A Direct Dark Matter Search with the MAJORANA
Low-Background Broad Energy Germanium Detector

Padraic Seamus Finnerty

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Approved by:
Dr. Reyco Henning
Dr. John F. Wilkerson
Dr. Albert Young
Dr. Jonathan Engel
Dr. Fabian Heitcsh
It is well established that a significant portion of our Universe is comprised of invisible, non-luminous matter, commonly referred to as dark matter. The detection and characterization of this missing matter is an active area of research in cosmology and particle astrophysics. A general class of candidates for non-baryonic particle dark matter is weakly interacting massive particles (WIMPs). WIMPs emerge naturally from supersymmetry with predicted masses between $1 - 1000$ GeV. There are many current and near-future experiments that may shed light on the nature of dark matter by directly detecting WIMP-nucleus scattering events.

The Majorana experiment will use p-type point contact (PPC) germanium detectors as both the source and detector to search for neutrinoless double-beta decay in $^{76}$Ge. These detectors have both exceptional energy resolution and low-energy thresholds. The low-energy performance of PPC detectors, due to their low-capacitance point-contact design, makes them suitable for direct dark matter searches.

As a part of the research and development efforts for the Majorana experiment, a custom Canberra PPC detector has been deployed at the Kimballton Underground Research Facility in Ripplemead, Virginia. This detector has been used to perform a search for low-mass ($< 10$ GeV) WIMP induced nuclear recoils using a 221.49 live-day exposure. It was found that events originating near the surface of the detector plague the signal region, even after all cuts. For this reason, only an upper limit on WIMP induced nuclear recoils was placed. This limit is inconsistent with several recent claims to have observed light WIMP based dark matter.
ACKNOWLEDGMENTS

There are too many people to list who have helped me along the way, so forgive me if I forget any of you. First, and foremost, I want to thank my future wife, Angela. She is my compass, and without her, I would be lost. Also, without the love and support of my family, I never would have made it this far — thank you for pushing me so hard.

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<tr>
<td>$0\nu\beta\beta$</td>
<td>Neutrinoless Double-Beta Decay</td>
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<tr>
<td>$2\nu\beta\beta$</td>
<td>Two Neutrino Double-Beta Decay</td>
</tr>
<tr>
<td>BEGe</td>
<td>Broad Energy Germanium</td>
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<tr>
<td>CC</td>
<td>Charged Current</td>
</tr>
<tr>
<td>CDM</td>
<td>Cold Dark Matter</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
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<tr>
<td>DM</td>
<td>Dark Matter</td>
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<tr>
<td>DDR</td>
<td>Diffusion Dominated Region</td>
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<td>DSP</td>
<td>Digital Signal Processing</td>
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<tr>
<td>FCCD</td>
<td>Full Charge Collection Depth</td>
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<td>GAT</td>
<td>Germanium Analysis Toolkit</td>
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<td>GERDA</td>
<td>GERmanium Detector Array</td>
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<tr>
<td>HPGe</td>
<td>High Purity Germanium</td>
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<tr>
<td>KURF</td>
<td>Kimballton Underground Research Facility</td>
</tr>
<tr>
<td>LH</td>
<td>Left Handed</td>
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<tr>
<td>LN$_2$</td>
<td>Liquid Nitrogen</td>
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<tr>
<td>MALBEK</td>
<td>MAJORANA Low-Background BEGe Detector at Kimballton</td>
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<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MGDO</td>
<td>MAJORANA-GERDA Data Objects</td>
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<td>MJOR</td>
<td>MAJORANA-ORCARoot</td>
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<tr>
<td>mBEGe</td>
<td>Modified Broad Energy Germanium</td>
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<tr>
<td>MSSM</td>
<td>Minimal Supersymmetric Standard Model</td>
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<tr>
<td>NSC</td>
<td>N-type Segmented Contact</td>
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<tr>
<td>NC</td>
<td>Neutral Current</td>
</tr>
<tr>
<td>ORCA</td>
<td>Object-oriented Real-time Control and Acquisition</td>
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<tr>
<td>Abbreviation</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PPC</td>
<td>P-type Point Contact</td>
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<tr>
<td>PL</td>
<td>Profile Likelihood</td>
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<tr>
<td>RDR</td>
<td>Recombination Dominated Region</td>
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<tr>
<td>RH</td>
<td>Right Handed</td>
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<tr>
<td>SCM</td>
<td>Standard Cosmological Model</td>
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<tr>
<td>SM</td>
<td>Standard Model</td>
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<tr>
<td>SSC</td>
<td>Slow-Signal Cut</td>
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<tr>
<td>SURF</td>
<td>Sanford Underground Research Facility</td>
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<tr>
<td>SUSY</td>
<td>Supersymmetry</td>
</tr>
<tr>
<td>UMxL</td>
<td>Unbinned Maximum Likelihood</td>
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<tr>
<td>WIMP</td>
<td>Weakly Interacting Massive Particle</td>
</tr>
<tr>
<td>WIMPs</td>
<td>Weakly Interacting Massive Particles</td>
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Chapter 1

Introduction

The work outlined in this dissertation is a subsidiary project of the MAJORANA experiment. The two main physics topics are (1) dark matter and (2) the nature of neutrinos. The main goal of this dissertation was to directly search for particle dark matter. In this chapter I will give an introduction to Dark Matter (DM) and follow up with a brief introduction to neutrinos and the MAJORANA experiment.

1.1 Dark Matter in the Universe

Isaac Newton’s theory of gravity works remarkably well at explaining the motions of celestial objects, for example the motions of planets around a star or a star around a galactic core. However, small deviations from the expected trajectories have been measured. When questions arose on anomalies in the motion of planets in the Solar system, the scientific community asked themselves: Is Newton’s theory wrong or is there something there that we just can’t see? In the case of Uranus’ orbit, the answer was that there was indeed something we couldn’t see: Neptune. However, by similar logic, the motion of Mercury led scientists to claim there had to be a planet nearby\(^1\). Astronomers found no evidence of a ‘missing planet’ near Mercury and it turned out that the solution would have to wait until Einstein’s theory of general relativity (GR) [1], i.e. the introduction of a more refined description of the laws of gravitation. The Dark Matter Problem is strikingly similar to the scenarios outlined above.

\(^1\)They called it Vulcan
Is something really there that we can’t see or is our understanding of the Universe still in its infant stages? After a brief introduction to cosmology and the history of the Universe, I will discuss how we have irrefutable evidence that there is in fact DM in the Universe and what it is thought to be comprised of.

1.1.1 The Standard Cosmological Model

Cosmology is theoretical astrophysics at its largest scales. It deals with the Universe as a whole – its origin, distant past, evolution, and structure. When looking at the world at such grand scales, locally ‘flat’ and ‘slow’ approximations (the realm of the Newtonian mechanics) are no longer justified. For more details on cosmological theory and the evidence that supports it, see any of these modern textbooks [2–6]. Additionally, throughout the remainder of this dissertation, we will use natural units (i.e. $c = 1$ and $h = 1$).

Theoretical Framework

The framework for understanding the evolution of our Universe is referred to as the Standard Cosmological Model (SCM, sometimes referred to as Λ-CDM) [7]. The SCM is deeply rooted in Einstein’s theory of GR and assumes that the Universe, on its largest scales, is homogenous and isotropic. These features have been confirmed observationally. The SCM has proven to be an excellent model, as it can satisfactorily explain several key features of the early Universe:

- **Thermal history**, how long a species of particle will take place in fundamental interactions before the interaction rate becomes negligible during various epochs, see Section 1.1.1.

- **Relic background radiation**, the cosmic microwave background (CMB) and relic neutrino background, see Section 1.1.2.

- **Abundances of the elements**, the amount of various elements present in the Universe today, see Section 1.1.2.

- **Large scale structure of the Universe**, meaning the observed density of galaxies present in the Universe today.
The SCM has three key foundations that help us explain the physical world [8]:

1. **Einstein equations**, relates the geometry of the Universe with its matter and energy content.

2. **Metrics**, describing the symmetries of the problem, *e.g.* what space-time are we in: flat? curved?

3. **Equation of state**, specifying the physical properties of the matter and energy content, *e.g.* what is the relation between pressures and densities?

The Einstein equations of motion are given by [7–9],

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = -\frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}, \tag{1.1} \]

where \( R_{\mu\nu} \) and \( R \) are the Ricci tensor and scalar respectively, \( g_{\mu\nu} \) is the metric tensor, \( G_N \) is Newton’s constant, \( T_{\mu\nu} \) is the stress-energy tensor and \( \Lambda \) is the cosmological constant or Dark Energy component. For now, if we ignore the term with the cosmological constant in it (\( \Lambda g_{\mu\nu} \)), this equation is easily understood. In simple terms, this equation states that the geometry of the Universe is determined by the energy content of the Universe.

To solve these equations, we need to specify a line element or metric, and the most common metric used is

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \tag{1.2} \]

where \( a(t) \) is the scale factor and the constant \( k \) describes the spatial curvature of space-time. The value of \( k \) can either be -1 (open), 1 (closed) or 0 (flat). In the simplest case, for a flat space-time, Equation 1.1 reduces down to ordinary Euclidean space. We can then solve the Einstein equations to derive the Friedmann equation [8, 10],

\[ \left( \frac{\dot{a}}{a} \right)^2 + k a^2 = \frac{8\pi G_N}{3} \rho_{\text{tot}}, \tag{1.3} \]
where $\rho_{\text{tot}}$ is the average energy density of the Universe. It is common to introduce a parameter referred to as the Hubble parameter given by

$$H(t) = \frac{a'(t)}{a(t)},$$

and is a measure of the rate at which space-time is expanding or contracting. The current value of the Hubble parameter is referred to as the Hubble constant and is denoted by $H_0$. A recent estimate of the Hubble constant with the 9-year WMAP data alone gives $H_0 = 70.0 \pm 2.2$ km s$^{-1}$ Mpc$^{-1}$ [11]. We can see from Equation 1.3 that the Universe is flat when the energy density equals a critical density $\rho_c$,

$$\rho_c = \frac{3H^2}{8\pi G_N}.$$  \hspace{1cm} (1.5)

Here we adopt the notation used by [8, 12], which expresses the abundance of a substance in the Universe (matter, radiation, vacuum energy, etc.) in units of the critical density $\rho_c$. Therefore, a quantity $\Omega_i$ of a substance of species $i$ is given as

$$\Omega_i \equiv \frac{\rho_i}{\rho_c},$$

it also follows that the mass-energy density of the Universe in these units is

$$\Omega = \sum_i \Omega_i \equiv \sum_i \frac{\rho_i}{\rho_c}.$$  \hspace{1cm} (1.7)

With these new definitions of $\Omega$ and $\rho_c$, the Friedmann equation (Equation 1.3) can be written as

$$\Omega - 1 = \frac{k}{H^2 a^2}.$$  \hspace{1cm} (1.8)

The sign of $k$ is thus determined by whether $\Omega$ is greater, less than or equal to unity. In order to understand how the energy content of the Universe evolves according to the Friedmann equation, we split $\Omega$ up into its individual components as mentioned previously. We now introduce the redshift parameter, $z$, which relates the observed wavelength ($\lambda_{\text{obs}}$) to the emitted wavelength ($\lambda_{\text{emitted}}$) from distant astronomical objects. Redshift is then directly
proportional to the distance of the object. The difference in scale factor $a(t)$ between the source and observation points can also be expressed in terms of $z$:

$$z \equiv \frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} - 1 = \frac{a(t_{\text{obs}})}{a(t_{\text{emitted}})} - 1. \quad (1.9)$$

The Friedmann equation can be rewritten in more general form as,

$$\frac{H^2(z)}{H_0^2} = \left[ \Omega_\Lambda + \Omega_K(1 + z)^2 + \Omega_M(1 + z)^3 + \Omega_R(1 + z)^4 \right], \quad (1.10)$$

where $\Omega_M$, $\Omega_R$ and $\Omega_\Lambda$ refer respectively to the present day matter, radiation and dark energy (fractional) densities and sum to $\Omega_0$. $\Omega_K = \frac{-k}{a_0^2 H_0^2}$ contains the curvature sensitive part of the equation.

**Thermal History**

The differing $z$ dependencies in Equation 1.10 for matter, radiation and dark energy provide a method for disentangling their respective contributions to $\Omega$ through astrophysical observations at different redshifts. As is illustrated in Figure 1.1, the history of the Universe divides into three distinct epochs during which a different component of $\Omega$ dominates the evolution of the scale factor [13]:

1. **Radiation Dominated** ($z \gtrsim 3265$): Photons and neutrinos dominated the evolution of the scale factor in the earliest moments following the Big-Bang. During this period, several events occurred:

   - **Neutrino Decoupling**: Neutrinos were in equilibrium until $\sim 0.1$ seconds after the Big Bang, at which time the rate of their interactions with other weakly-interacting matter dropped below the rate at which the scale factor was expanding. The temperature of the Universe at this time was $\sim 3$ MeV [14]. A direct consequence of neutrino decoupling is that the weak processes that maintain thermal equilibrium between protons and neutrons quickly turned off as the decoupled neutrinos continued to cool. Approximately one second later, the neutron-to-proton ratio
became fixed and played a critical role in the next stage.

- **Nucleosynthesis**: Light nuclei began to form once the temperature of the Universe cooled to less than 2.23 MeV, below which the average nucleon energy is less than the deuteron binding energy, allowing the $p + n \rightarrow D + \gamma$ reaction to occur. Due to the large density of the photon-background, it was not until the Universe cooled to 100 keV that the deuteron-dissociating photons fell below the number of nucleons and reaction could produce a stably increasing deuteron density [4]. Most nucleosynthesis occurred roughly 100 seconds after the Big Bang, and the abundances of light nuclei froze out after about a half an hour. These abundances place strong constraints on the baryonic contribution to $\Omega_M$. See Section 1.1.2 for more details.

- **Recombination**: The Universe cooled down to below the electron binding energy of hydrogen (13.6 eV) and neutral hydrogen began to form out of the plasma
of electrons and protons. The rate of photodissociation was superseded by the expansion rate at a temperature of about 0.3 eV, resulting in the recombination of protons and electrons into neutral hydrogen several hundred thousand years after the Big-Bang [4]. Prior to recombination, the Universe was opaque to electromagnetic radiation due to Thomson scattering of the photon background by free electrons. After recombination, the free-electron density dropped significantly, causing an equally dramatic increase in the photon mean free path and resulting in a Universe that is transparent to light. Recombination is often referred to as the surface of last scattering since the photons that emerge from this event traverse the Universe largely unscathed. Due to the expansion of the Universe, the surface of last scattering is seen today as a uniform glow with a characteristic temperature of 2.73 K. This glow is referred to as the Cosmic Microwave Background (CMB). See Section 1.1.2 for further details.

(2) **Matter Dominated** (0.5 $\lesssim z \lesssim$ 3265): Matter, or baryons and DM, dominated the evolution of the scale factor for redshifts between $\sim$3265 and $\sim$0.5. During this time, the Universe expanded more rapidly than during the radiation-dominated era. The density perturbations from the surface of last scattering continued to grow and eventually formed the first stars and galaxies. Between recombination and the formation of the first stars, there is a period referred to as the “dark ages” where the only significant source of light was the CMB radiation. The first celestial objects to form radiated sufficient heat to initiate a period of neutral hydrogen reionization that lasted for hundreds of millions of years ($z_{\text{reion}} = 10.1$) [11]. The expansion of the Universe had diluted the distribution of matter sufficiently enough such that the Universe remained largely transparent to light despite the reionized hydrogen. However, the reionization caused a $\sim$10% opacity that can be seen in the pattern of the CMB fluctuations today. Supernovae that occurred late in the matter dominated era provide some of the most convincing evidence that the expansion of the Universe deviates from Hubble’s law [15], in which the galaxy redshift is linearly proportional to its distance. In fact, the present-day expansion is
accelerating, indicating a need for a non-zero cosmological constant and leading to the following epoch.

(3) **Dark Energy Dominated** (\(z \lesssim 0.5\)): Around \(z \simeq 0.5\), the scale factor transitioned into a dark-energy-dominated era. The expansion rate is accelerating and will eventually yield an \(a \propto e^{Ht}\) behavior. If the Universe continues to expand in this nature, it will expand forever. The evolution of gravitationally bound structures will become increasingly more complicated and nonlinear, while unbound structures will gradually disperse until all of the matter in the Universe is effectively isolated.

### 1.1.2 Observational Evidence

By now it is well established that a significant portion of our Universe is invisible and non-luminous matter that we ignorantly refer to as DM [8, 12]. In Section 1.1.1, I showed that modern cosmology splits up the energy content of the Universe into three main components: matter, radiation and dark energy. The reasons for this are well motivated by experimental observations and in the following sections I will outline several of these. It is important to note that about 95% of the SCM’s energy content is comprised of an unknown or *dark* component (dark energy and dark matter).

#### Galaxies

The earliest indication for the possible presence of DM came from the dynamical study of our Galaxy. In 1922 British astronomer James Jeans [16] analyzed the vertical motions of stars near the plane of our Galaxy. From these data, Jeans calculated the density of matter near the Sun and also estimated the density due all stars near the Galactic plane. The results indicated that there was twice as much matter there than he could see. The second piece of evidence was provided by a Swiss astrophysicist, Fritz Zwicky, in 1933 [17]. He used the virial theorem to show that the observed (luminous) matter was not nearly enough to keep Coma cluster of galaxies together. This led Zwicky to infer that there was more matter in the Coma cluster than could be seen with optical instruments. The missing mass reported by
Zwicky was largely ignored for 40 years until Rubin & Ford [18–20] measured the rotational velocities of edge-on spiral galaxies. To the astonishment of the scientific community, they showed that most stars in spiral galaxies orbit the galactic core at roughly the same speed – this is shown in Figure 1.2. This observation suggested that the mass densities of the galaxies were uniform well out to the largest visible radii. This was consistent with the spiral galaxies being embedded in a much larger halo of invisible mass, a DM halo. The mathematical details are simple, the rotational velocity $v$ of an object on a stable Keplerian orbit with radius $r$ around a galactic core is given by

$$\sqrt{\frac{GM(r)}{r}}$$

(1.11)

where $M(r)$ is the total mass inside an orbit with radius $r$. They found that instead of a $v \propto r^{-1/2}$ relationship, the galactic rotational velocities were constant, or flat, out to large $r$ even beyond the edge of the visible disks, as shown in Figure 1.2. This holds true in most spiral galaxies, and in the case of the Milky Way, the rotational velocity in our solar neighborhood is $\sim 240$ km/s with little change out to the largest observable radius.

**Gravitational Lensing**

Gravitational lensing is a direct consequence of GR, in which the trajectory of a photon is affected by the curvature of space-time induced by a nearby massive object thus causing a lensing effect. In the weak field limit, the refractive index of a gravitational lens is directly proportional to its gravitational field. This allows one to extract a mass-density map from a lensing object, including DM.

Some of the most spectacular results obtained regarding the nature of DM using gravitational lensing are those on the ‘Bullet Cluster’ 1E0657-558 [22] and more recently on the lensing cluster MACS J0025.4-1222 [23]. In both clusters, the reconstruction of the mass distribution show two massive substructures that are offset with respect to the baryon distribution observed in X-rays. Figure 1.3 shows two images of the Bullet Cluster that indicate a large separation between the sub-cluster mass densities inferred by gravitational lensing.
Figure 1.2: Rotation curve of NGC 6503. The dotted, dashed and dash-dotted lines are the contributions of gas, disk and DM, respectively. Figure from Ref. [21].
but a smaller separation and a bow shock between clumps of baryonic gas, inferred by X-ray imaging. The conclusion is that the two clusters’ member galaxies and DM halos passed through one another relatively intact, while their intracluster gas clouds were stripped away by drag forces such that they appear to lag behind. The Bullet Cluster provides compelling evidence for the existence of particulate DM.

Figure 1.3: Left: Optical image of the Bullet Cluster from the Hubble Space Telescope with lensing contours indicating the mass-density distribution. Right: X-ray image of the Bullet cluster with the Chandra X-ray Observatory with lensing contours indicating the mass-density distribution. Figure from Ref. [22].

Big Bang Nucleosynthesis

According to the Big Bang model, the Universe began in an extremely hot and dense state. For about the first 0.1 seconds, it was so hot that atomic nuclei could not form – space was essentially filled with a hot soup of protons, neutrons, electrons and photons. Even if a proton and neutron collided to form a deuterium, the high temperatures caused the nucleus to be immediately broken up by photons [24].

From around times 0.1-10^4 seconds, the Universe cooled off enough so that light nuclei could form:

- $^2$H (Deuterium): p(n, $\gamma$)D;
- $^3$He: D(p, $\gamma$)$^3$He;
- $^4$He: $^3$He(D, p)$^4$He;
- $^7\text{Li}: \ ^3\text{He}(^4\text{He},\gamma)^7\text{Li}$.

This era is referred to the Big Bang Nucleosynthesis (BBN) era$^2$. The amount of baryonic matter in the Universe today is connected to the ratio of baryons to photons during the BBN era,

$$\eta = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim \frac{n_b}{n_\gamma},$$

(1.12)

where $n_b$, $n_{\bar{b}}$, and $n_\gamma$ are the baryon, antibaryon, and photon number density respectively. Models of BBN predict the primordial abundances of light nuclei ($A < 8$). One example being the mass fraction of $^4\text{He}$, given by

$$Y_p = \frac{2(n/p)}{1 + n/p},$$

(1.13)

where $n/p$ is the ratio of the neutron to proton number density, as a function of $\eta$. In summary, BBN refers to a set of highly constrained calculations that predict the abundances of light nuclei ($A < 8$) as a function of a single parameter, the baryon-to-photon ratio ($\eta$); $\eta$ is directly related to the fraction $\Omega$ contained in baryons, $\Omega_b$. According to Ref. [7], $5.1 \times 10^{-10} < \eta < 6.5 \times 10^{-10}$, and

$$\Omega_b = 3.66 \times 10^7 \eta h^{-2} \text{ or } 10^{10} \eta = 274 \Omega_b h^2,$$

(1.14)

where $h$ is the Hubble constant ($H_0$) in units of 100 km s$^{-1}$ Mpc$^{-1}$ [7]. The elemental abundances can then be calculated and plotted as a function of $\eta$, as shown in Figure 1.4. After a detailed analysis, it is possible to extract limits on the baryonic density in the Universe based on BBN,

$$0.019 \leq \Omega_b h^2 \leq 0.024.$$

(1.15)

According to these calculations, baryons cannot close the Universe. Furthermore, since $\Omega_M \sim 0.3$ [7], most of the matter in the Universe is not only DM, but it also is predominantly non-baryonic. These calculations agree quite well with what will be presented in the next section.

$^2$For a detailed review, please read [25, 26]
Figure 1.4: Abundances of $^4\text{He}$ and $^2\text{H}$ (D), $^3\text{He}$ and $^7\text{Li}$ as a function of $\eta$ as predicted by the standard model of BBN. The bands show the 95% C.L. range. Boxes indicate the observed light element abundances (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density (see Section 1.1.2) while the wider range indicates the $1\sigma$ confidence intervals associated with recent measurements (both at 95% C.L.), Figure from Ref. [7].
Cosmic Microwave Background

In 1964 the Cosmic Microwave Background (CMB) radiation was detected [27, 28]. This discovery was a powerful confirmation of the Big Bang theory, that the Universe started in a very hot and dense state and expanded rapidly. Several hundred thousand years after the Big Bang, the Universe reached a critical temperature in which the gas became neutral, causing the baryons and photons to become decoupled. Before this time, baryons and photons were tightly coupled (referred to as the *baryon-photon fluid*). After decoupling, the baryons were free to collapse into potential wells generated by gravitational instabilities (widely thought to be caused by DM [4]). The CMB is in essence a record of the conditions at the time of last scattering (surface of last scattering). The detailed pattern of the matter-density-fluctuation power spectrum anisotropies, shown in Figure 1.5, depends upon all of the cosmological parameters. The CMB and its anisotropies provide standard measures to characterize various cosmological parameters and are a critical component of the precision cosmology of today [29]. The relative amplitude of the peaks of the CMB angular power spectrum, shown in Figure 1.5, constrain the baryon density and non-baryonic DM density. The 9-year WMAP data [11] alone finds that the density of baryons in the Universe ($\Omega_b$) is given by

$$\Omega_b = 0.0463 \pm 0.0024,$$

while the density of physical matter ($\Omega_M$) is given by

$$\Omega_M = 0.279 \pm 0.025.$$

These values tell us that a significant amount of DM is non-baryonic. Furthermore, this non-baryonic DM cannot have coupled strongly to the baryon-photon fluid prior to last scattering or have been moving at relativistic speeds. This implies that DM is moving at non-relativistic speeds (COLD) and non-luminous (DARK) matter, or Cold Dark Matter (CDM).
Summary of Dark Matter Evidence

The evidence presented in this dissertation is not comprehensive; a full discussion on the evidence for DM is beyond the scope of this work. The existence of DM was established via observations of flat rotation curves in spiral galaxies. Additionally, the collisions of galaxy clusters and subsequent observations with gravitational lensing and X-ray imaging have given us the most visually stunning evidence that there is non-luminous and weakly interacting DM in the Universe. Big Bang Nucleosynthesis and CMB observations have shown us that the DM component need not only be weakly interacting, but the majority of it must be in non-baryonic form. In summary the current mass budget of our Universe can be expressed in one simple table, see Table 1.1. This table shows that CDM constitutes $\sim23\%$ of the total mass of the Universe, whereas the matter that we see, feel and touch (baryonic) constitutes only $\sim4.6\%$. 

Figure 1.5: The angular power spectrum of the CMB temperature anisotropies from WMAP-9 [11]. The solid line shows the fitting after incorporating all the cosmological parameters within the $\Lambda$CDM model. Figure from Ref. [7].
Table 1.1: The mass budget of the Universe according to WMAP-9 alone [11].

<table>
<thead>
<tr>
<th>Age of the Universe (Gyr)</th>
<th>$t_0$</th>
<th>13.74 ± 0.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hubble constant (km s$^{-1}$ Mpc$^{-1}$)</td>
<td>$H_0$</td>
<td>70.0 ± 2.2</td>
</tr>
<tr>
<td>Baryon density</td>
<td>$\Omega_b$</td>
<td>0.0463 ± 0.0024</td>
</tr>
<tr>
<td>Cold DM density</td>
<td>$\Omega_{cdm}$</td>
<td>0.233 ± 0.023</td>
</tr>
<tr>
<td>Matter density ($\Omega_b + \Omega_{cdm}$)</td>
<td>$\Omega_M$</td>
<td>0.279 ± 0.025</td>
</tr>
<tr>
<td>Dark Energy density</td>
<td>$\Omega_\Lambda$</td>
<td>0.721 ± 0.025</td>
</tr>
</tbody>
</table>

1.1.3 Dark Matter Candidates

The requirements of a viable DM candidate are listed below:

1. *Does it agree with the relic density?*

2. *Is it cold (non-relativistic)?*

3. *Is it neutral?*

4. *Compatible with BBN and CMB?*

5. *Consistent with direct DM searches?*

There are far too many DM candidates to describe in detail here. I will concentrate on the most popular and well motivated SM and exotic possibilities for which a large number of experimental efforts are now underway. For a detailed list of candidates see [7, 8, 12, 30] and references therein.

**Mirror Matter**

We know that parity is maximally violated in the weak sector [31]. However, if parity is a universally conserved quantity, then there could exist a parallel hidden (mirror) sector of the Universe composed of particles that are identical to SM particles. Mirror DM models [32–36] have proposed that in addition to the SM, a second identical SM exists and is coupled to the known SM particles through a new type of symmetry. The motivation for a second mirror SM is that parity is violated in SM weak interactions. The addition of a second mirror SM would allow for a global conservation of parity. In most mirror matter models, the only
fundamental SM boson identical to its mirror particle would be the graviton. Mirror matter and SM particles would therefore interact gravitationally, but otherwise be inert. Though the idea of a second mirror SM is exotic, mirror matter is a stable self-collisional DM candidate. According to [36], the mass of mirror DM would be < 52 GeV, and hence a viable candidate for low-mass DM.

**Axions**

Axions were first postulated to solve the strong CP problem of QCD [37–39] and are expected to be extremely weakly interacting, meaning that they were not in thermal equilibrium in the early Universe [8]. Since they would be produced athermally in the early Universe, they would be simultaneously light and cold. Although weakly interacting, axions are thought to be detectable through conversion to photons in a strong magnetic field [40]. Typical searches for axionic DM utilize resonant cavities to enhance the expected conversion rate and look for axion masses in the range of $\sim$1-100 $\mu$eV [41, 42]. For more information on axions, see Ref. [7, 12] and references therein.

**Weakly Interacting Massive Particles (WIMPs)**

The most studied class of DM candidates are a class of particles called WIMPs. WIMPs are well motivated by both cosmology and particle physics and have masses between 1–1000 GeV. WIMP particles, $\chi$, are hypothesized to have been created during the early Universe, < 1 ns after the Big Bang, and have been in equilibrium with other particles (e.g. photons and neutrinos). As the temperature of the Universe dropped below the mass of the WIMP ($T < M_W$), WIMPs fell out of equilibrium and production ceased, while annihilation continued. During this time period WIMPs cooled to non-relativistic speeds and the expansion of space-time diluted their numbers. Annihilation continued until its rate fell below the expansion rate of the Universe thereby ‘freezing-out’ the number density of WIMPs. For $M_W \sim$ 10 GeV, the present relic density is then approximately given by

$$\Omega_{\chi} h^2 \simeq \frac{0.1 \text{ pb}}{\langle \sigma_A v \rangle}$$

(1.18)
where $\sigma_A$ is the total annihilation cross section of the WIMPs, $v$ is the relative velocity of the WIMPs and $\langle \ldots \rangle$ denotes thermal averaging [7]. For $\Omega_\chi \sim 0.2$ and $h \sim 0.7$, we learn from the above equation that the annihilation cross section is characteristic of weak-scale interactions ($\sigma_{EW} \sim 10^{-2}$ pb). This is because the order of magnitude of the matrix element governing the annihilation process matches the weak force coupling, $G_F$. This coincidence, which is not tuned, represents one of the motivating factors for believing that WIMPs could provide the dominant contribution to the matter in the Universe [43].

There are many theories which extend the SM and naturally contain a WIMP. Perhaps one of the most favored theories is supersymmetry (SUSY) in which each elementary particle has a supersymmetric partner (for a more detailed overview of SUSY see e.g. [30, 44–49]). SUSY helps stabilize the masses of scalar particles, such as the Higgs boson, with the addition of particles with masses in the same range pointed to with cosmological arguments above.

It is possible to extend the SM with SUSY in any number of ways, however the most promising models contain the smallest possible number of fields necessary to replicate what we have experimentally measured for each SM particle. The minimal supersymmetric extension to the SM, or MSSM [46], is the most economical extension to the SM that satisfies the above constraints. The MSSM contains three neutral, colorless particles which are plausible candidates for WIMP DM: the gravitino, the lightest sneutrino and the lightest neutralino. DM could consist of any combination of these three particles, however present-day experiments are not sensitive to gravitinos, and likely never will be. Additionally, sneutrinos have been essentially excluded by the null results of direct DM searches [7, 12, 43]. The lightest neutralino is the most commonly considered SUSY DM candidate and is the focus of this dissertation. Experiments are needed, such as the work presented in this dissertation, to test the WIMP hypothesis and to determine which (if any) flavor of WIMP makes up the Universe’s missing mass.
1.2 Neutrinos

The MAJORANA experiment’s primary goal is to search for Neutrinoless Double-Beta Decay ($0\nu\beta\beta$) of $^{76}$Ge. Since the work presented in this dissertation is a subsidiary project of the MAJORANA experiment, a brief discussion of neutrinos is needed. In this section we will switch gears and discuss neutrinos in the context of the MAJORANA experiment.

1.2.1 A brief history

The observation of a continuous energy spectrum of electrons emitted in $\beta$-decay came as a surprise to researchers in 1927 [50]. They had expected to see a single, discrete, peak at a fixed energy. Due to conservation of energy and momentum, this would be exactly what we expect if $\beta$-decay proceeded as a two-body decay

$$M(A,Z) \rightarrow D(A,Z+1) + e^-, \quad (1.19)$$

where $M(A,Z)$ is the mother nucleus with mass number $A$ and atomic number $Z$, $D(A,Z+1)$ is the daughter nucleus and $e^-$ is the electron. In order to solve this problem, in 1930, Wolfgang Pauli proposed the existence of a light, neutral spin-$\frac{1}{2}$ particle ($\nu$)\(^3\) that carried away the missing energy resulting in a continuous spectrum,

$$M(A,Z) \rightarrow D(A,Z+1) + e^- + \nu, \quad (1.20)$$

This particle was given the name neutrino, which in Italian means ‘neutral one’. In 1934, Enrico Fermi developed the theory of $\beta$-decay [51] which incorporated neutrinos and agreed with experimental data extremely well. However, the neutrino proved to be elusive and we would have to wait until 1956 [52] before the first experimental evidence of the neutrino.

\(^3\)We would later learn that there are in fact at least three flavors or types of neutrinos, $\nu_e$, $\nu_\mu$ and $\nu_\tau$ with each having a corresponding anti-particle $\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. 

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1.2.2 Neutrinos in the Standard Model

The building blocks of the standard model (SM) of particle physics consist of six quarks and six leptons, all of them being spin-$\frac{1}{2}$ fermions. They interact with each other via the four fundamental forces: gravity, electromagnetic, strong and weak. The neutrino is one of the six leptons in Table 1.2. Neutrinos are the only SM fermions to only interact via gravity and the weak force. The weak interaction proceeds with the exchange of two types of bosons,

(1) $Z^0$ and

(2) $W^\pm$.

An interaction which exchanges a $Z^0$ is called a neutral current (NC) interaction. Similarly, an interaction which exchanges a $W^\pm$ is called a charged current (CC) interaction. In CC interactions, charge conservation requires that the a charged lepton exits the interaction. The neutrino always emits the $W^+$ and the anti-neutrino always emits the $W^-$. Within the context of the SM, neutrinos are massless and there are three (at least) neutrino types or flavors $\nu_e$, $\nu_\mu$ and $\nu_\tau$. These three neutrinos are named accordingly because they interact via weak interactions with electrons, muons and taus. The ‘flavor’ $(e, \mu, \tau)$ of these neutrinos\textsuperscript{4} is defined by the flavor of lepton emitted and so these three neutrinos constitute a basis of flavor eigenstates. There is also a basis of mass eigenstates $(1, 2, 3)$. The flavor and mass eigenstates are connected by a unitary mixing matrix, $U_{\alpha i}$

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle, \quad (1.21)$$

\textsuperscript{4}...and anti-neutrinos if in fact they are distinct from their anti-particles.
where for the case of 3 neutrinos, \( \alpha \) (flavor eigenstate) = \( \epsilon, \mu, \tau \); \( i \) (mass eigenstate) = 1, 2, 3; \( U_{\alpha i} \) is given by

\[
U_{\alpha i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}
\] (1.22)

where \( s_{ij} = \sin(\theta_{ij}) \) and \( c_{ij} = \cos(\theta_{ij}) \). The current best values for these parameters (\( \Delta m_{ij}, \theta_{ij} \)) are listed in Table 1.3, where \( \Delta m_{ij}^2 \equiv (m_i^2 - m_j^2), \ i \neq j \). Neutrino oscillation experiments are only sensitive to the mass squared differences, \( \Delta m_{ij}^2 \), but not the actual mass. The magnitudes of both mass square differences (\( \Delta m_{21}^2, \Delta m_{32}^2 \)) have been measured, but the sign of \( \Delta m_{32}^2 \) has not been measured. This leads to the neutrino-mass hierarchy problem. To simplify this, we do not know which mass eigenstate is heaviest, see Figure 1.6. If \( \Delta m_{32}^2 > 0 \), then \( m_3 > m_2 \) and that would indicate a ‘normal’ hierarchy. The term ‘normal’ alludes to the fact that this is the hierarchy observed in the quark sector. However, if \( \Delta m_{32}^2 < 0 \), then \( m_3 < m_2 \) and this is the ‘inverted’ hierarchy. In addition to the mass hierarchy problem, we do not have a direct measurement of the neutrino mass. The most stringent limits on the \( \bar{\nu}_e \) mass were obtained by Troitsk [53] and Mainz [54] with \(^3\)H \( \beta \)-decay:

\[
m_{\bar{\nu}_e} < 2.3 \text{ eV}
\] (1.23)

The KATRIN experiment [54] hopes to reach a sensitivity of \( m_{\bar{\nu}_e} \sim 0.20 \text{ eV} \). This mass is sometimes referred to as \( m_\beta \), which is given by \( m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2} \).

To fully understand the nature of neutrinos, we need to first understand the basics of helicity and chirality. For a spin-\( \frac{1}{2} \) fermion, helicity is the projection of a particles spin along its direction of motion. Helicity has two possible states for spin-\( \frac{1}{2} \) particles: (1) spin aligned opposite the direction of motion and (2) spin aligned along the direction of motion. If a particle is massive, then the sign of the helicity becomes frame dependent, meaning that it

---

5We assume \( \Delta m_{32}^2 \sim \Delta m_{31}^2 \).

6[7] recommends < 2.0 eV
Table 1.2: Summary of the properties of the leptons; $L_i$, flavor-related lepton number, $L = \sum_{i=e,\mu,\tau} L_i$. Table from [55].

<table>
<thead>
<tr>
<th>Lepton</th>
<th>$Q[e]$</th>
<th>$L_e$</th>
<th>$L_\mu$</th>
<th>$L_\tau$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^-$</td>
<td>$-1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>$-1$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\tau^-$</td>
<td>$-1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1.3: Neutrino mixing matrix parameters, from Ref. [7]. The limit quoted for $\sin^2(2\theta_{23})$ corresponds to the projection onto the $\sin^2(2\theta_{23})$ axis of the 90% C.L. contour in the $\sin^2(2\theta_{23})$-$\Delta m_{32}^2$ plane. The sign of $\Delta m_{32}^2$ is not known at this time. $\sin^2(2\theta_{13})$ was recently measured by the Daya Bay Collaboration [56]. We assume $\Delta m_{32}^2 \sim \Delta m_{31}^2$.

$\sin^2(2\theta_{12})$: $0.861^{+0.026}_{-0.022}$
$\Delta m_{21}^2$: $(7.59 \pm 0.21) \times 10^{-5} \text{ eV}^2$
$\sin^2(2\theta_{23})$: $> 0.92$
$|\Delta m_{32}^2|^2$: $(2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2$
$\sin^2(2\theta_{13})$: $0.092 \pm 0.016 \text{ (stat)} \pm 0.005 \text{ (syst)}$

Figure 1.6: Neutrino mass hierarchies for $\nu_1, \nu_2, \nu_3$. $m_1$ is less than $m_2$ based on solar neutrino observations, however $m_2$ may be less than $m_3$, which leads to the normal hierarchy, or $m_2$ may be greater than $m_3$ leading to the inverted hierarchy. Normal hierarchy is pictured on the left and inverted on the right. The absolute scaling is not known.
is not Lorentz invariant. For example, we could potentially boost to a frame where we are moving faster than the particle of interest and the sign of the momentum would change (spin stays the same) and the helicity would flip. For massless particles (neutrinos in the SM) boosting into such a frame is impossible and we could never change the helicity.

The terms helicity and chirality are often used interchangeably, however there is a difference. Unlike helicity, chirality is frame independent (Lorentz invariant). Chirality is the analogue to helicity for both massive and massless neutrinos. There are two states of chirality: left handed (LH) and right handed (RH). For the case of massless particles, helicity and handedness (chirality) are identical. A massless fermion is either purely LH or RH, and, in principle can appear in either state. Massive particles have both LH and RH components. A helicity eigenstate for a massive particle is then a superposition of states. We know from experiment that neutrinos are LH and anti-neutrinos are RH [57]. This indicates that the weak interaction maximally violates parity.

Following the formalism in [58], in order to enforce parity violation, we consider a fermion wavefunction, \( \psi \), split up into its LH and RH components:

\[
\psi = \psi_L + \psi_R.
\]  

(1.24)

We still need an operator to select out each component, this operator is given by:

\[
\gamma^5 \psi_{L,R} = \mp \psi_{L,R}.
\]  

(1.25)

Requiring a factor of \( (1 - \gamma^5) / 2 \) at every weak vertex involving a neutrino, we can enforce the correct handedness. This LH projection operator is the reason the charged weak interaction \( (W^\pm) \) exchange) is called left-handed.

In principle, RH neutrinos and LH anti-neutrinos could exist, but are never detected because they never interact through any of the fundamental forces. These neutrinos are aptly named sterile neutrinos. The SM does not contain any RH neutrinos (sterile neutrinos).
Since there is no RH partner in the SM, the neutrino can have no Dirac mass term in the Lagrangian. To see this, note that the free-particle Lagrangian for a massive, spin-$\frac{1}{2}$ particle is,

$$L = i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi - m\bar{\psi}\psi,$$  \hspace{1cm} (1.26)

where, $\bar{\psi}\psi$ can be written as

$$\psi_{L,R} = \frac{1}{2}(1 \mp \gamma^5)\psi$$  \hspace{1cm} (1.27)

and

$$\bar{\psi}_{L,R} = \frac{1}{2}\bar{\psi}(1 \mp \gamma^5)$$  \hspace{1cm} (1.28)

thus giving

$$\bar{\psi}\psi = \bar{\psi}\left[\frac{1 + \gamma^5}{2} + \frac{1 - \gamma^5}{2}\right]\left[\frac{1 + \gamma^5}{2} + \frac{1 - \gamma^5}{2}\right]\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$$  \hspace{1cm} (1.29)

Substituting Equation 1.29 into Equation 1.26 tells us that the $m\bar{\psi}\psi$ term (the mass term) mixes RH and LH states of the fermion. However, if the fermions have only one handedness (like neutrinos) then the Dirac mass term will automatically vanish. In the SM, there is no Dirac mass term for neutrinos.

### 1.2.3 Neutrinos beyond the Standard Model

By now it is well-established that neutrinos have mass and oscillate between flavor eigenstates [59–61]. These results have given us the first indication that the SM of nuclear physics is in need of revision. No matter how we revise the SM, we need to incorporate the fact that neutrinos have mass and that they oscillate between flavors. The most obvious way in which to do so is to require neutrinos to appear in the Lagrangian the same way as for the charged fermions – via a Dirac mass term. The ways in which a Dirac term is motivated are troubling and theorists have turned to other explanations for neutrino masses. One solution would be that neutrinos are their own anti-particles, *i.e.* Majorana particles. If neutrinos are Dirac particles then $\nu \neq \bar{\nu}$. The particle, $\nu$, has lepton number +1 and the anti-particle, $\bar{\nu}$, has lepton number −1. Lepton number is then conserved in an interaction with Dirac neutrinos.
However, for Majorana particles, the $\nu$ and $\bar{\nu}$ are simply two different helicity states of the same Majorana particle, which we call $\nu^{maj}$. This model can explain all of the data without invoking lepton number, however this sets neutrinos apart from all other SM fermions in being Majorana particles.

Saying that the neutrino is its own anti-particle is equivalent to saying that the neutrino is its own charge conjugate, $\psi^c = \psi$. The operators which appear in the Lagrangian for the neutrino in this case are the set $(\psi_L, \psi_R, \psi^c_L, \psi^c_R)$ and $(\bar{\psi}_L, \bar{\psi}_R, \bar{\psi}^c_L, \bar{\psi}^c_R)$. These terms can combine to give terms of the form $m\bar{\psi}_L \psi_R + \ldots$, which are Dirac mass terms. However, we also get terms of the form,

$$\frac{M_L}{2} (\bar{\psi}^c_L \psi_L) + \frac{M_R}{2} (\bar{\psi}^c_R \psi_R) + \ldots$$

which are the Majorana mass terms, and mix the pair of charge-conjugate states of the fermion. If the particle is not its own charge conjugate, then these terms vanish and we are left with only Dirac mass terms. Dirac particles have no Majorana terms, but Majorana particles will have Dirac mass terms.

These mass terms of the Lagrangian can be written in matrix form,

$$\frac{1}{2} (\bar{\psi}^c \psi) \begin{pmatrix} M_L & m \\ m & M_R \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi^c_R \end{pmatrix} + h.c.$$ 

with the Dirac mass terms on the off-diagonal elements, while Majorana mass constants $M_L$ and $M_R$ are on the diagonal. To obtain physical masses, one diagonalizes the matrix. One can now invoke a model that can motivate the smallness of the observed neutrino masses. The most popular of these models are the see-saw models, see [62] for a detailed discussion on see-saw models for neutrino mass.

### 1.2.4 The Nature of the Neutrino: Dirac or Majorana?

Imagine we have a source of only LH neutrinos from $\pi^+$ decays. These neutrinos may either be either Dirac or LH Majorana neutrinos. Let’s assume that we can flip the helicity of the
neutrinos produced. If the helicity-flipped neutrinos are Majorana particles and they would behave like anti-neutrinos when they interact. On the other hand, if the helicity-flipped neutrinos were Dirac, then only sterile RH neutrinos would be emitted. They do not interact at all. In an ideal world, we would be able to build the experiment described above, however it is currently impossible. Instead, we are pursuing another route: neutrinoless double-\(\beta\) decay \((0\nu\beta\beta)\), which is a beyond the standard model analogue to single \(\beta\)-decay.

**Neutrinoless double-beta decay**

Single-\(\beta\) decay is energetically forbidden for many even-even nuclei. However, a second order process that changes a nucleus atomic number, \(Z\), by two units is possible. In this process, two electrons are emitted along with two anti-neutrinos. This process is called two-neutrino double-\(\beta\) decay \((2\nu\beta\beta)\). A more interesting process is neutrinoless double-beta decay \((0\nu\beta\beta)\), which as the name states emits zero anti-neutrinos,

\[
M(A,Z) \rightarrow D(A,Z+2) + 2e^-.
\] (1.32)

This equation shows us that this process violates lepton number, but there is nothing in the SM that says lepton number must be conserved. This process can be visualized as an exchange of a virtual neutrino between two neutrons within a nucleus, see Figure 1.7. In the SM of weak interactions, the first neutron will emit a \emph{right}-handed anti-neutrino. But, the second neutron requires the absorption of a \emph{left}-handed neutrino. Several things need to happen for this to occur, (1) the neutrino must have mass (and we know it does) so that it is not in a pure helicity state and (2) the neutrino and anti-neutrino would have to be indistinguishable, hence a Majorana particle. Neutrinoless double-beta decay is the only viable way to show that neutrinos are either Majorana or Dirac. The discovery of this decay would signal that the neutrino is a Majorana fermion (its own anti-particle) and that lepton number is violated, having significant implications for our understanding of the nature of the neutrino and fundamental interactions. Also, since the decay rate (Equation 1.33) is

\footnote{For a recent review of \(0\nu\beta\beta\) theory see \[63\] and references therein.}
proportional to the effective Majorana mass (Equation 1.34) of the electron neutrino, the scale and hierarchy of the masses could potentially be set.

\[
(T_{1/2}^{0\nu})^{-1} = G^{0\nu} |\langle m_{\beta\beta} \rangle|^2 \left| M_F^{0\nu} - \left( \frac{g_A}{g_V} \right)^2 M_{GT}^{0\nu} \right|^2
\]

(1.33)

\[
\langle m_{\beta\beta} \rangle = \left| \sum_i U_{ei}^2 m_i \right| = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_2 e^{i\phi_3} \right|
\]

(1.34)

In Equation 1.33 \(G^{0\nu}\) is the two-body phase-space factor including coupling constant, \(M_F^{0\nu}\)

\[\begin{align*}
\text{Figure 1.7: The Feynman diagrams for } &0\nu\beta\beta\text{ decay (left) and }2\nu\beta\beta\text{ decay (right). }0\nu\beta\beta\text{ occurs} \\
&\text{with the exchange of a virtual Majorana neutrino, which is only possible if the neutrino is its} \\
&\text{own anti-particle. Figure from Ref. [64].}
\end{align*}\]

and \(M_{GT}^{0\nu}\) are the Fermi and Gamow-Teller matrix elements, respectively. The constants \(g_A\) and \(g_V\) are the axial-vector and vector relative weak coupling constants, respectively. The quantity \(\langle m_{\beta\beta} \rangle\) is the effective Majorana mass, which is given in Equation 1.34 where \(e^{i\phi_i}\) are the unknown Majorana phases. The term \(\langle m_{\beta\beta} \rangle\) contains all of the interesting physics of \(0\nu\beta\beta\) and is equivalent to zero if the neutrino is a Dirac particle. However, the presence of these unknown phases adds a bit of uncertainty to the determination of \(\langle m_{\beta\beta} \rangle\) since terms may cancel. Furthermore, the nuclear matrix elements, \(M_F^{0\nu}\) and \(M_{GT}^{0\nu}\), must be calculated and the current literature values have a spread of a roughly a factor of two [7]. Finally, there may be other processes that may contribute to \(0\nu\beta\beta\). However, if \(0\nu\beta\beta\) is observed,
independent measurements with several isotopes would allow for the extraction of \(\langle m_{\beta\beta} \rangle\) [65].

**Experimental Aspects of 0\(\nu\beta\beta\)**

The signal for 0\(\nu\beta\beta\) would be a peak in the kinetic energy spectrum corresponding to the two electrons. This peak is located at the endpoint energy as determined by the mass differences of the parent and daughter nuclei (the Q-value), see Figure 1.8. This is sharp contrast to the 2\(\nu\beta\beta\) energy spectrum, which would be a continuous spectrum since the energy of the decay is shared between four particles, the two neutrinos and two electrons. Experimental searches for 0\(\nu\beta\beta\) have been carried out using several nuclei, including but not limited to, \(^{48}\text{Ca}, {^{76}}\text{Ge}, \) \(^{82}\text{Se}, {^{96}}\text{Zr}, {^{100}}\text{Mo}, {^{116}}\text{Cd}, {^{128}}\text{Te}, {^{130}}\text{Te}, {^{136}}\text{Xe}\) and \(^{150}\text{Nd}\). In addition, a subset of the Heidelberg-Moscow collaboration claim to have measured \(T_{1/2}^{0\nu} = (1.19^{+2.99}_{-0.50}) \times 10^{25}\) years and \(\langle m_{\beta\beta} \rangle = 0.22 - 0.35\) eV in \(^{76}\text{Ge}\) [66]. Since there is such a large uncertainty in the nuclear matrix elements, convincing evidence of the Majorana nature of the neutrino will require several experiments to have observed 0\(\nu\beta\beta\). A number of experiments are coming online in the near future which can either confirm or refute the claim of an observation made by [66]. One of which is the MAJORANA Experiment [67–69], which will be discussed briefly in the next section and in more detail in Chapter 3.

1.3 The MAJORANA Experiment

The MAJORANA experiment [68–71] aims to detect 0\(\nu\beta\beta\) (see Section 1.2.4) in a phased approach with the ultimate goal of performing this search with a tonne\(^8\) of \(^{76}\text{Ge}\) using high purity germanium (HPGe) detectors\(^9\).

Recent direct search CDM experiments have hinted towards a low-mass WIMP [73–77]. The technology chosen for detection of 0\(\nu\beta\beta\) also makes MAJORANA an excellent tool for direct CDM measurements (low-mass WIMPs). The discussions outlined in Section 1.1 and Section 1.2 provide the physics background and basis for MAJORANA. The MAJORANA ex-

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*\(^8\)The tonne-scale experiment will be a joint effort with the European counterpart to MAJORANA, the GERDA collaboration [72].

*\(^9\)These detectors will be discussed in Chapter 2.
Figure 1.8: The kinetic energy of electrons from $2\nu\beta\beta$ and $0\nu\beta\beta$ decays is shown here for the $^{76}$Ge isotope ($Q = 2039$ keV). The hypothetical detector was assumed to have an energy resolution of 6 keV FWHM at 2039 keV. Note the two decays are not to scale in this plot.

1.4 Outline of this Dissertation

After alluding to the fact that MAJORANA will use HPGe detectors, Chapter 2 will discuss these detectors in detail, focusing on the fundamental mechanisms which allow them to function as gamma-ray detectors. Chapter 3 will then go into more detail on the MAJORANA experiment and discuss the types of HPGe detectors MAJORANA will use and why they were chosen. As a part of the research and development efforts for the MAJORANA experiment, we have deployed a custom low-background HPGe detector in an underground environment with the hopes to understand the low-energy backgrounds and also perform a direct DM search. Chapters 4 and 5 will focus on the characteristics of this detector and the analysis tools developed. Chapter 6 will then discuss the backgrounds associated with events originating near the surface of this detector. The DM results obtained with this detector are the primary
focus of this dissertation and will be presented in Chapter 7. \textsuperscript{10}

\textsuperscript{10}I would like to give two recommendations: (1) A great neutrino physics book can be found in [55], and (2) A comprehensive particle DM book can be found in Ref [12].
Chapter 2

Germanium Detectors

In the previous chapter, we briefly discussed how the MAJORANA experiment will use High Purity Germanium (HPGe) detectors to search for neutrinoless double-beta decay ($0\nu\beta\beta$) and weakly interacting massive particles (WIMPs). These detectors are widely used to detect gamma- and X-ray radiation. They are well known for their superior energy resolution and commercial availability. This chapter will serve as an introduction to HPGe detectors.

2.1 Basics of Semiconductor Gamma-Ray Detectors

Since the results from the direct dark matter search discussed in Chapter 7 were obtained with a semiconductor gamma-ray detector, we must first discuss how these detectors work. The three main gamma-matter interactions: photoelectric absorption, pair production and Compton scattering all create energetic electrons. In this section I will discuss how semiconductors can be used to detect these electrons.

2.1.1 Introduction

In solids there are essentially two energy bands for electrons to populate: a conduction band and a valence band, see Figure 2.1. Between these bands are energy regions that are forbidden to electrons. In insulators, the entire valence band is full and the energy required to jump to the conduction band (\(~10\) eV) is much higher than the typical thermal excitation energy. This results in insulators having no conductivity since electrons can not jump to the conduction
band. In conductive metals, the conduction band is partially filled allowing electrons to move freely throughout the bulk of the solid. Semiconductor materials are fundamentally different than conductive metals and insulators. In semiconductors, the valence band is completely full, however the energy required to jump to the conduction band (∼1 eV) is comparable to the thermal excitation energy. Figure 2.1 illustrates these band gaps for insulators and semiconductors. The probability that an electron will be promoted to the conduction band is strongly dependent on the temperature of the solid,

\[ p(T) \propto T^{3/2} \exp(-E_g/2kT), \]  

(2.1)

where \( T \) is the temperature, \( k \) is the Boltzmann constant and \( E_g \) is the band gap energy or the energy required to jump from the valence band to the conduction band [78–82].

We have seen that gamma-ray interactions within an absorbing material (a detector) liberates energetic electrons. The amount of electrons liberated is directly proportional to the gamma-ray energy. Since these electrons are able to jump to the conduction band in semiconductors, the application of an electric field provides a relatively simple way to collect and measure these liberated electrons. According to Equation 2.1, cooling the material will reduce the number of electrons in the conduction band, thereby reducing the background current (the leakage current) making it much easier to detect the extra excitation due to gamma-ray interactions [78–82]. This is the basis of semiconductor gamma-ray detectors. A more detailed description of semiconductor gamma-ray spectrometers is given in the following sections.

### 2.1.2 Electron and Hole Mobility

Whenever an electron jumps to the conduction band from the valence band, it leaves behind a vacancy in the otherwise full valence band. This vacancy is positively charged and is referred to as a hole. Holes and electrons are mobile in semiconductors. The vacancy left behind from the initial electron jump can be filled by another electron from deeper within the valence band. This will then leave another vacancy. Under the influence of an electric field, the hole
Figure 2.1: The band structure for insulators, semiconductors and conductors are shown. In insulators and semiconductors, the two bands are separated by the band gap energy, $E_g$. The size of $E_g$ determines whether a material is an insulator or semiconductor. In the absence of thermal excitation, electrons in the valence band cannot jump to the conduction band and the material will have no conductivity. Also, in conductors, the conduction and valence bands overlap, allowing for the conduction of electrons.
can appear to migrate towards the cathode. Since both electrons and holes carry charge, both contribute to the conductivity of the material [78, 83].

2.1.3 Charge Carrier Creation

The electrons produced from gamma-ray or neutron interactions within a semiconductor will have energies greater than thermal energies. This means that electrons from deep occupied bands can be raised to the conduction band. These deeply embedded holes and excited electrons will redistribute themselves within the available energy bands until the holes are at the top of the valence band and the electrons are at the base of the conduction band. During this redistribution process, further excitations can occur giving a cascade of electron-hole pairs for each primary electron interaction. In the presence of an electric field, both charge carriers will be migrated towards the anode (electrons) and cathode (holes). The number of electron-hole pairs created, \( n \), is directly proportional to the gamma-ray energy that is absorbed \( (E_{\text{abs}}) \),

\[
n = \frac{E_{\text{abs}}}{\langle \epsilon \rangle},
\]  

(2.2)

where \( \langle \epsilon \rangle \) is the average energy required to create an electron-hole pair. The energy required to generate an electron-hole pair depends upon which particular energy level the electron is promoted to within the conduction band. Therefore, there is an uncertainty in \( n \), that will lead to a distribution of signal amplitudes for a given gamma-ray energy. Materials with small \( \langle \epsilon \rangle \) will have superior energy resolution since more charge carriers will be created [78–82]. Additionally, the number of electron-hole pairs (Equation 2.2) is an average and this must be taken into account (see Section 2.2.6).

Furthermore, the charge carriers must be collected in a reasonable amount of time in order for a semiconductor to be practical. This translates into the requirement that the electrons and holes must have good mobility within the bulk in order for them to be able to reach the collecting electrodes. The liberated charge carriers can become trapped en route to the electrode by any of the following mechanisms: (1) impurities within the semiconductor lattice, (2) structural defects, and (3) radiation damage [84].
Quenching

Gamma-ray and neutron/WIMP interactions within an ionization detector are fundamentally different — neutrons/WIMPs recoil off of nuclei, while photons interact with the electrons in the detector. Since semiconductor gamma-ray detectors are typically calibrated with standard gamma-ray check sources, this calibration will not be valid for nuclear recoils. There exists a relationship, both measured experimentally [85] and theoretically motivated [86], between the observed ionization from a nuclear recoil and the observed ionization from an electron recoil (at the same energy); this is called quenching. Quenching will be discussed further in Chapter 7.

2.1.4 The Nature of Semiconductors

In an ideal semiconductor detector, the material would have no impurities and thermal excitation would promote a certain number of electrons to the conduction band, leaving behind an equal number of holes in the valence band. A material of this type is called an intrinsic semiconductor. It is difficult to fabricate intrinsic semiconductors, and the presence of impurities has a significant impact on the conductivity [80–82, 84, 87].

Take for example a germanium atom; it has four valence electrons and in a lattice will be connected to four other atoms, ideally four germanium atoms. If there are impurities present, and one of the four germanium atoms is replaced by an impurity atom with a different number of valence electrons, this will disturb the electronic balance of the crystal lattice. For example, gallium and boron have three valence electrons; if one of these atoms replaced a germanium atom then there would be one electron too few to maintain the overall electronic configuration of the lattice. We essentially have a hole in the crystal lattice. These impurities are referred to as acceptor impurities and when distributed throughout the lattice will give rise to extra energy states just above the valence band, called acceptor states. If we had germanium in this state, the material would be called p-type germanium (where ‘p’ stands for positive acceptor impurities) [80–82, 84, 87].

We could also have an impurity with extra valence electrons. The impurity atom is then
referred to as a donor atom and will introduce donor states just below the conduction band. Germanium in this state is called **n-type** germanium (where ‘n’ stands for negative donor impurities).

It is also possible to have an equal number of donor and acceptor atoms within the bulk. Each of these impurities will effectively cancel out and in this event we end up with what is called **compensated** germanium. However, it is very unlikely that the impurities will exactly cancel out.

The introduction of impurity atoms introduces extra states either just below the conduction band or just above the valence band. This, in effect, reduces the band gap energy. The doping of semiconductor materials by introducing impurities then directly affects the conductivity of the material [80–82, 84, 87].

Semiconductor gamma-ray spectrometers depend on the redistribution of charge when p-type and n-type germanium are produced in contact with one another. The p-type material has an excess of holes while the n-type material has an excess of electrons. The thermal diffusion of holes toward the n-type side and electrons toward the p-type side creates a region in the middle where excess charge carriers have effectively cancelled out as shown in Figure 2.2. This is called a **depletion** region, see e.g. Figure 2.2. The migration of the charge carriers gives rise to a space charge distribution, or a region in which the excess nuclear charge from the doping atoms is not neutralized by the movable carriers, in this region and the generation of a voltage across the junction called the **contact** or **diffusion voltage**, typically 0.4 V in germanium.

![Figure 2.2: A p-n diode junction in thermal equilibrium, with its parts separated (a) and brought together (b). Electrons are denoted by black circles and holes by open circles. Figure from Ref. [82].](image)

The depletion region is the active region, or region sensitive to electron or nuclear recoils, of
the detector. This region is extremely thin, but if a positive voltage is applied to the n-type side, the width of the depletion region grows as the electrons are removed from the material. Similarly, a negative voltage applied to the p-type side of the junction will withdraw holes. Since the positive voltage is applied to the n-type semiconductor, this is called a reverse bias junction. The width of the depletion region can be estimated as,

$$d \approx (2\epsilon\mu\rho(V_0 + V_b))^{1/2},$$

(2.3)

where $V_0$ and $V_b$ are the contact and bias voltages, respectively, $\epsilon$ is the dielectric constant ($\epsilon = 16\epsilon_0$ in Ge), $\rho$ is the resistivity of the material and $\mu$ is the mobility, or how easily charge carriers are able to move throughout the crystal lattice, of the majority charge carriers in the material [80–82, 84, 87]. In a p-type, holes are the charge carriers while in n-type, the electrons are the majority charge carriers. The resistivity of the material is given by,

$$\rho = \frac{1}{eN\mu},$$

(2.4)

where $N$ is the concentration of dopant atoms in the material [80–82, 84, 87]. In gamma-ray spectrometry, we want the depletion depth to encompass as much of the bulk material as possible and at the lowest bias voltage. According to Equations 2.3 and 2.4, this is achieved by minimizing $N$. In order to make a large detector, we then need very pure material in order to be able to keep the bias voltage in a reasonable range ($< 5000$ V). Germanium used to fabricate HPGe detectors is the purest material produced in bulk throughout the world (with only $\sim 10^{10}$ impurity atoms per cm$^3$) [80–82].

In the real world, the fabrication of semiconductor detectors is not achieved by placing two types of semiconductor materials in contact with each other. One end of a block of germanium is converted to the opposite type by evaporation and diffusion (or by ion implantation). In order to make a p-type HPGe detector, we would take a suitably high-purity chunk of p-type germanium and create on one face an n$^+$ layer (usually lithium), then apply a reverse bias to the detector to create the depletion layer throughout the p-type material. This is the basis
Table 2.1: Properties of materials suitable for semiconductor detectors. RT stands for room temperature (300 K). \(\langle \epsilon \rangle\) is given at the operating temperature. Table from [84].

<table>
<thead>
<tr>
<th>Material</th>
<th>(Z)</th>
<th>Operating (T)</th>
<th>(E_g) (eV)</th>
<th>(\langle \epsilon \rangle) (eV)</th>
<th>(\rho) (g cm(^{-3}))</th>
<th>Mobility (cm(^2) V(^{-1}) s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>14</td>
<td>RT</td>
<td>1.106</td>
<td>3.62</td>
<td>2.33</td>
<td>1350 (\text{Electrons}) 480 (\text{Holes})</td>
</tr>
<tr>
<td>Ge</td>
<td>32</td>
<td>Liquid N(_2) (77 K)</td>
<td>0.67</td>
<td>2.96</td>
<td>5.32</td>
<td>(3.6\times10^4) (\text{Electrons}) (4.2\times10^4) (\text{Holes})</td>
</tr>
<tr>
<td>CdZnTe</td>
<td>48,30,52</td>
<td>RT</td>
<td>1.57</td>
<td>4.64</td>
<td>5.78</td>
<td>1000 (\text{Electrons}) 50-80 (\text{Holes})</td>
</tr>
</tbody>
</table>

for all HPGe detector manufacturing [84, 87].

2.1.5 Practical Semiconductor Materials

A list of desirable characteristics for a practical gamma-ray spectrometer include:

- As large atomic number, \(Z\), as is possible in order to be able to stop incoming gamma-rays with the hopes of full absorption;

- As small an \(\langle \epsilon \rangle\) as possible in order to maximize charge carriers, \(n\), created (See Equation 2.2);

- Good electron and hole mobility;

- Be available in large mass crystals;

- Easy to fabricate and reproducible at a reasonable cost;

- If possible, operation at room temperature.

The parameters for three types of semiconductor materials are listed in Table 2.1. From Table 2.1, it is clear that germanium is a good choice for a gamma-ray detector. This is due to a relatively large \(Z\), making it useful for higher energy gammas. Also, germanium has excellent charge carrier mobility and low \(\langle \epsilon \rangle\) allowing for excellent charge collection properties and energy resolution, respectively. The only drawback in using germanium as a gamma-ray detector (aside from cost) is that it needs to be cooled with liquid nitrogen (LN\(_2\)) in order to minimize leakage current and FET noise (see Equation 2.1).
2.2 High-Purity Germanium Detectors

This section will continue the discussion of semiconductor gamma-ray detectors started in the previous section. However, we now focus solely on detectors fabricated out of HPGe. We will discuss the two main HPGe detector configurations relevant to this dissertation. Chapter 6 will discuss signals from the n$^+$-bulk boundary in detail, therefore in this section we will discuss signal generation, i.e. electric fields, induced charge, charge collection and dead layers in HPGe. We will conclude with a discussion of detector electronics, readout and energy resolution.

2.2.1 Configurations of HPGe Detectors

There are many configurations for HPGe detectors. For example, the coaxial configuration (cylindrical with a central bore hole) is the most commonly used in the field of nuclear physics or gamma-ray spectrometry. Additionally, planar (cylindrical - no bore hole) detectors are used for low-energy (< 100 keV) measurements. Since this dissertation is only concerned with P-type Point Contact (PPC) detectors, we will discuss these detectors in detail in the following sections.

PPC Detectors

PPC HPGe detectors [88, 89], a relatively new development in semiconductor detector technology, have been demonstrated to provide both exceptional energy resolution (∼150 eV FWHM at 10.36 keV) and low-energy thresholds (∼500 eV) [73, 89], see Figure 2.4. The low-energy threshold of PPC detectors, attributed to their low-capacitance point-contact design, makes them ideal for low-mass Weakly Interacting Massive Particle (WIMP) DM searches. Several successful prototypes have been commercially produced and successfully operated in an underground environment [73, 89]. PPC detectors range in mass from as low as 100 g to 1 kg. The n$^+$ contact on PPC detectors can either abruptly stop at the base of the detector (see Figure 2.3) or it can wrap around the base, but never extends all the way to the p$^+$ contact. The area between the n$^+$ and p$^+$ contacts is passivated by evaporating amorphous germa-
nium or silicon onto the surface to leave it relatively intrinsic \cite{90} since gases, \textit{e.g.} water vapor, can deposit onto the surface over time causing an un-passivated surface to become polarized making the surface slightly p-type. Passivation leaves the surface reasonably intrinsic. In summary, passivation makes the region ‘passive’ in terms of its response to the environment.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2_3.png}
\caption{A PPC detector is shown here (not to scale). The n\textsuperscript{+} contact usually does not wrap around the base of the detector, but in some PPC detectors it can wrap around the base but never all the way to the p\textsuperscript{+} contact. The area between the n\textsuperscript{+} and p\textsuperscript{+} contacts is passivated.}
\end{figure}

\textbf{Broad Energy Germanium (BEGe) Detectors}

PPC-like detectors are commercially available from Canberra as BEGe detectors \cite{91}. The BEGe detectors from Canberra typically have a thin front window, usually amorphous germanium, which allows lower energy gamma-rays to penetrate into the active region. The BEGe detector used for this dissertation (more details in Chapters 4 and 5) does not have this feature. Instead, a thick (~1.0 mm) n\textsuperscript{+} contact wraps all the way around the detector.

\textbf{2.2.2 Electric Field, Electric Potential and Induced Charge}

Signals arise in HPGe detectors due to the motion of charge carriers after they have been formed by incident radiation, see Section 2.1.3. The output signal forms immediately after the incident particle deposits its energy and liberates electrons in the active region of the
detector. Once the last of the charge carriers arrives at the electrode the process of charge induction ends and the pulse is fully developed.

**Electric Field and Potential**

The geometry of a HPGe detector determines the electric field and capacitance. The electric field directly determines the drift velocity of charge carriers, therefore the configuration is very important in considerations of pulse shape, timing behavior and the overall completeness of charge collection. In each configuration, to determine the electric potential within the detector, we need to solve Poisson’s equation,

$$\nabla^2 \varphi = -\frac{\rho}{\epsilon},$$  \hspace{1cm} (2.5)

where $\varphi$ is the electric potential in the presence of the charge density $\rho$ in a dielectric with a dielectric constant $\epsilon$. For p-type germanium, $\rho = -eN_A$, where $e$ is the electronic charge and
\[ N_A \] is the density of acceptor atoms. Once the electric potential is calculated, the electric field is simply,

\[ \mathbf{E} = -\nabla \varphi. \quad (2.6) \]

If we neglect diffusion, the charge carriers generated within the detector will follow the electric field lines (or direction of maximal gradient in the potential) from their point of origin to the electrode. Holes get collected at the p+ contact while electrons get collected at the n+ contact.

**Induced Charge**

The Shockley-Ramo Theorem [92, 93] provides a method for calculating the induced charge on the collecting electrodes due to the motion of charge carriers. The concept of a weighting field and weighting potential are crucial to the theorem. The instantaneous current induced on a given electrode is given by,

\[ i = q \mathbf{v} \cdot \mathbf{E}_0, \quad (2.7) \]

where \( q \) is the charge of the carrier, \( \mathbf{v} \) is the velocity and \( \mathbf{E}_0 \) is the weighting field. Similarly, the induced charge on the electrode is the product of the charge on the carrier multiplied by the difference in the weighting potential (unit-less), \( \varphi_0 \) from the start to the end of the charge carrier path length [87]:

\[ Q = q \Delta \varphi_0. \quad (2.8) \]

The weighting potential is calculated by solving the Laplace equation (\( \nabla^2 \varphi_0 = 0 \)) for the geometry of the detector with artificial boundary conditions [87]:

- The voltage on the electrode for which the induced charge is to be calculated is set to unity.
- The voltages on all other electrodes are set to zero.
- Ignore trapped charges (i.e. use Laplace not Poisson equation).

The solution under these conditions yields the weighting potential and the gradient of which is the weighting field. The weighting potential serves as a convenience that allows simple
determination of the induced charge and is not the actual potential within the detector. The induced charge is calculated by taking the differences in the weighting potential at the start and end of the carrier motion. The path of the carrier is determined by the electric field lines. After mapping out the position of a carrier as a function of time, the induced charge as a function of time can be traced out to determine the shape of the output pulse. Figure 2.5 shows the weighting potential for a modified BEGe detector and charge carrier (holes) drift paths.

![Figure 2.5: The weighting potential and charge carrier (holes) drift paths of a p-type modified BEGe detector is shown here (the one used in this dissertation). Notice that the weighting potential is maximum near the point contact and essentially zero everywhere else. Induced charge will not contribute to the output signal until the carriers are near the point contact. Figure generated with the M3DCR [94] and SIGGEN [95] software packages.](image)

In order to solve for the potential in complex geometries numerical techniques must be used. The Poisson equation is solved by applying a relaxation algorithm. Once the potential is known throughout the bulk, the electric field can be calculated. For example, the M3DCR software package [94] was developed to calculate weighting potential, capacitance and electric fields for complex geometries. The SIGGEN software package [95] was developed to calculate the induced charge and signals from complex geometries based on the M3DCR output.
Capacitance

In general, to minimize noise, the capacitance must be as small as possible. In the case of PPC or BEGe detectors, this translates into minimizing the size of the p$^+$ contact. According to Ref. [88], if the p$^+$ contact on a PPC is taken to be hemispherical in shape with radius $r$, the capacitance is given by,

$$C \approx 2\pi \epsilon \epsilon_0 r$$

where $\epsilon_0 = 8.85 \times 10^{-12}$ farad/m is the free space permittivity and $\epsilon$ is the dielectric constant of the material. For reference, the capacitance of planar and coaxial detectors is $>10$ pF, whereas PPC and BEGe detectors have capacitances $<2$ pF.

2.2.3 ‘Dead’ Layers

The n$^+$ and p$^+$ contacts do have an appreciable thickness and cannot be neglected, especially the n$^+$ contact. The most common technique used to create the n$^+$ contact is to diffuse lithium onto the crystal. In the case of both PPC and BEGe detectors, this is on the outside of the detector and will have an effect on the detector response to low-energy gamma-rays ($<200$ keV). Chapter 6 will discuss the effects of the n$^+$ layer in detail.

2.2.4 Charge Collection

Following the formalism outlined in Section 2.2.2, it is possible to calculate the expected signals from HPGe detectors. The contribution of a charge carrier to the output signal depends on the charge it carries and the length of the drift path. If an electron-hole pair is created near the positive collector, the electron, though arriving first, will make little contribution to the signal since it only travels a small length on the electric field lines. The hole created will be the main contribution to the signal. The weighting potential for a modified BEGe detector is shown in Figure 2.5 is essentially zero throughout the crystal except near the p$^+$ contact. This results in rapid charge collection around the p$^+$ contact.

Charge trapping in regions of weak field or sites of impurities can have a significant impact on the charge collection process. Charge trapping results in one of two scenarios: (1)
the charge will subsequently be released or (2) the charge will become trapped longer than the integration time of the electronics. Scenario (1) results in signals with a slow time component and (2) results in energy-degradation or pulse height deficit. The effects of trapping will be discussed further in Chapter 6.

2.2.5 Electronics and Readout

The signal from the detector needs to be processed so that energy (or time) information from gamma-ray interactions can be extracted.

Preamplifiers

A preamplifier is the first component in a signal processing chain. Despite the name, preamplifiers do not amplify the signal. The main purpose of a preamplifier is to extract the signal from the detector without significantly degrading the signal-to-noise ratio. For this reason, preamplifiers are placed as close to the detector as possible.

The most common preamplifier used in gamma-ray spectrometry is a resistive feedback preamplifier, see Figure 2.6. The charge is collected over a capacitor \( C_f \) which integrates the pulse giving a step-like voltage signal. The presence of a feedback resistor essentially creates an RC circuit with time constant, \( \tau_{RC} = R_f C_f \), which then causes the output voltage signal to decay away. The value of \( \tau_{RC} \) must be significantly longer than the rise time of the pulse (typical rise times from HPGe detectors range from \( \sim 100-500 \) ns) in order for the pulse height to be proportional to the energy deposited. There are two major limitations of this type of preamplifier: (1) pile up due to the requirement of long \( \tau_{RC} \)'s and (2) noise associated with the feedback resistor \( R_f \).

To compensate for the major drawbacks of a resistive feedback preamplifier, the feedback resistor can be removed entirely. Without a resistor, charge is simply accumulated on the feedback capacitor \( C_f \), see Figures 2.7 and 2.8. There are several techniques to drain this charge, but the most common connects an active circuit with a transistor to the input stage (transistor reset preamplifier). Of course, there will be dead time associated with resetting the voltage to zero. Reset preamplifiers provide an inhibit signal which is active when the
reset circuitry is active. In low-count-rate systems where noise is a major concern, reset preamplifiers are preferred. However, in extremely high-count-rate systems, the dead time associated with resetting can be a major limitation.

**Shaping**

In order to achieve better resolution, the output from the preamplifier is sent through a shaping amplifier to magnify the amplitude of the preamplifier output pulse from the mV range to the 0.1−10 V range. The amplifier also shapes the signal to optimize energy resolution and to minimize pile-up. The simplest concept for pulse shaping is the use of CR and RC circuits (with the same time constant). A CR circuit is essentially a low pass filter (differentiator) and an RC circuit can be thought of as a high pass filter (integrator). The most common filter used is a CR circuit followed by several RC circuits, *i.e.* CR-(RC)^n where *n* is the number of RC circuits the signal is sent through. The output signal somewhat resembles a Gaussian therefore this type of shaping is sometimes referred to as semi-Gaussian. The time constant used in the CR-(RC)^n circuit, $\tau_{sh}$, is called the shaping time. The output pulse rises to reach its maximum at $\tau_{sh}$. The time taken for the signal to rise from zero to maximum is also called the peaking time. We will see later that electronic noise from the preamplifier can be minimized by choosing an optimal shaping time. Figure 2.9 illustrates how shaping with CR...
Figure 2.7: (Top) Output from a transistor reset preamplifier is shown in black. The inhibit signal is shown in red (active low). (Bottom) The preamplifier output has been AC coupled with a CR circuit and is shown in black. This is effectively a high bandpass filter. The inhibit signal is shown in red (active low).
Figure 2.8: A zoomed-in plot of Figure 2.7. (Top) Output from a transistor reset preamplifier is shown in black. The inhibit signal is shown in red (active low). (Bottom) The preamplifier output has been AC coupled with a CR circuit and is shown in black. This is effectively a high bandpass filter. The inhibit signal is shown in red (active low).
and RC circuits works.

**Example Readout Chain**

Figure 2.10 shows a rudimentary signal processing chain for gamma-ray spectrometry. The high voltage is fed to the detector through the preamplifier. A waveform generator (pulser) is also being used to examine the electronic noise. The preamplifier signal is then shaped by a spectroscopy amplifier and the pulse height is binned by a multi-channel analyzer (MCA) to obtain an energy spectrum. Figures 2.11 and 2.12 show example energy spectra obtained using a modified BEGe detector with an MCA.

**2.2.6 Energy Resolution**

A gamma-ray peak is typically idealized as a Gaussian, however as we will see several factors can alter the peak to give it a low-energy tail (see Figure 2.12). The ratio between the full width at tenth maximum (FWTM) and full width at half maximum (FWHM), \( \frac{\text{FWTM}}{\text{FWHM}} \approx 1.82 \) for a Gaussian peak. The resolution in the above spectra (Figures 2.11 and 2.12) can be attributed to several sources of uncertainty that factor in between the actual energy deposition within the crystal and the eventual readout of the voltage signal. These sources of uncertainty add in quadrature to give to the FWHM (\( \omega_T \)) [81, 84, 87, 96],

\[
\omega_T^2 = \omega_E^2 + \omega_C^2 + \omega_P^2, \tag{2.10}
\]

where,

- \( \omega_P \) is the inherent statistical fluctuation in the number of electron-hole pairs created,
- \( \omega_E \) is due to broadening effects attributed to electronic noise, and
- \( \omega_C \) is due to the uncertainty in charge collection.
Figure 2.9: (a) An idealized input signal from the preamplifier; (b) Output from CR shaping with $\tau_{sh} = 3.0 \, \mu s$; (c) Output from RC shaping with $\tau_{sh} = 3.0 \, \mu s$; (d) Output from CR-RC shaping with $\tau_{sh} = 3.0 \, \mu s$; (e) Output from CR-(RC)$^n$ shaping with $\tau_{sh} = 3.0 \, \mu s$. As $n$ increases the pulse becomes more Gaussian.
Figure 2.10: A simple readout chain for gamma-ray spectrometry is shown here.

Figure 2.11: A modified BEGe detector response to a $^{133}\text{Ba}$ source placed 25 cm above the cryostat is shown here. The spectrum is normalized by the live time and the width of the bins on the x-axis.
Figure 2.12: A modified BEGe detector response to a $^{241}$Am source placed 25 cm above the cryostat is shown here. The spectrum is normalized by the live time and the width of the bins on the x-axis.

**Charge Production ($\omega_P$)**

In Section 2.1.3 we saw that it takes on average 2.96 eV to create an electron-hole pair in germanium (see also Table 2.1). The number of electron-hole pairs (Equation 2.2) is then an average and this must be taken into account. If we assume Poisson statistics, the uncertainty in $n$ is,

$$\sigma_n = \sqrt{n} = \sqrt{E/\epsilon}, \quad (2.11)$$

where $\epsilon = 2.96$ eV. If we put this in terms of energy,

$$\sigma_E = \epsilon \sqrt{n} = \sqrt{E \epsilon}. \quad (2.12)$$

Putting this in terms of FWHM and keV ($E \rightarrow E_{keV}$ means that $E$ is in units of keV), we introduce a factor of 2.355 and 1000,

$$\omega_P = \frac{2.355 \sqrt{E_{keV} \cdot 1000 \cdot \epsilon}}{1000} = 0.128 \sqrt{E_{keV}}. \quad (2.13)$$
If all other sources of uncertainty in Equation 2.10 are zero, then this is theoretically the best possible FWHM achievable. For the 1332 keV line in $^{60}$Co this corresponds to a FWHM of 4.68 keV. However, FWHM values for the 1332 keV line have been quoted as low as 1.6 keV, therefore one of our initial assumptions must be incorrect. The basic assumption that charge carrier creation is Poisson in nature is flawed. Poisson statistics are only valid if each individual event is independent of all the others. The charge creation process is inherently not Poisson in nature since creating an electron-hole pair alters the local electron distribution; this needs to be taken into account. The Fano factor, $F$, was introduced to account for this discrepancy [97]. Introducing $F$ then gives,

$$\sigma_E = \sqrt{F E \epsilon},$$

(2.14)

and it follows that,

$$\omega_p = 0.128 \sqrt{FE_{keV}}.$$  

(2.15)

The Fano factor has been measured to be between 0.057 and 0.129 [84].

**Charge Collection ($\omega_C$)**

As mentioned in Section 2.2.4, charge can become trapped and not collected. The $\omega_C$ term in Equation 2.10 takes this into account. Incomplete charge collection leads to a low-energy tail on the Gaussian peak (a skewed Gaussian) and should be taken into account when fitting for the FWHM [98]$^1$. There is no functional form for $\omega_C$, however it can be parameterized and subtracted out with fitting techniques. The most common (not motivated by theory) functional form used to extract the $\omega_C$ component is,

$$\omega_C = cE_{keV},$$

(2.16)

where $c$ is a proportionality constant and $E_{keV}$ is the gamma-ray energy in keV [84].

---

$^1$Incorrect pole-zero correction can also give a low-energy tail, but here we assume the detector electronics are set up correctly.
Electronic Noise ($\omega_E$)

The noise associated with the amplification and readout electronics is referred to as electronic noise. Electronic noise is dependent on the shaping time ($\tau_{sh}$) of the spectroscopy amplifier used (see Figure 2.10). The three sources of electronic noise are outlined below and sum to give $\omega_E$,

$$\omega_E^2 = (\omega_{\text{series}})^2 + (\omega_{\text{parallel}})^2 + (\omega_{\text{flicker}})^2.$$  \hfill (2.17)

- **Parallel Noise**

Parallel noise is the noise associated with current flowing in parallel with the detector. These noise sources are integrated over the feedback capacitor, $C_f$. The Johnson noise of the feedback resistor is a source of parallel noise. The noise in resistors arises from the thermal motion of charge carriers that lead to a fluctuating potential difference across the resistor. As alluded to in Section 2.2.5, Johnson noise can be eliminated by using a transistor reset preamplifier. The leakage current of the detector is also a source of parallel noise. According to Ref. [84],

$$(\omega_{\text{parallel}})^2 \propto \left( I_L + \frac{2kT}{R_f} \right) \times \tau_{sh},$$  \hfill (2.18)

where $I_L$ is the detector leakage current, $R_f$ is the feedback resistor, $T$ is the temperature of the feedback resistor, and $\tau_{sh}$ is the shaping time.

- **Series Noise**

Series noise is the noise associated with current flowing in series with the detector. Shot noise in the preamplifiers FET is the most significant source of series noise. Shot noise in the FET is proportional to the temperature of the FET, therefore the FET is usually kept suitably cold. The capacitance of the detector is also a source of series noise and therefore detectors with low capacitance have low noise characteristics (BEGe or PPC). According to Ref. [84],

$$(\omega_{\text{series}})^2 = C^2 \left( \frac{2kT}{g_m \times 2.1 \times \tau_{sh}} \right),$$  \hfill (2.19)
where $T$ is the temperature of the FET, $C$ is the sum of the detector and stray capacitances, $g_m$ is the transconductance of the FET, or the ratio of the current change at the output port to the voltage change at the input port, and $\tau_{sh}$ is the shaping time.

- **Flicker (1/f) Noise**

  Flicker noise is associated with variations in the direct current in all active devices. Charge trapping the FET channel is a source of Flicker noise (series 1/f noise). Also, dielectric noise or parallel 1/f noise is a source of Flicker noise. Flicker noise is independent of shaping time.

  We have seen that the electronic noise component is dependent upon the shaping time of the electronics. Measuring the FWHM of a pulser at several shaping times gives what is referred to as the noise curve. An example of a noise curve is shown in Figure 2.13. The three components outlined above are listed on the graph. The functional form used to fit the data is,

  $$(\text{FWHM})^2 = \left( \frac{A}{\tau_{sh}} \right) + (B \times \tau_{sh}) + C,$$

  where $\tau_{sh}$ is the shaping time, $A$ is the series component, $B$ the parallel component and $C$ is the flicker or non-white noise. The detector used to perform the measurement in Figure 2.13 was a modified BEGe detector. More information on this detector can be found in Chapters 4 and 5.

2.3 **Summary**

The Majorana experiment has chosen PPC and modified BEGe detectors as their detector technology to search for $0\nu\beta\beta$ and WIMP dark matter (DM). Several factors played a role in this decision:

- $^{76}$Ge (7.44% natural abundance) can Two Neutrino Double-Beta Decay ($2\nu\beta\beta$) decay to $^{76}$Se and possibly decay via $0\nu\beta\beta$;
- Commercially available at a reasonable cost;
- Technology is a tested and proven one, in that it has been around for many years;
Figure 2.13: The electronic noise curve for a modified BEGe detector is shown here. The measurement was made with an analog spectroscopy amplifier and an MCA. The electronic noise is dominated by flicker or non-white noise. The FWHM at $\tau_{sh} = 6$ $\mu$s is 164.508 eV. The minimum value for the total fit (red dashed) is located at $\tau_{sh} = 5.82$ $\mu$s and gives a FWHM of 166.3 eV (or $2.765 \times 10^{-2}$ keV$^2$). More information on this detector can be found in Chapters 4 and 5.
• Excellent energy resolution near the $Q$ value for $0\nu\beta\beta$;

• Low-thresholds, allowing a search for WIMP DM;

• Excellent multi-site event rejection with pulse shape analysis.

Chapter 3 will discuss the MAJORANA experiment and the role of PPC and BEGe detectors in more detail.
Chapter 3

The MAJORANA Experiment

This chapter will serve as a continuation of Section 1.3 focusing on the detector technology employed by the MAJORANA Experiment. In addition, the sensitivity of the MAJORANA DEMONSTRATOR to $0\nu\beta\beta$ and Weakly Interacting Massive Particles (WIMPs) will be briefly discussed. The motivation for this dissertation, relating to known backgrounds in germanium detectors and their implications for a direct dark matter search, will be covered as well.

3.1 Overview of the MAJORANA Experiment

The MAJORANA Collaboration [67–69] will search for the neutrinoless double-beta decay ($0\nu\beta\beta$) of $^{76}$Ge. The observation of this rare decay would indicate the neutrino is a Majorana fermion, demonstrate that lepton number is not conserved, and provide information on the absolute mass-scale of the neutrino (see Section 1.2.4). Reaching the neutrino mass-scale associated with the inverted mass hierarchy, 20 – 50 meV, will require a half-life sensitivity on the order of $10^{27}$ y. This corresponds to a signal of a few counts per tonne-year in the $0\nu\beta\beta$ region of interest (ROI) – a 4 keV wide window centered at 2039 keV endpoint energy for $^{76}$Ge. The MAJORANA Collaboration is constructing the DEMONSTRATOR, an array of high-purity germanium (HPGe) detectors at the 4850 foot level of the Sanford Underground Research Facility (SURF) in Lead, South Dakota. Figure 3.1 shows the present (December 2012) status of the MAJORANA DEMONSTRATOR laboratory at SURF. The DEMONSTRATOR will consist of a mixture of natural (10-15 kg) and >86% enriched $^{76}$Ge (25-30 kg) HPGe
detectors in two low-background cryostats. Each cryostat will contain seven closely packed stacks, called strings, and each string will have up to five crystals (see Figs. 3.2 and 3.3). The *Demonstrator* aims to:

1. demonstrate a background rate less than 3 counts t\(^{-1}\) y\(^{-1}\) in the ROI,

2. establish the technology required to build a tonne-scale germanium-based \(0\nu\beta\beta\) experiment,

3. test the recent claim [66] of the observation of \(0\nu\beta\beta\), see Figure 3.4,

4. and perform a direct search for light WIMPs (<15 GeV).

The *Majorana* and GERmanium Detector Array (GERDA) [72] Collaborations are working together to prepare for a single tonne-scale \(^{76}\)Ge experiment that will combine the best technical features of both experiments. There are two main differences between the experimental techniques employed by *Majorana* and GERDA. First, the *Demonstrator* array will be deployed in a custom vacuum cryostat, whereas GERDA is submerging theirs in liquid argon. The *Demonstrator* will use a compact shield with lead, oxygen-free high-thermal conductivity (OFHC) copper, electroformed copper, and an active muon veto, whereas GERDA is using liquid argon and high-purity water as a shield.

### 3.2 Detector Technology

The *Majorana* Collaboration will use p-type point contact (PPC) HPGe detectors (see Chapter 2 for an introduction to germanium detectors). PPC detectors [88, 89] have been demonstrated to provide both exceptional energy resolution (<2.0 keV FWHM at 1332 keV) and low energy thresholds (~500 eV) [73, 89] (see Figure 2.4). Several successful prototypes have been commercially produced and successfully operated in an underground environment [73–76, 85]. The PPC detectors used in the *Demonstrator* will each have a mass between 0.6 – 1.0 kg. There are two types of detectors that will be used in the *Demonstrator*: (1) natural PPC detectors, which are made with natural germanium and (2) enriched PPC
Figure 3.1: A panoramic view of the Majorana Demonstrator laboratory at SURF as of December 2012 (Picture taken by M. Green).
Figure 3.2: A cross sectional view of a **Majorana Demonstrator** cryostat. The strings within the cryostat hold a mixture of natural (smaller/light blue) and enriched (larger/dark blue) germanium detectors.

Figure 3.3: The **Majorana Demonstrator** is shown here with both active and passive shielding in place. One cryostat is in place inside the shield while the other is being positioned for insertion. For scale, the inner copper shield is 20" high and 30" in length.
Figure 3.4: The sensitivity of the MAJORANA DEMONSTRATOR at 90% C.L. as a function of exposure for 0νββ in 76Ge with varying background rate assumptions. The matrix elements used to convert half-life to neutrino mass were taken from [99] (see e.g. Equation 1.33). The controversial claim of 0νββ [66] is shown as a blue horizontal band. Figure from [100].

detectors, which are made with germanium enriched to greater than 86% in the 76Ge isotope. Figure 3.5 shows an R&D PPC detector fabricated by the Semiconductor Laboratory at Lawrence Berkeley National Lab (LBNL) for the MAJORANA Collaboration.

### 3.3 Background Mitigation Techniques

One of the technical goals of the DEMONSTRATOR is to show that a 1 count t⁻¹ y⁻¹ ROI⁻¹ background rate is achievable for a tonne-scale experiment, which is ~100 times lower than previous germanium experiments. One of the primary methods for achieving this rate is to deploy the detectors inside two independent cryostats that minimize the amount of interstitial material. In addition, the materials used to fabricate the DEMONSTRATOR have been screened and selected based on strict radiopurity requirements. The main structural material closest to the detectors within the DEMONSTRATOR is electroformed copper due to its intrinsically low-background and excellent physical properties [101]. Electroformed copper
Figure 3.5: A PPC detector is shown here. The p\(^+\) point contact is clearly visible and has a diameter of \(\sim 3\) mm. The passivated surface (top) and n\(^+\) contact (sides) are also visible. For scale, the detector is 5 cm high and 6 cm in diameter.

is made by electroplating from high-purity commercial OFHC copper nuggets onto stainless steel mandrels. The electroforming process greatly reduces the radioactivity due to U, Th and cosmogenically-produced \(^{60}\)Co. Electroformed copper is being used to fabricate both of the cryostats for the DEMONSTRATOR.

Further background reduction in the DEMONSTRATOR is achieved using the technological features of the PPC detectors. A \(0\nu\beta\beta\) event will deposit all of its energy within a \(\sim 1\) mm\(^3\) region inside the PPC detector. These are called single-site events (SSE). In contrast, gamma-rays from radioactive contaminants of sufficient energy to affect the 2039 keV ROI will typically Compton scatter several times with a scattering length of \(\sim 1\) cm. These are called multi-site events (MSE). The DEMONSTRATOR will reduce Compton scattered gamma-ray backgrounds by implementing pulse shape analysis (PSA) techniques to separate SSE from MSE within a single germanium crystal. In addition to signal PSA, background events that deposit energy in more than one detector can be removed.

Cosmogenically-produced isotopes represent another source of background. Cosmic rays can create \(^{60}\)Co in copper components as well as \(^{68,71}\)Ge, \(^{60}\)Co and tritium within the detectors
themselves. In an effort to reduce these backgrounds, the detectors and copper components will spend minimal amount of time above ground. Also, all copper components within the cryostats are being electroformed underground to reduce the amount of $^{60}\text{Co}$ activation. Reducing surface exposure is the only option for most cosmogenically-produced isotopes. However, the background due to $^{68}\text{Ge}$ is a special case. $^{68}\text{Ge}$ decays to $^{68}\text{Ga}$ which has a half-life of 67.71 minutes. The low-thresholds and excellent energy resolution of PPC detectors allow for a single-site time-correlation (SSTC) cut, which looks backward in time from the current event in the ROI to search for signatures of parent or daughter isotopes\cite{102, 103}. The DEMONSTRATOR will implement a SSTC cut for the $^{68}\text{Ge}$-$^{68}\text{Ga}$ coincident decay for $\sim$five $^{68}\text{Ga}$ half-lives following the 10.3 keV K-shell de-excitations. The SSTC method can also be used to reduce backgrounds due to $^{208}\text{Tl}$ ($T_{1/2} = 3.05 \text{ min}$) and $^{214}\text{Bi}$ ($T_{1/2} = 19.9 \text{ min}$) in the germanium crystals and the inner mount. The SSTC and SSE/MSE cuts will both be used in order to maximize background reduction.

### 3.4 DEMONSTRATOR Implementation

The MAJORANA DEMONSTRATOR prototype cryostat is expected to be completed by late Spring 2013 (around the time of defending this dissertation) and will contain two strings of natural germanium PPC crystals. Two electroformed cryostats, Cryostat 1 and Cryostat 2, are being fabricated in a phased approach. Cryostat 1 will contain seven strings of germanium crystals, four holding natural germanium crystals and three containing germanium crystals enriched to greater than 86% $^{76}\text{Ge}$. Cryostat 1 is expected to be completed towards the end of 2013. Cryostat 2 will contain seven strings of germanium crystals, all of which will be enriched to greater than 86% $^{76}\text{Ge}$. Cryostat 2 is expected to be completed by the end of 2014. The MAJORANA DEMONSTRATOR will search for $0\nu\beta\beta$ and low-mass WIMPs once fabrication is completed at SURF. In addition, the MAJORANA DEMONSTRATOR should be able to verify or refute the recent observational claim of $0\nu\beta\beta$\cite{66} within 2-3 years of commissioning Cryostat 1.
3.5 Dark Matter and Motivation for this Dissertation

The low energy performance of PPC detectors, due to their low-capacitance point-contact design, makes them suitable for dark matter searches. The direct detection of dark matter remains an active area of research. WIMPs, a generic class of potential dark matter candidates, are widely regarded as the most promising candidate [7, 43]. Direct search experiments typically report their results as a contour plot as in Figure 3.6 since the WIMP-nucleus interaction rate depends on two unknowns: the WIMP mass and the cross section of the WIMP-nucleus interaction. Recent direct searches for WIMPs have hinted towards a low-mass WIMP at $\sim 10$ GeV [74–77, 104], see e.g. Figure 3.6. It is worth noting that one of the direct searches claiming a signal consistent with WIMPs, the CoGeNT Collaboration [74, 75], uses the same detector technology as the MAJORANA Collaboration – PPC detectors. In fact, the success of CoGeNT was one of the motivating factors in deciding to use PPC detectors for MAJORANA [85, 89]. The favored $\sim 10$ GeV WIMP is also consistent with the theoretical model proposed by Refs. [105, 106] to explain the excess flux of gamma-rays from the Galactic center. On the other hand, several other experiments have reported null-results or show no signs of a low-mass WIMP [104, 107–110]. With several experiments claiming a signal and several others claiming no signal, the current situation is tense. Kelso et al. [111] and Kopp et al. [112] have reported that these disagreements can be brought into alignment after correcting for experimental and astrophysical uncertainties, however this may be a bit optimistic. What would be more enlightening is a direct comparison to one of the above experiments using the same detector technology – this would provide a model independent test of their results. The MAJORANA DEMONSTRATOR will be able to shed light upon this situation. The projected sensitivity of the DEMONSTRATOR can be calculated assuming that the major background at low energies arises from the decay of cosmogenically produced tritium and a 500 eV threshold. This has been calculated in Refs. [76, 85, 113] and the sensitivity of the DEMONSTRATOR to a 10 GeV WIMP is expected to reach $10^{-43}$ cm$^2$ (normalized to nucleon), well below current claims – see Figure 3.7.

Additionally, as a part of the research and developmental efforts for the DEMONSTRA-
Figure 3.6: The current status of low-mass WIMP searches is shown here. All results shown are at 90% CL unless otherwise noted. Filled in regions denote experiments that have reported results compatible with a WIMP in that region of parameter space. Upper limits (null results) are depicted by lines. The DAMA/LIBRA annual modulation results shown here are the first results as interpreted by Ref. [104]. The other experiments that have shown a positive result are CRESST II [77] (95% CL), CoGeNT 2010 [74] and CoGeNT again in 2011 (annual modulation) [75]. Additionally, the CoGeNT 2011 result as interpreted by Ref. [111] is also shown. Several experiments have reported null results, or show no signs of a low-mass WIMP. Two of these experiments include XENON10-LE [108] and CDMS II [109]. (Data obtained using DMTools [114].)
Figure 3.7: The 90% CL projected sensitivities of future experiments are plotted with the data from Figure 3.6. The projected WIMP sensitivity of the DEMONSTRATOR for a 100 kg-y exposure assuming a 500 eV threshold [76]. In addition, the projected sensitivities of SuperCDMS Phase A [115], LUX 300 [116] and DarkSide-50 [117] are also shown. (Data obtained using DMTools [114].)
The Majorana Collaboration has deployed a custom PPC detector (MALBEK) in the Kimballton Underground Research Facility (KURF) in Ripplemead, Virginia, at a depth of 1450 meters water equivalent [118–120]. The next two chapters will highlight the MALBEK detector providing an introduction and analysis overview. The MALBEK detector and the detector used by the CoGeNT Collaboration are nearly identical – see Table 3.1. Recall that the CoGeNT Collaboration has performed a direct search for dark matter and has reported results consistent with a $\sim 10$ GeV WIMP (Figure 3.6); the MALBEK detector then provides a model independent test of the results reported by CoGeNT [74, 75]. In addition, the CoGeNT Collaboration has recently published a comprehensive technical description of their experiment and analysis methods [121]. In this paper, they highlight the fact that surface events represent a formidable background near threshold that plague the energy spectrum after all cuts. The MALBEK detector has been used both to investigate these surface events (reported on in Chapter 6) and to perform a direct search for dark matter in much the same manner as CoGeNT (reported on in Chapter 7). The remainder of this dissertation will focus on the MALBEK detector in the context of surface events and dark matter in an effort to either confirm or refute the CoGeNT observations.
### Table 3.1: MALBEK and CoGeNT comparisons.

|                  | **MALBEK** | **CoGeNT**
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>natural Ge</td>
<td>natural Ge</td>
</tr>
<tr>
<td>Manufacturer</td>
<td>Canberra Industries USA</td>
<td>Canberra Industries USA</td>
</tr>
<tr>
<td>Detector Type</td>
<td>BEGe PPC</td>
<td>BEGe PPC</td>
</tr>
<tr>
<td>Operational Depth [m.w.e]</td>
<td>1450</td>
<td>2090</td>
</tr>
<tr>
<td>Total Mass [g]</td>
<td>465(^b)</td>
<td>443</td>
</tr>
<tr>
<td>Fiducial Mass [g]</td>
<td>404.2(^b)</td>
<td>(\sim 330)</td>
</tr>
<tr>
<td>Point contact diameter [mm]</td>
<td>4.0</td>
<td>6.3</td>
</tr>
<tr>
<td>Energy Threshold [eV]</td>
<td>600</td>
<td>450, 500(^c)</td>
</tr>
<tr>
<td>Pulser FWHM [eV]</td>
<td>158</td>
<td>150</td>
</tr>
<tr>
<td>Capacitance [pF]</td>
<td>1.55</td>
<td>1.8</td>
</tr>
<tr>
<td>Crystal Diameter [mm]</td>
<td>60.6</td>
<td>60.5</td>
</tr>
<tr>
<td>Crystal Length [mm]</td>
<td>30.0</td>
<td>31.0</td>
</tr>
<tr>
<td>‘Dead’ Layer Thickness [mm]</td>
<td>0.933(^b,d)</td>
<td>2.0</td>
</tr>
<tr>
<td>Preamplifier</td>
<td>integrated transistor reset</td>
<td>integrated transistor reset</td>
</tr>
</tbody>
</table>

\(^a\) CoGeNT values from Ref. [121].

\(^b\) Values from Ref. [122].

\(^c\) Refs. [73–75] use a 450 eV threshold, while Ref. [76] uses a 500 eV threshold.

\(^d\) See Chapter 6 for a detailed discussion of the MALBEK n\(^+\) contact region.
Chapter 4

MALBEK Hardware and Infrastructure

The previous chapter introduced the MALBEK detector, the subject of this dissertation. This chapter will focus on the MALBEK detector characteristics, shield design, data acquisition (DAQ) system and deployment in an underground laboratory.

4.1 Introduction

As a part of the research and developmental efforts of the Majorana Demonstrator, a customized 465 g p-type BEGe (modified BEGe) detector manufactured by Canberra Industries was deployed underground on 12 January 2010 at the Kimballton Underground Research Facility (KURF) in Ripplemead, Virginia. This detector is referred to as the Majorana Low-Background BEGe at Kimballton (MALBEK) hereafter. The MALBEK germanium crystal is housed in a low-background cryostat fabricated out of oxygen free high-thermal conductivity (OFHC) copper\(^1\). The MALBEK cryostat is shown in Figure 4.1. The experimental goals for MALBEK are to:

1. test a Majorana Demonstrator-like DAQ, especially at low-energies;

2. explore optimum PPC detector geometry;

3. systematically characterize the entire energy spectrum;

\(^1\)Juan Collar of the University of Chicago consulted with Canberra Industries on clean assembly techniques and materials while the cryostat was being fabricated.
4. perform a light WIMP search;

5. and validate the **Majorana Demonstrator** background model [122].

![MALBEK Detector](image.jpg)

Figure 4.1: The MALBEK detector is shown here. The HPGe crystal was mounted in the low-background OFHC copper housing shown on the left. The cryostat is in a ‘dipstick-style’ configuration. Other components are explained in the text.

## 4.2 MALBEK Characteristics

### 4.2.1 Dimensions and Distinguishing Features

The MALBEK detector is different from standard BEGe detectors produced by Canberra in several respects:

- the typical thin front entrance window is absent (the lithium diffused n$^+$ contact is the same thickness over the entire n$^+$ contact region),

- the crystal aspect ratio (height-to-width ratio) was optimized,

- the point contact size is nearly a factor of two smaller (4.0 mm),

- the cryostat (see Figure 4.1) was fabricated out of low-background components, and

- the MALBEK preamplifier different than those used in production BEGe detectors. It is an integrated transistor reset preamplifier for a positive bias detector (they are typically used on negative bias detectors). It is also separated and offset from the cryostat can in order to minimize radioactive backgrounds (see Figure 4.1).

The reduction in point contact size decreases the capacitance, and hence the electronic noise. Simulations (Figure 2.5) performed by David Radford at Oak Ridge National Laboratory...
calculated the aspect ratio which optimized charge collection within the crystal [123]. Various MALBEK properties, e.g. crystal dimensions and impurity profile, are listed in Table 4.1.

### 4.2.2 Operational Characteristics

The electronic noise and capacitance of MALBEK were measured upon delivery to UNC from Canberra in November 2009. The DAQ setup for these measurements was similar to that shown in Figure 2.10, utilizing a pulser, MCA and spectroscopy amplifier. The capacitance of a germanium detector can be thought of as the ratio of charge induced across the diode to the voltage applied across it, \( i.e. \ C = \frac{Q}{V} \). It takes 2.96 eV to create an electron-hole pair in germanium, therefore using a calibration source, we can then calculate the number of charge carriers created,

\[
Q = e \frac{E_{\gamma}}{E_{ehp}} \frac{P_{\text{pulser}}}{P_{\gamma}}
\]  

(4.1)

where \( E_{\gamma} \) is the energy of the known gamma line, \( E_{ehp} \) is again the 2.96 eV, \( P_{\text{pulser}} \) is the location of the pulser peak in the MCA spectrum and \( P_{\gamma} \) is the gamma line location in the MCA spectrum. The voltage in the capacitance calculation is simply the input voltage of the pulser. The detector was found to deplete at 2850 V corresponding to a capacitance of 1.55 pF (see Figure 4.2). The electronic noise (width of a pulser) was measured at several shaping times, with a minimum value of \( \sigma = 70.3 \) eV, and is shown in Figure 4.3. Additionally, the resolution
of MALBEK in response to various calibration sources is shown in Table 4.2. The values listed in Table 4.2 remained stable throughout various operational modes, *e.g.* above ground and below ground with two different DAQ systems (MCA- and digitizer-based), therefore these values are a good representation of the resolution of the MALBEK detector.

![Figure 4.2: The capacitance of the MALBEK detector as a function of bias voltage. The detector depletes at 2850 V which corresponds to a capacitance of 1.55 pF.](image)

### 4.3 The Kimballton Underground Research Facility (KURF)

After the initial acceptance measurements outlined in the previous section were performed, MALBEK was deployed underground at KURF on 12 January 2010. The KURF experimental hall is located on the mine’s 14th level at a depth of 1450 m.w.e (meters of water equivalent shielding) [118]. The overburden consists of 520 m of dolomite, limestone and other sedimentary rock. Experiments are housed in a 30 m × 11 m laboratory building that was completed in October 2007 (see Figure 4.4). The laboratory’s general infrastructure is maintained primarily by collaborators at Virginia Polytechnic Institute and State University with additional support from Triangle Universities Nuclear Laboratory (TUNL). KURF also
Figure 4.3: The electronic noise curve for MALBEK is shown here. The measurement was made with an analog spectroscopy amplifier and an MCA. The electronic noise is dominated by flicker or non-white noise. The FWHM at $\tau_{sh} = 6 \, \mu s$ is 164.5 eV. The minimum value for the total fit (red dashed) is located at $\tau_{sh} = 5.82 \, \mu s$ and gives a FWHM of 166.3 eV (or $2.765 \times 10^{-2} \, \text{keV}^2$).

Table 4.2: Full width at half-maximum (FWHM) and full width at tenth-maximum (FWTM) values for various sources.

<table>
<thead>
<tr>
<th>Shaping Time ($\mu s$)</th>
<th>Source</th>
<th>FWHM</th>
<th>FWTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>Pulser</td>
<td>158.0 eV</td>
<td>286.0 eV</td>
</tr>
<tr>
<td>4.0</td>
<td>$^{57}\text{Co}$ (122 keV)</td>
<td>484.0 eV</td>
<td>881.0 eV</td>
</tr>
<tr>
<td>4.0</td>
<td>$^{60}\text{Co}$ (1332.5 keV)</td>
<td>1.63 keV</td>
<td>3.15 keV</td>
</tr>
<tr>
<td>8.0</td>
<td>Pulser</td>
<td>167.0 eV</td>
<td>302.0 eV</td>
</tr>
<tr>
<td>8.0</td>
<td>$^{57}\text{Co}$ (122 keV)</td>
<td>489.0 eV</td>
<td>918.0 eV</td>
</tr>
<tr>
<td>24.0</td>
<td>Pulser</td>
<td>222.0 eV</td>
<td>Not Measured</td>
</tr>
<tr>
<td>24.0</td>
<td>$^{57}\text{Co}$ (122 keV)</td>
<td>522.0 eV</td>
<td>942.0 eV</td>
</tr>
</tbody>
</table>
has the advantage of drive in access, making it relatively simple to transport personnel and equipment to the experimental hall. Liquid Nitrogen (LN$_2$) used in the laboratory is stored in a 2.4 m$^3$ portable dewar that can be transported to the surface and refilled as necessary. The radon levels have been measured at KURF and were found to vary from 37 Bq/m$^3$ in the winter to 122 Bq/m$^3$ in the summer [118].

The MALBEK detector and DAQ are housed in separate sealed modified shipping containers (MSCs) within the laboratory building in order to minimize traffic in the detector MSC. The MSCs were purchased from Mobile Mini, Inc. and are 8’ × 10’ × 7’ 9.5” (L × W × H) in size. TUNL technical staff and the author outfitted each trailer with all of the required infrastructure: ethernet junction boxes, lighting, flooring, and electrical conduits and outlets. Figure 4.5 shows the detector trailer before and after the modifications were made (similar

![Figure 4.4: (a) KURF pad before construction of a laboratory enclosure, (b) during construction, (c) after construction (d) current status of KURF. The author assisted in construction and commissioning of the laboratory. Images from [128].](image)
modifications were made to the DAQ trailer). After remodeling of the MSCs, they were moved to KURF (see Figure 4.6) and 120 V AC power was run to each trailer. Additionally, the LN\textsubscript{2} manifold in the detector MSC, air handling system (HVAC), phone, internet and oxygen monitors were also installed.

(a) Before. \hspace{1cm} (b) After.

Figure 4.5: The MALBEK detector MSC before and after modifications.

4.4 Shielding

The MALBEK detector is shielded from external gamma-rays by a lead shield. The lead shield is comprised of an inner lining of ancient lead (2.54 cm) and a 20 cm outer layer of certified lead. Both of these materials were chosen to reduce backgrounds from \(^{210}\text{Pb}\) \((T_{1/2} = 22\ \text{years})\) and its progeny – \(^{210}\text{Pb}\) and its daughters produce a 46.5 keV gamma-ray, X-rays between 70 – 90 keV and a bremsstrahlung continuum that extends out to 0.5 – 1.0 MeV \cite{129}. Lead bricks made from modern lead can have \(^{210}\text{Pb}\) activities as high as 50,000 Bq kg\(^{-1}\), resulting in a significant low-energy background \cite{129}. The ancient lead\(^{2}\) was donated by Juan Collar and has a \(^{210}\text{Pb}\) activity of < 0.01 Bq kg\(^{-1}\) \cite{130}. This ancient lead was produced >5 half-lives of \(^{210}\text{Pb}\) ago, allowing a significant amount of the \(^{210}\text{Pb}\) within it to decay away.

\(^{2}\)Believed to be from the ballast of a sunken British ship in a Dutch Harbor \(\sim\)500 years ago.
Certified low-background lead was purchased from Sullivan Metals and has a $^{210}\text{Pb}$ activity of $< 2.5 \text{ Bq kg}^{-1}$ [131]. Surrounding the lead shield is a radon purge box which creates a hermetically sealed enclosure and is continuously purged with LN$_2$ boil-off to reduce radon backgrounds. Neutrons produced by $(\alpha, n)$ and fission reactions within rock walls are shielded by 25.4 cm of polyethylene (purchased from Dillon Supply Co.) which surrounds the radon purge box. For every 13 cm of polyethylene, this low-energy neutron background is reduced by an order of magnitude [132]. The chosen thickness of the polyethylene shield was limited by the internal dimensions of the detector MSC. Higher energy neutrons produced by cosmic-ray muons are shielded by the 1450 m.w.e. overburden. Figure 4.7 shows the lead shield surrounded by the radon purge box.

### 4.4.1 Shield Stand Design

The driving factor in the shield stand design was the desire to not have to unstack the lead shield in order to access the detector. Unstacking the shield is labor-intensive, increases
the risk of contamination, and damages the lead bricks, making the shield more difficult to re-stack. With that in mind, the shield stand seen in Figure 4.7 was designed by TUNL engineers and fabricated at the UNC machine shop. The detector and dewar sit inside an aluminum/steel frame that bolts into the larger shield stand which supports the lead shield. A work positioner (pallet jack) is used to raise and lower the detector/dewar stand into and out of the shield stand from below the modern lead shield. The MALBEK cryostat and a base of Sullivan and ancient lead bricks in the detector/dewar stand is shown in Figure 4.8a. The detector/dewar stand after the inner ancient lead has been stacked is shown in Figure 4.8b. This assembly consisting of the detector and ancient lead bricks can be inserted or removed from the bottom of the main lead shield without unstacking any of the lead bricks. After the ancient lead has been stacked, the detector/dewar stand is ready to be inserted into the shield stand. Figure 4.8c shows the detector/dewar stand after insertion along with the lead shield and radon purge. Lastly, Figure 4.8d shows the polyethylene shield stacked around the entire shield stand.

4.4.2 Lead Brick Cleaning

Prior to deployment at KURF, each of the 180 Sullivan and ancient lead bricks were etched and cleaned to remove surface contamination at Duke University with BDH Aristar A.C.S. grade nitric acid\(^3\). Figure 4.9 shows the differences between cleaned and as-shipped Sullivan lead bricks and ancient lead bricks. Each of the 180 lead bricks were double bagged in nylon bags (see Figure 4.10) prior to shipment to KURF.

4.4.3 Shield Calibration Track

To calibrate the detector within the shield, the shield stand and lead shield were designed to incorporate a source calibration track that ends directly atop the cryostat can (although, there is 1” of ancient lead between the source and detector). Several Sullivan lead bricks were machined at UNC to accommodate the calibration track which is shown in Figure 4.11. A

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\(^3\)This designates a high quality chemical for laboratory use. The abbreviation “A.C.S.” means the chemical meets the specifications of the American Chemical Society, 68.0-70.0% HNO\(_3\) having a molarity of 15.6 mol/L.
Figure 4.7: A cross section of the MALBEK shield and support stand is shown here. The modified BEGe crystal is in dark blue, ancient lead in red, Sullivan lead in green, dipstick cryostat in light blue, LN$_2$ dewar in orange and support frame in gray. The radon purge enclosure surrounds the entire lead shield. See text for details.
Figure 4.8: The various components of the MALBEK shield are shown here. (a) shows the detector/dewar stand with a base of Sullivan and ancient lead bricks, (b) shows the same stand with the ancient lead house stacked. This house can be removed from the aluminum/steel frame without unstacking the lead shield, (c) shows the end result of inserting the lead house and detector/dewar stand into the shield stand and bolting it in place. Also shown is the teflon source calibration tube (white tubing) and LN$_2$ supply hose. (d) shows the entire setup surrounded by polyethylene.
(a) A clean Sullivan brick (top) compared to an as-shipped brick (bottom).

(b) A clean ancient lead brick (right) compared to an as-shipped brick (left) from the University of Chicago.

Figure 4.9: Shown here are etched and as-shipped Sullivan and ancient lead bricks.

Figure 4.10: The majority of the 180 cleaned and bagged lead bricks are shown here before transport to KURF.
1 $\mu$Ci $^{133}$Ba capsule source at the end of a piano wire was purchased from Eckert & Ziegler Isotope Products for detector calibrations. The piano wire is housed inside of a teflon tube which is then inserted into a larger diameter teflon tube that runs the length of the calibration track. The larger diameter teflon tube can be seen in Figures 4.8c and 4.8d. This tube is always in place, while the tube with the radioactive source is kept sufficiently far away during normal background runs.

![Cross-sectional view of the MALBEK shield calibration track](image)

Figure 4.11: A cross-sectional view of the MALBEK shield calibration track is shown here. Sullivan bricks are green and ancient lead bricks are dark blue. Also shown is the radon purge box and detector cryostat.

### 4.5 The MALBEK DAQ and Slow-Control System

#### 4.5.1 Overview

The design of the MALBEK DAQ and slow-control system was driven by several experimental goals and requirements:
1. the KURF underground environment calls for a DAQ that is automated and remotely accessible;

2. the wide energy range of interest requires a DAQ capable of triggering on pulses (and digitizing) ranging from several hundred eV to greater than 2 MeV;

3. and the MALBEK experiment provides a test bed for electronics that may be used with the Majorana Demonstrator, requiring the DAQ to be modular and easily reconfigurable.

The MALBEK DAQ and slow-control system is controlled by a software package developed at UNC called Object-oriented Real-time Control and Acquisition (ORCA), an object oriented data acquisition application [133, 134]. ORCA is self-monitoring, sending email notifications and alarms to operators based on user configurable preferences. Some of these include detector and purge dewar LN$_2$ levels, oxygen levels within the detector and DAQ MSC, daily status reports with energy spectra, channel rates and the status of the high voltage supply (see Appendix A.1).

4.5.2 Signal Chain

A block diagram of the MALBEK DAQ is shown in Figure 4.13, the details of which will be discussed in this section. Charge signals from MALBEK are filtered by a custom Canberra Industries low-noise integrated transistor reset preamplifier (see Section 2.2.5 for an overview of this design). The MALBEK FET is an ultra-low noise JFET manufactured by MOXTEK, Inc., Model MX-120, with transconductance $g_{gs} = 16$ mS and a capacitance of $C_{gs} = 1.7$ pF, providing excellent low-noise performance [85, 89]. The preamplifier has two identical signal outputs and one inhibit output which fires when the reset circuitry of the preamplifier is active (see Figure 2.7). In order to reduce file sizes, a real-time ORCA data filter was developed and incorporated into the DAQ that vetoes the majority of inhibit-related charge signals based upon waveform characteristics, e.g. the ADC values of the first several digitized samples differ significantly from ionization signals (See Appendix A.2). This also allows us to choose the inhibit veto window width offline rather than using the hardware inhibit
signal, which has a duration of 700 $\mu$s. The two signal outputs are AC-coupled (using a 500 nF capacitor and the impedance of the signal cable) to remove their DC components, specifically the slow-rise in the output voltage as the preamplifier is charged. One of the signal outputs is immediately digitized by a Struck Innovative Systeme 3302 (SIS3302)\(^4\) ADC (high-energy/low-gain channel). The second signal output is sent through a Phillips Scientific 777 fast amplifier before being digitized by the SIS3302 (low-energy/high-gain channel). The SIS3302 is a VME64x based, 8 channel, 16-bit digitizer capable of digitization rates up to 100 MHz. The SIS3302 uses an internal trapezoidal triggering filter capable of triggering on low amplitude signals and allows for data read-out in parallel with acquisition. The low-energy channel has a dynamic range of $\sim 0.6 - 150$ keV, and the high-energy has a range of $\sim 0.8 - 2600$ keV (where the highest energy is limited by the range of the preamplifier). Both channels have 8192 16-bit samples (81.92 $\mu$s trace length), enabling off-line analysis of the signals. During normal production runs, which are one hour in length, the test input of the preamplifier is used to inject 100 mV ($\sim 35$ keV) signals at a frequency of 0.1 Hz. The location and width of the pulser peak in the energy spectrum is measured on a run-by-run basis, enabling a measurement of the electronic noise as a function of time. Additionally, attenuators are also used in conjunction with the pulser to investigate the triggering performance of the digitizer down to threshold when desired. Figure 4.12 shows the triggering efficiency for both channels.

**Digitizer Readout Methods**

Data read out from the VME64x bus is performed by a Concurrent Technologies VX 40x/04x single board computer (SBC) that interfaces directly with the DAQ computer via gigabit ethernet. High traffic on the VME64x bus, and subsequently the gigabit ethernet cable, can lead to noise in the DAQ system. Noise of this nature was observed at a frequency of 66 kHz after initial tests with the SIS3302 – Figure 4.14 illustrates how this noise manifests itself on digitized waveforms.

Mark Howe, the lead software developer for ORCA, developed a new read out method (aptly named *special mode*) to mitigate this noise. In short, this new read out method

\(^4\)http://www.struck.de/sis3302.htm
Figure 4.12: The trigger efficiency for the high-energy (solid) and low-energy channel (dashed).

reduces traffic on the VME backplane by polling the card less frequently and discarding data taken during polling. The implementation of special mode into the MALBEK DAQ greatly reduced the 66 kHz noise (referred to as polling noise), as is shown in Figure 4.15. The data presented in Chapter 7 were all taken in special mode. The next chapter will highlight some of the drawbacks to using special mode, focusing on the stability of the digitizer during data acquisition.

4.5.3 Liquid Nitrogen Auto-Fill System

Two 240 L LN$_2$ dewars sit between the detector and DAQ MSC which feed into the LN$_2$ manifold. An IP power switch and solenoid valves are used to control which dewar is feeding the LN$_2$ manifold. The IP power switch can be controlled remotely with any internet browser. LN$_2$ is delivered to the purge and detector 30 L dewars via the LN$_2$ manifold within the detector MSC. The manifold is controlled by an American Magnetics, Inc. Model 286 Liquid Level Controller (AMI286) which has two level sensors in both the purge and detector dewar. The AMI286 is controlled with ORCA and fills the dewars whenever the LN$_2$ level in either
Figure 4.13: MALBEK DAQ diagram - see text for details.
Figure 4.14: The 66 kHz polling noise (shaded regions) is shown here on a 6.5 keV digitized waveform.

Figure 4.15: Waveforms acquired in both normal mode (black) and special mode (red) are shown here. It is clear that the 66 kHz polling noise has been eliminated.
dewar reaches a user defined level. The levels in each dewar are polled every five minutes and shipped into the ORCA data stream. In order to reduce the temperature of the FET, and therefore leakage current, the detector dewar LN$_2$ level was only allowed to reach 80% before refilling. With the LN$_2$ levels in the ORCA data stream, it is possible to veto time periods during which either dewar filled.

4.6 Discussion

This chapter has introduced the MALBEK detector focusing on the operational performance, infrastructure at KURF, shield and DAQ. The DAQ was shown to be capable of efficiently triggering on events as low in energy as 550 eV, meeting the experimental requirements for a light-WIMP search. Additionally, a new method of read-out (special mode) for the SIS3302 has been developed and implemented in the MALBEK DAQ. The next chapter will focus on the data taken with MALBEK at KURF as well as the data cleaning and analysis methods developed.
Chapter 5

MALBEK Data and Analysis

The previous chapter presented the MALBEK detector, DAQ and shield. This chapter focuses on the data taken with MALBEK at KURF and the techniques used for analysis in the low-energy channel. In addition, this chapter will discuss the stability and possible systematic uncertainties associated with the long term operation of MALBEK.

5.1 Description of Data Acquired

After deployment to KURF on January 12, 2010, several data sets were acquired with MALBEK in various data acquisition (DAQ) and shield configurations, as summarized in Table 5.1. Within several days of running, an unexpected background due to $^{210}\text{Pb}$ (46.5 keV gamma-ray line, X-rays from 70 → 90 keV and a bremsstrahlung continuum) was observed, see e.g. the black histogram in Figure 5.1. It was decided to continue operating the detector underground while we attempted to identify the origin of the background from the data and simulation. During this time the noise in the DAQ system was being addressed, and the special mode SIS3302 read-out was developed. Additionally, data for the MALBEK background model comparison were taken (see Section 4.1 and Ref. [122]). Based on the observation of the $^{210}\text{Pb}$ 46.5 keV peak, X-rays and bremsstrahlung, we concluded that the origin of the $^{210}\text{Pb}$ must be in bulk lead very near the crystal. The MALBEK background model confirmed this hypothesis estimating the most likely origin to be lead shims within the cryostat adjacent to the crystal [122]. On October 24, 2011, the MALBEK detector was removed from its shield,
taken from the mine, and transported to Canberra Industries in Meriden, CT. The cryostat was opened and the lead foils were removed (see Figure 5.2). The lead foils were replaced with two low-background PTFE shims purchased from Goodfellow Corporation (Part No. FP301350) 30 × 40 × 0.25 mm in size. After reassembly, MALBEK was transported back to KURF. This change reduced the number of counts in the 46.5 keV full energy peak by a factor of 10.4, see Figure 5.1 for a comparison. Table 5.2 lists the dominant features observed in the MALBEK low-energy spectrum (Figure 5.1) as well as their sources. We should note that the lead foils were supposed to have been fabricated from ancient lead (see Section 4.4) with $^{210}$Pb activity less than 0.01 Bq kg$^{-1}$, however different lead was unknowingly used ($> 100$ Bq kg$^{-1}$) [122]. This was likely because of mislabeling at the University of Chicago, which provided the lead shims.

The total time above ground for the lead foil removal operation was 2 days 19 hours and 30 minutes. During this time period, the germanium crystal and copper cryostat were exposed to cosmic ray neutrons increasing the number of $^{71}$Ge ($T_{1/2} = 11.43$ days) and $^{68}$Ge ($T_{1/2} = 270.95$ days) atoms within the crystal (cosmogenic activation). It should be noted that several other isotopes were also created during surface exposure, however the contributions from $^{71}$Ge and $^{68}$Ge dominate the low-energy spectrum (see e.g. Table 2.2 in Ref. [135]).

The data taken with MALBEK have been split into distinct data sets – with and without lead shims, shown in Table 5.1. Each data set consists of a series of individual runs one hour in length. The runs in data set 1 (DS1) were acquired while the electronic noise in the MALBEK DAQ was being investigated. These data sets will not be covered in detail in this dissertation, aside from a comparison of the two eras (before/after lead shims). Data sets 2 (DS2) and 3 (DS3) represent the only data sets with backgrounds low enough to perform a light WIMP search, however this dissertation will focus solely on DS3 since the polyethylene shield was not in place during DS2.

---

1. Thank you to Mike Yocum and Jim Colaresi for assisting in the removal of the lead foils at Canberra.
Figure 5.1: MALBEK energy spectra before (black) and after (blue) the lead shims were removed. The dominant features are labeled (energies in keV) and are also itemized in Table 5.2. These spectra contain slow- and fast-pulses (distinction given in Section 5.1.3). The data cleaning methods used to generate these figures are discussed in Section 5.3. (a) full dynamic range of the low-energy channel (0.6 – 150 keV). (b) expanded scale showing lower energies (0.6 – 12 keV).
Figure 5.2: Two aluminized lead shims were extracted from the cryostat. These shims were between the HV copper foil and the Ge crystal. A date “8-7-08” has been scribed on the larger piece. The numbers “0” and “8” were scribed on the other shim. These shims have a combined mass of 3.01 g.

Table 5.1: The MALBEK data sets are listed here. See text for detailed descriptions.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Date Begin</th>
<th>Date End</th>
<th>Live Time (days)</th>
<th>VME64x Crate</th>
<th>SIS3302 Readout</th>
<th>Pb Shims</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>2011-03-08 14:40</td>
<td>2011-04-14 13:40</td>
<td>33.8542</td>
<td>Dawn</td>
<td>Normal</td>
<td>Yes</td>
<td>All shielding present</td>
</tr>
<tr>
<td>1b</td>
<td>2011-04-14 15:36</td>
<td>2011-05-27 10:28</td>
<td>34.8845</td>
<td>Dawn</td>
<td>Special</td>
<td>Yes</td>
<td>All shielding present</td>
</tr>
<tr>
<td>1c</td>
<td>2011-06-20 13:01</td>
<td>2011-08-16 07:12</td>
<td>40.0881</td>
<td>Linear</td>
<td>Special</td>
<td>Yes</td>
<td>All shielding present</td>
</tr>
<tr>
<td>2</td>
<td>2011-11-02 08:19</td>
<td>2011-11-14 16:31</td>
<td>12.0027</td>
<td>Dawn</td>
<td>Special</td>
<td>No</td>
<td>Polyethylene shield not stacked</td>
</tr>
<tr>
<td>3a</td>
<td>2011-11-15 17:06</td>
<td>2012-03-12 17:06</td>
<td>104.3214</td>
<td>Dawn</td>
<td>Special</td>
<td>No</td>
<td>All shielding present (before power outages)</td>
</tr>
<tr>
<td>3b</td>
<td>2012-04-09 11:25</td>
<td>2012-08-29 14:56</td>
<td>117.1725</td>
<td>Dawn</td>
<td>Special</td>
<td>No</td>
<td>All shielding present (after power outages)</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td></td>
<td><strong>342.3234</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2: Dominant features in the MALBEK low-energy spectrum. The last three lines listed were not observed prior to the lead shim removal due to high backgrounds.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.096</td>
<td>Zn L-capture</td>
</tr>
<tr>
<td>1.299†</td>
<td>Ge L-capture</td>
</tr>
<tr>
<td>4.966*</td>
<td>V K-capture</td>
</tr>
<tr>
<td>6.539†</td>
<td>Fe K-capture</td>
</tr>
<tr>
<td>8.979†</td>
<td>Zn K-capture</td>
</tr>
<tr>
<td>9.659†</td>
<td>Ga K-capture</td>
</tr>
<tr>
<td>10.367†</td>
<td>Ge K-capture</td>
</tr>
<tr>
<td>46.539</td>
<td>Pb</td>
</tr>
<tr>
<td>72.0→90.0</td>
<td>Pb and Bi X-rays</td>
</tr>
<tr>
<td>92.38, 92.80</td>
<td>Th</td>
</tr>
<tr>
<td>122.06</td>
<td>Co</td>
</tr>
<tr>
<td>136.47 + 7.06 (143.53)</td>
<td>Co γ + X-ray summing</td>
</tr>
</tbody>
</table>

† These peaks were used for an energy calibration.
* There is some uncertainty with this peak identification due to its low-intensity (< 2.4 µHz).

5.1.1 With Lead Shims

Data set 1 contains three sub-data sets (DS1a, DS1b, DS1c), all of which had lead shims in the cryostat. DS1a was unique in that that SIS3302 was operating in normal read-out mode since special mode had not yet been developed. The SIS3302 was run in special mode in all subsequent data sets. Approximately 34 days after special mode had been implemented, a VME64x crate with linear power supplies was installed in place of the Dawn 9U switching power supply VME64x crate in an effort to reduce noise. The hope was that by using a linear power supply instead of a switching power supply we would reduce noise in our DAQ. This is due to the fact that switching power supplies generate noise at harmonics of the switching frequency – see e.g. Figure 5.17. Graham Giovanetti re-worked a spare VME64x crate at UNC by replacing the switching power supplies with linear supplies purchased from International Power (ICP197 - 5V, 2x IHE12-10.2 - ±12 V). Approximately 40 days of data were acquired (DS1c) with the linear crate before transporting the detector to Canberra to remove the lead shims. The linear crate proved to be unstable, e.g. the high voltage supply would randomly ramp down, and did not improve noise as expected; therefore the linear crate was removed.
from the DAQ when the detector was moved back underground, post-lead shim removal.

5.1.2 Without Lead Shims

DS2 and DS3 were taken without the lead shims in place. DS2 was taken immediately after the detector was brought back underground, however the polyethylene shield was not yet installed. Approximately 12 days later the polyethylene shield was stacked and the data acquisition for the light WIMP search in Chapter 7 began (DS3). Figure 5.3 clearly illustrates the enhancement due to cosmogenic activation in the number of $^{68}$Ge and $^{71}$Ge decays within the MALBEK detector after it was transported to-and-from Canberra. The decay observed in Figure 5.3 is dominated by the 11.43 half-life of $^{71}$Ge. A fit to the observed exponential decay in the count rates of the $^{68,71}$Ge K-capture line has been performed and agrees with the data (0.931 P-value, $\chi^2$/NDF = 0.834), this fit is shown in Figure 5.4. DS3 has been split into two separate run periods due to unscheduled power losses at KURF. During the month between DS3a and DS3b, power interruptions occurred every 1–3 days. This caused the gain of the detector to shift significantly over short timescales, making a reliable energy calibration in this region impossible, therefore these data have been omitted from this analysis.

5.1.3 Slow Signal Backgrounds and Lead Shims

Slow, energy-degraded, signals in PPC detectors have been reported in the literature, specifically Refs. [67, 73–75, 121, 136–146], and represent a significant low-energy background in the MALBEK detector. These signals arise from interactions near the lithium diffused n$^+$ contact. Diffusion and recombination processes are hypothesized to dominate the charge transport in this region of the detector, as opposed to the electrostatic transport that dominates in the bulk of the crystal. As will be discussed later, this causes the ionization signal to take longer to reach its maximum and leads to incomplete charge collection (energy degradation) by the acquisition system. Signals from this region of the detector are often called slow-signals, while events in the bulk are referred to as fast-signals. For reference, Figure 5.5 shows both a $\sim$20 keV slow- and fast-signal. Chapter 6 will discuss the underlying physical mechanism behind these slow-signals in detail. For now, it will be sufficient to understand
Figure 5.3: The number of counts per day in the $^{68,71}$Ge K-capture line versus time has been plotted for DS1c, DS3a and DS3b (see legend). Day 0 corresponds to 2011-06-20. The increase in counts observed in DS3a can be attributed to cosmogenic activation during the trip to Canberra.

Figure 5.4: A fit to the exponential decay due to $^{71}$Ge: $T_{1/2} = 11.43$ days and $^{68}$Ge: $T_{1/2} = 270.95$ days as a function of DS3 run time. The decay rates were fixed to their known values for this fit and the amplitudes were allowed to float. The fit to the observed count rate in the 10.367 keV peak agrees with the data, with a $\chi^2$ of 126.04 and 151 degrees of freedom, corresponding to a P-value of 0.931.
that slow-signals represent a background in MALBEK and Section 5.2.3 will cover how to mitigate these events.

DS1 (with lead shims) and DS3 (without lead shims) provide an excellent illustration of the magnitude of the slow-signal background when the lead shims were present. The mean free path of a 46.5 keV gamma from $^{210}\text{Pb}$ in germanium is 0.47 mm, so many of these gammas interact in the n$^+$ contact region resulting in a significant source of slow-signals. The most common technique to quantify the slow-ness of an ionization signal is to calculate the time a signal takes to rise from 10% to 90% of its maximum, or $t_{10-90}$. The value of $t_{10-90}$ depends on the type of HPGe detector, e.g. in most BEGe detectors, $t_{10-90} \sim 200-400$ ns for fast-signals and $t_{10-90} > 600$ ns for slow-signals. Figure 5.6 shows the $t_{10-90}$ versus energy distributions before and after the lead shims were removed, where only the first $\sim 40$ days of DS3a have been plotted in order to compare similar live times. A significant reduction in the number of slow-signals was observed between these two eras. The slow-signal background was reduced by more than an order of magnitude after the lead shims were removed, see Figure 5.7, which shows the binned energies of slow signals in both eras normalized by live time. The following section will discuss how these plots were generated, highlighting the energy and $t_{10-90}$ calculations as well as a new method developed to measure the slowness of a signal.

5.2 Digital Signal Processing

5.2.1 Overview

Digital signal processing (DSP) of waveforms allows for the extraction of useful characteristics, such as rise-time, energy, baseline, extrema values, derivative maxima, integral, etc. These parameters are essential for removing non-physics backgrounds. Off-line event-by-event processing also enables further digital processing of the waveform, e.g. waveform smoothing by wavelet de-noising or averaging.

The DSP of MALBEK waveform data was implemented in a tiered approach. Figure 5.8 illustrates the three tiers of analysis. The analysis chain starts when Tier 0 data (raw ORCA
Figure 5.5: An example of a slow-signal is shown in red and a fast-signal is shown in black. Notice that the slow-signal takes much longer than a microsecond to reach 90% of its maximum value, while the fast-signal takes less than half a microsecond. Both events have an energy of $\sim 20$ keV.

binary) is sent from the KURF DAQ machine to the UNC KillDevil cluster [147]. This is done every night at midnight with a bash script based on rsync that runs with crontab. During normal production data acquisition, ORCA saves a data file for each one hour long run. Tier 0 data is then processed with Majorana-ORCARoot (MJOR) that translates the ORCA binary data into ROOT [148] compatible TTrees. After processing, the newly created ROOT file is moved to the Tier 1 directory. Tier 1 files are then processed with the Germanium Analysis Toolkit (GAT) software package which was developed by members of the Majorana collaboration to process Geant4 and waveform data. GAT is a collection of C++ objects (or processors) that process entries in ROOT TTrees. GAT processors can be used to process waveform data or results from waveform data analysis. Two passes of GAT processing are performed on MALBEK data, with the first pass geared towards the DSP of each waveform in the file, where the $t_{10-90}$, un-calibrated energy, baseline, etc. are computed for each waveform. The second pass deals with timing analysis and energy calibrations. The output from the first wave of GAT processing is stored in the Tier 2 directory and the output
Figure 5.6: The 10%-90% rise-time distributions before and after the lead shims were removed. It is clear that the source of $^{210}\text{Pb}$ was the cause of a significant low-energy background due to slow-signals (this is also shown in Figure 5.7). The $t_{10-90}$ calculation is discussed in Section 5.2.3.
Figure 5.7: The energy spectrum corresponding to slow-signals. Solid (black): with lead shims in place; solid (blue): after lead shims were removed. See Section 5.2.3 for details on slow-signal discrimination techniques.
from the GAT timing analysis is stored in Tier 3. Tier 3 files also contain the results from GAT DSP. In summary, ORCA saves a data file for each one hour long run and each run is sent through three tiers of analysis as illustrated in Figure 5.8. A list of parameters saved for each waveform has been given in Table B.1 in Appendix B. The next two sections will outline how the energy and rise-time are calculated for each event as well as describe the energy calibration.

5.2.2 Energy Calculation

A symmetric trapezoidal filter was implemented in software to calculate the un-calibrated energy of each waveform [149, 150]. A trapezoidal filter can be summarized as a filter that transforms the typical signal generated by a preamplifier into a trapezoid that presents a flat top whose height is proportional to the amplitude of the input pulse, i.e. the energy of the event. The time duration in which the trapezoid is increasing/decreasing is the peaking time and the time duration that the trapezoid is flat is the gap time. Before the trapezoidal filtering is performed, the baseline of each waveform was removed. This is accomplished by averaging the first several samples of each waveform for a set period of time, i.e. the baseline averaging time (see Table 5.3). This average value is then subtracted from each data point in the waveform. Additionally, a pole-zero correction is performed on each waveform to remove the exponential decay. The baseline removal, pole-zero correction and trapezoidal filtering process are illustrated in Figure 5.9. The un-calibrated energy of each waveform was set to the maximum of the trapezoidal filtered waveform. The parameters used for the trapezoidal filter are listed in Table 5.3. These parameters were optimized based on a scan of the available parameter space (gap and peaking times) with an ORCA script. In this script, the on-board SIS3302 energy filter peaking time and gap times were set to user defined values. After the values have been initialized, the width of a pulser signal was measured to evaluate the energy resolution of the chosen gap and peaking times. Figure 5.10 illustrates the results of this scan. It is clear that to optimize the energy resolution, a small gap time, on the order of one microsecond, and a peaking time around 10 microseconds should be chosen. This scan could have been done offline in software, however it was faster to have ORCA perform the analysis.
Figure 5.8: MALBEK analysis tiered approach diagram. Data is sent from the DAQ to the UNC KillDevil cluster with a bash script based on rsync. This script runs automatically every night at midnight. Tier 0 files are processed with MJOR (Majorana ORCA-ROOT) and sent to the Tier 1 directory. Tier 1 files are processed with GAT (Germanium Analysis Toolkit). Event-by-event waveform DSP is performed during this step. The output from GAT DSP is sent to the Tier 2 directory. Tier 2 files are processed again with GAT, focusing on timing analysis and energy calibrations. The output from GAT Timing is sent to the Tier 3 directory.
Table 5.3: Parameters used for the trapezoidal filter of MALBEK data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Averaging Time</td>
<td>1–20 µs</td>
</tr>
<tr>
<td>Waveform Decay Constant</td>
<td>72.31 µs</td>
</tr>
<tr>
<td>Trapezoidal Filter Gap Time</td>
<td>1.0 µs</td>
</tr>
<tr>
<td>Trapezoidal Filter Peaking Time</td>
<td>11.0 µs</td>
</tr>
</tbody>
</table>

in real-time.

**Energy Calibration**

A reliable energy calibration in the high-gain (low-energy) channel is challenging, because any low-energy calibration source will be greatly attenuated by the cryostat and internal components. However, the low-energy internal X-rays listed in Table 5.2 provide a means to calibrate the detector at the lowest energies (<12 keV). These X-rays have been used to calibrate MALBEK in both DS3a and DS3b. There were significantly more counts in the X-ray peaks in DS3a, because the detector had been cosmogenically activated shortly before data taking had started (see Figure 5.3). In each data set, a subset of the peaks listed in Table 5.2 were fit with a Gaussian and background function (linear or linear + exponential) to extract the un-calibrated peak centroid. The un-calibrated peak centroids were then compared to their known energy values and fit with a linear equation to extract the peak ADC to energy relationship:

\[ E(x) = A + Bx, \]  

(5.1)

where \( x \) is the un-calibrated peak centroid in ADC units, \( E(x) \) is the energy in keV, and \( A \) and \( B \) are the fit parameters. The un-calibrated energy peak fits for both data sets are shown in Appendix C. The linear calibration curves are shown in Figures 5.11 (DS3a) and 5.12 (DS3b). It should be noted that in these figures, there are no y-errors since the y-variable (known energy) was assumed to have zero error. However, the x-errors are included - they are taken from the uncertainty in the fit to the un-calibrated energy. These errors are too small to be observed in these figures. The values of \( A \) and \( B \) were determined for each data set separately.
Figure 5.9: The trapezoidal filtering process is shown here: (a) raw waveform with a decay constant of 70.0 µs and a baseline average of 0.2 - the averaging region is shown in hatched blue, (b) baseline removed waveform, (c) pole-zero corrected waveform, i.e. removal of exponential decay, (d) the resulting trapezoidal filter on the pole-zero corrected and baseline removed waveform. The blue hatched regions are the peaking times (20 µs) and the green hatched region is the gap time (10 µs). The values for the peaking time and gap time here were chosen for image clarity and are not the values used in the MALBEK analysis, these are listed in Table 5.3. Alternatively, items (c) and (d) could be done simultaneously.
Figure 5.10: Optimal peaking time and gap time parameter space scan.
allowing one to combine both data sets after calibration (DS3).

Figure 5.11: DS3a linear calibration curve along with the residuals is shown here. There are no y-errors since the ‘Energy (keV)’ values were taken from literature, however the x-errors are included and are too small to be seen. The fitted parameters are: $A = 5.52 \times 10^{-2} \pm 2.68 \times 10^{-2} \text{ keV}$, $B = 3.60 \times 10^{-6} \pm 1.20 \times 10^{-8} \text{ keV/ADC}$.

**Energy Resolution**

After calibration, the shape of all peaks listed in Table 5.2 were fit for all three data sets. The fitting results are listed in Tables 5.4 (DS3a), 5.5 (DS3b) and 5.6 (DS3). Most peaks were fit with a linear background and a Gaussian signal. The only exception being the L-capture lines at 1.096 keV ($^{65}\text{Zn}$) and 1.299 keV ($^{68,71}\text{Ge}$). For these lines, an exponentially decaying
Figure 5.12: DS3b linear calibration curve along with the residuals is shown here. There are no y-errors since the ‘Energy (keV)’ values were taken from literature, however the x-errors are included and are too small to be seen. The fitted parameters are: $A = 1.15 \times 10^{-1} \pm 8.67 \times 10^{-3}$ keV, $B = 3.68 \times 10^{-6} \pm 4.00 \times 10^{-9}$ keV/ADC.
function was added near threshold. All of the fits in these tables are shown in Appendix C; see Section C.3 for DS3a, Section C.4 for DS3b and Section C.5 for DS3. The widths of the peaks were used to calculate the energy dependence of the energy resolution ($\sigma$), which was expected to follow this equation,

$$\sigma(E) = \sqrt{\sigma_e^2 + \eta FE + cE^2},$$  \hspace{1cm} (5.2)

where $\sigma_e$ is the electronic noise of the detector, which was measured to be 69.8 ± 0.2 eV for MALBEK. Figure 5.22 in Section 5.2.3 illustrates this for a data set taken with a pulser signal of varying amplitude. Also, as previously mentioned, the width of a pulser signal was measured during every run in DS3 – this is shown in Figures 5.37 and 5.38 of Section 5.4.1. $\eta$ is the energy required to generate an electron-hole pair (2.96 eV), $F$ is the Fano factor (see Section 2.2.6), $c$ is a constant that takes into account gain drift over long periods of time as well as charge trapping effects and $E$ is the energy of the gamma-ray [84, 87]. It was found that the best fit results were achieved by setting $c = 0$. The widths and energies of all data sets were fit with this equation, however here we only show the results from DS3a in Figure 5.13, because DS3a provided the best statistics in each of the low-energy peaks (due to cosmogenic activation). It should be reiterated (see Chapter 2) that the Fano factor has been measured to be between 0.057 and 0.129 in germanium [84]. The discrepancy between the accepted values for $F$ and what was measured is most likely due to low statistics in the peaks used for the fit.

**Energy Linearity**

It is also important to investigate the linearity of the MALBEK DAQ system. This has been done by comparing the calibrated energies with their published energies. The results are shown in Figures 5.14 (DS3a), 5.15 (DS3b), and 5.16 (DS3). For all three data sets, the calibrated energies and published energies are in agreement, indicating a satisfactory energy calibration.
Energy (keV)

\[ \sigma(E) = \sqrt{\sigma_E^2 + \eta FE} \]
\[ \eta = 2.96 \text{ eV} \]
\[ \sigma_E = 69.86 \text{ eV (fixed)} \]
\[ F = 0.19 \pm 0.02 \]
\[ \chi^2/\text{DOF} = 4.826/5 (0.965) \]
\[ \text{P-Value} = 0.4375 \]

Figure 5.13: Fit to the MALBEK energy resolution in DS3a.

Table 5.4: DS3a peak fitting results in the low-energy channel.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Energy (keV)</th>
<th>Centroid (keV)</th>
<th>(\sigma) (eV)</th>
<th>Count Rate ((\mu)Hz)</th>
<th>(\chi^2/\text{DOF})</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{65}\text{Zn}) L</td>
<td>1.096</td>
<td>1.07 ± 0.04</td>
<td>123.63 ± 11.48</td>
<td>4.29 ± 0.47</td>
<td>117.1/92 (1.27)</td>
<td>3.982e-02</td>
</tr>
<tr>
<td>(^{68,71}\text{Ge}) L</td>
<td>1.299*</td>
<td>1.31 ± 0.01</td>
<td>85.35 ± 9.60</td>
<td>13.14 ± 1.97</td>
<td>117.1/92 (1.27)</td>
<td>3.982e-02</td>
</tr>
<tr>
<td>(^{49}\text{V}) K</td>
<td>4.966</td>
<td>4.96 ± 0.02</td>
<td>68.31 ± 18.36</td>
<td>2.45 ± 0.83</td>
<td>38.6/46 (0.84)</td>
<td>7.726e-01</td>
</tr>
<tr>
<td>(^{55}\text{Fe}) K</td>
<td>6.539*</td>
<td>6.57 ± 0.02</td>
<td>75.51 ± 14.54</td>
<td>5.62 ± 1.17</td>
<td>38.5/46 (0.84)</td>
<td>7.746e-01</td>
</tr>
<tr>
<td>(^{65}\text{Zn}) K</td>
<td>8.979*</td>
<td>8.97 ± 0.01</td>
<td>106.43 ± 7.41</td>
<td>26.24 ± 1.92</td>
<td>59.8/90 (0.66)</td>
<td>9.941e-01</td>
</tr>
<tr>
<td>(^{68}\text{Ga}) K</td>
<td>9.659*</td>
<td>9.65 ± 0.02</td>
<td>83.84 ± 17.84</td>
<td>3.58 ± 0.91</td>
<td>59.8/90 (0.66)</td>
<td>9.941e-01</td>
</tr>
<tr>
<td>(^{68,71}\text{Ge}) K</td>
<td>10.367*</td>
<td>10.37 ± 0.00</td>
<td>103.88 ± 3.42</td>
<td>80.38 ± 3.12</td>
<td>59.8/90 (0.66)</td>
<td>9.941e-01</td>
</tr>
</tbody>
</table>

* These peaks were used to calibrate.
Figure 5.14: The linearity of DS3a is shown here.

Table 5.5: DS3b peak fitting results in the low-energy channel. Again, non-linearities in the SIS3302 give rise to poor agreement with published energies above 12 keV. The gain more than likely drifted during data taking giving rise to the larger $\sigma$ values as compared to DS3a.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Energy  (keV)</th>
<th>Centroid (keV)</th>
<th>$\sigma$ (eV)</th>
<th>Count Rate ($\mu$Hz)</th>
<th>$\chi^2$ / DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{65}$Zn L</td>
<td>1.096</td>
<td>1.07 ± 0.04</td>
<td>123.63 ± 7.08</td>
<td>3.51 ± 0.65</td>
<td>166.2/92 (1.81)</td>
<td>3.457e-06</td>
</tr>
<tr>
<td>$^{68,71}$Ge L</td>
<td>1.299*</td>
<td>1.33 ± 0.00</td>
<td>106.99 ± 16.06</td>
<td>4.73 ± 1.65</td>
<td>166.2/92 (1.81)</td>
<td>3.457e-06</td>
</tr>
<tr>
<td>$^{55}$Fe K</td>
<td>6.539*</td>
<td>6.55 ± 0.02</td>
<td>87.48 ± 25.81</td>
<td>4.19 ± 1.20</td>
<td>25.3/46 (0.55)</td>
<td>9.945e-01</td>
</tr>
<tr>
<td>$^{65}$Zn K</td>
<td>8.979*</td>
<td>8.98 ± 0.01</td>
<td>127.79 ± 11.05</td>
<td>17.62 ± 1.56</td>
<td>54.5/90 (0.61)</td>
<td>9.989e-01</td>
</tr>
<tr>
<td>$^{68}$Ga K</td>
<td>9.659*</td>
<td>9.66 ± 0.02</td>
<td>53.78 ± 18.96</td>
<td>1.98 ± 0.65</td>
<td>54.5/90 (0.61)</td>
<td>9.989e-01</td>
</tr>
<tr>
<td>$^{68,71}$Ge K</td>
<td>10.367*</td>
<td>10.37 ± 0.01</td>
<td>111.98 ± 7.25</td>
<td>23.74 ± 1.71</td>
<td>54.5/90 (0.61)</td>
<td>9.989e-01</td>
</tr>
</tbody>
</table>

* These peaks were used to calibrate.
Figure 5.15: The linearity of DS3b is shown here.
Figure 5.16: The linearity of DS3 is shown here.
Table 5.6: DS3a and DS3b combined peak fitting results. After calibration in both data sets, the peaks listed in Table 5.2 were fit to extract the resolution and centroids. The combined resolution is slightly worse than fitting the individual data sets.

<table>
<thead>
<tr>
<th>Peak</th>
<th>Energy (keV)</th>
<th>Centroid (keV)</th>
<th>$\sigma$ (eV)</th>
<th>Count Rate ($\mu$Hz)</th>
<th>$\chi^2$ / DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{65}$Zn L</td>
<td>1.096</td>
<td>1.13 ± 0.04</td>
<td>123.63 ± 76.11</td>
<td>3.55 ± 0.36</td>
<td>154.8/92 (1.68)</td>
<td>4.582e-05</td>
</tr>
<tr>
<td>$^{68,71}$Ge L</td>
<td>1.299</td>
<td>1.33 ± 0.05</td>
<td>92.29 ± 10.62</td>
<td>8.27 ± 1.24</td>
<td>154.8/92 (1.68)</td>
<td>4.582e-05</td>
</tr>
<tr>
<td>$^{49}$V K</td>
<td>4.966</td>
<td>4.99 ± 0.02</td>
<td>71.54 ± 38.82</td>
<td>1.92 ± 0.78</td>
<td>92.4/46 (2.01)</td>
<td>5.948e-05</td>
</tr>
<tr>
<td>$^{55}$Fe K</td>
<td>6.539</td>
<td>6.56 ± 0.01</td>
<td>81.04 ± 13.17</td>
<td>4.85 ± 0.81</td>
<td>69.9/46 (1.52)</td>
<td>1.304e-02</td>
</tr>
<tr>
<td>$^{65}$Zn K</td>
<td>8.979</td>
<td>8.97 ± 0.01</td>
<td>115.17 ± 6.28</td>
<td>21.65 ± 1.22</td>
<td>73.2/90 (0.81)</td>
<td>9.014e-01</td>
</tr>
<tr>
<td>$^{68}$Ga K</td>
<td>9.659</td>
<td>9.65 ± 0.02</td>
<td>75.53 ± 14.35</td>
<td>2.78 ± 0.56</td>
<td>73.2/90 (0.81)</td>
<td>9.014e-01</td>
</tr>
<tr>
<td>$^{68,71}$Ge K</td>
<td>10.367</td>
<td>10.37 ± 0.00</td>
<td>105.89 ± 3.13</td>
<td>50.41 ± 1.72</td>
<td>73.2/90 (0.81)</td>
<td>9.014e-01</td>
</tr>
</tbody>
</table>

5.2.3 Rise-Time Discrimination Techniques

Mitigating the slow-signal backgrounds discussed in Section 5.1.3 requires the ability to discriminate between slow and fast rise-time events. As stated earlier, a standard metric used to perform such discrimination is the 10%-90% rise-time ($t_{10-90}$) of the waveform, which is the amount of time it takes for the signal to rise from 10% to 90% of its maximum. In our application of the $t_{10-90}$ method, the waveform is smoothed via wavelet de-noising and then scanned to find the locations where the waveform rises from 10% to 90% of its maximum amplitude, see e.g. Figure 5.17. The algorithm chosen for wavelet de-noising is similar to those used by Refs. [74–76] which were based on Refs. [151–155].

Before we proceed, a brief introduction to wavelets and de-noising is required\(^2\). The Fourier Transform (FT) gives information about which frequencies are present within a signal, however the FT assumes that these frequencies are present at all times within the signal. The Continuous Wavelet Transform (CWT) of a signal $f(t)$, however, gives information about the frequencies ($s$, the scale) and their location on the waveform ($\tau$, the translation). The CWT is defined as

$$CWT(s, \tau) = \int f(t) \psi^*_s,\tau(t) dt,$$

\(^2\)For a more detailed overview of wavelet signal analysis, see Ref. [156].
Figure 5.17: A de-noised 6.67 keV signal is shown in red. The two vertical dashed lines (blue) indicate the 10% and 90% of the signal maximum and correspond to $t_{10-90} = 403.17$ ns.

where the wavelet function $\psi_{s,\tau}(t)$ is given by

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}} \psi_0 \left( \frac{t - \tau}{s} \right),$$

and $\psi_0 \left( \frac{t - \tau}{s} \right)$ is referred to as the mother wavelet. A change in scale changes the frequencies that a wavelet samples, or is sensitive to, e.g. larger scale, larger frequency. The mother wavelet chosen for this analysis is the Haar wavelet, a single cycle of the square wave [157]. The Haar wavelet was chosen as the basis because ionization signals resemble square waves or step functions.

The CWT can be written in terms of wavelet functions, which then allows one to think of the CWT as an ensemble of convolutions of wavelets with differing scales (frequencies). The CWT power ($= CWT(s, \tau)^2$) is then large for values of scale (frequencies) that are important in the signal. The wavelet transform is done numerically by applying high pass ($H_0$) and low pass ($G_0$) filters to the input signal. This is done to split the signal into a high frequency component that captures fine details of the signal and a low frequency...
component that encapsulates its overall shape. These filters only allow half of the frequency band through, therefore the signal is reduced by half, i.e. $i = 8192$ samples before filtering and $i = 4096$ samples after filtering. This is referred to as down-sampling. In order to retain the same signal length, the signal is padded with zeros – that is, zeros are placed in between each sample to bring the length of the filtered signal back up to 8192. $H_0$ yields a filtered signal referred to as detail coefficients ($c_{D}^{(i)}$). Similarly, $G_0$ yields a filtered signal referred to as approximation coefficients ($c_{A}^{(i)}$). Detail coefficients hold the data’s detailed information and approximation coefficients hold the data’s smooth information. This filtering process can be repeated on the approximation coefficients $n$ times yielding $n$ sets of approximation and detail coefficients. For the analysis that follows, this was repeated eight times ($n = 8$).

Another type of wavelet transform, which is very similar to the CWT, is the Stationary Wavelet Transform (SWT). Unlike the CWT, the SWT performs transformations at all possible translations, and an inverse SWT effectively averages these together. It should be quite obvious now that since wavelet transformations pick out the most important components of a signal, they can be used to remove the unimportant components as well - this is referred to as de-noising. De-noising is accomplished by setting unimportant detail coefficients to zero and performing an inverse transformation to produce a de-noised signal. The algorithm can be outlined in three steps:

1. perform a SWT using a Haar mother wavelet to $n = 8$ levels, thus creating $n$ sets of detail and approximation coefficients ($c_{D}^{(i)}(n)$ and $c_{A}^{(i)}(n)$ respectively, where $i = [0, 1, \ldots, 8192]$ is the number of samples),

2. set unimportant detail coefficients $c_{D}^{(i)}(n)$ to zero, and

3. perform an inverse transformation resulting in a de-noised signal.

This process is illustrated and explained in Figure 5.18.

At very low energies ($< 4$ keV) the low signal-to-noise ratio (S/N) can lead to large fluctuations in $t_{10-90}$, limiting its utility in the very region where it is needed most to discriminate slow-signals from potential dark matter signals. This is illustrated in Figure 5.19 which shows
Figure 5.18: The first four sets of detail coefficients for an $n = 8$ decomposition are shown along with their corresponding thresholds for a 6.67 keV signal. Noise reduction (de-noising) is obtained by setting all $c_D^{(i)}(n)$ that fall between the dashed lines to 0. For reference, time is on the x-axis (0-81.92 µs).
two waveforms along with their calculated 10% and 90% values. It is clear that the $t_{10-90}$ algorithm failed. A more sophisticated method has been developed based upon a wavelet parameter pulse shape discrimination technique. In addition to wavelet de-noising to level $n = 8$, this technique performs a pulse shape analysis (PSA) on the wavelet power spectrum, which is defined as,

$$\text{Power} = \left| c_D^{(i)}(n = 0) \right|^2,$$  \hspace{1cm} (5.5)

where $n$ is the level and $i$ is the number of samples. An example of the wavelet power spectrum of an ionization signal is shown in Figure 5.20. The wavelet power spectrum picks out the most important components of the signal. In the case of ionization signals, this is related to the information contained in the rising edge of the pulse. A new parameter was found that is also sensitive to the rise-time of the pulse, $w_{par}$, and is defined as:

$$w_{par} = \frac{\max \left( \left| c_D^{(i)}(n = 0) \right|^2 \right)}{E^2},$$  \hspace{1cm} (5.6)

where $E$ is the energy of the event as calculated by a standard trapezoidal filter (see Section 5.2.2). This parameter can be thought of as a ‘smoothed out’ derivative normalized by the square of the energy of the event. Since $w_{par}$ does not rely on scanning the waveform to locate $x\%$ of the pulse height, it is robust to a poor signal-to-noise ratio. Figure 5.21 illustrates the correlation between $t_{10-90}$ and $w_{par}$ for energies $>4$ keV (only plotted for $E > 4$ keV since the $t_{10-90}$ metric fails below 4 keV, making a comparison meaningless).

To test the efficacy of the $t_{10-90}$ and $w_{par}$ metrics, the attenuators and arbitrary waveform generator mentioned in Chapter 4 were used, see e.g. Figure 4.13. A set of waveforms with known rise-time ($t_{10-90} = 403$ ns, chosen to match as closely as possible to the MALBEK signal rise-times) and amplitude ($0.3 \rightarrow 6.67$ keV) were injected into the test input of the MALBEK preamplifier. The amplitude of the pulse was initially set to 6.67 keV and ramped down in 35 steps, with the step-size continually decreasing so as to obtain more statistics near threshold. The energy resolution, $\sigma$, of each step was measured as well and is shown in Figure 5.22. This not only allows one to see the noise performance at the lowest energies, but
Figure 5.19: Illustration of how poor S/N leads to large fluctuations in $t_{10-90}$
Figure 5.20: The $n = 0$ wavelet power spectrum is dashed (red) and the raw signal is solid (black). The apparent offset between the two signals is due to the numerical filtering process.

Figure 5.21: An intensity plot of $t_{10-90}$ versus $w_{par}$ for data collected with an external $^{241}\text{Am}$ source and energies between 4 and 58 keV. Note the clear correlation between the two parameters. The color scale on the right is in units of counts.
also allows one to see the varying step size. The signals collected were processed off-line in

Figure 5.22: The width of the pulser peak in MALBEK for the efficiency test runs with the attenuators. The data were fit with a flat function $f(x) = A$, where $A$ was allowed to float and is the electronic noise of MALBEK. The resulting fit yielded a 0.35 P-value, $\chi^2/\text{NDF} = 1.10$ and $A = 69.8 \pm 0.2$ eV. A fit was attempted with a linear function, however the slope term was consistent with zero.

order to calculate both the $t_{10-90}$ and $w_{\text{par}}$ values. The resulting distributions were then used to generate acceptance curves that include 99% of the fast-signals, see Figures 5.23 and 5.24. The spread at low energies of both curves is shown in Figures 5.23 and 5.24, where it is clear that the $w_{\text{par}}$ parameter is more stable. The $w_{\text{par}}$ 99% exclusion curve will be used to discriminate between slow- and fast-signals for the remainder of this dissertation and referred to as the slow-signal cut (SSC).

### 5.3 Data Cleaning

Data set 3 has been used to perform a light WIMP search, the results of which are reported in Chapter 7. The remainder of this chapter will focus on the methods used to remove non-physics events from the data (data cleaning) as well as the stability of MALBEK during data
Figure 5.23: 99% exclusion curves for $t_{10-90} = 403$ ns pulser signals. A pulser/attenuator system has been used to train the slow-signal cuts. (a) $t_{10-90}$ distribution and the 99% exclusion curve for pulser data. All events above the curve will be considered slow-signals. This will be referred to as the $t_{10-90}$ cut hereafter. It is clear that the $t_{10-90}$ cut fails at low energies and grossly overestimates the rise-time. The color scale on the right is in units of counts. (b) $w_{par}$ distribution and the 99% exclusion curve for pulser data. All events below the curve will be considered slow-signals. The stability of the $w_{par}$ parameter relative to the $t_{10-90}$ at low energies is clearly illustrated. The color scale on the right is in units of counts.
Figure 5.24: 99% exclusion curves with DS1c (with lead shims in place). Blue hatched regions indicate slow-pulse regions. (a) $t_{10-90}$ distribution and the 99% exclusion curve. (b) $w_{\text{par}}$ distribution and the 99% exclusion curve.
acquisition. The chapter will close with a summary of the potential systematics that will each be covered below.

5.3.1 Pulser, Inhibit, and LN\textsubscript{2} Cuts

There are several known sources of non-physics events that are directly correlated with various DAQ components. First, the energy resolution (electronic noise) of MALBEK is measured regularly during normal background runs. To do this, a pulser is injected into the test input of the preamplifier at a rate of 0.1 Hz. In order to obtain a physics-only energy spectrum, events coincident with this pulser signal are removed. Secondly, since the MALBEK preamplifier is a transistor reset preamplifier (see Section 2.2.5), events coincident with the inhibit signal from the preamplifier are removed. Lastly, vibrations from filling the detector and purge dewars with LN\textsubscript{2} result in microphonics or noise on the output of the MALBEK preamplifier. To avoid this, a 15 minute window is vetoed surrounding each LN\textsubscript{2} fill. This window is sufficiently long to allow the LN\textsubscript{2} transfer lines within the detector trailer to thaw. These three cuts will be referred to as the timing cuts hereafter.

5.3.2 Microphonics and Noise Cuts

As alluded to above, microphonic noise is caused by mechanical vibration of electronic circuit components with respect to surfaces at different potentials. Aside from the microphonics due to LN\textsubscript{2} fills, we observe two major classes of microphonics in MALBEK:

- **HV micro-discharges:** Reverse polarity pulses related to HV micro-discharges [121, 158]. The following sections outline the DSP methods used to remove these events.

- **Ringing:** We also observe waveforms that are triggered by electronic ringing. Again, the following sections outline the DSP methods used to remove these events.

To eliminate ringing and HV micro-discharge induced signals, a suite of three cuts has been developed. These cuts are applied in a tiered approach, where the first tier removes waveforms that are relatively easy to identify as microphonics and the second and third remove the remaining events. The first cut eliminates microphonic waveforms based on the derivative
amplitude to energy ratio \((A/E)\). This cut is used by the GERDA collaboration \cite{72} to discriminate between single- and multi-site events. This metric can also be used to identify non-physics signals. Figure 5.25a shows the \(A/E\) value versus energy with only the timing cuts applied. An example of a waveform removed by this cut is also shown in Figure 5.25b. To avoid confusion with a multi-site event cut, this cut will be referred to as the \textbf{max-min cut} hereafter. The second cut implemented is based on the technique developed in Ref. \cite{159}, which analyzes the energy calculated with two separate shaping times. This is done off-line by performing two separate trapezoidal filters with differing peaking times (11 \(\mu s\) and 5 \(\mu s\)). This cut will be referred to as the \textbf{microphonics cut} hereafter\footnote{For historical reasons only - Ref. \cite{76} used the same cut and referred to it as his Microphonics Cut.}. Figure 5.26a shows the microphonics cut, where the max-min cut has already been applied illustrating that non-physics related backgrounds remain following the max-min cut. The curves shown have been calculated in the same manner as the \(w_{\text{par}}\) and \(t_{10–90}\) acceptance curves, injecting known waveforms into the test input of the preamplifier and calculating acceptance curves such that 99\% of pulser signals fall between them. The third cut applied to remove microphonics/noise is based upon the integral, or sum of the ADC values, of the waveform and will be referred to as the \textbf{integral cut} hereafter. Figure 5.27a shows the integral cut, where again the max-min cut as already been applied to the data in Figure 5.27a. The curves have been generated in the same manner as the microphonics cut. It should be noted that the integral and microphonics cuts both remove a small fraction of slow-signals (\(<3\%\) of the total slow-signal population) from the energy spectrum.

In addition to the max-min, integral and microphonics cuts, a cut based on waveform health has been implemented. The slope of each waveform baseline is calculated and the resulting distribution is fit with a Gaussian function of width \(\sigma\) for both DS3a and DS3b. Events with a baseline slope outside a \(\pm 3\sigma\) window are removed. This will be referred to as the \textbf{baseline slope cut} hereafter. This cut removes pile-up events, or events which occur close in time to a previous trigger. The baseline slope distributions for DS3a and DS3b are shown in Figure 5.28 – both fits have P-values of 0.998. Additionally, an example of a pile-up waveform is shown in Figure 5.29. This cut removes \(<0.05\%\) of all events.
Figure 5.25: (a) The derivative maximum divided by the energy of the waveform versus energy for DS3a is shown here. All events which fall above the blue line are microphonics. The location of the blue line was chosen such that the two classes of events here are completely separated for energies greater than 600 eV. (b) An example of a ringing waveform that is removed with this cut.
Figure 5.26: (a) The microphonics cut for DS3a is shown here. 99% of events fall between the curves as trained with a pulser. The max-min cut has already been applied to these data in advance. (b) An example of a HV micro-discharge waveform that is removed with this cut.
Figure 5.27: (a) The integral cut for DS3a is shown here. 99% of events fall between the curves as trained with a pulser. The max-min cut has already been applied to these data in advance. (b) An example HV micro-discharge waveform that is removed with this cut.
Figure 5.28: The baseline slope distributions are shown here for DS3a and DS3b. For both data sets, a Gaussian function (red) was used to fit the distribution. The vertical red dashed lines indicate the $\pm 3\sigma$ boundaries.

Figure 5.29: A typical pile-up event removed with the baseline slope cut.
5.3.3 Slow Signal Cuts

The SSC presented in Section 5.2.3 is used to discriminate between slow- and fast-signals. Figure 5.30 shows the SSC cuts for both DS3a and DS3b, the blue hatched region is the region of slow-signals. Any events falling in the blue hatched region are removed. The next section will summarize the cuts developed and the order in which they are applied to the data.

5.3.4 Order of Cuts Applied

The cuts presented in the previous sections have been applied to the data in the following order:

1. Apply pulser and inhibit cuts,
2. Apply LN\textsubscript{2} cut,
3. Apply max-min cut and baseline slope cuts (preliminary cut),
4. Apply integral and microphonics cut,
5. Apply SSC.

Item (3) will be referred to as the preliminary cut hereafter since these are done before applying the more sophisticated integral, microphonics and SSC cuts. The integral and microphonics cuts have been grouped together below since they are semi-degenerate, in that they both remove microphonic events, primarily those related to HV micro-discharge.

Cuts Applied to Data Set 3a

The effect of the cuts in the previous section on the DS3a energy spectrum is shown in Figure 5.31. Table 5.7 also shows the number of events that pass each cut and Figure 5.32 contains the energy spectra of cut events for each cut applied to the data.

Cuts Applied to Data Set 3b

The effect of the cuts in the previous section on the DS3b energy spectrum is shown in Figure 5.33. Table 5.8 also shows the number of events that pass each cut and Figure 5.34
Figure 5.30: The SSC cuts for DS3a and DS3b from 0.6→12 keV are illustrated here. Slow signals are located in the blue hatched region.
Table 5.7: Cut Percentages in DS3a.

### 0.6 - 2.0 keV

<table>
<thead>
<tr>
<th>Cut</th>
<th>N Processed</th>
<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>71386</td>
<td>23255</td>
<td>48131</td>
<td>32.58</td>
</tr>
<tr>
<td>Preliminary</td>
<td>23255</td>
<td>644</td>
<td>22611</td>
<td>2.77</td>
</tr>
<tr>
<td>Integral</td>
<td>644</td>
<td>628</td>
<td>16</td>
<td>97.52</td>
</tr>
<tr>
<td>Microphonics</td>
<td>644</td>
<td>608</td>
<td>36</td>
<td>94.41</td>
</tr>
<tr>
<td>SSC</td>
<td>644</td>
<td>394</td>
<td>250</td>
<td>61.18</td>
</tr>
</tbody>
</table>

### 0.6 - 12.0 keV

<table>
<thead>
<tr>
<th>Cut</th>
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<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>76513</td>
<td>28352</td>
<td>48161</td>
<td>37.06</td>
</tr>
<tr>
<td>Preliminary</td>
<td>28352</td>
<td>3072</td>
<td>25280</td>
<td>10.84</td>
</tr>
<tr>
<td>Integral</td>
<td>3072</td>
<td>3044</td>
<td>28</td>
<td>99.09</td>
</tr>
<tr>
<td>Microphonics</td>
<td>3072</td>
<td>2903</td>
<td>169</td>
<td>94.50</td>
</tr>
<tr>
<td>SSC</td>
<td>3072</td>
<td>2302</td>
<td>770</td>
<td>74.93</td>
</tr>
</tbody>
</table>

### 0.6 - 150.0 keV

<table>
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<th>N Processed</th>
<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>81684</td>
<td>28454</td>
<td>53230</td>
<td>34.83</td>
</tr>
<tr>
<td>Preliminary</td>
<td>28454</td>
<td>7638</td>
<td>20816</td>
<td>26.84</td>
</tr>
<tr>
<td>Integral</td>
<td>7638</td>
<td>7579</td>
<td>59</td>
<td>99.23</td>
</tr>
<tr>
<td>Microphonics</td>
<td>7638</td>
<td>7341</td>
<td>297</td>
<td>96.11</td>
</tr>
<tr>
<td>SSC</td>
<td>7638</td>
<td>6246</td>
<td>1392</td>
<td>81.78</td>
</tr>
</tbody>
</table>
Figure 5.31: The DS3a energy spectrum is shown here after various cuts.

contains the energy spectra of cut events for each cut applied to the data.

Stability of the L- and K- Capture Lines in Data Set 3

The ratios of the Ge and Zn L-to-K capture line amplitudes have been well understood since the early 1960s [160, 161] and provide a useful systematic check of the stability of an energy-dependent cut applied near these lines. The amplitudes of the Ge and Zn K and L lines were measured using an unbinned fit to the DS3 (DS3a + DS3b) energy spectrum with the following cuts applied:

- Preliminary,
- Preliminary + Integral + Microphonics,
- Preliminary + Integral + Microphonics + SSC.

Table 5.9 lists the ratios that one would expect if each cut retained all physics-related events. Low statistics in DS3 for the $^{65}$Zn L-capture line ($< 60$ counts over 221.49 days) led to little
Figure 5.32: Various energy spectra from cut events in DS3a. (a) shows the energy spectrum from cut events with the LN$_2$ cut; (b) shows the same for the preliminary cut; the feature shown at $\sim$5 keV is due to HV micro-discharges whose minimum ADC values exceed that available on the SIS3302 (see Figure 5.35), (c) shows the integral cut, (d) microphonics cut, (e) SSC cut.
Table 5.8: Cut percentages in DS3b.

### 0.6 - 2.0 keV

<table>
<thead>
<tr>
<th>Cut</th>
<th>N Processed</th>
<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>119688</td>
<td>32445</td>
<td>87243</td>
<td>27.11</td>
</tr>
<tr>
<td>Preliminary</td>
<td>32445</td>
<td>829</td>
<td>31616</td>
<td>2.56</td>
</tr>
<tr>
<td>Integral</td>
<td>829</td>
<td>710</td>
<td>119</td>
<td>85.65</td>
</tr>
<tr>
<td>Microphonics</td>
<td>829</td>
<td>714</td>
<td>115</td>
<td>86.13</td>
</tr>
<tr>
<td>SSC</td>
<td>829</td>
<td>397</td>
<td>432</td>
<td>47.89</td>
</tr>
</tbody>
</table>

### 0.6 - 12.0 keV

<table>
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<th>Cut</th>
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<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>125785</td>
<td>38515</td>
<td>87270</td>
<td>30.62</td>
</tr>
<tr>
<td>Preliminary</td>
<td>38515</td>
<td>2799</td>
<td>35716</td>
<td>7.27</td>
</tr>
<tr>
<td>Integral</td>
<td>2799</td>
<td>2673</td>
<td>126</td>
<td>95.50</td>
</tr>
<tr>
<td>Microphonics</td>
<td>2799</td>
<td>2539</td>
<td>260</td>
<td>90.71</td>
</tr>
<tr>
<td>SSC</td>
<td>2799</td>
<td>1762</td>
<td>1037</td>
<td>62.95</td>
</tr>
</tbody>
</table>

### 0.6 - 150.0 keV

<table>
<thead>
<tr>
<th>Cut</th>
<th>N Processed</th>
<th>N Passed</th>
<th>N Rejected</th>
<th>% Passed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN₂</td>
<td>133024</td>
<td>41104</td>
<td>91920</td>
<td>30.90</td>
</tr>
<tr>
<td>Preliminary</td>
<td>41104</td>
<td>8852</td>
<td>32252</td>
<td>21.54</td>
</tr>
<tr>
<td>Integral</td>
<td>8852</td>
<td>8712</td>
<td>140</td>
<td>98.42</td>
</tr>
<tr>
<td>Microphonics</td>
<td>8852</td>
<td>8437</td>
<td>415</td>
<td>95.31</td>
</tr>
<tr>
<td>SSC</td>
<td>8852</td>
<td>6938</td>
<td>1914</td>
<td>78.38</td>
</tr>
</tbody>
</table>
Figure 5.33: The DS3b energy spectrum is shown here after various cuts.

Information gained from this analysis. However, the $^{68}\text{Ge}$ K- and L-capture lines provided enough counts to perform this systematic check. Since the $^{68}\text{Ge}$ L-line and $^{65}\text{Zn}$ L-line are very close in energy (1.299 keV, 1.096 keV respectively), a cross-check with the expected $^{68}\text{Ge}$ L/K capture ratio was assumed to be sufficient. The measured Ge L/K capture ratio was found to be consistent with published values after each cut and the results are listed in Table 5.10. The fact that the measured $^{68}\text{Ge}$ L/K capture ratio was consistent with the expected value provides additional confidence that the SSC does not remove fast-signals (at least down to 1.29 keV).

Table 5.9: Expected L/K capture ratios for $^{68}\text{Ge}$ and $^{65}\text{Zn}$.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{68}\text{Ge}$</td>
<td>0.126</td>
<td>[160, 162, 163]</td>
</tr>
<tr>
<td>$^{65}\text{Zn}$</td>
<td>0.108</td>
<td>[161–163]</td>
</tr>
</tbody>
</table>
Figure 5.34: Various energy spectra from cut events in DS3b. (a) shows the energy spectrum from cut events with the LN$_2$ cut; (b) shows the same for the preliminary cut; the feature shown at $\sim 5$ keV is due to HV micro-discharges whose ADC values exceed that available on the SIS3302 (see Figure 5.35), (c) shows the integral cut, (d) microphonics cut, (e) SSC cut.
Figure 5.35: A typical $\sim$5 keV HV micro-discharge waveform is shown here.

Table 5.10: Measured L/K capture ratios for $^{68}$Ge. This is not an error, all of the measured ratios were found to be equal to their most significant digit. The theoretical value was taken to be 0.126, see Table 5.9.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Value</th>
<th>Theory/Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preliminary</td>
<td>0.14 ± 0.02</td>
<td>0.9 ± 0.1</td>
</tr>
<tr>
<td>Preliminary + Integral + Microphonics</td>
<td>0.14 ± 0.02</td>
<td>0.9 ± 0.1</td>
</tr>
<tr>
<td>Preliminary + Integral + Microphonics + SSC</td>
<td>0.14 ± 0.02</td>
<td>0.9 ± 0.1</td>
</tr>
</tbody>
</table>
5.3.5 Cut Efficiencies

The pulser/attenuator data discussed in Section 5.2.3 has also been used to calculate cut acceptance efficiencies for physics events as a function of energy. The true number of pulser events has been tracked by digitizing the sync output of the pulser while sending the other output through attenuators and eventually into the test input of the MALBEK preamplifier. Each cut outlined above has been applied to the pulser data. The efficiency is the ratio of the number of observed events after cuts to the true (expected) number of pulser events. The results from these calculations are shown in Figure 5.36. The conservative 600 eV threshold implemented for the light WIMP search in Chapter 7 is well above where the efficiencies begin to fall to zero near 550 eV. Additionally, setting the threshold to 600 eV allows us to completely ignore threshold drift effects, which were estimated to be \(\sim 6.8\) eV (see Section 5.4.1).

Figure 5.36: The cut efficiencies versus energy is shown here (results from a fit to data with an error function). Inhibits have been removed from the pulser efficiency runs. Also, the integral and microphonics cuts have been grouped together since they are semi-degenerate.
5.4 Stability

The ability to operate in steady-state for an extended period of time is crucial for low-mass WIMP searches, the primary reason being that the region of interest (near threshold) is also the most susceptible to environmental and noise fluctuations. As alluded to in the previous chapter, operating the SIS3302 in special read-out mode caused disruptions in data acquisition. The following sections outline the detector health versus time, SIS3302 special read-out mode and the Poisson nature of both data sets.

5.4.1 Detector Health Versus Time

Various parameter values have been monitored as a function of time throughout DS3a and DS3b in an attempt to quantify the stability of the MALBEK detector. Figures 5.37 and 5.38 show the pulser mean, pulser width, baseline RMS, baseline value, preamplifier reset rate, and LN$_2$ dewar levels for every one hour long run for DS3a and DS3b, respectively. It is clear that the pulser mean fluctuates, with maximal displacement of $\sim 400$ eV. It is impossible to determine whether the shift in pulser location is due to changes in the gain of the MALBEK signal chain, or changes in the gain of the pulser itself. Assuming that the shifts in pulser mean are entirely due to gain shifts of the signal chain results in a shift of $\sim 6.8$ eV at the 600 eV energy threshold of the detector. Therefore, it is safe to conclude that threshold drift is negligible. The pulser width remained very stable throughout operation, which indicates that the electronic noise in the system was constant. The preamplifier reset rate shows spikes as well as high-reset rate runs that appear to be outliers. The spikes are due to the detector being un-biased and re-biased (indicated by blue vertical lines in Figures 5.37 and 5.38). The reason for re-biasing will be discussed in Section 5.4.2. Since the electronic noise is stable (pulser width vs. time), the increase in preamplifier reset rate (leakage current) doesn’t affect the quality of the data. If the increased leakage current had been a problem, we would have seen a noticeable change in the pulser width as a function of time. The high-reset rate runs, which appear to be outliers, correspond to LN$_2$ fills. The LN$_2$ levels were kept as constant as possible in an effort to minimize temperature of the FET, and therefore the leakage current,
however due to unforeseen circumstances with LN$_2$ availability underground, a longer time period between fills was required on more than one occasion. The baseline value was also monitored even though the value is subtracted on an event-by-event basis. The baseline value remained stable throughout operation in DS3a and DS3b.

Figure 5.37: Various parameters as a function of run number for DS3a. Blue vertical lines indicate that the SIS3302 card needed to be reset at the beginning of that run. For reference, each run is one hour long. From the top down: (1) pulser mean, (2) pulser width, (3) baseline RMS, (4) baseline value, (5) preamplifier reset rate, (6) detector [black] and purge [red] dewar levels.
Figure 5.38: Various parameters as a function of run number for DS3b. Blue vertical lines indicate that the SIS3302 card needed to be reset at the beginning of that run. For reference, each run is one hour long. From the top down: (1) pulser mean, (2) pulser width, (3) baseline RMS, (4) baseline value, (5) preamplifier reset rate, (6) detector [black] and purge [red] dewar levels.
5.4.2 SIS3302 Special Mode Stability

The disadvantage to using special mode (see Section 4.5) is that occasionally the SIS3302 memory buffers need to be cleared. At the time of writing, the cause of this is unknown. Currently, the only way to clear the buffers is to perform a SYSRESET of the VME64x crate which resets all cards within the crate. Unfortunately, the MALBEK high voltage supply is also housed in the VME64x crate and is reset whenever the buffers are cleared. The MALBEK DAQ has been configured to send an email notification whenever the SIS3302 needs to be reset, allowing for the controlled ramp-down of the high voltage on MALBEK before performing a SYSRESET. It is thought that the MALBEK leakage current, which is directly proportional to the preamplifier reset rate, increases following a re-bias of the detector due to electrons being trapped within the crystal. It takes several hours before these electrons are swept out of the bulk allowing for the reset rate to return to its nominal level. The electronic noise, and hence the leakage current, has been monitored with a pulser during every run. The width of the pulser peak has been observed to be very stable during both DS3a and DS3b despite the frequent need to re-bias the detector, see the blue vertical lines in Figures 5.37 and 5.38. Therefore, it has been concluded that the effect of increased leakage current after a re-bias has little or no effect. In the future, in order to investigate the effect of leakage current on resolution, one could make noise measurements as a function of temperature.

5.4.3 Poisson Distribution of Event Timing in Data Sets 3a and 3b

The detector performance, specifically the electronic noise, has been shown to be stable throughout DS3a and DS3b. It is also important to study the event rates and time differences between events in various energy regions. The energy regions below were used to perform a comprehensive timing analysis:

\[
\text{Energy Regions} = \begin{cases} 
0.6 \rightarrow 1.0 \text{ keV (covers the threshold region),} \\
2.0 \rightarrow 8.0 \text{ keV (covers the } \sim \text{flat region),}
\end{cases}
\]
The time since last event (backwards timing), time until next event (forward timing), nearest time difference between events (lesser of the previous two) and the number of counts in an 8 hour time window (count rates) have been calculated in each of the energy regions listed above. Furthermore, each calculation was performed after each cut was applied. These analyses provide a way to search time correlated backgrounds. Provided that the cosmogenic backgrounds within the data have half-lives much longer than the live time of the data set (which is not always the case), the backwards timing, forward timing and lesser of the two are all expected to exhibit an exponential distribution,

\[ f(t) = Ce^{\lambda t}, \]  

(5.7)

where \( C \) and \( \lambda \) are floating parameters. Additionally, the last test (count rates) is expected to resemble a Poisson distribution whose mean is the only floating parameter. The full results from fitting the backwards timing, forward timing and lesser of the two with Equation 5.7 are listed in Tables 5.12 (DS3a) and 5.13 (DS3b). It is clear that due to the presence of \(^{71}\)Ge (\( T_{1/2} = 11.43 \) days) in DS3a during the first \( \sim 40 \) days of data taking, the fits were poor. However, satisfactory agreement was found in DS3b. The results from the count rate study are shown in Table 5.11. For illustration purposes, the results from the time since last event study for DS3a in the 2.0 → 8.0 keV energy region, which are representative of all energy regions studied in both DS3a and DS3b, are shown in Figure 5.39. Similarly, representative results from the count rate study for DS3b in both energy regions are shown in Figures 5.40 and 5.41. These results indicate that both data sets do not contain bursts of events arriving in a non-Poisson fashion.
Figure 5.39: Time since last event in the $2.0 \rightarrow 8.0$ keV energy region for DS3b after all cuts. The data were fit to Equation 5.7 and are plotted in (a) and the residuals are shown in (b).
Figure 5.40: The number of counts in a predefined time window in the 0.6 → 1.0 keV energy region for DS3b after various cuts are shown here. The data were fit to a Poisson distribution.
Figure 5.41: The number of counts in a predefined time window in the 2.0 → 8.0 keV energy region for DS3b after various cuts are shown here. The data were fit to a Poisson distribution.
Table 5.11: Results from the count rate study of DS3. The data were fit with a Poisson distribution. The Cuts column has been abbreviated as follows: SSC = Slow Signal Cut, Pre = Preliminary Cuts, LN = LN Cut, I&M = Integral and Microphonics. The cuts have been applied in the order outlined in Section 5.3.4, e.g. if only Preliminary is listed below, the LN cut has also been applied. Also, the integral and microphonics cuts have been grouped together.

**Data Set 3a**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Cuts</th>
<th>Mean (Counts / 12 Hours)</th>
<th>$\chi^2$</th>
<th>DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>SSC</td>
<td>5.47e-01 ± 5.29e-02</td>
<td>6.18</td>
<td>5</td>
<td>2.89e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>SSC</td>
<td>3.16e+00 ± 1.28e-01</td>
<td>4.09</td>
<td>10</td>
<td>9.43e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>I&amp;M</td>
<td>9.20e-01 ± 6.77e-02</td>
<td>1.98</td>
<td>7</td>
<td>9.61e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>I&amp;M</td>
<td>4.61e+00 ± 1.53e-01</td>
<td>4.72</td>
<td>12</td>
<td>9.67e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>Pre</td>
<td>1.01e+00 ± 7.04e-02</td>
<td>1.60</td>
<td>7</td>
<td>9.79e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>Pre</td>
<td>5.12e+00 ± 1.63e-01</td>
<td>5.42</td>
<td>12</td>
<td>9.43e-01</td>
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<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>LN</td>
<td>5.65e+01 ± 5.19e-01</td>
<td>35.13</td>
<td>100</td>
<td>1.00e+00</td>
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<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>LN</td>
<td>1.56e+01 ± 2.73e-01</td>
<td>6.97</td>
<td>40</td>
<td>1.00e+00</td>
</tr>
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</table>

**Data Set 3b**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Cuts</th>
<th>Mean (Counts / 12 Hours)</th>
<th>$\chi^2$</th>
<th>DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>SSC</td>
<td>5.20e-01 ± 5.02e-02</td>
<td>7.64</td>
<td>5</td>
<td>1.77e-01</td>
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<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>SSC</td>
<td>2.74e+00 ± 1.08e-01</td>
<td>3.28</td>
<td>10</td>
<td>9.74e-01</td>
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<td>0.6 $\rightarrow$ 1.0</td>
<td>I&amp;M</td>
<td>1.12e+00 ± 6.89e-02</td>
<td>0.95</td>
<td>7</td>
<td>9.96e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>I&amp;M</td>
<td>4.25e+00 ± 1.35e-01</td>
<td>2.68</td>
<td>12</td>
<td>9.97e-01</td>
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<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>Pre</td>
<td>1.59e+00 ± 8.28e-02</td>
<td>0.85</td>
<td>7</td>
<td>9.97e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>Pre</td>
<td>4.76e+00 ± 1.44e-01</td>
<td>7.42</td>
<td>12</td>
<td>8.29e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>LN</td>
<td>6.91e+01 ± 5.42e-01</td>
<td>42.43</td>
<td>100</td>
<td>1.00e+00</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>LN</td>
<td>1.87e+01 ± 2.82e-01</td>
<td>18.88</td>
<td>40</td>
<td>9.98e-01</td>
</tr>
</tbody>
</table>
Table 5.12: Results from the exponential fitting to the time since last, time to next and nearest event time distributions for DS3a over various energy ranges. The Cuts column has been abbreviated as follows: SSC = Slow Signal Cut, Pre = Preliminary Cuts, LN$_2$ = LN$_2$ Cut, I&M = Integral and Microphonics. The cuts have been applied in the order outlined in Section 5.3.4, e.g. if only Pre is listed below, the LN$_2$ cut has also been applied. Also, the integral and microphonics cuts have been grouped together.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Test</th>
<th>Cuts</th>
<th>$C$ (counts)</th>
<th>$\lambda$ (hr$^{-1}$)</th>
<th>$\chi^2$</th>
<th>DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 → 1.0</td>
<td>TTN</td>
<td>SSC</td>
<td>1.43e+00 ± 3.54e-01</td>
<td>-1.27e-02 ± 2.03e-02</td>
<td>9.67</td>
<td>44</td>
<td>1.00e+00</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TSL</td>
<td>SSC</td>
<td>2.39e+00 ± 4.76e-01</td>
<td>-6.99e-02 ± 3.79e-02</td>
<td>28.23</td>
<td>38</td>
<td>8.76e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>NT</td>
<td>SSC</td>
<td>2.42e+00 ± 4.91e-01</td>
<td>-6.92e-02 ± 4.07e-02</td>
<td>30.16</td>
<td>36</td>
<td>7.42e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TTN</td>
<td>I&amp;M</td>
<td>2.34e+00 ± 3.84e-01</td>
<td>-3.68e-02 ± 1.53e-02</td>
<td>39.51</td>
<td>69</td>
<td>9.98e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TSL</td>
<td>I&amp;M</td>
<td>6.51e+00 ± 9.14e-01</td>
<td>-1.65e-01 ± 2.98e-02</td>
<td>31.87</td>
<td>52</td>
<td>9.87e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>NT</td>
<td>I&amp;M</td>
<td>6.74e+00 ± 9.55e-01</td>
<td>-1.79e-01 ± 3.27e-02</td>
<td>39.73</td>
<td>47</td>
<td>7.65e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TTN</td>
<td>Pre</td>
<td>2.64e+00 ± 3.83e-01</td>
<td>-4.00e-02 ± 1.34e-02</td>
<td>48.41</td>
<td>73</td>
<td>9.88e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TSL</td>
<td>Pre</td>
<td>7.56e+00 ± 1.05e+00</td>
<td>-1.62e-01 ± 2.71e-02</td>
<td>46.71</td>
<td>52</td>
<td>6.81e-01</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>NT</td>
<td>Pre</td>
<td>5.26e+00 ± 7.56e-01</td>
<td>-1.26e-01 ± 2.76e-02</td>
<td>74.61</td>
<td>47</td>
<td>6.33e-03</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TTN</td>
<td>LN$_2$</td>
<td>1.39e+04 ± 1.98e+02</td>
<td>-4.79e+00 ± 4.71e-02</td>
<td>54.10</td>
<td>33</td>
<td>1.17e-02</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>TSL</td>
<td>LN$_2$</td>
<td>1.30e+03 ± 1.65e+01</td>
<td>-4.57e+00 ± 3.88e-02</td>
<td>639.49</td>
<td>68</td>
<td>8.01e-94</td>
</tr>
<tr>
<td>0.6 → 1.0</td>
<td>NT</td>
<td>LN$_2$</td>
<td>1.22e+03 ± 1.68e+01</td>
<td>-9.81e+00 ± 9.01e-02</td>
<td>289.58</td>
<td>73</td>
<td>1.43e-27</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TTN</td>
<td>SSC</td>
<td>3.34e+01 ± 2.28e+00</td>
<td>-2.52e-01 ± 1.45e-02</td>
<td>56.59</td>
<td>75</td>
<td>9.44e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TSL</td>
<td>SSC</td>
<td>4.43e+01 ± 2.95e+00</td>
<td>-5.05e-01 ± 2.84e-02</td>
<td>39.47</td>
<td>49</td>
<td>8.33e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>NT</td>
<td>SSC</td>
<td>3.34e+01 ± 2.24e+00</td>
<td>-7.07e-01 ± 4.15e-02</td>
<td>47.47</td>
<td>57</td>
<td>8.12e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TTN</td>
<td>I&amp;M</td>
<td>8.08e+01 ± 4.15e+00</td>
<td>-3.75e-01 ± 1.53e-02</td>
<td>50.04</td>
<td>59</td>
<td>7.91e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TSL</td>
<td>I&amp;M</td>
<td>5.08e+01 ± 2.67e+00</td>
<td>-5.67e-01 ± 2.41e-02</td>
<td>78.02</td>
<td>71</td>
<td>2.66e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>NT</td>
<td>I&amp;M</td>
<td>6.62e+01 ± 3.14e+00</td>
<td>-9.27e-01 ± 3.73e-02</td>
<td>75.14</td>
<td>58</td>
<td>6.45e-02</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TTN</td>
<td>Pre</td>
<td>1.00e+02 ± 4.69e+00</td>
<td>-4.33e-01 ± 1.52e-02</td>
<td>70.42</td>
<td>58</td>
<td>1.27e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TSL</td>
<td>Pre</td>
<td>6.45e+01 ± 3.09e+00</td>
<td>-5.62e-01 ± 2.11e-02</td>
<td>71.57</td>
<td>69</td>
<td>3.93e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>NT</td>
<td>Pre</td>
<td>8.22e+01 ± 3.85e+00</td>
<td>-9.68e-01 ± 3.41e-02</td>
<td>67.02</td>
<td>60</td>
<td>2.49e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TTN</td>
<td>LN$_2$</td>
<td>1.08e+03 ± 2.65e+01</td>
<td>-1.31e+00 ± 2.25e-02</td>
<td>38.41</td>
<td>39</td>
<td>4.96e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>TSL</td>
<td>LN$_2$</td>
<td>2.72e+02 ± 6.74e+00</td>
<td>-1.34e+00 ± 2.40e-02</td>
<td>71.96</td>
<td>70</td>
<td>4.13e-01</td>
</tr>
<tr>
<td>2.0 → 8.0</td>
<td>NT</td>
<td>LN$_2$</td>
<td>2.92e+02 ± 7.25e+00</td>
<td>-2.55e+00 ± 4.44e-02</td>
<td>169.95</td>
<td>69</td>
<td>1.73e-10</td>
</tr>
</tbody>
</table>
Table 5.13: Results from the exponential fitting to the time since last (TSL), time to next (TTN) and nearest event time (NT) distributions for DS3b over various energy ranges. The Cuts column has been abbreviated as follows: SSC = Slow Signal Cut, Pre = Preliminary Cuts, LN2 = LN2 Cut, I&M = Integral and Microphonics. The cuts have been applied in the order outlined in Section 5.3.4, e.g. if only Pre is listed below, the LN2 cut has also been applied. Also, the integral and microphonics cuts have been grouped together.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Test</th>
<th>Cuts</th>
<th>$C$ (counts)</th>
<th>$\lambda$ (hr$^{-1}$)</th>
<th>$\chi^2$</th>
<th>DOF</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TTN</td>
<td>SSC</td>
<td>1.96e+00 ± 3.70e-01</td>
<td>-3.16e-02 ± 1.80e-02</td>
<td>24.07</td>
<td>58</td>
<td>1.00e+00</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TSL</td>
<td>SSC</td>
<td>6.75e+00 ± 1.15e+00</td>
<td>-1.47e+00 ± 2.12e-02</td>
<td>34.24</td>
<td>43</td>
<td>8.27e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>NT</td>
<td>SSC</td>
<td>6.74e+00 ± 1.19e+00</td>
<td>-1.37e+00 ± 6.70e-02</td>
<td>45.64</td>
<td>39</td>
<td>2.15e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TTN</td>
<td>I&amp;M</td>
<td>3.59e+00 ± 5.69e-01</td>
<td>-6.80e+02 ± 1.72e-02</td>
<td>47.08</td>
<td>76</td>
<td>9.96e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TSL</td>
<td>I&amp;M</td>
<td>1.42e+01 ± 1.58e+00</td>
<td>-2.83e+01 ± 3.03e-02</td>
<td>29.46</td>
<td>44</td>
<td>9.55e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>NT</td>
<td>I&amp;M</td>
<td>1.48e+01 ± 2.04e+00</td>
<td>-5.09e+00 ± 4.57e-02</td>
<td>40.10</td>
<td>39</td>
<td>4.21e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TTN</td>
<td>Pre</td>
<td>4.88e+00 ± 6.28e-01</td>
<td>-7.84e+02 ± 1.41e-02</td>
<td>42.31</td>
<td>76</td>
<td>9.99e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TSL</td>
<td>Pre</td>
<td>1.52e+01 ± 1.64e+00</td>
<td>-2.87e+01 ± 2.80e-02</td>
<td>40.86</td>
<td>51</td>
<td>8.44e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>NT</td>
<td>Pre</td>
<td>1.69e+01 ± 1.76e+00</td>
<td>-4.62e+01 ± 3.83e-02</td>
<td>36.37</td>
<td>43</td>
<td>7.52e-01</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TTN</td>
<td>LN2</td>
<td>1.67e+01 ± 1.69e+02</td>
<td>-4.78e+00 ± 1.92e-02</td>
<td>55.80</td>
<td>38</td>
<td>3.12e-02</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>TSL</td>
<td>LN2</td>
<td>1.55e+03 ± 1.87e+01</td>
<td>-4.59e+00 ± 3.77e-02</td>
<td>525.68</td>
<td>67</td>
<td>2.34e-72</td>
</tr>
<tr>
<td>0.6 $\rightarrow$ 1.0</td>
<td>NT</td>
<td>LN2</td>
<td>2.13e+03 ± 2.94e+01</td>
<td>-9.82e+00 ± 8.84e-02</td>
<td>105.94</td>
<td>53</td>
<td>2.16e-05</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TTN</td>
<td>SSC</td>
<td>3.59e+01 ± 2.27e+00</td>
<td>-2.30e+01 ± 1.23e-02</td>
<td>62.67</td>
<td>70</td>
<td>7.21e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TSL</td>
<td>SSC</td>
<td>3.47e+01 ± 2.15e+00</td>
<td>-5.10e+01 ± 2.58e-02</td>
<td>49.38</td>
<td>60</td>
<td>8.34e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>NT</td>
<td>SSC</td>
<td>4.35e+01 ± 2.66e+00</td>
<td>-7.44e+01 ± 3.51e-02</td>
<td>49.79</td>
<td>55</td>
<td>6.73e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TTN</td>
<td>I&amp;M</td>
<td>8.67e+01 ± 4.27e+00</td>
<td>-3.43e+01 ± 1.34e-02</td>
<td>48.33</td>
<td>61</td>
<td>8.80e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TSL</td>
<td>I&amp;M</td>
<td>7.18e+01 ± 3.41e+00</td>
<td>-6.20e+01 ± 2.20e-02</td>
<td>77.57</td>
<td>56</td>
<td>2.98e-02</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>NT</td>
<td>I&amp;M</td>
<td>6.59e+01 ± 3.14e+00</td>
<td>-1.01e+00 ± 3.50e-02</td>
<td>87.51</td>
<td>69</td>
<td>6.56e-02</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TTN</td>
<td>Pre</td>
<td>1.07e+02 ± 5.02e+00</td>
<td>-3.83e+01 ± 1.40e-02</td>
<td>55.57</td>
<td>60</td>
<td>6.32e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TSL</td>
<td>Pre</td>
<td>8.26e+01 ± 3.09e+00</td>
<td>-6.35e+01 ± 2.12e-02</td>
<td>81.96</td>
<td>55</td>
<td>1.07e-02</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>NT</td>
<td>Pre</td>
<td>7.82e+01 ± 3.52e+00</td>
<td>-1.06e+00 ± 3.52e-02</td>
<td>88.66</td>
<td>64</td>
<td>2.24e-02</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TTN</td>
<td>LN2</td>
<td>1.31e+03 ± 2.92e+01</td>
<td>-1.47e+00 ± 2.34e-02</td>
<td>56.49</td>
<td>45</td>
<td>1.17e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>TSL</td>
<td>LN2</td>
<td>3.51e+02 ± 7.68e+00</td>
<td>-1.50e+00 ± 2.33e-02</td>
<td>84.76</td>
<td>74</td>
<td>1.84e-01</td>
</tr>
<tr>
<td>2.0 $\rightarrow$ 8.0</td>
<td>NT</td>
<td>LN2</td>
<td>4.00e+02 ± 8.71e+00</td>
<td>-2.84e+00 ± 4.28e-02</td>
<td>175.60</td>
<td>63</td>
<td>1.45e-12</td>
</tr>
</tbody>
</table>

5.5 Summary of Possible Systematic Uncertainties

The possible sources of systematic uncertainty have been outlined above and are summarized in Table 5.14 below. However, we have not discussed the uncertainty in the detector mass. The removal of slow-signals effectively reduces the active mass of the detector. This fiducial mass has an uncertainty associated with it that relates to one’s ability to determine the thickness of the diffusion and recombination regions, see Section 5.1.3. This measurement is discussed in detail in the following chapter, see Section 6.1.2 [122], however this error has
Table 5.14: Systematic studies summary.

<table>
<thead>
<tr>
<th>Source</th>
<th>Estimate</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiducial mass error</td>
<td>3.83%</td>
<td>See Section 6.1.2 [122], negligible/ignored.</td>
</tr>
<tr>
<td>Non-Poisson process contribution</td>
<td>≤ 0.57% of counts</td>
<td>Less than or equal to the error in the count rate in 0.6–1.0 keV after LN$_2$ cut, negligible/ignored.</td>
</tr>
<tr>
<td>Threshold drift</td>
<td>~6.8 eV</td>
<td>Estimated from shifts in pulser mean (Section 5.4.1), negligible/ignored.</td>
</tr>
<tr>
<td>SSC acceptance efficiency error</td>
<td>≲ 0.3%</td>
<td>Based on statistics from pulser efficiency runs, negligible/ignored.</td>
</tr>
<tr>
<td>SSC rejection efficiency error</td>
<td>Unknown</td>
<td>Attempts were made to quantify this, see Section 6.5.</td>
</tr>
</tbody>
</table>

A conservative upper limit to a non-Poisson process contribution can be calculated assuming the error is less than or equal to the rate measured in the 0.6–1.0 keV region after the LN$_2$ cut [76]. In DS3 this rate was measured to be $5.26 \pm 0.03 \text{ hr}^{-1}$, corresponding to 0.57% of the counts in the threshold region. This contribution was concluded to be negligible and ignored.

The threshold drift estimate was conservatively calculated assuming the drifts in the pulser mean are solely due to gain drifts in the detector. This uncertainty is negligible, however we have set our threshold to 600 eV, rather than 550 eV, as one might assume after examining Figure 5.36, in order to be able to completely ignore threshold drift effects.

The uncertainty associated with the SSC acceptance efficiency, as calculated by a pulser, is dominated by statistics and can be ignored. However, the next chapter will highlight the fact that slow-signals are known to leak after the SSC is applied. Several attempts were made to quantify this leakage of slow-signals and develop a SSC rejection efficiency curve, however these attempts were not successful. In an effort to account for this when calculating WIMP limits (see Section 7.4.2), we have performed WIMP fits both with and without an exponential PDF.
5.6 Discussion

The various conditions in which MALBEK has been used to acquire data has been discussed. Following the removal of the lead shims, the slow-signal backgrounds were reduced by more than an order of magnitude, see e.g. Figures 5.7 and 5.6. Additionally, a new method to discriminate slow- from fast-signals has been developed. In this method, the metric used to discriminate remains stable down to threshold, unlike the standard $t_{10-90}$ method. Section 5.4 has shown that MALBEK remained stable throughout DS3, agreeing with Poisson statistics. Lastly, Section 5.5 has listed the various systematics that could affect the result outlined in Chapter 7. All of these effects were found to be negligible, with the exception of slow-signal leakage after the SSC and the fiducial mass calculation. Slow-signal contamination after the SSC will be discussed in the next chapter, focusing on the various attempts made at quantifying this contamination. Additionally, the next chapter will discuss the the underlying physical mechanism responsible for slow-signals.
Chapter 6

Slow Signals

This chapter will cover the experimental and modeling efforts made to fully understand the nature of slow-signals. A complete discussion on the lithium diffused n$^+$-bulk boundary and its role in causing slow-signals will be presented. Additionally, efforts to quantify the slow-signal leakage after the Slow-Signal Cut (SSC) has been applied will also be presented.

6.1 Introduction

The Particle Data Group (PDG) [7] suggests that for an experiment to set a reliable limit for WIMP dark matter it should be sensitive to at least 1% of the total WIMP signal recoil spectrum. For this reason, low-energy thresholds are an experimental requirement for direct dark matter searches which aim to probe the low-mass ($<10$ GeV) WIMP parameter space. A PPC detector with a 600 eV threshold would meet this requirement for WIMP masses greater than 5.7 GeV. However, the previous chapter alluded to the fact that slow, energy-degraded signals present a potential large source of background near threshold. In any scientific experiment, one should strive to understand and quantify all possible backgrounds. The focus of this chapter will be to fully understand what causes these signals and how to quantify their contamination of the WIMP signal region.
6.1.1 The p-n Junction

In PPC detectors, the n⁺ contact is typically made by diffusing lithium onto the outer surface of the germanium crystal with the exception of the base (where the point contact is located). However, in some cases, diffusing lithium onto a small fraction of the base of the detector (lithium wrap-around) is desirable. The MALBEK detector has this lithium wrap-around. The diffusion of lithium ions into the germanium crystal is achieved by placing the crystal in a carefully controlled high-temperature furnace and passing a gas mixture of lithium ions through it for a specific time and temperature [164]. Manufacturers sometimes do this process more than once, which makes calculating the depth of the lithium ions more difficult. In the case of a single diffusion process, the depth of the lithium ions in the germanium crystal can be calculated by solving the diffusion equation:

\[
\frac{\partial C(x,t)}{\partial t} = D \frac{\partial^2 C(x,t)}{\partial x^2},
\]

where \( C(x,t) \) is the lithium concentration, \( t \) is time, \( x \) is depth within the crystal and \( D \) is the diffusion coefficient or diffusivity. In general, the higher the diffusivity (of lithium with respect to germanium), the faster the lithium ions will diffuse into the germanium. This of course depends on the temperature of the furnace described above. The diffusion coefficient can be expressed as,

\[
D = D_0 \exp \left( \frac{-E_g}{kT} \right),
\]

where \( E_g \) is the activation energy, or the energy required to move a dopant ion (lithium) from one interstitial site to another, which typically varies between 0.5 – 2 eV. \( D_0 \) is the diffusion coefficient extrapolated to infinite temperature, therefore the use of this equation is limited since such an extrapolation is not viable [82]. Equation 6.1 can be solved easily given a set of initial conditions. If the surface concentration is held constant, the initial conditions for diffusing lithium into a germanium crystal can be assumed to be:

- \( C(x,0) = 0 \)

\(^1\)And sometimes under the influence of an electric field.
\begin{itemize}
  \item $C(0, t) = C_s$ (constant surface concentration)
  \item $C(\infty, 0) = 0$.
\end{itemize}

The solution to Equation 6.1 is then given by,

$$C(x, t) = C_s \text{erfc} \left( \frac{x}{2\sqrt{Dt}} \right),$$

(6.3)

with $\sqrt{Dt}$ being the diffusion length. The diffusion length is the average length a lithium ion moves before it reaches its final location within the germanium crystal. This shows that the lithium concentration drops off gradually, and does not abruptly drop to zero after a certain depth [78–82].

The depth of the p-n junction can be calculated by finding the location at which the lithium concentration equals the impurity concentration ($N_A$) within the germanium crystal. The impurity concentration in MALBEK was measured by Canberra and reported in Table 4.1, showing that $N_A \simeq 10^{10}$ cm$^{-3}$. In order to calculate the depth of the p-n junction, both $D$ and $C_s$ are required. A good first order estimate of the surface concentration is given by the solid solubility of lithium in germanium at 300 °C, $C_s \simeq 4 \times 10^{16}$ cm$^{-3}$ [165]. The diffusion coefficient depends on temperature, $E_g$ and $D_0$. Ref. [164] measured the diffusion coefficient in both silicon and germanium at various temperatures and developed empirical functions for $D$ that only depend on temperature,

$$D \simeq 9.10 \times 10^{-3} \exp \left( \frac{-13100}{RT} \right) \text{[cm}^2\text{sec]}^{-1},$$

(6.4)

for germanium, where $T$ is in Kelvin and $R = 1.98$ cal/K. Assuming a 5 minute (300 s) diffusion at 300 °C (573 K), the lithium concentration as a function of depth has been calculated and is shown in Figure 6.1. The depth at which the two curves meet is known as the p-n junction depth [78–82].
Figure 6.1: The (+) lithium ion concentration (red) and (-) impurity concentration (blue) curves are shown. The point at which the two curves meet is the p-n junction (black).

6.1.2 Charge Collection Near the p-n Junction

There are several processes that dominate charge collection near the p-n junction. This section will provide a brief introduction to these processes and their hypothesized ranges.

The Depleted Region

The sensitive volume of a germanium detector is the region that has been depleted of charge, or the depletion region (see Section 2.1.3). The width of this depletion region, \( d \), is given by,

\[
d \simeq \sqrt{\frac{2\epsilon(V_{bi} - V)}{qN_B}},
\]

where \( V \) and \( V_{bi} \) are the contact and bias voltages, respectively, \( \epsilon \) is the dielectric constant (for Ge \( \epsilon = 16\epsilon_0 \)), \( q \) is the electronic charge \( (1.60218 \times 10^{-19} \text{ C}) \), and \( N_B \) is the impurity concentration within the crystal. Germanium is four valent – therefore, at impurity lattice sites in p-type germanium, a germanium atom is replaced by an impurity atom that is three valent (e.g. boron or gallium) resulting in a missing electron (or hole). This is referred to
as an acceptor impurity and the concentration of acceptor impurities throughout the bulk of the crystal is denoted by $N_A$. Similarly, for n-type germanium, lattice sites will be replaced with five valent donor impurities, e.g. arsenic or phosphorus. The concentration of donor impurities in n-type germanium throughout the bulk is denoted by $N_D$. In Equation 6.5, $N_B = N_A$ for a p-type germanium detector and $N_B = N_D$ for an n-type germanium detector [78–82].

As a detector is biased, depletion starts at the p-n junction and proceeds such that equal and opposite amounts of charge are left on either side of the p-n junction – this was illustrated in Figure 2.2. The depth at which depletion starts will be referred to as the Full Charge Collection Depth (FCCD) since any energy deposition beyond this point will result in full charge collection at the $p^+$ contact [146], see Figure 6.2. The slow, energy-degraded signals referred to in the previous chapters result from interactions that take place at depths < FCCD (will be discussed in next Section).

The FCCD depth of the MALBEK detector has been measured and reported in Refs. [122, 146]. The measurement of the MALBEK FCCD was performed by measuring the detector response to $^{133}$Ba gammas, specifically by measuring the ratio of peak areas of the 356 keV and 81 keV gammas. The MALBEK Geant4-based Monte Carlo model developed by Alexis Schubert was used by her to calculate the simulated peak area ratio as a function of FCCD [122, 166, 167]. The depth at which the simulation agreed with experiment was taken as the FCCD of MALBEK, see Figure 6.3. The FCCD of MALBEK was found to be $933 \pm 18 \text{ stat} \mu\text{m}$. The application of the SSC essentially removes events that occur at depths < FCCD. Therefore the fiducial mass, or mass of the active region, of the detector will become smaller once the SSC is applied. In the MALBEK detector, this amounts to a reduction in active volume (mass) from $85.6 \pm 3.0 \text{ cm}^3 \ (455.5 \pm 15.8 \text{ g})$ to $75.9 \pm 2.9 \text{ cm}^3 \ (404.2 \pm 15.5 \text{ g})$ [122].

Recall that the depth of the p-n junction depends upon the conditions under which the n$^+$ contact was made, i.e. temperature and time of diffusion. The depth of the p-n junction has been shown to change following temperature cycles of germanium detectors [168], therefore, ideally the FCCD should be measured following every temperature cycle of the detector. The MALBEK FCCD was measured after the trip to Canberra to remove the lead foils (see
Figure 6.2: The (+) lithium ion concentration (red) and (-) impurity concentration (blue) curves are shown. The point at which the two curves meet is the p-n junction (black). The depletion zone (FCCD) starts just before the p-n junction and extends to the p$^+$ contact.

Figure 6.3: The data points in the figure are simulated results of the peak area ratio as a function of FCCD. The horizontal band represents the ratio as measured in the data. The vertical band indicates where simulation agrees with experiment, FCCD = $933 \pm 18_{stat}^{92}_{sys}$ µm. Figure from Ref. [146], based on work done by Alexis Schubert.
Section 5.1), and the detector has been kept cold since. Therefore, it is safe to conclude that the FCCD reported above can be used in the analysis of DS2 and DS3 (see Table 5.1).

The Diffusion and Recombination Dominated Regions

At depths < FCCD, full charge collection will not occur. This has been reported in the literature extensively [67, 73–75, 121, 136–146], however a complete understanding of the physics taking place in this region is lacking. The hypothesis presented here, which was initially developed by David Radford at Oak Ridge National Laboratory, is that at depths < FCCD, the germanium is undepleted (no electric field) and a charge cloud created in this region will have to diffuse out of this region and into the bulk (depths ≥ FCCD) in order to be collected. A combination of two processes dominate the charge transport in this region: (1) recombination and (2) diffusion. Charge clouds created in this region are hypothesized to be more susceptible to recombination (mainly due to the fact that it takes longer for the charge cloud to diffuse out of this region), which removes holes from the signal and results in a lower apparent energy (energy-degraded). The time a charge cloud takes to diffuse out of the lithium layer and into the depleted bulk is much longer than the charge collection time in the bulk. This results in signals with longer drift-times. Additionally, the diffusion process also spreads out the charge distribution, resulting in events with longer rise times. The combination of these processes is believed to lead to the slow, energy-degraded signals referred to in the previous chapter. The waveforms shown in Figure 6.4 illustrate the difference in rise time between interactions occurring in these two regions. It is also hypothesized that a dead region of the detector may be present, i.e. a region in which any charge cloud created will completely recombine with electrons. This region will be referred to as the Recombination Dominated Region (RDR), whereas the region between the RDR and FCCD will be referred to as the Diffusion Dominated Region (DDR). Figure 6.5 illustrates the difference between RDR, DDR and FCCD. Equation 6.3 was used to generate this figure for the MALBEK detector, however the diffusion length has been adjusted such that the depth of the p-n junction is greater than the measured MALBEK FCCD (see Figure 6.2). This is justifiable due to the fact that the exact temperature and time of lithium diffusion in the MALBEK detector are unknown. The
widths of the DDR and RDR shown in Figure 6.5 are unknown, and are only labeled for illustrative purposes. The next two sections will highlight experimental results that support and led to the hypothesis outlined above.

Figure 6.4: The red signal is hypothesized to result from an interaction at a depth < FCCD (it failed the slow-signal cut (SSC) described in the previous chapter), while the black signal results from an interaction in the depleted bulk (it passed the SSC). Both events have an energy of \( \sim 20 \text{ keV} \).

### 6.2 Slow Signal Dependence on \( n^+ \) Contact Material

Lithium diffused \( n^+ \) hole-barrier contacts are used on the majority of p-type germanium detectors, \( e.g. \) MALBEK and all of the MAJORANA PPC detectors. These contacts are typically on the order of 1 mm thick (0.993 mm for MALBEK) and represent a barrier to incoming low-energy radiation and alphas. This is desirable in some circumstances, specifically in the search for \( 0\nu\beta\beta \), however in terms of the slow-signals discussed so far this greatly complicates matters at low-energy.

In 2011, PHDs Co. [169] demonstrated the use of thin (0.1\( \mu \)m) yttrium hole-barrier contacts for use in low-energy spectroscopy with germanium detectors [170]. Similarly, they have
Figure 6.5: The RDR, DDR, and depleted regions are shown here. The border between the depleted region and the DDR is the FCCD (933 ± 18 \text{ stat\ \mu m})\). The widths of the DDR and RDR are unknown, and are only labeled here for illustrative purposes. It is very likely that the width of the DDR is significantly less than the width of the RDR. The lithium concentration (red) and impurity concentration (blue) curves are shown as well.
also fabricated detectors with thin n\(^+\) contacts created with silver and nickel. Ethan Hull of PHDs Co. graciously allowed members of the MAJORANA collaboration to analyze data acquired with these types of detectors to investigate the effect of n\(^+\) contact material on signal rise times. These detectors were roughly 15 mm in diameter and 10 mm thick. The contact was segmented into a center- and guard-ring segment with photolithography [170]. The width of the gap between these segments for these detectors is \(\sim100\mu m\). A picture of one of these types of detectors (not one used in analysis) is shown in Figure 6.6. The geometry of the contacts are not believed to affect the measurements outlined below [170].

![Figure 6.6: A PHDs Co. detector with thin a n\(^+\) contact. The contact was segmented into a center and guard-ring segment with photolithography. The width of the gap in this detector is >100\(\mu m\). Figure adapted from Ref. [170].](image)

6.2.1 Data and Analysis of the PHDs Co. Detectors

\(^{241}\)Am data from two detectors with differing n\(^+\) contact material, both lithium and silver, has been analyzed. The data were acquired by personnel at PHDs Co. with their own customized digitizer. Events were not timestamped and live time was not reported (aside from the
fact that the two data sets had equivalent live times), therefore no timing analysis could be
performed. The signals were digitized at 50 MHz for 1024 samples, corresponding to a 20.48µs
trace length, see Figure 6.7 for an example waveform. For each signal, all parameters listed
in Table B.1 were calculated. Due to the coarse sampling period (20 ns) the locations of the
10% and 90% of each waveform were calculated by linear interpolation between neighboring
samples. The energy of each event was calculated with a trapezoidal filter (see Section 5.2.2)
whose parameters are listed in Table 6.1.

Figure 6.7: An example of a ∼40 keV PHDs Co. detector waveform

Table 6.1: Parameters used for the trapezoidal filter of PHDs Co. Detectors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Averaging Time</td>
<td>0.0 – 3.0 µs</td>
</tr>
<tr>
<td>Waveform Decay Constant</td>
<td>150.0 µs</td>
</tr>
<tr>
<td>Trapezoidal Filter Gap Time</td>
<td>0.5 µs</td>
</tr>
<tr>
<td>Trapezoidal Filter Peaking Time</td>
<td>2.5 µs</td>
</tr>
</tbody>
</table>
6.2.2 Results

After analysis of both data sets, it was clear that the lithium layer plays a very important role in slow-signal generation. This is illustrated in Figure 6.8. The red box in both figures contains known slow events. At lower energies the rise time calculation performed poorly, mainly due to poor signal-to-noise (see Section 5.2.3) and coarse sampling. These figures show that the silver contact reduced the number of slow-signals observed by a factor of \( \sim 600 \). The results reported here support the hypothesis that slow-signals arise from interactions near the lithium diffused n\(^+\) contact. Furthermore, the thinner silver contact also allows for very low-energy X-rays to penetrate the crystal. This is clearly illustrated in Figures 6.8b and 6.9 which show X-rays between 10 - 30 keV whose origin is believed to be neptunium. In summary, these results show that the material and thickness used to create the n\(^+\) contact is directly related to the amount of slow-signals. Therefore, these results locate the source of the slow-signal contamination. The next section will discuss how signals originating from this region are also delayed (long drift times).

6.3 Correlation Between Rise Time and Drift Time

The hypothesis outlined in Section 6.1.2 states that if a charge cloud is created in the DDR, a fraction of the cloud will diffuse into the active region of the detector since there is no electric field in the undepleted region. This would result in longer drift times, or simply put, it will take longer for the cloud to be collected at the p\(^+\) electrode than a charge cloud created in the depleted region of the detector. In order to investigate this, a measurement of the charge carrier drift time has been performed with the MALBEK detector.

6.3.1 Experimental Technique

The drift time was measured by determining the time difference between two coincident gamma-rays, one incident upon MALBEK, and the other incident upon a 3M3/3-X Saint-Gobain Crystals NaI[Tl] detector. NaI[Tl] was chosen as the second detector due to its intrinsic fast response time. A 1 \( \mu \)Ci \(^{133}\)Ba source was used for the coincident gammas. \(^{133}\)Ba
Figure 6.8: $^{241}$Am rise time distribution comparison between silver and lithium $n^+$ contacts. The red box in both figures contains known slow events, i.e. where the rise time calculation can be trusted. The silver contact reduced the number of slow-signals observed by a factor of $\sim600$. Both data sets have equivalent live times. Below 30 keV, poor signal-to-noise led to overestimates of the rise time (see text for details).
Figure 6.9: Energy spectra from PHDs Co. detectors with silver and lithium n$^+$ contacts. It is clear that the thinner silver contact allows lower energy X-rays (from neptunium and Ag fluorescence) to penetrate the bulk.

has two gammas that are of particular interest, a low-energy (81.9979 keV) and higher energy (356.0129 keV) gamma-ray, which have an average time interval of 6.28 ns between the two [171]. The NaI[Tl] detector was used to measure the 356 keV gamma, while MALBEK was used to measure the 81 keV gamma. In order to reduce file sizes, a hardware gate surrounding the 356 keV peak was implemented in the NaI[Tl] detector and from threshold to 83 keV in MALBEK. An event required a gamma in each detector to be in coincidence, with the 356 keV signal in the NaI[Tl] marking the start of an event. The charge carrier drift time was then taken to be the time difference between a NaI[Tl] and MALBEK signal. Since the 81 keV gamma will interact in the outer 1-3 mm of the BEGe crystal the majority of the time, this probes the DDR and tests the hypothesis that slow-signals have long drift-times.

The experiment was conducted with the $^{133}$Ba source in several locations, where one example is shown in Figure 6.10 and should be used to visualize the other locations outlined below:

- *Top Uncollimated*: source centered 5 cm from the top of the cryostat; 3.59 hours of data
(see Figure 6.10);

- **Top Collimated**: bottom of collimator centered 4.54 cm from the top of the cryostat; 7.23 hours of data;

- **Side Uncollimated**: source 5 cm from the side of the cryostat, 3 cm below top of cryostat (directs gammas in middle of BEGe crystal); 1.96 hours of data;

- **Side Collimated**: bottom of collimator 4.54 cm from the side of the cryostat, 3 cm below top of cryostat (directs gammas in middle of BEGe crystal); 17.26 hours of data.

For runs in which the source was collimated, a block of lead $5 \times 5 \times 5$ cm in size with a 5 mm through hole was used to collimate the 81 keV gammas towards the MALBEK cryostat (no collimation towards the NaI[Tl] detector). For runs in which the source was on the side of the cryostat, the source/collimator were positioned so that they were centered with respect to the height of the BEGe crystal (3 cm below the top of the cryostat).

![Image](image_url)

**Figure 6.10**: The NaI[Tl] and MALBEK detectors along with the $^{133}$Ba source are shown here. This setup was used to acquire the uncollimated data from the top of the cryostat.
6.3.2 Data Acquisition

The MALBEK DAQ was used for this measurement, see Figure 4.13. The only difference being the high-energy channel shown in Figure 4.13 was replaced with the NaI[Tl] signal. The NaI[Tl] signals were sent through an ORTEC Model 113 preamplifier and shaped with an ORTEC Model 474 timing filter amplifier (TFA) with 500 ns of integration and 500 ns of differentiation. The shaped NaI[Tl] signals were then digitized by the SIS3302 with a 10 μs trace length and 5 μs of baseline. Of course, shaping the NaI[Tl] signals resulted in a timing delay. The delay of the electronics chain was measured to be 55 ± 5 ns (with the NaI[Tl] signal occurring after the MALBEK signal) by simultaneously sending a pulser signal through both the NaI[Tl] and MALBEK signal chains. This delay was taken into account and will be discussed in the next section. We chose to digitize all events for the following reasons: (1) it was easier to use an existing DAQ, (2) it allows us to choose any timing gate for the MALBEK channel, and (3) the analysis outlined below requires the rise time \( t_{10-90} \) and location in time that the signal reached 50% of its maximum \( t_{50} \). In order to be able to digitize both the NaI[Tl] and MALBEK signal such that the rising edge occurred in the middle of the digitized window, we made use of the SIS3302’s pretrigger and buffer wrap delay features. These features delay the ADC conversion for a set period of time such that an arbitrary portion of the waveform can be digitized. These delay features also needed to be accounted for and will be discussed below.

6.3.3 Data Analysis

Several GAT processors were developed in order to analyze the data from both the NaI[Tl] and MALBEK detectors, one for calibrating digitizer timestamps and the other for the Digital Signal Processing (DSP) of the NaI[Tl] waveforms. The timestamps were calibrated to take into account delays associated with the pretrigger and buffer wrap delays as well as the delay caused by shaping the NaI[Tl] signal. For both the NaI[Tl] and MALBEK signals, since the timestamp marked the start of the digitization window, the location in time that the signal reached 50% of its maximum \( t_{50} \) was added to each timestamp. Once the timestamps were
calibrated, we calculated the time since last 356 keV peak event in the MALBEK signal chain.
This time difference was taken as the charge carrier drift time. This resulted in a significant
amount of accidental coincidences; this is clearly shown in Figures 6.13 and 6.14 as the
horizontal band at $t_{10−90} \sim 400$ ns.

6.3.4 Results

The results from this experiment were consistent with the hypothesis outlined in Section 6.1.2,
showing that events with long drift times are directly related to slow-signals, see *e.g.* Figures 6.13 and 6.14. It was also found that the maximal drift time (Figure 6.16), rise time (Figure 6.15) and spectral shape (Figure 6.12) depend significantly on the position of the source. This was expected since the distance to the collecting electrode and electric field change as a function of interaction position; see Ref. [172] for a detailed review of position dependence within germanium detectors. Perhaps this is better illustrated by examining the rise time energy dependence for various positions, as shown in Figure 6.11. This figure shows the rise time energy dependence for the collimated data only – again, it is clear that the source location greatly affects the rise time energy dependence. With this in mind, recall the cosmogenic backgrounds discussed in the previous chapter. These backgrounds are uniformly distributed throughout the germanium crystal, creating a uniform distribution of potential slow-signal sources. This is a drastically different scenario than placing a calibration source above or beside a detector. Due to attenuation, these external gamma-rays will have a preference towards interactions near the outer parts of the detector. Therefore, it can be argued that if one’s background spectrum is dominated by slow-signals from cosmogenics, then a source measurement cannot be used to determine an expected spectral shape due to slow-signals.

The shape of Figure 6.16 merits an in depth explanation. Consider for example the runs in which the source was uncollimated and located on top of the cryostat. The entire top face of the crystal was bathed in gamma-rays from $^{133}$Ba, even the corners. An interaction taking place in the corner of the crystal will have a longer distance to drift as well as a weaker field to drift through (slower drift velocity). Conversely, an interaction taking place directly in the
Figure 6.11: $^{133}$Ba rise time energy dependence for (a) a collimated source illuminating the side of MALBEK, (b) a collimated source illuminating the top of MALBEK.
middle of the top face of the detector will have the shortest drift path and strongest field to drift through. Therefore, the drift time peak in the top uncollimated measurement was not symmetric, and appeared to be a superposition of the collimated top and uncollimated side measurements. The distributions observed in Figure 6.15 can easily be understood by comparing them with Figure 6.11. It is clear that when the source is located on the side of the detector, the maximal rise time appears to be less than the maximal rise time from when the source is located on top of the detector.

These results highlight the need for a model that takes into account the physical processes taking place in the RDR and DDR. The following sections will present a simple model, initially developed by David Radford, that, after taking into account diffusion and recombination, reproduces to first order several experimental measurements.

Figure 6.12: The energy spectra from MALBEK during the drift time measurements (see legend). The SIS3302 was configured so as to not trigger on events with energy greater than \(~82\) keV. It is clear that the shape of the low-energy rise due to slow-signals is dependent upon the position of the source. Only the inhibit and preliminary cuts have been applied to these data (see Section 5.3.4).
Figure 6.13: $t_{10-90}$ versus drift time from above the cryostat, collimated (top panel), uncollimated (bottom panel). See text for explanation of the distributions observed. However, it is clear that an event’s rise time is proportional to its drift time.
Figure 6.14: $t_{10-90}$ versus drift time from the side of the cryostat, collimated (top panel), uncollimated (bottom panel). See text for explanation of the distributions observed. However, it is clear that slow-signals are directly correlated to events with long drift times.
Figure 6.15: MALBEK $t_{10-90}$ histogram from various source positions. Only events with energies between 4 keV and 81 keV have been plotted. See text for explanation.

Figure 6.16: MALBEK drift time histogram from various source positions with energies greater than 4 keV but less than 81 keV. See text for explanation.
6.4 Modeling Diffusion in the DDR

Since the experimental measurements outlined in Sections 6.2 and 6.3 supported the initial hypothesis that slow-signals arise from interactions near the n$^+$-bulk boundary, this section will present a model initially developed by David Radford that simulates the diffusion of charges out of the DDR and into either the RDR or depletion region.

6.4.1 Introduction

The model discretizes the DDR in 1-D using 20 $\mu$m bins and allows a charge carrier density $\rho(x,t)$ to diffuse in 1 ns time steps according the diffusion equation (ignoring recombination)$^2$,

$$\frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2}.$$  \hspace{1cm} (6.6)

This equation is a simple second order differential equation that is easily solvable numerically. Additionally, the Einstein relation relates the diffusivity to the charge carrier carrier mobility $^1$:

$$D = \frac{kT}{q \mu},$$  \hspace{1cm} (6.7)

where $k$ is the Boltzmann constant, $T$ is the temperature, $q$ is the electronic charge and $\mu$ is the hole mobility ($4.2 \times 10^4$ cm$^2$ V$^{-1}$ s$^{-1}$ at 80 K in germanium, see Table 2.1). The quantity $kT/q$ is equal to 0.00689 V at $T = 80$ K. Equation 6.6 requires taking derivatives numerically for realistic initial charge carrier densities. Let us adopt the notation where $\rho_i$ denotes the charge carrier density in a depth bin $i$. In order to solve the above equation, we make use of the Taylor series expansion in which the charge density in neighboring bins is given by:

$$\rho_{i \pm 1} = \rho_i \pm \delta x \frac{\partial \rho}{\partial x} + \frac{1}{2} \delta x^2 \frac{\partial^2 \rho}{\partial x^2} + \ldots.$$  \hspace{1cm} (6.8)

$^2$The choice of step size is somewhat arbitrary, however the time step was chosen so as to have a finer sampling period than the SIS3302, which is 10 ns.
where $\delta x$ is the depth bin size, $20\ \mu m$. Ignoring higher terms, the first derivative of the charge density is approximated by:

$$\frac{\partial \rho}{\partial x} \approx \frac{\rho_{i+1} - \rho_i}{\delta x} = \frac{\rho_i - \rho_{i-1}}{\delta x} = \frac{\rho_{i+1} - \rho_{i-1}}{2\delta x}. \quad (6.9)$$

Similarly, it is easy to show that the second order derivative is approximated by:

$$\frac{\partial^2 \rho}{\partial x^2} \approx \frac{(\rho_{i+1} - \rho_i) - (\rho_i - \rho_{i-1})}{\delta x^2} \quad (6.10)$$

From Equation 6.6 this shows

$$\frac{\partial \rho}{\partial t} \approx D \frac{\delta x^2}{\rho_{i+1} - \rho_i} - (\rho_i - \rho_{i-1}) \quad (6.11)$$

Taking another Taylor series expansion, the partial derivative with respect to $t$ can be calculated by taking the difference between $\rho$ evaluated at two different times, and dividing by $\delta t$ – this gives

$$\frac{\partial \rho}{\partial t} \approx \frac{D}{\delta x^2} ((\rho_{i+1} - \rho_i) - (\rho_i - \rho_{i-1})) \quad (6.12)$$

where $n$ is now the time step bin, 1 ns. This reduces down to

$$\rho(i, n + 1) \approx \rho(i, n) + \frac{D\delta t}{\delta x^2} (\rho(i + 1, n) + \rho(i - 1, n) - 2\rho(i, n)) \quad (6.13)$$

and defining $\lambda \equiv \frac{D\delta t}{\delta x^2}$ then gives,

$$\rho(i, n + 1) \approx (1 - 2\lambda)\rho(i, n) + \lambda\rho(i + 1, n) + \lambda\rho(i - 1, n). \quad (6.14)$$

This recursive relationship is the solution to the diffusion equation and is the fundamental equation for the models that will be presented in the following sections. In this equation, $\lambda$ can be thought of as the probability that a charge will be located in the neighboring bin on the next time-step. Since we have made quite a few assumptions (ignored higher terms in the series expansion), inserting the values ($\delta t$, $\delta x$ and $D$) into the equation for $\lambda$ will give a
slightly incorrect result. However, this probability is easily calculated with a toy Monte Carlo (not to be confused with the model being presented) that allows a Gaussian charge cloud \[82\] with a width defined as

\[
\sigma = \sqrt{\frac{2kT}{q} \mu t}
\]

(6.15)
to diffuse in 1 ns time steps. The probability, \( \lambda \), is given by the original concentration at the centroid minus the two neighboring concentrations at \( t \). Figure 6.17 shows the results of this simple toy Monte Carlo yielding a \( \lambda \) value of 0.07955. This value of \( \lambda \) will be used in Equation 6.14 hereafter. If we had simply plugged the numbers in the above equation we would have got 0.072345, which is close, but not correct.

Figure 6.17: Diffusion probability Monte Carlo results. After \( \sim 10^4 \) iterations, the value for \( \lambda \) settles at 0.07955. See Equation 6.14.
In both of the models presented in the following sections, the basic diffusion modeling algorithm is the same:

1. Define the thickness over which to model the diffusion, \textit{i.e.} the width of the DDR + RDR.

2. Discretize this thickness in 20 \( \mu \text{m} \) bins.

3. For each interaction depth, \( i \in (0, X) \):
   (a) Initialize the charge density to be localized in bin \( i \).
   (b) \textbf{Loop over time}, \( n \in (0, N) \):
      \rightarrow Allow the charge density to diffuse (Equation 6.14), be collected and recombine. The collection and recombination probabilities are depth-dependent, \textit{i.e.} they depend on \( i \). The depth-dependent recombination probability is the only difference between the models that follow.
   (c) Calculate and report results for this interaction position, \( i \).

6.4.2 Two-Plane Model

Equation 6.14 has been used to calculate the charge density as a function of time and position. We have implemented a simple two-plane model for the recombination probability for this study. In the two-plane model, the recombination probability is zero everywhere except for at the surface of the detector. Similarly, the charge collection probability is zero everywhere except for inside the depletion region (\( \geq \text{FCCD} \)), see Figure 6.18. In this model, a charge cloud created in the DDR will diffuse in both directions, towards the RDR and towards the depletion region, and only charges that make it to the depletion region will be collected and contribute to the signal. The inputs to the model can be summarized as follows:

- time step (\( \delta t \)), taken to be 1 ns;
- depth bin size (\( \delta x \)), taken to be 20\( \mu \text{m} \);
• width of the RDR ($w_{RDR}$) in units of depth bins, taken to be $w_{RDR} = 0$ (no truly dead region);

• width of the DDR ($w_{DDR}$) in units of depth bins, taken to be $w_{DDR} = 47$ (0.940 mm, as close to the FCCD of MALBEK as possible);

• initial concentration of charges, $\rho(i, 0) = 1000 \text{ cm}^{-3}$. This value was chosen so as to resemble a standard ADC with >8-bit resolution;

• number of time steps, taken to be 10000;

• depth-dependent recombination and collection probabilities, described above.

For each position, the charges are allowed to diffuse in 10000 time steps. Using these values as input, several parameters can be calculated that are experimentally verifiable:

• $t_{x-y}$, rise time, where $x, y$ can be any combination of percentages such that $x < y$;

• $t_{50}$, drift time;

• $f_C$, fraction of charge collected;

• $S_{\text{del}}(t)$, delayed ionization signal (fraction of charge collected as a function of time).

Results

Using the two-plane model for recombination and collection probabilities, the fraction of charge collected as a function of depth has been calculated and is shown in Figure 6.18. These results show that the fraction of charge collected linearly increases until the FCCD is reached. Additionally, the 10–90% and 20–80% rise times have been calculated as a function of depth, see Figure 6.20a. The drift time ($t_{50}$) is also shown in Figure 6.20a. While the assumption that the recombination probability is zero everywhere except at the detector surface is unrealistic, these results qualitatively agree with the initial assumption that slower signals, having longer drift times, originate near the n$^+$ contact. These promising initial results indicate that further investigation of the recombination probability is needed. The
next section will outline a more realistic assumption of the recombination probability and its affect on these results.

### 6.4.3 Probabilistic Recombination Model

The two-plane model showed promising results. To further our understanding of the processes taking place near the n⁺ contact we developed a more realistic depth-dependent recombination probability function. We assumed that the recombination probability is roughly proportional to the lithium concentration until a certain depth at which it was set to zero. The width of the non-zero recombination probability can be thought of as the thickness of the RDR. An upper limit on this thickness has been measured for MALBEK and reported in Ref. [146], in which the width of the RDR was found to be ≤0.44 mm. The recombination probability was set to zero for depths greater than this value. The recombination probability as a function of depth used is shown in Figure 6.19. Although this recombination probability is slightly more physically motivated than the two-plane model, it is still just an educated guess that assumes the recombination probability is proportional to an assumed lithium ion concentration.

**Results**

The results obtained with the new recombination probabilities are shown in Figures 6.19 and 6.20b, which show the fraction of charge collected and rise times respectively. It is clear that the maximum rise time is lower than the two-plane model results. This was expected since the slower signals near the n⁺ contact were now allowed to recombine and not contribute to the total signal. Additionally, the fraction of charge collected as a function of depth was found to be roughly linear between 0.44 mm and the FCCD and quickly drop to zero for depths less than 0.44 mm. These results again support the initial hypothesis and data presented thus far. The next logical step is to use these results in a Monte Carlo simulation to generate an energy spectrum and also to calculate slow ionization signals. The next two sections will discuss both of these, starting with the latter.
Figure 6.18: Two-plane diffusion model recombination probability, collection probability and fraction of charge collected versus depth. The plot is shown with both log (a) and linear y-axes (b) to illustrate the linear fraction collected curve. The recombination probability and collection probability are provided as input to the model. They are used to compute the shown fraction collected. The fraction of charge collected linearly increases with depth, as expected.
Figure 6.19: The recombination and collection probabilities used in the probabilistic recombination model are shown here. In addition, the calculated fraction of charge collected as a function of interaction depth is also shown. The fraction of charge collected rises linearly between 0.44 mm and the FCCD. At shallower depths, the charge collected drops faster, as expected, due to the non-zero recombination probability. See text for details.
Figure 6.20: (a) Calculated results for the two-plane diffusion model 10–90%, 20–80% and drift time versus depth. These values are larger than what is observed in data. See text for more details. (b) Calculated results for the probabilistic recombination diffusion model 10–90%, 20–80% and drift time versus depth.
6.4.4 Calculating the Shape of Slow-Signals

Using the siggen \cite{95} software package, which takes into account the signal generation mechanisms described in Section 2.2.2, along with the probabilistic recombination model output, the shapes of slow-signals as a function of depth have been calculated. The siggen package was used to generate a prompt ionization signal ($S_{\text{prompt}}(t)$) in the MALBEK detector. By convolving $S_{\text{prompt}}(t)$ with the delayed signal ($S_{\text{del}}(t)$),

$$S(t) = \sum_{x=1}^{t} (S_{\text{del}}(x) - S_{\text{del}}(x-1)) S_{\text{prompt}}(t-x)$$  \hspace{1cm} (6.16)

the shape of the slow-signals as a function of interaction depth was calculated. Figure 6.21 shows several ionization signals as a function of interaction depth. Energy degradation, long rise times and long drift times are clearly reproduced.

6.4.5 Implementing the Diffusion Model in Monte Carlo

In an effort to quantify the effect the probabilistic diffusion model has on an energy spectrum, a toy Monte Carlo model has been developed which makes use of the calculated fraction of charge collected vs. depth (Figure 6.19). By choosing a detector geometry and randomly sampling depths within the detector (correcting for attenuation of the gamma-rays in germanium), an energy spectrum from the $^{241}$Am 59.5 keV gamma line has been calculated for the MALBEK detector geometry (30.0 mm height). At each depth considered, the energy collected was taken as $59.5 \text{ keV} \times f_C(x)$, where $f_C(x)$ is the fraction of charge collected for an interaction occurring at depth $x$. Additionally, the energy spectrum was smeared according to the finite energy resolution of the MALBEK detector (see Section 5.2.2).

Results

The $^{241}$Am energy spectrum calculated with the toy Monte Carlo is shown in Figure 6.22. The diffusion code and toy Monte Carlo qualitatively reproduce the increase in counts at low-energy due to slow-signals. It should be noted that this has not been done with a model before. Additionally, the model also shows a continuum between threshold and the full-energy
Figure 6.21: Calculated slow-signals using the probabilistic recombination diffusion model. \( d \) here denotes the depth of the interaction simulated.
peak solely due to slow-signals (Compton scattering was not simulated). Figure 6.23 shows the rise times calculated by the diffusion code (red circles) compared to data obtained with the MALBEK detector; the results agree quite well with the rise times observed in the MALBEK detector. These results are very promising and merit further investigation and fine tuning of the probabilistic recombination model as well as the implementation in Monte Carlo.

![Energy Spectrum](image)

Figure 6.22: Energy spectrum from toy Monte Carlo based on probabilistic recombination diffusion model. Notice an increase in counts at low-energy and a slow-signal continuum (Compton scattering was not modeled).

### 6.4.6 Summary and Outlook

Based on the experimental measurements made in Sections 6.2 and 6.3, a numerical model for diffusion and recombination in germanium detectors has been developed. This model shows very promising preliminary results. The results qualitatively reproduce several critical phenomena:

- The rise in counts at low-energy (<5 keV), see *e.g.* Figure 6.22;
- Slow-signals are also delayed (correlated with long drift times), see *e.g.* Figure 6.20b;
- The mean rise time of slow-signals in the MALBEK detector, see *e.g.* Figure 6.23;
The shape of a slow-signal ionization pulse, see *e.g.* Figure 6.21.

In addition, the thickness of the DDR was found to be directly related to the average maximum rise time, *i.e.* \( \max(\langle t_{10-90} \rangle) \). This is easily visualized in Figure 6.23, where \( \max(\langle t_{10-90} \rangle) \approx 2900 \) ns. This relationship can be expressed as

\[
\max(\langle t_{10-90} \rangle) \sim \text{DDR}^2, \tag{6.17}
\]

which, upon further examination of Figure 6.11, implies that the DDR is roughly 1.6 to 2.0 times thicker on the top than on the side of MALBEK.

It should be noted that this model is far from complete. The recombination probability as a function of depth needs to be more fully understood. Additionally, the depth of the RDR and DDR change with location and this has not been taken into account. This could be due to non-uniform lithium diffusion and/or the depth of the depletion layer changing as a function of electric field. Furthermore, while the toy Monte Carlo reproduced, to first order, the low-energy rise, it does not take into account additional gamma-ray interactions that also
take place. In order for this to be done correctly, the $f_C(x)$ curve needs to be implemented in a Geant4 based Monte Carlo model. In summary, while the diffusion models both give only qualitative agreement thus far, this is an enormous improvement of what was available prior to these models, which were just educated guesses at the shape of the $f_C(x)$ curve (see e.g. Refs. [74, 75]). These models are represent a big first step towards fully understanding the processes taking place near the n$^+$-bulk boundary.

6.5 Attempts at Quantifying Slow-Signal Leakage After the SSC

Numerous attempts were made at quantifying the number of slow-signals remaining after the SSC has been applied, see Table 5.14. Additionally, an attempt was made to parameterize the slow-signal energy spectrum in order to incorporate this in a WIMP fit. However, the results from the analyses outlined below proved to be difficult to interpret. The reason for this is the shape of the slow-signal energy spectrum depends on the source of slow-signals, i.e. their energy and distribution. Section 6.3.4 showed that the shape of the rise time spectrum is strongly dependent upon the source location and energy, therefore a slow-signal energy spectrum from an external source cannot be used to parameterize a slow-signal energy spectrum from backgrounds (cosmogenic $^{68,71}\text{Ge}$) alone. Any attempts at quantifying the slow-signal leakage with a source measurement would also be difficult, but may be possible [174], for these same reasons. The following sections will outline the attempts made in an effort to document lessons learned.

6.5.1 Ratio Analysis

As mentioned in the previous chapter, the MALBEK detector collected data at KURF for several months with $^{210}\text{Pb}$ contamination in the cryostat (see Section 5.1). By using these data (DS1c) and the data acquired after the lead shims were removed (DS3) it is possible to determine if there was any slow-signal contribution after the SSC. In what follows, we will take the proof-by-contradiction approach by assuming that the SSC is 100% efficient at
rejecting slow-signals near threshold. If this were the case, then the rate of fast-signals near threshold before (DS1c) and after (DS3) the lead shim removal should be equal (ignoring bremsstrahlung and Compton scatters from $^{210}\text{Pb}$). Similarly, the rate of slow-signals with the lead shims in place would be more than an order of magnitude larger than without (see Section 5.1.3). A useful way to test this hypothesis is to count the number of slow- and fast-signals, as determined by the SSC, in two separate energy regions for both data sets. First, count the number of slow-signals ($N_S$) in an energy region that the SSC is known to be valid, e.g. $2\text{–}8 \text{ keV}$, in both data sets. Second, count the number of fast-signals ($N_F$) near threshold, e.g. $0.6\text{–}1.0 \text{ keV}$, in both data sets. Figure 6.24 illustrates this for DS1c. Lastly, construct the following ratios for the energy regions outlined above,

$$R_S = \left( \frac{N_S \text{ events WITH Pb shims}}{N_S \text{ events WITHOUT Pb shims}} \right)_{2\text{–}8 \text{ keV}},$$

and

$$R_F = \left( \frac{N_F \text{ events WITH Pb shims}}{N_F \text{ events WITHOUT Pb shims}} \right)_{0.6\text{–}1.0 \text{ keV}}.$$ (6.19)

From the arguments outlined above, $R_S$ should be $\simeq 10$ and $R_F \simeq 1$. Therefore, the ratio of $R_F/R_S$ should be $\sim 0.1$. However, this ratio has been measured to be $1.21 \pm 0.11$. Since we calculated the number of slow-signals in an energy region in which the SSC can be trusted, this points to $R_F$ being the cause of the discrepancy. In order to bring the ratio up to the measured value, $R_F$ would have to increase by an order of magnitude, bringing into question the rejection power of the SSC near threshold. Unfortunately, it is not possible to make a statement on the number of slow-signals that contaminate the low-energy region after cuts. This is again due to the fact that this background is strongly dependent on the nature of the source. With the lead shims in place, the background due to slow-signals was dominated by external gamma-rays, whereas once they were removed, the slow-signal background was dominated by $^{68,71}\text{Ge}$, which is internal to the germanium crystal. In summary, the only conclusion that can be drawn from this analysis is that the SSC does not remove all of the

\footnote{We were only interested in the slow-signal counts here since we were searching for a relationship that related the number of observed slow-signals at higher energies to the number of expected slow-signals at lower energies. However, the results here proved to be difficult to interpret.}
slow-signals and some remain after cuts.

Figure 6.24: The calculation of $N_S$ and $N_F$ for DS1c is shown here. See text for details.

### 6.5.2 Using a Slow Signal Dominated Source

A 10 $\mu$Ci $^{241}$Am source was used for this analysis with the MALBEK detector. The source was suspended 25 cm away from the top of the cryostat for 1.8 hours. It was clear from the analysis that the $^{241}$Am data was dominated by slow-signals at low-energy ($< 12$ keV) and the contribution due to fast-signals could be ignored – the background rate due to slow-signals was much greater than that of fast-signals, i.e. $> 80\%$ of the events were slow.

For the data acquired in the configuration outlined above, slow-signals comprised $\sim 90\%$ of the event population between $0.6 - 12$ keV. Therefore, a slow-pulse acceptance curve was generated in the same manner as in Figure 5.23. This slow-pulse acceptance curve is shown in Figure 6.25 (dashed) along with the SSC curve (solid). The clear overlap at low energies highlights the need to quantify the slow-signal rejection efficiency. But, once again, since this analysis depends on an external source, $^{241}$Am, it cannot be used to quantify slow-signal backgrounds from background data. In addition to the ratio analysis presented in the previous
section, this analysis clearly shows that slow-signals remain after the SSC cut is applied to background data (DS3).

6.6 Discussion

This chapter has discussed slow-signal backgrounds in depth. Several new qualities of slow-signals were experimentally measured, e.g. background dependence on n$^+$ contact material and rise time/drift-time correlations. The results from these measurements agreed with the hypothesized physical mechanism responsible for slow-signals, diffusion and recombination near the n$^+$-bulk boundary.

Additionally, two simple diffusion models were presented in this chapter: the two-plane diffusion model and the probabilistic recombination diffusion model. Both of these models show that slow-signals arise from interactions near the n$^+$-bulk boundary and, to first order, reproduce several experimental phenomena: rise in counts in the low-energy spectrum, shape of slow ionization signals, rise time/drift-time correlation, and the mean rise time of slow-signals in the MALBEK detector. While the initial results from these models look very promising, there is still a lot to be done before a complete description of the physical processes taking place can be developed. The recombination probability, which is arguably the most important input to the model, needs to be more fully studied and this is currently being pursued by collaborators within MAJORANA.

This chapter has also shown that slow-signals significantly contaminate the low-energy spectrum after the SSC has been applied. This has been known, see e.g. Ref. [121]; however, different analysis techniques and a different detector were used here. This additional evidence of slow-signal contamination after cuts underscores the need to fully quantify slow-signal backgrounds in any germanium detector whose primary region-of-interest is the low-energy region ($< 12$ keV).

In summary, it is important to reiterate the fact that the shape of the slow-signal energy spectrum depends on the source of slow-signals, i.e. their energy and distribution. For this reason, there are only two ways to quantify this source of background:
Figure 6.25: (a) A 98% slow-signal acceptance curve for $^{241}\text{Am}$ with the MALBEK detector is shown here (dashed). 98% of all events fall below the dashed curve drawn here and are slow. Also plotted here is the SSC curve (solid) from Figure 5.30, which is also shown below. There is a clear overlap of the two curves at low-energies. The intensity scale is in units of Hz. (b) The SSC curve from DS3a used in the top plot is shown here.
• **Data driven method**: data used to quantify slow-signal backgrounds must have the same distribution of slow-signal sources, *i.e.* similar energy and location. To describe slow-signal backgrounds due to cosmogenic $^{68,71}\text{Ge}$, this could be done by activating the detector in order to obtain higher statistics, and using methods similar to those presented in this chapter as well as in Ref. [121].

• **Model driven method**: once a complete model has been developed, this model could be used to generate a fractional charge collected versus depth curve, $f_C(x)$. This curve could then be input into a Geant4-based Monte Carlo to develop energy spectra PDFs due to slow-signals. These PDFs could then be input into a WIMP analysis and also provide a means to quantify the slow-signal leakage after the SSC has been applied.
Chapter 7

Results From a Search for Light WIMPs

This chapter will report on results from a direct search for low-mass particle dark matter using a 221.49 live day exposure with the MALBEK detector. The expected signal from WIMP dark matter in germanium detectors, analysis techniques and results will be presented.

7.1 Introduction

As discussed in Section 1.1.3, WIMPs are a class of particle with masses between $1 - 1000$ GeV that explain the observed amount of nonbaryonic dark matter in the Universe, a result referred to as the WIMP miracle. Direct detection experiments, such as the MALBEK detector, aim to detect the nuclear recoils caused by WIMPs elastically scattering off atomic nuclei. The event rate and energy of the recoil depend upon experimental, astrophysical and nuclear factors:

- **Experimental:**
  - Target material, detector efficiency, target mass, and atomic number,
  - Backgrounds that can mimic WIMP signals, such as neutrons.

- **Astrophysical:**
  - WIMP halo properties (*i.e.* particle density, particle distribution, velocity distribution),
The motion of the Earth relative to the WIMP halo.

- **Nuclear:**
  - Nuclear form factor corrections due to the finite size of the nucleus. This also differs for spin-independent and spin-dependent interactions.

The next section will outline the expected signal from WIMP dark matter in germanium detectors.

### 7.2 The Signal from WIMP Dark Matter

This section will outline the signal from WIMP dark matter given the considerations outlined in the previous section. Since we are only sensitive to spin-independent interactions\(^1\), there will be no discussion of spin-dependent interactions.

#### 7.2.1 Event Rate

The energy spectrum from WIMP-nuclear recoils arises from the kinematics of elastic scattering. In the center-of-momentum frame, we assume that the WIMP scatters off a nucleus through an angle \(\phi\), with \(\cos \phi\) uniformly distributed between \(-1\) and \(+1\) (isotropic scattering). Assuming the WIMPs are moving at non-relativistic velocities their kinetic energy is expressed as \(E_i = \frac{M_W v^2}{2}\), and the nucleus will recoil with an energy (in the lab frame) of

\[
E_R = E_i r \frac{(1 - \cos \phi)}{2},
\]

where \(r\) is a dimensionless kinematic parameter between 0 and +1 that is defined as \(r \equiv \frac{4M_W M_T}{(M_W + M_T)^2}\), \(M_T\) is the mass of the target nucleus and \(M_W\) is the mass of the WIMP. It's clear that the recoil energy is uniformly distributed between \(0 - E_i r\). It is interesting to note that \(r = 1\) only if \(M_W = M_T\). The differential event rate, \(R\), with dimensions

\(^1\)The only naturally occurring isotope of germanium with a net nuclear spin is \(^{73}\text{Ge} \ (J = 9/2^+)\). Unfortunately, \(^{74}\text{Ge}\) only makes up \(\sim 7\%\) of natural germanium, therefore we are essentially insensitive to spin-dependent reactions.
of inverse time and energy, can be shown to be

\[ \frac{dR}{dE_R} (E_R) = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dR(E_i)}{E_i r}. \]  \hspace{1cm} (7.2)

The maximum initial WIMP energy is based on the Galactic escape velocity, \( v_{\text{esc}} \): \( E_{\text{max}} = M_W v_{\text{esc}}^2 / 2 \). To cause a recoil of energy \( E_R \), the minimum WIMP energy is \( E_{\text{min}} = E_R / r \) \((\phi = \pi)\) and the corresponding WIMP velocity, \( v_{\text{min}} \) is \( v_{\text{min}} = \sqrt{2E_{\text{min}} / M_W} = \sqrt{2E_R / (rM_W)} \). In order to determine the rate of WIMP-nucleus scattering, imagine the target nucleus moving with velocity \( v \) relative to the WIMPs. In time \( dt \), each nucleus will interact with any WIMP inside a volume \( dV = \sigma_{\text{nucl}} v \ dt \), where \( \sigma_{\text{nucl}} \) is the WIMP-nucleus cross section. The number of WIMPs inside this volume moving with velocity \( v \) is then

\[ dN = n_0 f(v, v_E) \sigma_{\text{nucl}} v \ dt, \]  \hspace{1cm} (7.3)

where the local WIMP number density is related to the local mass density of WIMPs in our Galactic neighborhood, \( n_0 = \rho_D / M_W \). The above equation relies upon an assumption of the WIMP velocity distribution, which depends on the velocity of the Earth with respect to the Galaxy \((v_E)\) and the WIMP velocity with respect to the earth \((v)\). The most common assumption made is that the WIMPs follow a Maxwellian distribution [30],

\[ f(v, v_E) = \frac{e^{-(v + v_E)^2 / v_0^2}}{k}, \]  \hspace{1cm} (7.4)

where \( v_0 \) is the dark matter halo velocity dispersion and \( k \) is a normalization constant. Although this is an assumption, a Maxwellian distribution is a good approximation and provides a useful standard for experiments to abide by [30, 175]. Additionally, most experiments assume that \( \rho_D = 0.3 \text{ GeV cm}^{-3} \) [77, 107–109, 111, 112], despite the fact that recent measurements have shown this value to be closer to 0.4 GeV cm\(^{-3}\) [7, 176–180]. It is important to follow the standards already laid out in order to be able to compare results from several experiments. Therefore we have adopted a value of 0.3 GeV cm\(^{-3}\) for the local dark matter density.
The differential rate per kilogram of target mass is then the product of the number of interactions per nucleon with the number of nuclei per kilogram of material, or

\[ dR = \frac{N_0}{A} n_0 f(v, v_E) \sigma_{\text{nucI}} v \, d^3v \]  

(7.5)

where \( N_0 \) is Avogadro’s number, so that \( N_0/A \) is the number of nuclei per kilogram of material.

The full calculation of the energy spectrum for WIMP-nucleus elastic scattering, including the effects of both escape velocity and the Earth’s velocity, has been done by Lewin and Smith [181] and is given by,

\[ \frac{dR}{dE_R} = \frac{k_0}{k_1} \frac{R_0}{E_0} r \left\{ \frac{v_0}{4E} \left[ \text{erf} \left( \frac{v_{\text{min}} + v_E}{v_0} \right) - \text{erf} \left( \frac{v_{\text{min}} - v_E}{v_0} \right) \right] - e^{-v_{\text{esc}}^2/v_0^2} \right\} \]  

(7.6)

The parameters in the above equation are itemized as follows:

\( R \) – Event rate per unit mass,

\( R_0 \) – Total event rate,

\( E_R \) – Recoil energy of the nucleus,

\( v_0 \) – Dark matter halo velocity dispersion,

\( E_0 \) – Energy of WIMP moving with velocity \( v_0 \),

\( v_{\text{min}} \) – Minimum WIMP velocity which can give a recoil energy \( E_R \),

\( v_{\text{esc}} \) – Galactic escape velocity,

\( v_E \) – Velocity of the Earth (target) relative to the dark matter distribution,

\( M_W \) – Mass of a WIMP,

\( M_T \) – Mass of a nucleus in the detector material,

\( r \) – Reduced mass (kinematical factor) given by \( \frac{4M_W M_T}{(M_W + M_T)^2} \),

\( A \) – Atomic number of a nucleus in the detector material,
\( \rho_D \) – Local dark matter halo density,

\( N_A \) – Avogadro’s number,

\( \sigma_{\text{nucl}} \) – WIMP-nucleus cross section at zero velocity.

with

\[
\frac{k_0}{k_1} = \left[ \text{erf} \left( \frac{v_{\text{esc}}}{v_0} \right) - \frac{2}{\sqrt{\pi}} \frac{v_{\text{esc}}}{v_0} e^{-v_{\text{esc}}^2/v_0^2} \right]^{-1},
\]

(7.7)

and where the relationship between \( R_0 \) and the WIMP-nucleus cross section, \( \sigma_{\text{nucl}} \) is given by:

\[
R_0 = \frac{2}{\sqrt{\pi}} \frac{N_A \rho_D}{M_W} \sigma_{\text{nucl}} v_0.
\]

(7.8)

In the above equations, the parameter \( k \) is a normalization constant such that \( \int_0^{v_{\text{esc}}} dn = n_0 \).

The normalization constants \( k_0 \) and \( k_1 \) seen in Equation 7.6 correspond to the value of \( k \) truncated at differing escape velocities, \( v_{\text{esc}} = \infty \) and \( v_{\text{esc}} = |v + v_E| \), respectively. Given the assumptions about the WIMP density and velocity distributions, the interaction rate depends on two unknowns, the WIMP mass (\( M_W \)) and the cross section of the WIMP-nucleus interaction (\( \sigma_{\text{nucl}} \)). For this reason, experiments typically report results as a contour plot in the WIMP mass-cross section plane. An example of such a contour plot was shown in Figure 3.6 of Chapter 3.

**The WIMP-nucleon Cross Section**

In order to be able to compare results between experiments which use a different target nucleus, Lewin and Smith [181] suggest to scale \( \sigma_{\text{nucl}} \) to a WIMP-nucleon cross section, \( \sigma_{W-n} \), whose relationship is defined as [182]:

\[
\sigma_{W-n} = \left( \frac{\mu_1}{\mu_A} \right)^2 \frac{1}{A^2} \sigma_{\text{nucl}}
\]

(7.9)
where $\mu_A = \frac{M_W M_T}{M_W + M_T}$ is the reduced mass of the WIMP-target system and $\mu_1$ is defined at $A = 1$.

### 7.2.2 Nuclear Form Factor Correction

In a WIMP-nucleus recoil, if the momentum transfer $(q = \sqrt{2M_T E_R})$ is large enough so that the deBroglie wavelength $h/q$ is small compared to the size of the nuclear radius, then the effective cross section begins to fall with increasing momentum transfer. This is often referred to as ‘loss of coherence,’ and is due to the fact that the WIMP is no longer probing average nuclear properties. The distribution of nucleons within the nucleus is now important. This is accounted for by applying a nuclear form factor correction, $F(qr_n)$, such that $\sigma_{\text{nucl}} \rightarrow \sigma_{\text{nucl}} F^2$, where $r_n$ is the effective nuclear radius ($\approx 1.14A^{1/3}$) [181, 184]. As proposed by Engel et al. [185], and implemented by Lewin and Smith [181], the Woods-Saxon form factor [186] is an excellent approximation for spin-independent interactions,

$$F(qr_n) = \frac{3[\sin(qr_n) - qr_n \cos(qr_n)]}{(qr_n)^3} e^{-(qs)^2/2}, \quad (7.10)$$

where $s$ is a measure of the nuclear skin thickness (0.9 fm for Ge [181, 184]) and $qr_n$ is given as

$$qr_n = \frac{\sqrt{2M_T E_R}}{197.3} 1.14A^{1/3}. \quad (7.11)$$

This correction is implemented by replacing $\sigma_{\text{nucl}}$ by $\sigma_{\text{nucl}} F^2$.

### 7.2.3 Quenching in Germanium

Quenching in germanium was briefly discussed in 2.1.1. A more in depth discussion is needed since this is arguably the most important experimental input when calculating limits.

Gamma-ray and neutron/WIMP interactions within an ionization detector are fundamentally different — neutrons/WIMPs recoil off of nuclei, while photons interact with the electrons in the detector. Since semiconductor gamma-ray detectors are typically calibrated with standard gamma-ray check sources, this calibration will not be valid for nuclear recoils. The apparent observed nuclear recoil energy is less than the true value. The ratio, or relative


efficiency, \( f_n \) is determined by neutron scattering measurements. Consequently, the nuclear recoil energy in the above event rates should be replaced by the ionization energy, \( E_I \), using \( E_R = E_I / f_n \). Lindhard et al. [86] represent \( f_n \) by

\[
\frac{1}{f_n} = \frac{kg(\epsilon)}{1 + kg(\epsilon)}
\]

(7.12)

where, for a nucleus of atomic number \( Z \),

\[
\epsilon = 11.5 E_R(\text{keV}) Z^{-7/3},
\]

(7.13)

and \( g(\epsilon) \) has been well fit by

\[
g(\epsilon) = 3\epsilon^{0.15} + 0.7\epsilon^{0.6} + \epsilon.
\]

(7.14)

Barbeau et al. [85] have recently measured \( k \) to be \( \sim 0.2 \) in germanium for energies \(< 4 \text{ keV} \). To summarize, the relationship between nuclear and electronic recoils is expressed as follows,

\[
E_I = \frac{k E_R g(\epsilon)}{1 + kg(\epsilon)}
\]

(7.15)

Incorporating these results into a WIMP fit requires one to invert Equation 7.15, which can be done numerically, but not analytically. Here we take the same approach used in Refs. [76, 121] and use an empirical formula to describe this relationship

\[
E_I = \alpha E_R^\beta,
\]

(7.16)

where \( \alpha \) and \( \beta \) are 0.2 and 1.12, respectively [121].

### 7.2.4 Summary

In summary, the final observed event rate is given by

\[
\frac{dR}{dE_I} = \left( \frac{dR}{dE_R} \right) \left( \frac{dE_R}{dE_I} \right) F^2
\]

(7.17)
where \( \frac{dR}{dE_R} \) is given by Equation 7.6, \( \frac{dE_R}{dE_I} \) is obtained by inverting Equation 7.16 and taking the derivative with respect to \( E_I \) and \( F \) is given in Equation 7.10. Equation 7.17 will be used to fit the observed energy spectrum from Data Set 3 (see Table 5.1) and search for WIMP dark matter. Table 7.1 lists the parameters used in the WIMP fits shown in the following sections. In addition, for illustrative purposes, Figure 7.1 shows the final observed event rate as a function of energy (Eq. 7.17) in a germanium detector for a 5, 10 and 15 GeV WIMP.

Table 7.1: Astrophysical\(^{a} \) and experimental parameters used in the WIMP analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atomic Mass</td>
<td>72.61</td>
<td>amu</td>
<td>[187]</td>
</tr>
<tr>
<td>Local dark matter density, ( \rho_D )</td>
<td>0.3</td>
<td>GeV cm(^{-3} )</td>
<td>[77, 107–109, 111, 112]</td>
</tr>
<tr>
<td>Halo velocity dispersion, ( v_0 )</td>
<td>220</td>
<td>km s(^{-1} )</td>
<td>[7, 188–194]</td>
</tr>
<tr>
<td>Galactic escape velocity, ( v_{esc} )</td>
<td>544</td>
<td>km s(^{-1} )</td>
<td>[7, 188, 195]</td>
</tr>
<tr>
<td>Earth velocity, ( v_E )</td>
<td>244</td>
<td>km s(^{-1} )</td>
<td>[181]</td>
</tr>
<tr>
<td>Detector mass</td>
<td>0.4042</td>
<td>kg</td>
<td>[122]</td>
</tr>
<tr>
<td>Nuclear skin thickness, ( s )</td>
<td>0.9</td>
<td>fm</td>
<td>[181, 184]</td>
</tr>
<tr>
<td>Data Set 3 Specifics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threshold</td>
<td>0.6</td>
<td>keV</td>
<td>N/A</td>
</tr>
<tr>
<td>Max energy</td>
<td>4.0</td>
<td>keV</td>
<td>N/A</td>
</tr>
<tr>
<td>Live time</td>
<td>221.494</td>
<td>d</td>
<td>N/A</td>
</tr>
<tr>
<td>Exposure</td>
<td>89.528</td>
<td>kg-d</td>
<td>N/A</td>
</tr>
</tbody>
</table>

\(^{a}\) There remains significant uncertainty in the astrophysical parameters (\( \rho_D, v_0, v_{esc}, v_E \)), therefore we have used values used by other experimental programs, e.g. Refs. [77, 107–109, 111, 112].

\(^{b}\) The fiducial mass of the detector is smaller when the SSC is applied, see Section 6.1.2.

### 7.3 Fitting Technique

This section will cover the analysis techniques used to extract a WIMP dark matter signal limit from the MALBEK data. The MALBEK data used for this analysis is shown in Figure 7.2b along with the data from CoGeNT [75, 121]. The aim here is to test if the WIMP signal (Equation 7.17) is compatible with our experimental data given a specific background model. The method used in this analysis follows the prescription outlined in Ref. [196]. This method
Figure 7.1: Ionization spectrum induced from WIMP nuclear-recoils for various WIMP masses.
will be referred to as the Rolke method hereafter. The Rolke method was also used by members of the CoGeNT collaboration to place a limit on $\sigma_{W-n}$ as a function of $M_W$ \cite{76}, \textit{i.e.} calculation of a WIMP signal exclusion curve or signal region. The software package \texttt{pyWIMP} \cite{197} was developed by Michael Marino to calculate these limits with the Rolke method. The framework of \texttt{pyWIMP} was used in this analysis. The following section will introduce details of the Rolke method and how it was implemented.

### 7.3.1 Rolke Method

The Rolke method provides a means by which to calculate confidence intervals (or set limits) on a parameter from a data set with unknown backgrounds. In our case, this parameter is $\sigma_{\text{nucl}}$ (which is directly related to $\sigma_{W-n}$). Prior to the Rolke method, the standard method used by many in high-energy physics was the Feldman and Cousins method \cite{198}, which assumes that the background is exactly known. An alternative method used prior to the Feldman and Cousins method was the $\mathcal{L} + \frac{1}{2}$ method which relies on finding the points where the profile likelihood function (details to follow) increases by a factor defined by the required confidence level (CL). However, both of these methods have their problems:

- The $\mathcal{L} + \frac{1}{2}$ method performs poorly in cases with low-statistics, as is the case in most rare-event searches \cite{7, 196};

- The Feldman and Cousins method requires one to know their background exactly, which is rarely the case \cite{7, 196}.

Rolke et al. developed their method in order to calculate confidence intervals when sources of background are unknown, these backgrounds are referred to as a nuisance parameters. The Rolke method combines the $\mathcal{L} + \frac{1}{2}$ method with the profile likelihood approach. Prior to discussing the details of the profile likelihood approach, a discussion on the method of Maximum Likelihood (ML) is needed.

\begin{footnote}{\texttt{pyWIMP} documentation can be found in Ref. \cite{76}.}
Figure 7.2: (a) The MALBEK energy spectrum with the SSC applied is shown here. (b) The energy spectrum used for the CoGeNT analysis in Ref. [75] (blue) is shown here in comparison to the MALBEK energy spectrum with the SSC applied (red hatched). Both spectra have been corrected for cut acceptance efficiencies as well. In addition, the CoGeNT collaboration has made an attempt to correct for slow-signal contamination in Ref. [121]. The black/gray hatched spectrum shows the CoGeNT spectrum from Ref. [121] corrected for slow-signal leakage and with the L-lines removed. It is interesting to note that the flat part of the background is identical in both experiments, even after CoGeNT corrected for slow-signal contamination, a topic of future investigation.
The Method of Maximum Likelihood

Suppose that we have $N$ measured quantities $\mathbf{x} = (x_1, \ldots, x_N)$ that can be described by a probability density function (PDF) $f(\mathbf{x}; \theta)$ where $\theta = (\theta_1, \ldots, \theta_n)$ are a list of $n$ parameters that are unknown. The likelihood function is then given by the PDF evaluated with the data $\mathbf{x}$, but viewed as a function of the parameters, $L(\theta) = f(\mathbf{x}; \theta)$. The likelihood function is given by

$$L(\theta) = \prod_{i=1}^{N} f(x_i; \theta). \quad (7.18)$$

The ML method takes the estimators, $\hat{\theta}_T$, to be those values of $\theta$ that maximize $L(\theta)$, or equivalently, minimize $-\log(L(\theta))$. In practice, it is easier to work with $\log(L(\theta))$ and the ML estimators are then found by solving the likelihood equations

$$\frac{\partial \log(L)}{\partial \theta_i} = 0, \quad i = 1, \ldots, n. \quad (7.19)$$

The solutions to the likelihood equations are usually found numerically.

Profile Likelihood Approach

When searching for a signal in a data set, it is useful to perform a hypothesis test that compares the null hypothesis (no signal) to a model with signal included. In practice this is performed using a profile likelihood test which involves the construction of a likelihood ratio, $\lambda(\theta_0)$, over a set of parameters, $\theta_0$,

$$\lambda(\theta_0) = \frac{L_{\text{max}}}{L_{\text{max}}(\theta_0, \theta_n)} \quad (7.20)$$

which can be rewritten as

$$-2 \log(\lambda(\theta_0)) = 2 \left( \log(L_{\text{max}}(\theta_0, \theta_n)) - \log(L_{\text{max}}) \right) \quad (7.21)$$

where $\theta_0$ is a subset of $\theta$, or $\theta = \{\theta_0, \theta_n\}$. $L_{\text{max}}$ is the likelihood function that has been maximized over all parameters whereas $L_{\text{max}}(\theta_0, \theta_n)$ is the likelihood function maximized...
over all parameters except $\theta_0$. A standard result from Statistics [199] is that $-2\log(\lambda(\theta_0))$ resembles a $\chi^2$ distribution with $N_{dof}$ degrees of freedom. To find a $100(1 - \alpha)\%$ confidence interval, we start at the minimum of the $-2\log(\lambda(\theta_0))$ curve and then move to the left and right to find the points at which the function increases by the $\alpha$ percentile for a $\chi^2$ distribution. For example, if $N_{dof} = 1$, a 90% confidence interval corresponds to an increase of 2.71 and 95% corresponds to an increase of 3.84, see e.g. Figure 7.3 [7, 196].

![Figure 7.3: An example of a $-2\log(\lambda(\theta_0))$ curve with the 95% confidence intervals shown by vertical lines. Figure from Ref. [196].](image)

**Implementation**

The Rolke method is very similar to the profile likelihood approach, with the exception that it takes into account two scenarios:

1. Best fit occurs in an unphysical region, e.g. $\sigma_{nucl} < 0$;
2. There is no minimum of the $-2\log(\lambda(\theta_0))$ curve, e.g. signal and background very similar.

Rolke et al. proposed the following solution for dealing with these scenarios: If the best fit of the parameter of interest occurs in an unphysical region, then the parameter is forced to
its nearest boundary and this is used as the best fit (or starting point) to scan for confidence intervals. On the other hand, if the best fit occurs in a physical region, this method is identical to the profile likelihood method. For example, in Figure 7.3, if the best fit for the signal rate was <0, then only an upper limit could have been found. If this were the case, the upper limit is found by starting at a signal rate of 0 (nearest physical boundary) and scanning to the right until a desired confidence interval is reached. Rolke et al. refer to this method as the bounded-likelihood method and this method was used by Ref. [76] as well. In the following sections, all limits will have been found using this method. Additionally, Ref. [76] has performed a coverage test with their background model and has shown this method to have sufficient coverage. The background model presented in the following section is identical to the background model used in Ref. [76], therefore it is safe to conclude that all limits derived using the Rolke method have sufficient coverage.

### 7.4 WIMP Dark Matter Limits from MALBEK

The data presented and analyzed in Chapter 5, specifically DS3, were used to constrain the $\{M_W, \sigma_{W-}n\}$ parameter space. The Rolke method outlined above was used to calculate upper limits on $\sigma_{nucl}$, and hence $\sigma_{W-}n$, for a range of WIMP masses at 90% CL. The following sections will outline the data and fitting model, present the results and finish with a discussion of the results obtained.

#### 7.4.1 Data and Fitting Model

An unbinned extended maximum likelihood fit with the Rolke method has been implemented, thus making the best possible use of the recorded energy of each individual event. Each data point was multiplied by a weighting factor defined by the inverse of the cut efficiency at that energy, see Figure 5.36. The fitting PDFs were split up into signal and background. The components of each are listed in Table 7.2. An extended likelihood formalism was used since the primary concern was the number of signal, or WIMP-related, events in DS3 for each
WIMP mass. The total fit PDF then reduces to

\[ f_{\text{total}}(E) = N_S S(E) + N_B B(E), \]

(7.22)

where \( S(E) = f_W(E) \) is the signal PDF in Table 7.2 (Equation 7.17), \( B(E) \) is the sum of the background PDFs in Table 7.2, \( N_S \) is the number of signal events, and \( N_B \) is the number of background events. The above equation is normalized so that \( N_{\text{total}} = N_S + N_B \), where \( N_{\text{total}} \) is the total number of events in the sample (DS3). Additionally, each background PDF has with it an associated amplitude in units of number of events, \( i.e. \)

\[ N_B = N_{\text{flat}} + N_{\text{exp}} + N_{\text{ZnL}} + N_{\text{GeL}} \]

(7.23)

where \( N_x \) is the number of counts associated with PDF \( x \). The RooFit package \cite{200} was used to add these PDFs together and construct the likelihood ratio with the parameter of interest being \( \sigma_{\text{nuc}} \), or equivalently \( -2 \log(\lambda(\sigma_{\text{nuc}})) \). For each WIMP mass, ranging from 5.5 GeV to 100 GeV, a \( -2 \log(\lambda(\sigma_{\text{nuc}})) \) curve was constructed by scanning the \( \sigma_{\text{nuc}} \) parameter space so as to include the best fit (minimum of \( -2 \log(\lambda(\sigma_{\text{nuc}})) \)) and sufficient samples above the 90% CL, see \( e.g. \) Figure 7.4.

Table 7.2: Likelihood fitting PDF components - parameters are provided in Table 7.3.

<table>
<thead>
<tr>
<th>Description</th>
<th>Functional Form</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Background PDF Components</strong></td>
<td></td>
</tr>
<tr>
<td>Flat background</td>
<td>( f_{\text{flat}}(E) = 1 )</td>
</tr>
<tr>
<td>Exponential background(^a)</td>
<td>( f_{\text{exp}}(E) = \exp(c_1 E) )</td>
</tr>
<tr>
<td>( ^{65}\text{Zn} ) L-capture ( \gamma ) line</td>
<td>( f_{\text{ZnL}}(E) = \frac{1}{\sigma_{\text{ZnL}} \sqrt{2\pi}} \exp \left( -\frac{(E-\mu_{\text{ZnL}})^2}{2\sigma_{\text{ZnL}}^2} \right) )</td>
</tr>
<tr>
<td>( ^{68,71}\text{Ge} ) L-capture ( \gamma ) line</td>
<td>( f_{\text{GeL}}(E) = \frac{1}{\sigma_{\text{GeL}} \sqrt{2\pi}} \exp \left( -\frac{(E-\mu_{\text{GeL}})^2}{2\sigma_{\text{GeL}}^2} \right) )</td>
</tr>
<tr>
<td><strong>Signal PDF Components</strong></td>
<td></td>
</tr>
<tr>
<td>WIMP signal</td>
<td>( f_W(E) = \left( \frac{dR}{dE_R} \right) \left( \frac{dE_R}{dE_I} \right) F^2 )</td>
</tr>
</tbody>
</table>

\(^a\) The fit was performed both with and without the exponential component.

The parameters in the PDFs listed in Table 7.2 were given ranges of values for the fit –
Figure 7.4: An example of a $-2\log(\lambda(\theta_0))$ curve with $\theta_0 \equiv \sigma_{W-n}$. Vertical and horizontal lines intersect at the 90% CL upper limit on the cross section. $\sigma_{\text{nucl}}$ was converted into $\sigma_{W-n}$ after the fit was performed. This curve was generated for $M_W = 10.2$ GeV. It is clear that the lower limit at which 90% CL occurs is in an unphysical region, i.e. $\sigma_{W-n} < 0$.

Table 7.3: Allowed ranges and values for input parameters used in the WIMP fit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Background PDF Components</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential shape parameter, $c_1$</td>
<td>$-100 \rightarrow 5$</td>
<td>keV$^{-1}$</td>
</tr>
<tr>
<td>Zn L-capture mean, $\mu_{Zn,L}$</td>
<td>1.096 (fixed)</td>
<td>keV</td>
</tr>
<tr>
<td>Zn L-capture sigma, $\sigma_{Zn,L}$</td>
<td>7.22e-2 $\rightarrow$ 7.51e-2</td>
<td>keV</td>
</tr>
<tr>
<td>Ge L-capture mean, $\mu_{Ge,L}$</td>
<td>1.299 (fixed)</td>
<td>keV</td>
</tr>
<tr>
<td>Ge L-capture sigma, $\sigma_{Ge,L}$</td>
<td>7.31e-2 $\rightarrow$ 7.60e-2</td>
<td>keV</td>
</tr>
<tr>
<td>$N_{flat}$</td>
<td>$0 \rightarrow 10^5$</td>
<td>counts</td>
</tr>
<tr>
<td>$N_{exp}$</td>
<td>$0 \rightarrow 10^5$</td>
<td>counts</td>
</tr>
<tr>
<td>$N_{GeL} + N_{ZnL}$</td>
<td>$0 \rightarrow 10^5$</td>
<td>counts</td>
</tr>
<tr>
<td><strong>Signal PDF Components</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WIMP-nucleus cross section, $\sigma_{\text{nucl}}$</td>
<td>$-10 \rightarrow 100^a$</td>
<td>pb$^b$</td>
</tr>
</tbody>
</table>

$^a$ The upper limit was expanded dynamically to ensure that $\lambda(\sigma_{\text{nucl}})$ would exceed the desired $\chi^2$-quantile value.

$^b$ 1 pb $= 10^{-36}$ cm$^2$.

$^c$ The fit was performed both with and without the exponential component.
these parameters are summarized in Table 7.3. In addition, the $^{68,71}$Ge and $^{65}$Zn L-capture lines were grouped as one PDF during the fit. Results were obtained by fitting the energy spectrum with the SSC applied using Equation 7.17. As mentioned in the previous chapter, we were unable to parameterize the slow-signal energy spectrum since it is dominated by internal cosmogenics. If we were successful in doing so, we would incorporate a slow-signal PDF into the background PDFs. Members of the CoGeNT collaboration have taken an approach that utilizes an exponential PDF to encapsulate their ignorance of the shape of the slow-signal energy spectrum [76] – we have taken the same approach. In addition, Ref. [112] has reanalyzed CoGeNT data sets with this exponential. The WIMP and exponential PDFs closely resemble one another over various WIMP masses, therefore the inclusion of an exponential PDF can result in an unphysical enhanced sensitivity to WIMP-nucleus recoils. The enhanced sensitivity results from the fact that more events could be attributed to the exponential PDF rather than the WIMP PDF for a certain WIMP mass – this is illustrated in Figure 7.5 for a 7.4 GeV WIMP. A more conservative approach would be to eliminate the exponential PDF entirely, therefore the observed increase in counts at low-energy (< 2 keV) would be completely attributed to WIMP-nucleus recoils in the fit. Therefore, we have calculated results both with and without the exponential PDF. In summary, results have been calculated to constrain the $\{M_W, \sigma_{W-n}\}$ parameter space in the following scenarios:

1. With the SSC applied AND using the exponential PDF (SSC + Exp),

2. With the SSC applied AND without using the exponential PDF (SSC + No Exp),

We will see in the following sections that the removal of the exponential PDF has a significant effect on the fit at higher WIMP masses. This stems from the fact that at higher WIMP masses, the WIMP PDF does not resemble the observed low-energy rise.

### 7.4.2 Results

We have calculated WIMP exclusion limits at 90% CL under various background assumptions and with differing cuts applied. The following sub-sections will highlight these results.
Figure 7.5: Best fit result for a 7.4 GeV WIMP with the SSC applied. The WIMP signal is in dashed blue, \(^{68,71}\text{Ge}\) and \(^{65}\text{Zn}\) L-capture lines in solid red, exponential in dashed red and total fit result in solid blue. Notice how the exponential and WIMP components closely resemble one another.
Example Spectral Fits

It is important to examine the spectral fits for each WIMP mass. Fits for $M_W = 8.0$ and 60.0 GeV were chosen for each of the fit methods outlined above. Figure 7.6 shows spectral fits with the SSC applied and with the exponential PDF – it is clear that at 8.0 GeV, the 90% CL fit attributes most of the low-energy rise to WIMPs. However, this is not the case at $M_W = 60.0$ GeV, where the low-energy rise is attributed solely to the exponential PDF.

As alluded to previously, while the removal of the exponential PDF may be more conservative, this does not imply that is the correct avenue to pursue. This is illustrated in Figure 7.7b which shows fit results for $M_W = 60.0$ GeV – it is apparent that the signal and background models do not agree with the data. This can be attributed to the sharp increase in counts at low-energy; the shape of this feature is not encapsulated in the background PDFs after the removal of the exponential PDF. However, at lower WIMP masses ($M_W \lesssim 10.0$ GeV), the data is well fit without the exponential PDF as shown in Figure 7.7a. The features observed in the spectral fits both with and without the exponential PDF merit a more in depth study of the fit stability as a function of WIMP mass. This is covered in the following section.

Fitted Background Component Stability

The previous section illustrated that the removal of the exponential PDF leads to poor fits at higher WIMP masses, and subsequently less stringent exclusion limits. For this reason, a study of the background component amplitude as a function of WIMP mass was performed. The amplitude of each background component was traced as the WIMP mass was varied. This provided a means by which to search for fit systematics. Figure 7.8 shows the background amplitudes without the exponential PDF. It is clear that as $M_W$ increases, the flat background component approaches zero - this is caused by the WIMP signal taking over the flat background component, see e.g. Figure 7.7b. However, the L-capture lines amplitude remained quite stable throughout the entire fitting range.

Figures 7.9 and 7.10 illustrate these results with the exponential PDF included. These figures show that the flat and exponential backgrounds exhibit a give-and-take relationship –
Figure 7.6: 90% CL exclusion limit spectral fits with exponential component and SSC applied. The $^{68,71}$Ge and $^{65}$Zn L-capture lines are shown as solid red, exponential background as dashed red, WIMP signal as dashed blue and total fit as solid blue.
Figure 7.7: 90% CL exclusion limit spectral fits without exponential component and SSC applied. The $^{68,71}$Ge and $^{65}$Zn L-capture lines are shown as solid red, WIMP signal as dashed blue and total fit as solid blue.
as one of the components is decreased the other is increased. In regions like this, the WIMP signal and exponential background closely resemble each other. Taking Figure 7.10 as an example, this figure illustrates the fit stability as a function of WIMP mass with the SSC and the exponential PDF. This figure clearly shows the give-and-take relationship outlined above, however it should be noted that the number of counts attributed to the exponential PDF is negligible otherwise. Additionally, in all of these figures, 7.8 → 7.10, the L-capture line components were extremely stable, indicating that their contribution to the fit has little effect on the WIMP signal component.

90% CL Exclusion Limits

The limits derived from the analysis described above are shown in Figure 7.11. In addition, a zoom in to lower WIMP masses is shown in Figure 7.12. These results echo what we have stated previously, that the removal of the exponential PDF decreases the sensitivity. However, it should be noted that if we were able to correct for slow-signal contamination, similar to Ref. [121], then our sensitivity would be enhanced. This enhancement was illustrated for CoGeNT by Refs. [111, 112].

7.4.3 Discussion and Comparison to Other Experiments

Figures 7.13 and 7.14 show the 90% CL MALBEK exclusion limits compared to several other experimental programs. There is slight tension between the MALBEK results and those from CoGeNT [74, 75], DAMA/LIBRA [104] and CRESST-II [77]. The CoGeNT and MALBEK detectors are nearly identical (see Table 3.1), therefore one would naturally attribute the increased sensitivity to lower backgrounds. This is exactly the case as is shown in Figure 7.2b. The MALBEK backgrounds are lower than those of CoGeNT in Ref. [75] below the L-capture peaks. Despite a higher threshold (600 eV compared to ~500 eV), the decreased backgrounds allowed MALBEK to be more sensitive to the WIMP signal. However, it is clear from the energy spectrum shown in Figure 7.2b that a low-energy rise is still observed – this could be attributed to WIMPs or the known leakage from slow-signals (see Chapter 6). As previously stated, the CoGeNT collaboration has recently made an attempt to correct for slow-signal...
Figure 7.8: (SSC + No Exp) The 90% CL final floating parameter values are shown here in units of counts kg\(^{-1}\) d\(^{-1}\) versus WIMP mass. The SSC has been applied, whereas the exponential PDF was not included. The horizontal red line in the L-capture component panel indicates the expected number of counts based on the number of K-capture events\(^{(68\text{Ge} [10.367 \text{ keV}]: 965 \text{ events}; \ 65\text{Zn} [8.979 \text{ keV}]: 414 \text{ events})}\) – see Section 5.3.4. However, it should be noted that during the first 40 days of DS3 a non-negligible amount of \(^{71}\text{Ge}\) was present.
Figure 7.9: (SSC + Exp) The 90% CL final floating parameter values are shown here in units of counts kg$^{-1}$ d$^{-1}$ versus WIMP mass. The SSC has been applied and the exponential PDF was included. Again, the red line indicates the expected number of L-capture events.
Figure 7.10: (SSC + Exp) Zoomed in version of Figure 7.9.
Figure 7.11: The MALBEK WIMP exclusion limits at 90% CL are shown here. See text for detailed explanation.
Figure 7.12: The MALBEK WIMP exclusion limits at 90% CL are shown here zoomed in. See text for detailed explanation.
contamination in Ref. [121]. This correction to their energy spectrum is also shown in Figure 7.2b with the L-capture lines removed. Ref. [111] has re-analyzed the CoGeNT data in Ref. [75] taking into account the slow-signal leakage – their results moved the CoGeNT claim down in cross-section making it more sensitive, see Figure 7.14. If we were able to do the same with MALBEK, our limits would also become more sensitive, thus increasing the tension of our results with those of CRESST-II and DAMA/LIBRA. However, there would likely still be room for both CoGeNT and MALBEK to be compatible. We should also note that the MALBEK exclusion limits are in agreement with several other null-result experiments: CDMS II (low-energy analysis)[109] and XENON10-LE [108].

7.5 Concluding Remarks

We have performed a search for low-mass WIMPs with the MALBEK detector. It is very important to reiterate that we know that slow-signals contaminate the spectrum after the SSC. Since we were unable to quantify this background, we have performed an identical WIMP analysis to Ref. [76] due to the fact that the MALBEK and CoGeNT detectors are very similar (same material, roughly the same mass and energy threshold and similar dominant background components - slow signals and cosmogenics). The MALBEK results were found to be in agreement with null observations and in slight disagreement with CoGeNT, CRESST-II and DAMA/LIBRA. It is very important to note that there exist astrophysical and experimental uncertainties that could bring these disagreements into alignment [111, 112], most notably the proper inclusion of the slow-signal contamination. The investigation into this contamination will be followed up by other members of the MAJORANA Collaboration and we will hopefully have an answer in the near future.
Figure 7.13: The current status of low-mass WIMP searches with the 90% CL limits from MALBEK is shown here. All results shown are at 90% CL unless otherwise noted. The DAMA/LIBRA annual modulation results shown here are the first results as interpreted by Ref. [104]. The other experiments that have shown a positive result are CRESST II [77] (95% CL), CoGeNT 2010 [74] and CoGeNT again in 2011 (annual modulation) [75]. Additionally, the CoGeNT 2011 result as interpreted by Ref. [111] is also shown. Several experiments have reported null results, or show no signs of a low-mass WIMP. Two of these experiments are shown here: XENON10-LE [108] and CDMS II [109]. (Data obtained using DMTools [114].)
Figure 7.14: A zoomed in version of Figure 7.13.
Appendix A

ORCA Scripts

A.1 Status Script

The status script was written by Graham Giovanetti and me with a lot of help from Mark Howe.

```c
function main()
{
    // loop forever
    ln2_alarm = 0;
    while (ln2_alarm ==0){
        i = 0;
        j = 0;
        k = 0;
        l = 0;
        totalsize = 0;
        totalcount1 = 0;
        totalcount2 = 0;
        totalcount3 = 0;
        totalcount4 = 0;
        totalqdc triggers = 0;
        runLength = 3600;

        energySize = 64000;
        array runNum[24];
        array startTime[24];
        array fileSize[24];
        array counts1[24];
        array counts2[24];
        array counts3[24];
        array counts4[24];
        array qdc triggers[24];
        array energy[energySize];

        rc = find(ORRunModel,1);
        datafile = find(ORDataFileModel,1);
        ds = find(ORHistoModel,1);
        s = find(ORScriptTaskModel,6);
        ami = find(ORAmi286Model,1);
        vmecpu = find(ORVmecpuModel,0,0);
        sbc = [vmecpu sbcLink];
        hv_mod = find(ORVHQ224Model,0,2);
        qdc = find(ORCaen965Model,0,13);
}```
// loop until midnight
while((rc hour) < 23 || sent){
    sleep(5.0);

    // check to see if the detector level (channel 0) is
    // alarming, if so unbias the detector
    [ami pollLevels];
    LN2_low_alarm_level = [ami lowAlarmLevel:0];
    LN2_current_level = [ami level:0];

    if (LN2_current_level < LN2_low_alarm_level){
        ln2_alarm = 1;
        break; // get out of the rc run hour < 23 loop
    }

    sleep(1.0);

    [rc startRun];
    waitUntil([rc isRunning]);
    // Added this loop to sleep the SBC while running.
    // This will stop whenever the run stops.
    do{
        [sbc pauseRun];
        sleep(0.001);
        [sbc resumeRun];
    }while([rc isRunning]);
    waituntil(![rc isRunning]);

    sleep(10.0);
    // high gain = channel 0 = low energy channel
    // low gain = channel 1 = high energy channel
    data_lowG = [ds dataSetWithName:"SIS3302,Energy,0,10,1"];
    data_highG = [ds dataSetWithName:"SIS3302,Energy,0,10,0"];
    data_inhibit = [ds dataSetWithName:"SIS3302,Energy,0,10,2"];
    data_pulsar = [ds dataSetWithName:"SIS3302,Energy,0,10,5"];
    data_qdc = [ds dataSetWithName:"CAEN965QDC(L),0,13,0"];

    // add to array holding histogram data
    for(l=0; l < energySize; l++){
        energy[l] = energy[l] + [data_lowG value: l];
    }

    // fill array with run info
    runNum[i] = [rc runNumber];
    startTime[i] = [rc startTime];
    fileSiz[i] = [datafile dataFileSize];
    counts1[i] = [data_lowG totalCounts];
    counts2[i] = [data_highG totalCounts];
    counts3[i] = [data_inhibit totalCounts];
    counts4[i] = [data_pulsar totalCounts];
    qdc triggers[i] = [data_qdc totalCounts];

    if (counts1[i] < 1){
        content1 = "";
    }
content1 = strcat(content1, "Run\nNumber: ", runNum[i], "\n");  
[s sendMailTo: "FIXME@gmail.com, FIXME2@gmail.com" cc:nil subject: "NO\nLOW-GAIN\nCOUNTS! (ch_{1})" content:content1];
}
if (counts2[i] < 1) {
  content2 = "";
  content2 = strcat(content2, "Run\nNumber: ", runNum[i], "\n");
  [s sendMailTo: "FIXME@gmail.com, FIXME2@gmail.com" cc:nil subject: "NO\nHIGH-GAIN\nCOUNTS! (ch_{0})" content:content2];
}
if (counts3[i] < 1) {
  content3 = "";
  content3 = strcat(content3, "Run\nNumber: ", runNum[i], "\n");
  [s sendMailTo: "FIXME@gmail.com, FIXME2@gmail.com" cc:nil subject: "NO\nINHIBIT\nCOUNTS! (ch_{2})" content:content3];
}
if (counts4[i] < 1) {
  content4 = "";
  content4 = strcat(content4, "Run\nNumber: ", runNum[i], "\n");
  [s sendMailTo: "FIXME@gmail.com, FIXME2@gmail.com" cc:nil subject: "NO\nPULSER\nCOUNTS! (ch_{5})" content:content4];
}
sent = false;
i++;
}// end loop until midnight

// if the LN2 level is below the alarm level, unbias the detector and stop runs
if (ln2_alarm == 1){
  [hv_mod setRampRate:0 withValue:5];
  [hv_mod setVoltage:0 withValue:0];
  [hv_mod loadValues:0];
  postalarm("Detector_LN2_Level_Alarm",11);
  postalarm("Detector_Unbiased",11);
  content5 = "";
  content5 = strcat(content5, "Unbiased\ndetector\nafter\nrun\nnumber: ", [rc runNumber], "\n");
  subject5 = "";
  subject5 = strcat(subject5, "LN2\nLevel\nAlarm\nUnbiased\nDetector\nafter\nrun\n", [rc runNumber], "\n");
  [s sendMailTo: "FIXME@gmail.com, FIXME2@gmail.com" cc:nil subject:subject5 content:content5];
  break; // stops the status script
}

// combine run info
while(k < i){
totalsize += fileSize[k];
totalcount1 += counts1[k];
totalcount2 += counts2[k];
totalcount3 += counts3[k];
totalcount4 += counts4[k];
totalqdctrigger += qdctrigger[k];
k++;
}
avgsize = totalsize/i/1000000;
avgcount1 = totalcount1/i/runLength;
avgcount2 = totalcount2/i/runLength;
avgcount3 = totalcount3/i/runLength;
avgcount4 = totalcount4/i/runLength;
avgqdctriggers = totalqdctriggers/i/runLength;

// concat general run info
content = strcat("MALBEK\RUN_STATS\n");
content = strcat(content, "run\_numbers: ", runNum[0], " ", runNum[i-1],"\_avg\_file\_size: ", fixed(avgsize, 2), " MB");
content = strcat(content, "\_avg\_low\_gain\_rate: ", fixed(avgcount1, 3), " Hz");
content = strcat(content, "\_avg\_high\_gain\_rate: ", fixed(avgcount2, 3), " Hz");
content = strcat(content, "\_avg\_inhibit\_rate: ", fixed(avgcount3, 3), " Hz");
content = strcat(content, "\_avg\_pulsar\_rate: ", fixed(avgcount4, 3), " Hz");
content = strcat(content, "\_avg\_qdc\_trigger\_rate: ", fixed(avgqdctriggers, 3), " Hz");
content = strcat(content, "\_ndetector\_dewar\_level: " , [ami level: 0] , "\_npurge\_dewar\_level: " , [ami level: 1] , "") ;

// concat single run info
while(j < i){
size = fileSize[j]/1000000;
count1norm = counts1[j]/runLength;
count2norm = counts2[j]/runLength;
count3norm = counts3[j]/runLength;
count4norm = counts4[j]/runLength;
qdcnorm = qdctriggers[j]/runLength;
content = strcat(content, "run number starting at ", runNum[j], ", file size: ", fixed(size, 2), " MB" );
content = strcat(content, "\_low\_gain\_rate: ", fixed(count1norm, 3), " Hz");
content = strcat(content, "\_high\_gain\_rate: ", fixed(count2norm, 3), " Hz");
content = strcat(content, "\_inhibit\_rate: ", fixed(count3norm, 3), " Hz");
content = strcat(content, "\_pulsar\_rate: ", fixed(count4norm, 3), " Hz");
content = strcat(content, "\_qdc\_trigger\_rate: ", fixed(qdcnorm, 3), " Hz")
}

content = strcat(content, "Energy")
for(l =0; l < energySize ; l++){
if(energy[l] != 0)
{
content = strcat(content, l , " \_t " , energy[l] , "\n")
}
}

content = strcat(content, "\_low\_rate")
for(l =0; l < 24 ; l++)
if(runNum[l] != 0)
{
count1norm = counts1[l]/runLength;
ccontent = strcat(content, l , " \_t " , fixed(count1norm, 3), "\n")
}
}
content = strcat(content, "\n\nhigh rate\n\n")
for(l=0; l < 24; l++){
    if(runNum[l] != 0){
        count2norm = counts2[l]/runLength;
        content = strcat(content, runNum[l], "\t", fixed(count2norm,3), "\n");
    }
}
content = strcat(content, "\n\nInhibit rate\n\n")
for(l=0; l < 24; l++){
    if(runNum[l] != 0){
        count3norm = counts3[l]/runLength;
        content = strcat(content, runNum[l], "\t", fixed(count3norm,3), "\n");
    }
}
print "sending mail";
sendMailTo: "FIXME2@gmail.com, FIXME3@gmail.com" cc:nil subject: "MALBEK_STATUS_UPDATE" content :content];
sent = true;
}// end loop forever
}// end main
A.2 Data Filter

The data filter script was written by Graham Giovanetti and me with a lot of help from Mark Howe and Michael Marino.

```c
filter
{
  WaveformLimit0 = FIXME;
  WaveformLimit1 = FIXME;
  // Also, require the last value to be greater than zero since we chose our DAC offset to cause the
  // inhibit pulses to go below zero.
  WaveformLimit1 = FIXME;
  // Also, require the last value to be greater than zero since we chose our DAC offset to cause the
  // inhibit pulses to go below zero.

  // ORSIS3302DecoderForEnergy
  if (currentRecordIs (ORSIS3302DecoderForEnergy)) {
    firstValue = 0;
    lastValueValue = 0;
    recLen = extractRecordLen (CurrentRecordPtr [0]);
    channel = extractValue (CurrentRecordPtr [1], 0xFF00, 8);
    WfLengthFromOrca = CurrentRecordPtr [2];
    EnergyWfLengthFromOrca = CurrentRecordPtr [3];
    if (channel == 0 || channel == 1) {
      WrapModeOn = (CurrentRecordPtr [1] & 0x1);
      if (WrapModeOn == 1) {
        headerLen = 8;
        trailerOffset = headerLen + WfLengthFromOrca + EnergyWfLengthFromOrca;
        hardware_energy = CurrentRecordPtr [trailerOffset];
        WfLength = CurrentRecordPtr [6]; // # samples
        WfStartingIndex = CurrentRecordPtr [7]; // # samples offset to start
        s1 = WfStartingIndex;
        if (s1 != 0) s2 = s1 - 1;
        else s2 = WfLength - 1;
        recordOffset1 = headerLen + s1 / 2; // offset into record for first sample
        recordOffset2 = headerLen + s2 / 2; // offset into record for last sample
        if ((recordOffset1 < recLen) && (recordOffset2 < recLen)) {
          if (s1 % 2) firstValue = CurrentRecordPtr [recordOffset1] & 0xFFFF;
          else firstValue = extractValue (CurrentRecordPtr [recordOffset1], 0xFFFF0000, 16);
          if (s2 % 2) lastValue = CurrentRecordPtr [recordOffset2] & 0xFFFF;
          else lastValue = extractValue (CurrentRecordPtr [recordOffset2], 0xFFFF0000, 16);
        }
      } else {
        headerLen = 6; // orcaheader + sisheader
        trailerOffset = headerLen + WfLengthFromOrca + EnergyWfLengthFromOrca;
        hardware_energy = CurrentRecordPtr [trailerOffset];
        WfLength = CurrentRecordPtr [2];
        firstValue = CurrentRecordPtr [headerLen] & 0xFFFF;
        lastValue = extractValue (CurrentRecordPtr [headerLen + WfLength - 1], 0xFFFF0000, 16);
      }
    }
  } else {
    headerLen = 6; // orcaheader + sisheader
    trailerOffset = headerLen + WfLengthFromOrca + EnergyWfLengthFromOrca;
    hardware_energy = CurrentRecordPtr [trailerOffset];
    WfLength = CurrentRecordPtr [2];
    firstValue = CurrentRecordPtr [headerLen] & 0xFFFF;
    lastValue = extractValue (CurrentRecordPtr [headerLen + WfLength - 1], 0xFFFF0000, 16);
  }
  // Now get rid of inhibit related events, typically look like sloping lines or a bipolar pulse
  // whose last value is much less than the first value
```

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if (channel==0) checkLimit = WaveformLimit0;
else    checkLimit = WaveformLimit1;
if ((firstValue > checkLimit) && (lastValue > 1)) {
    shipRecord(CurrentRecordPtr);
}
else  shipRecord(CurrentRecordPtr);
else  shipRecord(CurrentRecordPtr);
## Appendix B

### Calculated Waveform Parameters

Table B.1: A list of the parameters that are saved for each waveform in the MALBEK analysis chain.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fSwEnergy</td>
<td>Uncalibrated peak height as calculated by averaging the pole-zero corrected tail of the waveform</td>
</tr>
<tr>
<td>fCalSwEnergy</td>
<td>Calibrated peak height as calculated by averaging the pole-zero corrected tail of the waveform</td>
</tr>
<tr>
<td>fTrapEnergyOne</td>
<td>Uncalibrated peak height of an 11 $\mu$s peaking time, 1 $\mu$s gap time trapezoidal filtered waveform</td>
</tr>
<tr>
<td>fCalTrapEnergyOne</td>
<td>Calibrated peak height of an 11 $\mu$s peaking time, 1 $\mu$s gap time trapezoidal filtered waveform</td>
</tr>
<tr>
<td>fTrapEnergyTwo</td>
<td>Uncalibrated peak height of a 5 $\mu$s peaking time, 1 $\mu$s gap time trapezoidal filtered waveform</td>
</tr>
<tr>
<td>fCalTrapEnergyTwo</td>
<td>Calibrated peak height of a 5 $\mu$s peaking time, 1 $\mu$s gap time trapezoidal filtered waveform</td>
</tr>
<tr>
<td>fSISTrapEnergy</td>
<td>Uncalibrated peak height as calculated by the SIS3302 on-board energy filter</td>
</tr>
<tr>
<td>fCalSISTrapEnergy</td>
<td>Calibrated peak height as calculated by the SIS3302 on-board energy filter</td>
</tr>
<tr>
<td>fRiseTime</td>
<td>10%-90% rise-time of the waveform</td>
</tr>
<tr>
<td>fStartRiseTime</td>
<td>location in time of the 10% peak height</td>
</tr>
<tr>
<td>fEndRiseTime</td>
<td>location in time of the 90% peak height</td>
</tr>
<tr>
<td>fSecondaryRiseTime</td>
<td>75%-95% rise-time of the waveform</td>
</tr>
<tr>
<td>fSecondaryStartRiseTime</td>
<td>Location in time of the 75% peak height</td>
</tr>
<tr>
<td>fSecondaryEndRiseTime</td>
<td>Location in time of the 95% peak height</td>
</tr>
</tbody>
</table>

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Table B.1 – continued from previous page

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fFitConstant</td>
<td>Parameter $B$ from a fit to the baseline using the formula: $f(x) = Ax + B$</td>
</tr>
<tr>
<td>fFitSlope</td>
<td>Parameter $A$ from a fit to the baseline using the formula: $f(x) = Ax + B$</td>
</tr>
<tr>
<td>fFitDecayTau</td>
<td>Parameter $\tau$ from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayAmplitude</td>
<td>Parameter $A$ from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayBaseline</td>
<td>Parameter $B$ from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayEnergy</td>
<td>Difference between parameters $A$ and $B$ from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayChiSq</td>
<td>$\chi^2$ from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayPVal</td>
<td>$P$-value from a fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitDecayNDF</td>
<td>Number of degrees of freedom in fit to the tail of the waveform using the formula: $f(x) = A e^{-(x-B)/\tau}$</td>
</tr>
<tr>
<td>fFitPZSlope</td>
<td>Parameter $A$ from a fit to the pole-zero corrected tail of the waveform using the formula: $f(x) = Ax + B$</td>
</tr>
<tr>
<td>fFitPZConstant</td>
<td>Parameter $B$ from a fit to the pole-zero corrected tail of the waveform using the formula: $f(x) = Ax + B$</td>
</tr>
<tr>
<td>fBaselineValue</td>
<td>Baseline average</td>
</tr>
<tr>
<td>fBaselineRMS</td>
<td>RMS of the baseline</td>
</tr>
<tr>
<td>fIntegral</td>
<td>The integral (or sum) of the waveform ADC values</td>
</tr>
<tr>
<td>fMaxTrapOneTLoc</td>
<td>Location in time of the maximum of an 11 $\mu$s peaking time, 1 $\mu$s gap time trapezoidal filtered waveform</td>
</tr>
<tr>
<td>fMaxRawWfVal</td>
<td>Global maximum of the raw waveform</td>
</tr>
<tr>
<td>fMaxRawWfTLoc</td>
<td>Location in time of the global maxima of the raw waveform</td>
</tr>
<tr>
<td>fMinRawWfVal</td>
<td>Global minimum of the raw waveform</td>
</tr>
<tr>
<td>fMinRawWfTLoc</td>
<td>Location in time of the global minima of the raw waveform</td>
</tr>
</tbody>
</table>

...continued on next page...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>fMaxBlRmWfVal</td>
<td>Global maximum of the baseline removed waveform</td>
</tr>
<tr>
<td>fMaxBlRmWfTLoc</td>
<td>Location in time of the global maxima of the baseline removed waveform</td>
</tr>
<tr>
<td>fMinBlRmWfVal</td>
<td>Global minimum of the baseline removed waveform</td>
</tr>
<tr>
<td>fMinBlRmWfTLoc</td>
<td>Location in time of the global minima of the baseline removed waveform</td>
</tr>
<tr>
<td>fMaxDerivativeVal</td>
<td>Global maximum of the derivative of the waveform (current pulse)</td>
</tr>
<tr>
<td>fMaxDerivativeTLoc</td>
<td>Location in time of the global minimum of the derivative of the waveform</td>
</tr>
<tr>
<td>fCDPowerFWHM</td>
<td>FWHM of the wavelet power spectrum (Equation 5.5)</td>
</tr>
<tr>
<td>fCDPowerFWHM_start</td>
<td>Location in time of the start of the FWHM of the wavelet power spectrum</td>
</tr>
<tr>
<td>fCDPowerFWHM_end</td>
<td>Location in time of the end of the FWHM of the wavelet power spectrum</td>
</tr>
<tr>
<td>fCDPowerFWTM</td>
<td>FWTM of the wavelet power spectrum (Equation 5.5)</td>
</tr>
<tr>
<td>fCDPowerFWTM_start</td>
<td>Location in time of the start of the FWTM of the wavelet power spectrum</td>
</tr>
<tr>
<td>fCDPowerFWTM_end</td>
<td>Location in time of the end of the FWTM of the wavelet power spectrum</td>
</tr>
<tr>
<td>fCDPowerAmplitude</td>
<td>Global maxima of the wavelet power spectrum (Equation 5.5)</td>
</tr>
<tr>
<td>fCDPowerBaselineValue</td>
<td>Baseline average of the wavelet power spectrum (Equation 5.5)</td>
</tr>
<tr>
<td>fCDPowerBaselineRMS</td>
<td>Baseline RMS of the wavelet power spectrum (Equation 5.5)</td>
</tr>
<tr>
<td>fCDNNonZeroCoeffs[n]</td>
<td>Number of non-zero $c_D^{(i)}(n)$ left after thresholding for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDAbsSumOfCoeffs[n]</td>
<td>Sum of the absolute value of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDSumOfCoeffs[n]</td>
<td>The regular sum of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDRMS[n]</td>
<td>RMS/STD of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDVariance[n]</td>
<td>Variance of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDMean[n]</td>
<td>Mean of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
<tr>
<td>fCDMin[n]</td>
<td>Global minima of $c_D^{(i)}(n)$ for each level of decomposition $n = 0,\ldots,8$</td>
</tr>
</tbody>
</table>

...continued on next page...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{CD\text{Min}TLoc}[n]$</td>
<td>Location in time of the global minima of $c_D^{(i)}(n)$ for each level of decomposition $n = 0, \ldots, 8$</td>
</tr>
<tr>
<td>$f_{CD\text{Max}[n]}$</td>
<td>Global maxima of $c_D^{(i)}(n)$ for each level of decomposition $n = 0, \ldots, 8$</td>
</tr>
<tr>
<td>$f_{CD\text{MaxTLoc}[n]}$</td>
<td>Location in time of the global maxima of $c_D^{(i)}(n)$ for each level of decomposition $n = 0, \ldots, 8$</td>
</tr>
</tbody>
</table>
Appendix C

Peak Fitting

An Unbinned Maximum Likelihood (UMxL) fit was performed on each of the peaks in this chapter using the RooFit package [200]. In each plot, the blue curve represents the total Probability Density Function (PDF), and where shown, the individual components are shown in red. The $\chi^2$ and P-values listed were calculated using the PSF_RooFitResultCalc.py program. For more details on PSF_RooFitResultCalc.py, see Appendix D.
C.1 Data Set 3a Uncalibrated Peak Fits

$^{65}$Zn (1.096 keV), $^{68,71}$Ge (1.299 keV) L-Capture

Figure C.1: DS3a fit to the $^{68,71}$Ge L-capture line. The $\chi^2$ was 83.1 with 94 degrees of freedom, corresponding to a 0.782 P-value. Note: only the $^{68,71}$Ge line was used for calibration (1.299 keV).
Figure C.2: DS3a fit to the $^{55}$Fe K-capture line. The $\chi^2$ was 11.0 with 96 degrees of freedom, corresponding to a 1.000 P-value.
$^{68,71}\text{Ge (10.367 keV)}, \, ^{68}\text{Ga (9.659 keV)}, \, ^{65}\text{Zn (8.979 keV)}$ K-Capture

Figure C.3: DS3a fit to the $^{65}\text{Zn}$, $^{68}\text{Ga}$ and $^{68,71}\text{Ge}$ K-capture lines. The $\chi^2$ was 81.2 with 90 degrees of freedom, corresponding to a 0.736 P-value.
C.2 Data Set 3b Uncalibrated Peak Fits

$^{65}\text{Zn} \ (1.096 \text{ keV}), \ ^{68,71}\text{Ge} \ (1.299 \text{ keV})$ L-Capture

Figure C.4: DS3b fit to the $^{68,71}\text{Ge}$ L-capture line. The $\chi^2$ was 75.9 with 94 degrees of freedom, corresponding to a 0.914 P-value. Note: only the $^{68,71}\text{Ge}$ line was used for calibration (1.299 keV).
$^{55}$Fe K-Capture (6.539 keV)

Figure C.5: DS3b fit to the $^{55}$Fe K-capture line. The $\chi^2$ was 14.1 with 96 degrees of freedom, corresponding to a 1.000 P-value.
$^{68,71}\text{Ge}$ (10.367 keV), $^{68}\text{Ga}$ (9.659 keV), $^{65}\text{Zn}$ (8.979 keV) K-Capture

Figure C.6: DS3b fit to the $^{65}\text{Zn}$, $^{68}\text{Ga}$ and $^{68,71}\text{Ge}$ K-capture lines. The $\chi^2$ was 51.6 with 90 degrees of freedom, corresponding to a 1.000 P-value.
C.3 Data Set 3a Calibrated Peak Fits

$^{65}\text{Zn} \ (1.096 \text{ keV}), \ ^{68,71}\text{Ge} \ (1.299 \text{ keV}) \ L$-Capture

Figure C.7: Calibrated fit for $^{65}\text{Zn} \ L \ (1.096 \text{ keV})$ and $^{68,71}\text{Ge} \ L \ (1.299 \text{ keV})$ in DS3a. The fit has a $\chi^2$/DOF $=117.1/92 \ (1.27)$, corresponding to a P-value of $3.982e-02$. 

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$^{49}$V K-Capture (4.966 keV)

Figure C.8: Calibrated fit for $^{49}$V K (4.966 keV) in DS3a. The fit has a $\chi^2$/DOF = 38.6/46 (0.84), corresponding to a P-value of 7.726e-01.
$^{55}$Fe K-Capture (6.539 keV)

![Calibrated fit for $^{55}$Fe K (6.539 keV) in DS3a. The fit has a $\chi^2$/DOF =38.5/46 (0.84), corresponding to a P-value of 7.746e-01.]

Figure C.9: Calibrated fit for $^{55}$Fe K (6.539 keV) in DS3a. The fit has a $\chi^2$/DOF =38.5/46 (0.84), corresponding to a P-value of 7.746e-01.
$^{68,71}\text{Ge (10.367 keV)}, \, ^{68}\text{Ga (9.659 keV)}, \, ^{65}\text{Zn (8.979 keV)}$ K-Capture

Figure C.10: Calibrated fit for $^{65}\text{Zn K (8.979 keV)}, \, ^{68}\text{Ga K (9.659 keV)},$ and $^{68,71}\text{Ge K (10.367 keV)}$ in DS3a. The fit has a $\chi^2/\text{DOF} = 59.8/90 (0.66)$, corresponding to a P-value of $9.941\times10^{-1}$. 
\( ^{210}\text{Pb} \) (46.539 keV)

Figure C.11: Calibrated fit for \( ^{210}\text{Pb} \) (46.539 keV) in DS3a. The fit has a \( \chi^2/\text{DOF} = 34.7/96 \) (0.36), corresponding to a P-value of 1.000e+00.
Figure C.12: Calibrated fit for $^{234}$Th (92.38, 92.80 keV) in DS3a. The fit has a $\chi^2$/DOF = 49.0/96 (0.51), corresponding to a P-value of 1.000e+00.

$^{57}$Co 122.06 keV)
Figure C.13: Calibrated fit for $^{57}$Co (122.06 keV) in DS3a. The fit has a $\chi^2$/DOF = 110.9/96 (1.15), corresponding to a P-value of 1.423e-01.
Figure C.14: Calibrated fit for $^{57}\text{Co }\gamma + \text{X-ray summing (136.57 + 7.06 (143.53) keV)}$ in DS3a. The fit has a $\chi^2/\text{DOF} = 104.5/96 (1.09)$, corresponding to a P-value of $2.594\times10^{-1}$. 

$^{57}\text{Co }\gamma + \text{X-ray summing (136.57 + 7.06 (143.53) keV)}$
C.4 Data Set 3b Calibrated Peak Fits

$^{65}$Zn (1.096 keV), $^{68,71}$Ge (1.299 keV) L-Capture

Figure C.15: Calibrated fit for $^{65}$Zn L (1.096 keV) and $^{68,71}$Ge L (1.299 keV) in DS3b. The fit has a $\chi^2$/DOF = 166.2/92 (1.81), corresponding to a P-value of 3.457e-06.
Figure C.16: Calibrated fit for $^{55}$Fe K (6.539 keV) in DS3b. The fit has a $\chi^2$/DOF = 25.3/46 (0.55), corresponding to a P-value of $9.945\times10^{-1}$. 
Figure C.17: Calibrated fit for $^{65}$ZnK (8.979 keV), $^{68}$Ga K (9.659 keV) and $^{68,71}$Ge K (10.367 keV) in DS3b. The fit has a $\chi^2$/DOF = 54.5/90 (0.61), corresponding to a P-value of 9.989e-01.
$^{210}\text{Pb}$ (46.539 keV)

Figure C.18: Calibrated fit for $^{210}\text{Pb}$ (46.539 keV) in DS3b. The fit has a $\chi^2$/DOF = 51.6/96 (0.54), corresponding to a P-value of 9.999e-01.
Figure C.19: Calibrated fit for $^{234}$Th (92.38, 92.80 keV) in DS3b. The fit has a $\chi^2$/DOF = 104.3/96 (1.09), corresponding to a P-value of 2.645e-01.
$^{57}$Co (122.06 keV)

Figure C.20: Calibrated fit for $^{57}$Co (122.06 keV) in DS3b. The fit has a $\chi^2$/DOF = 78.1/96 (0.81), corresponding to a P-value of 9.085e-01.
Figure C.21: Calibrated fit for $^{57}$Co $\gamma$ + X-ray summing (136.57 + 7.06 (143.53) keV) in DS3b. The fit has a $\chi^2$/DOF = 112.2/96 (1.17), corresponding to a P-value of 1.240e-01.
C.5 Data Set 3 Calibrated Peak Fits

$^{65}\text{Zn} \ (1.096 \text{ keV}), \ ^{68,71}\text{Ge} \ (1.299 \text{ keV})$ L-Capture

Figure C.22: Calibrated fit for $^{65}\text{Zn} \ L (1.096 \text{ keV})$ and $^{68,71}\text{Ge} \ L (1.299 \text{ keV})$ in DS3. The fit has a $\chi^2/\text{DOF} = 154.8/92 (1.68)$, corresponding to a P-value of 4.582e-05.
$^{49}$V K-Capture (4.966 keV)

![Calibrated fit for $^{49}$V K (4.966 keV) in DS3. The fit has a $\chi^2$/DOF = 92.4/46 (2.01), corresponding to a P-value of 5.948e-05.](image)

Figure C.23: Calibrated fit for $^{49}$V K (4.966 keV) in DS3. The fit has a $\chi^2$/DOF = 92.4/46 (2.01), corresponding to a P-value of 5.948e-05.
Figure C.24: Calibrated fit for $^{55}$Fe K (6.539 keV) in DS3. The fit has a $\chi^2$/DOF =69.9/46 (1.52), corresponding to a P-value of $1.304\times10^{-2}$. 

$^{55}$Fe K-Capture (6.539 keV)
Figure C.25: Calibrated fit for $^{65}$Zn K (8.979 keV), $^{68}$Ga K (9.659 keV) and $^{68,71}$Ge K (10.367 keV) K-Capture in DS3. The fit has $\chi^2$/DOF $= 73.2/90$ (0.81), corresponding to a P-value of 9.014e-01.
Figure C.26: Calibrated fit for $^{210}$Pb (46.539 keV) in DS3. The fit has a $\chi^2$/DOF = 121.5/96 (1.27), corresponding to a P-value of 4.054e-02.
Figure C.27: Calibrated fit for $^{234}\text{Th}$ (92.38, 92.80 keV) in DS3. The fit has a $\chi^2$/DOF = 195.0/96 (2.03), corresponding to a P-value of 1.001e-08.
$^{57}\text{Co}$ (122.06 keV)

Figure C.28: Calibrated fit for $^{57}\text{Co}$ (122.06 keV) in DS3. The fit has a $\chi^2$/DOF = 133.6/96 (1.39), corresponding to a P-value of 6.771e-03.
$^{57}\text{Co } \gamma + \text{ X-ray summing } (136.57 + 7.06 \ (143.53) \ \text{keV})$

Figure C.29: Calibrated fit for $^{57}\text{Co } \gamma + \text{ X-ray summing } (136.57 + 7.06 \ (143.53) \ \text{keV})$ in DS3. The fit has a $\chi^2/\text{DOF} = 97.4/96 \ (1.01)$, corresponding to a P-value of $4.423\text{e-01}$. 
Appendix D

RooFit Tools

Extracting $\chi^2$ and P-Values from ML Fits

Extracting a goodness-of-fit for an UMxL fit is not as straightforward as a least squares fit. Assuming we have an unbinned data set (a ROOT TTree), using the RooFit package [200], we can then fit the data set with an arbitrary PDF that best describes the data (model). The RooFit package performs the UMxL fit, and will plot the model on a binned histogram, despite the fact that the fit was performed by UMxL. Using the histogram (which is really a TGraphAsymmErrors object), we can then easily calculate the $\chi^2$ and P-values for the model and binned histogram. The code below provides a means in which to do this, handling the asymmetric bin errors using the same method that ROOT uses to fit TGraphAsymmErrors objects (Source Code Here).

```python
#!/usr/bin/env python
# encoding: utf-8

PSF_RooFitResultCalc.py

Created by paddy on 2012-12-14.
Copyright (c) 2012 UNC Chapel Hill. All rights reserved.

import sys
import os
import unittest
import numpy
from ROOT import *

class PSF_RooFitResultCalc:
    def __init__(self, m, d, nFitParams, fitMin=None, fitMax=None, verbose=False):
        m = xframe.getCurve("model1_plot")  # inherits from TGraph
        d = xframe.getHist("h_data")  # inherits from TGraphAsymmErrors (TGraph)
```
Grab the nFitParams from the PDF:

```python
flParams = RooArgSet(PDF.getParameters(RooArgSet(rfEnergy,bin)).selectByAttr("Constant", kFALSE))
nFitParams = flParams.getSize()
```

Returns the chiSq, p-value, n_data, redChiSq, and a NiceLookingGraph.

NiceLookingGraph is a TGraphAsymmErrors object that we can plot with later in case there are entries with zero content (and weighted zero, so not used in the fit).

```python
x = d.GetX()
y = d.GetY()
eyh = d.GetEYhigh()
eyl = d.GetEYlow()
exh = d.GetEXhigh()
exl = d.GetEXlow()

if not fitMin: fitMin = x[0]
if not fitMax: fitMax = x[d.GetN()-1]

n=0
for i in range(d.GetN()):
    if x[i] < fitMin or x[i] > fitMax: continue
    # Only consider data points with non-zero content
    if y[i] > 0.0: n+=1

self.graph = TGraphAsymmErrors(n-1)
self.graph.SetName(d.GetName())

# // (y - f(x))^2
# // c = SUM ------------------------------- = -------------------------------
# // ef^2 + (0.5*(eex+exh)*f'(x))^2 ey ey + eux eux
c = 0.0  # chisquare
rf_c = 0.0
n_data_pts = 0.0

n = 0
for i in range(d.GetN()):
    if x[i] < fitMin or x[i] > fitMax: continue
    # Only consider data points with non-zero content
    n_data_pts += 1.0

    if y[i] > 0.0:
        self.graph.SetPoint(n, x[i], y[i])
        self.graph.SetPointError(n, exl[i], exh[i], eyl[i], eyh[i])
        n+=1

    fsun = y[i] - m.Eval(x[i])

    # Check to see if the curve lies above or below the data point
    if fsun < 0: ey = eyh[i]
    else: ey = eyl[i]

    # Check to see if the error bars go below zero, if so set them to zero
    if (exl[i] < 0): exl = 0;
```

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if (exh[i] < 0): exh = 0;
if (ey < 0): ey = 0;

if (exh[i] > 0 or exh[i] > 0):
    # Calc derivative f'(x) = (f(x+h) - f(x-h)) / 2h
    # First, check if we are at the fit limits
    xl = x[i] - exh[i]
    if xl<fitMin: xl = fitMin
    xu = x[i] + exh[i]
    if xu>fitMax: xu = fitMax

    h = xu - xl
    der = 0.5*(m.Eval(xu) - m.Eval(xl)) / h

    eux = 0.5*(exh[i] + exh[i]) * der
    else: eux = 0.0

    eu = ey*ey + eux*eux
    if eu <= 0: eu = 1.
    c += fsum*fsum / eu

self.n dof = int(n.data pts - nFitParams)
self.chiSq = float(c)
if self.n dof: self.redChiSq = self.chiSq / float(self.n dof)
else: self.redChiSq = 0.0
self.p_value = TMath.Prob(self.chiSq, self.n dof)

if verbose:
    print '-------------------------------'
    print 'PSFRooFitResultCalc-----------'
    print 'Results-----------------------'
    print '-------------------------------'
    print '\tNDataPts\t\t%i' % n_data pts
    print '\tFitParams\t\t%i' % nFitParams
    print '\tchi2\t\%=\%4.4f' % self.chiSq
    print '\tDOF\t\%=\%4.4f' % self.n dof
    print '\tP-value\t\%=\%4.4f' % self.p_value
    print '-------------------------------'

def GetFitResults(self):
    return self.chiSq, self.n dof, self.p_value, self.redChiSq, self.graph

def GetPValue(self):
    return self.p_value

def GetChiSq(self):
    return self.chiSq

def GetRedChiSq(self):
    return self.redChiSq

def GetDOF(self):
    return self.n dof
def GetGraphWithoutEmptyBins(self):
    return self.graph
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