A CYCLIC COSMOLOGY

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ABSTRACT

LAURIS M. BAUM: A Cyclic Cosmology
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It is speculated how dark energy in a braneworld can help reconcile an infinitely cyclic cosmology with the second law of thermodynamics. A cyclic model featuring dark energy with a phantom ($w < -1$) equation of state leads to a turnaround at a time just before a would-be Big Rip at one end of the cycle and a bounce just before a would be crunch at the other. At the turnaround, both the volume and entropy of our universe decrease by a gigantic factor while very many independent small contracting universes are spawned. The entropy of our model decreases to nearly zero as it approaches the turnaround after which it increases by only a vanishing amount during the contracting stage, empty of matter. Shortly after the bounce, the entropy increases by a large factor during inflationary expansion. We next examine the content of the contracting universe (cu) and its entropy S_{cu}. We find that in addition to dark energy, the universe contains zero photons on average (with the unlikely single photon, if present immediately after the turnaround, having infinitesimal energy that blue shifts eventually to produce e^+e^- pairs). These statements are independent of the equation of state $\omega = p/\rho$ of dark energy provided $\omega < -1$. Thus $S_{cu} = 0$ and if observations confirm $\omega < -1$ the entropy problem is solved. We discuss the absence of a theoretical lower bound on $\phi = |\omega + 1|$ and then describe an anthropic fine tuning argument that renders unlikely an extremely small ϕ. The present bound $\phi < 0.1$ already implies a time until turnaround of $(t_T - t_0) \gtrsim 100$ Gy. The requirement that our universe satisfy a CBE-condition (*Comes Back Empty*) imposes a lower bound on the number $N_{\rm cp}$ of causal patches which separate at turnaround. This bound depends

on the dark energy equation of state $w = p/\rho = -1 - \phi$ with $\phi > 0$. More accurate measurement of ϕ will constrain N_{cp}. The critical density ρ_c in the model has a lower bound $\rho_c \geq (10^9\mathrm{GeV})^4$ or $\rho_c \geq (10^{18}\mathrm{GeV})^4$ when the smallest bound state has size 10^{-15}m, or 10^{-35}m, respectively.

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INTRODUCTION

1.1 General Background

One of the longest standing issues in cosmology has been what is the overall nature and history of the universe in the broadest sense? Did it spring forth from one initial big bang? Has it always existed more or less as it is today? Or has it perhaps gone through an infinite cycle of expansions and contractions? Or something altogether different?

In 1916, Albert Einstein published his theory of general relativity containing the famous Einstein equation $(G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = 8\pi G T_{\mu\nu}$. One of the popular assumptions of the time, based on the general observational data available and the lack of understanding of what nebulae actually were, was the notion of a static and largely uniform universe. When Einstein began to look into the implications of his equation for the universe as a whole, he quickly realized that it was not possible to obtain a static solution with a uniform distribution of matter so, in 1917, he introduced a cosmological constant term $\Lambda g_{\mu\nu}$ into his field equation to get $(G_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ (taking $\Lambda = 8\pi G\varrho$ in order to balance out the attractive force of gravity) and created a static, closed model of the universe. However, this proposal did not turn out to be tenable as it was unstable to even the smallest of perturbations and it was ruled out by later astronomical data which revealed an expanding universe.

Interestingly, over two centuries earlier similar discussions had occurred based upon Newtonian theory. In 1692 Richard Bentley gave the first series of Boyle Lectures. At the lectures, a consensus was reached among those present that the universe must be infinite if it was to have any chance of not collapsing into a point due to the force of gravity (although they were wrong because under Newtonian gravitation an infinite universe would actually collapse back on itself in the same time as a finite universe would) [1]. Furthermore, they agreed that it would have to be perfectly arranged otherwise even the smallest perturbation would lead to collapse. Newton concurred with their findings. Later in 1917, Willem de Sitter introduced an alternative construction for a static universe making use of the cosmological constant. His model contained no matter at all, much to the consternation of Einstein who had originally hoped that his theory of general relativity would have a single unique solution that was our universe. It was not until six years later that Arthur Eddington and Hermann Weyl were to discover that serious confusions (perhaps aided by the prejudice of the day for a static universe) had led to the mistaken conclusion that de Sitter's model actually represented a static universe when it was, in fact, a model of an expanding universe [1]. After considerable initial interest, de Sitter's theory eventually fell by the wayside as the static universe was disproven and the mass density was radically too low to be realistic. It did not re-emerge until decades later when, in 1980, interest began in inflationary theory.

In 1922 mathematician and meteorologist Alexander Friedmann came up with his famous solutions to the Einstein equation [2]. For calculational simplicity he took the universe to be homogenous and isotropic, neither of which appeared to be observationally troubling. In his 1922 paper he discovered cases for expanding universes having closed spatial geometries. In some of these cases the universe would expand to a maximum size before turning around and crunching back into a point. In a second paper, in 1924, he found cases for infinite open universes having a hyperbolic geometry which are know as the Friedmann world models. From the time-time component of the Einstein equation came the the Friedmann equation $(\frac{\dot{a}}{a})^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\varrho$ (sometimes written using Hubble's constant H, the expansion rate of the universe, with

$H \equiv \frac{\dot{a}}{a}$) and from the space-space components came $2\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{\kappa}{R^2} = -8\pi G\varrho$ where a(t) is the scale factor for the expansion, κ characterizes the geometry of the universe and is usually normalized to $\pm 1, 0$ ($\kappa = -1 \Leftrightarrow open, \kappa = 0 \Leftrightarrow flat, \kappa = 1 \Leftrightarrow closed$) and ϱ is the total energy density. Taking the difference between the two equations of Friedmann provides an equation for the acceleration: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varrho + 3p)$. Paired with the Robertson-Walker spacetime metric $ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2]$, we have the Friedmann-Roberston-Walker (FRW) model of the universe. If we include and separate out the cosmological constant term, the Friedmann equation gets modified to $(\frac{\dot{a}}{a})^2 + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\varrho + \frac{\Lambda}{3}$. We can also provide a measure of the rate of change of the expansion, known as the deceleration parameter, $q = -\frac{a\ddot{a}}{\dot{a}^2}$. Although he did not investigate the particular case, at a certain critical density the limiting case of his open hyperbolic or closed models produces a flat Euclidean model of infinite extent. The FRW model was not generally given much regard at first since most data appeared to show a generally static universe, but this would soon change.

In fact, there had already been hints. In 1868, Sir William Huggins had noted a Doppler shift appearing in some starlight. Then in 1912, Vesto Melvin Slipher had obtained data showing that spiral nebulae were red-shifted in a way consistent with expansion, however, since nobody knew what the nebulae were and nobody imagined, in particular, that they were actually distant galaxies, the data were not given much thought (although as far back as the 18th century some, such as Kant, Lambert, Swedenborg, and Wright, actually had philosophized that perhaps these nebulae were actually distant galaxies much like our own). Furthermore, Arthur Eddington already recognized in 1923 that Slipher's data would fit in with an expanding de Sitter model universe.

During the mid to late 1920's, astronomers began taking general notice that the nebulae were red-shifted. When Sir Edwin Hubble discovered the Cepheid variables within the Andromeda Nebulae in 1925, he conclusively demonstrated for the first time that the spiral nebulae were in fact extra galactic systems and thus settled at least the second half of what had become known as ‘The Great Debate’- what is the size of our galaxy and are the spiral nebulae members of our galaxy or separate ‘island universes’ well beyond it? This led the Belgian civil engineer, priest and later astrophysicist Georges Lemaître, in 1927, to independently derive the Friedmann solutions (they were not yet widely known) and to propose that the universe might not fit the static model but might instead have burst forth from a primeval atom. This was the original big bang proposal. None of Lemaître’s models for the Big Bang universe ultimately provided useful though as he ended up having to construct highly convoluted models in order to force them to be in agreement with both observational data for the age of the universe and an early and highly erroneous value for the Hubble constant which was off by an order of magnitude [1].

A couple of years later, in 1929, Hubble announced that almost all galaxies (the neighboring Andromeda is a notable exception) were moving away from us faster the farther away from us they were and that they were all receding away from each other. Then, in 1931, he went on to demonstrate this with data and Eddington brought to general light Lemaître’s work. It was then that the expanding universe solutions of the Friedmann equations finally began to attract considerable attention. One of the Eddington/Lemaître models described a closed but ever expanding universe that started out as an Einstein static universe and then later expanded at an ever increasing rate due to the presence of a positive cosmological constant. As objects within this universe were ultimately being expanded away from one another at faster than the speed of light, Eddington believed that the galaxies would eventually become casually disconnected from one another, each forming their own separate island universe [1,3]. Such ideas soon fell out of fashion since there was not even a hint of observational evidence suggesting the possibility, but they were to awaken again decades later when, in 1998, new observational data arrived on the scene [3].

It now appeared as if though the universe might be a dynamic one in which wherever one stood it would look as if everything else was receding away in all directions. Astronomers characterize the redshift of distant objects by $z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$ with $\lambda_{emitted}$ the wavelength of the light as emitted by the source and $\lambda_{observed}$ the wavelength of the light when it reaches the Earth. The scale factor of the universe at the time (t_1)light was emitted from an object of redshift z is given by $a(t_1) = \frac{1}{1+z}$. In the special relativistic Doppler shift formula one gets $1+z = (\frac{1+\frac{v}{c}}{1-\frac{v}{c}})^{\frac{1}{2}}$ and although the Doppler shift deals with relative velocity between two objects, the redshift is usually thought of as a Doppler shift in the sense that the wavelength as we receive it may appear to be longer or shorter than it was as it was emitted from the object. Some suggest to interpret it as infinite sums of infinitesimal shifts along the path [4]. For galaxies that are not too far off, astronomers can ignore the complications of general relativity and simply use $v = cz$ or, for the Hubble law, $v = Hd$ where $d(t) = a(t)r$ with r being the co-moving radial coordinate and $d(t)$ taken as the physical distance. This gives the linear formula whereby a galaxy's recession velocity is proportional to its distance to us. The value of H at the present time is $H_0 \simeq 70km/s/Mpc$.

It was at this time that Einstein suggested to remove the cosmological constant from his equation and told George Gamow that the greatest blunder of his life had been inserting it into his equation. However, the saga of the cosmological constant was far from over. It was resurrected in the 1940's and 1950's when it was discovered that the Hubble time was not consistent with the age of the Earth (producing an Earth older than the universe itself!). The cosmological constant was removed again, however, when it discovered that Hubble's original estimate of his constant had been off by an order of magnitude and the Hubble time actually had been consistent to begin with, although this was still not the last that was to be heard from the cosmological constant. It was ultimately discovered that it was a natural part of the equation. However, taken as a vacuum energy, the expectation would have been for it to have either a massively large value or, through some method of cancelation, a value of zero; anything but the vanishingly small and yet non-zero value that it is now known (since 1998) to have.

After the discovery of the expansion of the universe by Hubble, two main models for the universe emerged, the Steady State and the Big Bang.

The Big Bang model, which was promoted and expanded upon by Gamow, was a model of a dynamic, changing universe which started out as a singularity and then burst force across the entire volume of the universe from a primordial explosion after which it went on to pass through a very hot radiation dominated stage before proceeding to a matter dominated phase. In 1948, Gamow's coworkers Raplh Alpher and Robert Herman predicted, from their model, that there should exist a cosmic microwave background (CMB) radiation of about 5K leftover from the cooling of the surface of last scattering of the CMB (about 400,000 years after the alleged Big Bang). At this time the universe was cool enough for electrons and protons to combine into atoms and change from an opaque plasma to its present neutral and therefore transparent form.

On the other hand, in the Steady State model of Fred Hoyle, Thomas Gold, Hermann Bondi and others, new matter would be continually created as space expanded so that the universe would be, overall, time independent. This model met the Perfect Cosmological Principle which states the universe is homogenous and isotropic in both space AND time (on the largest scales, the universe has a surprisingly uniform distribution of matter and looks now the same as it always has and always will). Hoyle proposed a new C-Field that would have negative pressure to drive the expansion and that would also create new matter. Hoyle despised the Big Bang model and yet, ironically enough, was responsible for giving it its moniker, derisively referring to it as "that big bang idea" on various BBC programs and lecture series throughout 1949 and 1950. The moniker soon stuck, but without any of the negative connotation.

More recently, Andrei Linde tried to invoke eternal inflation featuring self-reproducing universes in order to avoid a beginning [5, 6]. However, Arvind Borde and Alexander Vilenkin demonstrated that a reasonable spacetime that eternally inflated into the future had to posses an initial singularity [7]. The idea was revisited by Aguirre in 2002 [8, 9]. In more modern forms, eternal inflation or chaotic inflation models of the universe have been introduced which while on smaller scales appear more in line with the Big Bang model appear to have similarities to the Steady State models when viewed on the grandest scales.

At first the community was split about fifty-fifty between the two competing models (the Big Bang and the Steady State), but by the 1960's the community had shifted largely behind the Big Bang model as more and more astronomical data hinted at a dynamic universe. For instance, quasars, seemed to show up in only very ancient galaxies. By 1964 it was realized that stellar production could never give the twenty-four percent by mass helium that was being seen throughout the universe while primordial nucleosynthesis could; this was actually shown by the calculations of Fred Hoyle and Roger Tayler in their 1964 paper in 'Nature'.

In early 1964 came a revolutionary event when two Bell Labs radio astronomers, Arno Penzias and Robert Wilson, just out of nearby Princeton graduate school, found a pesky 2.7K source of systematic error in the experiment they were carrying out at the Holmdel, NJ Bell Labs facility. By chance at a meeting they ran into someone familiar with work going on at Princeton and had it suggested to them that they get into contact with the gravitational physics group led by Robert H. Dicke.

Dicke was working on an oscillatory model for the universe and also independently arriving at the idea of a relic cosmic background radiation. Also a member of the group was Jim Peebles who worked out a clear prediction for a blackbody spectrum for the CMB in 1965 (they later came across Gamow's earlier little known 1950 research and prediction). It was the Princeton group that convinced Penzias and Wilson that is was the CMB that they had discovered (beating out the Princeton group's Peter G. Roll and David T. Wilkinson who had been setting up their own experimental search at the time).

This was, for the most part, the death knell for the Steady State models although there were still some further variants such as Zwicki's tired light model and the Quasi-steady State model introduced by Fred Hoyle and others in 1993. Both appeared problematic and never caught on.

The Penzias-Wilson discovery had an impact on competing Big Bang models as well. The uniformity of the CMB allowed it to rule out Oskar Klein and Hannes Alfven's localized explosion model for the Big Bang [1].

1.2 Cyclic Universe Background

However, one important class of cosmological model, of an entirely different nature, has been skipped over so far.

In the early 1930's, Tolman proposed an oscillatory model for the universe [10]. He, along with many others over the years, didn't like the idea of having an initial singularity or the requirement for initial conditions, both inherent with the Big Bang model, and considered the possibility of an ever existing, past and future complete, infinite cyclic cosmology. His proposal was based upon a mechanism naturally provided by a "closed" universe which provides a turnaround point after the universe has grown to a certain size, leading then to collapse back in on itself. The cyclic idea is it will then spring forth again and repeat an infinite number of times.

Tolman carried out the earliest extensive work on cyclic universes [10,11] and was for long their most ardent supporter although various others such as Markov and Dicke extensively investigated such models over the years too. However, these models all ran into various issues which rendered them problematic.

First, there is the second law of thermodynamics which leads to an entropy conundrum. The entropy of the universe would increase with each cycle. Since the greater the entropy in the universe the greater the maximum radius reached before turnaround, each cycle would grow larger and longer than the previous one. Thus tracing back in reverse from the current day one would get led back through ever smaller cycles until one reached a singularity, exactly what one had hoped to avoid [12]. These models of the universe would thus be future complete but could not be past infinitely cyclic. Second, even if the entropy conundrum were not present, there would still be the issue whereby large scale structure and black hole formation would lead to serious problems during the contracting phase. For instance, black holes could grow so large and copious during the contracting phase that it would cause a premature bounce or cause the equations to break down. Finally, mix-master issues can occur during the contracting stages which will cause wild oscillations of scale differentially along the dimensions of space and create serious anisotropy issues [13–16]. On a side note, in the late 1960's Charles Misner tried unsuccessfully to use mix-master effects to his advantage to get around the problems of causality and the uniformness of the CMB for which inflation would later be proposed [14].

Ultimately, all of these early cyclic models were dropped as being futile. The Big Bang emerged as the clear consensus model and, for all intents and purposes, all cyclic cosmological models appeared to be dead. There was a brief revival of such models by Markov in early to mid 1980's [12, 17] but they also proved to be problematic and the work on cyclic models was more or less abandoned once again. However, extra-dimensional theories slowly began coming into vogue again, such as String Theory, and then, in 1998, interesting astronomical data came to light [3] which would revolutionize physics. It appeared as if most of the universe was currently made up of an unknown energy called dark energy that was causing the universe to enter into a period of accelerated expansion.

At least since the time of Theodor Kaluza and Oskar Klein in the 1920's, there has been serious speculation that perhaps there are more dimensions than one of time and three of space (in fact Gunnar Nordstrom, in 1914, had already attempted to make use of an extra dimension when attempting to formulate his own theory of gravitation and combine it with electromagnetism [18]). In 1921, Kaluza proposed a theory [19] using an extra dimension of space, a 5th dimension, in an attempt to unify Einstein's theory of gravitation with Maxwell's electromagnetism. Miraculously, by simply adding an extra fifth dimension to Einstein's theory of gravitation it automatically produced the Maxwell equations of electromagnetism in addition to the Einstein equations for gravitation! However, since we don't observe any extra dimensions they must be hidden and their effects suppressed in some way.

In 1926 Klein proposed [20] that the extra 5th dimension might be a compactified dimension, a circular extra dimension appearing at each point in three dimensional space, this becoming known as Kaluza-Klein reduction or compactification. Imagining for simplicity only one instead of three standard spatial dimensions, the extra compactified dimension cam be envisioned like a garden hose of vanishingly small radius. Klein thereby solved the problem of why the extra dimension's effects appeared to be suppressed and why we don't notice them. Additionally, it explained why the electric charge is quantized [21].

However, Kaluza-Klein unification would not be the end of the story for numerous technical issues not to mention entirely new forces and particles which came to light. For decades it was thought that the only way in which extra dimensions could tenably arise was through compactification into exceedingly small size.

However, in 1998 Nima Arkani-Hamed, Savas Dimopoulos, and Gia Dvali (ADD) proposed a new model in an attempt to explain the relative weakness of gravity compared to the other forces. The ADD model made use of large, millimeter-scale extra dimensions [22]. The following year, Lisa Randall and Raman Sundrum introduced their "RS1" model featuring two branes and a large, warped extra dimension [23]. Shortly afterward, they introduced their "RS2" model which contained an extra dimension that was not only not compactified, but actually infinite in extent as they sent the second brane in RS1 to infinity [24]. From the five-dimensional RS model one can obtain an effective modified Friedmann equation in four-dimensions that contains a new ϱ^2 ($\varrho \equiv$ density) term in addition to the usual ϱ term. This form of modified Friedmann equation is not actually specific to this particular model but more general [25].

In braneworld scenarios like RS1 one takes the standard model particles to be constrained within a hyper-surface brane that lies within a higher-dimensional bulk. Gravity (and perhaps exotic particles) can travel throughout the bulk. Such models can arise, for instance, from within string theory such as through the strong coupling limit of the $E_8 X E_8$ Horava-Witten model [26]. Compacting six of eleven dimensions onto a Calabi-Yau manifold leaves an effectively five-dimensional spacetime with two four-dimensional boundary branes. In many models featuring two such branes, with the standard particles confined to one of the two, changing the moduli controlling the distance between the two branes within the bulk can effectively look, in four-dimensions, like an expansion or contraction of space. For example, in the RS models, if the brane moves as a function of $w(t)$ this induces a FRW scale factor of $a(t) = e^{-k|w|}$ [27, 28]. In some models the brane separation needs to be constrained to provide the proper spectrum of standard model particles and forces [25, 29, 30].

Also totally unexpected were the implications of new astronomical observations carried out in the late 1990's. In 1998 it was announced that the expansion rate of the universe was actually increasing at the current time instead of remaining constant or slowing down [3]. This was an entirely unexpected finding. The cause for the increasing expansion rate became referred to as dark energy. Dark energy is parameterized by an equation of state $w = \frac{p}{\varrho}$ with p the pressure and ϱ the density which goes as $\varrho \propto a^{-3(1+w)}$ in the Friedmann equation. More specifically, ϱ goes, depending upon whether it is radiation, matter, vacuum energy, or phantom energy as:

radiation: $p = \frac{\varrho}{3} \Rightarrow \varrho \propto \frac{1}{a^4}$

matter: p=0 $\Rightarrow \varrho \propto \frac{1}{a^3}$

vacuum energy: $p = -\varrho \Rightarrow \varrho \propto constant$

phantom energy: $p < -\varrho$ (taking, for one example, $p = -\frac{4}{3}\varrho \Rightarrow \varrho \propto a$)

with the latter phantom form capable of eventually leading to exceptional causal structure as will be employed in this dissertation.

With the discovery of dark energy and it's potentially exotic nature, and with renewed interest in extra-dimensions and the ensuing braneworlds and other constructions, it became natural to wonder if there might not be some way around the problems which had derailed prior attempts at constructing consistent cyclic models of the universe. Might the cyclic universe not be opened up again to possibility?

Work attempting to use some of the new possibilities began again, for instance in a 2001 model without the need for inflation, the Ekpyrotic model [31]. Bouncing braneworlds with an extra dimension of time such as [32] began to be investigated. Also making use of branes and phantom energy [33], which examined the consequences for cyclic cosmology.

After their initial Ekpyrotic model, Steinhardt and Turok began working on a derivative model [34–42] which could avoid a big bang and thereby made progress towards a cyclic model (although without resolving the entropy conundrum [43]).

1.3 Outline of a Cyclic Model

The rise of dark energy, only discovered in 1998 [3], proves to be a crucial ingredient required to create a viable cyclic model of the universe. If the equation of state is $w < -1$ (current observations are centered around -1 with some slightly favoring $w < -1$ [44]) then the universe may eventually enter a period of exceptional causal structure. Within the realms of models following the standard Friedmann equation, such a phantom dark energy would eventually lead to a Big Rip within a finite amount of time and tear apart the fabric of spacetime [45].

The Big Rip and replacement of dark energy by modified gravity were explored further in [46,47]. If one turns to a modified Friedmann equation ($H^2 = \frac{8\pi}{3M_p^2}(\varrho - \frac{\varrho^2}{2\sigma})$) where σ is a brane tension then at $\varrho_b = 2|\sigma|$ one finds $H = 0$ as required at a turnaround or bounce in cyclic cosmology. Such can arise in a variety of models, the Big Rip can be avoided as well as any crunch to a singularity at the other end of the cycle.

It is such ideas coupled to dark energy that provide new avenues to travel down in the hopes of being able to construct cyclic models of the universe which may be both future AND past complete.

We consider here a model where just before reaching the Big Rip a brane contribution creates a turnaround at which time the scale factor deflates to a very tiny fraction (f) of itself and only one casual patch is retained, while the other $\frac{1}{f^3}$ patches independently contract to separate universes.

The universe is causally fractionated into a tremendous number (more than one googol, 10^{100}) of independent patches. Contraction occurs with a much smaller universe which returns almost empty and eventually bounces just before a singularity would be hit and then enters into a brief period of standard inflationary expansion. Inflation injects a huge amount of entropy and so begins the next generically identical cycle and the new idea of how to overcome the seemingly impossible entropy conundrum.

CHAPTER 2

TURNAROUND IN CYCLIC COSMOLOGY

2.1 Basics of Model

If the dark energy has a super-negative equation of state, $\omega_\Lambda = \frac{p_\Lambda}{\varrho_\Lambda} < -1$, it leads to a Big Rip [45, 48] at a finite future time where there exist extraordinary conditions with regard to density and causality as one approaches the Rip. We next explore whether these exceptional conditions can assist in providing an infinitely cyclic model.

We consider a model where, as we approach, expansion stops just short of the Big Rip due to a brane-like contribution. There is a turnaround at time $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently to separate universes. Turnaround takes place an extremely short time $(< 10^{-27}s)$ before the Big Rip would have occurred, at a time when the universe is fractionated into many independent causal patches [47].

There follows contraction which occurs with a very much smaller universe than in expansion and with almost vanishing entropy because it is assumed empty of dust, matter and black holes and where even photons have become infinitely redshifted.

On the other end of a cycle, a bounce takes place a short time before a crunch into a singularity would have occurred (based on present day knowledge). After the bounce, entropy is injected by inflation [49], where it is assumed that an inflaton field is excited. Inflation is thus a part of the present model which is one distinction from the work of [34–37]. For cyclicity of the entropy ($S(t) = S(t+\tau)$) to be consistent with thermodynamics it is necessary that the deflationary decrease by f^3 will compensate for the entropy increase acquired during expansion including during inflation.

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies [43], but to our understanding for the model we outline this problem is avoided (provided that the time average of the Hubble parameter during expansion is equal in magnitude and opposite in sign to its average during contraction).

2.2 Friedmann Equation for Expansion Phase

Let the period of the Universe be designated by τ and the bounce take place at $t = 0$ and turnaround at $t = t_T$. Then we have the expansion phase for times $0 < t < t_T$ and the contraction phase corresponds for times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\varrho_\Lambda)_0}{a(t)^{3(\omega_\Lambda+1)}} + \frac{(\varrho_m)_0}{a(t)^3} + \frac{(\varrho_r)_0}{a(t)^4}\right) - \frac{\varrho_{total}(t)^2}{\varrho_c}\right] \tag{1}$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \simeq 14Gy$. To explain the notation, $(\varrho_i)_0$ denotes the value of the density ϱ_i at time $t = t_0$. The first two terms are the dark energy and total matter (dark plus luminous) satisfying

$$\Omega_\Lambda = \frac{8\pi G(\varrho_\Lambda)_0}{3H_0^2} = 0.72 \quad \text{and} \quad \Omega_m = \frac{8\pi G(\varrho_m)_0}{3H_0^2} = 0.28 \tag{2}$$

where $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the radiation density which is now $\Omega_r = 1.3 \times 10^{-4}$. The final term $\sim \varrho_{total}(t)^2$ is derivable from a braneworld construct [23,24,33,50]; we use a negative sign naturally arising from negative brane tension (the negative sign can arise also from a second time-like dimension that causes issues with closed time-like paths). In this third term, $\varrho_{total} = \Sigma_{i=\Lambda,m,r}\varrho_i$.

As the turnaround is approached, the only significant terms in Eq.(1) are the first (where $\omega_\Lambda < -1$) and the last. As the bounce is approached, the only important terms in Eq.(1) are the third and the last. (We shall later argue that the second term, for matter, is absent during contraction.) In particular, the final term of Eq. (1), $\sim \varrho_{total}(t)^2$, arising from the braneworld construct is insignificant for almost the entire cycle but becomes dominant as one approaches $t \to t_T$ for the turnaround and again for $t \to \tau$ as once approaches the bounce.

2.3 Turnaround

Let us assume for algebraic simplicity that $\omega_\Lambda = -4/3 = $ constant. This value is already almost excluded by WMAP3 [51] but to begin we are aiming only at consistency of infinite cyclicity. More realistic values are left for future investigation. With the value $\omega_\Lambda = -4/3$ we learn from [46] that the time to the Big Rip is $(t_{rip} - t_0) = 11\text{Gy}(-\omega_\Lambda - 1)^{-1} = 33\text{Gy}$ which is, as we shall discuss, within 10^{-27} second or less of when turnaround occurs at $t = t_T$. So if we adopt $t_0 = 14Gy$ then $t_T = t_0 + (t_{rip} - t_0) \sim (14 + 33)Gy = 47Gy$. From the analysis in [45–48] the time when a system becomes gravitationally unbound corresponds approximately to the time when the dark energy density matches the mean density of the bound system. For an object like the Earth or a hydrogen atom, water density ϱ_{H_2O} is a practical unit.

With this in mind, for the simple case of $\omega = -4/3$ we see from Eq.(1) that the dark energy density grows proportional to the scale factor $\varrho_\Lambda(t) \propto a(t)$ and so given that the dark energy at present is $\varrho_\Lambda \sim 10^{-29} g/cm^3$ it follows that $\varrho_\Lambda(t_{H_2O}) = \varrho_{H_2O}$ when $a(t_{H_2O}) \sim 10^{29}$. We can estimate the time t_{H_2O} by taking on the RHS of the Friedmann equation only dark energy $\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_\Lambda a^{-\beta}$ with $\beta = 3(1+\omega)$. When we specialize to $\omega = -4/3$ it follows that

$$\frac{a(t_{H_2O})}{(a(t_0) = 1)} = \left(\frac{(t_{rip} - t_0)}{(t_{rip} - t_{H_2O})} \right)^2 \tag{3}$$

so that $(t_{rip} - t_{H_2O}) = 33Gy \times 10^{-14.5} \simeq 10^{3.5}s \sim 1$ hour. [The value is sensitive to ω.] It is instructive to consider an approach to the Rip using a more general critical density $\varrho_c = \eta\varrho_{H_2O}$ and to compute the time $(t_{rip} - t_\eta)$ such that $\varrho_\Lambda(t_\eta) = \varrho_c = \eta\varrho_{H_2O}$. We then find, using $a(t_\eta) = 10^{29}\eta$, that[#1]

$$(t_{rip} - t_\eta) = (t_{rip} - t_0)10^{-14.5}\eta^{-\frac{1}{2}} \simeq \eta^{-\frac{1}{2}}\text{hours} \tag{4}$$

which is the required result. We shall see that $\eta > 10^{31}$ so the time in (4) is $< 10^{-27}s$.

To discuss the turnaround analytically we keep only the first and last terms, the only significant ones, on the RHS of Eq.(1) which becomes for the special case $\omega = 4/3$

$$\left(\frac{\dot{a}}{a}\right)^2 = \alpha_1 a - \alpha_2 a^2 \tag{5}$$

in which

$$\alpha_1 = \frac{8\pi G}{3}(\varrho_\Lambda)_0 \qquad \alpha_2 = \frac{8\pi G}{3}\frac{(\varrho_\Lambda)_0^2}{\varrho_c} \tag{6}$$

Writing $a = z^2$ and $z = (\alpha_1/\alpha_2)^{1/2} sin\theta$ gives

$$dt = \frac{2\sqrt{\alpha_2}}{\alpha_1}\frac{d\theta}{sin^2\theta} = \frac{2\sqrt{\alpha_2}}{\alpha_1}d(-cot\theta) \tag{7}$$

Integration then gives for the scale factor

$$a(t) = \left(\frac{\alpha_1}{\alpha_2}\right) sin^2\theta = \frac{\varrho_c}{(\varrho_\Lambda)_0}\left[\frac{1}{1 + \left(\frac{t_T - t}{C}\right)^2}\right] \tag{8}$$

where $C = -(3/2\pi G\varrho_c)^{1/2}$. At turnaround $t = t_T$, $a(t_T) = [\varrho_C/(\varrho_\lambda)_0] = (a(t))_{max}$. At the present time $t = t_0$, $a(t_0) = 1$ and $sin^2\theta_0 = [(\varrho_\Lambda)_0/\varrho_C] \ll 1$, increasing during subsequent expansion to $\theta_T = \pi/4$.

A key ingredient in our model is that at turnaround, $t = t_T$, our universe

[#1] note the correction to the exponent of η in Eq.(4) of [52] deflates dramatically with an effective scale factor $a(t_T)$ shrinking before contraction to $\hat{a}(t_T) = fa(t_T)$ where $f < 10^{-28}$. This jettisoning of almost all, a fraction $(1-f)$, of the accumulated entropy need be permitted by the exceptional causal structure of the universe in some fashion. We shall see later that the parameter η at turnaround lies in the range $\eta = 10^{31}$ to $\eta = 10^{87}$ which implies a dark energy density at turnaround (Planckian density of $\varrho_\Lambda \sim 10^{104}\varrho_{H_2O}$ can be avoided) such that, according to the Big Rip analysis of [46, 47], all known, and yet unknown smaller, bound systems have become unbound and the constituents causally disconnected. Recall that the density of a hydrogen atom is approximately ϱ_{H_2O} and we are reaching a dark energy density of from 31 to 87 orders of magnitude higher.

According to these estimates, at $t = t_T$ the universe has already fragmented into an astronomical number $(1/f^3)$ of causal patches, each of which independently contracts as a separate universe leading to an infinite multiverse. The entropy at $t = t_T$ is thus divided between these new contracting universes and our universe retains only a fraction f^3. Since our model universe has cycled an infinite number of times, the number of parallel universes is infinite.

2.4 Deflation

A central assumption in our cyclic model is that almost all of the entropy is jettisoned at turnaround by the retention of one causal patch. We cannot justify this step rigorously but hope to convince the reader by the following physical argument. Let us take a bounce temperature $T_B = 10^p$ GeV with $p > 3$. This gives (see below) $\eta = 10^{(19+4p)}$ and hence, from Eq.(4), $(t_{rip} - t_T) \sim 10^{-(19+4p)}$ hours. The dark energy density $\varrho_c \sim 10^{(19+4p)} g/cm^3$ at turnaround implies the prior disintegration of all bound systems with mean density $\varrho < \varrho_c$, which for $p > 3$ includes atoms, nuclei, nucleons ($10^{15} g/cm^3$) and even smaller bound systems, if any. As shown in [45, 46, 48], at a similar time, actually somewhat but not too much later, these constituents become causally disconnected. For such a density, a generic causal patch contains no quarks or leptons, only dark energy together with a small number of highly infra-red photons. Black holes are also absent, having been torn apart by the approach to the would-be Big Rip *e.g.* [33]. The entropy of such a patch is essentially zero, by which we mean $S = O(10^1)$ compared to the earlier $S > 10^{88}$. This dramatic decrease in entropy is called deflation for obvious reasons. During contraction, as we shall describe, the entropy remains constant at essentially zero because dark energy has zero entropy and radiation contracts adiabatically. Although these heuristic arguments about deflation seem clear, a more rigorous justification would certainly be most desirable.

2.5 Friedmann Equation for Contraction Phase

The contraction phase for our universe occurs for the period $t_T < t < \tau$. The scale factor during the contraction phase will be denoted by $\hat{a}(t)$ while we always use the same linear time t subject to the periodicity $t + \tau \equiv t$. At the turnaround we retain a fraction f^3 of the entropy with $\hat{a}(t_T) = f a(t_T)$. For the contraction phase, the Friedmann equation is

$$\left(\frac{\dot{\hat{a}}(t)}{\hat{a}(t)}\right)^2 = \frac{8\pi G}{3}\left[\left(\frac{(\hat{\varrho}_\Lambda)_0}{\hat{a}(t)^{3(\omega_\Lambda+1)}} + \frac{(\hat{\varrho}_r)_0}{\hat{a}(t)^4}\right) - \frac{\hat{\varrho}_{total}(t)^2}{\hat{\varrho}_c}\right] \tag{9}$$

where we have defined

$$\hat{\varrho}_i(t) = \frac{(\varrho_i)_0 f^{3(\omega_i+1)}}{\hat{a}(t)^{3(\omega_i+1)}} = \frac{(\hat{\varrho}_i)_0}{\hat{a}(t)^{3(\omega_i+1)}} \tag{10}$$

In contrast to Eq.(1), however, we have set $\hat{\varrho}_m = 0$ because our hypothesis is that the causal patch retained in the model contains only dark energy and radiation but no matter (including no black holes). This is necessary because during a contracting phase dust or matter would clump, even more readily than during expansion, and inevitably interfere with cyclicity. Perhaps more importantly, presence of dust or matter would require that our universe go in reverse through several phase transitions (recombination, QCD and electroweak to name a few) which would violate the second law of thermodynamics. We thus require that *our universe comes back empty!* Note that any tiny entropy associated with radiation will be constant during adiabatic contraction.

The contraction of our universe will proceed from one of the $1/f^3$ causal patches following Eq.(9) until the radiation balances the brane tension at the bounce.

2.6 Bounce

At the bounce, the contraction scale is given, using $\varrho_c = \eta \varrho_{H_2O}$, from Eq. (1) as

$$a(\tau)^4 = \left(\frac{(\varrho_r)_0}{\eta \varrho_{H_2O}} \right) \tag{11}$$

Now the model's bounce at $t = \tau$ must be before the electroweak transition at $t_{EW} = 10^{-10}s$ when $a(t_{EW}) = 10^{-15}$, and after the Planck scale when $a(t_{Planck}) = 10^{-32}$ in order to accommodate the well established weak transition and to avoid uncertainties associated with quantum gravity. With this in mind, here are three illustrative values (A, B, C) for the bounce temperature T_B:

- (A) At a GUT scale $T_B = 10^{17} GeV, a(t_B) = 10^{-30}$.
- (B) At an intermediate scale $T_B = 10^{10} GeV, a(t_B) = 10^{-23}$.
- (C) At a weak scale $T_B = 10^3 GeV, a(t_B) = 10^{-16}$.

From Eq.(11) and Eq.(4) for these three cases one finds

- (A) $\eta = 10^{87}$ and $(t_{rip} - t_T) = 10^{-87} hr$.
- (A) $\eta = 10^{59}$ and $(t_{rip} - t_T) = 10^{-59} hr$.
- (A) $\eta = 10^{31}$ and $(t_{rip} - t_T) = 10^{-31} hr$.

Immediately after the bounce, we assume that an inflaton field is excited and there is conventional inflation with enhancement $\mathrm{E} = a(\tau + \delta)/\hat{a}(\tau)$. Successful inflation requires $E > 10^{28}$. Consistency requires therefore $f < E^{-1}$ to allow for the entropy accrued during expansion after inflation. The fraction of entropy jettisoned from our universe at deflation is thus extremely close to one, being less than one and more than $(1 - 10^{-28})^3$.

2.7 Entropy

Consider first the present epoch $t = t_0$. The contributions of radiation to the entropy density s follow the relation

$$s = \frac{2\pi^2}{45} g_* T^3 \tag{12}$$

Photons contribute $g_* = 2$. The present CMB temperature is $T = 2.73K \equiv 0.235meV \sim 1.191(mm)^{-1}$. Substitution into Eq.(12) gives a present radiation entropy density of $s_\gamma(t_0) = 1.48(mm)^{-3}$. Using a volume estimate $V = (4\pi/3)R^3$ with $R = 10Gly \simeq 10^{29}mm$ gives a total radiation entropy of $S_\gamma \sim 6.3 \times 10^{87}$. Including neutrinos increase g_* in Eq.(12) from $g_* = 2$ to $g_* = 3.36 = 2 + 6 \times (7/8) \times (4/11)^{4/3}$. This increases $S_\gamma = 6.3 \times 10^{87}$ to $S_{\gamma+\nu} \sim \times 10^{88}$.

This total entropy is interpretable as $exp(10^{88})$ degrees of freedom, or in information theory [53] to a number I of qubits where $2^I = e^S$ so that $I = S/(ln2 = 0.693) \sim 10^{88}$. This is well below the holographic bound which is dictated by the area in terms of Planck units $10^{-64}mm^2$ which gives $S_{holog}(t_0) = 4\pi(10^{29}mm)^2/(10^{-32}mm)^2 \sim 10^{123}$ or about 10^{35} times bigger. In [53] it is suggested that at least some of this difference may come from super-massive black holes. The entropy contribution from baryons is smaller than S_γ by some ten orders of magnitude so, like that from the dark matter, it is negligible.

What is the entropy of the dark energy? If it is perfectly homogeneous and non-interacting it has zero temperature and entropy. Finally, as we have already estimated, the fourth term in Eq.(1), corresponding to the brane term, is negligible. The conclusion is that at present $S_{total}(t_0) \sim 10^{88}$.

Now consider the entropy approaching turnaround at $t = t_T$. We have estimated that $a(t_T) = 10^{29}\eta$ and representative values for $\eta = \varrho_c/\varrho_{H_2O}$ are $10^{31}, 10^{59}$ and 10^{87}. The temperature T_γ of the radiation scales as $T_\gamma \propto a(t)^{-1}$ so using the entropy density of Eq.(12) a co-moving 3-volume $\propto a(t)^3$ will contain the same total radiation entropy $S_\gamma(t_T) = S_\gamma(t_0)$ as at present; this is simply the usual adiabatic expansion. The expansion from $t = 0$ to t_T is not purely adiabatic because irreversible processes take place. The first is inflation which increases entropy by $> 10^{84}$. There are also phase transitions such as the electroweak transition at $t_{EW} \sim 100ps$, the QCD phase transition at $t_{QCD} \sim 100\mu s$, and recombination at $t_{rec} \sim 10^{13}s$. Furthermore, there are irreversible processes that occur during stellar evolution. Although the expansion of the radiation, the dominant contributor to the entropy, is adiabatic, the entropy of matter increases in accord with the second law of thermodynamics. In our model, the entropy of the matter increases between $t = 0$ and $t_T \sim 47Gy$. Setting the entropy of the dark energy to zero and with the the radiation acting adiabatically, it is the matter part, represented by ϱ_m, which would cause the entropy to rise from $S(t = 0)$ to $S(t_T) = S(t = 0) + \Delta S$. It is this ΔS that was one of the plagues of previous oscillatory model universes [2, 10, 11].

Our main point is that in order for entropy to be cyclic, the entropy which was enhanced by a huge factor $E^3 > 10^{84}$ at inflation must then be reduced dramatically at some point during the cycle so that satisfying $S(t) = S(t + \tau)$ becomes possible. Since entropy increases during regular expansion and contraction, the only logical possibility is to have the decrease occur right at the approach to and at turnaround as accomplished by phantom dark energy and our causal patch idea. The second law of thermodynamics continues to obtain for other causal patches taken as a whole, each with practically vanishing entropy at turnaround, but these are permanently removed from the consideration of any particular universe under inspection, as they all contract instead into separate universes.

During contraction, $t_T < t < \tau$, we are assuming the universe is empty of matter until the bounce so its entropy is vanishingly small. Immediately after the bounce, inflation arises from an inflaton field, assumed to be excited. We find the counterpoise of inflation at the bounce and deflation at turnaround an appealing aspect of the present model. And this would be furtherso, were the phantom dark energy and inflaton be related or interacting, especially in a self-perpetuating manner.

2.8 Conclusion

The standard cosmology based on a Big Bang augmented by an inflationary era is impressively consistent with the detailed data from WMAP3 [51] if dark energy, most conservatively taken as a cosmological constant, is included. Objections to this standard model are more aesthetic than motivated directly by observations (although certain polarizations of the CMB, were they to be discovered, might well lead to some discord). The first potential objection is the nature of the initial singularity and the initial conditions. A second potential objection, if not universally shared either, is that the predicted fate of the universe is an infinitely long expansion. Regardless of whether or not one finds these issues more troubling than the nature of an infinitely oscillatory cosmology, and there are many on either side of the aisle, we are driven by the curiosity as to whether -and it is instructive to see as well- or not such a model of the universe may even be possible even in theory. We have outlined here a cyclic cosmology resting on phantom dark energy where these potential aesthetic objections of some are ameliorated: the classical density and temperature never become infinite and future expansion is truncated. Also, our proposal of deflation naturally leads to a multiverse picture, somewhat reminiscent of that predicted in eternal inflation (and perhaps even including the infinities of eternal inflation within our model's own infinities). Here though our new proliferation of infinite universes originates at the opposite end of a cyclic cosmology, at its maximum rather than at its minimum size.

CHAPTER 3

ENTROPY OF CONTRACTING UNIVERSE IN CYCLIC COSMOLOGY

3.1 Entropy

To recapitulate, the most important new ingredient in our model is the idea that the contracting universe has essentially zero entropy and comes back empty of matter. Taking this model seriously [#2] we continue on to examine more critically some general features including its possibility of being tested.

The contracting universe of the cyclic model contains phantom dark energy with zero entropy and perhaps a small amount of radiation which could possess entropy. The deflation at turnaround reduces entropy from a gigantic value O($> 10^{88}$) to an extremely low value O(10^1). We will next proceed to study the entropy of the contracting universe in our speculative scenario more quantitatively, now taking an arbitrary $\omega = -1 - \phi$ with $\phi > 0$ such that $\varrho_\Lambda \propto a^{3\phi}$.

The quantity ϕ is the most important parameter for observational discrimination between our cyclic model which takes dark energy to be phantom in nature and one taking dark energy as the cosmological constant [#3]. The next test of $\phi \neq 0$ will likely come from the Planck Surveyor satellite [54]. One wonders, therefore, how different from zero ϕ is? There is no lower bound necessary on ϕ for our model to work other than it must be non-zero. We already know that $\phi \lesssim 0.1$ from the WMAP3

[#2]It was once memorably stated by Noble laureate Steven Weinberg [55] that "the problem is not that theorists take their model too seriously but that they do not take it seriously enough".

[#3]and from the Steinhardt-Turok cyclic model [34–36].

data [51]. If ϕ is truly infinitesimal, the test must, alas, await improved technology. To restore optimism, at the end of this section will be provided an anthropic fine tuning argument demonstrating that an extremely small value for ϕ is unlikely.

We have emphasized that the universe comes back empty of matter, including black holes. The presence of matter during contraction causes apparently insuperable problems because accelerated structure formation will precipitate a premature bounce or lead to other even more dire issues. Black holes, if present, will expand and proliferate with the same consequences. But the presence of radiation must also be carefully studied because, although at turnaround the photon energy is infinitesimal ($E_\gamma \lesssim 10^{-200} eV$), the blue shifting during contraction leads to production of e^+e^- pairs before the bounce. This is undesirable because, generically, they will create problems with continued contraction. As we shall proceed to show, there are fortunately no photons in the contracting phase of the cycle, only the presumably innocuous dark energy.

Our cyclic model contains but one free parameter, ϱ_C, the common density at which the universe both turns around and bounces. Since the bounce is independent of ω, we begin with it and take as bounce temperatures $T_B = 10^p$ GeV, so as to be above the weak and below the Planck scales, with $3 \leq p \leq 17$. Using the derivation from section 2, this gives $\varrho_C = \eta \varrho_{H_2O}$ where $\eta = 10^{(19+4p)}$ and $\varrho_{H_20} = 1 g/cm^3$ is the density of water. The density of water being an easily imaginable unit somewhere between the unimaginably small present mean cosmic density and the unimaginably large critical density ϱ_C at turnaround and bounce.

Going now to the turnaround at time $t = t_T$, the scale factor $a(t_T)$ is given by (since $a(t_0) = 1$ and taking $\varrho_0 = 10^{-29} \varrho_{H_2O}$)

$$a(t_T)^{3\phi} = 10^{29} \eta = 10^{48+4p} \tag{13}$$

The present radiation temperature is $(T_\gamma)_0 = 2 \times 10^{-4}$ eV, and so from Eq.(13) the radiation temperature at turnaround is

$$(T_\gamma)_T = 2 \times 10^{-4} \left(10^{(48+4p)}\right)^{-1/3\phi} \text{eV} \tag{14}$$

which is infinitesimal. Putting $\phi = 0.1$, Eq.(14) gives 10^{-200} eV for p=3 and 10^{-390} eV for p=17; with $\phi = 0.01$, the photon energy is 10^{-2000} eV for p=3 and 10^{-3900} eV for p=17. In all cases, the photon wavelength is an astronomical number of orders of magnitude longer than the present Hubble length.

To evaluate the entropy during contraction, we need to estimate how many such photons there will be per causal patch at turnaround. The deflationary factor multiplying entropy at turnaround must be much less than the inverse of the inflationary increase ($\gtrsim 10^{84}$) of the early universe. We take the huge number of causal patches to be $10^{90}\alpha$ where $\alpha \gg 1$ is a parameter to allow an arbitrarily larger number, and $\alpha = 1$ will give an overestimate of the contraction entropy.

At turnaround the scale factor is

$$a(t_T) = \left(10^{(48+4p)}\right)^{\frac{1}{3\phi}} \tag{15}$$

so taking the present volume as $10^{84} cm^3$ and the present radiation density to be $\varrho_r(t_0) = 10^{-33} g/cm^3 = 1eV/cm^3$ gives us the radiation energy in one causal patch:

$$(E_r)_{patch} = \frac{1}{(100\alpha)^3} \left(10^{(48+4p)}\right)^{-\frac{1}{3\phi}} \quad \text{eV} \tag{16}$$

Comparison with Eq.(14) then gives the number of photons per causal patch to be

$$n_\gamma = \frac{1}{200\alpha^3} \ll 1 \tag{17}$$

which is small even for the unrealistic case $\alpha = 1$ and essentially zero for $\alpha \gg 1$.

Thus, the entropy of the contracting universe (cu) vanishes $S_{cu} = 0$ for any value of the equation of state of the dark energy $\omega = p/\varrho = -1 - \phi$ since Eq.(17) has no ϕ dependence.

3.2 Anthropic Fine Tuning Argument About ϕ

The time until turnaround is given, *e.g.* [46], by

$$(t_T - t_0) \simeq \frac{t_0}{\phi} \tag{18}$$

so, if we take for simplicity the origin of life to have occurred at t_0, after the most recent bounce we see from Eq. (18) that given small $\phi \ll 1$ then ϕ will measure the fraction of the expansion phase taken to originate life. An anthropic argument is: it is unreasonable for the fraction ϕ, assuming it is non-zero, to be extremely close to zero.

The special case $\phi = 0$ is the standard cosmological model with a cosmological constant where there is no turnaround and the future lifetime is infinite so the origin of life necessarily takes place after a vanishing fraction of the expansion lifetime. Such an infinite expansion seems unaesthetic to some, but it is not a universal concern. The vanishing fraction coincidence is puzzling however.

As soon as one commits to $\phi \neq 0$, however, the anthropic type argument emerges and it is unlikely that $\phi <<< 1$. For example, if $\phi = 10^{-3}$ then the length of the expansion phase is 10^4 Gy whereas life originated after only about 10 Gy which is only 0.1% of the expansion time. If life plays a central role in our universe, as in our understanding is the spirit of the anthropic principle, such a tiny value of ϕ is strongly disfavored; one expects at least $\phi \gtrsim 0.01$ so the fraction before the origin of life is $\gtrsim 1.0\%$ of the total expansion time.

This encouraging argument makes it more optimistic that the next generation of observations such as the Planck Surveyor [54] will succeed in detecting a $\phi \neq 0$.

CHAPTER 4

CONSTRAINTS ON DEFLATION FROM THE EQUATION OF STATE OF DARK ENERGY

4.1 Background

Our model involves two key ideas: (i) that the universe deflate just prior to the turnaround from expansion to contraction by disintegrating into a very large number $N_{\rm cp}$ of causal patches (in our notation, note that $N_{\rm cp} = 1/f^3$); (ii) that the contracting universe be empty, meaning that one causal patch at turnaround must contain no matter or black holes, only dark energy. This is called the CBE condition (*Comes Back Empty*). Implementation of CBE requires, as we shall explain, a lower bound on N_{cp} which depends on the length scale L characterizing the smallest bound system. We shall consider both $L = 10^{-15}$m for a nucleon then $L \geq 10^{-35}$m for a PPP (*Presently Point Particle*) meaning a particle which at present is considered to be point-like, such as a quark or lepton, but which might potentially have a characteristic size greater than or equal to the Planck scale, if at least a few orders of magnitude smaller than a nucleon.

In the foreseeable future, it is expected that the equation of state of dark energy w, and hence ϕ, will be measured with higher accuracy by, for example, the Planck Surveyor satellite [54]. What we shall show is that this measurement can, within this model, constrain for a given w the number $N_{\rm cp}$ of causal patches at turnaround by imposing a lower bound thereon.

In the present chapter, the times at which unbinding, causal disconnection and turnaround occur are discussed. After that, the constraints on $N_{\rm cp}$ from measurement of ϕ are derived followed by more discussion (also see Appendices A-C for more technical material supplementing the main text here).

4.2 Times of Unbinding, Causal Disconnection and Turnaround

In this sub-section we analyze four relationships between cosmic times, in addition to the present time t_0, in the cyclic model expansion era: i) $t_{\rm unbound}$ (at which point a bound system will become unbound due to the large phantom dark energy force with $w < -1$); ii) $t_{\rm caus}$ (at which point a previously bound system becomes casually disconnected, meaning that no light signal could exchange before the would-be Big Rip; this is how we estimate $N_{\rm cp}$); iii) t_T (at which point the turnaround occurs); iv) $t_{\rm rip}$ (at which point a "would-be" Big Rip would have taken place).

There are three parameters: w (the equation of state of dark energy), ϱ_C (the critical density when the total density in the system $\varrho_{\rm tot}$ is reached at $t = t_T$), f (the deflation fraction parameter related to the number of causal patches by $N_{\rm cp} = (1/f^3)$). We proceed to analyze the model taking w to lie within the range

$$-1.10000 \leq \omega \leq -1.00001 \,, \tag{19}$$

and ϱ_C within

$$(10^3\,{\rm GeV})^4 \leq \varrho_C \leq (10^{19}\,{\rm GeV})^4 \,. \tag{20}$$

The choice of the lower bound on the range of w is motivated by current observations [51, 56] while we are led to the upper bound by the cosmic variance uncertainty in this measurement.

Reserving details of their derivation to Appendix A, we shall here refer to the resultant expressions:

- $(t_{\rm rip} - t_0)$

$$t_{\rm rip} - t_0 \simeq \frac{11\,{\rm Gyr}}{|1+w|}\,. \tag{21}$$

- $(t_{\rm rip} - t_{\rm unbound})$

$$t_{\rm rip} - t_{\rm unbound} = \alpha(w) P \tag{22}$$

where [45]

$$\alpha(w) = \frac{\sqrt{2|1+3w|}}{6\pi|1+w|} \tag{23}$$

and P denotes the period associated with the binding force which had been constraining objects into a certain bound system before $t = t_{\rm unbound}$.

- $(t_{\rm rip} - t_{\rm caus})$

$$t_{\rm rip} - t_{\rm caus} = \left|\frac{1+3w}{3(1+w)}\right| \left(\frac{L}{c}\right) \tag{24}$$

where c is the speed of light and L stands for the length scale of the bound system [45].

- $(t_{\rm rip} - t_T)$

$$t_{\rm rip} - t_T = \frac{11\,{\rm Gyr}}{|1+w|} 10^{-14.5} \eta^{-\frac{1}{2}} \tag{25}$$

where η is a scale factor of ϱ_C defined by $\varrho_C = \eta \varrho_{H_2O}$ with ϱ_{H_2O} being the density of water, $\varrho_{H_2O} = 1{\rm g} \cdot {\rm cm}^{-3}$.

The numerical analysis for these relationships is presented in Appendices A and B. As a result, we find the lower bound for ϱ_C,

$$\varrho_C \gtrsim (10^{18}\,\mathrm{GeV})^4, \tag{26}$$

which is obtained by imposing that a presently point particle (PPP), having a size $10^{-33}\mathrm{m} = \mathrm{L}$, satisfy for time: $t_{\mathrm{rip}} > t_T > t_{\mathrm{caus}}^{\mathrm{PPP}} > t_{\mathrm{unbound}}^{\mathrm{PPP}}$. It should be emphasized that this result is almost independent of the choice of w within the range of interest.

For a nucleon with $L \simeq 10^{-15}\mathrm{m}$, the corresponding lower bound is

$$\varrho_C \gtrsim (10^{9}\,\mathrm{GeV})^4\,. \tag{27}$$

4.3 Given w, the constraints on N_{cp}

In the chapter, as in [45], various bound systems have been discussed including galaxies, the Earth-Sun system, the hydrogen atom and a nucleon. Each may be characterized by a present length scale L_0.

For the CBE condition we must insist that the smallest bound systems, whose present length scale is $L(t_0) = L_0$, are disintegrated before turnaround which means that the size of a generic causal patch L_{cp} (to be defined below) must be smaller than $L(t_T)$ at the turnaround, namely

$$L_{\mathrm{cp}} \leq L(t_T) = L_0 \left(\frac{a(t_T)}{a(t_{\mathrm{unbound}})} \right). \tag{28}$$

We remind the reader that the CBE condition is mandatory because if the contracting universe contains matter it will not generally contract sufficiently and will undergo a premature bounce. Even if a causal patch contains only one very infra-red photon, this can blue-shift to an energy sufficient to create e^+e^- pairs before the bounce, again disallowing sufficient contraction for infinite cyclicity.

We have already demonstrated that the mean number of low-energy photons per causal patch is very much less than one and therefore almost every patch contains no photons. There will always be a vanishing but strictly non-zero number of patches which fail to cycle but it can clearly be seen that this will not overwhelm the overall infinity and will not ruin the model as was verified by direct calculation in [57, 58]. The probability of a successful universe is equal to one. The total number of universes has always been, and always will be constantly infinite and equal to $\aleph_0$ (Aleph-zero). $\aleph_0$ is a countable infinity, exemplified by the number of primes, integers or rational numbers.

To enable infinite cyclicity we must have the CBE condition, Eq.(28), for the smallest bound systems. The smallest bound systems we know of are nucleons with $L_0 = 10^{-15}$m.

To be general, we consider PPPs. We allow a bound state scale for PPPs to be anywhere between the present upper limit of about $(1\text{TeV})^{-1} = 10^{-18}$m and the Planck scale of 10^{-35}m. As we shall see shortly, the lower bound on N_{cp} is so sensitive to where L_0 is chosen within these twenty orders of magnitude that its presentation requires us to plot $\log_{10}\log_{10} N_{\text{cp}}$ against the equation of state of dark energy.

The present Hubble length $r_H(t_0)$ is given by

$$r_H(t_0) = \frac{1}{H_0} \tag{29}$$

which, at the turnaround, would naively become

$$r_H(t_T) = r_H(t_0)a(t_T) \tag{30}$$

since by definition $a(t_0) = 1$.

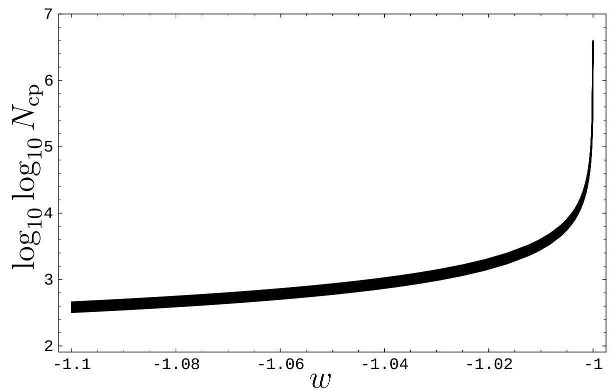

Figure 1: Constraint on w and $N_{\rm cp}$ coming from the CBE condition (Comes Back Empty), corresponding to inequality (28). The black band in this figure has been created by varying the length of smaller bound systems from $L_p(t_0) = 10^{-33}$m to $L_p(t_0) = 10^{-15}$m; the bottom edge corresponds to the lower value of $L_p(t_0)$. The region below the black band is forbidden by the CBE condition.

In our cyclic model, the size of a causal patch $L_{\rm cp}$ is instead defined by

$$L_{\rm cp} = \frac{r_H(t_T)}{N_{\rm cp}} \tag{31}$$

and therefore Eq.(31) can be calculated for different values of L_0, see Appendix C. The results are illustrated in Figure 1 where we plot $\log_{10}\log_{10} N_{\rm cp}$ versus $w = -1-\phi$.

From this figure we see that a measurement of w in the range anticipated for the Planck surveyor will provide a lower bound on $N_{\rm cp}$. For example $w = -1.05$ implies $N_{\rm cp} \gtrsim 10^{630}$ for disintegration of nucleons and $N_{\rm cp} \gtrsim 10^{1000}$ for disintegration of PPPs having a bound scale at the Planck length.

Since we know the entropy of the present universe is at least $S(t_0) \gtrsim 10^{102}$ [59, 60] one must impose

$$N_{\rm cp} \gtrsim 10^{102} \tag{32}$$

and, by requiring only the dissociation of nucleons, we see from Figure 2 that this

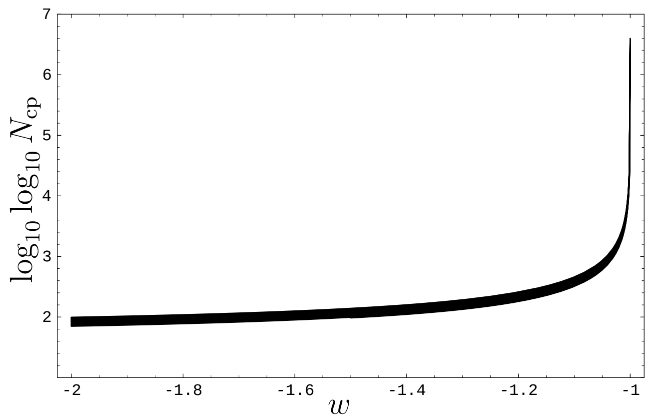

Figure 2: Constraint for $-2 \leq w \leq -1.1$ and $N_{\rm cp}$ coming from the CBE condition (Comes Back Empty), corresponding to inequality (28). The black band in this figure has been created by varying the length of smaller bound systems from $L_p(t_0) = 10^{-33}$m to $L_p(t_0) = 10^{-15}$m; the bottom edge corresponds to the lower value of $L_p(t_0)$. The region below the black band is forbidden by the CBE condition.

implies that

$$w \gtrsim -2. \tag{33}$$

Of course WMAP data [51] already guarantees this condition but it is interesting that our cyclic model would be impossible if Eq.(33) had been violated.

4.4 Discussion

We have deduced that the parameters ϱ_c, w and $N_{\rm cp}$ in cyclic cosmology are already constrained by existing data. For example, one requires $w \gtrsim -2$ for the CBE aspect to work, and this has already been experimentally verified. As better and more accurate cosmological data become available, further light shall be shed upon the viability of the theory.

In particular, the accurate measurement of the equation of state $w = -1 - \phi$ is of special interest. Fortunately the Planck Surveyor [54] is anticipated to acquire improved accuracy on w in the near future. As we have discussed, this will provide a lower bound on the number $N_{\rm cp}$ of causal patches necessary to dissociate the smallest bound systems at turnaround and hence to solve the entropy problem and, via CBE, enable the possibility of infinite cyclicity.

It is amusing that the physical conditions at the approach of deflation are so extraordinary that it is natural to ask whether the systems presently regarded as point particles may be composite because the phantom dark energy density grows to unimaginably large values and can disintegrate bound systems down to arbitrarily small scales. We have conservatively limited our attention to systems bigger than the Planck length. However, although this requirement seems dictated by considerations of quantum gravity, it is possible that the dark energy will dissociate even smaller systems should they exist.

A potential advantage of cyclic cosmology is that it removes the initial singularity associated with the Big Bang, about 13.7 billion years ago, and allows that time never began, although whether this seems more aesthetic and reasonable or not is open for debate. One might argue that is actually less aesthetic in some ways and might lead to even more puzzling questions. The previous attempts to create a consistent infinite cyclicity were stymied between about 1934 [11] and 2002 [34–37] primarily because of the entropy problem and the second law of thermodynamics. The discovery of the accelerated expansion rate of the universe and the concomitant necessity of dark energy have permitted somewhat more optimism that the cyclic cosmology might potentially be, after all, on the right track.

CHAPTER 5

OPEN QUESTIONS

One open question is whether or not phantom dark energy with equation of state $w = \frac{p}{\varrho} < -1$, a strict requirement for the model, is extant. Phantom energy violates the dominant energy condition[#4] and most attempts have met with instabilities, wrong sign spatial kinetic terms or other serious issues and to this day phantom energy remains problematic [61,62] at a microscopic level, at least unless their is only a limited time of $w < -1$ [63].

That said, the avenue is far from closed and there have been some promising recent attempts within string theory [64] and elsewhere appearing to demonstrate stability in certain cases [65] and most interestingly it was recently shown [66] that it is actually all but impossible to create an inflationary cosmology that agrees with current measurement if one accepts that theories need to rely upon more than four dimensions and they do not include phantom energy!!!

Another open issue is how to underwrite by a mathematical equation deflation from $a(t)$ to $\hat{a}(t) = f\ a(t)$ at turnaround that both avoids problems with fracturing spacetime geometry and concomitant issues while, with certainty, still preventing any patches from coming back into casual contact. The inverse f^{-1} must be larger than the cube root of a googol, 10^{34}. The partitioning of patches with specific boundaries and center points has long been an open issue as well for the author. It is hoped

[#4] The dominant energy condition includes the weak energy condition, $T_{\mu\nu}t^\mu t^\nu \geq 0\ \forall$ timelike vectors t^μ, as well as the additional constraint that $T^{\mu\nu}t_\mu$ is a non-spacelike vector ($T_{\mu\nu}T^\nu_\lambda t^\mu t^\lambda \leq 0$) or, more simply in the case when dealing with a perfect fluid as here, simply that we have $\varrho \geq |p|$.

that the BGV theorem [43] is violated due to the extraordinary nature of spacetime during approach to the turnaround and certain other overall conditions, allowing for a straightforward solution (as presented, but with a minor, yet significant, modification) to the open questions that arise with this specific version of deflation.

A further issue is how to calculate entropy in the extreme spacetime background at turnaround? No directly applicable mathematical framework exists in present literature.

The cyclic universe outlined in this dissertation has been in the scientific literature for over two years and so far no fatal flaw has been demonstrated.

APPENDIX A

DERIVATION OF FORMULAS AND NUMERICAL ANALYSIS

A.1 The formula for $(t_{\rm rip} - t)$

We begin by writing down the Friedman equation for times ($t_0 < t < t_T$ which correspond to the expansion phase [52],

$$H^2(t) \equiv \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\left[\frac{\varrho_\Lambda^0}{[a(t)]^{3(1+w)}} - \frac{[\varrho_{\rm tot}(t)]^2}{\varrho_C}\right], \tag{34}$$

where we have set $\Omega_r^0 = \Omega_m^0 = 0$. Taking into account the rapid acceleration when $t_0 < t < t_T$ (or $t_{\rm rip}$), we may neglect the last term proportional to $\varrho_{\rm tot}^2 \sim [a^{3(1+w)}]^2$, so that

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 \simeq H_0^2 \frac{\Omega_\Lambda^0}{[a(t)]^{3(1+w)}}. \tag{35}$$

Setting the boundary condition $a(t_{\rm rip}) = \infty$ and employing the equation of state $w < -1$, Eq.(35) can readily be solved for an arbitrary time t satisfying $t < t_{\rm rip}$ to get

$$t_{\rm rip} - t = (H_0\sqrt{\Omega_\Lambda^0})^{-1}\frac{2}{3|1+w|}a(t)^{-\frac{3|1+w|}{2}}. \tag{36}$$

A.2 The formula for $(t_{\rm rip} - t_0)$

Taking $t = t_0$ at which point $a(t_0) = 1$ and using current observational values [56], $H_0 = 73\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1}$ and $\Omega_\Lambda^0 = 0.76$, we find the time interval $(t_{\rm rip} - t_0)$ from Eq.(36) to be [45]

$$t_{\rm rip} - t_0 \simeq \frac{11\,\mathrm{Gyr}}{|1+w|}. \tag{37}$$

In Table 1 we list the values of $(t_{\rm rip} - t_0)$ for 37 specific choices of w in the range $-1.10000 \leq \omega \leq -1.00001$.

w	$(t_{\rm rip} - t_0)$ [Gyr]
-1.10000	110
-1.09000	122
-1.08000	137
-1.07000	157
-1.06000	183
-1.05000	220
-1.04000	275
-1.03000	366
-1.02000	550
-1.01000	1100
-1.00900	1222
-1.00800	1375
-1.00700	1571
-1.00600	1833
-1.00500	2200
-1.00400	2750
-1.00300	3666
-1.00200	5500
-1.00100	11000
-1.00090	12222
-1.00080	13750
-1.00070	15714
-1.00060	18333
-1.00050	22000
-1.00040	27500
-1.00030	36666
-1.00020	55000
-1.00010	110000
-1.00009	122222
-1.00008	137500
-1.00007	157143
-1.00006	183333
-1.00005	220000
-1.00004	275000
-1.00003	366667
-1.00002	550000
-1.00001	1100000

Table 1: Values of $(t_{\rm rip} - t_0)$ for $-1.10000 \leq \omega \leq -1.00001$.

A.3 The formula for $(t_{\rm rip} - t_T)$

Putting $t = t_T$ in Eq.(36) and dividing both sides by $(t_{\rm rip} - t_0)$, we find a relationship independent of both H_0 and Ω^0_Λ:

$$\frac{t_{\rm rip} - t_T}{t_{\rm rip} - t_0} = [a(t_T)]^{-\frac{3\phi}{2}} . \tag{38}$$

We shall recall here that the turnaround-time t_T is characterized by $\varrho_\Lambda(t_T) = \varrho_C$, derived from examining a solution $H^2 = 0$ of Eq.(34), which allows us to rewrite Eq.(38) as

$$\frac{t_{\rm rip} - t_T}{t_{\rm rip} - t_0} = \sqrt{\frac{\varrho^0_\Lambda}{\varrho_C}} , \tag{39}$$

where we have calculated the right-hand side using

$$[a(t_T)]^{-3\phi} = \frac{\varrho_\Lambda(t_0)}{\varrho_\Lambda(t_T)} = \frac{\varrho_\Lambda(t_0)}{\varrho_C} . \tag{40}$$

Following from the first chapter, we may introduce a unit of energy density, ϱ_{H_2O}, in such a way that the critical density ϱ_C is scaled by a factor of η

$$\varrho_C \equiv \eta \cdot \varrho_{H_2O} . \tag{41}$$

The present dark energy density ϱ^0_Λ can be expressed in terms of $\varrho_{H_2O} = 1{\rm g/cm}^3$ as

$$\varrho^0_\Lambda = 10^{-29} \varrho_{H_2O} . \tag{42}$$

This immediately leads us to

$$\frac{\varrho^0_\Lambda}{\varrho_C} = \frac{10^{-29}\varrho_{H_2O}}{\eta \cdot \varrho_{H_2O}} = \eta^{-1} 10^{-29} . \tag{43}$$

Hence we have

$$t_{\rm rip} - t_T = (t_{\rm rip} - t_0) \cdot 10^{-14.5} \cdot \eta^{-1/2} \,. \tag{44}$$

In Tables 2 to 4, choosing $\eta = 10^{29}, 10^{57}, 10^{93}$, respectively, corresponding to $\varrho_C \simeq (10^3\,{\rm GeV})^4$, $(10^{10}\,{\rm GeV})^4, (10^{19}\,{\rm GeV})^4$ in units of ϱ_{H_2O}, we list the values for the time interval $(t_{\rm rip} - t_T)$, which turn out to be at most of order $\mathcal{O}(10^{-7}\,{\rm s})$.

A.4 The formula for $(t_{\rm rip} - t_{\rm unbound})$

Let us next consider a time $t_{\rm unbound}$ at which a gravitationally bound system will become unbound due to an extraordinarily rapid expansion of the universe. Roughly speaking, a bound system in circular orbit at radius R with mass M becomes unbound when

$$\frac{4\pi}{3} R^3 \varrho_\Lambda(t_{\rm unbound})|1+3w| \simeq M \,, \tag{45}$$

where the left-hand side comes from the $T_{\mu\nu}$-term in the right-hand side of the Einstein equation.

Putting $t = t_{\rm unbound}$ in Eq.(36) and rewriting the overall factor $(H_0\sqrt{\Omega})^{-1}$ in terms of ϱ_Λ^0 and the gravitational constant G getting $(H_0\sqrt{\Omega})^{-1} = (8\pi G/3 \cdot \varrho_\Lambda^0)^{-1/2}$, we may express a time interval $(t_{\rm rip} - t_{\rm unbound})$ as

$$\begin{aligned}
(t_{\rm rip} - t_{\rm unbound}) &= (8\pi G/3\varrho_\Lambda^0)^{-1/2} \frac{2}{3|1+w|} [a(t_{\rm unbound})]^{-\frac{3|1+w|}{2}} \\
&= (8\pi G/3)^{-1/2} \sqrt{\frac{1}{\varrho_\Lambda^0}} \frac{2}{3|1+w|} \sqrt{\frac{\varrho_\Lambda^0}{\varrho_\Lambda(t_{\rm unbound})}} \\
&= (8\pi G/3)^{-1/2} \frac{2}{3|1+w|} \sqrt{\frac{1}{\varrho_\Lambda(t_{\rm unbound})}} \,.
\end{aligned} \tag{46}$$

$\eta = 10^{29}$

w	$(t_{\rm rip} - t_T)$ [s]
-1.10000	3.4×10^{-11}
-1.09000	3.8×10^{-11}
-1.08000	4.3×10^{-11}
-1.07000	4.9×10^{-11}
-1.06000	5.7×10^{-11}
-1.05000	6.9×10^{-11}
-1.04000	8.6×10^{-11}
-1.03000	1.1×10^{-10}
-1.02000	1.7×10^{-10}
-1.01000	3.4×10^{-10}
-1.00900	3.8×10^{-10}
-1.00800	4.3×10^{-10}
-1.00700	4.9×10^{-10}
-1.00600	5.7×10^{-10}
-1.00500	6.9×10^{-10}
-1.00400	8.6×10^{-10}
-1.00300	1.1×10^{-9}
-1.00200	1.7×10^{-9}
-1.00100	3.4×10^{-9}
-1.00090	3.8×10^{-9}
-1.00080	4.3×10^{-9}
-1.00070	4.9×10^{-9}
-1.00060	5.7×10^{-9}
-1.00050	6.9×10^{-9}
-1.00040	8.6×10^{-9}
-1.00030	1.1×10^{-8}
-1.00020	1.7×10^{-8}
-1.00010	3.4×10^{-8}
-1.00009	3.8×10^{-8}
-1.00008	4.3×10^{-8}
-1.00007	4.9×10^{-8}
-1.00006	5.7×10^{-8}
-1.00005	6.9×10^{-8}
-1.00004	8.6×10^{-8}
-1.00003	1.1×10^{-7}
-1.00002	1.7×10^{-7}
-1.00001	3.4×10^{-7}

Table 2: Values of $(t_{\rm rip} - t_T)$ for $-1.10000 \leq \omega \leq -1.00001$ with $\eta = 10^{29}$ fixed.

$\eta = 10^{57}$

w	$(t_{\rm rip} - t_T)$ [s]
-1.10000	3.4×10^{-25}
-1.09000	3.8×10^{-25}
-1.08000	4.3×10^{-25}
-1.07000	4.9×10^{-25}
-1.06000	5.7×10^{-25}
-1.05000	6.9×10^{-25}
-1.04000	8.6×10^{-25}
-1.03000	1.1×10^{-24}
-1.02000	1.7×10^{-24}
-1.01000	3.4×10^{-24}
-1.00900	3.8×10^{-24}
-1.00800	4.3×10^{-24}
-1.00700	4.9×10^{-24}
-1.00600	5.7×10^{-24}
-1.00500	6.9×10^{-24}
-1.00400	8.6×10^{-24}
-1.00300	1.1×10^{-23}
-1.00200	1.7×10^{-23}
-1.00100	3.4×10^{-23}
-1.00090	3.8×10^{-23}
-1.00080	4.3×10^{-23}
-1.00070	4.9×10^{-23}
-1.00060	5.7×10^{-23}
-1.00050	6.9×10^{-23}
-1.00040	8.6×10^{-23}
-1.00030	1.1×10^{-22}
-1.00020	1.7×10^{-22}
-1.00010	3.4×10^{-22}
-1.00009	3.8×10^{-22}
-1.00008	4.3×10^{-22}
-1.00007	4.9×10^{-22}
-1.00006	5.7×10^{-22}
-1.00005	6.9×10^{-22}
-1.00004	8.6×10^{-22}
-1.00003	1.1×10^{-21}
-1.00002	1.7×10^{-21}
-1.00001	3.4×10^{-21}

Table 3: Values of $(t_{\rm rip} - t_T)$ for $-1.10000 \leq \omega \leq -1.00001$ with $\eta = 10^{57}$ fixed.

$\eta = 10^{93}$

w	$(t_{\rm rip} - t_T)$ [s]
-1.10000	3.4×10^{-43}
-1.09000	3.8×10^{-43}
-1.08000	4.3×10^{-43}
-1.07000	4.9×10^{-43}
-1.06000	5.7×10^{-43}
-1.05000	6.9×10^{-43}
-1.04000	8.6×10^{-43}
-1.03000	1.1×10^{-42}
-1.02000	1.7×10^{-42}
-1.01000	3.4×10^{-42}
-1.00900	3.8×10^{-42}
-1.00800	4.3×10^{-42}
-1.00700	4.9×10^{-42}
-1.00600	5.7×10^{-42}
-1.00500	6.9×10^{-42}
-1.00400	8.6×10^{-42}
-1.00300	1.1×10^{-41}
-1.00200	1.7×10^{-41}
-1.00100	3.4×10^{-41}
-1.00090	3.8×10^{-41}
-1.00080	4.3×10^{-41}
-1.00070	4.9×10^{-41}
-1.00060	5.7×10^{-41}
-1.00050	6.9×10^{-41}
-1.00040	8.6×10^{-41}
-1.00030	1.1×10^{-40}
-1.00020	1.7×10^{-40}
-1.00010	3.4×10^{-40}
-1.00009	3.8×10^{-40}
-1.00008	4.3×10^{-40}
-1.00007	4.9×10^{-40}
-1.00006	5.7×10^{-40}
-1.00005	6.9×10^{-40}
-1.00004	8.6×10^{-40}
-1.00003	1.1×10^{-39}
-1.00002	1.7×10^{-39}
-1.00001	3.4×10^{-39}

Table 4: Values of $(t_{\rm rip} - t_T)$ for $-1.10000 \le \omega \le -1.00001$ with $\eta = 10^{93}$ fixed.

Using Eq.(45) we can further rewrite the right-hand side as

$$\begin{aligned}(t_{\rm rip} - t_{\rm unbound}) &= \frac{\sqrt{2|1+3w|}}{3|1+w|}\sqrt{\frac{R^3}{GM}} \\ &= \frac{\sqrt{2|1+3w|}}{6\pi|1+w|}P\,,\end{aligned} \tag{47}$$

where in the last line we have used a relationship from classical gravitational systems,

$$P = 2\pi\sqrt{\frac{R^3}{GM}}\,, \tag{48}$$

in which P denotes the period for a circular orbit of radius R around a system of mass M bound by a gravitational force. Thus we reach the expression [45] for the time interval $(t_{\rm rip} - t_{\rm unbound})$,

$$(t_{\rm rip} - t_{\rm unbound}) = \alpha(w)P\,, \tag{49}$$

where

$$\alpha(w) = \frac{\sqrt{2|1+3w|}}{6\pi|1+w|}\,. \tag{50}$$

Similarly to Eq.(45), even for binding forces other than gravity, we can roughly estimate an unbinding time $t_{\rm unbound}$. For simplicity, we shall derive here a relationship similar to Eq.(45) focusing on an electromagnetically bound system, e.g., a hydrogen atom H, in which the electron is constrained to a circular orbit of radius R by the Coulomb force around a proton. This can be done just by taking into account the balance problem between the Coulomb force F_C and the dark energy force F_w. We find that the system will become unbound when

$$\begin{aligned} & F_C \simeq F_w(t_{\rm unbound})\,, \\ \rightarrow \qquad & \frac{e^2}{4\pi\epsilon_0}\frac{1}{R^2} \simeq G\frac{m_e m_{\rm eff}(w, t_{\rm unbound})}{R^2}\,,\end{aligned} \tag{51}$$

where $m_{\rm eff}(w, t_{\rm unbound})$ denotes an effective "mass" arising from the dark energy density at $t = t_{\rm unbound}$,

$$m_{\rm eff}(w, t_{\rm unbound}) = \frac{4\pi R^3}{3} \varrho_\Lambda(t_{\rm unbound})|1 + 3w| \,. \tag{52}$$

Using the expression for the period associated with the electromagnetic force

$$P_{\rm em} = 2\pi \sqrt{\frac{m_e R^3}{\hbar c \alpha}}, \tag{53}$$

we can rewrite Eq.(51) as

$$\varrho_\Lambda(t_{\rm unbound})|1 + 3w| \simeq \frac{3\pi}{G P_{\rm em}^2} \,, \tag{54}$$

which leads immediately to the formula for $(t_{\rm rip} - t^{\rm H}_{\rm unbound})$ in the case of an H atom. As a result, we find it takes the same form as Eq.(49),

$$t_{\rm rip} - t^{\rm H}_{\rm unbound} = \alpha(w) P_{\rm em} \,. \tag{55}$$

It is straightforward to show that for other binding forces, e.g. strong forces, etc., the form of Eq.(55) is unchanged except for replacing $P_{\rm em}$ with the appropriate one associated with the binding force.

Choosing typical bound systems – galaxy, Sun-Earth, and hydrogen atom – and supplying the corresponding values for the period $P(P_{\rm em})$

Bound System	P	L
Typical Galaxy	2.0×10 yr	1.6×10^4 pc
Sun-Earth	1 yr	1.5×10^8 km
Hydrogen Atom	10^{-16} s	0.5×10^{-10} m

we calculate the values of $(t_{\rm rip} - t_{\rm unbound})$ for each bound system by taking values of

w from the range $-1.10000 \leq \omega \leq -1.00001$. The result is summarized in Table 5.

A.5 The formula for $(t_{\rm rip} - t_{\rm caus})$

After $t = t_{\rm unbound}$, objects that had been previously constrained within bound systems become free to move far apart and will end up causally disconnected starting at time $t = t_{\rm caus}$. Such a time, $t_{\rm caus}$, can be defined, taking $c = 1$, by

$$\frac{L}{a(t_{\rm caus})} = \int_{t_{\rm caus}}^{t_{\rm rip}} \frac{dt}{a(t)}\,, \tag{56}$$

where L denotes the length scale at which two objects separate at $t = t_{\rm caus}$, and the right-hand side stands for the co-moving distance of light which arises from traveling at light speed c during a time-interval $t_{\rm caus} < t < t_{\rm rip}$. Noting that $dt = da/(aH)$ and rewriting $H(a)$ from the Friedman equation in terms of a function of a, we calculate more explicitly the right-hand side of Eq.(56) as follows:

$$\begin{aligned}
\frac{L}{a(t_{\rm caus})} &= \int_{a(t_{\rm caus})}^{a(t_{\rm rip})} \frac{da}{a^2 H(a)} \\
&= (H_0 \cdot \Omega_\Lambda^0)^{-1} \int_{a(t_{\rm caus})}^{\infty} da\, a^{-1/2(1-3w)} \\
&= (H_0 \Omega_\Lambda^0)^{-1} \frac{2}{|1+3w|} [a(t_{\rm caus})]^{-|1+3w|/2}\,.
\end{aligned} \tag{57}$$

Taking $t = t_{\rm caus}$ in Eq.(36) and dividing both sides by the resultant expression, we can continue calculating to get

$$\begin{aligned}
\frac{L}{a(t_{\rm caus})} \frac{1}{(t_{\rm rip} - t_{\rm caus})} &= \frac{3|1+w|}{|1+3w|} \left(\frac{[a(t_{\rm caus})]^{3|1+w|}}{[a(t_{\rm caus})]^{|1+3w|}} \right)^{1/2} \\
&= \frac{3|1+w|}{|1+3w|} \frac{1}{a(t_{\rm caus})}\,.
\end{aligned} \tag{58}$$

w	Typical Galaxy [Gyr]	Sun-Earth [yr]	Hydrogen Atom [s]
	$(t_{\rm rip} - t_{\rm unbound})$		
-1.10000	0.22	1.13	1.13×10^{-16}
-1.09000	0.25	1.25	1.25×10^{-16}
-1.08000	0.28	1.40	1.40×10^{-16}
-1.07000	0.31	1.59	1.59×10^{-16}
-1.06000	0.36	1.84	1.84×10^{-16}
-1.05000	0.44	2.20	2.20×10^{-16}
-1.04000	0.54	2.73	2.73×10^{-16}
-1.03000	0.72	3.61	3.61×10^{-16}
-1.02000	1.07	5.38	5.38×10^{-16}
-1.01000	2.13	10.6	1.06×10^{-15}
-1.00900	2.37	11.8	1.18×10^{-15}
-1.00800	2.66	13.3	1.33×10^{-15}
-1.00700	3.04	15.2	1.52×10^{-15}
-1.00600	3.55	17.7	1.77×10^{-15}
-1.00500	4.26	21.3	2.13×10^{-15}
-1.00400	5.32	26.6	2.66×10^{-15}
-1.00300	7.08	35.4	3.54×10^{-15}
-1.00200	10.2	53.1	5.31×10^{-15}
-1.00100	21.2	106	1.06×10^{-14}
-1.00090	23.5	117	1.17×10^{-14}
-1.00080	26.5	132	1.32×10^{-14}
-1.00070	30.3	151	1.51×10^{-14}
-1.00060	35.3	176	1.76×10^{-14}
-1.00050	42.4	212	2.12×10^{-14}
-1.00040	53.0	265	2.65×10^{-14}
-1.00030	70.7	353	3.53×10^{-14}
-1.00020	106	530	5.30×10^{-14}
-1.00010	212	1061	1.06×10^{-13}
-1.00009	235	1179	1.17×10^{-13}
-1.00008	265	1326	1.32×10^{-13}
-1.00007	303	1515	1.51×10^{-13}
-1.00006	353	1768	1.76×10^{-13}
-1.00005	424	2122	2.12×10^{-13}
-1.00004	530	2652	2.65×10^{-13}
-1.00003	707	3536	3.53×10^{-13}
-1.00002	1060	5305	5.30×10^{-13}
-1.00001	2120	10610	1.06×10^{-12}

Table 5: Values of $(t_{\rm rip} - t_{\rm unbound})$ for $-1.10000 \leq \omega \leq -1.00001$.

In the end we reach the expression [45]

$$t_{\rm rip} - t_{\rm caus} = \left| \frac{1+3w}{3(1+w)} \right| \frac{L}{c} \,. \tag{59}$$

Similarly to the previous section, we take values of w in the range $-1.10000 \leq \omega \leq -1.00001$ and calculate the values of $(t_{\rm rip} - t_{\rm caus})$ for typical bound systems such as galaxies, the Sun-Earth, a hydrogen atom, and summarize in Table 6.

w	Typical Galaxy [Myr]	Sun-Earth [day]	Hydrogen Atom [s]
	$(t_{\rm rip} - t_{\rm caus})$		
-1.10000	0.40	0.044	1.27×10^{-18}
-1.09000	0.44	0.048	1.40×10^{-18}
-1.08000	0.49	0.054	1.55×10^{-18}
-1.07000	0.55	0.060	1.75×10^{-18}
-1.06000	0.64	0.070	2.01×10^{-18}
-1.05000	0.75	0.083	2.39×10^{-18}
-1.04000	0.93	0.102	2.94×10^{-18}
-1.03000	1.22	0.134	3.87×10^{-18}
-1.02000	1.81	0.198	5.72×10^{-18}
-1.01000	3.57	0.391	1.12×10^{-17}
-1.00900	3.96	0.434	1.25×10^{-17}
-1.00800	4.45	0.488	1.40×10^{-17}
-1.00700	5.08	0.557	1.60×10^{-17}
-1.00600	5.92	0.649	1.86×10^{-17}
-1.00500	7.09	0.777	2.24×10^{-17}
-1.00400	8.86	0.970	2.79×10^{-17}
-1.00300	11.7	1.29	3.72×10^{-17}
-1.00200	17.6	1.93	5.57×10^{-17}
-1.00100	35.2	3.86	1.11×10^{-16}
-1.00090	39.2	4.29	1.23×10^{-16}
-1.00080	44.0	4.83	1.39×10^{-16}
-1.00070	50.3	5.52	1.59×10^{-16}
-1.00060	58.7	6.44	1.85×10^{-16}
-1.00050	70.5	7.72	2.22×10^{-16}
-1.00040	88.1	9.65	2.78×10^{-16}
-1.00030	117	12.8	3.70×10^{-16}
-1.00020	176	19.3	5.56×10^{-16}
-1.00010	352	38.6	1.11×10^{-15}
-1.00009	391	42.9	1.23×10^{-15}
-1.00008	440	48.2	1.39×10^{-15}
-1.00007	503	55.1	1.58×10^{-15}
-1.00006	587	64.3	1.85×10^{-15}
-1.00005	704	77.2	2.22×10^{-15}
-1.00004	880	96.5	2.77×10^{-15}
-1.00003	1170	128	3.70×10^{-15}
-1.00002	1760	193	5.55×10^{-15}
-1.00001	3520	386	1.11×10^{-14}

Table 6: Values of $(t_{\rm rip} - t_{\rm caus})$ for $-1.10000 \leq \omega \leq -1.00001$.

APPENDIX B

CONSTRAINT ON ϱ_C FROM CAUSALITY

There exists a lower bound on the value of ϱ_C coming from the causal disconnection condition for smaller bound systems such as the hydrogen atom and nucleon. We study here the lower bound on ϱ_C calculating $(t_T - t_{\rm caus})$ numerically as a function of w and ϱ_C for each bound system with a size$\lesssim 10^{-10}$m.

Imposing the condition $t_T > t_{\rm caus}^{\rm H,N}$ where H, N denote the hydrogen atom and nucleon, respectively, we may obtain physical constraints on η and w from the following inequality:

$$\begin{aligned}(t_T - t_{\rm caus})^{\rm H,N} &= \frac{|1+3w|(L_{\rm H,N}/c)}{3|1+w|}\left[1 - \frac{3.12\times 10^{11.95}/L_{\rm H,N}}{\sqrt{\eta}|1+3w|}\right] \\ &\geq 0\,, \end{aligned} \tag{60}$$

which follows from Eqs.(44) and (59). In Fig 3 we show the constraints on the model parameters η and w, coming from the causal disconnection condition (60). Here we have taken $L_{\rm H} = 0.5\times 10^{-10}$m, $L_{\rm N} = 10^{-15}$m, and [56] $c = 2.9979\times 10^8\,{\rm m\,s^{-1}}$, ${\rm yr} = 3.1556\times 10^7$ s.

From this figure, we find a lower bound on the value of η, or equivalently on the critical density ϱ_C,

$$\eta \gtrsim 10^{54} \quad \leftrightarrow \quad \varrho_C \gtrsim (10^9\,{\rm GeV})^4\,, \tag{61}$$

where we have calculated $\varrho_C = (1.44\times 10^{x/4-4.5}\,{\rm GeV})^4$ with $x = \log_{10}\eta$.

If we extend a similar study to a bound PPP system, for which we set a scale that is slightly above the Planck scale, $L_{\rm PPP} \simeq 10^{-33}$m, we find a stronger lower

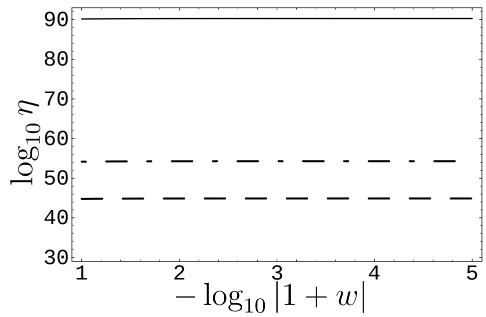

Figure 3: The lower bound on η coming from the causal disconnection conditions, $t_T > t_{\rm caus}^{\rm H}$ (dashed line), $t_T > t_{\rm caus}^{\rm N}$ (dashed-dotted line), $t_T > t_{\rm caus}^{\rm PPP}$ (solid line), for $10^{29} \leq \eta \leq 10^{93}$ and $-1.10000 \leq w \leq -1.00001$. The regions below these three lines are forbidden by causality.

bound on η (see the solid line illustrated in Fig. 3),

$$\eta \gtrsim 10^{90} \quad \leftrightarrow \quad \varrho_C \gtrsim (10^{18}\,{\rm GeV})^4\,. \tag{62}$$

It is interesting to note that the result on these lower bounds is fairly insensitive to the choice of w within the range of interest.

APPENDIX C

COMES-BACK-EMPTY CONDITION

At $t = t_T$ we require that at deflation, immediately prior to turnaround of the cyclic universe, almost very causal patch *comes back empty* which demands that the deflation factor f satisfies $f^{-3} \gtrsim 10^{102}$ in order to solve the entropy problem since the present entropy is at least 10^{102} [59, 60].

This requirement can be met by imposing that one causal patch be less than the size of the smallest bound systems ($L_p(t_T)$):

$$\frac{r_H(t_T)}{N_{\rm cp}} \leq L_p(t_T)\,, \tag{63}$$

where the subscript p denotes a bound system whose size L_0 at present lies in the range $10^{-33}\mathrm{m} \leq L_0 \leq 10^{-15}\mathrm{m}$. Noting that

$$r_H(t_T) = r_{H_0} a(t_T)\,, \tag{64}$$

$$L_p(t_T) = L_p(t_0)\frac{a(t_T)}{a(t_{\rm unbound})}\,, \tag{65}$$

we can rewrite the condition (63) as

$$a(t_{\rm unbound}) \leq N_{\rm cp}\frac{L_p(t_0)}{r_{H_0}}\,. \tag{66}$$

Looking at Eqs.(46), (49) and (55), we see that the left hand side of Eq.(66) can be reexpressed as

$$\left[\frac{\sqrt{2|1+3w|}}{4\pi}P_p(H_0\sqrt{\Omega^0_\Lambda})\right]^{\frac{2}{-3|1+w|}} \leq N_{\rm cp}\frac{L_p(t_0)}{r_{H_0}}\,, \tag{67}$$

where the period P_p, associated with a certain smaller bound system p we are concerned with, can be expressed as $P_p \simeq L_p(t_0)/c$.

Taking $\log_{10}\log_{10}$ of $N_{\rm cp}$ makes it easier to plot the inequality (67). A plot of the $(w,\, \log_{10}\log_{10} N_{\rm cp})$-plane varying the value of $L_p(t_0)$ in the range of interest, $10^{-33}\text{m} \leq L_p(t_0) \leq 10^{-15}\text{m}$ is shown in Figure 1 (see page 34). We have used [56] $r_{H_0} = 1.232 \times 10^{26}\text{m}$ and $\Omega^0_\Lambda = 0.76$.

APPENDIX D

CONVENTIONS AND ADDITIONAL BASIC FORMULAS

metric: -+++.

Greek indices such as μ and ν run over the 4 space-time dimensions [0,1,2,3 or t,x,y,z].

Latin indices such as i and j run over the 3 spacial dimensions [1,2,3 or x,y,z].

Full indices run over 5-dimensional space-time [0,1,2,3,4 or t,x,y,z,w].

$T^{\mu}_{\nu} = diag(\varrho, -p, -p, -p)$.

The Roberston-Walker metric: $-a(t)^2(\frac{dr^2}{1-\kappa r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)$.

$d(\varrho a^3) = -pd(a^3)$ from $\mu = 0$ of $T^{\mu\nu}_{;\nu} = 0$ by the first law of thermodynamics.

$ds^2 = -dt^2 + a^2(t)d\sigma^2$, where $d\sigma^2 = \gamma_{ij}(u)du^i du^j$ is the metric on the spatial coordinates and γ_{ij} is maximally symmetric.

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