

CHARACTERIZING AND FACILITATING PROSPECTIVE TEACHERS' ENGAGEMENT  
WITH STUDENT THINKING ABOUT FRACTIONS

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## **ABSTRACT**

Katherine Baker: Characterizing and Facilitating Prospective Teachers' Engagement with Student Thinking About Fractions  
(Under the direction of Susan N. Friel)

Reform-based mathematics instruction emphasizes that mathematics is learned through reasoning and sense-making rather than strict memorization and is taught through facilitation rather than telling (NCTM, 1989, 1991, 1995, 2000). Teachers' engagement with student thinking to inform instruction is central to such teaching. Engagement with student thinking involves eliciting and using evidence of student thinking in instruction (NCTM, 2014). This case study explored how three prospective teachers placed at 4<sup>th</sup> grade engaged with student thinking in mathematics and used this thinking to guide instruction during six weeks of their student teaching experiences. The prospective teachers were placed at the same school and same grade level and were supported through facilitated team meetings in a community of practice (Wenger, 1998) in which they were coached in the pedagogical and content needs of their instructional units that focused on fractions.

The study addressed four questions: (1) How might a prospective teacher's engagement with student thinking be characterized? (2) In what ways does context influence a prospective teacher's engagement with student thinking? (3) In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions? (4) How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade? Findings revealed that the prospective teachers aligned their beliefs and practices with reform-based mathematics teaching practices

and engaged with student thinking. Although their community of practice was initially facilitated by a participant-researcher, the prospective teachers voluntarily adopted and extended the community structure outside of the researcher-scheduled meetings.

The study highlights the importance of studying and supporting prospective teachers in the context of the student teaching experience in order to help transfer intentions for teaching learned during teacher preparation coursework into practice. The findings offer several contributions to the field, including a suggested model of support to encourage prospective teachers' engagement with student thinking in mathematics and a proposed model for characterizing the nature of such engagement. The models may be used to assess and support prospective teachers in the use of reform-based mathematics practices that focus on student thinking.

For the 4<sup>th</sup> and 5<sup>th</sup> graders who know me as Ms. Phelps, thank you for sharing your  
thinking with me.  
For all the mathematics teachers who listen and learn from children, my many thanks for making  
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## **LIST OF ABBREVIATIONS**

CGI	Cognitively Guided Instruction
CT	Cooperating Teacher
IMAP	Integrating Mathematics and Pedagogy
IRB	Institutional Review Board
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
OGAP	Ongoing Assessment Project
PLC	Professional Learning Community
PT	Prospective Teacher

## CHAPTER 1: INTRODUCTION

*To fail children in mathematics, or to let mathematics fail them, is to close off an important means of access to society's resources. (Schoenfeld, 2002, p. 13)*

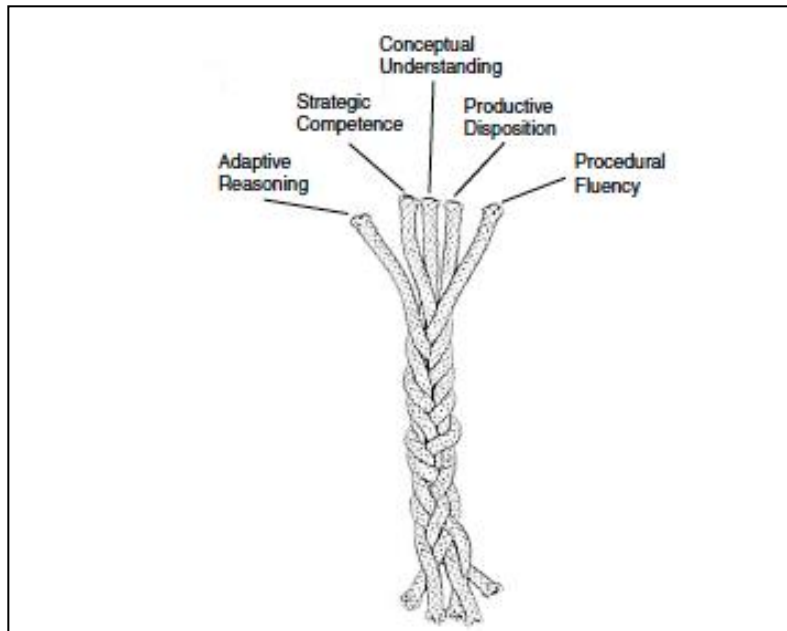
The study of teacher content knowledge in mathematics education has shifted from a study of how teachers understand procedural mathematics operations to a study of how they make sense of mathematics, which includes understanding operations and the concepts of mathematics (Olanoff, Lo & Tobias, 2014). This shift in teacher content knowledge expectations reflects the changing views of mathematics education in the United States. In the 1980s a shift in focus to the cognitive processing of content fueled the development of a research-base that examined children's thinking in mathematics and suggested how to facilitate instruction from a child's point of understanding (Carpenter, 1979; Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Carpenter, Hiebert & Moser 1979, 1983; Carpenter & Fennema, 1992; Clements & Battista, 1990; Hiebert et al., 1997; NCTM, 1991; Simon, 1995). This research, accompanied with dismal student mathematics scores reflecting the 1970s Back to Basics movement, helped to motivate the development of the National Council of Teachers of Mathematics (NCTM) Standards that proposed a new model of reform-based mathematics pedagogy (Berry, Ellis & Hughes, 2013; Schoenfeld, 2004).

NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) was the first set of standards to establish a view of mathematics instruction and learning equated with the reform-based movement. The reformed view asserts that mathematics is learned through reasoning and sense-making rather than strict memorization and is taught through facilitation

rather than telling. The *Standards* movement in mathematics education (NCTM, 1989, 1991, 1995, 2000) coupled with the new direction in research about teaching and learning addressed the content of mathematics, how students learn mathematics, and how mathematics is taught K-12.

Learning environments that support students' reasoning and sense making in mathematics focus on discourse and the use of pedagogy that emphasizes facilitation rather than direct instruction (NCTM, 1991). Principles that guide the creation of these environments were articulated within the framework of the standards movement in mathematics (NCTM 1989, 1995). The principles assert that all students deserve access to high quality mathematics instruction and the opportunity to do and understand mathematics. Reform-based mathematics instruction that is focused on problem solving supports the development of a foundation of mathematical competence that provides better opportunities in the life outside the classroom (NCTM, 2000).

NCTM (1991, 1995) focused attention on the characterization of effective mathematics instruction, which highlighted what students need to know and do mathematically with a commitment to challenging and supporting students to learn mathematics conceptually. Student mathematical success is a command of a subset of skills interwoven into a construct identified as mathematical proficiency (NRC; National Research Council, 2001). Mathematical proficiency includes five mathematical strands that can be pictured as interwoven and are conceptually related (see Figure 1.1).



*Figure 1.1. The Intertwined Strands of Proficiency (NRC, 2001)*

The NRC defined the five mathematical strands as follows:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p.

116)

The emphasis on mathematical proficiency implies a shift in how mathematics is learned and taught and highlights the use of student thinking to inform instructional decisions (NCTM, 2014). Effective mathematics teaching that leads to students' mathematical proficiency requires teachers to understand that students bring their own thinking and strategies into the classroom and to position this thinking as valuable to instruction. In short, supporting mathematical proficiency requires that teachers engage with student thinking to provide effective mathematics instruction.

Most recently, the content of K-12 mathematics in the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010) was conceived within a framework of learning progressions or trajectories (Battista, 2011; Clements & Sarama, 2004; Daro, Mosher, & Corcoran, 2011; Fosnot & Dolk, 2001). Again, students are expected to learn mathematics through reasoning and sense making as originally charged by the NCTM standards (1989, 1991, 1995, 2000). Teachers move students through the progression of the content through elicitation of student thinking and integration of this thinking into instruction. Environments that support this type of learning focus on productive discourse and on the use of pedagogy that emphasizes facilitation (NCTM, 1991, 2000, 2014).

The reformed view's historical shifts in the nature and kind of mathematics instruction punctuated the need for responsive teaching (Ball & Forzani, 2011; Empson, 2014; Jacobs & Empson, 2015; Richards & Robertson, 2016). Responsive mathematics teaching facilitates and supports engagement with children's mathematical thinking.

*Responsive teaching* entails taking into account the evidence provided during instruction about children's thinking and its advancement. To teach in ways that are responsive to children's mathematical thinking, teachers need to elicit children's thinking, interpret this thinking, and then 'respond helpfully.' (Empson, 2014, p. 24)

The emphasis on responsive teaching has implications for the design and implementation of teacher preparation programs. There is a need for teacher preparation programs to support a prospective teacher's engagement with student thinking in mathematics instruction.

### **Purpose of Study**

A time-honored practice in teacher preparation programs involves the prospective teacher's participation in field experiences, both as field-based experiences integrated into their coursework, and as a culminating student teaching experience toward the end of their preparation (Darling-Hammond, 2006; Hollins, 2011). The culminating student teaching experience is often a semester-long field-based placement in which the prospective teacher practices her craft in the setting of a practicing cooperating teacher's classroom while a university supervisor provides oversight of the prospective teacher's work. The triad among the prospective teacher, cooperating teacher, and university supervisor is the traditional format of student teaching mentorship and supervision (Borko & Mayfield, 1995; Nguyen, 2009; Rodgers & Keil, 2007; Slick, 1998). The triad members serve as the bridge for the prospective teacher's transition from the teacher preparation program's intended preparation to actual pedagogical practice.

Shifting the focus from the cooperating teacher and university supervisor as the sole bridge from the teacher preparation program to initial licensure, this study explored an additional support of a community of practice (Wenger, 1998). A community of practice is a group of people who share common concerns or passions for something they do and learn how to do this thing better by interacting (Wenger, 1998; Wenger, McDermott, & Snyder, 2002b). Although traditionally an interpersonal structure of practicing professionals rather than prospective professionals, it is within a community of practice that prospective teachers may become their own bridge from teacher preparation program vision to practice.

Wenger (1998) placed learning within the social context of the world (Greeno, 1997;

Lave & Wenger, 1991). The collective learning that occurs within a community of practice results in the development of shared practices that reflect the dynamics, pursuits, and social relations of the group (Wenger, 1998; Wenger, McDermott, & Snyder, 2002a). In this study, a community of practice structure was used as an innovation to the traditional prospective teacher preparation structure of the triad as sole support during the student teaching experience. The cooperating teacher, university supervisor, and prospective teacher still interacted in a traditional triad, but additionally, a group of prospective teachers engaged in a community of practice that provided additional support, clarification, and development related to working with student thinking as the focus of instruction. It is not the cooperating teacher or university supervisor's practice that was examined, but rather, if and how the community of practice structure supported and provided direction for the development of prospective teachers' beliefs and practices. This study investigated the community of practice as a vehicle to help prepare prospective teachers for the work of reform-based mathematics instruction, with specific attention to the work of revealing and using students' thinking in learning fractions.

Currently, there is a lack of research about how to influence prospective teachers' engagement with student mathematical thinking during the student teaching experience. There is an extensive body of research on how to influence prospective teachers' beliefs towards engaging in reformed-based mathematics teaching and learning (Ambrose, 2004; Philipp, 2008; Philipp et al. 2007; Swars, 2005; Thanheiser, Philipp, Fasteen, Strand & Mills, 2013; Wilkins & Brand, 2004). There is also research on how to influence prospective teachers' noticing of student mathematical thinking (Barnhart & van Es, 2015; Sherin & van Es, 2005; Star, Lynch & Perova, 2011; Star & Strickland, 2008; Stockero, 2008). Research also informs us regarding how a practicing teacher's ability to engage with student thinking may be characterized and

influenced (Fennema et al. 1996; Franke, Carpenter, Levi, & Fennema, 2001; Steinberg, Empson, & Carpenter, 2004). This study contributes to a missing area of research; research that investigates if and how context and supports can influence prospective teacher beliefs and actions towards engagement with student thinking, and if and how those influences impact prospective teachers' understanding and use of student thinking in instructional decision making.

### **Overview of Research Design and Questions**

In this study, three prospective elementary teachers formed a community of practice and participated in researcher-facilitated team meetings during the student teaching experience in order to develop and support one another's pedagogical strategies and assessment techniques that focused on the use of student thinking in learning fractions. A case study methodology was used to study the prospective teachers in their community of practice and to consider how the work around engagement with student thinking within the community of practice team meetings was made evident in their teaching practices. It was hypothesized that the intensive focus through case study would allow for new insights that could greatly impact future studies and teacher preparation program structures. The following four research questions were addressed in the context of a 4<sup>th</sup> grade student teaching experience focused on teaching and learning fractions:

1. How might a prospective teacher's engagement with student thinking be characterized?
2. In what ways does context influence a prospective teacher's engagement with student thinking?
3. In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?
4. How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?

The first research question considered a way to modify existing frameworks that characterize teacher engagement with student thinking for use with prospective teachers. The second research question considered the relevance of the prospective teachers' teacher preparation program and student teaching placement school to examine if and how context influenced their engagement with student thinking. Additionally, the question led to an examination of other aspects of context that may influence prospective teacher engagement with student thinking. These include, but were not limited to, cooperating teacher support, classroom dynamics, and/or prior course instruction. The third question delved into the examination of the prospective teacher community of practice and if and how this structure and its supports influenced their engagement with student thinking. The fourth question was dependent on the prior three questions and took into account if this manner of instruction was effective in promoting students' conceptual understanding of fractions.

### **Terminology**

Several key terms were used in the development and presentation of this dissertation research. The researcher and reader require a shared meaning of the following terminology.

*Community of practice* was defined as a group of people who share common concerns or passions for something they do, and learn how to do this thing better by interacting (Wenger, 1998; Wenger, McDermott, & Snyder, 2002b). This study used *team meetings* as required meeting time for the community of practice participants to unpack reform-based mathematics pedagogies. The voluntary aspect of a community of practice (Wenger, 1998) resulted from the participants' willingness to meet and extend their collaboration and cooperative learning outside of the required team meetings.

*Context* was defined as the real-world situation of the case (Yin, 2014). When used in this case study, context refers to the situation of the participants' student teaching experience in both

a university teacher preparation program and an elementary school site. The context introduced elements of potential influence that included participants' prior coursework, interpersonal relationships, and classroom dynamics.

*Engagement with student thinking* was defined as the elicitation and use of evidence of student thinking in instruction (NCTM, 2014). This study's focus was prospective teachers' elicitation and use of students' mathematical ideas and strategies around fractions to inform instruction.

*Fraction* was defined as a non-negative rational number that can be written in the form  $a/b$  where  $a$  and  $b$  are both integers and  $b$  does not equal zero (Behr, Lesh, Post, & Silver, 1983; Charalambous & Pitta-Pantazi, 2005; Kieran, 1976; Olanoff, Lo, & Tobias, 2014). This definition of fraction and its multiple mathematical interpretations and representations are explored in Chapter 2.

### **Dissertation Organization**

This dissertation is organized into five chapters. Chapter 2 provides a review of the literature that influenced the development of the study. Chapter 3 presents the study design and articulates the rationale for qualitative case study methodology. Chapters 4 and 5 present the study's findings, limitations, and implications for teacher preparation programs and future research.

## **CHAPTER 2: LITERATURE REVIEW**

Schoenfeld (2002) claimed that students' poor achievement in mathematics was a result of traditional direct instructional approaches to teaching and learning mathematics. The use of prescriptive curricula that permit teachers little discretion suffocates the intuitive mathematics that students bring to classrooms, yet these lockstep curricula are all too common (Schoenfeld, 2002, 2004). Instead of traditional methods of direct instruction, there is a need for instruction that attends to the ways students think, makes sense of this thinking, and integrates student thinking into the planning of mathematics instruction.

The purpose of this research was to study the community of practice as proposed method of support for elementary prospective teachers that encouraged their engagement with student thinking. The research was guided by the following research questions addressed in the context of a 4<sup>th</sup> grade student teaching experience focused on instruction of fractions:

1. How might a prospective teacher's engagement with student thinking be characterized?
2. In what ways does context influence a prospective teacher's engagement with student thinking?
3. In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?
4. How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?

The study of fractions was the content focus for this dissertation research because it was the topic of the prospective teachers' instructional units taught during their student teaching placements and was the content domain for their teacher preparation program professional portfolio requirements. In the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010), *Number and Operations in Fractions* is a significant content domain in the elementary curriculum. According to the North Carolina Assessment Specifications (North Carolina Department of Public Instruction, 2015) the weight distribution of the fractions content domain assessed on the 4<sup>th</sup> grade End of Grade Assessment is 27-32%. This distribution increases to 47%-52% by 5<sup>th</sup> grade. The NC Department of Public Instruction emphasizes the importance of understanding fraction concepts for elementary students. Additionally, the National Mathematics Advisory Panel (2008) asserted that proficiency with fractions should be a goal for K-8 education because fraction understanding is currently underdeveloped yet is foundational for algebraic understanding. A study focused around the enhancement of prospective teachers' instruction in this content area is justified.

The following literature review provides an overarching conceptual framework focused on teacher engagement with student thinking. Literature was centered on specific themes relevant to the formulation of the study. These areas of investigation included (a) the context of the student teaching experience including relationships formed within that context, (b) the content and pedagogical knowledge and beliefs held by the prospective teachers including those around fractions, and (c) elementary students' understanding of fractions. Taken together, literature within these themes provides the necessary backdrop and argument for the design and implementation of this dissertation research.

## Conceptual Framework

In the 1980s, research and professional development around students' problem-solving strategies with whole number operations emerged as an important focus in the Cognitively Guided Instruction (CGI) program (Carpenter, 1979; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter & Moser, 1984). CGI is grounded in how "teachers use research based knowledge about children's thinking and problem solving to make decisions as they plan and implement instruction, and how this instruction affects their students' learning" (Carpenter & Fennema, 1992, p. 458). CGI was developed as a model of professional development in which listening and attending to students' thinking was at the forefront of instructional decision-making in order to support children's mathematical development (Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2014). Teacher participants in CGI professional development learn the research-base around students' mathematical problem-solving strategies of word problems with various problem structures, and then learn how to use this knowledge to make instructional decisions. Students' understandings are elicited through problem posing, observation, questioning, and discussion. Students listen and learn from one another to further their mathematical development with the guidance of the teacher.

The *Levels of Engagement with Children's Mathematical Thinking* (Fennema et al., 1996; Franke, Carpenter, Levi, & Fennema, 2001; Franke, Fennema, Carpenter, & Ansell, 1992) was developed in order to describe the changes in beliefs and practices in teachers involved in CGI professional development. The levels are meant to capture the patterns of change within teachers' journeys as they engage in the use of CGI methodologies. The levels are used to characterize how responsive teachers are to students' mathematical thinking. Level 1 is the least aligned with reform-based beliefs and practices and Level 4b is the most aligned to reform-based instruction (see Table 2.1).

The *Levels of Engagement with Children's Mathematical Thinking* was originally developed to code observations of instruction, interviews, and assessments for a group of 21 practicing teachers to determine the extent of their beliefs around and use of cognitively guided pedagogy (Steinberg, Empson & Carpenter, 2004). The levels provide a means to monitor the changes in practices and beliefs of teachers. It is not assumed that all teachers move through the levels in the same timeframe or linearly (Franke et al., 2001). Instead, movement within the levels can vary by teacher. Exploration is needed to determine how the *Levels of Engagement with Children's Mathematical Thinking* might be used and perhaps adapted to monitor and characterize prospective teachers' engagement with student thinking. This study sought to examine its fit with prospective teachers. To do so, an understanding of literature around engagement with student thinking and the potential influences of prospective teachers' engagement with student thinking were examined.

Table 2.1. *Levels of Engagement with Children's Mathematical Thinking*

Level	Teacher Orientation (Beliefs)	Teacher Activity (Practices)
<b>Level 1</b>	Does not believe students in his or her classroom can solve problems unless they have been taught how	Does not provide opportunities for solving problems Does not ask students how they solved problems Does not use students' mathematical thinking in making instructional decisions
<b>Level 2</b>	Begins to view students as bringing mathematical knowledge to learning situations	Believes students can solve problems without being explicitly taught a strategy Talks about value of variety of solutions and expands types of problems they use Is inconsistent in beliefs and practices related to showing children how to solve problems Permits issues other than students' thinking to drive selection of problems and activities
<b>Level 3</b>	Believes it is beneficial for students to solve problems in own ways because they make more sense to them; wants the students to understand what they are doing	Provides a variety of different problems for students to solve Provides an opportunity for students to discuss their solutions Listens to students talk about their thinking
<b>Level 4A</b>	Believes students' mathematical thinking should determine evolution of curriculum and the ways in which teachers individually interact with students	Provides opportunities for students to solve problems and elicits their thinking Describes in detail individual students' mathematical thinking Uses knowledge of thinking of students as a group to make instructional decisions
<b>Level 4B</b>	Knows how what an individual student knows fits in with how children's mathematical understanding develops	Creates opportunities to build on student's mathematical thinking Describes in detail individual students' mathematical thinking Uses what he or she learns about individual students' mathematical thinking to drive instruction

*Note.* Adapted from "Capturing Teachers' Generative Change: A Follow-Up Study of Professional Development in Mathematics" by M. Franke, T. Carpenter, L. Levi and E. Fennema *American Educational Research Journal*, 38, 2001

## **Review of the Research around Engagement with Student Thinking**

The research relevant to the potential influences prospective teachers' engagement with student thinking was reviewed and is presented using the following themes: (a) the context of the student teaching experience and the relationships formed in this experience, (b) the content and pedagogical knowledge and beliefs held by the prospective teachers, and (c) student understanding of fractions. These bodies of literature worked in tandem to inform the study structure, specifically regarding how prospective teachers' engagement with student thinking could be captured. The literature also informed the structure and use of the community of practice team meetings that the prospective teachers were a part of during the student teaching experience.

### **Theme 1: Context of the Student Teaching Experience**

The term context is used in reference to the entirety of the student teaching experience situated in both a teacher preparation program and a school site where the internship takes place. The physical sites, along with their norms and people, introduced potential influences into a prospective teacher's practice. The research related to teacher preparation program structures and interpersonal relationships was explored to better examine the context of this study.

**Teacher preparation program structure.** Traditional United States undergraduate education degrees and certifications are typically earned in four or five year programs (Andrew, 1990; Darling-Hammond, 2006). The teacher preparation program is the last two to three years of the degree, and the student teaching experience is the culmination of the program. Traditional preparation experiences consist of three key elements: (a) subject matter knowledge along with general education structural knowledge, (b) pedagogy and methodology taught in a structured, integrated manner (c) experiential learning that generally commences with student teaching (Borman, Mueninghoff, Cotner, & Frederick, 2009). The student teaching experience is

regarded primarily as an independent showcase of teaching by the prospective teacher (Borman et al. 2009; Lortie, 1975). Research varies regarding teacher preparation program structures, duration, and strategies that best support prospective teachers and influence what they do in practice (Andrew, 1990; Borman et al., 2009; Darling-Hammond, 2006). Thus, an understanding of the structure of the particular teacher preparation program of the participants in this study was important to later analysis phases regarding the teacher preparation program's potential influences.

**Relationships formed during the student teaching experience.** Researchers agree that the student teaching experience is of utmost importance in the preparation of prospective teachers for the career (Ben-Peretz & Rumney, 1991; Borko & Mayfield, 1995; Darling-Hammond, 2006; Hollins, 2011; Yee, 1969), yet doubts have been raised about the effectiveness in transferring the university intended message to practice. This is in part due to the intricate relationships at play during student teaching. Learning to teach mathematics is a complex process, and complicating it is the interaction of ideas, beliefs, knowledge, and attitudes of those in the profession (Ball, 1990; Ball & Cohen, 1999; Ma, 1999).

***The triad.*** Central to this dialogue is the relationship among the practicum triad consisting of the prospective teacher, cooperating teacher and university supervisor (Nguyen, 2009). Each member functions as an individual while simultaneously functioning as a unit. This interplay of relationships impacts if and how a prospective teacher implements reform-based initiatives in mathematics instruction and whether student thinking is emphasized in the classroom. When relationship interactions are planned and implemented thoughtfully to include “feedback about specific lesson components, suggestions about new ways to think about teaching and learning, and encouragement to reflect on one's practice” (Borko & Mayfield, 1995,

p. 515) then prospective teacher transformation is realized. However, many relationship interactions with prospective teachers are not about content and pedagogy toward reform.

***Prospective teacher relationship with cooperating teacher.*** When comparing the members of the triad, the cooperating teacher has the most influence on the attitudes of the prospective teacher (Borman et al., 2009; Philipp et al., 2007; Yee, 1969). In one early landmark study (Yee, 1969), the attitudes of 124 cooperating teacher and prospective teacher pairs and their twelve university supervisors were measured before and after the student teaching semester. Influence was investigated across all directions in the triad (cooperating teacher to prospective teacher, prospective teacher to cooperating teacher, prospective teacher to university supervisor, and university supervisor to prospective teacher). The study found highly significant results for influence from the cooperating teacher to the prospective teacher, in that prospective teachers shifted their attitudes to more closely align with their cooperating teachers by the end of the student teaching semester. This seminal study set the stage for more recent studies in mathematics education about purposeful prospective teacher placements with cooperating teachers that utilized student thinking.

One prominent study in mathematics education to suggest cooperating teacher influence was an experimental study (Philipp et al., 2007) that examined purposeful cooperating teacher classroom placements. This examination was part of a research study on prospective teachers' beliefs about mathematical knowledge and instruction. The study was based upon the research team's assumption that prospective teachers knowledge and beliefs "will be enhanced if they are provided with opportunities to learn about children's mathematical thinking while they are learning the mathematics they will teach" (p.439).

To test their assumption, Philipp et al. (2007) randomly assigned 159 prospective teachers into one of the five groups that either: (a) learned about children's thinking through watching videos of children solving problems, (b) watched the same problem-solving videos and also directly interviewed children about the math problems, (c) visited elementary school classrooms of specially selected teachers trained in and using reform-based practices around children's mathematical thinking, (d) visited elementary classrooms close to campus as a matter of convenience, and (e) experienced the regular slated mathematics content course (control group). The prospective teachers assigned to groups 1-4 concurrently participated in the same mathematics content course as the control group.

The study found that prospective teachers who visited the specifically selected reform-based mathematics classrooms experienced belief changes towards reform ideals. The group with the least amount of change towards reform (change towards traditional practices) was that of prospective teachers visiting the conveniently located classrooms. In fact, these prospective teachers experienced even less change toward reform ideologies than the control group who received no additional supports. The results indicated that being paired with random cooperating teachers actually interfered with the prospective teachers' learning of reformed-based ideals, which implies that any work done in university coursework may be for naught if purposeful placements are not made.

More recently, prospective teacher and cooperating teacher relationships were explored through the lens of attending to student thinking in mathematics (Bieda, Sela, & Chazan, 2015). The purpose of the study was to monitor teachers' reactions to a video of a student sharing an alternate solution strategy in a mathematics class. Together, a group of cooperating teacher and their prospective teachers watched a vignette that featured a student suggesting an alternative

solution strategy that was dismissed by the teacher. The group watched the vignette in the fall and re-watched in the spring after the year of student teaching placements.

In the fall, the cooperating teachers were initially bothered that the teacher in the video missed a teachable moment by dismissing the student's alternative strategy, but they rationalized that the teacher did this to avoid confusion for others in the class. The prospective teachers were concerned about the teacher's dismissiveness of the individual child and wondered how the moment might affect his feelings. In the spring, the cooperating teachers' reactions to the video remained consistent, but the prospective teachers changed to agree with the cooperating teachers. They shared the belief that the alternative strategy should not be shared because it overly-complicates a lesson to introduce other strategies in a mathematics classroom. A group consensus was then made that the mathematics problem used by the teacher in the vignette was a faulty problem choice if it allowed for multiple solution strategies to emerge and thus student confusion. During the spring session, the prospective teachers were deemed better at their jobs by the cooperating teachers, even though they had adopted pedagogical moves that were less conducive to reformed-based ideals. The study suggested that if a cooperating teacher disregards or fails to elicit student thinking in the classroom, then the prospective teacher will most likely disregard and fail to elicit it as well. The study concluded that teacher preparation programs must attend to prospective teacher and cooperating teacher beliefs and help prospective teachers manage the tensions that might exist between themselves and their cooperating teachers.

***Prospective teacher relationship with university supervisor.*** Traditionally, the university supervisor's primary role is to ensure university requirements are met through evaluative observations and feedback to the prospective teacher (McDonnough & Matkins, 2010; Slick, 1998). An analysis of university supervisor post-conferences regarding prospective teacher

mathematics teaching episodes exposed that the guiding theme was university paperwork, such as prospective teacher assignments and completing the university official observation form (Borko & Mayfield, 1995). Slick (1998) termed this ‘supervisor as overseer’ in which the supervisor is asked to evaluate and judge the prospective teachers but little else is fleshed out by the university regarding the role and relationship of the university supervisor and prospective teacher.

When pedagogy was discussed in teaching post-conferences, it was rarely about the mathematics content or teaching methods and more often it was concerned about classroom management. This held true even when a prospective teacher taught a mathematics content error in the lesson. The researchers hypothesized that this was because the mathematics was not a featured component of the university observation and because university supervisors were not trained in mathematics content. When university supervisors were later interviewed about their goal for post-conferences, the primary goal was to maintain a positive tone and positive relationship with the prospective teachers, and not about content or pedagogy. The findings imply that the teacher preparation program and prospective teacher cannot rely on the university supervisor to be the sole bridge from coursework theory to practice.

***Relationships among prospective teachers.*** Since a teacher preparation program cannot rely on cooperating teacher and university supervisor mentorship alone to influence prospective teacher beliefs and practice toward reform, a teacher preparation program must also look to other supports and structures in the field (Goodnough, Osmond, Dibbon, Glassman, & Stevens, 2009; Nguyen, 2009). Community structures can support prospective teachers in the implementation of reform-based mathematics. Dinsmore and Wenger (2006) examined prospective teachers’ cohorts as a culture in which socialization into the profession takes place. Their research around

prospective teachers' perceptions of cohort models suggested that interactions in the cohort must foster a community spirit in order to enhance prospective teacher preparation. Negative relationships within prospective teacher cohorts, like cliques, negatively influenced teacher preparation program aims.

To conceptualize learning within communities is to acknowledge that learning is social (Wenger, 1998). Communities of practice are “groups of people who share a concern, a set of problems, or a passion about a topic, and who deepen their knowledge and expertise in this area by interacting on an ongoing basis” (Wenger, McDermott & Snyder, 2002a, p.4). Within any community of practice there exist varying levels of participation. The three main levels of community participation have been defined as core group members, active group members, and peripheral members (Wenger, 1998; Wenger, McDermott & Snyder, 2002b). The core group members are active participants in the community who identify the issues of the community and tackle those issues. Within this core group, a coordinator organizes and connects the community; but with maturation of the community, the core members also take upon leadership roles with the coordinator. Active members are involved in the issues of the community, but without the consistency and intensity of the core group members. Peripheral members are at the sidelines of the community and watch the interactions of the core and active members.

In the greater context of the schools and grade level teacher teams, prospective teachers often assume the role of peripheral members due to being the novices in the profession (Goodnough et al., 2008). Goodnough et al. examined the effects of placing pairs of prospective teachers in cooperating teachers' classrooms. This paired placement was meant as a support to ease the transition from peripheral membership to full involvement. While the study found that prospective teachers' viewed the paired placements as an opportunity to learn from one another

and an opportunity to better address the needs of children, the prospective teachers also viewed the paired placements as a loss of individuality, and as a means for competition, or as sense of “trying to outdo” one another (p.293). The current study attempted to build on the positive aspects of paired placements identified in Goodnough et al. while minimizing the negative aspects by placing prospective teachers together, but with different cooperating teachers at the same grade level. The intent was to allow prospective teachers to develop individuality of teaching styles while removing competition and providing the prospective teachers the opportunity to learn from and support one another.

There is a basic human need to belong, to contribute, and to feel significant (DuFour & Eaker, 1998). A professional learning community (PLC) model is the manifestation of this human need within the educational setting (DuFour, 2004). A PLC can be considered a specific model within the framework of community of practice. A PLC is structured around three tenets: ensure students will learn, create a culture of collaboration, and focus on results. In the PLC model “every teacher team participates in an ongoing process of identifying the current level of student achievement, establishing a goal to improve the current level, working together to achieve that goal, and providing periodic evidence of progress” (DuFour, 2004, p.5).

## **Theme 2: Prospective Teacher Content Knowledge, Pedagogical Knowledge, and Beliefs**

Important to understanding a prospective teacher’s instruction is perspective on the types of knowledge and beliefs teachers hold. Teacher knowledge is historically credited with the work of Shulman (1986) and his identification of three categories of teaching content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular content knowledge. Subject matter content knowledge is the knowing of a discipline and the variety of ways in which that discipline are organized. Pedagogical content knowledge is the knowledge of how to instruct in ways that make the discipline understandable to others, as well

as knowledge of the learners and learning process of that discipline. Curricular content knowledge is knowledge of the programs and resources available to teach a discipline and the discerning of how best to use them. Literature around knowledge for teachers was reviewed for the domain of mathematics and specifically fractions to better understand what knowledge the prospective teachers in this study may or may not hold.

Mathematical beliefs were also examined as they influence how a prospective teacher enacts her teaching practices. Regarding the terminology used throughout the next subsections, the word *belief* is interpreted using Pajares' (1992) view, which drew upon the work of past researchers (Rokeach, 1968; Bandura 1986; Nespor, 1987). From this view, a belief is a form of truth in the deeply personal sense; whereas knowledge is based on objective fact, belief is based on judgment. Beliefs influence how individuals make sense of the world and are strong predictors of behavior. *Self-efficacy* is used to address an individual's beliefs about his or her abilities to successfully accomplish a task that influences how he/she approaches goals, tasks, and challenges (Bandura, 1977; Swars 2004, 2005).

**Mathematical knowledge for teaching.** Ball and Bass (2000) established three overarching issues to address regarding mathematics teaching: identification of the content knowledge that matters for teaching, understanding ways the knowledge needs to be held, and unveiling what is required to learn in order to use knowledge in practice. Ball and Bass' second and third issues charge teacher preparation programs to look beyond traditional university course mathematics content to other aspects that may impact how prospective teachers understand, hold, and use this knowledge.

Shulman's (1986) categories of teacher knowledge were honed to develop a theoretical framework about the knowledge needed for teaching mathematics (Ball, Thames, & Phelps,

2008; Hill & Ball, 2004; Hill et al., 2008; Hill, Rowan & Ball, 2005; Hill, Schilling, & Ball, 2004). This framework is known as *Mathematical Knowledge for Teaching* and is the skill bundle needed for the work of effective mathematics teaching. Mathematical Knowledge for Teaching is comprised of subject matter content knowledge and pedagogical content knowledge (see Figure 2.1). Each domain contains subcategories that lend themselves specifically to teaching mathematics.

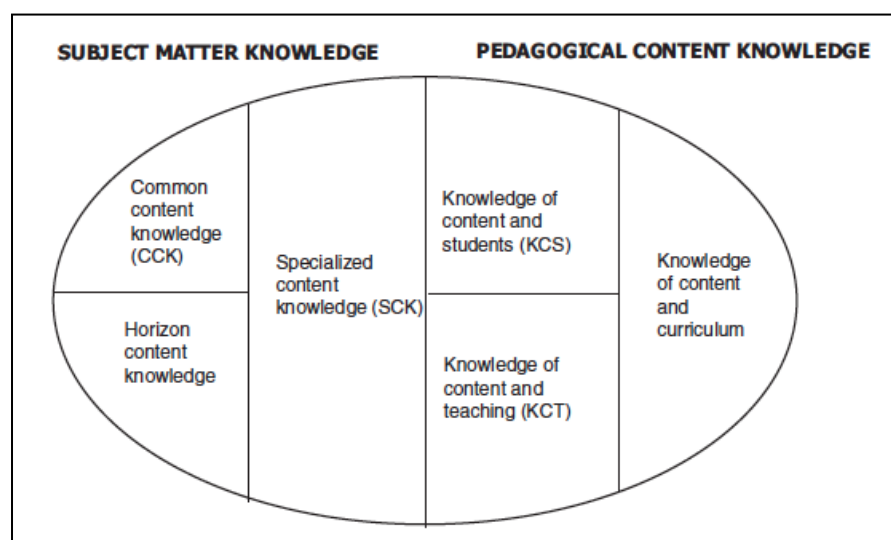


Figure 2.1. Mathematical Knowledge for Teaching (Ball, Thames, & Phelps, 2008)

Within the domain of subject matter content knowledge are the components of common content knowledge, specialized content knowledge, and knowledge of mathematics on the horizon. Common content knowledge is the knowledge of mathematics and mathematical skills used by the general learner. Specialized content knowledge is the content knowledge and skills unique to teaching mathematics. Put simply, common content knowledge is the knowledge of how to subtract, and specialized content knowledge is the knowledge of the various strategies and models of subtraction (Ball et al., 2008). Horizon content knowledge is the knowledge of the span of curriculum and pushes an educator to look to the entire future of mathematics for their students. Within the pedagogical content knowledge domain are the components of

knowledge of content and students, knowledge of content and teaching and Shulman's original curricular content knowledge. Knowledge of content and students means to have a command of the mathematics and a knowledge of one's students. Knowledge of content and teaching then combines the knowledge of the mathematics being taught with knowledge of how to instruct.

Mathematical Knowledge for Teaching provides a framework for how teacher educators might think about prospective teacher preparation. A reflective stance around Mathematical Knowledge for Teaching is important during coursework and the student teaching experience. Reflection around the domain of knowledge of content and students is especially important when considering a prospective teacher's work of engaging with student thinking.

***Subject matter knowledge.*** An examination of the research around fractions through the lens of Mathematical Knowledge for Teaching is warranted, because this is the instructional content matter of the prospective teachers in the current study. Lamon (2007), in the *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, 2007), claimed that of all the mathematical curriculum topics, it is fractions, ratios, and proportions that are "protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites" (p. 629). Pedagogical content knowledge in such a complex domain must target core cognitive structures such as students' ways of thinking and mechanisms for growth (Lamon, 2007).

Fractions can be used and interpreted with different meanings (Lamon, 2007). First, fractions can be used in everyday language, such as *a fraction of the people*. Often in everyday language, fraction is taken to mean 'a little' or 'a small amount.' This idea of fraction may well be ingrained in children when they enter a classroom. Fractions can also be considered as

bipartite symbols with  $a/b$  notation. In this interpretation, a fraction is a symbol of a notational system for writing numbers.

Fractions can also be interpreted as non-negative rational numbers and explored as the following (see Table 2.2): part-whole models, ratio, operator, quotient, and measures (Behr et al. 1983; Charalambous & Pitta-Pantazi, 2005; Kieran, 1976). However, fractions are typically taught conceptually as part-whole using area and set models. Lamon (2007) claims this limited focus leaves students “with an impoverished notion of the rational numbers” (p.635).

Instead, fractions must be recognized as numbers that represent the underlining multitude of forms of rational numbers (Lamon, 2007; Park, Güçler, McCrory, 2013; Petit, Laird, Marsden, Ebby, 2015). Within the part-whole, ratio, operator, quotient, and measure interpretations of fractions, various models can and should be used with students. Students should be exposed to fractions in representations that include symbolic notation, area/region, number line, and sets of objects. Figure 2.2 illustrates how each interpretation is represented with the various fractional models.

Table 2.2. *Interpretations of Fractions, Exemplified as Using 3/4*

Interpretation	Definition and Implication for Children
Part-whole	<p>In part-whole cases, a continuous quantity or a set of discrete objects is partitioned into a number of equal-sized parts. E.g. a pie is divided into four equal parts (quarters) and three are eaten, so <math>\frac{3}{4}</math> of the pie has been eaten.</p> <p>Children need to: understand that (a) the parts into which the whole is partitioned must be of equal size; (b) the parts, taken together, must equal to the whole; (c) the more parts the whole is divided into, the smaller the parts become; and (d) the relationship between the parts and the whole is conserved, regardless of the size, shape or orientation of the equivalent parts (Leung, 2009).</p>
Ratio	<p>A fraction can be seen as a ratio of two quantities; in this case it is seen as a part-part interpretation and it is considered to be a comparative index rather than a number (Carragher, 1996). E.g. Three parts out of every four are red.</p> <p>Children need to: understand the relative nature of the quantities. They also need to know that when two quantities in the ratio are multiplied by the same positive number, the value of the ratio is unchanged (Leung, 2009).</p>
Operator	<p>A fraction is interpreted as an operator when it is applied as a function to a number, set or object. E.g. showing <math>\frac{3}{4}</math> of a pie chart or finding <math>\frac{3}{4}</math> of 24.</p> <p>Children need to: interpret a fractional multiplier in a variety of ways; name a single fraction to describe a composite operation, when two multiplicative operations are performed; and relate outputs to inputs (Leung, 2009).</p>
Quotient	<p>The quotient interpretation is the result of a division. It results in a number that can be placed on a number line. E.g. <math>3 \div 4 = \frac{3}{4}</math>.</p> <p>Children need to: identify fractions with division and understand the role of the dividend and the divisor in this operation (Leung, 2009).</p>
Measure	<p>In a measure interpretation a unit fraction is identified (e.g. <math>\frac{1}{4}</math>) and how many units are used repeatedly determine a distance from a predetermined starting point (e.g. <math>3 \times \frac{1}{4} = \frac{3}{4}</math>).</p> <p>Children need to: use given unit interval to measure any distance from the origin; locate a number on a number line; and identify a number represented by a point on the number line (Leung, 2009) or region (Pantziara &amp; Philippou, 2012).</p>

*Note.* Adapted From the iTalk2Learn Project, 2014



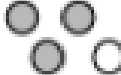


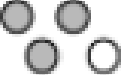



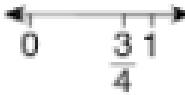



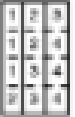






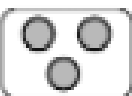
	Symbolic	Area/region	Number line	Sets of objects
Part-whole	$\frac{3}{4}$	 $\frac{3}{4}$ of the area is shaded	 $\frac{3}{4}$ of the number line is grey	 $\frac{3}{4}$ of the objects are shaded
Ratio	$\frac{3}{4}$	 3 out of 4 parts are shaded	 3 out of 4 parts have jumped along the number line ( $3 \times \frac{1}{4}$ )	 3 out of 4 objects are shaded
Operator	$\frac{3}{4}$	 Finding $\frac{3}{4}$ of the region gives: 	 Finding $\frac{3}{4}$ of the line segment gives: 	 Finding $\frac{3}{4}$ of the objects gives: 
Quotient	$\frac{3}{4}$	 $\leftarrow 3$ are shared by 4 $\rightarrow$  So each person receives $\frac{3}{4}$ each	e.g. Road relay String	 3 objects are shared by 4, so each person receives $\frac{1}{4}$ of each object $\frac{1}{4} \times 3 = \frac{3}{4}$ each
Measure	$\frac{3}{4}$	 The shaded object is $\frac{3}{4}$ the white object 	 The second line segment is $\frac{3}{4}$ the first 	A  B  Set B is $\frac{3}{4}$ of Set A

Figure 2.2. Fraction Interpretations and Representations Matrix (iTalk2Learn Project, 2014)

Much of the fractional content knowledge prospective teachers hold is based on their own experience with “harmful algorithms,” or those algorithms performed without sense making instead of requiring fraction number sense (Utley & Reeder, 2012, p.1). Reeder and Utley (2007) defined fraction sense as “an intuitive feel for fractions and fraction relationship; flexible thinking about fractions (e.g. can use benchmark fractions to determine reasonableness of fraction operations)” (p.2). It is important that prospective teachers have this sense if they are to nurture it in students. Many prospective teachers leave their teacher preparation program coursework with remaining misconceptions around fractions (Olanoff, Lo & Tobias, 2014; Utley & Reeder, 2012), thus the need for a study that continues to examine the support of prospective teachers’ instruction of this content domain once in the field.

***Pedagogical content knowledge.*** Once some of the content implications for elementary students have been considered and the prospective teacher is attuned to crucial aspects of the content, their energy can be freed towards understanding how they can elicit this mathematics from students. While student fraction strategies can be in written form, an educator learns the most about student understanding when she can attach this work on paper to the students’ explanations and conversations. In order to move a student towards a more sophisticated strategy, a prospective teacher must learn to listen to children, and thus purposeful classroom discussions are used.

In their research on productive talk about mathematics, Chapin, O’Connor, and Anderson (2013) found that meaningful classroom discussions are centered on four primary goals that are hierarchal in nature. The first of these goals is to help students clarify their own thinking. Once this occurs the next teaching goal is to help students orient to the thinking of others, the third goal is to help students deepen their own reasoning, and then finally, help students engage with

others' reasoning. In order to achieve these goals, a teacher elicits the discussions through purposeful teaching moves referred to as productive *talk moves* (Chapin, O'Connor, & Anderson, 2003, 2013). The set of productive talk moves are comprised of strategic questioning and conversational prompts. Teachers may elicit students to turn-and-talk to one another, or ask students to repeat, rephrase, or add on to what a classmate said. When used thoughtfully, the talk moves produce discussions rich with data about students' understanding.

Once a prospective teacher has the tools to elicit and engage in talk around mathematics, she must think about the mathematics content and structure that encourages it. The orchestration of a discussion-based lesson is elaborated upon in Stein, Engle, Smith, and Hughes' (2008) Five Practices Model. This model explicitly states that it is a teacher's role to: (1) anticipate student responses to challenging mathematical tasks; (2) monitor students' work on and engagement with the tasks; (3) select particular students to present their mathematical work; (4) sequence the student responses that will be displayed in specific order; and (5) connect different students' responses and connect the responses to key mathematical ideas. The Five Practices model gives structure to mathematics discussions by bringing to the forefront the selection of mathematical tasks that are cognitively worthy of student discussion. It is worth noting that this framework closely aligns with the recommended instructional implications of the CGI research (Carpenter et al., 2014; Empson & Levi, 2011).

### **Prospective teacher beliefs about mathematics and teaching mathematics.**

Prospective teachers' personal beliefs and values are undeniable factors that impact the student teaching experience (Borko and Mayfield, 1995). The average student spends 13,000 hours in direct contact with teachers before graduating high school (Lortie, 1975). This means that prospective teachers have influential fieldwork before ever entering their education coursework

at the university level. Lortie (1975) termed this the apprenticeship-of-observation and claimed that because of their extensive experience with school and teachers, many prospective teachers hold the belief that they already know everything they need to know in order to be experts in their jobs. Therefore, if teacher preparation programs want to instill change consistent with reform-based mathematics, prospective teachers' beliefs and the effects of the beliefs cannot be overlooked. In fact, much of the work of the teacher preparation program must revolve around confrontation of the apprenticeship of observation and shape beliefs about teaching and learning (Darling-Hammond, 2006).

Much of the research on prospective teachers' beliefs and self-efficacy in mathematics references Pajares' (1992) who asserted that researchers cannot look solely to content and teacher thinking to impact reform movements, but they must also examine what teachers believe and the ways teachers believe. Although educational researchers may not agree on a working definition of beliefs, they do agree that all teachers hold beliefs about the career and their roles, thus understanding beliefs is important to future education initiatives.

Battista (1994) researched beliefs specifically in the context of mathematics. Historically, mathematics was seen as computation, so teaching mathematics meant providing students with a set of skills and learning mathematics meant remembering and using these set skills. Battista asserted that when this is a teacher's belief system about mathematics they are "robbing their students of opportunities to 'do' mathematics" (p.467). These beliefs also block the reform movement from making headway, because they do not align with the reform philosophies of mathematical sense-making and reasoning.

Both Battista (1994) and Pajares (1992) echoed Lortie (1975) in their assessment that prospective teachers feel and believe they are insiders in the profession before they ever begin

official training. Most likely, prospective teachers were products of traditional mathematics curriculums themselves, thus traditional beliefs are ingrained when they enter their fieldwork. A teacher preparation program's work is to reveal and shift prospective teachers' beliefs about the subject in general and about the teaching and learning of mathematics.

The structure of coursework can be one mechanism to develop prospective teachers' beliefs that align with mathematics reform (Ambrose, 2004; Burton, 2012; Swars, 2004, 2005; Wilkins & Brand, 2004). Wilkins and Brand (2004) used the *Mathematics Belief Instrument* with 89 prospective teachers to evaluate a mathematics methods course in relation to beliefs. The methods course was modeled after the reform-based mathematics initiatives. It was student-centered and utilized an investigative approach to mathematics with manipulatives, hands-on explorations, and cooperative groups. Wilkins and Brand evaluated both magnitude and direction of change in beliefs to determine the success of the methods course. They found that after being a part of a methods course of this nature, prospective teachers' self-efficacy about mathematics was changed in a positive way and their beliefs about what mathematics is and what it entails became more consistent with the reform movement. The research affirmed that prospective teachers' underlying beliefs about mathematics must be determined and addressed alongside content and pedagogy in coursework to hope to spur change in practice.

Much like Wilkins and Brand, Ambrose (2004) studied her own university teaching to shed light on what can be done in coursework to prepare prospective teachers for their teaching. Ambrose challenged the typical teacher preparation program practice of ignoring or tearing down prospective teacher beliefs and instead implemented coursework that built upon prospective teachers' existing beliefs and helped them form new ones. She acknowledged that many prospective teachers enter the field with what is an "optimistic bias," which means they assume

they already know all they need to know to be teachers and will do well teaching due to their personal experience of being students in classrooms (p. 91). This optimistic bias must be acknowledged and addressed if prospective teachers are to make meaningful changes in their instruction.

Ambrose (2004) established four mechanisms to elicit belief change and belief formation in prospective teachers: (a) create emotion-filled experiences in courses, (b) develop a positive community that will instill positive beliefs in relation to mathematics, (c) reflect on beliefs so that hidden beliefs become overt, and (d) offer experiences or reflections that help prospective teachers connect beliefs to other beliefs. Ambrose's goal was to provide the four mechanisms for belief change within an iterative cycle of work with children, reflection, and instructor feedback.

As part of this cycle, prospective teachers submitted reflections and were interviewed about their experience in the course. Prospective teacher responses showed changes in their beliefs about teaching and student learning towards the reformed ideals, suggesting that an iterative cycle of work with children can spur belief evolution for prospective teachers. However, if this extensive work is done during the methods coursework, considerations must be made regarding how to continue the support into the student teaching experience

Swars (2004, 2005) also studied her own practice, but did so within the construct of teacher efficacy. Teacher efficacy represents a teacher's belief that teaching can bring about change regardless of external factors and a belief the teacher has the skills and abilities to influence student learning and behavior. Swars used the Mathematics Teaching Efficacy Beliefs Instrument (2005) companioned with interviews to provide insight in what contributes to prospective teachers' mathematics teacher efficacy. Prospective teachers were part of a mathematics methods course structured around the NCTM (2000) process standards that utilized

reform pedagogical techniques like manipulative use, hands-on learning of concepts and group work. Fieldwork experiences were also a part of the course during which participants taught three mathematics lessons that they had practiced earlier during their course time in local elementary schools.

Swars (2004, 2005) found that prospective teachers with the lowest mathematics teacher efficacy levels reported negative mathematics experiences in earlier years and felt that they had been denied sense-making opportunities in their own elementary experiences. However, a prospective teacher with high teaching efficacy levels also reported negative personal experience with mathematics but viewed this as a catalytic moment because her struggle could make her a more aware teacher. An encouraging finding from Swars was that prospective teachers with the lowest self-efficacy scores felt that they could eventually teach mathematics effectively; they would just require more time, support, and effort. This points to the responsibility of teacher preparation programs to find ways to embed continued content support during the student teaching experience and not just during the methods coursework.

Thanheiser, Philipp, Fasteen, Strand, and Mills (2013) took a slightly different approach to coursework when they interviewed prospective teachers about their understandings of mathematical operations. This interview was then used as a catalyst for reflection regarding the ways that elementary students should learn mathematical operations. Mathematics content interviews were administered to the prospective teachers before they began their mathematics content and methods coursework. The interviews allowed the instructors to gain insight into the prospective teachers abilities and shape course direction, and allowed the prospective teachers to gain insight into the mathematical processes they knew only as procedures instead of understood conceptually. The authors paired these interviews with instruction that fostered a sense-making

approach in order for prospective teachers to discover mathematics so that they could then help students do the same. The intention of the interviews was that prospective teachers would see the value of learning on a conceptual level rather than a rote level, so that they would carry this value forward into their classrooms during the student teaching experience.

Much of the literature on prospective teacher mathematical knowledge for teaching and their beliefs suggests how to unveil and influence the beliefs during teacher preparation program coursework. The current study's intent was to address how beliefs and knowledge transfer to and potentially change during the student teaching experience, especially with the support of a community of practice. Based on the proceeding literature, it was hypothesized that team meetings of the community of practice embedded in the student teaching experience would provide for the continued development of belief and knowledge development toward reformed-based mathematics.

### **Theme 3: Student Understanding of Fractions**

Previous research provided the background in order to better understand if and how the supports in this study would facilitate prospective teachers' engagement with student thinking. Additional literature was examined to take into account student understanding, specifically in the content domain of fractions, and to best understand how prospective teachers may elicit, use, and progress student understanding.

**Student understanding in reformed-based mathematics.** The NRC's (2001) definition of mathematical proficiency was considered in order to unpack the intended meaning of the term *student understanding*. Mathematical proficiency requires a command of five strands:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations

- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 116)

Mathematical proficiency emerged from mathematics research within the constructivist and cognitivist frameworks (Carpenter, 1979; Clements & Battista, 1990; Secada, 1991; Simon, 1995; Wheatley, 1991). The reformed-based mathematics movement is based on several tenets of constructivism including:

- Knowledge is actively created or invented by the child, not passively received from the environment.
- Children create new mathematical knowledge by reflecting on their physical and mental actions.
- No one true reality exists, only individual interpretations of the world.
- Learning is a social process in which children grow in the intellectual life of those around them.
- When a teacher demands that students use set mathematical methods, the sense-making activity of students is seriously curtailed. (Clements & Battista, 1990, p.34-35)

In constructivist learning environments, mathematics learning is structured around student thinking. Constructivist teachers who believe in mathematical proficiency for all children work towards “a problem solving environment in which developing an approach to thinking about mathematics is valued more highly than memorizing algorithms and using them to get right answers” (Schifter & Fosnot, 1993, p. 9).

**Student understanding about fractions.** Since the prospective teachers in this study provided instruction within the domain of fractions, an examination of student conceptual understanding within the domain was needed. The most common ways to instruct about fractions in elementary schools is through the part-whole model (Alajimi, 2012; Lamon, 2007). However, this model is limiting and counterintuitive to how children think about fractions. Children come into the classroom with natural abilities to partition objects and to think about fractions in the context of sharing items, e.g. *four students share six candy bars. How much candy bar do they each get if they share equally?* (Charles & Nason, 2000; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009; Empson, 1995, 1999; Empson and Levi, 2011; Lamon, 1996; Pothier & Swada, 1983; Steffe, 2001; Streefland, 1993). Equipartitioning is the act of dividing into equal-sized groups or parts. Fraction research in the area of equipartitioning is focused on children’s intuitive strategies and provides educators with a set of typical student understandings to look for and notice while teaching. If a prospective teacher is to maintain the dedication to student thinking and influencing student understanding, then attention must be given to students’ equipartitioning strategies.

Although there is a large body of research on children’s partitioning strategies, it was Empson and Levi (2011) who made significant contributions to classroom implications of equipartitioning. They present partitioning through the context of equal sharing problems and

provide a framework of typical student strategies that result from these types of problems.

Although the authors did not explicitly refer to their set of typical student partitioning strategies as a trajectory, they make clear that the evolution of these strategies follows a predictable pattern with children. The partitioning strategies children employ within the Equal Sharing framework are: Non-Anticipatory Sharing, Additive Coordination (either sharing one item at a time or sharing groups of items at a time), Ratio, and Multiplicative Coordination (see Figure 2.3).

The Equal Sharing strategies framework is considered the typical progression of strategies employed by children, however, the authors emphasized that the teacher plays a critical role in connecting and extending children's thinking through the number choices and discussions used in the Equal Sharing problems. This aligns with Sarama and Clements' view that, "students may skip levels altogether, or may revert to previous levels when faced with changes in tasks or even benign alterations in their instructional environment" (as cited in Blanton, Brizuela, Gardiner, Sawry, & Newman-Owens, 2015, p.5). The fact that children use different strategies at different times points to the need for prospective teachers to understand the typical strategies in order to look for and respond to them accordingly in the classroom environment.

If prospective teachers learn this framework of typical strategies and how strategies look when produced by students, they can then enter the classroom better equipped to anticipate and attend to the strategies in real-time. A honed focus in one mathematical content area within a domain, such as equipartitioning in fractions, allows prospective teachers to learn to be more intentional in their noticing of strategies and response to students.

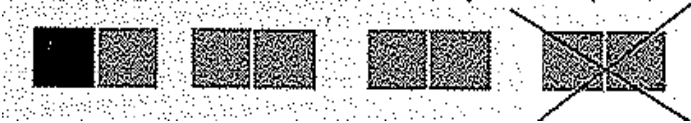


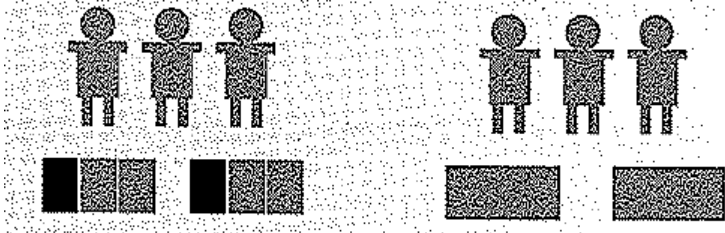
Types of Strategies – Equal Sharing Problems From Empson & Levi (2011). <i>Extending Children's Mathematics</i> . Portsmouth, NH: Heinemann, Figure 1-17, p. 25	
<b>Problem: 6 children are sharing 4 candy bars so that everyone gets the same amount. How much candy bar can each child have?</b>	
Strategy Name	Strategy Description
<b>Non-Anticipatory Sharing</b>	<p>Child does not think in advance of both number of sharers and amount to be shared. For example, child splits each candy bar into halves because halves are easy to make. Gives each person 1/2. Child may or may not decide to split the last candy bar into sixths.</p> <p><i>Each person gets <math>\frac{1}{2}</math> of a candy bar and a "little piece," if the last candy bar is split.</i></p> 
<b>Additive Coordination</b> Sharing one item at a time	<p>Child represents each candy bar. Splits first candy bar into sixths because that is the number of sharers. Each person gets 1 sixth piece. Repeats process until all 4 candy bars are shared.</p> <p><i>Each person gets <math>\frac{4}{6}</math> of a candy bar altogether.</i></p> 
<b>Additive Coordination</b> Sharing groups of items	<p>Child represents each candy bar. Realizes that splitting 2 candy bars each into thirds can create 6 pieces. Each person gets 1/3. Child moves to another group of items and continues similarly until all the candy bars are used up.</p> <p><i>Each person gets <math>\frac{2}{3}</math> of a candy bar altogether.</i></p> 
<b>Ratio</b> Repeated halving Factors	<p>Child may or may not represent all of the candy bars and people. Uses Knowledge for repeated halving or multiplication factors to transform the problem into a simpler problem, 3 children sharing 2 candy bars. Solves the simpler problem.</p> <p><i>Each child gets <math>\frac{2}{3}</math> of a candy bar.</i></p> 
<b>Multiplicative Coordination</b>	<p>Child does not need to represent each candy bar. Child understands that <math>a</math> things shared by <math>b</math> people is <math>a/b</math>, so 4 candy bars shared by 6 people means <i>each person gets <math>\frac{4}{6}</math> of a candy bar.</i></p>

Figure 2.3. Types of Strategies – Equal Sharing Problems (Empson & Levi, 2011)

The Equal Sharing strategies progression is one way for educators to think about classifying student thinking. However, it serves to examine student work only around one type of word problem context. Prospective teachers must also be equipped with ways to examine student thinking around other types of fractional situations. The Ongoing Assessment Project (OGAP) Fraction Progression (Petit et al., 2015, p. 196; Figure 2.4) can be used as a companion to the equal sharing problems to explore other types of student thinking.

The OGAP Fraction Progression portrays a developmental continuum of student strategies in key fraction concepts. It also draws attention to common student errors, as well as preconceptions and misconceptions. It is in this progression that student work in the topics of partitioning, comparison and ordering, equivalence, operations, and density is analyzed based on levels of strategy sophistication. The levels from least sophistication to most are: non-fractional understanding, early fractional strategies, transitional strategies, and fractional strategies. Taken together the OGAP Fraction Progression with the equal sharing strategies can be used to shed light on what a student understands or misunderstands about fractional concepts.

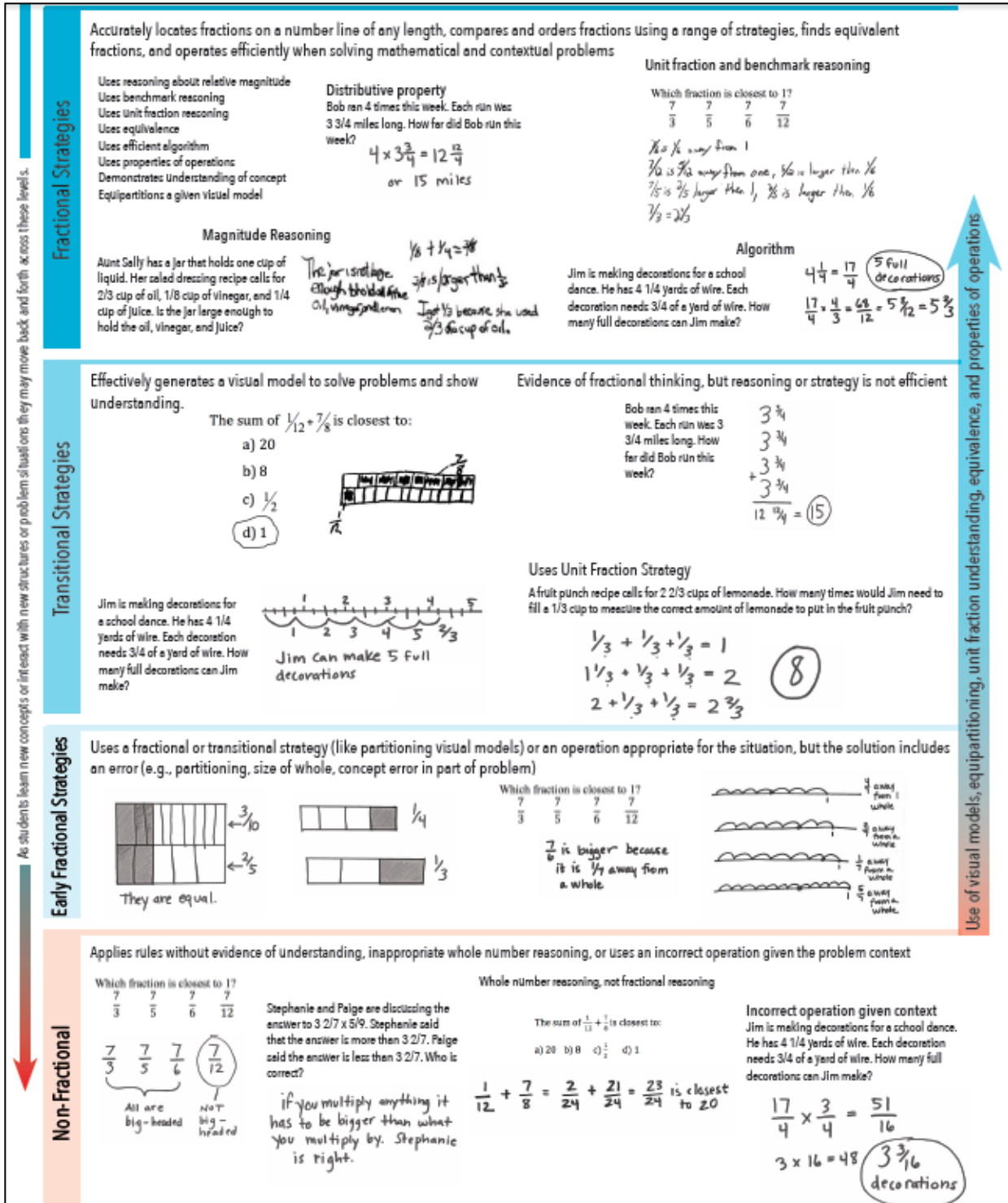


Figure 2.4. The OGAP Fraction Progression (Petit, Laird, Marsden, & Ebby, 2015)

**Eliciting, using, and progressing student understanding.** NCTM's (2014) *Principles to Actions: Ensuring Mathematical Success for All* defined eight effective mathematics teaching practices that align with the reform ideals (Figure 2.5). When used together, these practices support student understanding of the mathematics at hand.

Effective Mathematics Teaching Practices
<b>Establish mathematics goals to focus learning.</b> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
<b>Implement tasks that promote reasoning and problem solving.</b> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
<b>Use and connect mathematical representations.</b> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
<b>Facilitate meaningful mathematical discourse.</b> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
<b>Pose purposeful questions.</b> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
<b>Build procedural fluency from conceptual understanding.</b> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
<b>Support productive struggle in learning mathematics.</b> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
<b>Elicit and use evidence of student thinking.</b> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Figure 2.5. Effective Mathematics Teaching Practices (NCTM, 2014, p. 10)

While all eight of the mathematics teaching practices are essential to a reformed classroom, requiring mastery implementation of eight practices is overwhelming to a fledgling in the profession. Often, prospective teachers are coached in and collaborate around the development and use of learning goals and tasks in their coursework. Since these teaching practices are not independently their responsibilities, much of their energy can go into unveiling

student thinking in attempt to engage with it. Understanding and engaging with student's understanding of mathematics can be done through honing the following three practices: (a) *Facilitate Meaningful Mathematical Discourse*, (b) *Pose Purposeful Questions*, and (c) *Elicit and Use Evidence of Student Thinking*. In implementing the mathematics teaching practice of *Elicit and Use Evidence of Student Thinking*, teachers need to know how to elicit student thinking, interpret student understandings and misunderstandings, and respond by adjusting instruction to support and extend learning (Chamberlin, 2005; NCTM 2014). In preparing a prospective teacher to engage with student thinking in planning and instruction, attention to this teaching practice needs to be highlighted and thoughtful supports must be in place to help the prospective teacher learn to do the work of attending to, interpreting, and using student thinking regularly.

While NCTM (2014) advocated for instruction that engages with student thinking, the groundwork for attending to student thinking through responsive mathematics teaching was laid in the earlier work of mathematics educators and researchers who put student thinking at the forefront of their pedagogy (Ball, 1993; Chamberlin, 2005; Lampert, 2003; Paley, 1986). Most recently, Jacobs and Empson (2015) explored responsive teaching through the eyes of an upper elementary practitioner. Their case study featured a teacher that utilized CGI methodologies around fraction story problems.

The case followed the teacher's interactions with children around their solving and partitioning strategies. The study resulted in a framework of teacher moves that categorized the ways one can attend to student thinking. Empson and Jacobs (2015) found that a responsive teacher makes one or more of the following moves: (a) ensures the child is making sense of the story problem, (b) explores details of the child's existing strategy, (c) encourages the child to

consider other strategies, and (d) connects the child's thinking to symbolic notation. This categorization of teacher moves is helpful in that it creates indicators about what the responsive teacher attends to within the domain of student understanding of fractions, which was the focus of the prospective teachers' work in this study. However, because the framework was derived from a case of one veteran teacher, it alerted the need to be sensitive to what may or may not transfer to prospective teachers' practices.

### **Summary: Envisioning the Prospective Teacher's Mathematics Teaching Experience**

The review of the literature began with an understanding of the *Levels of Engagement with Children's Mathematical Thinking* and its articulation of teachers' engagement with student thinking. Then literature was explored around the description of the context of the student teaching experience including personal interactions during the experience. Next, literature was explored around mathematical content, pedagogical, and belief implications that could influence the prospective teachers' ability to engage with student thinking. Finally, the review closed with a description of what student understanding means to the content domain of fractions and how it might be elicited and used in order to consider how this ambitious work could be done by prospective teachers. The analysis of the literature allowed for a vision of what mathematics teaching that engages with student thinking may look like for prospective teachers during their student teaching experience, and how this teaching may be observed and supported. This dissertation research was focused on how to help prospective teachers move in such a direction that student thinking in mathematics is prioritized during the student teaching experience.

## **CHAPTER 3: STUDY METHODOLOGY**

### **Design Overview**

The current investigation used qualitative case study methodology to permit careful attention to prospective teachers' beliefs, concerns, communications, and interpretations of reform-based mathematics teaching. The study examined the prospective teachers' engagement with student thinking and case study methodology allowed for sensitivity to the prospective teachers' instructional changes because of its goal to richly describe a phenomenon. Yin (2013) stated that the utilization of case study inquiry provides honed focus to a situation in which otherwise numerous variables of interest could be exposed. An intensive focus on the prospective teachers' beliefs and actions around engagement with student thinking allowed for new discoveries and insights that could greatly impact future research and higher education practices. Qualitative case study methodology addressed the four research questions in the context of a 4<sup>th</sup> grade student teaching experience focused on the teaching and learning of fractions:

1. How might a prospective teacher's engagement with student thinking be characterized?
2. In what ways does context influence a prospective teacher's engagement with student thinking?
3. In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?

4. How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?

This chapter presents the rationale for study design, description of the study context and participants, and a detailed account of the data collection and analysis processes.

### **Design Rationale**

Glesne (2011) constructed a concise definition of a case study as a rigorous study of a case, but acknowledged “what a ‘case’ means can vary, from one person to a village or from an event to a set of procedures such as the implementation of a particular program” (p.22). Given this, a case must be bounded, considered as one system with its interacting parts being studied (Stake, 1995; 2005). It is left to the researcher to decide what and how something will be bounded and to make these decisions apparent. In this study, the case was bounded by one community of practice involving three prospective teachers placed in three different 4<sup>th</sup> grade classrooms at the same elementary school.

Stake (1995, 2005) fleshed out three approaches to case study – intrinsic, instrumental, and collective (see Table 3.1) – and this study was considered an instrumental case situated in an already occurring context for the prospective teachers. The researcher's interest in detailing the case drove the research. However, what was most important was that the detailing of the case resulted in the generalization of how a teacher preparation program may influence prospective teachers' instruction around engagement with student thinking. First priority was to provide what Geertz (1994) referred to as ‘thick description’ of the case. Thick description is the detailed account of field experiences in which social and relational interactions are contextualized. The intention of the documentation of this particular case was that through rich description, ideas for generalization to other contexts could emerge.

Table 3.1. *Types and Definitions of Case Studies*

Intrinsic	Case is dominant; case is naturally occurring and researcher's natural interest of the case drives study. Exploration of the case itself is of highest important rather than extending theory or generalizing across cases.
Instrumental	Issue is dominant; case is secondary to understanding a phenomenon. Exploration and understanding of the case is used to provide understanding of greater theory or issue.
Collective	Coordination of multiple cases; selection and understanding of multiple cases and/or generalizing across cases.

*Note.* Adapted from Stake, 2005

### Study Context

The setting for this study was an elementary school in the southeastern United States. Brookside Elementary School (pseudonym) is a Title I school serving 611 prek-5<sup>th</sup> grade students with 64.2% of the student population considered economically disadvantaged. As of the 2016-2017 school-year, the overall student population racial group percentages at Brookside Elementary were as follows: 41.2% White, 46 % Hispanic, and 8.5% African American<sup>1</sup>. Additionally, 13.3% of the school population was classified as Exceptional Children and 23.5% of the students were classified as Limited English Proficiency. Brookside is part of a district that did not have an adopted mathematics curriculum, but did have a district pacing guide for mathematics with suggested resources.

For the past three years, Brookside has been a partner school with the local university and hosted six or more prospective teachers during the student teaching experience. Brookside is deemed a mathematics focus school by the Elementary Education program of the university, meaning the prospective teachers placed there teach all core subjects, but complete a mathematics instructional portfolio for their certification purposes. Each year approximately three prospective teachers are placed in the K-2nd grade band, and three are placed in the 3rd- 5<sup>th</sup>

<sup>1</sup> School data provided from state and public reports as compiled by SchoolDigger (2016); demographic language reflects that of public reports

grade band. For the 2016-2017, three prospective teachers in the 3<sup>rd</sup> through 5<sup>th</sup> grade band were all placed in 4th grade. The 4th grade mathematics block is approximately 75 minutes in length.

The study was situated in the spring semester of a teacher preparation program. In the program, prospective teachers are engaged in full-time field placement settings as part of a fifteen-week student teaching experience. In this experience prospective teachers *lead teach* for a duration of four to six weeks, meaning they are responsible for the planning and instruction of all subject matters. They also complete a teaching portfolio around the planning, instruction, and assessment of one subject matter as part of their requirements for licensure. Both a cooperating teacher and a university supervisor mentor the prospective teacher during the student teaching experience. The university supervisor is typically a graduate student or clinical faculty member from the university that does weekly or biweekly check-ins and observations of varying lengths. The cooperating teacher must have at least three years of teaching experience and is with the student teacher on a daily basis. In this study, the university supervisor that oversaw the prospective teachers was not a mathematics content specialist and therefore did not mentor specifically to mathematics instruction. However, the cooperating teachers in this study had mentored past prospective teachers through their mathematics portfolio completion process. Because of this factor and because of their daily interaction with the prospective teachers, the instructional beliefs of the cooperating teachers were examined for their potential for influence.

### **Participants**

Three 4th grade prospective teachers from a large southeastern university and their corresponding cooperating teachers took part in this study. The cooperating teachers were a part inasmuch that their classrooms hosted the prospective teachers, and they participated in one interview about their mentorship and instructional beliefs. The prospective teachers and cooperating teachers all identified as female.

The study followed the prospective teachers' six-week lead teaching segment in their student teaching semester. This was the segment of student teaching in which the prospective teachers were fully responsible for the instruction in the classroom. Specifically to mathematics, the instructional content and strategies were part of a recommended sequence for fractions, derived from work in classrooms over the past five years by university instructors and researchers (Mojica & Friel, 2015). Appendix A provides the overview calendar of the instructional activities of the prospective teachers' unit. Additionally, the prospective teachers were required to complete a portfolio around their planning, instruction, and assessment for their teaching certification and university degree requirements. Since the study took place at Brookside Elementary, a mathematics focus school, the prospective teachers completed a mathematics portfolio for their program requirements. The portfolio required the same caliber of reflection as required by the community of practice team meetings, thus participating in the study did not place undue burden on them in the midst of their student teaching experience.

For the sake of time and to allow for the formation of a community of practice, it was important that the prospective teachers were at the same school and same grade level. This meant discussion and shared experience could bond the members of the community of practice. Although in the same grade level, each prospective teacher was a distinct individual who brought her own belief and knowledge package into the community, working with her own cooperating teacher. This allowed for both differences and commonalities in experience to be exposed, which opened the study to generalizations of learned results to others' practices.

The study utilized criterion sampling, in that the criterion was set by the researcher and met by participants before the beginning of the research. The three criteria were that participating prospective teachers were: placed at a mathematics focus school, completing a

mathematics portfolio, placed at the same grade level thereby teaching similar content.

Participants were recruited through an in-person information meeting that outlined the approved University of North Carolina at Chapel Hill Institutional Review Board (IRB) form (Appendix E).

### **Data Collection and Instruments**

In order to understand prospective teachers' engagement with student thinking during the student teaching experience, data collection focused on each prospective teacher and if and how her individual beliefs and practices changed. The community of practice team meetings provided a unifying context for the prospective teachers and gave them access to the same professional support around reform-based pedagogical strategies focused on the teaching and learning of fractions. The researcher then examined if and how each prospective teacher took up components of the team meetings in belief and practice. Since each prospective teacher fulfilled their student teaching requirements in separate classrooms, the case study allowed for data collection and analysis across participants in order to see how and if a community of practice structure impacted change differently for each individual prospective teacher.

In order to address the research questions and to identify and measure prospective teachers instructional change that aligned with reform ideals, data was collected from six sources: archival data, prospective teacher interviews, cooperating teacher interviews, prospective teacher observations and informal debrief sessions, prospective teacher community of practice team meetings, and student artifacts (see Table 3.2). These particular data sources and instruments were selected because they allowed for triangulation that supports the validity of this study. Triangulation allowed the researcher to analyze prospective teachers' beliefs and practices from instrument to instrument, and allowed for any discontinuity in a prospective teacher's practices or beliefs to be uncovered. For example, if the participant stated that she

planned to use a particular teaching method or mathematics problem while in the community of practice meetings, a subsequent observation noted use, misuse, or lack of use of the said method when working with children in the classroom. The data collection did not look only to confirm a data point, but to confirm or deny a data point, which thereby produced a more accurate depiction of the prospective teacher. The description of each data source and the data collection process follows Table 3.2.

### **Archival Data**

Archival data were collected in this study, and particular attention was given to data that aided in unpacking the prospective teachers' previous coursework and field experiences during the completion of their teacher preparation program. Archival data was collected in the form of teacher preparation program handbooks, teacher preparation program syllabi, prospective teacher mathematics notebooks, prospective teacher unit plans, and prospective teacher assignments that were completed prior to the beginning of the study. These data were requested and collected through the prospective teachers and analyzed with their permission and were used to address the second research question about the influence of context.

Table 3.2. *Data Collection Crosswalk*

Research Questions	Data Sources					
	Archival	PT pre and post Interview	CT Interview	PT Team Meetings	PT Observations	Student Pre/Post Tests
Question 1: How might a prospective teacher's engagement with student thinking be characterized?	X	X		X	X	
Question 2: In what ways does context influence pre-service teachers' engagement with student thinking?	X	X	X			
Question 3: In what ways does a community of practice structure facilitate pre-service teachers' engagement with student thinking in the area of fractions?		X		X	X	
Question 4: How does a pre-service teacher's engagement with student thinking influence student understanding of fractions in the 4 <sup>th</sup> grade?				X	X	X

*Note.* PT= prospective teacher, CT = cooperating teacher

## Interviews

**Prospective teacher pre and post interviews.** Prior to beginning their mathematics instruction in the student teaching experience, each prospective teacher participated in an interview focused on her beliefs about the instruction and learning of mathematics. This interview was the initial data collected that focused on the prospective teachers' engagement with student thinking. The interview protocol and questions are provided in Appendix B. All

listed questions were asked to each prospective teacher, but a flexible interview style was used to clarify responses, probe for more detailed responses, and to follow the individual.

The interview protocol included questions from the Integrating Mathematics and Pedagogy (IMAP) Web-Based Beliefs Survey (Philipp & Sowder, 2003). The IMAP Survey is a mixture of open-ended and Likert scale responses that are evaluated on normed rubrics. IMAP items that addressed general mathematics teaching beliefs and those specific to the fraction domain were selected for use in this interview. The IMAP was designed for administration through an online format with written response. Even though the IMAP survey is accessible online, the selected items were used in a face-to-face interview format in order to allow for additional probes into a participant's responses. Since participants tend to write differently than they discuss, the intention for the selected interview format was to reveal a more complete profile of each prospective teacher.

The prospective teacher interviews also included questions about the participant's personal history with mathematics learning and instruction. These interview probes were similar to those used for Mathematics Life Stories (Drake, 2006), which asked participants to share a narrative of themselves as learners and teachers of mathematics. The interview utilized these questions to open conversation about mathematics experiences, and then each prospective teacher was asked specifically about her previous experiences in the teacher preparation program's mathematics content and methods courses and student teaching placement sites. During these interviews, prospective teachers were asked to bring and share artifacts from their coursework (such as notebooks, lesson plans, and assignments) and to speak to their perceived usefulness of the artifacts. This protocol fleshed out the details of context and helped to

determine if and how the teacher preparation program and student teaching experience influenced their engagement with student thinking.

Another administration of this interview took place with each prospective teacher at the end of the study. The responses were compared to the beginning interview responses and to the data collected throughout the study.

**Cooperating teacher interview.** To further address the influence of context, an interview was scheduled with each cooperating teacher (Appendix B). The interview exposed each cooperating teacher's historical involvement in prospective teacher mentorship through the university, and provided insight into her current mentorship of the prospective teachers in the study. The data from these interviews also provided a historical context for the mathematics professional development done with Brookside Elementary School cooperating teachers in previous years. Brookside Elementary was a site for university-sponsored professional development in mathematics and literacy from the years of 2012-2014. Data about the professional development was obtained through the cooperating teachers shared experiences in interviews.

### **Community of Practice Team Meetings**

The community of prospective teachers participated in team meetings eight times over the course of an 11-week research project. Each meeting lasted for 60-90 minutes. Although the prospective teachers participated in other faculty and grade-level meetings at Brookside, these community of practice team meetings were comprised solely of the prospective teachers and the researcher. At these team meetings, the researcher acted as participant-researcher, facilitating professional learning and discussions around fraction instructional strategies that were part of the prospective teacher mathematics methods course. The facilitation plans that guided each meeting are provided in Appendix C.

Once the prospective teachers were situated in their cooperating teachers' classrooms and fully involved in lead instruction, they collected student work to bring to meetings. Their student work, along with their self-identified needs, acted as the catalyst for the prospective teachers' conversations and decision-making and took precedence over the researcher's suggested topics. The researcher remained active as participant-researcher though by questioning and supporting the prospective teachers instructional planning.

In order to monitor instructional and belief change, the participant-researcher collected field notes at the team meetings, documenting the ways that the prospective teachers discussed student thinking and learning and how they planned to use it in their instruction. The team meetings were also recorded for reviewing to expand the field notes. The researcher then observed whether the prospective teachers' expressed beliefs and ideas were enacted in classroom practice.

### **Observations and Informal Debrief Sessions**

The prospective teachers taught similar content that had been previously planned in their coursework or discussed in the community of practice team meetings, so their lesson implementation was similarly paced. Each prospective teacher had three scheduled in-person lesson observations over her six-week lead instructional period. Each observed lesson was also video-recorded for later analysis by the researcher. When the researcher was doing an in-person observation in one prospective teacher's classroom, the simultaneous lesson was video-recorded in the other two classrooms. This meant that a total of nine video-recorded lessons were collected for each prospective teacher. The lessons reflected instruction from the beginning (n=3), middle (n=3), and end (n=3) of the six-week sequence. This allowed the researcher to create a profile of instruction and instructional changes for each prospective teacher.

Observational field notes were collected during the in-person observations. As stated, special attention was given to watching if aspects discussed in the community of practice team meetings were integrated into practice. The observation protocol is provided in Appendix D. The field notes first captured a description of physical setting and context. Then, a running record style of observation was completed to showcase the timing and duration of different portions of each mathematics lesson. Because of the study's emphasis on the use of student thinking, to the greatest extent possible, the field note running records captured direct quotations and included moments of teacher-to-student questioning, student-focused interactions, and student-to-student discussions. This same approach for the observational field notes was used upon viewing the video footage of the others' subsequent lessons. This allowed for the exposition of instructional similarities and differences within the same content.

The most prominent difference between in-person and video-recorded observations was that the in-person observation was followed by an informal debrief session with the observed prospective teacher. Although the observation itself was not a participant observation, the debrief time transitioned into a participant-researcher model. The debrief session was not evaluative, and instead served as a discussion time for the prospective teacher to reflect upon her teaching with supporting questions and suggestions from the researcher. It was also a time that the researcher was able to gain additional insight into the prospective teacher's beliefs about her instruction and engagement with student thinking.

### **Student Pretests and Posttests**

Question 3 required an examination of student learning. A formalized pretest and posttest on fractions was administered to students through the prospective teachers. This was done prior to the fraction unit start date and at the end of the prospective teacher instructional segment. The assessment was administered to students as part of the natural classroom

procedures and did not disrupt instruction. It was the expectation at Brookside Elementary that pretests and posttests be administered to students. Copies of the student assessments were shared with the researcher with identifying student information removed.

The pretest/posttest used the same assessment form and is provided in Appendix F. The assessment was built around the constructivist view of student understanding. Rather than measure learning solely through a correct/incorrect item analysis, the assessment allowed for item analysis according to student learning progressions in fractions. Items 1-8 were from the Grade 4 Fractions Pre/Post from the Vermont Mathematics Partnership Ongoing Assessment Project (OGAP; Marge Petit Consulting, Hulbert, Laird, 2013) and emphasize student understanding of the density of fractions. Items 9-11 were equal sharing problems from past research projects (Lewis, Gibbons, Kazemi, & Lind, 2015; Petit, Laird, Marsden, & Ebby, 2015) and showcased the range in levels of conceptual sophistication for these problem types (more items than sharers, less items than sharers requiring halving, less items than sharers requiring fourths). The OGAP assessment items were grounded in research on formative assessment and student developmental progressions (Black & William, 1998; Daro, Mosher, & Corcoran, 2011). The equal sharing items were tied to a review of research on how students learn fractions and innately work with fractional quantities (Petit et al., 2015). The specific timeline for assessment administration and other data collection is presented in Table 3.3.

Table 3.3. *Data Collection Timeline*

	Months and Weeks of Research											
	Jan 11 <sup>th</sup> *	Jan 16 <sup>th</sup>	Jan 23 <sup>rd</sup>	Jan 30 <sup>th</sup>	Feb 6 <sup>th</sup>	Feb 13 <sup>th</sup>	Feb 20 <sup>th</sup>	Feb 27 <sup>th</sup>	Mar 6 <sup>th</sup>	Mar 13 <sup>th</sup>	Mar 20 <sup>th</sup>	Mar 27 <sup>th</sup> *
<b>Data Collection</b>												
Participant 1	c, a, i PT	-	-	-	o,v,v	d	v,o,v	d	v,v,o	d, a	i CT	iPT
Participant 2	c, a, i PT	-	-	-	v,o,v	d	v,v,o	d	o,v,v	d, a	i CT	iPT
Participant 3	c, a, i PT	-	-	-	v,v,o	d	o,v,v	d	v,o,v	d, a	i CT	iPT
<b>Community of Practice Meetings</b>												
All Participants		m1		m2	m3	m4	m5	m6	m7	m8		

Key:

PT= prospective teacher, CT= cooperating teacher

c = consent, i = interview (for PT or CT), a= student fraction assessment (pre/post)

m= community of practice team meetings scheduled, o = observation (in-person) & informal debrief/discussion,

v= video recording of mathematics lesson , d= data analysis of previous week's lessons

\*University holiday- prospective teachers off campus from 12/8/17 - 1/10/17; Spring Break for prospective teachers begins March 27<sup>th</sup>

*Note.* Table format adapted from Toles, Colón-Emeric, Naylor, Barroso, & Anderson, 2016 with permission from personal communication with Toles

## **Data Analysis**

Data analysis captured the prospective teachers' engagement with student thinking individually as well as holistically across the community of practice. Three analysis passes were done across the interview and observational data so as to ascertain an understanding of data through the lens of each research question. The first pass was done through directed content analysis (Hsieh & Shannon, 2005) and provided a holistic view of each prospective teacher and their engagement with student thinking. Next, a contextual analysis (MacDonald, Liben, Carnevale & Cohen, 2012) aided in understanding the sociocultural landscape of which the community of practice was embedded. This allowed for insight into contextual factors of influence on prospective teacher engagement with student thinking. A third pass was completed with open coding to lend insight into the manners of support the community of practice provided in the work of engagement with student thinking. Appendix H provides an overview with images of the systems of analysis that were used for research questions 1-3. Finally, student pretest and posttest data were analyzed with learning progression and strategy frameworks. Table 3.4 provides a data analysis crosswalk to show how each form of data was analyzed. Memoing was used throughout each step of the data analysis process to ensure the researcher remained close to the data to aid in accuracy of findings. A description of each analysis pass follows the table.

Table 3.4. *Data Analysis Crosswalk*

	Tools for Analysis				
	Memoing	Directed Analysis (pre-ordinate codes from the <i>Levels of Engagement and Principles to Action</i> )	Contextual Analysis	Thematic Analysis (open-coding)	Learning Progressions (OGAP, Equal Sharing)
<b>Data Sources</b>					
Archival Data	X	X	X		
PT pre and post interview	X	X	X	X	
CT interview	X	X	X		
PT Team Meetings	X	X	X	X	
PT Observations	X	X	X		
Student Pre/Post Test	X				X

*Note.* PT= prospective teacher, CT = cooperating teacher

### Question 1: Directed Content Analysis

Question 1 asked how a prospective teacher's engagement with student thinking might be characterized. This question necessitated the need for a mechanism of characterization of each prospective teacher's practice. It was critical that a systematic way to characterize prospective teacher engagement be developed and employed before questions 2, 3 and 4 were answered. Since these subsequent questions address influences on prospective teachers' engagement with student thinking and the impact of engagement with student thinking, it had to first be determined if and how each prospective teacher engaged with student thinking throughout the study.

A first pass of analysis was completed using data from the interviews, observations, and community of practice team meetings. Specifically, directed content analysis (Hsieh & Shannon, 2005) was employed to answer the first question. Directed content analysis aims to validate or extend an existing theoretical framework or theory. Key concepts of the theory are used to

determine coding categories and operational definitions for the codes are determined by the theory. For the current study, the existing framework used for analysis was the *Levels of Engagement with Children's Mathematical Thinking* (Franke, Carpenter, Levi, & Fennema, 2001) because it is a framework that allows for both a qualitative and quantitative characterization of a teacher's practice specifically in regards to engagement with student thinking. The qualitative characterization met the study aim in that it provided for rich description of the prospective teachers' practices, and the quantitative characterization assisted in the tool's efficiency for potential use for teacher preparation programs and future studies.

The *Levels of Engagement with Children's Mathematical Thinking* capture how teachers interact with student thinking to make their instructional decisions and also reveals when there is a lack of use of student thinking in instruction. The levels contributed to the mathematics education field in that they went beyond the dichotomy characterization of mathematics teaching as reformed-based or not (Steinberg, Empson & Carpenter, 2004) and described different stages of engagement with student thinking. Each level within the framework is distinguished by determining if and how teachers hold space for and use student thinking and student problem solving in the classroom.

The levels were originally developed for a longitudinal study by Fennema et al. (1996) to capture generative teacher change over years. They were built from past research that examined teaching through the constructivist perspective (Schifter & Fosnot, 1993). The five levels were established by a team of researchers interested in Cognitively Guided Instruction (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2014) using the following process: (1) identify fundamental beliefs of CGI, (2) define four instructional and four belief levels, and (3) trial categorizations of non-study teachers' instruction and beliefs within these levels over a two-

year period (Franke et. al, 1992). The process was cyclical and revision of the levels continued until there was agreement that the levels accurately captured non-study teachers.

The levels are set along a continuum of least to most cognitively guided, with Level 1 being the least aligned with reform-based beliefs and practices and Level 4B being the most aligned to reform-based instruction. At Level 4 teachers believe student thinking should drive the curriculum. The level is split into 4A and 4B to more accurately describe some of the variation seen among the most cognitively guided teachers (Fennema et al., 1996). At Level 4A the decisions about how to build on children's thinking are made at the classroom level and at Level 4B these decisions are made at the individual student level.

Each level and its corresponding orientation and practices became the coding labels and definitions in order to create a profile of each prospective teacher around their engagement with student thinking. Coded evidence from each source of data was collected within a table to allow for synthesis by instrument across all prospective teachers and by an individual prospective teacher across all the instruments. A process of code-recode (Poggenpoel & Myburgh, 2003; Saldana, 2009) was used to ensure accuracy of the coding scheme and accuracy of the characterization of each prospective teacher. Table 3.5 represents the *Levels of Engagement with Children's Mathematical Thinking* framework in its data synthesis format and is followed by a detailed description of each instrument's coding process for the directed content analysis pass.

Table 3.5. *Data Analysis System: Directed Content Analysis using the Levels of Engagement with Children's Mathematical Thinking*

[illegible]

**Directed content analysis: interviews.** As previously stated, interviews were recorded and after completion were listened to again in whole. Notes were taken to get a better sense of each interview. Then, interviews were transcribed and re-read in full. Upon the third exposure, an interview was analyzed with the codes listed in Table 3.5. The first prospective teacher interviews were held prior to observation of their practice, so the *Levels of Engagement with Children's Mathematical Thinking* were modified to use as a coding tool because it captures both beliefs and practices, but these first interviews only exposed the prospective teachers' intentions for practice. Thus, the teacher orientation descriptions of each level were used to operationalize the codes for the data from the first interviews. Text from each transcript that aligned to the language of the framework and that gave an impression of adherence to a certain orientation within the framework was highlighted. Highlighted excerpts were then coded with the corresponding level.

Additionally, because some of the interview questions were adapted from the IMAP Beliefs-Survey, they could be confirmed with the corresponding normed response rubrics of the instrument. The IMAP rubrics were designed to give a belief score for prospective teacher beliefs about mathematics, beliefs about learning or knowing mathematics, and beliefs about children's learning and doing mathematics. Because the instrument was not used in its original web form, the rubrics were not used to denote a quantitative measure. However, they did guide the coding of the interviews and helped affirm the prospective teachers' orientation characterization within the *Levels of Engagement with Children's Mathematical Thinking*.

**Directed content analysis: observations and community of practice team meetings.** Once observations and community of practice team meetings were underway, a more accurate profile for each prospective teacher was established. As mentioned, field notes and a recording

were collected at each observation and each team meeting. Field notes were expanded upon watching each recording and memoing was used to note possible commonalities in themes across the team meetings and observations/debriefs. After the observations and team meetings were understood holistically they were coded in the same process of the interviews in that actions and beliefs that represented a level were coded accordingly.

The observations and team meetings were also analyzed for evidences of reformed-based practices and languages in order to get a deeper understanding of the types of engagement techniques prospective teachers utilized. This was done through highlighting the observable teacher actions from one of the *Principles to Actions* (NCTM, 2014) teaching practices: *Elicit and Use Evidence of Student Thinking*. This principle aligned to this study's definition of engagement and the principle's corresponding teacher actions represent the teaching behaviors that highlight unveiling and utilizing student thinking. Because the *Levels of Engagement with Children's Mathematical Thinking* was designed for use with in-service teachers over extended time, the principle's teacher actions complemented the levels' descriptions and increased the sensitivity of the analysis instrument. The teacher actions of this principle are indicative of Levels 3 or 4 of the *Levels of Engagement with Children's Mathematical Thinking* and allowed for a finer tuned analysis into the types of actions the prospective teachers exhibit (or do not exhibit) when attempting to engage with student thinking. A lack of the principle's teacher actions in a prospective teacher's practice could also work to confirm any placement at Levels 1 or 2.

As stated, the actions of the principle align with Levels 3-4B in the *Levels of Engagement with Children's Mathematical Thinking*, thus giving more depth and breadth to the observable teaching behaviors in the study. Table 3.6 provides the principle's teacher actions. These

teacher actions became additional pre-ordinate sub-codes embedded within Levels 3-4B to use for analysis of the prospective teacher's observation and community of practice team meeting.

Table 3.6. *Principles to Actions* Mathematics Teaching Practice

<i>Elicit and Use Evidence of Student Thinking</i>	
Teacher Actions that align with Levels 3-4B from the Levels of Engagement	1. Identify what counts as evidence of student progress toward mathematics learning goals.
	2. Elicit and gather evidence of student understanding at strategic points during instruction.
	3. Interpret student thinking to assess mathematical understanding, reasoning, and methods.
	4. Make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.
	5. Reflect on evidence of student learning to inform the planning of next instructional steps.

*Note.* Adapted from *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014), p. 56; Teacher Actions may not occur in set order during instruction.

## Question 2: Contextual Analysis

After directed content analysis was used to create a profile of engagement for each prospective teacher, a contextual analysis was used to answer question two of this dissertation research (*In what ways does context influence a prospective teacher's engagement with student thinking?*). MacDonald et al. (2012) used the term contextual analysis to emphasize the attention to and reporting of the sociocultural environment of a study setting. Archival data were read to establish what Sandalowski (1995) referred to as “a sense of the whole” (p. 373). After reviewing each document in completion, a re-read was completed while memoing important programmatic notes, researcher ideas, and concepts. The participant background questions in the prospective teacher and cooperating teacher interviews, along with observation field notes explicitly around the site environment, helped to further flesh out the context of the study. Memos around the contextual features of the interviews and observations were then compared to memos from the archival data and all were analyzed across to understand the contextual

influence on the prospective teachers' ability to engage with student thinking. Special attention was given to prospective teacher/cooperating teacher dyads and any commonalities or discrepancies between their responses in interviews. Additionally, any quotations from the prospective teachers specifically about the contextual aspects of study (i.e. quotations that were school-based, classroom-based, mentor-based) were noted in the transcripts and field notes. All contextual-based evidence taken together was analyzed for themes of contextual influences.

### **Question 3: Thematic Analysis**

After the completion of the directed content analysis pass and contextual analysis, a third pass through the data was done in order to best address question three (*In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?*). This question necessitated an examination around the potential supports that facilitate prospective teacher engagement with student thinking. While directed content analysis was used to gain a profile on the prospective teachers regarding their engagement with student thinking, thematic analysis (Braun & Clarke, 2006; Hsieh & Shannon, 2005) was used to uncover the supports that facilitated the prospective teachers' engagement with student thinking. Community of Practice team meeting field notes and recordings, along with the final prospective teacher interviews were re-analyzed with an open coding strategy (Strauss & Corbin, 1994). Open coding allowed patterned responses within the data to emerge and then patterns were grouped as themes. The themes allowed for insight into what aspects of the study, specifically related to the community of practice structure, aided in prospective teacher engagement with student thinking in mathematics.

#### **Question 4: Analysis Through Student Strategy Framework and Learning Progression**

Finally, an analysis of student understanding was completed to answer the fourth research question (*How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?*). The pretest and posttest results were the data analyzed for this portion of the dissertation. The test forms were the same to analyze and measure changes in student understanding from the beginning to the end of the unit. Student understanding on items 1-8 was evaluated using the OGAP Fraction Progression (Chapter 2, Figure 2.4). Student understanding on items 9-11 was evaluated using Empson and Levi's (2011) Types of Strategies – Equal Sharing Problems (Chapter 2, Figure 2.3). Each prospective teacher first evaluated the pretest/posttest individually, as did the researcher. Then, assessments and their evaluations were brought to a future community of practice team meeting. The assessments were re-evaluated together at the community of practice team meeting. Disagreements were acknowledged and discussion ensued around a student understanding level until all participants agreed upon the students' level of understanding on each item. This assured inter-rater reliability for the student pretest and posttest data. Each student's responses were then analyzed for changes in understanding from pretest to posttest. Trends were looked for across each class and then across the grade level.

#### **Validity Through Quality and Rigor**

There are four basic indicators of quality that span both quantitative and qualitative research: truth value, applicability, consistency, neutrality (Lincoln & Guba, 1985, Whittemore, Chase, & Mandle, 2001). The indicators and how they are addressed in the different research styles are presented in Table 3.7. This dissertation was designed and analyzed as qualitative research and therefore research validity was interpreted as trustworthiness and established through methodological rigor with attention to the quality criteria of credibility, transferability,

dependability, and confirmability (Guba, 1981; Guba & Lincoln, 1989; Lincoln and Guba, 1985). Additionally, researcher positionality (Lincoln, 1995) was made transparent and is shared to further establish the validity of this dissertation research.

Table 3.7. *Indicators of Quality Research*

	<b>Truth Value</b>	<b>Applicability</b>	<b>Consistency</b>	<b>Neutrality</b>
<b>Quantitative</b>	Internal Validity	External Validity	Reliability	Objectivity
<b>Qualitative</b>	Credibility	Transferability	Dependability	Confirmability

Credibility was established through prolonged engagement in the research site, thick description of the site and participant experiences, and member checks. The member checks informally arose when the researcher clarified emerging understandings with the participants to ensure that experiences and perceptions were being accurately captured in the study. Member checks were formalized when the researcher's understanding of participants' beliefs and practices were inquired about in the final interviews. Transferability was established through detailed descriptions of the contextual aspects, such as research site and participants. Being that transferability is a shared responsibility between researcher and consumer, the contextual detail allows the consumer of information to make an informed decision on the feasibility of transfer to another site. Dependability was established through an audit trail (Halpern, 1983 as cited in Rodgers & Cowles, 1993) which consisted of instrument development records, audio and video recordings, student artifacts, field notes, coding records, and data analysis and synthesis products (Lincoln & Guba, 1985; Rodgers & Cowles, 1993). Confirmability was established due to the transparency and auditability of the data analysis procedures. The coding scheme and processes were shared in this chapter, and detailed memos were kept around data reduction and the interpretative process. Additionally contributing to confirmability was an understanding of the

original development and intention of the *Levels of Engagement with Children's Mathematical Thinking* with the acknowledgement of possible complications and accommodations for its transfer to this study.

### **Researcher Positionality**

Central to the qualitative case study is the researcher as the instrument (Denzin & Lincoln, 2005). This means that the researcher herself is the primary tool of data collection and analysis. Because the instrument is human, qualitative research assumes that no case study can be truly objective (Merriman, 1998). The researcher brings her positionality into each space and interaction. The need for thick description (Geertz, 1994), or the detailed account of field experiences in which social and relational interactions are contextualized, requires that the researcher be carefully immersed in the case setting. This detailed account of the field experiences allows the researcher to make explicit the patterns of cultural and social relationships (Holloway, 1997). The researcher needs to be a trusted member of the space and community of participants but also needs to examine and re-examine biases, limitations, and assumptions about the space and participants. The researcher also needs to describe the phenomenon in sufficient detail in order to evaluate the extent to which conclusions drawn are transferrable to other times, settings, situations, and people. The transparency of the researcher's positionality described next is meant to help induce trustworthiness and credibility in the research.

I am an educator with ten years of experience in the K-12 and university settings. I am interested in elementary mathematics education and this is the focus area of my doctoral program in Curriculum and Instruction. My personal history with learning mathematics was that of rote memorization without understanding. Exposure to reform-based ideals during my undergraduate education experience urged me to unpack mathematics and make sense of it for myself and

facilitate sense-making for students. In 2011, I completed a M.Ed. program focused on K-8 mathematics teaching and learning in a reform-based manner. As a classroom teacher, I utilized reformed pedagogy and my teaching philosophy was in line with that of the CGI research. As a teacher educator, I introduced and supported reformed pedagogies with elementary prospective teachers.

I believe that reform-based mathematics teaching and learning is equitable and just for all students. This is because of my personal history and experience with CGI in the elementary classroom. My classroom teaching experience was at a Title I elementary school similar to Brookside Elementary School. I taught 4<sup>th</sup> and 5<sup>th</sup> grade students and aspired to listen to their thinking and honor it at the forefront of planning and instruction. Students felt empowered by mathematics teaching in this manner. I used strategy frameworks like those in this study to assess student understanding and to progress their understanding of the mathematics. Not only did this form of mathematics instruction translate to in-class mathematics proficiency, but it also translated to proficiency on state assessments. In an educational era that places focus on testing and accountability, enabling and encouraging students to be confident and capable problem-solvers who take their skills and knowledge into other mediums is important to their futures.

### **Special Considerations and Issues of Ethic**

#### **Researcher Bias**

This qualitative case study positioned the researcher as instrument (Denzin & Lincoln, 2005; Poggenpoel & Myburgh, 2003), meaning that the researcher was the instrument that obtained and analyzed the data. This introduces researcher biases into the data collection, analysis, and interpretation of the study. Attention to quality indicators for qualitative research and strategies such as a researcher code-recode of data helped to ensure the trustworthiness of the findings. However, although the researcher's personal biases could be recognized and managed,

they could not be fully isolated from the research experience. The statement of positionality attempted to make transparent some of the researcher's biases about mathematics and teaching that may have potentially influenced findings. Yin (2014) asserted that bias in case study methodology can be addressed and minimized by a researcher remaining open to contrary evidence. In this study, the researcher looked for confirmation of participants' shared beliefs and intentions in actual practice, but also looked for discontinuity. Additionally, attention to "maintaining strong professional competence that includes keeping up with related research, ensuring accuracy, striving for credibility, and understanding and divulging the needed methodological qualifiers and limitations to one's work" can help to minimize bias (Yin, 2014). Researcher's bias in this study was managed through the detailed description of the study's data collection and analysis processes in Chapter 3, the statement of researcher involvement in Chapters 4 and 5, and the list of limitations in Chapter 5.

### **Assumptions**

One assumption in this study was that prospective teachers were honest about their beliefs and experiences in their interview responses and honest in their interactions with the researcher during their community of practice team meetings. Another assumption was that reform-based mathematics pedagogy was the best manner of instruction to support student learning. Because of these assumptions, instructional techniques aligned with traditional ideals (e.g. students using one specific fraction problem-solving approach dictated by teachers) were deemed insufficient and the community of practice team meetings worked to support prospective teachers' implementation of reformed approaches that honored student thinking and multiple approaches to learning mathematics.

## **Population of Interest**

The community of practice and its team meetings did not supersede or take the place of university coursework or graduation requirements. Participants were informed that there were additional requirements outside of coursework in order to participate and voluntarily elected to be a part of the study. Participants were guaranteed anonymity and assured that they could opt out at any time. Their participants' rights were protected under the Institutional Review Board (IRB). The adult consent form for this study is provided in Appendix E.

The prospective teachers' university expectations, portfolio completion, and grading were monitored by a site-based university supervisor and not by the researcher. The prospective teachers' work, discussions, interviews, and observations from this study were kept private and were not shared with their site-based university supervisor or cooperating teacher except in one instance when the researcher and university supervisor met jointly with a participant in a post-teaching debrief session. This session was at the dyad's request, and with prospective teacher permission. The cooperating teacher had also observed the participant's practice that day and joint feedback was requested around the prospective teacher's employed pedagogical techniques so that consistency of vision could be encouraged. This session is described in Chapter 4, and it is important to note that the conversation around practice did not continue outside of the time granted by the prospective teacher's permission. This attention to privacy protected the prospective teachers from the study directly impacting their grades. It was the hope that prospective teachers' practices would improve based upon membership in a community of practice and participation in the team meetings, so it was possible that grades were indirectly impacted simply through involvement. However, there was no direct discussion of the study's findings between the researcher and other members of the school and university community.

## **Place of Research**

Research took place with prospective teachers enrolled in the university's elementary education program, but nested in a cooperating elementary school. Elementary students and staff were protected through proper research approval channels with the district. The district's point of contact for research approval was made aware of process and approved the research prior to data collection taking place. Elementary students and cooperating teachers were not directly studied in this research. Any data collected about students was done through the prospective teachers and through the established pre and posttest protocol. Student assessment data was blinded for the researcher.

## **CHAPTER 4: FINDINGS**

The long-term goal of this research was to guide teacher educators in structuring programs to best support prospective teachers movement toward reform-based teaching behaviors that center on engagement with students' thinking. To do this, the current study exposed prospective teacher beliefs and instructional practices in order to better understand what supports influence changes in practice that focus on the use of student thinking in the planning and implementation of instruction. It was through the methods described in Chapter 3 that a depiction of prospective teachers' practices, both individually and as a community, was developed. Through qualitative analysis, several themes emerged about the supports and structures that facilitate prospective teachers' engagement with student thinking in mathematics. The findings are presented in relation to each research question:

1. How might a prospective teacher's engagement with student thinking be characterized?
2. In what ways does context influence a prospective teacher's engagement with student thinking?
3. In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?
4. How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?

## **Orientation to the Chapter**

Several tables and figures from Chapters 2 and 3 are key to understanding the findings of this study. They are as follows: Table 2.1 *Levels of Engagement with Children's Mathematical Thinking*, Table 3.3 Data Collection Timeline, Table 3.6 *Principles to Actions Mathematics Teaching Practice*, Figure 2.3 Types of Strategies – Equal Sharing Problems and Figure 2.4 The OGAP Fraction Progression. Each is referenced in this section and/or subsequent sections. The prospective teachers were placed at Brookside Elementary School in separate 4<sup>th</sup> grade classrooms and were part of a community of practice structure that included scheduled team meetings with the researcher. The facilitation plans that guided each team meeting are provided in Appendix C. These meetings served to reorient the prospective teachers to key pedagogical resources from their previous coursework and to support their prioritization of attending to student thinking when planning for instruction. It was hypothesized that the continual focus on children's thinking would support the prospective teachers' attention to its use even when juggling the responsibilities of full-time teaching. A clear understanding of the structure of this study, specifically its community of practice team meetings and timeline of events, will aid in the understanding of the findings. As a reminder, Table 3.3 Data Collection Timeline provides the review of that timeline.

### **Team Meeting Structure**

The current study addressed the implementation of a novel professional learning pathway for prospective teachers' that was intended to support their use of the mathematics teaching practices in working with 4th grade students' to develop their conceptual understanding of fractions (Mojica & Friel, 2015). The proposed 4th grade pathway featured the use of cognitively demanding tasks from the *Fraction Kit* (Burns, 2001), *Fraction Tracks* (NCTM, 2016; TERC, 2004), and *Extending Children's Mathematics* (Empson & Levi, 2011). An overview of this

instructional path is provided in Appendix A. In addition, reorientation and reinforcement to previous coursework initiatives such as *Classroom Discussions in Math* (Chapin, O'Connor, & Anderson, 2003; 2013) and its corresponding productive *talk moves* and Cognitively Guided Instruction (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2014) was addressed in the team meetings. The prospective teachers analyzed students' fraction work according to the frameworks presented in Figures 2.3 Types of Strategies – Equal Sharing Problems and 2.4. The OGAP Fraction Progression. The framework provided in Figure 2.3 (Empson & Levi, 2011, p. 25) regarding the types of strategies children use to solve equal sharing problems was the basis for the initial work in the analysis of student understanding during the community of practice team meetings. Prospective teachers were also trained to use the Ongoing Assessment Process (OGAP), Fraction Progression (Figure 2.4; Petit, Laird, Marsden, & Ebby, 2015, p. 196) which was another tool utilized for analysis of their students' work.

It was hypothesized that the prospective teachers working together in community of practice team meetings would develop their craft, specifically with the focus on engaging with student thinking around fractions. The construct of *professional noticing of children's mathematical thinking* (Jacobs, Lamb & Philipp, 2010) guided the development of the team meetings. Professional noticing of children's mathematics necessitates a teacher's expertise in attending to children's strategies, interpreting children's understandings, and deciding how to respond instructionally based on this attention and interpretation. Professional noticing involves the decisions made about how to respond based on what is noticed, but it is not an action; the action happens in the next stage, that of instruction that engages with student thinking.

The researcher facilitated the community of practice meetings. In that role, she prompted the prospective teachers to discuss their implementations of the tasks from the 4th grade

instructional pathway. She encouraged prospective teachers to bring student artifacts to the team meetings in order to give evidence of and be used to unpack their noticing and decision-making. It was intended that the prospective teachers would develop their noticing and understanding of student thinking through careful analysis of student work and through cooperative dialogue with each other. While the researcher prompted and extended conversation within the community of practice, it was the prospective teachers' own experiences, student work samples, and questions that guided the foci of the team meetings.

### **Researcher Involvement**

The pronoun "I" is used throughout the remainder of this chapter. This reflects the researcher's role of participant-researcher in the community of practice team meetings and the prospective teachers' instructional debrief sessions. This firsthand role provided insight into the prospective teachers' beliefs and practices that may not have been attained otherwise. The use of "I" acknowledges the researchers' potential influence in shaping some of these beliefs and practices. Additionally, claims of participants' beliefs made herein reflect participants' own statements made during study interactions. An instrument to ascertain belief and beliefs change quantitatively was not used; rather prospective teachers' beliefs were expressed through interviews and interactions and were filtered through researcher interpretation and portrayal in the results. While some interpretations of the findings are suggested in Chapter 4, the study's interpretations and their implications are formally addressed in Chapter 5.

### **Prospective Teachers' Stories**

Each prospective teacher's experience throughout the case study was first examined as a story of practice. It became apparent that each prospective teacher experienced a pivotal moment that shaped her practice and her ability to engage with student thinking in mathematics. While these pivotal moments were experienced individually, they became a part of the

community of practice in that they impacted how the prospective teachers moved forward with one another. These stories serve as an important prologue to the findings presented in relation to each question.

The following three subsections provide a story of each prospective teacher's engagement with student thinking comprised from interview and observational data. Each story is presented in narrative form with beginning, middle, and end sections in order to best reflect the study timeline (Table 3.3) and to note key moments of transition within the prospective teachers' practices. The beginning section captures a depiction through the initial interview, the first in-person observational period, and team meetings 1-3. The middle section captures the second observational period and team meetings 4-6. The end section captures the third observational period, team meetings 7-8 and the closing interview.

Throughout these subsections, the term *scaffolding* is used. Scaffolding is an educational term used to represent successive levels of support that help a learner reach a higher level of understanding or skill that would not be attainable without the assistance (Hidden Curriculum, 2014). When scaffolding moves are used by a prospective teacher in the classroom, they represent questions and prompts that support a student's thinking and help move the student to a higher level of thinking or to a more sophisticated fraction strategy. When scaffolding is used with the prospective teacher by the researcher, it represents the development of more supportive lesson materials in order for prospective teachers to do the in-the-moment work of the classroom.

### **Prospective Teacher 1: Kristi's Story**

**Kristi in the beginning.** Kristi claimed her view of mathematics instruction and learning was dependent on each individual student.

I definitely think it needs to start off with, like, them. Kind of seeing whether it's how they solve a problem or simply what they know about a given topic. So it starts off with what they know and what they can do, and then goes into what they're wondering or

what they're thinking or feeling or where they want to go from there. Which, like, I'm not going to say that that's where I'm always going to take them, but, like, at least I know that that's a parallel trajectory that they can be on as I lead them through whatever guided steps are necessary to get them to an eventual, like, efficient method of doing it.

Kristi seemed to be expressing the belief that the teacher has knowledge of the mathematical trajectory a child should move on towards computational knowledge or fluency, and that while helping the child progress on this path, the teacher must also value children's personal goals for themselves. In theory, Kristi was deeply committed to meet the students where they were and support and scaffold learning for each on an individualized path to understanding.

Once instruction began Kristi renegotiated her view of mathematics instruction as being driven by each individual child. She expressed that this may happen in "an ideal world" but that she was losing track of the needs of individual children in real-time. At the beginning of the six-week instructional period, she used the productive talk moves haphazardly. She commonly asked students "who can repeat, agree or disagree with, or add on to what was just said?" (See Appendix G for an overview of the productive talk moves). In the researcher debrief session, I shared that her implementation of talk moves was problematic for students because she was asking them to do too much at once. They could not engage in discussion, and therefore she could not engage further with their thinking. This deepened the real-time confusion about what to do and where to go next with the mathematics. Kristi made a goal to reorient herself and her students to the use of only one talk move at a time.

However, Kristi grew increasingly overwhelmed at balancing her goals for instruction and the in-the-moment needs of individual children. At the third community of practice team meeting, she expressed the need for a curriculum. When pressed as to what she meant by curriculum she qualified, "We need resources and to know where to go, and with the progression how to scaffold that thinking. I have no idea how to do that without some sort of guide." The

district had a pacing sequence with recommended resources, but did not have a set curriculum in place. The progressions we used to classify student work at team meetings did not have the scaffolding guidance Kristi requested. The prospective teachers had their mathematics unit plans that outlined what to teach according to the mathematics standards, but she felt this was not enough. Originally she thought knowing what to do with children's thinking in-the-moment would come through "just noticing to see how kids work best" but realized that without some sort of explicit pathway of scaffolding their thinking, she floundered. It was at this point in the study she had a career crisis. Kristi felt she was not connecting to the children and was not sure if she should continue teaching. She began to search for different career options.

**Kristi in the middle.** In response to Kristi's voiced feeling of helplessness and need for a curriculum, we used the team meetings to further develop existing lessons plans and explicitly discuss the questions the prospective teachers might ask at different points and consider what students might say in response. Additionally, the prospective teachers met on weekends outside of the planned team meetings to plan lessons and discuss the pace of their instruction. With the support of her community of practice, Kristi was able to reconcile her beliefs about individualized instruction to find a methodology that allowed her to manage the mathematics in addition to all the other components of student teaching. Her orientation shifted from considering each individual child to considering the whole group or small groups of students. She expressed that her goal was to find a balance between open explorations and planned instructional points.

True to this vision, Kristi's instructional observations began to showcase a mixture of whole group problem solving and discussion around a shared context, with movement into small groups around particular planned teaching points. Kristi used a majority of probing questions

when interacting with children either in circulating the room or in small groups. Probing questions serve the purpose of eliciting a students' thinking. These probing questions included, (a) "What did you do?" (b) "Why do you think that?" and (c) "Tell me what you did here." She also used scaffolding questions that were dependent on student work. In an interaction in which the student told her  $\frac{5}{8}$  is equal to  $\frac{4}{6}$ , Kristi pondered aloud "Hmm....I wonder, well, I wonder how many of these [showing the student a  $\frac{1}{8}$  fraction strip] are equal to  $\frac{1}{6}$ ?" Kristi provided a question that helped scaffold the student by referring her back to the student-made manipulative kits and back to the unit fractions.

Kristi focused her talk moves on the *repeat* and *rephrase* moves in which students restate peers' shared ideas. She also *revoiced* students' ideas and she explained that she hoped this helped everyone gain access to the important points of mathematics to move forward in understanding. At the midway instructional debrief session, she referred to the initial debrief moment around the erratic use of talk moves as when she realized that she had be intentional with what she was asking of children if she wanted them to be able to share and engage during instruction.

At team meetings 4 through 6 Kristi discussed individual children, specifically around what instructional moves were needed to help them progress into different equal share strategies from Empson and Levi's (2011; Figure 2.3) Types of Strategies – Equal Sharing Problems. She could identify what strategies students in her classroom used and took an interest in the models being employed. However, contrary to what she initially thought about being able to individually address all these aspects in real-time, she had to do the work of identification and interpretation outside of the instruction.

**Kristi in the end.** Ultimately, Kristi decided she would continue pursuing teaching as a career for the time being and credited the support of others as what helped her to this decision.

My CT helped me stay in the profession...and my family. And this group, and you [the researcher]. This group, well the support it gave, but also you just listening and telling us we weren't crazy for feeling the way we're feeling.

At study end, her expressed need for a curriculum remained in place, but she explained her belief that within set plans, "kids should work with their own meaning to a point that they can make understanding of a subject." Kristi said she needed a curriculum to move forward in her teaching, but to her, the term curriculum meant some path or guidance of what she should be doing to help move children forward in their understanding. She felt a provided curriculum would not constrain or restrict her ability to engage with children; rather, it would further it. She qualified, "I mean, a curriculum just allows you to know what's going on and gives some order. What are kids supposed to know and do? You still listen to your kids."

### **Prospective Teacher 2: Nora's Story**

**Nora in the beginning.** Nora's major academic concentration within the elementary education program was in Mathematics and Science. This meant that she completed other educational coursework in these content domains beyond the required core courses. She said she could not remember mathematics in her elementary grade years, but loved calculus and viewed herself as good at mathematics. She also appeared to enter the study with more traditional views of students' learning of mathematics.

Just having practice with problem sets. They learn best... um, practice, practice, practice. And working on their level, I guess. Kind of like if someone's higher in math, like, something more challenging. If they're lower, maybe like a smaller number set.

When given a selection of fraction problems and asked to rank the order of difficulty for children to understand (Interview questions 7-8, Appendix B) she thought the computational problem of  $1/5 \times 1/8$  would be the easiest for children to understand "because you're just multiplying

across” and a student would “be able to see, ok, so I multiply at the top and then the bottom.”

Nora thought this problem would be easier for children to understand than a contextualized equal sharing problem about children sharing candy bars. Her view of understanding mathematics was aligned with computational accuracy, perhaps because she could easily solve mathematical computations.

However, once Nora started to teach, she became actively involved with student thinking and consistently used probing questions. In a ten-minute interchange with various students she probed in numerous ways including: (a) “Explain your drawing to me.” (b) “How do you know  $1/10$ ?” (c) “How did you add your fractions?” (d) “Tell me what you did here.” (e) “Tell me how you solved this.” and (f) “How did you know to split the last one into tenths?”. In her debrief session with me, I commented on her use of probing questions and her attentiveness to various children’s strategies. Nora explained, “Well, I have to ask questions, otherwise what else would I do?” When I asked for clarification, she further explained that she would not be able to teach mathematics without the students’ contributions. She needed their explanations to figure out what to do and where to go next in the instruction. What appeared as specific questioning for her teaching point was actually open-ended questioning to find a teaching point she felt prepared enough to take up for the day. She understood that if she did not question students about their thinking, she would have to teach through direct instruction of a strategy, but knew that was not acceptable by her cooperating teacher. She also expressed that even if she wanted to use direct instruction, she would not know what to teach students, because they were solving problems differently than she would solve them. Nora was engaging with students, but, to some extent, she appeared to be doing so out of self-preservation and from loss of knowing what else to do.

**Nora in the middle.** By the midpoint of the study, Nora became overwhelmed trying to balance teaching, her portfolio expectations, and exploring different ways to teach mathematics. She tried different teaching approaches in mathematics (whole group, small group, partner work) and began to flesh out her philosophy of teaching. As part of her instructional goals, she also worked on her pacing within mathematics lessons. Nora became consumed with pacing and timing her lesson segments to the point that it superseded students' needs. Nora had to strike a balance, and it was at this point that her cooperating teacher spoke to her about being teacher-centered versus student-centered in her instruction. Even though the cooperating teacher recognized that Nora led discussions utilizing talk moves, she felt the moves and structures were now centered on Nora's pacing and not around student thinking. This was a turning point for Nora, as she had perceived herself as being student-centered.

However, just alerting Nora to teacher-centered versus student-centered instruction was not enough to change her practice; she needed specific support around what she could do. After the second in-person lesson observation, her cooperating teacher and I did a shared debrief session in which we opened by asking Nora when she felt best teaching mathematics. This session resulted from a request from the dyad to debrief around a shared vision for practice and took place with Nora's permission. The cooperating teacher and I had noticed moments and structures in which Nora seemed to excel in her teaching. These moments happened when she dug into the mathematics with the students for the sake of the mathematics and children. I wanted this revelation to come from Nora and not from our imposition of beliefs upon her.

Nora said she felt best instructing in small groups because she could better attend to students and could interact with them more easily to make decisions about the next steps in instruction. Her cooperating teacher reminded her that while timing of a lesson was important,

the students come first. From that moment on, Nora began to structure her mathematics lessons around a whole group launch and then break into small groups to circulate through the whole class. The groups were flexible and she analyzed student work at night to make decisions about how to work with students the following day.

Purposeful attention to student thinking became more prominent in Nora's discussions in the community of practice team meetings. At a meeting organized by the prospective teachers outside of our planned team meetings, Nora brought student work to analyze. I was invited to the meeting and while I was there, Nora highlighted some of the students' equipartitioning work and wanted to unpack the solution strategies that perplexed her. This spurred involvement from the other prospective teachers around student strategies for equipartitioning. It led to an hour-long discussion around number choices for equal share problems and around various students' equal sharing strategies according to levels of sophistication from the Types of Strategies – Equal Sharing Problems (Figure 2.3). We planned future problems to pose to students and mapped possible strategies to highlight in class for the following instructional week based on their students' previous work. At this meeting Nora also shared about her new home filing system for student work and about how she analyzed and organized work at home. She had a color-coded organizational system by subject and a note-taking system to look across multiple students for strategy trends and to also look within work samples from one student to note recurrence of strategies. Nora was outwardly proud by her effort and her emphasis on student thinking was becoming outwardly apparent.

**Nora in the end.** In the final series of observations, Nora's practice shifted in relation to her prompting and questioning of students. Nora began to reflect on how to move her questioning and teacher practices to extensions of student thinking. In previous observations,

she had utilized the teacher actions of probing and scaffolding questions, but in the end began to consider what extending questions and prompts would entail. At our last lesson debrief session, Nora explained her choice of problems, number choice within problems, and her questioning. She thought through possible levels of sophistication of fraction addition problems for 4th grade students. She took the work from the community of practice team meetings in which we considered levels of sophistication of student work, and applied this mindset to her lesson planning and implementation. Her cooperating teacher echoed that she felt Nora was just now ready to think about this next piece of extension focused on what extending student thinking looks like and how to challenge students without just giving them bigger numbers in problems.

The final interview helped clarify that Nora's reason for engaging with student thinking shifted; she wanted to engage with student thinking because it made mathematics class more engaging for the students.

I've learned that it's much better to have, like, those student-centered conversations because it's more engaging to the students and because it challenges them to be actively listening and participating in the discussions. Rather than just an 'oh, I know I'm not going to get called on, so I'm going to check out.'

Previously, Nora had engaged as a coping mechanism for not knowing where to go next in the mathematics teaching. While she still questioned students to make decisions about where to go next instructionally, she did so with more confidence around the content because of the intensive planning with her peers in the community of practice. Additionally, she probed student thinking in an aim to incorporate students into lessons and show them their ideas were needed and integrated.

### Prospective Teacher 3: Liz's Story

**Liz in the beginning.** Liz was vocal about her dislike of being a mathematics student in the opening interview. This dislike and self-proclaimed being “bad a math” transferred to a fear of not being able to know what and how to teach mathematics. She did however credit learning about CGI in her Mathematics Methods course as what helped her embrace mathematics teaching.

CGI and that stuff, that's what made me feel really confident in teaching math and that's the kind of direction I want to go in, for sure. It was, like, student led, so instead of all of the weight of their comprehension being on – well, it still kind of is on me – but I'm not the one spewing out facts and, like, saying, ‘This plus this is this.’

Liz went on to explain that when she asked students to solve CGI problems “it was really nice to hear what they had to say and then know from there, oh, I can just pick up here and kind of scaffold it in.” She expressed that she felt teaching and learning of mathematics came from giving students space to talk about mathematics and being thorough with the ways to solve one problem rather than going over answers for a battery of problems in the same amount of time.

Once in the classroom, Liz felt her students were missing some of the key norms and classroom set-up for productive discussions. However, during instruction she utilized the talk move of “Who would like to add on?” She immediately jumped to a talk move that asked students to share thinking, absorb a classmate's thinking, and add onto it, but the students needed time to develop the norms of active listening through *wait time* and the repeat/rephrase moves. Much like Kristi, Liz's debrief session was used to remind her that the students needed time to adjust to the talk moves and to discussion. Liz recognized this herself, but said that in the moment, got ahead of herself and her class.

**Liz in the middle.** In subsequent lessons, Liz used the repeat move and used probing and scaffolding questions such as: (a) “How do you know?” (b) “What will you do next?” (c) “What does this line represent?” and (d) “Tell me more.” However, during her second in-person observation, Liz had what she perceived a “failing moment” and began to tear up at her follow-up debrief session. During a discussion on equivalency, a student noticed that certain fractions like  $\frac{1}{2}$ ,  $\frac{2}{4}$ ,  $\frac{4}{8}$  and  $\frac{1}{5}$ ,  $\frac{2}{10}$  have a pattern. The student expressed the pattern as a numerator and denominator doubled made the next fraction. Liz acknowledged this observation, restated it for all of the class then questioned, “Do you think this is true of all numbers?” Some students replied no, some yes, and students grappled with supporting their claims. Another student offered that she thought the pattern had something to do with multiplication and division. Liz grabbed onto that offering and tried to tie the work together. She began to explain how  $\frac{1}{2}$  could be multiplied by the whole  $\frac{2}{2}$ , but stopped mid-sentence and paused for approximately four seconds. Relatively speaking, four seconds is a short amount of time, but it seemed long to Liz and apparently to some students as one jumped in and asked, “Did we just stump the teacher?” Liz replied, “I know what I want to say, just not how to say it.” Liz started again, took another two-second pause, and then asked students to think back to their division unit. She prompted, “If [the fraction bar] was interpreted as a division bar like it was in your division unit, do you notice any patterns or connections now?” This was enough of a scaffold to bring another student into the conversation that shared he noticed the fractions as being just like division and multiplication facts, but flipped because  $\frac{2}{1}=2$ ,  $\frac{4}{2}=2$ ,  $\frac{8}{4}=2$  and in multiplication they are all double. Liz affirmed his thinking and wrapped up the lesson by stating they would come back to the conversation tomorrow.

Later in the debrief session, Liz explained that she was trying to steer away from simply giving the students a rote procedure of “multiply the top and bottom by two” for finding an equivalent fraction. She did not feel the students were ready for to make conceptual sense of that computation and that if she introduced it they would not understand why the process worked to create an equivalent fraction. However, she did want to honor what the original student had noticed about a pattern, because she had overheard others starting to discuss this notion as well. Liz felt that she needed more time to think about the mathematics content before being able to connect it meaningfully to the students’ contributions. Liz saw her inability to think in the moment as a failure. I told Liz that she needed to reframe this from a failure to a moment of courage and that I was proud that she chose to pause instead of charge ahead into something the students were not ready for nor did she keep talking for the sake of talking. Liz and I shared this moment with the others at the next team meeting. During the remainder of the study, we came back to this moment and continued to embrace it, recognizing that teachers require think time just like students require think time.

**Liz in the end.** By the end of the study, Liz still thought mathematics was the most difficult subject to teach, but felt she had learned about teaching in general through the experience in the community of practice and that she had a clearer picture of how she wanted to teach mathematics moving forward. She expressed that facilitating group discussions with topical launch questions for her students was still something she had to work on.

Because I know they were purposeful, but I didn’t know where they were always leading. ‘What am I supposed to be getting from this?’ Like those kinds of things. So I don’t know like what from an activity they’re supposed to have command of. She also cemented her vision for mathematics teaching. She knew she did not want to group students before knowing what they capable of doing with the mathematics. She felt she had seen

mathematics instruction modeled that ability-grouped children as low, medium, and high for the duration of a unit.

Well, also just because ... I don't know...just because a kid is usually approaching skill there are things that they're extending skill on, so like going ahead and before they even begin the unit, dividing kids out I feel like is problematic in that way.

Instead of labeling children low, medium, or high, Liz wanted to acknowledge where children were with certain skills and teach according to those skillsets. She felt that regardless of skillset, students needed whole group discussion together so they had shared exposure to concepts and contexts.

### **The Relevance of the Prospective Teacher Stories**

The stories helped to showcase the prolonged engagement of the researcher and provided rich description of the case study. Each prospective teacher's story highlighted the uniqueness of each participant in the case study, but also worked to expose the influence of the study supports on the collective group. The purpose of the stories was to provide an overview of the case through each prospective teacher's practice so that these moments of practice may be referenced throughout the subsequent findings sections as supporting evidence. The stories serve as a bridge to the presentation of findings for each question because they provide insight and detail to enhance the analysis processes described in Chapter 3.

### **Question 1: How Might a Prospective Teacher's Engagement with Student Thinking Be Characterized?**

The first question of the study addressed the need to characterize prospective teachers' engagement with student thinking in order to better inform teacher preparation programs about the supports that may influence engagement. To do this, the combined use of two tools was proposed: *Levels of Engagement with Children's Mathematical Thinking* (Franke, Carpenter, Levi, & Fennema, 2001) and the mathematics teaching practice *Elicit and Use Evidence of*

*Student Thinking* as defined in the *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014, p.10) teaching and learning guiding principle. The two tools were used to analyze the prospective teachers' practices because both appeared to provide ways to identify evidence that characterized the prospective teachers' engagement with student thinking. As the stories suggest, each prospective teacher was an individual, who held individual beliefs and experienced individual changes in practice. In order to discuss the findings that address the contextual and support factors that influenced prospective teachers' engagement with student thinking, each prospective teacher's engagement with student thinking needed to be exposed and characterized using a coding scheme. This was done through aligning their stories with the *Levels of Engagement with Children's Mathematical Thinking* (Table 2.1). Levels 1-2 align with traditional mathematics beliefs and practices and Levels 3-4B align with reform mathematics beliefs and practices that give prominence to children's thinking. At Level 3, a teacher believes students should solve problems in their own ways and listens to their thinking. A transition to Level 4 means that the teacher believes that the student thinking determines the evolution of the curriculum. Note that Levels 4A and B require sophistication in the teaching practice and are not expected of a teacher early in her career or expected without sustained support in reform-based practices.

As reported in Chapters 2 and 3, the *Levels of Engagement with Children's Mathematical Thinking* (Table 2.1) were developed and used to measure generative change toward the use of cognitively guided instruction that engaged with children's thinking. While a student teaching experience does not provide the opportunity for longitudinal measure of change across years, there was still a great deal of change observed in the six-week lead teaching experience. To further refine the use of the *Levels of Engagement with Children's Mathematical Thinking* for

characterizing prospective teachers' engagement, a selected mathematics teaching practice and its corresponding teacher actions from the *Principles to Actions: Ensuring Mathematical Success for All* was also used. In the *Principles to Actions*, eight mathematics teaching practices are identified. Each mathematics teaching practice is accompanied with deliberate articulation of how this practice looks in action, that is, what teachers do and what students do. For this study, the focus is on the teacher actions for the one identified mathematics teaching practice *Elicit and Use Evidence of Student Thinking* (see Table 3.6). In combination, these tools provided a consistent and structured way to characterize each prospective teacher's practice related to their engagement with student thinking.

Table 4.1 and Figure 4.1 that follow provide the overview of each prospective teacher's level within the *Levels of Engagement with Children's Mathematical Thinking* at the beginning and end of the study and are followed by the justification for each prospective teacher's level placement. The prospective teachers appeared to range in their initial Levels of Engagement from Level 2 to Level 4A. These beginning levels were determined using data from the timeframe of the first interview through team meetings 1-2. These events took place before the six-week teaching experience, and therefore the beginning level is classified only with the *Orientation* portion of *Levels of Engagement with Children's Mathematical Thinking* and with any intended practices from the prospective teacher. The end levels at the completion of the study take into account the six-week teaching period, team meetings 3-8, and the final interview. These levels represent a coordination of *Orientation* and *Activity* portions of the *Levels of Engagement with Children's Mathematical Thinking*. The three prospective teachers appear to have arrived at and upheld the characteristics of Level 3 by the end of the study.

Table 4.1. *Overview of Prospective Teacher Profiles According to the Levels of Engagement with Children's Mathematical Thinking*

	Beginning Level: Orientation	End Level: Orientation and Practices Integrated
Kristi	4A	3
Nora	2	3
Liz	3	3

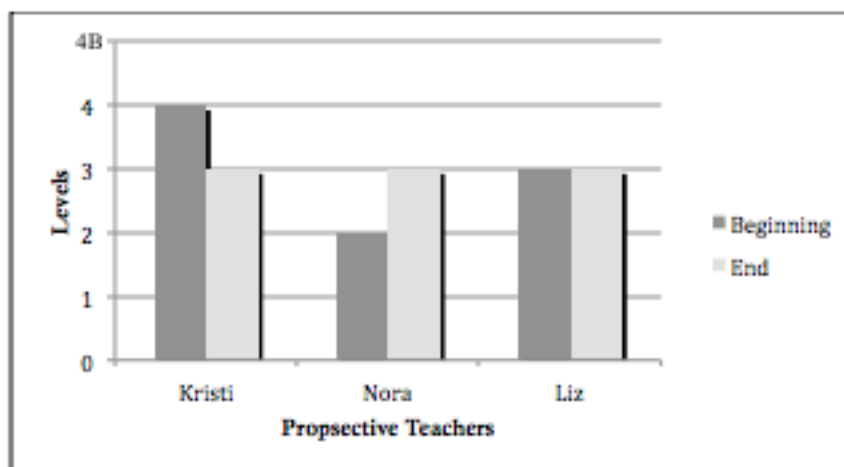


Figure 4.1. Overview of Prospective Teacher Profiles According to Levels of Engagement with Children's Mathematical Thinking

### Kristi's Engagement with Student Thinking

As Kristi's story exposed, Kristi's beliefs strongly aligned with reformed ideals at the beginning of the study. She expressed the beliefs of Level 4A in which a teacher *believes students' mathematical thinking should determine evolution of curriculum and the ways in which teachers individually interact with students*. In her first interview, Kristi's view of mathematics instruction and learning was dependent on each individual student, and she aimed to consider each child's needs as she planned a pathway for mathematics. After actual practice into consideration with her overall orientation, Kristi could no longer embody a Level 4A; however, she did still retain mathematics reform ideals. According to the teacher actions (Table 3.6) of the mathematics teaching practice *Elicit and Use Evidence of Student Thinking*, Kristi used a

majority of probing questions and prompts when interacting with children in whole group or small group (teacher action 4). Rather than consistently making in-the-moment interpretations of student thinking, Kristi often had to spend time outside the classroom considering her students' needs, and more time than she originally anticipated. However, because she continued to view student thinking as an important driver for instructional decisions, she dedicated the time to reflect on student learning to inform planning (teacher action 5). Her analysis of student work at team meetings around Types of Strategies – Equal Sharing Problems (Figure 2.3) and the OGAP Fraction Progression (Figure 2.4) showed that she was able to identify evidence of student progress toward mathematical goals (teacher action 2) and then interpret this evidence to consider student understanding and student methods (teacher action 3).

By the end of the study, Kristi's beliefs and practices were still aligned with reform mathematics ideals and focused on engagement with students, but not to the sophistication of Level 4A. Kristi ended the study most aligned with the *Levels of Engagement with Children's Mathematical Thinking* Level 3: *Believes it is beneficial for students to solve problems in own ways because they make more sense to them; wants the students to understand what they are doing*. Thus, her end-of-study characterization was Level 3.

### **Nora's Engagement with Student Thinking**

Nora's story exposed her inconsistent beliefs around the teaching and learning of mathematics at the beginning of study. Her view of learning mathematics was aligned with cooperative learning but understanding mathematics was aligned with computational fluency. Nora's view of teaching and learning was closely in line with the *Levels of Engagement with Children's Mathematical Thinking* Level 2 in which one of the descriptors is that the teacher is *inconsistent in beliefs and practices related to showing children how to solve problems*.

However, Nora became focused on student work and student thinking, and this shifted how she

discussed mathematics and planned for mathematics instruction. She presented student work to her colleagues and asked for their help in interpreting her students' thinking in order to make decisions about her next steps. In doing so, she exhibited the mathematics teaching practice *Elicit and Use Evidence of Student Thinking* teacher actions 3 and 5. These teacher actions paired for Nora, in that they require a teacher to interpret student thinking in order to assess understanding (teacher action 3) and to reflect on the student thinking to inform planning (teacher action 5). Nora even exhibited these actions outside of the participant researcher guidance of the team meetings and transferred them to the unplanned meeting setting. During her instruction she consistently attempted to elicit and gather evidence of student understanding (teacher action 2) through the use of questions and prompts that probe student thinking (teacher action 4). However, questions/prompts that probe student thinking is only one aspect of that specific teacher action. By the end of the study, Nora exhibited a balance of all the aspects of that teacher action when she employed probing, scaffolding, and extending questions and prompts during instruction (teacher action 4).

Nora's actions and beliefs appeared to have shifted from Level 2 to Level 3 of the *Levels of Engagement with Children's Mathematical Thinking* because she seemed to align to its orientation: *Believes it is beneficial for students to solve problems in own ways because they make more sense to them; wants the students to understand what they are doing*. Her openness to allow students to solve problems in their own ways and willingness to unpack the analysis of these ways to consider next problem choices affirmed this placement. To transition to Level 4A, Nora would have to showcase that the students' mathematical thinking determines the evolution of curriculum. However, by study end, she continued to make overarching weekly decisions based around a curriculum overview and lesson planning from the team meetings. While she did

adjust her teaching within the week to accommodate students, their individual thinking did not drive the planning and instruction as in accordance with Levels 4A and B.

### **Liz's Engagement with Student Thinking**

Liz's story exposed that at the beginning of the study, she felt teaching mathematics meant providing students space to discuss the mathematics and then showcasing the many ways to solve a mathematics problem. Her beliefs aligned with Level 3 in that she almost overtly stated its orientation in the interview of *Believes it is beneficial for students to solve problems in own ways because they make more sense to them*. Like the others, Liz exhibited the mathematics teaching practice *Elicit and Use Evidence of Student Thinking* by utilizing a majority of questions and prompts that probed for student thinking (teacher action 4). At first, probing questions were her primary means to elicit and gather evidence of her students' understandings of the mathematics (teacher action 2). However, her use of probing questions transitioned to a balance of in-the-moment scaffolding and extending questions as she became more comfortable with instruction (teacher action 4). She then used what she learned in lessons about student understanding to adapt lesson plans for the following day (teacher action 5).

At the end of the study, Liz's intention for mathematics instruction appeared to begin to approach Level 4A *Believes students' mathematical thinking should determine evolution of curriculum and the ways in which teachers individually interact with students*. However she did not yet have a depth of knowledge of the mathematics content to allow student thinking to be the primary drivers of the curriculum. She instead relied on the structured lesson planning done with Kristi and Nora at our team meetings to guide her mathematics in the classroom. Liz entered the study with the orientation of Level 3, and her practices of providing students with opportunities to solve problems and discuss their ways of thinking, and then listening to their thinking affirmed a continued placement at Level 3 at the end of the study.

## **Analysis Consideration with Respect to Engagement with Student Thinking**

Note that an end-of-study Level 3 was assigned with awareness and caution that it reflects a six-week teaching period. Although prospective teacher practices within their instructional unit were aligned to the practices within this level and their beliefs aligned with the orientation, the researcher acknowledges that six weeks provides only a glimpse at a teacher's mathematical practice. Therefore, a characterization of Level 3 is provisional yet optimistic; provisional in that this change was observed in one content domain of one subject area with the support of a community of practice yet optimistic in that if the prospective teachers showcased these beliefs and practices in this content domain, it is hopeful that they can and will uphold these beliefs going into other domains and into their induction year practice. Further, the prospective teachers' rationale behind why they held these beliefs and demonstrated practices in a reform manner influenced a characterization of a Level 3. Teacher rationale is currently not part of the *Levels of Engagement with Children's Mathematical Thinking* framework. The potential for inclusion of this aspect will be explained in greater detail in Chapter 5. This inclusion of rationale may contribute to the framework and contribute to how it is used with prospective teachers for future researchers.

### **Question 1 Summary**

In summary, it appears that the tools chosen to characterize a prospective teacher's engagement with student thinking work together to provide insight into how a prospective teacher might engage with student thinking during the student teaching experience. The teacher actions from the mathematics teaching practice of *Elicit and Use Evidence of Student Thinking* helped fine-tune the *Levels of Engagement with Children's Mathematical Thinking* to capture the ways in which the prospective teachers in this study engaged with student thinking during their six-week teaching period. The use of the tools together also exposed the sophistication of the

tools' expectations for prospective teachers. The mathematics teacher practice of *Elicit and Use Evidence of Student Thinking* groups several aspects of instruction together, as seen in teacher action 4's *questions and prompts that probe, scaffold, and extend*. However, the stories indicated that these types of questions and prompts are distinct mechanisms of engagement with student thinking that a prospective teacher needs to unpack and learn to utilize. The tools taken together also made it possible to characterize shifts in engagement with student thinking during an expedited timeframe rather than longitudinally over years. This is important to teacher preparation programs since their period of influence with prospective teachers is relatively brief. Since the tools allowed for a characterization of a prospective teacher's engagement with student thinking, they give way for a better understanding of the supports and structures that may possibly influence a prospective teacher's ability to engage with student thinking.

### **Question 2: In What Ways Does Context Influence a Prospective Teacher's Engagement with Student Thinking?**

Once it was determined that the prospective teachers were characterized at Level 3 within the *Levels of Engagement with Children's Mathematical Thinking*, analysis considered how reform ideals and engagement with student thinking were fostered and maintained during the student-teaching semester. Facilitation of a prospective teacher's engagement with student thinking was first explored through a contextual analysis. Acknowledging that various aspects of context may influence a lived experience at any given moment, the research sought to identify aspects of contextual influence that pertained to the prospective teachers' mathematics teaching and that were pervasive in their perspectives and conversations throughout the timeframe of the study. Themes related to context emerged from the prospective teachers in their interviews, instructional debrief sessions, and team meetings. The emergent themes condensed to three overarching themes related to the context's potential influence of the prospective teachers'

engagement with student thinking: (a) the teacher preparation program, (b) the classroom settings in which the prospective teachers were placed, (c) the relationships of the prospective teachers with their cooperating teachers. Within each theme contextual impacts for each prospective teacher were explored.

### **Teacher Preparation Program**

An understanding of the background in the prospective teachers' preparation permitted for a better understanding of how it may have influenced their ability to engage with student thinking once in their field placements. An overview of the teacher preparation program was established, followed by interviews that addressed the prospective teachers' perspectives on their teacher preparation program. The overview of the teacher preparation program was developed using a review of related program handbooks, syllabi, and university program website information, along with researcher's personal experiences with the teacher preparation program.

The prospective teachers were enrolled in a two-year Elementary Education program that occurred during the junior and senior years of their four-year undergraduate degree program. Their program was considered a traditional college/school of education experience. A School of Education in a large southeastern university was host to their teacher preparation program. The program was cohort-based meaning the prospective teachers enrolled in and completed all core coursework with the same cohort of peers (Dinsmore & Wenger, 1996). The teacher preparation program included once-a-week field visits that began as part of the junior year coursework and continued through fall semester of the senior year. Four content-based field experiences were also embedded in the literacy, science, and mathematics methods coursework in the fall of their senior year. The role of mathematical and pedagogical content knowledge was addressed in the context of two courses in the teacher preparation program; MATH 307: *Revisiting Real Numbers and Algebra* and EDUC 513: *Mathematics Methods for Teaching in the Elementary School*.

The study took place during a transition year for the university. The teacher preparation program was in its final year as an undergraduate program and would move to a Master's level degree program in future years. This structural transition was accompanied with faculty transitions. The elementary program had a new program coordinator and several new course instructors. The cohort members were aware of the changes underway and as determined through personal communication, seemed to attach to the identity of being "the final undergrad cohort."

The prospective teachers' insights on the program and its influences were examined through their interviews and conversations. The prospective teachers in this study were enrolled in the elementary mathematics content course during their junior year (MATH 307) and then the elementary mathematics methods course (EDUC 513) the fall of their senior year. Nora took an additional mathematics teaching content course that addressed measurement and geometry, MATH 411: *Developing Mathematical Concepts*. The prospective teachers' instructors for MATH 307 varied, but all had the same instructor for EDUC 513. MATH 307 and MATH 411 shaped how the prospective teachers came to view students' learning of mathematics based on their own experiences making sense of the mathematics and EDUC 513 shaped their views of instruction.

**The influence of MATH 307.** Kristi felt that MATH 307 is what sparked her interest in mathematics, specifically in how children learn and think about mathematics. Nora had always enjoyed mathematics, but took pause to consider how people other than herself solved problems during MATH 307. Both commented that this was the course that led to the realization that children learn differently than them. Prior to the course they had not thought about how children learned mathematics. Nora and Kristi both felt they had learned through traditional instruction

and memorization, but MATH 307 opened their eyes to the various strategies and paths of thinking that exist when solving problems.

Liz was much more succinct about her MATH 307 experience. She explained that she saw the value of MATH 307, but felt her experience was complicated by unclear instructor expectations. Because she saw herself as a struggling mathematician, the unclear expectations made her resent the experience. She felt she worked hard to persist and solve problems but that her instructor never viewed it as “enough.” Liz saw this as an opportunity to learn how not to instruct in the future, and left the course motivated to scaffold and support her students in problem-solving unlike her own experiences in MATH 307.

Even though their MATH 307 courses had different instructors, the prospective teachers mentioned common instructional experiences. One such pillar that all mentioned was an instructional activity named Xmania (Schifter & Fosnot, 1993). Xmania was an activity in which the prospective teachers revisited the base-10 number system structure created in an alternative number system (i.e., base-5). The prospective teachers were presented with an unknown counting system and were tasked with counting and operating within the system. All described the experience as “hard” or “difficult,” but all said it was “eye-opening.” Kristi commented that it made her realize how abstract the base-10 system is to children and “how hard it must be for them to comprehend what we’re doing.” Instructional experiences that cause the prospective teachers to take on a young learner’s perspective are a theme of MATH 307 and seem to be important to a prospective teacher’s interest in and ability to engage with children’s thinking in their student teaching experience.

**The influence of EDUC 513.** All three prospective teachers mentioned the importance of EDUC 513 in shaping their view of instruction of mathematics. Two specific mathematics pedagogy constructs and their corresponding assignments were described: Cognitively Guided Instruction (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 1999/2014) and the productive talk moves (Chapin, O'Connor, & Anderson, 2003; 2013; Appendix G).

Although two of the three prospective teachers had some exposure to CGI in their MATH 307 course, they digested the content provided in the text and accompanying videos in EDUC 513. They also completed a CGI interview assignment with two students in their placement classrooms. The interview asked the students to solve various CGI problem types, and the prospective teachers probed about the students' strategies. As stated in Liz's story of practice, she found CGI to be an empowering construct that shaped her teaching. To her, learning about CGI was a relief because instead of direct instruction and "spitting out facts" at students, CGI encouraged a manner of instruction that moved from the students' thinking and descriptions of their strategies. Both Kristi and Liz mentioned the CGI interview assignment as meaningful to their development as mathematics teachers. They credited this experience for helping them understand student thinking and reflecting upon the questions they could ask to investigate the thinking. Nora credited CGI as what helped her think about the various ways children think about problems.

All three prospective teachers also spoke about the use of productive talk moves and the value of these instructional tools not only for the mathematics teaching, but also across the school day for instruction of various content areas. They felt the productive talk moves helped them unveil children's thinking and encouraged participation in discussions. Specifically the *repeat*, *rephrase* and *revoice* talk moves were mentioned by all of the prospective teachers.

These moves occur when a teacher asks for a student to repeat or rephrase what a peer has shared, or the teacher herself revoices what a student shared. Kristi spoke of the value of these particular moves and felt they aided in equitable participation and access to the mathematics. She commented, “I use rephrase and repeat all of the time and all of my kids are involved in math discussions because everyone can rephrase or repeat.” The prospective teachers also discussed the value of an assignment around the use of the productive talk moves from their EDUC 513 course. The prospective teachers had to observe their cooperating teachers’ instructional blocks and take note of the types of talk moves that were used to gain access to student thinking. They felt that this assignment oriented them to the types of talk moves already in place in their classrooms, or in some cases, not in place. It allowed them to make goals about how they hoped to integrate the talk moves into their own instruction to give way to engage with student thinking.

### **The Influence of the Classroom Setting**

The demographics of the three 4<sup>th</sup> grade classrooms were representative of classrooms at Brookside as a whole. Table 4.2 presents relevant student information for each classroom to show how certain classrooms had clusters of educational subgroups of children, per district grouping procedures. Although all prospective teachers were at the 4<sup>th</sup> grade level, each classroom had its own diverse make-up and thus an even more pronounced need for the prospective teacher to engage with student thinking to most appropriately adapt mathematics instruction.

Table 4.2. *Prospective Teacher Classroom Information*

	Exceptional Children (with Individualized Education Plans)	Academically Gifted Students	English Language Learners (with Limited English Proficiency Plans)
Classroom 1: Kristi, n= 22	4	0	11
Classroom 2: Nora, n= 21	0	9	0
Classroom 3: Liz, n= 22	1	4	4*

\*3 were non-English speaking

*Note:* All classroom information provided through the prospective teachers.

While the prospective teachers structured their lessons plans around the same university-suggested fraction resources and coordinated instruction using the district pacing guide, they expressed the need for modifications based on their different classroom make-ups. Kristi expressed the need to have differentiated avenues of the topic at hand to accommodate Individualized Education Plans and Limited English Proficiency Plans. Nora expressed the need to ensure students had challenges for the topic to accommodate for her students' Academically Gifted plans. Liz had the most heterogeneous mix of students and felt she needed to consider a variety of avenues of access to accommodate the various learners. Liz also had to translate some documents and directions to Spanish to accommodate her non-English speaking students. Liz referred to her work of modifying our set plans as providing “different modes of access” for students.

It was Kristi who seemed to become the most bogged down by the modifications she felt she had to make for her students. As stated in her story of practice, she hit a professional “low point” when she felt she was not connecting with her students and did not know how to ensure the connections could happen. When she was upset over management and building relationships in the classroom, it impacted how she discussed the students outside the classroom. She became focused on what her students could not do, and would have to be reminded to reference what the

students did know and could do. As part of my facilitation role, I reminded her that I did not have a deficit mindset around her abilities in her practice; therefore, she had to work against harboring a deficit mindset and using deficit language about her students. Kristi entered the student teaching experience with a socially just lens to teaching, so re-centering her on this lens helped her mathematics planning and instruction. Although she may still have experienced internal frustrations about her students and vocalized these in other facets of her day, her language around the children changed in the presence of our team meetings. Rather than focus on students' lack of understanding she focused on what they knew and attempted to troubleshoot how we as a team could help progress their level of understanding.

### **The Influence of the Relationship with Cooperating Teacher**

All three cooperating teachers had 8+ years of teaching experience and mentored for the university in past years. Kristi and Nora's cooperating teachers were in their third year of mentorship roles and Liz's cooperating teacher was in her second year. This was also the second year that all three cooperating teachers taught together at the 4<sup>th</sup> grade level at Brookside. These were the only two years at Brookside for Liz's cooperating teacher, as she had previously taught at another school. Kristi and Nora's cooperating teachers had taught at Brookside for 5+ years and had been part of a school-based, university-facilitated book study in fractions around Empson and Levi's (2011) work. Nora's cooperating teacher had also been a part of a university-led mathematics professional development day around discussion-based mathematics. Liz's cooperating teacher was not a part of any of the university professional developments as she transferred to the building after their completion. She was familiar with mentoring prospective teachers through student teaching and familiar with the expectations of their mathematics instruction portfolio process.

Like the prospective teachers, all of the cooperating teachers felt the use of the productive talk moves was a valuable strategy that the prospective teachers had been equipped with and were using during instruction. However, the cooperating teachers also used their final interviews to express that this group of prospective teachers required support in content knowledge and planning, especially in the area of literacy and facilitating guided reading groups. The cooperating teachers felt that the prospective teachers' need for content and pedagogical knowledge impacted the ability to coach the prospective teachers around other aspects of teaching like management, and reduced the time they could spend to guide the prospective teachers through reflection of practice. To remedy the content and pedagogical knowledge need, the cooperating teachers created a coaching plan in which one cooperating teacher was point person for the planning for each subject matter. Kristi's cooperating teacher coached the prospective teachers in writing and science, Nora's cooperating teacher coached in reading, and Liz's cooperating teacher coached in mathematics.

With specific reference to mathematics, all prospective teachers considered the cooperating teacher coaching to be centered on the navigation of district and school documents. They felt the meetings were focused on the sharing of district and school resources, and the cooperating teacher outlined how to use the materials from previous years. The prospective teachers felt the intense unpacking of the content and consideration of the content in regards to a 4th grade student came from the community of practice and its team meetings. Perhaps their commitment to the community of practice and its team meetings impacted their ability to invest fully in the coaching offered through the school site.

Next, the influence of each dyad of prospective teacher and cooperating teacher is explored separately. The dyad relationships were examined specifically for influences on the prospective teachers' ability to engage with student thinking.

**Dyad 1: the influence of Kristi's cooperating teacher.** Kristi's cooperating teacher had participated in a previous university-facilitated book study of Empson and Levi's (2011) *Extending Children's Mathematics*. The cooperating teacher felt this book study was especially valuable because it helped teachers analyze their own students' work samples related to the levels of sophistication of equal sharing strategies. The book study group also developed resources for the district that aligned to the book. Kristi's cooperating teacher's experience with the book study and its focus on the utilization of children's thinking to inform instruction could be one of the aspects that encouraged Kristi to continue doing so, even when she became overwhelmed in the classroom setting.

During her beginning interview, Kristi expressed that mathematics was the subject she felt most prepared to teach because of prior coursework and purported a socially just lens to teaching. She placed value on the accessibility of the mathematics content and learning for all children. Once embedded in the classroom she struggled with how to connect to the students and to manage behaviors. The cooperating teacher referred to the classroom as a "very challenging class, behaviorally, so that can sometimes get in the way of the instruction." Additionally, the cooperating teacher expressed that she was not sure what she had done that allowed her to connect with the students and to gain their respect. She felt she had exhausted all of her ideas to help Kristi build her own relationships with students. The cooperating teacher wondered if perhaps a prospective teacher was not technically at the point of being able to navigate all the think-ahead components needed to manage her particular classroom.

With outreach to other district staff, the cooperating teacher and Kristi settled on attempting a new system of relationship building with the students. Kristi ran restorative circles each day in an attempt to build community among the students and between herself and students. Restorative practices, including the restorative classroom circles, reflect a shift away from punishment-oriented management styles as they are a tool intended to teach social skills and social problem solving while establishing community (Restorative Practices Working Group, 2014). Even with the restorative circles in place, Kristi felt she still struggled to manage all the dynamics of the classroom behaviors. However, she did feel she had the support of her cooperating teacher to assist when needed and to back-up her management decision-making. Kristi said she was able to persevere in the career because of her cooperating teacher's encouragement.

Since both Kristi and her cooperating teacher used the language of "challenge" to describe their students, it could be that this shared language created a mindset that shaped some of Kristi's sentiments around her students. Although she worked to use affirming, skill-based language around mathematics, if she were to perceive future students as challenging it could impact her ability to validate and productively engage with student thinking.

**Dyad 2: the influence of Nora's cooperating teacher.** Nora's cooperating teacher was part of the Empson and Levi (2011) book study and an additional university-led professional development on the use of productive talk moves. During her interview, the cooperating teacher recalled vividly Types of Strategies – Equal Sharing Problems (Figure 2.3) by page number and how she used it to think about student thinking. She also said her own familiarity with the talk moves made it possible to notice Nora's use and push Nora further into some of the more sophisticated classroom discussion goals.

Nora's cooperating teacher was also a Beginning Teacher mentor for the district. Through this role she received mentorship training and shared some of what that training involved. While she had not received specific mentorship training through the university, she felt the district training changed the way she mentored Nora and put a name to some of the tools she used to support Nora. One of the aspects of the training was learning about coaching styles and identifying one's self as either a directive or non-directive coach. The cooperating teacher felt that she was a non-directive coach and led someone to his/her own conclusions through a reflective process. However, she felt she had to be more direct with Nora because Nora did not move easily through the self-reflection process alone. She wondered if this was due to a lack of reflection time in the year's student teaching process or if Nora had not been given the tools on needed to reflect effectively. She also wondered if her use of a directive style of coaching would be a hindrance to Nora in future years, or if it meant Nora might need another directive mentor in her induction year.

As shared in Nora's story of practice, midway through the study her cooperating teacher spoke to her about the difference between being student-centered versus teacher-centered. From Nora's perspective, the conversation was direct and ultimately it was needed. Nora reported that, "the message wasn't delivered harshly, but it was harsh to hear." It was a transformative moment for Nora though, because after this episode, Nora placed emphasis on both engagement with student thinking and student engagement. She worked to keep her mathematical goal in mind, but also to ensure students were being heard and that their contributions were impacting her goal. She spoke about the challenge of this many times and candidly shared, "I don't know how to teach mathematics yet, I'm just doing the best I can." However, she preserved in her

efforts in response to feedback and in doing the best she could, she was teaching mathematics in a reformed-based manner.

**Dyad 3: the influence of Liz's cooperating teacher.** Liz's cooperating teacher was not part of the university-led book study or professional development initiatives. She had mentored a prospective teacher from the university in a previous year and was familiar with the expectations of the university through her specific site-based supervisor. She never had formal training with the university around her role of cooperating teacher. She felt her role was to "try to be helpful and a mentor" and that it was partially done through sharing past experiences with prospective teachers and answering their questions.

Liz and her cooperating teacher had a courteous relationship but not a particularly collaborative one. Both shared that there was a miscommunication incident before the student teaching semester began, and they never seemed to move into a place of comfortable interaction. The cooperating teacher felt she offered her experience and collaboration to Liz and wanted Liz to ask questions. Liz expressed that she knew she was offered support, but was not sure if support would be enacted in a manner that aligned with her vision for the classroom. Liz complimented her cooperating teacher as having a "math brain" and "knowing all the different ways students should go," but did not feel as if she saw her cooperating teacher model some of the elements of mathematics instruction strategies she had learned about in her coursework. Liz wanted to try some of the new methodologies during the six-week teaching period. The cooperating teacher perceived Liz's ownership of the teaching as not being interested in her suggestions or in a co-teaching relationship. Liz felt she acknowledged her cooperating teacher's suggestions, but again, was interested in testing some of the university methodologies and saw this as part of her duty and right for student teaching. Had I not individually interviewed both

the cooperating teacher and prospective teacher about their experience, this miscommunication and misconception around one another's roles would remain uncovered.

### **Question 2 Summary**

In summary, the themes related to the contextual influences on a prospective teacher's engagement with student thinking were the teacher preparation program, the classroom setting, and the relationship with the cooperating teacher. Within each theme, the impact of context for each prospective teacher was explored. The prospective teachers' prior teacher preparation courses of MATH 307: *Revisiting Real Numbers and Algebra* and EDUC 513: *Mathematics Methods for Teaching in the Elementary School* were prominent factors of contextual influence on their ability to engage with student thinking. Within the theme of classroom setting, each classroom's unique dynamics had the potential for influence on how the prospective teacher engaged with student thinking. Finally, the influence of the cooperating teacher was examined through the characteristics of the collective group of cooperating teachers and through the relationships of each cooperating teacher and prospective teacher dyad.

### **Question 3: In What Ways Does a Community of Practice Structure Facilitate Prospective Teachers' Engagement with Student Thinking in the Area of Fractions?**

This research question's findings focus on the facilitation of the prospective teachers' engagement with student thinking as provided through an analysis of the community of practice and its team meetings. The prospective teachers participated in their cooperating teachers' district-required grade-level professional learning community (PLC, DuFour, 2004; DuFour & Eaker, 1998) and its weekly meetings. The prospective teachers also attended building-level staff meetings with their cooperating teachers and school support staff. The PLC meetings and staff meetings had school and district norms and goals to follow. However, the intention of initializing the community of practice structure meant that team meetings of the community of

practice could be the prospective teachers alone in an effort to support their efforts to problematize and address reformed-based mathematics pedagogy. The community of practice they formed in this study was not structured around the PLC's prescribed model (DuFour, 2004), but instead structured in a manner to deepen their knowledge and skills to engage with student thinking (see Appendix C). This means that unlike a professional learning community, the community of practice was structured around the prospective teachers' instructional needs first and foremost.

As the study progressed, it became apparent that a true community of practice structure was in place for the three prospective teachers outside of the official team meetings facilitated by the researcher. Although each in their own classroom, the prospective teachers' shared experience of being in the same grade level at the same school spurred the community of practice model's true intention of being a voluntary group of people working with shared goals (Wegner, 1998; Wenger, McDermott, Snyder, 2002b). This section first explores the prominent themes of support that emerged within the team meetings. Then the additional supports of the community that emerged organically are explored. These are prospective teacher unplanned meetings and prospective teacher camaraderie.

### **The Influence of Support Within the Team Meetings**

The initial team meetings of the community of practice were focused on reorientation to core reform-based mathematics practices originally addressed in the prospective teachers' prior coursework. As the prospective teachers became more involved with the classroom, they brought student work and instructional dilemmas and successes into the meetings to discuss and reflect upon. The focus on student work aligned with mathematical teaching practice *Elicit and Use Evidence of Student Thinking* teacher action 1.) *Identify what counts as evidence of student progress toward mathematics learning goals*, teacher action 3.) *Interpret student thinking to*

*assess mathematical understanding, reasoning, and methods, and teacher action 5.) Reflect on evidence of student learning to inform the planning of next instructional steps* (Table 4.3). The intention was that alignment of the team meetings around these teacher actions would then allow for a honed ability of the in-the-moment teacher actions focused on attending to engaging with student thinking. While student learning was important at the team meetings, prospective teacher learning was central in the community of practice team meetings. Three themes emerged and gave shape to the content of the team meetings. The prospective teachers moved between figuring out students, figuring out content and curriculum, and figuring out themselves.

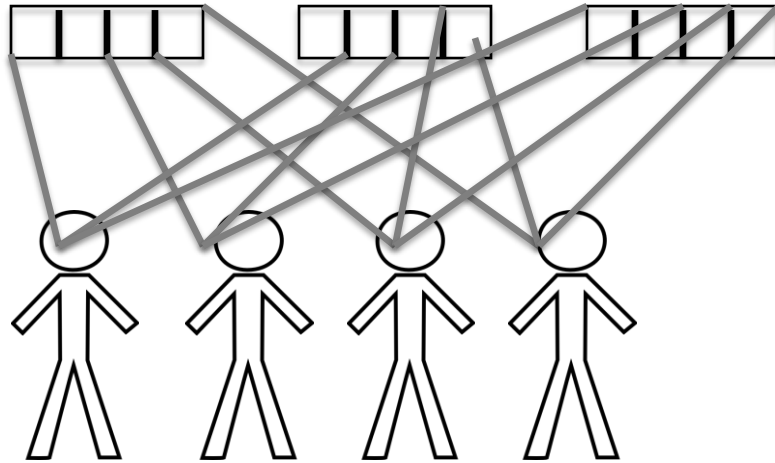
**Figuring out students.** The team meetings were launched with attention to student progression frameworks in fractions and discussion of prospective teachers' students' work on the fraction pretest. The intention was that this would establish an expectation that investigation of student learning was important, and analysis of student work should drive the mathematics planning and instruction. As described in Chapter 3, the pretest and posttest were analyzed according to Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression frameworks (Figures 2.3 & 2.4). The prospective teachers were most attached to the structure provided through Types of Strategies – Equal Sharing Problems (Figure 2.3) because they felt most comfortable analyzing work according to the student strategies provided as exemplars. This comfort may have resulted because the framework was introduced in their previous coursework while the OGAP Fraction Progression was first introduced at the team meetings. Alternatively, the focus of the Types of Strategies – Equal Sharing Problems on one single type of problem (equal sharing) and its corresponding student strategies may have been more supportive than the OGAP Fraction Progression that is used to classify several different fractional problem types in terms of student responses. Still yet, it may have resulted because it

was the framework their cooperating teachers were aware of thorough participation in the district book study. Perhaps because of all of these reasons, the prospective teachers were able to analyze equal sharing problems more deeply than other types of problems.

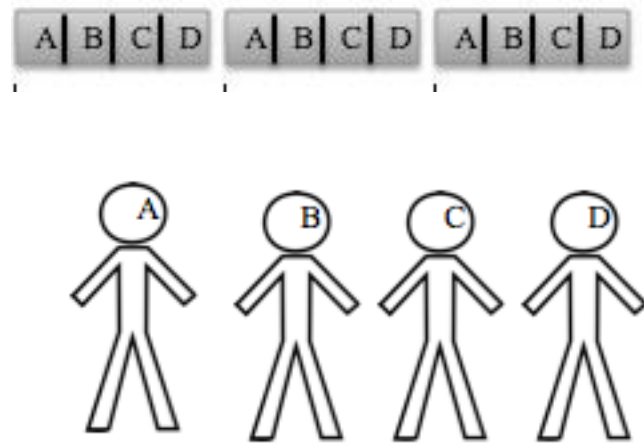
One particular discussion around student thinking resulted from the prospective teachers' noticing of the different pictorial representations that students used within a particular equal sharing strategy type. I expected the prospective teachers to classify student work by the strategies students used to solve an equal share problem (Non-Anticipatory, Additive Coordination, etc.), but I did not anticipate that they would dissect the strategies further. Kristi presented an example of three different students' pictorial representations of the same problem. All three students had the same answer and their strategies were classified as *Additive Coordination- sharing one item at a time*. Kristi noted, "I mean, surely, these all show different levels of understanding within the same strategy, right?" Kristi alluded to the fact that the students seemed to exhibit different understandings even though they were all classified the same way according to Types of Strategies – Equal Sharing Problems (Figure 2.3). Figure 4.2 shows the three student drawings Kristi shared.

**Student Additive Coordination models to represent 3 objects shared equally by 4 people**

Student 1: Depicts the 3 objects and 4 people. Draws lines to each portion of 4th to corresponding person.



Student 2: Depicts 3 objects and 4 people. Organizes each partition by person with a notation system (letter or number).



Student 3: Depicts 3 objects, no people. Organizes each partition by person with a notation system.



*Figure 4.2. Additive Coordination Pictorial Representations*

Once Kristi shared the three representations, the other prospective teachers said that they too were wondering about the various student pictures and if they represented different levels of sophistication within a strategy. The prospective teachers asked for a classification system for the student pictorial representations within each strategy from Types of Strategies – Equal Sharing Problems (Figure 2.3). Although Empson and Levi’s (2011) work provided a “progression of children’s use of fraction word and symbols” (p. 27), there was not a specific breakdown of possible pictorial representations by strategy. A group consensus was made that since there was not a sophistication of pictorial representations within each equal sharing strategy, the prospective teachers would use their own understanding of the student models to develop a pictorial progression. The order of Student 1 through Student 3 as presented in Figure 4.4 depict a hierarchy of student pictorial representations within an *Additive Coordination-sharing one item at a time* strategy, with Student 3 proposed as being the most sophisticated in her/his understanding.

Regarding the Student 1 pictorial strategy, the prospective teachers felt students utilizing this pictorial strategy often lost track of how many lines went to each sharer. Although Student 1 used the Additive Coordination strategy, this student sometimes missed a direction line and had to recount the shares. The prospective teachers likened this strategy to the CGI whole number strategy of directly modeling all parts of a problem context. They referred to the Student 1 strategy as “the rainbow strategy” because it often resulted in many arched lines from object to sharer. Student 2 also directly modeled the people and objects, but students employing this strategy used an organization system of numbering or lettering the portions each sharer received. The prospective teachers felt this was more advanced than the first strategy, but not as advanced as the Student 3 strategy. Student 3 used an organization system like Student 2, but did not need

to represent the people. This student still drew the objects, but partitioned each for the correct number of sharers without physically representing the people. The prospective teachers felt that they may be able to scaffold students utilizing the Student 3 strategy to a more sophisticated equal sharing strategy because these students no longer needed to directly model all parts of the problem. This detailed attention to student pictorial representations led to the prospective teachers planning how to lead discussions that specifically questioned students about why they used the models they did. They also thought about creating classroom charts with student names attached to the different pictorial representations so that other classmates could consider various representations and their affordances.

The prospective teachers' pictorial progression for the *Additive Coordination- sharing one item at a time* strategy resulted from a discussion of what each of Kristi's work samples, and any student who used a similar pictorial strategy, may be exhibiting conceptually. This discussion and targeted work around classification and scaffolding of student thinking aligned with the mathematics teaching practice of *Elicit and Use Evidence of Student Thinking* teacher action 1.) *Identify what counts as evidence of student progress toward mathematics learning goals* and teacher action 3.) *Interpret student thinking to assess mathematical understanding, reasoning, and methods*. Even though the prospective teachers were most engrossed and intrigued with discussions around the equal sharing problems, they used both the OGAP Fraction Progression and Types of Strategies – Equal Sharing Problems as the classification framework for their mathematics portfolio process. They categorized student work according to both frameworks and then analyzed it looking for trends in learning across their classrooms to reflect on their practice's successes and missed opportunities. Though each completed an individual portfolio, they brought the various dilemmas that were raised to team meetings. The prospective

teachers recognized that the various dynamics of their classrooms raised unique student challenges, but also looked for trends across the grade level to consider how to best support one another and support the children they taught.

**Figuring out content and curriculum.** As stated in Chapter 3, Brookside Elementary did not have an adopted mathematics curriculum, but did have a district pacing guide for mathematics with suggested resources. Additionally, the prospective teachers had university recommended resources for fraction instruction, and as part of their portfolio process, had extensively developed lesson plans for a one-week segment of their fraction unit. The cooperating teachers were supportive of the prospective teachers' instruction, but also felt obligated to stay aligned with other teachers at the 4th grade level. A district-required multiple choice mathematics benchmark test was scheduled at the end of the prospective teachers' six-week teaching experience. This test was comprehensive and based upon the district pacing guide.

The prospective teachers became overwhelmed about how to navigate the various resources and reconcile them with the pacing of content expected of their grade level. Because of this, much of the time in the team meetings was spent unpacking the Common Core State Standards (National Governors Association Center for Best Practices, 2010) fraction standards and the implications of the content for a 4th grade student. They began to ask about a curriculum and wondered aloud why there was no adopted curriculum for the district or school. Curriculum seemed to be synonymous with textbook during the initial prospective teachers' conversations because as Nora stated, "We're new teachers. We just need something that tells us what to do and why we're doing it." Engagement with student thinking had been positioned as important in the team meetings, but if the prospective teachers were to engage in the in-the-moment decision-

making, it would require explicit coaching on teacher action 5.) *Reflect on evidence of student learning to inform the planning of next instructional steps.*

An hour was allotted for each team meeting, but four of the eight meetings lasted 90 minutes to three hours. Some team meetings took place in a university classroom and some took place in a classroom at Brookside. Both team meeting 5 and team meeting 7 lasted for three hours. These two meetings occurred in the Brookside mathematics resource lab classroom. It was in this space that the community of practice had access to various mathematics manipulatives and books, and perhaps these factors encouraged the longer meeting duration.

Team meeting 5 lasted three hours because the prospective teachers prepared hands-on materials for their upcoming lessons and rehearsed with the materials. Together we unpacked the mathematical content behind the materials that included composition and decomposition of unit fractions and number line representations. The prospective teachers then rehearsed and considered the possible pitfalls of utilizing the materials in real-time and the questions they would ask students around the material usage. Liz pushed her peers to be explicit about their questions and asserted, “No, not detailed enough,” when too open-ended of a question was posed.

The rehearsals were important to the prospective teachers, but were not the formalized *lesson rehearsals* that are in current favor in teacher preparation programs. Formal lesson rehearsals are structured cycles that allow for coached practice of ambitious instructional activities before they are enacted in the classroom setting (Kazemi, Ghouseini, Cunard & Turrou, 2016; Lampert et al., 2013). In formal lesson rehearsals, prospective teachers first observe an instructional activity taught by a master teacher, then collectively analyze the observation and prepare to teach the instructional activity themselves. Prospective teachers

rehearse the instructional activity in their course setting with their peers and receive embedded coaching from their teacher educator. Lesson rehearsals provide a safe medium in which to hone teaching around reformed-based practices, such as orchestrating student discourse and facilitating rich problem solving. The commonality between formalized rehearsals and the impromptu rehearsals in team meeting 5 was that a safe space and immediate feedback from peers and the researcher were provided.

Team meeting 7 lasted three hours because the prospective mapped out a curriculum guide for fractional topics that covered their last two weeks of six-week instructional period and the fractional topics that would be covered in their phase-out when they began to turn over instructional responsibilities back over to their cooperating teachers. The guide was also set up to extend to the remainder of the fractional and decimal topics that their cooperating teachers would take over beyond the student teaching sequence (*Common Core State Standards: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number and Understand decimal notation for fractions and compare decimal fractions*). During the instructional day that led up to team meeting 7, the cooperating teachers participated in a grade level PLC around data and planning and the prospective teachers remained in their classrooms to teach. At the PLC, the cooperating teachers wrote “exit tickets” for each of the prospective teachers’ remaining lessons. The exit tickets were meant to assess students at the end of each lesson to determine what students learned from the lesson. The prospective teachers came to that afternoon’s team meeting with the exit tickets, but were left to decide how to best instruct in order to meet the expectations of each exit ticket. The prospective teachers were familiar with a launch, explore/explain, and extend lesson-planning template from their teacher preparation program coursework. We used this to backward map the remainder of their instructional unit

from the exit tickets. An example of one instructional day's curriculum guide is shown in Table 4.3.

Once the entire week was mapped out as in Table 4.3, the prospective teachers began to develop the corresponding questions and components of each day's segment. Each prospective teacher took the components of the guide and modified them for their respective students. As they worked, Nora's cooperating teacher visited the meeting space and complimented the format and said the district had never paced out the fractions unit like this before. After she left, Nora smiled and said, "We're leaders; like, this is a legacy for the district." Liz responded enthusiastically, "This is the best day ever; we're getting so much done." The guide was made into an electronic collaborative document and shared with the cooperating teachers and Brookside's mathematics coach. It became a living document with all parties leaving comments about possible edits or resources that could be used to support the lesson goals.

Table 4.3. *One-Day Excerpt from Prospective Teacher Curriculum Guide*

CCSS	4.NF.1 & 2
<b>Lesson Topic</b>	Comparing & Equivalency
<b>Launch: Warm-up, Number Talk, Mini-Lesson</b>	<p>Nikki and Josie ran laps for fun. Nikki ran <math>\frac{7}{8}</math> of a mile. Josie ran <math>\frac{7}{12}</math> of a mile. Did they run the same amount? How do you know? Who ran the most?</p> <p>What if Nikki ran <math>\frac{7}{8}</math> of a mile and Josie ran <math>\frac{5}{8}</math> of a mile? Who ran the most now? How do you know?</p>
<b>Explore &amp; Explain: Task, Game, Stations, etc.</b>	<p>Station 1:            Fraction cards            -cut and sort the fractions            -closer to 0, <math>\frac{1}{2}</math>, or 1?            -WHY?</p> <p>Station 2:            Division Review</p> <p>Station 3:            -comparing fractions with landmarks worksheet            -challenge problems if finished early</p>
<b>Extend (if applicable)</b>	N/A
<b>Exit Ticket</b>	<p>Which fraction is closer to <math>\frac{1}{2}</math>? Justify your reasoning.</p> <ol style="list-style-type: none"> <li>1. <math>\frac{16}{30}</math></li> <li>2. <math>\frac{2}{7}</math></li> <li>3. <math>\frac{8}{10}</math></li> <li>4. <math>\frac{4}{5}</math></li> </ol>

Team meeting 7 and the closing interviews cemented that the term curriculum did not simply mean textbook for the prospective teachers. They did not want a rote script to follow, but they did want some sort of guide that provided a scaffold to their learning about the instruction of fractions, just like they provided scaffolds to their students' learning in the classroom. At study end, I asked the prospective teachers what aspects of the team meetings they felt supported their instruction.

Kristi: I think that the curriculum [aspect] was, like, really big in general and that lent itself to the progression of what a 4<sup>th</sup> grader should know. Because if we had had a textbook or if we had had a curriculum, we would have known where they were supposed to go. But it's helpful and beneficial so that in the future when we do hopefully have a curriculum we'll know this is why the curriculum is doing what it's doing.

Nora: Well, having that, definitely having that day-by-day pacing with the launch, and then what we could do in each station. That was a lot more helpful because as a new teacher, we don't have all these resources. I struggle finding things to use, because I don't know yet.

Liz: I know I want to work in a place with a curriculum. Who would think that just because you have a curriculum means you won't listen to your kids? You have to listen to your kids to know where to take a lesson.

All of the prospective teachers specifically cited the curriculum guide development as one of the key supports of their mathematics instruction, along with unpacking the fractional content in the context of a 4<sup>th</sup> grade classroom.

**Figuring out selves.** The prospective teachers were not only in rehearsals of their pedagogy, but also in dress rehearsals of their educational philosophies. Some of the team meeting time was devoted to their exploration around their visions of mathematics teaching and how it could be enacted once in real-time. Sometimes their discussions centered on what they learned about in their EDUC 513: *Mathematics Methods for Teaching in the Elementary School* course and negotiating what the class topics looked like in the context of their schools and their individual classrooms. One of their spring semester assignments for another course was writing an educational philosophy. Discussions around their philosophies surfaced during our team meetings. Each had a differing "I believe" lead statement that portrayed their personalities, but also their unification around children being the focus of the classroom.

I believe in the power of children, not of adults to empower children, but of the extreme power a child holds by simply being a child. -- Kristi

I believe that students all come from different backgrounds with their own life experiences and funds of knowledge. They can offer their knowledge and experiences to contribute to the learning of all students because each student has a topic that they are experts in. -- Nora

I believe that the classroom should have a student-centered format, where the teacher acts more as a facilitator of discussion rather the sole discussor with all of the knowledge. -- Liz

I felt it important to maintain a safe space for the prospective teachers' discussions around teaching philosophies and their embodiment of mathematics instruction. I listened but refrained from contribution unless I was asked for an opinion. It was important that they had protected space to figure out who they were as practitioners in open and honest ways. Although I was at the team meetings as a facilitator for the mathematics at hand, ultimately this was their time to decide who they were as teachers and how they would move forward in the profession.

### **Support of Unplanned Meetings**

Perhaps some of the most important aspects of support were the prospective teachers' weekly, unplanned meetings. These meetings were unplanned in that they were not designated by the research design nor were they part of the set team-meeting schedule. While I facilitated the planned team meetings that took place once a week, the unplanned meetings were established and orchestrated by the prospective teachers. I was invited to two of these unplanned meetings and attended both. One meeting was in between team meetings 3 and 4 when the planning and instruction for mathematics along with the other subject matters became an overwhelming reality for the prospective teachers. The other unplanned meeting was at the end of the six-week teaching period in which the prospective teachers were finalizing their instructional units.

The invitations to these meetings made apparent that I was viewed a participating member of the community of practice, but the fact that the meetings were established and took

place regardless of my attendance revealed the voluntary nature of the community of practice. The ownership of the community of practice became that of the prospective teachers. When I attended the unplanned meetings, I acknowledged that they were not mine to lead; rather, I was invited to follow their leads and help accomplish their goals.

As stated, the first unplanned meeting was midway through the study. I attended for three hours and an overarching agenda of topics seemed to be enacted, even if the agenda was not formalized: Hour one- Review portfolio expectations, Hour two- Unpack fractions content and consider student work in regards to Types of Strategies – Equal Sharing Problems (Figure 2.3), Hour three- Revise future lesson plans based on students. Even without the structure of the scheduled team meetings, the prospective teachers adopted the *Elicit and Use Evidence of Student Thinking* teacher actions that reinforced attention to student work and thinking in planning for instruction. The prospective teachers informed me that they continued to meet about other content areas after the initial three-hour duration. Individually I asked each prospective teacher what caused her to start to meet outside our planned team meetings and each voiced a similar theme.

Kristi: Fear; overwhelming stress.

Nora: To keep sane with each other together.

Liz: Stress; not knowing.

Their responses made obvious that outside of the team meetings, a voluntary community of practice originated around shared concerns about practice. Because I was not a prospective teacher, there were certain parameters of the community I no longer met. I did not have need for a space of intensive pacing and planning around the curriculum as the three members of the community did.

## **Support of Camaraderie**

At the beginning of the study, one cooperating teacher remarked, “They’re not friends, but they’re here together.” She explained that she would be curious how this year would unfold and referenced other years in which the prospective teachers placed at the school site had pre-established friendships prior to the student teaching experience. However, as mentioned, the rationale for the prospective teachers’ unplanned meetings was due to their camaraderie of being new to a profession and having concerns about how to plan and instruct. In their final interviews, the prospective teachers shared that they began the study not knowing one other than as classmates at separate tables in the fall semester, but became connected over student teaching and their time at the team meetings.

It was perhaps because of their newness to the profession that the prospective teachers negotiated their place within the school setting together. They viewed themselves as part of their 4<sup>th</sup> grade PLC with their cooperating teachers but also as outsiders to it. At the second team meeting, the prospective teachers announced their idea for “teacher days.” Teacher days were a mechanism of support observed from their cooperating teachers. Each cooperating teacher was given a day of the week that was their day; on this day they were provided with support or encouragement from the other cooperating teachers. According to the prospective teachers, this could range from words of affirmation, to cards, to small gifts, to a school responsibility being offloaded. From the perspective of the prospective teachers, they could not be a part of this teacher day arrangement because it was a bond already in place for the cooperating teachers. However, the prospective teachers established their own teacher day arrangement amongst themselves and offered some of the same small tokens of appreciation on a team member’s teacher day.

The creation of a separate teacher day arrangement symbolized the coexistence of the teams of cooperating teachers and prospective teachers. The two groups coexisted in the space of Brookside, but did not always collaborate as a cohesive unit. The prospective teachers observed and took on behaviors of their cooperating teachers, but also remained their own entity. Whether this was because of the nature of the research design or because of the nature of the people is left undetermined. Possible interpretations and remediation of this separateness are explored in Chapter 5.

**The influence of the group.** As the prospective teachers strengthened their bond of camaraderie outside of the school setting, they became a group that influenced one another. Liz expressed this as, “Who else can I even talk to?” and explained that the three spent their weekends together and planned amongst themselves first because they “had to,” and then because they choose to. The prospective teachers felt it was easiest to be together in planning because they already understood one another’s context and the constraints and allowances that the context brought with it. Kristi expressed, “Honestly, I don’t know what I would have done if it weren’t for other people being in the same grade.”

Again, it was unclear if the required team meetings bred a co-dependence among the prospective teachers, or if a relationship would have been established regardless since they were placed at the same grade. What is known is that the aspect of camaraderie meant so much to the prospective teachers that two decided they would job search together. At the final team meeting, Kristi struggled over her decision to leave the state to teach. Liz was also leaving, but to a different state than Kristi. Both wanted the experience of stepping outside their comfort zone and living somewhere new, but both were concerned about how they would form a connection like the one they had generated in this experience. Liz commented that she did not know how

they could go onto teach if they did not have a support network like what was established in their community of practice. We discussed how to form bonds in their future school settings, and that content support could come from curriculum coaches and support specialists within their future schools. However, in the final interview, Kristi and Liz shared news that while they would still be leaving the state to teach, they would now be leaving together. Kristi and Liz had been hired to teach at the same grade level at the same school.

### **Question 3 Summary**

In summation, Question 3 exposed the prominent sub-themes of support that emerged within the team meetings as figuring out students, figuring out the content/curriculum, and figuring out selves. These supports aligned with several of the teacher actions from the mathematics teaching practice *Elicit and Use Evidence of Student Thinking* and this alignment may have influenced the prospective teachers' abilities to engage with student thinking in their classrooms and to be characterized at Level 3 on the *Levels of Engagement with Children's Mathematical Thinking*. The additional organic community of practice supports of the prospective teacher unplanned meetings and prospective teacher camaraderie emerged because of the study structure and these supports reinforced the study focus. Although the prospective teachers' camaraderie may not have been explicitly around the mathematics, it was developed in part due to the study structure and therefore worth unpacking. The unplanned meetings and camaraderie these meetings created may have supported the prospective teachers' adoption and maintenance of reform-based practices, specifically in regards to their engagement with student thinking in mathematics.

#### **Question 4: How Does a Prospective Teacher's Engagement with Student Thinking Influence Student Understanding of Fractions in the 4<sup>th</sup> Grade?**

Finally, the findings in regards to students' understanding about fractions are presented from analysis through the Types of Strategies – Equal Sharing Problems framework and the OGAP Fraction Progression (Figures 2.3 and 2.4). Once it was determined that the prospective teachers utilized reformed-based mathematics ideals and engaged with their students' thinking, the impact on student understanding was examined. The pretest to posttest (Appendix F) results showed growth in each prospective teacher's classroom in raw amount-correct percentages, and more importantly to this study, across all items on the frameworks. While individualized, blinded student data were provided to the researcher and analyzed for use with the prospective teachers at team meetings, individualized student achievement results are not presented in this dissertation in order to maintain protection of students' privacy.

The pretests and posttests were administered by and collected through the prospective teachers. General classroom dynamics were known about the clustering of subgroups in each classroom (Table 4.2), but no identifying data were provided as to which tests corresponded to which student or which subgroup. To protect student privacy and maintain confidentiality, results of the pretest and posttest item analyses are presented according to each classroom as a whole. Since this study was not framed around individual student proficiency, but instead around prospective teachers' instruction, the findings for Question 4 show how collective class proficiency changed from the beginning to the end of study. Table 4.4 presents the average of pretest/posttest items correct expressed as a percent for each classroom. Figure 4.3 presents this information in graphic form.

Table 4.4. Classroom Average Expressed as Percent

	Pretest	Posttest	Change
<b>Classroom 1: Kristi</b> <b>n=22</b>	22%	40%	<b>18%</b>
<b>Classroom 2: Nora</b> <b>n=20</b>	38%	58%	<b>20%</b>
<b>Classroom 3: Liz</b> <b>n=18</b>	34%	62%	<b>28%</b>

*Note.* Averages reflect students that took both pre and posttests.

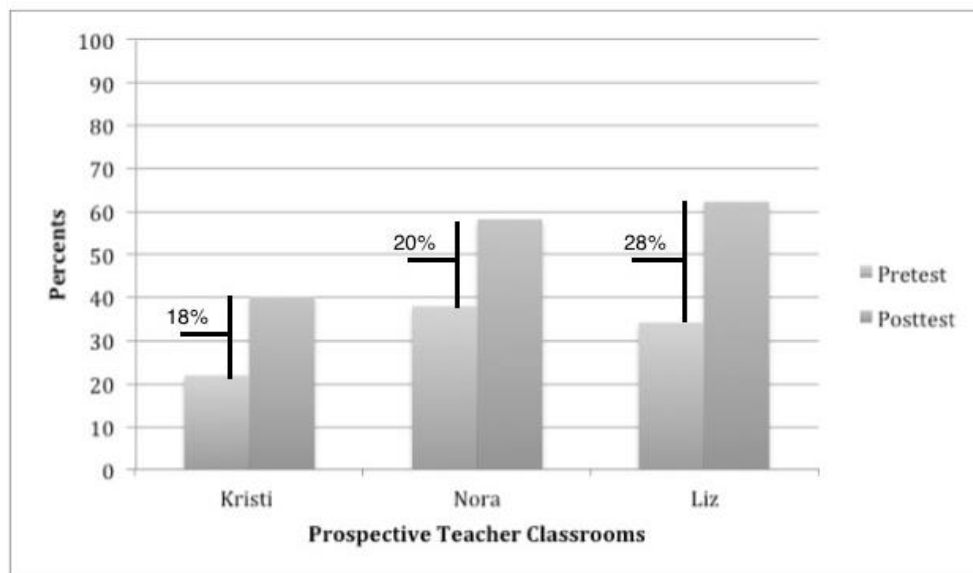


Figure 4.3. Classroom Average Expressed as Percent and Delta Growth

The table and figure show that each classroom experienced growth in percentage correct from pretest to posttest. This suggested that regardless of the subgroup cluster, engagement with student thinking is possible and beneficial for all children. Interestingly, Nora's academically gifted cluster, based on comparing test scores, appear to have grown less and had a lower posttest average (58%) than Liz's heterogeneously grouped classroom (62%). As a reminder, Liz did not have a dominant subgroup cluster in her classroom. Her higher growth and posttest average could be a result of numerous causes. It may also be the result of fewer students completing the test in comparison to the other classrooms (i.e.  $n = 18$  compared to  $n = 20$  or  $n = 22$ ). Or perhaps it may indicate that a heterogeneous grouping allowed for more diverse mathematical viewpoints,

which in turn benefitted learners within a reform-based mathematics classroom where various strategies were elicited and shared. It may also indicate that a heterogeneous group is a more accessible group to instruct for a prospective teacher than that of instruction of an academically gifted group of students. Perhaps the instruction of academically gifted students (as done by Nora) is difficult because it requires thoughtful extension beyond the basic curriculum. As explained in Nora's story, by the end of the six-week period she was just at the point of consideration of how to effectively extend the mathematics for her students, but it was not an initial undertaking with all the other managements of instruction. The focus and constraints of the study allow only for hypothetical causation as to why particular classrooms exhibited their results. However, the results point to the need for future studies around subgroups of learners and their response to a teacher's engagement with their thinking.

More important than percentage correct, the study sought to explore student understanding of fractions. Student understanding was based on the notion that "developing an approach to thinking about mathematics is valued more highly than memorizing algorithms and using them to get right answers" (Schifter & Fosnot, 1993, p. 9). This emphasis on understanding as thinking was relayed to the prospective teachers and encouraged through the use of progressions and strategy frameworks to analyze their student work rather than simply a correct/incorrect grading procedure.

As stated, the pretest and posttest for each student in each classroom was analyzed by each prospective teacher and by the researcher. Each item number was leveled according to its corresponding progression level or strategy. Results were then discussed at the team meetings. If any disagreements emerged between the leveling of an item for an item, conversation was had until there was 100% agreement.

One such disagreement was around a trend with students in Kristi's classroom. On pretest Item 1, six of her students transformed the whole number 1 on the number line to the numerator for the fraction  $\frac{1}{3}$  (see Figure 4.4). Four of these same students transitioned to placing  $\frac{1}{3}$  on the number line during the posttest, but still misplacing the fraction  $\frac{3}{4}$ . I classified both pretest and posttest answers as Non-Fractional on the OGAP Fraction Progression, because I felt they exhibited *an inappropriate whole number reasoning with fractions* per the progression.

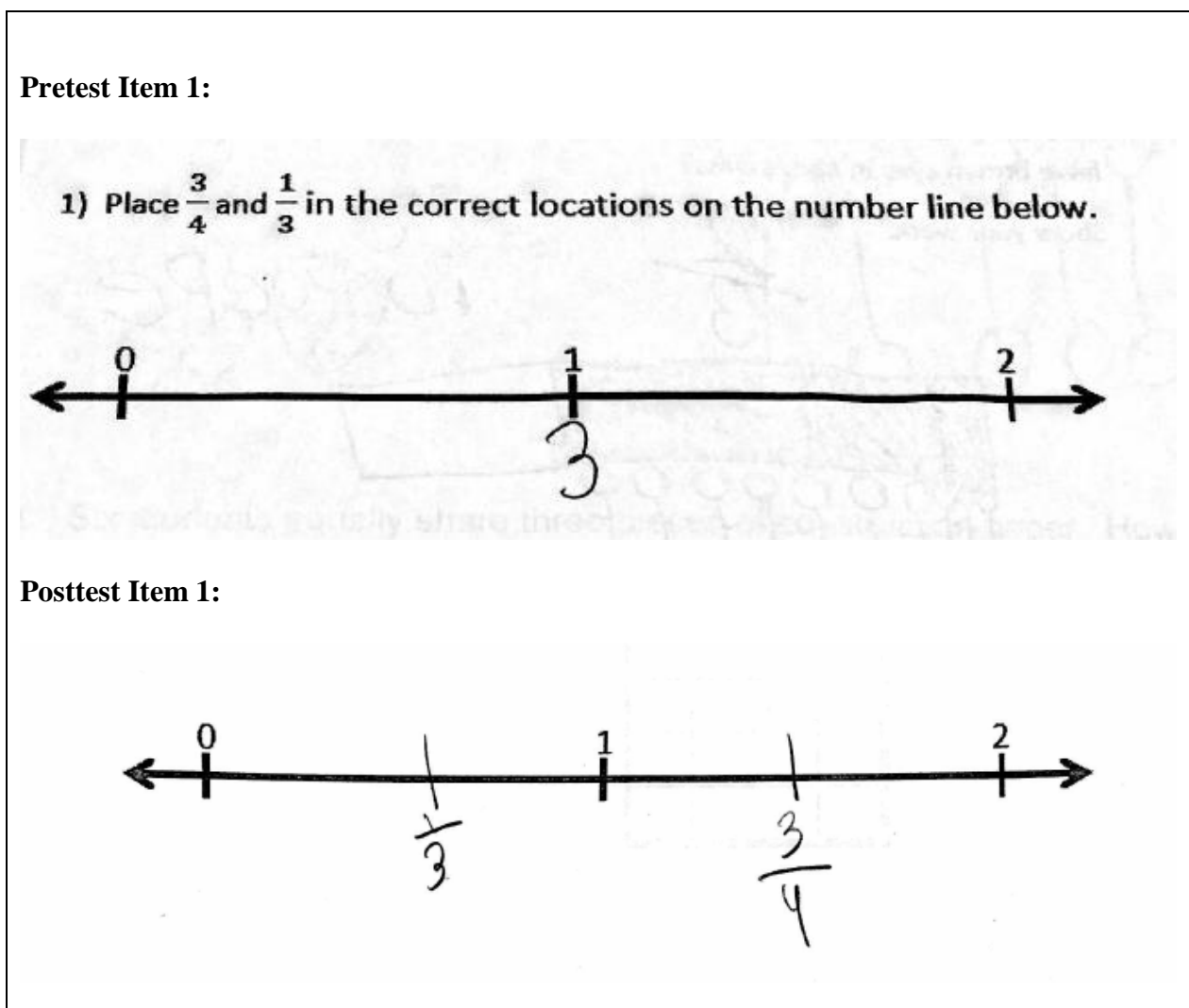


Figure 4.4. Student Response on Item 1 from Pretest to Posttest

However, Nora spoke up at this classification of the posttest item.

No, I think I know what they're doing. Some of my students did it in class, too. They're not looking at 0 to 1. They treat the line 0 to 2 as the whole. They are partitioning it into the fraction parts. See?

She then showed us how the students partitioned the entire line from 0 to 2 into sections by thirds and fourths, using 1 as their halfway point. All three prospective teachers felt the posttest item should be classified as Early Fractional because of the progression's qualifier of *uses a fractional or transitional strategy like partitioning visual models, but the solution includes an error (e.g., partitioning, size of whole)*. I had looked only at the misplacement of the fractions and not considered the student's accurate partitioning but inaccurate use of the size of the whole. I changed my posttest item classification to Early Fractional based on the prospective teachers' sense making of student work. Beyond discussions of classification, the prospective teachers were also beginning to notice student movement of thinking. Using the whole number 1 as the numerator on the pretest meant that a student could not place the fraction  $\frac{3}{4}$  because no 3 was represented on the number line. Even if a student had not represented the  $\frac{3}{4}$  on the posttest, the prospective teachers asserted that accurately writing  $\frac{1}{3}$  was a transition in thinking.

Table 4.5 shows the occurrence of items classified within each level of the Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression (Figures 2.3 and 2.4). In each classroom there was movement from the less sophisticated level or strategy to a more sophisticated from pretest to posttest. Notable is the movement across all classes from Non-Fractional levels on the pretest to Fractional levels on the posttest. Additionally, the use of Non-Anticipatory (guess and check) partitioning strategies decreased in all rooms and instead Additive Coordination by Groups of Items was the most commonly used strategy on the posttest. The Ratio strategy was not present in any classrooms for either pretest or posttest. This strategy

requires the child to *transform the problem into a simpler problem* through knowledge of multiplication factors. This is not a common strategy and because 4<sup>th</sup> grade students do not have explicit experience with ratios and proportional reasoning, it may not be an inherent strategy for them to use. Students do not necessarily move linearly through the equal sharing strategies (Empson & Levi, 2011), so it is reasonable that some 4th grade students could move to Multiplicative Coordination without first exhibiting Ratio strategies.

Table 4.5. *Progression/Strategy Occurrences Expressed as Percent*

		Classroom 1: Kristi n= 22		Classroom 2: Nora n= 20		Classroom 3: Liz n=18	
		Pre	Post	Pre	Post	Pre	Post
<b>OGAP Progression Least to most sophisticated ; Items 1-8</b>	Non-Fractional	63%	42%	48%	28%	41%	20%
	Early Fractional	14%	19%	6%	13%	17%	16%
	Transitional	4%	5%	14%	2%	6%	6%
	Fractional	11%	33%	24%	53%	27%	57%
<b>Strategies for Equal Sharing Least to most sophisticated ; Items 9-11</b>	Non-Anticipatory	20%	14%	53%	33%	39%	15%
	Additive Coordination (By 1)	17%	35%	12%	12%	4%	19%
	Additive Coordination (By Groups)	20%	41%	13%	38%	19%	35%
	Ratio	0%	0%	0%	0%	0%	0%
	Multiplicative Coordination	0%	4%	5%	7%	4%	11%

*Note.* Items left blank by student were not classifiable; total percentages for OGAP/Equal Sharing Strategies by class reflect this.

The classification data in Table 4.5 are presented by classroom and the table shows an overview of shifts for each class as a whole. The shaded percentages aligned with items 1-8 show the decrease in Non-Fractional classifications and the increase in Fractional classifications across all classes. A 4<sup>th</sup> grade student is not expected to be operating at the Fractional level on all items, but should be progressing away from Non-Fractional strategies. An increase in

Fractional classifications is important because fractional understanding at the elementary level allows for application of the understanding to middle school mathematics topics including, but not limited to, proportions, expressions, probability, and functions. The shaded percentages aligned with items 9-11 show the decrease in Non-Anticipatory equal sharing strategies across all rooms. This is important because it means that even if students did not answer an equal sharing item correctly, they anticipated a strategy for solving and used this more sophisticated strategy. Also highlighted for items 9-11 is the most implemented strategy across all classrooms which was Additive Coordination –sharing groups of items. This is worthwhile to note because it is the strategy the 4<sup>th</sup> grade prospective teachers sought to support through discussion and questioning with students during their instruction. Students using this strategy anticipate a method for solving an equal sharing problem and can use number relationships to partition and share more than one section of an item at a time. A Common Core State Standards (National Governors Association Center for Best Practices, 2010) mathematics goal for 5<sup>th</sup> grade students is to *interpret a fraction as division of the numerator by the denominator*. Students who gain a conceptual understanding of fractions through equal sharing and who utilize the Additive Coordination –sharing groups of items strategy in 4<sup>th</sup> grade are better equipped to meet this Common Core State Standards 5<sup>th</sup> grade goal.

Although the table provides student information across whole classrooms, with my facilitation, each prospective teacher analyzed their assessment classifications by student and by item in order to reveal student and classroom trends. The prospective teachers began to see that growth in understanding could be found within student work, even if the percentage correct did not reflect accuracy or proficiency. When Kristi looked at the posttest percent correct for her

students, she felt the score was low. However, when she began to look specifically at student strategies, she found shifts in students' understanding of fractions.

Kristi: I mean [sigh] they only grew 1 or 2 points.

Researcher: Wait a second. Don't look at the points. What do you know about these kids now? Why are we using a strategy scale if we're not going to use it?

Kristi: You're right, there are a lot more Transitional and Fractional now, and a lot more additive coordination on equal sharing. Kids have a strategy instead of just guess and check.

Researcher: Look at student 17. That student was all Non-Fractional and Non-Anticipatory answers for his pretest, and now is writing fractions. Look at equal sharing.

Kristi: Yeah, you're right! Look at that, he's drawing the numbers in the problem. He's drawing equal shares now, he's thinking about the numbers! I mean, you're right, you're right. This isn't about proficiency, it's about growth. And they grew.

Perhaps as important as capturing student growth on the progression and strategy levels, the assessments also captured students' growth in ability to explain their answers. Even though the items asked students to describe their thinking/show their work on both pretests and posttest, it was not until the posttest that more students followed this direction. The prospective teachers discussed how providing details and explanations became an expectation in the classroom. Kristi shared, "I told them they had to explain themselves, even if the explanation is just guessing."

Figure 4.5 shows some of the variation in student explanations within the different levels of the OGAP Fraction Progression and the utilization of different approaches to explanations.

Item 7, Non-Fractional, Word Explanation:

7)

$\frac{1}{8} + \frac{2}{8}$  is closest to

A) 1

B) 0

C) 3

D) 16

Explain your choice.

$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$  I guess

Item 7, Fractional, Word Explanation:

7)

$\frac{1}{8} + \frac{2}{8}$  is closest to

A) 1

B) 0

C) 3

D) 16

Explain your choice.

I think zero wholes because  
 $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$ , and I need 5 more  
eighths to get to 1 whole and I  
only need  $\frac{3}{8}$  to be taken  
away to get zero wholes.

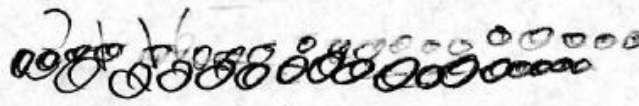
Item 7, Non-Fractional, Pictorial Explanation:

7)

$\frac{1}{8} + \frac{2}{8}$  is closest to

A) 1  
B) 0  
C) 3  
D) 16

Explain your choice.



Item 7, Fractional, Pictorial Explanation:

7)

$\frac{1}{8} + \frac{2}{8}$  is closest to

A) 1  
B) 0  
C) 3  
D) 16

Explain your choice.

$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

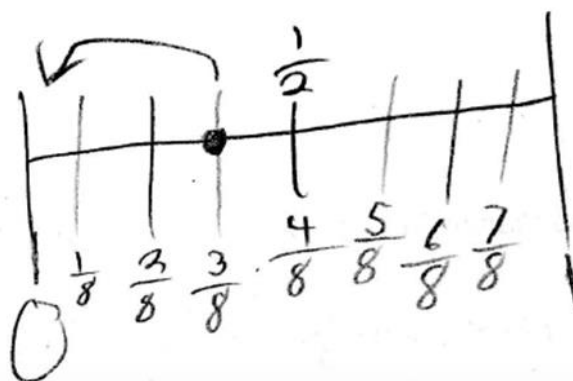


Figure 4.5. Student Descriptions on Item 8 of Posttest

The increase in number of explanations – either pictorially or in words – may indicate that when student thinking, discussion, and justifications become part of the culture of the mathematics classroom, it becomes part of their routine even in assessment contexts. Explaining oneself on paper is a useful student skill to have honed, especially for new practitioners. Not all student work can be captured through in-person interactions. Some analysis of student work must be done through their written form, and the more insight a student gives as to their thinking, the more apt fledging educators are to make impactful instructional decisions.

#### **Question 4 Summary**

In summary, because of the study's emphasis on students' conceptual understanding of fractions, the prospective teachers utilized progression and strategy frameworks to identify levels of student understanding. This emphasis on student progression and strategy levels rather than right or wrong answers kept the prospective teachers' planning focused on student thinking and encouraged the prospective teachers to make instructional decisions based on their students' levels of understanding. While the pretest to posttest results showed improvement in the raw percentages of items correct for each classroom, more importantly to this study and its definition of understanding, the strategies utilized by students changed in sophistication. No definitive conclusion can be made about the correlation between prospective teacher engagement with student thinking and an increase in student understanding of fractions. However, the findings suggest that the study's emphasis on engagement with student thinking in planning and instruction was cause for a shift in students' fractional thinking and how they approached fraction problem solving from pretest to posttest.

## Conclusion

In summary, the findings in this chapter were presented first as an overall profile of each prospective teacher according to their engagement with student thinking. Then findings were presented according to each research question in regards to what facilitated the prospective teachers' engagement with student thinking. The interpretations and implications of the findings are offered in the final chapter. These implications include a suggested model of support for teacher preparation and a revised envisioning of the *Levels of Engagement of Children's Mathematical Thinking* for use with prospective teachers.

## CHAPTER 5: DISUCSSION AND CONCLUSION

The current case study followed three prospective teachers' instruction of fractions during their student teaching experience in a cluster of 4<sup>th</sup> grade classrooms in one elementary school. The rich description of their teaching episodes and community of practice team meetings made data analysis possible and gave way to the themes that answered the research questions.

The questions that guided data collection and analysis were:

1. How might a prospective teacher's engagement with student thinking be characterized?
2. In what ways does context influence a prospective teacher's engagement with student thinking?
3. In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?
4. How does a prospective teacher's engagement with student thinking influence student understanding of fractions in the 4<sup>th</sup> grade?

Findings and some initial interpretations related to the four questions were presented in the previous chapter. In this chapter, interpretations and implications for teacher educators are formalized, including a proposed model of support for prospective teachers during their student teaching experience and a re-envisioning of a section of the *Levels of Engagement with Children's Mathematical Thinking* (Franke, Carpenter, Levi, & Fennema, 2001; Table 2.1) for use with prospective teachers. Finally the limitations, areas for future research, and concluding thoughts are shared.

## **Interpretations and Implications**

Interpretations related to the questions of this study were synthesized to suggest implications for teacher preparation and future research. The implications for practice resulted in two frameworks that may be used to guide teacher educators in their support of prospective teachers' engagement with student thinking as a way to plan and organize mathematics instruction. They are: (a) a model of support that occurs within the student teaching experience and (b) a model for the clarifying and characterizing of the nature of prospective teachers' engagement with student thinking during mathematics instruction. As in the previous chapter, the use of the pronoun "I" is integrated to reflect the researcher's direct interaction and facilitation of prospective teachers' engagement with student thinking.

### **Model of Support**

If prospective teachers' engagement with student thinking during mathematics instruction is valued then teacher educators must consider and provide a model of support that facilitates their engagement. The community of practice team meetings used in this study and the attention given to student thinking in mathematics curriculum planning indicated the importance of this work to prospective teachers' development in the profession. Based on the findings of the study, it is possible to articulate a model of support that serves this purpose. The suggested model of support focuses on three components: (1) eliciting prospective teachers' reflection on prior mathematics course experiences at they begin the student teaching experience, (2) building collaborative relationships among prospective and cooperating teachers, and (3) using learning progression and/or strategy analysis protocols with prospective teachers so they may make sense of students' understanding of mathematics. Each of these components is discussed and when considered together, may be used to frame a proposed model of support for the student teaching

experience designed to support prospective teachers' engagement with student thinking as a means to plan and implement mathematics instruction.

### **Eliciting Prospective Teachers' Reflections of Prior Mathematics Course**

**Experience.** One's beliefs shape how teaching is enacted in the classroom. Much of the research about prospective teachers' beliefs about mathematics teaching and learning is situated during their university coursework and does not continue into the student teaching experience (Ambrose, 2004; Burton, 2012; Hart, 2002; Hart, Oesterle, & Swars, 2013; Philipp et al. 2007; Thanheiser, Philipp, Fasteen, Strand, & Mills, 2013; Timmerman; Uusimaki & Nason, 2004; Wilkins & Brand, 2004). However, this study's initial interviews suggest the need for continuing to understand prospective teachers' past experiences and how the experiences have shaped beliefs about the instruction of mathematics during the student teaching experience. While beliefs are personal and prospective teachers may not always feel comfortable sharing their beliefs, if a relationship of trust is developed in which beliefs may be shared and unpacked, then university supervisors and cooperating teachers (i.e., support personnel) in direct contact during the student teaching experience can continue to support belief development towards the use of reform-based mathematics practices.

Each of the prospective teachers shared their beliefs about mathematics teaching and learning prior to beginning this study. For example, as discussed in Chapter 4, Liz shared her perspective on her experience in MATH 307: *Revisiting Real Numbers and Algebra*. Liz viewed mathematics as a challenging topic before her education coursework, and her experiences in MATH 307 continued to support this view. Liz deemed MATH 307 a space of confusion with unclear expectations about what is required from problem solving in reform-based mathematics classrooms. This could have cemented Liz's negative beliefs about mathematics further and

made it difficult for her to move into the use of reform-based mathematics teaching. Often, negative feelings about mathematics can manifest as mathematics anxiety, and when a teacher experiences mathematics anxiety it can manifest in the classroom instruction (Uusimaki & Nason, 2004).

Fortunately, Liz credited her EDUC 513: *Mathematics Methods for Teaching in the Elementary School* coursework and specifically Cognitively Guided Instruction (CGI; Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2014) for helping to reshape her beliefs about what mathematics instruction could be in the classroom and how she might approach it during her student teaching experience. Because Liz and I unpacked her personal history and relationship with mathematics, I was able to provide support that helped her maintain her reshaped beliefs in action during the student teaching experience.

Similar exploration and unpacking of beliefs about mathematics teaching and learning occurred with the other two participants in this study prior to the beginning of this study. This allowed for their beliefs to be addressed and perhaps aided in shaping their beliefs towards reform-based mathematics practices during the student teaching experience. There appears to be need for longitudinal attention to the prospective teachers' beliefs that are often revealed and addressed in coursework; attention to and addressing of beliefs should continue during the student teaching experience. This pertains to both negative and positive beliefs about the use of reformed-based mathematics instruction; being aware of prospective teachers' beliefs as they enter and continue through student teaching can help the support personnel make decisions about how best to provide support and coaching.

The current study also exposed the need for support personnel to understand and unpack a prospective teacher's prior exposure to curriculum during the student teaching semester. The

prospective teachers entered this study with lesson plans and a university-suggested set of lesson materials in preparation for teaching their units of instruction, but these tools proved insufficient for the prospective teachers to focus on student thinking during the mathematics instruction once practicing in real-time. Work completed in preparation for student teaching, in this case, was insufficient to sustain day-to-day planning for instruction. While it appears that prospective teachers do need thoughtful curriculum planning prior to their student teaching experience that supports their content knowledge of a topic, once engaged in student teaching, they need explicit help to unpack and use the planning materials from earlier coursework with the specific children in their placement classrooms. The results of this study indicated that once immersed in practice, the planning that had been completed during the prospective teachers' coursework was too far removed from the actual daily expectations of the classroom and their particular students. It appears that without the support of the team meetings, the prospective teachers may have resorted to more direct instruction pedagogy and minimized their engagement with student thinking, possibly for self-preservation. However, since engagement with student thinking was the norm of the team meetings and later the self-imposed expectation of the prospective teachers, they did not turn to direct instruction methods that did not consider their students' thinking in mathematics. The results of this study made clear that lesson plans are not lesson planning; it seems an in-depth curriculum pathway that explores both content and a progression of mathematical ideas in consideration of their students is needed by prospective teachers in order to be able to engage with student thinking in their instructional planning and implementation.

**Building collaborative relationships among prospective and cooperating teachers.** It

appears the structure and support provided during this study helped the prospective teachers bond with each other and encouraged them to engage in their own voluntary community of practice. Nora confirmed that she found value in the prospective teachers' work together:

Just hearing all their different perspectives on things and just being able to form this strong relationship, helping each other out, and then like dividing up all the work. And having that support system to be able to vent when we needed to, and then get feedback from one other, like, 'Hey does this lesson look okay, is there anything I should add?' or 'Does this make sense?'. Which was extremely helpful.

Prospective teachers need the space to make mistakes, challenge ideas, and question their practice during the context of the student teaching semester. They need to explore teaching in ways in which they are not merely being asked to replicate the practice of another teacher.

However important this community of practice was to the prospective teachers' development of practice, it is possible that the structure that fostered their bonding may have also fostered some separation between the prospective teachers and their cooperating teachers. Based on the findings in this study, providing protected space that allows for prospective teacher identity formation is important, but there is also need for group identity formation among the prospective teachers and cooperating teachers during the student teaching experience. Consideration must be given to achieving a balance among these needs and to supporting prospective teachers and cooperating teachers work together to maintain prospective teacher identity formation while also reinforcing development of collaborative relationships.

The possibility of providing shared bonding experiences among the prospective teachers and their cooperating teachers may need to be encouraged (Fede, Civil & Toscano, 2014). For example, a shared experience around classifying student work on the Ongoing Assessment Project (OGAP) Fraction Progression (Petit, Laird, Marsden, & Ebby, 2015) may have served as

the catalyst for relationship building and the identity formation of professional colleagues among the prospective teachers and cooperating teachers. The OGAP Fraction Progression was a new construct for the prospective teachers and would have been for the cooperating teachers as well; therefore, neither group would have been identified as immediate pedagogical experts in its use. Instead, they may have become a group of learners working together. A mechanism that involved building shared instructional knowledge may have bridged what was seemingly a relationship of coexistence but not collaborative existence among the two groups of the practitioners – the cooperating teachers and the prospective teachers. It may have helped each group maintain identities in certain spaces, as is important, but re-negotiate some of the roles of relating to each other within other spaces.

Another mechanism that may have strengthened and supported a collaborative bond came from insight into the relationships in the dyads. Nora and her cooperating teacher built a relationship that focused on instruction. Kristi and her cooperating teacher built a relationship around Kristi's struggles with remaining in the profession. Liz and her cooperating teacher maintained a polite relationship but not one of particular closeness either personally or pedagogically.

Nora's cooperating teacher was the only one to have had district-led beginning teacher mentorship training that was focused on coaching styles. Nora spoke to specific moments of intentional coaching, and her cooperating teacher corroborated these moments in her interview by explaining how and why she intentionally coached. At the same time, Liz's cooperating teacher pointed to an incidence of miscommunication that occurred before the student teaching experience even began. This incident likely impacted their relationship throughout the student teaching placement. The contrast between Nora and Liz's relationships with their cooperating

teachers suggests the value of some type of mentorship and communication training for cooperating teachers and possibly university supervisors. Although the cooperating teachers had participated in content and pedagogical professional development opportunities provided by the district and by faculty from the university in years previous to the study, any specific cooperating teacher training was focused on program handbooks, policies, and procedures. This assumes that cooperating teachers are equipped for their responsibilities in mentoring, but the findings of this study suggest a greater need for training focused on communication and mentoring. This could also support the collaborative relationship between the somewhat separate communities of prospective teachers and cooperating teachers, in that if the groups develop a relationship of open communication, it may increase the chance of needs being expressed and help to avoid a possible ‘us versus them’ mentality.

**Using learning progression and/or strategy analysis protocols to make sense of student understanding of mathematics.** The findings provide evidence in the value of prospective teachers analyzing student work throughout their student teaching experience if their mathematics pedagogy is to be centered on engagement with student thinking. In Kristi’s final interview, she commented that looking at student data together was one of the most important aspects of support. She indicated that she hoped that she would have someone in her future professional setting to help her navigate student work and its analysis. This points to the value of dedicating time analyzing student work with prospective teachers using their own students’ artifacts and not just through outside exemplars from other students. This finding echoes prior research in which teacher learning in mathematics content and pedagogy is enhanced by explicit and coached practice around the examination and analysis of their own students’ work (Ball & Cohen, 1999; Borko, 2004; Carpenter, Blanton, Cobb, Franke, Kaput, & McClain, 2004).

In my personal history of interaction and support of prospective teachers, I relied on their placement schools to offer mentorship around the examination and interpretation of student data. I rarely saw prospective teachers' actual student artifacts and instead used outside exemplar artifacts in coaching. The study findings revealed that even if school-based PLCs are coaching prospective teachers in the analysis of student work, prospective teachers need designated time outside of the school to question, consider, and plan what is next around the students' understandings. The prospective teachers in this study also needed a way to classify student work and a content expert to facilitate what do to with the data upon completion of analysis.

This study exposed that examination of student work was important to the prospective teacher development in reform-based mathematics ideals, even if they may not have overtly realized it was impacting their development. The attention to rubrics and progressions to analyze student work seemed to change the prospective teachers' language around students, which echoes past research findings (Sztajn, Wilson, Edgington, & Myers, 2014). Instead of proficiency language of "right" and "wrong," the prospective teachers began to discuss the sophistication of thinking the students showed in their problem solutions. Between weeks three and four of the study at an unplanned meeting, the prospective teachers realized that the language of labeling student groups as "low, medium, high" was not asset-based. The cooperating teachers used language of low, medium, high to group students in mathematics. The prospective teachers did not enter the student teaching experience with another language around what else they may label students, but expressed that labeling a student low was problematic. Their expressed joint belief was that an actual child was not low; merely the level of the progression at which the student appeared to be with a certain strategy was less sophisticated.

The prospective teachers expressed a desire for a system of categorizing trends in learning that allowed them to efficiently group students for small group and station learning, much like the cooperating teachers used low, medium, and high. The equal sharing strategy classifications (Empson & Levi. 2011; Figure 2.3) only worked in use in the context of one type of problem, and the OGAP Fraction Progression (Figure 2.4) was not easily transferred from team meetings to actual classroom practice, as aforementioned, perhaps because of the OGAP Fraction Progression's unfamiliarity and/or its more generalized applicability across fraction learning tasks. The prospective teachers seemed in search of a language system that would reframe the low, medium, high mindset within their community of practice, but that could still be easily understood to those outside the community.

At a mid-study team meeting, Liz asserted again that an individual child was not low; rather, a skill was missing or some misunderstanding was in place. Since their discussion was centered on children missing a particular skill, I suggested *approaching skill*, *on/with skill*, and *extending skill*. While still labels, the prospective teachers agreed this seemed a better option to label recognizing skillsets within particular topics because it allowed for flexibility of movement by skills, and remedied the silent messaging of labeling a child as low, medium, or high performing. Admittedly, a change in language does not equal a change in belief or practice. However, even an acknowledgement of language dissonance and the policing of one another in the community to utilize the new agreed upon language suggests that the supports helped the prospective teachers to begin to problematize embedded schooling practices that do not reflect the reform-based mathematics ideals.

**Proposed model of support.** A proposed model of support is offered for teacher educators taking into consideration these findings and their interpretations. This study used a placement of three prospective teachers at the same grade level and same school and supported their engagement with student thinking in mathematics instruction through researcher-facilitated team meetings and observational debrief sessions. The aspects of team meetings and debrief sessions appear to have been critical to the ongoing development of the prospective teachers' abilities to engage with student thinking, as elaborated in the stories of Chapter 4. However, part of the consideration in the proposed model of support is that there needs to be both boning of the prospective teachers around their shared identity and also building of prospective teacher and cooperating teacher shared identities.

The research findings and their interpretations suggest that certain nonnegotiable components should be included in a model of support of prospective teachers' engagement with student thinking in mathematics. The three key components are:

- (1) A community of practice structure in which the prospective teachers can practice their instructional ideas, take risks, make disagreements and receive feedback, both through a content expert's facilitation, and on their own is essential. This structure needs to produce a safe space that is neither a replication of a cooperating teacher's practice (Putnam & Borko, 2000) nor "style shows" in which each learn their own style without critique or challenge (Ball, 1994). A reminder that Appendix C provides the detail of what occurred in the team meetings for this study. This plan may be used as a template to guide others' work with prospective teachers. Attention needs to be given to mathematics standards and their instructional progressions. Time must be dedicated to unpacking curriculum materials and to lesson planning

that addresses the students' of the particular student teaching context and their thinking about the mathematics.

**(2)** Specialized mentorship training for cooperating teachers appears to be helpful based on the dyads' experiences; this training prepares cooperating teachers for the unique challenges of supporting prospective teachers and includes training in effective communication and coaching between a mentor and prospective teacher. The district-led beginning teacher mentor training around coaching styles that was experienced by one cooperating teacher seemed to help the cooperating teacher build awareness of her mentorship style with the prospective teacher and effectively communicate and support certain mathematics instructional goals. Additionally, the dyads' differing relationships and the prospective teachers' bonding seemed to indicate the need for planned opportunities of shared experiences to learn and explore mathematical content or pedagogical topics as a cohesive unit in order to further strengthen relationships and foster the identity of a collaborative group.

**(3)** Focused and substantive time is needed for prospective teachers to invest in the analysis of student work using identified progressions and strategy frameworks. Prospective teachers' practices benefit from the use of explicit progressions and strategy frameworks to examine their own students' work. This experience also serves as a catalyst for negotiation of language they might use to characterize the performance levels of children. Analysis of student work also emphasizes a continual focus on children's thinking being the primary driver for planning and instructional decisions.

## Envisioning the Use of the Levels of Engagement with Children's Mathematical Thinking with Prospective Teachers

As previously stated, the *Levels of Engagement with Children's Mathematical Thinking* (Franke et al., 2001) is a helpful tool for analysis of practice because the levels characterize teacher beliefs and practices on a spectrum rather than dichotomize a practice as 'reform or not'. However, because the *Levels of Engagement with Children's Mathematical Thinking* were developed and used with practicing teachers in attempt to measure generative change, they were accompanied by evidence of the use of the *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014) mathematics teaching practice *Elicit and Use Evidence of Student Thinking* and its corresponding teacher actions in attempt to better capture the orientation and practices of prospective teachers. This was still not a perfect fit and raised opportunity for envisioning how together, these frameworks might be even more fine-tuned so as to be used to characterize a reform-based mathematics prospective teacher. Since the prospective teachers in this study appeared to align to Level 3 in the *Levels of Engagement with Children's Mathematical Thinking*, that is the Level that was refined using the findings of this study.

Within Level 3, the prospective teachers in this study expressed transitioning rationales for why they chose to engage with student thinking. They also exhibited teacher actions from *Elicit and Use Evidence of Student Thinking* as they transitioned to and within Level 3. Since the *Levels of Engagement with Children's Mathematical Thinking* does not include teacher rationale for engagement with its beliefs and practices identifiers, a look into the rationales of the prospective teachers' of this study may assist others who work with prospective teachers. These various rationales for engagement were paired with the mathematics teaching practice *Elicit and Use Evidence of Student Thinking* teacher action of *Making in-the-moment decisions on how to respond to students with question and prompts that probe, scaffold, and extend* (Table 4.3).

Although presented as a single item to consider in terms of performance, this study found that the prospective teachers tended to move through probing, scaffolding, and extending questions and prompts in a progression of use and not engage in mixed use, at least initially.

The diagram in Figure 5.1 *Proposed Model of Engagement for Prospective Teachers* represents the prospective teachers' engagement with student thinking as a sort of cycle. It is hypothesized that this model may be repeated as a prospective teacher may experience a similar cycle when encountering a new content topic or set of lessons in mathematics. The cycle proposed in Figure 5.1 is focused on a depth of understanding in the use of prospective teachers' questioning to engage with their students' thinking about mathematics. It reflects a level of content understanding on the part of the prospective teachers that is needed in order for them to be able to engage in such questioning. It would make sense that as prospective teachers encounter new mathematics content, they need both to build their own understanding of the mathematics content and their understanding of the ways their students may make sense of this content.

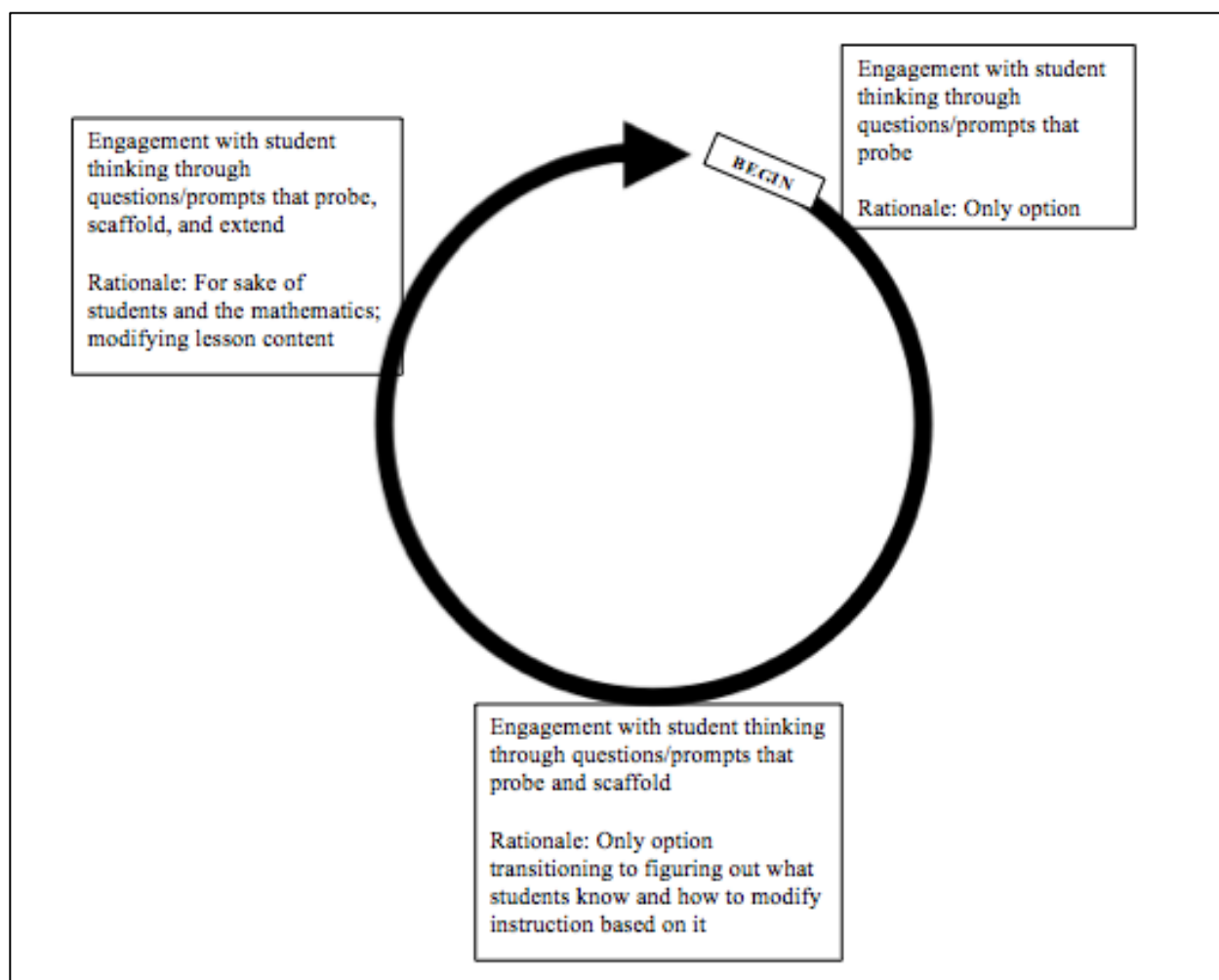


Figure 5.1. Proposed Model of Engagement for Prospective Teachers

The model suggests that prospective teachers who believe they should listen to children's thinking, may initially engage with student thinking because they feel it is their only option. The prospective teachers in this study revealed that they initially engaged with student thinking because of various reasons, but all reasons seemed mandatory. One reason the prospective teachers' seemed to engage with student thinking was because of the obligation to past coursework and its emphasis on eliciting student thinking through the use of productive *talk moves* (Chapin, O'Connor, & Anderson, 2003; 2013). This obligation was reinforced through the continual focus on student thinking as part of our community of practice team meetings. Another reason they engaged with student thinking was that they were not sure how else to

instruct, in that utilizing student thinking was the only model of instruction they had been exposed to or felt comfortable using. Finally, they engaged with student thinking simply because they were not sure where to move next after posing a problem or launching a lesson topic. In this final scenario, the prospective teachers felt they had to ask questions until they found a student contribution in which they were comfortable enough in both content and pedagogy to move forward with during the remainder of the mathematics instructional block. As Nora stated, “I don’t know how to teach math so I am having to ask kids what they think, and what they did, and what they drew, because I really don’t know.”

In this initial phase, the prospective teachers utilized a majority of questions and prompts that probed for student thinking. Probing questions and prompts aligned with certain productive talk moves that helped students to clarify their own thoughts (Appendix G), such as, “Can you say more?” or “Can you give an example?” However, the prospective teachers were not necessarily utilizing these talk moves as intended. They did not use the talk moves to allow students to process their own thoughts, but rather to provide themselves with time to process how to use a student contribution during a lesson. Perhaps a more apt description of the prospective teachers’ engagement with student thinking in this initial phase is what Jacobs and Empson (2015) classified as starter questions. Starter questions are used to invite the children to start a conversation about his and her thinking, and are questions such as, (a) “Tell me what you did.” (b) “What are you thinking about?” (c) “Tell me about your drawing.” or (d) “Explain your strategy.” These starter questions, similar in intent to the “can you say more” talk move, were used to probe for student contributions that were in the prospective teachers’ content comfort level.

After a mandatory “only option” phase of engagement with student thinking, the prospective teachers became more comfortable with instruction and the content (i.e., fractions) and they engaged with student thinking with the intention of being able to decide how to modify their instruction based on the students’ thinking. After this phase, it seemed the prospective teachers then transitioned to engaging with students’ thinking because they were genuinely intrigued by what they heard and could ‘in-the-moment’ manage what they learned from students with what they knew about the content and where they wanted to guide the instruction. In these two later phases, the comfort with content and teaching itself gave way to the prospective teachers’ use of questions and prompts that scaffold and extend student thinking. They were no longer just probing to make thinking transparent, but were starting to purposefully move student thinking along. The prospective teachers began to hone their use of productive talk moves to better accomplish teaching points and used them for the sake of student orientation to a classmate’s thinking or to deepen a student’s own reasoning. These talk moves included questions and prompts such as, (a) “Who can repeat or rephrase that?” (b) “Could someone put that in their own words?” (c) “Why do you think that?” and (d) “How did you figure that out?”.

This in-depth look at a Level 3 for prospective teachers may permit support personnel such as instructors, supervisors, and cooperating teachers to identify where prospective teachers are in their journey of use of reform-based mathematics teaching and to coach prospective teachers’ engagement with student thinking. It also provides a hypothesized model for analysis for future studies to see if this model holds true for prospective teachers outside of the bounds of this study. The model is a proposed cycle and future use in other contexts and with other prospective teachers to determine its usefulness is needed. It could be that this cycle also reflects the experiences of a teacher during an induction year, as presumably, the content and

instructional demands will be new to teachers once again. This suggests the need for an examination of other prospective teachers and induction year teachers who are committed to the orientation of Level 3 to examine if this model appears accurate, and if so, is the cyclic model repeated when encountering new content. Future application of the model may demonstrate that the cycle is instead steps in a process, and once a prospective teacher goes through the process she does not have to cycle from the beginning again. Finally, very possibly this model connects to the *Concerns-Based Adoption Model of Teacher Change* (Hall & Hord, 2000). A more substantive consideration of *Concerns-Based Adoption Model of Teacher Change* may help both to detail and to elaborate this possibly cyclic model.

### **Limitations and Future Research**

In this section, limitations of the study are discussed and future directions for research are provided that might better address the limitations. As with any study and its methodology, there are limitations which impact data collection, analysis, and findings. This design does not lend itself to generalization beyond the immediate setting of this study; however, the attention to methodological rigor as outlined in Chapter 3 suggests that study design and findings may be transferable to other teacher preparation programs and prospective teacher placement sites.

### **Study Duration**

The study lasted for a span of three months from the time of pre-instruction interviews through the instructional phase to post-instruction interviews. While this brevity in research could be concerning, the research began with an acknowledgement that if change in prospective teachers' practice was not recognizable within this time period it would be noted and would be important information for teacher preparation programs and future research. The timespan and observation period was justified in that it followed the prospective teachers' full-time teaching for a six-week period in the student teaching semester. The instructional observations began and

ended within this six-week period in order to observe the teaching of a mathematics instructional unit that was the sole responsibility of each prospective teacher. While change that aligned with reform-based mathematics was observed for each prospective teacher in this study, following prospective teachers for an extended time, such as from their previous semester into the student teaching semester, would provide even more insights into the changes that occur throughout a prospective teachers time in a teacher preparation program and insight into what facilitates these changes.

### **Study Size and Scalability**

The study followed three prospective teachers all teaching 4<sup>th</sup> grade and within the content domain of fractions. This focus allowed for rich description of the prospective teachers' experiences and immersion in the site even with the constraint of having only one researcher. If the participant numbers or study boundaries were expanded, it may not have resulted in such an intricate understanding and portrayal of the prospective teachers and their community of practice. This study sample size does, however, present certain complications for scalability. Scaling up such a detailed support model would require commitment from teacher preparation faculty on the programmatic and institutional level. A content expert would have to provide facilitation of the prospective teachers' meetings. Scalability of the proposed support model requires resource and time investment.

### **Participants**

The primary participants of the study were limited to three prospective teachers and only included pretests and posttests for the elementary students in their classrooms. All student data and artifacts were collected through the prospective teachers and provided to the researcher without student identification. As a result, the researcher did not know which pieces of student work belonged to which students, so student-specific information regarding growth as a result of

the fractions unit could not be ascertained. Student demographic information was provided by the prospective teachers but not in relation to individual children. Providing findings as holistic classroom percentages in Chapter 4 protected student privacy. However, these findings, especially those around potential groupings of students, point to the need for future research around instructional impact. Future studies could receive Institutional Review Board permission to collect identified student data on subgroup and demographic information. Then, student understanding could be monitored between and amongst certain groups of students.

### **Generalizability**

Generalizability of this study was considered both in terms of transfer of the prospective teachers' habits at study end to their future teaching lives and in relation to transfer of this study to other research sites. First, since the prospective teachers all taught 4<sup>th</sup> grade within the content domain of fractions, one can only hypothesize that the findings in the prospective teachers' beliefs and practices will transition across future teaching contexts and domains. Since the study took place in one site at one grade level, no conclusions can be drawn about how prospective teachers may or may not engage with student thinking if this model was introduced at other grade levels or schools. Specifically of benefit to this study was that Brookside Elementary was what the university deemed a mathematics focus school, so the cooperating teachers understood or were at least aware of the university's initiatives in mathematics. If a site does not have this asset, then it may be more difficult to transfer the study methodology and results without first preparing the school's cooperating teachers with professional learning that provides a similar background to that of the prospective teachers. Also unique to Brookside was that it did not have an adopted mathematics curriculum in place. The expressed need for a curriculum and a fleshed out pathway became a recurring theme for the prospective teachers. Perhaps if a required district or school curriculum had been in place, it would have influenced the findings of this study.

Adherence to an adopted curriculum is an aspect to consider in transfer of this study to another site.

### **Future Research**

Taking this study's limitations into consideration provides opportunity for moving forward in future research. First, in relation to the limitations of participants' content and grade level, it would be worthwhile to explore the support model including its team meetings, observational debrief sessions, and group placement of prospective teachers in other grade spans and within teaching in other content domains. Stories of practice, like those from this study, could be collected from other prospective teachers. It would also behoove the field to learn from the voices of the prospective teachers. Stories of engagement with student thinking from the perspective of a prospective teacher could be cross analyzed with the perspective of the researcher on the practices being observed.

It would be especially useful to follow the same set of prospective teachers in teaching through two different mathematics content domains in order to see if the same patterns of negotiating practice through the suggested engagement model occur. The *Proposed Model of Engagement for Prospective Teachers* in Figure 5.1 calls for future use in other contexts and with other prospective teachers to see if it holds true. If so, it can be then used as a tool for how to observe and scaffold prospective teachers' in their instruction and engagement with student thinking. It could also be transitioned to use with teachers new to the practice or teachers new to reform-based mathematics teaching.

Greater insight can also be gained as to the effectiveness of the study's structure and supports in instilling prospective teacher beliefs and practices that will transfer to their induction year classrooms. The induction year of teaching brings with it its own stressors and factors of change. It is hypothesized that when beliefs are cemented in line with reform, they will stay in

place even in a change of setting. However, when the prospective teachers are no longer being influenced and supported by one another within this context, they may regress in their focus on engagement with student thinking. This question of continued engagement or loss of engagement with student thinking may be addressed in other studies that follow prospective teachers not only through the student teaching semester, but continue into the induction year as well.

Finally, as presented in relation to Question 4, it would be helpful to have a study that follows the elementary students when their prospective teachers utilize engagement with their thinking in mathematics. It would behoove research to follow students who represent various educational subgroups and determine how they compare when prospective teachers and/or practicing teachers utilize engagement with their thinking in instruction. Not only could a study provide quantitative measures regarding various student subgroups and how their learning is influenced, but also research could qualitatively capture elementary students' perspectives on this type of instruction. Perhaps students do not even perceive or notice a difference in this type of instruction relative to other forms. Current research studies the educators' perspectives on reform-based mathematics practices, but missing are the students' voices, opinions, and feelings to ascertain what reform mathematics is like from the learner's point of view.

### **Concluding Thoughts**

In conclusion, the case study methodology effectively illuminated the prospective teachers' insights and instructional practices around engagement with student thinking in mathematics. The findings of this case study exposed the need for prospective teachers to have a support system that facilitates their understanding and instruction of mathematics in order to sustain engagement with student thinking. However, within this system of support in learning the content and pedagogy of mathematics, prospective teachers also required space to explore

and question their surroundings and their teaching styles and philosophies. They needed to do so in a space in which they felt safe in hopes that they would adopt the orientation of reform-based mathematics for themselves and not for the sake of the coursework or mentor. The voluntary supports that the prospective teachers created outside of the study's structure indicate the need for prospective teachers to shape their professional identities in context-specific way, yet do so outside of the context.

The findings also reveal the opportunity for future research in how to best utilize group placements of prospective teachers to best support their navigation of reform-based mathematics ideals. Schools need teachers who engage with students' thinking in a way that constructively uses the mathematics content in order for students to grow in their conceptual understanding of the topics at hand. Therefore, teacher preparation programs need support models in place that encourage and facilitate prospective teachers' work in engagement with student thinking.

## APPENDIX A: PROSPECTIVE TEACHER INSTRUCTIONAL UNIT OVERVIEW CALENDAR

Note: Prior to teaching sequence, each student made a Fraction Kit (Burns, 2001) and corresponding Cover It and Uncover games were played.

DAY 1 Equal Sharing Problems	DAY 2 Equal Sharing Problems	DAY 3 Equal Sharing Problems	DAY 4 Review of Fraction Kit with $\frac{1}{2}$ , $\frac{1}{4}$ , $\frac{1}{8}$ / $\frac{1}{16}$ & Cover Up Game	DAY 5 Cont. Cover Up Review
DAY 6 Equal Sharing Problems with Equivalent Fraction Situations	DAY 7 Construction of Fraction Kit $\frac{1}{3}$ , $\frac{1}{6}$ , $\frac{1}{12}$ & Making Connections to Rest of Kit	DAY 8 Whole-Class <i>Fraction Tracks</i> (TERC, 2014) Gameboard Introduction (Number-line model) & Comparison to Fraction Kits	DAY 9 Using Strategies to Compare Fractions (on <i>Fraction Tracks</i> Gameboard and with Fraction Kits)	DAY 10 Apply Equivalency and Comparison Ideas through <i>Fraction Tracks</i> as Whole Class
DAY 11 Intro <i>Fraction Tracks</i> as Partner Competition Game; Whole Group Debrief	DAY 12 <i>Fraction Tracks</i> as a Competition Continued; Discussion of Game Strategy Around Equivalency	DAY 13 Introduction of Submarine Sandwich Equal Sharing Task (Fosnot, 2007)	DAY 14 Work Day on Submarine Sandwich Task, with Challenge Component if Needed	DAY 15 Math Congress Set-up and Experience
DAY 16 Introduction to Common Denominators with Submarine Context; Share Out of Challenge Component (if applicable)	DAY 17 Readdressing the Submarine Sandwich Task with Common Denominators (Initial Student Exploration of Addition of Fractions)	DAY 18 Student Addition Exploration continued	DAY 19 Math Congress Around Addition Strategies	DAY 20 Formalize Class Addition Strategies
DAY 21 Student Exploration of Subtraction (Fraction Kit & Number Lines as Tools)	DAY 22 Student Exploration of Subtraction Continued	DAY 23 Formalize Class Subtraction Strategies	DAY 24 Practice with Subtraction Through Stations	DAY 25 Practice with Subtraction Through Stations
DAY 26 Mixed Operational Tasks and Stations	DAY 27 Mixed Operational Tasks and Stations	DAY 28 Mixed Operational Tasks and Stations	DAY 29 Mixed Operational Tasks and Stations	DAY 30 Posttest Administered

## APPENDIX B: INTERVIEW PROTOCOL AND INTERVIEW QUESTIONS

**I. PURPOSE.** To provide a uniform and standard way of conducting in-depth interviews with all the selected interviewees in the study area.

**II. RATIONALE.** A semi-structured interview will be conducted to gather information about participants' beliefs and ideas about teaching and learning mathematics and about their involvement in the Elementary Education program at the university. The information gathered will help to better understand the context of research study. Information will also help to reveal the level to which participants engage with student thinking during mathematics instruction.

### III. METHOD.

Face-to-face interviews will be conducted by the researcher with every selected participant who consents to participate in the study. This will be a one-time only involvement for cooperating teachers and twice occurring for pre-service teachers. The interviews will be semi-structured (question prompts provided), but will remain open to each participant's views and beliefs. Open-ended questioning probes will be used to better understand participants.

### V. SUPPLIES AND MATERIALS

- Signed Participant Consent
- Notepad
- Clipboard
- Pens
- Audio recording device

### VI. PROCEDURES

- Inform the participant that I will begin an in-depth interview that will request them to express their own views or opinions about mathematics teaching and learning, and about the context of school. Remind participant about their voluntary participation in the voice recording.

#### Introductory Statement:

FOR ALL:

*"Hello. I am Katherine Baker from University of North Carolina and am lead researcher in a study about the supports for prospective teachers to be able to engage with student thinking. Previously we completed a consent form for the study. Now we will ask you to express your own views and experiences with teaching mathematics and about your involvement with the university's Elementary Education program.*

FOR PROSPECTIVE TEACHERS: *It is very important for me to hear your views and experiences of being a student in the Elementary Education program and hear your views of mathematics teaching and learning.*

**FOR COOPERATING TEACHERS:** *It is very important for me to hear your views and experiences because you are a member of this student teaching team, and have a history as a cooperating teacher for the university's program. You can provide me with valuable insight about supporting a pre-service teacher in mathematics instruction.*

**FOR ALL:**

*I am going to turn on the voice recorder now and ask for your permission to audio-record this interview. [Turn on Recorder]. Do I have your permission to audio record this interview? [Wait for response].*

*You can ask me to end the interview at any time. You can also ask me to stop recording at any time. Do you have any questions before we start?"*

### **Turning on the voice recorder**

- Turn on the voice recorder and show the participant the color indicating it is recording. Speak clearly and loudly enough for the recording and encourage the participant to do so too. Ask them to repeat any quiet statements but try to allow them to speak freely without fear of the recording.

### **Administering the Interview**

- The researcher will follow the corresponding interview question guide for corresponding participant.
- The pace will be set by the interviewee.
- The researcher will practice active listening.
- Researcher should follow-up all general statements made by the respondent with a probe, particularly bearing in mind the purpose of the research.
- The researcher will take notes of the main items discussed during the interview, but writing will not detract from active listening. The audio record will be used to transcribe and expand interview notes.
- When the topic guide questions are finished, the researcher will ask for any additional comments the participant would like to give and remind them that all the information given will be kept confidential.
- Researcher will thank the participant at the end of the interview.

## Pre/Post Interview Question Guide for Prospective Teacher<sup>2</sup>

NOTE: Administer to PT before/after teaching to monitor if/how responses have changed.

1. Describe a memory about math from your childhood that stands out in your mind as especially important or significant. It may be a positive or negative memory.
  - a. Possible Probes: Who was involved? What did you do? What were you thinking and feeling? What impact has the event had on you? What does it say about who you were? Why is it important?
2. When you were a child, how did you feel about mathematics?
3. Is there something else about your experiences learning mathematics in elementary school that you would like to share?
4. What were your reactions when you were asked to solve a new kind of problem without the teacher's showing you how to solve it?
5. When you are a teacher, will you ever ask your students to solve a new kind of problem without first showing them how to solve it?
  - a. Possible Probes: Please elaborate on your answer. How often would you have students solve novel problems?
6. What are your beliefs about how to teach mathematics to elementary children? On how they learn mathematics?
7. Place the following four problems in rank order of difficulty for children to understand, and explain your ordering (you may rank two or more items as being of equal difficulty).

a.) Understand  $\frac{1}{5} + \frac{1}{8}$ .

b.) Understand  $\frac{1}{5} \times \frac{1}{8}$ .

c.) Which fraction is larger,  $\frac{1}{5}$  or  $\frac{1}{8}$ , or are they same size?

d.) Your friend Jake attends a birthday party at which five guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake each get the same amount of candy bar?

8. Please explain your ranking.

---

<sup>2</sup> Interview questions adapted from Philipp & Sowder (2003) Integrating Mathematics and Pedagogy (IMAP) Web-based Belief Survey Manual and from Drake (2006) Mathematics Life Stories

9. Why did you rank \_\_\_\_\_ as easiest for children to understand? Why did you rank \_\_\_\_\_ as most difficult for children to understand?
10. How are you thinking about student understanding?
  - a. Possible Probes: Explain further what you mean by understand. You indicated that student understanding is “getting the right answer.” Were you thinking about anything else?
11. Now, describe specific events from your adult years (age 20 and beyond) that stand out as being especially important or significant with respect to how you teach math.
  - a. Possible Probes: What in your math coursework stands out? What in your education coursework stands out? What were you thinking and feeling? What impact has the event had on you? What does it say about who you were? Why is it important?
12. Identify a person, group of persons, or organization/institution that has or have had positive influence on your perspective of teaching math. Please describe this person, group, or organization and the way in which he, she, it or they have had a positive impact on your role as a math teacher.
  - a. Possible Probes: How has your CT played a role in this perspective?

For post-interview only: Tell me about any supports that helped you engage with student thinking. Possible Probes: Why did they help? Who was helping?

### **Interview Question Guide for Cooperating Teacher**

1. Describe your historical experience as a cooperating teacher for the [University], including any professional development in mathematics that you have received through the role.
  - a. Possible Probes: When did you start? How many student teachers have you had? What do you think makes the experience successful?
2. Describe this current year’s experience as a cooperating teacher.
  - a. What is current mentorship style?
  - b. What supports do you have in place to help your student teacher in the area of mathematics instruction? In engaging with student thinking?
3. Do you ever ask your students to solve a new kind of problem without first showing them how to solve it?
  - a. Possible Probes: Please elaborate your answer. How often do you have students solve novel problems?
4. What are your beliefs about how to teach mathematics to elementary children? On how they learn mathematics? (For this question, the descriptors of the *Levels of Engagement with Children’s Mathematical Thinking* will be shown to the CT. Participants will be asked to identify the belief/practice descriptors that represent them and why).

5. Place the following four problems in rank order of difficulty for children to understand, and explain your ordering (you may rank two or more items as being of equal difficulty).

a.) Understand  $1/5 + 1/8$ .

b.) Understand  $1/5 \times 1/8$ .

c.) Which fraction is larger,  $1/5$  or  $1/8$ , or are they same size?

d.) Your friend Jake attends a birthday party at which five guests equally share a very large chocolate bar for dessert. You attend a different birthday party at which eight guests equally share a chocolate bar exactly the same size as the chocolate bar shared at the party Jake attended. Did Jake get more candy bar, did you get more candy bar, or did you and Jake each get the same amount of candy bar?

6. Please explain your ranking.
7. Why did you rank \_\_\_\_\_ as easiest for children to understand? Why did you rank \_\_\_\_\_ as most difficult for children to understand?
8. How are you thinking about student understanding?
- Possible Probes: Explain further what you mean by understand. You indicated that student understanding is “getting the right answer.” Were you thinking about anything else?

## APPENDIX C: COMMUNITY OF PRACTICE TEAM MEETING FACILITATION PLANS

Note: All Plans in the *Classroom Discussions in Math* template from Chapin, O'Connor, & Anderson (2013)

**Lesson Title:** Community of Practice Team Meeting 1

**Time:** 3:00-4:30

### Identifying the Mathematical Goals:

- Prospective teachers will review the content from *Classroom Discussions* (Chapin, O'Connor, & Anderson, 2013) and *Extending Children's Mathematics: Fractions and Decimals* (Empson & Levi, 2011) as introduced in their Math Methods course.
- Prospective teachers will be introduced to the Ongoing Assessment Project (OGAP) fraction progression of student solving strategies (Petit, Laird, Marsden, & Ebby, 2015)

### Anticipating Confusion:

- Prospective teachers may need re-orientation to the talk moves and how to integrate them into the various instructional tools learned about in the fall semester: Marilyn Burns Fraction Kit (2001), Fosnot Fraction Mini-Lessons (2001; 2007), Illuminations/Math Investigations Fraction Tracks (2004; 2016)
- Prospective teachers may not remember the Types of Strategies – Equal Sharing Problems table that outlines the typical strategies students use to solve Equal Sharing problems.
- Prospective teachers may not see how Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression compliment each other in regards to analyzing student work.

### Asking Questions:

- What talk move(s) do you feel you're integrating into instruction already?
- What talk moves would you like to begin to use this semester? Why?
- Why did you classify student work that way? (Defend your choice)

### Planning Implementation:

- Prospective teachers will be re-introduced to the Talk Moves and how they can be integrated into their pre-planned fraction units.
  - These pre-planned units give structure to the instruction, but the prospective teachers will also be reminded that they need to remain flexible to follow students' understanding.
  - Units include lessons from Burns, Fosnot, and Investigations
- Prospective teachers will be re-introduced to the Types of Strategies – Equal Sharing Problems student strategy table and will practice sorting sample student work.
- Prospective teachers will be introduced to the OGAP Fraction Progression table and will practice sorting sample student work.
- Time for questions, needs, and instructional planning.

- Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 2

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will use Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression to analyze their pre-assessment student data.

**Anticipating Confusion:**

- Prospective teachers may have difficulty classifying student work with the OGAP Fraction Progression because it is the newer instrument to them.

**Asking Questions:**

- Why did you classify student work that way? (Defend your choice)
- What does this student seem to understand?

**Planning Implementation:**

- Prospective teachers will come to meeting having initially analyzed and classified their pre-assessments.
  - Prospective teachers will re-analyze with researcher. They will defend their progression placements and adjust as necessary.
  - Prospective teachers and researcher will analyze pre-assessments until all raters agree on student Equal Sharing strategies and OGAP Fraction Progression level classifications. This will account for inter-rater reliability.
  - Address needs that arose at previous meeting.
  - Time for questions, needs, and instructional planning.
    - Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 3

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will reflect on day's instruction and set goals for remainder of unit.

**Anticipating Confusion:**

- Prospective teachers may be overwhelmed with their first day of full-time instruction of all subjects (including math) and first day of researcher video-taping/observing. They may need time to process full-time teaching responsibilities.
- Prospective teachers may be uncertain about how to adjust their pre-planned instructional unit based on student understanding.

**Asking Questions:**

- What was today's success? Today's stumble? Why? (Success/stumble is language used for a high point and point for improvement).
- What would you like to do moving forward? Why?
- How did you address student learning today?
- What student needs must still be met? How will you meet them?

**Planning Implementation:**

- Prospective teachers will discuss their self-perceptions and reflections about lessons and instruction today.
  - Prospective teachers will set personal goals for moving forward.
  - Researcher will address needs that arose at previous meeting.
  - Researcher will address any emerging content and pedagogical needs/themes that arose from today's observations.
  - Time for questions, needs, and instructional planning.
    - Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 4

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will review how to classify student work based Types of Strategies – Equal Sharing Problems and OGAP Fraction Progression.
- Prospective teachers will examine their own student work and classify it.
- Prospective teachers will adjust plans according to student understandings/misunderstandings.

**Anticipating Confusion:**

- Prospective teachers may need reminders about how to analyze student thinking based on progressions/strategies.
- Prospective teachers may be uncertain about how to adjust their pre-planned instructional unit based on student understandings.

**Asking Questions:**

- What were the week's successes? Stumbles? Why?
- What does this student/these students seem to understand? What question would you ask them next?
- What does this student/these students seem to need?
- How have you been addressing student learning this week?
- What student needs must still be met? How will you meet them?

**Planning Implementation:**

- Prospective teachers will practice their analysis of sample student work according to Types of Strategies – Equal Sharing Problems student strategy table and the OGAP Fraction Progression.
  - Prospective teachers will analyze and classify their own student work.
  - Researcher will address needs that arose at previous meeting.
  - Researcher will address any emerging content and pedagogical themes that arose from researcher video analysis (of previous week’s instruction).
  - Time for questions, needs, and instructional planning.
    - Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 5

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will set goals for the remainder of the unit instructional sequence as part of a mid-unit checkpoint.

**Anticipating Confusion:**

- Prospective teachers may be uncertain about how to adjust their pre-planned instructional unit based on student understanding.

**Asking Questions:**

- What were the week’s successes? Stumbles? Why?
- What would you like to do moving forward? Why?
- How have you addressed student learning to this point?
- What student needs must still be met? How do you know? How will you meet them?

**Planning Implementation:**

- Prospective teachers will share their self-perceptions and reflections about their instruction thus far as part of a mid-unit checkpoint.
  - Prospective teachers will set goals for the remainder of their unit sequence.
  - Address needs that arose at previous meeting.
  - Time for questions, needs, and instructional planning.
    - Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 6

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will examine their own student work and classify it according to Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression.

- Prospective teachers will adjust plans according to student understandings/misunderstandings.

### **Anticipating Confusion:**

- Prospective teachers may be uncertain about how to adjust their pre-planned instructional unit based on student understanding.

### **Asking Questions:**

- What were the week's successes? Stumbles? Why?
- What does this student/these students seem to understand? What question would you ask them next?
- What does this student/these students seem to need?
- How have you been addressing student learning this week?
- What student needs must still be met? How do you know? How will you meet them?

### **Planning Implementation:**

- Researcher will address needs that arose at previous meeting.
- Researcher will address any emerging content and pedagogical themes that arose from researcher video analysis (of previous week's instruction).
- Prospective teachers will bring work artifacts from classroom to analyze and classify according to Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression.
- Time for questions, needs, and instructional planning.
- Needs discussed here will be integrated into following meeting.

**Lesson Title:** Community of Practice Team Meeting 7

**Time:** 2:45-3:45

### **Identifying the Mathematical Goals:**

- Prospective teachers will examine their own student work and classify it according to Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression.
- Prospective teachers will adjust plans according to student understandings/misunderstandings.

### **Anticipating Confusion:**

- Prospective teachers may be uncertain about how to adjust their pre-planned instructional unit based on student understanding.

### **Asking Questions:**

- What were the week's successes? Stumbles? Why?
- What does this student/these students seem to understand? What question would you ask them next?
- What does this student/these students seem to need?
- How have you been addressing student learning this week?

- What student needs must still be met? How do you know? How will you meet them?

**Planning Implementation:**

- Researcher will address needs that arose at previous meeting.
  - Researcher will address any emerging content and pedagogical themes that arose from researcher video analysis (of previous week's instruction).
  - Prospective teachers will bring work artifacts from classroom to analyze and classify according to Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression.
  - Time for questions, needs, and instructional planning.
  - Needs discussed here will be integrated into following meeting.
- 

**Lesson Title:** Community of Practice Team Meeting 8

**Time:** 2:45-3:45

**Identifying the Mathematical Goals:**

- Prospective teachers will use Types of Strategies – Equal Sharing Problems and the OGAP Fraction Progression to analyze their post-assessment student data.
- Prospective teachers will share their final reflections about instruction of unit.

**Anticipating Confusion:**

- Prospective teachers may need to be re-orientation to analyzing assessments according to progressions.

**Asking Questions:**

- Why did you classify student work that way? (Defend your choice)
- What does this student seem to understand?
- What were you most proud of with this instructional unit? What would you most want to change?

**Planning Implementation:**

- Prospective teachers will come to meeting having initially analyzed and classified their post-assessments.
- Prospective teachers will re-analyze with researcher. They will defend their progression placements and adjust as necessary.
- Prospective teachers and researcher will analyze post-assessments until all raters agree on student Equal Sharing strategies and OGAP Fraction Progression level classifications. This will account for inter-rater reliability.
- Final discussion around perceptions, learning, and reflection about the instructional unit.

## APPENDIX D: OBSERVATION PROTOCOL

**I. PURPOSE.** To provide a uniform and standard way of conducting observations of pre-service teachers.

**II. RATIONALE.** Observations will provide insight into the in-the-moment instructional practices of the pre-service teachers and reveal if they are engaging with student thinking.

### III. METHOD.

Three in-person observations of mathematics lessons will be completed by researcher for each pre-service teacher. In these observations, the researcher will observe without interruption and complete a non-evaluative debrief with the pre-service teacher after instruction. Additionally, participant observations will be done at the team meetings with the pre-service teachers. At these meetings, the researcher will facilitate professional learning of mathematics content and pedagogy and aide the prospective teachers in analyzing their student work to make instructional decisions.

### V. SUPPLIES AND MATERIALS

- Signed Participant Consent
- Notepad
- Clipboard
- Pens
- Video recording device

### VI. PROCEDURES

- Schedule with participants when observations of instruction will take place. Team meetings will be scheduled together and video-recorded.
- Researcher will ask for verbal consent of video-recording at the beginning of team meetings.
- Field notes will be taken during the observations, specifically around the focus of if and how the pre-service teacher is eliciting and responding to student thinking.
- Researcher will follow the guidelines from the *Qualitative Research Methods: A Data Collector's Field Guide (2005)* regarding field notes:
  - Begin each field note entry with the date, time, place, and type of data collection event.
  - Leave space on the page for expanding your notes, or plan to expand them on a separate page.
  - Take notes strategically. It is usually practical to make only brief notes during data collection. Direct quotes can be especially hard to write down accurately. Rather than try to document every detail or quote, write down key words and phrases that will trigger your memory when you expand notes.
  - Use shorthand. Because you will expand and type your notes soon after you write them, it does not matter if you are the only person who can understand your shorthand system. Use abbreviations and acronyms to quickly note what is happening and being said.
  - Cover a range of observations. In addition to documenting events and informal conversations, note people's body language, moods, or attitudes; the general environment; interactions among participants; ambiance; and other information that could be relevant.

## APPENDIX E: INSTITUTIONAL REVIEW BOARD CONSENT FORM

University of North Carolina at Chapel Hill  
Consent to Participate in a Research Study  
Adult Participants

Consent Form Version Date: \_\_11-29-16\_\_

IRB Study # 16-3002

Title of Study: Revealing and Understanding Pre-service Teachers' Engagement with Student Thinking About Fractions

Principal Investigator: Katherine Baker

Principal Investigator Department: School of Education Deans Office

Principal Investigator Phone number: 919-428-0983

Principal Investigator Email Address: kaphelps@live.unc.edu

Faculty Advisor: Susan Friel

Faculty Advisor Contact Information: (919) 962-6605

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### **What are some general things you should know about research studies?**

You are being asked to take part in a research study. To join the study is voluntary.

You may choose not to participate, or you may withdraw your consent to be in the study, for any reason, without penalty.

Research studies are designed to obtain new knowledge. This new information may help people in the future. You may not receive any direct benefit from being in the research study. There also may be risks to being in research studies.

Details about this study are discussed below. It is important that you understand this information so that you can make an informed choice about being in this research study.

You will be given a copy of this consent form. You should ask the researchers named above any questions you have about this study at any time.

### **What is the purpose of this study?**

The purpose of this research study is to examine how context and a support of a community of practice influences pre-service teachers' ability to engage with student thinking in their instructional planning and implementation. This will be examined around the subject area of fractions. Additionally, the study will examine if and how the pre-service teachers' engagement with student thinking influences their students' understanding of fractions, as seen by the researcher through de-identified student work samples.

You are being asked to be in the study because you are a team of pre-service teachers and corresponding cooperating teachers through the elementary education program at the University of North Carolina-Chapel Hill. Additionally, you are a team of pre-service teachers who are completing their certification portfolio in the area of mathematics.

### **How many people will take part in this study?**

There will be up to 6 people in this research study.

### **How long will your part in this study last?**

The study will take place from January 2017 through March 2017 during the full-time student teaching expectations of the pre-service teachers. Cooperating teachers will be asked to complete a 30-minute interview about the student teaching semester context and supports in place. Pre-service teachers will be asked to complete a pre-and post study interview, 3 in-person lesson observations, up to 6 video-recorded lessons, and video-recorded community of practice team meetings around the implementation of mathematics best practices. Each of these components will

take approximately 30 minutes-90 minutes depending on the duration agreed-upon by the pre-service teacher and the researcher. There will be no follow-up to this study once the study duration is completed.

**What will happen if you take part in the study?**

As part of the cooperating and pre-service teachers in this study, you will all be asked to take place in an initial audio-recorded individual interview. You may choose not to answer any question for any reason. This will be the only study obligation for you as a cooperating teacher.

Next, as a pre-service teacher, you will be asked to take part in lesson observations, video-recorded lessons, and video-recorded community of practice team meetings around the implementation of mathematics best practices. Specifically, these observations and meetings will be around fraction instruction. These study components will be arranged with you. There will be no unannounced aspects of the study. Finally, as a pre-service teacher you will be asked to complete a post-study interview. Again, you may choose not to answer any question for any reason.

**What are the possible benefits from being in this study?**

Research is designed to benefit society by gaining new knowledge. You will not benefit personally from being in this research study. The new knowledge gained in this study about how pre-service teachers learn and instruct could impact teacher education programs and future professional development.

**What are the possible risks or discomforts involved from being in this study?**

The chance of risk for involvement in this study is unlikely. However, it is possible that a breach of confidentiality could occur. To mitigate this risk, several steps will be taken to protect participants' privacy. First, all participants will be given pseudonyms and the key to pseudonyms will be kept securely away from data. Next, all participants will be asked to keep anything shared in team meetings confidential so as to protect one another's privacy. Finally, all electronic records (video and audio) will be stored securely during study use and destroyed after study use.

Additionally, if you were to experience emotional discomfort resulting from this study (such as embarrassment), you should report this to the researcher and you have the right to withdraw at any time.

**How will information about you be protected?**

Your privacy and confidentiality will be protected.

- As participants you will be given pseudonyms. Your real names will be stripped from the data and replaced with pseudonyms. Your real names will not be stored with the data. It will be stored in a secure location with access only from the researcher.
- Video-recordings will be protected with Level II Security Requirements, which include:
- Access to study data must be protected by a username and password that meets the complexity and change management requirements of a [UNC ONYEN](#).
- Study data that are accessible over a network connection must be accessed from within a secure network (i.e., from on campus or via a VPN connection).
- All video-recorded data will be kept secured and separated from identifiers.
- Identifiers will be destroyed at the end of the study.
- Video-recordings will be destroyed at the end of the study according to the IT parameters for destroying video data.
- Additionally, you are advised to keep private anything learned in a team meeting. This will ensure the confidentiality of all participants.

Participants will not be identified in any report or publication about this study. Although every effort will be made to keep research records private, there may be times when federal or state law requires the disclosure of such records, including personal information. This is very unlikely, but if disclosure is ever required, UNC-Chapel Hill will take steps allowable by law to protect the privacy of personal information. In some cases, your information in this research study could be reviewed by representatives of the University, research sponsors, or government agencies (for example, the FDA) for purposes such as quality control or safety.

- Your video recordings will be stored securely away from identifying information until the end of the study, at which point they will be destroyed.
- Your audio recordings will be stored securely away from identifying information until the end of the study, at which point they will be destroyed.
- At any point, you may request that audio or video recordings can be turned off.

Check the line that best matches your choice:

\_\_\_\_\_ OK to record me during the study

\_\_\_\_\_ Not OK to record me during the study

**What if you want to stop before your part in the study is complete?**

You can withdraw from this study at any time, without penalty.

**Will you receive anything for being in this study?**

You will not receive anything for taking part in this study.

**Will it cost you anything to be in this study?**

It will not cost you anything to be in this study.

**What if you are a UNC student?**

You may choose not to be in the study or to stop being in the study before it is over at any time. This will not affect your class standing or grades at UNC-Chapel Hill. You will not be offered or receive any special consideration if you take part in this research.

**What if you have questions about this study?**

You have the right to ask, and have answered, any questions you may have about this research. If you have questions about the study, complaints, or concerns you should contact the researcher listed on the first page of this form.

**What if you have questions about your rights as a research participant?**

All research on human volunteers is reviewed by a committee that works to protect your rights and welfare. If you have questions or concerns about your rights as a research subject, or if you would like to obtain information or offer input, you may contact the Institutional Review Board at 919-966-3113 or by email to [IRB\\_subjects@unc.edu](mailto:IRB_subjects@unc.edu).

**Participant's Agreement:**

I have read the information provided above. I have asked all the questions I have at this time. I voluntarily agree to participate in this research study.

---

Signature of Research Participant

---

Date

---

Printed Name of Research Participant

---

Signature of Research Team Member Obtaining Consent

---

Date

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Printed Name of Research Team Member Obtaining Consent

## APPENDIX F: STUDENT PRETESTS AND POSTTESTS

Note: Items 1-8 from Marge Petit Consulting, Hulbert, Laird (2013)  
Items 9-11 from Lewis, Gibbons, Kazemi, & Lind (2015)

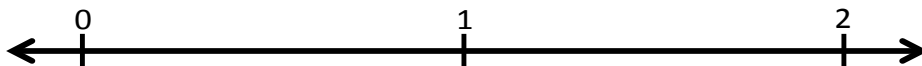
Grade 4 Fractions Pre/Post

[NAME \_\_\_\_\_]

DATE \_\_\_\_\_]

1)

1) Place  $\frac{3}{4}$  and  $\frac{1}{3}$  in the correct locations on the number line below.



2)

Ralph *correctly* drew this picture to represent  $\frac{3}{4}$  of the rectangle.



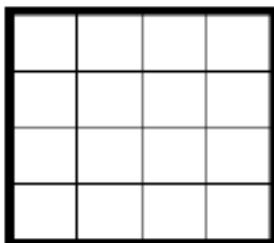
Kim said that  $\frac{3}{4}$  is equivalent to  $\frac{6}{8}$ .

Is Kim correct?

Use the picture above (or one of your own) and explanations to explain why Kim is correct or incorrect.

3)

Shade  $\frac{3}{8}$  of the figure.



---

4)

Which fraction is closest to 1? Show your work.

$$\frac{1}{2}$$

$$\frac{3}{4}$$

$$\frac{2}{3}$$

$$\frac{1}{5}$$

---

5)

There are 36 students in Abdi's class.

Two-thirds of the students in Abdi's class have brown eyes. How many students have brown eyes in Abdi's class?

Show your work.

6)

The distance from Billy's house to work is  $2\frac{1}{5}$  miles.

His car broke down  $\frac{3}{5}$  of a mile from work.

How far is Billy from his house?

Show your work.

7)

$\frac{1}{8} + \frac{2}{8}$  is closest to

- A) 1
- B) 0
- C) 3
- D) 16

Explain your choice.

8)

Place  $\frac{1}{3}$ ,  $\frac{4}{12}$ ,  $\frac{5}{3}$ , and  $2\frac{5}{6}$  in the correct locations on the number line below.



9.) 6 children are sharing 8 small sandwiches. They are sharing so each child gets the same amount. How many sandwiches will one child get? Show your work.

Solution: \_\_\_\_\_

10.) Six students equally share three pieces of construction paper. How much construction paper does each child get? Show your work.

Solution: \_\_\_\_\_

11.) Twelve students are sharing four pizzas equally. How much will each student get? Show your work.

Solution: \_\_\_\_\_

## APPENDIX G: OVERVIEW OF THE PRODUCTIVE TALK MOVES

### **Classroom Discussions Checklist** **Goals for Productive Discussions through the Productive Talk Moves**

Goal One: Help Students Clarify Their Own Thoughts		Notes/Frequency of Use
<input type="checkbox"/>	<b>Time to Think or Elaborate</b> <ul style="list-style-type: none"> <li>- Wait Time</li> <li>- Turn and Talk</li> <li>- Ask the Question Again</li> <li>- “Can you say more?”</li> </ul>	
<input type="checkbox"/>	<b>Revoicing Strategies</b> <ul style="list-style-type: none"> <li>- “So, let me see if I understand what you’re saying. Are you saying...?” (then leave space for the original student to agree or disagree or say more)</li> <li>- “I hear you saying..., say more about that.”</li> <li>- “Could you say that again?”</li> <li>- “Can you give an example?”</li> </ul>	
Goal Two: Help Students Orient to the Thinking of Others		Notes/Frequency of Use
<input type="checkbox"/>	<b>Who Can Rephrase or Repeat?</b> <ul style="list-style-type: none"> <li>- “Can someone rephrase or repeat that?”</li> <li>- “Could someone put that in their own words?”</li> <li>- “Who can restate what _____ said?”</li> <li>- After a Turn and Talk: “Please tell us what your partner said.”</li> </ul>	
Goal Three: Help Students Deepen their Own Reasoning		Notes/Frequency of Use
<input type="checkbox"/>	<b>Press for Reasoning</b> <ul style="list-style-type: none"> <li>- “Why do you think that?”</li> <li>- “What’s your evidence?”</li> <li>- “What convinced you that was the answer?”</li> <li>- “Why do you think that strategy would work?”</li> <li>- “How did you figure that out?”</li> </ul>	
Goal Four: Help Students Engage with Others’ Reasoning		Notes/Frequency of Use
<input type="checkbox"/>	<b>Prompt for Further Participation</b> <ul style="list-style-type: none"> <li>- “What do other people think?”</li> <li>- “Do you agree or disagree and why?”</li> <li>- “Can you add on?” or “Who can add on?”</li> <li>- “Does anyone have a different view?”</li> <li>- “What do you think about what _____ said? Who thinks they can explain what _____ means?”</li> </ul>	

K. Baker, October 2015; Checklist adapted from TERC The Inquiry Project (<http://inquiryproject.terc.edu/>) and Chapin et al. (2009, 2013) *Classrooms Discussions in Math*

## APPENDIX H: OVERVIEW OF THE SYSTEMS FOR ANALYSIS

The following provides a glimpse into the systems of analysis used for research questions 1-3. Chapter 3 provides the descriptions for the varying analysis processes used to answer each question and this appendix provides images to accompany the descriptions. The images are meant to provide an overview of the organization of each analysis pass through the data.

Question 1: How might a prospective teacher's engagement with student thinking be characterized?

		Level 3				Level 4A				Level 4B			
		Levels 1 & 2											
		Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
PT 1-3 Pre-Interview	PT 1-3 Interview 1	Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
	PT 1-3 Interview 2	Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
	PT 1-3 Interview 3	Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
PT 1-3 Beginning Observations	PT 1-3 Observation 1	Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
	PT 1-3 Observation 2	Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.
Observations cont., Post-Interviews, Team Meetings		Level 1: Observation - when asked to solve any problem, she only looks at the problem.	Level 2: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 3: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4A: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.	Level 4B: Observation - when asked to solve any problem, she only looks at the problem.

The table above highlights the analysis process for Question 1. The purpose of the image is not to be able to read the text, but to see how the directed content analysis (Hsieh & Shannon, 2005) codes were organized and how patterns and supporting evidence for the *Levels of Engagement with Children's Mathematical Thinking* (Franke, Carpenter, Levi, & Fennema, 2001) were examined. Coding for the Levels was done by hand on the interview transcripts, observation field notes, and team meetings notes. Then, quotations and actions were captured in a spreadsheet to analyze across one prospective teacher's practice, and also across all prospective teachers' practices. Note that rows are the different data collection methods for each participant for this particular question. Columns are the descriptors for the various Levels of Engagement. This organization system allowed for a visualization of where supporting evidence fell within the Levels, and in this case, the shaded block shows that a majority of evidence fell within Level 3. The sub-codes of the *Elicit and Use Evidence of Student Thinking* (NCTM, 2014) teacher actions were later integrated into this analysis system within the descriptors of Levels 3-4B.

Question 2: In what ways does context influence a prospective teacher's engagement with student thinking?

		Themes		
		CTP	Classroom	CT/PT relationship
Prospective Teachers	PT 1	307: Fried because learned the content (p. 8 x-marks as shaping experience); 312: Baker (CGI as shaping exp p.8); and Social Justice Course because learned how to put practice into action; PT 1: She felt the most prepared to teach math based on her previous coursework, even though she was an English major. (pg. 9); "Talk Moves"	PT 1: recurring theme- Speaks to the make-up of the classroom and how it is what makes it difficult to instruct - see CoP notes on her "crisis" day around interacting with students.	"CT helped me stay in the profession" (And family, and CoP, and me); "Personal Convo: Grateful for [u] what would I do without her? What if I didn't have a GT who supported me like some of the other GTs (convo came after finding one GT was having difficulty with CT and leaving program)"
	PT 2	X-marks; CGI: Talk Moves assignment - "Observation helped with What we see, or don't see in classrooms" "Negative experience in MATH 307"	Discussions of extending for students, how to extend, how to push them beyond basic understanding and keep engaged. (Unplanned) Meeting 1 Convo about student work- see pics & final observation debrief about student understanding- see pic in notebook)	"At CoP unpacking Fraction Tracks: "Delivery wasn't harsh, but message was harsh." -about student-centered teaching interaction with CT "Interview 2: Discussing how the kids love and always talk for CT."
	PT 3	X-marks; CGI: CGI empowering- see quotation from first interview "Talk Moves"	Classroom discussed in context of having a range of kids and them needing to be in mixed groups, and also sometimes in like groups (explains her ideal make-up of classroom in interview 2)	See both final interviews- the dyad discusses their perspectives on miscommunication between them.
Cooperating Teachers	CT 1	"More so than other years, they are coming in with the content work and I have to do that"- talks about especially in literacy and guided reading. Discusses lack in ability of PT to reflect: "We didn't do the reflection portion this year. We just didn't work in time for that. I wasn't a requirement this year. And by the time we got to it they were already teaching full-time and couldn't reflect until end of day, and can't remember then."	Regarding students: "This class" "This group" "Challenging" - language used by CT/PT the same	"Fraction training but no discussions training"
	CT 2	"Talks about past experience and being mentored by supervisor. Weekly emails from supervisor around specific teaching points each week. None of that this year."	Discussion around student levels - how to extend- how to make extension more than just harder numbers. Reflecting with me and GT- why do you choose the numbers you do? Why choose the extensions you do? (Says all of the GTs are just now really to start thinking about this at this level at point of final interview).	"Only CT with mentoring training and with both fraction and discussions training." "Just being able to prompt some of these goals to say, you know, 'I'd like, for - maybe red every time these are going to happen for you, but I'd like for some of these things to start happening, and you can see what that looks like.'"
	CT 3		Final interview: discussion around the Spanish speaking students and how to draw them into content, not just give separate work, how to hook students by connecting to real world context- Nasacar example. Nothing explicitly about students' or math levels.	"Never had the PD training of fraction book or discussions with"

The table above highlights the analysis process used for Question 2, with each prospective teacher then each cooperating teacher listed along the left as rows. Data were analyzed for mention of contextual features by the prospective teachers and/or cooperating teachers and for potential influences of context on the prospective teachers' ability to engage with student thinking. The recurring codes about context became themes and were listed in a spreadsheet with their supporting evidences. This process allowed for analysis by prospective teacher and also across and between the prospective teachers, cooperating teachers, and dyads. The table below features some of the key evidences within the themes of Question 2.

## Question 2 Themes and Evidence Examples

	<b>Teacher Preparation Program</b>	<b>Classrooms</b>	<b>CT/PT relationship</b>
PT 1	<p>*Influence of MATH 307 “because [Instructor] helped us learn the content” and describes Xmania as shaping experience; CGI in EDUC 513. – Pre-Interview</p> <p>*“I feel most prepared to teach math based on previous coursework, even though I’m an English major.” – Pre-Interview</p>	<p>*Continually speaks to the make-up of the classroom at our CoP meetings, and attributes the student make-up as what creates difficulty for instruction.</p> <p>*At CoP 4 discusses “most difficult day” around interacting with students and difficulty with relationship building.</p>	<p>*"[CT] helped me stay in the profession...and my family, and the CoP, and [researcher].” – Post-Interview</p> <p>*“I’m grateful for [CT], what would I do without her? What if I didn't have a CT who supported me like some of the other STs – CoP 6</p>
PT 2	<p>*Xmania in MATH 307; CGI in EDUC 513 – Pre-Interview</p> <p>*Talk Moves Observation assignment in EDUC 513 “helped with what talk moves we see, or don't see in classrooms” – Post-Interview</p>	<p>*Starts conversation about her student work and how to push students beyond basic understanding and keep them engaged. – Unplanned Meeting 1</p> <p>*Raises discussions about extending learning for her students; wonders how to best extend, and explains her reasoning behind lesson’s extensions. – Final observation debrief</p>	<p>*"The message wasn’t delivered harshly, but it was harsh to hear.” –CoP 5 regarding conversation with CT about using student-centered instruction over teacher-centered instruction</p> <p>*Discusses how the kids love CT and always seem to willingly talk for CT in discussions, but she has to work at it. – Post-Interview</p>
PT 3	<p>*Xmania in MATH 307; CGI in EDUC 513: “CGI and that stuff and that’s what made me feel really confident in teaching math and that’s what – the kind of direction I want to go in, for sure” – Pre-Interview</p> <p>*Discussed negative experiences in MATH 307</p>	<p>Classroom discussed in context of having a range of kids and that students sometimes need to be in mixed groups and also sometimes in homogenous groups and sometimes in whole group discussions- “a perfect balance.” – Post-Interview</p>	<p>“It was confusing because we were using their slides from last year. But, had to still completely re-create. They were critiquing us on what we’re teaching, but they were really critiquing us on how they had been teaching last year.” – Post-Interview,</p>

	as well. – Pre-Interview		regarding feeling the pressure to replicate CT practice instead of teach in ways learned in program
CT 1	"More so than other years, they aren't coming in with the content work and I have to do that, especially in literacy and guided reading. So I can't take the time to coach in other areas." –Interview	Regarding students- "It's this class", "It's just this group", "It's a challenging group" - language used by CT/PT is the same regarding students –Interview	Had fraction book study professional development around <i>Extending Children's Mathematics</i> , but no <i>Classroom Discussions</i> professional development
CT 2	"We didn't do the reflection portion this year [in the fall semester]. We just didn't work in time for that, it wasn't a requirement this year. And by the time we got to it they were already teaching full-time and couldn't reflect until end of day, and they can't remember everything that's happened then." –Interview	Discussion about how to extend the math and how to make extension something more than just harder numbers. Reflects with me about how to question ST and asks "Why do you choose the numbers you do? Why choose the extensions you do? All of the student teachers are just now ready to start thinking about this in math." –Interview	Only CT with mentorship training from district and with both <i>Extending Children's Mathematics</i> , and <i>Classroom Discussions</i> professional developments  *"Just being able to pinpoint some of these [Talk Moves] goals to say, you know, 'I'd like, for – maybe not every time these are going to happen for you, but I'd like for some of these things to start happening,' and you can see what that looks like." –Interview
CT 3	Discusses past experience and being trained/coached by site-based supervisor. Last year received weekly emails from supervisor around specific coaching points each week. Didn't receive those this year.	Final interview- discussion around the Spanish speaking students and how to draw them into content, not just give separate work; how to hook students by connecting to real world context- Nascar example. Nothing explicitly about students' or math levels.	Did not have fraction book study professional development around <i>Extending Children's Mathematics</i> or <i>Classroom Discussions</i> professional development

Question 3: In what ways does a community of practice structure facilitate prospective teachers' engagement with student thinking in the area of fractions?

		Codes				
Themes		Figuring Out Students	Figuring Out Content/Curriculum	Figuring Out Self-Praxis "by/on"	Coping & Camaraderie	Unplanned Meetings
		<p>"What I noticed is that even though 42 would be right, it's technically a misconception they're showing."</p> <p>"Yeah I noticed that too." Item 2 on pre-assessment CoP2</p> <p>"Item 3 on assessment most inconsistent between group in how to categorize. (Changes to item 1 on post-assessment and how to categorize.)</p> <p>CoP Meetings (see credit for early)</p>	<p>Links to all the Content Documents: Fractions: <a href="https://docs.google.com/presentation/d/1Q_3P6d7vU8LXGGbWkX0MM8u7vix0XAZQdJwEdW8d4v4p/edit">https://docs.google.com/presentation/d/1Q_3P6d7vU8LXGGbWkX0MM8u7vix0XAZQdJwEdW8d4v4p/edit</a></p> <p>Fractions Week 2: <a href="https://docs.google.com/presentation/d/1vGF5QCRmP6k5u0d8t8wemz8KCU1K3_g0X3dyw8Wd4v4p/edit">https://docs.google.com/presentation/d/1vGF5QCRmP6k5u0d8t8wemz8KCU1K3_g0X3dyw8Wd4v4p/edit</a></p> <p>"Field Trips &amp; Fundraisers: <a href="https://docs.google.com/presentation/d/14p-0k-DNPS5u0WvF3ZDm-c0BxKvUw2qDPenr8wEdW8d4v4p/edit">https://docs.google.com/presentation/d/14p-0k-DNPS5u0WvF3ZDm-c0BxKvUw2qDPenr8wEdW8d4v4p/edit</a></p> <p>"COY GUIDE: <a href="https://docs.google.com/document/d/1c2ekL3y0MUR2vH-RuX0G2lgnXGUCWFS2AGuM/edit">https://docs.google.com/document/d/1c2ekL3y0MUR2vH-RuX0G2lgnXGUCWFS2AGuM/edit</a></p>	<p>The midway breaking points of each of the PTs:</p> <p>"PT 1 over relating to students"</p> <p>"PT 2 over student-centered &amp; teacher-centered instruction"</p> <p>"PT 3 over having to pause during instruction. CoP 4-5"</p>	<p>Just hearing all their different perspectives on things and just being able to form this strong relationship, helping each other out, and then like dividing up all the work. And having that support system to be able to vent when we needed to, and then get feedback from one other, like, "Hey does this lesson look okay, is there anything I should add?" or "Does this make sense?". Which was extremely helpful. - PT 2 and interview 1</p> <p>"PT 1 &amp; 3 final interview about perceived feeling of not having support of CTs and not having others to turn to people outside don't get this)</p> <p>"This group helped them "survive", "stay in program" (all final interviews mention it)</p> <p>"Having someone to listen to us and tell us we're not crazy"</p>	<p>SEE BELOW ROW- unplanned meetings discussed at Team Meetings but prevalent enough to become own theme, not subtheme</p>
		<p>"Sophistication of the levels of student drawings for anticipatory sharing 1 item at time, not just chart. "I mean, surely, these are all different levels right?"</p> <p>"Figuring out different classification language for students rather than low, medium, high-- (used this different language for the remainder of the meetings and study duration and also mentioned in PT 1 and 3 interviews)</p> <p>One Meetings (in PT 1 and 3 interviews)</p>	<p>"WHOA I mean wait, when we flip the phrasing of 4 people sharing 3 items to 3 items being shared by 4 people then we get the fraction it's going to turn out to be at the end. That's crazy! I'm just making that connection now."</p> <p>"I don't either" - PT 1 and PT 3 (Referring to the wording of equal share problems and fractional representation. Also talked about level of sophistication on chart, and noticed how most the problem wordings are shared first then items, but wondered what would happen if students saw them the other way around.)</p> <p>"Unplanned Meeting 2: All PTs using the OGAP and Equal Sharing Framework for Task 3 (assessment commentary) in edTPA and using 1 for patterns across class learning and reflecting on their movement for instruction.</p>	<p>Language discussion- not low, medium, high. Approaching skill, on-skill, extending skill at Unplanned Meeting 1 field notes. (And See PT 3 interview about why this is important, PT 1 still using this new language at this meeting as well. ALL three PTs use in their edTPA portfolio Task 3 for analyzing work)</p>	<p>"Do all other assignments together as well for other course (EDUC 503 Leadership Seminar) break up assignments- See email code on and the</p> <p>"Also plan for other subjects together (after unplanned meetings that I'm there for math, go into their other subjects)</p> <p>"PT 1 &amp; 3 moving together- final interview</p>	<p>(These supports are in the site meetings horizontal row; were happening throughout the study duration once they got to know each other- specifically spoke about reason for them in final interview from PT 1 &amp; 3)</p>

Open coding was done across the data for potential influences of the community of practice structure. The table above shows that codes were listed in a spreadsheet and supporting evidences were listed for each code. Codes were then condensed to overarching themes. The team meetings became one of the themes of influence of the community of practice structure and sub-themes emerged of figuring out content/curriculum, figuring out students, and figuring out selves. The shaded section of the above table shows how the code of unplanned meetings became its own theme because evidences from each code were found within the prospective teachers' unplanned meetings. The table below shows the three themes that emerged regarding the community of practice structure and provides some of the quotations and evidences from each prospective teacher.

### Question 3 Themes and Evidence Examples

	Team Meetings	Unplanned Meetings	Camaraderie
Kristi	Figuring out Self: “But our CTs use that language...what do we use instead?” – CoP 3, regarding calling students “low, medium, or high”	"WHOA! I mean wait, when we flip the phrasing of 4 people sharing 3 items to 3 items being shared by 4 people then we get the fraction it's going to turn out to be at the end! That's crazy! I'm just making that connection now." – Unplanned 1	“Like the only people I see are Nora and Liz and Liz’s boyfriend, he just sits there while we plan...” – Post Interview
Nora	Figuring out Students: “Wait! I think I know what they’re doing. Look at the way they’re sectioning. They’re not treating the whole as 0 to 1, they’re treating the whole as the whole number line.” – CoP 8	Student work brought to unplanned meeting to analyze and ask questions about. – Unplanned 1	“Just hearing all their different perspectives on things and just being able to form this strong relationship, helping each other out, and then like dividing up all the work...” – Post Interview
Liz	Figuring out Curriculum: “This is the best day ever; we’re getting so much done.” – CoP 7, describing grade level curriculum guide that was created by prospective teachers	Raises question about her student’s (student 11) work samples and all members of team go through all items and discuss placement on rubrics – Unplanned 2	“Who else can I even talk to?” – Post Interview, describing student teaching experience and reliance on the group of three

## REFERENCES

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