Monetary Policy Evaluation and Inverse Control with Endogenous Policy Regimes

by
Zongqiang Liao

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Approved by:

Michael Salemi, Advisor

Richard Froyen, Reader

Neville Francis, Reader

Lutz Hendricks, Reader

William Parke, Reader

Abstract

ZONGQIANG LIAO: Monetary Policy Evaluation and Inverse Control with Endogenous Policy Regimes.
(Under the direction of Michael Salemi.)

This study applies a unified approach of econometric policy evaluation to investigate "endogenous" evolution of U.S. monetary policy during the post-war period. A policy regime is defined as a set of preference parameters which the Federal Reserve chooses for minimizing the value of loss function. The evolution of monetary policy is assumed to depend on a stochastic switching process that evolves according to a Markov chain. Preference parameters in the Federal Reserve's policy objective function together with parameters in the structure of economy are estimated simultaneously. This study uses forward-looking New Keynesian models as the structure of economy in which equilibrium values of output and inflation depend on their own future expected values.

The results suggest that three different policy regimes can be better used to describe U.S. monetary policy than two policy regimes and that in all policy regimes the Federal Reserve consistently placed far greater weight on inflation stabilization than on output and interest rate stabilizations. Estimating a baseline model with data from 1965 through 2001 shows that policy regime one is a special regime that only prevailed between 1979 and 1984, which is commonly known as the Volcker disinflation period. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during expansionary periods, and regime three is a regime which the monetary policy switched into during recessionary periods. The estimation with optimal policy restriction can also alter and sharpen the estimates of model parameters. Estimating the baseline model with extended data from 1965 through 2008 shows similar results to those obtained with the shorter sample. The results from the longer sample suggest that the Federal Reserve did follow an optimal policy during the post-war period. The Federal Reserve's monetary actions were very close to optimal during policy regimes two and three. A counterfactual analysis shows that the small value of preference parameter placed on output stabilization plays an important role in conducting monetary policy. Finally, estimating an augmented model with extended data from 1965 through 2008 suggest that the findings associated with the three-regime monetary policy are robust.

To my parents and my wife.

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Chapter 1

Introduction

1.1 Overview

In recent economic literature on monetary policy, it has become a common practice to analyze and evaluate central bank's policy behavior using a simple policy rule similar in nature to the one suggested in Taylor (1993), see Clarida, Galí and Gertler (2000). Although an estimated policy rule can capture the systematic relationship between interest rates and macroeconomic variables and can be viewed as an approximation to central bank's decision rule, it is not able to address questions on the policy formulation process. Since the policy rule function is only a reduced form, the feedback coefficients in the policy rule do not have a structural interpretation and can not draw conclusions on the policy preferences of central bank.

As shown by Svensson (1999), under an optimal monetary policy framework, one can formally derive a Taylor-type interest rate rule that solves the intertemporal optimization problem of a central bank for a specified loss function. In the framework, the coefficients in the optimal interest rate rule depend on the central bank's preferences as well as parameters describing the structure of economy. Therefore, if a policy rule is estimated under the optimal policy framework, a change of estimated parameters in the policy equation can be unequivocally interpreted as a modification of the central bank's preferences rather than a modification in the structure of economy, see Favero and Rovelli

(2003). This study follows the optimal monetary policy framework and describes monetary policy behavior at the level of policy objectives and not just at the level of reaction function. Therefore, this study defines a policy regime as a set of preference parameters which central bank chooses for minimizing the value of its loss function.

It is also widely believed that monetary policymakers should display time-varying behavior in conducting monetary policy, especially when considering a longer period of time. One potential source of changing monetary policy is that monetary policymakers would face different levels and variabilities of inflation and other economic variables in different time spans. In the case of United States, monetary policymakers in the Federal Reserve must contend with the high inflation that occurred in the 1970's, the disinflation in the 1980's, several large oil price shocks, the geopolitical events, and the recessions that occurred in the early 1980's and 1990's, early and late 2000's. Clarida, Galí and Gertler (2000) show that many studies credit the stabilization of macroeconomic volatility between 1970 and 1980 to the switch of U.S. monetary policy behavior. On the other hand, there are also studies showing that the Federal Reserve consistently conducted monetary policy throughout the post-war period. Therefore, it is interesting to study the evolution process of underlying policy preferences of monetary policymakers.

This study extends the existing literature by applying a unified approach of econometric policy evaluation to investigate "endogenous" evolution of monetary policy over time. The unified approach allows the structural parameters of the model to be estimated jointly with the parameters of the central bank's objective function. There are several advantages of being able to use the unified approach, see Atoian (2005), Dennis (2006), and Salemi (2006). First, it allows direct estimation of the preference parameters in central bank's objective function, and it is an example of inverse control. Therefore, this methodology is more appropriate when one's primary objective is to investigate evolution of policy preference parameters in the objective function over time. Second, the unified estimation approach allows one to test whether an estimated policy rule is the outcome of central bank's optimal behavior and assess whether observed economic outcomes can be reconciled and accounted for within an optimal policy framework. Indeed, the setup of the problem allows for a formal test of the hypothesis that the policy is implemented in a manner consistent with the loss-minimizing behavior of the central bank. Third, while assuming the policy rule is loss-minimizing, the unified approach to policy preference estimation also considers additional economic structure equations. This estimation approach can potentially sharpen estimates of the structural model parameters. The cost of the unified approach is the greater computational burden that results from nesting the optimal-policy algorithm of the central bank within the estimation algorithm of the model.

In this paper, the evolution of monetary policy is assumed to depend on a stochastic switching process that evolves according to a Markov chain. The Markov-switching framework is an appealing technique for evaluating monetary policy behavior over time. First, it reflects the common view on the likelihood that monetary policy may take the form of an abrupt shift from one policy regime to another[1]. Second, the technique reveals a new policy regime rather than making assumption on the timing of a switch in policy regime. At the same time this approach can utilize full-sample information to make accurate inferences[2]. Finally, using the Markov-switching approach respects the Lucas (1976) critique in monetary policy evaluation. According to the Lucas critique, policy changes are embedded in private agents' perception, and agents will change their

[1]In contrast to assuming abrupt policy switches hence using the Markov-switching models, some studies alternatively assume that monetary policy changes gradually and develop a functional form for the gradually time-varying policy changes.

[2]It is a common practice in the literature to assume changes in policy regime according to a priori information. However, Davig and Leeper (2007) find that splitting sample into distinct regimes can distort inferences when compared to inferences drawn on the full sample. Liu, Waggoner and Zha (2008) also suggest that caution needs to be taken in interpreting empirical results which are derived from a sub-sample covering only one of the policy regimes.

expectations and decisions accordingly with changes in policy. Markov-switching models are expected to fit data well if there were policy changes and important expectation effects of these policy changes on private sectors' behavior.

This study uses forward-looking New Keynesian structure model as the constraint for the central bank's optimization problem. The central bank's stabilization objectives are represented by a quadratic loss function that penalizes deviations of inflation and output from their targets as well as changes in the policy instrument referred to as interest rate smoothing. This study estimates the parameters of forward-looking structural model jointly with the preference parameters of the central bank's objective function under the restriction that monetary policy is a solution to the policymaker's loss minimization problem. With this restriction, the central bank follows a Taylor-type policy with coefficient values chosen optimally to minimize the policymaker's quadratic objective function. To examine how optimal monetary policy affects the empirical findings, this study also estimates the model separately without imposing the optimality restriction. This study assesses the model fit of these two policies by using statistical tests. The tests employed are appropriate for comparing model fit because optimal and non-optimal policies are nested policies in the sense that optimal policy imposes a set of restrictions on the model.

This study estimates two types of structural models subject to the condition that the policy equation minimizes a well-defined loss function. Both models are New Keynesian in spirit in that policy affects aggregate demand through a conventional interest rate channel and inflation through a Phillips curve specification. These structural New Keynesian models are becoming pervasive in macroeconomic analysis. The first model is a baseline model that determines the equilibrium relationship among the output gap, the inflation rate, and the short-term interest rate controlled by the central bank. While sharing some broad characteristics, the second model differs substantially from the baseline model by taking into account the additional inflationary effect generated by oil prices. The augmented model explicitly models an oil production sector and illustrates that the Phillips curve includes a term that is a measure of oil price that exerts extra pressures on inflation. The second model confirms that the standard cost push effects of oil prices are present in the economy. Therefore, the model with oil price implies a more complex relationship between the structural and reduced form parameters.

The important findings in this paper are the following. First, estimating the baseline model with data from 1965 through 2001 suggests three different policy regimes can be better used to describe U.S. monetary policy than two policy regimes. Policy regime one is a special regime that only prevailed between 1979 and 1984. During this period, the Federal Reserve successfully reduced high inflationary pressure in the U.S., and this period of time is commonly known as the Volcker disinflation period. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during expansionary periods, and regime three is a regime which the monetary policy switched into after recessionary periods. The results also show that in all policy regimes including the economic expansionary periods the Federal Reserve consistently placed greater weight on inflation stabilization than on output stabilization. Finally, the data can formally reject the hypothesis that U.S. monetary policy was loss-minimizing.

Second, estimating the baseline model with extended data from 1965 through 2008 shows similar results to those obtained with the shorter sample data. Indeed, three different policy regimes provide a better description of U.S. monetary policy than two policy regimes. Policy regime one is a special regime that only prevailed between 1979 and 1984 and is commonly known as the Volcker disinflation period. Regime two is a regime prevailed during the economic expansionary periods, and regime three is a regime prevailed during the economic recession periods. In all policy regimes, the inflation stabilization objective is found to dominate the Federal Reserve's preferences. Contrary to the previous findings, the results here suggest that the Federal Reserve did follow an optimal policy during the extended sample period. The Federal Reserve's monetary actions were very close to their optimal counterparts during policy regimes two and three.

Third, the results from estimating the augmented model support the presence of three policy regimes. Policy regime one is a regime that only prevailed during the deep post-war recessions began in 1973, 1980, 1981, and 2007. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during the post-1985 expansionary periods and mild recessions in 1990 and 2000. Policy regime three is a regime which the monetary policy switched into during the pre-1985 expansionary periods and mild recession in 1970. Therefore, the important findings associated with the three-regime monetary policy are robust to estimations using a variation of the structural model. The results here still suggest that three different policy regimes can provide a cogent description of U.S. monetary policy although the interpretation for each policy regime is different from that in the previous findings. In line with the previous findings, the results also suggest the Federal Reserve consistently placed greater weight on inflation stabilization than on output and interest rate stabilizations in all three policy regimes including the economic expansionary periods. Lastly, the Federal Reserve did follow an optimal policy during the extended sample period. This conclusion is also consistent with that in the previous findings. The Federal Reserve's monetary action was very close to their optimal counterpart during policy regime two.

1.2 Related Literature

This study is not the first one that applies inverse control on monetary policy to reveal preference parameters in policy objective function[3]. Salemi (1995) estimates a VAR

[3]Although the methodology of inverse control is applicable in studying the behavior of central bank, some studies use indirect approaches to estimate preference parameters in central bank policy objective function, see Ozlale (2003) and Assenmacher-Wesche (2006).

subject to the restriction that the equation with monetary policy variables minimizes expected loss. He finds that the Federal Reserve placed greater weight on stabilizing output prior to 1979 and greater weight on stabilizing inflation between 1982 and 1992, and he can not reject the hypothesis that the Federal Reserve's policy rule minimized expected loss. Korenko (1998) infers preferences of the Federal Reserve using a version of Dornbusch's overshooting model similar to that in Papell (1989). He finds that the Federal Reserve placed much higher weight on output stabilization than on other objectives during 1973 through 1995. Dennis (2006) jointly estimates the parameters of an IS equation and a Phillips curve in a backward-looking Rudebusch and Svensson (1999) model and Federal Reserve loss function subject to the restriction that the coefficients of the reaction function minimize loss function. He finds that for both pre-Volcker period $(1966-1979)$ and post-Volcker period $(1980-2000)$, the Federal Reserve placed more weight on stabilizing output than on stabilizing inflation and even more weight on stabilizing interest rate changes. Favero and Rovelli (2003) also use the backward-looking Rudebusch and Svensson model to estimate loss-function parameters[4]. In contrary to the findings in other studies, Favero and Rovelli find that the Federal Reserve placed more weight on stabilizing inflation than on stabilizing output in two sub-samples $1961-1979$ and $1980-1998$.

Although the previously-mentioned studies have produced plausible results, all of the work has employed backward-looking models that are not able to incorporate rational expectation and consequently can potentially be subject to the Lucas (1976) critique[5]. To overcome this concern, one would need to develop an algorithm where the unified

[4] Unlike the common practice in literature that assumes a central bank's optimization problem has an infinite horizon, Favero and Rovelli assume that the Federal Reserve is only concerned with the impact of its policy decision for four quarters.

[5] Many studies use statistical tests to show no evidence of parameter instability implied by Lucas critique. However, it is more attractive to explicitly consider households and firms as forward-looking agents.

estimation approach is applied in the framework of forward-looking model. However, the use of a forward-looking structure complicates the estimation algorithm dramatically because strategic interactions may occur between the economic agents in the model.

Salemi (2006) is known as the first attempt to conduct a full-scale estimation of central bank preference parameters with a forward-looking economic structure[6]. He imposes policy optimality hypothesis by requiring the coefficients of the central bank's reaction function to minimize policy loss function and uses an algorithm that re-computes the state transition equation for every contemplated change in monetary policy rule. In contrast to the previous studies, Salemi finds that stabilizing inflation was far more important than stabilizing output and interest rate in both periods $1965-1979$ and $1980-2001$. Dennis (2003) utilizes a forward-looking rational expectation model and finds that interest rate smoothing appeared to be the most important element in the Federal Reserve's objective function while the weight on consumption stabilization is found to be at zero, and the weight on interest rate smoothing became smaller after 1980. Atoian (2005) uses a small open-economy model with forward-looking agents to study policy preferences of central banks in Canada and Switzerland[7]. He finds that the Bank of Canada placed virtually no weight on inflation stabilization from 1970 to 1979, approximately equal weights on inflation and output stabilizations between 1980 and 1987, and a greater weight on inflation stabilization from 1988 to 2002; the Swiss National Bank assigned dominant weight on inflation stabilization objective in both sub-samples $1973-1987$ and $1988-2003$. Givens (2004) considers a representative agent model in which wages and prices are both sticky, and he finds that delegating policy authority to a central

[6]Given the difficulties of conducting a full-scale estimation in the forward-looking environment, some studies attempt to reveal central bank policy preference parameters using calibration method to match the stylized economic facts, see Söderström, Södelind, and Vredin (2002) and Castelnuovo (2003).

[7]Another known attempt to study central bank's preferences in an open-economy framework is a calibration exercise in Collins and Siklos (2004). They vary the weight parameters of the loss function to obtain a reaction function that is similar to the case without the optimal policy constraint.

banker who cares about wage stabilization produces policy outcomes almost as good as provided by the timeless perspective policy. Givens and Salemi (2008) estimate central bank preference parameters within several forward-looking models but focus on testing the efficiency of a proposed GMM strategy for joint estimation.

While the above-mentioned studies in this literature investigate whether central bank's policy preference parameters were time-varying over time, they all share one aspect in common. Each paper makes an a priori assumption about the timing of the shifts of monetary policy regime by associating the shifts with the terms of office of different chairpersons in the central bank. Then central bank's preference parameters are estimated in each sub-sample. By contrast, this study allows policy regimes to be determined endogenously. Therefore, this study systematically infers central bank's policy preference parameters that are governed by a stochastic process and presents empirical effects of stochastic policy regime changes.

In sum, this study employs forward-looking models that emphasize the role of private-sector expectations in the transmission of monetary policy and estimates central bank's preference parameters in various endogenously determined policy regimes using the unified approach.

1.3 The Remainder of the Study

The remainder of this study is organized as follows. Chapter 2 is dedicated to an application of the unified estimation approach to estimate a baseline model using U.S. quarterly data from 1965:Q1 to 2001:Q4. It first presents reduced form structural equations describing the evolution of model variables, outlines the major features of the model, and describes the policy objective function. Meanwhile, this chapter illustrates the approach to find the rational expectation model solution, describes the algorithm for computing the optimal monetary policy rule, and presents the econometric strategy for simultaneous estimation of model structure parameters, central bank preference parameters, and probability inferences of monetary policy regimes. Then Chapter 2 describes the data and briefly presents stylized facts for U.S. monetary policy. Finally this chapter estimates the model. The results are related to stylized facts of U.S. macroeconomic history and studies in literature. This chapter also provides a formal assessment of the monetary policy optimality hypothesis.

Chapter 3 applies the unified estimation methodology to estimate the baseline model using extended U.S. quarterly data from 1965:Q1 to 2008:Q4. Similar to the previous chapter, it starts with a brief description of the data and an overview of development in U.S. monetary policy during the sample period. Then this chapter reports the estimation results. Chapter 3 also evaluates the model's ability to replicate stylized facts of U.S. macroeconomic history and provides a formal assessment of the monetary policy optimality hypothesis. Finally, this chapter reports results of counterfactual experiment when alternative strategy of monetary policy conduct is considered.

Chapter 4 uses the methodology in previous chapters to estimate a model with oil price and analyzes monetary policy of the Federal Reserve from 1965 through 2008. The chapter first presents the major features of the new structural model that is used to represent the constraints on the Federal Reserve's optimization problem. Then it derives reduced form equations describing the evolution of model variables and presents the policy objective function that is to be estimated. This chapter then explains how the estimation algorithm set out previously must be revised to account for changes in the model. Chapter 4 describes the data and briefly presents their stylized facts. Finally it estimates simultaneously the model structure parameters, central bank preference parameters and probability inferences of monetary policy regimes. The results are related to stylized facts of U.S. macroeconomic history.

Chapter 5 concludes and outlines possible extensions for future research. The Appendices provide technical details on model solution, estimation strategies, and model derivations.

Chapter 2

Revealing U.S. Monetary Policy Regimes Between 1965 and 2001 Using a Baseline Model

This chapter presents a model based on the assumption that the Federal Reserve conducts policy in an optimal way. The Federal Reserve's optimal monetary policy is subject to constraints controlled by behavior of private agents in the economy, and the optimal monetary policy has a direct impact on the economy through the setting of policy instrument. Therefore, the macro-economy and the optimization problem of the Federal Reserve have to be estimated simultaneously. However, it is still useful to divide the descriptions of the model and optimal monetary policy into two separate parts. The macro-economy is first described without requiring the Federal Reserve to minimize its loss function. Then the optimization problem of the Federal Reserve is derived, taking the model economy as given.

2.1 The Model Economy

The baseline model used in this study is a forward-looking economic model used in Salemi (2006). In the model, a central bank's main objectives are to stabilize the time paths of output and inflation by using the interest rate as its policy instrument. This study denotes y as the output gap, which is the difference between output and its long term growth path. This study also denotes π and r as differences of inflation and the interest rate from their target values, respectively. The stabilization of output means keeping y close to zero. Stabilizations of inflation and the interest rate mean keeping them close to their target paths.

The baseline model for y, π, and r consists of an IS curve, a Phillips curve and a policy rule for the short term interest rate. The model can be derived from a micro-founded New Keynesian sticky-price model in which both households and firms are forward-looking. The parameters of the IS schedule and Phillips curve are non-linear combinations of the parameters of the micro-founded model. In line with Salemi (2006), the investigation taken up in this study does not concern the deeper structural parameters in the model.

The model economy consists of the following expectation-augmented linear equations

$$y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \tag{2.1}$$

$$\pi_t = \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \tag{2.2}$$

$$r_t = c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}. \tag{2.3}$$

y_t, π_t, and r_t are defined in the previous paragraph. $\varepsilon_{\pi,t}$, $\varepsilon_{y,t}$, and $\varepsilon_{r,t}$ are shock disturbances.

Equation (2.1) takes the form of a New Keynesian IS curve linking output during period t to its own expected future and lagged values, and to the value of ex ante real interest rate. Equation (2.1) is obtained by combining the linearized Euler equation that characterizes a representative household's optimal consumption level and the market clearing condition for output in a dynamic general equilibrium model. Clarida, Galí and Gertler (1999) show that the presence of expected future output in the IS equation can be explained by the desire of households to smooth consumption. When households expect higher consumption in the future, they consume more in the present, which raises aggregate demand and introduces a positive relationship between current output and its expected future value. The presence of lagged output in the IS equation can be explained by habit persistence. Habit formation and sticky price mechanism can cause delays between decision making and consumption. Hence aggregate demand and output adjust only partially such that output depends on a combination of its own lagged values, see Svensson (2000). The inertial movements of consumption and aggregate output have been commonly found to be important in rational expectation New Keynesian models, and the habit specification in the model can help explain the sluggish movements. Fuhrer (2000) finds that habit formation helps New Keynesian models replicate the observed effects on real spending shocks of various kinds. Smets and Wouters (2005) find point estimates for degree of habit formation are very close to unity in the U.S. and Euro area. From an econometric perspective, Lindé (2005) argues that the additional lags of output are required to make the disturbance term in equation (2.1) white noise.

Equation (2.2) is a Phillips curve, in which both expected future and past inflation help determine the current inflation rate. If the coefficient on lagged inflation becomes zero and drops out of the specification, the equation becomes a new Phillips curve as defined by Clarida, Galí and Gertler (2000), Svensson (2000) and many others. The new Phillips curve can be derived from a model where monopolistically competitive firms adjust their prices in a sluggish way as suggested by Calvo (1983). A representative firm adjusts its price to maximize expected profit by taking into account the restriction of future price adjustment and expected future prices of its competitors. With staggered price adjustment, the inflation rate depends on the representative firm's marginal cost of production and the expected future inflation rate. The new Phillips curve can be obtained when the output gap is used as a proxy for marginal cost.

If both the coefficients on expected and lagged inflation are non-zero, the equation becomes a hybrid Phillips curve that can be used to account for inflation persistence or inertia. Several foundations have been proposed to account for the presence of lagged inflation in the Phillips curve. Rotemberg and Woodford (1997) assume a fraction of the firms that re-optimize in the current period must wait until the next period to change their new price. Galí and Gertler (1999) extend the Calvo mechanism to allow a fraction of firms set their price at time t equal to last period's average reset price plus the lagged inflation rate. Lagged inflation may appear in the Phillips curve to account for other economic factors. Fuhrer and Moore (1995) derive a Phillips curve with lagged inflation as the result of wage contracts, and Clarida, Galí and Gertler (2000) use lagged inflation in the Phillips curve to account for serial correlation in supply shocks.

Equation (2.3) is the policy rule or reaction function that explains how the central bank sets the short term interest rate. The equation takes a form of the Taylor (1993) type reaction function and corresponds to the common practice of using the short term interest rate as monetary policy instrument[8]. It can be seen that a disturbance term $\varepsilon_{r,t}$ is present in the policy rule. Hansen and Sargent (1980) explain why an error may be in a policy rule. Dennis (2006) argues that the error term can capture some variables that are omitted by econometricians since they tend to have less information than the policymakers. Salemi (2006) interprets the error as a term that includes idiosyncratic wisdom of the monetary policy authority. As a result, this disturbance term is commonly referred to as the policy shock. The policy rule allows the central bank to react to all the variables in the state vector, but only to lagged values of those variables. McCallum (1997) argues that a central bank does not know the current output level and inflation rate when it sets the interest rate. This does capture some broad features of data availability on real GDP and CPI, which are released with a delay. A lagged interest rate term is also included in the central bank's reaction function to match the persistence in interest

[8] Sims and Zha (2006) argue that leaving money aggregate from the policy rule can be bias for studying U.S. monetary policy in the post-war period because various money aggregates were essential to monetary policymakers during the 1970s. Sims and Zha suggest that the Taylor-type rule may be valuable for characterizing policy after 1980, but such a policy rule is not appropriate for studying other historical spans.

rate. Such format of a policy rule allows the central bank to change the interest rate persistently therefore to spread "policy medicine" over time. This would allow for a moderate response of interest rate to shocks hitting the economy. In line with Clarida, Galí and Gertler (1999) and Salemi (2006), the coefficients of the reaction function are assumed to be fixed in a policy regime so that policy is time consistent. For now, this section defers discussion of how the coefficients of the policy rule are chosen in an optimal way by the central bank and turns attention to solution of the model.

The model with equations (2.1) – (2.3) is a system of linear expectation vector difference equations. The system can be rewritten equivalently in the following compact representation

$$A \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ X_t \end{bmatrix} = B \begin{bmatrix} y_t \\ \pi_t \\ X_{t-1} \end{bmatrix} + D \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \\ \varepsilon_{r,t} \end{bmatrix}, \tag{2.4}$$

where $X_t = [y_t \;\; \pi_t \;\; i_t \;\; y_{t-1}]'$ is the state vector, A, B, and D are 6×6, 6×6, and 6×3 coefficient matrices respectively. The elements in the coefficient matrices are non-linear functions of the parameters of equations (2.1) – (2.3).

Blanchard and Kahn (1980) and Klein (2000) describe methods for solving systems of linear expectation equations such as (2.4). The solution can express the model's endogenous variables in terms of the model's exogenous and predetermined endogenous variables[9]. Note that there are four predetermined variables in the state vector X_t. Thus, if four of the generalized eigenvalues of coefficient matrix A lie outside the unit circle, then the system (2.4) has a unique solution for the model that quite conveniently takes

[9] Appendix A provides details on using the Klein (2000) algorithm for computing a unique saddle path solution for the model.

the form as

$$X_t = GX_{t-1} + \varphi_t, \tag{2.5}$$

where G is a 4×4 matrix with elements that are convolutions of the eigenvectors of matrix A[10]. φ_t is the 4×1 vector of reduced-form errors with typical elements φ_{kt}, $k \in \{1, \dots, 4\}$ and $\varphi_{4t} = 0$. This study allows for the possibility that the errors may be correlated. Let Ω be the 4×4 covariance matrix of φ_t. As a result, the upper-left 3×3 block of Ω is the covariance matrix of vector φ_{kt}, $k \in \{1, \dots, 3\}$, and the rest elements of Ω are zeros. Let C be a 3×3 matrix that is the upper-triangular Choleski decomposition of the non-zero block of Ω, and the elements $C_{11}, \dots, C_{33}$ is used in the Log Likelihood estimation. Since the structure shocks are assumed to be serially uncorrelated and φ_{kt}, $k \in \{1, \dots, 3\}$ are linear combinations of the structure errors, φ_t is serially uncorrelated.

In summary, this study uses a rational expectation model that can be derived from a micro-founded dynamic general equilibrium model with deep parameters that govern private agents' behavior. The macroeconomic structure explicitly considers that private agents have rational expectation and they can react correspondingly when monetary policy changes, hence the model specification responds to the Lucas (1976) critique and the model parameters in equations (2.1) and (2.2) are structural.

2.2 The Optimal Simple Monetary Policy Rule

The model includes the central bank's policy rule for setting the short term interest rate, which is specified in equation (2.3). Conditional on the macroeconomic structure,

[10] If more than four of the generalized eigenvalues of A lie outside the unit circle, then the system has no solution; if less than four of the generalized eigenvalues of A lie outside the unit circle, the solution has multiple solutions. In the course of estimation, this study imposes the unique saddle-path solution via a penalty function. The detailed description of estimation procedure that incorporates the model solution algorithm and penalty function is shown in Appendix A.

the central bank sets monetary policy in an optimal way to fulfil its objectives. This study assumes that the monetary authority credibly commits to an optimal policy rule in the sense that the coefficients of the policy rule c_1 through c_4 are chosen to minimize the expected value of intertemporal loss function

$$\mathbf{L}_t = E_t \sum_{k=0}^{\infty} \delta^k X'_{t+k} W X_{t+k}, \tag{2.6}$$

where $\delta \in (0,1)$ is the central bank's time rate of discount. W is a 4×4 matrix of non-negative weights that determine the relative importance to the central bank on its various stabilization objectives. W is positive semi-definite. Dennis (2006) suggests that a discounted quadratic objective function together with linear policy constraints are extremely powerful tools for analyzing central bank's stochastic dynamic optimization problems. Since Barro and Gordon (1983), a loss function formulation similar to equation (2.6) is widely used as the objective function of central bank in literature. As a results, the estimates obtained in this study can be easily interpreted and compared with many others in the literature.

In this study, W is diagonal with non-zero elements w_y, w_π, and w_r on the main diagonal and zeros everywhere else. The diagonal elements are weights which the central bank places on stabilizing output, inflation, and the interest rate, respectively. Because only the relative sizes of weights are identified, w_π is normalized to one. Rogoff (1985) argues that it is desirable to appoint a more conservative central banker who places a higher emphasis on stabilizing inflation than on output. Assenmacher-Wesche (2006) illustrates that a central bank is commonly viewed as an independent one if the value of relative weight on output is small. For simplicity, the central bank's discount rate δ is fixed at 0.990, which is a reasonable value for a quarterly time rate of discount and is widely used in literature[11]. In this study, the central bank sets its policy optimally with full information on the true economic structure and model parameters[12]. In what follows, a policy regime is a set of weights in loss function and the corresponding coefficients in reaction function used to minimize expected loss. Södelind (1999) calls a policy rule with coefficients chosen to minimize expected loss function an "optimal simple rule".

Rotemberg and Woodford (1997) show that under certain conditions, a loss function similar to equation (2.6) is derived as the second order approximation to the discounted intertemporal utility function of a representative household, and the elements of W are nonlinear functions of deep parameters of the model. If the central bank sets the monetary policy that maximizes a representative household's utility, the parameters of the policy rule are also nonlinear functions of the model parameters. However, following Salemi (2006) this study does not impose the restriction of Rotemberg and Woodford (1997) in the model. Although interest rate smoothing is commonly viewed as an important objective of U.S. monetary policy, maximizing representative household utility does not require stabilizing the interest rate. Second, this study treats equations (2.1) – (2.3) as the structure of the economy rather than taking a stand on the specific model that gives rise to (2.1) – (2.3). Therefore, in this study the elements of W are parameters that carry information on the objectives of central bank.

Why does it make sense to assume that the central bank commits to an optimal simple rule? The advantages for a central bank to commit to a policy rule are well known. In a study that is important to time-consistency literature, Kydland and Prescott (1977) analyze distinction between policy rules and discretion. They refer a policy rule as the optimal solution to a dynamic optimization, and a discretionary policy as the

[11]Imposition of an ex ante value to the discount factor is common in both real business cycle and New Keynesian models since the discount factor is very difficult to be estimated directly.

[12]Svensson and Williams (2007b) study the optimization problem of a central bank when it can not observe the true economic structure and learns as time passes.

inconsistent solution. A central bank's commitment to a rule can facilitate public's expectation formations of future monetary policy and of future inflation or output. The public can understand and gain confidence in the central bank's stabilization policy. Therefore, monetary policy becomes more effective since its future course is predictable. Commitment to a policy rule would allow the central bank to follow a guideline to future decisions and adjust policy gradually. In a case with high inflation that is caused by a supply shock, if a central bank aims at containing and reducing the rate of inflation, it can raise interest rates moderately to disinflate gradually. For a central bank that lacks commitment to a rule, a higher initial interest rate increase is required since the public would not be sure that the central bank would maintain the increase.

Nonetheless, one should note that the optimal simple rule is different from the optimal commitment policy. The optimal commitment policy supposes that monetary policymakers guarantee that they will never default on any of their promises when choosing the interest rate to minimize expected loss function. To set an optimal commitment policy, at time zero the central bank evaluates all possible outcomes, decides how to react to each, and promises to stick with the chosen set of reactions. Södelind (1999) shows that a state-space representation for the optimal commitment plan exists provided that the state vector is augmented to include the co-state variables from the optimization. But writing the optimal commitment plan as a function only of lagged values of output, inflation, and the interest rate gives an equation for the interest rate that depends on the entire history of the state vector dating back to the time when policy is set.

This study assumes that the central bank follows an optimal simple rule rather than the optimal commitment policy rule for two reasons. First, Woodford (2003) argues that it is not feasible for a central bank to provide the public at the beginning with a complete listing of relevant contingencies and it is thus difficult for the public to verify that the central bank is following the optimal commitment policy rather than engaging in discretion. The public's ability to predict central bank policy is improved if the central bank commits to a rigid and simple rule. Therefore, it is more desirable for the central bank to follow an optimal simple rule rather than the optimal commitment policy, see McCallum (1997). Second, Atoian, Givens and Salemi (2003) study the relative performance of optimal simple rules versus optimal commitment policy rules in a structural model that is identical to the one used in this study. They find that when the central bank places primary emphasis on inflation stabilization, loss under the optimal simple rule is nearly as small as loss under optimal commitment.

Despite the benefits of commitment to a policy rule, Svensson (2003) cautions that assuming a central bank commits to a simple policy rule is not attractive. The simple rule does not allow for policy adjustment based on newly arrived information on the economic mechanisms or shocks, therefore it seems unlikely that a central bank would be willing to commit itself to an explicit and mechanical simple rule. In addition, although the simple rule may be a good approximation to optimal policy under normal circumstances, it would be sub-optimal under some unusual events. Svensson prefers an optimal commitment "targeting" rule that specifies the target variables, the targets, and the loss function to be minimized.

The policy rule considered in this study is less specific and mechanical than the one objected to by Svensson (2003). The values of the coefficients that appear on the right-hand-side of the policy rule are not specified in advance but depend on the economic structure, and they are chosen to minimize central bank's expected loss. Furthermore, the policy rule is not assumed to hold exactly, but includes a disturbance term. The form of the policy rule is also consistent with Svensson's idea that policy makers respond to forecasts of target variables since those forecasts lie in the space spanned by the state vector. Lastly, this study considers explicitly the objectives of the central bank, the target variables, and the targets, which are consistent with the ideas that motivate targeting as Svensson's recommendations. On the other hand, this study makes no attempt to allow for learning by the policymaker or to determine whether "unusual events" help explain the evolution of the interest rate.

2.3 Computing the Optimal Simple Rule

In the standard linear quadratic dynamic optimization problem, the state transition equation does not vary with the policy rule and one can compute the coefficients in the policy rule that minimize central bank's loss function by iterating the Ricatti equation to convergence, see Sargent (1987). A short list of algorithms and applications would include Backus and Driffill (1986), Södelind (1999), Svensson (2000), Dennis (2004). However, the appearances of forward-looking variables in the structure equations make it more complicated to derive the coefficients for the optimal policy rule.

This study follows the approach used in Salemi (2006) to compute the central bank's expected loss in terms of the forecast error variances of endogenous variables. The approach decomposes the expected loss (2.6) into two parts:

$$\begin{aligned}
\mathbf{L}_t &= trace\left[W\sum_{k=1}^{\infty}\delta^k E_t(X_{t+k}X'_{t+k})\right], \qquad (2.7)\\
&= trace\left[W\sum_{k=1}^{\infty}\delta^k (E_t(X_{t+k}-E_tX_{t+k})(X_{t+k}-E_tX_{t+k})' + (E_tX_{t+k})(E_tX_{t+k})')\right],\\
&= trace[W(M_t+N_t)],
\end{aligned}$$

where

$$M_t = \sum_{k=1}^{\infty}\delta^k E_t(X_{t+k}-E_tX_{t+k})(X_{t+k}-E_tX_{t+k})',$$

and

$$N_t = \sum_{k=1}^{\infty} \delta^k (E_t X_{t+k})(E_t X_{t+k})'.$$

M_t is the discounted sum of forecast error variances of X computed when policy is set. N_t is the discounted sum of quadratic terms in expected departures of X from its target. This study assumes that the monetary authority ignores N_t when it chooses its policy rule either because the economy begins on its target path or because the monetary authority follows the timeless perspective introduced by Woodford (1999). Because φ_t is serially uncorrelated, using backward substitution X_{t+k} can be expressed in terms of period t values of X and future shocks

$$X_{t+k} = G^k X_t + G^{k-1}\varphi_{t+1} + \ldots + G\varphi_{t+k-1} + \varphi_{t+k}.$$

Therefore, one can write the forecast error variances of X_t as

$$E_t(X_{t+k} - E_t X_{t+k})(X_{t+k} - E_t X_{t+k})' = \Omega + G\Omega G' + \ldots + G^{k-1}\Omega(G^{k-1})',$$

where Ω represents the covariance matrix of innovations φ_t in the model solution form. In this study, a policy regime is defined as a set of preference parameters which central bank chooses for minimizing the value of its loss function, and the covariance matrix of shocks is assumed to differ in different policy regimes. Thus one should note that W and Ω are both regime specific.

An expression for M_t takes the following form

$$\begin{aligned} M_t &= \delta\Omega + \delta^2(\Omega + G\Omega G') + \ldots \\ &\quad + \delta^{k+1}(\Omega + G\Omega G' + \ldots + G^{k-1}\Omega(G^{k-1})') + \ldots, \\ &= \frac{\delta}{1-\delta}[\Omega + \delta G\Omega G' + \delta^2 G^2\Omega(G^2)' + \ldots]. \end{aligned} \tag{2.8}$$

Therefore, the central bank's expected loss can be expressed in terms of the forecast error covariances of exogenous variables. Iterating on equation (2.8) provides a numerical value for M_t, which can be substituted into equation (2.7) to numerically evaluate the central bank's objective function.

To compute expected loss associated with a set of policy rule coefficients, this study computes the sequence of partial sum in (2.8) until it converges[13]. To find the optimal policy, this study searches for the policy-rule coefficients that can minimize $trace(WM_t)$. Salemi (2006) points out that the problem of choosing the optimal values of policy rule parameters is complicated by the fact that private agents in the economy are forward-looking. Equation (2.8) implies that the loss-minimizing policy-rule coefficients depend, through G, on the parameters of equations (2.1) – (2.3). Because private agents in the economy are forward looking, G depends on the policy-rule coefficients as well. Therefore, the parameters of equations (2.1) – (2.3) and the policy-rule coefficients must be computed simultaneously.

Atoian, Givens and Salemi (2003) provide an algorithm that is used to compute the optimal values of policy rule coefficients which are chosen by the central bank to minimize its loss function. First, for any given values of the structural parameter vector, the matrix describing the policymaker's preferences, and the variance-covariance matrix of the reduced form shocks, one can choose some arbitrary initial values of the policy rule

[13]Iterations on the sequence of partial sum are performed until a convergence criterion is met: the difference between two sequential iterations of the partial sum is less than 1×10^{-7}.

coefficients and use equation (2.4) to compute the reduced form model solution. Given G, one can evaluate the policy loss using equation (2.7). In the second step, one can numerically compute partial derivative of the loss function with respect to each policy rule coefficient while recomputing G for every change in policy coefficients to capture the adjustment of private agents' expectations to changes in monetary policy. In the final step, the algorithm updates policy rule coefficients if doing so reduces policy loss. The algorithm repeats the last two steps until it converges to the point in the parameter space that yields the minimum value for the policy loss.

2.4 Inferring Monetary Policy Regimes

It is widely believed that monetary policymakers should display time-varying policy preferences over time. One potential source of changing monetary policy is that monetary policymakers would face different levels and variabilities of inflation and other economic variables in different time spans. In the case of United States, monetary policymakers in the Federal Reserve must contend with the high inflation that occurred in the 1970's, the disinflation in the 1980's, several large oil price shocks, the geopolitical events, and the recessions that occurred in the early 1980's and 1990's, early and late 2000's. Thus, over time the Federal Reserve may adjust its focus on different objectives hence change the set of weights in loss function and the corresponding policy-rule coefficients. In order to infer the central bank's time-varying policy preferences, this study assumes that at each period t there are n_j monetary policy regimes and each policy regime can occur with a certain probability. In a given monetary policy regime, the central bank can choose a set of weights in the loss function and the corresponding coefficients in the reaction function to minimize its expected loss. Hence each weight in the loss function can take n_j different values in period t, and each value corresponds to the policy regime j_t, where $j_t \in \{1, 2, \ldots, n_j\}$.

The transitions between policy regimes are modeled to follow a Markov-switching process. The timing of each transition in the stochastic process of policy regimes is not determined ex ante, but is estimated from the data. The policy regimes are assumed to be independent of each other. An unobservable exogenous variable j_t determines the current regime. j_t follows a Markov chain with a constant $n_j \times n_j$ transition matrix

$$\mathbf{P} = \begin{bmatrix} p_{11} & \cdots & p_{n_j 1} \\ \vdots & p_{jk} & \vdots \\ p_{1 n_j} & \cdots & p_{n_j n_j} \end{bmatrix},$$

where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$. Furthermore, $\sum_{k=1}^{n_j} p_{jk} = 1$ and $p_{jk} \geq 0$, $\forall\, k,\, j \in \{1, \ldots, n_j\}$. Let the n_j vector $p_t \equiv (p_{1t}, \ldots, p_{n_j t})'$ denote the probability distribution of policy regimes in period t, where $p_{jt} \equiv Pr(j_t = j)$. Hence the probability distribution of policy regimes in period $t+1$ is

$$p_{t+1} = \mathbf{P} p_t. \tag{2.9}$$

Why does it make sense to assume that the transitions between policy regimes follow a stochastic process? Cooley, Leroy and Raymon (1984), Sims (1987), and Sims and Zha (2006) argue that if monetary policymakers can contemplate changing policy, then a monetary policy regime can recur. Given that there is a history of changing monetary policy and changes in monetary policy are difficult to predict, it is best for private agents to presume policy regimes can recur and ascribe probability distributions over the possible policy regimes. Private agents' expectations and decision rules should incorporate the beliefs that policy regimes are not once-and-for-all and that returning to the old policy regime is always possible. Beliefs of policy regime changes are used to study when and how policy regime changes occur.

Davig and Leeper (2007) and Chung, Davig and Leeper (2007) point out that modeling monetary policy regime changes stochastically is especially important for the United States. Monetary policy in the U.S. is largely determined by the confluence of economic and political conditions, which fluctuate all the time. Furthermore, there is no good mechanism that can bind the Federal Reserve's action in the future, so it is very hard to exclude the possibility of returning to inflationary policy even though the Federal Reserve successfully brought down inflation. As a result, stochastic policy regime switching is an assumption that is closer to the reality than the non-recurrent regime switching.

Although modeling the transitions between policy regimes with a stochastic process is useful for capturing the complex behavior of central bank, it does not address the issue as to what extent the probability distribution of a policy regime is affected by the economic environment or other non-economic factors. This issue deserves a separate investigation, which is beyond the scope of this study. Given that the high degree of uncertainty in understanding the stochastic properties of economic environment, incorrectly modeling the systematic relationship between economic structure and monetary policy is not necessarily better than modeling the policy regime changes with a stochastic process. The stochastic transitions between policy regimes can also help improve model tractability and permit straightforward interpretations. At this stage, the specification of Markov transitions between policy regimes is a starting point for building more sophisticated models of monetary authority's policy choice, and it can help develop intuitions of the appropriate policy design.

2.5 Maximum Likelihood Estimation: SURF and SORF

The parameters of the model are collected into two groups. Collect the structural parameters into vector $V = \{\lambda,\ a_1,\ a_2,\ b,\ \alpha_1,\ \alpha_2,\ \beta,\ C_{11},\ \ldots,\ C_{33}\}$, which contains the parameters of the IS equation, Phillips curve, and the elements in the variance-covariance matrix of the reduced-form shocks. Collect the coefficients of the monetary policy rule into vector $\Delta = \{c_1, \ c_2, \ c_3, \ c_4\}$.

In line with Salemi (2006), this study uses two estimation strategies. The first type of econometric policy evaluation in this study estimates V and Δ while treating Δ as unrestricted. This estimation strategy is referred as SURF: Structure estimation with an Unrestricted Reaction Function. In the following sections, it can be seen that this study reports SURF estimates of V and Δ and analyzes policy with these estimates.

The hypothesis that the policy-rule coefficients minimize expected loss induces a restriction function on the policy-rule coefficients, $\Delta = g(V, \ W)$. This study uses a unified approach that estimates V and W subject to the restriction, $\Delta = g(V, \ W)$. This estimation strategy is referred as SORF: Structure estimation with an Optimal Reaction Function. In the following sections, it can be seen that this study reports SORF estimates of V and W together with the implied value of Δ. Appendix A provides an explanation of the algorithm used to compute SORF estimates. SORF estimates can allow one to perform formal assessment of the monetary policy optimality hypothesis.

2.6 Data Description

The baseline model and the optimal-policy hypothesis are fitted to U.S. quarterly data from 1965:Q1 to 2001:Q4 on output, inflation, and interest rates. Since this study applies a rational expectation Keynesian model for policy analysis starting 1965, here the monetary policymakers in the Federal Reserve are assumed to understand the forward-looking nature of private agents' behavior two decades before Lucas published his famous critique on policy analysis[14].

[14] This assumption is in line with Salemi (2006).

The data used here are the same as those in Salemi (2006), the results of which establish a benchmark for this study. The raw data series, except for population, are drawn from FRED, the Federal Reserve Bank of St. Louis database. Quarterly population numbers are interpolations of annual values found in the 2002 *Economic Report of the President*.

Output measure is real GDP (in 1996 chained dollars) divided by civilian population. As illustrated in Salemi (2006), some undesirable effects were generated as real per capita GDP growth was depressed throughout the late 1970's, most of the 1980's, and the early 1990's. The economy restored its growth in the early 2000's. Empirical implementation of the model presented earlier requires stationary data. To induce stationarity, this study fits a constant-coefficient time trend to the natural log of real GDP per capita. The trend is used as a proxy for long run growth path of per capita output. Variable y_t, the output gap that is used in the structural model estimation, is the difference between the natural logarithm of real GDP per capita and its long run trend. Such transformation of the output variable is consistent with the assumption that the central bank aims to keep output close to its long-run growth path.

The inflation rate is the annualized percentage change in the chained GDP deflator. The inflation rate rose from 1965 to 1980, dropped dramatically between 1980 and 1985, and continued to trend gradually downward between 1986 and 2001. The quarterly interest rate is the annualized secondary market yield of three-month treasury bills. The interest rate also followed a trend path that is similar to that of inflation. Salemi (2006) suggests that it is not appealing to assume that there was only one constant trend throughout the sample period. Therefore, target values for the inflation rate and interest rate are obtained by fitting continuous, piecewise-linear trends that allow for trend coefficient changes in 1980:Q1 and 1986:Q1[15] but trend coefficients in the inflation

[15] Given the changes in the target paths that occurred in 1980 and 1986, it is possible that there is a policy regime prevailing between 1980 and 1985.

rate and interest rate piecewise regressions are constrained to be the same. Variables π_t and r_t are the residuals from the two trend regressions. These transformed series imply that the central bank tries to minimize the variations of inflation and the interest rate around their target paths.

The target-path assumptions on the inflation rate and interest rate imply that the Federal Reserve acclimated itself to rising inflation during the 1960's and 1970's. The paths also imply that the Federal Reserve rapidly decreased inflation target between 1980 and 1986 and then gradually lowered target inflation over the next 15 years. The target-path assumptions on the inflation rate and interest rate are consistent with many studies in the literature. Finally, the paths imply a constant target value for the real rate of interest.

There is no evidence of a unit root in the time series of y_t, π_t, and r_t, see results in Salemi (2006). For y_t, the hypothesis of a unit root can be rejected at the 5% level using the augmented Dickey-Fuller statistic with lag lengths 2 to 4. For π_t and r_t, the hypothesis of a unit root can also be rejected at the 5% level using either the augmented Dickey-Fuller or the Phillips-Perron test with lag lengths 0 through 4.

Because the model is forward-looking, changes in the policy rule will cause across-regime changes in the parameters of the reduced form equations. But the results in many studies, including Salemi (2006), suggest that there is not evidence of large changes in reduced form parameters that could indicate a structural break in the sample period. Salemi (2006) shows that there is no strong evidence that the coefficients of the y_t and π_t equations changed in 1980. The p-values associated with the relevant F statistics are 0.420 for equation (2.1) and 0.490 for equation (2.2). The p-value associated with the likelihood ratio statistic for the hypothesis of no shift in the parameters of either equation is 0.380.

As equations (2.7) and (2.8) indicate, loss-minimizing values of the policy-rule coefficients depend on the covariance matrix of shocks Ω as well as on the reduced form parameters G[16]. Therefore, it is important to ask whether the Federal Reserve faced different distributions of shocks over time. The results of linear projections in Salemi (2006) suggest that the covariance matrix of shocks should be different across policy regimes.

In summary, this study will estimate and test the model by varying the number of policy regime from two to three. The parameters of equations (2.1) – (2.2) will be held constant across policy regimes but the monetary policy and covariance matrix of shocks would be allowed to change.

2.7 Econometric Evaluation of U.S. Monetary Policy with Two Policy Regimes

This section applies the estimation approach described in the previous section to estimate the structural model parameters in equations (2.1) – (2.3) jointly with the policy preference parameters in equation (2.6). In this section, monetary policy is assumed to switch between two regimes during the sample period. Therefore, in the estimation algorithm, the policy parameters and policymaker's preference parameters are allowed to change in each regime.

Figures 2.1 and 2.2 present the probability inferences in SURF and SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 2.1, 2.2, and 2.3 report SURF and SORF estimates of model parameters along with the value of the log likelihood function. To make inferences about the significance of estimated parameters, this study reports the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is zero. A similar

[16] While certainty equivalence does not hold for the optimal simple rule, Svensson and Woodford (2003) make clear that it does hold for the optimal time-consistent policy.

approach is used by Gallant, Hsu, and Tauchen (1999) and Salemi (2006). The reason of using this approach is that the likelihood surface is highly non-smooth, so derivative-based procedures that calculate the inverse of information matrix can not produce reasonable estimates of standard errors.

2.7.1 Probability Inferences for Two Policy Regimes

From the estimations of the Markov-switching models one can obtain the probability inferences for the monetary policy prevailing in each regime for each date t in the sample. Figure 2.1 and Figure 2.2 plot the SURF and SORF estimated probability series of being in the two regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. One should note that these recession determinations are made informally on the basis of a large number of time series and are usually made some time after the event, see Hamilton (1994). Although these business cycle dates were not used in any way to estimate parameters or form inferences about the Markov transition process, it is interesting that the traditional business cycle dates correspond fairly closely to the switches of monetary policy as described by the SURF and SORF estimates in Figure 2.1 and Figure 2.2.

Regardless of whether the optimal-policy hypothesis is imposed, the estimates reveal that monetary policy regime one prevailed roughly after the recession started, and policy regime two prevailed during the expansionary periods. After the recession dates in 1970, 1975, 1980, and 2001, U.S. monetary policy is estimated to be in regime one, in which the Federal Reserve quickly adjusted its policy instruments to fight recession. When the economy got out of recession, U.S. monetary policy switched to regime two, in which the primary goal of the Federal Reserve was to maintain low inflation rate and sustainable output growth. As a result, it can be seen that regime two was dominant for most of the sample period including most of the late 1960's, the latter half of the 1970's, and the entire period from the late 1980's onward until the end of the sample period.

2.7.2 Estimates of Structural Parameters

The discussions here begin with the SURF structural parameters estimates, see Table 2.1. A sizable value of parameter λ implies that expected future output plays an important role in determining aggregate demand of household. Here the estimate of λ is 0.282 when the assumption of policy optimality is relaxed. While the estimate is sizable, the p-value of the likelihood ratio test indicates that one can not reject the null hypothesis that the true parameter value is very close to zero. Therefore, the estimate of λ in SURF estimates implies that expected future output may not be an important factor in affecting aggregate demand. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are found to be 0.914 and -0.207. The estimate of a_1 is statistically indifferent from one and the estimate of a_2 is significantly different from zero at any conventional confidence level. Hence these estimates imply that shocks have sustained effects on output. Third, the estimate of b is 0.022 without imposing the constraint of optimal reaction function. Despite the fact that b is estimated to be quantitatively small, the estimate appears to be significant at 10% level. The estimate implies that changes in the interest rate do cause changes in aggregate demand although the effect of interest rate on aggregate demand is relatively insensitive. Fourth, turning to the parameters of the Phillips curve, one can note that firms are substantially more backward-looking than forward-looking. The estimate of the relative weight α_1 is very small and is found to be highly insignificant, and estimate of α_2 is 0.534 and is found to be significantly different from zero. The findings here result in a backward-looking Phillips curve and cast doubt on the sticky price mechanism of Calvo. Fifth, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The estimate of β is 0.080 and is found to be significant which implies that a positive output gap raises the inflation rate.

The SORF estimates differ in several important ways. When the policy optimality is imposed, the estimate of λ is 0.184, which is smaller and still insignificant. Second, the SORF estimate of b is larger and more precisely estimated than the estimate under SURF, which imply that a change in the interest rate has a larger effect on aggregate demand. Third, SORF estimates of both α_1 and α_2 are positive and highly significant, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, one can not reject the hypothesis that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006), and SORF estimates were obtained with that restriction imposed. One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with Galí and Gertler (1999), who find that most of the U.S. firms are purely forward-looking but a fraction are following a backward-looking rule of thumb. Fourth, while still highly significant at any conventional confidence level, the SORF estimate of β is much smaller implying that it is more difficult for the Federal Reserve to control inflation by changing aggregate demand.

As the p-values reported in Table 2.1 indicate, some of the structural parameters, such as α_1 the parameter of the forward-looking term in the Phillips curve, are more precisely estimated under SORF than under SURF. It is believed that a unified approach of joint estimation of the structural model parameters with the parameters of the policymaker's objective function provides additional across equation restrictions that result in a more precise estimation of the model parameters. Hence the SORF estimates more strongly support a forward-looking Phillips curve. Castelnuovo (2003) argues that hybrid formulation of the Phillips curve is crucial for appropriate identification of policy preferences. Moreover, Salemi (2006) suggests that when the interest rate is highly persistent, the best way to reconcile a persistent interest rate process and policy optimality is to hypothesize that both occurred in an economy where private agents are forward looking. The finding of highly persistent interest rate in both regimes will be shown below.

2.7.3 Estimates of Preference Parameters and Test of Policy Optimality

Table 2.2 reports estimation results for the preference parameters in two regimes identified in U.S. monetary policy, and it also presents two sets of reaction function coefficients for each policy regime: SURF estimates (obtained while treating coefficients of the policy rule as free parameters), and SORF estimates (implied values of the policy rule coefficients consistent with the policy optimality hypothesis). To interpret the results on policy preference estimation, one should note that the relative weight on inflation stabilization objective is normalized to one.

The results in Table 2.2 support several conclusions. First, one can notice that under SORF the estimated values of w_y and w_r are very small implying that inflation stabilization was the dominant objective of the Federal Reserve in both policy regimes. The relative weights placed on output stabilization in two policy regimes are both 0.0020. The weight w_y in regime one is insignificant, while in regime two it is marginally significantly different from zero at 10% level. The weight placed on interest rate stabilization is small but significant in each regime, and the value of w_r is slightly larger in the second regime than that in the first regime. Given the normalization of the weights, the estimates of the central bank's preferences in both policy regimes suggest that in conducting its monetary policy the Federal Reserve was far more concerned with keeping the inflation close to its long run target path than it was with trying to stabilize output and interest rate around their targets. Unlike many other central banks, the Federal Reserve Bank refrained from attempting to fine-tune its policy to stabilize the real sector of the economy and focused instead on the price stability. Second, there is clear evidence that the estimated and implied policy-rule coefficient estimates in two regimes differ significantly and substantially from each other. The SURF coefficients on lagged inflation (c_2) and lagged outputs (c_1 and c_4) are larger in magnitude and insignificant in the first policy regime but smaller in magnitude and highly significant in the second regime. The SORF estimates of policy rule coefficients are larger in magnitude in the first policy regime than those in the second regime. Third, all SORF estimates of the policy rule coefficients are greater in magnitude than the SURF estimates in both regimes. This suggests that the Federal Reserve responded consistently to inflation and output, which is discussed shortly.

How different are the estimated preferences here from the ones reported in other studies? Compared to estimates of the weights in the Federal Reserve's loss function in the literature[17], the results here are at the lower end for both w_y and w_r. Salemi (2006) estimates a similar model, and he obtains a statistically insignificant estimate 0.0047 for w_y and a significant estimate 0.013 for w_r during the Volcker-Greenspan period $(1980 - 2001)$ and an insignificant estimate 0.0012 for w_y and 0.0062 for w_y during the Martin-Burns period $(1965-1980)$. Using a backward-looking model, Favero and Rovelli (2003) estimate w_y and w_r directly from the first-order condition of the Federal Reserve's optimization problem and obtain small but statistically significant weights of 0.00125 for w_y and 0.0085 for w_r during $1980 - 1998$, and 0.00153 for w_y and 0.0051 for w_r during $1961 - 1979$. Ozlale (2003) gets estimates[18] of 0.52 for output and 0.86 for interest rate smoothing for the Greenspan period $(1987 - 1999)$, 0.47 for output and 0.76 for interest rate smoothing for the Volcker period $(1979 - 1987)$, and 1.12 for output and 0.91 for interest rate smoothing for the Burns period $(1970 - 1978)$. For the Federal Reserve, Assenmacher-Wesche (2006) obtains estimates of 1.16 for w_y and 0.75 for w_r in one of

[17]One should note that the studies in literature use various economic models and empirical strategies varying from unified approach to two-step approach to estimate weights in the central bank's loss function.

[18]Since Ozlale (2003) does not normalize the weight on inflation stabilization w_π to one, this study reports the ratios of the weight on output to the weight on inflation (w_y/w_π) and the weight on interest rate smoothing to the weight on inflation (w_r/w_π) based on Ozlale's results.

the two policy regimes defined in her paper, and estimates of 1.29 for w_y and 0.94 for w_r in another policy regime. Dennis (2006) estimates a backward-looking model on U.S. data from 1982 to 2000, and he shows a statistically insignificant estimate 2.941 for w_y and a significant estimate of 4.517 for w_r. As a result, most of these studies report a high degree of inflation stabilization in U.S. monetary policy, and all these studies find statistically significant evidence of the Federal Reserve's concerns with interest rate behavior.

Did the Federal Reserve set monetary policy in a way consistent with the definition of optimal monetary policy used in this study? In light of Salemi (2006), a test of policy optimality is a test of the hypothesis that SORF fits the data as well as SURF, that is, the SORF and SURF policy rule coefficients are the same. The answer is no. In each policy regime, there are four freely estimated parameters in the unrestricted policy rule but only two freely estimated parameters, w_y and w_r, in the optimal policy rule, so SORF implies four restrictions in total. As a result, the relevant likelihood ratio statistic is 14.957 with a p-value 0.005. Thus the hypothesis that the policy rule coefficients that fit the data best were those that minimized quadratic expected loss as defined in (2.6) can be soundly rejected.

2.7.4 Interest Rate Smoothing

As can be seen in Table 2.2, the reaction-function coefficient for the lagged interest rate is always large compared to the coefficients for inflation and output so that, according to the definition provided by Clarida, Galí and Gertler (2000), the Federal Reserve engages in "interest rate smoothing". This finding supports the widely held viewpoint that most central banks, including the Federal Reserve, are very cautious when changing interest rates. Brainard (1967) argues that large movements in interest rates can lead to lost of credibility, if a policy intervention subsequently needs to be reversed. Cukierman (1989) and Goodfriend (1991) suggest that too much volatility in interest rates is undesirable because it may result in maturity mismatches between bank's assets and liabilities, which would be disruptive for financial sector. Sack and Weiland (2000) provide three competing explanations for interest rate smoothing. First, the Federal Reserve may realize that rules with persistence are more effective for stabilizing output and inflation than rules without persistence in economies where forward-looking expectations matter. Second, the Federal Reserve may be uncertain about the accuracy of the data on which they rely to conduct policy. Third, the Federal Reserve may be uncertain about the values of structural parameters. Lansing (2002) cautions that smoothing can arise spuriously if the lagged rate of interest is correlated with measurement errors in the data.

While the estimates in the reaction functions reported in Table 2.2 imply that the Federal Reserve smooths interest rates, SORF estimates suggest that the Federal Reserve placed little attention on interest rate stability in both policy regimes. Thus, in line with Salemi (2006), the estimates here are consistent with Sack and Weiland's first explanation for smoothing. The large coefficient on the lagged interest rate can be explained by the finding that forward-looking expectations are empirically important and the fact that optimal policies require persistent responses to shocks.

2.7.5 Estimated Transition Probabilities

Table 2.3 gives SURF and SORF estimates for the transition probabilities for switching in two monetary policy regimes. As the p-values indicate, all transition probabilities are precisely estimated. The probabilities on the diagonal of the transition matrix are close to unity, which means that both policy regimes show high persistence – a feature that is commonly seen in the estimation of Markov switching models, see Sims (1999) and Assenmacher-Wesche (2006). For instance, the SURF estimates indicate that, if the monetary policy is in regime one this period, there is a 89.6 percent probability that it will be in regime one next period; if the monetary policy is in regime two this period, there is a 94.4 percent probability that it will be in regime two next period. Similarly, the SORF estimates indicate that, if the monetary policy is in regime one this period, there is a 87.7 percent probability that it will be in regime one next period; if the monetary policy is in regime two this period, there is a 95.7 percent probability that it will be in regime two next period. These suggest that changes in the monetary policy regime will occur infrequently. Although these two policy regimes are persistent, the SURF and SORF estimates show that they are not degenerate regimes. The p-values for p_{11} and p_{22} are measured by the likelihood ratio test for the hypothesis that the true value of the transition probability is 0.995, meaning that there is nearly no switch back into the regime prevailing at the beginning of the sample period. The test statistics reject the hypothesis thus the SURF and SORF transition probabilities do not induce degenerate policy regimes.

2.7.6 Comparison of Impulse Response Functions for Structural Estimation

To check how much the model can explain the variation in the data, this section compares the impulse response functions (IRF's) implied by SURF and SORF. In line with Salemi (2006), this study makes the hypothesis that monetary policy is conditioned only on lagged values of output and inflation. In keeping with this hypothesis, this study chooses y-π-r as the within period causal ordering for the impulse response function. The ordering implies that a shock to output affects only output contemporaneously, a shock to inflation affects both inflation and output contemporaneously, and a shock to the interest rate affects all variables contemporaneously. The ordering makes difference because of the contemporaneous correlation between output and the interest rate in both policy regimes. Figures 2.3 and 2.4 present the IRF's for regimes one and two. The shade areas in the figures are 95% confidence intervals computed with parametric Bootstrap method[19].

The figures show clearly that the SURF and SORF estimates share many features of the impulse response functions. For instance, SURF and SORF estimates all imply that a positive shock to output keeps output above its trend for more than 30 quarters. Both imply that a positive shock to inflation produces moderate inflation persistence with inflation returning to trend in about six quarters. Both imply that a positive inflation shock causes output to fall below trend for a substantial period of time. SURF and SORF IRF's all show that the interest rate rises quickly after a positive output shock and remains above trend for at least ten quarters.

There are differences in the impulse response functions. SURF estimates imply that the interest rate increases then falls gradually after a positive inflation shock in regime one and rises then falls below trend after ten quarters in regime two. SORF estimates imply that the interest rate rises quickly then returns to trend after about ten quarters in regime one and rises then remains above trend for a long time after a positive inflation shock in regime two. A second difference is related to the first. SORF estimates imply that the inflation response to a positive output shock is far smaller than implied by the SURF estimates.

Comparing the IRF's suggest that actual and optimal responses of the interest rate to output shocks were nearly the same in policy regime one but they were quite different in regime two. Therefore, in policy regime one, SURF and SORF estimates imply that inflation resulting from positive output shocks would be dampened completely. In policy regime two, SURF estimates imply that a positive output shock causes interest rate to rise above trend for a very long time, and SORF estimates imply that a positive output shock first causes interest rate to rise quickly then fall below trend after about ten quarters.

[19]This study simulates data by supplying sequences of random disturbances to the reduced form solution of the estimated model. These shocks are drawn from the zero mean multivariate normal distributions with variance-covariance matrices identical to those estimated for each of the two policy regimes.

The comparison also implies that actual interest rate responses to inflation shocks were not optimal. A loss-minimizing policy would have raised interest rate more after inflation shocks in both regimes one and two.

Figure 2.5 compares the responses of output and inflation to an interest rate shock. The policy rule equation residuals are positively correlated with the IS and Phillips curve equation residuals in both policy regimes. On the maintained hypothesis that the monetary policy reacts to economic conditions only with a lag, this correlation between must be due either to responses of output and inflation to an interest rate surprise or to responses of output, inflation, and interest rate to an un-modeled shock. Following Salemi (2006), to obtain the model's prediction of responses of output and inflation to a pure interest rate shock, this study ignores the contemporaneous correlations when computing the IRF's reported in Figure 2.5.

Figure 2.5 shows that in both regimes SURF and SORF estimates can account for the "hump-shaped" response of output to an interest rate surprise. In policy regime one, SURF peak occurs about ten quarters after the interest rate shock, and SORF peak occurs about five quarters after the shock. In policy regime two, both SURF and SORF peaks occur about ten quarters after the interest rate shock. Both SURF and SORF estimates show that the inflation rate declines immediately after an interest rate shock and remains below trend for many quarters. SORF estimates imply a much more smaller inflation response than do SURF estimates, especially in policy regime one.

2.8 Econometric Evaluation of U.S. Monetary Policy with Three Policy Regimes

This section applies the estimation approach described earlier to estimate the structural model parameters in equations (2.1) – (2.3) jointly with the policy preference parameters in equation (2.6). In this section, monetary policy is assumed to switch among three regimes during the sample period. Therefore, in the estimation algorithm, the policy parameters and policymaker's preference parameters are allowed to change across three policy regimes. In addition, comparing results in this section and the previous section allows a test for the existence of a third policy regime.

Figures 2.6 and 2.7 present the probability inferences in SURF and SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 2.4, 2.5, and 2.6 report SURF and SORF estimates of model parameters along with the value of the log likelihood function. To make inferences about the significance of estimated parameters, this study reports the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is zero. A similar approach is used by Gallant, Hsu, and Tauchen (1999) and Salemi (2006). The reason of using this approach is that the likelihood surface is highly non-smooth, so derivative-based procedures that calculate the inverse of information matrix can not produce reasonable estimates of standard errors.

2.8.1 Probability Inferences for Three Policy Regimes

From the estimations of the Markov-switching models one also obtain the probability inferences for the monetary policy prevailing in each regime for each date t in the sample. Figure 2.6 and Figure 2.7 plot the SURF and SORF estimated probability series of being in the three regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. One should note that these recession determinations are made informally on the basis of a large number of time series and are usually made some time after the event, see Hamilton (1994). Although these business cycle dates were not used in any way to estimate parameters or form inferences about the Markov transition process, it is interesting that the traditional business cycle dates are fairly related to the switches of monetary policy between the three regimes as described by the SURF and SORF estimates in Figure 2.6 and Figure 2.7.

Both SURF and SORF estimates show that policy regime one is a special regime that only prevailed between 1979 and 1984. During this period, the Federal Reserve successfully reduced high inflationary pressure in the U.S., and this period of time is commonly known as the Volcker disinflation period. Both Figure 2.6 and Figure 2.7 show that the special regime one prevailed only during the disinflation period and nowhere else[20]. Regime two is a regime which the Federal Reserve's monetary policy switched into during the expansionary periods, and regime three is a regime which the monetary policy switched into after the recession happened. When the economy came out of recession, U.S. monetary policy switched to regime two, in which the primary goal of the Federal Reserve was to maintain low inflation rate and sustainable output growth. As a result, it can be seen that regime two was dominant for most of the sample period, including most of the late 1960's, the latter half of the 1970's, and the entire period from the late 1980's onward until the end of the sample period. During the recession periods in 1970, 1975, 1982, and 2001, monetary policy in the U.S. is estimated to switch into regime three, in which the Federal Reserve adjusted its policy instruments quickly to fight recession.

2.8.2 Estimates of Structural Parameters

The discussions here begin with the SURF structural parameters estimates, see Table 2.4. A sizable value of parameter λ implies that expected future output plays an important role in determining aggregate demand of household. Here the estimate of λ is 0.270 when the assumption of policy optimality is relaxed. While the estimate is sizable, the p-value of the likelihood ratio test indicates that one can not reject the null hypothesis that the true parameter value is zero. Therefore, the estimate of λ in SURF

[20] Although regime one only prevailed during $1979 - 1984$, estimates of the three-regime transition matrix in Table 2.6 do not suggest that regime one is a degenerate regime.

estimates implies that expected future output is not an important factor in affecting aggregate demand. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are found to be sizable and they are 0.940 and -0.221, respectively. The estimate of a_1 is statistically indifferent from one and the estimate of a_2 is significantly different from zero at any conventional confidence level. Hence these estimates imply that shocks have sustained effects on output. Third, the estimate of b is 0.022 without imposing the constraint of optimal reaction function. Despite the fact that b is estimated to be quantitatively small, the estimate appears to be significant at marginal 10% level. The estimate implies that changes in the interest rate do cause changes in aggregate demand although the effect of interest rate on aggregate demand is relatively insensitive. Fourth, turning to the parameters of the Phillips curve, one can note that firms are substantially more backward-looking than forward-looking. The estimate of the relative weight α_1 is very small and is found to be highly insignificant, and estimate of α_2 is 0.544 and is found to be significantly different from zero. The findings here result in a backward-looking Phillips curve and cast doubt on the sticky price mechanism of Calvo. Fifth, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The estimate of β is 0.066 and is found to be significant which implies that a positive output gap raises the inflation rate.

The SORF estimates differ in several important ways. When the policy optimality is imposed, the estimate of λ is 0.278, which is nearly the same as SURF estimate but is highly significant. Hence SORF estimate of λ implies that expected future output is an important factor in determining aggregate demand. Second, the SORF estimate of b is smaller but is more precisely estimated than the estimate under SURF, which imply that a change in the interest rate has a smaller effect on aggregate demand. Third, SORF estimates of both α_1 and α_2 are positive and highly significant, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, one can not reject the hypothesis that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006), and SORF estimates were obtained with that restriction imposed. One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with Galí and Gertler (1999), who find that most of the U.S. firms are purely forward-looking but a fraction are following a backward-looking rule of thumb. Fourth, while highly significant at any conventional confidence level, the SORF estimate of β is much smaller implying that it is more difficult for the Federal Reserve to control inflation by changing aggregate demand.

As the p-values reported in Table 2.4 indicate, some of the structural parameters, such as λ and α_1 the parameters of the forward-looking terms in the IS schedule and Phillips curve, are more precisely estimated under SORF than under SURF. It is believed that a unified approach of joint estimation of structural model parameters with the parameters of the policymaker's objective function provides additional across equation restrictions that result in a more precise estimation of the model parameters. Hence the SORF estimates more strongly support the assumption that private agents are forward looking. Castelnuovo (2003) argues that hybrid formulation of the Phillips curve is crucial for appropriate identification of policy preferences. Moreover, Salemi (2006) suggests that when the interest rate is highly persistent, the best way to reconcile a persistent interest rate process and policy optimality is to hypothesize that both occurred in an economy where private agents are forward looking. The finding of highly persistent interest rate in all regimes will be shown below.

2.8.3 Estimates of Preference Parameters and Test of Policy Optimality

Table 2.5 reports estimation results for the preference parameters in three policy regimes identified in U.S. monetary policy, and it also presents two sets of reaction function coefficients for each policy regime: SURF estimates (obtained while treating coefficients of the policy rule as free parameters), and SORF estimates (implied values of the policy rule coefficients consistent with the policy optimality hypothesis). To interpret the results on policy preference estimation, one should note that the relative weight on inflation stabilization objective is normalized to one.

One can notice that, under SORF the estimated values of w_y and w_r are very small implying that inflation stabilization was always the dominant objective of the Federal Reserve in all three policy regimes. Policy regime one is an interesting case, covering the period of 1979-1984, during the Volcker disinflation period. Looking at the estimates describing the Federal Reserve's preferences during policy regime one, the estimate of w_y is 0.0010 and the likelihood ratio test can not reject the null hypothesis that the true weight is zero. The estimate of w_r is 0.0017, and it is significantly different from zero. Given the normalization of the weights, this finding suggests that in conducting its monetary policy the Federal Reserve was far more concerned with keeping the inflation close to its long run target path than it was with trying to stabilize output and interest rate around their targets. Indeed, in policy regime one, both SURF and SORF estimates show that the policy rule coefficient on lagged inflation c_3 is very sizable, and the SURF estimate of c_3 is significantly different from zero. While being significant, the interest rate stabilization objective appears to be of secondary importance to the Federal Reserve in policy regime one. The estimates of preferences in policy regime one are consistent with the historical evidence. As described earlier, in this special regime the Federal Reserve faced high level of inflation, and its sole objective was to reduce inflation substantially.

In policy regime two, inflation stabilization was still the dominant objective. The estimates of the policy preferences in this policy regime suggest that the Federal Reserve Bank had been conducting its policy in the manner consistent with the policy used during economic expansionary periods. A large weight was assigned to inflation stabilization, and very small weights were placed on both output and interest rate stabilizations. Indeed, the estimate of w_y is 0.0045 which is statistically significant at any conventional confidence level. The estimated policy weight placed on the interest rate stabilization objective w_r is 0.0061 and is significantly different from zero. Interest rate smoothing became a more important objective in policy regime two than in policy regime one. As pointed out previously, in this expansionary regime the Federal Reserve would put more concerns on potential exacerbate inflation situation and use instrumental interest rates wisely. Switching into policy regime two would result in a low level of inflation.

In policy regime three, the normalization of the weights suggests that keeping the inflation close to its long run target path was far more important for the Federal Reserve than stabilizing output and the interest rate. The estimate of w_y in this regime is 0.0087 and is the largest among three policy regimes. The likelihood ratio test for w_y strongly rejects the null hypothesis that the true weight is zero. In addition, in policy regime three, both SURF and SORF estimates show that the policy rule coefficient on lagged output c_1 is sizable. The estimate of w_r in this regime is 0.0046 and is highly significant. However, the interest rate stabilization objective was not as important as the output stabilization objective for the Federal Reserve in policy regime three.

There is clear evidence that the estimated and implied policy-rule coefficient estimates in three regimes differ significantly and substantially from each other. First, the coefficient on lagged output (c_1) is negative in policy regime one but sizable and positive in regimes two and three. The coefficient on lagged inflation (c_2) is large and positive in regime one but small in regimes two and three. The coefficient on lagged two output (c_4) is sizable and positive in regime one but negative in policy regimes two and three. Second, SORF estimate of the policy rule coefficient on lagged inflation is positive in all regimes and larger than the SURF estimate in regimes two and three. This suggests that the Federal Reserve should have responded more strongly to inflation than it did in regimes two and three, which will be discussed shortly.

How different are the estimated preferences here from the ones reported in other studies? Compared to estimates of the weights in the Federal Reserve's loss function in the literature[21], the results here are at the lower end for w_y an w_r. Salemi (2006) estimates a similar model, and he obtains a statistically insignificant estimate 0.0047 for w_y and a significant estimate 0.013 for w_r during the Volcker-Greenspan period $(1980 - 2001)$ and an insignificant estimate 0.0012 for w_y and 0.0062 for w_y during the Martin-Burns period $(1965 - 1980)$. Using a backward-looking model, Favero and Rovelli (2003) estimate w_y and w_r directly from the first-order condition of the Federal Reserve's optimization problem and obtain small but statistically significant weights of 0.00125 for w_y and 0.0085 for w_r during $1980-1998$, and 0.00153 for w_y and 0.0051 for w_r during $1961-1979$. Ozlale (2003) gets estimates[22] of 0.52 for output and 0.86 for interest rate smoothing for the Greenspan period $(1987 - 1999)$, 0.47 for output and 0.76 for interest rate smoothing for the Volcker period $(1979-1987)$, and 1.12 for output and 0.91 for interest rate smoothing for the Burns period $(1970-1978)$. For the Federal Reserve, Assenmacher-Wesche (2006) obtains estimates of 1.16 for w_y and 0.75 for w_r in one of the two policy regimes defined in her paper, and estimates of 1.29 for w_y and 0.94 for w_r in another policy regime. Dennis (2006) estimates a backward-looking model on U.S. data from 1982 to 2000, and he shows a statistically insignificant estimate 2.941 for w_y and a significant estimate of 4.517 for w_r. As a result, most of these studies report a high degree of inflation stabilization in U.S. monetary policy, and all these studies find statistically significant evidence of the Federal Reserve's concerns with interest rate behavior.

[21] One should note that the studies in literature use various economic models and empirical strategies varying from unified approach to two-step approach to estimate weights in the central bank's loss function.

[22] Since Ozlale (2003) does not normalize the weight on inflation stabilization w_π to one, this study reports the ratios of the weight on output to the weight on inflation (w_y/w_π) and the weight on interest rate smoothing to the weight on inflation (w_r/w_π) based on Ozlale's results.

Did the Federal Reserve set monetary policy in a way consistent with the definition of optimal monetary policy used in this study? In light of Salemi (2006), a test of policy optimality is a test of the hypothesis that SORF fits the data as well as SURF, that is, the SORF and SURF policy rule coefficients are the same. The answer is no. In each policy regime, there are four freely estimated parameters in the unrestricted policy rule but only two freely estimated parameters, w_y and w_r, in the optimal policy rule, so SORF implies six restrictions. As a result, the relevant likelihood ratio statistic is 12.511 with a p-value 0.051. Thus the hypothesis that the policy rule coefficients that fit the data best were those that minimized quadratic expected loss as defined in (2.6) can be rejected.

2.8.4 Interest Rate Smoothing

As can be seen in Table 2.5, the reaction-function coefficient for the lagged interest rate is large compared to the coefficients for inflation and output so that, according to the definition provided by Clarida, Galí and Gertler (2000), the Federal Reserve engages in "interest rate smoothing". This finding supports the widely held viewpoint that most central banks, including the Federal Reserve, are very cautious when changing interest rates. Brainard (1967) argues that large movements in interest rates can lead to lost of credibility, if a policy intervention subsequently needs to be reversed. Cukierman (1989) and Goodfriend (1991) suggest that too much volatility in interest rates is undesirable because it may result in maturity mismatches between bank's assets and liabilities, which would be disruptive for financial sector. Sack and Weiland (2000) provide three competing explanations for interest rate smoothing. First, the Federal Reserve may realize that rules with persistence are more effective for stabilizing output and inflation than rules without persistence in economies where forward-looking expectations matter. Second, the Federal Reserve may be uncertain about the accuracy of the data on which they rely to conduct policy. Third, the Federal Reserve may be uncertain about the values of structural parameters. Lansing (2002) cautions that smoothing can arise spuriously if the lagged rate of interest is correlated with measurement errors in the data.

While the estimates in the reaction functions reported in Table 2.5 imply that the Federal Reserve smooths interest rates, SORF estimates suggest that the Federal Reserve placed small weight on interest rate stability in all three policy regimes. Thus, in line with Salemi (2006), the estimates here are consistent with Sack and Weiland's first explanation for smoothing. The large coefficient on the lagged interest rate can be explained by the finding that forward-looking expectations are empirically important and the fact that optimal policies require persistent responses to shocks.

2.8.5 Estimated Transition Probabilities

Table 2.6 gives SURF and SORF estimates for the transition probabilities for switching in three monetary policy regimes. As the p-values indicate, most transition probabilities are precisely estimated. The probabilities on the diagonal of the transition matrix are close to unity, which means that all policy regimes show high persistence[23]– a feature that is commonly seen in the estimation of Markov switching models, see Sims (1999) and Assenmacher-Wesche (2006). For instance, both SURF and SORF estimates indicate that, if the monetary policy is in regime one this period, there is a 94.1 percent probability that it will be in regime one next period; similarly, if the monetary policy is in regime three this period, there is a 94.1 percent probability that it will be in regime three next period. These suggest that changes in the monetary policy regime will occur infrequently. However, all policy regimes except one in both SURF and SORF estimates are not degenerate regimes. The p-values for SURF and SORF diagonal probabilities p_{11}, p_{22}, and p_{33} are measured by the likelihood ratio test for the hypothesis that the true value of the transition probability is 0.995, meaning that there is nearly no switch back into the regime prevailing at the beginning of the sample period. The test statistic for

[23]To guarantee that the diagonal transition probabilities are highly persistent, in the estimation they were restricted to be no less than 0.900.

SURF estimate of p_{22} can not reject the hypothesis thus the policy regime two in SURF estimates is a degenerate regime.

2.8.6 Test of a Third Regime in U.S. Monetary Policy

Was there a third regime in U.S. monetary policy? A comparison between models with two-regime and three-regime monetary policy is helpful in answering the question. This study uses likelihood ratio test statistic to test the hypothesis that the two-regime models fit the data as well as the three-regime models. Therefore, one likelihood ratio test statistic tests the two-regime SURF model against the three-regime SURF model, and the other likelihood ratio test statistic is a test of the two-regime SORF model against the three-regime SORF model.

There are eight more freely estimated parameters in the model with three-regime unrestricted policy rule than those in the two-regime SURF model. Thus the two-regime SURF model implies eight restrictions. The test statistic for a likelihood ratio test of the three-regime SURF against the two-regime SURF would lie in 29.939, which implies a p-value that is less than 0.001. For the test of the three-regime SORF model versus the two-regime SORF model, the two-regime SORF implies six restrictions, and the likelihood ratio test statistic is 32.385 implying a p-value that is less than 0.001. The tests thus support the three-regime models over the two-regime models. The hypothesis that the models that fit the data best were those in which the monetary policy follows a two-regime Markov-switching process can be soundly rejected.

2.8.7 Comparison of Impulse Response Functions for Structural Estimation

To check how much the model can explain the variation in the data, this section compares the impulse response functions (IRF's) implied by SURF and SORF. In line with Salemi (2006), this study makes the hypothesis that monetary policy is conditioned only on lagged values of output and inflation. In keeping with this hypothesis, this study chooses y-π-r as the within period causal ordering for the impulse response function. The ordering implies that a shock to output affects only output contemporaneously, a shock to inflation affects both inflation and output contemporaneously, and a shock to the interest rate affects all variables contemporaneously. The ordering makes difference because of the contemporaneous correlation between output and the interest rate in all policy regimes. Figures 2.8, 2.9, and 2.10 present the IRF's for all three regimes. The shade areas in the figures are 95% confidence intervals computed with parametric Bootstrap method[24].

The figures show clearly that the SURF and SORF estimates share many features of the impulse response functions. For instance, SURF and SORF estimates all imply that a positive shock to output keeps output above its trend for more than 30 quarters. Both SURF and SORF IRF's show that a positive shock to inflation produces moderate inflation persistence with inflation returning to trend in about six quarters.

There are significant differences in the impulse response functions across three policy regimes. In regimes one and two, both SURF and SORF estimates imply that the interest rate increases then falls gradually to trend after a positive inflation shock; in policy regime three, SURF estimates show that the interest rate falls then increases gradually to trend after a positive inflation shock, and SORF estimates show that interest rate falls first, increases slightly above trend then decreases gradually to trend after 20 quarters. Since policy regime one is the Volcker disinflation regime, the interest rate responds positively to a positive inflation shock, and the responses are much larger than those in regimes two and three. The comparisons also imply that the actual interest rate responses to inflation were not optimal. A loss-minimizing policy would have raised the interest rate more in all policy regimes. A second difference is somewhat related to the first. In all three regimes SORF estimates imply that the inflation response to a positive output

[24]This study simulates data by supplying sequences of random disturbances to the reduced form solution of the estimated model. These shocks are drawn from the zero mean multivariate normal distributions with variance-covariance matrices identical to those estimated for each of the three policy regimes.

shock is far smaller than implied by the SURF estimates.

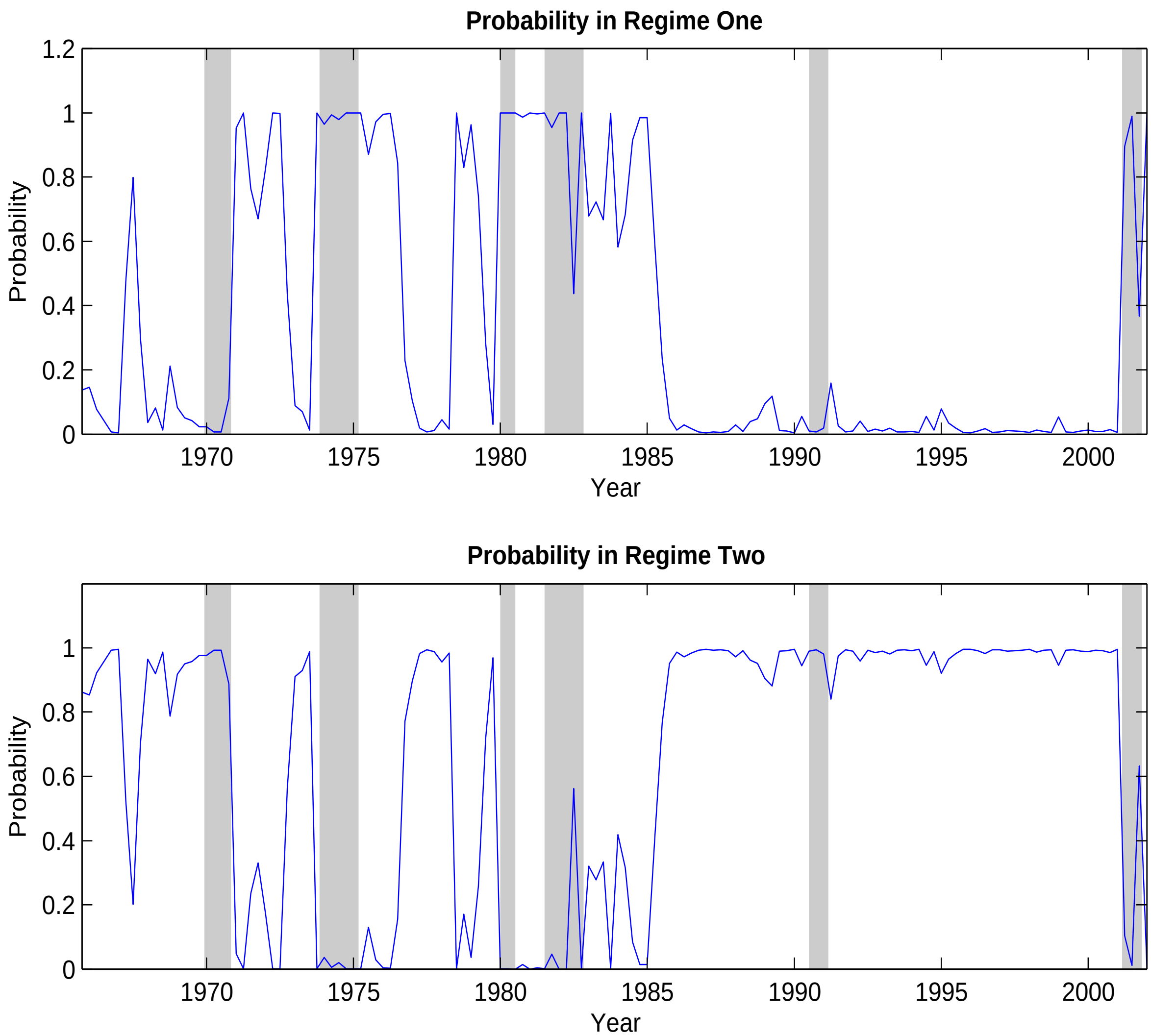
Comparing the IRF's suggest that actual and optimal responses of the interest rate to output shocks were not the same in all three policy regimes. In regime one the interest rate increases initially then falls and returns to trend after a negative output shock, while in regimes two and three the interest rate falls then rises to trend gradually after a negative output shock. Therefore, in policy regime one, SURF and SORF estimates imply that inflation would be dampened completely. In policy regime two, SURF estimates imply that a negative output shock causes interest rate to decrease below trend for 20 quarters. Because regime one is the special Volcker disinflation regime, the primary goal of the Federal Reserve was to decrease inflation. While regime three is a recessionary regime, the Federal Reserve would like to maintain output growth hence the interest rate responses are greater than those in regimes one and two.

A fourth difference is that, in regime one a positive inflation shock causes output to increase then fall to or under trend gradually; in policy regimes two and three, a positive inflation shock causes output to fall below trend for a substantial period of time.

Figure 2.11 compares the responses of output and inflation to an interest rate shock. The policy rule equation residuals are correlated with the IS and Phillips curve equation residuals in both policy regimes. On the maintained hypothesis that the monetary policy reacts to economic conditions only with a lag, this correlation between must be due either to responses of output and inflation to an interest rate surprise or to responses of output, inflation, and interest rate to an un-modeled shock. Following Salemi (2006), to obtain the model's prediction of responses of output and inflation to a pure interest rate shock, this study ignores the contemporaneous correlations when computing the IRF's reported in Figure 2.11.

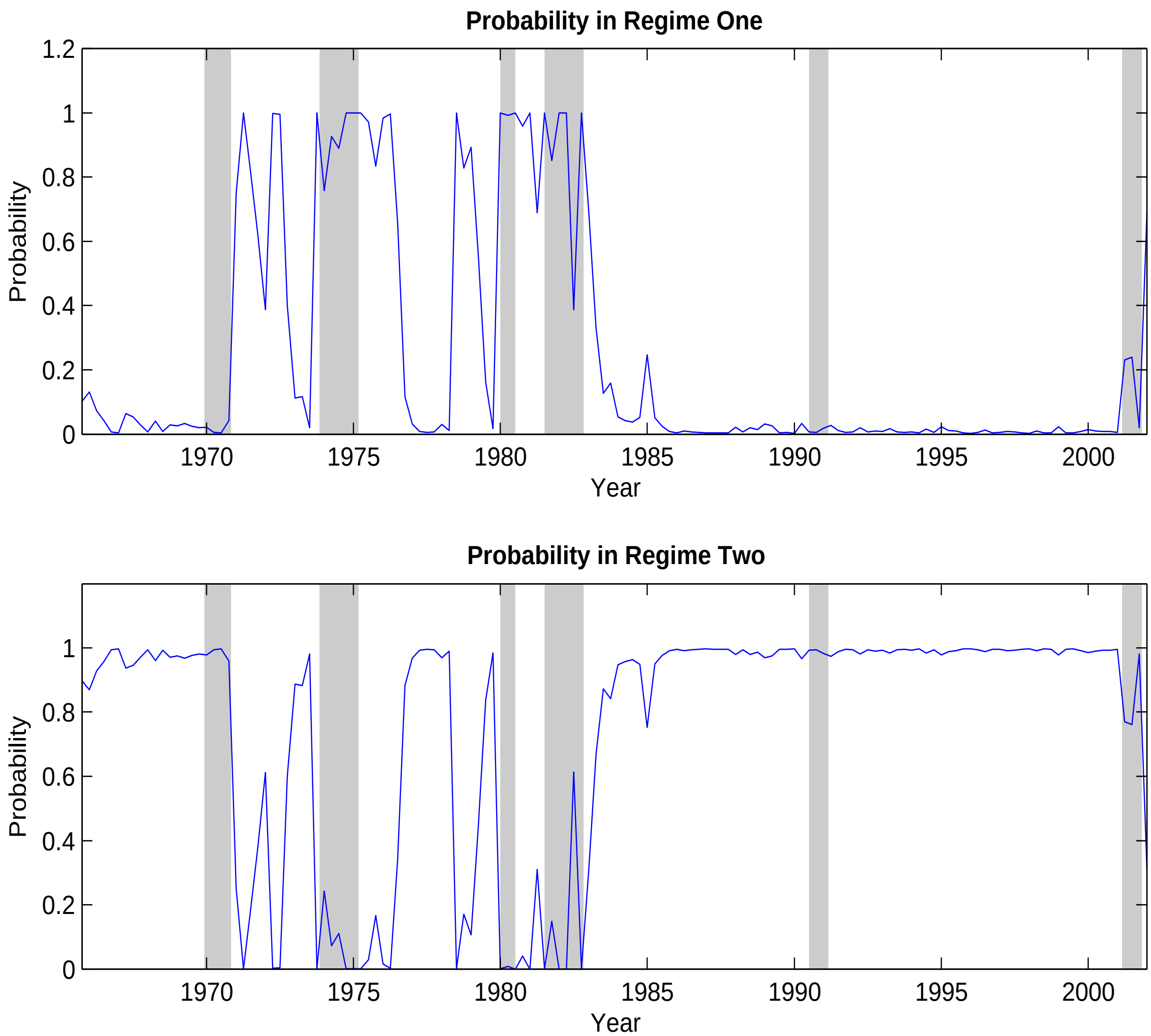
Figure 2.11 shows that in all three regimes SURF and SORF estimates can account for the "hump-shaped" response of output to an interest rate surprise. In policy regime one, SURF peak occurs about six quarters after the interest rate shock, and SORF peak occurs about eight quarters after the shock. In policy regime two, both SURF and SORF peaks occur about 13 quarters after the interest rate shock. In policy regime three, SURF peak occurs about 11 quarters after the interest rate shock, and SORF peak occurs about 18 quarters after the shock. Both SURF and SORF estimates show that the inflation rate declines immediately after an interest rate shock and remains below trend for many quarters. SORF estimates imply a much smaller inflation response than do SURF estimates, especially in policy regime three. Because regime three is a recessionary regime, the Federal Reserve would like to maintain output growth hence the inflation responses are smaller than those in regimes one and two.

Figure 2.1: Probabilistic Inference for the Two-Regime Unrestricted Monetary Policy



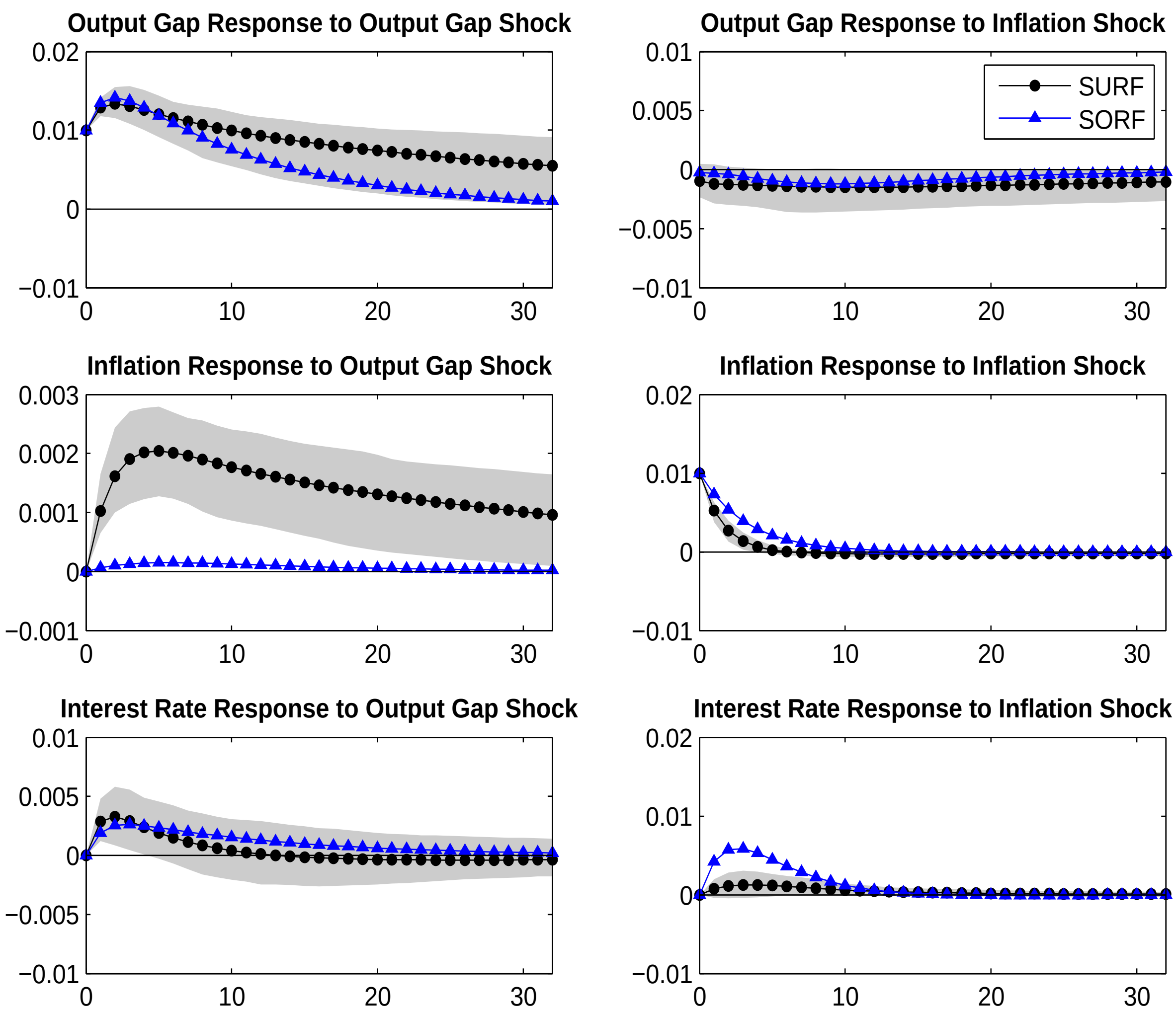
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 2.2: Probabilistic Inference for the Two-Regime Optimal Monetary Policy



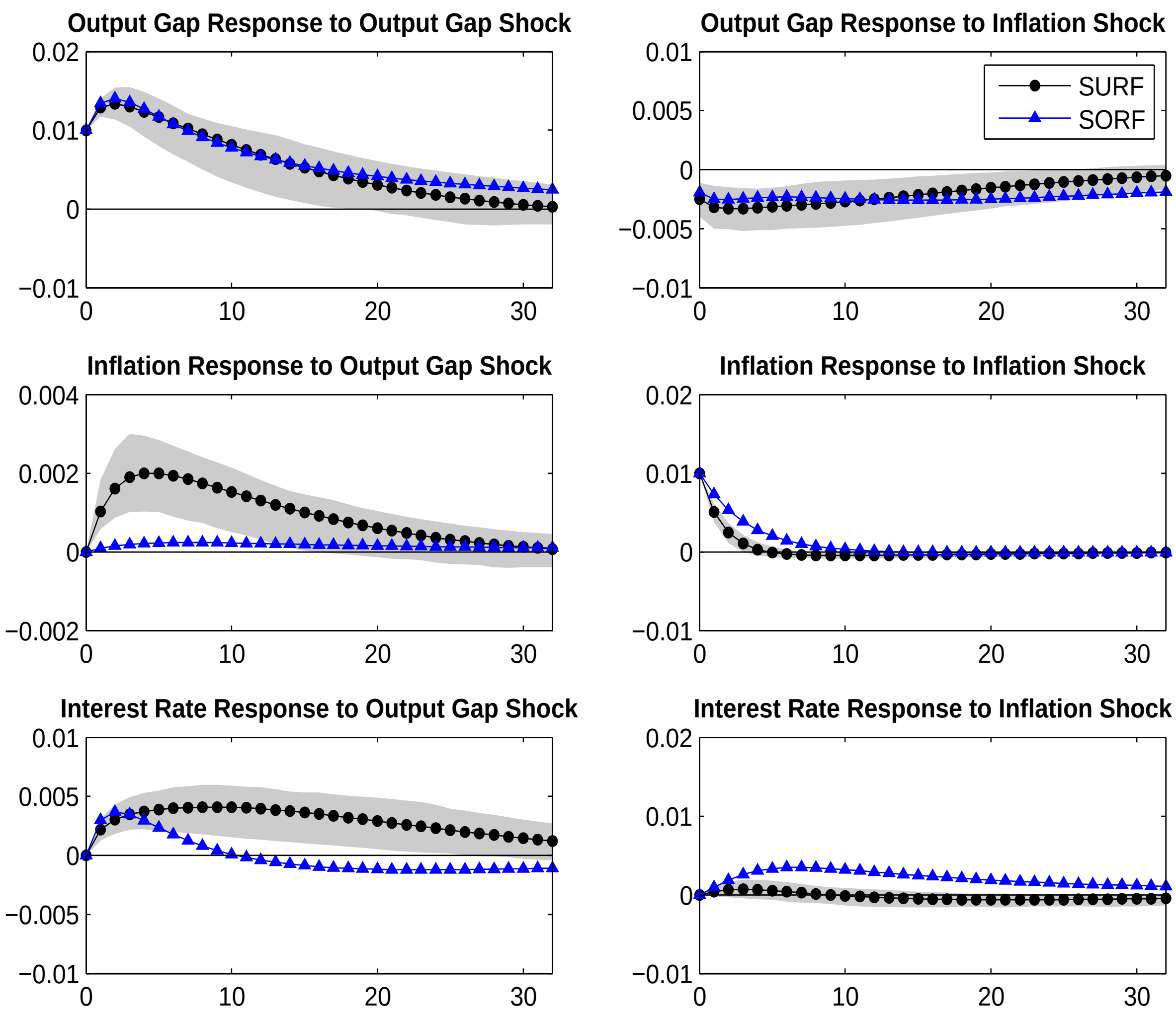
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 2.3: Comparison of Impulse Response Functions
for the Model with Two-Regime Monetary Policy
in
Regime One



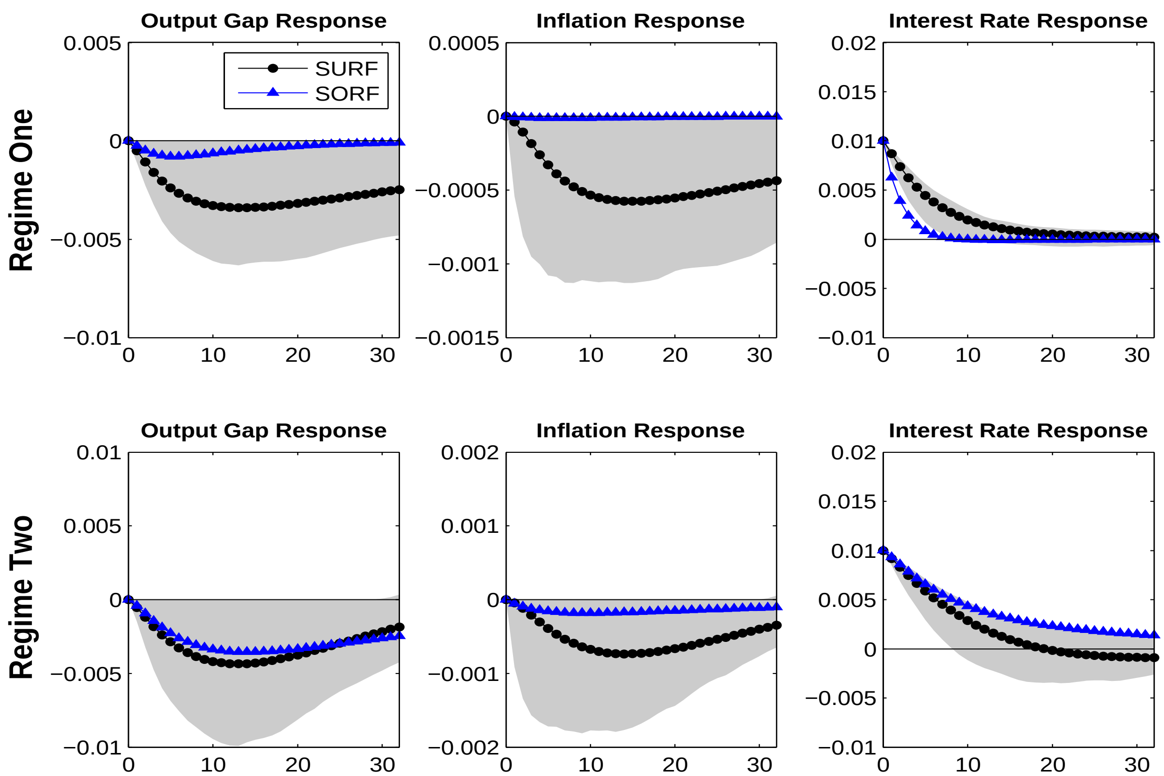
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.4: Comparison of Impulse Response Functions
for the Model with Two-Regime Monetary Policy
in
Regime Two



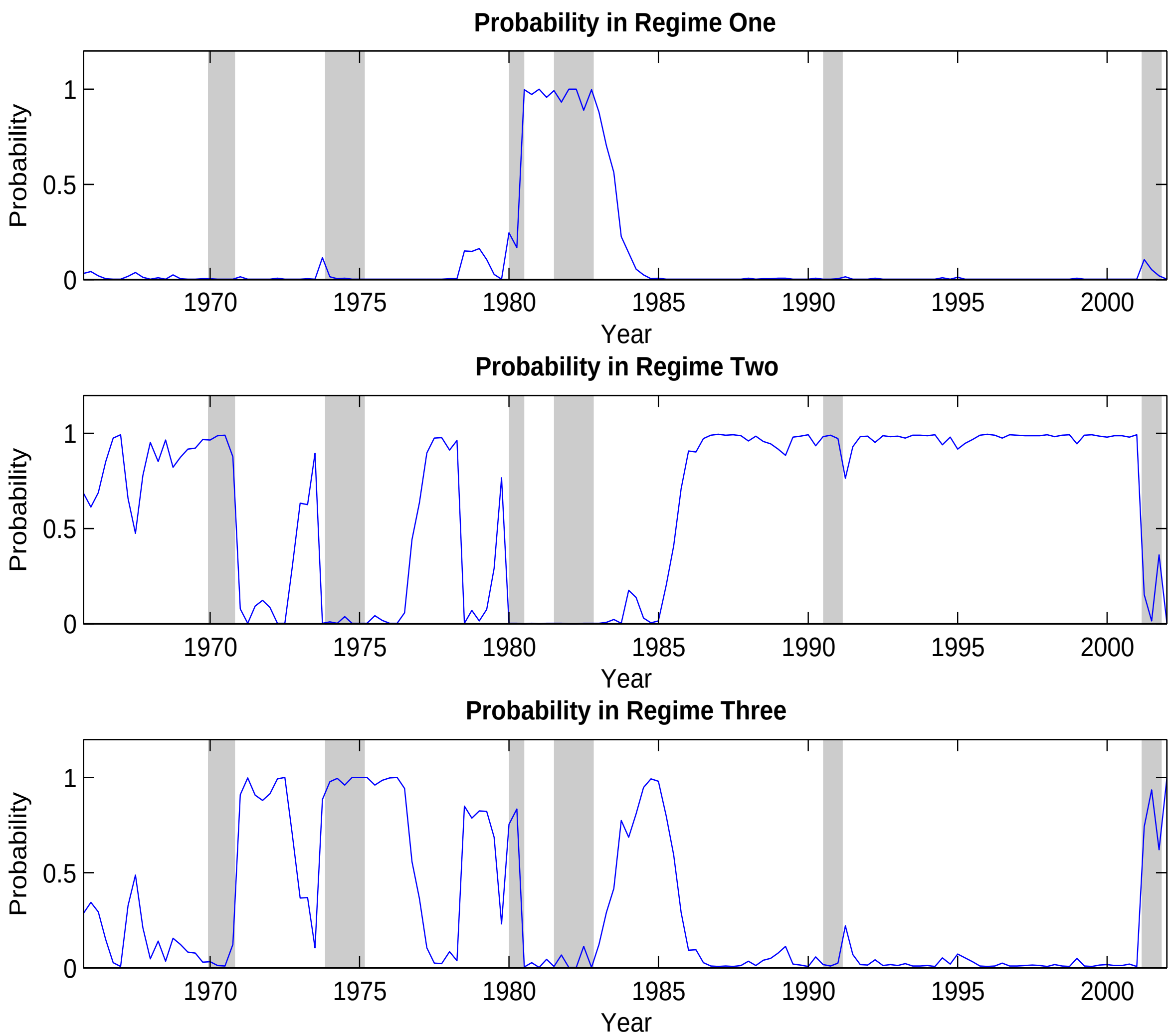
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.5: Comparison of Impulse Response Functions to an Interest Rate Shock for the Model with Two-Regime Monetary Policy



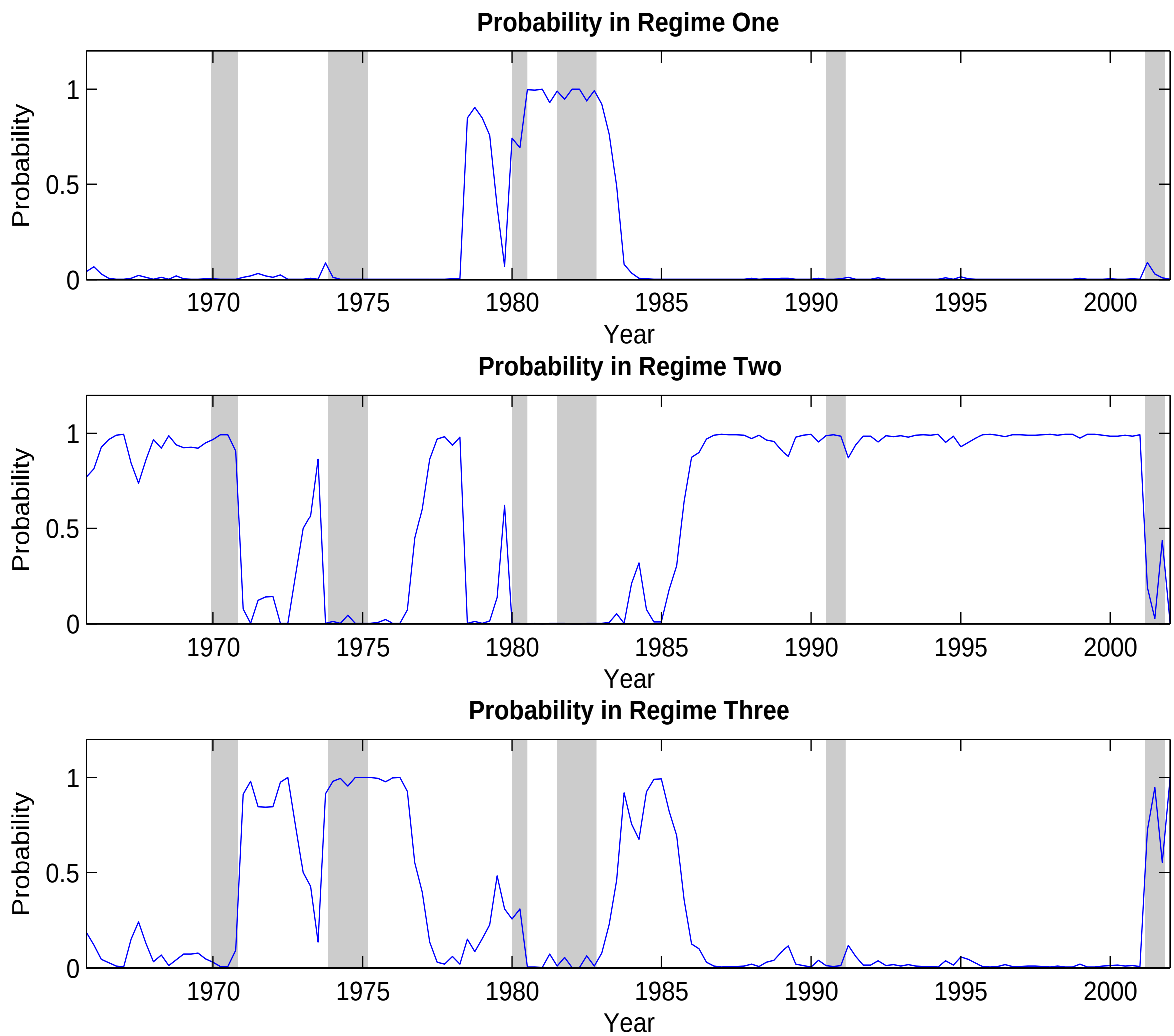
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.6: Probabilistic Inference for the Three-Regime Unrestricted Monetary Policy



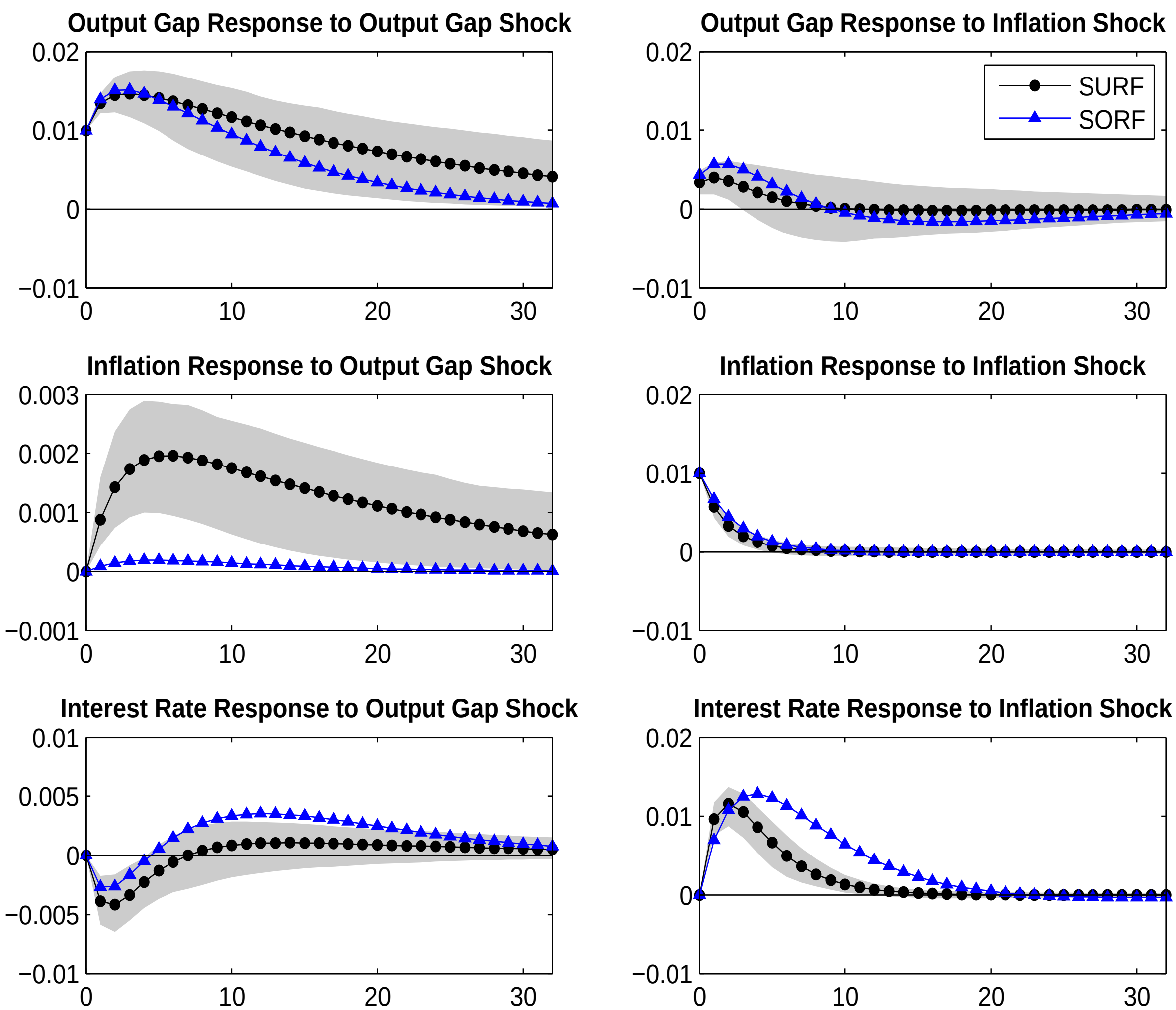
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 2.7: Probabilistic Inference for the Three-Regime Optimal Monetary Policy



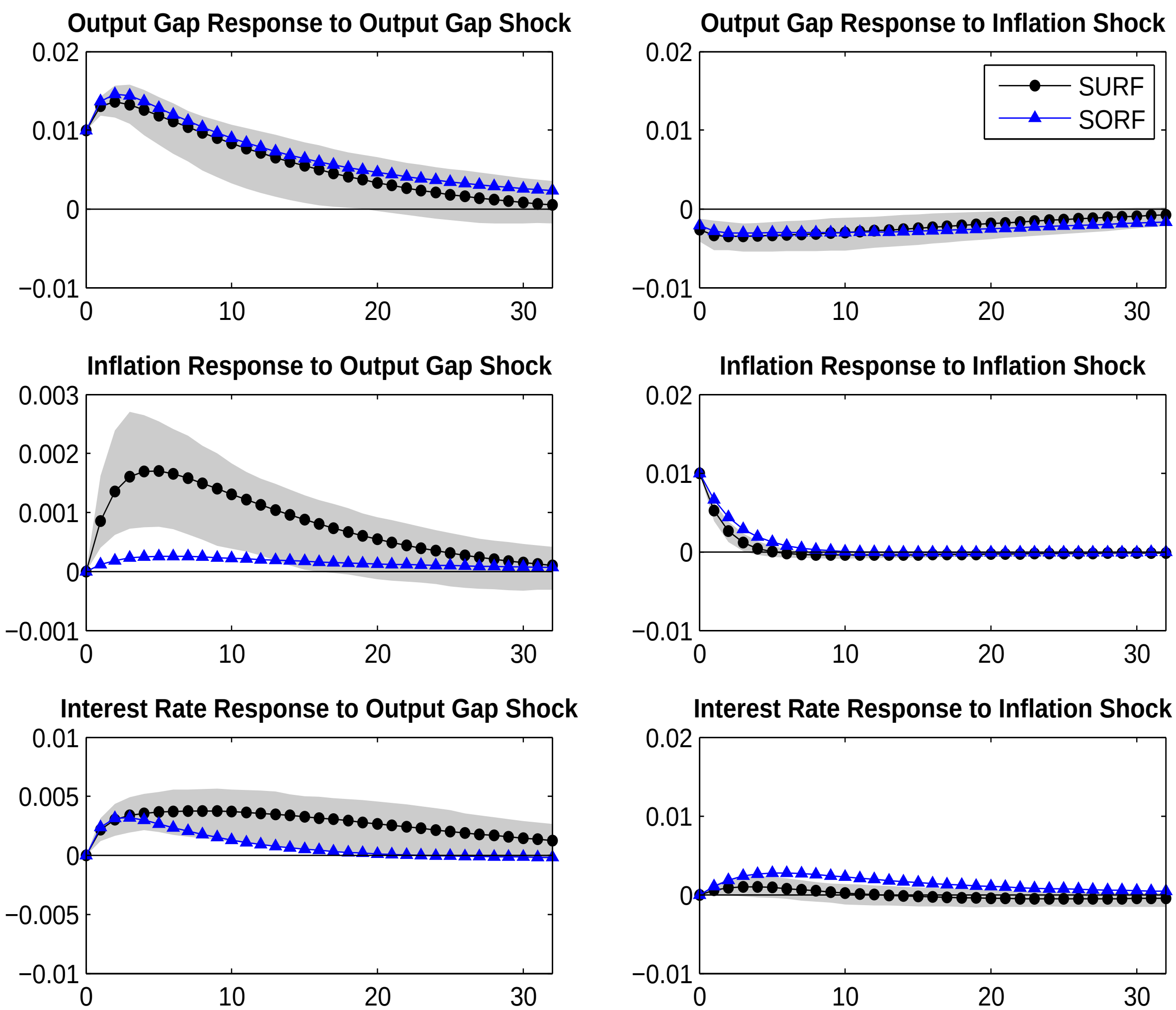
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 2.8: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime One



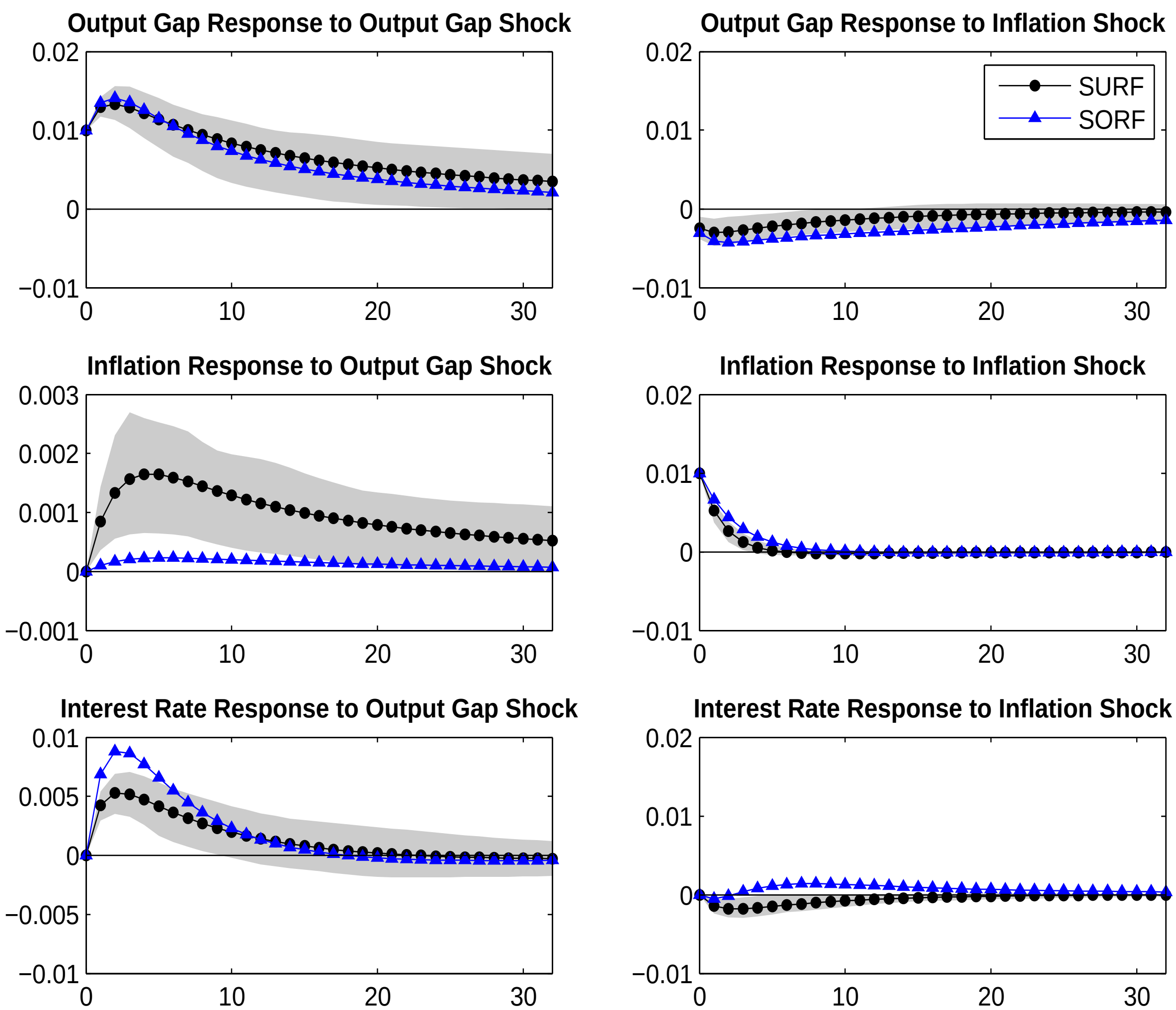
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.9: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Two



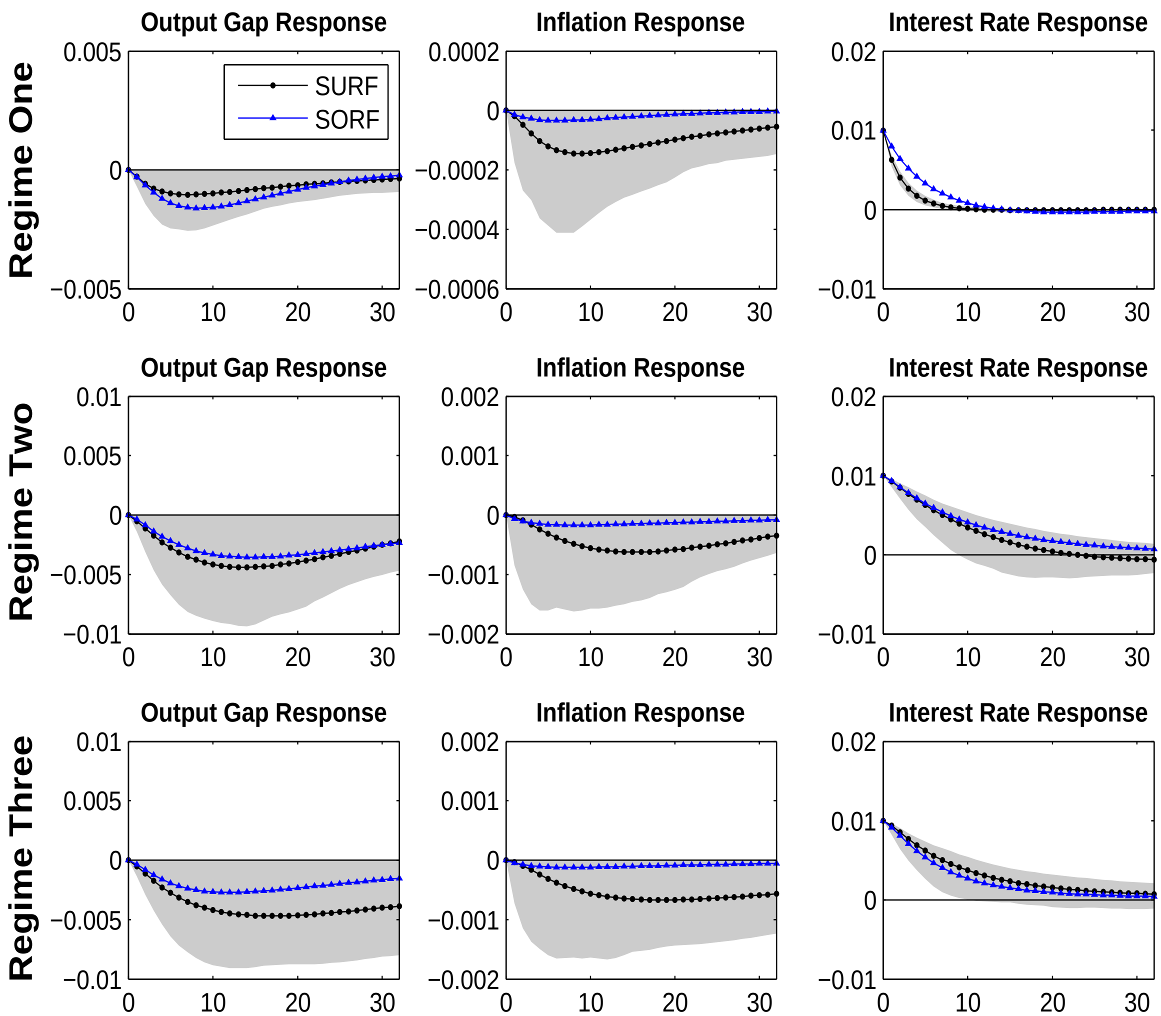
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.10: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Three



The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 2.11: Comparison of Impulse Response Functions to an Interest Rate Shock for the Model with Three-Regime Monetary Policy



The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Table 2.1: Estimates of Model Parameters for the Model with Two-Regime Monetary Policy

Model Parameters	Estimation with Unrestricted Reaction Function SURF		Estimation with Optimal Reaction Function SORF	
	Estimate	Significance (p-value)	Estimate	Significance (p-value)
λ	0.282	0.769	0.184	0.571
a_1	0.914	0.894^4	1.091	0.141^4
a_2	-0.207	< 0.001	-0.298	< 0.001
b	0.022	0.090	0.025	< 0.001
α_1	0.0003	1.000	0.577^5	< 0.001
α_2	0.534	< 0.001	0.423^5	< 0.001
β	0.080	0.002	0.0002	< 0.001
Log Likelihood	1513.114		1505.636	

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first two equations of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for two policy regimes. The policy rule coefficients of the interest rate equation ($c_1 - c_4$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 2.2.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. The p-value reported in this case is for a test of the hypothesis that $a_1 = 1$.

5. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

Table 2.2: Policy Rule Coefficient Estimates for the Model with Two-Regime Monetary Policy

	Reaction Function Coefficients	SURF Coefficient	SURF P-value	SORF Coefficient	SORF P-value
Regime One	c_1	0.288	0.157	0.392	—
	c_2	0.102	0.459	0.312	—
	c_3	0.868	< 0.001	0.877	—
	c_4	-0.306	0.130	-0.407	—
	Loss Function Wt. on y	—	—	0.0020	0.693
	Loss Function Wt. on r	—	—	0.0032	< 0.001
Regime Two	c_1	0.217	0.001	0.301	—
	c_2	0.097	0.050	0.151	—
	c_3	0.918	< 0.001	0.934	—
	c_4	-0.185	0.008	-0.314	—
	Loss Function Wt. on y	—	—	0.0020	0.130
	Loss Function Wt. on r	—	—	0.0064	< 0.001
	Log Likelihood	1513.114		1505.636	

1. The table shows Maximum Likelihood estimates of the policy rule coefficients that are in the third equation of the following model economy.

$$\begin{aligned} y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\ \pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\ r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}. \end{aligned}$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve. The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

Table 2.3: Transition Matrix Estimates for the Model with Two-Regime Monetary Policy

Transition Probabilities	SURF		SORF	
	Estimate	P-value	Estimate	P-value
p_{11}	0.896	0.001^4	0.877	0.001^4
p_{21}	0.056	< 0.001	0.043	< 0.001
p_{12}	0.104	< 0.001	0.123	< 0.001
p_{22}	0.944	0.001^4	0.957	0.011^4
Log Likelihood	1513.114		1505.636	

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{2} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2\}$. **SURF** estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for two policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. Parameter-estimate significance for an off-diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. Parameter-estimate significance for a diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.995.

Table 2.4: Estimates of Model Parameters for the Model with Three-Regime Monetary Policy

Model Parameters	Estimation with Unrestricted Reaction Function SURF		Estimation with Optimal Reaction Function SORF	
	Estimate	Significance (p-value)	Estimate	Significance (p-value)
λ	0.270	1.000	0.278	0.036
a_1	0.940	1.000[4]	0.963	0.290[4]
a_2	-0.221	< 0.001	-0.254	< 0.001
b	0.022	0.137	0.010	< 0.001
α_1	0.00005	0.986	0.595[5]	< 0.001
α_2	0.544	< 0.001	0.405[5]	< 0.001
β	0.066	0.010	0.0004	< 0.001
Log Likelihood	1528.084		1521.828	

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first two equations of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for three policy regimes. The policy rule coefficients of the interest rate equation ($c_1 - c_4$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 2.5.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. The p-value reported in this case is for a test of the hypothesis that $a_1 = 1$.

5. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

Table 2.5: Policy Rule Coefficient Estimates for the Model with Three-Regime Monetary Policy

Reaction Function		SURF		SORF	
Coefficients		Coefficient	P-value	Coefficient	P-value
Regime One	c_1	-0.388	0.160	-0.290	—
	c_2	1.088	< 0.001	0.760	—
	c_3	0.629	0.007	0.814	—
	c_4	0.250	0.008	0.347	—
Loss Function Wt. on y		—	—	0.0010	0.199
Loss Function Wt. on r		—	—	0.0017	< 0.001
Regime Two	c_1	0.220	1.000	0.237	—
	c_2	0.115	< 0.001	0.146	—
	c_3	0.927	< 0.001	0.931	—
	c_4	-0.197	1.000	-0.229	—
Loss Function Wt. on y		—	—	0.0045	< 0.001
Loss Function Wt. on r		—	—	0.0061	< 0.001
Regime Three	c_1	0.425	1.000	0.693	—
	c_2	-0.038	1.000	0.159	—
	c_3	0.934	< 0.001	0.930	—
	c_4	-0.414	< 0.001	-0.677	—
Loss Function Wt. on y		—	—	0.0087	< 0.001
Loss Function Wt. on r		—	—	0.0046	< 0.001
Log Likelihood		1528.084		1521.828	

1. The table shows Maximum Likelihood estimates of the policy rule coefficients that are in the third equation of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve. The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

Table 2.6: Transition Matrix Estimates for the Model with Three-Regime Monetary Policy

Transition Probabilities	SURF		SORF	
	Estimate	P-value	Estimate	P-value
p_{11}	0.941^4	$< 0.001^5$	0.941^4	$< 0.001^5$
p_{21}	0.010	< 0.001	0.010	< 0.001
p_{31}	0.000	1.000	0.000	1.000
p_{12}	0.000	1.000	0.000	1.000
p_{22}	0.956^4	1.000^5	0.965^4	0.005^5
p_{32}	0.059	1.000	0.059	< 0.001
p_{13}	0.059	1.000	0.059	< 0.001
p_{23}	0.034	0.024	0.025	0.012
p_{33}	0.941^4	$< 0.001^5$	0.941^4	$< 0.001^5$
Log Likelihood	1528.084		1521.828	

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{3} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2, 3\}$. **SURF** estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for three policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. Parameter-estimate significance for an off-diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. To guarantee that the diagonal transition probabilities are highly persistence, in the estimation they were restricted to be no less than 0.900.

5. Parameter-estimate significance for a diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.995.

Chapter 3

Revealing U.S. Monetary Policy Regimes Between 1965 and 2008 Using a Baseline Model

This chapter extends the previous chapter in a way which estimates the same model as the previous chapter with updated data for the period 1965:Q1 to 2008:Q4. Now there are seven more years of data which allow one to examine the Greenspan years more thoroughly. The longer time period covers the latest recession date in 2008, which was determined to begin by the National Bureau of Economic Research.

3.1 Data Description

This and the following sections describe what happens when this study takes the model and policy hypothesis to U.S. quarterly data from 1965:Q1 to 2008:Q4. Since this study applies a rational expectation Keynesian model for policy analysis starting 1965, in line with Salemi (2006) the monetary policymakers in the Federal Reserve are assumed to understand the forward-looking nature of private agents' behavior two decades before Lucas published his famous critique on policy analysis.

The raw data, except for population, come from FRED, the Federal Reserve Bank of St. Louis database. Quarterly population numbers are interpolations of annual values found in the 2010 *Economic Report of the President*. Output measure is real GDP (in

1996 chained dollars) divided by civilian population. The long run growth path of per capita output is estimated by fitting a constant-coefficient time trend to the natural log of real GDP per capita, see Figure 3.1. Variable y_t, the output gap, is the difference between the natural logarithm of real GDP per capita and its trend. Such transformation of the output variable is consistent with the assumption that the central bank aims to keep output close to its long-run growth path.

The measure of target output in this study reflects the assumptions that there is a normal average growth path for the economy and this linear trend tends to vary little over time. The computation of target output is based on purely statistical measure derived from historical data. One should notice that output gap can also be constructed using more involved statistical and econometric methods suggested by modern theory. Orphanides (2000), Orphanides and Williams (2002), and Ghysels, Swanson, and Callan (2002) argue that monetary policy should be analyzed using potential output that is estimated by central bank contemporaneously. They caution that if the Federal Reserve stimulated the economy in response to slow output growth only to conclude later that potential output growth had also slowed, one working with revised data would misunderstand Federal Reserve's motives. Nonetheless, measuring target output with a trend may avoid pitfalls that are associated with more complicated models of potential output. This study assumes that the Federal Reserve perceives potential output as a constant-coefficient time trend and it knows the true value of the trend coefficient. In reality, monetary policymakers may use varied trend coefficient through time or more complicated models to measure potential output, hence the estimate of y_t here may differ from that of the Federal Reserve.

The inflation rate is the annualized percentage change in the chained GDP deflator. Figure 3.1 shows that the inflation rate rose from 1965 to 1980, dropped dramatically between 1980 and 1985, and continued to trend gradually downward between 1986 and

2008. The quarterly interest rate is the annualized secondary market yield of three-month treasury bills. Figure 3.1 shows that the interest rate also followed a trend path that is similar to that of inflation. Target values for the inflation rate and interest rate are obtained by fitting continuous, piecewise-linear trends that allow for trend coefficient changes in 1980:Q1 and 1986:Q1 but trend coefficients in the inflation rate and interest rate piecewise regressions are constrained to be the same. The resulting target paths are displayed in Figure 3.1. Variables π_t and r_t are the residuals from the two trend regressions. These transformed series imply that the central bank tries to minimize the variation of inflation and the interest rate around their target paths.

The target-path assumptions on the inflation rate and interest rate imply that the Federal Reserve acclimated itself to rising inflation during the 1960's and 1970's, rapidly decreased inflation target between 1980 and 1986[25], and then gradually lowered target inflation through 2008. The paths imply a constant target value for the real rate of interest. Alternatively one can hypothesize that the Federal Reserve set constant target values for the inflation and interest rate over time, so target values for the inflation rate and interest rate were constant and equal to sample average values. Salemi (2006) cautions that this alternative hypothesis is unattractive because it implies that the actual values of inflation rate and interest rate were below their targets after the fourth quarter of 1991 and a superior policy thereafter would have raised the inflation rate. Hence, the approach used in this study to estimate target values for output, inflation and the interest rate is more reasonable. However, none of the previously-mentioned two hypotheses are perfect. It would be ideal to estimate the target values along with other model parameters, but it is beyond the scope of this study.

As equations (2.7) and (2.8) indicate, loss-minimizing values of the policy-rule coefficients depend on the covariance matrix of shocks Ω as well as on the reduced form

[25]Given the changes in the target paths that occurred in 1980 and 1986, it is possible that there is a policy regime prevailing between 1980 and 1985.

parameters G[26]. Therefore, it is important to ask whether the Federal Reserve faced different distributions of shocks over time. One can see clearly from Figure 3.1 that volatilities of variables were especially large in the middle of the sample from 1970's to 1980's, then they declined subsequently[27]. Therefore, the evidence suggests that the covariance matrix of shocks used to capture the volatilities should differ in different policy regimes. Sims and Zha (2006) and Liu, Waggoner and Zha (2007) suggest that allowing for heteroscedasticity in shock variances can improve the Markov-switching model fit considerably. In this study the covariance matrix of shocks is assumed to switch simultaneously with the monetary policy[28].

This study also assumes that the coefficients of the y_t and π_t equations remain unchanged in different monetary policy regimes. Because the model is forward-looking, changes in the policy rule will cause across-regime changes in the parameters of the reduced form equations. But this study maintains the assumption that there is not evidence of large changes in structural equation parameters because many studies in the literature, including Salemi (2006), use statistical tests to show no evidence of parameter instability in U.S. economy during the post-war period.

In summary, this study will estimate and test the model by varying the number of policy regime from two to three. The parameters of equations (2.1) – (2.2) will be held

[26]While certainty equivalence does not hold for the optimal simple rule, Svensson and Woodford (2003) make clear that it does hold for the optimal time-consistent policy.

[27]The decline in the volatilities of real GDP and inflation in the last two decades in the U.S. has been dubbed the Great Moderation, which may have been the most important macroeconomic event in the last half century.

[28]The covariance matrix of shocks and monetary policy thus follow the same Markov-switching process. Alternatively, one can follow Assenmacher-Wesche (2006) to assume that the covariance matrix follows a Markov-switching process which is independent of the Markov-switching process for monetary policy. Another less restrictive assumption would be that one or more shocks could be in one regime while one or more others could be in another regime, and such a setup would require specifying a transition matrix for each structural shock. Both alternative assumptions would increase the number of free parameters substantially and make estimation difficult. Thus, the assumption used in this study can keep the number of parameters to be estimated tractable and avoid over-fitting the data.

constant across policy regimes but the monetary policy and covariance matrix of shocks would be allowed to change.

3.2 Econometric Evaluation of U.S. Monetary Policy with Two Policy Regimes

This section applies the estimation approach described in the previous chapter to estimate the structural model parameters in equations (2.1) – (2.3) jointly with the policy preference parameters in equation (2.6). In this section, monetary policy is assumed to switch between two regimes during the sample period. Therefore, in the estimation algorithm, the policy parameters and policymaker's preference parameters are allowed to change in each regime.

Figures 3.2 and 3.3 present the probability inferences in SURF and SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 3.1, 3.2, and 3.3 report SURF and SORF estimates of model parameters along with the value of the log likelihood function. To make inferences about the significance of estimated parameters, this study reports the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is zero.

3.2.1 Probability Inferences for Two Policy Regimes

Figure 3.2 and Figure 3.3 plot the SURF and SORF estimated probability series of being in the two regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. Although these business cycle dates were not used in any way to estimate parameters or form inferences about the Markov transition process, the traditional business cycle dates correspond fairly closely to the switches of monetary policy as described by the SURF and SORF estimates in Figure 3.2 and Figure 3.3.

The findings here are consistent with those in the previous chapter. Regardless of whether the optimal-policy hypothesis is imposed, the estimates reveal that monetary policy regime one prevailed roughly after the recession started, and policy regime two prevailed during the expansionary periods. Both SURF and SORF continue to detect the switches from the expansionary regime to the recessionary regime in U.S. monetary policy, especially the switches that occurred in the recent recession dates of 2001 and 2008 during the newly included seven more years. Furthermore, policy regime two prevailed in most of the additional seven years. Therefore after the recession dates in 1970, 1975, 1980, 1990, 2001, and 2008, U.S. monetary policy is estimated to be in regime one, in which the Federal Reserve adjusted its policy instruments quickly to fight recession. When the economy got out of recession, U.S. monetary policy switched to regime two, in which the primary goal of the Federal Reserve is to maintain low inflation rate and sustainable output growth. As a result, it can be seen that regime two was dominant for most of the sample period including most of the late 1960's, the latter half of the 1970's, and the entire period from the late 1980's onward until the end of the sample period.

3.2.2 Estimates of Structural Parameters

Table 3.1 reports the structural parameters estimated with the extended dataset. Both SURF and SORF estimates are very close to their Table 2.1 counterparts. Log likelihood values are higher than those in the previous chapter but SURF log likelihood remains significantly larger than SORF log likelihood.

When the assumption of policy optimality is relaxed, the current estimate of λ is 0.337, which is a bit larger than its earlier estimate. While the estimate is sizable, the p-value of the likelihood ratio test indicates that one can not reject the null hypothesis that the true parameter value is zero. This is in line with the results in Table 2.1. Therefore, the estimate of λ again implies that expected future output may not be an important factor in affecting aggregate demand. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are currently found to be 0.844 and -0.190, which are smaller in magnitude than their counterparts in the previous chapter. The estimate of a_1 is statistically indifferent from one and the estimate of a_2 is significantly different from zero at any conventional confidence level, which are consistent with the earlier findings. The estimates here imply that shocks have sustained effects on output. Third, the estimate of b is 0.009. In contrast to its earlier estimate, the current estimate of b is not only quantitatively smaller, but it appears to be insignificant at any level. Thus, the estimate implies that changes in the interest rate may not cause changes in aggregate demand and the effect of interest rate on aggregate demand is relatively insensitive. Fourth, turning to the parameters of the Phillips curve, one can note that firms are substantially more backward-looking than forward-looking. The estimate of the relative weight α_1 is very small and is found to be highly insignificant, and the estimate of α_2 is 0.584 and is found to be significantly different from zero. The current estimates of α_1 and α_2 are a little larger than their earlier estimates. The findings here again result in a backward-looking Phillips curve and cast doubt on the sticky price mechanism of Calvo. Lastly, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The current estimate of β is 0.042 and is found to be not significant, which implies that a positive output gap may not raise the inflation rate. In contrast, the earlier estimate of β is larger and significantly different from zero implying that a positive output gap does raise the inflation rate.

SORF produces almost identical estimates of the structural parameters to their Table 2.1 counterparts. The current estimate of λ is 0.211, which is a little larger than its earlier estimate. The estimate of λ is still insignificant. Thus, the estimate of λ again implies that expected future output may not be an important factor in affecting aggregate demand. The current estimates of a_1 and a_2 are very close to their earlier counterparts. The estimate of a_1 is statistically indifferent from one, and the estimate of

a_2 is significantly different from zero at any conventional confidence level. The estimates again imply that shocks have sustained effects on output. Third, the current estimate of b is close to its earlier estimate. Furthermore, the estimate of b is larger and more precisely estimated than the estimate under SURF, which imply that a change in the interest rate has a larger effect on aggregate demand. Fourth, estimates of both α_1 and α_2 are positive and highly significant, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, the current estimate of α_1 is a bit larger than its earlier estimate, while α_2 is smaller than the earlier estimate. Again, one can not reject the hypothesis that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006), and SORF estimates were obtained with that restriction imposed. One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with the earlier finding. Finally, the current estimate of β is very close to its earlier estimate and still highly significant at any conventional confidence level. In line with the finding in the previous chapter, the SORF estimate of β is much smaller than its SURF estimate implying that it is more difficult for the Federal Reserve to control inflation by changing aggregate demand.

As the p-values reported in Table 3.1 indicate, some of the structural parameters, such as α_1 the parameter of the forward-looking term in the Phillips curve, are more precisely estimated under SORF than under SURF, which is consistent with the conclusion in the previous chapter. It is believed that a unified approach of joint estimation of structural model parameters with the parameters of the policymaker's objective function provides additional across equation restrictions that result in a more precise estimation of the model parameters. Hence the SORF estimates more strongly support a forward-looking Phillips curve. Castelnuovo (2003) argues that hybrid formulation of the Phillips curve is crucial for appropriate identification of policy preferences. Moreover, Salemi (2006) suggests that when the interest rate is highly persistent, the best way to reconcile a persistent interest rate process and policy optimality is to hypothesize that both occurred in an economy where private agents are forward looking. The finding of highly persistent interest rate in both regimes will be shown below.

3.2.3 Estimates of Preference Parameters and Test of Policy Optimality

Table 3.2 reports estimation results for the preference parameters of both two regimes identified in U.S. monetary policy, and it also presents two sets of reaction function coefficients for each policy regime: SURF estimates (obtained while treating coefficients of the policy rule as free parameters), and SORF estimates (implied values of the policy rule coefficients consistent with the policy optimality hypothesis). To interpret the results on policy preference estimation, one should note that the relative weight on inflation stabilization objective is normalized to one.

As Table 3.2 shows, both SURF and SORF produce similar estimates of the central bank's preference parameters and policy-rule coefficients to their earlier Table 2.2 counterparts. First, in both policy regimes SORF estimates of w_y and w_r are very small implying that inflation stabilization was still the most important policy objective of the Federal Reserve. The relative weights placed on output stabilization in two policy regimes are both 0.0022. The weight w_y in regime one is insignificant, while in regime two it is marginally significantly different from zero at 10% level. The weight placed on interest rate stabilization is small but significant in each regime, and the value of w_r is slightly larger in the second regime than that in the first regime. These findings are in accord with those in the previous chapter. Given the normalization of the weights, the estimates suggest that in conducting its monetary policy the Federal Reserve was far more concerned with keeping the inflation close to its long run target path than it was with trying to stabilize output and interest rate around their targets. Second, both SURF and SORF continue to show that the estimated policy-rule coefficient estimates in two regimes differ significantly and substantially from each other. The SURF coefficients on lagged inflation (c_2) and lagged outputs (c_1 and c_4) are larger in magnitude in the first policy regime but smaller in magnitude in the second regime. The SURF coefficient on lagged interest rate (c_3) is smaller in the first policy regime but larger in the second regime. Third, all SORF estimates of the policy rule coefficients are greater in magnitude than the SURF estimates in both regimes except c_3 in regime two. This suggests that the Federal Reserve responded consistently to inflation and output, which is in line with the earlier conclusion.

Did the Federal Reserve set monetary policy in a way consistent with the definition of optimal monetary policy used in this study? In light of Salemi (2006), a test of policy optimality is a test of the hypothesis that SORF fits the data as well as SURF, that is, the SORF and SURF policy rule coefficients are the same. The answer is no. In each policy regime, there are four freely estimated parameters in the unrestricted policy rule but only two freely estimated parameters, w_y and w_r, in the optimal policy rule, so SORF implies four restrictions in total. As a result, the relevant likelihood ratio statistic is 9.782 with a p-value 0.044. Thus the hypothesis that the policy rule coefficients that fit the data best were those that minimized quadratic expected loss as defined in (2.6) can be soundly rejected. This conclusion is consistent with the one in the previous chapter.

3.2.4 Interest Rate Smoothing

As can be seen in Table 3.2, the reaction-function coefficient for the lagged interest rate is always large compared to the coefficients for inflation and output so that, according to the definition provided by Clarida, Galí and Gertler (2000), the Federal Reserve engages in "interest rate smoothing". This finding supports the widely held viewpoint that most central banks, including the Federal Reserve, are very cautious when changing interest rates. While the estimates in the reaction functions reported in Table 3.2 imply that the

Federal Reserve smooths interest rates, SORF estimates suggest that the Federal Reserve placed little attention on interest rate stability in both policy regimes. Thus, in line with the previous chapter, the estimates here are consistent with Sack and Weiland (2000)'s explanation for interest rate smoothing. The large coefficient on the lagged interest rate can be explained by the finding that forward-looking expectations are empirically important and the fact that optimal policies require persistent responses to shocks.

3.2.5 Estimated Transition Probabilities

Table 3.3 presents SURF and SORF estimates for the transition probabilities for switching in two monetary policy regimes. Both SURF and SORF produce similar estimates of the transition matrices to the results in Table 2.3. The current estimates of probabilities on the diagonal of the transition matrix are a bit larger than their earlier estimates, and they are again close to unity implying that both policy regimes are highly persistent. The SURF estimates indicate that, if the monetary policy is in regime one this period, there is a 91.7 percent probability that it will be in regime one next period; if the monetary policy is in regime two this period, there is a 95.6 percent probability that it will be in regime two next period. Similarly, the SORF estimates indicate that, if the monetary policy is in regime one this period, there is a 91.8 percent probability that it will be in regime one next period; if the monetary policy is in regime two this period, there is a 97.1 percent probability that it will be in regime two next period. These suggest that changes in the monetary policy regime will occur infrequently.

Both SURF and SORF estimates show that these two policy regimes are not degenerate regimes, which is in line with the earlier findings. The p-values for p_{11} and p_{22} are measured by the likelihood ratio test for the hypothesis that the true value of the transition probability is 0.995, meaning that there is nearly no switch back into the regimes prevailing at the beginning of the sample period. The test statistic for SURF estimate of p_{11} is significant 1% level and the one for p_{22} is marginally significant at 10% level. By contrast, the earlier estimates show that SURF estimates of p_{11} and p_{22} are significant at 1% level, and the test statistics for SORF estimates of p_{11} and p_{22} are both significant at 1% level. The test statistics reject the hypothesis thus the SURF and SORF transition probabilities do not induce degenerate policy regimes. In sum, the important findings associated with the two-regime monetary policy are robust to estimations using the updated dataset with seven more years data.

3.2.6 Comparison of Impulse Response Function for Structural Estimation

To check how much the model can explain the variation in the data, this section compares the impulse response functions (IRF's) implied by SURF and SORF. In line with Salemi (2006), this study makes the hypothesis that monetary policy is conditioned only on lagged values of output and inflation. In keeping with this hypothesis, this study chooses y-π-r as the within period causal ordering for the impulse response function. The ordering implies that a shock to output affects only output contemporaneously, a shock to inflation affects both inflation and output contemporaneously, and a shock to the interest rate affects all variables contemporaneously. The ordering makes difference because of the contemporaneous correlation between output and the interest rate in both policy regimes. Figures 3.4 and 3.5 present the IRF's for regimes one and two. The shade areas in the figures are 95% confidence intervals computed with parametric Bootstrap method.

The figures show clearly that the SURF and SORF estimates share many features of the impulse response functions. For instance, SURF and SORF estimates all imply that a positive shock to output keeps output above its trend for more than 30 quarters. Both imply that a positive shock to inflation produces moderate inflation persistence with inflation returning to trend in about six quarters. SURF and SORF IRF's all show that the interest rate rises quickly after a positive output shock and remains above trend for at least ten quarters.

There are differences in the impulse response functions. In regime one, SURF and SORF estimates imply that the interest rate increases quickly then falls gradually after a positive inflation shock; in regime two, SURF and SORF estimates imply that the interest rate rises slightly then falls to trend slowly after a positive inflation shock. Because regime one is the recessionary regime, the primary goal of the Federal Reserve was to maintain output growth and the interest rate responded more strongly to an inflation shock than it did in regime two. A second difference is related to the first. SORF estimates imply that the inflation response to a positive output shock is far smaller than implied by the SURF estimates because the SORF interest rate response to a positive inflation shock is larger than the SURF interest rate response.

Comparing the IRF's suggest that actual and optimal responses of the interest rate to output shocks were nearly the same in policy regime one but they were quite different in regime two. Therefore, in policy regime one, SURF and SORF estimates imply that inflation resulting from positive output shocks would be dampened completely. In policy regime two, SURF estimates imply that a positive output shock causes interest rate to rise above trend for a very long time, and SORF estimates imply that a positive output shock first cause interest rate to rise quickly then fall below trend after about ten quarters. In regime two, SORF estimates imply that inflation caused by positive output shocks would be dampened completely.

The fourth difference is, both SURF and SORF estimates imply that in regime one a positive inflation shock causes output to increase slightly then fall under trend gradually; in policy regime two, a positive inflation shock causes output to fall below trend for a substantial period of time. The comparison also implies that actual interest rate responses to inflation shocks were not optimal. A loss-minimizing policy would have raised interest rate more after inflation shocks in both regimes one and two.

Figure 3.6 compares the responses of output and inflation to an interest rate shock. The policy rule equation residuals are positively correlated with the IS and Phillips curve equation residuals in both policy regimes. On the maintained hypothesis that the monetary policy reacts to economic conditions only with a lag, this correlation between must be due either to responses of output and inflation to an interest rate surprise or to responses of output, inflation, and interest rate to an un-modeled shock. Following Salemi (2006), to obtain the model's prediction of responses of output and inflation to a pure interest rate shock, this study ignores the contemporaneous correlations when computing the IRF's reported in Figure 3.6.

Figure 3.6 shows that in both regimes SURF and SORF estimates can account for the "hump-shaped" response of output to an interest rate surprise[29]. In policy regime one, SURF peak occurs about ten quarters after the interest rate shock, and SORF peak occurs about seven quarters after the shock. In policy regime two, both SURF and SORF peaks occur about ten quarters after the interest rate shock. Both SURF and SORF estimates show that the inflation rate declines immediately after an interest rate shock and remains below trend for many quarters. SORF estimates imply a more smaller inflation response than do SURF estimates, especially in policy regime one.

[29] As Table 3.1 shows, the estimates of the coefficients for the interest rate in the IS schedule and the output gap in the Phillips curve are very small, which makes one believe that monetary policy is not possible. Thus, this study also analyzes the effects of monetary policy rule if it would have not responded to lagged terms of output and inflation. In the reaction function, this study sets $c_i = 0$ for $i \in \{1, 2, 4\}$ and keeps c_3 at its estimated value. With the new parameterizations, both SURF and SORF estimates show that the impulse responses to an interest rate shock are very close to those obtained with the original monetary policy rule. The results suggest that output and inflation are still very responsive to a shock in the interest rate, and the small values of the coefficients on the interest rate in the IS schedule and on the output gap in the Phillips curve still play an important role in accounting for the responses. Therefore, it is possible for the Federal Reserve to conduct monetary policy effectively during the sample period.

3.3 Econometric Evaluation of U.S. Monetary Policy with Three Policy Regimes

This section applies the estimation approach described in the previous chapter to estimate the structural model parameters in equations (2.1) – (2.3) jointly with the policy preference parameters in equation (2.6). In this section, monetary policy is assumed to switch between three regimes during the sample period. Therefore, in the estimation algorithm, the policy parameters and policymaker's preference parameters are allowed to change in each policy regime. In addition, comparing results in this section and the previous section allows a test for the existence of a third policy regime.

Figures 3.7 and 3.8 present the probability inferences in SURF and SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 3.4, 3.5, and 3.8 report SURF and SORF estimates of model parameters along with the value of the log likelihood function. To make inferences about the significance of estimated parameters, this study reports the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is zero.

3.3.1 Probability Inferences for Three Policy Regimes

Figure 3.7 and Figure 3.8 plot the SURF and SORF estimated probability series of being in the three regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. Although these business cycle dates were not used in any way to estimate parameters or form inferences about the Markov transition process, the traditional business cycle dates are fairly related to the switches of monetary policy between the three regimes as described by the SURF and SORF estimates in Figure 3.7 and Figure 3.8.

The findings here are consistent with those in the previous chapter. Both SURF and SORF estimates show that policy regime one is still a special regime that only prevailed between 1979 and 1984. During this period, the Federal Reserve successfully reduced high inflationary pressure in the U.S., and this period of time is commonly known as the Volcker disinflation period. Both Figure 3.7 and Figure 3.8 show that the special regime one prevailed only during the disinflation period and nowhere else[30]. Regime two is a regime which the Federal Reserve's monetary policy switched into during the expansionary periods, and regime three is a regime which the monetary policy switched into after the recession happened. Both SURF and SORF continue to detect the switches from the expansionary regime to the recessionary regime in U.S. monetary policy, especially the switches that occurred in the recent recession dates of 2001 and 2008 during the newly included seven more years. Furthermore, policy regime two prevailed in most of the additional seven years. When the economy came out of recession, U.S. monetary policy switched to regime two, in which the primary goal of the Federal Reserve was to maintain low inflation rate and sustainable output growth. As a result, it can be seen that regime two was dominant for most of the sample period including most of the late 1960's, the latter half of the 1970's, and the entire period from the late 1980's onward until the end of the sample period. During the recession periods in 1970, 1975, 1982, 1990, 2001, and 2008, monetary policy in the U.S. is estimated to switch into regime three, in which the Federal Reserve adjusted its policy instruments quickly to fight recession.

3.3.2 Estimates of Structural Parameters

Table 3.4 reports the structural parameters estimated with the extended dataset. Both SURF and SORF estimates are close to their Table 2.4 counterparts. Log likelihood values are higher than those in the previous chapter, and SURF log likelihood remains larger than SORF log likelihood.

When the assumption of policy optimality is relaxed, the current estimate of λ is

[30] Although regime one only prevailed during $1979 - 1984$, estimates of the three-regime transition matrix in Table 3.8 do not suggest that regime one is a degenerate regime.

0.379, which is larger than its earlier estimate. While the estimate is sizable, the p-value of the likelihood ratio test indicates that one can not reject the null hypothesis that the true parameter value is zero. This is in line with the results in Table 2.4. Therefore, the estimate of λ in SURF estimates again implies that expected future output may not be an important factor in affecting aggregate demand. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are currently found to be sizable: 0.789 and -0.173 respectively, which are smaller in magnitude than their counterparts in the previous chapter. The estimate of a_1 is statistically indifferent from one and estimate of a_2 is significantly different from zero at any conventional confidence level, which are consistent with the earlier findings. Hence these estimates imply that shocks have sustained effects on output. Third, the estimate of b is 0.005 without imposing the constraint of optimal reaction function. The estimate of b is smaller than its earlier estimate. Despite the fact that b is estimated to be quantitatively small, the estimate appears to be significant at marginal 5% level. The current estimate implies that changes in the interest rate do cause changes in aggregate demand although the effect of interest rate on aggregate demand is relatively insensitive. Fourth, turning to the parameters of the Phillips curve, one can note that firms are substantially more backward-looking than forward-looking. The estimate of the relative weight α_1 is small and is found to be insignificant, and the estimate of α_2 is 0.581 and is found to be insignificantly different from zero. The current estimates of α_1 and α_2 are larger than their earlier estimates. The findings here again result in a backward-looking Phillips curve. Lastly, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The estimate of β is 0.026, which is smaller than the earlier estimate. The estimate of β is found to be significant at any conventional confidence level which implies that a positive output gap raises the inflation rate. This finding is in line with the results in Table 2.4.

SORF also produces close estimates of the structural parameters to their Table 2.4 counterparts. The current estimate of λ is 0.362. In contrast to its earlier estimate, the current estimate of λ is not only quantitatively larger, but it appears to be significantly different from zero at any conventional level. Furthermore, the SORF estimate of λ is nearly the same as its SURF estimate. Thus the current estimate of λ implies that expected future output is an important factor in determining aggregate demand. Second, the current estimates of a_1 and a_2 are smaller in magnitude than their earlier counterparts. The estimate of a_1 is now statistically different from one, and the estimate of a_2 is significantly different from zero at any conventional confidence level. The estimates again imply that shocks have sustained effects on output. Third, the current estimate of b is a little smaller than its earlier estimate. Furthermore, the estimate of b is a bit larger and more precisely estimated than the estimate under SURF, which imply that a change in the interest rate has a larger effect on aggregate demand. Fourth, SORF estimates of both α_1 and α_2 are positive and highly significant, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, the current estimates of α_1 and α_2 are very close to their earlier estimates. α_1 is a bit larger than its earlier estimate, while α_2 is smaller than the earlier estimate. Again, one can not reject the hypothesis that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006), and SORF estimates were obtained with that restriction imposed. One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with the earlier finding. Finally, the current estimate of β is the same as its earlier estimate and still highly significant at any conventional confidence level. In line with the finding in the previous chapter, the SORF estimate of β is much smaller than its SURF estimate implying that it is more difficult for the Federal Reserve to control inflation by changing aggregate demand.

As the p-values reported in Table 3.4 indicate, some of the structural parameters, such as λ and α_1 the parameters of the forward-looking terms in the IS schedule and Phillips curve, are more precisely estimated under SORF than under SURF, which is consistent with the conclusion in the previous chapter. It is believed that a unified approach of joint estimation of structural model parameters with the parameters of the policymaker's objective function provides additional across equation restrictions that result in a more precise estimation of the model parameters. Hence the SORF estimates more strongly support the assumption that private agents are forward looking. Castelnuovo (2003) argues that hybrid formulation of the Phillips curve is crucial for appropriate identification of policy preferences. Moreover, Salemi (2006) suggests that when the interest rate is highly persistent, the best way to reconcile a persistent interest rate process that is resulted from an optimal monetary policy is that both occurred in an economy where private agents are forward looking. The finding of highly persistent interest rate in all regimes will be shown below.

3.3.3 Estimates of Preference Parameters and Test of Policy Optimality

Table 3.5 reports estimation results for the preference parameters of three policy regimes identified in U.S. monetary policy, and it also presents two sets of reaction function coefficients for each policy regime: SURF estimates (obtained while treating coefficients of the policy rule as free parameters), and SORF estimates (implied values of the policy rule coefficients consistent with the policy optimality hypothesis). To interpret the results on policy preference estimation, one should note that the relative weight on inflation stabilization objective is normalized to one.

As Table 3.5 shows, both SURF and SORF produce similar estimates of the central bank's preference parameters and policy-rule coefficients to their earlier Table 2.5 counterparts. In all three policy regimes under SORF, the estimated values of w_y and w_r are very small implying that inflation stabilization was always the dominant objective of the

Federal Reserve. Policy regime one is a special regime that only prevailed during the Volcker disinflation period, covering the period of 1979-1984. Looking at the estimates describing the Federal Reserve's preferences in policy regime one, the estimate of w_y is 0.00003 and the likelihood ratio test can not reject the null hypothesis that the true weight is zero. The weight placed on interest rate stabilization is 0.0014 and significantly different from zero. The value of w_r in the first regime is the smallest among three policy regimes. These findings are in accord with those in the previous chapter. Given the normalization of the weights, this finding suggests that in conducting its monetary policy the Federal Reserve was far more concerned with keeping the inflation close to its long run target path than it was with trying to stabilize output and interest rate around their targets. Indeed, in policy regime one, both SURF and SORF estimates show that the policy rule coefficient on lagged inflation c_3 is very sizable, and the SURF estimate of c_3 is significantly different from zero. While being significant, the interest rate stabilization objective appears to be of secondary importance to the Federal Reserve in policy regime one. The estimates of preferences in policy regime one are consistent with the historical evidence. As described earlier, in this special regime the Federal Reserve faced high level of inflation, and its sole objective was to reduce inflation substantially.

In policy regime two, inflation stabilization was still the dominant objective. The estimates of the policy preferences in this policy regime suggest that the Federal Reserve Bank had been conducting its policy in the manner consistent with the policy used during economic expansionary periods. A large weight was assigned to inflation stabilization, and very small weights were placed on both output and interest rate stabilizations. Indeed, the estimate of w_y is 0.0025 which is statistically significant at any conventional confidence level. The estimated policy weight placed on the interest rate stabilization objective w_r is 0.0046 and is significantly different from zero. These findings are consistent with those in the previous chapter. Interest rate smoothing became a more important objective in policy regime two than in policy regime one. As pointed out previously, in this expansionary regime the Federal Reserve would put more concerns on potential exacerbate inflation situation and use instrumental interest rates wisely. Switching into policy regime two would result in a low level of inflation.

In policy regime three, the normalization of the weights suggests that keeping the inflation close to its long run target path was far more important for the Federal Reserve than stabilizing output and the interest rate. The estimate of w_y in this regime is 0.0079 and is the largest among three policy regimes. The likelihood ratio test for w_y strongly rejects the null hypothesis that the true weight is zero. In addition, in policy regime three, both SURF and SORF estimates show that the policy rule coefficient on lagged output c_1 is sizable. The estimate of w_r in this regime is 0.0042 and is highly significant. These findings are in accord with those in the previous chapter. The interest rate stabilization objective was not as important as the output stabilization objective for the Federal Reserve in policy regime three.

There is clear evidence that the estimated and implied policy-rule coefficient estimates in three regimes differ significantly and substantially from each other. First, the coefficient on lagged output (c_1) is negative in policy regime one but sizable and positive in regimes two and three. The coefficient on lagged inflation (c_2) is large and positive in regime one but small in regimes two and three. The coefficient on lagged two output (c_4) is sizable and positive in regime one but negative in policy regimes two and three. Second, SORF estimate of the policy rule coefficient on lagged inflation is positive in all regimes and larger than the SURF estimate in regime three. This suggests that the Federal Reserve should have responded more strongly to inflation than it did in regimes one and two, which is in line with the earlier conclusion.

One potential cause of changing policy-rule coefficient estimates is that monetary policymakers would face different variabilities of inflation and other economic variables in different policy regimes. Table 3.6 and Table 3.7 report SURF and SORF estimates of variance-covariance matrices of disturbances in three policy regimes respectively. One can see that structural shocks changed jointly across policy regimes. Both SURF and SORF estimates show that in the special Volcker disinflation regime, the variances of output and the policy shocks are the largest. This finding is consistent with the conclusion in Sims and Zha (2006). In the second regime during the expansionary periods, the shock variances of output, inflation, and the interest rate are the smallest among all policy regimes. In policy regime three, shock variance of inflation turns out the be the largest reflecting several economic disruptions, e.g. large oil shocks, happened during the beginning of the recessionary periods. By comparing the covariances of shocks, both SURF and SORF estimates show that the correlation between output and inflation was positive in policy regime one, but became negative in policy regimes two and three. SURF estimates show that inflation and the interest rate were negatively correlated in policy regime one, while they were positively correlated under SORF estimates. The comparison implies that the actual monetary policy used in policy regime one was not optimal.

Did the Federal Reserve set monetary policy optimally with three policy regimes? In light of Salemi (2006), a test of policy optimality is a test of the hypothesis that SORF fits the data as well as SURF, that is, the SORF and SURF policy rule coefficients are the same. In each policy regime, there are four freely estimated parameters in the unrestricted policy rule but only two freely estimated parameters, w_y and w_r, in the optimal policy rule, so SORF implies six restrictions. As a result, the relevant likelihood ratio statistic is 2.488 with a p-value 0.870. In contrast to the conclusion in the previous chapter, the hypothesis that the policy rule coefficients that fit the data best were those that minimized quadratic expected loss as defined in (2.6) can not be rejected. Hence, the estimations using the updated dataset show that the Federal Reserve did set monetary policy optimally with three policy regimes.

3.3.4 Interest Rate Smoothing

As can be seen in Table 3.5, the reaction-function coefficient for the lagged interest rate is always large compared to the coefficients for inflation and output so that, according to the definition provided by Clarida, Galí and Gertler (2000), the Federal Reserve engages in "interest rate smoothing". This finding supports the widely held viewpoint that most central banks, including the Federal Reserve, are very cautious when changing interest rates. While the estimates in the reaction functions reported in Table 3.5 imply that the Federal Reserve smooths interest rates, SORF estimates suggest that the Federal Reserve placed small weight on interest rate stability in all three policy regimes. Thus, in line with the previous chapter, the estimates here are consistent with Sack and Weiland's explanation for smoothing. The large coefficient on the lagged interest rate can be explained by the finding that forward-looking expectations are empirically important and the fact that optimal policies require persistent responses to shocks.

3.3.5 Estimated Transition Probabilities

Table 3.8 presents SURF and SORF estimates for the transition probabilities for switching in three monetary policy regimes. Both SURF and SORF produce similar estimates of the transition matrices to the results in Table 2.6. As the p-values indicate, all transition probabilities are precisely estimated. The probabilities on the diagonal of the transition matrix are close to unity, which means that all policy regimes show high persistence[31]. The SURF estimates indicate that, if the monetary policy is in regime one this period, there is a 93.4 percent probability that it will be in regime one next period; if the monetary policy is in regime three this period, there is a 93.5 percent probability

[31] To guarantee that the diagonal transition probabilities are highly persistent, in the estimation they were restricted to be no less than 0.900.

that it will be in regime three next period. Similarly, the SORF estimates indicate that, if the monetary policy is in regime one this period, there is a 94.1 percent probability that it will be in regime one next period; if the monetary policy is in regime three this period, there is a 92.3 percent probability that it will be in regime three next period. These suggest that changes in the monetary policy regime will occur infrequently.

Both SURF and SORF estimates show that all policy regimes except one are not degenerate regimes, which is in line with the earlier findings. The p-values for SURF and SORF diagonal probabilities p_{11}, p_{22}, and p_{33} are measured by the likelihood ratio test for the hypothesis that the true value of the transition probability is 0.995, meaning that there is nearly no switch back into the regimes prevailing at the beginning of the sample period. The test statistic for SURF estimate of p_{22} can not reject the hypothesis thus the policy regime two in SURF estimates is a degenerate regime. In sum, the important findings associated with the three-regime monetary policy are robust to estimations using the updated dataset with seven more years data.

3.3.6 Test of a Third Regime in U.S. Monetary Policy

Was there a third regime in U.S. monetary policy? A comparison between models with two-regime and three-regime monetary policy is helpful in answering the question. This study uses likelihood ratio test statistic to test the hypothesis that the two-regime models fit the data as well as the three-regime models. Therefore, one likelihood ratio statistic tests the two-regime SURF model against the three-regime SURF model, and the other likelihood ratio test statistic is a test of the two-regime SORF model against the three-regime SORF model.

There are eight more freely estimated parameters in the model with three-regime unrestricted policy rule than those in the two-regime SURF model. Thus the two-regime SURF implies eight restrictions. The test statistic for a likelihood ratio test of the three-regime SURF against the two-regime SURF would lie in 30.654, which implies a p-value that is less than 0.001. For the test of the three-regime SORF model versus the two-regime SORF model, the two-regime SORF implies six restrictions, and the likelihood ratio test statistic is 33.868 implying a p-value that is less than 0.001. The tests thus support the three-regime models over the two-regime models. The hypothesis that the models that fit the data best were those in which the monetary policy follows a two-regime Markov-switching process can be soundly rejected.

3.3.7 Comparison of Impulse Response Function for Structural Estimation

To check how much the model can explain the variation in the data, this section compares the impulse response functions (IRF's) implied by SURF and SORF. In line with Salemi (2006), this study makes the hypothesis that monetary policy is conditioned only on lagged values of output and inflation. In keeping with this hypothesis, this study chooses y-π-r as the within period causal ordering for the impulse response function. The ordering implies that a shock to output affects only output contemporaneously, a shock to inflation affects both inflation and output contemporaneously, and a shock to the interest rate affects all variables contemporaneously. The ordering makes difference because of the contemporaneous correlation between output and the interest rate in all policy regimes. Figures 3.9, 3.10, and 3.11 present the IRF's for all three regimes. The shade areas in the figures are 95% confidence intervals computed with parametric Bootstrap method.

The figures show clearly that the SURF and SORF estimates share many features of the impulse response functions. For instance, SURF and SORF estimates all imply that a positive shock to output keeps output above its trend for more than 30 quarters. Both SURF and SORF IRF's show that a positive shock to inflation produces moderate inflation persistence with inflation returning to trend in about six quarters.

There are significant differences in the impulse response functions across three policy regimes. In regimes one and two, both SURF and SORF estimates imply that the interest rate increases then falls gradually to trend after a positive inflation shock; in policy regime three, SURF estimates show that the interest rate falls then increases gradually to trend after a positive inflation shock, and SORF estimates show that interest rate increases slightly above trend then decreases gradually to trend after 20 quarters. Since policy regime one is the Volcker disinflation regime, the interest rate responds positively to a positive inflation shock, and the responses are much larger than those in regimes two and three. The comparisons also imply that the actual interest rate response to inflation was not optimal in regimes one and three but it was very close to optimal in regime two. A loss-minimizing policy would have raised the interest rate more in policy regimes one and three. A second difference is somewhat related to the first. In all three regimes SORF estimates imply that the inflation response to a positive output shock is smaller than implied by the SURF estimates.

Comparing the IRF's suggest that actual and optimal responses of the interest rate to output shocks were not the same in all three policy regimes. In regime one the interest rate increases initially then falls and returns to trend after a negative output shock, while in regimes two and three the interest rate falls then rises to trend gradually after a negative output shock. Therefore, in policy regime one, SURF and SORF estimates imply that inflation would be dampened completely. In policy regimes two and three, both SURF and SORF estimates imply that a negative output shock causes interest rate to decrease below trend for 20 quarters. Because regime one is the special Volcker disinflation regime, the primary goal of the Federal Reserve was to decrease inflation. While regime three is a recessionary regime, the Federal Reserve would like to maintain output growth hence the interest rate responses are greater than those in regimes one and two.

A fourth difference is that, in regime one a positive inflation shock causes output to increase then fall to or under trend gradually; in policy regimes two and three, a positive inflation shock causes output to fall below trend for a substantial period of time.

Figure 3.12 compares the responses of output and inflation to an interest rate shock. The policy rule equation residuals are correlated with the IS and Phillips curve equation residuals in both policy regimes. On the maintained hypothesis that the monetary policy reacts to economic conditions only with a lag, this correlation between must be due either to responses of output and inflation to an interest rate surprise or to responses of output, inflation, and interest rate to an un-modeled shock. Following Salemi (2006), to obtain the model's prediction of responses of output and inflation to a pure interest rate shock, this study ignores the contemporaneous correlations when computing the IRF's reported in Figure 3.12.

Figure 3.12 shows that in all three regimes SURF and SORF estimates can account for the "hump-shaped" response of output to an interest rate surprise[32]. In policy regime one, SURF peak occurs about five quarters after the interest rate shock, and SORF peak occurs about ten quarters after the shock. In policy regime two, both SURF and SORF peaks occur about 13 quarters after the interest rate shock. In policy regime three, SURF peak occurs about 11 quarters after the interest rate shock, and SORF peak occurs about ten quarters after the shock. Both SURF and SORF estimates show that the inflation rate declines immediately after an interest rate shock and remains below trend for many quarters. SORF estimates imply a larger inflation response than do SURF estimates in policy regime one. Because regime one is a special disinflation regime, the Federal

[32]As Table 3.4 shows, the estimates of the coefficients for the interest rate in the IS schedule and the output gap in the Phillips curve are very small, which makes one believe that monetary policy is not possible. Thus this study also analyzes the effects of monetary policy rule if it would have not responded to lagged terms of output and inflation. In the reaction function, this study sets $c_i = 0$ for $i \in \{1, 2, 4\}$ and keeps c_3 at its estimated value. With the new parameterizations, both SURF and SORF estimates show that the impulse responses to an interest rate shock are very close to those obtained with the original monetary policy rule. The results suggest that output and inflation are still very responsive to a shock in the interest rate, and the small values of the coefficients on the interest rate in the IS schedule and on the output gap in the Phillips curve still play an important role in accounting for the responses. Therefore, it is possible for the Federal Reserve to conduct monetary policy effectively during the sample period.

Reserve would like to decrease inflation hence the SORF inflation responses are larger than those implied by SURF estimate.

3.3.8 A Counterfactual Analysis

This section considers the possibility that the Federal Reserve does not put any emphasis on output stability. To check this alternative specification, this study first restricts the value of weight on output stabilization to zero in all three policy regimes[33]. This study then estimates the counterfactual model under the assumption that monetary policy is loss-minimizing. Figure 3.13 presents the probability inferences in SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 3.9, 3.10, and 3.11 report SORF estimates of model parameters along with the value of the log likelihood function. For convenience of comparison with estimates, the tables also show SORF estimates for the original three-regime model.

Figure 3.13 plots the SORF estimated probability series of being in the three regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. The SORF estimated probability series for the counterfactual model are consistent with those obtained in the original model. SORF estimates show that policy regime one is still a special regime that only prevailed during the Volcker disinflation period between 1979 and 1984, and nowhere else. Regime two is a regime which the Federal Reserve's monetary policy switched into during the expansionary periods, and regime three is a regime which the monetary policy switched into during the recessionary periods. SORF estimates for the counterfactual model continue to detect the switches from the expansionary regime to the recessionary regime in U.S. monetary policy.

[33] This study continues to impose $w_\pi = 1$ as a normalization. Thus there is only one free element in the symmetric weight matrix of the central bank.

Table 3.9 reports the structural parameters for the counterfactual model. One can notice that log likelihood value is significantly lower than that in the original model. The SORF estimates differ from their counterparts in the original model in several important ways. The current estimate of λ is 0.197, which is quantitatively smaller than its counterpart in the original model. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are currently found to be sizable: 1.099 and -0.300 respectively, which are larger in magnitude than their counterparts in the original model. Third, the estimate of b is 0.001, which is smaller than its counterpart. Fourth, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The estimate of β is 0.0006, which is a bit larger than its counterpart. SORF estimates of both α_1 and α_2 are positive, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, the current estimates of α_1 and α_2 are close to their counterparts. The estimate of α_1 is a little larger, while the estimate of α_2 is a bit smaller. Again, SORF estimates were obtained with the restriction that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006). One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with the earlier finding.

Table 3.10 reports estimation results for the preference parameters of three policy regimes identified in U.S. monetary policy, and it also presents implied values of the policy rule coefficients consistent with the policy optimality hypothesis. To interpret the results on policy preference estimation, one should note that in this counterfactual analysis the relative weight on output stabilization is restricted to zero and the weight on inflation stabilization objective is normalized to one. As Table 3.10 shows, in all three policy regimes, estimate of the weight placed by the Federal Reserve on interest rate stabilization is smaller than its counterpart in the original model and the estimated values of w_r is very small, which imply that inflation stabilization was always the dominant objective of the Federal Reserve. Looking at the Federal Reserve's preferences in policy regime one, the weight placed on interest rate stabilization is 0.0009, which is the smallest among three policy regimes. This finding is in accord with that in the original model. The estimate of preference in policy regime one is consistent with the historical evidence. As described earlier, in this special regime the Federal Reserve faced high level of inflation, and its sole objective was to reduce inflation substantially. In policy regime two, the policy preferences suggest that the Federal Reserve Bank had been conducting its policy in the manner consistent with the policy used during economic expansionary periods. A large weight was assigned to inflation stabilization, and a very small weight was placed on interest rate stabilization. The estimated policy weight placed on the interest rate stabilization objective w_r is 0.0015. This finding echoes the result in previous section. Interest rate smoothing became a more important objective in policy regime two than in policy regime one. As pointed out previously, in this expansionary regime the Federal Reserve would put more concerns on potential exacerbate inflation situation and use instrumental interest rates wisely. In policy regime three, the normalization of the weights suggests that keeping the inflation close to its long run target path was far more important for the Federal Reserve than stabilizing the interest rate. The estimate of w_r in this regime is 0.0009 and is the same as that in regime one.

There is clear evidence that the implied policy-rule coefficient estimates in all regimes differ substantially from their counterparts in the original model. First, in all three policy regimes, the policy rule coefficients on lagged output c_1 and c_4 become smaller in magnitude. This suggests that the Federal Reserve responded less strongly to output than it did in the original model, which is in line with the restriction imposed by the counterfactual analysis. Second, in all three policy regimes, the policy rule coefficient on the lagged interest rate c_3 becomes very sizable and is larger than its counterpart. In addition, the values of c_3 are nearly the same across policy regimes in the counterfactual model. This finding is consistent with the previous result that the estimated policy weight placed on the interest rate stabilization objective is reasonably close to each other in three regimes. Third, the coefficient on lagged inflation (c_2) is smaller in all three policy regimes.

Did the Federal Reserve stabilize output when conducting it optimal policy actions? A formal likelihood ratio test between the counterfactual and original models with three-regime monetary policy is helpful in answering the question. This study tests the hypothesis that the counterfactual model fits the data as well as the original model. There are three more freely estimated parameters in the original model with three-regime optimal policy rule than the counterfactual SORF model. Thus the counterfactual model places three restrictions. The test statistic for a likelihood ratio test of the original three-regime SORF against the counterfactual three-regime SORF would lie in 16.537, which implies a p-value that is 0.001. The test thus supports the original model over the counterfactual model. The hypothesis that the model that fits the data best was that in which the policymakers in the central bank do not wish to stabilize output can be soundly rejected.

Table 3.11 presents SORF estimates for the transition probabilities of the counterfactual model in three monetary policy regimes. SORF produces very similar estimates of the transition matrix to the results in the original model. The probabilities on the diagonal of the transition matrix are close to unity, which means that all policy regimes show high persistence[34]. The SORF estimates for the counterfactual model indicate that, if the monetary policy is in regime one this period, there is a 93.9 percent probability that it will be in regime one next period; if the monetary policy is in regime three this period, there is a 92.4 percent probability that it will be in regime three next period. In sum, these findings are consistent with those in the original model.

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[34] To guarantee that the diagonal transition probabilities are highly persistent, in the estimation they were restricted to be no less than 0.900.

This exercise also compares the impulse response functions (IRF's) implied by SORF estimations of the counterfactual and original models. Figures 3.14, 3.15, and 3.16 present the IRF's of output and inflation for all three regimes. The figures show clearly SORF estimates for the counterfactual model imply that in response to a positive output shock, output, inflation, and the interest rate react more strongly and persistently than their counterparts implied by the original model in all three policy regimes. Second, IRF's of the counterfactual model show that a positive shock to inflation produces almost identical inflation persistence as implied by the original model. Response of inflation to its own shock returns to trend in about six quarters.

There are significant differences in the impulse response functions across three policy regimes. In regime one, counterfactual SORF estimates imply that output responds more strongly and persistently to a positive inflation shock than implied by the original model. In the counterfactual model, output increases then remains above trend for a very long time after a positive inflation shock; while in the original model, output increases then falls below trend after about eight quarters. In policy regime two, both counterfactual and original SORF estimates imply that output falls immediately and remains below trend after a positive inflation shock, but the response of output in the counterfactual model is a bit larger and more persistent than in the original model. In regime three, counterfactual SORF estimates imply that output falls below trend after a positive inflation shock, and the response of output is almost identical to that implied by the original model. A second difference is related to response of the interest rate to a positive inflation shock. In regime one, counterfactual SORF estimates imply that response of the interest rate to a positive inflation shock is smaller than implied by the original model, but the interest rate remains above trend for a substantial period of time. In policy regime two, counterfactual SORF estimates imply that the interest rate responds less aggressively to a positive inflation shock than implied by the original model. In the counterfactual model, the interest rate falls gradually below trend after a positive inflation shock; while in the original model, output increases then falls gradually to trend. In policy regime three, both counterfactual and original SORF estimates imply that the interest rate rises and decreases gradually to trend after a positive inflation shock, and the response in the counterfactual model is very close to that in the original model.

Figure 3.17 compares the responses of output and inflation to an interest rate shock implied by the counterfactual and original models. Counterfactual SORF estimates show that in all three policy regimes the response of output to an interest rate surprise is much smaller than implied by the original model, and counterfactual SORF peak occurs a very long period time after the interest rate shock. The counterfactual SORF estimates also imply larger and more persistent inflation and the interest rate responses than do original model estimates in all policy regimes. These results are consistent with the restriction imposed by this counterfactual exercise. Because the policymakers in the central bank are assumed to put no emphasis on output stabilization, the SORF output responses are smaller and inflation responses are larger than those implied by the original SORF estimates.

Figure 3.1: Actual and Target Values of Variables

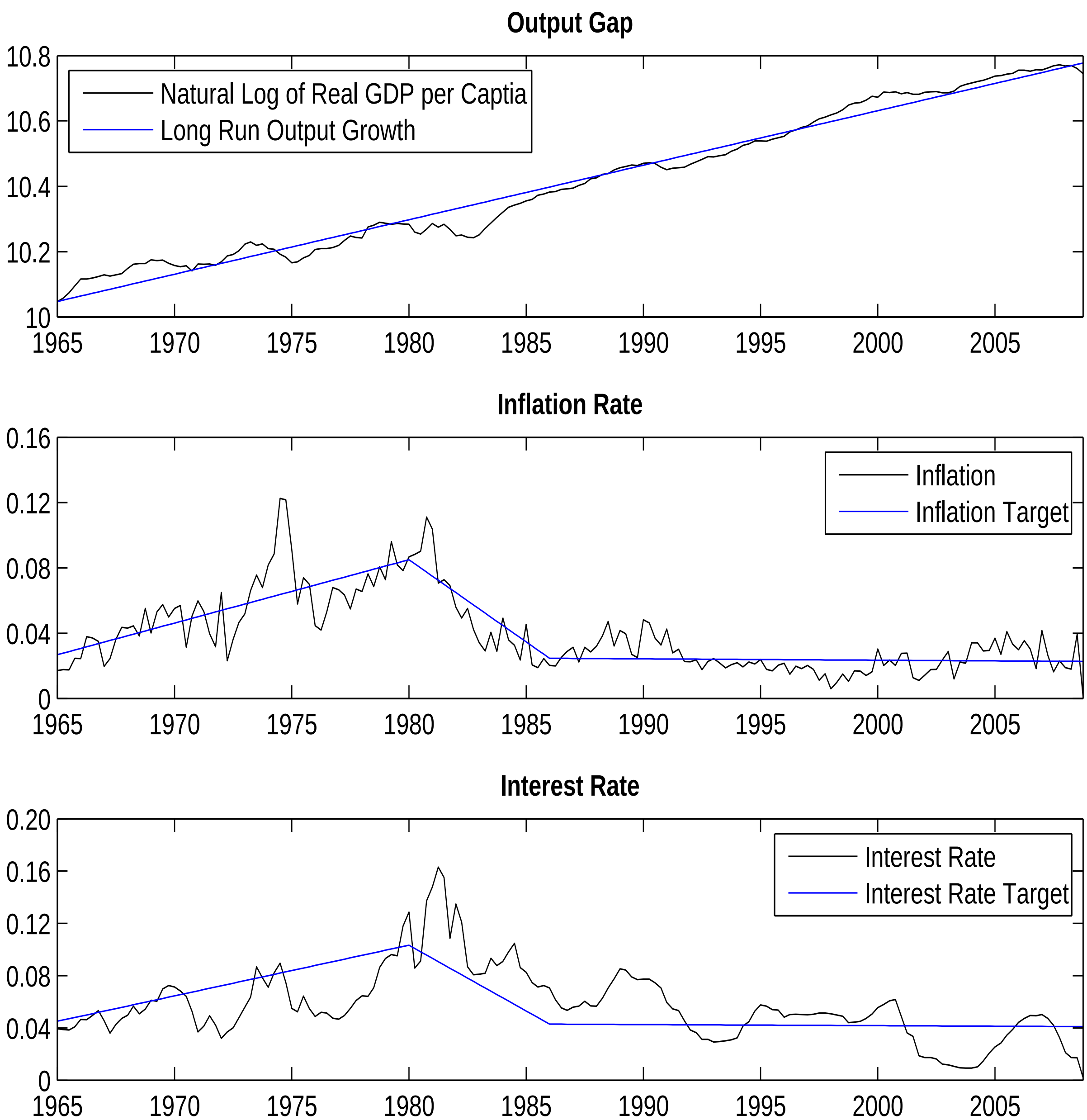
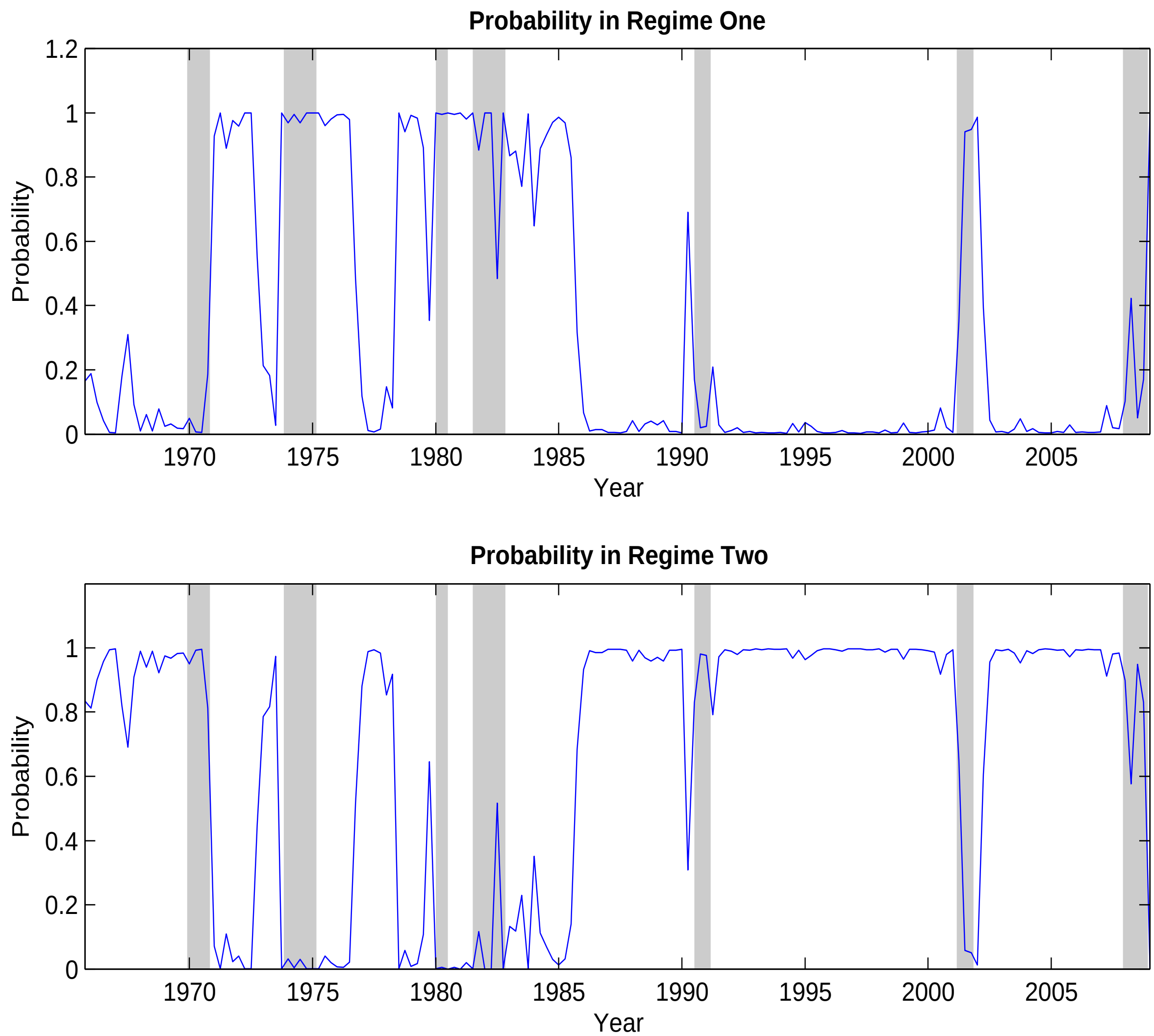
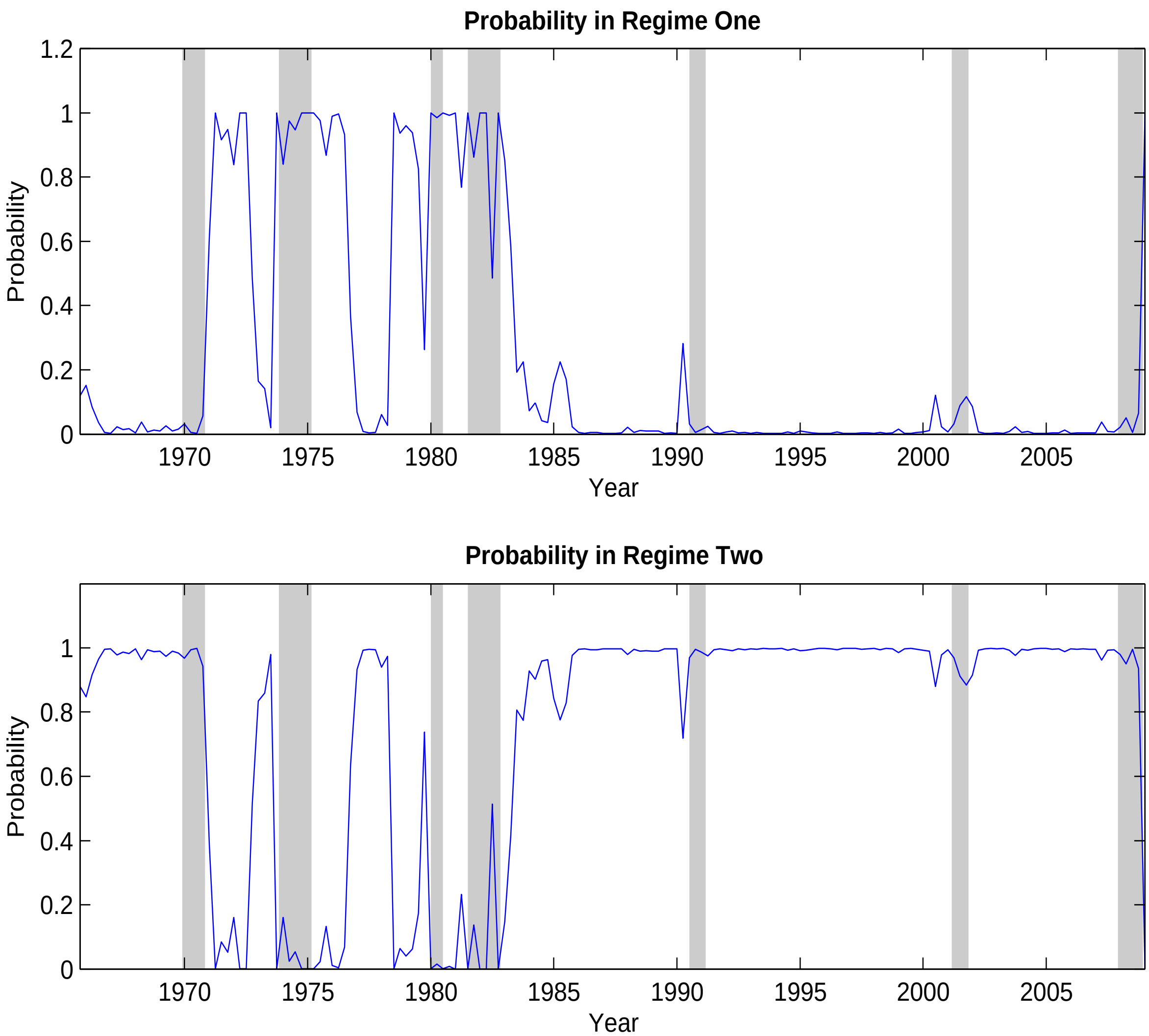


Figure 3.2: Probabilistic Inference for the Two-Regime Unrestricted Monetary Policy



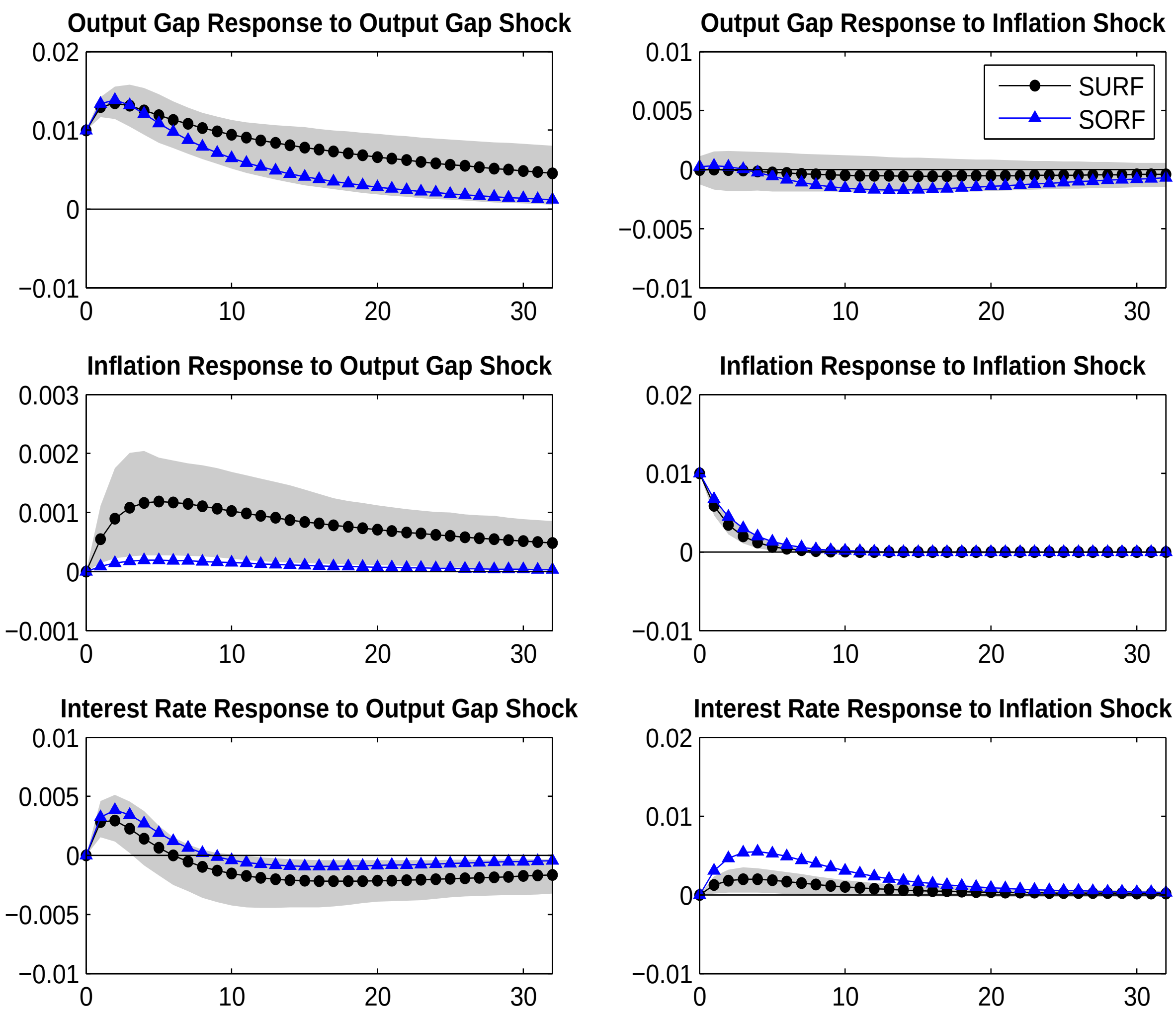
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 3.3: Probabilistic Inference for the Two-Regime Optimal Monetary Policy



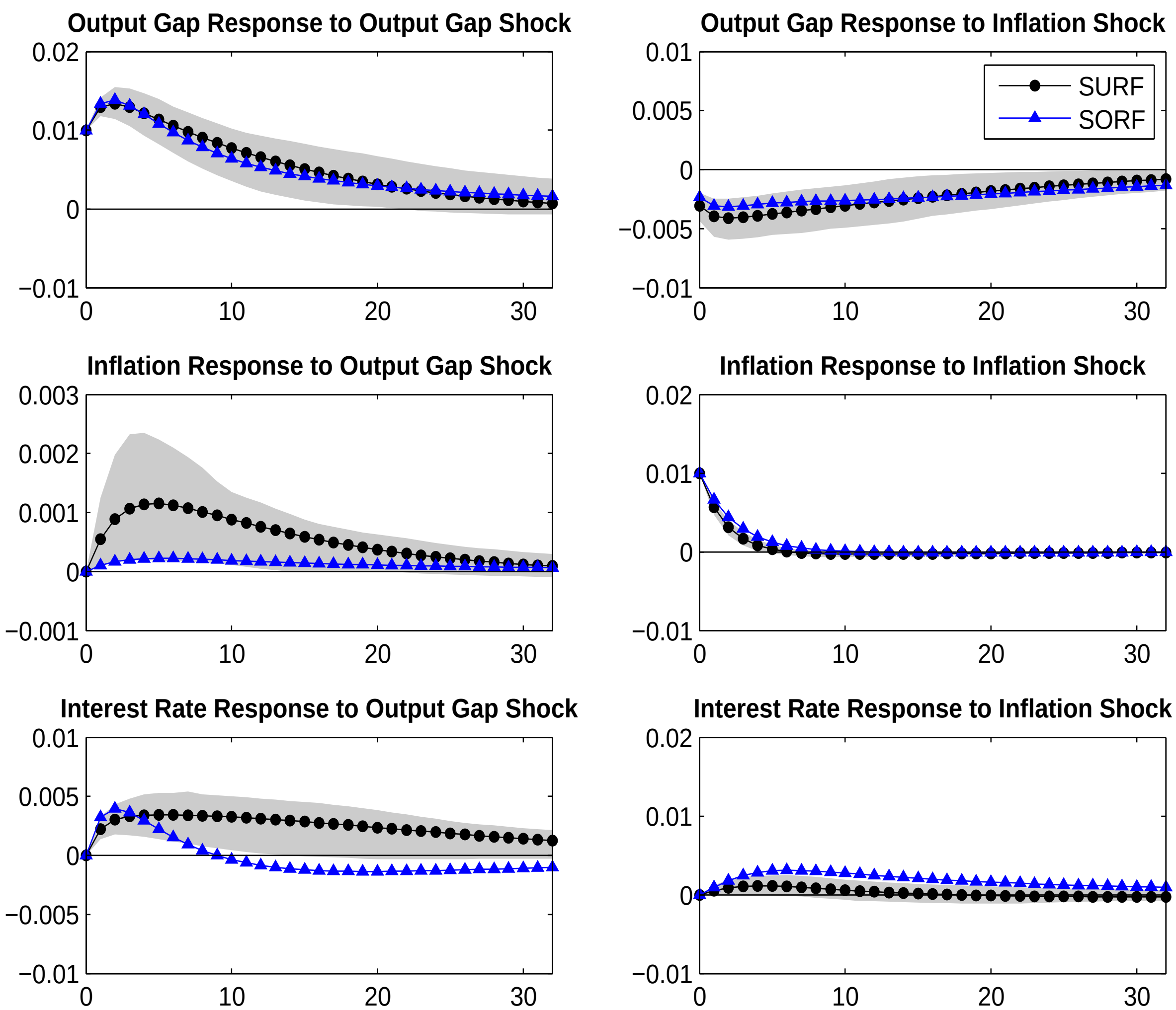
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 3.4: Comparison of Impulse Response Functions
for the Model with Two-Regime Monetary Policy
in
Regime One



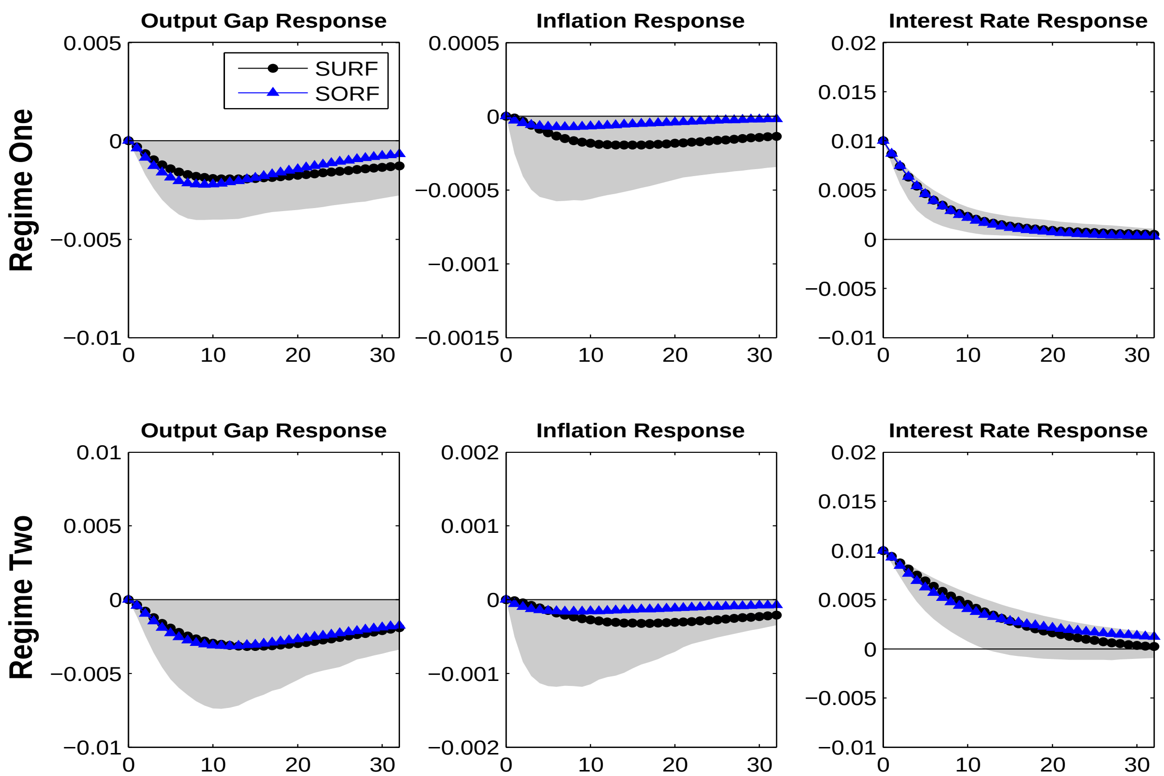
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.5: Comparison of Impulse Response Functions
for the Model with Two-Regime Monetary Policy
in
Regime Two



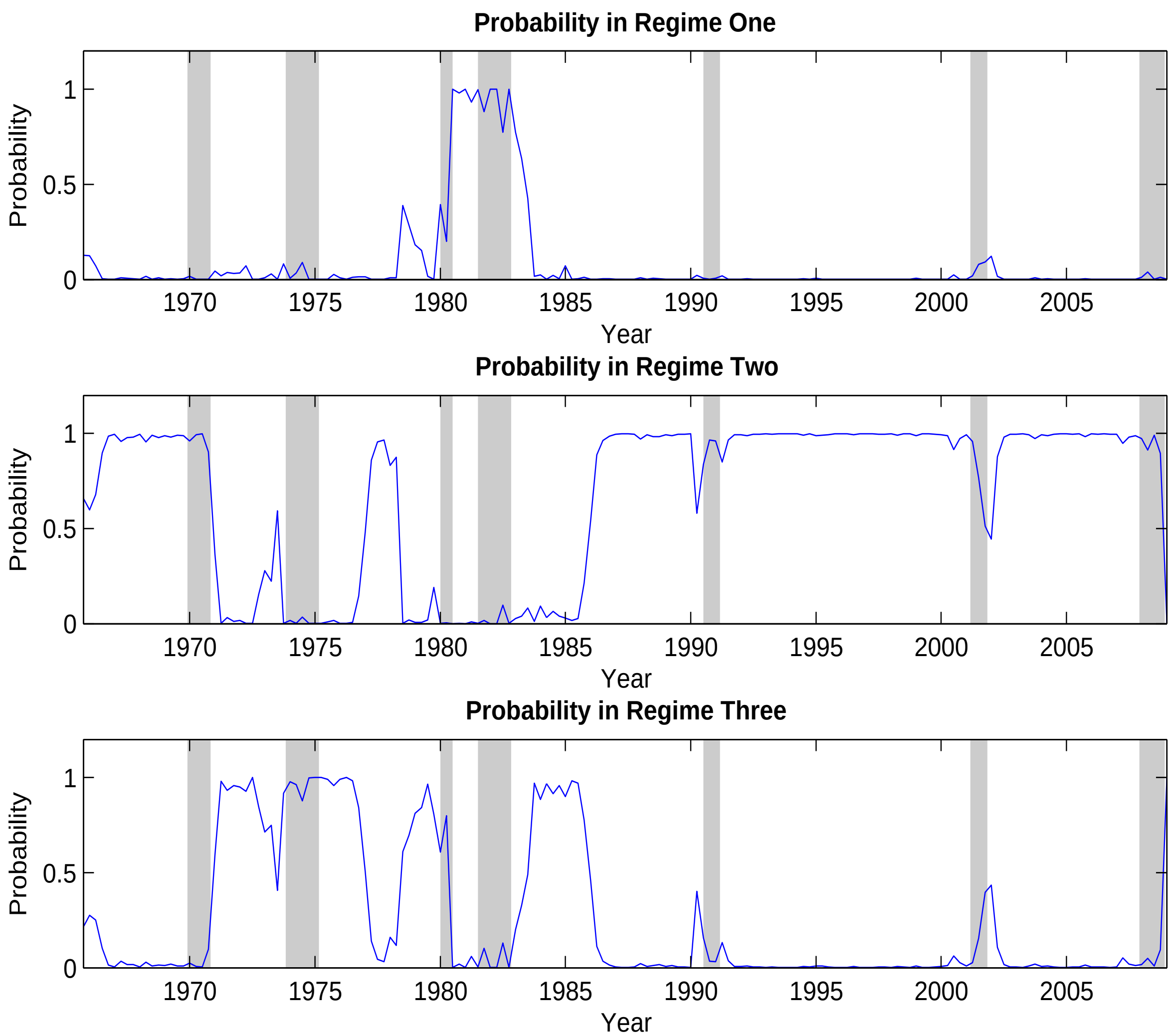
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.6: Comparison of Impulse Response Functions to an Interest Rate Shock for the Model with Two-Regime Monetary Policy



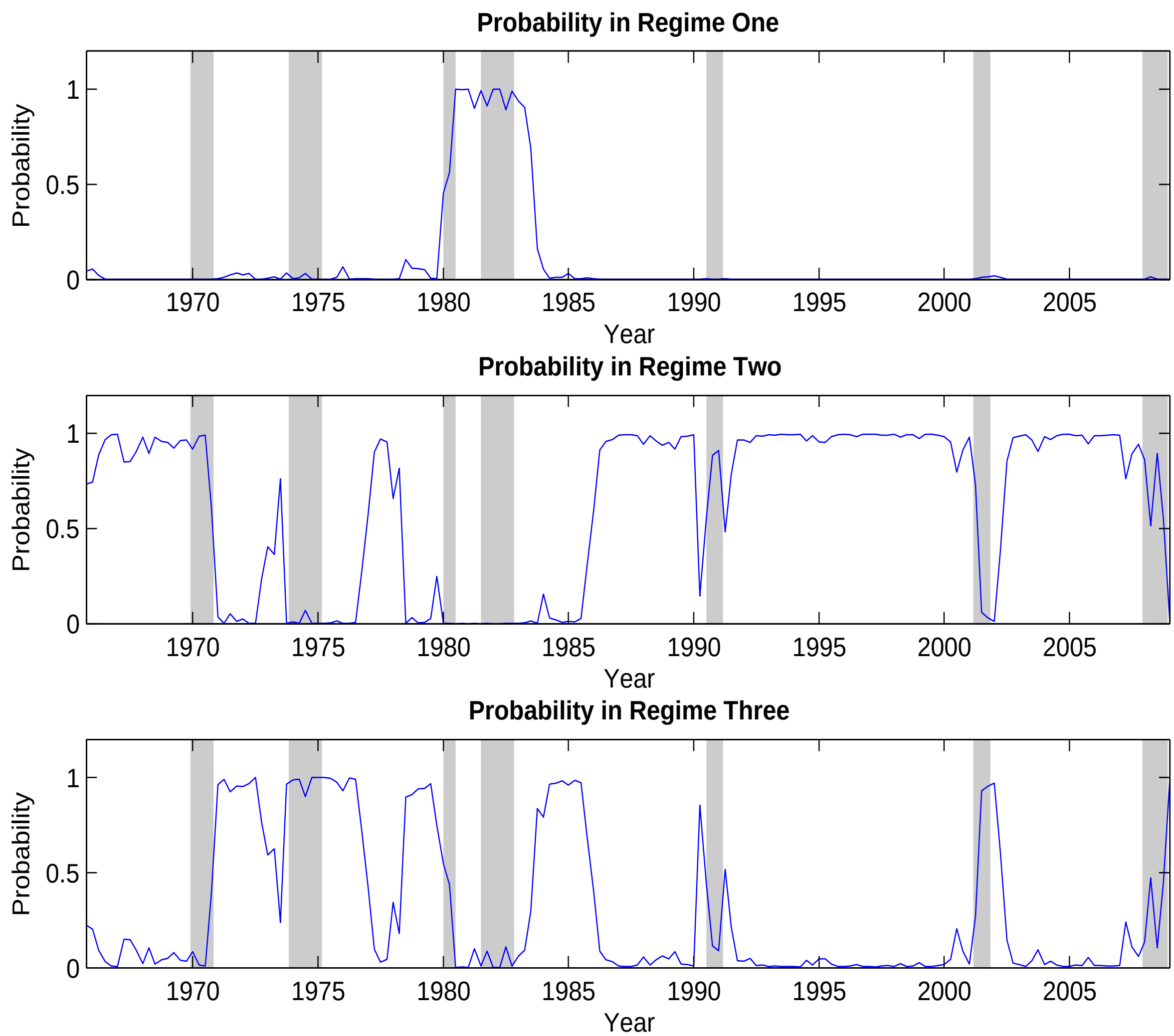
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.7: Probabilistic Inference for the Three-Regime Unrestricted Monetary Policy



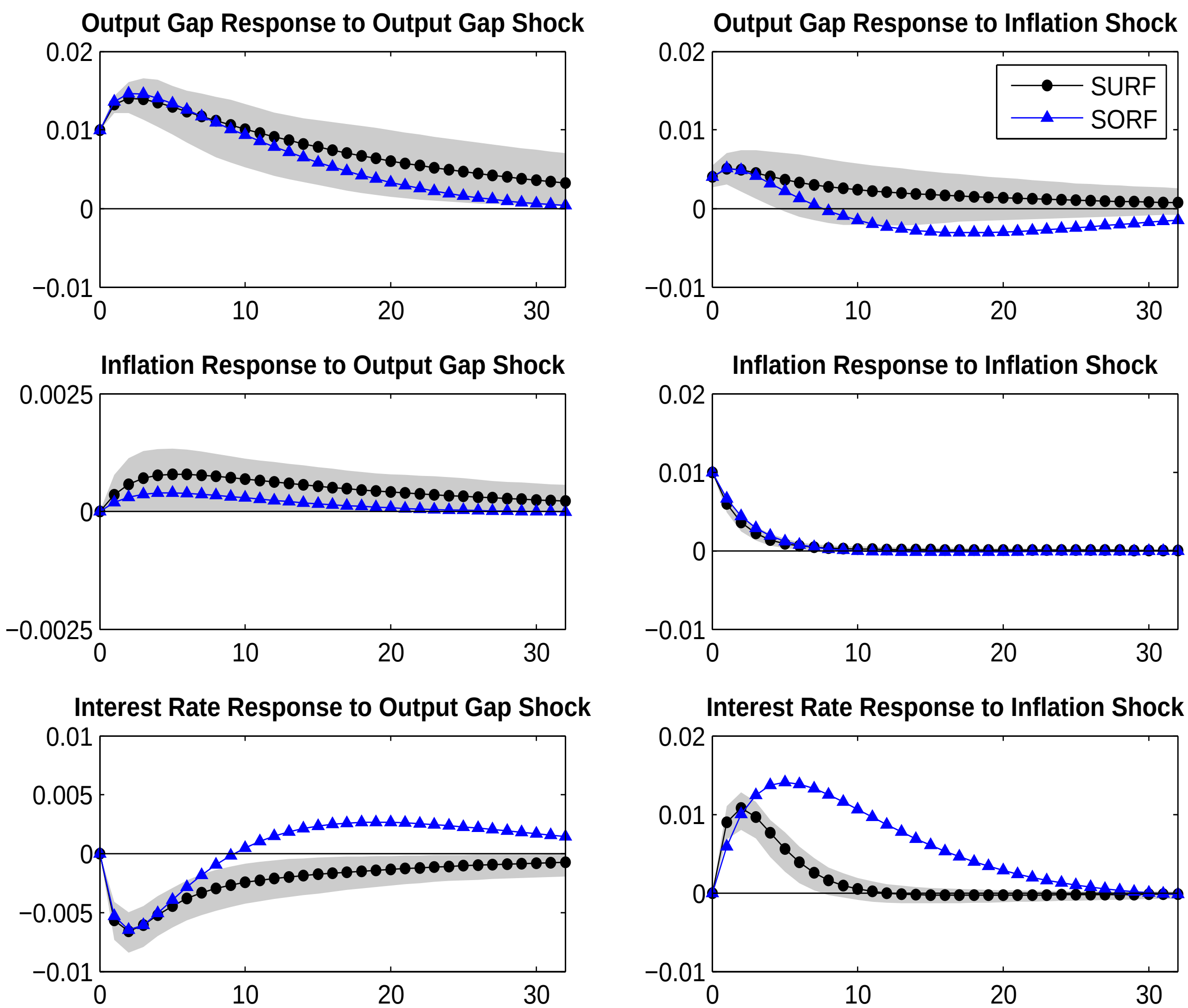
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 3.8: Probabilistic Inference for the Three-Regime Optimal Monetary Policy



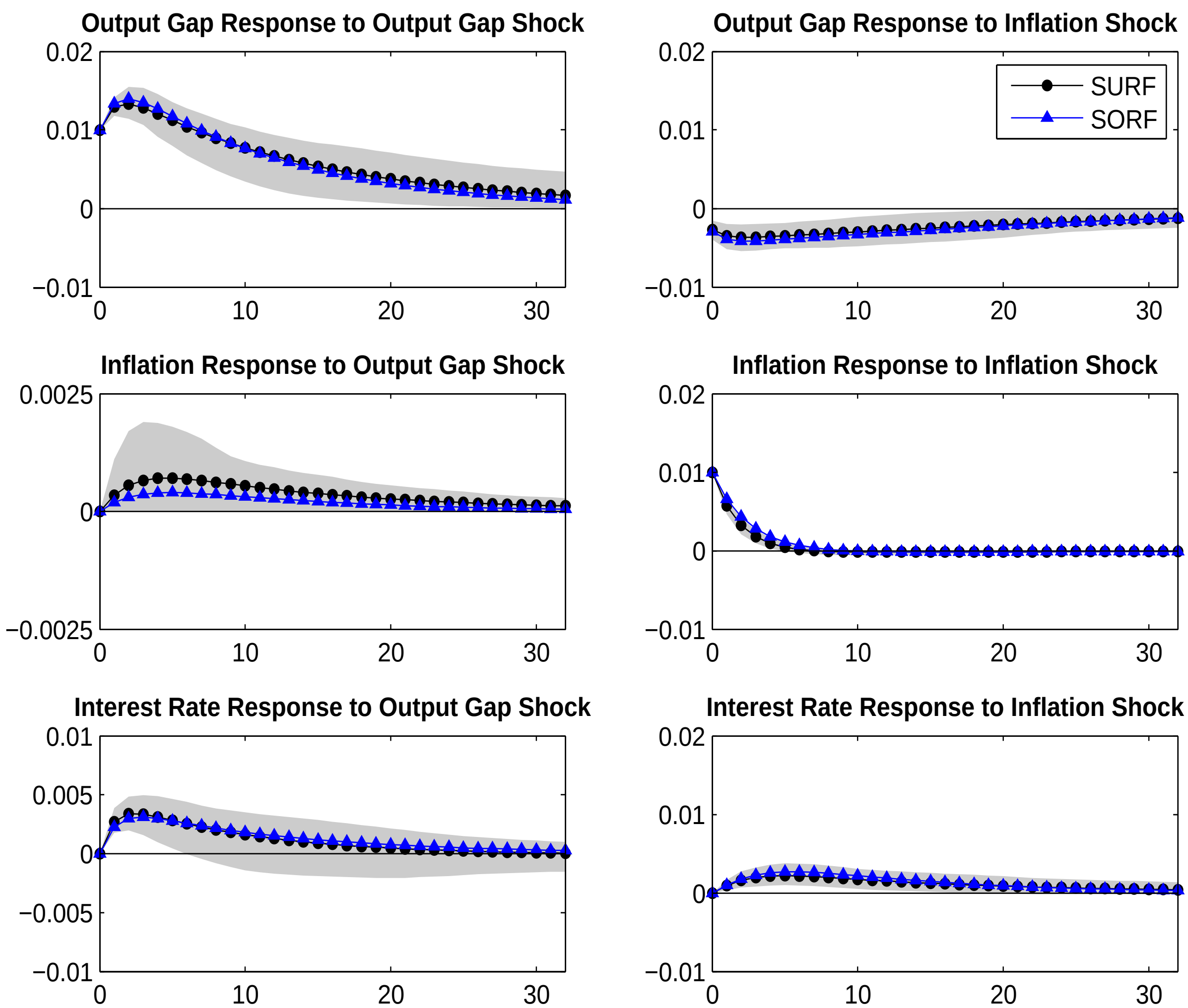
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 3.9: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime One



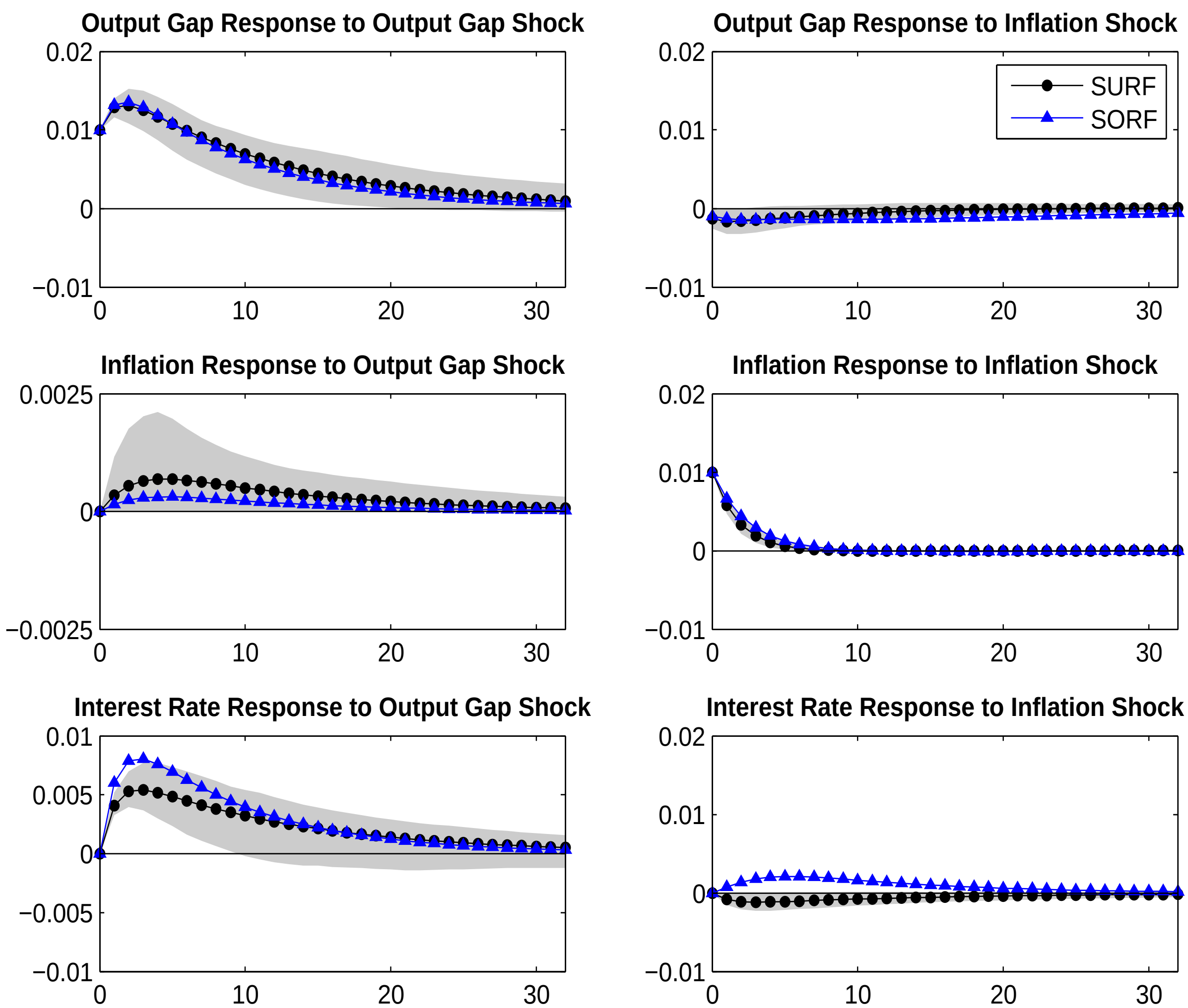
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.10: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Two



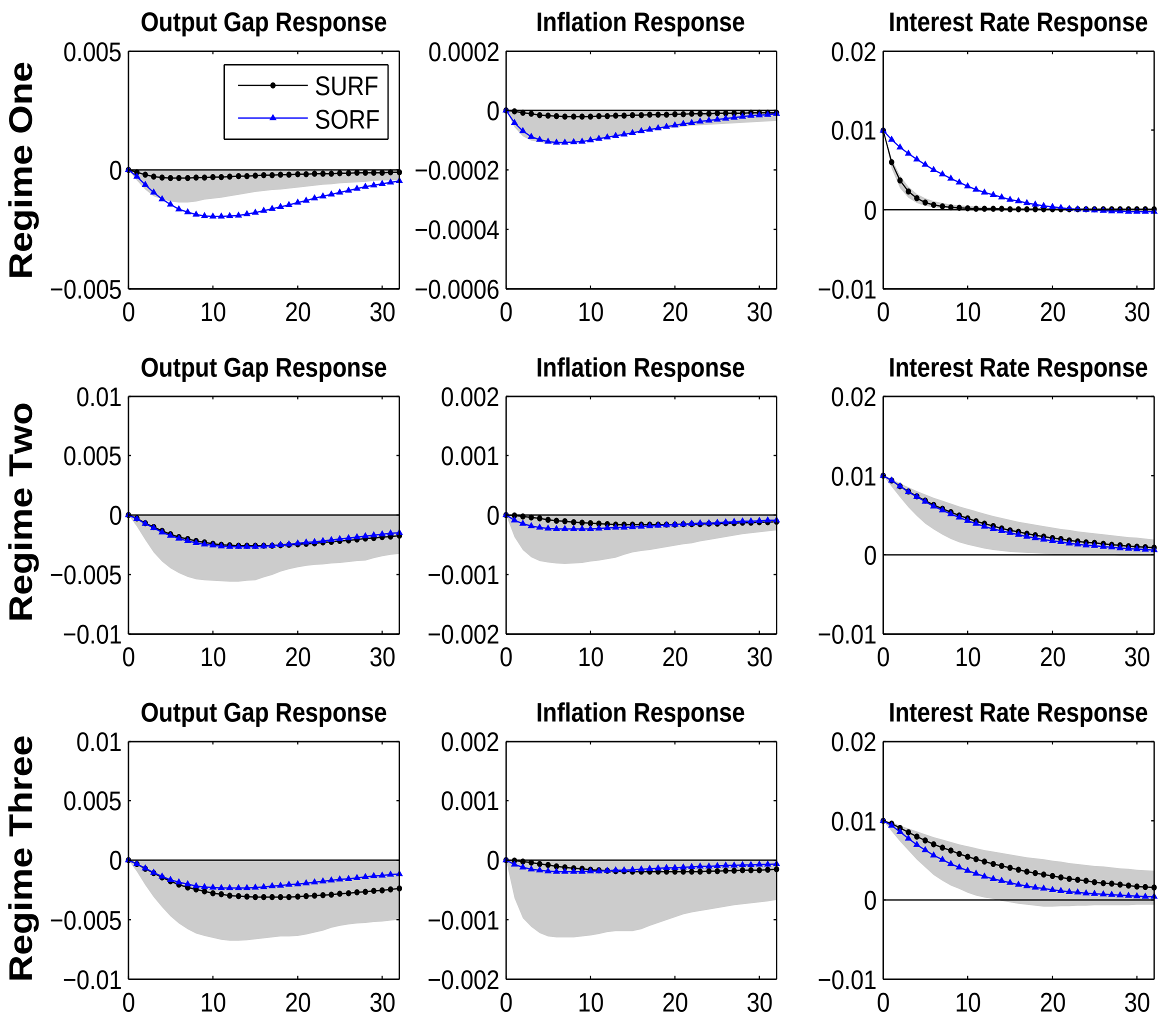
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.11: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Three



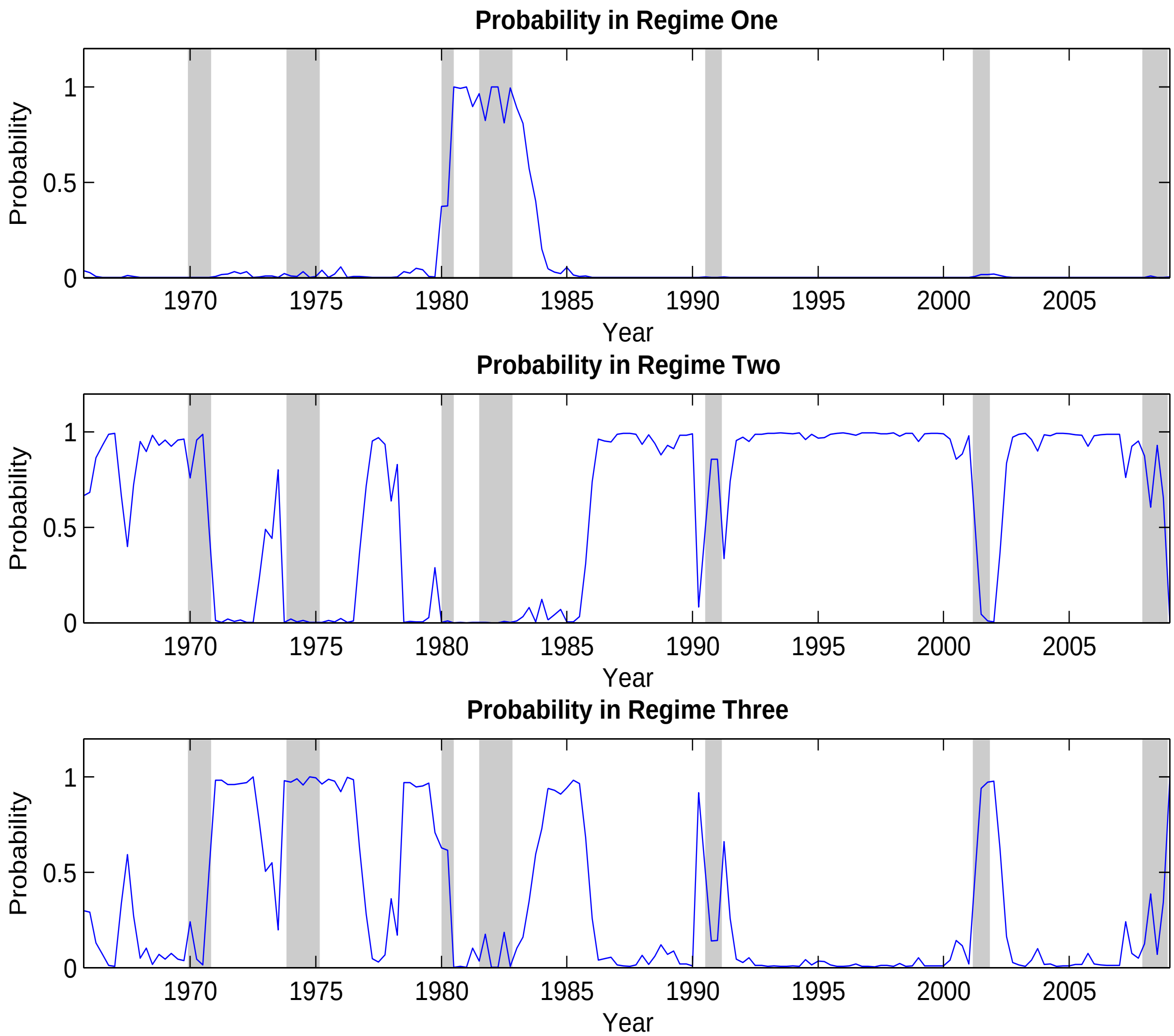
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.12: Comparison of Impulse Response Functions to an Interest Rate Shock for the Model with Three-Regime Monetary Policy



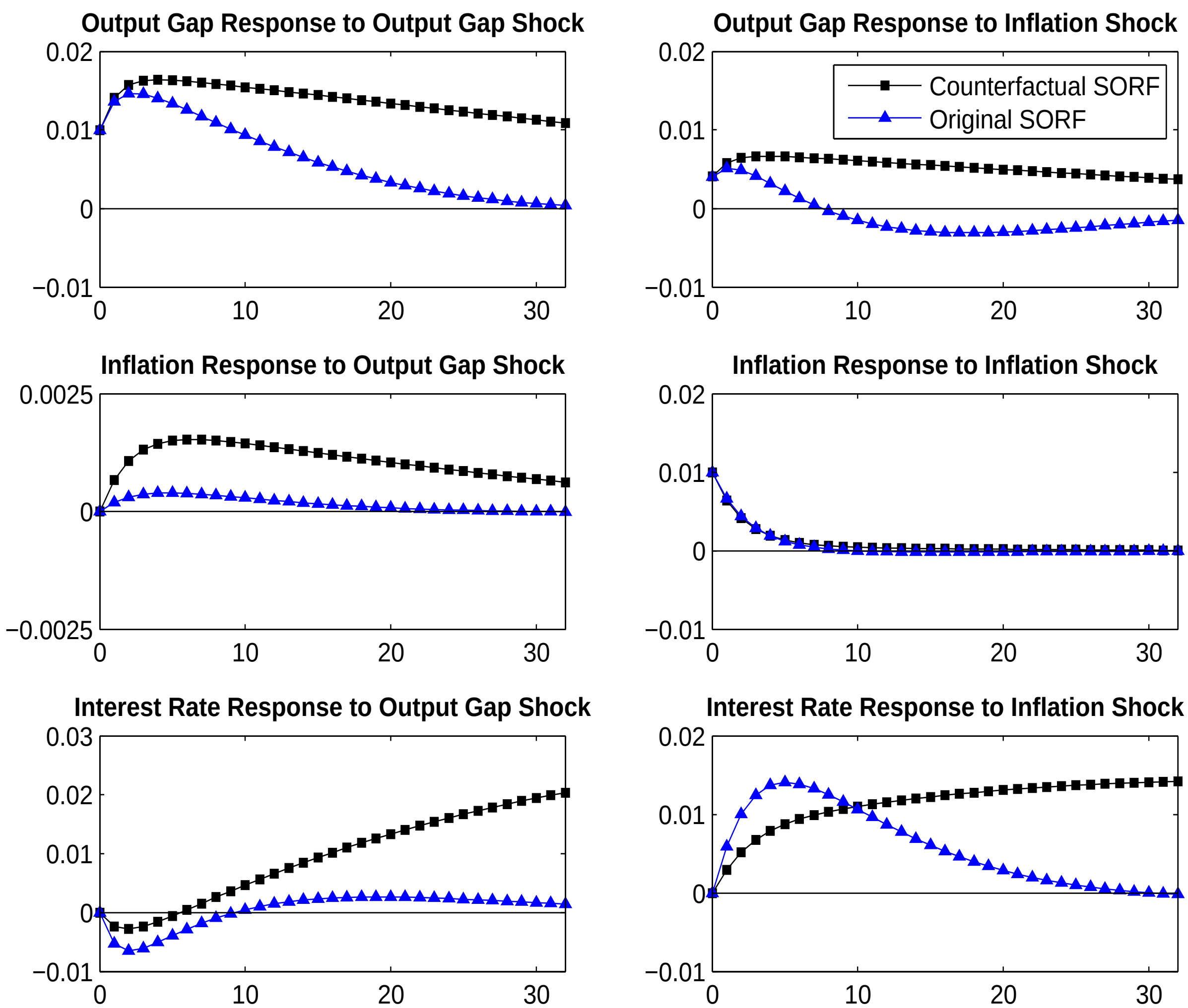
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 3.13: Probabilistic Inference for the Counterfactual Model
with Three-Regime Optimal Monetary Policy



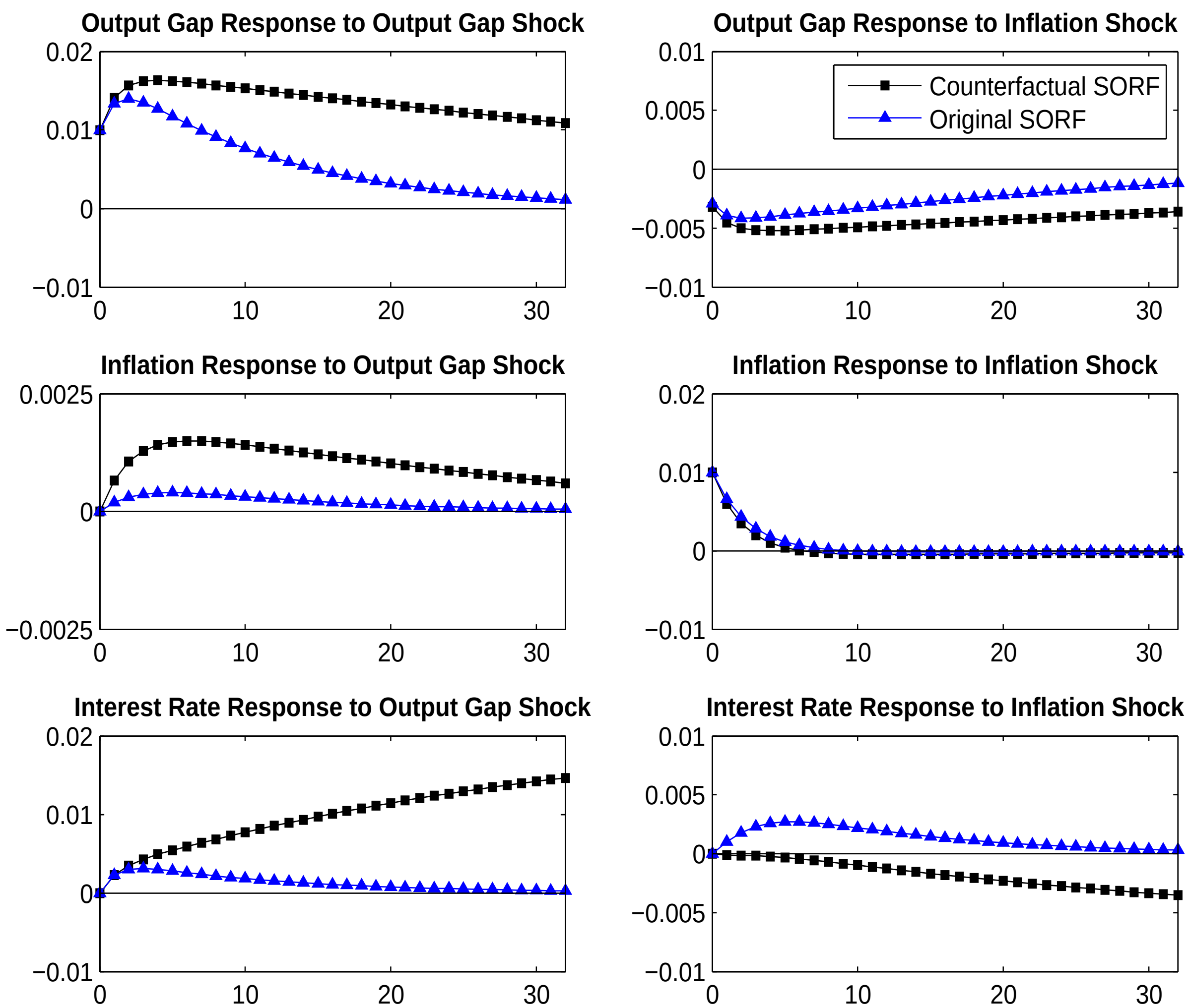
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the counterfactual New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 3.14: Comparison of Impulse Response Functions
for the Counterfactual Model with Three-Regime Optimal Monetary Policy
in
Regime One



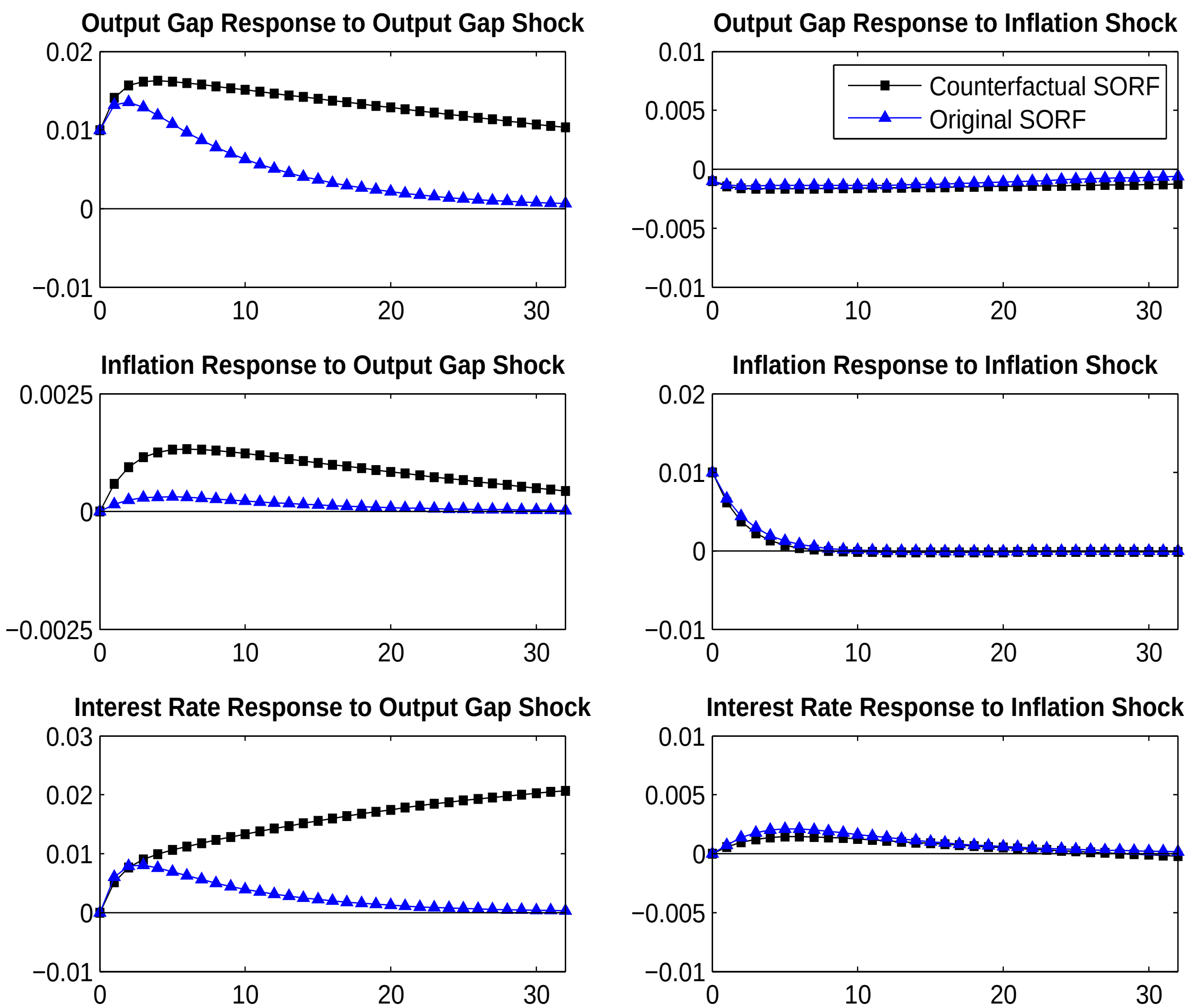
The figure presents impulse response functions implied by SORF estimates of the counterfactual model solution form, which are labeled counterfactual SORF. For convenience of comparison, the figure also shows impulse response functions implied by SORF estimates of the original three-regime model, which are labeled as original SORF. The IRFs are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank.

Figure 3.15: Comparison of Impulse Response Functions
for the Counterfactual Model with Three-Regime Optimal Monetary Policy
in
Regime Two



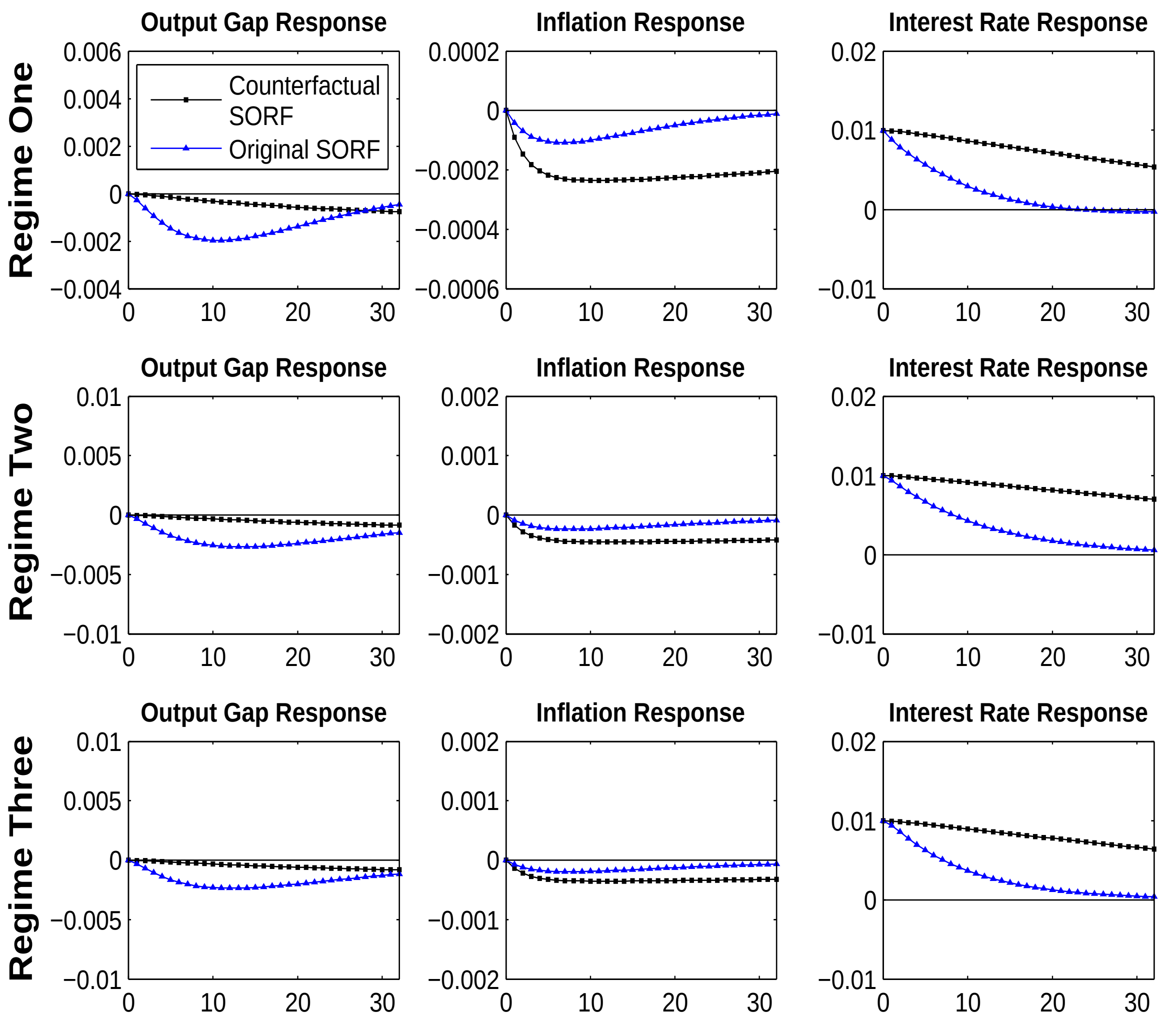
The figure presents impulse response functions implied by SORF estimates of the counterfactual model solution form, which are labeled counterfactual SORF. For convenience of comparison, the figure also shows impulse response functions implied by SORF estimates of the original three-regime model, which are labeled as original SORF. The IRFs are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank.

Figure 3.16: Comparison of Impulse Response Functions
for the Counterfactual Model with Three-Regime Optimal Monetary Policy
in
Regime Three



The figure presents impulse response functions implied by SORF estimates of the counterfactual model solution form, which are labeled counterfactual SORF. For convenience of comparison, the figure also shows impulse response functions implied by SORF estimates of the original three-regime model, which are labeled as original SORF. The IRFs are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank.

Figure 3.17: Comparison of Impulse Response Functions to an Interest Rate Shock for the Counterfactual Model with Three-Regime Optimal Monetary Policy



The figure presents impulse response functions implied by SORF estimates of the counterfactual model solution form, which are labeled counterfactual SORF. For convenience of comparison, the figure also shows impulse response functions implied by SORF estimates of the original three-regime model, which are labeled as original SORF. The IRFs are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank.

Table 3.1: Estimates of Model Parameters for the Model with Two-Regime Monetary Policy

Model Parameters	Estimation with Unrestricted Reaction Function SURF: Estimate	Significance (p-value)	Estimation with Optimal Reaction Function SORF: Estimate	Significance (p-value)
λ	0.337	1.000	0.211	0.306
a_1	0.844	0.879[4]	1.052	0.457[4]
a_2	-0.190	< 0.001	-0.291	< 0.001
b	0.009	1.000	0.023	< 0.001
α_1	0.0006	0.964	0.598[5]	< 0.001
α_2	0.584	< 0.001	0.402[5]	< 0.001
β	0.042	0.356	0.0003	< 0.001
Log Likelihood	1822.702		1819.851	

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first two equations of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for two policy regimes. The policy rule coefficients of the interest rate equation ($c_1 - c_4$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 3.2.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. The p-value reported in this case is for a test of the hypothesis that $a_1 = 1$.

5. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

Table 3.2: Policy Rule Coefficient Estimates for the Model with Two-Regime Monetary Policy

Reaction Function		SURF		SORF	
Coefficients		Coefficient	P-value	Coefficient	P-value
Regime One	c_1	0.284	0.097	0.325	—
	c_2	0.125	0.003	0.296	—
	c_3	0.866	< 0.001	0.872	—
	c_4	-0.326	1.000	-0.334	—
	Loss Function Wt. on y	—	—	0.0022	0.510
	Loss Function Wt. on r	—	—	0.0029	< 0.001
Regime Two	c_1	0.223	0.001	0.323	—
	c_2	0.118	0.033	0.170	—
	c_3	0.939	< 0.001	0.927	—
	c_4	-0.203	0.002	-0.338	—
	Loss Function Wt. on y	—	—	0.0022	0.103
	Loss Function Wt. on r	—	—	0.0045	< 0.001
	Log Likelihood	1822.702		1819.851	

1. The table shows Maximum Likelihood estimates of the policy rule coefficients that are in the third equation of the following model economy.

$$\begin{aligned} y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\ \pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\ r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}. \end{aligned}$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve. The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

Table 3.3: Transition Matrix Estimates for the Model with Two-Regime Monetary Policy

Transition Probabilities	SURF		SORF	
	Estimate	P-value	Estimate	P-value
p_{11}	0.917	0.014^4	0.918	0.002^4
p_{21}	0.044	0.021	0.029	< 0.001
p_{12}	0.083	< 0.001	0.082	0.002
p_{22}	0.956	0.119^4	0.971	0.011^4
Log Likelihood	1822.702		1819.851	

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{2} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2\}$. **SURF** estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for two policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. Parameter-estimate significance for an off-diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. Parameter-estimate significance for a diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.995.

Table 3.4: Estimates of Model Parameters for the Model with Three-Regime Monetary Policy

Model Parameters	Estimation with Unrestricted Reaction Function SURF		Estimation with Optimal Reaction Function SORF	
	Estimate	Significance (p-value)	Estimate	Significance (p-value)
λ	0.379	1.000	0.362	0.001
a_1	0.789	0.766[4]	0.830	0.031[4]
a_2	-0.173	< 0.001	-0.197	< 0.001
b	0.005	0.042	0.006	< 0.001
α_1	0.002	0.238	0.606[5]	< 0.001
α_2	0.581	1.000	0.394[5]	< 0.001
β	0.026	< 0.001	0.0004	< 0.001
Log Likelihood	1838.029		1836.785	

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first two equations of the following model economy.

$$\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for three policy regimes. The policy rule coefficients of the interest rate equation ($c_1 - c_4$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 3.5.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. The p-value reported in this case is for a test of the hypothesis that $a_1 = 1$.

5. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

Table 3.5: Policy Rule Coefficient Estimates for the Model with Three-Regime Monetary Policy

Reaction Function Coefficients		SURF Coefficient	SURF P-value	SORF Coefficient	SORF P-value
Regime One	c_1	-0.566	< 0.001	-0.539	—
	c_2	1.127	0.046	0.800	—
	c_3	0.598	0.069	0.871	—
	c_4	0.391	< 0.001	0.537	—
	Loss Function Wt. on y	—	—	0.00003	0.244
	Loss Function Wt. on r	—	—	0.0014	< 0.001
Regime Two	c_1	0.272	< 0.001	0.229	—
	c_2	0.167	0.004	0.164	—
	c_3	0.935	0.004	0.934	—
	c_4	-0.272	< 0.001	-0.224	—
	Loss Function Wt. on y	—	—	0.0025	0.002
	Loss Function Wt. on r	—	—	0.0046	< 0.001
Regime Three	c_1	0.409	0.013	0.588	—
	c_2	-0.028	1.000	0.133	—
	c_3	0.959	0.003	0.937	—
	c_4	-0.389	< 0.001	-0.563	—
	Loss Function Wt. on y	—	—	0.0079	0.001
	Loss Function Wt. on r	—	—	0.0042	< 0.001
Log Likelihood		1838.029		1836.785	

1. The table shows Maximum Likelihood estimates of the policy rule coefficients that are in the third equation of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve. The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

Table 3.6: Variance-Covariance Matrices of Shocks for the Model with Three-Regime Unrestricted Monetary Policy

(a) Regime One

	y	π	r
y	1.961×10^{-4}	0.595×10^{-4}	1.067×10^{-4}
π	0.595×10^{-4}	1.284×10^{-4}	-0.052×10^{-4}
r	1.067×10^{-4}	-0.052×10^{-4}	3.658×10^{-4}

(b) Regime Two

	y	π	r
y	0.299×10^{-4}	-0.100×10^{-4}	0.061×10^{-4}
π	-0.100×10^{-4}	0.499×10^{-4}	0.071×10^{-4}
r	0.061×10^{-4}	0.071×10^{-4}	0.185×10^{-4}

(c) Regime Three

	y	π	r
y	1.054×10^{-4}	-0.209×10^{-4}	0.099×10^{-4}
π	-0.209×10^{-4}	2.373×10^{-4}	0.278×10^{-4}
r	0.099×10^{-4}	0.278×10^{-4}	0.661×10^{-4}

1. The table shows Maximum Likelihood estimates of the variance-covariance matrices across three policy regimes when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate.

2. The values are in the upper-left 3×3 block of the covariance matrix of shocks, and the rest elements of the covariance matrix of shocks are zeros.

Table 3.7: Variance-Covariance Matrices of Shocks for the Model with Three-Regime Optimal Monetary Policy

(a) Regime One

	y	π	r
y	1.768×10^{-4}	0.534×10^{-4}	1.252×10^{-4}
π	0.534×10^{-4}	1.173×10^{-4}	0.003×10^{-4}
r	1.252×10^{-4}	0.003×10^{-4}	4.856×10^{-4}

(b) Regime Two

	y	π	r
y	0.298×10^{-4}	-0.106×10^{-4}	0.056×10^{-4}
π	-0.106×10^{-4}	0.472×10^{-4}	0.068×10^{-4}
r	0.056×10^{-4}	0.068×10^{-4}	0.154×10^{-4}

(c) Regime Three

	y	π	r
y	0.994×10^{-4}	-0.181×10^{-4}	0.205×10^{-4}
π	-0.181×10^{-4}	2.567×10^{-4}	0.468×10^{-4}
r	0.205×10^{-4}	0.468×10^{-4}	0.762×10^{-4}

1. The table shows Maximum Likelihood estimates of the variance-covariance matrices across three policy regimes when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss.

2. The values are in the upper-left 3×3 block of the covariance matrix of shocks, and the rest elements of the covariance matrix of shocks are zeros.

Table 3.8: Transition Matrix Estimates for the Model with Three-Regime Monetary Policy

Transition Probabilities	SURF		SORF	
	Estimate	P-value	Estimate	P-value
p_{11}	0.934[4]	0.088[5]	0.941[4]	0.021[5]
p_{21}	0.010	1.000	0.000	1.000
p_{31}	0.058	0.261	0.019	0.011
p_{12}	0.033	0.010	0.000	1.000
p_{22}	0.979[4]	1.000[5]	0.962[4]	0.016[5]
p_{32}	0.007	< 0.001	0.058	< 0.001
p_{13}	0.033	0.004	0.059	< 0.001
p_{23}	0.011	0.043	0.038	0.008
p_{33}	0.935[4]	0.042[5]	0.923[4]	< 0.001[5]
Log Likelihood	1838.029		1836.785	

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{3} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2, 3\}$. **SURF** estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for three policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. Parameter-estimate significance for an off-diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. To guarantee that the diagonal transition probabilities are highly persistence, in the estimation they were restricted to be no less than 0.900.

5. Parameter-estimate significance for a diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.995.

Table 3.9: Estimates of Model Parameters for the Counterfactual Model with Three-Regime Optimal Monetary Policy

Model Parameters	Original Model Estimated with Optimal Reaction Function SORF Estimate	Counterfactual Model Estimated with Optimal Reaction Function SORF Estimate
λ	0.362	0.197
a_1	0.830	1.099
a_2	-0.197	-0.300
b	0.006	0.001
α_1	0.606^3	0.616^3
α_2	0.394^3	0.384^3
β	0.0004	0.0006
Log Likelihood	1836.785	1828.516

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first two equations of the following model economy.

$$\begin{aligned} y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\ \pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\ r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}. \end{aligned}$$

   SORF estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for three policy regimes. The optimal policy rule coefficients of the interest rate equation ($c_1 - c_4$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 3.10.

3. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

4. For convenience of comparison with estimates, the table also reports SORF estimates for parameters of the original three-regime model as shown in Table 3.4.

Table 3.10: Policy Rule Coefficient Estimates for the Counterfactual Model with Three-Regime Optimal Monetary Policy

Reaction Function Coefficients		Original Model Estimated with Optimal Reaction Function SORF Coefficient	Counterfactual Model Estimated with Optimal Reaction Function SORF Coefficient
State One	c_1	-0.539	-0.212
	c_2	0.800	0.405
	c_3	0.871	0.993
	c_4	0.537	0.239
	Loss Function Wt. on y	0.00003	0.0^3
	Loss Function Wt. on r	0.0014	0.0009
State Two	c_1	0.229	0.228
	c_2	0.164	0.061
	c_3	0.934	0.994
	c_4	-0.224	-0.200
	Loss Function Wt. on y	0.0025	0.0^3
	Loss Function Wt. on r	0.0046	0.0015
State Three	c_1	0.588	0.514
	c_2	0.133	0.108
	c_3	0.937	0.994
	c_4	-0.563	-0.479
	Loss Function Wt. on y	0.0079	0.0^3
	Loss Function Wt. on r	0.0042	0.0009
	Log Likelihood	1836.785	1828.516

1. The table shows Maximum Likelihood estimates of central bank preference parameters and implied policy rule coefficients. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \varepsilon_{\pi,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Estimation reported here is obtained under the assumption that the policymakers in the central bank do not wish to stabilize output ($w_y = 0.0$).

4. For convenience of comparison with estimates, the table also reports SORF estimates for preference parameters and implied policy rule coefficients of the original three-regime model as shown in Table 3.5.

Table 3.11: Transition Matrix Estimates for the Counterfactual Model with Three-Regime Optimal Monetary Policy

Transition Probabilities	Original Model Estimated with Optimal Reaction Function SORF	Counterfactual Model Estimated with Optimal Reaction Function SORF
	Estimate	Estimate
p_{11}	0.941^3	0.939^3
p_{21}	0.000	0.000
p_{31}	0.019	0.019
p_{12}	0.000	0.002
p_{22}	0.962^3	0.957^3
p_{32}	0.058	0.058
p_{13}	0.059	0.059
p_{23}	0.038	0.043
p_{33}	0.923^3	0.924^3
Log Likelihood	1836.785	1828.516

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{3} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2, 3\}$. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for three policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. To guarantee that the diagonal transition probabilities are highly persistence, in the estimation they were restricted to be no less than 0.900.

4. For convenience of comparison with estimates, the table also reports SORF estimates for transition matrix of the original three-regime model as shown in Table 3.8.

Chapter 4

Revealing U.S. Monetary Policy Regimes Between 1965 and 2008 Using a Model with Oil Price

In order to check the robustness of the findings presented earlier to variation in the structural model, this study considers a more complex model economy with oil price and investigates how the Federal Reserve weighted the trade-offs among stabilizing inflation, output, and the interest rate when confronting oil price fluctuations. The new model can be derived from a micro-founded dynamic stochastic general equilibrium model with oil as an input in the production process.

Oil is an important production factor that is used in every industry to some extent, and it can not be easily substituted by other factors in production process. Furthermore, the market price of oil has fluctuated considerably throughout the post-war period. The price of oil was raised significantly during the energy crises of 1973 and 1979, then it fell dramatically starting in 1981. In the 2000's, and until the middle of 2008, the price of oil increased to a record high, but it collapsed by the end of the same year. In 2009, oil prices began to increase from a low point and returned to the level of the late 1970's. High volatility of oil prices can have serious effects on the economy: an increase in oil prices causes an increase in inflation and subsequently leads to a decrease in output. Hamilton (1983) suggests that oil price increases are responsible for the increased volatility of the U.S. economy during the 1970's, especially for the heightened inflation and the decline in output growth. The heavy dependence on oil and sustained changes in oil prices pose a difficult challenge for monetary policymakers on how to simultaneously achieve low inflation and stable output growth. Montoro (2010) shows that on one hand, if central bank focuses exclusively on the recessive effects of oil prices and tries to stabilize output, this would generate inflation. On the other hand, if central bank focuses exclusively on neutralizing the impact of the oil prices on inflation through a contractive monetary policy, the sluggish price responses to changes in output would imply a large drop in output. Work by Bernanke, Gertler and Watson (2004) suggests that monetary policymakers have historically leaned towards keeping inflation at bay, at the cost of a greater slowdown in economic activity following an oil price shock. Leduc and Sill (2004) show that theoretically in the absence of distortions other than price stickiness, price stability is the best overall policy prescription in response to oil price shocks, even if that leads to a large drop in output[35]. Acknowledging the existing literature discussed above, there appears to be little or no formal research that estimates preference parameters of central bank on different policy objectives in the dynamic responses of the economy to observable oil price changes.

This study presents an attempt to estimate explicitly central bank's preference parameters in an oil-producing economy that depends, to an important degree, on oil prices for its economic performance and stability. In doing so, this study first derives analytically a micro-founded New Keynesian dynamic stochastic general equilibrium model with oil as a production input for intermediate goods. The New Keynesian framework has become the standard workhorse of monetary analysis over the past decade and allows an explicit micro-founded modeling of the transmission channels. Since oil is difficult to substitute in

[35]The theoretical result stands out that optimal monetary policy should aim at replicating the real allocation under flexible prices, which features constant markups and no inflation. Blanchard and Galí (2007) suggest there is a "divine coincidence", an absence of trade-off between stabilizing inflation and stabilizing the "welfare relevant" output gap.

production, this study uses a constant-elasticity-of-substitution (CES) production function to capture the low substitutability of oil. A low elasticity of substitution between labor and oil also indicates a high dependence on oil. The analytical results then boil down to a model economy where monetary policy affects aggregate demand through a conventional interest rate channel and inflation through a Phillips curve specification. The model determines the equilibrium relationship among the output gap, the inflation rate, the real oil price, and the short-maturity interest rate controlled by the central bank. This study uses the model to estimate the preference parameters of the Federal Reserve that are assumed to vary across different policy regimes. While sharing many characteristics with the previously-studied baseline model, the new model with oil price has a more complex relationship between structural parameters. Furthermore, the structural parameters of the model are still subject to the condition that the policy equation minimizes a well-defined loss function. Therefore, the macro-economy and the optimization problem of the Federal Reserve are estimated simultaneously.

Since the Federal Reserve's optimal monetary policy is subject to constraints controlled by behavior of private agents in the economy and the optimal monetary policy has a direct impact on the economy through the setting of policy instrument, it is useful to divide the descriptions of the model and optimal monetary policy into two separate parts. The macro-economy is first described without requiring the Federal Reserve to minimize its loss function. Then the optimization problem of the Federal Reserve is derived, taking the model economy as given.

4.1 The Model Economy

In the model, a central bank's main objectives are to stabilize the time paths of output and inflation by changing the interest rate. This study denotes y as the output gap, which is the difference between output and its long term growth path. This study also denotes π and r as differences of inflation and the interest rate from their target values, respectively. The stabilization of output means keeping keep y close to zero. Stabilizations of inflation and the interest rate mean keeping them close to their target paths.

4.1.1 Summary of Model Equations

The model with oil price consists of an IS curve, a Phillips curve, an autoregressive process of oil price, and a policy rule for the short term interest rate. The model can be derived from an underlying dynamic stochastic general equilibrium model based on optimizing agents, and the model equations are the concentration of economic content in the DSGE model that is discussed shortly. In that sense, the model is structural. While the model emphasizes the role of forward-looking behavior and rational expectations, it also incorporates a substantial degree of persistence in the form of multiple lags of output and inflation. In line with Salemi (2006), the investigation taken up in this study does not concern the deeper structural parameters in the model.

The model economy consists of the following linearized equations

$$y_t = \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \tag{4.1}$$

$$\pi_t = \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \eta q_t + \varepsilon_{\pi,t}, \tag{4.2}$$

$$q_t = \rho_1 q_{t-1} + \varepsilon_{q,t}, \tag{4.3}$$

$$r_t = c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 q_{t-1} + c_5 y_{t-2} + \varepsilon_{r,t}. \tag{4.4}$$

The complete model characterizes the equilibrium dynamics of four variables: y_t, π_t, q_t, and r_t. q_t is defined as the real oil price. y_t, π_t, and r_t carry the same definitions presented in previous paragraph. Each variable is expressed as deviation from its own trend. The stochastic variables $\varepsilon_{\pi,t}$, $\varepsilon_{y,t}$, $\varepsilon_{q,t}$, and $\varepsilon_{r,t}$ are serially uncorrelated shocks that account for exogenous variations in aggregate supply, aggregate demand, oil price, and monetary policy, respectively.

Equation (4.1) is loosely consistent with a linearized Euler condition characterizing the optimal consumption in a dynamic general equilibrium setting. The equation links output during period t to its own expected future and lagged values, and to the value of ex ante real interest rate. b measures the inverse relationship between current output and the real interest rate which reflects intertemporal substitution on the optimization of households. Clarida, Galí and Gertler (1999) show that the presence of expected future output in the IS equation can be explained by the desire of households to smooth consumption. The presence of lagged output in the IS equation can be explained by habit persistence. Habit formation can cause delays between decision making and consumption. Hence aggregate demand and output adjust only partially such that output depends on a combination of its own lagged values, see Svensson (2000).

Equation (4.2) is a Phillips curve governing the dynamic behavior of inflation. The specification that oil is used as an input for producing intermediate goods makes current inflation depend on the real oil price[36]. The Phillips curve also has the appearance of inflation being partly "backward looking" in spite of the fact that firms are rational and forward looking. Therefore, equation (4.2) takes the form of a hybrid forward- and backward-looking New Keynesian Phillips curve. If the coefficients on lagged inflation and real oil price become zero and drop out of the specification, the equation reduces to the purely forward-looking New Keynesian Phillips curve analyzed by Galí and Gertler (1999) that links inflation to expected future inflation and output gap.

Equation (4.3) describes the process for the real oil price. In this study, oil price movement is derived to be in the form of a first-order autoregressive driving process, which is in line with many studies, e.g., Kim and Loungani (1992), Leduc and Sill (2004), Carlstrom and Fuerst (2005), and De Walque, Smets and Wouters (2005). Therefore,

[36]Details of deriving the Phillips curve as equation (4.2) can be found in Appendix B.

macroeconomic variables are affected by the oil supply disruption through higher oil prices. Hamilton (1983) suggests that oil price changes prior to 1973 were likely to be exogenous due to oil commissions that insulated the oil prices from movements in oil demand. Montoro (2010) also suggests that during the 1970's and through the 1990's the major source of oil price hikes seemed to be on the international supply side, either because of attempts to gain more oil revenue or supply interruptions due to geopolitical events, such as Iranian Revolution and the first Gulf war. However, many studies argue that in the 2000's the high price of oil is more related to international demand side[37]. It would be ideal to use an open economy model with endogenous oil price to estimate preference parameters of the Federal Reserve. This is a case worth exploring, which this study leaves for further research.

Equation (4.4) is the policy rule or reaction function that explains how the central bank sets the short term interest rate. The linearized equation takes a form of the Taylor (1993) type reaction function and corresponds to the common practice of using the short term interest rate as monetary policy instrument by most central banks. The reaction function also features intrinsic persistence in the policy rate, because such persistence allows monetary policy to achieve a given degree of stabilization with less volatile short-term interest rate. The persistence feature of policy rule is actually followed by many central bank, see Sack and Weiland (2000). Dennis (2006) argues that the error term $\varepsilon_{r,t}$ can capture some variables that are omitted by econometricians since they tend to have less information than the policymakers. Salemi (2006) interprets the disturbance as a term that includes idiosyncratic wisdom of the monetary policy authority. In line with Clarida, Galí and Gertler (1999) and Salemi (2006), the coefficients of the reaction

[37] Within a global economic setting, a list of articles find evidence in support of endogenous responses of the real price of oil to the global macroeconomic conditions, see e.g., Barsky and Kilian (2004), Hamilton (2005), Woodford (2007), Kilian (2008), and Kilian (2009). These studies argue that there are additional transmission channels that cause oil prices to affect the macroeconomic variables and all major real oil price increases since the mid-1970's can be traced to increased global aggregate demand.

function are assumed to be fixed in a policy regime so that policy is time consistent.

4.1.2 Theoretical Framework

The model equations described in the previous section can be derived from a micro-founded dynamic stochastic general equilibrium model. The theoretical model specification is similar to the New Keynesian models of Castillo, Montoro and Tuesta (2007) and Montoro (2010), which allow price stickiness and oil as a low-substitutable input in the production process of intermediate goods. Furthermore, oil supply is assumed to be exogenous, and it follows a first-order autoregressive process. In the context of this study, the model offers an explicit specification of the channels through which commodity prices affect the behavior of households and firms. In particular, an increase in the price of oil raises the price of factor input and production costs, hence reduces the purchasing power of household income and consumption. Therefore, an increase in the price of oil has a negative impact on both supply and demand. If central bank focuses exclusively on the recessive effects of oil prices and tries to stabilize output, this would generate inflation. On the other hand, if central bank focuses exclusively on neutralizing the impact of the oil prices on inflation through a contractive monetary policy, the sluggish price responses to changes in output would imply a large decrease in output.

The underlying New Keynesian DSGE model consists of a continuum of households, final goods producers, oil producers, and intermediate goods producers. Oil is used as an input in the production of intermediate goods, and oil is a homogenous commodity supplied to intermediate goods producers by oil producers. The oil producers produce oil only, and they take the oil price as given when choosing their production level. The intermediate goods are differentiated and imperfect substitutes for each other in production, so the intermediate goods firms are monopolistically competitive. Prices for intermediate goods are adjusted infrequently, which are set according a variant of the mechanism in Calvo (1983). It is assumed that at each period a fraction of firms are not able to re-optimize prices and they use a rule of thumb to set their prices. Final goods are produced with intermediate goods, and they are used for consumption and oil production. Households consume goods and supply labor. Each household has a specific labor skill that is only hired by the corresponding intermediate good producer. Labor market is assumed to be perfect competitive, so no household has market power over the wage. Firms and households maximize intertemporal profit and utility respectively over an infinite planning horizon.

The model illustrates the direct and indirect effects of oil price and of sticky pricing mechanism on production costs, and consequently on consumption, output, and inflation. In what follows, different sectors of the model are described briefly. Appendix B describes the full model and its linearization around the steady state in detail, and sums up a smaller and simpler set of equations as presented previously.

The Households

The economy has a continuum of households on the unit interval. A representative household $i \in [0,1]$ maximizes the expected inter-temporal discounted utility function defined over consumption C_t and labor L_t

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{C_{t-t_0}^{1-\sigma}}{1-\sigma} - \frac{L_{t-t_0}^{1+\nu}}{1+\nu} \right], \tag{4.5}$$

where $\beta \in (0,1)$ is the household's subjective discount factor, σ and ν are the household's preference parameters. σ is the intertemporal elasticity of substitution, $\sigma \in (0,\infty)$. ν is the elasticity of labor supply, $\nu \in (0,\infty)$. One can see that the household's utility is additively separable in consumption and labor supplied. The additive separability is needed to derive a conventional specification for the model's IS relationship that excludes term involving employment. Each household is endowed with one unit of potential work-time every period which it supplies inelastically to the labor market.

The budget constraint for a typical household is given by

$$P_t C_t + \frac{B_t}{R_t} = W_t L_t + B_{t-1} + \Gamma_t + T_t, \tag{4.6}$$

The household receives labor income $W_t L_t$, where W_t is the nominal wage. It also receives Γ_t the share of the representative household on total nominal profits created by firms, and T_t transfers from the government, respectively. The household uses these funds to finance consumption C_t, which costs P_t per unit. There is a private financial bond in the economy. In every period t, a household purchases bond B_t, where B_t is the end of period nominal bond holdings. R_t is the nominal gross interest rate.

Taking price and nominal wage as given, a representative household maximizes lifetime utility (4.5) with respect to C_t, L_t, and B_t, subject to budget constraint (4.6). The set of first order conditions from the household optimization problem is reported in Appendix B.

Final Goods Producers

There is a continuum of final goods producers of mass one, indexed by $f \in [0,1]$ that operate in an environment of perfect competition. Each final good producer uses intermediate goods as inputs, indexed by $z \in [0,1]$ to produce final consumption good with the following technology

$$Y_t(f) = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4.7}$$

where ε is the elasticity of substitution between intermediate goods.

Y_t represents the aggregate level of output

$$Y_t = \int_0^1 Y_t(f) df. \tag{4.8}$$

This final good producer's profit maximization leads to the demand for each intermediate good

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t, \tag{4.9}$$

where $P_t(z)$ is the price of intermediate good z, P_t is the price of the final good.

Since the final goods are sold in a perfectly competitive market. Competition drives the final good producer's profits to zero in the equilibrium. The aggregate price level is equal to the marginal cost of the final good producer and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}. \tag{4.10}$$

Oil Producers

A representative household owns a continuum of oil firms, indexed by $j \in [0,1]$ that operate in a perfect competitive market[38]. Each firm produces a quantity $X_t(j)$ of oil according to the technology

$$X_t(j) = Z_t I_t(j), \tag{4.11}$$

where Z_t is a component of aggregate productivity common to all oil firms, and the input $I_t(j)$ is purchased from the final goods producers as is the consumption of the representative household.

The aggregate productivity of the oil sector, Z_t, is assumed to vary according to a

[38] Alternatively, many studies assume that oil is extracted with no cost by the government, which supplies all demanded quantities of oil to the firms at the given price. The government then transfers the proceeds in a lump sum fashion to the households, see e.g. Castillo, Montoro and Tuesta (2007), Natal (2009), and Montoro (2010).

stationary AR(1) stochastic process in logs

$$\ln(Z_t) = ln(\overline{Z}) + \rho \ln(Z_{t-1}) + \varepsilon_{z,t}, \tag{4.12}$$

where coefficient ρ measures the persistence in the process, and $\varepsilon_{z,t}$ is the shock process. As a result, $1/Z_t$ measures the real marginal cost of all oil firms.

The produced oil is sold at the real price Q_t, which oil producers take as given. Hence, $Q_t = 1/Z_t$, and the real oil price, Q_t, also follows an AR(1) process in logs

$$\ln(Q_t) = ln(\overline{Q}) + \rho \ln(Q_{t-1}) + \varepsilon_{q,t}, \tag{4.13}$$

where $\overline{Q}$ is the steady state level of oil price and $\varepsilon_{q,t}$ is Gaussian white noise.

Intermediate Goods Producers

There is a continuum of intermediate goods producers indexed by $z \in [0,1]$ that operate in a monopolistic competitive market. Each firm specializes in production of a single differentiated good using CES technology with two inputs of production, labor L_t and oil M_t

$$Y_t(z) = \left[(1-\alpha)(L_t(z))^{\frac{\psi-1}{\psi}} + \alpha(M_t(z))^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{4.14}$$

where ψ represents the intratemporal elasticity of substitution between labor and oil inputs, α denotes the share of oil in the production function. Generally, ψ is assumed to be less than one. Following Castillo, Montoro and Tuesta (2007) and Montoro (2010), this study uses this production function in order to capture the fact that oil has few substitutes[39].

Since the intermediate goods substitute imperfectly for one another in producing final good, a representative firm in the intermediate goods sector can sell its output in a monopolistically competitive market and can set a price for its product by taking the demand schedule as given. Given a level of production, an intermediate good producer chooses labor and oil demands to minimize its total cost. When labor and oil markets clear, the firm's optimal choices for labor and oil lead to its marginal cost.

The intermediate goods producing firms' pricing decisions are subject to price friction. This study uses a sticky price mechanism that is extended from the Calvo (1983) model by Galí and Gertler (1999). Galí and Gertler assume that in each period t a fraction $\theta \in (0, 1)$ of firms are not able to re-optimize their prices $P_t(z)$ while the rest are. Among the firms that can reset their prices, a fraction $\omega \in (0, 1)$ of the price setters use a "rule of thumb", which is based on past price developments; a fraction $(1 - \omega)$ of the price setters optimize their prices in an optimal forward-looking manner and take into account that their prices may be effective for more than one period. Those firms that follow the "rule of thumb" set their prices equal to last period's average price set by their competitors plus the lagged inflation rate. The fractions θ and ω are the same for all firms, and they are constant over time.

The Linearized Model

This section presents the log-linear approximation of the model framework. The log-linearized model gives a more compact representation of the economic structure. The

[39]Since oil has few substitutes, an appealing functional form to capture this feature is the CES production function. Therefore, there is a complementary relationship between both inputs. As in Leduc and Sill (2004), the increase in oil price would exclude substitution and uniformly reduce both oil and labor demand. The CES production function also offers a flexible form for the degree of substitution between oil and labor. Notice that when $\psi = 1$, the production function collapses to the standard Cobb-Douglas function as the one used by Blanchard and Galí (2007): $Y_t(z) = (L_t(z))^{1-\alpha} M_t^{\alpha}$, which considers labor and oil to be substitutes. Hence an increase in oil price triggers a substitution of labor for oil under flexible relative factor prices.

optimal consumption path for households is determined by the Euler equation, which is obtained by combining the first order conditions for consumption, labor, and bond holdings together with market clearing condition for output. The aggregate demand component of the model is described by the IS equation. This study assumes a symmetric equilibrium, so consumption will be equal across all households. In log-linearized form, the IS curve is the following reduced form equation[40,41]

$$x_t \;=\; \lambda E_t x_{t+1} + a_1 x_{t-1} + a_2 x_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{x,t}. \tag{4.15}$$

The IS equation is a standard Euler equation relating the expected change in output gap to the interest rate, lags of output gap, as well as to the expected inflation depreciation. x_t is the efficient output gap, which is the difference between the sticky price level of output and its corresponding efficient level, $x_t = y_t - y_t^E$, where y_t^E denotes the log deviation of the efficient level of output defined in Appendix B. The IS equation also contains a stochastic disturbance $\varepsilon_{x,t}$ which relates to output shocks. In the equation, the endogenous variables are output gap x_t, inflation rate π_t, interest rate r_t.

Taking wages and final goods prices as given, intermediate goods producing firms set their prices to solve their profit maximization problems subject to the staggered pricing mechanism as in Galí and Gertler (1999). The price optimization problem determines the optimal price for firm z. In a symmetric equilibrium, all intermediate goods producing firms in period t make identical decisions. It can be shown that a representative intermediate good producer's optimal price setting decision and the final good price index lead

[40] In what follows, lower case letter denotes log deviation from steady state value of the corresponding upper case letter.

[41] Reduced form coefficients in the equation are defined in Appendix B.

to the log-linearized Phillips curve equation

$$\pi_t = \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta x_t + \eta q_t + \varepsilon_{\pi,t}. \tag{4.16}$$

The α_1 parameter determines the degree of firms' forward-looking behavior in their price settings. The specification of the Phillips curve allows for potential source of inflation persistence observed in the data. The equation states that inflation in the model depends on its own lagged value as well as inflation persistence coming from the expected future inflation, which appear to be capable of generating significant inflation inertia. This specification of the Phillips curve equation implies that inflation is higher when output is above its efficient level because firms start exploiting their monopolistic market power by demanding higher prices. The responsiveness of inflation to excess demand is measured by β. Inflation dynamics are also affected by the real oil price, and the magnitude of variation in the inflation in response to fluctuation in the real oil price is measured by η.

Monetary Authority

To complete the model, this study needs to specify a monetary policy rule for the central bank. It should be noticed that the model framework used here is different from the baseline setup of the model economy in previous chapters. Here oil is used as a raw-material input to intermediate goods production. Therefore, the economy is subject to oil shocks that can significantly affect business cycle fluctuations. This new feature can lead the central bank to face a more complicated problem than its counterpart in the baseline model. As a result, the complications cause the central bank to explicitly take into account real oil price when designing and implementing its monetary policy rule. Monetary policy also has an impact on the supply side of the economy, since variations in the real oil price affect the real cost of production.

The central bank's monetary policy rule is described by a reaction function after log-linearization

$$r_t \;=\; c_1 x_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 q_{t-1} + c_5 x_{t-2} + \varepsilon_{r,t}. \tag{4.17}$$

Note that the reaction function is amended with a term of output gap with two-period lag. The central bank adjusts its instrument interest rate in response to deviation of output, inflation, and real oil price from their target values. This study hypothesizes that, from the policymaker's point of view, variations of real oil price convey the complementary information on inflation. The specific format of the rule is chosen to allow the central bank to react effectively to shocks that originate within the economy.

One can also note that, to be fully operational, monetary policy is assumed to utilize only information that can be realistically considered as available to the monetary authority at the time of decision-making. McCallum (1997) argues that central bank does not know current output level and inflation when it sets the interest rate. This does capture some broad features of data availability on real GDP and CPI, which are released with a delay. Therefore, the policy rule allows the central bank to react only to lagged values of those variables. A lagged interest rate term is also included in the central bank's reaction function to match the persistence in interest rate. Such format of a policy rule allows the central bank to change the interest rate persistently therefore to spread "policy medicine" over time. This would allow for a moderate response of interest rate to shocks hitting the economy. $\varepsilon_{r,t}$, an exogenous policy shock, is added to the reaction function. The disturbance term can be interpreted as the unsystematic component of monetary policy.

4.1.3 Model Solution

The model with equations (4.1) – (4.4) is a system of linear expectation vector difference equations. It is straightforward to rewrite the system equations in the following form

$$A\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ X_t \end{bmatrix} = B\begin{bmatrix} y_t \\ \pi_t \\ X_{t-1} \end{bmatrix} + DS_t, \tag{4.18}$$

where $X_t = [y_t \;\; \pi_t \;\; r_t \;\; q_t \;\; y_{t-1} \;\; q_{t-1}]'$ is a vector of state variables, $S_t = [\varepsilon_{y,t} \;\; \varepsilon_{\pi,t} \;\; \varepsilon_{q,t} \;\; \varepsilon_{r,t}]'$ is a vector of structural shocks, and where matrices A, B, and D are 8×8, 8×8, and 8×4, respectively.

To find a reduced form solution for the model, this study applies the algorithm suggested by Klein (2000) where the solution for the model's endogenous variables is expressed in terms of the model's exogenous variables. If the number of unstable eigenvalues equals the number of non-predetermined variables, then there exists a unique saddle path solution for the model that may be written as a first-order vector autoregression

$$X_t = GX_{t-1} + \varphi_t, \tag{4.19}$$

where G is the reduced form model solution. φ_t is the 6×1 vector of reduced-form errors with typical elements φ_{kt}, $k \in \{1, \ldots, 6\}$, and $\varphi_{5t} = 0$, $\varphi_{6t} = 0$. For the model used in this study, a unique saddle path solution exists if there are six stable and two unstable eigenvalues. This study allows for the possibility that the errors may be correlated. Let Ω be the 6×6 covariance matrix of φ_t. As a result, the upper-left 4×4 block of Ω is the covariance matrix of vector φ_{kt}, $k \in \{1, \ldots, 4\}$, and the rest elements of Ω are zero. Let C be a 4×4 matrix that is the upper-triangular Choleski decomposition of the non-zero block of Ω, and the elements $C_{11}, \ldots, C_{44}$ is used in the Log Likelihood estimation. Since the structure shocks are assumed to be serially uncorrelated and φ_{kt}, $k \in \{1, \ldots, 4\}$ are linear combinations of the structure errors, φ_t is serially uncorrelated.

In summary, this study uses a rational expectation model that is derived from a micro-founded dynamic general equilibrium model with deep parameters that govern private agents' behavior. The macroeconomic structure explicitly considers that private agents have rational expectation and they can react correspondingly when monetary policy changes, hence the model specification responds to the Lucas (1976) critique and the model parameters in equations (4.1) and (4.2) are structural.

4.2 The Optimal Simple Monetary Policy Rule

The central bank uses policy rule specified in equation (4.4) to set the short term interest rate. Conditional on the macroeconomic structure, the central bank sets monetary policy in an optimal way to stabilize output, inflation, and the interest rate. This study assumes that the monetary authority credibly commits to an optimal policy rule in the sense that the coefficients of the policy rule c_1 through c_5 are chosen to minimize the expected value of intertemporal loss function

$$\mathbf{L}_t = E_t \sum_{k=0}^{\infty} \delta^k X'_{t+k} W X_{t+k}, \tag{4.20}$$

where $\delta \in (0,1)$ is the central bank's time rate of discount. W is a 6×6 matrix of non-negative weights that determine the relative importance to the central bank of its various stabilization objectives.

In this study, W is diagonal with non-zero elements w_y, w_π, and w_r on the main diagonal and zeros everywhere else. Because only the relative sizes of weights are identified, w_π is normalized to one. The central bank's discount rate δ is fixed at 0.990, which is a reasonable value for a quarterly time rate of discount. In this study, the central bank sets its policy optimally with full information on the true economic structure and model parameters.

This study uses the algorithm described previously to compute the optimal values of policy rule coefficients which are chosen to minimize the loss function.

4.3 Maximum Likelihood Estimation: SURF and SORF

The parameters of the model are collected into two groups. Collect the structural parameters into vector $V = \{\lambda,\ a_1,\ a_2,\ b,\ \alpha_1,\ \alpha_2,\ \beta,\ \eta,\ \rho_1,\ C_{11},\ \ldots,\ C_{44}\}$, which contains the parameters of the IS equation, Phillips curve, the evolution process of the real oil price, and the elements in the variance-covariance matrix of the reduced-form shocks. Collect the coefficients of the monetary policy rule into vector $\Delta = \{c_1,\ c_2,\ c_3,\ c_4,\ c_5\}$.

This study still uses two estimation strategies. The first type of econometric policy evaluation is SURF, which estimates V and Δ while treating Δ as unrestricted. The second estimation strategy is SORF, which maintains the hypothesis that the policy-rule coefficients are chosen optimally to minimize expected loss hence induces a restriction function on the policy-rule coefficients, $\Delta = g(V,\ W)$. As a result, SORF estimates V and W subject to the restriction, $\Delta = g(V,\ W)$.

Both SURF and SORF use the algorithm of Klein (2000) described earlier to compute the solution for the model, which is then used to compute predicted values for X_t in equation (4.19) and to compute residuals and log likelihood. Because the Klein algorithm may try values of V and Δ that do not have a unique solution for the rational expectation system, the maximand used in the SURF and SORF estimations is log likelihood minus a penalty value. Let J_1 through J_8 be the eigenvalues of matrix A in equation (4.18) sorted in ascending order. If a unique saddle path exists, then $|J_k| > 1$ for $k = 1$ and 2 and $|J_k| < 1$ for $k = 3, \ldots, 8$. Let I_k be the indicator function that equals 0 if the root condition is satisfied for the kth root and 1 if it is not. Thus the penalty function is

$$\textbf{Penalty} = \bar{p} \sum_{k=1}^{8} I_k(|J_k| - 1)^2,$$

where $\bar{p}$ is a constant chosen to guarantee that the value of **Penalty** is large relative to log likelihood. If the root condition is satisfied, **Penalty** $= 0$. If it is not satisfied, **Penalty** is a smooth, positive function of the difference between out-of-bounds roots and 1.

Having obtained the model solution, this study uses the Markov-switching algorithm described in previous chapters to conduct SURF and SORF estimations.

4.4 Data Description

The model and the optimal-policy hypothesis are fitted to U.S. quarterly data from 1965:Q1 to 2008:Q4. Since this study applies a rational expectation Keynesian model for policy analysis starting 1965, in line with previous chapters, the monetary policy-makers in the Federal Reserve are assumed to understand the forward-looking nature of private agents' behaviors two decades before Lucas published his famous critique on policy analysis.

The macroeconomic variables include output gap, inflation, the real oil price, and the short term interest rate. The raw data, except for population, come from FRED, the Federal Reserve Bank of St. Louis database. Quarterly population numbers are interpolations of annual values found in the 2010 *Economic Report of the President*. As described in the previous chapter, output measure is real GDP (in 1996 chained dollars) divided by civilian population. The long run growth path of output per capita is estimated by fitting a constant-coefficient time trend to the natural logarithm of real GDP per capita. Variable y_t, the output gap, is the difference between the natural logarithm of real GDP per capita and its trend. The inflation rate is the annualized percentage change in the chained GDP deflator. The quarterly interest rate is the annualized secondary market yield of three-month treasury bills. Target values for the inflation rate and interest rate are obtained by fitting continuous, piecewise-linear trends that allow for trend coefficient changes in 1980:Q1 and 1986:Q1 while trend coefficients in the inflation rate and interest rate are constrained to be the same. Variables π_t and r_t are the residuals from the two trend regressions.

The real oil price is computed by deflating the Producer Price Index in Fuels and Related Products by the inflation rate. Figure 4.1 plots the level of real oil price from 1965:Q1 to 2010:Q1[42]. As the figure shows, there is no obvious evidence of a trend for the real oil price, although one could argue that it will not return to the mean any time soon. The dynamics of inflation seem to be closely related to those of the real oil price. One can observe that persistent increases in inflation as well as increases in the real oil price following two oil shocks in the 1970's. From 1981, one can see a steady decline in inflation accompanied by a persistent drop in the real oil price. From the early 1990's until 1999, there is a small downward trend in both inflation and the real oil price. From 2000 on one can observe a remarkably upward trend in the real oil price and a moderate increase in inflation. As a result, the movements in the real oil price may yield some information on the dynamics of the inflation rate during the sample period. This evidence motivates the development of the mechanism that is highlighted in the previous section to generate a link between inflation and real oil price in a New Keynesian framework. Instead of measuring variable q_t by the level of real oil price, this study uses the difference between the percentage change of real oil price and its long run trend[43]. The growth rate of real oil price and its trend are also plotted in Figure 4.1.

[42] To show trend in the level of real oil price, this study plots a longer sample period than the one used in the estimation of the structural model and optimal monetary policy.

[43] Earlier estimations suggest that results obtained using the percentage change of real oil price are considerably better than those obtained using the level of real oil price. Measuring oil prices by the growth rate of real oil prices can also be found in e.g. Herrera and Pesavento (2009).

4.5 Econometric Evaluation of U.S. Monetary Policy with Three Policy Regimes

This section applies the estimation approach described earlier to estimate the structural model parameters in equations (4.1) – (4.4) jointly with the policy preference parameters in equation (4.20). In this section, monetary policy is assumed to switch among three regimes during the sample period. Therefore, in the estimation algorithm, the policy parameters and policymaker's preference parameters are allowed to change in each regime.

Figures 4.2 and 4.3 present the probability inferences in SURF and SORF estimates for the monetary policy prevailing in each regime for each date t throughout the sample period. Tables 4.1, 4.2, and 4.3 report SURF and SORF estimates of model parameters along with the value of the log likelihood function. To make inferences about the significance of estimated parameters, this study reports the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is zero.

4.5.1 Probability Inferences for Three Policy Regimes

Figure 4.2 and Figure 4.3 plot the SURF and SORF estimated probability series of being in the three regimes for the Markov process governing the monetary policy respectively, together with the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research as shown in the vertical grey areas. Although these business cycle dates were not used in any way to estimate parameters or form inferences about the Markov transition process, it is interesting that the traditional business cycle dates are fairly related to the switches of monetary policy between the three regimes as described by the SURF and SORF estimates in Figure 4.2 and Figure 4.3.

Both SURF and SORF estimates show that policy regime one is a special regime that only prevailed during the deep post-war recessions began in 1973, 1980, 1981, and

2007. These recessions were unusually deep and long[44]. The 1973, 1980, and 2007 recessions also featured large spikes in oil prices near the beginning of the recession[45]. Therefore, all these deep recessions took place in a context of high inflation that made the Federal Reserve set its primary policy goal to bring down high inflation and hesitate to aggressively reduce interest rates to stimulate economic activity. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during the post-1985 expansionary periods and mild recessions in 1990 and 2000. These two recessions were among the mildest and shortest recessions since the Second World War[46]. Policy regime three is a regime which the monetary policy switched into during the pre-1985 expansionary periods and mild recession in 1970[47]. As a result, it can be seen that regimes two and three were dominant for most of the sample. Regime two included the entire period from 1985 onward until the end of the sample. Regime three included most of the pre-1985 period. When U.S. monetary policy switched into regime two or three, the primary goal of the Federal Reserve was to maintain low inflation rate and sustainable output growth.

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[44] During the 1973, 1981, and 2007 recessions, output lost as measured by the decline in real gross domestic product (GDP) was at least a cumulative 2 percent. These deep recessions lasted at least 16 months, which were longer than the rest of post-war recessions. The median length of all post-war recessions is 9.5 months. The NBER declared that the 2007 recession ended at June 2009, so the most recent recession lasted 18 months, which turns out to be the longest in the post-war period.

[45] The recessions of 1973 and the early 1980's are remembered for their large oil shocks caused by the 1973 and 1979 energy crises. At the beginning of the 2007 recession, especially from January 2007 to July 2008, crude oil price rose from 51 dollars per barrel to a record high of 129 dollars per barrel.

[46] For the recession of early 1990's, real GDP declined by a cumulative 1.3 percent, and the length of recession is 6 months. During the recession beginning in 2000, real GDP fell by a cumulative 0.2 percent, and the duration is 8 months. Hence, these two recessions were brief and shallow.

[47] During the recession beginning in 1970, real GDP fell by a cumulative 0.6 percent, and the duration is 11 months. Hence, this recession was relatively mild.

4.5.2 Estimates of Structural Parameters

The discussions here begin with the SURF structural parameters estimates, see Table 4.1. A sizable value of parameter λ implies that expected future output plays an important role in determining aggregate demand of household. Here the estimate of λ is 0.432 when the assumption of policy optimality is relaxed. While the estimate is sizable, the p-value of the likelihood ratio test indicates that one can not reject the null hypothesis that the true parameter value is zero. Therefore, the estimate of λ in SURF estimates implies that expected future output is not an important factor in affecting aggregate demand. The next two structural parameters are linked to the persistent effects on output. The parameters on lagged outputs a_1 and a_2 are found to be sizable: 0.700 and -0.134, respectively. The estimate of a_1 is statistically indifferent from one and the estimate of a_2 is significantly different from zero at any conventional confidence level. Hence these estimates imply that shocks have sustained effects on output. Third, the estimate of b is 0.003 without imposing the constraint of optimal reaction function. Despite the fact that b is estimated to be quantitatively small, the estimate appears to be significant at any conventional confidence level. The estimate implies that changes in the interest rate do cause changes in aggregate demand although the effect of interest rate on aggregate demand is relatively insensitive. Fourth, turning to the parameters of the Phillips curve, one can note that firms are substantially more backward-looking than forward-looking. The estimate of the relative weight α_1 is very small and is found to be highly insignificant, and the estimate of α_2 is 0.564 and is found to be significantly different from zero. The findings here result in a backward-looking Phillips curve and cast doubt on the sticky price mechanism of Calvo. Fifth, the estimate of β is responsible for capturing the impact of business cycle fluctuations on real price. The estimate of β is 0.020 and is found to be not significantly different from zero which implies that a positive output gap may not raise the inflation rate. Sixth, a sizeable value of parameter η implies that real oil price can significantly affect the overall inflation level. The estimate of η is 0.090, which is relatively small but is found to be highly significant. Thus the estimate implies that an increase in real oil price can raise the inflation rate. Finally, in the evolution process of real oil price, it can be seen that real oil price follows a stationary autoregressive process. The estimate of persistence parameter ρ_1 is 0.108 and is just at the edge of 15% significance level, which implies that there is a small positive correlation between real oil prices from time period to time period.

When the policy optimality is imposed, the SORF estimate of λ is 0.422, which is nearly the same as the SURF estimate. The estimate of λ is still found to be highly insignificant implying that expected future output does not play an important role in determining aggregate demand of household. Second, the SORF estimates of a_1 and a_2 are 0.722 and -0.146 respectively, which are very close to the SURF estimates. The estimate of a_1 is statistically different from one and the estimate of a_2 is significantly different from zero at any conventional confidence level. Hence these estimates imply that shocks have sustained effects on output. Third, b is estimated to be 0.004, which is a bit larger than the estimate under SURF. The estimate appears to be significant at any conventional confidence level, which implies that changes in the interest rate do cause changes in aggregate demand although the effect of interest rate on aggregate demand is relatively small. Fourth, SORF estimates of both α_1 and α_2 are positive and highly significant, which support for the forward-looking Phillips curve implied by the sticky price-setting mechanism. In particular, one can not reject the hypothesis that α_1 and α_2 sum to one as in Fuhrer and Moore (1995) and Salemi (2006), and SORF estimates were obtained with that restriction imposed. One can note that the past and future prices' overlap in the current price is somewhat skewed toward the future price. This finding is in general accord with Galí and Gertler (1999), who find that most of the U.S. firms are purely forward-looking but a fraction are following a backward-looking rule of thumb.

Fifth, the SORF estimate of β is much smaller than its SURF estimate but is highly significant at any conventional confidence level, which imply that it is more difficult for the Federal Reserve to control inflation by changing aggregate demand. Sixth, the SORF estimate of η is smaller than the estimate under SURF and is significant at marginally 15% level, which imply that an increase in the real oil price has a smaller effect on raising the inflation rate. Finally, in the evolution process of real oil price, the SORF estimate of persistence parameter ρ_1 is a bit larger than its SURF estimate and is significant at marginally 10% level, which imply that there is a larger positive correlation between real oil prices from time period to time period and the real oil price still follows a stationary autoregressive process.

As the p-values reported in Table 4.1 indicate, the structural parameter α_1 is more precisely estimated under SORF than under SURF. It is believed that a unified approach of joint estimation of structural model parameters with the parameters of the policymaker's objective function provides additional across equation restrictions that result in a more precise estimation of the model parameters. Hence the SORF estimates more strongly support a forward-looking Phillips curve. Castelnuovo (2003) argues that hybrid formulation of the Phillips curve is crucial for appropriate identification of policy preferences. Moreover, Salemi (2006) suggests that when the interest rate is highly persistent, the best way to reconcile a persistent interest rate process and policy optimality is to hypothesize that both occurred in an economy where private agents are forward looking. The finding of highly persistent interest rate in both regimes will be shown below.

4.5.3 Estimates of Preference Parameters and Test of Policy Optimality

Table 4.2 reports estimation results for the preference parameters of three policy regimes identified in U.S. monetary policy, and it also presents two sets of reaction function coefficients for each policy regime: SURF estimates (obtained while treating coefficients of the policy rule as free parameters), and SORF estimates (implied values of the policy rule coefficients consistent with the policy optimality hypothesis). To interpret the results on policy preference estimation, one should note that the relative weight on inflation stabilization objective is normalized to one.

One can notice that, in all three policy regimes under SORF, the estimated values of w_y and w_r are very small implying that inflation stabilization was always the dominant objective of the Federal Reserve. Policy regime one is an interesting case, covering the post-war deep recessions began in 1973, 1980, 1981, and 2007. Looking at the estimates describing the Federal Reserve's preferences during policy regime one, the estimate of w_y is 0.0001 and significantly different from zero at marginally 5% level. The weight placed on interest rate stabilization is 0.0017 and significantly different from zero. The values of w_y and w_r in the first regime are the smallest among three policy regimes. Given the normalization of the weights, this finding suggests that in conducting its monetary policy the Federal Reserve was far more concerned with keeping the inflation close to its long run target path than it was with trying to stabilize output and interest rate around their targets. Indeed, in policy regime one, both SURF and SORF estimates show that the policy rule coefficient on lagged inflation c_3 is very sizable. The interest rate stabilization objective appears to be of secondary importance to the Federal Reserve in policy regime one. The estimates of preferences in policy regime one are consistent with the historical evidence. As described earlier, in policy regime one the Federal Reserve faced high level of inflation, and its sole objective was to reduce inflation substantially.

In policy regime two, inflation stabilization was still the dominant objective. The estimates of the policy preferences in this policy regime suggest that the Federal Reserve Bank had been conducting its policy in the manner consistent with the policy used during economic expansionary period. A large weight was assigned to the inflation stabilization objective and some weights were placed on output and interest rate stabilizations. Indeed, the estimate of w_y is 0.0059 which is statistically significant at any conventional confidence level. The estimated policy weight placed on the interest rate stabilization objective w_r is 0.0053 and is significantly different from zero. The objective of output stabilization was almost equally as important as interest rate smoothing in policy regime two. The estimated and impled policy rule coefficients on lagged output are sizable. As pointed out previously, during the expansionary regime the Federal Reserve would put more concerns on potential exacerbate inflation situation and use instrumental interest rates wisely. Switching into policy regime two would result in a low level of inflation.

In policy regime three, the normalization of the weights suggests that keeping the inflation close to its long run target path was far more important for the Federal Reserve than stabilizing output and the interest rate. The estimate of w_y in this regime is 0.0078 and is the largest among three policy regimes. The likelihood ratio test for w_y strongly rejects the null hypothesis that the true weight is zero. In addition, the estimated and impled policy rule coefficients on lagged output are sizable. The estimate of w_r in this regime is 0.0064 and and is highly significant. The estimate of w_r here is the largest among three policy regimes. However, the interest rate stabilization objective was not as important as output stabilization to the Federal Reserve in policy regime three.

There is clear evidence that the estimated and implied policy-rule coefficient estimates in three regimes differ significantly and substantially from each other. First, the coefficient on lagged output (c_1) is negative in policy regime one but sizable and positive in regimes two and three. The coefficient on lagged inflation (c_2) is large and positive in regime one but small in regimes two and three. The SURF coefficient on lagged real oil price (c_4) is positive in regime one but negative in regimes two and three. The coefficient on lagged two output (c_5) is positive in regime one but negative in policy regimes two and three. Second, SORF estimate of the policy rule coefficient on lagged inflation is positive in all regimes and larger than the SURF estimate in regimes two and three. This suggests that the Federal Reserve should have responded more strongly to inflation than it did in regimes two and three, which will be discussed shortly.

Did the Federal Reserve set monetary policy optimally with three policy regimes? In light of Salemi (2006), here a test of policy optimality is a test of the hypothesis that SORF fits the data as well as SURF, that is, the SORF and SURF policy rule coefficients are the same. In each policy regime, there are five freely estimated parameters in the unrestricted policy rule but only two freely estimated parameters, w_y and w_r, in the optimal policy rule, so SORF implies nine restrictions. As a result, the relevant likelihood ratio statistic is 11.873 with a p-value 0.221. Thus the hypothesis that the policy rule coefficients that fit the data best were those that minimized quadratic expected loss as defined in (2.6) can not be rejected. Hence, the estimations using a variation of the structural model show that the Federal Reserve did set monetary policy optimally with three policy regimes. This conclusion is in line with that in the previous chapter.

4.5.4 Interest Rate Smoothing

As can be seen in Table 4.2, the reaction-function coefficient for the lagged interest rate is always large compared to the coefficients for inflation and output so that, according to the definition provided by Clarida, Galí and Gertler (2000), the Federal Reserve engages in "interest rate smoothing". This finding supports the widely held viewpoint that most central banks, including the Federal Reserve, are very cautious when changing interest rates. While the estimates in the reaction functions reported in Table 4.2 imply that the Federal Reserve smooths interest rates, SORF estimates suggest that the Federal Reserve placed small weight on interest rate stability in all three policy regimes. Thus, in line with the previous chapter, the estimates here are consistent with Sack and Weiland's explanation for smoothing. The large coefficient on the lagged interest rate can be explained by the finding that forward-looking expectations are empirically important and the fact that optimal policies require persistent responses to shocks.

4.5.5 Estimated Transition Probabilities

Table 4.3 presents SURF and SORF estimates for the transition probabilities for switching in three monetary policy regimes. As the p-values indicate, all transition probabilities are precisely estimated. The probabilities on the diagonal of the transition matrix are close to unity, which means that all policy regimes show high persistence[48]– a feature that is commonly seen in the estimation of Markov switching models, see Sims (1999) and Assenmacher-Wesche (2006). The SURF estimates indicate that, if the monetary policy is in regime one this period, there is a 98.4 percent probability that it will be in regime one next period; if the monetary policy is in regime three this period, there is a 95.9 percent probability that it will be in regime three next period. Similarly, the SORF estimates indicate that, if the monetary policy is in regime one this period, there is a 94.8 percent probability that it will be in regime one next period; if the monetary policy is in regime three this period, there is a 95.9 percent probability that it will be in regime three next period. These suggest that changes in the monetary policy regime will occur infrequently.

SURF estimates show that all policy regimes except one are not degenerate regimes. The p-values for SURF and SORF diagonal probabilities p_{11}, p_{22}, and p_{33} are measured by the likelihood ratio test for the hypothesis that the true value of the transition probability is 0.995, meaning that there is nearly no switch back into the regimes prevailing at the beginning of the sample period. The test statistic for SURF estimate of p_{22} can not reject the hypothesis thus the policy regime two in SURF estimates is a degenerate regime. In sum, the important findings associated with the three-regime monetary policy are robust to estimations using a variation of the structural model.

[48] To guarantee that the diagonal transition probabilities are highly persistence, in the estimation they were restricted to be no less than 0.900.

4.5.6 Comparison of Impulse Response Function for Structural Estimation

To check how much the model can explain the variation in the data, this section compares the impulse response functions (IRF's) implied by SURF and SORF. This study makes two assumptions. The first assumption is that monetary policy is conditioned only on lagged values of output and inflation. The second assumption is that macroeconomic variables are affected by oil supply disruptions that are exogenous to U.S. economy[49]. The second assumption implies that the oil price is predetermined with respect to the Federal Reserve's instrument rate. In keeping with these assumptions, this study chooses the following within period causal ordering for the impulse response function: a shock to output affects only output contemporaneously, a shock to inflation affects both inflation and output contemporaneously, a shock to the interest rate affects the interest rate, inflation and output contemporaneously, a shock to the real oil price affects all variables contemporaneously. The ordering makes difference because of the contemporaneous correlation between output and the interest rate in all policy regimes. The ordering also implies that there is no contemporaneous feedback from monetary policy to oil price, while the real oil price contemporaneously affects the monetary policy. Figures 4.4, 4.5, and 4.6 present the IRF's for all three regimes. The shade areas in the figures are 95% confidence intervals computed with parametric Bootstrap method.

The figures show clearly that the SURF and SORF estimates share many features of the impulse response functions. For instance, SURF and SORF estimates all imply that a positive shock to output keeps output above its trend for more than 30 quarters. Both SURF and SORF estimates show that a positive shock to output keeps inflation above its trend for a substantial period of time. Both show that a positive shock to inflation

[49]This assumption is in line with many studies, see e.g. Kim and Loungani (1992), Leduc and Sill (2004), Carlstrom and Fuerst (2005), and De Walque, Smets and Wouters (2005). However, there are studies that argue that since 1973, oil prices have become increasingly responsive to demand conditions, e.g., Barsky and Kilian (2004), Hamilton (2005), Woodford (2007), Kilian (2008), and Kilian (2009).

produces moderate inflation persistence with inflation returning to trend in about six quarters. Lastly, both SURF and SORF estimates imply that a positive inflation shock causes output to fall below trend for a substantial period of time.

There are significant differences in the impulse response functions across three policy regimes. In regimes one and two, both SURF and SORF estimates imply that the interest rate increases then falls gradually to trend after a positive inflation shock; in policy regime three, SURF estimates show that the interest rate falls then increases gradually to trend after a positive inflation shock, and SORF estimates show that interest rate increases above trend then decreases gradually to trend after 30 quarters. Since policy regime one is the special policy regime that prevailed during the deep post-war recessions that were in a context of high inflation, the interest rate responds positively to a positive inflation shock, and the responses are much larger than those in regimes two and three. The comparisons also imply that the actual interest rate response to inflation was not optimal in regimes one and three but it was very close to optimal in regime two. A loss-minimizing policy would have raised the interest rate more in policy regimes one and three. A second difference is related to the first. SORF estimates imply that the inflation response to a positive output shock is smaller than implied by the SURF estimates because the SORF interest rate response to a positive inflation shock is larger than the SURF interest rate response.

Comparing the IRF's suggest that actual and optimal responses of the interest rate to output shocks were not the same in all three policy regimes. In regime one the interest rate increases initially then falls and returns to trend after a negative output shock, while in regimes two and three the interest rate falls below trend for a long time then rises to trend gradually after a negative output shock. Therefore, in policy regime one, SURF and SORF estimates imply that inflation would be dampened completely. Because regime one is the special deep recession regime with high inflation, the primary goal of the Federal Reserve was to decrease inflation. While regimes two and three are both expansionary regimes, the Federal Reserve would like to maintain stable inflation level hence the interest rate responses in regimes two and three are opposite to that in regime one.

The fourth difference is that, in regimes one and three SORF estimates imply output declines more after a positive inflation shock than implied by the SURF estimates, while in policy regime two SURF estimates imply approximate the same output response to a positive inflation shock as the SORF estimates.

Figure 4.7 plots and compares the response functions of output, inflation, and the interest rate to a positive oil price shock. As can be seen, in all three policy regimes SURF and SORF estimates imply that an unexpected increase in the price of oil results in a decline in output activity. In policy regime one, SURF estimates imply that the contractionary effect of the oil price shock on output reaches a peak four quarters after the shock, and SORF peak occurs about 12 quarters after the shock. In policy regime two, both SURF and SORF peaks occur about three quarters after the oil price shock. In policy regime three, SURF peak occurs about three quarters after the oil price shock, and SORF peak occurs about five quarters after the shock. Since policy regime one is the special deep recession regime with high inflation, the output responses are larger than those in policy regimes two and three.

In all three regimes, the SURF and SORF estimates also predict a tighter monetary policy reflected by an increase in the interest rate, possibly aiming at curbing inflation. The responses of interest rate in regime one are much larger than those in regimes two and three. However, the patterns in the response of the interest rate are different in three policy regimes. In policy regime one, both SURF and SORF estimates show that the interest rate increases gradually then declines slowly to trend after an oil price shock, while in policy regimes two and three SURF and SORF estimates imply the interest rate responds immediately after an oil price shock. These patterns in the response of the interest rate suggest that quicker policy adjustment in policy regimes two and three could have contributed to the smoother and faster adjustment of inflation to oil price shock. The comparisons also imply that the actual interest rate response to oil price was not optimal in regimes one and three but it was very close to optimal in regime two. A loss-minimizing policy would have raised the interest rate more in policy regimes one and three.

Another interesting result is the response functions of inflation in three policy regimes. Note that in all policy regimes the oil price shock generates an increase in inflation. However, during policy regimes two and three the increases in inflation are considerably smaller than that in regime one. The differences in the responses suggest that the monetary policy used during the expansionary regimes two and three might be more effective in controlling the expectation of higher inflation that follows an oil price shock.

Figure 4.8 compares the responses of output and inflation to an interest rate shock. The policy rule equation residuals are correlated with the IS and Phillips curve equation residuals in both policy regimes. On the maintained hypothesis that the monetary policy reacts to economic conditions only with a lag, this correlation between must be due either to responses of output and inflation to an interest rate surprise or to responses of output, inflation, and interest rate to an un-modeled shock. Following Salemi (2006), to obtain the model's prediction of responses of output and inflation to a pure interest rate shock, this study ignores the contemporaneous correlations when computing the IRF's reported in Figure 4.8.

Figure 4.8 shows that in all three regimes SURF and SORF estimates can account for the "hump-shaped" response of output to an interest rate surprise. In policy regime one, SURF peak occurs about five quarters after the interest rate shock, and SORF peak occurs about 12 quarters after the shock. In policy regime two, both SURF and SORF peaks occur about 13 quarters after the interest rate shock. In policy regime three, SURF peak occurs about 15 quarters after the interest rate shock, and SORF peak occurs about ten quarters after the shock. Both SURF and SORF estimates show that the inflation rate declines immediately after an interest rate shock and remains below trend for many quarters. SORF estimates imply a larger inflation response than do SURF estimates in policy regimes one and two. Because regime one is a special regime with high inflation, the Federal Reserve would like to decrease inflation hence the SORF inflation response is much larger than implied by SURF estimates.

Figure 4.1: Real Oil Price: Level and Growth Rate

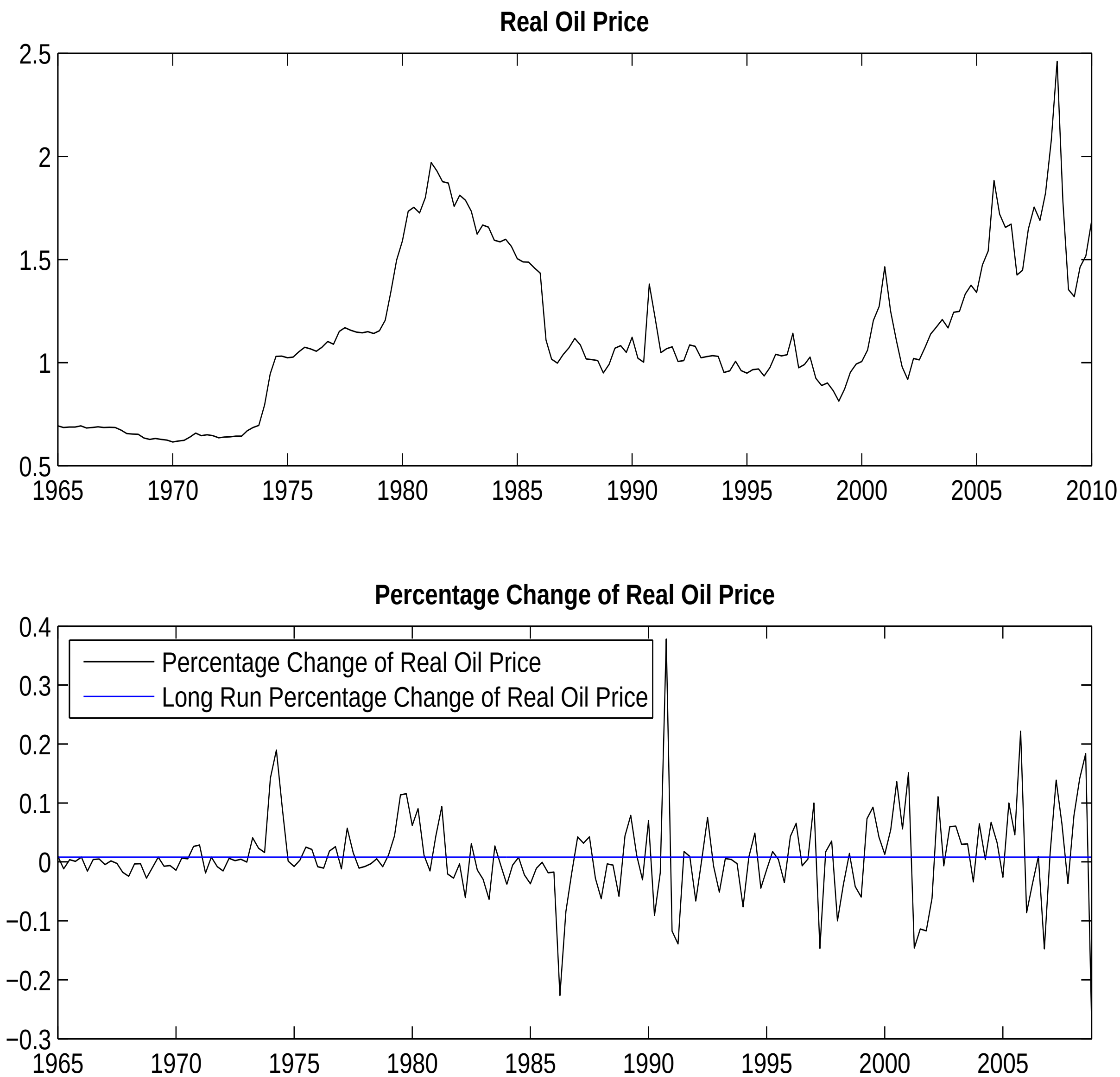
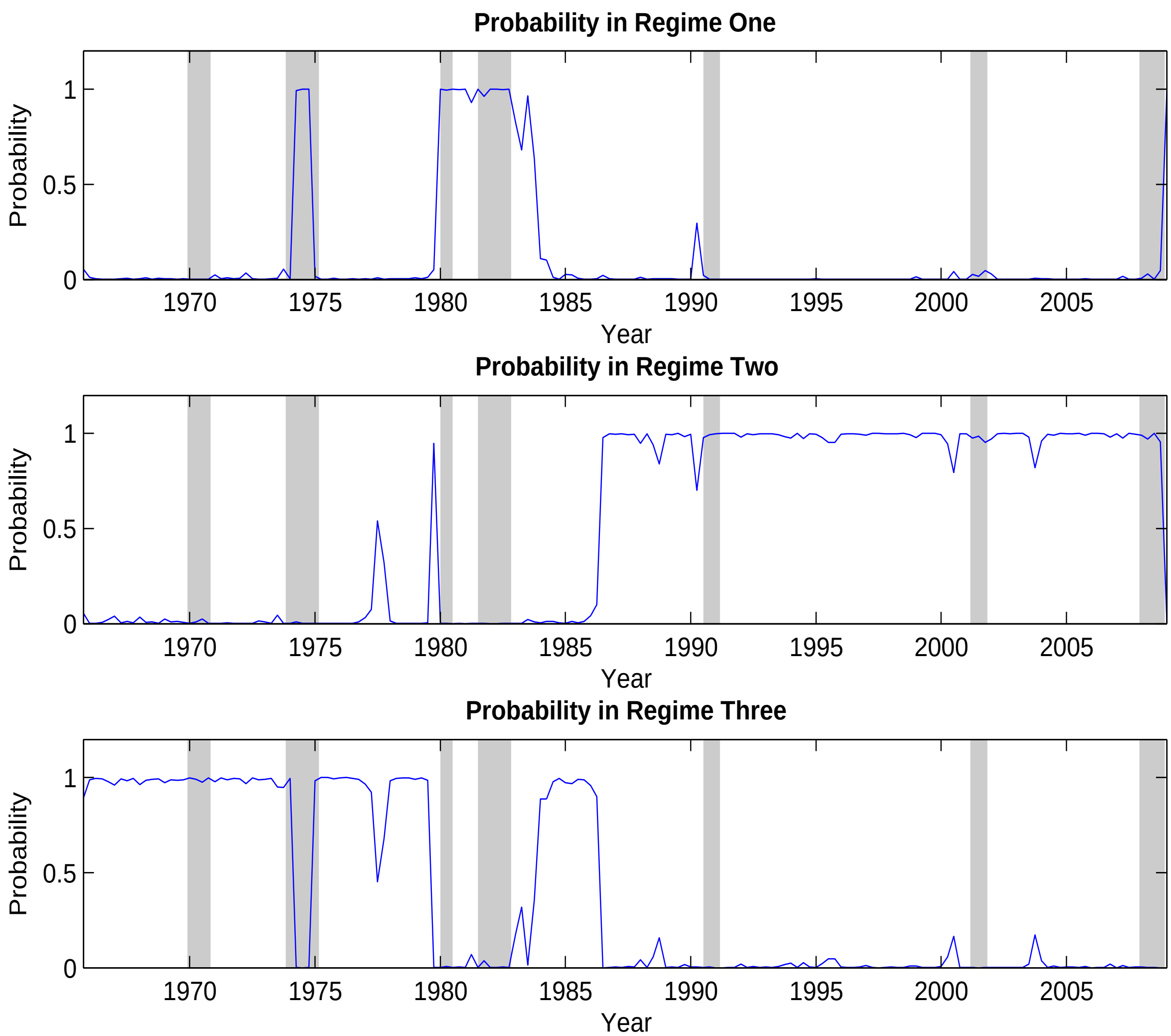
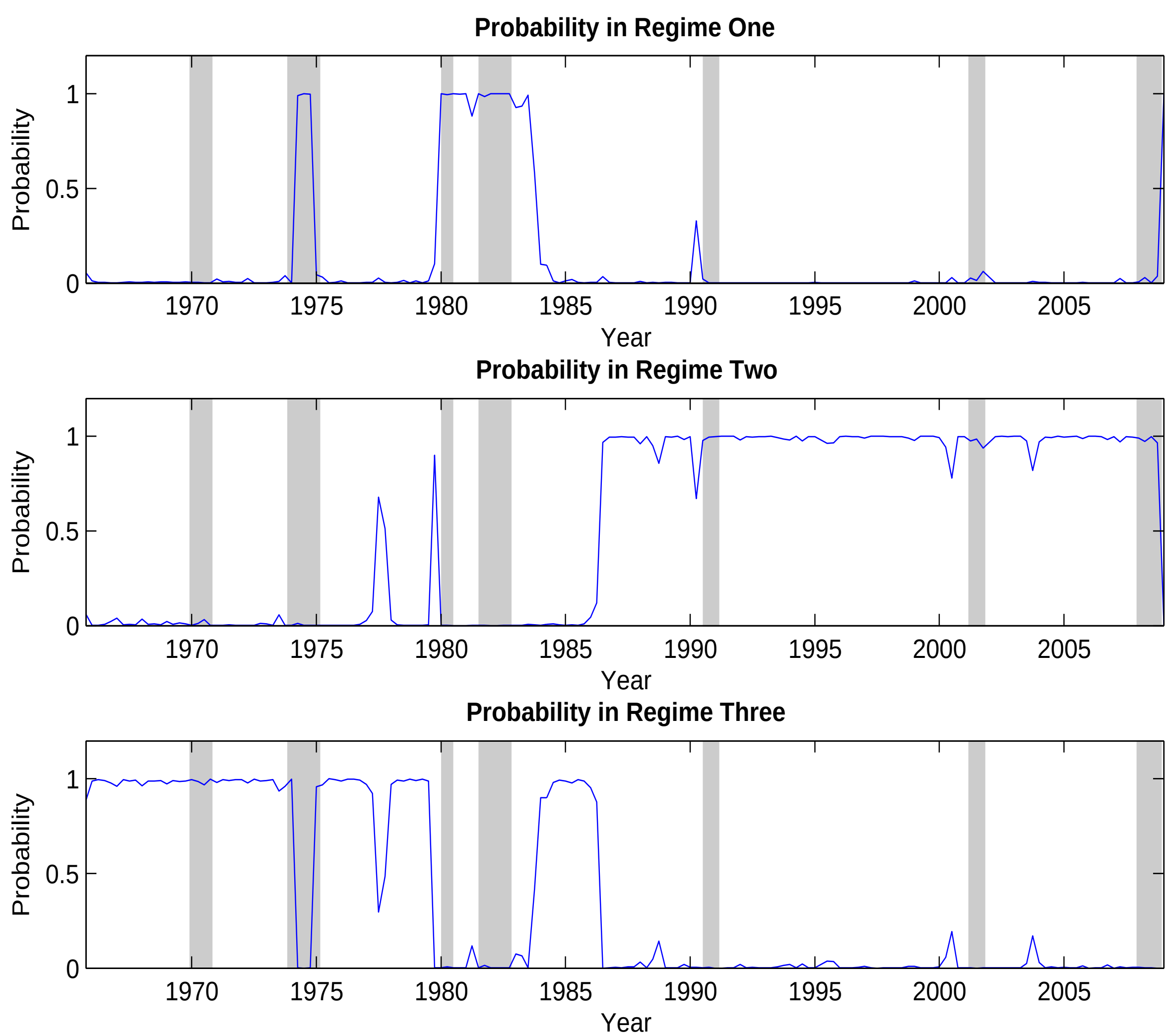


Figure 4.2: Probabilistic Inference for the Three-Regime Unrestricted Monetary Policy



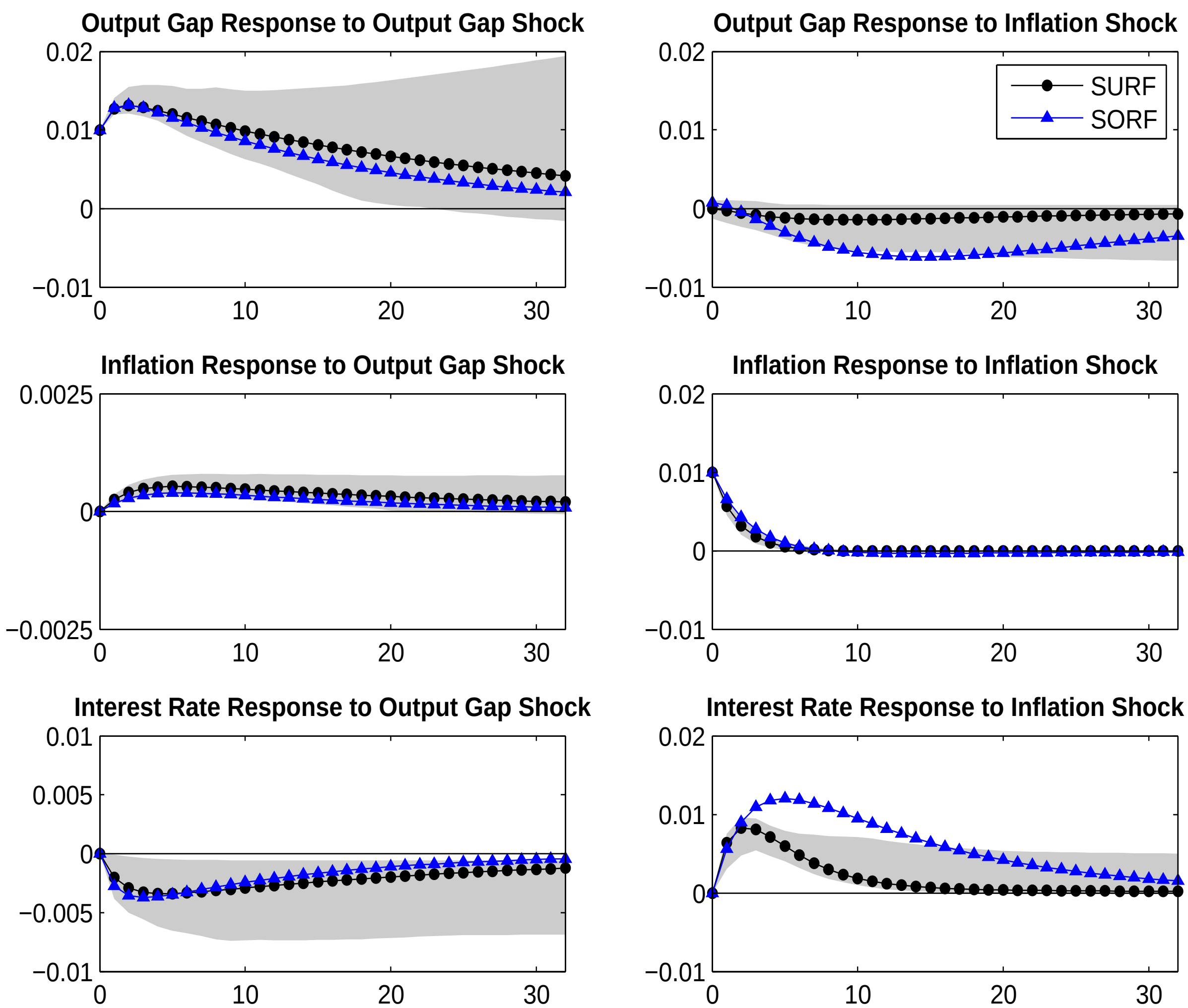
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated with an unrestricted reaction function for the interest rate. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 4.3: Probabilistic Inference for the Three-Regime Optimal Monetary Policy



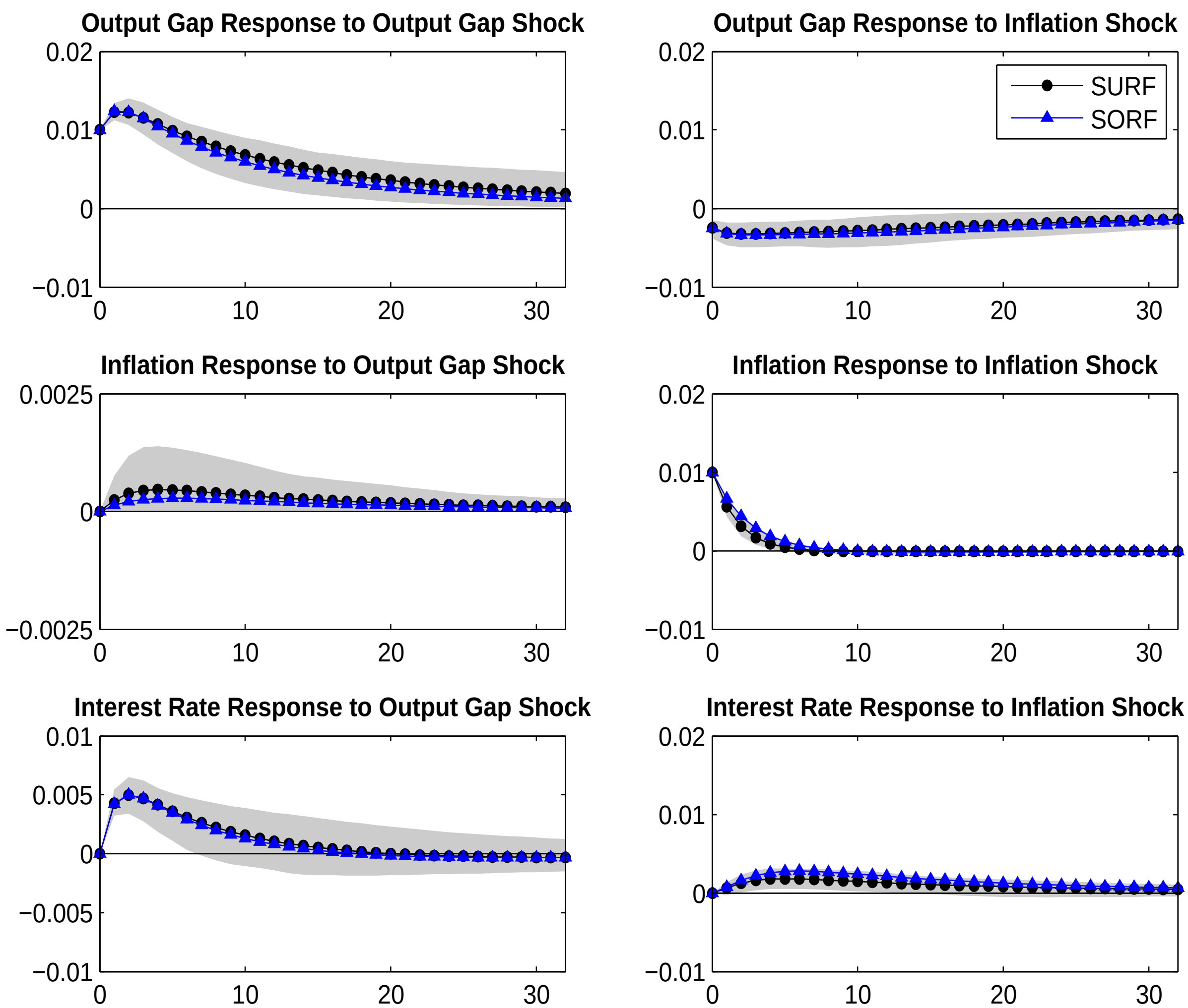
The figure presents the probability inference for the monetary policy prevailing in each regime for each date t in the sample when the reduced form of the New Keynesian model is estimated subjected to the restriction that the reaction function minimizes expected loss. The vertical shade areas in the figure indicate the dates at which economic recessions were determined to begin and end according to the National Bureau of Economic Research.

Figure 4.4: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime One



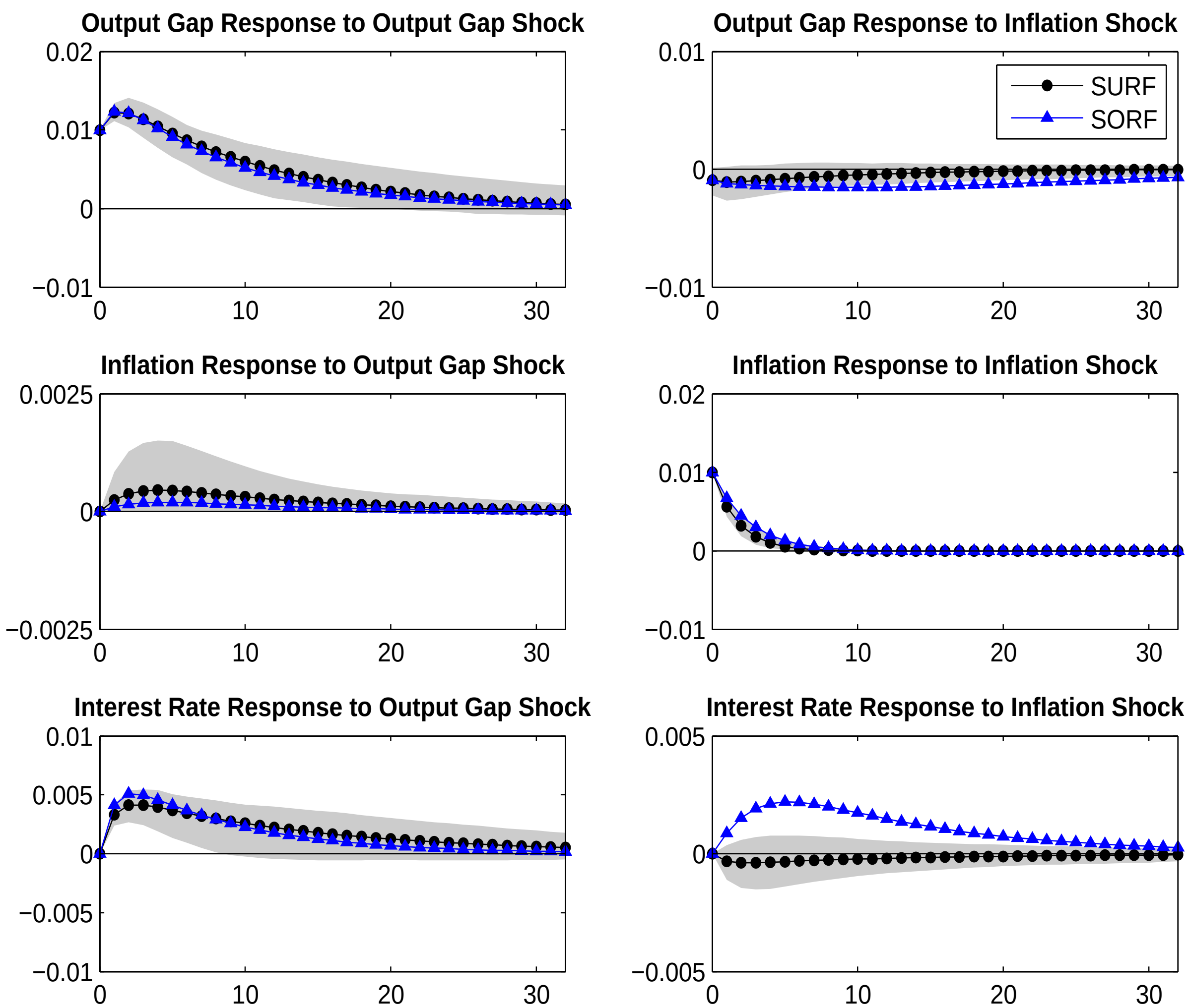
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 4.5: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Two



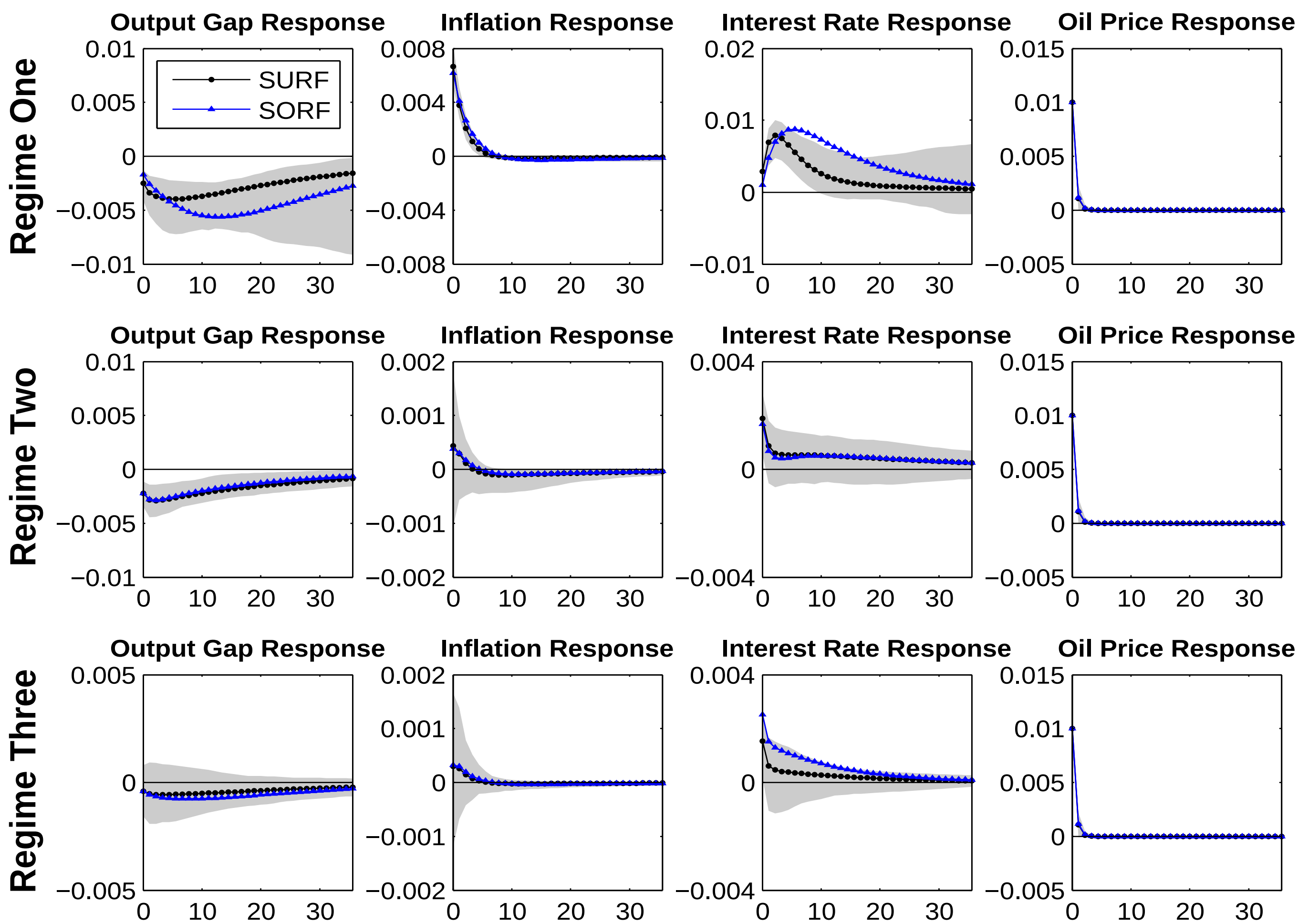
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 4.6: Comparison of Impulse Response Functions
for the Model with Three-Regime Monetary Policy
in
Regime Three



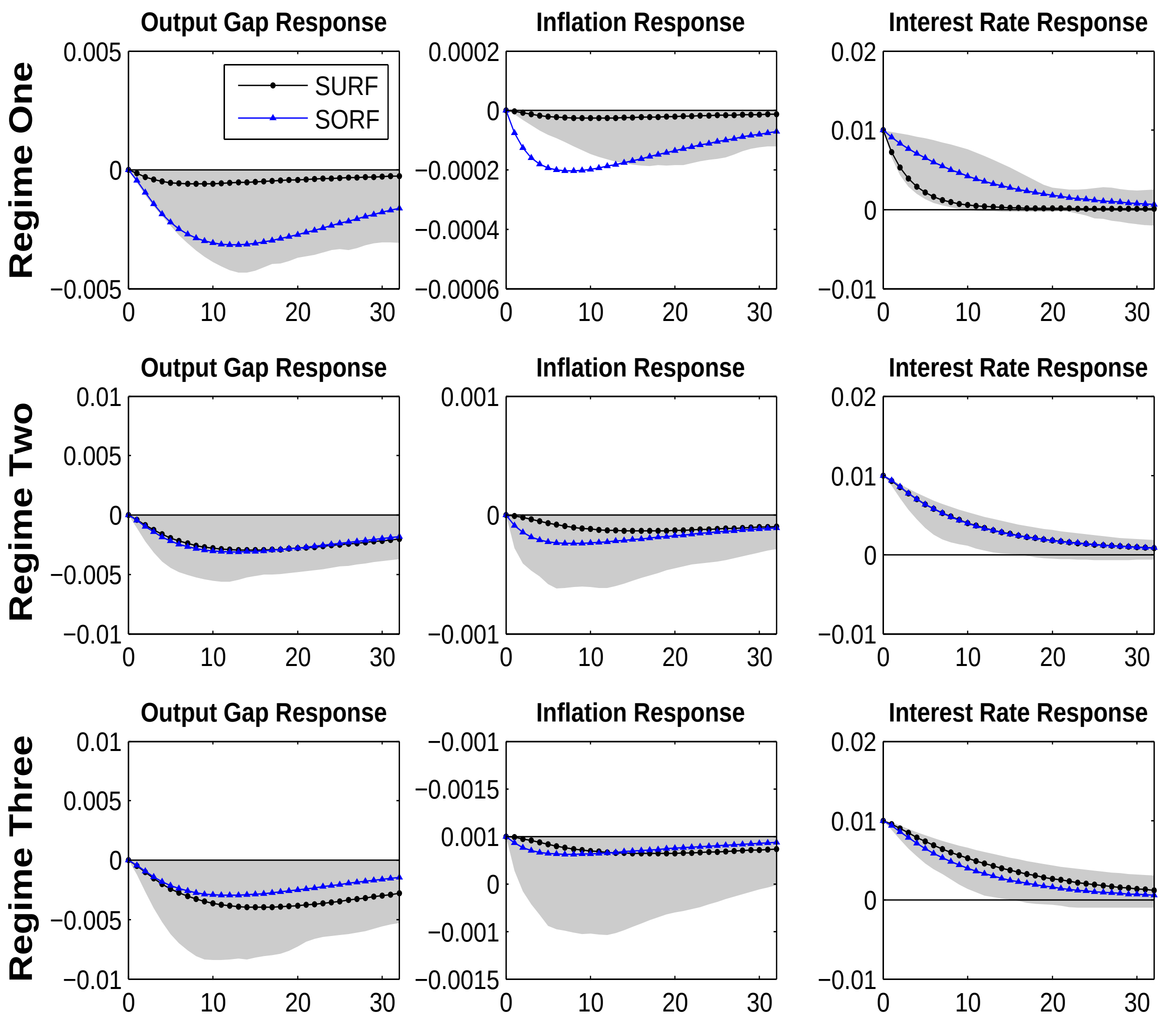
The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 4.7: Comparison of Impulse Response Functions to an Oil Price Shock for the Model with Three-Regime Monetary Policy



The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Figure 4.8: Comparison of Impulse Response Functions to an Interest Rate Shock for the Model with Three-Regime Monetary Policy



The figure presents impulse response functions implied by two estimates of the model solution form. The IRFs labeled SURF are based on the solution form of the New Keynesian model estimated with an unrestricted reaction function for the interest rate. The IRFs labeled SORF are based on the solution form of the New Keynesian model estimated subject to the restriction that the reaction function minimized expected loss of the central bank. The shade areas are 5% and 95% confidence intervals estimated by the parametric Bootstrap method described in the text.

Table 4.1: Estimates of Model Parameters for the Model with Three-Regime Monetary Policy

Model Parameters	Estimation with Unrestricted Reaction Function SURF: Estimate	SURF: Significance (p-value)	Estimation with Optimal Reaction Function SORF: Estimate	SORF: Significance (p-value)
λ	0.432	1.000	0.422	1.000
a_1	0.700	1.000^4	0.722	$< 0.001^4$
a_2	-0.134	< 0.001	-0.146	< 0.001
b	0.003	< 0.001	0.004	< 0.001
α_1	0.002	0.867	0.597^5	< 0.001
α_2	0.564	< 0.001	0.403^5	< 0.001
β	0.020	0.415	0.0004	< 0.001
η	0.090	< 0.001	0.043	0.188
ρ_1	0.108	0.146	0.116	0.119
Log Likelihood	1944.598		1938.662	

1. The table shows Maximum Likelihood estimates of the model parameters that are in the first three equations of the following model economy.

$$
\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \eta q_t + \varepsilon_{\pi,t}, \\
q_t &= \rho_1 q_{t-1} + \varepsilon_{q,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 q_{t-1} + c_5 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}
$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values reported in this table are assumed to remain the same for three policy regimes. The policy rule coefficients of the interest rate equation ($c_1 - c_5$) are allowed to change across regimes. Estimates for parameters of the reaction function are reported in Table 4.2.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. The p-value reported in this case is for a test of the hypothesis that $a_1 = 1$.

5. Estimation is conducted subject to the restriction $\alpha_2 = 1 - \alpha_1$.

Table 4.2: Policy Rule Coefficient Estimates for the Model with Three-Regime Monetary Policy

Reaction Function		SURF		SORF	
	Coefficients	Coefficient	P-value	Coefficient	P-value
Regime One	c_1	-0.204	1.000	-0.273	—
	c_2	0.636	1.000	0.583	—
	c_3	0.726	0.001	0.909	—
	c_4	0.006	< 0.001	-0.021	—
	c_5	0.046	< 0.001	0.231	—
	Loss Function Wt. on y	—	—	0.0001	0.077
	Loss Function Wt. on r	—	—	0.0017	< 0.001
Regime Two	c_1	0.427	< 0.001	0.422	—
	c_2	0.166	< 0.001	0.179	—
	c_3	0.931	< 0.001	0.936	—
	c_4	-0.001	< 0.001	-0.002	—
	c_5	-0.427	< 0.001	-0.421	—
	Loss Function Wt. on y	—	—	0.0059	< 0.001
	Loss Function Wt. on r	—	—	0.0053	< 0.001
Regime Three	c_1	0.332	< 0.001	0.414	—
	c_2	-0.001	< 0.001	0.126	—
	c_3	0.957	< 0.001	0.938	—
	c_4	-0.072	1.000	-0.073	—
	c_5	-0.312	< 0.001	-0.392	—
	Loss Function Wt. on y	—	—	0.0078	< 0.001
	Loss Function Wt. on r	—	—	0.0064	< 0.001
	Log Likelihood	1944.598		1938.662	

1. The table shows Maximum Likelihood estimates of the policy rule coefficients that are in the fourth equation of the following model economy.

$$\begin{aligned}
y_t &= \lambda E_t y_{t+1} + a_1 y_{t-1} + a_2 y_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{y,t}, \\
\pi_t &= \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta y_t + \eta q_t + \varepsilon_{\pi,t}, \\
q_t &= \rho_1 q_{t-1} + \varepsilon_{q,t}, \\
r_t &= c_1 y_{t-1} + c_2 \pi_{t-1} + c_3 r_{t-1} + c_4 q_{t-1} + c_5 y_{t-2} + \varepsilon_{r,t}.
\end{aligned}$$

   SURF estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve. The values of the reaction function coefficients reported for **SORF** are those derived jointly by the model parameters, the loss function weights, and the requirement that the policy rule minimizes loss. The policy rule coefficients are allowed to change across regimes.

2. Loss function weights measure the relative importance to the Federal Reserve of stabilizing output and the interest rate. Here, the weight on stabilizing inflation is normalized to 1.

3. Parameter-estimate significance is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

Table 4.3: Transition Matrix Estimates for the Model with Three-Regime Monetary Policy

Transition Probabilities	SURF		SORF	
	Estimate	P-value	Estimate	P-value
p_{11}	0.984^4	$< 0.001^5$	0.948^4	$< 0.001^5$
p_{21}	0.000	1.000	0.008	0.004
p_{31}	0.013	< 0.001	0.029	< 0.001
p_{12}	0.008	< 0.001	0.000	1.000
p_{22}	0.948^4	1.000^5	0.984^4	0.034^5
p_{32}	0.028	< 0.001	0.012	< 0.001
p_{13}	0.008	< 0.001	0.052	< 0.001
p_{23}	0.052	< 0.001	0.008	0.002
p_{33}	0.959^4	$< 0.001^5$	0.959^4	$< 0.001^5$
Log Likelihood	1944.598		1938.662	

1. The table shows Maximum Likelihood estimates of the probability parameters of the following transition matrix which the monetary policy follows.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} & p_{31} \\ p_{12} & p_{22} & p_{32} \\ p_{13} & p_{23} & p_{33} \end{bmatrix},$$

   where $p_{jk} = Pr\{j_{t+1} = k | j_t = j\}$ is the probability switching from regime j at time t to regime k at time $t+1$, $\sum_{k=1}^{3} p_{jk} = 1$, and $p_{jk} \geq 0$, $\forall\ k,\ j \in \{1, 2, 3\}$. **SURF** estimation imposes no restrictions on the parameters of the reaction function. **SORF** estimation requires that the parameters of the reaction function minimize the loss function of the Federal Reserve.

2. The model parameter values are assumed to remain the same for three policy regimes, while the policy rule coefficients of the interest rate equation are allowed to change across regimes.

3. Parameter-estimate significance for an off-diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.

4. To guarantee that the diagonal transition probabilities are highly persistence, in the estimation they were restricted to be no less than 0.900.

5. Parameter-estimate significance for a diagonal probability of the transition matrix is measured by using the p-value of the likelihood ratio test statistic for the hypothesis that the true value of the parameter is 0.995.

Chapter 5

Conclusion

This study implements a unified approach of econometric policy evaluation to investigate "endogenous" evolution of monetary policy over time. The unified approach allows the structural parameters of forward-looking model to be estimated simultaneously with the parameters of the central bank's objective function. A policy regime is defined as a set of preference parameters which central bank chooses for minimizing the value of its objective function. While being computationally challenging, the technique of policy preference identification exhibits several significant advantages. First, describing monetary policy behavior at the level of policy objectives, rather than at the level of reaction function coefficients, allows for direct estimation of the preference parameters in central bank's objective function. Therefore, this methodology is more appropriate when one's primary objective is to investigate evolution of policy preference parameters in the objective function over time. Second, the unified estimation approach allows one to test whether an estimated policy rule is the outcome of central bank's optimal behavior and assess whether observed economic outcomes can be reconciled and accounted for within an optimal policy framework. Indeed, the setup of the problem allows for a formal test of the hypothesis that the policy is implemented in a manner consistent with the loss-minimizing behavior of the central bank. Third, while assuming the policy rule is loss-minimizing, the unified approach to policy preferences estimation also considers additional economic structural equations. Therefore, by estimating monetary policy that was indeed loss-minimizing, the methodology could sharpen estimates of the model parameters.

This study assumes that the evolution of monetary policy depends on a stochastic switching process that evolves according to a Markov chain. The Markov-switching framework is an appealing technique for evaluating monetary policy behavior over time. First, it reflects the common view on the likelihood that monetary policy may take the form of an abrupt shift from one policy regime to another. Second, the technique reveals a new policy regime rather than making assumption on the timing of a switch in policy regime. At the same time this approach can utilize full-sample information to make accurate inference. Finally, using the Markov-switching approach respects the Lucas critique in monetary policy evaluation. According to the Lucas critique, policy changes are embedded in private agents' perception, and agents will change their expectations and decisions accordingly with changes in policy. Markov-switching models are expected to fit data well if there were policy changes and important expectation effects of these policy changes on private sectors' behavior.

This study estimates two different types of models subject to the condition that the policy equation minimizes a well-defined loss function. Both models are New Keynesian in spirit in that policy affects aggregate demand through a conventional interest rate channel and inflation through a Phillips curve specification. The first model is a baseline model that determines the equilibrium relationship among the output gap, the inflation rate, and the short-term interest rate controlled by the central bank. While sharing some broad characteristics, the second model differs substantially from the baseline model by introducing oil as an input in the production process. Therefore, the second model has a more complex relationship between the structural and reduced form parameters.

Estimating the baseline model with data from 1965 through 2001 suggests three different policy regimes can be better used to describe U.S. monetary policy than two policy regimes. Policy regime one is a special regime that only prevailed between 1979 and 1984. During this period, the Federal Reserve successfully reduced high inflationary pressure in the U.S., and this period of time is commonly known as the Volcker disinflation period. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during the expansionary periods, and regime three is a regime which the monetary policy switched into during the recessionary periods. The results provide evidence that inflation stabilization was a far more important policy objective of the Federal Reserve than output and interest rate stabilizations throughout the sample period. Finally, the data can formally reject the hypothesis that U.S. monetary policy was loss-minimizing.

Estimating the baseline model with extended data from 1965 through 2008 shows similar results to those obtained with the shorter sample data. Indeed, three different policy regimes provide a better description of U.S. monetary policy than two policy regimes. Policy regime one is a special regime that only prevailed between 1979 and 1984 and is commonly known as the Volcker disinflation period. Regime two is a regime prevailed during the economic expansionary periods, and regime three is a regime prevailed during the economic recession periods. In all policy regimes, the inflation stabilization objective is found to dominate the Federal Reserve's preferences. Contrary to the previous findings, the results here suggest that the Federal Reserve did follow an optimal policy during the extended sample period. The Federal Reserve's monetary actions were very close to their optimal counterparts during policy regimes two and three.

The results from estimating the augmented model support the presence of three policy regimes. Policy regime one is a regime that only prevailed during the deep post-war recessions began in 1973, 1980, 1981, and 2007. Policy regime two is a regime which the Federal Reserve's monetary policy switched into during the post-1985 expansionary periods and mild recessions in 1990 and 2000. Policy regime three is a regime which the monetary policy switched into during the pre-1985 expansionary periods and mild recession in 1970. Therefore, the important findings associated with the three-regime monetary policy are robust to estimations using a variation of the structural model. The results here still suggest that three different policy regimes can provide a cogent description of U.S. monetary policy although the interpretation for each policy regime is different from that in the previous findings. In line with the previous findings, the results also suggest the Federal Reserve consistently placed greater weight on inflation stabilization than on output and interest rate stabilizations in all three policy regimes including the economic expansionary periods. Lastly, the Federal Reserve did follow an optimal policy during the extended sample period. This conclusion is also consistent with that in the previous findings. The Federal Reserve's monetary action was very close to their optimal counterpart during policy regime two.

The analysis of this study suggests several avenues for the future research. One interesting extension would be to experiment with a broader class of central bank objective function. Central banks may respond to far-from-target realizations of inflation or output by raising the weight placed on stabilizing the variable that has strayed. For instance, a central bank might only change preference weights on objectives by responding to inflation that exceeds the target band and pursue different objectives when inflation stays within the band. Second, it would be interesting to experiment with different specifications of central bank's reaction function. For instance, one can model monetary policy as optimal discretion or commitment rather than the Taylor-type policy rule. Finally, it is interesting to apply the approach in this study to data for different countries and to different and larger economic models.

Appendix A

Descriptions of Baseline Model Solution and Estimation Algorithms

A.1 Model Solution Algorithm

It is straightforward to rewrite the reduced-form model equations in the text as the following form

$$A \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ X_t \end{bmatrix} = B \begin{bmatrix} y_t \\ \pi_t \\ X_{t-1} \end{bmatrix} + DS_t, \tag{A-1}$$

where $X_t = [y_t \quad \pi_t \quad r_t \quad y_{t-1}]'$ is a vector of state variables, $S_t = [\varepsilon_{y,t} \quad \varepsilon_{\pi,t} \quad \varepsilon_{r,t}]'$ is a vector of structural shocks, and where A, B, and D are 6×6, 6×6, and 6×3 matrices, respectively.

Since matrix A can be singular, Klein (2000) suggests computing a QZ decomposition to obtain generalized eigenvalues: for any pair of square matrices like A and B, there exist matrices Q and Z and upper triangular matrices S and T such that

$$\begin{aligned} A &= Q'SZ', & \text{(A-2)} \\ B &= Q'TZ', & \text{(A-3)} \\ QQ' &= ZZ' = I & \text{(A-4)} \end{aligned}$$

Furthermore, Q and Z are reordered so that all possible zeros of S occur in the lower right corner and such that the remaining ratios t_{ii}/s_{ii} of diagonal elements in T and S

are non-decreasing in absolute value as one moves down the diagonal. These ratios are the generalized eigenvalues of the pair A and B. If the number of unstable eigenvalues equals the number of non-predetermined variables, then there exists a unique saddle path solution for the model that may be written as a first-order vector autoregression

$$X_t = GX_{t-1} + \varphi_t, \tag{A-5}$$

where $G \equiv Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}$ is the reduced form model solution, Z_{11}, S_{11}, and T_{11} are the 4×4 lower left blocks of matrices Z, S, and T respectively. φ_t is the 4×1 vector of reduced-form errors with typical elements φ_{kt}, $k \in \{1, \ldots, 4\}$, and $\varphi_{4t} = 0$. For the model used in the text, a unique saddle path solution exists if there are four stable and two unstable eigenvalues.

A.2 Model Estimation Algorithm

The parameters of the model are divided into three groups: V, Δ, and W. Vector V contains the parameters of the IS equation, Phillips curve, and Choleski decomposition of the covariance matrix of the reduced-form errors. Δ is a vector that contains the coefficients of the monetary policy rule. W is a vector that contains the parameters of the central bank's loss function, which measure the importance to the central bank on inflation, output, and interest rate stabilizations, respectively. As a normalization, the weight on inflation stabilization is fixed at 1.

SURF algorithm estimates V and Δ without restricting Δ, while SORF algorithm estimates V and W subject to the restriction that the vector Δ is a function of vectors V and W, that is $\Delta = g(V,\ W)$. Hence SORF nests the central bank's minimization problem as described in equation (2.8) inside the model solution algorithm. For given V and Δ, the SURF and SORF algorithms use the algorithm of Klein (2000) described previously to compute the solution for the model, which is then used to compute predicted values for X_t in equation (2.5) and to compute residuals and log likelihood values. Because the algorithm of searching model solution may try values of V and Δ that do not have a unique solution for the rational expectation system, in light of Salemi (2006) the maximand used in the SURF and SORF estimations is log likelihood minus a penalty value. Let J_1 through J_6 be the eigenvalues of matrix A in equation (2.4) sorted in ascending order. If a unique saddle path exists, then $|J_k| > 1$ for $k = 1$ and 2 and $|J_k| < 1$ for $k = 3, \ldots, 6$. Let I_k be the indicator function that equals 0 if the root condition is satisfied for the kth root and 1 if it is not. The penalty function is

$$\textbf{Penalty} = \bar{p} \sum_{k=1}^{6} I_k(|J_k| - 1)^2,$$

where $\bar{p}$ is a constant chosen to guarantee that the value of **Penalty** is large relative to log likelihood. If the root condition is satisfied, **Penalty** $= 0$. If it is not satisfied, **Penalty** is a smooth, positive function of the differences between out-of-bound roots and 1. When the root condition is violated, the Klein formula for G in equation (2.5) can evaluate to complex numbers. In order to guarantee that the solution algorithm search all points in the parameter space, the elements of G are set to the real part of the corresponding solutions of Klein (2000) algorithm. When the saddle path condition is met, the conversion has no effect.

Having obtained the model solution, both SURF and SORF use the Markov-switching algorithm to estimate the model with n_j policy regimes. The Markov-switching algorithm takes the previous values of the probabilities as input, then uses these results to get the unconditional joint distribution of X_t, which is calculated by summing conditional distribution of X_t over possible number of regimes n_j.

$$f(X_t|I_{t-1}, \Theta) = \sum_{k=1}^{n_j} P(j_t = k|I_{t-1}, \Theta) \cdot f(X_t|j_t = k, I_{t-1}, \Theta), \tag{A-6}$$

where $I_{t-1} = (X_{t-1}, \ldots, X_1)$ denotes information set available at time $t-1$. Θ is the vector of parameters, including all parameters of the model and transition probabilities. The weights are probabilities of being in n_j policy regimes.

The conditional distribution $f(X_t|j_t = k, I_{t-1}, \Theta)$ is the conditional normal density function for the regime $j_t = k$:

$$f(X_t|j_t = k, I_{t-1}, \Theta) = (2\pi)^{-\frac{q}{2}} \cdot |\Sigma_k^{-\frac{1}{2}}| \cdot \exp\left\{ -\frac{u'_{t,k}\Sigma_k^{-1}u_{t,k}}{2}\right\}, \tag{A-7}$$

where q is the number of variables in X_t, Σ_k is the non-zero block of the variance-covariance matrix of φ_t in regime $j_t = k$, $u_{t,k}$ is a vector of errors between the actual values and the predicted values of X_t in regime k.

To get the values of parameters Θ, the algorithm maximizes the log value:

$$L(\Theta) = \sum_{t=1}^{T} \ln(f(X_t|I_{t-1}, \Theta)). \tag{A-8}$$

The Markov-switching algorithm also updates the probability of being in each policy regime at every period t based on the information up to that date, which is known as filtered probability. The filtered probability is computed using the iterative filter proposed by Kim and Nelson (1999):

$$P(j_t = k|I_t, \Theta) = \frac{P(j_t = k|I_{t-1}, \Theta) \cdot f(X_t|j_t = k, I_{t-1}, \Theta)}{\sum_{k=1}^{n_j} P(j_t = k|I_{t-1}, \Theta) \cdot f(X_t|j_t = k, I_{t-1}, \Theta)}. \tag{A-9}$$

To implement the maximization of the log likelihood in (A-8) with respect to model parameters, this paper utilizes the Hooke-Jeeves method that is a direct search hill-climber. This method maximizes the log likelihood function without any explicit or implicit derivative information, which is particularly robust in cases when the objective function is highly non-smooth[50]. The optimization procedure is sensitive to the initial step size, thus this study restarts the maximization algorithm several times. At each time the algorithm uses the previous terminating values of the parameters as the new starting points and reduces the search step size.

[50] All earlier attempts to use derivative based hill-climber were unsuccessful, which can be explained by the high degree of non-smoothness exhibited by the log likelihood function.

Appendix B

A New Keynesian Model with Oil Price

To make the study self-contained, this section lays out the detailed derivation of key structural equations implied by an oil-producing model economy that is similar to the dynamic stochastic general equilibrium models used in Castillo, Montoro and Tuesta (2007) and Montoro (2010). The model's dynamics are enriched by indexation of oil price and introducing oil as a production input. Here oil is represented by M, and the real price of oil is denoted by Q that can be derived to be exogenous.

B.1 Households

The economy is populated by a continuum of households on a unit interval. A representative household $i \in [0, 1]$ maximizes the expected inter-temporal discounted utility function defined over consumption C_t and labor L_t

$$U_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U\Big(C_{t-t_0}, L_{t-t_0}\Big), \tag{B-1}$$

where $\beta \in (0, 1)$ is the household's subjective discount factor, and $U_t(\cdot)$ is the instantaneous utility function assumed to be separable across consumption and labor.

The specific functional form of the period utility function is

$$U\Big(C_t, L_t\Big) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu}, \tag{B-2}$$

where σ and ν represent the coefficient of risk aversion and the inverse of the elasticity of labor supply, respectively.

The budget constraint for a typical household is given by

$$P_t C_t + \frac{B_t}{R_t} = W_t L_t + B_{t-1} + \Gamma_t + T_t. \tag{B-3}$$

The household receives labor income $W_t L_t$, where W_t is the nominal wage. It also receives Γ_t the share of the representative household on total nominal profits created by firms, and T_t transfers from the government, respectively. The household uses these funds to finance consumption C_t, which costs P_t per unit. There is a private financial bond in the economy. In every period t, a household purchases bond B_t, where B_t is the end of period nominal bond holdings. R_t is the nominal gross interest rate.

A representative household solves its utility maximization problem subject to the real budget constraint. The set of first order conditions from the household optimization problem can be written as follows

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right], \tag{B-4}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\nu} = MRS_t. \tag{B-5}$$

Equation (B-4) is the standard Euler equation that determines the optimal path of consumption. At the optimum, the representative consumer is indifferent between consuming today or tomorrow. Equation (B-5) describes the optimal labor supply decision. MRS_t denotes the marginal rate of substitution between labor and consumption.

In this study, labor markets are assumed to be competitive. Each household is endowed with one unit of potential work-time every period which it supplies inelastically to the labor market. Households work in each intermediate good producer $z \in [0, 1]$.

Therefore, L_t corresponds to the aggregate labor supply in period t

$$L_t = \int_0^1 L_t(z)dz. \tag{B-6}$$

B.2 Final Goods Producers

There is a continuum of final goods producers of mass one, indexed by $f \in [0,1]$ that operate in an environment of perfect competition. Each final good producer uses intermediate goods as inputs, indexed by $z \in [0,1]$ to produce final good using the following technology

$$Y_t(f) = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{B-7}$$

where ε is the elasticity of substitution between intermediate goods.

Y_t represents the aggregate level of output

$$Y_t = \int_0^1 Y_t(f)df. \tag{B-8}$$

Then the demand function of each type of differentiated good is obtained by aggregating the input demand of final goods producers

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t, \tag{B-9}$$

where the aggregate price level is equal to the marginal cost of the final goods producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{B-10}$$

B.3 Oil Producers

A representative household owns a continuum of oil firms, indexed by $j \in [0,1]$ that operate in a perfect competitive market. Each firm produces a quantity $X_t(j)$ of oil according to the technology

$$X_t(j) = Z_t I_t(j), \tag{B-11}$$

where Z_t is a component of aggregate productivity common to all oil firms, and the input $I_t(j)$ is purchased from the final goods producers as is the consumption of the representative household.

The aggregate productivity of the oil sector, Z_t, is assumed to vary according to a stationary AR(1) stochastic process in natural logs

$$\ln(Z_t) = ln(\overline{Z}) + \rho \ln(Z_{t-1}) + \varepsilon_{z,t}, \tag{B-12}$$

where coefficient $\rho \in (0,1)$ measures the persistence in the process, and $\varepsilon_{z,t}$ is the shock process. As a result, $1/Z_t$ measures the real marginal cost of all oil firms.

The produced oil is sold at the real price Q_t, which the oil producers take as given. Hence, $Q_t = 1/Z_t$, and the real oil price, Q_t, also follows an AR(1) process in logs

$$\ln(Q_t) = ln(\overline{Q}) + \rho \ln(Q_{t-1}) + \varepsilon_{q,t}, \tag{B-13}$$

where $\overline{Q}$ is the steady state level of oil price. $\varepsilon_{q,t}$ is an independent and identically distributed random variable with mean zero and variance σ_q^2.

The total amount of final goods demanded by competitive oil producers is

$$I_t = \int_0^1 I_t(j)dj. \tag{B-14}$$

The total amount of oil produced by the competitive oil producers as a whole is given by

$$X_t = \int_0^1 X_t(j)dj. \tag{B-15}$$

B.4 Intermediate Goods Producers

There is a continuum of intermediate goods producers indexed by $z \in [0,1]$ that operate in a monopolistic competitive market. Each firm specializes in production of a single differentiated good using CES technology with two inputs of production, labor L_t and oil M_t

$$Y_t(z) = \left[(1-\alpha)(L_t(z))^{\frac{\psi-1}{\psi}} + \alpha(M_t(z))^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{B-16}$$

where ψ represents the intratemporal elasticity of substitution between labor and oil inputs, α denotes the share of oil in the production function. Generally, ψ is assumed to be less than one.

From the cost minimization problem of the firm, one can obtain an expression for real marginal cost

$$MC_t(z) = \left[(1-\alpha)^{\psi}\left(\frac{W_t}{P_t}\right)^{1-\psi} + \alpha^{\psi}(Q_t)^{1-\psi}\right]^{\frac{1}{1-\psi}}, \tag{B-17}$$

where $MC_t(z)$ represents the real marginal cost, W_t is nominal wage, and P_t is consumer price index. Since technology has constant returns to scale and factor markets are competitive, marginal costs are the same for all intermediate firms, $MC_t(z) = MC_t$.

The individual firm's labor demand is given by

$$L_t^d(z) = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t(z), \tag{B-18}$$

and oil demand is given by

$$M_t(z) = \left(\frac{1}{\alpha}\frac{Q_t}{MC_t}\right)^{-\psi} Y_t(z). \tag{B-19}$$

B.5 Price Setting

Final goods producers operate under perfect competition and therefore take the price level P_t as given. In contrast, intermediate goods producers operate under monopolistic competition and face a downward-sloping demand curve for their products, whose price elasticity is positively related to the degree of competition in the market. As a result, intermediate goods producers enjoy monopoly power over the goods they produce, and they can treat the price of the good as a choice variable. Under the flexible pricing mechanism, each intermediate good producing firm's optimization problem is to choose P_t to maximize its profits while producing whatever quantity of its good demanded at this price.

Many studies have provided ample evidence for sluggish price adjustment in goods market. Producers do reset prices only from time to time. This study introduces a staggered pricing mechanism as in Galí and Gertler (1999), who use a variant of the mechanism formulated by Calvo (1983). In each period, a share of $1-\theta$ of the firms are allowed to change prices while the remaining firms keep prices fixed. Of the firms which are allowed to change their prices, a fraction $1-\omega$ do so in an optimal, forward-looking manner while a fraction ω instead set the new price using a rule of thumb, which is based on past price developments. The reason for firms to use the backward-looking manner is that there are some costs associated with the process of setting an optimal price, such as information gathering costs, decision making costs, and contractual obligations.

A firm who can reoptimize in a given period will choose $P_t^f(z)$ to maximize expected discounted profit

$$E_t \sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} \Gamma(P_t^f(z), P_{t+k}, MC_{t+k}, Y_{t+k}), \tag{B-20}$$

where $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor. The function $\Gamma(P_t^f(z), P_{t+k}, MC_{t+k}, Y$ $[(1-\tau)P_t^f(z) - P_{t+k}MC_{t+k}]\left(\frac{P_t^f(z)}{P_{t+k}}\right)Y_{t+k}$ is the after-tax nominal profits of the supplier of good z with price $P_t^f(z)$, where the aggregate demand and aggregate marginal cost are Y_{t+k} and MC_{t+k}, respectively. τ is the proportional tax on sale revenue and is assumed to be constant. The optimal price that is chosen to solve this firm's problem is given by

$$\frac{P_t^f(z)}{P_t} = \frac{\mu E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} MC_{t+k} F_{t+k}^{\varepsilon+1} Y_{t+k} \right]}{E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k} \right]}, \tag{B-21}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon - 1}/(1-\tau)$ is the price markup net of taxes, $F_{t+k} = \frac{P_{t+k}}{P_t}$ is the cumulative level of inflation. The optimal price is determined by the average of expected future marginal costs, and it can be rewritten as follows

$$\frac{P_t^f(z)}{P_t} = \mu E_t \left[\sum_{k=0}^{\infty} \varphi_{t,t+k} MC_{t+k} \right], \tag{B-22}$$

where

$$\varphi_{t,t+k} \equiv \frac{\theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon+1} Y_{t+k}}{E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{\varepsilon} Y_{t+k} \right]}. \tag{B-23}$$

A backward-looking firm is assumed to follow a rule of thumb as the following

$$\frac{P_t^b(z)}{P_{t-1}^b(z)} = \left(\frac{P_{t-1}^*}{P_{t-2}}\right)\left(\frac{P_{t-1}}{P_{t-1}^b(z)}\right), \tag{B-24}$$

where $P_t^b(z)$ is the price charged by a backward-looking firm z that is adjusting its price at t, P_{t-1}^* is an index of new prices posted at $t-1$. According to equation (B-24), the gross price adjustment at t under the rule, $\frac{P_t^b(z)}{P_{t-1}^b(z)}$, equals the average gross price adjustment by those who changed price in the previous period, $\frac{P_{t-1}^*}{P_{t-2}}$, times a factor $\frac{P_{t-1}}{P_{t-1}^b(z)}$ which corrects for the firm's relative price in that period.

Galí and Gertler (1999) show that $\frac{P_{t-1}^*}{P_{t-2}}$ provides a guide as how much the firm should change price at t since it reflects the average adjustment by firms that had to make a similar decision just one period earlier. This measure is adjusted by the firm's relative price at t to correct for the initial position of the firm's price relative to the average. Therefore, a firm that has its price above average in $t-1$ will increase it by less at t, everything else equal.

Equation (B-24) can be simplified to

$$P_t^b(z) = \left(\frac{P_{t-1}^*}{P_{t-2}}\right) P_{t-1}. \tag{B-25}$$

Thus, all rule of thumb adjusters choose the same price P_t^b at period t.

The aggregate price index, defined as the price of the final good that minimizes the cost of the final goods producers, can be expressed as

$$P_t = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{B-26}$$

with

$$P_t^* = \left[\omega (P_t^b)^{1-\varepsilon} + (1-\omega)(P_t^f)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{B-27}$$

These equations are used to derive the recursive representation of the Phillips curve.

B.6 Government

In the model, this paper abstracts from any other roles for the government and assumes that it runs a balanced budget every period. The government collects a proportional tax on sale revenues and transfers all the revenues to households. The budget constraint implies that total net transfer in period t in real terms is

$$\frac{T_t}{P_t} = \tau Y_t. \tag{B-28}$$

B.7 Monetary Policy

A central bank conducts monetary policy by targeting the nominal interest rate in the following way

$$R_t = R_{t-1}^{\phi_r} \left[\overline{R} \left(\frac{\Pi_{t-1}}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_{t-1}}{\overline{Y}} \right)^{\phi_y} \left(\frac{Q_{t-1}}{\overline{Q}} \right)^{\phi_q} \right]^{1-\phi_r} \exp(\varepsilon_{r,t}), \tag{B-29}$$

where $\phi_\pi > 0$, $\phi_y > 0$, and $\phi_q > 0$ measure the response of the nominal interest rate to lagged terms of inflation, output, and oil price, respectively. Also, the policy rule allows for an inertial adjustment of interest rates. The degree of interest rate smoothing is measured by $0 \leq \phi_r \leq 1$. The steady state values are expressed without a time subscript and with an upper bar. The term $\varepsilon_{r,t}$ is a shock to monetary policy and follows a normal distribution with mean zero and variance σ_r^2. The central bank's monetary policy rule is a Taylor-type reaction function, and it is used to close the model.

B.8 Market Clearing

In equilibrium, labor, oil, intermediate goods, and final goods markets clear. The output of final goods is demanded by two types of buyers: households that account for demand C_t, and oil producers that purchase I_t:

$$Y_t = C_t + I_t. \tag{B-30}$$

The oil market clearing condition is given by:

$$X_t = M_t, \tag{B-31}$$

where the demand for oil comes from the aggregation of individual intermediate producers

$$\begin{aligned} M_t &= \int_0^1 M_t(z)dz = \left(\frac{1}{\alpha}\frac{Q_t}{MC_t}\right)^{-\psi} \int_0^1 Y_t(z)dz & \text{(B-32)} \\ &= \left(\frac{1}{\alpha}\frac{Q_t}{MC_t}\right)^{-\psi} Y_t \Delta_t, & \text{(B-33)} \end{aligned}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well, implying that is not possible to use the usual representative firm assumption. Therefore, the price dispersion factor, Δ_t appears in the aggregate labor demand equation.

The labor market clearing condition is given by:

$$L_t = L_t^d, \tag{B-34}$$

where the demand for labor is the aggregation of individual intermediate producers in the same way as for the labor supply

$$\begin{aligned} L_t^d &= \int_0^1 L_t^d(z)dz = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} \int_0^1 Y_t(z)dz & \text{(B-35)} \\ &= \left(\frac{1}{1-\alpha}\frac{W_t P_t}{MC_t}\right)^{-\psi} Y_t \Delta_t. & \text{(B-36)} \end{aligned}$$

B.9 The Steady State

Here, variables in the steady state are denoted overlined (i.e. $\overline{X}$). The non-stochastic steady state of endogenous variables for $\overline{\Pi} = 1$ is provided as the following.

Interest rate in the steady state is

$$\overline{R} = \beta^{-1}. \tag{B-37}$$

Real marginal cost in steady state is

$$\overline{MC} = \frac{1-\tau}{\varepsilon/(\varepsilon-1)} \leq 1. \tag{B-38}$$

Real wage in steady state is given by

$$\overline{W/P} = (1-\alpha)\overline{MC}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1}{1-\psi}}, \tag{B-39}$$

where $\overline{\alpha} \equiv \alpha^{\psi}\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi}$ is the share of oil on total cost in the steady state.

Output in steady state is given by

$$\overline{Y} = ((1-\alpha)\overline{MC})^{\frac{1}{\sigma+\nu}}(1-\overline{\alpha}\overline{MC})^{-\frac{\sigma}{\sigma+\nu}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}. \tag{B-40}$$

Labor in steady state is

$$\overline{L} = ((1-\alpha)\overline{MC})^{\frac{1}{\sigma+\nu}}(1-\overline{\alpha}\overline{MC})^{-\frac{\sigma}{\sigma+\nu}}\left(\frac{1-\overline{\alpha}}{1-\alpha}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}. \tag{B-41}$$

Note that, from the definition of $\overline{\alpha}$, the steady state values of real wage, output, and labor depend on the steady state ratio of oil price with respect to the real marginal cost. This implies that a permanent change in oil price would generate a permanent increase in $\overline{\alpha}$ given $\psi < 1$, and it would also lead to changes in the steady state of real wage, output, and labor.

In standard New Keynesian models, the real marginal cost in steady state is equal to the inverse of the mark-up. Since monopolistic competition and tax affect the steady state of the model, output in steady state can be below the efficient level. Therefore, the steady state is distorted. In the special case that $\tau = -1/(\varepsilon - 1) < 0$, distortions are eliminated and the steady state is efficient. One can denote the steady state distortion by

$$\Phi = 1 - \frac{1-\tau}{\varepsilon/(\varepsilon - 1)}. \tag{B-42}$$

Hence $\Phi = 0$ when a subsidy on sales makes the steady state undistorted.

B.10 The Log-Linear Economy

The non-linear model can be approximated linearly. The linearization simplifies the model equations and illustrates the effects of oil price in the dynamic equilibrium of the economy. The approximate linear relations are obtained from taking a log-linear approximation of variables around their deterministic steady state values. This paper denotes variables in their log-deviations around the steady state with lower case letter, $x_t = \ln(X_t/\overline{X})$.

After imposing the goods, oil, and labor market clearing conditions and eliminating real wage from the system, the dynamics of the economy are determined by the following equations:

$$
\begin{aligned}
l_t &= y_t - [(\sigma+\nu)\zeta_y y_t - \zeta_q q_t], && \text{(B-43)}\\
mc_t &= \chi(\nu+\sigma)y_t + (1-\chi\xi)q_t, && \text{(B-44)}\\
\pi_t &= \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \kappa mc_t, && \text{(B-45)}\\
y_t &= E_t y_{t+1} - \frac{1}{\sigma\vartheta_y}[r_t - E_t\pi_{t+1} + \sigma\vartheta_q(q_{t+1}-q_t)], && \text{(B-46)}\\
q_t &= \rho q_{t-1} + \varepsilon_{q,t}, && \text{(B-47)}
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta_y &= \psi\Big[\Big(\frac{1}{1+\nu\psi}\Big)\Big(1+\frac{\psi}{1+\nu\psi}\varpi_y\chi\Big)-\chi\Big],\\
\zeta_q &= \psi\Big[\Big(\frac{\psi}{1-\nu\psi}\Big)(1-\chi\xi)\varpi_y-\varpi_q\Big],\\
\varpi_y &= \frac{\nu-\overline{\alpha}\overline{MC}(\nu+\sigma)}{1-\overline{\alpha}\overline{MC}},\\
\varpi_q &= \frac{\overline{\alpha}\overline{MC}(1-\psi)\sigma}{(1-\overline{\alpha}\overline{MC})(1+\nu\psi)},\\
\chi &= \frac{(1-\overline{\alpha})(1-\overline{\alpha}\overline{MC})}{(1-\overline{\alpha}\overline{MC})(1+\overline{\alpha}\nu\psi)+\overline{\alpha}\overline{MC}\psi\sigma(1-\overline{\alpha})},\\
\xi &= 1+\frac{\overline{\alpha}\overline{MC}\sigma}{(1-\overline{\alpha}\overline{MC})},\\
\gamma_b &= \frac{\omega}{\theta+\omega(1-\theta(1-\beta))},\\
\gamma_f &= \frac{\beta\theta}{\theta+\omega(1-\theta(1-\beta))},\\
\kappa &= \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta+\omega(1-\theta(1-\beta))},\\
\vartheta_y &= 1-\Big(\frac{\overline{\alpha}\overline{MC}}{1-\overline{\alpha}\overline{MC}}\Big)\psi\chi(\nu+\sigma),\\
\vartheta_q &= \Big(\frac{\overline{\alpha}\overline{MC}}{1-\overline{\alpha}\overline{MC}}\Big)\Big[1-\psi\chi\Big(1+\frac{\overline{\alpha}\overline{MC}\sigma}{1-\overline{\alpha}\overline{MC}}\Big)\Big].
\end{aligned}
$$

ζ_q and $(1-\chi\xi)$ account for the effects of oil price in labor and marginal cost, respectively.

κ is the elasticity of inflation respect to marginal cost.

Equation (B-45) is generated by the variant of the Calvo pricing structure. In the equation, inflation depends on its past value as well as its future value, this framework is capable of generating significant inflation inertia.

Interestingly, the effect of oil price on marginal cost, given by $(1 - \chi\xi)$ in equation (B-44), depends crucially on the quasi-share of oil in the production function α and on the elasticity of substitution between oil and labor ψ. Thus when α or ψ changes, marginal cost changes correspondingly. It is important to note that even though the quasi-share of oil in the production function α can be small, its impact on marginal cost $\overline{\alpha}$ can be magnified when oil has few substitutes, that is when ψ is low. Moreover, a permanent increase in real oil price $\overline{Q}$ or in the distortions of steady state such as a decrease in $\overline{MC}$, would increase $\overline{\alpha}$ therefore make the marginal cost of firm change its sensitivity to oil price.

B.11 Derivation of Efficient Output Level

Here output gap is denoted by x_t, and it is defined as the difference between the sticky price level of output and its corresponding efficient level, $x_t = y_t - y_t^E$, where y_t^E denotes the log deviation of the efficient level of output. In this economy, the efficient allocation is achieved when $\overline{MC} = 1$, which corresponds to the equilibrium where intermediate firms are perfectly competitive.

To derive the efficient level of output, it is convenient to first derive the level of output under flexible price. Rewrite equation (B-44) as

$$mc_t = \frac{\kappa_y}{\kappa_d} y_t + \frac{\kappa_q}{\kappa_d} q_t, \tag{B-48}$$

where $\kappa_y = (1-\overline{\alpha})(\nu+\sigma)(1-\overline{\alpha}\overline{MC})$, $\kappa_q = (1-\overline{\alpha}\overline{MC})\overline{\alpha}(1+\nu\psi) - \overline{\alpha}\overline{MC}\sigma(1-\psi)(1-\overline{\alpha})$, $\kappa_d = (1-\overline{\alpha}\overline{MC})(1+\overline{\alpha}\nu\psi) + \overline{\alpha}\overline{MC}\psi\sigma(1-\overline{\alpha})$.

Under flexible price, $mc_t = 0$. Condition that defines the natural level of output in terms of the oil price

$$y_t^F = -\frac{\kappa_q}{\kappa_y} q_t. \tag{B-49}$$

Notice that in this economy the flexible price level of output does not coincide with the efficient one since the steady state is distorted by monopolistic competition. In this study, the efficient level of output is defined as the level of output with flexible price under perfect competition, one can use equation (B-48) to calculate this efficient level of output under the condition that $\mu = 1$ as follows

$$y_t^E \approx -\left(\frac{\alpha^E}{1-\alpha^E}\right)\left(\frac{1-\overline{\alpha}}{\overline{\alpha}}\right)\frac{\kappa_q}{\kappa_y} q_t, \tag{B-50}$$

where $\alpha^E = \alpha^{\psi}(\overline{Q})^{1-\psi}$. This parameter can be also expressed in terms of the participation of oil under flexible price as follows

$$\alpha^E = \overline{\alpha}\mu^{\psi-1}. \tag{B-51}$$

Notice that when there is no monopolistic distortion or when $\psi = 1$, $\alpha^E = \overline{\alpha}$ and $y_t^E = y_t^F$.

Using the definition of efficient level of output, one can write the marginal cost equation in terms of the efficient output gap x_t, where $x_t = y_t - y_t^E$ in the following way

$$mc_t = \frac{\kappa_y}{\kappa_d}(y_t - y_t^E) + \frac{1}{\kappa_d}\mu_t, \tag{B-52}$$

where

$$\mu_t = \kappa_y\left[1 - \left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)\left(\frac{1-\alpha^E}{\alpha^E}\right)\right] y_t^E. \tag{B-53}$$

Using equations (B-45) and (B-52), the Phillips curve can be written as follows

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \frac{\kappa \kappa_y}{\kappa_d} x_t + \frac{\kappa}{\kappa_d} \mu_t, \tag{B-54}$$

One can further write μ_t in terms of the oil price using the definition of the efficient level of output

$$\mu_t = \kappa_q \left[\frac{\overline{\alpha} - \alpha^E}{(1-\alpha^E)\overline{\alpha}} \right] q_t. \tag{B-55}$$

The dynamic IS equation can also be written in terms of the efficient output gap

$$x_t = E_t x_{t+1} - \frac{1}{\sigma \vartheta_y} [r_t - E_t \pi_{t+1} - r_t^E], \tag{B-56}$$

where r_t^E is the natural interest rate that is the real interest rate consistent with y_t^E. r_t^E can be written as follows

$$r_t^E = \sigma \left[\vartheta_q + \left(\frac{\alpha^E}{1-\alpha^E} \right) \left(\frac{1-\overline{\alpha}}{\overline{\alpha}} \right) \frac{\kappa_q}{\kappa_y} \right] E_t[\Delta q_{t+1}], \tag{B-57}$$

where the expected variation in the level of real oil price is

$$E_t[\Delta q_{t+1}] = E_t q_{t+1} - q_t. \tag{B-58}$$

As a result, the economy can be represented by two equations in terms of the efficient output gap x_t and inflation π_t

$$x_t = E_t x_{t+1} - \frac{1}{\sigma \vartheta_y} [r_t - E_t \pi_{t+1} - r_t^E], \tag{B-59}$$

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \frac{\kappa \kappa_y}{\kappa_d} x_t + \kappa_\mu q_t. \tag{B-60}$$

where $\kappa_\mu = \frac{\kappa\kappa_q}{\kappa_d}\left[\frac{\overline{\alpha}-\alpha^E}{(1-\alpha^E)\overline{\alpha}}\right]$. One can notice that when there is no monopolistic distortion or when $\psi = 1$, $\alpha^E = \overline{\alpha}$, which implies that $\kappa_\mu = 0$.

The expression for efficient output level is used to compute the output gap, x_t, which introduces contemporaneous oil price term into the inflation equation. Thus, the model used in this study opens an additional channel for affecting inflation dynamics. This extra source is a special feature of the economy that uses oil as an input. Changes in the nominal oil price affect real oil price, which in turn influences the efficient output level. Variations in the efficient output level translate into fluctuations in the output gap, which feeds into the inflation dynamics.

B.12 Structural Model of the Economy

Since the investigation taken up in this study does not concern the deeper structural parameters in the model, the coefficients in equations (B-59) and (B-60) can be aggregated to obtain the model equations in the text. The following two equations are the IS curve and Phillips curve with reduced form coefficients, which are estimated simultaneously with the central bank's monetary policy.

$$x_t = \lambda E_t x_{t+1} + a_1 x_{t-1} + a_2 x_{t-2} - b(r_t - E_t \pi_{t+1}) + \varepsilon_{x,t}, \tag{B-61}$$

$$\pi_t = \alpha_1 E_t \pi_{t+1} + \alpha_2 \pi_{t-1} + \beta x_t + \eta q_t + \varepsilon_{\pi,t}, \tag{B-62}$$

where π_t, x_t, r_t, q_t can be measured by inflation rate, output gap, short term interest rate, real oil price. $\varepsilon_{x,t}$ and $\varepsilon_{\pi,t}$ are disturbances. Note that the IS equation is amended with two lags of output gap. The lag terms are added to the model since they can help explain the inertial movements of consumption and aggregate output. It is widely agreed that some adjustment process must be added to the equation in order to match the inertial responses of output that are apparent in the data, see e.g. Fuhrer (2000).

$\varepsilon_{x,t}$ represents an aggregate demand shock. One can notice that the natural interest rate r_t^E is incorporated into the disturbance term $\varepsilon_{x,t}$. As a result, it is highly possible that demand shock is correlated with the oil price shock. In the Phillips curve, $\varepsilon_{\pi,t}$ is used to account for supply shocks that shift the inflation-output trade-off. $\varepsilon_{\pi,t}$ is assumed to be an independent and identically distributed random variable with mean zero and variance σ_π^2.

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