COMPETITION IN MARKETS WITH NETWORK EXTERNALITIES

Fatma Busra Gunay

A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Economics.

Chapel Hill
2013

Approved by:

Gary Biglaiser

Helen Tauchen

Peter Norman R.

Vijay Krishna

Toan Phan

Abstract

FATMA BUSRA GUNAY: Competition in Markets with Network Externalities.
(Under the direction of Gary Biglaiser)

This paper analyzes the effects of network externalities on an incumbent's advantage in a static duopoly model where an entrant and an incumbent strategically set prices. A Global Games approach is used as an equilibrium refinement, where consumers receive both a public and a private signal about the entrant's quality. While a unique equilibrium is not guaranteed in all of the cases, the incumbent's advantage arises in specific cases depending on the relative precision of the signals. As an extension, I show in a model of endogenous advertisement choice that the multiple equilibria problem is resolved because the entrant prefers an advertisement level which makes the private signal precise enough to generate a unique equilibrium. I also investigate the effects of coordination on the pricing strategy in a dynamic duopoly market with network externalities. I use a Global Games approach as an equilibrium refinement to obtain a unique equilibrium. In a setting where the firms set prices in the first period, I find that if the consumers value the network externalities enough, the firm with the higher expected quality expects to sell less in the second period than the first period. In both periods, the firm with higher market share charges a higher price and receives higher profit.

Acknowledgments

I am grateful to Gary Biglaiser and R. Vijay Krishna for their guidance and support. I would also like to thank Peter Norman, Helen Tauchen and Toan Phan for being on my dissertation committee, and helpful comments and suggestions.

Table of Contents

List of Figures

Chapter 1

Introduction

Many markets such as wireless phone networks and financial exchanges exhibit consumer lock-in[1]. In this paper, I try to find the reason why we observe incumbency advantage in these markets. These markets do not have substantial switching costs like the standard examples given for lock-in, such as QWERTY keyboard and frequent flyer programs of airlines. I claim that the existence of uncertain network externalities[2] can create endogenous switching costs which creates an advantage for the first firm in the market. Consumers may not find it individually rational to buy from the incumbent even if a coordinated switch is beneficial for most consumers.

In such markets, the market structure is expected to be different than markets with a standard price competition. It is possible to have entry deterrence as in some markets with explicit switching costs, such as cable TV markets. If entry deterrence is possible how does it affect the welfare? What are the policy implications? Are there any strategies for the entrant to overcome the incumbency advantage? In this paper, I address these questions in an incomplete information framework, using two different models of network externalities.

Network externalities on the demand side cause coordination problems for the consumers because of strategic complementarities. Since there may exist many ways to coordinate, in network industry, there are potentially many equilibria. In this framework, an additional question arises: which equilibrium will be played?

---

[1] See Ramos and von Thadden, 2003

[2] Securities and exchanges exhibit network effects since trading volume increases liquidity. mobile phone networks charge lower prices for intra-network calls.

The two main approaches used in the literature to handle the multiple equilibria problem are: the higher order beliefs approach by Mertens and Zamir (1985) and the global games approach by Carlsson and van Damme (1993). This paper adopts the latter as an equilibrium refinement. Existence of imperfect information not only requires the players to form beliefs about other players but it also requires them to form beliefs about other players' beliefs, and about other players' beliefs about his belief, etc. The higher order beliefs approach by Mertens and Zamir (1985) makes the analysis more realistic in the sense that it takes higher order beliefs into account. But it also complicates the equilibrium analysis because of the same reason. On the other hand, the global games approach is sophisticated enough to capture the effects of higher order beliefs but simple enough to allow tractable analysis.

In my first model, I develop a static duopoly market with network externalities where two firms (an incumbent and an entrant) simultaneously set prices. After observing prices along with a public and a private signal about the entrant's quality, the consumers update not only their beliefs about the entrant's quality, but also their beliefs about the signals received by other consumers. Using the global games approach, this updating process generates a unique equilibrium in some cases by iterative elimination of strictly dominated strategies. I show that if the private signal is more precise relative to the public signal, the market has a unique perfect Bayesian Nash Equilibrium. In this unique PBNE, the incumbent charges a higher price and receives higher market share compared to a benchmark Hotelling model without any network externalities. I conclude that network externalities magnifies the market concentration, in the sense that the market is more unbalanced with network effects.

In the second model, I offer a model of endogenous advertising. The entrant sets price and the level of advertisement to strategically manipulate the coordination problem of the consumers. By choosing the level of advertisement, the entrant can determine the number as well as the characteristics of the equilibria. I show that the multiplicity problem is resolved for sufficiently small advertisement costs.

By using advertisement, the entrant can make the signal received by the consumers precise enough so that the consumers are coordinated on his product. Therefore, the advertisement

plays the role of a coordination device.

The originality of this research stems from the fact that there is no one-to-one association between switching costs and network effects. The size of network effects is endogenous since its magnitude depends on the result of the coordination problem of consumers. The literature about pricing in markets with switching costs and network externalities with perfect information is extensive but there are few papers about the effect of network externalities on the pricing in imperfect information setting.

The paper is organized as follows. I start with a brief literature review to combine 2 different literature this paper belongs. In section 3, I introduce a model of network externalities followed by a short discussion about the coordination problems and the relevant equilibrium concept. I characterize the sufficient conditions for uniqueness and analyze the unique equilibrium case. In section 4, I analyze a Bertrand model of differentiated types without network externalities, then compare it to the model with network externalities. In section 5, I discuss the multiplicity issue and offer a model of endogenous advertisement choice. Section 6 concludes.

In some markets, the optimal action of a consumer may depend on the actions taken by other consumers. For example, in the smart phone market, if a brand is more popular than the other, there will be more software applications specific to the brand. Similarly, the larger the demand for a computer program, the easier to find software compatible with it. In these type of decisions, the optimal action of a consumer is complementary to other consumers' actions in the sense that coordination among the consumers is beneficial to all.

This paper analyzes the decision making process of firms in a duopoly setting where there exists strategic complementarities for consumers. How do the firms compete if the consumers receive network externalities from the consumption of other consumers? The coordination problem embedded in the market creates multiple equilibria which in turn makes the analysis of the optimal pricing strategy impossible. This is a standard feature of the coordination problems.

Coordination problems generate multiple equilibria which is not easy to handle by the other agents in the market such as the firms and the policy makers. In order to predict the outcome of such games, an equilibrium selection procedure should be used. Carlsson and Van

3

Damme developed such an equilibrium selection theory which generates a unique equilibrium under specific conditions. In this paper, I follow their Global games approach to deal with multiplicity.

The global games approach has been used in many applications, both theoretical and empirical but mostly in a static context. Even though these models are necessary to understand the coordination games, they are not sufficient in the sense that many coordination problems are dynamic.

I develop a dynamic duopoly model with network externalities where both the firms and the consumers have imperfect information about the quality of the product. The quality of the product changes over time and I allow for the consumers receive a private imperfect signal about the entrant's quality which creates interesting dynamic issues.

While the firms set both period's prices in the first period, they need to consider the information that the consumers will receive in the second period. Therefore, the firms form a belief about the consumers' belief in both periods, using only the initial information that they have.

The model predicts that the firm with higher expected quality in the first period expects to sell more in the second period than in the first period if each firms must commit to the same price over both periods. On the other hand, in the equilibrium where the firms are allowed to set different prices in both periods, the firm with higher quality, expects to sell more in the first period. It is beneficial for the higher expected quality product to set a higher price in the second period, even if he loses some market share. This effect is common in models of differentiated products where one firm has an higher intrinsic quality for all consumers than another firm.

The paper is organized as follows. I start with a brief literature review to combine 2 different branches of literature in dynamic games that this paper belongs. In section 2, I introduce a dynamic duopoly model of strategic complementarity. I solve the model using backwards induction and I characterize the sufficient conditions for uniqueness of the equilibrium. I analyze the equilibrium in terms of pricing and the market share in both periods. Then, I compare it with a similar but static model under the assumption that the conditions for

4

unique equilibrium hold.

Chapter 2

Literature Review

This paper belongs to the global game literature originated by Carlsson and Van Damme (1993) as well as the network effects literature started with Katz and Shapiro (1985). This paper deals with the coordination problem associated with the network effects using "global games" perspective. Therefore, I will analyze these two branches separately.

Katz and Shapiro (1985) show that in an oligopoly, consumption externalities give rise to demand-side economies of scale which vary with consumer expectations. Their equilibrium notion is "Fulfilled Expectations Cournot Equilibrium" where consumers expectations of the network sizes are fulfilled at the equilibrium. The information structure is a simplified version of the one in this paper in the sense that consumer expectations about the network size are assumed to be fixed. They focus on the demand side of the market, comparing industry-wide externalities with firm specific network externalities in terms of output and efficiency without analyzing the pricing decision. Economides (1996) analyzes entry in a monopoly with network externalities where the externalities affect not only each firm's demand but also the total market demand. Assuming the expectations about the network size are fixed, he shows that the monopolist may have incentives to facilitate entry if the benefit from the increase in market demand with the entry is higher than potential loss in his own demand. Following Katz and Shapiro (1985), he also restricts the equilibrium to fulfilled expectations equilibrium. In the equilibrium, expected mean sales are realized. Cabral et al. (1999) show that in a durable goods market with network externalities, price increases over time if network externalities are strong enough. They have a two-period dynamic setting where differentiated consumers can

On the other hand, Chassang (2008) emphasizes the sustainability of the coordination outcome in a dynamic setting. He argues that in a dynamic global games model, existence of the possibility of future miscoordination hampers the sustainability of coordination. Therefore, there exist multiple equilibria.

make a purchase in only one of the periods. In a Perfect Bayesian Equilibrium, sales occur in the first period rather than the second because the seller sets a price lower than the expected second period price. In another dynamic model of network externalities, Doganoglu (2003) specifies conditions for the existence of a stable Markov Perfect Equilibrium in linear strategies. He finds that in equilibrium, a firm with a higher previous market share charges a higher price. He assumes that each period, consumers benefit from the previous period's network size. He finds that in the steady state, existence of network externalities generates a more competitive market compared to a market with no network externalities.

Despite the importance of network externalities in many markets, such as telecommunication networks and financial investments, little work has been done to explain price competition in such markets. Biglaiser and Cremer (2012) offers a static model which captures the incumbency advantage and the generalize it to an infinite horizon model with free entry. They define "sedentary consumers" equilibria in which consumers will only switch to a new network if they believe others will also do so. They show that incumbency advantage is limited in the infinite horizon model. Cabral (2011) considers a dynamic model of competition between two networks. Consumers die and are replaced with a constant hazard rate, and firms compete for new consumers by offering lock-in prices. He considers equilibria in Markov strategies. He shows that larger networks set higher prices.

The second branch of literature this paper relates to deals with Global Games literature. Carlsson and Van Damme (1993) define a global game as a game of incomplete information where the uncertainty arises from the payoff structure. Each player observes a signal about the actual payoff structure of the game. As the noise vanishes, they show that the unique equilibrium of the game satisfies Harsanyi and Selten's risk dominance criterion. In a binary game, they show that a rational individual will always choose the risk dominant equilibrium even if there exist other Pareto dominant equilibria. Frankel et al. (2001) generalize this result to an arbitrary number of players and actions. They prove the limit uniqueness, i.e., they prove that there exists a unique strategy that survives iterative dominance. On the other hand, Morris and Shin (2001) look at different information structures (private and public information and private information alone) and show that the unique equilibrium result of

Carlsson and Van Damme (1993) holds under specific informational assumptions. In all of these papers the focus is on coordination games and on the generation of uniqueness condition. The model closest to my paper is Argenziano (2011). She analyzes a duopoly model with product differentiation and network effects in terms of efficiency. She characterizes the conditions for a unique equilibrium and she finds that the equilibrium allocation is not efficient not only because network externalities are not fully internalized but also due to the strategic pricing decision of the firms. The market shares are too balanced because the firm with the higher quality product charges higher prices.

This paper tries also to characterize equilibrium in a dynamic coordination game under the assumption that a uniqueness condition similar to the one in Carlsson and Van Damme (1993) holds. Since I analyze a dynamic model with network externalities in the global games framework, I need to go over both the literature on network externalities as well as on dynamic global games.

Farrell and Shapiro (1988) analyze an OLG model of duopolistic competition with switching costs in a perfect information setting. They look at Markov Perfect equilibria and conclude that in each period the incumbent serves only its previously attached consumers and encourages entry by charging a higher price so that the new consumers purchase from the entrant. They also generalize this result to markets with network externalities. After this, many papers tried to incorporate imperfect information to this setting, but they had to assume simpler market structures to analyze, such as monopoly.

One of these papers is by Cabral, Salant and Woroch (1999). They develop a model of a durable good monopoly with network externalities on the consumer' side. They find that the existence of imperfect information assures the implementation of "introductory pricing" to reach a "critical mass" of consumers. Bensaid and Lesne (1996) analyzed the pricing in a similar setting and conclude that if network externalities are strong enough, equilibrium prices increase over time. Their result is contradictory to the Coase (1972) conjecture that consumers' intertemporal substitution effect forces the monopoly to lower its prices overtime, unless it can credibly commit itself not to do so. They say that the existence of strong network externalities is such a tool in the commitment to higher future prices.

A paper which covers both imperfect information and competition is by Cabral (2011). He considers a dynamic model of competition between two networks. He assumes there is one new consumer in each period. Existing consumers cannot switch networks. Therefore, in each period, networks compete for the new consumer. Since the initial distribution of consumers is given and there is only one new consumer in each period, coordination failure is not a problem in the model which makes the analysis much easier but at the same time not very comprehensive. He concludes that there exists a unique equilibrium where larger networks charge higher prices.

On the other hand, Doganoglu (2003) overcomes the coordination problem by assuming that the consumers receive network externalities from the previous period's market share. He considers a duopoly model and characterizes the Markov Perfect equilibrium in linear strategies. He finds that the firm with higher market share charges higher market prices, which is consistent with the literature. Implementation of "Introductory pricing" in order to reach the critical mass is possible in some periods.

Chamley (1999) uses a rational learning from the observation of aggregate activity to capture the effect of imperfect information on coordination problems. He analyzes a dynamic model of regime change with one-period-lived agents. He finds that there exists a unique equilibrium under specific conditions. In this equilibrium, the economy randomly evolves between low and high activity. His paper analyzes the coordination problem alone. There exist no other decision makers (e.g. firms, government) other than the agents deciding the regime change. Even if Chamley does not use the word Global games in his paper, the mechanism works exactly as in Carlsson and Van Damme (1993), in the sense that he uses iterated elimination of strictly dominated strategies to obtain the unique equilibrium.

A second branch of literature I need to consider is the dynamic global games literature. The literature about the Global games emphasizes the coordination problem along with multiplicity of equilibria. My paper benefits from the tools of Global games to analyze the price competition in a dynamic duopoly model.

The following papers are considered dynamic because of various intertemporal links. This link is not always related to strategic complementarities but they are useful in understanding

the effects of different intertemporal variables on the equilibria. For instance, in Giannitsarou and Toxvaerd (2012), the economic fundamental follows a Markov process, each player's productivity depends on the previous period's action and the players' types can be thought of as a state variable which affects future payoffs. They provide conditions for uniqueness of equilibria. They show that as the noise of the players' signal vanishes, there exists a unique Markov perfect equilibrium. I cannot implement their result to my paper because they prove uniqueness by induction using the fact that the last period's game is supermodular and it has a unique equilibrium. They provide sufficient conditions required in order to be able to consider the full game as a set of independent stage games.

Another interpretation of a Dynamic Global game is by Heidhues and Melissas (2005). They consider a model of strategic waiting where the agents make or delay an irreversible investment decision. By delaying, agents receive better information about the state of the world. They differentiate intra-period network effects from inter-period network effects. They define dynamic increasing differences and posit that dynamic increasing differences requires no inter-temporal network effects. They show that dynamic increasing differences is a necessary condition for a unique equilibrium.

Similar to Heidhues and Melissas (2005), Kovac and Steiner (2008) emphasize the importance of the irreversibility of actions on the agents' decision making when there exist strategic complementarities. In an environment where the agents dynamically receive information about other agents, they generalized the Laplacian property in order to characterize the effects of the irreversibility of actions on the coordination outcome.

On the other hand, Chassang (2008) emphasizes the sustainability of the coordination outcome in a dynamic setting. He argues that in a dynamic global games model, existence of the possibility of future miscoordination hampers the sustainability of coordination. Therefore, there exist multiple equilibria.

Chapter 3

Static Competition

3.1   The Model

I consider a one period model of price competition between an incumbent (I) and an entrant (E). I has been in the market at least one period before the start of the game along with the consumers. The incumbent's product quality is common knowledge and it is normalized to 0. The entrant's product quality is θ_E, unknown to both firms and consumers . The quality can be thought of as the degree of match between the product and the consumer, where a higher θ_E implies a better match with the consumer's taste. Therefore the products are horizontally differentiated.

3.1.1 Preferences & Profits

There is a continuum of risk neutral consumers of measure 1. Consumers form expectations about the quality of the entrant's product as well as the size of the network while making purchase decisions. Consumers are required to buy one of the two products. Consumers have linear preferences over the quality and network size. Let $U(E, \lambda, \theta_E)$ be the utility function of a consumer who buys from E whose quality level is θ_E and whose ex-post network size is λ. $U(I, 1 - \lambda)$ is the utility of a consumer who buys from I with quality 0 and ex-post network

I consider symmetric switching strategies in the sense that the consumers switch to E's product if they believe that E's quality is higher than some level k (same for all consumers). I define symmetric switching strategy for consumers in terms of the expected quality of the entrant's product. The benchmark switching strategy models in the literature usually defines the cutoff level as the level of signal which makes the consumer indifferent between two products. But, in my model, since there exist two independent signals it is more convenient to conduct analysis in terms of posterior expectations of quality. Specifically, I will define k as the level of the entrant's expected quality which makes the consumer indifferent between E and I.

size $(1 - \lambda)$ and c is the relative marginal value of quality over externality

$$U(E, \lambda, \theta_E) = -p_E + c\theta_E + \lambda$$

$$U(I, 1 - \lambda) = -p_I + (1 - \lambda)$$

Firms are expected profit maximizers with zero marginal cost[1]. Firms' profit functions are given by

$$\Pi_E = \lambda p_E$$

$$\Pi_I = (1 - \lambda)p_I$$

where λ is the network size of E, and p_i is the price charged by firm $i \in \{I, E\}$

Firms and consumers observe the same public signal. Then, the firms set prices p_I and p_E simultaneously. Consumers observe prices along with a private signal and choose the product to buy. This purchase decision depends not only on prices, but also on the consumer's beliefs about E's quality and network sizes.

3.1.2 Beliefs and Bayesian Updating

Firms observe a noisy public signal θ_0 about E's quality. If we interpret the quality as the degree of match between the product and the consumer, then it is reasonable to assume that E has incomplete information about his own quality. The public signal is of the form: $\theta_0 = \theta_E + \eta$ where η is normally distributed with mean 0 and standard deviation τ.

Consumers are differentiated by a private signal about E's quality. They observe two signals: the public signal and a private signal $x_i = \theta_E + \epsilon_i$ where each ϵ_i is independently normally distributed with mean 0 and standard deviation σ. I assume that consumers bought from the incumbent previously, therefore they are perfectly informed about the incumbent's quality.

---

[1] It is a realistic assumption in the sense that many industries with network externalities exhibit zero marginal cost (wireless phone networks, software, internet, etc...

Beliefs About The Quality

I consider symmetric switching strategies in the sense that the consumers switch to E's product if they believe that E's quality is higher than some level k (same for all consumers). I define symmetric switching strategy for consumers in terms of the expected quality of the entrant's product. The benchmark switching strategy models in the literature usually defines the cutoff level as the level of signal which makes the consumer indifferent between two products. But, in my model, since there exist two independent signals it is more convenient to conduct analysis in terms of posterior expectations of quality. Specifically, I will define k as the level of the entrant's expected quality which makes the consumer indifferent between E and I.

After observing the public signal θ_0 and the private signal x_i, consumer i believes that the entrant's quality θ is normally distributed with mean $\bar{\theta}$ and variance $\frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}$ where[2]

$$\bar{\theta} = E[\theta_E \mid x, \theta_0] = \frac{\sigma^2 \theta_0 + \tau^2 x}{\sigma^2 + \tau^2} \tag{3.1}$$

Now, we can define the consumers' strategy as a function of expected quality of E for the consumer.

Definition 1. *A pure symmetric switching strategy around* k *is* $s(\bar{\theta})$ *such that*

$$s(\bar{\theta}) = \begin{cases} E & if \ \bar{\theta} \geq k \\ I & if \ \bar{\theta} < k \end{cases} \tag{3.2}$$

Beliefs of a Consumer About Other Consumers

The coordination problem analyzed in this model has two aspects of incomplete information: incomplete information about the quality of the entrant's product and incomplete information about the network sizes arising from higher order beliefs. The presence of higher order beliefs may cause tractability problems. The approach offered by Carlsson and Van Damme (1993) is rich enough to capture the effects of higher order beliefs and simple enough to allow tractable analysis.

---

[2]See DeGroot (2005) for a complete derivation.

For consumer i , consumer j's private signal x_j satisfies the following; $x_j = \theta + \epsilon_j$. Since i knows that the entrant's quality θ is normally distributed with mean $\bar{\theta}$ and variance $\frac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}$, i believes that $x_j \backsim \mathcal{N}(\bar{\theta} \ , \ \frac{2\sigma^2 \tau^2 + \sigma^4}{\sigma^2 + \tau^2})$. In accordance with the symmetric strategy, consumer i believes that player j will purchase entrant's product if his expectation about the quality is at least k, in other words, if $\frac{\sigma^2 \theta_0 + \tau^2 x_j}{\sigma^2 + \tau^2} > k$ or, $\quad x_j > k + \frac{\sigma^2}{\tau^2}(k - \theta_0)$ Since the distribution of x_j is known by i, the probability of other consumers buying the entrant's product (which is also equal to the percentage of consumers buying the entrant's product (λ)) will be:

$$\lambda = 1 - \Phi \left(\frac{k + \frac{\sigma^2}{\tau^2}(k - \theta_0) - \bar{\theta}}{\sqrt{\frac{2\sigma^2 \tau^2 + \sigma^4}{\sigma^2 + \tau^2}}} \right) \tag{3.3}$$

At the cutoff k, $\bar{\theta} = k$, therefore;

$$\lambda = 1 - \Phi \left(\sqrt{\gamma}(k - \theta_0) \right)$$

where γ is defined as:

$$\gamma = \frac{\sigma^2}{\tau^4} \left(\frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2} \right)$$

The combined parameter γ measures the relative precision of the public and private signals. It is increasing in the variance of the private signal and decreasing in the variance of the public signal. A large γ implies a less precise private signal compared to the public signal. The equilibrium will depend crucially on γ and I conduct comparative statics with respect to γ to analyze the effect of the signals on the equilibrium.

3.2 Results

I analyze the model in two steps by backward induction. I start with the analysis of the "induced game", which is the coordination game of the consumers for a given set of prices. Then I analyze the "full game, where firms strategically choose price anticipating the later coordination game of consumers.

3.2.1 Induced Game

The induced game is the continuation game starting from the decision node of the consumers. Therefore, in this subsection, prices have already been announced and signals have been observed by the consumers.

Proposition 1. *The induced game has a symmetric switching strategy equilibrium around cutoff k, where k solves*

$$ck = p_E - p_I - 1 + 2\Phi\left(\sqrt{\gamma}(k - \theta_0)\right)$$

Formally, define the expected net payoff of buying entrant's product over buying the incumbent's product as $v(\bar{\theta}, k)$.

$$v(\bar{\theta}, k) = p_I - p_E + 1 + c\bar{\theta} - 2\Phi\left(\sqrt{\gamma}(k - \theta_0)\right) \tag{3.4}$$

The consumer will be indifferent between the two products when $\bar{\theta} = k$. Then, the value (s) of k which satisfies $v(k, k) = p_I - p_E + 1 + ck - 2\Phi\left(\sqrt{\gamma}(k - \theta_0)\right) = 0$ is the cutoff value (s) associated with the symmetric strategy aforementioned. Existence of such k's follows by the intermediate value theorem using the fact that for any $p_I - p_E$, $v(k, k) < 0$ for k small enough and $v(k, k) > 0$ for k large enough and continuity of v. Figure 3.1 shows $v(k, k)$ for 2 different levels of γ, fixing the prices and the parameters. As Figure 3.1 shows, depending on the value
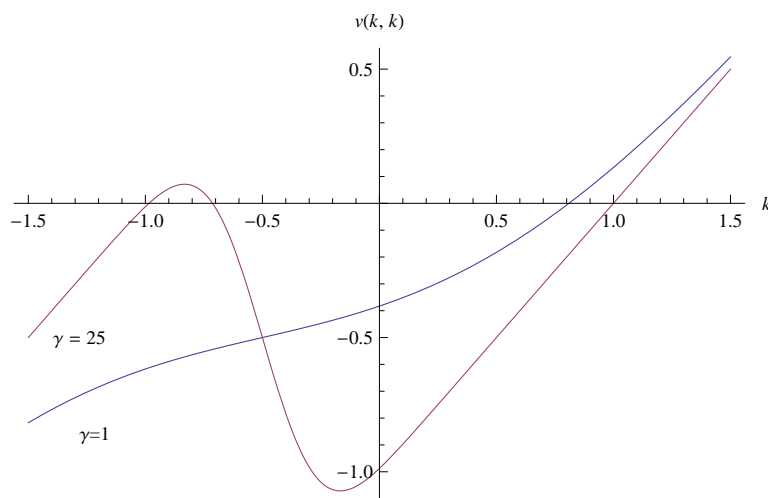

Figure 3.1: Consumer's Best Response Correspondence

15

of γ, there may exist multiple induced game equilibria. The equilibrium cutoff levels are values where the curves hit the $x-axis$. If the private signal is precise enough (γ is small), then there exists a unique equilibrium. The next proposition gives sufficiency condition for uniqueness.

Proposition 2. *The following statements hold.*

i) For any price difference $p_I - p_E$, if $\gamma < \frac{\pi c^2}{2}$, then there exists a unique k^* for which $v(k^*, k^*) = 0$ holds.

ii) For any γ, c such that $\gamma \geq \frac{\pi c^2}{2}$, There exist price differences $p_I - p_E$ where there exist multiple induced game equilibria. Specifically, there exist 3 equilibria iff $h(k_1) < p_I - p_E < h(k_2)$ where $k_1 < k_2$ are roots of $h'(k) = 0$ and h is defined as $h(k) = 2\Phi\left(\sqrt{\gamma}(k - \theta_0)\right) - 1 - ck$

Figure 3.2 shows a multiple equilibria case with $\gamma = 4$, for a fixed set of price difference and parameter values.   The induced game has a unique or three equilibria[3] depending on
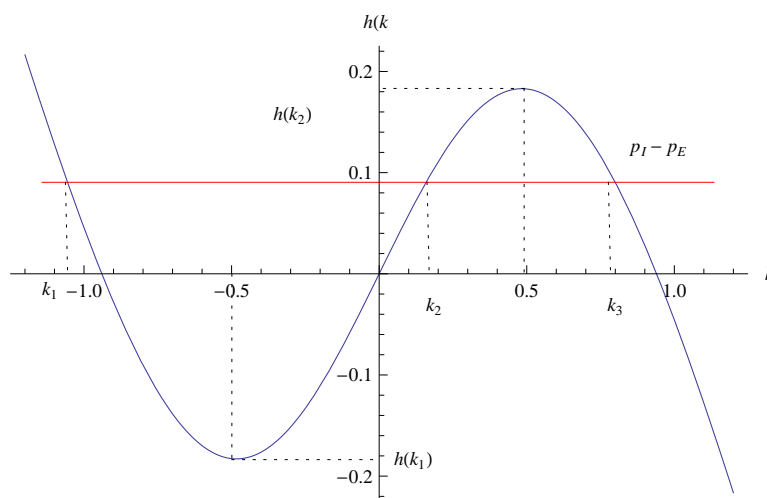

Figure 3.2: Multiple Induced Game Equilibria

the parameter level γ. In the next two sections, I analyze them separately along with some comparative statics with respect to γ.

---

[3] See Appendix for 2 equilibria case

Multiple Induced Game Equilibria

Proposition 3. *If there exist three induced game equilibria at a price difference $p_I - p_E$ for a given γ_0, then there exist three induced game equilibria for any $\gamma > \gamma_0$.*

Proposition 3 shows the importance of the relative precision of public and private signals in coordination problems. Carlsson and Van Damme (1993) showed that even a small uncertainty about payoffs is enough to solve the multiple equilibria problem in coordination games. But, in order to reach a unique equilibrium, they assumed an information structure where there is no other coordination device such as a public signal. However in most markets, consumers not only have private signals about the quality of the products, but they may also have noisy public signals such as internet reviews, commercials or brand recognition. In my model, the induced game equilibrium depends on the relative precision of these signals. A very noisy public signal relative to the private signal (a small γ) causes the consumers to put more weight on the private signal when forming their expectations about the quality of the entrant's product. If the private signal is precise enough, then as Carlsson and Van Damme (1993) suggest, by iterated dominance the induced game will have a unique equilibrium cutoff. Conversely, a very precise public signal compared to the private signal generates possibilities of coordination at different products. Take the limiting case where $\tau = 0$, (γ will be very large). Consumers publicly observe the real quality of the entrant's product. They can coordinate on either of the firms. If the public signal is very noisy, the weight of the private signal in the expected quality of the entrant's product will be zero. Therefore, even if a very high private signal is observed, consumers may not switch to the entrant's product, or even if a very low private signal is observed, they may still switch to the entrant.

Unique Induced Game Equilibrium

In this section, I assume that there exists a unique equilibrium of the induced game, i.e. $\gamma < \frac{\pi c^2}{2}$.

Proposition 4. *If $\gamma < \frac{\pi c^2}{2}$, for given price difference $p_I - p_E$, more people buy from the incumbent as γ increases, if initially $k > \theta_0$.*

17

Proposition 4 states that as the precision of the public signal increases, consumers value the public signal more than the private signal (they put more weight on it). Therefore, even if they receive a very high private signal, they may still decide to stay with the incumbent. Since the public signal is very accurate compared to the private signal, consumers who received a very high private signal believe that other consumers will receive lower private signals and not switch.

For example, assume the public signal is 1 and consumers decide to switch if they expect the quality to be greater than 2. In this case, a consumer who believes that the quality is 3 is going to buy from the entrant. Now, suppose they learn that the public signal is more precise than they think (τ is smaller). Now, the same consumer may decide to stay with the incumbent since right now, he believes that the public signal is more probable. Figure 3.3 shows the equilibrium cut-off k for different levels of γ.

If initially $k > \theta_0$, there is a private signal x associated with k such that $x > k > \theta_0$. As γ increases, the private signal becomes less precise compared to public signal. Therefore, consumers put higher weight to the public signal. Consider consumer j which observed $x_j > k > \theta_0$ and decide to buy E's product since he expects the quality to be higher than k, as γ increases, his expectation will be lower and he may buy I's product. As γ increases, consumers believe that more people will buy from incumbent since demand for I's product is $\Phi\left(\sqrt{\gamma}(k - \theta_0)\right)$. The level of k which makes the consumer indifferent between I and E will be higher [4].

3.2.2 Strategic Pricing Decision

This section analyzes the price competition of the firms. Since the underlying coordination problem of the induced game may generate multiple equilibria, the full game may have a multiple equilibria issue. I start with the profit maximization problem of the firms and without

---

[4] The equilibrium condition in the induced game of the no-network externality model is: $k = \frac{p_I - p_E}{c}$. As γ increases, k does not change. I sells more since the expected value of the quality decreases. Therefore demand to I's product will increase: $\Phi\left(\sqrt{\gamma}(k - \theta_0)\right)$
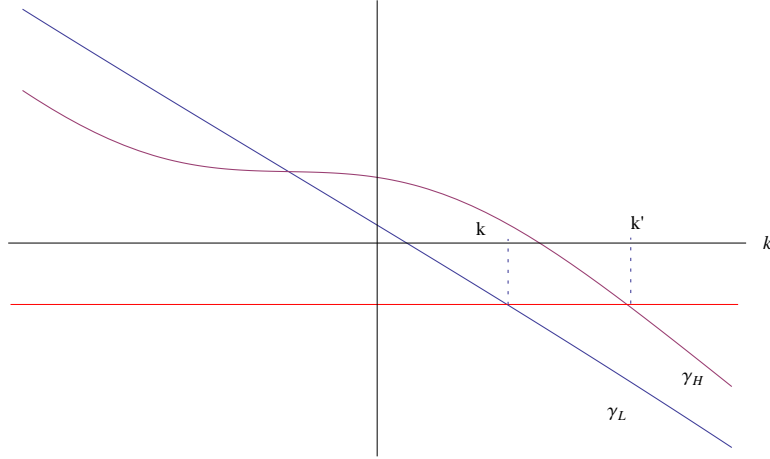
Figure 3.3: Induced Game Equilibrium for Different γ Levels

making any further assumption about their beliefs, I characterize conditions for profit maximization. Once I provide the sufficient condition for uniqueness, in later subsections, I will make further assumptions about consumers' beliefs.

Both E and I maximize their expected profit subject to the belief that the expected quality of E is $\bar{\theta} \sim N(\theta_0, \tau)$.

Proposition 5. *The conditions characterizing the equilibrium prices as a function of the cutoff k are*

$$1 - \Phi\left(\frac{k - \theta_0}{\tau}\right) = p_E \frac{\phi\left(\frac{k - \theta_0}{\tau}\right)}{\tau[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)]} \tag{3.5}$$

$$\Phi\left(\frac{k - \theta_0}{\tau}\right) = p_I \frac{\phi\left(\frac{k - \theta_0}{\tau}\right)}{\tau[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)]} \tag{3.6}$$

where k is a function of the prices.

For non-negative prices to exist, I need to assume that $c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right) > 0$ holds at the equilibrium. This condition also ensures that the law of demand is satisfied. By the

condition for induced game equilibrium, the effect of price changes on demand is

$$\frac{\partial k}{\partial p_E} = \frac{1}{c - 2\sqrt{\gamma}\phi(\sqrt{\gamma}(k - \theta_0))} > 0$$

$$\frac{\partial k}{\partial p_I} = -\frac{1}{c - 2\sqrt{\gamma}\phi(\sqrt{\gamma}(k - \theta_0))} < 0$$

Combining Equations 3.4, 3.5 and 3.6 gives the following equilibrium condition in the full game :

$$F(k^*, \gamma) = 2\Phi\left(\sqrt{\gamma}(k^* - \theta_0)\right) - ck^* - 1 - A(k^*)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k^* - \theta_0)\right)] = 0 \qquad (3.7)$$

where $A(k) = \frac{\tau\left[2\Phi\left(\frac{k-\theta_0}{\tau}\right)-1\right]}{\phi\left(\frac{k-\theta_0}{\tau}\right)}$.

Theorem 1 (Sufficiency Condition for Uniqueness). *There exists a unique k^* and price pair (p^*_E, p_I^*) satisfying the equilibrium condition and equation 3.5 and 3.6 if $\gamma < \frac{\pi}{2}\frac{c^2}{2}$,*

We need to keep in mind that this unique equilibrium condition is only a sufficiency condition therefore, it does not include all possible unique equilibrium cases.

Figure 3.4 shows the equilibrium cutoff level k^* for different levels of γ The equilibrium
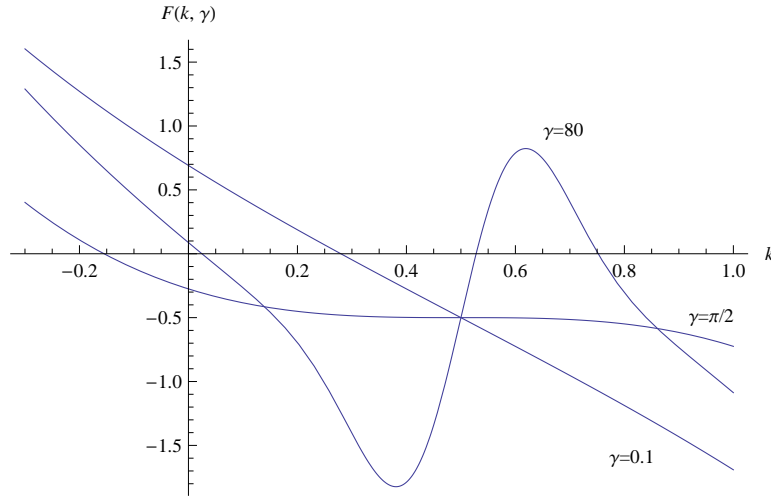

Figure 3.4: Equilibrium cutoff level for Different γ Levels

cutoff levels are values where the curves hit the $x - axis$.

The following lemmas will be helpful in comparing equilibrium prices and in conducting

comparative statics on the market shares.

Lemma 1. *For all γ, the lines $F(k, \gamma)$ intersect at the point $(\theta_0, -c\theta_0)$.*

Lemma 2. *For any γ, $F(k, \gamma)$ is symmetric around the point $(\theta_0, -c\theta_0)$*

The symmetry property[5] of the equilibrium condition ensures that analyzing one of many possible equilibria is enough to infer characteristics of other equilibria. In the next subsection, I study the unique equilibrium case using the sufficiency condition for uniqueness.

Unique Full Game Equilibrium

In this section, I assume that there exists unique solution to equation 3.5 and 3.6

Firms, by choosing prices, determines the equilibrium level of k. Since for any price level there exists unique k which satisfies the induced game equilibrium condition, there exists a unique k which satisfies the full equilibrium condition.

Proposition 6. *If the public signal (θ_0) is smaller (larger) than 0, I will have higher (lower) market share than E at the equilibrium.*

If the public signal is smaller than 0, more than half of the consumers believe that E's quality is lower than I's quality. Therefore, even without taking the network externality into account, more than half of the consumers will buy from I. Moreover, they will expect higher network benefits if they buy from I which will make I's market share higher.

The next proposition states the effect of changes in parameter values on equilibrium statistics.

Proposition 7 (Comparative Statics)**.** *For a given γ, if the incumbent has a higher market share than the incumbent, then as γ increases,*

   i) The k increases and the incumbent sells more.

   ii) The ratio of equilibrium prices, $\frac{p_E}{p_I}$, decreases.

---

[5] The proof to Lemma 2 includes a wide treatment of the symmetry property.

iii) The incumbent charges higher prices and gets higher profits.

A higher γ implies relatively less precise private signal compared to the public signal. If I has higher market share for a given γ, then the public signal is lower than 0. As γ increases, private signal becomes less informative. The consumers put more weight to the public signal. Then more consumers will buy from I because the public signal indicates that the incumbent's quality is higher than the entrant's quality. On the other hand, a higher market share for the incumbent implies that $p_E < p_I$[6]. The entrant must undercut I on price in order to compete with the network advantage I has.

One of the main foci of this paper is to evaluate the effect of network externalities on the strategic pricing decision of firms. Hence it is useful to define a benchmark model without network externalities.

No-Network Externalities Model

In this benchmark model, I have the same informational assumptions as the main model, except that there exists no network externalities, hence consumers do not need to form expectations about other consumers' strategies. In this case, the preferences and the profits will be the following:

$$U(E,\theta) = -p_E + c\theta \qquad\qquad U(I) = -p_I$$

$$\Pi_E = \lambda p_E \qquad\qquad \Pi_I = (1-\lambda)p_I$$

Proposition 8. *The equilibrium cutoff level, k_{nn} satisfies the following*

$$A(k_{nn}) + k_{nn} = 0$$

where $A(k) = \dfrac{\tau[2\Phi\left(\frac{k-\theta_0}{\tau}\right)-1]}{\phi\left(\frac{k-\theta_0}{\tau}\right)}$

In this benchmark model, the equilibrium condition does not depend on the variance of

---

[6] By firms' problem, $\dfrac{p_E}{p_I} = \dfrac{1-\Phi\left(\frac{k-\theta_0}{\tau}\right)}{\Phi\left(\frac{k-\theta_0}{\tau}\right)} < 1$ if $k > \theta_0$

the private signal. Consumers do not value network externalities therefore, the updating process of consumers' belief does not play a role in the consumption decision of the consumers. Naturally, expected quality depends on the private signal, but its determination depends only on the public signal.

Lemma 3. *There exists a unique equilibrium price pair (p_I, p_E) and cutoff level k_{nn}. They satisfy the following:*

i) *If $\theta_0 < 0$ then, $\theta_0 < k_{nn} < 0$ and I has a higher equilibrium market share than E and $p_E < p_I$[7].*

ii) *If $\theta_0 > 0$ then, $0 < k_{nn} < \theta_0$ and E has a higher equilibrium market share than I and $p_I < p_E$.*

Without network externalities, quality is the main determinant of the market structure. If I's quality is ex-ante expected to be better than E's quality, I will charge higher prices and get higher market share compared to E.

3.3 Comparison with The No-Network Externality Model

In order to isolate the effect of network externalities on the market structure, I compare the benchmark model with the network-externality model in terms of market shares and prices.

Proposition 9. *If $\theta_0 < 0$, the incumbent has a higher market share in the externality model than the no-externality model. The reverse is true if $\theta_0 > 0$.*

The next figure shows the comparison of the equilibrium cutoff levels for $\theta_0 < 0$. Network externalities have a magnifying effect on the advantage because of the difference in quality. For instance if E's quality is expected to be lower than I's quality, even without taking network effects into account, I's market share will be higher. Network externalities, which is higher on I's side, creates an extra incentive for consumers to buy from I. Therefore, network externalities increase the asymmetry in the market structure.

---

[7] If $p_E = p_I$, $k_{nn} = \theta_0$ and the firms share the market equally.
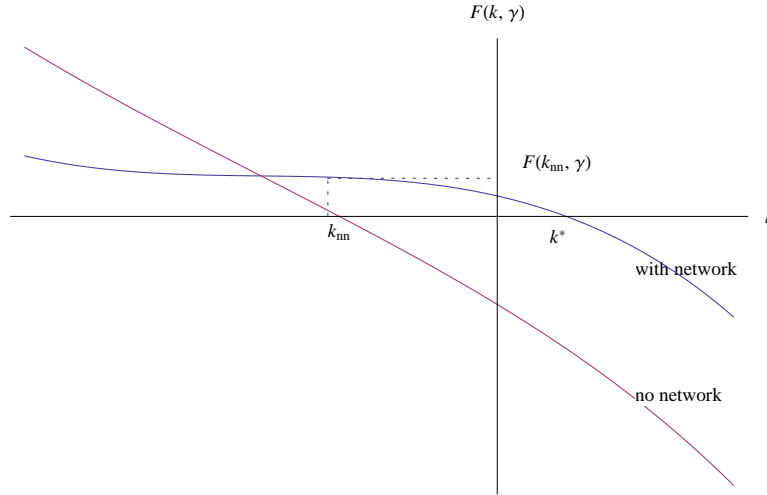
23

Figure 3.5: Comparison of The Market Shares

If there were no network externalities, an equilibrium cut-off level of $k^* = 0$ can be obtained if and only if the public signal is extremely noisy ($\tau \to \infty$). But in model with network externalities, even if τ is very large, depending on the public signal, there is be an advantage for the firm with better quality. In other words, if the public signal is higher than the incumbent's quality, existence of network externalities assures that k^* is higher than the one in the model without network externalities. The incumbent will sell more.

This proposition states that if public information signals that the entrant's quality is better than the incumbent's quality, then the consumers are willing to switch at expected qualities lower than the incumbent's quality. A slightly higher public signal can increase demand to the entrant's product a lot. This is due to the existence of network externalities.

Multiple Full Game Equilibria

In the case of a unique induced game equilibrium, I do not have multiple full game equilibria problem. Therefore, solving firms' profit maximization problem directly gives the equilibrium prices. For other cases where multiple equilibria may arise, I assume an additional belief system for firms to prevent issues arising from multiple equilibria. A possible multiple equilibria issue (if it exists) arises because of the coordination problem in the subgame. Each firm sets its price based on the belief that the consumers will choose the "k" that the firm wants. But

actually, for any price pair, there exists 3 best response k's for consumers. In the following section, I assume that the consumers are pessimistic about other consumers' switching decision. Meaning that out of the three possibilities, each consumer believes that other consumers will choose the one which gives the entrant the lowest profit (i.e. lowest market share). Therefore in the following section, if γ is such that there may be multiple full game equilibria, consumers are assumed to be pessimistic about other consumers' switching decision.

Firms, by choosing price level, determines the equilibrium level of k. The equilibrium selection problem arises in multiple induced game equilibria cases. The next figure illustrates this coordination issue for the entrant.   The blue line shows the best response function of
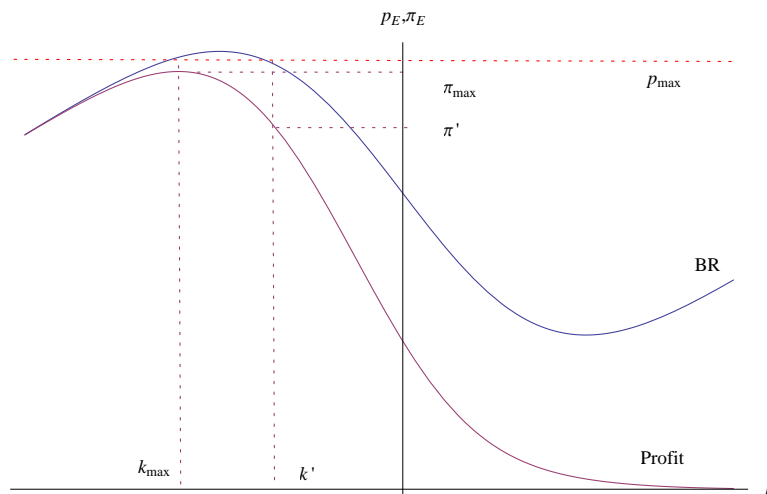

Figure 3.6: Coordination Problem in Full Game

the consumers and the red line shows the the entrant's profit function. As a profit maximizer, a rational entrant may choose $p_{\max}$, expecting to get $\pi_{\max}$. But consumers may respond $p_{\max}$ by playing the switching strategy around $k_{\max}$ or k' . If they choose $k_{\max}$, E will get the maximum profit, $\pi_{\max}$. On the other hand, if they choose k', E's profit will be π'.

Theorem 2. *if there exist multiple equilibria for $\gamma = \gamma^*$, then there exist multiple equilibria for all $\gamma > \gamma^*$*

By the symmetry property of F, $\mid k_{\max} - \theta_0 \mid > \mid k' - \theta_0 \mid$ where $k_{\max} < k'$ are equilibrium values for a given parameter level γ. Using the FOC condition for the entrant, I know that the

entrant prefers the $k_{\max}$ to k' since he gets higher profit by selling more and charging more. Therefore, an interesting question is whether there are tools which make the equilibrium the one the entrant prefers. Advertising may be such a tool if it gives the entrant the opportunity of changing the noise structure. An advertisement level which generates a γ such that there exists a unique full game equilibrium, may make E better off. In the next section, I introduce a model of endogenous advertising.

3.4   Advertisement Decision

In this section, I assume that the entrant can undertake advertisement in order to increase the consumers' perception about his quality. Advertisement is assumed to be free and it increases the precision of the private signal.

The advertisement does not affect the public signal due to the definition of quality throughout the paper. Quality is the degree of match between the product and the consumers' preferences. Assuming that the advertisement increases the amount of information available to consumers, the private signal (either good or bad) will be more precise for the consumers. On the other hand, the public signal will be unaffected because the advertisement does not give any information about the degree of match to the firms. Following the smart phone example, an advertisement can be an informative newspaper article about the characteristics of the new phone. By reading this article, consumers receive more accurate information about the match of the product with their own preferences.

Instead of introducing a new variable for the advertisement, I assume that the entrant chooses the parameter level γ by determining the advertisement level along with the price level p_E. A higher advertisement level will result a lower level of γ since it increases the precision of the private signal.

Cases with a Unique Equilibrium

The following proposition indicates that if the public signal is greater than zero, then the entrant does not want to disturb a market with a unique equilibrium by undertaking advertisement.

Proposition 10. *For $\theta_0 > 0$, As advertisement increases, the entrant sells less if without advertisement $\gamma(k - \theta_0)^2 < 1$.[8] In other words, if the market shares are balanced[9].*

Since the advertisement increases the precision of the private signal, the consumers put more weight on the private signal compared to the public signal. In the case where $\theta_0 > 0$, this means that the consumers believe the bad, private signal more than the good, public signal. Therefore, their expected quality decreases and they switch harder than before.

Cases with Multiple Equilibria

Proposition 11. *For a given γ such that there exists multiple equilibria without advertising, if there exists an equilibrium with a positive level of advertising , then $(k^* - \theta_0) < 0$, where k^* is the equilibrium cutoff level.*

In the model with multiple equilibria, there are 2 equilibria $k's$: such that $k_1 < \theta_0 < k_2$. Using Proposition 11, $k^* < 0 < k_2$.[10] In other words; if consumers are pessimistic about the switching decision of other consumers, then undertaking advertisement generates higher market share for the entrant. Therefore, the entrant is better off with advertising than the worst equilibrium he can reach without advertising. Depending on the parameter levels, advertisement may be better than any equilibria he can reach without advertising. The next proposition characterizes these conditions.

---

[8] If $\gamma < \frac{\pi c^2}{2}$, the maximum value $\gamma(k - \theta_0)^2$ can take is smaller than 1 and it is at the point where γ approaches to $\frac{\pi c^2}{2}$.

[9] If the market shares are at most 1-standard deviation from the mean.

[10] Depending on the parameter γ and θ_0, k^* can be smaller or larger than k_1

Optimal level of advertisement

Proposition 12. *If $\theta_0 > 0$, the entrant prefers an advertisement level which generates a unique equilibrium.*
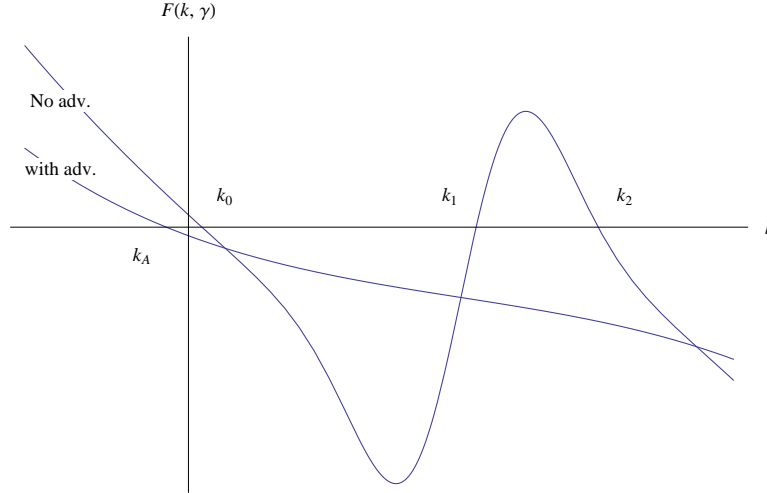

Figure 3.7: Multiple vs a Unique Equilibrium

The figure above shows the difference in market shares with and without advertisement. With advertising, the entrant receives a higher market share than any equilibria he can reach without advertising. Since, the firm with higher market share charges higher price, the entrant increases his profit by advertising.

Proposition 13. *If $\theta_0 > 0$ for some parameter values of γ such that there exist multiple equilibria without advertising, then undertaking advertisement generates higher market share to the entrant compared to the best multiple equilibria the entrant can reach. Specifically, the following is satisfied at the optimal level of advertisement*

$$\gamma(k - \theta_0)^2 = 1 + \frac{k - \theta_0}{A(k)}$$

By using the level of advertisement, the entrant may be able to eliminate the "bad" equilibrium. The equilibrium with advertisement may be better than the best equilibrium he can reach without advertisement.

28

3.5   Conclusion

In this paper, I analyze an imperfect information models of duopoly with network externalities on the demand side and a model of endogenous advertisement. Using a global games approach, I predict that there exists a unique equilibrium in the former if the private signal is relatively more precise compared to the public signal. The Incumbency advantage arises in the sense that in the equilibrium, the incumbent charges higher price and has a higher market share compared to the entrant in cases where the incumbent has an expected quality advantage to begin with. In order to isolate the effect of network externalities on the equilibrium statistics, I use a benchmark Bertrand model of differentiated types without network externalities. The comparison shows that the network externalities have a magnifying effect on the incumbency advantage. Incumbent's price and market share is higher in the model with network externalities.

In case where there exist multiple equilibria, I show that endogenizing the precision of the private signal may generate a unique equilibrium. If the entrant uses the level of advertisement to increase the precision of the private signal, I show that he can manipulate the game by setting a level of precision which generates the equilibrium he prefers. I show that for if the public signal is greater than zero, the entrant chooses to undertake advertisement in order to obtain a unique equilibrium rather than multiplicity.

Chapter 4

Dynamic Competition

4.1 The Model

This is a dynamic model of network externalities where consumers make a purchase decision in each period by choosing between an incumbent's and an entrant's product.

4.1.1 Firms

I consider a 2-period dynamic model of price competition between 2 firms, E and I. The entrant (E) sells a product of quality θ_t, in each period $t = 1, 2$ whereas the incumbent (I) sells the product of quality normalized to 0, in each period.[1]Firms maximize the discounted total profit.[2] They are assumed to have zero marginal costs. The profit functions of the firms are as follows.

$$\Pi_E = \lambda_1 p_{1E} + \delta \lambda_2 p_{2E}$$

$$\Pi_I = (1 - \lambda_1)p_{1I} + \delta(1 - \lambda_2)p_{2I}$$

where λ_t is the entrant's market share in period t, p_{tE} and p_{tI} are prices charged at t by E and I, respectively. δ is the discount factor for both firms.

Following Biglaiser and Cremer (2013), I assume that the firms are committed to provide a certain level of quality in both periods. Interpreting the prices as the physical level of quality, I therefore assume that the firms are committed to the second period prices, which is actually a "quality assurance pledge".

---

[1] If either firm has zero market share in the first period, the firm exits. But the information structure will prevent such a case.

[2] With some abuse of notation, I denote the entrant firm at $t = 1$ by E and I keep calling it the "entrant firm" in period 2 even though he has been in the market since the first period.

Firms set both period's prices at t=1 and commit to them. Biglaiser and Cremer (2013) interpret prices in a different way.[3] Prices can be thought as "negative quality" in the sense that committing to a price level means that the firm is committed to offer a certain level of quality in both periods.

4.1.2 Consumers

There is a continuum of consumers of size normalized to 1. Consumers make a binary purchase decision in each period to maximize the total discounted expected utility. The per-period utility functions are

$$U(E, \lambda_t) = -p_{tE} + c\theta_t + \lambda_t$$

$$U(I, \lambda_t) = -p_{tI} + (1 - \lambda_t)$$

Each consumer's utility is given by two components: $c\theta_t$ is the direct benefit from the consumption of E's product and λ_t is the network externality benefit arising from other consumers' consumption. Here, c is the marginal value of quality over externality for the consumers.

4.1.3 Information Structure

The quality of the entrant changes overtime. The firms and the consumers have imperfect information about both period's quality. Following the definition of quality, consumers are better informed than the firms about the quality of the entrant's product, since they are better informed about the degree of match between the product characteristics and their preferences. In order to capture this difference in information, I assume that the firms observe only the previous period's quality but the consumers observe both the previous period's quality and a private signal, different for each consumer. By observing previous period's quality, both the firms and the consumers learn the degree of match in the previous period. The firms use this information to estimate the degree of match in the current period. The consumers use this

---

[3] Biglaiser and Cremer, (2013), Value of Incumbency in Heterogenous Networks.

information too, but they also have private signal about the new product's degree of match with their preferences. Therefore, the consumers have a better estimate about the product quality as well as the resulting level of network externalities.

I assume that the quality of the entrant's product before the game starts is θ_0. The game starts at $t = 1$. The entrant's quality in each period evolves following a Gaussian random walk, $\theta_1 = \theta_0 + \mu_1$, where μ_1 is standard normal. At the end of the first period, the entrant's quality is revealed to all.[4] The second period's quality then, satisfies $\theta_2 = \theta_1 + \mu_2$ where μ_2 is standard normal. The private signal for a consumer i satisfies $x_{ti} = \theta_t + \epsilon_{ti}$ where $\epsilon_{ti} \sim N(0, \sigma^2)$ in each $t = 1, 2$.

4.1.4   Timing

In period 1, all observe the initial quality $\theta_1 = \theta_0 + \mu_1$, $\mu_1 \sim N(0, 1)$ and consumer i observes a private signal $x_{1i} = \theta_1 + \epsilon_{1i}$ where $\epsilon_{1i} \sim N(0, \sigma^2)$. After updating their beliefs about the quality and the magnitude of the network externality, firms set prices $(p_{tI}, p_{tE})_{t=1,2}$ for both periods.

Following Biglaiser and Cremer (2013), I assume that the firms are committed to provide a certain level of quality in both periods. Interpreting the prices as the physical level of quality, I therefore assume that the firms are committed to the second period prices, which is actually a "quality assurance pledge".

Consumers observe prices and choose between the entrant's and the incumbent's product. At the end of the first period, consumers observe market share. At $t = 2$, consumers observe a new private signal $x_{2i} = \theta_2 + \epsilon_{2i}$ about the new quality where $\epsilon_{2i} \sim N(0, \sigma^2)$.

The firms maximize their expected total profits from $t = 1$ and 2 by setting prices at $t = 1$. Consumers choose between the entrant and the incumbent. No consumer affects any other consumer's decision since there is continuum of agents and each consumer is of zero measure. Since there are no switching costs, the decision made at t=1 does not affect the purchase decision at $t = 2$. Therefore, maximizing the total utility is identical to maximizing

---

[4] Since the signals are normally distributed, when consumers observe the $t = 1$ market shares, they deduce the entrant's quality.

each period's utility separately.

4.2 Purchase Decision

After observing the previous period's quality[5] and the private signal, consumers update their beliefs about the quality of the product as well as the network externality associated with the market shares.

4.2.1 Information Updating

I assume that the consumers use a symmetric switching strategy around a cutoff level k_t. Consumer i believes that the consumer j has the following type of strategy in period t.

$$
s(\bar{\theta}) = \begin{cases} E & \text{if } \bar{\theta} \geq k_t \\ I & \text{if } \bar{\theta} < k_t \end{cases} \tag{4.1}
$$

Consumer i believes that consumer j will buy from the entrant if j expects the quality to be higher than k_t. Since consumers are myopic, they consider other consumers' current period's strategy. In other words, in the first (second) period, consumer i does not need to hold a belief about consumer j's second (first) period strategy. The expected market share of the entrant in period t is then $\lambda_t = prob(\bar{\theta}_t > k_t)$

After observing the private signal, i believes that j believes $\bar{\theta}_t = \frac{x_{tj} + \sigma^2 \theta_{t-1}}{1+\sigma^2}$ where $x_{tj} \sim N(\bar{\theta}_t, \frac{1\sigma^2}{1+\sigma^2} + \sigma^2)$. Therefore at the symmetric cutoff value where consumer i is indifferent between the two products, the expected market share of the entrant is

$$
\lambda_t = prob(\bar{\theta}_t > k_t) = 1 - \Phi[\frac{\sigma(k_t - \theta_{t-1})}{\sqrt{\frac{1}{1+\sigma^2} + 1}}]
$$

$$
\lambda_t = 1 - \Phi[\sqrt{\gamma}(k_t - \theta_{t-1})]
$$

where $\gamma = \sigma^2(\frac{1+\sigma^2}{2+\sigma^2})$ is a combined parameter of the precisions of the signals, decreasing in

---

[5] At $t = 2$, the first period's quality is a public signal about the second period's quality, satisfying $\theta_2 = \theta_1 + \mu_2$.

the precision of the private signal. Throughout the paper, I use this combined parameter to analyze the effect of relative precision of the signals on the equilibrium. For instance, a relatively small level of γ implies a more precise private signal than the public signal. The best response correspondence of the consumers at period t is then,

$$p_{tE} - p_{tI} = 1 + ck_t - 2\Phi[\sqrt{\gamma}(k_t - \theta_{t-1})]$$

Proposition 14. *There exists a unique BR for any given price pair* (p_{tI}, p_{tE}) *if* $\gamma < \frac{\pi c^2}{2}$

The right hand side of the best response correspondence of the consumers is the net expected utility of buying the entrant's product over the incumbent's product. As the cutoff value increases, less people will buy from the entrant, therefore the benefit from network externalities on the entrant will decrease. On the other hand a higher equilibrium cutoff value implies a higher expected quality which increases the benefit from the entrant's quality. Depending on the parameter level, either of the effects can dominate. The sufficiency condition above characterizes the parameter values where the effect of network externalities always outweighs the effect of increased expected quality. Technically in this interval, the right hand side of the best response correspondence is always decreasing, which generates a unique best response.

Note that the sufficiency condition is the same as the one in the static model. Even though the necessary condition for uniqueness depends on the public signal in the first period and on the realization of the quality at the end of the first period, the sufficiency conditions are independent of the period in which the decision is made.

4.3 Profit Maximization Problem

The entrant's and the incumbent's problems are similar. Therefore, I only analyze the entrant's problem.

The entrant makes his pricing decision based on the information he has at $t = 1$. He estimates the quality in both periods after observing the initial quality. He knows that the

second period's quality will be a normally distributed around θ_1. Even if he does not know the quality θ_1 when he sets the price, he has a belief about it. Therefore he uses the information he has to estimate the quality at $t = 1$ and then uses this new belief to estimate the quality at $t = 2$. The entrant maximizes the total expected profit, $\Pi_E = \lambda_1 p_{1E} + \delta \lambda_2 p_{2E}$.

Both the entrant and the incumbent believe that the consumers will use a symmetric switching strategy. Therefore, they believe that the consumers will buy from the entrant if they expect the entrant's quality to be higher than some cutoff level k_t, otherwise the consumers will buy from the incumbent.

4.3.1 Entrant's Belief About The Network Size at t=1

The entrant believes that the consumers will buy from him if their expected quality, $\bar{\theta}_1$ is greater than k_1. Therefore, the entrant expects a market share of λ_1 where

$$\lambda_1 = prob(\bar{\theta}_1 > k_1) = prob(\frac{x_j + \sigma^2 \theta_0}{1 + \sigma^2} > k_1)$$

After observing the previous period's quality, he believes that $x_{1j} = N(\theta_0, 1 + \sigma^2)$. Therefore the entrant expects a market share of size

$$\lambda_1 = 1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))$$

where $\gamma_{1F} = 1 + \sigma^2$

The entrant's expectation about the market share is different than the consumers', since consumers are better informed about the entrant's quality.

4.3.2 Entrant's Belief About The Network Size at t=2

After observing the previous period's quality, the entrant believes that the private signal that will be observed by the consumer in the next period is going to be $x_{2j} \sim N(\theta_0, 2 + \sigma^2)$ and $\theta_1 \sim N(\theta_0, 1)$. Therefore, he believes that the consumers believe that the expected quality is

$$\frac{x_{2j} + \sigma^2 \theta_1}{1 + \sigma^2} \sim N(\theta_0, (\frac{1}{1 + \sigma^2})^2 (2 + \sigma^2) + (\frac{\sigma^2}{1 + \sigma^2})^2)$$

35

Since he also believes that the consumers will use a symmetric switching strategy around k_2 in the second period, similar to the first period, he expects his market share to be λ_2 with

$$\lambda_2 = prob(\bar{\theta}_2 > k_2) = prob(\frac{x_{2j} + \sigma^2\theta_1}{1 + \sigma^2} > k_2)$$

$$\lambda_2 = 1 - \Phi[\frac{k_2 - \theta_0}{\sqrt{(\frac{1}{1+\sigma^2})^2(2 + \sigma^2) + (\frac{\sigma^2}{1+\sigma^2})^2}}]$$

$$\lambda_2 = 1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))$$

where $\gamma_{2F} = \frac{1}{(\frac{1}{1+\sigma^2})^2(2+\sigma^2)+(\frac{\sigma^2}{1+\sigma^2})^2}$

The entrant's expected market share in the first period is different than his expected market share in the second period because, while estimating the consumers' switching strategy in the second period, the entrant does not know the quality θ_1 (because this estimation is made in the first period) but he holds a belief about it. But while estimating the consumers' switching strategy in the first period, he knows θ_0.

Once he estimates both periods' market shares, the entrant solves

$$max_{k_1,k_2}\Pi_E = [1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]p_{1E} + \delta[1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))]]p_{2E}$$

where k_1 and k_2 satisfy

$$p_{1E} - p_{1I} = 1 + ck_1 - 2\Phi[\sqrt{\gamma}(k_1 - \theta_0)]$$

$$p_{2E} - p_{2I} = 1 + ck_2 - 2\int \Phi[\sqrt{\gamma}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1$$

When considering the consumers' choice at $t = 2$, the entrant believes that they will believe to be indifferent at k_2 where k_2 satisfies

$$p_{2E} - p_{2I} = 1 + ck_2 - 2\Phi[\sqrt{\gamma}(k_2 - \theta_1)]$$

But at $t = 1$, E does not know θ_1. He believes that $\theta_1 \sim N(\theta_0, 1)$. Therefore E believes

36

that the consumers believe k_2 satisfy:

$$p_{2E} - p_{2I} = 1 + ck_2 - 2\int \Phi[\sqrt{\gamma}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1$$

Before moving on with the full game, I want to compare price differences in 2 periods for a fixed cutoff level k. The following proposition shows that the firm with quality advantage in the first period, increases his price in the second period relative to the other firm.

Proposition 15. *If firms must choose the same prices in both periods with $p_I - p_E > -c\theta_0$, the entrant expects to sell more at $t = 2$ compared to $t = 1$.*

Denote that $p_I - p_E > -c\theta_0$ is actually the comparison of consumers' utility without taking the network externalities into consideration. The firms believe that the consumers pay p_E and get direct benefit from consumption of the entrant's product, $c\theta_0$. [6] Therefore, $-p_E + c\theta_0 > -p_I$ implies the cases where the firms expect the consumers to be better off by buying the entrant's product without considering the indirect benefit from the network. In such a case, the entrant expects to have higher market share than the incumbent. He expects to have even higher market share in the second period because he is less informed about the second period compared to the first period and assigns higher probabilities to the states where he has higher quality compared to the incumbent. The following proposition characterizes the equilibrium in full game.

Proposition 16. *Equilibrium in full game is characterized by the following conditions*

$$2\Phi[\sqrt{\gamma}(k_1 - \theta_0)] - 1 - ck_1 - A_1(k_1)[c - 2\sqrt{\gamma}\phi[\sqrt{\gamma}(k_1 - \theta_0)]] = 0$$

$$2\int \Phi[\sqrt{\gamma}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1 - 1 - ck_2 - A_2(k_2)[c - 2\sqrt{\gamma}\int \phi[\sqrt{\gamma}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1] = 0$$

---

[6]The expected quality of the entrant's product is θ_0. I need to use the firms' belief about the quality because the comparison reflects the belief of the firms about the utility received by the consumers.

where

$$A_1(k) = \frac{[2\Phi[\sqrt{\gamma_{1F}}(k - \theta_0))] - 1]}{\sqrt{\gamma_{1F}}\phi[\sqrt{\gamma_{1F}}(k - \theta_0))]}$$

$$A_2(k) = \frac{[2\Phi[\sqrt{\gamma_{2F}}(k - \theta_0)] - 1]}{\sqrt{\gamma_{2F}}\phi[\sqrt{\gamma_{2F}}(k - \theta_0))]}$$

Denote that the condition for the second period is different than the one for the first period in the sense that the second period's condition incorporates the ex-ante belief of the firms about the second period's quality.

Proposition 17. *In each period, the firm with the higher expected market share charges a higher price.*

For given prices, the firm with higher expected quality has higher expected market share. The firm with the lower expected quality has to compensate by charging a lower price.
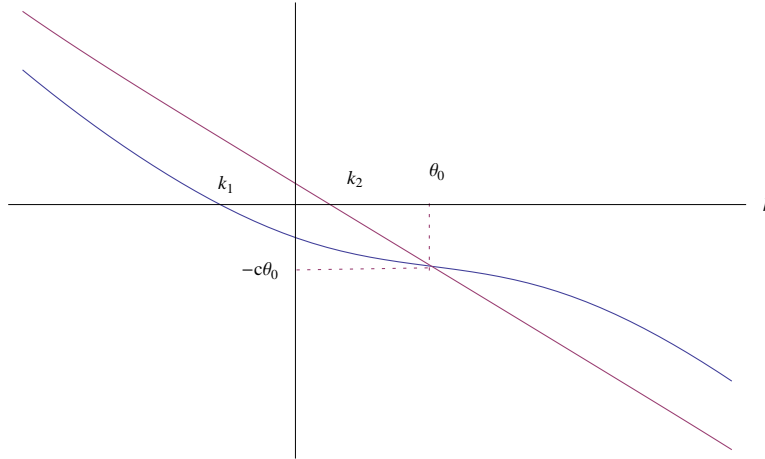

Figure 4.1: Equilibrium Cutoff Levels in Different Periods

Proposition 18. *The entrant expects to sell less in the second period if the consumers value the network externalities enough. Specifically, $k_2 > k_1$, if $c < \sqrt{\frac{24}{5\pi}}$ and $\theta_0 > 0$*

In other words, the entrant has a lower market share at $t = 2$ compared to the first period, if he has an expected quality advantage in the first period. The figure above shows the equilibrium cutoff levels in both periods. The opposite is true if the incumbent has an

expected quality advantage in the first period. The red line and the blue line illustrate the equilibrium condition at $t = 1$ and $t = 2$ respectively. If the expected quality of the entrant's product is higher than the incumbent's, in other words if $\theta_0 > 0$, the entrant expects to sell less in the second period than the first period.

4.4   Conclusion

I investigated the effects of coordination on the pricing strategy in a duopoly market with network externalities. I used a Global Games approach as an equilibrium refinement to obtain a unique equilibrium. I characterize the conditions for uniqueness in the dynamic game. In a setting where the firms set prices in the first period, I find that if the consumers value the network externalities enough, the entrant expects to sell less in the second period compared to the first period. In both periods, the firm with higher market share charges higher price and receives higher profit. Price commitment is a crucial assumption here, in the sense that the firms have to use their ex-ante (first period) beliefs about the second period's market share, while setting prices.

Appendix A

Appendix to Chapter 3

A.1 Proofs of Propositions, Lemmas and Theorems

Both the entrant and the incumbent believe that the consumers will use a symmetric switching strategy. Therefore, they believe that the consumers will buy from the entrant if they expect the entrant's quality to be higher than some cutoff level k_t, otherwise the consumers will buy from the incumbent.

Proof of Proposition 2. Use the properties of the normal distribution to prove the statement.

i) $v(k,k)$ is strictly increasing if $\gamma < \frac{\pi c^2}{2}$, therefore there exists a unique k^* for which $v(k^*, k^*) = 0$ holds.

Formally, $v'(k,k) = c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right) >$ implies $\frac{c}{2\sqrt{\gamma}} > \phi\left(\sqrt{\gamma}(k - \theta_0)\right) > \frac{1}{\sqrt{2\pi}}$ since the PDF of a normal distribution reaches its maximum value of $\frac{1}{\sqrt{2\pi}}$ at 0.

ii) Redefine the equilibrium condition in induced game as

$$p_I - p_E = 2\Phi\left(\sqrt{\gamma}(k^* - \theta_0)\right) - 1 - ck^* = h(k^*).$$

The function h has a local minimum at k_1 and a local maximum at k_2. Therefore, for any $p_I - p_E$ such that $h(k_1) < p_I - p_E < h(k_2)$, There exists three $k^{*'}s$ for which the equilibrium condition holds. Moreover, from the symmetry of the normal distribution,

$$k_1 - \theta_0 = -(k_2 - \theta_0)$$
$$\Phi\left(\sqrt{\gamma}(k_1 - \theta_0)\right) = 1 - \Phi\left(\sqrt{\gamma}(k_2 - \theta_0)\right)$$

Then, $h(k_1) < p_I - p_E < h(k_2)$ becomes;

$$2\Phi\left(\sqrt{\gamma}(k_1 - \theta_0)\right) - 1 - ck_1 < p_I - p_E < -[2\Phi\left(\sqrt{\gamma}(k_1 - \theta_0)\right) - 1] - ck_2$$

Proof of Proposition 3. Assume there exist three $k^{*'}s$ at price difference $p_I - p_E$ for a given γ_0. By Proposition 2, $h(k_1, \gamma_0) < p_I - p_E < h(k_2, \gamma_0)$. Denote that $h(k_1, \gamma) < h(k_1, \gamma_0)$ and $h(k_2, \gamma) > h(k_2, \gamma_0)$ for any $\gamma > \gamma_0$. Then, $h(k_1, \gamma_0) < p_I - p_E < h(k_2, \gamma_0)$ for all $\gamma > \gamma_0$. Finally, by Proposition 2, there exist three $k^{*'}s$ at any $\gamma > \gamma_0$.

Proof of Proposition 4. Using the equilibrium condition in induced game,

$$\frac{\partial k}{\partial \gamma} = -\frac{\gamma^{-1/2}(k - \theta_0)\phi\left(\sqrt{\gamma}(k - \theta_0)\right)}{2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right) - c} > 0 \ if \ k > \theta_0$$

Incumbent sells $\Phi\left(\sqrt{\gamma}(k - \theta_0)\right)$ which increases as γ increases.

Proof of Proposition 5. Firms' problem is

$$\max_{p_E} \Pi_E = \max_{p_E} \lambda p_E$$

$$\max_{p_I} \Pi_I = \max_{p_I} (1 - \lambda) p_I$$

where $\lambda = 1 - \Phi\left(\frac{k - \theta_0}{\tau}\right)$ Using IFT on Equation 2,

$$\frac{\partial k}{\partial p_E} = \frac{1}{c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)}$$

$$\frac{\partial k}{\partial p_I} = -\frac{1}{c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)}$$

Denote that $\frac{\partial k}{\partial p_E}$ and $\frac{\partial k}{\partial p_I}$ are not continuous at k_1 and k_2 defined before as the local minimum and the local maximum of the RHS of Equation 2. Therefore k is not universally continuous in prices. But, it is locally continuous on the interval $(-\infty, k_1) \cup (k_1, k_2) \cup (k_2, \infty)$. FOCs give Equation 3.

Proof of Theorem 1. $\lim_{k \to -\infty} F(k, \gamma) = \infty$ and $\lim_{k \to \infty} F(k, \gamma) = -\infty$. It is enough to prove that $F_k(k, \gamma) < 0$ for all $\gamma < \frac{\pi c^2}{2}$.

$$F_k(k, \gamma) = 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right) - c - A'(k)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)] + A(k)[2\gamma\phi'\left(\sqrt{\gamma}(k - \theta_0)\right)]$$

using

$$\phi'(x) = -x\phi(x)$$

$$A'(k) = 2 + \frac{k - \theta_0}{\sigma^2} A(k)$$

and the fact that $A(k) > 0$ for all $k > \theta_0$ and $A(k) < 0$ for all $k < \theta_0$,

$$F_k(k, \gamma) = [2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right) - c]$$

$$-A'(k)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)] - A(k)[2\gamma^{3/2}(k - \theta_0)\phi\left(\sqrt{\gamma}((k - \theta_0))\right] < 0$$

for all k if $\gamma < \frac{\pi c^2}{2}$.

Then, by IVT there exists a unique k^* such that $F(k^*, \gamma) = 0$. Finally, Equation 2 along with the sufficiency condition for unique equilibrium in induced game prove that there exists unique price pair (p_E^*, p_I^*)

Proof of Lemma 1. For any γ;

$$F(\theta_0, \gamma) = 2\Phi\left(\sqrt{\gamma}(\theta_0 - \theta_0)\right) - c\theta_0 - 1 - A(\theta_0)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(\theta_0 - \theta_0)\right)] = -c\theta_0$$

Proof of Lemma 2. The following figure gives an overall idea about the symmetry of F, For symmetry to be satisfied, the following condition must hold:

$$F(2\theta_0 - x) - (-c\theta_0) = (-c\theta_0) - F(x)$$

for any x

$$F(2\theta_0 - x) = 2\Phi\left(\sqrt{\gamma}(\theta_0 - x)\right) - c(2\theta_0 - x) - 1 - A(2\theta_0 - x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(\theta_0 - x)\right)]$$
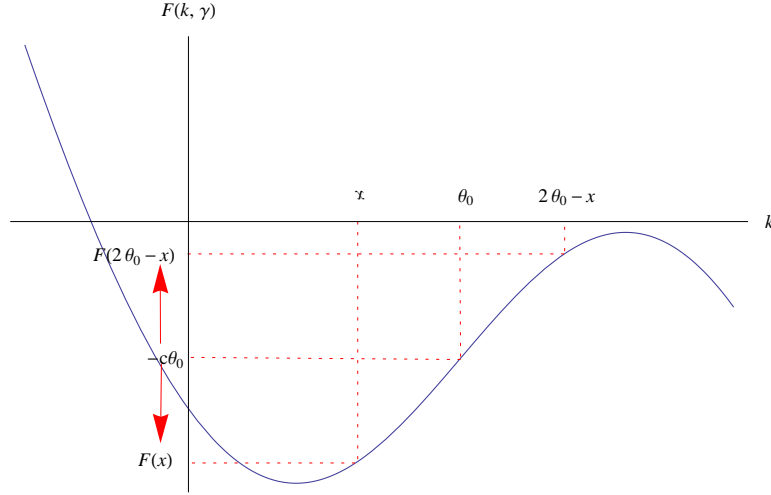
Figure A.1: Symmetry of The Equilibrium Condition in Full Game

Using $\Phi(x) = 1 - \Phi(x)$ and $\phi(x) = \phi(-x)$

$$F(2\theta_0 - x) = 2[1 - \Phi\left(\sqrt{\gamma}(x - \theta_0)\right)] - c(2\theta_0 - x) - 1 - A(2\theta_0 - x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta_0)\right)]$$

$$1 - 2\Phi\left(\sqrt{\gamma}(x - \theta_0)\right) + cx - 2c\theta_0 - A(2\theta_0 - x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta 0)\right)]$$

$$-[2\Phi\left(\sqrt{\gamma}(x - \theta_0)\right) - cx - 1] - 2c\theta_0 - A(2\theta_0 - x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta_0)\right)]$$

$$-F(x) - A(x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta_0)\right)] - A(2\theta_0 - x)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta_0)\right)] - 2c\theta_0$$

$$-F(x) - [c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(x - \theta_0)\right)][A(x) + A(2\theta_0 - x)] - 2c\theta_0$$

Since $A(x) = -A(2\theta_0 - x)$

$$F(2\theta_0 - x) + F(x) = -2c\theta$$

Therefore, the symmetry condition holds.

Proof of Proposition 6. Since we are still assuming that there exists a unique equilibrium, $\gamma < \frac{\pi c^2}{2}$. Equation 4 is strictly decreasing in k and $F(\theta_0, \gamma) = -c\theta_0 < 0$ Since,

$$\lim_{k \to -\infty} F(k, \gamma) = +\infty, k^* < \theta_0$$

Then, I's market share is $\Phi\left(\frac{k^* - \theta_0}{\tau}\right) > 0.5$

43

Proof of Proposition 7. Check the following for a complete treatment of the properties mentioned in the proposition.

i) Using IFT, $\frac{\partial k}{\partial \gamma} = -\frac{\partial F/\partial \gamma}{\partial F/\partial k}$ Which is equal to

$$\frac{-\gamma^{-1/2}\phi[\sqrt{\gamma}(k-\theta_0)] \atop (k-\theta_0) + A(k)[1 - \gamma(k-\theta_0)^2]}{-[1+A'(k)][c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k-\theta_0)\right)] - A(k)(k-\theta_0)[2\gamma^{3/2}\phi\left(\sqrt{\gamma}(k-\theta_0)\right)]}$$

where $\partial F/\partial k < 0$ by the unique equilibrium assumption. Check: $\partial F/\partial \gamma > 0$ Assume $\partial F/\partial \gamma > 0$ at some $k' > \theta_0$ then,

$$(k'-\theta_0) + A(k')[1 - \gamma(k'-\theta_0)^2] > 0$$
$$\gamma < \frac{1}{A(k')(k'-\theta_0)} + \frac{1}{(k'-\theta_0)^2}$$

then for all k s.t $\theta_0 < k < k'$, $\partial F/\partial \gamma > 0$ Therefore, it is enough to find a k' large enough so that our k falls within the range above.

ii) By firms' problem:
$$\frac{p_E}{p_I} = \frac{1 - \Phi\left(\frac{k-\theta_0}{\tau}\right)}{\Phi\left(\frac{k-\theta_0}{\tau}\right)}$$

As γ increases, k increases, then $\frac{p_E}{p_I}$ decreases.

iii) By I's problem:
$$\Phi\left(\frac{k-\theta_0}{\tau}\right) = p_I^* \frac{\phi\left(\frac{k-\theta_0}{\tau}\right)}{\tau[c - 2\sqrt{\gamma}\phi(\sqrt{\gamma}(k-\theta_0))]}$$

I need to prove that $[c - 2\sqrt{\gamma}\phi(\sqrt{\gamma}(k-\theta_0))]$ increases as γ increases. Define $Q(\gamma, k) = c - 2\sqrt{\gamma}\phi(\sqrt{\gamma}(k-\theta_0))$

$$\frac{dQ(\gamma, k)}{d\gamma} = \frac{\partial Q(\gamma, k)}{\partial \gamma} + \frac{\partial Q(\gamma, k)}{\partial k}\frac{dk}{d\gamma}$$
$$\frac{dQ(\gamma, k)}{d\gamma} = \sqrt{\gamma}\phi(\sqrt{\gamma}(k-\theta_0))[-1 + \gamma(k-\theta_0)^2] + 2\gamma^{3/2}(k-\theta_0)\phi(\sqrt{\gamma}(k-\theta_0))\frac{dk}{d\gamma} > 0$$

44

Proof of Proposition 8. At the equilibrium cutoff k^*, consumers will be indifferent between firms, given prices.

$$k^* = \frac{p_E - p_I}{c}$$

Firms' maximization problems are the same as in the main model.

$$\max_{p_E} \Pi_E = \max_{p_E} \lambda p_E$$

$$\max_{p_I} \Pi_I = \max_{p_I} (1 - \lambda) p_I$$

where $\lambda = 1 - \Phi\left(\frac{k - \theta_0}{\tau}\right)$ except that:

$$\frac{\partial k^*}{\partial p_E} = \frac{1}{c}$$

$$\frac{\partial k}{\partial p_I} = -\frac{1}{c}$$

Therefore, the equilibrium conditions are:

$$1 - \Phi\left(\frac{k - \theta_0}{\tau}\right) = p_E \frac{1}{\tau c} \phi\left(\frac{k - \theta_0}{\tau}\right)$$

$$\Phi\left(\frac{k - \theta_0}{\tau}\right) = p_I \frac{1}{\tau c} \phi\left(\frac{k - \theta_0}{\tau}\right)$$

$$k = \frac{p_E - p_I}{c}$$

Combining the equilibrium conditions below gives:

$$A(k) + k = 0$$

Proof of Proposition 9. Let the equilibrium cutoff level in the benchmark model is k_{nn}, Then

$$A(k_{nn}) + k_{nn} = 0$$

Rearranging the equilibrium condition in the externalities model,

$$F(k_{nn}, \gamma) = 2\Phi\left(\sqrt{\gamma}(k_{nn} - \theta_0)\right) - 1 - c[k_{nn} + A(k_{nn})] + 2A(k_{nn})\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k_{nn} - \theta_0)\right)]$$

$$2\Phi\left(\sqrt{\gamma}(k_{nn} - \theta_0)\right) - 1 + 2A(k_{nn})\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k_{nn} - \theta_0)\right)]$$

If $\theta_0 > 0$, by Lemma 3, $k_{nn} < \theta_0$. Then, $F(k_{nn}, \gamma) < 0$. Since F is decreasing in k, $k^* < k_{nn}$ where $F(k^*, \gamma) = 0$

Proof of Lemma 3. Since $A(k)$ is strictly increasing in k, there is a unique k_{nn} which satisfies the equilibrium equation.

i) Assume by contradiction, $\theta_0 < 0$ and $k_{nn} < \theta_0$, then $A(k_{nn}) < 0$ and $k_{nn} < 0$ which imply $A(k_{nn}) + k_{nn} < 0$. Therefore, if $\theta_0 < 0$, $k_{nn} > \theta_0$.

ii) Similarly, assume $\theta_0 > 0$ and $k_{nn} > \theta_0$, Then $A(k_{nn}) > 0$ and $k_{nn} > 0$ which imply $A(k_{nn}) + k_{nn} > 0$. Therefore, if $\theta_0 > 0$, $k_{nn} > \theta_0$.

Market shares are followed by the equilibrium conditions.

Proof of Theorem 1. If there exists multiple best response k's in induced game, then there will be multiple k's in full game.[1]

Proof of Proposition 10.

$$\frac{\partial k}{\partial \gamma} = -\frac{(k - \theta_0) + A(k)[1 - \gamma(k - \theta_0)^2]}{[1 + A'(k)][c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)] - 2\gamma^{3/2}A(k)(k - \theta_0)\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)}$$

Since $\theta_0 > 0$ implies $k < \theta_0$, the denominator is smaller than zero. A sufficient condition for $\frac{\partial k}{\partial \gamma} < 0$ is then $[1 - \gamma(k - \theta_0)^2] > 0$ or,

$$\gamma(k - \theta_0)^2 < 1$$

---

[1] See the proof in induced game.

Therefore, if $\gamma(k - \theta_0)^2 < 1$, as advertisement increases: γ decreases and k increases which means that the entrant's market share decreases.

Proof of Proposition 11. Assume that a denotes the level of advertisement. Using the consumer's problem,

$$\frac{\partial k}{\partial A} = \frac{a^{-1}\sqrt{\frac{\gamma}{a}}(k - \theta_0)\phi\left(\sqrt{\frac{\gamma}{a}}(k - \theta_0)\right)}{c - 2\sqrt{\frac{\gamma}{a}}\phi\left(\sqrt{\frac{\gamma}{a}}(k - \theta_0)\right)} > 0 \ if(k - \theta_0) > 0$$

But the FOC from the maximization problem of the entrant, $\frac{\partial \Pi_E}{\partial a} = 0$, cannot be satisfied unless $(k - \theta_0) \leq 0$

Proof of Proposition 12. Let's start with a level of γ which generates multiple equilibria. Then, $\gamma(k - \theta_0)^2 > 1$ But, then $\frac{\partial k}{\partial \gamma} > 0$, which means that increasing the advertisement (decreasing γ) will increase the entrant's market share. Therefore, it is optimal to decrease γ.

Proof of Proposition 13. Suppose that k^* is a unique cutoff level generated by an advertisement choice if,

$$F(k^*, \gamma) = 2\Phi\left(\sqrt{\gamma}(k^* - \theta_0)\right) - ck^* - 1 - A(k^*)[c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k^* - \theta_0)\right)] > 0$$

for some γ, then the proof is complete.

$$\lim_{\gamma \to \infty} F(k^*, \gamma) = -ck^* - 1 - A(k^*)c > 0$$

There exists such a k^* level since, $k^* + A(k^*) < -\frac{1}{c}$ holds for many k^*. $F(k^*, \gamma) > 0$ and $F(\theta_0, \gamma) < 0$ along with $\theta_0 > 0$ imply that there exists at least one $k > k^*$ such that $F(k, \gamma) > 0$. Then the advertisement level generates a higher market share.

$$\frac{\partial k}{\partial \gamma} = -\frac{(k - \theta_0) + A(k)[1 - \gamma(k - \theta_0)^2]}{[1 + A'(k)][c - 2\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)] - 2\gamma^{3/2}A(k)(k - \theta_0)\sqrt{\gamma}\phi\left(\sqrt{\gamma}(k - \theta_0)\right)} = 0$$

$$(k - \theta_0) + A(k)[1 - \gamma(k - \theta_0)^2] = 0$$

$$\gamma(k - \theta_0)^2 = 1 + \frac{k - \theta_0}{A(k)}$$

Appendix B

Appendix to Chapter 4

B.1 Proofs of Propositions, Lemmas and Theorems

Firms observe a noisy public signal θ_0 about E's quality. If we interpret the quality as the degree of match between the product and the consumer, then it is reasonable to assume that E has incomplete information about his own quality. The public signal is of the form: $\theta_0 = \theta_E + \eta$ where η is normally distributed with mean 0 and standard deviation τ.

Proof of Proposition 14. Define $g(k) = -1 - ck_t + 2\Phi[\sqrt{\gamma}(k_t - \theta_{t-1})]$ The function g is strictly decreasing if $-c + 2\sqrt{\gamma}\phi[\sqrt{\gamma}(k_t - \theta_{t-1})] < 0$ The maximum value the PDF of a normal distribution can take is $\frac{1}{\sqrt{2\pi}}$ Therefore a sufficient condition for $g'(k) < 0$ is $\sqrt{2\gamma}\frac{1}{\sqrt{\pi}} < c$ or,

$$\gamma < \frac{\pi c^2}{2}$$

Proof of Proposition 15. Using Lemma 5, for any $k < \theta_0$, $\Phi[\quad \sqrt{\gamma}(k-\theta_0)] - \int \Phi[\sqrt{\gamma}(k-\theta_1)]\phi[\theta_1 - \theta_0]d\theta_1 < 0$, Therefore $[-1 - ck + 2\int \Phi[\sqrt{\gamma}(k-\theta_1)]\phi[\theta_1 - \theta_0]d\theta_1] - [-1 - ck + 2\Phi[\sqrt{\gamma}(k-\theta_0)]] > 0$ Then, the best response function of the consumers at $t = 1$ lies below the one at $t = 2$. This implies that for any fixed price level $p_I - p_E > -c\theta_0$, $k_2 < k_1$.

Proof of Proposition 16. E solves: $max_{k_1, k_2}\Pi_E = [1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]]p_{1E}(k_1) + \delta[1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0)]]p_{2E}(k_2)$ The partial derivatives that are used in the maximization problem are

$$\frac{\partial p_{1E}}{\partial k_1} = c - 2\sqrt{\gamma}\phi[\sqrt{\gamma}(k_1 - \theta_0)]$$
$$\frac{\partial p_{2E}}{\partial k_2} = c - 2\sqrt{\gamma}\int \phi[\sqrt{\gamma}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1$$

using the partial derivatives, the FOCs are:

$$[1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]][c - 2\sqrt{\bar{\gamma}}\phi[\sqrt{\bar{\gamma}}(k_1 - \theta_0)]] = \sqrt{\gamma_{1F}}\phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]p_{1E}$$

$$[1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0)]][c - 2\sqrt{\bar{\gamma}} \int \phi[\sqrt{\bar{\gamma}}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1] = \sqrt{\gamma_{2F}}\phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))]p_{2E}$$

Similarly, the FOCs of the incumbent are

$$\Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))][c - 2\sqrt{\bar{\gamma}}\phi[\sqrt{\bar{\gamma}}(k_1 - \theta_0)]] = \sqrt{\gamma_{1F}}\phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]p_{1I}$$

$$\Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0)][c - 2\sqrt{\bar{\gamma}} \int \phi[\sqrt{\bar{\gamma}}(k_2 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1] = \sqrt{\gamma_{2F}}\phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))]p_{2I}$$

Proof of Proposition 17. Solving the FOCs simultaneously,

$$\frac{p_{1E}}{p_{1I}} = \frac{1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]}{\Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]} > 1 if 1 - \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))] > \Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))]$$

$$\Phi[\sqrt{\gamma_{1F}}(k_1 - \theta_0))] < 0.5$$

In other words, if the incumbent's market share is smaller than the entrant's market share at $t = 1$. $\frac{p_{2E}}{p_{2I}} = \frac{1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0)]}{\Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))]} > 1$ if $1 - \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0)] > \Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))]$ $\Phi[\sqrt{\gamma_{2F}}(k_2 - \theta_0))] < 0.5$ In other words, if the incumbent's market share is smaller than the entrant's market share $t = 2$.

Proof of Proposition 18. Assume that k_1 is the equilibrium cutoff level at $t = 1$. Then,

$$2\Phi[\sqrt{\bar{\gamma}}(k_1 - \theta_0)] - 1 - ck_1 - A_1(k_1)[c - 2\sqrt{\bar{\gamma}}\phi[\sqrt{\bar{\gamma}}(k_1 - \theta_0)]] = 0$$

Since $\theta_0 > 0$,

$$2 \int \Phi[\sqrt{\bar{\gamma}}(k_1 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1 - 1 - ck_1 - A_1(k_1)[c - 2\sqrt{\bar{\gamma}}\phi[\sqrt{\bar{\gamma}}(k_1 - \theta_0)]] > 0$$

if the consumers value network externalities enough,[1]

$$2\int \Phi[\sqrt{\gamma}(k_1-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 - 1 - ck_1 - A_2(k_1)[c-2\sqrt{\gamma}\int \phi[\sqrt{\gamma}(k_1-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1] > 0$$

Since the function characterizing the equilibrium condition at $t=2$ is decreasing for all k, the equilibrium cutoff level $k_2 > k_1$.

B.2   Additional Lemmas Used in Proofs

Lemma 4. *The following comparison about the precision parameters holds.*

   i) $\gamma_{2F} < \gamma_{1F}$

   ii) $\gamma < \gamma_{1F}$

Proof. Denote that the comparison of these parameters is crucial in the uniqueness of the equilibrium.

   i) $\frac{\gamma_{2F}}{\gamma_{1F}} = \frac{(1+\sigma^2)}{(2+\sigma^2)+\sigma^4} < 1$

   ii) $\gamma = \sigma^2(\frac{1+\sigma^2}{2+\sigma^2}) < \sigma^2 < \sigma^2 + 1 = \gamma_{1F}$

Lemma 5. $\Phi[\sqrt{\gamma}(k-\theta_0)] < (>) \int \Phi[\sqrt{\gamma}(k-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1$ *for all* $k < (>)\theta_0$

Proof. If $k = \theta_0$ $\int \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 = \int_{-\infty}^{\theta_0} \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 + \int_{\theta_0}^{\infty} \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1$ $\int \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 = \int_{-\infty}^{\theta_0} \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 + \int_{\theta_0}^{\infty} \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1$ $\int \Phi[\sqrt{\gamma}(\theta_0-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1 = \int_{-\infty}^{\theta_0} [[\Phi[\sqrt{\gamma}(\theta_0-\theta_1)]+1-\Phi[-\sqrt{\gamma}(\theta_0-\theta_1)]]\phi[\theta_1-\theta_0]d\theta_1$ $\int \Phi[\sqrt{\gamma}(\theta_0 - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1 = \int_{-\infty}^{\theta_0} \phi[\theta_1 - \theta_0]d\theta_1 = 0.5$ Since $\Phi[\sqrt{\gamma}(\theta_0 - \theta_0)] = 0.5$, $\Phi[\sqrt{\gamma}(k-\theta_0)] = \int \Phi[\sqrt{\gamma}(k-\theta_1)]\phi[\theta_1-\theta_0]d\theta_1$ at $k = \theta_0$ If $k < \theta_0$, when calculating $\int \Phi[\sqrt{\gamma}(k-\theta_1)]\phi[\theta_1 - \theta_0]d\theta_1$, the integral expression assigns higher weights to higher values. Therefore

---

[1] Check Remark 2 for a detailed explanation.

$\int \Phi[\sqrt{\gamma}(k - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1 > \Phi[\sqrt{\gamma}(k - \theta_0)]$ Similar argument holds if $k > \theta_0$. Check the following figure for an intuitive representation. The red line shows $\Phi[\sqrt{\gamma}(k - \theta_1)]$ and blue line
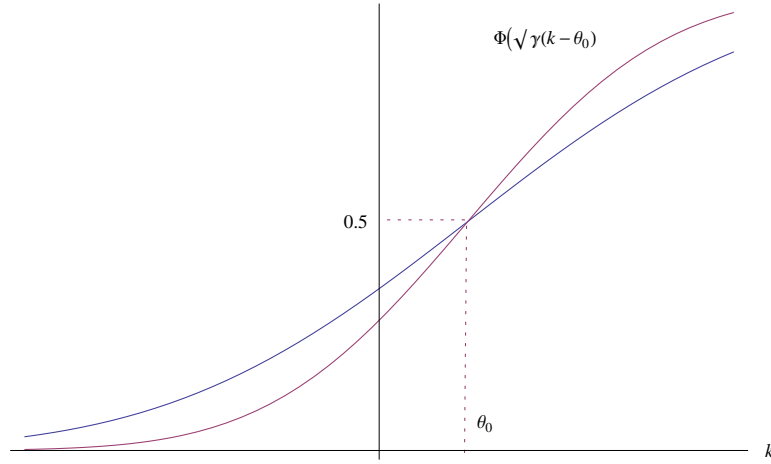

Figure B.1: Comparison of The Precision Parameters

shows $\int \Phi[\sqrt{\gamma}(k - \theta_1)]\phi[\theta_1 - \theta_0]d\theta_1$ for different values of k

Lemma 6. $A_1(k) < A_2(k)$ for all k, if $\gamma_{2F} > \frac{1}{(k - \theta_0)^2}$

Proof. Check the derivative of A with respect to γ.

B.3 Remarks About Parameters

Remark 1. $\frac{\partial \gamma_{2F}}{\partial \sigma} > 0$ *if $\sigma^2 < 3$ Remember that in order to have uniqueness, I need the private signal to be more precise than the public signal. Therefore this assumption is plausible.*

Remark 2. *The comparison of the two following expressions is crucial in determining how realistic the uniqueness assumption is.*

$$\gamma = \sigma^2\left(\frac{1 + \sigma^2}{2 + \sigma^2}\right) < \frac{\pi c^2}{2}$$

$$\sigma^2 < 3$$

If $\frac{24}{5\pi c^2} > 1$, then $\gamma < \frac{\pi c^2}{2}$ is more restrictive. Since I already assume that it holds for uniqueness, I do not need to make an extra assumption about σ^2 while proving Proposition 18. I just

need to assume that the marginal value of quality over externality is small enough, or in other words consumers value the network externalities enough.

Bibliography

Ahdieh, R. B. (2002). Making Markets: Network Effects and the Role of Law i n the Creation of Strong Securities Markets. S. Cal. L. Rev., 76, 277.

Argenziano, R. (2008). Differentiated networks: equilibrium and efficiency. RAND Journal of Economics, RAND Corporation, vol. 39(3), 747-769

Bensaid, B., and Lesne, J. P. (1996). Dynamic monopoly pricing with network externali-ties. International Journal of Industrial Organization, 14(6), 837-855.

Biglaiser, G.,and Cremer, J.(2013) Value of Incumbency in Heterogenous Networks.

Cabral, L., Salant, D. J., and Woroch, G. A. (1999). Monopoly pricing with network externalities. International Journal of Industrial Organization, 17(2), 199-214.

Cabral, L. (2011). Dynamic price competition with network effects. The Review of Economic Studies, 78(1), 83-111.

Carlsson, H., and Van Damme, E. (1993). Global games and equilibrium selection. Econo-metrica: Journal of the Econometric Society, 989-1018.

Chamley, C. (1999). Coordinating regime switches. The Quarterly Journal of Economics, 114(3), 869-905.

Chassang, S. (2008). Uniform selection in global games. Journal of Economic Theory, 139(1), 222-241.

Chassang, S. (2010). Fear of miscoordination and the robustness of cooperation in dynamic global games with exit. Econometrica, 78(3), 973-1006.

DeGroot, M. H. (2005). Optimal statistical decisions (Vol. 82). Wiley-Interscience.

Doganoglu, T. (2003). Dynamic price competition with consumption externalities. net-nomics, 5(1), 43-69.

Economides, N. (1996). Network externalities, complementarities, and invitations to enter. European Journal of Political Economy, 12(2), 211-233.

Farrell, J., and Shapiro, C. (1988). Dynamic competition with switching costs. The RAND Journal of Economics, 123-137.

Farrell, J., & Klemperer, P. (2007). Coordination and lock-in: Competition with switching costs and network effects. Handbook of industrial organization, 3, 1967-2072.

Federal Communications Commission. (2011). 15th Mobile Wireless Competition Report. Commercial Mobile Radio Services (CMRS) Competition Reports.

Fudenberg, D., & Tirole, J. (2000). Pricing a network good to deter entry. The Journal of Industrial Economics, 48(4), 373-390.

Giannitsarou, C., and Toxvaerd, F. (2007). Recursive global games.

Heidhues, P., and Melissas, N. (2006). Equilibria i n a dynamic global game: the role of cohort effects. Economic Theory, 28(3), 531-557.

Hellwig, C. (2002). Public information, private information, and the multiplicity of equi-libria in coordination games. Journal of Economic Theory, 107(2), 191-222.

Hotelling, H. (1990). Stability in competition (pp. 50-63). Springer New York.

Katz, M. L., & Shapiro, C. (1985). Network externalities, competition, and compatibility. The American economic review, 75(3), 424-440.

Kov´ac, E., and Steiner, J. (2008). Reversibility in dynamic coordination problems. CERGE-EI Working Paper Series, (374).

Lee, R. (2002). The future of securities exchanges. Brookings-Wharton Papers on Finan-cial Services, 2002(1), 1-33.

Mertens, J. F., & Zamir, S. (1985). Formulation of Bayesian analysis for games with incomplete information. International Journal of Game Theory, 14(1), 1-29.

Morris, S., and Shin, H. (2001). Global games: theory and applications.

Morris, S., & Shin, H. S. (2003). Global games: theory and applications. ECONOMETRIC SOCIETY MONOGRAPHS, 35, 56-114.

Ramos, S. B., & von Thadden, E. L. (2003). Stock exchange competition in a simple model of capital market equilibrium (No. rp109). International Center for Financial