CONSUMER OPTIONS AND FORWARD PRICING:
THEORY AND EMPIRICAL ANALYSES IN TICKET MARKETS

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ABSTRACT

Preethika Seshasainam: Consumer Options and Forward Pricing - Theory And Empirical Analyses In Ticket Markets
(Under the direction of Sridhar Balasubramanian and Barry L. Bayus)

Sports markets are lucrative. Most sports markets have one or more elimination-style tournament in which individuals or teams compete. Currently tickets to these games are sold in advance. Inherently, in these games there is uncertainty regarding the teams that will ultimately participate in the event. Because tickets are sold in advance, fans who want to wait and buy tickets after they know which teams are playing in the game need to head to scalpers. Since scalpers can charge exorbitant prices, many fans are irate at the leagues for not doing anything about them. Further, the presence of scalpers suggests that the leagues are leaving money on the table. Thus, it seems that there are reasons why both fans and leagues might not be happy with the current practice. In this dissertation I propose alternative pricing mechanisms which give the fan the choice of waiting before buying the ticket while simultaneously giving the league more profits.

Essay 1 suggests that the league offer “consumer options”. Initially defined in the finance literature, buying an option confers a right, but not an obligation, to follow through on some course of action. I examine this pricing mechanism both from the fans’ and the league’s perspectives.
Essay 2 is aimed at empirically understanding fan behavior in a real-life forwards market for sports tickets. A consumer forward is different from a consumer option in that it is team-specific (i.e., if the team for which the forward is bought does not make it to the final game, the forward expires). Given that forwards expire, it becomes crucial to ‘pick’ the right team when buying forwards. I study the problem of how fans can maximize their chances of attending the final game while simultaneously minimizing their cost and suggest recommendations based on fan type.

Essay 3 studies the concept of consumer forwards using an analytical model and experimental data. Further I contrast the profitability of option versus forward pricing.

Taken together the three essays suggest alternate ways of pricing tournament tickets and assess their profitability for the league while simultaneously understanding fan behavior in these markets.
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CHAPTER 1 – INTRODUCTION:

This dissertation addresses new pricing mechanisms in markets where (a) there is temporal uncertainty regarding the quality of the purchased offering, and (b) consumers are affected differently by the uncertainty. In situations where purchase is separated from consumption, consumers are uncertain of the valuation for the product at the time of purchase. Under some situations, such uncertainty is more at the market-level as opposed to the consumer-level (i.e., the uncertainty is not in the hands of the consumer). For example, consumers may be uncertain if a particular technology may succeed. Further, this market-level uncertainty could affect different types of consumers differently. For the purposes of the dissertation I focus on the context of sports markets, which can exhibit all of these properties.

Sports markets are lucrative. Total ticket revenues for the 2000 Sydney Olympic Games were almost $500 million. Most sports markets have one or more elimination-style tournaments in which individuals or teams compete. Currently tickets to these games are sold in advance. Inherently, in these games there is uncertainty regarding the teams that will ultimately participate in the event. Because tickets are sold in advance, fans who want to wait and buy tickets after they know which teams are playing in the game need to head to scalpers. Because most scalpers charge exorbitant prices, many fans are irate at the leagues for not doing anything about them. Further, the presence of scalpers suggests that the leagues are leaving money on the table. Thus, it seems that there are reasons why both fans and leagues might not be happy with the current pricing practice. In this dissertation, I propose and study the viability of alternative pricing
mechanisms which give fans the opportunity to decide whether to attend the game after the uncertainty of the teams playing in the final is resolved. Simultaneously, these new pricing mechanisms provide the league with greater profit than the currently employed mechanism of advance selling.

In chapter 1, I propose that the league offer ‘consumer options.’ Initially defined in the finance literature, buying an option confers a right, but not an obligation, to follow through on some course of action. With consumer options, the firm decides on the option and exercise prices before the final outcome is known using estimates of consumers’ expected values for the offering. Given these prices, a consumer can then take an option on a ticket by paying the option price before the uncertainty is resolved. Later, once the final outcome is known, the consumer can choose to exercise the option by paying the exercise price. If she does not exercise the option, it expires and the firm retains the option price. In this chapter I analytically model the viability of consumer options and contrast it with (a) advance selling and (b) pricing after the uncertainty has been resolved (“full information” pricing). I demonstrate that when the league uses option pricing, profits can equal or exceed those from the current practice of advance selling. I also demarcate conditions under which option pricing delivers profits that are equal to, and in some cases higher than profits that would be obtained under full information pricing. Further, I conduct a study to examine fan reactions to the newly proposed pricing mechanism. I also obtain their willingness-to-pay for tickets under the different pricing mechanisms and study the behavioral determinants of their willingness-to-pay. Fans gained more surplus when they bought an option ticket as opposed to a ‘regular’ ticket, because of the protection and flexibility offered by the former. I empirically validate the profitability conditions outlined in the analytical model. The percentage improvement in profits under option pricing over those under advance
selling was found to be the highest when the probability of the favorite team making it to the tournament was the lowest. This finding highlights the value of consumer options in protecting the fan from making a substantial investment upfront when the probability of the preferred outcome is low.

Chapter 2 is aimed at empirically understanding fan behavior in a real-life forward market for sports tickets. A consumer forward is team-specific. If a consumer is interested in attending a game in which her favorite team is playing, she could purchase the team’s forward by paying a preset “forward price.” Similar to the option ticket, if the fan’s favorite team makes it past the preliminary rounds and all the way to the tournament, she then pays an additional exercise price to buy the ticket. However, unlike in the option pricing case, if the specific team for which the forward was purchased does not make it to the tournament the fan loses the forward price. In other words, if the team for which the forward is bought does not make it to the final game, the forward expires. I obtained data from a large, well-known firm that sells consumer forwards for the Final Four market in 2006. This firm has a proprietary market; at any time in between the purchase of the forward and until the uncertainty is revealed fans have access to this market in which they can resell their forwards. I account for the presence of different types of fans in the market and estimate different models to understand their behaviors. While some fans engage in transactions on a single team, others buy and resell forwards across many different teams. The first group of fans tends not to resell their team’s forward, irrespective of its performance. I assume the objective of this group is to reduce its costs when purchasing forwards. Consequently, I estimate a regression model that provides insights on cost-minimization strategies for this group. For the second group that transacts across many different teams, I study factors that help the fans pick the ‘right’ teams to buy forwards on. This is
especially important given that forwards expire if the specific team does not make it to the final. If the forward expires fans lose: (a) the forward price that they paid and (b) the chance to attend the game. I study the factors that help the second group maximize their chances of attending the final game while simultaneously minimizing their cost. Further, I draw from closely related literature on risk-management in financial markets to create independent variables that determine whether some of the accepted results on risk-management for individual investors in financial markets apply to fans minimizing risk in sports markets. I identify the risk-minimization strategies that transfer from financial to consumer markets and those that do not.

In chapter 3 I study the concept of forward pricing using an analytical model. As stated earlier forwards are tied to a specific team and expire if the given team does not make it. In this chapter, I allow for some fans to buy multiple forwards to maximize their chances of going to the game. I determine the conditions under which profits from forward pricing exceed that from pricing later in the season. Further, profits from forward pricing always exceed that from advance selling. I conduct a study across two universities to obtain fans willingness-to-pay under the different pricing mechanisms. Empirically, I demonstrate that profitability of forward pricing is always greater than advance selling. This is interesting because buying the regular ticket in advance permits the fan to attend any game of her choice whereas the forward ticket is a ‘lesser’ ticket in that it only permits the fan to attend the game if the team that is associated with the forward makes it to the final game. To better understand fans’ perceptions of forward pricing, I obtain open-ended feedback from them. Surprisingly, I find that one group of fans values the fact that a forward is a ‘lesser’ ticket. Because forwards expire if the team associated with it does not make it through, it relieves stress for fans who do not have to go through the hassle of resale. I compare profits from forward pricing with those from pricing later in the season and find that
when the probability of at least one of the two teams making it to the final is low, the profitability of forwards is higher. This is intuitive because when the probability of a team making it to the final is low, the value of the forward is higher. Finally, I contrast the profitability of option versus forward pricing. I find that if the market is composed entirely of fans who have no utility for any team except their favorite one, then forward pricing is just as profitable as option pricing. Such fans will not exercise either the option or the forward if their favorite team does not play in the final game. If, however, the market has another type of fan – one who is willing to attend any game, then the profitability of forward versus option pricing depends on the utility of the fans in this group. If their utility is high enough they could purchase multiple forwards to increase their chances of attending the game – something they do not need to do under option pricing. In this case profits from forward pricing are higher; else profit from option pricing could be higher than those from forward pricing.

Taken together the three chapters suggest new and interesting ways of incorporating fans’ uncertainties into the sport league’s ticket pricing decisions and help us better understand fan behavior. Specifically, I demonstrate that the proposed pricing mechanisms are more profitable, and more general, than the current practice of advance selling. Further, using option and forward pricing, the league can continue to set prices early in the season. I have also studied these pricing mechanisms from the perspective of the fans and found that they value the flexibility these new pricing mechanisms provide. Analyzing fan behavior by studying their transactions in a real-life market for forwards yields some new perspectives on their behavior. Overall, my dissertation employs a combination of analytical and empirical methodologies to demonstrate that option and forward pricing mechanisms are preferred – from the perspectives of both the sport league and the fans.
CHAPTER 2:

CONSUMER OPTIONS: THEORY AND EMPIRICAL APPLICATION TO TOURNAMENT TICKET PRICING

2.1 INTRODUCTION

Sports markets are lucrative. Depending on the set of events taken into account, estimates of the size of the total annual market for sports and event tickets vary from $7 billion to $60 billion (Happel and Jennings 2002). As an example, ticket revenues for the 2000 Sydney Olympic Games were almost $500 million (Preuss 2002). The National Collegiate Athletic Association (NCAA) had over $500 million in revenues during the 2005-6 season with Division I men’s basketball tickets alone accounting for more than $27 million (Alesia 2006).

Most sports markets have one or more elimination style tournaments in which individuals and/or teams compete. For example, “March Madness” hits college campuses across the United States every year as basketball teams compete to reach the Final Four tournament on their way to the NCAA Championship. A key characteristic of this setting is the considerable uncertainty about which competitors (teams, hereafter) will actually make it through the tournament to the final game\(^1\). Consumers (fans, hereafter) have uncertain future valuations for tournament tickets because they do not know which teams will be playing until the tournament unfolds. Today, the

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\(^1\) For example, of the twenty-eight teams that made it through to the men’s Final Four tournament in the last seven years only six teams have made it there more than once (i.e., it was the first appearance in the Final Four for twenty-two teams). Of the twenty teams that made it to the Super Bowl in the last ten years, only three teams have appeared more than once. And, of the twenty-four teams that have made it to the Soccer World Cup semi-finals in the last twenty years only five teams have appeared more than once.
firm (league, hereafter) prices and sells tournament tickets at the beginning of the season, which is well in advance of the actual tournament that is at the end of the season. Thus, the league’s current behavior is consistent with the “advance pricing” literature (Geng, et al. 2007; Shugan and Xie 2000; 2004; 2005; Xie and Shugan 2001). Unfortunately, this practice means that many fans who would like to attend the final championship game once they know the teams that are playing will be disappointed because tickets are typically sold out in advance.

In this chapter, I propose an alternative pricing arrangement whereby the league can earn greater profits than through its current practice of advance selling. I borrow some ideas from the literature on real options to introduce the concept of consumer options. Specifically, an option confers upon the buyer the right, but not the obligation, to follow through on some course of action (Amran and Kulatilaka 1999). Whereas the literature on real options generally deals with the situation in which a firm strategically creates options for itself (e.g., Luehrman 1998; Amran and Kulatilaka 1999), the concept of consumer options involves a firm pricing and selling an option to consumers. With consumer options, the firm decides on the option and exercise prices before the final outcome is known using estimates of consumer expected value. Given these prices, a consumer can then take an option on a ticket by paying the option price before the uncertainty is resolved. Later, once the final outcome is known, the consumer can choose to exercise the option by paying the exercise price. If she does not exercise the option, it expires and the firm retains the option price. Table 1 further highlights the concept of consumer options and how it differs from real options².

² Similar to two-part tariffs, our concept of consumer options involves separate option and exercise prices. However, unlike in the case of communication service usage which can involve a price for receiving the service as well as a charge based on usage (e.g., Lambrecht and Skiera 2006; Lambrecht, et al. 2007), consumer options do not involve a lump-sum fee and a per-unit charge.
Table 1: The Concept of Consumer Options

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<th>Consumer Options</th>
<th>Real Options</th>
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<td></td>
<td>Consumers purchase the right, but not the obligation, to make a future purchase from the firm</td>
<td>A firm makes an investment to preserve the right, but not the obligation, to make a future investment</td>
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| Pricing the Option | Option price and exercise price are endogenously determined by the firm | Option price is endogenously determined by the firm; exercise price is exogenous |

| Valuing the Option | Heterogeneous consumers value the option based on their (known) preferences for uncertain outcomes; the firm values the option based on expected consumer demand | A firm values the option based on an estimate of what its incremental investment would trade for in the capital markets |

| Exercising the Option | Consumers’ exercise decisions are made after the final outcome is revealed | The firm’s exercise decision is triggered by its estimates of market-priced risk |

To illustrate the concept, consider the following simple, stylized example. Suppose there are two (risk neutral) fans in the market, each having the same favorite team. Let us assume that the probability of this favorite team playing in the final championship game is 0.5. For one fan, the value of attending the final game is $200 if her favorite team is playing and $170 if it is not (e.g., the event is still “the best party in town”). For the other fan, the value is $250 if the
favorite team makes it to the big game, but she will not attend (i.e., her valuation is $0) if her favorite team suffers an early defeat. It is straightforward to calculate the expected valuations: for the first fan it is $185 and for the second fan it is $125. I assume that the fan will purchase the ticket if the expected value equals the price. With advance selling, the league will generate profits of $250 by selling to both fans at a price of $125. If the league has full information of the outcome before it sets prices, it can do better by charging $200 if the favorite team plays in the final (so that both fans will buy a ticket) and $170 otherwise—leading to expected profits of $285.

Now, consider pricing with consumer options as shown in Figure 1. Fans can advance purchase a non-refundable option ticket by paying $65 (the option price, \( P_o \)) before the tournament teams have been announced and then decide later once the matchups are known whether they want to attend the final game for an additional $120 (the exercise price, \( P_e \)). In this case, the first fan will buy the option and exercise it since her expected valuation is the same as the ticket price (profits = $185). The other fan will buy the option (profit=$65) and exercise it if her favorite team is playing. Thus, there is a 50% chance this fan will also end up buying a ticket (yielding another $60 in expected profits). Using consumer options in this manner, the expected total profits for the league is $310. In this simple example, the league is able to extract more from the first fan with option pricing than with advance selling ($185 instead of $125) and can expect to extract the same amount from the other fan ($125) as under advance selling. Thus, option pricing can be more profitable than advance selling or even full information pricing.
The extant literature on advance selling assumes that all consumers have positive valuations for both the preferred and non-preferred outcomes. However, the existence of fan heterogeneity in the example means that some fans are very sensitive to the outcome—their valuation for the non-preferred outcome might be zero. In this case, additional profits can be earned by offering consumer options. If all fans have positive valuations for both the preferred and non-preferred outcomes, the league can always replicate advance selling using option pricing. This is done by setting the exercise price to be zero and the option price to be equal to the price under advance selling. Therefore, at worst, option pricing can do as well as advance selling. Because pricing with consumer options is relatively new, I first consider the profitability of consumer options from the firm’s perspective in the next section. To do this, I formulate and analyze a stylized model of the league’s ticket pricing decision.
Then in the following section, I take the fan’s perspective by empirically exploring fan responses to consumer options for NCAA Division I men’s Final Four basketball tickets. Most of the fans in the sample find that consumer options are easy to understand. Moreover, fans do not believe that consumer options are an unfair pricing mechanism, even if they chose to not exercise the option ticket. In agreement with the insights from the analytical model, I find that some fans do indeed have zero valuation for a ticket to the championship game without their favorite team—suggesting that fans are heterogeneous in their valuations for the (uncertain) championship game. I further find that basketball fans are willing to pay more for the flexibility that options afford, especially when uncertainty about the teams playing the final game is high. Most important, I provide empirical evidence that consumer options can create a win-win situation for the league and fans—pricing with consumer options generates more profits for the league than their current practice of advance selling, and at the same time, consumer options allows positive surplus for fans.

I conclude by discussing the potential benefits of consumer options to the league given their institutional constraints. I also outline some promising directions for future research.

2.2 MODELS AND ANALYSES

I first briefly detail the institutional considerations that leagues face in setting prices for tournament tickets and discuss why it is important for the leagues to address fans’ concerns.

Currently most major sports leagues advance sell tickets to the public through a lottery. The question that arises is: Given the uncertainty about the teams that will play in the final game, why does the league not set ticket prices after the matchups are known? First, given the possibility that less popular teams may end up playing, it could just be less risky for leagues to
set ticket prices upfront. Competition between popular, powerhouse teams is strongly desired in sports markets. For example, demand and prices dropped for the 2006 men’s Final Four when it became known that some non-traditional, “Cinderella” teams would be playing (Drew 2006). Second, leagues may be subject to “fairness constraints” (Kahneman et al. 1986). Specifically, the league could be viewed as an unfair hoarder if it priced and sold tickets right before the game. Researchers argue that this is why leagues price tickets before the uncertainty has been resolved (Happel and Jennings 1995; Krueger 2001a). Accordingly, in the proposed model, I set both the option and the exercise price before the uncertainty is resolved.

While I believe the concept of consumer options is new to the academic literature, it is interesting that some enterprising (third-party) firms have already implemented a version of consumer options for sports tickets. For example, FirstDibz.com (formerly Ticketreserve) sells “forward” tickets that are tied to a specific team making it through the tournament—if this team loses early, then the forward expires. Ticket forwards are intriguing because the third-party seller would not only earn the exercise price from the fans whose team makes it to the big game, but would also collect the initial forward price from every fan thinking their team had a chance to make it through the tournament. My conversations with NCAA league officials, however, suggest that they are not interested in offering ticket forwards due to the potential for fan backlash (e.g., being seen as profit-hungry organizations that do not really care about the fans). On the other hand, my discussions with league representatives indicate that they are open to the idea of selling consumer options—tickets that do not expire if a particular team does not make it to the final game. Thus, I hope that this research will help inform the league’s tournament ticket pricing decision.
Accommodating the sentiments of fans is important because a sizeable fraction of leagues’ incomes accrue from other revenue streams, including network broadcasting rights. For example, the NCAA’s 11-year contract with CBS Sports (which expires in 2014) brought in $6 billion - one of the largest contracts in U.S. sports history (Baade and Matheson 2004). Without loyal fans, networks would lose viewers, and the leagues could lose bargaining power over the stations. Given the critical importance of fan support for both ticket sales and broadcasting revenues, upsetting fans is a risk that the leagues are not willing to take. The NFL’s position is consistent with this reasoning. Greg Aiello, the league’s vice president for public relations, states that the league tries to set a “fair, reasonable price” to maintain an “ongoing relationship with fans and business associates” (Krueger 2001b).

2.2.1 Aggregate Demand Model

In this section, I formulate and analyze a stylized model of the league’s ticket pricing decision. Using this model, I identify the key conditions under which pricing with consumer options generates higher profits than advance selling and full information pricing.

Two models of the league setting the ticket price to a tournament game before the uncertainty is resolved are developed – these capture advance selling and option pricing. Further, one model of the league setting prices after the uncertainty is resolved is developed – this captures the case of full information pricing. For simplicity, I model the situation in which there are two possible final outcomes: (1) when a “popular game” occurs and (2) when an “unpopular” game occurs. Whereas there are certainly some idiosyncratic team preferences across individual fans (e.g., students and alumni are often loyal to their college teams), the reasonable assumption that some matchups are more desirable than others is made. For example, Paul and Weinbach (2007) find that fans prefer games with a quality matchup between teams with winning records.
and those that are high-scoring – I refer to games with these traits as popular games. I assume that the probability of a popular game occurring is $\gamma$. This “consensus” probability is exogenously determined and is known to both fans and the league.\footnote{Upon comparing men’s college basketball team rankings from various sources (Associated Press Poll, Coaches Poll, ESPN Poll etc) we find that the pre-season rankings for the various teams across many years are very similar. We assume that the league and fans can use this readily available information to reach a consensus estimate of the probability with which a popular game will occur in a tournament like the Final Four. Our model is robust to some error being associated with this consensus estimate across fans – in this case, the consensus estimate represents the expected probability.}

It is assumed that fans have stable temporal preferences and prefer the popular game over the unpopular game. "Team-based" fans are defined as those who have positive utility for the game only if it is popular (they have zero utility for an unpopular game). "Game-based" fans, however, have positive utility for the game irrespective of whether the matchup is popular or unpopular (e.g., even if the game is unpopular, the event is still "the best party in town"). Across the pricing mechanisms I model fans’ valuations of the game using a standard vertical differentiation model. The valuation of the game for the different fans is represented by $\theta_i \in (0, \alpha_i)$, where $i=\{G,T\}$ represents game- or team-based fans. Within each segment, a fan with a higher $\theta$ values the game more than a fan with a lower $\theta$. Fan valuations are distributed uniformly within the interval $[0, \alpha]$; therefore, $\theta_G \sim [0, \alpha_G]$ and $\theta_T \sim [0, \alpha_T]$ depict the game- and team-based fans’ valuations of the game. The net utility of the fan is given by:

$$U = \theta_i \lambda_j - P$$  \hspace{1cm} (1)

where $\lambda_j$ represents the quality of a popular or unpopular game (depending on whether $j=\{1,2\}$) and $P$ is the price paid under a given pricing mechanism. Consistent with the vertical differentiation logic (see Shaked and Sutton 1982), I assume that the quality of the popular game is higher than that of the unpopular game (i.e., $\lambda_1 > \lambda_2$).
In any ticket pricing problem, capacity constraints are an issue. For ease of presentation I do not include capacity constraints in our formulation. However, I have verified that the results hold when capacity constraints are considered. Details are available in the Appendix. The central results remain unchanged when the presence of a resale market is accounted for. I analyze the option pricing case first.

**ANALYSIS**

**Pricing an option ticket**

Under option pricing, a fan pays the option price \( P_o \) before the uncertainty has been resolved (time 1, hereafter) and the exercise price \( P_g \) once the final outcome is known (time 2, hereafter). However, the league sets both prices at time 1. In equilibrium, depending on the exercise price and the utility for the unpopular game, some game-based fans may attend only the popular game (Case A) or all game-based fans could attend either game (Case B). For example, if their utility for the unpopular game is low and the exercise price is high, some game-based fans may only attend the popular game (if it occurs) and not the unpopular game (if it occurs). Therefore, game-based fans may either engage in three behaviors (see Figure 2A) or in two behaviors (see Figure 2B). The difference between Figures 2A and 2B is the existence of a group of fans who will attend only the popular game. Note that the exercise price is paid only if a fan who has purchased the option decides to attend the game at time 2.

---

**Figure 2: Different demand patterns from the game-based fan segment**

*Figure 2A (Three Behaviors)*

- 0
- \( \theta_1 \)
- \( \theta_2 \)
- \( \alpha_G \)
- Not buy the option
- Buy the option and attend only the popular game
- Buy the option and watch either game

*Figure 2B (Two Behaviors)*

- 0
- \( \theta_1 \)
- \( \alpha_G \)
- Not buy the option
- Buy the option and attend either game
Figure 3: Demand from the team-based segment

![Diagram showing demand from the team-based segment]

Across the cases the behavior of the team-based fan stays the same (see Figure 3) - they attend only the popular game. This is because, by definition, these fans have no utility for the unpopular game.

The league can decide if it will serve fans from one or both segments through the price that it sets. That is, the league can end up serving: (a) only game-based fans, (b) only team-based fans, or (c) both fan types. Typically the league will ignore one segment if it is small relative to the other segment. I focus on the more interesting case where the segments are of comparable size, so that the league serves both segments\(^\text{4}\). The behaviors in Figure 2A and 2B are considered separately as Case A and Case B.

**Case A: Some game-based fans may watch only the popular game (3 behaviors)**

In this case the game-based fans can engage in one of three behaviors (a) buy the option and attend either game (BE), (b) buy the option and attend only the popular game (BO) or (c) not buy the option (NB). It can be shown that in equilibrium, fans who engage in the BE behavior have a higher utility than those who adopt the BO behavior, who have a higher utility than those who do not buy the option (NB). Let \(q_{GE}^A\) represent the demand of game-based fans who attend either game, \(q_{GO}^A\) represent the demand of game-based fans who attend only the popular game.

---

\(^4\) In addition, when serving one segment the league can always use option pricing to replicate advance selling, by setting the exercise price to zero. Therefore, this case is not of theoretical interest. Details are available from the authors.
The fan with the lowest valuation who adopts the BO behavior is located at

\[ \theta_1 = \alpha_G - q_{GE}^A - q_{GO}^A \] (see Figure 2A). From eqn. (1) the utility of the game-based fan who attends only the popular game is

\[ \gamma (\alpha_G - q_{GE}^A - q_{GO}^A) \lambda_1 - (P_o^A + \gamma P_g^A) \] where \( q_{GE}^A, q_{GO}^A \) represent the demands and \( \lambda_1 \) represents the quality of the popular game. The utility of the game-based fan who does not buy the option is zero. Equating the utilities from the BO and the NB strategies the demand of the game-based fans who attend only the popular game is obtained:

\[ q_{GO}^A = (\alpha_G - q_{GE}^A) - (P_o^A + \gamma P_g^A / \gamma \lambda_1) \]  

Equation (2)

I set \( \lambda_1 = 1 / \beta_{G1} \) - this implies that the game quality is inversely related to the price sensitivity of the fans for that game. This captures the fact that as the game-based fans’ utility (\( \theta_G \lambda_1 \)) increases their price sensitivity decreases. Therefore, eqn. (2) can be simplified as:

\[ q_{GO}^A = (\alpha_G - q_{GE}^A) - (\beta_{G1} / \gamma)(P_o^A + \gamma P_g^A) \]  

Equation (3)

The location of the fan with the lowest valuation for the BE behavior is at \( \theta_2 = \alpha_G - q_{GE}^A \) (see Figure 2A). The utility of the fan who attends either game is

\[ \gamma (\alpha_G - q_{GE}^A) \lambda_1 + (1 - \gamma) (\alpha_G - q_{GE}^A) \lambda_2 - (P_o^A + P_g^A) \] . Equating the utilities of the marginal fan who is indifferent between the BE and BO behaviors and substituting \( \lambda_1 = 1 / \beta_{G1} \) and \( \lambda_2 = 1 / \beta_{G2} \), I obtain the demand of fans who attend either game:

\[ q_{GE}^A = \alpha_G - \beta_{G2} P_g^A \]  

Equation (4)

Substituting eqn.(4) into eqn.(3) yields the demand of the game-based fans who attend only the popular game:

\[ q_{GO}^A = \beta_{G2} P_g^A - (\beta_{G1} / \gamma)(P_o^A + \gamma P_g^A) \]  

Equation (5)
Next, consider the team-based fans. The team-based fan chooses between (a) buying the option and attending the popular game (BO) or (b) not buying the option (NB). It can be shown that in equilibrium, fans who engage in the BO behavior have a higher utility than those who adopt the NB behavior. The team-based fan with the lowest valuation who adopts the BO strategy is located at \( \theta_1 = \alpha_T - q_{BO}^T \) (see Figure 3). From eqn.(1), the utility of the team-based fan who attends only the popular game is \( \gamma(\alpha_T - q_{BO}^T)\lambda_1 - (P_o^A + \gamma P_G^A) \), and the utility of the team-based fan who does not buy the option is zero. Equating the utilities from the BO and the NB behaviors I obtain the demand of the team-based fans who attend only the popular game:

\[
q_{BO}^T = \alpha_T - (P_o^A + \gamma P_G^A) / \gamma \lambda_1
\]  

(6)

As before, I set \( \lambda_1 = 1/\beta_{T1} \) - this implies that the game quality is inversely related to the price sensitivity of the fans for that game. Therefore, the demand is given by:

\[
q_{BO}^T = \alpha_T - (\beta_{T1}/\gamma)(P_o^A + \gamma P_G^A)
\]  

(7)

Because the option price is already paid when game-based fans decide whether or not to attend the unpopular game, the option price (\( P_O^A \)) does not play a role in the demand in eqn. (4). However, fans would not purchase the option unless they intend to attend at least the popular game. Therefore, the demand of game- and team-based fans who attend only the popular game (in eqns. (5) and (7)) is affected by both option and the exercise prices. Profits (\( \pi_{OB}^A \)) are denoted by:

\[
\pi_{OB}^A = (q_{GO}^A + q_{BO}^A)(P_o^A + \gamma P_G^A) + q_{GE}^A (P_o^A + P_G^A).
\]  

(8)

In eqn. (8) the league obtains both prices (\( P_O^A + P_G^A \)) from the game-based fans who attend either game (\( q_{GE}^A \)) whereas it obtains the exercise price with a certain probability (\( \gamma \)) from game- and
team-based fans who attend only the popular game \((q^A_{GO}, q^A_{TO})\). The league chooses the optimal option and exercise prices after substituting the demands from eqns. (4), (5) and (7) into the profit function in eqn. (8). Differentiating this profit expression with respect to \(P^A_O\) and \(P^A_G\), equating those differentials to zero, and solving the resulting first order conditions yields the optimal outcomes in Table 2.

### Table 2: Optimal prices, quantities, and profits under option pricing – Case A

<table>
<thead>
<tr>
<th>Prices</th>
<th>Quantities</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P^A_O = \gamma (\frac{\alpha_T + \alpha_G}{2(\beta_G + \beta_T)}) - \frac{\alpha_G}{2\beta_G})</td>
<td>(q^A_{GO} = \frac{\alpha_G}{2})</td>
<td>(\pi^A_{GO} = \gamma (\frac{\alpha_G + \alpha_T}{4(\beta_G + \beta_T)}) + (1-\gamma) \frac{\alpha_G^2}{4\beta_G})</td>
</tr>
<tr>
<td>(P^A_G = \frac{\alpha_G}{2\beta_G})</td>
<td>(q^A_{TO} = \frac{\alpha_T(2\beta_G + \beta_T) - \alpha_G\beta_G}{2(\beta_G + \beta_T)})</td>
<td></td>
</tr>
</tbody>
</table>

Note that as fans’ utilities increase, their price sensitivities decrease. As an example, an increase in the overall utility of the game-based segment for the popular game can be associated with an increase in \(\alpha_G\) or a decrease in \(\beta_G\). This interpretation is used to explain the intuitions for the optimal prices\(^5\).

Note that the price of the option \((P^A_O)\) increases with respect to \(\beta_G\) in Table 2:

\[
P^A_O > 0 \text{ if } \beta_G > \alpha_G\beta_T/\alpha_T
\]

Intuitively, as the utility for the unpopular game decreases (i.e., as \(\beta_G\) increases), the value of the option increases. Conversely, as the utility of the unpopular game increases (i.e., as \(\beta_G\) decreases), the exercise price \((P^A_G)\) increases.

---

\(^5\) This is consistent with the individual utility model. For example, as game-based fans’ valuations for the popular game \((\theta_G)\) increase, their price sensitivities \((\beta_G)\) decrease because \(\lambda_G = 1/\beta_G\).
Some game-based fans attend only the popular game when the utility of the game-based segment for the popular game is sufficiently high (i.e., when $\beta_{G1}$ is low):

$$q_{GO}^A > 0 \text{ if } \beta_{G1} < \alpha_G \beta_{T1}/\alpha_T$$

(10)

To the right of the threshold in eqn. (10), as the game-based fan’s valuation increases, all game-based fans who buy the option attend either game (i.e., the number of fans who attend just one game, $q_{GO}^A$, equals zero). Therefore, Case A no longer applies. Profits in Case B where all game-based fans who purchase the option attend either game (i.e., the case when $\beta_{G1} > \alpha_G \beta_{T1}/\alpha_T$) are computed next.

**Case B: All game-based fans watch either game (2 Behaviors)**

I begin Case B by deriving the demand from the game- and team-based fans. Here, the game-based fans can engage in either BE or NB (the corresponding utility is zero). The fan with the lowest (positive) valuation for the BE behavior is located at $\theta_1 = \alpha_G - q_{GE}^B$. The utility of a fan who adopts BE is $\gamma(\alpha_G - q_{GE}^B)\lambda_1 + (1 - \gamma)(\alpha_G - q_{GE}^B)\lambda_2 - (P_{O}^B + P_{G}^B)$. Equating the utilities from the BE and NB behaviors, the demand from fans who watch either game is:

$$q_{GE}^B = \alpha_G - (\beta_{G1}\beta_{G2}(P_{O}^B + P_{G}^B))/(\gamma \beta_{G2} + (1 - \gamma)\beta_{G1})$$

(11)

The team-based segment’s behavior does not change between Cases A and B. Therefore, the demand from the team-based segment is derived exactly as in Case A (eqn.(6)):

$$q_{TO}^B = \alpha_T - (\beta_{T1}/\gamma)(P_{O}^B + \gamma P_{G}^B)$$

(12)

The demand functions are denoted by eqns. (11) and (12). Profits are denoted by:

$$\pi_{OB}^B = q_{GE}^B (P_{O}^B + P_{G}^B) + q_{TO}^B (P_{O}^B + \gamma P_{G}^B).$$

(13)
Note that all game-based fans who purchase the option end up exercising the option by paying $P^B_G$ to watch either the popular game or the unpopular game. Maximizing these profits with respect to $P^B_G$ and $P^B_O$ (and straightforward substitution of the derived optimal prices) yields the optimal outcomes in Table 3.

Table 3: Optimal prices, quantities, and profits under option pricing – Case B

<table>
<thead>
<tr>
<th>Prices</th>
<th>Quantities</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^B_o = \gamma \left( \frac{\alpha_T \beta_{g1} - \alpha_G \beta_{T1} \gamma}{2 \beta_{T1} \beta_{g1}} - \frac{\alpha_G}{2 \beta_{g2}} \right)$</td>
<td>$q^B_{OE} = \frac{\alpha_G}{2}$</td>
<td>$\pi^B_{OB} = \alpha_G \beta_{g2} \gamma + \alpha_G \beta_{T1} (\beta_{g1} (1 - \gamma) + \beta_{g2} \gamma)$</td>
</tr>
<tr>
<td>$P^B_g = \frac{\alpha_G}{2 \beta_{g2}} - \gamma \left( \frac{\alpha_T \beta_{g1} - \alpha_G \beta_{T1}}{2 \beta_{T1} \beta_{g1}} \right)$</td>
<td>$q^B_{TO} = \frac{\alpha_T}{2}$</td>
<td>$4 \beta_{g2} \beta_{g1} \beta_{T1}$</td>
</tr>
</tbody>
</table>

Note that:

$$P^B_o > 0 \text{ if } \beta_{g2} > \alpha_G \beta_{T1} / \alpha_T$$  
(14)

$$P^B_g > 0 \text{ if } \beta_{g2} < \alpha_G \beta_{g1} \beta_{T1} (1 - \gamma) / (\alpha_T \beta_{g1} - \alpha_G \beta_{T1}) \gamma$$  
(15)

That is, for the optimal option and exercise price to be positive, $\beta_{g2}$ should lie within certain bounds (i.e., $\alpha_G \beta_{T1} / \alpha_T < \beta_{g2} < \alpha_G \beta_{g1} \beta_{T1} (1 - \gamma) / (\alpha_T \beta_{g1} - \alpha_G \beta_{T1}) \gamma$). The left bound in the expression ($\alpha_G \beta_{T1} / \alpha_T < \beta_{g2}$) indicates the condition required for the option price ($P^B_o$) to be positive. Similar to the intuition in Case A as the utility for the unpopular game decreases, the value of the option increases. The right bound ($\beta_{g2} < \alpha_G \beta_{g1} \beta_{T1} (1 - \gamma) / (\alpha_T \beta_{g1} - \alpha_G \beta_{T1}) \gamma$) indicates the condition needed for the exercise price ($P^B_g$) to be positive. Together, the bounds indicate that game-based fans should have a minimum value for the unpopular game so that the optimal exercise price is positive and a maximum value for the unpopular game so that the optimal option price is positive.
When the probability of the popular game occurring is low (i.e., if $\gamma < 1/2$) the value of the option is high and the game-based fans’ utility for the unpopular game can lie within the required bounds (i.e., when $\gamma < 1/2$ the right bound is greater than the left bound). However, if $\gamma > 1/2$, the value of the option is low and a higher utility for the popular game is required to ensure that the right bound is greater than the left bound:

$$\beta_{G1} < \alpha_G \beta_T \gamma / \alpha_T (2\gamma - 1)$$

(16)

I compare the optimal prices and profits across option Cases A and B. The exercise price in Case A is lower than that in Case B (i.e., $P_G^B < P_G^A$). Intuitively, the lower exercise price ensures that game-based fans who buy the option attend either game in Case B. However, this also means they are willing to pay a lower price for the option. Consistent with this intuition, the option price in Case B is higher than in Case A ($P_O^B > P_O^A$). Further, it can be seen that profits are higher in Case B ($\pi_{OB}^B > \pi_{OB}^A$). The intuition for this is as follows. In Case A, much like the team-based fans, some game-based fans attend only the popular game. Setting prices at levels which ensure that team-based fans purchase the option implies leaving game-based fans with considerable surplus. In Case B, because the behaviors of the two segments are different (i.e., all team-based fans attend only the popular game, all game-based fans attend either game) more surplus can be extracted from both groups, leading to higher profits in Case B than in Case A.

Depending on the fans’ valuations the league sets the prices, which in turn, determines whether some game-based fans watch only the popular game (Case A applies) or if all watch either game (Case B applies).

**Advance selling**
Advance selling refers to the conventional approach – here the league sets the price at the beginning of the season before the uncertainty has been resolved (time 1). Fans purchase tickets based on forward-looking expected utility. The advance ticket price is in the nature of a sunk cost that is paid up front. Because game-based fans obtain positive utility from attending either game at time 2 without making any additional payments they attend either the popular or unpopular game - whichever event happens. Therefore, the game-based fans will (a) attend either game (BE) or (b) not buy the ticket (NB). Equating the expected utility of the game-based fan from buying the ticket under advance pricing (i.e., under BE) to zero (the utility under NB), the demand of the game-based fans who attend either game is:

\[ q_G = \alpha_G - (P \beta_{G1}\beta_{G2})/(\gamma \beta_{G2} + (1-\gamma)\beta_{G1}) \]  

(17)

The team-based fans obtain zero utility from attending the unpopular game. Therefore, they will watch only the popular game if it occurs (BO), or not buy the ticket (NB). Equating the utilities from the BO and the NB behaviors, I obtain the demand of the team-based fans who attend only the popular game:

\[ q_T = \alpha_T - (P \beta_{T1}/\gamma) \]  

(18)

Here, \( P \) represents the advance selling price, \( q_G \) is the quantity demanded by the game-based segment and \( q_T \) is the quantity demanded by the team-based segment. The profit function (\( \pi_{Ab} \)) is given by:

\[ \pi_{Ab} = P(q_G + q_T) \]  

(19)

Substituting the demands from eqns. (17) and (18) into eqn. (19) yields the profit expression. Differentiating this profit expression with respect to \( P \), equating that differential to zero, and solving the resulting first order condition yields the optimal outcomes in Table 4.
Table 4: Optimal price, quantities, and profits under advance selling

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantities</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{\gamma(a_g + \alpha_f)((1-\gamma)\beta_{g1} + \gamma\beta_{g2})}{2(\beta_{r1}\beta_{g2}\gamma + \beta_{g1}(\beta_{r1}(1-\gamma) + \beta_{g2}\gamma))} ]</td>
<td>[ q_{G1} = \frac{2\alpha_{G2}\beta_{r1}\beta_{g1}}{2(\beta_{r1}\beta_{g2}\gamma + \beta_{g1}(\beta_{r1}(1-\gamma) + \beta_{g2}\gamma))} ]</td>
<td>[ \pi_{AB} = -\frac{\gamma(a_g + \alpha_f)^2(\beta_{g1}(1-\gamma) + \beta_{g2}\gamma)}{4(\beta_{r1}\beta_{g2} + \beta_{g1}(\beta_{r1}(1-\gamma) + \beta_{g2}\gamma))} ]</td>
</tr>
<tr>
<td>[ q_{r1} = \alpha_{r1} - \frac{(a_g + \alpha_f)(\beta_{g1}(1-\gamma) + \beta_{g2}\gamma)}{2(\beta_{r1}\beta_{g2}\gamma + \beta_{g1}(\beta_{r1}(1-\gamma) + \beta_{g2}\gamma))} ]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Given these profits, which pricing mechanism should the league implement? Proposition 1 provides the answer:

**PROPOSITION 1:** Profits under option pricing are always greater than or equal to those under advance selling.

**Proof of Proposition 1**

I first compare profits under option pricing Case A with those under advance selling:

\[
\pi_{G1} - \pi_{AB} = \frac{(1-\gamma)(a_g^2\beta_{r1}(\beta_{g1}(1-\beta) - (\beta_{g1} - \beta r1\beta_{g2} + \beta_{g1}(\beta_{r1} - 2\beta_{g2}))\gamma) - \gamma\alpha_{r1}\beta_{g2}\beta_{g1}^2(2a_g + \alpha_f)}{\beta_{g2}(\beta_{g1} + \beta_{r1})(\beta_{g2}\beta_{r1}\gamma + \beta_{g1}(\beta_{r1}(1-\gamma) + \beta_{g2}\gamma))}
\]

(20)

The denominator is positive. To prove the proposition, I therefore need to show that the numerator is positive as well. I rewrite the numerator as:

\[
(1-\gamma)(a_g^2\beta_{g1}\beta_{r1}(1-\gamma)(\beta_{g1} + \beta_{r1}) - \beta_{g2}(\alpha_{r1}\beta_{g1} - \alpha_{r1}\beta_{g1})(\alpha_{r1}\beta_{g1} + \alpha_{g}(2\beta_{g1} + \beta_{r1}))\gamma
\]

(21)

It is easy to see that eqn. (21) is positive when \( \beta_{r1} < \alpha_{r1}\beta_{r1}/\alpha_{r1} \). From eqn. (10) this is the condition that ensures \( q_{r1} > 0 \). Therefore, in the parametric region relevant to the analysis, profits under option pricing Case A exceed those from advance selling.

Next, I compare profits under option pricing Case B with those under advance selling:
\[ \pi^B_{ob} - \pi_{AB} = (\alpha_t \beta g_1 \beta g_2 \gamma + \alpha_g \beta r_1 (\beta g_1 (1 - \gamma) + \beta g_2 \gamma)) / 4 \beta g_1 \beta g_2 \beta r_1 (\beta g_2 \beta r_1 \gamma + \beta g_1 (\beta r_1 (1 - \gamma) + \beta g_2 \gamma)) \]

(22)

Note that eqn. (22) is always positive. Therefore, profits under option pricing Case B exceed those from advance selling.

The intuition for proposition 1 is simple. Prices under both option and advance selling are set early in the season. The league can *always* use option pricing to mimic advance selling by setting the option price equal to the price under advance selling, and the exercise price equal to zero. Therefore, option pricing can do no worse than advance selling.

A more detailed picture can be obtained by comparing the differences in the way that option pricing and advance selling extract surplus from fans. Across the pricing mechanisms, the expected utility of the game-based fans are higher, on average, than that of the team-based fans. When the expected utility of the game-based fans is higher, the price under advance selling is set low enough so that a sufficient number of team-based fans are captured. In that process, game-based fans are left with substantial surplus. Under option pricing, in contrast, the league extracts more surplus from the team-based segment by increasing the option price and reducing the exercise price. The reduction in exercise price ensures the game-based fans who buy the option are willing to watch whichever game occurs. Further, the high option price can be used to extract the surplus of more game-based fans as well. Therefore, by effectively balancing the option and exercise prices, option pricing succeeds in extracting greater surplus from the fans compared to advance selling and consequently yields the higher profits.
Proposition 1 is depicted graphically using Figure 4\textsuperscript{6}. The x-axis represents the price sensitivity of game-based fans for the popular game ($\beta_{G_1}$). As we move along the x-axis (as the price sensitivity increases) the utility of game-based fans for the popular game decreases. Therefore as we move along the x-axis the relative preference of the team-based fan over that of the game-based fan increases. The y-axis represents the price sensitivity ($\beta_{G_2}$) of the game-based fans for the unpopular game. Similar to the intuition in the x-axis, as we move along the y-axis the utility for the unpopular game decreases. Therefore, the y-axis can be seen as indicating the preference of the game-based fan for the unpopular game. Consistent with the assumption in the vertical differentiation model the quality of the popular game is higher than that of the unpopular game (or $\beta_{G_2} > \beta_{G_1}$). Consequently the region where $\beta_{G_2} < \beta_{G_1}$ is ignored because in this region the quality of the unpopular game is higher than that of the popular game.

I begin by explaining the thresholds on the two axes. On the x-axis, to the left of threshold $a$, the league serves only the game-based fans under option pricing. To the right of Curve A, where the expected utility of the game-based segment is sufficiently low, the league serves only the team-based fans. The striped region indicates the area where both segments are served under option pricing and advance selling. As stated earlier, our interest lies in the striped region. Threshold $b$ is defined as the ‘main condition’ – to the left of $b$ the relative utility of the game-based segment over the team-based segment is sufficiently high, therefore some game-based fans watch just the popular game (Case A applies). To the right of $b$ all the game-based fans who buy the option watch either game (Case B applies). The threshold on the y-axis

\textsuperscript{6} For expository convenience, we set $\alpha_G = 5$, $\alpha_T = 6$, $\beta_{T_1} = 1$ and $\gamma = 0.7$ for drawing the curves in the figures that follow. However, we characterize all the results in analytical terms.
corresponding to $\beta_{G2} > \alpha_G \beta_{T1}/\alpha_T$ indicates the level above which the option prices $(P^A_o, P^R_o)$ are positive.

**Figure 4: Optimal league strategy under option pricing and advance selling**

Because profits under option pricing exceed those from advance selling (from Proposition 1) the league should implement option pricing in the striped region. As discussed in Case B when the probability of the popular game occurring is high $(\gamma > 1/2)$ threshold $c$ ensures that the utility for the unpopular game is within the bounds that ensures positive option and exercise prices. To the right of threshold $c$ option pricing can be used to replicate advance selling (by setting the option price equal to the price under advance selling and the exercise price at zero). When the probability of the popular game occurring is low $(\gamma < 1/2)$ the value of the option is high and threshold $c$ does not exist - Case B extends to the right. This implies profits under
option pricing exceed those from advance selling over a larger region when \( \gamma < 1/2 \). Intuitively, the role of the option in protecting the fans from uncertainty becomes more salient as the likelihood of the popular game occurring decreases.

In summary, profits under option pricing always equal or exceed those from advance selling. With both option pricing and advance selling the league sets prices before the uncertainty is resolved. Therefore, it is intuitive that option pricing can be made to mimic advance selling. However, with full information pricing the league prices tickets \textit{after} the uncertainty is resolved. Therefore, it is unclear how profits under option pricing will compare with those under full information pricing. The full information pricing mechanism is discussed next.

**Full information pricing**

With full information pricing, the league can set different prices \((P_1 \text{ or } P_2)\) depending on whether the popular or unpopular game occurs\(^7\). Note that when the unpopular game occurs the league will always serve only the game-based fans because this is the only segment with positive valuation in that state. I derive the demand of the game-based fans first. Consider the case where the popular game occurs. The utility of the game-based fan who attends this game is \((\alpha_g - q_{G1})\lambda_1 - P_1\) and the utility from not attending the game is zero. Equating these utilities, the demand of game-based fans for the popular game is:

\[
q_{G1} = \alpha_g - \beta_{G1} P_1
\]  
(23)

If the unpopular game occurs, the utility of the fan from attending this game is \((\alpha_g - q_{G2})\lambda_2 - P_2\) and the utility from not attending is zero. Equating these utilities, the demand of game-based fans is:

\[^7\text{As discussed earlier, full information pricing is challenging for the leagues to implement.}\]
\[ q_{G2} = \alpha_G - \beta_{G2}P_2 \]  

Next, I derive the demand of the team-based fans. Consider the case where the popular game occurs (team-based fans will not see the unpopular game). The utility of the team-based fan attending the popular game is \((\alpha_T - q_{T1})\lambda_T - P_1\) and the utility of not attending the game is zero. Equating these utilities, the demand of team-based fans for the popular game is:

\[ q_{T1} = \alpha_T - \beta_{T1}P_1 \]  

The demand of the game- and team-based segments (from eqns. (23) and (25)) are denoted by:

\[ q_{G1} = \alpha_G - \beta_{G1}P_1; \quad q_{T1} = \alpha_T - \beta_{T1}P_1 \]  

Expected profits \((\pi_{FB})\) are:

\[ \pi_{FB} = \gamma P_1(q_{T1} + q_{G1}) + (1 - \gamma)P_2q_{G2}. \]  

Substituting the demands from eqn. (25) and (26) into eqn. (27) yields the optimal expected profits. Differentiating this profit expression with respect to \(P_1\) and \(P_2\), equating those differentials to zero, and solving the resulting first order conditions yields the optimal outcomes in Table 5.

**Table 5: Optimal prices, quantities, and profits under full information pricing**

<table>
<thead>
<tr>
<th>Prices</th>
<th>Quantities</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = \frac{(\alpha_G + \alpha_T)}{2(\beta_{G1} + \beta_{T1})} )</td>
<td>( q_{G1} = \alpha_G - ((\alpha_G + \alpha_T)\beta_{G1}/2(\beta_{G1} + \beta_{T1})) )</td>
<td>( \pi_{FB} = \frac{\gamma(\alpha_G + \alpha_T)^2}{4(\beta_{G1} + \beta_{T1})} + \frac{(1 - \gamma)\alpha_G^2}{4\beta_{G2}} )</td>
</tr>
<tr>
<td>( P_2 = \frac{\alpha_G}{2\beta_{G2}} )</td>
<td>( q_{T1} = \alpha_T - ((\alpha_G + \alpha_T)\beta_{T1}/2(\beta_{G1} + \beta_{T1})) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( q_{G2} = \frac{\alpha_G}{2} )</td>
<td></td>
</tr>
</tbody>
</table>

The comparison of profits under option pricing in Cases A and B with full information profits are summarized in the propositions below:
PROPOSITION 2: Profits under option pricing when some game-based fans may watch only the popular game (Case A) equal expected profits under full information pricing.

Proof of Proposition 2

Comparing the profitability equations from above, it is straightforward to show that these profits are equal.

The intuition for proposition 2 is as follows. When the average utility of the game-based fan for the unpopular game is high (i.e., when $\beta_{G_2}$ is low) the value of the option is low because fans do not need a strong protection against the downside. For the option price to be positive the utility for the unpopular game should be low (from eqns. (9) and (14)). In this region because the average utility of the game-based fan for the unpopular game is low (i.e., $\beta_{G_2}$ is high) fans gain more by protecting themselves from the downside. Consequently, the value of the option is high and profits under option pricing equal expected profits under full information pricing.

PROPOSITION 3: Profits under option pricing when all game-based fans watch either game (Case B) exceed expected profits under full information pricing.

Proof of Proposition 3

I compare profits under option pricing (Case B) with expected profits under full information pricing:

$$\pi^n_{OB} - \pi^n_{FB} = (\alpha_T \beta_{G_1} - \alpha_G \beta_{T_1})^2 \gamma / 4 \beta_{G_1} \beta_{T_1} (\beta_{G_1} + \beta_{T_1})$$  \(28\)
Note that eqn. (28) is always positive. Therefore, profits under option pricing in Case B (corresponding to the parametric region to the right of the main condition in Figure 4) exceed expected profits under full information pricing.

The intuition for proposition 3 is as follows. Under Case B, the game-based fans have a lower utility for the popular game. At the same time, under Case B, game-based fans have a higher expected utility than the team-based fans (see eqn.(15)). Under full information pricing, when serving both segments and the popular game occurs, the price will be set low to capture a sufficient number of game-based fans who have a lower utility for that game on average than team-based fans. Therefore, team-based fans are left with substantial surplus. As discussed earlier, under option pricing, the league extracts more surplus from the team-based segment by increasing the option price and reducing the exercise price. By effectively balancing between the option and exercise price, option pricing succeeds in extracting surplus from more of the game-based and team-based fans compared to full information pricing. Note that in this case, the league can potentially obtain higher profits by setting the option and exercise prices before uncertainty has been revealed, compared to profits obtained if it set prices after uncertainty was revealed. This result has implications for other markets where it may not be possible to implement full information pricing due to fairness constraints or other restrictions.

To complete the analysis, I compare profits under full information with those from advance selling. The results are summarized in the following proposition:

**PROPOSITION 4a:** To the left of the main condition (threshold b in Figure 4), expected profits under full information pricing always exceed those under advance selling.
Proof of Proposition 4a

I compare profits from full information and advance selling. The difference between the profits is given by:

$$\pi_{FB} - \pi_{AB} = \frac{(1-\gamma)(\alpha_G^2 \beta_{T1}(\beta_{G1} + \beta_{T1}) - (\beta_{G1}^2 - \beta_{T1} \beta_{G1} + \beta_{G1} (\beta_{T1} - 2 \beta_{G1}))\gamma) - \gamma \alpha_T \beta_{G2} \beta_{G1}^2 (2 \alpha_G + \alpha_T)}{\beta_{G2}(\beta_{G1} + \beta_{T1})(\beta_{G2} \beta_{T1} \gamma + \beta_{G1}(1-\gamma) + \beta_{G2} \gamma)}$$  \hfill (29)

The denominator is positive. To prove the proposition, I therefore need to show that the numerator is positive as well. I rewrite the numerator as:

$$(1-\gamma)(\alpha_G^2 \beta_{G1} \beta_{T1}(1-\gamma)(\beta_{G1} + \beta_{T1}) - \beta_{G2}(\alpha_T \beta_{G1} - \alpha_G \beta_{T1})(\alpha_T \beta_{G1} + \alpha_G (2 \beta_{G1} + \beta_{T1}))\gamma$$  \hfill (30)

It is easy to see that eqn. (30) is positive when $\beta_{G1} < \alpha_G \beta_{T1} / \alpha_T$. From eqn. (10) this is the condition that ensures $q_{GA}^A > 0$. Therefore, to the left of the ‘main condition’ (when $\beta_{G1} < \alpha_G \beta_{T1} / \alpha_T$), expected profits under full information exceed those from advance selling.

The intuition is as follows: to the left of threshold $b$ the average utility of the game-based segment for the popular game is higher than that of the team-based segment (from eqn.(10)). Further, the game-based segment can derive additional utility from watching the unpopular game. Consequently, the average expected utility of the team-based segment is low compared to the game-based segment. This low level of expected utility for the team-based segment drags down the advance selling price, thereby doing a poor job of surplus extraction across the segments and leaving the game-based fans with a substantial surplus. In contrast, the full information pricing mechanism collects profits separately from the states of nature where the popular or unpopular games occur. Therefore, it can capture additional surplus from the game-based segment in case the unpopular game occurs. Further, in the event that the popular game occurs, the gap between the average utilities across the segments is narrower, and the
corresponding full information price can do a better job of extracting surplus across the segments.

**PROPOSITION 4b:** To the right of the main condition (threshold \( b \) in Figure 4), expected profits under full information pricing exceed those under advance selling only when the average utility of the game-based fans for the unpopular game is high.

**Proof of Proposition 4b**

The difference between full information and advance selling profits is:

\[
\pi_{FB} - \pi_{AB} = \frac{(1-\gamma)(\alpha_G^2\beta_{T_1}(\beta_{G_1} + \beta_{T_1}) - (\beta_{G_1} - \beta_{T_1}\beta_G^2 + \beta_{G_1}(\beta_{T_1} - 2\beta_{G_2}))\gamma) - \gamma\alpha_T\beta_{G_2}\beta_{G_1}^2(2\alpha_G + \alpha_T)}{\beta_{G_2}(\beta_{G_1} + \beta_{T_1})(\beta_{G_2}\beta_{T_1}\gamma + \beta_{G_1}(\beta_{T_1}(1-\gamma) + \beta_{G_2}\gamma))}
\]  

(31)

The denominator is always positive. To the right of the main condition, the numerator is negative and profits under full information exceed those under advance selling if:

\[
\beta_{G_2} < \alpha_G^2\beta_{G_1}\beta_{T_1}(1-\gamma)(\beta_{G_1} + \beta_{T_1})/(\alpha_T\beta_{G_1} - \alpha_G\beta_{T_1})(2\alpha_G\beta_{G_1} + \alpha_T\beta_{G_1} + \alpha_G\beta_{T_1})\gamma
\]  

(32)

Stated differently, profits under advance selling exceed those under full information pricing when \( \beta_{G_2} \) is sufficiently high, i.e., when the utility of the game-based fans for the unpopular game is relatively low.

The intuition is as follows. Pricing under full information separately extracts surplus from the game-based fans when the unpopular game occurs – therefore, it can extract this surplus efficiently. To the extent that the utility of the game-based fans associated with the unpopular game is higher, the advantage of full information pricing over advance pricing increases. In contrast, if the average utility for the unpopular game is low (i.e., \( \beta_{G_2} \) is high), the power of full information pricing to separately extract profits in the case of the unpopular game is less
important. Therefore, expected profits under full information exceed profits under advance selling when the utility for the unpopular game is high\(^8\).

**Optimal league pricing strategies**

I summarize the discussion using Figure 5. I focus on the regions where both segments are served and the option and exercise prices are positive. This is indicated by regions \(S_1\) and \(S_2\) in Figure 5.

As in Figure 4 the x-axis represents the price sensitivity of game-based fans for the popular game \(\beta_{G_1}\) and the y-axis represents the price sensitivity of the game-based fan segment \(\beta_{G_2}\) for the unpopular game. As a reminder, the ‘main condition’ differentiates between when some game-based fans may watch only the popular game (Case A) versus when all game-based fans watch either game (Case B). To the left of the threshold \(\alpha_0 \beta_1 \gamma / \alpha_0 (2 \gamma - 1)\) on the x-axis, the utility for the unpopular game is such that option and exercise prices can be positive in Case B. Threshold \(i\) indicates the threshold above which the option prices \(P^A_0, P^B_0\) are positive.

In region \(S_1\) profits under option pricing equal expected full information profits (Proposition 2 applies). It has been shown earlier that profits under option pricing exceed those under advance selling in this region. Therefore, in \(S_1\) the league should implement either option pricing or full information pricing. In the presence of fairness constraints, option pricing is the more viable alternative.

In region \(S_2\), option pricing is more profitable than full information pricing (Proposition 3 applies). It has already been shown that profits under option pricing Case B exceed those from advance selling. Therefore profits under option pricing Case B exceed profits under advance

\(^8\) Xie and Shugan (2001) have demonstrated that, under some conditions, advance selling can be more profitable than pricing closer to the event. Our results confirm this finding.
sells and full information pricing in this region. As discussed earlier, the region where profitability of options exceeds those from other pricing mechanisms ($S_2$) extends to the right when probability of the popular game occurring is low (or if $\gamma < 1/2$). Profitability from options can be further enhanced if the league resells the unexercised options once the uncertainty has been revealed.

Figure 5: Optimal league strategies across pricing mechanisms

In order to clarify the intuitions underlying the profitability of option pricing, I next consider a simple model involving two fans, each having different valuations for attending the

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9 Xie and Gerstner (2003) suggest that stadiums with fixed capacity can profit by making the cancellation process easier for fans so that leagues can resell the limited seats. In our case, if fans do not exercise the option by a given cut-off date the league could resell the unexercised options as discounted regular tickets. The league would collect an option price from those who did not exercise their options and could resell the ticket at a discount, further disadvantaging scalpers.
uncertain championship game. This individual level model allows us to identify the conditions under which pricing with consumer options can indeed generate higher profits for the league than advance selling or pricing with full information. Importantly, I demonstrate that fan heterogeneity (i.e., one fan has zero valuation for the non-preferred outcome) is a necessary condition for consumer options to be valuable to consumers and firms.

2.2.2 Individual Level Model

Similar to the assumptions in the aggregate demand model I assume tickets (options) are only sold at the beginning of the season before the final teams are known (tickets are unavailable at the end of the season when there is no uncertainty in the final matchup). I assume that the favorite team is common across fans and both fans prefer a game in which the favorite team is playing (the preferred game) to one in which it is not (the non-preferred game). However, one fan is assumed to be very sensitive to who is playing and thus is much less interested in attending the final game than the other fan if the favorite team is not playing. I label the first a team-based fan and the second a game-based fan. I assume that the team-based fan is willing to pay $U_T^+$ for a game involving the favorite team and $U_T^-$ otherwise ($U_T^+ > U_T^-$); whereas the game-based fan is willing to pay $U_G^+$ for the preferred game and $U_G^-$ for the non-preferred game ($U_G^+ > U_G^-$). Fans are assumed to be risk neutral and thus will make their ticket purchase decisions by maximizing their expected utility\textsuperscript{10}. I further assume the probability of the preferred game occurring ($\gamma$) is exogenous and known to both the fans and the league\textsuperscript{11}. Given this notation, the team-based

\textsuperscript{10} We note that this is a conservative approach for examining the profitability of consumer options. If fans are risk averse, their valuation of an option ticket would only be greater than that discussed in this section.

\textsuperscript{11} Experts often provide these estimates early in the season. In comparing men’s college basketball team rankings from various expert sources (Associated Press Poll, Coaches Poll, ESPN Poll, etc.), we find that the preseason rankings for the various teams across many years are very similar. This is because most experts use the same
fan’s expected valuation for the game is $\mathcal{BV}_f = \gamma u_f^* + (1 - \gamma)u_f^-$ and the game-based fan’s expected valuation is $\mathcal{BV}_g = \gamma u_g^* + (1 - \gamma)u_g^-$. Finally, I assume there are no additional fixed or variable costs associated with implementing consumer options.

In the remainder of this section, I focus on the situation in which the league sets ticket prices so that both fans participate. In this general situation, there is a price at which each fan will purchase a ticket and attend the championship game irrespective of the teams. To decide on ticket prices, the league must consider the relative sizes of the fans’ expected valuations. The league will set prices to equal the lower valuation to ensure that both fans participate: (1) if $\mathcal{BV}_g > \mathcal{BV}_f$, then the league sets price equal to $\mathcal{BV}_f$ and (2) if $\mathcal{BV}_f > \mathcal{BV}_g$, then the league sets price equal to $\mathcal{BV}_g$. In either case, the optimal advance selling price is equal to the optimal consumer options price ($p_{AS}^* = p_{o}^* + p_{g}^*$), and consequently, profits are the same ($\pi_{AS} = \pi_o$). In this general situation, consumer options offer no additional benefits over the current practice of advance selling.

However, suppose the team-based fan is so sensitive to the teams in the final game that their willingness to pay for the non-preferred game is zero (i.e., $u_T^- = 0$). In this case, the league cannot ensure that the team-based fan will attend the final game—here, the team-based fan will only exercise the option if the preferred game occurs. Unlike the more general situation above, consumer options now offers the team-based fan a valuable way to manage the risk associated with the final game. Moreover, if $\mathcal{BV}_g > \mathcal{BV}_f$, the league can obtain higher profits from consumer options than advance selling\textsuperscript{12}.

\textsuperscript{12} If $\mathcal{BV}_g > \mathcal{BV}_f$, then the league will set price equal to $\mathcal{BV}_g$ to ensure that the game-based fan will purchase a ticket. Once again, the advance selling price is equal to the consumer options price and profits are equivalent.
There are two cases to examine\(^{13}\): (1) \(U^+_g > U^+_c > U^-_c > 0\) and (2) \(U^+_g > U^+ _c > U^-_c > 0\).

In the first case, the league sets the consumer options price so that the individual rationality of the team-based fan is satisfied: \(P^*_o + \gamma P^*_o = \gamma U^+_c\). To ensure that the game-based fan exercises the option, the league sets the exercise price equal to the game-based fan’s willingness to pay for the non-preferred game: \(P^*_o = U^-_c\). Using these together to solve for the optimal option price gives \(P^*_o = \gamma(U^+_c - U^-_c)\). Here, the game-based fan will purchase the option and exercise it, while the team-based fan will purchase the option and only exercise it if the preferred game occurs. Expected league profits are:

\[
\pi^*_o = 2P^*_o + (1 + \gamma)P^*_o = 2\gamma(U^+_c - U^-_c) + (1 + \gamma)U^-_c = 2\gamma U^+_c + (1 - \gamma)U^-_c \tag{33}
\]

When \(U^+_c > U^+_g > U^-_c > 0\), the league sets the consumer options price equal to the expected valuation of each fan: \(P^*_o + P^*_c = \gamma U^-_c + (1 - \gamma)U^-_c\) and \(P^*_o + \gamma P^*_o = \gamma U^+_c\). Simultaneously solving these two equations yields optimal option and exercise prices:

\[
P^*_o = \frac{\gamma}{1 - \gamma}(U^+_c - (\gamma U^-_c + (1 - \gamma)U^-_c)) \quad \text{and} \quad P^*_c = U^-_c - \frac{\gamma}{1 - \gamma}(U^+_c - U^-_c).\]

Note that as the utility for the non-preferred game increases, the price of the option decreases. This is intuitive because as the utility for the non-preferred game increases the value of protecting against the downside decreases. Therefore the value of the option, and consequently the price of the option, reduces. In this case, expected league profits are:

\[
\pi^*_o = 2P^*_o + (1 + \gamma)P^*_c = \gamma U^+_c + \gamma U^-_c + (1 - \gamma)U^-_c \tag{34}
\]

The profits from consumer options in (33) and (34) can be compared with profits from advance selling. When \(RV_g > RV_c\), the league implements an advanced selling strategy by setting

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13 If \(U^+_g > U^+_c > U^-_c > 0\), the league follows an advanced selling strategy.
the optimal price equal to the team-based fan’s expected valuation \((P_{1d}^* = \gamma U^+_T)\), generating profits of

\[
\pi_{1d}^* = 2\gamma U^+_T
\]  

(35)

Keeping in mind that \(\gamma U^+_G + (1 - \gamma)U^-_G > \gamma U^+_T\) in this case, I find that expected profits from consumer options are always greater than profits from advance selling.

The analytical results to this point can be condensed into two conditions necessary for profits from consumer options to be higher than profits from advance selling.

**Condition 1:** *Fan valuations for the game are heterogeneous. Specifically, team-based fans have zero valuation for the non-preferred game.*

**Condition 2:** *The game-based fan’s expected valuation for the game is higher than the team-based fan’s expected valuation.*

Further, I compare these profits to those from full information pricing\(^{14}\). When \(U^+_T > U^+_G > U^-_G > 0\), the league can implement full information pricing as follows\(^{15}\). If the preferred game occurs, the league sets price equal to the game-based fan’s willingness to pay for the preferred game and if the non-preferred game occurs, it sets price equal to the game-based fan’s willingness to pay for the non-preferred game. In this case, expected league profits are

\[
\pi_{1f}^* = 2\gamma U^+_G + (1 - \gamma)U^-_G
\]  

(36)

---

\(^{14}\) The comparison of profits with full information pricing is done more for theoretical interest; as stated earlier, due to fairness constraints, the league would not set ticket prices after knowing which teams were playing in a final game.

\(^{15}\) If \(U^+_G > U^+_T > U^-_G > 0\), the league sets price equal to the team-based fan’s willingness to pay for the preferred game if the preferred game occurs. If the non-preferred game occurs, it sets price equal to the game-based fan’s willingness to pay for the non-preferred game. In this case, profits from full information pricing equal profits from consumer options in (1).
Comparing (34), (35) and (36), I find that profits from consumer options will be higher than profits from advance selling and full information pricing if the team-based fan’s willingness to pay for the preferred game is higher than the game-based fan’s willingness to pay for the preferred game.

The analytical findings indicate that consumer options can indeed generate more profits for the league than the current practice of advance selling. The conditions under which expected profits from consumer options dominate profits from advance selling are summarized in Conditions 1 and 2. Intuitively, as the game-based fan's gap in utility between the preferred and the non-preferred outcomes increases the value of the option increases. Further, as the profitability of the favorite team making it to the final decreases the profit gap between option pricing and advance selling increases. This speaks to the value of option pricing in uncertain environments. The league can always mimic advance selling profits using option pricing by setting the exercise price equal to zero. Therefore advance selling is a special case of option pricing. Option pricing enables higher profits through: (a) better price discrimination and (b) leveraging uncertainty.

In the next section, I empirically demonstrate the face validity of these analytical conditions and that expected profits from consumer options (based on stated willingness to pay) are higher than other pricing arrangements.

### 2.3 AN EMPIRICAL STUDY

In this section, I empirically explore fans’ willingness to pay (WTP) for different types of tickets to the NCAA Final Four men’s basketball championship game under various level of uncertainty in the team matchups. I use Final Four tickets as the stimulus because of its
relevance to the student population (94% of the respondents were familiar with the Final Four tournament). To elicit more accurate WTP measures, I offered fans the opportunity to enter a drawing to buy a real ticket to a Final Four championship game based on their stated WTP for a ticket.  

2.3.1 Data and Measures

One hundred and fifty-five undergraduate students from a large university were recruited to participate in a 20-minute within-subject computer study in return for $5 cash. At the beginning of the study, these respondents were asked to indicate their favorite men’s college basketball team (over 90% of respondents listed the home university as their favorite team). Subsequent questions were then automatically adjusted so that they were pertinent to that particular team. To be eligible for inclusion in the study, respondents had to answer a set of questions designed to test their understanding of the options concept; four students were dropped from the sample because they did not pass this screen, resulting in a final sample size of one hundred and fifty-one fans.

Fans indicated their WTP for tickets to the final game for three different pricing conditions: (1) advance selling, (2) consumer options, and (3) full information pricing. Across the pricing conditions, I offered fans the same broad range of prices ($0 to $400, in increments of $20). In the advance selling condition, fans indicated their WTP for a “regular ticket” given three uncertainty levels (i.e., probability of the favorite team making it to the final game; $\gamma=0.25, 0.50,$

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Of the fans in our study, 63% volunteered to participate in the drawing. Participants were informed that if their name was drawn, they would be required to buy the ticket by paying the average price they had indicated in our study. The winner of the drawing paid $200 for an upper-level ticket to the 2007 Championship game at the RCA Dome in Indianapolis.

We use a within-subject design because the consumer options concept is new. To help ensure that this concept was understood, fans had to think about the option ticket as well as the more familiar regular ticket. Pretests further indicated that fans were not overwhelmed with the three pricing conditions because their responses simply involved clicking on their preferred dollar amount.
and 0.75) of their favorite team making it through the tournament. Based on extensive pre-
testing, in the consumer options condition I fixed the levels of the option price (\( P_o \)) for these
three probabilities at $20, $40 and $60, respectively\(^{18}\). Given a certain option price, fans
indicated the exercise price they were willing to pay (\( \hat{P}_e \)) at the three different uncertainty levels
(thus, expected WTP for the option ticket is \( P_o + \gamma \hat{P}_e \))\(^{19}\). In the full information condition, fans
indicated their WTP for a “regular” ticket after they were told whether or not their favorite team
actually made it to the final game. Importantly, fans had the choice of not buying a ticket under
all pricing conditions—they could simply indicate a WTP of $0.

Because this is a within-subject design, the presentation order of tickets was counter-
balanced to control for possible order effects\(^{20}\). To control for differences in payment timing
across the pricing conditions (e.g., fans pay an exercise price in the future, whereas they pay in
the present under advance selling), I asked fans to indicate their WTP the exercise price in the
present should their favorite team make it through the tournament. To control for the possibility
that WTP may be influenced by the mere fact that choices become unavailable in the future (e.g.,
Ariely and Shin 2004), I asked fans to assume that both ticket types (options and regular) were

\(^{18}\) Fans were explicitly instructed that a WTP of $0 for the exercise prices across all the uncertainty levels would be
treated as if they did not want to purchase the option ticket.

\(^{19}\) We do not allow fans to choose both the option and the exercise price because we want to obtain their WTP
without confounding our measures with individual differences in risk preferences. For example, suppose we allow
subjects to state their willingness to pay for both the option and the exercise price, and one subject was willing to
pay an option price of $50 and an exercise price of $100 given a 0.25 probability of her favorite team making it to
the big game. Suppose another subject was willing to pay $25 for the option price and $100 as the exercise price
given a 0.50 probability of her favorite team making it through the tournament. In this case, the expected value of
both subjects’ willingness to pay is the same ($75). However, it can be argued that the first fan who was willing to
pay more for the option when there was a lower probability of the team playing in the final game is more risk-averse
than the latter.

\(^{20}\) Because we find no significant order effects in any of our analyses, we ignore this factor in the remainder of this
chapter.
only available at the start of the season (before the uncertainty of the final team matchups has been resolved).

Finally, because I also want to better understand the behavioral determinants of consumer willingness to pay under various pricing approaches, I follow Padmanabhan and Rao (1993) and measure fans’ risk preferences based on their responses to a lottery question (fans who are willing to pay less for the lottery than its expected value are considered to be risk-averse; these fans represent 81% of the sample).

2.3.2 General Fan Reactions to Consumer Options

To better understand fans’ perceptions of consumer options, I obtained open-ended feedback from respondents. Overall, 90% of the sample thought that consumer options were simple to understand and 88% deemed the pricing mechanism to be fair. For example, one respondent thought the option ticket was fair “because it allows you to risk a small amount to gain something of much larger value.” I also asked a separate sample of forty fans how fair they thought consumer options were after it was known that their favorite team was not going to be in the final game. I find that fans continue to view consumer options as a fair pricing mechanism ex post, even if their favorite team does not make it to the final game and they lose the option price.21

2.3.3 Willingness to Pay

The sample distribution of fans’ valuations for a championship ticket not involving the favorite team is graphically depicted in Figure 6. These WTP values are based on responses to the full information condition when the final outcome is known with certainty. This distribution suggests that there are two fan segments. Consistent with Condition 1 in the analytical model,

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21 When \( \gamma \) is relatively small, a repeated measures ANOVA indicates that fans believe that consumer options are significantly more fair than regular tickets, even if they eventually do not exercise the option (\( F(2,40)=4.26, p<0.02 \)).
respondents can be divided into team-based fans that are willing to pay zero to watch a team other than their favorite (representing 29% of the sample) and game-based fans indicating some non-zero valuation for attending a game without their favorite team. Figure 7 shows the distribution of fans’ valuations for the preferred game. Team-based fans are willing to pay an average price of $102 for a ticket to the championship game when it is known with certainty that their favorite team is playing, while game-based fans are willing to pay an average price of $179—this difference is statistically significant (t=5.07; p<0.01). These results imply that the game-based fans’ expected valuation for the game will be significantly higher than the team-based fans’ expected valuation at any given uncertainty level\(^\text{22}\). Consequently, these findings are consistent with Condition 2 in the analytical model. Further, a significantly greater proportion of the team-based fans are risk-averse as compared to the game-based fans (93% and 76%, respectively; t=3.08, p<0.01). Thus, team-based fans are generally more risk-averse, have zero valuation for the non-preferred game, and value the preferred game significantly less than game-based fans.

\(^{22}\text{At } \gamma=0.25, 0.50 \text{ and } 0.75 \text{ the } t \text{ values are 10.08, 8.04 and 6.30 respectively.}\)
Figure 6: Fans’ Stated Valuation for the Non-Preferred Game under Full Information

Game-Based Fans ($\mu=76$)
Figure 7: Fans’ Stated Valuation for the Preferred Game under Full Information

Team-Based Fans ($\mu=$$102)

Game-Based Fans ($\mu=$$179)

Willingness to Pay (preferred game)
Fans’ average WTP for Final Four tickets under various pricing arrangements are in Table 6. The results from the associated 3 x 3 repeated measures ANOVA is in Table 7. Not surprisingly, the WTP values in Table 7 are significantly different across pricing condition and uncertainty levels. More interesting are the statistical results involving the interaction terms which can be interpreted by examining the relative magnitudes of the values in Table 7. The significant interaction involving pricing condition and uncertainty level implies that fans in general are willing to pay more for consumer options when uncertainty is high ($\gamma$ is small). This result is entirely intuitive as consumer options give the most protection to fans when the final outcome is most uncertain. The significant interaction between pricing condition and fan type indicates that team-based fans are willing to pay more for consumer options than game-based fans. Again, this is very reasonable since team-based fans are much more sensitive to the teams playing in the final game than game-based fans. Finally, the significant 3-way interaction involving pricing condition, uncertainty level and fan type implies team-based fans are willing to pay more for consumer options at every uncertainty level while game-based fans are most interested in consumer options only when uncertainty is high.
Table 6: Fans’ Average Stated Willingness to Pay under Different Pricing Arrangements

<table>
<thead>
<tr>
<th></th>
<th>Game-Based Fans</th>
<th>Team-Based Fans</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Information</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=0.25$</td>
<td>$101.8$</td>
<td>$25.6$</td>
</tr>
<tr>
<td>$\gamma=0.50$</td>
<td>$127.4$</td>
<td>$51.1$</td>
</tr>
<tr>
<td>$\gamma=0.75$</td>
<td>$152.9$</td>
<td>$76.7$</td>
</tr>
<tr>
<td><strong>Advanced Selling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=0.25$</td>
<td>$89.0$</td>
<td>$27.7$</td>
</tr>
<tr>
<td>$\gamma=0.50$</td>
<td>$110.1$</td>
<td>$48.2$</td>
</tr>
<tr>
<td>$\gamma=0.75$</td>
<td>$136.3$</td>
<td>$67.7$</td>
</tr>
<tr>
<td><strong>Consumer Options</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=0.25$</td>
<td>$116.8$</td>
<td>$64.6$</td>
</tr>
<tr>
<td>$\gamma=0.50$</td>
<td>$125.1$</td>
<td>$82.3$</td>
</tr>
<tr>
<td>$\gamma=0.75$</td>
<td>$129.3$</td>
<td>$111.6$</td>
</tr>
</tbody>
</table>
### Table 7: Repeated Measures ANOVA (with fan type as a between-subject factor)
*(significance level in parentheses; n=151)*

<table>
<thead>
<tr>
<th>Source</th>
<th>Type III Mean Square</th>
<th>F-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Pricing Condition</em></td>
<td>60113.1</td>
<td>18.35 (0.000)</td>
</tr>
<tr>
<td><em>Uncertainty Level</em></td>
<td>161362.5</td>
<td>128.42 (0.000)</td>
</tr>
<tr>
<td><em>Pricing Condition x Uncertainty Level</em></td>
<td>3694.2</td>
<td>5.36 (0.000)</td>
</tr>
<tr>
<td><em>Pricing Condition x Fan Type</em></td>
<td>36501.3</td>
<td>11.14 (0.000)</td>
</tr>
<tr>
<td><em>Uncertainty Level x Fan Type</em></td>
<td>2009.7</td>
<td>1.60 (0.204)</td>
</tr>
<tr>
<td><em>Pricing Condition x Uncertainty Level x Fan Type</em></td>
<td>4209.7</td>
<td>6.11 (0.000)</td>
</tr>
</tbody>
</table>

#### 2.3.4 Firm Profits and Consumer Surplus

My empirical observations to this point suggest that fans have very different valuations for tickets to an uncertain championship game and consumer options represent a mechanism which affords fans flexibility in obtaining tickets in the face of this uncertainty. Moreover, fans are generally very positive towards consumer options for sports tickets and are willing to pay more for consumer options than regular tickets. I next consider the profit implications of this fan
behavior for the league. Because I find empirical support for Conditions 1 and 2 in the analytical model, I expect profits from consumer options to be higher than profits from advance selling.

To compute profits, I calculate total demand at various prices using the individual-level WTP information. I then select the optimal price under each pricing strategy to be the price yielding the highest total profits. Profits per fan under advance selling and consumer options are reported in Table 8.

<table>
<thead>
<tr>
<th>Game-Based Fans</th>
<th>Advanced Selling</th>
<th>Consumer Options</th>
<th>Profit Premium from Consumer Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ=0.25</td>
<td>$39.3</td>
<td>$71.5</td>
<td>1.8</td>
</tr>
<tr>
<td>γ=0.50</td>
<td>$48.6</td>
<td>$81.7</td>
<td>1.7</td>
</tr>
<tr>
<td>γ=0.75</td>
<td>$62.6</td>
<td>$88.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Team-Based Fans</th>
<th>Advanced Selling</th>
<th>Consumer Options</th>
<th>Profit Premium from Consumer Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ=0.25</td>
<td>$12.3</td>
<td>$54.0</td>
<td>4.4</td>
</tr>
<tr>
<td>γ=0.50</td>
<td>$18.2</td>
<td>$53.2</td>
<td>2.9</td>
</tr>
<tr>
<td>γ=0.75</td>
<td>$25.0</td>
<td>$68.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>

As expected, profits from consumer options are considerably higher than those from advance selling (the profit premium or ratio of profit per fan from consumer options to advance

---

23 For γ=0.25, 0.50 and 0.75, optimal regular ticket prices are $60, $80, $100, respectively; optimal prices under consumer options are $P_o=$60 and $P_e=$20, $40 and $40, respectively. The profit difference between option pricing and advance selling was $5290, $5080 and $4730 at uncertainty levels of 0.25, 0.50 and 0.75 respectively.
selling is greater than one). And, the profit premium from consumer options for both game-based and team-based fans is highest when there is greater uncertainty (γ is small). More interesting, the profit premium per team-based fan from consumer options over advance selling is much higher than that of game-based fans. This is consistent with the finding that team-based fans are generally willing to pay more for consumer options than other pricing arrangements (see Table 6). Further, the calculated profit per fan in Table 8 shows that the consumer options profit premium from team-based fans is the highest when the final outcome is most uncertain.

Clearly, some of the additional profits from consumer options come from both the game-based and team-based fans’ higher willingness to pay for the flexibility that consumer options affords them over purchasing a regular ticket at the beginning of the season. In addition, I find evidence of demand expansion: fifty-four fans will only participate when offered consumer options (i.e., they are not willing to purchase a regular ticket in advance). Interestingly, these fifty-four fans have a very low valuation for the non-preferred game (on average, they are only willing to pay $24.4 for the non-preferred game while the rest of the sample is willing to pay $72.1). Moreover, of these fifty-four fans, twenty-three are team-based fans. Thus, a disproportionate share of the extra fans attracted by consumer options are team-based fans (43% as compared to the 29% of the sample that are team-based fans). Therefore option pricing is more profitable, at a given capacity, by changing the optimal mix of fans in the stadium. Further from the study it is clear that fans derive some positive externality from the presence of additional team-based fans. One fan commented: “The team ticket is fair because only loyal fans will pay in advance to reserve a seat. As a fan I would be upset if the arena was full of people who did not care about the outcome of the game and just came there because they got the ticket for a good price.” Intuitively, as seen in Figure 8, consumer options seem to be particularly
desirable by risk-averse fans (i.e., fans that really have a preference for avoiding the non-preferred outcome)\textsuperscript{24}. Nineteen fans do not participate under either advance selling or option pricing. These differences in fan valuations for the preferred and non-preferred games throw light on fans’ choice behaviors.

The interaction of pricing condition (advance selling, option pricing, full information) with the risk aversion measure (more or less) is significant ($F=12.28$, $p<0.001$). The average WTP of fans across the different probabilities is shown for more versus less risk averse fans in Figure 8.

The empirical analyses to this point demonstrate that the league can earn a profit premium by pricing with consumer options. But, how will fans fare under consumer options? To calculate fan surplus, I subtract the optimal prices used to compute profits in Table 8 from each fan’s WTP.

\textsuperscript{24} The interaction of pricing condition (advance selling, option pricing, full information) with the risk aversion measure (more or less) is significant ($F=12.28$, $p<0.001$). The average WTP of fans across the different probabilities is shown for more versus less risk averse fans in Figure 8.
under the alternative pricing strategies. Surplus per fan under advance selling and consumer options is reported in Table 9.

<table>
<thead>
<tr>
<th></th>
<th>Advanced Selling</th>
<th>Consumer Options</th>
<th>Surplus Premium from Consumer Options</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Game-Based Fans</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=0.25$</td>
<td>$40.2$</td>
<td>$42.1$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\gamma=0.50$</td>
<td>$35.7$</td>
<td>$47.9$</td>
<td>1.3</td>
</tr>
<tr>
<td>$\gamma=0.75$</td>
<td>$53.3$</td>
<td>$56.2$</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>Team-Based Fans</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma=0.25$</td>
<td>$7.7$</td>
<td>$12.3$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\gamma=0.50$</td>
<td>$19.5$</td>
<td>$21.8$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\gamma=0.75$</td>
<td>$22.7$</td>
<td>$29.0$</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Importantly, consumer surplus under both pricing arrangements is positive and greater than zero. More interesting however, is the finding that fan surplus is always higher when pricing with consumer options than advanced selling (the surplus premium or ratio of surplus per fan from consumer options to advance selling is greater than one). Further, I find that the consumer options surplus premium for team-based fans is the highest when the final outcome is most uncertain ($\gamma$ is small). Thus, consumer options do not unreasonably exploit fans’ higher willingness to pay even when the option ticket is most valuable.

Given that option pricing is more profitable than advance selling how does option pricing result in more fan surplus as well? The profit difference between option pricing and advance
selling is the highest when the probability of the preferred game is the lowest. While option pricing can extract more surplus than advance selling the optimal price under option pricing does not increase enough to capture all of the increased WTP. This is because under option pricing both the option and the exercise prices are set early in the season. If the league could set the exercise price later in the season it can extract fan’s surplus more effectively. However, the league needs to set both the option and the exercise price early on in the season due to fairness constraints. This implies that, under lower probabilities, fans have an enhanced willingness-to-pay for an option ticket which is not completely extracted by the optimal prices, leaving them with more surplus than under advance selling. Under higher probabilities fans are WTP more for the ticket under both advance selling and option pricing - this is the main effect associated with an increased likelihood of the preferred outcome. However, because the uncertainty is low under higher probabilities, the value of the option is low. Correspondingly, the optimal price that is charged under option pricing is even lower than the advance selling price. Therefore, across the probabilities, surplus from option pricing remains higher than that under advance selling. With full information pricing because ticket prices are set after knowing the teams that are playing in the game it extracts fan surplus most efficiently. Therefore fan surplus under option pricing is always higher than that under full information pricing.

Together, these empirical results suggest that consumer options creates a win-win situation for the league and fans—pricing with consumer options generates more profits for the league than advance selling, as well as more surplus for fans. Even if the league is not solely interested in maximizing its profits (e.g., if enhancing fan surplus is desirable), pricing with consumer options will outperform advance selling.
2.4 DISCUSSION

In this chapter, I empirically validated and modeled the concept of option pricing in a sports market setting. I compare the profitability of three pricing mechanisms: (a) advance selling, where the league sets the price before the uncertainty regarding the teams that will play in the tournament is resolved; (b) full information pricing, where the league sets prices after this uncertainty is resolved; and (c) option pricing, where the league charges an upfront option price that offers fans the right but not the obligation to see the game, and an exercise price that fans who purchased the option can choose to pay once the uncertainty is resolved. I determine the conditions under which the league can use option pricing to obtain profits that can equal or exceed those from advance selling and full information pricing. With a simple two-fan analytical model I show that option pricing can completely extract the surplus of both fans. Any other pricing mechanism can at best equal profits from option pricing.

I empirically verified the profitability of option pricing over the other pricing mechanisms. I found that (a) fans are willing to pay most for option pricing under the lower probabilities when the value of option pricing is the highest (b) risk-averse fans are willing to pay more for option pricing (c) profitability from option pricing is the highest across the probabilities (d) the profit gap between option pricing and advance selling is the highest when the probability of the favorite team making it to the tournament was the lowest and (e) fan surplus is higher under option pricing compared to the other pricing mechanisms.

One possible concern that arises under option pricing is what to do with the unexercised options (if any). The league certainly does not want to have empty seats in the stadium. Practically, the league can set a date within which the options need to be exercised. The league can then resell the unexercised options. Gerstner and Xie (2003) suggest making customer
cancellations easier so league can profit from resale of advance tickets. I believe that getting fans
to inform the league of their cancellations of ‘regular’ tickets (especially to events like the Final
Four or the Super bowl) will be hard, no matter how easy the cancellation is made. However, if
fans do not exercise their option within a certain date then they lose the right to the option and
the league now has possession of the ticket which it can resell. Note that the current profitability
of option pricing does not take into account the potential additional profitability from resale.
Further, with option pricing the league can continue to set prices at time 1 as it has always done.

These findings highlight the value of consumer options in protecting the league and the
fans. It protects the fan from making a substantial upfront investment when the probability of the
preferred outcome is low. Option pricing provides team-based fans, who have value in waiting to
purchase their tickets, with a viable alternative to scalpers. Option pricing leaves some surplus
with fans by charging a lower price than the created surplus. Further, from the open-ended
feedback I received it is clear that fans perceive option pricing will generate positive externality
from bringing in more fans who are passionate about their teams. For the league option pricing is
an innovative way to continue selling tickets early in the season and make more profits. Fans
who pay the reserve deposit upfront do not need to approach scalpers to purchase a ticket after
the uncertainty about the game is resolved. Profits from scalping do not flow into the league’s
coffers – therefore, option pricing can help shore up profits and lessen the league’s vulnerability
to scalping. Option pricing can come in handy as an alternate way of pricing to season ticket
holders. Because season tickets are sold in advance and for all games in the season, fans are
forced to buy season tickets fearing that tickets may become unavailable for the games they
would like to attend. A byproduct of this is that fans who may not want to attend all the games
end up reselling the tickets in the secondary market. The league can reduce the secondary market
activity by using option pricing. With option pricing fans can buy tickets to the games that they really care to attend. Fans can wait and watch the teams’ performances before deciding whether or not they want to attend a particular game. Buying the option gives the fans access to a ticket later in the season.

2.5 CONCLUSION

Because profits under option pricing are greater than expected profits under full information pricing, this implies that although the league sells tickets at the beginning of the season, it can obtain the same profits as it would if it priced after the uncertainty has been revealed. This result has implications for a range of markets in which consumers face uncertainties but where the firm may find it difficult to set prices after the uncertainty is resolved.

The concept of the consumer option can be extended, with appropriate modifications, to other situations where consumers face uncertain market-level outcomes. For example, consider the market for High Definition Television sets (HDTV). Consumer uncertainty about the variety of programming available in HD format has been a key impediment to the growth of HDTV sales over the past few years (Bryant 2007). This uncertainty can also suppress market prices and sales for large-screen, non-HDTV sets because consumers do not want to be saddled with a large, sunk investment in outdated technology when HDTV arrives. In this situation, the TV manufacturer could encourage the consumer to purchase a conventional large screen TV along with an option. The option, which would expire at a certain point in time in the future, would allow the consumer to pay an exercise price and buy a new HDTV-capable set at a reduced price (or alternatively, pay the exercise price and buy a HD module that can be integrated into the
existing TV set). Consumers who did not feel that there was sufficient HD programming available to make the exercise of the option worthwhile would let it expire, whereas others would exercise it.

Likewise, consumers who would like to protect themselves against the risk of sold-out flights to Florida in the winter can buy a flight ticket option (with an expiration date by which time it needs to be exercised) that covers flights for a fixed set of dates. If the weather in Florida turns out to be pleasant close to the exercise date the consumer can exercise the option and buy the ticket. The firm can charge a higher option price if the consumer wants more flexibility – i.e., if the consumer is willing to exercise the option, say, three weeks before the travel date the price of the option can be lower than if she is willing to exercise it, say, three days before the travel date. Therefore, while I have focused primarily on sports markets there are other applications for the option pricing concept.

The current analysis has some limitations. First, fans and the league could have different estimates of the popular game occurring in the tournament. Having a distribution of the probability estimates across fans could imply that the fans’ valuations of a specific pricing mechanism will not just be a function of their utilities and the prices but also of their probability estimates. Second, I did not consider hybrid pricing strategies that simultaneously offer a mix of option, advance selling and full information pricing. Depending on the circumstances, such a menu of pricing mechanisms may yield higher profits than any of the mechanisms implemented on a stand-alone basis. Finally, adding modeling the presence of scalpers explicitly could result in lower demand across all pricing mechanisms. I hope to consider hybrid pricing mechanisms and scalpers in future research.
This chapter is but an early step towards examining the performance of consumer options from an analytical and empirical perspective. The idea of option ticket pricing resonated with both league officials and university athletic officials I interviewed during the course of the study. In the empirical study, the majority of the respondents thought that option pricing was a good idea – one that was both fair and easy to understand. Both analytically and experimentally, I demonstrated that option pricing could enhance league profits over existing pricing mechanisms. Despite all these positives, much remains to be done to fully understand the benefits and the drawbacks of consumer options. I hope this chapter sparks further research and managerial interest in this area.
CHAPTER 3:
AN EXPLORATION OF CONSUMER BEHAVIOR IN AN ONLINE MARKET FOR FORWARD TICKETS

3.1 INTRODUCTION

Recently a number of firms are introducing ‘consumer forwards’ as a way of helping consumers minimize their exposure to risk in a variety of markets. Some firms (firstdibz.com) have introduced forwards for consumer products that are popular and hard to obtain (e.g., 3G iPhones, special blue-ray compatible versions of PlayStation games etc). Other firms (yoonew.com, ticketoption.com) have introduced ‘forward tickets’ to help fans obtain tickets to tournament games in sports markets. A consumer forward is different from a consumer option (defined in chapter 1) in that it is tied, in the context of sports tickets, to a particular team. If the team does not make it through to the final, the forward expires. Tickets to tournament games are typically sold in advance when fans are unaware of the teams that will be playing in them. If fans choose to wait and buy tickets after the uncertainty regarding the competing teams is resolved, tickets can be scarce and exorbitantly priced. Buying a forward early on gives the fan the right to exercise it later should she decide to attend the game. In this chapter I estimate models to understand fan behaviors in a consumer forwards market for sports tickets.
The most closely related literature to the consumer forwards market is that of individual investors mitigating portfolio risk in financial markets. However, consumer markets are structured differently from financial markets. For example, assumptions of no arbitrage or frictionless markets that are used to price forwards in financial markets do not apply to consumer markets. Further, the individuals who participate in these markets are different. Investors in financial markets tend to be more savvy than the average fan buying a sports ticket. Also, whereas investors in financial markets may be indifferent between two stocks offering them the same returns, in sports markets fans have preferences towards certain teams. Given the differences between consumer and financial markets I study whether conventional wisdom on risk management in financial markets applies to consumer markets.

I study factors that mitigate fans’ risk in the Final Four forwards market. In the traditional finance literature different types of investor behavior are not studied - the focus is more on the ‘typical’ investor. In this chapter, however, I recognize the presence of different types of fans and estimate different models to capture their differences in behaviors.

Because the consumer forwards market is still fairly new the specifics of how the market works is first detailed. Fans objectives are outlined in the conceptual framework section. I then hypothesize and operationalize consumer risk mitigation strategies in this environment. Finally I present the estimation results and implications for future research.

3.1.1 How the consumer forwards market works

The process is depicted in Figure 9. If a consumer (hereafter, a “fan”) is interested in attending a game in which her favorite team is playing, she could purchase the team’s forward by paying a

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Further, in financial markets individual investors are advised to invest in riskless assets (such as index funds or t-bills) to reduce portfolio volatility. In ticket markets there could be less risky teams but there are no ‘risk free’ teams. For example, in the last seven years, of the 28 teams that made it through to the men’s Final Four tournament only 6 teams had made it there more than once. A team is rarely a ‘sure shot’!
preset “forward price.” Fans can buy forwards for multiple teams across many athletic conferences. Forward prices vary by team and over time. If the fan’s favorite team makes it past the preliminary rounds and all the way to the tournament, she then pays an additional exercise price (set by the National Collegiate Athletic Association – hereafter ‘the league’) to buy the ticket. The exercise price varies from year to year and is the same as the face value of the ticket. For example, the league has set the exercise price for the Final Four tournament between $140-$160 in the previous years. The exercise price is set in advance and the fans are aware of this price at the time that they purchase the forward. The forward price (set by the firm that offers forwards for sale) that was paid earlier is the price of a reservation to buy a ticket should the team play in the tournament. If the specific team for which the forward was purchased does not make it to the tournament the fan loses the forward price. At any time in between the purchase of the forward and until the uncertainty is revealed fans have access to the firm’s proprietary market in which they can resell their forwards. The firm obtains revenue from its sale of forwards and a certain percentage of the transactions between the fans in its marketplace (a buyer and seller fee of 5%). However, once the fans exercise their forwards the league gets the associated revenue.

**Figure 9: Consumer Decision Sequence in a Forwards Market**

Fan buys team-specific forward (pays ‘forward price’ set by firm)

Team makes it to the game

- Fan pays the exercise price = face value (league)

Team does not make it to the game

- Fan loses the forward price

Between the time the forward is bought and the uncertainty is resolved, the fan can resell the forward

In the next section I present a detailed description of the data.
3.2 DATA DESCRIPTION AND ANALYSIS

I obtained data from a large, well-known, firm that offers team-specific forwards for various events. On the firm’s website forwards are defined as “tradable reservations for a face value ticket to see your favorite team if they participate in an event.” Due to non-disclosure agreements the name of the firm cannot be revealed and it will simply be referred to as ‘the firm’. The firm initially offered forwards only to the NCAA Final Four market. Now, along with the Final Four, it offers forwards for events like the Super Bowl, the AFC and NFC Championships, College Football Bowl games, BCS National Championship, the NBA Conference Semi-Finals and the NBA Finals.

The firm shared data on all the transactions that occurred in the Final Four market in 2006 (from April 2005 to March 2006). Each fan was allowed to purchase a maximum of eight forwards per team. Forward prices ranged from $5 to $800 per forward depending on the team and the time that the forward was bought. Each bidder has a unique id; giving us the ability to track each bidders’ purchase and resale behavior, if resale occurred. The data is organized at the transaction-level. In each transaction, details on the following issues are available: (a) which team the bidder bought the forward for, (b) the number of units bid on, (c) whether the forward was bought from the firm (termed an ‘issuance’) or from another bidder (‘trade’), (d) the price the forward was purchased for, (e) the day and time of purchase. If the forward was subsequently sold by the bidder there is information on: (a) the sale price, (b) the timing of the sale and (c) the identity of the buyer. Forwards could be sold to other bidders; it cannot be sold back to the firm.

I complemented the existing data with Associated Press (AP) polls on the ranking for the various teams across all the weeks of the regular season\textsuperscript{26}. Note that better performing teams

\textsuperscript{26} Team rankings (between ranks 1 and 50) are available through the length of the regular season (till week 18).
have lower ranks. The objective of the analysis is to understand fans’ behaviors in terms of minimizing their risk in this environment – therefore scalpers are removed from the dataset. The procedure that was used to eliminate scalpers is described next.

### 3.2.1 Removing Scalpers From The Dataset

A team is considered to be ‘performing well’ in a given week if it has a better rank in that week than in the previous week. Scalpers are defined as those who sell their forwards even when the team is performing well. This implies that those fans who never resold their forwards were automatically exempted from being classified as scalpers. On average, across all teams, fans received a higher resale price when the team was performing well. Therefore, if fans resold their forwards when their teams’ were not performing well they were not considered to be scalpers; merely fans who were minimizing their losses. For instance these may be fans who, due to a schedule conflict, are unable to attend the game and would like to sell their forward. Before eliminating scalpers the dataset had 3196 transactions with 582 unique bidders who bought forwards of 83 unique teams. Upon removing the scalpers we are left with 431 unique fans who engaged in a total of 2129 transactions across all 83 unique teams.

Among the 431 bidders there were clearly two groups of people: (a) those who bought (and sold) forwards on only one team (labeled ‘team-based’ fans) and (b) fans who bought and sold multiple teams’ forwards (labeled ‘game-based’ fans). In the data there were 216 unique game-based fans. Most game-based fans bought forwards for 2 to 10 unique teams; few bought forwards for more than 10 unique teams. The frequency plot of the number of unique teams

---

27 As a robustness check I estimated a logit model of resale behavior with teams’ performance as one of the independent variables. When the analysis was run on the entire data the performance of the teams was insignificant. However when the sample was restricted to scalper-free data, teams’ performance was significant (p = 0.014) and negative (the worse the team, the higher the likelihood of resale), as expected. This confirms that we removed those who did not consider team performance as a factor in resale, typically scalpers, from the dataset.
game-based fans bought is shown in Figure 10. On average, game-based fans bought 5 unique teams’ forwards.

![Figure 10: Number of Unique Teams Bought By Game-Based Fans](image)

Of all the fan transactions only 20% of the transactions were made on teams that eventually made it to the final game. The fact that 80% of the transactions were done on the ‘wrong’ teams increases the value of understanding fan behavior in this market. Because scalpers have been removed from the dataset I assume that the objective of the remaining consumers (fans) who buy the ticket is to attend the final game. I discuss the components of the fans’ utility next.

### 3.3 Conceptual Framework

Consistent with standard economic theory we assume that game- and team-based fans maximize their utility (Jehle and Reny 2003). Fans’ utility maximization problem is given by:

\[
Max \ (U \ (\text{attending game}) - \text{Price})
\]  

(37)
As stated in eqn. (37) fans have utility in attending the game (the utility of not attending the game is normalized to be zero). Forwards expire if the team that it is associated with does not make it to the tournament. Therefore it becomes important for the fans to buy forwards on the ‘right’ teams. Further, even if the team makes it to the tournament, unless the fan retains a forward she cannot attend the game (i.e., if she resells all her forwards she cannot attend the game). Therefore, for the fan, the utility of attending the game is composed of two components: (a) picking the right teams and (b) retaining at least some of the purchased forwards on the winning teams.

The other aspect in the utility maximization problem relates to the price paid. We assume, ceteris paribus, that fans prefer to spend less on the purchase of a forward. In this market fans have the ability to resell their forwards and ‘offset’ their expenses. Therefore, after accounting for the money obtained through resale and the money spent on purchase, the fan’s objective would be to be in the black (so as to not spend disproportionately more than the amount received).

Though all fans buy forwards so that they can attend the final game, we allow for the fact that different types of fans may have different objectives. Game-based fans have a portfolio of different teams’ forwards; therefore we assume that they have a dual objective of ‘picking the right teams to maximize chances of attending the game’ while simultaneously ‘minimizing the associated costs.’ I estimate an ordered logit model that identifies factors that affect the dual objectives of game-based fans. Most team-based fans (156 of the 215 fans) do not resell their teams’ forwards irrespective of their teams’ performance. Because team-based fans choose to buy forwards to one team and hold onto them irrespective of their teams’ performance, I make the assumption that their choice of team is driven by their loyalty (and not by a conscious
decision to ‘pick’ any one team)\textsuperscript{28}. Therefore their only objective is to minimize the amount that they spend on the forward. I estimate a regression model that identifies factors that reduce the amount paid by the team-based fan. Next, I discuss the dependent and independent measures that will be used in the estimations.

3.4 Measures

Based on the utility maximization framework, different dependent variables for the game- and team-based fans are created.

3.4.1 Dependent Variables:

Team-based fans seek to minimize their spending. Because this market has both purchase and resale behavior, I matched each bidder’s purchase of a given team forward with its sale (if applicable). The variable ‘net dollars’ which indicates the difference between the total amount obtained through resale and the total amount spent on purchasing the forward in a given transaction is created. In a market with resale, minimizing costs amounts to maximizing the ‘net dollars’ variable. Therefore, the dependent variable in the team-based fans’ regression is the ‘net dollars’ variable. For game-based fans there is an additional objective: of picking the ‘right’ teams to purchase forwards on. Because these transactions have occurred in the past, I retrospectively know which teams eventually made it to the Final Four tournament in 2006. Therefore I have the unique ability to classify a transaction (\textit{ex post}) as being ‘good’ (if the transaction was on a team that eventually made it to the final game) or ‘bad’ (if the transaction was on a team that eventually did not make it to the final game)\textsuperscript{29}. If the game-based fan had forwards on teams that eventually made it to the final game \textit{and} the fan held onto at least one

\textsuperscript{28} This assumption is relaxed later.

\textsuperscript{29} Teams that made it to the Final Four in 2006 were Louisiana State Tigers, UCLA Bruins, Florida Gators and George Mason Colonials. That year the National Champions were the Florida Gators.
forward from the winning team, we assume the fan attends the game (and the ‘attend game’ variable is set to 1). The dependent variable for the game-based fans analysis is a variable termed ‘rank’ which combines game-based fans dual objective of attending the game at the least possible cost. Game-based fans who attend the final game are classified as being ranked better than those who do not. Further, among those fans who attended the final game, those who spent the least amount of money (i.e., those with higher ‘net dollars’) are ranked better than those who spent more. Specifically, the following cutoffs were established to compute the ‘rank’ variable:

- If the forward was bought on a team that ultimately made it to the final game and it was held till the end (i.e., if ‘attend game’ = 1):
  - If, on a given transaction, the fan spent less on forwards than she obtained through resale (i.e., if ‘net dollars’ >= $0) her out of pocket expenses are minimized. In this case the best possible rank (i.e., rank 1) was assigned.
  - If, on a given transaction, the fan spent more than she obtained through resale (i.e., if ‘net dollars’ < $0), rank 2 was assigned.

- If the forward was bought on a team that did not make it to the final or if the forward was resold (i.e., if ‘attend game’ = 0):
  - If, on a given transaction, the fan spent less on forwards than she obtained through resale (i.e., if ‘net dollars’ >= $0), rank 3 was assigned.
  - If, on a given transaction, the fan spent more than she obtained through resale (i.e., if ‘net dollars’ < $0), rank 4 was assigned.

Because we construct the ‘rank’ variable as a discrete (but ordered) categorical variable we estimate the model using an ordered logit model. As stated earlier most of the transactions were not made on winning teams; this implies that most of the transactions were assigned ranks 3 or 4.
3.4.2 Independent Variables:

Because their objectives are different, I create different independent variables for the game- and team-based fans. Game-based fans have portfolios of different forwards; therefore understanding strategies that enable them to create and manage ‘winning’ portfolios is of value (and interest). Stated in terms of the dependent variable, the goal is to identify and understand factors that enable game-based fans to have a better rank (rank 1 being the best). Helping the game-based fans meet their objectives consists of two components: (1) managing the risk in their portfolios and (2) minimizing their costs. For team-based fans I seek to identify factors that minimize costs. I begin with the cost-minimization variables that affect both fan types.

Cost minimization strategies:

One strategy that fans could use to minimize their cost is to time their purchase (and sale). The operationalization of the corresponding independent variables are discussed below:

(a) **Purchase Timing:** A cross-tabulation reveals that on average fans spend $60 to buy a forward if they buy early versus $138 if they buy late. Buying early can therefore help the fan by minimizing her cost. If the forward was bought in the pre-season or within the first 10 weeks of the regular season it is classified as an early buy (if the forward was bought after week 10 and into the tournament season it is classified as a late buy). Around the end of the first 10 weeks, the new year sets in and a newer, more frenzied chapter in the season begins. Therefore I use the 10 week mark as the natural division between early and late buys.

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30 While it can be argued that buying late allows fans to have more information which may guide their buying choice better, it can be counter argued that even if they bought the forward early fans could always sell the forward based on the changing team performance (new information).
(b) **Sale Timing:** Resale, per se, helps increase the fans’ ‘net dollars’. Further selling early helps the fan minimize her risk. Whereas selling late, on average, commands a higher price ($153 versus $130) if the team performs well, there is a risk associated with waiting for too long. If the team is performing badly the fan may not be able to recoup the amount she paid to buy the forward. Worse still, if the fan waits till the tournament begins, the team may not make it past a certain round and the forward becomes worthless instantaneously. If the forward is resold in the pre-season or within the first 10 weeks of the regular season, it is classified as an early sale (forwards that are resold later are classified as late sales).

(c) **Type of purchase (Issuance versus Trade):** The average sellers’ price per forward is higher for a trade ($172) than an issuance ($97). This reveals that the firm is pricing more conservatively compared to what the market is willing to bear. From a fan’s perspective buying an issuance seems to be a cost minimization strategy. I create a ‘trade type’ variable that is coded ‘1’ or ‘0’ depending on whether or not the transaction is a trade or an issuance respectively.

Extant literature in finance offers some insights on managing individual investors’ portfolio risk. We use these insights to motivate game-based fans’ risk-management strategies.

**Risk-management strategies:**

(d) **Portfolio Breadth:**

i. **Linear effects:** In line with the existing literature in finance (Markowitz 1952), diversifying a game-based fan’s portfolio is hypothesized to be a better risk-management strategy. In this setting a diversified portfolio amounts to having many unique teams’ forwards. As the number of unique teams in a
fan’s portfolio increases, this increases the fans’ chances of attending the game. For each bidder, I create a measure of portfolio breadth: a count of the total number of unique teams that the fan held in her portfolio till the end of the regular season. On average game-based fans held about 4 teams in their portfolio till the very end. Team-based fans, by definition, have a single team in their portfolio. Therefore even though they could be left with zero or one team at the end of the regular season, adding this variable to the team-based fan analysis is of limited value. This is because the recommendation of having more (or less) unique teams does not apply to team-based fans who have a single team in their portfolio.

ii. **Non-linear effects:** As the number of unique teams increase so does the cost associated with the purchase of multiple fan forwards. As hypothesized in the finance literature (Berger and Ofek 1995, Kennon 2007), beyond a certain threshold the incremental value of an additional portfolio investment (buying an additional team, in this case) is lesser than the cost involved. To capture this effect for game-based fans I introduce a quadratic term that represents the square of the number of unique teams a game-based fan possesses. As with the linear case, this variable is not applicable for the team-based fan analysis.

(e) **Portfolio Risk:** As the number of unique teams in the portfolio increases, the variance of teams in terms of their rankings also increases. Consistent with the wisdom in portfolio management (Stein and DeMuth 2008) as the portfolio risk increases it has a negative effect on the fans’ chances of attending the game. I create a “spread” variable that captures the difference in ranking at the end of the regular
season between the highest and lowest ranked teams in each game-based fan’s portfolio. In other words the spread captures the gap in the best and worst teams’ performance in a fan’s portfolio. If the portfolio has better ranked teams the spread measure is adjusted (and renamed as ‘portfolio risk’) to reflect this. The portfolio risk measure is created to reflect the differences between portfolios with the same gap (the gap between teams with rankings 1 and 11 is the same as the gap between teams ranked 21 and 31; in the first case the portfolio risk is lower than in the second). Because the team-based fan has a single team in her portfolio, the portfolio risk variable is not applicable for team-based fans.

(f) Team Performance (‘Rank Deviation’): Investors have been cautioned against chasing recent performers (Stein and DeMuth 2008). In our context, I check to see if buying a team that is coming off a recent win can be a better strategy to pick the right teams (than buying one that is coming off a loss). In other words I check the effectiveness of the strategy of buying a team that at week $t$ is performing better relative to its performance at week $t-1$. I create a ‘rank deviation’ variable which is the difference in team ranking at the week of purchase from that in the previous week. If the rank deviation is positive it implies the team is performing worse in the current week. Though this variable is classified as capturing a risk-minimization strategy it is plausible that this strategy can be used to minimize costs as well. So I use this variable in the team-based fan analysis as well.

(g) Expert Predictions (‘Pre-season picks’ and ‘Mid-season picks’): Finally, a recommended risk-mitigation strategy for individual investors involves composing the majority of their portfolio with stocks from reputed firms (Stein and DeMuth
To draw a parallel to the sports teams context I obtained the ‘top 5 winningest teams of all time’ from the NCAA website (Johnson 2007). At least in the last few years (since 2004) it seems experts have picked most of their recommendations for the Final Four teams from this list: Duke, Kansas, Kentucky, UCLA, UNC – all storied teams in college basketball history. So in this context the hypothesis is that fans would be increasing their chances of attending the game by heeding expert predictions.

Expert predictions are made twice during the season – once before the season begins (week 1) and the other about mid way through the season (week 11). Pre-season picks reflect experts’ predictions before the teams start playing in the regular season. These predictions are based on how the teams performed in the previous season. Mid-season picks reflect an updating by experts; depending on the teams’ performance in the current season they make revisions to their pre-season forecasts. I hypothesize that both pre- and mid-season picks help fans make the purchases on the ‘right’ teams – the mid-season picks possibly help the fan more than pre-season picks (in terms of coefficient size). I obtained expert predictions from the Entertainment and Sports Programming Network (ESPN) website’s Coaches Poll during the 2005-2006 season. The week 1 predictions were labeled “pre-season picks” and their week 11 predictions were labeled “mid-season picks”\(^\text{31}\).

Because heeding expert predictions could be used as a strategy to reduce losses, this strategy also has cost implications. Therefore I use this variable in the team-based fan analysis as well.

\(^{31}\) The pre-season favorites were Duke, Villanova, UConn, Mich. St. and Texas. The mid-season favorites were UConn, Duke, Memphis, Florida and Texas.
3.4.3 Control Variables:

(h) Conference: I control for the fact that there may be certain conferences that people would be willing to spend more on because of their history in making it to the championship games. Dummy variables indicating the team’s conference are created. Since all conferences are not uniformly represented in the data I choose some that are well-represented. These include the SEC, Pac 10, Missouri Valley, Conference USA, Big East, Big Ten, Big Twelve, Atlantic 10 and the ACC.

(i) Units Bid On: The number of units bid on for a given team typically tends to be a function of how many friends the fan is going to the game with. If the fan purchases multiple forwards for a single team, the dollars spent increases without increasing the chances of attending the game. Therefore this variable needs to be controlled for.

3.5 MODEL ESTIMATION AND RESULTS

Summary statistics for game- and team-based fans are presented in Table 10. On average most game- and team-based fans buy and sell late. Most team-based fans, on average, buy from others whereas game-based fans buy from the firm. Portfolio breadth indicates the number of unique teams held by fans at the end of the regular season. Because team-based fans have only one unique team in their portfolio they could be left with no teams or one team at the end of the regular season. The majority of team-based fans (77%) hold onto their one team till the very end; game-based fans hold about four unique teams till the end. A higher portfolio risk value indicates a higher risk – for game-based fans the average portfolio risk is low indicating that most game-based fans held teams that were, on average, performing well. Team-based fans’ buy more risky teams (p=0.001). Finally, on average, team-based fans buy two forwards; game-based fans buy three forwards.
Table 10: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>GB Fans</th>
<th></th>
<th></th>
<th>GB Fans</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
<td>Min</td>
<td>Max</td>
<td>Mean</td>
</tr>
<tr>
<td>Bidder Buys Early</td>
<td>0</td>
<td>1</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
<td>0.42</td>
</tr>
<tr>
<td>Bidder Sells Early</td>
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<td>1</td>
<td>0.06</td>
<td>0</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>Trade type</td>
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<td>1</td>
<td>0.49</td>
<td>0</td>
<td>1</td>
<td>0.60</td>
</tr>
<tr>
<td>Rank Deviation</td>
<td>-24</td>
<td>42</td>
<td>0.62</td>
<td>-9</td>
<td>42</td>
<td>0.55</td>
</tr>
<tr>
<td>Portfolio Breadth</td>
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<td>33</td>
<td>4.39</td>
<td>0</td>
<td>1</td>
<td>0.77</td>
</tr>
<tr>
<td>Portfolio Risk</td>
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<td>3</td>
<td>1.23</td>
<td>0</td>
<td>3</td>
<td>1.78</td>
</tr>
<tr>
<td>Units Bid On</td>
<td>1</td>
<td>8</td>
<td>3.22</td>
<td>1</td>
<td>8</td>
<td>2.42</td>
</tr>
</tbody>
</table>

Spearman correlations among the independent variables are presented in Table 11.

Table 11: Correlations

<table>
<thead>
<tr>
<th>Variable</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidder Buys Early</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bidder Sells Early</td>
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<td>1.00</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade type</td>
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<td>-0.12</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rank Deviation</td>
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<td>-0.00</td>
<td>-0.05</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio Breadth</td>
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<td>0.01</td>
<td>-0.06</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Portfolio Risk</td>
<td>0.39</td>
<td>0.08</td>
<td>-0.44</td>
<td>0.26</td>
<td>-0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

3.5.1 Results

Estimation results of regression models I and II are presented in Table 12, results of the game-based fan’s probit model (model II) is presented in Table 13, results of the ordered logit models (models IV and V) are reported in Table 14. Because the data is at the transaction-level standard errors are clustered to control for the effect of the same bidder engaging in many transactions. I begin with the team-based fan regression model in model I.

---

32 Rank deviation = Current week rank – Previous week rank. Therefore higher rank deviation implies the team is performing worse at the week of purchase compared to the week before. The ‘-24’ case, for instance, represents a current week ranking of 15 and a previous week ranking of 39. Similarly the ‘42’ ranking represents the case where the current rank is 42 and the previous week the team was not ranked in the top 50 (these teams are coded as rank 0).

33 I have verified that the results of the regression model with the ‘amount paid’ as the dependent variable (instead of the difference in amount obtained through resale minus amount paid) are similar to those presented in Model I.
Table 12: Results of Estimations of Regression Models (I and II)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model I: TB Fan Regression Model</th>
<th>Model II: GB Fan Regression Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable:</strong></td>
<td>Net dollars</td>
<td>Net dollars</td>
</tr>
<tr>
<td>constant</td>
<td>181.23*</td>
<td>179.19*</td>
</tr>
<tr>
<td><strong>Independent Variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy Early</td>
<td>62.34</td>
<td>50.72</td>
</tr>
<tr>
<td>Sell Early</td>
<td>138.09*</td>
<td>-17.95</td>
</tr>
<tr>
<td>Trade type</td>
<td>-117.18*</td>
<td>-247.91*</td>
</tr>
<tr>
<td>Rank Deviation</td>
<td>4.80*</td>
<td>1.76</td>
</tr>
<tr>
<td>Pre-season picks</td>
<td>N/A</td>
<td>298.95*</td>
</tr>
<tr>
<td>Mid-season picks</td>
<td>N/A</td>
<td>-110.21*</td>
</tr>
<tr>
<td>Portfolio Breadth Linear</td>
<td>N/A</td>
<td>-23.65*</td>
</tr>
<tr>
<td>Portfolio Breadth Quadratic</td>
<td>N/A</td>
<td>0.566*</td>
</tr>
<tr>
<td>Portfolio Risk</td>
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<td>45.49</td>
</tr>
<tr>
<td><strong>Control Variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conference SEC</td>
<td>-37.14</td>
<td>81.47*</td>
</tr>
<tr>
<td>Conference PAC10</td>
<td>-1.19</td>
<td>72.15</td>
</tr>
<tr>
<td>Conference ACC</td>
<td>-191.45*</td>
<td>-42.20</td>
</tr>
<tr>
<td>Conference Big East</td>
<td>-114.02*</td>
<td>-19.47</td>
</tr>
<tr>
<td>Conference Big Ten</td>
<td>-127.99*</td>
<td>-82.92*</td>
</tr>
<tr>
<td>Conference Big Twelve</td>
<td>-111.49*</td>
<td>39.84</td>
</tr>
<tr>
<td>Conference Atlantic10</td>
<td>-72.41</td>
<td>68.44</td>
</tr>
<tr>
<td>Conference USA</td>
<td>-106.21*</td>
<td>121.07</td>
</tr>
<tr>
<td>Conference Missouri Valley</td>
<td>-80.22</td>
<td>56.65</td>
</tr>
<tr>
<td>Units bid on</td>
<td>-70.64*</td>
<td>-68.57*</td>
</tr>
<tr>
<td>Number of observations</td>
<td>205</td>
<td>1195</td>
</tr>
<tr>
<td>Number of unique bidders</td>
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<td>168</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.23</td>
<td>0.39</td>
</tr>
</tbody>
</table>

*: significant at 0.05
Table 13: Results of Estimations of GB Fans’ Probit Model (III)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model III: GB Fan Probit Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable:</td>
<td>Attend game</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.15</td>
</tr>
<tr>
<td>Independent Variables:</td>
<td></td>
</tr>
<tr>
<td>Buy Early</td>
<td>-0.86*</td>
</tr>
<tr>
<td>Trade type</td>
<td>-1.42*</td>
</tr>
<tr>
<td>Rank Deviation</td>
<td>-0.12*</td>
</tr>
<tr>
<td>Mid-season picks</td>
<td>0.82*</td>
</tr>
<tr>
<td>Portfolio Breadth Linear</td>
<td>0.05</td>
</tr>
<tr>
<td>Portfolio Breadth Quadratic</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Portfolio Risk</td>
<td>-0.572*</td>
</tr>
<tr>
<td>Control Variables:</td>
<td></td>
</tr>
<tr>
<td>Conference SEC</td>
<td>2.25*</td>
</tr>
<tr>
<td>Conference PAC10</td>
<td>1.99*</td>
</tr>
<tr>
<td>Units bid on</td>
<td>-0.08</td>
</tr>
<tr>
<td>Number of observations</td>
<td>353</td>
</tr>
<tr>
<td>Number of unique bidders</td>
<td>116</td>
</tr>
<tr>
<td>Pseudo R²</td>
<td>0.47</td>
</tr>
</tbody>
</table>

*: significant at 0.05
Table 14: Results of Estimations of the Ordered Logit Models (IV, V)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model IV: GB Fan ordered Logit Model</th>
<th>Model V: Entire Sample Ordered Logit Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable:</strong></td>
<td>Rank</td>
<td>Rank</td>
</tr>
<tr>
<td><strong>Independent Variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Buy Early</td>
<td>1.616*</td>
<td>1.453*</td>
</tr>
<tr>
<td>Sell Early</td>
<td>1.785*</td>
<td>1.798*</td>
</tr>
<tr>
<td>Trade type</td>
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<td>-0.689*</td>
</tr>
<tr>
<td>Rank Deviation</td>
<td>0.090*</td>
<td>0.086*</td>
</tr>
<tr>
<td>Pre-season picks</td>
<td>1.093*</td>
<td>0.899*</td>
</tr>
<tr>
<td>Mid-season picks</td>
<td>0.816*</td>
<td>0.867*</td>
</tr>
<tr>
<td>Portfolio Breadth Linear</td>
<td>-0.148*</td>
<td>-0.064</td>
</tr>
<tr>
<td>Portfolio Breadth Quadratic</td>
<td>0.002</td>
<td>0.0005</td>
</tr>
<tr>
<td>Portfolio Risk</td>
<td>-0.189</td>
<td>-0.201</td>
</tr>
<tr>
<td><strong>Control Variables:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conference SEC</td>
<td>-2.145*</td>
<td>-2.142*</td>
</tr>
<tr>
<td>Conference PAC10</td>
<td>-1.756*</td>
<td>-2.269*</td>
</tr>
<tr>
<td>Conference ACC</td>
<td>-0.182</td>
<td>-0.255</td>
</tr>
<tr>
<td>Conference Big East</td>
<td>0.139</td>
<td>0.012</td>
</tr>
<tr>
<td>Conference Big Ten</td>
<td>-0.390</td>
<td>-0.463</td>
</tr>
<tr>
<td>Conference Big Twelve</td>
<td>0.253</td>
<td>-0.001</td>
</tr>
<tr>
<td>Conference Atlantic10</td>
<td>-0.483</td>
<td>-0.159</td>
</tr>
<tr>
<td>Conference USA</td>
<td>0.585</td>
<td>0.347</td>
</tr>
<tr>
<td>Conference Missouri Valley</td>
<td>0.815</td>
<td>0.702</td>
</tr>
<tr>
<td>Units bid on</td>
<td>-0.128</td>
<td>-0.160*</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-5.69</td>
<td>-5.25</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-5.49</td>
<td>-5.07</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.60</td>
<td>0.744</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1195</td>
<td>1400</td>
</tr>
<tr>
<td>Number of unique bidders</td>
<td>168</td>
<td>355</td>
</tr>
<tr>
<td>Pseudo R$^2$</td>
<td>0.30</td>
<td>0.27</td>
</tr>
</tbody>
</table>

*: significant at 0.05
As stated earlier, the team-based fan’s objective is to minimize out of pocket expenses (i.e., amount got through resale minus the amount paid on purchase); therefore, I estimate a regression model for team-based fans with ‘net dollars’ as the dependent variable (model I). The independent variables that have a positive coefficient increase net dollars (and minimize costs) and are therefore deemed to be helpful. Selling early helps team-based fans because the amount obtained through resale is about the same, on average, irrespective of when she resells. Buying from other fans (engaging in a ‘trade’ as opposed to an ‘issuance’) hurts ‘net dollars’ because of the higher cost of trades. If a team is coming off a bad performance relative to the previous week (‘rank deviation’ variable is increasing) its resale price does not drop immediately. Because selling off the forward can still command a healthy price it helps increase ‘net dollars’. Expert picks are not applicable in the team-based fan analysis because team-based fans do not base their purchase of a forward on expert recommendations. Also, as stated earlier, portfolio variables are not applicable in the team-based fan analysis.

If the objective of the game-based fans is only to minimize their costs, a regression model needs to be estimated – model II represents this case. The effects of the trade type variable is the same as in model I. There seems to be a negative effect of having many unique teams (linear portfolio breadth variable); this implies that having more teams increases the amount spent more than the amount obtained through resale. The quadratic portfolio breadth variable however is significant and positive – this implies that beyond a certain threshold (approximately 9 unique teams) having more teams helps increase net dollars. To understand why this is the case I ran two separate regression models: one with the ‘total amount paid’ as the dependent variable and the other with the ‘total amount obtained through resale’ as the dependent variable. The results indicate that the quadratic portfolio breadth variable decreases the total amount paid and increases
the total amount obtained through resale. This implies that beyond a certain threshold of unique teams fans buy and sell forwards more efficiently. In comparing the results of model II with that of model I it can be seen that selling early is non-significant in the game-based fan model (model II) whereas it is significant and helps in the team-based fan model (model I). Most team-based fans tend to hold onto their forwards longer; this implies that if their team does not make it to the final they lose out on recouping even the price they paid. Therefore selling early helps team-based fans. The ‘rank deviation’ variable is insignificant in the game-based fan model (model II) while it is significant and helps in the team-based fan model (model I). Rank deviation helps because a small change in ranking (in either direction) increases bidder’s willingness-to-pay for the team. Since many transactions occur towards the end of the season, team-based fans (who resell) reap the benefits of the changes in ranking. Finally, pre-season picks are significant and help increase net dollars whereas mid-season picks are significant and hurt net dollars in the game-based fan model (model II). This is because while the amount that is obtained through resale is almost the same for both pre- and mid-season picks, the amount paid for pre-season picks tends to be significantly lower than that paid for mid-season picks.

In model III (Table 13) the factors that affect the game-based fans’ probability of attending the game are presented. First off, selling early, pre-season picks and all conference variables other than SEC and PAC10 were automatically dropped because they predicted failure perfectly. Conferences SEC and PAC10 produced 3 of the 4 teams that made it to the Final Four in 2006 – LSU, UCLA and Florida (George Mason is in the ‘Colonial Athletic Association’ conference). This is the reason why only 353 observations are used in the estimation of the probit model. Clearly buying early decreases the game-based fans’ chances of attending the game. With team rankings changing every week, it is over time that there is more information on which
teams have a chance of making it through. Because one of the mid-season picks (Florida) made it to the Final Four, heeding experts mid-season picks helps game-based fans. So while mid-season picks hurt the game-based fan’s net dollars (mode II) they increase the fans chances of attending the game. A negative coefficient on the ‘rank deviation’ variable indicates that a bad performance in the short-term is typically associated with reducing the fans’ chances of attending the game. Further a risky portfolio also hurts game-based fans chances of attending the game.

Model IV (Table 14) denotes the factors that affect the game-based fan’s dual objective of maximizing her chances of attending the game at a minimum cost. Recall that the fan prefers to be in rank 1; therefore a negative coefficient in model IV indicates that the variable helps the fan. First, and most important, we see that the coefficient of the linear portfolio breadth variable is statistically significant. This implies that having many unique teams helps the game-based fan’s chances of attending the game. However because the quadratic portfolio breadth variable is insignificant it is unclear whether the benefit of having unique teams reduces at some point.

Buying early decreases the probability of attending the game (model III) and therefore hurts the rank variable as well (model IV). Selling early also hurts the fans’ chances of attending the game. Further, as a strategy, buying a team that is coming off a recent bad performance seems to hurt the fan (the coefficient of the ‘rank deviation’ variable is positive). It hurts the probability of attending the game (see model III). Expert picks (pre- and mid-season) hurt the game-based fan’s dual objective. In the probit model (model III) pre-season picks hurt and mid-season picks help the game-based fan’s chances of attending the game. In the regression model (model II) pre-season picks hurt while mid-season picks help. When combining these effects in model V, it seems the negative effects of both pre- and mid-season picks prevail. The lambdas are the estimated cutoff points on the latent variable used to differentiate between the low and
high ranks. Subjects with a value of -5.69 or lower on the underlying latent variable that gave rise to the rank variable would be classified as rank 1 given the other variables are evaluated at zero. Similarly, subjects with values between -5.69 and -5.49 would be classified as rank 2, those with values between -5.49 and 0.602 would be classified as rank 3 and those with values higher than 0.602 would be classified as rank 4.

Finally, we estimate the factors that affect both team- and game-based fans if they have the same dual objective (in model V – Table 14). In other words, if team-based fans also made a conscious decision to pick the right team and their choice was not driven by loyalties, this model would apply. Note though, that all the indications we have point to the fact that team-based fans do choose teams based on loyalties – for instance team-based fans don’t resell their team’s forward irrespective of the its performance. Therefore, model V is estimated for the sake of completeness. We contrast the results of the individual models with that of the combined model.

In the game-based fan model (model IV), the linear portfolio breadth term is significant and reveals that diversifying the portfolio of game-based fans help. However in the combined model (model V) the effects of both the portfolio breadth variables (linear and quadratic) are insignificant. Therefore when combining the sample of team- and game-based fans the effects of portfolio diversification seems to get washed out. Further, the effect of the trade type variable is curious – it is insignificant in the game-based fan model while it hurts team-based fans. Combining these two samples however makes the trade type variable help in model V. Also, while selling early and a change in ranking (rank deviation) help team-based fans (model I), they hurt game-based fans (model IV). In the combined model, it seems like the effect of game-based fans supersedes that of team-based fans (it hurts in the combined model). Therefore assuming the same objective for the entire sample as done in model V helps clarify that the two groups of fans
are indeed different and that combing them either tilts the results in favor of the game-based fans (because they engage in the majority of the transactions) or the effects get washed out. This speaks to the importance of having different models to estimate different fan behaviors.

3.5.2 Robustness check

As a justification for estimating the regression, probit and the ordered logit models I consider an alternate specification. For all the previously estimated models I have assumed that each of the factors of the dual objective do not affect the other – in other words, the possibility of a simultaneous system has not been accounted for. It could however be argued that the fans’ probability of attending the game is a function of the amount spent for the ticket and vice versa. To rule out this possibility I estimate a simultaneous system using a standard two-stage estimation process. Because there is a binary and a continuous dependent variable, care must be taken in estimating the standard errors (Maddala 1983, pp 244-245). Fortunately, Keshk (2003) has recently developed a Stata program (CDSIMEQ procedure) that implements the two stage probit least squares (2SPLS) procedure and corrects the standard errors.

In the first stage, I regress the two dependent variables (‘net dollars’ and ‘attend game’) on a set of exogenous variables. In the second stage, the fitted variable for a given dependent variable is used as the exogenous variable for the other dependent variable. In particular, I use the first-stage regression to estimate net dollars*, the fitted value of the ‘net dollars’ variable. Then, in the second stage probit, it is analyzed whether net dollars* is significantly related to the ‘attend game’ variable. To identify the simultaneous system I include the “units bid on” variable. I estimated the simultaneous system with (a) the entire sample and (b) with individual

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34 Note that buying one forward as opposed to ten forwards (on a given team) influences the amount of money spent (and therefore the ‘net dollars’ variable) but does not increase the probability of attending the game.
models for the team- and game-based fans. The results of the CDSIMEQ procedure indicate that the system is not simultaneous.

3.6 CONCLUSION

This chapter gives us a better understanding of the factors that affect fans’ decision-making in ticket markets. Accounting for fan heterogeneity, I allow fans to have different objectives in this market and consequently estimate different models to understand their behaviors.

An ordered logit model that parses out the effects of risk minimization and cost reduction is estimated for game-based fans; a regression model that identifies cost reduction strategies is estimated for team-based fans. The question of whether conventional wisdom on risk management from financial markets can be applied to consumer markets is addressed. Some of the results from the finance literature do carry through. Consistent with the idea of diversifying one’s portfolio in financial markets, it is demonstrated that it is a good strategy for fans to buy forwards of many unique teams (to increase their chances of attending the game). Further, using a short-term indicator of performance (seeing if the team performs better at the week of purchase compared to its performance in the previous week) as a strategy to ‘pick’ the right teams does not pay off for game-based fans; it does seem like a good strategy for team-based fans who want to decrease their out-of-pocket expenses. More long-term performance measures should be used to guide buying decisions. Interestingly, heeding expert forecasts does not seem to be a good strategy for game-based fans.

The sports market setting used was for various reasons. Firstly, in this context the two forces at play (maximizing the probability of attending the game while minimizing the cost) can be clearly seen. Further, given that it is known which teams made it to the final game in the
previous years I have the unique ability to classify fans’ transactions as being ‘good’ or ‘bad’. Consumers tend to have dual objectives (of maximizing their chance and minimizing their cost) in other environments with market-uncertainties as well. For instance, till recently, there was uncertainty regarding which format of DVD (HD or Blue ray) would emerge the winner. As of February 2008 Toshiba dropped out of the HD race leaving the market for Sony’s Blue ray partly because the fight between the two formats was keeping many consumers out of the market. According to a technology analyst “.consumers had held off investing in the latest recorders and players because they didn’t know which format would emerge dominant” (Kageyama 2008). In this market, before the uncertainty was resolved, consumers would have faced a similar dilemma - to pick the ‘right format’ to invest in while simultaneously seeking to minimize costs. Our methodology could be used to study the factors that would satisfy consumers’ dual objectives in such settings.

There are many interesting directions for future research even in the sports market context. For example, it would be interesting to study the value of having similar teams in a portfolio. Finance literature suggests a low correlation among portfolio holdings can help. The is because when one of the investments does not perform well the rest of the portfolio can remain balanced. In the context of sports teams, similarity could, arguably, be a function of which conference the teams belong to. It is unclear that in a highly volatile market that operates for less than twenty-two weeks in a year, the recommendation of a dissimilar portfolio would continue to hold. Another issue of interest is to study what causes the difference in the amount paid by ‘successful’ fans (those who eventually attend the game). One could compare the effectiveness of strategies such as ‘buy and hold’ versus ‘buy and sell when team is not performing well’. While reselling a forward clearly helps the fan offset the total amount spent, given that there is a
tradeoff in not holding the ‘right’ team until the end, it is unclear which strategy will be preferred. Finally even though I used existing procedures to estimate the simultaneous models, a new procedure needs to be developed to be able to cluster standard errors in a system with a binary and a continuous dependent variable. Because clustering for standard errors cannot be properly accounted for in this setting it is recognized that the significance of some of the independent variables could be understated (Chaney et al 2007).
CHAPTER 4:

CONSUMER FORWARDS: THEORY AND EMPIRICAL APPLICATION TO TOURNAMENT TICKET PRICING

4.1 INTRODUCTION

In chapter 1 option pricing was studied as a new pricing mechanism that could be implemented in markets with uncertainty. This chapter considers a different type of pricing mechanism – forward pricing - that can be implemented under the same conditions. Given that option pricing was a more profitable pricing mechanism that elicited positive reactions from fans, why is another new pricing mechanism required? Further how does option pricing (proposed in chapter 1) compare with the newly proposed forward pricing mechanism? This chapter answers these questions. To begin with, the concept of forward pricing is introduced. Then, the differences between option pricing and forward pricing are highlighted. Fans’ willingness to pay (WTP) are elicited using a study. Finally a simple analytical model is formulated and conditions under which profitability of forward pricing is higher than those from the other pricing mechanisms are discussed.

Unlike an option, a forward ticket is bought for a specific team by paying the ‘forward price’. If that specific team makes it to the final game the fan is eligible to exercise the forward by paying the exercise price. If the team does not make it to the final game the forward expires and the fan loses the forward price.
Because the forward expires the fan does not need to pay the exercise price. With option pricing because the option is not tied to a particular team, fans who buy an option could choose to attend any game. With forwards however if fans seek to increase their chances of attending the game they should purchase multiple forwards to different teams. Another point of differentiation is that option pricing was used to price tickets in a local market where there was one preferred team. With forward pricing the league can set prices when there are many preferred teams. Therefore this chapter can be seen as an extension to the option pricing chapter (chapter 1).

This pricing mechanism has not yet been implemented by the league. One reason could be the possible negative fan perceptions of this practice. Leagues are sensitive to fan perceptions of their pricing practices partly because the league gets most of its revenues (directly through ticket sales, or indirectly through network broadcasting rights which are affected by fan viewership) from the fans. If the league had a hundred seats to sell to a game it could, theoretically, collect the forward price across all fans and the exercise price from those fans who make it to the final. The fact that leagues could profit from ‘reselling one ticket many times over’ could create some resentment among fans. But forward pricing has been implemented by some third-party firms which offer forwards on tickets to various tournament games. So this research could be viewed as an attempt to catch up with business practices.

Because the forward pricing concept is relatively new I begin with an exploration of fans’ willingness to pay (WTP) for this type of ticket. From the open-ended feedback that was collected it can be seen that fans agree forward pricing would be valuable offering. I analyze an analytical model with two types of fans with different valuations for the final game and I demonstrate the conditions under which profits from forward pricing are higher than those from the other pricing mechanisms.
4.2 EMPIRICAL ANALYSIS

I begin by exploring fans’ willingness-to-pay (WTP) for tickets to the Final Four game under different levels of uncertainty in team matchups. Final Four tickets are used as our stimulus because of its relevance to the student population. Using a computer study I ascertain students’ willingness to pay for a ticket to the National Collegiate Athletic Association (NCAA) Final Four men’s basketball game (hereafter ‘the final game’) at two college campuses. I conduct the study at the student union buildings of two large public universities in the south. At each campus I obtain students WTP for tickets to the final game under different probabilities that their college will make it to the game. To elicit more accurate WTP measures students at both campuses were offered the opportunity to enter a drawing to buy a real ticket to the championship game based on their stated WTP for the ticket. Participants were informed that if their name was drawn they would be required to buy the ticket by paying the average price they had indicated in the study. The winner of the ticket paid us $200 for an upper-level ticket to the 2007 Championship game at the RCA Dome in Indianapolis.

4.2.1 Method

Twenty-five undergraduate students were recruited from each of the two college campuses to participate in a 20-minute within-subject computer study in return for $5 cash. At the beginning of the study participants were asked to indicate their favorite team – over 90% of the respondents listed their home university as their favorite team. Subsequent questions were automatically adjusted so that they were pertinent to that particular team. To be eligible for inclusion in our study students had to answer a set of ‘test your understanding’ questions correctly. This indicated that they understood the forward pricing concept. Four students did not pass the test and our sample is comprised of forty six students (twenty three from each campus).
Fans indicated their WTP under three pricing conditions (1) advance selling (2) forward pricing and (3) full information pricing. In the advance selling condition, subjects indicated their willingness to pay for a “regular ticket” given three different probabilities (25%, 50%, and 75%) of their favorite team making it through the tournament. In the forward pricing condition, based on extensive pre-testing, the levels of the forward price \( P_F \) were fixed at $20, $40 and $60. Given a certain forward price, subjects then indicated their willingness to pay for the “exercise price” \( P_E \). In the full information condition, subjects indicated their willingness to pay for the ticket once they knew whether or not their favorite team made it to the Final Four (0% or 100%). Across the pricing conditions the fans were offered the same broad range of prices to choose from ($0 to $400 in increments of $20). Fans had the choice of not buying a ticket under all pricing conditions – they could simply indicate a WTP of $0. Because this is a within-subject design, the presentation order of tickets was counterbalanced in order to control for possible order effects. No significant order effects were found; therefore this factor is ignored in the remainder of the chapter.

To better understand what fans thought about the concept of forward pricing I obtained some open-ended feedback from respondents. Finally, because I wanted to understand the behavioral determinants of consumer willingness to pay under various pricing approaches, I follow Padmanabhan and Rao (1993) and measure fans’ risk preferences based on their

\[35\] We do not allow fans to choose both the forward and the exercise price because we want to obtain their WTP without confounding our measures with individual differences in risk preferences. For example, suppose we allow subjects to state their willingness to pay for both the forward and the exercise price, and one subject was willing to pay a forward price of $50 and an exercise price of $100 given a 0.25 probability of her favorite team making it to the big game. Suppose another subject was willing to pay $25 for the forward price and $100 as the exercise price given a 0.50 probability of her favorite team making it through the tournament. In this case, the expected value of both subjects’ willingness to pay is the same ($75). However, it can be argued that the first fan who was willing to pay more for the forward when there was a lower probability of the team playing in the final game is more risk-averse than the latter.
responses to a lottery question (fans who are willing to pay less for the lottery than its expected value are considered to be risk-averse; these fans represent 83% of the sample).

4.2.2 Results

Overall, the majority (57%) of the respondents thought that this pricing mechanism would succeed and that they would buy a forward ticket if offered. For example one respondent commented: “The forward ticket concept relieves a lot of the stress that comes with buying a ticket for a game. A person buys it ahead of time with the chance that their team will go to the championships. This way if they don't, they don't have to deal with the stress of reselling the tickets. I would buy it”. Another one said he liked the concept because “it would mean more fans of whichever team was playing present, and I would rather have that than people who attend because their company got them tickets.”

About 33% of the sample were fans who were WTP zero to attend a game in which their favorite team was NOT playing – these fans are labeled as team-based fans. Other fans with positive valuations for any game are labeled as game-based fans. Further a greater proportion of team-based fans are generally more risk averse compared to the game-based fans. Thus team-based fans are more risk-averse (t=3.23, p<0.001) and they have zero-valuation for the non-preferred game.

The average expected valuations of the team-based and game-based fans can be calculated for the various uncertainty levels using the stated WTP for the preferred and non-preferred game under full information. Table 15 reports these expected valuations for three probability levels (0.25, 0.50 and 0.75). I find that game-based fans’ expected valuation for the game is always significantly higher than the team-based fans’ expected valuation.
Table 15: Fans expected valuation under different uncertainty levels

<table>
<thead>
<tr>
<th>Probability</th>
<th>Game-Based Fan</th>
<th>Team-Based fan</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.25$</td>
<td>$121.61$</td>
<td>$31.33$</td>
<td>$5.307$</td>
</tr>
<tr>
<td>$\gamma = 0.50$</td>
<td>$146.45$</td>
<td>$62.67$</td>
<td>$4.352$</td>
</tr>
<tr>
<td>$\gamma = 0.75$</td>
<td>$171.29$</td>
<td>$94.00$</td>
<td>$3.389$</td>
</tr>
</tbody>
</table>

The main effect of pricing is significant ($F = 4.50$, $p<0.015$). This implies that fans are WTP differently under the different pricing mechanisms. The interaction between pricing and risk aversion is not significant with forward pricing\(^{36}\). With forward pricing because forwards expire if the specific team does not make it to the final game, fans are WTP lesser for it than other pricing mechanisms irrespective of risk aversion. Therefore while risk averse fans are more WTP for forward pricing than less risk averse fans, they are NOT WTP more for forward pricing than the other pricing mechanisms.

From the willingness to pay information, the demand (at various prices), was calculated. I then compute profits at the various prices and use the price that yields the highest profit as the optimal price under each pricing approach. I calculate the profitability of the different pricing mechanisms under three different probabilities (0.25, 0.50 and 0.75) of each team making it to the final game. The grouping of the probabilities of profits in all possible scenarios is given in the table below:

\(^{36}\) With option pricing the interaction between risk aversion and pricing was significant. This is because fans are WTP more for option pricing (than for any other pricing mechanism) if they are risk averse.
Table 16: Scenarios Used in Profit Computation

<table>
<thead>
<tr>
<th>Combination</th>
<th>Probability of Team A making it to the Final Four ($\gamma_A$)</th>
<th>Probability of Team B making it to the Final Four ($\gamma_B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.50</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
<td>6</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>7</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.25</td>
</tr>
</tbody>
</table>

In combinations 1 through 6 independent of the probability of team A making it ($\gamma_A$), team B can have a high or low probability of making it. However combinations 7 through 9 represent the cases where there could be correlation between the probabilities of teams A and B (i.e., $\gamma_A$ and $\gamma_B$). For example, in the extreme case of $\gamma_A = 0$, $\gamma_B$ has a high probability of playing in the final game. This concept is captured by combinations 7, 8 and 9 – when $\gamma_A$ is low $\gamma_B$ is high and vice versa.

Profits from all pricing mechanisms under the nine combinations are depicted in Figure 1. From Figure 1, as would be expected, we see that profits from full information pricing are always higher than those from advance selling. In combinations 7, 8 and 9 profits from forward pricing exceed those from full information pricing. These are the combinations in which there is a direct correlation between team A’s chance and team B’s chances of making it to the final game (i.e., if team A had a 25% chance of making it to the game, team B has a 75% chance). Therefore the value of full information is lower in these combinations. In combinations 2 through 6 profits from full information are higher than those from forwards. In combination 1 the probability of both teams making it to the Final game is really low ($\gamma_A = 0.25$, $\gamma_B = 0.25$). In this
case profits from forward pricing exceed that from full information pricing. Therefore
uncertainty helps forward pricing.

To summarize, if the probability of both teams making it is low (Case 1) or if the
probabilities are correlated (Cases 7 through 9) profits from forward pricing exceed those from
full information pricing. For the other combinations profits from full information pricing exceed
those from forward pricing. Across the combinations profit from forward pricing and full
information pricing are always higher than that from advance selling (see Figure 11).

**Figure 11: Profitability Across Pricing Conditions**

![Figure 11: Profitability Across Pricing Conditions](image)

### 4.3 MODEL

In this section I consider a stylized individual-level model to study the conditions under which
forward pricing is more profitable than the other pricing mechanisms. As stated earlier, unlike
option pricing, forward pricing can be studied when there is more than one preferred team in the
market. Therefore I study the case where three teams (A, B and C) compete amongst themselves
and two (of the three teams) make it to the final game. The three team case is the simplest way to capture the case of multiple teams with different probabilities of making it to the final.

An example of when three teams compete amongst each other and two (of the three) reach the final game is the Wildcard weekend in professional football games (NFL). The lower ranked teams play amongst themselves; the winner plays with the top-ranked team. To clarify the case that is being considered Figure 12 is presented.

**Figure 12: Three Teams Competing To Reach the Final Game**

At time 1 \((t=1)\) three teams \((A, B\) and \(C)\) are competing to reach the final game. This is when forward tickets and advance selling tickets are sold. At time 2 it is known which of the team combinations will be playing in the final game. This is when tickets are sold under full information pricing. I begin the analysis with a simplification: I consider only one of the three possible final matchups (that of team \(A\) and team \(B\) playing in the final). I later extend the analysis by allowing for the other possibilities as well.
I consider two types of fans: game- and team-based. Teams A and B have their own team and game-based fans. Team A’s team-based fans have zero utility for team B (and vice versa). Game-based fans on the other hand have positive utility for both team A and team B. The utility of Team A’s team-based fan is represented by $U_{TA}$ and that of team B’s team-based fan is represented by $U_{TB}$. The utility of the game-based fan for team A is represented by $U_{GA}$ (and that of team B is represented by $U_{GB}$). When the data that was collected from the two colleges was combined I find two different types of game-based fans in the market – some with $U_{GA} > U_{GB}$ and others with $U_{GB} > U_{GA}$. Therefore four different fans are considered (two team-based fans and two types of game-based fans). Empirically, on average, $U_{GB} > U_{GA} > U_{TA} > U_{TB}$. Therefore this is the ordering that is analyzed. The probability with which a given team $i$ makes it to the final game is given by $\gamma_i^{37}$. I define the forward and exercise prices to be $P_F$ and $P_G$ respectively. The advance selling price is given by $P_A$; the full information price when teams A and B make it to the final game is given by $P_{F(AB)}$. Because team-based fans have no utility in watching any team except their favorite one play, we assume they buy a single forward (to the team of their choice). Game-based fans can buy multiple forwards to increase their chances of attending the game.

In the remainder of this section, I focus on the situation in which the league sets ticket prices so that all fans participate. I have ensured that, under certain conditions, it is indeed optimal for the league to serve all fans under all three pricing mechanisms. I compute the profitability across forward pricing, advance selling and full information pricing.

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37 Substituting $\gamma_A = 1 - \gamma_B$ ensures that the probabilities of team A and B are correlated and it does not change any of the results.
4.3.1 Case I: Teams A and B can make it to the final game

**Utility Ordering:** $U_{GB} > U_{GA} > U_{TA} > U_{TB}$

**Forward Pricing:**

With forward pricing, a fan pays the forward price ($P_F$) at time 1 and the exercise price ($P_G$) once the final outcome is known (at time 2). The league sets both the forward and exercise prices at time 1. The league can extract the surplus from either the game- or the team-based fans. I begin by ensuring the individual rationality constraint of both team-based fans hold (in Sub case A). I later check to see if this is optimal.

**Sub case A:**

Setting the individual rationality constraint of both team-based fans results in the equations below:

$$P_F + \gamma_A P_G - \gamma_A U_{TA} = 0 \quad (38)$$

$$P_F + \gamma_B P_G - \gamma_B U_{TB} = 0 \quad (39)$$

Solving eqns. (38) and (39) gives the optimal forward and exercise price:

$$P_F = \gamma_A \gamma_B (U_{TB} - U_{TA}) / (\gamma_A - \gamma_B) \quad (40)$$

$$P_G = (U_{TA} \gamma_A - U_{TB} \gamma_B) / (\gamma_A - \gamma_B) \quad (41)$$

From the optimal forward price it is easy to see that it is decreasing with respect to $\gamma_B$. Therefore as the probability of team B making it to the final decreases the league charges a low forward price from team B’s team-based fan. This ensures her participation. However as the probability of team A making it ($\gamma_A$) increases, the forward price increases. This is because the utility of team A’s team-based fan is higher than that of team B’s team-based fan (i.e., $U_{TA} > U_{TB}$) – therefore she has enough reason to participate as it is. For the optimal forward price to be
positive the probability of team A making it should be lower than that of team B (i.e., $\gamma_A < \gamma_B$).

For the optimal exercise price to be positive, given that $\gamma_A < \gamma_B$, the expected utility of team A’s team-based fan should be lower than that of team B’s team-based fan (i.e., $U_{TA}\gamma_A < U_{TB}\gamma_B$).

The game-based fan could engage in one of the following strategies: (a) she could buy team A’s forward (b) she could buy team B’s forward or (c) she could buy both A and B’s forwards and exercise either (depending on which of these occur). She chooses the strategy that gives her the highest surplus. Because the entire surplus is extracted from the team-based fans (see eqns. (38) and (39)), the game-based fan receives a surplus of $\gamma_A(U_{GA} - U_{TA})$ if she chooses to buy only team A’s forward, a surplus of $\gamma_B(U_{GB} - U_{TB})$ if she buys only team B’s forward and $\gamma_A(U_{GA} - U_{TA}) + \gamma_B(U_{GB} - U_{TB})$ if she buys both forwards. Because buying both forwards gives her the highest surplus she buys both teams’ forwards, paying an expected price of

$$2P + (\gamma_A + \gamma_B)P_G.$$

Profits from forward pricing ($\pi_F$) are obtained from the two types of game-based fans and the two team-based fans. They are given by:

$$\pi_F = 2(2P + (\gamma_A + \gamma_B)P_G) + P_f + \gamma_A P_G + P_f + \gamma_b P_G$$

Substituting the optimal prices from eqns. (40) and (41), gives the optimal profits:

$$\pi_F = 3(\gamma_A U_{TA} + \gamma_B U_{TB})$$

**Advance Selling:**

As stated earlier, $U_{TA}\gamma_A < U_{TB}\gamma_B$ ensures that the exercise price is positive. Given this ordering of expected utility the advance selling price is set at $U_{TA}\gamma_A$ (i.e., $P_a = U_{TA}\gamma_A$) to ensure all four fans participate. Therefore profits ($\pi_A$) are given by:
\[ \pi_A = 4\gamma_A U_{TA} \] (44)

**Full Information Pricing:**

In this type of pricing, the league sets the ticket price *after* it knows which teams are playing in the final game. Therefore, given that teams A and B are playing the league sets the price at \( U_{TB} \) (i.e., \( P_F(AB) = U_{TB} \)) such that all fans (both types of game-based fans and the two team-based fans) will attend the game. Therefore expected full information profits \( (\pi_{FI}) \) are given by:

\[ \pi_{FI} = 4\gamma_B U_{TB} \] (45)

**Comparing Profits Across Pricing Mechanisms:**

Given that \( U_{TA}\gamma_A < U_{TB}\gamma_B \), the difference between the profits in the full information case (eqn. (45)) and advance selling (eqn. (44)) is positive. Further, the difference in profits between forward pricing and advance selling (eqns. (43) and (45)) which is given below, is also positive:

\[ \pi_F - \pi_A = 3\gamma_B U_{TB} - \gamma_A U_{TA} \] (46)

Given that the league sets both the advance selling price and the price under forward pricing at time 1 it can always set the forward price equal to the advance selling price (and the exercise price to be zero). Therefore it is intuitive that forward pricing can do no worse than advance selling. As \( \gamma_B \) increases profits from forward pricing increase more than they do with advance selling. This is because the \( \gamma_B U_{TB} \) term plays a bigger role in forward profits than does \( \gamma_A U_{TA} \) (as stated earlier for the exercise price to be positive \( \gamma_A U_{TA} \) should be less than \( \gamma_B U_{TB} \)).

The difference in profits between forward pricing and full information pricing (eqns. (43) and (45)) is given by:

\[ \pi_F - \pi_{FI} = 3(\gamma_A U_{TA} + \gamma_B U_{TB}) - 4\gamma_B U_{TB} \] (47)
Profits from forward pricing can exceed those from full information pricing if
\[ 3\gamma_A U_{TA} > \gamma_B U_{TB}. \]
Therefore, if the expected utility of team A’s team-based fan is high enough, the
profit ordering in Case 1 is given by:
\[ \pi_F > \pi_{FI} > \pi_A \]  \hspace{1cm} (48)

Profits from forward pricing were computed when the individual rationality of the team-based fans were satisfied. It remains to be shown that this is indeed optimal. Next, the
profitability from forward pricing when the individual rationality constraints of a game-based
fan and a team-based fan are satisfied is computed in Sub case B.

**Sub case B:**

Setting the individual rationality constraint of team A’s team-based fan and a game-based
fan results in the equations below:
\[ P_F + \gamma_A P_G - \gamma_A U_{TA} = 0 \]  \hspace{1cm} (49)
\[ 2P_F + (\gamma_A + \gamma_B) P_G - (\gamma_A U_{GA} + \gamma_B U_{GB}) = 0 \]  \hspace{1cm} (50)

As shown in the earlier section (Sub case A), given the utility ordering in the current case
\( U_{GB} > U_{GA} > U_{TA} > U_{TB} \) game-based fans buy both forwards. Solving eqns. (38) and (39), the
optimal forward and exercise prices are obtained:
\[ P_F = \gamma_A (\gamma_A U_{GA} + \gamma_B U_{GB} - U_{TA}(\gamma_A + \gamma_B)) / (\gamma_A - \gamma_B) \]  \hspace{1cm} (51)
\[ P_G = (2U_{TA} \gamma_A - (U_{GA} \gamma_A + U_{GB} \gamma_B)) / (\gamma_A - \gamma_B) \]  \hspace{1cm} (52)

Given these optimal prices the league needs to ensure the participation of the other team-based fan. Team B’s team-based fan participates if the price she pays is less than her expected
utility i.e., if \( P_F + \gamma_B P_G < \gamma_B U_{TB} \). Substituting the optimal prices the inequality reduces to
\[ \gamma_A U_{GA} + \gamma_B U_{GB} < \gamma_A U_{TA} + \gamma_B U_{TB}. \]  This is never true given the current utility ordering. This implies
that team B’s team-based fan never participates if the prices are set such that the game-based fan
and team A’s team-based fan’s individual rationality constraints are satisfied.

Finally, what if the individual rationality constraint of one game-based fan and team B’s
team-based fan is satisfied? This scenario is considered in Sub case C.

**Sub case C:**

The two individual rationality constraints that would need to be solved are:

\[ P_{F} + \gamma_{b}P_{G} - \gamma_{b}U_{TB} = 0 \quad (53) \]
\[ 2P_{F} + (\gamma_{A} + \gamma_{b})P_{G} - (\gamma_{A}U_{GA} + \gamma_{b}U_{GB}) = 0 \quad (54) \]

Solving eqns. (53) and (54) the optimal forward and exercise prices are obtained:

\[ P_{F} = \gamma_{b}(U_{TB}(\gamma_{A} + \gamma_{b}) - (\gamma_{A}U_{GA} + \gamma_{b}U_{GB}))/(\gamma_{A} - \gamma_{b}) \quad (55) \]
\[ P_{G} = ((U_{GA}\gamma_{A} + U_{GB}\gamma_{b}) - 2U_{TB}\gamma_{b})/(\gamma_{A} - \gamma_{b}) \quad (56) \]

Given these optimal prices the league needs to ensure the participation of the other team-
based fan. Team A’s team-based fan participates if the price she pays is less than her expected
utility i.e., if \( P_{F} + \gamma_{A}P_{G} < \gamma_{A}U_{TA} \). Substituting the optimal prices the inequality reduces to
\[ \gamma_{A}U_{GA} + \gamma_{b}U_{GB} < \gamma_{A}U_{TA} + \gamma_{b}U_{TB} \]. Again, this is never true given the current utility ordering. This
implies that team A’s team-based fan never participates if the prices are set such that the game-
based fan and team B’s team-based fan’s individual rationality constraints are satisfied.
Therefore it is optimal for the league to extract the surplus of both the team-based fans as done in
Sub case A.

When considering two teams’ making it to the final (A and B) we see that profits from
forward pricing always exceeds that from advance selling and under some conditions it can
exceed profits from full information pricing as well. Specifically, if the estimated probability of a
team A making it to the final is low the required condition for the profitability of forward pricing

over full information pricing is that the expected utility of A’s team-based fan should be high. In the next section I extend the analysis to include all three combinations in Figure 2.

4.3.2 Case II: Multiple Team Combinations

I compute the profitability of forward pricing, advance selling and full information pricing when three teams compete and any two make it past the tournament to the final game. In other words, given that there are three teams, there are three combinations with which two teams can play in the final game (i.e., when A, B and C are competing to reach the final any combination of (A, B), (B, C) or (A, C) could end up playing at the final game).

Because there are multiple combinations probabilities are assigned to each of these combinations: $\gamma_1$ is the probability with which teams A and B will play in the final, $\gamma_2$ is the probability with which teams B and C will play in the final and $1 - \gamma_1 - \gamma_2$ is the probability with which teams A and C will play in the final. Therefore the probability with which team A will play in the final game is $\gamma_1 + 1 - \gamma_1 - \gamma_2 = 1 - \gamma_2$, team B will play in the final is $\gamma_1 + \gamma_2$ and team C will play in the final is $\gamma_2 + 1 - \gamma_1 - \gamma_2 = 1 - \gamma_1$.

Team-based fans utilities for teams A, B and C are $U_{TA}$, $U_{TB}$ and $U_{TC}$ respectively. As a simplification I assume there is one type of game-based fan having the same utility for all three teams (i.e., $U_{GA} = U_{GB} = U_{GC} = U_G$).

I consider the ordering given below. Just as in the previous case I believe the utility of the game-based fans across the three teams will, on average, be greater than that of the team-based fans.
Utility Ordering: $U_G > U_{TA} > U_{TB} > U_{TC}$

As done previously, I begin the analysis by ensuring the individual rationality of team A and team B’s team-based fans.

Forward Pricing:

Setting the individual rationality constraint of team A and team B’s team-based fans results in the equations below:

$$P_F + (1 - \gamma_2)P_G - (1 - \gamma_2)U_{TA} = 0 \quad (57)$$

$$P_F + (\gamma_1 + \gamma_2)P_G - (\gamma_1 + \gamma_2)U_{TB} = 0 \quad (58)$$

Solving eqns. (57) and (58) the optimal forward and exercise price are obtained:

$$P_F = (1 - \gamma_2)(\gamma_1 + \gamma_2)(U_{TA} - U_{TB})/(\gamma_1 + 2\gamma_2 - 1) \quad (59)$$

$$P_G = U_{TB}(\gamma_1 + \gamma_2) - U_{TA}(1 - \gamma_2)/(\gamma_1 + 2\gamma_2 - 1) \quad (60)$$

From the optimal forward price it is easy to see that it is decreasing as the probability with which team B makes it to the final increases (i.e., as $(\gamma_1 + \gamma_2)$ increases). Therefore as the probability with which team B making it to the final decreases the league charges a low forward price from team B’s team-based fan. This ensures her participation. However, as the probability of team A making it $(1 - \gamma_2)$ increases the forward price increases. This is because the utility of team A’s team-based fan is sufficiently high to ensure her participation $(U_{TA} > U_{TB})$. Further, as the probability of (A, B) making it to the final increases (i.e., as $\gamma_1$ increases) the forward price, which is also collected from the team-based fans of team C, decreases. But the exercise price increases as the probability of the (A, B) combination making it increases $(\gamma_1)$ because team C’s team-based fan will not participate anyway.
For the optimal forward price to be positive it is required that $\gamma_1 + 2\gamma_2 - 1 > 0$. Further, for the optimal exercise price to be positive, given that $\gamma_1 + 2\gamma_2 - 1 > 0$, the expected utility of team A’s team-based fan should be lesser than that of team B’s team-based fan (i.e., $U_{TA}(1-\gamma_2) < U_{TB}(\gamma_1 + \gamma_2)$).

Given that the optimal prices have been set, I check whether team C’s team-based fan will participate. She will participate if the expected price she pays is lower than her expected utility. In other words she will participate if $P_F + (1-\gamma_1)P_G < (1-\gamma_1)U_{TC}$. This condition will be satisfied if the utility of the team-based fan is high enough, as given below:

$$U_{TC} > U_{TA}(1-\gamma_2)(2\gamma_1 + \gamma_2 -1) + U_{TB}(\gamma_2^2 - \gamma_1^2)/(\gamma_1 + 2\gamma_2 - 1)(1-\gamma_1)$$

(61)

The game-based fan could do one of the following things: (a) she could buy team A’s forward (b) team B’s forward (c) team C’s forward or (d) she could buy all teams to ensure she gets to attend any game. She chooses the strategy that gives her the most surplus. Given that A and B’s team-based fan’s surplus have been completely extracted, if she buys team A’s forward she gets a surplus of $(1-\gamma_2)(U_G - U_{TA})$; if she buys team B’s forward she gets a surplus of $(\gamma_1 + \gamma_2)(U_G - U_{TB})$. Also, given that team C’s team-based fan participates this implies that the game-based fan gets a surplus when she buys only team C’s forward as well (because $U_G > U_{TC}$). Therefore she gets the highest surplus when she buys and exercises all three teams’ forwards thereby paying $3P_F + P_G$.

Profits from forward pricing ($\pi_F$) are collected from one game-based fan and three team-based fans. They are given by:

$$\pi_F = 3P_F + P_G + P_F + \gamma_A P_G + P_F + \gamma_B P_G + P_F + \gamma_C P_G$$

(62)

Substituting the optimal prices from eqns. (59) and (60), the optimal profits are obtained:
\[ \pi_f = (1 - \gamma_2)(2\gamma_1 + 2\gamma_2 - 1)U_{TA} - (1 - 2\gamma_2)(\gamma_1 + \gamma_2)U_{TA}/(\gamma_1 + 2\gamma_2 - 1) \] 

(63)

**Advance Selling:**

In the forward pricing case the expected utility of team A’s team-based fan should be lower than that of team B’s team-based fan (i.e., \( U_{TA} (1 - \gamma_2) < U_{TB} (\gamma_1 + \gamma_2) \)) for the optimal exercise price to be positive. If the expected utility of team A’s team-based fan is also lower than that of team C (i.e., \( U_{TA} (1 - \gamma_2) < U_{TC} (1 - \gamma_1) \)) then the advance price will be set at \( U_{TA} (1 - \gamma_2) \) to ensure that all fans participate. First, I check if \( U_{TA} (1 - \gamma_2) < U_{TC} (1 - \gamma_1) \).

Eqn. (61) gives us the condition that will ensure the participation of team C’s team-based fan. If \( U_{TA} (1 - \gamma_2) < U_{TA} (1 - \gamma_2)(2\gamma_1 + \gamma_2 - 1) + U_{TB} (\gamma_2^2 - \gamma_1^2)/(\gamma_1 + 2\gamma_2 - 1)(1 - \gamma_1) \), (the rhs of eqn. (61)), then it implies that \( U_{TC} (1 - \gamma_1) > U_{TA} (1 - \gamma_2) \). Is

\[ U_{TA} (1 - \gamma_2)(\gamma_1 + 2\gamma_2 - 1)(1 - \gamma_1) < U_{TA} (1 - \gamma_2)(2\gamma_1 + \gamma_2 - 1) + U_{TB} (\gamma_2^2 - \gamma_1^2) \] ? Rearranging the terms and multiplying through reduces the inequality to \( U_{TA} (1 - \gamma_2) < U_{TB} (\gamma_1 + \gamma_2) \). This inequality is satisfied (from the forward pricing case this inequality ensures the exercise price is positive).

Therefore \( U_{TA} (1 - \gamma_2) < U_{TC} (1 - \gamma_1) \) and the advance selling price is set equal to \( U_{TA} (1 - \gamma_2) \).

Profits from advance selling are obtained from three team-based fans and one game-based fan. They are given by:

\[ \pi_a = 4(1 - \gamma_2)U_{TA} \] 

(64)

**Full Information Pricing:**

The league sets the price after knowing which of the team combinations will be playing in the final game. Depending on the teams that make it to the final the respective teams’ team-based fans will attend the game. The game-based fan will attend any final game. If (A, B) make it to the final game the league sets the price at \( U_{TB} \) such that one game-based fan and two team-based
fans (of A and B) attend the game. However, if either (A, C) or (B,C) occurs then the league sets
the price equal to \( U_{TC} \) to ensure that all three fans in question will participate. Therefore expected
full information profits \( \pi_{FI} \) are given by:

\[
\pi_{FI} = 3(\gamma_1 U_{TB} + (1-\gamma_1)U_{TC})
\]  

(65)

**Comparing Profits Across the Pricing Mechanisms:**

The difference between the profits in the forward pricing case (eqn.(63)) and advance selling
(eqn.(64)) is given by:

\[
\pi_F - \pi_A = ((1-\gamma_2)U_{TA} (1+2\gamma_1 - 2\gamma_2) + 3(2\gamma_2 - 1)(\gamma_1 + \gamma_2)U_{TB})/(\gamma_1 + 2\gamma_2 - 1)
\]  

(66)

The denominator is positive (this ensures that the forward price is positive). Therefore if
the numerator is positive then forward profits are higher than those from advance selling. In
other words, after rearranging the terms, the condition that is required for the numerator to be
positive is given below:

\[
(1-\gamma_2)U_{TA} < 3(2\gamma_2 - 1)(\gamma_1 + \gamma_2)U_{TB}/(2\gamma_2 - 2\gamma_1 - 1)
\]  

(67)

For the exercise price to be positive \( U_{TA}(1-\gamma_2) < U_{TB}(\gamma_1 + \gamma_2) \). Therefore if
\[
3(2\gamma_2 - 1)(\gamma_1 + \gamma_2)U_{TB}/(-1-2\gamma_1 + 2\gamma_2) > U_{TB}(\gamma_1 + \gamma_2) \]  

then \( U_{TA}(1-\gamma_2) < U_{TB}(\gamma_1 + \gamma_2) \) implies that
the condition in eqn. (67) holds true. Is \( 3(2\gamma_2 - 1)(\gamma_1 + \gamma_2)U_{TB}/(-1-2\gamma_1 + 2\gamma_2) > U_{TB}(\gamma_1 + \gamma_2) \)?

Rearranging the terms and cross multiplying results in the inequality above reducing to the
question of whether \( \gamma_1 + 2\gamma_2 - 1 > 0 \) which is known to be true (this is the required condition for
the forward price to be positive). Therefore, because the condition in eqn. (67) is satisfied profits
from forward pricing exceed those from advance selling.

Next, profits from full information pricing and advance selling are compared. The
difference in profits is given by:
\[ \pi_A - \pi_{FI} = 4(1 - \gamma_2)U_{TA} - 3(U_{TB} \gamma_1 + U_{TC}(1 - \gamma_1)) \]  

(68)

Profits from advance selling exceed those from full information pricing if the condition below holds:

\[ (1 - \gamma_2)U_{TA} > \frac{3}{4}(\gamma_1 U_{TB} + (1 - \gamma_1)U_{TC}) \]  

(69)

It has already been demonstrated that profits from forward pricing are greater than those from advance selling. Therefore profits from forward pricing exceed those from full information pricing when the condition in eqn. (69) is satisfied.

Comparing the conditions under which forward pricing is more profitable than full information pricing we see that in Case 1, (under the same ordering \( U_{GB} > U_{GA} > U_{TA} > U_{TB} \)) profits from forward pricing could exceed those from full information pricing provided the expected utility of team A’s team-based fan was high enough (i.e., when \( \gamma_A U_{TA} > 3 \gamma_B U_{TB} \)). The intuition behind this is as follows. Forward pricing is more profitable when there is uncertainty in the market place. A required condition for the prices to be positive in both cases is that the probability of team A making it to the final game is lower than that of team B (i.e., \( \gamma_A < \gamma_B \)). Therefore the expected utility of team A’s team-based fan should be high such that it compensates for the lower probability with which team A makes it to the final game. This ensures that team A’s team-based fan is valuable enough to be served.

The reason that profitability from forward pricing can be higher than that from full information pricing is because forward pricing is able to extract the surplus from team-based fans of teams A and B while ensuring that the other fans participate. With full information however the best the league can do is it can extract \( U_{TB} \) when it ensures all fans participate. So how is forward pricing able to get team C’s team-based fan to participate? By lowering the forward
price sufficiently when the probability of team C making it is low (and extracting more using the exercise price that is paid later).

Again, to ensure that extracting the surplus of one team-based fan and one game-based fan is optimal I calculate the optimal profits from satisfying the individual rationality constraint of the game-based fan and any team-based fan. It results in one of the team-based fans not participating. Therefore, similar to Case I, it is most optimal for the league to extract the surplus from both team-based fans in Case II.

Across the cases profits from forward pricing exceed those from advance selling. This is validated empirically as well. This is interesting because a forward ticket is a ‘lesser’ ticket than an advance ticket – when a fan buys an advance ticket she can choose to attend any game she wants. With a forward ticket however, a fan can ONLY attend the game if the team associated with the forward makes it to the final. Therefore the fact that fans prefer this kind of pricing mechanism because it protects them from having to resell the ticket demonstrates that there are indeed fans who have a valuation of zero for the non-preferred game. This is demonstrated empirically as well.

4.3.3 Comparing Profits Across Option and Forward Pricing Mechanisms:

If the market is composed entirely of team-based fans, then forward pricing will be just as profitable as option pricing. These fans will not exercise either the option or the forward if their favorite team does not play in the final game. If, however, the market is composed of both game- and team-based fans, then the profitability of forward versus option pricing depends on the utility of the game-based fans. With option pricing a game-based fan who buys an option can use it at no extra charge irrespective of whether or not a given team makes it to the final. With forward
pricing however, given that forwards expire, game-based fans could buy multiple forwards if they would like to maximize their chances of attending the game. If the game-based fan gets higher surplus from buying multiple forwards (as is the case given the current utility ordering), the league gets multiple forward prices from the game-based fan which it would not under option pricing. This would result in higher profitability for the league.

However, if the utility of the game-based fan is lower, because the game-based fan has to pay the additional forward price for every team she buys, her optimal strategy could change from buying multiple forwards to buying any one team’s forward. This implies her behavior would be similar to that of the team-based fan’s behavior. In this case profits from option pricing could be higher. This is because game-based fans value an option more than they value a forward (which is a ‘lesser’ product). Therefore, if the market is composed of both team- and game-based fans, the profitability of option pricing over forward pricing depends on the utility of the game-based fans.

4.4 CONCLUSION

A mechanism to manage uncertainty in markets where purchase is separated from consumption is proposed. This type of pricing is applicable when there are heterogeneous consumers with different valuations for the outcomes and there are many preferred outcomes. With option pricing (proposed in chapter 1) fans had the same preferred team within the local market. Forward pricing can be used, for example, when in a given market there is more than one preferred team. I analytically model the forward pricing methodology and compare it with some other, more traditional, pricing methodologies. Profits from forward pricing can exceed those from expected full information profits. They always exceed those from advance selling. The
profitability of forward pricing is empirically verified using a study conducted on two college campuses. I obtain fans willingness-to-pay for tickets under different probabilities of their favorite team making it to the final game. Then, the profitability across different pricing mechanisms (full information, advance selling, forward pricing) is computed at different probabilities of the two teams making it to the final game. Profits from full information pricing are lower than those from forward pricing when the probability of both teams making it to the final game is really low or when the probabilities are correlated. Further profits from forward pricing always exceed those from advance selling.

Though the concept of forward pricing is relatively new to the marketing academic literature, some third-party firms (such as firstdibz.com, yoonew.com) have already implemented this type of pricing. This work can be seen as an attempt to catch up with practice.
APPENDIX

CAPACITY CONSTRAINTS MODEL

We restrict our analysis to the case where the league serves both segments and all game-based fans watch either game (Case B). In this region, profits under option pricing exceed those under other pricing mechanisms. We examine whether this finding continues to hold in the presence of capacity constraints. Let the capacity of the stadium be $C$.

Advance Selling

Similar to the derivations in the aggregate demand model in chapter 2, we obtain the quantity demanded by game- and team-based fans using a standard vertical differentiation model:

\[
q_G = \alpha_G - (P\beta_{G1}\beta_{G2}/(\beta_{G1}(1-\gamma) + \beta_{G2}\gamma)); \quad q_T = \alpha_T - (P\beta_{T1}/\gamma)
\]

The demand is constrained by the available capacity, $C$:

\[
C = q_G + q_T
\]

We find the price that the league can charge, given that the capacity constraint ($C$) is binding.

\[
\alpha_G + \alpha_T - (P\beta_{T1}/\gamma) - (P\beta_{G1}\beta_{G2}/(\beta_{G1}(1-\gamma) + \beta_{G2}\gamma)) = C
\]

Solving for the price that satisfies the capacity yields,

\[
P_{AB}^C = (\alpha_G + \alpha_T - C)\gamma(\beta_{G1}(1-\gamma) + \beta_{G2}\gamma)/((\beta_{G1}\beta_{T1}(1-\gamma) + (\beta_{G1} + \beta_{T1})\beta_{G2}\gamma)
\]

By comparing the price in the capacity constraint case with the price in the unconstrained capacity case (eqn. (73) and the price from Table 4) we see that the price is higher in the capacity constrained case if the following condition holds:

\[
P_{AB}^C > P \text{ if } C < (\alpha_G + \alpha_T)/2
\]

From Table 4 note that the total quantity served in the advance selling case ($q_G + q_T$) equals $(\alpha_G + \alpha_T)/2$. Therefore, when the capacity constraint binds (i.e., when $C < (\alpha_G + \alpha_T)/2$) price
under capacity constraints is greater than the unconstrained price. Substituting the price into the
profit function \( \pi_{AB}^C = P_{AB}^C C \) yields the expression for profits under advance selling under a
capacity constraint:

\[
\pi_{AB}^C = C(\alpha_G + \alpha_T - C)\gamma (\beta_{G1}(1-\gamma) + \beta_{G2}\gamma)/(\beta_{G1}\beta_{T1}(1-\gamma) + (\beta_{G1} + \beta_{T1})\beta_{G2}\gamma)
\]  

(75)

**Pricing an option ticket**

**Case B:**

Similar to the derivations in the aggregate demand model in chapter 2, we obtain the quantity
demanded by game- and team-based fans using a standard vertical differentiation model:

\[
q_{GE}^b = \alpha_G - (\beta_{G1}\beta_{G2}(P_o^b + P_g^b))/((\gamma\beta_{G2} + (1-\gamma)\beta_{G1}) ; \quad q_{TO}^b = \alpha_T - (1/\gamma)(P_o^b + \gamma P_g^b)
\]

(76)

Solving for the option and exercise price simultaneously yields:

\[
P_o^C = \gamma(\alpha_T\beta_{G1}\beta_{G2} - q_{GE}^b\beta_{G1}\beta_{G2} + (q_{GE}^b - \alpha_G)\beta_{T1}(\beta_{G1}(1-\gamma) + \beta_{G2}\gamma))/\beta_{T1}\beta_{G1}\beta_{G2}(1-\gamma)
\]

(77)

\[
P_g^C = (\gamma(q_{TO}^b - \alpha_T) \beta_{G1}\beta_{G2} - (q_{GE}^b - \alpha_G)(\beta_{G2} - \beta_{G1})\beta_{T1}\gamma)-(q_{GE}^b - \alpha_G)\beta_{G1}\beta_{T1}/\beta_{T1}\beta_{G1}\beta_{G2}(1-\gamma)
\]

(78)

Under option pricing, the league can determine the optimal number of seats to serve to each fan
segment through the price it sets. Said differently, we assume that the league serves \( \delta C \) game-
based fans and \( (1-\delta)C \) team-based fans, where the optimal number of game-based fans \( (\delta) \) is
determined by the league. Option profits are given by:

\[
\pi_{OB}^C = \delta C(P_o^C + P_g^C) + (1-\delta)C(P_o^C + \gamma P_g^C).
\]

(79)

Substituting \( q_{GE}^b = \delta C \) and \( q_{TO}^b = (1-\delta)C \) into eqn. (79) and solving for the optimal fraction of
game-based fans, \( \delta \), we obtain:
\[
\delta^* = \frac{(2C - \alpha_T) \beta_{g1} \beta_{g2} \gamma + \alpha_g \beta_{t1}(\beta_{g1}(1-\gamma) + \beta_{g2} \gamma)}{2C(\beta_{g2} \beta_{t1} \gamma + \beta_{g1}(\beta_{t1}(1-\gamma) + \beta_{g2} \gamma))}.
\]

This implies that when there are capacity constraints the league will serve a positive number of game-based fans only if there is enough capacity to serve a minimum number of team-based fans (i.e., \( \delta^* > 0 \) if \( C > \alpha_T/2 \)). Substituting the optimal value of \( \delta^* \) into the option and exercise prices we obtain the optimal option and exercise prices to be:

\[
p_c^e = \frac{\gamma(\alpha_T \beta_{g1} \beta_{g2}(\beta_{t1} \beta_{g2}(2-\gamma) \gamma + \beta_{g1}(\beta_{t1}(2-\gamma)(1-\gamma) + \beta_{g2} \gamma) - (\beta_{g1}(1-\gamma) + \beta_{g2} \gamma))}{2C\beta_{g1} \beta_{g2} \beta_{t1}(1-\gamma) + \alpha_g \beta_{t1}(\beta_{g2} \beta_{t1} \gamma + \beta_{g1}(\beta_{t1}(1-\gamma) - \beta_{g2}(1-2\gamma)))}
\]

\[
p_G^e = \frac{\alpha_g}{2\beta_{g2}} - \frac{\gamma(\alpha_T \beta_{g1} - \alpha_g \beta_{t1})}{2\beta_{t1} \beta_{g1}(1-\gamma)}
\]

Compared to the case without capacity constraints, the price of the option increases whereas the exercise price remains the same as in the unconstrained case. Intuitively, the value of a "reservation" on a seat is higher in the presence of capacity constraints – therefore the value of the option increases when there is limited capacity. However, because the fan already holds a reservation, the increased value of which is reflected in the option price, the exercise price remains unchanged.

After substituting in the optimal prices and the optimal fraction of game-based fans who are served \( (\delta^*) \), option profits are:

\[
\pi_{ob}^e = \frac{\alpha_g^2 \beta_{t1}^2(\beta_{g1}(1-\gamma) + \beta_{g2} \gamma)^2 - 2\alpha_g(\alpha_T - 2C) \beta_{g1} \beta_{g2} \beta_{t1} \gamma(\beta_{g1}(1-\gamma) + \beta_{g2} \gamma) + \beta_{g1} \beta_{g2} \gamma(4C(\alpha_T - C) \beta_{g1} \beta_{t1} + (\alpha_T \beta_{g1} \beta_{g2} + 4C(\alpha_T - C)(\beta_{g2} - \beta_{g1}) \beta_{t1} \gamma))}{4\beta_{g2} \beta_{g1} \beta_{t1}(\beta_{g2} \beta_{t1} \gamma + \beta_{g1}(\beta_{t1}(1-\gamma) + \beta_{g2} \gamma))}
\]
Full Information Pricing

Similar to the derivations in the aggregate demand model in chapter 2, we obtain the quantity demanded by game- and team-based fans using a standard vertical differentiation model:

\[ q_{G_1} = \alpha_g - P_1 \beta_{G_1} \gamma, \quad q_{T_1} = \alpha_t - P_1 \beta_{T_1} \gamma \tag{84} \]

The league can a maximum of \( C \) seats, or till the capacity is filled:

\[ C = q_{G_1} + q_{T_1} \tag{85} \]

We find the price that maximizes the capacity:

\[ P_1 = (\alpha_g + \alpha_t - C) / (\beta_{G_1} + \beta_{T_1}) \tag{86} \]

Substituting eqn. (86) into the profit function of the league we obtain the optimal profits to be:

\[ \pi = C (\alpha_g + \alpha_t - C) / (\beta_{G_1} + \beta_{T_1}) \tag{87} \]

If the unpopular game occurs the capacity is filled with game-based fans. Price and profits are:

\[ P_2 = (\alpha_g - C) / \beta_{G_2}, \quad \pi = C (\alpha_g - C) / \beta_{G_2} \tag{88} \]

As in the advance selling case, full information prices are also higher in the presence of capacity constraints. Expected profits under full information pricing are given by:

\[ \pi_{FB}^C = C(\alpha_g - C)(\beta_{G_1} + \beta_{T_1}) + C(\alpha_f \beta_{G_2} - (\alpha_g - C)(\beta_{G_1} - \beta_{G_2} + \beta_{T_1})) \gamma / \beta_{G_2}(\beta_{G_1} + \beta_{T_1}) \tag{89} \]

Profit comparisons

The difference in profits between the option and advance selling the case \((\pi_{OF}^C - \pi_{AB}^C)\) is:

\[ (\alpha_f \beta_{G_1} - \beta_{G_2} \gamma + \alpha_g \beta_{T_1} (\beta_{G_1}(1-\gamma) + \beta_{G_2} \gamma))/4 \beta_{G_1} \beta_{G_2} \beta_{T_1} \beta_{T_1} (\beta_{G_2} \beta_{T_1} \gamma + \beta_{G_1} (\beta_{T_1}(1-\gamma) + \beta_{G_2} \gamma)) \tag{90} \]

Note that eqn. (90) is always positive – therefore, profits under option pricing when capacity constraints are considered always exceed profits under advance selling. Therefore, our basic result regarding the superiority of options in this region is unchanged in the presence of capacity constraints.
constraints. Next we compare profits under option pricing with expected profits under full information pricing:

\[ \pi_{OB}^C - \pi_{FB}^C > 0 \text{ if } \beta_{G2} > \alpha_G \beta_{G1} \beta_{G2} (\beta_{G1} + \beta_{T1})(1 - \gamma)/\left(\alpha_T \beta_{G1} \gamma - 2 \alpha_G \beta_{G1} \beta_{T1} \gamma - \alpha_G \beta_{T1}^2 \gamma\right) \]

and \[ \alpha_T > \alpha_G \beta_{T1} (2 \beta_{G1} + \beta_{T1})/\beta_{G1}^2 \] (91)

The intuition for this is as follows: If the utility of the game-based fan for the unpopular game is low (i.e., \( \beta_{G2} \) is high) full information pricing loses the advantage of being able to extract profits separately in the case that the unpopular game occurs. Simultaneously, if the utility of the game-based fan for the unpopular game is low the value of the option is high. Further, if there are sufficient team-based fans (i.e., \( \alpha_T \) is high) who do not purchase the ticket in the case that the unpopular game occurs, profits under full information pricing are low. Therefore, under these conditions, profits under option pricing can exceed expected profits under full information even in the presence of capacity constraints.
REFERENCES


