MONETARY POLICY IN A ZERO LOWER BOUND ENVIRONMENT

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ABSTRACT

LAURA E. JACKSON: MONETARY POLICY IN A ZERO LOWER BOUND ENVIRONMENT.
(Under the direction of Neville Francis.)

In the wake of the Great Recession, the Federal Reserve lowered the federal funds rate (FFR) target to zero and resorted to unconventional monetary policy. With the nominal FFR constrained by the zero lower bound (ZLB), empirical monetary models cannot be estimated as usual. First, in joint work with Neville Francis and Michael Owyang, we consider whether standard models of monetary policy can be preserved without breaks. We consider whether alternative policy instruments can be considered substitutes for the FFR over the ZLB period. Furthermore, we construct a shadow rate via the methods proposed in Krippner (2012) and Wu and Xia (2014) to proxy for the stance of policy. We ask whether the shadow rate is a sufficient representation of the policy instrument or if the financial crisis requires other modifications. We find that, if using a dataset that spans the pre-ZLB and ZLB environments, the shadow rate acts as a fairly good proxy for monetary policy by producing impulse responses similar to what we’d expect based on the non-ZLB benchmark. However, the linear model exhibits a structural break at the onset of the ZLB and the shadow rate may be insufficient for examining the ZLB period in isolation.

Second, I describe the joint dynamics of bond yields, monetary policy and macroeconomic variables within a no-arbitrage affine term structure framework while explicitly modeling the ZLB using the shadow rate. I include data on the unemployment gap and inflation to build a more comprehensive stance of policy, incorporating the influences of unconventional instruments introduced to combat the Great Recession. I find that shadow rate models incorporating macroeconomic factors suggest a more negative shadow rate and a longer expected duration of the ZLB episode than models including only financial data. Also, including the macro data
allows the model to better capture the shift in policy focus towards targeting longer-term yields. Finally, the shadow rate produces a proxy for the stance of policy that suggests the unconventional programs achieved a substantial accommodation, in excess of that prescribed by a standard policy rule.
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CHAPTER 1 FOREWORD

The global recession of 2008-2009, driven mainly by the financial crisis in the U.S., triggered unprecedented responses in both fiscal and monetary policy. Traditional economic theory suggests that, in normal times, the short-term interest rate (Federal Funds rate) provides a clear indicator of the stance of monetary policy and allows consumers and investors to discern the intent of the central bank. The Federal Reserve targets a lower (higher) policy rate in order to stimulate (contract) the economy in response to business cycle fluctuations. To combat the recent recession, the Fed lowered short-term interest rates to near-zero levels to make it easier for businesses and households to borrow and invest. Nominal interest rates cannot become negative if currency exists since money pays zero interest and investors would rather hold cash than earn a negative return. Therefore, the Federal Funds rate (henceforth referred to as FFR) being near zero for an extended period forced the adoption of alternative policies to drive economic growth.

The Fed introduced programs such as quantitative easing and large-scale purchases of long-term assets, including mortgage-backed securities. In doing so, the Fed sought to lower interest rates in the housing market and on other long-term financial assets to ease borrowing costs and encourage investment. These unconventional programs represent a diversion from traditional policy as described by models of monetary economics. Therefore, these models must either include more policy instruments or introduce some sort of transitional structural change in the operating procedures of the Fed during this time.

Economists are tasked with redefining the “policy rate” to absorb the effects of these alternative programs and thus give a more accurate description of the stance of monetary policy. To this end, my research expands upon current models of estimating a shadow policy rate which incorporates the unconventional aspects of policy and underlies the behavior of the
central bank. Although market interest rates are necessarily nonnegative, the shadow rate may become negative during periods of extremely loose monetary policy. One can construct this shadow rate by estimating a model of the term structure of interest rates, anchored by the Fed’s policy rate. Typical term structure models do not control for the inability of short-term rates to fall below zero. This dissertation estimates various models accounting for this constraint and analyzes how the alternative programs affected the true and perceived stances of monetary policy throughout the Great Recession.

In Section 1.1, I summarize the existing literature on measuring monetary policy at the ZLB. Many studies have sought to discern the effects of unconventional policy instruments by conducting event-studies around their introduction.

In Chapter 2, joint work with Neville Francis and Michael Owyang, we consider whether the standard empirical model of monetary policy can be preserved without breaks. We test whether alternative policy instruments (e.g., the size of the balance sheet) can be considered substitutes for the FFR over the ZLB period. That is, we ask whether, during the ZLB period, there exists a relationship between the FFR and the alternative instruments so that the standard monetary policy vector autoregression (VAR) dynamics can be preserved. We propose controlling for the size of the Fed’s balance sheet and information on alternative policy announcements to represent policy action during the ZLB period from 2009:Q1 through 2012:Q4. We included these additional variables in a VAR to determine whether we could produce the same impulse responses of key macroeconomic indicators in the ZLB environment as those generated in baseline post-WWII macroeconomic models. We found that the balance sheet and policy events appear to produce an accurate representation of policy during this period but may be inefficient to use for future empirical work once we return to a normal, non-ZLB environment and these policies are no longer in use. Next, we test whether the shadow rate is a sufficient representation of the policy instrument or if the financial crisis requires other modifications to the monetary model. We find that the shadow rate acts as a fairly good proxy for monetary policy, if using a dataset that spans the pre-ZLB period throughout the ZLB environment, and produces impulse responses of macro indicators similar to what we’d expect based on the post-WWII, non-ZLB benchmark. However, it may still be insufficient
for examining the ZLB period in isolation.

In Chapter 3, I expand this framework using economic indicators to establish the relationship between interest rates, monetary policy and macroeconomic fluctuations. I construct a macro-finance shadow rate model which extends the current shadow rate methodologies to incorporate information on key macroeconomic indicators when constructing the perceived stance of policy through the ZLB period. An expansive literature has developed which recognizes the value of modeling the macroeconomy and financial markets together. The Fed sets policy to influence the availability of money and credit, thus establishing a direct channel linking policy and financial market activity. Furthermore, actions of the Federal Open Markets Committee (FOMC) are closely watched by financial markets while the FOMC itself extracts information about the current state of the economy from interest rates before setting policy.

I construct three different versions of the shadow rate model, differing in the extent to which I allow the macro data to influence term structure dynamics. First, in the “Macro-Finance model”, I incorporate macro fluctuations as factors which directly influence the shadow short-term rate. According to this interest rate reaction function, the Fed sets monetary policy in response to the current state of the economy and makes adjustments according to changes in inflation and real activity. Including a latent factor to represent unobserved shocks to policy, I anchor the term structure with a shadow short rate directly influenced by macro fluctuations. Secondly, in the “Macro-Monetary model”, I restrict the observable dynamics to describe the shadow rate as only a function of latent financial factors, prohibiting the macro factors from directly feeding back into the term structure. Based upon a standard monetary VAR, I model the unemployment gap and inflation as responding to lagged policy rates. I extract additional information from observations on macro indicators each period to filter the path of the shadow rate. Finally, for comparison to existing studies, I construct the shadow rate using only interest rate data in the “Finance model”.

The results suggest that it is inappropriate to use only the Federal Funds rate to represent policy in a dataset which spans both normal and ZLB environments. The shadow rate model produces a more comprehensive measurement of monetary policy at the ZLB by capturing how the Fed has changed focus over time in targeting short-, medium-, or long-term interest rates.
Furthermore, utilizing macro data in a reactionary way, as in the Macro-Monetary model, is more appropriate than models allowing the macro fluctuations to feedback into the term structure. Finally, the shadow rate produces a proxy for the stance of policy that suggests the unconventional policy programs achieved a substantial accommodation, in excess of that prescribed by a standard policy rule.

1.1 Literature on Monetary Policy in a ZLB Environment

Bernanke & Reinhart (2004) discuss various strategies for stimulating the economy facing the ZLB when the short-term policy rate cannot be lowered any further. These strategies include the use of forward guidance to assure investors that the Fed will hold short-rates low for a longer period into the future than may be expected by financial markets, changing the composition of the Fed’s balance sheet to adjust the relative supplies of securities of different maturities available in the market, and quantitative easing to increase the size of the balance sheet beyond that required to keep the policy rate near zero. The use of these alternative tools signals to the public that the Fed can still enact effective policy even when traditional policy in not applicable. In related work, Bernanke et al. (2004) use a variety of empirical finance methods to gauge the potential effectiveness of unconventional monetary policy programs in a ZLB environment. The authors conduct an event-study analysis to measure the response of financial markets to various central bank announcements. Also, they estimate different no-arbitrage VAR models of bond yields using only observable macroeconomic factors to describe the term structure. In the VAR models, they establish a direct link between the term structure and observable economic conditions which makes it easy to see how unconventional policies can be effective. They find that policymakers still can influence expectations of future policy even when the current FFR falls to zero. By convincing financial markets that it will maintain low policy rates for longer than expected, the Fed can reduce long-term interest rates and stimulate real economic activity.

Williams (2010) discusses the implications of extended ZLB episodes for the efficacy of monetary policy. In the wake of severe, prolonged recessions accompanied by deflation, standard open-market operations are insufficient to return inflation to its target. As a result, the
monetary authority must seek alternative sources of stimulus. In response to a sluggish recovery and deflationary scare, the Fed incorporated recommendations from mainstream research and cut the policy rate to a very low level. Once exhausting its traditional policy instrument, the FOMC used policy statements to communicate expectations for the contour of the future path of policy. Also, the FOMC communicated its patience in removing policy accommodation by stating that it anticipated economic conditions would warrant exceptionally low levels of the FFR for an extended period. This language remained vague enough to avoid committing to specific values of the policy rate and to avoid forcing market expectations to abide by a particular timeline.

A key motivation of the large-scale asset purchase (LSAP) programs was to lower interest rates paid by households and businesses to support consumption and investment spending. Wright (2011) conducts an event-study analysis to determine how financial markets responded to the FOMC news. After November 2008, estimated monetary policy shocks significantly affected 10-year Treasury yields and long-maturity corporate bond yields but the effects lasted only a few months. Furthermore, policy shocks had a small effect on 2-year Treasury yields. The existence of a relationship between aggregate demand and long-term interest rates would suggest that unconventional monetary policy at the ZLB had a stimulative effect on the economy, albeit rather modest.

Swanson & Williams (2013) measure the effects of the ZLB on interest rates of all maturities by examining the sensitivity of interest rates to announcements about macroeconomic conditions. Over the period from 2008 through 2010, yields on Treasuries with more than one year to maturity were very responsive to macro news, indicative of how monetary and fiscal policy were as effective as in normal times. By late 2011, once the ZLB environment had persisted for an extended period, the sensitivity fell considerably and almost entirely muted the responsiveness of interest rates to macro news. The authors offer the explanation that market participants may have begun to expect the FFR target to be above zero four quarters in the future so medium- and long-term yields ceased to respond to cyclical news. The unconventional policy actions of the Fed may also have helped to offset the effects of news announcements. If the central bank can commit to future values of the policy rate, it can circumvent the ZLB
constraint by promising accommodative monetary policy in the future once the constraint no longer binds.

Bauer (2011) performs another event-study analysis of U.S. policies implemented in the ZLB environment. He constructs a simple three-factor GATSM that accounts for heterogeneity of shocks to interest rates stemming from different types of macro news announcements and policy surprises. Daily interest rate changes are driven mostly by news announcements due to changes in expectations of future policy rates and unexpected changes in risk-premia. Also, news about inflationary pressures can move the long end of the term structure. The event-study of Gagnon et al. (2011a) examines changes in interest rates within one-day windows around official communications regarding asset purchases. The authors find evidence that LSAP’s led to economically meaningful and persistent reductions in longer-term rates. This was likely driven by lower risk premiums and lower expectations of future short rates. Furthermore, Neely (2013) finds that announcements of LSAP’s by the Fed had substantial global effects and reduced international long-term yields and the spot value of the dollar. This further suggests that central banks can enact effective monetary policy even when constrained by the ZLB.
CHAPTER 2 MONETARY POLICY ANALYSIS AFTER THE FINANCIAL CRISIS

2.1 Introduction

Since the onset of the Financial Crisis and Great Recession, monetary policy in the U.S. and around the world has taken unprecedented measures in an effort to stimulate the economy. The Federal Reserve, for example, lowered its primary policy instrument—the federal funds target—essentially to zero.\footnote{Friedman (2010) describes a detailed timeline of the sequence of steps taken by the Fed along with significant market events. Williams (2011) presents a review of the unconventional monetary policy tactics employed to combat the financial crisis.} At that point, this instrument became ineffective due to the nominal bound at zero and the Fed was forced to resort to unconventional monetary policy.\footnote{Unconventional monetary policy used in the U.S. included quantitative easing (QE), large scale asset purchases (LSAPs), and forward guidance. Walsh (2010) discusses the channels through which quantitative easing could stabilize the economy. Wright (2011) analyzes how long-term interest rates respond to LSAP’s in a ZLB environment. Gagnon et al. (2011b) present the mechanisms through which these purchases affect the overall macroeconomy. Campbell et al. (2012) discuss the effects of forward guidance.}

From the standpoint of academics, this period presents an important problem for assessing the effects of monetary policy. Many monetary models use the effective nominal fed funds rate as the primary policy instrument. With the nominal funds rate constrained by the zero lower bound (ZLB) for an extended period, empirical monetary models cannot be estimated as usual.

The empirical literature offers a number of remedies. First, we could treat the ZLB period as special, using either breaks or dummies to represent changes in economic relationships.\footnote{This option could be considered the most extreme as it suggests that the effect of monetary policy is potentially time-varying [see, for example, Aastveit et al. (2014)]. If the effectiveness of policy varies, the Fed must reconsider at each moment the conduct of policy and the appropriate instruments.} Second, we could include alternative policy instruments, such as the size of the balance sheet or dummies representing the implementation of unconventional policies. These two alternatives...
have the disadvantage of increasing the number of estimated parameters for a period that, presumably, represents a short sample. Third, we could replace the FFR as the conventional stance of monetary policy with a proxy that is allowed to violate the ZLB and that captures the effects of both conventional and unconventional policy.

Since the financial crisis, academics have proposed such measures of the accommodation in monetary policy when the short rate is at the zero lower bound. Recently, Krippner (2012) and Wu & Xia (2014) have used the shadow rate methodology to construct alternative measures of the stance of policy. Krippner (2012) builds on Black (1995) and Gorovoi & Linetsky (2004), modeling interest rates as options by calculating the value of a call option to hold cash. The modifications in Krippner (2012) generate closed-form solutions for bond prices and yields. Rather than describing yields in a ZLB environment directly, Wu & Xia (2014) construct an analytical approximation of forward rates in discrete time. This allows for a more straightforward estimation approach than the other shadow rate methodologies and produces closed-form expressions for the shadow forward rates. Both models calculate a shadow short-term interest rate which would be seen in financial markets if the cash option did not exist. In principle, the Fed may have dropped the fed funds rate further if not for the nominal bound at zero. The shadow rate has been considered a proxy for the stance of monetary policy in an environment in which the zero lower bound is binding. From this foundation one can develop a full model of the shadow term structure based upon the shadow short rate depicting the fundamental policy objectives.

Most previous empirical models of the effects of policy were linear. If these shadow short rates are proper measures of the monetary accommodation, the underlying model would, in the best of worlds, still be linear and consistent across sub-periods. In this paper, we compare some of these new measures of monetary policy. We consider whether the standard empirical

\[4\] Black (1995) modified the typical Gaussian affine term structure model (GATSM) to eliminate the occurrence of negative interest rates by introducing the idea of interest rates as options. Nominal interest rates cannot take on negative values in a world in which cash is a viable alternative to standard short-term assets. Since cash essentially pays zero interest, should nominal bonds pay a negative interest rate, investors would have an arbitrage opportunity by issuing bonds at negative interest rates and using the borrowed funds to purchase cash, earning a zero rate of interest. Black (1995) introduced the concept of a shadow instantaneous interest rate which is free to take on negative values while the nominal interest rate is the positive part of the shadow rate due to the always present option to convert an asset to currency.
model of monetary policy can be preserved without breaks by using these measures. That is, we ask whether, during the ZLB period, there exists a linear relationship between the alternative instruments and standard macroeconomic variables so that the standard linear VAR can be preserved.

The question going forward is whether these new alternative shadow short rates are sufficient representations of monetary policy or if the financial crisis requires other modifications to the monetary model. We ask the following questions: (1) How large are the biases in the estimated impulse responses if one uses the FFR for the full sample? (2) Does adding policy dummies and the balance sheet of the Fed mitigate these biases? (3) Does replacing the effective funds rate a shadow short rate mitigate these biases? (4) Which shadow short rate does a better job at mitigating these biases?

We find that the shadow rate acts as a fairly good proxy for monetary policy, if using a dataset that spans the pre-ZLB period throughout the ZLB environment, by producing impulse responses of macro indicators similar to what we’d expect based on the post-WWII, non-ZLB benchmark. However, the linear model exhibits a significant structural break at the onset of the ZLB and the shadow rate may still be insufficient for examining the ZLB period in isolation.

The balance of the paper is organized as follows: Section 2.2 establishes the benchmark specification using standard empirical models to describe Fed policy in a normal, pre-ZLB environment. Section 2.3 examines how we can model some of the actions taken by the Fed at the zero lower bound in these standard models. In this section, we consider whether standard empirical models of monetary policy can be salvaged, either using new measures of policy, allowing for breaks in the effects of policy, and/or accounting for unconventional policy instruments. Finally, Section 2.4 concludes.

2.2 The Benchmark Specification

The short rate—often, an overnight rate—is one of the primary instruments for conducting monetary policy. When adverse shocks are large, monetary accommodation can drive the short rate close to zero. In practice, no-arbitrage conditions prevent nominal rates from falling below
zero since agents can substitute out of bonds into cash. This feature of nominal interest rates can prevent a proper evaluation of the stance of monetary policy when the short rate is at or near zero and other instruments must be relied upon to conduct policy. In this section, we estimate a standard monetary VAR for the pre-crisis period (1960:I-2007:IV) and then naively extend the analysis with data for the financial crisis period.

Before we can determine whether empirical models of monetary policy have changed, we must first establish a baseline. We estimate a quarterly four-variable VAR(4) in output, inflation, commodity prices, and a policy instrument. For the baseline model, we include the effective FFR as the policy instrument:

\[ Y_t = A(L) Y_{t-1} + \varepsilon_t, \]

where \( A(L) \) is a polynomial in the lag operator, \( \varepsilon_t \sim N(0, \Sigma) \), and we suppress the constant and any trends for notational simplicity. The monetary shock is identified by assuming that FFR can react to macro variables but the macro variables cannot contemporaneously react to shocks to the FFR. Partitioning \( A(L) \) into blocks will facilitate exposition: Let \( X_t \) represent the three macro variables of interest and \( R_t \) represent the FFR. Then, we can rewrite the VAR as:

\[
\begin{bmatrix} X_t \\ R_t \end{bmatrix} = \begin{bmatrix} A^X(L) & A^{RX}(L) \\ A^{XR}(L) & A^R(L) \end{bmatrix} \begin{bmatrix} X_{t-1} \\ R_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^X_t \\ \varepsilon^R_t \end{bmatrix},
\]

where \( A^X(L) \) represents the effects of changes in the lagged macro variables on each other, \( A^{RX}(L) \) represents the effects of policy on the macro variables, \( A^{XR}(L) \) represents the feedback from macro variables to policy, and \( A^R(L) \) represents the possible persistence in FFR.

The baseline data sample covers the period from 1960:I to 2007:IV, which starts after the Korean War price control period and ends prior to the financial crisis and generally corresponds with the standard VAR used for monetary analysis prior to the FFR hitting the zero lower

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5Our measure of output is the annualized quarterly difference in the log of GDP taken from the BEA. Inflation is taken as the difference in the log of the CPI taken from the BLS. Commodity prices are the log differences in the Producer Price Index: All Commodities. All data are seasonally adjusted.
The first column of Figure B.1 shows the impulse responses for the VAR(4) outlined above using the effective nominal FFR as the policy instrument and data for the period ending in 2007:IV. The responses shown are to a 25 basis point shock to the effective nominal FFR ordered last in the VAR and identified using the Cholesky decomposition. The responses are computed for each draw of the sampler, generating the posterior coverage. The plots show the median response (black line) as well as the 95-percent posterior coverage intervals (blue shaded regions). The impulse responses are as expected: An increase in the policy rate causes output to fall and inflation to rise in the short run.

We next examine one of the challenges faced by academics posed by the ZLB period. We first estimate a linear VAR for the full sample (1960:I-2013:III) to show how the impulse responses would change if one did not account for the use of alternative monetary instruments. This VAR is what one would obtain by naively extending the sample through the ZLB period without accounting for the use of alternative monetary instruments.

The second column of Figure B.1 shows the impulse responses for the VAR(4) outlined above using the effective nominal FFR as the policy instrument through the ZLB period. The plots show the median responses (dark green line) as well their associated 68-percent posterior coverage intervals (green shaded region) for the naive full-sample VAR extending the data through 2013:III. The thick dark blue lines and blue shaded regions give the impulse response median point estimates and their 68-percent posterior coverage for the benchmark specification in which the data end in 2007:IV. The responses resemble those of the baseline in both quantitative and qualitative terms—a contractionary policy shock results in a decrease in output, a recognizable price puzzle with increasing inflation, and increasing commodity prices.

The nominal funds rate does not move during the ZLB period. The ZLB period does not qualitatively change the resulting impulse responses to a monetary shock but does produce a slight quantitative bias. However, this exercise does not tell us much about the ZLB period.

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6To preserve consistency with results in later sections, the VAR presented here is estimated with Bayesian methods. The prior is a zero mean Normal-inverse Wishart distribution. The posterior distributions are simulated using the Gibbs sampler.
itself since the conventional policy instrument effectively does nothing during this period. While the estimates are slightly biased, we can still effectively describe what happens in normal times but we do not know what happens in the ZLB period.

2.2.1 Testing for Parameter Instability

To model the ZLB period, one could impose a break at the time that the effective nominal funds rate hit the bound.\(^7\) We compare the pre- and full-sample VAR by conducting formal tests to determine the extent to which the model changed during the financial crisis. First, we compare Bayes Factors to determine the likelihood of parameter instability between the pre-ZLB and the crisis/ZLB period. We construct a dummy variable to indicate data from the crisis and post-crisis recovery ZLB period and test for varied responses of macro variables to the policy instrument over the transition to the ZLB environment. In each of the first three VAR equations, we include an interaction between the ZLB dummy variable and all lags of the policy rate. Therefore, the break model allows for a change in the VAR coefficients on the policy rate.\(^8\)

We take twice the log of the Bayes Factor comparing the break model to the no-break model in order to convert the test statistic into a scale comparable to that of the likelihood ratio test statistic. Let \(\pi(Y|M_i)\) be the marginal likelihood of the data, given model \(M_i\), and define model \(M_0\) and \(M_1\) as the no-break and break model models, respectively.\(^9\) Therefore, the Bayes Factor is computed as:

\[
B_{01} = \frac{\pi(Y|M_1)}{\pi(Y|M_0)},
\]

\(^7\)We assume that the onset of any potential parameter instability would take place after 2007:IV. Throughout 2008, the U.S. economy experienced substantial negative shocks, forcing the Fed to make drastic cuts in the FFR towards the ZLB. Once dropping the FFR target as low as possible at the end of 2008, the Fed sought unconventional measures to produce additional stimulus. We seek to measure how the overall macroeconomic dynamics may have changed throughout this transition.

\(^8\)This treatment is equivalent to modeling \(A^{RX}(L)\) of the VAR in Section 2.2 differently in each period.

\(^9\)We compute the marginal likelihood using the output of the Gibbs sampler with the method described in Chib (1995).
and we compute

\[ B_{01} = 2 \ln (B_{01}). \]  

(2.2)

We use the scale suggested in Jeffreys (1961) to interpret the strength of evidence against model \( M_0 \) and in favor of model \( M_1 \), where values of \( B_{01} > 6 \) are considered strong evidence in favor of the break model. Negative values of \( B_{01} \) indicate that the no-break model is preferred.

The first line of Table B.1 shows \( B_{01} \) for the model comparison using the effective FFR as the monetary instrument for the entire post-war sample (1960:I-2013:III) and for the post-Great Moderation sample (1984:I-2013:III). We find strong evidence against the model with constant parameters using each of the two sub-samples, respectively. The results favor the model with parameter instability over the baseline model, thus suggesting some added explanatory value by allowing the parameters to change when the economy encounters the ZLB.

2.3 Monetary Policy at the ZLB

Ideally, we want to be able to account for the effects of the Fed’s unconventional policy action during the times in which the FFR does not fluctuate. Using the FFR alone to represent policy would suggest that the Fed was inactive during the depths of the financial crisis and did little to stimulate the recovery. Therefore, we need a way to incorporate the policy accommodation associated with the balance sheet liquidity programs and the use of forward guidance. In the next section, we augment the VAR with announcement effects and the Fed’s balance sheet to determine whether accounting for alternative policy instruments are sufficient to preserve the dynamic responses suggested by the benchmark VAR model. Finally, we estimate a shadow rate and use this as a proxy for the policy instrument during the ZLB period.

2.3.1 Adding Alternative Monetary Instruments

As we mentioned above, during the ZLB period, the Federal Reserve began to utilize alternative policy measures. These measures were intended to provide temporary injections of liquidity and often targeted yields for longer maturity assets. These policies also represented a substantial increase in the Fed’s balance sheet. One way to model the effects of these unconventional
policies is include them directly in the VAR by using dummy variables.

Augmenting the standard VAR model to account for the use of these instruments is not straightforward. The policies are often thought to have both implementation effects and announcement effects—that is, the stance of policy could be thought to change both at the times the programs were announced and at the times the balance sheet actually changed. One way to add these instruments into the model is to include event dummies for announcements and to include changes in the size of the Fed’s balance sheet. Let $B_t$ represent the difference in the size of the Fed’s balance sheet from $t-1$ to $t$ and let $P_t$ be a dummy variable that indicates the announcement of a future Fed action. Then, the VAR becomes

$$
\begin{bmatrix}
X_t \\
R_t
\end{bmatrix} =
\begin{bmatrix}
A^X (L) & A^{RX} (L) \\
A^{XR} (L) & A^R (L)
\end{bmatrix}
\begin{bmatrix}
X_{t-1} \\
R_{t-1}
\end{bmatrix} +
\begin{bmatrix}
A^{BX} (L) & A^{PX} (L) \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
B_{t-1} \\
P_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_t^X \\
\varepsilon_t^R
\end{bmatrix},
$$

where the zero restrictions impose orthogonality between the unconventional policies and the effective funds rate.

We can then determine whether accounting for alternative policy instruments is sufficient to preserve the structure of the VAR into the ZLB period: Does including $A^{BX} (L)$ and $A^{PX} (L)$ make the VAR consistent across the ZLB period? The second line of Table B.1 shows the results of the Bayes Factor comparing VAR’s with exogenous controls for $B_t$ and $P_t$ in the equations for $X_t$, with and without parameter instability at the ZLB. The evidence against the model with constant parameters is even stronger than when using only the FFR, as in the previous section. The additional $B_t$ and $P_t$ terms come into play primarily around the early stages of the financial crisis and over the period witnessing drastic cuts in the FFR towards the ZLB. Incorporating these additional dynamics emphasizes the variation underlying the structural form of the model and amplifies the importance of allowing for parameter instability. This very strong evidence favoring changing coefficients over constant coefficients suggests that accounting for the unconventional policy via event dummies is not sufficient to maintain

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10 One might even argue that markets anticipated the announcements, either independently or through speeches made by FOMC members, further complicating the identification of these effects.
linearity.

Figure B.1 compares the impulse responses to a shock to the FFR in the benchmark to the VAR that includes $B_t$ and $P_t$. As in the previous analysis, the right-hand column shows the median point estimate and 68-percent posterior coverage intervals for the impulse responses estimated using the full-sample VAR, with $B_t$ and $P_t$. The dark blue lines and blue shaded regions replicate the benchmark results. The median point estimates of the full-sample seem to deviate only slightly from the benchmark, with a more significant change for the response of commodity prices, but the posterior coverage is considerably wider. Introducing additional structure into the model and requiring estimation of the coefficients on $B_t$ and $P_t$ degrade the precision with which the rest of the model parameters are estimated.

Similar to the results above, accounting for the FFR, the balance sheet, and significant policy events produces a sufficient representative of policy for the full post-war period, including a majority of non-ZLB data. However, augmenting the model in this way may be an inefficient approach for future empirical work once the policy environment returns to a normal, non-ZLB environment and these unconventional policies are no longer in use. The Fed has a variety of alternative policy programs in its arsenal but does not need to use them when it can adjust the FFR effectively. Including $B_t$ and $P_t$ introduces more parameters to estimate and more structure in the model, especially if imposing identifying restrictions in terms of their relationships with the other variables in the model. In response to this, we pose the question: can we find a proxy measurement of $R_t$ that captures the stance of policy across all periods? We attempt to answer this question in the remainder of the paper.

2.3.2 Shadow Short Rates

One of the Fed’s stated objectives in conducting unconventional monetary policy was to affect interest rates for longer maturity assets, suggesting that examining the term structure of interest rates could uncover a potential alternative policy instrument. Because the nominal short rate is constrained during the ZLB period, Black (1995) proposed a model with a fictitious shadow bond with the same maturity as the policy instrument and an unconstrained shadow interest rate. The nominal short rate, $R_t$, can then be expressed as the maximum of the
shadow short rate, \( r_t \), and zero:

\[
R_t = \max \{ r_t, 0 \}. \tag{2.3}
\]

When the nominal rate binds at the ZLB, the shadow rate is unconstrained and can fall below zero. Krippner (2012), Wu & Xia (2014), and Christensen & Rudebusch (2013) estimate versions of this shadow rate using financial market data spanning the full term structure. Krippner (2012) modifies the Black (1995) framework of modeling interest rates as options by calculating the value of a call option to hold cash. This methodology includes two latent factors with a series of restrictive normalizations in order to apply the option-pricing framework. Krippner (2012) uses numerical integration to generate closed-form solutions for bond prices and yields within the shadow term structure. Christensen & Rudebusch (2013) apply the option-based pricing approach formalized by Krippner (2012) to estimate the first three-factor shadow rate model using data on Japanese government bond yields. Alternatively, Wu & Xia (2013) construct an analytical approximation of forward rates in discrete time. This allows for a more straightforward estimation approach than the other shadow rate methodologies. The authors include three latent factors and apply the normalization technique introduced by Joslin et al. (2011).

Krippner (2012) and Wu & Xia (2014) argue that the shadow rate can be used to measure the stance of monetary policy when nominal rates hit the ZLB. The shadow rate, however, is a purely financial construct that does not take into account its effects on macro variables. If we are to use the shadow rate in empirical models of monetary policy, we need to know whether standard VAR models can be extended through the ZLB period by replacing the effective FFR with the exogenously constructed shadow rate or if the ZLB period, in and of itself, requires an alternative model.

Following the methodology of Krippner (2012), we construct the shadow rate use data on the full yield curve, out to the 30-year Treasury bond. The dataset spans 1986:IV through 2013:III so, in principle, we can derive a shadow rate for this entire period. However, we will only need the shadow rate values to proxy for the FFR once hitting the ZLB. The details of Krippner’s model and the estimation procedure are described in the appendix. Krippner’s
method generates a monthly shadow rate. However, in order to use this as the policy instrument in the VAR(4) analysis, we aggregate over the quarter by averaging the estimated values for the shadow rate over each quarter. The shadow rate developed in Wu & Xia (2014) is made publicly available on Federal Reserve Bank of Atlanta website. We convert the monthly frequency to a quarterly frequency in the same way as with our version of Krippner’s shadow rate.

Figure B.3 shows a sub-sample from 2006:I-2013:III of the quarterly policy instruments used for estimation in the VAR. Prior to 2009:I, all policy instruments use observed values of the nominal FFR. From 2009:I through the end of the sample, we substitute the two shadow rate measures for the effective fed funds rate in separate VAR’s. By construction, when the nominal FFR is sufficiently far from zero, it and the shadow rate move consistently together. The shadow rate should be equal to the nominal short rate when the nominal short rate is positive and the model preserves this relationship up to a small measurement error. However, once the FFR effectively reaches the ZLB in 2008:IV, the shadow rate becomes increasingly more negative as the Fed took action to jump-start the economy.

Therefore, as a third alternative treatment option for the ZLB period, we estimate the same VAR(4) as above with this new hybrid policy measurement. Lines 3 and 4 of Table 1 show results of the Bayes Factor model comparisons using both the Krippner (2012) and Wu & Xia (2014) shadow rates as the policy instrument, respectively. Similar to the case with using the FFR, the linear VAR using the Wu & Xia (2014) shadow rate still favors the model incorporating parameter instability around 2008:I, after the onset of the financial crisis. There is very strong evidence in favor of the model allowing for a shift in the macro responses over the entire post-war sample. The evidence is still positive, but much less strong for the post-Great Moderation subsample. Conversely, when using the Krippner (2012) shadow rate, the results favor the constant parameter model for both the full post-war or the post-Great Moderation samples. This shadow rate exhibits much richer dynamics over the crisis and ZLB periods, taking on greater negative values. It appears that this variation, in contrast to the FFR or the smoother path of the Wu & Xia (2014) shadow rate, allows the Krippner (2012) shadow rate to provide a better proxy for the policy instrument in the VAR setting.
These results elicit the question: is the shadow rate a sufficient proxy for the stance of policy to preserve the linearity in the monetary models? Figure B.4 shows the impulse responses of the VAR using the shadow rates substituted for the effective funds rate at the ZLB. The first column repeats the benchmark using the FFR from 1960:1-2007:IV from Figure B.1. The center column estimates the VAR with the Krippner (2012) shadow rate as the policy instrument from 2009:I-2013:III and the right column estimates the VAR with the Wu & Xia (2014) shadow rate as the policy instrument. Using either shadow rate to represent policy generates posterior coverages for the responses of all macro variables similar to the benchmark. The median benchmark responses fall within the posterior coverage for the full-sample analyses in all cases except for a small bias on the persistence of the policy shock’s effect on the policy rate itself. Again, the ZLB period itself is very short compared to the entire sample and the lack of significant differences in responses may be due to the stronger influence of pre-ZLB data. Even with the subtle differences noted here, the shadow rates seem to preserve the qualitative (and much of the quantitative) relationships between our macroeconomic indicators and the effects of monetary policy actions.

We are interested in whether the shadow rate provides a sufficient proxy for monetary policy in the ZLB environment, in particular to explain the effects of policy throughout the economic contraction of the recent recession. The objective is to use the shadow rate to represent the significant policy stimulus associated with unconventional policies when the FFR does not deviate from the ZLB. The substantial downward movement of the shadow rate occurs in the early stages of the Great Recession, with increasingly negative shadow rates while the FFR is near zero.

Wu & Xia (2014) treat this period differently than the subsequent ZLB period once the economy is no longer in recession but the FFR is still near zero and unconventional policies are still in use. They argue for using the shadow rate to model policy action only after the recessionary conditions subside. We would like to construct a comprehensive measure of policy even during the recessionary period. The large negative shocks that pushed the economy into recession and drove the FFR towards zero are important for determining the validity of the shadow rate approach to modeling policy in these abnormal environments.
We re-estimate the VAR and include only the period after the onset of the ZLB to look specifically at economic conditions when the shadow rate should provide more information than the FFR alone.\textsuperscript{11} Figure B.5 shows the impulse responses of the benchmark VAR(4) and then using the shadow rates over the period from 2008:I through 2013:III, isolating the period during which we model the ZLB as binding. During this time the nominal FFR hardly fluctuated from zero and thus could not effect much change itself on the overall economy. The shadow rates incorporate other external influences on both current policy as well as market expectations of future policy and future economic conditions. Thus, the responses of macro aggregates to shocks to this alternative policy measurement illustrate more comprehensive policy action during the severe economic contraction.

The impulse responses are estimated with much less precision and the posterior coverage is considerably wider for the VAR shadow rate estimation using only ZLB data than for the benchmark. As a result, the median benchmark responses tend to fall within the considerably wide posterior coverage. The median point estimate of the response of output is comparable when examining the benchmark and using the Wu-Xia shadow rate over the ZLB period. The Krippner shadow rate does not induce a contractionary response until after one period but then moves in a similar fashion to the benchmark. Not surprisingly, the median responses of inflation and commodity prices still fluctuate from the benchmark. The response of inflation implies a price puzzle in the benchmark but neither of the shadow rates replicate this type of response during the ZLB period. Contractionary policy shocks are associated with falling inflation using either shadow rate. As we previously discussed, the shadow rate may incorporate future expectations as it extracts data from interest rates and investment decisions. Finally, while the response of the policy rate to its own shocks dissipates more quickly with the shadow rates than the benchmark, the qualitative nature of the response matches that of the FFR in normal times. \textsuperscript{12}

\textsuperscript{11}Of course, given the data limitations for this period, error bands are expected to be large and results will be only suggestive.

\textsuperscript{12}For robustness, we repeat these impulse response comparisons after adjusting the benchmark to begin after the end of the Great Moderation (1984:I-2007:IV) rather than using the entire post-war sample. We reach the same qualitative conclusions regarding the deviation from the benchmark for full-sample and ZLB sub-sample.
2.3.3 Model Performance

Finally, we test whether there are appreciable differences in the impulse responses to a shock to the shadow short rate estimated over the full sample versus the responses to a shock to the FFR in our benchmark pre-ZLB sample. In order to do this, we construct the Kullback-Leibler Divergence (KLD) between the posterior distribution of the benchmark VAR parameters and the posterior distribution of the VAR parameters estimated using the alternative policy instruments.\(^{13}\) We can think of the KLD as a type of loss function that measures deviations between distributions. We take the benchmark, pre-ZLB posterior parameter distributions as the truth and measure the extent to which the posterior distributions differ from this when using the shadow rate proxies. Ideally, we want to look at the difference in the distributions of the impulse responses themselves. However, since there is a one-to-one mapping between the impulse responses functions and the VAR coefficient and covariance matrices, we can use the output of the Gibbs sampler to analyze the posterior distribution of the parameter estimates directly.

Table B.2 gives the values of the KLD between the post-war, pre-ZLB benchmark (1960:I-2007:IV) and the full-sample (1960:I-2013:III) when using the FFR and each of the two shadow rates. When including both the pre-ZLB and ZLB periods within the sample, the model in which the FFR is used to represent policy the entire time produces the smallest KLD, thus showing the smallest deviation from the benchmark. This is in agreement with our previous results using the full-sample with a majority of non-ZLB data. However, when looking specifically at the ZLB period, the posterior distribution of VAR parameters estimated with the shadow rate of Krippner (2012) has the smallest KLD, thus exhibiting less variation than the distributions using the FFR or the shadow rate of Wu & Xia (2014). The results are the same if we adjust the benchmark to only consider data after the end of the Great Moderation, therefore estimating the VAR using data from 1984:I-2007:IV to establish our basis for comparison.

\(^{13}\)The Kullback-Leibler Distance is a metric to assess the deviation of one distribution from another. See Kullback & Leibler (1951) for more details regarding how to construct this distance.
Again, the KLD produced from the FFR is smaller than those of the other two models for both the full, post-Great Moderation (1984:I-2013:III) sample. Furthermore, the KLD using the Krippner (2012) shadow rate is smallest for the ZLB sub-sample. Removing the influence of pre-ZLB data in the dataset by excluding the first 24 years of data allows for the shadow rate modifications at the ZLB to achieve greater success at merging a comprehensive, continuous representation of policy between these two periods.

We cannot compute the impulse responses of macroeconomic variables to the FFR during the ZLB period alone as the policy instrument did not exhibit meaningful variation over this time. When at the ZLB, the shadow rate of Krippner (2012) more closely recovers some of the benchmark macro dynamics of the pre-ZLB period and these are further preserved if we employ a full dataset over the entire post-war period, including the years at the ZLB. We have also found that controlling for the size of the Fed’s balance sheet and significant policy announcements regarding alternative policy programs allows for recovering our baseline dynamics and provides options to researchers seeking to model economies in which the central bank is constrained by the ZLB. Having these anomalous years of data between extended episodes of normal economic activity does not seem to prohibit the use of standard VAR analyses for the effects of monetary policy.

2.3.4 The Bottom Line

The ZLB period poses an interesting dilemma for empirical researchers. At the outset, we posed a series of questions for the future empirical study of monetary policy, assuming that the FFR is the policy instrument when normalcy returns. First, how large are the biases in the estimated IRF’s if one uses the FFR for the full sample? When estimating the linear VAR model over a long time span, inclusive of a period at the ZLB, we find that it is sufficient to simply use the FFR as the policy instrument. While the results will be slightly biased, the biases appear to be small. Once the economy lifts-off from the ZLB and returns to normal conditions, this bias should be mitigated and the majority of non-ZLB data should dominate the results.

Second, does adding policy dummies and the balance sheet of the Fed mitigate these
biases? Representing the policy instrument with a combination of measurements of the FFR, changes in the size of the Fed’s balance sheet, and indicator variables for significant policy events within the VAR over the full post-war period still produces similar, if not larger, biases to those produced by the model using only the FFR. This exercise requires estimation of additional parameters associated with policies unique to the ZLB environment and thus may introduce inefficiencies when using a dataset consisting of predominantly non-ZLB data.

Third, does exchanging either shadow short rate mitigate these biases? We find that should one wish to substitute a shadow rate proxy for the policy instrument during the ZLB period, hoping to maintain consistent model dynamics throughout the time series, one must recognize that the shadow rate methodology does not achieve a consistent model in all circumstances. The choice over which shadow rate to incorporate will dictate whether or not accounting for structural change is required.

Finally, which shadow short rate does a better job at mitigating these biases? Interestingly, should one attempt to examine particular ZLB periods in isolation, neither the FFR nor the shadow rate serve as an adequate representation of policy and do not produce the expected relationship between effective monetary policy and macroeconomic fluctuations. Therefore, modeling this unique period requires further adjustments and a linear model may not suffice.

2.4 Conclusions

Researchers attempting to measure the effects of monetary policy during the financial crisis and subsequent recession beginning in 2008 have encountered difficulties when trying to use the FFR which essentially flatlines at zero for much of the period under consideration. We have proposed using the shadow rate as a measurement of policy which is able to fluctuate to negative values when the effective central bank policy rate faces a binding constraint at zero. Our results suggest that the shadow rate acts as a good proxy for monetary policy throughout the ZLB environment only if using a dataset that spans the pre-ZLB period throughout the ZLB environment. However, the shadow rate is an insufficient proxy for a comprehensive measurement of monetary policy when examining the ZLB period in isolation.

Examining the FFR alone may suggest that policy has become inactive or ineffective but
the monetary authority has indeed been successful at implementing expansionary policy albeit through alternative mechanisms. An important point to note is that the economy has witnessed a break in the instrument used to enact policy but not a break in the effects of monetary policy on the macroeconomy. Economic researchers use the FFR as a measurement of the policy instrument for the post-WWII era even though the Fed targeted non-borrowed reserves from 1979-1982 and borrowed reserves from late 1982 through the mid-1980’s. It did not stop targeting M1 until 1987 and M2 until 1993 and began announcing formal targets for the FFR only in 1994. Similarly to that change in the behavior of central bankers, the ZLB period beginning in December 2008 has rendered the traditional policy tool impotent for stimulating economic activity. The Fed has successfully utilized balance sheet items as instruments and introduced a much more expansive period of alternative policy measures than the time spent targeting non-borrowed/borrowed reserves. In order to accurately represent monetary policy during this period, we need a surrogate measurement such as the shadow rate.
CHAPTER 3 MONETARY POLICY AND MACRO FACTORS AT THE ZLB

3.1 Introduction

The impaired functioning of financial markets, leading to the financial crisis and recession of 2008 and 2009, triggered substantial policy action in major economies around the world. For the first time since the Great Depression, U.S. interest rates faced the binding constraint of the zero lower bound (ZLB), stifling the efficacy of efforts of the Federal Reserve to stimulate economic activity using its standard policy instrument. This forced the adoption of alternative monetary policy instruments to combat the recession. The Fed broadened its lender-of-last resort policies, implemented multiple rounds of quantitative easing and used carefully selected language as forward guidance in its FOMC statements to steer market expectations.

The FOMC policy rate has been pegged effectively at zero since December 2008. Short-term nominal interest rates have traditionally been considered an indicator of the stance of monetary policy - looser policy coincides with lower interest rates. However, when interest rates face a binding constraint at zero, the true policy stance becomes obscured. Tallman & Zaman (2012) highlight that most Taylor-type rules for monetary policy would have recommended a negative policy rate starting in 2009 through 2012. This suggests that since the nominal FFR could not fall below zero, policymakers faced substantial constraints regarding the deviation of the desired path of the policy rate from that which was feasible. Bernanke & Reinhart (2004) discuss various strategies for stimulating an economy facing the ZLB. In order to mitigate this restriction, the Fed began targeting lower long-term interest rates through purchases of longer-term securities with quantitative easing (QE1 and QE2) and the Maturities Extension Program (Operation Twist).1 Such unconventional policy measures can ease the perceived

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1See Wright (2011), Bauer (2011), Gagnon et al. (2011a), Neely (2013), and Swanson & Williams (2013) for
stance of policy but we then need a different mechanism to describe the overall level of policy accommodation.  

In this paper, I describe the joint dynamics of monetary policy, bond yields, and macroeconomic variables, while explicitly modeling the ZLB. Many researchers work from the assumption that the Fed uses the FFR as its primary instrument to set policy in response to fluctuations of real output and inflation. When the policy rate is constrained near zero, the Fed still must act to provide stimulus upon observing stagnant or contractionary economic activity. One of the Fed’s stated objectives in conducting unconventional monetary policy is to affect interest rates for longer maturity assets. This suggests there could be some explanatory value gleaned from examining the overall term structure to construct a measurement of the stance of policy during the ZLB period. Unfortunately, most models of the term structure do not constrain the short rate to be positive and would have suggested negative short-term rates throughout the crisis. Many researchers have contributed to the literature on term structure models that respect the ZLB constraint. Some contemporary approaches include applying a quadratic term structure model or a model with square-root (CIR) processes.  

More recently, Monfort et al. (2014) use the autoregressive gamma process to establish a strictly positive probability of zero interest rates. Alternatively, Black (1995) established the shadow rate approach and proposed a solution by introducing the idea of interest rates as options, modifying the typical GATSM to eliminate the occurrence of negative interest rates. In doing so, Black considered a hypothetical instantaneous shadow interest rate which is free to take on negative values. As a result, any time the shadow interest rate takes on negative values we censor those at zero and establish the event-study analyses of the effects of FOMC announcements and unconventional policies on financial markets.

Francis et al. (2014) propose controlling for the size of the Fed’s balance sheet and information on alternative policy announcements in a standard monetary policy vector autoregression (VAR) to represent policy action during the ZLB period from 2008:I through 2012:IV. They find that this distorts the impulse responses of key macroeconomic indicators to policy shocks and thus may be an inefficient approach for future empirical work once we return to a normal, non-ZLB environment and these policies are no longer in use.

binding constraint observed in financial markets. Bullard (2012) and Bullard (2013) recommend using the shadow rate to represent the stance of policy and highlight how this approach can shed some light on whether the Fed's alternative policies achieved their desired levels of stimulus. This makes it possible to examine the effects of monetary policy in an environment which solves the ZLB issue.


I build upon the foundation established in Krippner (2012), Krippner (2013b), Christensen & Rudebusch (2013), and Wu & Xia (2014) which use only data on interest rates to extract the shadow rate. Contemporary research on interest rate dynamics recognizes the importance of accounting for the role played by macroeconomic indicators and monetary policy in

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5Francis et al. (2014) try using the Krippner (2012) and Wu & Xia (2014) shadow rates as measurements of policy during the ZLB period associated with the Great Recession. The results suggest that while the shadow rates can be a good proxy for monetary policy within a sample spanning both non-ZLB and ZLB environments, they may be insufficient for preserving the expected responses of macro variables to policy during this time.
Influencing financial markets.\textsuperscript{6} In most industrialized countries, it appears that the monetary authority can move the short end of the yield curve, and yields of all maturities respond to unanticipated changes in the policy rate.\textsuperscript{7} Additionally, longer-term interest rates seem linked to aggregate demand and economic output. Many term structure models adopt latent factors to explain yield curve fluctuations, some making an effort to interpret the factors but rarely in terms of how they compare to macroeconomic factors. Some empirical applications use the vector autoregression (VAR) framework establish a direct link between interest rates and macroeconomic variables and taking this into account would likely add value to studies of the yield curve (see Christiano et al. (1996) and [1999] for a survey). Many of the previous shadow rate applications do not preserve the empirical connections between monetary policy and the macroeconomy.

Bauer & Rudebusch (2013) are the first to include macroeconomic factors within the shadow rate model. The authors apply the basic specification of Black (1995) to replace the affine short rate, censored at zero, with the shadow rate. Bond prices are defined based upon the expected future path of the nominal short rate, which is determined by the expected probabilities of negative future shadow rates. Bauer & Rudebusch (2013) apply the econometric methods of Joslin et al. (2011) to describe fluctuations in the yield data using linear combinations of observed interest rates and data on macro aggregates. Their purpose is not to describe the stance of monetary policy as they are of the belief that the shadow rate does not give an accurate depiction of the current policy stance.\textsuperscript{8} Rather, they use the model to get an idea of how restrictive is the ZLB and when markets anticipate a lift-off from the ZLB.

In this paper, the goal is to generate a unified representation of the stance of policy during the ZLB episode, incorporating the influences of unconventional policy instruments. Throughout the ZLB environment, we observe fluctuations in macro aggregates and thus use the nature

\textsuperscript{6}See Piazzesi (2001), Ang & Piazzesi (2003), Ang et al. (2007), and De Pooter et al. (2010) for various applications describing the dynamics of the macroeconomy within the affine term structure methodology.

\textsuperscript{7}See Fleming & Remolona (1999), Gurkaynak et al. (2005), and Staker (2009).

\textsuperscript{8}Bernanke et al. (2004) caution against directly interpreting short-term interest rates as representative of the current stance of monetary policy. A given policy rate may be associated with many different configurations of asset prices and returns.
of their behavior, in addition to the financial data, to get a more comprehensive picture of the true stance of policy. Analyzing the path of the shadow rate allows for tracing out fluctuations in the short end of the yield curve corresponding to unconventional policy stimulus. This provides some insight into whether these alternative programs continued to ease the stance of policy once the FFR hit the ZLB.

I apply the general framework introduced in Bauer & Rudebusch (2013), but I incorporate the macro data in two different ways. First, like the Macro-Finance model of Bauer & Rudebusch (2013), I use the macro data as explanatory factors to directly influence the path of the shadow rate. I adopt a specification for the policy rate in which the central bank reacts to the current state of the economy and incorporate a latent factor to capture unobserved shocks to policy. By describing the shadow rate using a simple interest rate reaction function, I establish an easily interpretable relationship between observable macro and latent factors and the policy rate. Policymakers can thus use the model-implied path of the shadow rate as a metric to test the effectiveness of unconventional policies once the FFR hits zero. The empirical exercise here differs from that of Bauer & Rudebusch (2013) in that I consider fluctuations in the shadow rate to be representative of fluctuations in the stance of policy and examine the systematic and unsystematic components underlying its dynamics. Secondly, my next innovation involves shutting down one direction of the endogeneity between macro factors and the shadow rate where the macroeconomy simply responds to the policy rate and there is no direct feedback mechanism from macro fluctuations into policy. Here, I build a shadow short-rate-augmented VAR in which the macro aggregates can respond to policy fluctuations, but do not feed back into the term structure. The macro dynamics provide additional information to more accurately extract the path of the shadow rate. Finally, I perform a similar exercise to the other existing shadow rate methodologies and estimate the model excluding the macro data. I use Bayesian Markov-Chain Monte Carlo estimation techniques to obtain the model parameters.

I find that the estimated path of the shadow rate suggests that the Fed’s unconventional instruments achieved accommodative policy effects, despite the FFR being near zero for such an extended time. Models of the term structure that only use data on interest rates, excluding macro data, suggest a shadow short rate that stays much closer to the ZLB. Incorporating
macro data through a factor-augmented monetary VAR framework instead allows for predicting fluctuations in medium- and longer-term yields with greater accuracy. Furthermore, using reaction functions to utilize the macro data in constructing the shadow rate generates a proxy for the stance of policy that suggests the unconventional policy programs achieved a substantial accommodation, in excess of that prescribed by a standard policy rule.

Finally, I assess how market expectations of future policy rates changed with the introduction of various unconventional instruments. I look at how this expected duration changed throughout the financial crisis and through the first quarter of 2014. Incorporating macroeconomic data into the model of the yield curve produces expectations of the future short rate being constrained by the ZLB into 2015 if the macro data simply respond to the shadow rate, or into 2016 if macro factors influence yield curve dynamics. These expectations are in agreement with the Fed’s policy commitments suggested by the use of forward guidance in FOMC statements.

The remainder of this paper is laid out as follows. Section 3.2 outlines the empirical model. Section 3.3 describes the data and Section 3.4 describes the estimation methods. Section 3.5 presents the results of the estimated shadow rates. Section 3.6 analyzes the systematic and shock components of the policy rule underlying the Macro-Finance shadow rate. Section 3.7 describes simulation exercises to assess future expectations of the stance of policy. Section 3.8 concludes.

### 3.2 Empirical Methodology

Standard macro models for the post-war period in the U.S. usually include the effective FFR as the policy instrument. Take, for example, the monetary vector autoregression (VAR). Let $Z_t$ represent the macro variables of interest, $R_t$ represent the FFR, and define $Y_t = [Z_t, R_t]^T$:

$$Y_{t+1} = A(L) Y_t + \varepsilon_{t+1}, \quad (3.1)$$

See Stock & Watson (2005) and Bernanke et al. (2005) for a thorough discussion of the factor augmented VAR (FAVAR) methodology.
where $A(L)$ is a polynomial in the lag operator and $\varepsilon_t \sim N(0, \Sigma)$. The monetary shock is identified by assuming that FFR can react to shocks to macro variables but the macro variables cannot contemporaneously react to shocks to the FFR.

At the ZLB, the shadow short rate represents the effective policy rate when all we observe is a censored version of the nominal rate. The nominal short rate $R_t$ can be expressed as the maximum of the shadow short rate and zero:

$$R_t = \max \{r_t, 0\}.$$  \hfill (3.2)

Based upon this approach, the shadow term structure describes the dynamics of interest rates and bond prices in a realistic way, accounting for the impossibility of negative returns on nominal bonds of all maturities. The general multi-factor GATSM specification describes the shadow short rate as a linear function of an $N$-dimensional state vector [Dai & Singleton (2000); Duffie & Kan (1996)]:

$$r_t = a_0 + b_0^t x_t,$$  \hfill (3.3)

where $a_0$ is a constant, $b_0$ an $N \times 1$ vector of coefficients, and $x_t$ is the $N \times 1$ vector of latent and/or observable state variables driving the dynamics of the yield curve.\(^\text{10}\)

### 3.2.1 Central Bank Policy Rate and Observable Macroeconomic Indicators

Krippner (2012) and Christensen & Rudebusch (2013) construct the shadow rate by extracting information from financial market data only. All elements of $x_t$ are unobserved factors describing fluctuations in the term structure.\(^\text{11}\) Wu & Xia (2014) also construct the shadow rate using only data on interest rates and then include this in a FAVAR with other macro

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\(^{10}\)Piazzesi (2010) reviews the literature on affine term structure methodologies.

\(^{11}\)Their methods produce closed-form expressions for bond prices and yields in the ZLB environment but are not easy to apply with observable macro factors. Incorporating macro data as state variables requires allowing the states to depend upon not only their own first lags, but also the lags of the other states. This VAR interdependence is not possible in the construction of shadow option pricing as described by Chen (1995) and implemented in Krippner (2012).
variables to represent policy effects. That exercise builds $r_t$ as completely exogenous to the macro framework and then inserts this series into the VAR, assuming it is a comprehensive representation of the policy instrument. The shadow rate model alone produces a time series for $r_t$ where what we observe, $R_t$, is censored at zero should $r_t$ take on negative values. This $r_t$ describes financial conditions but omits the the effects of monetary policy and the feedback between macro dynamics and policy behavior. I incorporate data on real activity and inflation to establish a direct relationship between the macroeconomy, monetary policy, and financial markets. Therefore, the dynamics of $r_t$ are not exogenous to the macro model.

This endogeneity between the shadow rate mechanism and the macro model can be seen by first illustrating the state-space system, within which one can construct the unobserved shadow rate based on observable macro fluctuations. The state transition equation describes the evolution of the $N$-dimensional state vector $x_t$:

$$x_t = \mu + \rho(L)x_{t-1} + \epsilon_t, \quad (3.4)$$

where $\rho(L)$ is a lag polynomial of order $p$ and $\epsilon_t \sim N(0, \Sigma)$. One can partition the observable dynamics into blocks: one describing the relationship between macroeconomic indicators and the policy stance in a standard monetary VAR framework, and the second allowing the term structure dynamics to feed into measures of inflation and real activity through the short-term policy rate, $r_t$, which is a function of $x_t$ according to equation (3.3). It will facilitate exposition to partition the VAR expression from (3.1) above into these blocks. One can rewrite this as a FAVAR:

$$
\begin{bmatrix}
Z_{t+1} \\
r_{t+1}
\end{bmatrix} =
\begin{bmatrix}
A^Z(L) & A^{rZ}(L) \\
A^{Zr}(L) & A^r(L)
\end{bmatrix}
\begin{bmatrix}
Z_t \\
r_t
\end{bmatrix}
+ 
\begin{bmatrix}
Z_{t+1} \\
r_{t+1}
\end{bmatrix},
$$

where $A^Z(L)$ represents the effects of changes in the lagged macro variables on each other, $A^{rZ}(L)$ represents the effects of policy on the macro variables, $A^{Zr}(L)$ represents the feedback from macro variables to policy, and $A^r(L)$ represents the possible persistence in policy.

\footnote{Francis et al. (2014) perform a similar exercise using the shadow rate of Krippner (2012) in a FAVAR.}
The third block of the observable dynamics includes the term structure relationships between observed nominal interest rates, explicitly censored at zero, and the vector of factors $x_t$. See Section A.1 in the Technical Appendix for a detailed discussion of the GATSM model. I account for the ZLB constraint on nominal interest rates using the methodology discussed in Bauer & Rudebusch (2013) in which expectations of future nominal short rates are based upon the probability that the shadow rate will stay above zero. Assuming that the shadow rate follows a normal distribution, the observed short rate follows a truncated distribution, bounded below at zero. Bond prices, and their corresponding interest rates, along the term structure are based upon expectation of future short rates. Section A.2 of the Technical Appendix discusses the details of the Bauer & Rudebusch (2013) shadow rate methodology in detail. In constructing these expectations, one must impose the lower bound by accounting for the probability that the shadow rate may be negative:

$$E(R_{t+h} | X_t) = P(r_{t+h} > 0) \times E(r_{t+h} | X_t, r_{t+h} > 0) + P(r_{t+h} < 0) \times 0,$$

where the final term on the right-hand side illustrates how when the shadow rate is negative, we see the observed nominal rate censored at zero.

Each month, indexed by $t$, measurements of observable interest rates $R_t$ become available for $K$ different times to maturity. The nominal rates observed in financial markets are related to the state vector through the following equation, where $\widehat{R}_t$ is the estimate of equation (A.22) in the appendix:

$$ \begin{bmatrix} R_{t1}^1 \\ \vdots \\ R_{tK}^K \end{bmatrix} = \begin{bmatrix} \widehat{R}_{t1}^1 \\ \vdots \\ \widehat{R}_{tK}^K \end{bmatrix} + \begin{bmatrix} v_{t1}^1 \\ \vdots \\ v_{tK}^K \end{bmatrix} $$

$$R_t = \widehat{R}_t + v_t$$

(3.7)

I assume that all nominal rates are predicted with error and that the innovations $v_{tj}^j$ for $j = 1, \ldots, K$ are normally distributed with mean zero and diagonal covariance matrix: $\Sigma_R$. Absence of cross-correlations between the innovations to interest rates of different maturities
allows for the model to capture any systematic relationship in movements along the term structure and any measurement errors outside of that are independent from one another across different τ and across time.

Combining these blocks, the complete measurement equation becomes:

\[
\begin{bmatrix}
Z_{t+1} \\
r_{t+1} \\
R_t
\end{bmatrix} = \tilde{A} \cdot \begin{bmatrix}
Z_t \\
r_t \\
\tilde{R}_t
\end{bmatrix} + \begin{bmatrix}
u_{t+1}^Z \\
u_{t+1}^r \\
v_t
\end{bmatrix},
\]

(3.8)

where \(\tilde{A}\) collects the appropriate coefficient matrices.

The standard approach to estimating such a state-space system is to apply Kalman filtering techniques with a prediction and updating step where the estimate of the unobservable states is updated by observable information that occurs within the state-space. This simple exercise incorporates additional information on observable macro dynamics when constructing the model-implied path of the shadow rate, and provides a solid foundation for interpreting the shadow rate as the stance of policy.

I estimate three different versions of this model, differentiated by the extent to which I allow the macro data to influence the construction of \(r_t\). First, I incorporate macro fluctuations as factors which directly influence the shadow short-term rate. In this model, \(x_t\) includes both observable macro factors and a latent factor. I refer to this as the "Macro-Finance Model". Secondly, I allow the macro variables to be functions of their own lags, lags of the other macro variable, as well as the lagged shadow policy rate. The shadow short rate, however, is defined analogously to previous exercises as only a function of latent factors. This amounts to restricting the coefficient matrix \(A^{Zr}(L)\) to all zeros. I refer to this as the "Macro-Monetary Model". Finally, for comparison to earlier shadow rate applications, I exclude the macro data completely and estimate the model using only data on interest rates. Here, I exclude the \(Z_t\) and \(r_t\) blocks from the observation equation. I refer to this as the "Finance Model". 

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3.2.2 Macro-Finance Model

The Fed sets monetary policy in response to the current state of the economy and makes adjustments according to changes in inflation and real activity. Since the end of 2008, when the Fed embarked on the initial series of large-scale asset purchases (QE1), FOMC statements have explicitly recognized the second pillar of the dual mandate in terms of a maximum employment objective, abandoning the equivocal reference to economic growth. While pointedly monitoring inflationary pressures, the FOMC has also emphasized the importance of labor market conditions in assessing overall economic slack and the necessity for policy accommodation. As a result, future policy action has been tied to economic conditions, specifically related to the level of the unemployment rate. Like Ang & Piazzesi (2003), I incorporate macroeconomic fluctuations through data on the unemployment gap and inflation as factors which directly influence the shadow short-term rate and in turn explain movements in the yield curve. In addition to this, I allow the shadow policy rate to depend on a latent state variable \( f \) capturing transient movements of the yield curve not explained by observable macroeconomic factors. One can interpret this latent factor as an unobservable shock to monetary policy. This will affect yield spreads and portray how business cycle fluctuations affect investors’ attitudes about pricing bonds as well as gauge the tightness of monetary policy. The short-term interest rate is a function of both observable macro data and this unobservable factor. Letting \( x_t = [y_t, \pi_t, f_t] \), \( r_t \) is defined as:

\[
    r_t = a_0 + b_y y_t + b_\pi \pi_t + b_x f_t. \tag{3.9}
\]

I normalize \( b_x = 1 \) since the latent factor can be arbitrarily scaled. Therefore, one can rewrite the policy rule as:

\[
    r_t = a_0 + b_y y_t + b_\pi \pi_t + f_t. \tag{3.10}
\]

I describe \( r_t \) in this way in order to achieve greater flexibility in how interest rates respond to the factors in the model and introduce richer dynamics in the relationship between macro
indicators and financial market fluctuations.

I include the macro data directly in the factor transition equation and describe the joint dynamics of $y_t$, $\pi_t$, and $f_t$ over time by allowing for correlation in the innovations to each macro data series and the latent factor. Using monthly data, I include 12 lags of the macro variables and only one lag of the latent factor for parsimony:

$$
\begin{bmatrix}
y_t \\
\pi_t \\
f_t \\
x_t
\end{bmatrix} = 
\begin{bmatrix}
\mu_y \\
\mu_\pi \\
\mu_f \\
\mu
\end{bmatrix} + 
\rho_1 
\begin{bmatrix}
y_{t-1} \\
\pi_{t-1} \\
f_{t-1}
\end{bmatrix} + 
\rho_2 
\begin{bmatrix}
y_{t-2} \\
\pi_{t-2}
\end{bmatrix} + \cdots + 
\rho_{12} 
\begin{bmatrix}
y_{t-12} \\
\pi_{t-12}
\end{bmatrix} + 
\begin{bmatrix}
e^y_t \\
e^\pi_t \\
e^f_t
\end{bmatrix}
$$

The error terms can possibly be correlated:

$$
e_t = 
\begin{bmatrix}
e^y_t \\
e^\pi_t \\
e^f_t
\end{bmatrix} \sim N(0, \Sigma).
$$

Since the macro data are included directly in the transition equation, the measurement equation includes only the term structure relationship between $R_t$ and $x_t$, defined in Section A.2 of the Technical Appendix:

$$R_t = \tilde{R}_t + v_t.
$$

### 3.2.3 Macro-Monetary Model

In the Macro-Monetary VAR model, I restrict the observable dynamics to describe the shadow rate as only a function of latent factors, prohibiting the macro factors from directly feeding back into the state-transition equation. Instead, the term structure dynamics feed into measures of inflation and real activity through the short-term policy rate. I construct the shadow short rate to represent the true stance of monetary policy and allow this to propagate through the VAR. When modeling the joint dynamics of $y_t$ and $\pi_t$ over time, I allow each macro variable to be a function of it’s own lags, lags of the other macro variable, as well as the lagged shadow
Following traditional monetary policy applications, the policy rate does not have a contemporaneous effect on inflation and real output. The shadow short rate, serving as a proxy for the true stance of monetary policy, is the sum of three latent yield curve factors and the long-run mean short rate level, $a_0$:

$$r_t = a_0 + f_{1t} + f_{2t} + f_{3t}.$$  \hspace{1cm} (3.14)

Here, let $x_t = [f_{1t}, f_{2t}, f_{3t}]'$. The three factor structure is often used to construct a parsimonious model while still accounting for varied effects on different components of the yield curve. This state vector evolves according to a similar transition equation as that described by (3.11), except that I only include one lag of $x_t$ ($p = 1$). For identification, I adopt the canonical form of Dai & Singleton (2002) and assume that $\rho$ is a lower-diagonal coefficient matrix, $\Sigma$ is diagonal, and each factor is mean zero under the physical measure, $\mu = [0, 0, 0]'$:

$$x_t = \mu + \rho x_{t-1} + e_t, \ e_t \sim N(0, \Sigma).$$  \hspace{1cm} (3.15)

Define the vector of observed macro variables as $Z_t = [y_t, \pi_t]'$. Macroeconomic fluctuations are modeled by the following system:

$$\begin{bmatrix} Z_{t+1} \\ r_{t+1} \end{bmatrix} = \begin{bmatrix} A^Z (L) & A^Z r (L) \\ 0 & A^r (L) \end{bmatrix} \begin{bmatrix} Z_t \\ r_t \end{bmatrix} + \Sigma^M \begin{bmatrix} u^Z_{t+1} \\ u^r_{t+1} \end{bmatrix},$$  \hspace{1cm} (3.16)

where $u^Z_t = [u^y_t, u^\pi_t]'$, $u^y$ and $u^\pi$ are i.i.d. $N(0, 1)$, and $\Sigma^M$ is lower triangular. I include $r_t$ in the VAR as it influences the macro variables in the subsequent period, but the macro variables do not influence $r_t$, $A^{Zr} (L) = 0$. Since the vector $x_t$ is a function of its first lag, and $r_t$ is simply a linear function of $x_t$, $r_t$ can be described as a function of its own first lag. Therefore, the term $u^r_{t+1}$ incorporates the innovations from the factor transition equation, $e_t$, and $A^r (L)$ picks up the autoregressive nature of $x_t$. The coefficient matrices $A^Z (L)$ and $A^{Zr} (L)$ are lag polynomials with 12 lags of each macroeconomic indicator and/or policy rate. In this way, real activity and inflation respond to lagged fluctuations in the term structure through their
responses to the perceived policy rate. Allowing feedback from the latent financial factors into
the macroeconomy makes this a factor-augmented VAR (FAVAR) that can be estimated within
the same state-space as the term structure relationships. For these purposes, I am interested
in the response of the macro variables to the shadow rate, rather than to each individual
factor itself. Adding this dimension introduces feedback from the macroeconomy, through the
measurement errors, when extracting the shadow rate from financial data. In this way, the
model incorporates how overall macro dynamics respond to the stance of policy. Even when
the observed nominal FFR is constrained at zero, fluctuations in the shadow rate can explain
fluctuations in macro dynamics resulting from the various stimulus programs introduced by
the Fed throughout the Great Recession.

In addition to (3.16), the observation equation of the Macro-Monetary VAR model includes
$R_t$. Collecting the appropriate coefficient matrices into $\tilde{A}$, the observation equation takes the
following form:

$$
\begin{bmatrix}
Z_{t+1} \\
r_{t+1} \\
\mathcal{R}_t
\end{bmatrix} = \begin{bmatrix}
A^Z (L) & A^r Z (L) & 0 \\
0 & A^r (L) & 0 \\
0 & 0 & I_K
\end{bmatrix} \begin{bmatrix}
Z_t \\
r_t \\
\tilde{\mathcal{R}}_t
\end{bmatrix} + \begin{bmatrix}
u^Z_{t+1} \\
u^r_{t+1} \\
v_t
\end{bmatrix},
$$

(3.17)

$$
= \tilde{A} \begin{bmatrix}
Z_t \\
r_t \\
\tilde{\mathcal{R}}_t
\end{bmatrix} + \begin{bmatrix}
u^Z_{t+1} \\
u^r_{t+1} \\
v_t
\end{bmatrix}. 
$$

(3.18)

### 3.2.4 Finance Model

Finally, I also construct the shadow rate using only financial data and include three latent
factors. The shadow rate is described by equation (3.14) and the latent states evolve according
to equation (3.15). The difference between this and the Macro-Monetary Model is that the
observation equation now only includes the term structure relationship between $\mathcal{R}_t$ and $x_t = [f_{1t}, f_{2t}, f_{3t}]'$, defined in Section A.2 of the Technical Appendix.
3.2.5 State-Space Representation

The nonlinear state space of the shadow rate model consists of a discrete time estimation problem in which the state variables evolve according to the system transition equations described by the previous three sections. Depending on the model specification, the observation equations place certain restrictions on the components of the coefficient matrix $\tilde{A}$. The remaining details required for application of the Kalman Filter with all three models are discussed in Section A.1.1 of the Technical Appendix.

3.3 Data

The interest rate data includes end-of-month observations on 3- and 6-month U.S. Treasury bill rates (from the FRED database on the St. Louis Reserve website) and 1-, 2-, 3-, 4-, and 5-year smoothed zero-coupon U.S. Treasury bond rates as developed in the data set used for Gurkaynak et al. (2006) (from the Federal Reserve Board Research Data website). The data span the period from 01/1985-03/2014. I start the dataset in 1985 in order to ensure the data come from a period of relatively stable monetary policy. In order to interpret the short-term interest rate as the main policy tool, it is appropriate to exclude the period in the late 1970’s and early 1980’s during which the Fed targeted non-borrowed reserves. I utilize a two-step estimation procedure. For the first step, I estimate the model parameters using the data from 01/1985-12/2007, before the depth of the financial crisis and the onset of the ZLB environment. Then using these parameter values, I filter the states using the extended Kalman filter (EKF) through the nonlinear state space.

Figure B.6 plots the monthly time series of macro data from 01/1985-03/2014. To measure real economic activity, I construct the unemployment gap by taking the difference between the unemployment rate reported by the Bureau of Labor Statistics and the Congressional Budget Office estimate of the natural rate of unemployment. Negative values correspond to a contemporaneous unemployment rate larger than the natural rate and thus represent contractionary economic conditions. To measure inflation, I calculate the annual log difference in the consumer price index for all urban consumers, excluding food and energy (core CPI):
log (yt) − log (yt−12). All price data are obtained from the St. Louis FRED database. Like Bauer & Rudebusch (2013), I use measurements of inflation and the unemployment gap as they have been found to be closely linked to the Fed’s target for the FFR.13 The observations cover the same time period as the interest rate data. I convert the interest rate data into yields per month and therefore divide the macro data by 12 in order to represent all data in monthly terms. Finally, I demean the macro data and impose that the monetary policy shock is mean zero in order to omit the intercept term from the state transition equation: μ is a vector of zeros.

3.4 Estimation

As mentioned previously, I adopt a two-step method by first estimating the model parameters from the standard linear state space model under conditions where the ZLB doesn’t bind (01/1985-12/2007, prior to the financial crisis and Great Recession). I apply an iterative Bayesian Markov Chain Monte Carlo (MCMC) algorithm to filter the state vector, conditional on the model parameters, for the period in which all nominal interest rates are observed.14 Then conditioning on the filtered state values, I estimate the model parameters with Gibbs sampling or Metropolis-Hastings steps, depending on if the model generates a closed-form posterior distribution [Carter & Kohn (1994); Casella & George (1992)]. I repeat this procedure until convergence and consider the parameter values generated from this algorithm to be appropriate for empirical analysis of the remaining ZLB period. I set diffuse prior hyper-parameters and initialize the Gibbs sampling paths based on OLS estimates of macro dynamics with the observed FFR as the short-term interest rate. To include macroeconomic dynamics in the measurement equation of the Macro-Monetary model, I estimate a Bayesian VAR of the unemployment gap and inflation which both respond to lagged fluctuations in the shadow rate. See Appendix Section A.3 for a description of the MCMC application and Table B.3 for details on the prior parameterization.

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13 See Rudebusch (2009).

14 Sanford & Martin (2005), Blais (2009), Bauer (2012), Ang et al. (2007), and Chib & Ergashev (2008) also adopt Bayesian methods to estimate dynamic term structure models.
The second step involves making the assumption that the model parameters do not change once nominal rates hit the ZLB. When this occurs, the shadow rate may fall below zero but the fundamental model dynamics are preserved. Krippner (2012) and Bauer & Rudebusch (2013) also take this approach, and estimate the affine model prior to the onset of the ZLB environment. In the appendix, Bauer & Rudebusch (2013) justify using the affine model parameters for the shadow rate model by comparing model-implied yields over the pre-2008 period within each model framework. Using the same set of parameter values, the root mean squared errors from the affine and shadow rate models are very similar and thus the cross-sectional fit is almost identical. This suggests that parameter instability in the fundamental term structure relationships does not seem to be an issue. Under this assumption, I estimate the latent state vector using the EKF to propagate the states forward through the ZLB period. See Section A.4 in the Appendix for a brief overview of the steps involved in the EKF.

3.5 Shadow Rate Estimation Results

3.5.1 Macro-Finance Model

Figure B.7 shows the effective FFR and all three filtered shadow rates from 01/2006-03/2014, beginning prior to the financial crisis and capturing the descent towards the ZLB. The red line traces out the model-implied time series for the Macro-Finance shadow rate. For the earlier period in which the ZLB did not present a binding constraint, the path of the shadow rate followed that of the FFR and differences from the effective policy rate are due to measurement error resulting from the filtering application. The shadow rate crossed zero between 10/2008 and 11/2008, around the same time that the FFR hit the ZLB. Right around the announcement of QE2 in 11/2010, the shadow rate reached a local minimum and spiked afterwards. The reaction function for the short rate suggests the interpretation that the announcement of QE2 had expansionary effects by stimulating inflation and real activity, thus increasing the short rate. While still significantly below zero, the shadow rate declines at a lesser rate throughout the implementation of MEP (09/2011) and QE3 (09/2012) and does not fall as steeply as during the previous period. The shadow rate reached its lowest value of −8.81% in 07/2012,
leading up to the start of QE3 and prior to the extension of the Fed’s forward guidance in the FOMC statement later that year (12/2012) in which the committee promised to keep interest rates low, and near zero, until economic conditions improve. After QE3, the shadow rate steadily increased through the end of the sample. As a result of the construction of the policy reaction function, this is representative of improved economic conditions in response to extensive policy stimulus. By the end of the sample in 07/2013, the shadow rate remained negative at −2.99%. This suggests the short end of the yield curve was still substantially constrained by the ZLB.

3.5.2 Macro-Monetary Model

The blue line of Figure B.7 plots the filtered time series for the Macro-Monetary shadow rate. This shadow rate did not fall completely to zero when the FFR hit the ZLB. Rather, the shadow rate did not cross the zero threshold until 11/2009 and stayed between −0.31% and 0.75% throughout the financial crisis and the Great Recession, rarely dipping below zero. This filtering exercise finds that the shadow rate was only slightly negative by the end of the sample in 03/2014 at −0.11%.

Inference from this model suggests that real activity and inflation react to a policy measurement which more resembles the observed stance of policy, as represented by the nominal FFR. The effective FFR obviously remained positive throughout the entire period, facing the barrier at zero. Filtering the shadow rate through the monetary VAR in the measurement equation implies a near zero, yet still mostly positive value for the policy rate propagating through the VAR mechanism. This may help explain why the empirical exercises of Francis et al. (2014) find that using the observed FFR as the measurement of policy, even including the ZLB environment, still produces results close to the expected impulse responses of overall economic indicators in the baseline period.

Bauer & Rudebusch (2013), Christensen & Rudebusch (2013), and Krippner (2013b) emphasize how shadow short rates are often sensitive to the characteristics of the model used to construct them, as well as the data and estimation technique. Furthermore, these model-implied shadow rates are not the effective rates actually faced by agents in the economy making
investment and consumption decisions. For example, investors are not compensated for borrowing in the case of a negative shadow rate. Macro aggregates respond to the observed policy rate in that they cannot react to a perceived negative short rate. This would explain why the Macro-Monetary shadow rate remained mostly positive, but very close to zero, throughout the period in which the Fed exercised zero interest rate policy. As a result, this specific model does not require the added flexibility of possibly negative short-term rates but rarely violates the ZLB without necessarily imposing it as a constraint.

3.5.3 Finance Model

The orange line in Figure B.7 shows the estimated time series for the Finance shadow rate from 01/2006-03/2014. In the pre-ZLB environment, the model achieves considerably close fit to the true data. Around the same time when the Fed dropped the target for the FFR essentially to zero in 12/2008, the shadow rate dropped down near zero and fluctuated between 0% and 1.10% until 11/2010 when it fell below zero. The shadow rate reached its lowest value of −1.82% at the end of the sample in 03/2014, one-and-a-half years after the implementation of QE3. The 03/19/2014 FOMC statement\(^{15}\) expressed the Fed’s commitment to maintain its highly accommodative stance of policy, with low interest rates, and to continuously assess the progress "towards its objectives of maximum employment and 2 percent inflation," thus avoiding any date-based guidance or tying policy action to a specific threshold for unemployment.

The shadow rate appears highly sensitive to the characteristics of the given model used to construct it. One can compare the three-factor Finance shadow rate with estimates from Wu & Xia (2014) using the normalizations described by Joslin et al. (2011) to construct an alternative three-factor shadow rate model also using only yield curve data. Whereas I estimate the model using Bayesian methods, Wu & Xia (2014) use the maximum likelihood technique with robust standard errors. The authors splice together the effective FFR time series through 12/2008 with the estimated shadow from 01/2009 onwards and define this latter period as the that during which the ZLB binds and the shadow rate methodology becomes relevant. I

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\(^{15}\)See FOMC (2014).
do the same with the Finance shadow rate produced here in order to construct comparable series for the policy rate. Figure B.8 plots the effective FFR and the Wu & Xia (2014) and the Finance policy rate series from 01/2006 through 03/2014. The model restrictions and estimation method differ but these three-factor models using only financial data produce qualitatively similar shadow rate paths over the ZLB period, decreasing and increasing over virtually the same periods, but the Finance shadow rate shows a level shift above the Wu-Xia rate. These two models have comparable implications regarding the extent to which the ZLB represented a significant constraint from 2008 through 2014.

3.5.4 In-Sample Yield Curve Forecasting

To evaluate the effectiveness in improving model estimates by incorporating the ZLB into the GATSM, one can assess how well the model captures the cross-sectional behavior of observed yields along the length of the term structure. Bauer & Rudebusch (2013) suggest the idea that a sufficiently flexible shadow rate model should be able to fit the cross-section of longer-term yields without pushing the short-term interest rate far below zero, and far below the values observed in financial markets. Models that do not fit the data well may then compensate for the discrepancy by pushing the shadow rate far into negative territory in order to come closer to matching the medium- and long-term rates. Table B.4 shows the root mean squared-errors (RMSE’s) for all seven yields in the dataset and the spread between the 5-year Treasury bond and the 3-month T-bill, as implied by each of the three models. For comparison, I compute the RMSE’s between the observed data and model predictions over the pre-ZLB period when the standard GATSM would apply (01/1985-12/2008), and over the ZLB period when the modifications of the shadow rate model are required (01/2009-03/2014: the end of the sample). I construct model-implied interest rates using the latent factor(s) filtered using the EKF and the observed macro factors to compute: $\hat{R}_t = -\frac{1}{2} \log \left( \hat{P}_t^p \right)$, where $\hat{P}_t^p$ is described by equation (4.25) in the Technical Appendix. The RMSE values are on annual yields and are expressed in percentage points.

As the top panel of Table B.4 shows, during the pre-ZLB period the Finance model produces the smallest RMSE’s for the shortest maturity yield in the model. Therefore, the term
structure dynamics at the short-end are best explained by looking at a panel of financial data. However, the Macro-Monetary model improves estimation for medium-term yields and produces a smaller RMSE for the 6-month T-bill and the 1-, 2-, and 3-year T-bonds. For the longest maturities and the term spread, the Finance model regains its superior performance. Including the macro data in a reactionary way allows for more accurately modeling interest rates over the time span during which lagged policy effects may surface.

For the 3-month through 4-year yields, the Macro-Finance model achieves comparable accuracy across maturities but is less accurate than the other two models. For the longest maturity and the spread, the performance of the Macro-Finance model deteriorates considerably. Assuming the short-term rate responds directly to observed inflation and the unemployment gap, along with a 12-lag structure of the macro dynamics, may not be the most appropriate way to incorporate macro data into a term structure model in the typical, non-ZLB environment.

The bottom panel of Table B.4 reports the RMSE’s specifically for the ZLB period (01/2009-03/2014). For the shortest maturity, all three models achieve comparable accuracy and produce RMSE’s around 10 basis points. The Macro-Monetary and Finance models maintain this accuracy out to the 2-year bond, outperforming the Macro-Finance model. Like in the pre-ZLB period, the accuracy of the Macro-Finance model deteriorates for medium- and long-term yields, reaching over one percentage point on the 5-year yield. The Macro-Monetary model performs well at matching the medium-term yields within a ZLB environment, producing smaller RMSE’s than those from the Finance model on the 2-, 3-, and 4-year bonds. Finally, for the longer end of the yield curve, the Finance model now slightly outperforms the Macro-Monetary model, but the two models suggest similar interpretations regarding the extent of the effects of the ZLB constraint. Moreover, the Macro-Monetary model produces a substantial reduction in RMSE’s when compared with the Macro-Finance model for these maturities. Incorporating macro fluctuations in the model in a reactionary way, through the monetary VAR, rather than allowing the macro factors to directly influence the term structure, generates more accurate predictions of rates for the medium- and long-term. The response of macro indicators to lagged short rates is informative for discerning expectations about economic and financial fluctuations over at least the medium-horizon, thus supporting the notion
that fluctuations in the stance of policy have lagged effects on the macroeconomy.

All three shadow rate models come closer to matching the true yields during the ZLB period than during the years prior to the financial crisis. More specifically, with the exception of the 2- and 3-year yields from the Finance model, the RMSE's across the entire term structure are smaller during the ZLB period and particularly for the longer-term yields. Furthermore, all three models match the spread between the 5-year and 3-month Treasury yields more accurately during the ZLB period. Allowing for additional flexibility within the shadow rate model produces more accurate model-implied estimates of yields across the term structure, but this flexibility may be distortionary during normal times and impede the predictive ability of the models.

Furthermore, all three models generate slightly different implications for the stance of monetary policy at the ZLB and the projected duration for which the economy will remain constrained with prohibitively low interest rates. These results motivate the question: does a closer model fit suggest a more accurate representation of the stance of policy and how is this reconciled with the proposition of Bauer & Rudebusch (2013) suggesting that a shadow rate closer to the observed short-term interest rate is indicative of greater predictability of the full term structure. While the FFR anchors the short-end of the yield curve, when it is constrained by the ZLB, the Macro-Monetary model appears to capture how the constrained short rate elicits a new focus on targeting rates with longer maturities. Real activity and inflation exhibit stronger responsiveness to the observed FFR while the model picks up the shifted emphasis of policy towards longer term rates and matches this data more closely. Intuitively, traditional macro models in the non-ZLB period should match the shorter-end of the yield curve better as policymakers clearly focus on directly influencing the FFR. Under these circumstances, the Finance model produces a better model fit. However, once at the ZLB, the policy consensus shifts towards influencing medium- and longer-term interest rates. As a result, we need to change the measurement of the overall stance of policy itself. The Macro-Monetary model provides a mechanism for capturing this institutional change and produces results comparable to the Finance model, without requiring the shadow rate to substantially violate the ZLB.

It is inconsistent to think that the short end of the yield curve will be most accurately
modeled in a ZLB environment when the Fed abandons traditional policy objectives and the goal shifts towards moving the long end, distorting the slope of the yield curve as the short end ceased to show substantial variation. As a result, fluctuations in real economic activity and inflation may become less important for influencing the short end of the yield curve. The intention of policy has been to target long-term rates directly in the face of stagnant real growth and little risk of inflation. Traditional monetary policy is inapplicable under these circumstances and the Fed has continued expansionary policy efforts not by targeting a new level for the FFR, but rather influencing investment and consumption decisions through longer rates. In doing so, the short-term interest rate is less responsive to macro fluctuations, as evidenced by the less accurate model predictions from the Macro-Finance model where the policy rate responds directly to observed macro data. The Macro-Monetary model, in which macro aggregates simply respond to fluctuations in interest rates, gives a more accurate description of the break in policy directives at the ZLB.

3.6 Macro-Finance Shadow Rate and Policy Rules

Underlying the shadow rate model is the assumption that the standard macro model doesn’t change once we hit the ZLB. The fundamental relationships between macro aggregates, financial markets, and monetary policy exhibit the same behavior in the ZLB environment as in normal times, as long as the correct measurement of monetary policy is used. Previous shadow rate models that exclude macro data maintain consistent dynamics in financial markets but do not address what is going on in the macroeconomy. This paper makes the additional assumption that the consistency applies to macro dynamics, where the shadow rate provides a comprehensive representation of the stance of monetary policy.

By construction, the Macro-Finance model embeds a policy rule through which the Fed pursues its dual mandate of maximum employment and price stability. Incorporating the unemployment gap to represent real economic activity aligns the interest rate reaction function with FOMC statements specifically related to forward guidance. While consistently monitoring inflationary pressures, policymakers have recently acknowledged the maximum employment mandate directly. Seeking to guide expectations of future policy action, in particular regarding
when the FFR may be lifted away from the ZLB, the Fed mentioned targets for the unemployment rate beginning in December 2012. Due to the importance of labor market conditions in determining slack in the overall economy, I define the policy rule as a function of the difference between the CBO estimate of the natural rate of unemployment and the observed unemployment rate, along with the inflation rate of core CPI. I extract the systematic component of the Macro-Finance shadow rate to illustrate what the policy rate should be based on economic conditions. I decompose the shadow rate expression in equation 3.10 into the policy rule component, \( \text{policy}_t = a_0 + b_y y_t + b_{\pi} \pi_t \), and the unobserved component, \( f_t \). This allows one to differentiate the policy rule from the latent factor, capturing unexplained shocks to the stance of policy.

Figure B.9 shows plots of the effective FFR, the Macro-Finance shadow rate, the policy rule component of the shadow rate (\( \text{policy}_t \)), and the unobserved factor (\( f_t \)) from 01/2000-03/2014. Over the period preceding the financial crisis, the difference between the systematic policy prescription and both the FFR and the shadow rate suggests that policy was more accommodative than would have been recommended by a strict rule. Part of this deviation may be due to less emphasis on unemployment over this earlier time.\(^{16}\) Once hitting the ZLB, the latent factor falls substantially to incorporate the effects of unconventional policies that did not necessarily rely on historic policy rules to dictate the appropriate level of accommodation. Even through the strenuous recovery, the policy rule prescribes much less accommodation than that depicted by the shadow rate, where the enhanced accommodation is apparent in the unobserved factor. In setting policy, the Fed was likely reacting to a more diverse set of economic conditions than simply the unemployment gap.

I focus on the later part of the sample to capture the unique environment surrounding the ZLB. As shown in Figure B.9, the policy rule recommends a negative rate from mid-2009 through 2011, leading up to QE2 in 11/2010, and even suggests possible tightening from 2012 through 2014. Lombardi & Zhu (2014) reach a similar conclusion with their shadow suggesting

\(^{16}\)Lombardi & Zhu (2014) also examine the gap between their model-implied shadow rate and the levels of the FFR suggested by simple Taylor Rules. They also find that the stance of policy was too loose between 2001 and 2006.
a tightening in the stance of policy after 08/2011. Inflationary pressures and reductions in labor market slack support moderating the extent of policy accommodation. Interestingly, the unobserved factor reaches its lowest values between the implementation of the MEP (09/2011) and QE3 (09/2012). In comparison to the estimated policy rule, this period represents the most substantial unconventional policy accommodation.

3.7 What Do Markets Expect?

In addition to past or contemporaneous values of the shadow rate, expectations of future shadow rates also have important economic implications, especially during times in which the Fed seeks to influence expectations of future policy action. I use all three models to analyze what one can discern about market expectations of future shadow rates and, thus, the extent to which markets expect the economy remain in a ZLB environment. Taking the filtered values for each of the factors at the end of the sample, I perform Monte Carlo (MC) simulations to construct possible paths for these factors using the risk-neutral data generating process described by equation (A.6) in the Technical Appendix. I utilize the expression from the model which relates the expectation of the short rate at horizon \( h \) to the expectation of the vector of factors \( E_t^Q (r_{t+h}) = E_t^Q (a_0 + b_0' X_{t+h}) \) and simulate 10,000 candidate paths for each factor five years into the future. At each horizon \( h \), I construct the model-implied shadow rate and the corresponding short rate \( E_t^Q (R_{t+h}) = \max \{ E_t^Q (r_{t+h}), 0 \} \). Finally, I look at the distribution of future shadow rates to describe what markets expect regarding future policy.

Initially, I use the full data sample to construct expectations based on the market’s knowledge as of 03/2014. Secondly, I consider the economic environment around the announcements of two rounds of quantitative easing: QE1 in 11/2008 and QE2 in 11/2010, and the issuance of two FOMC statements in which the Fed used forward guidance to steer policy expectations. The first use of forward guidance that I consider focuses on the 08/2011 FOMC statement when the Fed promised to keep interest rates low, and near zero, until at least mid-2013. The second use of forward guidance (12/2012) suggested the FFR target will only be lifted when
economic conditions improve enough to warrant the increase. I conduct the future simulations both using data up through the month preceding and the month following each of these policy announcements. The model-implied paths of the shadow rate throughout the era of unconventional policy measures exhibit substantial variation, suggesting high sensitivity to the given model specifications. For specific details of the timing and results from these simulation exercises, see Table B.5.

Figure B.10 plots the projected path of the shadow rate produced by each model specification over the next five years, constructed by simulating forward many possible paths of the macro and/or latent factors beginning in 04/2014. The plot includes both the MC estimate of the mean as well as the 95% coverage interval over all simulations. By the end of the sample, all three models suggest a shadow rate representative of continued policy easing with values still below zero (−2.99% for the Macro-Finance shadow rate, −0.11 for the Macro-Monetary shadow rate, and −1.82% for the Finance shadow rate). Basing the simulated forecasts on the full sample, the expected Macro-Finance and Finance shadow rates remain significantly below zero until late-2015 and until late-2014, respectively. Expectations based on the Macro-Finance model suggest that markets anticipate the ZLB to significantly constrain the short end of the yield curve for a considerable time into the future while the ZLB represents a minimal threat going forward in the Finance model. Interestingly, using expectations based only on financial data suggests the ZLB environment is projected to persist for a shorter time than expectations incorporating macro factors directly. Therefore, the Finance model gives a description of financial markets as being relatively less constrained by the ZLB at this point in the recovery. Alternatively, the expected future path of the Macro-Monetary shadow rate never falls significantly below zero within the next five years. The Macro-Monetary model tries to capture movements in the longer end of the yield curve without pushing the shadow rate far below the zero threshold. There is much less support in this model for a negative future shadow rate and thus incorporating the monetary VAR into the model illustrates how macro aggregates respond to the observed stance of policy.

While the quantitative nature of the paths differ, they exhibit similar qualitative characteristics around the timing of policy announcements. For example, around the announcement
of QE1, all three models suggest a perceived policy easing with their respective shadow rates falling between the months before and after the announcement. QE1 appears to have had some expansionary effects by adjusting market expectations of future short rates downward for at least the next year. Basing the simulated forecasts on data up through the months preceding and following the announcement of QE1, only the Macro-Finance shadow rate was expected to be significantly below zero after the policy was put in place.

Regarding QE2, both models incorporating macro data suggest a slight tightening of policy in the months around the announcement. The Macro-Finance and Macro-Monetary shadow rates increase around the announcement. In contrast, the shadow rate from the Finance model best captures the impact effect of a perceived policy easing with a decline in the contemporaneous shadow rate around this time. Forecasts based on data through the announcement of QE2 also illustrate that market expectations in only the Macro-Finance and Finance models suggested a significantly negative path for the shadow rate in future months.

All three models accurately interpreted the first use of forward guidance in the 08/2011 FOMC statement as a promise for continued policy accommodation. All three shadow rates fell between the months before and after the release of this statement. Forecasts using data up through 08/2011 show that markets expected the Macro-Finance and Finance shadow rates to be significantly negative in the coming months. Markets may have interpreted this policy action as a commitment by the central bank to maintain expansionary policy for a considerable amount of time, despite the fact that traditional interest rate policy couldn’t be used effectively. By the second use of forward guidance in 12/2012, now linking policy action to future economic conditions, market expectations seem to have stabilized and participants expected easy policy for a significant time into the future. The FOMC statement may not have contained any sort of policy surprise as the model-implied shadow rates changed only very slightly. However, forecasts based on data through this point in time suggest that markets anticipated significantly negative shadow rates based upon the two models including macro data. The 12/2012 FOMC statement helped to temper future expectations regarding the expected duration of the ZLB episode and maintain a successfully accommodative policy stance.
3.8 Conclusion

In this paper, I incorporate macroeconomic information into a model of the term structure, capturing dynamics in the relationship between monetary policy, overall economic conditions, and financial markets. The findings suggest that it is inappropriate to use only the FFR to represent policy in a monetary policy analysis at the ZLB. The shadow rate model produces a more comprehensive measurement of monetary policy in a ZLB environment by capturing how the Fed has changed focus over time by targeting short-, medium-, or long-term interest rates. Using only interest rate data, the Finance shadow rate stayed fairly close to zero while the FFR hovered just above zero throughout the Great Recession and its recovery. Including macro data produces a description of how the economy responded to the Fed’s arsenal of unconventional policies. The Macro-Finance shadow rate dipped below zero in the very early stages of the financial crisis and remained substantially below zero through the end of the sample. A widening unemployment gap and declining inflation characterize the persistent economic contraction and keep the shadow rate well below zero through the Great Recession and its subsequent arduous recovery. The Macro-Monetary shadow rate suggests that real activity and inflation react more to the observed stance of policy, as discerned by the nominal effective FFR. This approach acknowledges the relationship between the macroeconomy and short-term interest rates, as is usually done in a standard monetary VAR, and stabilizes fluctuations in the shadow rate once the ZLB binds. Utilizing macro data in this manner is more effective at predicting medium- and longer-term yields than allowing the macro data to feed into the term structure itself and thus better captures the shift in policy focus to targeting the long end of the yield curve with unconventional policy programs.

The Fed seemed to have been successful at effectively communicating information about these programs and guiding future expectations when the well-understood method for setting policy could no longer be used effectively. With this substantial accommodation, market expectations of future shadow rates indicate that the economy is believed to stay near the ZLB for the foreseeable future, in line with the language associated with forward guidance in FOMC statements. Excluding the macro factors results in an interpretation of a tighter
stance of policy and the expected lift-off from the ZLB would occur considerably earlier in future projections.
A.1 Standard GATSM Methodology

To preserve the functional form of the standard GATSM, I combine all lags of each state variable in the vector $X_t$ and write the VAR in companion form:

$$ x_t = \mu + \rho (L) x_{t-1} + e_t, \quad (A.1) $$

where $\rho$ and $\mu$ collect the appropriate mean and autocorrelation coefficients. The factor innovation volatilities are normally distributed, $e_t \sim N (0, \Sigma)$, where either $\Sigma^{1/2}$ is lower-triangular or $\Sigma$ is diagonal, depending on the model specification.

I can write the factor dynamics in companion form, stacking all $p$ lags of $x_t$ into one vector, $X_t$:

$$ X_t = \tilde{\mu} + \rho X_{t-1} + \varepsilon_t, \quad (A.2) $$

$$ \tilde{\mu} = \begin{bmatrix} \mu \\ 0 \end{bmatrix}_{\Sigma \times (N \times (p-1))}, \quad (A.3) $$

$$ \varepsilon_t = \begin{bmatrix} e_t \\ 0 \end{bmatrix}_{\Sigma \times (N \times (p-1))} \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma^{1/2} & 0_{N \times (N \times (p-1))} \\ 0_{(N \times (p-1)) \times N} & 0_{(N \times (p-1)) \times (N \times (p-1))} \end{bmatrix}. \quad (A.4) $$

The coefficient matrix $\rho$ collects the lag polynomial coefficients for $p$ lags. Written in this form, the residual terms in the equations for lagged factors are identically equal to zero.

Typically, authors differentiate between the physical, observed dynamics and risk-neutral dynamics of the state variables. The risk neutral-dynamics illustrate how a risk-neutral investor would view the evolution of the factors over time. Assuming an absence of arbitrage implies the existence of this risk-neutral measure, which is used for pricing assets and determining yields. I assume time-varying market prices of risk which are linear in the state vector:

$$ \Gamma_t = \gamma_0 + \gamma_1 X_t, \quad (A.5) $$
Under the risk-neutral measure, $X_t$ evolves according to the following expression:

$$X_t = \mu^Q + \rho^Q X_{t-1} + \Sigma_{e_t}^Q,$$  \hspace{1cm} (A.6)

where

$$\mu^Q = \mu - \Sigma \gamma_0,$$ \hspace{1cm} (A.7)

$$\rho^Q = \rho - \Sigma \gamma_1.$$ \hspace{1cm} (A.8)

To identify the latent factor, I assume a zero mean under the physical measure ($\mu = 0$). Like Ang et al. (2007), to reduce the number of parameters within the model, I assume that the constant component of the prices of risk, $\gamma_0$, depends only on current factors:

$$\gamma_0 = \begin{bmatrix} \bar{\gamma}_0 \\ 0_{(N \cdot (p-1)) \times 1} \end{bmatrix},$$ \hspace{1cm} (A.9)

where $\bar{\gamma}_0$ is of dimension $N \times 1$. The coefficient on $X_t$, $\gamma_1$, relates the time-varying components to the risk prices and places weight on contemporaneous and lagged values of the macro factors but only on the current value of the latent factor:

$$\gamma_1 = \begin{bmatrix} \bar{\gamma}_1 \\ 0_{(N \cdot (p-1)) \times (N \cdot p)} \end{bmatrix},$$ \hspace{1cm} (A.10)

where $\bar{\gamma}_1$ has zeros in the columns that correspond to higher lags of $x^{U}$ in the Macro-Finance model. The VAR dynamics under the risk-neutral measure incorporate fluctuations of all state variables, adjusted for the risk associated with innovations to the factors.

A.1.1 Estimation of Parameters with the Kalman Filter

The standard GATSM specification can be written in state space form, with state variables driving the shadow short rate. Introducing the extensions in the shadow rate models to account for episodes in which interest rates reach the ZLB makes the linear state space representation infeasible. Therefore, I follow Bauer & Rudebusch (2013) and Krippner (2012) by estimating
the model in linear state space form with data where interest rates do not hover near the ZLB
and then using these parameter values to estimate a system of equations representing interest
rates in the shadow rate framework.

The state transition equations takes on the standard from when writing the VAR for vector
$X_t$ under the physical measure in companion form. Recall, I demean the macro data and impose
that the latent monetary policy shock is mean zero and therefore omit the intercept term:

$$X_t = \rho X_{t-1} + e_t. \quad (A.11)$$

Thus the covariance matrix for the transition equation of latent yield curve state variables is
equal to $Q = \Sigma$.

The standard GATSM combines interest rates for $K$ different maturities at time $t$ and
allows for constructing the measurement equation of the linear state space as:

$$R_t = a + bX_t + v_t, \quad (A.12)$$

where $R_t$ is the $K \times 1$ vector of yield curve data, $A$ is the $K \times 1$ vector $[a^{\tau_1}, ..., a^{\tau_K}]'$, $B$ is the
$K \times N$ matrix $[b^{\tau_1}, ..., b^{\tau_K}]$ and $\tau_1, ..., \tau_K$ represent the times to maturity of the yield curve
data. The covariance matrix for the measurement equation takes the form:

$$\Sigma^R = diag [\sigma_v^2(\tau_1), ..., \sigma_v^2(\tau_K)], \quad (A.13)$$

where the diagonality is standard in this literature under the assumption that all other con-
temporaneous and inter-temporal covariances between interest rates of different maturities are
zero.

### A.2 Constructing the Shadow Term Structure

With the shadow policy rate described by equation (3.10) and the nominal policy rate by
equation (3.2), I follow the approach of Bauer & Rudebusch (2013) and focus on the conditional
expectation and variance of future shadow rates, taking as given the time $t$ values of the factors
\[ \tilde{r}_{t+h} = E (r_{t+h} \mid X_t) = a_0 + b'_0 E (X_{t+h} \mid X_t) = a_0 + b'_0 \left[ \left( I - \rho^h \right) E (X_t) + \rho^h X_t \right], \quad (A.14) \]

\[ \left( \Sigma^h \right)^2 = Var (r_{t+h} \mid X_t) = b'_0 Var (X_{t+h} \mid X_t) b_0 = b'_0 \left( \sum_{i=0}^{h-1} \rho^i \Sigma \rho^i \right) b_0. \quad (A.15) \]

Expectations of future nominal short rates are based upon the probability that the shadow rate will stay above zero. Assuming that the shadow rate follows a normal distribution, the observed short rate follows a truncated distribution, bounded below at zero. In constructing expectations of future short rates, I impose the lower bound by accounting for the probability that the shadow rate may be negative, resulting in the observed rate being censored at zero:

\[
E (R_{t+h} \mid X_t) = P (r_{t+h} > 0) \times E (r_{t+h} \mid X_t, r_{t+h} > 0) + P (r_{t+h} < 0) \times 0
\]

\[
\begin{align*}
&= \Phi \left( \frac{\tilde{r}_{t+h}}{\Sigma^h} \right) \left[ \tilde{r}_{t+h} + \Sigma^h \phi \left( \frac{\tilde{r}_{t+h}}{\Sigma^h} \right) \right] \\
&= \Phi \left( \frac{\tilde{r}_{t+h}}{\Sigma^h} \right) \tilde{r}_{t+h} + \Sigma^h \phi \left( -\frac{\tilde{r}_{t+h}}{\Sigma^h} \right),
\end{align*}
\quad (A.16)
\]

where \( \Phi (\cdot) \) is the standard normal cumulative distribution function and \( \phi (\cdot) \) is the associated density function.

To constrain a normally distributed random variable, \( r \sim N \left( \tilde{r}, \Sigma^2 \right) \), to lie within a specific interval, \( r \in (\alpha, \beta) \), one can consider \( r \) to follow a truncated normal distribution. In the case regarding observed short rates, \( \alpha = 0 \) and \( r \) is unbounded above \( (\beta = \infty) \). Since we only observe \( R_t = r_t \) if \( r_t > 0 \), the first line of \( (A.16) \) illustrates that we must consider the portion of the distribution for the shadow rate when \( r \) takes on positive values. The first term of the second line gives the probability that the shadow rate is positive: \( P (r_{t+h} > 0) = \Phi \left( \frac{\tilde{r}_{t+h}}{\Sigma^h} \right) \).

---

\(^1\) For the Macro-Finance model, \( X_t \) includes twelve lags of \([y_t, \pi_t, f_t]\). For the Finance and Macro-Monetary models, \( X_t = [f_{1t}, f_{2t}, f_{3t}] \).
where the standard normal CDF is evaluated at

\[
\frac{E (r_{t+h} \mid X_t)}{(Var (r_{t+h} \mid X_t)))^{0.5} = \frac{\tilde{r}_{t+h}}{\Sigma^h}. \quad (A.17)
\]

The second term of this line uses the one-sided truncated normal distribution to describe the expectation of the shadow rate, given that it is above zero:

\[
E (r_{t+h} \mid X_t, r_{t+h} > 0) = E (r_{t+h} \mid X_t) + \sum^h \phi \left( \frac{\alpha-E(r_{t+h} \mid X_t)}{\Sigma^h} \right) \frac{\Phi \left( \frac{-\tilde{r}_{t+h}}{\Sigma^h} \right)}{1 - \Phi \left( \frac{-\tilde{r}_{t+h}}{\Sigma^h} \right)}, \quad (A.18)
\]

where the arguments for \( \phi (\cdot) \) and \( \Phi (\cdot) \) in (A.18) come from the fact that the lower bound \( \alpha = 0 \) and \( E (r_{t+h} \mid X_t) = \tilde{r}_{t+h} \).

In the standard GATSM, bond prices are exponential affine functions of the underlying factors \([X_t]\). Let \( P_t^\tau \) be the price of a zero-coupon bond at time \( t \) with time to maturity \( \tau \):

\[
P_t^\tau = \exp (A^\tau + B^\tau X_t), \quad (A.19)
\]

where the coefficients are functions of the state vector autoregressive coefficients, the market prices of risk of each factor, and the factor volatilities:

\[
A^{\tau+1} = A^\tau + B^\tau \mu^Q + \frac{1}{2} B^\tau \Sigma \Sigma' B^\tau' - a_0, \quad (A.20)
\]

\[
B^{\tau+1} = B^\tau \rho^Q - b_0. \quad (A.21)
\]

The meta parameters – like many deep parameters in macro models – produce cross-equation restrictions in the measurement equation of the state space. Bond yields are related to prices by the following:

\[
R_t^\tau = -\frac{1}{\tau} \log (P_t^\tau) = a^\tau + b^\tau X_t, \quad (A.22)
\]

where \( a^\tau = -\frac{A^\tau}{\tau} \) and \( b^\tau = -\frac{B^\tau}{\tau} \). Let \( \mathcal{R}_t = [R_t^{\tau_1}, ..., R_t^{\tau_K}]' \) be a \( K \times 1 \) vector of period–t
nominal interest rates for bonds of different maturities. Since each element of $\mathcal{R}_t$ is related to the state variables by the affine function (A.22), the observation equation of the state space system in the standard GATSM takes a linear form:

$$\mathcal{R}_t = a + bX_t + v_t,$$

where $v_{it} \sim iidN(0, \sigma^2_{v}(\tau_i))$, and $a$ and $b$ are vectors of the $a^\tau$ and $b^\tau$ coefficients for $\tau = \{1, ..., K\}$.

However when observed short rates are censored at the ZLB, I instead focus on expectations of the future path of the shadow rate which in turn guide expectations about the nominal short rate based upon whether markets expect the shadow rate to stay below zero. For a set of factors at time $t$, the bond price can be expressed as an expectation of future nominal short rates:

$$P_t^\tau = E_t^Q \left[ \exp \left( -\sum_{i=0}^{\tau-1} R_{t+i} \right) \right].$$

The affine relationship of equation (A.23) deteriorates at the ZLB and (A.24) establishes a nonlinear relationship between yields and the underlying factors. One cannot describe bond prices and yields with analytic closed-form expressions once accounting for the probability of negative $r_t$ and censored $R_t$. Therefore, I adopt the same methodology applied in Bauer & Rudebusch (2013) and use Monte Carlo simulations. To evaluate the expectation in (A.24), I simulate $W = 500$ paths of the macro and latent factors, each of length $\tau$, based on the risk-neutral dynamics described by equation (A.6). Like Bauer & Rudebusch (2013) and others, I use antithetic sampling to construct the sequences of shocks to each of the factors in iteration $j$ by taking the negative of the shocks from iteration $j - 1$. The resulting negative dependence between sets of consecutive iterations helps reduce the variance of the estimators.\(^2\) I use these simulated factor values to construct a time-series of expected shadow rates, equation (3.10), and nominal rates, equation (3.2). Collecting all $W$ paths allows for approximating bond prices

\(^2\)Using antithetic sampling, each draw of shock sequences is no longer i.i.d.. Therefore, the covariance between two consecutive draws is negative. This reduces the variance of the estimated yields, which are constructed by averaging across the simulations.
as:

\[ \hat{P}_t^\tau = \frac{1}{W} \sum_{j=1}^{W} \exp \left( -\sum_{i=0}^{\tau-1} \hat{R}_{t+i}(j) \right), \quad (A.25) \]

where \( \hat{R}_{t+i}(j) \) is the simulated path of short rates at iteration \( j \). In the shadow model, yields are still related to prices through \( R_t^\tau = -\frac{1}{\tau} \log (P_t^\tau) \) but now there is no closed-form expression for \( P_t^\tau \). The measurement equation of the state space no longer maintains the linear structure of equation (A.23), which is easily dealt with by Kalman filtering methods. However, by construction, the shadow rate model collapses to a standard GATSM when interest rates are sufficiently far from zero so that \( P(r_{t+h} < 0) \) is very small and the ZLB does not present a concern. Therefore, I utilize the linear expressions to estimate the parameters of the model over a time when financial markets operated normally away from the ZLB and the FFR could be used effectively as the primary monetary policy tool. From there, I use those parameter values and filter through the nonlinear state space to extract the shadow rate when the ZLB binds and nominal rates are censored. For these purposes, I define \( \hat{R}_t^\tau = -\frac{1}{\tau} \log (\hat{P}_t^\tau) \) and \( \hat{R}_t = [\hat{R}_t^{1\tau}, ..., \hat{R}_t^{\ell\tau}]' \).

### A.3 MCMC Algorithm for Drawing Parameters and Latent Factors

The full parameter set spanning all three models is organized into six different blocks: \( \rho \) and \( \mu, \rho^Q \) and \( \mu^Q, \Sigma, \tilde{A}, \Sigma^R, \Sigma^M \), and the short rate coefficients \( a_0 \) and \( b_0 \). I implement Bayesian Gibbs sampling steps when the priors are standard conjugates and thus generate conjugate posteriors from which one can easily make draws. In the other cases, I use a Metropolis-Hastings procedure to construct acceptance probabilities for the proposed values of each parameter block. After discarding 50,000 draws to achieve convergence, I use the following 50,000 post-burn-in draws to approximate the full joint posterior for all model parameters. The next subsections present details on the prior parameterization. See Mikkelsen (2001) for an overview of the methodology used to apply MCMC estimation of term structure models.
A.3.1 Likelihood

Let $Y = [Z, R]$, $Z = [Z_1, ..., Z_T]'$, $R = [R_1, ..., R_T]'$, and $X = [X_1, ..., X_T]'$. The set of model parameters to be estimated for all models is $\theta = (\mu, \mu^Q, p, p^Q, \Sigma, \Sigma^R)$ where $\Sigma$ and $\Sigma^R$ are the variance-covariance matrices of the error terms in the state equation for $X$ and the measurement equation for nominal interest rates, $R$. In the Macro-Monetary model, I must also estimate $\tilde{A}, \text{and } \Sigma^M$. Additionally, the Macro-Finance model requires estimation of the short rate coefficients $a_0$ and $b_0$. The full likelihood of the model is:

$$p(Y | \theta) = p(Y | \theta, X) \times p(X | \theta), \quad \text{(A.26)}$$

where the likelihood of the factors, $p(X | \theta)$, can be expressed as the likelihood of a Gaussian VAR due to the structure of the state transition equation. The data generating process of $X$ depends on the physical dynamics of the state vector and the variance-covariance matrix of the state innovations, $\Sigma$. The likelihood of the observed nominal interest rate data, conditional on the factors $X$, depends upon the following parameters:

$$p(R | \theta, X) = p(R | \mu^Q, p^Q, \Sigma, \Sigma^R, X).$$

This is a function of the risk-neutral parameters of the state dynamics and the covariance matrix $\Sigma$.

Finally, for the Macro-Monetary VAR model, the likelihood of the macro data conditional on $X$ depends upon the loadings linking inflation and real activity to the shadow rate:

$$p(Y | \theta, X) = p(Y | \tilde{A}, \Sigma^M, X).$$

This is also the likelihood of a Gaussian VAR due to the FAVAR nature of the relationship between the policy rate and macro fluctuations.
A.3.2 Drawing the Latent Factors ($f_t$)

Bayesian estimation of dynamic state systems involves constructing the probability density function (pdf) of the state variables, based on all available information. When the system is linear, the Kalman filter allows for easy prediction and updating of the state each time observable information becomes available since the pdf will be Gaussian at each step. However, when we see a nonlinear relationship between the states and the observable variables, there is generally no closed-form expression for the required pdf. Nonlinear Bayesian state space estimation requires the EKF algorithm to recursively generate an approximation to the pdf of the unobservable state variables. Here, I initially use the standard Kalman filter with the linear state space representation of the state transition equation (3.4) and the measurement equation (3.8). I iterate through the MCMC algorithm, alternating between filtering the latent states and then estimating the model parameters, given these state values using data prior to the ZLB period for this first-step estimation. Since I identify the latent factor as having mean zero, I impose this by centering each draw of $f_t$ around zero. After obtaining the parameter estimates, I use the EKF to propagate the latent states forward through the ZLB period and extract the model-implied shadow rate over the remainder of the sample. The EKF is detailed in Appendix Section A.4.

A.3.3 Drawing the State Transition VAR Dynamics

One can draw the physical dynamics of $X_t$ using standard Gibbs sampling with conjugate normal priors for $\rho$, imposing that $\mu$ is a vector of zeros. Like Ang et al. (2007), I follow Johannes & Polson (2005) in differentiating between $\{\mu, \rho\}$ and $\{\mu^Q, \rho^Q\}$. Since the state vector follows the VAR described by equation (A.1), the draws are standard Gibbs sampling and I draw $\rho$ separately for each equation in the state VAR. I specify the prior $p(\rho)$ to be $N(0, 1000)$ which implies a natural conjugate normal posterior distribution.

I assume a dispersed inverse Wishart (IW) prior for the factor shock covariance matrix, $\Sigma$. Like the previous blocks of parameters, the conditional posterior of the factor shock covariance
matrix $\Sigma$ takes the following form:

$$
P\left( \Sigma \mid \theta_-, X, Y \right) \propto p\left( Y \mid \theta, X \right) p\left( X \mid \theta \right) p\left( \theta \right), \quad \text{(A.27)}$$

where now $\theta_-$ denotes all parameters except for $\Sigma$. Given an IW natural conjugate prior for the factor shock covariance matrix, I draw the entire matrix in one Independence Metropolis step from a conjugate IW proposal density: $q\left( \Sigma \right) = p\left( X \mid \Sigma, \theta_- \right) p\left( \Sigma \right)$ where $p\left( \Sigma \right)$ is an IW prior. Therefore, the proposal draw $\Sigma^{(i)}$ is accepted with probability:

$$
\alpha\left( \Sigma^{(i)}, \Sigma^{(i-1)} \right) = \min\left\{ \frac{p\left( Y \mid \Sigma^{(i)}, \theta_-, X \right) p\left( X \mid \Sigma^{(i)}, \theta_- \right) p\left( \Sigma^{(i)}, \theta_- \right) q\left( \Sigma^{(i-1)} \right)}{p\left( Y \mid \Sigma^{(i-1)}, \theta_-, X \right) p\left( X \mid \Sigma^{(i-1)}, \theta_- \right) p\left( \Sigma^{(i-1)}, \theta_- \right) q\left( \Sigma^{(i)} \right)}, 1 \right\},
$$

$$
= \min\left\{ \frac{p\left( Y \mid \Sigma^{(i)}, \theta_-, X \right)}{p\left( Y \mid \Sigma^{(i-1)}, \theta_-, X \right)}, 1 \right\}
$$

The transition density is then just the ratio of likelihoods of the new draw, $\Sigma^{(i)}$, versus the old draw, $\Sigma^{(i-1)}$.

### A.3.4 Drawing the Risk Neutral Dynamics

I assume flat priors for the risk neutral parameters $\mu^Q$ and $\rho^Q$. I draw each element of $\mu^Q$ and the relevant sections of each row of $\rho^Q$ separately with a Random Walk Metropolis algorithm and scale factors adjusted to tune the acceptance probabilities. Since the proposal draw comes from a Random Walk step, the acceptance probabilities take the following forms:

$$
\alpha\left( \mu^{Q(i-1)}, \mu^{Q(i)} \right) = \min\left\{ \frac{p\left( Y \mid \mu^{Q(i)}, \theta_-, X \right) p\left( \mu^{Q(i)} \right)}{p\left( Y \mid \mu^{Q(i-1)}, \theta_-, X \right) p\left( \mu^{Q(i-1)} \right)}, 1 \right\}, \quad \text{(A.28)}
$$

and

$$
\alpha\left( \rho^{Q(i-1)}, \rho^{Q(i)} \right) = \min\left\{ \frac{p\left( Y \mid \rho^{Q(i)}, \theta_-, X \right) p\left( \rho^{Q(i)} \right)}{p\left( Y \mid \rho^{Q(i-1)}, \theta_-, X \right) p\left( \rho^{Q(i-1)} \right)}, 1 \right\}. \quad \text{(A.29)}
$$

These probabilities are both the minimum of one and the posterior ratio of the model likelihood times the ratio of priors for the new draw versus the old draw.

I do not specify separate priors for the market prices of risk. Instead I estimate the
physical and risk-neutral dynamics separately and then invert to obtain $\gamma_0$ and $\gamma_1$ based on the relationships in equations (A.7) and (A.8).

### A.3.5 Drawing the Observable Dynamics

Assumed to follow a Gaussian system in the Macro-Monetary model, the evolution of observable macro variables over time is described by equation (3.16), where inflation and output growth are functions of 12 of their own lags and 12 lags of the shadow monetary policy rate. The macro block of the measurement equation also assumes a Gaussian VAR form and thus I assume conjugate normal priors for $\tilde{A}$, given values for the other parameters. In addition, I assume dispersed priors for the covariance matrices— an inverse Wishart (IW) prior for the covariance of shocks in the macro VAR, $\Sigma^M$, and an inverse gamma (IG) prior for the measurement errors associated with the observed nominal interest rates, $\Sigma^R$. Following from conjugate normal priors for $\tilde{A}$, I again can implement standard Gibbs sampling steps to draw from the conjugate normal posteriors for both lag polynomial coefficient matrices. The variances of the two blocks of measurement errors can be drawn directly from the conditional posterior distribution for $\Sigma^R$ and $\Sigma^M$. I use the standard Gibbs sampling algorithm since conditional on the data, the latent factors, and the other parameters in the model, the measurement errors are orthogonal to the model. Thus the conditional posterior for $\Sigma^R$ is the natural conjugate IG distribution and the conditional posterior for $\Sigma^M$ is the natural conjugate IW distribution, from both of which I easily make draws:

\[
\Sigma^R \mid \theta, X, Y \sim IG \left( \frac{T + a_0}{2}, b_0 + \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t^2 \right) ; \tag{A.30}
\]

\[
\Sigma^M \mid \theta, X, Y \sim IW \left( \hat{\Sigma}^M, T + c_0 \right) . \tag{A.31}
\]

### A.3.6 Estimation of the Short Rate Dynamics

The policy rate reaction function gives a preliminary basis for how much of the yield curve movements are associated with macro factors and thus how much is attributed to unobservable
factors. In the Macro-Finance model, I initialize $b_y$ and $b_r$ at their OLS estimates using the effective FFR as the policy rate and then propose a random walk Metropolis Hastings algorithm to determine the loadings on inflation and real activity in the short rate expression individually. The acceptance probability resembles that described by expression (A.28). I then allow the MCMC iterations to gauge whether the macroeconomic data contributes to describing yield curve fluctuations and to disentangle the feedback mechanism between policy-setting, macroeconomic dynamics, and financial markets. Rather than separately drawing $a_0$, I construct the model implied short rate to match the sample mean of the FFR. In each Gibbs iteration, I set the value of $a_0$ according to: $a_0 = \bar{R} - b_0' \bar{X}$, where $\bar{R}$ is the mean of the FFR and $\bar{X}$ includes the time-series means of the factors. Since $f$ and $b_0$ are drawn within each iteration, $a_0$ is updated at each step.

A.4 Extended Kalman Filter

Estimating the two latent state variables underlying the yield curve dynamics requires use of the extended kalman filter (henceforth EKF). The state variables evolve according to a Gaussian VAR transition equation but the relationship between the states and the observable interest rates is nonlinear and prohibits the use of standard kalman filtering methods. The nonlinearity is introduced due to the added complexity of the shadow rate model accounting for censoring at zero and thus introduces a type of truncated distribution for observed rates. I find it appropriate in this case to apply the EKF rather than more complicated nonlinear particle filtering methods because when the observed short rate is positive, the model collapses to an affine term structure model based upon a linear foundation. The EKF constructs a local linearization of the transition and measurement equations to approximate the nonlinearity encountered with the equations relating market interest rates to $X_t$ through the expressions for the shadow bond prices.

The EKF proceeds like the standard Kalman filter with a series of predicting and updating steps for the state variables, incorporating new observed information at each point in time. One can generalize the state space as a combination of the state transition and measurement
equations:
\[ X_t = \rho X_{t-1} + \varepsilon_t, \tag{A.32} \]
\[ Y_t = h(X_t) + v_t, \quad v_t \sim N(0, \Sigma_Y) \tag{A.33} \]
\[ R_t^r = -\frac{1}{\tau} \log \left( \hat{P}_t^r \right). \tag{A.34} \]

This format illustrates the nonlinear nature of the function relating \( X_t \) to the observable interest rates, \( h(X_t) \). This function also includes the Gaussian VAR linking macro dynamics to the latent state variables when macro variables are included in the measurement equation in the Macro-Monetary Model. In the general EKF algorithm, the pdf of state variables given the observable data is approximated by a Gaussian for each iteration \( t = 1, \ldots, T \):

\[ p(X_{t-1} \mid Y_{1:t-1}) = N(X_{t-1}; X_{t-1|t-1}, P_{t-1|t-1}) \tag{A.35} \]
\[ p(X_t \mid Y_{1:t-1}) = N(X_t; X_{t|t-1}, P_{t|t-1}) \tag{A.36} \]
\[ p(X_t \mid Y_{1:t}) = N(X_t; X_{t|t}, P_{t|t}) \tag{A.37} \]

where \( X_{t:t} \) and \( P_{t:t} \) are the updated estimates of the state and state covariance matrix, respectively, and \( X_{t:t-1} \) and \( P_{t:t-1} \) are the predictions. Since I model the state dynamics as a Gaussian VAR, this step follows exactly as in the standard Kalman filter. The only nonlinearity is introduced in the measurement equation for interest rates. Here, the updating steps will require an approximation. The steps proceed as follows:

\[ X_{t|t-1} = \rho X_{t-1}, \tag{A.38} \]
\[ P_{t|t-1} = \Sigma + \rho P_{t-1|t-1} \rho, \tag{A.39} \]
\[ X_{t|t} = X_{t|t-1} + K_t \left( Y_t - h(X_{t|t-1}) \right), \tag{A.40} \]
\[ P_{t|t} = P_{t|t-1} - K_t M_t P_{t|t-1}, \tag{A.41} \]
where the local linearizations of the nonlinear function \( h(X_t) \) are:

\[
H_t = \left. \frac{dh(X)}{dX} \right|_{X=X_{t|t-1}}, \quad \tag{A.42}
\]

\[
K_t = P_{t|t-1} H_t' Z_t^{-1}, \quad \tag{A.43}
\]

\[
Z_t = H_t P_{t|t-1} H_t' + \Sigma^y. \quad \tag{A.44}
\]

I approximate the derivative of \( \frac{1}{2} \log \left( \hat{P}_t^2 \right) \) numerically with respect to each of the state variables. In the Macro-Monetary Model specification, I also need to consider the derivative of the added convolution from the macro VAR dynamics:

\[
\frac{\delta y_t}{\delta x_{t,i}} = \left[ A^{RZ} (L) \right]_{1,i}, \quad \tag{A.45}
\]

\[
\frac{\delta \pi_t}{\delta x_{t,i}} = \left[ A^{RZ} (L) \right]_{2,i}, \quad \tag{A.46}
\]

for \( i = \{1, 2, 3\} \). Stacking these derivatives produces the approximate coefficient matrix for the extended Kalman filter algorithm.
Table B.1: Bayes Factor Comparison for Parameter Instability. Comparison of log marginal likelihoods of models allowing for parameter instability in the response of macro variables to various measures of the policy rate: 1. the effective nominal FFR, 2. the FFR with controls for changes in the size of the Fed balance sheet and policy announcement effects, 3. the shadow rate of Krippner (2012), and 4. the shadow rate of Wu and Xia (2014). We compare one model assuming constant parameters in the pre-ZLB and ZLB periods with another allowing for shifts in the response of all variables to lags of the policy rate. We compute an adjusted Bayes Factor which can be interpreted as follows: * indicates positive evidence in favor of the model with parameter instability, ** indicates strong evidence, and *** indicates very strong evidence.
Table B.2: Kullback-Leibler Divergence

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative:</td>
<td>Full-Sample</td>
<td>ZLB</td>
<td>Full-Sample</td>
<td>ZLB</td>
</tr>
<tr>
<td>Federal Funds Rate</td>
<td>28.70</td>
<td>182.74</td>
<td>55.68</td>
<td>110.93</td>
</tr>
<tr>
<td>Krippner Shadow Rate</td>
<td>32.10</td>
<td>123.45</td>
<td>60.30</td>
<td>93.79</td>
</tr>
<tr>
<td>Wu-Xia Shadow Rate</td>
<td>30.20</td>
<td>187.49</td>
<td>63.64</td>
<td>121.15</td>
</tr>
</tbody>
</table>

Table B.2: Kullback-Leibler Divergence between the posterior distributions of the VAR parameters estimated with the benchmark data, using the FFR, and the posterior distributions when estimating the model using full-sample instrument combining the FFR and shadow rate policy measurements. We compare the distributions to either the full pre-ZLB benchmark (1960:I-2007:IV) or the Post-Great Moderation benchmark (1984:I-2007:IV).

Table B.3: Priors for MCMC estimation algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Hyper-parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$N(0,1000)$</td>
<td></td>
</tr>
<tr>
<td>$\mu^Q, \rho^Q$</td>
<td>$N(0,1)$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\Sigma}$</td>
<td>$N(0,10)^*$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$IW(\Sigma_0, c_0)$</td>
<td>$\Sigma_0 = I_N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c_0 = N^{**} + 2$</td>
</tr>
<tr>
<td>$\Sigma^M$</td>
<td>$IW(\Sigma^M_0, d_0)$</td>
<td>$\Sigma^M_0 = 10 \times I_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_0 = M^{***} + 2$</td>
</tr>
<tr>
<td>$\Sigma^R$</td>
<td>$IG(e_0, f_0)$</td>
<td>$e_0 = 0.01$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_0 = 0.01$</td>
</tr>
<tr>
<td>$b_0$</td>
<td>$N(0,1)$</td>
<td></td>
</tr>
</tbody>
</table>

Table B.3: Prior parameterization for MCMC estimation algorithm. With this prior specification, we initialize the state equation dynamics in all models and the Macro-Monetary VAR portion of the measurement equation with OLS estimates of the coefficient matrices.

**N= number of state variables (3)
***M= number of observable macro variables in VAR (2)
Table B.4: Root-MSE of Term Structure Estimates

<table>
<thead>
<tr>
<th></th>
<th>3 mo.</th>
<th>6 mo.</th>
<th>1 yr.</th>
<th>2 yr.</th>
<th>3 yr.</th>
<th>4 yr.</th>
<th>5 yr.</th>
<th>Spread: 5 yr.–3 mo.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-ZLB (Filtered 01/1985-12/2008)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macro-Finance</td>
<td>0.68</td>
<td>0.51</td>
<td>0.51</td>
<td>0.62</td>
<td>0.56</td>
<td>0.66</td>
<td>1.70</td>
<td>1.38</td>
</tr>
<tr>
<td>Macro-Monetary</td>
<td>0.38</td>
<td>0.23</td>
<td>0.08</td>
<td>0.19</td>
<td>0.51</td>
<td>1.10</td>
<td>1.91</td>
<td>1.71</td>
</tr>
<tr>
<td>Finance</td>
<td>0.29</td>
<td>0.25</td>
<td>0.25</td>
<td>0.35</td>
<td>0.53</td>
<td>0.96</td>
<td>1.59</td>
<td>1.38</td>
</tr>
<tr>
<td><strong>ZLB (Filtered 01/2009-03/2014)</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Macro-Finance</td>
<td>0.11</td>
<td>0.18</td>
<td>0.31</td>
<td>0.52</td>
<td>0.46</td>
<td>0.24</td>
<td>1.08</td>
<td>1.18</td>
</tr>
<tr>
<td>Macro-Monetary</td>
<td>0.08</td>
<td>0.09</td>
<td>0.14</td>
<td>0.11</td>
<td>0.32</td>
<td>0.60</td>
<td>1.03</td>
<td>1.04</td>
</tr>
<tr>
<td>Finance</td>
<td>0.10</td>
<td>0.08</td>
<td>0.08</td>
<td>0.39</td>
<td>0.60</td>
<td>0.72</td>
<td>0.87</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table B.4: Root mean-squared errors between the model-implied yields and the observed data over the pre-ZLB period (01/1985 - 12/2008) and ZLB period (01/2009 - 03/2014). The model-implied yields are computed using the filtered latent state values and the observed macro factors over this period.
### Table B.5: Model Simulations of Expected Future Shadow Rates

<table>
<thead>
<tr>
<th>Below Zero:</th>
<th>Macro-Finance</th>
<th></th>
<th>Macro-Monetary</th>
<th></th>
<th>Finance</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulations Beginning:</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Before QE1</td>
<td>.</td>
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<td>.</td>
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<td>.</td>
<td></td>
</tr>
</tbody>
</table>

Table B.5: Monte Carlo simulations of expected future shadow rate paths using parameter estimates from 1985-2007. I initialize the simulations using values of the filtered latent factors and observed macro factors at the specified date and simulate forward 10,000 possible paths five years into the future, using the risk neutral process described by equation (25). The "MC Mean" is the mean over all 10,000 paths. The second column of each panel specifies the dates for which the estimated mean path takes on negative values. The third column of each panel specifies the dates for which the 95% posterior coverage interval lies completely below zero. A dot indicates that the simulated shadow rate path never satisfies this criterion. "FG1" refers to the FOMC statement in which the Fed committed to keeping interest rates low through mid-2013. "FG2" refers to the FOMC statement in which the Fed committed to keeping interest rates low until economic conditions improved.
Figure B.1: Impulse Responses Over the Full Sample with the FFR - Left panel: Benchmark IRF with FFR 1960:I-2007:IV: VAR(4) with GDP, CPI, Commodity Prices, and FFR as the policy instrument. This column gives the VAR results using data from 1960:I-2007:IV and serves as our baseline "truth" during normal, non-ZLB environments. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the policy instrument.

Right panel: IRF of full-sample VAR which extends the dataset from 1960:I-2013:III and continues using the FFR as the policy instrument through the ZLB period. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, incorporating data through the ZLB period without accounting for any potential changing macroeconomic dynamics. The thick blue line and the blue shaded region replicate the plots from the benchmark model.
Figure B.2: Impulse Responses Over the Full Sample with the FFR and Balance Sheet and Policy Announcement Effects - Left panel: Benchmark IRF with FFR 1960:I-2007:IV: VAR(4) with GDP, CPI, Commodity Prices, and FFR as the policy instrument. This column gives the VAR results using data from 1960:I-2007:IV and serves as our baseline "truth" during normal, non-ZLB environments. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the policy instrument. Right panel: IRF of full-sample VAR which extends the dataset from 1960:I-2013:III and uses the FFR to represent the policy instrument during the ZLB period but also includes controls for changes in the size of the Fed’s balance sheet as well as policy announcement effects over this time. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, incorporating data through the ZLB period without accounting for any potential changing macroeconomic dynamics. The thick blue line and the blue shaded region replicate the plots from the benchmark model.
Figure B.3: Plot of quarterly nominal FFR and estimated shadow rates over the period from 2006:I - 2013:III. The shaded area highlights the Great Recession period in the US.
Figure B.4: Impulse Responses Over the Full Sample with Shadow Rates - Left panel: Benchmark IRF with FFR 1960:I-2007:IV: VAR(4) with GDP, CPI, Commodity Prices, and FFR as the policy instrument. This column gives the VAR results using data from 1960:I-2007:IV and serves as our baseline "truth" during normal, non-ZLB environments. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the policy instrument. Center panel: IRF of full-sample VAR which extends the dataset from 1960:I-2013:III and uses the Krippner (2012) shadow rate as the policy instrument through the ZLB period. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, incorporating data through the ZLB period. The thick blue line and the blue shaded region replicate the plots from the benchmark model. Right panel: IRF of full-sample VAR which uses the Wu and Xia (2014) shadow rate as the policy instrument through the ZLB period.
Figure B.5: Impulse Responses in the ZLB Environment with Shadow Rates- Left panel: Benchmark IRF with FFR 1960:I-2007:IV: VAR(4) with GDP, CPI, Commodity Prices, and FFR as the policy instrument. This column gives the VAR results using data from 1960:I-2007:IV and serves as our baseline "truth" during normal, non-ZLB environments. The thick blue line and the blue shaded area show the median point estimate and the 68% posterior coverage, respectively, for the impulse responses to a 25 basis point shock to the policy instrument. Center panel: IRF of VAR(4) using data from the ZLB period only, 2008:I-2013:III, and using the Krippner (2012) shadow rate as the policy instrument. The thick green line and the green shaded area give the median IRF point estimates and 68% posterior coverage, respectively, using only data from the ZLB period. The thick blue line and the blue shaded region replicate the plots from the benchmark model. Right panel: IRF of VAR(4) using data from the ZLB period only, 2008:I-2013:III, and using the Wu and Xia (2014) shadow rate as the policy instrument.
Figure B.6: Monthly macro data on real activity, measured by the unemployment gap (CBO estimate of the natural rate of unemployment minus the observed unemployment rate), and core CPI inflation from 01/1985-03/2014.

Figure B.7: Plot of effective FFR and all three filtered shadow rates from prior to the financial crisis through the end of the sample: 01/2006 through 03/2014. The shaded area indicates the NBER-dated recession period. The vertical bars highlight specific dates on which the Fed announced its unconventional policy programs.
Figure B.8: Effective FFR with the model-implied shadow rate from the Finance model and the Wu & Xia (2014) shadow rate. Both models include three latent factors and no macro data. The shadow rate time series splice together the observed values of the effective FFR from 01/2000 through 12/2008 with the shadow rate values from 01/2009 through 03/2014.

Figure B.9: Plot of effective FFR, Macro-Finance shadow rate, the systematic component of the shadow rate describing the policy rule, and the unobserved factor representing shocks to policy from 01/2000 through 03/2014. The shaded areas indicates the NBER-dated recession periods.
Figure B.10: Simulated future paths of the shadow rate from each model specification. The projections are based upon market expectations of the shadow rate at the end of the sample, 03/2014, and the factors are simulated forward for five years, based upon the risk neutral dynamics in equation (26). The expected shadow rate is described by equation (3). The blue line represents this MC estimated mean and the bands give the 95% posterior coverage.
REFERENCES


