Competitive Supply Chain Strategies in the Retail Sector

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ABSTRACT

YEN-TING LIN: Competitive Supply Chain Strategies in the Retail Sector
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In response to increasing competition, quick response (QR) and vertical integration are commonly used strategies in the retail industry to gain competitive edge. While the benefits of these strategies have been well studied in a monopoly setting, their value under competition has received less attention. It is therefore essential to understand the competitive value of these strategies. In the first chapter, we investigate the value of an additional in-season replenishment opportunity provided by QR in a supply chain with a manufacturer serving two competing retailers. We find offering QR to only one of the ex-ante symmetric retailers may be the optimal policy for a manufacturer, rather than offering QR to both of them or refraining from offering it at all. Moreover, we show QR may prove detrimental to a retailer when retail competition is taken into account. In the second chapter, we examine the value of vertical integration for a manufacturer under channel competition. We build a model with two competing supply chains, each with a supplier, a manufacturer and a retailer. The manufacturer considers three strategies: (1) forward integration, (2) backward integration, and (3) no integration. We show backward integration benefits a manufacturer while forward integration can be harmful to it. For manufacturers’ competitive choice of integration strategy, we find manufacturers encounter prisoner’s dilemma: every manufacturer chooses to vertically integrate, making them and the entire channel worse off than they would be if none of them vertically integrate. Finally, vertical integration can result in a better quality product sold at a lower price. In the third chapter, we examine the impact of having strategic customers on firms’ profitability and the performance of the entire channel. Interestingly, we show that having strategic customers benefits a supplier from higher sales. It also benefits a retailer when a product is sufficiently, but not overly, fashionable. The total supply chain profit can be higher with strategic customers. A decentralized channel with strategic customers can perform better than a centralized one.
with strategic customers or a decentralized one with myopic customers. That is, decentralized decision making and having strategic customers can improve channel performance.
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CHAPTER 1

Introduction

1.1 Motivation

Retailing is highly relevant to our daily life, and it also is a vital sector of most developed economies. In the U.S. the retail business represents about 40% of the Gross Domestic Product (GDP) and is the largest domestic employer (Fisher and Raman 2001). Many operational strategies have been developed in the retail industry to gain competitive edge. Among these strategies, quick response (QR) and vertical integration have drawn much attention from academic circles for their popularity, and the value of these strategies has been shown within a monopoly setting.

The retail industry, however, is facing two challenges: growing competition and smarter customers. Many retailers extend their reach beyond domestic markets after their domestic success. For instance, the Japanese apparel retailer Uniqlo invaded the U.S. market in 2006 (Alexander 2009). Such globalization draws more players into the arena and intensifies competition faced by retailers. While fierce competition affects the value of operational strategies, the value of QR and vertical integration has received less attention. Moreover, customers are trained to be strategic: many shoppers intentionally delay their purchase until discounts are offered in holiday seasons. This strategic move potentially lowers sales at the full price and undermines retailers’ margins.
In this dissertation, we focus on retail competition and customers’ strategic behavior. We examine how retail competition affects the value of quick response and vertical integration, as well as the impact of customers’ strategic behavior on firm performance. Toward this end, we draw from literature in operations, strategy, marketing, and economics to develop analytical models. First, in Chapter 2, we study firms’ QR offerings and adoption decisions in competition. In Chapter 3, we shift our focus to vertical integration, addressing a manufacturer’s choice between forward and backward integration. Finally, in Chapter 4 we analyze how having strategic customers affects the performance of a supply chain.

1.2 Dissertation Overview

1.2.1 Quick Response under Competition

Quick response was initiated by the U.S. apparel industry during the mid-1980s (Hammond and Kelly 1990). It aims to increase production flexibility and therefore drastically reduces supply lead times. This directly benefits retailers by enabling additional inventory replenishment as updated demand information becomes available. The benefits of quick response have gained much attention, and more and more firms have adopted QR to gain a competitive edge. Nevertheless, the growing popularity of QR also has intensified competition, potentially diminishing QR’s value. While the benefit of QR in a monopoly setting has been studied extensively, its value under retail competition has received less attention. Moreover, it is not uncommon to see a manufacturer serving competing customers. For example, Hot Kiss, a California based manufacturer serves junior fashion retailers Hot Topic and deLia’s as well as upscale department stores like Dillard’s and Nordstrom (Bhatnagar 2009). Nevertheless, literature has focused only on single-manufacturer-single-retailer relationships (e.g., Fisher and Raman 1996; Iyer and Bergen 1997; Caro and Martínez-de-Albéniz 2010). The value of QR when a manufacturer serves multiple competing customers has not been well understood.

In Chapter 2, we develop a stylized model with a manufacturer serving two competing retailers. The retailers place their regular (initial) orders before the selling season starts. In
addition to an initial order, quick response allows a retailer an additional replenishment opportunity after demand uncertainty is resolved. The manufacturer sets the price for replenishment opportunities while the retailers decide their inventory order quantities. We characterize retailers’ ordering decisions and the manufacturer’s pricing decisions in equilibrium when none, only one, and both of the retailers have QR capability. We then compare firms’ decisions and profits across scenarios to reveal the value of QR.

We first confirm the popular belief that QR capability is beneficial to all firms in a monopoly setting. However, in the presence of retail competition, we find that offering QR to only one of the ex-ante symmetric retailers may be the optimal policy for a manufacturer, rather than offering QR to both of them or refraining from offering it at all. Furthermore, we find that QR may prove detrimental to a retailer when retail competition is taken into account. Interestingly, a retailer that does not have QR capability may benefit from its competitor’s QR capability. In addition, we confirm that the above insights continue to hold for the following extensions: (1) when the manufacturer has full control over the price for all ordering opportunities; (2) alternative sequences of events; (3) when the manufacturer has a capacity limit for QR replenishment; and (4) numerical study for other demand distributions.

A key message of this research is that firms need to pay attention to retail competition before they make their QR offering/adoption decisions. A manufacturer does not necessarily benefit from offering QR service to all of its customers. On the other hand, adopting QR actually may hurt a retailer.

1.2.2 Competitive Vertical Integration Strategies in the Fashion Industry

Vertical integration extends a firm’s operational reach in a supply chain. In chapter 3, we examine manufacturers’ choices among no integration, forward integration and backward integration. Forward integration extends a manufacturer’s reach over product distribution, tightening its grip on the demand side, while backward integration allows stronger control over the quality on the supply side. Both types of integrations are seen commonly in practice, and firms present inconsistent choice of integration strategies. In the apparel industry, for example, Chinese
manufacturer Esquel chooses backward integration for control over its cotton supply (Peleg-Gillai 2007), while the Taiwanese manufacturer Tainan Enterprise chooses forward integration, launching its own brand and establishing its distribution channel (Ho 2002). It is intriguing to observe that in practice manufacturers - even in the same industry - demonstrate inconsistency in the direction of vertical integration. Thus, in the presence of channel competition, we analyze (1) the impact of vertical integration on firm profit, product quality and retail price; (2) the effect of a manufacturer’s vertical integration on others choice of integration strategy; and (3) manufacturers’ equilibrium integration strategies.

Vertical integration has received significant attention in marketing, economics and strategy literature. Yet, previous studies consider only manufacturer-retailer integration (forward integration). We contribute to this line of research in two dimensions. First, by considering both forward and backward integrations, we capture more options available in practice and are able to examine a firm’s trade-off between them. Second, we endogenize firms’ investment in quality improvement and investigate the effect of vertical integration on product quality.

We build a stylized model with two competing supply chains, each with a supplier, a manufacturer and a retailer. The supplier can attempt to improve the quality of material it supplies to the manufacturer. The manufacturer makes a product and sells it exclusively through the retailer. The manufacturer considers three strategies: (1) forward integration, (2) backward integration, and (3) no integration. We analyze firms’ equilibrium decisions and profitability under various supply chain structures.

Interestingly, we find that backward integration benefits a manufacturer while forward integration can be harmful. A manufacturer is more inclined toward forward integration when its competitor integrates vertically. For manufacturers’ competitive choice of integration strategy, we find that manufacturers encounter a prisoner’s dilemma: in equilibrium, every manufacturer chooses to integrate vertically, and their performance will be worse off than if none of them integrates vertically. Finally, vertical integration can result in a better quality product sold at a lower price.
1.2.3 Are Strategic Customers Bad for a Supply Chain?

Customers today are trained to wait for sales. They anticipate deep discounts on, for example, the day after Thanksgiving, and therefore, intentionally delay their purchase. This behavior limits retailers’ demand at full price and increases their challenges during sales seasons. Customers’ strategic delay of purchase has gained growing attention to operations management, and many remedies have been proposed to counteract customers’ strategic behavior. While it is expected that customers’ strategic behavior would have an adverse effect on firm profits, the impact of that behavior on the performance of the entire supply chain has not been studied. Therefore, our goal is to understand the impact of customers’ strategic behavior on the performance of the firms and that of the entire supply chain, as a whole.

To that end, we build a model with a single supplier serving a single retailer who sells a product over two periods. The supplier sets the unit wholesale price it charges to the retailer; the retailer determines its order quantity and the retail price in each period. To understand the impact of customers’ strategic behavior, we compare firm profit and the total supply chain profit between two scenarios: (1) when customers are myopic, and (2) when they are strategic. Myopic customers make their purchase decisions based solely on the current retail price without considering future change in price. In contrast, strategic customers consider future change in price and time their purchase; they may postpone their purchase in anticipation of future discount.

Our results show that firm profits can be higher when customers are strategic. By holding less inventory, a retailer eliminates customers’ incentive to delay their purchase, increasing their willingness to buy at full price. As a result, this benefits a supplier with increased revenue. Moreover, when the product is sufficiently, but not overly, fashionable, a supplier charges a lower wholesale price to encourage its sales, and this benefits a retailer with lower costs.
CHAPTER 2

Quick Response under Competition

2.1 Introduction

Quick response (QR) is an operational lever that aims to provide better response to variations in demand. One of its benefits is to enable in-season replenishment through lead time reduction. The success of QR has received much attention (Hammond and Kelly 1990), and its benefits have been studied extensively in literature (e.g., Fisher and Raman 1996; Iyer and Bergen 1997). Naturally, more and more firms have adopted QR to gain a competitive edge. For example, a burgeoning British apparel retailer, Primark, uses QR for faster product turnover, and it fetched 10.1% market share, while the market leader Marks & Spencer garnered 11.4% market share in the U.K. in 2008 (Vickers 2008). Nevertheless, the growing popularity of QR has also intensified competition, which can potentially diminish the value of QR. For instance, after its domestic success with quick response, the Japanese retailer, Uniqlo invaded the U.S. market in 2006 (Alexander 2009), and a similar move is taken by the British retailer Topshop whose New York flagship opened in 2009 (Resto 2010). The effect of competition on QR, however, has received less attention and not been fully understood, and it is our main area of focus in this paper.
Despite the extensive studies on the benefits of QR for retailers (e.g., Caro and Martínez-de-Albéniz 2010; Cachon and Swinney 2009), there has been less focus on the value of QR to a manufacturer. When should a manufacturer offer QR? What is its optimal supply chain structure? Should a manufacturer serving competing retailers offer QR? Indeed, it is not uncommon to see a manufacturer serving competing customers. For example, Hot Kiss, a California-based manufacturer serves junior fashion retailers Hot Topic and deLia’s as well as upscale department stores like Dillard’s and Nordstrom (Bhatnagar 2006). Hot Kiss achieves quick response by taking advantage of local production in California. Similarly, Makalot, a leading Taiwanese apparel manufacturer serves Kohl’s, Target, JC Penny and Gap. In addition to regular deliveries, Makalot also provides faster in-season deliveries to its clients, and achieves quick response by flexible capacity allocation and improved information sharing with its clients.¹ In the footwear industry, Yue Yuen, a major sportswear manufacturer that provides a shorter lead time than its competitors, supplies brand names like Nike, Puma and Adidas (Taylor 2008).

In this paper, we model a supply chain with a single manufacturer supplying homogeneous products to two competing retailers. The retailers sell their products in a consumer market with a single selling season. Prior to the selling season, the manufacturer sets the QR price for QR replenishment, and then each retailer places a regular (initial) order at an exogenous wholesale price. We allow the manufacturer to determine this price in Section 2.6.1. After observing the actual demand, each retailer with QR ability places a second order at the QR price. Finally, the selling season starts and the retailers compete in the consumer market, following Cournot quantity competition. Quantity based competition is appropriate for industries with long supply lead time (e.g., apparel and footwear); in these industries, price competition is less likely because

¹Chou, L., Personal interview with the president of Makalot. February 2009.
it requires instant adjustment of production quantity (Feng and Lu 2010). We consider three scenarios, with zero, one, and both retailers having QR ability, respectively. We derive the equilibrium for each of these scenarios and their comparison leads to a number of interesting results.

As a result of interplay between demand variability and retail competition, we find that the manufacturer may find it optimal to offer QR to only one of the ex-ante symmetric retailers, rather than both of them. When a retailer attains QR ability, the tendency is to reduce the initial order quantity and use the QR order to fulfill any additional demand. The manufacturer’s value of QR therefore, depends on the trade-off between the initial order loss and additional QR profit gain. As demand variability decreases, the expected size of QR orders and therefore the manufacturer’s QR profit decreases as well. Furthermore, due to intensifying effect on retail competition, the manufacturer’s QR profit from offering QR to the second retailer is less than that of the first. Thus, when demand variability is sufficiently small, although the manufacturer’s QR profit from the first retailer outweighs the profit loss in its regular orders, its QR profit from the second retailer is insufficient to compensate the profit loss in its regular order. Thus, it is more advantageous for the manufacturer to offer QR exclusively to one of the retailers. When demand variability is sufficiently large, the manufacturer offers QR to both of the retailers. Moreover, the total channel profit can also be maximized with only one retailer with QR option instead of both, as retail competition hinders the value of having a second retailer with QR option.

When retail competition is ignored, QR always benefits a retailer. Surprisingly, however, we find that in the presence of retail competition having QR ability can be detrimental to a retailer.

Furthermore, quantity based competition keeps the problem tractable, thus, it is commonly used in competitive models in the operations literature (e.g., Ha et al. 2008; Anand and Girotra 2007; Goyal and Netessine 2007; Mendelson and Tunca 2007).
when demand variability is sufficiently small. When competing against a competitor without QR ability, the competitor increases its order quantity to compensate for its lack of QR ability by ordering a high amount, threatening to deflate the price. This in turn forces the retailer with QR option to reduce its initial order. When demand variability is small, the benefit of using QR to match additional demand is insignificant. Consequently, gaining QR ability hurts the retailer due to potential loss from the initial order. In contrast, when the demand variability is sufficiently large, QR benefits the retailer. Similarly, when competing against a competitor who already has QR ability, not having QR ability enables a retailer to force its competitor to reduce its initial order quantity. When demand variability is small, committing to such a threat as a result of not having QR ability dominates the benefit of reducing mismatch between supply and demand using QR ability.

We demonstrate our results can continue to hold for a number of extensions by: (i) Allowing the manufacturer to set the wholesale price endogenously; (ii) Considering alternative sequence of events such as allowing the QR price to be set after retailers place their regular orders or after demand uncertainty is resolved; (iii) Considering normally distributed demand through numerical studies; and, (iv) Studying the outcomes when the manufacturer has limited capacity for fulfilling QR orders. Overall, our results demonstrate how retail competition changes the value of QR, and provide managerial insights to a manufacturer’s QR offering decision as well as a retailer’s QR adoption decision.

These extensions also yield some additional results. Specifically, when the QR price is determined after the retailers place their initial orders, the manufacturer may find it optimal not to offer QR to any of the retailers. In addition, when there is limited QR capacity, the manufacturer may always find it optimal to offer QR to only one of the retailers due to capacity
limit even when demand variability is sufficiently high.

The remainder of this paper is organized as follows. In Section 2.2, we present our literature review. Section 2.3 describes our model. Section 2.4 derives the equilibrium. Section 2.5 discusses the value of QR both from the manufacturer’s and retailers’ perspective. Section 2.6 presents several extensions to the base model. Section 2.7 offers our concluding remarks. We present the monopoly retailer benchmark in Section 6.1.1, and all proofs appear in Section 6.1.7.

2.2 Literature Review

Understanding the value of QR has attracted growing attention in academic circles since the early 1990s. Here, we first summarize papers that consider QR in the monopoly setting. In their seminal paper, Fisher and Raman (1996) show how early sales information can be used to improve demand forecasts and better manage production decisions. Iyer and Bergen (1997) evaluate the effect of lead time reduction enabled by QR in a two-level supply chain, and find that QR benefits the retailer while it may be detrimental to the manufacturer. Eppen and Iyer (1997) examine the value of backup agreements. Under this agreement, a retailer can place an additional order using QR up to a certain percentage of its initial order at the original cost, but any additional order in excess of that fraction is charged a higher cost. They show that backup agreements can benefit both the retailer and the manufacturer. Cachon and Swinney (2009) identify the sufficient conditions under which QR benefits a retailer when it faces strategic consumers. Fisher et al. (2001) propose a heuristic that determines both ordering quantities and in-season replenishment time for a catalog retailer, finding this procedure offers the potential to double that retailer’s profit. These papers do not consider the effect of competition, and
they treat the price for QR replenishment as exogenously determined. By contrast, we study the effect of competition on the value of QR, allowing the manufacturer to set endogenously the price for QR replenishment.

While the above studies are restricted to monopoly, recently, Caro and Martínez-de-Albéniz (2010), Li and Ha (2008) and Krishnan et al. (2010) have examined the competitive value of QR. Caro and Martínez-de-Albéniz (2010) and Li and Ha (2008) focus on retailer competition like our paper whereas Krishnan et al. (2010) focus on manufacturer competition. Specifically, Caro and Martínez-de-Albéniz (2010) and Li and Ha (2008) consider duopoly retailers competing for spill-over demand where consumers seek the other retailer only when their first choice retailer runs out of stock. In contrast, we adopt Cournot competition, in which inventory competition has a direct impact on the retail price. In addition, both Caro and Martínez-de-Albéniz (2010) and Li and Ha (2008) treat the retailers’ cost for QR replenishment as exogenous, whereas we allow the manufacturer to set that price. This allows us to study vertical interactions between the manufacturer and the retailers in addition to horizontal retail competition. Such variances in modeling approaches also leads to different results. Assuming identical prices for all replenishment opportunities, Caro and Martínez-de-Albéniz (2010) demonstrate that QR always benefits a retailer. Similarly, Li and Ha (2008) find a firm always benefits from having reactive capacity that enables replenishments after better demand information is observed. In contrast, we find QR may hurt a retailer when demand variability is too small.

Krishnan et al. (2010) consider a manufacturer selling its product through a retailer who also carries a competing product from another manufacturer. The retailer can exert sales effort to switch demand from one product to another. Therefore, their model studies the competition between two manufacturers’ products sold by a single retailer. In contrast, we consider the
competition between two retailers selling products supplied by a single manufacturer. Krishnan et al. (2010) find that QR can hurt the manufacturer’s sales because it reduces the retailer’s commitment to promote the product.

QR enables additional order placement after better demand information becomes available, and there exists a rich literature studying how firms can make use of updated demand information in their procurement decisions. Gurnani and Tang (1999) analyze a situation in which a retailer can place a second order when it receives better demand information. But at the time of the first order, the price for the second order is uncertain. Weng (2004) considers a single-buyer single-manufacturer channel in which the manufacturer is able to dictate its price for the buyer’s second order. He presents a quantity discount scheme that coordinates the channel. Milner and Kouvelis (2005) study the effect of demand characteristics on the value of supply chain flexibility, which is characterized by the timing or quantity flexibility for the second ordering opportunity. Donohue (2000) shows that a buy-back contract can achieve channel coordination for a supply chain with one manufacturer supplying a single retailer. Cvsa and Gilbert (2002) examine a manufacturer’s trade-off between offering early and delayed purchases. In their model, a retailer places an order either before or after demand uncertainty is realized, whereas we allow a retailer to place orders both before and after the uncertainty is resolved. In addition, Cachon (2004), Dong and Zhu (2007), and Erhun et al. (2008a) examine the impact of push, pull, and advance-purchase discount contracts. Although these models incorporate the idea of making use of updated demand information in procurement decision, only Milner and Kouvelis (2005), Erhun et al. (2008a) and Cvsa and Gilbert (2002) study the value of this ability. Moreover, only Cvsa and Gilbert (2002) consider the effect of competition.

Models with multiple ordering decisions, albeit without demand information updates, are
also of particular interest to operations management. Martínez-de-Albéniz and Simchi-Levi (2007) and Erhun et al. (2008b) examine the effect of multiple procurement opportunities before an uncertain selling season starts. Both find more frequent procurement decreases double marginalization while increasing profitability for all supply chain participants. Anand et al. (2008) present a two-period model with identical deterministic demand curves and endogenous wholesale prices. They remove classical motivations and highlight strategic implications for carrying inventory. This model is extended by Keskinocak et al. (2008) to incorporate capacity limitations. Works that highlight competition with multiple ordering opportunities include Hall and Porteus (2000), Netessine et al. (2006), and Liu et al. (2007). All of these study products sold in multiple periods, whereas we are concerned with a short life cycle product that is sold over a single period. Furthermore, these works do not study the effect of improved demand information on inventory decisions.

In addition to using quick response, researchers have identified a number of operational strategies to cope with demand uncertainty. For example, a firm that produces and sells its product directly to consumers may invest in reactive capacity to allow for additional production after better demand information is obtained (e.g., Raman and Kim, 2002; Li and Ha, 2008). Although both QR and reactive capacity enable a second replenishment opportunity, the second replenishment is limited by the capacity level set beforehand in the case of reactive capacity. Delayed product differentiation provides firms with another instrument to respond to demand uncertainty (e.g., Lee and Tang, 1997; Anand and Girotra, 2007). It allows a firm to configure an intermediate good into different products after demand uncertainty is resolved, whereas QR considers a firm’s ability to order additional inventory. Finally, spot trading is also another commonly used strategy and its value is studied by Mendelson and Tunca (2007). While spot
trading allows retailers to trade among themselves, in our model QR only allows them to buy additional units from the manufacturer. Mendelson and Tunca (2007) show spot trading can adversely affect a firm, and similarly we find that QR ability can be harmful in a competitive environment.

2.3 The Model

First we introduce the demand model, followed by detailed descriptions of the firms’ decisions. We consider a manufacturer supplying homogeneous products to two competing retailers, indexed by \( i = 1, 2 \). All firms are risk neutral and seek to maximize their individual expected profits. The retailers sell their products in an uncertain consumer market with a linear demand curve:

\[
p = A - \sum_{i=1}^{2} X_i,
\]

where \( p \) is the clearing price, \( X_i \) is the quantity sold by retailer \( i \), and \( A \) is the demand state that takes values \( m + v \) and \( m - v \) with equal probabilities, i.e., \( P(A = m + v) = P(A = m - v) = 0.5 \), where \( m \) is the mean demand, and \( v \) is a measure of demand variability. We also discuss what happens when \( A \) is normally distributed in Section 2.6.4. The distribution of \( A \) is public information. We assume \( 0 < v < m \) to avoid non-positive demand state. We refer to \( A = m + v \) as “high market,” and similarly \( A = m - v \) as “low market.”

There are two types of retailers: slow (S) and fast (F). They differ in their ordering opportunities. A slow retailer has only one ordering opportunity: it places its regular order before the demand uncertainty is resolved. In addition to this initial order, a fast retailer has QR
ability to place a second order after the demand uncertainty is resolved. Each retailer places its regular order \( Q_i \) at a wholesale price \( c_w \) per unit, and each fast retailer places its QR order \( q_i \) at a price \( c_q \) per unit. We assume the order quantities are public information, which is common in models of inventory competition (e.g., Li and Ha 2008; Netessine et al. 2006; Olsen and Parker 2008). The products are sold in a single selling season. We assume that the salvage value of the products is insignificant, and the retailers sell out all of their inventory in the selling season, that is, \( X_i = Q_i + q_i \). This is a common assumption in literature (e.g., Goyal and Netessine 2007; Chod and Rudi 2005; Anand et al. 2008). Therefore, a retailer’s profit \( \pi_i \) is given as follows. Note that \( q_i = 0 \) for a slow retailer \( i \), and it only chooses \( Q_i \).

\[
\pi_i = (A - \sum_{j=1}^{2} (Q_j + q_j))(Q_i + q_i) - c_w Q_i - c_q q_i, \quad i = 1, 2, 
\]

(2.1)

As (2.1) shows, retailer \( i \)’s profit consists of three parts: the first part represents retailer \( i \)’s revenue; the second, its cost for the initial order; and the last part captures its cost for the QR order.

The wholesale price \( c_w \) for regular orders is exogenously determined. However, the manufacturer determines its unit price \( c_q \) for QR orders. This mimics the situation in which many other manufacturers are able to deliver the products when given sufficiently long lead time, determining that the wholesale price \( c_w \) is dictated by competition. By contrast, few other manufacturers are able to offer quick response as it requires additional capabilities. This makes it possible to dictate its QR price \( c_q \). Note that we also study what happens when the wholesale price \( c_w \) is set endogenously in Section 2.6.1.

The manufacturer’s production cost for regular orders is normalized to zero. Because imple-
menting QR requires additional costs (e.g., overtime expenses and more costly transportation methods) however, the manufacturer incurs a cost premium \( \delta > 0 \) per unit for QR replenishments. We assume \( \delta < v \) to eliminate trivial cases in which the QR cost \( \delta \) is so high, QR is never used. Thus, given the retailers’ order quantities, the manufacturer’s profit \( \pi_M \) is calculated as:

\[
\pi_M = c_w \left( \sum_{i=1}^{2} Q_i \right) + (c_q - \delta) \left( \sum_{i=1}^{2} q_i \right).
\]  

(2.2)

To avoid an additional trivial case, we assume \( c_w < m \). When \( c_w \geq m \) the product is not feasible (i.e., no unit will be sold). This can be seen clearly from (2.3).

Figure 2.1 shows the order of events: First, the manufacturer announces the QR price \( c_q \). Retailers then place their regular orders simultaneously for delivery before the beginning of the selling season. The demand state \( A \) is revealed completely to the retailers. Next, each fast retailer places its QR order, which will also be delivered before the selling season. Finally, the selling season ensues during which the retailers sell their inventory, and profits are realized.
2.4 Competition

We consider three competition scenarios, denoted by SS (two competing slow retailers), FS (one fast retailer versus one slow retailer), and FF (two competing fast retailers). In this section, we solve for the firms’ subgame perfect Nash equilibrium (SPNE) strategies in each scenario. We will compare these scenarios to characterize the value of QR in the next section.

2.4.1 SS Scenario (Two Competing Slow Retailers)

We consider the SS scenario as a benchmark. In this scenario, none of the retailers has QR ability, i.e., each retailer can place only a single order that must be decided prior to the resolution of demand uncertainty. Consequently, this problem reduces to a single stage standard Cournot duopoly model (Tirole 1988). In this scenario, retailer $i$’s expected profit is given by $E[\pi_i]$, where $E$ is the expectation with respect to the demand intercept $A$ and $\pi_i$ is given in (2.1) with $q_1 = q_2 = 0$. It is straightforward to show that the unique equilibrium is given by:

$$Q_i = \frac{m - c_w}{3}, \quad i = 1, 2.$$  

(2.3)

2.4.2 FS Scenario (A Fast Retailer versus a Slow Retailer)

We now study competition between a fast (1) and a slow retailer (2): In this scenario, as described by the sequence of events given in Figure 2.1, QR ability allows the fast retailer to place an additional order after demand uncertainty is revealed. In the following, we derive the firms’ equilibrium decisions by applying backward induction.

In the last stage game, the demand state $A$ is revealed to the retailers. The fast retailer
determines its QR order quantity \( q_1 \) to maximize its profit \( \pi_1 \) that is given by (2.1). It is straightforward to show that \( \pi_1 \) is concave in \( q_1 \), and, following the first order condition, retailer 1’s optimal QR order quantity is given by:

\[
q_1 = \left( \frac{A - c_q - Q_2}{2} - Q_1 \right)^+,
\]

where, \( A \) is the demand state, \( c_q \) is the unit QR ordering cost, and \((x)^+ = \max(0, x)\). As (2.4) shows, retailer 1 places its QR order following a base-stock policy and the base-stock level decreases in both the QR price and the competing retailer’s regular order quantity.

In the second stage game, the retailers determine their regular order quantities to maximize their expected profits \( E[\pi_i] \). The following lemma characterizes the retailers’ equilibrium regular and QR order decisions.

**Lemma 1** There exists a unique equilibrium for the retailers’ regular order quantity game in the FS scenario. The retailers’ equilibrium actions are described below and the equilibrium regular order quantities are given in Section 6.1.2 in the Appendices.

(i) For \( \bar{\theta}_{FS} \leq c_q \): \( Q_2 = Q_1 \geq 0 \), and retailer 1 does not place a QR order for any market outcome.

(ii) For \( \underline{\theta}_{FS} \leq c_q < \bar{\theta}_{FS} \): \( Q_2 > Q_1 \geq 0 \), and retailer 1 places a QR order only in a high market.

(iii) For \( c_q < \underline{\theta}_{FS} \): \( Q_2 > Q_1 = 0 \), and retailer 1 places QR orders in both high and low market outcomes, where

\[
\bar{\theta}_{FS} = c_w + v \text{ and } \underline{\theta}_{FS} = \min(c_w, \frac{3}{4}m + \frac{4}{7} c_w - \frac{5}{7}v, m - v).
\]
A higher QR price, \( c_q \), reduces the attractiveness of QR ability. As a result, when \( c_q \) is sufficiently high, as in case (i) of Lemma 1, QR is never used and thus the retailers’ behavior is identical to that of the SS scenario. On the other hand, QR is used only in a high market for \( \bar{\theta}^{FS} \leq c_q < \tilde{\theta}^{FS} \). In this case, the slow retailer places a larger regular order than its fast competitor to compensate for the lack of QR option. Finally, when \( c_q < \bar{\theta}^{FS} \), the QR price is extremely low, and the fast retailer relies only on QR for inventory replenishment, it does not place a regular order.

In the first stage game, the manufacturer sets the QR price, \( c_q \), to maximize its expected profit \( E[\pi_M] \). We characterize the manufacturer’s optimal \( c_q \) in the following proposition:

**Proposition 1** Let

\[
\beta^{FS} = \begin{cases} 
\frac{18m + \sqrt{21(3m-5v+5\delta)}}{9\delta} & \text{for } v \leq \frac{3}{5}m + \delta \\
 m - \frac{5}{6}(v - \delta) & \text{otherwise}
\end{cases}
\]

(i) When \( c_w < \beta^{FS} \), the manufacturer sets \( c_q = c_w + \frac{v + \delta}{2} \), the retailers order \( Q_1 = \frac{3}{10}(m - c_w) - \frac{1}{4}(v - \delta) \), \( Q_2 = m - c_w \).

(ii) When \( c_w \geq \beta^{FS} \), the manufacturer sets \( c_q = \min(\frac{8c_w - 3m}{5} + v, \frac{3m + 8c_w + 7(v + \delta)}{14}) \), the retailers order \( Q_1 = 0 \), \( Q_2 = \left( \frac{45m - 48c_w - 7(v - \delta)}{48} \right)^+ \).

In both cases, the fast retailer places a QR order only in a high market, and its QR order quantity is given by (2.4).

When \( c_w \geq \beta^{FS} \), the wholesale price is extremely high and this results in a trivial case, where the fast retailer never places a regular order, whereas when \( c_w < \beta^{FS} \) both retailers
place a regular order. In comparison to the SS scenario, equation (2.3) and Proposition 1 show
the fast retailer chooses a smaller regular order quantity because it has a second replenishment
opportunity.

2.4.3 FF Scenario (Two Competing Fast Retailers)

The FF scenario concerns competition between two fast retailers. Here both retailers can
place a QR order after the market uncertainty is resolved, as shown in Figure 2.1. We derive
the firms’ equilibrium decisions by applying backward induction. In the last stage game, the
retailers determine their QR order quantities. It is straightforward to show that each retailer
i’s profit, as given in (2.1), is concave in its QR order quantity \( q_i \). Therefore, retailer i’s best
response QR order quantity, \( q^{BR}_i \), can be derived using the first order condition:

\[
q^{BR}_i(A, Q_i, Q_j, q_j) = \left( \frac{A - c q - q_j - Q_j}{2} - Q_i \right)^+, 
\]

where, \( i = 1, 2 \) and \( j = 3 - i \). Without loss of generality, we assume that retailer i places a
larger regular order, i.e., \( Q_i \geq Q_j \). Let \( q^{FF}_i \) be retailer i’s equilibrium QR order quantity in the
FF scenario. By using the fact that the equilibrium should satisfy \( q^{BR}_i(A, Q_i, Q_j, q^{FF}_j) = q^{FF}_i \),
we obtain the following equilibrium QR order quantity pair:

\[
(q^{FF}_i, q^{FF}_j) = \begin{cases}
(\frac{A - c q - Q_i}{3} - Q_i, \frac{A - c q}{3} - Q_j), & \text{if } A - c q \geq 3Q_i, \\
(0, (\frac{A - c q - Q_i}{2} - Q_j)^+), & \text{otherwise}
\end{cases}
\]  

(2.5)

Thus, a retailer places a QR order only when its regular order quantity \( Q_i \) relative to the
demand \( A \) is sufficiently small.
In the second stage game, the retailers determine their regular order quantities simultaneously to maximize their expected profits prior to observing the actual demand state. The following lemma describes the retailers’ equilibrium actions:

**Lemma 2** There exists a unique equilibrium for the retailers’ regular order quantity game in the FF scenario. In equilibrium, \( Q_1 = Q_2 \) and they are given in Section 6.1.2 in the Appendices. The retailers’ equilibrium actions are given below:

(i) For \( \bar{\theta}^{FF} \leq c_q \): the retailers do not place a QR order for any market outcome.

(ii) For \( \bar{\theta}^{FF} \leq c_q < \bar{\theta}^{FF} \): the retailers place QR orders only in a high market.

(iii) For \( c_q < \bar{\theta}^{FF} \): \( Q_1 = Q_2 = 0 \), and the retailers place QR orders in both high and low markets, where

\[
\bar{\theta}^{FF} = c_w + v \quad \text{and} \quad \underline{\theta}^{FF} = \min(c_w, m - v).
\]

Note that Lemma 2 is structurally similar to Lemma 1. In the FS scenario, Lemma 1 establishes the slow retailer initially orders more than its fast counterpart due to asymmetric QR ability. In contrast, Lemma 2 shows the retailers choose equal regular order quantities in the FF scenario as both of them have symmetric QR ability.

In the first stage game, the manufacturer sets its QR price to maximize its expected profit \( E[\pi_M] \). The following proposition summarizes the equilibrium.

**Proposition 2** Let

\[
\beta^{FF} = \begin{cases} 
\frac{m + \sqrt{v^2 + 2m^2 - 2m\delta}}{2} & \text{for } v \leq (\sqrt{2} - 1)(m - \delta) \\
\frac{2m + \sqrt{2}(m - v + \delta)}{4} & \text{otherwise}
\end{cases}
\]
(i) When \( c_w < \beta F F \), the retailers order \( Q_1 = Q_2 = m - c_w - \frac{1}{6} (v - \delta) \), the manufacturer sets \( c_q = c_w + \frac{v + \delta}{2} \), and the retailers place QR orders only in a high market.

(ii) When \( c_w \geq \beta F F \), the retailers choose \( Q_1 = Q_2 = 0 \),

a. for \( v \leq (\sqrt{2} - 1)(m - \delta) \), the manufacturer sets \( c_q = \frac{m + \delta}{2} \), and the retailers place QR orders in both high and low markets.

b. for \( v > (\sqrt{2} - 1)(m - \delta) \), the manufacturer sets \( c_q = \frac{m + v + \delta}{2} \), and the retailers place QR orders only in a high market.

In all cases retailers’ QR order quantities are given by (2.5).

Retailer behavior in the FF scenario is similar to that of the fast retailer in the FS scenario—they place regular orders only when the wholesale price, \( c_w \), is not extremely high. In this case, the retailers place QR orders if the market turns out to be high, but do not place any QR order if the market turns out to be low. Also, equation (2.3) and Proposition 2 show a fast retailer in the FF scenario chooses a smaller regular order quantity due to QR, in comparison to the SS scenario. With a solid understanding of the firms’ equilibrium actions, we proceed to evaluate the value of QR.

### 2.5 The Value of QR

Here we study the impact of having QR ability on the profitability of all channel participants. This allows us to address numerous questions of managerial interest, including: Should the manufacturer offer QR ability to all, some or none of the retailers? How does retail competition affect the value of QR? Does QR improve the performance of the supply chain as a whole? What
is the impact of demand uncertainty? Section 2.5.1 considers the monopoly retailer benchmark. Sections 2.5.2, 2.5.3, and 2.5.4 consider duopoly competition and explore the value of QR for the manufacturer, the retailers and the whole channel.

2.5.1 Monopolist Retailer Benchmark

To tease out the effect of competition, we first consider a monopolist retailer. We will contrast monopoly and duopoly results to understand the effect of retail competition. When the manufacturer serves a monopolist retailer, the firms’ pricing and ordering decisions are described in Section 6.1.1. Let $\Pi^a_R$ and $\Pi^a_M$ be the expected equilibrium profits for the monopolist retailer and the manufacturer, respectively, when the retailer is type $a$, where $a = F, S$ stands for fast and slow. The following proposition summarizes the effect of QR on the profitability of the manufacturer, the retailer and the channel:

**Proposition 3**

(i) $\Pi^F_M > \Pi^S_M$.

(ii) $\Pi^F_R > \Pi^S_R$.

(iii) $\Pi^F_M + \Pi^F_R > \Pi^S_M + \Pi^S_R$.

Proposition 3 shows that QR increases the profitability of the manufacturer, the monopolist retailer and the entire channel. This is intuitive, because both the manufacturer and the retailer can always match their no-QR profit. The manufacturer can nullify QR options by setting a sufficiently high QR price $c_q$. Similarly, the monopolist retailer utilizes QR only if it will increase profitability.
2.5.2 Impact of QR on the Manufacturer’s Equilibrium Profit

Next we turn our attention back to duopoly retailers. For example, how many retailers should receive QR offers from the manufacturer? Most strikingly, we find that offering QR ability to only one of the ex-ante symmetric retailers may be the optimal choice. Let $\Pi_{ab}^M$ show the manufacturer’s expected equilibrium profit when retailers 1 and 2 are types $a$ and $b$, where $a, b = F, S$. We define thresholds for demand variability parameter $v$ to illustrate our results in this section, these thresholds are displayed in Table 6.1 in Section 6.1.3.

The following proposition identifies the supply chain configuration that maximizes the manufacturer’s profit:

**Proposition 4**

(i) For $v \leq v_M$, $\Pi_{FS}^M \geq \Pi_{FF}^M > \Pi_{SS}^M$.

(ii) For $v > v_M$, $\Pi_{FF}^M > \Pi_{FS}^M > \Pi_{SS}^M$.

Figure 2.2 illustrates the optimal scenario for the manufacturer as Proposition 4 describes for $m = 1$ and $\delta = 0.5$. Note that the shape of $v_M$ boundary in the figure depends on $c_w \gtrless \beta_{FS}, \beta_{FF}$ following Propositions 1 and 2.

A retailer with QR option decreases its regular order as seen in Propositions 1 and 2. Furthermore, the expected size of the QR order decreases as demand variability gets smaller. The manufacturer exchanges loss from regular orders for gain from QR orders which increases in demand variability. When demand variability is high, as in case (ii), the manufacturer prefers offering QR to both retailers. When it is small, however, as in case (i), surprisingly, the manufacturer is better off by offering QR ability to only one of the retailers as opposed to both.
of them, because the $FS$ scenario generates a larger profit for the manufacturer from regular orders than the FF scenario. In this case, such profits outweigh the additional QR profit for the manufacturer in the FF scenario. Due to retail competition, the manufacturer’s QR profit from the addition of a second fast retailer (FS to FF) is smaller than that of the first (SS to FS). Thus, even when QR profit from the first fast retailer (SS to FS) outweighs the profit loss in its regular orders, QR profit from the second (FS to FF) may not be sufficient to compensate the profit loss in its regular order. Finally, the FS scenario always yields a higher profit than the SS scenario as the manufacturer sets the QR price endogenously: it can always nullify QR option through pricing.

![Diagram](image)

**FIGURE 2.2: The Scenarios that Maximize the Manufacturer’s Profit for $m = 1, \delta = 0.5$**

In sum, the manufacturer does not always benefit from offering QR to both of the retailers. This is in contrast to the monopoly benchmark in Section 2.5.1, where the manufacturer always benefits from offering QR to the monopolist retailer. Our results in Propositions 3 and 4 demonstrate the manufacturer’s optimal policy critically depends on (i) the competition in retail market (monopoly vs. duopoly), (ii) the demand variability, (iii) its wholesale price for regular orders (dictated by the level of competition in the supply market), and (iv) the cost premium for QR replenishments.
2.5.3 Impact of QR on the Retailers’ Equilibrium Profits

Turning to the impact of QR on retailer equilibrium profits, we now explore the value of QR for a retailer under competition. Let \( \Pi_{ab}^i \) be retailer \( i \)’s expected equilibrium profit when retailers 1 and 2 are types \( a \) and \( b \) respectively, where \( i = 1, 2 \) and \( a, b = F, S \). The following proposition describes a retailer’s value of QR as well as the impact of gaining QR ability on the competitor’s profitability. It shows having QR ability can be detrimental to a retailer while benefiting its competitor. (All of the threshold values used in this section are provided in Table 6.1 in Section 6.1.3.)

Proposition 5

(i) \( \Pi_{1S}^F < \Pi_{1S}^S \) if and only if \( v < v_1^S \), and \( \Pi_{1F}^F < \Pi_{1S}^F \) if and only if \( v < v_1^F \), furthermore \( v_1^S \geq v_1^F \).

(ii) \( \Pi_{2S}^F > \Pi_{2S}^S \), and \( \Pi_{2F}^F > \Pi_{2S}^F \) if and only if \( v < v_2^F \).

(iii) \( \Pi_{1FS}^F < \Pi_{2FS}^F \) if and only if \( v < v_{FS}^F \).

Contrary to basic intuition, Proposition 5.(i) demonstrates having QR ability can hurt a retailer regardless of its competitor’s type when demand variability is sufficiently small, due to the impact of QR ability on the competitor’s actions. For intuition, consider a fast retailer, Retailer A, (who has QR option) competing against a slow retailer, Retailer B (who does not). Acquiring QR option can be harmful to Retailer A in this case, because the slow competitor, Retailer B, can credibly threaten to deflate the price by ordering a high amount to compensate its lack of QR opportunity. Deflation of the price forces Retailer A to reduce its regular order quantity. When demand variability is low, there is little to be gained from a QR order, and
thus, Retailer A’s loss due to regular orders dominates, making QR ability harmful.

By the same token, when demand variability is high, mismatch between supply and demand is also high, and Retailer B benefits from having QR ability even if this means giving up forcing the fast competitor to reduce its regular order quantity. Note that Proposition 5.(i) also shows \( v_1^S \geq v_1^F \). For QR to be beneficial, a higher level of demand variability is required when competing against a slow competitor compared to a fast competitor. In other words, a retailer whose competitor already has QR option is more likely to benefit from having QR opportunity compared to a retailer whose competitor does not have QR option.

Ignoring competitive factors, our monopoly benchmark and existing work show that QR always benefits the retailer (for example, Fisher et al., 1997; Iyer and Bergen, 1997). In contrast, Proposition 5 demonstrates how competition can actually make QR unattractive to a retailer.

In addition, part (ii) of Proposition 5 shows when a retailer gains QR ability, it can actually benefit its slow competitor. In particular, a slow competitor always fares better as it enjoys a larger order quantity over the fast retailer. The fast competitor only fares better if demand variability is small. Likewise, if both firms have QR opportunity, the competition in a high market is intensified and this makes the fast competitor fare worse when demand variability is high. In addition, part (iii) of Proposition 5 compares the retailers’ profits in the FS scenario, showing the slow retailer achieves a higher profit only when the demand variability is sufficiently low.

Comparing Propositions 4 and 5 also reveals that when a retailer is given QR option, this can benefit all supply chain members. In particular, all of the firms are strictly better off in the FS scenario than in the SS scenario when \( v_1^S < v \). In the next proposition, we describe what happens when both of the retailers gain QR ability simultaneously:
FIGURE 2.3: The Boundaries Given in Proposition 5 for $m = 1$ and $\delta = 0.5$

**Proposition 6** $\Pi_{i}^{FF} > \Pi_{i}^{SS}$ for $i = 1, 2$.

Proposition 6 shows that both retailers reap greater benefits if both gain QR ability simultaneously. When they all have QR opportunity, no retailer can threaten to place a higher regular order quantity.

One might wonder what the equilibrium would be if retailers choose to adopt QR themselves rather than having it dictated to them by the manufacturer. This is studied in detail in Section 6.1.6 in the Appendices. We find that the equilibrium is always symmetric, either both (FF) or none (SS) of the retailers choose to adopt QR. Specifically, when demand variability is low, none of the retailers adopt QR (SS), when demand variability is high, both of them adopt QR (FF), and when demand variability is moderate both SS and FF scenarios can be equilibria.

### 2.5.4 Impact of QR on the Channel’s Equilibrium Profit

Next, we analyze which channel configuration, namely the number of fast retailers, is the most profitable for the entire channel. Let $\Pi_{C}^{abh}$ be the expected channel profit in equilibrium, i.e., the total expected profit achieved by the manufacturer and both of the retailers in equilibrium...
when retailers 1 and 2 are types \( a \) and \( b \) respectively, \( a, b = F, S \):

\[
\Pi_{ab}^C = \Pi_M^{ab} + \sum_{i=1}^{2} \Pi_{i}^{ab}.
\]

The following proposition compares the expected channel profits across the three scenarios.

**Proposition 7** \( \Pi_F^C \geq \max(\Pi_S^C, \Pi_S^C) \) for \( \nu \geq \nu_C \), and \( \Pi_F^C > \max(\Pi_F^C, \Pi_S^C) \) otherwise.

Propositions 4 and 5 demonstrate QR ability benefits the manufacturer and the retailers when the demand variability is sufficiently high but can be detrimental when it is low. Proposition 7 is in agreement. This is intuitive, since the channel profit is the sum of the manufacturer’s and retailers’ profits. Proposition 7 shows the channel profit is maximized with two fast retailers when demand variability is sufficiently high, otherwise the channel might be better off with only one fast retailer.

Overall, the expected channel profit can be maximized by granting QR options exclusively to a single retailer. In contrast to the monopoly benchmark where having a QR retailer always benefits the entire channel, retail competition extends the optimal channel configuration to a continuum: the total channel profit may be maximized by having one or two retailers with QR ability.

### 2.6 Extensions

We now consider a number of extensions to our base model that suggest our key insights continue to hold in various settings, and illustrate the robustness of our results.
2.6.1 Endogenous Wholesale Price

First, we extend the base model given in Section 2.3 by allowing the manufacturer to dictate the wholesale price at the beginning of the timeline. Specifically, now it chooses the wholesale price $c_w$ to maximize its expected profit in equilibrium. In the following, we present the optimal wholesale price the manufacturer would choose, then discuss the value of QR.

Lemma 3 Suppose the manufacturer can dictate the wholesale price, it chooses $c_w = \frac{m}{2}$ to maximize its expected profit in all scenarios (SS, FS and FF).

Knowing the manufacturer’s choice of the wholesale price, we are able to derive the firms’ equilibrium profits in each scenario. Comparing these profits across scenarios reveals the firms’ value of QR as the following proposition summarizes.

Proposition 8

(i) $\Pi_{FS}^M > \Pi_{FF}^M$ if and only if $v < \bar{v}_M$, and $\Pi_{SS}^M < \max(\Pi_{FS}^M, \Pi_{FF}^M)$.

(ii. a) $\Pi_{FS}^1 < \Pi_{SS}^1$ if and only if $v < \bar{v}_1^S$, and $\Pi_{FF}^1 < \Pi_{SF}^1$ if and only if $v < \bar{v}_1^F$, furthermore $\bar{v}_1^S > \bar{v}_1^F$.

(ii. b) $\Pi_{FS}^2 > \Pi_{SS}^2; \Pi_{FF}^2 > \Pi_{SF}^2$ if and only if $v < \bar{v}_C$.

(iii) $\Pi_{CS}^F > \Pi_{CF}^F$ if and only if $v < \bar{v}_C$, and $\Pi_{SS}^C < \max(\Pi_{CS}^F, \Pi_{CF}^F)$.

The threshold values $\bar{v}_M$, $\bar{v}_1^F$, $\bar{v}_1^S$ and $\bar{v}_C$ are provided in Section 6.1.4 of the Appendices.

Proposition 8 shows our results in Section 2.5 continue to hold when the manufacturer is able to choose the wholesale price in addition to the QR price. Specifically, Proposition 8.(i)
extends Proposition 4, showing the manufacturer’s optimal policy is to offer QR to only one of
the retailers when demand variability is low. Proposition 8.(ii. a) and (ii. b) echo Proposition
5. They demonstrate how QR ability can hurt a retailer when demand variability is sufficiently
low, and gaining QR can actually benefit the competing retailer. Finally, Proposition 8.(iii)
mimics Proposition 7 showing that the total channel profit can be maximized by having only
one QR-enabled retailer when demand variability is small.

2.6.2 Alternative Sequence of Events

In our base model, the QR price is set at the beginning of the timeline before the retailers place
their regular orders. Here, we discuss two alternative models with regard to timing of the QR
price and analyze the value of QR for the manufacturer, the retailers, and the channel as a
whole. Specifically, we consider the following models:

(E1): The QR price is set after the regular orders are placed, but before the realization of
demand uncertainty.

(E2): The QR price is set after the demand uncertainty is resolved. The remaining events are
the same as our base model.

Models E1 and E2 actually yield identical equilibrium outcomes in our setup. This is
because of the binary nature of demand distribution. In particular, in equilibrium, a fast
retailer places a QR order only in a high market. Therefore, the manufacturer always sets the
QR price for a high market, and the timing of the QR price (whether before or after demand
realization) becomes irrelevant.

We impose an additional assumption, $c_w \leq \delta$, in this subsection. If this assumption is vio-
lated, it demonstrates the manufacturer’s chosen QR price would be smaller than the wholesale price (i.e., $c_q < c_w$).\footnote{The proof of Proposition 9 in Section 6.1 shows how $c_w > \delta$ implies $c_q < c_w$.} Thus, retailers always place QR orders regardless of the demand outcome, which is inconsistent with practice. Furthermore, relaxing this assumption creates a region with no pure-strategy equilibrium in the FS scenario, which would complicate our analysis.

The following proposition summarizes the firms’ equilibrium actions for the models $E1$ and $E2$.

**Proposition 9** For the models $E1$ and $E2$:

(i) The FS scenario has a unique equilibrium in which $Q_1 \leq Q_2$ and

a. For $v \leq \epsilon_1$, the fast retailer does not place a QR order for any market outcome.

b. For $v > \epsilon_1$, the fast retailer places a QR order only in a high market and it does not place a QR order in a low market.

(ii) The FF scenario has a unique equilibrium only for $v \leq \epsilon_1$ and $v \geq \epsilon_2$, but there does not exist a pure-strategy equilibrium for $\epsilon_1 < v < \epsilon_2$. When the equilibrium exists, $Q_1 = Q_2$ and

a. for $v \leq \epsilon_1$, the retailers do not place a QR order for any market outcome.

b. for $v \geq \epsilon_2$, the retailers place QR orders only in a high market and they do not place any QR order in a low market.

The threshold values $\epsilon_1$ and $\epsilon_2$ are given in Section 6.1.4.

Note that the SS scenario in $E1$ and $E2$ models is same as our base model—QR is not offered and thus QR price is not relevant. When the retailers have QR ability, Proposition 9
shows QR is only used in a high market as in the base model. Notice however, a pure-strategy equilibrium in the \( FF \) scenario for \( \epsilon_1 < v < \epsilon_2 \) does not exist, because having the QR price set after the regular orders are placed results in piecewise concave profit functions for the retailers. Retailer profit functions may contain multiple maxima, which leads to discontinuity in the retailers’ best response functions.

Building on Proposition 9, we characterize the value of QR for the manufacturer, retailers and the entire channel in Section 6.1.5 in the Appendices. These are formally stated in Propositions 30 to 32 in that section. We find that our results of the base model continue to hold even when the QR price is determined after retailers place their regular orders. In particular, the profits of the manufacturer and the entire channel can still be maximized by granting QR ability to only one of the retailers, rather than both of them (Propositions 30 and 32). Furthermore, having QR ability can still be detrimental to a retailer while benefitting the opponent (Proposition 31).

We also find additional results. In models \( E1 \) and \( E2 \), the manufacturer may find it optimal not to offer QR at all when the demand variability is too low (Proposition 30). In contrast, in our base model, the QR price is set at the beginning of the timeline and the manufacturer enjoys the first mover advantage, consistently offering QR to at least one of the retailers (Proposition 4). When the QR price is set after retailers place regular orders, the manufacturer loses the first mover advantage, and this reduces the value it can extract from the retailers due to QR. Similarly, the total channel profit can also be maximized with no QR-enabled retailer at all (Proposition 32). Finally, we compare retailers’ profitability in our base and \( E1 \) and \( E2 \) models in Proposition 33 in Section 6.1.5 in the Appendices. We find that demand variability is the key factor; competing fast retailers are better off in models \( E1 \) and \( E2 \) if and only if demand
variability is sufficiently small.

2.6.3 Limited QR Capacity

In this section, we study what occurs when the manufacturer has limited QR capacity to grant. Specifically, we assume the manufacturer can fulfill at most \( k \) units using QR. When the retailers’ total QR order quantity exceeds the manufacturer’s QR capacity, the manufacturer allocates its capacity evenly among the retailers. Any unused capacity by one retailer can be reallocated to the other retailer. In addition to the assumptions for the base model, we further restrict our analysis to \( k < (m - \delta)/6 \) to ensure the QR capacity is indeed limited and binds in both FS and FF scenarios. Moreover, given any QR capacity level \( k \), we focus only on \( c_w < m - 5k/3 \) to eliminate the unrealistic scenario in which retailers do not place a regular order due to a high wholesale price. We derive SPNE for FF and FS scenarios and subsequently examine the value of QR. The following proposition summarizes the effect of limited capacity on the value of QR.

**Proposition 10** When the manufacturer has a total capacity \( k \) for QR replenishment:

(i) Manufacturer:

a. \( \Pi_{M}^{FF} > \max(\Pi_{M}^{FS}, \Pi_{M}^{SS}) \) for \( c_w < \tilde{w}_M \) and \( v > \tilde{v}_M \).

b. \( \Pi_{M}^{FS} > \max(\Pi_{M}^{FF}, \Pi_{M}^{SS}) \) otherwise.

(ii) Retailers:

a. \( \Pi_{i}^{FS} > \Pi_{i}^{SS} \) if and only if \( c_w > \tilde{w}_i \) and \( v > \tilde{v}_i \); \( \Pi_{i}^{FF} > \Pi_{i}^{SF} \) if and only if \( c_w > \tilde{w}_i \) and \( v > \tilde{v}_i \).
b. The imposition of QR capacity limit increases (weakly) the regular order size of a fast retailer.

(iii) Channel:

a. \( \Pi_{CS}^{FS} > \max(\Pi_{CC}^{FF}, \Pi_{CS}^{SS}) \) for \( c_w > \tilde{w}_C \).

b. \( \Pi_{CC}^{FF} > \max(\Pi_{CS}^{FS}, \Pi_{CS}^{SS}) \) otherwise.

Note that all of the threshold values are summarized in Section 6.1.4 in the Appendices.

Imposing QR capacity limit induces a fast retailer to increase (weakly) the size of its regular order. We find our key insights continue to hold for this extension. This extension also yields an additional insight. In our main model without the capacity limitation, the manufacturer prefers having two fast retailers when demand variability is sufficiently high (Proposition 4). With limited QR capacity, Proposition 10.(i) implies having only one fast retailer maximizes the manufacturer’s profit when the wholesale price is sufficiently high (i.e., \( c_w \geq \tilde{w}_M \)). This result shows the QR capacity limit can be also another reason for not offering QR option to both of the retailers. Intuitively, given a high wholesale price, a fast retailer with QR option decreases its initial order and relies more heavily on its QR order. In this case, however, the manufacturer does not have sufficient capacity to satisfy QR orders of two fast retailers. Thus, the manufacturer is better off by offering QR option to only one of the retailers which alleviates the reduction in their initial order quantities.

2.6.4 Numerical Study: Normally Distributed Demand

In this section, we use computational studies to explore an alternative demand distribution. Specifically, we allow the demand intercept \( A \) to follow a truncated normal distribution with
mean 1 and standard deviation $\sigma$. We consider all combinations of the following parameters:

\[
c_w \in \{0.1, 0.2, 0.3, 0.4, 0.5\},
\]
\[
\delta \in \{0.02, 0.04, 0.06, 0.08\},
\]
\[
\sigma \in \{0.1, 0.2, 0.3, 0.4, 0.5\}.
\]

For each parameter combination $(c_w, \delta, \sigma)$, we numerically search for the firms’ equilibrium decisions in the $SS$, $FS$ and $FF$ scenarios, i.e., when there are zero, one and two fast retailers respectively, and this generates a total of 300 instances for our study.

We define the value of QR (VQR) for a retailer as the percentage increase in its profit after adopting QR. Specifically, it is given by

\[
\frac{\Pi_{Fb} - \Pi_{SS}}{\Pi_{SS}} \times 100\%, \, b = F, S, \tag{2.6}
\]

where $\Pi_{Fb}$ is retailer 1’s equilibrium profit when the competitor’s type is $b$. Similarly, we define the value of QR for the manufacturer and the entire channel as the percentage increase in their profits compared to the $SS$ scenario, which is given by

\[
\frac{\Pi_{Fb} - \Pi_{SS}}{\Pi_{SS}} \times 100\%, \, b = F, S \text{ and } i = M, C, \tag{2.7}
\]

where $\Pi_{Fb}$ and $\Pi_{SS}$ are the equilibrium profits of the manufacturer ($M$) or the channel ($C$) in the scenarios $Fb$ and $SS$ respectively.

We find our key results continue to hold in our numerical studies. Table 2.1 reports our findings for $c_w = 0.2, 0.3$ and $\delta = 0.02$, which is representative, and other parameter combinations

36
TABLE 2.1: Value of QR for $c_w = 0.2$, $0.3$ and $\delta = 0.2$

(a) Manufacturer and Channel VQR (%) in 2.7

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>$c_w = 0.2$</th>
<th>$c_w = 0.3$</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>FS</td>
<td>FF</td>
<td>FS</td>
</tr>
<tr>
<td>0.1</td>
<td>10.71</td>
<td>0.23</td>
<td>9.43</td>
</tr>
<tr>
<td>0.2</td>
<td>11.52</td>
<td>3.22</td>
<td>10.83</td>
</tr>
<tr>
<td>0.3</td>
<td>14.88</td>
<td>9.02</td>
<td>13.31</td>
</tr>
<tr>
<td>0.4</td>
<td>16.30</td>
<td>16.10</td>
<td>15.31</td>
</tr>
<tr>
<td>0.5</td>
<td>23.74</td>
<td>26.87</td>
<td>22.54</td>
</tr>
</tbody>
</table>

(b) Retailer VQR (\%) in 2.6

<table>
<thead>
<tr>
<th>Competitor Type</th>
<th>$c_w = 0.2$</th>
<th>$c_w = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>FS</td>
<td>FF</td>
</tr>
<tr>
<td>0.1</td>
<td>-30.92</td>
<td>-10.33</td>
</tr>
<tr>
<td>0.2</td>
<td>-9.75</td>
<td>-4.37</td>
</tr>
<tr>
<td>0.3</td>
<td>5.24</td>
<td>2.56</td>
</tr>
<tr>
<td>0.4</td>
<td>11.40</td>
<td>4.81</td>
</tr>
<tr>
<td>0.5</td>
<td>28.84</td>
<td>13.46</td>
</tr>
</tbody>
</table>

considered in our studies also yield similar results. As expected, Table 2.1 shows the value of QR for the retailers and the manufacturer increases in the demand standard deviation $\sigma$. For $c_w = 0.3$, the manufacturer prefers having only one fast retailer ($FS$) when $\sigma \leq 0.3$, and two fast retailers ($FF$) otherwise. In other words, the manufacturer’s optimal policy is to offer QR to only one of the retailers when the demand variability is not sufficiently high. Similarly, Table 2.1.(a) also demonstrates the total channel profit can be maximized with only one QR-enabled retailer when demand variability is not sufficiently high ($\sigma \leq 0.4$ for $c_w = 0.3$). Furthermore, Table 2.1.(a) shows the manufacturer and entire channel are always better off offering QR to at least one of the retailers. Moreover, Table 2.1.(b) confirms having QR ability can hurt a
retailer if the demand variability is not sufficiently high: Adopting QR hurts a retailer when $\sigma \leq 0.2$.

### 2.7 Conclusions

In this paper we examine the value of QR under retail competition. For this purpose, we consider a market served by two competing retailers and compare the equilibrium profits for the manufacturer, the retailers and the entire supply chain as a whole, when QR is available to one, both, or none of the retailers. We allow the manufacturer to set the prices for regular and QR replenishments. We also consider a higher cost for implementing QR, thereby quantifying the tradeoff between benefits and additional costs of QR.

We demonstrate offering QR ability to a retailer may harm the manufacturer when the demand variability is not sufficiently high. In particular, we find a manufacturer may find it attractive to offer QR to only one of the ex-ante symmetric retailers. This happens because a retailer reduces its regular (initial) order quantity when it can place a QR order. Furthermore, when the demand is not sufficiently volatile, offering QR can generate insufficient QR profit to balance the loss that results from a retailer’s reduction in its regular order. Moreover, the manufacturer’s additional QR profit gain from offering QR to the second retailer is less than that from the first retailer, as a consequence of retail competition. Therefore, the manufacturer does not necessarily benefit from having two retailers with QR ability. The total channel profit can also be maximized with only one retailer with QR ability, instead of two, when demand variability is not sufficiently high.

We also highlight the potential strategic peril of QR ability for a retailer in the presence of
retail competition. As expected, QR ability benefits a monopolist retailer with better response to variation in demand. However, retail competition undermines the value of QR, and obtaining QR ability can actually harm a retailer when the demand variability is low and we explicitly characterize when this happens.

We recognize our model has several limitations. We assume retailers who aim to maximize their expected profits are risk neutral. Unlike a regular order, a QR order faces no demand risk, thus it has a lower risk than a regular order. A risk-averse retailer will be more inclined to use QR to decrease its demand risk. We expect a risk-averse retailer to increase its allocation of QR order (and hence decrease its allocation of regular order), making QR more valuable than our model predicts. Quantifying the impact of risk-aversion on the value of QR could be a fruitful avenue for future work. Furthermore, our model assumes QR lead time is relatively short compared to the selling season. However, this lead time can be significant and also later arriving units may suffer from drops in sales price over the selling season. These factors will degrade the attractiveness of QR and firms will shift their allocations from QR to regular orders. Thus, when such factors are accounted, we expect the outcome to fall between our fast (QR) and slow (no QR) firm scenarios. Nonetheless, our model cannot fully address these extensions, and it would be worthwhile to generalize our setting to a multi-period model to allow for long QR lead time and declining prices and study their impact.

We study a single supplier serving two retailers. While this is not uncommon in practice (introduction provides some examples), we recognize other supply chain scenarios are possible, e.g., a retailer having multiple suppliers, or each retailer having a distinct supplier and so on, and some of our results may not apply to these scenarios. Thus, future work can study the impact of supply chain configuration on the value of QR considering various scenarios.
Furthermore, our numerical study in Section 2.6.4 suggests our results can continue to hold for other more general demand distribution functions, however, showing this extension analytically would be worthwhile for future work. We also note that, in practice, retailers may not observe each other’s order quantities. In this case, the manufacturer’s pricing would provide a signal about order quantities and retailers would choose their best actions accordingly. Additionally, our model assumes the manufacturer incurs an identical unit QR cost $\delta$ for each retailer, making it indifferent between them. In practice, however, due to geographic dispersion, one retailer may actually result in a higher expediting cost, and thus the manufacturer may prefer offering QR to the less costly retailer. Finally, generalizing our duopoly model to oligopoly retailers is another possible extension. We expect with many competitors, reactions to a retailer’s gaining of QR ability may not be as strong, thus, a retailer may be more likely to benefit from QR in an oligopoly.
CHAPTER 3

Competitive Vertical Integration Strategies in the Fashion Industry

3.1 Introduction

Vertical integration, a 100-year-old strategy, is regaining a place in the spotlight amid recent economic turmoil. This revival of vertical integration does not portend the formation of vertical conglomerates, who exercise full control over material supply, manufacturing and distribution, like Ford and Carnegie did in the early 20th century (Worthen et al. 2009). Instead, manufacturers present diversity in their directions of vertical integration: some choose to forward integrate distribution operations, while others opt to backward integrate supply activities. In this paper, we study a manufacturer’s choice of vertical integration strategy under competition and look at its implications on profitability, product quality and price.

Forward integration extends a manufacturer’s operational reach to product distribution, tightening its grip on the demand side. For instance, Pepsi purchased its bottlers for better control over the distribution of its growing product offerings (Collier 2009). This control over product distribution allows for better response to change in demand, making forward integration common in the fashion industry. For example, European fashion giant Zara, and American
Apparel, a Los Angeles based apparel retailer, manufacture products and sell them through their own retail channels. Tainan Enterprise, a Taiwan based manufacturer, established its own brand, Tony Wear, in China in the late 1990’s (Ho 2002). Conversely, backward integration stretches a manufacturer’s operations toward the source of raw materials, seizing a stronger control over quality on the supply side, one of the top reasons that motivate backward integration. For instance, steelmaker ArcelorMittal is moving deeper into the mining business to ensure stable material supply (Worthen et al. 2009); likewise, the Chinese apparel manufacturer Esquel, backward integrates supply functions such as cotton farming to improve the quality of its raw material (Peleg-Gillai 2007).

Forward and backward integrations benefit firms in different ways, and a firm’s choice between them is unclear. In the apparel industry, we observe both types of integration strategies. We are interested in the reasons behind firms’ selection of one direction or the other. This is complicated by the competition among supply chains, which affects the value of vertical integration. Furthermore, it is unclear how one firm’s integration affects the choices of others in selecting forward and backward integration.

In this paper, we consider two competing supply chains, each consisting of a supplier, a manufacturer and a retailer. The supplier can exert effort to improve the quality of material it supplies to the manufacturer. The manufacturer then makes a product and sells it through the retailer exclusively. The product is sold in two periods and its popularity, thereby the market potential, decreases in time.

Each manufacturer chooses one of the following strategies: (1) forward integration, (2) backward integration, and (3) no integration. We examine the effect of vertical integration on firm decisions and study the equilibrium choice of vertical integration strategy for a manufacturer.
There is a great body of research on the choice of distribution channels (e.g., Jeuland and Shugan 1983; McGuire and Staelin 1983; Gupta and Loulou 1998). Yet, the current research considers only forward integration (manufacturer-retailer integration) and the effect of product quality is absent. We contribute new findings to this line of research twofold. First, by considering both forward and backward integrations, we capture more options that occur in practice. Second, we endogenize firm investment on quality improvement.

Our model addresses the following questions:

• When does vertical integration benefit a manufacturer? Can it hurt a manufacturer’s profitability?

• How does a manufacturer’s selection of forward integration, backward integration or no integration at all depend on its product fashionability, quality cost and competitor’s supply chain structure?

• What is the resulting equilibrium supply chain structure when firms can (1) only forward integrate, or (2) only backward integrate, or (3) choose to either forward or backward integrate? What is the effect of vertical integration on product quality and retail price?

Our study shows that backward integration always benefits a manufacturer. However, forward integration can hurt a manufacturer because it intensifies retail competition, dropping the retail price, which in turn hurts the manufacturer’s margin. Such a drop is less severe when the competing supply chain has fewer intermediaries. Therefore, when a competitor vertically integrates, a manufacturer is more inclined to favor forward integration over backward. In addition, this effect is more pronounced when the product is highly fashionable, i.e., when product popularity decreases more significantly over time. This reflects the control over product
distribution dominates the control over quality for highly fashionable products.

We also study competitive choice of integration strategies by manufacturers, finding disintegration in both supply chains can never be an equilibrium. This is contrary to the celebrated result in prior studies that disintegration can be an equilibrium when only manufacturer-retailer (forward) integration is considered (e.g., McGuire and Staelin 1983; Gupta and Loulou 1998). The inclusion of backward integration drives our departure from prior results. Additionally, manufacturers can fall into prisoner’s dilemma: in equilibrium, all manufacturers vertically integrate while achieving lower profits.

Interestingly, we find that vertical integration results in a higher quality product sold at a lower retail price. Vertical integration lowers the retail price of a product because it reduces the number of intermediaries profiting from it. This benefit alleviates double-marginalization and encourages more investment in quality improvement.

We also analyze what happens when forward integration results in pricing advantage by reducing consumer price sensitivity. This advantage increases the attractiveness of forward integration. When the competitor is already forward integrated, the potential benefit of backward integration can be nullified by the competitor’s pricing advantage. Consequently, despite the gain of control over quality, backward integration can actually decrease profitability and it can lower product quality and sales.

The remainder of this paper is organized as follows. In Section 3.2, we present our literature review. Section 3.3 describes our model and Section 3.4 derives firm quality and price decisions. Section 3.5 discusses firm profitability and manufacturer equilibrium integration strategies while Section 3.6 presents extensions to the base model. Finally, Section 3.7 offers our concluding remarks.
3.2 Related Literature

Our work is most relevant to the literature on the competitive choice of distribution channels. This stream of literature begins with the seminal work of McGuire and Staelin (1983). In that paper, the authors consider duopoly channels, each with a manufacturer distributing its product through an exclusive retailer. It is well recognized that the profit of a manufacturer and the entire channel is maximized when the manufacturer vertically integrates, thereby achieving centralized decision making, in the absence of competition. Interestingly, however, McGuire and Staelin (1983) find that in the presence of channel competition, manufacturers may choose not to vertically integrate, and this may actually yield the highest profit for the manufacturers and the entire channel. Moorthy (1988) further investigates the driver for this result, owing it to the rise in manufacturer demand caused by the strategic interaction between channels.

A number of following works have extended the model of McGuire and Staelin (1983), confirming vertical integration is not the profit-maximizing strategy for a manufacturer under various extensions. For example, Coughlan (1985) extends the model of McGuire and Staelin (1983) by adopting a general demand function, and Trivedi (1998) considers retailers carrying products of multiple manufacturers. Gupta and Loulou (1998) allow a manufacturer to invest in research and development to reduce unit production cost, and Gupta (2008) further extends this work by incorporating involuntary knowledge spillovers. Wu et al. (2007) investigate the effect of demand uncertainty on the equilibrium distribution structure and identify cases in which demand variability affects, and does not affect, equilibrium design. In addition to intensifying price competition, Wu et al. (2007) find that disintegration can also intensify other dimension of competition, such as advertising.
Similarly, Corbett and Karmarkar (2001) investigate the effect of vertical integration, finding that it lowers the total channel profit. They focus on the impact of variable and fixed costs on serial multi-tier supply chains. Our work is similar in spirit to Corbett and Karmarkar (2001), however, we also consider the effect of vertical integration on product quality in addition to prices. Furthermore, we study competing firms’ choice of vertical integration strategy while they do not. Boyaci and Gallego (2004) consider two supply chains, each with a wholesaler and a retailer, and the supply chains compete strictly on service levels. They identify prisoner’s dilemma in the choice of vertical control: coordinated decision making in each supply chains is the dominant strategy even though it results in lower overall supply chain performance. While the above studies consider only manufacturer-retailer integration, we also consider supplier-manufacturer integration, allowing for either forward or backward integration. In addition, Savaskan et al. (2004) and Savaskan and Wassnhove (2006) examine channel designs in the context of closed-loop supply chains. They analyze the performance of various channel structures with products that can be recycled and remanufactured, whereas we consider products that cannot.

There exists a rich literature on supply chain contracting and coordination. This stream of research focuses on the design of contractual agreements among supply chain members to maximize supply chain efficiency. Cachon (2003) and Lariviere (1999) provide excellent reviews of this literature. Instead of using contracts to coordinate decisions of individual entities, vertical integration achieves centralized decisions by extending a firm’s operational capability. In addition, in our model, a manufacturer chooses its direction to vertically integrate (i.e., backward or forward). This differs from the current literature on supply chain contracts where firms do not choose their contract partners.
In addition to vertical integration, a great variety of operational strategies are also studied under supply chain competition. For example, Caro and Martínez-de-Albeniz (2010) and Lin and Parlaktürk (2010) study the value of quick response, and Li and Ha (2008) examine the impact of inventory cost and reactive capacity on firm competitiveness. Ha and Tong (2008) and Ha et al. (2008) investigate the value of vertical information sharing in the presence of supply chain competition. Anand and Girotra (2007) analyze the value of delayed product differentiation, finding delayed product differentiation can be detrimental in a competitive environment. The strategies studied by those works focus on operations performed within a firm, whereas we consider using vertical integration to extend the operational capability of a firm. We identify circumstances under which this extension of operational capability is actually harmful to a firm. Finally, our work is also relevant to quality management in the realm of supply chain (e.g., Baiman et al. 2000; Balachandran and Radhakrishnan 2005; Zhu et al. 2007). However, none of these papers consider quality improvement in the presence of supply chain competition, which is the focus of our research.

3.3 Model

We consider two supply chains \((i = 1, 2)\) selling fashionable products competitively to a consumer market. These products are sold over two periods \((t = 1, 2)\), and their consumer popularity decreases over time. In the following, we introduce the model of consumer choice, firm decisions and manufacturer vertical integration strategies.

Following Salop’s (1979) spatial differentiation model, we assume consumers are utility maximizers and they are uniformly distributed along a circle at \(\frac{1}{2}\) units of density in each
TABLE 3.1: Parameters and Decision Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time period</td>
</tr>
<tr>
<td>$k$</td>
<td>Consumer population in the second period</td>
</tr>
<tr>
<td>$m$</td>
<td>Consumer reservation value</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Consumer quality sensitivity</td>
</tr>
<tr>
<td>$d$</td>
<td>Consumer disutility per unit deviation from the ideal product</td>
</tr>
<tr>
<td>$\psi_{i,t}$</td>
<td>Distance between product $i$ and a consumer’s ideal product in period $t$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Quality of product $i$</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Raw material price charged by supplier $i$</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Wholesale price charged by manufacturer $i$</td>
</tr>
<tr>
<td>$p_{i,t}$</td>
<td>Retail price of product $i$ in period $t$</td>
</tr>
<tr>
<td>$Q_{i,t}$</td>
<td>Sales for product $i$ in period $t$</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost coefficient for quality improvement</td>
</tr>
<tr>
<td>$N, F, B$</td>
<td>No integration, forward integration and backward integration respectively</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Manufacturer $i$’s vertical integration strategy</td>
</tr>
</tbody>
</table>

Each consumer is identified by a point on the circle which represents his or her ideal product. The size of the market in period 1 is normalized to 1, whereas the size of the market shrinks to $k < 1$ in period 2. Thus, in $t = 2$, there are fewer consumers and they are distributed on a smaller circle as seen in Figure 3.1 (a). Here, $k$ measures the product degree of fashion: a smaller $k$ indicates faster decrease in product popularity over time, thereby a more fashionable product. The two competing products are located at the two ends of the diameter.

Each consumer has a unitary demand and buys a product only when the purchase generates a positive utility. Specifically, a consumer derives utility

$$U(\theta_i, p_{i,t}, \psi_{i,t}) = m + \alpha \theta_i - p_{i,t} - d \psi_{i,t}$$

(3.1)

from purchasing product $i$ (i.e., the product of supply chain $i$) in period $t$, where $m$ is a consumer’s reservation value, $\theta_i$ represents the quality of product $i$, and $p_{i,t}$ is the retail price.

1Changing the density affects equilibrium decisions but it does not alter our insights.
2It is well known in equilibrium, symmetric duopoly firms locate at each end of the diameter of a circular market (c.f., Salop 1979). Thus, our circular market is identical to using Hotelling’s (1929) model of spatial product differentiation with duopolists located on each side of the Hotelling line.
of product $i$ in period $t$. Here, $\alpha$ captures consumer sensitivity to product quality. We fix the consumer price sensitivity, i.e., the coefficient to $p_{i,t}$ in (3.1), to 1. This base model makes key results clear and easy to understand. However, we relax this assumption and present additional insights in Section 3.6. Finally, $\psi_{i,t}$ is the shortest distance between product $i$ and the consumer’s ideal product as Figure 3.1 (b) illustrates, and a consumer incurs disutility $d > 0$ per unit of distance due to mismatch of her preference. Table 3.1 summarizes the parameters and decision variables of our model.

FIGURE 3.1: Circular Model of Competition and Arc Distances

Each supply chain $i$ consists of a supplier ($L_i$), a manufacturer ($M_i$) and a retailer ($R_i$), and all firms are risk neutral profit maximizers. A supplier provides raw materials to its downstream manufacturer. Supplier $i$ invests in material quality which in turn determines the product quality $\theta_i$, and it supplies manufacturer $i$ at a unit material price $r_i$. This mimics the situation in which product quality directly depends on material quality. For instance, the quality of a T-shirt is determined by its fabric quality: an all-cotton shirt provides better sweat absorption and a greater feeling of airiness (Levinson 2000). Manufacturer $i$ produces each product $i$ with a unit of raw material and sells it to retailer $i$ at a unit wholesale price $w_i$. Finally, retailer $i$ determines the retail price $p_{i,t}$ for product $i$, in each period $t$, and sells it in the consumer market. The material price $r_i$ and wholesale price $w_i$ do not change across periods because firms often sign relatively long term contracts with their suppliers. On the
other hand, we allow the retail price $p_i$ to be adjusted from period to period, reflecting the fact that a retailer has more flexibility in pricing.

To focus on the effect of competition, we assume firms do not incur variable costs for production and retailing. Nevertheless, supplier $i$ incurs a fixed cost $c\theta_i^2$ for achieving quality level $\theta_i$, where $c$ determines how expensive it is to improve quality. We note that some literature (like this paper) regards quality improvement as a one-time investment that does not affect marginal cost of production (e.g., Bonanno 1986; Demirhan et al. 2007; Bhaskaran and Krishnan 2009; Kaya and Özer 2009). At the same time, some others argue quality improvement accompanies an increase in marginal production cost (e.g., Mussa and Rosen 1978; Desai 2001; Heese and Swaminathan 2006; Netessine and Taylor 2007). It is not uncommon to see quality improvement as a one-time investment in the apparel industry. For example, Esquel, a major Chinese apparel manufacturer, provides its supplying cotton farmers with training in process improvement techniques, such as seed selection and impurities elimination, to ensure their quality for high-end cotton (Peleg-Gillai 2007). In addition, advances in spinning and knitting technologies improve the production process, allowing yarns to produce fabric with superior quality (Bainbridge 2009). Following these observations, we focus on the cases where quality improvement is achieved through a one-time process improvement, characterized by a fixed cost investment. We also make the common assumption that firm decisions are common knowledge (e.g., McGuire and Staelin 1983; Trivedi 1998; Tsay and Agrawal 2000), and firms have sufficient capacity to fulfill any demand.

![FIGURE 3.2: A Manufacturer’s Vertical Integration Strategies](image-url)
As Figure 3.2 indicates, we envision three integration strategies for each manufacturer: no integration (N), forward integration (F) and backward integration (B). When a manufacturer does not integrate, its material supply and product retail are accomplished through other independent firms. In that case, the manufacturer has control only over the wholesale price it charges to the retailer.

When a manufacturer forward integrates, it sells the product through its own company stores, and therefore the manufacturer controls the retail price of its product. For instance, Tainan Enterprise, a leading Taiwanese manufacturer, established its own brand, Tony Wear, one of the most popular menswear brands in China (Ho 2002). Alternatively, a manufacturer can backward integrate by performing supply operations in-house, thereby allowing the manufacturer to dictate the quality \( \theta_i \) in addition to its wholesale price \( w_i \). For example, Esquel gradually expands its operational scope by developing yarn spinning, cotton ginning and farming abilities traditionally provided by other suppliers (Peleg-Gillai 2007).

We use \( S_i \in \{N,F,B\} \) to denote manufacturer \( i \)'s integration strategy, and \( S_1S_2 \) to denote different scenarios of supply chain structures in the industry. We restrict our analysis to \( c > \frac{2m^2}{27d} \) to ensure the concavity of supplier profit; otherwise, quality improvement is too cheap, firms invest overly on quality and they do not make any profit when both manufacturers vertically integrate. In addition, we assume \( d < \frac{2m}{3(5+9k)} \) to avoid trivial cases where the firms form local monopolies and do not compete. Finally, we restrict our analysis to \( k > \frac{1}{11} \); otherwise, in the \( FN \) scenario, the market size in the second period is too small and all consumers buy product 1 while product 2 does not survive in that period. Under these assumptions, products compete and the market is covered in each period.

Let \( Q_{i,t} \) be the sales quantity for product \( i \) in period \( t \). Then the profit functions for a
retailer $\pi^N_{Ri}$, manufacturer $\pi^N_{Mi}$ and supplier $\pi^N_{Li}$ in a disintegrated supply chain $i$ are given by:

$$\pi^N_{Ri} = \sum_{t=1}^{2} (p_{i,t} - w_i) Q_{i,t}, \quad (3.2)$$

$$\pi^N_{Mi} = (w_i - r_i) \sum_{t=1}^{2} Q_{i,t}, \quad (3.3)$$

$$\pi^N_{Li} = r_i \sum_{t=1}^{2} Q_{i,t} - c \theta_i^2. \quad (3.4)$$

When manufacturer $i$ forward integrates, it sets the retail price itself. In this case, the profit function for manufacturer $i$ becomes:

$$\pi^F_{Mi} = \sum_{t=1}^{2} (p_{i,t} - r_i) Q_{i,t}. \quad (3.5)$$

On the other hand, backward integration allows manufacturer $i$ to dictate its quality level. This yields the following profit function for manufacturer $i$:

$$\pi^B_{Mi} = w_i \sum_{t=1}^{2} Q_{i,t} - c \theta_i^2. \quad (3.6)$$

For any given channel arrangement $S_1 S_2$ of the industry, decisions are made as follows. First, firms who control material supply (a supplier or a backward integrated manufacturer) competitively determine their quality levels. Contingent on the quality levels, these firms set the unit price they charge to their downstream customers. Thereafter, a manufacturer sets its wholesale price if it does not vertically integrate. Finally, the selling season begins, and firms that sell products to consumers (a retailer or a forward integrated manufacturer) set their retail prices for each period and demand is realized. Following this sequence of events, we will solve for a Subgame Perfect Nash Equilibrium (SPNE) by applying backward induction.
3.4 Quality and Price Decisions

In this section, we examine the effect of vertical integration on price and quality decisions and sales. To begin, we analyze firms’ decisions under any given supply chain structures $S_1S_2$. Subsequently, we contrast these decisions across different scenarios of supply chain structures to reveal the impact of vertical integration.

3.4.1 Characterization of Equilibrium Quality, Prices and Sales

Contingent on the price and quality specifications offered by upstream firms, each firm considers the response of rival firms in determining the best approach to maximizing its own profit. Let $\theta$, $r$, $w$ and $p$ be the vectors for product qualities, material prices, wholesale prices and retail prices respectively. Based on the decision sequence described in Section 3.3, a SPNE when none of the manufacturers vertically integrates ($NN$) satisfies the followings for $i = 1, 2$ and $t = 1, 2$:

$$\theta_i^* = \arg \max_{\theta_i} r_i \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(r^*, \theta)) - c\theta_i^2,$$  \hspace{1cm} (3.7)

$$r_i^* = \arg \max_{r_i} r_i \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(r, \theta)) - c\theta_i^2,$$  \hspace{1cm} (3.8)

$$w_i^* = \arg \max_{w_i} (w_i - r_i) \sum_{t=1}^{2} Q_{i,t}(\theta, p^*(w)),$$  \hspace{1cm} (3.9)

$$p_i^* = \arg \max_{p_{i,1}, p_{i,2}} p_{i,t} - w_i \sum_{t=1}^{2} (p_{i,t} - w_i) Q_{i,t}(\theta, p).$$  \hspace{1cm} (3.10)

Equations (3.7) to (3.10) formulate the optimization problems for the suppliers, manufacturers and retailers. Essentially, problems (3.7) and (3.8) state a supplier chooses the quality and material price to maximize its profit. Problem (3.9) shows a manufacturer sets the wholesale
price to maximize its profit. (3.10) states a retailer maximizes its profit by setting the retail price.

When manufacturer $i$ vertically integrates, it no longer solves problem (3.9) and a new problem arises: When manufacturer $i$ forward integrates, it sets retail prices and solves problem (3.10) with $w_i$ replaced by $r_i$. Alternatively, when manufacturer $i$ backward integrates, it determines its quality $\theta_i$, solving problems (3.7) and (3.8) with $r_i$ replaced by $w_i$. We derive the equilibrium decisions by applying backward induction, essentially solving problems (3.7) to (3.10) in reverse order. This procedure leads to the equilibrium product qualities, retail prices and sales as follows:

**Proposition 11** The unique SPNE product quality $\theta_i$, retail price $p_{i,t}$ and total sales $Q_i$ for each scenario $S_1S_2$, $S_i \in \{N, F, B\}$, are as follows:

(i) When none of the manufacturers vertically integrates, $NN$:

$$\theta_1 = \theta_2 = \frac{(1+k)\alpha}{6c}, \ p_{1,1} = p_{2,1} = d(7 + 6k), \ p_{1,2} = p_{2,2} = d(6 + 7k), \ Q_1 = Q_2 = \frac{1+k}{2}.$$  

(ii) When only manufacturer 1 vertically integrates, $FN$ or $BN$:

$$\theta_1 = \frac{(1+k)\alpha(63cd-2\alpha^2)}{6c(54cd-2\alpha^2)} \ , \ p_{1,1} = \frac{d(9cd(55+43k)-4(4+3k)\alpha^2)}{4(27cd+\alpha^2)}, \ p_{1,2} = \frac{d(9cd(43+55k)-4(3+4k)\alpha^2)}{4(27cd+\alpha^2)}, \ Q_1 = \frac{1+k(108cd-4\alpha^2)}{108cd-4\alpha^2}.$$  

$$\theta_2 = \frac{(1+k)\alpha(45cd-2\alpha^2)}{6c(54cd-2\alpha^2)}, \ p_{2,1} = \frac{d(18cd(14+11k)-(11+9k)\alpha^2)}{2(27cd+\alpha^2)}, \ p_{2,2} = \frac{d(18cd(11+14k)-(9+11k)\alpha^2)}{2(27cd+\alpha^2)}, \ Q_2 = \frac{1+k(108cd-2\alpha^2)}{108cd-2\alpha^2}.$$  

(iii) When both manufacturers vertically integrate, $FF$, $BF$, $FB$ or $BB$:

$$\theta_1 = \theta_2 = \frac{(1+k)\alpha}{6c}, \ p_{1,1} = p_{2,1} = \frac{d(5+3k)}{2}, \ p_{1,2} = p_{2,2} = \frac{d(3+5k)}{2}, \ Q_1 = Q_2 = \frac{1+k}{2}.$$  

A manufacturer can choose to forward integrate ($F$), backward integrate ($B$) or not integrate
at all \((N)\). When only manufacturer 1 is vertically integrated, Proposition 11.(ii) states that \(FN\) and \(BN\) scenarios share the same equilibrium quality, price and sales outcomes. This is because the sequence of events and decisions are identical in these scenarios. Manufacturer 1, however, achieves different profits: in the \(BN\) scenario, it collects the profit of a supplier in a two-tier supply chain, whereas it collects the profit of a retailer in the \(FN\) scenario. Proposition 13 compares these profits. Likewise, when both manufacturers are vertically integrated as in Proposition 11.(iii), \(FF\), \(BF\), \(FB\) and \(BB\) scenarios have the same equilibrium quality, price and sales outcomes. Knowing firms’ decisions in each scenario, we next compare the decisions across scenarios to examine the effect of vertical integration on product quality, retail price and sales.

3.4.2 The Effect of Integration on Product Quality, Sales and Retail Price

It is a well-established notion that decisions made by self-profit-maximizing firms generate sub-optimal profit for the entire channel, producing the effect of double-marginalization. Therefore double-marginalization is viewed as a frictional cost within a channel due to disintegration inefficiency. Vertical integration centralizes decisions and alleviates double-marginalization. The gain of this benefit for a supply chain is reflected on the change of quality, price and sales as the following proposition describes. In the following, we drop the subscript \(t\) in \(p_{i,t}\) for ease of notation and remind that the superscript \(S_1S_2\) denotes the structure of supply chains.

**Proposition 12** For \(S_1 \in \{F, B\}\) and

(i) any \(S_2\), \(Q_1^{S_1S_2} > Q_1^{NS_2}\), \(\theta_1^{S_1S_2} > \theta_1^{NS_2}\) and \(p_1^{S_1S_2} < p_1^{NS_2}\).

(ii) any \(S_2\), \(Q_2^{S_1S_2} < Q_2^{NS_2}\), \(\theta_2^{S_1S_2} < \theta_2^{NS_2}\) and \(p_2^{S_1S_2} < p_2^{NS_2}\).
(iii) $S_2 \in \{N, B\}$, $w_{2}^{S_1S_2} < w_{2}^{NS_2}$.

(iv) $S_2 \in \{N, F\}$, $r_{2}^{S_1S_2} < r_{2}^{NS_2}$.

Interestingly, Proposition 12.(i) shows manufacturer 1’s vertical integration (either forward or backward) results in the sale of a better quality product 1 at a lower retail price. Vertical integration alleviates double-marginalization, reducing the frictional cost within supply chain 1, and thus, it encourages more quality investment for product 1. The improved product quality elevates consumers’ valuation of product 1, encouraging an increase on the retail price. On the other hand, vertical integration removes intermediaries who add their margins to the retail price, leading to an opposite force which lowers the retail price. As a result, the latter force dominates, and the quality of product 1 increases while its retail price drops.

On the other hand, reduced double-marginalization in supply chain 1 hurts the competitiveness of supply chain 2, resulting in less investment on the quality of product 2 as in Proposition 12.(ii). In addition, when manufacturer 1 vertically integrates, the reduced $p_1$ forces the competing product to lower its retail price $p_2$ to remain competitiveness. As a result, the competing manufacturer reduces its wholesale price and the competing supplier lowers its material price as in cases (iii) and (iv).\footnote{Part (iii) is irrelevant for $S_2 = F$ because a forward integrated manufacturer does not set the wholesale price. Similarly, Part (iv) is irrelevant for $S_2 = B$ because there is no material price when a manufacturer backward integrates.}

### 3.5 Profitability and Equilibrium Integration Strategies

Having characterized equilibrium decisions in each scenario, we now turn our focus to the effect of vertical integration on profitability. Here, we provide answers to the following questions:
Does vertical integration always benefit a manufacturer? How does vertical integration affect the total profit of the entire supply chain? What is the equilibrium structure if manufacturers determine their integration strategies competitively?

3.5.1 Manufacturer’s Value of Vertical Integration

This section analyzes the effect of vertical integration on a manufacturer’s profitability. Forward integration gives a manufacturer better control over demand while backward integration improves its control over quality. When channel competition is absent, it can be shown that vertical integration always benefits a manufacturer and the entire channel. In the presence of channel competition, we now examine how the competitor’s reaction affects the value of vertical integration for a manufacturer. Let $\Pi^{S_1 S_2}_{M_1}$ be the equilibrium profit of manufacturer 1 when manufacturer $i, i = 1, 2$, chooses strategy $S_i$. Then the following proposition summarizes the value of forward and backward integrations for a manufacturer.

Proposition 13

(i.a) $\Pi^{F N}_{M_1} < \Pi^{NN}_{M_1}.$

(i.b) $\Pi^{F S_2}_{M_1} > \Pi^{NS_2}_{M_1}$ for $S_2 \in \{F, B\}$ if and only if $k < \frac{4(27\gamma - 1)\sqrt{8 - 32\gamma + 315\gamma^2 - 3(45\gamma - 2)^2}}{4 - 108\gamma + 243\gamma^2}$

where $\gamma = \frac{c d}{\alpha^2}$.

(ii) $\Pi^{BS_2}_{M_1} > \Pi^{NS_2}_{M_1}$ for $S_2 \in \{F, N, B\}$.

Vertical integration reduces the effect of double-marginalization in a channel, and one may expect it also improves a manufacturer’s profitability. In contrast, although backward integration always benefits the manufacturer as case (ii) illustrates, forward integration can go either way: It is always detrimental when the competitor is not vertically integrated as in case (i.a),
but it can be beneficial when facing a vertically integrated competitor as case (i.b) shows.

Forward integration can hurt manufacturer 1 because it overly reduces the retail price $p_1$ for product 1. Specifically, there are two causes to the drop of $p_1$: (1) alleviated double-marginalization in supply chain 1, and (2) the drop in the competing product’s retail price $p_2$. When the competing manufacturer 2 is not integrated, the drop in $p_1$ is significant due to the strong effect of the latter force. As a result, forward integration reduces the profit margin for manufacturer 1, producing an adverse effect that outweighs the benefit of increased sales.

On the other hand, facing an vertically integrated competitor ($S_2 \in \{F, B\}$), the reduction in $p_2$ is less pronounced because there are fewer firms in the competing supply chain removing their margins from $p_2$. Consequently, the reduction in $p_1$ is smaller and forward integration can be beneficial when $k$ is small. That is, forward integration benefits a manufacturer when the product is highly fashionable, because the change in demand is drastic, emphasizing the benefit of flexible pricing ability.

Backward integration always benefits a manufacturer due to alleviation of double-marginalization. In this case, the competing firms also drop their prices, forcing the manufacturer to reduce its profit margin as it backward integrates. However, compared to forward integration, the reduction is less pronounced due to the gain of Stackelberg leadership in setting quality. Therefore, backward integration always benefits a manufacturer with increased sales.

The previous proposition discusses the change in manufacturer profitability when it moves from disintegration to vertical integration. In the following proposition, we compare a manufacturer’s profit when it forward integrates to its profit when it backward integrates. It shows forward integration is more likely to be favorable as the competitor vertically integrates.
Proposition 14

(i) $\Pi_{M_1}^{RN} > \Pi_{M_1}^{FN}$.

(ii) $\Pi_{M_1}^{FS_2} > \Pi_{M_1}^{BS_2}$ for $S_2 \in \{F, B\}$ if and only if $k < \delta$.

(iii) $\Pi_{M_1}^{FS_2} - \Pi_{M_1}^{BS_2} > \Pi_{M_1}^{FN} - \Pi_{M_1}^{BN}$ for $S_2 \in \{F, B\}$,

where $\delta = \frac{(27\gamma - 1) + 6\sqrt{\gamma (18\gamma - 1)}}{9\gamma - 1}$ and $\gamma = \frac{cd}{\alpha^2}$.

Forward integration provides a manufacturer flexibility of setting its retail price for each period.\(^4\) On the other hand, backward integration grants a manufacturer Stackelberg leadership in controlling product quality. Proposition 14 demonstrates that a manufacturer’s choice between these benefits is contingent on the structure of its competing supply chain. When the competing channel is not integrated as in Proposition 14.(i), backward integration is always more favorable. This happens because vertical integration leads to severe drop in retail price, hurting a manufacturer’s profit margin. However, this adverse effect is less severe when the manufacturer backward integrates due to its Stackelberg leadership.

When the competing manufacturer is already vertically integrated as in case (ii), forward integration can be more favorable. In this case, the pressure of dropping the retail price is reduced because there are fewer firms in the competing channel, resulting in smaller drop in the competing retail price and increasing the attractiveness of forward integration. As a result, forward integration is more favorable when the product is highly fashionable: The product popularity drops significantly in time, making flexible pricing ability more valuable as case (ii) shows. Following the same token, a manufacturer is more likely to favor forward over backward integration when its competitor moves from disintegration to vertical integration as in case (iii).

\(^4\)Recall that the retail price is set independently for each period while the wholesale and material prices remain the same across periods.
Previous results describe a manufacturer’s best response integration strategy given the channel structure of its competitor. We are also interested in the equilibrium $S^*_1 S^*_2$ when manufacturers choose their integration strategies competitively. We describe the equilibrium in the following proposition.

**Proposition 15**

(i) When manufacturers consider no integration ($N$) and only backward integration ($B$),

$$S^*_1 S^*_2 = BB.$$

(ii) When manufacturers consider no integration ($N$) and only forward integration ($F$),

$$S^*_1 S^*_2 = NN.$$

(iii) When manufacturers consider no integration ($N$), and both forward ($F$) and backward ($B$) integration, then

$$S^*_1 S^*_2 = \begin{cases} 
FF & \text{for } k < \delta, \\
BB & \text{for } k > \delta,
\end{cases}$$

where $\delta$ is given in Proposition 14.

Parts (i) and (ii) of Proposition 15 describe the equilibrium when a disintegrated manufacturer considers only backward or only forward integration. When only backward integration is considered, both manufacturers choose to vertically integrate, but they stay rather disintegrated when only forward integration is considered.\(^5\) Such discrepancy occurs because forward integration overly reduces manufacturer profit margins. This negative effect is less pronounced

\(^5\)In this case, there may exist multiple equilibria. Specifically, $FF$ can be another equilibrium. However, we focus on $NN$ because it is Pareto optimal as Proposition 16 shows.
with backward integration because manufacturers gain leadership in setting quality. When a manufacturer can choose to either forward or backward integrate, Proposition 15.(iii) demonstrates that both of the manufacturers choose to forward integrate when the product is highly fashionable \((k < \delta)\), because the value of flexible pricing ability is significant in that case. On the other hand, both manufacturers choose to backward integrate if product fashionability is low. For \(k = \delta\) boundary, each manufacturer is indifferent between forward and backward integration, and all possible integration scenarios \(BB, FB, BF\) and \(FF\) are equilibria.

Previous literature on distribution channels has focused only on manufacturer-retailer (forward) integration (e.g., McGuire and Staelin 1983; Gupta and Loulou 1998; Trivedi 1998), finding disintegration in all channels can be an equilibrium. We contribute to this stream of literature by considering backward integration and highlight its strategic implication: while \(NN\) can be equilibrium when firms consider only forward integration, it cannot be equilibrium when backward integration is also considered. This happens because, when a manufacturer does not vertically integrate, its competitor is always better off by integrating backward for channel leadership.

McGuire and Staelin (1983) and Gupta and Loulou (1998) argue that manufacturers may prefer disintegration because the insertion of an independent retailer mitigates competition faced by a manufacturer. This buffering effect is also present in our model. In particular, it can be shown that a manufacturer’s derived consumer sensitivity to quality and price is weakly higher (lower) as it forward (backward) integrates. In other words, the further a manufacturer is away from the market, the smaller the competition intensity it faces.

Having characterized manufacturer equilibrium strategies, the following proposition compares a manufacturer profit before and after integration.
Proposition 16 $\Pi_{M_1}^{NN} > \Pi_{M_1}^{FF}$ and $\Pi_{M_1}^{NN} > \Pi_{M_1}^{BB}$.

Contrary to common belief, Proposition 16 shows manufacturers achieve lower profits when both of them vertically integrate. In this case, a manufacturer’s benefit of vertical integration is outweighed by the same benefit gained by the competitor. Recall that when only manufacturer 1 vertically integrates, the drop in $p_2$ (retail price of product 2) is one of the causes driving down $p_1$. When both manufacturers 1 and 2 vertically integrate, reduced double-marginalization of supply chain 2 constitutes another force driving down $p_2$, further intensifying retail competition and hurting the profit margin of manufacturer 1. Proposition 15 together with Proposition 16 suggest the presence of prisoners’ dilemma: each manufacturer attempts to benefit from vertical integration, but that benefit is actually nullified by the competitor’s gain from also integrating.

3.5.2 Effect of Vertical Integration on Channel Profitability

The previous section shows how vertical integration can hurt a manufacturer. It is unclear whether this indicates another channel participant retains potential benefit from integration. To see if this is the case, we examine the effect of vertical integration on the total channel profitability. Let $\Pi_{C_1}^{S_iS_2}$ be the total equilibrium profit achieved by supply chain 1 when manufacturer $i$, $i = 1, 2$, chooses strategy $S_i$, and let $\Pi_{j_1}^{S_1S_2}$ be the equilibrium profit for firm $j$ in supply chain 1, where $j = L$ (Supplier), $M$ (Manufacturer) and $R$ (Retailer). The next proposition shows vertical integration is detrimental to the profitability of the entire supply chain.

Proposition 17

(i) $\Pi_{M_1}^{FS_2} < \Pi_{M_1}^{NS_2} + \Pi_{R_1}^{NS_2}$ and $\Pi_{M_1}^{BS_2} < \Pi_{M_1}^{NS_2} + \Pi_{L_1}^{NS_2}$ for any $S_2$. 

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(ii) $\Pi_{C_1}^{FS_2} < \Pi_{C_1}^{NS_2}$ and $\Pi_{C_1}^{BS_2} < \Pi_{C_1}^{NS_2}$ for any $S_2$.

(iii) $\Pi_{C_1}^{S_1S_2} \leq \Pi_{C_1}^{NN}$.

One would expect vertical integration to improve the profitability of the entire supply chain. Rather, Proposition 17 states that vertical integration always lowers the total supply chain profit due to channel competition. When manufacturer 1 vertically integrates, again, the drop in the retail price of product 2 leads to a significant drop in the retail price of product 1, hurting the profit margin for the entire channel. Thus, Proposition 17.(i) shows that an integrated manufacturer makes less than the total profit achieved by two individual firms combined. While the manufacturer profit can be improved, vertical integration is detrimental to the total supply chain profit as demonstrated by Proposition 17.(ii). Consequently, Proposition 17.(iii) states in equilibrium, each supply chain achieves a lower profit than it does if none of the manufacturers consider vertical integration. Note the equality for case (iii) holds only when manufacturers consider either forward integration or no integration at all. In that case, $S_1^*S_2^* = NN$ as in case (ii) of Proposition 15.

### 3.6 Forward Integration and Price Sensitivity

So far we have assumed vertical integration does not affect consumer price sensitivity, and consumers have identical price sensitivity for each product. In this section, we relax this assumption and consider what happens when forward integration reduces consumer price sensitivity. Intuitively, direct contact with consumers can improve a manufacturer’s pricing advantage because company stores provide better brand perception which increases consumer willingness to pay.

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6When supply chain competition is absent, it can be shown an integrated manufacturer always achieves a higher profit than two separated firms, i.e., $\Pi_{M_1}^{FS} > \Pi_{M_1}^{N} + \Pi_{R_1}^{N}$ and $\Pi_{M_1}^{BS} > \Pi_{M_1}^{N} + \Pi_{S_1}^{N}$, due to alleviation of double-marginalization.
Indeed, it is not uncommon to see higher retail prices in company stores than in general retailers, and some examples are provided in Table 3.2.\footnote{The prices are collected on February 11, 2010 from both physical and online stores.}

3.6.1 Forward Integration: Symmetric Reduction to Price Sensitivity

Now we investigate what happens if manufacturers enjoy identical reduction in consumer price sensitivity when they forward integrate. Specifically, let $\beta_i^S$ be the consumer price sensitivity to product $i$ when manufacturer $i$ chooses strategy $S_i, S_i \in \{F, N, B\}$. Then the base model described in Section 3.3 entails $\beta_i^F = \beta_i^B = \beta_i^N = 1$ as (3.1) shows. In this section, we relax that assumption, allowing $\beta_i^F = \beta_i^B = \beta_i^F \leq 1$ while keeping the assumption $\beta_i^B = \beta_i^N = 1$. That is, forward integration reduces consumer price sensitivity while backward integration does not.

We need to revise our parametric assumption that ensures the concavity of supplier profit in $FB$ scenario, specifically, in accordance with $\beta_i^F \leq 1$, we now require $c > \frac{2a^2}{27d\beta^F}$. In addition, we further restrict our analysis to $d > \frac{a^2(1-\beta_i^F+k(3+\beta_i^F))}{54ck\beta_i^F}$ to eliminate the trivial case where product 1 covers all demand while product 2 does not survive in the second period under $FB$ scenario. We present the resulting equilibrium quality, prices and sales for all possible supply chain configurations in Table 6.3 in Section 6.2.1.

While our discussion in the following will focus on additional findings due to relaxing $\beta_i^F = 1$, we want to point that most of the key results of the base model continue to hold. Specifically, Proposition 18 shows vertical integration can still improve product quality and sales while it reduces the retail price as in Proposition 12. Proposition 19 shows vertical integration can be detrimental to a manufacturer, which is consistent with Proposition 13. Proposition 19 also characterizes the conditions that make forward integration more attractive than backward inte-
igration similar to Proposition 14. However, allowing $\beta^F < 1$ complicates the analysis, making characterization of manufacturers’ equilibrium vertical integration strategies as in Proposition 15 intractable. This difficulty arises because firm profit expressions involve high order polynomials, and characterizing equilibrium regions requires comparing multiple high order polynomials. Even though, we cannot fully characterize equilibrium integration strategies, Proposition 20 confirms that no integration cannot still be an equilibrium integration strategy when $\beta^F < 1$ too.

<table>
<thead>
<tr>
<th>TABLE 3.2: Examples of Difference in Retail Prices</th>
</tr>
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<tbody>
<tr>
<td>Product</td>
</tr>
<tr>
<td>Columbia Steep Slop Parka Mens’s Ski Jacket</td>
</tr>
<tr>
<td>North Face Denali Thermal Women’s Jacket</td>
</tr>
<tr>
<td>Nike Dri-Fit UV Men’s Stripe Golf Polo</td>
</tr>
<tr>
<td>Apple iPod Nano 5th Gen 8GB</td>
</tr>
<tr>
<td>Sony DSC-T90 Digital Camera</td>
</tr>
</tbody>
</table>

We first examine the effect of vertical integration on product quality and sales. When forward integration reduces consumer price sensitivity, the next proposition demonstrates that vertical integration does not necessarily improve quality and sales.

**Proposition 18** Let $S_1, S_2 \in \{F, N, B\}$:

(i.a) For $(S_1, S_2) = (B, F)$: $\theta_1^{S_1S_2} < \theta_1^{NS_2}$, $Q_1^{S_1S_2} < Q_1^{NS_2}$, $\theta_2^{S_1S_2} > \theta_2^{NS_2}$, $Q_2^{S_1S_2} > Q_2^{NS_2}$ if and only if $\beta^F < \frac{4}{\sigma^{S_2}}$.

(i.b) For $(S_1, S_2) \neq (B, F)$: $\theta_1^{S_1S_2} \geq \theta_1^{NS_2}$, $Q_1^{S_1S_2} \geq Q_1^{NS_2}$, $\theta_2^{S_1S_2} \leq \theta_2^{NS_2}$, $Q_2^{S_1S_2} \leq Q_2^{NS_2}$.

(ii.a) $p_1^{FS_2} > p_1^{NS_2}$ if and only if $\beta^F < \sigma^{S_2}$.

(ii.b) $p_1^{BS_2} < p_1^{NS_2}$; $p_2^{S_1S_2} \leq p_2^{NS_2}$ for $S_1 \neq B$.  

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The threshold $\sigma^{S_2}$ is stated in the proof in Section 6.2.2 and the equalities in (i.b) and (ii.b) holds only for $S_1 = N$.

When manufacturer 1 backward integrates, Proposition 18.(i.a) shows a new result: manufacturer 1’s anticipation of quality improvement can be nullified by the competitor’s pricing advantage. Specifically, as manufacturer 1 moves from disintegration to backward integration, the quality of product 1 gets lower if the competitor is already forward integrated. In this case, consumers are less price sensitive to the competing product 2 ($\beta^F_1 = 1$ and $\beta^F_2 = \beta^F < 1$). When $\beta^F$ is small, the competitor uses this advantage and improves its quality to offset product 1’s competitive gain from backward integration. Additionally, contrary to the previous result where vertical integration always lowers retail price, case (ii.a) shows forward integration can increase the retail price of product 1, when $\beta^F$ is small, because consumers become very price insensitive in that case.

Proposition 18 (i.b) and (ii.a) imply a new insight: forward integration increases both product quality and retail price if it significantly improves a manufacturer’s pricing advantage, i.e., $\beta^F < \sigma^{S_2}$. For example, a forward integrated firm can frequently redesign it storefronts to keep the store at the pinnacle of world fashion. This promotes consumer willingness to pay and increases the sale of premium products.

The previous proposition identifies new results for product quality, price and sales. In the next proposition, we present new findings regarding the value of vertical integration to a manufacturer.

Proposition 19

(i) $\Pi^{F_{S_2}}_{M_1} > \Pi_{M_1}^{N_{S_2}}$ if and only if $\beta^F < \tau_1^{S_2}$ for $S_2 \in \{F, N, B\}$. 
(ii) $\Pi_{M_1}^{BF} < \Pi_{M_1}^{NF}$ if and only if $\beta_F < \tau_2$; $\Pi_{M_1}^{BS_2} > \Pi_{M_1}^{NS_2}$ for $S_2 \in \{N, B\}$.

(iii) $\Pi_{M}^{FS_2} > \Pi_{M}^{BS_2}$ if and only if $\beta_F < \tau_3$ for $S_2 \in \{F, N, B\}$.

(iv) $\Pi_{M_1}^{BF} - \Pi_{M_1}^{FF} > \Pi_{M_1}^{BN} - \Pi_{M_1}^{FN}$ if and only if $\beta_F < \tau_4$; $\Pi_{M_1}^{FB} - \Pi_{M_1}^{FB} > \Pi_{M_1}^{BN} - \Pi_{M_1}^{BN}$.

The thresholds $\tau_1, \tau_2$ and $\tau_3$ are characterized in the proof in Section 6.2.2.

When forward integration improves a manufacturer’s pricing advantage, manufacturers are more likely to adopt this strategy. Consequently, regardless of the competitor’s integration strategy, forward integration can be beneficial and it can be favored over backward integration when the pricing advantage is great enough as in Proposition 19.(i) and (iii). In contrast, when the effect of vertical integration on pricing advantage is ignored, Propositions 13 and 14 state that forward integration benefits manufacturer 1 only when manufacturer 2 is already vertically integrated (i.e., $S_2 \in \{F, B\}$).

Interestingly, Proposition 19.(ii) demonstrates an additional insight: the potential benefit of backward integration can be nullified by the competitor’s superior pricing advantage. This is in contrast to our base model where backward integration is always beneficial. Specifically, moving from disintegration to backward integration harms manufacturer 1’s profitability if the competing manufacturer is already forward integrated ($S_2 = F$) and $\beta_F$ is small. In that case, the competitor uses its lower consumer price sensitivity advantage ($\beta_F^2 < \beta_F^B$), improving its product quality to undermine product 1’s potential quality advantage due to backward integration.

Finally, Proposition 19.(iv) finds an opposite result to Proposition 13: a manufacturer can be more likely to be better off by being backward integrated rather than being forward integrated as the competitor moves from disintegration to forward integration. In other words,
the manufacturer can be more likely to favor backward over forward integration as its competitor moves from disintegration to forward integration. The competitor gains the pricing advantage as it forward integrates which intensifies the competition on the retail level. When $\beta^F$ is low, this effect is strong, making a manufacturer favor backward integration which helps avoiding the intensified competition on the retail level.

Next, we summarize the equilibrium when manufacturers competitively determine their vertical integration strategies.

**Proposition 20** When $\beta^F \leq 1$ and manufacturers consider either no integration ($N$), forward ($F$), or backward ($B$) integration, then $NN$ cannot be an equilibrium.

Proposition 20 confirms when both forward and backward integrations are available to a manufacturer, disintegration in every supply chain cannot be an equilibrium outcome. This is because backward integration does not affect consumer price sensitivity. When a manufacturer chooses to stay disintegrated ($N$), its competitor always has incentive to move from disintegration to backward integration as Proposition 13 shows, i.e., $\Pi_{M_1}^{BN} > \Pi_{M_1}^{NN}$.

### 3.6.2 Forward Integration: Asymmetric Reduction to Price Sensitivity

In the previous section, we have assumed manufacturers have identical price sensitivity when they forward integrate, i.e., $\beta_1^F = \beta_2^F$. Now, we relax this assumption, allowing for $\beta_1^F \neq \beta_2^F$ where $0 < \beta_i^F < 1$, $i = 1, 2$, while maintaining the assumption $\beta_i^B = \beta_i^N = 1$. This mimics the situation in which manufacturers gain different pricing advantage when they forward integrate.

The resulting equilibrium quality, prices and sales for each scenario are summarized in Table 6.3 in Section 6.2.1. Because the model discussed in Section 3.6.1 is a special case with $\beta_1^F = \beta_2^F \leq 1,$
all of the results continue to hold for $\beta_1^F \neq \beta_2^F$. As in section 3.6.1, however, characterizing manufacturers’ equilibrium integration strategy is intractable for $\beta_1^F \neq \beta_2^F$ because firm profit expressions are high order polynomials, making comparisons across scenarios difficult.

**FIGURE 3.3: Qualities and Sales in Scenarios NF and FF**

(a) $\beta_1^F = 0.5$, $\beta_2^F = 0.7$

(b) $\beta_1^F = 0.9$, $\beta_2^F = 0.7$

In the following proposition, we summarize the impact on sales and quality when manufacturer 1 moves from disintegration to forward integration while manufacturer 2 is already forward integrated (i.e., NF to FF). We focus on the move from NF to FF because, as we will explain later, other cases (i.e., manufacturer 1 moves from disintegration to backward integration, or manufacturer 2 is already backward integrated) are its special cases.

**Proposition 21** There exists $\xi_i^\theta$ and $\xi_i^Q$, $i = 1, 2$, such that:

(i) For $\beta_1^F \leq \beta_2^F$, $Q_1^{NF} < Q_1^{FF}$, $\theta_1^{NF} < \theta_1^{FF}$, $Q_2^{NF} > Q_2^{FF}$, and $\theta_2^{NF} > \theta_2^{FF}$.

(ii) For $\beta_1^F > \beta_2^F$,

(a) $Q_1^{NF} > Q_1^{FF}$ if and only if $c < \xi_1^Q$, and $\theta_1^{NF} > \theta_1^{FF}$ if and only if $c < \xi_1^\theta$. 

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(b) $Q_2^{NF} < Q_2^{FF}$ if and only if if $c < \xi_2^Q$, and $\theta_2^{NF} < \theta_2^{FF}$ if and only if $c < \xi_2^\theta$.

(iii) $\xi_1^\theta$ and $\xi_1^Q$ increase in $\beta_1^F - \beta_2^F$,

where $\xi_1^\theta$ and $\xi_1^Q$ are given in the proof in Section 6.2.2.

When forward integration gives manufacturer 1 a superior pricing advantage ($\beta_1^F \leq \beta_2^F$), it improves both its quality and sales quantity while reducing those of the competitor. Figure 3.3 depicts the product quality and sales quantity for each firm in $NF$ and $FF$ scenarios. The solid curves represent product 1 while the dashed ones represent product 2. In Figure 3.3(a), $\beta_1^F < \beta_2^F$ and in this case quality and sales quantity increase for product 1 when the market moves from $NF$ to $FF$ scenario.

Interestingly, if forward integration leads to inferior pricing advantage for manufacturer 1 ($\beta_1^F > \beta_2^F$), forward integration hurts both quality and sales quantity for product 1 when the quality cost $c$ is low, as shown in Figure 3.3(b). In that case, superior pricing advantage allows supply chain 2 to improve its product quality, discouraging supply chain 1 from improving its quality. Nevertheless, if $c$ is high, this suppression is too costly and vertical integration improves quality and sales quantity for product 1.

![FIGURE 3.4: The Effect on Quality when Changing from NN to FN](image-url)
The results in Proposition 21.(i) and (ii) can be understood as follows: Whether manufacturer 1’s move from disintegration to forward integration increases the quality for product 1 is determined by (i) manufacturer 1’s benefit from forward integration or (ii) the effect of pricing advantage. As Figure 3.4 illustrates, when the quality cost $c$ is high, as in the high region of Figure 3.4, manufacturer 1’s benefit from forward integration dominates, so forward integration increases its product quality. On the other hand, when $c$ is low, as in the low region of Figure 3.4, pricing advantage dominates: product 1 quality is improved only when manufacturer 1 has superior pricing advantage (i.e., $\beta_{1F}^F < \beta_{2F}^F$).

Moreover, part (iii) of proposition 21 shows that $\xi_i^\theta$ and $\xi_i^Q$ increase in product 2’s pricing advantage $\beta_{1F}^F - \beta_{2F}^F$. In other words, the greater pricing advantage the competitor has, the more likely it reacts to manufacturer 1’s forward integration by improving its product quality.

While Proposition 21 is concerned with the effect of moving supply chain structures from $NF$ to $FF$, it can also explain the effect of manufacturer 1’s vertical integration when its competitor is already vertically integrated (i.e., $NB$ to $FB$, $NB$ to $BB$, or $NF$ to $BF$), as these are just special cases for Proposition 21 with $\beta_{1S1}^S = 1$ or $\beta_{2S2}^S = 1$.

### 3.7 Concluding Remarks

In this paper we examine vertical integration strategies under channel competition. To this end, we analyze two competing three-tier supply chains, each with a supplier, a manufacturer and a retailer. The retailers sell vertically differentiated products in a consumer market. Each manufacturer considers three vertical integration strategies: forward, backward and no integration. Forward integration enables a manufacturer to control its retail price, whereas backward
integration allows it to control product quality.

We show backward integration is always beneficial to a manufacturer due to its quality leadership advantage. Forward integration, however, can be detrimental because intensified supply chain competition drops the retail price, leading to a lower profit margin. This negative effect is alleviated when the competing channel is shorter (i.e., already integrated), and thus a manufacturer is more likely to forward integrate when its competitor moves from disintegration to vertical integration.

Regarding manufacturers’ competitive choice of integration strategies, we highlight the implication of having the option of backward integration. When manufacturers consider solely moving from disintegration to forward integration, no integration in every channel can be an equilibrium outcome, consistent with McGuire and Staelin (1983), Gupta and Loulou (1998) and Trivedi (1998). This is no longer an equilibrium, however, when backward introduction is also considered. Finally, we encounter prisoner’s dilemma: in equilibrium, all manufacturers choose to vertically integrate, and this lowers their as well as the entire channel’s profitability.

Finally, we find a manufacturer’s vertical integration can result in a higher quality product sold at a lower price. This occurs because vertical integration encourages quality improvement and it results in fewer firms adding their margins to the retail price. On the other hand, the competitor always responds to a manufacturer’s vertical integration by lowering both its retail price and product quality.

We identify additional results when forward integration reduces consumer price sensitivity. First, we show backward integration can be detrimental if the competitor is already forward integrated. When a manufacturer backward integrates, the potential benefit is nullified by the competitor’s reaction of increased quality. As a result, the manufacturer lowers its quality and
suffers from having a smaller sales quantity. In addition, forward integration can both increase retail price and product quality.

Our model has several limitations. We assume products are sold through exclusive retailers and therefore ignore product line pricing. While it is common for company stores to sell products exclusively for a manufacturer, general retailers usually carry products of multiple brands. In that case, an individual retailer conducts product line pricing, further intensifying retail competition, and we expect forward integration to be less valuable than our model predicts. It is recognized that consumers may strategically delay their shopping in anticipation of price drop. Nonetheless, our model considers long term strategic decisions and thus, it does not take the short term strategic actions of consumers into account. Finally, we ignore the effect of demand uncertainty to focus on the choice between control over quality from backward integration versus control over retail price from forward integration.
CHAPTER 4

Are Strategic Customers Bad for a Supply Chain?

4.1 Introduction

Retailers frequently use discounts as a means for increasing sales. Customers also learn to act strategically, and many holiday shoppers plan to wait for the year-end sales. Moreover, fierce competition during holiday seasons drags retailers into price wars, forcing them to offer deeper and deeper discount (Bynes and Zellner 2004). Thinner profit margins and great demand uncertainty challenge retailers’ ability to provide adequate service level and, therefore, profitability. Thus, it is commonly believed that customers’ strategic behavior may posture an adverse effect on retailers’ profitability. However, is this really the case? If so, does it also have a negative impact on the performance of a supplier and the entire supply chain as a whole? These are the questions we try to answer.

Recognizing the importance of strategic customer behavior, there is a growing body of research studying its implication in operations management, and many remedies have been proposed to mitigate the adverse effect of strategic customers. Examples of those remedies include: using price or quantity commitment to reduce markdown (Su and Zhang 2008; Aviv
and Pazgal 2008; Su and Zhang 2009); satisfying only a fraction of the demand at a lower price to increase willingness to pay (Zhang and Cooper 2008); creation of scarcity to eliminate strategic behavior with low inventory (Liu and van Ryzin 2008; Su and Zhang 2008) or adjusting display format (Yin and Tang 2006); using posterior price match (Lai et al. 2010); better alignment of supply and demand with quick response (Cachon and Swinney 2009). There are also works centering on a firm’s optimal pricing decision in the presence of strategic customers (Levin et al. 2009; W. Elmaghraby and Keskinocak 2008; Levin et al. 2010) or on customers’ learning process to anticipate future sales (Popescu and Wu 2007). Despite the growing attention on strategic customers, none of the literature actually examines its effect on firm profitability.

As such, we propose the following research questions and attempt to answer them in this chapter:

- Does customers’ strategic behavior deteriorate the profitability of every firm in a supply chain?
- How does it affect the profitability of the entire supply chain as a whole?
- How does the strategic behavior impact firms’ pricing and ordering decisions? Does the presence of strategic behavior encourage a retailer to restrict product availability?

### 4.2 The Model and Assumptions

We consider a supply chain with a supplier serving a retailer. The retailer sells a product over two periods, $t = 1, 2$. Every firm is risk neutral and aims to maximize its profit over two periods. Before the selling season starts, the supplier first sets a unit wholesale price $w$ it charges to the retailer, and then the retailer determines its order quantity $Q$ which will be
FIGURE 4.1: Sequence of Events

delivered before the selling seasons start. At the beginning of each selling season \( t \), the retailer sets the retail price \( p_t \), and then customers make their purchase decisions. The sequence of events is depicted in Figure 4.1 and we assume all prices (\( w \) and \( p_t \)) are public information.

Any inventory leftover at the end of period 1 will be carried over to period 2 at no inventory holding cost. Inventory has zero salvage value at the end of period 2. We assume firms do not incur variable production cost for ease of exposition. The retailer and the supplier know the order quantity \( Q \), but customers do not observe that quantity and they form a common belief \( \hat{Q} \) about it for decision making, i.e., they do not know a retailer’s inventory level.

We assume there is a unit mass of customers, each of whom has a unitary demand. Every customer has an individual valuation \( V \) known only to himself, and \( V \) is publicly known to distribute uniformly on \((0, 1)\). However, a customer’s interest in the product decreases over time and its valuation in \( t = 2 \) is decreased by \( 1 - \delta \) where \( 0 < \delta < 1 \). Thus, a customer’s surplus from a purchase in period \( t \), \( t = 1, 2 \), is given by

\[
U_t = \delta^{t-1} V - p_t. \tag{4.1}
\]
Customers have two types: Myopic (M) and strategic (S). A *myopic* customer buys a product in the first period if and only if $U_1 > 0$. Otherwise he waits until the second period and buys the product if and only if $U_2 > 0$. On the other hand, a *strategic* customer times his purchase to maximize his surplus. Specifically, based on his belief, a strategic customer conjectures the retail price $p_2(\hat{Q})$ in the second period. Then he buys a product in the first period if and only if

$$U_1(p_1) > \text{Max}[0, U_2(p_2(\hat{Q}))]. \quad (4.2)$$

Thus, a strategic customer looks forward into future, whereas a myopic customer makes his purchase decision solely based on the current retail price.

We solve for the firms’ and customers’ subgame perfect Nash equilibrium (SPNE) pricing and purchasing decisions. When customers are strategic, they anticipate future price based on their belief $\hat{Q}$. In that case, we look for rational expectations equilibrium which assumes customers’ belief turns out to be correct. This is a common assumption in the strategic customer literature (e.g., Su and Zhang 2008; Cachon and Swinney 2009; Su and Zhang 2009). Let $\theta$ be the valuation of the marginal customer who is indifferent between purchasing in period 1 and period 2. Then a SPNE satisfies the following:
\[ p_2^*(\theta, Q) = \arg \max_{p_2} p_2 \min[(\theta - p_2/\delta), \bar{Q}(\theta, Q)] \]  
\[ \bar{Q}(\theta, Q) = \max[Q - (1 - \theta), 0] \]  
\[ \theta^* = \inf \{ \theta : \theta - p_1 > \bar{V} \}, \]  
\[ \bar{V} = \begin{cases} 
0 & \text{for myopic customers,} \\
\delta \theta - p_2^*(\theta, \bar{Q}) & \text{for strategic customers,} 
\end{cases} \]  
\[ (p_1^*, Q^*) = \arg \max_{(p_1, Q)} p_1 \min[1 - \theta^*, Q] + p_2^*(\theta^* - p_2^*/\delta) - w Q, \]  
\[ \bar{Q} = Q^*, \]  
\[ w^* = \arg \max_w w Q^*. \]  

Basically, (4.3) states the retailer sets \( p_2 \) to maximize its revenue in the second period. \( \bar{Q}(\theta, Q) \) is defined in (4.4) and it denotes the inventory carried over from period 1 to period 2. The minimum operator in (4.3) indicates the retailer cannot sell more than what it has. (4.5) defines the marginal customer who is indifferent between purchasing and not purchasing in period 1. Next, (4.5) and (4.6) show for myopic customers, the marginal customer generates zero utility from purchasing in period 1 and a customer buys the product in period 1 if \( V > \theta^* \). On the other hand, when customers are strategic, the marginal customer is indifferent between purchasing the product in period 1 and 2. A strategic customer conjectures the retail price in period 2 and waits for period 2 if his utility is lower than the marginal value \( \theta^* \). (4.7) states the retailer chooses its order quantity \( Q \) and the retail price \( p_1 \) in the first period to maximize its profit. Furthermore, we consider the rational equilibrium which requires the belief to be correct as (4.8) shows. Finally, (4.9) states the supplier selects the wholesale price \( w \) to maximize its profit.
4.3 Equilibrium Decisions

In this section, we derive firms’ equilibrium decisions for each customer type and contrast these decisions between strategic and myopic customers. As Figure 4.2 shows, the supplier first determines the wholesale price \( w \). Then the retailer places an order \( Q \), and then it sets the retail price \( p_t \) at the beginning of each period \( t \). We derive the equilibrium decisions by solving equations (4.3-4.9) in three stage games using backward induction. First, we characterize the retailer’s decision on the retail price for the second period \( p_2 \). Next, we derive its order size \( Q \) and the retail price \( p_1 \) for the first period. Finally, we analyze the supplier’s choice of the wholesale price \( w \).

Following backward induction, first we examine the retailer’s pricing decision for the second period, which leads to the following lemma.

**Lemma 4** Let \([0, \theta)\) be the segment of customers remains in the market at the beginning of \( t = 2 \) and \( \bar{Q} \) be the retailer’s remaining inventory at that time. Then

(i) For \( \bar{Q} \leq \frac{\theta}{2} \), \( p_2 = \delta (1 - Q) \) and the retailer sell \( \bar{Q} \) in \( t = 2 \).

(ii) For \( \bar{Q} > \frac{\theta}{2} \), \( p_2 = \frac{\delta \theta}{2} \) and the retailer does not sell all of \( \bar{Q} \) in \( t = 2 \).

The retailer sells the product in period 1, and it carries remaining inventory, if any, to period 2. Since inventory is not replenished in period 2, Lemma 4 shows the retailer sells all of the remaining inventory if the inventory level is not too high. Otherwise, the retailer sells only some of its inventory in period 2.
In the second stage game, the retailer determines its order quantity $Q$ and the retail price $p_1$ for the first period. When customers are myopic, the marginal customer is $\theta = p_1$. However, when they are strategic, the marginal customer is indifferent between purchasing in the first or second period, i.e., $\theta - p_1 = \delta \theta - p_2(\hat{Q})$, where $p_2(\hat{Q})$ is given by Lemma 4. Note that customers do not observe the true inventory level, and they conjecture the retail price $p_2$ in the second period based on their belief $\hat{Q}$. Having the marginal customer characterized for both customer types, the retailer’s profit is given by

$$\pi_R = p_1(1 - \theta) + p_2(Q - (1 - \theta)) - wQ,$$  \hspace{1cm} (4.10)

where $p_2$ is a function of $\hat{Q}$ given by Lemma 4. The retailer then chooses its order quantity $Q$ and retailer price $p_1$ for period one to maximize its profit. Specifically, first we derive the optimal retail price for any given $Q$ (and $\hat{Q}$ if customers are strategic). Then we characterize the optimal order quantity for the retailer. When customers are strategic, the equilibrium order quantity satisfies $\frac{\partial \pi_R}{\partial Q} = 0$ and $\hat{Q} = Q$. This process leads to the following lemma:

**Lemma 5** When customers are strategic, the retailer’s equilibrium order quantity and retail price are:

(i) For $w < \frac{\delta}{2}$, $Q = \frac{\delta^2 - 2w}{4\delta}$ and $p_1 = \frac{2 + 2w - \delta}{4}$. The product is sold in $t = 1, 2$.

(ii) For $\frac{\delta}{2} \leq w < \frac{\delta}{2 - \delta}$, $Q = \frac{1 - w}{2 - \delta}$ and $p_1 = \frac{1 + w - \delta}{2 - \delta}$. The product is sold only in $t = 1$.

(iii) For $\frac{\delta}{2 - \delta} \leq w$, $Q = \frac{1 - w}{2}$ and $p_1 = \frac{1 + w}{2}$. The product is sold only in $t = 1$.

When customers are myopic, the retailer’s optimal order quantity and retail price are:

(iv) For $w < \frac{\delta}{2 - \delta}$, $Q = \frac{(3 - \delta)\delta - 2w}{(4 - \delta)^2}$ and $p_1 = \frac{2 + w}{4 - \delta}$. The product is sold in $t = 1, 2$. 

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(v) For \( \delta \leq w \), \( Q = \frac{1-w}{2} \) and \( p_1 = \frac{1+w}{2} \). The product is sold only in \( t = 1 \).

Cases (i) to (iii) of Lemma 5 describe the retailer’s decisions when customers are strategic. In this case, the retailer orders more and the product is sold in both periods when the wholesale price \( w \) is low. But when \( w \) is sufficiently high, as in cases (ii) and (iii), the retailer orders less and it sells the product only in the first period. In this case, strategic customers recognize that the product will not be available in \( t = 2 \), which increases their willingness to buy in \( t = 1 \). Consequently, this encourages the retailer to order more when \( w \) is not overly high as in case (ii) (i.e., \( \frac{1-w}{2} > \frac{1+w}{2} \)). On the other hand, when the wholesale price \( w \) is too high, as in case (iii), the increased willingness to pay does not encourage the retailer’s order size. In contrast, when customers are myopic as in cases (iv) and (v), the retailer is not encouraged to order more as in case (ii) because customers do not take future product availability into account.

Next we characterize the supplier’s choice of wholesale price \( w \). The supplier sets \( w \) to maximize its profit:

\[
\pi_S = wQ.
\]

We summarize the supplier’s choice of \( w \) as well as the retailer’s decisions in the following proposition.

**Proposition 22** When customers are strategic, the retailer sells the product only in \( t = 1 \) in equilibrium. When customers are myopic, the retailer sells the product in both periods for \( \delta > 2 - \sqrt{2} \) and it sells the product only in \( t = 1 \) otherwise. The firms’ equilibrium decisions are summarized in Table 4.1.
Customer Type | Strategic | Myopic
---|---|---
Range | $\delta \leq 0.4247$ | $\delta \leq 2 - \sqrt{2}$
 | $0.4247 < \delta \leq \frac{5}{4}$ | $2 - \sqrt{2} < \delta$
 | $\frac{2}{3} < \delta$ | $2 - \sqrt{2} < \delta$

TABLE 4.1: Firms’ Equilibrium Decisions

Consistent with prior study (e.g., Xu and Zhang 2008), Proposition 22 states that the product is only sold in the first period when customers are strategic. To unveil the impact of having strategic customers, we compare firms’ equilibrium decisions across customer types, which leads to the following corollary. Note the superscript $S$ denotes strategic customers and $M$ refers to myopic ones.

**Corollary 1**

(i) $p_1^S \leq p_1^M$.

(ii) $Q^S \geq Q^M$.

(iii) $w^S < w^M$ if and only if $0.4247 < \delta < 2 - \sqrt{2}$.

As expected, Corollary 1 (i) shows the retail price is lower when customers are strategic. However, the total sales are higher when customers are strategic. This is due to a lower retail price and the customers’ recognition that the product will not be available in the second period. Interestingly, case (iii) states the wholesale price $w$ is lower with strategic customers when $\delta$ is sufficiently, but not overly, high. The reason is as follows. When $\delta$ is low, the product’s value in the second period is negligible. In this case, customer type does not affect decisions and the product is sold only in the first period. But when $\delta$ is sufficiently, but not overly,
high, the product becomes sufficiently attractive to customers in the second period. As such, the shortage in that period increases strategic customers’ willingness to buy in the first period, motivating the supplier to lower the wholesale price for a higher sales. But when $\delta$ is very high, the wholesale price is lower with myopic customers, because $w$ is dropped in that case so the product is sold in both periods, benefiting the supplier from a higher sales.

### 4.4 Impact of Customer Type and Channel structure on Profitability

We have shown how firm decisions depend on customer type. Now we shift our focus to profitability. First we examine the impact of customer type on a firm’s profitability. Next, we discuss how the total channel profitability depends on its structure (centralized versus decentralized channel) and customer type.

#### 4.4.1 Impact of Customer Type on Firm Profit

In this section, we examine how customer type affects firm profit? Does having strategic customers decrease profitability? To this end, we use the decisions characterized in Proposition 22 to derive firm profits described by equations (4.10) and (4.11). Let $\Pi_S$ and $\Pi_R$ be the supplier’s and the retailer’s equilibrium profits respectively. We use superscripts $SD$ and $MD$ to denote the scenarios with strategic customers and myopic customers, respectively. The next proposition summarizes the impact of having strategic customers on firm profits:

**Proposition 23**

(i) $\Pi_S^{SD} > \Pi_S^{MD}$ for $\delta > 0.4247$, and $\Pi_S^{SD} = \Pi_S^{MD}$ otherwise.
(ii) $\Pi^S_D > \Pi^M_D$ if and only if $0.4247 < \delta < 2 - \sqrt{2}$.

Customers’ product valuation increases in $\delta$, and so does strategic customers’ tendency to wait. When $\delta \leq 0.4247$, the product value is too low in the second period. The product is only sold in the first period and the customer type does not affect profitability. Interestingly, when $\delta$ is sufficiently high, Proposition 23 (i) shows that the supplier benefits from having strategic customers. In this case, the product is only sold in the first period, and the supplier benefits from a higher sales because the shortage in the second period increases strategic customers’ willingness to buy.

Surprisingly, Proposition 23 (ii) states the retailer can be strictly better off when customers are strategic. The benefit is driven by a lower wholesale price. Specifically, for $0.4247 < \delta < 2 - \sqrt{2}$, the supplier charges a lower wholesale price than it would with myopic customers to encourage order quantity (Corollary 1). As a result, the retailer benefits from a lower cost.

Combining cases (i) and (ii) suggests that both firms are strictly better off when customers become strategic for $0.4247 < \delta < 2 - \sqrt{2}$. That is, firms can actually take advantage of customers’ strategic behavior by selling a product only in the first period to increase customers’ willingness to purchase at a higher price. When the product is sufficiently, but not overly, fashionable, both firms lower their prices and reap a greater profit from increased sales.

### 4.4.2 Supply Chain Performance

The last section demonstrates that customers’ strategic behavior can actually benefit a firm. Does it also benefit the total profit achieved by the entire supply chain? Does the entire supply chain perform better in a centralized system? We provide answers to these questions in this
section. Let $\Pi_T = \Pi_S + \Pi_R$ be the total profit achieved by a supply chain. Our base model is a decentralized system where each firm chooses decisions in its own favor, and we use superscripts $SD$ and $MD$ to denote scenarios with strategic and myopic customers, respectively. Here, we also define superscripts $SC$ and $MC$ similarly for a centralized system where decisions are made by a central planner who aims to maximize the total channel profit. We describe the equilibrium decisions for $SC$ and $MC$ scenarios in Appendix 6.3.

First, we discuss the effect of having strategic customers on channel profitability. The following proposition shows a decentralized supply chain reaps a higher profit when customers are strategic. Note the subscript $T$ denotes the total profit achieved by the entire channel.

**Proposition 24**

(i) $\Pi_{T}^{SC} < \Pi_{T}^{MC}$.

(ii) $\Pi_{T}^{SD} \geq \Pi_{T}^{MD}$.

Strategic customers take future price and inventory availability into account and time their purchase. Compared to myopic customers, intuitively this should adversely affect channel profitability, and Proposition 24 (i) confirms this belief in a centralized supply chain. Interestingly, Proposition 24 (ii) illustrates a decentralized supply chain is more profitable when customers are strategic. This is due to a higher sales when customers are strategic as Corollary 1 shows: the shortage of product in the second period encourages the sales in the first period, outweighing the disadvantage of a lower retail price with strategic customers. In other words, making a product available only when it is “hot” benefits the entire channel when customers become strategic.

Next, we examine the effect of channel structure, i.e., centralized or decentralized supply
chain, on channel profitability. The following proposition shows a decentralized supply chain can outperform a centralized one when customers are strategic.

**Proposition 25**

(i) \( \Pi^M_D < \Pi^M_C \).

(ii) \( \Pi^S_D > \Pi^S_C \) if and only if \( \delta > 0.4247 \).

(iii) \( Q^S_D < Q^S_C \) and \( p^S_D > p^S_C \).

It is generally believed that a centralized supply chain achieves a higher profit than a decentralized one, and Proposition 25 (i) confirms this belief when customers are myopic. In contrast, case (ii) states that having strategic customers benefits a decentralized channel when \( \delta \) is sufficiently high. To see why, first observe from case (iii) that a decentralized system achieves smaller sales but a higher retail price. When \( \delta \) is small, the impact of having two selling periods is insignificant as the product value is low in the second period, making a centralized channel outperform a decentralized one.

Nevertheless, when \( \delta \) is large enough, a decentralized channel performs better: customers’ strategic behavior is significant and a decentralized channel benefits from reducing inventory level in exchange for a higher margin. Note that a centralized channel cannot credibly reduce its sales for a higher margin, because customers do not believe the retailer would order a small amount in the absence of a wholesale price.

Common intuition suggests that both decentralization and strategic customers deteriorate channel profitability. Drawing from the previous two propositions, the following corollary actually finds the channel does not necessarily perform the worst in the \( SD \) scenario.
Corollary 2

(i) $\Pi^{MC}_{I} > \text{Max}(\Pi^{MD}_{I}, \Pi^{SC}_{I}, \Pi^{SD}_{I})$.

(ii) $\Pi^{SD}_{I} > \text{Max}(\Pi^{MD}_{I}, \Pi^{SC}_{I})$ if and only if $\delta > 0.4247$.

Essentially, Corollary 2 compares channel profit across four possible scenarios from the two dimensions we discussed: myopic versus strategic customers and centralized versus decentralized systems. First note that a centralized system with myopic customer achieves the highest profit as one would expect. However, case (ii) finds that a decentralized system with strategic customer (SD) outperforms other two possible supply chain setups (MD and SC). That is, the scenario with both seemingly adverse effects, strategic customers and decentralized system, can achieve a higher total channel profit than scenarios where only one of them takes place. Decentralization lowers the inventory level of a channel, and strategic behavior increases customers’ willingness to purchase at $p_1$. As a result, the entire channel benefits from higher sales at the regular price $p_1$.

4.5 Extensions

We have shown that having strategic customers does not necessarily imply lower firm profitability. In contrast, having strategic customers can Pareto-improve firm profits. In this section, we consider several extensions and demonstrate these key results continue to hold in various situations.
4.5.1 Capacity Limit

In the base model described in section 4.2, we assume the supplier does not have any capacity limit and it can meet any demand. Suppose now the supplier does have a production capacity limit which it determines in advance. Do our results continue to hold? Specifically, we assume in this section that the supplier sets its production capacity and it incurs a cost $c$ per unit capacity. The supplier first determines its capacity level $k$, and it chooses the wholesale price $w$. The retailer then places an order subject to the capacity limit $k$ and the rest of events unfold as Figure 4.2 shows. Thus, the supplier’s profit becomes:

$$\Pi_S = w \min(Q, k) - ck.$$  \hspace{1cm} (4.12)

We derive firms’ equilibrium decisions using backward induction which are summarized in Appendix 6.3. Basically, we find when there is production capacity limit, product sales are decreased but firm behavior is not changed: the product is sold only in the first period when customers are strategic, and it can be sold in both periods when customers are myopic. In the next proposition, we summarize the impact of having strategic customers under capacity limit. Recall that $\Pi_i$ denotes the equilibrium profit for firm $i$, which refers to a supplier ($S$), a retailer ($R$) or the entire channel ($C$).

**Proposition 26** When the supplier invests production capacity at unit cost $c$:

(i) $\Pi^{SD}_S \geq \Pi^{MD}_S$.

(ii) $\Pi^{SD}_R > \Pi^{MD}_R$ if and only if $\bar{c} < c < c$.

(iii) $\Pi^{SD}_C \geq \Pi^{MD}_C$, 

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where

\[ \bar{c} = \max(1 - \frac{8(1 - \delta)\sqrt{1 - \delta}}{(2 - \delta)^2}, 1 - \frac{(1 - \delta)(2 + \sqrt{(4 - \delta)\delta})}{2 - \delta}), \text{ and } c = \frac{\delta^2 - 4(1 - \delta)(1 - \sqrt{2\delta})}{(2 - \delta)^2}. \]

Proposition 26 shows that our key results continue to hold. Specifically, a supplier and the entire supply chain achieve higher profits when customers are strategic. Also, a retailer can reap a higher profit with strategic customer when the capacity cost \( c \) is sufficiently, but not overly, high.

### 4.5.2 Exogenous Wholesale Price

In the base model we assume the supplier has dominant bargaining power and acts as the Stackelburg leader. In practice, however, this may not be the case. In this section, we assume the wholesale price \( w \) is exogenous, and firm decisions are characterized in Lemma 5. One can interpret \( w \) as the bargain outcome of a retailer and its supplier: a smaller \( w \) favors the retailer and it implies stronger bargaining power to the retailer. In the following proposition, we show the supplier and the entire supply chain can still be better off when customers are strategic.

**Proposition 27** When the wholesale price \( w \) is exogenously determined:

1. \( \Pi_{RD}^{SD} \leq \Pi_{RD}^{MD} \).
2. \( \Pi_{SD}^{SD} > \Pi_{SD}^{MD} \) if and only if \( w < \frac{\delta}{2 - \delta} \).
3. \( \Pi_{CD}^{SD} > \Pi_{CD}^{MD} \) if and only if \( \frac{\delta}{2} < w < \frac{\delta}{2 - \delta} \).

Proposition 27 contrasts firm profits between having myopic and strategic customers. When \( w \) is exogenously determined, the retailer’s ordering cost does not differ by the customer type.
Thus, when customers are strategic, the retailer does not benefit from having a lower ordering cost as in Corollary 1, and customers’ strategic behavior decreases its profitability as case (i) shows. Interestingly, cases (ii) and (iii) illustrate that profitability of the supplier and the entire channel can still be higher when customers are strategic. It can be shown that the retail price \( p_t, t = 1, 2 \), is lower when customers are strategic, benefiting the supplier from a higher sales in this case. This benefit is the most significant for medium \( w \), making the entire channel benefit from having strategic customers.

4.5.3 Quick Response

Suppliers often offer quick response service to provide retail customers with additional replenishment opportunities during a selling season. In our base model, a retailer has only one ordering opportunity. In this section, we consider what happens if a retailer can place a second order \( q \) at the same wholesale price \( w \) after demand in the first period is realized, but before the retail price \( p_2 \) of the second period is determined. The sequence of events is depicted in Figure 4.2.

![Sequence of Events with Quick Response](image)

**FIGURE 4.2: Sequence of Events with Quick Response**

We derive firms’ equilibrium decisions using backward induction, and they are summarized in the following lemma.
Lemma 6 Suppose a retailer has a second replenishment opportunity and

(i) customers are myopic, then the equilibrium decisions are given in Table 4.1.

(ii) customers are strategic, then the equilibrium decisions are given in Table 4.2.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>(\frac{1}{4})</th>
<th>(\frac{2-\delta}{4-3\delta})</th>
<th>(\frac{6-5\delta}{2(4-3\delta)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w)</td>
<td>(\frac{1}{4})</td>
<td>(\frac{2(1-\delta)}{4-3\delta})</td>
<td>(\frac{(6-5\delta)\delta}{4(4-3\delta)})</td>
</tr>
<tr>
<td>(Q+q)</td>
<td>(\frac{1}{4})</td>
<td>(\frac{2-\delta}{4-3\delta})</td>
<td>(\frac{2+\delta}{4})</td>
</tr>
<tr>
<td>(p_1)</td>
<td>(\frac{3}{4})</td>
<td>(\frac{2-\delta}{4-3\delta})</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>(p_2)</td>
<td>n/a</td>
<td>n/a</td>
<td>(\frac{(10-7\delta)\delta}{4(4-3\delta)})</td>
</tr>
</tbody>
</table>

TABLE 4.2: Firm Decisions with Quick Response when Customers are Strategic

Lemma 4.2 (i) states that having quick response ability does not alter the equilibrium outcome when customers are myopic. This is because the wholesale price is the same for both ordering opportunities, which nullifies the value of having another ordering opportunity in our deterministic setting. On the other hand, when customers are strategic as in case (ii), the retailer does not use the second replenishment when $\delta < \frac{2}{3}$ as the product value is too low in the second selling period. Thus, the product is sold only in the first period in this case. But when $\delta \geq \frac{2}{3}$, high product value in the second period justifies a second order, and the product is sold in both periods. Using the decisions in Lemma 6, we compare firms’ equilibrium profits and summarize the impact of having strategic customers in the following proposition:

Proposition 28 When the retailer has an additional ordering opportunity:

(i) $\Pi_{SD}^R > \Pi_{MD}^R$ if and only if $0.5245 < \delta < 2 - \sqrt{2}$.

(ii) $\Pi_{SD}^S > \Pi_{MD}^S$ if and only if $0.5245 < \delta < 0.842$.

(iii) $\Pi_{SD}^C > \Pi_{MD}^C$ if and only if $0.5245 < \delta < 0.6062$. 

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Essentially, Proposition 28 shows that firm profits can still be higher when customers are strategic. For a retailer, it benefits from a lower wholesale price, as in our base model, when $\delta$ is sufficient, but not overly, high. However, Proposition 28 (ii) demonstrates the supplier can be worse off by having strategic customers when $\delta$ is high enough (Recall that, in the base model, a supplier is better off by having strategic customers). Specifically, quick response allows a retailer to achieve its ideal sales through the additional ordering opportunity, enabling it to credibly maintain a low inventory level and benefit from a higher retail price. When $\delta$ is high, customers are more likely to postpone their purchase and, in response, a retailer orders less to keep a high profit margin, hurting its supplier with reduced sales. As a result, the reduced supplier profit hurts the total channel profit when $\delta$ is sufficiently high.

### 4.5.4 Price and Quantity Commitments

Prior studies have shown the use of price or quantity commitments can effectively mitigate the adverse effect of strategic customers (e.g., Su and Zhang 2008; Aviv and Pazgal 2008; Su and Zhang 2009). However, their impact on the profitability of a supplier and the entire channel has received less attention. Thus, in this section, we derive equilibrium decisions under these commitments, and examine their impact on firm profitability. Specifically, when price commitment is implemented, a retailer announces its retail prices $p_1$ and $p_2$ before any demand is realized. On the other hand, when quantity commitment is applied, it announces its inventory level before the retail price $p_1$ is determined. In other words, price and quantity commitments eliminate customers’ uncertainty about future price and inventory level, respectively. Recall that the superscript $SD$ refers to a decentralized channel with strategic customers, and let $SDC$ denote that scenario with price or quantity commitments. In the following proposition,
we show that price or quantity commitments do not necessarily benefit a firm.

**Proposition 29** When a retailer commits on its future retail price or order quantity:

(i) In either case, the supplier sets \( w = \frac{1}{2} \) and the retailer orders \( Q = \frac{1}{4} \). The retail prices are \( p_1 = \frac{3}{4} \) and \( p_2 = \frac{3\delta}{4} \), and the product is only sold in \( t = 1 \).

(ii) \( \Pi_{SDC}^R > \Pi_{SD}^R \) if and only if \( 0.4247 < \delta < 0.6135 \).

(iii) \( \Pi_S^{SDC} \leq \Pi_S^{SD} \) and \( \Pi_C^{SDC} \leq \Pi_C^{SD} \).

When a retailer commits on its future retail price, customers can correctly infer the retailer’s inventory level. Similarly, quantity commitment enables customers to correctly anticipate future price. Thus, both commitments eliminate customers’ uncertainty on price and quantity, and they lead to the same equilibrium outcome as Proposition 29 (i) shows. Under these commitments, a retailer sets a high \( p_2 \) so its product is sold only in the first period at a higher margin. Interestingly, case (ii) finds that the retailer profit can be lower with price or quantity commitments, because the wholesale price is higher when a commitment is made. Specifically, the revelation of price or order quantity dampens customers’ uncertainty and thus their motivation to purchase in the first period. Consequently, the supplier lacks the motivation to lower its wholesale price when a commitment is made. In addition, we can show that these commitments lower the retailer’s order size, hurting the supplier’s profit as well as the total channel profitability as case (iii) shows.
4.6 Concluding Remarks

Strategic customers have attracted a growing attention in operations literature. Many remedies have been proposed to mitigate its adverse impact on a retailer. However, the impact of having strategic customers on other channel participants has received less attention. Thus, in this chapter, we investigate what happens to firm profitability when customers become strategic.

We study a supply chain with a supplier serving a retailer who sells a product over two periods. The supplier determines the wholesale price it charges to the retailer, while the retailer chooses its order quantity and retail price. Customers have heterogeneous product valuation which decreases over time. There are two types of customers: strategic customers are those who take future price into account when making their purchase decision, while myopic customers ignore those factors. We characterize firms’ subgame perfect equilibrium decisions for each customer type. We contrast the equilibrium decisions and firm profits under each customer type to understand the impact of having strategic customers.

Our results show that firms can be more profitable when customers are strategic. In that case, a retailer sells its product only in the first period, i.e., when the product is “hot”. Limiting product availability increases strategic customers’ willingness to buy, benefiting a supplier and the entire channel from a higher sales. In addition, due to customers’ increased willingness to buy, the supplier lowers its wholesale price to encourage sales when the product is sufficiently, but not overly, perishable. Consequently, this benefits the retailer by a lower purchase cost in that case. We also extend our model and show firm profits can be higher with strategic customers when (1) the wholesale price is exogenously determined, (2) a supplier has a production capacity limit, and (3) a retailer has an additional ordering opportunity.
We also examine the impact of channel structure on the performance of the entire supply chain. Common intuition suggests that decentralized decision making or strategic customers decrease the total channel profitability. Surprisingly, we show that the total channel profit can be higher when both of them hold. That is, firms can actually take advantage of these seemingly disadvantageous factors to improve channel performance: firms can stimulate sales by selling a product to strategic customers only when it is “hot”, and decentralization lowers inventory level for higher profit margin. In sum, having strategic customers does not necessarily decrease firms’ profitability, and a product’s perishability plays a pivotal role on the impact of having strategic customers.

Our model also has several limitations. For instance, we assume that each customer has identical discount $\delta$ to product value. In practice, this discount may vary by person, and one may investigate whether our results continue to hold in this situation. Also, in some cases, customers’ product valuation may increase in the number of total customers who own the product. This phenomenon is not captured in our model, and reexamining our results in this situation can be an interesting future avenue. As our model shows, many commonly accepted results in the literature can be different when customers are strategic. It will be an prospective avenue to examine whether commonly used contracts (e.g., two-part tariff and revenue sharing contracts) can still coordinate a supply chain when customers are strategic.
CHAPTER 5

Conclusions and Future Research

Retailing is a vital industry in most developed economies, and it is facing fiercer competition and smarter customers. Many operational strategies have been developed to gain competitive edge. However, retail competition complicates firms’ decision making, and the value of these strategies becomes less predictable. Moreover, customers have become smarter and more strategic: they learn to time their purchase for the best bargain. Understanding the impact of these challenges is critical to retailers’ survival. To this end, we develop analytical models to study these challenges in three essays. The first two essays focus on two commonly used strategies, quick response and vertical integration, and we examine their value under retail competition. Then in the third essay, we investigate how customers’ strategic behavior affects firms’ decisions and profitability. In the following, we summarize our findings in each essay and propose avenues for future research.

The first essay of this dissertation is titled “Quick Response under Competition”, and it investigates the impact of retail competition on the value of quick response. It is common to see a supplier serving multiple competing retailers, but the value of quick response has not been studied in this situation. For example, Hot Kiss, a California based manufacturer serves junior
fashion retailers Hot Topic and deLia’s as well as upscale department stores like Dillard’s and Nordstrom (Bhatnagar 2009). In this case, should a supplier offer quick response service to none, some, or all of its retail clients? For a retail client, should it adopt this service? Also, what is the optimal strategy to maximize the profitability of the entire supply chain?

To answer these questions, we develop a stylized model with a manufacturer serving two competing retailers. In our model, each retailer places an initial order before a selling season starts, and quick response allows a retailer to replenish inventory after demand uncertainty is resolved. The manufacturer determines the unit price it charges to the retailers and each retailer chooses its order quantity. The products are sold in a selling season and the retailers are engaged in Cournot competition. We derive the subgame perfect equilibrium when no, one, or two retailers have quick response ability, and compare firm profitability across these scenarios. We first show that, in the absence of competition, quick response alleviates the mismatch between supply and demand, thereby improving the profitability of every channel participant.

Nevertheless, we find that the value of quick response is undermined by retail competition. First, under retail competition, the manufacturer’s optimal strategy is to offer quick response to none, only one, or both of the retailers as demand uncertainty increases. In other words, the manufacturer does not always benefit from offering quick response to all of its clients. This result shows that retail competition diminishes the marginal value of offering quick response, and higher demand uncertainty is needed to justify its value to a manufacturer. Retailers also may experience a decrease in the value of quick response. We find that QR may prove detrimental to a retailer when demand uncertainty is low. Similarly, competition may erode the value of quick response to the entire channel; depending on demand uncertainty, the total
channel profit can be maximized with zero, one, or two retailers with quick response.

In addition, the above insights continue to hold for the following situations: (1) when the manufacturer has full control over the price for all ordering opportunities; (2) alternative timing of pricing decision; (3) when the manufacturer has capacity limit for QR replenishment; and (4) numerical studies of other demand distributions. Overall, the degree of demand uncertainty and competition are critical determinants for the value of quick response. Thus, managers should take them into account in their quick response decisions.

The second essay of this dissertation, titled “Competitive Vertical Integration Strategies in the Fashion Industry”, is motivated by the apparel manufacturing industry. We focus on two key characteristics of this industry: fashion and quality differentiation. Apparel products are fashionable: they have short life cycle and their value decreases over time. This characteristic highlights the importance of timely response to customers’ change in taste. Thus, some apparel manufacturers choose forward integration to extend their reach toward product distribution and improve their influence over demand. For example, the Taiwanese manufacturer Tainan Enterprise forward integrates by launching its own brand and selling products through its own distribution channel (Ho 2002). Apparel products are also differentiated by quality. For example, the quality of a T-shirt is determined by its fabric quality: an all-cotton shirt provides better sweat absorption and a greater feeling of airiness (Levinson 2000). A better quality product often suggests the need for better raw materials, and some apparel manufacturers choose backward integration to tighten their grip on material quality. For example, the Chinese manufacturer Esquel chooses backward integration to improve its cotton quality as well as to assure material supply (Peleg-Gillai 2007).

It is intriguing that manufacturers, even in the same industry, demonstrate inconsistent
direction of vertical integration. Moreover, these apparel products compete in the same apparel market and it is not clear how competition affects manufacturers’ choice of integration strategies. Thus, we ask the following research questions: (1) When does vertical integration benefit? Can it hurt a manufacturer’s profitability? (2) How does a manufacturer’s choice between forward and backward integration depend on its degree of fashion, quality cost and competitor’s supply chain structure? (3) What is the resulting equilibrium supply chain structure under channel competition?

To investigate these questions, we build a model with two competing supply chains, each with a supplier, a manufacturer and a retailer. Each supplier controls the quality of raw material, which determines the quality of a product, and retailers sell products competitively in a market over two periods. Each firm also determines the unit price it charges to the downstream party. Products are fashionable, and therefore the firms’ potential market size reduces over time. The manufacturer considers three strategies: (1) forward integration, (2) backward integration, and (3) no integration. We analyze firms’ equilibrium decisions and profitability under various supply chain structures.

Among other results, our key findings are as follows: First, we find that backward integration benefits a manufacturer while forward integration can be harmful. Specifically, vertical integration leads to a lower retail price, and thereby reduced margins, due to intensified competition. This erosion on profit margin can outweigh the benefit of reduced double-marginalization when a manufacturer forward integrates. However, when a manufacturer backward integrates, its Stackelberg leadership in setting quality alleviates the hurt of a lower profit margin, making backward integration remain attractive.

We also examine manufacturers’ competitive choice of integration strategies, showing it
depends greatly on the degree of product fashion. When products are highly fashionable, i.e.,
when their popularity drops significantly over time, every manufacturer chooses to forward inte-
grate in equilibrium; otherwise, all of them choose to backward integrate. In other words, when
a product is highly fashionable, the importance of controlling demand dominates, motivating a
manufacturer to forward integrate. But the benefit of stronger control over quality dominates
when a product is more durable. Thus, the degree of product fashion is a key determinant of
supply chain structures.

We also find that a manufacturer’s choice between forward and backward integration de-
pends on the structure of its competing supply chain. When the competing channel disinte-
grates, backward integration always is more favorable due to the gain of Stackelberg leadership.
But when the competing manufacturer already is vertically integrated, forward integration can
be more favorable. In this case, the pressure of dropping the retail price is lessened due to
fewer firms in the competing channel. In sum, we characterize a manufacturer’s choice between
the benefit of forward and backward integrations. Forward and backward integrations provide
different competitive edges, and managers need to consider the structure of their competing
channels.

In the previous two essays, we examine the impact of competition on quick response and
vertical integration strategies. In addition, customers today are strategic: They anticipate
deep discounts, for example, the day after Thanksgiving and therefore intentionally delay their
purchase. It is a common belief that strategic customers erode a retailer’s profitability. However,
the full impact of strategic customers on the profitability of every supply chain participant is
unclear. Thus, in the third essay, titled “Are Strategic Customers Bad for a Supply Chain?”, we
answer the following research questions: Does it really harm a retailer when customers become
strategic? If so, is its negative impact passed on to a supplier as well as the performance of the entire supply chain as a whole?

We study these questions using a model with a single supplier serving a single retailer who sells a product over two periods. The supplier sets the unit wholesale price it charges to the retailer; the retailer determines its order quantity and the retail price in each period. We consider two customer types: Strategic customers take future price into account when making their purchase decisions, while myopic customers do not consider future price in their decisions. Comparing firm profitability under these two customer types allows us to understand the impact of strategic customers.

Surprisingly, our key findings show that firms can be more profitable when customers are strategic. Firms can exploit customers’ strategic behavior by selling a product only in the first period, i.e., when customers value it highly. This strategy increases strategic customers’ willingness to pay the full price which in turn benefits a supplier with higher sales. Moreover, when a product is sufficiently, but not overly, fashionable, the supplier charges a lower wholesale price to encourage sales, and this benefit a retailer with a lower unit cost. In that case, interestingly, the profitability of both firms becomes higher when customers are strategic. We extend our model and find these results continue to hold when (1) the wholesale price is exogenously determined, (2) the supplier has a production capacity limit, and (3) a retailer has an additional ordering opportunity.

Moreover, it is believed that decentralized decision making and/or strategic customers diminish the total channel profitability. Interestingly, we show that the total channel profit can be higher when both of them hold. In other words, these seemingly disadvantageous factors actually can work together to improve channel performance. Decentralization lowers the inven-
tory level of a channel, and strategic behavior increases customers’ willingness to purchase at the full price. As a result, the entire channel benefits from higher sales at the full price.

This dissertation contributes to operations literature by examining the competitive value of quick response and vertical integration, and it demonstrates the theoretical benefit of strategic customers. There are several avenues for future research. For example, horizontal integration is another commonly used strategy. Horizontal integration extends the potential market size, providing economies of scale and bargaining power for a retailer. On the other hand, vertical integration streamlines a supply chain, improving its cost efficiency. Examining a retailer’s choice between vertical and horizontal integration can be a prospective future direction. Our models assume that a retailer carries only one product and therefore, we ignore product line pricing. One may consider extending our model by allowing for product line pricing and investigate its impact on the value of quick response and vertical integration strategies. In addition, it would be interesting to examine the value of commonly used contracts, for example, a buy-back contract or a revenue sharing contract, when customers are strategic. While the value of these contracts has been studied when customers’ strategic behavior is ignored, investigating the impact of strategic customers on the value of these contracts may be a worthwhile future study.
CHAPTER 6

Appendices

6.1 Appendix I

In this section, we present additional results, threshold values and proofs for Chapter 2.

6.1.1 Monopoly Retailer Benchmark

Here we characterize firms’ decision in the monopoly setting where the supply chain is comprised of a manufacturer serving a monopoly retailer. Let \( q^H \) and \( q^L \) be the QR order quantities for the monopoly retailer in the high and low markets respectively. The following lemma characterizes the supply chain participants’ equilibrium strategies:

**Lemma 7**

(i) When the monopoly retailer does not have QR ability, the unique equilibrium order quantity for the retailer is \( Q = \frac{m - c_w}{2} \).

(ii) When the monopoly retailer can place a QR order, there exists a unique equilibrium as follows:

\[
\begin{align*}
(a) & \quad \text{For } c_w \leq c_F: \quad Q = \frac{m - v - 2c_w + c_q}{2}, \quad c_q = \frac{2c_w + v + \delta}{2}, \quad q^H > 0 \text{ and } q^L = 0. \\
(b) & \quad \text{For } c_w > c_F: \quad Q = 0, \quad q^H \geq 0, \quad q^L \geq 0 \text{ and} \\
1. & \quad \text{For } v \leq \frac{m - \delta}{2}: \quad c_q = \frac{m + \delta}{2}. \\
2. & \quad \text{For } \frac{m - \delta}{2} < v: \quad c_q = m - v.
\end{align*}
\]

6.1.2 Addendum to Lemmas

**Lemma 1:** This lemma describes the retailers’ equilibrium actions after \( c_q \) is chosen in the \( FS \) scenario. The following describes their equilibrium regular order quantities:

(i) For \( \hat{c}_FS \leq c_q \): \( Q_1 = Q_2 = \frac{m - c_w}{3} \).
(ii) For \( \hat{\theta}^{FS} \leq \alpha_q < \hat{\theta}^{FS} \): \((Q_1, Q_2) = (\frac{3m-5\nu-8c_w+5c_q}{10}, \frac{2(m-c_w)}{5})\) for \( c_w \leq \alpha_1 \); \((Q_1, Q_2) = (0, \frac{3m-\nu-4c_w+c_q}{6})\) for \( \alpha_1 < c_w \leq \alpha_2 \); \((Q_1, Q_2) = (0, 0)\) otherwise, where \( \alpha_1 = \frac{3m-5\nu+5c_q}{4} \) and \( \alpha_2 = \frac{6m-6\nu+c_q}{4} \).

(iii) For \( c_q < \hat{\theta}^{FS} \): \((Q_1, Q_2) = (0, \frac{m+c_q}{2} - c_w)\).

**Lemma 2:** This lemma describes the retailers’ equilibrium actions after \( c_q \) is chosen in the FF scenario. The following describes their equilibrium regular order quantities:

(i) For \( \hat{\theta}^{FF} \leq c_q \): \( Q_1 = Q_2 = \frac{m-c_q}{\beta} \).

(ii) For \( \hat{\theta}^{FF} \leq c_q < \hat{\theta}^{FF} \): \( Q_1 = Q_2 = \frac{m-\nu-2c_w+c_q}{3} \) for \( c_w < \frac{c_q+m-\nu}{2} \), and \( Q_1 = Q_2 = 0 \) otherwise.

(iii) For \( c_q < \hat{\theta}^{FF} \): \( Q_1 = Q_2 = 0 \).

### 6.1.3 Demand Variability \( v \) Threshold Values for the Base Model

The following table describes the threshold values in section 2.5 for \( c_w < \min(\beta^{FS}, \beta^{FF}) \) and \( c_w \geq \min(\beta^{FS}, \beta^{FF}) \), where \( \beta^{FS} \) and \( \beta^{FF} \) are given in Propositions 1 and 2 respectively.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Threshold Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_w &lt; \min(\beta^{FS}, \beta^{FF}) )</td>
<td>( v_M = 2\sqrt{c_w(m-c_w)} + \delta ), ( v_1^S = \frac{2\sqrt{19}}{15} (m-c_w) + \delta )</td>
</tr>
<tr>
<td>( v_2^F = \frac{2}{5} \sqrt{19} (m-c_w) + \delta ), ( v_1^F = \frac{2\sqrt{2}}{5} (m-c_w) + \delta )</td>
<td></td>
</tr>
<tr>
<td>( v_2^C = \frac{2\sqrt{52c_w(m-41c_w+11m^2)}}{5\sqrt{5}} + \delta )</td>
<td>( v_2^F ) is irrelevant in this case</td>
</tr>
<tr>
<td>( v_M = \min(x_1, x_2) )</td>
<td>( v_1^S = v_1^F = v^{FS} = \min(\beta^{FS}, \beta^{FF}) )</td>
</tr>
<tr>
<td>( c_w \geq \min(\beta^{FS}, \beta^{FF}) )</td>
<td>( v_2^C = \min(x_3, \max(\beta^{FS}, \beta^{FF})) )</td>
</tr>
</tbody>
</table>

\[
x_1 = \delta - 5m + 8\sqrt{\frac{2}{7}(m^2 + 3c_m m - 3c^2_w)}
\]

\[
x_2 = \frac{1}{3}m + \frac{4}{3\sqrt{7}}(2c_w - m) + \delta
\]

\[
x_3 = \frac{194m+7(18(\delta-v)+\sqrt{6(57m^2-326m(v-\delta)+561v^2)}))}{416}
\]

### 6.1.4 Demand Variability \( v \) Threshold Values for the Extensions
6.1.5 Value of QR in Models E1 and E2

Here, we discuss the value of QR to the manufacturer, retailer, and the entire channel for the extended models E1 and E2 described in Section 2.6.2. Since a pure-strategy equilibrium may not exist in FS scenario of these extended models (see Proposition 9), in this section we only compare FS to FF scenarios for \( v \leq c_1 \) and \( v \geq c_2 \) in which a pure-strategy equilibrium exists in both scenarios.

The following proposition characterizes the value of QR for the manufacturer.

**Proposition 30** For the models E1 and E2:

(i) \( \Pi^S_S > \max(\Pi^F_S, \Pi^F_F) \) for \( v < \hat{v}^1_M \).
(ii) \( \Pi^F_S \geq \max(\Pi^S_S, \Pi^F_F) \) for \( \hat{v}^1_M \leq v < \hat{v}^2_M \).
(iii) \( \Pi^F_F \geq \max(\Pi^S_S, \Pi^F_S) \) for If \( \hat{v}^2_M \leq v \).

The threshold values \( \hat{v}^1_M \) and \( \hat{v}^2_M \) are given in Appendix 6.1.4.

The next proposition characterizes retailers’ value of QR. It indicates having QR ability can still be detrimental to a retailer and it can benefit its rival.

**Proposition 31** For the models \( E_1 \) and \( E_2 \):
(i) \( \Pi^F_S < \Pi^S_S \) if and only if \( v < \hat{v}^S \), and \( \Pi^F_F > \Pi^S_F \).
(ii) \( \Pi^F_S > \Pi^S_S \) if and only if \( v < \hat{v}^S \), and \( \Pi^F_F > \Pi^S_F \) if and only if \( v < \hat{v}^F \).

The threshold values \( \hat{v}^S \) and \( \hat{v}^F \) are given in Appendix 6.1.4.

In the base model, we show that QR ability can hurt a retailer regardless of its competitor’s type (fast or slow). In contrast, when the regular orders are placed at the beginning of the timeline, retailers become the first mover, increasing the value extractable from QR. As a result, Proposition 31 indicates that gaining QR ability is now always beneficial to a retailer when its competitor already has QR ability. Nevertheless, QR ability can still be harmful to a retailer against a competitor who does not have QR option. In addition, gaining QR ability can still benefit a competing retailer.

The third proposition addresses the effect of QR on the channel profitability for the models \( E_1 \) and \( E_2 \). It shows the channel profit can still be maximized with only one fast retailer and the demand variability is the key determinant.

**Proposition 32** For the models \( E_1 \) and \( E_2 \):
(i) \( \Pi^F_F > \max(\Pi^F_S, \Pi^S_S) \) for \( \hat{v}^1_C < v \).
(ii) \( \Pi^F_S \geq \max(\Pi^F_F, \Pi^S_S) \) for \( \hat{v}^2_C < v \leq \hat{v}^1_C \).
(iii) \( \Pi^S_S \geq \max(\Pi^F_F, \Pi^F_S) \) for \( v \leq \hat{v}^2_C \).

The threshold values \( \hat{v}^1_C \) and \( \hat{v}^2_C \) are given in Appendix 6.1.4.

Different from our base model, Proposition 32 shows the total channel profit can also be maximized with no QR-enabled retailer at all. This result reflects the effect of the retailers’ gain of first mover advantage: placing regular orders before the QR price is set. The first mover advantage encourages the excess use of QR. When demand variability is sufficiently low, there is little value to QR and it does not justify the cost for the entire channel.

Finally, we compare the retailers’ profits in our base model and alternative \( E_1 \) and \( E_2 \) models. Let \( \Pi^a_i \) show retailer \( i \)'s equilibrium profit in the base model when retailers 1 and 2 are types \( a \) and \( b \), where \( a, b = F, S \) and \( i = 1, 2 \). Similarly, let \( \Pi^b_i \) be retailer \( i \)'s equilibrium profit in the alternative models. Recall that \( E_1 \) and \( E_2 \) models result in the same outcome.
Proposition 33

(i) $\Pi_{i,B}^{FF} < \Pi_{i,E}^{FF}$ if and only if $v < \frac{24c_\omega \sqrt{5}m}{13} + \frac{4\sqrt{2(788c_\omega - 376c_\omega m + 63m^2)}}{63}$ for $i = 1, 2$.

(ii) $\Pi_{i,B}^{FS} < \Pi_{i,E}^{FS}$ and $\Pi_{2,B}^{FS} > \Pi_{2,E}^{FS}$.

Note that since QR option is not used in SS scenario, our base and E1 and E2 models do not differ.

In models E1 and E2, the QR price is set after initial orders are placed. Therefore, when choosing their initial order quantity, retailers take into account the impact on the QR price whereby a larger initial order quantity results in a lower QR price. When the demand variability $v$ is high, the QR option is more valuable, thus a retailer indeed orders larger initial order quantities to receive a lower QR price. However, in the FF scenario increased order quantities of both retailers results in more intense competition making the retailers worse off. In contrast, when the demand variability $v$ is low, the QR option is less valuable, retailers do not have a strong incentive to order a large quantity initially, and they enjoy the first mover advantage, which makes them better off compared to the base scenario.

In the FS scenario, the fast retailer enjoys a higher profit in E1 and E2 models due to its first mover advantage. Thus, not surprisingly, the slow retailer is worse off in in E1 and E2 models.

6.1.6 When the Retailers Can Decide Whether to Adopt QR

Here we describe what happens when the retailers can simultaneously determine whether to adopt QR. Let $a$ and $b$ be retailer 1 and 2’s QR decision, $a, b = F, S$. Then there are three possible scenarios for equilibrium outcome: FF, FS and SS. A scenario is an equilibrium if none of the retailers is better off by deviating to another decision (changing its decision from $F$ to $S$ or $S$ to $F$). Using the SPNE derived in section 4, we compare the retailers’ profits across scenarios, and obtain the following result:

Proposition 34 When the retailers choose whether to adopt QR simultaneously, the equilibrium choices $(a, b)$ is

$$(a, b) = \begin{cases} (S, S) & \text{for } v \leq v_1^F \\ (S, S) \text{ or } (F, F) & \text{for } v_1^F < v \leq v_1^S \\ (F, F) & \text{for } v_1^S < v \end{cases}$$

$v_1^S > v_1^F$, and they are given in Table 6.1.

Figure 6.1 describes the equilibrium region given in Proposition 34. As the figure shows, both of the retailers choose not to have QR ability when demand variability is too low, and both adopt QR when demand variability is too high. Nevertheless, both the SS and FF scenarios can be equilibria when the demand variability is moderate.
6.1.7 Proofs for Chapter 2

In this section we provide proofs of lemmas and propositions in Chapter 2.

Proof of Lemma 1.

The retailers’ expected profits are

\[ \pi_1 = E[(A - Q_1 - q_1 - Q_2)(Q_1 + q_1) - c_w Q_1], \]
\[ \pi_2 = E[(A - Q_1 - q_1 - Q_2)Q_2] - c_w Q_2, \]

where \( q_1 \) is given by (2.4). It can be shown \( \frac{\partial^2 \pi_i}{\partial Q_i^2} < 0 \). Let \( q^H \) and \( q^L \) denote the fast retailer’s QR order quantities in a high and low market, respectively. The n solving the first order conditions \( \frac{\partial \pi_i}{\partial Q_i} = 0 \), for \( i = 1, 2 \), yields the following initial order quantities:

a. For \( \theta_{FS} \leq c_q < \bar{\theta}_{FS} \):
\[ (Q_1, Q_2) = \begin{cases} (0, \frac{2m - c_q}{3}), & \text{for } c_w \leq \alpha_1 \\ (\frac{2m - c_q}{3}, 0), & \text{for } \alpha_1 < c_w \leq \alpha_2, \text{ and } q^H > 0 \text{ while } q^L = 0 \\ (0, 0), & \text{for } \alpha_2 < c_w \end{cases} \]

b. For \( \bar{\theta}_{FS} < c_q < \bar{\theta}_{FS} \):
\[ (Q_1, Q_2) = (0, \frac{m + c_q}{2} - c_w), \text{ } q^H \geq 0 \text{ and } q^L \geq 0. \]

\( \alpha_1 = \frac{3m - 5v - 5c_q}{10} \) and \( \alpha_2 = \frac{3m - v + c_q}{6} \) correspond to the thresholds such that \( Q_1 = 0 \) and \( Q_2 = 0 \) for \( \bar{\theta}_{FS} \leq c_q < \bar{\theta}_{FS} \), while \( \bar{\theta}_{FS} \) and \( \bar{\theta}_{FS} \) correspond to the thresholds such that \( q_1 = 0 \) in a high market and \( q_1 = 0 \) in a low market.

Proof of Proposition 1.
The manufacturer solves the following problem to maximize its profit:

$$\max_{c_q} \mathbb{E}[\pi_M] = \mathbb{E}[(c_q - \delta)q_1] + c_w(Q_1 + Q_2),$$

where $q_1$ is given by (2.4) and $Q_i$, $i = 1, 2$, is characterized in Lemma 1. It can be shown that $\mathbb{E}[\pi_M]$ is a piecewise concave function: it is continuous and concave in $c_q$ for $c_q > \bar{\theta}^{FS}$ and $c_q < \bar{\theta}^{FS}$ respectively, but is discontinuous at $c_q = \bar{\theta}^{FS}$, because in equilibrium $Q_1 = 0$ for $c_q < \bar{\theta}^{FS}$ and $Q_1 > 0$ otherwise. In other words, the discontinuity is due to the fast retailer’s change in behavior: it places an initial order only when the QR price is sufficiently high, but it does not place any initial order when the QR price is too low. Since $c_q$ changes in behavior: it places an initial order only when the QR price is sufficiently high, but it does not place any initial order when the QR price is too low. Since $\mathbb{E}[\pi_M]$ is concave in $c_q$ for $c_q > \bar{\theta}^{FS}$, we obtain the optimal QR price by applying the first order conditions, and the threshold $\bar{\theta}^{FS}$ is given by the solution to $Q_1 = 0$ in this case. Furthermore, following Lemma 1, this optimal price is feasible only for $c_q < \bar{\theta}^{FS}$ which translates to $\delta < \nu$. Otherwise, demand uncertainty is too low and QR is never used. Similarly, for $c_q < \bar{\theta}^{FS}$ we derive the optimal QR price for this case by applying the first order conditions. Comparing the manufacturer’s profit for $c_q > \bar{\theta}^{FS}$ and $c_q < \bar{\theta}^{FS}$ with the optimal QR price for each of these cases reveals the manufacturer is always better off by using the optimal $c_q$ for $c_q > \bar{\theta}^{FS}$. That is, the manufacturer induces the fast retailer to place a QR order only in a high market.

**Proof of Lemma 2.**

Retailer $i$ maximizes its expected profit

$$\pi_i = \mathbb{E}[(A - Q_i - q_i - Q_j - q_j)(Q_i + q_i) - c_wQ_i - c_q q_1],$$

where $q_i$ and $q_j$ are given by (2.5). It can be shown $\frac{\partial^2 \pi_i}{\partial Q_i^2} < 0$. Let $q_i^H$ and $q_i^L$ be retailer $i$’s QR order quantities in a high and low market respectively. Then the equilibrium order quantities can be obtained by solving $\frac{\partial \pi_i}{\partial Q_i} = 0$, for $i = 1, 2$, leading to the following results:

(i) For $\bar{\theta}^{FF} < c_q$: $Q_1 = Q_2 = \frac{\nu - c_w}{\nu}$ and $q_i^H = q_i^L = 0$.

(ii) For $\bar{\theta}^{FF} < c_q < \bar{\theta}^{FF}$:

$$Q_1 = Q_2 = \begin{cases} \frac{m - \nu - 2c_w + c_q}{4}, & \text{for } c_w < \frac{c_w + m - \nu}{2}, \\ 0, & \text{otherwise} \end{cases} \text{, and } q_i^H > 0 \text{ while } q_i^L = 0.$$

(iii) For $c_q < \bar{\theta}^{FF}$: $Q_1 = Q_2 = 0$, $q_i^H > 0$ and $q_i^L > 0$.

The threshold $\bar{\theta}^{FF}$ is derived from the condition $q^H = 0$ for the cases (i) and (ii), and $\bar{\theta}^{FF}$ is derived from the condition $q^L = 0$ for the cases (ii) and (iii).

**Proof of Proposition 2.**

The procedure of this proof essentially follows that of Proposition 1. The manufacturer solves the following problem to maximize its profit:

$$\max_{c_q} \mathbb{E}[\pi_M] = \mathbb{E}[(c_q - \delta)(q_1 + q_2) + c_w(Q_1 + Q_2)],$$

where $q_i$ is given by (2.5) and $Q_i$, $i = 1, 2$, is characterized in Lemma 2. It can be shown that $\mathbb{E}[\pi_M]$ is piecewise concave in $c_q$ but discontinuous at $c_q = \bar{\theta}^{FF}$ because the wholesale price is
sufficiently small and the retailers do not place any QR order for \( c_q \leq \theta^{FF} \). Since \( E[\pi_M] \) is concave in \( c_q \) for \( c_q > \theta^{FF} \), we solve \( \max_{c_q > \theta^{FF}} E[\pi_M] \) by applying the first order conditions, which leads to the optimal \( c_q \) in which \( Q_1 = Q_2 \geq 0, q_H \geq 0 \) and \( q_L = 0 \). Similarly, we obtain the optimal \( c_q \) for \( c_q \leq \theta^{FF} \) using the first order conditions, leading to another optimal \( c_q \) in which \( Q_1 = Q_2 = 0, q_H > 0 \) and \( q_L > 0 \). Finally, comparing the manufacturer’s profits between these two cases reveals the boundary \( \beta^{FF} \).

**Proof of Proposition 3.**

Parts (i) and (ii) of this proposition are straightforward by showing that \( \Pi^F_M \geq \Pi^S_M \) and \( \Pi^F_R \geq \Pi^S_R \). In addition, combining (i) and (ii) leads to (iii) of this proposition.

**Proof of Proposition 4.**

The results are straightforward from comparing the manufacturer’s expected profit \( \Pi_M \) across the scenarios, and \( v_M \) is derived by solving \( \Pi^FS_M = \Pi^FF_M \).

**Proofs of Propositions 5 and 6.**

The results are derived by comparing each of the retailer’s expected profit across the scenarios.

**Proof of Proposition 7.**

The results are derived by comparing the expected total channel profit across the scenarios.

**Proof of Lemma 3.**

The optimal wholesale price is given by the solution to the following problem

\[
\max_{c_w} E[\pi_M]
\]  

(6.1)

for SS, FS and FF scenarios. It can be shown that \( E[\pi_M] \) is concave in \( c_w \) in each of these scenarios, and the optimal wholesale price \( c_w = \frac{m}{2} \) can be derived by solving the first order conditions.

**Proof of Proposition 8.**

First, we obtain the firms’ expected profits using the wholesale price \( c_w = \frac{m}{2} \) given in Lemma 3. Then part (i) of the proposition appears straightforward in comparing \( \Pi^FS_M, \Pi^F_M \) and \( \Pi^FF_M \). Similarly, parts (ii) and (iii) of the proposition are straightforward from comparing the expected profits of a retailer and the entire channel respectively across the scenarios.

**Proof of Proposition 9.**

We provide the proof for the model \( E_1 \). We first consider the FS scenario and next the FF scenario. In each scenario, following backward induction, we first derive the manufacturer’s choice of \( c_q \) which is described by Lemma 8, followed by the retailers’ equilibrium regular order decisions which are given by Lemma 9. The results for the model \( E_2 \) can be derived following the same procedure, which yields the same results as \( E_1 \).

In the last stage game in the FS scenario for the model \( E_1 \), the fast retailer determines its QR order quantity as given in (2.4). Using this QR order quantity, in the second stage game, the manufacturer determines its QR price to maximize its expected profit \( E[\pi_M] \), which
is piecewise concave in $c_q$. The manufacturer’s optimization of QR price leads to the following pricing scheme:

**Lemma 8** The optimal QR price for the manufacturer in the FS scenario for model E1 is given below:

1. For $0 \leq Q_1 \leq \sigma_1$: $c_q = \frac{m-Q_2+\delta}{2} - Q_1$, and the fast retailer places a QR order for both high and low market outcomes;
2. For $\sigma_1 < Q_1 < \sigma_2$: $c_q = \frac{m-Q_2+v+\delta}{2} - Q_1$, and the fast retailer places a QR order only in a high market;
3. For $\sigma_2 \leq Q_1$: $c_q = \delta$, and the fast retailer does not place a QR order for any market outcome;

where $\sigma_1 = \frac{m-(1+\sqrt{2})v-\delta-Q_2^2}{2}$ and $\sigma_2 = \frac{m+v-\delta-Q_2^2}{2}$.

Next, in the first stage game, each of the retailers places an initial order to maximize its expected profit $E[\pi_i]$, which is piecewise concave in $Q_i$ as $c_q$ is discontinuous on $Q_1 = \sigma_1$. Observe that the equilibrium initial order quantities must satisfy one of the cases stated in Lemma 8, and $E[\pi_i]$ is concave in $Q_i$ for each of the cases in that lemma. Therefore, we apply the first order conditions to derive the expressions for equilibrium order quantities (if it exists). Nevertheless, we need to verify that no retailer has incentive to deviate from these quantities so that they are equilibrium. This procedure leads to the following results:

**Lemma 9** There exists a unique equilibrium for the FS scenario in model E1:

1. For $v \leq \delta - c_w$, $Q_i = \frac{m-c_w}{3}$ for $i = 1, 2$. The fast retailer does not place a QR order for any market outcome.
2. For $\delta - c_w < v$, $Q_1 = \left(\frac{7m-8c_w-v+\delta}{22}\right)^+$ and $Q_2 = \left(\frac{4(7m-8c_w-v+\delta)}{77}\right)^+$. The fast retailer places a QR order only in a high market.

**proof:** We derive cases (1) and (2) in this lemma as follows. Case (1) concerns an equilibrium in which $q_1 = 0$ in all market outcomes, corresponding to case (1) of Lemma 8, and solving the first order conditions yields $Q_i = (m-c_w)/3$ for $i = 1, 2$. Since $E[\pi_i]$ is piecewise concave in $Q_i$, the first order condition only provides a necessary condition for an equilibrium; we also need to confirm that no retailer has incentive for unilateral deviation. For the quantities derived in this case, it suffices to ensure that the fast retailer has no incentive to place a QR order in a high market even when $c_q = \delta$, i.e.,

$$\frac{d\pi_1}{dq_1} \bigg|_{Q_1=Q_2=m-v,q_1=0,A=m+v \leq 0},$$

which implies $v \leq \delta - c_w$.

Case (2) concerns an equilibrium in which QR is used only in a high market, corresponding to case (2) of Lemma 8. Solving the first order condition yields $(Q_1, Q_2) = \left((\frac{7m-8c_w-v+\delta}{22})^+, (\frac{4(7m-8c_w-v+\delta)}{77})^+\right)$. Moreover, $q_1^* < 0$ implies $v > \delta - c_w$. 

\[111\]
Now we have to ensure no retailer has incentive to deviate. For the fast retailer, deviation such that \( Q_1 \geq \sigma_2 \) is unattractive, because \( \mathbb{E}[\pi_1] \) is concave in \( Q_1 \) for \( Q_1 \geq \sigma_2 \) and
\[
\frac{d\mathbb{E}[\pi_1]}{dQ_1}_{Q_1=\sigma_2,Q_2=0} = \frac{\sigma_2 - \sigma_1}{2} \leq 0.
\]
Now consider retailer 1’s deviation so that \( Q_1 \leq \sigma_1 \). Since \( \mathbb{E}[\pi_1] \) is concave in \( Q_1 \) for \( Q_1 \leq \sigma_1 \),
\[
\frac{d\mathbb{E}[\pi_1]}{dQ_1}_{Q_1=\sigma_1} \geq 0.
\]
and deviating to \( Q_1 = \sigma_1 \) is unattractive, we conclude that retailer 1 has no incentive to deviate to \( Q_1 \leq \sigma_1 \). Applying similar analysis reveals that the slow retailer has no incentive to deviate either.

Similar analysis can be applied to examine what happens when QR is used in both low and high markets, i.e., corresponding to case (3) of Lemma 8. This analysis reveals that \( c_q = \frac{2c_w + \delta}{3} \) in equilibrium, implying that \( c_q < c_w \) for \( c_w > \delta \). Moreover, \( q_1^L > 0 \) implies \( c_w > \frac{2c_w}{3} + \delta \), and therefore assuming \( c_w \leq \delta \) eliminates an equilibrium in which QR is used in both of the market outcomes.

Now consider the FF scenario. We apply the same procedure described above to derive the SPNE for this scenario. In the last stage game, the retailers determine their QR order quantities as given in (2.5). Next in the second stage game, the manufacturer determines \( c_q \) to maximize its expected profit \( \mathbb{E}[\pi_M] \). Using the QR order quantities described in (2.5), the manufacturer’s expected profit \( \mathbb{E}[\pi_M] \) is again piecewise concave in \( c_q \), and the manufacturer’s optimization problem leads to the following result:

**Lemma 10** The optimal QR price for the manufacturer in the FF scenario for model E1 is given by:

\[(a)\text{ For } \min(\sigma_4, \sigma_5, \sigma_7) \leq Q_1 \leq \min(\sigma_3, \sigma_6): c_q = \frac{2m-3(Q_1+Q_2)+2(\epsilon+\delta)}{4}, \text{ which yields } q_1^H > 0, q_2^H > 0, q_1^L = 0, q_2^L = 0;\]

\[(b)\text{ For } \min(\sigma_3, \sigma_{15}) \leq Q_1 \text{ and } \frac{m-(1+\sqrt{2})n-\delta}{3} \leq Q_2 \leq \frac{m+\sqrt{2}n-\delta}{3}: c_q = \frac{m-3Q_1+\epsilon+\delta}{2}, \text{ which yields } q_1^H = 0, q_2^H > 0, q_1^L = 0, q_2^L = 0;\]

\[(c)\text{ For } \sigma_8 \leq Q_1 \leq \min(\sigma_5, \sigma_{10}, \sigma_{11}): c_q = \frac{7m-12Q_1-9Q_2+7\epsilon+7\delta}{14}, \text{ which yields } q_1^H > 0, q_2^H > 0, q_1^L > 0, q_2^L = 0;\]

\[(d)\text{ For } \min(\sigma_9, \sigma_{10}) \leq Q_1 \leq \min(\sigma_4, \sigma_{16}): c_q = \frac{m-2Q_1-Q_2+\epsilon+\delta}{2}, \text{ which yields } q_1^H > 0, q_2^H > 0, q_1^L = 0, q_2^L = 0;\]

\[(e)\text{ For } \sigma_{11} \leq Q_1 \leq \min(\sigma_7, \sigma_{12}): c_q = \frac{2m-3Q_1-3Q_2+2\epsilon+2\delta}{4}, \text{ which yields } q_1^H > 0, q_2^H > 0, q_1^L > 0, q_2^L > 0;\]

\[(f)\text{ For } Q_1 \leq \min(\sigma_8, \sigma_9): c_q = \frac{m-2Q_1-Q_2+\epsilon+\delta}{2}, \text{ which yields } q_1^H > 0, q_2^H = 0, q_1^L > 0, q_2^L = 0;\]

\[(g)\text{ For } \sigma_{13} \leq Q_1 \text{ and } Q_2 \leq \frac{m-(1+\sqrt{2})n-\delta}{3}: c_q = \frac{m-3Q_1+\epsilon+\delta}{2}, \text{ which yields } q_1^H = 0, q_2^H > 0, q_1^L = 0, q_2^L > 0;\]

\[(h)\text{ For } \min(\sigma_6, \sigma_{12}) \leq Q_1 \leq \min(\sigma_{13}, \sigma_{15}): c_q = \frac{3m-3Q_1-6Q_2+3\epsilon+3\delta}{6}, \text{ which yields } q_1^H > 0, q_2^H > 0, q_1^L = 0, q_2^L > 0;\]
(i) For \( \sigma_1 \leq Q_1 \) and \( Q_2 \geq \frac{m + v - \delta}{3} \): \( c_q = \delta \), which yields \( q_1^H = 0 \), \( q_2^H = 0 \), \( q_1^L = 0 \), \( q_2^L = 0 \); where \( \sigma_3 \) to \( \sigma_{16} \) are given in Table 6.2.

\[
\begin{align*}
\sigma_3 &= (2m - 3Q_2 + 2v - \sqrt{2}(m - 3Q_2 + v - \delta) - 2\delta)/3 \\
\sigma_4 &= Q_2 - (m - 3Q_2 + v - \delta)/\sqrt{3} \\
\sigma_5 &= (14m - 15Q_2 - 10v - \sqrt{7}(m - 3Q_2 + 7v - \delta) - 14\delta)/27 \\
\sigma_6 &= Q_2 + (4v + \sqrt{6}(m - 3Q_2 + v + \delta))/3 \\
\sigma_7 &= (2m - 3Q_2 - 2(v + \sqrt{2}v + \delta))/3 \\
\sigma_8 &= Q_2 - v/2 - \sqrt{7}/6(m - 3Q_2 + v - \delta)/2 \\
\sigma_9 &= (m - Q_2 - v - \sqrt{2}v - \delta)/2 \\
\sigma_{10} &= (21m - 33Q_2 - 15v - \sqrt{21}(m - 3Q_2 + 5v + \delta) - 21\delta)/30 \\
\sigma_{11} &= Q_2 + (4v + \sqrt{14}(m - 3Q_2 + v + \delta))/6 \\
\sigma_{12} &= (3m - 3Q_2 - v - \sqrt{3}(m - 3Q_2 + v - \delta) - 3\delta)/6 \\
\sigma_{13} &= (3m - 6Q_2 + v + \sqrt{6}(m - 3Q_2 + \delta) - 3\delta)/3 \\
\sigma_{14} &= (m - (1 + \sqrt{2})v - \delta)/3 \\
\sigma_{15} &= m + (6Q_2 + v - \sqrt{3}(m - 3Q_2 + v - \delta) - 3\delta)/3 \\
\sigma_{16} &= (m - Q_2 + v - \delta)/2
\end{align*}
\]

TABLE 6.2: Threshold Values for \( c_q \) in the FF scenario of the model E1

![Regions Characterized in Lemma 10](image_url)

FIGURE 6.2: Regions Characterized in Lemma 10 \((m = 1, v = 0.7, c_w = 0.5, \delta = 0.5)\)

Note: Some regions may not exist, depending on \( m, v, c_w \) and \( \delta \).

Figure 6.2 depicts the regions described in Lemma 10 for \( m = 1, v = 0.7, c_w = 0.5, \delta = 0.5 \). In the first stage game, the retailers determine their initial order quantities to maximize their expected profits. Similar to the FS scenario, the manufacturer’s chosen \( c_q \) described in Lemma 10 is discontinuous on some of the boundaries due to piecewise concavity of \( \mathbb{E}[\pi_i] \). As a result, a retailer’s expected profit \( \mathbb{E}[\pi_i] \) is piecewise concave in \( Q_i \), and discontinuity occurs on some of the boundaries given in Table 6.2. Nevertheless, \( \mathbb{E}[\pi_i] \) is concave in each of the cases (a) to (i) described in Lemma 10. Since an equilibrium must satisfy one of these cases, we can apply the first order conditions to derive the order quantities for an equilibrium. Then we check
for retailers’ incentive for deviation to characterize an equilibrium. This process leads to the following symmetric result, i.e., \( Q_1 = Q_2 \):

**Lemma 11** There exists a unique equilibrium for the FF scenario in model \( E_1 \) only for \( v \leq \epsilon_1 \) and \( v \geq \epsilon_2 \), and there does not exist a pure-strategy equilibrium otherwise. The unique equilibrium is given below:

1. For \( v \leq \epsilon_1 \), \( Q_i = \frac{m-c_w}{3} \) for \( i = 1, 2 \). The retailers do not place a QR order for any market outcome.
2. For \( v \geq \epsilon_2 \), \( Q_i = \left( \frac{19m-24c_w-5v+5\delta}{60} \right)^+ \) for \( i = 1, 2 \). Each retailer places a QR order only in a high market,

where \( \epsilon_1 = \delta - c_w \) and \( \epsilon_2 = \frac{13m-168c_w+155\delta}{155} \).

**Proof:** We derive cases (1) and (2) in this Lemma as follows. Case (1) concerns an equilibrium in which \( q_i = 0 \) in all market outcomes, corresponding to case (i) of Lemma 10. Solving the first order conditions yields \( Q_i = \frac{m-c_w}{3} \) for \( i = 1, 2 \). This quantity is an equilibrium only if \( q_i^H \leq 0 \), which implies \( v \leq \delta - c_w \).

Case (2) concerns an equilibrium in which QR is used only in a high market, corresponding to case (a) of Lemma 10. Solving the first order condition yields \( Q_i = \left( \frac{19m-24c_w-5v+5\delta}{60} \right)^+ \). This is an equilibrium only if no retailer has incentive to deviate, and it can be shown that deviation is attractive for \( v < \frac{13m-168c_w+155\delta}{155} \). In that case, a retailer has incentive to deviate by purchasing more initially but not using QR at all.

Finally, applying the analysis described above reveals that there does not exist an equilibrium (asymmetric) corresponding to the other cases described in Lemma 10. Therefore cases (1) and (2) characterize the unique equilibrium for \( v \leq \delta - c_w \) and \( v \geq \frac{13m-168c_w+155\delta}{155} \), and there is no pure-strategy equilibrium otherwise.

Note that there does not exist an equilibrium for the FF scenario for \( \delta - c_w < v < \frac{13m-168c_w+155\delta}{155} \). This happens because the retailers’ profit functions are piecewise concave in their regular order quantities, leading to multiple local maxima and hence the discontinuity of their best response functions. Finally, Proposition 9 for the model \( E_1 \) proceeds by combining Lemmas 9 and 11.

**Proof of Proposition 10.**

The proof of this proposition involves two parts: (1) obtaining the SPNE of each scenario, and (2) comparing profits across scenarios. We illustrate the derivation and the results of the first part; the latter part is straightforward after the first part is obtained.

Basically, the derivation of SPNE follows the steps shown in sections 2.4.2 and 2.4.3. The key difference is driven by the introduction of the QR capacity limit \( k \), which results in additional cases to be analyzed in each stage game.

For the FS scenario, using the first order conditions we derive the fast retailer’s QR order quantity:

\[
q_1 = \min\left( \frac{A-c_q-2Q_1-Q_2}{2} , k \right)
\]

Next we proceed to solve for the retailers’ equilibrium regular order quantities with this QR
ordering policy. This yields a result similar to Lemma 1 with one additional case: For \( v \geq k \) and \( \min(c_w + k - v, \frac{2c_w + k + 2m - 4v}{4}) < c_q < \min(c_w + k + v, \frac{2c_w + 7k + 2m + 4v}{4}) \), the fast retailer orders \( q_i^H = k \) and \( q_i^L = 0 \). That is, when the demand variability is large enough and \( c_q \) is not overly high, the QR capacity is fully used in a high market. It can also be shown that in equilibrium \( Q_2 > Q_1 \), and \( Q_1 > 0 \) implies \( c_w > m - \frac{4k}{3} \). Next we derive the manufacturer’s optimal QR price for \( Q_1 > 0 \), which yields

\[
c_q = \begin{cases} 
    c_w + v & \text{for } v \leq \delta, \text{ and } q_i^H = q_i^L = 0, \\
    \frac{2c_w + v + \delta}{2} & \text{for } \delta < v < 2k + \delta, \text{ and } 0 < q_i^H < k, q_i^L = 0, \\
    c_w + v - k & \text{for } 2k + \delta \geq v, \text{ and } q_i^H = k, q_i^L = 0.
\end{cases}
\]

For the \( FF \) scenario, first we solve for the retailers’ equilibrium QR order quantities. Without loss of generality, we assume that \( Q_1 \geq Q_2 \). Recall that we assume that when the retailers’ total QR order quantity exceeds the manufacturer’s QR capacity, the manufacturer allocates its capacity evenly between the retailers. This complicates the analysis and the equilibrium is characterized in seven regions. Using this result, next we derive the retailers’ equilibrium regular order quantities. This yields a result similar to Lemma 2 with one additional case: For \( \min(c_w + \frac{3k}{2} - v, m - v) < c_q < \min(c_w + \frac{3k}{2} + v, m + v - 3k) \), the retailers order \( q_i^H = k \) and \( q_i^L = 0 \). This case is relevant only for \( v \geq \frac{3k}{2} \), and \( Q_i > 0 \) implies \( c_w < m - \frac{3k}{2} \). Recall that \( Q_1 > 0 \) in the \( FS \) scenario requires that \( c_w < m - \frac{5k}{3} \), and therefore \( Q_i > 0 \) for both of the \( FS \) and \( FF \) scenarios requires that \( c_w < m - \frac{5k}{3} \). Knowing the retailers’ ordering policies, finally we study the manufacturer’s QR pricing decision for \( c_w < m - \frac{5k}{3} \), which yields:

\[
c_q = \begin{cases} 
    c_w + v & \text{for } v \leq \delta, \text{ and } q_i^H = q_i^L = 0, \\
    \frac{2c_w + v + \delta}{2} & \text{for } \delta < v < \frac{3k}{2} + \delta, \text{ and } 0 < q_i^H < k, q_i^L = 0, \\
    c_w + v - \frac{3k}{2} & \text{for } \frac{3k}{2} + \delta \leq v, \text{ and } q_i^H = k, q_i^L = 0.
\end{cases}
\]

The above results implies that the QR capacity is fully utilized in both \( FF \) and \( FS \) scenarios only for \( v \geq \max(2k + \delta, \frac{3k}{2} + \delta) \). Also note we assume \( v < m \), and hence \( m > \max(2k + \delta, \frac{3k}{2} + \delta) \) is the necessary condition for QR to be fully used, which implies \( k < \frac{m - \delta}{2} \). Finally, we obtain the firms’ equilibrium profits with the above results, and comparing these profits across the scenarios yields the results described in this proposition. ■

**Proof of Proposition 30.**

The results are derived by comparing the manufacturer’s expected profit across the scenarios. ■

**Proof of Propositions 31.**

The results are derived by comparing each of the retailer’s expected profit across the scenarios. ■

**Proof of Proposition 32.**

The results are derived by comparing the channel’s total expected profit across the scenarios. ■
Proof of Lemma 7. In this case, the retailer’s profit is given by
\[ \pi_R = (A - Q - q)(Q + q) - c_q q - c_w Q, \]
where \( Q \) and \( q \) are the initial and QR order quantities respectively. It is straightforward that \( \pi_R \) is concave in \( q \), and the retailer’s optimal QR order quantity is given by
\[ q = \left( \frac{A - c_q - 2Q}{2} \right)^+. \]
Given the QR ordering policy, the retailer determines its initial order \( Q \) to maximize its expected profit \( E[\pi_R] \). Simple algebra reveals that \( \frac{\partial E[\pi_R]}{\partial Q} \leq 0 \), and applying the first order condition yields the retailer’s optimal initial order quantity as follows:
\[
Q = \begin{cases} 
\frac{m - c}{2} & \text{for } c_w + v < c_q, \text{ and } q^H = q^L = 0, \\
\frac{m - v - 2c_w + c_q}{2} & \text{for } c_w < c_q \leq c_w + v, \text{ and } q^H > 0 q^L = 0, \\
0 & \text{for } m - v < c_q \leq c_w, \text{ and } q^H \geq 0 q^L \geq 0, \\
\frac{m - v - c_q}{2} & \text{for } c_q \leq \min(m - v, c_w), \text{ and } q^H \geq 0 q^L \geq 0.
\end{cases}
\]
Anticipating the retailer’s initial and QR order quantities as described above, the manufacturer chooses its QR price, \( c_q \), to maximize its expected profit
\[ E[\pi_M] = E[(c_q - \delta)q] + c_w Q. \]
It can also be confirmed that \( E[\pi_M] \) is piecewise concave in \( c_q \), and solving \( \frac{\partial E[\pi_M]}{\partial c_q} = 0 \) leads to the results given in the proposition with
\[
c_F = \begin{cases} 
\frac{m + \sqrt{(2m - \delta)(m - \delta)}}{2} & \text{for } v \leq \min(\delta, \frac{m - \delta}{2}), \\
\frac{m + \sqrt{m^2 - 4mv + 4v(\delta + \delta)}}{2} & \text{for } \frac{m - \delta}{2} < v \leq \delta, \\
\frac{m + \sqrt{m^2 - 4mv + 5v^2 + 2\delta + \delta^2}}{2} & \text{for } \max(\delta, \frac{m - \delta}{2}) < v, \\
\frac{m + \sqrt{v^2 + 2mv - 2\delta^2}}{2} & \text{for } \delta < v \leq \frac{m - \delta}{2}.
\end{cases}
\]

Proof of Proposition 33. The result is straightforward by comparing retailer profit across different sequence of events.

Proof of Proposition 34. The result is established by showing that no retailer has incentive to deviate from these decisions.

6.2 Appendix II

In this section, we first present the equilibrium decisions in sections 3.6.1 and 3.6.2 and then we provide the proofs for Chapter 3.
6.2.1 Equilibrium Decisions in Sections 3.6.1 and 3.6.2

In these sections, we consider $\beta^F_i < 1$ when manufacturer $i$ forward integrates. Following the procedure described in the proof of Proposition 11, we derive the following equilibrium decisions.

### Table 6.3: Equilibrium Quality, Retail Price and Sales in Section 3.6.1

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Quality</th>
<th>Period 1 Retail Price</th>
<th>Period 2 Retail Price</th>
<th>Total Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 = N, S_2 = N$</td>
<td>$\theta_1 = \theta_2 = \frac{(1+k)\alpha}{6c}$</td>
<td>$p_1 = p_2 = d(7 + 6k)$</td>
<td>$p_1 = p_2 = d(6 + 7k)$</td>
<td>$Q_1 = Q_2 = \frac{1+k}{2}$</td>
</tr>
<tr>
<td>$S_1 = F, S_2 = N$</td>
<td>$\theta_1 = (63\gamma - 2)A$</td>
<td>$p_1 = B_1$</td>
<td>$p_1 = B_1 - \frac{d(1-k)}{\beta}$</td>
<td>$Q_1 = (63\gamma - 2)C$</td>
</tr>
<tr>
<td>&amp; $\theta_2 = (45\gamma - 2)A$</td>
<td>$p_2 = B_2$</td>
<td>$p_2 = B_2 - d(1-k)$</td>
<td>$Q_2 = (45\gamma - 2)C$</td>
<td></td>
</tr>
<tr>
<td>$S_1 = B, S_2 = N$</td>
<td>$\theta_1 = (63\gamma - 2)D$</td>
<td>$p_1 = E_1$</td>
<td>$p_1 = E_1 - d(1-k)$</td>
<td>$Q_1 = (63\gamma - 2)F$</td>
</tr>
<tr>
<td>&amp; $\theta_2 = (45\gamma - 2)D$</td>
<td>$p_2 = E_1 + E_2$</td>
<td>$p_2 = E_1 + E_2 - d(1-k)$</td>
<td>$Q_2 = (45\gamma - 2)F$</td>
<td></td>
</tr>
<tr>
<td>$S_1, S_2 \in {F, B}$</td>
<td>$\theta_1 = (27\gamma\beta - 2)G$</td>
<td>$p_1 = H_1$</td>
<td>$p_1 = H_1 - \frac{d(1-k)}{\beta}$</td>
<td>$Q_1 = \beta_1(27\gamma\beta - 2)I$</td>
</tr>
<tr>
<td>&amp; $\theta_2 = (27\gamma\beta - 2)G$</td>
<td>$p_2 = H_2$</td>
<td>$p_1 = H_2 - \frac{d(1-k)}{\beta}$</td>
<td>$Q_2 = \beta_2(27\gamma\beta - 2)I$</td>
<td></td>
</tr>
</tbody>
</table>

### Equations

\[
A = \frac{(1+k)\alpha}{6c}, \quad B_1 = \frac{d(k^2 - 15\beta_1 - 13\gamma \beta_1 + (495\gamma + 387k)\gamma)}{2\beta_1}, \quad B_2 = \frac{d((504\gamma - 1)\beta_1 - 21 + k(396\gamma \beta_1 + \beta_1 - 19) - 21)}{27},
\]
\[
C = \frac{(1+k)}{2(54\gamma - 1)\beta_1 - 2}, \quad D = \frac{(1+k)\alpha}{12(27\gamma - 1)}, \quad E_1 = \frac{d(495\gamma + 36(129\gamma - 4) - 16)}{4(27\gamma - 1)}, \quad E_2 = \frac{3d(1+k)(3\gamma - 2)}{4(27\gamma - 1)}, \quad F = \frac{(1+k)}{4(27\gamma - 1)}.
\]
\[
G = \frac{(54\gamma - 1)\beta_1 - 1, \quad \gamma = \frac{cd}{\alpha}}, \quad H_i = \frac{d(\beta_1 \beta_2 \gamma(135 + 81k) - (9 + 7k)\beta_1 + (k - 1)\beta_1 - 1)}{2\beta_1(27\gamma \beta_1 - \beta_1 - \beta_2)}, \quad I = \frac{(1+k)}{2(27\gamma \beta_1 - \beta_1 - \beta_2)}, \quad J = \frac{(1+k)}{2(27\gamma \beta_1 - \beta_1 - \beta_2)}.
\]
\[
\beta_i = \begin{cases} 
1 & \text{if } S_i = B \\
\beta^F_i < 1 & \text{if } S_i = F
\end{cases}
\]

6.2.2 Proofs

**Proof of Proposition 11.**

We present the proof for NN scenario; the equilibrium for other scenarios can be derived following the same procedure. Following backward induction, first we derive the retailers’ equilibrium retail prices for each period. Retailers $i$’s sales $Q_{i,t}$ can be obtained by solving $U(\theta_i, p_{i,t}, Q_{i,t}) = U(\theta_j, p_{j,t}, p_t - Q_{i,t})$ for $j = 3 - i$, which yields

\[
Q_{i,t} = \frac{\alpha(\theta_i - \theta_j) - p_{i,t} + p_{j,t} + dp_t}{2d}, \quad (6.2)
\]

where $p_1 = 1$ and $p_2 = k$ are the market sizes in each period. Using the sales given by (6.2), retailers determine their retail price $p_{i,t}, t = 1, 2$, competitively to maximize their profits. It is straightforward to show retailer profit in the NN scenario $\pi_{NN}$ is concave in $p_{i,1}$ and $p_{i,2}$. Since inventory is not carried over to the second period, we can solve the retailer pricing problem
separately for each period using the first order conditions, and obtain the equilibrium prices and sales:

\[ p_{i,t}^* = \frac{\alpha (\theta_i - \theta_j) + 2w_i + w_j}{3}, \]

\[ Q_{i,t}^* = \frac{\rho_i}{2} + \frac{\alpha (\theta_i - \theta_j) + (w_j - w_i)}{6d}. \]

Having known the retail sales, each manufacturer sets the wholesale price \( w_i \) to maximize its profit \( \pi_{Mi}^N \) given in (3.3). It is straightforward to show \( \pi_{Mi}^N \) is concave in \( w_i \), and the equilibrium for the wholesale price game can be derived by solving \( \frac{\partial \pi_{Mi}^N}{\partial w_i} = 0 \) simultaneously for \( i = 1, 2 \), which yields:

\[ w_i^* = \frac{3d(1 + k)}{2} + \frac{(2r_i + r_j) + \alpha (\theta_i - \theta_j)}{3}. \]

Given the manufacturer response for wholesale prices, the suppliers then determine their material prices. Each supplier sets its material price \( r_i \) to maximize its profit \( \pi_{Si}^N \) given in (3.4). Again, it is straightforward that the profit function is concave in \( r_i \). Therefore the equilibrium satisfies \( \frac{\partial \pi_{Si}^N}{\partial r_i} = 0 \) for \( i = 1, 2 \), which yields:

\[ r_i^* = \frac{27d(1 + k) + 2\alpha (\theta_i - \theta_j)}{6}. \]

Finally, we consider the supplier quality game. Each supplier determines its quality \( \theta_i \) to maximize profit \( \pi_{Si}^N \). It can be shown that \( \frac{\partial^2 \pi_{Si}^N}{\partial \theta_i^2} < 0 \) \( \Leftrightarrow \) \( \frac{\alpha^2}{81d} \). Thus, we need \( c > \frac{\alpha^2}{81d} \) to ensure the concavity of \( \pi_{Si}^N \) with respect to \( \theta_i \). Otherwise, quality improvement is too cheap and competition drives both suppliers to overly invest on quality, making no profit. Assuming \( c > \frac{\alpha^2}{81d} \), we solve for the suppliers’ equilibrium quality decision following the first order conditions and obtain:

\[ \theta_i^* \left( \frac{1 + k + \alpha}{6} \right). \]

Finally, the equilibrium prices and sales in Proposition 11 follow using this equilibrium quality. Note the above SPNE is derived for the case where the retailers compete in each market, and we need to find the parametric conditions for this case. One can find that retailer \( i \)’s best response retail price is given as follows:

\[ p_{i,t}^{BR} = \begin{cases} \frac{m + w_i + \alpha \theta_i}{2}, & \text{for } Max(\sigma_1, \sigma_2) \leq p_{j,t}. \text{ Local monopoly;} \\ 2m - dk - p_{j,t} + \alpha (\theta_i + \theta_j), & \text{for } Min(\sigma_2, \sigma_3) \leq p_{j,t} < Max(\sigma_1, \sigma_2). \text{ Local monopoly;} \\ \frac{2 + w_i + \alpha (\theta_i - \theta_j)}{2}, & \text{for } p_{j,t} < Min(\sigma_2, \sigma_3). \text{ Retailers compete, market is cleared;} \end{cases} \]

where

\[ \sigma_1 = \frac{1}{2}(3m - 2dk - w_i + \alpha (\theta_i + 2\theta_j)), \]

\[ \sigma_2 = (\sqrt{2m - dk} + (1 - \sqrt{2})(w_i - \alpha \theta_i) + \alpha \theta_j), \]

\[ \sigma_3 = \frac{1}{2}(4m - 3dk - w_i + \alpha (\theta_i + 3\theta_j)). \]

To ensure that the retailers always engage in competition, for each stage game analyzed above, we check for conditions under which manufacturer and supplier decisions always result in \( p_{j,t}^* < Min(\sigma_2, \sigma_3) \) in equilibrium for the retail pricing game, and this procedure leads to the condition \( d < \frac{\alpha^2 (5 + 9k)}{54m} \). In sum, we need two parametric assumptions for the NN scenario: (1) \( c > \frac{\alpha^2}{81d} \) for concavity of suppliers’ profit functions, and (2) \( d < \frac{\alpha^2 (5 + 9k)}{54m} \) to ensure retail competition in each.
period. In addition, under these assumptions, plugging the equilibrium prices into consumers utility function reveals that every consumer earns positive utility from their purchase.

We apply the same approach to derive the equilibrium for other scenarios. Likewise, each scenario generates two conditions: one for concavity of profit functions and the other for ensuring retail competition. Comparing these conditions across scenarios, we find the FF, BF and FB scenarios generate the highest lower bound \( c > \frac{2a^2}{27d} \) to ensure concavity of profit functions, and the NN scenario gives the smallest upper bound \( d < \frac{a^2(5+9k)}{364m} \) to ensure retail competition. Finally, in the FN scenario, it can be shown \( Q_{2,2} = \frac{9cd(11k-1)-4k\alpha^2}{216cd-8\alpha^2} \), and \( 216cd-8\alpha^2 > 0 \) in the parameter space we consider. Thus, we also need \( k > \frac{1}{16} \) so that product 2 survives in \( t = 2 \).

**Proof of Proposition 12.**

The proof proceeds by comparing the equilibrium qualities and sales given in Proposition 11 across scenarios in the parameter space we consider. For example, \( \theta_{1,1}^{FN} - \theta_{1,1}^{NN} = \frac{3d(1+k)\alpha}{108cd-4\alpha^2} \) and \( Q_{1,1}^{FN} - Q_{1,1}^{NN} = \frac{9d(1+k)\alpha}{108cd-4\alpha^2} \). We assume \( c > \frac{2a^2}{27d} \) which implies \( 108cd - 4\alpha^2 > 0 \), and thereby \( \theta_{1,1}^{FN} > \theta_{1,1}^{NN} \) and \( Q_{1,1}^{FN} > Q_{1,1}^{NN} \). Other results in this proposition can be derived following the same procedure.

**Proof of Proposition 13.**

The proof proceeds by comparing manufacturer profit across scenarios using the equilibrium quality and prices given in Proposition 11. For part (i.a), we have \( \Pi_{M_1}^{NN} - \Pi_{M_1}^{FN} = \frac{\epsilon_1}{16(27\gamma^2-1)^2} \) where \( \epsilon_1 = 4 - 180\gamma + 1863\gamma^2 + k^2(4 - 180\gamma + 1863\gamma^2) + 6k(4 - 204\gamma + 2565\gamma^2) \) and \( \gamma = \frac{cd}{a^2} \). Note the assumption \( c > \frac{2a^2}{27d} \) implies \( \gamma > \frac{2}{27} \), for which it can be shown \( \epsilon_1 > 0 \), and thereby \( \Pi_{M_1}^{NN} > \Pi_{M_1}^{FN} \). The proof for (i.b) is straightforward by solving \( \Pi_{M_1}^{FS_2} - \Pi_{M_1}^{NS_2} = 0 \). Part (ii) proceeds following the same procedure as in part (i.a).

**Proof of Proposition 14.**

Part (i) proceeds by Proposition 13 (i.a) and (ii): \( \Pi_{M_1}^{BN} > \Pi_{M_1}^{NN} > \Pi_{M_1}^{FN} \). Part (ii) compares \( \Pi_{M_1}^{BS_2} \) and \( \Pi_{M_1}^{FS_2} \). It can be shown \( \Pi_{M_1}^{BS_2} - \Pi_{M_1}^{FS_2} = \frac{\epsilon_2}{\alpha^2} \) where \( \epsilon_2 = 9(1+k+k^2)\gamma \gamma - (1+k)^2 \). Solving \( \Pi_{M_1}^{BS_2} - \Pi_{M_1}^{FS_2} = 0 \) is equivalent to solving \( \epsilon_2 = 0 \), which yields two roots with \( \delta \) being the larger one. \( \delta \) is the only relevant root because \( \frac{\partial^2 \epsilon_2}{\partial k^2} = 18\gamma - 2 > 0 \) and the smaller root is negative. Part (iii) proceeds by the fact \( \Pi_{M_1}^{FS_2} - \Pi_{M_1}^{BS_2} = \Pi_{M_1}^{FN} - \Pi_{M_1}^{BN} = \frac{(1+k)^2+\alpha^2}{16c(27\gamma^2-1)^2} > 0 \) where \( \epsilon_3 = 16 - 945\gamma + 14013\gamma^2 > 0 \).

**Proof of Proposition 15.**

Parts (i) and (ii) follow from \( \Pi_{M_1}^{BS_2} > \Pi_{M_1}^{FS_2} \) and \( \Pi_{M_1}^{FN} < \Pi_{M_1}^{NN} \) by Proposition 13. In particular, when a manufacturer can only choose to forward integrate or not integrate at all, it can be shown that FF is another equilibrium for \( k < \frac{5405+6075\gamma^2-12+3(27\gamma^2-1)^2+3159\gamma^2}{4-108\gamma+234\gamma^2} \). However, Proposition 16 shows \( \Pi_{M_1}^{FF} > \Pi_{M_1}^{NN} \), and therefore NN is Pareto optimal.

The proof for part (iii) proceeds by showing no firm has incentive to deviate. First we show
**Proof of Proposition 16.**

The proof is straightforward because $\Pi^{NN}_{M_1} - \Pi^{FF}_{M_1} = \frac{d(1+6k+k^2)}{4} > 0$ and $\Pi^{NN}_{M_1} - \Pi^{BB}_{M_1} = \frac{(1+k)^2\sigma^2}{4} > 0$. ■

**Proof of Proposition 17.**

First consider part (i) of this proposition. For $S_2 \in \{B, F\}$, the proof is straightforward because $\Pi^{NS_2}_{R_1} + \Pi^{NS_2}_{M_1} - \Pi^{FS_2}_{M_1} = \frac{3d(1+k)^2\epsilon_5}{16(27\gamma-1)^2}$ where $\epsilon_5 = 4 - 156\gamma + 1377\gamma^2 > 0$. Other cases for part (i) can be shown following the same procedure.

Now consider part (ii) of this proposition. For $S_2 \in \{B, F\}$, we have $\Pi^{NS_2}_{C_1} - \Pi^{FS_2}_{C_1} = \Pi^{NS_2}_{C_1} - \Pi^{BS_2}_{C_1} = \frac{(1+k)^2(18\gamma-1)\alpha\gamma(47\gamma-20)}{16(27\gamma-1)^2} > 0$. For $S_2 = N$, we have $\Pi^{NS_2}_{C_1} - \Pi^{FS_2}_{C_1} = \Pi^{NS_2}_{C_1} - \Pi^{BS_2}_{C_1} = \frac{(1+k)^2\alpha^2(20-92\gamma+10125\gamma^2)}{16(27\gamma-1)^2} > 0$.

For part (iii), first note Proposition 15 states that $S^*_1S^*_2 = NN, FF$ or $BB$ depending on the strategies that are considered. Then part (iii) follows because $\Pi^{NN}_{C_1} - \Pi^{FF}_{C_1} = \Pi^{NN}_{C_1} - \Pi^{BB}_{C_1} = \frac{3}{4}d(1+k)^2 > 0$. ■

**Proof of Proposition 18.**

We use $\beta$ to denote $\beta^F$ in this proof for ease of notation. The proof proceeds by comparing the equilibrium quality, price decisions and sales given in Table 6.3. For part (ii.a), solving $p^{FF}_{S_2} - p^{NS_{S_2}}_{S_2} = 0$ reveals that $p^{FS_{S_2}}_{S_2} > p^{NS_{S_2}}_{S_2}$ if and only if $\beta < \sigma S_2$, where $\sigma S_2$ is the largest root to $\chi S_2 = 0$, where $S_2 \in \{F, N, B\}$, and

$$\chi^F = \begin{cases} 
\alpha^2(1 - \beta - k + 14\beta^2 + 12k\beta^2) - 9cd\beta(84\beta - 55 + k(72\beta - 43)), & \text{for } t = 1 \\
\alpha^2(1 - \beta + 12\beta^2 + k(14\beta^2 - 1 - \beta)) - 9cd\beta(72\beta - 43 + k(84\beta - 55)), & \text{for } t = 2 
\end{cases}$$

$$\chi^N = \begin{cases} 
18cd\beta(28\beta - 15 + k(22\beta - 9)) + \alpha^2(5 - 16\beta - \beta^2 + k(3 - 16\beta + \beta^2)), & \text{for } t = 1 \\
18cd\beta(22\beta - 9 + k(28\beta - 15)) + \alpha^2(3 - 16\beta + \beta^2 + k(5 - 16\beta - \beta^2)), & \text{for } t = 2 
\end{cases}$$
and (3) υ procedure. In addition, we use derivation forrium quality and prices given in Table 6.3. In the following, we characterize the existence and

\[\chi^B = \begin{cases} 
\alpha^4(1 - k - 2\beta - 2k\beta - 11\beta^2 - 9k\beta^2) + 243c^2d^2\beta(15 - 28\beta + k(9 - 22\beta)) & \text{for } t = 1 \\
-9cda^2(3 + 14\beta - 61\beta^2 - k(3 - 8\beta + 49\beta^2)), & \\
\alpha^4(k(1 - 2\beta - 11\beta^2) - 1 - 2\beta - 9\beta^2) - 243c^2d^2\beta(22\beta - 9 + k(28\beta - 15)) & \text{for } t = 2 \\
+9cda^2(3 - 8\beta + 49\beta^2 - k(3 + 4\beta - 61\beta^2)), & 
\end{cases}\]

**Proof of Proposition 19.**

The proof proceeds by comparing manufacturer profit across scenarios using the equilibrium quality and prices given in Table 6.3. In the following, we characterize the existence and derivation for \(\tau_N^N\). The derivation for \(\tau_1^B\), \(\tau_1^F\), \(\tau_2^B\), \(\tau_2^F\) and \(\tau_4\) can be obtained following the same procedure. In addition, we use \(\beta\) to denote \(\beta^F\) in this proof for ease of notation. It can be shown that \(\Pi_{M_1}^{B,N} - \Pi_{M_1}^{N,N} = \frac{d}{d\beta}(v_2 - v_1v_3)\), where \(v_1 = 3(1 + k)^2\), \(v_2 = 81c^2d^2(85 + 26k + 85k^2)\beta^2 - 36cda^2\beta(3 + 10\beta + k(8\beta - 6) + k^2(3 + 10\beta)) + \alpha^4(1 + 2\beta + 5\beta^2 + k^2(1 + 2\beta + 5\beta^2) - k(2 + 4\beta - 6\beta^2)\), \(v_3 = \beta(54cd\beta - \alpha^2(1 + \beta))^2\). Since \(v_3 > 0\), solving \(\Pi_{M_1}^{B,N} - \Pi_{M_1}^{N,N} = 0\) is equivalent to solving \(\epsilon_5 = v_2 - v_1v_3 = 0\). It can be shown that \(\epsilon_5 = 0\) has only one real root for \(0 < \beta < 1\). Then \(\tau_1^N\) is given by this root because of the following facts: (1) \(\epsilon_5 > 0\) for \(\beta = 1\), (2) \(\epsilon_5 < 0\) for \(\beta = 0\), and (3) \(\frac{d\epsilon_5}{d\beta} > 0\).

**Proof of Proposition 20.**

\(NN\) cannot be an equilibrium, because proposition 13 (ii) states \(\Pi_{M_1}^{B,N} > \Pi_{M_1}^{N,N}\), showing manufacturer 1 has incentive to deviate by choosing backward integration.

**Proof of Proposition 21.**

We use \(\beta_i\) to denote \(\beta_i^F\) in this proof for ease of notation. First, we obtain firm equilibrium decisions given in Appendix 6.2.1 following the procedure described in the proof of Proposition 11. Then the results in this proposition proceeds by comparing equilibrium quality and sales across scenarios, which leads to the following threshold values

\[\xi_1^Q = \xi_2^Q = \xi_2^Q = 0\]

Part (iii) of this proposition follows by replacing \(\beta_2^F = \beta_1^F - \Delta\) and show \(\frac{d^Q}{d\Delta} > 0\) and \(\frac{d^Q}{d\Delta} > 0\).
6.3 Appendix III

6.3.1 Equilibrium Decisions for Extensions

In the following lemma, we describe the equilibrium decisions for a centralized supply chain for both customer types.

**Lemma 12** In a centralized supply chain, the product is sold in both periods. When customers are strategic, the equilibrium order quantity is $Q = \frac{5-\delta}{2-\delta}$, the retail prices are $p_1 = \frac{(2-\delta)^2}{8-6\delta}$ and $p_2 = \frac{(2-\delta)\delta}{8-6\delta}$. When customers are myopic, the equilibrium order quantity is $Q = \frac{3-\delta}{4-\delta}$, the retail prices are $p_1 = \frac{2}{2-\delta}$ and $p_2 = \frac{\delta}{4-\delta}$.

The next lemma summarizes the equilibrium decisions when the supplier has a capacity limit.

**Lemma 13** When there is capacity limit and customers are strategic, the product is sold only in $t=1$ in equilibrium. The supplier’s optimal capacity level and wholesale price are:

1. For $c < \frac{3\delta-2}{2-\delta} : k = \frac{1-c}{2-\delta}$ and $w = \frac{1+c}{2}$.
2. For $\frac{3\delta-2}{2-\delta} \leq c < \frac{4-4\delta-\delta^2+4\sqrt{(\delta-1)(2-\delta)^2}}{(2-\delta)^2} : k = \frac{2(1-\delta)}{(2-\delta)^2}$ and $w = \frac{\delta}{2-\delta}$.
3. For $\frac{4-4\delta-\delta^2+4\sqrt{(\delta-1)(2-\delta)^2}}{(2-\delta)^2} \leq c : k = \frac{1-c}{4} and w = \frac{1+c}{2}$.

When there is capacity limit and customers are myopic, the product is sold only in $t=1$ in equilibrium. The supplier’s optimal capacity level and wholesale price are:

1. For $c < \frac{\delta-(1-\delta)\sqrt{(4-\delta)\delta}}{2(4-\delta)\delta} : k = \frac{(3-\delta)\delta-2c}{2(4-\delta)\delta}$ and $w = \frac{(3-\delta)\delta-2c}{4}$.
2. Otherwise: $k = \frac{1-c}{4}$ and $w = \frac{1+c}{2}$.

6.3.2 Proofs

**Proof of Lemma 4.** The retailer solves

$$\max_{p_2} \quad \pi_R = p_2(\theta - \frac{p_2}{\delta})$$

$$s.t. \quad \theta - \frac{p_2}{\delta} \leq Q - (1-\theta)$$

Since $\pi_R$ is concave in $p_2$, KKT conditions are sufficient to characterize the optimal $p_2$, which leads to cases (i) and (ii) of this lemma. Following the same procedure, we derive the supplier’s optimal capacity level and wholesale price when customers are myopic.

**Proof of Lemma 5.** First consider when customers are strategic. There are there possible cases in equilibrium: $Q \geq (1-\theta/2)$, $(1-\theta) \leq Q < (1-\theta/2)$ and $Q \leq (1-\theta)$. Note the first case is impossible, because the retailer has leftover inventory at the end of $t=2$ in that case and it can be strictly better off by reducing its order quantity. Next, we discuss the equilibrium decisions for the other two cases.
(1) For $(1 - \theta) \leq Q < (1 - \theta/2)$: In this case, $\theta = \frac{p_1 - \delta(1 - \tilde{Q})}{1 - \delta}$. Using this $\theta$, it can be shown $\pi_R$ given by (4.10) is concave in $p_1$. Thus, by applying the first order condition with respect to $p_1$, we obtain the optimal price

$$p_1 = \text{Max} \left( \frac{1 + \delta}{2} - \frac{(Q + \tilde{Q})\delta}{1 - \delta}, 1 - Q(1 - \delta) - \tilde{Q}\delta \right).$$

Using this optimal $p_1$, we derive the rational equilibrium order quantity by solving $\frac{\partial \pi_R}{\partial Q} = 0$ and $Q = \tilde{Q}$, which yields

$$Q = \begin{cases} 
\frac{3\delta - 2w}{43} & \text{for } w < \frac{\delta}{2} \\
\frac{1 - w}{2 - \delta} & \text{otherwise} 
\end{cases}.$$

(2) For $Q \leq (1 - \theta)$: In this case, customers believe that the retailer does not have any inventory at the end of $t = 1$, and thus the marginal customer is $\theta = p_1$. With this $\theta$, the retailer’s profit $\pi_R$ is joint concave in $p_1$ and $Q$. Thus the first order condition yields the optimal retail price $p_1 = \frac{1 + w}{2}$ and order size $Q = \frac{1 - w}{2}$. It can be shown that $\pi_R$ is higher with this order quantity for $w > \frac{\delta}{2}$. However, it can be shown that for $w < \frac{1 - \delta}{2}$, the retailer has incentive to deviate by ordering more and sells the product in $t = 2$. Thus, $Q = \frac{1 - w}{2}$ is an equilibrium only for $w > \frac{1 - \delta}{2}$. Finally, cases (i) to (iii) of this lemma follow by combining (1) and (2). When customers are myopic, cases (iv) and (v) can be derived similarly using $\theta = p_1$.

Proof of Proposition 22. The supplier maximizes its profit by solving

$$\max_{w} \pi_S = wQ$$

where $Q$ is given by Lemma 5. It can be shown that $\pi_S$ is concave in $w$ and the optimal $w$ can be characterized by the first order condition, leading to $w$ in Table 4.1. Finally, the equilibrium $Q$, $p_1$ and $p_2$ can be obtained through Lemma 5.

Proof of Corollary 1. The results proceed by comparing the decisions given in Table 4.1.

Proof of Proposition 23. First we derive firm profits by applying the equilibrium decisions in Table 4.1 to profit functions given by equations (4.10) and (4.11). Then the results are straightforward by comparing firm profits.

Proof of Proposition 24. The results are straightforward by deriving total channel profit as $\Pi_C = \Pi_R + \Pi_S$ and comparing it across scenarios.

Proof of Proposition 25. The results are straightforward by comparing the total channel profit and equilibrium decisions across scenarios.

Proof of Corollary 2. The results are straightforward by comparing the total channel profit across scenarios.

Proof of Proposition 26. The results are straightforward by comparing firm profits using the equilibrium decisions given in Lemma 13.

Proof of Proposition 27. First we derive firm profits by plugging the equilibrium decisions given in Lemma 5 into the profit functions given by equations (4.10) and (4.11). Then the results in this proposition proceed by comparing firm profits across scenarios.
Proof of Lemma 6. We present the proof when customers are strategic. The equilibrium decisions when customers are myopic can be derived following the same procedure. When the supplier offers quick response, the retailer profit is

\[ \pi_R = p_1(1 - \theta) + p_2(q + \bar{Q}) - w(Q + q), \]

where \( \bar{Q} = Q - (1 - \theta) \) is the inventory carried over from \( t = 1 \) to \( t = 2 \), and \( \theta \) is the marginal customer who is indifferent between buying in \( t = 1 \) or \( t = 2 \). The supplier profit is given by

\[ \pi_S = w(Q + q). \] (6.3)

Following backward induction, first we derive the retailer’s ordering decision in \( t = 2 \). In this case, the retailer solves

\[ \max_{p_2, q} \pi_R = p_2 \text{Min}(q + \bar{Q}, \theta - p_2 \delta) - wq, \]

This problem is jointly concave in \( p_2 \) and \( q \). Therefore the optimal solution can be derived using KKT conditions, which leads to the following result:

1. For \( \frac{\theta - w/2}{\delta} \leq \bar{Q} \leq \frac{\theta}{2} \): \( p_2 = \frac{\theta - w/2}{2} - \bar{Q} \).
2. For \( \frac{\theta - w/2}{\delta} \leq \bar{Q} < \frac{\theta}{2} \): \( p_2 = \delta(\theta - \bar{Q}) \) and \( q = 0 \).
3. For \( \frac{\theta}{2} \leq \bar{Q} \): \( p_2 = \frac{\theta}{2} \) and \( q = 0 \).

Next, we derive the retailer’s equilibrium order quantity and retail price \( p_1 \). When customers believe \( \bar{Q} \geq 0 \), the marginal customer satisfies \( \theta - p_1 = \delta \theta - p_2 \). Using \( p_2 \) given in case (1), the marginal customer is

\[ \theta = \frac{p_1 - p_2}{1 - \delta} = \frac{2p_1 - w}{2 - \delta}. \] (6.4)

We characterize firm decisions using rational expectation equilibrium, seeking for an equilibrium satisfying \( \frac{\partial \pi_R}{\partial Q} = 0 \) and \( Q = \bar{Q} \). This procedure leads to the following result.

**Lemma 14** When the supplier offers quick response, the retailer’s equilibrium order quantity and retail price in \( t = 1 \) are

1. For \( w < \frac{(2 - \delta)\delta}{4 - 3\delta} \): \( Q = \left\{ Q : \frac{2(1-\delta)}{4 - 3\delta} \leq Q \leq \frac{6 - 5\delta}{2(1 - \delta)} - \frac{w}{2} \right\} \), \( q \geq 0 \) and \( p_1 = \frac{(2 - \delta)^2 + w(4 - 3\delta)}{8 - 6\delta} \).
2. For \( \frac{(2 - \delta)\delta}{4 - 3\delta} \leq w < \frac{\delta}{2 - \theta} \): \( Q = 1 - \frac{w}{\delta} \), \( q = 0 \) and \( p_1 = \frac{w}{8} \).
3. For \( \frac{\delta}{2 - \theta} \leq w \): \( Q = \frac{1-w}{2} \), \( q = 0 \) and \( p_1 = \frac{1+w}{2} \).

Next, knowing the retailer’s ordering decision, the supplier chooses \( w \) to maximize its profit given by equation (6.3), which leads to the result of this proposition. ■

**Proof of Proposition 28.** The proof proceeds by comparing \( \pi_R \) and \( \pi_S \) given in equations (6.3.2) and (6.3) with decisions characterized in Lemma 6.

■

**Proof of Proposition 29.** First we consider quantity commitment where customers observe \( Q \). Following backward induction, first we characterize the retailer’s pricing decision \( p_2 \) in the second period and the result of this problem is given by Lemma 4.
Next, we derive the retailer’s equilibrium order quantity and retail price for \( t = 1 \). In this case, \( \theta = \frac{p_1 - \delta (1 - Q)}{1 - \delta} \) and \( \hat{Q} = Q \). The retailer maximizes its profit \( \pi_R \) given by (4.10). We first characterize the optimal retail price for \( t = 1 \) for any given \( Q \) as follows:

\[
p_1^*(Q) = \begin{cases} 
1 - Q & \text{for } Q < \frac{1}{2}, \\
\left(\frac{1 + \delta}{2} - \frac{\delta Q}{1 - \delta}\right) & \text{for } \frac{1}{2} \leq Q < \frac{1 - 3\delta + \sqrt{1 - 7\delta + 3\delta^2}}{8 - 6\delta}, \\
\text{otherwise.} & 
\end{cases}
\]

Given the optimal price \( p_1^*(Q) \), we maximize \( \pi_R \) over \( Q \), leading to the optimal quantity \( Q = \frac{1 - w}{2} \). Finally, the supplier chooses \( w \) to maximize its profit \( \pi_S = w \left( 1 - \frac{w}{2} \right) \). It is straightforward to show the optimal wholesale price is \( w = \frac{1}{2} \) and thereby the equilibrium decisions.

Now we consider price commitment where \( p_1 \) and \( p_2 \) become common knowledge. Note that customers do not need to form a belief \( \hat{Q} \) because \( p_2 \) is common knowledge. In this case, the marginal customer is characterized by \( \theta = \frac{p_1 - p_2}{1 - \delta} \).

Let \( \tilde{Q} \) be the inventory carried from \( t = 1 \) to \( t = 2 \). For any given \( p_1 \) and \( p_2 \), we have the following cases for customer behavior:

1. For \( Q (1 - \delta) - (1 - p_1 - \delta) < p_2 \leq p_1 \delta \): \( \tilde{Q} = 0 \) and the product is sold only in \( t = 1 \).
2. For \( p_2 \leq \min(Q(1 - \delta) - (1 - p_1 - \delta), \delta (1 - Q)) \): \( \tilde{Q} = 0 \) and the product is sold in both periods.
3. For \( \delta (1 - Q) < p_2 \leq p_1 \delta \): \( \tilde{Q} > 0 \) and the product is sold in both periods with inventory unsold at the end of \( t = 2 \).

The retailer profit is jointly concave in \( p_1 \) and \( p_2 \). Thus KKT conditions is sufficient to characterize the optimal solution which is given as follows:

\[
(p_1, p_2) = \begin{cases} 
(1 - Q, \delta (1 - Q)) & \text{for } Q < \frac{1}{2}, \\
\left(\frac{1}{2}, \frac{\delta}{2}\right) & \text{otherwise.} 
\end{cases}
\]

Using these optimal prices, it can be shown that the retailer’s profit is concave in \( Q \), and the first order condition yields the optimal order quantity \( Q = \frac{1 - w}{2} \). Next the supplier chooses \( w \) to maximize its profit \( \pi_S = w \left( 1 - \frac{w}{2} \right) \). It is straightforward that the optimal wholesale price is \( w = \frac{1}{2} \) and thereby the equilibrium decisions.

Finally, cases (ii) and (iii) of this proposition proceed by comparing firm profits against those in the SD scenario using the equilibrium decisions given in case (i) .

**Proof of Lemma 13.** First consider when customers are strategic. Given any capacity level \( k \), the supplier in this case solves

\[
\max_w \pi_S = w \min(Q, k) - ck, \quad (6.5)
\]

where the order quantity \( Q \) is given by Lemma 5. This problem leads to the following result:
(1) For $\delta < 0.4247$: The product is sold only in $t = 1$ and

1. a) For $k \leq \frac{1}{4}$: $Q = k, w = 1 - 2k$.

1. b) For $k > \frac{1}{4}$: $Q = 1/4, w = \frac{1}{2}$.

(2) For $0.4247 \leq \delta < \frac{4}{9}$: The product is sold only in $t = 1$ and

2. a) For $k \leq \frac{1}{4}$: $Q = k, w = 1 - 2k$.

2. b) For $\frac{1}{4} < k \leq \frac{2 - \delta}{8 \delta}$: $Q = \frac{1}{4}, w = \frac{1}{2}$.

2. c) For $\frac{2 - \delta}{8 \delta} < k \leq \frac{2(1 - \delta)}{(2 - \delta)^2}$: $Q = k, w = \frac{\delta}{2 - \delta}$.

(3) For $\frac{4}{9} \leq \delta$: The product is sold only in $t = 1$ and

3. a) For $k \leq \frac{1 - \delta}{2 - \delta}$: $Q = k, w = 1 - 2k$.

3. b) For $\frac{1 - \delta}{2 - \delta} < k \leq \frac{2(1 - \delta)}{(2 - \delta)^2}$: $Q = k, w = \frac{\delta}{2 - \delta}$.

3. c) For $\frac{2(1 - \delta)}{(2 - \delta)^2} < k \leq \frac{1}{4 - 2\delta}$: $Q = k, w = 1 - k(2 - \delta)$.

3. d) For $\frac{1}{4 - 2\delta} < k$: $Q = \frac{1}{4 - 2\delta}, w = \frac{1}{2}$.

Using these results, the supplier then maximize its profit with respect to $k$:

$$\max_k \pi_S = w \min(Q, k) - ck,$$

(6.6)

The supplier profit $\Pi_S$ is piecewise concave in $k$. The result comes straightforward by solving for the optimal $k$ for each given $\delta$.  

\[\blacksquare\]
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