MAKING THE INVISIBLE VISIBLE: AN EXAMINATION OF AFRICAN AMERICAN STUDENTS’ STRATEGY USE DURING MATHEMATICAL PROBLEM SOLVING

Crystal Antoinette Hill

A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the School of Education

Chapel Hill
2008

Approved by:
Advisor: Carol E. Malloy
Reader: Judith Meece
Reader: George Noblit
Reader: Olof Steinthorsdottir
Reader: James Trier
ABSTRACT

CRYSTAL ANTOINETTE HILL: Making the Invisible Visible: An Examination of African American Students’ Strategy Use During Mathematical Problem Solving
(Under the direction of Carol E. Malloy)

By focusing on the voices of African American mathematics learners, this dissertation seeks to address a gap in the problem solving and gender literature. Using a preexisting data set from the Mathematical Identity and Development Project (MIDDLE), I used mixed methods to provide a rich description of African American students as mathematical problem solvers and mathematics learners through the examination of students’ strategy use on items assessing rational number and measurement concepts and student responses from mathematical autobiographies and student interviews. In this study, components of Schoenfeld (1985, 2006) and Polya’s (1957) frameworks for successful problem solving was the tool of analysis.

Eight major conclusions emerged from the study. Two conclusions address strategy use and success and nonsuccess during problem solving two address students’ understanding, two address gender differences, and two address the profiles of student learners. A major finding in the analysis of students’ profiles and interviews showed that males and females described an understanding of mathematics as doing mathematics, but the majority of females discussed the importance of helping others understand while the majority of males discussed applying their knowledge to real world situations. African American males and females employed similar strategies when problem solving and there were no gender differences in students’ demonstration of conceptual understanding, but small differences in
strategy use. Success on problems did not always indicate students having a procedural or conceptual understanding of the concepts assessed in the conceptual understanding items and nonsuccess on a conceptual understanding items did not always mean students lacked understanding.
DEDICATION

To the I AM THAT I AM

and

In Loving Memory of Ara Jacobs
(A Great Pillar of Strength)
ACKNOWLEDGMENTS

My academic journey has been one of many trials and triumphs; this is a journey I could not have made alone. I would first like to thank my Lord and Savior Jesus Christ, because without Him none of this would be possible. Jesus is truly the “Best Thing” that ever happened to me. Secondly, I would like to thank my mother and father, Hazel and James Hill. I thank my mother for her prayers, encouragement, numerous trips to cook and look after me, and for being an example of what it means to persevere against all odds. I thank my father for being there when I needed him most. To Pastor P.O. and Barbara Rodgers and members of DaySpring Christian Center, thank you for your prayers, financial support, and for always being by my side encouraging me to press forward. I would like to thank Pastor Patrick L. Wooden Sr., First Lady Pamela Wooden, and members of the Upper Room Church of God in Christ for their prayers and support.

I am extremely grateful to the members of my committee: Dr. Judith Meece, Dr. Carol Malloy, Dr. George Noblit, Dr. Olof Steinthorsdottir, and Dr. Jim Trier. I benefited tremendously from their encouragement, advice and support. A former UNC-School of Education Alum stated that I had the “Dream Team” for my committee and I must say that I agree. Thank you Dr. Meece for pushing me in the area of my quantitative analyses and providing me the support I needed to understand repeated measures MANCOVA. Dr. Noblit, thank you for giving of your time to help me figure out the best explain how I coded my data. You provided invaluable qualitative support. Dr. Olof Steinthorsdottir, thank you
for meeting to discuss rational number concepts and for the meals you provided during our meetings. I would also like to thank you for making it possible for me to visit your beautiful home country of Iceland to work on my dissertation this past summer. Dr. Trier, thank you for nurturing my cultural studies side and helping me to find ways to merge mathematics education and cultural studies.

I would especially like to thank Dr. Carol Malloy who has been so much more than an academic advisor and mentor; she has been a mother and a friend. Dr. Malloy, thank you for your words of encouragement and affirmation when I began to doubt myself. Thank you for cheering me on when I really wanted to quit. When you were my M.A.T advisor, I mentioned that I wanted to be you one day and you have done a tremendous job in shaping and molding me into a wonderful mathematics educator. I could never repay you for all that you have done and have been in my life. Hopefully, I can make a small repayment to you as I mentor students the way you have mentored me. So to the “Dream Team” I say thank you again. As I move forward in my new academic journey, every task I put my hands towards completing will contain a small portion of each of you. You have all played an important role in my completing this doctoral journey and preparing for what lies ahead.

I wish to acknowledge Dr. Henry Frierson, the Research Education Support Program (RES) and Tracey Joseph, a former RES program coordinator, for giving me the opportunity to experience research while I was an undergraduate student, which motivated me to pursue my graduate degrees. I thank Dr. Lynda Stone and members of the School of Education administrative staff for all your words of encouragement and the help you provided along the way. I would also like to thank Iris Weiss and the staff at Horizon Research Incorporated for granting me unique opportunities to grow professionally and for all your supportive and
helpful words. I would also like to thank my mentors and friends BJ Berent, Beth Newman, and Sue Shepanek. BJ I would like to thank you for being there during good and tough times and opening your home to me. Beth thank you for the breakfast meetings to check and to give me encouragement and Sue thank you for always letting me know that your home was open for me to visit whenever I needed to get away.

I am forever grateful to my entire family, especially my sisters and my nieces and nephews who would often sit and listen to me practice job talks and other presentations, and my wonderful network of friends. It would take another dissertation to name and personally thank all of you; therefore I am not able to do that but please know that I appreciate and am forever indebted to all of you. I would like to acknowledge some friends who have been instrumental during the dissertation process. I would like to acknowledge Shante’ Martin and Erica Green for giving of their time, prayers, words of encouragement, money, and many, many meals to help me through this process. To Erica McMillan who, though she was in another state, would call to say “Chrissy, I am so proud of you.” and her calls were so timely. I thank Shameeka Latham and Cecil Outlaw who made many calls of encouragement. To my prayer partners Aiko Bethea and Monica Gibbs, I could not have finished this course without our early morning prayer. To Dr. Jan Yow and Dr. Mark Ellis, thank you for sharing your words of wisdom. To my fellow MIDDLE co-workers both former and current, thank you for all your supportive words and sound advice as we worked hard in the MIDDLE office also known as “The Bat Cave”. I would especially like to thank Spike Peterson for his quantitative analysis support. To member and co-founder of the Dissertation Anonymous Group (DAG), Daniella Cook, I could not have made it without you. Thank you for praying with me, traveling to the beach for writing trips, encouraging
and supporting me through some not so happy times, celebrating with me during the happy times, reading and critiquing my work, providing great meals, and introducing me to wonderful new restaurants. I would like to thank Jason Mendez for his help with formatting and introducing DAG to “New York Style”. I would also like to thank an honorary member of DAG, Brad Hinson, who unselfishly gave of his time, talent, and monies to help both Daniella and I through this dissertation process. Thank you for fixing computers, recovering lost documents, formatting tables, figures, and appendices, providing meals, and just being there as a source of support. I would like to thank Renee Sterling, Howie Machtinger, Brad Hinson, Daniella Cook, and Stephanie Hawkins for sitting through my practice dissertation defense and providing invaluable feedback.

To all of you who have helped me make it through this doctoral journey, thank you from the bottom of my heart. God truly blessed me with a wonderful support network and I will never forget the things you all have done for me. So as I close these acknowledgements, I want to again say thank you.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES</th>
<th>xiv</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF FIGURES</td>
<td>xv</td>
</tr>
</tbody>
</table>

**Chapter**

1. INTRODUCTION ................................................................. 1
   - Rationale ................................................................. 1
   - Purpose of Study ...................................................... 2
   - Contribution to the Literature .................................... 4

2. REVIEW OF LITERATURE ..................................................... 5
   - Problem Solving ...................................................... 5
     - What is Problem Solving? ........................................ 5
     - The Nature of Problem Solving ............................... 7
     - Problem Solving Strategies .................................... 8
     - The Process of Problem Solving ............................. 9
     - Characteristics of Problem Solvers ....................... 11
   - African American Student Learner ............................ 13
   - African American Culture ........................................ 14
   - African American Learning ....................................... 16
   - Gender Differences in Mathematics and Cognition ........ 17
   - Cognition ................................................................. 19
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Numbers and Percentages of Students at Different Grade Levels</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>Conceptual Framework Codes and Descriptions</td>
<td>49</td>
</tr>
<tr>
<td>3</td>
<td>Successful Strategies Categories and Subcategories</td>
<td>51</td>
</tr>
<tr>
<td>4</td>
<td>Unsuccessful Strategies Categories and Subcategories</td>
<td>52</td>
</tr>
<tr>
<td>5</td>
<td>Categories and Subcategories for Changes in Strategy Use and</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Demonstration of Conceptual Understanding Over Time</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Codes for Level II Student Interviews and Autobiographies</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>Spring Conceptual Understanding Score Distribution</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>Correlation Between Students’ Conceptual Understanding</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Scores and Fifth Grade End-of-Grade Scores</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Description of Conceptual Understanding Items</td>
<td>43</td>
</tr>
<tr>
<td>2</td>
<td>Descriptions of Successful Strategies for Conceptual Understanding Item 6-3</td>
<td>66</td>
</tr>
<tr>
<td>3</td>
<td>Descriptions of Unsuccessful Strategies for Conceptual Understanding Item 6-3</td>
<td>69</td>
</tr>
<tr>
<td>4</td>
<td>Descriptions of Successful Strategies for Conceptual Understanding Item 7-7</td>
<td>73</td>
</tr>
<tr>
<td>5</td>
<td>Descriptions of Unsuccessful Strategies for Conceptual Understanding Item 7-7</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>Special Case 4026</td>
<td>78</td>
</tr>
<tr>
<td>7</td>
<td>Descriptions of Successful Strategies for Conceptual Understanding Item 8-3</td>
<td>82</td>
</tr>
<tr>
<td>8</td>
<td>Descriptions of Unsuccessful Strategies for Conceptual Understanding Item 8-3</td>
<td>84</td>
</tr>
<tr>
<td>9</td>
<td>Special Cases for 8-3</td>
<td>86</td>
</tr>
<tr>
<td>10</td>
<td>Special Cases Work Samples</td>
<td>103</td>
</tr>
<tr>
<td>11</td>
<td>Ambiguous Student Solutions</td>
<td>112</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION

Rationale

I undertook this project as a means to legitimize the mathematical thinking of African American students. My “desire to be a megaphone for these [students’] voices” —a medium through which to amplify their stories as mathematical learners” (Jay, 2006, p.52), reflects my commitment to exploring and sharing the mathematical experiences of African American students. This dissertation is about increasing the voices of African American mathematics learners in the problem solving and gender literature because it is dominated by the voices of the mainstream White population. Malloy and Jones (1998) found studies that focus on mathematical problem solving have traditionally examined white students and typically do not report for African American students. Thus, there is little research addressing African American students’ mathematical learning or mathematical problem solving (Malloy & Jones, 1998). Kahle and Meece (1994) contend that since the majority of the conclusions reached in the gender literature are based on predominantly White samples, it is questionable as to whether these findings can be generalized to African American students. Research (Malloy & Jones, 1998; Kahle & Meece, 1994) has identified a gap in the literature and therefore the need for specifically focusing on African American students in the gender and problem solving literature. Problem solving is an important tool in students’ development as
mathematics learners, and successful problem solving is necessary for the development of mathematical understanding (NCTM, 2000). Understanding how African American students problem solve could assist educators in better serving this often underserved population.

Although a study by the National Assessment of Educational Progress (2003) showed African American students’ average mathematics scores increased by 13 points in 4th grade and 8 points in 8th grade, there was still a statistically significantly difference observed between the performance of African American and White students. Increasing the voices of African American students as mathematical learners could provide educators with valuable information about how to further help students develop as mathematical problem solvers and learners, thus result in their increased mathematical understanding.

Purpose of Study

The purpose of this study is two-fold. First, this study provides a description of African American students as mathematical problem solvers and mathematics learners through the examination of students’ strategy use on items assessing rational number and measurement concepts and student responses from mathematical autobiographies and student interviews. I examined students’ work to categorize students as successful or unsuccessful. Successful students were those who reached a correct numerical solution and unsuccessful students were those who could not reach a correct numerical solution. After categorizing students as successful and unsuccessful, I examined students’ work to not only categorize and describe their strategy use but also identify students’ common errors, misconceptions, level of conceptual understanding for each item, and determined if gender differences emerged from this examination. Profiles of student learners were built from student interviews and mathematical autobiography responses addressing the way African American
students describe what it means to understand mathematics, how they best learn and prefer to
learn mathematics, and how their teachers help them to understand mathematics better.

Second, this study addressed major gaps in the problem solving and gender literature
concerning African American students as mathematical learners. Similar to the problem
solving literature, the majority of the gender research was based on predominantly White
student samples. Also, within gender research, if ethnicity is a variable in a study, very few
comparisons within ethnic groups were conducted. The mainstream gender literature reveals
gender differences between White males and females, but these differences may look
different among African American students. In order to address gaps within the gender
literature, this study examines whether gender differences exist between students’
demonstration of conceptual understanding and the strategies they employ during problem
solving. Fennema and Peterson (1985), using a predominately White sample, hypothesized
that females are less autonomous learners than males. A comparison between African
American males and females was conducted to confirm or refute this hypothesis. It is my
hope that the results will help promote the successful development of African American
students as problem solvers and mathematical learners. The questions guiding my research
are

1) What strategies do African American students employ during mathematical problem
   solving?

2) Do differences exist between African American male and female’s strategy use
during mathematical problem solving and the student’s demonstration of conceptual
   understanding?
Contribution to the Literature

This study was built upon Malloy’s (1994) study of African American students and problem solving. Malloy (1994) analyzed how twenty-four African American students solve mathematics problems to build a foundation for understanding how these students successfully solve mathematics problems. In addition, Malloy’s (1994) work considered the influence of students’ culture on their students’ approach to mathematical problem solving. One limitation in her study was not comparing strategy use based on gender. Her student sample was not selected based on gender, leading to an uneven number of males and females and preventing an appropriate gender comparison. While building on the work of Malloy (1994), a comparison of strategy use by gender was incorporated into this study. Research has shown gender differences in student’s mathematics achievement and their strategy use during problem solving (Hembree; 1992; Kahle & Meece; 1994; Hyde, Fennema, & Lamon, 1990). This research inspired me to explore if these findings are consistent among African American students. It is important to remember that differences do not equate to deficits.

The results of this study add vital information to the literature concerning the mathematical thinking of African American students. This research revealed African American students’ strengths, weaknesses, and gaps in mathematical knowledge, common errors and misconceptions, and levels of conceptual understanding on assessment items. An understanding of how African American students think and reason during problem solving has major instructional implications because teachers’ knowledge of what students know and think impacts instructional practice and hence student learning (Cai, 2000). This information can inform teacher instruction and help to create strategies to assist students in avoiding common errors and developing misunderstandings about mathematical concepts.
CHAPTER 2
REVIEW OF LITERATURE

National Assessment of Educational Progress data continues to show that African American students are not reaching the expected levels in mathematics, but this group of students is often ignored in the mathematics literature. This chapter provides a general literature review related to problem solving, African American learners, conceptual and procedural understanding, and gender differences as they relate to mathematics and cognition. This literature review will end with an explanation of the conceptual and theoretical frameworks used for this study.

Problem Solving

*What is Problem Solving?*

Researchers and educators agree that problem solving is a crucial part of mathematical understanding and the substance of the mathematics discipline (Polya, 1981; NCTM, 2000). In their 1989 evaluation standards, the National Council of Teachers of Mathematics stated that problem solving should be the center of the mathematics curriculum. Problem solving has meant different things to different people, but it is often defined as a means by which an individual uses prior knowledge, skills, and understanding to find a solution to an unfamiliar situation (Mayer & Wittrock, 2006). According to Polya (1949), “To solve a problem is to find a way where no way is known off-hand, to find a way out of a difficulty, to find a way around and obstacle, to attain a desired end, that is not immediate
attainable, by appropriate means” (p. 1). In Mathematical Discovery, Polya (1981) states that problem solving is the “know-how” and “the doing” (pp. xi-xii) in mathematics. Polya suggests that a teacher’s main goal is the successful development of students as problem solvers. Problem solving is more than completing numerical operations; it is a way of thinking and involves using one’s knowledge to generate new knowledge and solve a problem that does not have an obvious solution (Malloy & Jones, 1998). According to Lester and Kahle (2003), “problem-solving is an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are non-routine” (p. 510). Successful problem solving involves using prior knowledge, experiences, and familiar representations to make sense and attempt to obtain new information about the problem solving situation. When the solver can not directly apply his or her mathematical knowledge to a problem-solving situation they must transform or find a new perspective on their knowledge so it can be applied to the problem situation. This process of transforming their knowledge and finding new perspective could lead to the development of new mathematical ideas (Lester & Kahle, 2003; Nunokawa, 2005).

Problem solving is a principle reason for studying mathematics because students are encouraged to construct their own mathematical knowledge, and this action can stimulate students’ desire to learn and appreciate mathematics. Problem solving enables students to construct and test mathematical theories, solve their own problems, and discuss these theories, leading to an increase in students’ mathematical understanding. Problem solving also provides a context through which students can connect complex mathematical concepts to real life situations (Malloy & Jones, 1998). Problem solving serves as a vehicle to introduce and develop an understanding of mathematical concepts while enhancing student
learning (Taplin, 2004). When students are allowed to grapple with situations that involve important mathematical concepts, they construct a clearer understanding of the mathematics (Kroll & Miller, 1993). When describing trends, key elements, and perspective within the research agenda of mathematical problem solving, Lesh and Zawojewski (2007) suggest that problem solving is important to the development of mathematical understanding. The National Council of Teachers of Mathematics (NCTM) states that “problem solving is an integral part of all mathematics learning” (2000, p 52).

**The Nature of Problem Solving**

Students’ problem solving abilities are affected by cognitive and non-cognitive factors. Lester, Garofalo, and Kroll (1989) place these factors into five categories: knowledge, control, affects, beliefs, and context. Knowledge represents the resources a problem solver uses to complete a task. Students must posses and activate the appropriate knowledge to successfully solve a problem. Knowledge may consist of rules, algorithms, definitions, schema knowledge (the ability to understand the structure of the problem), and non-routine algorithms. Students’ knowledge is often referred to as procedural and/or conceptual in nature, but the importance lies in students’ ability to organize and call upon their knowledge. The distinctions and relationships between conceptual and procedural knowledge are explored in following sections. Control entails the regulation of behavior during problem solving and deals with the decisions and courses of action used during problem solving. Students must explore the problem, plan a course of action to tackle the problem, implement a strategy, and reflect both on their strategy and the entire problem solving process. Belief systems held by students about mathematics, their emotions, and their perception of themselves as mathematics learners has an effect on students’ problem
solving performance. Since problem solving is not exclusively developed in school, contextual factors must be considered. Students bring informal problem solving strategies to school, and these informal strategies are developed within the students’ social and cultural environments. Students’ knowledge, control, affect, and beliefs are all influenced by contextual factors. Therefore, students’ problem solving strategies are influenced by cognitive, social, and cultural factors.

Problem Solving Strategies

Problem solving strategies serve as a guide in the problem solving process; although they do not guarantee a solution, they may provide a pathway to solutions (Gick, 1986). Problem solving strategies are “rules of thumb for successful problem solving, general suggestions to help an individual to understand a problem better or make progress towards a solution” (Schoenfeld, 1985 p.23). In essence, problem solving strategies are “cognitive or behavioral activities” (Siegler, p. 191) employed by students to reach a problem solution. Student reasoning in mathematics has been analyzed based on the strategies they use when problem solving (Steinthorsdottir, 2003). Hembree’s (1992) meta-analysis of 487 articles and reports related to problem solving reported strategies students used when solving problems. They included drawing a picture or diagram, using algorithms, trial and error, and guess and check. Other strategies include the use of patterns, making lists and charts, disregarding extra data, solving a simpler problem, working backwards, logical deduction, problem decomposition (breaking down a problem), and a combination of strategies (English, 1993; Malloy & Jones, 1998). When examining the strategy flexibility of high school junior and seniors on SAT mathematics problems, Gallagher, et al. (2000) generated strategy categories listed above in addition to insight with algorithm; logic, estimation, or
insight; no strategy; and misinterpretation. Insight with algorithm includes solution in which students realized they did not need to use an algorithm to solve the problem. Logic, estimation, or insight involves applying mathematical principles or logic with or without estimation or insight. No attempt at a solution was labeled no strategy and misinterpretation categorized solutions generated based on misreading of misinterpreting the problem.

A study of 200 middle school students’ strategies use when solving an augmented-quotient division-with-remainders problem showed that all student responses involved the use of an algorithm or a combination of algorithms (Silver, Shapiro, & Deutsch, 1993). English (1993) investigated the strategies employed by 96 children when solving novel combinatorial problems. She found that students who used more sophisticated strategies were more successful in correctly solving the problem. More sophisticated strategies involved identifying and applying patterns to solve the combinatorial problem. The literature documents a variety of strategies employed by students during mathematical problem solving. While some strategies are labeled as more sophisticated than others, the strategies students used and the successful application of these strategies provide insight into students’ mathematical thinking and level of understanding (Cai, 2000).

The Process of Problem Solving

In the literature, problem solving is sometimes described as a process taught to students (Krulick, 1996), but Polya (1957) saw problem solving as more than a process taught to students. He conceived of problem solving as a way of thinking. Polya’s heuristics provide strategies that help students think about, interpret, and reflect on problem situations rather than simply what “to do” when tackling a problem situation. Problem solving involves the problem solver developing an effective way of thinking about a given problem situation.
For example, drawing a picture when solving a problem depends on the problem solver’s ability to interpret the problem rather than his or her ability to make a drawing. Problem solving becomes the process of mathematically interpreting a situation involving iterative cycles of understanding given information and the need to solve a problem situation. Mathematical problem solving is about interpreting, describing, explaining—not simply executing rules and algorithms (Lesh & Zawojewski, 2007).

The cognitive processes employed during mathematical problem solving and students’ performance as mathematical problem solvers are usually examined utilizing a framework built on the foundation of Polya’s four-step heuristics model (Lester, Garofalo, & Kroll, 1989; Wilson, Fernandez, & Hadaway, 2005). Polya’s (1957) model includes four phases: understanding, planning, carrying out the plan, and looking back. Polya’s model focuses on how to do mathematics and how to reflect on problems solved. Polya’s model does not explicitly focus on metacognitive processes, but it expects students to think about strategies, tactics, and patterns available to them to solve problems (Wilson, Fernandez, & Hadaway, 2005).

Schoenfeld (1985, 2006) built upon the work of Polya by devising a scheme for dividing problem solving protocols into episodes and executive decisions to analyze cognitive moves made during problem solving. Schoenfeld’s framework had a more explicit focus on metacognitive processes (Lester, F., Garofalo, J., & Kroll, D.L., 1989). Schoenfeld identified knowledge and behaviors necessary for successful characterization of mathematical problem solving. He placed knowledge and behaviors into four categories: knowledge base, use of problem solving strategies, metacognitive aspects of behavior, and beliefs. These categories were built on Polya’s work but defined more broadly. Schoenfeld
(2006) took his framework further by proposing a grand theory of problem solving to characterize and explain the decisions humans make during problem solving with routine and non-routine problems. His theory suggests that:

an individual’s beliefs, in interaction with the context, shape the formation and prioritization of goals. Given a particular constellation of goals, the individual looks for and implements knowledge that is consistent with his or her belief systems and is designed to satisfy one or more high-priority goals. As goals are satisfied (or not), or as the context changes, new goals take on high priority, and actions are then taken in the pursuit of these goals (p. 41)

This theory builds upon Schoenfeld’s (1985) framework for analysis of mathematical problem solving behaviors. As with Polya, these frameworks are derived primarily from work with the mainstream general population.

**Characteristics of Problem Solvers**

In the mainstream literature, a good problem solver is able to analyze situations mathematically in order to generate problems based on what is seen and then employ a range of strategies for solving problems. Good problem solvers possess the following attributes: basics skills, analytic reasoning ability, strategy use, verification, confidence, field independence, and self-esteem (Hembree, 1992). Other qualities include the ability to understand mathematical concepts, to note irrelevant details in problems, to generalize on the basis of a few examples, and to switch methods readily (Suydam, 1980). Good problem solvers have a greater ability to recall information from mathematical problems and they are able to forget the details of the problem while remembering the structural features (Krulik & Rudnick, 1996). Characteristics of good problem solvers are traditionally derived from research about the mainstream middle class White population (Malloy, 1997).

Taken as a whole, there has been a great deal of educational research on mathematical problem solving in the last three decades, but the actual amount of research decreased each
decade. The number of problem solving articles in the Journal for Research in Mathematics Education (JRME) was 31 in the 1980’s, 22 in the 1990’s, and 4 between 2000 and 2003. This change in the number of articles may signify that the amount of research is on the decline, but there are reasons indicating a shift back to focusing on problem solving. These reasons are 1) emphasis of problem solving at the international level; 2) recent research emphasizing how mathematics is used in fields related to engineering, medicine, and business management; 3) the current areas of research highly related to problem solving including situated cognition, community of practice, and representational fluency (Lesh & Zawojewski, 2007, pp. 764-765). Even though shifts have occurred and are occurring in the field, problem solving research has deepened our understanding of problem solving and related pedagogical issues immensely. As we reflect on the research trends in mathematical problem solving, we realize just how dynamic research on mathematical problem solving is. This is hardly surprising when one considers some of the fundamental questions that the field has to address: What is mathematical problem solving? What are the cognitive processes used in solving mathematical problems? What are the purposes of problem solving? . . . . What is the teacher’s role in implementing problem solving in mathematics classroom? The view of the mathematics community on each of these questions have [sic.] evolved over time and are still in flux (Cai, Mamona-Downs, & Weber, 2005, p. 217).

What is missing from their assessment of the problem solving literature is the lack of studies focusing specifically on African American students and the generalizability of research findings to African American learners. Within the mainstream literature, the characteristics of a good problem solver have been traditionally derived from mainstream White middle class samples and therefore these characteristics are not indicative of a more heterogeneous population. In a study examining how twenty-four African American students solve mathematical problems, it was found that overall these African American students solved problems similar to White students reported
on in the problem solving literature, but there was documentation of distinct differences in some African American students’ approach to problem solving (Malloy & Jones, 1998). Students approached solving problems in a holistic manner, were more creative in their problem solving approaches, and used cultural knowledge when solving problems (Malloy & Jones, 1998). This study does not suggest that African American students are not successful in solving problems, but that some African American students may solve mathematical problems differently than the characteristics reported in the mainstream literature.

A review of the literature resulting in few articles focusing solely on African American is evidence for the importance of this study. This study will help to fill the tremendous gap within the problem solving literature as it relates to African American learners.

**African American Student Learner**

A definition of learning and factors influencing learning and learning styles will set the stage for this discussion on African American mathematics learning. Learning has been defined as an expansion of one's knowledge. Research has found seven major influences on learning and learning style development. Those factors are ethnicity, culture, social differences (which may include factors such as SES), individual differences (which may include factors such as learning disabilities), attitudes, beliefs, and information processing approaches (Dunn-Briggs, 1995). A student’s preferred way of learning is defined as their learning style preferences (Shade, 1992). Bennett (2001) and Shade (1992) have described culture as a framework for the development of learning style preferences. It follows that in order to study the mathematical learning of African American students, it is important to briefly examine African American culture.
African American Culture

The term ‘culture’ is now one of the most frequently used terms in the social sciences, its meaning normally being taken for granted. This tends to create an impression of universal consensus—to some extent justified in so far as there is probably a core of common meaning, but somewhat misleading because different writers, even within the social sciences, vary quite considerably in their handling of the concept. There is not single agreed definition of ‘culture’ (Jahoda, 1993, p.4).

It is difficult to define culture since it is the essence of one’s being. Culture is understanding derived from one’s experiences about how to live together and interact within a community (Hollins, 1996). Culture encompasses patterns of knowledge, skills, behaviors, beliefs, and attitudes. Hall (1977) defines culture as

[hu]man’s medium; there is not one aspect of human life that is not touched and altered by culture. This means personality, how people express themselves (including shows of emotion), the way they think, how they move, how problems are solved, how their cities are planned and laid out, how transportation systems function and are organized as well as how economic and government systems are put together and function” (p.16)

Culture influences how meaning, values, and significance are assigned to things, events, and behaviors (Hollins, 1996 & Pai, Adler, & Shadiow, 2006). Culture consists of maps of meaning, which are frameworks that allow us to make sense of the world. Sense making among various groups of people arises from shared cultural maps. In Representation and Media, Stuart Hall defines cultural maps as a system of representation, or a system of different concepts and ways of classifying, sorting, and arranging concepts to establish relations between various concepts. Meaning emerges from the relationships between concepts. He describes culture as shared meaning and shared cultural maps because belonging to the same culture means building similar cultural meaning and interpreting the
world in a similar manner. This perspective makes culture more than a set of core values, attributes, attitudes, and beliefs—it is rather the way we make sense of the world. Culture is a site through which we construct, experience, and interpret social realities in the world (Jhally, 1997).

The literature states that African American culture is rooted in West Africa and has a strong orientation towards religion, emotions, and interdependence (Boykin & Toms, 1985; Hale-Benson, 1986). While examining African American communities and African American children socialization, Boykin and Toms (1985) describe at least nine interconnected dimensions of African American cultural experiences. The nine dimensions included:

1. Spirituality is the conviction that non-material forces influence people’s everyday lives;
2. Harmony addresses the notion that people are interrelated with other elements; humankind and nature are harmonically conjoined;
3. Movement emphasizes the interweaving of movement, rhythm, pulsation, music, and dance;
4. Verve is a propensity for relatively high levels of simulation, to action that is energetic and lively;
5. Affect focuses on emotions, feelings, and nurturing;
6. Communalism is an awareness that social bonds and responsibilities transcend individual privileges;
7. Expressive individualism is the cultivation of a distinct personality and proclivity for spontaneous, genuine personal expression;
8. A social time perspective in an orientation in which time is treated as passing through a social space rather than a material one;
9. Oral tradition is a preference for oral modes of communication in which both speaking and listening are treated as performances (Boykin & Toms, 1985, p.41).

These dimensions were not characterized as learning styles but were observed during learning situations. The dimensions put forth by Boykin and Tom were also more prevalent in lower SES families and were attributed to their lack of interaction with the dominant culture when compared to higher SES families (Boykin & Toms, 1985; Hale-Benson, 1986).

While reviewing studies comparing African American and Whites on different dimensions, Shade (1982) found characteristics similar to Boykin and Toms (1985). She
found that African Americans have an affective focus; in social interaction they focused more on the people. African Americans prefer stimulus variety, which is similar to Boykin’s verve, and are found to be more field dependent (Willis, 1992), meaning they focus on a global picture. Their holistic approach to problems often includes a combination of factual and emotional aspects of the situation (Pai, et al. 2006). Learners who are field dependent “need cues from the environment, prefer external structure, are people-oriented, are intuitive thinkers, and remember material in social context” (Berry, 2002, p.30). Hillard (1976) found that African Americans prefer to view information holistically rather than in individual parts. They also prefer inferential reasoning over deductive reasoning and approximate time and space. African Americans also demonstrate proficiency in verbal and non-verbal communication. The characteristics found by Hillard are similar to Boykin and Toms’ (1985) dimensions and field dependent characteristics.

The characteristics put forth by Boykin and Toms does not serve as a comprehensive description of African American culture. Culture in general and specifically African American culture is not static, because components of culture may be learned and unlearned. The characteristics of African American culture presented in the literature are not intended to stereotype all African American students into one group; rather, they are an attempt to shed light on the importance and necessity of considering culture in the discourse on African American learning.

African American Learning

African American students tend to be holistic thinkers attuned to verbal and non-verbal communication. They also tend to be intuitive, tactile-kinesthetic learners who posses auditory, tactile and aural modes of perception, preferring to process information top down
and in context (Shade, 1992). They are creative in their approach to problem solving (Malloy & Jones, 1998). Some of these characteristics lead to the classification of African American students as field dependent learners (Shade, 1992).

As holistic learners, African American students prefer to focus on the whole picture instead of isolated parts. They process information better when presented with the big idea, and they continue to revisit the larger ideas as they learn. Optimal learning occurs for African Americans when they are able to connect new knowledge with the existing knowledge in their cognitive structures (Shade 1992; Hale-Benson, 1986). This connection of ideas and new knowledge is more successful when African Americans learn information in context and are able to see the relevance of this new knowledge. As intuitive thinkers, they use a combination of intuition and analytical skills when learning (Malloy & Malloy, 1998). The body of literature studying the influence of African American learning styles on their approach to problem solving is at best minimal; African American students and their mathematics problem solving are almost invisible in the literature.

**Gender Differences in Mathematics and Cognition**

In response to growing concern over the mathematics achievement of females in the United States and other parts of the world during the 1970’s and 1980’s, educational researchers began to produce research on gender differences within the field of mathematics. Comprehensive reviews and meta-analysis of this body of research from the 1970s to late 1990s (Hyde, Fennema, & Lamon, 1990; Leder, 1992; Kahle & Meece, 1994) report trends in the areas of achievement, attitudes, strategy use, cognition, and attribution. Researchers have found small but consistent gender differences in the areas of strategy use when solving mathematical problems and mathematical tasks involving computations, rational numbers,
measurement, and spatial visualization. Reported differences in these areas oftentimes but not always favor males (McGraw & Lubienski, 2007).

Hyde, Fennema, and Lamon (1990) conducted a meta-analysis of 100 studies to assess the magnitude of gender differences in mathematics performance. Out of 100 studies, Hyde et al. reported on four studies in which African American students were a subset of the sample. Two of these four studies were reports from achievement test data. Gilah Leder (1992) conducted a survey of the gender literature from the 1970’s to the 1980’s. She reviewed about 250 articles from the Journal for Research in Mathematics Education (JRME) and found 38 that dealt with the theme of gender. Three of the 38 studies examined race as a factor for gender differences. Gender literature from the 1990s to the present continues to be based upon predominately White samples; thus, African American students have limited representation within the gender literature.

The research summarized in this literature review discusses trends within the mathematics gender literature as it relates to students’ cognition and strategy use. The gender literature reveals little support for the global conclusion that males excel in their mathematical ability (Hyde, Fennema, & Lamon, 1990); however, small differences do exists between males and females in some areas, and these differences become more apparent around the age of 13 (Hyde, Fennema, & Lamon, 1990; Kahlé & Meece, 1994). Gender differences in mathematics and cognition emerge more strongly as the sample becomes more selective. When samples are more ability-mixed, gender differences lessen or disappear all together. Item-format (e.g. multiple choice, free response) on assessments, course selection, and ethnicity also influence the emergence of gender differences (Hyde, Fennema, & Lamon, 1990; Halpern, 2006; Casey, Nuttall, Perzaris, & Benbow, 1995).
Schratz (1978) studied gender differences in mathematical skills and spatial reasoning skills of 120 Hispanic, African American, and White preadolescent and adolescent (grades 3, 4, 5 and 9) students through the Modern Mathematics Supplement to the Iowa test. The study found no significant differences in the preadolescent group. Differences in the scores between White males and females were not significant, but males had a higher mean score. In contrast, differences between the scores of African American males and females were not significant, but females had the higher mean score. Hispanic females had a significantly higher score than Hispanic males.

Since the late 1970s, research studies have continued to find that spatial ability influences mathematics performance and achievement on standardized tests; the type of spatial reasoning was a major predictor of achievement. It was found that males have higher spatial visualization skills than females (Ethninton and Wolfe, 1984). Males were found to have higher mental rotation ability, and there was a significant relationship between mental rotation ability and females’ performance on standardized exams. (Casey, Nuttal, Perzaris & Benbow, 1995; Halpern, 2006). Females were found to perform better on computational tasks (those requiring quick retrieval from long-term memory), and tasks closely related to those performed by teachers in the classroom. Males performed better on tasks requiring mental representation and mental rotation (Halpern, 2006). Males outperformed females on standardized tests, but females had higher mathematics grades and performed better on tests that modeled material covered in the classroom. Even though the literature reports differences favoring males in spatial ability and favoring females in computational ability, there was no gender difference in overall intelligence or support for the conclusion that males

Schratz’s (1978) study is one of few that focuses on differences within ethnic groups. The results found and the studies above share the common finding that males have stronger spatial ability skills, which is assumed to lead to higher performance on standardized test measures. However, outside of Schratz (1978) these studies were based on predominately White samples, which call into question the validity of applying these conclusions to African American students.

**Strategy Use**

Recent gender research has focused on differences in students’ strategy use and performance when solving various mathematical problems. Marshall (1984) examined the performance of sixth graders on computations and story problem tasks. The data for this study was the responses of about 300,000 sixth graders from the California Survey of Basic Skills Test. Computational tasks consisted of equations with symbols indicating the operations needed to be performed, and the operations were not explicitly stated for the story problem tasks. Girls were more successful on the computational tasks and boys were more successful on the story problem tasks.

Gallagher and DeLisi (1994) studied 25 male and 22 female high ability\(^1\) students from four high schools and reported differences in strategy use with problems selected from the SAT-Mathematics Exam. There were no differences found in overall performance, but high-ability girls were shown to use more conventional strategies (e.g. application of computational algorithms) and high-ability males used less conventional strategies. Seventy percent of high-ability females’ strategies were conventional compared to 57% of high-

---

\(^1\) Juniors and seniors who scored higher than 670 on the SAT-Mathematics
ability males. Twenty-five percent of female strategies were unconventional compared to 37% of males. When examining problem type, there was a significant difference favoring females on the percent correct with conventional problems.\(^2\) There was no significant difference for non-conventional problems\(^3\), but males had a higher percentage of correct responses. They concluded that the results of this study support the “rote versus autonomous learner” and “novelty versus familiarity” hypotheses reviewed by Kimball (1989). These hypotheses suggest that girls are more likely than males to use strategies similar to those presented by their teacher and are less likely to do well on novel problem situations.

Gallagher et al. (2000) conducted a replication of Gallagher and Delisi’s (1994) study with an added component of manipulating the problem format and two additional studies. The first study involved 14 male and female juniors and senior who scored 650 or better on the SAT-Mathematics. With multiple-choice problems, females performed better on conventional than unconventional problems and there were no differences with males. On free-response items, females performed better with unconventional items and males with conventional items. Strategy use was measured by correct number of strategy hits. A strategy hit was defined as using an algorithmic strategy to correctly solve a conventional problem and using an intuitive strategy to solve an unconventional problem. Males had higher strategy hits than females on conventional and unconventional problems. The authors concluded that the results of this study support the conclusions presented in Gallagher and DeLisi (1994). When a more diverse sample was used in the second study, there was no significant difference in strategy use on either type of problem. Males and females used algorithmic and intuitive strategies equally. In general, high-ability females performed better

\(^2\) Problems primarily answered by algorithmic methods

\(^3\) Problems requiring estimation, atypical strategies, and insight
on conventional tasks and used more conventional strategies. When items were presented in
a free-response format, females performed better than males on unconventional items but
performed poorly on conventional items.

Carr and Jessup (1997) documented gender differences in strategies used by first
graders. Through analyzing 30 males and 28 female first graders’ strategies for addition and
subtraction, it was concluded that females attempted and used more overt strategies (e.g.
counting on fingers or manipulatives) than males, while males used more retrieval strategies
(e.g. form of mental calculations). Males did not have better metacognitive knowledge and
there were no gender differences in the total number of correct responses. Fennema et al.
(1998) studied 44 males and 38 females in grades 1-3 and found that starting in grade 1 and
continuing to grade 3, there were significant differences in males’ and females’ strategy use.
Females tended to use more concrete strategies such as modeling and counting and males
used more abstract strategies reflecting conceptual understanding. These results were similar
to the findings of Carr and Jessup (1997) and Carr and Davis (2001). In Fennema et al.
(1998), there were no gender differences in the number of correct solutions and more
complex extension problems were the only type of problem where gender differences arise.
Several questions surrounded the interpretation of these results (Sowder, 1998; Hyde &
strategies could potentially lead to diminished understanding of important mathematical
concepts. Hyde and Jaffee (1998) suggest that the use of a prescribed curriculum activated
stereotypes on the part of the teacher and student that lead to the use of different strategies
when solving problems. Noddings (1998) recommends considering that girls could be less
interested in mathematics, which could lead to differences in performance.
The common trend between these studies is they show that differences exist in males’ and females’ strategy use and that student performance on mathematical tasks varies based upon item format. These studies suggest that females have a less sophisticated and more rote approach to solving mathematical problems. However, these studies were based on findings from predominately-White samples and it is important to conduct research with predominately African American samples to determine if these conclusions are valid for African American students.

Given that the majority of the conclusions reached in the gender literature are based on predominantly White sample it is questionable as to whether these findings can be generalized to African American students (Kahle & Meece, 1994). When performance differences are broken down by ethnicity, gender differences are smaller and sometimes non-existent between African American males and females (Leder, 1992; McGraw & Lubienski, 2007). More research is needed with predominantly African American samples to determine if similar trends found in the mainstream literature emerge and what this information means for the learning and teaching of African American students.

The most recent NAEP data reveal small differences in the mathematical performance of male and female students that favor males, but there were no significant gender gaps found for the mathematical performance of African American students (McGraw & Lubienksi, 2007). The burning question becomes, then, why are gender differences in overall performance not found between African American students? Could an answer to this question change the way mathematics is approached and taught to all students? Does it speak to the culture of the mathematics classroom, the students’ culture, or issues of testing? Sufficient answers are not provided to these questions because of the limited amount of
gender research with a focus on within ethnic group comparisons. Conclusions from the gender literature explaining differences in males’ and females’ performance in mathematics are based on predominantly White samples, which present a huge gap in the gender literature as it relates to the gender research focusing on African American male and female learners. This study contributes to helping close this gap in the literature.

**Procedural and Conceptual Understanding**

Procedural and conceptual understanding are necessary for students to develop mathematical understanding, and the distinction between these two types of understanding plays an important role in knowledge acquisition. Procedural understanding involves understanding the formal or symbol language in mathematics, algorithms, and rules for completing mathematical tasks. Procedural understanding is often referred to as surface-level understanding. Conceptual understanding consists of an understanding of governing principles, models, and relationships in the domain of mathematics. Conceptual understanding is rich in relationships and connections (Carpenter & Lehrer, 1999; Hibert & Lefevre, 1986).

Kroll and Miller (1993) suggest that to solve problems efficiently students must possess appropriate knowledge and be able to coordinate their use of appropriate skills. Research has shown that students struggle with deciding what to do with problem solving when they lack conceptual understanding. Malloy and Jones (1998) found a strong relationship between students’ strategy use and conceptual score. Conceptual score measured students’ understanding of the problem and students’ solutions. Students with higher conceptual scores used strategies more successfully than students with lower conceptual scores. Conceptual understanding is an important factor in students’ strategy
development, even though conceptual knowledge does not always predate the emergence of new strategies and is not always prerequisite to strategy development (Carr & Hettinger, 2002). Conceptual understanding is necessary for the development of mathematical thinking and successful problem solving. Successful mathematical problem solving involves the interplay of procedural and conceptual understanding. Conceptual understanding is flexible and generative and therefore students are able to transfer this knowledge to different domains (Carpenter & Lehrer, 1999). The flexibility in students’ knowledge results in flexibility in strategy use and the successful implementation of these strategies (Malloy & Jones, 1998; Carr & Hettinger, 2002).

Conceptual understanding for this study is defined according to students ability to “(a) apply concepts to new situations; (b) connect new concepts with existing information; and (c) use mathematical principles to explain and justify problem solutions” (Malloy & Meece, under review, p. 8). This definition is directly related to Carpenter and Lehrer’s (1999) model that proposes that students must engage in five interrelated mental activities to develop conceptual understanding. These five dimensions are: (a) constructing relationships, (b) extending and applying mathematical knowledge, (c) reflecting about experiences, (d) articulating what one knows, and (e) making mathematical knowledge one’s own (Malloy & Meece, under review).

In this study, students demonstrate their level of conceptual understanding during problem solving moments by their ability to apply concepts to new situations, to connect new concepts with existing information, and to use mathematical principles to explain and justify problem solutions. Students’ level of conceptual understanding serves as a lens to examine
the development of problem solving strategies and the students’ decision to employ certain strategies during problem solving.

Frameworks

This section provides a description of the conceptual framework used to analyze the cognitive progression of students as they problem solve and the theoretical framework for examining gender difference.

Conceptual Framework

A framework for examining what students know and do as they solve mathematical problems is a necessary guide for this study. The work of Polya (1957) and Schoenfeld (1985, 2006) serve as tools for analyzing the cognitive processes of middle school students as they problem solve.

Polya’s four-step model is used throughout the literature to examine cognitive processes that occur during successful problem solving and serves as a framework for the teaching and learning of problem solving (Malloy, 1994). Polya’s model focuses on how to do mathematics and how to reflect on problems solved. Polya’s model does not explicitly focus on metacognitive processes, but the model does anticipate student will think about strategies, tactics, and patterns available to them to solve problems (Wilson, Fernandez, & Hadaway, 2005). Polya’s model for problem solving consists of the following steps:

1. **Understand** - Students must understand the verbal information and the data presented in the problem;

2. **Devise a plan** - Students should find the connection between the data and the unknown and should eventually generate a solution path to solve the problem;

3. **Carry out the plan** - Students should correctly carry out their solution path;
4. **Looking Back** - Students should look back to make sure their thinking was correct as they carry out their solution path. This process helps students solidify their thinking (Polya, 1957).

In his study of college students’ performance with non-routine problems, Schoenfeld (1985) built upon the work of Polya by devising a scheme for dividing problem solving protocols into episodes and executive decisions to analyze cognitive moves made during problem solving. Schoenfeld’s framework had a more explicit focus on metacognitive processes (Lester, F., Garofalo, J., & Kroll, D.L., 1989). Schoenfeld examined students’ solution processes as they were taking place to explain what occurs during problem solving. His framework answers the following questions: What knowledge is accessible to the problem solver? How is it chosen and used? Why does the solution evolve the way it does? How does the student’s approach to the problem reflect the student’s individual understanding (Schoenfeld, 1985, p 11)?

Schoenfeld identified knowledge and behaviors necessary for successful characterization of mathematical problem solving. Knowledge and behaviors were placed into four categories: (a) knowledge base, (b) use of problem solving strategies, (c) metacognitive aspects of behavior, and (d) beliefs. These categories were defined more broadly than Polya’s steps. Schoenfeld describes the four categories as follows:

1. **Knowledge base** is the knowledge possessed by the individual that can be used to solve the problem at hand. This knowledge may include intuition, informal knowledge about the domain, facts, algorithms, routine procedures and an understanding of agreed upon rules;
2. **Problem solving strategies** are the techniques for making progress on unfamiliar and non-standard problems. Some strategies include drawing a picture, exploring related problems, working backwards, testing and verifying procedures;

3. **Metacognition** are the global decisions regarding the selection and implementation of resources and strategies. These decisions concern what solution path to take and therefore what solution paths not to take. These decisions may include planning, monitoring and assessment, and decision-making;

4. **Beliefs** make up one’s “mathematical worldview” which is a set of determinants that determines one’s mathematical behavior. Beliefs influence how one approaches a problem and what techniques will be used or avoided. One’s mathematical worldview may contain beliefs about self, the environment, the specific topic, and mathematics in general. Beliefs provide the context for which one’s knowledge base, strategies, and metacognitive processes operate. (Schoenfeld 1985).

It has been argued in multiple fields that these types of knowledge and behaviors are determinants of success or failure in problem solving (Schoenfeld, 2006).

Pieces of Polya’s and Schoenfeld’s frameworks will serve as a conceptual framework in this study for analyzing cognitive processes evoked during mathematical problem solving. This study’s framework consists of five categories:

1. **Understand**–Students must understand the written information and the data presented in the problem.

2. **Knowledge base**–The knowledge possessed by the individual that can be used to solve the problem at hand. This knowledge may include intuition, informal knowledge about the domain, facts, algorithms, routine procedures and an understanding of agreed upon rules.
3. **Devise a Plan**- Students should find the connection between the data and the unknown and should eventually generate a solution path utilizing appropriate problem solving strategies to solve unfamiliar and non-routine problems. The decisions students make concerning the selection and implementation of resources and strategies represent the metacognitive processes of their problem solving behavior.

4. **Carry out the plan**- Students carry out their solution path and problem solving strategies. The carrying out of this plan may result in a correct or incorrect solution.

5. **Looking Back**- Students should look back to make sure their thinking was correct as they carry out their solution paths.

This metacognitive process helps to solidify students thinking (Schoenfeld 1985, 2006; Polya, 1957).

This framework will serve as a tool for organizing analyses and interpreting finding from this study. If students are unable to solve a problem successfully in this study, this framework will help to determine at what level of cognitive processing a student’s problem solving stopped.

*Theoretical Framework*

Theoretical models for examining gender differences are generally built on the assumption that males outperform females on mathematical tasks (Hyde, Fennema, & Lamon, 1990). Fennema and Peterson’s (1985) Autonomous Learning Behavior model (ALB) suggests a relationship between variables within students’ internal beliefs and the development of gender differences in mathematics achievement and participation. The ALB attempts to explain why gender differences exist in achievement on higher-level cognitive mathematics tasks. Higher-level cognitive tasks are the focus because gender differences are
more apparent at this level and skills necessary for higher-level cognitive tasks are associated with problem solving (Meyer & Koehler, 1990). The ALB model was built on the hypothesis that one must participate in autonomous learning behaviors to develop higher level cognitive skills and therefore gender difference exists based on differential participation of females and males in autonomous learning behaviors (Fennema & Peterson, 1985; Meyer & Koehler, 1990).

Autonomous learning behaviors serve as the mediator between internal and external influences and gender related differences in mathematics achievement. Internal influences consist of internal motivational beliefs such as attributional style, confidence in one’s ability to learn mathematics, perceived usefulness of the mathematics, and congruency with sex-role identity. External factors include family, peers, media, and the mathematics classroom (Fennema & Peterson, 1985).

The literature defines an autonomous learner as one who assumes control of the learning process, chooses to work independently and chooses to work on cognitively challenging tasks. When faced with difficult tasks, autonomous learners persistent. The autonomous learning behaviors of the autonomous learner result in success on higher-level cognitive tasks (Meyer & Koehler, 1990). Fennema and Peterson (1985) use this model to suggest that gender differences in mathematics achievement exist because females are less autonomous learners than males. Females are described as exhibiting learned helplessness and an avoidance to difficult tasks, while males are described as having a mastery orientation towards mathematics. Based on this hypothesis males' more autonomous approach to learning helps them to perform better on tasks requiring one to apply mathematical knowledge to unfamiliar or new problems. Females’ more rote approach to learning
mathematics helps them perform better on tasks requiring the use of more routine rules and algorithms learned in the class. If this hypothesis is true, then differences between males and females will be evident in the types of strategies students employ during mathematical problem solving. Females will use more algorithmic strategies as documented by Marshall (1984), Gallagher and DeLisi (1994), Carr and Jessup (1997), and Gallagher et al. (2000).

This study will explore the hypothesis underlying the Autonomous Learning Behavior Model. This hypothesis suggests that gender differences in mathematics are due to differences in autonomous learning behaviors. Fennema and Peterson’s Autonomous Learning Behavior model is based upon the mainstream White population.

As a conceptual framework, a combination of Polya (1957) and Schoenfeld’s (1985, 2006) schema for successful problem solving was fitting for this study because it supported the use of qualitative analysis. The story of African American students as mathematics problem solvers was found in examining students’ problem solving strategies and their ability to explain the reasoning behind how and why their strategies led to their solutions. This framework identified the cognitive processes students employed during mathematical problem solving. As a theoretical framework, Fennema and Peterson’s (1985) Autonomous Learning Behavior (ALB) was fitting for this study because it supported the use of quantitative and qualitative analyses to examine gender differences between African American males’ and females’ demonstration of conceptual understanding and approach to mathematics. The study did not test the ALB model itself, but rather the hypothesis Fennema and Peterson proposed using the ALB model. This hypothesis suggests that females are less autonomous learners than males. This hypothesis was examined to determine if it is confirmed or refuted for an African American sample.
CHAPTER 3

RESEARCH METHODOLOGY

National data shows that African American students are not performing at expected achievement levels. While a plethora of factors may explain this discrepancy, the purpose of this study was not to investigate and interrogate those factors nor to isolate specific reasons African American students are not reaching their full potential in mathematics. I simply wanted to add their voices to the literature because they can have implications for mathematics instruction and therefore educators need to understand how these students think and reason about mathematics. I did not want to add a voice of deficiency, a voice which only focuses on the underachievement of African American students, because that voice has been heard loud and clear, but rather I wanted to add a voice of understanding African American students as mathematical problem solvers. I believed that a mixed methods analysis approach was the appropriate tool for amplifying this voice because the combination of quantitative and qualitative methods worked together to provide a deeper understanding of African American as mathematical problem solvers.

Mixed methods analysis, sometimes called “mixed analysis” is praised for yielding more complete answers to research questions and a deeper understanding of a given phenomena. Mixed methods data analysis is defined as “the use of quantitative and qualitative analytical techniques, either concurrently or sequentially….from which interpretation is made (Tashakkori & Teddlie, 2003, pp.17-18). Mixed methods data analysis is not dependent on the research design that is employed but rather stems from the research
purpose. When using this analytic approach the researcher must determine if qualitative and quantitative will be used equally or if one will be dominant; this decision is also contingent upon the research purpose (Tashakkori & Teddlie, 2003).

Two major rationales for using mixed methods analysis are the concepts of representation and legitimation. Representation is the ability to gather adequate information from the data, while legitimation refers to the validity of data interpretation. Derry, Levin, Jones, and Peterson (2000) used mixed methods analysis to examine undergraduate students’ scientific statistical skills after taking an innovative statistics course. Students’ growth was assessed with pre- and post-course interviews designed to assess students’ ability to reason with statistics. Quantitative analyses revealed improvement in students’ statistical thinking. Qualitative analyses were incorporated to shed light on a full description of students’ statistical thinking based on interview responses. The use of mixed methods increased their ability to extract meaning from the data, enhancing representation (Derry et al., 2000). Crone and Teddlie (1995) established how themes could be quantitized and analyzed statistically. In their study of teachers in efficient schools, qualitative analyses were used to identify emerging theme. One theme was the “cohesiveness of faculty working together towards goals.” The quantitative data analysis found that teachers were significantly different from each other in terms of “cohesiveness with which they worked together (pp 355-356.).” The quantitative analysis added to the legitimacy of the qualitative results (Tashakkori & Teddlie, 2003). These two studies provide examples to support the two major rationales for the use of mixed methods analysis.

---

4 Quantitization is defined as converting qualitative data into numerical codes that can be statistically analyzed (Tashakkori & Teddlie, 2003, p.714).
I was attracted to mixed methods analysis because it utilized the strengths of both qualitative and quantitative analysis to better understand a phenomenon. “The ability to ‘get more out the data’ provides the opportunity to generate more meaning, thereby enhancing the quality of data interpretation” (Tashakkori & Teddlie, 2003, p. 353). Below, I explain the necessity and benefit of using both qualitative and quantitative analysis techniques in my research.

Quantitative analysis methods involve statistical manipulations ranging from descriptive accounts to complex statistical analyses. Quantitative results are often generalized to the larger population (Meadow, 2003). I used quantitative analysis because it allowed me to efficiently analyze large amounts of data and to test existing hypotheses about students’ mathematical learning. A critique of quantitative methods is the validity of results is dependent upon assumptions about the data. Violation of these assumptions can lead to significant results that are meaningless. Quantitative methods provide a quantitative description of the data but do not provide the story behind the numbers. According to Patton (2002, p. 163), “qualitative data can put flesh on the bones of quantitative results, bringing the results to life through in-depth case elaboration.”

Qualitative methods permit an examination and understanding of a phenomenon in depth. They shed light on understanding in a way that increases knowledge about a particular story (Patton, 2002). I used qualitative methods because they allowed me to understand how people construct their meaning (Glesne, 2006) and the opportunity to add the voice of the other to the mathematics literature (Noblit, Flores & Murillo, 2004). Another attracting point of qualitative methods was their usefulness when “research is lacking in an
area” (Meltzoff, 1998, p.229). Critiques of qualitative research are the labor-intensive nature and the inability to generalize qualitative findings (Patton, 2002).

Using a preexisting data set from the Mathematical Identity and Development Project (MIDDLE), I employed a mixed but primarily qualitative analysis to provide a rich description of African American students’ strategy use, specifically focusing on whether there are differences between male and female students’ strategy use and conceptual understanding. Qualitative analysis made it possible to reach the goal of describing the strategies African American male and female students used while problem solving and to build profiles of procedural and conceptual learners. Qualitative and quantitative analysis aided in the examination of differences in males and females’ strategy use, demonstration of conceptual understanding, and confirmation or refutation of Fennema and Peterson’s (1985) hypothesis given an African American sample. Results emerging from quantitative analysis were given meaning through qualitative analysis. Quantitative analyses were also used to determine whether differences existed in students’ strategy use over time. Strauss and Corbin (1990) state that qualitative methods help to explain and understand what lies behind any phenomena about which little is known. Combining qualitative and quantitative analysis provided a more detailed examination of the strategies employed by African American students and what these strategies reveal about the students’ approach to mathematics.

Below, I provide a description of the MIDDLE project to provide a context for the sample and data sources used in my study.

**Background of MIDDLE**

The Mathematical Identity Development and Learning (MIDDLE) research project was a three-year longitudinal (2002-2005) study with the following purposes:
(1) to better understand how mathematics reform affects students’ development as mathematics knowers and learners; (2) to provide a longitudinal analysis of students’ mathematical development during the middle school years; and (3) to identify the processes that explain changes in students’ mathematical learning and self-conceptions.

The MIDDLE project was conducted in an urban school district located in mid-size city in the southeastern United States. The population of this district was roughly 30,500 students. The district and participating schools served an economically and ethnically diverse community composed of primarily African American and White families. The student population was approximately 54% African American, 24.3% White, 15.7% Latino, 3.4% multiracial, 2.4% Asian, and 0.2% Native American (Malloy, 2004).

MIDDLE data was collected at two levels: Level I looked at classrooms and Level II looked at individual teachers and students. Level I data consisted of conceptual understanding items used to assess students’ levels of conceptual understanding and two student surveys used to assess students’ perceptions of teacher practice, self-conceptions, motivational orientations, and peer group norms and affiliations in the classroom. The measures for Level II included observations of students and teachers, individual student and teacher interviews, and student mathematical autobiographies. Level II data provided an in-depth examination of students’ mathematical understanding, motivation, identity, peer pressures, and mathematical experiences (Malloy, 2004).

The MIDDLE longitudinal dataset consisted of 363 students, 233 females and 140 males. All students had Level I data, and 40 of those students had Level II data. Level II students were selected from classrooms of teachers who volunteered to participate in Level II
data collection. These teachers used reform-teaching practices, traditional teaching practices, or varied their teaching practices based on their students.

**MIDDLE Data Collection**

I only discuss the collection data sources used in my study: conceptual understanding items, mathematical autobiographies, and student interviews. Conceptual understanding items were administered each year from 2002 to 2005 in both the fall and spring semesters. Students completed six items in the fall and ten items during the spring semester. In year one and year two of the study, a purposeful sub-sample of Level I students completed Level II mathematics autobiographies. Autobiographies were not completed in year three (2004-2005) because the same students were retained from year two (2003-2004). During the spring semester of year one, two, and three, these students completed Level II student interviews.

Conceptual understanding items

The MIDDLE conceptual understanding items measured students’ understanding of major strands within the middle school mathematics curriculum. These strands included rational numbers, geometry and measurement, and problem solving. These strands were stressed in NCTM (2000) *Principles and Standards of School Mathematics*, state standards, and local pacing guides. These items were selected from released Trends in International Mathematics and Science Study (TIMSS) (1994) and National Assessment of Educational Progress (NAEP) (1990 & 1992) items and then modified into open middle tasks. These items had one solution, but students could use multiple strategies to reach that solution. MIDDLE had a total of ten conceptual understanding items for sixth, seventh, and eighth grades. Four items were within grade items meaning they measured students’ growth within
a specific grade level. The within grade items were different for each grade level. The six remaining were longitudinal items that measured growth over time and were the same for each grade level (Malloy, 2004).

A team of five to seven people scored each item. Each person in the team scored the items independently with one person assigned to each item. A second rater scored 25 percent of the papers to gain inter-rater agreement, which allowed for the calculation of percent agreement using the more conservative Cohen’s kappa measure of agreement. The reliability score for within grade items was .63 for the sixth grade item and .793 for the eighth grade item. The reliability score for the longitudinal item was .855 for year one and .609 for year two. A reliability score was not calculated for year three because the reliability was established in a prior year with the same scorers. When interpreting Cohen’s kappa, a 0.40 is a minimum acceptable value and 0.60 or above indicates a good to excellent reliability (Landis & Knoch, 1977). Thus, all three items demonstrate good to excellent levels of agreement in scoring (Malloy & Meece, under review).

All conceptual understanding data was collected via students’ written responses because “[w]riting has been viewed as ‘thinking-aloud’ on paper” (Pugalee, 2004, p.29). In order to understand students’ cognitive processes during mathematical problem solving, researchers often use “think-aloud” protocols. Verbal protocols are powerful to gain information about students’ cognitive processes, but research has shown the feasibility and validity of using written responses from open-ended tasks to assess students’ cognitive processes during mathematical problem solving (Cai, 1997).

*Conceptual understanding rubric*
A general conceptual understanding rubric that applied to all items was developed. See Appendix A for a general rubric with descriptors for each level. Item-specific rubrics were developed for each of the ten items to evaluate students’ understanding of key concepts in middle grades mathematics. Broad descriptors from the general rubric were refined for each item through a three-step process: (1) important concepts required to solve the item were identified; (2) a list of indicators, statements built from the general rubric, was generated for each item; and (3) student responses that reflected the constructed indicators were placed alongside indicators in a student response section (Ellis, Burg, Gould, Joyner, & Sylvester, 2004, p. 9).

The scores in the conceptual understanding rubric ranged from 0 to 4, with 4 being the highest. The rubric scores were the following: 0 demonstrates no attempt; 1 demonstrates no conceptual understanding; 2 demonstrates no to limited conceptual understanding, 3 demonstrates procedural understanding, but conceptual understanding is not demonstrated or incomplete, and 4 demonstrates conceptual understanding. The qualitative data collected from the conceptual understanding items were quantified into a numerical score of 0-4, based upon the item-specific rubric. See Appendix B for item-specific rubrics. This numerical score represented the student’s level of conceptual understanding on each item.

Mathematical autobiographies

Level II students completed mathematical autobiographies during the spring semester in year one and two (2003 & 2004). The mathematical autobiographies provided a sketch of students’ mathematical experience and their sense of self as mathematics learners. The
autobiographies were given to students to complete at home and return to their classroom teachers. It took about 30 minutes to complete the autobiographies.

Interviews

Over the three years of the study, MIDDLE staff conducted interviews with students during their class period, after school, or free period. The amount of time needed for the interviews ranged from 30 to 45 minutes. Interviews were digitally recorded and transcribed by an outside transcriber. In the interviews students were asked to reflect on how they learn mathematics, their classroom structure, and their classroom experiences.

Current Study

Participants

For my study I used a subset of MIDDLE’s longitudinal dataset. My sample consisted of all the African American students in the longitudinal dataset (n=191) selected from 21 classrooms in year one, 33 classrooms in year two, and 46 in year three. Each student had participated for at least two years in the MIDDLE project. The sample consisted of 72 males and 119 females (Table 1 contains descriptions of the participants). Fifth grade End of Grade (EOG) scores showed that 86.6% of the females and 86.2% of the males entered middle school at or above grade level. Each student had Level I data consisting of conceptual understanding items and scores. Twenty (10 males and 10 females) of the 191 students had additional Level II data consisting of narrative autobiographies and individual student interviews.
Table 1

*Numbers and Percentages of Students at Different Grade Level*

<table>
<thead>
<tr>
<th>Year</th>
<th>Sixth Grade</th>
<th></th>
<th>Seventh Grade</th>
<th></th>
<th>Eighth Grade</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td>Male</td>
<td>Total</td>
<td>Female</td>
<td>Male</td>
<td>Total</td>
</tr>
<tr>
<td>One</td>
<td>87(64)</td>
<td>50(36)</td>
<td>137</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Two</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>112(65)</td>
<td>60(35)</td>
<td>172</td>
</tr>
<tr>
<td>Three</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Procedures

For my study, I used mixed method analysis to conduct a secondary analysis of a subset of MIDDLE longitudinal dataset. I analyzed students’ written solutions and explanations from the Level I conceptual understanding items to determine the type of strategy employed by students and if students demonstrated conceptual understanding of the concepts addressed in the item. I used Level II students’ data to create profiles of student learners based on students’ mathematics autobiographies and student interviews. These profiles include how students think during problem solving, how they write about how they best learn mathematics, how their teacher helps them to understand mathematics, their description of what is means to understand mathematics, and their mathematical learning preferences. I used quantitative analyses of the Level I conceptual understanding item scores to determine whether differences exist between conceptual understanding scores of males and females and to determine if there is a difference in students’ strategy use over time.
Data Sources

Conceptual understanding items

For my study, I used three MIDDLE conceptual understanding items: one item from sixth, seventh, and eighth grades. See Figure 1 for a description of each item.

The sixth and eighth grade items were within grade items, which assessed growth in conceptual understanding within a given year. The seventh grade item was a longitudinal item, which assessed growth in conceptual understanding over time and was administered in the sixth, seventh, and eighth grade. The within grade items were administered in both fall and spring semesters each year of the study, and the longitudinal item was administered only in the spring semester each year of the study. The sixth and seventh grade items assessed rational number concepts, and the eighth grade item assessed measurement concepts. I used the conceptual understanding items in both the qualitative and quantitative analyses.
Sixth Grade Conceptual Understanding Item
6-3. Write a fraction that is greater than 2/7.
Explain why you think your answer is correct.

Seventh Grade Conceptual Understanding Item
7-7. A class has 28 students. The ratio of girls to boys is 4 to 3.
How many girls are in the class?
Explain why you think your answer is correct.

Eighth Grade Conceptual Understanding Item
8-3. A certain rectangle has its area equal to the sum of the areas of the four rectangles shown below. If its length is 4, what is its width?

Explain why you think your answer is correct.

Figure 1. Description of Conceptual Understanding Items

Mathematical autobiographies

For the purposes of my study, I was granted permission to examine four of nine starter prompts from the mathematical autobiographies. These starter prompts were:

1. The person who helps me understand mathematics best is …because….
2. When I think of my best mathematics teacher, the reason I think he/she was the best is…
3. When I think of my worst mathematics teacher, the reason I think he/she was the worst is….
4. My mathematics teachers could help me learn mathematics better by…
Students completed mathematical autobiographies in the spring semester of year one and year two resulting in a total of 20 mathematical autobiographies. I used both years of autobiography data.

Student interviews

For the purposes of my study, I was granted permission to analyze two out of the fourteen questions in the student interview. These questions were:

1. What does it mean to have an understanding of mathematics?
2. What things does your teacher do to help you understand mathematics?

Student interviews were collected each year of the project, resulting in a total of 42 student interviews over the three years of the study.

Data Analysis

Quantitative and qualitative analyses were used to answer my research questions. I used spring semester conceptual understanding scores when conducting data analysis for the following reasons. First, this study was not examining growth within a given year; therefore, it was not necessary to examine the fall and spring scores within a given year. Secondly, during the spring semester in which these items were completed, it was believed that students would have received the necessary instruction to successfully complete the item. Thirdly, the longitudinal items were only administered in the spring semester of each year of the study.

Quantitative

Quantitative analyses were used to answer the following research question:
(1) Do differences exist between African America male and female strategy use during mathematical problem solving and the students’ demonstration of conceptual understanding?

I used SPSS version 16.0 to complete all quantitative analyses. Descriptive statistics (mean, median, and standard deviations) and frequency counts were conducted to describe the data. Since the students in this sample were not randomly assigned and prior knowledge can affect students’ demonstration of conceptual understanding (Carpenter & Lehrer, 1999), I ran univariate correlations to determine if students conceptual understanding scores were correlated with the students’ fifth grade End-Of-Grade (EOG) scores. In this study, the students’ fifth grade End-Of-Grade (EOG) raw scores were used as an indicator of students’ prior knowledge, because they provide a baseline of student’s prior knowledge before entering middle school mathematics. An Analysis of Variance (ANOVA) showed no gender differences in students’ fifth grade EOG scores. Because EOG scores influenced students’ demonstration of conceptual understanding on all the conceptual understanding items, I performed an analysis of covariance (ANCOVA) with gender as the independent variable, conceptual understanding score as the dependent variable, and fifth grade EOG as the covariate.

ANCOVA evaluates whether the group means on the dependent variable varies across different levels of the independent variables, adjusting for differences on the covariate. In other words, ANCOVA controls for the effects of the covariate so that the researcher is better able to investigate the primary effects on the independent variables (Hinkle, Wiersman & Jurs, 2003; Green & Salkind, 2005). In this study the ANCOVA determined the covariation between prior knowledge and conceptual understanding scores and removed that
variance associated with the prior knowledge and conceptual understanding scores before
determining whether differences between gender and conceptual understanding scores were
significant. I used the ANCOVA to determine whether gender differences existed in
students’ performance on conceptual understanding items to determine if Fennema and
Peterson’s (1985) hypothesis based on their Autonomous Learning Behavior Model would be
confirmed or refuted. In this study, students using strategies demonstrating conceptual
understanding would have a higher mean conceptual understanding score than a student
using a procedural/rote strategy.

I performed a multivariate analysis of covariance (MANCOVA) for repeated
measures to examine differences in students’ performance in measures of conceptual
understanding over time and to examine gender differences in measures of conceptual
understanding over time. In this study, there were three dependent variables (conceptual
understanding 6-7, 7-7, 8-7), one independent variable (gender), and one covariate (EOG
mathematics scores).

I counted the frequency of categories/themes which emerged from the qualitative
analysis of students’ strategies to determine similarities and differences between strategies
employed by males and females. The process of counting themes is referred to as
“quantitizing” data, or representing it “by numerical codes the can be represented
statistically” (Tashakkori & Teddlie, 2003, p. 355). Quantitizing qualitative data helps to
more fully describe and/or interpret a phenomenon (Tashakkori & Teddlie, 2003).
Quantitizing will aid me in describing how African American students’ problem solve and
help to interpret any differences that may emerge between male and female students. A Chi-
Square test was conducted to assess whether differences exist between strategies used by males and females on the conceptual understanding items.

Qualitative

I used qualitative analysis to answer two questions

(1) What strategies do African American students employ during mathematical problem solving?

(2) Do differences exist between African American male and female strategy use during mathematical problem solving and the students’ demonstration of conceptual understanding?

Conceptual understanding items

An analysis of qualitative data starts with the identification of themes and patterns, which capture something important in relation to the research purpose (Coffey & Atkinson, 1996). In this study, I first coded each student response on the five conceptual understanding items (n=739) from the Level I dataset according to the general codes of correct, incorrect, or no solution/attempt. I coded blank items and those stating some variation of “don’t know” or “don’t understand” as no solution/attempt. A response with the correct numerical answer was coded as correct. A code of incorrect included all responses with incorrect numerical solutions.

After documenting the number of correct and incorrect numerical solutions, I examined and coded each student response using my conceptual framework\(^5\) as the analyzing tool to determine the cognitive processes employed by students. My framework included

---

\(^5\) I modified and then combined the frameworks from Polya (1957) and Schoenfeld (1995, 2006).
five categories: (1) understanding the problem; (2) students’ knowledge to solve the problem (3) devising a solution plan; (4) carrying out the solution plan; (5) looking back. I wanted to determine if students progressed through the five processes noting where the cognitive progression stopped if they did not progress through all five. For example, students who did not understand the item were coded “UP.” Students who demonstrated progression through the five processes was coded “AF.” See Table 2 for list of codes and descriptions. Analysis of students’ problem solving strategies, which were defined as the mathematical operations and processes employed to solve the item occurred when examining students’ solution plan and the carrying out of the solution plan.
Table 2

*Conceptual Framework Codes and Descriptions*

<table>
<thead>
<tr>
<th>COGNITIVE PROCESS</th>
<th>CODE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the problem</td>
<td>UP</td>
<td>Student did not understand the verbal information presented in the item.</td>
</tr>
<tr>
<td>Knowledge base</td>
<td>KB</td>
<td>Student did not possess or was unable to recall the necessary knowledge to solve the item.</td>
</tr>
<tr>
<td>Devising a plan</td>
<td>DP</td>
<td>Student provided no indication of a solution plan to solve the item.</td>
</tr>
<tr>
<td>Carrying out the plan</td>
<td>CP</td>
<td>Student was able to generate a solution plan but was not able to successful carry it out to solve the item.</td>
</tr>
<tr>
<td>Looking back</td>
<td>LB</td>
<td>Students did not provide an explanation for why their thinking was correct or students provided an incomplete explanation.</td>
</tr>
<tr>
<td>Progression through all</td>
<td>AF</td>
<td>Students demonstrated progression through the five processes.</td>
</tr>
</tbody>
</table>

I read and reread through each written response and used open coding to code and classify the different strategies students generated to solve the items. Open coding involved comparing problem solving strategies and searching for similarities and differences between the different types of strategies employed (Liamputtong & Ezzy, 2005). Codes were generated from the data rather than a predetermined coding schema. Students were grouped based on the codes. Within the groups, students were separated by gender. I then looked for patterns among the codes to generate categories/themes. Axial coding was used to further scrutinize each category to determine if subcategories were needed (Lianputtong & Ezzy, 2005). Axial coding led to the generation of subcategories (Coffey & Atkinson, 1996), which provided greater detail about how African American students solve problems. For example, when asked to write a fraction greater than 2/7, many students used a drawing. Further analysis showed that some students used the drawing to compare 2/7 to a benchmark value.
(1/2 or a whole) while other students made a drawing of a fraction with an increased numerator. Table 3 lists the categories and subcategories of successful strategies and Table 4 list the categories and subcategories for unsuccessful strategies.
### Table 3

**Successful Strategies Categories and Subcategories**

<table>
<thead>
<tr>
<th>Conceptual understanding item</th>
<th>Successful Strategy Categories and Subcategories</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sixth grade item (6-3)</strong></td>
<td>I) Numerator and Denominator Relationship</td>
</tr>
<tr>
<td></td>
<td>A) Increase numerator-denominator the same</td>
</tr>
<tr>
<td></td>
<td>B) Same numerator-smaller denominator</td>
</tr>
<tr>
<td></td>
<td>C) Larger numerator-smaller denominator</td>
</tr>
<tr>
<td></td>
<td>D) Larger numerator-larger denominator</td>
</tr>
<tr>
<td></td>
<td>E) Equivalent fractions</td>
</tr>
<tr>
<td></td>
<td>F) Compared magnitude of numerator and denominator</td>
</tr>
<tr>
<td></td>
<td>II) Pictorial Representation</td>
</tr>
<tr>
<td></td>
<td>A) Comparison</td>
</tr>
<tr>
<td></td>
<td>1) Whole the same</td>
</tr>
<tr>
<td></td>
<td>a) benchmark value</td>
</tr>
<tr>
<td></td>
<td>i) 1/2</td>
</tr>
<tr>
<td></td>
<td>ii) a whole</td>
</tr>
<tr>
<td></td>
<td>b) Numerator-denominator relationship</td>
</tr>
<tr>
<td></td>
<td>i) larger numerator</td>
</tr>
<tr>
<td></td>
<td>2) Whole not the same</td>
</tr>
<tr>
<td></td>
<td>a) benchmark value</td>
</tr>
<tr>
<td></td>
<td>i) 1/2</td>
</tr>
<tr>
<td></td>
<td>3) Did not partition correctly</td>
</tr>
<tr>
<td></td>
<td>a) unique</td>
</tr>
<tr>
<td></td>
<td>III) Benchmark comparison without pictorial representation</td>
</tr>
<tr>
<td></td>
<td>A) 1/2</td>
</tr>
<tr>
<td></td>
<td>B) Whole</td>
</tr>
<tr>
<td></td>
<td>IV) Numerical conversion and compare</td>
</tr>
<tr>
<td></td>
<td>A) Decimal</td>
</tr>
<tr>
<td></td>
<td>B) Percent</td>
</tr>
<tr>
<td></td>
<td>V) Cross multiplication</td>
</tr>
<tr>
<td></td>
<td>A) Unique</td>
</tr>
<tr>
<td></td>
<td>VI) Guess</td>
</tr>
<tr>
<td><strong>Seventh grade item (7-7)</strong></td>
<td>I) Guess</td>
</tr>
<tr>
<td>(longitudinal item)</td>
<td>A) Guess and check</td>
</tr>
<tr>
<td></td>
<td>B) Random</td>
</tr>
<tr>
<td></td>
<td>II) Build Up</td>
</tr>
<tr>
<td></td>
<td>III) Multiplicative relationship</td>
</tr>
<tr>
<td></td>
<td>A) Between ratios</td>
</tr>
<tr>
<td></td>
<td>IV) Equivalent fractions</td>
</tr>
<tr>
<td></td>
<td>V) Pictorial representation</td>
</tr>
<tr>
<td></td>
<td>A) Build up</td>
</tr>
<tr>
<td><strong>Eight grade item (8-3)</strong></td>
<td>I) Find total area</td>
</tr>
<tr>
<td></td>
<td>A) Divide by 4 to get width</td>
</tr>
<tr>
<td></td>
<td>B) Guess and check to get width</td>
</tr>
</tbody>
</table>
### Table 4

**Unsuccessful Strategies Categories and Subcategories**

<table>
<thead>
<tr>
<th>Conceptual understanding item</th>
<th>Unsuccessful Strategy Categories and subcategories</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sixth grade item (6-3)</strong></td>
<td>I) Numerator and Denominator Relationship</td>
</tr>
<tr>
<td></td>
<td>A) Larger denominator-same numerator</td>
</tr>
<tr>
<td></td>
<td>B) Smaller numerator-same denominator</td>
</tr>
<tr>
<td></td>
<td>C) Larger numerator-larger denominator</td>
</tr>
<tr>
<td></td>
<td>D) Equivalent fractions</td>
</tr>
<tr>
<td></td>
<td>1) computational error</td>
</tr>
<tr>
<td></td>
<td>E) Compared magnitude of numerator and denominator</td>
</tr>
<tr>
<td></td>
<td>II) Pictorial Representation</td>
</tr>
<tr>
<td></td>
<td>A) Comparison</td>
</tr>
<tr>
<td></td>
<td>1) Did not partition correctly</td>
</tr>
<tr>
<td></td>
<td>III) Benchmark comparison without pictorial representation</td>
</tr>
<tr>
<td></td>
<td>A) 1/2</td>
</tr>
<tr>
<td></td>
<td>i) real world context</td>
</tr>
<tr>
<td></td>
<td>B) Whole</td>
</tr>
<tr>
<td></td>
<td>i) real world context</td>
</tr>
<tr>
<td></td>
<td>IV) Random operations with numbers in the item</td>
</tr>
<tr>
<td></td>
<td>V) Guess</td>
</tr>
<tr>
<td><strong>Seventh grade item (7-7)</strong> (longitudinal item)</td>
<td>I) Guess</td>
</tr>
<tr>
<td></td>
<td>II) Numerical operations with numbers in item</td>
</tr>
<tr>
<td></td>
<td>A) Division</td>
</tr>
<tr>
<td></td>
<td>B) Multiplication</td>
</tr>
<tr>
<td></td>
<td>C) Subtraction</td>
</tr>
<tr>
<td></td>
<td>D) Addition</td>
</tr>
<tr>
<td></td>
<td>III) Build up</td>
</tr>
<tr>
<td></td>
<td>A) Inverted ratio</td>
</tr>
<tr>
<td></td>
<td>B) From wrong value</td>
</tr>
<tr>
<td></td>
<td>IV) Multiplicative relationship</td>
</tr>
<tr>
<td></td>
<td>A) Between ratios</td>
</tr>
<tr>
<td></td>
<td>V) Equivalent fractions</td>
</tr>
<tr>
<td></td>
<td>VI) Proportions</td>
</tr>
<tr>
<td></td>
<td>A) Set up incorrectly</td>
</tr>
<tr>
<td></td>
<td>VII) No solution</td>
</tr>
<tr>
<td></td>
<td>A) “Don’t know”</td>
</tr>
<tr>
<td></td>
<td>B) Blank</td>
</tr>
<tr>
<td><strong>Eight grade item (8-3)</strong></td>
<td>I) Find total area</td>
</tr>
<tr>
<td></td>
<td>A) Only</td>
</tr>
<tr>
<td></td>
<td>B) Guess and check to find width</td>
</tr>
<tr>
<td></td>
<td>C) Computational error when finding width or area</td>
</tr>
<tr>
<td></td>
<td>D) Approximation</td>
</tr>
<tr>
<td></td>
<td>II) Guess</td>
</tr>
<tr>
<td></td>
<td>III) No solution</td>
</tr>
<tr>
<td></td>
<td>C) “Don’t know”</td>
</tr>
<tr>
<td></td>
<td>D) Blank</td>
</tr>
<tr>
<td></td>
<td>IV) “Same as pattern”</td>
</tr>
<tr>
<td></td>
<td>A) Same width</td>
</tr>
<tr>
<td></td>
<td>B) Same width and length</td>
</tr>
<tr>
<td></td>
<td>1) unique</td>
</tr>
<tr>
<td></td>
<td>V) Incorrect operation</td>
</tr>
<tr>
<td></td>
<td>A) Perimeter</td>
</tr>
<tr>
<td></td>
<td>B) ½ length</td>
</tr>
</tbody>
</table>
Students’ written responses were further analyzed using five themes that emerged from the coding: reoccurring strategies; unique strategies (strategy used by no more than one student); reoccurring errors (computational, incorrect formulas); misconceptions (i.e. large denominator always means a larger fraction); and changes in student strategies over time (longitudinal item). These were themes that emerged from the data.

The longitudinal item was examined for changes in students’ strategy use and demonstration of conceptual understanding over time. This item was administered in the spring semester each year of the study, resulting in up to three different scores for this item. Because I was looking at changes in strategy use overtime, I examined the written responses of students with three years of data. There were a total of 82 (58 female and 24 male) students with at least two conceptual understanding scores for this item. There were a total of 256 student responses over the three-year period. Each item was read and coded as maintained (using the same strategy over time) or changed (using different strategies over time).

Each response in the ‘maintained’ category was coded as (1) successful (led to correct solution) or (2) unsuccessful (led to incorrect solution). The successful strategies were coded for (1) demonstrating conceptual understanding or (2) demonstrating procedural understanding. Each response in the ‘change’ category was coded to determine the type of change. This examination led to the emergence of various subcategories: (1) successful to unsuccessful; (2) unsuccessful to successful; (3) unsuccessful to successful and back to unsuccessful; (4) successful to unsuccessful back to successful; (5) unsuccessful to unsuccessful; (6) successful to successful. Responses within the ‘change’ category were
further analyzed to determine changes in students’ demonstration of conceptual understanding. Table 5 shows the full list of codes.

I examined students’ written responses to describe the strategies used by the entire sample and to compare the written strategies of male and female students. Written solutions were analyzed to compare male and female students’ changes in strategy use and demonstration of conceptual understanding over time.

Table 5

*Categories and Subcategories for Changes in Strategy Use and Demonstration of Conceptual Understanding Over Time*

<table>
<thead>
<tr>
<th>Category</th>
<th>Description of Strategy and Demonstration of Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maintained</td>
<td>I) Successful (led to correct numerical solution)</td>
</tr>
<tr>
<td></td>
<td>A) Demonstrated conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>B) Demonstrated procedural understanding</td>
</tr>
<tr>
<td></td>
<td>II) Unsuccessful (led to incorrect numerical solution)</td>
</tr>
<tr>
<td></td>
<td>A) No conceptual or procedural understanding</td>
</tr>
<tr>
<td>Change</td>
<td>I) Successful to unsuccessful (led to correct numerical solution)</td>
</tr>
<tr>
<td></td>
<td>A) Conceptual understanding to limited or no conceptual or procedural understanding</td>
</tr>
<tr>
<td></td>
<td>B) Procedural understanding to limited or no conceptual or procedural understanding</td>
</tr>
<tr>
<td></td>
<td>II) Unsuccessful to successful</td>
</tr>
<tr>
<td></td>
<td>A) No conceptual to conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>B) No conceptual to procedural understanding</td>
</tr>
<tr>
<td></td>
<td>C) No conceptual to procedural to conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>III) Successful to unsuccessful back to successful</td>
</tr>
<tr>
<td></td>
<td>A) No conceptual understanding to conceptual understanding back to no conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>IV) Successful to successful</td>
</tr>
<tr>
<td></td>
<td>A) Procedural to conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>V) Unsuccessful to unsuccessful</td>
</tr>
<tr>
<td></td>
<td>A) No conceptual to no conceptual understanding</td>
</tr>
</tbody>
</table>
Mathematical autobiographies and interviews

I examined the designated questions from the interview and prompts from the mathematical autobiographies using the same analytic procedures used to code and analyze students’ problem solving strategies. Table 6 shows the full list of codes. I combined information from student interviews and mathematical autobiographies to create profiles of student learners by gender. These profiles include how students think during problem solving, how they write about how they best learn mathematics, how their teacher helps them to understand mathematics, their description of what it means to understand mathematics, and their mathematical learning preferences.

Table 6

<table>
<thead>
<tr>
<th>Codes for Level II Student Interviews and Autobiographies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question</td>
</tr>
<tr>
<td>----------</td>
</tr>
</tbody>
</table>
| Interview question #1 | I) Doing the mathematics  
II) Apply real life context  
III) Understand concepts  
IV) Ability to explain to others  
V) Good grades |
| Interview question #2; Autobiography questions #1, 2, 3, and 4 | I) Teaching style  
A) “Break down”  
B) Extra help  
II) Instructional strategies  
A) Use manipulatives  
B) Group work  
C) Modeling  
D) Provide notes, examples, and homework  
E) Connect to real world  
III) Teacher attributes  
A) Attitude  
B) Personality  
C) Organized  
D) Teacher content knowledge  
E) Ability to teach |
In conclusion, I used mixed methods analysis to investigate the research questions:

(1) What strategies do African American students employ during mathematical problem solving?

(2) Do differences exist between African America male and female strategy use during mathematical problem solving and the students’ demonstration of conceptual understanding?

Findings from these research questions are presented in the next chapter.
CHAPTER 4
FINDINGS

This chapter has two major sections: the first presents findings from quantitative analyses, and the second presents findings from the qualitative analyses. Specifically, section one presents descriptive statistics and univariate and multivariate analysis employed to examine the data for gender differences and changes in students’ performance over time; and section two describes students’ strategy use, changes in their strategy use and demonstration of conceptual understanding over time, students’ errors and misconceptions, and profiles of student learners. The findings I discuss in this chapter address the following research questions:

(1) What strategies do African American students employ during mathematical problem solving?

(2) Do differences exist between African America male and female strategy use during mathematical problem solving and the students’ demonstration of conceptual understanding?

Quantitative Analyses

Gender Differences

The gender literature reports differences in the strategies employed by male and female students during mathematical problem solving (Gallagher & DeLisi, 1994; Marshall, 1984; Carr & Jessup, 1977; Fennema et al., 1998). Fennema and Peterson (1985) proposed
the Autonomous Learning Behavior (ALB) Model and hypothesized that females are less
autonomous learners than males. This hypothesis implies that females would use more
algorithmic/procedural problem solving strategies (Marshall, 1984; Gallagher & DeLisi,
1994; Carr & Jessup, 1997; Gallagher et al., 2000). To answer my first question, I used
univariate and multivariate statistical analyses to determine if differences exist between the
strategies African American male and female students employ during mathematical problem
solving and what these strategies indicate about their approach to learning mathematics and
demonstration of conceptual understanding.

Descriptive Analyses: Student data

Table 7 shows the univariate statistics and distributions of students’ scores on the
conceptual understanding items for the combined sample and by gender. Students’
conceptual understanding scores were screened for normality. Univariate statistics showed
that the skewness values were within +/- 2 and the kurtosis was less than 7. Scatterplots were
used to verify linearity, and data were assessed for univariate and multivariate outliers.
Screening the data indicated that the data were normally distributed and there were no
problems with skewness, kurtosis, and outliers.
Table 7

*Spring Conceptual Understanding Score Distribution (percentages)*

<table>
<thead>
<tr>
<th>Item</th>
<th>Female N</th>
<th>M</th>
<th>SD</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6-3</td>
<td>87</td>
<td>2.39</td>
<td>1.21</td>
<td>0</td>
</tr>
<tr>
<td>7-7</td>
<td>113</td>
<td>1.32</td>
<td>1.14</td>
<td>19</td>
</tr>
<tr>
<td>8-3</td>
<td>92</td>
<td>1.18</td>
<td>1.19</td>
<td>28</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Male</th>
<th>N</th>
<th>M</th>
<th>SD</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6-3</td>
<td>50</td>
<td>2.40</td>
<td>1.11</td>
<td>0</td>
</tr>
<tr>
<td>7-7</td>
<td>60</td>
<td>1.37</td>
<td>1.26</td>
<td>22</td>
</tr>
<tr>
<td>8-3</td>
<td>56</td>
<td>1.21</td>
<td>1.27</td>
<td>30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total</th>
<th>N</th>
<th>M</th>
<th>SD</th>
<th>Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>6-3</td>
<td>137</td>
<td>2.39</td>
<td>1.17</td>
<td>0</td>
</tr>
<tr>
<td>7-7</td>
<td>173</td>
<td>1.34</td>
<td>1.19</td>
<td>20</td>
</tr>
<tr>
<td>8-3</td>
<td>148</td>
<td>1.20</td>
<td>1.22</td>
<td>29</td>
</tr>
</tbody>
</table>
Since students in this sample do not represent a random sample and prior knowledge can affect students’ performance on conceptual understanding items, some preliminary statistical analyses were run using SPSS 16.0. Correlations were calculated between conceptual understanding items and students’ fifth grade mathematics End-Of-Grade (EOG) test raw scores to determine if EOG scores influenced students’ conceptual understanding scores. Results revealed that fifth grade mathematics EOGs were significantly correlated with the five conceptual understanding items. Table 8 shows the correlation between EOG scores and conceptual understanding items.

Table 8

<table>
<thead>
<tr>
<th>Item</th>
<th>6-3</th>
<th>6-7</th>
<th>7-7</th>
<th>8-7</th>
<th>8-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOG Score</td>
<td>.245*</td>
<td>.159*</td>
<td>.387*</td>
<td>.465*</td>
<td>.534*</td>
</tr>
</tbody>
</table>

*Correlation is significant at .01 level

**Univariate Analysis**

A one-way analysis of covariance (ANCOVA) was conducted to compare the mean conceptual understanding scores of male and female students. The independent variable gender included two levels, male and female, and the dependent variable was the students’ conceptual understanding scores with the covariate fifth grade EOG scores. An alpha level of .05 was used to determine statistical significance during this analysis. An ANCOVA was run for each conceptual understanding item to determine if a statistically significant difference existed among male and female conceptual understanding scores. A preliminary analysis evaluating homogeneity of slopes assumption indicated that the relationship between the fifth grade EOG scores and conceptual understanding scores did not differ significantly as a function of gender. APPENDIX C shows the results evaluating the homogeneity of
slopes assumption. The ANCOVA was not significant for the five conceptual understanding items. The p-values were (p=.729) for 6-3; (p=.160) for 6-7; (p=.347) for 7-7; (p=.422) for 8-7; and (p=.220) for 8-3. Results reveal no significant gender differences in the mean conceptual understanding scores of African American male and female students.

**Multivariate Analysis**

Multivariate analysis of covariance (MANCOVA) for repeated measures procedures was used to examine differences in students’ performance in measures of conceptual understanding. There were three dependent variables (item 6-7, 7-7, and 8-7), one independent variable gender, and one covariate fifth grade EOG score. An alpha level of .05 was used to determine statistical significance during analysis. The sample sizes for this analysis were unequal but they were sufficiently large to ensure normality of the sampling distribution of the means (Tabachnick & Fidell, 2001). The univariate statistics showed skewness within +/- 2 and kurtosis less than 7 for each dependent variable. Screening the date revealed no multivariate outliers. The Levene’s test of equality of error variance was not significant, therefore there is no problem with the homogeneity of variance, yet Box’s M was significant, p=.023. The significant Box’s M test suggests that the homogeneity of variance and covariance assumption was rejected, suggesting that there were differences in the variance and covariance matrices. An examination of the covariance matrices using SAS 9.1 revealed that male scores on the three dependent variables were not correlated unlike the female scores which were correlated. Although the Box M was violated, the majority of the sample was females, 57 females and 24 males. Hence, although homogeneity was violated, the majority of the sample was homogeneous. Thus, although not of primary concern for this study, identifying the cause of the difference in the covariance matrix will be explored in a
subsequent study. There were no multivariate effects for independent variables with time. Repeated measures analysis showed no significant differences in students’ performance on the longitudinal item over time and no gender differences in the students’ performance on the longitudinal item over time. The p-values were p=.086 and p=.095 respectively.

Nonparametric Procedures

The chi-square test was used to examine gender differences in students’ specific strategy use on the different conceptual understanding items. Differences were found in students’ strategy use on the sixth grade, seventh grade, and eighth grade items. The sixth grade item asked students to find a fraction greater than 2/7. Both successful male and female students used a combination of six different strategies when solving the sixth grade item. More successful females than males used the strategies of changing the value of the numerator and denominator (p=.011) and using a pictorial representation (p=.019) to generate a fraction greater than 2/7. When examining differences in unsuccessful student strategies on the sixth grade item, more males changed the values of the numerator and denominator (p=.039). In the seventh grade item, students were asked to find the number of females in a class of 28 students when the ratio of females to males was 4 to 3. For the seventh grade item, differences in strategy use were only found among unsuccessful students. More unsuccessful males than females performed numerical operations with numbers in the problem (p=.023). On the eighth grade item, students were asked to find the width of a rectangle with a length of 4, given that the total area of four rectangles in the problem was 9. The only significant difference that emerged with this item was between male and female unsuccessful students. A greater percentage of unsuccessful females used an incorrect
formula (p=.013) to solve the item. This included finding the perimeter instead of the area, or using the wrong formula to find the area.

Univariate and multivariate statistical analyses provided no evidence to support gender differences in males’ and females’ strategy use or demonstration of conceptual understanding, but non parametric analysis showed some small gender differences in students’ problem solving strategies. These findings did not indicate that females had a more procedural approach to mathematics than males. In summary, there is no statistical evidence to confirm Fennema and Peterson’s (1985) hypothesis that females have a less autonomous approach to mathematics than males.

**Qualitative Analyses**

**Student Strategy Use**

This section is divided into two parts: student strategy use and student profiles. The student strategy section contains problems within this content strand included the sixth grade within grade item and seventh grade longitudinal item. The longitudinal item was administered the spring of students sixth, seventh, and eighth grade years. These problems assessed concepts related to fractions, ratios, and proportionality. The second part contains findings from qualitative analysis of student interviews and autobiographies. These findings are presented under the student profile section by gender.

**Rational number content strand**

This section provides findings from analysis of students’ written solutions on the conceptual understanding items. These findings are organized by content strand. A description of successful and unsuccessful students’ cognitive progression, strategy use,
demonstration of conceptual and procedural understanding, errors, and misconceptions are provided. Within the rational number section, there is a description of students’ strategy use and demonstration of conceptual understanding over time. Descriptions of students designated as special cases are provided. Special cases are defined as students who did not reach a correct solution but demonstrated an understanding of the concepts assessed.

Sixth Grade Item

**Problem 6-3**
Write a fraction that is greater than 2/7.
*Explain why you think your answer is correct.*

This problem assessed students’ understanding of a fraction as a part-to-whole relationship, of the relationship between the numerator and denominator of a fraction, and of how to compare fractions. One hundred and thirty-seven students (87 females and 50 males) completed this problem. One hundred and thirteen students were successful on this problem and 24 students were unsuccessful. Thirty-seven students used strategies demonstrating complete conceptual understanding, 20 students used strategies demonstrating procedural understanding, and 80 students demonstrated limited to no understanding because they did not explain why their answer was correct. All but two students appeared to demonstrate an understanding of the verbal information presented in this problem, but 19 students appeared to not understand the relationship between the numerator and denominator. Fifty-six students appeared to understand the problem, possess the necessary knowledge to solve the problem, and clearly indicated and carried out a solution plan but were unable to articulate why their solution was correct. Fifty-one students appeared to progress successfully through all five cognitive processes. One student was not able to carry out her

---

6 Concepts defined by the middle project’s Principal Investigators
7 See Table 5 for distribution of conceptual understanding scores
solution plan, and the remaining eight students provided no indication of their cognitive processes.

_The Successful._ Six categories\(^8\) emerged from the strategies employed by successful students. These six categories were:

1. Guessing the correct answer.
2. Changing the value of the numerator and/or denominator: students reasoned directly about those values. For example, with \(\frac{2}{7}\) and \(\frac{3}{7}\), students conclude \(\frac{3}{7}\) is greater because 3 is greater than 2.
3. Use of a pictorial representation: students visually showed the relationship between the numerator and denominator or visually compared \(\frac{2}{7}\) to a benchmark value
4. Comparing to a benchmark without a pictorial representation: students compared to \(\frac{2}{7}\) to a benchmark without the use of a pictorial representation.
5. Numerical conversion and comparing: students converted fractions to decimal and/or percents and comparing those values
6. Cross multiplication: students multiply bottom up (denominator * numerator) and place the product above the numerator. The numerator with the largest product above it is the larger fraction

Figure 2 provides the category name, percent of students who employed the strategy and a sample of student work.

The most common strategy among the successful students was comparing the fraction \(\frac{2}{7}\) to a benchmark value either verbally, pictorially, or both. The second most common strategy involved changing the numerator and denominator to create a larger fraction. The

\(^8\) See Table 3 for a full list of categories and subcategories
use of cross multiplication to compare fractions was an uncommon approach to solving this problem.

Figure 2. Descriptions of successful strategies for conceptual understanding item 6-3. The percentages are based upon the total number of successful student (total male and total female)

<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Student</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td>Male</td>
</tr>
<tr>
<td>Changing the value of the numerator and denominator</td>
<td>23(33)</td>
<td>10(23.2)</td>
</tr>
<tr>
<td>Pictorial representation</td>
<td>19(27.2)</td>
<td>9(20.9)</td>
</tr>
<tr>
<td>Benchmark comparison with out pictorial representation</td>
<td>25(35.7)</td>
<td>21(48.8)</td>
</tr>
<tr>
<td>Method</td>
<td>Value 1</td>
<td>Value 2</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Conversion and compare</td>
<td>3(4.3)</td>
<td>0(0.0)</td>
</tr>
<tr>
<td>Cross multiplication</td>
<td>0(0.0)</td>
<td>1(2.4)</td>
</tr>
<tr>
<td>Guess</td>
<td>0(0.0)</td>
<td>2(4.7)</td>
</tr>
</tbody>
</table>
The Unsuccessful. Unsuccessful students employed strategies similar to successful students, but they did not use the strategies effectively. Unsuccessful students employed one additional strategy, random operations with numerical values in the problem. With this strategy students multiplied the numerator and denominator of the fractions and compared the resulting products. Figure 3 provides the strategy category name, percent of students who employed the strategy, and a sample of student work.
Figure 3. Descriptions of unsuccessful strategies for conceptual understanding item 6-3. The percentages are based upon the total number of unsuccessful student (total male and total female)

<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Female</th>
<th>Male</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changing the value of the numerator and denominator</td>
<td>13(76.5)</td>
<td>6(85.7)</td>
<td>19(79)</td>
</tr>
<tr>
<td>Pictorial representation</td>
<td>1(5.9)</td>
<td>0(0.0)</td>
<td>1(4.2)</td>
</tr>
<tr>
<td>Benchmark comparison with out pictorial representation</td>
<td>1(5.9)</td>
<td>0(0.0)</td>
<td>1(4.2)</td>
</tr>
</tbody>
</table>

3. Write a fraction that is greater than \( \frac{3}{4} \).
   \( \frac{4}{4} \)
   Explain why you think your answer is correct.
   All I did was multiply \( \frac{3}{4} \) by \( \frac{2}{2} \)
   and \( \frac{3}{4} \) and \( \frac{4}{4} \) is \( \frac{4}{4} \) is bigger than \( \frac{3}{4} \).

3. Write a fraction that is greater than \( \frac{3}{4} \).
   \( \frac{3}{4} \)
   Explain why you think your answer is correct.
   \( \frac{2}{2} \) is right because if a pie were split into 7 slices and another one in 4 slices. The 2 slices from the first pie would be less than the one slice from the second one.
3. Write a fraction that is greater than \( \frac{1}{7} \).

Random operations with numbers in problem

<table>
<thead>
<tr>
<th>Guess</th>
<th>0(0.0)</th>
<th>1(4.3)</th>
<th>1(4.2)</th>
</tr>
</thead>
</table>

Because: \( 2 \times 7 = 14 \)
Errors and Misconceptions. Both successful and unsuccessful students drew inaccurate pictorial representations to compare fractions. In successful students’ drawings the whole was not the same, but students were able to generate a correct answer. Since fractions are relations, the whole must be the same in order to compare fractions. An understanding that the whole matters when comparing fractions is critical to students’ development of understanding fraction equivalence (Fosnot & Dolk, 2002).

Two common misconceptions emerged from the written solutions of unsuccessful students: (1) equivalent fractions are larger (larger numerator and larger denominator); (2) larger denominator means larger fraction. Students’ misconceptions stemmed from a lack of understanding the relationship between the numerator and the denominator of a fraction.

Seventh Grade Item

Problem 7-7
A class has 28 students. The ratio of girls to boys is 4 to 3.
How many girls are in the class?

Explain why you think your answer is correct.

The seventh grade item was a longitudinal item that students completed in the sixth, seventh, and eighth grades. This item assessed students’ understanding of a fraction always representing part-whole relationships and a ratio\(^9\) representing part-to-part or part-to-whole relationships. Additionally, this item assessed students’ understanding and application of proportional reasoning in scaling. One hundred and seventy-two students (112 females and 60 males) completed this item in the seventh grade. Thirty-one students were successful, 100 students were unsuccessful, and 41 students stated “don’t know” or made no attempt. Of the successful and unsuccessful students, twenty students used strategies demonstrating complete conceptual understanding, eight students used strategies demonstrating procedural

---

\(^9\) I am defining ratio as the relationship between two different quantities.
understanding, and 103 students demonstrated limited to no understanding. Of the 131 students who completed the problem, all students except one appeared to understand the verbal information presented in this problem, but 76 students appeared to lack the necessary knowledge of the concepts assessed by the problem. Some students appeared to understand ratios but not proportionality, and other students did not understand ratios or proportionality. Eight students appeared to understand the problem, possess the necessary knowledge of the concepts assessed to solve the problem, and clearly indicated and carried out a solution plan but were unable to articulate why their solution was correct. Thirty-six students progressed through all five cognitive processes. Three students could not carry out their solution plan. Seven students only gave an incorrect numerical answer, providing no clear indication of cognitive progression on this item.

*The Successful.* Five categories\(^{10}\) emerged from the strategies employed by successful students on this problem. These five categories were:

1. **Guessing:** students found two numbers that sum to 28
2. **Build up strategies:** students build up from the ratio 4:3 to the desired ratio 16:12
3. **Build up strategy with pictorial representation:** students draw a pictorial representation to illustrate building up from the ratio 4:3 to the desired ratio 16:12
4. **Multiplicative relationship:** students found the multiplicative relationship between ratios
5. **Equivalent fractions:** students multiplied each part of the ratio by 4.

Figure 4 provides the strategy category name, percent of students who employed the strategy, and a sample of student work. The use of a build up strategy pictorially and/or verbally (numerically) was the most common strategy.

\(^{10}\) See Table 3 for a full list of categories and subcategories
Figure 4. Descriptions of successful strategies for conceptual understanding item 7-7. The percentages are based upon the total number of successful student (total male and total female).

<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Female</th>
<th>Male</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guess</td>
<td>2(10)</td>
<td>1(9.1)</td>
<td>3(9.7)</td>
</tr>
<tr>
<td>Build up strategy</td>
<td>7(35)</td>
<td>5(45.5)</td>
<td>12(38.7)</td>
</tr>
<tr>
<td>Build up with pictorial representation</td>
<td>5(25)</td>
<td>3(27.3)</td>
<td>8(25.8)</td>
</tr>
</tbody>
</table>
7. A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class?

Explain why you think your answer is correct.

I said 4 : 3 and x each side by 4 to get 16 girls and 12 boys.
The Unsuccessful. Students who were unsuccessful ineffectively employed similar strategies to successful students, but additional strategies emerged from unsuccessful students’ written responses. These additional strategies were:

(1) Setting up a proportion: students set up incorrect missing value proportion and solved.

(2) Numerical operations with numbers in problem: students performed various arithmetic operations with values listed in the problem

Figure 5 provides the strategy category name, percent of students who employed the strategy, and a sample of student work.

The most common strategy employed among unsuccessful students was guessing or performing various arithmetic operations with numerical values in the problem.
Figure 5. Descriptions of unsuccessful strategies for conceptual understanding item 7-7. The percentages are based upon the total number of unsuccessful student (total male and total female).

<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Student Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
</tr>
<tr>
<td>Guess</td>
<td>30(44.1)</td>
</tr>
<tr>
<td>Build up strategy</td>
<td>5(7.4)</td>
</tr>
<tr>
<td>Proportion</td>
<td>6(8.8)</td>
</tr>
<tr>
<td>Numerical operation with numbers in problem</td>
<td>26(38.2)</td>
</tr>
<tr>
<td>--------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Equivalent fractions</td>
<td>1(1.5)</td>
</tr>
</tbody>
</table>
Special Cases. Students demonstrating an understanding of the concepts assessed in this problem but who were unable to reach a correct solution because of misinterpreting the problem or computational flaws were categorized as special cases. Four special cases emerged from the seventh grade item. Two students successfully used the build up strategy, one found the multiplicative relationship between ratios, and the other used an equivalent fraction. Instead of reading the problem as 4 (girls) to 3 (boys), they inverted the ratio to read 4 (boys) to 3 (girls). A sample of student work is below. The written work of the special case 4026 demonstrates the student’s understanding of the concepts assessed by the item.

Figure 6. Special case 4026

Errors and Misconceptions. A misinterpretation of ratio in the problem as four boys to three girls was the most common error. Generating and solving a missing value proportion problem with the three numbers in the problem was a common misconception; that is, students set up a proportion comparing the number of girls to boys with an “unknown” to
total number of students. A correct proportion would compare four girls to seven students with “unknown” to total number of students.

The above analysis shows that fewer students were successful on this problem in comparison to the sixth grade items. Students in the sixth, seventh, and eighth grades completed this item which provided information about changes in students performance over time. The next section presents findings from analyzing the longitudinal item for changes in strategy use over time.

Changes over time. This analysis included students with three years of data for item seven, resulting in 82 students (58 females and 24 males). Of the 82 students, 41 (34 females and 7 males) had conceptual understanding scores of only 0’s and 1’s in each administration of the item. These students were excluded from further analyses because students either consistently made no attempt to solve the problem, wrote a correct or incorrect solution only, or wrote “I don’t know/understand” and demonstrated no conceptual understanding over time. A total of 41 students (24 females and 17 males) were analyzed for changes in strategy use and demonstration of conceptual understanding. One hundred and twenty-six items were analyzed.

Two students maintained the same unsuccessful strategy, demonstrating no conceptual understanding, over time. The remaining 39 students changed the methods they used to solve the problem resulted in movement from successful strategies to unsuccessful strategies, unsuccessful strategies to successful strategies, successful strategies to successful strategies, or fluctuated back and forth between successful and unsuccessful strategies. Two students in this study used a successful strategy each year, but employed different types of successful strategies. The longitudinal analysis also revealed that students changed the types
of strategies they used to solve problems over time, but changes in strategies resulted in little change in the students’ demonstration of conceptual understanding. At the end of the three years, twenty students ended with strategies demonstrating no or limited conceptual understanding and 21 students ended with strategies demonstrating procedural and complete conceptual understanding.

Measurement content strand

The problem within this content strand included the eighth grade item assessing students’ understanding of the relationship between length, width, and the area of a rectangle and understanding decimal/fraction as part to whole.

Eighth Grade Item

Problem 8-3
A certain rectangle has its area equal to the sum of the areas of the four rectangles shown below. If its length is 4, what is its width?

Explain why you think your answer is correct

One hundred and forty-eight students (92 females and 56 males completed this problem. Fifteen students were successful, 69 students were unsuccessful, and 64 students made no attempt or stated “I don’t know”. Of the 84 students who attempted the problem, 17 used strategies demonstrating complete conceptual understanding, 4 used strategies demonstrating some procedural understanding, 63 students used strategies demonstrating limited to no understanding. Of the students who attempted the problem, all except two
appeared to understand the verbal information presented in this problem; however, 50 students appeared to lack knowledge pertaining to the concepts assessed in the problem. Of these 50 students, 35 appeared to not understand the relationship between length, width, and area, and fifteen students appeared to not understand decimal fraction as part of a whole. Four students appeared to understand the problem, possess the necessary knowledge to solve the problem, clearly indicated a solution plan, but were unable to carry out their solution plan. Sixteen students progressed successfully through all five cognitive processes. Twelve students provided no indication of their cognitive progression.

The Successful. Two categories emerged from the strategies employed by successful students. These categories included:

1. Finding the total area and dividing by four: students found the sum of the total area of the rectangles and divided by four to get the width

2. Finding total area and using guess and check: students found the sum of the total area of the rectangle and used guess and check to find the width

Figure 7 provides a description of each category, percent of students who employed the strategy, and a sample of student work. The most common strategy was finding the total area and dividing by four to obtain the width.
<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Student</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total area divided by four</td>
<td>13(86.7)</td>
<td></td>
</tr>
<tr>
<td>Total area and guess and check</td>
<td>2(13.3)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7. Descriptions of successful strategies for conceptual understanding item 8-3. The percentages are based upon the total number of successful student (total male and total female)
The Unsuccessful. Five categories emerged from the strategies of unsuccessful students. These categories were:

(1) Guessing: students provide unsupported incorrect numerical answer

(2) “Same as” pattern: students presumed all widths were 1; incorrectly saw other widths as 1

(3) Total area: students found the total area only, found the total area and guessed incorrect width, made a computational error when finding total area, or divided incorrectly by four

(4) Incorrect formula: students found the perimeter or 1/2 length

Figure 8 provides a description of each category, percent of students who employed the strategy, and a sample of student work.
<table>
<thead>
<tr>
<th>Strategy Category</th>
<th>Student</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td>Male</td>
</tr>
<tr>
<td>Guess</td>
<td>10(22.3)</td>
<td>8(33.3)</td>
</tr>
<tr>
<td>“Same as” pattern</td>
<td>13(28.9)</td>
<td>10(41.7)</td>
</tr>
<tr>
<td>Total area</td>
<td>11(24.4)</td>
<td>4(16.7)</td>
</tr>
<tr>
<td>Incorrect formula</td>
<td>11(24.4)</td>
<td>2(8.3)</td>
</tr>
</tbody>
</table>

Figure 8. Descriptions of unsuccessful strategies for conceptual understanding item 8-3. The percentages are based upon the total number of unsuccessful student (total male and total female)
“Special Cases”. “Special cases” were students who demonstrated an understanding of the concepts, but were unable to reach the correct solution because of problems with division. See Figure 9 for student work samples. Student 3567 found the area of each rectangle, found the sum and appeared to want to divide but could not. The students used approximation to find the width. Student 8221 was similar to student 3567. The students knew the width of the rectangle was between 1 and 3 but the student does not divide to find the exact answer. Student 4038 knew and attempted to divide but was not able to reach the correct solution.
Figure 9. Special cases for 8-3
Errors and Misconceptions. The lack of division skills resulted in some students’ inability to reach correct solutions. Some students appeared to assume that width must be a whole number and not a decimal. Students also misinterpreted the problem as perimeter and added together the sides to get the width. The largest number of students assumed the width was one because the width of each rectangle was one.

In summary, the analysis of the conceptual understanding items showed that African American students employed a variety of strategies when problem solving. The strategies employed by African American students were consistent with those reported in the mainstream literature. Male and female students employed similar strategies when solving the conceptual understanding items. Qualitative analysis revealed the highest percentage of students were successful on the sixth grade item (82.48%), followed by the seventh grade item (18.02%) and the eighth grade item (10.14%). Unsuccessful students were often hindered by computational errors, misconceptions in their mathematical knowledge, or misinterpretation of the written information in the problem. Unsuccessful students often performed calculations with numbers in the problem which reflected their lack of understanding.

The highest percentage of students used strategies demonstrating complete conceptual understanding and procedural understanding on the sixth grade within grade item. The lowest percentage of students used strategies demonstrating complete conceptual understanding or procedural understanding on the sixth grade longitudinal item. Students’ demonstration of conceptual or procedural understanding did not always mean that students were able to reach a correct numerical solution on the conceptual understanding. Students defined as special cases demonstrated a clear procedural or conceptual understanding of the
concepts assessed in the item but were unable to reach a correct numerical solution because of computational errors, misinterpretation of the problem, or the inability to perform arithmetic operations.

The next section presents learning profiles for African American male and female learners.

*Learner Profiles*

The questions analyzed for Level II students’ interviews and mathematical autobiographies were used to create student profiles discussing: (1) how students think during problem solving; (2) how they write about how they best learn mathematics; (3) how their teacher helps them to understand mathematics; (4) their description of what it means to understand mathematics; and (5) mathematical learning preferences. I originally planned to create profiles for students at each level of the general conceptual understanding rubric (See rubric Appendix A); however, students at the different levels had similar responses to the student interviews and mathematical autobiographies. Students’ differences in approaches to problem solving, evidenced by their problem solving strategies, did transfer to their responses to student interview questions and autobiography responses. Therefore two profiles were created, one describing male voices and another describing female voices. These profiles are based upon the responses from 10 female and 10 male students’ interviews and autobiographies.

The Females

In this study females demonstrated various levels of conceptual understanding and generated both successful and unsuccessful problem solving strategies, but there was agreement among students when asked what it means to have an understanding of
mathematics. Understanding mathematics was about understanding mathematics concepts and doing mathematics. Being able to understand and do mathematics was not only for individual benefit, but for being able to explain the mathematics to someone else. One voice echoed many others when she responded that to understand mathematics was,

\[\ldots\text{to know the concept of what you’re doing, not only to do it, but to be able to explain it to somebody else (6063).}\]

In addition to explaining mathematics to others, someone who understands mathematics should be able to apply what they know to real life situations:

\[\text{It means to me that… it means like if you get a job, you have to know how to count money. You have to calculate calculations (8248).}\]

When it comes to how teachers can help females understand mathematics and learn mathematics better, teaching style was important. Females understood mathematics better when teachers used a variety of instructional strategies such as hands-on activities and group work. Female students spoke more about the benefit of teachers modeling, providing multiple examples, providing notes, and assigning homework for practice.

\[\text{We take notes in class. And we have little sets that help us understand it more. And we have groups we work in. And if we don’t understand we can ask her and she’ll try to explain it more in a way that we’ll understand it (6049).}\]

Females understand better when teachers “break down” the material so they can understand and provide extra help when they are having trouble. One female summarized how teachers helped develop their understanding of mathematics when she said:

\[\text{She gives us plenty of notes on the particular item we are studying. And she also goes over it, and if half the class didn’t do well on the quiz or even if they do well on the quiz, she still goes over it with us so we will know how to do it later. She also has after-school tutorial for those who need it and sometimes we do hands on activities and group work (8248).}\]

Female students preferred to learn and learn mathematics best in an environment where their teachers helped them understand mathematics through varied methods. Females preferred to
work in groups, participated in peer teaching, but also enjoyed taking notes and following the teacher as he/she modeled various examples.

The Males

As with the females, males used a variety of problem solving strategies and demonstrated different levels of conceptual understanding, and there was agreement among students when asked what it means to understand mathematics. Understanding mathematics was about doing the mathematics, understanding the concepts, but mainly about the ability to apply the mathematics to real life contexts. The following male voices echoed those of other males when asked what it means to understand mathematics:

- It means that you know how to do it and you know how to solve problems and you know how to solve problems and work it out. It means like you can be able to get better jobs. You can learn how to do more things. You can build buildings with a math education (3544).
- It means you will have a better life. You can organize your money when you get older. It’s easier to count things and all sorts of things like that (8023).

When it comes to how teachers can help males understand mathematics and learn mathematics better, teaching style was at the forefront. Male students stated that they learned and understood better when teachers used different strategies to help “break down” the content student understanding. They said they learned and understood better when their teachers modeled problems, provided notes, and reviewed homework. Two voices spoke similar responses to others when they stated how teachers helped them to learn and understand mathematics better.

- She’ll come over there and she tries to help us. She solves on a piece of paper or something for us. And she gives homework like every mathematics class (3544).
- Like he just break it down for us. How something works or how to do something. He breaks it down and then asks us if we understand it. If you
don’t understand it then he’ll explain it to us. And if we do understand it he’ll give us a worksheet and see how good we can do (3999).

Males also learned and understood better when the teacher provided extra help when they were having trouble understanding the content. One male stated,

She lets me come to her if I need help. She doesn’t make you feel uncomfortable about asking her for help. Like sometimes when she’s up at the board, and she has to teach it to the whole class, and teach it in a way that some people understand it and some people don’t. And the people who don’t understand it can ask her about it (3567).

Male students preferred to learn in a fun and engaging environment with a fun, nice, and organized teacher. They preferred the teacher to use different teaching strategies to help them understand the mathematics and pass tests.

Responses from mathematical autobiographies and student interviews of the 20 Level II students resulted in various themes. Two major themes addressed, first how African American males and females described what it means to have an understanding of mathematics and second a description of how these students prefer to learn and best understand mathematics. African American students provided different descriptions for what it meant to understand mathematics but “doing” mathematics was the most frequent responses for both males and females. Females focused on learning to do the mathematics so they can help others and males more than females focused on applying their understanding of mathematics to real world situations (i.e. counting money and careers).

Quantitative and qualitative analyses revealed that African American males and females employed similar strategies when problem solving and there were no gender differences in students’ demonstration of conceptual understanding. Other findings showed that students’ strategy use was consistent with that of the mainstream literature; some strategies included using diagrams, lists, algorithms, guess and check, and trial and error,
which are successful strategies reported in the mainstream literature and students’ success and nonsuccess on varied based on the conceptual understanding item. Success on problems did not always indicate students having a procedural or conceptual understanding of the concepts assessed in the conceptual understanding item and nonsuccess on a conceptual understanding item did not always mean students lacked understanding. Analysis of students’ profiles and interviews showed that males and females described an understanding of mathematics as doing mathematics, but the majority of females discussed the importance of helping others understand while the majority of males discussed applying their knowledge to real world situations. The following chapter will provide details about the conclusions drawn from the findings in this chapter.
CHAPTER 5
DISCUSSION AND CONCLUSIONS

The goal of this study was to provide a detailed description of African American students’ problem solving strategies. I specifically looked for commonalities and differences between males and females, errors and misconceptions that emerged while problem solving, how these students prefer to learn and most effectively learn mathematics, and their thoughts on what it means to have an understanding of mathematics.

For this study, a synthesis of Polya’s (1957) and Schoenfeld’s (1995, 2006) frameworks for problem solving served as the conceptual framework and provided a lens through which to analyze the cognitive processes that should occur during problem solving. Since I wanted to test the hypothesis on which this model is built with an African American sample, Fennema and Peterson’s (1985) Autonomous Learning Behavior (ALB) served as the theoretical framework for examining gender differences in strategy use and demonstration of conceptual understanding. A mixed methods approach allowed me to explore phenomenon through both quantitative and qualitative lenses. Mixed methods analysis has the potential to provide a more complete answer to research questions and a deeper understanding of a phenomenon (Tashakkori & Teddlie, 2003).

Students’ strategies provide insight into their mathematical thinking and their level of understanding (Steinthorsdottir, 2003; Cai, 2000). I chose to examine the strategies African American middle school students used during mathematical problem solving. In addition, student interviews and mathematical autobiographies provided insight into: (1) how students
write about how they best learn and prefer to learn mathematics; (2) how their teacher helps
them to understand mathematics better; and (3) their description of what it means to
understand mathematics. Written solutions, student interviews, and mathematical
autobiographies provided rich information about African American students as problem
solvers and mathematical learners. Conceptual understanding scores were assigned to
students’ written solutions. Percentages of strategy use provided information about African
American students’ problem solving strategies and demonstration of conceptual
understanding.

Rich descriptions of African American students’ strategy use, demonstration of
conceptual understanding, approach to mathematics and characteristics as mathematical
learners emerged from this study. I reached eight major conclusions from these descriptions.
Two conclusions address strategy use and success and nonsuccess during problem solving,
two conclusions address students’ understanding, one speaks to gender differences, and two
address the profiles of student learners.

*Strategy Use and Success and Nonsuccess*

1. Student problem success and nonsuccess varied based on the conceptual
understanding item.

2. Students’ strategy use was generally consistent with the mainstream literature.

*Students’ Understanding*

1. Success on conceptual understanding items did not always mean students understood
the mathematical concepts assessed in the items

2. Nonsuccess on problems did not always mean students lacked understanding of the
concepts assessed in the items.

*Gender Differences*
1. There were no gender differences in performance and demonstration of conceptual understanding on the within grade and longitudinal items.

2. There were small gender differences in strategy use.

Profiles of Student Learners

1. An understanding of mathematics was seen as “doing” mathematics and “applying” mathematics to real world situations, such as counting money, paying bills, and having mathematics-related careers.

2. Teachers’ style and use of various instructional strategies were important to students’ preferences for learning and understanding mathematics best.

In this chapter, I present a discussion of the findings, conclusions, limitations, and implications for further research.

Discussion

The 191 students in this study employed problem solving strategies similar to those presented in the mainstream literature. The literature reports a variety of strategies students use when problem solving, including drawing a picture, using algorithms, guessing and checking, making lists and diagrams, estimating, employing problem decomposition, using logic, and using no strategy (Hembree, 1992; English, 1993; Gallagher, 2000). When solving the conceptual items in this study, both male and female students utilized a mixture of problem solving strategies discussed in the problem solving literature. Overall, students did not employ strategies deviating from those reported in the mainstream literature. The seven conclusions resulting from this study are discussed below.

Gender Differences

Although the literature reports that differences exist between the strategies employed by male and female students, these studies were often based on predominately white samples (Gallagher and DeLisi, 1994; Gallagher, 2000; Carr and Jessup, 1997; Fennema, Carpenter,
Jacobs, Franke, & Levi, 1998). Studies in the gender literature tend to suggest that females use less sophisticated strategies than males and have a more rote approach/less autonomous approach to mathematics than males. Fennema and Peterson (1989) constructed the Autonomous Learning Behavior model from the hypothesis that females are less autonomous learners than males. This hypothesis suggests that females perform better on tasks using routine rules and algorithms presented in class, which indicates they have a more procedural approach to mathematics. Gallagher and DeLisi (1984) found that girls used more conventional strategies (i.e. application of computational algorithms) and concluded from these results that there was enough evidence to support the hypothesis of girls being more procedural learners than males.

Using qualitative and quantitative analyses, my study revealed no gender difference in students’ conceptual understanding scores, but there were some differences in male and female strategy use. On the sixth grade item, more successful females changed the value of the numerator and denominator and used a pictorial representation to compare fractions when compared to males. When examining differences in unsuccessful student strategies, more males changed the values of the numerator and denominator. For the seventh grade item differences in strategy use were only found among unsuccessful students. More unsuccessful males than females performed numerical operations with numbers in the problem. On the eighth grade item, more unsuccessful females used an incorrect formula to solve the item. These findings did not indicate that females have a more procedural approach towards mathematics as evident in the strategies they used in problem solving. Results of repeated measures analyses showed that there was no difference in male and female students’ performance/demonstration of conceptual understanding over time.
The results of my study did not provide any evidence to support gender differences between African American male and female students’ demonstration of conceptual understanding and females’ use of more rote strategies. The question now becomes: Why are the differences not found with this group of students? Research has shown that gender differences depend on the selectivity of the sample and the type of assessment items students complete. Many of the studies that found differences in strategy use were conducted with high ability students (Hyde, Fennema, & Lamon, 1990); these differences disappeared when a mixed-ability sample of students were used. Students in this study were a mixed-ability group, which may account for the disappearance of gender differences. The items used in this study were open-ended items, which are found to generate fewer differences between male and female students (Garner & Engelhard, 1999). Gender differences are often reported from studies using multiple choice items. Gender differences in mathematics and cognition emerge more strongly as the sample becomes more selective. When samples are more ability-mixed, gender differences lessen or disappear all together. Item-format (e.g., multiple choice & open-ended) on assessments, course selection, and ethnicity also influence the emergence of gender differences (Hyde, Fennema, & Lamon, 1990; Halpern, 2006; Casey, Nuttall, Perzaris, & Benbow, 1995).

_strategy use and success and nonsuccess_

Success and nonsuccess as function of conceptual understanding item. The degree of student success and nonsuccess varied based on the conceptual understanding item. The highest percentage of students (82.48%) was successful on the sixth grade item followed by the seventh grade item (19.77%), and then eighth grade item (10.14%). Results showed that problems with lower percentage success rates also generate a high percentage of students’
who did not understand the concepts assessed on the item. This lack of knowledge could have also attributed to the higher percentages of “no attempts.”

A characteristic of successful problem solvers includes the ability to understand mathematical concepts (Suydam, 1980). Polya’s (1957) and Schoenfeld’s (1995, 2006) frameworks for successful problem solving include the importance of students’ content knowledge. Kroll and Miller (1993) suggest that to solve problems efficiently, students must possess appropriate knowledge and be able to coordinate their use of appropriate skills. Research has shown that students struggle with deciding what to do with problem solving when they lack conceptual understanding. In a study examining African American students’ mathematical problem solving, Malloy & Jones (1998) found a strong relationship between students’ strategy use and conceptual understanding scores, which measure students’ understanding of the problem and students’ solutions. In this study, students with higher conceptual scores used strategies more successfully than students with lower conceptual scores. Similarly, students in the present study with higher conceptual understanding scores were more successful on the items than those with lower conceptual understanding scores.

Strategy use. An examination of success and nonsuccess rates on problems revealed that African American students employed strategies consistent with the mainstream problem solving literature, a literature predominately based on the mainstream White population. In discussing the strategies employed by students, each item will be focused on individually.

When students solved the sixth grade item comparing fractions, their strategy use mirrored the research on reasoning about fractions. When examining how upper elementary, middle, and high school students reason about fractions, Smith (2002) found that students think about fraction order and equivalence in four ways: (1) using diagrams to compare; (2)
reasoning directly about the numerator and denominator; (3) determining where the fraction lies in relation to reference numbers (0,1/2, 1) for proper fractions; (4) numerical transformations for fractions, conversion to common denominator, conversion to decimals, and cross multiplication (pp.11-12). Students in this study reasoned about fraction comparison in one or a combination of the above ways of thinking. Results showed that the most common strategy was comparing 2/7 to a benchmark values (e.g.1 and 1/2) with and without a drawing, and the second most common strategy was reasoning directly about the given numerator and denominator. Only 3.54% or 4 out of 113 students who were successful on this problem used strategies that Smith (2002) states can be used without an understanding of why it works. These strategies were converting fractions decimals or percents and the use of the cross multiplication algorithm to solve problems.

When solving the seventh grade item addressing concepts related to proportionality, students most commonly used the additive build up strategy. Some students used pictures and tally marks to help their reasoning through the build up strategies. Other successful strategies included finding the multiplicative relationship between ratios, the use of equivalent factions, and guessing to find two numbers that equaled the total number of students in the problem.

In regards to their ability to reason with proportions, students use three common strategies: qualitative, additive, and multiplicative (Behr, Post, & Lesh, 1992). Additive strategy, also known as build up strategy, is the dominant strategy used during childhood and adolescence, and it was the dominate strategy among this group of adolescents (Lamon, 1993; Tourniare & Pulos, 1985). Students sometimes used tally marks and other visual representations to help support their reasoning (Lamon, 1993). Multiplicative strategies
Involving finding the multiplicative relationship within or between ratios (Behr, Post, & Lesh, 1992).

In the eighth grade items in this study, students used algorithms, diagrams, guess and check, approximation, and a combination of these strategies to solve the problem. These strategies were reported by Hembree (1992) as strategies for successful problem solving.

Interestingly, unsuccessful students used the same strategies as successful students but were not effective because they did not have the same conceptual understanding and skills. Unsuccessful students were often hindered by computational errors, misconceptions in their mathematical knowledge, or misinterpretation of the written information in the problem. Unsuccessful students often performed calculations with numbers in the problem, which reflected their lack of understanding. Guessing was frequently used among unsuccessful students. These students are similar to those reported by Romberg and Collins (1985). Romberg and Collins (1985) found that students did not have the ability to reason about addition and subtraction and did not have the skills and counting strategies to solve verbal problems; thus, although they attempted to employ strategies similar to students who were able to solve verbal addition and subtraction problems, they were not successful in doing so (Romberg & Collins, 1985).

In summary, on these three conceptual understanding items, African American students employed strategies consistent with those in the mainstream problem solving literature.

Students’ Understanding

In this study, students’ level of conceptual understanding was measured by the conceptual understanding items that were (but were not solely) based upon students reaching
a correct numerical solution. Conceptual understanding for this study is defined according to students ability to “(a) apply concepts to new situations; (b) connect new concepts with existing information; and (c) use mathematical principles to explain and justify problem solutions” (Malloy & Meece, under review, p. 8).

Research suggests that procedural and conceptual understanding are both important to the development of students’ mathematical understanding. Procedural understanding involves understanding the formal or symbolic language in mathematics, algorithms, and rules for completing mathematical tasks. Conceptual understanding consists of an understanding of governing principles, models, and relationships in the domain of mathematics. The interplay between procedural and conceptual understanding is involved in successful problem solving (Carpenter & Lehrer, 1999; other authors (Hibert & Lefevre, 1986).

In this study, special cases emerged wherein students who demonstrated a clear procedural or conceptual understanding of the concepts assessed in the items but were unable to reach a correct numerical solution. See Figure 10 for special cases’ work samples. Students were unable to reach a correct solution because of computational errors, misinterpretation of the problem, and inability to perform arithmetic operations, such as division. Special cases where students were unable to divide demonstrate the importance of the interplay between procedural and conceptual understanding for successful problem solving. When solving the eight grade item special case 3567 clearly understood the concepts assessed in the item, but he could not complete the algorithmic procedure of dividing. Special cases where students were hindered by computational errors special case 8221 and misinterpretation of the seventh grade item special case 4026 demonstrated the
importance of students’ explaining and justifying their solutions. By looking back at the solution, special case 4026 may have noticed their misread of the 4 girls to 3 boys ratio. The process of explaining and justifying solutions helps solidify and clarify students’ thinking (Schoenfeld 1985, 2006; Polya, 1957).
In this study, there was another group of students who generated a correct numerical solution but did not explain or justify their thinking. In this study, students’ explaining and justification of their thinking occurred when looking back. Forty-nine percent of successful students on the sixth grade item, 26% on the seventh grade item, and 21% on the eighth grade item did not explain and/or justify their thinking. Their failure to look back could be
attributed to their lack of understanding, their personal decision not to explain their strategies and solutions, or students could have interpreted the prompt, “explain your thinking” to mean “show your work.” Even though there was evidence to show that not all successful students explained thinking, more research is needed before conclusions can be drawn about why students chose not to.

In summary, reaching a correct numerical solution on the conceptual understanding item was not always a clear indication of students’ demonstration or lack of demonstration of conceptual or procedural knowledge. More research is needed to determine why successful students in this study did not provide evidence of procedural or conceptual understanding beyond a correct solution.

Profiles of Student Learners

Responses from mathematical autobiographies and student interviews of the 20 Level II students resulted in various themes. Two major conclusions were emerged from this study. First, there was a difference in how African American males and females describe what it means to have an understanding of mathematics. Second, these students description of their learning preferences included: teachers using a variety of instructional strategies, particularly modeling, teachers breaking down the content for them to understand, and the teacher providing extra help when needed. Students in this study preferred to work interdependently with both teacher and other students. These characteristics reflect descriptions of African American learners found in the literature (Hale-Benson, 1986; Shade 1992).

Understanding as “doing” and “applying.” When asked the question what it means to understand mathematics, the most common response was “doing” and “applying” mathematical knowledge to a real world applications. Students defined real world
applications as counting money, paying bills, and having mathematics-based careers.

Females and males had similar and dissimilar responses. Females and males both spoke about the “doing” of mathematics, but only females mentioned that the ability to do mathematics was not only for oneself but also to help others. In addition to “doing” mathematics, more males than females spoke of mathematical understanding being attached to their ability to apply mathematical knowledge to situations in real world. These real world situations included counting money, paying bills, and the ability to have mathematical based careers. This difference in how male and female students think about what it means to understand mathematics could potentially be explained by results in the gender literature.

In her review of the gender literature, Gilah Leder (1992) found that schooling affects students’ gender role perceptions. Students often see females in roles caring for younger students and males are seen managing the schools and staff. Media in larger society tend to show more males in mathematically dominated careers (Leder, 1992). The issue of gender roles may help to explain this difference in how students describe what it means to understand mathematics, but more research is needed to examine the root(s) of this difference in males’ and females’ thinking.

This notion of “doing” could imply that these students maintain a procedural instead of conceptual approach to mathematics, or it may address the students being holistic learners. As holistic learners, African American students prefer to focus on the whole picture instead of isolated parts (Shade 1992; Hale-Benson, 1986). A holistic pattern of cognition includes elements of learning-by-doing (Malloy, 1994). Therefore, more research is needed to pinpoint exactly what students meant by “doing,” since some students specifically referenced solving problems when talking about understanding as “doing,” while others simply stated
being able to do the mathematics. Teachers’ teaching styles and use of diverse instructional strategies assisted in students’ understanding and learning mathematics.

*Teaching style and diverse instructional strategies.* Teaching style and instructional strategies could have played an integral part in African American understanding as “doing.” Student responses centered on teaching style and instructional strategies when asked about how they best learn to do mathematics, what teachers can do to help them learn mathematics, and the qualities of their best teacher. Students stated that they learn mathematics better when teachers use a variety instructional strategies such as hands-on activities and group work. Interestingly, male and female students talked more about the benefits of taking notes and having the teacher model/show them how to do mathematics. This connects back to students defining an understanding of mathematics as “doing.” The combination of teachers modeling style and students doing mathematics points to the African Americans’ holistic approach to learning. “A holistic pattern of cognition is associated an entire ‘observation-learning’ complex that includes elements of observing first, and thus gaining competence before performance, learning-by-doing, visual representational structures an children’s involvement in adult activity (Tharp, 1989, p. 54).” Both male and female students discussed the teacher “breaking down” mathematics content for them and providing extra help when needed. The use of existing data makes it difficult to interpret what students meant by “breaking it down.” Based on my experiential knowledge as classroom teacher of African American students, I interpreted “breaking it down” to mean that the teacher modeled the material in a manner in which students are provided all the steps they need to make to complete the tasks assigned to them. This is not strictly a procedural approach because “breaking it down” involves students understanding why specific steps must be taken to
solve a problem. The dependence on the teacher and the desire to participate in hands-on activities and group work is characteristic of African Americans as field dependent learners. Learners who are field dependent “need cues from the environment, prefer external structure, are people-oriented, are intuitive thinkers, and remember material in social context” (Berry, 2002, p.20). Field dependent learners prefer to learn interdependently as evidenced by a desire to work with other students and their desire to work with their teachers. This desire to work with the teacher emerged when students discussed wanting their teacher to provide extra help.

The culture of the student may play a role in the development of how a student understands mathematics and his or her learning preferences. Students’ desire to learn interdependently is related to the harmony tenet of African American culture presented in the literature (Boykin & Toms, 1985). However, because this was not a comparative study between African American students and other ethnic groups, I was not able to drawn any conclusion based on culture.

In summary, African American students provided different meanings for what it means to understand mathematics, but “doing” mathematics was the most frequent response for both males and females. Females focused on learning to do the mathematics so they can help others, and males more than females focused on applying their understanding of mathematics to real world situations (i.e. counting money and careers). More research is needed to pinpoint exactly what students meant by “doing” mathematics and what causes the gender differences in the students’ response. The way African American students described what it means to learn mathematics and their desire for teachers to use a variety of
instructional strategies aligns with characteristics discussed in the African American learning literature (Shade, 1982; Hale-Benson, 1986).

**Implications for Teaching**

The descriptions of African American students’ problem solving in this study offer suggestions for mathematics instruction for all students but specifically for African American students. The National Assessments of Educational Progress (NAEP) 2003 results show that African American students’ overall mathematics scores were significantly higher than scores reported in 2003, but African American 4th and 5th graders continued to perform below basic proficiency levels. In 4th grade 46% of African American students\(^\text{11}\) scored below basic proficiency, increasing to 61% in 8th grade. The results mean that in 8th grade, 61% of African American students assessed did not exhibit evidence of conceptual or procedural understanding in the areas of arithmetic operations on whole numbers, fractions, decimals, and percents and estimation (Kenney & Kloosterman, 2007; Lubienski & Crokett, 2007). Changes in mathematics teaching could make a difference in the performance of African American and all students. The results of this study have important implications for promoting mathematical understanding of concepts within the middle school curriculum.

*Rational Number*

The results of this study show that students successfully employed strategies using diagrams to compare fractions. Even though this is a successful strategy, it becomes difficult to use when the denominators are prime numbers and/or close together in value, because it is difficult to draw equal partitions. Research suggests that use of diagrams to compare fractions should not be the primary strategy taught to students because it may slow students’

\(^{11}\) Approximately 190,000 4th graders and 153,000 8th graders participated in the 2003 NAEP assessment. African Americans made up about 17% of the 4th graders and 16% of sixth graders.
ability to reason directly with the numerators and denominators in the problem (Lamon, 1999). Teachers should use a variety of strategies that involve:

1. reasoning with divided quantity diagrams;
2. reasoning directly about the numerators and denominators in the problem;
3. thinking about where the fractions lie relative to important numerical markers or “reference points”, such as 1 or ½;
4. numerical transformations for fractions, conversion to common denominator, conversion to decimals, and cross multiplication (Smith, 2002, pp.10-11).

Smith (2002) warns that students can use numerical transformations to compare fractions without understanding why it works. It is important for teachers to have students explain why numerical transformations are a successful strategy for comparing fractions. It is important for teachers and students to talk about the progression from diagrams to other successful methods when comparing fractions.

When solving problems that assessed concepts related to proportionality, results from this study showed that the most common successful strategy was the build-up strategy. Build-up strategies involve additive reasoning, but although they may lead to correct solutions they become inefficient when ratios are made up of non-integers (Tourniaire & Pulos, 1985). Build-up strategies also allow students to solve ratio problems without understanding the multiplicative relationship within or between ratios. Since proportionality is defined as multiplicative, it is important that students transition from additive to multiplicative reasoning (Steinthorsdottir, 2003).

Measurement

To help develop students’ understanding of the concept of area, it is helpful that teachers provide students with the opportunity to investigate the relationship between the
length and width. Students can explore the relationship between the length and width by measuring physical objects, drawing and measuring objects, and covering objects with square inch tiles. It is important for students to understand the concept of area before moving to the area formula (NCTM, 2000). Students in this study struggled with finding the width of a rectangle that was non-integer. To help combat this problem, teachers can expose students to area problems that include simple and complex fractions and where the answer can be an integer or a non-integer.

Teaching for Understanding

The National Council of Teachers of Mathematics proclaimed that, “students must learn mathematics with understanding, actively building new knowledge from experiences and prior knowledge” (p. 20). One of the greatest hindrances to students successfully solving problems was their lack of knowledge of the concepts assessed. In the middle grades, Sowder and Philipp (1999) suggest that to promote understanding in middle school classroom, teachers must understand the interrelated nature of concepts and how students develop understanding of these concepts. These concepts include:

(1) extending whole-number concepts and reasoning to reasoning about rational number concepts and reasoning; (2) recognition of situations that are multiplicative rather than additive in nature, and, therefore, demand a different type of reasoning plays in central role and mathematics at this level; (3) another central but closely related idea is understanding the role of proportionality in mathematical situations (p.89).

Along with understanding these concepts and how students develop these concepts, teachers must provide challenging appropriate tasks and allow students the opportunity to
wrestle with those tasks. The climate of the classroom should be one that is non-threatening and where the ideas of all students are valued (Sowder & Philip, 1999). Within classrooms that promote understanding, students should also have the opportunity construct relationships, extend and apply the knowledge they acquire, reflect on their leaning experiences, articulate what they know, and make the knowledge their own (Carpenter & Lehrer, 1999).

In summary, a combination of the above suggestions can help to develop an understanding of middle school mathematical concepts for African Americans and all students. Results of this study also suggest that African American students benefit from learning by doing, learning interdependently, and through the use of various instructional strategies.

**Limitations of the Current Study**

This study presented two major limitations. The first limitation was the use of an existing dataset. Only having student’s written solutions to problems limited the interpretations that could be made about their thinking. Some written solutions were ambiguous, making it difficult to interpret the students’ explanation. Figure 11 shows an example of a student who reached a correct solution but whose explanation is not clear. It appears that the student is talking about writing 1/2 in 7ths would be between 3/7 and 4/7 but instead the student states 1/2 of 2/7 which equals 1/7 which is not a correct explanation.
There was also difficulty in interpreting students’ responses to the interviews and autobiographies. When students described understanding as “doing” and wanting teachers to “break down” the content, their responses did not provide a clear meaning of those statements. I was not able to provide precise interpretations of what students meant by these statements. Not having the students present to answer questions about their written solutions on the conceptual understanding items or to explain the meaning behind the responses made it difficult to draw conclusions from the student data. To address this limitation, verbal protocols could be incorporated. The use of verbal protocols would allow the researcher the opportunity to assess the mental processes of the student, as students share their thoughts as they work through the problem. Researchers could also question students about how and why they are solving a problem using a particular strategy (Pugalee, 2004). Verbal protocols could provide a deeper and richer understanding of how African American students think during mathematical problem solving and what factors influence their thinking.

Secondly, the participants in this study represented a voluntary sample. A voluntary sample may bias the sample and make the sample unrepresentative of the population. The generalizability of findings is threatened by using a voluntary sample (Meltzoff, 1998).
Within the sample of 191 students, only 20 students had level II data, which may not represent the views of the entire sample.

**Recommendations for Future Research**

This study was secondary analysis of an existing longitudinal dataset to explore the strategies employed by African American students during mathematical problem solving and to determine whether gender difference arose in students’ strategy use and demonstration of conceptual understanding. Profiles of student learners were built to describe: how they write about how they best learn and prefer to learn mathematics, how their teachers help them to understand mathematics, their descriptions of what is means to understand mathematics.

Findings from the student profiles showed a difference in male and females’ description of what it means to understand mathematics. Females stated that understanding mathematics was about helping others to do the mathematics and males talked about the understanding of mathematics as the ability to apply their mathematics knowledge to real life situations (e.g. paying bills, counting money, and getting jobs). A study of these students using interview protocols asking to students to explain their description of the meaning of understanding mathematics and why they use this description could help explore the emergence of this difference between males and females.

When students discussed how they best learn and prefer to learn mathematics, how their teacher helps them to understand mathematics, and their description of what it means to understand mathematics, students in this study consistently discussed the importance of “doing” the mathematics and their preference for their teacher to use a modeling teaching style. Understanding mathematics as “doing” and a preference for a modeling style of teaching are characteristic of a holistic approach to learning (Tharp, 1989). In this study,
students’ conception of what it means to understand mathematics and their preference for learning may be influenced by the students’ culture. According to Shade (1992) and Bennett (2001), students’ culture plays an important role in their approach to learning and learning preferences. A comparative study of students from different cultures could be used to investigate the role of culture in African American students’ mathematical learning.

It is my hope that the current study will serve to motivate other researchers to focus on the mathematical learning and problem solving of African American students. In addition, this study could serve as resource for educators and researchers interested in learning and investigating African American students’ problem solving and mathematics learning.

Conclusion

The purpose of this dissertation was two-fold. The first purpose was to examine the strategies employed by African American middle school students as they solve problems involving measurement and rational numbers and to determine if differences exist in students’ strategy use and demonstration of conceptual understanding. In answering these questions, I examined African American students’ strategy use and demonstration of conceptual understanding and determined that they employed strategies consistent with the mainstream literature. The errors and misconceptions which emerged from analyzing students’ strategies were also consistent with those of students reported in the mainstream literature. Results of this study showed and there were no gender differences in students’ demonstration of conceptual understanding and some small differences in students’ strategy use.
After analyzing students’ strategy use and demonstration of conceptual understanding, I created profiles of male and female student learners. Creating profiles of African American student learners showed that both male and female students describe understanding mathematics as “doing.” Females associated understanding mathematics with the ability to help others and males associated an understanding of mathematics to being able to apply the mathematics to real world situations. Males talked about being able to count money, paying bills and having mathematics-based careers. Both male and females students prefer to learn in environment where teachers use a variety of teaching strategies such as modeling, group work, and hands of activities where the students are able to understand the content. It was important for teachers to provide extra help to students when they were having difficulty with the content.

The second purpose was to address major gaps in the problem solving and gender literature concerning African American students as mathematical learners. Findings from the problem solving and gender literature are based on predominately-White samples and the research on African American students as mathematics learners are minimal and often dated. If problem solving can act as a mediator to mathematical understanding and “mathematics for all,” a shift must occur in the current research to study the problem solving strategies employed by all mathematical learners. Educators must understand how students problem solve, what factors shape their problem solving behavior, how to develop their problem solving skills, and how student approach mathematical problem solving and learning.

The mainstream literature in the arena of problem solving and gender differences needs to mirror the diverse population of students in America’s schools. No minority
student, teacher of that student, or parent of that student should look in the mirror of literature and not see their image starring back. Even though this dissertation revealed that students problem solving strategies were consistent with those in the mainstream literature, it is still important to legitimize the voices of people who are often silenced. We can not know how African American students problem solve if the time is not taken to find out. There is still work to do to make the invisible visible.
APPENDIX A: CONCEPTUAL SCORING RUBRIC WITH CRITERIA

Conceptual Scoring Rubric with Criteria

0 points: There is no response.
- No work on the problem.
- For multi-part problems, circled a letter (as if multiple choice) and gave no explanation or “guessed.”

1 point: The response demonstrated no conceptual understanding of the problem.
- Wrote some information related to the problem but did nothing with the information.
- Showed no indication of how the work was accomplished.
- Provided an explanation for a solution that was not related to concepts involved.

2 points: The response demonstrated limited conceptual understanding and/or had significant errors.
- Showed limited understanding of the concepts involved in reaching a solution, but demonstrated a calculation error.
- Showed limited understanding of concepts involved in solution, but did not complete the solution.

3 points: The response demonstrated some conceptual understanding but was incomplete.
- Showed some conceptual understanding that led to the solution but explanation of key concepts was not complete.

4 points: The response demonstrated complete conceptual understanding of the problem.
- Mentioned in words the key concepts related to the solution of problem.
- Showed mathematically the key concepts related to the solution of problem.
## Conceptual Scoring Rubric for 6-3

<table>
<thead>
<tr>
<th>Level</th>
<th>Identifiers</th>
<th>Examples of student responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No work or states they do not understand with no answer given.</td>
<td>“I don’t understand.”</td>
</tr>
</tbody>
</table>
| 1     | No evidence of understanding relationship between numerator and denominator as part of a whole. Misconceptions and shallow knowledge are evident. | “1/3 because the higher the denominator the lower the fraction, so I picked 1/3 because 3 is lower than 7.”
|       |                                                                             | “1/2 because whenever the denominator is bigger the fraction is smaller.”                       |
| 2     | Demonstrates some evidence of knowledge numerator/denominator relationships to determine size relationships between fractions verbally or visually. Answer often involves unsupported speculation. | “5/7 because it is a larger number.”
<p>|       |                                                                             | “Because, 2/7 is like getting a piece of cake and ½ is like getting half the cake so ½ is larger than 2/7.” |
|       |                                                                             | ½ because 2/7 is small when you draw it and ½ is bigger (no drawing given). [#6190 W3]       |
|       |                                                                             | ½ the smaller the denominator the bigger the fraction. [#8147 W3]                            |
| 3     | Evidence of complete understanding is not provided. Mechanistic feel to the explanation. | “5/11 because if you put 2/7 and 5/11 beside each other and cross multiply, you can tell that 5/11 is bigger than 2/7 and see how big 5/11’s numerator is compared to 2/7’s numerator.” |
|       | Visual comparisons give evidence of understanding parts and whole relationships but are not drawn appropriately to support conclusions. | 5/7 more parts are shaded [#1221 W3]                                                         |
|       |                                                                             | ![Image of shaded parts]                                                                      |</p>
<table>
<thead>
<tr>
<th>4</th>
<th>Clear understanding of relationship between numerator and denominator as part of a whole (visually or verbally). Alternatively, students might provide thinking that demonstrates a comparison to a benchmark (typically ½). Draws and correctly shows relationships between the two fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>“3/7 since 2 sevenths is 2 out of 7, so I just put three out of seven.” “3/7 since 2/7 would be [diagram showing 2/7 of one circle shaded] and 3/7 would be [3/7 of another circle is shaded]. “2/3. The denominator is smaller in 2/3 so each part is larger.” “3/7 because 3 is bigger then 2” [#3027] “7/7=1 whole. 1 whole is always greater than a fraction.” [#6028] “4/5 I think my answer is correct because in 2/7 2 is a lot less than 7 &amp; in 4/5 the 4 is VERY close to the 5. In fact that fraction is almost a whole. [#1255 W3]</td>
</tr>
<tr>
<td>Level</td>
<td>Identifiers</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>0</td>
<td>No work or states they do not understand with no answer given.</td>
</tr>
<tr>
<td>1</td>
<td>No evidence of understanding concepts related to fractions or proportionality.</td>
</tr>
<tr>
<td>2</td>
<td>Written or symbolic explanation shows an understanding the meaning of a ratio of 4:3, but does not apply the ratio to solve the problem. Written work provides no explanation of how the answer was found or explanation is solely procedural.</td>
</tr>
<tr>
<td>3</td>
<td>Explanation is accurate does not thoroughly explain the rationale used in solving the problem. In some cases the explanation is procedural rather than conceptual.</td>
</tr>
<tr>
<td>4</td>
<td>Evidence of full understanding of proportionality either verbally or visually (scaling 4:3 or using and explaining the proportion 4/7 = 16/28).</td>
</tr>
</tbody>
</table>
## Conceptual Scoring Rubric for 8-3

<table>
<thead>
<tr>
<th>Score</th>
<th>Indicator</th>
<th>Student Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>• No work or states they do not understand with no answer given.</td>
<td>• “I don’t understand.”</td>
</tr>
</tbody>
</table>
| 1     | • Gives an answer (correct or incorrect) but gives unrelated or no explanation.  
• No evidence of understanding of finding area. | • “The width is 1 because the width on every other rectangle is 1.” (8940)         |
| 2     | • Limited understanding of finding width.                                   | • 5; “I think my answer is correct because I added all the areas together and the subtracted the length.” (8942)  
• Describes a correct method to start the problem but fails to carry through to find the correct width. |
| 3     | • Demonstrates mechanical understanding of finding the width.  
• Does not provide sufficient explanation of strategy used in solving problem.  
• Gives an answer not close to 2.25. | • 2.1; “I think I’m right because I added all the areas up and I did everything I was taught to do.” (8951)  
• “Its width is about 1.6. I know this b/c the area of the four other rectangles combined is 9 and 9 divided by four is about 1.6” (8945) |
| 4     | • Demonstrates understanding of relationship between area, length, and width.  
• Explains why the method they chose works in this case.  
• (Accept if process and explanation are correct, but gives an incorrect answer, which is close to 2.25, due to computational error.) | • 2.25 “I solved this by adding by the rectangles area (9). I then set up the equation 4 x=9. I divided 4 into 9 and got 2.25, then check my work by multiplying 4 by 2.25.” (8931)  
• 2.25; “The sum of all the areas in the problem is 9. If the length is 4, 4·2.25=9.” (8932) |
### APPENDIX C. HOMOGENEITY OF SLOPES

<table>
<thead>
<tr>
<th>Item</th>
<th>6-3</th>
<th>6-7</th>
<th>7-7</th>
<th>8-7</th>
<th>8-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction</td>
<td>$F(1,128)=3.688$, p=.057</td>
<td>$F(1,129)=1.466$, p=.228</td>
<td>$F(1,154)=.002$, p=.963</td>
<td>$F(1,134)=.268$, p=.606</td>
<td>$F(1,131)=.418$, p=.519</td>
</tr>
</tbody>
</table>

* .05 significance level
REFERENCES


of Educational Progress (pp. 1-22). Reston, VA: National Council of Teachers of Mathematics.


Malloy, C. & Meece, J. (in press). The Relation between Reform Instruction and Student Achievement in Middle Grades Mathematics Classrooms.


131