A CASE STUDY IN PARTICPATION OF STUDENTS WITH DISABILTIES IN AN INCLUSION REFORM-BASED MATHEMATICS CLASSROOM

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ABSTRACT

Matthew Joseph Miller: A Case Study in Participation of Students with Disabilities in an Inclusion Reform-Based Mathematics Classroom
(Under the direction of Linda Mason)

In recent years, The *National Council for the Teachers of Mathematics* (NCTM) and *Common Core State Standards of Mathematics* (CCSS-M, 2011) have placed greater emphasis on students developing a deeper understanding of conceptual mathematics. However, research regarding educational supports for students with disabilities in inclusive classrooms that highlight conceptual mathematics learning is limited. The use of Cognitively Guided Instruction (CGI), in which students explain and defend their strategies and solutions of novel word problems to others in a group setting, is one approach to foster this skill. This descriptive case study, consisting of qualitative and quantitative data, investigated the interactions of 3rd grade students receiving general education services (n=8), students referred for special education (n=5), and students receiving special education services (n=4) in the inclusive group setting over a six-week period. Data analyzed included video analysis of student participation and visual inspection of mathematical journal entries and teacher behaviors intended to support the learning of all students. Findings suggested that within in this context, students receiving special education services verbally participated with a similar frequency and duration as students in general education and referred for special education. Furthermore, data from student journals suggested that the three groups of students participated with similar frequency, but students without a disability scored higher, on average, in the areas of understanding, appropriate strategies, and communication than the other two groups.

This work is dedicated to my family, both blood related and other.
I could not have done this without your unwavering support.
I love and thank you.

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In lieu of writing an acknowledge section, I have instead decided to write personalized individual letters to all of those who have helped me through this process to express my gratitude and appreciation. Although this may seem unorthodox, my educational journey at the University of North Carolina at Chapel Hill has been many things, except customary and typical.

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CHAPTER 1: INTRODUCTION

The mathematics reform movement, which began over 20 years ago, led by the National Council of Teachers of Mathematics (NCTM), envisioned a future where *all* students have the opportunity to learn important mathematics, with deep conceptual understanding (NCTM, 1989, 2000). Current national initiatives, such as the *Common Core State Standards for Mathematics* (CCSS-M), reflect a similar goal as well (Council of Chief State School Officers & National Governor's Association (CCSSO), 2010). Mathematics instruction that supports designing learning environments where all students have the opportunity to engage in significant mathematics requires a shift in instruction from focusing solely on developing procedural fluency to student-centered instruction that promotes a deeper understanding of mathematical concepts that provides more flexibility for learners.

In addition to emphasizing deeper mathematical conceptual understanding in which students move from concrete to abstract thoughts, many states are requiring students to demonstrate higher levels of mathematics coursework in order to earn a high school diploma. *The Center for Public Education* (2013) reports currently 32 states, plus the District of Columbia, is now, or in the process of requiring all students to pass Algebra 1 in order to receive a diploma. Students who struggle in learning conceptual mathematics may be at risk for failing the Algebra 1 required for a diploma. A plethora of research has illuminated the negative long-term effects of failing to graduate from high school from both personal and societal implications (Cairns & Cairns, 1994; Rumberger, Ghatak, Poulos, Ritter, & Dornbusch, 1990). Therefore, it is imperative that researchers, math educators, special education teachers, and paraprofessionals gain a better understanding of how low-achieving students with and without disabilities perform in classrooms that emphasize the teaching and learning of conceptual understanding of mathematics.

Within the same timeframe of the mathematics reform movement, students with high-incidence disabilities have been increasingly receiving instruction in general education classrooms (Obiakor, Utley, Banks, & Algozzine, 2014). The current literature related to students with disabilities and special education postulates that possibly the most significant barrier to success for these students in the inclusion setting is quality instruction that produces reasonable progress (McIntosh, Vaughn, Schumm, Hager, & Lee, 1994; Zigmond et al., 1994). With this in mind, educators have begun to focus on evidence-based practices that are effective for all students of varying abilities levels; as well as, cultural and linguistic differences in the general education setting.

Research has shown that typically developing students who participate in classrooms that emphasize the ideals of the mathematics reform movement make fewer mistakes in calculation, retain algorithms longer, and perform better on problem solving exercises (Fennema & Carpenter, 1992; Hiebert, 1999; Villasenor & Kepner, 1993). However, educators, researchers, and policy makers have yet to reach a consensus on the components of mathematics instruction that supports *all* students, including students with high-incidence disabilities, in learning meaningful mathematics as embodied by the philosophies and practices of the mathematics reform movement. The purpose of this study was to examine how students with high-incidence disabilities participate in inclusion reform-based elementary mathematics classrooms and to identify the extent to which instructional practices support these learners. More specifically, this study examined the participation of students with high-incidence disabilities within classrooms that implement Cognitively Guided Instruction (CGI). Researchers developed CGI to assist practicing teachers in constructing understandings of the development of children's mathematical thinking in specific mathematics domains. These domains include addition/subtraction, multiplication/division, fractions, and ratios (Carpenter, Fennema, & Franke, 1996). Furthermore, CGI emphasizes student discourse, collaboration, and deeper conceptual mathematical thinking. While researchers have established benefits to student learning in CGI classrooms, the majority of these studies have focused on typically developing students. With the exception of work by Behrend in 1992 and Moscardini in 2010, little research has focused on students with high-incidence disabilities within CGI classrooms.

Students with High-incidence Disabilities

The National Center for Education Statistics (NCES) reported that 6.4 million students received special education services during the 2014-2015 academic year. Nearly 70% of these students fall under the category of high-incidence disability. These disabilities include specific learning disabilities (LD), speech or language impairments (SLI), emotional disturbance (ED), mild/moderate intellectual disabilities (ID), and other health impaired (OHI). Examples of manifestations of theses disabilities include impaired articulation, language impairment, poor reading fluency and comprehension skills, dyslexia, dyscalculia, inability to build and maintain interpersonal relationships, mental health issues, poor organizational skills, difficulties in written and oral expression, and complications with acquiring social skills (Smith, Polloway, Patton, Dowdy, & Doughty, 2015). Due to the subjective nature of many of these characteristics, accurate diagnosis of disabilities is a difficult task (Donovan & Cross, 2002). Friend and Bursuck (2002) describe members of this population as possessing similarities to non-disabled peers that make distinguishing differences between groups difficult, particularly in non-school settings.

They add that students with high-incidence disabilities are often able to meet the same student standards as non-disabled students while possibly having a combination of academic, behavioral, and social difficulties.

Past research that relates to mathematics instruction and achievement of students with high-incidence disabilities often uses the category of math learning disability (MLD). Generally, researchers use the discrepancy of scores between a standardized mathematics achievement test and an intelligence test (IQ) as the criteria for qualification for MLD (Geary, 2003). Typically, students that score lower than the 20th or 25th percentile and fall within the average to below-average range on an intelligence test meet the criteria. Within this scale, scholars estimate that between 4% and 8% of students possess some form of memory or cognitive disability that affects the process of learning mathematics (Badian, 1983; Geary, 2003; Gross-Tsur, Manor, & Shalev, 1996). Students that possess the aforementioned discrepancy between measured intelligence and achievement may qualify for special education services under the category of specific learning disability (SLD) and not MLD.

Students with High-incidence Disabilities Mathematical Performance

Over the past 40 years, researchers have investigated the similarities and differences between typically developing students, students with reading disabilities, students with MLD, and those with comorbidity of both mathematics and reading difficulties. It is beyond the scope of this dissertation to inspect the relationship between reading disability and MLD, in regards to learning mathematics. Instead, the focus in study is on students that struggle in learning mathematics and how that relates to the recent trends in reform classrooms.

Research concerning the academic achievement of students with mathematical challenges has lagged behind the study of students with reading disability, due in part to the complexity and depth of the field of mathematics (Geary & Hoard, 2002). Since mathematics requires a wide range of skills (e.g., computation skills, pattern recognition, problem solving, spatial reasoning, symbol recognition, creation of models and organizers), a wide range of variation in performance of tasks exists in students that struggle in developing mathematical skills. However, a growing amount of literature suggests that students with MLD struggle specifically with number concepts and number systems (Gilmore, McCarthy, & Spelke, 2010; Lyons & Beilock, 2011; Neider & Dehaene, 2009). In addition, many students in this population may also have challenges in counting, in understanding abstract concepts of temperature, time, and directions, in retrieving computation facts, estimation, and in solving word problems (Bryant, Bryant, & Hammill, 2000; Cawley, Parmar, Lucas-Fusco, Kilian, & Foley, 2007; Geary, 2003; Watson & Gable, 2012). Traditionally, educators have seen the manifestation of these difficulties originating from inside the child, either stemming from neurological or cognitive deficits. More recent attention focuses on the possibility of limited exposure, inadequate instruction or an inappropriate setting as a factor for poor understanding and achievement (Baroody, Bajwa, & Eiland, 2009; Fennell, 2011).

Statement of the Problem and Research Questions

Over the course of the last 35 years, a plethora of research has examined children's mathematical thinking in reform-based classrooms (e.g., Baker & Harter, 2015; Kleickmann, Trobst, Jonen, Vehmeyer, & Moller, 2016; Sood & Mackey, 2014). However, a dearth of research still exists in regards to the academic participation and outcomes for students with high-incidence disabilities in the inclusion reform setting. One possible explanation for the lack of research in this field is due to the differing epistemological foundations of scholars, researchers, and clinicians in mathematics educations and special education. As classrooms become more diverse in regards to student ability, an inspection of the interaction between students with and without disabilities in reform-based classrooms is paramount and would contribute greatly to the knowledge base in area where there is little research.

Many educators and pre-service teachers who attend CGI professional development trainings are exposed to strategies designed to facilitate group discussions where students show and explain solutions to novel mathematical problems. For educators who teach in the inclusive setting, this can be challenging due to the wide range of academic and behavioral skills of students. However, due to the versatile nature of CGI, participation from all students during group sharing is possible (Christenson & Wager, 2012).

Within in the framework of CGI, few studies have investigated the participation of students with and without disabilities in inclusive classrooms that implement CGI philosophies (viz. Behrend, 1992; Moscardini, 2010). Therefore it is logical to investigate the characteristics of student engagement and teacher support within this setting. Student participation, for the purpose of this study, includes both the daily independent completion of novel mathematical problems in student journals and verbal participation during whole class discussions.

The purposes of this case study were to (1) bridge the gap in research literature between the fields of mathematics education and special education, (2) examine the extent to which students with disabilities participated in the group setting in solving novel mathematical problems, and (3) investigate the methods in which the classroom teacher supported the learning of all students. The researcher addressed the following questions:

1. To what extent do third grade students with high-incidence disabilities participate in inclusion reform-based mathematics classrooms?
2. What are the characteristics of student participation among students with high-incidence disabilities in reform-based inclusion mathematics classrooms?

3. How does the teacher support student learning for all students while implementing CGI philosophies in an inclusive classroom?

Organization of the Dissertation

Chapter 1 presents a statement of the problem, the purpose of the study, defines the participant population, and lists the study research questions. Chapter 2 reviews relevant literature focusing on important aspects of this proposed study: interventions and outcomes designed to support mathematical learning for students with high-incidence disabilities, and philosophies and approaches of CGI. Chapter 3 consists of the conceptual framework for the study, along with a description of the design of the study, which includes the participants, context, data collection and analysis. Chapter 4 of consists of student demographic information, quantitative data regarding student participation, and qualitative data of contextual information and teacher classroom behavior. Chapter 5 entails of the discussion of student participation, teacher behaviors intended to support the learning of all students, connections to literature, study limitations, and significance to the field of study.

CHAPTER 2: REVIEW OF LITERATURE

The purpose of this study was to investigate how students with high-incidence disabilities participate in inclusion CGI elementary mathematics classrooms and to identify the extent to which instructional practices support these learners. As indicated in Chapter 1, the standards and goals for learning and understanding mathematical concepts is becoming increasing more complex for elementary aged students. National and state standards have placed more emphasis on problem solving, on metacognition, and on analyzing others' thinking in mathematics classrooms over the past 30 years. At the same time, classrooms in public schools have become more inclusive towards serving students with a variety of disabilities within the regular educational setting. Currently, little research investigating classrooms where the combination and interaction of these two educational trends exist.

After identifying the relevance of the topic, this literature review begins with framing of mathematics instruction philosophies from a historical context. Following this brief summary, the review continues with an examination and analysis of interventions and outcomes designed to support mathematical learning for students with high-incidence disabilities. Although much of the existing research concerning this population relates to explicit instruction of procedural skills, relatively recent trends in national education policy, standards, and pedagogical recommendations (Council of Chief State School Officers & National Governor's Association, 2010; NCTM, 1989, 1991, 2000) demand that teachers support all students in developing a deeper mathematical understanding. Therefore, this literature review concludes with an examination of relevant literature pertaining to philosophies and approaches of CGI, previous research that involves reform based mathematics approaches and students with disabilities, as well as conjectures on the participation and performance of students diagnosed with a high-incidence disability in the inclusion setting.

Relevance of Topic

Long-term outcomes for low-achieving students who have a disability that interfere with learning mathematics are problematic. Research suggests that students who struggle with mathematics achievement throughout school are more likely to be unemployed or receive lower wages (Rivera-Baitz, 1992), even if they possess relatively higher reading skills (Parsons & Bynner, 1997). These outcomes are especially troublesome for students who struggle in the United States, where current trends in employment show growth in technology and sales based careers in which the frequent use of mathematical skills is common, and a decrease in repetitive skills based positions.

The intent of many overarching long-term goals of public education is to support independence, self-efficacy in higher education and the workplace, and adaptability to changing environmental factors. Problem solving in mathematics is a factor in all these areas. Students in special education and general education, for example, need proficient skills in budgeting personal finances, organizing time, describing and following spatial directions, and adaptability in solving unforeseen dilemmas in the place and home. Therefore, it is appropriate for educators to emphasize the generalizing of problem solving skills through classrooms instruction and activities.

Due to recommendations from the NCTM (1989, 1991, 2000, 2006) reform documents on standards, evaluation, curriculum, and professional standards for teaching, as well as recent changes in educational policies such as the CCSS-M (CCSSO, 2010) and No Child Left Behind, researchers and practitioners continue to collect and analyze data concerning the implementation and overall success of the mathematical reform movement. As noted previously, researchers in mathematics education field are still in the process of gaining ground in comparison to early reading interventions. Therefore, at this time, there is sparse information on the best methods to foster mathematical proficiency for students with high incidence disabilities in the inclusion setting (Baroody, 2011). However, quality instruction, engaging lessons, connections to previous experiences regarding mathematics, and drawing on intuitive knowledge increases the chances of success for these students from an academic and classroom management perspective.

Instructional Strategies Used for Students with High-Incidence Disabilities in Mathematical Classrooms: A Historical Perspective

For centuries, academics and educators have debated over the most effective and efficient method for teaching mathematics (Wilson, 2003). In the early part of the twentieth century, behavioral psychologist Edward Thorndike (1922) set forth the position that explicit and organized instruction of procedures through memorization and repetition developed habits of problem solving for learners. Educational reformer, William Brownell (1935) opposed this method of repetition as a means to understanding mathematical concepts and instead proposed that deeper understanding develops for learners through meaningful connections to the subject matter. Although educational researchers generally promoted Brownell's "new mathematics instruction" philosophies, practitioners of mathematics largely acted on the teachings of Thorndike (Ginsberg, Klein, & Starkey, 1998). Many teachers of mathematics still emphasize explicit instruction and procedural repetition in their pedagogy (Fennell, 2011). In contrast, the NCTM (1989, 1991, 2000, 2006) has continued to advocate for teaching methods that emphasize practical applications, meaningful connections to subject matter, and multiple methods for finding solutions, as well as supporting students in the development of both procedural fluency and conceptual understanding.

Mathematics Teachers for Students with High-Incidence Disabilities

Often, a content area teacher, special educator, paraprofessional, or combinations of the three provide instruction to students with high-incidence disabilities in a variety of settings. Examples of this include a general education classroom, inclusion-based classroom, resource room, or self-contained setting. Due to the wide range of variability of severity in both academic and behavioral needs for students and resources available within a particular school, predicting the location of instruction for a "typical student" with a high incident disability, or comorbidity of disabilities, is difficult. However, under the *Individuals with Disabilities Education Act* (IDEA, 2004), students who qualify for special education services have the opportunity to participate with non-disabled peers to the greatest extent that is appropriate in the least restrictive environment (LRE). Therefore, students with fewer and less severe academic or behavioral difficulties typically receive instruction alongside non-disabled peers.

Traditionally, many pre-service general education teacher programs emphasize content knowledge and pedagogy. Conversely, special education pre-service teachers receive training in modifying curricula to best support student learning in a variety of settings. Examples of this include extending time on assignments, preferential seating, and increase in opportunities to respond, reduction of tasks, as well as smaller class sizes. Therefore, students with high-incidence disabilities may receive instruction in a classroom in which the teacher has few experiences in altering lessons to meet student needs, or has limited proficiency in the subject matter. A recent study by Rosas and Campbell (2010) illustrates this dilemma. They found that a high percentage of graduate students seeking licensure in special education had limited experience with mathematics, lacked proficiency, and often had negative experiences as a student in mathematics. These findings highlight the importance of collaboration between general education and special education teachers concerning content knowledge and curriculum modification techniques.

Instructional Techniques for Students with High-incidence Disabilities

Educators that teach mathematics to students with high-incidence disabilities have traditionally used direct instruction, drill, and rote methods to address specific skill deficits (Fennell, 2011). Carnine and colleagues (1994) argue that explicit instruction for low-achieving students reduce ambiguity and confusion of concepts for students, and they advocate teaching one strategy to solve a specific type of problem. According to Baroody (2001), there are multiples explanations for why many current practitioners emphasize a procedural approach in teaching mathematics in special education. He argues that educators have lower expectations for students that struggle with learning basic skills, and therefore believes students are less capable of learning higher order thinking skills. He also indicates that research and development have given an inadequate amount of attention to effective instructional strategies in promoting higher order thinking skills for students who struggle in mathematics (cf. Giordano, 1993; Mallory, 1994). Finally, current teacher training programs lack rigorous components intended to support and promote deeper mathematical thinking from all students.

Students that qualify for special education services receive an Individual Education Plan (IEP) to address specific academic or behavioral challenges. Embedded in the IEP are observable and measurable goals. Additionally, IEP team members create objectives, guided by these goals, to support students in meeting both short-term and long-term benchmarks. Often, educators tailor interventions to address specific learning and behavioral challenges to fit the needs of a student or group of students. In order to collect data on observable goals, explicit instruction of tasks is common to provide quantitative data to show progress obtaining these benchmarks.

Unsurprisingly, research in the area of mathematics instruction for students with disabilities reflects the explicit and direct instruction of procedures for data collection purposes.

Although the debate on teaching practices remains, some agreement exists on the goals of mathematics instruction. In 2001, the National Research Council defined mathematical proficiency as five intertwined strands composed of the following: conceptual understanding, procedural fluency (e.g., speed and accuracy with computational skills), strategic reasoning, adaptive reasoning (e.g., problem solving), and productive disposition (e.g., confidence to use mathematics effectively in everyday life). Standards documents from the NCTM (1989, 2000) produced during the reform movement and the more recent CCSS-M (CCSSO, 2010), also suggest that mathematics instruction should focus on developing both procedural fluency and conceptual understanding, as well as problem solving. In accordance with the NRC and NCTM, the U.S. Department of Education (2008) states that classroom instruction should foster conceptual understanding, procedural fluency, strategic and adaptive thinking, and a productive disposition to achieve mathematical proficiency. However, no global, national, or local consensus exists on the how to reach this goal.

Mathematics Interventions for Students with High-incidence Disabilities

Over the past 30 years, researchers and teachers have devised and implemented a variety of programs and strategies aimed to support mathematical thinking and performance in students of all ages. Typically, these educational approaches fit into the category of “teacher-directed,” which emphasize explicit instruction of procedures or “student-centered,” which focuses on generating mathematical knowledge through active involvement in activities (Clements & Battista, 1990). Both of these approaches have shown varying degrees of success for specific targeted skills and populations (Morgan, Farkas, & Maczuga, 2014). However, due to a vast amount of diversity in mathematics domains, reviewing all literature associated with the combination of students with disabilities and skills associated with mathematics would be a daunting task. Therefore, the scope of this literature review will focus on previous research in the areas of fraction instruction, problem solving, and mathematical discourse for students with high-incidence disabilities.

Fraction Instruction

The National Mathematics Advisory Panel (NMAP: 2008) identified specific areas of concern in mathematics education in the United States. With this charge, the NMAP highlighted the domain of fraction instruction due to a foundational nature in other areas of mathematics and connections to the workplace. In a recent literature review of fraction instruction for struggling learners, Misquitta (2007) described and analyzed 11 studies that involved students with disabilities and fraction instruction. This meta-analysis revealed multiple themes. First, three types of intervention repeatedly showed improvements for low-achieving students' performance in learning fractions: graduated sequence (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Jordan, Miller, & Mercer, 1994), direct instruction (Bottge & Hasselbring, 1993; Flores & Kaylor, 2007; Gersten & Kelly, 1992; Kelly, Gersten, & Carnine, 1990), and strategy instruction (Joseph & Hunter, 2001; Test & Ellis, 2005). Secondly, implicit approaches designed to improve problem solving, with real-life problems involving fractions, showed no gains in computational skills, but did show improvement in problem-solving (Bottge, 1999; Bottge, Heinrichs, Mehta, & Hung, 2002). Finally, Misquitta (2011) asserts that both procedural and conceptual knowledge are necessary for learners to be successful in facing fractional problems in a variety of settings.

Findings from these studies support the rationale for using explicit instruction techniques in teaching procedural fraction skills to low achieving and students with various disabilities. However, data also supports the rationale for using implicit, student-centered pedagogical techniques in generating and generalizing knowledge across settings (Botte, 1999; Bottge et al., 2002). Thus, students with high incidence disabilities may benefit more from a balance of both direct instruction addressing procedural tasks and implicit activities with connections to real life situations to promote generalization. Educators are able to provide this balance of instruction in general education classrooms, resource rooms, or the self-contained setting depending on behavioral factors and resources available.

Problem Solving

Mathematical problem solving requires students to draw on skills to solve novel questions (Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004). Students with high-incidence disabilities often struggle particularly in this domain due to possible challenges in attention, working memory, background knowledge, vocabulary, language processing ability, lack of strategy knowledge, and self-regulation (Baker, Simmons, & Kame'enui, 1995; Geary, 2003; Jitendra & Star, 2011). Addressing the needs of learners in mathematical problem solving activities is a daunting task due to the number of variables associated with type of skill. According to Cooper and Sweller (1987) three of these variables that contribute to problem solving success are mastering rules to arrive at solutions accurately, developing categories for sorting problems with similar attributes, and awareness that novel problems have similarities to previously attempted problems. With this in mind, researchers, using the schema construction theory, have designed interventions to increase the probability that learners will recognize connections to familiar and new problems.

Two examples of programs designed to increase the mathematical schema of students with specific learning disabilities are the schema-based transfer instructions (SBTI) and schema-based instruction (SBI). These programs both use explicit instructions to guide students into using appropriate means to solving word problems. Although these approaches have strikingly similar acronyms, SBTI directly teaches students methods of altering familiar problems to appear novel, in an effort to increase exposure to new ways of solving problems thereby increasing self-efficacy and problem solving techniques in students (Fuchs et al., 2004). Results from this study showed it had positive impact on problem solving tasks compared to control groups. Meanwhile, the intent of SBI is to encourage middle school students with a learning disability to grasp underlying mathematical concepts and structures in ratio and proportion problems (Jitendra & Star, 2011). At this time, no data is available supporting the use of SBI at the middle school level.

Mathematical Discourse

Peer-to-peer and peer-to-teacher mathematical discussions are an integral part of the current trend in reform-based classrooms (Kazemi & Stipek, 2001; Nathan & Knuth, 2003). Emphasis on discourse has changed the role of mathematics educators from conveyors of knowledge that assess accuracy in student performance to builders of learning environments that challenge and support student understanding in mathematics through the sharing of ideas (Stein, Engle, Smith, & Hughes, 2008). The promotion of mathematical discourse within reform-based classrooms supports the learning of procedural and conceptual understanding by having students construct and evaluate their own understanding as well as others (Forman, McCormick, & Donato, 1998). In other words, having students share their strategies and reasoning with others is another method intended to increase students' mathematical schemas, self-regulation in accuracy efficiency, and generalization of skills with novel problems.

Mathematical discourse is possible between two individuals or a large group of people. It may involve sharing one's own reasoning, procedural thinking, final products, sequential computation, connections to practical or theoretical practices, listening to others' thinking, and assimilating others' thinking into existing schemas. Previous research using the peer-assisted learning strategies (PALS), investigated the effects of dyadic work groups using discourse techniques on high achieving, typically achieving, low achieving, and students with disabilities (Fuchs, Fuchs, Hamlett, Phillips, Karns, & Dutka, 1997; Fuchs, Fuchs, & Karns, 2001). In these studies, teacher rank ordered students in each class according to teachers' judgments on mathematical competences, did a median split, and paired the top students from each half. Teachers paired the remaining students in each group using the same pattern. Higher achieving students acted as the tutor for the first half of each lesson, and students traded roles at the halfway point. Thereby, both students served as the student and teacher during each lesson. At the end of the each 2-weeks cycle, students change partners in an effort to expose students to a variety of partners and procedures. Results from these studies showed improvements in all groups, compared to control groups, but particularly those with disabilities and high-achieving students. This gives merit to the conjecture that students with high variability of performance levels are able to benefit from explaining their own thinking and listening to peers discuss mathematical solutions.

For class-wide discussions, Stein and colleagues (2008) provide a framework for novice and experienced teachers in facilitating mathematics based discussions intended to address student needs and support student development. The five practices intended to support orchestrating productive mathematical discourse are as follows: (a) anticipating likely answers from cognitively demanding tasks, (b) monitoring students' responses while they explore tasks, (c) selecting specific students or groups of students to present responses, (d) purposeful sequencing of selected student work, and (e) leading a whole class discussion to summarize and assist students in making connections between different students' responses, key mathematical ideas, and, practical uses.

CGI

CGI is a professional development program that blends four areas of research instruction intended to support students' mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 1999). The components of CGI are (a) the development of students' mathematical thinking, (b) instructional techniques and methods that influence development, (c) the beliefs and knowledge of teachers that influence instruction and (d) the way in which student's mathematical thinking influence the beliefs and instructional practices of teachers. Initial research for this program began with the investigation of techniques that children use to solve addition/subtraction and multiplication/division world problem using whole numbers without explicit instruction. Research has extended into investigating children's means of problem solving concerning fractions, algorithms, and algebraic reasoning.

Focus on Student Thinking

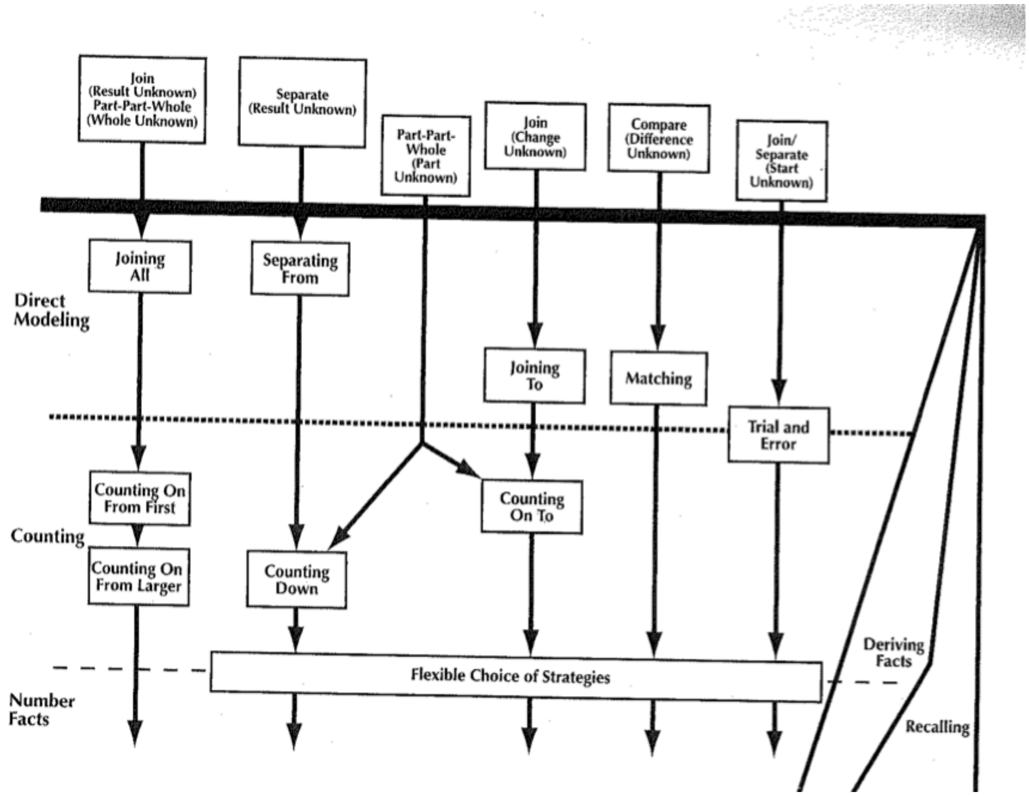
The foundation of the CGI teaching approach stems from the notion that children come to school with a pre-existing informal and intuitive knowledge of mathematic problem solving (Carpenter, Fennema, & Franke, 1996). This intuitive knowledge serves as the basis for development of formal mathematics. Teachers should build on this intuitive knowledge to support students in developing thinking that is more sophisticated and efficient. In opposition to rote memorization as a method to achieve higher thinking skills, CGI encourages children as young as kindergarteners to use their own strategies to make sense of and reason about word problems in computation, fractions, and algebraic reasoning by creating their own models without the teacher initially providing a procedure. The teacher monitors students as they work, providing scaffolds and support as needed. This also gives the teacher the opportunity to observe the written and/or oral expression of children's solutions. After children make sense of word problems using their own strategies, teachers provide opportunities for children to share their strategies and reasoning through discussions. This may involve an individual or group of students explaining their thinking to the teacher, a peer, a small group of peers, or the entire class.

Part of implementing CGI involves numerous and continued opportunities for students to engage in problem solving, as well as other individualized student supports determined by the classroom teacher. In other words, students do no engage in problem solving in one class period or just a few, children must be actively engaged in problem solving on a regular basis; it should be part of the classroom culture. Multiple studies have demonstrated that children are able to solve problems with accuracy in this manner (e.g., Carpenter, 1985; Carpenter, Fennema, & Franke, 1996).

Results from these studies have led to the formation of a learning progression that describes how student thinking becomes more sophisticated over time. Student progression typically moves from *Direct Modeling* (i.e., the use of a manipulative or other representations, such as drawings) to *Counting* to reasoning about *Number Relationships and/or Number Facts*, which increases the amount of flexibility in possible solutions. For example, consider the following word problem: Millie has 6 stickers. How many more stickers does she need to collect to have 13 altogether? A direct modeler would likely use a *Joining To* strategy by counting out 6 objects, “1, 2, 3, 4, 5, 6.” Then, the child would add one object at a time, “7, 8, 9, 10, 11, 12, 13.” The direct modeler would than point to the objects that were added and count each object one at time, “1, 2, 3, 4, 5, 6, 7. She needs 7 more.” A student who is a counter would likely use a *Counting On* strategy, with or without his or her fingers or objects. The child would likely state, “6.” The student would count on from 6, “7, 8, 9, 10, 11, 12 13.” The student may use his or her fingers to show this and would conclude that Millie needs 7 more. Finally, a child who uses number relationships and/or facts might reason about relationships among the quantities and reason using a *Doubles Plus 1* strategy, “6 plus 6 is 12, and 1 more is 13. So, she needs 6 plus 1, which is 7.” Within this framework, students practice the same type of problems so that reliance on manipulative use decreases, efficiency increases, and knowledge of concepts become more concrete.

Carpenter et al. (1999) identified a typical learning progression for solving various types of addition/subtraction word problems with varying unknowns. Figure 1 summarizes many important components of CGI in relation to how student thinking becomes more sophisticated over time in relation to reasoning about addition/subtraction word problems. Over multiple studies, Carpenter and colleagues identified how children perceive addition/subtraction word problem structures (e.g., Join, Separate, Part-part-whole, Compare), which is illustrated in Figure 1. CGI researchers also found that children at different points along the learning progression will use specific strategies to solve certain problem types. For example, the word problem used as an example in the previous paragraph is a Join, Change Unknown word problem. As previously stated, a Direct Modeler would use a Joining To strategy to solve a Join problem where the Change is unknown.

Figure 1. *Carpenter et al.'s (p.31, 1999) framework for illustrating children's solution strategies*



Instructional Methods

Often teachers who utilize CGI in their classrooms create word problems for children to solve using a classification system as a guide to work on specific types of problems. Teachers develop mathematical problems that combine intuitive student knowledge, real world connections, and current levels of performance. Carpenter et al. (1996) state that these problems arise naturally from subject matter in other classes or activities (e.g., readings from social studies or English, combining experiments from science class, or deciding how to share snacks among classmates) or situations that children experience in everyday life. Moreover, classroom teachers are able to manipulate variables in the problems in order to increase or decrease complexities in problem solving activities. Work by Carpenter and colleagues (1992) has shown that a Join, Result Unknown problem is less challenging for children than a Join, Change Unknown. Therefore, the curriculum in CGI classrooms stems from teacher analysis of preferred and observable student thinking processes (Fennema, Carpenter, Franke, & Carey, 1992). Furthermore, this work has also shown that varying number quantities will also increase or decrease the complexity of problem solving. For example, consider the following word problem: Mario has 8 books. Keisha has 13 books. How many more books does Keisha have than Mario? CGI teachers often provide alternative numbers to their students as they engage in problem solving. In addition to the numbers in the word problem above, teachers allow children to alternatively select (3,7) to decrease complexity or (17, 24) to increase complexity. Thus, allowing the teacher to differentiate instruction to meet the needs of all learners, as they work on the same type of word problem.

Teacher Beliefs

In a study that compared the attitudes, beliefs, and instructional practices of 20 first-grade teachers using CGI to a control group; results indicated that teachers participating in CGI listened to students more, knew more about children's thinking, and placed a greater emphasis on problem solving and multiple-solution strategies (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Fennema and colleagues (1996) extended the knowledge in this field with a three-year longitudinal study of 21 teachers to investigate changes in instructional practices and teacher beliefs of those using the CGI professional development program. Researchers identified and described multiple levels of progression. *Level 1* teachers hold the belief that explicit instruction of mathematical procedures and concepts is the most suitable for children. Teachers in this level generally explain and model a single procedure clearly to students. Students then practice the procedure with little or no discussions of alternative methods. *Level 2* teachers start to question whether students need explicit instruction to solve problems or that children can solve problems without receiving a strategy. The "turning point" for many educators is during *Level 3* (Carpenter, Fennema, Franke, Levi, & Empson, 2000). Teachers in *Level 3* classrooms believe that children can solve problems without receiving an explicit strategy and provide no strategy to students. Common characteristics in *Level 3* classrooms are peer-to peer and peer-to-teacher discussions about mathematics, students solving the same problem through a variety of self-discovered methods, and comparison and contrast of strategies demonstrated by students. In *Level 4*, teachers have more stable and less fragmented view of student thinking. They use this information to guide the creation of instruction to bolster deeper mathematical thinking.

Teaching Response to Student Thinking

In order for teachers to make instructional decisions based on observed student performance, researchers constructed conceptual maps to assist classroom teachers to track student development in specific mathematical domains (Carpenter, Fennema, & Franke, 1996). These conceptual maps have multiple purposes. Initially, conceptual mapping allows teachers to reframe student thinking and track learning trajectories in specific content areas. This knowledge has the capability to create a more complete and robust understanding of the development with in specific domains of individual students. Secondly, it affords teachers the opportunity to modify instructional practices to meet the needs of needs of students quickly and efficiently. Examples of this include providing more scaffolding and time for students who struggle with particular skills, altering activities and number sets to challenge students who grasp concepts more rapidly and creating word-problems with connections to student interests. Finally, the emphasis on student discourse in CGI classrooms provides students with multiple means to provide mathematical solutions, as well as providing teachers more opportunities to observe and respond to student thinking.

Research Findings on Students' Mathematical Thinking in CGI Classrooms

Results from empirical studies support the claim that classrooms that implement the CGI approach increase deeper mathematical thinking. Carpenter and colleagues (1989) found kindergarteners scored significantly higher on achievements tests than control groups in regards to problem solving. Moreover, although scores related to computation skills remained similar between the two groups, the CGI group did not emphasize those skills during instruction. Replications of this study to urban and traditionally under-achieving groups reported similar increases of accuracy in regards to written problem-solving, oral expression of solutions, and oral expression of number facts (Franke & Carey, 1997; Peterson, Fennema, & Carpenter, 1997; Villasenor & Kepner, 1993). Outcomes from these studies support the promotion of the CGI program in schools and classrooms that wish to increase problem-solving skills in students.

Extension of CGI

Initially, researchers developed CGI in an effort help support young students develop a deeper conceptual understanding of mathematics by drawing their intuitive mathematical knowledge as a foundation for solving word problems (Carpenter, 1985; Carpenter, et. al, 1989). Early literature on this topic focused on the methods and thinking of students entering elementary school in the areas of counting, adding, subtracting, multiplying, and dividing. Since that time, research regarding CGI and children's thinking has grown to include the areas of fractions and decimals instruction (Empson & Levi, 2011) and algebraic reasoning (Carpenter & Levi, 2000; Jacobs, Franke, Carpenter, Levi, & Battey, 2007).

Extensions of CGI research in the areas of decimal, fraction, and algebraic reasoning usage has shown that students go through a similar progression of learning compared to the acquisition of counting and computation skills (Empson et al., 2011; Jacobs et al., 2007). A characteristic of this process shows a shift in students using intuitive knowledge in solving complex problems to gradually developing and assimilating more efficient and sophisticated methods of problem solving through an emphasis in student discourse and connections between student thinking and key mathematical ideas. Moreover, research in CGI classrooms have reported that students develop trajectories of learning that vary from student to student according to many different factors. From these trajectories, teachers are able to modify pacing, members of groups, and specific questions with the purpose of increasing or decreasing the amounts of necessary scaffolding.

Philosophy of CGI

The central pillar for CGI is the belief that students' thinking guides the instruction of the teacher. Teachers using this program need to take an active and respectful role in observing and responding to intuitive knowledge that students bring to school, the methods and processes students employ to arrive at solutions and student interests in order to form meaningful connections between of mathematics and real life experiences, which is the basis for meaningful understanding. In the foreword of Empson and Levi's *Extending Children's Mathematics* (2011), Carpenter describes two fundamental principles of learning with understanding as "rich in connections and its generative" (p. xii). The applicability of these principles overarches subject matter ranging from counting single digit numbers to algebra, an immense range of student abilities, and variety of cultural backgrounds.

Other prominent members of the mathematics education field echo similar philosophical characteristics put forth by CGI researchers. Clements and Sarama (2014) discuss in detail the importance of tying young children's intuitive mathematical knowledge to classroom practices. They argue that all students have a developmental progression of learning conceptual and procedural mathematical skills called learning trajectories. According to the authors, these learning trajectories consist of a goal, a developmental path in which children develop, and a group of activities to support the development of mathematical skills and thinking. However, CGI incorporates other characteristics such as teacher development, interactions between teachers and students, discourse between students, and assimilation of others thinking in a broader framework.

As typical classrooms become more heterogeneous in regards to cultural, linguistics, and ability, the role of educators is becoming more complex (Sowder, 2007). Therefore, one of the goals of the CGI framework is to provide supports for teachers to reduce the amount of complexity in supporting students' mathematical thinking. In this framework, teachers are required to reflect on student thinking in order to plan and respond. Students are encouraged to reflect on thinking of others in order to defend and streamline their own processes. This emphasis on metacognition supports self-regulation in student strategies and assimilation of new concepts.

Student Participation within the CGI framework

Previous research regarding student participation during group sharing solutions of CGI problems has centered on the complexity of student responses (Carpenter, Fennema, Franke, Levi, & Empson, 1999) teacher behaviors to illicit student responses (Buschman, 2001), teacher decision making in displaying student responses (Franke, Webb, Chan, Ing, Freund, & Battey, 2009), and the creation of novel mathematical problems designed to challenge and engage students (Borko, 2004). However, no literature was found to date on the characteristics of whole group discussion in classrooms with a wide range of academic and behavioral abilities.

According to Hung (2015), student participation during group-sharing time is defined as "students' verbal contributions to public mathematical discourse" (p.259). Verbal contributions, within the group setting can very widely in complexity. Student responses can range from one-word retorts in which they agree or disagree with other students or answer a direct question from a teacher (i.e. “yes”, “no”, “four”, “more”, etc.) to extended explanations of complex solutions using symbols and mathematical terms.

The most common observable student behaviors that demonstrate participation within the group-sharing portion of CGI lessons are verbal and written expressions. Student verbal expression, within the group-sharing context, can take the form of students explaining their thinking to other individuals or the entire group, asking and answering questions, and defending mathematical conclusions. At times, one or two students working on the same problem and demonstrating their strategies with the other members of the class generally present written expression participation, within the group context.

Projected Participation and Outcomes for Students with High-Incidence Disabilities in Inclusive Classrooms Rooms

Klingner and Vaughn (1999) conducted research on both disabled and non-disabled students’ perception of instruction in inclusion classroom. A synthesis of 20 articles, which includes 4,659 students (760 with a high-incidence disability) ranging in age from kindergarten to 12th grade demonstrated that students with disabilities desired the same activities, instructional materials, grading criteria, and assignments as non-disabled classmates. Students without disability agreed on these feelings, believing that it promotes equality in the classroom. Furthermore, both groups valued teachers who slow down instruction when needed, explained expectations clearly, taught the same subject matter in different ways to accommodate student leaning of students; and they recognized that not all students learn at the same rate or through the same method of instruction. These findings underscore the idea that both disabled and non-disabled students value educators that take into consideration the academic attributes of individual students and adjust instructional techniques to support learning.

With the recent push to include students with high-incidence disabilities in the general education classroom, many schools have implemented a collaborative model. This model entails combing the expertise of a general education and special education teacher in a joint effort to provide services to students through mutually defined classroom dilemmas (Langone, 1998). Recommendations for achieving student success in a collaborative model requires, mutual goals, voluntary participation, coequality among participants, and shared responsibilities in planning, decision making, resources, and student outcomes (Friend & Cook, 1996). Theoretically, this model provides a framework to support the learning and service delivery of all students in the inclusion setting.

Research on the participation of students with high-incidence disabilities in classrooms that use CGI, however, is limited. In a mixed method descriptive study, Behrend (1994) investigated the problem solving characteristics of six elementary-aged students identified as LD in a resource room that used the CGI framework. Findings indicated that students with LD were able to solve mathematical word problems using similar strategies as non-disabled peers, with the addition of extended time and instructional scaffolding. Furthermore, nearly all students were willing to share their mathematical thinking strategies to other students and the entire group, as well listen to the strategies of others. Finally, data showed that students were able to complete multi-step tasks, such as multiplication and division, while still displaying difficulties with computation skills in addition and subtraction. This finding is particularly notable because it suggests that higher order thinking skills in mathematic maybe not dependent on a prerequisite skill set.

Findings from Behrend's (1994) research helped guide future research in the field of mathematics instruction for students with disabilities. Because many classrooms now include students with disabilities that manifest behavioral or academic challenges for teachers, research investigating the performance of students in the inclusion setting is crucial. Moreover, for students who struggle with appropriate social skill acquisition, characteristics of CGI classrooms have the potential to have a positive influence the behavioral development of students, through increases in opportunities to practice social skills. Suggestions to extend research in this area include increasing the sample size of participants, observing classroom behavior and mathematical thinking in heterogeneous classrooms, the inclusion of students with other types of disabilities in the classroom, and conducting research in CGI inclusion classrooms in middle and secondary schools.

A recent study investigated the participation and performance of 24 students with mild/ moderate cognitive disabilities on CGI type word problems in Scotland (Moscardini, 2010). Findings from this study demonstrated that children with disabilities were able to invent, transfer, and retain strategies in arithmetic. However, the author noted that some students demonstrated difficulties with recalling the problems in their entirety. The classroom teacher remediated this difficulty by restating the problem in smaller sections. Additionally, teachers reported an increase in the perception of student engagement; meanwhile, nearly all students responded that they enjoyed the reform-based type of learning environment more in contrast to the traditional classroom setting.

Embedded Differentiation of Instruction

Within the CGI philosophical framework, teachers are able to reflect on the learning progressions of a student, group of students, or the class as a whole. This reflection enables teachers to alter lesson plans in order to meet the needs of individual students, or groups of students, within the setting of the entire class. Differentiated instruction (DI) concept of altering and customizing pedagogy, activities, assessment methods, and student product criteria. Traditionally, special education teachers have utilized this method of instruction, particularly in self-contained classrooms where students display significant variability in academic and behavioral performance, as a means of best supporting the benchmarks and goals of individual learners in a group setting.

Subban (2006), discusses the theoretical and conceptual basis for differentiated instruction at length. He argues that the foundation for this philosophical approach of teaching is the blending of Vygotsky's (1980) general theory of cognitive development, which includes the central premise of the zone of proximal development, and Gardner's (1999) theory of multiple intelligences. Briefly stated, Vygotsky's zone of proximal development (ZPD) is the distance between current developmental level in independent problem solving trials and the potential development of problem solving through a more able-bodied peer or adult guidance. Furthermore, Gardner theorizes that learners possess specific inclinations in problem solving methods. These inclinations, or types of intelligences, are visual, kinesthetic, musical, interpersonal, intrapersonal, linguistic, and mathematical (Gardner, 1999). Thus, differentiated instruction is the practical application and extension of cognitive development through social interactions, when educators take into account students' learning propensities.

As the population of students has increasingly become more diverse in recent years (Gable, Hendrickson, Tonelson, & Van Acker, 2000; McCoy & Ketterlin-Geller, 2004), so has the need to address the cultural, linguistic and ability differences in the classroom. Tomlinson (1999) defines differentiating instruction as teaching philosophy that is based on the idea that students learn best when teachers take into consideration student readiness, interest, learning profile and affect; and accommodate instruction accordingly. She explains that differentiated instruction is not a teaching strategy or curriculum, but instead a philosophy that drives the way educators think about teaching and learning (Tomlinson, 2000). Although special education teachers, who serve students with a wide range of abilities across multiple settings, have implemented many of these practices for years, this philosophy is now gradually seeping into the general education pedagogy.

Teachers who incorporate differentiated instructional philosophies into the classroom are able to accommodate the wide variability of learner characteristics in a multiple ways. Pedagogical differentiation takes place when educators alter the amount content covered, pacing of lessons, and delivery of instruction, activities, and student grouping. Furthermore, differentiation may also occur in assessment of student work (Tomlinson, 2003). The flexible nature of differentiated instructional techniques and variability among students' characteristics in classrooms makes comparisons between classrooms that use traditional approaches problematic. However, there is a dearth of group design empirical research on the effects of differentiation of instruction due to its relative newness and comparisons of groups, as opposed to individuals, with significant variability to each other. Therefore, it appears that teasing out the effectiveness of differentiated instruction in terms of student achievement in a group setting is sparse at this time.

Although the amount of research investigating the degree to which differentiated instruction affects the learning all students is limited at this time, reports of positive impacts exist. Hodge (1997) reported higher scores on standardized mathematics tests for students who participated in classrooms where teachers used differentiated instruction with fidelity. Surprisingly, in this same study, scores for reading fluency showed no gains from this intervention. In a similar study at a district-wide level, McAdamis (2001) reported significant improvements on standardized tests for low-achieving students following the use of differentiated instruction. An additional component of this study revealed that teachers perceived that their students were more motivated and enthusiastic in regards to subject matter and activities after using differentiated instruction techniques. Finally, the addition of enrichment activities to differentiated instruction techniques increased both reading fluency and comprehension when compared to whole class basal approach (Reis, McCoach, Little, Muller, & Kaniskan, 2011). These findings support the notion that altering lessons in regards to individual student characteristics in the inclusion setting have an impact on both student learning and teacher perception.

Social Supports for Students with High-incidence Disabilities

Students diagnosed with a high-incidence disability, particularly those with EBD, may exhibit behaviors that are problematic in building and maintaining appropriate interpersonal relationships with peers and teachers. Examples of these challenging behaviors include physical and/or verbal aggression, inappropriate language, defiance, theft, and vandalism. Yell and colleagues (2004) report that inclusion classrooms serve students with EBD at a less frequent rate compared to all other groups of students with disabilities. Although including students from this population in the general education classroom needs to be determined on an individual student basis, based on severity and frequency of undesired behaviors, classrooms that use the CGI approach have the potential to incorporate behavioral supports, as well.

Prior research conducted on students with EBD in the inclusion setting shows that clearly defined rules and expectations and reinforcements for meeting those expectations decreased inappropriate behaviors and increased academic achievement (Mayer, 1999; Shores, Jack, Gunter, Ellis, DeBriere, & Wehby, 1993). Reinforcements for students may come in the form of preferred activities, choices in partner or groups for assignments, or specific public praise for meeting expectations. Although CGI does not address classroom management directly, embedded flexibility in instructional procedures allows teachers to provide academic, as well as behavioral supports when needed.

Points of Contact

Students that participate in CGI classrooms are able to demonstrate and share their thinking and strategies at numerous points in time in a variety of ways. All students, regardless of ability, may show their work through oral and written expression, with a variety of representations, such as the use of manipulatives, tables and figures, drawings/pictures, and the manipulation of symbols and syntax. Furthermore, students can share their strategies and reasoning with another student, a group of students, the teacher, or the entire class. For students who have difficulty speaking in front of others, they can practice using mathematical language with others before sharing with larger groups to lessen anxiety. The connections between differentiated instruction and CGI are plentiful. Historically, special education teachers in self-contained and inclusion classrooms have used these strategies to meet the needs of students with a wide range of academic and behavioral abilities. Due to the flexible nature of CGI classrooms, teachers that facilitate these classrooms have the opportunity to observe, assess, support, and respond to student needs accordingly.

The vision put forth by the NCTM (1989, 2000) to promote opportunities for all learners to participate in deeper mathematical thinking has slowly influenced pedagogical practices in many classrooms. The CCSS-M (CCSSO, 2010) supports the efforts put forth decades ago during the mathematics reform movement. As classrooms have become more diverse in culture, language, and ability (Sowden, 2007), future research is needed to investigate the most effective and efficient methods available for supporting the procedural and conceptual understanding of all students in heterogeneous classrooms, including inclusion classrooms.

Manifestations of academic and behavioral disabilities are diverse in inclusion classrooms in regards to severity, frequency, and impact on learning. Students diagnosed with a high-incidence disability may have difficulties with working memory, reading comprehension, reading fluency, organization, written expression, listening comprehension, an inability to maintain appropriate relationships with peers and adults, and inappropriate responses to normal situations. Since mathematical reform classrooms stress conceptual understanding through student discovery and discourse, teachers have the opportunity to differentiate classrooms to meet the diverse behavioral and academic needs of students.

A plethora of research exists on programs and interventions for students who struggle with mathematics. Oftentimes, the aim is to address specific domain deficiency in order to support sequential learning of procedures and concepts. Moreover, the use of explicit teacher directed instruction is a common trait among these programs. However, as the reform mathematics movement becomes more widely accepted, a need exists to examine the participation and performance of students' with high-incidence disabilities in the area of mathematics.

Previous research regarding the mathematical thinking of students with high-incidence disabilities suggests that they are able to participate and have success in classrooms with proper supports. The embedded differentiated instruction of this approach allows teachers to repeatedly and consistently investigate students' thinking and respond quickly to social and academic challenges in appropriate manners. Although previous research has shown that low-income and minority students have benefitted from reform-based mathematics programs (e.g., Fennema, et.al., 1993), there is little documentation on benefits of such programs for students with high-incidence disabilities (Jitendra & Star, 2011) at this time.

This study addressed the gaps in the literature that exist concerning teacher involvement in CGI classrooms that promote solving mathematical word problems by students with high-incidence disabilities in classrooms that serve both students with and without high-incidence disabilities. Furthermore, this study sought to detail the degree and dimensions to which students with high-incidence disabilities participate in CGI inclusive classrooms, as well as the nature of this participation, and instructional strategies that support participation. Finally, this study sought to help bolster communication between the fields of mathematics education and special education.

CHAPTER 3: METHOD

The author reviewed the relevant literature focusing on two major aspects of this study in Chapter 2: interventions and outcomes designed to support mathematical learning for students with high-incidence disabilities, and philosophies and approaches of Cognitively Guided Instruction (CGI). Over the past 30 years, researchers have created an extensive body of literature investigating CGI philosophies and pedagogical approaches in relation to mathematical thinking for a variety of different student populations (Carpenter, Fennema, Franke, Levi, Jacobs, Empson, 1996; Waxman & Pedron, 1995; Xin, Jitendra, & Deatline-Buchman, 2005). However, few studies have examined the extent to which students with disabilities participate within inclusion classrooms that employ CGI strategies. Moreover, during the same span of time, general education classrooms have seen an increase in the number of students who receive special education services through educational philosophies that promotes inclusive practices (Fuchs & Fuchs, 1994). Therefore, the purpose of this study is twofold. First, it aims to add to the growing body of literature regarding the participation of students with disabilities (i.e., learning disabled, emotional/behavioral disorders, speech and language impairments, and mild intellectual disabilities) in classrooms that use CGI. In addition, to illuminate the teaching practices of a teacher in an inclusive setting, where explicit instruction for solving mathematical problems is not solely provided, in order to better inform current and future classroom teachers.

In order to explore the two purposes, the researcher selected a case study methodology. Chapter 3 consists of the conceptual framework for this study, the case study method rationale, the case study design, participant information, the context, as well data collection and analysis methodology.

Conceptual Framework

Reform documents (National Council of Teachers of Mathematics [NCTM], 1989, 2000) and the recent Common Core State Standards for Mathematics (CCSS-M, 2010) call for teachers to support students in developing both procedural and conceptual knowledge. In order to connect procedures to concepts in a meaningful way, these documents describe the processes that encourage this development. Thus, these documents provide a lens to view effective instruction.

In the *Principles and Standards for School Mathematics* (2000), the NCTM highlighted key processes to engage students in meaningful, high-quality mathematics. The Process Standards include problem solving, reasoning and proof, communication, connections, and representation. In 2003, the National Research Council (NCR) described the framework for student to understanding in mathematics. Five strands of mathematical proficiency were identified as (p. 5): (1) conceptual understanding—comprehension of mathematical concepts, operations, and relation, (2) procedural fluency—skill in carrying out procedures: flexibly, accurately, efficiently, and appropriately, (3) strategic competence—ability to formulate, represent, and solve mathematical problems, (4) adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; and, (5) productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Grounded in the NCTM's Process Standards (2000) and the NRC's (2003) definition of mathematical proficiency, writers of the CCSS-M recently described the Standards for Mathematical Practice (SMP) as (CCSSO, 2010): (1) make sense of problems and preserver in solving them; (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; (4) model with mathematics; (5) use appropriate tools strategically; (6) attend to precision; (7) look for and make use of structure; and, (8) look for and express regularity in repeated reasoning. By using CGI, teachers attempt to address these standards with mathematical lessons.

Multiple entry points exist for student interaction within the mathematical and social context of the whole-group sharing portion of CGI style lessons. The entry points for student lesson engagement in CGI lessons, include verbally sharing methods, models, constructing and defending arguments, as well as questioning the conclusions, accuracy and efficiency of classmates while problem solving novel dilemmas in group setting (Duncan & Hmelo-Silver, 2009). Often times the teacher, within this setting, becomes a facilitator of conversation and the debate moderator as opposed to a procedural instructor (Borko, 2004). For educators in inclusion classrooms, facilitating meaningful whole group conversations for all students is challenging due to wide range of academic and behavioral student abilities. Therefore, differentiation of complexities within tasks that address the same mathematical concept is appropriate for all learners.

Due to the complexities of academic, physical, and behavioral disabilities and within the context of current mathematical standards, addressing the diversity of student needs in inclusion classrooms is challenging. Tomlinson and McTighe (2006) recommend differentiated or responsive teaching where teachers "make modifications in how students get access to important ideas and skills, and in the learning environment-all with an eye to supporting maximum success for each learner" (p.18). Therefore, the conceptual framework for this study aims to measure and describe the verbal and written participation of each student, within setting of an elementary inclusion classroom that endorses reform mathematical practices.

Case Study Rationale

Past research, investigating the phenomenon of student performance and behavior in classrooms that employ a reform-based mathematic approach that include both disabled and non-disabled peers is scant. Therefore, the initial proposal of this project planned to investigate multiple classrooms that incorporate CGI in inclusion classrooms. However, due to low recruitment numbers of students with disabilities in possible participating classrooms, the researcher opted to look more closely at the classroom with highest rate of student recruitment (95%). Therefore, the researcher shifted the focus and methodology of the initial study from a mixed methods approach across multiple classrooms to a case study design of a particular classroom to gain a deeper understanding of the phenomena.

Due to the unique variables and dearth of previous research concerning this phenomenon, Towne and Shavelson (2002) recommend the use of a detailed case study in order to provide rich descriptions of the procedures, behaviors, and context of student participation and achievement. Stake (1995) argues, “Seldom is an entirely new understanding reached but refinement of understanding is” (p. 7) when referring to conclusions based on case study research. Therefore, generalizations concerning teacher and student behavior within this context are problematic. However, the purpose of this case study is to document, with empirical evidence, an approach to supporting the learning of students with and without disabilities within inclusion classroom that aligns with current reform mathematical practices.

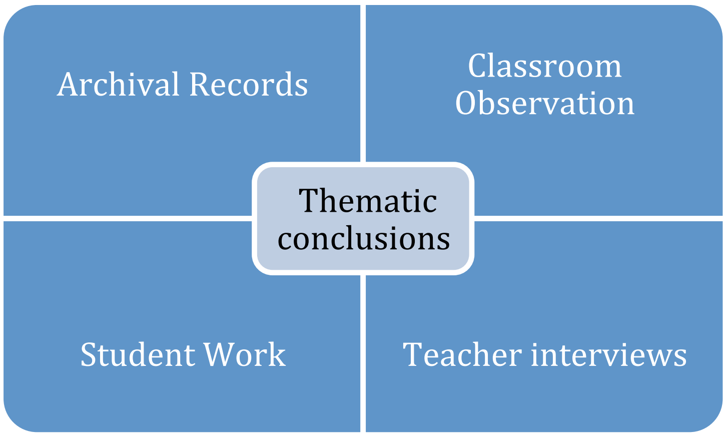
An investigation of the interactions between students with and without disabilities and the most recent mathematical standards (CCSS-M, 2010) aligns most succulently with the constructs of the descriptive case study. In contrast to other case study constructs (e.g., experimental, survey, and historical) the descriptive case study aims to answer the “how” and “why” of a specific phenomenon within a setting where distinct boundaries are difficult to observe (Yin, 2014). Nebulous boundaries for this case study exist in multiple domains. These include time allotment for mathematical discussion (i.e., the lack of predetermined time exists for group-sharing time), addressing the needs of students labeled and/or disabled, (i.e., inclusion classroom population) and the challenges of teaching students whose specific learning and behavioral disabilities in elementary aged yet to be determined (i.e., students who have been referred for special education assessment, but do not currently qualify for special education services).

According to Yin (2014), one the rationales for using a descriptive case study is to focus on contemporary events in which the researcher has minimal control over the study participants. Therefore, the researcher in this case study made diligent efforts to observe and record classroom behaviors (i.e., video recording of lessons), student demographic information (i.e., IEP and school record information), and classroom contextual information (i.e., semi-structured teacher interviews) without interfering with the typical school day course of events.

Study Design

In order to provide a detailed account of (a) the intersections of participation of students with and without a disability and (b) teacher behavior within a classroom that promotes reform mathematics practices, both qualitative and quantitative data were collected within Yin’s (2014) convergence of evidence model for case studies (see Figure 2.). Thematic conclusions were based on the analysis of archival records (i.e., student IEP’s, 504 plans, and school records), 11 classroom observations of group sharing time sessions, student work (e.g., student journal entries for novel mathematical problems), and teacher interviews of the classroom and the academically or intellectually gifted (AIG) enrichment teacher. The researcher collected and analyzed classroom sessions and student worked in sequential order. Furthermore, the researcher conducted teacher interviews after all classroom data had been collected.

Figure 2. *Convergence of evidence model*



A single-case, or holistic, design was used for this study (Figure 2).   This model, to be described more fully in the Current Case Study section, represents the chain of evidence gathered for this study as well as the convergence analysis of both quantitative and qualitative data to support thematic conclusions. The researcher's use of both quantitative and qualitative data is to highlight the width and depth of students with disabilities' participation within the CGI inclusion setting.

The quantitative data for this study consisted of timing the duration and frequency of student verbal participation within the group-sharing context. Furthermore, the researcher measured student written work using a Likert-type scale ranging in answers from 0-4 in regards to written responses in communication, reasoning and strategies for novel word problems. Qualitative data for this study consisted of semi-structured interviews of the classroom teacher and the academic support staff, observations of teacher behaviors, and analysis of classroom interactions between all students and the classroom teacher.

Current Case Study

Context

This investigation was an extension of a larger a year long qualitative study investigating the use professional development (PD) with pre-service and in-service teachers in regards to implementing CGI word problems for 3^{rd} through 5^{th} grade classrooms. Participants of this year long PD consisted of eleven cooperating elementary school teachers and six prospective elementary school teachers from five different elementary schools. In this study, the researchers led 12 PD sessions after school or on in-service workdays for the school district.

Details of the professional development involved the implementation of CGI word problems, teacher rationale for selecting student work as examples, and analysis of student understanding demonstrated in discussion (Mojica & Wright, 2012). Furthermore, it examined, through qualitative analysis the interaction between cooperating teachers, student teachers, and elementary aged students involving fraction instructional practices from Empson & Levi's (2011).

Additionally, researchers utilized philosophies from the *Five Practices* (Smith & Stein, 2011) in an effort to connect mathematical teaching theory and practice in promoting meaningful classroom discussion. Smith and Stein (2011) depict these practices as *skillful improvisation* in facilitating rich mathematical discourse: anticipating students' responses, monitoring how students respond to questions, selecting student work to share with the entire class, sequencing student examples in regards to sophistication or popularity, and connecting student responses to other class members.

Over the past 3 years, local university professors, along with research assistants, have worked with the local school personnel in supporting continuing mathematics professional development. Therefore, school personnel are familiar and comfortable with researchers leading on-going CGI professional development and collecting data in the mathematics classrooms.

Group Sharing

The researcher collected all data for verbal student participation and teacher behavior during group-sharing time of CGI lessons. During this portion, of the CGI lesson, all students gather around the classroom smart board in a semicircle to discuss teacher selected novel mathematical problems from the students' journals. During this study, group-sharing time always took place during the late morning. Furthermore, there was no predetermined set amount of time for teacher led discussion, however the classroom teacher extended or shortened group-sharing time depending multiple factors (e.g., school-wide activities, other planned classroom activities, abbreviated school schedules).

Participants

The extent to which general education and students diagnosed with a disability participated in an inclusion classroom that promotes reform based mathematical philosophies was investigated. Furthermore, this study investigated the participation of all students with and without disabilities in elementary mathematics classrooms. Therefore, the researcher observed students in the classroom assigned by the elementary school faculty and administration at the beginning of the school year. Thus, neither the researcher nor the participating teacher reassigned students either into or out of the classroom during the duration of this study.

Teachers

The classroom teacher selected to participate in this study had previously been a participant in the aforementioned CGI professional development fraction study. Therefore, the researcher employed criterion sampling to select participants based on several key factors (Patton, 2001). Firstly, the classroom teacher in this study held a current state elementary teaching license and had a minimum of two years teaching experience. Secondly, the classroom teacher taught in a classroom that had both general education and students with a disability. Thirdly, the participating teacher took part in CGI professional development with researchers and implemented reform-based mathematics teaching practices, like CGI. In addition, the researcher interviewed (AIG) support teacher to gain a better of understanding of enrichment supports for students who participate in the inclusion classroom.

Students

The researcher recruited 3rd grade students for this study through the classroom teacher. The researcher invited l9 students currently enrolled in the classroom to be a part of this study. However, one student chose not to participate and another student received her mathematics instruction in another room as required in her IEP. Thus, the total number of students participating was 17. The researcher gained parental or guardian consent as well as student assent for all student participants. The researcher used pseudonyms to protect the identity of all participants.

Archival Records

The researcher gathered student information regarding disability classification through the classroom teachers' search of current IEP's and 504 plans. Current data from the archival records included primary and secondary classification status as well as present levels of performance. To maintain confidentiality for all other student information contained within the IEPs and 504 plans, the researcher opted to collect through the classroom teacher.

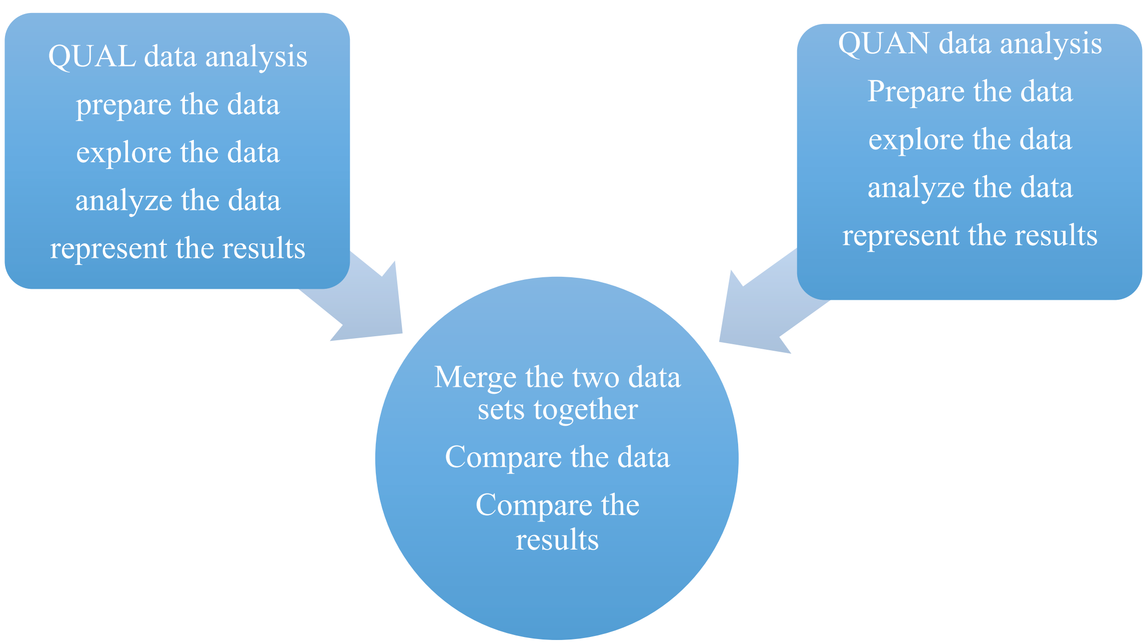
Data Collection

The researcher collected both qualitative and quantitative data in this case study. Data included the following: video recordings of classroom observations, interviews of the classroom and AIG support teacher, written work from student journal, and vignettes of classroom interactions involving the classroom teacher and students with varying academic and social abilities.

All classroom observations of mathematics group discussion were video recorded and transcribed. In order to maintain participant confidentiality, video data from classroom observations were stored on a secure server maintained by the researcher. Furthermore, the researcher recorded all video data in this study.

As previously stated, the researcher collected separate sets of data through this investigation using a concurrent embedded design. Creswell (2007) states guidelines that apply for effective data analysis in this design which include conducting separate initial data analysis for each quantitative and qualitative data sets, merging the data sets together to reinforce or refute data from a specific data set, and explaining and discussing the extent the data converge as well as confirm or counter each other. Figure 3 displays visual representation of the triangulation of data analysis.

Figure 3. *Triangulation of data analysis*



This study involved 3rd grade students enrolled in a classroom where the teacher promoted the use of CGI practices. Student participants in this study consisted of students who received general education services, students diagnosed with a disability, and those referred for special education assessments. Prior to engaging in this study, the researcher obtained parental or guardian consent and student assent. In order to protect the identity of all participants, participants were be given pseudonyms. The researcher then created a key to match participants with pseudonyms, and the key was stored separately from data.

Duration

In order to achieve a better understanding of interactions between educators, students with disabilities, and non-disabled peers in classrooms that promote reform-based mathematics pedagogy, the researcher collected data in a 3rd grade inclusion classroom. The researcher collected data, by video recording all student group-sharing sessions during a period of 6 weeks. The researcher collected all data for this study during school hours over the time span. The 3rd grade classroom in this study consisted of students diagnosed with a disability (n=4), students who have been referred for special education assessment (n=5), and non-disabled students (n=8). The researcher gathered student's demographic data through semi-structured interviews from school personnel, teacher reported information from current Individual Education Plan (IEP) of student participants to determine age, gender, race, primary and secondary diagnosed disability. The AIG teacher verified student demographic information and reported to the researcher.

Data Collection and Analysis

In order to demonstrate the degree to which all students participated and how the teacher attempted to support the needs of diverse learners in the inclusive reform mathematics setting, the researcher collected both quantitative and qualitative to provide the breath and depth of this phenomenon. Cohen and colleagues (2011) argue that combining both quantitative and qualitative data within the case study framework enables readers to form a better understanding of how theory and practice align. Therefore, the following sections will break down the data collection methodology and analysis for investigating student participation and teacher support behaviors.

Student Participation

To gain a broad perspective of student verbal participation, the researcher used Rubric 1 (see Appendix A) of the Instructional Quality Assessment (IQA). The definition for participation, defined by the authors of this scale, count as any verbal contribution, including brief comments by an individual as evidence of participation. However, student responses given chorally or one-word responses do not qualify as participation within this measurement tool. When using the rubric, the researcher determined the percentage of all students verbally participating during group sharing time. Data was collected for all 11 group-sharing discussions over a six-week period. This IQA utilizes a 5-point scale which measured the extent to which widespread participation of all students exists and scores the discussion into quartiles, with zero as a score of no student participation. The researcher selected this assessment because it is designed to be as low inference as possible (Junker et al., 2006) and lends itself to portraying a broad perspective of how many students are verbally participating with extended answers during classroom discussions.

A more detailed inspection of verbal classrooms interactions was analyzed using video footage of group-sharing lesson. The researcher measured verbal student interactions for duration and frequency in regards to student classification (i.e. general education, special education, or referred for special education assessment status). Student verbal interactions, which addressed the mathematical topic and were longer than one-word answers, were timed and added together. The researcher then separated the timed interactions into sub groups of (a) students in general education, (b) students referred for special education and c) students with and active IEP or 504 plan and compared to each other. Moreover, frequency of verbal and written participation (i.e., which students were called on to share their work at the smart board) was measured for each lesson.

Participation Characteristics

In order to answer research question 2, the researcher analyzed video recordings of mathematics group discussions to identify frequency and duration of verbal participation of all students. For analysis purposes, again used the groups of (a) students receiving general education services, (b) referred students for special education assessment, and (c) students with a current IEP or a 504 plan.

In addition, to gain a deeper understanding of the performance characteristics of students with disabilities in the inclusive setting, the researcher gathered data on student thinking and performance through an examination of students' mathematical journals. Within the participating classroom, students attempted to solve CGI word problems individually before they discuss their strategies as a whole class. Students showed their strategies as a representation of their thinking using pictures/drawings, symbols, and words in their mathematics journals. Their solutions also include an explanation of their strategies. The researcher collected data from the student journals for all participating students with a diagnosed high incidence disability. Work from the journals also consisted of notes, drawings, procedures, attempts, and solutions by their peers.

The researcher conducted an analysis of each student's work completed in a journal using a scoring rubric currently used by classroom teachers in the participating school district. (See Appendix B). The conceptual framework basis for this rubric aligns with Standards of

Mathematical Practices (SMP) of the Common Core State Standards of Mathematics (CCSS-M, 2010). Specific academic areas of interest include the categories of problem solving, reasoning and proof, communication, and representation of mathematical concepts. Furthermore, school personnel participating in this study are currently used this rubric to score student to test for mastery. Therefore, the rationale for using it in this study was to use the tool, which teachers currently employ in this school district, as a means to measure the degree to which all students participate and perform within the writing portion of CGI lessons.

Student and teacher interactions, in the form of vignettes, were also analyzed to investigate the characteristics of student participation. Data from the vignettes were analyzed for commonalities and differences among the three sub groups of students. The researcher focused on how students interacted with the novel mathematical problems, other students, and the classroom teacher as opposed to the accuracy of solutions.

Teacher Supports

To answer question 3, the researcher provided demographic, instructional, and experiential context within the framework of both the NCTM's *Principals to Action Checklist* (2014) as well as differentiated instruction methods (Tomlinson, 2001). These methods of differentiating include altering the content, process, product, and the learning environment to support student learning.

The researcher used a semi-structured interview protocol to acquire the classroom teacher and support teacher perspectives on addressing the needs of a diverse student population. During the interview, the researcher asked opened-ended questions allowing participants opportunities to express their own views on their terms, and allowed researchers the opportunity to compare answers of different participants. Interviews were audio recorded and transcribed, and analyzed for consistent and reoccurring themes.

The researcher analyzed the teacher interview transcripts to identify significant themes, as well as patterns in the data (Strauss & Corbin, 1998). Furthermore, the researcher examined transcripts for emerging themes across participants. Riessman (2008) recommends using *thematic analysis* to identify common elements across research participants, phenomenon they observe, and actions they take in similar settings. Furthermore, this method of analysis allowed flexibility in location, time, and number of participants in order to establish common themes regarding educational views, perceived classroom barriers, and individual perspectives on student success and progress.

The semi-structured interview protocol was use the following questions:

1) How long have you been a certified classrooms teacher? What certification do you hold?
2) How much (either in-service or pre service) training have you had in working with students with disabilities?
3) What types of disabilities do students have in your classroom this year?
4) How do these students generally perform?
5) In what ways do they excel in class?
6) In what ways do they struggle in class?
7) In what ways do you modify the math curriculum to accommodate the needs of students with disabilities?
8) How much time do you spend having to modify the curriculum?
9) In what ways do you get feedback on the participation or performance of students with disabilities in your classroom?

10) In your opinion, what are some of the barriers in having students with and without disabilities work together?

In order to ensure that participants were using reform-oriented practice, the researcher conducted 11 classroom observations to determine the quality of instruction and the extent to which the teacher implemented specific practices based on NCTM recommendations.

Furthermore, the researcher used the *National Council of Teachers of Mathematics' 8 Principles to Actions Executive Summary* (2014) as a framework to illustrate critical pedagogical moments. These recommendations for classrooms teachers are the most current suggestions on supporting "the mathematical understanding and self-confidence in *all* students" (p.3). The eight principles include: (1) establishing mathematical goals to focus learning, (2) implementing tasks that promote reasoning and problem solving, (3) using and connecting mathematical representations, (4) facilitating meaningful mathematical discourse, (5) posing purposeful questions, (6) building procedural fluency from conceptual understanding, (7) supporting the productive struggle in learning mathematics, and (8) eliciting and using evidence of student thinking. The researcher observed, recorded, transcribed, and coded for the previously stated practices for each group-sharing lesson to investigate for pedagogical themes.

In order to provide appropriate and effective instruction to students with a singular disability or comorbidity of disabilities, teachers must often differentiate their instruction to accommodate the needs of students. Often times, through teacher creativity, lessons are altered to increase or decrease difficulty, extend or limit time constraints, or provide greater or fewer supports on a specific task. Therefore, the researcher employed this model in order to capture specific moments of instruction that included class wide facilitation of discussion as well differentiation to accommodate a diversity of learners. The researcher identified critical events that meet the criteria described in this paragraph.

Of the 11 group-sharing lessons recorded and transcribed, the researcher included six vignettes of teacher-to-student and student-to-student interactions in the results sections. The aim of the vignettes was to capture the both verbal interactions of students, with and without a disability, that share their thoughts and written solutions in the group setting, as well as the teacher's behavior. Spalding and Phillips (2007) argue that use of vignettes provide a single account of the events that contributed to the improvement of practice in similar settings.

Triangulation

Cohen and colleagues (2011) describe *methodological triangulation* as the use of using two or more methods of data collection to investigate the same phenomenon. They contend that it is beneficial in the social sciences in an effort reduce researcher biases in environments where multiple variables are interacting as well as increasing researcher and reader confidence in findings associated with a particular study. Lincoln and Guba (1985) echo this perspective in spirit, in a naturalistic setting; triangulation of data is intended to reinforce findings through the use of multiple methods. Therefore, as stated previously, quantitative and qualitative data regarding student participation, characteristics of student participation and teacher supports in the classroom were collected and analyzed.

The researcher reported results in chapter 4 in the order of (a) student demographic information, (b) frequency and duration group participation data, (c) participation characteristic data, (d) teacher support data. Both quantitative and qualitative data was merged together (see Figure 3) to draw thematic conclusions on the characteristics of student with disabilities participation with the group-sharing portion of CGI lessons.

Validity and Reliability

To ensure reliability and validity of the data, multiple steps were be taken. The researcher address reliability, inter-rater reliability (IRR) for scoring rubrics from the IQA, as well as scoring frequency and duration of student verbal participation in the group-sharing portion of lessons. The researcher, and a current graduate student in elementary mathematics education and an expertise in CGI instruction, rated 2 of the 11 lessons (18%), for agreeability as well as 4 of the 18 participating (22%) student work journals. In scoring student journal, the researcher used a kappa score, using Cohen's D statistical to check for the possibility of scoring by random chance.

Furthermore, the researcher and, the aforementioned graduate student, coded together until IRR of 85% was established. Once the raters met the threshold, scoring occurred independently. At the conclusion of the study, the researchers returned recorded interviews to the participating teachers and school staff interviews for member checking purposes.

CHAPTER 4: FINDINGS

This study focused on the participation of 3rd grade students in an inclusion classroom that utilizes reform based mathematics practices. The conceptual framework for this study intertwined Cognitively Guided instruction (CGI) student learning philosophy and differentiated instruction that take into consideration student ability, interests, and production. The researcher posed the following questions for this study:

1. To what extent do 3th grade students with high-incidence disabilities participate in inclusion reform-based mathematics classrooms?
2. What are the characteristics of student participation among students with disabilities in reform-based inclusion mathematics classrooms?
3. How does the teacher support the learning for all students while implementing CGI philosophies in an inclusive classroom?

The researcher explored these questions by examining the oral expression and written work of students diagnosed with a disability and typically developing students, as well as teacher behaviors in the inclusion setting. This chapter will present the findings for the aforementioned questions.

In sequential order, chapter 4 consists of describing participant demographics, classroom participation data (i.e., Instructional Quality Index (IQA) and student verbal and written participation frequency and duration), student participation characteristic data (i.e., student journal data and classrooms discussion vignettes), followed by data regarding teacher supports (i.e., teacher behavior checklist and teacher interviews). Chapter 4 concludes with a discussion of data reliability and a summary of the data presented in this chapter.

Participant Demographics

The researcher recruited participants for this study from an elementary school located in a suburban area in the southeastern United States. The teacher who participated in this study was an active participant in previous mathematical based professional development sessions in a partnership between the local school district and the current university of the principal investigator.

After obtaining parental consent, and student assent, the researcher collected data through school records and reported by the classroom teacher concerning the age, gender, race, and current diagnosed disability of each child participating in the study. Participating students (n=18) agreed to be included in the study, with one choosing not participate and another received her math lesson in a small group setting in accordance with the accommodations in her IEP. Student participants in this study were all 3rd graders who received their mathematics lessons in the inclusion classroom. The researcher placed students in groups according to current educational services received by each student. These include general education, referred for special education, and receiving special education services. Table 1 details student demographics for participants in this study.

Table 1. *Student Demographics*

Participant	Grade	Gender	Race	Classification	Primary Disability
Amy	3rd	F	White	Gen Ed	
Anna	3rd	F	White	Gen Ed	
Cary	3rd	F	White	504 Plan	ASD
Billy	3rd	M	White	Referred	
Bonnie	3rd	F	White	Referred	
Carol	3rd	F	Asian	Special Education	VI
Chester	3rd	M	White	Special Education	SLD
Abraham	3rd	M	Latino	Gen Ed	
Andre	3rd	M	White	Gifted	
Alex	3rd	M	White	Gen Ed	
Bart	3rd	M	African American	Referred	
Brenna	3rd	F	White	Referred	
Barney	3rd	M	White	Referred	
Aaron	3rd	M	White	Gen Ed	
Adam	3rd	M	White	Gen Ed	
Chris	3rd	M	White	Special Education	OHI
Adele	3rd	F	White	Gen Ed	

Note. ASD= autism spectrum disorder; SLD= specific learning disability; VI= Vison Impairment; OHI=other health impaired.

Student Participation Data

For each group sharing session recorded, the researcher coded student behavior using the Instructional Quality Assessment (IQA) to measure and record class-wide student verbal participation in the group-sharing portion of the CGI lesson (See Table 2.).

Table 2. *Instructional Quality Assessment Data (IQA)*

Session Number	Total Number of Students	Percentage of Class Who Verbally Participated	IQA Participation Quartile
1	18	67	3
2	18	56	3
3	18	33	2
4	18	50	3
5	17	53	3
6	18	44	2
7	18	39	2
8	18	61	3
9	18	56	3
10	18	56	3
11	18	61	3

The researcher video recorded and analyzed 11 group-sharing lessons for both verbal and written participation in the group setting (See Table 3.). Student group classifications in Table 4 shows those receiving only general education services, those who have been referred for possible special education assessment, and those with either an IEP or 504 plan. Frequency of verbal participation occurrences (VPO) was indicated for each instance that a student discussed the relevant mathematical topic within the group setting for duration longer than a one-word answer. Furthermore, frequency of written participation occurrences (WPO) of students was indicated when a student drew or solved a problem on the classroom smart board for all students to observe.

Table 3. *Verbal and Written Student Participation*

	Gen Ed.		Referred		Spec Ed	
Session #	VPO	WPO	VPO	WPO	VPO	WPO
1	8	0	6	1	6	1
2	5	1	5	1	2	1
3	3	2	4	1	1	0
4	3	1	3	0	3	1
5	4	1	3	1	2	0
6	5	1	2	1	5	0
7	6	2	3	1	6	2
8	5	2	2	1	4	2
9	3	1	3	1	4	0
10	2	1	6	0	2	0
11	1	0	2	0	6	2
Totals	45	12	39	8	41	9

Note. VPO = verbal participation occurrence; WPO = written participation occurrence.

The researcher measured the duration of student participation verbal participation using video analysis. The researcher timed student responses, longer than one-word answers or monosyllabic responses in seconds. Again, the researcher classified student groups were as general education, referred for special evaluation, and students with an IEP or 504 Plan. The researcher used a hand calculator in totaling duration of student participation. The researcher totaled verbal participation duration for each sub-group divided by the total number of groups to calculate the percentage that individuals from each group contributed to the overall discussion. The researcher used the IQA to document the amount of *Accountable Talk* between and among students and the teacher in solving mathematic word problems. For the purpose of this study, the definition of accountable talk a verbal communication that relates to subject matter that consists of more than a one-word answer. Table 3 displays the data for group sharing student participation for all students using the IQA.

Table 4. *Verbal Participation Duration During Group Share Time*

	Group Sharing Duration in Minutes	Total Student Participation in Seconds	General Education Group Percentage	Referred Group Percentage	Special Education/504 Group Percentage
1	16	412	31.067	31.067	37.864
2	13	278	33.812	32.374	33.812
3	15	287	67.595	31.010	1.393
4	16	367	27.792	24.250	52.043
5	12	301	29.900	41.528	28.571
6	20	460	46.086	38.913	15.000
7	22	592	36.148	10.641	53.209
8	17	624	39.743	14.743	45.512
9	25	881	42.678	37.570	19.750
10	9	193	36.787	48.186	15.025
11	13	326	1.226	4.294	94.478
M			35.70	28.59	36.05
SD			15.81	13.73	25.43

Verbal participation of student subgroups during group sharing time data was subjected to one-way analysis of variance (ANOVA) for all subgroups with a .05 significance level. Results from this analysis, $F\ (2,30) = .54$, $p = .59$, were found to be statistically non-significant.

Participation Characteristic Data

The researcher collected, copied, and scored written work from the student's mathematics journals. Word problems from the student journals coincide with the problems students discuss during group sharing time; however, Mrs. Hoover did not discuss all problems during this time. Mrs. Hoover assigned 22 problems to the class during the 6-week duration of this study. Table 6, 7, and 8 displays the individual number of problems attempted for each student as well as the mean score for each student in the categories of the scoring rubric. The researcher calculated mean scores for each sub-group by diving the number of points earned under each category divided by the number total number of opportunities for each student. Therefore, students who did not attempt a particular problem received no score in that trial.

Table 5. *General Education Group*

Name	Attempted CGI Problems	Understanding/Reasoning	Strategies/Procedures	Communication
Amy	22	4.0	4.0	3.72
Anna	19	3.94	3.94	3.57
Abraham	21	2.71	3	1.76
Andre	20	3.15	3.2	1.35
Alex	17	2.76	2.70	2
Aaron	16	2.43	2.68	1.25
Adam	19	3.21	3.26	2.47
Adele	20	3.1	3.2	2
Totals	154	25.32	26	18.13
M	19.25	3.16	3.25	2.26

Table 6. *Students Referred for Special Education*

Name	Attempted CGI Problems	Understanding/Reasoning	Strategies/Procedures	Communication
Billy	21	3	3.09	1.47
Bonnie	18	2.83	2.77	1.27
Bart	17	2.23	2.41	1.35
Brenna	21	3.42	3.52	2.95
Barney	19	2.36	2.36	1.63
Totals	96	13.86	14.14	8.68
M	19.2	2.77	2.83	1.73

Table 7. *Students with an IEP or 504 Plan*

Name	Attempted CGI Problems	Understanding/Reasoning	Strategies/Procedures	Communication
Cary	18	3.27	3.11	2.16
Carol	22	3.86	3.68	2.59
Chester	17	2.47	2.47	1.70
Chris	12	2.666	2.666	1.333
Totals	69	12.27	11.928	7.794
M	17.25	3.067	2.982	1.948

The researcher scored the student journal scores for understanding and reasoning, strategies and procedures, and communication to a one-way analysis of variance (ANOVA) for all subgroups with a .05 significance level. Results of the analysis for the category of understanding and reasoning $F(2,14) = .79$, $p = .47$; strategies and procedures $F(2,14) = 1.13$, $p = .35$; and communication $F(2,14) = .71$, $p = .50$ were found to be statistically non-significant**.**

Reliability

In order to ensure reliability, the researcher took random samples from both video recordings of group-sharing lessons and student journals. The research used stratified sampled criteria in order to ensure that representation of all subgroups. The researcher and a fellow graduate student, who has extensive knowledge of CGI, viewed and analyzed two selected lessons independently. The researcher and the graduate student agreed on 97% of verbal and written occurrences, 89% of student verbal and written participation duration. Furthermore, the researcher took student samples of writing journals from four students (22%). These students included two from the general education group, one from the referral group, and one from the group receiving special education services. The researcher and graduate student analyzed 75 total problems for three domains (i.e., understanding and reasoning, strategies/procedures, and communication). The raters agreed on 192 of 225 (85%) scoring opportunities. Furthermore, to control for the possibility of random agreement among raters, the researcher used Cohen's kappa test. The test revealed a kappa score of .73.

Group Sharing Vignettes

The following accounts are descriptions of student and teacher interactions taken from transcriptions of video recordings of classroom mathematic lessons and researcher observations. The researcher chose these six vignettes because they include interactions between students in the general education group and students in either the referred or special education group working on the same novel problem during group sharing time, teacher behavior that included differentiated methods of instruction or assessment, or a combination of both. The following vignettes are designed to illuminate the commonalities and differences in the interactions between students of varying academic and communicative skill levels while sharing their problem solving processes and solutions, and not a definitive description of student behavior to generalize in others settings.

Vignette 1

Mrs. Hoover began the lesson by instructing all the 3rd grade students in class to sit on the large section of carpeting which lays in the front of the classroom near the smart board.

She informed the class, "Today, we are going to be working on solving for the unknown using an array model. So we are going to be talking about what we don't know in division and how we can use an array model to help us." She then typed the following problem on the smart board and read the problem to the students.

> "Stacey has 18 bracelets. After she organizes her bracelets by color, she has 3 equal groups. How many bracelets are in each group?"

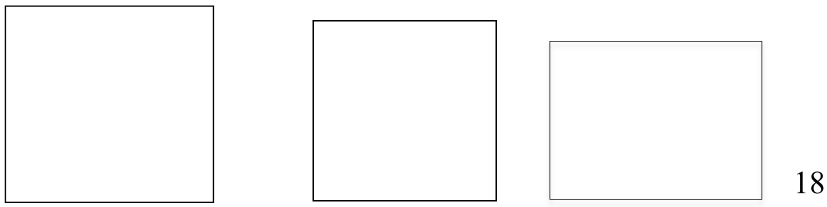
After reading the problem to the entire class, Mrs. Hoover asked the students if any of them would like to share a method that they used in order to solve the problem. Six students raised their hands and she called on Amy to come to the board and share her work. Amy, a student in general education who volunteered often to share her work with the group, arose from the carpeting and walked to the front of the room.

As Amy prepared to demonstrate to the class how she solved the question, Mrs. Hoover asks the class, "Can you only solve this problem one way?"

The class responded, "No!"

Mrs. Hoover continued, "Good, we know that there are multiple ways to solve the same problem, so lets see what Amy did." Amy begins by drawing 3 boxes approximately the same size with the 18 next to them (Figure 4).

Figure 4. *Amy's sample work a*



Amy explained that she has 3 groups and needs to place the eighteen bracelets equally within the three groups.

Mrs. Hoover asked, "How did you know that?"

Amy responded by pointing to the portion of the problem on the board that stated the there are 3 equal parts and said, "it says that there are 3 equal parts here."

Mrs. Hoover then turned to the entire class and said, "So she decided to show squares. Did anyone else use the same strategy?" Seven students responded by raising their hands.

Mrs. Hoover continued by asking Amy, "I noticed that you wrote the number 18 next to your boxes, can you explain why you did that?"

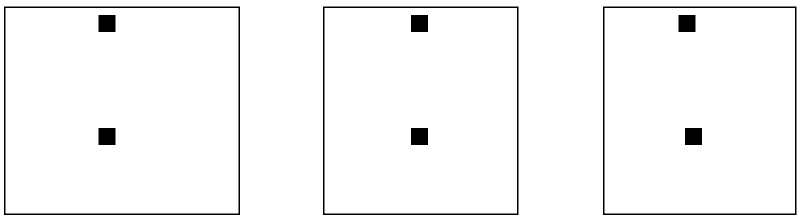
Amy replied by telling Mrs. Hoover and the entire class, "because that's what we have to put in the 3 equal groups, and it helps me remember." The group-sharing lesson continued with Amy demonstrating how she distributed the bracelets equally in the 3 groups. Amy explained, "I passed them out by 2's until I didn't have any left."

Mrs. Hoover, turning to the whole group asked, "How many of you passed them 1 at a time?" and 2 students raised their hands. "That's ok!" she exclaimed. "Amy decided to pass them out by 2's and how did you know that 2's would work?"

Alex, a student in general education raised his hand and said, "I know that it will fit because 2 and 18 are both even numbers."

“How many people agree?” Mrs. Hoover responded. The entire class answered chorally, with an enthusiastic, “Yes!”

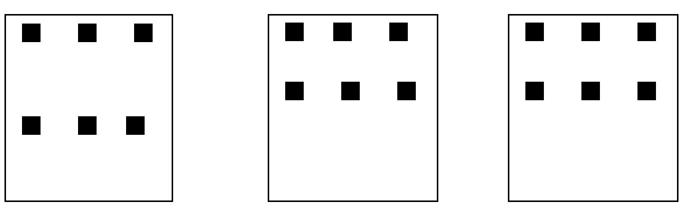
Figure 5. *Amy’s sample work b*



After getting other students’ input on different methods of distributing the bracelets in this problem, Mrs. Hoover asked, “Amy can you please share with us what your answer is for this problem?”

Amy then shared the 3 boxes, each with 6 dots inside of them, representing the eighteen total bracelets evenly distributed between 3 groups (Figure 6).

Figure 6. *Amy’s sample work c*



Mrs. Hoover then transitioned the class from Amy’s solution to giving another student an opportunity to share their response to the problem. She asked the class, “Who else would like to share how they solved the problem that is different from Amy’s method?”

Cary, a student who receives special education services for Autism Spectrum Disorder (ASD), raised her hand and is called to the front of the class to share her work. Cary draws a number bond on the board (Figure 7), and stands silently when finished.

Mrs. Hoover asked the class as a whole, “Do you think that is the way she solved the problem or the way she showed her solution?" A few students responded with saying that she showed her solution and not her method for solving the problem. Mrs. Hoover then asked Cary, “Can you talk about this?”

Cary, looking at Mrs. Hoover explained, “Um. Ok. I did a number bond.”

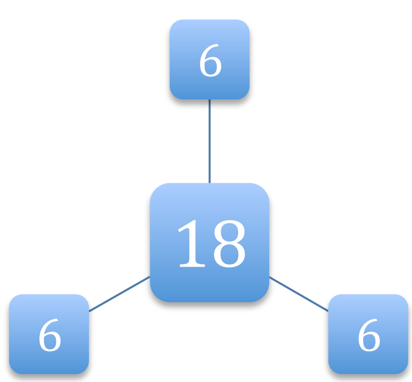
“And how did that help you solve this problem?” Mrs. Hoover asked.

Cary responded, “Because, um. I already knew the answer.”

Mrs. Hoover, attempting to rephrase and provide support for Cary said, “So you already knew that 3 groups of 6 was going to give you 18? Ok, if you already knew that, would it be ok to solve it this way?”

“Yes”, replied Cary.

Figure 7. *Cary’s sample work a*



After Cary and Amy shared their work, and all students were seated on the carpeting again, Mrs. Hoover talked about how using mental math is appropriate at many different times, but being able to check the solution of work is always important.

Vignette 2

Mrs. Hoover began the group-sharing lesson by asking all students in her class to gather on the carpeting in front of the smart board and to bring their math student journals with them. Once the students found their place on the floor, she wrote the following question on the board:

> "Eli has some candy. If there are ____ pieces of candy in each bag and there are _____ total pieces, how many bags does Eli have? (2,10) (4,36) (8,11)"

Mrs. Hoover then asked Adam, a student in general education, to read the problem to the entire class. She stops Adam in the middle of reading the problem and redirects a group of students from talking to each other and to listen to the problem. Once the students quieted down, Adam reread the problem for the class, without any numbers included in the problem.

Mrs. Hoover then asked, "What are you thinking about this problem? Don't tell me the answer, because I didn't even put any numbers up there. I did that for a reason. So what are you thinking right now, with no numbers?" She paused and then continued, "Cary, what are you thinking?"

Cary, a student with ASD, stands in the back of the group and faces the class. Cary states, "I need to know the number of groups".

Mrs. Hoover then asked the class if they agree by responding to Cary with a hand gesture. Nearly all of the students hold up their fist and stick out their pinky and thumb and twist it. Mrs. Hoover continues, "Cary, tell me what are the clues that helped you understand that you need to know the number of groups?"

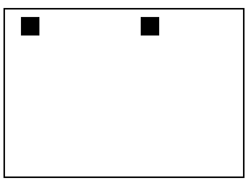
Cary replied, "Um, there are 'some' bags and the bags are the groups."

"So what do you need to know in the problem?" asked Mrs. Hoover.

Cary replied, "Ah…I need to know the number of groups and the total number of pieces."

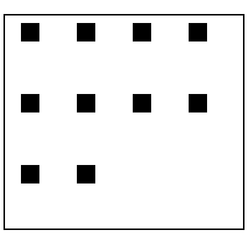
After discussing the mathematical problem, sans numbers, with the entire group, Mrs. Hoover inserted the numbers 2 and 10 into the problem. She then calls on Bart, to come up to the front of the group and read the problem with numbers out loud. Bart, a student who has been referred for special education assessment, drew a single box with 2 black dots inside of it (Figure 8).

Figure 8. *Bart's sample work a*



Mrs. Hoover, after seeing Bart's work, reread the problem out loud for him. Again, after listening to the problem for a second time, Bart then drew a single box with 10 black dots inside it (shown in Figure 9).

Figure 9. *Bart's sample work b*

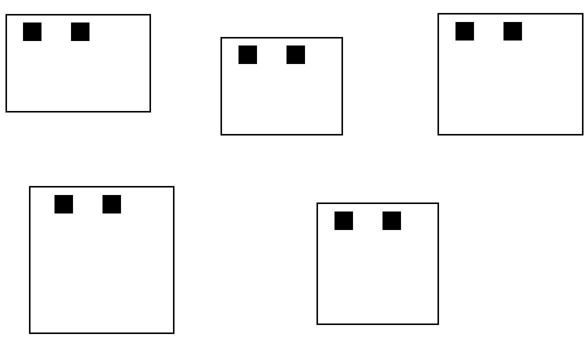


Mrs. Hoover responded to Bart by saying, "I see you have a bag with 10 pieces of candy in it. Lets read the question again together and check our work." Mrs. Hoover emphasized the word "two" in a louder voice as she and Bart read the problem from the smart board. After rereading the problem, Mrs. Hoover, asked, "Bart, can you think of anything else that you can do in this problem?" Bart stood up, looking at the problem written on the board for approximately 30 seconds. Mrs. Hoover, allowing time for Bart to think about a new approach to problem, praised the rest of the students for waiting patiently and not "shouting the answer out." As Bart stared at the question, Mrs. Hoover picked up 10 plastic blocks located in a box next to the smart board. She hands the cubes to Bart and asked, "How many blocks do we need?"

He responded by saying "Ten. We need ten total blocks."

She responded by saying, "Great. Ok, and how many go in each group?" Bart does not respond verbally, but takes the blocks and separates them into five different groups of two on the carpeting. Bart then stood up and drew five separate boxes, each filled with two black dots (shown in Figure 10).

Figure 10. *Bart's sample work c*



After Bart displayed his work for the class, Mrs. Hoover proceeded to ask him questions about his answer. "How did that help you know how many bags Eli has," asked Mrs. Hoover.

"Once I passed out the blocks by two, I ran out of blocks and counted the number of groups," replied Bart.

Mrs. Hoover continued, "Great, and how many groups were there?"

"Five," replied Bart.

Mrs. Hoover then thanked Bart for his effort and directed him to sit among the other students on the carpeting again.

With the remaining time left during group-sharing time, Mrs. Hoover asked the students if they would like to make an attempt to solve the same problem with a different set of numbers. She proceeded to then write the sets of numbers (4, 36) and (11, 88) on the smart board for the students to see. "Who would like to try the problem with one of these sets of numbers? Adele, would like to try?"

Adele, a student in general education, walks to the front of the room and begins to draw a large circle with the number 36 in middle of it. She then drew eight spokes off of the larger circle capped with smaller circles. Within each of the smaller circles, Adele drew the number 3. As Adele finished drawing the 3's within the smaller circles students on the carpeting began to raise their hands.

Mrs. Hoover waited for Adele to finish filling in all the smaller circles with the number 3, and then calls on Bonnie. "Bonnie do you have something you would like to share or add to Adele?" asked Mrs. Hoover.

Bonnie, a student referred for special education assessment, said "Adele, I think you may have put 3's in for 4's in that problem".

"You're right, you're right! Oops, thank you," said Adele.

As Adele turns around and changes all of the 3's to 4's in the smaller circles, Mrs. Hoover asked Adele, "Could you tell us about what you drew on the board."

Adele responded by saying, "Um. Yeah. So I knew I had 36 pieces of candy and I put 4 pieces in each bag. I think I got confused when I wrote down 3 instead of 4. But I just counted up all the bags and got 8."

“Okay, great,” replied Mrs. Hoover. “I think you all did a great job talking about the different ways to solve this problem.”

Vignette 3

“Ok Anna, would you like to read the problem in your loudest teacher voice?” asked, Mrs. Hoover as the students gathered again on the carpeting to discuss a novel mathematical word problem for the day. After waiting for the students to find their spaces on the carpeting, Anna read the following problem out loud:

> “John has ___ Tomatoes on each plant. If he has ____ tomatoes altogether, how many tomato plants does he have now? (6, 24) (10, 80) (17, 51)”

Mrs. Hoover continued by saying, “We are going to start with putting the numbers 6 and 24 in this problem although I heard some of you say that the second set of numbers is actually easier.”

Many of the students responded by loudly saying, “It is easier!”

Smiling, Mrs. Hoover says, “Ok, if you are going to tell me that, I want you to tell me the reason why. Why do you think some of the sets of numbers are easier to use that the other ones?”

Chester, a student diagnosed with a learning disability (LD), said to the group, “Because one of the numbers is 10 and 10 can go into both numbers.”

Mrs. Hoover replied, “And you know your multiples of 10 pretty easily, right? And how long have you been counting by 10’s, Chester?”

Chester replied, “A long time.”

“Well, lets start with the 6 and the 24, who would like to try it?” Amy, a student in general education raises her and is directed to the front of the room. She writes 24/6 = 4. “Can you explain this, Amy?”

"Yes," replied Amy, "24 divided by 6 is 4 and 4 times 6 is 24."

"Ok. Great. Is there another way you could have done it?"

Amy nods and said, "Yes." Amy then turns and draws 6 circles on the smart board (shown in Figure 11). Amy rereads the problem again and when she finished Mrs. Hoover asked her to explain what the circles represent. "They represent each group of 4." Amy starred at the circles on the board for a few moments.

Figure 11. *Amy's work sample d*

O O O O O O

"So what are you trying to find, Amy?" asked Mrs. Hoover.

"The total number of plants." replied Amy, while looking frustrated.

"Right, and you are being a great learning example for everyone in her right now."

Amy then exclaimed, "Oh, we are looking for how many groups!" She then drew 4 boxes with the numbers 6, 12, 18, and 24 in them (Figure 12).

Barney, a student referred for special education, raises his hand and says "I like how you counted by 6's in each box because it tells you where to stop."

Mrs. Hoover then thanked Amy for sharing her work and asks her to retake her seat on the carpeting.

Figure 12. *Amy's work sample e*

6	12	18	24

“Ok, who hasn’t been up here to share their work in the last few days? Barney, come on up and share your work with some new numbers. Would you like to use the harder one or the hardest ones?” Mrs. Hoover said.

“I want to try the hardest ones,” Barney replied.

“Ok, lets do it! What information do we already have?” Mrs. Hoover stated with enthusiasm.

“Well, we know that there are 17 tomatoes on each plant and there are total of 51 total tomatoes,” replied Barney.

Mrs. Hoover asked, “Very good, how many people think Barney is correct?”

Nearly all of the students raised their hand in the air with their thumb and pinky extended in a twisting motion. Barney then proceeds to draw 3 boxes with the numbers 17, 34, and 51 in them respectively (Figure 13).

“Can you explain this, Barney?” asked Mrs. Hoover.

“Yes, I started counting by 17 and when I got to the total number, I just quit counting. Then I added up the number of groups I had and I have 3 groups. So the answer is 3.”

Mrs. Hoover continued, “He used a ‘count by’, does anyone have anything they want to add to this? Wait Barney, did you use mental math or did you add them together using a pencil and paper?”

Barney replied, “I added 10 and 10 and 14 to get 34. I decomposed them. Then I added 10 more and 7 again on my paper.”

Mrs. Hoover restated Barney’s thought by saying “So you wrote 17 + 17 + 17 is 54. Can anyone else think of another way to write this?”

Andre, A student qualified for the gifted program, said, “17 times 3 is the same thing.”

“Great”, replied Mrs. Hoover.

Figure 13. *Barney’s sample work*

$\boxed{17}$ $\boxed{34}$ $\boxed{51}$

Mrs. Hoover, transitioned from one student’s work to another by asking “Carol, will you come up here and show us how you solved this problem?” Carol, a student who receives special education for a vision impairment (VI) and also consultation services for the gifted and talented program made her way to the smart board.

Quietly, she asked Mrs. Hoover to move the problem down from the top of the screen so that she could reread the problem when standing close to the board and said, “I can do this with mental math.”

Mrs. Hoover replied, “what if you couldn’t do this with mental math, could you think of another way to do it?”

Without speaking, Carol wrote 17 X 3 and decomposed the 17 into a 10 and a 7. She then rewrote the equations as 7 times 3 equals 21 and 10 times 3 equals 30. Continuing, she added both of the products together to arrive at the answer of 51 (Figure 14).

“Great job, Carol, Would anyone else like to pick some number out that we could get when we decompose 17?”

Students throughout the room yell “15 and 2, 12 and 5, and 11 and 6.”

“Lets 11 and 6 before we run out of time,” Mrs. Hoover said hurriedly. She then asked Abraham, a student in general education, to explain how this would work.

"Well, if we multiply 11 times 3 we will get 33 and then we need to multiply 6 times 3 and we get 18. When we add them together we should get 51" (Figure 15).

"You all did a great job today. Please put away your notebooks and take your seat in your desks again."

Figure 14. *Carol's sample work*

7 X 3 =21

<u>10 X 3 = 30</u>

51

Figure 15. *Abraham's sample work*

11 X 3 = 33

<u>6 X 3 = 18</u>

51

Vignette 4

"Today's problem is going to have two different answers." Mrs. Hoover stated as the students were finding their places on the floor. She continued, "Alex, when everyone gets situated, will you please the read the problem up on the smart board for everyone to hear." Alex, a student in general education, reads the following problem:

> "Anna has 4 Christmas trees at home. Each tree has 14 ornaments. How many ornaments does she have? Two of the trees fall down. How many ornaments does Anna have left standing?"

"Chris, do you think you could help us do this problem? Did you do it?" asked Mrs. Hoover.

"I think I did a different one," he replied. Chris, a student diagnosed with Attention Deficit with Hyperactivity Disorder (ADHD), stood in front of the class. "Can I try it anyway?" he asked he Mrs. Hoover.

She then responded, "Ok, you want to try it. Do you need me to reread the problem?"

"No, I can do it," he replied.

Mrs. Hoover then instructed him to do the first part of the problem and to save the second part of the problem for someone else. After a few moments of staring at the problem Chris stated, "I don't know how to do this, can I get help from someone else?"

"Ok, Chris," Mrs. Hoover replied. "Andre, will you come up and show us what you did?"

As Andre, a student in general education, makes his way to the smart board, Chris finds his seat on the carpeting. Andre then proceeded to write four boxes each containing the number 14 in it on the board (Figure 16).

Figure 16. *Andre's sample work a*

$\boxed{14}$ $\boxed{14}$ $\boxed{14}$ $\boxed{14}$ $\boxed{14}$

"Can you tell us what you did," asked Mrs. Hoover.

"I know that there are five trees and that each tree has 14 ornaments. So I need to just count them up," stated Andre.

"Whoa," Mrs. Hoover said. "That's going to be a big number."

"It's 70," he replied without hesitation.

Mrs. Hoover continued, "How did you get 70?"

"Well, I multiplied 5 times 14 and got 70," he said. As he starts to write on the smart board again, the smart board suddenly stopped working. Mrs. Hoover then directed him to use the large easel next to smart board and he draws five boxes, each with a number inside of it, increased by 14, ranging from 14 to 70 (Figure 17).

Figure 17. *Andre's work sample b*

$\boxed{14}$ $\boxed{28}$ $\boxed{42}$ $\boxed{56}$ $\boxed{70}$

"That's right! I thought you would write 14 + 14 + 14 + 14 + 14 +14 = 70. This is great mental math, Andre. Did anyone use a different way of getting to 70? First of all, how many people got 70?" stated Mrs. Hoover. Nearly all of the students raised their hands. "Ok, great, he used a 'count by' strategy would anyone else like to share?" She continued, "Anna go." As Anna made her way to the front of the room, Mrs. Hoover instructed the general education student, "Anna, tell everyone about your strategy and what you did."

"Um...ok." Anna began, "I decomposed the 14 into 10 and 4 and I know that 10 times 5 is 50 and that 4 times is 5 is 20. Then I added them together."

Mrs. Hoover then walked to the smart board and pointed to Anna's model and asked her what property she used. Anna stated that she didn't know. "Can anyone help her, maybe?" asked Mrs. Hoover.

Billy, a student referred for special education assessment, yelled out "'dis something property!"

"Yes! The distributive property." "Are you checking your work now Anna? And how are you doing that?" asked Mrs. Hoover.

"I'm dividing 70 by 5." "So what answer are you expecting?" replied Mrs. Hoover.

"It should come out to 14," replied Anna.

"Great," responded Mrs. Hoover.

“So what are we going to do with the rest of our problem? If we know the first part of our problem is 70. Aaron will you come up here and finish it for us?” asked Mrs. Hoover. Aaron arose from his seat on the carpeting and walked to the smart board. On the board, he drew two separate equations (Figure 18). “Can you explain this for us, Aaron?” asked Mrs. Hoover.

“Yeah, so I added up two trees and then subtracted that from 70. And, 70 is the total number of ornaments I had in the beginning.” explained Aaron.

Figure 18. *Aaron’s sample work*

70 – 28 = 42

14 + 14 = 28

Vignette 5

After calling of all of the students onto the carpeting, Mrs. Hoover read the following problems for the entire class.

> “The student council holds a meeting in Mr. Chang’s classroom. They arrange the chairs in 3 rows of 5 chairs. How many chairs are used in all? Use the Read, Draw, and Write (RDW) process.”

Mrs. Hoover instructed the students to work on the problem silently and independently in their notebooks while seated on the floor. As they worked, she walked around and between students observing their work. “I see some interesting drawings, I see a lot of people working very hard, I see some different ways to draw the problem,” Mrs. Hoover stated as she walks through the students working. She continued, “Again, just a reminder, please do not draw each individual chair, because we will be here all day. Please draw circles or squares in place of the actual chairs.” After a few more minutes, Mrs. Hoover has the students stop working and calls Amy up to the board to share her strategy. Amy, a student in general education, drew 15 circles in 3 rows of 5 on the board (Figure19) and the students raise their hands twist their fists in agreement.

Figure 19. *Amy's sample work f*

O O O O O
O O O O O
O O O O O

"Amy drew an array. Can you tell us about it Amy?" asked Mrs. Hoover.

"Sure. It has 3 rows and 5 columns and I can count them up or multiply them if I want," replied Amy.

Mrs. Hoover then asked if anyone else had something they would like to add to Amy's Model. Billy, a student referred for special education, raised his hand and makes his way to the front of the class. He writes the equation 3 times 5 equals and 5 plus 5 plus 5 equals 15 (shown in Figure 20).

Figure 20. *Billy's sample work*

3 X 5 =15

5 + 5 + 5

Mrs. Hoover then asked the entire class, "Does anyone know what he doing on the bottom? What is that called?"

A group of students replied with "repeated addition."

Mrs. Hoover, then returned to Billy and asked, "Billy, why did you write two separate equations?"

He replied, "Well, I thought it was a good way to show that I checked my work."

Turning back to the group, Mrs. Hoover then asked Cary, a student with (ASD), to explain how the repeated addition related to the array model that Amy created.

"Well, it shows that there are 3's and 5's. Well, 3 groups of 5," answered Amy.

Mrs. Hoover then asked Adam, a student in general education, to come a show his model for the class. Adam arose from the carpet and drew the same array model that Amy had shown earlier, with the addition of lines underneath each section (Figure 21). Adam went on to describe to the class that he understands that he could have 3 groups of 5 or 5 groups of 3 but he prefers adding by 5 instead of 3.

Figure 21. *Adam's sample work*

OOOOO
OOOOO
OOOOO

After thanking Adam for showing his model, Mrs. Hoover then drew a model that showed two sides of the array that both Adam and Amy drew, but without all of the circles filled in the middle (Figure 22). She explained, "This is called a 'fast array'. We will be using this later on when we larger sets of numbers. And when talk about 'area'. How many of you have heard of 'area' in math?" Approximately half the class raised their hands.

Figure 22. *Mrs. Hoover's solution example*

```
   O
3  O
   O O O O O
     5
```

Mrs. Hoover concluded this portion of the group-sharing by asking Barney, a student referred for special education, what the difference is between his equation (shown in Figure 23) and Billy's equation of 3 times 5 equals 15.

Barney explained, while seated, that the two equations are the same but written "backwards." He tells the class, and Mrs. Hoover, that because you would get the same answer for both of them, "just like an addition problem too."

Figure 23. *Barney's work sample*

5 X 3 =15

Vignette 6

After waiting for the students to gather on the carpeting and quiet down, Mrs. Hoover read the following problem to the class.

Dana has some cupcakes. She eats 43 of them. Now she has 20 left. How many did she start with?

Mrs. Hoover asked the class, "Tell me what you are thinking when you read that first sentence."

Chester, a student in special education diagnoses with a specific learning disability (SLD) raised his hand and said, "that she has some cupcakes and that some is more than none."

Mrs. Hoover then continued to question Chester and asked, "So what do we call something in math when we don't what it is?"

"A hypothesis," replied Chester.

"Not quite. Amy, do you know?" asked, Mrs. Hoover.

Amy, a student in general education, takes a moment and then says, "an unknown."

"Excellent!" Mrs. Hoover replied. "Billy, please read the next sentence in our problem."

Billy, a student referred for special education assessment proceeded to read the problem out loud for the class. When he finished, Mrs. Hoover asked him to think about the first two sentences together of the problem.

After a few moments of silence he says, “I don’t know.”

Amy, without raising her hand says, “I think it means we are going to have to subtract.”

Mrs. Hoover turns to Amy and asked her, “What words tell you that?”

“Well, because if you eat them they are going away. I think we should add 43 and 20 to find the total. Oh, wait. No.”

Mrs. Hoover then asked the students to raise their hand if they have a strategy that they would like to share if it helped them figure out the problem.

Cary, a student with autism, raised her hand and shared her strategy. “Can I just tell you what to write?”

Without any hesitation, Mrs. Hoover says “Absolutely! Tell me what you like me to write on the board.”

Cary, began by saying, “I decomposed the numbers.”

Mrs. Hoover replied, “Ok. Wait a second, tell me what you what you did before that. How did you know what to do?”

Cary, paused for a moment and replied, “Um, because it says how many cupcakes did Dana have to start with.”

Mrs. Hoover continued her line of questioning to Cary, “Ok. So did you write an equation at all?”

Again, there are a few moments of silence before Cary gives her response. She said, “I wrote a vertical equation.”

Mrs. Hoover then replied, “Tell what the vertical equation that you wrote.” (Figure 24)

Figure 24. *Cary’s work sample*

$$\begin{array}{r} 20 \\ \underline{\underline{+43}} \\ 63 \end{array}$$

“Did anyone write something different than this? asked Mrs. Hoover, while pointing to Cary’s equation. Two students raised their hands. “What did you do?” asked Mrs. Hoover to Aaron.

Aaron, a student in general education, described how created a “math mountain” to solve the problem. Aaron headed to board and drew an inverted “V” with the numbers 20 and 43 placed at each leg at the base.

“Tell us about this, Aaron,” Mrs. Hoover said for the entire class to hear. “Well, I know that 43 and 20 are my two partners.”

“Did anyone do it this way?” asked Mrs. Hoover, while drawing a box on the board (Figure 25).

“I did it that way,” says a Bonnie, a student referred for special education assessment.

Figure 25. *Mrs. Hoover’s work sample b*

$$\boxed{}$$

“What did you put next to the box?” asked Mrs. Hoover.

“I wrote the box minus 20 equals equal 43.” (Figure 26)

Some of the students voice their disagreement with Bonnie’s answer.

“You are getting some feedback from some other people. Would you like to change anything about your answer?” asked Mrs. Hoover to Bonnie.

Bonnie stood for a moment, staring at the problem, and replied, “Yes, I want to change the 43 and the 20.” Mrs. Hoover then rewrote the problem as Bonnie stated (Figure 27).

Figure 26. *Bonnie’s work sample a*

$\boxed{}$ - 20 =43

Figure 27. *Bonnie’s work sample b*

$\boxed{}$ - 43 =20

“Is there something we can put in that box instead of having a box? Andre, what do you think?”

Andre responded quickly by a saying “a letter.”

“What letter?” Mrs. Hoover replied.

“Any letter,” said Andre.

“Pick one, Andre,” continued Mrs. Hoover.

“An ‘A’”, said Andre.

“So this called an um…,” Mrs. Hoover said slowly.

The class replied as a group, “known!”

“Great! Could this be a ‘B’”? Mrs. Hoover asked as she pointed to box.

“Yes!” replied the students.

“Could this be a ‘T””? continued Mrs. Hoover.

“No!” replied the students.

Adele, a student in general education, then explained how it could be a “T” but it would not be a good idea because it looks too much like a plus sign and that might confuse a student.

“Ok, so lets not pick a 'T’ then,” responded Mrs. Hoover.

Cary then raised her hand and added that using an ‘x’ wouldn’t be a good idea either because it looks too much like a multiplication sign.

“That’s a good point, Cary,” said Mrs. Hoover.

Mrs. Hoover concluded the lesson by calling Anna to the board to have her demonstrate how she would solve the problem. Anna drew a vertical equation of 43 + 20 = 63. Mrs. Hoover then asked her if that if she did not have a piece of paper, how she might solve it?

Anna replied, “I would count by 10’s. Starting at 43 and going to 53 and then 63.”

“Good” said Mrs. Hoover, “Barney, how would you do it?”

“I did the same thing, except I started at 20 and then added 43. 20, 30, 40, 50, 60 and then 3 more.”

Mrs. Hoover continued, “Did it work when you reversed it?” “Oh yeah,” replied, Barney enthusiastically.

Teacher Support Data

In order to gain a better understanding of the educational and professional experiences which shape the current pedagogical perspectives of the classroom teacher, the researcher interviewed the classroom teacher, Mrs. Hoover and the academically intellectually gifted (AIG) teacher, Mrs. Bronte in face-to-face format. The interviews were transcribed and coded for contextual information in regards to teacher training, professional development in regards to CGI training, and classroom expectations for students with and without disabilities in her classroom.

Mrs. Hoover Interview

During the interview, Mrs. Hoover, explained that she currently holds a K-5 teaching license and is in her 19th year being a classroom teacher. Furthermore, she described her pre-service teacher-training program in relation to working with students with disabilities as "three or four classes," as well as receiving professional development during her career regarding supporting students with "autism and spectrum disorders". In addition, she also mentioned that informal trainings with behavior specialists and special education teachers over the years had also shaped her current perspective on ways to support student learning in her classroom.

Within the school, Mrs. Hoover's classroom has the highest concentration of students who have an IEP, 504 Plan, or referral for additional academic or behavioral supports. She explained the process of student placement as:

> "The principal, the administration sets it up. They will make clusters of students based on either their EC (Exceptional Children) diagnosis or their ELL (English Language Learner) status. With EC, at least up until this year, they have taken into account how far they are below grade level and the amount of time that they need to need to be pulled out rather than being pushed in with in inclusion. The question has been, who would benefit from the inclusion classroom?"

Moreover, Mrs. Hoover on to explained that her classroom is considered an "inclusion" classroom within the setting of her specific school, even though not all students receive instruction in her classroom due to either ability or behavior. One student, who agree to be a part of this study receive her mathematics instructions in a "pull out" classroom in which the school is able to provide more one-on-one instruction and attention. Mrs. Hoover explains the details that shape the decision making process for placing students in specific locations in for mathematics instruction:

> "There are other students with disabilities that are not in this classroom who receive their services by being pulled out. I do have one child in here who does not really fit the inclusion model that we have done and she doesn't receive math in here. She's in here

> based on personality of the teacher who would be able to handle her the best so that goes into it too."

Mrs. Hoover believes that the current inclusion model that the school uses is a product of past difficulties in addressing the individual needs of students in the inclusion setting. She explained how the inclusion model in her school has evolved during her time there:

> "I have been doing the inclusion classroom since I first got here. I think it was I just ended up with it at first. Then they realized I was good at it and I could work well with the EC teacher. It's changed, the model. That year they put about 11 kids that had special needs in one class and it was just overwhelming. There were just way too many. This year it seems to be there are kids sprinkled throughout the other classrooms that have special needs but need more of a pull out model because they're two or more grade levels behind."

When asked to describe the academic and behavioral characteristics of students in her class that either qualify for special education services or are in the process of referral for additional services, Mrs. Hoover described both student attributes and challenges in her classroom. Her perspective reflects the complex nature of solving novel word problems in a group setting where a wide range of ability exists in reading fluency, reading comprehension, procedural fluency, computation, and writing and oral expression skills. She discussed students who receive special education services as being "smartest," "excellent," "brilliant," and "making huge gains" in regards to working with others, problem solving, and communicating. Conversely, she described specific difficulties for these same students as being "easily distracted" at times, struggling with reading comprehension, "getting thoughts down on paper" and computing errors.

Apart from the individual goals that Mrs. Hoover has for specific students in her classroom, she also described the desired learning atmosphere for all students. She spoke about the possibilities of friction among students who possess a wide range of academic skill sets who are working cooperatively to solve novel mathematical word problems. She stated:

> "I have kids that are more struggling in math than kids who are labeled EC in here. I mean I have kids that are not EC that just either have a low IQ or just working with what they have. It sometimes takes twice as long to get a concept than with one of my children who are labeled as EC."

Furthermore, she goes on to explain that students who receive special education services in her classroom still hold anonymity among general education students. She explains:

> "I don't think they even know who they are. I don't think anyone knows in here, I mean other than the one little girl who was born at 23 weeks who looks obviously different and talks differently. I don't think any of them have any clue who's an EC child in here. That's what was my biggest goal for somebody to come in here and not have any idea. Yeah, I don't think they know because we do mixed abilities so much that nobody would say like 'Oh, he's the smart kid,' or 'Oh he's not the smart kid in math.'"

Mrs. Bronte Interview

Two of the students who attend in Mrs. Hoover's math class, Dahlia and Gary, also receive additional supports through the AIG specialist. Mrs. Bronte, who earned a political science degree, a K-6 generalist certificate, 6-9 English Language leaner teaching certificate, and a Master's degree in gifted education, works with each of the students once a week on logic and problem solving in her office. Although most students do not receive AIG services until 4^{th} grade within this school district, Gary moved to this school already receiving services, and Dahlia received a parent nomination in the first grade. Mrs. Bronte explains "students who are K-2 that are showing a demonstrated need of something different than is being offered in the general education classroom we will do individual ID's younger than third grade. Dahlia was one of those students."

Dahlia, who qualifies for special education services under the label of vision impairment (VI), exceeds in reasoning, logic, and number sense, according to Mrs. Bronte. These skills are transferable to problem solving within context of written journals within the reform mathematics classroom. However, Mrs. Bronte is aware of some of the difficulties that she has in the group setting of mathematical discourse. She explained these challenges as:

> "Dahlia, some of her struggles are situational. It's hard to tell how much of that is connected to her very severe vision impairment…When you're talking through the math with her, or she's sharing her thinking, which sometimes takes a while to get her to share because she's not always, in the past, has not always been super eager to volunteer. Maybe that's going to come with being more comfortable, more mature, as she grows."

Teacher Behavior Codes

The researcher collected and coded classroom video data for teacher behaviors within the framework of the National Council of Teachers of Mathematics' *Principles to Actions Executive Summary* (NCTM, 2015) recommended teaching practices. A sample of 6 classroom sessions were recorded, transcribed, coded, and analyzed. The most recent teacher behavior recommendations, the *National Council of Teachers of Mathematics' Principles to Actions* (NCTM, 2014) provided the framework for this section. The eight behaviors they recommend include: establishing mathematical goals, implementing tasks that promote reasoning, using and connecting mathematical reasoning, facilitating meaningful discourse, asking purposeful questions, building conceptual understanding, supporting the struggle in learning mathematics, and extracting and using the evidence of student thinking. Table 8 displays the occurrence of teacher behavior behaviors that support each recommended teacher practice from a sample of six group-sharing lessons.

Table 8. *Checklist for NCTM's Principals to Actions Teacher Behaviors*

Session	Established Mathematical Goals	Promoted Reasoning and Problem Solving	Used and Connect Mathematical Representations	Facilitated Meaningful Discourse	Posed Purposeful Questions	Built Procedural Fluency	Supported the Productive Struggle of Students	Showed Evidence of Student Thinking
1		X	X	X	X	X	X	X
2	X	X	X	X	X		X	X
3		X	X	X	X	X	X	X
4		X	X	X	X	X		X
5		X	X	X	X		X	X
6		X	X	X	X	X	X	X

Teacher Behavior

Evidence of teacher behavior, which aligns with the National Council of Teacher of Mathematics' *Principles to Action Executive Summary*, were direct quotes taken from the transcripts of a sample six lessons during group-sharing time. The duration of these lessons ranged from nine to 25 minutes in length and incorporated the possible discourse of 18 students. The researcher checked for occurrence each practice through teacher facilitation, but did not measure for frequency or duration of during the lessons. The following data are examples of teacher phrases that support each of the eight teacher practices.

Established mathematical goals. Of the six sample lessons examined, the only example of the teacher addressing either short-term lesson goals or long-term unit goals is "Today we going to be working on interpreting the unknown using a division and an array model". It is possible that the classroom teacher discussed individual and class-wide mathematical goals outside of the group-sharing environment, but the researcher did not collect data during those times. Furthermore, the classroom teacher's perspective of "solving novel problems" may have influenced her lack of discourse in explaining the purpose for solving mathematical dilemmas during group discussions or explicitly tying those student solutions to larger instructional goals.

Promoted reasoning and problem solving. Mrs. Hoover encouraged her students often, throughout classroom discussion, to explain their rationale and proof in working toward mathematical solutions. The researcher observed her questioning students at the onset of student receiving a problem where as she asked students to deconstruct the words and clues embedded in the problems. Examples of this included, "What did the problem tell you to do there?" and "What words tell you that?" Moreover, transcripts of group lessons also revealed that Mrs. Hoover asked students to explain their reasoning during the process of solving (e.g., "So you already knew that three of six was going to give you the answer. So if you already knew that would it be ok would it be ok to show the answer in a different way?")   Finally, at the conclusion of problems, Mrs. Hoover often addressed improving students "number sense" concepts through challenging students to inspect a familiar problem with fewer tools available. An example of this is "If you didn't have a piece of paper and pencil, what might be a mental strategy you could use?"

Used and connected mathematical thinking. Through student questioning, Mrs. Hoover connected the thinking of students to one another. She addressed this teaching practice in both the written and verbal components of group-sharing time. She asked students to incorporate others processes into their own that they found useful. "This is what I would like you to do. If there is something that you learned from someone else while we were doing this. I would like you to add it to your math notebook." Furthermore, as she elicited student discourse, she attempted to tie student thinking to one another through conversation. She stated, "A two-step word problem, I see Cary saying ‘yes’. She agrees. Anybody else agree?” Because multiple methods exist for solving problems, Mrs. Hoover did not appear to be looking for student consensus in techniques in solving problems, but instead, shared common ground of student thinking. This is evident in her statement and question, “So she decided to draw circles. Did anyone use the same strategies?”

Facilitated meaningful discourse. During group sharing time, Mrs. Hoover directed the discourse during group sharing time towards solving the mathematical task. Mrs. Hoover, directed questions to the entire group, for example, “How many people decided to pass them out one at a time…That’s ok…Amy decided to pass them out by two’s. How do you know 18 would work with two’s?” as well as, “Did anyone write anything different?” In addition, she also directed questions to individuals to illuminate the similarities and differences of student thinking and methods. Asking the question repeatedly during group sharing time, “Is there anything else someone would like to add or share that you did?” Furthermore, when students disagreed on a solution or had difficulty grasping a concept, she often asked students to rephrase their thinking, or the thinking of another in order maintain the classroom discourse. Examples of this include, “Can you tell me more about that?” “Bart, can you tell me what Barney just said in another way”, and “Can I help you say that again, so that everyone can hear it?”

Posed purposeful questions. For many educators using philosophies, there are many purposes and goals for each lesson. During group sharing time, Mrs. Hoover provided students with the opportunity to answer and ask questions publically, demonstrate their thinking through writing and oral discourse, disagree respectfully, and advocate for their own strategies. Examples of the public practice of these skills included, “Can you draw what you just did on the board? Ok. Go for it!” and, “Can someone tell me what to do? You tell me what to do and I’ll write and listen" Moreover, Mrs. Hoover repeatedly asked students collectively and individually "why does/doesn't this work" and "how do you know that" more specific examples of the lines of questions are as follows: "Why does he have a three here and a six here?" "How does that help you help you know what many bags Eli has?"

Built procedural fluency. On multiple occasions, Mrs. Hoover led students in choral counting during the group-sharing portion of the lesson. This action deviates from many CGI classrooms, where student thinking is the centerpiece and student find their own solutions errors in problem solving activities. An example of this is, "Beautiful, he said that you could count up by 10's….so let's count together 43, 53, 63...beautiful." Besides, attempts to build computation fluency, Mrs. Hoover made efforts to expose students more efficient means of solving the same problem. An example of this is as follows; "It's the same thing. We are getting the same answer. But why is one of them a more efficient strategy?"

Supported the productive struggle of students. Due to the diversity of ability in verbal and oral expression, computation skills, and number sense of students in Mrs. Hoover's classroom, she often differentiates numbers within problems to accommodate student needs and align with student skills. An example of Mrs. Hoover's approach to this phenomenon is, "She wants to put blank marbles in each jar. In addition, it is asking us, how many jars do we need. Or does she need? So, you had three different choices and numbers, yes. You had three sets. You could either choose (9 and 3), (21 and 3) or (63 and 9)." Students are encouraged, through Mrs. Hoover's instruction, to choose the set of numbers that they feel most comfortable solving. Therefore, the class still discusses the same mathematical concept, with differing variables in order to challenge students without overly frustrating them. Furthermore, at times when an individual student struggled with a computation or concept publically, Mrs. Hoover explained that some students require more time than others to arrive at a final solution. She explained, "You guys [referring to entire class] are really being supportive of your friend by not shouting out and giving him the answer. You are giving him some nice time to think."

Showed evidence of student thinking. During each lesson, Mrs. Hoover asked individual students to share their methods and solutions in front of the entire class. The number of students who shared their procedures and solutions ranged from one to four throughout the 11 lessons. While at the front room, the student had the opportunity to draw, write, model and explain their thinking; while other students were encouraged to share the similarities and differences in their own thinking for a specific problem. Mrs. Hoover, facilitated classroom interactions through direct questions to individual students (e.g., "Tell me the vertical equation that you wrote." "Tell me the rest of it." "Can you go and say or draw something that helped you solve this problem?"). In addition, she asked questions to the entire class. Examples of this include "Could she have turned it around to check it?" "Did anyone do it differently?" And finally, she also rephrased student thinking (e.g. "What do you want me to write to help you figure out the answer? I see a place value drawing. I see her checking her work. I see a math mountain [referring to strategy]. I see her putting her hundreds, tens, and ones in order. Rock star!"

Summary

The researcher collected the aforementioned data from 18 students over a 6-week period in a 3rd grade elementary inclusion classroom that endorses reform mathematics philosophies. All classroom group discussion sessions (n=11) were video-recorded, transcribed, and analyzed in regards to teacher behavior within the context of current guiding principles of the *National Council of Teachers of Mathematics* (2015). Furthermore, the Instructional Quality Index (IQA) measured the overall participation of all students. To gain more clarity on the extent to which students with disabilities participate in mathematical discourse within the group setting, the researcher recorded, transcribed, and analyzed students' written and oral expressions for comparison across of students in receiving services in general education, special education, and those besting evaluated for special education services. Finally, the researcher collected and analyzed student-writing samples using the modified version of the NAPE scoring rubric, which aligns with the National Association of Governors Mathematical Standards (CCSS-M, 2010).

CHAPTER 5: DISCUSSION

The researcher listed and described the qualitative and quantitative data collected for this descriptive case study in Chapter 4. Moving forward, the researcher will discuss the significance of the presented data within the framework of answering, the degree to which students with disabilities participate in an 3rd grade inclusive classroom that employ reform based mathematics teaching practices. In addition, the characteristics of student participation of students with disabilities within the 3rd grade inclusive setting, and teacher behaviors that support student participation in the inclusive reform mathematics setting. Following the discussion of the presented data, the researcher will then discuss the significance of the findings, connections to previous literature, limitations of this study, and recommended future lines of research.

Student Participation

1. To what extent do 3rd grade students with disabilities participate in an inclusive reform-based mathematics classroom?

Of the 18 student participants in this study, only one student missed one session of group sharing time (see Table 1); therefore, student absenteeism was not a factor. Data collected from the 11 classrooms sessions, in which students have the opportunity to share their mathematical strategies in a group setting, shows students with a disability verbally participated in each session with occurrences ranging from one to six times (see Table 2). Furthermore, students diagnosed with a disability shared their thinking and strategies through written expression to the entire group in 6 of the 11 (54%) of sessions.

In addition to frequency of oral and written participation occurrences, data of verbal participation duration for students in general education, students referred for special education assessment, and students with a current 504 or IEP plan. Data collected from the 11 classroom sessions, demonstrates that students with a disability, within in this setting, participated verbally (35.6%) as often as students without a disability (35.7%) and slightly more often than the students who have been referred for special education assessment (28.5%)(see Table 3). Data taken from the 11 classroom sessions, revels that the percentage of verbal participation for students with a disability ranged from 1.3% to 94.4% of total classroom student verbal participation per session. The wide range verbal student participation percentage within individual group sessions is also apparent within the general education (1.2% to 67.5%) and referred for special education assessment groups (4.2% to 48.1%). Therefore, no visible differences were found in the frequency of verbal participation between the subgroups.

Students with Disabilities Participation Characteristics

Question 2. What are the characteristics of participation among 3rd grade students with disabilities in a reform-based inclusive mathematics classroom?

Data from student journals, in which students individually solved novel word problems prior to sharing their solution in the group setting, revealed that students with a disability averaged attempting 17.25 problems over a 6-week period. One student in the group (Chris) completed 12 problems, the lowest total of any student participating in the study, thus lowering the overall group average due in part to the limited sample size (n=4). In comparison, students without a diagnosed disability, and students referred for assessment for special education services averaged 19.25 and 19.2 problems respectively. The relatively limited size of the gap between problems attempted between groups suggests all three sub groups demonstrated sustained engagement in problem solving within the confines of using their individual journals throughout the study.

An inspection of written response characteristics from student journals revealed that all students in this inclusive setting demonstrated mathematical understanding and reasoning, as well as appropriate strategies and procedures in solving novel word problems with more proficiency than communicating their answers through written expression. Mean group scores in the area of understanding and reasoning for all three groups ranged from 2.772 to 3.165 (see Tables 4, 5, and 6), with students with a disability scoring 3.067 on average, showed little variability across groups. Furthermore, student scores across groups for the use of appropriate strategies and procedures showed a similar lack of variability. Mean scores ranged from 2.834 to 3.25. Again, the students diagnosed with a disability mean scores (2.982) fell between the mean scores of the other two groups.

Within this study, all groups of students scored lower in communicating their answers through written expression of mathematical explanations compared to demonstrating understanding and reasoning or appropriate strategies and procedures. Students without a disability displayed a mean score of 2.267, students with a disability scored on average 1.948, while those students referred for special education assessment demonstrated a mean of 1.737.

One possible explanation for the discrepancy in mean totals across student skill groups (i.e., understanding and reasoning, strategies and procedures, and communication) is the due to the amount of exposure and practice students have had to date in writing about their mathematical thinking. Although students from this study had performed writing exercises and tasks associated explaining, persuading, or describing topics through written expression in previous grades, the undertaking of writing about their own meta-cognitive process generally does not begin until the beginning of third grade for these particular students. Furthermore, the lack of exposure and daily use of mathematical vocabulary (e.g., divide, product, area, array, commutative, operation, sum, et al.) may well have had an impact on the overall class-wide communication mean scores.

Interestingly, group mean scores were slightly higher in the all student performance areas for students currently receiving special education services compared to those students referred for special education testing. Possible explanations for this include small samples sizes for both groups (n=5, n=4), the extent of impact of unknown possible disabilities for students who have been referred for assessment, as well as the omission of students of students with a moderate or severe cognitive disability from the group receiving special education services.

Teacher Behavior

Question 3. How does the teacher support the learning for all students while implementing CGI philosophies in an inclusive classroom?

Data collected, using semi-structure teacher interviews, revealed that Mrs. Hoover and Mrs. Bronte possessed extensive information on the strengths, challenges, and preferences of individual students with whom they work. Furthermore, both teachers commented on their perceived academic or behavioral progression of individual students within the context of the school year. This qualitative data suggests that both teachers incorporate the present levels of participation of current student into lesson and task planning. Furthermore, their reflective nature of teacher planning in order maximize student participation fits squarely within the theoretical framework of differentiated instructional (DI) techniques as well the anticipatory planning strategies of cognitively guided instruction (CGI).

Teacher behavior during group sharing time, exclusively led by Mrs. Hoover, generally demonstrated the promotion of reasoning and problem solving through purposeful questioning, connecting student mathematical thinking to others' thinking as well as representations, and having students show and defend their thinking. Data from the sample of six lessons indicates the aforementioned teaching principals occurred in each of the classroom lessons. In over half of the sample lessons Mrs. Hoover supported computational fluency and the productive struggle of students. Moreover, in only one lesson, taken from the sample of six, did Mrs. Hoover establish or address the specific mathematical goals for group sharing time. Possible explanations for her tendency towards promoting meaningful student discourse through purposeful questions and omission of addressing lesson or unit goals during group-sharing time are teacher preference during the lesson or the influence of professional development on her current use of CGI. Because Mrs. Hoover took part in the CGI professional development sessions prior to the most current *National Council of Teachers of Mathematics* (NCTM) teacher behavior recommendations for supporting student learning (2015) it is probable she did not receive instruction or guidance on the most common standards.

Furthermore, data collected from the 6 vignettes gives a more detailed description of how the teacher attempted to support student learning for all students and in what ways the students responded. Data from the vignettes demonstrated common teacher directed occurrences designed to support learning and engagement of all students as well as displayed critical omissions of teacher and student behavior.

Before asking any students to come to share their answer verbally, Mrs. Hoover asked one student to read the question aloud. This served multiple purposes. First, it gave a marker to the entire class that group sharing would begin. Secondly, it provided the information of the days' problem verbally so that student who may struggle with reading fluency or vocabulary could listen to the problem. Moreover, the teacher used the smart board to project the problem on the screen at the front of the class so that students who may struggle with audio processing could reference the problem. Thus increasing the accessibility of the problem and the lesson concept for the entire class.

Data from Vignette 2 and 3 shows that the classroom teacher aimed to engage the class in solving word problems where in the students needed to divide or multiply respectively. In an effort to engage the entire class, which consisted of students with a wide range of mathematical abilities, she offered the students three sets of numbers from which to choose. After giving each student a choice of which number set they wanted to work with, she led a group discussion on their perception of which set was the "easiest" and "hardest". The teacher offered no praise, nor criticism toward individual students, groups of students, or the class as a whole in regards to their choice of problem or accuracy of the solution. This differentiation of problem complexity allowed students allowed students of varied skill levels to engage in and discuss the common mathematical concept of each lesson.

Moreover, data from the vignettes showed few instances of either specific praise for desired student behavior or teacher reprimands for undesired student behavior during group sharing time. Instead, Mrs. Hoover tended to use the phrases "great" and "superstar" to reinforce the efforts of individuals or the class as a whole. Furthermore, she rarely asked students for the answer to a specific problem. Rather she opted to ask questions in which allowed students to explain their thinking and strategies to the class. Examples of this included, "Can you tell us what you did?" (Vignette 4), "Did anyone do it differently?" (Vignette 3), and "Tell me what you are thinking. (Vignette 5)." The absence of frequent undesired behaviors, during group sharing, in addition to high rates of student journal problem completion time suggests that class wide engagement remained consistently high.

Finally, data collected from the vignettes suggested that students possessed the understanding that missteps and inaccuracies in solving novel mathematical problems was expected. Examples of student responses to inaccuracies or misunderstandings are found in Vignettes 2, 4, and 6. These responses include saying “I don’t know” “Oh right! Thank you!” and, “Can I get some help with this?” The researcher did not perceive that in any of these instances a student or group of students appeared frustrated with the question being asked or with not knowing the correct answer. Moreover, data collected from the teacher interviews suggests that one of the goals for this classroom is to promote the abilities of all students during activities creating a classroom climate where “productive struggle” is the norm.

Connection to Literature

Over the course of the past 30 years, researchers involved in the reform mathematics movement have argued for a greater emphasis in teaching conceptual understanding of mathematical concepts. Kilpatrick (2014) states that a majority of student learning in mathematics occurs alongside other students, where the teacher and other students influence their thinking. CGI is one method for students to share their thinking process, models, and solutions in a group setting (Carpenter, Fennema, & Franke, 1996). However, previous research in the use of CGI has generally focused on the progression of student thinking (Carpenter et al., 1999), teacher beliefs (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), instructional practices (Fennema et al., 1996), and student learning in specific mathematical areas (Carpenter & Levi, 2000; Empson & Levi, 2011; Jacobs, Franke, Carpenter, Levi, & Battey, 2007). However, the author has not found any published article to date investigating the participation of students with disabilities in an inclusive setting that employs CGI philosophies.

Historically, research regarding mathematics interventions for students in special education or struggling students is often explicit, scripted instructional procedures (Hunt, Valentine, Bryant, Pfannenstiel, & Bryant, 2015). The intention of standardizing teaching procedures for supporting struggling students in mathematics is largely bases in achieving consistency in instructional practices, which increased student mathematical achievement (Gersten, Chard, Jayanthi, Baker, Morphy, & Flojo, 2009). Furthermore, many instructional interventions designed to increase student performance target a specific deficit skills areas (Coyne, Kame'enui, & Carnine, 2011). These areas include number sense, arithmetic, measurement, order of operations, as well, as speed and accuracy.

Although literature regarding the intersection of reform mathematics practices and inclusive teaching practices is limited, the need for research in this area is essential. The emphasis of current mathematical standards (CCSS-M, 2010) in regards to conceptual understanding for all students places new pressures on classroom teachers. Furthermore, recent trends in state policies requiring students to pass Algebra I in order to receive a high school diploma (*The Center for Public Education*, 2013) have also placed a greater emphasis on the important of student understanding of conceptual mathematics.

Significance

Data from this descriptive case study reinforced previous findings concerning the interaction of students with disabilities with CGI word problems. Behrend (1994) and Morascandi (2009) found that children with a disability were able to participate in classrooms that employ novel mathematical word-problems without teacher provided procedural methods. However, this descriptive case study differed from the aforementioned studies in four significant ways. The following section describes particular characteristics of this study that differ from previous studies that have investigated the convergence of current reform based mathematical practices and inclusive teaching practices.

First, this study quantitatively measured the degree to which all students verbally and through written expression participated within the group setting, comparing students with and without disabilities. Of the 11 group-sharing sessions, data from this study suggested that given the opportunity to address novel mathematical word problems, students with a disability, students without a disability, and students referred for special education assessment were able to participate through written and oral expression, given the opportunity through teacher direction.

Secondly, this study described some of attributes and challenges of student participation for all students in regards to solving and explaining mathematical word problems through writing in an inclusive setting, where there is a wide range of academic skills sets amongst all students. The attributes included verbally describing mathematical models, using arithmetic, questioning others, defending solutions derived from word problems, reading mathematical word problems and using current technological tools in the group setting.

Thirdly, the inclusive setting for this study differed from previous studies conducted in in more homogenous setting (i.e., resource and self-contained classrooms). Previous studies regarding the performance of students with a disability in classrooms that promote reform based mathematical practices (e.g., Behrend, 1994; Morascandi, 2009) took place in self-contained setting or school respectively.

Finally, this study described specific teacher behaviors, aligned with current recommended best practices, with the intention of including the verbal participation of all students within the inclusive setting. These teacher behaviors have the possibility of providing a frame of reference for future inquiries for teachers that employ reform-based mathematics pedagogy in the inclusive setting.

Data from the IQA reveled that in 8 of the 11 group-sharing lessons, at least half of the students verbally participated. This data combined with student attendance and number of problems attempted from student work journal (see Tables 5, 6, and 7) suggests that student engagement in the group-sharing portion of lessons was high for all students. Furthermore, data from both verbal teacher behavior examples and vignettes demonstrating the characteristics of interactions between students and the teacher supports the notion through the consistent absence of teacher redirection of inappropriate student behavior. The use of differentiated instructional techniques (i.e., presenting multiple sets of numbers for use in problems, allowing students to present mathematical models and solutions in various ways, giving students opportunities to respond in areas that best fit their academic and social abilities) is one possible explanation for the lack of frequent or consistent inappropriate student behavior.

Another significant contribution from this study comes Mrs. Hoover's classroom behavior in her effort to support the "productive struggle" of all students. Mathematics education literature has a extended tradition of promoting student disequilibrium in order to gain a deeper understanding of mathematical concepts (Dewey, 1933; Festinger, 1957; Handa, 2003; Warshauer, 2014). Therefore, supporting academic productive struggles among students in a classroom with a wide range of academic skills is significant challenge for even the most experienced teachers. Information gathered from semi-structured interviews of the classroom and AIG teacher revealed the teachers working with students in this classrooms possessed extensive knowledge of the strengths and challenges of individual students. Furthermore, qualitative data from Mrs. Hoover's classroom behavior using the NCTM's best practices framework as well as classroom vignettes demonstrated that she praised students flexible thinking, efforts instead of solutions, and students behavior for offering and accepting feedback appropriately.

Furthermore, vignettes describing student behaviors and classroom interactions in the group sharing setting portrayed particular students modifying and altering their processes and solutions to solving individual word problems in a public space, often with the help of others. This process of working towards a solution, while using previously gained skills and knowledge, with the assistance of peers and educators fits squarely with in the framework of self-directed learning (SDL) (Knowles, 1975). Knowles (1990) argues that learning does not take place in a vacuum, but in combination with others, such as peers and teachers. The process of teacher facilitated and students sharing mathematical models, arithmetic, strategies, and solutions of novel mathematical word problems lends itself to students making possible errors in an effort to find a solution to the assigned problem. The possibility of students making errors publically, coupled with the opportunity to ask for assistance or feedback, provided the students to not only demonstrate mathematical skills, but also take risk in solving equations with a reduced amount of stigma attached to not achieving at the correct answer. Data from teacher interviews, teacher behavior examples, and classroom vignettes, supports the posture that within Mrs. Hoover's classroom, unsuccessful attempts by all students in attempting to solve mathematical problems is perceived as a typical part of the process. Furthermore, data from classroom vignettes, classroom participation measurements, and student journals supports the stance that students are engaged with the material and show few instances of inappropriate behavior due to frustration.

Furthermore, this study intended to offer several significant contributions concerning the teaching and learning interactions of students in the inclusive setting that employs reform based mathematics practices. First, although many national organizations such as the NCTM (1989,

2000) and current standards documents like the CCSS-M (CCSSO, 2010) recommend using reform-based approaches, such as CGI, few studies have investigated the effects on student learning in inclusion classrooms that focus on students with high-incidence disabilities. Secondly, by documenting the degree to which all students interact with the material and each other in inclusion, it is the goal of this study to present a more refined view of the challenges and attributed students with disabilities bring to mathematical in discussions. Thirdly, this study aimed to help fill this gap in the mathematics education literature, as well as the special education literature in order to better support the learning of all students. Through this study, the researcher brought together two fields of education that serve the same student population, but often use different methodologies to build theory about how students develop mathematical reasoning. Finally, this study hoped to illuminate implications for designing and implementing classroom instruction that supports students with disabilities as they develop mathematical understanding in light of current higher academic standards.

Limitations

Although this study aims to contribute to the fields of mathematics education and special education, there are also some limitations. The most significant limitations of this study are as follows. First, the student population participating in this study does not reflect the larger national population in terms of race or socio-economic status. The majority of students in this study were white and lived in an upper middle class area. Secondly, the duration of this study did not allow researchers the opportunity to draw conclusion on the rate in which all students, both with and without disabilities, progress in their mathematical thinking in relation to one another. Therefore, the reader should avoid generalizing on the rate at which students in this setting progress from concrete to abstract thinking. Finally, generalizations on the performance and participation of students with disabilities not included (e.g., severe and profound intellectual disability, hearing impairment, emotional/behavioral disorders, et al.) in the study should also be avoided.

Future Research Considerations

As stated previously, making grand generalizations from case studies is problematic due to specific and unique variables and small sample sizes. However, due to wide range of characteristics (e.g., academic, social/behavioral, physical) and severity (e.g., mild, moderate, severe/profound) of disabilities, gaining access to large populations of homogenous groups within special education is doubtful. Therefore, the use of case studies involving students with disabilities interacting with CGI problems may be the most appropriate for future research endeavors. The following section discusses four future directions of research involving both CGI philosophies and teaching students with a disability in order to support the deeper understanding of mathematical concepts.

Although this study employed a criteria reference in recruiting teachers and students, it would be sensible for researchers in the future to investigate the interaction between students with disabilities and CGI with a narrower focus on specific disabilities. Due to the socially interactive nature of CGI (e.g., group-sharing time), a future investigation would be prudent on documenting the effects this classroom method has students that struggle with social skills. This population would include students that struggle with externalized behaviors (i.e., inappropriate language, impulsiveness, verbal and physical aggression towards peers and adults, and poor coping skills) and internalized behaviors (i.e., withdrawing, depression, anxiety, and excessive shyness). Typically, students that exhibit the aforementioned behaviors may be categorized under emotional/behavioral disorder (E/BD) or autism spectrum disorder (ASD).

The descriptive case study model used for this study demonstrated the degree to which students with disabilities participated in one specific classroom using CGI strategies. However, this study did not take into account the rate of learning mathematical concepts compared to either non-disabled students in the same classroom or students who learned mathematics in classrooms that do not employ CGI practices. Therefore, the author recommends that in order to compare these groups to one another, randomized control groups study in multiple classrooms is the ideal.

Due to the time and logistical constraints of conducting individual interviews of student participants, no student opinions regarding solving novel mathematical problems in an inclusive setting were in this study. Furthermore, the author recommends that future research endeavors, in this area, include the opportunity for all students to express their thoughts and feelings in regards to working with other students in an inclusive classroom. Information gathered through student interviews may reveal student preferences and frustrations of student who work with others that possess a wide range of challenges and attributes their own.

The researcher found no literature, regarding the instruction, opinions, implementation difficulties, or professional development challenges of training special education teachers in the use of CGI. Current policy trends suggest that there is a need for training current and future special education and general education teachers in emphasizing deeper mathematical thinking, solving novel problems, and defense and explanation of student thinking (CCSS-M, 2011) for all students. Due to the diverse range of classroom and school settings (e.g., inclusive, self-contained, resource, general education) in which students with disabilities attend, as well as the wide range of academic or behavioral impairment severity degree (e.g., mild/moderate, severe/profound, et al.) suggests that combining differentiation of instruction techniques and CGI philosophies is prudent.

Conclusion

Recent educational policies, led by the NCTM, regarding mathematical standards (CCSSO, 2010) have placed a greater emphasis on deeper conceptual understanding for all learners. Furthermore, the recent statewide policy trend (*The Center for Public Education*, 2013) requiring all students to successfully complete Algebra 1 in many states, in order to receive a high school diploma, have placed a higher level of attention for educators to support the learning of all students conceptual understanding of mathematics. Therefore, to address current mathematical standards aligned with the conceptual understanding of mathematics, many practitioners are beginning to use reform-based teaching approaches, like CGI.

The focus of this study was to gain a more detailed description of the performance and participation of students with disabilities in the inclusive setting than previous studies (*rf*., Behrend, 1993, Moscardini, 2010) in a classroom that employs reformed based mathematics practices. The researcher compare the verbal and written participation of students in general education, those referred for special education assessment, and students currently receiving special education services in the inclusive setting. Over the course of six weeks, the researcher collected data from 11 group sharing session, as well journal entries for all participating students. Participants included students in general education (n=9), students referred for special education assessment (n=5), and students with a current IEP or 504 plan (n=4), as well the classroom teacher and a support teacher.

Data from this case study revealed that students diagnosed with a disability and those receiving general education services attended class and attempted nearly the same amount of journal problems in the inclusive setting. Furthermore, data from this study showed that students with disabilities, students referred for special education, and students in general education verbally participated nearly the same percentage of time within the group-sharing portion of the lesson over the course of 11 sessions. However, an analysis of student writing samples revealed that students receiving general education services, as a group, outperformed both students in special education, and those referred for special education in the areas of understanding and reasoning, the use of strategies, and written communication.

Data from this study supports the notion that teaching conceptual mathematics to students with disabilities in the inclusive setting offers multiple entry points for student participation. Moreover, because of the emphasis on group discourse, multiple entry points exist within this setting for educators to assess student reasoning, verbal communication skills, creativity, and social skills. Data from this study, suggests that extra supports may needed for students who struggle with conveying thoughts about mathematics through writing.

Researcher Confessional

The researcher played a significant role in the collection and analysis of qualitative data in this study. Instead of viewing the researcher as an unemotional, impartial observer who lacks prejudices, preconceptions, and assumptions about the subject matter, this study clearly states that the researcher is human and, therefore, is subject to circumstance and past experiences. Birks and Mills (2014), echo the view that researchers are the sum of all that they have experienced. Therefore, the following section frames the researcher's personal and professional views in order to provide context and transparency for the reader.

As a student, I have attended both public and private schools. Although I have never qualified for special education placement, I recall receiving pull out services while attending primary school for speech and language difficulties. Throughout my time as a student, I enjoyed studying social studies, literature, and mathematics. I generally did not excel nor enjoy my interactions with language arts or science. I earned a bachelors of arts in history and anticipated teaching social studies at the secondary level, however while working towards my teaching license I was required to work with students with disabilities and found it more enjoyable and rewarding. Therefore, I made the decision to pursue a teaching certificate in special education.

After teaching special education for 10 years in the public school system in both rural and urban areas, I have a strong connection to promoting inclusive practices for all students. As a classroom teacher, I spent the majority of my career working with secondary students diagnosed with emotional/behavioral disorders (E/BD) that exhibit externalized behaviors in the self-contained setting. Many of the classrooms I taught had a wide range of variability in regards to academic student motivation and achievement. Furthermore, many of my former students diagnosed with having a comorbidity of a disability; the most common of these were the diagnosis of learning disability (LD). During the course of my teaching career, I held a K-12 teaching license in five different states. While teaching in the self-contained and inclusion setting, I taught a wide range of topics and skills in regards to mathematics. However, as a classroom teacher I knew little about promoting current reform mathematical practices. While working as a research assistant during my time pursing a doctorate degree, I was drawn to the engagement and motivation of students who participated in classrooms that embraced reformed based mathematics pedagogy. In part, due to the ways in which the teacher could showcase the strengths and talents of all students, while creating a culture where not achieving the correct answer was seen as the only value of the learning process.

APPENDIX A: RUBRIC 1: PARTICIPATION

Rubric 1:
Participation

Rubric 1: Participation

Was there widespread participation in teacher-facilitated discussion?

	Rubric 1: Participation
4	Over 75% of the students participated throughout the discussion.
3	50-75% of the students participated in the discussion.
2	25-49% of the students participated in the discussion.
1	Less than 25% of the students participated in the discussion.
0	None of the students participated in the discussion.
N/A	Reason:

GUIDELINES FOR SCORING <u>PARTICPATION</u>:

Any verbal contribution, including one-word responses or very brief comments, by an individual student counts as evidence of participation. When students respond to a teacher's question chorally this does not count as evidence of participation.

APPENDIX B: RUBRIC FOR SCORING STUDENT EXPLANATIONS OF STRATEGIES

(K-5 Math Rubric Teacher Use, 2012)
Standards for Mathematical Practices (SMP) from CCSS-M.

	Level 1 Novice	Level 2 Apprentice	Level 3 Practitioner	Level 4 Expert
Understanding and Reasoning (SMP 1,2,8)	*There is no correct solution or the solution has no relationship to the task. *Inappropriate concepts are applied. *The solution addresses none of the mathematical components presented in the task. *No evidence of mathematical reasoning.	*The solution is not complete indicating that parts of the problem are not understood. *The solution addresses some, but not all of the mathematical components presented in the task. *Some evidence of mathematical reasoning.	*The solution shows that the student has a broad understanding of t he problem and the major concepts nece ssary for its solution. *The solution addresses all of the mathematical components presented in the task*St udent uses effective mathematical reasoning.	The solution shows a deep understanding of the pr oblem and can identify the appropriate information necessary for the solution. *The solution completely addresses all mathematical components presented in the task *Student uses refined and complex reasoning.
Strategies and Procedures (SMP 4, 5,7)	*No evidence of a strategy or procedure. Or the use of strategy that does not help solve the problem. *There were many errors in the mathematical procedures and the problem was not solved correctly.	Use of a strategy that is partially correct, may lead toward a solution, but not to a full solution. *Could not completely carry out mathematical procedures needed for p roblem. *Some parts may be correct, but a correct answer is not achieved.	*Use of a strategy that leads to a solution of the problem. *Mathematical procedures used correctly. *All parts are correct and a correct ans wer is achieved.	Use of an efficient and sophisticated strategy leading directly to a solution AND s hows/explains other ways t hat the problem could be solved. *Applies accurate procedures to correctly solv e the problem. *Verifies solution and/or ev aluates the reasonableness of the soluti on. Makes mathematically relevant observations/ conne ctions.
Communication (SMP 3,6)	*There is no explanation or the explanation cannot be understood or is unrelated to the problem. *There is no use or inappropriate use of math representation (e.g. figures, symbols, diagrams, graphs, tables, etc.) *No use or inappropriate use of math language.	There is an incomplete explanation; it may not be clearly described. *There is some use of appropriate math representations. *There is some use of m ath language.	*There is clear and detailed explanation. There is appropriate use of accurate/relevant math representations. There is use of math language.	There is a clear effective explanation detailing how the problem is solved. All the steps are included so that the reader does not need to infer how the decision were made. *Math representation is actively used as means of communication ideas related to the solution of the problem. *There is a precise and appropriate use of math language.

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