# Applications of Game Theory to Topics in Political Economy

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## Abstract

**TIMOTHY J. MOORE: Applications of Game Theory to Topics in Political Economy.**
**(Under the direction of Peter Norman.)**

In the first chapter, *Political Business Cycles with Policy Compromise*, I consider a dynamic model of political decision-making where policy decisions are the result of bargaining between two political parties. The focus of this paper is on how unobservable actions by parties (e.g., the whip encourages certain voting behavior in private strategy sessions) generate inefficient political business cycles. The paper supposes that parties place a high value on future policy outcomes and considers the set of (constrained) surplus-maximizing equilibria. As the future surplus, or government budget, to be divided amongst the parties is stochastically determined by parties' hidden actions, a moral hazard problem arises. Due to this moral hazard problem, any constrained surplus-maximizing equilibria is necessarily inefficient. Furthermore, if efforts towards cooperation are complementary, constrained surplus-maximizing equilibria generate policy outcomes that exhibit political cycles. This result therefore provides a rationale for political business cycles in an environment where cooperative, patient parties negotiate policy. In the second chapter, *Gridlocks, Extreme Policies, and the Proximity of an Upcoming Election*, I analyze how the proximity of an upcoming election affects the path of policy proposals before that election. Policy outcomes before an election date depend on the proximity of this election date and on current and discounted expected future political power. When there is a common expectation that policy outcomes immediately after the election will generate high social surplus, phases of legislative gridlock, where agreement is infeasible, will either be nonexistent or occur immediately before the election. When the distribution of political power is highly asymmetric, implemented policies favor the party with higher power. When the distribution of power is fairly symmetric, implemented policies can favor either party and intervals of disagreement, and thus legislative gridlock, often occur more frequently.

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## 1.1 Introduction

The literature on political business cycles argues that the election cycle affects policy outcomes. This hypothesis is based on the idea that once political considerations, such as a party's desire to be in power or to implement policies that are consistent with its views, are taken into account, elections influence policy decisions. Empirical analysis supports the notion that there is a relationship between the election cycle and fluctuations in policy variables, citing evidence of cycles in government budget deficits (Alesina, Cohen, and Roubini, 1992; Shi and Svensson, 2006), tax policy (Persson and Tabellini, 2003), and economic growth (Drazen, 2000).

In representative democracies where policy decisions are made by a legislature, negotiations amongst political parties determine policy outcomes and the nature of political business cycles. When considering legislative policy-making, the policies that emerge are not only a function of the observable actions that parties take (e.g., attending legislative hearings, writing legislation, or voting on a proposed bill), but also the result of parties' actions that are unobservable or "hidden." For instance, often in legislatures such as the United States Congress, private strategy sessions amongst the members of a party occur multiple times a week. A specific example of these meetings are the party lunches amongst members of the US Senate that often occur Tuesday, Wednesday and Thursday of each week. Similarly, in the UK Parliament or the US Congress, the party leadership, such as the whip, will have private conversations with members of the party to encourage certain voting behavior or attendance at important policy debates. These private meetings amongst the members of a party often shape the party's stance on particular issues, and hence, influence the policy outcomes that result from negotiations between political parties.

I develop a dynamic model that considers how parties' hidden action choices affect policy outcomes and ultimately generate inefficient political business cycles. I consider the set of policies that maximize welfare while also being consistent with equilibrium behavior. I find that in these "constrained" surplus-maximizing equilibria, due to the hidden nature of parties' actions, political cycles can arise. This result suggests that even when parties engage in cooperation, with efforts at bipartisanship, inefficient political cycles can emerge.

The existing literature that focuses on how negotiations between political parties produce political cycles (Alesina, 1987) or policy distortions (Acemoglu, Golosov, and Tsyvinski, 2010) demonstrates that in constrained efficient equilibria, political cycles or distortions are largely insignificant if parties' actions are observable and parties place a high value on future policy outcomes (i.e., parties have high discount factors). My paper therefore complements this literature by showing that, in an environment where parties take hidden actions that influence policy outcomes, even when parties are arbitrarily patient and coordinate on a constrained surplus-maximizing equilibrium, inefficient political cycles can emerge. Thus, with patient parties, political cycles are not only consistent with the notion of myopic political parties that choose policies that are only optimal from a short-run perspective. Indeed, political cycles can even be generated by strategies that aim to maximize expected welfare.

I model government policy-making as an infinite-horizon game played by two political parties. Each period, one party is "in power," where the evolution of political power is taken as given and political power may change hands every other period. At the beginning of each period, the party in power determines how the government budget should be allocated between two public projects, where each party has a preferred project and receives no payoff from spending on the other's preferred project. In the same period, after this allocation decision has been made, parties simultaneously choose an action ("investment effort") that stochastically affects economic growth, where higher growth means, in expectation, a larger government budget in the future. This action choice has two key features. One, each party's choice is unobservable to the other party. Two, when making this choice, a party faces a tradeoff between securing itself a high benefit today or generating a large government budget tomorrow. Given any pair of investment efforts, there is a common expectation amongst parties regarding the expected size of tomorrow's budget. The identity of the party in power, the size of the government budget, and the allocation of that budget are all commonly known.

I consider political business cycles in the expected size of the government budget. In any surplus-maximizing policy, parties efforts towards cooperation are time-invariant. This implies that, in expectation, the size of the government budget will be identical across periods. Hence, in any surplus-maximizing policy, there are no cyclical fluctuations in the size of the budget or in parties' investment efforts. Political cycles are therefore necessarily inefficient in the model.

Inefficient political cycles surface in constrained surplus-maximizing equilibria as the unobservable effort choice creates a moral hazard problem (the game is one of *imperfect public monitoring*). As parties' efforts today stochastically determine the size of the budget tomorrow, if a party observes a small government budget at the beginning of the period, it cannot be sure if this is due to a bad shock to the economy or to the other party focusing its efforts not on future growth, but on its own interests.[1] The nature of cooperation, whether in the form of parties sacrificing current partisan objectives for the sake of budget growth or through the party in power implementing more equitable divisions of the budget, is affected by the severity of this moral hazard problem. This is best illustrated by considering a variant of the model where the investment effort is observable. If parties are sufficiently patient, it is possible to construct an equilibrium that sustains the surplus-maximizing policy. In this equilibrium, by simply threatening to revert to a "bad" equilibrium with low payoffs in the event that a party deviates to a non surplus-maximizing effort choice. In contrast, when effort is unobservable, such a strategy will no longer work as it is not possible to observe when deviations have occurred. Cooperation on "good" policies each period is therefore more difficult to sustain and inefficient policy outcomes, and hence political cycles, can arise.

The main result of the paper establishes that, if parties' efforts toward cooperation are complimentary, constrained surplus-maximizing equilibria display political cycles. These strategies are based on the idea that nontrivial intertemporal incentives can be used to enforce certain effort levels, where parties are "punished" or "rewarded" based on whatever information is publicly available about parties' past effort choices. In my model, this implies that the future payoffs (the punishments and rewards) that parties receive must be conditioned on the size of the government budget that is realized at the beginning of a period. Consider a constrained surplus-maximizing equilibrium where both parties must choose to exert high effort in order

[1]Note that if the pair of investment choices today determined, with probability one, the size of the budget tomorrow, each party would be able to figure out exactly what the effort choice of the other party was, and there would be no moral hazard problem.

to maximize aggregate surplus. In order to provide parties with an incentive to put aside partisan differences and choose high effort levels, the equilibrium must use strategies where "bad", or low, realizations of the budget trigger a reversion to a punishment phase where both parties receive a low payoff. In this punishment phase, the size of the government budget is, in expectation, inefficiently low. Furthermore, the budget can exhibit high frequency cyclical fluctuations where the budget is, in expectation, smaller immediately after an election, compared to the budget in the middle of the term. Hence, it is possible for constrained surplus-maximizing equilibria to exhibit political cycles.

I now discuss some features of my approach. In the model, political parties cannot make any binding commitments to either future allocations or investment efforts. Hence, the only constraint on policies at any date is that they are consistent with parties acting rationally given the current state of the environment (i.e., the date, which party is in power, and the size of the budget) and any information about past play. Regarding the notion of a political party, though parties disagree on how to allocate the budget, they share a common view regarding what generates budget growth. If one considers a party as a group from the same geographically-defined district, the model is consistent with the idea that parties compete to secure funding for local public goods (e.g., local infrastructure and parks), but agree on what types of investments stimulate budget growth. Alternatively, one can consider parties with ideological differences that compete to secure funding for "pet projects" (e.g., farm subsidies, museums, local public goods), and while they may disagree in general on what investments stimulate growth, there is agreement on a subset of investment policies (e.g., tax reform and education) that foster growth.

Two features of the investment effort choice deserve further attention. First, the assumption concerning the tradeoff parties face (i.e., securing immediate benefits today or increasing the expected size of future budgets) captures the idea that a party can focus attention on legislation that may either favor the party directly, through an immediate payoff, or indirectly, through higher economic growth, and thus, a larger expected future budget. For instance, a party can focus attention on gathering support for legislation that is loaded with pork-barrel spending or on legislation that invests in national infrastructure, where this investment increases productivity, and thus the pool of taxable income. Likewise, a party can work towards passing legislation that includes tax breaks and subsidies for the group it represents, or on tax code reform that generates higher government revenues directly, or indirectly by increasing productivity, and hence increasing the national tax base. Second, the assumption that each party's effort choice is unobservable to the other party is consistent with the ideas discussed earlier. Policy outcomes, and specifically budget growth, are often influenced by the unobservable actions that parties take during the policy-making process, whether during the whip process, in small committee meetings or in partisan strategy sessions. If parties are unwilling to engage in the bipartisan cooperation that is required for "good" policies, then wasteful programs will persist and budget-growing policies, such as a tax code free of loopholes and credits, will fail to be implemented.

The remainder of the paper is structured as follows. Section 2 reviews the related literature in greater detail. The model is introduced in Section 3. Section 4 considers surplus-maximizing policies and constrained surplus-maximizing equilibria under perfect monitoring. Section 5 considers constrained surplus-maximizing equilibria under imperfect public monitoring. Finally, Section 6 concludes. The Appendix contains omitted proofs.

### 1.2 Related Literature

Broadly speaking, there have been two types of theories that aim to rationalize political business cycles: a theory that focuses on politicians' "opportunistic manipulation" of voters and a "partisan" theory that considers how differences in political parties' policy preferences produce cycles. I now review each strand of literature in turn. Before doing so, it is important to note that in all of the papers mentioned below, Pareto efficient policies do not exhibit cyclical dynamics; hence, if the political economy friction that is introduced causes political cycles, these cycles are inefficient.

Many models that contribute to the theoretical research on political business cycles are motivated by the robust empirical observation that economic conditions before an election heavily influence voters' decisions.[2] Politicians will then, to the best of their ability, influence the economy or other policy variables in an effort to be reelected. This is the basic observation that motivates the literature on opportunistic manipulation.

One of the first models of opportunistic political cycles, developed by Nordhaus (1975), considers an economy that is represented by a downward-sloping Phillips curve with (homogenous) voters with (irrational) adaptive expectations. Given that voters prefer low inflation and low unemployment, the office-seeking politician can secure reelection by depressing the economy for most of her term until right before the election, where the economy is stimulated with expansionary monetary policy. The economy then follows a cyclical pattern with expansionary policy right before the election and contractionary policy right after the election.

In an effort to model opportunistic political cycles with fully rational voters, Rogoff (1990), along with much of the subsequent literature focusing on opportunistic manipulation, considers a model based on an informational asymmetry between voters and the politician. More specifically, Rogoff supposes that politicians differ in their competence, where highly competent politicians can provide more public goods at a lower level of taxes. Moreover, an information structure is assumed where (homogenous) voters are uninformed about one element of fiscal policy but can perfectly monitor the politician with a one-period lag. In the first half of the term policy is efficient, while in the second half of the term a competent politician sets taxes too low and spending too high in order to communicate her ability. Hence, a political business cycle may be generated due to a politician attempting to signal her private information.

Other papers of opportunistic manipulation based on signaling include Shi and Svensson (2006), Martinez (2009), and Drazen and Eslava (2006). Shi and Svensson (2006) consider a model similar to Rogoff's, and find that the size of a pre-election spending boom, and hence the political cycle, depends positively on the portion of "uninformed" voters and the ego-rent collected by the politician.[3] While Rogoff and Shi and Svensson assume an information structure where voters are only uninformed at the end of the term—thus limiting the role

[2] See, for instance, Drazen (2000) and the references therein.

[3] They are ultimately concerned with an empirical study on how political cycles differ across developed and developing countries and use the model to provide a theoretical explanation for some of their findings.

of signaling to this time period—Martinez (2009) considers a political-agency model where voters are imperfectly informed at all times and is able to show that politicians may have a greater incentive to generate "good" economic conditions near the end of the term. In contrast to these papers, Drazen and Eslava (2006) consider a model of voter manipulation with heterogeneous voters that can view all aspects of fiscal policy and politicians that are all equally "competent." Voters are differentiated based on the types of government spending they prefer and a politician has private information regarding her preferences over spending. They show that there exists an equilibrium where there is a political cycle in the composition of the budget, with higher targeted spending for the group of voters that is more likely to swing the election.

Now, I consider papers on the partisan theory, where these models focus on how negotiations amongst political parties with different policy preferences generate political cycles. Hibbs (1977) presents one of the first partisan models. As in Nordhaus's model, the economy is represented by a downward-sloping Phillips curve and voters do not have rational expectations.[4] Two political parties have different preferences over inflation and unemployment, and due to irrational expectations, the party in power can cause inflationary surprises. This implies that cycles then arise in the economy, with fluctuations coming due to changes in which party is in power.

Towards developing a partisan theory with fully rational agents, Alesina (1987) develops a partisan model that focuses on cycles in macroeconomic outcomes, but allows for voters to have rational expectations. In such a model, only unanticipated monetary policy can affect real variables. Policy-making is modeled as a dynamic game of perfect information, where the party currently in power chooses policy. The uncertainty caused by the election, coupled with an assumption on the rigidity of nominal wage contracts, implies that there will be surprise inflation at the beginning of a party's term in power. There will then be the following cycle: in the first part of the term, under the left-wing party policies will be relatively expansionary while under the right-wing party policies will be relatively contractionary; in the second part

[4]Like Nordhaus's model, it is assumed that voters have adaptive expectations.

of the term parties implement the same policy.

Closely related to my paper are the recent papers by Dixit, Grossman, and Gul (2000) and Acemoglu, Golosov, and Tsyvinski (2010). Though not specifically models of political business cycles, these papers consider the closely related question of how fluctuations in political power may distort policy outcomes. Dixit, Grossman, and Gul consider a two-player game with perfect information, where political power may change every period and changes in power follow an exogenously given Markov process (as in my paper) with multiple degrees of political power. As in my paper, political power takes the form of being able to allocate some surplus each period, where, unlike in my paper, this size of this surplus is exogenously determined in each period. Acemoglu, Golosov, and Tsyvinski (2010) consider a $n$-player game with perfect information, where, as in Dixit, Grossman, and Gul (2000), the party in power determines the allocation of some surplus, political power may change every period and changes in power follow an exogenously given Markov process (as in my paper). Besides considering a game with more than two political parties, the main departure from Dixit, Grossman, and Gul is the introduction of a production economy, where parties can exert productive efforts in order to increase the size of the government budget. Hence, just as in my paper, the size of the surplus that is allocated by the party in power is determined endogenously. Both papers study the set of constrained efficient allocations and find that the sequence of constrained efficient allocations is such that the current policy depends not only on which party is in power but on how that party arrived to power. Dixit, Grossman and Gul also consider how voting rules, such as majority or supermajority rule, affect constrained efficient allocations, while Acemoglu, Golosov, and Tsyvinski consider how the frequency of power switches affect these allocations.

A fundamental difference between my paper and Alesina (1987), Dixit, Grossman, and Gul (2000), and Acemoglu, Golosov, and Tsyvinski (2010) is that I consider a game with imperfect public monitoring. Drawing a direct comparison to Acemoglu, Golosov, and Tsyvinski (2010), when considering each party's productive effort in each period, they assume this is public information, while I assume that a party's effort is unobservable to the other party. This assumption changes the nature of constrained surplus-maximizing equilibria. One finding that emerges from the models considered by Alesina and Acemoglu, Golosov, and Tsyvinski, is that if parties are sufficiently patient (as I assume in my paper), in constrained surplus-maximizing equilibria, policy distortions—and hence, political cycles—will vanish. Policy distortions (and any associated political cycles) are therefore on the equilibrium path when considering inefficient equilibria (i.e., equilibria that are not constrained efficient) or constrained surplus-maximizing equilibria in a model where parties are impatient. In contrast, in my paper even when parties are patient and play a constrained surplus-maximizing equilibrium, political cycles can arise and are persistent.

### 1.3 The Model

Time is discrete and indexed by $t \in \{1, 2, \ldots\}$. Let $N = \{1, 2\}$ denotes the set of parties. At each date, exactly one party is the *party in power*. Political power potentially changes hands at the beginning of every odd period, where the evolution of political power follows an exogenously given, time invariant, irreducible Markov process. For any period $2z$, where $z \in \{1, 2, \ldots\}$, with party $k \in N$ currently in power, let $m(i \,|\, k) \in (0, 1)$ denote the probability that party $i$ is in power at the beginning of the next period.

At the beginning of each period $t$, there is income $y_t \in Y = \{0, y\}$, where $y > 0$. I assume there is zero income at the beginning of period 1. In period $t$, the party in power determines the allocation of the income $y_t$, denoted $c_t$, where $c_t \in C(y_t) = \{c_t^1, c_t^2 \in [0, y_t]^2 : c_t^1 + c_t^2 \leq y_t\}$. After the income $y_t$ has been allocated, parties simultaneously choose actions, where party $i \in N$ chooses an action $a^i \in A^i = [0, 1]$.

The action profile $a \in A = A^1 \times A^2$ affects $r$ and $\pi$, where $r$ is a payoff vector and $\pi$ is the probability distribution that stochastically determines the income level at the beginning of the next period. Specifically, given the action profile $a \in A$, party $i$ gets a payoff of $r^i(a^i)$ and the probability of obtaining the high income realization of $y$ is $\pi(a)$, while the probability of obtaining the low income realization of 0 is $1 - \pi(a)$.

The flow payoff for party $i$ given the consumption allocation $c$ and the action profile $a$ is $c^i + r^i(a^i)$. Parties discount payoffs using the common discount factor $\delta \in (0, 1)$.

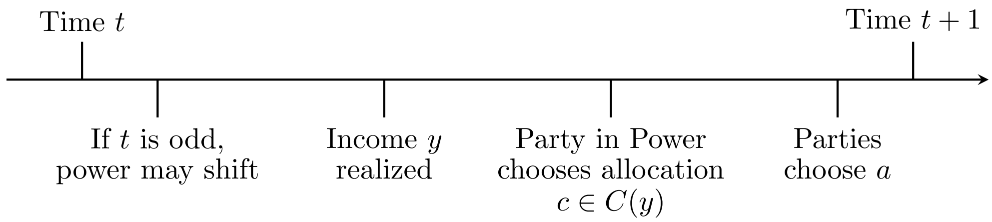


Figure 1.1: **Timeline of Period t**

The following assumptions on the payoff function $r$ and the probability $\pi$ are made throughout the entire paper.

**Assumption 1** Given $i \in N$, $r^i(a^i) = -a^i$ for any $a^i \in A^i$, and $\pi(\cdot, a^{-i})$ is increasing in $a^i$ for any $a^{-i} \in A^{-i}$.

Thus, higher actions give each party $i$ a lower immediate payoff $r$, but generate higher expected income in the future. Each party then faces a tradeoff when choosing an action: lower actions generate a higher payoff today, while higher actions generate a higher expected return tomorrow.

**Assumption 2** $\pi$ is concave, differentiable and satisfies the following Inada conditions

$$\lim_{a^i \to 0} \frac{\partial \pi(a^1, a^2)}{\partial a^i} = \infty \quad \text{and} \quad \lim_{a^i \to 1} \frac{\partial \pi(a^1, a^2)}{\partial a^i} = 0 \qquad \forall a^j \in [0,1]$$

Assumption 2 implies that there are decreasing marginal returns to efforts at cooperation. The differentiability of $\pi$ and the Inada conditions ensure that a simple set of first order conditions can be used when characterizing the surplus-maximizing policies and the set of surplus-maximizing equilibria.

### 1.3.1 An Imperfect Monitoring Game

In what follows, I consider a stochastic game of imperfect public monitoring (see, for instance, Fudenberg and Yamamoto, 2011, and Hörner et al, 2011). Specifically, for any period $t$, while the party in power $k_t \in N$, the income realization $y_t \in Y$, and the allocation decision $c \in \mathbb{R}^2$ are public information, the action $a^i \in A^i$ taken in any period $t$ is private information for party $i \in N$. The upcoming analysis also applies to the case where the action profile $a \in A$ for any period $t$ is public information, with comparisons to this case being made occasionally below.

The public history at the beginning of period $t$ is

$$h_t = (k_1, y_1, (c_1^1, c_1^2), \ldots, k_{t-1}, y_{t-1}, (c_{t-1}^1, c_{t-1}^2), k_t, y_t).$$

The set of public histories at the beginning of period $t$ is then $H_t = (N \times Y \times \mathbb{R}^2)^{t-1} \times (N \times Y)$ and $H = \cup_{t \geq 1} H_t$ denotes the set of all public histories. The private history for party $i$ at the beginning in period $t$ is a sequence

$$h_t^i = (k_1, y_1, (c_1^1, c_1^2), a_1^i, \ldots, k_{t-1}, y_{t-1}, (c_{t-1}^1, c_{t-1}^2), a_{t-1}^i, k_t, y_t).$$

The set of private histories at the beginning of period $t$ is then $H_t^i = H_t \bigcup (A^i)^{t-1}$ and $H^i = \cup_{t \geq 1} H_t^i$ denotes the set of all private histories.

A (behavior) strategy for party $i$ is given by $\sigma^i = \{\gamma^i, \omega^i\}$, where $\gamma^i : H^i \to \cup_{y \in Y} \Delta(C(y))$ and $\omega^i : H^i \to A^i$. As parties utility is linear in consumption and $\pi$ is concave, it is without loss of generality to limit attention to the set of pure actions $A^i$ for each $i \in N$. Parties seek to maximize the average discounted sum of their expected flow payoffs, where given the initial party in power $k_1$, initial income $y_1$, and strategy profile $\sigma$, this payoff is

$$\sum_{t=1}^{\infty} (1-\delta)\delta^{t-1} \mathbb{E}_{k_1, y_1, \sigma}[-a_t^i + c_t^i].$$

I only consider a special class of equilibria. A strategy $\sigma^i$ for party $i$ is *public* if it only depends on the public histories $H$. A *perfect public* equilibrium (henceforth, PPE) is a profile of public strategies such that, for any period $t$ and public history $h_t$, $\sigma|_{h_t}$ (the continuation strategy induced by $h_t$) is a Nash equilibrium from that period on. Note that the set of PPE is a subset of the set of sequential equilibria.

## 1.4 Surplus-Maximizing Policies and Perfect Information

Before considering the (constrained) surplus-maximizing PPE of the imperfect monitoring game, it is useful to consider both the surplus-maximizing polices and the (constrained) surplus-maximizing equilibrium with perfect monitoring.

### 1.4.1 Surplus-Maximizing Policies

Given any state $s' \in S$, the set of surplus-maximizing policies are characterized by considering the following problem

$$\max_{\{a_t, c_t\}_{t=1}^{\infty}} \quad \mathbb{E}_{k_1, y_1} \sum_{t=1}^{\infty} (1-\delta)\delta^{t-1} \Big[ \Big( - a_t^1 + c_t^1 \Big) + \Big( - a_t^2 + c_t^2 \Big) \Big],$$

where $a_t \in A$ and $c_t \in C(y_t)$. As this problem is stationary and parties' utility from income is linear, the action profile in any surplus-maximizing policy is time-invariant, i.e., $a_t^* = a_{t'}^* \equiv a^*$ for any periods $t$ and $t'$. The action profile $a^*$ is found by considering the action profile that maximizes expected surplus over two periods. Formally, $a^*$ is the solution the problem

$$\max_{a \in [0,1]^2} \; -a^1 - a^2 + \delta \pi(a) y.$$

Given that $\pi$ is concave and differentiable, as $\pi$ is assumed to satisfy the Inada conditions outlined in Assumption 2, the solution to this problem is characterized by the optimality condition

$$-1 + \delta \frac{\partial \pi(a)}{\partial a^i} y = 0 \quad \forall i \in N.$$

Thus, in each period $t$, $a_t^i = a^{i*}$ where $a^{i*}$ satisfies the above optimality condition. As parties' utility from income is linear, in any period $t$, in the event of a high income realization, any allocation $c_t$ such that $c_t^1 + c_t^2 = y$ is consistent with a surplus-maximizing policy. Note that the government budget is identical, in expectation, entering any period in any surplus-maximizing policy. Hence, there are no political cycles in any surplus-maximizing policy.

### 1.4.2 (Constrained) Surplus-Maximizing Equilibria Under Perfect Information

The following lemma establishes that, if parties are sufficiently patient and monitoring is perfect, it is possible to construct an equilibrium that delivers the highest possible social surplus of $-a^{1*} - a^{2*} + \delta\pi(a^*)y$.

**Lemma 1.** *Suppose monitoring is perfect. There exists a $\bar{\delta} \in (0, 1)$ such that for any $\delta \geq \bar{\delta}$ there is an equilibrium that sustains a surplus-maximizing policy.*

*Proof.* See the Appendix.

If parties are sufficiently patient and monitoring is perfect, political cycles will only arise on the equilibrium path in "bad" equilibria, where parties coordinate on an inefficient equilibrium. Hence, under perfect monitoring, cycles will only arise on the equilibrium path if parties are "short-sighted" in a sense, whether it is due to parties simply being impatient or playing an inefficient equilibrium where parties myopically optimize. In the next section, I consider (constrained) surplus-maximizing equilibria under imperfect public monitoring and show that, even when parties are not short-sighted in this sense, inefficient political cycles can arise on the equilibrium path. This result therefore illustrates that even with political parties that aim to maximize welfare when negotiating policy, political cycles emerge. As argued earlier, the assumption of imperfect monitoring is consistent with the notion that the unobserved actions that parties take during the political decision-making process, such as debating policies in private meetings in order to find consensus amongst the party, often have important implications on what policies are implemented.

## 1.5 (Constrained) Surplus-Maximizing PPE Under Imperfect Monitoring

In this section, I consider (constrained) surplus-maximizing PPE and, more specifically, the equilibria when parties are sufficiently patient. As illustrated in the previous section, if monitoring is perfect and if parties are sufficiently patient, it is possible to construct an equilibrium that delivers the highest possible surplus. In contrast, with imperfect public monitoring, this is not possible. The following lemma establishes that, regardless of how patient parties are, the surplus from any (constrained) surplus-maximizing equilibrium is necessarily less than the highest possible surplus.

**Lemma 2.** *Given any $\delta \in (0,1)$, the surplus from any (constrained) surplus-maximizing PPE is strictly less than the surplus from any surplus-maximizing policy.*

*Proof.* See the Appendix.

The rest of this section focuses on the strategies that deliver the (constrained) surplus-maximizing PPE payoff. The argument used to analyze (constrained) surplus-maximizing PPE, as parties become arbitrarily patient, is as follows. First, as a benchmark, consider the set of equilibria where after the income realization at any date $t$, the expected surplus from the equilibrium is independent of the income realization $y_t$. It follows that, in these equilibria, the expected surplus from the continuation payoffs in equilibrium is time-invariant and these equilibria do not use "punishments." Next, note that, if there do exist equilibria that provide higher expected surplus than any of these equilibria without punishments, these equilibria must involve punishments where the expected surplus from the continuation payoffs falls in the event of a low income realization. Essentially, if a low income realization triggers a punishment, where parties receive low payoffs, then, as the probability of getting a high income realization is increasing in parties efforts, these equilibria will feature higher efforts initially until the (inevitable) punishment is administered. When considering the form surplus-maximizing equilibria may take, the following question then arises: is the expected surplus higher if parties choose high efforts initially, with an eventual reversion to a punishment phase with low payoffs, or is the surplus higher with (relatively) moderate efforts each period and no punishment?

The main result, Proposition 1, establishes that if parties efforts are complementary (in the sense described in Assumption 3 below), then as parties become sufficiently patient, it is always possible to construct an equilibrium with high initial efforts and a punishment phase, that improves on any "no punishment" equilibrium. Hence, surplus-maximizing equilibria require a punishment phase, and thus, as I will discuss shortly, political cycles.

I use the following assumption for some of the results stated in the rest of the paper.

**Assumption 3** Suppose $a^i > a^j$. Then $\pi(a^i, a^j) < \pi(a^i - \varepsilon, a^j + \varepsilon)$, where $\varepsilon \in (0, \min\{a^i, a^i - a^j\})$.

Assumption 3 implies that the probability $\pi$ is symmetric and that parties efforts towards cooperation are complements. The following lemma characterizes the benchmark equilibrium discussed above that does not use punishments.

**Lemma 3.** *Under Assumption 3,*

1. *There exists a $\bar{\delta} \in (0,1)$ such that for any $\delta \geq \bar{\delta}$, there is an equilibrium where, in each period $t$, each party $i \in N$ chooses an action $a_t^i$ where*

$$-1 + \delta \frac{\partial \pi(a_t)}{\partial a_t^i} \frac{y}{2} = 0;$$

2. *This equilibrium generates the highest surplus amongst all equilibria that do not use punishments.*

*Proof.* See the Appendix.

The equilibrium characterized in Lemma 3 provides a lower bound on the surplus from any (constrained) surplus-maximizing equilibrium. It is important to note that, in the equilibrium in Lemma 3, the actions chosen each period are inefficiently low. Hence, in order to provide incentives for higher, more efficient actions, a strategy must rely on punishments where the expected surplus from the continuation payoffs in the event of a high income realization is strictly greater than the surplus in the event of a low income realization. In the main result, Proposition 1, I show that, if $\delta$ is sufficiently large, then any (constrained) surplus-maximizing equilibrium requires these higher actions initially.

**Notation and Preliminaries.** The following notation and terminology is used is some of the results in the remainder of the paper. It is useful to introduce the following state space $S$. Let $T = \{t_1, t_2\}$ and $S = N \times T$, where $s \in S$ specifies the identity of the party in power and whether it is the first part of the term (date $t_1$ ) or the second part of the term (date $t_2$). Given a state $s' \in S$ at the beginning of the period, let $p(s \,|\, s')$ denote the probability that state is $s \in S$ at the beginning of the next period, where

- Given $(j, t_1) \in S$, $p(j, t_2 \,|\, j, t_1) = 1$;
- Given $(j, t_2) \in S$, $p(i, t_1 \,|\, j, t_2) = m(i \,|\, j)$ for any $j \in N$.

Define party $i$'s minmax payoff for initial state $s$, initial income 0 and discount factor $\delta$

$$\underline{v}_s^i = \min_{\sigma^{-i}} \max_{\sigma^i} \sum_{t=1}^{\infty} (1-\delta)\delta^{t-1} \mathbb{E}_{s,\sigma}[-a_t^i + c_t^i].$$

As $\delta \to 1$, $\underline{v}_s^i$ converges to party $i$'s limit-average minmax payoff with initial state $s$ and income 0. (see Mertens and Neyman, 1981). As the Markov chain over $S$ is irreducible, as $\delta \to 1$, $\underline{v}_s^i$ is independent of the initial state $s$ (see, for instance, Dutta 1995).

Before stating Proposition 1, the following lemma characterizes the "worst" equilibrium for each party $i \in N$, where in the worst equilibrium for party $i$, party $i$ receives its minmax payoff. These equilibria (or equilibrium if there is one equilibrium that gives each party its minmax payoff) are used when constructing an equilibrium that gives higher surplus than any "no punishment" equilibrium.

**Lemma 4.** *Given an initial state $s \in S$, there exists a $\bar{\delta} \in (0, 1)$ such that for any $\delta \geq \bar{\delta}$ there is an equilibrium with the payoff vector $\hat{v}_s$ such that $\hat{v}_s^i = \underline{v}_s^i$*

*Proof.* See the Appendix.

The equilibrium constructed in Lemma 4 features a temporary phase of mutual minmaxing, followed by a return to an equilibrium that gives each party $i$ a payoff strictly higher then the payoff $\hat{v}_s^i$. The behavior during the phase of mutual minmaxing can be described as follows. The party in power takes all the income at the beginning of each period. In the first part of the term, the party out of power $j$ chooses the lowest possible action $a^j = 0$, while the party in power $i$ chooses the action that solves

$$\max_{a^i \in [0,1]} -a^i + \delta\pi(a^i, 0)y.$$

In the second part of the term, both parties chooses the lowest possible action 0. Hence, there are high frequency cyclical fluctuations during the phase of mutual minmaxing, as the size of the government budget is, in expectation, smaller at the beginning of the first part of the term than at the beginning of the second part of the term.

The statement of Proposition 1 relies on the following strategy. Consider *Strategy SM*:

- There are the following eight phases: Phases $(A_s)$ and Phases $(B_s)$. Transitions between phases may only occur after the party-in-power's allocation decision in a particular period. Given an initial state $s \in S$ at date 1, begin in Phase $A_s$.

- Given an initial state $s \in S$, in Phase $A_s$, the action profile $a^*$ is played. If in Phase $A_s$ and there is a positive income realization, each party receives $y/2$.

- If in Phase $A_{s'}$ and there is a high income realization, if the next state is $s$, move to Phase $A_s$. If in Phase $A_{s'}$ and there is a low income realization, if the next state is $s$, with probability $\rho_s \in (0, 1)$ move to Phase $A_s$ and with probability $1 - \rho_s$ transition to Phase $B_s$.

- If currently in Phase $B_s$ and the previous phase was Phase $A_{s'}$, with probability $\xi_s$ revert to the worst equilibrium for party 1 and with probability $1 - \xi_s$ revert to the worst equilibrium for party 2.

- If currently in Phase $A_s$ and there is a positive income realization, if the consumption allocation $c_s \neq (y/2, y/2)$, revert to the worst equilibrium for the party currently in power.

**Proposition 1.** *Under Assumption 3, there exists a* $\bar{\delta} \in (0,1)$ *such that for any* $\delta \geq \bar{\delta}$*, Strategy SM is an equilibrium that generates higher expected surplus than any "no punishment" equilibrium.*

*Proof.* See the Appendix.

The intuition for the proof of Proposition 1 is as follows. The strategy featured in Proposition 1 (Strategy SM) begins with parties choosing an action profile that yields a higher social surplus than the action profile in the benchmark equilibrium constructed in Lemma 3. If $\delta$ is sufficiently large, then the transition probabilities ($\rho_s$) required for the action profile $a^*$ to be incentive compatible get close to 1. Thus, the probability of transitioning to the punishment phase becomes sufficiently small so that an equilibrium generates higher surplus than the equilibrium constructed in Lemma 3 that does not rely on punishments.

It is important to note that without Assumption 3, the problem of characterizing (constrained) surplus-maximizing equilibria is less tractable. Essentially, if parties' efforts towards cooperation are not complementary and symmetric, it may be too costly to provide incentives for both parties to choose high actions initially. Indeed, this may not be consistent with a (constrained) surplus-maximizing equilibrium. Characterizing these equilibria then involves comparing different "punishment" and "no punishment" equilibria, where it is not straightforward to determine which yield a greater expected surplus.

Proposition 1 illustrates that if efforts towards cooperation are complementary and symmetric, then surplus is maximized by a strategy where both parties are choosing high effort initially, with an eventual (stochastic) reversion to a punishment phase where a bad equilibrium with low payoffs is played. Therefore, when joint efforts at budget growth are sufficiently productive (as is the case with complementary efforts), then surplus-maximizing equilibria will involve fluctuations with periods of time where the (expected) size of the budget is high and periods where the (expected) size of the budget is low. Furthermore, during a stretch of bad policy where parties are minmaxing each other, recalling the worst equilibrium constructed in Lemma 4, the equilibrium can display high frequency political cycles. In these political cycles the (expected) size of the government budget is smaller at the beginning of the first part of the term as compared to at the beginning of the second part of the term.

### 1.6 Conclusion

This paper analyzes a model of government policy-making where political business cycles arise due to parties' hidden actions, fluctuations in political power and differences in parties' views over the optimal allocation of the government budget. I suppose political parties have a high discount factor and consider the set of constrained surplus-maximizing equilibria, where the assumption that each party's investment effort choice is unobservable generates a moral hazard problem that has key implications on how close the payoffs from these equilibria are to the Pareto frontier. Equilibrium payoffs are bounded away from the Pareto frontier. When PPE payoffs are necessarily inefficient, if efforts toward bipartisanship are complementary, constrained surplus-maximizing equilibria have cyclical dynamics. Hence, there are inefficient political cycles on the equilibrium path.

My paper illustrates that inefficient political cycles can be consistent with constrained surplus-maximizing equilibria. Hence, political cycles will not only arise with impatient political parties, with parties that coordinate on a "bad" equilibrium, or with short-sighted parties that only choose policies that are optimal from a short-run perspective. Essentially, political cycles, through the association with a punishment phase where both parties receive low payoffs, may be consistent with providing parties nontrivial intertemporal incentives that are instrumental in sustaining political compromise.

## 1.7 Appendix

### 1.7.1 Characterization of Minmax payoffs

The strategy profile that party $j$ uses to minimize the other party $i$'s payoff is described as follows.

- When out of power, $j$ chooses $a^j = 0$.
- When in power, in the first part of the term, party $j$ chooses any action $a^j \in [0, 1]$ and takes all the income that is realized at the beginning of the period.
- When in power, in the second part of the term, party $j$ chooses $a^j = 0$.

First, when in power, party $j$ minimizes party $i$'s payoff by giving $j$ none of any income that is realized at the beginning of the period. In regards to party $j$'s action choice in the first part of the term, as $j$ will be in power in the next period, and thus can ensure that $i$ will receive an expected payoff of zero (regardless of what action profile is played in the first part of the term), any action $a^j \in [0, 1]$ is consistent with $j$ minmaxing $i$. When $j$ is in power and it is the second part of the term, or if $j$ is not in power, $j$ minimizes $i$'s payoff by choosing the lowest possible effort $a^j = 0$.

In response to this strategy, party $i$ will find the following optimal

- When out of power, in the first part of the term, $i$ chooses $a^i = 0$.
- When out of power, in the second part of the term, $i$ solves

$$\max_{a^i \in [0,1]} \; -a^i + \delta\pi(0, a^i)m(i \,|\, j)y.$$

- When in power, in the first part of term, take all the realized income and solve

$$\max_{a^i \in [0,1]} \; -a^i + \delta\pi(0, a^i)y.$$

- When in power, in the second part of term, take all the realized income and solve

$$\max_{a^i \in [0,1]} \; -a^i + \delta \pi(0, a^i) m(i \mid i) y.$$

Given the strategy $j$ uses to minimize $i$'s payoff, when out of power, in the first part of the term, $i$ knows that it will receive non of the income in the next period; hence, $a^i = 0$ is optimal. When out of power, in the second part of the term, considering that $a^j = 0$ and the probability of gaining power (and hence getting all the income) is $m(j \mid i)$, party $i$ solves the problem outlined above. Similarly, when in power, in the second part of the term, considering that $a^j = 0$ and the probability of gaining power (and hence getting all the income) is $m(i \mid i)$, party $i$ solves the problem outlined above. Finally, when in power, in the first part of the term, considering that $a^j = 0$ and the probability of gaining power (and hence getting all the income) is 1, party $i$ chooses the action outlined above. This strategy will yield the minmax payoff vector.

### 1.7.2 Omitted Proofs

**Proof of Lemma 1**

Let $a^*$ denote the action profile that maximizes social surplus. Consider the following strategy:

- Begin in Phase $A$.
- If in Phase $A$, the action profile $a^*$ is played and each party receives $y/2$ in the event of a positive income realization.
- If $a^i_t \neq a^{i*}$ or if $c_t \neq (y/2, y/2)$, with the party in power $i$, revert to the worst equilibrium for party $i$; otherwise, remain in Phase $A$.

Define the payoff vector $v$ as follows

$$v^{i*}_{s'} = -(1-\delta)a^{i*} + \delta \sum_{s \in S} p(s \mid s') \Big[\pi(a^*)\Big((1-\delta)\frac{y}{2} + v^{i*}_s\Big) + (1 - \pi(a^*)v^{i*}_s\Big].$$

In order for the action $a^{i*}$ to be consistent with equilibrium, given the state $s' \in S$, the following constraint must be satisfied

$$v_{s'}^{i*} \geq -(1-\delta)a^i + \delta \sum_{s \in S} p(s \,|\, s') \Big[\pi(a^i, a^{j*})\Big((1-\delta)\tilde{c}_s^i + \underline{v}_s^i\Big) + (1-\pi(a^*)\underline{v}_s^i\Big]$$

for each $i \in N$, any $a^i \neq a^{i*}$ and $\tilde{c}_s^i = y$ if $i$ is the party in power and $\tilde{c}_s^i = 0$ is not the party in power. Also, the allocation constraint for the party in power $i$

$$(1-\delta)\frac{y}{2} + v_s^{i*} \geq (1-\delta)y + \underline{v}_s^i.$$

As surplus under this strategy is maximized and, as $\delta \to 1$, $\underline{v}_s^1 \to \underline{v}_s^2$ for any $s \in S$, if $\delta$ is large enough, both the incentive constraint for actions and the party-in-power's allocation constrained are satisfied in each period $t$. Hence, there exists an equilibrium with payoff vector $v_s^*$ in state $s$ where $v_s^{1*} + v_s^{2*} = -a^{1*} - a^{2*} + \delta\pi(a^*)y$. □

**Proof of Lemma 2**

By contradiction. Suppose that the surplus-maximizing action profile $a^*$ can be supported in each period. Then there exists continuations $(w_s(0), w_s(y))$ and allocations $(c_s)$ such that

$$a^{i*} = \arg\max_{a^i \in [0,1]} \; -(1-\delta)a^i + \delta \sum_{s \in S} p(s \,|s') \Big[\pi(a^*)\Big((1-\delta)c_s^i + w_s^i(y)\Big) + (1-\pi(a^*))w_s^i(0)$$

This implies the following optimality conditions for $i$'s action

$$(1-\delta) = \delta \sum_{s \in S} p(s \,|s') \frac{\partial \pi(a^*)}{\partial a^i} \Big[(1-\delta)c_s^i + w_s^i(y) - w_s^i(0)\Big].$$

Noting that

$$\delta \frac{\partial \pi(a^*)}{\partial a^i} y = 1$$

and summing the two optimality conditions, we have

$$2(1-\delta) = \frac{1}{y}\sum_{s\in S} p(s\,|s')\Big[(1-\delta)y + w_s^1(y) + w_s^2(y) - (w_s^1(0) + w_s^2(0)\Big].$$

If a PPE existed that gave the same surplus as the surplus-maximizing policy, then $w_s^1(y) + w_s^2(y) = w_s^1(0) + w_s^2(0)$. Considering the condition immediately above, this implies

$$2(1-\delta) = (1-\delta)$$

giving the contradiction. □

**Proof of Lemma 3**

**Part 1.** Given the initial state $s' \in S$, if party $i$ finds it optimal to choose the $a^i$ in the statement of the Lemma, then

$$-(1-\delta) + \delta\frac{\partial\pi(a)}{\partial a^i}\sum_{s\in S} p(s\,|\,s')\Big[(1-\delta)c_s^i + w_s^i(y) - w_s^i(0)\Big] = 0$$

where, as $\frac{\partial\pi(a)}{\partial a^i} = \frac{2}{y\delta}$,

$$\sum_{s\in S} p(s\,|\,s')\Big[(1-\delta)c_s^i + w_s^i(y) - w_s^i(0)\Big] = (1-\delta)\frac{y}{2}$$

for each $i \in N$.

Note that the payoff vector $v_{s'}$ satisfies the following for each $i \in N$

$$v_{s'}^i = -(1-\delta)a^i + \delta\sum_{s\in S} p(s\,|\,s')\pi(a)\Big[(1-\delta)c_s^i + w_s^i(y) - w_s^i(0)\Big] + \delta\sum_{s\in S} p(s\,|\,s')w_s^i(0);$$

hence, using the equality immediately above, we have

$$v_{s'}^i = (1-\delta)\Big[-a^i + \pi(a)\frac{y}{2}\Big] + \delta\sum_{s\in S} p(s\,|\,s')w_s^i(0).$$

As each period $a^1 = a^2$ and each party $i$ receives the flow payoff $-a^i + \pi(a)\frac{y}{2}$, it must be the case that the equilibrium payoff, regardless of the income realization at the beginning of the period, is

$$v^i_{s'} = -a^i + \delta\pi(a)\frac{y}{2} \quad \forall s' \in S, i \in N.$$

This implies that $c^i_s = y/2$ for any $s \in S, i \in N$.

In order to guarantee that this is an equilibrium, the allocation constraint for the party in power $i$ needs to be checked. With $v^i_s = v^i$ for each $s \in S$, given an initial state $s \in S$, this allocation constraint is

$$(1-\delta)\frac{y}{2} + v^i \geq (1-\delta)y + \underline{v}^i_s.$$

First, note that, under Assumption 3, the surplus from the strategy outlined in Lemma 3 yields an expected surplus that is strictly greater than the surplus $\underline{v}^1_s + \underline{v}^2_s$ for any $s \in S$. Second, $v^1 = v^2$ and $\underline{v}^1_s \to \underline{v}^2_s$ for any $s \in S$ as $\delta \to 1$. These two pieces imply that if $\delta$ is large enough then the allocation constraint is satisfied for any Markov process that satisfies the assumptions outlined in The Model presented in Section 3. Hence, the strategy proposed in Lemma 3 is an equilibrium for $\delta$ large enough.

**Part 2.** It remains to show that this equilibrium generates the highest expected surplus amongst all equilibria that do not use punishments. The argument makes use of the following lemma.

**Lemma 5.** *Under Assumption 3, there exists a $\bar{\delta} \in (0,1)$ such that for any $\delta \geq \bar{\delta}$, in any constrained surplus-maximizing equilibrium both parties choose the same action in the first period.*

*Proof.* By contradiction. Fix a constrained surplus-maximizing equilibrium $\sigma$. Given the initial state $s'$, suppose $a$ is the action profile in the current period, where $a^1 \neq a^2$, $c_s$ is the consumption allocation in the next period if there is a positive income realization and the next state is $s \in S$, and $\{w_s(0), w_s(y)\}$ are the continuation values if the next state is $s \in S$. It is important to note that for any constrained surplus-maximizing equilibrium where one party receives all the income in the event of a positive income realization, there is an alternative equilibrium that delivers the same payoff and gives both parties positive consumption. With this in mind, suppose $c_s^i \in (0, y)$ for each $i \in N$.

Without loss of generality, suppose $a^1 > a^2$. Consider the strategy $\hat{\sigma}$, with the action profile $\hat{a}$, with $\hat{a}^1 = a^1 - \varepsilon$ and $\hat{a}^2 = a^2 + \varepsilon$, where $\varepsilon > 0$ is very small. The consumption profile $(\hat{c}_s)$ will be defined below and the continuations $\{\hat{w}_s(0), \hat{w}_s(y)\}$ are such that $\hat{w}_s(0) = w_s(0)$ and $\hat{w}_s(y) = w_s(y)$ for each state $s \in S$.

Given the actions $(\hat{a}^1, \hat{a}^2)$, for each $s$, I now show that there exists a consumption allocation $\hat{c}_s$ such that

$$-\hat{a}^i + \delta\pi(\hat{a})\hat{c}_s^i \geq -a^i + \delta\pi(a)c_s^i$$

for each $i \in N$. First, there exists a $\bar{\varepsilon} > 0$ such that for any $\varepsilon \in (0, \bar{\varepsilon})$, there exists a $\hat{c}_s^1 \in (0, y)$ where

$$-a^1 + \delta\pi(a)c_s^1 = -\hat{a}^1 + \delta\pi(\hat{a})\hat{c}_s^1.$$

Choose $\varepsilon \in (0, \bar{\varepsilon})$ and such a $\hat{c}_s^1$. Noting that

$$-\hat{a}^2 + \delta\pi(\hat{a})\hat{c}_s^2 = -\hat{a}^2 + \delta\pi(\hat{a})(y - \hat{c}_s^1)$$

we have

$$\begin{aligned}
-\hat{a}^2 + \delta\pi(\hat{a})\hat{c}_s^2 &= -\hat{a}^2 + \delta\pi(\hat{a})y + \varepsilon - \delta\pi(a)c_s^1 \\
&> -a^2 + \delta\pi(a)(y - c_s^1) \quad \text{(by Assumption 3)} \\
&= -a^2 + \delta\pi(a)c_s^2,
\end{aligned}$$

and we have established, under the action profile $\hat{a}$, there is a Pareto improvement. It remains to show that the consumption allocation $\hat{c}_s$ is incentive compatible for the party in power for each $s \in S$. Noting that the allocation $c_s$ is incentive compatible and that $c_s^i \in (0, y)$ for any $i \in N$, it follows that when $\varepsilon > 0$ is sufficiently small, for $\delta$ large $\hat{c}_s$ will also be incentive compatible. Hence, if $\delta$ is sufficiently large, there exists an alternative equilibrium $\hat{\sigma}$ that Pareto dominates $\sigma$. A repeated application of this argument establishes the desired result. $\square$

Fix $\delta \in (0,1)$ such that, under Assumption 3, in any constrained surplus-maximizing equilibrium parties choose the same action profile. Towards a contradiction, suppose there exists an equilibrium that does not use punishments with an action profile $a' \neq a$, where $a$ is the action profile characterized in Part 1. The optimality condition for this action choice for party $i$ is

$$-(1-\delta) + \delta \frac{\partial \pi(a)}{\partial a^i} \sum_{s \in S} p(s \mid s') \Big[ (1-\delta) c_s^i + w_s^i(y) - w_s^i(0) \Big] = 0.$$

As $a^1 = a^2$, under Assumption 3, adding these two optimality conditions and noting that

$$\sum_{s \in S} p(s \mid s') \Big[ w_s^1(y) + w_s^2(y) \Big] = \sum_{s \in S} p(s \mid s') \Big[ w_s^1(0) + w_s^2(0) \Big],$$

we obtain

$$\delta \frac{\partial \pi(a)}{\partial a^i} (1-\delta) y = 2(1-\delta).$$

This implies that $\frac{\partial \pi(a)}{\partial a^i} = \frac{2}{y\delta}$ for each $i \in N$. Hence, parties choose the same action as outlined in Part 1 and a contradiction is obtained. Therefore, for $\delta$ sufficiently large, the equilibrium constructed in Part 1 offers the highest expected surplus amongst all equilibria that do not use punishments. $\square$

**Proof of Lemma 4**

The proof of Lemma 5 relies on the following lemma.

**Lemma 6.** *Given $\delta \in (0,1)$ and $s \in S$, there exists an equilibrium with payoff vector $v$ such that $v_s^i > \underline{v}_s^i$ for any $i \in N$.*

*Proof.* Consider the following equilibrium

- In any period, the party in power receives all the income in the event of a positive realization.

- If party $j$ is out of power and it is the first part of the term, party $i$ chooses $a^j = 0$.

- If party $i$ is in power and it is the first part of the term, party $i$ chooses an action that solves

$$\max_{a^i \in [0,1]} -a^i + \delta \pi(a^i, 0) y.$$

- If party $i$ is out of power and it is the second part of the term, party $i$ chooses an action that solves

$$\max_{a^j \in [0,1]} -a^j + \delta \pi(a^i, a^j) m(j \mid i) y.$$

- If party $i$ is out of power and it is the second part of the term, party $i$ chooses an action that solves

$$\max_{a^i \in [0,1]} -a^i + \delta \pi(a^i, a^j) m(i \mid i) y.$$

Notice that the expected payoff that each party receives from the action choice in the first part of each term is the same as the strategy that implements the minmax payoff profile. On the other hand, the expected payoff that each party receives from the action choice in the second part of the term is strictly greater than that received from the strategy that implements the minmax payoff profile. Hence, under this equilibrium each party receives an expected discounted payoff that is strictly greater than its minmax payoff. □

Now for the proof of Lemma 5. Consider the following strategy, denoted $\underline{\sigma}_i$, that generates the payoff vector $\underline{\underline{v}}_s$ with $\underline{\underline{v}}^i_s = \underline{v}^i_s$.

- There are two phases: Phases A and B. Play starts in Phase A. In Phase B, the equilibrium constructed in Lemma 6 is played.

- In Phase A, the party in power receives all income in the event of a positive realization. In the first part of the term, the party out of power $j$ chooses $a^j = 0$, while the party in power chooses an action that solves

$$\max_{a^i \in [0,1]} -a^i + \delta \pi(a^i, 0) y.$$

In the second part of the term, each party $i \in N$ chooses $a^i = 0$.

- If there is a low income realization in the second part of the term, with probability $\rho_s$ play shifts to Phase B while with probability $1 - \rho_s$ play remains in Phase A. If there is a high income realization in the second part of the term, play remains in Phase A. In the first part of the term, regardless of the income realization, play remains in Phase A.

First, let $\hat{v}_s$ denote the payoff vector for the equilibrium constructed in Lemma 6. Second, note that there always exists $(\rho_s)$, where $\rho_s \in (0,1)$ for each $s \in S$, such that the strategy $\underline{\sigma}_i$ generates exactly the payoff $\underline{v}_s^i$. Third, consider the action and allocation incentive constraints. As the party in power simply takes all the income in the event of a positive realization, the allocation constraint is always satisfied. In the first part of the term, as continuations are independent of the income realization, the prescribed action profile is incentive compatible. In the second part of the term, as each party's payoff $\hat{v}_s^i$ from the equilibrium in Lemma 6, is strictly greater than the payoff vector $\underline{\underline{v}}_s^i$ for any $i \in N$, if $\delta$ is large enough,

$$(1-\delta)c_s^i + \rho_s(\hat{v}_s^i - \underline{\underline{v}}_s^i) < 0$$

for each $i \in N$. Hence, party $i$'s is decreasing in $a^i$ and $a^i = 0$ for each $i \in N$ is incentive compatible. It follows that the proposed strategy is an equilibrium and, thus, there exists an equilibrium that delivers the minmax payoff $\underline{v}_s^i$ if $\delta$ is sufficiently large. □

### 1.7.3 Proof of Proposition 1

Let $\bar{\delta}$ be such that for any $\delta \geq \bar{\delta}$, any constrained surplus-maximizing equilibria has both parties choosing the same action in the first period. Consider *Strategy SM*, where the surplus-maximizing profile $a^*$ is played in the first period. The payoff vector $v_{s'}$ from this strategy is defined by

$$v_{s'}^i = -(1-\delta)a^{i*} + \delta\pi(a^*)\sum_{s\in S} p(s\,|\,s')\Big[(1-\delta)c_s^i + (1-\rho_s)(v_s^i - \hat{v}_s^i)\Big] + \delta\sum_{s\in S} p(s\,|\,s')\Big[\rho_s v_s^i + (1-\rho_s)\hat{v}_s^i\Big],$$

where $(\hat{v}_s)$ is the payoff vector from the equilibrium outlined in *Strategy SM* where with probability $\xi_s$ the worst equilibrium for party 1 and with probability $1-\xi_s$ the worst equilibrium for party 2. Note that there always exists a $\xi_s \in (0,1)$ such that $\hat{v}_s^1 = \hat{v}_s^2$.

The optimality conditions required for the action profile $a^*$ to be incentive compatible are

$$\delta \sum_{s \in S} p(s \,|\, s') \frac{\partial \pi(a^*)}{\partial a^i} \Big[ (1-\delta) c_s^i + (1-\rho_s)(v_s^i - \hat{v}_s^i) \Big] = (1-\delta).$$

Noting that $\frac{\partial \pi(a^*)}{\partial a^i} = \frac{1}{y\delta}$ and combining these conditions gives

$$v_{s'}^i = (1-\delta) \Big[ - a^{i*} + \pi(a^*) y \Big] + \delta \sum_{s \in S} p(s \,|\, s') \Big[ \rho_s v_s^i + (1-\rho_s) \hat{v}_s^i \Big],$$

Noting that $-a^{1*} + \pi(a^*)y = -a^{2*} + \pi(a^*)y$, parties receives the same flow payoff during the any reward phase. This, along with the result that $\hat{v}_s^1 = \hat{v}_s^2$ for any $s \in S$, implies that $v_s^1 = v_s^2$ for any $s \in S$. This implies that $c_s^i = y/2$ for any $s \in S, i \in N$. Substituting $c_s^i = y/2$ and $\frac{\partial \pi(a^*)}{\partial a^i} = \frac{1}{y\delta}$ into the optimal conditions that must hold under $a^*$, we obtain

$$\sum_{s \in S} p(s \,|\, s') \Big[ (1-\delta) \frac{y}{2} + (1-\rho_s)(v_s^i - \hat{v}_s^i) \Big] = (1-\delta) y$$

where, as $\hat{v}_s^1 = \hat{v}_s^2$ and $v_s^1 = v_s^2$ for any $s \in S$, the two optimality conditions (one for each party) reduce to one condition.

It must now established that there does exist a $\rho_s \in (0,1)$ for each $s \in S$ such that the single optimality condition for the action profile $a^*$ does hold. As $v_s$, $\hat{v}_s$, and $c_s$ are each independent of the state $s$, it is without loss of generality to consider a transition probability $\rho$ that is independent of the state.

Let $v^i \equiv v_s^i$ and $\hat{v}^i \equiv \hat{v}_s^i$ for any $s \in S$. Considering the single optimality condition above, if exists a $\rho \in (0,1)$ such that

$$(1-\rho)(v^i - \hat{v}^i) = (1-\delta) \frac{y}{2}$$

then the optimality condition above is satisfied. The payoff vector $v$ is such that

$$v^i = \frac{(1-\delta)\Big[-a^{i*} + \delta\pi(a^*)\frac{y}{2}\Big] + \delta(1-\pi(a^*))(1-\rho)\hat{v}^i}{1-\delta\pi(a^*) - \delta(1-\pi(a^*))\rho}.$$

Let $\tilde{v}$ denote the payoff vector from the best no-punishment equilibrium constructed in Lemma 4. Noting that $v^i$ is increasing in $\rho$, with $v^i_{|\rho=1} > \tilde{v}^i$ and $v^i_{|\rho=0} < \tilde{v}^i$, there exists a $\bar{\rho} \in (0,1)$ such that $v^i_{|\rho=\bar{\rho}} = \tilde{v}^i$. It then follows that

$$v^i - \hat{v}^i > v^i_{|\rho=\bar{\rho}} - \hat{v}^i = \tilde{v}^i - \hat{v}^i \quad \forall \rho \in (\bar{\rho}, 1).$$

For each $\rho \in (\bar{\rho}, 1)$, there exists a $\delta \in (0,1)$ large enough such that

$$(1-\rho)(v^i - \hat{v}^i) = (1-\delta)\frac{y}{2}.$$

This implies that there exists a $\bar{\bar{\delta}} \in (0,1)$ such that for any $\delta \geq \bar{\bar{\delta}}$, there exists a $\rho$ such that $(1-\rho)(v^i - \hat{v}^i) = (1-\delta)\frac{y}{2}$ with $\rho \in (\bar{\rho}, 1)$. Hence, if $\delta \geq \bar{\bar{\delta}}$, then the single optimality condition above for the action profile $a^*$ does hold.

Suppose $\delta \geq \bar{\bar{\delta}}$. In order to establish that this strategy is indeed an equilibrium, it must be checked that the allocation constraint for the party in power $i$ is satisfied. Given the state $s \in S$, this constraint is

$$(1-\delta)\frac{y}{2} + \rho_s v^i_s + (1-\rho_s)\hat{v}^i_s \geq (1-\delta)y + \underline{v}^i_s.$$

As $\rho_s v^i_s + (1-\rho_s)\hat{v}^i_s > \underline{v}^i_s$, there exists a $\hat{\delta} \in (0,1)$ such that for any $\delta \geq \hat{\delta}$, this allocation constraint is always satisfied.

Take $\delta \geq \max\{\bar{\delta}, \bar{\bar{\delta}}, \hat{\delta}\}$. Then, if the sufficient condition is satisfied, the strategy stated in the proposition is a surplus-maximizing equilibrium. □

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## 2 Gridlocks, Extreme Policies and the Proximity of an Upcoming Election

### 2.1 Introduction

When considering democratic governments where policy decisions are made by a legislature, such as the United States Congress or the British Parliament, it is often asserted that proposed legislation, or lack thereof, depends on the proximity and expected outcome of an upcoming election. Considering how an election ultimately affects policy outcomes before that election, a few questions come to mind. When will policy negotiations end in a stalemate, giving legislative gridlock? When will policy outcomes tend to be moderate or extreme? How does the distribution of power amongst the political parties bargaining over policy influence policy outcomes?

In this paper, I analyze these questions in a simple dynamic model of legislative policy-making. The policy outcome at any date is a function of current political power, the proximity of the next election, and the expected outcome of that election. The model offers predictions on when delay in the policy-making process, interpreted as legislative gridlock, will occur and considers when relatively moderate or extreme policy outcomes are likely. In addition to focusing on how an upcoming election and other factors in the environment influence policy outcomes at dates before an election, this paper also considers the welfare consequences associated with various policy decisions.

The policy-making process is modeled as a perfect information, infinite-horizon bargaining game. Two parties bargain over the allocation of the government budget, where a proposed division of the budget is only implemented when both parties agree. If parties have not yet agreed on an allocation of the budget, a party is randomly selected to propose an allocation. At a single exogenous date $T$, the recognition process that determines which party proposes policy at any given date may change, where this change is taken as given. In the model, the ability to propose policy is one of the key determinants of "political power," where a party with high political power is more likely to successfully implement its preferred legislation. Hence, at date $T$ there is a shift in a factor instrumental to the distribution of political power in the model. Consistent with the idea that elections bring shifts in political power, I would like to interpret the date $T$ as an election date.

The government budget can be spent on indivisible public goods or distributed back to the parties in the form of cash transfers. Parties disagree on which of two public goods projects is optimal and, by assumption, the budget is not sufficient to finance both projects. At most, one party's preferred project will therefore be implemented when there is agreement. Furthermore, it is assumed that each party would strictly prefer to implement its preferred project as opposed to receiving a transfer worth the cost of the project. Indeed, it is the surplus generated by implementing a project that fosters disagreement in the model.

As a benchmark, in any Pareto efficient outcome, there is immediate agreement and one party's preferred project is implemented. Hence, legislative gridlock is necessarily inefficient in the model.

When considering policy outcomes after the "election" occurs at the beginning of date $T$, at each date $t \geq T$ there always exists at least one policy that both parties are willing to accept. Thus, after the election date parties prefer implementing policy to legislative gridlock. Given the expected policy outcomes at date $T$, I consider bargaining dynamics before date $T$, i.e., in the build up to the election date.

Equilibrium dynamics depend critically on the distribution of political power in the model. If political power is sufficiently asymmetric, these dynamics take a simple form. There is either agreement at each date $t < T$, or there are two phases, $\{1, \ldots, \hat{t}\}$ and $\{\hat{t}+1, \ldots, T-1\}$, where there is agreement at each date $t \in \{1, \ldots, \hat{t}\}$ and disagreement at each date $t \in \{\hat{t}+1, \ldots, T-1\}$. The intuition for this result is fairly straightforward. Near the election date $T$, as political power may change hands, parties may prefer to wait until after the election to agree on a policy. As the time until the election increases, parties' impatience dampens the incentive to delay agreement until after the election, thus making a compromise feasible at date $\hat{t}$. If political power is quite asymmetric, the party with low power expects a low payoff from this agreement. This implies that this party is then easy to negotiate with at each date $t \leq \hat{t}$. Agreement is then feasible at each date $t \leq \hat{t}$.

This line of reasoning suggests that, if political power is rather asymmetric, the policy implemented at date $\hat{t}$ will heavily favor the party with high power. Indeed, this is the case.

This, in turn, preserves this party's advantage, as if the party expects to receive a high payoff tomorrow it can credibly demand its preferred policies today. It follows that, when political power is sufficiently asymmetric, the policy implemented at each date $t < \hat{t}$ will be relative "extreme" in the sense that it generates a high payoff for the party with high political power, while leaving the other party with very little.

Next, consider the length of disagreement spells before the election date $T$ when the expected policy outcome immediately after the election generates high social surplus. Under some mild restrictions on the parameters, if there are multiple spells of disagreement, and hence multiple episodes of legislative gridlock, the longest disagreement interval will occur in the dates immediately before the election. Hence, though it is possible for an interval of time with legislative gridlock to occur at any point before the election, a relatively lengthy interval of gridlock will only occur close to the election.

When the distribution of political power is fairly asymmetric, this distribution affects the path of potential policy outcomes in an unsurprising way: the party with high power will be able to successfully implement its preferred project more often; thus policies will be more extreme and represent the preferences of the party with more political power. In contrast, when bargaining power is symmetric, the path of potential policy outcomes becomes more volatile, with potentially multiple transitions between intervals of time when agreement is feasible and those when it is not.

The intuition for why the equilibrium may cycle between agreement and disagreement "regimes" is as follows. Consider a period (tomorrow) where agreement is feasible, and furthermore, agreement involves the implementation of a public goods project. In the period before (today), if parties have an equal chance of being recognized to propose an allocation of the budget and if a project generates a sufficient amount of surplus, then each party's discounted expected payoff at the beginning of the next period will be high, making agreement today infeasible. Working backwards, as parties are assumed to be impatient, there will exist a date where parties will be able to agree. The cycle will then repeat itself, as agreement tomorrow implies that agreement today is impossible.

It is important to consider exactly why there are periods where agreement on a policy proposal is not feasible. Note that if the recognition process that determines which party proposes policy at a given date remained fixed for the entire game, then agreement is feasible in each period. Given the assumption that recognition probabilities change at some fixed date $T$, agreement is also feasible in each period if there is enough room in the government budget to fund both party's preferred project, parties have the same preferences over public projects, or perfectly divisible investments into public projects can be made. Thus, disagreement regarding which public project is optimal, indivisible public goods, and the existence of a known date where political power shifts combine to make disagreement feasible in equilibrium.

Broadly speaking, this paper is part of the literature that uses dynamic models to study political decision making, and specifically, how changes in political power affect policy choices.[1] While many papers explore the impact of electoral outcomes on policy-making, elections are typically assumed to occur every period or every other period, thereby precluding the analysis of policy-making between elections, where there are multiple periods in which parties can negotiate policy. By allowing multiple policy-making periods before an election, I can analyze how bargaining outcomes depend on an upcoming election date.

This paper is also related to a strand of literature that provides explanations for disagreement and delays in agreement in bargaining situations that take place in perfect information environments. A more in depth discussion of how this paper relates to this bargaining literature is given in Section 5.

The remainder of the paper is structured as follows. The model is introduced in Section 2. Section 3 considers the continuation values in equilibrium. In Section 4, the main results regarding equilibrium dynamics are presented as well as some examples to illustrate what dynamics can arise in equilibrium. Section 5 analyzes the related bargaining literature in greater detail. Finally, Section 6 concludes. The Appendix contains omitted details that are used in the equilibrium analysis.

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[1] See for instance Alesina (1988), Baron and Ferejohn (1989), Merlo and Wilson (1995, 1998), Baron (1996), Banks and Duggan (2000), Dixit et al. (2000), Eraslan and Merlo (2002), and Battaglini and Coate (2007).

## 2.2 The Model

Time is discrete and indexed by $t \in \{1, 2, \ldots\}$. Let $N = \{1, 2\}$ denote the set of parties. Parties meet to determine the allocation of a unit of income, where this income can be spent on public goods, with any remaining income being divided between the parties.

There are two possible indivisible public goods projects, project 1 and project 2, where party 1's preferred project is 1 and party 2's preferred project is 2. The cost of implementing one project is $c > 1/2$; thus, only one project can be implemented. When parties reach an agreement there are two possibilities: one, implement a project and divide the remaining $1 - c$ of income, or two, divide the unit of income. Let $Z = \{1, X(1-c)\} \bigcup \{2, X(1-c)\} \bigcup \{0, X(1)\}$ denote the set of potential policy outcomes, where "0" means that no project is implemented, "$j$" means that project $j \in \{1, 2\}$ is implemented, and $X(y) = \{(x^1, x^2) \in [0, y]^2, x^1 + x^2 \leq y\}$, with $y \in \{1 - c, 1\}$

Each party $i \in N$ cares about the project that is implemented and how much of the remaining income she receives. Party $i$ gets a payoff of $\theta$ from project $i$ and a payoff of 0 from project $j \neq i$. Given a division $(x^1, x^2) \in X(y)$, with $y \in \{1 - c, 1\}$, party $i$ gets a payoff $x^i$. I assume that $\theta > c$ so implementing a public goods project creates a higher aggregate surplus as opposed to implementing no project and dividing the unit of income. Let $u^i(z)$ denote the flow payoff $i$ receives from the policy $z \in Z$. Payoffs are discounted by the common discount factor $\delta \in (0, 1)$.

Let $S$ denote a finite state space. Given $s \in S$, $\{p^0(s), p^1(s), p^2(s)\}$ are *recognition probabilities*, with $p^k(s) \in [0, 1]$ for each $k \in \{0, 1, 2\}$ and $p^0(s) + p^1(s) + p^2(s) = 1$. The state can only possibly transition at the very beginning of period $T$, where $T > 1$ is exogenously given. Given the state $s' \in S$, where $s'$ is the state for each date $t < T$, with probability $m_s \in (0, 1)$ the state transitions from $s'$ to $s$, where $s$ is then the state for each date $t \geq T$. The state is public information in each period.

Policy outcomes are determined by the following perfect information bargaining game. At the beginning of each period $t$ (after the potential state transitions if $t = T$), if the parties have not agreed yet, given the state $s \in S$, with probability $p^i(s)$ party $i \in N$ is *recognized* to make a proposal and with probability $p^0(s) < 1$ neither party is recognized to make a proposal. When a party is recognized to make a proposal, the proposer makes an offer that specifies how the unit of income will be spent. If the other party accepts the policy, parties receive the payoffs from the policy and the game ends. If the other party rejects the policy, each party gets an instantaneous utility of zero, and the game proceeds to period $t+1$. On the other hand, when neither party is recognized to propose, each party gets an instantaneous utility of zero, and the game proceeds to period $t+1$.

The history at the beginning of period $t$ is $h_t = (s_1, a_1, \ldots, s_{t-1}, a_{t-1}, s_t)$, where $a_\tau$, with $\tau \leq t-1$, contains the following information: whether or not a party was recognized to make a proposal in period $\tau$, and if a party was recognized, the identity of the proposer; the proposal that was made; and the vote cast by the party not proposing. Let $H_t$ denote the set of all such time $t$ histories and $H = \bigcup_{t\geq 1} H_t$ denote the set of all such histories. Let $h_t^i$ denote a history when it is party $i$'s turn to vote in period $t$, where $h_t^i$ lists the information in $h_t$, the identity of the proposer and the proposal. Let $H_t^i$ denote the set of all such time $t$ histories and $H^i = \bigcup_{t\geq 1} H_t^i$ denote the set of all such histories.

A (behavior) strategy for party $i \in N$ is given by $\sigma^i = \{r^i, v^i\}$, where $r^i : H \to \Delta(Z)$ and $v^i : H^i \to \Delta(\{\text{yes, no}\})$. I only consider Markov perfect equilibria (henceforth, MPE), a special class of subgame perfect equilibria. In the model, a *Markov perfect* equilibrium is a profile of strategies $\sigma$ that is a subgame perfect equilibrium and satisfies the following

- At any date $t < T$, the proposal only depends on the date $t$, the state, and the identity of the proposer $i$, while party $j$'s, with $j \neq i$, vote only depends on the date $t$, the state, the identity of the proposer, and the proposal;

- At any date $t \geq T$, the proposal only depends on the state and the identity of the proposer $i$, while party $j$'s vote depends only on the state, the identity of the proposer, and the proposal.

### 2.2.1 Remarks

A few features of the model deserve further attention. First, the assumption that in a given state $s \in S$, with probability $p^0(s)$ neither party is recognized to propose policy captures the idea that there are occasionally disturbances to the policy-making process that prevent parties from meeting to negotiation policy. For instance, another issue may become of primary importance, distracting parties from bargaining over the issue of initial concern, or a hostile political climate may prevent fruitful discourse over the issue at hand.

Second, I do not allow both parties to commit to lotteries over which public good to implement. Allowing such binding lotteries would facilitate compromise and affect some of the results of the paper, particularly those pertaining to legislative gridlock. Essentially, depending on the way that these binding lotteries are modeled, it is possible that there would no longer be any legislative gridlock on the equilibrium path. The assumption to rule out these lotteries captures the idea that often times political parties cannot commit to future policies.

Third, by limiting my attention to Markov perfect equilibria, I rule out equilibria that may rely on history-dependent punishment strategies in order to enforce certain outcomes. It should be noted that, if parties are sufficiently patient, it is possible to construct equilibria of this form in the model.

### 2.2.2 Pareto Efficient Policies

First, in any Pareto efficient policy there is immediate agreement, whether the implemented proposal involves the implementation of a project or not. Second, amongst the proposals that involve immediate agreement, it is possible to have efficient policies that involve the implementation of a project and efficient policies that do not. Note first that any policy involving the implementation of a project is Pareto efficient. Now consider policies that do not involve the implementation of a project. Any division $(x^1, x^2)$ of the unit of income, where $x^1 + x^2 = 1$, such that $x^i \in (1 - c, c)$ for all $i \in N$ is Pareto efficient.[2] This follows from the assumption that $\theta > c$. Observe that though there are policies involving divisions of the unit

[2] The interval $(1 - c, c)$ is nonempty due to the assumption that $c > 1/2$.

of income that are Pareto efficient, as $\theta > c$, policies involving the implementation of a project always generate a higher aggregate surplus.

### 2.3 Continuation Values in a MPE

In this section, I characterize the continuation values for any MPE. When considering Proposition 1 below, in a particular MPE, the proposals and acceptance rules at each date $t \geq T$ will be identical. Hence, the continuations values, at each date $t \geq T$ will be identical and will not be indexed by $t$. On the other hand, the continuation values at each date $t < T$ will typically depend on the date and will therefore be indexed by the date $t$. Before stating the proposition, the follow notation and background is needed.

**Preliminaries.** Given the period $t \geq T$ and the state $s \in S$, let $V^i(s)$ denote the continuation value for party $i$ at the beginning of the period. Let the aggregate surplus be $P(s) = V^1(s) + V^2(s)$. Given the period $t < T$ and the state $s \in S$, let $V_t^i(s)$ denote the continuation value for party $i$ at the beginning of period $t$. Let $P_t(s) = V_t^1(s) + V_t^2(s)$. In period $T$, let $V_T^i(s)$ be the continuation value after the state has transitioned to $s \in S$.

First, consider any date $t \geq T$. Given state $s \in S$, let $A^i(s) = \{z \in Z : u^i(z) \geq \delta V^i(s)\}$ denote the set of policies that party $i$ is willing to accept. The set $A(s) = A^1(s) \cap A^2(s)$ is then the set of policies that are acceptable to both parties. Though I do not allow parties to commit to binding lotteries over which public good to implement, I do allow parties to randomize over policy proposals when indifferent. Let $\pi^i(\cdot; s)$ denote the probability measure over the policy space $Z$ that governs party $i$'s proposal decision. This probability measure satisfies the following

- $\pi^i(\cdot; s)$ has full support on the set $\arg\max_{z \in A(s)} u^i(z)$ if $\max_{z \in A(s)} u^i(z) > \delta V^i(s)$;
- $\pi^i(\cdot; s)$ has full support on the set $\arg\max_{z \in A(s)} u^i(z) \cup Z \backslash A^j(s)$ if $\max_{z \in A(s)} u^i(z) = \delta V^i(s)$;
- $\pi^i(\cdot; s)$ has full support on $Z \backslash A^j(s)$ if $\max_{z \in A(s)} u^i(z) < \delta V^i(s)$ or if the set $A(s)$ is empty.

These mixed-proposals can be described as follows. If $\max_{z \in A(s)} \; u^i(z) > \delta V^i(s)$, then there exists at least one policy that party $i$ can propose that will be accepted by party $j$ and that gives $i$ a payoff strictly greater than its discounted continuation value. Hence, party $i$ will randomize over the set $\arg\max_{z \in A(s)} \; u^i(z)$. If $\max_{z \in A(s)} \; u^i(z) = \delta V^i(s)$, then party $i$ is indifferent between proposing a policy from the set $\arg\max_{z \in A(s)} \; u^i(z)$ and proposing a policy that will surely be rejected by party $j$ (i.e., proposing a policy from the set $Z \backslash A^j(s)$), where such a policy gives party $i$ its discounted continuation value $\delta V^i(s)$. Thus, party $i$ will randomize over the set $\arg\max_{z \in A(s)} \; u^i(z) \; \cup Z \backslash A^j(s)$. Finally, if $\max_{z \in A(s)} \; u^i(z) < \delta V^i(s)$, then party $i$ strictly prefers to propose a policy that will surely be rejected by party $j$, where such a proposal gives party $i$ its discounted continuation value $\delta V^i(s)$. Party $i$ will then randomize over the set $Z \backslash A^j(s)$.

Second, suppose it is any date $t < T$. Given state $s \in S$, let $A^i_t(s) = \{z \in Z : u^i(z) \geq \delta V^i_{t+1}(s)\}$ denote the set of policies that party $i$ is willing to accept. The set $A_t(s) = A^1_t(s) \cap A^2_t(s)$ is then the set of policies that are acceptable to both parties. Let $\pi^i_t(\cdot; s)$ denote the probability measure over $Z$ that governs party $i$'s proposal decision. This probability measure satisfies the following

- $\pi^i_t(\cdot; s)$ has full support on the set $\arg\max_{z \in A_t(s)} \; u^i(z)$ if $\max_{z \in A_t(s)} \; u^i(z) > \delta V^i_{t+1}(s)$;
- $\pi^i_t(\cdot; s)$ has full support on the set $\arg\max_{z \in A_t(s)} \; u^i(z) \; \cup Z \backslash A^j_t(s)$ if $\max_{z \in A_t(s)} \; u^i(z) = \delta V^i_{t+1}(s)$;
- $\pi^i_t(\cdot; s)$ has full support on $Z \backslash A^j_t(s)$ if $\max_{z \in A_t(s)} \; u^i(z) < \delta V^i_{t+1}(s)$ or if the set $A_t(s)$ is empty.

The logic underlying the mixed proposals at any date $t < T$ and the proposals at any date $t \geq T$ is identical. The following proposition considers the continuation values in a particular MPE.

**Proposition 2.** *In any MPE, given the state $s \in S$, at each date $t \geq T$ the continuation value* *for party $i \in N$ at the beginning of any period $t \geq T$ is*

$$V^i(s) = p^i(s)\Big[\int_{A(s)} u^i(z)\ d\pi^i(z;s) + \int_{Z\setminus A^j(s)} \delta V^i(s)\ d\pi^i(z;s)\Big]$$
$$+ p^j(s)\Big[\int_{A(s)} u^i(z)\ d\pi^j(z;s) + \int_{Z\setminus A^i(s)} \delta V^j(s)\ d\pi^j(z;s)\Big] + p^0(s)\delta V^i(s)$$

*where $j \neq i$. The continuation value for party $i \in N$ at the beginning of any period $t < T$ is*

$$V_t^i(s) = p^i(s)\Big[\int_{A_t(s)} u^i(z)\ d\pi_t^i(z;s) + \int_{Z\setminus A_t^j(s)} \delta V_{t+1}^i(s)\ d\pi_t^i(z;s)\Big]$$
$$+ p^j(s)\Big[\int_{A_t(s)} u^i(z)\ d\pi_t^j(z;s) + \int_{Z\setminus A_t^i(s)} \delta V_{t+1}^j(s)\ d\pi_t^j(z;s)\Big] + p^0(s)\delta V_{t+1}^i(s)$$

*where $j \neq i$. Note that if $t = T-1$, replace $V_{t+1}^i(s)$ for each $i \in N$ with $\sum_{s' \in S} m_{s'} V^i(s')$.*

*Proof.* First, consider the continuation game that begins at date $T$, after the state has transitioned. Consider why the continuation values as stated in the proposition are consistent with a MPE. Given the continuations payoffs $(V^1(s), V^2(s))$, in equilibrium, party $i$ will accept any policy $z \in Z$ such that $u^i(z) \geq \delta V^i(s)$. Given the continuations payoffs $(V^1(s), V^2(s))$, in equilibrium, if $\max_{z\in A(s)}\ u^i(z) > \delta V^i(s)$, then the proposer $i$ will randomize over the set $\arg\max_{z\in A(s)}\ u^i(z)$; if $\max_{z\in A(s)}\ u^i(z) = \delta V^i(s)$, then the proposer $i$ will randomize over the set $\arg\max_{z\in A(s)}\ u^i(z) \cup Z\setminus A^j(s)$; and, if $\max_{z\in A(s)}\ u^i(z) < \delta V^i(s)$, then the proposer $i$ will find it optimal to propose a policy that will be rejected by party $j$, i.e., a policy in $Z\setminus A^j(s)$. This argument implies that the probability measures $(\pi^1(\cdot;s), \pi^1(\cdot;s))$ are as stated in the proposition. In a MPE, the proposals and acceptance rules just described must induce the continuation values that are as stated in the proposition. Inspection of the continuations values stated in the proposition confirms that these proposals and acceptance rules do induce these values.

In order to complete this part of the proof, consider the following strategies. When party $i$ is not recognized to propose, she accepts any policy $z \in Z$ such that $u^i(z) \geq \delta V^i(s)$. When party $i$ is recognized to propose, the proposed policy is governed by $\pi^i(\cdot\ ;s)$. This strategy implements the continuation values stated in the proposition and no party has an incentive for a one-shot deviation; hence, this strategy is a MPE.

Now, consider the continuation values at each date $t < T$ and check that they are consistent with MPE. First, given a state $s \in S$, at each date $t < T$, the continuation values at the beginning of period $T$ (before the state transitions) are given by $\sum_{s' \in S} m_{s'} V^i(s')$. Given these continuation values, the MPE of the game is determined by backwards induction. For each date $t < T$, given the continuation payoffs $(V^1_{t+1}(s), V^2_{t+1}(s))$ (as stated in the proposition if $t = T-1$, replace $V^i_{t+1}(s)$ for each $i \in N$ with $\sum_{s' \in S} m_{s'} V^i(s')$), in equilibrium, party $i$ will accept any policy $z \in Z$ such that $u^i(z) \geq \delta V^i_{t+1}(s)$. Given the continuations payoffs $(V^1_{t+1}(s), V^2_{t+1}(s))$, in equilibrium, if $\max_{z \in A_t(s)} \; u^i(z) > \delta V^i_{t+1}(s)$, then the proposer $i$ will randomize over the set $\arg\max_{z \in A_t(s)} \; u^i(z)$; if $\max_{z \in A_t(s)} \; u^i(z) = \delta V^i_{t+1}(s)$, then the proposer $i$ will randomize over the set $\arg\max_{z \in A_t(s)} \; u^i(z) \cup Z \backslash A^j_t(s)$; and, if $\max_{z \in A_t(s)} \; u^i(z) < \delta V^i_{t+1}(s)$, then the proposer $i$ will find it optimal to propose a policy that will be rejected by party $j$, i.e., a policy in $Z \backslash A^j_t(s)$. This argument implies that the probability measures $(\pi^1_t(\cdot; s), \pi^1_t(\cdot; s))$ are as stated in the proposition. In a MPE, the proposals and acceptance rules just described must induce continuation values at date $t$ that are as stated in the proposition. Inspection of the continuation values stated in the proposition confirms that these proposals and acceptance rules do induce these values.

In order to complete this part of the proof, consider the following strategies. At date $t < T$, when party $i$ is not recognized to proposer, she accepts any policy $z \in Z$ such that $u^i(z) \geq \delta V^i_{t+1}(s)$. At date $t$, when party $i$ is recognized to propose, the proposed policy is governed by $\pi^i_t(\cdot \; ; s)$. This strategy implements the continuation payoffs stated in the proposition and no party has an incentive for a one-shot deviation; hence, this strategy is a MPE. □

Proposition 1 says nothing about the existence of a MPE. Notice that if there exists a MPE in the continuation game beginning at date $T$, then there exists a MPE in the game. Therefore, one starts by considering existence in the continuation game beginning at date $T$. As the policy space $Z$ is not convex, the probability measure $\pi^i(\cdot; s)$ that governs proposals for party $i$ is discontinuous in $\delta V^j(s)$.[3] It is therefore not possible to establish existence of a MPE by appealing to some of the standard fixed point arguments.

Fortunately, under certain parametric restrictions (see Condition D below), it is possible to prove existence of a MPE in the continuation game beginning at date $T$ by simply constructing an equilibrium. Though this is a limitation, as we cannot be sure if a MPE exists in the continuation game, if Condition D is satisfied, then the necessary condition for the existence of the MPE that are of primary interest in this paper—those where delay is possible on the equilibrium path—is satisfied (Claim 1 below). The following proposition establishes that under certain parametric restrictions, it is possible to construct MPE in the continuation game beginning at date $T$. It makes use of the following condition

**Condition D**: In each state $s \in S$, suppose $\frac{\delta(1-p^0(s))}{1-\delta p^0(s)}(\theta + 1 - c) > 1$.

**Proposition 3.** *Under Condition D, there exists a MPE in the continuation game beginning at date $T$ (after the state has transitioned). Furthermore, given the state $s \in S$, suppose (without loss of generality) that $p^1(s) \geq p^2(s)$. There always exists a MPE where both parties offer the same project with strategies*

- *For party 1: when recognized to propose, offer project 1 and the division $(x, 1-c-x)$ of the remaining income of size $1-c$; accept any offer $z \in Z$ if $u^1(z) \geq \delta V^1(s)$,*
- *For party 2: when recognized to propose, offer project 1 and the division $(y, 1-c-y)$ of the remaining income of size $1-c$; accept an offer $z \in Z$ if and only if $u^2(z) \geq \delta V^2(s)$,*

*where the values $x$ and $y$ are uniquely determined given $(\delta, p(s), c, \theta)$.*

[3] Consider the proposal strategy $\pi^i(\cdot; s)$ for party $i$, as a function of $\delta V^j(s)$. If $\delta V^j(s) \leq 1-c$, then it can be verified that it is optimal for party $i$ to propose the policy "implement project 1 and offer party $j$ the share $\delta V^j(s)$ of the remaining $1-c$ if income" with probability one. If $\delta V^j(s) \in (1-c, c)$, then it can be verified that it is optimal for party $i$ to propose the policy "implement no project and offer party $j$ the share $\delta V^j(s)$ of the remaining unit of income" with probability one. Observe that $\pi^i(\cdot; s)$ is discontinuous at $\delta V^j(s) = 1-c$ due to the assumption $\theta > c$.

*Proof.* Without loss of generality, for the proof let $i = 1$ and $j = 2$. In the proposed MPE, the continuation values at the beginning of any period $t \geq T$ are

$$V^1(s) = p^1(s)(\theta + x) + p^2(s)(\theta + y) + p^0(s)\delta V^1(s)$$
$$V^2(s) = p^1(s)(1 - c - x) + p^2(s)(1 - c - y) + p^0(s)\delta V^2(s),$$

where $x, y \in [0, 1 - c]$; these equalities can be simplified to

$$V^1(s) = \frac{p^1(s)(\theta + x) + p^2(s)(\theta + y)}{1 - \delta p^0(s)}$$
$$V^2(s) = \frac{p^1(s)(1 - c - x) + p^2(s)(1 - c - y)}{1 - \delta p^0(s)}.$$

Note that the second equation implies $1 - c > \delta V^2(s)$; this implies, that in equilibrium $x \in (0, 1 - c]$ and $1 - c - x = \delta V^2(s)$, as otherwise there exists a profitable deviation for party 1. Also, note that $\theta + 1 - c > \delta V^1(s)$; this implies that in equilibrium either $y = 0$ and $\theta \geq \delta V^1(s)$ or $y \in (0, 1 - c)$ and $\theta + y = \delta V^1(s)$, as otherwise there exists a profitable deviation for party 2.

I now consider when $y = 0$ is consistent with equilibrium. In this case,

$$\theta + x \geq \delta V^1(s)_{|y=0}, 1 - c - x = \delta V^2(s)_{|y=0} \quad \text{and} \quad \theta \geq \delta V^1(s)_{|y=0}, 1 - c \geq \delta V^2(s)_{|y=0};$$

notice that this set of equations reduces to $\theta \geq \delta V^1(s)_{|y=0}$ and $1 - c - x = \delta V^2(s)_{|y=0}$, as $x \in (0, 1 - c]$. Expanding the equality $1 - c - x = \delta V^2(s)_{|y=0}$, one has

$$1 - c - x = \delta\Big[\frac{p^1(s)(1 - c - x) + p^2(s)(1 - c)}{1 - \delta p^0(s)}\Big],$$

which gives

$$x = \frac{(1 - c)(1 - \delta)}{1 - \delta(p^0(s) + p^1(s))}.$$

Substituting this value for $x$ into $\theta \geq \delta V^1(s)_{|y=0}$, one has

$$\theta \geq \frac{\delta}{1-\delta p^0(s)}(1-p^0(s))\theta + \frac{\delta}{1-\delta p^0(s)}\Big[\frac{(1-c)(1-\delta)}{1-\delta(p^0(s)+p^1(s))}\Big], \tag{2.1}$$

where the right-hand side of (1) evaluated at $p^1(s) = 1$ is $\delta\theta + \delta(1-c)$. Noting that $V^1(s)_{|y=0}$ is increasing in $p^1(s)$, it follows that for each $\delta \in (0,1)$, there exists a $\bar{p}_\delta(s) \in [p^2(s), 1]$ such that for any $\bar{p}_\delta(s) \geq p^1(s)$, (1) is satisfied.

Given $\delta \in (0,1)$, suppose $\bar{p}_\delta(s) \geq p^1(s)$. I now show that, under Condition D, the strategies above with $y = 0$ are consistent with equilibrium. This is done by checking that there are no profitable one-shot deviations. First, consider deviations to policies that implement project 2. As $\theta > 1-c$, party 1 will never deviate by proposing to implement project 2. If party 2 deviates by proposing to implement project 2, such an offer will not be accepted by party 1 if $\delta V^1(s) = \frac{\delta p^1(s)(\theta+x)+\delta p^2(s)\theta}{1-\delta p^0(s)} > 1-c$. Notice that if $\delta V^1(s) \leq 1-c$, then, as $\delta V^2(s) \leq 1-c$,

$$\delta(V^1(s) + V^2(s)) = \frac{\delta}{1-\delta p^0(s)}\Big((1-p^0)(\theta+1-c)\Big) \leq 2(1-c) \leq 1,$$

which contradicts Condition D. Hence, under Condition D, there are no one-shot deviations of this kind. Second, consider deviations to policies involving no project, with parties agreeing on a division of the unit of income. As $\theta > c$, and $1-c > \delta V^2(s)$, party 1 will not choose such a deviation. As far as party 2 is concerned, if $\delta P > 1$, there does not exist a profitable one-shot deviation for party 2; under Condition D, as $\delta P = \frac{\delta}{1-\delta p^0(s)}\Big((1-p^0)(\theta+1-c)\Big) > 1$, no such one-shot deviation is possible. Thus, the proposed strategies constitute an MPE.

Given $\delta \in (0,1)$, suppose $p^1(s) > \bar{p}_\delta(s)$. In this case $y > 0$. Note that for any $y \in (0, 1-c)$ there exists an $x \in (0, 1-c]$ such that $1-c-x = \delta V^2(s)$. The value $y$ is then chosen so $\theta + y = \delta V^1(s)$; note that, as $\theta + 1 - c > \delta V^1(s)$ and $\theta < \delta V^1(s)$, such a $y \in (0, 1-y)$ always exists. Using arguments analogous to the case with $y = 0$, under Condition D there are no one-shot deviations and the proposed strategies constitute an equilibrium. □

Though Proposition 2 establishes that, under Condition D, there always exists an equilibrium in the continuation game beginning at date $T$ of the form described, it is also possible that there are multiple MPE in this continuation game.

**Multiple MPE.** In the following example, it is possible to construct three MPE in the continuation game game beginning at date $T$. Suppose $\delta \in (2(1-c), 1)$, the state $s \in S$ is such that $p^1(s) = p^2(s) = 1/2$ and Condition D is satisfied. First, there are MPE as constructed in Proposition 2, with one MPE that implements project 1 each period and one MPE that implements project 2 each period. Second, there is an MPE where parties agree each period on a policy where no project is implemented and the unit of income is divided. It is straightforward to verify that the continuation values are $V^1(s) = V^2(s) = 1/2$ and that the following strategy is a MPE: when recognized to propose, a party offers $\delta\frac{1}{2}$ to the other party; when not recognized to propose, accept $i$ accepts any offer $z \in Z$ where $u^i(z) \geq \delta V^i(s)$.

This example illustrates that it is possible for there to be multiple MPE in the continuation game beginning at date $T$. This implies that when MPE exist they need not be unique. Hence, continuation payoffs at any date may also not be unique.

## 2.4 Equilibrium Dynamics Before Date $T$

In this section, I consider equilibrium dynamics in the interval of time before date $T$ and focus on when it is possible for parties to agree on policy. As at the beginning of date $T$ one primary determinant of political power—the recognition probabilities—changes, this analysis considers how dynamics before an "election" depend both on the proximity and the expected outcome of that election.

**Types of Agreement.** In the event that there is agreement at date $t < T$, it is possible to put additional structure on both proposals and continuations values at that date. Section A.1 in the Appendix provides a formal characterization of the five possible types of agreement and the proposals and continuation values in each of these phases of agreement. The five types of agreement in a given period $t < T$ can be described as follows, where, without loss of generality, suppose $\delta V^1_{t+1}(s) \geq \delta V^2_{t+1}(s)$ given the state $s \in S$.

In two of the five possible agreement scenarios (in the Appendix, Type 1 and 2 agreement), parties agree on a proposal that implements project 1 regardless of which party is recognized to propose. This occurs when the gap between parties' discounted continuation values is significant enough that the only type of policy that party 1 will accept is one that implements her preferred project. As party 2 has such a small discounted continuation value, it will accept and propose such a policy.

In another type of agreement (in the Appendix, Type 3 agreement), if recognized to propose, party 1 proposes a policy that implements project 1 and this is accepted by party 2. If party 2 is recognized to propose, it proposes a policy that does not implement a project, with parties instead dividing the unit of income, and this is accepted by party 1. This occurs when party 2's discounted continuation value is small, while party 1 has a relatively moderate continuation value. As party 2 requires a small payoff from any policy, party 1 is able to successfully implement its preferred project when recognized to propose. On the other hand, as party 1 has a moderate continuation value, it will accept policies that do no implement project 1, with agents dividing the unit of income. Party 2 finds it optimal to propose such a policy.

In yet another type of agreement (in the Appendix, Type 4 agreement), regardless of which party is recognized to propose, parties agree on a proposal where no project is implemented, with parties dividing the unit of income. In this case, both parties have relatively moderate discounted continuation values, and hence, either party will reject a policy that implements the other party's preferred project, as it provides too small of a payoff. Though it is not possible to agree on a policy that implements a project, parties are willing to agree on a policy that simply divides the unit of income.

Finally, in the last type of agreement (in the Appendix, Type 5 agreement), the party that is recognized to propose offers a proposal that implements her preferred project and this proposal is accepted by the other party. In this case, parties' continuation values are so small that they are each willing to accept a policy that implements the other party's preferred project.

**Agreement and Disagreement in MPE.** Fix a MPE $\sigma$. At any date $t < T$, using the terminology in Yildiz (2003), there are two possibilities. First, there is an *agreement regime in $\sigma$ at date* $t$, if parties have not yet reached an agreement before date $t$, and as long as a party is recognized to propose, at date $t$ there will be agreement. Second, there is a *disagreement regime in $\sigma$ at date* $t$, if parties have not reached an agreement before date $t$, and regardless of which (if any) party is recognized to propose, at date $t$ there will be disagreement. Given the state $s \in S$ for each date $t < T$, if the following two conditions are satisfied at date $t$, then there is a disagreement regime in $\sigma$ at date $t$

1. $\delta V^i_{t+1}(s) > 1 - c$ for all $i \in N$;

2. $\delta P_{t+1}(s) = \delta(V^1_{t+1}(s) + V^2_{t+1}(s)) > 1$.

If either of these two conditions does not hold, then there is an agreement regime in $\sigma$ at date $t$. This can be explained as follows. If $\delta P_{t+1}(s) \leq 1$, then it is always possible that parties agree on a policy where no project is implemented, the proposer $i$ offers party $j$ exactly $\delta V^j_{t+1}(s)$, which will be accepted by party $j$, and the proposer $i$ receives $1-\delta V^i_{t+1}(s)$. As $1-\delta V^i_{t+1}(s) \geq \delta V^i_{t+1}(s)$, it is always possible that party $i$ will make such a proposal, as opposed to proposing a policy that will surely be reject by party $j$. Now, suppose that $\delta P_{t+1}(s) > 1$. In this case, agreement on any proposal where no project is implemented and parties agree on some division of the unit of income is not possible. Instead, consider proposals involving the implementation of project. If there exists a party $i$ such that $\delta V^i_{t+1}(s) \leq 1 - c$, then it is feasible for the parties to agree on a proposal of the following form: project $j \neq i$ is implemented, party $i$ receives $x$ of the remaining $1 - c$ of income and party $j$ receive $1 - c - x$ of this income.[4]

For each $t < T$, let $\bar{P}_t$ denote the upper bound on the surplus in period $t$. Keeping in mind that the surplus is maximized under a policy that implements a project, this upper bound is defined recursively as follows. Given a state $s \in S$ at each date $t < T$, the upper bound on the expected surplus at the very beginning of period $T$ is

$$\bar{P}_T \equiv \sum_{s' \in S} m_{s'} \frac{(1 - p^0(s'))}{1 - \delta p^0(s')} (\theta + 1 - c)$$

[4]Suppose such a proposal was not feasible. Then $\delta P_{t+1}(s) > \theta + 1 - c$ and a contradiction obtains.

The upper bound on the expected surplus at date $T-1$ is then

$$\bar{P}_{T-1}(s) \equiv (1-p^0(s))(\theta+1-c) + \delta p^0(s)\bar{P}_T; \tag{2.2}$$

it follows that the upper bound on expected surplus at each $t < T-1$ is

$$\bar{P}_t(s) \equiv (1-p^0(s))(\theta+1-c) + \delta p^0(s)\bar{P}_{t+1}. \tag{2.3}$$

A necessary condition for the existence of a disagreement regime at date $T-1$ is then $\delta\bar{P}_T > 1$, while a necessary condition for the existence of a disagreement regime at date $t < T-1$ is $\delta\bar{P}_{t+1}(s) > 1$. As $\theta > c$, these conditions will be satisfied when the positive surplus generated by implementing a project is sufficiently large, and the probability that neither party is recognized to propose policy each period ($p^0(s)$) is sufficiently small. Essentially, disagreement regimes will only arise when projects are highly valuable and, in expectation, parties are able to make policy proposals sufficiently often. Of course, parties must also be sufficiently patient.

The following claim verifies that if Condition D is satisfied, then a necessary condition for the existence of a MPE where there are periods of disagreement before date $T$ is satisfied.

**Claim 1.** *Suppose Condition D is satisfied. Then $\delta\bar{P}_T > 1$ and $\delta\bar{P}_{t+1}(s) > 1$ for each $t < T$.*

*Proof.* If Condition D is satisfied, then, for any $s \in S$,

$$\frac{1-p^0(s)}{1-\delta p^0(s)}\delta(\theta+1-c) > 1.$$

As $\frac{1-p^0(s)}{1-\delta p^0(s)} \leq 1$, this implies that $\delta(\theta+1-c) > 1$. It then follows that $\delta\bar{P}_T > 1$ and $\delta\bar{P}_{t+1}(s) > 1$ for each $t < T$. □

When considering the dynamics exhibited by a particular MPE in the interval of time before date $T$, given the pair of continuation values at the beginning of period $T$ (before the state transitions), determining proposals and acceptance rules at each date $t < T$ amounts to inducting backwards. In general, it is not clear in what periods there will be agreement regimes and what exactly accepted proposals will be in each of these agreement regimes. The following lemma considers equilibrium dynamics when a party is able to implement her preferred project in a given period, regardless of which party is recognized to propose.

**Lemma 7.** *Let $s \in S$ denote the state at each date $t < T$ and suppose Condition D holds. Fix a MPE where there is an agreement regime at date $\hat{t} < T$, and furthermore, regardless of which party is recognized to propose, the proposed policy implements project $k$. Then, in this MPE, there exists an agreement regime at each date $t < \hat{t}$, where, regardless of which party is recognized to propose, the proposed policy implements project $k$*

*Proof.* Consider date $\hat{t}$. If both parties $j$ and $k$ propose a policy (with probability one) that implements project $k$, it must be the case that $\delta V^k_{\hat{t}+1}(s) > c$ and $\delta V^j_{\hat{t}+1}(s) \leq 1 - c$. Hence, there is either Type 1 or Type 2 agreement (see the Appendix). It follows that continuation values at the beginning of date $\hat{t}$ are

$$V^k_{\hat{t}}(s) = p^k(s)(\theta + x) + p^j(s)(\theta + y) + p^0(s)\delta V^k_{\hat{t}+1}(s)$$
$$V^j_{\hat{t}}(s) = p^k(s)(1 - c - x) + p^j(s)(1 - c - y) + p^0(s)\delta V^j_{\hat{t}+1}(s),$$

where $j \neq k$ and $x, y \in [0, 1 - c]$. Note that $V^j_{\hat{t}}(s) \leq 1 - c$. I now show that $\delta V^k_{\hat{t}}(s) > c$. Observe that $V^k_{\hat{t}}(s) > (1-p^0(s))\theta + p^0(s)c$. Next, one can show that $\delta[(1-p^0(s))\theta+p^0(s)c] > c$. Towards a contradiction, suppose otherwise. This implies $\frac{\delta(1-p^0(s))}{1-\delta p^0(s)}\theta \leq c$, and consider

$$\underbrace{1 < \frac{\delta(1 - p^0(s))}{1 - \delta p^0(s)}(\theta + 1 - c)}_{\text{Condition D}} \leq c + \frac{\delta(1 - p^0(s))}{1 - \delta p^0(s)}(1 - c).$$

This implies

$$1 < \frac{\delta(1 - p^0(s))}{1 - \delta p^0(s)},$$

giving a contradiction. Thus, $\delta V^k_{\hat{t}}(s) > c$ and $V^j_{\hat{t}}(s) \leq 1 - c$. It follows that proposals at date $\hat{t} - 1$ will be as in period $\hat{t}$: both parties $j$ and $k$ propose a policy (with probability one) that implements project $k$. When consider proposals at each date $t \leq \hat{t} - 1$, an argument identical to one used when considering period $\hat{t} - 1$ establishes the desired result. □

If a party $i$ is able to implement her preferred project in a given period $\hat{t} < T$, regardless of which party is actually proposing policy, this gives party $i$ a high expected payoff at the beginning of period $\hat{t}$. Under Condition D, party $i$ will also be able to successfully implement its preferred project at date $\hat{t}-1$. Essentially, if a party knows that it will receive a high payoff tomorrow, it can credibly demand a high share today. It follows that party $i$ will be able to implement her preferred project at each date $t < \hat{t} - 1$.

The following proposition builds on Lemma 1 and illustrates how asymmetries in political power influence equilibrium dynamics and allow for a fairly straightforward characterization of when agreement regimes will occur. It is useful to note that, due to discounting, when $T$ is sufficiently large, in any MPE, there will exist a date $t < T$ where there is an agreement regime.

**Proposition 4.** *Let $s \in S$ denote the state at each date $t < T$ and suppose Condition D holds. Fix a MPE. Assuming $T$ is large enough, let $t_1 < T$ denote the first date, counting backwards from date $T$, where there is an agreement regime in this MPE. Given the surplus at the beginning of period $t_1$, denoted $P_{t_1+1}(s)$,*

1. *If $\delta P_{t_1+1}(s) > 1$, then there is an agreement regime in this MPE at each date $t < t_1$ with a project implemented in each period whenever some party is recognized to propose;*

2. *If $\delta P_{t_1+1}(s) \leq 1$, then there exists a $\bar{p}(s) \in (0,1)$ such that if $p \geq \bar{p}(s)$, with $p \in \{p^1(s), p^2(s)\}$, there is an agreement regime in this MPE at each date $t < t_1$ with a project implemented in each period whenever some party is recognized to propose.*

*Proof.* First, suppose $\delta P_{t_1+1}(s) > 1$. In this case, continuation values are such that $\delta V^i_{t_1+1}(s) > c$ and $\delta V^j_{t_1+1}(s) \leq 1 - c$. Then there is either Type 1 or Type 2 agreement (see the Appendix) and, if recognized to propose, either party will propose a policy that implements project $i$ with probability one. Lemma 1 then applies and the desired result is obtained.

Next, suppose $\delta P_{t_1+1}(s) \leq 1$. Considering how continuation values are defined in Proposition 1, there exists a $\bar{p}(s) \in (0,1)$ such that if $\max\{p^1, p^2\} \geq \bar{p}(s)$, then at each date $t \leq t_1$, $\min\{\delta V_t(s)^1, \delta V_t^2(s)\} \leq 1 - c$. It follows that there is agreement at each date $t < t_1$. □

Proposition 3 demonstrates that if political power is sufficiently asymmetric then the path of predicted policy outcomes takes a simple form. In a given MPE $\sigma$, if there is an interval of disagreement, it occurs immediately before the election, and whenever there is an agreement regime in $\sigma$ at date $t < t_1$, as long as some party is recognized to propose, the party with higher political power will implement her preferred project. Hence, implemented policies at dates before $t_1$ are "extreme" in a sense.

It is useful to consider where political power in the time before date $T$ comes from in model. There are two primary determinants of a party's political power. One, the recognition probabilities at each date $t < T$. In any MPE, a party's continuation value is nondecreasing in its recognition probability. This follows from the fact that whenever a party is recognized to propose, as parties are impatient, the recognized party is able to extract rents from the other party. The second factor that affects a party's political power are the continuation values at the very beginning of date $T$, which depend on the recognition probabilities at each date $t \geq T$, and, if there are multiple MPE, the equilibrium that parties coordinate on. If there are multiple MPE, these equilibria can be ranked according to the payoff each delivers to a particular party. In short, equilibria can be "bad" or "good" from a party's perspective. Whether due to a party $i$ having a high recognition probability or parties coordinating on a MPE that gives party $i$ a high payoff, these factors translate into $i$ having a high continuation value. This party then has high political power as it is able to successfully demand its preferred policies.

The distribution of political power is instrumental in both part 1 and 2 of Proposition 3. In part 1, if parties are willing to agree at date $t_1$ with $\delta P_{t_1+1}(s) > 1$, it must be the case that $\delta V^j_{t_1+1}(s) \leq 1 - c$ and $\delta V^i_{t_1+1}(s) > c$; this only occurs when political power, arising from the continuation values at the very beginning of date $T$, is sufficiently asymmetric. Notice that, as parties have not agreed at any date $t \in \{t_1 + 1, \ldots, T - 1\}$ (where this interval may be empty), the current recognition probabilities have no impact on the continuation values $(\delta V^1_{t_1+1}(s), \delta V^2_{t_1+1}(s))$. Due to Lemma 1, irrespective of the current recognition probabilities, there is an agreement regime at each period $t < t_1$, where party $i$ is able to implement her preferred project in each period, assuming some party has been recognized to propose. Hence, though current recognition probabilities may play a part in determining the shares of the $1-c$ of income that remains after implementing a project, the continuation values at the beginning of date $T$ introduce the asymmetry in political power that is key for the result.

In part 2 of Proposition 3, the influence that the distribution of political power has on agreement is more transparent: when there is agreement at date $t_1$, if current recognition probabilities are such that one party $j$ has a very low chance of being recognized to propose policy, then party $j$ will have a low continuation value at the beginning of date $t_1$, and at each earlier date. If party $j$ expects a low payoff tomorrow, then it is willing to agree on a policy that gives it a low payoff today; this implies that there is an agreement regime at each date $t < t_1$ with party $i$ able to implement its preferred project.

The proposition below considers the timing and duration of phases of legislative gridlock, under the assumption that parties play a particular type of MPE in the continuation game beginning at date $T$. Recall that, under Condition D, for any $s' \in S$ realized at the beginning of period $T$, there exists a MPE of the continuation game beginning at date $T$ where there is an agreement regime each period $t \geq T$ with accepted policies implementing a project.

**Proposition 5.** *Let $s \in S$ denote the state at each date $t < T$. Suppose Condition D is satisfied and for any $s' \in S$ realized at the beginning of period $T$, parties play a MPE in the continuation game beginning at date $T$ where a project is implemented whenever some party is recognized to propose. Then, in this MPE,*

1. *There will be agreement at each date $t < T$, or a disagreement regime of the form $\{\hat{t}, \ldots, T\}$, with $\hat{t} \leq T-1$;*
2. *Assume $p^0(s') = p^0(s'')$ for any $s', s'' \in S$. If there exists multiple phases of disagreement, there will not exist a phase longer than the disagreement interval $\{\hat{t}, \ldots T\}$.*

*Proof.* First, consider part 1. Let $s \in S$ denote the state at each date $t < T$. Given the equilibrium parties play at each state $s' \in S$ in the continuation game beginning at date $T$, the expected surplus at the very beginning of date $T$ is

$$\delta \sum_{s' \in S} m_{s'} P_T(s) = \underbrace{\sum_{s' \in S} m_{s'} \left[ \frac{\delta(1 - p^0(s'))}{1 - \delta p^0(s')} (\theta + 1 - c) \right] > 1}_{\text{By Condition D}} .$$

Notice that if there exists an agreement regime at date $T - 1$, it must be the case that

$$\delta \sum_{s' \in S} m_{s'} V_T^i(s') > c \quad \text{and} \quad \delta \sum_{s' \in S} m_{s'} V_T^j(s') \leq 1 - c.$$

In this case, there is an agreement regime at date $T - 1$ and, regardless of which party is recognized to propose, proposed policies implement party $i$'s preferred project (project $i$). Arguments from Lemma 1 imply that $\delta V_{T-1}^i > c$ and $\delta V_{T-1}^j \leq 1 - c$, and thus there is an agreement regime at date $T - 2$ and proposed policies implement project $i$. Inducting backwards, it follows that, for each $t < T - 2$, $\delta V_{t+1}^i > c$ and $\delta V_{t+1}^j \leq 1 - c$, and thus there is an agreement regime at date $t$ and proposed policies implement project $i$.

If it not the case that $\delta \sum_{s' \in S} m_{s'} V_T^i(s') > c$ and $\delta \sum_{s' \in S} m_{s'} V_T^j(s') \leq 1 - c$, then there is a disagreement regime at date $T - 1$. Depending on $T$ and $\delta$, there will either exist a disagreement regime at each date $t < T$, or, if $T$ is large enough, due to discounting, there will exist a date $1 \leq \hat{t} < T$, where there is an agreement regime.

Next, consider part 2. Let $t_1$ denote the first date, counting backwards from date $T$, where there is an agreement regime. If $\delta P_{t_1+1}(s) > 1$, then $\delta V_{t_1+1}^i(s) > c$ and $\delta V_{t_1+1}^i(s) \leq 1 - c$ and there is either Type 1 or Type 2 agreement (see Appendix A.1). Just as argued above, at date $t_1$, regardless of which party is recognized to propose, proposed policies implement party $i$'s preferred project (project $i$). Arguments from Lemma 1 imply that $\delta V_{t_1}^i(s) > c$ and $\delta V_{t_1}^j(s) \leq 1 - c$. Thus there is an agreement regime at date $t_1 - 1$ and proposed policies implement project $i$. Inducting backwards, it follows that, for each $t < t_1 - 1$, $\delta V_{t+1}^i(s) > c$ and $\delta V_{t+1}^j(s) \leq 1 - c$, and thus there is an agreement regime at date $t$ and proposed policies implement project $i$. This implies that the only, and hence the longest, disagreement regime occurs from date $t_1 + 1$ to date $T - 1$.

Now, suppose $\delta P_{t_1+1}(s) \leq 1$. Note that the surplus at the beginning of date $T$ is

$$P_T = \underbrace{\sum_{s' \in S} m_{s'} \frac{1 - p^0(s')}{1 - \delta p^0(s')} (\theta + 1 - c)}_{> \frac{1}{\delta} \text{ Due to Condition D}}.$$

The expected surplus at the beginning of date $t_1$ is

$$P_{t_1} \leq (1 - p^0(s))(\theta + 1 - c) + \delta p^0(s) P_{t_1+1}(s).$$

As

$$P_T = (1 - p^0(s))(\theta + 1 - c) + p^0(s) \underbrace{\delta P_T}_{>1},$$

it follows that $P_{t_1}(s) \leq P_T$.

To complete the proof, consider the second agreement date $t_2$ (counting backwards from date $T$) where there is an agreement regime. If $\delta P_{t_2+1}(s) > 1$, then using the same arguments as above, there is agreement at each date $t \leq t_2$. As $P_{t_1}(s) \leq P_T$, the (possibly empty) disagreement interval from date $t_2 + 1$ to date $t < t_1$ is no longer than the disagreement interval immediately before date $T$ and one has the desired result. If $\delta P_{t_2+1}(s) \leq 1$, then as $P_{t_1}(s) \leq P_T$, the disagreement interval from $t_2 + 1$ to date $t < t_1$ is shorter than the disagreement interval immediately before date $T$. The expected surplus at the beginning of date $t_2$ is

$$P_{t_2}(s) \leq (1 - p^0(s))(\theta + 1 - c) + \delta p^0(s) P_{t_2+1}(s).$$

As

$$P_T = (1 - p^0(s))(\theta + 1 - c) + p^0(s) \underbrace{\delta P_T}_{>1},$$

it follows that $P_{t_2}(s) \leq P_T$.

A repeated application of the same arguments establishes the desired result. □

Recall that any policy that implements a project maximizes aggregate surplus. Thus, when considering the equilibrium selection in Proposition 4, this proposition considers when disagreement regimes will arise when there is a common expectation that a policy that is highly valuable to one party (as its preferred project is implemented) will be passed immediately after the election. In other words, this proposition considers when disagreement regimes will arise in an equilibrium where parties are "optimistic" regarding the policy outcome immediately after the election. In such an equilibrium, though there may be multiple spells of disagreement, the lengthiest period of legislative gridlock occurs right before the election. Essentially, once the election is in relatively close proximity, the expectation of implementing a highly valuable policy right after the election makes parties' continuation values too high to allow for compromise. Agreement is then infeasible at dates close to the election. The disagreement phase immediately before the election is longest due to Condition D, the assumption that the probability that no party is recognized to propose is state-invariant, and the focus on an equilibrium where parties implements a project at date $T$. These conditions ensure that the expected aggregate surplus for any policy outcome at date $t < T$ is no more than the expected surplus from the policy outcome at date $T$. This implies that an interval of disagreement building up towards an agreement regime at some date $t < T$ cannot be longer than the disagreement interval before the agreement regime at date $T$.

There are many interesting cases that Propositions 3 and 4 do not cover. Though it is difficult to completely characterize bargaining dynamics in general, it is possible to construct different types of Markov perfect equilibria that illustrate what dynamics are feasible in the model. Before considering the examples that show the various kinds of dynamics that are consistent with MPE, it is important to note that, while Proposition 4 demonstrates that it is possible to construct a MPE where there is a disagreement regime close to date $T$ that gives way to an agreement regime of length $\{1, \ldots, \hat{t}\}$ for $\hat{t} < T$, it is also possible to construct a MPE where there are multiple transitions between agreement and disagreement regimes in the time before date $T$. The following provides an illustration of why this can occur. Suppose there is agreement at date $t' < T$, where, if a party is recognized to propose, that party will successfully implement her preferred project. Now, consider date $t'-1$. If parties are sufficiently patient and parties' recognition probabilities are not highly asymmetric, then there will exist a disagreement regime at date $t'-1$. Essentially, if there is common knowledge that there will be agreement at date $t'$, and furthermore, the proposer will be able to implement her preferred project, then both parties' continuation values at the beginning of period $t'$ are too large to allow for agreement at date $t'$. Though there may be a disagreement regime at date $t'-1$, if $T$ is sufficiently large, due to discounting, there will be exist a date $t'' < t'-1$ where there is an agreement regime. It is therefore possible to transition back and forth from agreement and disagreement regimes in a particular MPE.

### 2.4.1 Examples

The following examples aim to illustrate what dynamics are possible in equilibrium and how the implemented policies depend on the proximity until date $T$, where a shift in the recognition probabilities, and hence political power, may take place.

Examples 1 and 2 aim to illustrate how the MPE that parties coordinate on in the continuation game beginning at date $T$ has implications both on the length of any disagreement regime and the expected surplus $P_1(s_0)$ at the beginning of the bargaining game. Example 3 shows how it is possible to cycle between agreement and disagreement regimes in the time before date $T$. Example 4 illustrates, in contrast to Example 1, that it is possible for the identity of the party with the highest continuation value to change with the proximity of the election. Finally, Example 5 considers how the expected social surplus is affected by changes in the parameters $\{\theta, c, \delta\}$.

In each of the examples, the state space $S = \{s_0, s_1, s_2\}$, where in state $s_0$ recognition probabilities, with $p \in [0, 1/2]$, are $\{p^0(s_0), p^1(s_0), p^2(s_0)\} = \{1-2p, p, p\}$, in state $s_1$ recognition probabilities are $\{p^0(s_0), p^1(s_0), p^2(s_0)\} = \{1-2p, 2p, 0\}$, and in state $s_2$ recognition probabilities are $\{p^0(s_0), p^1(s_0), p^2(s_0)\} = \{1-2p, 0, 2p\}$. Therefore, in state $s_0$ parties' recognition probabilities are equal and in state $s_1$ (resp. state $s_2$), whenever a party is recognized to propose, it is party 1 (resp. party 2). Also, assume that Condition D is satisfied, where, given these parameters, this condition is $\frac{\delta 2p}{1-\delta(1-2p)}(\theta + 1 - c) > 1$.

Given these parametric restrictions, before considering the dynamics in the time before $T$, consider the possible MPE in the continuation game beginning at date $T$. In state $s_1$ (resp. state $s_2$) it is easy to show that there is a unique MPE where party 1 proposes a policy that implements project 1 (resp. project 2) and takes all of the $1-c$ of income. Continuations values are $(V_T^1(s_0), V_T^2(s_0)) = ((\theta+1-c)/(1-\delta(1-2p)), 0)$.

In state $s_0$, there are multiple MPE. First, as Condition D is satisfied and $p = p^1(s_0) = p^2(s_0)$, there exists a MPE with strategies (henceforth, MPE E1)

- For party 1: when recognized to propose, offer project 1 and the division $(x, 1-c-x)$ of the remaining income of size $1-c$; accept an offer $z \in Z$ if and only if $u^1(z) \geq \delta V^1(s_0)$.
- For party 2: when recognized to propose, offer project 1 and the division $(y, 1-c-y)$ of the remaining income of size $1-c$; accept an offer $z \in Z$ if and only if $u^2(z) \geq \delta V^2(s_0)$.

and a MPE with strategies (henceforth, MPE E2)

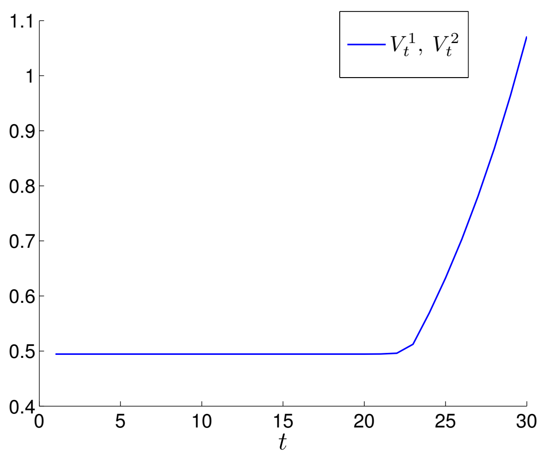
- For party 1: when recognized to propose, offer project 2 and the division $(1-c-y, y)$ of the remaining income of size $1-c$; accept an offer $z \in Z$ if and only if $u^2(z) \geq \delta V^2(s_0)$;
- For party 2: when recognized to propose, offer project 2 and the division $(1-c-x, x)$ of the remaining income of size $1-c$; accept an offer $z \in Z$ if and only if $u^1(z) \geq \delta V^1(s_0)$.

where $y = 0$ and $x = \frac{(1-\delta)(1-c)}{1-\delta(1-p)}$. Note that in each of these MPE there is an agreement regime in each period $t \geq T$. There may also be a MPE where no policy is implemented and parties agree on a division of the unit of income (henceforth, MPE E0). In this MPE, parties use the following strategies: if recognized to propose, party $i$ offers $\frac{1-\delta+\delta p}{1-\delta(1-2p)}$,while party $j \neq i$ accepts any offer $z \in Z$ where $u^j(z) \geq \delta V^j(s_0)$. In this MPE, there is an agreement regime at each date $t \geq T$. In order for these strategies to be consistent with equilibrium it must be the case that, given $(\delta, p, c)$,

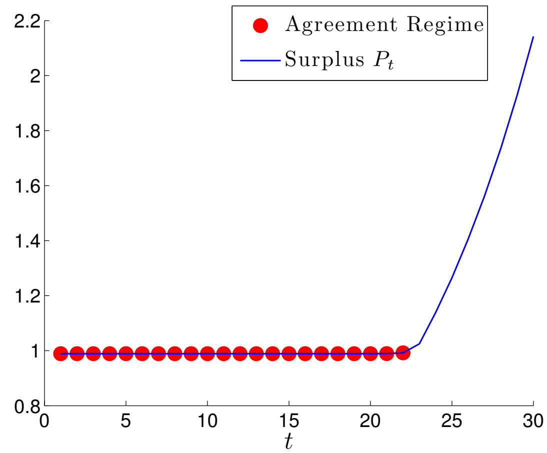
$$\delta V^1(s_0) = \delta V^2(s_0) = \frac{\delta p}{(1-\delta(1-2p))} > 1-c. \tag{2.4}$$

**Example 1**

Suppose the state at each date $t < T$ is $s_0$ and consider the following parameters: $T = 30$, $\delta = 0.90$, $\theta = 2$, $c = 0.60$, $p = 0.45$, $m_{s_1} = m_{s_2} = 0.35$, and $m_{s_0} = 0.30$. Condition D and inequality (4) are both satisfied; hence, in the continuation game beginning at date $T$, MPE E0, E1, and E2 all exist.




(a)




(b)

Figure 2.1: **A Symmetric Model with MPE E0 Played at Date $T$**

Examples 1 and 2 aim to illustrate how the MPE that parties coordinate on in the continuation game beginning at date $T$ has implications both on the length of any disagreement regime and the expected surplus $P_1(s_0)$ at the beginning of the bargaining game.

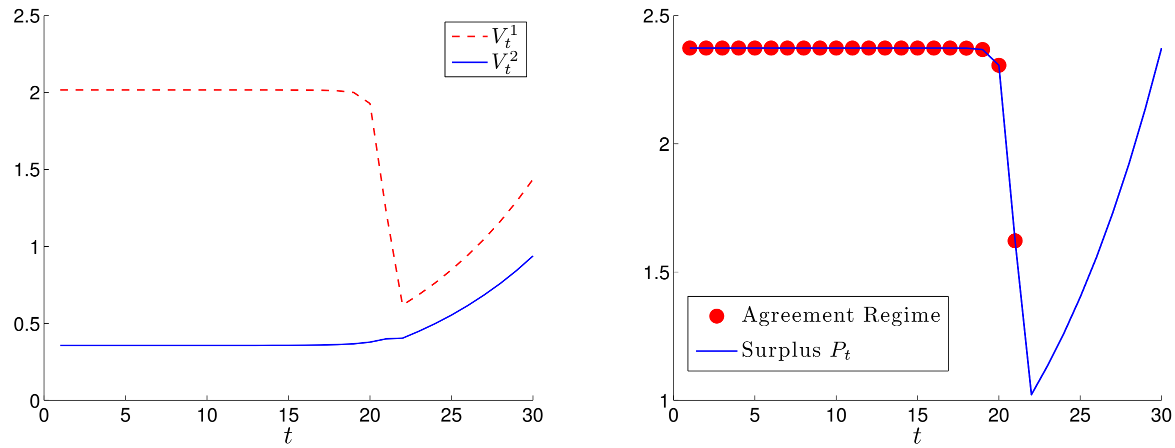
Suppose that in the continuation game beginning at date $T$, if the state transitions to $s_0$, parties play MPE E0. Consider the equilibrium dynamics in Figure 1. Given the symmetry of the example, with parties having the same recognition probability at each date $t < T$ and the same continuation value at the beginning of period $T$ (before the state transitions), as Figure 1 (a) shows, parties have identical continuation values at each date $t < T$. Consider when agreement takes place and on what policies parties are willing to agree on. Starting at date 1, though parties are willing to agree, they are only able to strike a deal on policy that divides the unit of income. Thus, no project is implemented. While such an agreement is Pareto efficient, it clearly does not maximize aggregate surplus. As the election date $T$ draws near, the parties enter a disagreement regime until transitioning to an agreement regime at date $T$ for the remainder of the game. Notice that as in each period with probability 0.1 neither party is recognized to make a proposal; therefore, the disagreement regime immediate before date $T$ is on the equilibrium path.

Due to discounting, it is clear that as the election date $T$ becomes further away, there exists some date $\hat{t} < T$ where parties are willing to agree. But why must parties only agree on a division of the unit of income, with no project being implemented, at each date $t \leq \hat{t}$ (with $\hat{t} = 23$)? As parties have the same continuation value at date $\hat{t} + 1 < T$, and these continuation values are not too small, the only type of agreement that is feasible is one where parties agree on a division of the unit of income. Given that parties have the same recognition probability, continuation values are identical in period $\hat{t}$. If parties' discount factor and the probability that some party gets recognized to propose ($2p = 0.90$) are both sufficiently high, then the continuation values $(V^1_{\hat{t}}(s_0), V^2_{\hat{t}}(s_0))$ are relatively moderate; specifically, for each $i \in N$, $\delta V^i_{\hat{t}}(s_0) > 1 - c$, with $\delta P_{\hat{t}}(s_0) \leq 1$. This implies that, just as in period $\hat{t}$, in period $\hat{t} - 1$ the only feasible type of agreement involves dividing the unit of income. An identical argument implies that parties only agree on a division of the unit of income at each date $t \leq \hat{t}$.

**Example 2**

Consider the same parameters as in Example 1. Suppose that in the continuation game beginning in date $T$, if the state transitions to $s_0$, instead of playing MPE E0, parties play MPE E1. Consider the equilibrium dynamics in Figure 2. In this case, there is an asymmetry in political power, relative to when parties play MPE E0, due to party 1 being able to implement her preferred project with a higher likelihood immediately after the election. Starting at date 1, parties are able to agree, and furthermore, this agreement maximizes aggregate surplus, as it involves implementing a project (specifically, project 1). Just as in the first case, as the election date approaches, the parties enter a disagreement regime until transitioning to an agreement regime at date $T$ for the remainder of the game.

It may not be clear why party 1 is able to implement its preferred project, regardless of which party is recognized to propose, during the agreement regime at dates $\{1, \ldots, 20\}$. The asymmetry created by parties coordinating on MPE E1 in the continuation game beginning at date $T$ with state $s_0$, allows party 1 to implement her preferred project at date 21 when recognized to propose. This creates a large asymmetry in continuation values at the beginning of period 21, and hence, a political advantage for party 1. This asymmetry in political power is preserved if parties' discount factor and the probability that some party gets recognized to



(a) (b)

Figure 2.2: **An Asymmetric Model with MPE E1 Played at Date $T$**

propose ($2p = 0.90$) are both sufficiently high, despite the symmetric recognition probabilities at each date $t < T$.

In comparison to Example 1, it is clear that the equilibrium that parties coordinate on in the continuation game beginning at date $T$ in state $s_0$ has important welfare consequences. Plainly, the MPE that parties play in this continuation game can cause asymmetries in political power that influence equilibrium dynamics, and thus, the expected social surplus from the bargaining game.

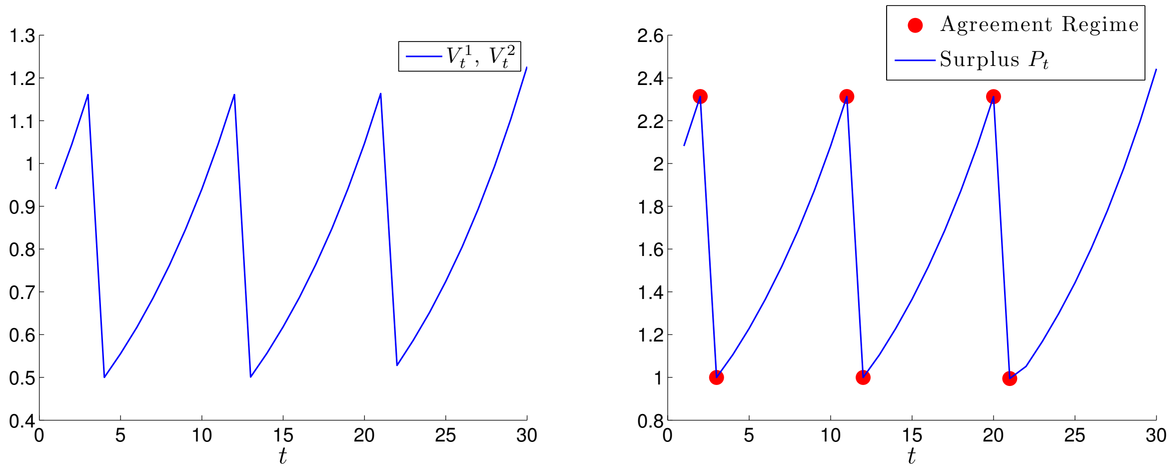
Before moving on to the next example, consider why, in both Examples 1 and 2, immediately before the election there is an interval of time where there is disagreement in each period. At date $T$, each party has a 35% chance of getting a recognition probability of 0.90, and hence the political advantage. This increase in political power allows a party to implement its preferred project, and thus, secure a high payoff. As the value of the project, relative to the cost, is sufficiently great, parties' continuation values at the beginning of period $T$ (before the state transitions) are too high to facilitate compromise. Hence, there is a disagreement regime for multiple periods before the election. As the date $T$ becomes further away, due to discounting, continuations values decrease and parties are able to agree.

**Example 3**

Suppose the state at each date $t < T$ is $s_0$ and consider the following parameters: $T = 30$, $\delta = 0.90$, $\theta = 2$, $c = 0.53$, $p = 0.45$, $m_{s_1} = m_{s_2} = 0.35$ and $m_{s_0} = 0.30$. Condition D is satisfied but inequality (4) is not; hence, in the continuation game beginning at date $T$, MPE E0 does not exist, while MPE E1 and E2 do.

Suppose that in the continuation game beginning at date $T$, if the state transitions to $s_0$, parties randomize over MPE E1 and E2. Specifically, parties play MPE E1 with probability 1/2 and MPE E2 with probability 1/2. As parties have the same recognition probability before date $T$ and the same continuation value at the beginning of period $T$, this example is symmetric. Consider the equilibrium dynamics illustrated in Figure 3, which can be described as follows. After a length of time where there is a disagreement regime, discounted continuation values are small enough to allow agreement at date $t' < T$, with this agreement taking place on a policy that implements no project and divides the unit of income. As the cost of implementing a project is relatively small, party $i$'s discounted continuation value $\delta V^i_{t'}(s_0) \leq 1-c$ for any $i \in N$. When this is the case, the party that is recognized to propose can successfully implement its preferred project (see Type 5 agreement in the Appendix). Hence, at date $t'-1$, if recognized to propose, a party will propose a policy that implements its preferred project, and this will be accepted by the other party. This implies that each party will have a fairly high continuation value at the beginning of period $t'-1$, which makes agreement at date $t'-2$ impossible. There is then a disagreement regime for multiple periods before discounted continuation values are small enough to allow agreement.

This example illustrates how it is possible to cycles between agreement and disagreement regimes in the time before date $T$. Notice that at date 1, in this MPE, it is common knowledge amongst the parties as to when the agreement regimes will arise. Furthermore, the timing of agreement is determined endogenously by the proximity of the election date $T$. When considering the welfare implications associated with cycling between agreement and disagreement regimes, though there is only one period of inefficient delay in this example, it is possible to construct examples where there are multiple periods of costly delay at the beginning of the

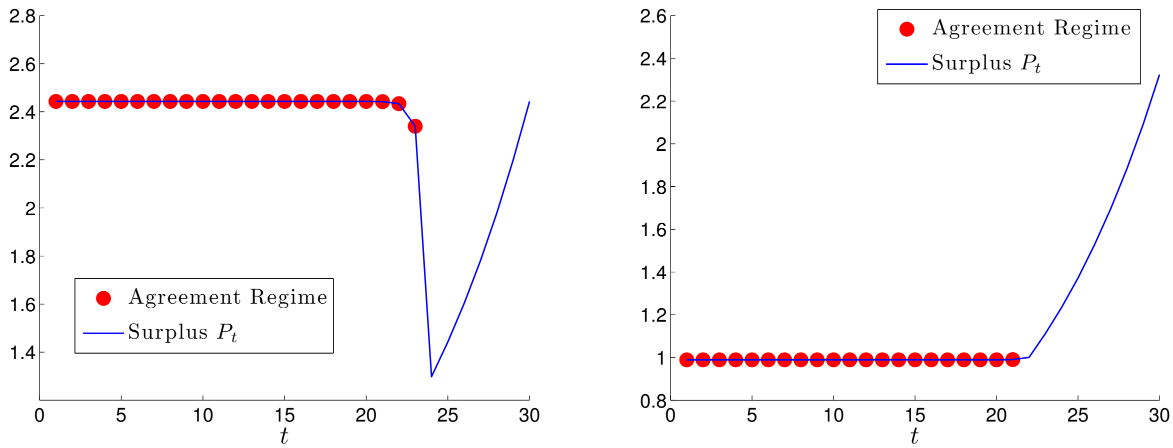



(a) (b)

Figure 2.3: **Cycling Between Agreement and Disagreement Regimes**

game.

In a particular MPE, whether there is cycling or not depends critically on the distribution of political power and the cost of implementing a project. The cycling illustrated in this example will only occur when there exists periods before date $T$ where any party that is recognized to propose will be able to implement her preferred project. As mentioned above, in order for there to exist periods that satisfy this restriction, there must be periods $T' \subset \{1, \dots, T-1\}$ where $\delta V_{t+1}^i(s_0) \leq 1 - c$ for any $i \in N$, $t \in T'$. Under Condition D, this only occurs when, as in the MPE considered in this example, political power is symmetric and the cost is relatively low. Figure 4 shows how if either of these properties are not satisfied, this cycling property vanishes. Figure 4 (a) illustrates the path of expected surplus and when agreement regimes arise when political power is asymmetric. Specifically, instead of parties randomizing with equal probability over MPE E1 and MPE E2, parties coordinate on MPE E1 in the continuation game that begins at date $T$. This gives an advantage to party 1. The rest of the parameters are as stated above. Figure 4 (b) illustrates the path of expected surplus and when agreement regimes arise with higher costs. Specifically, the cost is $c = 0.65$ instead of $c = 0.53$. The rest of the parameters are as stated above, and, when considering the equilibrium played in the continuation game beginning at date $T$, parties are still assumed to randomize over MPE E1




(a) Asymmetric Political Power

(b) Higher Costs

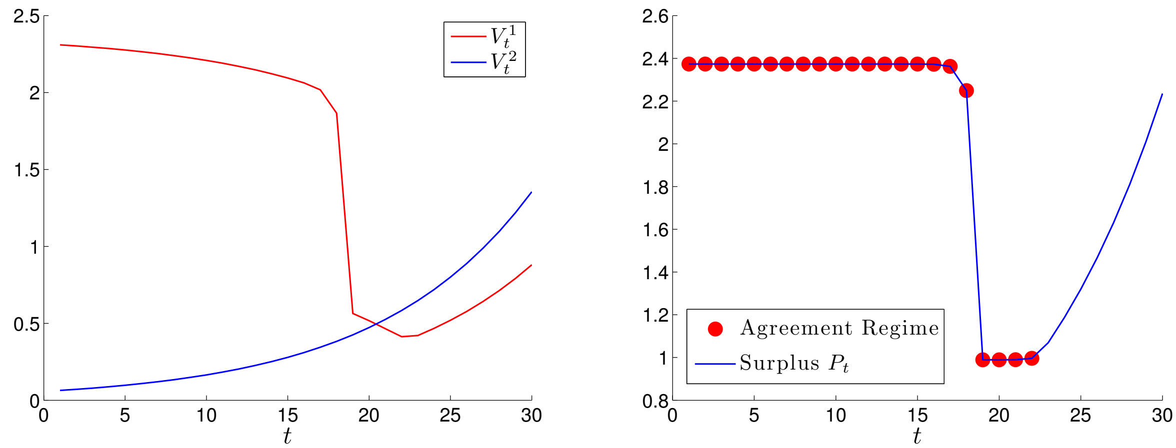
Figure 2.4: **No Cycling with Asymmetric Political Power or High Costs**

and MPE E2 with equal probability.

**Example 4**

Suppose the state at each date $t < T$ is $s_1$ and consider the following parameters: $T = 30$, $\delta = 0.90$, $\theta = 2$, $c = 0.60$, $p = 0.45$, $m_{s_1} = 0.35$, $m_{s_2} = 0.55$ and $m_{s_0} = 0.10$. Condition D and inequality (4) are both satisfied; hence, in the continuation game beginning at date $T$, MPE E0, E1, and E2 all exist. Suppose parties play MPE E0 in the continuation game beginning at date $T$. Also, notice that as $p = 0.45$ and the state at each date $t < T$ is $s_1$, at each date $t < T$, party 1 is recognized to propose with probability $2p = 0.90$, while with probability $1 - 2p = 0.90$, neither party is recognized.

This example illustrates, in contrast to Example 1, that it is possible for the identity of the party with the highest continuation value to change with the proximity of the election. Consider Figure 5. At dates close to the election date $T$, with $m(s_2 \,|\, s_0) > m(s_1 \,|\, s_0)$, party 2 is more likely to grab proposal rights, and hence, be able to implement its preferred project. This implies that at each date $t \in \{23, \ldots, 29\}$, though there is a disagreement regime, party 2 has a higher continuation value tha party 1. As date $T$ becomes farther away, the advantage that party 1 has from having a much higher recognition probability at each date $t < T$ begins




(a) (b)

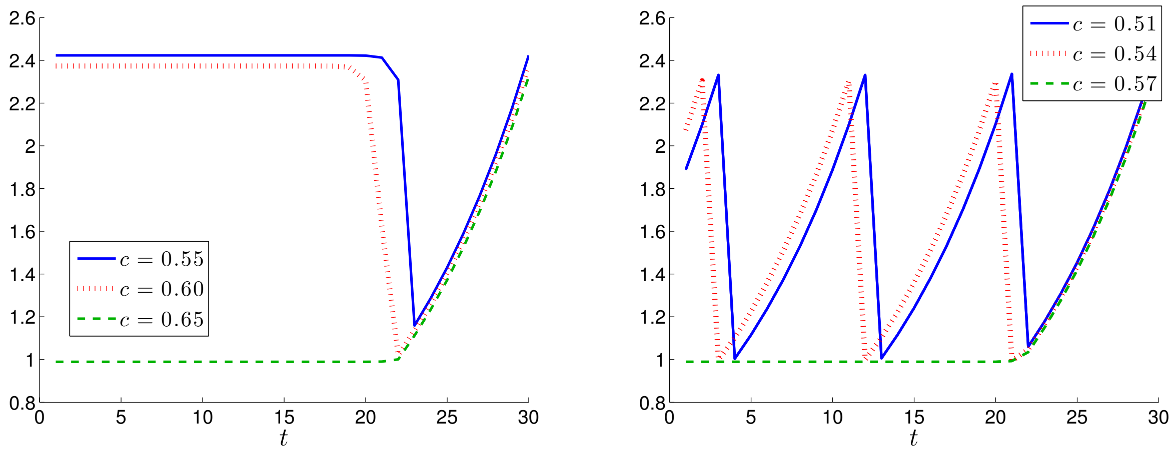
Figure 2.5: **A Shift in Political Power**

to overwhelm the advantage that party 2 had close to the election. Party 1 is then able to implement her preferred project at each date $t < 19$ giving her a higher continuation value, compared to party 2, at each of these dates.

### Example 5

This last example illustrates how the expected social surplus is affected by changes in the parameters $\{\theta, c, \delta\}$. Consider the same parameters as in Example 1, except with the costs as specified in Figure 6 (a) and (b). In the MPE illustrates in Figure 6 (a), it is assumed that in the continuation game beginning at date $T$, if the state transitions to $s_0$ parties play MPE E1. In the MPE illustrated in Figure 6 (b), it is assumed that in the continuation game beginning at date $T$, parties randomize over MPE E1 and E2, playing MPE E1 with probability 1/2 and MPE E2 with probability 1/2.

First, consider how changes in the cost of the project $c$ impact the expected social surplus. The cost impacts this expected surplus in two ways: one, when the cost of a project increases there is less surplus generated by any policy outcome; and two, when the cost of a project changes, there are implications on what type of agreement is feasible, and either positive or negative welfare consequences are possible. Hence, when considering an increase in the cost




(a) (b)

Figure 2.6: **The Social Surplus with Different Costs**

of a project, it is not entirely clear what the implications are on the expected social surplus. Figure 6 (a) illustrates the rather unsurprising case where an increase in the cost of a project leads to a decrease in the aggregate surplus. In contrast, Figure 6 (b) shows that it is possible for the surplus to be nonmonotonic in the cost of a project.

Second, consider how changes in the discount factor $\delta$ or the value of the project $\theta$ impact social surplus. While it is clear that an increase in the value of a project leads to an increase in the surplus generated by any policy outcome, just as when considering changes to the cost of a project, changes in both the value of a project and the discount factor may have ambiguous welfare consequences. Indeed, it is possible to construct examples that demonstrate how the expected surplus may be nonmonotonic in either the discount factor or the value of the project.

## 2.5 Relation to Previous Bargaining Papers

In this section, I consider the related papers from the bargaining literature. I start by discussing two recent papers that offer results similar to those presented in this paper, though the driving force behind these results is very different. Like my paper, one focus of Simsek and Yildiz (2009) is on how the anticipation of a change at an exogenous date influences behavior in earlier periods. Therefore, their model also lends itself to the study of political decision-making around election time. Briefly, they consider a continuous time bilateral bargaining model, where the recognition process follows some stochastic process and each party may have optimistic beliefs regarding her future bargaining power. If at some exogenous date bargaining power becomes sufficiently "durable"—that is, the stochastic process governing the distribution of power does not change very much—the probability of agreement is high around that date. Hence, if bargaining power is expected to be durable right after an election date, their model would offer predictions similar to my paper: agreement is difficult at dates close to and before the election. Though the predictions are similar in spirit, the details of the models are quite different, unsurprisingly producing disagreements through different channels. In their paper, optimism about the stochastic recognition process and the characteristics of that process produce disagreements; in my paper, where parties have a common prior regarding the recognition process, limitations on the set of feasible policy decisions foster disagreements. In both Simsek and Yildiz and my paper, changes in the recognition process are instrumental in generating periods with disagreement.

Ali (2006) considers a bargaining model where it is possible for an equilibrium to have a cyclical property with repeated transitions between dates when agreement is feasible and dates where it is not. Ali considers a multilateral bargaining game, where parties may have optimistic beliefs regarding the recognition process. If parties are sufficiently optimistic, then the cycling result obtains. In Ali, the equilibrium dynamics arise due to each party holding overly optimistic beliefs about the likelihood that she will be recognized to propose policy. In contrast, in my paper, parties have a common expectation regarding the likelihood that each party will be recognized. Cycling between agreement and disagreement regimes occurs in my model due both to parties' expectations regarding how the recognition process will fluctuate in the future and constraints on the set of feasible policy decisions.

It must be noted that a number of bargaining papers provide different rationales for disagreement and delay. The classic explanation for delays is due to asymmetric information (Rubinstein, 1985; Admati and Perry, 1987; Kennan and Wilson, 1993). Essentially, delays can be used to transmit information. a party may delay agreement to signal strength, or use delay as a screening device to determine whether the other player is weak or strong. A number of papers consider how delays may arise in environments with perfect information. This literature therefore aims to provide an explanation for delays that complements those based on asymmetric information. My paper is ultimately related to this strand of literature that focuses on developing explanations for disagreement and delay in models with perfect information. Regarding models with perfect information, delays may be produced if offers can be made simultaneously (Perry and Reny, 1993; Sákovics, 1993), if the asset being bargained over is stochastic (Avery and Zemsky, 1994; Merlo and Wilson, 1995; Eraslan and Merlo, 2002), or if parties are optimistic due to the lack of common knowledge (Yildiz, 2003, 2004; Ali, 2006; Simsek and Yildiz, 2009).

## 2.6 Conclusion

This paper studies when particular policy outcomes, such as stalemates in policy talks or the implementation of centrist policies, will occur in relation to an approaching election date. I consider a simple bargaining game played by two political parties, where the parties must determine the allocation of the government budget, where this budget can be used to finance only one of two public goods projects with the remainder redistributed to the parties.

I find that when political power is sufficiently asymmetric, at dates close to the upcoming election date $T$ it is most difficult for parties to reach agreement on a policy. Indeed, if there is disagreement at any date $t < T$, then there must be a disagreement interval $\{t', \dots, T-1\}$. Thus, legislative gridlock is more likely to occur immediately before the election. Furthermore, when parties are "optimistic" about expected policy outcomes after the election, if gridlock does occur before the election, the duration of these disagreement spells is longest right before the election. If one party has significantly more political power that the other before date $T$, agreement at a date $\bar{t} < T$ implies agreement at any earlier date $t < \bar{t}$ and implemented policies maximize aggregate surplus. Furthermore, as the gap $T - t$ increases, at date $t$ the party with the political advantage is able to pass legislation that is more favorable to the party. On the other hand, if political power is evenly distributed, then the path of policy outcomes before the elections becomes more volatile. In this situation, parties may agree on policies that do not maximize social surplus and it is feasible for there to be long stretches of disagreement, with cycling between agreement and disagreement phases.

The analysis may be relevant when considering negotiations in the United States Congress, and in particular, two recent policy debates. The model suggests two reasons that, after a year of negotiations with Republicans, the Democrats were able to pass a health care reform bill in March of 2010: one, the Democrats possessed a significant advantage over the Republicans in both the House of Representatives and the Senate; and two, the debate started well before the next midterm election in 2010. Likewise, when considering the recent impasse over how to handle the debt ceiling, the model suggests that it would be easier for Democrats and Republicans to reach an agreement if the distribution of seats in the 112th US Congress was less symmetric and if the next presidential election was far into the future.

## 2.7 Appendix

### 2.7.1 Types of Agreement at Date $t < T$

*In what follows, the dependence of the* $(V^1_{t+1}, V^2_{t+1})$ *and* $(p^0, p^1, p^2)$ *on the state at each date* $t < T$ *is suppressed. Also, if* $t = T-1$*, replace "*$V^i_{t+1}(s)$*" for each* $i$ *with* $\sum_{s' \in S} m(s' \mid s) V^i_T(s')$*, where again the dependence on the initial state* $s$ *suppressed.*

For $t < T$, given the continuations $(V^1_{t+1}, V^2_{t+1})$, there are five types of agreement

- *Type 1 Agreement.* Suppose $\delta V^i_{t+1} > \theta$ and $\delta V^j_{t+1} \leq 1 - c$. At date $t$, party $i$'s preferred project is implemented regardless of which party is recognized to propose, if party $i$ is recognized to propose, she will offer party $j$ exactly $\delta V^j_{t+1}$ of the income $1 - c$ that remains, and if party $j$ is recognized to propose, she will offer party $i$ exactly $\delta V^i_{t+1} - \theta$ of the income $1 - c$ that remains. Continuation values at the beginning of period $t$ are then

$$V^i_t = p^i(\theta + 1 - c - \delta V^j_{t+1}) + p^j \delta V^i_{t+1} + p^0 \delta V^i_{t+1}$$
$$V^j_t = p^i \delta V^j_{t+1} + p^j (1 - c - (\delta V^i_{t+1} - \theta)) + p^0 \delta V^j_{t+1}.$$

- *Type 2 Agreement.* Suppose $\delta V_{t+1}^i \in (c, \theta]$, and $\delta V_{t+1}^j \leq 1 - c$. At date $t$, party $i$'s preferred project is implemented regardless of which party is recognized to propose, if party $i$ is recognized to propose, she will offer party $j$ exactly $\delta V_{t+1}^j$ of the income $1 - c$ that remains, and if party $j$ is recognized to propose, she will offer party $i$ none of the income $1 - c$ that remains. Continuation values at the beginning of period $t$ are then

$$V_t^i = p^i(\theta + 1 - c - \delta V_{t+1}^j) + p^j\theta + p^0\delta V_{t+1}^i$$
$$V_t^j = p^i\delta V_{t+1}^j + p^j(1 - c) + p^0\delta V_{t+1}^j.$$

- *Type 3 Agreement.* Suppose $\delta V_{t+1}^i \in (1 - c, c]$, and $\delta V_{t+1}^j \leq 1 - c$. First, suppose $\delta V_{t+1}^i \in (1 - c, c)$. At date $t$, if party $i$ is recognized to propose, she implements her preferred project and offers party $j$ exactly $\delta V_{t+1}^j$, and if party $j$ is recognized to propose she implements a division of the unit of income and offers party $i$ exactly $\delta V_{t+1}^i$. Continuation values at the beginning of period $t$ are then

$$V_t^i = p^i(\theta + 1 - c - \delta V_{t+1}^j) + p^j\delta V_{t+1}^i + p^0\delta V_{t+1}^i$$
$$V_t^j = p^i\delta V_{t+1}^j + p^j(1 - \delta V_{t+1}^i) + p^0\delta V_{t+1}^j.$$

Next, suppose $\delta V_{t+1}^i = c$. At date $t$, if party $i$ is recognized to propose, she implements her preferred project and offers party $j$ exactly $\delta V_{t+1}^j$. If party $j$ is recognized to propose, she is indifferent between a policy that implements $i$'s preferred project and one that does not. With probability $\alpha_t^j \in [0, 1]$ she offers a policy that implements $i$'s preferred project and with probability $1 - \alpha_t^i$ she offers a policy with no project, and offers $i$ the share $\delta V_{t+1}^i$ of the unit of income. Continuation values at the beginning of period $t$ are then

$$V_t^i = p^i(\theta + 1 - c - \delta V_{t+1}^j) + p^j[\alpha_t^i\theta + (1 - \alpha_t^i)\delta V_{t+1}^i] + p^0\delta V_{t+1}^i$$
$$V_t^j = p^i\delta V_{t+1}^j + p^j(1 - \delta V_{t+1}^i) + p^0\delta V_{t+1}^j.$$

- *Type 4 Agreement.* Suppose $\delta V^1_{t+1}, \delta V^2_{t+1} > 1 - c$, $\delta S_{t+1} \leq 1$. At date $t$, parties agree on a division of the unit if income, if party $i$ is recognized she offers party $j$ exactly $\delta V^j_{t+1}$, and if party $j$ is recognized she offers party $i$ exactly $\delta V^i_{t+1}$. Continuation values at the beginning of period $t$ are then

$$V^i_{t_1} = p^i(1 - \delta V^j_{t+1}) + p^j \delta V^i_{t+1} + p^0 \delta V^i_{t+1}$$
$$V^j_{t_1} = p^i \delta V^j_{t+1} + p^j(1 - \delta V^i_{t+1}) + p^0 \delta V^j_{t+1}.$$

- *Type 5 Agreement.* Suppose $\delta V^1_{t+1}, \delta V^2_{t+1} \leq 1 - c$. At date $t$, if party $i$ is recognized to proposed she implements her preferred project and offers party $j$ exactly $\delta V^j_{t+1}$ of the remaining $1 - c$ of income, and if party $j$ is recognized to proposed she implements her preferred project and offers party $i$ exactly $\delta V^i_{t+1}$ of the remaining $1 - c$ of income.

$$V^i_t = p^i(\theta + 1 - c - \delta V^j_{t+1}) + p^j \delta V^i_{t+1} + p^0 \delta V^i_{t+1}$$
$$V^j_t = p^i \delta V^j_{t+1} + p^j(\theta + 1 - c - \delta V^i_{t+1}) + p^0 \delta V^j_{t+1}.$$

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