Acknowledgements

I am grateful to Dr. Klara Peter for helpful comments and suggestions, as well as for instilling in me a proactive approach towards the completion of this project. I am especially grateful to Dr. Lutz Hendricks for his immeasurable contributions at every stage of this honors thesis. If this paper does not embarrass UNC's undergraduate economics department, it will be due to Dr. Hendricks' willingness to share his expertise and his patience while doing so.

Without their guidance, this work would not have been possible.

Abstract

This paper explores a possible limitation of generalized human capital models that operate by relaxing the assumption that skilled and unskilled labor are perfect substitutes. We make use of the notion that, when skilled and unskilled labor are rather inelastic, countries with relatively few skilled workers should offer large skill premiums compared to countries where skilled labor is relatively abundant. We introduce the *a priori* assumption that the price of skilled labor should be higher in developed countries than in undeveloped ones, and see that as skilled and unskilled labor become increasingly inelastic, generalized human capital models can attribute the majority of cross-country income differences to human capital variation only at the cost of violating this *a priori* assumption. We argue that this implies a bound on the substitution elasticity between skilled and unskilled labor that effectively bounds the contribution of human capital to cross-country income differences. This supports the notion that a theory of Total Factor Productivity is necessary to explain income variation across countries.

I. Introduction

Traditional neoclassical growth models feature a production function that maps a country's labor and capital inputs to its economic output. However, models focused only on these input factors cannot explain cross-country income variation, as capital and labor stocks between rich and poor countries are not wide enough to justify the differences in their per capita output. Standard growth models are forced to account for the unexplained residual with a catchall variable commonly referred to as Total Factor Productivity (TFP). The existence of this residual has motivated the creation of a variety of theories meant to more accurately model input factors and chip away at this "measure of our ignorance."

Human capital models have traditionally attempted to reduce reliance on TFP by weighting labor stocks according to education levels, effectively widening the difference in labor inputs between rich and poor countries. While this is an intuitive approach, the effectiveness of human capital models are limited for reasons including the difficulty of measuring human capital, the relatively small amount of cross-country output variation that measurable human capital differences can account for, and the difficulty in proving causality between education levels and output.

Recently, human capital models have attempted to increase their explanatory power by amplifying the effect of human capital variation on output, rather than increasing cross-country human capital variation per se. This has been accomplished by relaxing the assumption that workers of different skill levels are perfect substitutes and proceeding to show that structural differences in a country's economy, such as the relative quantity of skilled and unskilled labor, can affect the productivity of workers depending on their education levels. This approach has been shown to decrease reliance on TFP when the substitution elasticity between worker types falls within a desired range.

While most human capital approaches focused on the substitution elasticity between labor inputs assume that labor is divided into one skilled group and one unskilled group, Jones (2014) produces a generalized human capital aggregator with *n* skill types to create a more adaptable model. His general approach extends the literature by producing an aggregator that takes advantage of the developments discussed above, while also nesting standard human capital models as a special case. This allows him to compare the two approaches directly, and establish that current human capital models provide only a lower bound on cross-country human capital variation.

Although this approach is mathematically sound and can decrease reliance on TFP, no work has been done to establish the limits of this approach with regards to the manipulation of the substitution elasticity between labor inputs. If such a limit is found to exist, then human capital models may not be able to reduce reliance on TFP and another route will have to be pursued to explaining cross-country income differences; otherwise, cross-country income differences may be fully explained within a human capital framework. With this in mind, we see that there is value in determining whether an upper bound on human capital variation exists inside a generalized framework.

Our paper finds that, although generalized human capital models can accurately model cross-country income differences when the substitution elasticity between labor inputs falls within a desired range, such accuracy comes at the cost of predicting the existence of skill premiums in undeveloped countries that far outstrip such premiums in developed countries. This forces generalized human capital models to conclude that human capital flight should be occurring in the opposite direction of what we see today: educated workers in developed countries such as the U.S. should be migrating to undeveloped countries like Indonesia or

Uruguay in order to “cash in” on those countries’ skill premiums. This introduces a possible upper bound on the accuracy of generalized human capital models; these models may only be able to manipulate the substitution elasticity between skill groups until the relative price of skilled labor between rich and poor countries reaches parity. If this result is correct, it implies that a theory of TFP is necessary to accurately model cross-country income differences.

The order of this paper, then, is as follows. We first review the literature and discuss the rationale behind generalized human capital accounting. We then produce a generalized human capital aggregator under CES specifications and conduct an accounting exercise, imposing weak restrictions on skill prices implied by the model to derive an upper bound on the contribution of human capital to income gaps. These restrictions will follow from the *a priori* assumption that no skill class of workers should have a higher wage in a poor country than in a rich one.

II. Literature Review

While human capital models have expanded in several directions, they are all in pursuit of the same goal: find a way to increase variation in cross-country capital and human capital stocks, such that rich countries get more of these inputs or poor countries get less. For instance, Francesco Caselli (2005) provides an overview of attempts to augment human capital’s role in explaining cross-country income variation based on the efficiency with which factor inputs are used. Metrics such as the quality of physical capital, the health of a nation’s workforce, and the quality of schooling are considered as possible candidates to decrease reliance on TFP.

While TFP has been shown to be robust to many of the “efficiency” approaches, much work has been done to show that unobserved parameters do affect the relation between a country’s inputs and output. Caselli and Coleman (2006) relax the assumption that skilled and unskilled workers are perfect substitutes, and find that there is a skill bias in the way rich and poor countries implement technologies. This leads to their assertion that rich countries use highly educated workers more effectively than poor countries, but use less educated workers relatively and, possibly, absolutely less effectively.

This is an important result, as it provides evidence that the *relative* amount of skilled or unskilled labor can affect wage rates, at least insofar as it obliges countries to implement technologies that augment either skilled or unskilled labor. It also asserts that assumptions made by standard human capital models may lead to biased results, and that adjusting labor inputs for quality increases can reduce the contribution of TFP.

Bowlus and Robinson (2012) present a different approach to extending Human Capital models by producing a more accurate measure of the components of human capital: its price and quantity. They identify these components separately by breaking down the labor input into three types, dependent on their education levels. This alone is a break from previous literature, which has tended to compare only low-skilled and high-skilled workers, and had usually defined "low skilled" workers as only those possessing no education.

Bowlus and Robinson continue their approach by looking at the composition of the labor input in macro models. While many previous models have defined the labor input by aggregate labor hours, Bowlus and Robinson use a weighted measure of labor hours to take into account differences in productivity that are not related to education, such as intrinsic ability. With this weighted labor input, they maintain that reliance of growth models on TFP can only decrease. Similarly to Caselli and Coleman, their findings also imply that the relative quantity of human capital is more important than its relative price when explaining why the observed wage rate of skilled and unskilled workers differ.

This research is insightful, because it provides firmer proof that a country's output may be dependent not only on its capital stock and the quality of its workers, but again on the relative amount of skilled and unskilled workers in an economy. The paper also provides a way to more accurately measure human capital, and takes a step in the direction of generalizing the number of labor classes in an economy. The findings also note that as the number of labor classes increases from two to three, the effect of relaxing the assumption of perfect substitution between labor classes to augment cross-country human capital variation becomes greater.

These approaches have been expanded by Jones (2014), who has produced a generalized human capital model that is able make use of recent advances in the literature while nesting standard human capital models as a special case – allowing a direct comparison between the two approaches. Jones' generalized human capital aggregator splits the labor input into *n* subgroups, divided with reference to observed education levels. He then builds off the work of Caselli by making two claims: That workers with different levels of human capital are imperfect substitutes, and that the marginal product of uneducated workers can be augmented by complementary effects between high and low skilled workers.

Jones' second claim provides his model with much of its bite. By pointing out that uneducated workers can see their productivity increase as the number of educated workers increases, Jones introduces a possible new bias in previous human capital models. This assumption is also rather intuitive; one might consider a factory producing widgets: workers will see their productivity increase if a talented manager organizes the production process, effectively divides the labor of his workers and makes use of state-of-the-art technologies and production methods.

Jones’ model provides the most direct evidence that standard human capital models are biased, and that they provide only a lower bound on human capital variation. He finds that when skilled and unskilled labor have a substitution elasticity of 1.5, reliance on TFP is eliminated. If it is found that Jones’ model is theoretically sound, then human capital may provide the way to explaining TFP. However, if an upper bound on human capital variation can be established, then another explanation must be found to explain cross-country income differences.

With the recent mathematical success of generalized human capital aggregators, one might think that these models are ever closer to eliminating reliance on TFP. But the notion that the cross-country income gaps we see today are caused by inelastic labor groups presents a variety of problems. For instance, if human capital is the primary driver behind productivity, then skilled workers who immigrate to rich countries from poor ones should hardly see an earnings gain. The fact that this implication contradicts what we see in the world today implies that there are other important factors that must be responsible for income differences, such as social and legal institutions that are better accounted for by TFP than Human Capital.

A similar critique of human capital models comes from Hendricks (2002), who makes use of the fact that comparing US immigrants with workers from their source countries allows one to bypass the difficult task of measuring human capital while also controlling for unseen factors. With this insight, Hendricks has shown that the wage increases experienced by US immigrants, from a variety of countries and across education levels, cannot be explained by human capital models unless immigrants have an enormous self-selection bias based on intrinsic ability. But there is no solid theory to suggest that immigrants experience this self-selection, and empirical studies have provided conflicting evidence on the matter.

Finally, the assumption that high and low skilled workers are not substitutable, combined with the assumption that highly skilled workers can increase the marginal productivity of human capital services, implies that there must be very large skill premiums in poor countries. This raises two problems for human capital models: What is preventing human capital accumulation in poor countries, and why are educated workers not migrating to poor countries to “cash in” on these large skill premiums?

III. Theoretical Model

A Review of Jones' Generalized Approach

Having provided an overview of the existing literature surrounding generalized human capital models, there is still value in reviewing the work of Jones with more mathematical rigor. As our CES aggregator must be of the same spirit as Jones' generalized aggregator, we commit ourselves now to reviewing his approach in some depth.

Jones begins by creating a human capital aggregator, where the human capital stock is equal to the sum total of the capital of the N different classes of workers. This provides the equation

$$H = G(H_1,\ H_2,\ .\ .\ .\ ,\ H_n)$$

Where $G(H_1,\ H_2,\ .\ .\ .\ ,\ H_n)$ is some aggregator such as CES. Jones shows that the above equation is equivalent to

$$(1) \qquad H = G_1(H_1,\ H_2,\ .\ .\ .\ ,\ H_n)\hat{H}$$

Where $\hat{H}$ is an unskilled worker equivalent, and G_1 is the marginal increase in total human capital services from an additional unit of unskilled human capital services. $\hat{H}$ can be thought of as a common measure of labor inputs, such that units of different types of labor are equivalent to different amounts of $\hat{H}$. We see that increasing/decreasing the term G_1 has the effect of increasing/decreasing the marginal value of a unit of unskilled labor $\hat{H}$.

By rewriting (1), Jones provides a simple equation that allows us to see what inputs we need to measure the human capital stock:

$$H = G_1 * h_1 \sum_{i=1}^{N} \frac{w_i}{w_1} L_i$$

where h_1 denotes the human capital of unskilled workers and L_i denotes the quantity of workers with the i^{th} level of human capital. This result points to the fact that, once we have used relative wages in an economy to convert workers into equivalent units of unskilled labor, we have to consider how the productivity of an unskilled worker depends on the skills of other workers, an effect encapsulated by G_1(Jones, 2014).

According to Jones, this shows why traditional human capital models have such trouble shaving down the total factor productivity scalar: variation in unskilled labor units is modest, so without accounting for the effect of the skills of other workers on the marginal output of unskilled workers, relatively little of the large income variation we see between countries can be explained with human capital.

Jones continues by introducing a variable that can give a measurement of the bias associated with current human capital models, defining $\Lambda = \frac{H^R / H^P}{\hat{H}^R / \hat{H}^P}$ "as the ratio of true human capital differences to the traditional calculation of human capital differences." Using (*), this allows him to write the equivalent relationship

$$\Lambda = G_1^R / G_1^P$$

This measurement indicates that standard growth models may suffer from biased results caused by the use of efficiency units when $\Lambda \geq 1$.

Jones introduces one final lemma with the introduction of a human capital aggregator

$$H = G(H_1, Z(H_2, \dots, H_N)),$$

where Z is a function representing the division of skilled labor. This provides Jones with evidence that traditional human capital accounting methods provide biased results, as he establishes that

$$(2) \qquad \Lambda = {G_1^R}\big/{G_1^P} \geq 1$$

If and only if

$$(3) \qquad {Z^R}\big/{H_1^R} \geq {Z^P}\big/{H_1^P}$$

According to Jones, because (2) can be satisfied under a number of broad conditions, we see that previous growth models are likely biased and have yet to account for the actual human capital variation that exists across countries.

With this assertion providing a possible answer to the question of cross-country income variation, the type of model we must create to test Jones' work is clear. Our model must be capable of separating the labor input into an arbitrary number of subclasses (i.e., skill classes), and our model must also be able to separate these skill classes into an arbitrary number of classes again (we refer to these "sub-subclasses" as human capital classes). Finally, our model must be capable of directly comparing cross-country income differences across a range of substitution elasticity values.

Producing a Human Capital Aggregator

With a prescribed direction that our model must follow, we now begin to develop our production function and human capital aggregator. Our first assumption is that there exists an aggregate production function of the form

$$Y = F(K, H, A),$$

where H is aggregate human capital, K is aggregate physical capital, and A is a scalar. We assume that aggregators have constant returns to scale with regards to their capital inputs and provide a Cobb-Douglas production function for each country C:

$$y_C = {k_C}^{\alpha}(A_C h_C)^{1-\alpha}$$

We note that α, representing capital's share of labor, is set equal to one third. Variables expressed as lowercase letters denote the fact that they are measured per worker.

Borrowing from the literature, we will assume that there exist human capital classes $H_{1,C},\ H_{2,C},\ .\ .\ .\ ,\ H_{N,C}$, with workers in these classes possessing distinct levels of schooling.

We will use a constant elasticity of substitution (CES) specification to carry out our aggregation procedures; the CES production function we will use is of the form:

$$CES(x_i, \mu, \theta) = \left[\sum_i (\mu_i x_i)^{\theta}\right]^{1/\theta}$$

where μ_i is a weight on factor x_i and θ is the elasticity of substitution parameter. When this CES production function aggregates distinct groups X and Y, with inputs x_i and y_i respectively, we have the equation

$$CES(X, Y;\ \mu, \theta) = \left[(\mu_1 X)^{\theta} + (\mu_2 Y)^{\theta}\right]^{1/\theta}$$

Because the purpose of this paper is to investigate the implications of relaxing the assumption of perfect substitutability between worker classes on cross-country human capital variation, we will define H as the aggregate human capital possessed by the class of unskilled workers Z_1 and the class of skilled workers Z_2. This provides the equation

$$H_C = CES(Z_{1,C}, Z_{2,C}; \rho, \varepsilon)$$

with ρ denoting the skill weight and ε the elasticity of substitution between the two classes of workers. We note that the skilled and unskilled human capital stocks themselves are aggregations of the human capital of worker classes $H_{1,C}, H_{2,C}, \ldots, H_{N,C}$, with

$$Z_{1,C} = CES(H_{1,C}, \ldots, H_{S,C}; \mu, \theta)$$

and

$$Z_{2,C} = CES(H_{S+1,C}, \ldots, H_{N,C}; \phi, \theta),$$

with μ denoting the skill weight for unskilled workers in the worker classes 1-S, and ϕ denoting the skill weight for skilled workers in the worker classes S+1 - N. We divide workers into these groups with reference to observed education levels, so that a worker in class i has less schooling than a worker in class $i+1$. With the aggregators in place, we note that the elasticity parameters for Z_1 and Z_2 are the same, such that the substitution elasticity between skilled workers is the same as the elasticity between unskilled workers. The elasticity of substitution between skilled *and* unskilled workers, however, is able to differ from the substitution elasticity within the two classes.

Assumptions Regarding the Labor Input

With a human capital aggregator specified, we move on to assumptions regarding the labor input. As is standard in neoclassical models, we assume factors are paid their marginal products, with the marginal product of capital input X_j described by the equation

$$\frac{\partial Y_C}{\partial X_{i,C}} = w_{i,C}$$

This is a standard assumption that makes intuitive sense; workers are hired (and capital is acquired) until the marginal products of workers (capital) equals their wage rate. This assumption allows us to see that the unobserved price of labor from the i^{th} worker class,

$1 \leq i \leq S$, is

$$w_{i,C} = \frac{\delta Y_C}{\delta H_C} \frac{\delta H_C}{\delta Z_{1,C}} \frac{\delta Z_{1,C}}{\delta H_{i,C}}$$

A parallel equation describes the unobserved price of type j labor, with type j labor belonging to the class of skilled workers. For $S + 1 \leq j \leq N$, we have

$$w_{j,C} = \frac{\delta Y_C}{\delta H_C} \frac{\delta H_C}{\delta Z_{2,C}} \frac{\delta Z_{2,C}}{\delta H_{j,C}}$$

For each worker class $H_{1,C}, H_{2,C}, \ldots, H_{N,C}$, workers in the class $H_{i,C}$ possess an average level of human capital $h_{i,C}$. Because of our assumption that factors are paid their marginal products, we can define the wage bill of worker class $H_{i,C}$, i.e., the sum of yearly earnings of all workers in class $H_{i,C}$, with the equation

$$wagebill_{i,C} = w_{i,C} h_{i,C} L_{i,C} = w_{i,C} H_{i,C}$$

This equation states that the wage bill of class $H_{i,C}$ is the unseen price of that labor type $w_{i,C}$, weighted by the average skill premium $H_{i,C}$ and total labor hours $L_{i,C}$. As many economists note, while it is easy to measure the quantity of each labor type $L_{1,C}, L_{2,C}, \ldots, L_{N,C}$, neither the quality of each labor type $h_{1,C}, h_{2,C}, \ldots, h_{n,C}$ nor the unseen price of labor can be easily observed. With $w_{i,C}$ and $h_{i,C}$ unknown, we will have to make additional assumptions to proceed. But once we do, we will be able to measure $h_{i,C}$ for different countries by solving the equations for each worker class's wage bill that have just been described.

IV. Data

With a theoretical model described, we now discuss the data that will be used in our model. The data on wages, labor hours, and education levels that we will use to measure human capital stocks come from IPUMS (Integrated Public Use Microdata Series) and IPUMS International, which provide harmonized census data. The censuses that will be used are the 2000 US Census, the 2000 Brazil Census, the 1995 Indonesia Census, and the 2006 Uruguay Census; these weighted samples cover 5% of each nation's population. These countries are included because they possess per capita income levels and capital stocks across a range of values, while still providing relatively complete information regarding schooling levels and employment statistics.

Of the data sets, the US census is by far the most extensive. It totals 14,081,466 weighted observations and provides information on income from wages and privately owned businesses, hours worked per week, and months worked in the past year. Included in the data is whether a person works for the government (either at the local, state, or federal level) or for a private business (either a for-profit or not-for-profit firm).

Foreign data sets possess varying degrees of detail; for instance, none rival the detail with which the U.S. categorizes income. No foreign data set has information on weeks worked, and so an average of 49 weeks is assumed. Unfortunately, Uruguay has information only on hours worked at one's main job (hrsmain), as opposed to total hours worked per week (HRSWRK1). However, because the two measurements produce such close results in the aggregate, Uruguay's labor hours are totaled using the variable hrsmain as if it is representative of total hours worked per week.

To assemble an adequate data set, we measure only those aged 20-69 who are working for wages in private firms. We measure only those who are working full time, in this paper defined as those who work 30 hours or more per week. Additionally, we will only consider workers who worked at least one quarter of the year, so that the reported number of months respondents worked is greater than or equal to 3. Unfortunately, only the U.S. data set has information on months worked.

To account for inaccuracies and outliers in the self-reported wage data, respondents will be dropped if their income is below 5% of the median income, or if their income exceeds 100 times the median income. As censuses range in year from 1994 to 2005, inflation figures are calculated with the Bureau of Labor Statistics, and monetary values are measured in PPP using the Penn World Tables. We also drop all respondents who are not native to the country in question, so that the workers we measure have all received similar educations for the years they were in school.

As no data set provides information on weeks worked, a figure of 49 is assumed. Aggregate labor hours will be derived by multiplying the average weekly hours worked by 49 weeks. Aggregate wage bills will be derived in a similar way, by multiplying weekly wages by 49 weeks. Table 4 provides a cross-country comparison of aggregate wage bills and labor hours for skilled and unskilled labor.

Following the literature, we will assume that there are 7 worker classes, with the first comprising workers with no schooling, the second comprising workers with up to five years of schooling, the third workers who did not begin secondary education, the fourth those with some secondary education, the fifth those who completed secondary education, the sixth those with some university education, and the seventh comprising those with a college degree and above. We divide workers in each data set into these seven groups based on their education levels.

While most countries have similar progressions of schooling, the Brazilian school system ended its secondary education at grade 11 at the time the survey we are using was taken. To account for this, the fourth worker class in Brazil consists of workers with no more than 10 years of education, and Brazilian workers who have completed their secondary education are noted to have just 11 years of education, in contrast to the 12 years of education possessed by the fifth class of workers native to the other countries.

With the above data facilitating the construction of each country's human capital stocks, we turn now to our Cobb-Douglas production function. Labor and physical capital stocks are taken from the Penn World Tables – allowing us to determine the per worker physical capital stock. The Penn World Tables' RGDPe measurement gives us information on per capita income for each country in 2005 US$. Information on these measurements is provided in Table 5.

With the Penn World Tables providing data for each country's income level and capital and labor stocks, all have only to derive each country's human capital stock from the census data described above. Once we have found each country's human capital stock, we will lack information only on each country's residual A_C. This will be found simply by solving our Cobb-Douglas production function.

V. Empirical Model and Procedure

Overview of our Accounting Approach

With our theoretical model and data described, we provide an overview of our accounting exercise, including the rationale behind the steps we take. At this early stage, we observe each country's per capita income y_C, per capita capital stock k_C, and the labor stocks $L_{i,C}$ and wage bills for each worker class. Our goal is to use this information to derive A_C, $h_{i,C}$, and $w_{i,C}$ for each country.

We begin by assuming that $h_{1,US} = h_{1,c} = 1$, which is equivalent to assuming that the quality of an uneducated person's labor is the same no matter what his nationality. This assumption allows us to measure human capital levels for $h_{i,C}$ and $h_{i,US}$ relative to the common base $h_1 = 1$, and the normalization of h_1 facilitates the human capital aggregation. We also normalize the skill weights μ and ϕ, from our CES aggregators for Z_1 and Z_2, respectively. We do this for all countries so as to give our human capital model the "benefit of the doubt," as it implies that cross-country income differences between workers within the same class will be due to differences in labor quality, rather than structural differences in each nation's economy.

With these assumptions in hand, we can proceed to derive $h_{i,C}$ by using information on the wage bill of worker class i relative to the wage bill of worker class 1. We will show that this method produces an equation for the value of $h_{i,C}$ provided we know the substitution elasticity between labor inputs. With the relationship between $w_{i,C}, h_{i,C}, L_{i,C}$, and $wagebill_{i,C}$ described above, we see that obtaining measurements for $h_{2,C}, \ldots, h_{n,C}$ easily provides measurements for $w_{1,C}, \ldots, w_{n,C}$.

With values of $H_{i,C}$ procured in this way, we can proceed to aggregate $Z_{1,C}$ and $Z_{2,C}$, in turn allowing us to produce a value for $H_C = CES(Z_{1,C}, Z_{2,C};\ \rho, \varepsilon)$. With values for H_C known, our Cobb-Douglas production function provides us an equation with each country's residual as the only unknown. Solving our production function for A_C allows us to compare cross-country income gaps.

Because this entire process depends on us assuming certain values for the substitution elasticity between labor inputs, we see that H_C and A_C are variables dependent on theta and epsilon. Following Jones (2014), we will show that, by assuming a relatively high level of theta, we can push down the value of epsilon such that cross-country residuals eventually reach parity.

However, our use of the term $w_{i,C}$ allows us to introduce a restriction that is critical to this paper: that the unobserved price of labor for any worker class in rich countries must be greater than the unobserved price of labor for any worker class in poor countries, or $w_{i,US} \geq w_{i,C}$. This restriction is implied by our model, as supposing otherwise would imply that workers in rich countries should be emigrating to poor countries with relatively few educated workers, in order to cash in on those countries' skill premiums.

Measuring Human Capital Stocks

To begin the empirical procedure, we first measure each country's human capital stocks. We recall that the price of labor i is given by the equation

$$(4) \qquad w_i = \frac{\delta Y}{\delta H_C} \frac{\delta H_C}{\delta Z_{1,C}} \frac{\delta Z_{1,C}}{\delta H_i}, \qquad 1 \leq i \leq S$$

We see that the price of labor within the unskilled labor class, relative to the uneducated labor price w_1, is given by

$$(5) \qquad \frac{w_{i,C}}{w_{1,C}} = \frac{\delta Z_{1,C} / \delta H_{i,C}}{\delta Z_{1,C} / \delta H_{1,C}} = \left(\frac{H_{i,C}}{H_{1,C}}\right)^{\theta-1}, \qquad 1 \leq i \leq 4$$

Similarly, the price of labor within the skilled class relative to the least uneducated labor price w_5 is given by

$$(5') \qquad \frac{w_{j,C}}{w_{5,C}} = \frac{\delta Z_{2,C} / \delta H_{j,C}}{\delta Z_{2,C} / \delta H_{5,C}} = \left(\frac{H_{j,C}}{H_{5,C}}\right)^{\theta-1}, \qquad 5 \leq j \leq 7$$

With data on each country's labor supply $L_{i,C}$ and wage bills $w_{i,C}H_{i,C}$, we proceed by normalizing the quality of uneducated US laborers and the quality of uneducated foreign workers, so that $h_{1,US} = h_{1,C} = 1$. This is equivalent to assuming that the quality of an uneducated person's labor is the same no matter what his nationality.

Finally, we normalize both skill weights in our CES equations for $Z_{1,C}$ and $Z_{2,C}$, so that μ and ϕ are equal to 1.

Having taken these steps, our unknowns remain $h_{2,C}, \dots, h_{N,C}$ and $w_{2,C}, \dots, w_{N,C}$ for all countries. To proceed, we note that (5) and (5′) are equivalent to the equations

$$(6) \qquad \frac{H_{i,C}w_{i,C}}{H_{1,C}w_{1,C}} = \left(\frac{H_{i,C}}{H_{1,C}}\right)^{\theta}$$

and

$$(6') \qquad \frac{H_{j,C}w_{j,C}}{H_{5,C}w_{5,C}} = \left(\frac{H_{j,C}}{H_{5,C}}\right)^{\theta}$$

Because (6) is simply the wage bill of worker class i relative to the wage bill of uneducated workers, and because we have normalized h_1 so that $H_1 = L_1$, we see that we can derive the capital stock of unskilled workers such that

$$H_{i,C} = L_{1,C}\left(\frac{H_{i,C}w_{i,C}}{H_{1,C}w_{1,C}}\right)^{1/\theta}$$

A parallel approach allows us to calculate $H_{j,C}$ relative to $H_{5,C}$, although it requires us to normalize $h_{5,C}$ and take it to be equal to one. This presents a problem, as normalizing $h_{5,C}$ implies that $h_{5,C} = h_{1,C}$; however, we will compensate for this dilemma – and the fact that skilled and unskilled human capital are being measured in different "units" – by utilizing our weight ρ in the CES aggregator $H_{US} = CES(Z_{1,US}, Z_{2,US}; \rho, \mathcal{E})$.

With the capital stocks of each worker class known, their labor prices quickly follow, providing us with information on $h_{2,C}, \dots, h_{N,C}$ and $w_{2,C}, \dots, w_{N,C}$ for all countries. We can now solve for

$$Z_{1,C} = CES(H_{1,C}, H_{2,C}, H_{3,C}, H_{4,C}; \mu, \theta),$$

$$Z_{2,C} = CES(H_{5,C}, H_{6,C}, H_{7,C}; \phi, \theta),$$

These equations now provide us with aggregate human capital for unskilled workers and for skilled workers, respectively.

With $Z_{1,C}$ and $Z_{2,C}$ known, we can solve for $H_C = CES(Z_{1,C}, Z_{2,C}; \rho, \mathcal{E})$ and in the process account for the fact that we have set $h_{5,C} = h_{1,C} = 1$. We have a system of two equations with three unknowns H_{US}, ρ_1, and ρ_2:

$$\text{(7)} \qquad H_{US} = \left[\left(\rho_1 Z_{1,US}\right)^{\varepsilon} + \left(\rho_2 Z_{2,US}\right)^{\varepsilon}\right]^{1/\varepsilon}$$

$$\rho_1 + \rho_2 = 1$$

We solve these equations for one country only, and apply whatever values have been found to the weights for all other countries as well. We do this so that differences in output will not be due to differences in the weights ρ_1 and ρ_2, but rather due to differences in human capital stocks.

We solve (7) using the US data set, and with an algebraic trick similar to (5). We see that the wage bill of unskilled workers in the US (denoted WB_U), divided by the wage bill of skilled workers in the US (denoted WB_S), is equal to

$$\text{(8)} \qquad \left(\frac{\delta H_{US} / Z_{1,US}}{\delta H_{US} / Z_{2,US}}\right) \frac{Z_{1,US}}{Z_{2,US}} = \frac{\rho_1^{\varepsilon} Z_{1,US}^{\varepsilon}}{\rho_2^{\varepsilon} Z_{2,US}^{\varepsilon}}$$

This relationship between the wage bills of skilled and unskilled workers and the weights of our CES aggregator allows us to solve a system of equations equivalent to (4):

$$(7') \qquad \frac{\rho_1}{\rho_2} = \frac{Z_{2,US}}{Z_{1,US}} \left(\frac{WB_U}{WB_S}\right)^{1/\varepsilon}$$

$$\rho_1 + \rho_2 = 1$$

With the system of equations (7) defined, it's clear that we have two equations and two unknowns, and hence the values of ρ_1 and ρ_2 can be found. It is also clear that the weights are dependent on both epsilon and theta. As both $Z_{1,C}$ and $Z_{2,C}$ are dependent on theta, we see that we have a clearly defined function for each country's aggregate human capital, dependent on values of theta and epsilon:

$$H_C = \left[\left(\rho_1 Z_{1,C}\right)^{\varepsilon} + \left(\rho_2 Z_{2,C}\right)^{\varepsilon}\right]^{1/\varepsilon}$$

Measuring Total Factor Productivity

With information on H, and with information on each country's capital stock *K*, we turn to our Cobb-Douglas production function

$$(9) \qquad y_C = {k_C}^{\alpha}(A_C h_C)^{1-\alpha}$$

With y_C and k_C taken from the Penn World Tables, and α set equal to one third as is standard in the literature, we see that every term in the production function is defined for each value of theta and epsilon save for our residual A_C. We remedy this by solving (9) for A_C, once again giving us a variable whose value is dependent on theta and epsilon.

With A_C defined in this way, we see that we can compare cross-country income differences, and that the relative level of these countries' residuals are dependent on theta and epsilon. We can compare the relative values of A_C, defining TFP's contribution to explaining cross country income variation by the relative residual $\frac{A_{US}}{A_C}$. To facilitate our comparisons, we create a table featuring each countries' residual at levels of epsilon ranging from .09 to .99, reflecting a substitution elasticity between skilled and unskilled labor that ranges from approximately 1.1 to 100. We note that this is due to the relationship between the elasticity parameter in our CES model (ε) and the substitution elasticity between skill groups, which is defined as $\frac{1}{1-\varepsilon}$.

VI. Results

Comparing cross-country levels of A_C, we see that when we assume the substitution elasticity between worker classes within the same skill group is set equal to 3 ($\theta = {}^2/_3$), human capital models become more accurate and reduce their dependence on TFP as skilled and unskilled labor becomes increasingly inelastic. For varying levels of θ, relative residuals for Brazil, Indonesia, and Uruguay all reach parity with the United States when the substitution elasticity between skilled and unskilled labor is within the range of 1.5 – 1.8. This is as expected, and supports the conclusion that accounting for inelastic labor inputs can allow human capital to account for the majority of cross-country income variation.

However, our model also predicts that when the substitution elasticity between skilled and unskilled workers is low enough to make our model accurate, the price of skilled labor in foreign countries becomes greater than the price of skilled labor in the US. This violates our *a priori* assumption that the price of labor for educated workers should be higher in the US than in underdeveloped countries. So we see that manipulating the elasticity parameters to erase our model's dependence on TFP also results in our model making the prediction that educated US workers would be better off immigrating to countries like Indonesia, Brazil and Uruguay.

Our model was able to reduce the relative residual between the U.S. and Uruguay to 3.2 before predicting that the price of labor for workers in the sixth labor class should be higher in Uruguay than in the U.S. Our model was able to reduce the relative residual to 1.9 before predicting that the price of labor for the seventh labor class should be higher in Uruguay than in the United States. Even giving our model the benefit of the doubt and accepting 1.9 as the upper bound on our model's accuracy, our model still over predicts Uruguay's output by a factor of 2.

Similarly, our model was unable to reduce the relative residuals of the US and Brazil to 1 before breaking our assumption regarding the relative price of skilled labor. Our model reduced the relative residual between the U.S. and Brazil to 2.46 before predicting that the price of labor for workers in the sixth labor class should be higher in Brazil than in the U.S. Giving our model the benefit of the doubt once more and continuing, we find that our model can reduce the relative residual to 1.85 before predicting that the seventh labor class should also receive a higher price for their labor in Brazil than in the U.S. Once again, our model over predicts output by close to a factor of 2.

Surprisingly, our model predicts that both the sixth and seventh labor classes should receive a higher price for their labor in Indonesia than in the U.S. at all values of epsilon. This implies that regardless of the substitution elasticity between skilled and unskilled labor, educated workers would be better off immigrating to Indonesia from the United States. Although the price of educated labor is roughly equal in the U.S. and Indonesia when skilled and unskilled labor are highly substitutable, lowering the substitution elasticity so that our model can fully explain Indonesia's output without relying on TFP implies that U.S. workers would see their wages more than double by moving to Indonesia.

These results suggest that human capital models can indeed decrease their reliance on TFP by assuming skilled and unskilled labor are inelastic – but only to a point. As skilled and unskilled workers become more and more inelastic, the price of skilled labor in poor countries increases far more rapidly than the price of skilled labor in the United States. The result is that our model breaks our *a priori* assumption long before it is able to explain the majority of cross-country income variation.

So we see that the same manipulation of elasticity parameters that can allow our model to increase its accuracy, also requires our model to predict that educated workers in rich countries should be immigrating to poor countries in order to "cash in" on high skill premiums. This contradiction suggests that we cannot manipulate elasticity parameters with abandon, and that there exists an upper bound in our model that prevents it from modeling income variation with complete (or even near) accuracy.

VII. Conclusion

In this paper, we have created a human capital model in the spirit of Jones' generalized aggregator and derived an upper bound on the contribution of human capital to income gaps by imposing weak restrictions on skill prices. We have established that models relying on the substitution elasticity between labor inputs cannot manipulate elasticity parameters to fit an arbitrarily desired range. Instead there exists a bound on the substitution elasticity between skilled and unskilled labor that, when crossed, forces our model to confront difficult questions regarding its implications.

This paper highlights a critical problem human capital models must solve if they hope to explain cross-country income variation without a theory of TFP. If human capital can explain the bulk of income variation by relying on a low substitution elasticity between skilled and unskilled labor, then why are skill premiums offered by countries with relatively low levels of skilled labor not enough to attract foreign professionals? What is preventing these relatively poor countries from accumulating human capital?

Our results support the possibility of an upper bound existing with regard to the substitution elasticity between skilled and unskilled labor, effectively bounding the role of human capital in explaining income variation. This bound prevents current generalized human capital models from divorcing themselves completely from TFP, and suggest that other factors are necessary to explain cross-country income variation.

VIII. Appendix

Figure 1

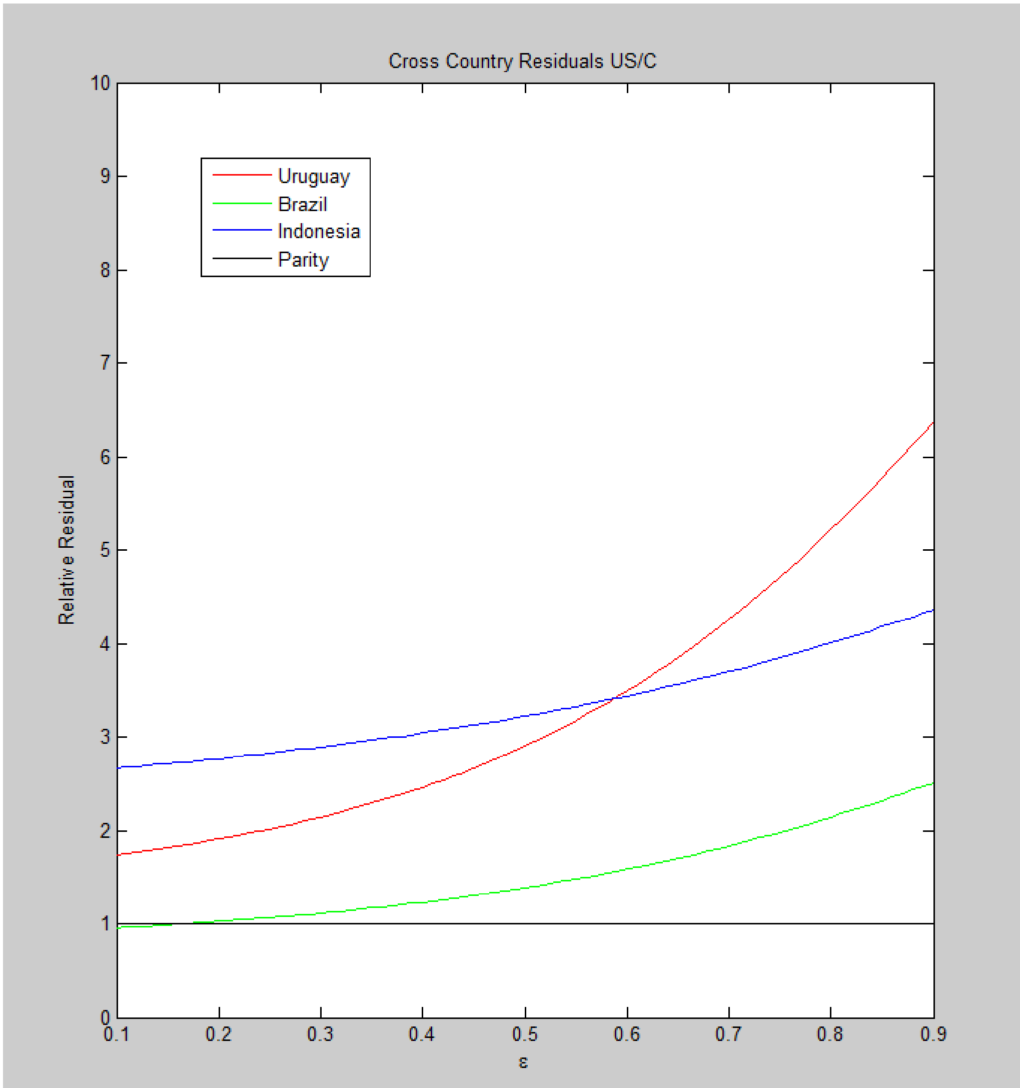

Figure 1 shows the relative residual (TFP) for the United States compared to Brazil, Indonesia and Uruguay. Here we have set theta to a third, representing a substitution elasticity of 3 between worker classes within each skill group. As epsilon approaches zero we see that our human capital model becomes more accurate, and correctly predicts the output of Brazil when epsilon is .17, representing a substitution elasticity of 1.2 between skilled and unskilled labor.

Note: Substitution elasticity between skilled and unskilled labor given by $\frac{1}{1-\varepsilon}$

Figure 2

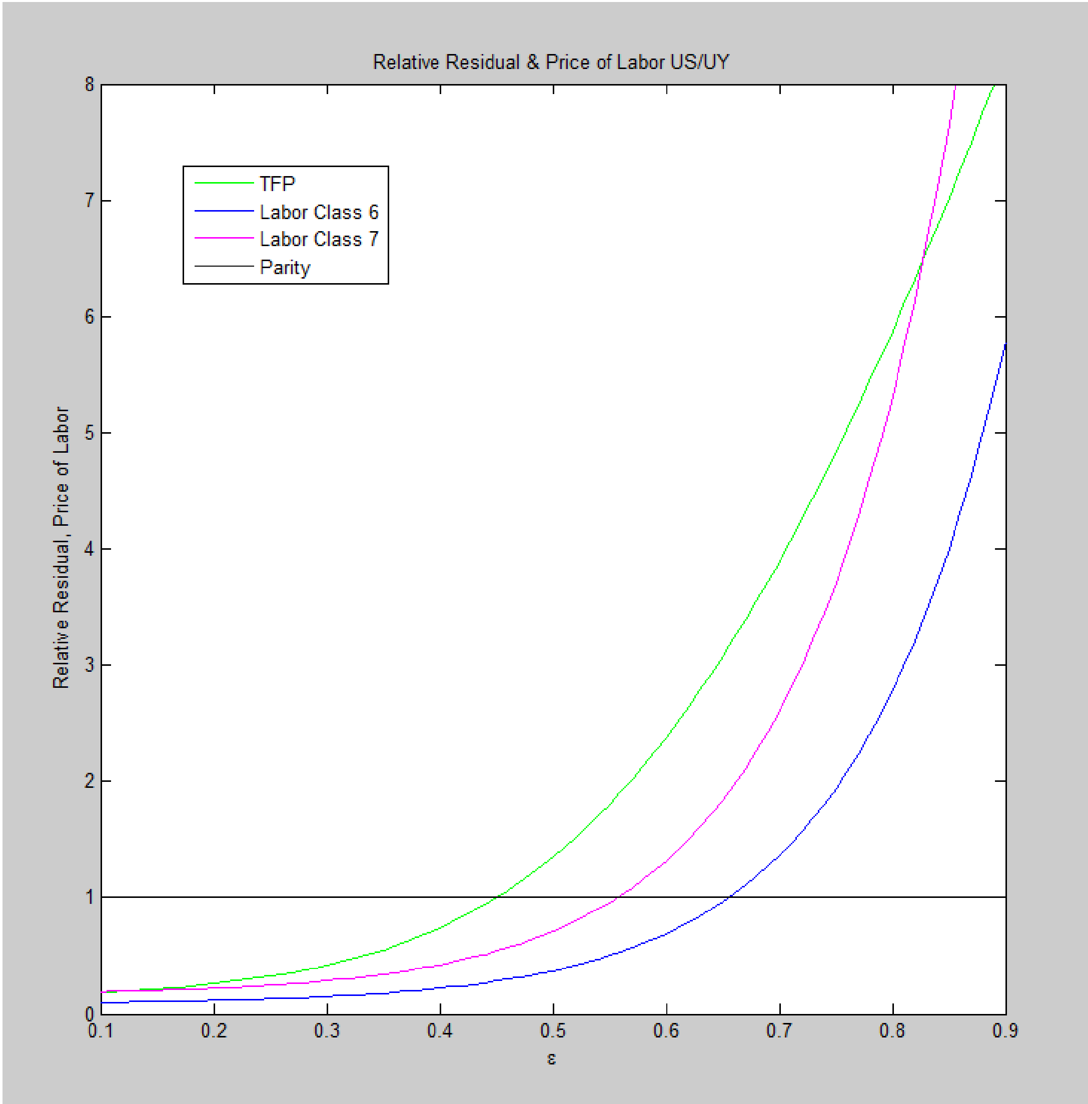

Figure 2 compares the relative residual between the U.S. and Uruguay to the relative price of skilled labor between those two countries. Here the elasticity within skill groups is assumed to be 1.5. The results show that our intuition regarding the relative price of skilled labor was correct. As skilled and unskilled labor become increasingly inelastic, our model implies the existence of increasingly large skill premiums in "skill-starved" countries. Our model breaks our *a piori* assumption long before explaining income variation.

Table 1

Elasticity	TFP	Labor Class 6	Labor Class 7
1.1	0.177737	0.098537	0.188398
1.16	0.207048	0.104573	0.199938
1.23	0.248774	0.113351	0.21672
1.32	0.30864	0.126107	0.24111
1.4	0.394919	0.144647	0.276557
1.52	0.518885	0.171592	0.328074
1.64	0.694898	0.210752	0.402946
1.79	0.939722	0.267666	0.511762
1.96	1.270918	0.350382	0.66991
2.17	1.704589	0.470597	0.899754
2.44	2.253137	0.645311	1.233798
2.78	2.923703	0.899233	1.719282
3.23	3.717666	1.268271	2.42486
3.85	4.631126	1.804612	3.450314
4.76	5.656025	2.584105	4.94066
6.25	6.781504	3.716982	7.106656
9.1	7.995184	5.363451	10.25461
16.67	9.284203	7.756349	14.82969
100	10.63597	11.23407	21.4789

Table1 compares the relative residual and price of labor between the U.S. and Uruguay across a range of substitution elasticity values. With this, we see that our model can explain income variation only by predicting that U.S. workers with some college education (labor class 6) or with a college, professional, or doctoral degree (labor class 7) should receive a two to threefold increase in their earnings by immigrating to Uruguay. We also see that, at its most accurate while still in line with our assumption regarding the relative price of labor, our model under predicts Uruguay's output by a factor of 3.

Figure 3

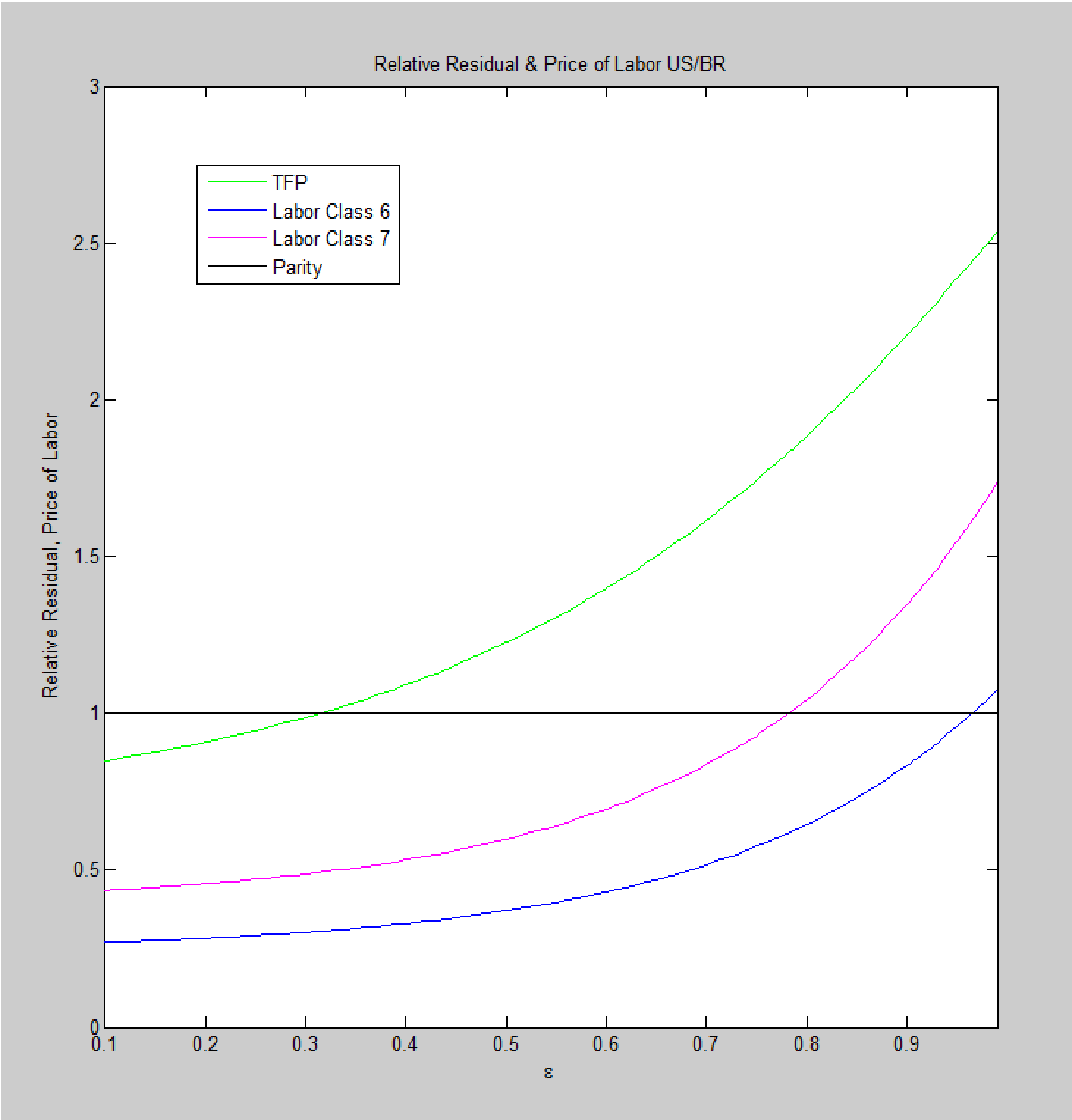

Figure 3 compares the relative residual between the U.S. and Brazil to the relative price of skilled labor between those two countries. Here the inner elasticity is set equal to 4. This result further supports our notion that the price of skilled labor can serve as a bound on the substitution elasticity between skilled and unskilled labor. Once again, our *a priori* assumption is broken long before our model is accurate enough to remove its reliance on TFP.

Table 2

Elasticity	TFP	Labor Class 6	Labor Class 7
1.1	0.843533	0.268617	0.434066
1.16	0.870684	0.274306	0.443259
1.23	0.901803	0.281188	0.45438
1.32	0.937488	0.289512	0.467832
1.4	0.978409	0.299583	0.484105
1.52	1.025308	0.311765	0.50379
1.64	1.078994	0.326501	0.527603
1.79	1.140331	0.344327	0.556409
1.96	1.21022	0.365891	0.591254
2.17	1.289575	0.391976	0.633406
2.44	1.379287	0.423531	0.684397
2.78	1.480186	0.461702	0.746079
3.23	1.592998	0.507877	0.820694
3.85	1.718302	0.563734	0.910955
4.76	1.856486	0.631303	1.020142
6.25	2.007726	0.713039	1.152222
9.1	2.171958	0.811914	1.311997
16.67	2.348884	0.931521	1.505273
100	2.537974	1.076207	1.739076

Table 2 compares the relative residual and price of labor between the U.S. and Brazil across a range of substitution elasticity values. As with Uruguay, we see that our model can explain income variation only by predicting that skilled U.S. workers should see a two to threefold increase in their earnings by immigrating to Brazil. We also see that, at its most accurate while still in line with our assumption regarding the relative price of labor, our model under predicts Brazil's output by a factor of 2.

Figure 4

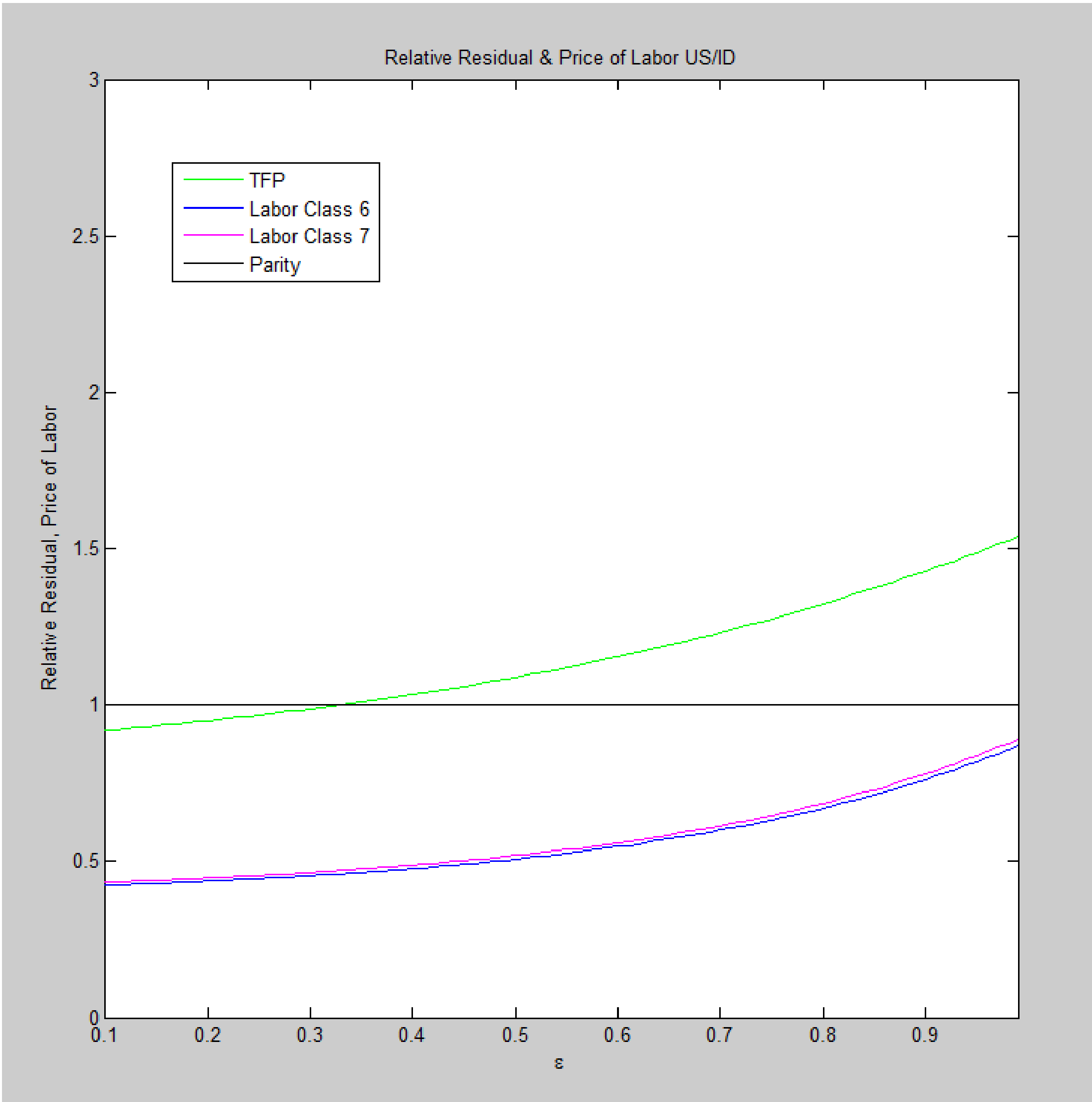

Figure 4 compares the relative residual between the U.S. and Indonesia to the relative price of skilled labor between those two countries. Here the inner elasticity is set equal to $\frac{5}{3}$. We see that our model predicts that Indonesia offers skill premiums that outstrip those in the U.S. for all levels of epsilon. Assuming this (surprising) result is correct, it once again supports the notion that generalized human capital models face restrictions when manipulating the substitution elasticity between skilled and unskilled labor.

Table 3

Elasticity	TFP	Labor Class 6	Labor Class 7
1.1	0.914915	0.4228	0.432515
1.16	0.92972	0.428706	0.438557
1.23	0.945965	0.435497	0.445504
1.32	0.963805	0.443309	0.453495
1.4	0.983395	0.452292	0.462685
1.52	1.004896	0.462624	0.473255
1.64	1.02848	0.474508	0.485411
1.79	1.054326	0.488174	0.499392
1.96	1.082617	0.503893	0.515471
2.17	1.11354	0.521971	0.533965
2.44	1.147281	0.542762	0.555234
2.78	1.184021	0.566674	0.579696
3.23	1.223931	0.594176	0.607829
3.85	1.267169	0.625806	0.640186
4.76	1.313873	0.662184	0.6774
6.25	1.364155	0.704023	0.7202
9.1	1.4181	0.752142	0.769424
16.67	1.475756	0.807483	0.826038
100	1.537135	0.871132	0.891149

Table 3 compares the relative residual and price of labor between the U.S. and Indonesia across a range of substitution elasticity values. Our model can explain income variation only by predicting that skilled U.S. workers should receive a twofold increase in their earnings by immigrating to Indonesia. Our model contradicts our *a priori* assumption at all elasticity values, motivating the question why, if human capital is responsible for the majority of cross-country income variation, why do countries with deficits in skilled labor have such trouble attracting foreign professionals despite the skill premiums offered?

Table 4

Class	US Hours	US Wage Bills	Brazil Hours	Brazil Wage Bills	Indonesia Hours	Indonesia Wage Bills	Uruguay Hours	Uruguay Wage Bills
1	$3.02*10^8$	$4.25*10^9$	$3.87*10^9$	$5.69*10^8$	$2.25*10^9$	$2.34*10^8$	$5.28*10^6$	$7.67*10^5$
2	$1.63*10^9$	$2.38*10^{10}$	$2.12*10^{10}$	$4.72*10^9$	$6.59*10^9$	$8.41*10^8$	$8.72*10^7$	$1.38*10^7$
3	$1.66*10^9$	$2.41*10^{10}$	$1.16*10^{10}$	$3.44*10^9$	$1.36*10^{10}$	$2.09*10^9$	$4.09*10^8$	$7.62*10^7$
4	$1.00*10^{10}$	$1.51*10^{11}$	$2.80*10^9$	$9.34*10^8$	$6.35*10^9$	$1.23*10^9$	$3.70*10^8$	$8.67*10^7$
5	$4.11*10^{10}$	$7.10*10^{11}$	$1.34*10^{10}$	$5.58*10^9$	$1.39*10^{10}$	$3.79*10^9$	$1.23*10^8$	$3.99*10^7$
6	$3.96*10^{10}$	$8.27*10^{11}$	$1.49*10^9$	$1.37*10^9$	$2.17*10^9$	$9.92*10^8$	$7.30*10^7$	$3.25*10^7$
7	$2.81*10^{10}$	$1.01*10^{12}$	$4.87*10^9$	$7.06*10^9$	$2.13*10^9$	$1.23*10^9$	$6.27*10^7$	$5.24*10^7$
Unskilled	$1.36*10^{10}$	$2.04*10^{11}$	$3.95*10^{10}$	$9.67*10^9$	$2.88*10^{10}$	$4.40*10^9$	$8.71*10^8$	$1.77*10^8$
Skilled	$1.1*10^{11}$	$2.55*10^{12}$	$1.98*10^{10}$	$1.40*10^{10}$	$1.82*10^{10}$	$6.01*10^9$	$2.59*10^8$	$1.25*10^8$

Wage bills in 2005 US$

Table 5 compares aggregate labor hours and wage bills for each labor class across countries, as accounted for in our IPUMS datasets. This highlights the relative scarcity of skilled labor in poor countries compared to the U.S. The aggregate labor hours of college educated workers in the U.S. is 100 times greater than the aggregate labor hours of workers with no education. Overall, skilled labor in the U.S. is almost ten times as plentiful as is unskilled labor. In Brazil and Indonesia, workers who have not completed primary school produce five times as many labor hours as college educated workers, and aggregate unskilled labor hours are more than three times as large as aggregate skilled labor hours in Uruguay.

Table 5

Measure	U.S.	Brazil	Indonesia	Uruguay
Panel A: Accounting Measurements				
Real GDP (mil. 2005 US$)	10711100	1146806	512643.3	28369.6
y_{US}/y_C	1	5.058204	12.94489	3.114636
Capital Stock (mil. 2005 US$)	28316298	3622159	831277.3	106252.8
k_{US}/k_C	1	4.233712	21.10426	2.198479
Persons Engaged (mil.)	136.3844	73.8613	84.4976	1.1251
L_{US}/L_C	1	1.846493	1.614062	121.2198
Panel B: TFP's Contribution to Income Gaps				
A_{US}/A_C $(w_{6,US} \leq w_{6,C})$	1	2.46	1.53	3.1
A_{US}/A_C $(w_{7,US} \leq w_{7,C})$	1	1.83	1.53	1.85

Lower case variables denote per worker measurements.

Table 5 lists accounting measurements and the lowest relative residuals before the price of workers with some college education (labor class 6) and the price of workers with a college degree or higher (labor class 7) becomes higher in foreign countries relative to the U.S.

Table 6

Variable	Description	U.S.	Brazil	Indonesia	Uruguay
age	Age of respondent	39.9	35.2	34.67	39.1
classwk	Field respondent works in. This variable includes local and federal government, private (nongovernment) for profit and not for profit firms, and domestic labor (0 = "private sector firm for wages")	0	0	0	0
HRSWRK1	Number of hours worked per week.	43.58	47.67	47.88	N/A
incwage	Wage income.	37357	N/A	219430	6080.48
incself	Income earned as independent business owner.	0	0	0	0
incearn	Earned income	37328	565.92	N/A	N/A
school	Whether respondent is currently in school. (0 = "No")	0	0	0	0
wrkmths	Months worked in the past year.	11.38	N/A	N/A	N/A
yrimm	Year immigrated. (0 = "Not immigrant")	0	N/A	N/A	N/A
educus	Highest education level attained.	Some College	N/A	N/A	N/A
edattan	Highest education level attained.	Secondary	Primary	Primary	Primary
yrschl	Number of years of schooling.	N/A	7.08	9.06	8.46
nativty	Respondent's country of birth (0 = "Native").	0	0	0	0
hrsmain	Hours per week worked in main job.	N/A	46.87	47.29	48.67

1) Average value for each discrete variable, median value for categorical variables included under country name
2) Income in national currencies

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