^L) > \frac{\pi^n-\pi^r+\delta(\alpha R+(1-\alpha)p\min(R,R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}$

Any policy parameter vector $(F, R, R_1^L, R_2^L, \alpha)$ satisfies at least one condition, but which condition would hold for a given policy depends also on firms' strategy (π^p, π^r), Nash equilibrium payoff (π^n) and shock occurrence probability (p). For example, if firms use trigger strategy $(\pi^n = \pi^p = \pi^r)$ and $R_1^L = 0$, Condition 1 is applied. So all firms make leniency application in ($State$ 3) under this strategy. But if firms retaliate mildly against deviation ($\pi^n < \pi^p$, $\pi^n < \pi^r$) and the expected discounted fine is small enough compared with reduced fine, it might be the case that Condition 2 holds.

Leniency Decision in ($State\ 4 : No\ Shock + No\ Deviation$) Similar to other states, I need to derive the expected discounted continuation payoff in all combinations of pure strategies in order to characterize the leniency decision in ($State$ 4).

As before, non-trivial case is to get the expected discounted continuation payoff of each firm when no firms apply leniency in ($State$ 4). After no leniency, each firm can choose either collusion output or deviation output. First, look at the expected discounted continuation payoff from collusion given all other firms also stick to collusion continuously. Then each firm's expected discounted continuation payoff, denoted by $E\Pi^{NC}$, comes to the following.

$$\begin{aligned} E\Pi^{NC} \quad = \quad & \pi^c + \delta[(1-\alpha)(1-p)\pi^c + \alpha(1-p)(\pi^n - F) + (1-\alpha)p(\pi^{ns} - R_2^L) + \alpha p(\pi^{ns} - F)] \\ & + \delta^2[(1-\alpha)^2(1-p)^2\pi^c + \alpha(1-\alpha)(1-p)^2(\pi^n - F) \\ & \quad + (1-\alpha)^2 p(1-p)(\pi^{ns} - R_2^L) + \alpha(1-\alpha)p(1-p)(\pi^{ns} - F) \\ & \quad + \alpha(1-p)^2\pi^n + \alpha(1-p)p\pi^{ns} + (1-\alpha)p\pi^{ns} + \alpha p\pi^{ns}] + \cdots \end{aligned}$$

Each firm earns π^c at period t because the industry still colludes. The payoff at period $(t+1)$ again depends on cartel conviction at period t and shock occurrence at period $(t+1)$. So the firm earns π^c with probability $(1-\alpha)(1-p)$, $(\pi^n - F)$ with probability $\alpha(1-p)$, $(\pi^{ns} - R_2^L)$ with probability $(1-\alpha)p$, and $(\pi^{ns} - F)$ with probability αp, respectively. Here, the payoff $(\pi^{ns} - R_2^L)$ that the firm obtains when cartel is not convicted and shock takes place reflects Claim 19 because the industry is in (*State* 1) at period $(t+1)$ and all firms apply leniency. Then, the expected sum of payoffs in these four cases amounts to the expected payoff at period $(t+1)$. I can derive the expected payoff in the following periods similarly. Claim 19 is used at all the subsequent histories which end up with (*State* 1) when $E\Pi^{NC}$ is calculated. With the notation $E\Pi^C(p)$ in equation (1), $E\Pi^{NC}$ is simplified into

$$\begin{aligned} E\Pi^{NC} \quad = \quad & E\Pi^C(\alpha + p - \alpha p) - \frac{\delta(\alpha F + (1-\alpha)pR_2^L)}{1 - \delta(1 - (\alpha + p - \alpha p))} \\ & + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1 - \delta(1-p))(1 - \delta(1-\alpha)(1-p))} \end{aligned} \tag{4.10}$$

Suppose, on the other hand, that a firm unilaterally deviate after no cartel member applies leniency in (*State* 4). If Condition 1 holds, then leniency application would be made for sure in one period after the deviation from Claim 20 and 21 (a). Given these leniency decisions in (*State* 2) and (*State* 3), the expected discounted continuation payoff of a firm who deviates after no firm applies leniency in (*State* 4), denoted by $E\Pi^{ND_1}$, comes to the following infinite sum.

$$\begin{aligned} E\Pi^{ND_1} \quad = \quad & \pi^d + \delta[(1-\alpha)(1-p)(\pi^n - \min\{R, R_2^L\}) + \alpha(1-p)(\pi^n - R) \\ & \quad + (1-\alpha)p(\pi^{ns} - \min\{R, R_2^L\}) + \alpha p(\pi^{ns} - R)] \\ & + \delta^2[(1-p)^2\pi^n + (1-(1-p)^2)\pi^{ns}] + \cdots \end{aligned}$$

After some algebra, $E\Pi^{ND_1}$ is simplified into

$$E\Pi^{ND_1} = E\Pi^D(p) - \delta(\alpha R + (1-\alpha)\min\{R, R_2^L\}) + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} \tag{4.11}$$

$E\Pi^{NC}$ and $E\Pi^{ND_1}$ are the same in every period $t \geq 1$ because they only depend on the states in the subsequent periods and are constructed in a recursive way. Then, $E\Pi^{NC} > E\Pi^{ND_1}$ holds because it is an incentive constraint for firms to agree on collusion in period 1.

If Condition 2 holds, leniency application in the subsequent histories would be made only at the period when there is a shock from Claim 20 and 21 (b). Given these leniency decisions in (*State* 2) and (*State* 3), the expected discounted continuation payoff of a firm who deviates after no leniency application in (*State* 4), denoted by $E\Pi^{ND_2}$, comes to

$$\begin{aligned} E\Pi^{ND_2} \quad = \quad & \pi^d + \delta[(1-\alpha)(1-p)\pi^r + \alpha(1-p)(\pi^n - R) \\ & \quad + (1-\alpha)p(\pi^{ns} - \min(R, R_2^L)) + \alpha p(\pi^{ns} - R)] \\ & + \delta^2[(1-\alpha)^2(1-p)^2\pi^r + \alpha(1-\alpha)(1-p)^2(\pi^n - R) \\ & \quad + (1-\alpha)^2 p(1-p)(\pi^{ns} - \min(R, R_2^L)) + \alpha(1-\alpha)p(1-p)(\pi^{ns} - R) \\ & \quad + \alpha(1-p)^2\pi^n + \alpha(1-p)p\pi^{ns} + (1-\alpha)p\pi^{ns} + \alpha p\pi^{ns}] + \cdots \end{aligned}$$

$E\Pi^{ND_2}$ is simplified into the following equation.

$$\begin{aligned} E\Pi^{ND_2} \quad = \quad & E\Pi^D(\alpha + p - \alpha p) - \frac{\delta(\alpha R + (1-\alpha)p\min(R, R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))} \\ & + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \end{aligned} \tag{4.12}$$

Similar to $E\Pi^{NC}$ and $E\Pi^{ND_1}$, $E\Pi^{ND_2}$ is also the same in every period $t \geq 1$ for the same reason. So $E\Pi^{NC} > E\Pi^{ND_2}$ holds from the incentive constraint of cartel agreement.

Next, consider the strategy to apply leniency in ($State$ 4). Clearly, cartel cannot be sustained from period t. So the firm's expected discounted payoff, denoted by $E\Pi^{L_j}$ for $j \in \{1, 2\}$, becomes the following.

$$E\Pi^{L_j} = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - R_j^L$$

As before, $R_j^L = R_1^L$ if $s_{-i} = \Pi_{j\neq i} s_j^2$ and $R_j^L = R_2^L$ if $s_{-i} \neq \Pi_{j\neq i} s_j^2$. Since $E\Pi^{NC} > \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))}$ is just a participation constraint for firms to agree on collusion in period 1, $E\Pi^{NC} > E\Pi^{LD_j}$ must hold for all $j \in \{1, 2\}$ because leniency decision requires cartel agreement at period 1.

Finally, when a firm does not apply leniency while at least one other firm self-reports, its expected discounted continuation payoff yields the following.

$$E\Pi^{L^{-i}} = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - F$$

Having these continuation payoffs $(E\Pi^{NC}, E\Pi^{ND_1}, E\Pi^{ND_2}, E\Pi^{L_j}, E\Pi^{L^{-i}})$, leniency decision in ($State$ 4) can be characterized as follows.

Claim 22. *(a) The set of Nash equilibria in $G_4(M) = \left(N, \Sigma, E\Pi^4\right)$ includes (1) all firms report ($\beta_*^4 = 0$), (2) no firm reports ($\beta_*^4 = 1$) and (3) a symmetric mixed strategy with* Pr(*not report*) $= \beta_*^4 \in [0, 1)$ *for all firms.*

(b) The unique efficient Nash equilibrium is the one that no firm report ($\beta_*^4 = 1$).

Proof. See the appendix. □

Claim 22 says that although there are multiple Nash equilibria on leniency decision in active cartel, no reporting equilibrium is most efficient in that this equilibrium gives the highest expected discounted payoff to every cartel member.

Table 4.1 summarizes leniency decision in each state. Since Claim 19 to Claim 22 characterize leniency decision in all possible states, I can pin down the equilibrium outcome path on leniency decision.

Table 4.1: Leniency Decision in Each State

	State	Leniency Decision
State 1	Shock + No Deviation	All apply
State 2	Shock + Deviation	All apply
State 3 (Condition 1)	No Shock + Deviation	All apply
State 3 (Condition 2)	No Shock + Deviation	None applies
State 4	No Shock + No Deviation	None applies

Proposition 6. *Leniency application is made only in* (*State* 1) *on the equilibrium path.*

Proof. Claim 22 implies that leniency application is not made in (*State* 4). Since $E\Pi^{NC} > E\Pi^{ND_j}$ is satisfied when Condition j holds for $j \in \{1, 2\}$, deviation from cartel agreement is not on the equilibrium path. So (*State* 2) and (*State* 3) is not reached on the equilibrium path. Hence, the only state that firms apply leniency is (*State* 1) on the equilibrium path. □

Proposition 6 and Claim 22 imply that any firm in "active cartel" would not reveal cartel evidence to antitrust authority because it can get the highest expected payoff by sticking to cartel agreement. Put it differently, "active cartel" has an incentive device to block whistle-blow by cartel members in itself. On the other hand, Proposition 6 and Claim 19 say that leniency application is made when there happens a shock on the equilibrium path, and this leniency notice is applied by all cartel members. In other words, "dying cartel" cannot sustain the incentive device to block leniency any longer. These results explain two facts that I addressed in the introduction: why most leniency applications are made by "dying cartels" and there is a simultaneous "rush to report" if cartel collapses. In short, these two facts are equilibrium outcomes of firms' leniency decision in this duration model.

Given Proposition 6, the result on the detection rate follows.

Corollary 7. *If leniency program does not affect α, it increases the discovery rate of cartel in each period from α to $(\alpha + (1 - \alpha)p)$.*

Proof. From Proposition 6 and assumption on α, cartel is discovered when it is detected and convicted by antitrust authority or it collapses due to shock. So, the discovery rate of a cartel in each period comes to $(\alpha + (1 - \alpha)p)$. □

There are some remarks on Corollary 7. First, if α is fixed, leniency program does not affect the expected duration of an individual cartel. So in this particular case, we can evaluate the effect of leniency program on social welfare - sum of consumer surplus and the aggregate profit of firms - by looking at whether the incentive constraint becomes harder to satisfy or not.

Second, let me compare the number of discovered cartels without and with leniency program. Just for comparison, assume that the number of cartelized industries in the economy, denoted by K, is the same without and with leniency program and that α is also the same in both environments (i.e. leniency is neutral on social welfare). Then, the sequence of the number of discovered cartels is given by $\left\langle (1-\alpha)^{t-1}\alpha * K \right\rangle_{t=1}^{\infty}$ without leniency program and $\left\langle \alpha K, \left((1-\alpha)^{t-1}(1-p)^{t-2}(\alpha+p-\alpha p) * K\right)_{t=2}^{\infty} \right\rangle$ with leniency program, respectively. So, the discovered cartel cases become concentrated in the early periods when there is leniency program. Even if K gets smaller with leniency program, the same pattern can be observed when the number of cartelized industries under leniency program is not too small compared with that without leniency program. If K is larger with leniency program, this pattern becomes more conspicuous.

Finally, consider the effect of policy change that arises at the period when leniency program is introduced. Suppose that leniency program is installed at period t unexpectedly, leniency does not affect α and each firm's incentive constraint for cartel sustainability. Then, all dead cartels which were not caught until period $(t-1)$ are now in ($State$ 1) after the introduction of leniency program, so leniency application would be made from those dead cartels. So if I add the transition issue on top of Corollary 7, the steep increase in the number of discovered cartels becomes more remarkable right after the introduction of leniency program. These observations imply that the increase in the number of discovered cartels does not necessarily mean that leniency program is effective on cartel deterrence.

Recall that Assumption 3 is the restriction for policy parameters of these results. Differently put, leniency is not applied at all if law enforcement against cartel is not strong (i.e low α and F) and R_1^L is high or uncertain for some reason, say antitrust authority's discretion. This kind of situation seemed to be prevalent under the 1978 version of leniency program in the U.S. which failed to induce colluding firms to come forward. Now Assumption 3 is more easily satisfied in any country having leniency program because it is a common practice that the first reporting firm gets full exemption for sure.[4] So three facts on leniency application pattern are quite robust to the change in other policy parameters $\left(F, R, R_2^L, \alpha\right)$. This robustness explains why most countries who adopted leniency program experience similar application pattern even though each country has a different law enforcement and leniency program.

4.4 The Effect of Leniency Program

4.4.1 Cartel Decision and Optimal Leniency Program

Since I obtain each cartel member's leniency decision in every period $t \geq 2$, cartel agreement decision in period 1 and optimal leniency program is characterized in this subsection. To this end, I suppose that conviction rate α does not change due to the presence of leniency program. Given fixed α, optimal leniency program is one that makes this incentive constraint most difficult to be satisfied. Then, the best policy that antitrust authority can do is to choose the parameters $\left(R, R_1^L, R_2^L\right)$ optimally. In addition, I introduce one assumption on firms' strategy before the characterization of optimal leniency program.

Assumption 4. $\pi^p < \pi^n + \delta(\alpha + p - \alpha p)F$ *and* $\pi^r < \pi^n + \delta(\alpha + p - \alpha p)F$

This assumption implies that firms do not retaliate too mild against deviation from cartel agreement. When F is big enough or δ, α, p are not too small, this assumption is satisfied in general.

Now we need to derive incentive compatibility condition for cartel agreement in period 1. Given law enforcement with leniency program $\left(F, R, R_1^L, R_2^L, \alpha\right)$, the expected discounted cartel payoff of each firm, denoted by $E\Pi_{t=1}^{CLL}$, is given by $E\Pi_{t=1}^{CLL} = E\Pi^{NC}$ in equation (4.10).

[4] This practice does not necessarily mean $R_1^L = 0$ in reality because there may exist other type of sanctions, such as private damage suit. In addition, $\frac{\alpha\delta F}{1-\delta(1-\alpha)}$ is over-estimation for the expected discounted fine because it is derived from the assumption that every cartel is discovered in the end. Hence, there still exists the possibility that some cartels fail to satisfy Assumption 3 under this practice.

In contrast, the expected discounted payoff from unilateral deviation, denoted by $E\Pi_{t=1}^{DLL}$, depends on whether Condition 1 or Condition 2 holds.

If Condition 1 holds, $E\Pi_{t=1}^{DLL}$ is given by $E\Pi^{ND_1}$ in equation (4.11). Then, cartel is supported as stationary Markov perfect equilibrium if and only if $E\Pi_{t=1}^{CLL} > E\Pi_{t=1}^{DLL}$ under law enforcement with leniency program $\left(F, R, R_1^L, R_2^L, \alpha\right)$, or equivalently

$$\begin{aligned} E\Pi^C(\alpha+p-\alpha p) \quad > \quad & E\Pi^D(p) + \frac{\delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))} - \delta(\alpha R + (1-\alpha)\min\{R, R_2^L\}) \\ & + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} + \frac{\delta\alpha(1-p)(\pi^{ns}-\pi^n)}{(1-\delta(1-p))(1-\delta(1-(\alpha+p-\alpha p))} \end{aligned} \tag{4.13}$$

When Condition 2 holds, $E\Pi_{t=1}^{DLL}$ comes to $E\Pi^{ND_2}$ in equation (4.12). So cartel is supported as stationary Markov perfect equilibrium if and only if

$$E\Pi^C(\alpha+p-\alpha p) > E\Pi^D(\alpha+p-\alpha p) + \frac{\delta[\alpha(F-R) + (1-\alpha)p(R_2^L - \min\{R, R_2^L\})]}{1-\delta(1-(\alpha+p-\alpha p))} \tag{4.14}$$

Let D_1 and D_2 be the right-hand side of inequality (4.13) and (4.14), respectively, and define sets of feasible parameter vectors as X_1, X_2, Y_1 and Y_2 reflecting two possible conditions.

$$\begin{aligned} W &= \left\{(R, R_1^L, R_2^L) \mid R_1^L < \frac{\alpha\delta F}{1-\delta(1-\alpha)}\right\} \\ X_1 &= W \cap \left\{(R, R_1^L, R_2^L) \mid R_1^L < \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}\right\} \\ X_2 &= W \cap \left\{(R, R_1^L, R_2^L) \mid R_1^L > \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}\right\} \\ Y_1 &= W \cap \left\{(R, R_1^L, R_2^L) \mid \min(R_1^L, R) < \frac{\pi^n - \pi^r + \delta(\alpha R + (1-\alpha)p\min(R, R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}\right\} \\ Y_2 &= W \cap \left\{(R, R_1^L, R_2^L) \mid \min(R_1^L, R) > \frac{\pi^n - \pi^r + \delta(\alpha R + (1-\alpha)p\min(R, R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}\right\} \\ & \quad , where\ R \in [0, F], R_1^L \in [0, F), R_2^L \in [R_1^L, F] \end{aligned}$$

$X_1 \cup Y_1$ $(X_2 \cap Y_2)$ is all feasible parameter vector (R, R_1^L, R_2^L) that satisfies Condition 1 (Condition 2). Neither X_1 nor Y_1 is an empty set because $(R, R_1^L, R_2^L) = (R, 0, F) \in X_1$ and $(R, R_1^L, R_2^L) = (F, 0, F) \in Y_1$ from Assumption 4. Then, optimal leniency program

$\left(R^*, R_1^{L*}, R_2^{L*}\right)$ solves the following optimization problem.

$$\max \left\{ \max_{(R,R_1^L,R_2^L)\in X_1\cup Y_1} D_1, \max_{(R,R_1^L,R_2^L)\in X_2\cap Y_2} D_2 \right\} \tag{4.15}$$

Let $\left(R^1, R_1^{L1}, R_2^{L1}\right)$ be the argument maximizer of D_1 and $\left(R^2, R_1^{L2}, R_2^{L2}\right)$ be the argument maximizer of D_2, respectively. Clearly, $\left(R^*, R_1^{L*}, R_2^{L*}\right) \in \left\{\left(R^1, R_1^{L1}, R_2^{L1}\right), \left(R^2, R_1^{L2}, R_2^{L2}\right)\right\}$ holds.

Given that α is fixed, $\left(R^1, R_1^{L1}, R_2^{L1}\right)$ solves

$$\max_{(R,R_1^L,R_2^L)\in X_1\cup Y_1} \left\{ \frac{\delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))} - \delta(\alpha R + (1-\alpha)\min\{R, R_2^L\}) \right\}$$

because (R, R_1^L, R_2^L) does not affect the other terms in D_1. Since there exists some $(R, R_1^L, R_2^L) \in X_1$ from $X_1 \neq \emptyset$, $(0, R_1^L, F) \in X_1$ for the same R_1^L because $R_1^L < \frac{\alpha\delta F}{1-\delta(1-\alpha)}$ and $R_1^L < \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))} \leq \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pF)}{1-\delta(1-(\alpha+p-\alpha p))}$ holds. Then, it is easy to see that $\left(R^1, R_1^{L1}, R_2^{L1}\right) = \left(R^*, R_1^{L*}, R_2^{L*}\right) = (0, R_1^L, F)$ if $X_2 \cap Y_2 = \emptyset$.

Now suppose that $X_2 \cap Y_2 \neq \emptyset$, and let me denote the closure of $X_2 \cap Y_2$ by set Z. Then the maximization problem

$$\max_{(R,R_1^L,R_2^L)\in Z} \left\{ \frac{\delta[\alpha(F-R) + (1-\alpha)p(R_2^L - \min\{R, R_2^L\})]}{1-\delta(1-(\alpha+p-\alpha p))} \right\}$$

has solution $z' = \left(R', R_1^{L\prime}, R_2^{L\prime}\right) \in Z$ because the objective function is continuous and Z is compact. So $z' = \left(R^2, R_1^{L2}, R_2^{L2}\right)$ if $z' \in X_2 \cap Y_2$. While $\left(R^2, R_1^{L2}, R_2^{L2}\right)$ does not exist if $z' \notin X_2 \cap Y_2$, there exists $z \in X_2 \cap Y_2$ in $B_\varepsilon(z')$ for all $\varepsilon > 0$. Hence, I can almost treat $z' = \left(R^2, R_1^{L2}, R_2^{L2}\right)$. From this discussion, I can characterize optimal leniency program.

Proposition 7. *Given that α is fixed, optimal leniency program is given by*

(a) $\left(R^*, R_1^{L*}, R_2^{L*}\right) = \left(0, R_1^L, F\right)$ *if either (1)* $X_2 \cap Y_2 = \emptyset$ *or (2)* $X_2 \cap Y_2 \neq \emptyset$ *and* $D_1\left(0, R_1^L, F\right) > D_2\left(R', R_1^{L\prime}, R_2^{L\prime}\right)$.

(b) $\left(R^*, R_1^{L*}, R_2^{L*}\right) = \left(R', R_1^{L\prime}, R_2^{L\prime}\right)$ *if* $X_2 \cap Y_2 \neq \emptyset$ *and* $D_1\left(0, R_1^L, F\right) < D_2\left(R', R_1^{L\prime}, R_2^{L\prime}\right)$.

Note that leniency decision under optimal leniency program in (*State* 3 : *No Shock* + *Deviation*) is different between case (a) and (b) in Proposition 7.

4.4.2 The Effect of Optimal Leniency Program and Policy Implication

In this subsection, I analyze the effect of leniency program on cartel deterrence when law enforcement with leniency program is $\eta^* = \left(F, 0, R_1^L, F, \alpha\right)$. From the previous discussion, η^* is optimal leniency program if either (1) $X_2 \cap Y_2 = \emptyset$ or (2) $X_2 \cap Y_2 \neq \emptyset$ and $D_1\left(0, R_1^L, F\right) > D_2\left(R', R_1^{L\prime}, R_2^{L\prime}\right)$ holds, otherwise it is at least sub-optimal leniency program in the sense that $\left(0, R_1^L, F\right)$ is a maximizer of D_1. Comparison is made between optimal law enforcement without leniency program $(F, R, \alpha) = (F, 0, \alpha)$ and optimal leniency program (or sub-optimal leniency program) $\eta^* = \left(F, 0, R_1^L, F, \alpha\right)$ for consistency. Recall that welfare effect of leniency program is equivalent to the cartel deterrence effect when α is fixed.

Using condition (4.7) and (4.13), I obtain the necessary sufficient condition that optimal leniency program η^* increases cartel deterrence.

$$
\begin{aligned}
&E\Pi^D(p) - E\Pi^D(\alpha + p - \alpha p) + \frac{\delta(\alpha + p - \alpha p)F}{1 - \delta(1 - (\alpha + p - \alpha p))} - \frac{\delta \alpha F}{1 - \delta(1 - \alpha)} \\
&+ \frac{\delta(1 - p)(\pi^n - \pi^r)}{1 - \delta(1 - p)} + \frac{\delta \alpha (1 - p)(\pi^{ns} - \pi^n)}{(1 - \delta(1 - p))(1 - \delta(1 - (\alpha + p - \alpha p)))} \\
&> 0
\end{aligned}
$$

With some algebra, this condition is simplified into

$$
\left[\frac{(\alpha + p - \alpha p)}{1 - \delta(1 - (\alpha + p - \alpha p))} - \frac{\alpha}{1 - \delta(1 - \alpha)}\right] \delta F + \frac{(1 - (\alpha + p - \alpha p))(\pi^n - \pi^r)}{1 - \delta(1 - (\alpha + p - \alpha p))} > 0 \qquad (4.16)
$$

The first term in left side of equation (4.16) is positive and reflects the increase in the expected fine under η^*. The reason why the expected fine increases is that cartel is discovered by antitrust authority at least until cartel collapses under leniency program, whereas cartel may not be discovered even after cartel collapses under law enforcement without leniency program. In contrast, the second term depends on firms' strategy. Under trigger ($\pi^p = \pi^r = \pi^n$) or more severe strategy ($\pi^r < \pi^n$), optimal leniency program increases the effectiveness of law enforcement. In addition, equation (4.16) shows that if monetary fine F is high enough, optimal leniency program is effective irrespective of firms' strategy.

One interesting aspect of optimal leniency program in Proposition 7 (a) is not to give any reduction to the simultaneous applications. Proposition 7 (b) also implies that optimal leniency program gives the least reduction to the simultaneous applications among the policy vector $(R, R_1^L, R_2^L) \in X_2 \cap Y_2$. This is because no or least reduction to simultaneous applicants maximizes the expected fine when firms make cartel decision. However, it is not technically easy to set $R_2^L = F$ because R_2^L is an expected fine when the leniency equilibrium is application by multiple firms in reality. Because antitrust authority need to maintain R_2^L as high as possible given this characterization of optimal leniency program, it is a questionable practice to give significant reduction to the second or subsequent applicants.

It is also worthwhile to note that the first reporting firm does not necessarily get full exemption under optimal leniency program. In fact, optimal leniency program only requires that firms choose to report when cartel collapses, or equivalently Assumption 3 is satisfied for R_1^L. In our model, the deterrent effect of leniency program comes mainly from increased expected sanctions, which arises because cartel is discovered at least until it collapses. Since Assumption 3 holds for wide range of positive R_1^L if law enforcement is strong enough (i.e. high F and α), a partial reduction for the first reporting firm can be a way to improve the efficacy of optimal leniency program because it has an effect to raise R_2^L. Put differently, while optimal leniency program reinforces the effectiveness of law enforcement under high enough F, strong law enforcement enlarges the room to design leniency program more effectively.

4.4.3 Discussion

Now I consider a law enforcement with leniency program $\left(F, R, R_1^L, R_2^L, \alpha\right)$, and assume that $(F, R, R_1^L,\ R_2^L, \alpha)$ is more effective to deter cartel than (F, R, α). I also assume that [Condition 1] holds for $\left(F, R, R_1^L, R_2^L, \alpha\right)$, which is a more realistic case. Then, it must be the case that condition (4.13) fails for p^L, the threshold shock occurrence probability under law enforcement. In other words, if I let p^{LP} be the threshold shock occurrence probability under law enforcement with leniency program, then $p^{LP} < p^L$ holds when $\left(F, R, R_1^L, R_2^L, \alpha\right)$ is more effective to deter cartel than (F, R, α). Hence, given α, the average duration of cartelized industries becomes longer under the law enforcement with effective leniency program. In addition, the effective leniency program has an immediate effect to collapse unstable cartels when it is introduced because any cartel such that $p \in [p^{LP}, p^L)$ fails to maintain the incentive compatibility condition under leniency program. These cartels are discovered by leniency applications and expected to have a shorter duration on average because they collapse due to the introduction of leniency program.

On the other hand, the effectiveness of leniency program may be affected by possible change in α. Because leniency program increases the number of discovered cartels at least in the early periods after its introduction, antitrust authority can use fewer personnel and less budget for non-leniency "active" cases given its resource constraint. Antitrust authority's case burden is likely to reduce α after the introduction of leniency program. On the other hand, while antitrust authority deals with more cartel cases, it may accumulate more information on cartel and its personnel may improve their investigation skill thanks to more experiences of cartel cases. If the first effect dominates the learning effect, the expected duration of individual cartel would get longer after the introduction of leniency program. In addition, lower α offsets the effect that leniency program increases the expected discounted fine because $\frac{\delta(\alpha F+(1-\alpha)pR_2^L)}{1-\delta(1-p)(1-\alpha)}$ in equation (4.10) gets smaller. Hence, the effectiveness of leniency program gets worse and the average duration of cartelized industries becomes longer as α becomes lower.

Endogeneity of firms' strategy may also affect the effectiveness of leniency program. If leniency program is introduced, then firms may change the payoff in punishment phase (π^p, π^r) when they agree on cartel agreement in a direction where incentive constraint can be satisfied with leniency program. This effect, if exists, weakens the effect of leniency program.

Admitting the possibility that α and firm's strategy may change, I can get some clue to whether the adopted leniency program enhanced effectiveness of law enforcement from leniency cases right after the introduction of leniency program. If leniency program is introduced unexpectedly enough, cartels that are active at the period of introduction are likely to be agreed on without full consideration about the new policy. Due to the unexpected introduction of leniency program, the incentive compatibility constraint of each firm in those cartels converts from $E\Pi^{CL}(F, R, \alpha\ ; \pi^p, \pi^r) > E\Pi^{DL}(F, R, \alpha\ ; \pi^p, \pi^r)$ to $E\Pi^{CLL}_{t=1}(F, R, R_1^L, R_2^L, \alpha^L\ ; \pi^{p\prime}, \pi^{r\prime}) > E\Pi^{DLL}_{t=1}(F, R, R_1^L, R_2^L, \alpha^L\ ; \pi^{p\prime}, \pi^{r\prime})$. So if the new incentive constraint is more binding, some active cartels may fail to satisfy the new incentive constraint whereas they satisfied the old one. Then active cartels may collapse only because of policy change, and then cartel members apply the leniency in that case. Hence this type of leniency will take the form of a “dying cartel” purely caused by introduction of leniency program. This line of argument implies that the collapsing rate of cartel is likely to be higher at the period when leniency program was introduced than other periods. If the old incentive constraint is more binding and leniency program does not bring higher deterrence, then the collapsing probability of cartel would not change at the period when leniency was introduced or later. In that case, leniency cases would only report “dead” or “dying cartel” caused by some shock, not caused by leniency program. Hence, an effective leniency is likely to imply that significantly more cartels collapse right after its introduction than the other periods.

4.4.4 Relation to the Literature

There have been many studies on the effect of leniency program. The first paper addressing this issue is Motta and Polo (2003). They mainly focus on the value of Section B[5]: whether firms that report information while already under investigation by antitrust authority should also be eligible to leniency. Their central result is that leniency program may increase deterrence by making prosecution more effective although it has a negative effect by reducing overall sanctions. My model deals with Section A and the prosecution technology is simplified by assuming that only one leniency application is sufficient to convict cartel for sure.

Spagnolo (2004) examined the ability of Section A to deter cartels. After he divides “courageous” leniency programs, which reward the first reporting party with the fines paid by all other members, and “moderate” leniency programs that only reduce or cancel sanctions, he shows that the former achieves the first best of complete and costless deterrence while the latter may deter cartels under restrictive conditions. Other researches also confirm that positive rewards provide stronger tools for the prevention of cartels. [Brisset and Thomas (2004), Aubert, Rey, and Kovacic (2006), etc.] In particular, Brisset and Thomas (2004) illustrate

[5]Section A of leniency policy is to grant leniency to a corporation reporting illegal activity before an investigation has begun, while section B is to give it to a firm reporting after antitrust authority's investigation began.

that the moderate leniency does not prevent cartel formation under the first price sealed bid auction setting with the asymmetric information on costs. In contrast, Chen and Harrington (2007) and Harrington (2008) show that the maximum moderate leniency makes collusion more difficult whereas softer leniency programs may have pro-collusive effects. While I focus on the pattern of leniency notice under "moderate" leniency program, I show that optimal leniency program among moderate leniency program may have an effect to deter cartels and it may includes a partial reduction for the first reporting firm.

Motchenkova (2004) is closely related to this research in that both papers deal with the "rush to report" phenomenon and cartel duration explicitly. But the timing of firm's leniency application is quite different in two papers. Cartel members either self-report simultaneously and immediately after cartel is formed or never apply leniency in Motchenkova (2004), whereas this paper predicts that they apply leniency simultaneously right after cartel collapses. The timing of leniency application in Motchenkova (2004) is related to the feature of his model that firms stop colluding because of the potential sanctions from law enforcement. My paper adds unexpected shocks to the sources of cartel instability, examines the incentive compatibility constraint of each firm under this environment, and hence provides more realistic leniency decision outcome.

In contrast, Harrington and Chang (2009a) develops a rich model that endogenizes the birth and death process for cartels given a population of heterogeneous industries. While they focus on how one can infer the impact of competition policy on the population of cartels by measuring its impact on the population of discovered cartels, they show that the average duration of discovered cartels rises in the short run in response to a more effective competition policy because the marginally stable cartels tend to be of relatively short duration and they exit from the cartel population due to the new policy. This paper confirms that they pointed out one important effect of an effective leniency program on the average duration of discovered cartels in a different way, but I also add three additional immediate or short-run effects caused by the introduction of leniency program. First, if leniency program is introduced, all dead cartels satisfying Assumption 3 would be discovered by self-report. Their average duration $\left(\frac{1}{p}\right)$ is longer than that of the cartels discovered by antitrust authority's investigation $\left(\frac{1}{\alpha+p-\alpha p}\right)$

because law enforcement did not affect their collapse. Second, leniency program may lower detection rate for non-leniency cases at least in the short run as discussed above. Harrington and Chang (2009b) also points out this possibility based on antitrust authority's incentive structure.[6] In that case, the average duration gets longer in the short run. Finally, cartels that exit from the cartel population due to the installation of an effective leniency program would also be discovered by self report. These cartels have shorter average duration because they did not collapse due to an innate shock or antitrust authority's conviction. The overall effect of leniency program on discovered cartels' average duration should be evaluated considering all these effects besides the inference of Harrington and Chang (2009a).

In an empirical side, Miller (2009) develops a dynamic model which predicts that leniency program increases the detection rate and decreases the cartel formation rate if the number of detected cartels temporarily increases and then decreases in the long term. Using Poisson estimation method, he assessed that the 1993 version of leniency program in the U.S. is effective to both detect and deter cartels. I show that leniency program increases the detection rate if it reduces the sanctions of the first reporting firm sufficiently. In contrast, using the E.U. cartel data and hazard model, Brenner (2009) found that the 1996 version of leniency program in the E.U. did not change the average duration of discovered cartels, and interpreted that leniency program did not affect cartel's instability based on the inference of Harrington and Chang (2009a). Because of the same reason mentioned above, careful interpretation is required about this result.

4.5 Cartel Deterrence by Selective Law Enforcement

This section considers the environment where there is no shock nor leniency program, and tries to find a way that antitrust authority spends its resources available for cartel conviction most effectively. The idea to examine here is whether the principle of "selection and concentration" works in antitrust policy area.

[6]Given the resource constraint, antitrust authority has higher incentive to deal with leniency cases than non-leniency cases because antitrust authority can win easily in the court and enhance its reputation. So the law enforcement for non-leniency cases becomes loose.

4.5.1 Model

The model consists of industry, law enforcement, and timing of the game. As before, I assume that antitrust authority sets and commits policy parameters and then firms interact in the oligopolistic supergame.

■ **Industry** : There are 2 representative industries (sectors). Each industry $i \in \{1, 2\}$ consists of $n_i \geq 2$ risk-neutral symmetric firms interacting repeatedly in the infinite, discrete time $t = 1, 2, \cdots$. All firms in the same industry i discount future profit with the common discount factor $\delta_i \in (0, 1)$.

As in the baseline model, a stage game has a unique symmetric Nash equilibrium in each industry, and each firm gets payoff π_i^n at Nash equilibrium. Let π_i^c denote payoff of each firm when industry i colludes and π_i^d the static payoff from unilaterally deviating and choosing the static best response. Finally, π_i^r will denote payoff that a defector in industry i would get in the punishment phase. $\pi_i^d > \pi_i^c > \max(\pi_i^n, \pi_i^r) > 0$ holds for each industry i.

■ **Law enforcement** : Antitrust authority can set and execute the policy parameter vector $(F, R, \alpha_o, \alpha_e)$.

1. Monetary fine $F > 0$ that a cartel member has to pay when cartel is convicted,

2. A reduced fine $R \in [0, F]$ that a deviator from cartel agreement can pay instead of F when cartel is convicted,

3. Probability $\alpha_o \in [0, \bar{\alpha}]$ with which cartel in industry 1 (industry 2) is convicted at odd (even) period $t \geq 1$ provided that it was not until period $(t - 1)$,

4. Probability $\alpha_e \in [0, \bar{\alpha}]$ with which cartel in industry 2 (industry 1) is convicted at odd (even) period $t \geq 1$ provided that it was not until period $(t - 1)$

Here, $\bar{\alpha}$ represents probability to convict a cartel of an industry at period t when antitrust authority spends all the resources in the industry. I assume that antitrust authority's budget for cartel conviction is fixed in every period ($\bar{B} = B_o + B_e$) and that $\alpha_o \leq \alpha_e$ holds without loss. (α_o, α_e) captures the concept of selective law enforcement if $\alpha_o < \alpha_e$ is satisfied. Antitrust authority's conviction technology is given by an increasing concave function of budget in both industries.

Assumption 5. $\alpha_j = f(B_j)$ *where* $f(0) = 0,\ f(\bar{B}) = \bar{\alpha},\ f^{'} > 0,\ f^{''} \leq 0$ *for* $j \in \{o, e\}$

So I can let $\alpha_o = f(\frac{\bar{B}}{2} - \varepsilon)$ and $\alpha_e = f(\frac{\bar{B}}{2} + \varepsilon)$ for $\varepsilon \in [0, \frac{\bar{B}}{2}]$. Then $\varepsilon > 0$ represents a selective law enforcement while $\varepsilon = 0$ represents non-selective law enforcement. I let $\alpha_m = f(\frac{\bar{B}}{2})$, then $\alpha_o = \alpha_e = \alpha_m$ under non-selective law enforcement. Crackdown policy can be represented by $(\alpha_o, \alpha_e) = (0, \bar{\alpha})$, or equivalently $\varepsilon = \frac{\bar{B}}{2}$.

Antitrust authority convicts cartel at the end of each period. If cartel is convicted, fines are charged at period (t+1) and the industry turns into Nash equilibrium from period (t+1) and on.

■ **Timing of the game** : The game proceeds as follows.

1. Antitrust authority commits the policy vector $(F, R, \alpha_o, \alpha_e)$ at period 0.

2. Each industry decides whether it colludes or not at the beginning of period 1.

3. Each firm chooses its output in every period $t \geq 1$.

4.5.2 The Effect of Selective Law Enforcement

The following analysis focuses on the cartel deterrent effect of selective law enforcement in industry 1. The same inference is possible for industry 2 because of the symmetric structure of the model. Let the cartel payoff of industry 1 at odd period be V_{1o}^c and that at even period V_{1e}^c. Then, V_{1o}^c and V_{1e}^c are obtained from these simultaneous equations.

$$\begin{aligned} V_{1o}^c &= \pi_1^c + \delta_1 \left((1 - \alpha_o) V_{1e}^c + \alpha_o \left(\frac{\pi_1^n}{1 - \delta_1} - F \right) \right) \\ V_{1e}^c &= \pi_1^c + \delta_1 \left((1 - \alpha_e) V_{1o}^c + \alpha_e \left(\frac{\pi_1^n}{1 - \delta_1} - F \right) \right) \end{aligned}$$

Each firm in industry 1 earns π_1^c in the current period when industry 1 colludes. The future value of the cartel consists of two parts. The cartel is not convicted with probability $1 - \alpha_o$ $(1 - \alpha_e)$ at odd (even) period, and the future payoff of this cartel becomes V_{1e}^c (V_{1o}^c) in this case because the industry is still cartelized at even (odd) period. The cartel is convicted with probability α_o (α_e), and the future payoff of this case is simply the discounted sum of Nash equilibrium payoff minus monetary fine. So V_{1o}^c and V_{1e}^c are given by

$$\begin{aligned} V_{1o}^c &= \frac{1+\delta_1(1-\alpha_o)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\pi_1^c + \frac{\delta_1\alpha_o+\delta_1^2\alpha_e(1-\alpha_o)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\left(\frac{\pi_1^n}{1-\delta_1}-F\right) & (4.17) \\ V_{1e}^c &= \frac{1+\delta_1(1-\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\pi_1^c + \frac{\delta_1\alpha_e+\delta_1^2\alpha_o(1-\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\left(\frac{\pi_1^n}{1-\delta_1}-F\right) & (4.18) \end{aligned}$$

I also need the deviation payoff in each period in order to evaluate incentive compatibility condition for cartel formation. V_{1o}^d (V_{1e}^d) denotes the deviation payoff of a firm in industry 1 when it deviates at odd (even) period. Then, I can let

$$\begin{aligned} V_{1o}^d &= \pi_1^d + \delta_1\left((1-\alpha_o)V_{1e}^r + \alpha_o(\frac{\pi_1^n}{1-\delta_1}-R)\right) \\ V_{1e}^d &= \pi_1^d + \delta_1\left((1-\alpha_e)V_{1o}^r + \alpha_e(\frac{\pi_1^n}{1-\delta_1}-R)\right) \end{aligned}$$

Here, $V_{1o}^r(V_{1e}^r)$ represents each firm's continuation payoff at odd (even) period in the punishment phase. The above equations reflects that if a firm deviates at odd (even) period and the cartel is not convicted, then the industry would be in the punishment phase at the next even (odd) period. If I solve V_{1o}^r and V_{1e}^r in a similar way and substitute the solutions for V_{1o}^r and V_{1e}^r, V_{1o}^d and V_{1e}^d yield

$$\begin{aligned} V_{1o}^d &= \pi_1^d + \frac{\delta_1(1-\alpha_o)(1+\delta_1(1-\alpha_e))}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\pi_1^r & (4.19) \\ &\quad + \frac{\delta_1\alpha_o+\delta_1^2\alpha_e(1-\alpha_o)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\left(\frac{\pi_1^n}{1-\delta_1}-R\right) \\ V_{1e}^d &= \pi_1^d + \frac{\delta_1(1-\alpha_e)(1+\delta_1(1-\alpha_o))}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\pi_1^r & (4.20) \\ &\quad + \frac{\delta_1\alpha_e+\delta_1^2\alpha_o(1-\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}\left(\frac{\pi_1^n}{1-\delta_1}-R\right) \end{aligned}$$

Under selective law enforcement (α_o, α_e), cartel is supported as subgame perfect equilibrium if and only if $V_{1o}^c \geq V_{1o}^d$ and $V_{1e}^c \geq V_{1e}^d$ hold. We can obtain these conditions from equation

(4.17) to (4.20), which are equivalent to

$$\begin{aligned}
\pi_1^c \geq & \frac{1}{1+\delta_1(1-\alpha_o)} * \left(\left(1-\delta_1^2(1-\alpha_o)(1-\alpha_e)\right)\pi_1^d \right. \qquad (4.21)\\
& \left. + \left(\delta_1(1-\alpha_o)(1+\delta_1(1-\alpha_e))\right)\pi_1^r + \left(\delta_1\alpha_o + \delta_1^2\alpha_e(1-\alpha_o)\right)(F-R)\right) \\
\pi_1^c \geq & \frac{1}{1+\delta_1(1-\alpha_e)} * \left(\left(1-\delta_1^2(1-\alpha_o)(1-\alpha_e)\right)\pi_1^d \right. \qquad (4.22)\\
& \left. + \left(\delta_1(1-\alpha_e)(1+\delta_1(1-\alpha_o))\right)\pi_1^r + \left(\delta_1\alpha_e + \delta_1^2\alpha_o(1-\alpha_e)\right)(F-R)\right)
\end{aligned}$$

If antitrust authority commits non-selective law enforcement ($\alpha_o = \alpha_e = \alpha$) instead, condition (4.21) and (4.22) become the same and are simplified into

$$\pi_1^c \geq (1-\delta_1(1-\alpha))\pi_1^d + \delta_1(1-\alpha)\pi_1^r + \delta_1\alpha\,(F-R) \qquad (4.23)$$

The incentive compatibility conditions from (4.21) to (4.23) imply that the optimal reduced fine is $R = 0$ in any law enforcement. Then, the optimal non-selective law enforcement is given by $(R, \alpha) = (0, \alpha_m)$. In order to compare the effect of selective law enforcement with the optimal non-selective law enforcement, let $(R, \alpha) = (0, \alpha_m)$, and let E_1, E_2, and E be the right side of (4.21), (4.22) and (4.23), respectively. With some algebra, we can obtain

$$\begin{aligned}
E - E_1 &= \frac{\delta_1\left[(\alpha_m-\alpha_o) - \delta_1(1-\alpha_o)(\alpha_e-\alpha_m)\right]}{1+\delta_1(1-\alpha_o)} * (\pi_1^d - \pi_1^r + F) \qquad (4.24)\\
E_2 - E &= \frac{\delta_1\left[(\alpha_e-\alpha_m) - \delta_1(1-\alpha_e)(\alpha_m-\alpha_o)\right]}{1+\delta_1(1-\alpha_e)} * (\pi_1^d - \pi_1^r + F) \qquad (4.25)
\end{aligned}$$

It is easy to see that $E > E_1$ holds from equation (4.24) and Assumption 5. Hence, the sign of equation (4.25) determines the effectiveness of selective law enforcement (α_o, α_e) (equivalently, selective law enforcement ε such that $\alpha_e = f(\frac{\bar{B}}{2} + \varepsilon)$).

Proposition 8. *Either (a) every selective law enforcement is more effective than non-selective law enforcement, or (b) there is $\varepsilon^* \in (0, \frac{\bar{B}}{2}]$ such that for all $\varepsilon \in (0, \varepsilon^*)$ selective law enforcement ε is more effective than non-selective law enforcement (i.e. $E < E_2$ either for all $\varepsilon > 0$ or for $\varepsilon \in (0, \varepsilon^*)$).*

Proof. See the appendix. □

Proposition 8 shows that there is a continuum of selective law enforcement in which even period's incentive constraint of industry 1 gets more binding for every ε close enough to 0. Symmetrically, there is a continuum of selective law enforcement such that odd period's incentive constraint of industry 2 gets more binding for all ε sufficiently close to 0. So, selective law enforcement ε is more effective to deter cartel in both industries if ε is close enough to 0.

Using equation (4.25), I can evaluate the effect of crackdown policy as well. The sign of $(E_2 - E)$ is equal to that of $[(\alpha_e - \alpha_m) - \delta_1(1 - \alpha_e)(\alpha_m - \alpha_o)]$. Let me define a function $g : [0, \frac{\bar{B}}{2}] \mapsto R$ as

$$\begin{aligned} g(\varepsilon) &= \alpha_e(\varepsilon) - \alpha_m - \delta_1(1 - \alpha_e(\varepsilon))(\alpha_m - \alpha_o(\varepsilon)) \\ &= f(\frac{\bar{B}}{2} + \varepsilon) - f(\frac{\bar{B}}{2}) - \delta_1(1 - f(\frac{\bar{B}}{2} + \varepsilon))(f(\frac{\bar{B}}{2}) - f(\frac{\bar{B}}{2} - \varepsilon)) \end{aligned}$$

Since $\varepsilon = \frac{\bar{B}}{2}$ under crackdown policy, it is more effective than non-selective law enforcement if and only if $g(\frac{\bar{B}}{2}) > 0$, or equivalently

$$\bar{\alpha} > \alpha_m(1 + \delta_1(1 - \bar{\alpha})) \tag{4.26}$$

Condition (4.26) depends on the curvature of conviction technology and the discount factor of industry 1. It is more likely to be satisfied when conviction technology is less concave and the discount factor is smaller. Suppose conviction technology is convex instead. Then $\alpha_e - \alpha_m \geq \alpha_m - \alpha_o$ holds for any $\varepsilon \in (0, \frac{\bar{B}}{2}]$, and $E < E_2$ is satisfied from equation (4.25). So any selective law enforcement is more effective than non-selective law enforcement under convex conviction technology.

4.5.3 Selective Law Enforcement under Linear Conviction Technology

In this subsection, I assume that conviction technology is linearly correlated to the budget in both industries as a special case of concave conviction function.

Assumption 6. $\alpha_j = k * B_j$ *for* $j = 1, 2$ *where* $k > 0$

With this conviction technology, $\alpha_o + \alpha_e = \bar{\alpha}$ holds for any selective law enforcement

(α_o, α_e). The optimal non-selective law enforcement is given by $(\alpha_o, \alpha_e) = \left(\frac{\bar{\alpha}}{2}, \frac{\bar{\alpha}}{2}\right)$ because $\alpha_m = \frac{k\bar{B}}{2} = \frac{\bar{\alpha}}{2}$. Then for any (α_o, α_e) such that $\alpha_o < \alpha_e$, I obtain

$$\begin{aligned} E_2 - E &= \frac{\delta_1 \left[(\alpha_e - \frac{\bar{\alpha}}{2}) - \delta_1(1-\alpha_e)(\frac{\bar{\alpha}}{2} - \alpha_o)\right]}{1 + \delta_1(1-\alpha_e)} * (\pi_1^d - \pi_1^r + F) \\ &= \frac{\delta_1(2\alpha_e - \bar{\alpha})(1 - \delta_1(1-\alpha_e))}{2(1+\delta_1(1-\alpha_e))} * (\pi_1^d - \pi_1^r + F) > 0 \end{aligned}$$

The second equality holds from $\alpha_e - \frac{\bar{\alpha}}{2} = \frac{\bar{\alpha}}{2} - \alpha_o$ from Assumption 6. So antitrust authority can enhance deterrence to cartel with any selective law enforcement under linear conviction technology. In order to find an optimal selective law enforcement in this case, let me define a function $h : [\frac{\bar{\alpha}}{2}, \bar{\alpha}] \mapsto R$ as $E_2 - E$. Then I obtain

$$h(\alpha_e) = \frac{\delta_1(2\alpha_e - \bar{\alpha})(1 - \delta_1(1-\alpha_e))}{2(1+\delta_1(1-\alpha_e))}$$

The first derivative of this function yields

$$h^{'}(\alpha_e) = \frac{\delta_1(1 - \delta_1^2(1-\alpha_e)^2 + \delta_1(2\alpha_e - \bar{\alpha}))}{(1+\delta_1(1-\alpha_e))^2}$$

Hence, $h^{'}(\alpha_e) > 0$ holds for all $\alpha_e \in [\frac{\bar{\alpha}}{2}, \bar{\alpha}]$. Since the optimal selective law enforcement is the one that maximizes $(E_2 - E)$, the optimal selective law enforcement comes to $\alpha_e = \bar{\alpha}$ from $h^{'}(\alpha_e) > 0$.

Corollary 8. *If conviction technology is linear, crackdown policy is the optimal.*

4.5.4 The Effect of Randomized Selective Law Enforcement

I now consider a cartel agreement decision when firms do not know which detection rate between α_o and α_e would be applied at period 1. Then, V_{1o}^c (V_{1e}^c) can be interpreted as the continuation payoff from cartel of a firm in industry 1 at the period when antitrust authority applies α_o (α_e). Similarly, V_{1o}^d (V_{1e}^d) can be interpreted as the continuation payoff from deviation at the period when α_o (α_e) is applied. So a firm's expected payoff from cartel is given by $V_1^c = \frac{1}{2}(V_{1o}^c + V_{1e}^c)$ while the expected payoff from deviation comes to $V_1^d = \frac{1}{2}(V_{1o}^d + V_{1e}^d)$ at the beginning of the game. Hence, cartel can be supported as subgame perfect equilibrium if and only if $V_1^c \geq V_1^d$, $V_{1o}^c \geq V_{1o}^d$ and $V_{1e}^c \geq V_{1e}^d$ hold simultaneously. Equation (4.24) and (4.25) implies that E_2 is greater than E_1, so $V_{1e}^c \geq V_{1e}^d$ is more binding than $V_{1o}^c \geq V_{1o}^d$. The remaining thing is to examine which condition is more binding between $V_1^c \geq V_1^d$ and $V_{1e}^c \geq V_{1e}^d$.

For the purpose, I need to derive V_1^c and V_1^d using equation (4.17) to (4.20).

$$\begin{aligned} V_1^c &= \frac{2+\delta_1(2-\alpha_o-\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)} * \frac{\pi_1^c}{2} + \frac{\delta_1(\alpha_o+\alpha_e)+\delta_1^2(\alpha_o+\alpha_e-2\alpha_o\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)} * \frac{\pi_1^n - F}{2} \\ V_1^d &= \pi_1^d + \frac{\delta_1(2-\alpha_o-\alpha_e)+2\delta_1^2(1-\alpha_o)(1-\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)} * \frac{\pi_1^r}{2} \\ &\quad + \frac{\delta_1(\alpha_o+\alpha_e)+\delta_1^2(\alpha_o+\alpha_e-2\alpha_o\alpha_e)}{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)} * \frac{\pi_1^n - R}{2} \end{aligned}$$

So $V_1^c \geq V_1^d$ is equivalent to

$$\begin{aligned} \pi_1^c \geq\; & \frac{2\left(1-\delta_1^2(1-\alpha_o)(1-\alpha_e)\right)\pi_1^d}{2+\delta_1(2-\alpha_o-\alpha_e)} + \frac{\left(\delta_1(2-\alpha_o-\alpha_e)+2\delta_1^2(1-\alpha_o)(1-\alpha_e)\right)\pi_1^r}{2+\delta_1(2-\alpha_o-\alpha_e)} \\ & + \frac{\left(\delta_1(\alpha_o+\alpha_e)+\delta_1^2(\alpha_o+\alpha_e-2\alpha_o\alpha_e)\right)(F-R)}{2+\delta_1(2-\alpha_o-\alpha_e)} \end{aligned} \tag{4.27}$$

Let E_3 be the right-hand side of condition (4.27) and $R = 0$, then it is easy to show that $E_1 < E_3 < E_2$ holds. Hence $V_{1e}^c \geq V_{1e}^d$ is the most binding constraint. So, randomization between α_o and α_e at the beginning of the game does not enhance the efficacy of selective law enforcement (α_o, α_e).

Another exercise is to see the effect of a randomized crackdown policy where antitrust authority chooses industry 1 as a target with probability $(1-q)$ and industry 2 with probability q every period. Again I need to derive the (continuation) payoff from collusion in industry 1 denoted by V_{1R}^c. In order to get V_{1R}^c, I have to introduce the continuation payoff from collusion when industry 1 is not selected as a target industry, denoted by V_{1N}^c, and that when industry 1 is selected, denoted by V_{1T}^c. Then V_{1R}^c is obtained from these simultaneous equations.

$$\begin{aligned} V_{1R}^c &= qV_{1N}^c + (1-q)V_{1T}^c \\ V_{1N}^c &= \pi_1^c + \delta_1 V_{1R}^c \\ V_{1T}^c &= \pi_1^c + \delta_1\left((1-\bar{\alpha})V_{1R}^c + \bar{\alpha}(\frac{\pi_1^n}{1-\delta_1} - F)\right) \end{aligned}$$

So I have

$$V_{1R}^{c} = \frac{\pi_1^c}{1-\delta_1(1-\bar{\alpha}(1-q))} + \frac{\delta_1\bar{\alpha}(1-q)}{1-\delta_1(1-\bar{\alpha}(1-q))}\left(\frac{\pi_1^n}{1-\delta_1} - F\right)$$

Using the similar method, I can get the (continuation) payoff from deviation in industry 1 denoted by V_{1R}^d.

$$V_{1R}^{d} = \pi_1^d + \frac{\delta_1(1-\bar{\alpha}(1-q))}{1-\delta_1(1-\bar{\alpha}(1-q))}\pi_1^r + \frac{\delta_1\bar{\alpha}(1-q)}{1-\delta_1(1-\bar{\alpha}(1-q))}\left(\frac{\pi_1^n}{1-\delta_1} - R\right)$$

Then cartel can be formed under the randomized crackdown policy if and only if $V_{1R}^c \geq V_{1R}^d$, or equivalently

$$\pi_1^c \geq (1-\delta_1(1-\bar{\alpha}(1-q)))\,\pi_1^d + \delta_1(1-\bar{\alpha}(1-q))\pi_1^r + \delta_1\bar{\alpha}(1-q)\,(F-R) \tag{4.28}$$

Let E_4 be the right-hand side of condition (4.28), then E_4 is monotonically decreasing in q. Hence E_4 is maximized (minimized) at $q=0$ ($q=1$). Suppose $q=\frac{1}{2}$ as a special case, where antitrust authority chooses a target industry by tossing a fair coin. Let $R=0$ as usual. Then I have the following result.

Claim 23. *Under concave conviction technology, non-selective law enforcement is more effective than a randomized crackdown policy in which each industry is selected with probability* $\frac{1}{2}$ *in every period.*

Proof.

$$\begin{aligned} E - E_4 &= \Big[(1-\delta_1(1-\alpha_m))\pi_1^d + \delta_1(1-\alpha_m)\pi_1^r + \delta_1\alpha_m F\Big] \\ &\quad - \Big[(1-\delta_1(1-\frac{\bar{\alpha}}{2}))\pi_1^d + \delta_1(1-\frac{\bar{\alpha}}{2})\pi_1^r + \delta_1\frac{\bar{\alpha}}{2}F\Big] \\ &= \delta_1\left(\alpha_m - \frac{\bar{\alpha}}{2}\right)\left(\pi_1^d - \pi_1^r + F\right) \end{aligned}$$

So $E \geq E_4$ holds under Assumption 5 while $E = E_4$ holds under Assumption 6. □

Comparing this result with condition (4.26) shows that randomization weakens the efficacy of crackdown policy. Condition (4.26) implies that the commitment crackdown policy is more effective than non-selective law enforcement if conviction technology is not too concave.

Claim 23 also shows that the randomized crackdown policy is more effective only if conviction technology is convex.

4.6 Conclusion

In this research, I studied leniency program and crackdown policy focusing on their deterrence effect to cartels. This research gives some policy implications on how these policies should be designed and operated given antitrust authority's resource constraint.

Regarding to leniency program, antitrust authority should not be enraptured by its success in discovering more cartels because the number of discovered cartels explodes even under ineffective leniency program. As most of collapsed cartels are reported to antitrust authority, the number of discovered cartels cannot be taken as an indicator of leniency program's effectiveness. Given antitrust authority's resource constraint, the following factors are crucial to construct an effective law enforcement scheme with leniency program: full reduction to a deviator irrespective of its leniency application; to restrict the leniency benefits to the first reporting firm and to decrease the amount of reduction in sanctions; to maintain the level of sanctions as high as possible.

Selective law enforcement can enhance the effectiveness of antitrust law enforcement with the same amount of resources if it is well-designed. The optimal degree of selectivity depends on the curvature of conviction technology function. In particular, crackdown policy is more effective than non-selective law enforcement only when more resource on one industry increases the conviction probability by much.

Both leniency program and crackdown policy are "double-bladed sword" in that they may have an effect to deter cartel or may not. So it is important to design and operate these policy tools prudently. Antitrust authority has to keep in mind that too lenient leniency program may give higher incentive to collude and too selective law enforcement may lower the efficacy of antitrust law enforcement.

Appendix A

Appendix of Chapter 2

A.1 Identical Constant Marginal Cost Model

The Relation between δ^* and L

(Nash Reversion Strategy)

Under this strategy, the deviation payoff of each firm comes to $\pi^D(L) = \left(\frac{L+1}{4L}\right)^2 + \frac{\delta}{(1-\delta)(L+1)^2}$. So, collusion is sustainable if and only if $\delta \geq \frac{L^2+2L+1}{L^2+6L+1}$. Given $\delta^* = \frac{L^2+2L+1}{L^2+6L+1}$, $f'(L) > 0$ holds.

(Optimal Punishment Strategy)

Let $f_n(N)$ be the threshold discount rate under Nash reversion strategy and $f_o(N)$ be that under optimal punishment strategy $\sigma(\bar{q}, \tilde{q})$. [Abreu (1986), Mailath and Samuelson (2006)] Define $\mu_N(q) = q(1-Nq)$. Then, under optimal punishment strategy $\sigma(\bar{q}, \tilde{q})$, I can have $\pi^D(N) = \left(\frac{N+1}{4N}\right)^2 + \frac{\delta}{(1-\delta)} v_N^*$, where $v_N^* = \mu_N(\tilde{q}) + \frac{\delta}{1-\delta}\mu_N(\bar{q})$. Since firms compete *à la* Cournot pre-merger, the following inequality holds:

$$\pi^C(N) < \pi^D(N) \Leftrightarrow \frac{1}{4N} < \left(\frac{N+1}{4N}\right)^2 + \delta v_N^*$$

Differently put, $\delta < f_o(N)$ holds. Note that $v_N^* \leq \pi_\infty(N) \equiv \frac{1}{(1-\delta)(L+1)^2}$ holds because v_N^* is the minimum payoff for each player possible under a (strongly symmetric) subgame perfect equilibrium and $\pi_\infty(N)$ is one feasible subgame perfect equilibrium payoff. So, $f_o(N) \leq f_n(N)$ is satisfied. Furthermore, we already know that $f_n(N)$ is strictly increasing function.

Now suppose that $\delta > f_n(L)$ post-merger for some $L < N$. Since $\delta > f_n(L)$ and $v_L^* \leq \pi_\infty(L)$, we have

$$\frac{1}{4L} > \left(\frac{L+1}{4L}\right)^2 + \delta\pi_\infty(L) \geq \left(\frac{L+1}{4L}\right)^2 + \delta v_L^*$$

Hence, optimal punishment strategy supports perfect collusion as subgame perfect equilibrium, which means that $f_o(L) < \delta < f_o(N)$ for such $L < N$. Even when $\delta < f_n(L)$ post-merger, it may be the case that $\frac{1}{4L} \geq \left(\frac{L+1}{4L}\right)^2 + \frac{\delta}{(1-\delta)} v_L^* $ from $v_L^* \leq \pi_\infty(L)$, which also means that $f_o(L) < \delta < f_o(N)$ for such $L < N$.

Proof of Claim 3

[Part (a)]For $M > 0$, $g_2(N, M) = \frac{1}{4(N-M)} - \frac{M+1}{(N+1)^2} = \frac{(N-2M-1)^2}{4(N-M)(N+1)^2}$. It holds trivially that $g_2(N, \frac{N-1}{2}) = 0$ and $g_2(N, M) > 0$ if $M \neq \frac{N-1}{2}$.

[Part (b)]From definition, $g_2(N, N-1) = \frac{1}{4} - \frac{N}{(N+1)^2}$ while $g_2(N, 1) = \frac{1}{4(N-1)} - \frac{2}{(N+1)^2}$. Hence, $g_2(N, N-1) - g_2(N, 1) = \frac{(N-2)(N^2-2N+5)}{4(N-1)(N+1)^2} > 0$ for all $N \geq 3$.

Proof of Corollary 1

[Part (a)]$g_2(N, M) - g_1(N, M) = \frac{1}{4(N-M)} - \frac{1}{(N-M+1)^2} = \frac{(N-M-1)^2}{4(N-M)(N-M+1)^2} > 0$.

[Part (b)]$\frac{\partial S_1}{\partial M} = -\frac{1}{(N-M+1)^3} < 0$ and $\frac{\partial^2 S_1}{\partial M^2} = -\frac{1}{(N-M+1)^4} < 0$, so $S_1(N, M)$ is decreasing and concave in M. Since $S_2(N, N-1) = \frac{4-(N+1)^2}{8(N+1)^2}$, $S_1(N, M) > S_2(N, M)$ for $\forall\ M \in (0, N-1)$

Proof of Proposition 1

Recall that $\hat{M}$ be such that $g_1(N, \hat{M}) = 0$ and $g_1(N, M) < 0$ ($g_1(N, M) > 0$) for all $M < \hat{M}$ ($M > \hat{M}$ *resp.*). M^* is either (1) $M^* < \hat{M}$ or (2) $M^* \geq \hat{M}$. If $M^* < \hat{M}$, then $g_1(N, M^*) < 0$. Hence, $g_3(N, M^*) = g_2(N, M^*) \geq 0$ and $g_3(N, M) = g_1(N, M) < 0$ for all $M \in (0, M^*)$. So, $M^{**} = M^*$ in case (1). If $M^* \geq \hat{M}$, then $g_1(N, M^*) \geq 0$. Hence, $g_3(N, \hat{M}) = g_1(N, \hat{M}) = 0$ and $g_3(N, M) = g_1(N, M) < 0$ for all $M < \hat{M}$. So, $M^{**} = \hat{M}$ in case (2). So $M^{**} \leq \hat{M}$ holds in any case.

Efficiency-Gaining Merger's Profitability & Welfare Effect in [Case 3]

[Case 3-E-1 : $\hat{g}_3(N, 1) \geq 0$, $\hat{S}_3(N, 1) > 0$] Define $D_M \equiv [1, M^* - 1]$, then D_M is compact in R^+ and $\hat{S}_3(N, M)$ is continuous in D_M. Hence, there exists $\tilde{M} \in D_M$ such that $\tilde{M} \in \{\arg \max_{M \in D_M} \hat{S}_3(N, M)\}$. Since $\hat{S}_3(N, 1) > 0$, $\hat{S}_3(N, \tilde{M}) > 0$. So $\hat{S}_3(N, M)$ is maximized at either $\tilde{M}$ if $\hat{S}_3(N, \tilde{M}) \geqslant \hat{S}_3(N, N-1)$, or $N-1$ if $\hat{S}_3(N, \tilde{M}) < \hat{S}_3(N, N-1)$. Moreover, merger is privately profitable at $\tilde{M}$ from $\hat{g}_3(N, 1) \geq 0$ and convexity of $g_1(N, M)$, whereas merger to monopoly is also privately profitable. Therefore, the optimal size of merger is compatible with the incentive of firms in this case.

[Case 3-E-2 : $\hat{g}_3(N,1) < 0,\ \hat{S}_3(N,1) \geq 0$**]** There exists $\tilde{M} \in D_M$ such that $\tilde{M} \in \{\arg\max_{M \in D_M} \hat{S}_3(N,M)\}$ and $\hat{S}_3(N,\tilde{M}) > 0$ as before. Contrary to [Case 3-E-1], however, it can be the case that $\hat{g}_3(N,\tilde{M}) < 0$. In fact, for $\forall\ M \in D_M$ such that $\hat{S}_3(N,M) \geq 0$, $\hat{g}_3(N,M)$ may be negative. Moreover, $\hat{S}_3(N,M) \leq \hat{S}_3(N,N-1) < \frac{4-(N+1)^2}{8(N+1)^2} + (N-1)\left[\frac{2}{(N+1)^2} - \frac{1}{N^2}\right] < 0$ for all $N \geq 4$. So $\hat{S}_3(N,M) < 0$ holds for $M \geq M^*$. Then, if $\hat{g}_3(N,M) < 0$ for all $M \in D_M$ such that $\hat{S}_3(N,M) \geq 0$, there is no M that is both privately profitable and socially beneficial. But, if there is $M \in D_M$ such that $\hat{g}_3(N,M) \geq 0$ and $\hat{S}_3(N,M) \geq 0$, then there exists $\check{M}$ which gives the highest $\hat{S}_3(N,M)$ among privately profitable $M's$.

[Case 3-E-3 : $\hat{g}_3(N,1) < 0,\ \hat{S}_3(N,1) < 0$**]** Since $S_1(N,M)$ is concave in M and $M * F$ is linear in M, $\hat{S}_3(N,M) < 0$ for $\forall\ M \geq 1$ in this case. Hence, every size of merger is socially injurious in this case while any merger such that $M \geq M^*$ is privately profitable.

A.2 Asymmetric Increasing Marginal Cost Model

Proof of Claim 4

Take a partial derivative of $g_R^{1+2}(e)$ with respect to e_j for some $j \geq 3$, then I have

$$\begin{aligned}
\frac{\partial}{\partial e_j} g_R^{1+2}(e) &= \frac{1}{(1+e_j)^2}\Bigg[\frac{e_1(1+2e_1)}{(1+e_1)^2(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^3} + \frac{e_2(1+2e_2)}{(1+e_2)^2(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^3} \\
&\qquad - \frac{(e_1+e_2)(1+2(e_1+e_2))}{(1+e_1+e_2)^2(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^3}\Bigg] \\
&= \frac{2}{(1+e_j)^2}\Bigg[\frac{\pi_1^* + \pi_2^*}{1+\sum_{k=1}^{N}\frac{e_k}{1+e_k}} - \frac{\pi_M^{1+2}}{1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k}}\Bigg]
\end{aligned}$$

Given $g_R^{1+2}(e) = \pi_M^{1+2} - (\pi_1^* + \pi_2^*) > 0$, $\frac{\partial}{\partial e_j} g_R^{1+2}(e) < 0$ holds from $\frac{e_1+e_2}{1+e_1+e_2} < \frac{e_1}{1+e_1} + \frac{e_2}{1+e_2}$.

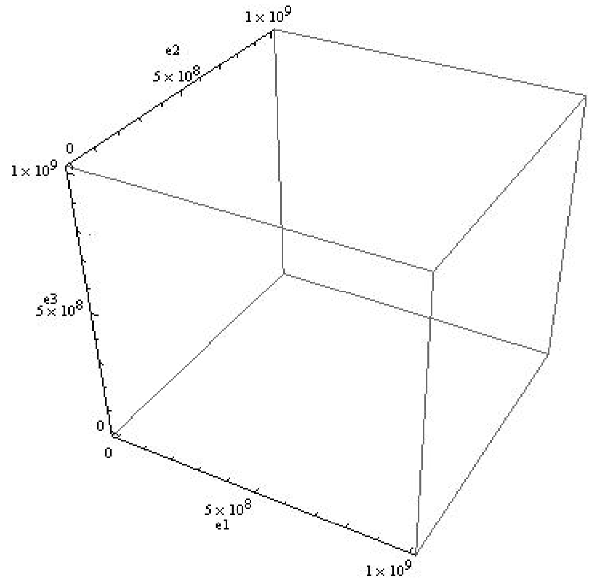


Figure A.1: Comparative Statics - Outsider's Efficiency

Numerical Proof of $\frac{\partial}{\partial e_3} g_R^{1+2}(e) < 0$ when $N = 3$

Taking a partial derivative of $g_R^{1+2}(e)$ with respect to e_3 when $N = 3$, then I have

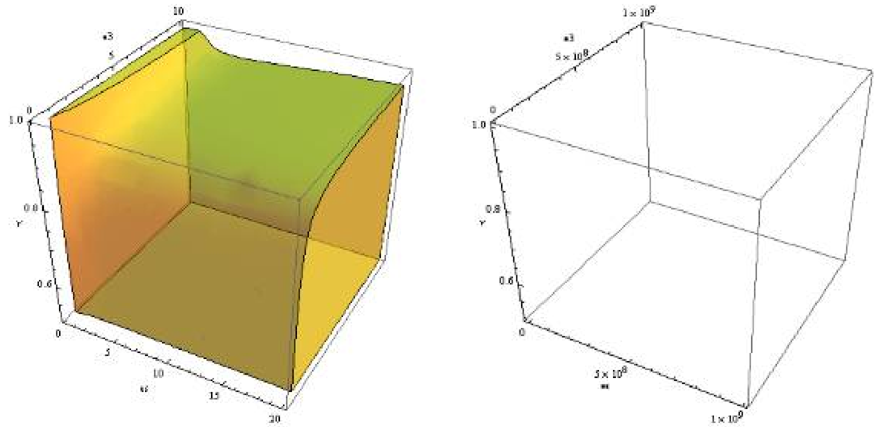
$$\begin{aligned}\frac{\partial}{\partial e_3} g_R^{1+2}(e) \quad = \quad & \frac{1}{(1+e_3)^2}\Big[\frac{e_1(1+2e_1)}{(1+e_1)^2(1+\sum_{k=1}^{3}\frac{e_k}{1+e_k})^3} + \frac{e_2(1+2e_2)}{(1+e_2)^2(1+\sum_{k=1}^{3}\frac{e_k}{1+e_k})^3} \\ & - \frac{(e_1+e_2)(1+2(e_1+e_2))}{(1+e_1+e_2)^2(1+\frac{e_1+e_2}{1+e_1+e_2}+\frac{e_3}{1+e_3})^3}\Big]\end{aligned}$$

So the sign of $\frac{\partial}{\partial e_3} g_R^{1+2}(e)$ can be checked for any $e \in R_+^3$ with the help of mathematica. Figure A.1 shows that the area such that $\frac{\partial}{\partial e_3} g_R^{1+2}(e) \geq 0$ is empty, which means that $\frac{\partial}{\partial e_3} g_R^{1+2}(e) < 0$ for all $e \in [0.1^6, 10^9]^3$.

Numerical Proof that $\frac{\partial}{\partial \nu} g_R^{1+2}(e_s, e_3, \nu) > 0$ if $g_R^{1+2}(e_s, e_3, \nu) \leq 0$

If I take a partial derivative of $g_R^{1+2}(e_s, e_3, \nu)$ with respect to ν, then I obtain

$$\begin{aligned}\frac{\partial}{\partial \nu} g_R^{1+2}(e_s, e_3, \nu) = & \frac{e_s^3(1+e_3)^2(2\nu-1)}{2} * \\ & \frac{(1+2e_s(1-2\nu)^2+e_s^2(\nu-1)\nu+e_3(-2+4e_s^2(\nu-1)\nu+e_s(1-16\nu+16\nu^2)))}{(-1-2e_s+3e_s^2(\nu-1)\nu+e_3(-2-3e_s+4e_s^2(\nu-1)\nu))^3}\end{aligned}$$



(A) (B)

(A) : Not-Profitable Merger

(B) : Not-Profitable Merger such that $\frac{\partial}{\partial \lambda} g^{1+2}(e_s, e_3, \lambda) \leq 0$

Figure A.2: Comparative Statics - Asymmetry

Although it looks complicated, the sign of $\frac{\partial}{\partial \nu} g_R^{1+2}(e_s, e_3, \nu)$ can be checked for any $(e_s, e_3, \nu) \in R_+^2 \times (0.5, 1.0)$ with the help of mathematica. Panel (A) in Figure A.2 plots the area such that $g_R^{1+2}(e_s, e_3, \nu) \leq 0$ for $e = (e_s, e_3) \in [0.1, 10]^2$. Panel (B) in Figure A.2 shows that there is no $e \in [0.1^6, 10^9]^2$ such that $g_R^{1+2}(e_s, e_3, \nu) \leq 0$ and $\frac{\partial}{\partial \nu} g_R^{1+2}(e_s, e_3, \nu) \leq 0$.

Derivation of Condition (2.21)

$$
\begin{aligned}
w_R^{1+2}(e) &= g_R^{1+2}(e) + \sum_{k=3}^{N} (\pi_k^{1+2} - \pi_k^*) - \int_{P_N^*}^{P_N^{1+2}} (1-P)dP \\
&= \frac{(e_1+e_2)(1+2(e_1+e_2))}{2(1+e_1+e_2)^2(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^2} \\
&\quad -\frac{1}{2(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^2}[\frac{e_1(1+2e_1)}{(1+e_1)^2}+\frac{e_2(1+2e_2)}{(1+e_2)^2}] \\
&\quad +\sum_{k=3}^{N}\frac{e_k(1+2e_k)}{2(1+e_k)^2}[\frac{1}{(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^2}-\frac{1}{(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^2}] \\
&\quad -[\frac{1}{1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k}}-\frac{1}{2}(\frac{1}{1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k}})^2 \\
&\qquad -\frac{1}{1+\sum_{k=1}^{N}\frac{e_k}{1+e_k}}+\frac{1}{2}(\frac{1}{1+\sum_{k=1}^{N}\frac{e_k}{1+e_k}})^2]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^2}[\frac{(e_1+e_2)(1+2(e_1+e_2))}{(1+e_1+e_2)^2}+\sum_{k=3}^{N}\frac{e_k(1+2e_k)}{(1+e_k)^2} \\
&\qquad -2(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})+1] \\
&\quad -\frac{1}{2(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^2}[\frac{e_1(1+2e_1)}{(1+e_1)^2}+\frac{e_2(1+2e_2)}{(1+e_2)^2}+\sum_{k=3}^{N}\frac{e_k(1+2e_k)}{(1+e_k)^2} \\
&\qquad -2(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})+1] \\
&= \frac{1}{2}[\frac{1}{(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^2}(\frac{(e_1+e_2)(1+2(e_1+e_2))}{(1+e_1+e_2)^2}-\frac{2(e_1+e_2)}{1+e_1+e_2} \\
&\qquad +\sum_{k=3}^{N}\left(\frac{e_k(1+2e_k)}{(1+e_k)^2}-\frac{2e_k}{1+e_k}\right)-1) \\
&\quad -\frac{1}{(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^2}(\sum_{k=1}^{N}\left(\frac{e_k(1+2e_k)}{(1+e_k)^2}-\frac{2e_k}{1+e_k}\right)-1)] \\
&= \frac{1}{2}[\frac{1}{(1+\sum_{k=1}^{N}\frac{e_k}{1+e_k})^2}\left(1+\sum_{k=1}^{N}\frac{e_k}{(1+e_k)^2}\right) \\
&\quad -\frac{1}{(1+\frac{e_1+e_2}{1+e_1+e_2}+\sum_{k=3}^{N}\frac{e_k}{1+e_k})^2}\left(1+\frac{e_1+e_2}{(1+e_1+e_2)^2}+\sum_{k=3}^{N}\frac{e_k}{(1+e_k)^2}\right)]
\end{aligned}
$$

Using equation (2.13), (2.14) and cost function, the above expression becomes

$$
\begin{aligned}
w_R^{1+2}(e) &= \frac{1}{2}[\{(P_N^*)^2+\sum_{k=1}^{N}q_k^* MC^k(q_k^*)\} - \\
&\qquad \{(P_N^{1+2})^2+q_M^{1+2}MC^{1+2}(q_M^{1+2})+\sum_{k=3}^{N}q_k^{1+2}MC^k(q_k^{1+2})\}]
\end{aligned}
$$

Since a merger is welfare-increasing if and only if $w_R^{1+2}(e)>0$, we can get condition (2.21). Condition (2.27) can be similarly derived.

Proof of Claim 5

By the construction of $g_S^{1+2}(e, e_M)$, it is enough to show that $e_M < e'_M$ implies $\pi_M^{1+2}(e_M, e_3, \cdots, e_N) < \pi_M^{1+2}(e'_M, e_3, \cdots, e_N)$.

First, note that outsider's post-merger output is smaller with higher synergies.

$$q_o^{1+2}(e'_M) = \frac{\frac{e_o}{1+e_o}}{1 + \frac{e'_M}{1+e'_M} + \sum_{k=3}^N \frac{e_k}{1+e_k}} < \frac{\frac{e_o}{1+e_o}}{1 + \frac{e_M}{1+e_M} + \sum_{k=3}^N \frac{e_k}{1+e_k}} = q_o^{1+2}(e_M)$$

The inequality comes from $\frac{e'_M}{1+e'_M} > \frac{e_M}{1+e_M}$. Let the merged firm's output be $q_M^{1+2}(e_M)$ given that e_M is synergies of the merger. Then,

$$\begin{aligned}
\pi_M^{1+2}(e_M, e_3, \cdots, e_N) &= (1 - q_M^{1+2}(e_M) - \sum_{k=3}^N q_k^{1+2}(e_M)) * q_M^{1+2}(e_M) - \frac{\left(q_M^{1+2}(e_M)\right)^2}{2e_M} \\
&< (1 - q_M^{1+2}(e_M) - \sum_{k=3}^N q_k^{1+2}(e'_M)) * q_M^{1+2}(e_M) - \frac{\left(q_M^{1+2}(e_M)\right)^2}{2e'_M} \\
&\leq (1 - q_M^{1+2}(e'_M) - \sum_{k=3}^N q_k^{1+2}(e'_M)) * q_M^{1+2}(e'_M) - \frac{\left(q_M^{1+2}(e'_M)\right)^2}{2e'_M} \\
&= \pi_M^{1+2}(e'_M, e_3, \cdots, e_N)
\end{aligned}$$

The first inequality comes from $\sum_{k=3}^N q_k^{1+2}(e_M) > \sum_{k=3}^N q_k^{1+2}(e'_M)$ and $e_M < e'_M$, and the second inequality comes from the revealed preference argument.

Proof of Claim 6

If I take a partial derivative on equation (2.26) with respect to e_M, I have

$$\frac{\partial}{\partial e_M} w_S^{1+2}(e, e_M) = \frac{1 + 4e_m + (e_m - 1)\sum_{k=3}^N \frac{e_k}{1+e_k} + 2(1+e_m)\sum_{k=3}^N \frac{e_k}{(1+e_k)^2}}{(1 + 2e_m + \sum_{k=3}^N \frac{e_k}{1+e_k} + e_m \sum_{k=3}^N \frac{e_k}{1+e_k})^3}$$

Then, $\frac{\partial}{\partial e_M} w_S^{1+2}(e, e_M) > 0$ is equivalent that numerator of the above expression is strictly positive, which yields condition (2.28).

Proof of Claim 7

CS-increasing merger decreases equilibrium price after merger. From equation (2.13) and (2.25), I can obtain

$$P_N^* > P_N^{1+2}(e_M) \Leftrightarrow \frac{1}{P_N^*} < \frac{1}{P_N^{1+2}(e_M)} \Leftrightarrow \lambda_1 + \lambda_2 < \lambda_M$$

Proof of Claim 8

I want to show that condition (2.29) implies $g_S^{1+2}(e, e_M) > 0$. Define $\delta = \frac{e_M}{1+e_M} - \frac{e_1}{1+e_1} - \frac{e_2}{1+e_2}$, then $\delta > 0$ holds for a CS-increasing merger. Now, $g_S^{1+2}(e, e_M)$ can be rewritten by

$$g_S^{1+2}(e,\delta) = \frac{(\delta + \frac{e_1}{1+e_1} + \frac{e_2}{1+e_2})(1+\delta+\frac{e_1}{1+e_1} + \frac{e_2}{1+e_2})}{2(1+\delta+\sum_{k=1}^N \frac{e_k}{1+e_k})^2} - \frac{\frac{e_1}{1+e_1}(1+\frac{e_1}{1+e_1}) + \frac{e_2}{1+e_2}(1+\frac{e_2}{1+e_2})}{2(1+\sum_{k=1}^N \frac{e_k}{1+e_k})^2}$$

Evaluate $g_S^{1+2}(e,\delta)$ at $\delta = 0$, then I get

$$g_S^{1+2}(e,0) = \frac{\frac{e_1 e_2}{(1+e_1)(1+e_2)}}{(1+\sum_{k=1}^N \frac{e_k}{1+e_k})^2} > 0$$

Moreover, the partial derivative of $g_S^{1+2}(e,\delta)$ with respect to δ yields

$$\frac{\partial}{\partial \delta} g_S^{1+2}(e,\delta) = \frac{1+\delta+\sum_{k=1}^N \frac{e_k}{1+e_k} + \frac{2e_M}{1+e_M}\sum_{k=3}^N \frac{e_k}{1+e_k}}{2(1+\delta+\sum_{k=1}^N \frac{e_k}{1+e_k})^3}$$

Hence, $\frac{\partial}{\partial \delta} g_S^{1+2}(e,\delta) > 0$ for all $\delta > 0$, which completes the proof.

Proof of Claim 8 under General Setting Revealed preference argument implies

$$P(q_M^{1+2} + \sum_{k=3}^N q_k^{1+2})\ q_M^{1+2} - C^{1+2}(q_M^{1+2}) \geq P(q + \sum_{k=3}^N q_k^{1+2})\ q - C^{1+2}(q) \tag{A.1}$$

for all $q > 0$. Suppose $\pi_1^* + \pi_2^* \geq \pi_M^{1+2}$ to the contrary, then I have

$$P(q_1^* + q_2^* + \sum_{k=3}^N q_k^*)\ (q_1^* + q_2^*) - C^1(q_1^*) - C^2(q_2^*) \geq P(q_M^{1+2} + \textstyle\sum_{k=3}^N q_k^{1+2})\ q_M^{1+2} - C^{1+2}(q_M^{1+2})$$

Since outsider firm j's $\lambda_j = -\frac{dq_j}{dQ} = -\frac{P' + q_i P''}{\frac{d^2}{dq_i^2}C^i - P'} > 0$ holds for all j ≥ 3 from condition (3) and (4) in Farrell-Shapiro, $\sum_{k=3}^N q_k^* > \sum_{k=3}^N q_k^{1+2}$ is satisfied in CS-increasing merger. Moreover, $C^{1+2}(q_1^* + q_2^*)$

$< C^1(q_1^*) + C^2(q_2^*)$ holds from rationalization and synergy effect. Then, I can get

$$
\begin{aligned}
P(q_1^* + q_2^* + \sum_{k=3}^{N} q_k^{1+2})\ (q_1^* + q_2^*) - C^{1+2}(q_1^* + q_2^*) \\
> P(q_1^* + q_2^* + \textstyle\sum_{k=3}^{N} q_k^*)\ (q_1^* + q_2^*) - C^1(q_1^*) - C^2(q_2^*) \\
\geq P(q_M^{1+2} + \textstyle\sum_{k=3}^{N} q_k^{1+2})\ q_M^{1+2} - C^{1+2}(q_M^{1+2}),
\end{aligned}
$$

which contradicts equation (A.1).

Using similar method, I can show that CS-increasing merger decreases outsider's profit and CS-neutral merger is welfare increasing under condition (3) and (4) in Farrell-Shapiro.

CS-increasing Merger's Effect on Outsider's Profit Given condition (3) and (4) in Farrell-Shapiro, outsider firm o's $\lambda_o = -\frac{dq_o}{dQ} > 0$ holds. Since $Q_N^{1+2} > Q_N^*$ holds in CS-increasing merger, $q_o^{1+2} < q_o^*$ and $q_{-o}^{1+2} > q_{-o}^*$ is satisfied. Revealed preference argument implies

$$
\begin{aligned}
P(q_o^* + q_{-o}^*)\ q_o^* - C^o(q_o^*) &\geq P(q + q_{-o}^*)q - C^o(q) \qquad \text{(A.2)} \\
P(q_o^{1+2} + q_{-o}^{1+2})\ q_o^{1+2} - C^o(q_o^{1+2}) &\geq P(q + q_{-o}^{1+2})q - C^o(q)
\end{aligned}
$$

for all $q > 0$. Suppose $\pi_o^{1+2} \geq \pi_o^*$ to the contrary, then I have

$$
P(q_o^{1+2} + q_{-o}^{1+2})\ q_o^{1+2} - C^o(q_o^{1+2}) \geq P(q_o^* + q_{-o}^*)q_o^* - C^o(q_o^*)
$$

Since $P(q_o^{1+2} + q_{-o}^{1+2}) < P(q_o^{1+2} + q_{-o}^*)$ from $q_{-o}^{1+2} > q_{-o}^*$, I obtain

$$
\begin{aligned}
P(q_o^{1+2} + q_{-o}^*)\ q_o^{1+2} - C^o(q_o^{1+2}) &> P(q_o^{1+2} + q_{-o}^{1+2})q_o^{1+2} - C^o(q_o^{1+2}) \\
&\geq P(q_o^* + q_{-o}^*)\ q_o^* - C^o(q_o^*),
\end{aligned}
$$

which contradicts equation (A.2).

Welfare Effect of CS-neutral Merger Contraposition of Farrell-Shapiro's Proposition 2 is that a merger generates synergies if it doesn't cause price to rise. So CS-neutral merger creates synergy effect. Since the welfare effect of CS-neutral merger is given by $w^{1+2} = \pi_M^{1+2} - \pi_1 - \pi_2$, I have

$$
\begin{aligned}
w^{1+2} &= P_N^{1+2} q_M^{1+2} - C^{1+2}(q_M^{1+2}) - (P_N^* q_1^* - C^1(q_1^*)) - (P_N^* q_2^* - C^2(q_2^*)) \\
&= C^1(q_1^*) + C^2(q_2^*) - C^{1+2}(q_M^{1+2}) > 0,
\end{aligned}
$$

where the second equality comes from $P_N^{1+2} = P_N^*$ and $q_M^{1+2} = q_1^* + q_2^*$, and the final inequality reflects that CS-neutral merger generates synergies and $q_M^{1+2} = q_1^* + q_2^*$.

Proof of Claim 9

$g^{1+2}(e_1, e_2, e_M, y) = 0$ is given. Claim 8 implies that $\lambda_1 + \lambda_2 > \lambda_M$ holds if $g^{1+2}(e_1, e_2, e_M, y) = 0$. Hence $\frac{\partial}{\partial y} g^{1+2}(e_1, e_2, e_M, y) < 0$ holds, so $g^{1+2}(e_1, e_2, e_M, y') > 0$.

A.3 Free Entry-Exit Model

Proof of Claim 11

We know that $N^\dagger \geq N^*$ holds in this case. If $N^\dagger = N^*$, $P_{N^\dagger}^{i+j} > P_{N^*+1}^*$ is satisfied because $P_{N^\dagger}^{i+j} > P_{N^*}^*$ from Claim 7 and $P_{N^*}^* > P_{N^*+1}^*$ holds. If $N^\dagger = N^* + 1$, $P_{N^\dagger}^{i+j} > P_{N^*+1}^*$ is also satisfied from Claim 7.

Now let $N^\dagger \geq N^* + 2$ and suppose $P_{N^\dagger}^{i+j} \leq P_{N^*+1}^*$ for contradiction. Since $N^\dagger$ is incumbent post-merger, $\pi_{N^\dagger}^{i+j} > F$ should be satisfied.

$$
\pi_{N^\dagger}^{i+j} = \frac{e_{N^\dagger}(1 + 2e_{N^\dagger})}{2(1 + e_{N^\dagger})^2 (1 + \frac{e_M}{1+e_M} + \sum_{k \neq i,j}^{N^\dagger} \frac{e_k}{1+e_k})^2} > F
$$

From $P_{N^\dagger}^{i+j} \leq P_{N^*+1}^*$, I have

$$
\frac{1}{P_{N^\dagger}^{i+j}} \geq \frac{1}{P_{N^*+1}^*} \Leftrightarrow 1 + \frac{e_M}{1 + e_M} + \sum_{k \neq i,j}^{N^\dagger} \frac{e_k}{1 + e_k} \geq 1 + \sum_{k=1}^{N^*+1} \frac{e_k}{1 + e_k}
$$

So I get

$$\begin{aligned} F &< \pi^{i+j}_{N^\dagger} \leq \frac{e_{N^\dagger}(1+2e_{N^\dagger})}{2(1+e_{N^\dagger})^2(1+\sum_{k=1}^{N^*+1}\frac{e_k}{1+e_k})^2} \\ &< \frac{e_{N^*+1}(1+2e_{N^*+1})}{2(1+e_{N^*+1})^2(1+\sum_{k=1}^{N^*+1}\frac{e_k}{1+e_k})^2} = \pi_{N^*+1} \end{aligned}$$

The last strict inequality comes from $e_{N^\dagger} < e_{N^*+1}$, which holds because $N^\dagger \geq N^*+2$ and $\langle e_n \rangle_{n=1}^{\infty}$ is a decreasing sequence. This result contradicts that N^* is the equilibrium number of firms before merger.

Proof of Claim 12

We know that $N^\dagger \leq L$ holds in this case. If $N^\dagger = L$, (a) $P^{i+j}_{N^\dagger} < P^*_{N^*}$ holds from Claim 7. Now suppose that $N^\dagger < L$ and (a) $P^{i+j}_{N^\dagger} < P^*_{N^*}$ does not hold. Then, I need to show that (b) $P^{i+j}_{N^\dagger+1} < P^*_{N^*}$ holds in order to prove the result. If $(N^\dagger+1) = L$, then $P^{i+j}_{N^\dagger+1} < P^*_{N^*}$ trivially holds from Claim 7.

Now suppose $(N^\dagger+1) < L$ and $P^{i+j}_{N^\dagger+1} \geq P^*_{N^*}$ for contradiction. Then we know that $\pi^*_{N^\dagger+1}(N^*) > F$ because it is an incumbent pre-merger from $(N^\dagger+1) < N^*$, which is equivalent to

$$\pi^*_{N^\dagger+1}(N^*) = \frac{e_{N^\dagger+1}(1+2e_{N^\dagger+1})}{2(1+e_{N^\dagger+1})^2(1+\sum_{k=1}^{N^*}\frac{e_k}{1+e_k})^2} > F$$

In contrast, $P^{i+j}_{N^\dagger+1} \geq P^*_{N^*}$ implies

$$\frac{1}{P^{i+j}_{N^\dagger+1}} \leq \frac{1}{P^*_{N^*}} \Leftrightarrow 1 + \frac{e_M}{1+e_M} + \sum_{k\neq i,j}^{N^\dagger+1} \frac{e_k}{1+e_k} \leq 1 + \sum_{k=1}^{N^*}\frac{e_k}{1+e_k}$$

Hence, we obtain

$$\pi^{i+j}_{N^\dagger+1} = \frac{e_{N^\dagger+1}(1+2e_{N^\dagger+1})}{2(1+e_{N^\dagger+1})^2(1+\frac{e_M}{1+e_M}+\sum_{k\neq i,j}^{N^\dagger+1}\frac{e_k}{1+e_k})^2} \geq \pi^*_{N^\dagger+1}(N^*) > F$$

Then, firm $(N^\dagger+1)$ does not exit, which is contradiction because firm $N^\dagger$ is the least efficient incumbent at post-merger equilibrium.

Appendix B

Appendix of Chapter 3

Proof of Lemma 1

The first order condition implies that best response function $q_i^* = q^d(q_{-i}, \gamma)$ satisfies

$$\gamma \frac{\partial}{\partial q_i} \pi_i(q_i^*, q^c) + (1-\gamma) \frac{\partial}{\partial q_i} \pi_i(q_i^*, q_{-i}) = 0$$

Denote $\pi_{i1}(q_i^*, q^c) \equiv \frac{\partial}{\partial q_i} \pi_i(q_i^*, q^c)$ and $\pi_{i1}(q_i^*, q_{-i}) \equiv \frac{\partial}{\partial q_i} \pi_i(q_i^*, q_{-i})$. Using implicit function theorem, I can obtain

$$\frac{\partial}{\partial \gamma} q^d(q_{-i}, \gamma) = -\frac{\pi_{i1}(q_i^*, q^c) - \pi_{i1}(q_i^*, q_{-i})}{\gamma \pi_{i11}(q_i^*, q^c) + (1-\gamma)\pi_{i11}(q_i^*, q_{-i})}$$
$$, where\ \pi_{i11}(q_i^*, q^c) \equiv \frac{\partial^2}{\partial q_i^2} \pi_i(q_i^*, q^c) \text{ and } \pi_{i11}(q_i^*, q_{-i}) \equiv \frac{\partial^2}{\partial q_i^2} \pi_i(q_i^*, q_{-i})$$

$\pi_{i11} < 0$ because π_i is concave and $\pi_{i1}(q_i^*, q^c) - \pi_{i1}(q_i^*, q_{-i}) > 0$ for all $q_{-i} > q^c$. Hence, $\frac{\partial}{\partial \gamma} q^d(q_{-i}, \gamma) > 0$ holds for all $q_{-i} > q^c$. Let the fixed output be $q^d = q^d(q^d(\gamma), \gamma) \equiv q^d(\gamma)$. Then

$$\frac{dq^d}{d\gamma} = \frac{\partial q^d}{\partial q_{-i}} \frac{dq^d}{d\gamma} + \frac{\partial q^d}{\partial \gamma} \Leftrightarrow \frac{dq^d}{d\gamma} = \frac{\frac{\partial q^d}{\partial \gamma}}{1 - \frac{\partial q^d}{\partial q_{-i}}}$$

Since I know $\frac{\partial}{\partial \gamma} q^d(q_{-i}, \gamma) > 0$ holds for $q_{-i} > q^c$, it is sufficient to show $\frac{\partial q^d}{\partial q_{-i}} < 0$ in order to prove the lemma. From implicit function theorem, I have

$$\frac{\partial}{\partial q_{-i}} q^d(q_{-i}, \gamma) = -\frac{(1-\gamma)\pi_{i12}(q_i^*, q_{-i})}{\gamma \pi_{i11}(q_i^*, q^c) + (1-\gamma)\pi_{i11}(q_i^*, q_{-i})}$$

So $\frac{\partial}{\partial q_{-i}} q^d(q_{-i}, \gamma) < 0$ at $q_{-i} = q^d$ is equivalent to $\pi_{i12}(q^d, q^d) < 0$. From the assumption, I can get

$$\pi_{i12}(q^d, q^d) = p'(q^d + q^d) + q^d p''(q^d + q^d) < 0,$$

which completes the proof.

Proof of Proposition 2

Given $s_{-i}(\delta_{-i}) = \bar{s}_{-i}(\delta_{-i})$, I can also derive the expected payoff of firm i when he chooses $s_i(\delta^H) \neq (Join, q^c)$ as follows;

$$\begin{aligned}
\Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) &= \pi^n \\
\Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) &= \gamma\left[(1-\delta^H)\pi_i(q_i, q^c) + \delta^H \pi^r\right] \\
&\quad +(1-\gamma)\left[(1-\delta^H)\pi_i(q_i, q^d(\gamma)) + \delta^H \pi^r\right], \\
where\ a_i &\in A_i \diagdown \{(Join, q^c), (Not\ Join)\}.
\end{aligned}$$

Similarly, the expected payoff of firm i when he chooses $s_i(\delta^L) \neq (Join, q^d(\gamma))$ is given by the following equations;

$$\begin{aligned}
\Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) &= \pi^n \\
\Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^L) &= \gamma\left\{1_{q_i \neq q_i^c}\left[(1-\delta^L)\pi_i(q_i, q^c) + \delta^L \pi^r\right] + 1_{q_i = q_i^c}\pi^c\right\} \\
&\quad +(1-\gamma)\left[(1-\delta^L)\pi_i(q_i, q^d(\gamma)) + \delta^L \pi^r\right], \\
where\ a_i &\in A_i \diagdown \{(Join, q^d(\gamma)), (Not\ Join)\}.
\end{aligned}$$

Given these deviation payoffs, it is required to show condition (3.1) holds for strategy profile $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2))$. First, check the condition (3.1) for type δ^H. If $a_i = (Not\ Join)$, then

$$\begin{aligned}
& \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) \\
= \ & \gamma(\pi^c - \pi^n) + (1-\gamma)\left[(1-\delta^H)\pi_i(q^c, q^d(\gamma)) + \delta^H\pi^r - \pi^n\right] \\
\geq \ & \gamma(\pi^c - \pi^n) + (1-\gamma)\left[(1-\delta^H)\pi^{d-} + \delta^H\pi^r - \pi^n\right]
\end{aligned}$$

The inequality comes from $\pi_i(q^c, q^d(\gamma)) \geq \pi_i(q^c, q^d(1)) = \pi^{d-}$ from Lemma 1. Since $\pi^c > \pi^n$ for all $i \in N$, there exists $\gamma_1 \in (0,1)$ such that the right side of the inequality is non-negative for $\forall$ $\gamma \in [\gamma_1, 1)$. But then, there must exists $\gamma_1^* \in (0, \gamma_1]$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(Not$

$Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) \geq 0$ for $\forall\ \gamma \in [\gamma_1^*, 1)$ and $\forall$ firm i with $\delta_i = \delta^H$. Similarly, if $a_i = (Join, q_i)$ such that $q_i \neq q^c$, then I have

$$
\begin{aligned}
& \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) \\
= \ & \gamma \left[\pi^c - (1-\delta^H)\pi_i(q_i, q^c) - \delta^H \pi^r\right] + (1-\gamma)(1-\delta^H)\left[\pi_i(q^c, q^d(\gamma)) - \pi_i(q_i, q^d(\gamma))\right] \\
\geq \ & \gamma \left[\pi^c - (1-\delta^H)\pi_i(q_i, q^c) - \delta^H \pi^r\right] + (1-\gamma)(1-\delta^H)\left[\pi^{d-} - \pi^m\right]
\end{aligned}
$$

The inequality comes from $\pi_i(q^c, q^d(\gamma)) \geq \pi^{d-}$ and $\pi_i(q_i, q^d(\gamma)) \leq \pi^m$, where π^m is the profit that firm i would get if he were a monopolist. Because $\pi^c - (1-\delta^H)\pi_i(q_i, q^c) - \delta^H\pi^r \geq \pi^c - (1 - \delta^H)\pi^{d+} - \delta^H\pi^r > 0$ for any $q_i \in R_+$ and $i \in N$, there exists $\gamma_2 \in (0,1)$ such that the right side of the inequality is non-negative for $\forall\ \gamma \in [\gamma_2, 1)$, $i \in N$ and $\forall\ q_i \in R_+$. But then, there must exists $\gamma_2^* \in (0, \gamma_2]$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) \geq 0$ for $\forall\ \gamma \in [\gamma_2^*, 1)$, $\forall$ firm i and any $q_i \in R_+$.

Next examine the condition (3.1) for type δ^L. Note that $\pi^{d+} = \pi_i(q^d(1), q^c)$ and $\pi^{d+} \geq \pi_i(q^d(\gamma), q^c)$ for all $\gamma \in (0,1)$ because of Lemma 1 and concavity of profit function. Since $(1 - \delta^L)\pi^{d+} + \delta^L\pi^r > \pi^c$, either there exists $\gamma^* \in (0,1)$ such that $(1-\delta^L)\pi_i(q^d(\gamma^*), q^c) + \delta^L\pi^r = \pi^c$ or $(1-\delta^L)\pi_i(q^d(0), q^c) + \delta^L\pi^r \geq \pi^c$ holds. If $(1-\delta^L)\pi_i(q^d(0), q^c) + \delta^L\pi^r \geq \pi^c$ holds, let $\gamma^* = 0$.

If $a_i = (Not\ Join)$, then

$$
\begin{aligned}
& \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) \\
= \ & \gamma[(1-\delta^L)\pi_i(q^d(\gamma), q^c) + \delta^L\pi^r - \pi^n] + (1-\gamma)\left[(1-\delta^L)\pi_i(q^d(\gamma), q^d(\gamma)) + \delta^L\pi^r - \pi^n\right] \\
\geq \ & \gamma[(1-\delta^L)\pi_i(q^d(\gamma^*), q^c) + \delta^L\pi^r - \pi^n] + (1-\gamma)\left[(1-\delta^L)\hat{\pi} + \delta^L\pi^r - \pi^n\right] \\
& \text{for all } \gamma \in [\gamma^*, 1)
\end{aligned}
$$

The inequality holds from $\pi_i(q^d(\gamma), q^c) \geq \pi_i(q^d(\gamma^*), q^c)$ for all $\gamma \in [\gamma^*, 1)$ and $\pi_i(q^d(\gamma), q^d(\gamma)) \geq \hat{\pi} \equiv \min_{\gamma \in [0,1]} \pi_i(q^d(\gamma), q^d(\gamma))$. Note that $\pi^{d+} = \pi_i(q^d(1), q^c) \geq \pi_i(q^d(\gamma), q^c)$ for all $\gamma \in (0,1)$. So for all $\gamma \in [\gamma^*, 1)$, $\pi_i(q^d(\gamma), q^c) \geq \pi_i(q^d(\gamma^*), q^c)$ holds from Lemma 1 and concavity of profit function. Then $(1-\delta^L)\pi_i(q^d(\gamma), q^c) + \delta^L\pi^r \geq \pi^c > \pi^n$ holds for all $\gamma \in [\gamma^*, 1)$. Hence, there exists $\gamma_3 \in [\gamma^*, 1)$ such that the right side of the inequality is non-negative for $\forall\ \gamma \in [\gamma_3, 1)$, $i \in N$. But then, there must exists $\gamma_3^* \in (0, \gamma_3]$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) \geq 0$ for $\forall$ $\gamma \in [\gamma_3^*, 1)$ and $\forall$ firm i. If $a_i = (Join, q^c)$, then

$$\begin{aligned}
& \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i((Join, q^c), \bar{s}_{-i}(\delta_{-i}); \delta^L) \\
= \; & \gamma \left[(1-\delta^L)\pi_i(q^d(\gamma), q^c) + \delta^L \pi^r - \pi^c\right] + (1-\gamma)(1-\delta^L)\left[\pi_i(q^d(\gamma), q^d(\gamma)) - \pi_i(q^c, q^d(\gamma))\right] \\
\geq \; & \gamma[(1-\delta^L)\pi_i(q^d(\gamma^* + \varepsilon), q^c) + \delta^L \pi^r - \pi^c] + (1-\gamma)(1-\delta^L)\,[\hat{\pi} - \pi^m] \\
& \quad \text{for all } \gamma \in (\gamma^* + \varepsilon, 1) \text{ given } \varepsilon > 0 \text{ and } \gamma^* + \varepsilon < 1
\end{aligned}$$

Since $(1-\delta^L)\pi_i(q^d(\gamma^*), q^c) + \delta^L \pi^r \geq \pi^c$ by definition, $(1-\delta^L)\pi_i(q^d(\gamma^* + \varepsilon), q^c) + \delta^L \pi^r > \pi^c$ holds. So by the same argument, there exists $\gamma_4^* \in (0, 1)$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i((Join, q^c), \bar{s}_{-i}(\delta_{-i}); \delta^L) \geq 0$ for $\forall$ $\gamma \in [\gamma_4^*, 1)$ and $\forall$ firm i. Finally, consider $a_i = (Join, q_i)$ such that $q_i \in R_+ \setminus \{q^d(\gamma), q^c\}$, then

$$\begin{aligned}
& \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^L) \\
= \; & \gamma(1-\delta^L)\left[\pi_i(q^d(\gamma), q^c) - \pi_i(q_i, q^c)\right] + (1-\gamma)(1-\delta^L)\left[\pi_i(q^d(\gamma), q^d(\gamma)) - \pi_i(q_i, q^d(\gamma))\right] \\
= \; & (1-\delta^L)[\{\gamma\pi_i(q^d(\gamma), q^c) + (1-\gamma)\pi_i(q^d(\gamma), q^d(\gamma))\} - \{\gamma\pi_i(q_i, q^c) + (1-\gamma)\pi_i(q_i, q^d(\gamma))\}] \\
\geq \; & 0
\end{aligned}$$

Last inequality holds because $q^d(\gamma) \in \arg\max_{q_i} \gamma\pi_i(q_i, q^c) + (1-\gamma)\pi_i(q_i, q^d(\gamma))$. Hence, if I define $\gamma^* = \max\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*\}$, then the condition (3.1) holds for every $\gamma \in (\gamma^*, 1)$.

Proof of Proposition 3

When firm i with δ^H chooses $s_i(\delta_i) \neq (Join, q^c)$ given $\bar{s}_{-i}(\delta_{-i}) = (\bar{s}_1(\delta_1), \cdots, \bar{s}_{i-1}(\delta_{i-1}), \bar{s}_{i+1}(\delta_{i+1}),$ $\cdots, \bar{s}_n(\delta_n))$, the expected payoff of it yields

$$\begin{aligned}
\Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) &= \pi^n \\
\Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) &= \sum_{j=0}^{n-1} {}_{n-1}C_j \; \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^H)\pi_i^{j-i}(q_i) + \delta^H \pi^r], \\
where\ a_i &\in A_i \setminus \{(Join, q^c), (Not\ Join)\}.
\end{aligned}$$

Similarly, when firm i with type δ^L chooses $s_i(\delta_i) \neq (Join, q^d(\gamma))$, the expected payoff of firm i comes to

$$\begin{aligned}
\Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) &= \pi^n \\
\Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^L) &= \gamma^{n-1}[1_{q_i \neq q_i^c}\left[(1-\delta^L)\pi_i(q_i, q_{-i}^c) + \delta^L \pi^r\right] + 1_{q_i = q_i^c}\pi^c] \\
&\quad + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^L)\pi_i^{j-i}(q_i) + \delta^L\pi^r], \\
where\ a_i &\in A_i \diagdown \{(Join, q^d(\gamma)), (Not\ Join)\}
\end{aligned}$$

First, check the condition (3.5) for type δ^H. Then,

$$\begin{aligned}
&\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) \\
=\ & \gamma^{n-1}(\pi^c - \pi^n) + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^H)\pi_i^{j-i}(q^c) + \delta^H\pi^r - \pi^n] \\
\geq\ & \gamma^{n-1}(\pi^c - \pi^n) + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^H)\pi^{d-} + \delta^H\pi^r - \pi^n]
\end{aligned}$$

Let me denote $\pi^{d-} \equiv \pi_i(q^c, q_{-i}(q_{-i}^c))$, where $q_{-i}^c = (q_1^c, \cdots, q_{-i-1}^c, q_{-i+1}^c, \cdots, q_n^c)$. Then, the inequality comes from $\pi_i^{j-i}(q^c) \geq \pi^{d-}$. Since $\pi^c > \pi^n$, there exists $\gamma_1 \in (0,1)$ such that the right side of the inequality is non-negative for $\forall\ \gamma \in [\gamma_1, 1)$ and $i \in N$. But then, there must exist $\gamma_1^* \in (0, \gamma_1]$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^H) \geq 0$ for $\forall\ \gamma \in [\gamma_1^*, 1)$ and $\forall$ firm i with type δ^H. Similarly,

$$\begin{aligned}
&\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) \\
=\ & \gamma^{n-1}[\pi^c - (1-\delta^H)\pi_i(q_i, q_{-i}^c) - \delta^H\pi^r] \\
&\ + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j(1-\delta^H)[\pi_i^{j-i}(q^c) - \pi_i^{j-i}(q_i)] \\
\geq\ & \gamma^{n-1}[\pi^c - (1-\delta^H)\pi_i(q_i, q_{-i}^c) - \delta^H\pi^r] + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j(1-\delta^H)[\pi^{d-} - \pi^m]
\end{aligned}$$

The inequality comes from $\pi_i^{j-i}(q^c) \geq \pi^{d-}$ and $\pi_i^{j-i}(q_i) \leq \pi^m$, where π^m is the profit that firm i would get if he were a monopolist. Because $\pi^c - (1-\delta^H)\pi_i(q_i, q_{-i}^c) - \delta^H\pi^r \geq \pi^c - (1-\delta^H)\pi^{d+} - \delta^H\pi^r > 0$

for any $q_i \in R_+$ and $i \in N$ for type δ^H, there exists $\gamma_2 \in (0,1)$ such that the right side of the inequality is non-negative for $\forall\, \gamma \in [\gamma_2, 1)$, $i \in N$ and $\forall\, q_i \in R_+$. But then, there must exist $\gamma_2^* \in (0, \gamma_2]$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^H) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^H) \geq 0$ for $\forall\, \gamma \in [\gamma_2^*, 1)$, $\forall\, a_i \in A_i$ and $\forall$ firm i with type δ^H.

Next examine the condition (3.5) for type δ^L. If $a_i = (Not\ Join)$, then

$$\begin{aligned} & \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) \\ = \ & \gamma^{n-1}[(1-\delta^L)\pi_i(q^d(\gamma), q^c_{-i}) + \delta^L \pi^r - \pi^n] \\ & + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j[(1-\delta^L)\pi_i^{j-i}(q^d(\gamma)) + \delta^L\pi^r - \pi^n] \end{aligned}$$

Notice that $\pi_i(q^d(\gamma), q^c_{-i})$ converges to π^{d+} as γ approaches 1. Since $(1-\delta^L)\pi^{d+} + \delta^L\pi^r > \pi^c > \pi^n$ for all $i \in N$ with type δ^L, there exists $\gamma_3^* \in (0,1)$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(Not\ Join, \bar{s}_{-i}(\delta_{-i}); \delta^L) \geq 0$ for $\forall\, \gamma \in [\gamma_3^*, 1)$ and $\forall$ firm i with type δ^L. Similarly, if $a_i = (Join, q^c)$, then

$$\begin{aligned} & \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i((Join, q^c), \bar{s}_{-i}(\delta_{-i}); \delta^L) \\ = \ & \gamma^{n-1}[(1-\delta^L)\pi_i(q^d(\gamma), q^c_{-i}) + \delta^L \pi^r - \pi^c] \\ & + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j(1-\delta^L)[\pi_i^{j-i}(q^d(\gamma)) - \pi_i^{j-i}(q^c)] \end{aligned}$$

Again $\pi_i(q^d(\gamma), q^c_{-i})$ converges to π^{d+} as γ approaches 1. Because $\pi^{d+} > \pi_i(q_i, q^c_{-i})$ for any $q_i \in R_+ \diagdown \{q_i(q^c_{-i})\}$ and $(1-\delta^L)\pi^{d+} + \delta^L\pi^r > \pi^c$, there exists $\gamma_4^* \in (0,1)$ such that $\Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L)$ $-\Pi_i((Join, q^c), \bar{s}_{-i}(\delta_{-i}); \delta^L) \geq 0$ for $\forall\, \gamma \in [\gamma_4^*, 1)$ and $\forall$ firm i with type δ^L. Finally, consider $a_i = (Join, q_i)$ such that $q_i \notin \{q^d(\gamma), q^c\}$, then

$$\begin{aligned} & \Pi_i(\bar{s}_i(\delta_i), \bar{s}_{-i}(\delta_{-i}); \delta^L) - \Pi_i(a_i, \bar{s}_{-i}(\delta_{-i}); \delta^L) \\ = \ & \gamma^{n-1}(1-\delta^L)\left[\pi_i(q^d(\gamma), q^c_{-i}) - \pi_i(q_i, q^c_{-i})\right] \\ & + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j(1-\delta^L)\left[\pi_i^{j-i}(q^d(\gamma)) - \pi_i^{j-i}(q_i)\right] \end{aligned}$$

$$
\begin{aligned}
&= (1-\delta^L)[\{\gamma^{n-1}\pi_i(q^d(\gamma), q^c_{-i}) + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j \pi_i^{j-i}(q^d(\gamma))\} \\
&\qquad - \{\gamma^{n-1}\pi_i(q_i, q^c_{-i}) + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j \pi_i^{j-i}(q_i)\}] \\
&\geq 0
\end{aligned}
$$

Last inequality holds from $q^d(\gamma) \in \arg\max_{q_i} \gamma^{n-1}\pi_i(q_i, q^c_{-i}) + \sum_{j=1}^{n-1} {}_{n-1}C_j\ \gamma^{n-1-j}(1-\gamma)^j \pi_i^{j-i}(q_i)$. Hence, if I define $\gamma^* = \max\{\gamma_1^*, \gamma_2^*, \gamma_3^*, \gamma_4^*\}$, then the condition (3.5) holds for every $\gamma \in (\gamma^*, 1)$.

Proof of Proposition 4

It is required to show that two conditions in (3.9) holds for strategy profile $\bar{s} = (\bar{s}_1(\delta_1), \bar{s}_2(\delta_2))$and the system of belief $\gamma = (\gamma_{1t}, \gamma_{2t})_{t=0}^{\infty}$ constructed in subsection 3.4.2.

In order to show the first condition, I will use the principle of optimality. For that purpose, consider one shot deviation and its expected payoff or continuation payoff in each history. If a firm deviates not to join the collusion at the beginning of the game, then its payoff is simply given by $\Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}\ ; \delta_i, \gamma_0) = \pi^n$ for each type of firm. Here, $\bar{s}_i(Not\ Join)$ denotes the same strategy with $\bar{s}_i$ except that $\bar{s}_i(Not\ Join)$ chooses not to join the collusion at $t = 0$. If a firm with δ^H chooses $q_{i1} \neq \bar{q}^c$ at $h_1 = (Join, Join)$, then its expected continuation payoff comes to

$$
\begin{aligned}
\Pi_{i1}(\bar{s}_i(q_{i1})|h_1, \bar{s}_{-i}|h_1\ ; \delta^H, \gamma_0) &= \gamma_0[(1-\delta^H)\pi(q_{i1}, \bar{q}^c) + \delta^H \pi^r] \\
&\quad +(1-\gamma_0)[(1-\delta^H)\pi(q_{i1}, q^d(\gamma_0, \bar{q}^c)) + \delta^H \pi^r]
\end{aligned}
$$

$\bar{s}_i(q_{i1})|h_1$ denotes the same continuation strategy with $\bar{s}_i|h_1$ except that high type firm chooses $q_{i1} \neq \bar{q}^c$ at $h_1 = (Join, Join)$. If a firm with δ^L chooses $q_{i1} = \bar{q}^c$ at $h_1 = (Join, Join)$, then its expected continuation payoff comes to

$$
\begin{aligned}
\Pi_{i1}(\bar{s}_i(\bar{q}^c)|h_1, \bar{s}_{-i}|h_1\ ; \delta^L, \gamma_0) &= \gamma_0[(1-\delta^L)\bar{\pi}^c + (1-\delta^L)\delta^L \pi^{d+} + (\delta^L)^2 \pi^r] \\
&\quad +(1-\gamma_0)[(1-\delta^L)\pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c)) + \delta^L \pi^r]
\end{aligned}
$$

$\bar{s}_i(\bar{q}^c)|h_1$ denotes the same continuation strategy with $\bar{s}_i|h_1$ except that low type firm chooses $q_{i1} = \bar{q}^c \neq q^d(\gamma_0, \bar{q}^c)$ at $h_1 = (Join, Join)$. If a firm with δ^L chooses $q_{i1} \in A_{i1} \setminus \{q^d(\gamma_0, \bar{q}^c), \bar{q}^c\}$ at $h_1 = (Join, Join)$, then its expected continuation payoff comes to

$$\begin{aligned}\Pi_{i1}(\bar{s}_i(q_{i1})|h_1, \bar{s}_{-i}|h_1\ ; \delta^L, \gamma_0) &= \gamma_0[(1-\delta^L)\pi(q_{i1}, \bar{q}^c) + \delta^L \pi^r] \\ &\quad +(1-\gamma_0)[(1-\delta^L)\pi(q_{i1}, q^d(\gamma_0, \bar{q}^c)) + \delta^L \pi^r]\end{aligned}$$

Finally, if a firm with δ^H chooses $q_{it} \neq q^c$ or a firm with δ^L chooses $q_{it} \neq q^{d+}$ at h_t such that $h_1 = (Join, Join)$ and $q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, q^c, \cdots, q^c)$ for $t \geq 2$, its continuation payoff comes to

$$\Pi_{it}(\bar{s}_i(q_{it})|h_t, \bar{s}_{-i}|h_t\ ; \delta_i, \gamma_{it}{=}\, 1) = (1-\delta_i)\pi(q_{it}, q^c) + \delta_i \pi^r, \text{ where } \delta_i \in \Delta$$

Now, check the sequential rationality of $\bar{s}$ for type δ^H. If $t = 0$ and deviation action $a_{i0} = (Not\ Join)$, then I get

$$\begin{aligned}&\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^H, \gamma_0) - \Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}; \delta^H, \gamma_0) \\ = \ &\gamma_0[(1-\delta^H)\bar{\pi}^c + \delta^H \pi^c - \pi^n] + (1-\gamma_0)[(1-\delta^H)\pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c)) + \delta^H \pi^r - \pi^n]\end{aligned}$$

Define $f_1 : X \longmapsto R$ such that $X = [0,1] \times [q^c, q^n]$ and

$$f_1(\gamma, q) = \gamma[(1-\delta^H)\pi(q,q) + \delta^H \pi^c - \pi^n] + (1-\gamma)[(1-\delta^H)\pi(q, q^d(\gamma, q)) + \delta^H \pi^r - \pi^n]$$

Clearly, $f_1(\gamma, q)$ is continuous and $f_1(1, q^c) = \pi^c - \pi^n > 0$. So $U_1(0) = \{(\gamma, q) \in X \mid f_1(\gamma, q) > 0\}$ is non-empty and open. Since $(1, q^c) \in U_1(0)$, there exists $\epsilon_1^* > 0$ such that $f_1(\gamma, q) > 0$ for all $(\gamma, q) \in B_{\epsilon_1^*}(1, q^c)$. From the construction of $f_1(\gamma, q)$, $\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^H, \gamma_0) - \Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}; \delta^H, \gamma_0) > 0$ holds for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_1^*}(1, q^c)$.

If $t = 1$ and deviation action of δ^H type firm is $a_{i1} = q_{i1}$, then I get

$$\begin{aligned}
& \Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1; \delta^H, \gamma_{i1}) - \Pi_{i1}(\bar{s}_i(q_{i1})|h_1, \bar{s}_{-i}|h_1 \ ; \delta^H, \gamma_0) \\
= \ & \gamma_0[(1-\delta^H)\bar{\pi}^c + \delta^H \pi^c - (1-\delta^H)\pi(q_{i1}, \bar{q}^c) - \delta^H \pi^r] \\
& +(1-\gamma_0)(1-\delta^H)[\pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c)) - \pi(q_{i1}, q^d(\gamma_0, \bar{q}^c))] \\
\geq \ & \gamma_0[(1-\delta^H)\bar{\pi}^c + \delta^H \pi^c - (1-\delta^H)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) - \delta^H \pi^r] + (1-\gamma_0)(1-\delta^H)\left[\pi^{d-} - \pi^m\right]
\end{aligned}$$

Similarly, define $f_2 : X \longmapsto R$ such that

$$f_2(\gamma, q) = \gamma[(1-\delta^H)\pi(q,q) + \delta^H \pi^c - (1-\delta^H)\pi(q^d(\gamma, q), q) - \delta^H \pi^r] + (1-\gamma)(1-\delta^H)[\pi^{d-} - \pi^m]$$

Clearly, $f_2(\gamma, q)$ is continuous and $f_2(1, q^c) = \pi^c - (1-\delta^H)\pi^{d+} - \delta^H \pi^r > 0$. So $U_2(0) = \{(\gamma, q) \in X \mid f_2(\gamma, q) > 0\}$ is non-empty and open. Since $(1, q^c) \in U_2(0)$, there exists $\epsilon_2 > 0$ such that $f_2(\gamma, q) > 0$ for all $(\gamma, q) \in B_{\epsilon_2}(1, q^c)$. From the construction of $f_2(\gamma, q)$, the right side of the inequality is positive for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_2}(1, q^c)$. But then, there must exists $\epsilon_2^* \geq \epsilon_2 > 0$ such that $\Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1; \delta^H, \gamma_{i1}) - \Pi_{i1}(\bar{s}_i(q_{i1})|h_1, \bar{s}_{-i}|h_1 \ ; \delta^H, \gamma_0) > 0$ holds for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_2^*}(1, q^c)$.

Next, check the sequential rationality of $\bar{s}$ for type δ^L. If $t = 0$ and deviation action $a_{i0} = (Not\ Join)$, then I have

$$\begin{aligned}
& \Pi_i(\bar{s}_i, \bar{s}_{-i} \ ; \delta^L, \gamma_0) - \Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}; \delta^L, \gamma_0) \\
= \ & \gamma_0[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) + \delta^L \pi^r - \pi^n] \\
& +(1-\gamma_0)[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) + \delta^L \pi^r - \pi^n]
\end{aligned}$$

Define $f_3 : X \longmapsto R$ such that

$$\begin{aligned}
f_3(\gamma, q) \ = \ & \gamma[(1-\delta^L)\pi(q^d(\gamma, q), q) + \delta^L \pi^r - \pi^n] \\
& +(1-\gamma)[(1-\delta^L)\pi(q^d(\gamma, q), q^d(\gamma, q)) + \delta^L \pi^r - \pi^n]
\end{aligned}$$

Clearly, $f_3(\gamma, q)$ is continuous and $f_3(1, q^c) = (1-\delta^L)\pi^{d+} + \delta^L \pi^r - \pi^n > 0$. So $U_3(0) = \{(\gamma, q) \in X \mid f_3(\gamma, q) > 0\}$ is non-empty and open. Since $(1, q^c) \in U_3(0)$, there exists $\epsilon_3^* > 0$ such that $f_3(\gamma, q) >$

0 for all $(\gamma, q) \in B_{\epsilon_3^*}(1, q^c)$. From the construction of $f_3(\gamma, q)$, $\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta^L, \gamma_0) - \Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}; \delta^L, \gamma_0) > 0$ holds for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_3^*}(1, q^c)$.

If $t = 1$ and $a_{i1} = \bar{q}^c$ is a deviation action of δ^L type firm, I have

$$\begin{aligned}
& \Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1; \delta^L, \gamma_{i1}) - \Pi_{i1}(\bar{s}_i(\bar{q}^c)|h_1, \bar{s}_{-i}|h_1\ ; \delta^L, \gamma_0) \\
= \ & \gamma_0[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) + \delta^L\pi^r - (1-\delta^L)\bar{\pi}^c - (1-\delta^L)\delta^L\pi^{d+} - (\delta^L)^2\pi^r] \\
& +(1-\gamma_0)(1-\delta^L)[\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) - \pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c))]
\end{aligned}$$

Note that $\Pi_i(t^* = 1; \delta^L, \gamma_0) - \Pi_i(t^* = 2; \delta^L, \gamma_0) > 0$ holds from $f(\gamma_0, \bar{q}^c) > 0$, which is equivalent with the following;

$$\begin{aligned}
& \Pi_i(t^* = 1; \delta^L, \gamma_0) - \Pi_i(t^* = 2; \delta^L, \gamma_0) \\
= \ & \gamma_0[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) + \delta^L\pi^r - (1-\delta^L)\bar{\pi}^c - (1-\delta^L)\delta^L\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) - (\delta^L)^2\pi^r] \\
& +(1-\gamma_0)[(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) + \delta^L\pi^r \\
& \qquad - (1-\delta^L)\bar{\pi}^c - (1-\delta^L)\delta^L\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) - (\delta^L)^2\pi^r] \\
> \ & 0
\end{aligned}$$

Let $\alpha(\gamma_0, \bar{q}^c)$ be defined as

$$\begin{aligned}
\alpha(\gamma_0, \bar{q}^c) = \ & (1-\gamma_0)\{(1-\delta^L)\bar{\pi}^c + (1-\delta^L)\delta^L\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) + (\delta^L)^2\pi^r \\
& - (1-\delta^L)\pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c)) - \delta^L\pi^r\}
\end{aligned}$$

If $\alpha(\gamma_0, \bar{q}^c) \geq 0$, then $\Pi_i(t^* = 1; \delta^L, \gamma_0) - \Pi_i(t^* = 2; \delta^L, \gamma_0) + \alpha(\gamma_0, \bar{q}^c) > 0$. If $\alpha(\gamma_0, \bar{q}^c) < 0$ instead, define $f_4 : X \longmapsto R$ such that

$$\begin{aligned}
f_4(\gamma, q) = \ & (1-\gamma)\{(1-\delta^L)\pi(q, q) + (1-\delta^L)\delta^L\pi(q^d(\gamma, q), q^d(\gamma, q)) + (\delta^L)^2\pi^r \\
& - (1-\delta^L)\pi(q, q^d(\gamma, q)) - \delta^L\pi^r\}
\end{aligned}$$

Clearly, $f_4(\gamma, q)$ is continuous and $f_4(1, q^c) = 0$. If I let $\beta(\gamma_0, \bar{q}^c) \equiv \Pi_i(t^* = 1; \delta^L, \gamma_0) - \Pi_i(t^* = 2; \delta^L, \gamma_0)$ and pick $\kappa \in (0, \beta(\gamma_0, \bar{q}^c))$, then $U_4(-\kappa) = \{(\gamma, q) \in X \mid f_4(\gamma, q) > -\kappa\}$ is non-empty and open for all $\kappa \in (0, \beta(\gamma_0, \bar{q}^c))$. So, there exists $\epsilon_4^*(\kappa) > 0$ such that $f_4(\gamma, q) > -\kappa$ for all $(\gamma, q) \in B_{\epsilon_4^*(\kappa)}(1, q^c)$. Define $\zeta(\gamma_0, \bar{q}^c) \equiv \Pi_i(t^* = 1; \delta^L, \gamma_0) - \Pi_i(t^* = 2; \delta^L, \gamma_0) + \alpha(\gamma_0, \bar{q}^c)$, then $\zeta(\gamma_0, \bar{q}^c) > 0$ holds for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_4^*(\kappa)}(1, q^c)$. Note that

$$\begin{aligned}\zeta(\gamma_0, \bar{q}^c) = \gamma_0[&(1-\delta^L)\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) + \delta^L\pi^r - (1-\delta^L)\bar{\pi}^c \\ &- (1-\delta^L)\delta^L\pi(q^d(\gamma_0, \bar{q}^c), \bar{q}^c) - (\delta^L)^2\pi^r] \\ &+ (1-\gamma_0)(1-\delta^L)[\pi(q^d(\gamma_0, \bar{q}^c), q^d(\gamma_0, \bar{q}^c)) - \pi(\bar{q}^c, q^d(\gamma_0, \bar{q}^c))]\end{aligned}$$

Define $f_5 : B_{\epsilon_4^*(\kappa)}(1, q^c) \longmapsto R$ such that $f_5(\gamma, q) = \gamma(1-\delta^L)\delta^L[\pi(q^d(\gamma, q), q) - \pi^{d+}]$. Then $f_5(\gamma, q)$ is continuous and $f_5(1, q^c) = 0$. Finally, define $f_6 : B_{\epsilon_4^*(\kappa)}(1, q^c) \longmapsto R$ such that

$$\begin{aligned}f_6(\gamma, q) &= \zeta(\gamma, q) + f_5(\gamma, q) \\ &= \gamma[(1-\delta^L)\pi(q^d(\gamma, q), q) + \delta^L\pi^r - (1-\delta^L)\pi(q, q) - (1-\delta^L)\delta^L\pi^{d+} - (\delta^L)^2\pi^r] \\ &\quad +(1-\gamma)(1-\delta^L)[\pi(q^d(\gamma, q), q^d(\gamma, q)) - \pi(q, q^d(\gamma, q))]\end{aligned}$$

Clearly, $f_6(\gamma, q)$ is continuous and $f_6(1, q^c) = \zeta(1, q^c) + f_5(1, q^c) = \zeta(1, q^c) > 0$. So $U_6(0) = \{(\gamma, q) \in B_{\epsilon_4^*(\kappa)}(1, q^c) \mid f_6(\gamma, q) > 0\}$ is non-empty and open. Since $(1, q^c) \in U_6(0)$, there exists $\epsilon_6^*(\kappa) > 0$ such that $f_6(\gamma, q) > 0$ for all $(\gamma, q) \in B_{\epsilon_6^*(\kappa)}(1, q^c)$. From the construction of $f_6(\gamma, q)$, $\Pi_{i1}(\bar{s}_i|h_1, \bar{s}_{-i}|h_1; \delta^L, \gamma_{i1}) - \Pi_{i1}(\bar{s}_i(\bar{q}^c)|h_1, \bar{s}_{-i}|h_1\,; \delta^L, \gamma_0) > 0$ holds for all $(\gamma_0, \bar{q}^c) \in B_{\epsilon_6^*(\kappa)}(1, q^c)$.

For all h_t such that $h_1 \neq (Join, Join)$, each firm chooses stage Nash output q^n. So there is no profitable deviation for such h_t, which implies that sequential rationality condition is satisfied. If h_t is such that $t \geq 2$ and $h_1 = (Join, Join)$, each subgame that starts from h_t is a perfect information game because $\gamma_{it} \in \{0, 1\}$ for all $i \in N$ and $t \geq 2$. Moreover, all continuation strategy $\bar{s}_i|h_t$ in equation (3.12) and (3.13) for such h_t is constructed in a way that can be supported as subgame perfect equilibrium in a dynamic game with perfect information. Hence, sequential rationality condition holds trivially for those histories. Since all possible histories were checked, I can conclude that $(\bar{s}, \gamma)$ is sequentially rational if $(\gamma_0, \bar{q}^c) \in B_{\epsilon^*(\kappa)}(1, q^c)$ if I define $\epsilon^*(\kappa) \equiv \min\{\epsilon_1^*, \epsilon_2^*, \epsilon_3^*, \epsilon_6^*(\kappa)\}$.

For the second condition of (3.9), γ is updated by Bayes rule along all the equilibrium outcome path whereas $\gamma_{it} = 0$ for at least one firm on every off-equilibrium path, which completes the proof.

Proof of Proposition 5

As in the proof of Proposition 4, consider one shot deviation and its expected payoff or continuation payoff in all possible histories. If a firm chooses not to join the collusion at the beginning of game, then its payoff is simply given by $\Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}\ ; \delta_i, \gamma_0) = \pi^n$. If a firm with δ_i chooses $q_{it} \neq \bar{q}^c$ at $h_t \in H_t$ such that $q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ for $t \geq 2$ given with $h_1 = (Join, Join)$, then its expected continuation payoff comes to

$$\Pi_{it}(\bar{s}_i(q_{it})|h_t, \bar{s}_{-i}|h_t\ ; \delta_i, \gamma_{it} = \gamma_0) = (1-\delta_i)\pi(q_{it}, \bar{q}^c) + \delta_i \pi^r,$$

where $\bar{s}_i(q_{it})|h_t$ is similarly defined.

First, check the sequential rationality of $\bar{s}$. If $t = 0$ and deviation action is $a_{i0} = (Not\ Join)$, then

$$\Pi_i(\bar{s}_i, \bar{s}_{-i}\ ; \delta_i, \gamma_0) - \Pi_i(\bar{s}_i(Not\ Join), \bar{s}_{-i}; \delta_i, \gamma_0) = \bar{\pi}^c - \pi^n > 0$$

for all $i \in N$, $\delta_i \in \Theta_i$. If deviation action is $a_{it} = q_{it} \neq \bar{q}^c$ at history h_t such that (1) either $h_1 = (Join, Join)$ for $t = 1$ or (2) $h_1 = (Join, Join)$ and $q_i^{t-1} = q_{-i}^{t-1} = (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ for $t \geq 2$, then I get

$$\begin{aligned}\Pi_{it}(\bar{s}_i|h_t, \bar{s}_{-i}|h_t; \delta_i, \gamma_{it} &= \gamma_0) - \Pi_{it}(\bar{s}_i(q_{it})|h_t, \bar{s}_{-i}|h_t\ ; \delta_i, \gamma_{it} = \gamma_0)\\ &= \bar{\pi}^c - (1-\delta_i)\pi(q_{it}, \bar{q}^c) - \delta_i \pi^r\\ &\geq \bar{\pi}^c - (1-\delta_i)\pi(q^d(\bar{q}^c), \bar{q}^c) - \delta_i \pi^r\end{aligned}$$

Define $f : [q^c, q^n] \mapsto R$ such that $f(q) = \pi(q,q) - (1-\delta_i)\pi(q^d(q), q) - \delta_i \pi^r$. Then, $f(q)$ is continuous and $f(q^n) = \delta_i(\pi^n - \pi^r) > 0$. So $U(0) = \{q \in [q^c, q^n] \mid f(q) > 0\}$ is non-empty and open. Hence, there exists ϵ^* such that $f(q) > 0$ for all $q \in (q^n - \epsilon^*, q^n]$.

For all h_t such that $h_1 \neq (Join, Join)$, each firm chooses stage Nash output q^n. So there is no profitable deviation for such h_t, which implies that sequential rationality condition is satisfied. Finally, suppose that h_t is such that $q_i^{t-1} \neq (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ or $q_{-i}^{t-1} \neq (\bar{q}^c, \bar{q}^c, \cdots, \bar{q}^c)$ for $t \geq 2$ given with $h_1 = (Join, Join)$. Then, either γ_{it} is updated into 0 if firm $-i$ deviates from $\bar{q}^c$ at period $(t-1)$ or γ_{-it} is updated into 0 otherwise. Moreover, the continuation strategy $\bar{s}_i|h_t$ in equation (14) for such

h_t describes punishment phase when there is a deviation in a dynamic game with perfect information. Hence, sequential rationality condition holds trivially for those histories. Since all possible histories were checked, I can conclude that $(\bar{s}, \gamma)$ is sequentially rational if $\bar{q}^c \in (q^n - \epsilon^*, q^n]$.

For the second condition of PBE in (3.9), the system of belief γ is updated by Bayes rule along all the equilibrium outcome path whereas $\gamma_{it} = 0$ for at least one firm on every off-equilibrium path, which completes the proof.

Appendix C

Appendix of Chapter 4

C.1 Cartel Duration Model: Leniency Program

Claim 20

Suppose that firm $j \in N$ deviated from cartel agreement at period $s < t$. Then, firm $j's$ expected discounted continuation payoff in $(State\ 2)$, denoted by $E\Pi_j^2$, comes to

$$E\Pi_j^2(s_j^1, s_{-j}) = \begin{cases} \frac{\pi^{ns}}{1-\delta} - \min(R_1^L, R) & if \quad s_{-j} = \Pi_{k \neq j} s_k^2 \\ \frac{\pi^{ns}}{1-\delta} - \min(R_2^L, R) & if \quad s_{-j} \neq \Pi_{k \neq j} s_k^2 \end{cases}$$

$$E\Pi_j^2(s_j^2, s_{-j}) = \begin{cases} \frac{\pi^{ns}}{1-\delta} - \frac{\alpha \delta R}{1-\delta(1-\alpha)} & if \quad s_{-j} = \Pi_{k \neq j} s_k^2 \\ \frac{\pi^{ns}}{1-\delta} - R & if \quad s_{-j} \neq \Pi_{k \neq j} s_k^2 \end{cases}$$

But all other firms $i \in N$ except firm j would have the same expected discounted continuation payoff with that in $(State\ 1)$. Hence, the dominant strategy of all firms but firm j is to apply leniency as well in $(State\ 2)$. Given that non-defector's dominant strategy, firm $j's$ best response is to report (s_j^1) if $R_2^L < R$ while it is indifferent for firm j whether to report or not if $R_2^L \geq R$. Hence, leniency application to AA is at least weakly dominant strategy for a defector after eliminating $s_{-j} = \Pi_{k \neq j} s_k^2$.

Claim 21

[Part (a)] Given the set of pure strategy $S_i = \{s_i^1, s_i^2\}$, the expected discounted continuation payoff of unilateral defector, denoted by $E\Pi_{i=d}^3$, can be derived as follows;

$$E\Pi_{i=d}^3(s_i^1, s_{-i}) = \begin{cases} E\Pi_{i=d}^{L_1} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi_{i=d}^{L_2} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

$$E\Pi_{i=d}^3(s_i^2, s_{-i}) = \begin{cases} E\Pi_{i=d}^{NL} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi_{i=d}^{L^{-i}} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

Likewise, the expected discounted continuation payoff of non-defector, denoted by $E\Pi_{i=n}^3$, can be derived as follows;

$$E\Pi_{i=n}^3(s_i^1, s_{-i}) = \begin{cases} E\Pi_{i=n}^{L_1} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi_{i=n}^{L_2} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

$$E\Pi_{i=n}^3(s_i^2, s_{-i}) = \begin{cases} E\Pi_{i=n}^{NL} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi_{i=n}^{L^{-i}} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

It is easy to see that $E\Pi_{i=d}^3(s_i^1, s_{-i}) \geq E\Pi_{i=d}^3(s_i^2, s_{-i})$ and $E\Pi_{i=n}^3(s_i^1, s_{-i}) \geq E\Pi_{i=n}^3(s_i^2, s_{-i})$ if $s_{-i} \neq \Pi_{j\neq i} s_j^2$. When $R_1^L < \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ holds, $E\Pi_{i=n}^{L_1} > E\Pi_{i=n}^{NL}$ is satisfied for non-defector. Hence, s_i^1 is a dominant strategy of all non-defectors. In that case, the best response of defector includes to report leniency.

Similarly, when $\min(R, R_1^L) < \frac{\pi^n - \pi^r + \delta(\alpha R + (1-\alpha)p\min(R, R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}$ holds, $E\Pi_{i=d}^{L_1} > E\Pi_{i=d}^{NL}$ is satisfied for a defector. Hence, s_i^1 is a dominant strategy of the defector. But then, the best response of non-defector includes to report leniency.

[Part (b)] If $R_1^L > \frac{\pi^n - \pi^p + \delta(\alpha F + (1-\alpha)pR_2^L)}{1-\delta(1-(\alpha+p-\alpha p))}$ holds,

$$\begin{aligned} E\Pi_{i=n}^{NL} &> \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - R_1^L \\ &\geq \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - R_2^L = E\Pi_{i=n}^{L_2} \end{aligned}$$

Similarly, if $\min(R, R_1^L) > \frac{\pi^n - \pi^r + \delta(\alpha R + (1-\alpha)p\min(R, R_2^L))}{1-\delta(1-(\alpha+p-\alpha p))}$ holds,

$$\begin{aligned} E\Pi_{i=d}^{NL} &> \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \min(R_1^L, R) \\ &\geq \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \min(R_2^L, R) = E\Pi_{i=d}^{L_2} \end{aligned}$$

So, the unique efficient Nash equilibrium is that no firm applies leniency.

Claim 22

[Part (a)] Firm $i's$ expected discounted continuation payoff in $(State\ 4)$, denoted by $E\Pi_i^4(s_i, s_{-i})$, can be derived as follows;

$$E\Pi_i^4(s_i^1, s_{-i}) = \begin{cases} E\Pi^{L_1} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi^{L_2} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

$$E\Pi_i^4(s_i^2, s_{-i}) = \begin{cases} E\Pi^{NC} & if \quad s_{-i} = \Pi_{j\neq i} s_j^2 \\ E\Pi^{L^{-i}} & if \quad s_{-i} \neq \Pi_{j\neq i} s_j^2 \end{cases}$$

Here, when other firms' strategy profile is given by $s_{-i} = \Pi_{j\neq i} s_j^2$, $E\Pi_i^4(s_i^2, s_{-i}) = E\Pi^{NC}$ holds from $E\Pi^{NC} > E\Pi^{ND_j}$ depending on [Condition j], where $j \in \{1, 2\}$. Hence, when all other firms do not report ($s_{-i} = \Pi_{j\neq i} s_j^2$), firm $i's$ best response is also not to report because $E\Pi^{NC} > E\Pi^{LD_1}$. But when there is at least one firm who apply leniency ($s_{-i} \neq \Pi_{j\neq i} s_j^2$), then firm $i's$ best response is to report to AA because $E\Pi^{L_2} \geq E\Pi^{L^{-i}}$ from $R_2^L \in [R_1^L, F]$. To find out a symmetric mixed strategy in $(State\ 4)$, let $\Pr(not\ report) = \beta$ for all other firms (i.e. $s_{-i} = \Pi_{j\neq i}\beta$) and define $f(\beta) \equiv E\Pi_i^4(s_i^2, s_{-i}) - E\Pi_i^4(s_i^1, s_{-i})$, then

$$f(\beta) = \beta^{n-1}(E\Pi^{NC} - E\Pi^{L_1}) + (1-\beta^{n-1})(R_2^L - F)$$

Clearly, $f(0) \leq 0$ - the inequality is strict if $R_2^L < F$ - and $f(1) > 0$ holds. Moreover, $f(\beta)$ is monotonically increasing in β. Therefore, there exists a unique $\beta_* \in [0, 1)$ such that $f(\beta_*) = 0$. Again, if $R_2^L < F$, $\beta_* \in (0, 1)$.

[Part (b)] Compare reporting equilibrium ($\beta_*^4 = 0$) by all firms with no repoting equilibrium ($\beta_*^4 = 1$). In each equilibrium, the continuation payoff of each firm, denoted by $E\Pi_i^4(0)$ and $E\Pi_i^4(1)$, will be $E\Pi_i^4(0) = E\Pi^{L_2}$ and $E\Pi_i^4(1) = E\Pi^{NC}$ respectively. Since $E\Pi^{NC} > \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \geq E\Pi^{L_2}$, $E\Pi_i^4(1) > E\Pi_i^4(0)$ holds for all firms.

Next, compare the symmetric mixed strategy Nash equilibrium $\beta_* \in (0,1)$ with the equilibrium that no firm reports. In each equilibrium, the continuation payoff of each firm, denoted by $E\Pi_i^4(\beta_*)$ and $E\Pi_i^4(1)$ respectively, will be

$$\begin{aligned}
E\Pi_i^4(\beta_*) &= \beta_*[(\beta_*)^{n-1} * E\Pi^{NC} + (1-(\beta_*)^{n-1}) * E\Pi^{L^{-i}}] \\
&\quad +(1-\beta_*)[(\beta_*)^{n-1} * E\Pi^{L_1} + (1-(\beta_*)^{n-1}) * E\Pi^{L_2}] \\
E\Pi_i^4(1) &= E\Pi^{NC}
\end{aligned}$$

Again $E\Pi^{NC} > \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))}$ and $E\Pi^{NC} > \max\{E\Pi^{L_1}, E\Pi^{L_2}\}$ implies that $E\Pi_i^4(1) > E\Pi_i^4(\beta_*)$ for all firms. Hence, the equilibrium that no firm notice is a unique efficient equilibrium in leniency decision. This proof also show that the not-reporting equilibrium is more efficient than any non-symmetric mixed strategy equilibrium, if it exists.

Equation (4.5)

Let $E\Pi^{CL} = V_1$, then

$$V_1 = \pi^c + \delta[(1-\alpha)(1-p)V_1 + \alpha(1-p)(V_2 - F) + (1-\alpha)pV_3 + \alpha p(V_4 - F) \tag{C.1}$$

For V_2, I have

$$\begin{aligned}
V_2 &= \pi^n + \delta[(1-p)V_2 + \frac{p\pi^{ns}}{1-\delta}] \\
&\Rightarrow \quad V_2 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))}
\end{aligned}$$

For V_3, I obtain

$$
\begin{aligned}
V_3 &= \pi^{ns} + \delta[(1-\alpha)V_3 + \alpha(\frac{\pi^{ns}}{1-\delta} - F)] \\
V_3 &= \frac{\pi^{ns}}{1-\delta(1-\alpha)} + \frac{\delta\alpha\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha))} - \frac{\delta\alpha F}{1-\delta(1-\alpha)} \\
&= \frac{\pi^{ns}}{1-\delta} - \frac{\delta\alpha F}{1-\delta(1-\alpha)}
\end{aligned}
$$

For V_4, I get $V_4 = \frac{\pi^{ns}}{1-\delta}$. Plug V_2, V_3 and V_4 into (C.1), then I have

$$
\begin{aligned}
(1-\delta(1-\alpha)(1-p))V_1 &= \pi^c + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \delta p\pi^{ns}[\frac{1}{1-\delta} + \frac{\delta\alpha(1-p)}{(1-\delta)(1-\delta(1-p))}] \\
&\quad -\delta\alpha F[1 + \frac{\delta(1-\alpha)p}{1-\delta(1-\alpha)}] \\
(1-\delta(1-\alpha)(1-p))V_1 &= \pi^c + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p(1-\delta(1-\alpha)(1-p))\pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\
&\quad -\frac{\delta\alpha(1-\delta(1-\alpha)(1-p))F}{1-\delta(1-\alpha)}
\end{aligned}
$$

Hence,

$$
\begin{aligned}
V_1 &= \frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
&\quad +\frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta\alpha F}{1-\delta(1-\alpha)} \\
&= \frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} - \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} \\
&\quad +\frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} + \frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta\alpha F}{1-\delta(1-\alpha)} \\
&= \frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} - \frac{\delta\alpha F}{1-\delta(1-\alpha)} \\
&\quad +\frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
&\quad +\frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))},
\end{aligned}
$$

where

$$
\frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} = E\Pi^C(\alpha+p-\alpha p)
$$

and

$$
\begin{aligned}
& \frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} \\
= \ & \frac{\delta\pi^{ns}}{(1-\delta)}[\frac{p(1-\delta(1-\alpha)(1-p)) - (\alpha+p-\alpha p)(1-\delta(1-p))}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}] \\
= \ & \frac{\delta\pi^{ns}}{(1-\delta)} * \frac{-\alpha(1-p)(1-\delta)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
= \ & -\frac{\delta\alpha(1-p)\pi^{ns}}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}
\end{aligned}
$$

Hence, V_1 comes to

$$
V_1 = E\Pi^C(\alpha+p-\alpha p) - \frac{\delta\alpha F}{1-\delta(1-\alpha)} + \frac{\delta\alpha(1-p)(\pi^n-\pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}
$$

Equation (4.8)

Let $E\Pi_{i=d}^{NL} = V_1$, then

$$
\begin{aligned}
V_1 &= \pi^r + \delta[(1-\alpha)(1-p)V_1 + \alpha(1-p)(V_2-R) + \alpha p(\frac{\pi^{ns}}{1-\delta} - R) \\
&\quad +(1-\alpha)p(\frac{\pi^{ns}}{1-\delta} - \min\{R, R_2^L\})]
\end{aligned} \tag{C.2}
$$

For V_2, I get

$$
\begin{aligned}
V_2 &= \pi^n + \delta[(1-p)V_2 + \frac{p\pi^{ns}}{1-\delta}] \\
&\Rightarrow \quad V_2 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))}
\end{aligned}
$$

Plug V_2 into (C.2), then I have

$$
\begin{aligned}
(1-\delta(1-\alpha)(1-p))V_1 &= \pi^r + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p\pi^{ns}}{1-\delta}[1 + \frac{\delta\alpha(1-p)}{1-\delta(1-p)}] \\
&\quad -\delta\alpha R - \delta(1-\alpha)p\min\{R, R_2^L\} \\
(1-\delta(1-\alpha)(1-p))V_1 &= \pi^r + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p(1-\delta(1-\alpha)(1-p))\pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\
&\quad -\delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}]
\end{aligned}
$$

Hence,

$$\begin{aligned}V_1 &= \frac{\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\ &+ \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)}\end{aligned}$$

Here, notice that

$$\frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} = \pi^n[\frac{1}{1-\delta(1-p)} - \frac{1}{1-\delta(1-\alpha)(1-p)}]$$

Therefore, V_1 comes to

$$V_1 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\pi^n - \pi^r + \delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)}$$

Equation (4.10)

Let $E\Pi^{NC} = V_1$, then

$$\begin{aligned}V_1 &= \pi^c + \delta[(1-\alpha)(1-p)V_1 + \alpha(1-p)(V_2 - F) \\ &\quad + (1-\alpha)p(\frac{\pi^{ns}}{1-\delta} - R_2^L) + \alpha p(\frac{\pi^{ns}}{1-\delta} - F)]\end{aligned} \tag{C.3}$$

For V_2, I obtain

$$\begin{aligned}V_2 &= \pi^n + \delta[(1-p)V_2 + \frac{p\pi^{ns}}{1-\delta}] \\ &\Rightarrow \quad V_2 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p\pi^{ns}}{(1-\delta)(1-\delta(1-p))}\end{aligned}$$

Plug V_2 into (C.3), then I have

$$\begin{aligned}(1-\delta(1-\alpha)(1-p))V_1 &= \pi^c + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p\pi^{ns}}{1-\delta}[1 + \frac{\delta\alpha(1-p)}{1-\delta(1-p)}] \\ &\quad -\delta\alpha F - \delta(1-\alpha)pR_2^L \\ (1-\delta(1-\alpha)(1-p))V_1 &= \pi^c + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p(1-\delta(1-\alpha)(1-p))\pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\ &\quad -\delta[\alpha F + (1-\alpha)pR_2^L]\end{aligned}$$

Hence,

$$\begin{aligned} V_1 &= \frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\ &\quad + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta[\alpha F + (1-\alpha)pR_2^L]}{1-\delta(1-\alpha)(1-p)} \end{aligned}$$

Here, notice that

$$\begin{aligned} & \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\ = \; & \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} - \frac{\delta\alpha(1-p)\pi^{ns}}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \end{aligned}$$

Therefore,

$$\begin{aligned} V_1 &= \frac{\pi^c}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} \\ &\quad + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} - \frac{\delta[\alpha F + (1-\alpha)pR_2^L]}{1-\delta(1-\alpha)(1-p)} \\ &= E\Pi^C(\alpha+p-\alpha p) + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} - \frac{\delta[\alpha F + (1-\alpha)pR_2^L]}{1-\delta(1-\alpha)(1-p)} \end{aligned}$$

Equation (4.11)

Let $E\Pi^{ND_1} = V_1$, then

$$\begin{aligned} V_1 &= \pi^d + \delta[(1-\alpha)(1-p)(V_2 - \min\{R, R_2^L\}) + \alpha(1-p)(V_2 - R) \\ &\quad + \alpha p(\frac{\pi^{ns}}{1-\delta} - R) + (1-\alpha)p(\frac{\pi^{ns}}{1-\delta} - \min\{R, R_2^L\})] \end{aligned} \tag{C.4}$$

For V_2, I get

$$\begin{aligned} V_2 &= \pi^n + \delta[(1-p)V_2 + \frac{p\pi^{ns}}{1-\delta}] \\ &\Rightarrow \quad V_2 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \end{aligned}$$

Plug V_2 into (C.4), then I have

$$\begin{aligned} V_1 &= \pi^d + \frac{\delta(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{1-\delta}[1 + \frac{\delta(1-p)}{1-\delta(1-p)}] - \delta\alpha R - \delta(1-\alpha)\min\{R, R_2^L\} \\ &= \pi^d + \frac{\delta(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}] \\ &= \pi^d + \frac{\delta(1-p)\pi^r}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}] \\ &\quad + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} \\ &= E\Pi^D(p) - \delta[\alpha R + (1-\alpha)p\min\{R, R_2^L\}] + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} \end{aligned}$$

Equation (4.12)

Let $E\Pi^{ND_2} = V_1$, then

$$\begin{aligned} V_1 &= \pi^d + \delta[(1-\alpha)(1-p)V_2 + \alpha(1-p)(V_3 - R) \\ &\quad + \alpha p(\frac{\pi^{ns}}{1-\delta} - R) + (1-\alpha)p(\frac{\pi^{ns}}{1-\delta} - \min\{R, R_2^L\})] \end{aligned} \tag{C.5}$$

Here, V_2 is given by

$$\begin{aligned} V_2 &= \pi^r + \delta[(1-\alpha)(1-p)V_2 + \alpha(1-p)(V_3 - R) \\ &\quad + \alpha p(\frac{\pi^{ns}}{1-\delta} - R) + (1-\alpha)p(\frac{\pi^{ns}}{1-\delta} - \min\{R, R_2^L\})] \end{aligned} \tag{C.6}$$

and V_3 is given by

$$V_3 = \frac{\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))}$$

Plug V_3 into (C.6), then I have

$$\begin{aligned} (1-\delta(1-\alpha)(1-p))V_2 &= \pi^r + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{1-\delta}[1 + \frac{\delta\alpha(1-p)}{1-\delta(1-p)}] \\ &\quad - \delta\alpha R - \delta(1-\alpha)p\min\{R, R_2^L\} \end{aligned}$$

So V_2 comes to

$$\begin{aligned}
V_2 &= \frac{\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
&\quad + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta[\alpha R + (1-\alpha) p \min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)}
\end{aligned}$$

Now, plug V_2 and V_3 into (C.5), then I have

$$\begin{aligned}
V_1 &= \pi^d + \frac{\delta(1-\alpha)(1-p)\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{1-\delta(1-p)}[1 + \frac{\delta(1-\alpha)(1-p)}{1-\delta(1-\alpha)(1-p)}] \\
&\quad + \frac{\delta p \pi^{ns}}{1-\delta}[1 + \frac{\delta(1-p)}{1-\delta(1-p)}] \\
&\quad - \delta[\alpha R + (1-\alpha) p \min\{R, R_2^L\}][1 + \frac{\delta(1-\alpha)(1-p)}{1-\delta(1-\alpha)(1-p)}] \\
&= \pi^d + \frac{\delta(1-\alpha)(1-p)\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta\alpha(1-p)\pi^n}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
&\quad + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} - \frac{\delta[\alpha R + (1-\alpha) p \min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)}
\end{aligned}$$

Using

$$\begin{aligned}
& \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\
&= \frac{\delta(\alpha + p - \alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} - \frac{\delta\alpha(1-p)\pi^{ns}}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}
\end{aligned}$$

, then I get

$$\begin{aligned}
V_1 &= \pi^d + \frac{\delta(1-\alpha)(1-p)\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha + p - \alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))} \\
&\quad + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} - \frac{\delta[\alpha R + (1-\alpha) p \min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)} \\
&= E\Pi^D(\alpha + p - \alpha p) \\
&\quad + \frac{\delta\alpha(1-p)(\pi^n - \pi^{ns})}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} - \frac{\delta[\alpha R + (1-\alpha) p \min\{R, R_2^L\}]}{1-\delta(1-\alpha)(1-p)}
\end{aligned}$$

Equation (4.16)

$$
\begin{aligned}
& E\Pi^D(p) - E\Pi^D(\alpha + p - \alpha p) \\
& + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} + \frac{\delta\alpha(1-p)(\pi^{ns} - \pi^n)}{(1-\delta(1-p))(1-\delta(1-(\alpha+p-\alpha p)))} \\
= \; & \pi^d + \frac{\delta(1-p)\pi^r}{1-\delta(1-p)} + \frac{\delta p \pi^{ns}}{(1-\delta)(1-\delta(1-p))} \\
& -[\pi^d + \frac{\delta(1-\alpha)(1-p)\pi^r}{1-\delta(1-\alpha)(1-p)} + \frac{\delta(\alpha+p-\alpha p)\pi^{ns}}{(1-\delta)(1-\delta(1-\alpha)(1-p))}] \\
& + \frac{\delta(1-p)(\pi^n - \pi^r)}{1-\delta(1-p)} + \frac{\delta\alpha(1-p)(\pi^{ns} - \pi^n)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
= \; & \pi^n[\frac{\delta(1-p)}{1-\delta(1-p)} - \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}] \\
& - \frac{\delta(1-\alpha)(1-p)\pi^r}{1-\delta(1-\alpha)(1-p)} \\
& + \pi^{ns}[\frac{\delta p}{(1-\delta)(1-\delta(1-p))} - \frac{\delta(\alpha+p-\alpha p)}{(1-\delta)(1-\delta(1-\alpha)(1-p))} \\
& \qquad + \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))}],
\end{aligned}
$$

where

$$
\begin{aligned}
& \frac{\delta(1-p)}{1-\delta(1-p)} - \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
= \; & \frac{\delta(1-p)}{1-\delta(1-p)}[1 - \frac{\alpha}{1-\delta(1-\alpha)(1-p)}] \\
= \; & \frac{\delta(1-p)}{1-\delta(1-p)} * \frac{1-\alpha-\delta(1-\alpha)(1-p)}{1-\delta(1-\alpha)(1-p)} = \frac{\delta(1-\alpha)(1-p)}{1-\delta(1-\alpha)(1-p)}
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\delta p}{(1-\delta)(1-\delta(1-p))} \\
& - \frac{\delta(\alpha+p-\alpha p)}{(1-\delta)(1-\delta(1-\alpha)(1-p))} + \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
= \; & \frac{\delta(\alpha+p-\alpha p)}{(1-\delta)(1-\delta(1-\alpha)(1-p))} - \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
& - \frac{\delta(\alpha+p-\alpha p)}{(1-\delta)(1-\delta(1-\alpha)(1-p))} + \frac{\delta\alpha(1-p)}{(1-\delta(1-p))(1-\delta(1-\alpha)(1-p))} \\
= \; & 0
\end{aligned}
$$

Hence, I get

$$\begin{aligned}
& E\Pi^D(p) - E\Pi^D(\alpha + p - \alpha p) \\
& + \frac{\delta(1-p)(\pi^n - \pi^r)}{1 - \delta(1-p)} + \frac{\delta\alpha(1-p)(\pi^{ns} - \pi^n)}{(1 - \delta(1-p))(1 - \delta(1 - (\alpha + p - \alpha p)))} \\
= \ & \frac{\delta(1-\alpha)(1-p)(\pi^n - \pi^r)}{1 - \delta(1-\alpha)(1-p)}
\end{aligned}$$

C.2 Two-Industry Model: Selective Law Enforcement

Proposition 8

The sign of $(E_2 - E)$ is equal to that of $[(\alpha_e - \alpha_m) - \delta_1(1-\alpha_e)(\alpha_m - \alpha_o)]$. Define a function $g : [0, \frac{\bar{B}}{2}] \mapsto R$ such that

$$\begin{aligned}
g(\varepsilon) &= \alpha_e(\varepsilon) - \alpha_m - \delta_1(1 - \alpha_e(\varepsilon))(\alpha_m - \alpha_o(\varepsilon)) \\
&= f(\frac{\bar{B}}{2} + \varepsilon) - f(\frac{\bar{B}}{2}) - \delta_1(1 - f(\frac{\bar{B}}{2} + \varepsilon))(f(\frac{\bar{B}}{2}) - f(\frac{\bar{B}}{2} - \varepsilon))
\end{aligned}$$

Then, $g(0) = 0$ and the derivative of this function yields

$$g'(\varepsilon) = f^{'}(\frac{\bar{B}}{2} + \varepsilon) - \delta_1[(1 - f(\frac{\bar{B}}{2} + \varepsilon))f^{'}(\frac{\bar{B}}{2} - \varepsilon) - f^{'}(\frac{\bar{B}}{2} + \varepsilon)(f(\frac{\bar{B}}{2}) - f(\frac{\bar{B}}{2} - \varepsilon)]$$

So $g'(0) = f^{'}(\frac{\bar{B}}{2})(1 - \delta(1 - f(\frac{\bar{B}}{2}))) > 0$. Since $g(\varepsilon)$ is continuous, it must be the case that either $g(\varepsilon) > 0$ for all $\varepsilon \in (0, \frac{\bar{B}}{2}]$ or there exists $\varepsilon^* \in (0, \frac{\bar{B}}{2}]$ such that $g(\varepsilon) > 0$ for all $\varepsilon \in (0, \varepsilon^*)$.

Equation (4.24)

$$\begin{aligned}
E - E_1 = {} & (1 - \delta_1(1-\alpha_m))\pi_1^d + \delta_1(1-\alpha_m)\pi_1^r + \delta_1\alpha_m F \\
& - [\frac{1 - \delta_1^2(1-\alpha_o)(1-\alpha_e)}{1 + \delta_1(1-\alpha_o)}\ \pi_1^d + \frac{\delta_1(1-\alpha_o)(1 + \delta_1(1-\alpha_e))}{1 + \delta_1(1-\alpha_o)}\pi_1^r \\
& + \frac{\delta_1\alpha_o + \delta_1^2\alpha_e(1-\alpha_o)}{1 + \delta_1(1-\alpha_o)}F]
\end{aligned}$$

Here,

$$
\begin{aligned}
(1-\delta_1(1-\alpha_m)) - \frac{1-\delta_1^2(1-\alpha_o)(1-\alpha_e)}{1+\delta_1(1-\alpha_o)} &= \frac{\delta_1(\alpha_m-\alpha_o)-\delta_1^2(1-\alpha_o)(\alpha_e-\alpha_m)}{1+\delta_1(1-\alpha_o)} \\
\delta_1(1-\alpha_m) - \frac{\delta_1(1-\alpha_o)(1+\delta_1(1-\alpha_e))}{1+\delta_1(1-\alpha_o)} &= -\frac{\delta_1(\alpha_m-\alpha_o)-\delta_1^2(1-\alpha_o)(\alpha_e-\alpha_m)}{1+\delta_1(1-\alpha_o)} \\
\delta_1\alpha_m - \frac{\delta_1\alpha_o+\delta_1^2\alpha_e(1-\alpha_o)}{1+\delta_1(1-\alpha_o)} &= \frac{\delta_1(\alpha_m-\alpha_o)-\delta_1^2(1-\alpha_o)(\alpha_e-\alpha_m)}{1+\delta_1(1-\alpha_o)}
\end{aligned}
$$

Hence, I have

$$
E - E_1 = \frac{\delta_1\left[(\alpha_m-\alpha_o)-\delta_1(1-\alpha_o)(\alpha_e-\alpha_m)\right]}{1+\delta_1(1-\alpha_o)} * (\pi_1^d - \pi_1^r + F)
$$

Equation (4.25) can derived in a similar way.

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