Endogenous Credit Market Incompleteness: RBC Approach to Emerging Markets Crises

by
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Abstract

SERGIY PEREDRIY: Endogenous Credit Market Incompleteness: RBC Approach to Emerging Markets Crises.
(Under the direction of Stanley Black.)

An endogenous liquidity constraint is applied to a Small open economy Real Business Cycle model driven by shocks to productivity and the world interest rate. The approach allows the reproduction of distinctive features of the crises in the short run, without introducing significant distortions in the long-term properties of the economy. Two variants of the model are considered: a one-sector model with a single tradable good, and a two-sector model which features tradable and non-tradable goods (both of which are produced). The level of the capital stock was found to have a significant effect on the short-run reaction of the liquidity constrained economy to the shocks in both the one- and two-sector models. For the two-sector economy, the other new effect that is reproduced and analyzed is a significant decrease in the tradables production when the constraint becomes binding. The effect of the tightness of the constraint on the long-term properties of the model is analyzed; the major difference between the models is the direction of the GDP change.
To Olesya,

for her endless patience, constant encouragement, and sincere interest.
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Chapter 1

Introduction

The last decade of the 20th century has witnessed a wave of economic crises in several emerging economies around the world. These include the Mexican crisis of 1994 followed by the infamous “Tequila Effect”, then the East Asian crisis of 1997-98 with echo effects as far as Russia, South America, and equity market in the U.S., and more recent Argentinean default of 2001. Emerging-markets crises are marked by empirically distinctive features (labeled “Sudden Stop” by Calvo, 1998), such as a sudden inability to access international credit markets, sudden reversals in capital inflows, corresponding dramatic decrease of current account deficits, collapse in the domestic real sector, and sharp declines in domestic equity prices and prices of non-tradables (Arellano and Mendoza, 2002). These empirical regularities contrast sharply with the outcomes of standard complete-markets real business cycle models. International risk sharing guarantees consumption smoothing – a country hit by a negative shock can borrow internationally if needed and thereby minimize the effect of an idiosyncratic shock on domestic economy. The assumptions of the standard theories of optimal international credit allocation, such as frictionless real business cycle models, cannot explain a sudden inability to access international credit markets or the large magnitude of collapses in the real sector and in relative prices in emerging economies during a crisis.
From the analytical standpoint, these empirical observations call for modification of a standard theory of frictionless international credit markets. A steady stream of literature on emerging market crises refers to various informational frictions as a distinctive feature of international credit markets relevant to emerging economies. The current work aims to contribute to a growing literature on financial-frictions transmission channels of recent crises in emerging economies and integrate this into an equilibrium business cycle model for a small open economy. The role of credit frictions in explaining Sudden Stops was emphasized by Calvo (1998). The majority of the models proposed to explain Sudden Stops can be classified into two categories, according to an approach used by Calvo and Reinhart (1999). First is a Keynesian concept of price and/or wage downward rigidity. The papers exploiting this setup are Cespedes, Chang and Velasco (2000) and Christiano, Gust and Roldos (2000), among others. The second popular approach is Fisherian debt-deflation analysis using collateral constraints proposed by Kiyotaki and Moore (1997). The papers in this class include Paasche (2001), Edison, Luangaram, and Miller (2000), and Izquierdo (2000), among others.

There are several features, which are considered increasingly important in the current development of literature on Sudden Stops (Mendoza, 2006a). First is the modeling of a Sudden Stop (SS) as an endogenous result of the stochastic dynamics of a model with shocks of moderate size\(^1\) and with agents who build the possibility of SS into their expectations. In contrast, several models consider current-account reversals triggering SS as large unexpected exogenous shocks (Chari, Kehoe, and McGrattan, 2005). The major drawback of such approaches is an inability to explain the current account reversal. Second, a SS model should be able to produce an output drop (in contrast, some models such as Chari et al., 2005, predict an output increase as a result of a SS). Third, quantitative results are important. The model in the current proposal attempts

\(^1\)such as one-standard-deviation shocks estimated from the data
to include all three features mentioned above. It is in the class of dynamic stochastic general equilibrium models of a small open economy with credit constraint of the following type: \( b_{t+1} \geq -\varphi f(t, t+1) \). Here \( b_{t+1} \) is borrowing of one-period international bonds, \( \varphi \) is a constant, and \( f(\cdot) \) is a function of current- or next-period prices, income, assets, or existing debt. The papers in this class include Christiano et al. (2000) and Mendoza and Smith (2006) using margin requirement \( (f(\cdot) = q_t \alpha_{t+1} K) \), where \( q_t \) is the price of equity, and \( \alpha_{t+1} \) are equity shares); Kocherlakota (2000) \( (f(\cdot) = q_t K) \); Mendoza (2001) using liquidity constraint \( (f(\cdot) = Y^T_t + pY^N_t) \), where \( Y^T \) and \( Y^N \) are incomes from tradable and non-tradable sector, respectively, and \( p \) is the relative price of non-tradables); and Mendoza (2006b) (with \( b_{t+1} \) being working capital loans and \( (f(\cdot) \) the value of a firm’s assets), among others.

I follow the liquidity-constraint strategy to model endogenous incompleteness of international credit markets. Mendoza (2001) proposes an endogenous liquidity constraint by which households are required to finance a fraction of their current expenses out of their current income.\(^2\) The constraint is added into a stochastic equilibrium RBC model of small open economy. The model in Mendoza (2001) is a two-sector model with a stochastic endowment of tradable goods and a production of non-tradable goods using inelastically supplied capital and variable labor supply. As a first step in my paper, a single-commodity model is assumed. The model is an extension of a frictionless Real Business Cycle model for a small open economy studied by Mendoza (1991). The model features a single internationally tradable good, which is produced with labor and flexible capital with depreciation and costs of adjustment. The model is extended here by introducing an endogenous liquidity constraint as used in Mendoza (2001).

\(^2\)The same type of constraint was introduced earlier by Ludvigson (1999). In his study, individuals face a debt constraint that limits next period debt amount not to exceed a fixed portion of their current income, with a stochastic disturbance. This constraint, according to the author, is consistent with the lending practices of banks. The framework helps in explaining the correlation between the consumption growth and predictable credit growth observed in U.S. aggregate data.
The one-sector model is then extended to include a non-tradable commodity. In this two-sector model, both goods are produced; therefore, the model has more flexibility than the one-sector model. Foreign assets are assumed to be denominated in units of tradables, but are partially leveraged on income generated in the non-tradable sector (Mendoza, 2001). This may lead to potentially large short-term fluctuations caused by production shocks and by variability of the price of non-tradables. The model considered here is an extension of Mendoza (2001) model. In the original model, only non-tradable goods are produced, with a stochastic endowment process for tradable goods. This limits the analysis of inter-sectoral interaction during a crisis. Output drops, observed during most of the recent emerging market crisis episodes, are driven in the model only by the slowdown in non-tradable sector (together with the swings of the relative price of nontradables). It is a well-known empirical observation, however, that the crises adversely affected both tradable and non-tradable sector (In the case of the Mexican crisis of 1994-1995, GDP of non-tradables fell short of its trend by 6.63% in Q1 1995; the corresponding figure for tradables GDP is a 10.14% drop in the same quarter (Mendoza 2001); 53% larger than for non-tradables). In the two-sector model laid out in the current thesis, an attempt is made to model the negative effects of Sudden Stops on both sectors of production. Tradable goods are produced utilizing flexible sector-specific capital with depreciation and adjustment costs. Numerical simulations are able to reproduce a drop in production of both sectors during a Sudden Stop.

One important feature of the models in the paper is an endogenous rate of time preference, which is critical in supporting the models’ occasionally binding constraint. If credit constraints are applied to a typical small open economy RBC model featuring an exogenous rate of time preferences, the result is either permanent binding of the constraint along the equilibrium path (if the interest rate is higher than the rate of time preference), or non-binding constraints in the long run (if the interest rate is
below or equal to the set rate of time preference). Therefore, such a model is not suitable for analyzing Sudden Stops as infrequently occurring dramatic events nested within regular business cycles. An endogenous rate of time preference, in contrast, provides a mechanism to approach Sudden Stops from such a viewpoint, allowing for occasionally binding credit constraints.

In the model, the utility function with an endogenous rate of time preference is in the form of Epstein’s (1983) stationary cardinal utility (SCU), but used in a stochastic setting. Under certain set of conditions (to be defined later, in Chapter 3), this framework generates a unique, invariant limiting distribution for a stochastic model of small open economy. Endogenous discounting is not the only method with such properties; among other methods are, for instance, foreign assets with transaction costs, interest rate as a function of the stock of foreign debt, and finitely-living economic agents (Blanchard preferences, see Blanchard, 1985). The major limitation of the above-mentioned methods is that they only permit the analysis near the steady state after a log-linear approximation. They are not suitable for capturing large (and short-lived) deviations from the steady state featured in Sudden Stops, which requires non-linear analysis. Another advantage of using SCU relevant to the model considered is its ability to support stationary equilibria with permanently binding credit constraints (Mendoza, 2006b).³ Moreover, Epstein’s SCU is the only specification in line with the following standard assumptions of RBC models: (a) infinitely-living economic agents, (b) equality in the long run of the interest rate and the rate of time preference, and (c) only economic agents’ preferences interacting with the real interest rate (and not ad-hoc formulations such as imposed interest-rate functions or assumptions about transaction-costs specifications) determine the amount of foreign borrowing in the long-run (Arellano and

³This feature is exploited in the sensitivity analysis, when high values of the liquidity constraint parameter make the constraint permanently binding for all shock realizations.
The rest of the thesis is organized as follows: Chapter 2 provides a literature review on credit market incompleteness in general, and on approaches towards analyzing recent emerging markets crises, with a particular concentration on credit-frictions transmission mechanisms. Chapter 3 presents the one-sector model with endogenous borrowing constraint, including the model description, dynamic programming solution to the model and calibration, analysis of the results, and sensitivity analysis. The two-sector model is considered next in Chapter 4. Chapter 5 provides empirical analysis of countries which have experienced a Sudden Stop event. Chapter 6 concludes and discusses venues for further research.

\textsuperscript{4}On a side note, as Uribe (2007) points out, endogenous discount factor ensures stationary dynamics of a model, whereas a log-linearization around the steady state of a model with exogenous discount factor would lead to a random-walk dynamics. This would make it impossible to compute unconditional second moments; moreover, log-linearization is no longer suitable for an approximation of a non-linear model. Alternative ways to induce a stationarity in small open economy RBC models are analyzed in Schmitt-Grohé and Uribe (2003).
Chapter 2

Literature Review

Empirical evidence on recent episodes of economic crises in emerging markets suggests that the assumption of perfect international credit markets should be reconsidered. In several cases, countries in crises lost access to the international credit market. These episodes clearly demonstrate that there exist various frictions in the international credit market, making it incomplete. In view of this, the literature review chapter consists of two parts. In the first part, the development and the classification of research efforts on emerging markets crises (Sudden Stops) is presented. The second part considers several typical examples of literature on credit market incompleteness, including literature on debt repudiation, and the role of non-tradable goods in creating credit market frictions.

2.1 Literature on Emerging Markets Crises

The term “Sudden Stops” was introduced by Calvo (1998). His model is highly stylized, and it is aimed at identifying the issue. Calvo (1998) considers a non-monetary endowment model of a small open economy featuring tradable and non-tradable goods, in a perfect credit market environment. Representative economic agents derive utility from consumption of both goods (the goods are utility-separable), and the time horizon is three periods, with perfect foresight. Non-tradable goods can only be produced
from tradables using linear technology (one-to-one, without loss of generality), and the agents receive an endowment of tradables only at the last period. At time 0, the firms make plans for the production of non-tradable goods, with tradables used as inputs. Since tradables can only be imported (in periods 0 and 1), firms borrow at time 0, taking into account a period-0 perceived relative price of non-tradables. At period 1, non-tradables are produced and sold, debt is repaid, agents borrow to import tradables for consumption, and both goods are consumed. At period 2, the agents obtain an (expected) endowment of tradables, part of which is consumed, and part is used to repay the debt from period-1 contract. At equilibrium, the actual relative price of non-tradables at period 1 equals the relative price of non-tradables perceived at period 0, and the debt is always repaid. If we assume that the agents unexpectedly cannot borrow at period 1 as planned (after production of non-tradables have occurred), then (relative to perfect credit market case) the consumption of tradables falls at period 1. This, in turn, reduces period-1 marginal rate of substitution between non-tradable and tradable goods; hence, the period-1 relative price of non-tradables is reduced. Therefore, firms go bankrupt as their actual profits (which are negative) are less than expected (zero). Without bankruptcy costs, the Pareto-optimal equilibrium would still hold, since individuals could borrow to prevent firms’ bankruptcies. However, when bankruptcies are costly, the unexpected loss of access to the international credit market leads to firms’ bankruptcies.

The literature on Sudden Stops can be classified in several ways, depending on the classification approach: what major theoretical concept is used, what type of credit friction is assumed, etc. In the next few paragraphs, I will introduce major classifications, and later I will discuss several models in detail.

The majority of the models proposed to explain Sudden Stops can be classified into

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1constant real interest rate of zero is assumed
two categories, according to an approach used in Calvo and Reinhart (1999). The first is a Keynesian concept of price and/or wage downward rigidity. This approach provides the following transmission channel by which Sudden Stops negatively affect output and employment. A decrease in capital inflows during Sudden Stops has to be met by either a decrease in current account (CA) deficit, or by a loss of international reserves, or both. International reserves are not infinite, and in most cases, CA deficit deteriorates. By national accounting, CA deficit is the difference between aggregate demand and GNP. Therefore, unless GNP goes up by the equal amount, there is a decrease in aggregate demand. Due to downward rigidity of prices and/or wages, the drop in aggregate demand induces a fall in employment and output. The papers exploiting this setup are Cespedes, Chang and Velasco (2000), among others. The second popular approach is Fisherian debt-deflation analysis using collateral constraints proposed by Kiyotaki and Moore (1997). According to this analysis, the drop in aggregate demand leads to a decline in the demand for tradable and non-tradable goods. Excess supply of non-tradable goods, in contrast to tradables, cannot be met by foreign demand; therefore, due to the relative excess supply of non-tradable goods, the relative price of non-tradables (in units of tradables) falls. With extra assumptions of a fixed price of tradables, the decrease in the relative real price of non-tradables is reflected in the fall of the nominal price of nontradables. If production of non-tradables is financed by borrowing from abroad, and the nominal exchange rate is fixed, then the producers of non-tradable goods face higher ex post real interest rate, which may lead to their bankruptcies. To protect themselves, the lenders impose a constraint: foreign assets cannot be higher than next-period-discounted liquidation value of capital. The papers along these lines include Paasche (2001), Edison, Luangaram, and Miller (2000), and Izquierdo(2000), among others. Other studies use collateral constraints different from those proposed by Kiyotaki and Moore. For instance, Aiyagari and Gertler (1999)
propose a collateral constraint that depends on the current liquidation value of assets. This “margin requirement” constraint is utilized in such papers as Mendoza and Smith (2001), Mendoza and Smith (2006), and Christiano, Gust and Roldos (2000), among others.

From the point of view of the enforcement mechanism employed to enforce foreign debt repayment, the significant part of the recent literature on emerging market crises can be divided into two categories, according to Arellano and Mendoza (2002). In particular, lenders, in order to reduce their exposure to the risk of borrower’s default, may impose certain conditions on borrowers, such as liquidity or collateral requirements. The assumption here is that borrowers are willing to repay their debts, but when negative disturbances hit an economy, their ability to pay may be endangered. Alternatively, lenders may question borrowers willingness to pay; they may incur monitoring costs to check the soundness of borrower’s default claim. Therefore, the two groups of studies are classified as ability-to-pay and willingness-to-pay models, respectively, according to Arellano and Mendoza (2002). Ability-to-pay models are explored in Calvo (1998), Mendoza (2001), Valderrama (2002), Mendoza and Smith (2001), Arellano (2002), Izquierdo (2000), Paasche (2001), Chari, Kehoe, and McGrattan (2005), and others. Willingness-to-pay models include papers on debt repudiation (such as Alvarez and Jermann (1981) and Kehoe and Perri (2000); both papers are considered in the next section), as well as others, such as Hamann (2002), Kletzer and Wright (2000), and Atkeson (1991).

The two types of classifications presented do not differentiate between modelling Sudden Stops as events triggered by unexpected (and often large) exogenous shocks versus incorporating the possibility of Sudden Stops into agents’ expectations. There are several features which are considered increasingly important in the current development of literature on Sudden Stops (Mendoza, 2006a). First is modelling of Sudden
Stops (SS) as an endogenous result of the stochastic dynamics of a model with shocks of moderate size\(^2\) and with agents who build the possibility of SS into their expectations. In contrast, several models consider current-account reversals triggering SS as large unexpected exogenous shocks (Chari, Kehoe, and McGrattan, 2005).\(^3\) The major drawback of such approaches is an inability to explain the current account reversal. Second, a SS model should be able to produce an output drop (in contrast, some models such as Chari et al., 2005, predict an output increase as a result of a SS). Third, quantitative results are important. The model in the current proposal attempts to include all three of the features mentioned above. It is in the class of dynamic stochastic general equilibrium models of a small open economy with a credit constraint of the following type: \(b_{t+1} \geq -\varphi f(t, t+1)\). Here \(b_{t+1}\) is borrowing of one period international bonds, \(\varphi\) is a constant, and \(f(\cdot)\) is a function of current- or next-period prices, income, assets, or existing debt. The papers in this class include Christiano et al. (2000) and Mendoza and Smith (2006) using margin requirement \((f(\cdot) = q_t\alpha_{t+1}K, \text{where } q_t \text{ is the price of equity, and } \alpha_{t+1} \text{ are equity shares}); Kocherlakota (2000) \((f(\cdot) = q_tK)\); Mendoza (2001) using liquidity constraint \((f(\cdot) = Y^T_t + pY^N_t, \text{where } Y^T_t \text{ and } Y^N_t \text{ are incomes from tradable and non-tradable sector, respectively, and } p \text{ is the relative price of non-tradables}); and Mendoza (2006b) (with \(b_{t+1}\) being working capital loans and \((f(\cdot) \text{ the value of a firm’s assets}), \text{ among others.}\)

Chari, Kehoe, and McGrattan (2005) develop a model of a small open economy with a collateral constraint. In the model, a representative domestic agent maximizes expected life-time utility, which is a function of a consumption stream and a labor input. The life-time utility features a fixed rate of time preference, \(\beta\). The agents

\(^2\)such as one-standard-deviation shocks estimated from the data

\(^3\)similarly, in Calvo (1998), Sudden Stops are triggered by an unexpected loss of access to international credit market
can trade one-period state-contingent bonds $b(s^t)$ with the rest of the world. Here $s^t = (s_0, \ldots, s_t)$ is the history of events $s_i$ from period 0 up to and including period $t$; the probability of realization of $s^t$ is given by $\pi(s^t)$. Domestically-owned firms produce a single internationally tradable good using a Cobb-Douglas technology with capital and labor as inputs. Capital is flexible and depreciates at a rate $\delta$. The collateral constraint imposed on the domestic agents is: $b(s^{t+1}) \leq V(s_{t+1})$. The maximum amount of borrowing, $V(s_{t+1})$, depends on the $(t+1)$-period shock and is uniformly bounded from above by a no-Ponzi-game condition. Shocks to the collateral constraint are interpreted as changes to the country’s financial reputation.

A sudden stop is defined as an unexpected large increase in the country’s net exports, which in the model is equivalent to a sharp (and unexpected) decrease in net borrowing. If this decrease in borrowing makes the collateral constraint binding, then the intertemporal marginal rate of substitution in consumption is affected. The authors argue that the ultimate result is an increase in the output.

Mendoza and Smith (2006) construct a stochastic general-equilibrium model of asset pricing with two sets of agents. Domestic agents are representative agents in a small open economy with idiosyncratic productivity shocks. The agents can engage in international trade in bonds and equity. The bonds are non-contingent, one-period financial instruments that pay an exogenously determined world real interest rate. Foreign agents are of two types: foreign securities firms trading equities of the small open economy, and the global market for the bonds. The trading costs are higher for foreign equity traders than for domestic traders.

Domestic firms produce a single tradable good with Cobb-Douglas technology using a fixed stock of capital, $K$, and variable labor. Labor demand is given by equating the marginal product of labor with the real wage. Dividend payments each period are equal to the marginal product of capital. Production assumes shocks to total factor
productivity (TFP). Households maximize expected lifetime utility, which has SCU form with an endogenous rate of time preference. In addition to a budget constraint, the domestic agents face two additional constraints. First is a margin requirement, which is borrowed from Aiyagari and Gertler (1999). It requires the borrowing $b_{t+1}$ not to exceed a fraction $\kappa$ of the value of a capital $K$ offered as collateral:

$$b_{t+1} \geq -\kappa q_t \alpha_{t+1} K, \quad 0 \leq \kappa \leq 1 \quad (2.1)$$

where $\alpha_{t+1}$ is end-of-period share of the domestic capital stock owned by domestic households, and $q_t$ is the price of the equity. Additionally, a short-selling constraint is imposed: $\alpha_t \geq \chi$ for $-\infty < \chi < 1$ and $t = 1, \ldots, \infty$. Here $\alpha_t$ are beginning-of-period domestic capital shares. The constraint is needed to enforce the margin requirement. Without this constraint, the domestic agents could nullify the effect of the margin constraint by unlimited short-selling of the equity.

The model is calibrated to Mexican data; in particular, the shocks to TFP correspond to one-standard-deviation productivity shocks from the data. The authors show that the model produces Sudden Stops quantitatively when the leverage ratio (i.e., debt-to-equity ratio, $-\frac{b_t}{q_t \alpha_t K}$) is sufficiently high and the short-selling restriction does not bind. In this case, in order to comply with the collateral constraint that binds, the domestic agents engage in fire sales of their assets to foreign traders. This lowers the price of the assets, since the traders adjust their portfolios slowly due to trading costs. This triggers a Fisherian debt-deflation “spiral” mechanism: the drop in asset prices tightens the collateral constraint, forcing the agents into further fire sales of the assets, which tightens the constraint even more. In the case of a high debt-to-equity ratio, this ultimately leads to a correction in foreign asset position of the economy, which results in a current account reversal and a drop in consumption.
Mendoza (2006b) extends the previous model by relaxing the fixed-capital assumption and incorporating capital adjustment costs \( \Psi \left( \frac{k_{t+1} - k_t}{k_t} \right) \), where \( k_t \) is time-\( t \) capital stock. In the model, infinitely-lived representative agents in a small open economy select a sequence of consumption, labor supply, investment in domestic capital, and borrowing of one-period international bonds in order to maximize their expected lifetime utility function. The utility is a SCU-type function of consumption and labor supply. The return on foreign bonds is an exogenously determined gross world interest rate \( R \) subject to a stochastic Markov disturbance \( \exp(\varepsilon_t^R) \). In addition to the budget constraint, the domestic agents face an Aiyagari-Gertler margin requirement, by which the agents cannot borrow more than a fraction \( \kappa \) of the market value of their capital: \( b_{t+1} \geq -\kappa q_t k_{t+1} \). Domestically-owned firms produce a single tradable good using domestic capital \( k_t \) with capacity utilization \( m_t \), labor \( L_t \), and imported inputs \( \vartheta_t \). Capital depreciation is an increasing function of capacity utilization, \( \delta(m_t) \). The cost of imported inputs, \( p \), is determined exogenously in the world market, and it is subject to a stochastic shock \( \exp(\varepsilon_t^p) \). Stochastic shocks also affect total factor productivity, so that the production function is \( \exp(\varepsilon_t^A)F(m_t k_t, L_t, \vartheta_t) \). The price of the firm’s output is the model’s numeraire. Firms use the working capital to pay for a fraction \( \phi \) of their expenses on wage payments, purchases of imported inputs, and capital depreciation. Working capital financing cannot be higher than a fraction \( \kappa^f \) of the value of firms’ assets. That is, the firms face the following collateral constraint:

\[
\exp(\varepsilon_t^R) R\phi \left( \omega_t L_t + \exp(\varepsilon_t^p) p \vartheta_t + \delta(m_t) k_t \right) \leq \kappa^f q_t k_{t+1}.
\]

In this model, the binding collateral constraint sets off a Fisherian debt-deflation mechanism similar to the mechanism in the previous model; in this case, however, the deflation operates by lowering Tobin’s Q, which negatively affects investment, future capital, and output. The model is calibrated to Mexican data; the shocks to gross interest rate, TFP, and the price of imported inputs correspond to one-standard-deviation

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shocks estimated from the data. When the agents are in a high-debt state, the collateral constraint can become binding due to the adverse shocks to $R$, TFP, or $p$, causing large (relative to perfect credit markets) current account reversals and drops in consumption and investment.

2.2 Literature on Credit Market Incompleteness

There is significant interest in incomplete market models in recent international real business cycle literature. One possible explanation for this increased interest is that complete market models often produce quantitative results inconsistent with the data. For example, Backus, Kehoe, and Kydland (1992), and Baxter and Crucini (1995) point out that, contrary to data, standard complete-markets models produce high cross-country correlations of consumption, which are also much higher than those for output. Indeed, with complete markets, the international risk sharing assumption results in large correlation between consumption fluctuations across countries and a small (or even negative) correlation of output. The discrepancy between theoretical and observed cross-country correlation of consumption relative to output is robust to changes in parameter values and model specification. The other such robust discrepancy is negative cross-country correlations for both employment and investment produced by the standard complete-market models, whereas these variables comove positively in the data.

The inconsistency between standard RBC theory with complete markets and empirical evidence from both developed and emerging economies may be the result of frictions present in actual international financial markets, as suggested, among others, by Kehoe and Perri (2000) and Calvo and Mendoza (1996). Credit market frictions can be modeled using different methods. Two possible strategies are exogenous and endogenous credit market incompleteness. An example of the exogenous frictions comes from
restricting international trade to one non-contingent bond instead of having a full set of state-contingent claims. This restriction limits international risk sharing by hindering the country’s ability to offset the effect of an idiosyncratic income shock when markets are incomplete. The expected outcome is a reduction in consumption correlation across countries when Arrow-Debreu securities are replaced with simple (one-period) non-contingent bonds. Papers exploring implications of this assumption are, for instance, the works of Baxter and Crucini (1995), Kollmann (1995), and a section in Kehoe and Perri (2000). All three papers consider a two-country, general equilibrium model with one-sector production of a homogeneous tradable good, where international trade is restricted to a one-period risk-free non-contingent bond. With this assumption, the corresponding models are able to come much closer to replicating real data population moments than their complete-markets counterparts.

Baxter and Crucini (1995) consider a symmetric two-country model where individuals of home and foreign countries each maximize expected life-time utility, which is a function of two goods: consumption and leisure. In each country, the total time endowment each period that can be used for labor and leisure is normalized to one. Technology assumes production of a single final good using a Cobb-Douglas production function of two inputs: capital and labor. Labor is internationally immobile. One particular feature of the production function is labor-augmented technological change with a growth rate that is constant and common in both countries. Production is subject to productivity shocks, assumed to follow a vector autoregressive (VAR) process. These are modeled according to Backus, Kehoe, and Kydland (1992):

\[
\begin{bmatrix}
\log A_t \\
\log A_t^*
\end{bmatrix} =
\begin{bmatrix}
\rho & \nu \\
\nu^* & \rho^*
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A_{t-1}^*
\end{bmatrix} +
\begin{bmatrix}
\epsilon_t \\
\epsilon_t^*
\end{bmatrix}
\]  

(2.2)
where $\rho$ and $\rho^*$ are persistence, and $\nu$ and $\nu^*$ are spillover parameters\(^4\) for stochastic processes for productivity in home and foreign countries, respectively. Innovations $\epsilon$ and $\epsilon^*$ have zero mean, constant variance, and contemporaneous correlation $E(\epsilon_t, \epsilon^*_t) = \psi$.

The capital accumulation equation exhibits depreciation and adjustment costs (both the depreciation rate and the capital adjustment cost function are the same for home and foreign countries). The authors first consider a complete-markets case with a full set of state-contingent claims that can be freely traded by individuals in the two countries. Then, this assumption is modified by assuming that trade is restricted to goods and non-contingent claims (one-period discount bonds). In this setting, the interest rate adjusts to clear the bond market each period. For each model, two possible scenarios are considered with regard to the stochastic productivity process: (1) trend-stationary shocks with correlated innovations and substantial international spillovers: $0 < \rho < 1$, $\psi > 0$, $\nu > 0$; and (2) a random walk without spillovers and with correlated innovations: $\rho = 1$, $\psi > 0$, $\nu = 0$. The difference between complete markets and bond economy simulation results depends on the stochastic process assumed. For trend-stationary shocks with spillovers, the structure of financial markets has only a minor impact on the business cycle properties of the simulated economy. The authors report that both models fail to replicate several important empirical regularities found in the international business cycles of major developed countries, such as low international consumption correlation, output correlation being higher than consumption correlation, and positive comovements of investment and labor between two countries. The failings are slightly more pronounced in the complete-markets model. With highly persistent (random-walk) productivity shocks without international transmission, the two models show significantly different results. The complete-market model continues

\(^4\)Unless stated otherwise, symmetry is assumed when these parameters are referenced, i.e., $\rho = \rho^*$ and $\nu = \nu^*$. 

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to produce counterfactual results, which are, in some cases, even more at odds with the data than when an alternative shock structure is assumed. In particular, cross-country correlation of output becomes negative. In contrast, the bond economy demonstrates positive international output correlation, and negative international consumption correlation (the latter result, though not common, is reported for several country pairs by Baxter and Crucini, 1993). Still, both models show negative cross-country labor and investment correlations, which is contrary to the data.

Kollman (1995) considers a similar two-country symmetric representative-agent model. In both countries, agents select an optimal stream of consumption and labor to maximize expected life-time utility. The total time endowment in each period is normalized to unity, and it can be spent on labor and leisure by an agent. Each country produces a single internationally tradable good that can also be used for investment. Output is produced with constant-returns-to-scale technology using capital and internationally immobile labor. The law of motion of capital includes depreciation and costs of adjustment (as in the previous model, symmetry in the capital accumulation equation is assumed in the two countries). The only source of uncertainty in the model, as with the previous one, is provided through exogenous shocks to productivity. Shocks to productivity follow a vector autoregressive process according to equation 2.2 with high persistence \( \rho = \rho^* = 0.95 \), no international transmission of productivity shocks (no “spillovers”: \( \nu = \nu^* = 0 \)), and with contemporaneous correlation of innovations (\( \psi > 0 \)). Credit market incompleteness takes the form of risk-free one-period real bonds, which are the only instrument allowed in international financial transactions. The interest rate is determined endogenously each period to clear the bond market. The author considers three specifications of the model in simulations.
The baseline specification assumes variable labor in both countries and a moderate coefficient of relative risk aversion. Alternative specifications are: (i) fixed labor supplies and (ii) high risk aversion. For each specification, the simulation results for complete and incomplete markets are compared to real data for a sample of G-7 countries. In the baseline specification, the incomplete-markets assumption improves the predictions of certain international comovements relative to the case of complete markets: cross-country correlation of consumption is reduced to a level comparable to the data, and net exports become counter-cyclical as the data suggest. Still, cross-country correlations of output, investment, and employment are under-predicted by the incomplete-markets model (with the latter two being negative, contrary to the data), even though they are higher than those produced by the model with complete markets. The specification with a fixed labor supply produces perfectly correlated consumption in two countries for the complete-markets model, which is to be expected from consumption smoothing. When financial frictions are assumed, the cross-country consumption correlation is significantly reduced. One particular result of the high risk aversion experiment is that it reduces cross-country consumption correlation and increases output correlation for both complete- and incomplete-markets models, relative to the baseline case. All three specifications, regardless of the financial market structure, produce the counterfactual result of cross-country consumption correlation being higher than that of output.

A model constructed by Kehoe and Perri (2000) is very similar to that of Kollman (1995), with one additional structure of financial friction. In addition to considering (i) a full set of state-contingent claims and (ii) one-period non-contingent bonds, as in the two above-mentioned papers, the authors present (iii) an enforcement constraint. Under this constraint, a country can renego on its international obligations, and it will

\[ \sigma = 2 \] for instantaneous utility of the form

\[ u(C, L) = \frac{1}{1-\sigma} \left[ (C(1-L)^\mu)^{1-\sigma} - 1 \right]. \]

\[ \sigma = 5. \]
be banned from international trade from that time on (analysis of this specification and its simulation results will be considered later, in the discussion of endogenous financial frictions). With regard to the productivity disturbances process, four sets of parameter values for equation 2.2 are considered: (i) baseline, with $\rho = 0.95$ and $\nu = 0$, which corresponds to Kollman (1995); (ii) high persistence: $\rho = 0.99$ and $\nu = 0$; (iii) high spillover: $\rho = 0.85$ and $\nu = 0.15$, and (iv) original estimates of Backus, Kehoe, and Kydland (1992), labeled BKK: $\rho = 0.906$ and $\nu = 0.088$. The capital accumulation equation has two versions: with and without costs of adjustment. This allows the authors to evaluate whether costs of adjustment influence investment and net export variability.

The results of the models are compared with the data for the United States and the aggregate of 15 European Countries for the time period from 1970:Q1 to 1998:Q4. The baseline parameters for the stochastic process for productivity are considered first. Comparing the results of the complete market model without adjustment costs to the data, the authors report three major discrepancies. First, contrary to the data, cross-country consumption correlation in the model is much higher than output correlation. Next, the model produces negative cross-country correlations of investment and employment, while they are positive in the data. Third, net exports and investment are much more variable in the model than in the data (85 and 8 times, respectively). Changing the structure of the asset market from complete market to non-contingent bonds does not change the results substantially, with quantitative differences being marginally smaller for some population moments. With adjustment costs added, the variability of both investment and net exports are reduced to levels close to the data, for both complete market and bond economies. The rest of the discrepancies remain.

\footnote{This result is a symmetrized version of the estimated matrix of total factor productivity (as Solow residuals) for the United States and an aggregate of six European countries spanning the time period from 1970:Q1 to 1986:Q4.}
with reduced data-to-model differences for bond vs. complete-market economies and economies with adjustment costs vs. those without. The authors also report the results of sensitivity analysis – changing the parameter values for the productivity stochastic process – for the bond economy with adjustment costs. The high persistence experiment does not change the results of the baseline model significantly. The gap between the cross-country correlations of consumption and output is reduced, while both employment and investment become more negatively correlated between two countries. In the high spillover setting, cross-country correlations for inputs are positive (though much smaller than in the data), while the gap between consumption and output cross-country correlations widens. This gap is even wider in the BKK experiment; the experiment also produces counterfactual cross-country correlations of inputs.

All three papers discussed above employ exogenous credit market incompleteness in the form of a one-period non-contingent bond in order to improve performance of the standard two-country RBC model. An alternative approach is employed by Stockman and Tesar (1995). They incorporate non-tradable goods in a regular two-country RBC model with complete markets. Since there is no credit friction in the model, this paper is a useful reference for observing how the inclusion of internationally non-tradable goods changes the results of an open-economy RBC model. The authors argue that non-traded goods may be the missing component without which traditional models exhibit consumption over-smoothing, as well as too-high investment variability and cross-country consumption correlation relative to data (from developed countries). When a significant portion\(^8\) of a country’s output is non-tradable, it greatly reduces the country’s ability to smooth consumption and to respond to productivity shocks by means of international trade. The structure of the model is considered next.

Two countries are structurally identical – each equation that holds for the home

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\(^8\)About 50 percent, according to the authors.
country has a foreign counterpart, and each parameter in one country’s economy has its identical twin in another. Each country produces two goods – tradable and non-tradable. The goods can be consumed or used as sector-specific investment goods. Each good is produced using Cobb-Douglas technology with two inputs – capital and labor. Capital is sector-specific, and it depreciates at a constant rate $\sigma$, which is the same for both sectors. The economy features labor-augmented technological progress with a constant growth rate. Labor is mobile between two sectors, but immobile internationally. The total time endowment of a representative individual is normalized to one, and can be used for working in either industry and for leisure. Shocks to total factor productivity are viewed as transitory deviations from a steady-state growth path.

The shocks to technology follow a VAR process, which is an extension of equation 2.2. The productivity vector now has four elements, $[A^T, A^{NT}, A^{T*}, A^{NT*}]$. The utility of a representative household is derived from consumption and leisure, where consumption comes from three sources: traded goods produced by domestic ($C^T_1$) and foreign ($C^{T*}_1$) firms, and non-traded goods ($D$). As an alternative specification, the authors add taste disturbances to the basic form of the utility function – $\tau_1$ and $\tau_2$, for the traded- and nontraded-good components of consumption, respectively. An equilibrium with a full set of state-contingent claims implies that in each country the output of a good in each sector equals total world expenditure on this good. That results in four equilibrium conditions:

$$Y^T_t = C^T_t + C^{T*}_t + I^T_t$$

9T and NT are labels for traded and non-traded sectors, and asterisks denote foreign-country variables.

10So that the general form of one-period utility (with corresponding foreign counterpart) is

$$u(C_1, C_2, D, L) = \frac{1}{1-\sigma} \left[ (\tau_1 C_1^{\theta} C_2^{1-\theta} - \mu) + (\tau_2 D)^{-\mu} \right]^{1-\sigma} L^{\sigma},$$

where $L$ is leisure, and $\tau_1 > 0$ and $\tau_2 > 0$ are taste shock random variables with $E(\tau_1) = E(\tau_2) = 1$. 

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\[ Y_t^{Ts*} = C_{2t} + C_{2t}^* + I_t^{Ts} \]  

(2.4)

for traded sector, and

\[ Y_t^{NT} = D_t + I_t^{NT} \]  

(2.5)

\[ Y_t^{NT*} = D_t^* + I_t^{NT*} \]  

(2.6)

for non-traded sector. The model is calibrated to mimic steady-state behavior of an average economy based on annual data from five industrialized countries\textsuperscript{11} for the 1970-1986 period.

The model with technology shocks alone is consistent with certain business-cycle properties of developed countries. In particular, variability of output, labor, investment, and consumption are reasonably close to the data, as is cross-country total output correlation. The model produces results that are close to the data for savings/investment, trade balance/output, and aggregate consumption/output correlations. Consumption correlation between two countries, however, is higher than the data suggest; it is also higher than output correlation. The traded-goods component of consumption shows an even higher cross-country correlation (whereas in the data, that correlation is smaller than for aggregate consumption). Two other major failings of the model are: (i) perfectly negative correlation between relative price and relative consumption of non-traded goods (comparing to -0.22 in the data), and (ii) insufficient variability of trade balance and terms of trade. The authors do not report results for cross-country correlations for investment or employment, which are of a particular interest, for they often

\textsuperscript{11}Canada, Germany, Italy, Japan, and the United States
comove counterfactually to data.

Technology shocks shift the relative supply curve, while the demand curve remains stable. Moreover, the model is solved via the linearization around its steady state, which produces linear demand curves. These two features are responsible for the perfect (negative) correlation between relative price and relative consumption of non-traded goods, as well as the high negative correlation between relative price and relative output of non-traded goods. In an effort to improve that part of the model performance, the authors introduce country-specific demand shocks in the form of taste disturbances, as mentioned above. In order to add a disturbance to demand for non-traded goods relative to traded goods, the authors fix $\tau_2$ at its mean value, while assuming an AR(1) process for $\tau_1$. The shocks are uncorrelated between countries. With this additional feature the model is able to cope with most inconsistencies demonstrated by the model with the technological shocks only. In particular, close to data results are generated for cross-country correlations for both aggregate and traded-good consumption; also, the latter is smaller than the former, as the data imply. The correlation between relative price and relative consumption of non-traded goods is improved (it is -0.66 vs. -1 in the previous model vs. -0.22 in the data). Moreover, the standard deviation of the trade balance is increased to a value more consistent with data, and correlation between consumption of traded and non-traded goods is reduced to become closer to the data. The only part in which the new model performs noticeably worse is the correlation between the trade balance and output (-0.05 vs. -0.42 in the previous model vs. -0.47 in the data). This, according to the authors, may be due to the assumption that only domestically produced capital is used in the traded-goods industry in each country.

\[12\] Standard deviation of $\tau_1$ is set to 85 percent of the magnitude of productivity shocks in the traded-good sector in order to match standard deviation of traded-good consumption in the data. The autocorrelation of the taste shock is the same as that for the technology shocks in the traded-good sector, which is 0.15.
Another approach to modelling financial frictions is an endogenous restriction on international trade. Imperfect enforceability of international debt contracts has drawn attention of researchers. Several authors explore the implications of permanent exclusion from international capital markets when a country defaults on its international debt obligations (e.g., Eaton and Gersovitz, 1981, Kehoe and Perri, 2000). Eaton and Gersovitz (1981) is a classical paper exploring the risk of potential repudiation of a borrower – a small open economy – in the global capital market. The borrower can use international credit to smooth consumption. It may choose to default on its debt obligations if it finds it optimal to do so, in which case it will be permanently excluded from international credit markets. The borrower interacts with risk-neutral lenders who know all relevant characteristics of the borrower. The setup of the model and major findings are presented next. The authors consider a general model, and two specialized versions – deterministic and stochastic, each with its own set of assumptions.

The borrower is endowed with a (generally) random amount of perishable output each period. The output, combined with international borrowing, is used for consumption and debt repayment. Debt $d_t$ is a one-period bond with a rate of interest that includes an endogenously determined default-risk premium over the risk-free asset’s interest rate. In the case of a default at time $t$, a penalty $P_{\tau}, \tau \geq t$ may be imposed on a reneged borrower,\textsuperscript{13} in addition to the international borrowing ban. In this case, consumption each period will equal the country’s endowment less the penalty. The borrower maximizes life-time expected discounted utility as a function of consumption. Each period, the borrower will default if the value of the objective function of autarky, $V^D_t$, is higher than the value of continuing repaying debts, $V^R_t$. Thus, the probability of a default at time $t$ given the information set at time $t - 1$ is given by a function $\lambda (d_t) = \Pr \left( V^D_t > V^R_t \right)$. International private lenders are competitive and risk neutral,

\textsuperscript{13}An example of the penalty may be an international aid cutoff.
they know the borrower’s function $\lambda(d_t)$, and they can invest into an alternative risk-free asset that pays interest rate $\bar{r}$. The total supply of loanable funds is finite. The major conclusions derived from this general model are that (1) higher debt obligations increase probability of default, and (2) there is an upper bound $\bar{b}_t$ on the borrowing at time $t$. These two findings imply that credit rationing may occur so that the actual borrowing may be less than desired.

A deterministic model with few specific assumptions is explored next in order to obtain analytical solutions for comparative statics analysis. The borrower’s income is assumed to alternate between high and low values equidistant from a trend. Borrowing for consumption-smoothing purposes occurs during low-income periods, and the debt must be fully repaid in the next period, when the income is high. The borrower is also allowed to lend (save) during high-income periods, with a subsequent collection of the whole investment amount and the interest in the next low-income period.\footnote{Borrowing and lending interest rates are, respectively, $R$ and $R'$.}

Consumption and trend income grow at a constant rate $g$; utility of consumption is of the constant relative risk aversion form. The model arrives at the following major conclusions:

1. desired borrowing increases with higher income variability or higher income growth rate;

2. there exists a maximum sustainable debt level, or credit ceiling, $\bar{b}$;

3. the credit ceiling increases with higher income variability; the effect of the income growth rate on the credit ceiling is ambiguous;

4. if a default penalty $P$ is assumed, then a higher $P$ increases the credit ceiling.\footnote{since the higher default penalty reduces the value of autarky, $V^D$.}
In this full-foresight model, a default does not occur if we dismiss the possibility that lenders mistakenly make a loan in excess of $b$.

An alternative specification is considered next that allows a different reason for a default. In this stochastic setting, income may take one of two values: $[1 + \sigma, 1 - \sigma]$, with equal probability. The current realization of the income does not depend on its previous value; there is no growth and no savings opportunity. Borrowing is only available to non-defaulted borrowers during low-income periods preceded by a no-borrowing period, and the debt must be repaid next period. For a discount factor $\beta$ close to 1, an increase in income variability $\sigma$ raises the credit ceiling $\overline{b}$, as in deterministic case. However, if the future is discounted heavily (i.e., $\beta$ approaches 0), the opposite effect is observed. In an empirical part of the paper, the authors analyze borrowing by a set of developing countries in 1970’s and find that most countries from the set are credit rationed.

An important implication of the paper is that a credit ceiling is set by a lender, and this ceiling is endogenously determined. If the country wants to borrow more than the ceiling, it will be rationed and unable to smooth consumption fully. As a side note, it is worth mentioning that the credit ceiling is determined by several factors, one of which is the borrower’s probability of a default. The probability of a default is affected, among other factors, by the borrower’s income variability.

Kehoe and Perri (2000) construct a two-country model with risk of repudiation (in addition to complete-markets and bond economies, as mentioned earlier). In their model, each country can default on its international debt obligations if the value of autarky starting from some period appears to be higher than the value of continuing trade (and repaying debts). The punishment for a default, as in Eaton and Gersovitz (1981), is an exclusion of this country from all future intertemporal and interstate trade. Although the assumption of exclusion from international financial markets as a
punishment for the defaulting country is extreme and far from reality, it helps the model come closer to the real-world data pattern than complete-market models or models of markets with exogenous frictions.

As was mentioned previously, several features are considered increasingly important in the recent literature on Sudden Stops. The first is using shocks of a moderate size, assuming that agents build the possibility of a SS into their expectations, and modeling Sudden Stop events as endogenous results of the stochastic dynamics. The second important feature is the model’s ability to reproduce stylized facts of observed Sudden Stops, such as a drop in output and aggregate demand. Third, quantitative results are important. Moreover, it would be desirable to reproduce both short-term, as well as long-term effects of a SS (in many cases, Sudden Stops had severe economic impact on a country in question, but the impact was limited to a few quarters, followed by a fast recovery). One more important feature is the presence of a non-tradable sector, in addition to the tradable sector. The non-tradable sector constitutes a non-trivial part of virtually every economy that has experienced a Sudden Stop, and ignoring this component would not only limit the model’s applicability, but would also miss important transmission mechanisms of Sudden Stops.
Chapter 3

One-Sector Model

In this chapter, a one-sector model with the liquidity constraint is constructed and analyzed. Section 3.1 describes the structure of the model; section 3.2 spells out a solution approach and the model parametrization; section 3.3 presents and discusses the model results for both the long and short term; section 3.4 checks the model’s robustness and sensitivity to changing parameter values, such as magnitude of the shocks, their correlation, and tightness of the liquidity constraint.

3.1 Structure of the Model

As a starting and reference point, the one-sector model is constructed first. The structure of the model corresponds to Mendoza (1991) with inclusion of an endogenous liquidity constraint following Mendoza (2001). Representative, infinitely-lived households choose the optimal time path for consumption $C_t$ and labor $L_t$ in order to maximize lifetime utility with Uzawa – Epstein – type preferences (cardinal stationary utility):

$$U = E_0 \left\{ \sum_{t=0}^{\infty} u(C_t, L_t) \cdot \exp \left[ - \sum_{\tau=0}^{t-1} \nu(C_{\tau}, L_{\tau}) \right] \right\}$$

(3.1)

The instantaneous utility function is given by
\[ u(C_t, L_t) = \frac{(C_t - \frac{L_t}{\omega})^{1-\gamma} - 1}{1 - \gamma} \]  

and the time preference function is given by

\[ \nu(C_t, L_t) = \beta \ln \left( 1 + C_t - \frac{L_t}{\omega^\omega} \right) \]  

The specification above conforms to the following conditions: both the utility function \( u(\cdot) < 0 \) and the time preference function \( \nu(\cdot) > 0 \) are increasing, concave, and twice continuously differentiable in their arguments. The utility structure with an endogenous rate of time preference, \( \exp[\nu(\cdot)] \), creates an “impatience effect” (Mendoza, 1991; Gomme and Greenwood, 1990): an increase in the current consumption leads to a subjective devaluation of the future consumption stream. It is also important to note that the following conditions must be satisfied:\(^1\) \( u'(\cdot) \exp[\nu(\cdot)] \) is non-increasing while \( \exp[\nu(\cdot)] \) is increasing.\(^2\) These conditions ensure that the argument of the utility function at each period \( t \) is a normal good, and the model produces a unique invariant limiting distribution of state variables. In addition, the model with an endogenous rate of time preference can support occasionally binding equilibria\(^3\) (Mendoza, 2000), which is a critical feature due to nonlinearities present in the model. The household’s coefficient of relative risk aversion and the inverse of its intertemporal elasticity of substitution are given by \( \gamma > 1 \). The disutility associated with labor is \( \frac{L_t}{\omega^\omega}, \omega > 0 \), where \( \frac{1}{\omega-1} \) is the elasticity of labor with respect to the real wage. The elasticity of the

\(^1\)The conditions are satisfied by setting \( \beta \leq \gamma \).

\(^2\)This prevents a Ponzi scheme from happening, as will be discussed later, in the Numerical Solution Section.

\(^3\) Indeed, if we had an exogenous discount factor, \( \beta \), and an exogenous world interest rate \( R \), then the country’s asset position would depend on the sign of \( (\beta R - 1) \). If it were less than zero, then the country would borrow infinitely (or up to imposed lower bound). If the expression were positive, then the country would tend to lend infinitely.
endogenous rate of time preference to changes in the instantaneous utility function is
determined by $\beta > 0$. The structure of $u(\cdot)$ and $\nu(\cdot)$ allows the marginal rate of substi-
tution between consumption and labor to be a function of the latter only (Greenwood,
Hercowitz, and Huffman, 1988). This formulation simplifies the first-order condition
(Equation 3.12) by removing the interaction between consumption and employment
(Mendoza, 1991) and allows us to concentrate our attention explicitly on the optimal
allocation of savings between foreign assets and physical capital.

An internationally tradable composite good is produced using the Cobb-Douglas
production function:

$$F(K_t, L_t) = e_t AK_t^\alpha L_t^{1-\alpha}$$

(3.4)

An exogenous disturbance $e_t$ follows the stochastic process to be specified later. Capital
evolves according to the following accumulation equation:

$$K_{t+1} = (1 - \delta)K_t + I_t - \frac{\phi}{2} (K_{t+1} - K_t)^2$$

(3.5)

Equation 3.5 features the capital adjustment cost $\frac{\phi}{2} (K_{t+1} - K_t)^2$; $I_t$ is gross investment,
and $0 < \delta < 1$ is the capital depreciation rate.

Individuals trade non-contingent one-period bonds $b_t$ in the perfectly competitive
international financial market. The exogenously determined mean gross interest rate
$R$ is subject to stochastic shocks $n_t$. The structure of the exogenous interest shock $n_t$
will be defined later. The combined resource constraint for the economy is

$$C_t + K_{t+1} + b_{t+1} = e_t AK_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t - \frac{\phi}{2} (K_{t+1} - K_t)^2 + n_t Rb_t$$

(3.6)
In addition to credit market incompleteness caused by non-contingent bonds, the individuals face an endogenous liquidity constraint. This is the requirement to finance a fixed fraction $\varphi \in [0, 1]$ of their current expenses on consumption, investment, and debt repayment out of their current income:

$$e_t AK_t^\alpha L_t^{1-\alpha} \geq \varphi \cdot (C_t + I_t - n_t R_b)$$

Equations 3.5, 3.6, and 3.7 can be combined to give the following:

$$b_{t+1} \geq -\frac{1 - \varphi}{\varphi} e_t AK_t^\alpha L_t^{1-\alpha}$$

Equation 3.8 gives a lower limit on borrowing at any time, which depends on the current income. The strictness of the limit is determined by $\varphi$: for $\varphi \to 0$ we approach the case of unlimited borrowing, and for $\varphi$ equal to unity we have a no-borrowing case (for all $t$, $b_{t+1} = 0$).

Endogenous credit market incompleteness leads to precautionary savings: agents save in “good” states (positive shocks to production and negative shock to interest rate), anticipating future “bad” states, when the constraint may bind and allowed borrowing will be less than desired.

Productivity and interest-rate shocks are assumed to follow two-point symmetric Markov chains with simple persistence (see, e.g., Mendoza 1995). Each shock may take one of two values: $e \in \{e^H, e^L\}$ and $n \in \{n^H, n^L\}$. Therefore, there are four possible realizations of shocks, i.e., four possible states of nature $\xi_t = (e_t, n_t)$. The evolution of the shocks is governed by the 4x4 transition matrix $P$, with $P_{ij}$ being the probability of going from a current state $\xi_i^t$ to the next-period state $\xi_j^{t+1}$, for $i, j = 1, 4$. The elements of the transition matrix are:
\[ P_{ij} = (1 - \theta)\Pi_j + \theta\delta_{ij} \]  

(3.9)

where

- \( P_{ij} = \Pr(\xi_{t+1} = \xi^j|\xi_t = \xi^i) \) is the conditional probability of going from state \( i \) in this period to state \( j \) next period, with \( i = 1 \ldots 4 \) and \( j = 1 \ldots 4 \);

- \( \theta \) is the persistence parameter governing both shocks, set to mimic their 1st-order autocorrelation: \( \theta = \rho_eA = \rho_eR \) (“simple-persistence” assumption);

- \( \Pi_j \) is the unconditional limiting probability of state \( \xi^j \);

- \( \delta_{ij} \) is Kronecker symbol;

- \( \Pi_1 \) is the probability that both shocks are positive;

- The Markov chain is assumed to be symmetric; this includes the following conditions: \( \Pi_1 = \Pi_4, \Pi_2 = \Pi_3, \sum_{i=1}^4 \Pi_i = 1 \), and the condition that the magnitude of high and low shocks are equal (\( \log e^H = -\log e^L \) and \( \log n^H = -\log n^L \));

- \( \rho_{eA,eR} = 4\Pi_1 - 1 \) where \( \rho_{eA,eR} \) is the cross-correlation of the shocks;

- the following regular conditions for transition probabilities must be met: \( \sum_{j=1}^4 P_{ij} = 1 \) and \( 0 \leq P_{ij} \leq 1 \).

The competitive equilibrium in the small open economy with an endogenous liquidity constraint is defined in a standard way – as a sequence \( \{C_t, L_t, b_{t+1}\}_{t=0}^\infty \) and \( \{w_t\}_{t=0}^\infty \) such that, given the initial amount of borrowing \( b_0 \), the initial shocks and transition probabilities \( P_{ij} \):

1. firms maximize their profits;

2. households maximize lifetime utility subject to their budget constraint;
3. labor market clears.

3.2 Numerical Solution

The social planner's problem assumes the following Lagrangean:

\[
L = U + \sum_{t=0}^{\infty} \left\{ \lambda_t \left[ e_t A K_t^\alpha L_t^{1-\alpha} + (1 - \delta) K_t - \frac{\phi}{2} (K_{t+1} - K_t)^2 - K_{t+1} + n_t R b_t - b_{t+1} - C_t \right] + \mu_t \left[ b_{t+1} + \frac{1 - \varphi}{\varphi} e_t A K_t^\alpha L_t^{1-\alpha} \right] \right\}
\]  

First-order conditions are:

\[
U_C(t) \cdot \left( 1 - \frac{\mu_t}{\lambda_t} \right) = \exp(-\nu(t)) RE_t U_C(t + 1) \tag{3.11}
\]

\[
L_t^{\alpha-1} = (1 - \alpha)e_t A K_t^\alpha L_t^{1-\alpha} \left[ 1 + \frac{\mu_t}{\lambda_t} \frac{1 - \varphi}{\varphi} \right] \tag{3.12}
\]

plus complementary slackness conditions. The current level of capital stock, \( K \equiv K_t \) and current asset holdings \( b \equiv b_t \), together with a shock variable \( \xi \equiv \{ e_t, n_t \} \), are the state variables for the dynamic programming approach. Consumption \( C_t \), labor \( L_t \), next-period capital stock and asset position, \( K' \equiv K_{t+1} \) and \( b' \equiv b_{t+1} \), are controls.

First, for every point \((K, K', b, b', \xi)\), we find \( C^* \) and \( L^* \) that solve the following system:

---

\textsuperscript{4}This Euler equation clearly shows why Ponzi schemes are ruled out by the specification of and assumptions on stationary cardinal utility. Indeed, as was noted above, the expression \( u'() \exp[\nu()] \) is non-increasing in its argument; therefore, increasing time-\( t \) consumption will devalue the future lifetime consumption stream starting from next period, \( t + 1 \).
\[ C^* + K' + b' = eAK^\alpha L^{s1-\alpha} + (1 - \delta)K - \frac{\phi}{2}(K' - K)^2 + nRb \]  
(3.13)

\[ b' \geq -\frac{1 - \varphi}{\varphi}eAK^\alpha L^{s1-\alpha} \]  
(3.14)

If 3.14 is slack, then the system includes equations 3.12 and 3.13.5 If 3.14 is binding, then the system is 3.13 and 3.14 written as equality.

Next, the dynamic programming problem for the social planner is: for every point \((K, b, \xi)\) in the state space, choose the optimal \(K'\) and \(b'\) as solutions to

\[ V(K, b, \xi) = \max_{K', b'} \{ u(C^*, L^*) + \exp(-\nu(C^*, L^*)) \cdot E[V(K', b', \xi')] \} \]  
(3.15)

Value function iteration is selected as a solution method due to the potentially high degree of non-linearity caused by an occasionally binding liquidity constraint.

The model is calibrated to make it approximately consistent with the empirical regularities of the Indonesian economy. Indonesia is considered to be a typical small open economy; it has experienced a Sudden Stop event as a part of the 1997 East Asian financial crisis. Stylized facts of Indonesian Real Business Cycles are summarized in Table 3.1.6 GDP is reported for 9 sectors, according to the kind of activity. The following sectors are considered tradable:7 1. agriculture, 2. mining and quarrying, and

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5In this case, the multiplier \(\mu_t\) is zero.

6The statistics are based on data from the Central Bank of Indonesia, which covers the period from 2000:1-2007:3. For comparison, Table C.1 of Appendix C has the population moments for the Indonesian Economy based on the International Financial Statistics (IMF) data for the period of 1990:1-2007:3. However, IFS data lacks information on sectoral GDP (which is needed for parametrizing the two-sector model); moreover, the data exhibits unusually low correlation between consumption and GDP (0.064). Therefore, Central Bank of Indonesia data is used as a main source for reporting population moments and calibrating the model.

7The sectors are classified as tradable or non-tradable according to World Bank.
### Table 3.1: Statistical Moments of Indonesian Business Cycles

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviation</th>
<th>First-order Autocorrelation</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>4.44</td>
<td>0.563</td>
<td>1.000</td>
</tr>
<tr>
<td>Tradable GDP</td>
<td>5.79</td>
<td>0.696</td>
<td>0.977</td>
</tr>
<tr>
<td>Nontradable GDP</td>
<td>3.33</td>
<td>0.251</td>
<td>0.901</td>
</tr>
<tr>
<td>Consumption</td>
<td>4.17</td>
<td>0.485</td>
<td>0.756</td>
</tr>
<tr>
<td>Government Spending</td>
<td>7.62</td>
<td>0.171</td>
<td>0.294</td>
</tr>
<tr>
<td>Net Exports</td>
<td>6.64</td>
<td>0.431</td>
<td>-0.103</td>
</tr>
<tr>
<td>Investment</td>
<td>15.06</td>
<td>0.583</td>
<td>0.496</td>
</tr>
<tr>
<td>Savings</td>
<td>10.20</td>
<td>0.514</td>
<td>0.586</td>
</tr>
<tr>
<td>World real interest rate</td>
<td>0.91</td>
<td>0.882</td>
<td>-0.291</td>
</tr>
</tbody>
</table>

**Notes:** The Indonesian data contains quarterly observations in 2000 constant prices for the period 2000:1-2007:3. Data are de-seasonalized, divided by total population, logged (except world real interest rate), and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600. Population is annual observations from *International Financial Statistics (IMF)*, interpolated into quarterly data using linear trend (annual data is almost perfectly linear). For each variable, standard deviation is a percentage standard deviation from HP trend. Investment is gross fixed capital formation plus change in stock. Net exports is defined as detrended exports minus detrended imports. Savings is defined as investment plus net exports. World real interest rate is London quote of the Eurodollar 3-month nominal interest rate minus the consumer price inflation of industrial countries (both available from *IFS*) for the same period; the rate is expressed as a gross rate. Correlation between world real interest rate and GDP is computed using *IFS* data on real GDP volume (2000=100) for the period 1997:1-2007:3.

**Data Source (unless noted otherwise):** Central Bank of Indonesia.

3. manufacturing industry (both oil/gas and non oil/gas industries\(^8\)). Non-tradable sectors are: 1. electricity, gas and clear water, 2. construction, 3. trade, hotel and restaurant, 4. transportation and communication, 5. finance, leasing and business services, 6. services. The parameter values of this one-sector model are summarized in Table 3.2. Capital share of output, \(\alpha\), is computed according to an approach used in

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\(^8\)These two activities are reported separately by the Central Bank of Indonesia.
Sarel (1997). This is done for consistency between the one-sector and two-sector models. The depreciation rate, $\delta$, is set at 10%, which is in line with estimates published by Central Bureau of Statistics (CBS 1997). The adjustment-cost parameter, $\phi$, is set to 0.025 to help bring the variability of investment close to the value observed in the data. The value of the world gross interest rate, $R = 1.04$, is a typical value used in RBC literature (see, for instance, Prescott, 1986). The value of $\omega = 1.455$ is taken from Mendoza (1991) (Rochjadi and Leuthold (1994) report the estimates of $\omega$ for Indonesia, which range from 1.82 to 4.23). The coefficient of relative risk aversion, $\gamma = 1.5$, is an average of two values considered by Mendoza (1991). The value of $\beta$ is determined by solving a system of equations that describe steady-state equilibrium of the model given the other parameter values. The equations are listed in the Appendix A. Equation A.1 sets the rate of time preference equal to gross real interest rate; return on capital net of depreciation should be equal to the return on foreign assets as expressed by equation A.2; labor market equilibrium is given by A.3; finally, A.4 gives equilibrium in the market for goods. One additional condition used to compute $\beta$ is the value of the external debt-to-GDP ratio for Indonesia (from Lane and Milesi-Ferretti, 1999). Also, $A$ is normalized to 1.

The value of the liquidity constraint, $\varphi = 0.6475$ is set below the critical value that

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9 The approach reports estimated capital shares for each of 9 kinds of activity for 5 ASEAN countries. These numbers, together with the relative intensities of nine major economic activities derived from the Indonesian data, are used to compute $\alpha$.

10 Other papers report different estimated $\alpha$ for Indonesian economy. For instance, Senhadji (2000) reports $\alpha = 0.47$.

11 They publish depreciation rate for 17 types of capital goods, with the range from 5% (for buildings) to 35% (for livestock and manufacture of furniture and fixtures).

12 The numbers are computed from their estimates of labor supply elasticity with respect to real wage, $\epsilon_W$, as follows: $\omega = 1 + \frac{1}{\epsilon_W}$.

13 He considers two values of $\gamma$: 1.001 and 2.
would bind in the deterministic steady state: \( \varphi_{cr} = 0.6485 \). Given the magnitude of shocks to productivity and the interest rate, \( \varphi \) is binding only for some realizations of shocks.

To complete the parametrization, the values of \( e, n, \theta, \) and \( \Pi_1 \) need to be supplied. Values of \( e \) and \( n \) are chosen to mimic variability of GDP. \( \theta \) is selected to make GDP persistence (as measured by first-order autocorrelation) consistent with the data. The value of \( \Pi_1 \) is determined from the equation \( \rho_{e,n} = 4\Pi_1 - 1 \), where \( \rho_{e,n} \) is a sample correlation between GDP and the world interest rate.

<table>
<thead>
<tr>
<th>Table 3.2: Parameter Values Used for Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology</td>
</tr>
<tr>
<td>( \alpha = 0.318 )</td>
</tr>
<tr>
<td>( A = 1 )</td>
</tr>
<tr>
<td>( \delta = 0.1 )</td>
</tr>
<tr>
<td>( \phi = 0.025 )</td>
</tr>
</tbody>
</table>

The model is solved by value function iteration over a discrete state grid. To find the centers of the grid, the deterministic steady state is computed. The values for the deterministic steady state are listed in Table 3.3, and the derivations are explained in Appendix I. The range of the capital stock is \([3.15, 3.57]\), and the range of the asset position is \([-1.45, 0.05]\). Both states have 42 equally spaced points over the corresponding range. The shock to productivity is one of the set \( \{\exp(0.0191), \exp(-0.0191)\} \), and the shock to the gross interest rate is one of the set \( \{\exp(0.0005), \exp(-0.0005)\} \). Therefore, the state space contains \( 42 \times 42 \times 4 = 7056 \) points.

\(^{14}\) The value of \( \varphi_{cr} \) can be determined by solving the equation 3.14 as an equality in the steady state: \( b = -\frac{1}{\varphi_{cr}} Y \), which gives \( \varphi_{cr} = \frac{Y}{b - \theta} \), where \( b \) and \( Y \) are steady-state values of asset position and output.
Table 3.3: Deterministic Steady State for the Economy without Liquidity Requirement

<table>
<thead>
<tr>
<th>K</th>
<th>L</th>
<th>b</th>
<th>C</th>
<th>Y</th>
<th>NX</th>
<th>I</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3285</td>
<td>0.9996</td>
<td>-0.7942</td>
<td>1.1008</td>
<td>1.4654</td>
<td>0.0318</td>
<td>0.333</td>
<td>0.365</td>
</tr>
</tbody>
</table>

3.3 Results and Discussion

The results for population moments for the one-sector model are shown in Table 3.4. The model without the liquidity constraint is able to match closely many of the population moments of the actual data. In particular, the standard deviations of the following main national indicators – output, consumption, investment, and savings – are in good correspondence with the data, even though they somewhat underestimate the actual values. This artifact may be corrected by increasing the variability of productivity shock, \( e \) and/or shock to the world interest rate, \( n \). Next, persistence (as measured by first-order autocorrelation) of the variables mentioned above is also close to the data, except for investment (-0.045 vs. 0.583 in the data). Furthermore, the model generates procyclical consumption, employment, capital, savings, and investment, whereas net exports and foreign asset holdings show almost no correlation with GDP (-0.006 and 0.053, correspondingly). Savings-investment correlation is higher in the data (0.833 vs. 0.586 in the model).

Introducing the liquidity constraint does not change the model population moments significantly (except for foreign asset position and net exports, relative changes in the population mean range from 0.2% (for labor) to 2.0% (for savings)). This observation can be interpreted in the following fashion: even if a country hits a borrowing limit in the short run, the long-run effects of this are insignificant, as economic agents adjust to the constraint. This is consistent with empirical evidence regarding “Sudden Stops”: despite their serious effect on the macroeconomic behavior of a typical small open economy, this effect is short-lived. The country is able to recover rather fast, and there
Table 3.4: Population Moments for One-Sector Model

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>First-Order Autocorr.</th>
<th>Correlation with GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Economy with Perfect Credit Markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.102</td>
<td>3.035</td>
<td>0.624</td>
<td>0.941</td>
</tr>
<tr>
<td>Capital</td>
<td>3.330</td>
<td>1.637</td>
<td>0.667</td>
<td>0.518</td>
</tr>
<tr>
<td>Labor</td>
<td>1.000</td>
<td>2.751</td>
<td>0.538</td>
<td>1.000</td>
</tr>
<tr>
<td>Output</td>
<td>1.467</td>
<td>4.002</td>
<td>0.538</td>
<td>1.000</td>
</tr>
<tr>
<td>Foreign Assets</td>
<td>-0.794</td>
<td>23.355</td>
<td>0.984</td>
<td>-0.006</td>
</tr>
<tr>
<td>Net Exports</td>
<td>0.032</td>
<td>109.54</td>
<td>0.130</td>
<td>0.053</td>
</tr>
<tr>
<td>Investment</td>
<td>0.333</td>
<td>12.780</td>
<td>-0.045</td>
<td>0.597</td>
</tr>
<tr>
<td>Savings</td>
<td>0.365</td>
<td>8.091</td>
<td>0.550</td>
<td>0.923</td>
</tr>
<tr>
<td><strong>Economy with Liquidity Constraint</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.105</td>
<td>3.170</td>
<td>0.649</td>
<td>0.961</td>
</tr>
<tr>
<td>Capital</td>
<td>3.312</td>
<td>2.044</td>
<td>0.795</td>
<td>0.520</td>
</tr>
<tr>
<td>Labor</td>
<td>0.998</td>
<td>2.780</td>
<td>0.554</td>
<td>1.000</td>
</tr>
<tr>
<td>Output</td>
<td>1.462</td>
<td>4.062</td>
<td>0.557</td>
<td>1.000</td>
</tr>
<tr>
<td>Foreign Assets</td>
<td>-0.660</td>
<td>16.877</td>
<td>0.963</td>
<td>0.016</td>
</tr>
<tr>
<td>Net Exports</td>
<td>0.026</td>
<td>118.40</td>
<td>0.028</td>
<td>-0.041</td>
</tr>
<tr>
<td>Investment</td>
<td>0.331</td>
<td>12.589</td>
<td>-0.043</td>
<td>0.648</td>
</tr>
<tr>
<td>Savings</td>
<td>0.358</td>
<td>7.693</td>
<td>0.508</td>
<td>0.936</td>
</tr>
</tbody>
</table>

Notes: Standard deviations are percentages of the corresponding means.

is practically no observed permanent exclusion from international financial markets of a country that experienced a default on its sovereign financial obligations. The only significant long-term effect of the liquidity constraint is a 16.9% decrease of foreign asset holdings (from -0.794 to -0.660). This can be explained as precautionary savings by economic agents in an anticipation of a realization of unfavorable shocks, in which they may not be able to borrow enough in order to smooth consumption.

Figure 3.1 shows joint distributions of capital and foreign asset position for the constrained and unconstrained economies. Marginal distributions for the constrained vs. unconstrained cases are depicted on Figures 3.2 for foreign bond and 3.3 for capital, respectively. As can be inferred from the graphs, the major impact of the constraint is on the probability distribution of the foreign bond holdings. For each $K_i$, there is
Figure 3.1: Joint Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (One-Sector Model)
Figure 3.2: Asset Limiting Probability Distribution (One-Sector Model)

Figure 3.3: Capital Limiting Probability Distribution (One-Sector Model)

Notes: Solid line corresponds to the unconstrained economy, and the dashed line is for the economy with the liquidity constraint.
a minimum foreign asset position (i.e., maximum level of borrowing), below which the long-term probability is zero. The corresponding line in \((K, b)\) space is defined as a set at which the liquidity constraint becomes binding for at least one realization of the shocks.\(^{15}\) The alternative definition of the set extends the explanation provided in Mendoza (2001) to the two-variable case: for a given \(K_i\), there is a maximum sustainable level of borrowing \(b_j\), for which the liquidity constraint is (marginally) non-binding if the worst state of nature (low productivity and high interest rate) persists. Depending on the value of \(K_i\), this set is at the following coordinates on \(b\) grid: 19\(^{th}\) (for \(K\) higher than 10\(^{th}\) coordinate), 20\(^{th}\) (for \(K\) between 7\(^{th}\) and 9\(^{th}\) coordinates), or 21\(^{st}\) (for \(K\) lower than 7\(^{th}\) coordinate). Correspondingly, for the marginal probability distribution of foreign assets in the constrained economy (lower part of Figure 3.2), there is a zero probability of foreign debt below 19\(^{th}\) coordinate.

The results of the analysis suggest that the long-term effects of the financial friction are insignificant: the only numerically large changes are observed for the mean and standard deviation of the net foreign asset position. To analyze the short-term effects of imposing the liquidity constraint, an experiment was conducted on the impact of switching from the best state of nature (positive shock to technology, negative shock to the world interest rate) at time \(t\) to the worst state (negative technological shock and positive shock to \(R\)) at time \(t+1\). The \(t \rightarrow t+1\) transition was computed according to the optimal decision rule for each economy: for each point \((K_t, b_t)\) in the state space, the optimal decision rule dictates a transition to a certain \((K_{t+1}, b_{t+1})\), given the values of the shocks at times \(t\) and \(t+1\), \(\xi_t\) and \(\xi_{t+1}\). Control variables such as consumption and labor are functions of state variables and shocks: \(C_t = C(K_t, b_t, \xi_t)\) and \(L_t = L(K_t, b_t, \xi_t)\); therefore, next-period consumption and employment can be

\(^{15}\)In Mendoza (2001), where capital is fixed, the set reduces to a single point of maximum sustainable foreign debt.
found in the same manner. The results of the impact of the switch are presented on Figures F.2 through F.5, for CA-output ratio, consumption, labor, and output, correspondingly. For each variable \( X \in \{ CA/Y, C, L, Y \} \), the impact of the switch is calculated as \( \frac{X_{t+1} - X_t}{X_t} \), for each point in \((K_t, b_t)\) grid. For visualization purpose, the transitions are illustrated by two-dimensional graphs: for each point \( K_i \) in the grid, min, mean, and max values of the switch impact are computed along the \( b \) dimension. The results for the unconstrained economy – min, mean, and max impact for each \( K_i \) – are three solid lines on the left graph; constrained economy results are on the right graph, with a dotted line for the mean value of the impact switch. The same dotted line (the mean value of the switch for the constrained economy) is superimposed on the left graph, for comparison purposes. Two more graphs for each variable of interest are produced by repeating the exercise for each point \( b_j \) in the grid (and finding min, mean, and max along the \( K \) dimension). As can be inferred from the graphs, such a switch has more dramatic effects for the constrained economy. The most illustrative graphs are the lower left graphs for each variable (min, mean, and max impact of the switch in the unconstrained economy plus mean impact in the constrained economy, plotted against the foreign asset position, \( b_t \)). The graph of the impact for GDP against the foreign asset position is illustrated on Figure 3.4.

There are several transmission channels through which shocks to productivity and interest rate affect intertemporal and atemporal decisions in the economy without the liquidity constraint. First, a shock to productivity affects the marginal reward to labor. In particular, an adverse productivity shock decreases the marginal product of labor, leading to a drop in employment. Second, a negative productivity shock results in a lower marginal product of capital, which affects agents’ investment decisions and

\[16\] If the change in the shock values had no effect on a certain variable, we would observe a horizontal line that goes through zero as a shift impact for that variable.
Figure 3.4: Switch Impact for Output, Foreign Asset Schedule (One-Sector Model)

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (with favorable shocks) to state $t+1$ (with unfavorable shocks), according to the optimal decision rule. For each point $b_i$ on the foreign asset grid, min, mean, and max values of the switch impact are computed along the $K$ (capital) dimension. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.

leads to a savings redistribution between capital and foreign assets. Third, a shock to the interest rate induces an intertemporal consumption-substitution effect. As the interest rate goes up, households save more to substitute current consumption for future consumption, in accordance with the Euler equation. Therefore, current consumption drops. Fourth, a shock to the interest rate induces a savings redistribution between capital and foreign assets, since the interest rate is an opportunity cost of capital as a rate of return of foreign assets. Fifth, there is a wealth effect associated with an interest rate shock. Since the economy is a net borrower, the spike in the interest rate decreases the economy’s total wealth (by increasing debt repayments).

For the liquidity constrained economy, there are two additional effects if the constraint becomes binding, according to Mendoza (2000). First, the binding constraint may prevent households from borrowing a sufficient amount to smooth consumption. This forces them to decrease consumption relative to the unconstrained case. Second,
increasing labor supply allows households to borrow more, according to the equation 3.8. Therefore, the effective marginal return to labor supply increases in the case of a binding constraint.

Figure F.2 shows the impact of the switch on the current account-output ratio as a function of $b_t$. The figure clearly demonstrates one of the distinctive features of Sudden Stops – current account reversal. For the unconstrained economy, the drop in the current account-GDP ratio varies from 30% to 100%, depending on the initial foreign asset position. In the liquidity-constrained economy, the impact of the switch is almost identical to the unconstrained case for the right portion of the graph (i.e., low initial debt); we see a drop in the ratio. Around the region of the graph where the constraint can become binding, however, we see an increase (up to 50%) in the CA/Y ratio. This current account reversal cannot be reproduced by the unconstrained economy model.

The major real macroeconomic variables which are relevant for the model – consumption, labor, and output – all show similar behavior during the switch. As lower left graphs of Figures F.3 through F.5 demonstrate, all three variables drop (on average) almost uniformly for any value of the initial (time-$t$) borrowing in the unconstrained economy. The mean value of the drop is around 4.4% for consumption, 4.5% for labor, and 6.5% for GDP. Qualitatively, these results are in line with the simulation results reported in Mendoza (2001). The current setup, however, allows us to study not only magnitude of the impact as a function of $b_t$, but also the variability of the impact, due to the flexible stock of capital present in the current model. It is worth mentioning that the variability of the impact for the unconstrained economy does not depend significantly on the level of initial borrowing, as shown by the min and max lines for each variable. The min and max lines are relatively flat and equidistant from the mean.

\^17\,again, taken along $K$ dimension for a fixed $b_t$ position
line\textsuperscript{18}, approximately $\pm 1.3\%$ for consumption and labor, and $\pm 1.9\%$ for output. The variability of the impact observed for the real variables of interest is driven by the initial (time-$t$) position on the capital grid. Upper left graphs of Figures F.3 through F.5 show the impact of the switch for the unconstrained economy as a function of time-$t$ capital. As the graphs indicate, consumption, employment, and output are depressed more if the initial stock of capital is higher. There are two different intertemporal adjustment mechanisms to consider when explaining the observed relationship between the current stock of capital and the relative magnitude of the impact of the adverse shock to the economy. First, the shock to productivity affects the marginal reward to labor. The marginal product of labor, $F'_L(K, L) = (1 - \alpha)eAK^\alpha L^{-\alpha}$, increases with the level of capital; therefore, for the same productivity shock, the marginal reward to labor changes more the higher is the capital. Hence, the higher is the current stock of capital, the more significant is the fall in demand for labor, for a given adverse shock to productivity. The drop in demand affects real wage adversely, and labor supply falls as well.

The average magnitude of the drop (middle solid line on the upper left graphs) for all three variables depends roughly linearly on the capital stock.

The effect of the switch on the constrained economy relative to the unconstrained case depends on the value of $b_t$. For low initial foreign debt ($b_t$ above 21\textsuperscript{st} coordinate), the impact is identical for both economies. In this region, the liquidity constraint does not bind; therefore, the two economies are identical in their dynamics. For the region with higher time-$t$ foreign debt (at or below 23\textsuperscript{rd} coordinate for $b$), the constrained economy reveals three distinctive features. First, the average drop in real variables is more dramatic for the constrained economy. Second, the higher is the initial foreign debt (the lower is $b_t$), the more significant is the drop, on average. Third, with higher

\textsuperscript{18}except for the border cases (minimum or maximum value of $b_t$)
initial debt, the variability of the impact on real variables increases. The variability of the impact is estimated as a vertical distance between min and max impact lines as functions of $b_t$ (lower right graphs for each of Figures F.3 through F.5. In the extreme case (high foreign debt, the left part of $b$ schedule), the magnitude of the drop in real variables reaches double digits, with an almost 19% drop in the employment, a 16% drop in output, and a 20% drop in consumption.

The lower left graph of Figure F.4 shows that there is a region in the $(K_t, b_t)$ space where the drop in the labor supply is lower in the constrained economy, relative to the unconstrained case. For this region, the positive effect on labor supply (caused by an increase in the effective marginal return to labor when the constraint binds) dominates the negative effect.

In addition to the impact-switching experiment from the best to the worst state of nature, in which both shocks change their values, the following experiments were conducted:

a. while keeping the shock to the interest rate at the same value, change the shock to technology from positive at time $t$ to negative at time $t + 1$;

b. while keeping the same technological shock, change the interest rate shock from negative at time $t$ to positive at time $t + 1$.

Note: since the stochastic structure of the model assumes a two-point symmetric Markov chain, the only feasible states for each shock are positive and negative; there is no neutral, or zero, shock.

For the case where productivity only was shocked (decrease), with the interest rate remaining the same (low), the results are almost identical to the baseline experiment. The results are presented graphically on Figures F.6 through F.9, for CA/Y ratio, consumption, labor, and output, correspondingly. In the next experiment, where the interest rate experienced an adverse shock, and productivity remained high, the results
were different. The results of the switch are depicted on Figures F.10 through F.13.

There is no current account reversal (even though the drop in CA/Y ratio in the constrained case is smaller than in the unconstrained economy around the Sudden Stop region). Moreover, the major variables can experience smaller drop (and sometimes even an increase), around the Sudden Stop region. The results of the experiments suggest that the major economic impact of a Sudden Stop in the model is due to the shocks in productivity rather than the interest rate disturbances.

As simulations suggest, Sudden Stops are relatively infrequent in the long run. The only relevant region in \((K, b)\) space is the set defined above where the liquidity constraint switches from always non-binding to binding in some states of nature (coordinates 19 through 21 on grid, depending on the value of \(K\)). Even if the economy finds itself on this set at some point during long-run stochastic simulations, it will move away from it next period due to the precautionary savings motive of economic agents. Although there is zero probability that the constrained economy will, in the long run, be in a state to the left of the 21st point on the \(b\) scale as seen from the lower portion of Figure 3.2, the economy has a non-zero probability of having larger foreign debt during the transition period, depending on the initial conditions. Let us assume the following time-0 distribution: the economy has a maximum amount of foreign assets allowed by the grid, \(b = -1.45\), with equal probability of realization of any initial shock and capital stock. That is, the initial probability distribution \(P_0(i, j, k)\) is

- \(\frac{1}{Knbm}\) if \(j = 1\)
- 0 otherwise,

where \(i \in 1 \ldots K_n, j \in 1 \ldots b_n, k \in 1 \ldots m\), and the state space is \((K_n \times b_n \times m) = (42 \times 42 \times 4)\). Given this time-0 distribution and the optimal decision rule, we can obtain transitional probability distributions as the economy moves forward. The result,
Figure 3.5: Transition Marginal Distributions of Foreign Bond Holdings for the Constrained (Top) and Unconstrained (Bottom) Economy

Notes: In both cases, the economy starts with the maximum amount of foreign assets allowed (-1.45), with equal probability of realization of any initial shock and the capital stock. Given this time-0 distribution and the optimal decision rule, we can obtain transitional probability distributions as the economy moves forward.
summarized as a distribution of foreign assets, is presented in Figure 3.5 (top). As can be inferred from the graph, the economy adjusts over time towards its long-run limiting distribution (the dotted line on the graph). After the 1st quarter (the solid line on the graph) the economy is in the Sudden Stop region, and, depending on the realization of the shocks, this can lead to a collapse of real economic variables, as was discussed above.

The economy moves away from this region rather quickly: after 4 quarters, there is a zero probability of being in the Sudden Stop region. The speed of adjustment, however, is highly dependent on the coefficient of relative risk aversion, $\gamma$, as discussed in Arellano and Mendoza (2002). By increasing the risk aversion coefficient, we can obtain a slower transition, with the economy staying in the Sudden Stop region for several quarters.

For comparison, a similar experiment was conducted for the unconstrained economy, with the results for the evolution of the foreign assets probability distribution shown on the bottom part of Figure 3.5. As is evident from the graph, in the absence of the liquidity constraint, the economy adjusts to its long-run distribution (dotted line) rather slowly.

This analysis suggests that, even though in the long run Sudden Stops are almost ruled out due to economic agents’ expectations, the economy can still experience dramatic effects of Sudden Stops in the short run due to unanticipated shocks.

### 3.4 Sensitivity Analysis

In this section, we analyze the long-run adjustment of the model to changes in two of the driving forces: (1) interest rate variability and (2) tightness of the liquidity constraint. In addition, the model’s robustness to a change in the correlation between the productivity and the interest rate shocks is analyzed. Figures F.14 through F.19 summarize the results of the following experiments on population moments of the unconstrained and constrained economies:
1. value of a shock to the (net) world interest rate was varied from 0 to $\exp 0.2$ with a step of 0.005, which constitutes 40 experiments. For a constrained economy, the liquidity constraint was at a baseline level: $\varphi = 0.715$.

2. value of the liquidity constraint, $\varphi$, was changed from 0.5 to 0.999, with 40 equally-spaced values total. The baseline value for interest rate shock was used for each simulation.

For each experiment, the economies are simulated for 1,000,000 periods.

Figure F.14 shows the population means of major macroeconomic variables as a function of the world interest rate shock. The figure clearly demonstrates a precautionary savings behavior of economic agents in response to increased uncertainty. Foreign asset holdings in the long run are decreased due to higher variability of R. The precautionary savings may be explained by uncertainty introduced by stochastic shocks to the interest rate, combined with credit-market incompleteness due to an absence of Arrow securities. Even if no other credit constraints are present in the model, the result of this setup is equivalent to imposing an endogenous credit constraint on the agents. The agents have access only to non-contingent foreign bonds; thus, they are not fully insured against idiosyncratic shocks, which causes fluctuations in their wealth. Since CRRA form of instantaneous utility is assumed, consumption varies with changes in wealth, and agents engage in precautionary savings in order to reduce consumption fluctuations.

The world interest rate, R, is a price not only for borrowing, but also for investing in physical capital. Increased variability of R, together with capital adjustment costs present in the model, are two factors responsible for the decline in investment in response to higher $\sigma^R$. Investment decline leads to a long-term decrease in the stock of physical capital. Lower capital stock reduces the marginal productivity of labor;
therefore, demand for labor falls. As a result, the real wage goes down, and employment drops in the long-run. GDP experiences a decline, since both production inputs drop. In turn, the decrease in income has a negative effect on the consumption level. On the other hand, the trade balance deteriorates due to a long-term decrease in the level of foreign asset holdings. Since both investment and the trade balance decrease, the consumption share of GDP goes up. As the lower top panel of the Figure F.14 demonstrates, the positive effect on consumption dominates the negative effect. The magnitude of the effect is not very significant, however. If we compare the population mean values of employment, investment, capital, output, and consumption for the smallest (0) and the largest (\(\exp 0.2 \approx 122\%\)) values of \(\sigma^R\), the difference is less than 0.5%.

Another important observation is that the increased variability of the world interest rate reduces the difference in population means for the two economies. Even for foreign asset holdings, the only variable that had a relatively large difference in population means between the economies, the difference becomes negligible for high variability of \(R\). For the constrained economy, an extra unit of labor increases the borrowing capacity of the household; therefore, the effective marginal reward to labor supply goes up when the constraint binds (relative to the unconstrained case). However, with the increased uncertainty regarding the debt repayment amount, this effect becomes less important. Therefore, the difference between the two economies in terms of population means deteriorates.

Figure F.15 demonstrates that the increased variance of world interest rate makes both economies more variable.

Population means as functions of a liquidity constraint parameter, \(\varphi\) are presented in Figure F.17. As can be seen from the graphs, there is a threshold value of \(\varphi\), below which the liquidity constraint is not binding in any state of nature, and the economy
becomes unconstrained. Below a value of $\varphi = 0.691$, the mean for each variable is a horizontal line. When $\varphi$ is above this value, the constraint becomes binding, and at the value of $\varphi = 1$ the economy becomes closed, as no borrowing from abroad is allowed. The mean values of foreign asset positions and net exports move towards zero as the liquidity constraint is tightened. We observe small decreases in labor, capital, output, investment, and savings. The economy was calibrated as a net borrower for the unconstrained case; therefore, as the liquidity constraint becomes tighter, the amount of allowed borrowing from the external sources decreases, which, in turn, reduces debt repayment each period. As a result, less output needs to be used for debt repayment, and consumption increases as a share of output.

To analyze the model’s robustness to the correlation between shocks to productivity and the world interest rate, the model (for both constrained and unconstrained economies) was simulated with the value of $\rho_{eA,eR}$ ranging from -0.9 to 0.9. The results of the experiment are presented in Table 3.5. The results indicate that the value of $\rho_{eA,eR}$ does not have a significant impact on the model. As the correlation between the two shocks is increased from -0.9 to 0.9, the standard deviation and first-order autocorrelation of output decreases slightly; investment becomes less variable as well. This observation holds for both economies. This result is consistent with the experiments on the impact switching in Section 3.3: since the major impact of a Sudden Stop is due to the productivity shocks rather then the interest rate shocks, then the correlation between the two shocks should have a minor effect on the model’s results.
Table 3.5: Changes in the Correlation of Productivity and Interest-Rate Shocks, One-Sector Model

<table>
<thead>
<tr>
<th>$\rho_{eA,eR}$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_I$</th>
<th>$\rho_Y$</th>
<th>$\rho_{eA,eR}$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_I$</th>
<th>$\rho_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>3.966</td>
<td>12.002</td>
<td>0.528</td>
<td>0.9</td>
<td>4.029</td>
<td>11.892</td>
<td>0.548</td>
</tr>
<tr>
<td>0.6</td>
<td>3.977</td>
<td>12.210</td>
<td>0.530</td>
<td>0.6</td>
<td>4.033</td>
<td>12.116</td>
<td>0.553</td>
</tr>
<tr>
<td>0.3</td>
<td>3.985</td>
<td>12.419</td>
<td>0.532</td>
<td>0.3</td>
<td>4.047</td>
<td>12.309</td>
<td>0.553</td>
</tr>
<tr>
<td>0.1</td>
<td>3.992</td>
<td>12.573</td>
<td>0.535</td>
<td>0.1</td>
<td>4.052</td>
<td>12.414</td>
<td>0.555</td>
</tr>
<tr>
<td>0</td>
<td>3.995</td>
<td>12.614</td>
<td>0.537</td>
<td>0</td>
<td>4.057</td>
<td>12.464</td>
<td>0.557</td>
</tr>
<tr>
<td>-0.1</td>
<td>3.999</td>
<td>12.698</td>
<td>0.538</td>
<td>-0.1</td>
<td>4.058</td>
<td>12.509</td>
<td>0.557</td>
</tr>
<tr>
<td>-0.3</td>
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<td>4.066</td>
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</tr>
<tr>
<td>-0.6</td>
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<td>13.057</td>
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<td>-0.6</td>
<td>4.075</td>
<td>12.765</td>
<td>0.562</td>
</tr>
<tr>
<td>-0.9</td>
<td>4.021</td>
<td>13.249</td>
<td>0.545</td>
<td>-0.9</td>
<td>4.085</td>
<td>12.948</td>
<td>0.566</td>
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</tbody>
</table>
Chapter 4

Two-Sector Model

In this chapter, a two-sector model with production in both tradable and non-tradable sectors is considered. The model features a liquidity constraint similar to the one used in the one-sector model. Section 4.1 describes the structure of the two-sector two-shock model (productivity shocks in both sectors are perfectly correlated); the solution approach and the parametrization are covered in section 4.2; section 4.3 analyzes the model’s results, which are available for both long and short run; section 4.4 provides an analysis of the model’s robustness and its sensitivity to parameter values. Section 4.5 is an extension of the two-sector model, in which productivity shocks for tradable and non-tradable sector are differentiated. The section describes the specifics of the solution approach for this two-sector three-shock model, and presents the results that are particular to the model.

4.1 Model Structure

In this chapter, the one-sector model is extended to account for a non-tradables sector. The model is a modified version of Mendoza (2000). In the model to be described, (1) both tradable and non-tradable goods are produced; (2) the model features flexible capital with depreciation and adjustment costs in the production of tradables; and (3)
there are only two shocks driving the economy’s business cycles – a shock to productivity, and a shock to the interest rate. In the original model, there is a policy shock as well.

Infinitely-lived representative households choose optimal intertemporal allocations of consumption of tradable and non-tradable goods, $C^T_t$ and $C^N_t$, and labor $L_t$, in order to maximize life-time utility:

$$U = E_0 \left[ \sum_{t=0}^{\infty} u \left( C(C^T_t, C^N_t), L_t \right) \cdot \exp \left\{ - \sum_{\tau=0}^{t-1} \nu \left( C(C^T_{\tau}, C^N_{\tau}), L_{\tau} \right) \right\} \right]$$  (4.1)

The one-period utility function is given by

$$u \left( C(C^T_t, C^N_t), L_t \right) = \frac{C(C^T_t, C^N_t) - \frac{L^\omega_t}{\omega}}{1 - \gamma} - 1$$  (4.2)

and the time preference function is

$$\nu \left( C(C^T_t, C^N_t), L_t \right) = \beta \ln \left[ 1 + C(C^T_t, C^N_t) - \frac{L^\omega_t}{\omega} \right]$$  (4.3)

The consumption aggregator is of the CES functional form:

$$C(C^T_t, C^N_t) = \left[ \sigma \left( C^T_t \right)^{-\eta} + (1 - \sigma) \left( C^N_t \right)^{-\eta} \right]^{-\frac{1}{\eta}}$$  (4.4)

and total labor is allocated between traded and non-traded sectors:

$$L_t = L^T_t + L^N$$  (4.5)

Firms produce tradables and nontradables using capital and labor. Labor is assumed to be inelastically supplied to the traded sector, and capital is inelastically supplied to the non-traded sector. This assumption is adopted from Mendoza (1995): it is an extreme representation of empirical evidence that the K/L ratio has larger variance in
the traded sector than in the non-traded sector. Tradables are produced according to:

\[ Y_t^T = x_t A \left( K_t^T \right)^{\alpha_T} \left( L_t^T \right)^{1-\alpha_T} \]  

(4.6)

with the capital accumulation equation:

\[ K_{t+1}^T = \left( 1 - \delta \right) K_t^T + I_t - \frac{\phi}{2} \left( K_{t+1}^T - K_t^T \right)^2 \]  

(4.7)

Here, \( x_t \) is the shock to productivity, and \( \delta, I_t, \) and \( \frac{\phi}{2} \left( K_{t+1}^T - K_t^T \right)^2 \) are constant depreciation rate (same for tradable and non-tradable sectors), gross investment, and capital adjustment costs, correspondingly. Non-tradables are produced using time-invariant capital and variable labor:

\[ Y_t^N = e_t A \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{1-\alpha_N} \]  

(4.8)

where \( e_t \) is the productivity shock.

Firms’ demand for labor is given by:

\[ p_t (1 - \alpha_N) e_t A \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{1-\alpha_N} = w_t \]  

(4.9)

where \( w_t \) is the real wage in the non-tradables sector, and \( p_t \) is the price of non-tradables. Both are in units of tradables (which is the model’s numeraire).

Agents trade a non-contingent, one-period bond \( b_t \) that pays the gross real interest rate in units of tradables. The asset accumulation equation is:

\[ b_{t+1} = TB_t + n_t Rb_t \]  

(4.10)

\( TB_t \) is the trade balance measured in units of tradables, and \( n_t \) is a shock to the mean value of the gross world real interest rate \( R \). The household budget constraint for the
tradables sector is:

\[
(1 + \tau)C_t^T + \tau p_t C_t^N = x_t A \left( K_t^T \right)^{\alpha_T} \left( L_t^T \right)^{1-\alpha_T} + (1 - \delta) K_t^T - \frac{\phi}{2} \left( K_{t+1}^T - K_t^T \right)^2 - K_{t+1}^T + n_t R b_t - b_{t+1} - T^T
\]

and nontradables sector:

\[
C_t^N = e_t A \left( K_t^N \right)^{\alpha_N} \left( L_t^N \right)^{1-\alpha_N} - T^N
\]

where \( \tau \) is consumption tax rate (the same for both sectors), and \( T^T \) and \( T^N \) are lump-sum taxes in the tradable and non-tradable sectors, in corresponding units. The government budget constraint is:

\[
G_t^T + p_t G_t^N = \tau C_t^T + \tau p_t C_t^N + T^T + p_t T^N \text{ with } G_t^N = T^N
\]

In addition to the budget constraint, households face a liquidity constraint, requiring that a fixed fraction of their current expenses (on consumption, investment, and debt repayment) be financed out of their current income:

\[
Y_t^T + p_t Y_t^N \geq \varphi \left[ (1 + \tau) \left( C_t^T + p_t C_t^N \right) + I_t^T + p_t I_t^N - n_t R b_t + T^T + p_t T^N \right]
\]

which can be rewritten as:

\[
Y_t^T + p_t Y_t^N \geq \varphi \left[ Y_t^T + p_t Y_t^N - b_{t+1} \right],
\]
or:

\[ b_{t+1} \geq -\frac{1 - \phi}{\phi} \left( Y_t^T + p_t Y_t^N \right) \]  \hspace{1cm} (4.15)

The relative price of aggregate consumption in units of tradables is given by \( p^C \). To obtain \( p^C \), we solve a problem of minimizing expenditure for a given level of one-period sub-utility (CES aggregator \( C \left( C_t^T, C_t^N \right) \) in our case; time subscripts are omitted):

\[
\min_{C_t^T, C_t^N} Z = C^T + p C^N \text{ s.t. } C(C_t^T, C_t^N) = C_0
\]

According to Frenkel and Razin (1987), the optimal solution for the expenditure is of the following form:

\[ Z = p^C(p) \cdot C_0 \]

In our case, the solution implies:

\[ p^C = \left[ \sigma^{\frac{1}{1+\eta}} + (1 - \sigma) \frac{1}{1+\eta} p^{\frac{\sigma}{1+\eta}} \right]^{\frac{1+\eta}{\eta}} \]  \hspace{1cm} (4.16)

### 4.2 Numerical solution

The competitive equilibrium can be represented as the solution to a social planner problem. The corresponding first-order conditions are:

\[
U_C(t) \left( 1 - \frac{\mu_t}{\lambda_t} \right) = \exp(-\nu(t)) \cdot R \cdot E_t \left[ \frac{p_t^C}{p_{t+1}} U_C(t + 1) \right] \]  \hspace{1cm} (4.17)

\[
\frac{C_{CN}(t)}{C_{CT}(t)} = p_t \]  \hspace{1cm} (4.18)
\[(L_t^N + L^T)^{\omega^{-1}} = \frac{w_t}{p_t^T(1 + \tau)} \left[ 1 + \frac{\mu_t}{\lambda_t} \frac{1 - \varphi}{\varphi} \right] \] (4.19)

plus complementary slackness conditions following the standard Kuhn-Tucker approach. First-order conditions are interpreted as follows. (4.17) is the Euler equation, where \(\lambda_t\) and \(\mu_t\) are multipliers for the aggregate budget constraint and liquidity constraint, correspondingly. (4.18) equates MRS between consumption of tradables and nontradables with the relative price of nontradables. (4.19) is equilibrium in the labor market.

Assume that \(x_t = e_t\) (shocks in both sectors are perfectly correlated). The state variables are: \(K^T = K_{t}^T, b = b_t,\) and \(\xi = \{e_t, n_t\}\). The dynamic programming approach is as follows: for every point \(K_t, b,\) and \(\xi,\) choose \(K_{t+1}^T = K_{t+1}^T\) and \(b' = b_{t+1}\) that solve the following Bellman equation:

\[
V(K^T, b, \xi) = \max_{K^T', b'} \left\{ u \left( C(C^T*, C^N*), L^{N*} \right) + \exp \left( -\nu \left( C(C^T*, C^N*), L^{N*} \right) \right) \cdot E \left[ V(K^T', b', \xi') \right] \}
\] (4.20)

First, we solve for \(C^T*, C^N*, L^{N*},\) and \(p^*\) as functions of \(K^T, K^T', b, b',\) and \(\xi:\)

\[
(1 + \tau)C^T* + \tau p^* C^N* = 
xA \left( K^T \right)^{\alpha_T} \left( L^T \right)^{1-\alpha_T} + (1 - \delta)K^T - \frac{\phi}{2}(K^T' - K^T)^2 - K^T' + nRb - b' - T^T
\] (4.21)

\[
C^N* = eA \left( K^N \right)^{\alpha_N} \left( L^{N*} \right)^{1-\alpha_N} - T^N - \delta K^N
\] (4.22)
\[ b' \geq - \frac{1 - \varphi}{\varphi} \left( xA(K^T)^{\alpha_T} (L^T)^{1-\alpha_T} + peA(K^N)^{\alpha_N} (L^N)^{1-\alpha_N} \right) \] (4.23)

If (4.23) is not binding, then the system is completely described by (4.9), (4.18), (4.19), (4.21), and (4.22). If (4.23) binds, then the system is described by (4.18), (4.21), and (4.22), and (4.23) (written as equality). Then, Bellman equation (4.20) iteration is performed. Value function iteration can account for the possibility of non-linearity due to occasionally binding constraints.

The shock structure is a two-point symmetric Markov chain with simple persistence; for a detailed definition, please see Section 3.1. Table 4.1 gives the parameter values used for the simulations. Deterministic steady-state values for major macroeconomic variables are presented in Table 4.2. For the details of finding the deterministic steady state, please refer to Appendix B.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Credit market</th>
<th>Preferences</th>
<th>Fiscal Policy</th>
<th>Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_T = 0.355 )</td>
<td>( R = 1.04 )</td>
<td>( \beta = 0.094 )</td>
<td>( \tau = 0.10 )</td>
<td>( \ln e = {-0.0118, 0.0118} )</td>
</tr>
<tr>
<td>( \alpha_N = 0.277 )</td>
<td>( \varphi = 0.880 )</td>
<td>( \omega = 2.9 )</td>
<td>( T^T = 0.055 )</td>
<td>( \ln n = {-0.0005, 0.0005} )</td>
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<tr>
<td>( A = 1 )</td>
<td>( \gamma = 1.1 )</td>
<td>( T^N = 0.056 )</td>
<td>( \rho_e = \rho_n = \theta = 0.36 )</td>
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</tr>
<tr>
<td>( \delta = 0.1 )</td>
<td>( \sigma = 0.5 )</td>
<td>( \rho_{e,n} = 4\Pi_1 - 1 = -0.291 )</td>
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<td></td>
</tr>
<tr>
<td>( \phi = 0.028 )</td>
<td>( \eta = 0.316 )</td>
<td></td>
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<td></td>
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Table 4.1: Parameter Values Used for Simulations (Two-Sector Model)

<table>
<thead>
<tr>
<th>Technology</th>
<th>Credit market</th>
<th>Preferences</th>
<th>Fiscal Policy</th>
<th>Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^T = 2.703 )</td>
<td>( L^T = 0.639 )</td>
<td>( C^T = 0.610 )</td>
<td>( Y^T = 1.066 )</td>
<td>( b = -0.237 )</td>
</tr>
<tr>
<td>( K^N = 1.642 )</td>
<td>( L^N = 0.639 )</td>
<td>( C^N = 0.610 )</td>
<td>( Y^N = 0.830 )</td>
<td>( NX = 0.0095 )</td>
</tr>
<tr>
<td>( I = 0.270 )</td>
<td>( S = 0.280 )</td>
<td>( C = 1.219 )</td>
<td>( Y = 1.896 )</td>
<td>( p = 1 )</td>
</tr>
</tbody>
</table>

Table 4.2: Deterministic Steady State for the Two-Sector Economy

As in the one-sector model, the two-sector model is calibrated to the Indonesian economy. Some of the parameter values are identical (or close) to one-sector model. There are several parameters specific to the two-sector model, and some parameters require a different set of conditions to determine their values. Capital shares of output
in the tradable \((\alpha_T = 0.355)\) and non-tradable \((\alpha_N = 0.277)\) sectors are determined according to the same procedure as in the one-sector case, using the method described in Sarel (1997); in this case, calculations are done separately for the two sectors (the list of economic activities defined as tradable or non-tradable is specified in Section 3.2). The consumption tax rate \(\tau\) is 10\%. The parameter \(\eta\) determines the elasticity of substitution between \(C^T\) and \(C^N\), which is expressed as \(\frac{1}{1+\eta}\). The value of \(\eta\) for developing countries as estimated by Ostry and Reinhart (1992) is 0.316. Given the other parameter values, the value of \(\beta\) is determined by solving the set of steady-state equilibrium conditions listed in Appendix B. The values of lump-sum taxes in tradable \((T^T)\) and non-tradable \((T^N)\) sectors are determined from the market-clearing conditions for each sector (equations B.4 and B.5, correspondingly).

### 4.3 Results and Discussion

In this two-sector model, the state variables are the current level of capital stock in the tradable sector, \(K_T^T\), and the current amount of foreign assets, \(b_t\). Joint limiting distributions of the state variables are presented in Figure F.20 for unconstrained (top) and constrained (bottom) economies. The corresponding marginal distributions are shown in Figures F.21 (for foreign bonds) and F.22 (for capital in tradable sector). Similarly to the one-sector economy, the liquidity constraint limits the maximum level of the foreign asset position (and that maximum depends on the level of capital in tradable sector). It is interesting to note that due to optimal borrowing decisions of the economic agents in the liquidity-constrained economy, the constraint imposed to

---

2. The same value is used in Mendoza (2001).
3. Current shock realizations are two more state variables.
limit the debt-to-income ratio effectively limits the level of borrowing in the long run. In the short run, however, the economy can find itself with levels of borrowing higher that in the limiting distributions. This can trigger the dynamics of a Sudden Stop, and the speed with which the economy moves out of the Sudden Stop region depends on the values of the shocks over time and on the agents’ preferences towards risk.\footnote{expressed in the model by the coefficient of a relative risk aversion, $\gamma$.}

To analyze the short-term reaction of the economy to the shocks, and how this reaction is modified when the liquidity constraint is added, an experiment on impact switching similar to the one conducted for the one-sector model was carried out.\footnote{For the detailed description of the experiment, please refer to Section 3.3.} The results of the transition from the best state of nature (high productivity and low interest rate) at time $t$ to the worst state (low productivity and high rate of interest) at time $t + 1$ are summarized in Figures F.24 through F.31. The impact of the switch on the current account-output ratio as a function of the time-$t$ foreign asset position is depicted in Figure F.24. The unconstrained economy produces a drop in the CA/GDP ratio of around 50\% (ignoring border cases), as a result of the adverse shock. The constrained economy behaves identically to the unconstrained case for high values of $b_t$ (low borrowing, right portion of the graph), where the constraint is not binding. Near the region where the constraint is engaged, however, the drop in the CA/GDP ratio is smaller, with the smallest value (the peak for the dotted line on the graph) a 0\% drop. This result is qualitatively similar to that of the one-sector model. Note that the liquidity-constrained two-sector economy does not produce the positive change in the CA/GDP ratio near Sudden Stop region. This result is due to the fact that for the two-sector economy, the mean ratio of foreign interest to GDP is set to just 0.5\% (it is 2.2\% in the one-sector model). It is possible to reproduce a current account reversal similar to the one-sector model. For this, the calibration should assume a higher ratio
of foreign interest payments to GDP, which is a frequent occurrence in an emerging economy.

<table>
<thead>
<tr>
<th>Table 4.3: Population Moments for Two-Sector Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td><strong>Economy with Perfect Credit Markets</strong></td>
</tr>
<tr>
<td>Consumption of tradables</td>
</tr>
<tr>
<td>Consumption of nontradables</td>
</tr>
<tr>
<td>Capital in tradable sector</td>
</tr>
<tr>
<td>Labor in nontradable sector</td>
</tr>
<tr>
<td>Tradables GDP</td>
</tr>
<tr>
<td>Nontradables GDP</td>
</tr>
<tr>
<td>Net foreign assets</td>
</tr>
<tr>
<td>Net exports</td>
</tr>
<tr>
<td>Price of nontradables</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>Savings</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>GDP</td>
</tr>
</tbody>
</table>

| **Economy with Liquidity Constraint**        |      |
| Consumption of tradables                     | 0.611 | 0.769 | 0.767 | 0.959 |
| Consumption of nontradables                  | 0.610 | 1.805 | 0.377 | 0.882 |
| Capital in tradable sector                   | 2.702 | 0.535 | 0.694 | 0.594 |
| Labor in nontradable sector                  | 0.639 | 0.213 | 0.677 | 0.986 |
| Tradables GDP                                | 1.066 | 1.267 | 0.464 | 0.892 |
| Nontradables GDP                             | 0.830 | 1.326 | 0.377 | 0.882 |
| Net foreign assets                           | -0.181 | -30.655 | 0.988 | 0.498 |
| Net exports                                  | 0.0072 | 0.0091 | 0.342 | 0.186 |
| Price of nontradables                        | 1.0033 | 1.754 | 0.347 | -0.640 |
| Investment                                   | 0.270 | 4.007 | -0.040 | 0.472 |
| Savings                                      | 0.277 | 3.718 | 0.464 | 0.660 |
| Consumption                                  | 1.223 | 0.704 | 0.898 | 0.857 |
| GDP                                          | 1.899 | 0.655 | 0.681 | 1.000 |

*Notes: Standard deviations are percentages of the corresponding means (except for net exports).*

In addition to the transmission channels driving business cycles in the one-sector model, the two-sector model has a price mechanism that determines the allocation of consumption between tradable and non-tradable goods in the same period. When
the adverse shock hits the *unconstrained* economy, production is decreased in both sectors. The households can smooth consumption of tradable goods by importing tradable goods from abroad to compensate for the drop in the supply of tradables. The decrease in the supply of non-tradable goods cannot be offset by external sources. Therefore, the relative supply of non-tradable goods drops as a result of the adverse shock to productivity and the interest rate. This leads to an increase in the relative price of non-tradables as seen on the lower left graph of Figure F.29. With the higher non-tradables relative price, the households contemporaneously substitute away from non-tradables consumption towards consumption of tradables. This can explain the fact that non-tradables consumption falls more (the average drop on the lower left graph of Figure F.27 is 3.9%) than the consumption of tradables (0.7% drop on average, as seen in Figure F.26). It can also explain why non-tradables consumption falls more than the production of non-tradables (2.45% average drop from Figure F.31), whereas the impact of the switch on production in both sectors is comparable (production in tradable sector falls by 2.25% on average, according to the Figure F.30). Note that consumption smoothing in the tradable sector is not complete, since the increase in imports needs to be financed by borrowing from abroad, and the price of borrowing is increased due to the adverse shock to the interest rate.

When the households face a borrowing constraint, they may not be able to borrow a sufficient amount in order to smooth their consumption of tradables. Therefore, if the constraint binds, the consumption of tradables falls more, relative to the unconstrained economy. This is clearly illustrated by two bottom graphs of Figure F.26). Near the region where the constraint becomes binding, the average drop in the consumption of tradables is deeper for the constrained vs. the unconstrained economy (0.9% vs. 0.7%). The difference between the two economies is even more pronounced for the total magnitude of the impact. The deepest drop in $C^T$ for the unconstrained economy
is 1.0%, and this value is the same for any position of $b_t$. For the constrained case, the drop in $C^T$ reaches 3.0%.

For the constrained economy, the relative price of non-tradable goods, $p$, increases less than in the unconstrained case (Figure F.29). Still, $p$ goes up for any combination of $(K^T_t, b_t)$. This is in line with the results reported by Mendoza (2001) for the case without policy shocks.6

As in the one-sector model, the following experiments were conducted in addition to the baseline impact-switching experiment:

a. while keeping the shock to the interest rate at the same value, change the shock to technology from positive at time $t$ to negative at time $t + 1$;

b. while keeping the same technological shock, change the interest rate shock from negative at time $t$ to positive at time $t + 1$.

Similarly to the one-sector case, if the productivity only is impacted (a drop), with the interest rate remaining the same (low), the results are almost identical to the baseline experiment. In the next experiment, the interest rate experienced an adverse shock, and the productivity remained high. The results of the switch are depicted in Figures F.32 through F.39. The effect on the current account is very similar to the previous experiment and the baseline scenario; however, the impact on all major variables is different than in the baseline scenario. For all variables in the unconstrained economy, the average drop (on the asset schedule, lower left portions of the graphs) is close to zero, with the maximum and the minimum impact lines being equidistant from the average impact line. This result is consistent with the one-sector case.

Adding a non-tradable sector with production allows an analytical exploration of several important aspects. First, the non-tradable sector is an important part of many

---

6To obtain a drop in the relative price of non-tradables, Mendoza (2001), in addition to a positive interest rate shock and a negative productivity shock, utilizes a shock to the consumption tax rate, $\tau$, where the tax rate increases from 2.1% to 11.8%.
economies\textsuperscript{7}. Second, it allows a comparative analysis of the SS impact on tradable and non-tradable sectors, in terms of output, sectoral demand for goods, and employment. Third, it allows an analysis of the price of non-tradable goods\textsuperscript{8}.

4.4 Sensitivity Analysis

Simulation experiments analogous to those performed with the one-sector model have been conducted, with the results summarized in Figures F.40 through F.45 and Table 4.4.

The experiments on changing interest rate variability indicate that the two-sector economy is almost neutral to the size of interest-rate disturbances. As mentioned by Mendoza (1991), this may be the result of a low average interest rate and a small ratio of foreign interest payments to GDP (0.5\% in the two-sector model). In this case, the wealth and intertemporal consumption-substitution effects induced by the shocks can be small.

For the second group of experiments, the value of $\varphi$ was changed from 0.6 to 0.999. Certain results are robust between one- and two-sector economies. Among these results is a decrease (in absolute terms) of foreign asset holdings and a decrease in net exports. Both variables approach zero as the economy approaches a closed state ($\varphi \rightarrow 1$). Similarly to the one-sector case, there is a decrease in the investment, capital in tradable sector, output of tradables, and an increase in the tradables consumption. One of the

\textsuperscript{7}Based on the Central Bank of Indonesia data (quarterly, 2000:1 – 2007:3), non-tradable sector accounts for 46.9\% of GDP; according to estimates of Stockman and Tesar (1995), non-tradable sector accounts for about 50\% of GDP for developed countries.

\textsuperscript{8}the question of liability dollarization and a collapse of the price of non-tradables in terms of tradables is an important theme of several papers on Sudden Stops; see, for instance, Chue and Cook (2007).
results particular to the two-sector economy is an increase in the relative price of non-tradable goods. As the economy becomes more closed, the productive resources are reallocated towards the non-tradable sector, and the production point on the economy’s production possibilities frontier (PPF) moves accordingly. The relative price of non-tradable goods equals the marginal rate of transformation between the production of the two goods; therefore, $p$ is higher for a relatively more closed economy. Labor is reallocated accordingly, from the tradable sector to the non-tradable sector. The combined effect of a decrease in $Y^T$ and an increase in both $Y^N$ and the relative price of non-tradables is an increase in GDP as ($\varphi \to 1$). This result is the major difference for the experiment on changing $\varphi$ between one-sector and two-sector economies. Even though tradables output falls in both models, this result shows that including non-tradable goods in the model can change its predictions.

The results of an experiment on model robustness to changes in the correlation between productivity and world interest rate shocks are summarized in Table 4.4 for both the unconstrained and the constrained economies. As can be inferred from the tables, the results are only marginally affected by the changes in $\rho_{eA,eR}$.

Table 4.4: Changes in the Correlation of Productivity and Interest-Rate Shocks, Two-Sector Model

<table>
<thead>
<tr>
<th>$\rho_{eA,eR}$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_I$</th>
<th>$\rho_Y$</th>
<th>$\rho_{eA,eR}$</th>
<th>$\sigma_Y$</th>
<th>$\sigma_I$</th>
<th>$\rho_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.612</td>
<td>2.827</td>
<td>0.671</td>
<td>0.9</td>
<td>0.626</td>
<td>2.938</td>
<td>0.668</td>
</tr>
<tr>
<td>0.6</td>
<td>0.618</td>
<td>3.138</td>
<td>0.673</td>
<td>0.6</td>
<td>0.636</td>
<td>3.227</td>
<td>0.673</td>
</tr>
<tr>
<td>0.3</td>
<td>0.626</td>
<td>3.427</td>
<td>0.677</td>
<td>0.3</td>
<td>0.641</td>
<td>3.491</td>
<td>0.674</td>
</tr>
<tr>
<td>0.1</td>
<td>0.631</td>
<td>3.619</td>
<td>0.680</td>
<td>0.1</td>
<td>0.645</td>
<td>3.671</td>
<td>0.677</td>
</tr>
<tr>
<td>0</td>
<td>0.631</td>
<td>3.705</td>
<td>0.678</td>
<td>0</td>
<td>0.648</td>
<td>3.754</td>
<td>0.679</td>
</tr>
<tr>
<td>-0.1</td>
<td>0.637</td>
<td>3.800</td>
<td>0.682</td>
<td>-0.1</td>
<td>0.651</td>
<td>3.841</td>
<td>0.681</td>
</tr>
<tr>
<td>-0.3</td>
<td>0.641</td>
<td>3.982</td>
<td>0.683</td>
<td>-0.3</td>
<td>0.655</td>
<td>4.014</td>
<td>0.682</td>
</tr>
<tr>
<td>-0.6</td>
<td>0.648</td>
<td>4.243</td>
<td>0.686</td>
<td>-0.6</td>
<td>0.661</td>
<td>4.256</td>
<td>0.684</td>
</tr>
<tr>
<td>-0.9</td>
<td>0.654</td>
<td>4.488</td>
<td>0.687</td>
<td>-0.9</td>
<td>0.668</td>
<td>4.479</td>
<td>0.687</td>
</tr>
</tbody>
</table>
4.5 Three-Shock Model

In the previous sections, the two-sector model assumes that the productivity shocks in the tradable ($x_t$ from Equation 4.6) and non-tradable sectors ($e_t$ from Equation 4.8) are perfectly correlated. In the current section, this assumption is relaxed. The model remains largely the same; the stochastic process needs to be modified, however. The approach of symmetric Markov chains with simple persistence, used previously for the one- and two-sector models, is not expandable beyond the case of two shocks. For this model, the method of approximating vector autoregressions with finite-state Markov chains developed by Tauchen (1986)\(^9\) is utilized. The method is employed to approximate the following VAR(1) process with a Markov chain:

$$
\begin{bmatrix}
Y^T_t \\
Y^N_t \\
R_t
\end{bmatrix} = A
\begin{bmatrix}
Y^T_{t-1} \\
Y^N_{t-1} \\
R_{t-1}
\end{bmatrix} + \begin{bmatrix}
\epsilon^1_t \\
\epsilon^2_t \\
\epsilon^3_t
\end{bmatrix}
\quad (4.24)
$$

In equation 4.24, $Y^T_t$, $Y^N_t$, and $R_t$ are HP-smoothed\(^10\) series for output of tradables, non-tradables, and the world interest rate\(^11\), correspondingly. The estimated coefficients matrix, $\hat{A}$, is:

$$
\hat{A} = \begin{bmatrix}
1.063 & -0.937 & 2.206 \\
0.284 & -0.233 & 1.100 \\
-0.044 & 0.047 & 0.881
\end{bmatrix}
\quad (4.25)
$$

For a Markov chain approximation, a symmetric two-point shock is assumed for each

\(^9\)The method offered in Tauchen and Hussey (1991) is more widely used; however, for the case of symmetric two-point Markov chains, both methods offer similar results.

\(^10\)\(Y^T_t\) and \(Y^N_t\) are logged; smoothing parameter is set to 1600.

of the shock variables; therefore, there are 8 possible states of nature. The estimated standard deviations of HP-smoothed series for $Y^T$, $Y^N$, and $R$, as well as $\hat{A}$ are used to determine the 8x8 transition probability matrix $P$.

The model is described by the same set of equations as the baseline two-sector model (equations 4.1 through 4.14); the steady-state values are the same as in Table 4.2. The same numerical method of value-function iteration is applied for solving the model. The corresponding state grid consists of 30 x 30 x 2 x 2 x 2 = 7200 points, which are equally spaced for each dimension. The range for $K^T$ is [2.645, 2.721], and the range for the foreign asset position is [-0.512, 0.038].

Population moments and the correlation tables for both the constrained and unconstrained economies are summarized in Appendix E. Joint limiting distributions of capital and foreign asset positions for both economies are depicted in Figure F.46; marginal distributions are shown in Figure F.47 for the foreign assets, and in Figure F.48 for the capital in the tradable sector.

To analyze the short-term implications of imposing the liquidity constraint on the model, impact-switching experiments were conducted. In the current setup with three shocks and two values for each shock, there are eight possible states of nature. It would be interesting to compare results of impact switching with the results for the two-shock two-sector model; also, the 3-shock model allows distinguishing between productivity shocks in the tradable and non-tradable sectors. With that in mind, four experiments on impact-switching from time $t$ to $t+1$ were conducted. For all cases, the same time-$t$ state of nature was selected: positive productivity shocks in both tradable and non-tradable sectors and a negative shock to the world interest rate ($\{x^H_t, e^H_t, n^H_t\}$ – the best state). The experiments were:

1. $\{x^L_{t+1}, e^L_{t+1}, n^H_{t+1}\}$ – the worst state; all shocks change their value to the opposite;

2. $\{x^H_{t+1}, e^H_{t+1}, n^H_{t+1}\}$ – adverse (i.e., positive) shock to the world interest rate only;
3. $\{x^L_{t+1}, e^H_{t+1}, n^L_{t+1}\}$ – negative productivity shock in the tradable sector only;

4. $\{x^H_{t+1}, e^L_{t+1}, n^L_{t+1}\}$ – negative productivity shock in the non-tradable sector only.

For each variable (GDP, $C^T$, $C^N$, $L^N$, $p$, $Y^T$, and $Y^N$), the summary of the impact results along the foreign asset position schedule for all four experiments (for comparison purposes) is presented in Figures F.49 through F.56. The summary is done in the same manner as before: solid lines represent the max, mean, and min impact calculated for each foreign asset position along the $K^T$ schedule for the unconstrained economy; dashed lines represent the corresponding results for the liquidity-constrained case. In contrast with the one-sector and two-sector two-shock models, some variables experience an increase for any initial combination of foreign assets and tradable-sector capital, for all four impact-switching experiments. This is observed for GDP (Figure F.50) and the price of non-tradables (Figure F.54). It is interesting to note that the impact for the output of tradables (Figure F.55) is the same for all four scenarios. For all other variables, the biggest drop (or the smallest increase) is observed for the case no. 4 (negative productivity shock in non-tradable sector only). The smallest drop (or the biggest increase, depending on the variable) is generated by scenario no. 3 (negative productivity shock in the tradable sector only); this is followed by the baseline scenario (no. 1, best to worst state), and then by scenario no. 2 (adverse shock to the world interest rate).

The model with three shocks provides an analytical tool for the discriminatory analysis of disturbances to tradable and non-tradable sectors of production, for evaluating their relative role in triggering Sudden Stops, and for comparing their impact on the major economic variables.
Chapter 5

Data Analysis

This chapter provides an empirical cross-country analysis of Sudden Stops. Section 5.1 analyzes whether the SS countries have recovered from the event, by comparing pre- with post-SS growth; section 5.2 looks at the countries' foreign debt as a factor that can trigger Sudden Stops; section 5.3 discusses observed post-SS reactions of several countries; in particular, foreign currency reserve accumulation.

5.1 Growth before and after a Sudden Stop

The model assumes that an economy completely recovers from a Sudden Stop. For an empirical justification of this claim, data on annual GDP growth of 13 countries were collected and analyzed.¹ The main goal is to find out whether the countries which went through a Sudden Stop have recovered from it. All of the countries from the sample have experienced at least one Sudden Stop event (Calvo and Reinhart, 1999, Arellano and Mendoza, 2002). The data covers a period from 1980 until 2006. The time series of GDP growth for each country are presented in the appendix, graphs F.57 through F.60. For the purpose of the analysis,

¹Source: World Economic Outlook
Sudden Stop years were identified for each country, as presented in Table 5.1. For each country, pre-SS and post-SS sub-series were identified from the time series. Then, the average was computed for pre- and post-SS GDP growth. For each country, a t-test was conducted with a null hypothesis that average growth after the SS is the same as before the SS. The results of the analysis are presented in Table 5.2. Argentina, Philippines, and Turkey have demonstrated significantly higher (at 5% significance level) post-SS growth, and Ecuador at 10% level. Indonesia and Thailand have slowed down (significant at 5% level), as well as Korea and Malaysia (10% level). For the rest of the sample (5 countries), the post-SS growth is insignificantly different from the pre-SS growth.

<table>
<thead>
<tr>
<th>Country</th>
<th>Sudden Stop Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1994-95, 2001-02</td>
</tr>
<tr>
<td>Hong Kong</td>
<td>1997-98</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1997-98</td>
</tr>
<tr>
<td>Korea</td>
<td>1997-98</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1997-98</td>
</tr>
<tr>
<td>Philippines</td>
<td>1997-98</td>
</tr>
<tr>
<td>Thailand</td>
<td>1997-98</td>
</tr>
<tr>
<td>Mexico</td>
<td>1995</td>
</tr>
<tr>
<td>Colombia</td>
<td>1998-99</td>
</tr>
<tr>
<td>Ecuador</td>
<td>1998-99</td>
</tr>
<tr>
<td>Brazil</td>
<td>1998-99</td>
</tr>
<tr>
<td>Turkey</td>
<td>1994, 1997-98, 2001</td>
</tr>
<tr>
<td>Chile</td>
<td>1999</td>
</tr>
</tbody>
</table>

This result could be influenced by the world economic trend. To find out how the same countries perform relative to the world, the difference between a country’s growth and the world growth was calculated for each year. Then, the same analysis was performed on pre-SS.

---

2For most countries, the impact of a Sudden Stop event was felt not only during the same year, but also the next year as well (Argentina and countries of East Asia are good examples).

3For instance, a pre-SS period ends the year before the SS as identified in Table 5.1. For countries with more than one occurrence of the SS, the pre-SS period ends before the first SS, and the post-SS period starts after the last SS.

4Source of world GDP growth: World Economic Outlook
vs. post-SS growth, with the results summarized in table 5.3. The results are qualitatively similar to those from the previous table, with the growth for 5 countries being insignificantly different for two periods (although not all of the countries are the same as before). However, only Argentina (at 5% level) and Colombia (at 10% level) have demonstrated significantly higher growth after the SS, and 6 countries have slowed down (at 5% significance level for Indonesia, Korea, Malaysia, and Thailand, and 10% level for Hong Kong and Chile).

<table>
<thead>
<tr>
<th>Table 5.2: Growth Before and After SS: T-Test</th>
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<tbody>
<tr>
<td><strong>Country</strong></td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Argentina</td>
</tr>
<tr>
<td>Hong Kong</td>
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<tr>
<td>Indonesia</td>
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<tr>
<td>Korea</td>
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<td>Malaysia</td>
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<td>Philippines</td>
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<td>Thailand</td>
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<tr>
<td>Mexico</td>
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<td>Colombia</td>
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<td>Ecuador</td>
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<tr>
<td>Brazil</td>
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<tr>
<td>Turkey</td>
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<td>Chile</td>
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</table>

<table>
<thead>
<tr>
<th>Table 5.3: Growth Before and After SS relative to the World: T-Test</th>
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</thead>
<tbody>
<tr>
<td><strong>Country</strong></td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Argentina</td>
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</tbody>
</table>
The next question being analyzed was whether the pre- and post-SS growth are related. To answer this question, cross-country analysis on average growth was performed. The results are summarized graphically at the top section of graph 5.1, with average pre-SS growth on the X axis, and average post-SS growth on the Y axis (and 45-degree line). The data on the country’s growth relative to the world is plotted on the lower section of graph 5.1.

Table 5.4: Growth Before and After SS: Cross-Country Regression

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2</td>
<td>-0.02</td>
</tr>
<tr>
<td>(1.1)</td>
<td>(0.20)</td>
</tr>
</tbody>
</table>

$R^2 = 0.001$

AStandard errors in parentheses

Regression: (Mean GDP Growth before SS) = $\alpha + \beta$ (Mean GDP Growth after SS)

Table 5.5: Growth Before and After SS: Outliers Removed

<table>
<thead>
<tr>
<th>Intercept</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2</td>
<td>0.26</td>
</tr>
<tr>
<td>(0.2)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

$R^2 = 0.543$

$^a$Standard errors in parentheses

$^b$Outliers: Argentina and Turkey

Average GDP growth after the SS was regressed on average GDP growth before the SS, and the results of the OLS regression are summarized in Table 5.4. The slope is insignificantly different from zero, and $R^2$ is very low. Both portions of the graph 5.1 clearly indicate two outliers: Argentina and Turkey. After the outliers were removed from the sample, the OLS regression was re-run, with the results in Table 5.5. In both cases, $R^2$ has increased to over 50%, and the slope is significant in both cases (and almost equal). The results of the first regression (the estimate of the slope equals $0.26 < 1$) suggest that relatively higher growth
Figure 5.1: Growth Before and After Sudden Stop: Cross-Country Analysis

Notes: Solid line is a 45-degree line: points to the left are countries with the average post-SS growth higher than the pre-SS growth, and vice versa.
before SS is associated with the relatively lower post-SS growth. The “break-even” point (where pre-SS growth is equal to post-SS growth) suggested by the regression is 4.4%.\textsuperscript{5} For the second regression (where the GDP growth is relative to the world), the estimated “break-even” point is -0.2%, which is insignificantly different from zero, taking into account that the numerator (the estimate of the slope) is insignificantly different from zero. This result suggests that the economies which demonstrated higher-than-the-world growth before SS have slowed down below the world growth after a SS event (and vice versa).

5.2 Indebtedness as a Sudden Stop Factor

During the recent wave of economic crises in emerging economies, some countries have experienced a Sudden Stop, and others have not. The question arises: what distinguished the two groups of countries, and are these distinguishing features captured within the framework of the proposed models? One of the key parameters in each of the models considered is a liquidity constraint parameter, $\varphi$. It is set just below the critical value $\varphi_{cr}$ that would bind in the deterministic steady state:

$$\varphi_{cr} = \frac{1}{1 - \frac{b}{Y}} \tag{5.1}$$

In equation 5.1, $\frac{b}{Y}$ is a steady-state ratio of foreign assets to GDP. The relationship is depicted graphically in Figure 5.2. As can be seen from Figure 5.2, higher foreign debt (lower $\frac{b}{Y}$ values) means lower values for $\varphi_{cr}$. This, in turn, leads to a wider range of values for $\varphi \geq \varphi_{cr}$ that would bind in the steady state. If $\varphi$ is set above $\varphi_{cr}$, then (depending on the size of stochastic shocks) a country could find itself constrained in all states of nature, thus increasing the possibility of a Sudden Stop.

Therefore, we would expect to find a negative relationship between a country’s ratio of net foreign assets to GDP and the probability of a Sudden Stop. To test this hypothesis, the data

\textsuperscript{5}the “break-even” point is calculated as $\frac{\alpha}{(1-\beta)}$, where $\alpha$ and $\beta$ are estimates of the intercept and the slope, correspondingly.

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Table 5.6: Developing Countries: Net Foreign Asset Position

<table>
<thead>
<tr>
<th>Country</th>
<th>NFA</th>
<th>Country</th>
<th>NFA</th>
<th>Country</th>
<th>NFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algeria</td>
<td>-49.1</td>
<td>Equador</td>
<td>-57.3</td>
<td>Paraguay</td>
<td>-21.2</td>
</tr>
<tr>
<td>Argentina</td>
<td>-32.9</td>
<td>Guatemala</td>
<td>-27.8</td>
<td>Peru</td>
<td>-46.5</td>
</tr>
<tr>
<td>Bolivia</td>
<td>-52.0</td>
<td>India</td>
<td>-16.8</td>
<td>Philippines</td>
<td>-31.7</td>
</tr>
<tr>
<td>Botswana</td>
<td>120.2</td>
<td>Indonesia</td>
<td>-54.2</td>
<td>Singapore</td>
<td>210.2</td>
</tr>
<tr>
<td>Brazil</td>
<td>-30.1</td>
<td>Israel</td>
<td>-12.1</td>
<td>South Africa</td>
<td>15.5</td>
</tr>
<tr>
<td>Chile</td>
<td>-47.7</td>
<td>Korea</td>
<td>-4.6</td>
<td>Sri Lanka</td>
<td>-38.1</td>
</tr>
<tr>
<td>China</td>
<td>-8.0</td>
<td>Malaysia</td>
<td>-44.9</td>
<td>Syria</td>
<td>-21.7</td>
</tr>
<tr>
<td>Colombia</td>
<td>-31.6</td>
<td>Mauritius</td>
<td>-32.7</td>
<td>Taiwan</td>
<td>51.2</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>-37.4</td>
<td>Mexico</td>
<td>-43.2</td>
<td>Thailand</td>
<td>-47.3</td>
</tr>
<tr>
<td>Dominican Rep.</td>
<td>-35.9</td>
<td>Morocco</td>
<td>-40.9</td>
<td>Tunisia</td>
<td>-43.0</td>
</tr>
<tr>
<td>Egypt</td>
<td>-19.3</td>
<td>Oman</td>
<td>15.1</td>
<td>Turkey</td>
<td>-29.8</td>
</tr>
<tr>
<td>El Salvador</td>
<td>-9.1</td>
<td>Pakistan</td>
<td>-50.3</td>
<td>Uruguay</td>
<td>11.4</td>
</tr>
</tbody>
</table>

*Source of NFA position: Lane and Milesi-Ferretti (1999)*

*Sudden-Stop countries are highlighted in bold*

on net foreign asset positions for 36 developing countries is used. Lane and Milesi-Ferretti (1999) computed the net foreign asset position as a ratio to GDP for a wide range of countries for the period from 1970 – 1997. The countries used for the analysis are listed in Table 5.6. The value of NFA is borrowed from Lane and Milesi-Ferretti (1999)⁶; the countries which have experienced a Sudden Stop event are highlighted in bold.

To estimate the marginal effect of an increase in net foreign asset position on the probability of a Sudden Stop event, a probit regression was used. The dependent variable is a Sudden Stop indicator (1 if a country experienced a Sudden Stop, 0 otherwise), and the explanatory variable is the NFA position. The results are summarized in Table 5.7. The results suggest that, for a cross-country analysis, a 1% drop in the steady-state NFA position leads to a 0.8% increase in the probability of a Sudden Stop event⁷.

⁶NFAs in their paper is computed as sum of net FDI, net equity, reserves, estimated assets, and negative of external debt.

⁷Average NFA position for 12 countries from Table 5.6 which experienced a Sudden Stop is -37.9%; average for the other 24 countries is -5.8%.
Figure 5.2: Critical value of the liquidity constraint as a function of net external position

\[ \phi_{cr} = \frac{Y_e}{Y} \]

Table 5.7: Marginal Effect of Change in NFA position on a Probability of a Sudden Stop, Probit Regression

<table>
<thead>
<tr>
<th>Marginal Effect</th>
<th>Robust Std. Err.</th>
<th>z-statistic</th>
<th>P-value</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.00796</td>
<td>.00295</td>
<td>-2.09</td>
<td>.036</td>
<td>-.01375, -.00218</td>
</tr>
</tbody>
</table>

Log pseudo-likelihood = -19.631

5.3 Post-Sudden Stop Measures

One of the assumptions of the model with the liquidity constraint is that the economic agents anticipate the possibility of a Sudden Stop and optimize their behavior accordingly. The agents engage in precautionary savings in the anticipation of unfavorable states of nature, when the constraint becomes binding and they would not be able to borrow (from abroad) enough to smooth consumption intertemporally. As a result, the borrowing from abroad decreases on average, compared to the case of the unconstrained economy. For instance, the amount of borrowing expressed as a ratio to GDP changes by 6.0 percentage points (from -39.8% to -33.8%) for the one-sector model, and by 2.9 percentage points (from -12.4% to -9.5%) for the two-sector model. This change can be attributed to the precautionary savings.
After the Asian crises, the Asian economies have accumulated significant foreign currency reserves. In part, this can be explained by comparing unconstrained vs. constrained economies. Before the crisis, the behavior of economic agents in the countries could be described by the unconstrained model. The agents did not have the possibility of a Sudden Stop built into their expectations. After a Sudden Stop episode, the agents’ behavior modified, which has led to the precautionary savings, according to the model. The amount of foreign asset holdings accumulated by a number of Asian countries, however, is much larger than the precautionary savings predicted by the model. For instance, official reserve assets in Thailand are 33.9% of GDP, and in Malaysia 60.3% of GDP as of Aug. 2007 (see Table 5.8). To explain this, it is worthwhile to recall that a Sudden Stop event in each country had significant short-term effects on different sectors of the economy; moreover, the crisis has caused repercussions in the rest of the world, affecting many countries seemingly unrelated to the Asian economies that were first affected. Therefore, the Asian countries consider a Sudden Stop as an unfavorable event, which needs to be avoided in the future. In order to insure themselves against future SS events, the countries have accumulated foreign reserves.

According to the model’s analysis, this shifts the steady state of an economy away from the Sudden Stop region predicted by the model.

<table>
<thead>
<tr>
<th>Country</th>
<th>Official Reserve Assets, millions $US</th>
<th>Assets/GDP ratio, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thailand</td>
<td>74,439.16</td>
<td>33.9</td>
</tr>
<tr>
<td>Indonesia</td>
<td>51,426.42</td>
<td>12.6</td>
</tr>
<tr>
<td>Malaysia</td>
<td>96,788.00</td>
<td>60.3</td>
</tr>
<tr>
<td>Philippines</td>
<td>30,485.13</td>
<td>22.9</td>
</tr>
<tr>
<td>Korea</td>
<td>255,302.00</td>
<td>27.1</td>
</tr>
</tbody>
</table>

a The data on official reserve assets is as of August 2007; GDPs are forecasts for 2007.

b Source: IMF; World Economic Outlook.
5.4 Data Analysis: Conclusion

The data analysis conducted in this chapter provides partial support for the models offered in the thesis. For instance, section 5.1 suggests that some countries have recovered from a Sudden Stop episode (in terms of economic growth), while others have not. In particular, the South-East Asian group demonstrates lower post-SS growth (relative to the pre-SS one), whereas in the Latin American group, post-SS growth generally exceeds pre-SS growth (see Figure 5.1). However, the countries from the South-East Asian group experienced periods of much higher growth before a Sudden Stop episode than other countries, with average pre-SS growth from 6.4% (for Indonesia) to 7.8% (For Korea and Thailand). In contrast, the Latin American group had slower pre-SS growth (from 1.5% for Argentina to 5.3% for Chile, Table 5.2). Moreover, even though the South-East Asian countries have slowed down after the corresponding Sudden Stop episodes, their post-SS growth is still at the same level or higher (with the exception of Argentina) than that of the Latin American group.
Chapter 6

Conclusion

The research conducted in the current thesis concentrates on the financial-frictions mechanism of recent economic crises in emerging economies (labeled “Sudden Stops”). The crises are approached as infrequent, high-variability events nested within regular business cycles. Two variants of the model of a small open economy are considered: (1) a model with a single tradable commodity and (2) a two-sector model with production of both tradable and non-tradable goods. Financial friction takes the form of a liquidity requirement. The models are able to reproduce certain features of Sudden Stops without significantly affecting the long-run characteristics of the corresponding economies.

A one-sector model is used to analyze the effect of the crisis on the overall performance of the economy. The model produces current account reversal, as well as a dramatic but short-lived economic slowdown, indicated by drops in GDP, employment, and consumption. The model also demonstrates that the amount of physical capital plays a non-trivial role in determining the short-term impact of the shocks on the economy.

The two-sector model features production of both tradable and non-tradable commodities. The model is able to reproduce the negative effect of underlying shocks on production in the tradable sector, while maintaining other features relevant to both sector-specific and whole-economy reactions to adverse shocks near the Sudden Stop region. These effects include (1) current account reversal, (2) economic slowdown in the non-tradable sector (drop in the production of non-tradables and employment in the non-tradable sector) and in the whole
economy (as indicated by GDP drop), and (3) weakening of consumer demand in both sectors (as consumption of both tradable and non-tradable goods falls). As in the one-sector case, the amount of physical capital is important in determining the severity of the crisis in the short-run.

6.1 Proposed Extensions

The two-sector model can be improved in the following ways. First, the short-term effect of an adverse shock in the current model is an increase in the relative price of non-tradables. This increase is smaller for the liquidity-constrained economy near the Sudden Stop region, relative to the unconstrained-economy case, for which the price increase is uniform. Therefore, the model reproduces the drop in the relative price of non-tradables, although this drop is relative to the unconstrained case. This is due to the fact that the only shocks driving the economy’s business cycles are shocks to productivity and the world interest rate. One possible extension is introducing an uncertainty to the value of the liquidity constraint. This can be interpreted as a sudden tightening of an access to the international credit market. The possible reasons for this tightening include a negative informational signal regarding the creditworthiness of the country’s economic agents, or any other informational signals that increase the subjective riskiness for international lenders to the small open economy in question. Alternatively, a shock to the mean value of the consumption tax can be introduced, as in Mendoza (2001).

Another drawback is the small variability of major economic indicators in the two-sector model. The potential solutions are (1) relaxing the assumption of fixed employment in the tradable sector and (2) decreasing the value of $\omega$, which determines the intertemporal elasticity of substitution in labor supply. In the current setup, the production-side adjustment to the shocks in the tradable sector is limited, since the capital cannot be changed until next period, and the labor is fixed by the setup. Therefore, the major adjustment mechanism is changing the amount of foreign asset holdings. This argument is supported by an observation that foreign assets and trade balance are the only variables that have higher variability in the
two-sector than in the one-sector model, for the same magnitude of shocks to both models. On the other hand, a high value of $\omega$ inhibits the employment variability in the non-tradable sector.
Appendix A

Steady State in One-Sector Model

The deterministic steady state is defined by the following equations:

\[
\left(1 + C - \frac{L^\omega}{\omega}\right)^\beta = R \quad (A.1)
\]

\[
\alpha AK^{\alpha-1}L^{1-\alpha} - \delta = R - 1 \quad (A.2)
\]

\[
L^{\omega-1} = (1 - \alpha)AK^{\alpha}L^{-\alpha} \quad (A.3)
\]

\[
C + B = AK^{\alpha}L^{1-\alpha} - \delta K + Rb \quad (A.4)
\]

Equation A.1 sets gross rate of time preference equal to gross real interest rate; return on capital net of depreciation should be equal to the return on foreign assets (equation A.2); labor market equilibrium is given by A.3; finally, A.4 gives equilibrium in the market for goods.

Additionally, the following external debt-to-GDP ratio is used for calibration purposes (from Lane and Milesi-Ferretti, 1999):

\[
\frac{b}{Y} = -0.542 \quad (A.5)
\]
Appendix B

Steady State in Two-Sector Model

Endogenous rate of time preference equals gross interest rate:

\[
1 + \left\{ \sigma(C_T)^{-\eta} + (1 - \sigma)(C_N)^{-\eta} \right\}^{-\frac{1}{\eta}} - \frac{(L^N)^{\omega}}{\omega} = R \quad (B.1)
\]

Net marginal product of capital equals interest rate in tradable sector:

\[
\alpha_T A(K^T)^{\alpha_T - 1}(L^T)^{1 - \alpha_T} - \delta = R - 1 \quad (B.2)
\]

Labor demand equals labor supply:

\[
(L^N + L^T)^{\omega - 1} = \frac{p}{\rho^T} \frac{1 - \alpha_N}{1 + \tau} A(K^N)^{\alpha_N}(L^N)^{-\alpha_N} \quad (B.3)
\]

MRS between nontradables and tradables equals relative price of nontradables:

\[
\frac{1 - \sigma}{\sigma} \left( \frac{C_T}{C_N} \right)^{\eta + 1} = p \quad (B.4)
\]

Supply-demand equilibrium in the market of tradables and nontradables:

\[
(1 + \tau)C^T + \tau p C^N = A \left( K^T \right)^{\alpha_T} \left( L^T \right)^{1 - \alpha_T} - \delta K^T + (R - 1)b - T^T \quad (B.5)
\]

\[
C^N = A \left( K^N \right)^{\alpha_N} \left( L^N \right)^{1 - \alpha_N} - T^N - \delta K^N \quad (B.6)
\]

In addition, there are several calibrational ratios that are used to compute steady state.

Labor is equally split between sectors (Mendoza, 1995):
\[ L^T = L^N \]  \hspace{1cm} (B.7)

Debt payment-to-GDP ratio\(^1\):

\[ \frac{(R - 1)b}{Y^T + pY^N} = -0.005 \]  \hspace{1cm} (B.8)

Consumption-to-GDP ratio is the average ratio for the period 2000:1 – 2007:3 computed from Central Bank of Indonesia data:

\[ \frac{C^T + pC^N}{Y^T + pY^N} = 0.643 \]  \hspace{1cm} (B.9)

Also, assume \( A = 1, \ p = 1, \) and \( \sigma = 0.5. \)

---

\(^1\)This value (0.5\%) is smaller in magnitude than the corresponding value for the one-sector model (from equation A.5 – 2.2\%). The only reason is to have a grid of foreign assets that reaches positive values, so that the country is not forced to be in the borrower’s state. Simulations with the same debt payment-to-GDP ratio as for the one-sector model produce very similar results; the only major difference is the mean of foreign asset position.
Appendix C

Statistical Moments of Indonesian Business Cycles from IFS data

This section reports statistical moments of Indonesian business cycles using *International Financial Statistics (IMF)* quarterly data for the period 1990:1-2007:3. Data are deseasonalized, divided by total population, logged, and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600. Population is annual observations for the period 1990-2007, interpolated into quarterly data using a linear trend. For each variable, $\sigma_x$ is a percentage standard deviation from the HP trend, $\rho_{xt,xt-1}$ is first-order autocorrelation, and $\rho_{xt,GDP_t}$ is a contemporaneous correlation with GDP. Investment is Gross fixed capital formation plus Change in stock. Net exports is defined as detrended exports minus detrended imports. Savings is defined as investment plus net exports.

Table C.1: Statistical Moments of Indonesian Business Cycles (IFS Data)

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_x$</th>
<th>$\rho_{xt,xt-1}$</th>
<th>$\rho_{xt,GDP_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>4.69</td>
<td>0.762</td>
<td>1.000</td>
</tr>
<tr>
<td>Consumption</td>
<td>4.72</td>
<td>0.272</td>
<td>0.064</td>
</tr>
<tr>
<td>Govt. Spending</td>
<td>9.42</td>
<td>0.213</td>
<td>0.358</td>
</tr>
<tr>
<td>Net Exports</td>
<td>7.48</td>
<td>0.526</td>
<td>-0.117</td>
</tr>
<tr>
<td>Investment</td>
<td>26.23</td>
<td>0.693</td>
<td>0.807</td>
</tr>
<tr>
<td>Savings</td>
<td>19.62</td>
<td>0.629</td>
<td>0.818</td>
</tr>
</tbody>
</table>
## Appendix D

### Correlation Tables

#### Table D.1: Correlations between Variables in One-Sector Model

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Capital</th>
<th>Labor</th>
<th>Output</th>
<th>Bonds</th>
<th>NX</th>
<th>I</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Economy with Perfect Credit Markets</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.537</td>
<td>0.941</td>
<td>0.941</td>
<td>0.318</td>
<td>-0.022</td>
<td>0.529</td>
<td>0.737</td>
</tr>
<tr>
<td>Capital</td>
<td>0.537</td>
<td>1.000</td>
<td>0.518</td>
<td>0.518</td>
<td>0.131</td>
<td>0.724</td>
<td>-0.299</td>
<td>0.422</td>
</tr>
<tr>
<td>Labor</td>
<td>0.941</td>
<td>0.518</td>
<td>1.000</td>
<td>1.000</td>
<td>-0.006</td>
<td>0.053</td>
<td>0.597</td>
<td>0.923</td>
</tr>
<tr>
<td>Output</td>
<td>0.941</td>
<td>0.518</td>
<td>1.000</td>
<td>1.000</td>
<td>-0.006</td>
<td>0.053</td>
<td>0.597</td>
<td>0.923</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.318</td>
<td>0.131</td>
<td>-0.006</td>
<td>1.000</td>
<td>-0.124</td>
<td>1.000</td>
<td>-0.726</td>
<td>0.131</td>
</tr>
<tr>
<td>NX</td>
<td>-0.022</td>
<td>0.724</td>
<td>0.053</td>
<td>0.053</td>
<td>-0.124</td>
<td>1.000</td>
<td>-0.726</td>
<td>0.131</td>
</tr>
<tr>
<td>I</td>
<td>0.529</td>
<td>-0.299</td>
<td>0.597</td>
<td>0.597</td>
<td>-0.157</td>
<td>-0.726</td>
<td>1.000</td>
<td>0.586</td>
</tr>
<tr>
<td>S</td>
<td>0.737</td>
<td>0.422</td>
<td>0.923</td>
<td>0.923</td>
<td>-0.372</td>
<td>0.131</td>
<td>0.586</td>
<td>1.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Capital</th>
<th>Labor</th>
<th>Output</th>
<th>Bonds</th>
<th>NX</th>
<th>I</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Economy with Liquidity Constraint</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1.000</td>
<td>0.665</td>
<td>0.964</td>
<td>0.966</td>
<td>-0.049</td>
<td>-0.033</td>
<td>0.330</td>
<td>0.823</td>
</tr>
<tr>
<td>Capital</td>
<td>0.665</td>
<td>1.000</td>
<td>0.631</td>
<td>0.634</td>
<td>-0.110</td>
<td>0.458</td>
<td>-0.248</td>
<td>0.529</td>
</tr>
<tr>
<td>Labor</td>
<td>0.964</td>
<td>0.631</td>
<td>1.000</td>
<td>0.999</td>
<td>-0.240</td>
<td>0.010</td>
<td>0.332</td>
<td>0.943</td>
</tr>
<tr>
<td>Output</td>
<td>0.966</td>
<td>0.634</td>
<td>0.999</td>
<td>1.000</td>
<td>-0.234</td>
<td>0.012</td>
<td>0.329</td>
<td>0.943</td>
</tr>
<tr>
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Table D.2: Correlations between Variables in Two-Sector Model

**Economy with Perfect Credit Markets**

**Economy with Liquidity Constraint**
## Appendix E

### Results for Two-Sector Model with 3 Shocks

#### Table E.1: Population Moments for Two-Sector Model with 3 Shocks

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*Notes: Std. dev. is percent of the corresponding mean (except for net exports).*
Table E.2: Correlations between Variables in Two-Sector Model with 3 Shocks

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| **Economy with Liquidity Constraint** |        |        |         |        |        |        |     |        |       |        |       |       |       |
| CT  | 1.000  | 0.576  | 0.343   | 0.975  | 0.237  | 0.576  | 0.763| -0.169 | -0.265| 0.125  | -0.112| 0.926 | 0.308 |
| CN  | 0.576  | 1.000  | 0.029   | 0.743  | 0.020  | 1.000  | 0.021| -0.165 | -0.941| 0.039  | -0.165| 0.223 | -0.273|
| KT  | 0.343  | 0.029  | 1.000   | 0.290  | 0.097  | 0.029  | 0.278| 0.243  | 0.108  | -0.459 | -0.027| 0.395 | 0.208 |
| LN  | 0.975  | 0.743  | 0.290   | 1.000  | 0.200  | 0.743  | 0.630| -0.183 | -0.473 | 0.113  | -0.136| 0.818 | 0.178 |
| YT  | 0.237  | 0.020  | 0.097   | 0.200  | 1.000  | 0.020  | 0.236| 0.739  | 0.074  | 0.121  | 0.939 | 0.273 | 0.897 |
| YN  | 0.576  | 1.000  | 0.029   | 0.743  | 0.020  | 1.000  | 0.021| -0.165 | -0.941| 0.039  | -0.165| 0.223 | -0.273|
| b   | 0.763  | 0.021  | 0.278   | 0.630  | 0.236  | 0.021  | 1.000| -0.008 | 0.291  | -0.047 | -0.041| 0.900 | 0.502 |
| NX  | -0.169 | -0.165 | 0.243   | -0.183 | 0.739  | -0.165 | -0.008| 1.000  | 0.125  | -0.511 | 0.816 | -0.125| 0.613 |
| p   | -0.265 | -0.941 | 0.108   | -0.473 | 0.074  | -0.941 | 0.291| 0.125  | 1.000  | 0.005  | 0.149 | 0.120 | 0.449 |
| I   | 0.125  | 0.039  | -0.459  | 0.113  | 0.121  | 0.039  | -0.047| -0.511 | 0.005  | 1.000  | 0.080 | 0.130 | 0.130 |
| S   | -0.112 | -0.165 | -0.027  | -0.136 | 0.939  | -0.165 | -0.041| 0.816  | 0.149  | 0.080  | 1.000 | -0.057| 0.798 |
| C   | 0.926  | 0.223  | 0.395   | 0.818  | 0.273  | 0.223  | 0.900| -0.125 | 0.120  | 0.130  | -0.057| 1.000 | 0.494 |
| Y   | 0.308  | -0.273 | 0.208   | 0.178  | 0.897  | -0.273 | 0.502| 0.613  | 0.449  | 0.130  | 0.798 | 0.494 | 1.000 |
Appendix F

Figures

Figure F.1: Value Function Difference for One-Sector Model

$V(\text{constrained}) - V(\text{unconstrained})$
Figure F.2: Switch Impact for Current Account-Output Ratio (One-Sector Model)

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (with favorable shocks) to state $t + 1$ (with unfavorable shocks), according to the optimal decision rule. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.

Figure F.3: Switch Impact for Consumption (One-Sector Model)

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (with favorable shocks) to state $t + 1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (with favorable shocks) to state $t+1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.6: Switch Impact for CA/Y Ratio (One-Sector Model), A Drops

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t+1$ (low interest rate, low productivity), according to the optimal decision rule. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.

Figure F.7: Switch Impact for Consumption (One-Sector Model), A Drops

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t+1$ (low interest rate, low productivity), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.8: Switch Impact for Labor (One-Sector Model), A Drops

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (low interest rate, low productivity), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).

Figure F.9: Switch Impact for Output (One-Sector Model), A Drops

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (low interest rate, low productivity), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.10: Switch Impact for CA/Y Ratio (One-Sector Model), R Increases

Notes: Switch impact is computed as \( \frac{X_{t+1} - X_t}{X_t} \), when the economy advances from state \( t \) (low interest rate, high productivity) to state \( t + 1 \) (high interest rate, high productivity), according to the optimal decision rule. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.11: Switch Impact for Consumption (One-Sector Model), R Increases

Figure F.12: Switch Impact for Labor (One-Sector Model), R Increases

Notes: Switch impact is computed as \( \frac{X_{t+1} - X_t}{X_t} \), when the economy advances from state \( t \) (low interest rate, high productivity) to state \( t + 1 \) (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point \( K_i \) on the capital grid, min, mean, and max values of the switch impact are computed along the \( b \) (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point \( b_i \) on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Notes: Switch impact is computed as $\frac{X_{t+1}-X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point $K_i$ on the capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.14: Means as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.15: Standard Deviations as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.16: First-Order Autocorrelations as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.17: Means as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\varphi$ is on the horizontal axis: $\varphi = 0$ means unlimited borrowing (unconstrained economy), and $\varphi = 1$ means no borrowing allowed.
Figure F.18: Standard Deviations as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\phi$ is on the horizontal axis: $\phi = 0$ means unlimited borrowing (unconstrained economy), and $\phi = 1$ means no borrowing allowed.
Figure F.19: First-Order Autocorrelations as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\varphi$ is on the horizontal axis: $\varphi = 0$ means unlimited borrowing (unconstrained economy), and $\varphi = 1$ means no borrowing allowed.
Figure F.20: Joint Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model)
Figure F.21: Asset Limiting Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model)
Figure F.22: Capital Limiting Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model)
Figure F.23: Value Function Difference for Two-Sector Model

$V(\text{constrained}) - V(\text{unconstrained})$
Figure F.24: Impact of a Switch for Current Account-GDP Ratio (Two-Sector Case)

Notes: Switch impact is computed as $X_{t+1} - X_t / X_t$, when the economy advances from state $t$ (with favorable shocks) to state $t+1$ (with unfavorable shocks), according to the optimal decision rule. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.

Figure F.25: Impact of a Switch for GDP (Two-Sector Case)

Notes: Switch impact is computed as $X_{t+1} - X_t / X_t$, when the economy advances from state $t$ (with favorable shocks) to state $t+1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K^T_i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Notes: Switch impact is computed as $X_{t+1} - X_t$, when the economy advances from state $t$ (with favorable shocks) to state $t + 1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K_{iT}$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.28: Impact of a Switch for Labor in Non-Tradable Sector

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (with favorable shocks) to state $t+1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K_T^i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).

Figure F.29: Impact of a Switch for Price of Non-Tradables
Figure F.30: Impact of a Switch for Output of Tradables

Figure F.31: Impact of a Switch for Output of Non-Tradables

Notes: Switch impact is computed as $X_{t+1} - X_t$, when the economy advances from state $t$ (with favorable shocks) to state $t + 1$ (with unfavorable shocks), according to the optimal decision rule. Top two subgraphs: For each point $K^T_i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.32: Impact of a Switch for Current Account-GDP Ratio, R Increases

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (high interest rate, high productivity), according to the optimal decision rule. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.33: Impact of a Switch for GDP, R Increases

Figure F.34: Impact of a Switch for Consumption of Tradables, R Increases

Notes: Switch impact is computed as $X_{t+1} - X_t$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point $K^T_i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.35: Impact of a Switch for Consumption of Non-Tradables, R Increases

Figure F.36: Impact of a Switch for Labor in Non-Tradable Sector, R Increases

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t+1$ (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point $K^T_i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.37: Impact of a Switch for Price of Non-Tradables, R Increases

![Graph](image1)

**Notes:** Switch impact is computed as \( \frac{X_{t+1} - X_t}{X_t} \), when the economy advances from state \( t \) (low interest rate, high productivity) to state \( t + 1 \) (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point \( K^T \) on the tradable capital grid, min, mean, and max values of the switch impact are computed along the \( b \) (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point \( b_i \) on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).

Figure F.38: Impact of a Switch for Output of Tradables, R Increases

![Graph](image2)
Figure F.39: Impact of a Switch for Output of Non-Tradables, R Increases

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$, when the economy advances from state $t$ (low interest rate, high productivity) to state $t + 1$ (high interest rate, high productivity), according to the optimal decision rule. Top two subgraphs: For each point $K^T_i$ on the tradable capital grid, min, mean, and max values of the switch impact are computed along the $b$ (foreign asset) dimension. Top right is unconstrained economy, and top left is for the constrained economy, with the mean for unconstrained as a dashed line. Two bottom subgraphs: the same exercise is done for each point $b_i$ on the foreign asset grid. Bottom right is the unconstrained economy, and bottom left is the constrained (solid lines) and unconstrained (dashed lines).
Figure F.40: Means as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.41: Standard Deviations as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.42: First-Order Autocorrelations as a Function of Interest Rate Shock (in percent)

Notes: The magnitude of the interest rate shock (as a percent of the world real gross interest rate) is on the horizontal axis. Solid line is for the constrained economy, and the dotted line is for the unconstrained.
Figure F.43: Means as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\varphi$ is on the horizontal axis: $\varphi = 0$ means unlimited borrowing (unconstrained economy), and $\varphi = 1$ means no borrowing allowed.
Figure F.44: Standard Deviations as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\varphi$ is on the horizontal axis: $\varphi = 0$ means unlimited borrowing (unconstrained economy), and $\varphi = 1$ means no borrowing allowed.
Figure F.45: First-Order Autocorrelations as a Function of Liquidity Constraint

Notes: The value of the liquidity constraint $\varphi$ is on the horizontal axis: $\varphi = 0$ means unlimited borrowing (unconstrained economy), and $\varphi = 1$ means no borrowing allowed.
Figure F.46: Joint Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model with 3 Shocks)
Figure F.47: Asset Limiting Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model with 3 Shocks)
Figure F.48: Capital Limiting Probability Distribution for Unconstrained (Top) and Constrained (Bottom) Economies (Two-Sector Model with 3 Shocks)
Figure F.49: Impact of a Switch for Current Account-GDP Ratio (3-Shock Model)

Notes: Switch impact is computed as $X_{t+1} - X_t$ for four different $t \rightarrow t + 1$ transitions. State $t$ (for all 4 cases) is low interest rate and high productivity in both sectors (best state). Top left subplot: $t + 1$ is high interest rate and low productivity in both sectors (worst state); top right: the only $t \rightarrow t + 1$ change is an increase in the interest rate; bottom left: the only change is a drop in the tradable sector productivity; bottom right: the only change is a drop in the non-tradable sector productivity. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.50: Impact of a Switch for GDP (3-Shock Model)

Figure F.51: Impact of a Switch for Consumption of Tradables (3-Shock Model)

Notes: Switch impact is computed as $X_{t+1} - X_t$ for four different $t \rightarrow t+1$ transitions. State $t$ (for all 4 cases) is low interest rate and high productivity in both sectors (best state). Top left subplot: $t+1$ is high interest rate and low productivity in both sectors (worst state); top right: the only $t \rightarrow t+1$ change is an increase in the interest rate; bottom left: the only change is a drop in the tradable sector productivity; bottom right: the only change is a drop in the non-tradable sector productivity. For each point $b_i$ on the foreign asset grid, min, mean, and max values of the switch impact are computed along the $K^T$ (tradable capital) dimension. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.52: Impact of a Switch for Consumption of Non-Tradables (3-Shock Model)

Figure F.53: Impact of a Switch for Labor in Non-Tradable Sector (3-Shock Model)

Notes: Switch impact is computed as $\frac{X_{t+1} - X_t}{X_t}$ for four different $t \rightarrow t + 1$ transitions. State $t$ (for all 4 cases) is low interest rate and high productivity in both sectors (best state). Top left subplot: $t + 1$ is high interest rate and low productivity in both sectors (worst state); top right: the only $t \rightarrow t + 1$ change is an increase in the interest rate; bottom left: the only change is a drop in the tradable sector productivity; bottom right: the only change is a drop in the non-tradable sector productivity. For each point $b_i$ on the foreign asset grid, min, mean, and max values of the switch impact are computed along the $K^T$ (tradable capital) dimension. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.54: Impact of a Switch for Price of Non-Tradables (3-Shock Model)

![Graph showing the impact of a switch for the price of non-tradables.]

Figure F.55: Impact of a Switch for Output of Tradables (3-Shock Model)

![Graph showing the impact of a switch for the output of tradables.]

**Notes:** Switch impact is computed as \( \frac{X_{t+1} - X_t}{X_t} \) for four different \( t \rightarrow t + 1 \) transitions. State \( t \) (for all 4 cases) is low interest rate and high productivity in both sectors (best state). Top left subplot: \( t + 1 \) is high interest rate and low productivity in both sectors (worst state); top right: the only \( t \rightarrow t + 1 \) change is an increase in the interest rate; bottom left: the only change is a drop in the tradable sector productivity; bottom right: the only change is a drop in the non-tradable sector productivity. For each point \( b_i \) on the foreign asset grid, min, mean, and max values of the switch impact are computed along the \( K^T \) (tradable capital) dimension. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.56: Impact of a Switch for Output of Non-Tradables (3-Shock Model)

Notes: Switch impact is computed as \( \frac{X_{t+1} - X_t}{X_t} \) for four different \( t \rightarrow t + 1 \) transitions. State \( t \) (for all 4 cases) is low interest rate and high productivity in both sectors (best state). Top left subplot: \( t + 1 \) is high interest rate and low productivity in both sectors (worst state); top right: the only \( t \rightarrow t + 1 \) change is an increase in the interest rate; bottom left: the only change is a drop in the tradable sector productivity; bottom right: the only change is a drop in the non-tradable sector productivity. For each point \( b \) on the foreign asset grid, min, mean, and max values of the switch impact are computed along the \( K^T \) (tradable capital) dimension. Solid lines correspond to the unconstrained economy, and dashed lines are for the economy with the liquidity constraint.
Figure F.57: GDP Growth: Argentina, Turkey, Philippines

Notes: Top (blue) line is GDP growth rate, and the bottom (pink) line is the growth relative to the world.
Figure F.58: GDP Growth: Ecuador, Mexico, Colombia

Notes: Top (blue) line is GDP growth rate, and the bottom (pink) line is the growth relative to the world.
Figure F.59: GDP Growth: Thailand, Korea, Malaysia

Notes: Top (blue) line is GDP growth rate, and the bottom (pink) line is the growth relative to the world.
Figure F.60: GDP Growth: Indonesia, Hong Kong, Chile

**Notes:** Top (blue) line is GDP growth rate, and the bottom (pink) line is the growth relative to the world.


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